SONDERFORSCHUNGSBEREICH 504

Rationalitätskonzepte,
Entscheidungsverhalten und
ökonomische Modellierung

No. 04-47

CEU Preferences and Dynamic Consistency

Simon Grant*
and Jürgen Eichberger**
and David Kelsey***

November 2004

Research supported by ESRC grants no. R000222597 and RES-000-22-0650. We would like to thank the referees and editor of this journal for comments and discussion.

*Department of Economics Rice University, email: sgrant@rice.edu

**Sonderforschungsbereich 504, email: juergen.eichberger@awi.uni-heidelberg.de

***Department of Economics, The University of Birmingham, email: D.KELSEY@BHAM.AC.UK

Universität Mannheim
L 13,15
68131 Mannheim

CEU Preferences and Dynamic Consistency*

Jürgen Eichberger
Alfred Weber Institut,
Universität Heidelberg, Germany.

Simon Grant
Department of Economics
Rice University, Texas, USA.

David Kelsey
Department of Economics, University of Exeter
and University of Birmingham, England.

13th July 2004

Abstract

This paper investigates the dynamic consistency of CEU preferences. A decision maker is faced with an information structure represented by a fixed filtration. If beliefs are represented by a convex capacity, we show that a necessary and sufficient condition for dynamic consistency is that beliefs be additive over the final stage in the filtration.

Address for Correspondence David Kelsey, Department of Economics, School of Business and Economics, University of Exeter, Rennes Drive, Exeter, Devon, EX4 4PU, ENGLAND.

e-mail Juergen.Eichberger@awi.uni-heidelberg.de

Keywords, Ambiguity, Choquet Integral, Capacity, Dynamic consistency.

JEL Classification: D81.

*Research supported by ESRC grants no. R000222597 and RES-000-22-0650. We would like to thank the referees and editor of this journal for comments and discussion.

1 Introduction

This paper finds necessary and sufficient conditions for dynamic consistency of Choquet Expected Utility preferences. Schmeidler (1989) proposed Choquet Expected Utility (henceforth CEU) as a theory of choice under ambiguity. However it also has other applications, for instance Wu (1999) has used it to model anxiety. Schmeidler's theory did not involve time. To make it more generally applicable it is desirable to extend it to an intertemporal model. Multi-period decisions present new problems. Firstly individuals will receive information as time progresses. It is necessary to model how they update their beliefs as this information is received. Secondly it is not clear whether individuals with non-additive beliefs will be dynamically consistent. We consider all updating rules which satisfy a property which we consider to be reasonable.

In Epstein and LeBreton (1993) and Eichberger and Kelsey (1996) it is shown that, under some assumptions, dynamic consistency of CEU preferences implies that beliefs must be additive. However these papers imposed conditions, which required consistency between different decision trees. In many economic problems, we only need to consider decision-making in a single tree, for instance any model based on an extensive form game. Hence it is not clear what implications the earlier results have in this context. Sarin and Wakker (1998) show that for a fixed decision tree, a necessary condition for dynamic consistency is that beliefs be additive except at the final stage. We provide a partial converse to their result by showing that if beliefs are represented by a convex capacity, this condition is also sufficient. In a recent paper Hanany & Kilbanoff (2004) show under alternative axioms how dynamic consistency can be maintained in a fixed decision tree.

It has been argued that non-expected utility preferences are difficult to apply, since they may be dynamically inconsistent, see for instance Green (1987) or Hammond (1988). We show that dynamic consistency does not imply beliefs should be additive, however it does impose some restrictions. How acceptable these restrictions are would depend on the context.

2 CEU Preferences and Dynamic Consistency

In this section we introduce CEU preferences and find conditions for them to be dynamically consistent. We consider a finite set of states of nature S. The set of outcomes is a convex set $X \subseteq \mathbb{R}^n$. An act is a function from S to X. The set of all acts is denoted by $A(S)$. In this paper we shall restrict attention to the case where beliefs are represented by convex capacities.

Definition 2.1 *A* convex capacity *on S is a real-valued function ν on the subsets of S which satisfies the following properties:,*

1. $A \subseteq B \Rightarrow \nu(A) \leqslant \nu(B)$;

2. $\nu(\varnothing) = 0,\ \nu(S) = 1.$

3. $\nu(A) + \nu(B) \leqslant \nu(A \cup B) + \nu(A \cap B)$, *for all $A, B \subseteq S$.*

Schmeidler (1989) argues that convex capacities represent ambiguity-aversion. However alternative definitions of ambiguity-aversion due to Epstein (1999) and Ghirardato and Marinacci (2002) have cast doubt on the relationship between convexity and ambiguity-aversion.

If beliefs are represented by a capacity ν on S, the expected utility of a given act can be found using the Choquet integral.

