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Asset/Liability Management of German Life Insurance Companies: A Value-at-Risk Approach in the Presence of Interest Rate Guarantees

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### Asset/Liability Management of German Life Insurance Companies: A Value-at-Risk Approach in the Presence of Interest Rate Guarantees

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**Abstract:** This contribution analyses the implications of two major determinants influencing the asset allocation decision of German life insurers, which are the capital market development on the one hand and the interest rate guarantees of the traditional life insurance policies on the other hand. The adverse development of the stock prices between 2000 and 2002 asks for a consideration of not only the "normal" volatility but also the worst-case developments in an asset/liability management. In order to meet the latter requirement, we technically apply the risk measures of Value-at-Risk and Conditional Value-at-Risk. German life insurance policies incorporate interest rate guarantees, which are granted on an annual basis. This specific "myopic" nature of guarantees creates – beyond the control of the shortfall risk in general – the necessity to manage the asset allocation on an annual basis to match the time horizon of assets and liabilities.

A quantitative approach analyses the impacts on the asset allocation decision. In our research we do not only consider market valuation, but also institutional peculiarities (such as hidden reserves and accounting norms) of German life insurers. We reveal the possibility of a riskless one-year investment, either based on market values or on book values, to be crucial for guaranteeing interest rates on an annual basis.

JEL classification: G 22

**Keywords.** asset allocation, interest rate guarantees, Value-at-Risk, Conditional Value-at-Risk.

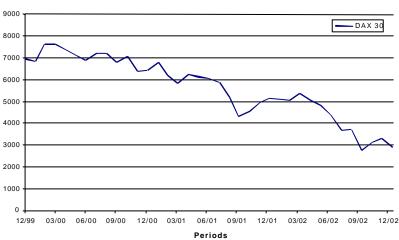
#### Introduction

Life insurers have to manage their assets while regarding both the risk/return profiles of their capital market investments (asset management), and their liabilities, which emerge from the insurer's range of offered products (asset/liability management). Thus, an implemented asset/liability management has to consider a realistic view of the capital markets as well as the company's specific product designs. Besides, the relevant institutional framework and legislation plays an important role regarding asset/liability management.

In this paper we narrow our analysis to the liabilities of German life insurers, which incorporate a peculiar structure of successive interest rate guarantees on an annual basis and to the German institutional framework. Since ecent negative stock market developments caused a lot of trouble, we extend the traditional Value-at-Risk approach by adding a new dimension of risk, the so-called worst-case risk.

Looking at the performance of the German stock market in the years 2000, 2001 and 2002 in figure 1 provides a first insight into the volatile capital markets.

Figure 1: Performance of DAX 30 from 12/1999 to 12/2002 (values taken at the end of a month)



Performance of DAX 30 between 12/1999-12/2002

The graph points out an extraordinary dramatic depreciation of the German DAX 30. Historically a similar development occurred the last time during the world economic crisis of the late twenties and early thirties. Critical numbers are summarized in table 1:

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Date	Levels of Dax 30	
Ultimo 1999:	6958.14	
07.03.2000:	8064.97	(all time high)
Ultimo 2000:	6433.61	[return of 2000: -7.54%]
21.09.2001:	3787.23	(low of 2001)
Ultimo 2001:	5160.10	[return of 2001: - 19.79%]
09.10.2002:	2597.88	(low of 2002)
Ultimo 2002:	2892.63	[return of 2002: -43.94%]

Table 1: Striking levels of the declining DAX 30

Obviously, the total loss of the last three years between 2000 and 2002 adds up to a terrifying 58.43%. Although stock markets are well-known to be highly volatile, this unique decline can hardly be explained by ordinary volatilities. It is rather characterized as a worst-case development, which has already been introduced in the academic literature<sup>1</sup>. The decrease of the DAX 30 emphatically highlights, that such worst-case scenarios are no longer only theoretical phenomena. They are an empirical fact, which undoubtedly has to influence a company's investment management in general and the asset management of life insurers in particular.

