Essays in Banking Regulation

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Chapter 1

Introduction

Banking regulation has drown special attention of policy makers, institutions and the economists since years. The integration of banks and financial markets made financial stability extremely dependent on the banking systems. The systemic crises in the past decades motivated forms of regulation which are more closely associated with prevention than with compensation. The need to increase stability and to help protect the international financial system led to the Bank for International Settlements (BIS) Accord in 1988 and the United States of America's new Federal Deposit Insurance Corporation Improvement Act of 1991. The successor accord, Basel II, introduced in 2004, aimed to improve upon the risk insensitivity of the Basel I. However, the recent crisis raised new challenges for banking regulation in addition to what had been considered previously.

The banking theory and in particular the theories of banking regulation have focused on the riskiness of banks' portfolio. The risk-taking behavior of banks and different situations of moral hazard problem have been the main concentration. This thesis is a collection of three essays which apply a different perspective to questions in the literature of banking regulation. Chapters two and three analyze the risk-taking behavior of a bank which can choose between two regimes of operation. Chapter four adds the regulator into the setup and surveys the banking theories for regulating not only an individual bank but also systemic risk-taking among banks. In the first two chapters, two portfolios different in their expected return and the risk level are available to the bank. When the bank changes the portfolio it is called regime switching. Both chapters study the bank's risk-return preferences in the absence of outsider intervention. While in chapter two the relationship between the cash-flow and the regime choice is investigated in a continuous time setup, in chapter three the analysis is carried out a discrete time setup. The regime choice is examined with respect to the capital level in static and also dynamic setups. Moreover, chapter three questions the standard theory of "gambling for resurrection". Chapter four covers regulatory policies to control a bank's risk of failure and collects macro-prudential regulatory proposals for different risk-taking issues in a banking system.

The literature generally defines a bank-regulator game. The bank optimizes its equity value. The regulator plays as a social planer and optimizes the social value of the bank, including both the equity value and deposits value. The key feature is that the regulator's decision need to be incentive compatible for the equity holders. Higher risk can increase the equity value of the bank under distress. However, the risky operating of the bank may have negative net present value. A strong regulator forces bank-closure before the bank's net present value becomes negative. In case that the regulator can commit to a policy, she plays first and announces the regulatory policies and the closure threshold. Given the closure threshold, the bank makes the decision of capital structure. The banking regulation theories solve the game by backward induction to find the optimal regulatory policies.

In contrast to most of other studies, this research allows for regime switching during a bank's lifetime. An agent changes its investment portfolio depending on preferences for risk and return, and the cash-flow of the investment. Chapter two analyzes this switching behavior for a bank optimizing equity value on behalf of its shareholders. Having deposits, the bank can choose one of the two regimes of operation in each moment of time. If the cash-flow is below the deposit payment, the bank has to inject money or go bankrupt. However, bankruptcy and liquidating assets are costly in the sense that the deposit insurer has to repay to depositors. The riskier regime returns a higher outcome but with lower probability. This regime raises the equity value when the cash-flow is low. Since the equity value is still positive, the bank has the chance to operate with low cash-flow, what creates risk-incentives for the bank under distress. Nevertheless, the switching involves some cost and it is not always a rational decision to bear this cost to increase the risk. For a high level of the cash-flow, the bank is able to pay the cost. Thus, the bank switches to the less risky projects (what offers a higher expected return with less uncertainty). This characteristic increases the equity value comparing to the riskier regime for large levels of the cash-flow. Therefore, the bank with high cash-flow has sufficient incentives for risk-reduction and always takes the opportunity regardless of a large switching cost.

This result is unlike the standard asset substitution opportunity, which insists on banks' risk-taking. If the switching cost reduces and tends to zero, the model above converges to the standard asset substitution theory. The other contribution of the second chapter is to bridge the gap between the stochastic switching models and the banking theories. This produces thresholds in terms of the state variable, the cash-flow, at which the regime of operation changes.

