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Partnerships and Double Auctions with Interdependent Valuations

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Abstract

In a symmetric independent private values setting a sealed-bid double auction dissolves a partnership efficiently. This well known result remains valid in a model with interdependent valuations. However, if the interdependent components of valuations are large agents might prefer not to participate in a double auction. Therefore a simple extention of the rules of double-auctions is suggested that ensures participation. Even though these modified double auctions are not incentive efficient, they still realize gains from trade and can be implemented without knowledge about the model's specifications.

1 Introduction

When two (or more) partners own a firm together and want to dissolve the partnership they face the problem of choosing a "good" way to do so. A common situation is one where different partners are responsible for different parts or departments of their firm. It is natural to assume that they gain different information that helps them valuing their partnership as a whole. A dissolution mechanism therefore has to take into account the fact that at the end of a partnership each of the partners might neither know the other partner's valuation nor her own valuation for the entire firm. The latter is due to the fact that a partner's valuation for the entire firm also depends on private information of the other partner who has gained more accurate information about the part of the firm she was supervising. Obviously no partner is willing to reveal her information for free because this may lead to disadvantages in negotiations about the conditions of the dissolution. Therefore agents have to be paid an informational rent to reveal their information truthfully. If the agents are not subsidized by a third party these rents have

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to be generated by the trading possibilities due to differences in valuations. An important question in this context is whether the partnership can be dissolved efficiently, i.e. whether a trading mechanism exists that allocates the entire partnership to the partner who (ex-post) values it most. In a sufficiently symmetric setting with independent private values an ex-post efficient dissolution can be achieved by a simple mechanism, the so called $k-$double auction[1]. In the $k-$double auction the partners each submit a sealed bid and the entire partnership is allocated to the partner with the highest bid. The highest bidder's payment to the other partners is a convex combination of the highest bid (b_H) and second highest bid (b_L), i.e. the payment for the entire partnership is given by $kb_L + (1-k)\, b_H$ where $k \in [0,1]$. Since the rules of the $k-$double auction are independent of the valuation structure[2], a mechanism designer does not need to know the details of a given trading situation in order to implement this mechanism. Furthermore the $k-$double auction is ex-post budget-balanced which means that it never has to be subsidized by the mechanism designer. This suggests that the $k-$double auction is a favorable dissolution mechanism in an independent private values framework. In particular it is better than a Clark-Groves-Vickrey mechanism (which is not ex-post budget balanced) or a widely used shoot-out mechanism in which one side submits an offer and the other side has the choice of either buying or selling at the offered price. Note that because the partner who makes the initial offer will (in general) not bid her true valuation, the shoot-out mechanism is not ex-post efficient.

In this paper I concentrate on the case of an equal partnership with two partners and a symmetric valuation structure. In contrast to most of the existing literature, I allow for interdependent valuations. In my model the agents' valuations depend on own private information and on private information available to the other agent. Therefore the agents' valuations are correlated[3]. I restrict attention to the case of a positive correlation between agents' valuations. This structure of the valuations does (in contrast to an independent private values model) take into account that both partners might have gathered private insight in the firm which determines the valuations of both partners.

I show that there exists a unique equilibrium in pure strategies of the $k-$double auction. This equilibrium is symmetric and results in an efficient allocation. An important difference compared to an independent private values model is that agents no longer have the opportunity to ensure themselves a positive payoff by participating in the $k-$double auction. This is due to the fact that agents do not know their valuations on the interim stage. Therefore they do not have a rescue strategy, as in the private values case, where submitting the valuation always guarantees a positive payoff[4]. I show that in environments where no budget-balanced, individually rational and incentive compatible

[1] A double auction is ex-post efficient in a sufficiently symmetric setting with independent private values in case of risk neutral partners (see Cramton et al. [1987]) or partners with CARA-utility functions (see McAfee [1992]).

[2] i.e. independent of valuation functions and distributions of types

[3] The agent's information is assumed to be independently distributed.

[4] Note that bidding her own valuation can never be unprofitable for a partner since she sells her share only if payments exceed her valuation and buys the other agent's share only if payments are below her valuation.

mechanism that dissolves the partnership in an ex-post efficient way exists, the efficient equilibrium of the $k-$double auction is not individually rational. The intuition behind this shortcoming of the $k-$double auction is given by the fact that a bidder has to take into account a winner's and a looser's curse. If a partner wins the double auction this is "bad news" for her since this indicates that her partner's and therefore her own valuation are likely to be low. On the other hand, if she looses this again is "bad news" since this suggests that her partner's information indicates a high value of the firm. If we cannot force agents to participate, the rules of the $k-$double auction have to be extended in a way such that voluntary participation is guaranteed. This is done by allowing for non-participating or vetoing against the dissolution: If at least one agent vetoes or does not participate, the mechanism designer (auctioneer) implements the initial allocation of the partnership thus creating inefficiencies. However the $k-$double auction with the opportunity to veto results in an ex-post efficient allocation whenever it is possible to dissolve the partnership efficiently in general, i.e. whenever an ex-post efficient, individually rational, budget-balanced and incentive compatible dissolution mechanism exists. Given further assumptions on the valuation functions, I show that the possibility to veto guarantees that there always exist equilibria that still realize gains from trade. It turns out that partners who expect to have an average valuation for the partnership prefer to veto in the auction. Their chances of becoming buyer or seller are almost the same whereby trading opportunities are worse than for partners with either high valuation (who mainly buy) or low valuation (who primarily sell).

The model of this paper is based on Cramton et al.[1987] and generalizes their setting to interdependent valuations. In McAfee[1992] special $k-$double auctions are compared to other simple dissolution mechanisms for the equal partnership case. Even though McAfee allows for CARA-utility functions, both papers restrict attention to an independent private values framework. Keeping the independent private values framework and assuming equal distribution of the partnership, de Frutos [2000] compares efficiency and revenue of the $k-$double auction for $k = 0, 1$ if valuations are distributed asymmetrically. In addition to the literature that derives properties of double auctions for the equal partnership there exists a vast literature on $k-$double auctions in the case of a buyer and a seller (which can be seen as an extreme case of a partnership where property rights belong to one agent, the seller). Leininger et al.[1989] and Satterthwaite and Williams[1989] show that in the buyer/seller setting $k-$double auctions possess a continuum of pure strategy equilibria[5] if $k \in (0, 1)$. These can be ranked from equilibria that realize no gains from trade to equilibria that are incentive efficient[6]. Note that the uniqueness result for equilibria in this paper shows that multiplicity of equilibria is not necessarily present in $k-$double auctions, even if $k \in (0, 1)$. Bulow et al.[1999] analyze special cases of the $k-$double auction in a common values model and uniform distribution of types. They analyze the effects of an unequal distribution of ownership rights on bidders advantages in a first-price ($k = 0$) or second price ($k = 1$) double auction. In addition they show the general uniqueness of

[5]If $k = 1$ or $k = 0$ there exist an unique equilibrium.

[6]For the existence of incentive efficient equilibria the assumption of uniform distributed valuations is needed.

the equilibrium of double auctions if $k = 0$ or $k = 1$ in their common value model. Note that in this paper I show uniqueness of equilibria for all $k-$double auctions. Engelbrecht-Wiggans [1994] computes equilibria of a first- and second-price double auction in a model with affiliated values.

Neglecting the problem of financing the agents' informational rents, Jehiel and Moldovanu [1999] show that as long as agents' private information is one-dimensional, an efficient mechanism to dissolve the partnership can always be found[7]. They also show that a refinement of the Clarke-Groves-Vickrey approach can be used to get an efficient and incentive compatible direct mechanism (this refinement has also been done in Dasgupta and Maskin [1999]). In Fieseler at al.[2000] this mechanism is used to analyze whether in a partnership model with interdependent valuations there exist mechanisms that are (ex-post) efficient, incentive compatible, individually rational and budget balanced. They show that in contrast to a private values setting, it might be impossible to find a distribution of ownership rights such that the partnership can be dissolved efficiently, i.e. by an efficient, incentive compatible individually rational and budget balanced mechanism. In particular they show that it might not be possible to dissolve the equal partnership efficiently if the interdependence of valuations is too strong.

This paper is organized as follows: In section 2, I introduce the model of interdependent valuations. In section 3, I generalize the optimality result for symmetric $k-$double auctions to environments with interdependent values in which incentive compatible, (ex-post) efficient, budget balanced and individually rational mechanisms exist. I also compute the symmetric bidding equilibrium which is the unique equilibrium in pure strategies. In section 4, I analyze those cases of separable valuation functions in which a double auction is not individually rational. By giving each bidder the possibility to veto against a dissolution, individual rationality of the double auction (with veto) is assured. Furthermore this auction is ex-post efficient in cases where ex-post efficient, budget balanced, individually rational and incentive compatible mechanism exists. I compute the symmetric equilibrium bidding strategies of this auction and show that there always exist a symmetric equilibrium in which gains from trade are realized. In addition I give an example in which the double auction with veto is not the optimal (i.e. incentive efficient) mechanism, which it is in the independent private values model. For that example I give an indication of the performance of the double auction with veto. Section 5 is the conclusion. The proofs can be found in the appendix.

