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Substitution Elasticities in a CES Production Framework

An Empirical Analysis on the Basis of Non-Linear Least Squares Estimations

Simon Koesler and Michael Schymura

ZEW
Zentrum für Europäische Wirtschaftsforschung GmbH
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EXECUTIVE SUMMARY

Many of today's challenges require regulative interventions by policymakers. From an economic perspective, effectiveness, cost-efficiency and distribution issues are crucial for any form of future regulation. Ultimately, this results in the need for capable and above all reliable instruments to assess environmentally motivated regulations ex ante, for example Computable General Equilibrium (CGE) models. Elasticities are key parameters for such analyses. But despite the central role of elasticities, the experience regarding of elasticities is rather unsatisfying and there exist only few estimates of the required elasticities. As a consequence, modellers frequently feel impelled to employ in their models elasticities from various originally unrelated sources or to use elasticities derived from different conceptual frameworks, thereby exposing themselves to criticism with respect to the usage of potentially inconsistent parameters estimates.

In this paper we seek to contribute to the solution of this problem and aim at overcoming the lack of adequate estimates. To this end, we consistently estimate substitution elasticities for CES production functions on the basis of different non-linear least squares estimation procedures. Thereby, we focus on the well-established nested CES KLEM production structure. Thus on three level production functions, in which capital and labour are combined in the bottom nest, energy is added in the middle nest and finally intermediates enter the production in the top nest. In the process we take advantage of the World Input-Output Database (WIOD). The new WIOD database allows for the first time to use one consistent dataset for the estimation process and gives us the opportunity to derive elasticities from the same data which researchers can also use to calibrate their simulations.

Our results show that compared to standard linear estimations using Kmenta approximations, non-linear estimation techniques perform significantly better in this context. Moreover, no significant change in input substitutability takes place during the time period we consider. Hence for most sectors we do not observe technological change through this channel. Although technological progress in the form of changing substitution elasticities may potentially be an issue when studying longer time periods. On the basis of our estimations, we demonstrate that the common practice of using Cobb-Douglas or Leontief production functions in economic models must be rejected for the majority of sectors. As a consequence we object a simplified approach to the choice of substitution elasticities in the framework of policy oriented economic modelling. In particular in response to this result, we provide a comprehensive set of consistently estimated substitution elasticities covering 35 sectors. Therewith we hope to make a valuable contribution to making instruments designed to evaluate policy measures ex-ante more reliable and support policy makers in their efforts to cope with global environmental problems such as climate change.

DAS WICHTIGSTE IN KÜRZE

Viele Herausforderungen des 21. Jahrhunderts erfordern regulative Eingriffe in das Wirtschaftsgeschehen. Aus ökonomischer Sicht sind dabei Effektivität, Kosteneffizienz und Verteilungsgerechtigkeit zentrale Aspekte, die berücksichtigt werden müssen. Letzten Endes verlangt dies eine ex-ante Evaluation geplanter Maßnahmen. Elastizitäten sind entscheidende Parameter für solche Analysen. Aber, trotz der immensen Bedeutung von Elastizitäten, existieren nur wenige konsistente und umfassende empirische Studien dieser Parameter für Modelle. Dies gilt insbesondere für Elastizitäten im Rahmen von CES-Produktionsfunktionen (constant elasticities of substitution) und für Substitutionselastizitäten. Dementsprechend müssen sich Modellierer entweder auf Werte aus viele verschiedene, voneinander unabhängige Quellen verlassen oder treffen Entscheidungen bezüglich der Elastizitäten gar "aus dem Bauch heraus". Offenkundig sind beide Varianten nicht optimal und eine häufige Ursache dafür, dass die Ergebnisse aus Simulationsmodellen skeptisch betrachtet werden.

Mit dieser Arbeit möchten wir zur Lösung dieses Problems beitragen und Modellierern verlässliche Schätzungen für die benötigten Substitutionselastizitäten anbieten. Wir konzentrieren uns dabei auf CES-Funktionen mit einer dreifach geschachtelten KLEM-Produktionsstruktur. Also Produktionsfunktionen in denen in der untersten Stufe Arbeit und Kapital verknüpft werden, in der mittleren Stufe Energie hinzugeführt wird und zuletzt in der obersten Stufe weitere Zwischenprodukte in die Produktion einfließen. Für die Schätzung nutzen wir die neue umfassende World-Input-Output Database (WIOD). Dies ermöglicht es zum ersten Mal nur einen konsistenten Datensatz für die Schätzungen zu verwenden und erlaubt Elastizitäten auf Basis der gleichen Daten zu bestimmen, die später auch für die Kalibrierung von Simulationsrechnungen verwendet werden können.

Unsere Ergebnisse deuten darauf hin, dass verglichen mit linearen Schätzverfahren unter Verwendung von Kmenta-Approximationen, nicht-lineare Schätzverfahren deutlich bessere Ergebnisse erzielen. Darüber hinaus zeigen wir, dass sich die geschätzten Substitutionselastizitäten über den von uns betrachteten Zeitraum nicht verändern. Folglich beobachten wir für die Mehrheit der betrachteten Sektoren keinen technologischen Wandel durch diesen Kanal. Sollten längere Zeiträume betrachtet werden, könnte die Form von technologischem Fortschritt dennoch eine Rolle spielen. Auf Basis unserer Ergebnisse können Cobb-Douglas- und Leontief-Produktionsfunktionen für die Mehrzahl der untersuchten Sektoren nicht bestätigt werden. Insbesondere als Antwort auf dieses Ergebnis bieten wir im Rahmen dieser Arbeit für 35 Sektoren einen Satz von konsistent geschätzten Substitutionselastizitäten für CES-Funktionen mit einer KLEM-Produktionsstruktur an und erhoffen uns damit, die ex-ante Bewertung von regulativen Eingriffen verlässlicher zu machen.

Substitution Elasticities in a CES Production Framework

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Simon Koesler† and Michael Schymura‡

Abstract

Effectiveness, cost-efficiency and distribution issues are crucial for any form of future regulation. This results in the need for reliable instruments to assess regulations ex ante. Elasticities are key parameters for such instruments. We consistently estimate substitution elasticities for a three level nested CES KLEM production structure on the basis of non-linear least squares estimation procedures. Thereby we take advantage of the new World-Input-Output Database. This allows us for the first time to use one consistent dataset for the estimation process and gives us the opportunity to derive elasticities from the same data which researchers can use to calibrate their simulations. Our results show that compared to standard linear estimations using Kmenta approximations, non-linear estimation techniques perform significantly better. Moreover, during the time period we consider, no significant change in input substitutability takes place over time. Furthermore, we demonstrate that the common practice of using Cobb-Douglas or Leontief production functions in economic models must be rejected for the majority of sectors. In response to this result, we provide a comprehensive set of consistently estimated substitution elasticities covering 35 sectors.

