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Asset Prices in an Overlapping Generations Model with Case-Based Decision Makers with Short Memory

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I am indebted to my advisor Juergen Eichberger for his helpful guidance. I would like to thank Hans Gersbach, Verena Liessem, Johannes Becker, Itzhak Gilboa, David Easley, Larry Blume, Ho-MouWu and Klaus Ritzeberger for helpful discussions and comments. Financial support by the DFG is greatefully acknowledged.

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I consider an economy, populated by case-based decision makers with one-period memory. Consumption can be transferred between the periods by the means of a riskless storage technology or a risky asset with iid dividend payments. I analyze the dynamics of asset holdings and asset prices. Whereas an economy in which the investors have low aspiration levels exhibits constant prices and asset holdings, investors with high aspiration levels create cycles, which may be stocahstic or deterministic. Arbitrage possibilities, deviation of the price from the fundamental value, predictability of returns and excessive volatility are shown to obtain in a market with case-based investors.

Keywords: Case-Based Decision Theory, financial markets, asset pricing.

JEL classification: D81, G12

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1 Introduction

The case-based decision theory has been proposed by Gilboa and Schmeidler (1995) as an alternative theory for decision making under uncertainty. Differently from the expected utility theory, it models decisions in situations of structural ignorance, in which neither states of the world, nor their probabilities can be naturally derived from the description of the problem. It is assumed that a decision maker can only learn from experience, by evaluating an act based on its past performance in similar circumstances. An aspiration level is used as a bench-mark in the evaluation process. It distinguishes results considered satisfactory, i.e. those exceeding the aspiration level, which make an act more attractive, from the unsatisfactory ones, which influence negatively the evaluation of an act.

Since the works of Arrow (1970, p. 98), it has been assumed that the expected utility framework naturally fits the description of an asset in terms of a probability distribution over state-contingent outcomes. However, a thorough consideration of this framework shows that the problem of formulating states of nature in the context of financial market might have no natural solution. Indeed, besides the problem of deciding, which payoffs of a security should be considered possible, the question of correlation among the payoffs of different assets arises. Hence, it is not a solution of the problem to identify the states of the world with the payoffs an asset renders[2].

Moreover, in a market environment payoffs are determined by capital gains, hence by equilibrium prices, which themselves depend on the expectations of the market participants. The well known beauty contest used by Keynes (1936) to describe the expectation formation in asset markets illustrates this point. As Arthur (1995, p. 23) notes ”[w]here forming expectations means predicting an aggregate outcome that is formed in part from others’ expectations, expectation formation can become self-referential. The problem of logically forming expectations then becomes ill-defined, and rational deduction finds itself with no bottom ground to stand upon”. Moreover, in economies with heterogenous investors the determination of an equilibrium might turn out to be a complex (unsolvable) computational problem. Since the case-based decision theory does not rely on the definition of states and state-contingent outcomes, it allows to address

[2] See Bossert, Pattanaik and Xu (2000, p.296) for a discussion of the problems connected with the construction of states.

these problems in a formal model.

Although the case-based decision theory has been applied in several economic contexts[3], it has not been used to model decision making in financial markets up to now. A model of financial markets, in which expected utility maximization is replaced by case-based reasoning is of interest for several reasons. First, it allows to gain a better understanding of the case-based decision theory itself. Second, the application of the case-based decision theory to financial markets contributes to the literature on behavioral finance, by describing the dynamics of portfolio holdings and asset prices in a market with case-based investors. The analysis of the behavior implied by case-based reasoning allows for comparisons to the predictions of the standard financial theory, as well as to the empirical findings.

This paper presents a first attempt to apply the theory of case-based decisions to a model of financial markets. First, a definition of a market equilibrium with case-based decision-makers is provided and studied. The existence of an equilibrium is shown under quite general conditions. Nevertheless, degenerate equilibria with 0 asset prices can emerge if the aspiration levels in the economy are not sufficiently low.

However, conditions can be stated under which at least one non-degenerate equilibrium exists in each period of time. In the sequel, such conditions are introduced to study the price dynamics in an economy with case-based decision-makers. By considering an economy with heterogenous consumers who differ in their aspiration levels, it is not only possible to analyze how the aspiration level determines the behavior of a given investor, but also to explore the interaction between investors with different aspiration levels and to indicate their influence on the asset price. The relationship which is found between the aspiration level and the investment behavior in an individual portfolio choice problem, see Guerdjikova (2001) reappears in a market environment, but the results also depend on the interaction of prices and portfolio choices in equilibrium.

In particular, it is found that investors with relatively low aspiration levels behave in a satisficing manner, choosing constant, but possibly suboptimal portfolios over time. An economy populated by such investors, therefore, exhibits constant prices, which in general differ from prices

[3] As for instance, in the consumer theory, Gilboa and Schmeidler (1997, 2001), Gilboa and Pazgal (2001), theory of voting, Aragones (1997), production theory, Jahnke, Chwolka and Simons (2001), social learning, Blonski (1999), cooperation in games, Pazgal (1997), herding behavior, Krause (2003), choices among lotteries, Gayer (2003).

under rational expectations. In contrast, investors with high aspiration levels constantly switch between the portfolios available. Hence, they cause stochastic or deterministic cycles, excessive price volatility and predictability of returns. Moreover, their behavior is suboptimal, since they buy at high prices and sell at low prices, incurring losses. These findings are consistent with empirical evidence from financial markets and demonstrate that the presence of case-based decision-makers in a financial market may account for some of the phenomena inconsistent with expected utility maximization combined with rational expectations.

The approach usually chosen in the literature to address these issues consists in studying the behavior of investors who satisfy the hypothesis of expected utility maximization but have biased beliefs about the distribution of future returns. For instance, Daniel, Hirshleifer and Subrahmanyam (1998) and Gervais and Odean (2001) provide an explanation of excessive trading frequency by assuming that traders suffer from a self-attribution bias. De Long, Shleifer, Summers and Waldmann (1990b), Barberis, Shleifer and Vishny (1998) and Cutler, Poterba and Summers (1990) assume that traders condition their behavior on past returns and induce positive correlation of asset returns. De Long, Shleifer, Summers and Waldmann (1990a), as well as Shleifer and Vishny (1997) show how noise traders can generate arbitrage possibilities in a theoretical model.

In contrast to this work, in this paper the framework of the expected utility theory is completely abandoned and replaced by the framework of the case-based decision theory.

The paper is structured as follows. In section 2, I introduce the model of the economy and derive the individual demand for assets. In section 3, I define an equilibrium for the market populated by case-based decision makers and prove existence under quite general conditions. Section 4 analyzes the evolution of the economy and determines the long-run distribution of the asset-price, as well as the evolution of asset holdings for different groups of investors. In section 5, it is discussed how empirically observed phenomena such as bubbles, predictability of returns, excessive volatility and arbitrage possibilities can emerge in a market populated by case-based decision makers. Section 6 concludes. The proofs of the results are stated in the appendix.

2 The Economy

Consider an economy consisting of a continuum of investors uniformly distributed on the interval $[0; n]$. For each $i \in [0; n]$ and some constant $\bar{u}^0 \in \mathbb{R}$ denote by $\bar{u}^i = \bar{u}^0 + i$ the aspiration level of investor i. The continuum of investors can, therefore, be identified with the continuum of aspiration levels $[\bar{u}^0; \bar{u}^n]$.

Each investor lives for two periods. Investors derive utility only from consumption in the second period of their life. The utility function $u(\cdot)$ is identical for all investors, strict monotonically increasing and continuous. There is one consumption good in the economy. The initial endowment of the investors consists of one unit of the consumption good in the first period and is 0 in the second period of life.

The investors can transfer consumption between two periods by using a risky asset a and a riskless asset b. The riskless asset delivers $(1 + r)$ units of consumption per unit invested. It is available in perfectly elastic supply at a price of 1.

