Competitive Search Markets with Adverse Selection*

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Abstract

In a seminal paper, Rothschild and Stiglitz (1976) show that competitive markets with incomplete information in which firms offer contracts to screen privately informed agents may have no equilibrium. In this paper, we argue that frictions in the form of delay or congestion provide a natural solution to the nonexistence problem. To show this, we extend the concept of competitive search equilibrium by Moen (1997) to markets with incomplete information. Our main result is that a separating equilibrium always exists. In particular, the separating equilibrium cannot be broken by a profitable pooling offer as the latter attracts only the lowest types in the population due to the ensuing congestion.

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1 Introduction

Since the seminal work by Rothschild and Stiglitz (1976), it is known that competitive markets with incomplete information in which firms offer contracts to screen privately informed agents may have no equilibrium. While a pooling equilibrium can always be broken by an offer that attracts only the highest types in the population, a separating equilibrium may not be viable because it can be broken by a pooling deviation if the proportion of high types in the population is sufficiently high. Several authors have suggested that the "flaw" in Rotschild and Stiglitz's analysis lies in their notion of competitive equilibrium. In particular, if firms are allowed to react to a deviating offer either by making an additional offer (Riley 1979), or by withdrawing previous offers (Wilson 1977), the nonexistence problem vanishes.[1]

In this paper, we argue that frictions such as delay or congestion constitute a natural solution to the nonexistence problem. To show this, we extend the concept of competitive search equilibrium by Moen (1997) to markets with incomplete information. Moen models a competitive labor market in which the "grand" market is divided into various submarkets. Each submarket consists of a prevailing wage, a subset of unemployed workers searching for a job, and firms with vacancies searching for job applicants. Search is costly to both firms and workers. When deciding whether to enter a particular submarket, agents compare the prevailing wage with the expected search cost, which depends on the ratio of searching workers to vacancies (or "labor market tightness") in the respective submarket. An equilibrium is characterized by two conditions: i) agents must choose optimally between submarkets, and ii) it must not be worthwile to open up new submarkets in addition to the already existing submarkets.

Unlike Moen (1997), we assume that workers engaging in job search are privately informed about their ability. Submarkets are now characterized by contracts, which consist of a wage and an additional screening variable (e.g. education). In any particular submarket, all firms offer the same contract. We show that a pooling equilibrium in which a submarket attracts different types of workers never exists. Moreover, we show that there always exists a fully separating equilibrium in which workers of different ability enter different submarkets. In this equilibrium, high-ability workers choose higher education levels and receive higher wages than low-ability workers. Most importantly, however, the separating equilibrium cannot be broken by a pooling deviation due to the ensuing congestion (or "delay") prevailing at the pooling offer.

As an illustration, consider the case where there are only two types of workers. The critical issue in the model by Rothschild and Stiglitz is that a (least-cost) separating

[1] For a detailed analysis and further references, see Mas-Colell, Whinston, and Green (1995), Ch. 13.

equilibrium can be broken by a profitable pooling deviation if the fraction of high-type workers in the population is sufficiently high.[2] Implicitly, this assumes that the pooling offer attracts both high- and low-type workers in proportion to the population. In a model with frictions where the number of vacancies per firm is limited, this need not be the case though. To see why, consider an equilibrium that is least-cost separating and suppose that a small fraction of firms deviates by making a pooling offer. When deciding whether to follow the offer, workers trade off the net gains (wage minus cost of education) against the expected congestion prevailing at this offer. Since low-ability workers gain relatively more from the pooling offer than high-ability workers, they are willing to accept a higher congestion and thus a greater delay. As more and more low-ability workers follow the offer, the congestion rises until at some level low-ability workers are indifferent between the pooling offer and the original equilibrium allocation. At this congestion level, however, high-ability workers strictly prefer the original equilibrium allocation, which implies that the pooling offer is not viable.

The rest of the paper is organized as follows. Section 2 extends the concept of competitve search equilibrium to incomplete information. Section 3 contains the main result. All proofs are provided in the Appendix.

