SONDERFORSCHUNGSBEREICH 504

Rationalitätskonzepte,
Entscheidungsverhalten und
ökonomische Modellierung

No. 03-36

A Utilitarian Approach to the Provision and Pricing of Excludable Public Goods

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December 2003

I am very grateful for helpful comments from Felix Bierbrauer and Peter Norman and for research support from the Deutsche Forschungsgemeinschaft through Sonderforschungsbereich 504 at the University of Mannheim.

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December 10, 2003

Abstract

The paper studies utilitarian welfare maximization in a model with an excludable public good where individual preferences are private information. If inequality aversion is large, optimal allocations involve the use of admission fees and exclusion to redistribute resources from people who benefit a lot from the public good to people who benefit little. If inequality aversion is close to zero, optimal admission fees are zero. These results are robust if earning abilities provide an additional source of heterogeneity and income taxation an additional policy instrument.

Key Words: Public-Good Provision, Entry Fees for Excludable Public Goods, Utilitarian Welfare Maximization

JEL Classification: D61, D63, H21, H41

1 Introduction

Following the seminal contribution of Samuelson (1954), the theory of public-goods provision has mainly focussed on goods that exhibit nonexcludability as well as nonrivalry in consumption. Less attention has been paid to goods that exhibit nonrivalry in consumption and yet allow for the possibility of individual exclusion. This neglect reflects the assessment that in a first-best world people should not be excluded from the enjoyment of a good that exhibits nonrivalry and whose use involves no externalities from crowding, mutual entertainment or mutual annoyance of participants. If it does not cost anything to admit another person to the enjoyment of the public good, the ability to exclude individuals should not be used. Samuelson (1958) actually used this argument to justify his

*I am very grateful for helpful comments from Felix Bierbrauer and Peter Norman and for research support from the Deutsche Forschungsgemeinschaft through Sonderforschungsbereich 504.

focus on public goods exhibiting nonrivalry and nonexcludability at the same time.

The present paper challenges this view. In a *second-best setting* involving private information about individual preferences, *equity concerns* can justify the active use of the ability to exclude individuals in spite of the efficiency losses that are thereby induced. If information about individual preferences is private, people with different preferences will end up with different payoffs. In particular, people who care a lot for the public good will end up being better off than people who do not care for the public good at all. Because people can always dissimulate their preferences, their contributions towards the provision of the public good cannot be made commensurate with the benefits they obtain. People who care a lot for the public good will therefore earn information rents. The distributional impact of these rents is reduced if admission to the public good is conditioned on the payment of a fee. By paying for admission, people who care a lot for the public good contribute more to its provision than people who do not care for it at all. For a given level of public-good provision, the information rents of people who care a lot for the public good are then lower, and the welfare of people who care little for the public good is higher. This equity gain may outweigh the efficiency loss from excluding people who care a little for the public good, but not enough to pay the admission fee.[1]

The paper studies this *equity-efficiency tradeoff* in a model of public-good provision by an inequality averse utilitarian planner. The choice of an optimal admission rule is shown to depend on the planner's degree of inequality aversion. If inequality aversion is low, an optimal incentive-compatible allocation involves completely open admissions and zero admission fees. If inequality aversion is high, an optimal incentive-compatible allocation involves positive admission fees and the exclusion of anybody who refuses to pay the fee. People who do not care for the public good at all then contribute less towards its provision. For extreme levels of inequality aversion, these people's net contribution towards the provision of the public good will actually be negative: an excess of admission fee revenues over provision costs, which is shared equally among the population, provides a tool by which the provision of the public good benefits even those people who do not care for the public good at all.

The underlying logic of the analysis is well known from the theory of optimal income taxation initiated by Mirrlees (1971). If people differ in some unobservable characteristic, first-best incentive-compatible allocations can have undesirable distributional properties. A utilitarian planner may therefore prefer to forego first-best efficiency in order to improve on the distributional properties of the allocation. In the theory of optimal income taxation, people are assumed to differ in their earnings abilities. In this paper, they differ in their ability to benefit from the enjoyment of the public good. Though politically less relevant than differences in earning abilities or in wealth, these differences in preferences - and the differences in outcomes they induce - are an important aspect

[1] For a nonexcludable public good, this tradeoff has previously been discussed by Ledyard and Palfrey (1999).

of public-good provision when there is incomplete information about individual preferences.

In challenging the view that the ability to exclude individuals from the enjoyment of a public good should never be used, the paper complements the recent work of Schmitz (1997) and Norman (2003). In their papers, excludability is useful because, in bargaining under incomplete information with voluntary participation, the threat of individual exclusion makes participants more willing to contribute to the financing of the public good. Given that each agent can veto the arrangement and prevent the public goods from being provided at all, a version[2] of the impossibility theorem of Myerson and Satterthwaite (1983) shows that first-best allocations cannot be implemented; second-best allocations involve admission fees and exclusion of anybody who fails to pay the fees. The argument is akin to the Ramsey-Boiteux argument concerning the desirability of consumer prices exceeding marginal costs to finance production in industries with significant overhead costs or otherwise increasing returns to scale when these industries are subject to budget constraints.[3]

Like the Ramsey-Boiteux argument, the Schmitz-Norman argument on the desirability of admission fees is subject to the critique of Atkinson and Stiglitz (1976) that divergences of consumer prices from marginal costs are undesirable if one can use lump sum taxes to provide the requisite funds. In Schmitz (1997) and Norman (2003), anything like lump sum taxation is unavailable because agents must participate voluntarily and any agent who has no use for the public good at all would veto any arrangement that requires him to make a lump sum payment towards its provision.[4] However, the voluntary-participation requirement is problematic if public-good provision is the subject of government activity rather than multilateral bargaining.

The government has a power of coercion, which it can use to impose lump sum taxes. In this case, the Atkinson-Stiglitz critique would seem to put us right back into a first-best world where there is no point in using admission fees to finance the public good. From d'Aspremont and Gérard-Varet (1979), it is well known that, in the absence of interim participation constraints, first-best allocations can be implemented through Bayesian mechanisms even though each agent's preferences are his own private information.

However, the first-best allocations that are thus obtained have unattractive distributive properties. Coercion to pay lump sum taxes hits those people most who do not care for the public good at all. As mentioned above, under the given

[2]See Güth and Hellwig (1986), Mailath and Postlewaite (1990).

[3]The link between excludable public goods and the Ramsey-Boiteux pricing problem has been pointed out by Samuelson (1958, 1969) and Laffont (1982/1988), see also Drèze (1980). If one phrases the excludable-public-good provision problem in terms of the services obtained by the different participants upon admission, one actually has a private-good provision problem with positive fixed costs and zero marginal costs, where the "quantity" of the public good appears as a parameter determing the common "quality" of the private good, i.e. the services obtained by the individual participants.

[4]An explicit account of the relation between interim participation constraints in the multilateral-bargaining mechanism design problem and the government budget constraint à la Ramsey-Boiteux in the absence of lump sum payments is given in Hellwig (2003).

information assumptions, these are the people in the economy who are worst off. The inequality between agents who care and agents who do not care for the public good is therefore exacerbated if agents who do not care have to pay lump sum taxes to contribute towards financing the public good. Interim participation constraints, which are problematic on efficiency grounds, can actually be seen as a crude device to protect those participants who are least well off against the imposition of a mechanism that worsens their position even further.

Utilitarian analyses of equity-efficiency tradeoffs usually focus on distributional issues arising from unobservable differences in *earning abilities*. Thus, in their critique of the Ramsey-Boiteux approach, Atkinson and Stiglitz (1976) allow for differences in earning abilities and find that deviations from first-best pricing rules for different consumption goods are unwarranted *unless* the demands for those goods vary systematically with the unobservable characteristic and hence with equilibrium labour supplies. Golosov et al. (2003) use wealth differences induced by differences in earning abilities to justify capital income taxation in a model of precautionary savings while reaffirming the Atkinson-Stiglitz critique to rule out any differential tax treatment of goods in the same period. Boadway and Keen (1993) consider the implications of differences in earning abilities for the rules governing public-good provision, but, like Atkinson and Stiglitz (1976), they do not consider other sources of heterogeneity. In particular, they do not allow for incomplete information and heterogeneity with respect to people's preferences for the public good.

The focus of the literature on differences in earning abilities or differences in wealth reflects the fact that, empirically, these are the most obvious reasons for differences in well being that we observe. However, within the utilitarian model, earning ability is just one parameter, and there is no reason to assume that it is the only parameter in which people differ. If people differ in other parameters, the utilitarian approach must take these differences into account. Given the importance of cardinal properties of utility representations of preferences in the utilitarian approach, one may feel uneasy about focussing on differences in preferences over private-good versus public-good consumption, but that is an argument against the utilitarian approach altogether. There does not seem to be any reason why the reliance on cardinal properties of utility representations of preferences should be any more suspect for preferences over private-good and public-good consumption than for the consumption-leisure choices that are treated in the income tax literature.

The importance of sources of heterogeneity other than differences in earning abilities has been pointed out in assessments of the Atkinson-Stiglitz results by Laffont and Tirole (1994, pp. 194 ff.) and, more recently, by Cremer, Pestieau and Rochet (2001), who study a model with differences in endowments as well as labour productivity. In the present context, their critique is particularly germane because much of the interest of the public-good provision problem itself comes from the heterogeneity of participants with respect to the benefits that they draw from the public good and from the incompleteness of information about individual preferences. In this line of argument, Cremer and Laffont (2003) consider utilitarian provision and pricing of an excludable public good when participants differ not only in terms of their incomes, but also in terms of access costs which they must incur (on top of any admission fees) in order to benefit at all from the public good. Their analysis focusses on the conflict between the requirements of incentive compatibility and the desire to subsidize the poor, a conflict which is particularly pronounced if access costs of the poor are higher than access costs of the rich.

In the following, Section 2 lays out the basic model and formulates the utilitarian welfare problem. The model involves one excludable public good and one private good in an economy with a continuum of agents. The cross-section distribution of tastes in the economy is fixed. However, individual agents' tastes are their private information. The utilitarian planner must therefore take account of incentive compatibility constraints as well as feasibility constraints. Sections 3 and 4 study optimal utilitarian allocations under the additional assumption that, conditional on people's types, there is no randomization over whether they are admitted to the enjoyment of the public good or not. Incentive compatible admission policies then involve the use of admission fees so that people are admitted if and only if they pay the stipulated fee. Section 3 discusses the equity-efficiency tradeoff involved in the choice of these fees. Section 4 shows how optimal admission fees and optimal provision levels for the public good depend on the utilitarian planner's inequality aversion and the individual participants' risk aversion. For the case of risk neutral consumers and an inequality averse planner, Section 5 studies the allocation problem without requiring admission policies to be deterministic. An example is given to show that randomized admissions may indeed be desirable. However, the considerations underlying the equity-efficiency tradeoff are the same as in the model without randomized admissions. Section 6 shows that the desirability of admission fees at high levels of inequality aversion is robust to the introduction of consumer risk aversion and to the introduction of differences in earning abilities as a second source of heterogeneity.

2 The Allocation Problem

I study a large-economy version of the model considered by Schmitz (1997) and Norman (2003). There is an atomless continuum of consumers with total mass equal to one. The economy has an aggregate production capacity Y, which can be used to provide an amount C of aggregate consumption of a private good and a level Q of a public good subject to the resource constraint

$$C + K(Q) \leq Y. \tag{2.1}$$

The cost function $K(.)$ is strictly increasing, strictly convex, and twice continuously differentiable, with $K(0) = 0$, $K'(0) = 0$, and $\lim_{Q\to\infty} K'(Q) = \infty$.

The public good is excludable. For each individual in the economy, an allocation must determine how much of the private good he gets to consume and whether he is admitted to the enjoyment of the public good or not. Consumers are assumed to be risk neutral. Given an expected consumption level c of the private good and a probability π of being admitted to the enjoyment of the public good at the level Q, a consumer obtains the payoff

$$c + \pi\theta Q. \tag{2.2}$$

The parameter θ is taken to be the consumer's private information. From the perspective of the other consumers, or of the system as a whole, θ is the realization of a random variable $\tilde{\theta}$, which takes values in the unit interval and has a probability distribution $F(.)$ with a strictly positive, continuously differentiable density $f(.)$. The mean of $\tilde{\theta}$ is denoted as $\bar{\theta}$. Assuming a large-numbers effect, $F(.)$ is also the cross-section distribution, and $\bar{\theta}$ is also the cross-section mean of the preference parameter in the population.

In this setting, an *allocation* corresponds to a level Q of public-good provision and a pair of functions $\pi(.), c(.)$ so that, for any $\theta \in [0,1]$, $\pi(\theta)$ is the probability that a consumer with preference parameter θ is admitted to the enjoyment of the public good, and $c(\theta)$ is his expected consumption of the private good. For $\theta \in [0,1]$,

$$v(\theta) := c(\theta) + \pi(\theta)\theta Q \tag{2.3}$$

is the expected payoff of a consumer with preference parameter θ under the allocation $(Q, \pi(.), c(.))$.

An allocation $(Q, \pi(.), c(.))$ is said to be *incentive compatible* if

$$v(\theta) \geq c(\theta') + \pi(\theta')\theta Q \tag{2.4}$$

for all θ and $\theta' \in [0,1]$. An allocation is *feasible*, if it satisfies the constraint

$$\int_0^1 c(\theta)f(\theta)d\theta + K(Q) \leq Y, \tag{2.5}$$

which is (2.1) with $C = \int_0^1 c(\theta)f(\theta)d\theta$.