The supply of a is fixed at A. Its payoff per unit is:

$$\delta_t = \left\{ \begin{array}{ll} \delta D & \text{with probability } q \\ 0 & \text{with probability } 1 - q \end{array} \right\},$$

identically and independently distributed in each period[4]. Denote the price of a at time t by p_t.

2.1 Portfolio Choice Problem

The decision of a young investor in terms of the case-based decision theory is described as a problem to be solved, by choosing an act out of a given set. In the present context the problem can be formulated as: ”Choose a portfolio of assets to enable consumption tomorrow”.

I consider only the case in which there are only two portfolios possible for a single investor — the whole initial endowment is invested either in a or in b. Short sales are prohibited[5].

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[4] q is interpreted as the objective probability of high returns known to an external observer, but not to the investors in the economy. Hence, q will be irrelevant for the investors' decisions. However, the specification of q makes it possible to analyze the long-run behavior of the economy.

[5] The model can be easily generalized for the case of allowed diversification by introducing a similarity function defined on problem - act / price - portfolio pairs. The results are robust in this respect as long as the similarity function is strictly decreasing in the Euclidean distance between such points. Since the investors in the economy are infinitesimally small, it is plausible that short sales are impossible for a single investor because of high transaction costs and legal requirements. The restriction of short sales is also necessary to guarantee existence of an equilibrium.

Let $\alpha_t^i \in \{a; b\}$ be the act chosen by a young investor with an aspiration level $\bar{u}^i$ in period t. Normalizing the price of the consumption good to 1 in each period, the indirect utility of consumption of an old investor in period t becomes:

$$v_t\left(\alpha_{t-1}^i\right) = \left\{ \begin{array}{ll} u\left(\frac{p_t+\delta_t}{p_{t-1}}\right), & \text{if } \alpha_{t-1}^i = a \\ u\left(1+r\right), & \text{if } \alpha_{t-1}^i = b \end{array} \right\}.$$

In the spirit of the case-based decision theory, I assume that the decision-makers have almost no information about the problem they are facing. They do not know the structure of the economy, nor the process of price formation. They have no information about possible prices and returns of the assets and their distribution. Hence, they can only learn from the experience of subjects, who have lived before them. This experience is summarized in the memory which is represented by a vector of cases. Each case consists of a past choice made and a utility realization consequently observed:

$$\left(\alpha_{\tau-1}; v_\tau\left(\alpha_{\tau-1}\right)\right).$$

Assume that the memory of a young investor i at time t consists only of the last case, experienced by the investor from the current old generation with the same aspiration level[6] i:

$$M_t^i = \left(\alpha_{t-1}^i; v_t\left(\alpha_{t-1}^i\right)\right)$$

The cumulative utility of an asset $\alpha \in \{a; b\}$ for investor i is, therefore:

$$U_t^i\left(\alpha\right) = \left\{ \begin{array}{l} \left[v_t\left(\alpha\right) - \bar{u}^i\right], \text{ if } \alpha_{t-1}^i = \alpha \\ 0, \text{ else} \end{array} \right\}.$$

According to the case-based decision theory, the investor chooses in each period the asset with the highest cumulative utility.

2.2 Individual Demand for Assets

Consider a young investor i living in period t, whose direct predecessor holds a. His decision is given by:

$$\alpha_t^i \in \left\{ \begin{array}{l} \{a\}, \text{ if } u\left(\frac{p_t+\delta_t}{p_{t-1}}\right) > \bar{u}^i \\ \{a; b\}, \text{ if } u\left(\frac{p_t+\delta_t}{p_{t-1}}\right) = \bar{u}^i \\ \{b\}, \text{ if } u\left(\frac{p_t+\delta_t}{p_{t-1}}\right) < \bar{u}^i \end{array} \right\}.$$

The continuity and strict monotonicity of u imply that in general there is a unique price $\tilde{p}_t\left(i\right)$

[6] One possibility to relax this assumption is by introducing social learning — the possibility to learn from investors with different characteristics, see the works of Blonski (1999) and Krause (2003) on this issue.

such that the demand of i for a, x_t^i satisfies:

$$x_t^i \in \left\{ \begin{array}{cc} 0, & \text{if } 0 \leq p_t < \tilde{p}_t\left(i\right) \\ \left\{\frac{1}{\tilde{p}_t(i)};0\right\}, & \text{if } p_t = \tilde{p}_t\left(i\right) \\ \frac{1}{p_t}, & \text{if } p_t > \tilde{p}_t\left(i\right) \end{array} \right\}$$

x_t^i is plotted in figure 1.

Demand for a of a single investor i

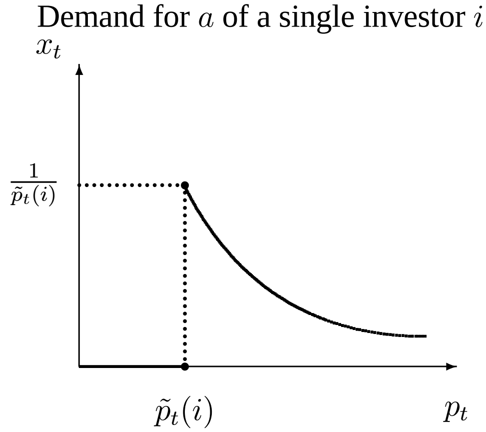

Figure 1

If , however,

$$u\left(\frac{\delta_t}{p_{t-1}}\right) - \bar{u}^i > 0, \tag{1}$$

for the given realization of δ_t, then $\tilde{p}_t\left(i\right) \geq 0$ as defined above does not exist. In this case, x_t^i is given by:

$$x_t^i = \left\{ \begin{array}{l} \frac{1}{p_t} \text{ for all } p_t > 0 \\ \infty \text{ for } p_t = 0 \end{array} \right\}, \tag{2}$$

see figure 2.

Now consider a young investor j, whose predecessor holds b. Since the comparison between

Demand for a of a single investor i with $\tilde{p}_t(i) < 0$

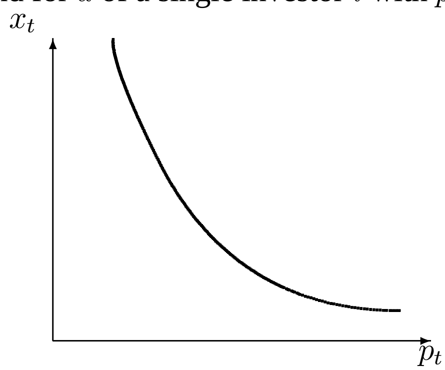

Figure 2

$u\,(1+r)$ and $\bar{u}^j$ does not reverse over time, the choice of this investor does not depend on p_t and is given by:

$$\alpha_t^i \in \left\{ \begin{array}{l} \{a\}, \text{ if } u\,(1+r) < \bar{u}^i \\ \{a; b\}, \text{ if } u\,(1+r) = \bar{u}^i \\ \{b\}, \text{ if } u\,(1+r) > \bar{u}^i \end{array} \right\}.$$

His demand for a is, therefore, constant in p_t.

The arguments above show that the individual demand for the risky asset in general depends non-monotonically on its current price. It increases for relatively low prices and decreases for high prices. In the case illustrated in figure 1, the individual demand for the risky asset is insensitive to price changes near 0. These characteristics of the individual demand determine the properties of the aggregate value of demand for the risky asset, which is increasing in p_t and can be 0 for low prices.

3 Temporary Equilibrium

Definition: A temporary equilibrium at time t is defined by:

- portfolio choices of the young investors $\alpha_t^i \in \{a; b\}$ for each $i \in [0; n]$;
- utility of consumption derived by the old investors $v_t\left(\alpha_{t-1}^i\right)$ for each $i \in [0; n]$;
- a price of the asset a, p_t

such that following conditions are fulfilled:

1. the young investors obey the case-based decision rule:
$$\alpha_t^i = \left\{ \begin{array}{ll} a, & \text{if } U_t^i(a) \geq U_t^i(b) \\ b, & \text{if } U_t^i(a) \leq U_t^i(b) \end{array} \right\}$$
at the equilibrium price p_t.