2 The Model

2.1 General Framework

This section extends the concept of competitive search equilibrium to incomplete information. To accommodate a larger contracting space, our notation differs occasionally from that in Moen (1997). Consider a labor market with a continuum of workers and firms. There is a fixed measure of workers who are privately informed about their ability. Workers can be of different types, which are denoted by subscripts $i \in I = \{1, ..., I\}$. The measure of workers with type i is denoted by $\mu_i^0 > 0$, where $\sum_{i \in I} \mu_i^0 = 1$. The measure of vacancies is determined endogenously by free entry. When opening a vacancy, firms incur a sunk cost $k \geq 0$. Labor contracts between workers and firms consist of a wage w and a screening variable $y \geq 0$, which is purely dissipative. For instance, y may represent the level of education of an individual worker. The model takes place in continuous time. The flow utility of a worker of type i from a contract $\omega = (w, y)$ is $u_i(\omega) = w - y/i$, and the corresponding flow utility of a firm employing a worker

[2] In a least-cost separating equilibrium, the contract chosen to attract a particular type maximizes the utility of that type subject to the incentive compatibility constraints of the lower types and the firm's zero-profit condition. Any separating equilibrium in the sense of Rothschild and Stiglitz is least-cost separating.

of type i is $v_i(\omega) = i - w$. Note that the specification of the utility functions is only for convenience. All our results continue to hold under more general utility functions if $v_i(\omega)$ is strictly increasing in i and $u_i(\omega)$ satisfies the single-crossing condition, which states that $u_i(\omega) \geq u_i(\omega')$ implies $u_j(\omega) > u_j(\omega')$ whenever $j > i$ and $y > y'$.

2.2 Matching Technology

The matching technology is identical to Moen (1997). Contracts between workers and firms are traded in a search market where unemployed workers are matched with firms offering a vacancy. Denote the measure of vacancies by F and the measure of unemployed workers of type i by L_i, where $L = \sum_{i \in I} L_i$. Accordingly, the labor market tightness is $\theta = F/L$, and the probability that an unemployed worker is of type i is $\mu_i = L_i/L$. The measure of worker-firm matchings per unit of time is expressed by the matching function $x(F, L)$, which is continuous and homogeneous of degree one in both arguments. The matching function captures the frictions in the labor market, which can be thought of as the costs and time delays due to the processing of applications. Let $p(\theta) = x(F, L)/L = x(\theta, 1)$ denote the transition rate from unemployment to employment for an unemployed worker, and $q(\theta) = x(F, L)/F = x(1, 1/\theta)$ the arrival rate of workers for a vacancy. Both transition rates are strictly monotonic with limits $\lim_{\theta \to 0} p(\theta) = \lim_{\theta \to \infty} q(\theta) = 0$, and $\lim_{\theta \to \infty} p(\theta) = \lim_{\theta \to 0} q(\theta) = \infty$. Moreover, each match faces a constant probability rate s of job destruction. The search market is thus fully characterized by the prevailing contract ω, the labor market tightness θ, and the distribution of types $\mu = \{\mu_i\}_{i \in I}$.

2.3 Asset Value Equations

Denote by $U_i(\omega, \theta)$ the utility of an unemployed worker of type i who is seeking employment. The asset value equation for $U_i(\omega, \theta)$ is

$$rU_i(\omega, \theta) = z + p(\theta)\left[E_i(\omega, \theta) - U_i(\omega, \theta)\right], \tag{1}$$

where $z \geq 0$ is the unemployment income, $r \geq 0$ is the discount rate, and $E_i(\omega, \theta)$ is the expected income if the worker is employed under the contract ω. The expected income satisfies

$$rE_i(\omega, \theta) = u_i(\omega) - s\left[E_i(\omega, \theta) - U_i(\omega, \theta)\right]. \tag{2}$$

Inserting (2) in (1) gives

$$rU_i(\omega, \theta) = \frac{(r + s)z + p(\theta)u_i(\omega)}{r + s + p(\theta)}. \tag{3}$$

Observe that (3) is only valid if $u_i(\omega) \geq z$. If $u_i(\omega) < z$, workers do not search and their expected income is z/r. Similarly, the value of a vacancy $V(\mu, \omega, \theta)$ is determined by the asset value equation

$$rV(\mu, \omega, \theta) = -c + q(\theta) [J(\mu, \omega, \theta) - V(\mu, \omega, \theta)], \tag{4}$$

where $c > 0$ denotes a constant search cost, and $J(\mu, \omega, \theta)$ denotes the asset value of a filled vacancy. The corresponding asset value equation for $J(\mu, \omega, \theta)$ is

$$rJ(\mu, \omega, \theta) = \sum_{i \in I} \mu_i v_i(\omega) - sJ(\mu, \omega, \theta). \tag{5}$$

Substituting (5) in (4), we obtain

$$[r + q(\theta)] V(\mu, \omega, \theta) = q(\theta) \frac{\sum_{i \in I} \mu_i v_i(\omega)}{r + s} - c. \tag{6}$$

Observe the similarity between (1)-(6) and the asset value equations in Moen (1997).