Therefore, a systematic risk management of life insurers has to cover

- not only normal volatility of the capital markets,
- but also worst-case developments as the preceding paragraph documents.

The liabilities of German life insurers emerge from the design of the traditional German life insurance contract, such as endowment and annuity policies, which contain interest rate guarantees due to mandatory fixed actuarial interest rate. These guarantees have to be covered first. Thus, the asset/liability management of German life insurers must focus on the control of the risk of not matching the predetermined interest rate guarantee.

Apart from this general requirement, the typical myopic structure of the liability portfolio of German life insurers has to be incorporated into the asset/liability management. If interest rate guarantees did mature at the expiration date of the underlying insurance policy, it would be possible for insurers to average periodic returns. But indeed, they mature annually at the end of each fiscal year during the contract duration<sup>2</sup>. Traditional German life insurance policies contain extraordi-

<sup>&</sup>lt;sup>1</sup> See Albrecht/Maurer/Ruckpaul (2001).

<sup>&</sup>lt;sup>2</sup> Each fiscal year the respective guaranteed capital gain must be added to the actuarial reserves.

nary investment guarantees consisting of successive annual guarantees. The necessity of covering the actuarial interest rate annually leads to major consequences for the asset management of the affected insurers. In fact, the investment horizon shortens to one year despite principally long-term insurance policies.<sup>3</sup> During this time horizon of one year, distinct minimum returns have to be obtained according to the date of issue of the policy<sup>4</sup>. Thus, the asset management cannot take advantage of the effects of diversification over long time horizons, that lower the volatility of the investment. As an implication for the asset/liability management of German life insurers the traditional strategic asset allocation, which primarily focuses on a long-term optimal position in the main asset classes, in particular independent of business cycles, has to be supplemented with an asset allocation on an annual basis.

Therefore, this paper deals with an analysis of such an asset allocation on an annual basis

- explicitly considering the impacts on the risk exposure
- considering the liabilities (given the annual target returns of the actuarial interest rate)
- considering the specific legislation for German life insurers (in particular we allow for book and market values of assets).

#### The Model and its Calibration

In order to become able to draw basic conclusions for an asset/liability management, we consider a simplified double-asset portfolio, which is limited to an investment in the German stock market (DAX 30) and bond market (REXP)<sup>5</sup>. Further evaluations within the model require an identification of the parameters "average return", "volatility of the returns" and "correlation between the returns" for each asset. Table 2 summarizes the applied constellation of parameters.

<sup>&</sup>lt;sup>3</sup> Thus, an immunization of the liabilities cannot be achieved by applying the concept of duration matching since this methodology is based on the sensitivity of the present value of the liabilities rather than controlling the risk during each predetermined fiscal year.

 $<sup>^4\,</sup>$  German life insurers guarantee 4% on contracts issued before July 2000 and 3,25% on those issued thereafter.

<sup>&</sup>lt;sup>5</sup> In the later chapter we will extend the portfolio in our analysis introducing a riskless asset.

Table 2: Calibration of the model

average return on DAX 30	8%
average return on REXP	5.5%
volatility of DAX	20%
volatility of REXP	5%
correlation between DAX and REXP	0.2

Average returns of DAX 30 and REXP are chosen rather prospectively and match with an economic scenario, that assumes moderate growth rates and moderate inflation. Volatilities and correlation are determined retrospectively from statistical data.

The following evaluation is based on the average return (of the portfolio) as the standard measure for performance on the one hand and different risk measures on the other hand. The first is an adaptation of the Value-at-Risk and may be interpreted as the probable minimum return (PMR) with respect to a distinct level of confidence  $\mapsto$ , e.g. 5% or 10%<sup>6</sup>. Intuitively, the 5%-PMR equals the distinct return, that will be fallen short by other random returns in an average of 5 of 100 cases<sup>7</sup>, which means that in 95 of 100 cases at least the 5%-PMR will be earned.