2. the old investors consume their whole income:
$$v_t\left(\alpha_{t-1}^i\right) = \left\{ \begin{array}{ll} u\left(\frac{p_t}{p_{t-1}} + \frac{\delta_t}{p_{t-1}}\right), & \text{if } \alpha_{t-1}^i = a \\ u(1+r), & \text{if } \alpha_{t-1}^i = b \end{array} \right\}$$

3. the market for the risky asset is cleared:
$$s_t =: Ap_t = \int\limits_{\left\{i:\alpha_t^i = a\right\}} di =: d_t$$
Here, d_t denotes the mass of the young investors, who choose to hold a in period t[7] and s_t is the value of supply of a.

Assume:

$(A1)$ The utility function $u(\cdot)$ is strict monotonically increasing and continuous.

$(A2)$ At $t = 1$ the population of the old investors can be partitioned into a finite number of intervals such that all the investors of the same interval hold the same asset.

The following proposition insures the existence of an equilibrium in each period $t \geq 1$.

Proposition 1 *Assume that $(A1)$ and $(A2)$ hold. Then a temporary equilibrium of the economy exists in every period $t \geq 1$.*

[7] Or, in other words — the value of demand for a in period t.

The equilibrium condition is illustrated in figure 3.

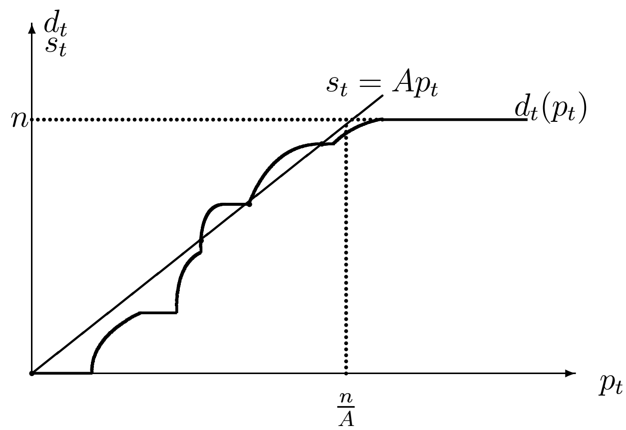

Figure 3

The figure shows that the uniqueness of the equilibrium is not guaranteed. This is due to the fact that the value of demand for the risky asset is an increasing function of the price. The argument of the proof also shows that it is not possible to rule out equilibria in which the price of the risky asset is equal to 0 and the demand for it is 0.

Nevertheless, as figure 4 illustrates, conditions which exclude equilibria with 0-prices can be stated.

The condition, which excludes non-degenerate equilibria should insure that the value of demand is strictly positive at $p_t = 0$ for each t. Hence, the aspiration levels of the young investors with memory $(a; v_t(a))$ should be sufficiently low so that they choose $\alpha_t^i = a$ even for $p_t = 0$. Since the lowest possible return of a is 0, this is only possible if $\bar{u}^0 < 0$ and a positive fraction of the old investors with aspiration levels below 0 are endowed with a at $t = 1$.

It is, however, questionable whether the assumption $\bar{u}^0 < 0$ is economically meaningful. In contrast, insuring that at least one equilibrium with non-zero price exists does not require such

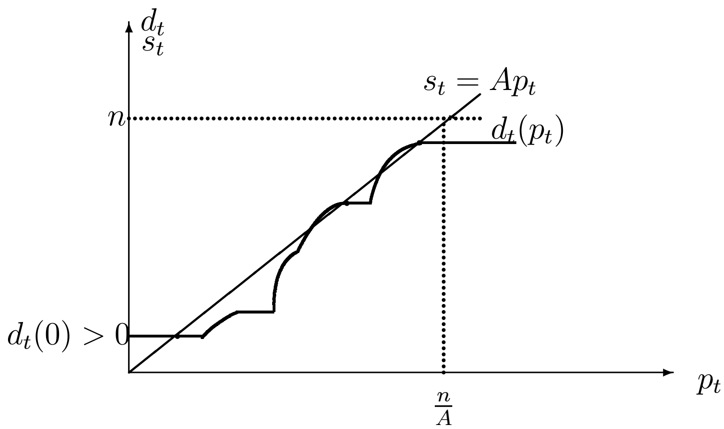

Figure 4

strong assumptions on the aspiration levels. Therefore, in the following section I state conditions which insure that for all $t \geq 1$, there is at least one equilibrium with $p_t > 0$.

4 Price-Dynamics

After introducing the notion and guaranteeing the existence of a temporary equilibrium, I proceed to investigate the dynamics of asset prices and asset holdings in the economy. The main factor determining the evolution of the economy is the distribution of aspiration levels.

I make the following assumptions which simplify the analysis:

$(A3)$ Let the utility functions of the investors be linear, i.e.:

$$\begin{aligned} u\left(\frac{p_t}{p_{t-1}} + \frac{\delta_t}{p_{t-1}}\right) &= \frac{p_t}{p_{t-1}} + \frac{\delta_t}{p_{t-1}} \\ u\,(1+r) &= 1+r. \end{aligned}$$

$(A4)$ Suppose that the initial holdings[8] of the old consumers at time $t = 1$ be such that the investors $[0; \bar{u}^a - \bar{u}^0]$ hold the risky asset a, whereas those on the interval $[\bar{u}^a - \bar{u}^0; n]$ hold the riskless

[8] Since at $t = 0$ the memory of all investors is empty, the choice between the acts is based on chance.

asset b for some $\bar{u}^a \in (\bar{u}^0; 1)$.

$(A5)$ Let $A = 1$ and assume that

$$p_0 = \frac{[\bar{u}^a - \bar{u}^0]}{A} = \left[\bar{u}^a - \bar{u}^0\right].$$

The assumption $(A3)$ about the linearity of the utility function is not substantial: it allows for an explicit computation of the equilibria, but does not influence the qualitative implications, as long as the utility function is continuous and strictly increasing. $(A4)$ specifies the initial holdings in the economy. It insures that a positive mass of investors with relatively low aspiration levels initially hold a, which is necessary for an equilibrium with $p_t > 0$ to exist in each period of time. An additional condition introduced below insures that the price of a remains positive along the equilibrium path. $(A5)$ specifies the price at $t = 0$ as the equilibrium price for the initial allocation, specified in $(A4)$.

Given these assumptions, three possible cases have to be distinguished, which will influence the dynamics of the price p_t qualitatively: either the highest aspiration level in the economy is lower than $(1 + r)$, or it lies in the interval between $(1 + r)$ and the return of the risky asset, when it brings positive dividends, or it is higher than the return of the risky asset when the dividends are positive.

Denote by h and l the following states of the economy characterized by an asset allocation and an equilibrium price:

$$\begin{aligned} \alpha_h^i &= a, \text{ if } \bar{u}^i \in \left[\bar{u}^0; \bar{u}^a\right] \cup \left[(1+r); \bar{u}^n\right] \\ \alpha_h^i &= b, \text{ if } \bar{u}^i \in \left[\bar{u}^a; (1+r)\right] \\ p_h &= \left[\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0\right] \end{aligned}$$

$$\begin{aligned} \alpha_l^i &= a, \text{ if } \bar{u}^i \in \left[\bar{u}^0; \bar{u}^a\right] \\ \alpha_l^i &= b, \text{ if } \bar{u}^i \in \left[\bar{u}^a; \bar{u}^n\right] \\ p_l &= \left[\bar{u}^a - \bar{u}^0\right]. \end{aligned}$$

Let s denote a typical equilibrium path characterizing the evolution of the economy. In the following it is shown that only those paths on which the two states h and l occur will play a role for the evolution of the economy. Hence, a typical equilibrium path s is (in general) a random sequence of h and l and can be written as

$$s = (s_t)_{t=0}^{\infty}$$

with $s_t \in \{h; l\}$. Let S denote the set of all such paths and Σ denote the σ-algebra on S. $C_t(h)$ and $C_t(l)$ describe the set of periods in which the economy is in state h and l, respectively (on a path s, where this dependence is omitted in the notation for convenience). Denote by π_h and π_l the limit frequencies of states h and l:

$$\begin{aligned} \pi_h &= \lim_{t \to \infty} \frac{|C_t(h)|}{t} \\ \pi_l &= \lim_{t \to \infty} \frac{|C_t(l)|}{t} \end{aligned}$$

if these limits exist. Usually, these limits will depend on the path s as well. However, the following proposition shows that for the economy at hand these frequencies are well defined and independent of the chosen path s.