Notation 2.1 *Since S is finite, one can order the utility from a given act $a : u\left(a^1\right) > u\left(a^2\right) > ... > u\left(a^{r-1}\right) > u\left(a^r\right)$, where $u\left(a^1\right), \ldots, u\left(a^r\right)$ are the possible utility levels yielded by action a. Denote by $A^k(a) = \{s \in S|\ u(a(s)) \geqslant u\left(a^k\right)\}$ the set of states that yield a utility at least as high as a^k. By convention, let $A^0(a) = \emptyset$.*

Definition 2.2 *The* Choquet expected utility *of u with respect to capacity ν is:*

$$\int u(a(s))d\nu(s) = \sum_{k=1}^{r} u\left(a^k\right)[\nu(A^k(a)) - \nu(A^{k-1}(a))].$$

Schmeidler (1989), Gilboa (1987) and Sarin and Wakker (1992) provide axioms for representing preferences by a Choquet integral of utility. Another advantage of assuming convexity is that it implies that CEU preferences also have an intuitive multiple priors representation. If beliefs are represented by a convex capacity, ν, there exists a closed convex set $\mathcal{C}$ of probability distributions on S, such that: $\int u(a(s))d\nu(s) = \min_{p\in\mathcal{C}} \mathbf{E}_p u(a)$.[1] In addition, we shall assume that the utility function is continuous.

Assumption 2.1 *The utility function $u : X \to \mathbb{R}$ is continuous.*

Assumption 2.2 (Strong Monotonicity) *For two acts $a, b \in A(S)$, if $\exists \hat{s} \in S$, such that $u(a(\hat{s})) > u(b(\hat{s}))$ and $\forall s \in S, u(a(s)) \geqslant u(b(s))$ then $a \succ b$.*

This says that no state is null in the sense that increasing the utility in any state will lead to a strictly preferred option.[2]

To apply CEU in an intertemporal context it is necessary to specify how beliefs will be updated as new information is received. There have been a number of proposals for updating CEU preferences, see, for instance, Gilboa and Schmeidler (1993). Instead of focusing on a specific rule we prove results for any updating procedure which satisfies the following assumption.

Assumption 2.3 *Let ν be a convex capacity on S and let E be an event. Then if ν_E denotes the update of ν conditional on E, we assume that, $\nu(E) + \nu(\neg E) = 1$ implies, $\nu_E(A) = \nu(A \cap E)/\nu(E)$ for $A \subseteq S$.*

[1] This is proved in the Proposition in Schmeidler (1989).

[2] We do not use the full strength of this assumption. In fact we only need it to apply to the events C and D in the proof of Theorem 2.1. There may be some null states, provided these events are non-null.

The strongest motivation for studying Assumption 2.3 is that it is satisfied by the three commonest rules for updating CEU preferences, the Optimistic update, the Dempster-Shafer update, and the Generalised Bayesian Update, (defined below). Thus using this assumption enables us to prove results for these three rules simultaneously. Assumption 2.3 was motivated by the desire to ensure that the updating rule agrees with Bayesian updating when there is no ambiguity. Since Bayesian updating is agreed to be correct for additive beliefs, it seems reasonable that an updating rule for non-additive beliefs should have this property. If $\nu(E) + \nu(\neg E) = 1$, Lemma 2.1 (below) implies that $\nu(A) = \nu(A \cap E) + \nu(A \cap \neg E)$. If E is observed $\nu(A \cap \neg E)$ is not relevant. Thus it does not seem unreasonable to take $\nu(A \cap E)$ as a measure of the likelihood of A. Dividing by $\nu(E)$ is a normalisation. The Dempster-Shafer update, see Shafer (1976) may be defined as follows.

Definition 2.3 *Let ν be a capacity on S. The* Dempster-Shafer update *(henceforth DS-update) of ν conditional on $E \subseteq S$ is given by:*

$$\nu_E(A) = \frac{\nu((A \cap E) \cup \neg E) - \nu(\neg E)}{1 - \nu(\neg E)}.$$

The DS-update has been axiomatised in Gilboa and Schmeidler (1993), where it is shown to be equivalent to a maximum likelihood updating procedure. An alternative is the Optimistic update defined below.

Definition 2.4 *Let ν be a capacity on S. If E is observed and $A \subseteq E$, the* Optimistic update *of ν conditional on E is given by:* $\nu_E(A) = \frac{\nu(A \cap E)}{\nu(E)}$.

This rule assumes that the worst possible outcome occurred on the complement of E, hence the term optimistic. The Generalised Bayesian Update (henceforth GBU) (see Jaffray (1992), Fagin and Halpern (1991) and Walley (1991)) is defined as follows.

Definition 2.5 *Let ν be a capacity on S and let $E \subseteq S$. If E is observed and $A \subseteq E$,* *the GBU of* ν *conditional on* E *is given by:*

$$\nu_E(A) = \frac{\nu(A)}{1 - \nu(\neg E \cup A) + \nu(A)}.$$

The GBU can be interpreted as the willingness to pay p for a lottery which pays 1 on A and 0 on $E \backslash A$ and is called off if $\neg E$ occurs:

$1 - p$	on	A
$-p$	on	$E - A$
0	on	$\neg E$

.