Index Terms

Keywords:	Substitution elasticity
	CES production function
	Policy evaluation
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†Centre for European Economic Research (ZEW), P.O.Box 103443, 68034 Mannheim, Germany; mail: koesler@zew.de, phone: +49 (0)621 1235-203, fax: +49 (0)621 1235-226

‡Centre for European Economic Research (ZEW), P.O.Box 103443, 68034 Mannheim, Germany; mail: schymura@zew.de, phone: +49 (0)621 1235-202, fax: +49 (0)621 1235-226

I. INTRODUCTION

Many of today's challenges require regulative interventions by policymakers. As a consequence, researchers as well as policymakers are discussing worldwide how polices should be designed to deal with a designated problem. From an economic perspective, environmental effectiveness, distribution issues and cost-efficiency are crucial for any form of future regulation. This hold particularly true in times of turbulent economic outlook and scarce financial resources. Ultimately, this results in the need for capable and above all reliable instruments to asses environmental motivated regulation ex ante. In modern applied economics and most notably in the field of environmental and climate policy, Computable General Equilibrium (CGE) models have proven to be one of the leading instruments to evaluate alternative policy measures (Devarajan and Robinson , 2002; Böhringer et al. , 2003; Sue Wing , 2004). As is true also for other policy-oriented models, elasticities are key parameters for CGE models since they are crucial for determining the comparative static behaviour and thereby strongly influence the results of any counterfactual policy analysis undertaken with the help of these models (Dawkins et al. , 2001). A good illustration of this is provided by Jacoby et al. (2006), who perform a sensitivity analysis of structural parameters of their MIT-EPPA model. They conclude that assumptions with respect to technical progress and in particular elasticities of substitution between energy and value added are the main drivers of model results. But despite the central role of elasticities within the framework of applied quantitative simulations, the current situation of elasticities is rather unsatisfying and although the lack of adequate elasticities has been acknowledged for a surprisingly long time (Mansur and Whalley , 1984; Dawkins et al. , 2001) the problem seems to persist. This holds particularly true for the constant elasticity of substitution (CES) framework commonly employed in CGE modelling and substitution elasticities (Okagawa and Ban , 2008). In this context, only few consistent estimates of the required elasticities exist. Those available are limited to a narrow set of sectors, rely on a combination of data from different sources, build on standard linear estimation procedures or focus on the substitutability between specific production inputs. Moreover their results are in parts contradictory.

Examples of studies having estimated substitution elasticities designated for the use in quantitative models building on a CES framework are Kemfert (1998), Balistreri et al. (2003), van der Werf (2008) and Okagawa and Ban (2008). Kemfert (1998) studies whether the CES framework is adequate to characterise the German industry and estimates the substitution elasticities between capital, energy and labour inputs for three CES production functions, each having a different nesting structure. Her findings suggest that CES production functions, ideally having a (KL)E nesting structure, can be used to describe Germany's industrial production behaviour. Balistreri et al. (2003) focus on the input substitutability between capital and labour and estimates the respective substitution elasticity for 28 US sectors. For the majority of sectors their results support the usage of Cobb-Douglas specification in the nest including capital and labour. van der Werf (2008) supplies estimated parameters for a set of two-level nested CES function with capital, labour and energy as inputs. Regarding substitution elasticities he also comes to the conclusion that the usage of a (KL)E nesting structure is justified and criticises the widespread use of Cobb-Douglas functions as his results imply that substitution elasticities are commonly smaller than one. Okagawa and Ban (2008) estimate CES production functions using panel data from the EU KLEMS dataset. They argue that higher values for substitution elasticities are closely related to energy inputs for energy-intensive industries. Moreover, according to them, substitution elasticities for other sectors are commonly overestimated in existing climate policy models.

Resulting from the lack of adequate estimates, modellers frequently feel impelled to employ in their models elasticities from various originally unrelated sources, thereby exposing themselves to criticism with respect to the usage of potentially inconsistent parameters estimates. Another issue regarding the problematic usage of elasticity estimates in CGE models relates to the inappropriate usage of elasticities and the conceptual mismatch between the estimation results and the policy experiment explored in the CGE framework. McKitrick (1998) for example deplores the usage of elasticities estimated for commodity classifications which are in disaccord with those represented in the model or for countries the model does not cover. Browning et al. (1999) in turn highlight the difficulties possibly arising due to the mismatch of definitions, for instance the disregard of the differences between short-term and long-term substitution elasticities. In some extreme cases, when estimates are not available altogether, modellers even resort to the usage of rather arbitrary values. In this regard Dawkins et al. (2001) most fittingly term the frequent usage of elasticities of unity the "idiot's law of elasticities" or the usage of rather arbitrary values as "coffee table elasticities".

In this paper we seek to contribute to the solution of this problem and aim at overcoming the lack of adequate estimates. To this end, we consistently estimate substitution elasticities specifically for the usage in CGE models building on CES production functions. In the process we take advantage of the new World Input-Output Database (WIOD). More specifically, we estimate elasticities of substitution for the well-established three level nested KLEM production structure on the basis of different non-linear least squares estimation procedures. Thereby, the new WIOD database allows us for the first time to use one consistent dataset for the estimation process and gives us the opportunity to derive elasticities from the same data which researchers can use to calibrate their simulations.

The remainder of this paper is organised as follows. After presenting in Section II the production structures for which the elasticities of substitution are estimated, we describe the data and outline the estimation procedure in Section III. The estimation results are presented and discussed in Section IV. Finally, we summarise and conclude in Section V

II. Specification of Production Structures

Not only in general equilibrium models but also in other economic applications with a micro-consistent basis, so called Constant Elasticity of Substitution (CES) functions have become very popular among programmers. The question to what extent factors of production are substitutable in a production process has become a main issue of economic research. It originates in the fundamental work of Solow (1956). Solow has considered three cases of production functions. He called the first "Harrod-Domar" (Solow , 1956, p. 73) with an elasticity of substitution equal to zero, the "Cobb-Douglas" case (Solow , 1956, p. 76) with an elasticity of one and a third, not explicitly named possibility with a flexible elasticity (Solow , 1956, p. 77). Solow elaborated the idea of CES production functions concept for the first time, and, five years later, together with his co-authors (Arrow et al. (1961)) he conceptualized the general form of the two-factor constant-elasticity-of-substitution (CES) production function (see e.g. Klump and de La Grandville (2000)). This new-developed CES production function can be seen as a generalization of the two older concepts of the Harrod-Domar-Leontief production function, which is based on the assumption that there is no substitutability between factors, and the Cobb-Douglas production function, which assumes unitary factor substitution elasticity. Since the introduction of the CES production function in 1956, a multitude of extensive studies on the elasticities of substitution between production inputs have been published. One of the latest analysis is in this regard is the work of León-Ledesma et al. (2010), who investigate if a simultaneous identification of the capital-labour substitution elasticity and the direction of technical change is feasible. For the n-input case the basic CES function takes the form:

$$y = \gamma \left(\sum_{i=1}^{n} \alpha_i x_i^{-\rho} \right)^{\frac{1}{-\rho}}, \tag{1}$$

where y is the output, x_i is input i, $0 \leq \alpha_i \leq 1$ with $\sum_{i=1}^{n} \alpha_i$ is the distribution parameter related to input i, $\gamma \geq 0$ represents the efficiency parameter and $\rho = \frac{1-\sigma}{\sigma}$ the substitution parameter whereas $\sigma = \frac{1}{1+\rho} \geq 0$ gives the elasticity of substitution and $\rho \geq -1$ must hold.

But in such a basic CES framework the production structure is limited to feature equal substitution elasticities between all inputs. To overcome this Sato (1967) extended the CES functional form and suggests the usage of nested CES functions. The general idea behind Sato's approach is to construct a separate CES function for each group of inputs that share the same substitution elasticity and to combine the different CES functions in different levels or nests of the overall CES function. This allows to easily implement even complicated production structures and is one of the main advantages of the CES functional form. Following Sato a four-input three-level nested CES function can be specified as:

$$\begin{aligned} y = \gamma \Bigg[& \alpha_1 x_1^{-\rho_1} + (1-\alpha_1) \Bigg(\Bigg(\alpha_2 x_2^{-\rho_2} + (1-\alpha_2) \\ & \left(\left(\alpha_3 x_3^{-\rho_3} + (1-\alpha_3) x_4^{-\rho_3} \right)^{\frac{1}{-\rho_3}} \right)^{-\rho_2} \Bigg)^{\frac{1}{-\rho_2}} \Bigg)^{-\rho_1} \Bigg]^{\frac{1}{-\rho_1}}, \end{aligned} \tag{2}$$

where α_n and ρ_n are the distribution and substitution parameters on the n-th nest of the CES function.