Thus, the PMR results from the predefined probability of falling short the target, which the asset management is willing to tolerate. PMR and the Value-at-Risk of assets respectively, serve as standard risk measures in order to control normal volatility. But these risk measures neglect the extent of the returns that have fallen short and do not consider worst-case scenarios. These factors are only included in the worst-case average return (WCAR)<sup>8</sup>, which is an adaptation of the Conditional Value-at-Risk, derived by Artzner et al.<sup>9</sup>. Like the PMR, the WCAR has an intuitive interpretation. The WCAR is the average of the returns that have fallen short the distinct level of confidence. Remembering the intuitive interpretation of the 5%-PMR as a level that random returns will fall short in an average of 5 of 100 cases, the 5%-WCAR is the average of these worst 5% returns.

#### The Case of German Life Insurance Companies

At first, we assume that target returns are solely earned from the assets which are directly linked to the corresponding liabilities. For instance, the actuarial interest rate on the actuarial reserve is supposed to be covered by the investment of the respective actuarial reserve fund. Thus, in our analysis the annual asset allocation

<sup>&</sup>lt;sup>6</sup> For a formalization of the PMR, see Appendix A.

<sup>&</sup>lt;sup>7</sup> Considering annual returns, this corresponds to an expected shortfall of one per twenty years.

<sup>&</sup>lt;sup>8</sup> For a formalization of the WCAR, see Appendix B.

<sup>&</sup>lt;sup>9</sup> See Artzner et al. (1999).

to the main asset classes, which we initially limited to stocks and bonds, is aour main control variable.

In a more general approach, the risk based capital, which is allocated to the asset management, may also be integrated into the model, since it serves as a buffer for extreme stock market developments<sup>10</sup>. Although it would be very simple to integrate risk based capital, we do not consider it in our approach but concentrate on the tactical asset allocation instead.

#### Pure market values of assets

In the following section we analyse various asset allocations in respect to their risk attitudes<sup>11</sup>. The calculation of underlying financial data (average returns, vola-tilities and correlation) is based on market values of the assets.

First, we only consider an investment in stocks (DAX 30). The level of confidence is supposed to be 10%. The 10%-PMR turns out to be -17.63% and the 10%-WCAR comprises -27.10%, which means that in 10% of possible outcomes, thus, in one of ten years, the portfolio return will even fall short of a return of as high as -17.63% (assuming the model calibration of the previous chapter). Furthermore, the average return of those returns of less than -17.63% is -27.10%. Hence, in these bad years, the return will on average be 1000 basis points less than 10%-PMR. Therefore, the worst-case risk, which is measured by the WCAR, turns out to be substantially high.

Switching to a pure bond portfolio (REXP) und again supposing the same level of confidence, we discover a value of -0.91% for the 10%-PMR and a value of -3.27% for the WCAR. This means on average in one of ten years the return on the bond portfolio falls short of a return of -0.91% and in these worst scenarios the average of the returns will amount to -3.27%. In particular, it is not possible to generate a positive target return for a given level of confidence of 10%. Therefore, an investment in bonds is not as riskless as it seems at first glance. The volatility of interest rates is empirically on such a high level that negative target returns cannot be excluded, if the level of confidence is reasonably high<sup>12</sup>. Thus, even a pure bond investment does not guarantee positive interest rates, if market values are considered.

<sup>&</sup>lt;sup>10</sup> Currently, risk based capital allocated to the asset management is calculated with no respect to the asset allocation (4% of the mathematical reserves plus the disposable fraction of the provision of premium refunds). From the perspective of risk management it might be more appropriate to explicitly link asset allocation and risk based capital as US legislation and European "Solvency II" suggest.

<sup>&</sup>lt;sup>11</sup> Our results are always based on the assumption of lognormal distributed returns.

