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On Bargaining with Endogenous Information

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Abstract

Two ex ante identically informed agents play a double auction over the division of a trading surplus with endogenous information and common values. This paper shows that if information acquisition is not observable, three types of inefficiencies can arise. If the information cost is in an intermediate range, no pure strategy equilibrium with trade exists although the agents maintain symmetric information. If the information cost is low, any trading equilibrium exhibits costly information acquisition. If the agents face asymmetric information cost the Akerlof's lemons problem arises as a self-fulfilling equilibrium and only partial trade occurs.

Journal of Economic Literature Classification Numbers: C78, D82, D83

Key words: double auction, endogenous lemons problem, information acquisition, speculation

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1. Introduction

Agents trading financial assets might face significant common values uncertainty because of the uncertain underlying cash flow stream. In particular, in secondary markets the seller of a financial asset must not necessarily posses better information about the value of the asset than a potential buyer but both agents can spend resources to obtain information. This paper analyzes double auction with endogenous information acquisition in such an environment. A buyer and a seller seek to agree on a price at which to trade an asset whose true value is unknown to both parties ex ante. The common value component (quality) of the asset can be either high or low. It is common knowledge that for both realizations the buyer's private valuation for the asset is higher than the seller's private valuation by a fixed margin. Prior to the auction stage the agents can acquire information about the true quality.

Given that there are always gains form trade and that acquiring information is costly, any socially efficient outcome must have trade taking place with probability one and without either party acquiring information irrespective of how the surplus is divided. This paper shows that if information acquisition is not observable endogenous information can cause three types of inefficiencies in double auction. No pure strategy equilibrium with efficient trade may exist. Any full trading equilibria may exhibit costly information acquisition by both agents. The Akerlof's lemons problem may arise as a self-fulfilling equilibrium and only partial trade occurs. The intuition for these observations is the following.

To get started, does the trading at the expected quality and the equal-split outcome constitute an equilibrium? Suppose the buyer acquires information and speculates. If the quality of the asset is low the informed buyer submits a low bid and no trade occurs. If the quality is high trade occurs and the seller suffers an endogenous lemons problem while the buyer makes some speculative profits. However, speculation causes an opportunity cost in the sense that the buyer forgoes some surplus in the low state. The seller faces an analogous incentive problem. So the more surplus the buyer is to obtain the higher the buyer's opportunity cost of speculation but the lower the seller's opportunity cost of speculation.

The following cases can arise. (1) If the information cost equals the speculative profit net the maximum of the opportunity cost of speculation of both agents then the equal-split trading equilibrium is the unique efficient equilibrium. Denote c_{min} as this critical (minimum allowable) level of the information cost. (2) If the information cost is smaller than c_{min} *but* larger than the trading surplus, no pure strategy equilibrium with trade exists. In the no trade equilibrium no agent acquires too expensive and non-exploitable information. Because of the endogenous lemons problem the buyer submits a low offer and the seller submits a high offer.

In equilibrium there is symmetric information and yet no trade occurs. This result is different from Chatterjee and Samuelson (1983) and Myerson and Satterthwaite (1983) since the trading gain is common knowledge. It is also different from Akerlof (1970) and Gresik (1991) since there is no asymmetric information about the common value component of the asset.

(3) If the information cost is lower than c_{min} *as well as* the trading surplus, any trading equilibrium exhibits costly information acquisition. It is the desire of the agents to transact on one hand and their concerns about making bad deals due to endogenous lemons problems on the other hand, that induce both the buyer and the seller to acquire information. This observation is related to Matthews (1984) and Hausch and Li (1993) who show that bidders acquire excessive information in common values auctions. (4) If the agents face asymmetric information cost the Akerlof's lemons problem arises as a self-fulfilling equilibrium. The agent with low information cost acquires information and only partial trading equilibria exist.

The remainder of the paper is organized as follows. The next section introduces the model. Section 3 shows that if information acquisition is not observable the set of efficient equilibria depends on the information cost and three types of inefficiencies can arise. Section 4 shows that if information acquisition is observable endogenous information has no adverse consequences for efficient trade. Section 5 concludes. Appendix A contains the proofs and Appendix B extends the basic model to the case where the quality of the asset is a continuous random variable and the agents can acquire $n \in N$ units of information.

2. The Model

A risk neutral buyer and a risk neutral seller seek to agree on a price p at which to trade an asset. The asset is worth $v+\Delta$ to the buyer and $v-\Delta$ to the seller where v is the uncertain common value component (quality) of the asset and $\Delta>0$ is the private value component (idiosyncratic taste parameter). If the agents agree on a price the trading surplus 2Δ is realized and $U^B=v+\Delta-p$ and $U^S=p-(v-\Delta)$. If no trade occurs the payoff is normalized to zero.