Moreover, the basic CES functional form can easily be extended to be able to account for technological change in the CES framework. In this spirit, for example Henningsen and Henningsen (2011) suggests the CES function

$$y_t = \gamma e^{t\lambda} \left(\sum_i \alpha_i (x_{i,t})^{-\rho} \right)^{\frac{1}{-\rho}} \tag{3}$$

to account for Hicks-neutral technological change and the CES function

$$y_t = \gamma \left(\sum_i \alpha_i (e^{t\lambda_i} x_{i,t})^{-\rho} \right)^{\frac{1}{-\rho}} \tag{4}$$

to incorporate factor augmenting (non-neutral) technological change. In both equations t is a time variable and $\lambda \geq 0$ is the rate of technological change, although in the case of factor augmenting technological change λ_i is specific for input i.

In the estimation exercise in this paper, we focus on estimating elasticity of substitutions for a three-level CES approach including the inputs capital (K), labour (L), energy (E) and other intermediates (M). Besides, during our analysis we concentrate on a $((KL)\,E)\,M$ nesting structure. This structure is probably the most popular CES form employed in CGE models evaluating environmental and climate policy and has been confirmed to be a good approximation of the production behaviour in several studies (e.g. Kemfert , 1998; van der Werf , 2008). With regard to technological progress, we estimate two specifications, one including Hicks-neutral technological change and one on condition that $\lambda = 0$.[1]

A three-level CES nesting structure with capital and labour in the lowest nest, where energy joins the capital-labour composite in the middle nest and intermediates enter in the top nest has the functional form:

$$Y_t = \gamma e^{t\lambda} \Big(\alpha_{KLEM} (M_t)^{-\rho_{KLEM}} + (1 - \alpha_{KLEM}) \Big(\big(\alpha_{KLE} (E_t)^{-\rho_{KLE}} + (1 - \alpha_{KLE}) \, (VA_t)^{-\rho_{KLE}} \big)^{\frac{1}{-\rho_{KLE}}} \Big)^{-\rho_{KLEM}} \Big)^{\frac{1}{-\rho_{KLEM}}} \tag{5}$$

with

$$VA_t = \left(\alpha_{KL} (K_t)^{-\rho_{KL}} + (1 - \alpha_{KL}) \, (L_t)^{-\rho_{KL}} \right)^{\frac{1}{-\rho_{KL}}} \tag{6}$$

III. Data and Estimation Procedure

A. Data

For our analysis we make use of the World Input-Output Database (WIOD).[2] The WIOD database has been constructed on the basis of national accounts data and harmonisation procedures were applied in order to ensure international comparability

[1] In practice, however, it is sometimes hard to distinguish between factor price induced innovation (technological change) and factor substitution. Suppose for example the "putty-clay" situation where a firm is unable to substitute factors for each other in the short run, for instance because of high costs of changing the production technology, and also research and development takes time so that factor input relations remains constant despite changing relative input prices. (von Weizsäcker , 1966, p. 245) argues that in such a case "[...] substitution takes time and it can therefore not strictly be distinguished from technical progress." Salter (1966) arrives at even a stronger conclusion, stating that "it is simply a matter of words whether one terms new techniques of this character inventions or a form of factor substitution" (Salter , 1966, p. 43).

[2] The WIOD database is are available at http://www.wiod.org. We use data from December 2011 in this paper.

of the basic data. The dataset covers 40 regions (27 EU countries and 13 other major countries), which together account for approximately 85 % of world's GDP in 2006. The WIOD data is disaggregated in 35 industries and provides detailed information on primary (raw materials), secondary (manufacturing) as well as tertiary (services) sectors. In addition, it offers annual data which ranges from 1995 to 2006 and in parts to 2009. Beside its broad country coverage, detailed sectoral disaggregation and time period character, the dataset has another important feature: it covers various aspects of economic activity and for example involves accounts for energy and environmental issues or socioeconomic and bilateral trade data. Employing the WIOD dataset in our estimation process involves three main benefits. We can estimate substitution elasticities using one consistent dataset and do not have to merge potentially incompatible data. The comprehensive sectoral coverage of WIOD allows us to estimate substitution elasticities for a broad set of sectors. Last but not least, for the first time we can derive elasticities from the same data which researchers can use also to calibrate their simulations.

In our analysis, we use in particular the WIOD Socio-Economic Accounts (SEA files) and the WIOD Energy Use tables (EU files). Taken together, they form a balanced panel covering 40 regions and 35 sectors over a period of 12 years (1995 to 2006) and include detailed information on production in- and outputs. More specifically WIOD supplies us with data regarding the number of employees for the variable labour L, gross value added at basic prices for the variable value added VA, intermediate inputs at purchasers' prices for the variable materials M, gross energy use for the variable energy E and gross output at basic prices for the variable output Y. But WIOD does not include the capital stock information required four our estimation.

Generally, the Achilles' heel of any empirical investigation of capital and labour substitutability is the capital stock variable. That is because it is usually hard to obtain reliable (physical) capital stock data from official sources, especially when requiring the data for a comprehensive set of regions and sectors. Unfortunately, to this regard WIOD is no exception. However, WIOD nevertheless offers a solution to circumvent this problem. It supplies quantity indices for physical capital stocks and information on gross fixed capital formation for each country, sector and year. We use this information in order to construct our own capital stock data using the perpetual inventory method (e.g. Caselli , 2005).[3] We are fully aware of the problem that the construction of own capital stock data and using it subsequently in our analysis is not the first best solution. Therefore we check very carefully the consistency and robustness of the capital stock data we construct and compare our estimates with the capital stock data provided in the Extended Penn World Tables 3.0 (Marquetti and Foley , 2008). The correlation of our estimate with the data from the EPWT 3.0 is 0.9235. Hence we are convinced that our estimated values are sufficiently reliable. For the estimation, all monetary values have been transformed to U.S. Dollars using

[3] At large, the perpetual inventory method consists of two steps. First, one estimates the initial capital stock $K_0 = \frac{I_0}{(g-\delta)}$, where I_0 is the value of the investment series (in our case gross fixed capital formation in the first year (e.g. 1995 for many countries), g is the average geometric growth rate for the investment series, and δ the depreciation rate. We arbitrarily set the denominator to $(g-\delta) = 0.05 = 5\%$, although different values will not affect the outcome significantly (Caselli , 2005). Next one applies a modified version of the perpetual equation $K_t = I_t + K_{t-1} \cdot (1-\delta)$ by using the sectoral and regional quantity indices in order to finally construct the capital stocks.

the Penn World Table (Heston et al. , 2011) and are reported at 1995 prices. Energy is in Terajouls. Labour is given in thousand persons. Table I gives an overview of the variables used in the estimation process. A complete list of the regions and sectors covered by this analysis is given in the Appendix.

Variable	**Definition**	**Source**	**Unit**
Output	Gross output at basic prices	WIOD SEA Files	million 1995 USD
Capital	Capital stock	Derived using WIOD SEA Files	million 1995 USD
Labour	Number of employees	WIOD SEA Files	thousand persons
Value Added	Gross value added at basic prices	WIOD SEA Files	million 1995 USD
Energy	Gross energy use	WIOD EU Files	Terajouls
Materials	Gross output at basic prices	WIOD SEA Files	million 1995 USD

Table I: Choice of Instruments

B. Estimation Procedure

CES functions are non-linear in parameters and hence parameters can initially not be estimated using standard non-linear estimation techniques. For this reason and due to the so far rather tricky implementation of non-linear estimation procedures, most researchers estimating elasticities of substitution within a CES framework work with CES functions that have been linearised in some form or the other. Thereby, the so-called Kmenta approximation (Kmenta , 1967) has been very popular. However, the original CES function cannot be linarised analytically and using approximation methods to linearise the CES function can have drawbacks. Kmenta (1967) himself notes that if in the production function under investigation the input ratio as well as the elasticity of substitution are either very high or very low, his approximation method may not perform well. Maddala and Kadane (1967) and Thursby and Lovell (1978) confirm this problem and shows that the standard Kmenta procedure may not lead to reliable estimates of parameters in a CES framework.