From the CEU of this lottery, $\nu(A)(1-p) + [1 - \nu(\neg E \cup A)](-p) = 0$, one can compute the price p as the likelihood of event A conditional on the event E obtaining.

The following lemma is provides a key step in the proof of the main result.

Lemma 2.1 *Let* $\mathcal{E} = E_1, ..., E_K$ *be a partition and let* ν *be a convex capacity on* S *such that* $\sum_{i=1}^{K} \nu(E_i) = 1$ *then for any* $B \subseteq S$, $\nu(B) = \sum_{i=1}^{K} \nu(B \cap E_i)$.

Proof. First consider the case where $K = 2$. Define sets C and D by $C = (B \cap E_1) \cup E_2, D = E_1 \cup (B \cap E_2)$. By convexity, $\nu(C) \geqslant \nu(B) + \nu(E_2) - \nu(B \cap E_2), \nu(D) \geqslant \nu(B) + \nu(E_1) - \nu(B \cap E_1)$ and $1 = \nu(S) \geqslant \nu(C) + \nu(D) - \nu(B)$. Substituting we obtain $1 \geqslant \nu(B) + \nu(E_2) - \nu(B \cap E_2) + \nu(B) + \nu(E_1) - \nu(B \cap E_1) - \nu(B) = 1 + \nu(B) - \nu(B \cap E_2) - \nu(B \cap E_1)$ or $\nu(B \cap E_2) + \nu(B \cap E_1) \geqslant \nu(B)$. However the opposite inequality follows directly from convexity, which establishes the result in this case. The general result follows by repeated application of the result for $K = 2$. ■

The following Proposition shows that the DS, optimistic and GBU updates agree with Bayesian updating if a non-ambiguous event is observed.

Proposition 2.1 *The DS-rule, the optimistic update and GBU satisfy Assumption 2.3.*

Proof. The result is trivial for the optimistic update. Now consider the DS-rule. Let E be an event such that $\nu(E)+\nu(\neg E)=1$. For $A \subseteq E, \nu_E(A) = \frac{\nu(A\cup\neg E)-\nu(\neg E)}{1-\nu(\neg E)}$. Let $F = A \cup \neg E$. By Lemma 2.1, $\nu(F) = \nu(F \cap E) + \nu(F \cap \neg E) = \nu(A) + \nu(\neg E)$. Hence $\nu_E(A) = \frac{\nu(A)}{\nu(E)}$.

Now consider the GBU, $\nu_E(A) = \nu(A)/\left[1 - \nu(\neg E \cup A) + \nu(A)\right]$. By Lemma 2.1, $\nu(\neg E \cup A) = \nu(\neg E) + \nu(A)$. Hence $\nu_E(A) = \frac{\nu(A)}{1-\nu(\neg E)}$. If $\nu(E) + \nu(\neg E) = 1$ this implies $\nu_E(A) = \frac{\nu(A)}{\nu(E)}$. ■

Next we shall find a necessary and sufficient condition for CEU preferences to be dynamically consistent. Let $\mathcal{E} = \langle \mathcal{E}_0, ..., \mathcal{E}_T \rangle$ be a filtration Let $\mathcal{E}_T = \langle E_1^T, ..., E_{K_T}^T \rangle$, let $A(E_k^T)$ be the set of acts available after event E_k^T is observed, i.e. $A(E_k^T)$ is a set of functions from E_k^T to X. If $E^t \in \mathcal{E}_t$, define $A(E^t) = \times_{E^T \in \mathcal{E}_T, E^T \subseteq E^t} A(E^T)$ to be the set of acts available at time t, conditional on event E^t being observed.

Assumption 2.4 *We assume that the partition* $\mathcal{E}_T = \langle E_1^T, ..., E_{K_T}^T \rangle$ *is* non-trivial, i.e. $\left| E_j^T \right| \geqslant 2$, *for* $1 \leqslant j \leqslant K_T$.

Definition 2.6 *We denote CEU preferences conditional on* $E^t \in \mathcal{E}_t$ *by* $\succeq_{E^t}$ *. They are defined by:* $a \succeq_{E^t} b \Leftrightarrow \int u(a(s))d\nu_{E^t}(s) \geqslant \int u(b(s))d\nu_{E^t}(s)$.

The individual has to choose an act from a set $A(S)$ of acts available at $t = 0$. At time $t = \tau$ (s)he receives a signal, that tells him/her in which element of the partition $E \in \mathcal{E}_\tau$ the state of nature lies. Beliefs are then updated and the individual has an opportunity to reconsider his/her decision. If the signal says that the true state of nature is in $E \in \mathcal{E}_\tau$, then (s)he may choose any act from $A(E)$.