A utilitarian planner assesses allocations according to the welfare functional[5]

$$\int_0^1 W(v(\theta))\ f(\theta)\ d\theta. \tag{2.6}$$

His problem is to maximize (2.6) over the set of feasible and incentive compatible allocations. The welfare function $W(.)$ on (2.6) is assumed to be strictly

[5]As written, the specification (2.6) presumes that the planner is concerned about each agent's *expected* utility. An alternative specification would have the planner concerned about each agent *ex post* utility, e.g., with an integrand of the form $(1 - \pi(\theta))W(c_0(\theta)) + \pi(\theta)W(c_1(\theta) + \theta Q)$, where $c_0(\theta)$ is private-good consumption of a consumer who is excluded and $c_1(\theta)$ is private-good consumption of a consumer who is admitted to the public good. Given the strict concavity of $W(.)$ though, under this alternative specification, the planner would find it optimal to set $c_0(\theta) = v(\theta)$ and $c_1(\theta) = v(\theta) - \theta Q$ so that (2.6) is again the appropriate specification. Because consumers care only about the expected consumption $c(\theta) = (1 - \pi(\theta))c_0(\theta) + \pi(\theta)c_1(\theta)$, there is no difficulty in arranging $c_0(\theta)$ and $c_1(\theta)$ so that conditional on θ, there is no further risk in consumers' payoffs.

increasing, strictly concave, and twice continuously differentiable. Following Atkinson (1973), I refer to the relative curvature $\rho_W(c) := -\frac{W''(c)}{W'(c)}$ as a measure of *inequality aversion.*

If the preference parameters of the different consumers were publicly observable, the planner would not have to worry about incentive compatibility. Given the strict concavity of $W(.)$, he would choose an allocation satisfying $\pi(\theta) = 1$ and $c(\theta) = c(0) - \theta Q$ for all θ. Everybody would be admitted to the enjoyment of the public good, and the payoff levels $v(\theta)$ would all be equal to $v(0) = c(0)$. Feasibility would imply $c(0) = Y - K(Q) + \bar{\theta}Q$, so Q would therefore be chosen to maximize $W(Y - K(Q) + \bar{\theta}Q)$. This requires that $K'(Q) = \bar{\theta}$, i.e., at the chosen Q, the marginal provision cost should just equal the expected marginal benefit *per capita.*

However, *the egalitarian first-best allocation is not incentive compatible.* With $\pi(\theta) = 1$ and $c(\theta) = c(0) - \theta Q$ for all θ, any consumer with $\theta > 0$ has an incentive to understate his preference for the public good in order to raise his consumption of the private good without having to reduce his enjoyment of the public good. If the preference parameter θ is each consumer's private information, the egalitarian first-best allocation cannot be implemented.

Therefore the planner faces an equity-efficiency tradeoff. *Efficiency considerations* call for open admissions to the enjoyment of the public good, with $\pi(\theta) = 1$ for all θ. But then incentive compatibility implies $c(\theta) = c(0)$ and $v(\theta) = Y - K(Q) + \theta Q$ for all θ. Agents with high θ are strictly better off than agents with low θ. Indeed, since $K(Q) > \theta Q$ for θ close to zero, the latter are made strictly worse of by the provision of the public good. *Equity considerations* call for some redistribution of the private good from agents with high θ to agents with low θ, providing some compensation for differences in the impact of the provision of the public good.

Such redistribution requires a self-selection device which induces agents to reveal their information by the choices they take. The use of such a device induces an inefficiency. If an agent with high θ is to accept a lower level of consumption of the private good, he must be compensated by a higher probability of being admitted to the enjoyment of the public good. This is only possible if agents claiming a low θ cannot be sure of being admitted, i.e. if $\pi(\theta) < 1$ for θ close to zero. Such exclusion of low-θ agents is inefficient. The question is to what extent the utilitarian planner wants to impose it anyway in order to screen high-θ agents from low-θ agents and provide for some redistribution of the private good from the former to the latter.

3 Nonrandomized Admissions, Admission Fees, and the Equity-Efficiency Tradeoff

The utilitarian allocation problem is first studied under the additional restriction that $\pi(.)$ takes only the values zero and one, i.e. that there is no randomization over whether someone is admitted to the enjoyment of the public good or not. This restriction is natural if the planner is unable to control the personal identities of people claiming admission to the use of the public good and the allocation must satisfy the additional constraint of not providing incentives for trading admission permits *ex post*.

If $\pi(.)$ takes only the values zero and one, incentive compatible allocations have a simple structure: A person is admitted to the enjoyment of the public good if and only if he or she pays an admission fee p. The fee is paid and admission is gained if the benefit θQ that a consumer draws from the enjoyment of the public good exceeds p; the fee is not paid if θQ is less than p. An incentive compatible allocation $(Q, \pi(.), c(.))$ with $Q > 0$ and $\pi(\theta) \in \{0, 1\}$ for all θ thus takes the form

$$c(\theta) = c_0 \text{ and } \pi(\theta) = 0, \text{ if } \theta Q < p, \tag{3.1}$$

$$c(\theta) = c_0 - p \text{ and } \pi(\theta) = 1, \text{ if } \theta Q > p \tag{3.2}$$

for some c_0 and $p \geq 0$. For $\theta = p/Q$, the consumer is indifferent about paying p to enjoy the public good, and one may have either $c(p/Q) = c_0$ and $\pi(p/Q) = 0$ or $c(p/Q) = c_0 - p$ and $\pi(p/Q) = 1$.

An incentive compatible allocation with nonrandomized admissions is thus characterized by the public-good provision level Q, the base consumption c_0, and the admission fee p. The planner's assessment of the allocation is therefore:

$$\int W(c_0 + \max(\theta Q - p, 0))\; f(\theta)d\theta, \tag{3.3}$$

and the feasibility condition (2.5) takes the form

$$c_0 \leq Y - K(Q) + p(1 - F(\frac{p}{Q})). \tag{3.4}$$

The term $p(1 - F(\frac{p}{Q}))$ in (3.4) represents the aggregate revenue from admission fees. This revenue provides scope to raise c_0 above $Y - K(Q)$, the *per capita* amount that is available after deduction of the cost $K(Q)$. The counterpart of c_0 exceeding $Y - K(Q)$ by the amount $p(1 - F(\frac{p}{Q}))$ is found in $c_0 - p$ falling short of $Y - K(Q)$ by the amount $pF(\frac{p}{Q})$.

The allocation problem is to choose Q, c_0, and p so as to maximize (3.3) subject to (3.4). In this problem, one can replace the admission fee p by the product $\hat{\theta}Q$, where $\hat{\theta}$ is the value of the preference parameter at which a consumer is just indifferent as to whether to pay the admission fee p. Because the constraint (3.4) is obviously binding, one can also replace c_0 by

$$c_0(Q, \hat{\theta}) := Y - K(Q) + \hat{\theta}Q(1 - F(\hat{\theta})). \tag{3.5}$$

The problem then is to choose Q and $\hat{\theta} \in [0, 1]$ so as to maximize

$$W^*(Q, \hat{\theta}) := \int W(Y - K(Q) + \hat{\theta}Q(1 - F(\hat{\theta})) + Q\max(\theta - \hat{\theta}, 0))\; f(\theta)d\theta. \tag{3.6}$$

In the remainder of this section and in the following section, I study the solutions to this maximization problem. I am particularly interested in their dependence on the welfare function $W(.)$. Suppressing the role of the other exogenous data, I write $(Q(W), \hat{\theta}(W))$ for a pair $(Q, \hat{\theta})$ which maximizes (3.6) for the given W.

The first-order conditions for maximizing (3.6) are:

$$\int_0^1 W' \left[-K'(Q) + \hat{\theta}(1 - F(\hat{\theta})) + \max(\theta - \hat{\theta}, 0) \right] f(\theta) d\theta = 0 \qquad (3.7)$$

and

$$Q \left[(1 - F(\hat{\theta}) - \hat{\theta} f(\hat{\theta})) \int_0^1 W' \, f(\theta) d\theta - \int_{\hat{\theta}}^1 W' \, f(\theta) d\theta \right] = 0, \qquad (3.8)$$

where in each integral, the derivative W' is evaluated at the point $c_0(Q, \hat{\theta}) + \max(\theta Q - \hat{\theta} Q, 0)$.

Inspection of (3.7) and (3.8) shows that, trivially, $(0, 1)$ is a critical point of the welfare function W^*. However, this critical point cannot correspond to a maximum of W^* : Whereas $W^*(0, 1) = W(Y)$, because $K'(0) = 0$, one finds that for some small $Q > 0$ and $\hat{\theta} = \frac{1}{2}$, aggregate entry fees $\frac{1}{2} Q(1 - F(\frac{1}{2}))$ exceed the cost $K(Q) < QK'(Q)$ by some $\varepsilon > 0$. For such Q and $\hat{\theta} = \frac{1}{2}$, the integrand in (3.6) exceeds $W(Y + \varepsilon)$ for all θ, and one has $W^*(Q, \frac{1}{2}) > W(Y + \varepsilon)$. Since $W^*(Q(W), \hat{\theta}(W)) \geq W^*(Q, \frac{1}{2})$ for any Q and, by inspection of (3.6), $W^*(Q(W), \hat{\theta}(W)) \leq W(Y + Q(W))$, this observation yields:

Lemma 3.1 *There exists $\varepsilon > 0$, such that, for any welfare function $W(.)$, $W^*(Q(W), \hat{\theta}(W)) \geq W(Y + \varepsilon)$ and $Q(W) \geq \varepsilon$.*

No matter what $W(.)$ may be, the level of public-good provision is bounded away from zero, and the entry fee is bounded away from being prohibitive. Relying on this information, the following lemmas provide additional bounds on $Q(W)$ and $\hat{\theta}(W)$.

Lemma 3.2 *Under the given assumptions about W,*

$$Q_*(\hat{\theta}(W)) < Q(W) < Q^*, \qquad (3.9)$$

where, for any $\hat{\theta}$,

$$Q_*(\hat{\theta}) := \arg\max_Q [Q \, \hat{\theta}(1 - F(\hat{\theta})) - K(Q)], \qquad (3.10)$$

and

$$Q^* := \arg\max_Q [Q \, \bar{\theta} - K(Q)]. \qquad (3.11)$$

Lemma 3.2 follows directly from the first-order condition (3.7), which equates the marginal welfare benefits and costs of an increase in Q. Marginal welfare benefits of an increase in Q arise from the increase in the information rent $\theta Q - \hat{\theta}(W)Q$ of consumers with preference parameter $\theta > \hat{\theta}(W)$ and from the increase in entry fee revenues $\hat{\theta}(W)Q(1-F(\hat{\theta}(W)))$ raising the base consumption $c_0(Q,\hat{\theta})$. With $\hat{\theta}(W) < 1$, aggregate marginal benefits from additional information rents are strictly positive, and $Q(W)$ is strictly greater than the provision level $Q_*(\hat{\theta})$ which maximizes "profits", i.e. the excess of entry fee revenues over provision costs.

Further, since $W'(.)$ is a decreasing function, the marginal increases in information rents of high-θ consumers receive less weight in (3.7) than the marginal increases in information rents of low-θ consumers. The benefits from additional information rents in (3.7) are therefore smaller than the difference between $\bar{\theta}\int_0^1 W' dF(\theta)$ and $\hat{\theta}(W)(1-F(\hat{\theta}(W)))\int_0^1 W' dF(\theta)$. The aggregate marginal benefits from an increase in Q altogether are no greater than $\bar{\theta}\int_0^1 W' dF(\theta)$. Therefore $Q(W)$ is less than the public-good provision level Q^* which maximizes expected surplus in the absence of risk aversion or inequality aversion when everybody is admitted to the enjoyment of the public good.

Lemma 3.3 *Under the given assumptions about W, $\hat{\theta}(W) < \theta^*$ where*

$$\theta^* := \min \arg\max_{\theta} \theta(1-F(\theta)). \tag{3.12}$$

Lemma 3.3 is derived from the first-order condition (3.8), which equates the marginal welfare benefits and costs of an increase in $\hat{\theta}$. On the benefit side, there is the possibility of the increase in $\hat{\theta}$ raising the aggregate revenue $Q\hat{\theta}(1-F(\hat{\theta}))$ from admission fees and permitting an increase in the base consumption $c_0(Q,\hat{\theta})$. On the cost side, there is the additional burden imposed on consumers who pay the fee. If the marginal revenue $Q(1-F(\hat{\theta}) - \hat{\theta}f(\hat{\theta}))$ is zero or negative, a small increase in $\hat{\theta}$ fails to raise aggregate revenue, and there is no benefit to compensate for the cost increase imposed on the users of the public good. For such $\hat{\theta}$, (3.8) cannot be satisfied. More generally, it is undesirable to have an admission fee which is so high that it can be lowered without an appreciable negative effect on aggregate admission fee revenue.

The tradeoff between the marginal benefits and costs of an increase in $\hat{\theta}$ can be interpreted as an *equity-efficiency tradeoff.* To show this, I rewrite condition (3.8) in the form

$$\begin{aligned} \hat{\theta}Qf(\hat{\theta}) \quad &= \quad \frac{Q\,(1-F(\hat{\theta}))\int_0^{\hat{\theta}} W'(c_0)\,dF(\theta)}{\int_0^1 W' dF(\theta)} \\ &\quad - \frac{Q\,F(\hat{\theta})\,\int_{\hat{\theta}}^1 W'(c_0+\theta Q-\hat{\theta}Q)\,dF(\theta)}{\int_0^1 W' dF(\theta)}\,. \end{aligned} \tag{3.13}$$

The *left-hand* side of (3.13) measures the *efficiency loss* associated with a marginal increase in the admission fee: If the fee is raised from $p = \hat{\theta}Q$ to $p + dp = (\hat{\theta} + d\hat{\theta})Q$, the set of participants who prefer to forego the enjoyment of the public good is expanded to include all those whose preference parameter θ lies between $\hat{\theta}$ and $\hat{\theta} + d\hat{\theta}$. The private-good equivalent of the surplus that is thereby lost is approximately $\hat{\theta}Q\ f(\hat{\theta})\ d\hat{\theta}$: the set of consumers involved has approximately the mass $f(\hat{\theta})\ d\hat{\theta}$ and each of these consumers loses an enjoyment that he considers to be worth approximately $\hat{\theta}Q$ units of the private good. This loss $\hat{\theta}Q\ f(\hat{\theta})\ d\hat{\theta}$ is a true deadweight loss because, in the absence of crowding or other external effects in the enjoyment of the public good, there is no real cost of admitting these consumers to the enjoyment of the public good.