Proposition 2 *Assume* $(A4)$, $(A5)$ *and* $(A6)$.

1. Suppose that

$$(1 + r) \geq \bar{u}^n > 1 \geq \bar{u}^a > \bar{u}^0. \tag{3}$$

holds. Then l is a stationary state of the economy and $\pi_l = 1$.

2. Suppose that

$$1 + \frac{\delta D}{\bar{u}^n - (1 + r) + \bar{u}^a - \bar{u}^0} > \bar{u}^n > 1 + r > 1 > \bar{u}^a > \bar{u}^0. \tag{4}$$

and

$$\frac{\bar{u}^a - \bar{u}^0}{\bar{u}^n - (1 + r) + \bar{u}^a - \bar{u}^0} > \bar{u}^a, \tag{5}$$

hold. Then the Markov process with states h and l and transition matrix $\bar{P}$:

$$\bar{P} = \left(\begin{array}{|c|c|c|} \hline & s_{t+1} = h & s_{t+1} = l \\ \hline s_t = h & q & 1 - q \\ \hline s_t = l & 1 & 0 \\ \hline \end{array} \right)$$

describes an equilibrium path of the economy. The limit frequencies π_h and π_l almost surely coincide with the invariant probability distribution of $\bar{P}$ and can be computed to be:

$$\begin{aligned} \pi_h &= \frac{1}{2 - q} \\ \pi_l &= \frac{1 - q}{2 - q}. \end{aligned}$$

3. Suppose that

$$\bar{u}^n > 1 + \frac{\delta D}{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0} > 1 + r > 1 > \bar{u}^a > \bar{u}^0 \tag{6}$$

and (5) hold. Then the deterministic cycle with two states h and l, such that $s_t = h$ for $t = 2k+1$ $s_t = l$ for $t = 2k$ with $k \in \mathbb{Z}_0^+$ is an equilibrium path of the economy. $\pi_h = \pi_l = \frac{1}{2}$ holds.

The specification of the aspiration levels is illustrated in figure 5 a)-c) for the three cases discussed in proposition 2. Part 1. of the proposition concerns the case of an economy with relatively low aspirations. Since in this case all investors in the market are satisfied with their initial holdings, the initial allocation is a stationary one.

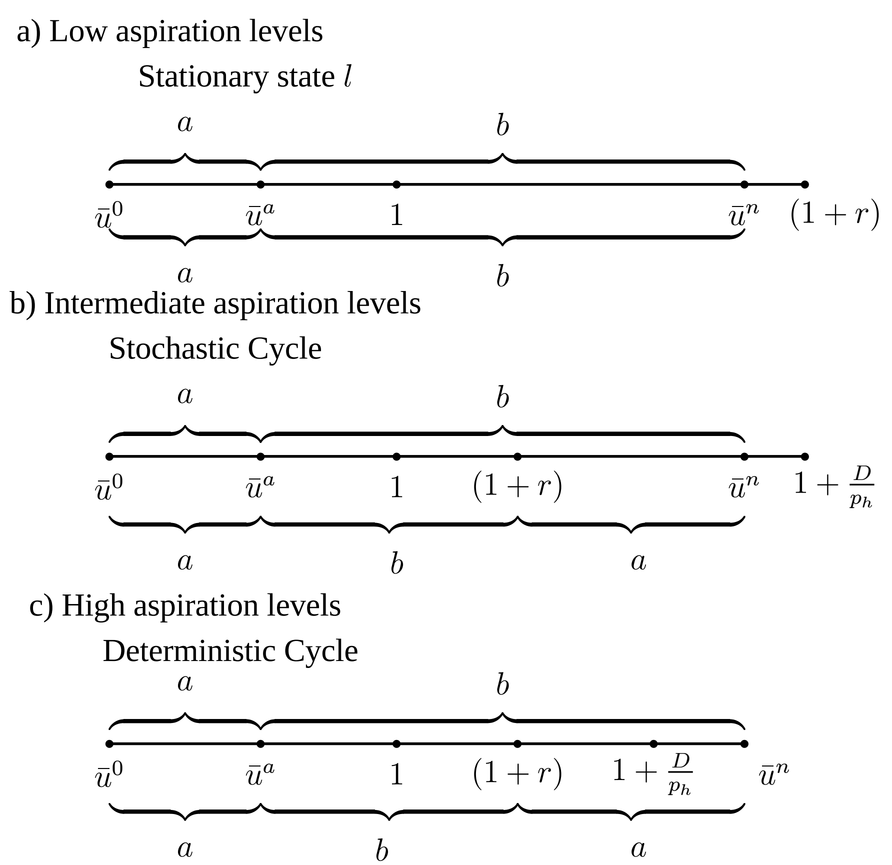

Figure 5

$p_h = [\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0]$

In case 2., the highest aspiration level is chosen in such a way that some of the investors are dissatisfied with the constant return of b and are also dissatisfied with the return of a as long as its dividend realization is low. Hence, these investors choose a as long as its dividend is positive, switch to b, once the dividend realization becomes 0 and switch back to a in the next period, thus causing the economy to evolve according to a stochastic cycle. Assumption (5) implies that the investors with low aspiration levels, $[\bar{u}^0; \bar{u}^a]$, are ready to hold the risky asset, even if its dividend is 0 and its price falls by the highest possible amount[9]. This condition insures the existence of an equilibrium path with $p_t > 0$ in each t.

In case 3., the highest aspiration level is chosen so as to exceed any possible return realization. Since this assumption renders some of the investors dissatisfied with any possible return realization, these investors switch in each period between the available assets and cause a deterministic cycle of the economy.

5 Discussion of the Results

The results derived in the previous section seem to be consistent with empirical results from real and experimental financial markets which have detected significant violations of the rational expectations hypothesis in financial markets. First, in a market populated by case-based decision-makers prices are not necessarily arbitrage-free:

Corollary 3 *Under the assumptions of proposition 2, arbitrage possibilities are present in the market if:*

1. (3) holds and $\frac{\delta D}{\bar{u}^a - \bar{u}^0} < r$. Then, asset b dominates asset a.

2. (4) and (5) hold and
$$\frac{[\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0]}{\bar{u}^a - \bar{u}^0} > 1 + r.$$
Then, asset a dominates asset b in state l.

3. (5) and (6) hold and
$$\frac{[\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0]}{\bar{u}^a - \bar{u}^0} > 1 + r.$$
Then, asset a dominates asset b in state l.

9 This happens if all investors with aspiration levels $[(1+r)\,;\bar{u}^n]$ switch from a to b in a single period t. Then $p_{t-1} = \bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0$, whereas $p_t = \bar{u}^a - \bar{u}^0$ obtain and the return of a becomes
$$\frac{\bar{u}^a - \bar{u}^0}{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0}.$$

Violations of the no-arbitrage restrictions have been found in empirical data. The works of Rosenthal and Young (1990), Lamont and Thaler (2001), Shleifer (2000, Chapter 3) document arbitrage opportunities in real markets. It has been argued that these phenomena can be traced back to short-sales limitations and other imperfections of real financial markets, see Cochrane (2002). However, in an experiment in which a perfect market is simulated, Oliven and Rietz (1995) and Rietz (1998) also record violations of arbitrage. They comment on the difficulties they encounter to enforce arbitrage free prices, even after explaining the subjects how to identify and use arbitrage opportunities.