The individual formulates a complete contingent plan of action at time $t = 0$. After the receipt of new information (s)he updates his/her beliefs. A new contingent plan is formulated for the remaining time periods. Acts are evaluated by a Choquet integral of utility with respect to the new beliefs. The individual is said to be dynamically consistent if (s)he keeps to his/her original plan. Below we formally define dynamic consistency with respect to a given filtration.

Definition 2.7 *Preferences are said to be* dynamically consistent *with respect to a filtration* $\mathcal{E}$, *if whenever* $\tau > t$, $a \succeq_{E^\tau} b$, *for all* $E^\tau \subseteq E^t$ *implies* $a \succeq_{E^t} b$.

This definition says that if conditional on any piece of information which might be received, b is not preferred to a then b is initially not preferred to a.

The following lemma establishes that, when restricted to non-ambiguous events, any updating-rule satisfying Assumption 2.3 is independent of the order in which information is received.

Lemma 2.2 *Let* $\mathcal{E} = \langle \mathcal{E}_0, ..., \mathcal{E}_T \rangle$ *be a filtration and let* ν *be a capacity, such that* $\sum_{E \in \mathcal{E}_T} \nu(E) = 1$. *Let* $E_t \in \mathcal{E}_t$ *and* $E_\tau \in \mathcal{E}_\tau$, *where* $\tau > t$. *Then if Assumption 2.3 is satisfied* $\nu_{E^t_{E^\tau}} = \nu_{E_\tau}$.

Proof. Consider $A \subseteq E_\tau$. By Assumption 2.3, $\nu_{E_{tE_\tau}}(A) = \frac{\nu_{E_t}(A)}{\nu_{E_t}(E_\tau)} = \frac{\frac{\nu(A)}{\nu(E_t)}}{\frac{\nu(E_\tau)}{\nu(E_t)}} = \frac{\nu(A)}{\nu(E_\tau)} = \nu_{E_\tau}(A)$. ■

Now we present our main result, which establishes a necessary and sufficient condition for CEU preferences to be dynamically consistent. Beliefs must be additive between different members of the finest partition in the filtration. They may however be non-additive within a member of this partition.

Theorem 2.1 *Let* $\mathcal{E} = \langle \mathcal{E}_0, ..., \mathcal{E}_T \rangle$ *be a given filtration on* S, *which satisfies Assumption 2.4. If a decision-maker has CEU preferences with beliefs represented by a convex capacity, which satisfy Assumptions 2.1 and 2.2 and (s)he uses an updating rule which satisfies Assumption 2.3 then the following conditions are equivalent:*

1. *(s)he will be dynamically consistent with respect to* $\mathcal{E}$,

2. $\sum_{E \in \mathcal{E}_T} \nu(E) = 1$.

Proof. **1⇒2** Suppose that the decision-maker is dynamically consistent. Consider first the case $K = 2$. Since the partition is non-trivial, we may find events, A, B, C, and D such that, $E_1 = A \cup B$, $E_2 = C \cup D$, where $A \cap B = C \cap D = \varnothing$. Consider acts a, b, c, d, e and f as described in the following table:

	E_1		E_2	
	A	B	C	D
a	1	1	1	1
b	1	1	β	0
c	0	0	1	1
d	0	0	β	0
e	β	β	1	1
f	β	β	β	0

We can ensure that acts with these values exist by appropriately normalising the utility function. Note that $\int ad\nu_{E_1} = \int bd\nu_{E_1}, \int cd\nu_{E_1} = \int dd\nu_{E_1}, \int ed\nu_{E_1} = \int fd\nu_{E_1}$; $\int ad\nu_{E_2} = \int cd\nu_{E_2} = \int ed\nu_{E_2}$ and $\int bd\nu_{E_2} = \int dd\nu_{E_2} = \int fd\nu_{E_2}$. By continuity and strong monotonicity we may choose $\beta > 1$ so that $\int ad\nu_{E_2} = \int bd\nu_{E_2}$. Dynamic consistency then implies that $a \sim b$, $c \sim d$ and $e \sim f$. By evaluating the Choquet integrals we find: $1 = (\beta - 1)\nu(C) + \nu(E_1 \cup C)$, $\nu(E_2) = \beta\nu(C)$ and $\beta\nu(E_1 \cup C) = \beta\nu(E_1) + 1 - \nu(E_1)$. These equations imply $\nu(E_1) + \nu(E_2) = 1$.

The general case can be established as follows. If $\sum_{E \in \mathcal{E}_T} \nu(E) < 1$, then we can apply the above argument to $F_1 = E_1$ and $F_2 = \bigcup_{E \in \mathcal{E}_T, E \neq E_1}$ to deduce that dynamic consistency implies $\nu(F_1) + \nu(F_2) = 1$, which is a contradiction.