The *right-hand* side of *(3.13)* provides a measure of the *utilitarian redistribution gain* from having all participants with $\theta > \hat{\theta}$ reduce their consumption of the private good by $dp = Qd\hat{\theta}$ units and using the proceeds to raise c_0 by $Q(1-F(\hat{\theta}))d\hat{\theta}$ units. For consumers with $\theta < \hat{\theta}$, this change entails a net increase of private-good consumption by $Q(1-F(\hat{\theta}))d\hat{\theta}$, for consumers with $\theta > \hat{\theta}$, a net decrease by $QF(\hat{\theta})d\hat{\theta}$. In the brackets on the right-hand side of (3.13), the first term represents the welfare gain from the increase of private-good consumption of agents with $\theta < \hat{\theta}$, the second term represents the welfare loss from the decrease of private-good consumption of agents with $\theta > \hat{\theta}$. Through deflation by $\int_0^1 W' dF(\theta)$, these terms are translated into an equivalent number of units of the private good. The difference between them provides a measure of the net welfare gains from this redistribution.

To see that there are net welfare gains rather than losses, observe that the right-hand side of (3.13) can be rearranged as

$$\frac{QF(\hat{\theta})}{\int_0^1 W' dF(\theta)} \int_{\hat{\theta}}^1 [W'(c_0) - W'(c_0 + \theta Q - \hat{\theta}Q)]\ dF(\theta), \tag{3.14}$$

which is never negative. Indeed, with $Q > 0$ and $\hat{\theta} < 1$, the strict concavity of $W(.)$ implies that the integral on the right-hand side of (3.14) is positive, so the net welfare effect of redistributing private-good consumption from agents with $\theta > \hat{\theta}$ to agents with $\theta < \hat{\theta}$ is positive if $F(\hat{\theta}) > 0$. The first-order condition for $\hat{\theta}$ just balances this gain against the marginal efficiency loss from having more agents forego the enjoyment of the public good.

The mathematical structure of this equity-efficiency tradeoff is the same as the mathematical structure of the tradeoff inherent in the standard first-order condition for optimal utilitarian income taxation, e.g., equation (27) in Mirrlees (1971) or equation (6.20) below. The different formalisms have different interpretations, but the underlying logic is the same.

4 Inequality Aversion and Optimal Admission Fees

The preceding discussion of the equity-efficiency tradeoff has *not* actually shown that optimal admission fees are positive. Indeed at $\hat{\theta} = 0$, the first-order condition (3.8) is *always* satisfied, the left-hand sides and right-hand sides of (3.13) and (3.14) are all equal to zero. The reason is that if one starts from $p = \hat{\theta}Q = 0$, then *both* the efficiency loss *and* the redistribution gain from a small increase in $\hat{\theta}$ are of the second order of smalls. The efficiency loss is of the second order of smalls because the few agents with $\theta \in [0, 0 + d\hat{\theta})$ suffer only a small loss if they forego the enjoyment of the public good. The redistribution gain is of the second order of smalls because the set of agents with $\theta \in [0, 0 + d\hat{\theta})$, who obtain a net increase of their private-good consumption, is small.[6] On agents with $\theta > 0 + d\hat{\theta}$, the introduction of the small admission fee has hardly any effect. For them, the increase in the base consumption c_0 provides close to a full compensation for the increase in the fee.

Even so, a zero admission fee is not always optimal. If $Q > 0$ and

$$W'(c_0(Q, 0)) > 2 \int_0^1 W'(c_0(Q, 0) + \theta Q) \, dF(\theta), \tag{4.1}$$

a zero admission fee yields a local *minimum* of W^* with respect to $\hat{\theta}$ because the second derivative

$$\frac{\partial^2 W^*}{\partial \hat{\theta}^2}(Q, 0) = Qf(0) \left[W'(c_0(Q, 0)) - 2 \int_0^1 W'(c_0(Q, 0) + \theta Q) dF(\theta) \right] \tag{4.2}$$

is positive. To see why (4.1) might hold, go back to the equity-efficiency tradeoff in (3.14). At $p = \hat{\theta}Q = 0$, the marginal efficiency loss on the left-hand side and the marginal redistribution gain on the right-hand side are both equal to zero. A marginal increase $dp = Qd\hat{\theta}$ in the admission fee raises the marginal redistribution gain (3.14) by

$$\frac{\int_0^1 [W'(c_0) - W'(c_0 + \theta Q)] \, dF(\theta)}{\int_0^1 W' \, dF(\theta)} f(0) Q d\hat{\theta},$$

which is strictly positive if $Q > 0$. An increase in the admission fee above zero thus *raises* the marginal redistribution gain that is to be expected from a further increase in the fee. Whereas a "first" small increase in the admission fee above zero has no first-order effects on the distribution of private-good consumption at all because the people paying the fee are roughly the same as the people benefitting from the induced increase in c_0, a "further" small increase does induce a redistribution gain because the "first" small increase in the fee above

[6] In the theory of optimal income taxation, this consideration implies the desirability of zero marginal tax rates at the top and at the bottom of the earnings distribution, see Seade (1977).

zero has created a clearly identified, nonnull set of agents who benefit from the redistribution involved in the "further" fee increase. If this increase in the marginal redistribution gain on the right-hand side is greater than the increase $f(0)Qd\hat{\theta}$ in the marginal efficiency loss on the left-hand side of (3.14), then (4.1) holds, and a further increase in $\hat{\theta}$ is desirable for the utilitarian planner.

These considerations suggest that the desirability of a positive admission fee should depend on the curvature $\rho_W(c) = -\frac{W''(c)}{W'(c)}$ of the welfare function $W(.)$, i.e. the planner's inequality aversion. Given that $Q(W)$ is bounded away from zero, (4.1) is likely to hold, if the curvature of $W(.)$ is large, and to fail, if the curvature of $W(.)$ is small. This intuition is confirmed by the following two results.

Proposition 4.1 *There exists $A > 0$ such that, if $\rho_W(c) < A$ for all c, then $\hat{\theta}(W) = 0$. Moreover, if $\{W_k\}$ is any sequence of increasing, concave, and twice continuously differentiable functions on $\Re_+$ such that $\lim_{k\to\infty} \rho_{W_k}(c) = 0$, uniformly for $c \in [Y - K(Q^*), Y - K(Q^*) + Q^*]$, then $\lim_{k\to\infty} Q(W_k) = Q^*$.*

Proposition 4.2 *If $\{W_k\}$ is any sequence of increasing, concave, and twice continuously differentiable functions on $\Re_+$ such that $\lim_{k\to\infty} \rho_{W_k}(c) = \infty$, uniformly in c, then $\lim_{k\to\infty} \hat{\theta}(W_k) = \theta^*$ and $\lim_{k\to\infty} Q(W_k) = Q_*(\theta^*)$. In particular $\hat{\theta}(W) > 0$ if $\rho_W(c)$ is uniformly large.*

If inequality aversion is uniformly[7] small, the pair $(Q(W), \hat{\theta}(W))$ is close to the pair $(Q^*, 0)$ which maximizes the expected aggregate surplus $\int_{\hat{\theta}}^1 \theta dF(\theta)\, Q - K(Q)$. There is thus no discontinuity in the dependence of the planner's choice on the welfare function as inequality aversion goes to zero. Indeed, the optimal admission fee isn't just small and converging to zero as overall inequality aversion goes to zero, but from some point onwards optimal admission fees are actually equal to zero if inequality aversion is sufficiently small (though still positive).

In contrast, optimal admission fees are positive if inequality aversion is uniformly large. As inequality aversion goes out of bounds, the planner's choices converge to the pair $(Q_*(\theta^*), \theta^*)$ which would be chosen by a monopolist maximizing the profit $Q\hat{\theta}(1 - F(\hat{\theta})) - K(Q)$ from installing the public good and charging admission fees for its use. By following the monopolist's strategy, the planner maximizes the base consumption $c_0(Q, \hat{\theta})$ and hence the welfare assessment $W(c_0(Q, \hat{\theta}))$ that is attached to the position of the worst-off individuals in the system, namely the individuals with $\theta = 0$. Proposition 4.2 thus confirms the well known principle that utilitarian welfare maximization with a high degree of inequality aversion yields results similar to the Rawlsian approach of maximizing the payoff of the worst-off individuals in the economy.[8]

From the utilitarian perspective, public-good provision serves two purposes: First, it benefits the people who enjoy the public good. Second, admission fee

[7] Uniformity of smallness or largeness of inequality aversion is not really needed, but helps avoid the need for complicated epsilon-delta arguments.

[8] See, e.g., Arrow (1973), Atkinson (1973).

revenues in excess of provision costs can be used to improve the position of low-θ consumers. If overall inequality aversion is small, only the first concern matters: The public good is provided *solely* because it benefits users. If overall inequality aversion is large, the second concern also matters. This concern actually becomes paramount when overall inequality aversion goes out of bounds and the welfare weight of high-θ consumers goes to zero.

Revenues from optimal admission fees will usually differ from provision costs. If overall inequality aversion is small, revenues from admission fees are zero because the fees themselves are zero. In this case, following the logic of Atkinson and Stiglitz (1976), the public good is paid for by a lump sum tax $K(Q(W))$ on all people in the economy, whether they draw utility from the public good or not. In contrast, if overall inequality aversion is large, revenues from admission fees exceed provision costs, providing a profit which is used to raise the base consumption of agents with low θ. In either case, if overall inequality aversion is small *and* if overall inequality aversion is large, there is no room for a budget constraint à la Ramsey-Boiteux. However, whether there is a surplus or a deficit from the provision of the public good subject to the payment of admission fees, depends on how strong a concern for redistribution the planner has.

The limits in Propositions 4.1 and 4.2 satisfy $Q^* > Q_*(\theta^*)$ and $0 < \theta^*$. These inequalities reflect the fact that an increase in Q or a decrease in $\hat{\theta}$ enhance the sensitivity of $c_0(Q, \hat{\theta}) + Q \max(\theta - \hat{\theta}, 0)$ with respect to θ and hence the inequality[9] inherent in the cross-section distribution of expected payoffs. Given this observation, one suspects that $Q(W)$ might be decreasing and $\hat{\theta}(W)$ nondecreasing with respect to changes in W that raise inequality aversion. For welfare functions exhibiting constant inequality aversion, this conjecture can actually be proved. For arbitrary welfare functions, I have only been able to obtain the weaker result that at least one of the two monotonicity properties must hold.

Proposition 4.3 *If two welfare functions W_1, W_2 are such that $\rho_{W_2}(c) > \rho_{W_2}(c)$ for all c, then $Q(W_2) < Q(W_1)$ or $\hat{\theta}(W_2) > \hat{\theta}(W_1)$.*

5 Solving the General Allocation Problem

I now return to the general version of the utilitarian allocation problem of maximizing (2.6) over the set of incentive compatible and feasible allocations, without the restriction that the admission rule must not involve any genuine randomization. By standard arguments, in this general setting, incentive compatibility of the allocation $(Q, \pi(.), c(.))$ is equivalent to the requirement that $\pi(.)$ be nondecreasing and that

$$c(\theta) = c_0 - Q \int_0^\theta \eta d\pi(\eta) = c_0 - \theta Q \pi(\theta) + Q \int_0^\theta \pi(\eta) d\eta \tag{5.1}$$

[9] In the sense of a mean-utility-preserving spread; see Diamond and Stiglitz (1974).

for some constant c_0 and all θ. At an incentive compatible allocation therefore, the welfare functional (2.6) takes the form

$$\int_0^1 W\left(c_0 + Q\int_0^\theta \pi(\eta)d\eta\right) f(\theta)d\theta, \tag{5.2}$$

and the feasibility constraint (2.5) takes the form:

$$K(Q) + c_0 + Q\int_0^1 \pi(\theta)[1 - F(\theta) - \theta f(\theta)]d\theta \leq Y. \tag{5.3}$$

The utilitarian allocation problem thus is equivalent to the problem of choosing a public-good provision level Q, a base consumption c_0, and a nondecreasing admission rule $\pi(.)$ so as to maximize (5.2) subject to the feasibility constraint (5.3).

To simplify the presentation, I further require that the admission rule be continuous from the right. This requirement eliminates the trivial multiplicity of solutions that arises because any modification of an admission rule $\pi(.)$ on a null set leaves the validity of the feasibility constraint (5.3) and the value of the objective (5.2) unchanged and, moreover, at discontinuity points of $\pi(.)$, such modifications are possible without upsetting monotonicity. By imposing continuity from the right, I pick exactly one member of any equivalence class of admission rules that can be generated by such modifications.

The following result shows that, apart from this trivial multiplicity arising from the arbitrariness of modifications at discontinuity points of admission rules, the solution to the utilitarian allocation problem is unique. In contrast to the problem with nonrandomized admissions, here the curvature assumptions on $W(.)$ and $K(.)$ are sufficient to rule out multiple solutions. The result also shows that the solution shares some of the properties that were established in Lemmas 3.1 - 3.3 for the problem with nonrandomized admissions.

Proposition 5.1 *The utilitarian allocation problem has a unique solution. The solution satisfies $Q \in (0, Q^*)$ and $\pi(\theta) = 1$ for $\theta \in [\theta^*, 1]$.*

To characterize the optimal admission rule, I use the control theoretic approach of Mirrlees (1971). For any θ, let

$$\hat{v}(\theta) = v(\theta) - v(0) = Q\int_0^\theta \pi(\eta)d\eta \tag{5.4}$$

be the information rent obtained by a consumer with preference parameter θ and note that the integrand in (5.2) can be written as $W(c_0 + \hat{v}(\theta))f(\theta)$. Let λ be a Lagrange multiplier λ for the feasibility constraint and consider the Lagrangian expression

$$\int_0^1 W(c_0 + \hat{v}(\theta))f(\theta)d\theta + \lambda[Y - K(Q) - c_0 - Q\int_0^1 \pi(\theta)[1 - F(\theta) - \theta f(\theta)]d\theta. \tag{5.5}$$

For a suitable choice of λ, the utilitarian allocation problem is equivalent to the problem of maximizing (5.5) with respect to Q, c_0, and the functions $\pi(.)$ and $\hat{v}(.)$ under the constraints that $\pi(.)$ be nondecreasing and that (5.4) hold for all θ. The requirement that (5.4) hold for all θ in turn is equivalent to the requirements that $\hat{v}(0) = 0$ and that $\hat{v}(.)$ be absolutely continuous with Radon-Nikodym derivative

$$\hat{v}'(\theta) = Q\pi(\theta). \tag{5.6}$$

As usual in problems of this type, it is convenient to initially neglect the second-order condition for incentive compatibility and to consider the *relaxed problem* of maximizing (5.5) subject to (5.6) without the requirement that $\pi(\theta)$ be nondecreasing in θ. If a solution to the relaxed problem happens to satisfy the additional constraint on $\pi(.)$, it is also a solution to the planner's original problem. Otherwise the solutions to the relaxed problem provide a starting point for the ironing procedure of Guesnerie and Laffont (1984) to obtain a solution to the planner's original problem.