The result derived for a market populated by case-based decision-makers shows that the notion of arbitrage crucially depends on the knowledge of the investors about the economy. If the investors do not know (or do not believe) that one asset is dominated by another one in each state of nature, then no arbitrage possibilities exist for them in the market, although they might be present from the viewpoint of an external observer.

Bubbles and significant deviations of the prices from fundamental values are often observed in financial markets, see Kindelberger (1978), Sunder (1995) and Camerer (1989). In a market populated by case-based decision-makers, the price of the risky asset need not coincide with the fundamental value, even if the investors have a linear utility function and a riskless asset exists:

Corollary 4 *Denote by FV the fundamental value of a:*

$$FV = \frac{q\delta D}{r}$$

and let μ_a denote the mean price of a. Under the assumption of proposition 2:

1. if (3) holds, $\mu_a \gtreqless FV$, iff $\bar{u}^a - \bar{u}^0 \gtreqless \frac{q\delta D}{r}$;

2. if (4) and (5) hold, $\mu_a \gtreqless FV$, iff[10]

$$D \lesseqgtr \frac{r\,(2-q)\,(\bar{u}^a - \bar{u}^0) + r\,(\bar{u}^n - (1+r))}{(2-q)\,q\delta};$$

[10]

$$D^*\delta > (\bar{u}^n - 1)\left(\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0\right),$$

as required in condition (4) holds, for instance, if

$$r > (2-q)\,q\,(\bar{u}^n - 1)\,.$$

If

$$D^*\delta < (\bar{u}^n - 1)\left(\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0\right),$$

then $D > D^*$ always holds and the risky asset is undervalued, $\mu_a < FV$.

3. if (5) and (6) hold, $\mu_a \gtreqless FV$, iff
$$\frac{q\delta D}{r} \lesseqgtr \frac{1}{2}\left[\bar{u}^n - (1+r)\right] + \left[\bar{u}^a - \bar{u}^0\right].$$

Moreover, when investors with aspiration levels exceeding $(1+r)$ are present in the market, we observe price upward movements, which do not depend on the dividend paid by a. Hence, they cannot be attributed either to changes in fundamentals, or to changes in the dividends. As long as (4) holds, the downward movements are in contrast conditioned on the asset paying a low dividend in the second period. As a whole, the structure of the asset-price movements reminds of a small bubble, which emerges without visible reasons (apart from the fact that a large part of the investors is dissatisfied by the returns of the safe technology) and then bursts, because of low dividend payments. Of course, if the asset continues to pay high dividends, then the high price of the asset will persist, until in some period t the dividend becomes low. As long as the probability $1-q$ of a low dividend is positive, the bubble will burst with probability one.

For case 3. of proposition 2, the bubbles become deterministic, since the downward movement also occurs independently of the dividend paid.

Corollary 5 *Under the assumption of proposition 2,*

1. if (3) holds, then the standard deviation of the price σ_a satisfies $\sigma_a = 0$;

2. if (4) and (5) hold,
$$\sigma_a = \sqrt{1-q}\frac{\left[\bar{u}^n - (1+r)\right]}{(2-q)};$$

3. if (6) and (5) hold,
$$\sigma_a = \frac{1}{2}\left[\bar{u}^n - (1+r)\right].$$

In cases 2. and 3. when investors have relatively high aspiration levels, the fluctuation of the price is neither due to changes in the fundamental value of the asset, nor to new information nor even necessarily to changes in the dividend payments. Hence, the asset price exhibits extreme volatility, which cannot be explained by the characteristics of the asset, but which is consistent with the empirical evidence on price volatility, see Roll (1984, 1989) and Shiller (1981, 1990).

The price fluctuation in the model can be explained by the decision-making process of the investors. The risk, faced by the investors in the market consists of two parts: the random fluctuation of the dividends described by q and the ignorance of the investors and their reliance on the cumulative utility when predicting returns. Especially, the volatility depends positively on the mass of investors with high aspiration levels $[\bar{u}^n - (1 + r)]$. In Guerdjikova (2001), it has been shown that investors with relatively high aspiration levels usually trade too much in the sense of Odean (1999). The results derived here demonstrate that in a market environment increasing the number of case-based decision-makers with high aspiration levels leads to an increase of the risk faced by the economy, as well as to frequent change of asset holdings. By switching too often between the available portfolios, not acting on information, these investors not only cause the price of the risky asset to fluctuate without changes of the fundamental value but also lower their profits, since they buy when prices are high and sell at low prices[11].

The behavior of investors with relatively high aspiration levels furthermore renders the price movements in the model predictable. Observe that if (4) and (5) hold, the upward price movements are predictable, whereas if conditions (6) and (5) hold, both the upwards and the downwards price movements are predictable to some extent. Predictability of asset returns has been recorded in financial markets by De Bondt and Thaler (1985), Chopra, Lakonishok and Ritter (1992), Bernard (1992), Bernard and Thomas (1989,1990), Loughran and Ritter (1995). Significant violations of the efficiency market hypothesis are found. Factors like past performance, market-to-book ratios and capitalizations of stocks, as well as seasonality can help predict future returns. Especially, negative short-run correlation of price movements for individual assets has been observed by Blume and Friend (1978), Lo and MacKinlay (1988) and Jegadeesh (1990).

A similar result obtains in the present model. Assume that the aspiration levels of the investors remain constant, but the dividend paid by the asset increases[12]. It is easy to see that the correlation between the returns of the asset a is small if the aspiration levels are relatively low as compared to δD (the capitalization of the firm is large)[13] and rises, as the capitalization diminishes, rendering the aspiration levels of the investor relatively high as compared to δD.

[11] Excessive frequency of trades which cannot be justified by changes in fundamentals or by profitability considerations is recorded by Odean (1999) and Barber and Odean (2001 a, 2001 b).

[12] The value of the dividend δD can be used as a proxy for the capitalization of the firm — D.

[13] In the case of low aspiration levels (δD is relatively high) there is no correlation between the returns. As δD decreases (the case of intermediate aspiration levels) the correlation increases (downward price movements are certainly followed by upward price movements), whereas in the case of high aspiration levels, upward and downward price movements alternate in each period.

6 Conclusion

In this paper, I define a market equilibrium for an economy populated by case-based decision-makers and study the asset price dynamics. The dynamic of asset prices and asset holdings is determined by the position of the highest aspiration level relative to the highest possible return of the risky asset. If the aspiration levels in the economy are relatively low, the price of the risky asset and the holdings of the investors remain constant over the time. Nevertheless, the price of the asset might deviate significantly from its fundamental value and arbitrage possibilities may be present in the market.

Higher aspiration levels induce cycles, which may be stochastic or deterministic. The risky asset exhibits excess volatility, which depends positively on the mass of investors with high aspiration levels in the economy. These investors trade too much, lowering their own profits and increasing the variance of prices. Hence, they exhibit the characteristic behavior of ”overconfident” investors, whose presence in the market was documented by Odean (1998). Since case-based investors base their decisions on past information, price movements are forecastable to some extent and exhibit negative correlation in the short-run. Hence, in a market populated by case-based investors, it is possible to identify phenomena which are inconsistent with expected utility maximization combined with rational expectations, but which might help to explain empirical findings from financial markets.

Although the assumptions made to derive the results might seem too severe, they can be easily relaxed and the results generalized accordingly. Allowing for diversification necessitates the introduction of a similarity function among price - portfolio pairs[14]. If the similarity is defined as a strictly decreasing function of the Euclidean distance between such pairs, the limit results derived in this paper remain unchanged.