Ex ante the agents have identical information about the uncertain common value v which is either v_L or v_H with equal probability, where $v_L, v_H \in R_+$ and $v_H>v_L>\Delta$. Prior to the trading stage agent i can buy $n_i \in \{0,1\}$ unit of information where n_B and n_S are chosen simultaneously. If no information is bought the agent has the prior information. If agent i pays the cost $c\geq 0$, agent i knows the true value. Then the agents play a double auction, i.e. the buyer submits a bid price b and the seller submits an ask price s simultaneously. If $b\geq s$ then the asset changes hand at the mean price $p=(b+s)/2$. Otherwise there is no trade. To save on some case distinctions it is assumed that $\Delta<(v_H-v_L)/8$ (*Assumption A*).

Assumption A implies that the potential lemons problem is important relative to the realization of the trading gain. Before proceeding to the analysis, the following definitions are introduced. An equilibrium in which trade occurs with probability one (less than one) is called a *full* (*partial*) *trade* equilibrium. An equilibrium is *efficient* if trade occurs with probability one and no agent acquires costly information. An equilibrium exhibits a k-sharing of the total surplus 2Δ if the buyer gets the expected surplus $\Delta-k$ and the seller gets the expected surplus $\Delta+k$. Such an equilibrium is called a *k-sharing* equilibrium. Note a set of (trivial) no trade equilibria is given by $B=(0,b)$ and $S=(0,s)$ with $b\leq v_L-\Delta$ and $s\geq v_H+\Delta$.

3. Equilibria When Information Acquisition Is Not Observable

3.1 The Set of Efficient Equilibria

Given that there are always gains form trade any socially efficient outcome must have trade taking place with probability one and without either party acquiring costly information. This section shows that if information acquisition is not observable by the other party the set of efficient equilibria depends on information cost. Since the outside option of the agents is normalized to zero any price $p\in[E[v]-\Delta,E[v]+\Delta]$ is acceptable for both (uninformed) agents. So the set of strategies leading to the set of efficient outcomes satisfying individual rationality is $B=(0,b)$ and $S=(0,s)$ with $b=s=E[v]+k$ for $k\in[-\Delta, \Delta]$. In the k-sharing outcome the buyer gets $EU^B=\Delta-k$ and the seller gets $EU^S=\Delta+k$. When do these strategies constitute best responses?

Suppose the buyer acquires information and speculates. If the buyer sees that $v=v_L$, he chooses a bid price $b_L<s$ and no trade occurs. Otherwise he chooses $b_H=s$. This response yields $EU^B=0.5[(v_H+\Delta)-(E[v]+k)]-c=(v_H-v_L)/4+0.5(\Delta-k)-c$. If $(v_H-v_L)/4+0.5(\Delta-k)-c>\Delta-k$, the efficient k-sharing outcome does not constitute an equilibrium. Analogously, if $(v_H-v_L)/4+0.5(\Delta+k)-c>\Delta+k$, the best response of the seller to $B=(0,b)$ with $b=E[v]+k$ is to choose $S=(1,s_L,s_H)$ with $s_L=b$ and $s_H>b$. So if $c<\min\{(v_H-v_L)/4-0.5(\Delta-k), (v_H-v_L)/4 -0.5(\Delta+k)\}=(v_H-v_L)/4-0.5\cdot\min\{\Delta-k,\Delta+k\}$, no efficient k-sharing equilibrium exists.

Proposition 1

Suppose that information acquisition is not observable and $k\in[-\Delta,\Delta]$.

Iff $c<(v_H-v_L)/4-0.5(\Delta-|k|)$ then no efficient k-sharing Nash equilibrium exists.

Proposition 1 has a simple economic interpretation. $(v_H-v_L)/4=\pi$ is the expected speculative profit an informed agent makes and $0.5(\Delta-|k|)$ is the expected opportunity cost of speculation. If trade is supposed to be carry out at the price $E[v]+k$ for $k\in[-\Delta, \Delta]$, the buyer gets $EU^B=\Delta-k$ and the seller gets $EU^S=\Delta+k$. If the buyer acquires information and speculates, no trade occurs in the low state and ex ante he forgoes the expected surplus $0.5(\Delta-k)\equiv c_B^{Sp}(k)$. If the seller speculates his opportunity cost of speculation is $c_S^{Sp}(k)=0.5(\Delta+k)$. Since the total surplus is fixed the opportunity costs of speculation of the buyer and the seller are perfectly negatively correlated. If $\pi>c+\min\{c_B^{Sp}(k),c_S^{Sp}(k)\}=c+0.5(\Delta-|k|)$ one agent has an incentive to speculate and trade at the price $p=E[v]+k$ is not an equilibrium.

Corollary 1

(a) If $c\geq(v_H-v_L)/4$ any efficient k-sharing outcome is attainable as a Nash equilibrium.