The relaxed problem is a standard optimal-control problem with state variable $\hat{v}$ and control variable π. The Hamiltonian for this control problem is given as

$$W(\hat{v}(\theta))f(\theta) + \lambda[(Y - K(Q) - c_0)f(\theta) - Q\pi(\theta)(1 - F(\theta) - \theta f(\theta))] + \varphi(\theta)\pi(\theta)Q, \tag{5.7}$$

where φ is the costate variable associated with the state variable $\hat{v}$. Pontryagin's conditions for a solution to this control problem are:

$$\pi(\theta) = 0 \;\text{ if }\; \varphi(\theta)Q < \lambda Q(1 - F(\theta) - \theta f(\theta)), \tag{5.8}$$

$$\pi(\theta) \in [0,1] \;\text{ if }\; \varphi(\theta)Q = \lambda Q(1 - F(\theta) - \theta f(\theta)), \tag{5.9}$$

$$\pi(\theta) = 1 \;\text{ if }\; \varphi(\theta)Q > \lambda Q(1 - F(\theta) - \theta f(\theta)) \tag{5.10}$$

as the Kuhn-Tucker conditions for $\pi(\theta)$,

$$\varphi'(\theta) = -W'(c_0 + \hat{v}(\theta))f(\theta) \tag{5.11}$$

as the condition for the "dynamics" of the costate variable, and

$$\varphi(1) = 0 \tag{5.12}$$

as the transversality condition. The first-order conditions for Q and c_0 take the form

$$\int_0^1 \pi(\theta)[\varphi(\theta) - \lambda(1 - F(\theta) - \theta f(\theta))]d\theta - \lambda K'(Q) = 0 \tag{5.13}$$

and

$$\int_0^1 W'(c_0 + \hat{v}(\theta))f(\theta)d\theta = \lambda. \tag{5.14}$$

Given that $Q > 0$, conditions (5.8) - (5.10) imply that the choice of $\pi(\theta)$ depends on whether

$$g(\theta) := \varphi(\theta) - \lambda(1 - F(\theta) - \theta f(\theta)) \tag{5.15}$$

is negative, zero, or positive. Because (5.11) and (5.12) imply

$$\varphi(\theta) = \int_\theta^1 W'(c_0 + \hat{v}(\theta))f(\eta)d\eta \tag{5.16}$$

for all θ, one has

$$g(\theta) = \int_\theta^1 W'(c_0 + \hat{v}(\theta))f(\eta)d\eta - (1 - F(\theta) - f(\theta)\theta)\int_0^1 W'(c_0 + \hat{v}(\theta))f(\eta)d\eta, \tag{5.17}$$

indicating that the choice of $\pi(\theta)$ involves the same equity-efficiency tradeoff as the choice of an optimal admission fee in the previous analysis. In particular, the first-order condition for an interior choice of $\pi(\theta)$ is exactly the same as the first-order condition (3.8) for the critical $\hat{\theta}$. Like (3.8), it also has the same mathematical structure as the usual first-order condition in the theory of optimal income taxation.

Not surprisingly therefore, the desirability of restricting admissions in order to redistribute the private good from high-θ agents to low-θ agents again depends on the planner's inequality aversion. The following results extend Propositions 4.1 and 4.2 to the present, more general formulation.

Proposition 5.2 *There exists $A > 0$ such that, if $\rho_W(c) \leq A$ for all c, then the solution $(Q, \pi(.), c(.), \hat{v}(.))$ to the utilitarian allocation problem satisfies $\pi(\theta) = 1$, $c(\theta) = Y - K(Q)$, and $v(\theta) = Y - K(Q) + \theta Q$ for all $\theta \in (0, 1]$.*

Proposition 5.3 *For $k = 1, 2, ...$, let $(Q^k, \pi^k(.), c^k(.), \hat{v}^k(.))$ be the solution to the utilitarian allocation problem for the welfare function W_k. If $\lim_{k\to\infty} \rho_{W_k}(c) = \infty$, uniformly in c, then $\lim_{k\to\infty} Q_k = Q_*(\theta^*)$ and $\lim_{k\to\infty} \pi^k(\theta) = 0$ for $\theta < \theta^*$. In particular, for $\theta < \theta^*$, the solution to the utilitarian allocation problem satisfies $\pi(\theta) < 1$, if $\rho_W(c)$ is sufficiently large, uniformly in c.*

At one end of the spectrum, having a richer set of admission rules with randomized as well as nonrandomized admissions does *not* eliminate the desirability of keeping admissions completely open if inequality aversion is small. At the other end of the spectrum, if inequality is very large, it is desirable to implement an allocation close to the one that arises if the level of public-good provision and entry fee maximize profits. The latter result is due to the fact that the level of base consumption c_0 that is compatible with the resource constraint (5.3) is never larger than the maximal level $c_0^* = c_0(Q_*(\theta^*), \theta^*)$ with nonrandomized admissions and is indeed maximal if the admission rule stipulates exclusion for

$\theta < \theta^*$ and admission for $\theta > \theta^*$. Indeed, using integration by parts, one can rewrite (5.3) in the form

$$c_0 \leq c_0(Q, \pi) := Y - K(Q) + Q \int_0^1 \theta(1 - F(\theta)) d\pi(\theta); \tag{5.18}$$

the right-hand side of (5.18) is obviously maximized if $\pi(.)$ is such that $\pi(\theta) = 0$ for $\theta < \theta^*$ and $\pi(\theta) = 1$ for $\theta > \theta^*$ and if $Q = Q_*(\theta^*)$. The richer set of admission rules with randomized as well as nonrandomized admissions does *not* provide for a greater ability to raise the level base consumption.

Even so, there are circumstances in which it is desirable to use randomized rather than nonrandomized admission rules.[10] Because of the nonconvexity inherent in the equity-efficiency tradeoff in Sections 3 and 4, it is *not* always desirable to charge a single admission fee and admit people if and only if they pay the fee. Proposition 5.4 below gives an example where the optimal admission rule involves some nondegenerate randomization. Subsequently, Proposition 5.5 gives a sufficient condition for nonrandomized admission rules to be optimal.

Proposition 5.4 *Assume that the welfare function $W(.)$ and the density function $f(.)$ take the forms $W(v) = -e^{-\rho v}$, with $\rho > 0$, and $f(\theta) = Ae^{-B\theta}$, with $B > 0$ and $A = B/(1-e^{-B})$. Then there exists $\hat{\theta} \in [0, \theta^*)$ such that the solution to the utilitarian allocation problem satisfies*

$$\pi(\theta) = \frac{B}{\rho Q(2 - B\theta)} \in (0,1) \quad \textit{if} \quad \theta \in [0, \hat{\theta}), \tag{5.19}$$

and

$$\pi(\theta) = 1 \quad \textit{if} \quad \theta \in [\hat{\theta}, 1]. \tag{5.20}$$

Moreover, $\hat{\theta} > 0$ if and only if

$$1 > 2\ A\ \frac{1 - e^{-(\rho Q + B)}}{\rho Q + B}. \tag{5.21}$$

Proposition 5.5 *Assume that the elasticity $\frac{\theta f'(\theta)}{f(\theta)}$ is increasing in θ. Then there exists $\hat{\theta} \in [0, 1)$ such that the solution to the utilitarian allocation problem satisfies*

$$\pi(\theta) = 0 \quad \textit{if} \quad \theta \in [0, \hat{\theta}), \tag{5.22}$$

and

$$\pi(\theta) = 1 \quad \textit{if} \quad \theta \in [\hat{\theta}, 1]. \tag{5.23}$$

Morever, $\hat{\theta} > 0$ if and only if

$$W'(c_0) > 2 \int_0^1 W'(c_0 + Q\theta)\ f(\theta)\ d\theta \tag{5.24}$$

[10] In the context of optimal income taxation, the potential desirability of randomized schemes has been pointed out by Stiglitz (1982).

Before I turn to the difference between these two propositions, I note that they give the *same* conditions for *not* having completely open admissions: For $W'(.)$ and $f(.)$ as given in Proposition 5.4, (5.24) specializes to (5.21). These conditions also coincide with condition (4.1), i.e. the condition under which a zero entry fee provides a local minimum rather than a maximum for the allocation problem with nonrandomized admissions.

To understand this common feature of optimal admission rules in the different settings, observe that (5.17) and (5.11) imply

$$g(0) = 0, \tag{5.25}$$

and

$$g'(0) = (2\lambda - W'(c_0))f(0). \tag{5.26}$$

Equation (5.25) indicates that in the planner's relaxed problem the choice of the admission probability $\pi(0)$ is a matter of indifference. This observation corresponds to the previous finding that, at $\hat{\theta} = 0$, a small increase in the critical $\hat{\theta}$ has no first-order effects on the utilitarian planner's assessment of the allocation. Equation (5.26) then implies that for θ close to zero, one has a strict preference for setting $\pi(\theta) = 1$ if $2\lambda > W'(c_0)$ and a strict preference for setting $\pi(\theta) = 0$ if $2\lambda < W'(c_0)$. This is the common structure behind conditions (5.24) and (5.21) as well as (4.1).

Propositions 5.4 and 5.5 stipulate *different admission rules* if inequality aversion is large. The difference is due to the difference in assumptions about the behaviour of the elasticity $\frac{\theta f'(\theta)}{f(\theta)}$ of the density function $f(.)$.[11] In Proposition 5.4, this elasticity is equal to $-B\theta$, which is *decreasing* in θ. In Proposition 5.5, the same elasticity is assumed to be *increasing* in θ. To see why this matters, observe that (5.17) and (5.11) imply

$$g'(\theta) = \left[\lambda(2 + \frac{\theta f'(\theta)}{f(\theta)}) - W'(c_0 + \hat{v}(\theta))\right] f(\theta). \tag{5.27}$$

If $\frac{\theta f'(\theta)}{f(\theta)}$ is *increasing* in θ, then, regardless of the function $\hat{v}(.)$, the term in brackets is increasing in θ, and there is at most one θ at which the derivative (5.27) is zero; moreover this θ corresponds to a local *minimum* of $g(\theta)$. If $2\lambda < W'(c_0)$, i.e. if (5.27) is negative at $\theta = 0$, there is exactly one $\hat{\theta} > 0$ at which $g(\theta) = 0$, and one must have $\pi(\theta) = 0$ for $\theta < \hat{\theta}$ and $\pi(\theta) = 1$ for $\theta \geq \hat{\theta}$. The assumption that $\frac{\theta f'(\theta)}{f(\theta)}$ is increasing in θ thus leads back to a world where the admission probabilities $\pi(\theta)$ are all either zero or one.

In contrast, if the elasticity $\frac{\theta f'(\theta)}{f(\theta)}$ is *decreasing* in θ, it is possible to calibrate the information rent $\hat{v}(\theta)$ so that the decrease in $W'(c_0 + \hat{v}(\theta))$ exactly matches

[11] In Manelli and Vincent (2002), the behaviour of the elasticity $\frac{\theta f'(\theta)}{f(\theta)}$ plays a similar role, albeit in a problem of multidimensional mechanism design involving monopoly profit maximization rather than welfare maximization.

the decrease in $\lambda\frac{\theta f'(\theta)}{f(\theta)}$ so as to keep the derivative (5.27) constant at zero. In the given example, this is actually possible without running afoul of the boundary and monotonicity conditions on $\pi(.)$. Specifically, with $\frac{\theta f'(\theta)}{f(\theta)} = -B\theta$, one has $g(\theta) = g'(\theta) = 0$ for all $\theta \in [0, \hat{\theta})$ if and only if

$$W'(c_0 + \hat{v}(\theta)) = \lambda(2 + \frac{\theta f'(\theta)}{f(\theta)}) = (2 - B\theta)\lambda \tag{5.28}$$

for all $\theta \in [0, \hat{\theta})$, i.e., if and only if $W'(c_0) = 2\lambda$ and $W''(c_0 + \hat{v}(\theta))\hat{v}'(\theta) = -B\lambda$ for all $\theta \in [0, \hat{\theta})$. By (5.6), the latter equation becomes

$$W''(c_0 + \hat{v}(\theta))\pi(\theta)Q = -B\lambda. \tag{5.29}$$

Upon combining (5.28) and (5.29) and noting that, for the specified welfare function $W''(c) = -\rho W'(c)$ for all c, one obtains (5.19). For $\theta < \hat{\theta}$, the admission probability $\pi(\theta)$ is calibrated precisely to ensure that (5.28) and (5.29) hold.

In Propositions 5.4 and 5.5, the requirement that $\pi(.)$ be nondecreasing, i.e. the second-order condition for incentive compatibility, is automatically satisfied, so the solution to the relaxed problem is also a solution to the original allocation problem. The monotonicity constraint on $\pi(.)$ may however be binding, e.g., if the function $\theta \to \frac{\theta f'(\theta)}{f(\theta)}$ exhibits a sufficiently pronounced sine wave pattern, inducing the function $g(.)$ to have multiple critical points and to cross the axis several times. In this case, conditions (5.8) - (5.10) yield a nonmonotonic control function $\pi(.)$, and the solution to the planner's relaxed problem is not also a solution to his original problem. The solution to the planner's original problem is then obtained by the ironing procedure of Guesnerie and Laffont (1984), see also Fudenberg and Tirole (1991), pp. 303 ff. If (θ_1, θ_2) is the interval where the monotonicity constraint is binding, then $g(\theta_1) = g(\theta_2) = 0$, and on the interval (θ_1, θ_2), the admission rule satisfies $\pi(\theta) = \hat{\pi}$ where

$$\hat{\pi} = 0 \text{ only if } \int_{\theta_1}^{\theta_2} g(\theta)d\theta \leq 0, \tag{5.30}$$

$$\hat{\pi} \in (0,1) \text{ only if } \int_{\theta_1}^{\theta_2} g(\theta)d\theta = 0, \tag{5.31}$$

$$\hat{\pi} = 1 \text{ only if } \int_{\theta_1}^{\theta_2} g(\theta)d\theta \geq 0. \tag{5.32}$$

The requirement for an interior choice of $\hat{\pi}$, i.e. for nondegenerate randomization, looks nongeneric, but one has to keep in mind that the functions $\varphi(.)$ and $g(.)$ are endogenous, being jointly determined with $\pi(.)$ and $\hat{v}(.)$.