<sup>&</sup>lt;sup>12</sup> During the last twenty years the annual return on the REXP turned out to be negative twice, in 1999 (-1.94%) and in 1994 (-2.51%). Thus, empirically the 10%-PMR is negative as well.

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But how does the Markowitz diversification affect the PMR, if we vary the asset allocation? Given the level of confidence constitutes 10%, the portfolio, which leads to the maximum 10%-PMR of -0.84% and the respective 10%-WCAR of -3.21%, consists of 4% of stocks (DAX 30) and 96% of bonds (REXP). Still, the PMR is negative and our results did not improve significantly, although we allowed for diversification. Results are summarized in table 3.

Table 3: PMR and W	CAR of various	portfolios
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	10%-PMR	10%-WCAR
Pure stock portfolio at market values	-17.63%	-27.10%
Pure bond portfolio at market values	-0.91%	-3.27%
Maximum PMR portfolio at market values	-0.84%	-3.21%
Maximum PMR portfolio at book values (hidden reserve of 20% and level of disso- lution of 75%)	0.07%	-2.35%
Portfolio consisting of 75% of bonded loans, 20% of stocks and 5% of bonds	3.83%	3.02%
Portfolio consisting of 75% of bonded loans, 20% of stocks and 5% of bonds	3.28% (5%-PMR)	2.66% (5%-WCAR)

Finally, we conclude:

- Even a pure bond investment does not guarantee a non-negative target return assuming a high level of confidence. The PMR of the bond portfolio is slightly negative.
- An investment in a portfolio of stocks and bonds, which generates the maximum PMR, while taking advantage of the Markowitz diversification, does not improve the PMR significantly. The PMR of the maximum PMR portfolio stays negative.
- If only stocks and bonds are considered, non-negative interest rate guarantees are not realistic with high confidence.

Keeping all this in mind, how do life insurers manage to provide interest rate guarantees at all, what they persistently do from an empirical perspective? Are there additional investment options or is the framework not adequate? The presence of hidden reserves due to a valuation in book values is a major issue, which we treat in the following section.

#### Book values of assets

In principle, (ex ante) returns that are based on market values of assets convert to respective returns that are calculated on a book value basis, if the amount of the hidden reserve at the beginning of the period is known and a maximum level of dissolution of this reserve is defined<sup>13</sup>. To simplify matters we concentrate on an example. We adopt the optimal 10%-PMR portfolio allocation, but assume an induced initial hidden reserve on stocks of 20%, which means that the stocks' book values range 20% below its market values. In addition, we suppose that the reserve may be dissolved up to 75% in order to buffer losses, which leads to a targeted hidden reserve of 5% on stocks. The bonds still appear in market values, thus their initial and targeted reserve equals to zero. Given the level of confidence of 10%, the PMR rises to 0.07% and the WCAR to -2.35%. The results have improved compared to a valuation based on pure market values, but the extent is still unsatisfactory. For a direct comparison see table 3.

We conclude:

- In principle, the incorporation of hidden reserves and their defined dissolution leads to an improvement of the risk/return ratio and to higher attainable target returns.
- Reachable target returns depend on the amount of hidden reserves and on the level of maximum dissolution.
- Regarding our example, effects are rather moderate.

Furthermore, hidden reserves are of temporary nature. They heavily depend on capital market developments and can only be consumed once. Consequently, a controlled dissolution of reserves can be utilized to guarantee minimum interest rates during adverse capital market developments, but it is unrealistic to identify them as a key component for persisting interest rate guarantees. But still, how do life insurers nevertheless provide policies based on interest rate guarantees? The missing link in the chain of arguments is the company's possibility to invest in a riskless asset, which we take into account in the next section.

#### The riskless asset

First, it has to be noted that the term "riskless asset" is only related to the volatility of asset value. Credit risk is explicitly excluded.