(b) If $c=c_{min}\equiv(v_H-v_L)/4-0.5\Delta$, only the equal-split ($k=0$) outcome is attainable as an efficient Nash equilibrium. If $c<c_{min}$ then no efficient Nash equilibrium exists.

<u>Remark 1</u>

Suppose the probability for v_L is $q\in(0,1)$. Analogously, one can derive the following results.

(a) Iff $c<q(1-q)(v_L+v_H)-\max\{q(\Delta-k),(1-q)(\Delta+k)\}$, no efficient k-sharing equilibrium exists.

(b) If $c=c_{min}\equiv q(1-q)(v_L+v_H)-2q(1-q)\Delta$, only the $k=(2q-1)\Delta$-sharing outcome constitutes an efficient equilibrium. If $c<c_{min}$ then no efficient equilibrium exists.

3.2 An Information Acquisition Dilemma

This section shows that endogenous information can cause an information acquisition dilemma. If the information cost of both agents is low the desire of the agents to transact on one hand and their concerns about making bad deals due to endogenous lemons problems on the other hand induce both agents to acquire information in any full trade equilibrium. If only one agent faces low information cost, the Akerlof's lemons problem arises as a self-fulfilling equilibrium and no full trade equilibrium exists.

Proposition 2

Suppose that information acquisition is not observable and $k\in[-\Delta, \Delta]$. If $c\leq\Delta-|k|$, then a full trade k-sharing Nash equilibrium exists. In any full trade equilibrium both agents acquire information and $EU^B=\Delta-k-c$ and $EU^S=\Delta+k-c$.

This observation is related to Matthews (1984) and Hausch and Li (1993) who analyze information acquisition in common values auctions. They show that the bidders acquire excessive information. The key difference between this bargaining model and most auction models (see also Milgrom (1981) and Persico (2000)) is that their auctioneer (seller) is non-strategic and ignores the lemons problem.[1] Bergemann and Valimäki (2002) employ a mechanism design approach and show that any ex post efficient allocation mechanism causes an ex ante information acquisition inefficiency. However, they point out that their Theorem 2 is a local result and state (p.1027) "*In particular, the theorem is not a statement about the (Nash) equilibrium decisions of agents*." In this model $u^B(\cdot)$ and $u^S(\cdot)$ are in some sense supermodular in v. In equilibrium the agents acquire too much information although their local prediction is that the agents acquire too little information relative to the social optimum.

Proposition 3

Suppose that information acquisition is not observable and $k \in [-\Delta,\Delta]$. If $c_S \leq 0.5(\Delta+k)$ and $c_B > 2\Delta$, only partial trade k-sharing Nash equilibria exist. In any such equilibrium the seller acquires information and $EU^B=0.5(\Delta-k)$ and $EU^S=0.5(\Delta+k)-c$.

The intuition behind Proposition 3 is similar. Since $c_B>2\Delta$ there exists no equilibrium in which the buyer acquires information. Suppose that the uninformed buyer is willing to offer full surplus to the seller and chooses $b=E[v]+\Delta$. Yet the best response of the seller is to acquire information and to speculate because $c_S \leq 0.5(\Delta+k) < (v_H-v_L)/4-\Delta$ (Assumption A). In order to avoid the speculative loss $-(v_H-v_L)/4$, the buyer submits a defensive offer which an uninformed seller would not "accept". Therefore, in any equilibrium in which trade is to occur with positive probability the seller "must" acquire information and the Akerlof`s lemons problem arises as a self-fulfilling equilibrium. If no agent acquires information then no trade occurs in equilibrium.

[1] Hirshleifer (1971) states "*When private information fails to lead to improved productive alignments....,it is evident that the individual's source of gains can only be at the expense of his fellow."(p.567) "In a world of pure exchange, there will in general be private overinvestment in information: resources committed to acquisition and to dissimination are both wasted from the social point of view*." (p.574). Cremer and Khalil (19829 analyze information acquisition in a principal-agent setting where the agent can acquire information about his disutility of production before signing a contract. They show that in equilibrium no information may be acquired although the information cost is very low. Reny and Perry (2003) analyze large double auction with strategic buyers and strategic sellers but information is exogenous.

3.3 A No-Efficient Trade Result

This section establishes the main result of the paper and shows that no pure strategy equilibrium in double auction may exist in which two identically informed agents agree on how to divide a surplus. It is not only actual asymmetric information but potential information asymmetry due to endogenous information acquisition can already render efficient trade unattractive.

Proposition 4

Suppose that information acquisition is not observable. If $\Delta<c<(v_H-v_L)/4-0.5\Delta$ then no pure strategy Nash equilibrium with trade exists.