To complete the discussion of optimal allocations with randomized admissions, I briefly discuss the level of public-good provision. If the solution to the planner's relaxed problem is also a solution to his original problem, i.e. if $\pi(.)$ is nondecreasing, conditions (5.8) - (5.10) can be used to rewrite (5.13) as:

$$\lambda K'(Q) = \int_{\hat{\theta}}^{1} g(\theta)d\theta, \tag{5.33}$$

where again $\hat{\theta} := \inf\{\theta \in [0,1] | \; v'(\theta) = Q\}$. If one substitutes for λ and $\varphi(\theta)$ from (5.14) and (5.16) and uses integration by parts, one can rewrite (5.33) as

$$\int_0^1 W'(v(\theta))(-K'(Q) + \hat{\theta}(1 - F(\hat{\theta})) + \max(\theta - \hat{\theta}, 0))dF(\theta) = 0, \qquad (5.34)$$

which is the same as the first-order condition (3.7) for the optimal level of public-good provision with nonrandomized admissions. If the solution to the planner's relaxed problem is *not* also a solution to his original problem, one can use (5.30) - (5.32) to show that the solution to the original problem still has to satisfy (5.33) and (5.34).

For the specification of Proposition 5.4, with constant inequality aversion, one can actually show that the optimal level of public-good provision is decreasing in the degree of inequality aversion ρ, with limits Q^* when ρ goes to zero and $Q_*(\theta^*)$ when ρ goes out of bounds. At the same time, the critical $\hat{\theta}$, above which consumers are always admitted to the enjoyment of the public good, is nondecreasing in ρ. More precisely, there exists some critical $\bar{\rho} > 0$ such that $\hat{\theta} = 0$ for $\rho \leq \bar{\rho}$, and above $\bar{\rho}$, $\hat{\theta}$ is increasing in ρ, rising to the revenue-maximizing level θ^* when ρ goes out of bounds. This is the analogue of the strengthening of Proposition 4.3 mentioned above which is available when inequality aversion is constant.

6 Robustness Considerations

6.1 Risk Aversion

Throughout the paper so far, I have assumed that consumers are risk neutral. This assumption is problematic - not just because risk neutrality is very special, but because risk plays an important role in the analysis. The treatment of incomplete information rests on a model involving prior uncertainty about each individual's preference parameter θ. From an *ex ante* point of view, the assessment of a given allocation by a consumer is likely to depend on his risk attitudes. Therefore it is important that the conclusions of the analysis should be robust to the introduction of risk aversion.

Risk aversion of consumers affects the utilitarian allocation problem in two ways. First, it introduces an additional element of inequality aversion. Even if the welfare function $W(.)$ were linear, equity considerations might play a role as way of reducing the consequences of *ex ante* uncertainty about preferences. Second, risk aversion of consumers affects incentive constraints, raising, e.g., the possibility that randomization over private-good consumption might be used to alleviate some incentive compatibility conditions. With risk aversion of consumers, conditions (2.3) and (2.4) take the form

$$v(\theta) = (1 - \pi(\theta)) \int u(c_0(\omega, \theta))d\nu(\omega) + \pi(\theta) \int u(c_1(\omega, \theta) + \theta Q)d\nu(\omega) \quad (6.1)$$

and

$$v(\theta) \geq (1-\pi(\theta'))\int u(c_0(\omega,\theta'))d\nu(\omega) + \pi(\theta')\int u(c_1(\omega,\theta') + \theta Q)d\nu(\omega) \quad (6.2)$$

for all θ and θ', where $c_0(.,\theta)$ and $c_1(.,\theta)$ are private-good consumption random variables defined on some probability space $(\Omega, \mathcal{F}, \nu)$.

In the absence of additional restrictions on the von Neuman-Morgenstern utility function $u(.)$, I don't know how to put the incentive compatibility condition (6.2) into an analytically tractable form. For the special case of constant absolute risk aversion $\rho_u(c) = \delta$ for all c, one can show that the first-order and second-order necessary conditions for incentive compatibility, namely the condition that

$$v'(\theta) = \delta Q \pi(\theta) \int e^{-\delta c_1(\omega,\theta)} d\nu(\omega) e^{-\delta\theta Q} \quad (6.3)$$

and the condition that $\pi(\theta)\int e^{-\delta c_1(\omega,\theta)}d\nu(\omega)$ be nondecreasing in θ, are in fact sufficient for incentive compatibility. Given this result, one can also show that randomization of private-good consumption is undesirable because, with constant absolute risk aversion, such randomization does not serve any screening function and merely raises the resources that are required to implement a given expected-payoff function $v(.)$.

Given that randomization of private-good consumption is undesirable, incentive compatible allocations with nonrandomized admissions have exactly the same structure as in the analysis of Sections 3 and 4: There is a base consumption c_0 and an entry fee p so that people with $\theta Q < p$ consume c_0 and are not admitted to the enjoyment of the public good, people with $\theta Q > p$ consume $c_1 = c_0 - p$ and are admitted to the enjoyment of the public good. The formalism for determining the optimal p and Q is exactly the same as before, *except* that the welfare function W has to be replaced by $V = W \circ u$, the composition of the welfare function with the consumer's utility function. With this modification, the results of Sections 3 and 4 go through unchanged; the optimal entry fee now depends on the curvature of V,

$$\rho_V(c) = \rho_W(u(c)) + \rho_u(c), \quad (6.4)$$

rather than just the curvature of W. The consumers' risk aversion $\rho_u(c) = \delta$ is simply added to the inequality aversion inherent in the welfare function W. As for the analysis of Section 5, Propositions 5.1 and, with $\rho_W(u(c))$ replaced by $\rho_V(c) = \rho_W(u(c)) + \rho_u(c)$, Propositions 5.2 and 5.3 go through unchanged.

6.2 Heterogeneity in Public-Good Preferences *and* Earning Abilities

As discussed in the introduction, utilitarian analyses of equity-efficiency tradeoffs have mostly been concerned with differences in earning abilities. While introducing differences in preferences for the public good as a new source of heterogeneity to be considered in such tradeoffs, this paper has neglected differences in earning abilities altogether. One must therefore ask whether the analysis remains valid if one allows for heterogeneity in *both*, earning abilities and in public-good preferences.

For a first approach to this question, I modify the model studied in this paper by assuming that the aggregate production capacity Y is not given exogenously, but is the aggregate of output contributions of the individual participants, and I assume that, in addition to the preference parameter θ, any participant is characterized by his earning ability n, which determines his cost $k(y,n)$ of exerting himself to make the output contribution y. The cost function $k(.,.)$ is twice continuously differentiable, with first and second partial derivatives satisfying $k_y > 0$, $k_n < 0$, $k_{yy} > 0$, and $k_{yn} = k_{ny} < 0$. An *allocation* now corresponds to a public-good provision level Q and a triple of functions $(\pi(.,.), c(.,.), y(.,.))$, such that for any θ and n, $\pi(\theta,n)$, $c(\theta,n)$, $y(\theta,n)$ are the admission probability, the level of private-good consumption, and the output contribution of an agent with public-good preference parameter θ and earning ability n. The payoff equation (2.3) and incentive compatibility constraint (2.4) are replaced by:

$$v(\theta,n) = c(\theta,n) + \pi(\theta,n)\theta Q - k(y(\theta,n),n) \tag{6.5}$$

and the requirement that

$$v(\theta,n) \geq c(\theta',n') + \pi(\theta',n')\theta Q - k(y(\theta',n'),n) \tag{6.6}$$

for all θ' and n'. The feasibility constraint (2.5) and the welfare functional (2.6) are replaced by

$$\int_0^1 \int_{\Re_+} c(\theta,n)f(\theta,n)dnd\theta \leq \int_0^1 \int_{\Re_+} y(\theta,n)f(\theta,n)dnd\theta - K(Q) \tag{6.7}$$

and

$$\int_0^1 \int_{\Re_+} W(v(\theta,n))f(\theta,n)dnd\theta. \tag{6.8}$$

The utilitarian allocation problem now is to maximize (6.8) subject to (6.5) - (6.7).

At this point, a solution to this two-dimensional mechanism design problem is not available. However, a solution is available for the simpler problem which is obtained if the utilitarian planner is unable to control the personal identities of people claiming admission to the use of the public good and the allocation must not provide any incentives for exchanging admission permits against the private good *ex post*. By standard arguments, *ex post* efficiency of the final allocation of consumption of the private good and the public good implies the existence of a relative price p such that, for any pair (θ,n), $\pi(\theta,n) = 0$ if $\theta Q < p$ and $\pi(\theta,n) = 1$ if $\theta Q > p$. With this restriction on admission rules, the two-dimensional mechanism design problem behaves like two interrelated one-dimensional problems.

To simplify the presentation, I again impose right-continuity, this time on the output contribution function $y(.,.)$ as well as the admission rule $\pi(.,.)$. As in Section 5, this assumption serves to eliminate the expositional complications that are due to the fact that, at discontinuity points of the admission probability and output contribution functions some modifications of the values of these functions are always possible without changing anything.

Lemma 6.1 *In the model with heterogeneity in earning abilities as well as preferences for the public good, let the admission rule* $\pi(.,.)$ *be such that*

$$\pi(\theta, n) = \bar{\pi}(\theta), \tag{6.9}$$

for all θ *and* n*, where, for some* $p \geq 0$,

$$\bar{\pi}(\theta) = 0 \quad \textit{if} \quad \theta Q < p \quad \textit{and} \quad \bar{\pi}(\theta) = 1 \quad \textit{if} \quad \theta Q \geq p. \tag{6.10}$$

Then an allocation $(Q, \pi(.,.), c(.,.), y(.,.))$ *with this admission rule is incentive compatible if and only if the functions* $c(.,.)$ *and* $y(.,.)$ *satisfy*

$$c(\theta, n) = c(0, n) - \bar{\pi}(\theta)\ p, \tag{6.11}$$

$$y(\theta, n) = y(0, n) \tag{6.12}$$

for all θ *and* n*, and moreover,*

$$c(0, n) - k(y(0, n), n) \geq c(0, n') - k(y(0, n'), n) \tag{6.13}$$

for all n' *and all* n.

Because the admission probability $\pi(\theta, n)$ is independent of n, the output contribution $y(\theta, n)$ is independent of θ, and the private-good consumption $c(\theta, n)$ is additively separable in θ and n. The two components of $c(\theta, n)$ correspond to the two dimensions of the incentive problem. They are determined by the incentive compatibility conditions for θ and n, each one considered on its own terms, without regard for the other.

By standard arguments, the incentive compatibility condition (6.13) is equivalent to the requirements that the payoff function $\bar{v}(.)$, which is defined by setting

$$\bar{v}(n) = c(0, n) - k(y(0, n), n), \tag{6.14}$$

be an absolutely continuous with derivative $\bar{v}'(n) = -k_n(y(0, n), n)$ for all n, and that $\bar{v}(.)$ be convex or, equivalently, that $y(0,.)$ be nondecreasing.

Under the restriction that the admission rule take the form (6.9), (6.10), the utilitarian allocation problem is therefore equivalent to the problem of choosing a public-good provision level Q, a critical $\hat{\theta}$, a payoff function $\bar{v}(.)$, and a nondecreasing output provision function $\bar{y}(.)$ so as to maximize the welfare functional

$$\int_0^1 \int_{\Re_+} W(\bar{v}(n) + \max(\theta - \hat{\theta}, 0)Q) f(\theta, n) dn d\theta \tag{6.15}$$

under the constraints that

$$\int_{\Re_+} (\bar{v}(n) + k(\bar{y}(n), n)) f^n(n) dn \leq \int_{\Re_+} \bar{y}(n) f^n(n) dn - K(Q) + \hat{\theta} Q(1 - F^\theta(\hat{\theta})) \tag{6.16}$$

and that

$$\bar{v}'(n) = -k_n(\bar{y}(n), n) \tag{6.17}$$

for all n. In (6.16), $f^n(.)$ denotes the density of the marginal distribution of n, and $F^\theta(.)$ denotes the marginal distribution of θ.

The solutions to this allocation problem have the same structure as the solutions to the corresponding one-dimensional problems. In particular, an optimal solution must satisfy the first-order conditions

$$\int_0^1 \int_{\Re_+} W' \left[-K'(Q) + \hat{\theta}(1 - F(\hat{\theta})) + \max(\theta - \hat{\theta}, 0) \right] f(\theta, n) dn d\theta = 0, \tag{6.18}$$

$$Q \left[(1 - F(\hat{\theta}) - \hat{\theta} f(\hat{\theta}))\lambda - \int_{\hat{\theta}}^1 \int_{\Re_+} W' \, f(\theta, n) dn d\theta \right] = 0, \tag{6.19}$$

and, for each n,

$$(1 - k_y) f^n(n) = (-k_{ny}) \int_n^\infty \int_0^1 \left(1 - \frac{W'}{\lambda} \right) f(\theta, n') d\theta dn', \tag{6.20}$$

where

$$\lambda = \int_{\hat{\theta}}^1 \int_{\Re_+} W' \, f(\theta, n) dn d\theta \tag{6.21}$$

is the Lagrange multiplier of the constraint (6.16) and, in each integral, W' is evaluated at the point $\bar{v}(n) + \max(\theta - \hat{\theta}, 0)Q$.

Conditions (6.18) and (6.19) are the same as (3.7) and (3.8), except that one is now averaging across earning abilities, i.e., W' in (3.7) and (3.8) is replaced by the conditional expectation $\int_{\Re_+} W' \, \frac{f(\theta,n)}{f^\theta(\theta)} dn$. Similarly, except for averaging across public-good preferences, condition (6.20) is the same as the standard first-order condition in the one-dimensional income tax problem[12] adapted to the additive utility specification considered here. The interpretation of these

[12] See again equation (27) in Mirrlees (1971).

conditions is the same as in the corresponding one-dimensional models. In particular, the characterization of optimal entry fees that was given in Sections 3 and 4 can be adapted to the two-dimensional model without change.