A primary example for such a riskless asset on an annual basis may then be a 12-month money market investment. Typically these financial instruments pay off no coupons and mature to par. Excluding any credit risk, an (in our terminology)

<sup>&</sup>lt;sup>13</sup> For a formalization, see Appendix C.

riskless interest payment has thus been generated. On the one hand, money market investments are very flexible and reduce risk already on a market value basis, but on the other hand, money market returns average at 6.08% between 1981 and 2000, which is fairly low compared to the average return on bonds (REXP) of 7.81% during the same period.

To take this into account, we consider such a riskless asset on a book value basis. Analysing the asset portfolio of German life insurers, we find the asset class of bonded loans to be dominant <sup>14</sup>. Bonded loans are loans, that are not traded at exchanges. Due to German law bonded loans enter the balance sheet at their initial value and will not be depreciated during the holding period, if credit risk is excluded. Therefore, the certain interest rate payments and the fixed value in the balance sheet render the investment riskless during the remaining time to maturity. Supposing that there is no necessity to sell bonded loans prior to maturity for liquidity reasons, which is a very plausible assumption for life insurers due to their long-term insurance policies, book values of assets should play a crucial role within the insurer's asset management rather than only market values<sup>15</sup>. In contrast to the money market investment only the supplementary purchases of bonded loans are variable on an annual basis, but the average return on bonded loans is significantly higher reaching about the return on treasury investments with a similar maturity.

In the recent past, German legislation created an alternative to bonded loans by introducing paragraph 341b HGB, which allows regular bonds to be considered as fixed assets. This perspective offers the possibility to list regular bonds in the balance sheet at their initial value. Thus, the above mentioned can be applied to regular bonds as well.

Such riskless assets, either on a book or market value basis, allow German life insurers to provide annual interest rate guarantees, because those reduce the high volatility of the capital markets to a satisfactory extent. This is why bonded loans play such a dominant role in the insurers' asset management.

In order to illustrate the consequences of an investment in those riskless assets, we look at the following example. 75% of the assets at book value are assumed to be invested in bonded loans, which yield 5.75%, and of the remaining 25% 5% are invested in stocks and 20% in bonds. Maintaining the model parameters of the previous sections, we obtain a 5%-PMR of 3.28% and a 5%-WCAR of 2.66%<sup>16</sup>. For a grading of the results see table 3. The dominant allocation to bonded loans establishes substantial positive target returns, even if a higher level of confidence of 5% is considered, which only allows for one shortfall every 20 years. If higher

<sup>&</sup>lt;sup>14</sup> We summarize bonded loans (*verbriefte Darlehen*), promissory note bonds (*Schuldscheindarlehen*), registered bonds (*Namensschuldverschreibungen*) and some other loans in this term.

<sup>&</sup>lt;sup>15</sup> Both, the allocation to the actuarial reserves and the bonus system of life insurance policies are based on book values as well.

<sup>&</sup>lt;sup>16</sup> Given the previously applied level of confidence of 10%, the 10%-PMR is 3.83% and the 10%-WCAR is 3.02%.

levels of confidence are chosen, positive target returns are still obtainable, provided the share of the riskless asset is further increased. Then, a perfectly immunized position<sup>17</sup> consists of bonded loans only (on a book value basis) or of money market only (on a market value basis). But the latter riskless portfolios result in a far less average return on life insurance policies. From a policy holder's perspective and in respect to the level of reachable interest rate guarantees and bonus payments, an insurer's investment in bonded loans rather than in the money market might be more preferable. In general, either on a book or market value basis, an active asset management might be defined as a designed deviation from an immunized position, in order to gain higher profits while accepting higher risks. The following section summarizes our results on riskless assets and draws some interesting conclusions.

The incorporation of bonded loans and other bonds, which belong to the fixed assets according to paragraph 341b HGB, in terms of a single-period riskless asset

- · decreases the portfolio volatility significantly and
- increases the obtainable PMR and WCAR values.

The extent of the effect depends heavily on the amount of bonded loans in the existing asset portfolio and its associated return on a book value basis respectively.