To avoid issues related to Kmenta approximations without having to use cumbersome non-linear estimation procedures, researchers also make use from the cost function approach (e.g. van der Werf , 2008; Okagawa and Ban , 2008). Thereby one can take advantage of the cost function associated with a specific production function and derive a linear system of equations from the corresponding optimal input demand. This can subsequently be used be used to estimate the function coefficients in question. But this approach requires comprehensive price data, which in most cases is rather difficult to come by, especially when undertaking sector specific analysis.

In contrast to the majority of other studies investigating the substitutability of inputs within a CES production structure, we estimate substitution elasticities directly from the CES production function and primarily building non-linear least-squares estimation procedures. Thereby we employ a set of different optimisation algorithms, namely the Levenberg-Marquardt algorithm (LM) (Marquardt , 1963), PORT routines (Gay , 1990), the Differential Evolution algorithm (DE) (Storn and Price , 1997; Price et al. , 2005), Nelder-Mead routines (NM) (Nelder and Mead , 1965), the Simulated Annealing algorithm (SANN) (Kirkpatrick et al. , 1987; Cerny , 1985) and the so called BFGS algorithm (Broyden , 1970; Fletcher , 1970; Goldfarb , 1970; Shanno , 1970). In some estimation runs we make use of starting values compiled by means of a preceding grid search for the substitution parameter ρ involving LM.[4] A detailed overview of all the estimations we run is given in Table X in the Appendix. However, after having shown that except for SANN and DE our results are robust with regard to the choice of the employed optimisation algorithm, we continue our analysis on the basis of the estimation process producing the best fit to the our data. Id est estimations relying on LM and PORT with starting values.

For the actual estimation process we use the programming environment R with the package micEconCES developed by Henningsen and Henningsen (2011). But the micEconCES package in its current version only allows to estimate parameters for a two-level nested CES production function. Yet we would like to derive the substitution elasticities for a three-level nested CES function. To overcome this limitation, we benefit from the separability implied by the CES framework and split the originally three-level nested KLEM CES function we would like to investigate given by Equation (5) into two individual CES functions. Accordingly we estimate the substitution elasticities first for the non-nested CES function

$$VA_t = \gamma_{KL} e^{t\lambda} \left(\alpha_{KL} (K_t)^{-\rho_{KL}} + (1 - \alpha_{KL}) (L_t)^{-\rho_{KL}} \right)^{\frac{1}{-\rho_{KL}}}, \tag{7}$$

with the substitution elasticity $\sigma_{KL} = \frac{1}{1+\rho_{KL}}$. Subsequently we do the same for the two-level CES function

$$\begin{aligned} Y_t = {} & \gamma_{KLEM} e^{t\lambda} \left(\alpha_{KLEM} (M_t)^{-\rho_{KLEM}} + (1 - \alpha_{KLEM}) \left(\left(\alpha_{KLE} (E_t)^{-\rho_{KLE}} + \right.\right.\right. \\ & \left.\left.\left. (1 - \alpha_{KLE}) (VA_t)^{-\rho_{KLE}} \right)^{\frac{1}{-\rho_{KLE}}} \right)^{-\rho_{KLEM}} \right)^{\frac{1}{-\rho_{KLEM}}}, \end{aligned} \tag{8}$$

with the substitution elasticities $\sigma_{KLE} = \frac{1}{1+\rho_{KLE}}$ and $\sigma_{KLEM} = \frac{1}{1+\rho_{KLEM}}$. Taken together, Equation (7) and Equation (8) represent the overall CES function in question, whereas, as already indicated by Equation (5), Equation (7) is the bottom nest and Equation (8) corresponds to the middle and upper nests of the production function under investigation.

The substitution elasticities are estimated specifically for each of the 35 sectors available in the WIOD dataset. Thereby, we pool all sectoral data across all regions. As indicated by Equation (7) and (8), initially we assume that elasticities are constant over time. Hence, in our setting technological progress can only take place through changes in overall productivity. Though, this assumption is relaxed at a later stage.[5]

4 For more information on how adequate starting values are derived applying a preceding grid search, the interested reader is kindly referred to Henningsen and Henningsen (2011).

5 It is also planned to relax the implicit assumption we have made by pooling all regions, i.e. that substitution elasticities are equal across all regions, in a later version of this paper.

IV. Estimation Results

Unsurprisingly the estimates for the substitution parameters ρ_{KL}, ρ_{KLE} and ρ_{KLEM} and hence for σ_{KL}, σ_{KLE} and σ_{KLEM}differ across different estimation methods. But all in all and in view of the respective standard errors, deviations are rather minor. Nevertheless we observe a small divergence between gradient-based local opimisation algorithms (BFGS, LM and PORT) and algorithms targeting global minima (e.g. NM). Robustness across estimation techniques decreases for smaller estimated values of ρ_{KL}, ρ_{KLE} and ρ_{KLEM} and increases when adequate starting values from a prior grid search are used in the estimation process. The SANN and DE technique however are exceptions and lead to notable different results in several cases, mainly suggesting smaller values for ρ_{KL}, ρ_{KLE} and ρ_{KLEM} than other methods. Convergence is tends also to be ann issue when applying these solvers. Given the overall robustness of the different estimations, we choose to continue our analysis on the basis of only one estimation process. Evaluated on the basis of R-squared, the estimations relying on PORT routines and to a slightly smaller extent those using the LM and BFGS methodologies perform best. Without starting values from a preceding grid search, SANN generates the poorest fit. When using starting values, DE appears to be the least powerful method. Furthermore, by the same measure, estimations using starting values from a preceding grid search generally have a better fit than estimations without. This holds true for the investigation of Equation 7 as well as for Equation 8. Given the benefit of the usage of starting values and on the whole very similar estimation results, in the following we focus on the estimations with the best fit to the data, id est estimations relying on PORT routines and which use starting values. The corresponding estimation results for the substitution parameter ρ are summarised in Table II. Note that for the middle and top nest of sector 5 we do not achieve convergence for any acceptable convergence criteria. For this reason the respective estimates are put in brackets and we do not report the corresponding standard deviation. Moreover, for sector 20 $\rho_{KL} < 1$ and hence violates the basic assumptions of the standard CES framework which requires $\sigma \geq 0$ respectively $\rho \geq -1$. While so far we have applied an unrestricted estimation approach, this indicates the need to incorporate the three parameter constraints implied by the CES framework $\gamma > 0$, $0 \geq \alpha \geq 1$ and $\sigma \geq 0$ into our estimations.

Table III summarises the results for ρ when applying the restricted estimation. The corresponding results for σ are given in Table XI in the Appendix. For obvious reasons the fit for the restricted model is not as good as before. Also the fact that for some estimates standard errors are disproportionate, in particular for such sectors in which the CES side constraints now incorporated in the estimation appear to be binding, indicates that the restricted estimation only provides a poor fit for some estimation structures. However, as for the big majority of sectors and nests our estimation results seem to be reliable and as the usage of of CES functions has proven to be very popular in particular in CGE models, we proceed with our estimation process and continue including the constraints on γ, α and σ.

Next we investigate whether the common simplification of using Cobb-Douglas or Leontief functions in CGE models can be rejected by our estimation results. Table IV presents our findings to this regard.