The intuition behind Proposition 4 is the following. Since $c>\Delta$ no equilibrium exists in which both agents acquire information. Suppose only one agent acquires information. Because of the lemons problem trade occurs with probability 0.5. Although the informed agent is to get full surplus his expected payoff is $EU=\Delta-c<0$. Therefore, in any equilibrium no agent acquires too expensive and non-exploitable information. However, since information cost is smaller than the speculative profit net the maximum of opportunity cost of speculation, an uninformed agent is concerned about the endogenous lemons problem. The buyer submits a low offer and the seller submits a high offer. There is symmetric information and yet no trade occurs.

The reason for this inefficiency result is different from Chatterjee and Samuelson (1983) and Myerson and Satterthwaite (1983). Their result is driven by the assumption that trading gain is not common knowledge. In this setting the gain from trade is common knowledge. There is also a difference between this observation and Akerlof (1970), Samuelson (1985) or Gresik (1991) who assume that one agent is better informed about the common valuation than the other agent. In this setting it is common knowledge that both agents have identical prior information about the common valuation. The threat of potential lemons problems due to endogenous information acquisition is the cause for the inefficiency. Note, if the agents play mixed strategies (over information acquisition), the no efficient trade result may be mitigated in the sense that trade may occur with positive probability.

4. Equilibira When Information Acquisition Is Observable

This section shows that if information acquisition is observable by the other agent at the auction stage endogenous information has no adverse consequences in the sense that any efficient outcome satisfying individual rationality is attainable as an equilibrium.

Proposition 5

Suppose that information acquisition is observable and $k \in [-\Delta, \Delta]$. Any efficient k-sharing outcome is attainable as a perfect Bayesian Nash equilibrium irrespective of information cost.

Since information acquisition is observable at the bargaining stage, an agent can also condition his offer strategy on the fact whether the other agent is better informed or not. Suppose ex ante the agents agree to trade at E[v] but the buyer acquires information and tries to exploit the seller. At the bargaining stage it is common knowledge that the buyer is better informed. So the seller does not submit the offer E[v] anymore. In order to account for the lemons problem the seller submits a high offer. Since the buyer anticipates the lemons problem he himself creates by acquiring information, his best response is not to acquire information. So no agent has an incentive to acquire more information than the counter party. However, if information acquisition is not observable, the agents cannot target their offer strategies appropriately and are concerned about the endogenous lemons problem.[2]

5. Conclusion

This paper analyses double auction between an ex ante identically informed buyer and seller over the division of a trading surplus with endogenous information and common values and derives the following results. If information acquisition is not observable three types of inefficiencies can arise. No pure strategy equilibrium with efficient trade may exist. Any full trade equilibrium may exhibit costly information acquisition by both agents. The Akerlof's lemons problem may arise as a self-fulfilling equilibrium and only partial trade occurs. If information acquisition is observable endogenous information has no adverse consequences for efficient trade.

The basic model can be extended along different dimensions. (1) If trade is conducted via take-it-or-leave-offer bargaining, it is straightforward to show that for $2\Delta<c<(v_H-v_L)/4-\Delta$ no (pure and mixed strategy) equilibrium with agreement exists.[3] (2) Suppose there is a large

[2] Proposition 5 is somewhat similar in flavor to Perry and Reny (2002) who show that two-stage bidding can achieve allocative efficiency in common values auctions with (exogenous) private information.

[3] Dang (2005) analyses two-period alternating offer bargaining with endogenous information and common values and shows that because of endogenous lemons problems and endogenous outside options perfect equilibira can have the following properties. (i) For the one period case, the agent responding to a take-it-or-leave-offer can capture full surplus. (ii) A low discounting of trading surplus, a positive externality of information acquisition, and an endogenous lemons problem can cause delay. (iii) The equilibrium payoffs of the agents are non-monotonic in the discount factor of trading surplus.

number of agents with real trading motives (liquidity traders). Dang (2005a) shows that if the liquidity traders behave strategically they themselves acquire information. In particular, the large liquidity traders are the ones who determine equilibrium prices. Furthermore, strategic liquidity trading constitutes one channel which breaks the inverse relationship between informationally efficient prices and the compensation for information cost since the trading surplus is fixed. This provides a counterexample to the impossibility result in Grossman and Stiglitz (1980). (3) The basic model can be used as an ingredient to discuss financial contracting in delegated portfolio management in which the liquidity traders hire portfolio managers to trade on behalf of them. In such a setting an optimal contract between investor A and manager A may also have to take into account the strategic effect on the contract between investor B and manager B and vice versa so that portfolio net returns may be endogenous.

Appendix A

Remark

Assumption A ($\Delta<(v_H-v_L)/8$) implies that $v_H-\Delta>v_L+\Delta$.

Proof of Proposition 2

Claim A : Suppose $r,t\in[-\Delta, \Delta]$. If $c\leq0.5\cdot(\Delta+\min\{r,-t\})$ then $B=S=(1, v_L+r, v_H+t)$ are best responses and $EU^B=\Delta-0.5(r+t)-c$ and $EU^S=\Delta+0.5(r+t)-c$.