Obviously, there is a close relationship between the present legislation for German insurance companies and the reachable interest rate guarantees. Therefore, a modification of German accounting legislation might have severe impacts on product variety. In particular, a required individual financial statement at pure market values will severely affect the German insurance market. In this case it will still be possible for insurers to provide interest rate guarantees, since a riskless asset on a market value basis exists, e.g. the money market investment, but the guaranteed interest rate might decline significantly according to the difference of returns on money market investments and on capital market investments.

Finally, our calculations display an other interesting argument. In order to provide substantial single-period interest rate guarantees, a large allocation of the insurer's assets to stocks is not suitable, at least if we apply our model parameters. Only if we assume a much higher average return on stocks, the stocks become more relevant. Since the critical average return were above the historical long-term average of about 10-11%, the resulting higher allocation to stocks will only be valid for "above-average" periods and needs to be reversible, in times of "below-average" periods.

Against the background of single-period interest rate guarantees, stocks possess a volatility based on market values, which is "too high". In order to keep German life insurers able to guarantee reasonable interest rates, the option to value stock

<sup>&</sup>lt;sup>17</sup> In a sense of an absolute riskless position with no volatility risk attached. An immunization related to the liabilities, e.g. the interest rate guarantees, is achieved, if the return on the riskless asset ranges above the interest rate guaranteed.

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market investments within the financial statement at their book value needs to be maintained.

Particularly, a more flexible interpretation of the criterion of a "permanent depreciation" of stock investments due to paragraph 341b HGB will considerably improve the position of the insurers. Since the asset management of life insurers primarily focuses on their long-term liabilities and the attractive long-term returns on stocks, asset/liability management should not be dominated by effects of shortterm volatility and short-term adverse capital market developments.

#### Summary

In this section, we present the summarized results of our research.

- An advanced risk management regarding every kind of asset investment is crucial for life insurers. Especially, the following aspects have to be contained<sup>18</sup>:
  - the control of the normal volatility of the capital markets as well as the risk of worst-case developments.
  - the control of the risk of failing interest rate guarantees.
- Because of the myopic structure of the interest rate guarantees of life insurance policies, the asset allocation on an annual basis turns out to be a decisive factor.
- This annual asset allocation has to accommodate to the institutional requirements of the life insurance policies and accounting legislation.
- Non-negative interest rate guarantees are not reachable for a pure stock and bond portfolio, given a sufficient level of confidence.
- Results improve only if a riskless asset and hidden reserves are taken into account.
- Modifying German accounting legislation and regulating an individual financial statement at market values only, will not inhibit the provision of interest rate guarantees itself, but reduce the amount of the guarantee.
- The scope of an active asset management<sup>19</sup> regarding the annual asset allocation is predominantly determined by the amount of hidden reserves and the re-

<sup>&</sup>lt;sup>18</sup> Besides, other components such as an efficient solvency planning are important as well.

<sup>&</sup>lt;sup>19</sup> An active asset management is again defined as a designed deviation from an immunized position.

turn on the existing bonded loans on a book value basis of each individual insurer.

- The scope of an active asset management<sup>20</sup> widens, if other reserves are considered, such as a systematic dissolution of the provision for premium refunds, a profit-taking of reserves on property and, in general, risk based capital.
- In order to provide substantial single-period interest rate guarantees, a large allocation of the insurer's assets to stocks is not suitable, at least if we apply our model parameters.
- A more flexible interpretation of the criterion of a "permanent depreciation" of stock investments due to paragraph 341b HGB will considerably improve the position of the insurers.