2⇒1 Now suppose that at time $\hat{t}$ event $\hat{E}$ is observed and at time $\tau > \hat{t}, a \succcurlyeq_{\tilde{E}} b$, for all $\tilde{E} \in \mathcal{E}_\tau, \tilde{E} \subseteq \hat{E}$. Let $V\left(a|\tilde{E}\right) = \int u(a)\, d\nu_{\tilde{E}}$ denote the (Choquet) expected utility of a conditional on $\tilde{E}$. By hypothesis and Assumption 2.3, $\nu_{\tilde{E}}\left(A_i \cap \tilde{E}\right) = \frac{\nu_{\hat{E}}(A_i \cap \tilde{E})}{\nu_{\hat{E}}(\tilde{E})}$.

Hence

$$\nu_{\hat{E}}\left(\tilde{E}\right)V\left(a|\tilde{E}\right)=u(x_1)\nu_{\hat{E}}(A_1\cap\tilde{E})+\sum_{i=2}^{m}u(x_i)\left[\nu_{\hat{E}}\left(A_i\cap\tilde{E}\right)-\nu_{\hat{E}}\left(A_{i-1}\cap\tilde{E}\right)\right]. \tag{1}$$

Now consider the decision at time $\hat{t}$. By definition, the (Choquet) expected utility of any given act a is equal to $V\left(a|\hat{E}\right)=u(x_1)\nu_{\hat{E}}(A_1)+\sum_{i=2}^{m}u(x_i)\left[\nu_{\hat{E}}\left(A_i\right)-\nu_{\hat{E}}\left(A_{i-1}\right)\right].$ Assumption 2.3 implies $\nu_{\hat{E}}\left(A\right)=\nu\left(A\right)/\nu\left(E\right)$, since ν is convex it follows that $\nu_{\hat{E}}$ is also convex. Lemma 2.1 implies that (1) may be rewritten as

$$\sum_{\tilde{E}\in\mathcal{E}_\tau,\tilde{E}\subseteq\hat{E}}\left\{u(x_1)\nu_{\hat{E}}(A_1\cap\tilde{E})+\sum_{i=2}^{m}u(x_i)\left[\nu_{\hat{E}}\left(A_i\cap\tilde{E}\right)-\nu_{\hat{E}}\left(A_{i-1}\cap\tilde{E}\right)\right]\right\}.$$

Thus $V\left(a|\hat{E}\right)=\sum_{\tilde{E}\in\mathcal{E}_\tau,\tilde{E}\subseteq\hat{E}}\nu_{\hat{E}}\left(\tilde{E}\right)V\left(a|\tilde{E}\right)$. A similar formula holds for the Choquet integral of b. Since for all $\tilde{E}\in\mathcal{E}_\tau,\tilde{E}\subseteq\hat{E}$, $V\left(a|\tilde{E}\right)\geqslant V\left(b|\tilde{E}\right)$, we have $V\left(a|\hat{E}\right)\geqslant V\left(b|\hat{E}\right)$ equivalently $a\succcurlyeq_{\hat{E}}b$, which establishes dynamic consistency. ■

Remark 1 *The strategy of the proof of 1⇒2 is similar to that of Theorem 3.1 in Sarin and Wakker (1998). Some of the assumptions may be relaxed slightly. The proof that 2⇒1 does not make use of the assumptions that utility is strongly monotonic or continuous. The proof that 1⇒2 does not use convexity.*

Remark 2 *From the proof of Theorem 2.1 we can see that Lemma 2.1, which requires beliefs to be represented by a convex capacity, is the most important step. The following example demonstrates that this result is no longer true if we do not assume convexity.*

Example 2.1 *Suppose there are two outcomes Win or Lose, where $u\left(Win\right)=1>0=u\left(Lose\right)$. Consider a six element state space $S=\{s_1,...,s_6\}$.*

The filtration $\mathcal{E}$ on S is $\langle\{S\},\{\{s_1,s_3,s_5\},\{s_2,s_4,s_6\}\}\rangle$. Consider a capacity ν on S

defined by:

$$\nu\left(\{s_1\}\right)=\nu\left(\{s_2\}\right)=0.16,\quad \nu\left(\{s_i\}\right)=0.15\quad \textit{for}\;\; i\notin\{1,2\}\,.$$

$$\begin{aligned}
&\nu\left(\{s_1,s_3\}\right)=\nu\left(\{s_1,s_5\}\right)=\nu\left(\{s_2,s_4\}\right)=\nu\left(\{s_2,s_6\}\right)=0.31,\\
&\nu\left(\{s_3,s_6\}\right)=\nu\left(\{s_4,s_5\}\right)=0.34,\\
&\nu\left(\{s_i,s_j\}\right)=0.32,\qquad \textit{otherwise.}
\end{aligned}$$

$$\begin{aligned}
&\nu\left(\{s_1,s_2,s_3\}\right)=\nu\left(\{s_2,s_4,s_6\}\right)=0.5,\\
&\nu\left(\{s_i,s_j,s_k\}\right)=0.49\qquad \textit{otherwise.}
\end{aligned}$$

$$\nu\left(\{s_i,s_j,s_k,s_\ell\}\right)=0.68\quad \textit{for all}\;\; i,j,k,l\in\{1,...,6\}.$$

$$\nu\left(\{s_i,s_j,s_k,s_\ell,s_m\}\right)=0.84\quad \textit{for all}\;\; i,j,k,l,m\in\{1,...,6\}.$$