Introducing long memory allows for learning, but only for investors whose aspiration levels are appropriately chosen. Even if all past cases are included in the memory, the investors might not be able to learn the optimal portfolio if their aspiration levels are too high or extremely low.

A major criticism of the model presented in this paper is that the economy consists only of case-

[14] See Gilboa and Schmeidler (1997) for an axiomatzation of the case-based decision theory, in which similarity perceptions among problem - act pairs are present. The introduction of similarity becomes necessary, if the number of available acts is infinite.

based investors and that their initial endowment does not change over the time and is, therefore, independent of their previous returns, given the OLG structure of the model. To address these issues a model of an economy with both case-based decision makers and expected utility maximizers should be considered. Should it be found that the share of wealth the case-based decision makers own shrinks to 0, as the time evolves, their influence on prices and returns would become negligible, prices and returns would behave as under rational expectations. The claim that the presence of case-based decision-makers can explain empirically observed phenomena such as bubbles, predictability of returns or arbitrage possibilities would then be unfounded. However, in Guerdjikova (2004) it is shown that the case-based decision-makers are not necessarily driven out of the market and that their behavior may have a significant influence on prices.

Appendix

Proof of Proposition 1: It is first stated that the value of demand for a of those intervals of investors, who remember a case of the type $(b; u(1+r))$ at time t is a constant function of the price p_t:

Lemma 6 *Consider an interval of investors with aspiration levels* $[\bar{u}^m; \bar{u}^p] \subset [\bar{u}^0; \bar{u}^n]$ *with identical memories. Let* $\alpha^i_{t-1} = b$*. Then*

- *if*
$$\left[u(1+r) - \bar{u}^i\right] < 0 \text{ for each } \bar{u}^i \in [\bar{u}^m; \bar{u}^p], \quad (7)$$
then $\alpha^i_t = a$ *for every* $i \in [\bar{u}^m - \bar{u}^0; \bar{u}^p - \bar{u}^0]$.

Proposition 7
- *if*
$$\left[u(1+r) - \bar{u}^i\right] < 0 \text{ for each } \bar{u}^i \in [\bar{u}^m; \bar{u}^p], \quad (8)$$
then $\alpha^i_t = b$ *for every* $i \in [\bar{u}^m - \bar{u}^0; \bar{u}^p - \bar{u}^0]$.

- *if neither (7), nor (8) are satisfied, then there is a critical aspiration level* $\hat{u} \in (\bar{u}^m; \bar{u}^p)$*, such that:*
$$\hat{u} = u(1+r)$$
and
$$\alpha^i_t = a, \text{ if } \bar{u}^i \geq \hat{u}$$

$$\alpha_t^i \;=\; b\text{, if } \bar{u}^i \leq \hat{u}.$$

Proof of Lemma 6:

Follows directly from the argument in section 2.2.■

Figure 6 illustrates the three cases.

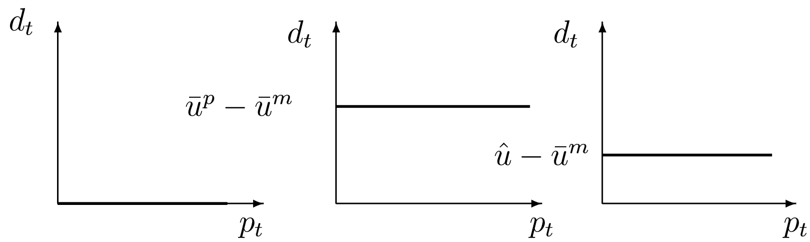

Figure 6

The following corollary characterizes the value of demand of these investors:

Corollary 8 *Consider an interval of investors with aspiration levels* $[\bar{u}^m; \bar{u}^p] \subset [\bar{u}^0; \bar{u}^n]$ *with identical memories. Let* $\alpha_{t-1}^i = b$ *for all* $\bar{u}^i \in [\bar{u}^m; \bar{u}^p]$*. Then the value of demand for* a *of the interval* $[\bar{u}^m; \bar{u}^p]$ *—* $d_t^A [\bar{u}^m; \bar{u}^p]$ *is a constant function of* p_t *and can obtain (depending on the memory and the aspiration levels* $[\bar{u}^m; \bar{u}^p]$*) only values between* 0 *and* $(\bar{u}^p - \bar{u}^m)$*.*

Now consider the investors whose predecessors hold a.

Proposition 9 *Consider an interval of investors with aspiration levels* $\left[\bar{u}^j; \bar{u}^l\right] \subset [\bar{u}^0; \bar{u}^n]$ *with identical memories, containing the case* $(a; v_t(a))$*. Define* $\hat{p}_t$ *by:*

$$\hat{p}_t = \min \left\{ p_t \in \mathbb{R}_0^+ \mid u \left(\frac{p_t + \delta_t}{p_{t-1}} \right) - \bar{u}^i \geq 0 \right\}$$

for every $\bar{u}^i \in \left[\bar{u}^j; \bar{u}^l\right]$ *and* $\check{p}_t$ *by:*

$$\check{p}_t = \max\left\{p_t \in \mathbb{R}_0^+ \mid u\left(\frac{p_t + \delta_t}{p_{t-1}}\right) - \bar{u}^i \leq 0\right\},$$

for every $\bar{u}^i \in \left[\bar{u}^j; \bar{u}^l\right]$ *if a maximum exists and set* $\check{p}_t = 0$*, else. Then the individual choices of the young investors in this interval are given by:*

- *if* $p_t \leq \check{p}_t$*, then* $\alpha_t^i = b$ *for all* $\bar{u}^i \in \left[\bar{u}^j; \bar{u}^l\right]$*;*
- *if* $p_t \geq \hat{p}_t$*, then* $\alpha_t^i = a$ *for all* $\bar{u}^i \in \left[\bar{u}^j; \bar{u}^l\right]$*;*
- *if* $\hat{p}_t > 0$ *and* $p_t \in [\check{p}_t; \hat{p}_t]$*, then there exists a critical aspiration level* $\bar{u}^* \in \left(\bar{u}^j; \bar{u}^l\right)$*, such that*

  $$\bar{u}^* = u\left(\frac{p_t + \delta_t}{p_{t-1}}\right) \tag{9}$$

  and

  $$\alpha_t^i = b, \text{ if } \bar{u}^i \geq \bar{u}^*$$

  $$\alpha_t^i = a, \text{ if } \bar{u}^i \leq \bar{u}^*.$$

Proof of Proposition 9:

Write the cumulative utility of a is given by:

$$U_t^i(a) = u\left(\frac{p_t + \delta_t}{p_{t-1}}\right) - \bar{u}^i, \tag{10}$$

whereas

$$U_t^i(b) = 0.$$

$\hat{p}_t$, as defined in the proposition, then denotes the lowest possible price of a, for which the cumulative utility of a exceeds the cumulative utility of b for all investors with aspiration levels $\left[\bar{u}^j; \bar{u}^l\right]$, whereas $\check{p}_t$ is the highest possible price, for which the inverse relation holds. The continuity and the strict monotonicity of the utility function with respect to p_t insures that $\hat{p}_t$ and $\check{p}_t$ are well defined and unique. The three cases listed in the proposition emerge naturally, when comparing the cumulative utilities of a and b. The definition of $\hat{p}_t$ and $\check{p}_t$ implies immediately that if $p_t \geq \hat{p}_t$, then act a is preferred by all investors with aspiration levels $\left[\bar{u}^j; \bar{u}^l\right]$, whereas $p_t \geq \check{p}_t$ implies that everyone chooses b. If $p_t \in [\check{p}_t; \hat{p}_t]$, some of the investors will choose a and some b. The critical aspiration level $\bar{u}^*$ (which of course depends on the price p_t) is determined by setting the two cumulative utilities equal and solving for $\bar{u}^i$. $(A1)$ insures that $\bar{u}^*$ is continuous and increasing in p_t. Therefore, the result of the proposition obtains.■