2.4 Equilibrium Conditions

The "grand" market is divided into N submarkets, where each submarket constitutes an independent search environment. Submarkets are denoted by superscripts $n \in N = \{1, 2, ..., N\}$. The measures of firms and unemployed workers of type i in submarket n are F^n and L_i^n, respectively, and the "characteristics" of submarket n are the distribution of types μ^n, the prevailing labor contract ω^n, and the labor market tightness θ^n.[3] Our equilibrium conditions extend those presented in Moen (1997) to accommodate incomplete information. First, we require that in equilibrium the decision of agents to enter any particular submarket must be optimal. In particular, this implies that the value of a vacancy in each submarket must equal the cost of opening a vacancy k. For convenience, define $U_i^n = U_i(\omega^n, \theta^n)$, $V^n = V(\mu^n, \omega^n, \theta^n)$, and $U_i = \max_{n \in N} U_i^n$.

(E.1) Optimal Choice of Submarket: *i)* $V^n = k$ *for all* $n \in N$, *and ii)* $U_i^n = U_i$ $(U_i^n \leq U_i)$ *if* $\mu_i^n > 0$ $(\mu_i^n = 0)$ *for all* $n \in N$, $i \in I$.

Second, we require that under any equilibrium set of submarkets, firms cannot be made better off if a new submarket is opened up in addition to the already existing submarkets. Whether firms are better off under a new submarket depends on both the labor market tightness θ and the distribution of types μ prevailing at this submarket. Given a

[3] In principle, one can think of a larger space of submarket characteristics in which submarkets are characterized by menues of contracts $\Omega^n \{\omega_1^n, ..., \omega_I^n\}$ instead of single contracts ω^n. As can be shown, however, the restriction to single contracts is without loss of generality.

new ("deviating") contract $\omega \notin \{\omega^n \mid n \in N\}$ and a set of equilibrium utilities $\{U_i\}_{i\in I}$, denote the set of pairs (θ, μ) that are "consistent" with $(\omega, \{U_i\}_{i\in I})$ by $\Gamma(\omega, \{U_i\}_{i\in I})$. By "consistent", we mean any pair of labor market tightness and distribution of types prevailing at the new submarket satisfying $U_i(\omega, \theta) = U_i$ if $\mu_i > 0$ and $U_i(\omega, \theta) \leq U_i$ if $\mu_i = 0$, given that the new contract ω satisfies $\max_{i\in I}[u_i(\omega) - rU_i] > 0$. In words, if a particular type of worker is attracted by the new submarket, we require that in equilibrium he must be indifferent between the new submarket and the set of already existing submarkets. Conversely, any type of worker that is not attracted by the new submarket must weakly prefer the set of existing submarkets to the new submarket. Observe that any new contract ω satisfying $\max_{i\in I}[u_i(\omega) - rU_i] \leq 0$ can be safely ruled out. For convenience, we specify for these contracts that $\theta = \infty$.

(E.2) Competitiveness: *Given an equilibrium set of submarkets with corresponding utilities $\{U_i\}_{i\in I}$, there must not exist a contract $\omega \notin \{\omega^n \mid n \in N\}$ such that $V(\mu, \omega, \theta) > 0$ for some $(\theta, \mu) \in \Gamma(\omega, \{U_i\}_{i\in I})$.*

3 Equilibrium Analysis

This section contains our main result. For any given utility level $U \geq z/r$ and any type of worker i, define the function $\theta_i(\omega, U)$ by

$$p(\theta_i(\omega, U)) = \begin{cases} \frac{rU-z}{u_i(\omega)-rU}(r+s) & \text{for } u_i(\omega) - rU > 0 \\ \infty & \text{otherwise.} \end{cases} \tag{7}$$