The set of admissible acts are bets on events of the form $\{s_i, s_j\}$*, where* $(i+j) \bmod 2 = 1$*. That is, the individual receives the outcome* Win *if a state from the event* $\{s_i, s_j\}$ *obtains, otherwise (s)he receives the outcome Lose.*

Clearly, a maximal ex ante strategy is to make a bet on an event $\{s_i, s_j\}$, for which $i + j = 9$. Now assume the decision-maker is allowed to make his/her bet on a state in the element of the partition $\mathcal{E}_1 = \{\text{Odd}, \text{Even}\}$, where $\text{Odd} = \{s_1, s_3, s_5\}$, $\text{Even} = \{s_2, s_4, s_6\}$. By Assumption 2.3 the updated beliefs are given by:

$$\begin{aligned}
\nu_{\text{Odd}}\left(\{s_i\}\right) &= \begin{cases} 0.32 & i=1 \\ 0.30 & i\in\{3,5\}\,, \end{cases}, \ \nu_{\text{Odd}}\left(\{s_i,s_j\}\right) = \begin{cases} 0.62 & \min\{i,j\}=1 \\ 0.64 & \min\{i,j\}>1, \end{cases}\\
\nu_{\text{Even}}\left(\{s_i\}\right) &= \begin{cases} 0.32 & i=2 \\ 0.30 & i\in\{4,6\}\,, \end{cases}, \ \nu_{\text{Even}}\left(\{s_i,s_j\}\right) = \begin{cases} 0.62 & \min\{i,j\}=2 \\ 0.64 & \min\{i,j\}>2. \end{cases}
\end{aligned}$$

A maximal interim strategy measurable with respect to the partition $\mathcal{E}_1$ is, bet on s_1 if Odd and bet on s_2 if Even.

Here is a decision-maker who has CEU preferences with beliefs represented by a capacity which satisfies $\nu(\{s_1, s_3, s_5\}) + \nu(\{s_2, s_4, s_6\}) = 0.5 + 0.5 = 1$. The updating rule satisfies Assumption 2.3, yet a maximal ex ante plan must involve a bet on an event $\{s_i, s_j\}$, for which $i + j = 9$. But this is *not* dynamically consistent, since the strategy which maximizes his updated CEU preferences involves betting on $\{s_1\}$ if $E =$ Odd and betting on $\{s_2\}$ if $E =$ Even. Theorem 2.1 does not apply since the capacity is not convex,

$$\begin{aligned} & \nu(\{s_1, s_3, s_5, s_6\}) + \nu(\{s_2, s_3, s_4, s_6\}) - \nu(\{s_3, s_6\}) \\ = \; & 0.68 + 0.68 - 0.34 = 1.02 \\ > \; & 1 = \nu(S) = \nu(\{s_1, s_3, s_5, s_6\} \cup \{s_2, s_3, s_4, s_6\}), \end{aligned}$$

and so Lemma 2.1 does not hold. To see this note that $\nu(\{s_3, s_6\}) = 0.34$ but $\nu(\{s_3, s_6\} \cap \text{Odd}) + \nu(\{s_3, s_6\} \cap \text{Even}) = \nu(\{s_3\}) + \nu(\{s_6\}) = 0.3$. ■

3 Conclusion

One of the more common ways to model ambiguity-aversion is to use CEU preferences with a convex capacity. This paper has found conditions under which such preferences will be dynamically consistent. As we have shown dynamic consistency does impose restrictions on CEU preferences. How acceptable these are would depend on the particular application being considered. There are a number of ways in which we could respond to this result.

We could relax dynamic consistency. There is very little experimental evidence which supports the hypothesis that individuals are dynamically consistent. To be convincing this approach would need to advance strong reasons why individuals might not mind apparent dynamic inconsistencies. Preliminary arguments along these lines can be found in Kelsey and Milne (1999) and Wu (1999).

Another possible reaction is to replace CEU with a different model of ambiguity. The leading contender is the multiple priors model, Gilboa and Schmeidler (1989). As shown in Sarin and Wakker (1998), dynamic consistency imposes a less stringent restriction on the multiple priors model. Pires (2002) has axiomatised an updating rule for such preferences.