2 The Model

Two risk neutral agents each own an equal share in a partnership. Each agent i has private information represented by a type θ_i which influences her own and her partner's valuation for the partnership. By θ_{-i} I denote the type of the agent other than i. Agent $i's$ valuation for the entire partnership is given by $v_i(\theta_1, \theta_2)$, which I assume to be contin-

[7] They also show that, in general, efficiency is inconsistent with information revelation if private information is multidimensional.

uously differentiable in every argument. I assume (unless otherwise stated) a symmetric environment: the types of the agents are drawn from the same distribution function F, and valuation functions are symmetric:

$$v_1(\theta_1, \theta_2) = v_2(\theta_2, \theta_1). \tag{1}$$

Note that symmetry assumptions of this type are necessary to directly compute equilibria of the considered auctions and can also be found in Cramton et al. [1987] or McAfee[1992]. The distribution function F is strictly increasing and differentiable with derivative f. The support of f is given by $\left[\underline{\theta}, \overline{\theta}\right] \subset \mathbb{R}$. I further assume that agents' types are independent. The valuation function v_i is strictly increasing in θ_i and increasing in θ_{-i} . I denote the partial derivative of v_i with respect to its j'th component with $v_{i,j}$ and I assume that

$$v_{1,1} > v_{2,1}. \tag{2}$$

Note that, because of (1), this is equivalent to $v_{2,2} > v_{1,2}$. (2) is a common assumption in interdependent valuation environments. It ensures the existence of efficient and incentive compatible mechanisms[8]. Given a realization of types the utility of agent i who owns β_i in the entire partnership and has money m_i is quasilinear and given by

$$u_i = \beta_i v_i(\theta_1, \theta_2) + m_i. \tag{3}$$

Characteristic functions are defined as follows:

$$\mathbf{1}(\text{statement}) := \begin{cases} 1, & \text{if statement is true} \\ 0, & \text{if statement is false.} \end{cases}$$

3 The $k-$Double Auctions

The k- *double auction* is a Bayesian game where the strategy spaces of the agents are given by the set of functions $b : \left[\underline{\theta}, \overline{\theta}\right] \mapsto \mathbb{R}$. Given her type θ_i, agent i submits a bid $b_i(\theta_i) \in \mathbb{R}$. Denote the index of the agent who submits the higher bid by H and the index of the other agent by L. Given the bids b_L and b_H and the parameter $k \in [0, 1]$, the agent with the higher bid gets the entire partnership and pays to the other agent the amount $\frac{1}{2}((1-k)b_H + kb_L)$. In case both agents submit the same bid the partnership is given to agent i with probability $\frac{1}{2}$ and the "winning bidder" pays 0 (or any other fixed amount) to the other agent. Note that such an auction is always ex-post budget balanced since agent L gets what agent H pays. Assume that agent $-i$ bids according to $b_{-i}(\theta_{-i})$. The interim utility of a type θ_i agent who bids b_i is given by

$$\begin{aligned} U_i(\theta_i, b_i) \quad : \quad &= \frac{1}{2} E_{\theta_{-i}} \left[\left(v_i(\theta_i, \theta_{-i}) - (1-k) b_i - k b_{-i}(\theta_{-i}) \right) \quad \mathbf{1}\left(b_i > b_{-i}(\theta_{-i})\right) \right] \\ &+ \frac{1}{2} E_{\theta_{-i}} \left[\left((1-k) b_{-i}(\theta_{-i}) + k b_i - v_i(\theta_i, \theta_{-i}) \right) \quad \mathbf{1}\left(b_i < b_{-i}(\theta_{-i})\right) \right]. \end{aligned}$$

[8] For a discussion of this assumption see Dasgupta and Maskin [1999] or Jehiel and Moldovanu [1999].

The equilibrium concept used is that of pure Bayesian-Nash-Equilibrium (*BNE*). A BNE $(b_1(\theta_1), b_2(\theta_2))$ is individually rational if for $i = 1, 2$

$$U_i(\theta_i, b_i(\theta_i)) \geq 0, \qquad \forall \theta_i. \tag{IR}$$

A BNE $(b_1(\theta_1), b_2(\theta_2))$ is ex-post efficient if for all $(\theta_1, \theta_2) \in [\underline{\theta}, \overline{\theta}]^2$ we have :

$$v_1(\theta_1, \theta_2) > v_2(\theta_1, \theta_2) \Rightarrow b_1(\theta_1) > b_2(\theta_2) \tag{EF}$$

which, because of $\theta_1 > \theta_2 \Leftrightarrow v_1(\theta_1, \theta_2) > v_2(\theta_1, \theta_2)$, is equivalent to

$$\theta_1 > \theta_2 \Rightarrow b_1(\theta_1) > b_2(\theta_2).$$

The next Theorem characterizes the possible outcomes of the $k-$double auction. It turns out that there exists a unique BNE (in pure strategies). This equilibrium is symmetric.

For simplicity I use the following notation:

$$V_i(\theta_i) := v_i(\theta_i, \theta_i), \quad V_i'(\theta_i) := \frac{dV_i(\theta_i)}{d\theta_i}.$$

Theorem 1 *The $k-$double auction has a unique equilibrium bidding strategy in pure strategies given by*

$$b(\theta_i) = V_i(\theta_i) - \frac{\int_{F^{-1}(k)}^{\theta_i} V_i'(u)(F(u) - k)^2 du}{(F(\theta_i) - k)^2}. \tag{4}$$

Note that this strategy is strictly increasing and therefore the $k-$double auction is ex-post efficient[9].

In the private values case, i.e. if $v_{i,-i} = 0$, $i = 1, 2$, any BNE of a k-double auction must be individually rational because by bidding exactly her valuation each agent can guarantee herself a positive outcome of the auction regardless of the bid of the other bidder. Independent of k, she never pays more than her valuation for the other agent's share if she wins and if she looses she never gets less than her valuation for the part of the partnership she sells[10]. In general, it is not possible for a partner to bid her true valuation, which depends on private information of the other partner. Therefore a partner might risk to loose her share for a payment that is smaller than her valuation. As shown below, this is exactly what happens if the influence of the other agent's information on the own valuation is high. The intuition behind this observation is that a bidder faces a winner's and a looser's curse. If she wins she risks to pay too much for the partnership since winning indicates a low partner's type and therefore a low valuation of the partnership. If she looses, this is again "bad news" for her since this indicates a high type of the partner

[9] This Theorem generalizes results in Cramton et al [1987] and furthermore shows that there cannot exist other pure strategy equilibria.

[10] Note that this argument does not depend on the assumption of equal distribution of ownership rights.

and therefore a high value of the partnership. Since a bidder has to take these winner's and looser's curses into account at the same time, she cannot correct for these in a way that prevents her from making losses.

The following Theorem characterizes environments in which $k-$double auctions are not individually rational.

Theorem 2 *A $k-$double auction is individually rational if and only if there exists an ex-post efficient, incentive compatible, individually rational and budget balanced direct revelation mechanism.*

An existence condition for ex-post efficient, incentive compatible, individually rational and budget balanced mechanisms is given in Fieseler et al. [2000] for the class of separable valuation functions:

$$v_i\left(\theta_i,\theta_{-i}\right)=g\left(\theta_i\right)+h\left(\theta_{-i}\right). \tag{5}$$

Given their result and the above Theorem we get:

Theorem 3 *Given valuation functions of the form* (5), *the $k-$double auction is individually rational if and only if*

$$2\int_{F^{-1}\left(\frac{1}{2}\right)}^{\overline{\theta}} g(\theta)\ f(\theta)\ d\theta-2\int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\ f(\theta)\ F\left(\theta\right)\ d\theta+$$
$$+\int_{\underline{\theta}}^{\overline{\theta}} h'\left(\theta\right)\ \left(F^2\left(\theta\right)-F(\theta)\right)\ d\theta\geq 0.$$

Note that this existence condition depends on v and F whereas the $k-$double auction is a simple mechanism in a sense that it does not depend on the specifications of the agents' valuations and can therefore be applied universally. Nevertheless if partners are able to refuse to participate in a $k-$double auction a mechanism designer who is not familiar with these specifications does not know whether the partners will participate and how those behave who are participating but expect others not to participate. In the next section I extend the rules of the $k-$double auction to obtain a mechanism that is always individually rational, does not depend on specifications of the valuation structures and is ex-post efficient whenever there exist an ex-post efficient, individually rational, budget balanced and incentive compatible mechanism.

4 Double Auctions with voluntary participation

To ensure that the equilibria of the $k-$double auction are individually rational I extend the strategy spaces in such a way that every agent has the right to say "No" (write "No" in the sealed bid). The agents' strategy spaces are given by the set of functions:

$$\{b\mid b:\left[\underline{\theta},\overline{\theta}\right]\mapsto\mathbb{R}\cup\{\mathrm{N}\}\}.$$

The outcome of the game is defined as follows: If $b_1 =$N or $b_2 =$N then the partnership is not dissolved (or, equivalently, each agent gets the partnership with probability $\frac{1}{2}$). In any other case, the partnership is given to the agent with the higher bid. He pays $p = \frac{1}{2}\left((1-k)\max\{b_1, b_2\} + k\min\{b_1, b_2\}\right)$, $k \in [0,1]$, to the other agent. I call this Bayesian game the *$k-$double auction with veto*.

Note that the $k-$double auction with veto is always individually rational, because every type can veto and therefore assure that she never makes losses by participating in the auction. Furthermore if the $k-$double auction (without veto) is individually rational its equilibrium is also an equilibrium of the $k-$double auction with veto. It is easy to see that the $k-$double auction with veto has at least one further equilibrium which does not realize any gains from trade: always vetoing.

In the following I restrict attention to the more elaborate environments where the $k-$double auction is not individually rational. To get a precise characterization of symmetric Bayesian Nash equilibria I also restrict the analysis to the case of additively separable valuation functions[11], i.e. agent i's valuation for the entire partnership is given by the function $v_i(\theta_1, \theta_2) = g(\theta_i) + h(\theta_{-i})$. I assume g, h to be twice differentiable with $g' > h' \geq 0$. I assume the existence condition in Theorem 3 not to hold, i.e.

$$2\int_{F^{-1}\left(\frac{1}{2}\right)}^{\overline{\theta}} g(\theta)\ f(\theta)\ d\theta - 2\int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\ f(\theta)\ F(\theta)\ d\theta + \\ + \int_{\underline{\theta}}^{\overline{\theta}} h'(\theta)\ \left(F^2(\theta) - F(\theta)\right) d\theta < 0.$$

I show that apart from the equilibrium where all types veto there exist further symmetric equilibria that realize gains from trade if we choose $k = \frac{1}{2}$. In these equilibria the types close to the boundaries of the support of agents' types want the partnership to be dissolved whereas types around $F^{-1}\left(\frac{1}{2}\right)$ prefer to veto. The intuition behind these equilibria is as follows. Agents with types close to $F^{-1}\left(\frac{1}{2}\right)$ are the "worst off" types in the $k-$double auction mechanism, i.e. these types have the lowest interim utility of participating in the $k-$double auction. This is due to the fact that these types are (almost) equally likely to be buyer or seller of a share. In each case the expected gains from trade (i.e. the expected differences in agents' valuations) are small compared to types close to the boundaries of the support of types. Therefore the types around $F^{-1}\left(\frac{1}{2}\right)$ are vetoing in the $\frac{1}{2}-$double auction with veto and types close to $\underline{\theta}$ or $\overline{\theta}$ do not veto. Indeed the following Theorem shows that all types in an interval $[c, d]$ around $F^{-1}\left(\frac{1}{2}\right)$ prefer to veto. This interval is determined by the fact that agents with type c or d are indifferent between vetoing and non-vetoing. The next Theorem summarizes these results and formulates necessary conditions for c and d. These can always be fulfilled, as shown in Theorem 5.

[11] This restriction on the valuation functions is equivalent to requiring that the influence of the other agent's type on an agent's valuation does not depend on his own type, i.e. $\frac{\partial v_i}{\partial \theta_{-i}} = 0$.