Sector	N	ρ_{KL}-Est.	Std. Dev.	ρ_{KLE}-Est.	Std. Dev.	ρ_{KLEM}-Est.	Std. Dev.
1	312	1.6574	0.2171	10.8130	75.7547	-0.1203	0.2209
2	372	-0.2188	0.1583	-0.0653	0.3649	3.5476	0.9281
3	396	3.1117	1.5527	3.1388	3.1956	-0.2518	0.1954
4	386	1.4786	0.3157	9.2629	31.2252	0.6197	0.1257
5	366	1.6643	0.6634	(-0.5546)		(-0.3927)	
6	394	3.2492	1.3513	0.5652	0.3250	0.5715	0.3533
7	396	3.5233	1.8517	0.4236	0.1574	0.3953	0.3127
8	337	2.3792	2.0202	-0.8347	0.7198	2.2180	0.4394
9	396	0.9589	0.3316	-0.1870	0.2446	0.3105	0.4263
10	390	3.3969	0.9186	1.0095	0.5815	0.7903	0.5737
11	396	3.2845	0.7915	1.9604	15.5046	0.5797	0.2962
12	396	2.7039	0.6915	1.8853	12.3101	4.8660	0.8455
13	396	0.2231	0.0941	1.2727	0.6970	0.4808	0.2159
14	389	2.7392	3.3729	-0.0769	0.0892	0.1532	0.0710
15	378	0.7306	0.2182	1.9326	0.3736	0.6558	0.3050
16	380	0.6927	0.0874	0.3499	2.4342	0.8877	0.2266
17	393	2.4729	0.4808	2.3294	0.5525	-0.1704	0.1074
18	392	2.2705	0.5488	-0.1752	0.3118	0.3909	0.3536
19	394	3.5020	4.9176	1.2664	0.9783	0.3510	0.1588
20	396	-1.0961	0.1880	2.7734	0.6418	0.2937	0.1214
21	388	4.1859	1.2322	0.0006	0.5003	0.1705	0.0961
22	369	0.7799	0.1680	-0.0629	0.1491	-0.0250	0.0805
23	369	0.1533	0.1894	3.2972	1.4051	0.0127	0.3082
24	357	2.3662	1.5474	-0.1319	0.0979	0.0834	0.1774
25	333	0.2148	0.1828	10.0938	85.3278	-0.6582	0.0634
26	380	1.9993	0.7192	0.4409	0.2469	0.4031	0.0499
27	393	-0.6836	0.1271	0.3861	0.2780	-0.0998	0.4191
28	396	-0.1468	1.2603	1.8526	0.1484	-0.1149	0.1536
29	395	1.6551	0.3485	3.3117	0.3129	-0.2091	0.0616
30	392	2.2470	0.2941	-0.5723	0.4418	0.4738	0.1800
31	382	2.2569	0.4637	2.2032	0.3677	0.1004	0.1141
32	348	0.3012	0.2169	1.8523	0.6869	-0.3570	0.0607
33	377	1.3003	0.1534	0.0351	0.3132	-0.1804	0.0511
34	379	3.4606	0.8897	1.1655	3.8947	0.0993	0.3067
36	396	2.3914	0.4256	1.1437	0.9563	-0.1319	0.3495

Table II: Estimation Results for ρ (Unrestricted PORT Routine with Starting Values)

Sector	N	ρ_{KL}-Est.	Std. Dev.	ρ_{KLE}-Est.	Std. Dev.	ρ_{KLEM}-Est.	Std. Dev.
1	312	1.9298	0.2978	-0.8088	0.9315	0.2795	0.2484
2	372	-0.2188	0.1583	(0.8531)	>10	2.7609	0.7642
3	396	2.8095	1.4562	9.4384	7.2847	0.6105	0.1871
4	386	1.4056	0.3035	7.2389	4.1483	0.5954	0.1253
5	366	1.6772	0.6669	4.8139	2.1139	1.2755	0.1373
6	394	3.5327	1.5375	0.5652	0.3250	0.5715	0.3533
7	396	3.4305	1.7580	(1.7828)	>10	0.4749	0.4541
8	337	2.2196	1.9217	-0.8727	0.5669	2.2518	0.4492
9	396	0.9419	0.3288	-0.1297	0.2485	0.3066	0.4298
10	390	3.4475	0.9363	(4.7844)	>10	0.7958	0.5684
11	396	2.4896	0.5306	(0.1507)	>10	0.5555	0.2853
12	396	2.2881	0.6016	(6.1628)	>10	5.2304	0.9733
13	396	0.2231	0.0941	(-1.0000)	>10	0.2674	0.1926
14	389	2.4643	3.0854	2.4279	1.3171	(-1.0000)	5.5666
15	378	0.7100	0.2133	2.0989	0.3833	0.8870	0.3074
16	380	0.6959	0.0877	-0.2106	1.7929	0.8503	0.2241
17	393	2.3098	0.4449	2.6268	0.5932	-0.1639	0.1072
18	392	2.2601	0.5455	(16.8981)	>10	0.4158	0.3699
19	394	4.0696	6.6469	(3.8731)	>10	0.4687	0.1509
20	396	(-1.0000)	0.1608	2.6537	0.5869	0.1992	0.1205
21	388	3.4819	0.9534	0.0457	0.4964	0.1517	0.0961
22	369	0.7606	0.1655	-0.0874	0.1585	0.2134	0.0970
23	369	0.1533	0.1894	3.4865	1.3732	0.0883	0.3103
24	357	2.3490	1.5388	-0.1663	0.0969	0.1943	0.1794
25	333	0.2155	0.1828	2.6182	1.9587	0.0289	0.0933
26	380	(1.7505)	>10	0.4313	0.2419	0.4110	0.0505
27	393	-0.6323	0.1289	(33.9831)	>10	-0.1487	0.3828
28	396	-0.2431	>10	1.8621	0.1505	-0.0335	0.1571
29	395	1.5531	0.3350	3.0418	0.2863	-0.2492	0.0604
30	392	2.1703	0.2795	(-0.1913)	>10	0.5218	0.1925
31	382	2.8469	0.6145	(12.9564)	>10	-0.1097	0.1126
32	348	(0.2123)	>10	(-1.0000)	0.3746	-0.1290	0.0778
33	377	1.3197	0.1556	0.0726	0.3500	0.0302	0.0578
34	379	3.7657	1.0627	2.1341	3.3036	0.1336	0.3071
36	396	3.0092	0.5861	1.1437	0.9563	-0.1319	0.3495

Table III: Estimation Results for ρ (Restricted PORT Routine with Starting Values)

Sector	H0: $\sigma_{KL}=0$	H0: $\sigma_{KL}=1$	H0: $\sigma_{KLE}=0$	H0: $\sigma_{KLE}=1$	H0: $\sigma_{KLEM}=0$	H0: $\sigma_{KLEM}=1$
1	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
2	<0.01	<0.01			<0.01	<0.01
3	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
4	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
5	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
6	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
7	<0.01	<0.01			<0.01	<0.01
8	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
9	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
10	<0.01	<0.01			<0.01	<0.01
11	<0.01	<0.01			<0.01	<0.01
12	<0.01	<0.01			<0.01	<0.01
13	<0.01	<0.01		<0.01	<0.01	<0.01
14	<0.01	<0.01	<0.01	<0.01	<0.01	
15	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
16	<0.01	<0.01	<0.01		<0.01	<0.01
17	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
18	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
19	<0.01	<0.01			<0.01	<0.01
20	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
21	<0.01	<0.01	<0.01		<0.01	<0.01
22	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
23	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
24	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
25	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
26			<0.01	<0.01	<0.01	<0.01
27	<0.01	<0.01			<0.01	<0.01
28			<0.01	<0.01	<0.01	<0.01
29	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
30	<0.01	<0.01			<0.01	<0.01
31	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
32			<0.01	<0.01	<0.01	<0.01
33	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
34	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
36	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

Table IV: Evaluation of Cobb-Douglas and Leontief Specification for CGE models (two-sided p-values for H0)

For all three nests the assumption of a Cobb-Douglas function ($\sigma_{KL} = 1$, $\sigma_{KLE} = 1$ or $\sigma_{KLEM} = 1$) can be dismissed for almost all sectors. A similar picture emerges for the assumption of a Leontief functional form ($\sigma_{KL} = 0$, $\sigma_{KLE} = 0$ or $\sigma_{KLEM} = 0$). To be exact, in the bottom nest neither the Leontief nor the Cobb-Douglas framework can be rejected for the sectors 26, 28 and 32. While in the middle nest the assumption of a Leontief-like production structure can not be discarded for the sectors 2, 7, 10, 11, 12, 13, 19, 27 and 30, a Cobb-Douglas production function can not be excluded in the sectors 2, 7, 10, 11, 12, 16, 21, 27 and 30. A Leontief framework in the top nest can be rejected for all sectors. In the same the Cobb-Douglas production structure can only not be discarded for sector 14. Overall, this suggest that a simplified approach to the choice of substitution elasticities including only Cobb-Douglas or Leontief production functions is not appropriate and will eventually lead to misguiding results of any counterfactual analysis.