Proof : The following arguments show that B is a best response to S. (1) If the buyer chooses B as stated above then $EU^B=0.5[v_L+\Delta-(v_L+r)]+0.5[v_H+\Delta-(v_H+t)]-c=\Delta-0.5(r+t)-c\geq0$ since $\Delta-0.5(r+t)\geq0.5\cdot(\Delta+\min\{r,-t\})\geq c$. (2) If the buyer chooses (0,b) with $b=v_H+t$ then $EU^B=0.5[v_L+\Delta-(v_L+r+v_H+t)/2]+0.5[v_H+\Delta-(v_H+t)]=\Delta-(v_H-v_L)/4-(r+3t)/4$. For $r=t=-\Delta$, the buyers' expected payoff is maximal and yet $EU^B=-(v_H-v_L)/4+2\Delta<0$ (Assumption A). (3) If the buyer chooses (0,b) with $b=v_L+r$ then $EU^B=0.5(\Delta-r)$. This response is weakly dominated by response (1) since $c\leq0.5(\Delta-t)$. So B is a best response to S.[4]

Analogously for the seller, if he chooses (0,s) with $s=v_L+r$ then $EU^S=\Delta+(3r+t)/4-(v_H-v_L)/4<0$. If the seller chooses (0,s) with $s=v_H+t$ then $EU^S=0.5(\Delta+t)\leq\Delta+0.5(r+t)-c$ since $c\leq0.5(\Delta+r)$. So S is a best response to B. For $c\leq\min\{0.5(\Delta-t),0.5(\Delta+r)\}=0.5\cdot(\Delta+\min\{r,-t\})$, (B,S) constitutes a Nash equilibrium.

[4] It is easy to see that the strategy (0,b) with $b\in(s_L,s_H)$ is strictly dominated by the strategy (0,b) with $b=s_L$. The strategy (0,b) with $b>s_H$ is (weakly) dominated by the strategy (0,b) with $b=s_H$. The strategy (0,b) with $b<s_L$ implies no trade and is weakly dominated by B.

Claim B : If $c \leq \Delta - |k|$ for $k \in [-\Delta, \Delta]$, there exists an information acquisition-full trade equilibrium with the payoffs $EU^B = \Delta - k - c$ and $EU^S = \Delta + k - c$.

Proof : For a fixed $k \in [-\Delta, \Delta]$, define $k = 0.5(r+t)$ then $r = 2k - t$ and $t = 2k - r$. The maximal allowable information cost c_k for the payoffs $EU^B = \Delta - 0.5(r+t) - c$ and $EU^S = \Delta + 0.5(r+t) - c$ being equilibrium payoffs is $c_k = 0.5 \cdot (\Delta + \min\{r, -t\})$. To find the maximum allowable cost, one solves the following problem

(1) $$\max_{\substack{r,t \in [-\Delta,\Delta] \\ \text{s.t. } r+t=2k}} 0.5 \cdot (\Delta + \min\{r, -t\}) = \max_{\substack{r,t \in [-\Delta,\Delta] \\ \text{s.t. } r+t=2k}} 0.5 \cdot (\Delta + \min\{2k - t, -2k + r.\}).$$

It is easy to see that the $\min\{r, -t\}$ in (1) is non-decreasing in r and $-t$. Therefore, one should start with $r = \Delta$ and $t = -\Delta$. Then

(2) $$\max_{\substack{r,t \in [-\Delta,\Delta] \\ \text{s.t. } r+t=2k}} 0.5 \cdot (\Delta + \min\{2k - t, -2k + r.\}) = 0.5 \cdot (\Delta + \min\{2k + \Delta, -2k + \Delta\}) = 0.5 \cdot (2\Delta - 2|k|).$$

So suppose $c = c_k = \Delta - |k|$. A corresponding equilibrium strategies pair $B = S = (1, v_L + r, v_H + t)$ leading to the payoffs $EU^B = \Delta - k - c$ and $EU^S = \Delta + k - c$ is given as follows: If $k \in [-\Delta, 0]$ then set $r = \Delta + 2k$ and $t = -\Delta$. If $k \in [0, \Delta]$ then set $r = \Delta$ and $t = -\Delta + 2k$.[5]

Claim C :If $c \leq \Delta - |k|$, no full trade equilibrium exists in which (i) no agent acquires information or (ii) only one agent acquires information.