Chapter three studies how banks' willingness to engage in risky investments relates to their capital. The fact that the continuous time model does not give a closed form solution motivates applying a discrete time setup for further analysis of the banks' risk-taking behavior. The other difference from chapter two is that the bank has to put effort to monitor its creditors in order to incur less risk. Making a safer portfolio by monitoring creditors is costly and the bank may optimally stop exerting effort. This means that the bank takes the more risky project which brings a higher outcome in the less probable case of success. In case of failure the risky project does not return more than the less risky regime. The striking result is that there is a non-monotonic relationship between a bank's risk level and its capital. Though bankers invest in riskier projects in distress, risky but efficient projects are also attractive for "relatively wealthy" banks. When the capital decreases, at the intermediate level the bank stops monitoring creditors and chooses riskier investments since the effort cost exceeds the expected loss of failure. Nevertheless, as the capital decreases further, the bank reverses its strategy in order to benefit from higher likelihood of success before falling into extreme distress, where the capital level is so low that the bank can only survive by taking more risk. The effort cost of monitoring is the main source of such a non-monotonic risk optimization. The robustness check on non-monetary effort cost confirms the result. In a dynamic setup the non-monotonicity vanishes due to the bankruptcy at failure. This finding is in line with the result of chapter two. For an intertemporal investment decision, where reinvestment rate is low, a bank highly in debt prefers the riskier investment in the hope of high profit in short-term.

Chapter four reviews the process in which banking regulation theories evolved from the individual bank regulation towards macro-prudential regulation. The chapter begins with a basic setup to explain how the risk-taking behavior cannot be dealt without regulatory actions. The chapter introduces the shareholders' risk taking in the way discussed in chapter three. In addition, there is managerial moral hazard such that a bank's manager takes more risky projects because of a pecuniary private benefit. If the capital is not sufficiently large, the shareholders do not offer an incentive compatible contract to the manager. The deposit insurance guarantees the depositors. Taxing the bank's operation prevents subsidizing banks by taxpayers. However, the insurance or tax system increase the shareholders' incentives for risk-taking, and the insurance does not reduce the managerial moral hazard. The intuition is that the bank has to pay for the tax or insurance out of its profit that decreases its equity value. These market failures require a strong supervisory agent to optimize ex-ante policies against risk-taking and ex-post resolution policies in case of bank failure.

It is shown that the capital adequacy and closure policy are not as effective as expected due to the social cost of asset liquidation. Partial deposit insurance creates incentives for the uninsure depositors to monitor the bank closely. A likely liquidity provision by the regulator increases the bank's charter value and thus reduces incentives for risk-taking. Despite these policies focused on individual banks, allowing a healthy bank take over a failed financial institute motivates banks to avoid risky speculative investments. In fact, this idea connects the individual bank regulation to the systemic risk regulation.

The second part in chapter four argues that if a bank is large, or if it is interconnected to many other institutes and/or many banks take risk together, then failure transmit into the entire banking system. In that situation, healthy institutes in the private sector do not have enough endowment to take over the failed banks. These externalities have been seen in the 2007-2009 crisis, what is studied in chapter four with statistics of huge payments by the government. This inspires banking regulation theories to concentrate on the systemic risk issues. The risk can originate from an exogenous economic shock or from an endogenous risk-taking of banks and their systemic failures. This part of research emphasizes the risk-taking due to moral hazard problems that arises from the asset side of banks' balance-sheet.

Rewarding schemes such as granting the healthy banks to take over the failed banks are proven to mitigate the moral hazard problem. Redefining the capital adequacy by taking into account banks' contribution to the collective risk-shifting (among banks) can effectively improve upon the risk-based capital requirement for an individual bank in the Basel II. A systemic tax policy is confirmed to be optimal for regulating a large bank which cannot be closed in the circumstance that the supervisory authority has power to expropriate the shareholders' ownership and the management. Indeed, for each regulatory policy requires an optimal implementation that takes into account specific conditions.

These three essays represent a relevant contribution to the literature once they combine several aspects of banking regulation theories. The assumptions are general and, thus, the results can be applied to the real world situations. For instance, statistical evidences are provided for the systemic failure situations in the last chapter. Instead of usual one-sided attitude of the literature to the risk-taking behavior of banks, this research takes an unbiased approach to examine the advantages and disadvantages of risk for banks. Thus, the findings suggest incentive mechanisms concerning different perspectives of the regulation problems. In a nutshell, this thesis first emphasizes the complex problems regarding risk-taking in the banking systems and demonstrates nonmonotonic relation of the risk choices to the banks' decisive factors. Next, the optimal methods to deal with the risk issues in both micro and macro scales are analyzed.