Theorem 4 *Let $c, d \in [\underline{\theta}, \overline{\theta}]$ be a solution of the following equations:*

$$\begin{aligned} 1 &= F(c) + F(d) \\ 0 &= \frac{1}{2F(c)} \int_{\underline{\theta}}^{c} (g(t) + h(t))(F(t) - F(c)) f(t)\, dt \\ &+ \frac{1}{2F(c)} \int_{d}^{\overline{\theta}} (g(t) + h(t))(F(t) - F(d)) f(t)\, dt \\ &+ \frac{1}{2} \int_{\underline{\theta}}^{c} g(t) f(t)\, dt - \frac{1}{2} \int_{d}^{\overline{\theta}} g(t) f(t)\, dt. \end{aligned} \tag{6}$$

Then the following bidding strategy constitutes a symmetric Bayesian Nash equilibrium of the $\frac{1}{2}-$double auction with veto:

$$b(\theta_i) = \begin{cases} g(\theta_i) + h(\theta_i) - \frac{\int_c^{\theta_i} (g'(t)+h'(t))(F(t)-F(c))^2 dt}{(F(\theta_i)-F(c))^2} & \text{if } \theta_i \in [\underline{\theta}, c) \\ N & \text{if } \theta_i \in [c, d] \\ g(\theta_i) + h(\theta_i) - \frac{\int_d^{\theta_i} (g'(t)+h'(t))(F(t)-F(d))^2 dt}{(F(\theta_i)-F(d))^2} & \text{if } \theta_i \in (d, \overline{\theta}]. \end{cases} \tag{7}$$

Instead of directly verifying that a deviation of the given strategy cannot be profitable if the other agent sticks to it, I use the Revenue-Equivalence-Theorem (Theorem 6, see the Appendix) for an indirect proof. Given an allocation rule s, the Revenue-Equivalence-Theorem determines (up to a type-independent constant) the payments (depending on the agents' reported types) necessary and sufficient to implement s in a truthtelling equilibrium. By the revelation principle in any (indirect) mechanism that implements s the expected payments to agents in equilibrium have to equal those given by the Revenue-Equivalence-Theorem (up to a type independent constant). Furthermore if the expected payments to agents induced by a given strategy-profile in an indirect mechanism that implements s equal those given by the Revenue-Equivalence-Theorem we know that imitating the strategy of a different type cannot be profitable. Therefore I show that the payments induced by (7) equal those of a direct mechanism that implements the allocation resulting from (7) in truthtelling. In addition I can show that deviating to a bid outside the range of (7) cannot be profitable which completes the proof.

Instead of vetoing, it is also possible to extend the rules of the $k-$double auction such that the agents can choose not to participate in the $k-$double auction. In this case the k-double auction is modelled as a two stage game. In the first stage each agent decides whether she participates in the 2nd stage or not. If at least one agent decides not to participate in the 2nd stage, the partnership is not dissolved. Otherwise in the 2nd stage a $k-$double auction (without veto) is run.

Obviously the concept of a double auction with veto is only meaningful if there exist equilibria that realize gains from trade and therefore do not sustain the status quo like the always vetoing equilibrium.

Theorem 5 *Every $\frac{1}{2}-$double auction with veto has a symmetric equilibrium where not vetoing occurs with positive probability.*

An important feature of the $k-$double auction with veto is the independence of its rules of v and F. A mechanism designer can run the auction and she gets the best possible outcome (in terms of efficiency) if in general this outcome can be obtained by some budget balanced and individually rational mechanism. If this is not possible, the set of types that do not want to dissolve the partnership is determined by the agents themselves, depending on their knowledge about v and F. As shown in the next section, a mechanism designer who knows v and F might find a more efficient dissolution mechanism. This reflects the intuition that a mechanism with simple rules that are independent of the specifications of the model is unlikely to be always optimal. The mechanism designer might loose efficiency if she has not full insight in the trading environment.

4.1 An Example for the performance of the double auction with veto

In this section I examine the performance of the symmetric equilibria of the $\frac{1}{2}-$double auction with veto in the case of linear valuation functions

$$v_1(\theta_1,\theta_2) = a\theta_1 + b\theta_2, \quad a > b \geq 0$$

and uniform distribution of types on the unit interval, i.e. $f(\theta) = 1_{[0,1]}(\theta)$.

Theorem 3 states that the $\frac{1}{2}-$double auction is individually rational (and efficient) if $2b \leq a$. The equilibrium of the $\frac{1}{2}-$double auction is given by $b(\theta) = (a+b)\left(\frac{2}{3}\theta + \frac{1}{6}\right)$. If $2b > a$ the only symmetric equilibrium of the $\frac{1}{2}-$double auction with veto that realizes some gains from trade is given by

$$b(\theta_i) = \begin{cases} (a+b)\left(\frac{2}{3}\theta + \frac{1}{3}\left(\frac{1}{2} - \varepsilon\right)\right) & \text{if} \quad \theta_i \in [0, \frac{1}{2} - \varepsilon) \\ \text{N} & \text{if} \quad \theta_i \in [\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon] \\ (a+b)\left(\frac{2}{3}\theta + \frac{1}{3}\left(\frac{1}{2} + \varepsilon\right)\right) & \text{if} \quad \theta_i \in (\frac{1}{2} + \varepsilon, 1]. \end{cases}$$

$$\text{where} \quad \varepsilon = -\frac{1}{2}\frac{a-2b}{2a-b},$$

Playing according to this equilibrium results in an ex-post allocation of the partnership which can be described by the probability that agent 1 gets the entire partnership (depending on the types of the agents). Figure 1 shows the resulting allocation. The performance of this equilibrium is measured by the ex ante gains from trade, i.e. the unweighted sum of the agents' ex-ante utilities. Because of budget balancedness this is given by:

$$G^{DA} := (a-b)\left(\frac{1}{6} - \frac{1}{2}\varepsilon + \frac{2}{3}\varepsilon^3\right).$$

The gains from trade that could be realized if all information were common knowledge is given by $\frac{1}{6}(a-b)$. It can be seen that the inefficiencies that arise in the $\frac{1}{2}-$double auction with veto become arbitrarily small if $2b \approx a$, i.e. if ε is small. To measure the performance of the $\frac{1}{2}-$double auction with veto in a more convincing way it should be compared with other mechanisms that are individually rational and budget-balanced.

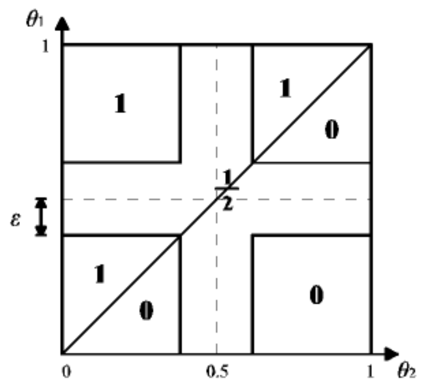

Figure 1: Allocation of $\frac{1}{2}-$double auction

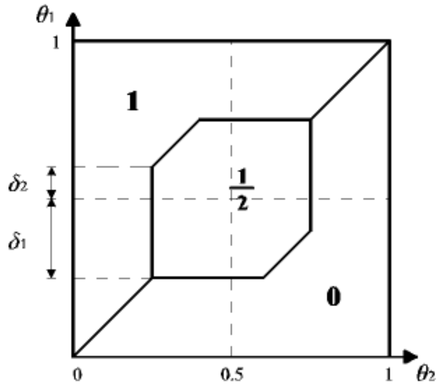

Figure 2: Welfare improving allocation

Unfortunately it is not known which mechanism performs best in the class of all IR and (ex-ante) budget balanced mechanisms. The general program of constructing the best performing mechanism is given in the appendix. This optimization program suggests that a mechanism might be able to extract more gains from trade if only for type combinations (θ_1, θ_2) around $\left(\frac{1}{2}, \frac{1}{2}\right)$ the partnership is not dissolved efficiently. Therefore I consider allocation rules as given in Figure 2. Most gains from trade can be realized if we chose the parameters $\delta_1 \in [0, \delta_2]$ and $\delta_2 \in [0, \frac{1}{2}]$ such that[12] $\delta_1 = \delta_2$. A comparison of the performance of this "third best" mechanism (denoted by G^{TB}) with the $\frac{1}{2}-$double auction is given in Table 1 for some parameters a, b. Such mechanisms perform better than the double auction with veto but a mechanism with "simple" rules independent of v_i and F

[12]These values of δ_1 and δ_2 are chosen such that the inequality condition of the maximization problem given in the appendix (8) is binding. This is the case for

$$\delta_1 = \delta_2 = \frac{1}{2\,(16a - 8b)} \left(-2a + 4b + 2\sqrt{(-15a^2 + 36ab - 12b^2)} \right).$$

a	b	G^{DA}	G^{TB}
1	.9	0.00169	0.00775
1	.7	0.02765	0.04459
1	.6	0.05248	0.06477
1	.55	0.06725	0.07437
1	.51	0.08002	0.08162

Table 1: Performance of $\frac{1}{2}$-double auction

performing better than the $\frac{1}{2}-$double auction could not be obtained. Further calculations show that the mechanisms implementing the allocation given in Figure 2 perform better, especially if $2b$ is significantly greater a. Nevertheless if parameters are such that we almost have $2b = a$ the differences in performance compared to the $\frac{1}{2}-$double auction become small.

5 Conclusion

The $k-$double auction is a favorable mechanism to dissolve a partnership since it has simple rules that do not depend on specifications of the agents' valuations. If the interdependent components of the valuation functions are small it can be applied without worrying about agents' participation decisions. Since this is not true any more if the influence of the other agent's information becomes larger, it might not be possible to successfully run a $k-$double auction. This paper suggests a modification of the $\frac{1}{2}-$double auction that is individually rational. Symmetric equilibria of this auction are derived and it is shown that even though the mechanism is not always optimal, it succeeds in realizing gains from trade. The rules remain simple and the mechanism designer does not need to know the distribution of types to determine those types not willing to dissolve the partnership. This is done by the participating agents themselves. An exemplarily comparison with another dissolution mechanism shows that (in contrast to the i.p.v. model) the mechanism designer can construct more efficient mechanisms if she is familiar with specifications of the valuations.

Unfortunately it was not possible to derive the incentive efficient mechanism in the model of this paper. This is subject to further research.