Proof : (i) For $c \leq \Delta - |k| < (v_H - v_L)/4 - 0.5\Delta$ (Assumption A), Corollary 1 implies that no equilibrium exists in which no agent acquires information and full trade occurs. (ii) Suppose that only the buyer acquires information and chooses $(1, b_L, b_H)$ with $b_L = v_L + r$ and $b_H = v_H + t$ where $r, t \in [-\Delta, \Delta]$. If there is to be full trade the seller must choose $(0, s)$ with $s = v_L + r$. For any $r, t \in [-\Delta, \Delta]$, $EU^S = \Delta + (3r + t)/4 - (v_H - v_L)/4 < 0$. Analogously for $n_B = 0$ and $n_S = 1$. So no full trade occurs if only one agent acquires information. **QED**

Proof of Proposition 3

Since the proof is completely analogous to the proof of Proposition 2 it is omitted.

Proof of Proposition 4

Corollary 1 (b) shows that if $c < (v_H - v_L)/4 - 0.5\Delta$, no equilibrium exists in which no agent acquires information and full trade occurs. It is easy to see that since $c > \Delta$, there exists no equilibrium in which both agents acquire information. Two things remain to be shown.

[5] If $c < \Delta - |k|$, one can show that there exists a continuum of equilibrium (offer) strategies which lead to $EU^B = \Delta - k - c$ and $EU^S = \Delta + k - c$. See Dang [5].

(1) There exists no equilibrium in which no agent acquires information and partial trade occurs. Only prices $p \in [E[v]-\Delta, E[v]+\Delta]$ are acceptable for both uninformed agents. Proposition 1 shows that no uninformed agent makes any such an offer because of the endogenous lemons problem. So no trade occurs if both agents are uninformed.

(2) There is no equilibrium in which one agent acquires information and full trade as well as partial trade occurs. Suppose that the buyer chooses $B=(1,b_L,b_H)$ with $b_L=v_L+r$ and $b_H=v_H+t$ where $r,t \in [-\Delta, \Delta]$. Claim C shows that no full trade exists. It is easy to see that B as stated above and $S=(0,s)$ with $s=v_H+t$ do not constitute best responses either. Trade occurs if $v=v_H$ and $EU^B=0.5(\Delta-t)-c$. For $t=-\Delta$, the expected payoff of the informed buyer is maximal and yet $EU^B=\Delta-c<0$. An analogous argument holds if only the seller acquires information. So no equilibrium in pure strategies with trade exists. **QED**

<u>Proof of Proposition 5</u>

Suppose $k \in [-\Delta,\Delta]$, $r \leq k$, and $t \geq k$ then

$$B=(0,b) \text{ with } b(\cdot)=\begin{cases} E[v]+k & \text{if } n_S=0 \\ v_L+r & \text{if } n_S=1 \end{cases}$$

$$S=(0,s) \text{ with } s(\cdot)=\begin{cases} E[v]+k & \text{if } n_B=0 \\ v_H+t & \text{if } n_B=1 \end{cases}$$

constitute best responses (and a perfect Bayesian Nash equilibrium).

<u>Step 1 (Best responses at the bargaining stage)</u>

(a) If $n_B=n_S=0$, it is common knowledge that no agent knows the true value. So any $b=s=E[v]+k$ for $k \in [-\Delta, \Delta]$ are best responses and $EU^B=\Delta-k$ and $EU^S=\Delta+k$.

(b) If $n_B=n_S=1$, it is common knowledge that both agents know the true value. Since the information cost is sunk at the trading stage, any $b=s=(v_L+r,v_H+t)$ for $r,t \in [-\Delta,\Delta]$ are best responses and $EU^B=\Delta-(r+t)-c$ and $EU^B=\Delta+(r+t)-c$

(c) If $n_B=1$ and $n_S=0$, it is common knowledge that the buyer knows the true value. The informed buyer is willing to choose any $b=(v_L+r, v_H+t)$ where $r,t \in [-\Delta, \Delta]$. If there is to be full trade the seller must choose $s=v_L+r$. For $r=t=\Delta$, the seller' expected payoff is maximal and yet $EU^S=-(v_H-v_L)/4+2\Delta<0$ (Assumption A). So no full trade occurs. It is easy to see that $b=(v_L+r, v_H+t)$ and $s=v_H+t$ for any $r \leq \Delta$ and $t \in [-\Delta,\Delta]$ are best responses and trade occurs with probability 0.5 and $EU^B=0.5(\Delta-t)-c$ and $EU^S=0.5(\Delta+t)$.

(d) If n_B=0 and n_S=1, analogously $b=v_L+r$ and $s=(v_L+r,\ v_H+t)$ for any $r\in[-\Delta, \Delta]$ and $t\geq-\Delta$ are best responses and trade occurs with probability 0.5 and $EU^B=0.5(\Delta-r)$ and $EU^S=0.5(\Delta+r)-c$.