Therefore, if inequality aversion is high, optimal admission fees are again positive. In the two-dimensional model with heterogeneity in public-good preferences *and* earning abilities, utilitarian concerns for redistribution are *not* fully met by income taxation. Income taxation meets those redistribution concerns which relate to differences in earning abilities. However, unless public-good preferences are perfectly correlated with earning abilities, differences in public-good preferences give rise to additional concerns for redistribution. In a second-best world, these latter concerns call for the use of admission fees and exclusion of anybody who fails to pay the fees whenever inequality aversion is high.

A Appendix: Proofs

The arguments for Lemmas 3.1 - 3.3 are sketched in the text; the details of the proofs are left to the reader.

Proof of Proposition 4.1. Let $A > 0$ be such that

$$\sup_{\hat{\theta}\in[0,1]} \frac{F(\hat{\theta})}{F(\hat{\theta}) + \hat{\theta} f(\hat{\theta})} < e^{-AQ^*}. \tag{A.1}$$

If $W(.)$ is such that $\rho_W(c) \leq A$ for all c, a straightforward integration yields

$$W'(c_0 + Q(W)) \geq W'(c_0)e^{-AQ(W)} \geq W'(c_0)e^{-AQ^*}. \tag{A.2}$$

Upon combining (A.1) and (A.2), one obtains

$$-(F(\hat{\theta}) + \hat{\theta} f(\hat{\theta}))\, W'(c_0 + Q(W)) + W'(c_0)F(\hat{\theta}) < 0 \tag{A.3}$$

for all $\hat{\theta} \in (0, 1]$. Because $W'(.)$ is nondecreasing, it follows that

$$-(F(\hat{\theta}) + \hat{\theta} f(\hat{\theta})) \int_0^1 W' dF + W'(c_0)F(\hat{\theta}) < 0 \tag{A.4}$$

for all $\hat{\theta} \in (0, 1]$, which is equivalent to

$$(1 - F(\hat{\theta}) - \hat{\theta} f(\hat{\theta})) \int_0^1 W' dF - \int_{\hat{\theta}}^1 W' dF < 0. \tag{A.5}$$

Because $Q(W) > 0$, it follows that no $\hat{\theta} \in (0, 1]$ satisfies the first-order condition (3.8). Therefore $\hat{\theta}(W) = 0$.

As for the second statement of the proposition, if $\hat{\theta}(W) = 0$, the first-order condition (3.7) for the public-good provision level takes the form

$$\int_0^1 W' \left[-K'(Q(W)) + \theta\right]\, dF(\theta) = 0. \tag{A.6}$$

If $\rho_W(c) \leq A$ for all c, then by (A.2), it follows that

$$-K'(Q(W))e^{-AQ^*}W'(c_0) + \bar{\theta}W'(c_0) \geq 0,$$

and hence that

$$Q(W) \geq K'^{-1}(\bar{\theta}e^{AQ^*}). \tag{A.7}$$

Given that $K'^{-1}(\bar{\theta}) \geq Q(W)$, it follows that $Q(W)$ is close to $Q^* = K'^{-1}(\bar{\theta})$ if A is close to zero. ■

Proof of Proposition 4.2. Fix $\bar{A} > 0$ and let $W(.)$ be such that $\rho_W(c) \geq \bar{A}$ for all c. I will show that, if $\bar{A}$ is large, then $\bar{c}(Q(W), \hat{\theta}(W))$ must be close to $c_0^* := \max_{(Q,\hat{\theta})} c_0(Q, \hat{\theta})$. More precisely, for any $\bar{\varepsilon} > 0$, I will show that $c_0^* \geq c_0(Q(W), \hat{\theta}(W)) \geq c_0^* - \bar{\varepsilon}$ if $\bar{A}$ is sufficiently large. The first of these inequalities is trivial. To prove that the second inequality holds if $\bar{A}$ is sufficiently large, I will show that for any pair $(Q, \hat{\theta})$ with $c_0(Q, \hat{\theta}) < c_0^* - \bar{\varepsilon}$ and $Q \leq Q^*$, one has $W^*(Q, \hat{\theta}) < W(c_0^*)$ if $\bar{A}$ is sufficiently large. By the concavity of $W(.)$, for such $(Q, \hat{\theta})$, one has

$$\begin{aligned} W(c_0 + Q\max(\theta - \hat{\theta}, 0)) - W(c_0^* + \delta) &\leq W'(c_0^* + \delta)(c_0(Q, \hat{\theta}) + \theta Q - c_0^* - \delta) \\ &\leq W'(c_0^* + \delta)(-\bar{\varepsilon} + \theta Q^* - \delta) \end{aligned}$$

for all $\delta \geq 0$ and all θ. In particular,

$$W(\bar{c} + Q\max(\theta - \hat{\theta}, 0)) - W(c_0^*) \leq W'(c_0^*)(-\bar{\varepsilon} + \theta Q^*)$$

for all θ. Therefore

$$W^*(Q, \hat{\theta}) - W(c_0^*) \leq \int_0^{\theta_0} (W(c_0 + Q\max(\theta - \hat{\theta}, 0)) - W(c_0^*))dF(\theta)$$

$$+ \int_{\theta_0}^1 (W(c_0 + Q\max(\theta - \hat{\theta}, 0)) - W(c_0^* + \delta))dF(\theta) + W(c_0^* + \delta) - W(c_0^*)$$

$$\leq W'(c_0^*) \int_0^{\theta_0} (-\bar{\varepsilon} + \theta Q^*)dF(\theta) + W'(c_0^* + \delta) \int_{\theta_0}^1 (-\bar{\varepsilon} + \theta Q^* - \delta)dF(\theta)$$
$$+ (W(c_0^* + \delta) - W(c_0^*)),$$

for all $\delta \geq 0$, where $\theta_0 := \min[-\bar{\varepsilon}/Q^*, 1]$. If $\bar{\varepsilon} \geq Q^*$, one has $\theta_0 = 1$ and $W^*(Q, \hat{\theta}) < W(c_0^*)$ without any need for further analysis. If $\bar{\varepsilon} < Q^*$, one has $\theta_0 < 1$ and

$$\begin{aligned} W^*(Q, \hat{\theta}) - W(c_0^*) &\leq W'(c_0^*) \int_0^{\theta_0} (-\bar{\varepsilon} + \theta Q^*)dF(\theta) + W'(c_0^* + \delta)(Q^* - \bar{\varepsilon}) + W'(c_0^*)\delta \\ &\leq W'(c_0^*) \left(\int_0^{\theta_0} (-\bar{\varepsilon} + \theta Q^*)dF(\theta) + e^{-\bar{A}\delta}(Q^* - \bar{\varepsilon}) + \delta \right) \end{aligned} \tag{A.8}$$

for all $\delta \geq 0$. If $\delta = \frac{1}{2}\int_0^{\theta_0}(\bar{\varepsilon} - \theta Q^*)dF(\theta)$ and $\bar{A}$ is sufficiently large, the right-hand side of (A.8) is negative. Thus, for $\bar{\varepsilon} < Q^*$ as well as $\bar{\varepsilon} \geq Q^*$, $\bar{c}(Q,\hat{\theta}) < \bar{c}^* - \bar{\varepsilon}$ and $Q \leq Q^*$ imply $W^*(Q,\hat{\theta}) < W(\bar{c}^*)$ if $\bar{A}$ is sufficiently large.

For the given sequence $\{W_k\}$ with inequality aversion $\rho_{W_k}(c)$ going out of bounds, uniformly in c, it follows that $\lim_{k\to\infty} c_0(Q(W_k),\hat{\theta}(W_k)) = c_0^*$. Any limit point (Q^∞,θ^∞) of the sequence $\{(Q(W_k),\hat{\theta}(W_k))\}$ must therefore be a maximizer of $c_0(.,.)$. By inspection of (3.5), such a limit point satisfies $\theta^\infty \in \arg\max \theta(1-F(\theta))$ and $Q^\infty = Q_*(\theta^\infty)$. By Lemma 3.3, θ^* is the only maximizer of $\theta(1-F(\theta))$ to which the sequence $\{\hat{\theta}(W_k)\}$ can converge. ■

Proof of Proposition 4.3. If the proposition is false, there exist W_1, W_2 satisfying $\rho_{W_2}(c) > \rho_{W_1}(c)$ for all c such that for $(Q_i,\hat{\theta}_i) = (Q(W_i),\hat{\theta}(W_i))$, one has

$$Q_2 \geq Q_1 \text{ and } \hat{\theta}_2 \leq \hat{\theta}_1. \tag{A.9}$$

For $i = 1, 2$, and any $\theta \in [0,1]$, let

$$x_i(\theta) = c_0(Q_i,\hat{\theta}_i) + Q_i \max(\theta - \hat{\theta}_i, 0), \tag{A.10}$$

and note that, by Lemma 3.2, $x_i(\theta) \in [Y - K(Q^*), Y - K(Q^*) + Q^*]$ for all θ. Note also that

$$\frac{\partial x_i(\theta)}{\partial\theta} = 0, \quad \text{if} \quad \theta < \hat{\theta}(W_i),$$

and

$$\frac{\partial x_i(\theta)}{\partial\theta} = Q_i, \quad \text{if} \quad \theta > \hat{\theta}(W_i),$$

so $(A.9)$ implies

$$\frac{\partial x_2(\theta)}{\partial\theta} \geq \frac{\partial x_1(\theta)}{\partial\theta} \geq 0, \tag{A.11}$$

where the first inequality is strict if $\theta \in (\hat{\theta}_2,\hat{\theta}_1)$ or if $\theta > \hat{\theta}_1$ and $Q_2 > Q_1$.

By the definition of $x_2(.)$ and the concavity of W_2 one has

$$\int_0^1 W_2'(x_2(\theta))[x_2(\theta) - x_1(\theta)]dF(\theta) \geq 0. \tag{A.12}$$

Since $W_2'(x)$ is decreasing in x and, by (A.11), $x_2(\theta)$ and $x_2(\theta) - x_1(\theta)$ are both nondecreasing in θ, (A.12) implies

$$\int_0^1 [x_2(\theta) - x_1(\theta)]dF(\theta) \geq 0, \tag{A.13}$$

i.e. the mean of the random variable $x_2(\tilde{\theta})$ cannot be less than the mean of the random variable $x_1(\tilde{\theta})$.

By the definition of $x_1(.)$, one also has

$$\int_0^1 W_1(x_1(\theta))dF(\theta) \geq \int_0^1 W_1(x_2(\theta))dF(\theta). \tag{A.14}$$

By (A.13) and (A.14), there exists $\lambda \in [0,1]$ such that

$$\int_0^1 W_\lambda(x_1(\theta))dF(\theta) = \int_0^1 W_\lambda(x_2(\theta))dF(\theta), \tag{A.15}$$

where W_λ is given by $W_\lambda(c) = (1-\lambda)W_1(c) + \lambda c$. One easily computes

$$\rho_{W_\lambda}(c) = -\frac{(1-\lambda)W_1''(c)}{(1-\lambda)W_1'(c)+\lambda} \leq \rho_{W_1}(c) < \rho_{W_2}(c)$$

for all c. By Theorem 1, p. 128, of Pratt (1964), it follows that there exists a strictly increasing, strictly concave function φ_λ so that $W_2 = \varphi_\lambda \circ W_\lambda$. From (A.11), in combination with Lemmas 3.1 and 3.3, one also finds that, if one of the inequalities in (A.9) is strict, then the random variable $W_\lambda(x_2(\tilde{\theta}))$ is given by a mean-preserving spread of the random variable $W_\lambda(x_1(\tilde{\theta}))$.[13] Given the strict concavity of φ_λ, it follows that

$$\int_0^1 \varphi_\lambda(W_\lambda(x_1(\theta)))dF(\theta) > \int_0^1 \varphi_\lambda(W_\lambda(x_2(\theta)))dF(\theta),$$

contrary to the definition of $x_2(.)$. The assumption that at least one of the inequalities in (A.9) is strict thus leads to a contradiction and must be false.

Alternatively, suppose that $Q_2 = Q_1 = Q$ and $\hat{\theta}_2 = \hat{\theta}_1 = \hat{\theta}$, and consider the first-order condition (3.7) for $Q_2 = Q$. Using the fact that, again by Pratt's theorem, $W_2 = \varphi_1 \circ W_1$, this first-order condition can be written in the form

$$\int_0^1 \varphi_1'(W_1(x_2(\theta)))W_1'(x_2(\theta))[-K'(Q) + \hat{\theta}(1-F(\hat{\theta})) + \max(\theta - \hat{\theta}, 0)]dF(\theta) = 0. \tag{A.16}$$

By Lemma 3.3, $\hat{\theta} < 1$, so (A.16) implies that there exists $\check{\theta}$ so that $-K'(Q) + \hat{\theta}(1-F(\hat{\theta})) + \max(\theta - \hat{\theta}, 0) \gtreqless 0$ as $\theta \gtreqless \check{\theta}$. Given that $\varphi'(.)$ is a strictly decreasing function, it follows that

$$\varphi'(W_1(x_2(\check{\theta}))) \int_0^1 W_1'(x_2(\theta))[-K'(Q) + \hat{\theta}(1-F(\hat{\theta})) + \max(\theta - \hat{\theta}, 0)]dF(\theta) < 0. \tag{A.17}$$

Since $Q(W_2) = Q(W_1) = Q$ and $\hat{\theta}(W_2) = \hat{\theta}(W_1) = \hat{\theta}$ imply $x_1(\theta) = x_2(\theta)$ for all θ, (A.17) is incompatible with the first-order condition (3.7) for $Q_1 = Q$. The

[13] Equivalently, in the terminology of Diamond and Stiglitz (1974), $x_2(\tilde{\theta})$ is given by a mean-W_λ-utility preserving spread of $x_1(\tilde{\theta})$.

assumption that $Q_2 = Q_1$ and $\hat{\theta}_2 = \hat{\theta}_1$ thus also leads to a contradiction and must be false. ■

Proof of Proposition 5.1. Trivially, a solution to the utilitarian allocation problem satisfies the feasibility constraint (5.3) with equality, i.e. one has $c_0 = c_0(Q, \pi)$, where $c_0(Q, \pi)$ is given by (5.18). If the set of nondecreasing, right-continuous functions from the unit interval into itself is given the topology of pointwise convergence at continuity points, then using Lebesgue's bounded-convergence theorem, the map

$$(Q, \pi) \to \int_0^1 W\left(c_0(Q, \pi) + \int_0^\theta \pi(\eta) d\eta\right) f(\theta) d\theta$$

is continuous. Moreover, by Helly's selection theorem, the space of nondecreasing right-continuous functions from the unit interval into itself is compact. To prove the existence of a solution to the utilitarian allocation problem, it therefore suffices to show that there is no loss of generality in supposing that $Q \in [0, Q^*]$.