6 Appendix

For simplicity of presentation I need the following notations and definitions in this appendix. In a direct revelation mechanisms (DRM) agents report their types, relinquish their share of the good, and then receive a payment $t_i(\theta)$ and a share $s_i(\theta)$ of the entire good. A DRM is therefore a game form $\Gamma = \left(\left[\underline{\theta}_1, \overline{\theta}_1\right], \left[\underline{\theta}_2, \overline{\theta}_2\right], s, t\right)$, where $s(\theta) = (s_1(\theta), s_2(\theta))$ is a vector with components $s_i : \times_{j=1}^{2}\left[\underline{\theta}_j, \overline{\theta}_j\right] \mapsto [0,1]$ such that

$s_1(\theta) + s_2(\theta) = 1$ for all θ, and $t(\theta) = (t_1(\theta), t_2(\theta))$ is a vector with components $t_i : \times_{j=1}^{2} \left[\underline{\theta}_j, \overline{\theta}_j\right] \mapsto \mathbb{R}$. I call s the allocation rule and t the payments. I refer to the pair (s,t) as a DRM if it is clear which strategy sets $\left[\underline{\theta}_i, \overline{\theta}_i\right]$ are meant.

A mechanism (s,t) *implements* the allocation rule s if truth-telling is a Bayes-Nash equilibrium in the game induced by Γ and the agents' utility functions. Such a mechanism is called *incentive compatible* (IC).

The interim utility of agent i given his type θ_i and his announcement $\widehat{\theta}_i$ (and truthtelling of the other agent) is given by:

$$\begin{aligned} U_i\left(\theta_i, \widehat{\theta}_i\right) &= E_{\theta_{-i}}[v(\theta_i, \theta_{-i})\left(s_i\left(\widehat{\theta}_i, \theta_{-i}\right) - \frac{1}{2}\right) + t_i\left(\widehat{\theta}_i, \theta_{-i}\right)] \\ &= : E_{\theta_{-i}}[v(\theta_i, \theta_{-i})\left(s_i\left(\widehat{\theta}_i, \theta_{-i}\right) - \frac{1}{2}\right)] + T_i(\theta_i). \end{aligned}$$

I use the following notation:

$$U_i(\theta_i) := U_i(\theta_i, \theta_i).$$

A DRM is called (ex-ante) budget balanced (BB) if $T_1 + T_2 = 0$. A DRM is individually rational (IR) if

$$U_i(\theta_i) \geq 0 \quad \text{for all } \theta_i, \quad i = 1, 2.$$

Performance is measured by realized gains from trade, i.e. by the unweighted sum of the agents' ex-ante utilities. An IC, IR, BB mechanism is called *incentive efficient* if there exists no IC, IR and BB mechanism that performs better.

For some proofs I need a generalization of the revenue equivalence theorem to environments with interdependent valuations of the form $v_i(\theta_1, \theta_2) = g(\theta_i) + h(\theta_{-i})$.

Theorem 6 *(Revenue-Equivalence-Theorem) A DRM (s,t) is incentive compatible if and only if the following holds for $i = 1, 2$:*
a) $\overline{s}_i(\theta_i) := \int_{\underline{\theta}}^{\overline{\theta}} \left(s_i(\theta_i, \theta_{-i}) - \frac{1}{2}\right) f(\theta_{-i})\, d\theta_{-i}$ *is increasing in θ_i,*
b) For all $\widetilde{\theta}_i, \theta_i \in \left[\underline{\theta}, \overline{\theta}\right]$ we have: $U_i(\theta_i) = U_i\left(\widetilde{\theta}_i\right) + \int_{\widetilde{\theta}_i}^{\theta_i} g'(t)\, \overline{s}_i(t)\, dt$

Proof. The proof is almost identical to the independent private values case. ■

6.1 The general problem an incentive efficient mechanism has to solve

Given the model of section 4.1 the general problem an incentive efficient mechanism has to solve is given by:

$$\max_{k_1} \int_0^1 \int_0^1 (U_1(\theta_1) + U_2(\theta_2))\, d\theta_1 d\theta_2$$
$$\text{s.t. } BB,\ IR,\ IC.$$

To get IC we must have for some type $\widetilde{\theta}_i$ (according to Theorem 6):

$$U_i(\theta_i) = U_i\left(\widetilde{\theta}_i\right) + a\int_{\widetilde{\theta}_i}^{\theta_i}\int_0^1 \left(k_i(t,\theta_{-i}) - \frac{1}{2}\right) d\theta_{-i}dt, \quad i = 1,2.$$

Therefore we have (using integration by parts and BB):

$$\begin{aligned}
& U_1\left(\widetilde{\theta}_1\right) + U_2\left(\widetilde{\theta}_2\right) \\
= & \int_0^1\int_0^1 \left(U_1(\theta_1) + U_2(\theta_2)\right) d\theta_1 d\theta_2 - a\int_0^1\int_{\widetilde{\theta}_1}^{\theta_1}\int_0^1 \left(k_1(t,\theta_2) - \frac{1}{2}\right) d\theta_2 dt d\theta_1 \\
& -a\int_0^1\int_{\widetilde{\theta}_2}^{\theta_2}\int_0^1 \left(\frac{1}{2} - k_1(\theta_1,t)\right) d\theta_1 dt d\theta_2 \\
= & \int_0^1\int_0^1 \left[(2a-b)(\theta_1-\theta_2) + a\left(1\left(\theta_2 > \widetilde{\theta}_2\right) - 1\left(\theta_1 > \widetilde{\theta}_1\right)\right)\right]\left(k_1(\theta_1,\theta_2) - \frac{1}{2}\right) d\theta_2 d\theta_1
\end{aligned}$$

The "worst off" types, i.e. the types for which the individual rationality constraints are binding, are given by

$$\widetilde{\theta}_i = \arg\min_{\theta_i} U_i(\theta_i) = \arg\min_{\theta_i} a\int_0^{\theta_i}\int_0^1 \left(k_i(t,\theta_{-i}) - \frac{1}{2}\right) d\theta_{-i}dt$$

and we therefore get

$$a\int_0^1 \left(k_i\left(\widetilde{\theta}_i,\theta_{-i}\right) - \frac{1}{2}\right) d\theta_{-i} = 0 \Rightarrow \int_0^1 k_i\left(\widetilde{\theta}_i,\theta_{-i}\right) d\theta_{-i} = \frac{1}{2}.$$

This is also sufficient for being the "worst off" type, because IC implies that $\int_0^1 k_i(t,\theta_{-i})\, d\theta_{-i}$ is increasing in t.

For IR to hold we need

$$U_1\left(\widetilde{\theta}_1\right) + U_2\left(\widetilde{\theta}_2\right) \geq 0.$$

The general problem of finding an incentive efficient mechanism can therefore be formulated as[13]:

$$\max_{k_1}\int_0^1\int_0^1 (a-b)(\theta_1-\theta_2)\left(k_1(\theta_1,\theta_2) - \frac{1}{2}\right) d\theta_1 d\theta_2 \tag{8}$$

[13]This formulation can be easily extended to the general case of separable valuation functions and general (strictly increasing and continuous) distribution functions.

s.t.

$$\max_{k_1} \int_0^1 \int_0^1 (a-b)(\theta_1 - \theta_2)\left(k_1(\theta_1,\theta_2) - \frac{1}{2}\right) d\theta_1 d\theta_2$$

$$\int_0^1 \int_0^1 \left[(2a-b)(\theta_1-\theta_2) + a\left(1\left(\theta_2 > \widetilde{\theta}_2\right) - 1\left(\theta_1 > \widetilde{\theta}_1\right)\right)\right]\left(k_1(\theta_1,\theta_2) - \frac{1}{2}\right) d\theta_2 d\theta_1 \geq 0,$$

$$k_1(\theta_1,\theta_2) \in [0,1],$$

$$\int_0^1 k_1\left(\widetilde{\theta}_1,\theta_2\right) d\theta_2 = \int_0^1 k_1\left(\theta_1,\widetilde{\theta}_2\right) d\theta_1 = \frac{1}{2},$$

$$\int_0^1 k_1(t,\theta_2)\, d\theta_2, \quad \int_0^1 k_1(\theta_1,t)\, d\theta_1 \text{ are increasing in } t.$$

Given a solution k_1^* of this problem we get $k_2^* = 1 - k_1^*$ and can calculate the according payments t_1, t_2 using Theorem 6. Note that the program above is much more complex than in the bilateral trade case, where one agent owns the entire good. In that setting the worst off types do not depend on the allocation and the last condition turns out to be not binding (given certain assumptions on F).

6.2 Proofs

Proof of Theorem 1:

In this proof I will denote an equilibrium of the $k-$double-auction by $(b_1(\theta_1), b_2(\theta_2))$ where $b_i(\theta_i)$ denotes the equilibrium bidding strategy of agent i. The agent other than i is denoted by $-i$. Throughout the proof I will assume $k \in (0,1)$. The cases $k = 1$ and $k = 2$ are indeed simpler to prove and can be found for a similar model in [1999].

I summarize the different steps to illustrate the logic behind the whole proof: In the first step I show that the equilibrium has to fulfill a (symmetric) system of differential equations if it is continuous and strictly increasing. In the 2nd step I show that an equilibrium bidding strategy $b_i(\theta_i)$ can only be decreasing if there is a gap in $b_{-i}(\theta_{-i})$ at $\theta_{-i} = F^{-1}(k)$. In step 3 I show that there cannot be atoms (i.e. a positive measure of types submitting the same bid) in the equilibrium bidding functions of both agents at the same bid. In the 4th step I show that the bids of the highest types have to be the same for both bidders and that this is also the case for the bids of the lowest types, i.e. $b_1(\underline{\theta}) = b_2(\underline{\theta})$ and $b_1\left(\overline{\theta}\right) = b_2\left(\overline{\theta}\right)$. We also get that $b_i(\underline{\theta}) > v_i(\underline{\theta},\underline{\theta})$ and $b_i\left(\overline{\theta}\right) < v_i\left(\overline{\theta},\overline{\theta}\right)$. I derive conditions that must hold to allow equilibrium bidding functions to have atoms (step 5) or gaps (step 6). Step 7 and 8 show that the differential equations determine a unique solution if starting from an initial condition $b_i(\underline{\theta}) = \underline{b}$ (or $b_i\left(\overline{\theta}\right) = \overline{b}$) we increase (decrease) θ_i as long as either $\theta_i = F^{-1}(k)$ or $b_i(\theta_i) = v_i(\theta_i,\theta_i)$. In the 9th step I show that for $\theta_i = F^{-1}(k)$ we get that $b_i(\theta_i) = v_i(\theta_i,\theta_i)$ and furthermore that even at $\theta_i = F^{-1}(k)$ the equilibrium bidding strategies are continuous. Hence the equilibrium bidding strategies are strictly increasing (have no atoms) and are continuous (have no gaps). This shows that the equilibrium has to fulfill the symmetric system of differential equations derived in step 1 and therefore is symmetric. In the last step (10) I show that it is unique, i.e. only one possible initial condition $b_i(\underline{\theta}) = \underline{b}$ and $b_i\left(\overline{\theta}\right) = \overline{b}$ can be fulfilled.