Table V compares the result of our estimations to the findings of Okagawa and Ban (2008), van der Werf (2008) and Kemfert (1998). It must be noted however that for several reasons a direct comparison of the results is difficult. First, non of the studies uses the same data. Second, all of researchers undertake estimations for a different set of sectors. Hence their findings can only be compared on the basis of a specific (possibly arbitrary) sectoral mapping. Third, Okagawa and Ban (2008) as well as Kemfert (1998) do not supply information on the standard error of their results. Fourth, the studies employ different estimation techniques. As a consequence we can not truly test whether our results differ from their findings. Nevertheless, keeping this in mind, we can observe that for a majority of sectors our estimates for σ_{KL} and

σ_{KLEM} tend to be higher than the substitution elasticities supplied by Okagawa and Ban , although this does not hold true for our estimate of σ_{KLE}. Compared to the elasticities derived by van der Werf (2008) or Kemfert (1998), our estimates seem not to be systematically smaller or bigger.

Besides the more fundamental issues mentioned above, there are potentially several reasons for the differences between our estimates and those from other studies. Our data may not be up to the task or our choice of instruments as illustrated in I is inappropriate. However, as Okagawa and Ban (2008), van der Werf (2008) as well as Kemfert (1998) use similar data and instruments, these issues do not immediately suggest themselves as the main reasons for the deviations. Alternatively, the differences may arise due to the usage of different estimation approaches, in particular with regard to linear or and non-linear estimation techniques. In effect, while Okagawa and Ban (2008), van der Werf (2008) and Kemfert (1998) estimate substitution elasticities using a linear estimation process, the elasticities derived in this paper stem from a non-linear estimation process using the original functional form of a CES production function.

Sector				σ_{KL}-Est.				σ_{KLE}-Est.				σ_{KLEM}-Est.	
Own	O	W	K	Own	O	W	K	Own	O	W	K	Own	O
1	AGR			0.3413	0.023			5.2298	0.516			0.7815	0.392
2	MIN		Stone and earth	1.2801	0.139		0.21	0.5396	0.553		0.56	0.2659	0.729
3	FOO	Food and tob.	Food	0.2625	0.382	0.4597	0.66	0.0958	0.395	0.399	0.78	0.6209	0.329
4	TEX	Textiles etc.		0.4157	0.161	0.2737		0.1214	0.637	0.2944		0.6268	0.722
5				0.3735				0.1720				0.4395	
6	WOO			0.2206	0.087			0.6389	0.456			0.6363	0.695
7	PPP	Paper etc.	Paper	0.2257	0.381	0.4103	0.35	0.3593	0.211	0.4489	0.73	0.6780	0.187
8				0.3106				7.8576				0.3075	
9	CHM		Chemical industry	0.5150	0.334		0.37	1.1491	-0.065		0.97	0.7654	0.848
10				0.2248				0.1729				0.5569	
11	NMM	Non-metal. min		0.2866	0.358	0.4541		0.8691	0.411	0.2546		0.6429	0.306
12	BME	Basis metals	Iron	0.3041	0.22	0.619	0.5	0.1396	0.644	0.6454	0.4	0.1605	1.173
13	MAC			0.8176	0.295			0.0000	0.292			0.7890	0.13
14	EEQ			0.2887	0.163			0.2917	0.524			0.0000	0.876
15	TEQ	Transport eq.	Vehicle	0.5848	0.144	0.4638	0.1	0.3227	0.519	0.1705	0.35	0.5299	0.548
16	MAN			0.5896	0.046			1.2667	0.529			0.5405	0.406
17	EGW			0.3021	0.46			0.2757	0.256			1.1960	-0.04
18	CON	Construction		0.3067	0.065	0.2242		0.0559	0.529	0.2892		0.7063	1.264
19				0.1973				0.2052				0.6809	
20				0.0000				0.2737				0.8339	
21				0.2231				0.9563				0.8682	
22				0.5680				1.0958				0.8241	
23	TRN			0.8671	0.31			0.2229	0.281			0.9189	0.352
24	TRN			0.2986	0.31			1.1994	0.281			0.8373	0.352
25	TRN			0.8227	0.31			0.2764	0.281			0.9719	0.352
26	TRN			0.3636	0.31			0.6986	0.281			0.7087	0.352
27	TEL			2.7200	0.37			0.0286	0.518			1.1747	0.654
28	FBS			1.3211	0.264			0.3494	0.32			1.0346	0.492
29				0.3917				0.2474				1.3319	
30				0.3154				1.2365				0.6571	
31				0.2600				0.0717				1.1232	
32				0.8249				0.0000				1.1481	
33				0.4311				0.9323				0.9707	
34	PSE			0.2098	0.316			0.3191	0.784			0.8821	0.902
36				0.2494				0.4665				1.1520	
			Non-ferrous				-0.22				0.83		

Table V: Comparison of Results with Other Studies, OB: Okagawa and Ban (2008), W: van der Werf (2008), K: Kemfert (1998)

To test whether in our setting a linear estimation approach would yield different results, we once more estimate the substitution elasticities for σ_{KL}. But this time use a standard linear least-squares estimation process by applying Kmenta approximations to Equation 7. Subsequently we contrast the results of this estimation with the results of a non-linear estimation process. For the non-linear estimation process we focus on an application using unrestricted LM algorithms and PORT routines with starting values from a preceding grid search, as these methods have proven to be robust and advantageous with regard to low values of sum of squared residuals. However, in contrast to the previous estimation exercises in this paper, this time we do not account for technological change because the Henningsen and Henningsen (2011) implementation of the Kmenta methodology does currently not support this and hence the findings of the estimations would else not be comparable.