Chapter 2

A Switching Model in Banking

2.1 Introduction

During its lifetime, a bank may change its portfolio several times. For instance, an under-capitalized bank which is likely to default may choose a riskier portfolio to increase its equity value. This behavior is known as “gambling for resurrection” and may be a rational strategy if this is the only chance to survive. Apart from this type of motivation to change the portfolio, a bank may generally change its regime of operation. A new regime of operation means that both return and risk of cash-flow generated by the bank’s assets are different from the initial asset allocation. Although the initial choice of strategy has been studied extensively in the literature, regime switching has been discussed only restrictively.

In this paper I investigate optimal switching strategies of a bank having the choice between two regimes of operation at each moment in time in a continuous time model. First, I discuss a bank’s operation under each regime separately. In my basic setup, a bank is insolvent when it cannot pay the deposit coupon out of the cash-flow. An insolvent bank closes down if the equity value is zero. This no-asset-substitution set-up is similar to Decamps et al. (2004)’s model without the regulatory part. This chapter studies the relationship between the equity value and the regime choice at each point in time. The main contribution is that the whole characteristics of the portfolio changes in a regime switch.

I find that less risk increases the equity value for the higher cash-flow, as a result of larger expected return. The effect is reversed for the low cash-flow case such that less risk decreases the equity value. I interpret the low risk regime as when the bank monitors its creditors to keep the net present value (NPV) of investments positive. This follows from the ”delegated monitoring” idea of Diamond (1984) where banks monitor investments on behalf of the depositors. Subsequently, the bank in distress shirks and chooses a riskier portfolio with lower expected return. The assumption of the higher risk and lower expected return rules out the first-order stochastic dominance problem. Next, knowing about the advantage of each regime conditional on the cash-flow I examine the switching strategies of a bank in the absence of outsider intervention. I borrow the assumption of Dangl and Lehar (2004) regarding reversibility of costly switching at each point of time.

To highlight the intuition of switching, I follow Decamps and Djembissi (2005) who show how the trade off between return and risk influences asset substitution behavior in firms. Banks finance their investments in large parts by deposits. Imperfect transferability of banks' assets make their liquidation costly. In addition, profitability of bank's investment requires costly monitoring by the bank. Without the incentive for the banker to monitor, the NPV of the investment becomes negative. This illustrates that insufficiently capitalized banks do not have the incentive to monitor and they switch to the higher risk regime in order to increase the equity value.

To analyze the bank's strategies I choose a continuous time framework which is generally applied in financial literature studying the switching behavior. To solve the switching model, I apply the stochastic control techniques and the general approach of the dynamic programming principle. Since the existing mathematical models do not combine the optimal switching strategy and the bank's stopping problem, I have to fill the gaps in the analytical solution with economical intuitions. In the setup of this paper, the outflow of the switching problem is the cash-flow net of the deposit payment that can be negative. Therefore, the basic assumptions of Vath and Pham (2007) are not satisfied. Still, their explicit solution provides an intuition for my case. The free boundary problem related to the variational inequalities divide the cash-flow state space into the stopping region and the continuation region. Pham (2005b) considers the smooth-fit principle for the value function through boundaries of switching regions. Having all his assumptions, this principle gives the boundary conditions to find closed forms of value functions. Yet, Pham (2005b)'s results are applicable to my model only when the closure level of the cash-flow is given. Hence, where the analytical solution is not available I present an intuitive conjecture about the missing characteristic of the objective function (NPV of an operating bank) and the boundary conditions in terms of the state variable cash-flow.