The steps in detail:

1. Assume that for a given range $\left(b^L, b^H\right)$ b_1, b_2 are continuous and strictly increasing on $b_1^{-1}\left(\left(b^L, b^H\right)\right)$ and $b_2^{-1}\left(\left(b^L, b^H\right)\right)$ respectively and all types of player i that are lower than all types in $b_i^{-1}\left(\left(b^L, b^H\right)\right)$ bid below b^L and all types of player i that are higher than all types in $b_i^{-1}\left(\left(b^L, b^H\right)\right)$ bid above b^H.
   The utility of a type θ_i-bidder submitting a bid $b \in \left(b^L, b^H\right)$ is given by

$$\begin{aligned} U_i\left(\theta_i, b\right) &= \frac{1}{2}\int_{\underline{\theta}}^{b_{-i}^{-1}(b)} \left(v_i\left(\theta_1, \theta_2\right) - \left(\left(1-k\right)b + kb_{-i}\left(\theta_{-i}\right)\right)\right) f\left(\theta_{-i}\right) d\theta_{-i} \\ &+\frac{1}{2}\int_{b_{-i}^{-1}(b)}^{\overline{\theta}} \left(kb + \left(1-k\right)b_{-i}\left(\theta_{-i}\right) - v_i\left(\theta_1, \theta_2\right)\right) f\left(\theta_{-i}\right) d\theta_{-i}. \end{aligned}$$

   Because b_1, b_2 are continuous and strictly increasing and therefore a.e. differentiable on $b_1^{-1}\left(\left(b^L, b^H\right)\right)$ and $b_2^{-1}\left(\left(b^L, b^H\right)\right)$ respectively, the same is true for the inverse functions $b_i^{-1}(b)$ on $\left(b^L, b^H\right)$. Differentiating with respect to b yields the following local first order conditions:

$$\begin{aligned} \left(v_1\left(b_1^{-1}(b), b_2^{-1}(b)\right) - b\right) f\left(b_2^{-1}(b)\right) \frac{\partial b_2^{-1}(b)}{\partial b} - \frac{1}{2}\left(F\left(b_2^{-1}(b)\right) - k\right) &= 0, \quad (9) \\ \left(v_2\left(b_1^{-1}(b), b_2^{-1}(b)\right) - b\right) f\left(b_1^{-1}(b)\right) \frac{\partial b_1^{-1}(b)}{\partial b} - \frac{1}{2}\left(F\left(b_1^{-1}(b)\right) - k\right) &= 0. \end{aligned}$$

   Note that this system of differential equations is given for the inverse functions of the equilibrium bidding functions. Since it is Lipschitz-continuous if $v_1\left(b_1^{-1}(b), b_2^{-1}(b)\right) \neq b$ and $v_2\left(b_1^{-1}(b), b_2^{-1}(b)\right) \neq b$ it uniquely determines b_i^{-1} uniquely given an "initial condition" on intervals where b_i^{-1} are strictly increasing, continuous and $v_1\left(b_1^{-1}(b), b_2^{-1}(b)\right) \neq b$, $v_2\left(b_1^{-1}(b), b_2^{-1}(b)\right) \neq b$. In particular the b_i are also uniquely determined on the range of such an interval. If we have $b_1^{-1} = b_2^{-1}$ we have Lipschitz-continuity if $v_1\left(b_1^{-1}(b), b_1^{-1}(b)\right) \neq b$ or equivalently $b_1(\theta) \neq v_1(\theta, \theta)$.

2. If b_i is (locally) decreasing, i.e. if we have $b_i\left(\theta_i^*\right) > b_i\left(\theta_i^{**}\right)$ for some $\theta_i^* < \theta_i^{**}$ the following holds:

$$\Pr_{\theta_{-i}}\{b_{-i}\left(\theta_{-i}\right) \in \left[b_i\left(\theta_i^{**}\right), b_i\left(\theta_i^*\right)\right]\} = 0$$

   and

$$\Pr_{\theta_{-i}}\{b_{-i}\left(\theta_{-i}\right) < b_i\left(\theta_i^{**}\right)\} = k.$$

   The proof of this statement follows standard revealed preferences arguments.

3. It is impossible that a positive measure of types of agent 1 and 2 submit the same bid, i.e. for all b we have:

$$\Pr_{\theta_i}\{b_i(\theta_i) = b\} > 0 \Rightarrow \Pr_{\theta_{-i}}\{b_{-i}(\theta_{-i}) = b\} = 0.$$

   If the contrary statement were true, a bidder would prefer to increase or decrease her bid slightly since this would hardly change payments but significantly change her probability of winning or loosing.

4. We have

$$b_1(\underline{\theta}) = b_2(\underline{\theta}) \neq b_1\left(\overline{\theta}\right) = b_2\left(\overline{\theta}\right),$$

$$b_i(\theta) \in \left[b_1(\underline{\theta}), b_1\left(\overline{\theta}\right)\right]$$

   and

$$b_1(\underline{\theta}) > v_1(\underline{\theta}, \underline{\theta}), \quad b_1\left(\overline{\theta}\right) < v_1\left(\overline{\theta}, \overline{\theta}\right).$$

   Assume without loss in generality that $\inf_\theta b_1(\theta) > \inf_\theta b_2(\theta)$ (I allow the last value to be $-\infty$). Then it is profitable for a type of player 2 who bids below $\inf_\theta b_1(\theta)$ to increase her bid such that she still looses against all types of player 1. Therefore we must have

$$\inf_\theta b_1(\theta) = \inf_\theta b_2(\theta). \tag{10}$$

   The monotonicity condition 2. implies that we must have

$$b_1(\underline{\theta}) = b_2(\underline{\theta}) = \inf_\theta b_1(\theta) = \inf_\theta b_2(\theta).$$

   An analogues argument sows that

$$b_1\left(\overline{\theta}\right) = b_2\left(\overline{\theta}\right) = \sup_\theta b_1(\theta) = \sup_\theta b_2(\theta).$$

   We also obtain $b_1(\underline{\theta}) \neq b_1\left(\overline{\theta}\right)$ from 2.
   In addition we know that $b_1(\underline{\theta}) > v_1(\underline{\theta}, \underline{\theta})$. This is because we cannot have an atom at $b_1(\underline{\theta})$ in both agents' strategies and therefore if we had $b_1(\underline{\theta}) \leq v_1(\underline{\theta}, \underline{\theta})$ at least for one agent raising her bid by a small ε gains $\frac{1}{2}k\varepsilon$ when she sells (with probability close to one) and loses less then $\frac{1}{2}(1-k)\varepsilon$ when she buys (with arbitrarily small probability). A similar reasoning shows that $b_1\left(\overline{\theta}\right) < v_1\left(\overline{\theta}, \overline{\theta}\right)$.

5. It is only possible to have an atom at $\widetilde{b}$ (i.e. a positive measure of types bidding $\widetilde{b}$) in the bidding function of agent i if there is either a gap in the equilibrium bidding function of the other agent below or above $\widetilde{b}$ or if for $\theta_{-i} := b_{-i}^{-1}\left(\widetilde{b}\right)$ we have $E_{\theta_i}[v_{-i}(\theta_1, \theta_2) \mid b_i(\theta_i) = \widetilde{b}] = \widetilde{b}$. This is because a small change in the bid for types bidding close to $\widetilde{b}$ does hardly change payments but significantly changes the probability of winning and losing. Therefore it is profitable to increase the bid slightly above $\widetilde{b}$ instead of bidding just below $\widetilde{b}$ if the expected value for the partnership is higher than its price, i.e. if $E_{\theta_i}[v_{-i}(\theta_1, \theta_2) \mid b_i(\theta_i) = \widetilde{b}] > \widetilde{b}$, or to lower the bid from just above $\widetilde{b}$ to just below $\widetilde{b}$ if $E_{\theta_i}[v_{-i}(\theta_1, \theta_2) \mid b_i(\theta_i) = \widetilde{b}] < \widetilde{b}$.

6. I show that if there is a gap between b^* and b** in the equilibrium bidding function of agent i and we have

$$(1-k) \Pr_{\theta_i}\{b_i(\theta_i) \leq b^*\} < k \Pr_{\theta_i}\{b_i(\theta_i) \geq b^{**}\}.$$

then we must have

$$(1-k) \Pr_{\theta_{-i}}\{b_{-i}(\theta_{-i}) \leq b^*\} \geq k \Pr_{\theta_{-i}}\{b_{-i}(\theta_{-i}) \geq b^{**}\}.$$

I assume without loss in generality that $\Pr_{\theta_i}\{b_i(\theta_i) \in (b^*, b^{**})\} = 0$ and $\Pr_{\theta_i}\{b_i(\theta_i) \in (b^* - \varepsilon, b^*]\} > 0$, $\Pr_{\theta_i}\{b_i(\theta_i) \in [b^{**}, b^{**} + \varepsilon)\} > 0$ for all $\varepsilon > 0$. Note that $(1-k)\Pr_{\theta_i}\{b_i(\theta_i) \leq b^*\} < k \Pr_{\theta_i}\{b_i(\theta_i) \geq b^{**}\}$ implies that there is also a gap in the bidding function of agent $-i$ between b^* and b^{**} since a sufficiently small increase of a bid within the interval (b^*, b^{**}) of agent $-i$ leads to higher expected payments without changing the winning (and loosing) probability. Because of 3. we cannot have atoms at b^* in the equilibrium bidding strategies of both players. Therefore if we had $(1-k)\Pr_{\theta_{-i}}\{b_{-i}(\theta_{-i}) \leq b^*\} < k \Pr_{\theta_{-i}}\{b_{-i}(\theta_{-i}) \geq b^{**}\}$ at least one player could gain by increasing her bid from b^* to just below b^{**} since this leads to higher expected payments without changing the winning (and loosing) probability.
Similar arguments show that if we have

$$(1-k) \Pr_{\theta_i}\{b_i(\theta_i) \leq b^*\} > k \Pr_{\theta_i}\{b_i(\theta_i) \geq b^{**}\}$$

then we must have

$$(1-k) \Pr_{\theta_{-i}}\{b_{-i}(\theta_{-i}) \leq b^*\} \leq k \Pr_{\theta_{-i}}\{b_{-i}(\theta_{-i}) \geq b^{**}\}.$$