For this purpose, I note that, for fixed $\pi(.)$, one has

$$\begin{aligned}
& \frac{d}{dQ} \int_0^1 W\left(c_0(Q, \pi) + Q \int_0^\theta \pi(\eta) d\eta\right) f(\theta) d\theta \\
\leq & \int_0^1 W' f(\theta) d\theta \left[-K'(Q) + \int_0^1 \theta(1 - F(\theta))\, d\pi(\theta) + \int_0^1 \int_0^\theta \pi(\eta) d\eta\, f(\theta) d\theta\right] \\
= & \int_0^1 W' f(\theta) d\theta \left[-K'(Q) + \int_0^1 \theta \pi(\theta) f(\theta) d\theta\right] \qquad \text{(A.18)} \\
\leq & \int_0^1 W' f(\theta) d\theta \left[-K'(Q) + \bar{\theta}\right];
\end{aligned}$$

the first inequality holds because $W'(.)$ is decreasing in $Q \int_0^\theta \pi(\eta) d\eta$, the subsequent equation follows from an integration by parts, and the last inequality holds because $\pi(\theta) \leq 1$ for all θ. Moreover, if $Q > 0$, the first inequality is strict unless $\pi(\theta) = 0$ for all θ, and the last inequality is strict unless $\pi(\theta) = 1$ for all θ, i.e. at least one of the two inequalities is strict. Utilitarian welfare is therefore decreasing in Q whenever $K'(Q) \geq \bar{\theta}$, i.e. whenever $Q \geq Q^*$. There is therefore no loss of generality in supposing that Q belongs to the compact interval $[0, Q^*]$.

In fact, the argument just given shows that an optimal Q must be strictly less than Q^*. To see that an optimal Q is also strictly positive, one notes that the first inequality in (A.18) holds as an equation if $Q = 0$. (A.18) therefore implies that, e.g. if $\pi(\theta) = 1$ for all θ, then an increase in Q above zero *raises* welfare so $Q = 0$ cannot be optimal.

To prove uniqueness, suppose that $(Q^1, c_0^1, \pi^1(.))$ and $(Q^2, c_0^2, \pi^2(.))$ both solve the utilitarian allocation problem, and consider the triple $(Q^\lambda, c_0^\lambda, \pi^\lambda(.))$

where $\lambda \in (0,1)$ and

$$(Q^\lambda, c_0^\lambda, Q^\lambda \pi^\lambda(.)) = \lambda(Q^1, c_0^1, Q^1\pi^1(.)) + (1-\lambda)(Q^2, c_0^2, Q^2\pi^2(.)).$$

By the convexity of $K(.)$, $(Q^\lambda, c_0^\lambda, \pi^\lambda(.))$ satisfies the feasibility constraint (5.3), and $\pi^\lambda(.)$ is nondecreasing. By the concavity of W, it follows that $(Q^\lambda, c_0^\lambda, Q^\lambda\pi^\lambda(.))$ also solves the utilitarian allocation problem and that

$$c_0^1 + Q^1 \int_0^\theta \pi^1(\eta)d\eta = c_0^2 + Q^2 \int_0^\theta \pi^2(\eta)d\eta$$

for almost all θ. Because of the strictness of the convexity of $K(.)$ and the concavity of $W(.)$, one actually must have $Q^1 = Q^2$, $c_0^1 = c_0^2$, and $Q^1\pi^1(\theta) = Q^2\pi^2(\theta)$ for almost all θ. Because $Q^1 = Q^2 > 0$, it follows that $\pi^1(\theta) = \pi^2(\theta)$ for almost all θ. Because $\pi^1(.)$ and $\pi^2(.)$ are both nondecreasing and right-continuous, it follows that $\pi^1(\theta) = \pi^2(\theta)$ for all θ.

To complete the proof, suppose that $\pi(\theta^*) < 1$. By the right-continuity of $\pi(.)$, it follows that for some $\theta' > \theta^*$, one has $\pi(\theta) < 1$ for all $\theta \in [\theta^*, \theta']$. If a new admission rule $\hat{\pi}(.)$ is given by setting $\hat{\pi}(\theta) = \pi(\theta)$ for $\theta < \theta^*$ and $\hat{\pi}(\theta) = 1$ for $\theta \geq \theta^*$, then $\hat{\pi}(.)$ is nondecreasing, and, by (5.18), the associated base consumption satisfies

$$\begin{aligned} c_0(Q,\hat{\pi}) - c_0(Q,\pi) &= Q\left(\int_0^1 \theta(1-F(\theta))d\hat{\pi}(\theta) - \int_0^1 \theta(1-F(\theta))d\pi(\theta)\right) \\ &= Q\theta^*(1-F(\theta^*))(1-\pi(\theta^*)) - Q\int_{\theta^*}^1 \theta(1-F(\theta))d\pi(\theta) \\ &= Q\int_{\theta^*}^1 [\theta^*(1-F(\theta^*)) - \theta(1-F(\theta))]d\pi(\theta). \end{aligned}$$

By the definition of θ^* as maximizing the product $\theta(1-F(\theta))$, the right-hand side is nonnegative, so replacing $(Q, c_0(Q,\pi), \pi(.))$ by $(Q, c_0(Q,\hat{\pi}), \hat{\pi}(.))$ has no adverse effects on base consumption. However, for $\theta \in (\theta^*, \theta')$ the information rent $Q\int_0^\theta \hat{\pi}(\eta)d\eta$ under the admission rule $\hat{\pi}(.)$ is greater than the information rent under the admission rule $\pi(.)$, which contradicts the optimality of $\pi(.)$. The assumption that $\pi(\theta^*) < 1$ thus leads to a contradiction and must be false. ■

Proof of Proposition 5.2. If $\rho_W(c) \leq A$ for $A > 0$ given by (A.1) and all c, then the same argument as in the proof of Proposition 4.1 shows that $g(\hat{\theta}) < 0$ for all $\hat{\theta} \in (0,1]$. By (5.11) - (5.16) and by right-continuity, it follows that the solution to the planner's relaxed problem satisfies $\pi(\hat{\theta}) = 1$ for all $\hat{\theta} \in [0,1]$. Because the specified $\pi(.)$ is nondecreasing, the solution to the planner's relaxed problem is also the solution to the planner's original problem. Therefore the solution to the planner's original problem also satisfies $\pi(\hat{\theta}) = 1$ for all $\hat{\theta} \in [0,1]$. ■

Proof of Proposition 5.3. The same argument as in the proof of Proposition 4.2 shows that as k goes out of bounds, the base consumption levels

$c_0(Q^k, \pi^k)$ converge to $c_0^{**} := \max_{(Q,\pi)} c_0(Q, \pi)$, the maximal base consumption level that is at all feasible. Any limit point (Q^∞, π^∞) of the sequence $\{(Q^k, \pi^k)\}$ must therefore be a maximizer of $c_0(Q, \pi)$. By inspection of (5.18), it follows that any such limit point involves a provision level Q^∞ of the public good which maximizes $Q \max_\theta \theta(1-F(\theta)) - K(Q)$ and an admission rule π^∞ which is a step function whose discontinuity points coincide with the maximizers of $\theta(1-F(\theta))$. By Proposition 5.1, it follows that any limit point (Q^∞, π^∞) of the sequence $\{(Q^k, \pi^k)\}$ satisfies $\pi^\infty(\theta) = 0$ for $\theta < \theta^*$ and $\pi^\infty(\theta) = 1$ for $\theta \geq \theta^*$ as well as $Q^\infty = Q_*(\theta^*)$. ■

Proof of Proposition 5.4. The proof of Proposition 5.4 proceeds in several steps. The overall idea is to show that a solution to the planner's relaxed problem exists, that any such solution exhibits the properties claimed in the proposition, and that any such solution is also a solution to the planner's original problem. Given that the solution to the planner's original problem is unique, it follows that any solution to the planner's original problem exhibits the properties claimed in the proposition.

Step 1: The planner's relaxed problem has a unique solution (Q^R, c_0^R, π_R). Moreover $Q^R > 0$.

Because the set of admissible triples (Q, c_0, π) in the planner's relaxed problem is larger than in the original problem, this claim does not follow from Proposition 5.1. Moreover, because the proof of Proposition 5.1 makes essential use of the monotonicity of $\pi(.)$, one needs a different argument here. For this reason, consider the information rent function defined in (5.4) and note that this function can be used to rewrite the planner's objective function and feasibility constraint as

$$\int_0^1 W(c_0 + \hat{v}(\theta))f(\theta)d\theta \tag{A.19}$$

and

$$c_0 \leq \hat{c}_0(Q, \hat{v}) := Y - K(Q) + \hat{v}(1)f(1) - \int_0^1 \hat{v}(\theta)(2f(\theta) + \theta f'(\theta))d\theta. \tag{A.20}$$

In this formulation, the relaxed problem depends only on Q and on the information rent function $\hat{v}$. By the same argument as before, the choice of Q can be restricted to the compact interval $[0, Q^*]$. As for the choice of $\hat{v}$, condition (5.4) implies that $\hat{v}$ is Lipschitz with constant Q. Because $\hat{v}(0) = 0$, $\hat{v}$ is also bounded between zero and Q. For $Q \in [0, Q^*]$, the choice of $\hat{v}$ may therefore be taken to be restricted to (a subset of) the space of Lipschitz continuous functions with common Lipschitz constant Q^* and common upper bound Q^* and lower bound zero. When endowed with the topology of uniform convergence, the latter space is compact. The existence of a maximizing pair $(Q, \hat{v})$ follows by standard arguments. Uniqueness follows by the same argument as before.

Given the solution (Q^R, c_0^R, π_R) to the planner's relaxed problem and given the associated information rent function $\hat{v}_R$, I define

$$\varphi(\theta) = \int_\theta^1 W'(c_0^R + \hat{v}_R(\eta))f(\eta)d\eta, \tag{A.21}$$

$$\lambda = \int_0^1 W'(c_0^R + \hat{v}_R(\theta))f(\theta)d\theta, \tag{A.22}$$

and

$$g(\theta) = \varphi(\theta) - \lambda(1 - F(\theta) - \theta f(\theta)). \tag{A.23}$$

From the first-order conditions (5.8) - (5.13) in combination with the positivity of Q^R, it follows that $\pi_R(\theta) = 0$ if $g(\theta) < 0$ and $\pi_R(\theta) = 1$ if $g(\theta) > 0$. By (5.6), one therefore has $\hat{v}_R'(\theta) = 0$ if $g(\theta) < 0$ and $\hat{v}_R'(\theta) = Q_R$ if $g(\theta) > 0$. For the given specification of $W(.)$ and $f(.)$, these conditions can be used to yield:

Step 2: There exists $\hat{\theta} \in [0, 1)$ such that $g(\theta) > 0$ for $\theta \in (\hat{\theta}, 1]$ and $g(\theta) = 0$ for $\theta \in [0, \hat{\theta}]$.

Define $\hat{\theta}$ as the supremum of the set $\{\theta \in [0,1] | \; g(\theta) \leq 0\}$. By (A.23), $g(0) = 0$, so this set is nonempty, and $\hat{\theta}$ is well defined. Also by (A.23), $g(1) = \lambda f(1) > 0$, so $\hat{\theta} < 1$. Because $g(.)$ is continuous, one also has $g(\hat{\theta}) = 0$ and $g(\theta) > 0$ for all $\theta \in (\hat{\theta}, 1]$.

To establish the claim that $g(\theta) = 0$ for $\theta \in [0, \hat{\theta}]$, I first show that there is no $\theta' \in [0, \hat{\theta}]$ so that $g(\theta') < 0$. For suppose that $g(\theta') < 0$ for some $\theta' \in [0, \hat{\theta}]$. Then the minimum of $g(\theta)$ for $\theta \in [0, \hat{\theta}]$ is strictly negative. Suppose that this minimum is attained at $\check{\theta}$. Because $g(0) = g(\hat{\theta}) = 0$, $g(\check{\theta}) < 0$ implies $\check{\theta} \in (0, \hat{\theta})$. Therefore $\check{\theta}$ satisfies the necessary conditions for an interior minimum of g, i.e. $g'(\check{\theta}) = 0$ and $g''(\check{\theta}) \geq 0$. However, using (A.21) and the specification $f(\theta) = Ae^{-B\theta}$, one computes

$$g'(\theta) = -W'(c_0^R + \hat{v}_R(\theta))Ae^{-B\theta} + \lambda(2 - B\theta)Ae^{-B\theta} \tag{A.24}$$

and

$$g''(\theta) = -Bg'(\theta) - W''(c_0^R + \hat{v}_R(\theta))\hat{v}_R'(\theta)Ae^{-B\theta} - \lambda BAe^{-B\theta} \tag{A.25}$$

for any θ. By (5.8) and (5.6), $g(\check{\theta}) < 0$ implies $\pi_R(\theta) = \hat{v}_R'(\check{\theta}) = 0$. Therefore $g(\check{\theta}) < 0$ and $g'(\check{\theta}) = 0$ imply $g''(\check{\theta}) = -\lambda BAe^{-B\theta}$, contrary to the second-order condition $g''(\check{\theta}) \geq 0$ for a minimum of g at $\check{\theta}$. The assumption that $g(\theta') < 0$ for some $\theta' \in [0, 1]$ thus leads to a contradiction and must be false.