Sector	Kmenta ρ_{KL}-Est.	Std. Dev.	r^2	LM-nTP ρ_{KL}-Est.	Std. Dev.	r^2	PORT-nTP ρ_{KL}-Est.	Std. Dev.	r^2
1	-0.0975	0.0512	0.7188	2.5709	0.6335	0.9301	2.7753	0.7486	0.9302
2	0.1479	0.1551	0.7483	-0.3388	0.1583	0.7790	-0.3388	0.1583	0.7790
3	-0.2328	0.0942	0.7781	2.9479	1.4164	0.9393	5.8768	2.6447	0.9427
4	-0.1101	0.0838	0.9076	1.1809	0.2534	0.9375	1.1809	0.2534	0.9375
5	(-1.2913)	0.9627	-6.8546	2.0952	0.7796	0.8566	2.0957	0.7798	0.8566
6	-0.3295	0.0542	-0.7801	3.5345	1.5014	0.8758	3.7635	1.6540	0.8758
7	-0.4825	0.1944	-1.7921	3.5692	1.7306	0.9516	5.3638	3.5857	0.9522
8	0.2591	0.1383	0.1066	1.9984	2.0939	0.7362	3.8117	3.3731	0.7405
9	-0.4286	0.1236	0.8955	0.5048	0.3207	0.9782	0.5049	0.3207	0.9782
10	-0.5023	0.2860	0.9298	3.2378	0.8941	0.9682	5.2990	1.5307	0.9686
11	-0.2415	0.1018	0.7893	3.2151	0.7777	0.9348	5.3529	1.7959	0.9366
12	-0.4881	0.1863	-0.4663	3.1623	0.8161	0.9633	3.6084	0.9377	0.9633
13	-0.0288	0.1188	0.8369	0.1247	0.0886	0.9439	0.1247	0.0886	0.9439
14	-0.3954	0.1159	0.2269	(-24.5419)	>10	0.4876	(-0.4290)	>10	0.4876
15	0.7852	1.4001	0.1832	2.8643	0.9524	0.9673	4.5096	1.3205	0.9676
16	(-2.7733)	6.5722	-2.8190	0.6971	0.0985	0.9708	0.6971	0.0985	0.9708
17	0.0885	0.6150	0.8815	2.3392	0.4698	0.9643	4.3153	0.9222	0.9686
18	-0.2422	0.0324	-2.9987	3.7564	1.0406	0.9755	5.7376	1.8229	0.9756
19	-0.3754	0.1876	0.7667	-0.6303	0.3866	0.8442	-0.6304	0.3867	0.8442
20	2.8902	3.9597	-0.0362	(-1.5664)	0.2828	0.9872	-1.0000	0.1166	0.9869
21	-0.0840	0.1610	0.7598	4.2021	1.2766	0.9709	6.6483	2.3976	0.9731
22	-0.2815	0.0464	-0.2446	0.8019	0.1730	0.9882	0.8019	0.1730	0.9882
23	-0.5170	0.2903	-1.8622	0.0890	0.1813	0.8931	0.0890	0.1813	0.8931
24	0.4366	0.4751	0.4011	2.2233	1.3725	0.7364	4.2390	5.5695	0.7397
25	-0.2693	0.2378	0.9189	0.7668	0.4022	0.9524	0.7674	0.4024	0.9524
26	-0.1807	0.0194	-8.4357	1.5332	0.6465	0.9295	(1.4075)	>10	0.9150
27	-0.3549	0.1531	0.9488	(-1.0177)	0.1902	0.9758	(-1.0000)	0.1886	0.9758
28	0.2161	0.0563	0.2677	-0.4158	0.8492	0.9723	(-0.2856)	>10	0.9711
29	-0.2285	0.0869	0.9718	1.9512	0.4056	0.9803	2.9891	0.6782	0.9810
30	0.0803	0.1089	0.8420	2.5427	0.3550	0.9889	2.5425	0.3549	0.9889
31	0.1379	0.2258	0.7453	3.1363	0.5745	0.9742	5.7789	1.3288	0.9758
32	1.1659	0.9564	0.0575	0.3664	0.2205	0.5195	(0.1538)	>10	0.1772
33	-0.3098	0.0349	-5.4857	1.4023	0.1715	0.9905	1.4022	0.1715	0.9905
34	-0.5547	0.1587	0.3724	3.1975	0.7226	0.9696	3.4592	0.8509	0.9697
36	-0.1341	0.0780	0.9120	3.0612	0.6883	0.9836	5.3392	1.1823	0.9847

Table VI: Comparison Standard Linear Estimation and Non-Linear Estimation

Table VI summaries the results of the three estimations and contrasts their findings. Apart of few exceptions, estimates for ρ_{KL} using the non-linear estimation techniques are higher than those relying on the standard linear estimation approach using Kmenta approximations. As noted in Section III, the usage of the Kmenta approximation itself may lead to biased results, although it remains unclear whether elasticities are over- or underestimated (Thursby and Lovell , 1978). In this context the former potentially seems to be the case. Furthermore, valued on the basis of the multiple R-squared, non-linear estimations perform clearly better than those relying on standard linear procedures. In some cases of the Kmenta apporach R-squared is even negative, indicating that using the simple average would perform better than an estimation building on a Kmenta approximation of the CES function in question. According to our estimations, the Kmenta approach performs in particular bad in cases where ρ is relatively low, respectively σ relatively high. This result is also in line with the observations of Thursby and Lovell (1978). Overall, the comparatively poor performance of the non-linear estimation approach supports our preference with respect to non-linear estimation procedures.

The time series character of our data allows us to engage in an additional analysis and makes it possible to investigate whether substitution elasticities change over time. In the economic literature, technological progress within the CES framework is mainly understood as a change in input productivity and researchers focus primarily on determining λ in Equations 9 and 4. But in principle the CES framework for production functions leaves room for technological change affecting not only productivity but also the substitutability between different production inputs. In this case a modified CES function which takes into account changes of the substitution parameter over time and incorporates Hicks-neutral technological change would take the form:

$$y = \gamma e^{\lambda t} \left(\sum_i \alpha_i (x_i)^{-\rho_t} \right)^{\frac{1}{-\rho_t}} . \tag{9}$$

The textile industry at the end of the 18th century provides an excellent example of this form of technological change. As looms became more and more advanced, human labour could be replaced more easily in the production process. Eventually this had a huge effect on business and society in that period.

Embarking on a simple approach, we test whether we can observe a change in input substitutability over time by reestimating and comparing σ for two different time periods (1995 to 1997 and 2004 to 2006). Table VII summarises the results to this regard. In the bottom and middle nest, the hypothesis that the substitution elasticities do not change over time can be rejected for about a third of the sectors under investigation. The inverse holds true for the the top nest and a significant change in the substitutability between materials and the labour-capital-energy composite can be observed for two thirds of the sectors. Hence, although significantly changing substitution elasticities appear not to be a problem for the majority of our estimations, the issue is potentially important. As a consequence, in future research this particular dimension of technological progress needs be taken into account and should be investigated with more rigour. Ultimately this will require studying longer time periods as those under investigation so far in studies on the substitutability of inputs and also a formalisation of the issue within the CES framework.

Sector	H0: $\sigma_{KL_{95-97}} = \sigma_{KL_{04-06}}$	H0: $\lvert\sigma_{KL_{95-97}} - \sigma_{KL_{04-06}}\rvert > 1$	H0: $\sigma_{KLE_{95-97}} = \sigma_{KLE_{04-06}}$	H0: $\lvert\sigma_{KLE_{95-97}} - \sigma_{KLE_{04-06}}\rvert > 1$	H0: $\sigma_{KLEM_{95-97}} = \sigma_{KLEM_{04-06}}$	H0: $\lvert\sigma_{KLEM_{95-97}} - \sigma_{KLEM_{04-06}}\rvert > 1$
1		<0.01			<0.01	<0.01
2	<0.05				<0.01	<0.01
3		<0.01			<0.01	<0.01
4	<0.01	<0.01	<0.01		<0.01	<0.01
5	<0.1	<0.01			<0.01	<0.01
6		<0.01		<0.01	<0.01	<0.01
7		<0.01			<0.05	<0.01
8			<0.01	<0.01	<0.01	<0.01
9		<0.01	<0.01	<0.01	<0.01	<0.01
10		<0.01				<0.01
11		<0.01		<0.01		<0.01
12	<0.01	<0.01			<0.01	<0.01
13	<0.01	<0.01			<0.01	<0.01
14		<0.01				<0.01
15	<0.1	<0.01			<0.01	<0.01
16	<0.01	<0.01				<0.01
17		<0.01				<0.01
18		<0.01			<0.01	<0.01
19		<0.01			<0.01	<0.01
20			<0.01	<0.01		<0.01
21	<0.1	<0.01				<0.01
22		<0.01	<0.01	<0.01	<0.01	<0.01
23		<0.01		<0.1		<0.01
24		<0.01	<0.01	<0.01	<0.01	<0.01
25	<0.01	<0.01			<0.01	<0.01
26				<0.01	<0.01	<0.01
27	<0.01	<0.01				<0.01
28			<0.01	<0.01	<0.01	<0.01
29		<0.01	<0.01			<0.01
30	<0.01	<0.01			<0.01	<0.01
31		<0.01			<0.01	<0.01
32						<0.01
33	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
34		<0.01			<0.05	<0.01
36	<0.01	<0.01	<0.01	<0.05	<0.01	<0.01

Table VII: Comparison of the Substitution Elasticities for the Periods 1995-1997 and 2004-2006 (p-values for H0)

V. Summary and Conclusion

Elasticities, in particular substitution elasticities, are vital parameters for any micro-consistent economic model and crucially influence the results of counterfactual policy analysis. But so far only few consistent estimates of elasticities exist. With this paper we aim at overcoming this problem. Building on a rich dataset based on the WIOD data, we systematically estimate substitution elasticities for a comprehensive set of sectors using different non-linear estimation procedures.