7. This part shows that starting from an initial bid $b_1(\underline{\theta}) = \underline{b} > v_i(\underline{\theta}, \underline{\theta})$, it is possible to uniquely continue the solution of the differential equation (9) by increasing θ til either $\theta = F^{-1}(k)$ or $b_1(\theta) = v_i(\theta, \theta)$.
Define $\theta^* = \arg\sup\{\theta < F^{-1}(k) \mid b_1(x) = b_2(x) > v_i(x, x)$ for all $x \leq \theta^*\}$ (note

that because of 4. θ^* is well defined). I show that either $\theta^* = F^{-1}(k)$ or $b_1(\theta^*) = v_i(\theta^*, \theta^*)$. If this were not the case we could find θ_ε arbitrarily close to θ^* with $\theta_\varepsilon \in (\theta^*, F^{-1}(k))$ and $v_i(\theta, \theta) < b_i(\theta) < b_{-i}(\theta)$ for all[14] $\theta \in (\theta^*, \theta_\varepsilon]$. Note that we cannot have a gap after[15] $b \leq b_i(\theta_\varepsilon)$.In addition we cannot have atoms in the equilibrium bidding functions at $b \in [b_i(\theta^*), b_i(\theta_\varepsilon)]$ if θ_ε is sufficiently close to θ^*. Assume there existed an interval $[\theta_j^D, \theta_j^U]$, $\theta_j^D \in [\theta^*, \theta_\varepsilon]$ of agent j bidding $\widetilde{b} \in [b_i(\theta^*), b_i(\theta_\varepsilon)]$. Since there are no gaps in the bidding function of $-j$ after or before $\widetilde{b}$ we have (because of 5.) for $\widetilde{\theta}_{-j} = b_{-j}^{-1}\left(\widetilde{b}\right) \leq \theta_\varepsilon$ that

$$E_{\theta_j}[v_{-j}\left(\theta_j, \widetilde{\theta}_{-j}\right) \ \mathbf{1}\left(b_j(\theta_j) = \widetilde{b}\right)] = \widetilde{b}.$$

Since we have $v_{-j}(\theta_\varepsilon, \theta_\varepsilon) < b_i(\theta_\varepsilon) < b_{-i}(\theta_\varepsilon)$ this implies $v_{-j}\left(\theta_j^U, \widetilde{\theta}_{-j}\right) > \widetilde{b}$. Therefore if θ_ε is chosen arbitrarily close to θ^* (which is possible) we have $\theta_j^U > \widetilde{\theta}_{-j}$ which implies $v_j\left(\theta_j^U, \widetilde{\theta}_{-j}\right) > v_{-j}\left(\theta_j^U, \widetilde{\theta}_{-j}\right) > \widetilde{b}$. On the other hand we have $\widetilde{\theta}_{-j} < F^{-1}(k)$ hence type θ^U of agent j wins with a probability smaller k and looses with a probability greater k and can therefore improve by raising her bid from $\widetilde{b} < v_j\left(\theta_j^U, \widetilde{\theta}_{-j}\right)$ (and winning against types close to $\widetilde{\theta}_{-j}$ where winning is profitable because of $\widetilde{b} < v_j\left(\theta_j^U, \widetilde{\theta}_{-j}\right)$). Since there are neither gaps nor atoms equilibrium in $[b_i(\theta^*), b_i(\theta_\varepsilon)]$ (9) prescribes a symmetric solution[16] for $\theta \in (\theta^*, \theta_\varepsilon)$ which is a contradiction to the definition of θ^* and this part of the proof is complete.
Since b_1, b_2 can have neither gaps nor atoms in this range (because of 3. and 6.) and are strictly increasing (because of 2.) the same holds for b_1^{-1}, b_2^{-1}.We can therefore, starting from an initial bid $b_1(\underline{\theta}) = \underline{b} > v_i(\underline{\theta}, \underline{\theta})$, uniquely continue the solution of the differential equation (9) by increasing θ til either $\theta = F^{-1}(k)$ or $b_1(\theta) = v_i(\theta, \theta)$. The same reasoning shows that starting with $b_1\left(\overline{\theta}\right) = b_2\left(\overline{\theta}\right) < v\left(\overline{\theta}, \overline{\theta}\right)$ if we decrease the type θ_i, b_1 and b_2 are uniquely determined by (9) (and therefore symmetric) as long as $\theta_i > F^{-1}(k)$and $b_i(\theta_i) < v_i(\theta_i, \theta_i)$.

8. If we can exclude for equilibrium bidding strategies $b_1(\theta_1)$, $b_2(\theta_2)$ that $b_i(\theta_i) = v_i(\theta_i, \theta_i)$ is possible if $\theta_i \neq F^{-1}(k)$ we have shown that any equilibrium is given by $b_1(\underline{\theta})$ and $b_1\left(\overline{\theta}\right)$ and the differentiable solution of (9) for $\theta_1 \neq F^{-1}(k)$. Assume without loss in generality that $b_i(\theta_i) > v_i(\theta_i, \theta_i)$ for all $\theta_i < \theta_i^*$ and we have $b_i(\theta_i^*) = v_i(\theta_i^*, \theta_i^*)$ and $\theta_i^* < F^{-1}(k)$. Arguments similar to those used in 7. show that there are neither gaps nor atoms in a small environment around $b_i(\theta_i^*)$ which implies that (9) is valid. Even though its solution is not necessarily unique any more (because $b_i(\theta_i^*) = v_i(\theta_i^*, \theta_i^*)$) we can deduce from (9) that at least for $\theta_i > \theta_i^*$

[14] Note that because of 2. bidding strategies cannot decrease in a neighborhood of θ^*.

[15] If we had a gap one of the bidders could improve by increasing her bid from b into the gap.

[16] In fact (9) is a differential equation b_1^{-1}, b_2^{-1} have to fulfill locally, if b_1^{-1}, b_2^{-1} are differentiable (a.e.). This is the case since b_1 and b_2 are strictly increasing and continuous (in the cinsidered range) and therefore the same is true for b_1^{-1}, b_2^{-1}.

and close to θ_i^* the derivatives of $b_i(\theta_i)$ are decreasing which is in contrast to 2. and therefore not possible. Again a similar argument shows that we cannot have $b_i(\theta_i^*) = v_i(\theta_i^*, \theta_i^*)$ and $\theta_i^* > F^{-1}(k)$.
Because of continuity of b_i and v we have $b_i(\theta_i) > v_i(\theta_i, \theta_i)$ for $\theta_i < F^{-1}(k)$ and $b_i(\theta_i) < v_i(\theta_i, \theta_i)$ for $\theta_i > F^{-1}(k)$ which implies (because of 2.) that $b_i(\theta_i)$ is continuous at $\theta_i = F^{-1}(k)$ and we have $b_i(F^{-1}(k)) = v_i(F^{-1}(k), F^{-1}(k))$.

9. From the previous steps we know that any equilibrium (b_1, b_2) has to be symmetric, strictly increasing and must be a solution of the symmetric system of differential equations given by (9). Furthermore an equilibrium is uniquely determined by (9) and the initial conditions $\underline{\theta} = b_i^{-1}(\underline{b})$ and $\overline{\theta} = b_i^{-1}(\overline{b})$ where $\underline{b}$ and $\overline{b}$ denote the lowest and highest bid since $v_i(\theta, \theta) = b(\theta) \Leftrightarrow \theta = F^{-1}(k)$. Therefore any equilibrium must also be a solution of the following differential equation, which is directly derived from (9) by using the symmetry property of the equilibrium:
$$(v_i(\theta, \theta) - b(\theta)) - \frac{1}{2}\frac{F(\theta) - k}{f(\theta)}\frac{db(\theta)}{d\theta} = 0.$$

   This is a linear differential equation and it is easy to verify that its solutions for $\theta \neq F^{-1}(k)$ must have the following form:
$$b(\theta) = v_i(\theta, \theta) - \frac{\int_c^\theta \frac{dv_i(\theta,\theta)}{d\theta}(F(t) - k)^2 dt}{(F(\theta) - k)^2}, \quad c \in \mathbb{R}.$$
   Since for any equilibrium we have $b(F^{-1}(k)) = v(F^{-1}(k), F^{-1}(k))$ we must have $c = F^{-1}(k)$ and therefore the only possible candidate for an equilibrium is given by 4. Checking the second order condition (which can be done by straight forward calculations) reveals that $(b_1(\theta), b_2(\theta))$ with $b_1(\theta) = b_2(\theta) = b(\theta)$ according to (4) indeed constitutes an equilibrium.

Q.E.D.

Proof of Theorem 2:
Because of Theorem 6 we know that the agents' interim utilities by participation in a mechanism that implements the efficient allocation of the partnership must have the following representations, where
$$Q(\theta_i) = \int_{\widetilde{\theta}_i}^{\theta_i} g'(t)\,\overline{s}_i(t)\,dt \quad \text{and} \quad R(\theta_i) = E_{\theta_{-i}}[v_i(\theta_i, \theta_{-i})\left(s_i\left(\widehat{\theta}_i, \theta_{-i}\right) - \frac{1}{2}\right)]$$
do not depend on payments[17]:

1. $U_i(\theta_i) = U_i\left(\widetilde{\theta}_i\right) + Q(\theta_i)$ where $\widetilde{\theta}_i$ denotes the type for which participation is most costly/ least profitable[18]

[17] Because of symmetry Q and R are independent of i.

[18] Note that $\widetilde{\theta}_i$ is the same for all efficient mechanisms. It can easily be shown that $\widetilde{\theta}_i = F^{-1}\left(\frac{1}{2}\right)$.

2. $U_i(\theta_i) = R(\theta_i) + T_i(\theta_i)$ where $T_i(\theta_i)$ are the expected payments to a type θ_i agent.

If there exists an incentive compatible, efficient, budget balanced and individually rational mechanism this mechanism satisfies

$$U_i^M\left(\widetilde{\theta}_i\right) = E_{\theta_i}[R(\theta_i) - Q(\theta_i) + T_i^M(\theta_i)] \geq 0$$

and because of budget balancedness we have $E_\theta[T_1^M(\theta_1) + T_2^M(\theta_2)] \leq 0$ and therefore

$$U_1^M\left(\widetilde{\theta}_1\right) + U_2^M\left(\widetilde{\theta}_2\right) \leq 2E_{\theta_i}[R(\theta_i) - Q(\theta_i)].$$

Since the $k-$double auction is budget balanced and efficient (as a result of Theorem 1) the interim utilities of the "worst-off" types $U_1^{DA}\left(\widetilde{\theta}_1\right) = U_2^{DA}\left(\widetilde{\theta}_2\right)$in the double auction must satisfy:

$$\begin{aligned} U_1^{DA}\left(\widetilde{\theta}_1\right) + U_2^{DA}\left(\widetilde{\theta}_2\right) &= E_\theta[2R(\theta_1) - 2Q(\theta_1) + T_1^M(\theta_1) + T_2^M(\theta_2)] \\ &= 2E_{\theta_i}[R(\theta_1) - Q(\theta_1)] \\ &\geq U_1^M\left(\widetilde{\theta}_1\right) + U_2^M\left(\widetilde{\theta}_2\right) \geq 0. \end{aligned}$$

Q.E.D.