Step 2 (Best responses at the information acquisition stage)

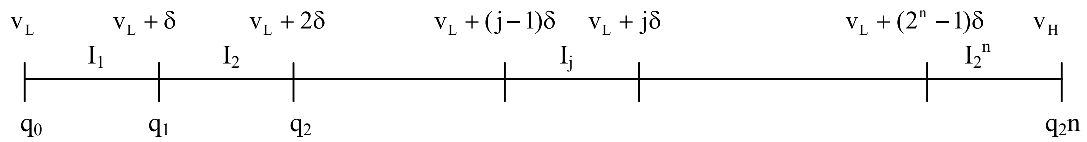
If the buyer chooses n_B=0, then $EU^B=\Delta-k$. If n_B=1, then $EU^B=0.5(\Delta-t)-c<\Delta-k$ since $k\leq t$. So n_B=0 is a best response. Analogously, n_S=0 is a best response of the seller. **QED**

Appendix B

This Appendix provides an extension of the binary state setting to a setup where the quality of the asset is uniformly distributed between $[v_L,v_H)=V$ with $\Delta<v_L,v_H\in R_+$. The agents can acquire $n\in N$ units of information at the cost $C(n)$ where $C(n+1)\geq C(n)$. If an agent acquires n units of information then his information set (filter) is as follows: The state space V is partitioned in 2^n disjoint and equidistant subsets I_j ($j=1,....2^n$) or intervals with length $\delta=(v_H-v_L)/2^n$. Formally, the information filter has the following structure (see also Figure A):

$$I(n,v)=\{I_1,I_2,....,I_{2^n}\}=\left\{I_j=\left\{v: v\in[q_{j-1},q_j)\,\middle|\,q_{j-1}=v_L+\tfrac{j-1}{2^n}(v_H-v_L)\right\}\middle|\, j=1,2,..,2^n\right\}$$

Figure A (The information filter)



Proposition A

Suppose that information acquisition is unobservable, $C(n)=nc$, and $\Delta<(v_H-v_L)/2^N$ where N is some integer. Define $n^*=\max\{0,\eta\}$ where

$$\eta=\text{round}\left[\frac{1}{\ln 2}\cdot\ln\left(\frac{v_H-v_L}{c+0.5\Delta}\right)-3\right].$$

If $n^*\geq1$, $c\leq(1-0.5^{n^*})\Delta/n^*$, and $n^*\leq N-2$, then in a symmetric full trade equal-spilt equilibrium the agents choose $n_B=n_S=n^*$ and $b=s=E[v|I(n^*,v)]$ and $EU^B=EU^S=\Delta-C(n^*)$.

Proof of Proposition A

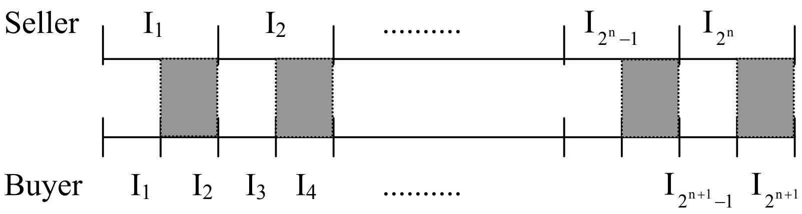
The information acquisition equilibrium is proven by construction.

Suppose $n_B=n_S=n$. If $v\in I_j(v)$ for $j=1,\ldots,2^n$ then

$$E[v|I(n,v)] = \frac{v_L + \frac{j-1}{2^n}(v_H - v_L) + v_L + \frac{j}{2^n}(v_H - v_L)}{2} = v_L + \frac{(2j-1)(v_H - v_L)}{2^{n+1}}$$

If the buyer and the seller choose $n_B=n_S=n$ and play the following symmetric offer strategy $b=s=E[v \mid I(n,v)]$ then $EU^B=EU^S=\Delta-C(n)$. Suppose the buyer deviates to $n_B=n+1$. His information advantage is illustrated in Figure B.

Figure B (Impact of acquiring more information)



Suppose that the buyer chooses the following offer strategy as a response to s,

$$b'(\cdot) = \begin{Bmatrix} v_L + \dfrac{(2j-1)(v_H - v_L)}{2^{n+2}}, & \text{if } v \in I_j(v) \text{ where } j = 1,3,5... \\ v_L + \dfrac{(2j-1)(v_H - v_L)}{2^{n+1}}, & \text{if } v \in I_j(v) \text{ where } j = 2,4,6... \end{Bmatrix}.$$