The function $\theta_i(\omega, U)$ specifies how the labor market tightness must adjust when the utility of a worker of type i is to be held constant at some given utility level U while ω varies. Note that $\theta_i(\omega, U)$ is uniquely defined for $u_i(\omega) - rU > 0$ and continuous in both arguments. Next, let us define the program $P_i^*(U)$ which solves $\max_\omega V(\mu, \omega, \theta)$ for the case where $\mu_i = 1$ and $\theta = \theta_i(\omega, U)$. Since $P_i^*(U)$ has no incentive compatibility constraints, any solution to $P_i^*(U)$ must have $y = 0$. Finally, it follows from our specification of the utility functions that we can restrict attention to a compact subset of contracts. Continuity of $V(\mu, \omega, \theta)$ in ω then implies that a solution always exists. Moreover, at any optimal solution, $u_i(\omega)$ must be bounded away from rU as $c > 0$. The following assumption ensures that the gains from employing the lowest type of worker are sufficiently high, i.e. that $V_1^*(z/r) > k$, where $V_i^*(\cdot)$ denotes the maximum value function associated with $P_i^*(\cdot)$.

(A.1) $1/(r+s) > k$.

We now show that any equilibrium must be fully separating, i.e. each submarket must attract exactly one type of worker. Intuitively, if some submarket attracted more than one type of worker, a new submarket could be constructed which skims off the highest type in the pooling submarket and which is profitable in the sense of (E.2).

Lemma 1: *Any equilibrium must be fully separating, i.e. for each submarket $n \in N$ there exists a type of worker $i \in I$ such that $\mu_i^n = 1$ and $\mu_j^n = 0$ for $j \neq i$.*

To derive our main result, we must construct a new set of programs which takes into account incentive compatibility. Hence, for any $i > 1$ and any two values $\tilde{U}_i, \tilde{U}_{i-1} \geq 0$, let us define the program $P_i^S(\tilde{U}_i, \tilde{U}_{i-1})$ which solves $\max_{\omega} V(\mu, \omega, \theta)$ subject to the incentive compatibility constraint of the adjacent lower type $U_{i-1}(\omega, \theta) \leq \tilde{U}_{i-1}$ for the case where $\mu_i = 1$ and $\theta = \theta_i(\omega, \tilde{U}_i)$. First, observe that the set of contracts satisfying the constraint is not empty. To see this, consider an arbitrary contract ω where $w - y/i > r\tilde{U}_i$. By the single-crossing condition, we can vary ω by moving along the indifference curve of type i such that the constraint is satisfied at some $y' > y$ and $w' = w + (y' - y)/i$. Again, it follows from our specification of the utility functions that we can restrict attention to a compact subset of contracts. Together with the fact that $V(\mu, \omega, \theta)$ is continuous in ω, this then implies that a solution to $P_i^S(\tilde{U}_i, \tilde{U}_{i-1})$ exists. Additionally, at any optimal solution, $u_i(\omega)$ must be bounded away from $r\tilde{U}_i$ as c is strictly positive. Denote the maximum value function associated with $P_i^S(\cdot)$ by $V_i^S(\cdot)$.

Proposition 1: *An equilibrium always exists. By Lemma 1, this equilibrium is fully separating. Moreover, the equilibrium set of submarkets has the following properties:*

i) Type $i = 1$: For any $n \in N$ with $\mu_1^n = 1$, the contract ω^n specifies $y^n = 0$ and solves the program $P_1^*(U_1)$, where U_1 is uniquely determined by $V_1^*(U_1) = k$.

ii) Type $i > 1$: For any $n \in N$ with $\mu_i^n = 1$, the contract ω^n solves the program $P_i^S(U_i, U_{i-1})$, where U_i is uniquely determined by $V_i^S(U_i, U_{i-1}) = k$.

iii) For any two submarkets $n, m \in N$ with $\mu_i^n = 1$ and $\mu_j^m = 1$, $j > i$ implies that $y^m > y^n$.

By Proposition 1, each type of worker receives a different contract in equilibrium. In particular, the equilibrium level of education is strictly increasing in the worker's type. Most importantly, however, the separating equilibrium cannot be broken by a new pooling submarket. For some heuristic arguments why a pooling deviation cannot be profitable, we refer the reader to the argument presented in the Introduction.