Proof of Theorem 4:

Instead of directly verifying that a deviation of the given strategy cannot be profitable if the other agent sticks to it, I use the Revenue-Equivalence-Theorem (Theorem 6) for an indirect proof. Given an allocation rule s, the Revenue-Equivalence-Theorem determines (up to a type-independent constant) the payments (depending on the agents' reported types) necessary and sufficient to implement s in a truthtelling equilibrium. By the revelation principle in any (indirect) mechanism that implements s the expected payments to agents in equilibrium have to equal those given by the Revenue-Equivalence-Theorem (up to a type independent constant). Furthermore if the expected payments to agents induced by a candidate of an equilibrium (i.e. (7)) of an indirect mechanism that implements s equal those given by the Revenue-Equivalence-Theorem we know that imitating the strategy of a different type cannot be profitable. Therefore I have to show that the payments induced by the given strategies of the double-auction with veto equal those of a direct mechanism that implements the same allocation as the suggested equilibrium strategies. If in addition I can show that deviating to a bid outside the range of (7) cannot be profitable these have to constitute an equilibrium.

I split the proof in four steps:

1. I show that the condition

$$F(c) + F(d) = 1$$

is necessary for the induced allocation to result from equilibrium bidding behavior.

2. For general $c, d \in [\underline{\theta}, \overline{\theta}]$ with $F(c) + F(d) = 1$ I calculate the expected payments of a direct mechanism that implements the allocation that would result from bidding according to (7).
3. For general $c, d \in [\underline{\theta}, \overline{\theta}]$ with $F(c) + F(d) = 1$ I calculate the expected payments induced by (7) and show that these equal the payments derived in step 3. if (and only if) (6) holds.
4. I show that no type has an incentive to bid outside the range of $b(\theta_i)$ defined by (7).

Step 1:
If the agents would bid according to (7) this would result in the following allocation:

$$s_{i,c,d}(\theta) := \begin{cases} 1 & if \quad \theta_i > \theta_{-i} \text{ and } \theta_i, \theta_{-i} \notin [c,d] \\ \frac{1}{2} & if \quad \theta_i \in [c,d],\ \theta_{-i} \in [c,d] \\ 0 & if \quad \theta_i \leq \theta_{-i} \text{ and } \theta_i, \theta_{-i} \notin [c,d], \end{cases} \qquad i = 1, 2.$$

Because of the Revenue-Equivalence-Theorem this allocation can only be implemented if

$$\overline{s}_{i,c,d}(\theta_i) = \int_{\underline{\theta}}^{\overline{\theta}} \left(s_{i,c,d}(\theta_i, \theta_{-i}) - \frac{1}{2} \right) f(\theta_{-i})\, d\theta_{-i}$$

is increasing in θ_i. This is the case iff $F(c) + F(d) = 1$. To see this note that for $\theta_i \in [\underline{\theta}, c]$ we have

$$\begin{aligned} & \int_{\underline{\theta}}^{\overline{\theta}} \left(s_{i,c,d}(\theta_i, \theta_{-i}) - \frac{1}{2} \right) f(\theta_{-i})\, d\theta_{-i} \\ = \;& F(\theta_i) + \frac{1}{2}(F(d) - F(c)) - \frac{1}{2}. \end{aligned}$$

Similarly for $\theta_i \in [d, 1]$ we have

$$\begin{aligned} & \int_{\underline{\theta}}^{\overline{\theta}} \left(s_{i,c,d}(\theta_i, \theta_{-i}) - \frac{1}{2} \right) f(\theta_{-i})\, d\theta_{-i} \\ = \;& F(\theta_i) - \frac{1}{2}(F(d) - F(c)) - \frac{1}{2}. \end{aligned}$$

Therefore $\overline{k}_{i,c,d}(\theta_i)$ is increasing iff

$$\begin{aligned} F(\theta_i) + \frac{1}{2}(F(d) - F(c)) - \frac{1}{2} &\leq 0 \quad \forall \theta_i \leq c \Leftrightarrow F(c) + F(d) \leq 1 \\ F(\theta_i) - \frac{1}{2}(F(d) - F(c)) - \frac{1}{2} &\geq 0 \quad \forall \theta_i \geq d \Leftrightarrow F(c) + F(d) \geq 1 \end{aligned}$$

Which means that $F(c)+F(d)=1$.

Step 2: Because of step 1 and Theorem 6 (s,t) is IC iff we have for an arbitrary type $\widetilde{\theta}_i$:

$$U_i(\theta_i)=U_i\left(\widetilde{\theta}_i\right)+\int_{\widetilde{\theta}_i}^{\theta_i} g'(t)\,\overline{s}_{i,c,d}(t)\,dt.$$

Note that this implies that all agents of type $\theta_i \in [c,d]$ must get the same interim utility, which we denote by K. Because of

$$T_i(\theta_i)=U(\theta_i)-E_{\theta_{-i}}\left[(g(\theta_i)+h(\theta_{-i}))\left(s_{i,c,d}(\theta_i,\theta_{-i})-\frac{1}{2}\right)\right]$$

it follows immediately that $T_i(\theta_i)=K$ if $\theta_i \in [c,d]$. If $\theta_i \in [\underline{\theta},c]\ \cup[d,\overline{\theta}]$ straight forward calculations result in

$$\begin{aligned}T_i(\theta_i) &= K-(g(\theta_i)+h(\theta_i))F(\theta_i)+\int_c^{\theta_1} g'(t)F(t)\,dt+\frac{1}{2}\int_c^{\theta_i} h'(t)F(t)\,dt \\ &\quad+\frac{1}{2}\int_{\underline{\theta}}^{\theta_1} h'(t)F(t)\,dt-\frac{1}{2}\int_d^{\overline{\theta}} h'(t)F(t)\,dt \\ &\quad+\frac{1}{2}h\left(\overline{\theta}\right)-\frac{1}{2}h(d)F(d)+g(c)F(c)+\frac{1}{2}h(c)F(c),\end{aligned} \tag{11}$$

if $\theta_i \in [c,d]$ and if $\theta_i \in [d,\overline{\theta}]$ then we get

$$\begin{aligned}T_i(\theta_i) &= K-(g(\theta_i)+h(\theta_i))F(\theta_i)+\int_d^{\theta_1} g'(t)F(t)\,dt+\frac{1}{2}\int_{\underline{\theta}}^{c} h'(t)F(t)\,dt \\ &\quad+\frac{1}{2}\int_d^{\theta_1} h'(t)F(t)\,dt-\frac{1}{2}\int_{\theta_1}^{\overline{\theta}} h'(t)F(t)\,dt \\ &\quad+\frac{1}{2}h\left(\overline{\theta}\right)+\frac{1}{2}h(d)F(d)+g(d)F(d)-\frac{1}{2}h(c)F(c).\end{aligned} \tag{12}$$

Step 3: We have to check whether the expected payments induced by bidding according to $b_i(\theta_i)$ defined by (7) equal those derived in the previous step. If this is the case we know that no agent can profit by deviating to another bid in the range of the given bidding function or by vetoing in case he has not vetoed before.

First note that $b(\theta_i)$ is strictly increasing in $\theta_i \in [\underline{\theta},c)\cup(d,\overline{\theta}]$. Using l'Hôpital's rule we get

$$\lim_{\theta_i\to c} b_i(\theta_i)=g(c)+h(c)$$

and

$$\lim_{\theta_i\to d} b_i(\theta_i)=g(d)+h(d).$$

In a next step I show that the expected payments to the agents equal those derived in step 2. For $\theta_i \in [c, d]$ this is obviously the case iff $K = 0$. Consider first the case $\theta_i \in [\underline{\theta}, c)$. Given the rules of the auction we have:

$$\begin{aligned}
T_i(\theta_i) &= -\int_{\underline{\theta}}^{\theta_i} \frac{b(\theta_i)+b(\theta_{-i})}{4} f(\theta_{-i})\, d\theta_{-i} + \int_{\theta_i}^{c} \frac{b(\theta_i)+b(\theta_{-i})}{4} f(\theta_{-i})\, d\theta_{-i} \\
&\quad + \int_{d}^{\overline{\theta}} \frac{b(\theta_i)+b(\theta_{-i})}{4} f(\theta_{-i})\, d\theta_{-i} \\
&= -\frac{1}{2}(g(\theta_i)+h(\theta_i))F(\theta_i) + \frac{1}{2}(g(\theta_i)+h(\theta_i))F(c) \\
&\quad + \frac{1}{2}\frac{\int_c^{\theta_i}(g'(t)+h'(t))(F(t)-F(c))^2\, dt}{F(\theta_i)-F(c)} \\
&\quad - \frac{1}{4}\int_{\underline{\theta}}^{\theta_i}(g(t)+h(t))f(t)\, dt + \frac{1}{4}\int_{\theta_i}^{c}(g(t)+h(t))f(t)\, dt \\
&\quad + \frac{1}{4}\int_{d}^{\overline{\theta}}(g(t)+h(t))f(t)\, dt \\
&\quad + \frac{1}{4}\int_{\underline{\theta}}^{\theta_i}\frac{\int_c^{\theta_{-i}}(g'(t)+h'(t))(F(t)-F(c))^2\, dt}{(F(\theta_{-i})-F(c))^2} f(\theta_{-i})\, d\theta_{-i} \\
&\quad - \frac{1}{4}\int_{\theta_i}^{c}\frac{\int_c^{\theta_{-i}}(g'(t)+h'(t))(F(t)-F(c))^2\, dt}{(F(\theta_{-i})-F(c))^2} f(\theta_{-i})\, d\theta_{-i} \\
&\quad - \frac{1}{4}\int_{d}^{\overline{\theta}}\frac{\int_d^{\theta_{-i}}(g'(t)+h'(t))(F(t)-F(c))^2\, dt}{(F(\theta_{-i})-F(d))^2} f(\theta_{-i})\, d\theta_{-i} \Bigg)
\end{aligned}$$