I also show that there is no $\theta' \in (0, \hat{\theta})$ such that $g(\theta') > 0$. For suppose that $g(\theta') > 0$ for some $\theta' \in (0, \hat{\theta})$. Then the maximum of $g(\theta)$ for $\theta \in [0, \hat{\theta}]$ is strictly positive. If the maximum is reached at $\ddot{\theta} \in (0, \hat{\theta})$, one has $g(\ddot{\theta}) > 0$, hence by (5.10) $\pi_R(\theta) = 1$ and $\hat{v}_R'(\ddot{\theta}) = Q_R$, as well as $g'(\ddot{\theta}) = 0$, and $g''(\ddot{\theta}) \leq 0$, so (A.25) implies

$$-W''(c_0^R + \hat{v}_R(\ddot{\theta}))Q_R \leq \lambda B. \tag{A.26}$$

Since $g(\theta) \geq 0$ for all θ and $g(\hat{\theta}) = 0$, one also has a minimum of $g(.)$ at $\hat{\theta}$, hence $g'(\hat{\theta}) = 0$, and $g''(\hat{\theta}) \geq 0$. By (A.25), it follows that

$$-W''(c_0^R + \hat{v}_R(\hat{\theta}))\hat{v}_R'(\hat{\theta}) \geq \lambda B,$$

and hence, since $\hat{v}_R'(\hat{\theta}) \leq Q$ and $-W''(c_0^R + \hat{v}_R(\hat{\theta})) > 0$,

$$-W''(c_0^R + \hat{v}_R(\hat{\theta}))Q_R \geq \lambda B. \tag{A.27}$$

Upon combining (A.26) and (A.27), one obtains $-W''(c_0^R + \hat{v}_R(\ddot{\theta})) \leq -W''(c_0^R + \hat{v}_R(\hat{\theta}))$, hence $\rho^2 e^{-\rho\hat{v}_R(\ddot{\theta})} \leq \rho^2 e^{-\rho\hat{v}_R(\hat{\theta})}$. Since $\hat{v}_R(.)$ is nondecreasing, it follows that $\hat{v}_R(\ddot{\theta}) = \hat{v}_R(\hat{\theta})$ and $\hat{v}_R'(\theta) = 0$ for all $\theta \in [\ddot{\theta}, \hat{\theta}]$, contrary to the assumption that $g(\ddot{\theta}) > 0$ and therefore $\hat{v}_R'(\theta) = Q_R > 0$ for θ close to $\ddot{\theta}$. The assumption that $g(\theta') > 0$ for some $\theta' \in (0, \hat{\theta})$ thus leads to a contradiction and must be false.

Step 3: For $\hat{\theta}$ given by Step 2, the admission rule $\pi_R(.)$ satisfies (5.19) and (5.20).

For $\theta \in (\hat{\theta}, 1]$, (5.20) follows from (5.10) because Step 2 implies $g(\theta) > 0$. Given the validity of (5.20) for $\theta \in (\hat{\theta}, 1]$, its validity for $\theta = \hat{\theta}$ follows from the right-continuity of π_R.

If $\hat{\theta} = 0$, there is nothing more to prove. If $\hat{\theta} > 0$, Step 2 implies $g(\theta) = g'(\theta) = g''(\theta) = 0$ for all $\theta \in (0, \hat{\theta})$. By (A.24) and (A.25), it follows that

$$\rho e^{-\rho(c_0^R + \hat{v}_R(\theta))} = \lambda(2 - B\theta) \tag{A.28}$$

and

$$\rho^2 e^{-\rho(c_0^R + \hat{v}_R(\theta))}\hat{v}_R'(\theta) = \lambda B \tag{A.29}$$

for all $\theta \in (0, \hat{\theta})$, which implies

$$\hat{v}_R'(\theta) = \frac{B}{\rho(2 - B\theta)}. \tag{A.30}$$

By (5.6) and the positivity of Q_R, it follows that $\pi_R(.)$ satisfies (5.19) for $\theta \in [0, \hat{\theta})$.

Step 4: If $\hat{\theta} = 0$, then (5.21) does not hold.

If $\hat{\theta} = 0$, one has $g(\theta) > 0$ for $\theta > 0$ and $g(0) = 0$. It follows that $g'(0) \geq 0$, i.e., by (A.24),

$$-W'(c_0^R) + 2\lambda \geq 0. \tag{A.31}$$

In this case, one also has $\hat{v}_R(\theta) = \theta Q^R$ for all θ, so (A.22) becomes

$$\lambda = \int_0^1 \rho e^{-\rho(c_0^R + \theta Q)} A e^{-B\theta} d\theta = W'(c_0^R)\; A\; \frac{1 - e^{-(\rho Q^R + B)}}{\rho Q^R + B}. \tag{A.32}$$

From (A.31) and (A.32), one obtains

$$1 \leq 2A \frac{1 - e^{-(\rho Q^R + B)}}{\rho Q^R + B}, \tag{A.33}$$

which is the reverse of (5.21).

Step 5: If $\hat{\theta} > 0$, then (5.21) holds.

If $\hat{\theta} > 0$, then (A.28) and the equation $\hat{v}_R(\theta) = \hat{v}_R(\hat{\theta}) + (\theta - \hat{\theta})Q^R$ for $\theta > \hat{\theta}$ can be used to rewrite condition (A.22) in the form

$$\lambda = \int_0^{\hat{\theta}} \lambda(2 - B\theta)Ae^{-B\theta}d\theta + \int_{\hat{\theta}}^1 \lambda(2 - B\hat{\theta})e^{-\rho(\theta-\hat{\theta})Q^R}Ae^{-B\theta}d\theta. \tag{A.34}$$

Upon dividing by λ, computing the integrals on the right-hand side and rearranging terms, one can rewrite this equation as

$$H(\hat{\theta}, Q^R) = 0, \tag{A.35}$$

where, for any $\theta \in [0, 1]$ and any $Q \geq 0$,

$$H(\theta, Q) := A \left[\frac{e^{-B} - e^{-B\theta} + B\theta e^{-B\theta}}{B} + (2 - B\theta)\, e^{-B\theta}\, \frac{1 - e^{-(\rho Q + B)(1-\theta)}}{(\rho Q + B)} \right]. \tag{A.36}$$

I claim that (A.35) implies

$$H(0, Q^R) = A \left[-\frac{1}{A} + 2\frac{1 - e^{-(\rho Q^R + B)}}{(\rho Q^R + B)} \right] < 0,$$

which is equivalent to (5.21). For this purpose, I compute

$$\frac{\partial H}{\partial \theta} = A[(2 - B\theta)\rho Q^R - B]\, e^{-B\theta}\, \frac{1 - e^{-(\rho Q^R + B)(1-\theta)}}{(\rho Q^R + B)} \tag{A.37}$$

for any θ. By a tedious rearrangement of terms, it follows that

$$\begin{aligned} \frac{\partial H}{\partial \theta} &= -BH(\theta, Q^R) + Ae^{-B\theta}[e^{-B(1-\theta)} - e^{-(\rho Q^R + B)(1-\theta)} \\ &\quad + (1 - \frac{B}{(\rho Q^R + B)})(1 - e^{-(\rho Q^R + B)(1-\theta)}) + B\hat{\theta}e^{-(\rho Q^R + B)(1-\theta)}] \\ &\geq -BH(\theta, Q^R) \end{aligned} \tag{A.38}$$

for any θ. Indeed, since $Q^R > 0$, the inequality is strict unless $\theta = 0$. For any $\theta > 0$, one therefore has $\frac{\partial H}{\partial \theta}(\theta, Q^R) > 0$ if $H(\theta, Q^R) = 0$. The validity of (A.35) for $\hat{\theta} > 0$ therefore implies $H(\theta, Q^R) < 0$ for all $\theta < \hat{\theta}$; in particular, $H(0, Q) < 0$, as claimed.

Step 6: The solution (Q^R, c_0^R, π_R) to the planner's relaxed problem, with associated information rent function $\hat{v}_R$, is also a solution to the planner's original problem.

To establish this claim, it suffices to show that the solution to the planner's relaxed problem satisfies the additional constraint of the original problem, i.e. that π_R is a nondecreasing function. From (5.19) and (5.20), one immediately sees that π_R is increasing on $[0, \hat{\theta})$ and constant on $[\hat{\theta}, 1]$. Moreover, if $\hat{\theta} > 0$, then from (A.37), (A.38), and (A.35), one has

$$\frac{\partial H}{\partial \theta}(\hat{\theta}, Q^R) = A[(2 - B\hat{\theta})\rho Q^R - B]\, e^{-B\hat{\theta}}\, \frac{1 - e^{-(\rho Q^R + B)(1-\hat{\theta})}}{(\rho Q^R + B)} > -BH(\hat{\theta}, Q^R) = 0,$$

hence $(2 - B\hat{\theta})\rho Q^R - B > 0$ and, by (5.19)

$$\lim_{\theta \uparrow \hat{\theta}} \pi_R(\theta) = \frac{B}{\rho Q^R(1 - B\hat{\theta})} < 1 = \pi_R(\hat{\theta}).$$

■

Proof of Proposition 5.5. Equation (5.27) shows that if the elasticity $\frac{\theta f'(\theta)}{f(\theta)}$ is increasing in θ, then $g'(\theta)$ is increasing in θ at any point at which $g'(\theta) = 0$. The function $g(.)$ is then strictly quasi-convex and has exactly one minimum on the interval $[0, 1]$. If $g'(0) \geq 0$, the minimum is a boundary minimum at $\hat{\theta} = 0$, and one has $g(\theta) > g(0) = 0$ for all $\theta \in (0, 1]$. In this case, (5.10) implies $\pi(\theta) = 1$ for almost all $\theta \in (0, 1]$.

If $g'(0) < 0$, the minimum of $g(.)$ occurs at some $\check{\theta} > 0$, with $g(\check{\theta}) < g(0) = 0 < \lambda f(1) = g(1)$. In this case, $\check{\theta} < 1$, and, as θ rises from $\check{\theta}$ to one, the value of $g(.)$ rises monotonically from $g(\check{\theta}) < 0$ to $g(1) = \lambda f(1) > 0$. There is then a unique $\hat{\theta} \in (\check{\theta}, 1)$ so that $g(\theta) < 0$ for $\theta \in (0, \hat{\theta})$ and $g(\theta) > 0$ for $\theta \in (\hat{\theta}, 1]$. In this case, (5.8) and (5.10) imply $\pi(\theta) = 0$ for almost all $\theta \in (0, \hat{\theta})$ and $\pi(\theta) = 1$ for almost all $\theta \in (\hat{\theta}, 1]$. Since $\pi(.)$ must be nondecreasing, one actually has $\pi(\theta) = 0$ for all $\theta \in [0, \hat{\theta})$ and $v'(\theta) = Q$ for all $\theta \in (\hat{\theta}, 1]$.

To complete the proof, one notes that, in the preceding argument, $\hat{\theta} = 0$ if $g'(0) \geq 0$ and $\hat{\theta} > 0$ if $g'(0) < 0$. By (5.26), $g'(0) \gtreqless 0$ as $W'(c_0) \lesseqgtr 2\lambda$. Thus $g'(0) \geq 0$ implies $\hat{\theta} = 0$ and $W'(v(0)) \leq 2\lambda$. By (5.14), one then has $W'(c_0) \leq 2 \int_0^1 W'(c_0 + \theta Q) f(\theta) d\theta$, which is the reverse of (5.24). Similarly, $g'(0) < 0$ implies $\hat{\theta} > 0$ and $W'(c_0) > 2\lambda$. (5.24) then follows by (5.14) and the strict concavity of $W(.)$. ■

Proof of Lemma 6.1. By standard arguments, for any n, θ_1, and $\theta_2 > \theta_1$, (6.5), (6.6), and (6.9), (6.10) imply that

$$v(\theta_2, n) - v(\theta_1, n) = \int_{\theta_1}^{\theta_2} \pi(\eta, n) Q d\eta = \int_{\theta_1}^{\theta_2} \bar{\pi}(\eta) Q d\eta \qquad \text{(A.39)}$$

for all n, θ_1, and $\theta_2 > \theta_1$. Thus if $\pi(.,.)$ satisfies (6.9) and (6.10), the function

$$(\theta, n) \to \hat{v}(\theta, n) := v(\theta, n) - \int_0^\theta \bar{\pi}(\eta) Q d\eta \tag{A.40}$$

must take the form

$$\hat{v}(\theta, n) = \bar{v}(n), \tag{A.41}$$

and one can write

$$v(\theta, n) = \bar{v}(n) + \int_0^\theta \bar{\pi}(\eta) Q d\eta. \tag{A.42}$$

By standard arguments, (6.5) and (6.6) also imply that, for any θ, the function $n \to y(\theta, n)$ is nondecreasing, and that

$$v(\theta, n_2) - v(\theta, n_1) = \bar{v}(n_2) - \bar{v}(n_1) = -\int_{n_1}^{n_2} k_n(y(\theta, \nu), \nu) d\nu \tag{A.43}$$

for all θ, n_1, and $n_2 > n_1$. Thus, for all n_1 and $n_2 > n_1$, $\int_{n_1}^{n_2} k_n(y(\theta, \nu), \nu) d\nu$ must be independent of θ, which implies that for any θ, one has $k_n(y(\theta, \nu), \nu) = k_n(y(0, \nu), \nu)$ for almost all ν. Because $k_n(y, \nu)$ is decreasing, it follows that, for any θ, one has $y(\theta, \nu) = y(0, \nu)$ for almost all ν. Because $y(\theta, .)$ is nondecreasing, it follows that $y(\theta, \nu) = y(0, \nu)$ at all ν at which the function $\nu \to y(\theta, \nu)$ is continuous. Because $y(\theta, .)$ has also been assumed to be right-continuous, it follows that $y(\theta, \nu) = y(0, \nu)$ at all ν.

As for the function, $c(.,.)$, (6.5) and (A.42) imply

$$\begin{aligned} c(\theta, n) &= \bar{v}(n) + \int_0^\theta \bar{\pi}(\eta) Q d\eta - \bar{\pi}(\theta)\theta Q + k(y(\theta, n), n) \\ &= \bar{v}(n) - \int_0^\theta \eta Q d\bar{\pi}(\eta) + k(y(\theta, n), n), \end{aligned}$$

so (6.11) follows from (6.10) and (6.12). Finally, (6.13) is the specialization of (6.6) to the case $\theta = \theta' = 0$. The proof that (6.11) - (6.13) are necessary for an allocation satisfying (6.9) and (6.10) to be incentive compatible is thus complete. The proof that these conditions are also sufficient is trivial and is left to the reader. ■

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