The following corollary obtains:

Corollary 10 *Consider an interval of investors with aspiration levels* $\left[\bar{u}^j; \bar{u}^l\right] \subset \left[\bar{u}^0; \bar{u}^n\right]$ *with identical memories. Let* $\alpha_{t-1}^i = a$ *for all* $\bar{u}^i \in \left[\bar{u}^j; \bar{u}^l\right]$*. The value of demand for* a *(or the mass of investors, who wish to hold* k*) of this interval —* $d_t\left(\left[\bar{u}^j; \bar{u}^l\right]\right)$ *is a monotone increasing, continuous function of the price* p_t*. The function consists of at most three segments:*

- *for* $p_t > \hat{p}_t$
$$d_t\left(\left[\bar{u}^j; \bar{u}^l\right]\right) = \bar{u}^l - \bar{u}^j = const,$$
- *for* $p_t < \check{p}_t$
$$d_t\left(\left[\bar{u}^j; \bar{u}^l\right]\right) = 0 = const,$$
- *for* $p_t \in [\check{p}_t; \hat{p}_t]$ $d_t\left(\left[\bar{u}^j; \bar{u}^l\right]\right)$ *is strongly increasing in* p_t *and convex, concave or linear if* $u(\cdot)$ *is convex, concave or linear, respectively.* $d_t\left(\left[\bar{u}^j; \bar{u}^l\right]\right)$ *is bounded above*[15] *by* $\left[\bar{u}^l - \bar{u}^j\right]$ *and below by* 0*.*

The result of the corollary is illustrated in figure 7 for a concave utility function $u(\cdot)$:

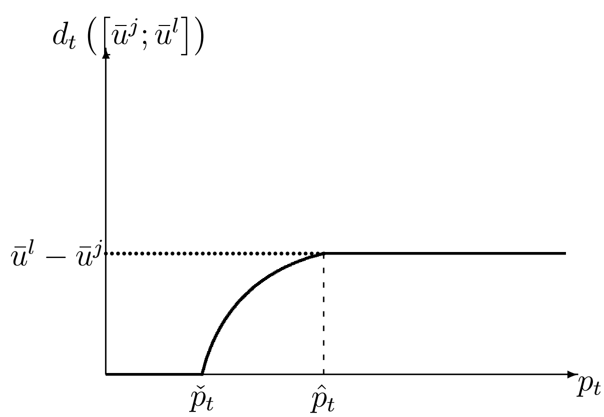

Figure 7

[15] The upper boundary results from the budget constraint of each investor.

Corollaries 8 and 10 demonstrate that each interval of investors with identical memories and choices at time $(t-1)$ is divided into at most two intervals such that all investors in the same interval have identical memories and make identical choices at time t. Hence, if $(A2)$ is satisfied in period $t=1$, then it is also satisfied in each period of time thereafter.

Corollaries 8 and 10 further demonstrate that the value of demand for a of each interval of investors with identical memories is a continuous function of p_t. The aggregate value of demand (obtained as the sum of the interval-values of demand) has therefore the same properties. Moreover, it maps the interval of possible prices $\left[0; \frac{n}{A}\right]$ into the interval of possible values of demand $[0; n]$ and therefore, according to the fixed-point theorem of Brouwer, see Mas-Collel, Whinston and Green (1995, p. 952), the aggregate value of demand has a fixed point. Such a fixed-point satisfies:

$$d_t(p_t) = Ap_t = s_t(p_t),$$

or for $p_t \neq 0$,

$$\frac{d_t(p_t)}{p_t} = x_t(p_t) = A,$$

which is the market-clearing condition for the risky asset.

For $p_t = 0$, the value of demand equals the value of supply and the demand for the asset is 0, whereas the supply is positive, hence the fixed-point again represents a market equilibrium.■

Proof of Proposition 2:

1. (3) implies that, for investors with $\bar{u}^i \in [\bar{u}^a; \bar{u}^n]$ and $\alpha_0^i = b$ the cumulative utilities satisfy:

$$U_1^i(b) = 1 + r - \bar{u}^i > 0 = U_1^i(a).$$

Hence, $\alpha_1^i = b$ for $i \in [\bar{u}^a - \bar{u}^0; \bar{u}^n - \bar{u}^0]$. By induction, suppose that the investors with $\bar{u}^i \in [\bar{u}^a; \bar{u}^n]$ choose b in some period t and consider the decision of the young investors in period $(t+1)$. Since

$$U_{t+1}^i(b) = 1 + r - \bar{u}^i > 0 = U_{t+1}^i(a)$$

for all $\bar{u}^i \in [\bar{u}^a; \bar{u}^n]$, it follows that $\alpha_t^i = b$ obtains in each period of time for $i \in [\bar{u}^a - \bar{u}^0; \bar{u}^n - \bar{u}^0]$.

Now consider the investors with $\bar{u}^i \in [\bar{u}^0; \bar{u}^a]$ and $\alpha_0^i = a$. In $t = 1$, the cumulative utilities they observe satisfy:

$$U_1^i(a) = 1 + \frac{\delta_t}{\bar{u}^a - \bar{u}^0} - \bar{u}^i \geq 1 - \bar{u}^i > 0 = U_1^i(b),$$

as long as $p_1 = \bar{u}^a - \bar{u}^0$, hence as long as $\alpha_1^i = a$ for $i \in [0; \bar{u}^a - \bar{u}^0]$. By induction, $U_t^i(a) >$

$U_t^i(b)$ holds for each $t \geq 1$ if $\alpha_t^i = a$ for all $i \in [0; \bar{u}^a - \bar{u}^0]$. Hence, in each $t \geq 1$, there is a temporary equilibrium in which $s_t = l$. Therefore, l is a stationary state of the economy.

2. Now assume (4) and (5). Suppose that at some $t-1$ the state of the economy is l.

   In period t, the investors with aspiration levels $\bar{u}^i \in [\bar{u}^a; (1+r)]$ observe cumulative utilities:

   $$U_t^i(b) = 1 + r - \bar{u}^i > 0 = U_t^i(a)$$

   and choose $\alpha_t^i = b$. The investors with aspiration levels $\bar{u}^i \in [(1+r); \bar{u}^n]$ observe cumulative utilities:

   $$U_t^i(b) = 1 + r - \bar{u}^i < 0 = U_t^i(a)$$

   and choose $\alpha_t^i = a$, regardless of the price p_t. Since their mass is $[\bar{u}^n - (1+r)]$, as long as the investors with aspiration levels $\bar{u}^i \in [\bar{u}^0; \bar{u}^a]$ choose $\alpha_t^i = a$, the cumulative utilities observed by these investors are

   $$U_t(a) = \frac{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0 + \delta_t}{\bar{u}^a - \bar{u}^0} - \bar{u}^i > 1 - \bar{u}^i > 0 = U_t^i(b),$$

   by assumption (4). Hence, the investor on the interval $[0; \bar{u}^a - \bar{u}^0]$ indeed choose $\alpha_t^i = a$ in equilibrium. Hence, in t, state h is an equilibrium.

   Now consider a period $t-1$ in which the state of the economy is h. Two cases must be considered:

   - $\delta_t = \delta D$

     The young investors with aspiration levels $[\bar{u}^a; (1+r)]$ observe cumulative utilities:

     $$U_t^i(b) = 1 + r - \bar{u}^i > 0 = U_t^i(a)$$

     and choose $\alpha_t^i = b$. As long as the investors with aspiration levels $\bar{u}^i \in [\bar{u}^0; \bar{u}^a] \cup [(1+r); \bar{u}^n]$ choose $\alpha_t^i = a$, they observe cumulative utilities:

     $$U_t^i(a) = 1 + \frac{\delta D}{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0} - \bar{u}^i > 0 > U_t^i(b),$$

     by assumption (4). Hence, it is indeed optimal for them to choose a. It follows that in period t with $\delta_t = \delta D$, $s_t = h$ is an equilibrium.