4 Appendix

Proof of Lemma 1: First, note that (A.1) in conjunction with (E.2) implies $U_i > z/r$ for all $i \in I$. Moreover, from our specification of $u_i(\omega)$ and (E.1) it follows that $U_i \geq U_j$ for all $i > j$. Suppose now that some equilibrium set of submarkets contains a pooling submarket. Denote this pooling submarket by n and define $I^n = \{i \mid \mu_i^n > 0\}$ and $i^M = \max I^n$. Consider a new contract ω in the neighborhood of ω^n, where $y > y^n$ such that $\theta_{i^M}(\omega, U_{i^M})$ is still finite. Accordingly, any feasible pair $(\theta, \mu) \in \Gamma(\omega, \{U_i\}_{i \in I})$ must satisfy

$$p(\theta) = (r+s) \min_{i \in I} \frac{rU_i - z}{\min\{u_i(\omega) - rU_i, 0\}}. \tag{8}$$

Observe that the unique labor market tightness θ solving (8) is continuous in ω. By the single-crossing condition and the fact that $U_{i^M}^n = U_{i^M}$ and $U_i^n \leq U_i$ for all $i \in I^n$, any distribution μ which is consistent with ω must have $\mu_i = 0$ for all $i < i^M$. From this it follows that $(\theta, \mu) \in \Gamma(\omega, \{U_i\}_{i \in I})$ implies $\mu_i = 0$ for all $i < i^M$. Since $V(\cdot)$ is increasing in i and since $V^n = k$ by (E.1), we know that $V(\mu', \omega^n, \theta^n) > k$ for all distributions μ' with $\mu'_i = 0$ and $i < i^M$. From the continuity of $V(\cdot)$ and the continuity of the unique labor market tightness defined in (8), it thus follows that there exists a profitable deviation ω in the neighborhood of ω^n such that (E.2) is violated, which contradicts our assumption that the existing set of submarkets (which contains the pooling submarket) constitutes an equilibrium.[4] ■

Proof of Proposition 1: We first prove an auxiliary claim.

Claim 1: The family of utilities $\{U_i\}_{i \in I}$ in parts i)-ii) of Proposition 1 is unique and satisfies $U_i > U_j$ for $j < i$ and $U_1 > z/r$. Moreover, any family of solutions $\{\omega_i\}_{i \in I}$ to the programs $P_1^*(U_1)$ and $P_i^S(U_i, U_{i-1})$ in parts i)-ii) of Proposition 1 is globally incentive compatible, i.e. $U_i \geq \max_{j \in I} U_i(\omega_j, \theta_j(\omega_j, U_j))$ for all $i \in I$, where the optimal screening variable satisfies $y_1 = 0$ and $y_j > y_i$ for all $j > i$.

Proof: Consider the family of utilities $\{U_i\}_{i \in I}$ defined in parts i)-ii) of Proposition 1. We first prove existence and uniqueness for $i = 1$. This follows from the fact that i) $V_1^*(U)$ is continuous in U (by the maximum theorem), ii) $V_1^*(U)$ is strictly decreasing in U,[5] iii) $\lim_{U \to \infty} V_1^*(U) < \infty$, and iv) $V_1^*(z/r) > k$ by (A.1). This also immediately implies that $U_1 > z/r$. Next, we prove existence and uniqueness for $i > 1$. We proceed by induction and assume that the claim regarding $\{U_i\}_{i \in I}$ holds for all types up to some type $i - 1 < I$. Consider therefore the next higher type i. For all $U \leq U_{i-1}$ the constrained program $P_i^S(U, U_{i-1})$ reduces to the unconstrained program $P_i^*(U)$. From (A.1) and the fact that $V(\cdot)$ is increasing in i, we know that $V_i^*(z/r) > k$. This then implies that $V_i^S(z/r, U_{i-1}) > k$ as $U_{i-1} > z/r$ by our inductive assumption. Since $\lim_{U \to \infty} V_i^S(U, U_{i-1}) < k$, and since $V_i^S(U, U_{i-1})$ is continuous in U by the maximum theorem, there exists some U which solves $V_i^S(U_i, U_{i-1}) = k$. Note that this also implies that $U_i > z/r$. To prove uniqueness, it remains to show that $V_i^S(U, U_{i-1})$

[4] Note that ω is profitable for all pairs $(\theta, \mu) \in \Gamma(\omega, \{U_i\}_{i \in I})$. Hence, even under a stronger version of (E.2) which requires that a new ("deviating") submarket must be profitable for *all* pairs $(\theta, \mu) \in \Gamma(\omega, \{U_i\}_{i \in I})$, a pooling equilibrium can still be ruled out.