Integration by parts and using the fact that

$$\lim_{\theta_i \to c} \frac{\int_c^{\theta_i}(g'(t)+h'(t))(F(t)-F(c))^2\, dt}{F(\theta_i)-F(c)} = \lim_{\theta_i \to c} \frac{(g'(\theta_i)+h'(\theta_i))(F(\theta_i)-F(c))^2\, dt}{f(\theta_i)} = 0$$

gives:

$$\begin{aligned}
T_i(\theta_i) &= -(g(\theta_i)+h(\theta_i))F(\theta_i) + \int_c^{\theta_i} g'(t)F(t)\, dt + \frac{1}{2}\int_c^{\theta_i} h'(t)F(t)\, dt \\
&\quad + \frac{1}{2}\int_{\underline{\theta}}^{\theta_i} h'(t)F(t)\, dt + \frac{1}{2}\int_{\underline{\theta}}^{c} g'(t)F(t)\, dt \\
&\quad + \frac{1}{4}(g(\underline{\theta})+h(\underline{\theta}))F(c) + \frac{1}{2}(g(c)+h(c))F(c) \\
&\quad - \frac{1}{4}\frac{\int_c^{\underline{\theta}}(g'(t)+h'(t))(F(t)-F(c))^2\, dt}{F(c)} + \frac{1}{4}\int_d^{\overline{\theta}}(g(t)+h(t))f(t)\, dt \\
&\quad + \frac{1}{4}\frac{\int_d^{\overline{\theta}}(g'(t)+h'(t))(F(t)-F(d))^2\, dt}{F(c)} - \frac{1}{4}\int_d^{\overline{\theta}}(g'(t)+h'(t))(F(t)-F(d))\, dt
\end{aligned}$$

This equals the expected payments given in (11) iff

$$\begin{aligned}
& \frac{1}{2}\int_{\underline{\theta}}^{c} g'(t)\,F(t)\,dt+\frac{1}{4}\left(g(\underline{\theta})+h(\underline{\theta})\right)F(c)+\frac{1}{2}\left(g(c)+h(c)\right)F(c) \\
& -\frac{1}{4}\frac{\int_{c}^{\underline{\theta}}\left(g'(t)+h'(t)\right)\left(F(t)-F(c)\right)^{2}dt}{F(c)}+\frac{1}{4}\int_{d}^{\overline{\theta}}\left(g(t)+h(t)\right)f(t)\,dt \\
& +\frac{1}{4}\frac{\int_{d}^{\overline{\theta}}\left(g'(t)+h'(t)\right)\left(F(t)-F(d)\right)^{2}dt}{F(c)}-\frac{1}{4}\int_{d}^{\overline{\theta}}\left(g'(t)-h'(t)\right)\left(F(t)-F(d)\right)dt \\
= \; & \frac{1}{2}\int_{d}^{\overline{\theta}} h(t)\,f(t)\,dt+g(c)\,F(c)+\frac{1}{2}h(c)\,F(c) \\
\Leftrightarrow \; & \frac{1}{4}\frac{\int_{c}^{\underline{\theta}}\left(g'(t)+h'(t)\right)\left(F(t)-F(c)\right)^{2}dt}{F(c)}-\frac{1}{4}\frac{\int_{d}^{\overline{\theta}}\left(g'(t)+h'(t)\right)\left(F(t)-F(d)\right)^{2}dt}{F(c)} \\
& -\frac{1}{2}\int_{\underline{\theta}}^{c} g'(t)\,F(t)\,dt+\frac{1}{2}\int_{d}^{\overline{\theta}} g'(t)\,F(t)\,dt \\
& -\frac{1}{4}\left(\left(g(\underline{\theta})+h(\underline{\theta})\right)F(c)+\left(g(\overline{\theta})+h(\overline{\theta})\right)F(d)\right) \\
& -\frac{1}{4}\left(g(\overline{\theta})-h(\overline{\theta})\right)+\frac{1}{2}g(d)\,F(d)+\frac{1}{2}g(c)\,F(c) \\
= \; & 0. \qquad (13)
\end{aligned}$$

Similar calculations reveal that the expected payments a player of type $\theta_i \in (d,\overline{\theta}]$ can expect by participating in the auction equals the expected payments given by (12) under the same condition. Therefore the expected payments in the double auction with veto equal those derived in the previous step iff (13) holds which is equivalent to (6).

Step 5: It remains to show that no type has an incentive to change his bid to a number out of the set: $(-\infty, b(\underline{\theta})) \cup [b(c), b(d)] \cup (b(\overline{\theta}), \infty)$ (I define $b(c) := \lim_{\theta\to c} b(c) = g(c)+h(c)$and $b(d) := \lim_{\theta\to d} b(d) = g(d)+h(d)$). A bidder would always prefer $b(\underline{\theta})$ to any bid in $(-\infty, b(\underline{\theta}))$ because in either case he never gets the partnership but he receives more money if he bids $b(\underline{\theta})$ instead of bidding a number in $(-\infty, b(\underline{\theta}))$. For a similar reason he would never bid a number in $(b(\overline{\theta}), \infty)$. To see why it is never profitable to bid $\widetilde{b} \in [b(c), b(d)]$ note first that the utility of a bidder having type θ_i and bidding $\widetilde{b} \in [b(c), b(d)]$ gives him utility

$$U_i\left(\theta_i,\widetilde{b}\right)=\frac{1}{2}\int_{\underline{\theta}}^{c}\left(h(\theta_{-i})-\frac{b(\theta_{-i})}{2}\right)f(\theta_{-i})\,d\theta_{-i}-\frac{1}{2}\int_{d}^{\overline{\theta}}\left(h(\theta_{-i})-\frac{b(\theta_{-i})}{2}\right)$$

which does not depend on $\widetilde{b}$ as long as $\widetilde{b} \in [b(c), b(d)]$. On the other hand we know from the calculations above, that the bidder has no incentive to deviate to bidding $b(c)$ or $b(d)$ (because the above calculations do not use the fact that types c and d veto instead of bidding $b(c)$ and $b(d)$). Therefore he has no incentive to bid $\widetilde{b} \in [b(c), b(d)]$.

Q.E.D.

Proof of Theorem 5:
It has to be shown that there always exists a solution $c, d \in (\underline{\theta}, \overline{\theta})$ to the equations:

$$\begin{aligned}
1 &= F(c) + F(d) \\
0 &= \frac{1}{2F(c)} \int_{\underline{\theta}}^{c} (g(t) + h(t))(F(t) - F(c)) f(t)\,dt \\
&\quad + \frac{1}{2F(c)} \int_{d}^{\overline{\theta}} (g(t) + h(t))(F(t) - F(d)) f(t)\,dt \\
&\quad + \frac{1}{2} \int_{\underline{\theta}}^{c} g(t) f(t)\,dt - \frac{1}{2} \int_{d}^{\overline{\theta}} g(t) f(t)\,dt
\end{aligned}$$

Because of the strict monotonicity of F we can combine these equations to:

$$\begin{aligned}
Q(c) \quad : &= \frac{1}{2F(c)} \int_{\underline{\theta}}^{c} (g(t) + h(t))(F(t) - F(c)) f(t)\,dt \\
&\quad + \frac{1}{2F(c)} \int_{F^{-1}(1-F(c))}^{\overline{\theta}} (g(t) + h(t))(F(t) - 1 + F(c)) f(t)\,dt \\
&\quad + \frac{1}{2} \int_{\underline{\theta}}^{c} g(t) f(t)\,dt - \frac{1}{2} \int_{F^{-1}(1-F(c))}^{\overline{\theta}} g(t) f(t)\,dt \\
&= 0
\end{aligned}$$

As a next step I calculate the value of $Q\left(F^{-1}\left(\frac{1}{2}\right)\right)$.

$$\begin{aligned}
Q\left(F^{-1}\left(\frac{1}{2}\right)\right) &= \int_{\underline{\theta}}^{\overline{\theta}} (g(t) + h(t))\left(F(t) - \frac{1}{2}\right) f(t)\,dt + \frac{1}{2} \int_{\underline{\theta}}^{F^{-1}\left(\frac{1}{2}\right)} g(t) f(t)\,dt \\
&\quad - \frac{1}{2} \int_{F^{-1}\left(\frac{1}{2}\right)}^{\overline{\theta}} g(t) f(t)\,dt \\
&= \int_{\underline{\theta}}^{\overline{\theta}} g(t) F(t) f(t)\,dt - \int_{F^{-1}\left(\frac{1}{2}\right)}^{\overline{\theta}} g(t) f(t)\,dt - \frac{1}{2} \int_{\underline{\theta}}^{\overline{\theta}} h(t) f(t)\,dt \\
&\quad + \int_{\underline{\theta}}^{\overline{\theta}} h(t) F(t) f(t)\,dt \\
&= \int_{\underline{\theta}}^{\overline{\theta}} g(t) F(t) f(t)\,dt - \int_{F^{-1}\left(\frac{1}{2}\right)}^{\overline{\theta}} g(t) f(t)\,dt + \frac{1}{2} \int_{\underline{\theta}}^{\overline{\theta}} h'(t) F(t)\,dt \\
&\quad - \frac{1}{2} \int_{\underline{\theta}}^{\overline{\theta}} h'(t) F^2(t)\,dt
\end{aligned}$$

Because I assumed the existence condition in Theorem 3 not to hold we have

$$Q\left(F^{-1}\left(\frac{1}{2}\right)\right) > 0.$$

On the other hand we have $Q(\underline{\theta}) = 0$. Because of the continuity of $Q(c)$ we have proved the statement if we can show that for an arbitrary $\varepsilon > 0$ we have $Q'(c) < 0$ for $c \in (\underline{\theta}, \varepsilon)$. Using

$$\frac{dF^{-1}(1 - F(c))}{dc} = -\frac{f(c)}{f(F^{-1}(1 - F(c)))}$$

we get

$$\begin{aligned}
Q'(c) &= -\frac{f(c)\int_{\underline{\theta}}^{c}(g(t) + h(t))F(t)f(t)\,dt}{2F^2(c)} \\
&\quad -\frac{f(c)\int_{F^{-1}(1-F(c))}^{\overline{\theta}}(g(t) + h(t))(F(t) - 1)f(t)\,dt}{2F^2(c)} \\
&\quad +\frac{1}{2}g(c)f(c) - \frac{1}{2}g\left(F^{-1}(1 - F(c))\right)f(c)
\end{aligned}$$

and hence using l'Hôpital's rule (for which we need that f'' exists):

$$\begin{aligned}
\lim_{c \to \underline{\theta}} Q'(c) &= -\frac{1}{4}(g(\underline{\theta}) + h(\underline{\theta}))f(\underline{\theta}) + \frac{1}{4}\left(g(\overline{\theta}) + h(\overline{\theta})\right)f(\underline{\theta}) \\
&\quad +\frac{1}{2}g(\underline{\theta})f(\underline{\theta}) - \frac{1}{2}g(\overline{\theta})f(\underline{\theta}) \\
&= \frac{1}{4}f(\underline{\theta})\left[\left(g(\underline{\theta}) - g(\overline{\theta})\right) - \left(h(\underline{\theta}) - h(\overline{\theta})\right)\right] \\
&< 0
\end{aligned}$$

where the last inequality results because of the assumption that $g' > h'$.

Q.E.D.

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