In other words, if $v \in I^B{}_j(v)$ where $j=1,3,5,...2^{n+1}-1$, then $b'(\cdot)<s(\cdot)$ and no trade occurs. If $v \in I^B{}_j(v)$ where $j=2,4,6,....,2^{n+1}$, then the buyer chooses the lowest bid where trade still occurs, i.e. $b'(\cdot)=s(\cdot)$. The buyer makes an (*interim*) expected speculative profit π from trade when $v \in I_j(v)$ with $j=2,4,6,...2^{n+1}$, which is given by

$$E[\pi] = E[v] - p = \frac{v_L + \frac{j-1}{2^{n+1}}(v_H - v_L) + v_L + \frac{j}{2^{n+1}}(v_H - v_L)}{2} - \left(v_L + \tfrac{2j-1}{2^{n+1}}(v_H - v_L)\right)$$

$$= \frac{v_H - v_L}{2^{n+2}}$$

Trade occurs with probability 0.5 (namely at all $v \in I^B{}_j(v)$ for j even) and the *ex ante* expected gross payoff GEU from and the benefit MBI of acquiring one more unit of information are,

$$GEU^B(n+1) = 0.5E[\pi] + 0.5\Delta = \frac{v_H - v_L}{2^{3+n}} + \frac{1}{2}\Delta \quad \text{and}$$

$$MBI^B(n) = GEU^B(n+1) - GEU^B(n) = \frac{v_H - v_L}{2^{3+n}} + \frac{1}{2}\Delta - \Delta = \frac{v_H - v_L}{2^{3+n}} - \frac{1}{2}\Delta.$$

The *ex ante* expected speculative profit is $\pi=(v_H-v_L)/2^{3+n}$ and the opportunity cost of speculation is 0.5Δ (at the equal-split agreement). The reason why for n=0, $\pi=(v_H-v_L)/8$ is smaller than the profit in the binary case is that in the binary case the informed buyer knows for sure that $v=v_H$. Since v is uniformly distributed the informed buyer here only knows that $v\in[E[v],v_H]$. If the "marginal" benefit of information acquisition is larger than the "marginal" cost, the buyer has an incentive to acquire n+1 units of information given that the seller plays $n_S=n$ and $s=E[v|I^S(n,v)]$. The seller faces an analogous incentive problem. There is no incentive to acquire more information if $MBI(n)\leq[C(n+1)-C(n)]$, i.e. if $(v_H-v_L)/2^{3+n}-0.5\Delta\leq c$

$$\Leftrightarrow n=\frac{1}{\ln 2}\cdot\ln\left(\frac{v_H-v_L}{c+0.5\Delta}\right)-3.$$

Due to the integer constraint, n is to round down to the next smallest integer denoted with η. For $n_B=n_S=\eta\geq1$ and $b=s=E[v|I(\eta,v)]$ being an equilibrium, one has to show that acquiring less information than η is not a profitable deviation.

(i) Suppose, the seller deviates to $n_S=\eta-1$. (a) If he adjusts his offer for the lemons problem, trade occurs with probabililty 0.5 (see Figure B). His expected payoff is $EU^S=0.5\Delta-C(\eta-1)$. This deviation is not profitable if $\Delta-C(\eta)>0.5\Delta-C(\eta-1)\Leftrightarrow c<0.5\Delta$. This deviation is not profitable since $c<(1-0.5^{\eta})\Delta/\eta$. (b) The less informed seller does not adjust for the lemons problem and wants trade to occur with probability 1. Therefore, he chooses $s=E[v|I^B_{2j-1}]$ if $v\in I^S_j$ (see Figure B). If $v\in I^S_j$ for j even, the seller suffers a loss of $(v_H-v_L)/2^{\eta+1}$. Conditional on observing a trade, this occurs with probability 0.5. The seller's expected payoff is $EU^S=\Delta-C(\eta-1)-(v_H-v_L)/2^{\eta+2}<0$ since by construction $\eta\leq N-2$ and $\Delta<(v_H-v_L)/2^N$.

(ii) Suppose, $n_S=0$. Again, because of the lemons problem the uninformed seller has to choose $s=E[v|\ I^B{}_{2^n}]$. Trade occurs with probability $1/2^{\eta}$, i.e. if $v\in I^B{}_{2^{\eta}}$. If This deviation is not profitbale if $\Delta-\eta c\geq0.5^{\eta}\Delta\Leftrightarrow c\leq(1-0.5^{\eta})\Delta/\eta$. (Note if $s=E[v|\ I^B{}_k]$ for $k<2^n$ then $EU^S<0$.) An analogous argumentation holds for the buyer. **QED**

<u>Numerical Example</u>

Suppose $v_L=100$, $v_H=200$, N=6, $\Delta=1$, and c=0.2. In a symmetric full trade equal-split equilibrium both agents acquire four units of information and $EU^B=EU^S=\Delta-C(4)=0.2$. If v=140, then $I(n=4,v=140)=I_7=[137.5,143.75)$ and trade is carried out at $p=E[v|I_7]=140.125$.

Corollary A

(a) If $c<(v_H-v_L)/8-0.5\Delta$ then no efficient outcome is attainable as an equilibrium.

(b) If $2\Delta<c<(v_H-v_L)/8-0.5\Delta$ then no pure strategy equilibrium with trade exits.

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