[5] To see this, suppose that ω solves $P_i^*(U)$ and consider the corresponding program for $U' < U$. By construction of the function $\theta_i(\cdot)$, we can choose a contract ω' such that $y = y'$, $w < w'$, and $\theta = \theta'$, where $\theta = \theta_i(\omega, U)$ and $\theta' = \theta_i(\omega', U')$. This immediately implies that $V_i^*(U') \geq V(\mu, \omega', \theta') > V_i^*(U)$.

is strictly decreasing in U. Suppose ω' is a solution to $P_i^S(U', U_{i-1})$ for some given value $U' > z/r$. By construction of the function $\theta_i(\cdot)$, there exists for any U'' with $z/r < U'' < U'$ some contract ω'' with $y'' = y'$ and $w'' < w'$ such that $\theta_i(\omega'', U'') = \theta_i(\omega', U')$. Since ω' is incentive compatible, ω'' must also be incentive compatible. Accordingly, strict monotonicity of $V_i^S(U, U_{i-1})$ in U follows from $V(\mu, \omega'', \theta) > V(\mu, \omega', \theta) = V_i^S(U', U_{i-1})$, where $\mu_i = 1$ and $\theta = \theta_i(\omega', U')$. Thus, by induction we have shown existence and uniqueness for all $i \geq 1$.

Similarly, we now prove by induction that $U_i > U_{i-1}$ and therefore make the inductive assumption that $U_{i'} > U_{i''}$ holds for all types $1 \leq i'' < i' \leq i-1$. To obtain a contradiction, suppose that $U_i \leq U_{i-1}$. For all $U \leq U_{i-1}$ the constrained program $P_i^S(U, U_{i-1})$ reduces to the unconstrained program $P_i^*(U)$, which, for $U_i \leq U_{i-1}$, implies that $V_i^*(U_i) > V_{i-1}^*(U_{i-1}) \geq k$.[6] As $V_i^*(U)$ is strictly decreasing in U, this contradicts $V_i^S(U_i, U_{i-1}) = k$.

Finally, consider the screening variable y. Strict monotonicity of y_i in a family of solutionsfollows from strict monotonicity of the family $\{U_i\}_{i \in I}$ in conjunction with the single-crossing condition, while any solution to the unconstrained program $P_1^*(\cdot)$ must obviously have $y_1 = 0$. Recall from the definition of $P_i^S(U_i, U_{i-1})$ that the family of solutions $\{\omega_i\}_{i \in I}$ in Proposition 1 satisfies local downwards incentive compatibility. In conjunction with the single-crossing condition, strict monotonicity of y_i then implies that $\{\omega_i\}_{i \in I}$ must also satisfy global incentive compatibility. ■