   - $\delta_t = 0$

     The young investors with aspiration levels $[\bar{u}^a; (1+r)]$ observe cumulative utilities:

     $$U_t^i(b) = 1 + r - \bar{u}^i > 0 = U_t^i(a)$$

     and choose $\alpha_t^i = b$.

     Even if all of the young consumers with aspiration levels $[\bar{u}^0; \bar{u}^a] \cup [(1+r); \bar{u}^n]$ choose a, the cumulative utilities they observe are given by:

$$U_t^i(a) = 1 - \bar{u}^i$$
$$U_t^i(b) = 0.$$

Whereas $U_t^i(a) > U_t^i(b)$ holds for all $i \in [0; \bar{u}^a - \bar{u}^0]$, it is obviously violated for $i \in [(1+r) - \bar{u}^0; n]$, because of condition (4). Hence, $\alpha_t^i = b$ for all $i \in [(1+r) - \bar{u}^0; n]$. The cumulative utilities observed by the investors $i \in [0; \bar{u}^a - \bar{u}^0]$ then become:

$$U_t^i(a) = \frac{\bar{u}^a - \bar{u}^0}{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0} - \bar{u}^i > 0 = U_t^i(b),$$

according to assumption (5) and these investors choose $\alpha_t^i = a$. Hence, in period t with $\delta_t = 0$, the economy returns to state l.

Since the economy starts in state l by assumptions $(A4)$ and $(A5)$, it follows that if in period t the economy is in state l, the state in $(t+1)$ is h with probability 1. If in period t the state is h, then in $(t+1)$ the economy moves to state l if the dividend realization is 0, hence, with probability $(1-q)$ and stays in state h if the dividend realization is δD, or with probability q. It follows that the economy evolves according to a Markov process with two states h and l and a transition matrix:

$$\bar{P} = \left(\begin{array}{|c|c|c|} \hline & p_{t+1} = p_h & p_{t+1} = p_l \\ \hline p_t = p_h & q & 1-q \\ \hline p_t = p_l & 1 & 0 \\ \hline \end{array} \right).$$

Now compute the invariant probability distribution of the Markov chain described by $\bar{P}$:

$$\begin{pmatrix} \pi_h \\ \pi_l \end{pmatrix} = \begin{pmatrix} \pi_h \\ \pi_l \end{pmatrix}' \bar{P},$$

which simplifies to

$$q\pi_h = (2 - \pi_h).$$

It follows that the invariant probabilities satisfy:

$$\pi_h = \frac{1}{2-q}$$
$$\pi_l = \frac{1-q}{2-q}.$$

These probabilities are obviously strictly positive for $q \in (0; 1)$ and therefore the Markov chain described by $\bar{P}$ is positive recurrent. Since any positive recurrent chain on a countable space is also positive Harris recurrent, see Meyn and Tweedie (1996, p. 208), it follows that the Law of Large Numbers applies for this chain. Hence, let ι_h denote the indicator function for state h:

$$\iota_h(t) = \left\{ \begin{array}{ll} 1, & \text{if } s_t = h \\ 0, & \text{if } s_t = l \end{array} \right\}.$$

According to theorem 17.1.7 in Meyn and Tweedie (1996, p. 425),

$$\lim_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t}\iota_h(t)=\int\iota_h(t)\,d\pi=\pi_h$$

holds almost surely for any initial distribution over the states h and l. Since $\frac{1}{t}\sum_{\tau=0}^{t}\iota_h(t)$ describes the mean time up to period t that the economy spends in state h, it follows that the frequency of state h equals π_h almost surely in the limit. Analogous arguments show that the frequency of state l equals π_l on almost each path $s\in S$.

3. Assume now that (6) and (5) hold. Let the state of the economy be l at some time $t-1$. Then, as shown in part 2. of this proof, the equilibrium of the economy is state h at time t. If the state of the economy is h at $t-1$ and the dividend payment is 0, then the arguments used in part 2. of this proof demonstrate that the state of the economy is l at time t. Assume, therefore, that $s_{t-1}=h$ and $\delta_t=\delta D$. The proof would be completed if it were shown that $s_t=l$ is an equilibrium.

   At t, the young investors with aspiration levels $[\bar{u}^a;(1+r)]$ observe cumulative utilities:

$$U_t^i(b)=1+r-\bar{u}^i>0=U_t^i(a)$$

   and choose $\alpha_t^i=b$.

   Even if all of the young consumers with aspiration levels $[\bar{u}^0;\bar{u}^a]\cup[(1+r);\bar{u}^n]$ choose a, the cumulative utilities they observe are given by:

$$U_t(a)=1+\frac{\delta D}{\bar{u}^n-(1+r)+\bar{u}^a-\bar{u}^0}-\bar{u}^i.$$

   Since

$$1+\frac{\delta D}{\bar{u}^n-(1+r)+\bar{u}^a-\bar{u}^0}-\bar{u}^n<0,$$

   it follows that there is a subinterval of investors with aspiration levels between $[(1+r);\bar{u}^n]$ for whom

$$U_t^i(a)<U_t^i(b)$$

   holds and who, therefore, choose $\alpha_t^i=b$.

   I will show that if (5) holds, there is an equilibrium, in which all investors with aspiration levels $[(1+r);\bar{u}^n]$ choose $\alpha_t^i=b$. Indeed, suppose to the contrary that if all investors from $[(1+r)-\bar{u}^0;n]$ choose $\alpha_t^i=b$, for some of them

$$U_t^i(a)=\frac{\bar{u}^a-\bar{u}^0+\delta D}{\bar{u}^n-(1+r)+\bar{u}^a-\bar{u}^0}-\bar{u}^i>0=U_t^i(b)$$

holds. This implies that

$$\frac{\bar{u}^a - \bar{u}^0 + \delta D}{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0} - (1+r) > 0,$$

since $(1+r)$ is the lowest aspiration level in this interval. Hence,

$$\frac{\delta D}{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0} > (1+r) - \frac{\bar{u}^a - \bar{u}^0}{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0}. \tag{11}$$

On the other hand, the first inequality of condition (6) implies that

$$\frac{\delta D}{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0} < \bar{u}^n - 1, \tag{12}$$

whereas condition (5) requires

$$\bar{u}^a - \bar{u}^0 > \bar{u}^a \left(\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0\right)$$

and since $\bar{u}^a - \bar{u}^0 < \bar{u}^a$, this means that

$$\left(\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0\right) < 1,$$

or

$$2 + r - \bar{u}^n - \bar{u}^a + \bar{u}^0 > 0$$

must hold.

Now compare the left hand sides of (11) and (12). It is easy to see that since

$$\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0 > 0,$$

$$(1+r) - \frac{\bar{u}^a - \bar{u}^0}{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0} > \bar{u}^n - 1$$

is equivalent to

$$\left[\bar{u}^n - (1+r)\right]\left[2 + r - \bar{u}^n - \bar{u}^a + \bar{u}^0\right] > 0,$$

which is always satisfied, as long as (6) and (5) hold. Hence, δD cannot satisfy (11) and (12) simultaneously and, therefore,

$$\frac{\bar{u}^a - \bar{u}^0 + \delta D}{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0} < (1+r)$$

obtains. But then

$$U_t^i(a) = \frac{\bar{u}^a - \bar{u}^0 + \delta D}{\bar{u}^n - (1+r) + \bar{u}^a - \bar{u}^0} - \bar{u}^i < 0 = U_t^i(b)$$

holds for all $\bar{u}^i \in [(1+r)\,; \bar{u}^n]$. Hence, state l is indeed an equilibrium at time t. It follows that $s_t = l$ implies $s_{t+1} = h$ and $s_t = h$ implies $s_{t+1} = l$.

Since the economy starts at state l, it follows that the two states h and l indeed define a deterministic cycle of the economy as described in the proposition.■

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