If uncertainty is resolved over a period of time, individuals will typically not be indifferent about the time at which uncertainty is resolved. This is related to the issues discussed in the present paper. Grant, Kajii, and Polak (2000) found that additivity over the final partition was also sufficient for CEU preferences to be information-loving. Wu (1999) has shown that a plausible model of preferences concerning the resolution of uncertainty can lead to preferences of the CEU form.

References

[1]

Eichberger, J., Kelsey, D., 1996. Uncertainty Aversion and Dynamic Consistency. International Economic Review 37, 625–640.

Epstein, L. G., 1999. A Definition of Uncertainty Aversion. Review of Economic Studies 66, 579–606.

Epstein, L. G., LeBreton M., 1993. Dynamically Consistent Beliefs Must Be Bayesian. Journal of Economic Theory 61, 1–22.

Fagin, R., Halpern J., 1991. A New Approach to Updating Beliefs. Uncertainty in Artificial Intelligence 6, 347–374.

Ghirardato, P., Marinacci M., 2002. Ambiguity Made Precise: A Comparative Foundation. Journal of Economic Theory 102, 251–289.

Gilboa, I., 1987. Expected Utility with Purely Subjective Non-additive Probabilities. Journal of Mathematical Economics 16, 65–88.

Gilboa, I., Schmeidler D., 1989. Maxmin Expected Utility with a Non-Unique Prior. Journal of Mathematical Economics 18, 141–153.

——— 1993. Updating Ambiguous Beliefs. Journal of Economic Theory 59, 33–49.

Grant, S., Kajii,A., Polak B., 2000. Temporal Resolution of Uncertainty and Recursive Non-Expected Utility Models. Econometrica 68, 425–434.

Green, J. C., 1987. Making Book Against Oneself: The Independence Axiom and Non-Linear Utility Theory. Quarterly Journal of Economics 102, 785–796.

Hammond, P., 1988. Consequentialist Foundations for Expected Utility. Theory and Decision 25, 25–78.

Hanany, E. Kilbanoff, P. 2004., Updating multiple prior preferences, working paper, Northwestern University .

Jaffray, J. Y., 1992. Bayesian Updating and Belief Functions. IEEE Transactions Systems, Manufacturing and Cybernetics 22, 1144–1152.

Kelsey, D., Milne F., 1999. Induced Preferences, Non-Additive Probabilities and Multiple Priors. International Economic Review, 40, 455–477.

Pires, C. P., 2002. A Rule for Updating Ambiguous Beliefs. Theory and Decision 53, 137–152.

Sarin, R., Wakker P., 1992. A Simple Axiomatization of Non-Additive Expected Utility. Econometrica 60, 1255–1272.

——— 1998. Dynamic Choice and Non-Expected Utility. Journal of Risk and Uncertainty 17, 87–120.

Schmeidler, D., 1989. Subjective Probability and Expected Utility without Additivity. Econometrica 57, 571–587.

Shafer, G., 1976. A Mathematical Theory of Evidence, Princeton University Press, New Jersey.

Walley, P., 1991. Statistical Reasoning with Imprecise Probabilities, Chapman and Hall, New York.

Wu, G., 1999. Anxiety and Decision Making with Delayed Resolution of Uncertainty. Theory and Decision 46, 159–198.

Nr.	Author	Title
04-43	Fabian Bornhorst Andrea Ichino Oliver Kirchkamp Karl H. Schlag Eyal Winter	How do People Play a Repeated Trust Game? Experimental Evidence
04-42	Martin Hellwig	Optimal Income Taxation, Public-Goods Provision and Public-Sector Pricing: A Contribution to the Foundations of Public Economics
04-41	Thomas Gschwend	Comparative Politics of Strategic Voting: A Hierarchy of Electoral Systems
04-40	Ron Johnston Thomas Gschwend Charles Pattie	On Estimates of Split-Ticket Voting: EI and EMax
04-39	Volker Stocké	Determinants and Consequences of Survey Respondentsí Social Desirability Beliefs about Racial Attitudes
04-38	Siegfried K. Berninghaus Marion Ott Bodo Vogt	Restricting the benefit flow from neighbors: Experiments on network formation
04-37	Christopher Koch	Behavioral Economics und die Unabhängigkeit des Wirtschaftsprüfers - Ein Forschungsüberblick
04-36	Christopher Koch	Behavioral Economics und das Entscheidungsverhalten des Wirtschaftsprüfers - Ein Forschungsüberblick
04-35	Christina Reifschneider	Behavioral Law and Economics: Überlegungen zu den Konsequenzen moderner Rationalitätskonzepte für die Gestaltung informationellen Kapitalmarktrechts
04-34	Siegfried K. Berninghaus Karl-Martin Ehrhart Marion Ott Bodo Vogt	Searching for "Stars" - Recent Experimental Results on Network Formation -