Recall that by Lemma 1 any equilibrium set of submarkets must be separating and that for each type of worker there must open at least one active submarket. We now prove that for any equilibrium set of submarkets, (E.1)-(E.2) implies parts i)-ii) of Proposition 1. By Claim 1, this also automatically proves part iii). For $i = 1$, this follows immediately from the construction of the program $P_1^*(\cdot)$. For types $i > 1$, we proceed again by induction and assume that for all types up to some type $i - 1 < I$, any submarket opening up in equilibrium for type j with $1 \leq j \leq i - 1$ must solve $P_j^S(U_j, U_{j-1})$. By Claim 1, this uniquely determines the (equilibrium) utilities for all types up to type $i - 1$. Consider now the next higher type i. To obtain a contradiction, suppose that there exists some submarket n that attracts type i but where the corresponding contract ω^n does not solve $P_i^S(U_i, U_{i-1})$, where U_i is uniquely determined by $V_i^S(U_i, U_{i-1}) = k$. For expositional clarity, we denote type i's utility in this submarket by $\bar{U}_i$. The equilibrium condition (E.1) now implies that $V(\mu^n, \omega^n, \theta^n) = k$, where $\mu_i^n = 1$, but it also implies that $U_{i-1}(\omega^n, \theta^n) \leq U_{i-1}$. Consequently, the contract ω^n must satisfy the constraint of the program $P_i^S(\bar{U}_i, U_{i-1})$. Since $V(\mu^n, \omega^n, \theta^n) = k$, and since $V_i^S(U, U_{i-1})$ is strictly decreasing in U as was shown earlier, it cannot be true that $\bar{U}_i > U_i$ as this would contradict $V_i^S(U, U_{i-1}) = k$. Therefore, it must be true that $\bar{U}_i < U_i$. We will now use this result to argue that there must exist a new contract ω which yields $V(\mu^n, \omega, \theta) > k$ whenever $\theta = \theta_i(\omega, \bar{U}_i)$. In particular, consider as a possible deviation the contract ω solving $P_i^S(U_i, U_{i-1})$. This contract yields $V(\mu^n, \omega, \theta) = k$ if $\theta = \theta_i(\omega, U_i)$ and therefore $V(\mu^n, \omega, \theta) > k$ if $\theta = \theta_i(\omega, \bar{U}_i)$. It is obvious that the local incentive compatibility constraint $U_{i-1}(\omega, \theta) \leq U_{i-1}$ for $\theta = \theta_i(\omega, U_i)$ implies $U_{i-1}(\omega, \theta) \leq U_{i-1}$ if $\theta = \theta_i(\omega, \bar{U}_i)$, where $\bar{U}_i < U_i$. By Claim 1, incentive compatibility must then hold for all lower types, i.e. $U_j(\omega, \theta) \leq U_j$ for all $j \leq i-1$ whenever $\theta = \theta_i(\omega, \bar{U}_i)$.[7] Hence, given ω, there exists a feasible pair (θ, μ) such that $V(\mu^n, \omega, \theta) > k$, which shows that in any submarket that attracts type i the corresponding contract ω^n must solve $P_i^S(U_i, U_{i-1})$.

[6] The last inequality follows from the fact that $k = V_{i-1}^S(U_{i-1}, U_{i-2}) \leq V_{i-1}^*(U_{i-1})$ for all $i - 1 > 1$ (note that by definition, $V_{i-1}^*(U_{i-1}) = k$ for $i - 1 = 1$).

[7] If this were not the case, there must exist some type $i' < i - 1$ and contracts ω', ω'' with $y > y'' > y'$ such that $u_{i-1}(\omega'') \geq u_{i-1}(\omega)$, $u_{i-1}(\omega'') \geq u_{i-1}(\omega')$, $u_{i'}(\omega') \geq u_{i'}(\omega'')$, and $u_{i'}(\omega') < u_i(\omega)$, contradicting the single-crossing condition.

Accordingly, the equilibrium utility for type i is uniquely determined by $V_i^S(U_i, U_{i-1}) = k$. This completes the proof that any equilibrium must satisfy parts i)-ii) of Proposition 1 (and by Claim 1 therefore also part iii).

Finally, we show that any set of submarkets satisfying parts i)-iii) of Proposition 1 constitutes an equilibrium. First, note that (E.1) holds because i) all firms realize exactly k in any submarket, and ii) all submarkets are globally incentive compatible by Claim 1. With regard to (E.2), note that by construction of the programs in parts i) and ii) of Proposition 1, opening a new separating submarket cannot be strictly profitable. It therefore remains to check whether there exists a profitable pooling deviation. To see that this is not the case, consider a pooling deviation ω and a feasible pair $(\theta, \omega) \in \Gamma(\omega, \{U_i\}_{i \in I})$ with $\bar{i} = \max\{i \mid \mu_i > 0\} > 1$. Recall that (θ, ω) must satisfy $U_{\bar{i}}(\omega, \theta) = U_{\bar{i}}$ and $U_{\bar{i}-1}(\omega, \theta) \leq U_{\bar{i}-1}$. Consequently, ω must satisfy the local incentive compatibility constraint of the program $P_{\bar{i}}^S(U_{\bar{i}}, U_{\bar{i}-1})$. By construction of the equilibrium and the fact that employing a high-type worker is strictly profitable, $V(\mu, \omega, \theta) \leq V_{\bar{i}}^S(U_{\bar{i}}, U_{\bar{i}-1}) = k$ then implies that the deviation cannot be strictly profitable. ■

5 References

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