Our results show that compared to standard linear estimations using Kmenta approximations, non-linear estimation techniques perform significantly better in this context. Moreover, no significant change in input substitutability takes place over during the time period we consider. Hence for most sectors we do not observe technological change through this channel. Although technological progress in the form of changing substitution elasticities may potentially be an issue when studying longer time periods. On the basis of our estimations, we demonstrate that the common practice of using Cobb-Douglas or Leontief production functions in economic models must be rejected for the majority of sectors. As a consequence we object a simplified approach to the choice of substitution elasticities in the framework of policy oriented economic modelling. In particular in response to this result, we provide a comprehensive set of consistently estimated substitution elasticities covering 35 sectors. Therewith we hope to make a valuable contribution to making instruments designed to evaluate policy measures ex-ante more reliable and support policy makers in their efforts to cope with global environmental problems such as climate change.

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Appendix

Countrycode	Country	Countrycode	Country
AUS	Australia	ITA	Italy
AUT	Austria	JPN	Japan
BEL	Belgium	KOR	Korea
BRA	Brazil	LTU	Lithuania
CAN	Canada	LUX	Luxembourg
CYP	Cyprus	LVA	Latvia
CZE	Czech Republic	MEX	Mexico
DEU	Germany	MLT	Malta
DNK	Denmark	NLD	Netherlands
ESP	Spain	POL	Poland
EST	Estonia	PRT	Portugal
FIN	Finland	RUS	Russia
FRA	France	SVK	Slovakia
GBR	United Kingdom	SVN	Slovenia
GRC	Greece	SWE	Sweden
HUN	Hungary	USA	United States
IRL	Ireland		

Table VIII: List of Regions Included in the Analysis

Sector Description	NACE	Code
Agriculture, hunting, forestry and fishing	AtB	1
Mining and quarrying	C	2
Food, beverages and tobacco	15t16	3
Textiles and textile	17t18	4
Leather, leather and footwear	19	5
Wood and of wood and cork	20	6
Pulp, paper, paper, printing and publishing	21t22	7
Coke, refined petroleum and nuclear fuel	23	8
Chemicals and chemical	24	9
Rubber and plastics	25	10
Other non-metallic mineral	26	11
Basic metals and fabricated metal	27t28	12
Machinery, nec	29	13
Electrical and optical equipment	30t33	14
Transport equipment	34t35	15
Manufacturing nec; recycling	36t37	16
Electricity, gas and water supply	E	17
Construction	F	18
Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of fuel	50	19
Wholesale trade and commission trade, except of motor vehicles and motorcycles	51	20
Retail trade, except of motor vehicles and motorcycles; repair of household goods	52	21
Hotels and restaurants	H	22
Inland transport	60	23
Water transport	61	24
Air transport	62	25
Supporting and auxiliary transport activities; activities of travel agencies	63	26
Post and telecommunications	64	27
Financial intermediation	J	28
Real estate activities	70	29
Renting of m&eq and other business activities	71t74	30
Public admin and defence; compulsory social security	L	31
Education	M	32
Health and social work	N	33
Other community, social and personal services	O	34
Total industries	TOT	36

Table IX: List of Sectors Included in the Analysis

Solver	Starting Values from Grid Search	Restricted Coefficients ($\gamma \geq 0, 0 \leq \alpha_i \leq 1, \sum_{i=1}^{n} \alpha_i, \rho \geq -1$)	Technological Progress (Hicks-neutral)
BFGS	no	no	yes
	yes	no	yes
DE	no	no	yes
	no	yes	yes
KM	no	no	no
LM	no	no	yes
	yes	no	yes
	no	no	no
	yes	no	no
NM	no	no	yes
	yes	no	yes
PORT	no	no	yes
	no	yes	yes
	yes	no	yes
	yes	yes	yes
	no	no	no
	no	yes	no
	yes	no	no
	yes	yes	no
SANN	no	no	yes
	yes	no	yes

Table X: List of Estimations Procedures Included in the Analysis

Sector	N	σ_{KL}-Est.	Std. Dev.	σ_{KLE}-Est.	Std. Dev.	σ_{KLEM}-Est.	Std. Dev.
1	312	0.3413	0.0347	(5.2298)	>10	0.7815	0.1517
2	372	1.2801	0.2594	(0.5396)	>10	0.2659	0.0540
3	396	0.2625	0.1003	0.0958	0.0669	0.6209	0.0722
4	386	0.4157	0.0525	0.1214	0.0611	0.6268	0.0492
5	366	0.3735	0.0930	0.1720	0.0625	0.4395	0.0265
6	394	0.2206	0.0748	0.6389	0.1327	0.6363	0.1431
7	396	0.2257	0.0896	(0.3593)	>10	0.6780	0.2087
8	337	0.3106	0.1854	(7.8576)	>10	0.3075	0.0425
9	396	0.5150	0.0872	1.1491	0.3281	0.7654	0.2517
10	390	0.2248	0.0473	(0.1729)	>10	0.5569	0.1762
11	396	0.2866	0.0436	(0.8691)	>10	0.6429	0.1179
12	396	0.3041	0.0556	(0.1396)	>10	0.1605	0.0251
13	396	0.8176	0.0629	(0.0000)	(0.0000)	0.7890	0.1199
14	389	0.2887	0.2571	0.2917	0.1121	(0.0000)	(0.0000)
15	378	0.5848	0.0729	0.3227	0.0399	0.5299	0.0863
16	380	0.5896	0.0305	1.2667	2.8770	0.5405	0.0655
17	393	0.3021	0.0406	0.2757	0.0451	1.1960	0.1533
18	392	0.3067	0.0513	0.0559	0.2692	0.7063	0.1846
19	394	0.1973	0.2586	(0.2052)	>10	0.6809	0.0700
20	396	(0.0000)	(0.0000)	0.2737	0.0440	0.8339	0.0838
21	388	0.2231	0.0475	0.9563	0.4540	0.8682	0.0724
22	369	0.5680	0.0534	1.0958	0.1904	0.8241	0.0659
23	369	0.8671	0.1424	0.2229	0.0682	0.9189	0.2620
24	357	0.2986	0.1372	1.1994	0.1394	0.8373	0.1258
25	333	0.8227	0.1237	0.2764	0.1496	0.9719	0.0881
26	380	(0.3636)	>10	0.6986	0.1181	0.7087	0.0254
27	393	2.7200	0.9540	(0.0286)	>10	1.1747	0.5283
28	396	(1.3211)	>10	0.3494	0.0184	1.0346	0.1682
29	395	0.3917	0.0514	0.2474	0.0175	1.3319	0.1072
30	392	0.3154	0.0278	(1.2365)	>10	0.6571	0.0831
31	382	0.2600	0.0415	0.0717	0.4378	1.1232	0.1421
32	348	(0.8249)	>10	(0.0000)	(0.0000)	1.1481	0.1025
33	377	0.4311	0.0289	0.9323	0.3042	0.9707	0.0545
34	379	0.2098	0.0468	0.3191	0.3363	0.8821	0.2390
36	396	0.2494	0.0365	0.4665	0.2081	1.1520	0.4638

Table XI: Estimation Results for σ (Restricted PORT Routine with Starting Values)