Nr.	Author	Title
04-33	Christopher Koch	Haftungserleichterungen bei der Offenlegung von Zukunftsinformationen in den USA
04-32	Oliver Kirchkamp J. Philipp Reiß	The overbidding-myth and the underbidding-bias in first-price auctions
04-31	Alexander Ludwig Alexander Zimper	Investment Behavior under Ambiguity: The Case of Pessimistic Decision Makers
04-30	Volker Stocké	Attitudes Toward Surveys, Attitude Accessibility and the Effect on Respondentsí Susceptibility to Nonresponse
04-29	Alexander Ludwig	Improving Tatonnement Methods for Solving Heterogeneous Agent Models
04-28	Marc Oliver Rieger Mei Wang	Cumulative Prospect Theory and the St.Petersburg Paradox
04-27	Michele Bernasconi Oliver Kirchkamp Paolo Paruolo	Do fiscal variables affect fiscal expectations? Experiments with real world and lab data
04-26	Daniel Schunk Cornelia Betsch	Explaining heterogeneity in utility functions by individual differences in preferred decision modes
04-25	Martin Weber Jens Wuestemann	Bedeutung des Börsenkurses im Rahmen der Unternehmensbewertung
04-24	Hannah Hörisch	Does foreign aid delay stabilization
04-23	Daniel Schunk Joachim Winter	The Relationship Between Risk Attitudes and Heuristics in Search Tasks: A Laboratory Experiment
04-22	Martin Hellwig	Risk Aversion in the Small and in the Large When Outcomes Are Multidimensional
04-21	Oliver Kirchkamp Eva Poen J. Philipp Reiß	Bidding with Outside Options

Nr.	Author	Title
04-20	Jens Wüstemann	Evaluation and Response to Risk in International Accounting and Audit Systems: Framework and German Experiences
04-19	Cornelia Betsch	Präferenz für Intuition und Deliberation (PID): Inventar zur Erfassung von affekt- und kognitionsbasiertem Entscheiden
04-18	Alexander Zimper	Dominance-Solvable Lattice Games
04-17	Volker Stocké Birgit Becker	DETERMINANTEN UND KONSEQUENZEN DER UMFRAGEEINSTELLUNG. Bewertungsdimensionen unterschiedlicher Umfragesponsoren und die Antwortbereitschaft der Befragten
04-16	Volker Stocké Christian Hunkler	Die angemessene Erfassung der Stärke und Richtung von Anreizen durch soziale Erwünschtheit
04-15	Elena Carletti Vittoria Cerasi Sonja Daltung	Multiple-bank lending: diversification and free-riding in monitoring
04-14	Volker Stocké	The Interdependence of Determinants for the Strength and Direction of Social Desirability Bias in Racial Attitude Surveys
04-13	Christopher Koch Paul Fischbeck	Evaluating Lotteries, Risks, and Risk-mitigation Programs Ño A Comparison of China and the United States
04-12	Alexander Ludwig Torsten Sløk	The relationship between stock prices, house prices and consumption in OECD countries
04-11	Jens Wüstemann	Disclosure Regimes and Corporate Governance
04-10	Peter Albrecht Timo Klett	Referenzpunktbezogene risikoadjustierte Performancemaße: Theoretische Grundlagen
04-09	Alexander Klos	The Investment Horizon and Dynamic Asset Allocation - Some Experimental Evidence

Nr.	Author	Title
04-08	Peter Albrecht Cemil Kantar Yanying Xiao	Mean Reversion-Effekte auf dem deutschen Aktienmarkt: Statistische Analysen der Entwicklung des DAX-KGV
04-07	Geschäftsstelle	Jahresbericht 2003
04-06	Oliver Kirchkamp	Why are Stabilisations delayed - an experiment with an application to all pay auctions
04-05	Karl-Martin Ehrhart Marion Ott	Auctions, Information, and New Technologies
04-04	Alexander Zimper	On the Existence of Strategic Solutions for Games with Security- and Potential Level Players
04-03	Alexander Zimper	A Note on the Equivalence of Rationalizability Concepts in Generalized Nice Games
04-02	Martin Hellwig	The Provision and Pricing of Excludable Public Goods: Ramsey-Boiteux Pricing versus Bundling
04-01	Alexander Klos Martin Weber	Portfolio Choice in the Presence of Nontradeable Income: An Experimental Analysis
03-39	Eric Igou Herbert Bless	More Thought - More Framing Effects? Framing Effects As a Function of Elaboration
03-38	Siegfried K. Berninghaus Werner Gueth Annette Kirstein	Trading Goods versus Sharing Money - An Experiment Testing Wether Fairness and Efficiency are Frame Dependent
03-37	Franz Urban Pappi Thomas Gschwend	Partei- und Koalitionspräferenzen der Wähler bei der Bundestagswahl 1998 und 2002
03-36	Martin Hellwig	A Utilitarian Approach to the Provision and Pricing of Excludable Public Goods
03-35	Daniel Schunk	The Pennsylvania Reemployment Bonus Experiments: How a survival model helps in the analysis of the data