#### Appendix A: Probable Minimum Return

The probable minimum return (PMR) is an adaptation of the Value-at-Risk. Given a distinct level of confidence  $\alpha$  (e.g.  $\alpha = 0.05$ ), the PMR of the random annual return R with respect to  $\alpha$  is defined to be

$$P[R < PMR_{\alpha}] = \alpha \tag{1}$$

which can easily be converted into the following equation containing the complementary:

$$P[R \ge PMR_{\alpha}] = 1 - \alpha.$$
<sup>(2)</sup>

Obviously, R falls short the PMR  $_{\alpha}$  (on average) only in 100  $\cdot \alpha$  % of possible outcomes of R and exceeds the PMR  $_{\alpha}$  (on average) only in 100  $\cdot (1-\alpha)$ % of possible outcomes of R.

Assuming a normal distribution for R, the  $\text{PMR}_{\alpha}\,$  can be analytically obtained as

$$PMR_{\alpha} = E(R) - N_{1-\alpha} \sigma(R), \qquad (3)$$

while  $N_{1-\alpha}$  denotes the  $(1-\alpha)$  - percentile of the standard normal distribution.

Assuming a lognormal distribution like  $ln(1+R) \sim N(m, v^2)$  instead, the PMR  $_{\alpha}$  can be calculated as follows:

<sup>&</sup>lt;sup>20</sup>An active asset management has the purpose to optimise the company's risk/return profile.

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$$PMR_{\alpha} = \exp\left(m - N_{1-\alpha} v\right) - 1.$$
(4)

#### Appendix B: Worst Case-Average Return

The Worst Case-Average Return (WCAR) is an adaptation of the Conditional Value-at-Risk<sup>21</sup>. Again, given a distinct level of confidence  $\alpha$ , the WCAR of the random annual return R with respect to  $\alpha$  is defined to be

$$WCAR_{\alpha} \coloneqq E[R | R < PMR_{\alpha}].$$
(5)

Evidently, the WCAR  $_{\alpha}$  is the conditional expectation of R, provided that R has fallen short the PMR  $_{\alpha}$ .

Subject to several conditions<sup>22</sup> the WCAR satisfies the characteristic of a  $\infty$ herent risk measure of Arzner et al. (1999), which is an approved criterion of the quality of a risk measure.

Assuming a normal distribution for R, the  $WCAR_{\alpha}$  can be analytically obtained as

WCAR<sub>$$\alpha$$</sub> = E(R) -  $\frac{\phi(N_{1-\alpha})}{\alpha}\sigma(R)$ , (6)

while  $N_{1-\alpha}$  denotes the  $(1-\alpha)$  - percentile of the standard normal distribution and  $\varphi(x)$  represents the density function of the standard normal distribution.

Assuming a lognormal distribution like  $\ln (1+R) \sim N(m, v^2)$  instead, the WCAR<sub> $\alpha$ </sub> can be calculated as follows:

WCAR<sub>$$\alpha$$</sub> =  $\left[1 + E(R)\right] \frac{\Phi(-N_{1-\alpha} - v)}{\alpha} - 1.$  (7)

# Appendix C: Conversion of market values into book values

Ex ante the following equation must be satisfied:

<sup>&</sup>lt;sup>21</sup> See Albrecht (2004).

<sup>&</sup>lt;sup>22</sup> See Albrecht (2004).

$$1 + R_{BW}(\alpha) = (1 + R_{MW}) \left[ \frac{1 - (1 - \alpha)h_0}{1 - h_0} \right],$$
(8)

while  $R_{MW}$  and  $R_{BW}$  represent the random annual returns on a market value or book value basis respectively.  $h_0$  denotes the hidden reserve quota at the beginning of the year and  $0 \le \alpha \le 1$  the desired degree of dissolution of the hidden reserve during the year.

#### Reference

Albrecht P (forthcoming) Risk Measures. To appear in: *Encyclopaedia of Actuarial Science*, Wiley.

Albrecht P, Maurer R, Ruckpaul U (2001) Shortfall-Risks of Stocks in the Long Run. Journal of Financial Markets und Portfolio Management 15: 481 - 499.

Artzner P, Delbaen F, Eber JM, Heath D (1999) Coherent Measures of Risk. Mathematical Finance 9: 203 – 228.

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