Finally, I apply a numerical method to solve three examples: the costly switching case, an extreme case of too expensive switching, and the other extreme case of cost-free switching. The simulations confirm my analytical results regarding the uniqueness of switching points for each regime. When capital drops below a cash-flow threshold the bank operating under less risky regime switches to the riskier regime. If the capital decreases further and falls below a minimum cash-flow level the bank closes. If the capital increases, above another threshold the bank operating in the risky regime switches to the less risky regime. The three thresholds are feasible in the way that the switching regions do not intersect. However, if there is no cost of switching there is a unique threshold below which the bank does not monitor creditors and operates in riskier regime as long as the cash-flow is above the closure threshold. If the switching is too costly, the bank does not shirk (from monitoring) when the capital decreases but stops operation. This happens while risk-reduction and operating in Good regime is more profitable for a bank when the capital raises sufficiently. My findings are in line with Dangl and Lehar (2004) if the expected returns do not change from one regime of operation to the other. Because in the setup of this chapter the expected returns play an important role, the extreme case of too costly switching is totaly different from the result of Dangl and Lehar (2004).

The literature on the bank risk-taking is closely linked to the literature on banking regulation. Decamps et al. (2004) study the three pillars of Basel II (Basel Committee (2001)) and attempt to clarify how market discipline and supervisory action can complement capital adequacy. They assume that a bank chooses one of two different regimes of operation, i.e. one with a higher return and another one with a higher risk, at the very beginning and follows it without switching. They show how the regulatory system can affect the initial decision of a bank to choose a safer portfolio.

In the setup of Dangl and Lehar (2004) bankers on behalf of equity-holders[1] can switch the regime of operation through asset substitution. In their model switching is costly and reversible such that only the risk level can change at each point of time. They assume that the regulator who audits the bank at random time intervals wants to prevent asset substitution for a higher risk. The regulatory closure thresholds allow a well-capitalized bank to lower its risk and continue to operate even if the cash-flow is smaller than it would be when the high risk portfolio were chosen. They compare the power of two exogenous regulations, i.e. Building Block (BB) regulation and a Value at Risk (VaR) regulatory capital adequacy. Since VaR regulation is risk sensitive, it is more efficient than risk insensitive BB capital adequacy in preventing gambling for resurrection.

Leland (1994) follows the asset substitution argument of Jensen and Meckling (1976). He studies the optimal capital structure and finds that equity-holders prefer to make the firm's activities riskier in order to increase the firm's equity value at the expense of debt value. In his paper the debt-holders are hurt by higher risk in the case of unprotected debt in which the equity value is enhanced by greater risk. But the opposite is true when the debt is protected by a positive net worth covenant. In this case, increasing the risk lowers the equity value as well as the debt value.

Leland and Toft (1996a) extend Leland (1994)'s model and show that risk shifting disappears when the time to maturity of debt is shortened, confirming that short-term debt facilitates the disciplining of bank managers. Likewise, Leland (1998) includes a single switching to risky portfolio without any cost. Erricson (1997) assumes a constant switching cost and allows for an irreversible switch. Both these papers focus on the optimal capital structure while the asset substitution opportunity causes agency costs.

[1] In this literature, the possible frictions between bankers and equity-holders and between depositors and the Deposit Insurer Foundation are ignored.

To obtain analytical solutions, I borrow the methods from the literature in stochastic switching models. Dangl and Lehar (2004) consider the elementary stopping models in which an agent decides on continuing or stopping the operation generating a stochastic outcome. Since these types of entry/exit models are not directly applicable to the switching model, the solution of Dangl and Lehar (2004) to the switching problem is by conjectures on the control limits policies.[2] The stopping model in the leading paper by Brekke and Oksendal (1994) has been developed into a computational model by Fackler (2004). Bayraktar and Egami (2007) apply a probabilistic approach towards the optimal switching problem in which the value function is characterized directly. They rely on the so called coupled optimal stopping problems instead of the dynamic programming principle.

Analytically the switching models of Pham (2005b) and Vath and Pham (2007) are the closest to my model. Vath and Pham (2007) solve the general switching model in which a regime is basically replaced by another regime in order to maximize an objective function. They use a viscosity solutions approach to determine the optimal investment decision for a multi-activity firm. Their method involves a sequence of stopping times with regime shifts. They find the explicit solution for the two regimes case of Brekke and Oksendal (1994).

The following section introduces the model, explains the no-asset-substitution cases. Section 2.3 presents the switching model and the optimal switching-stopping strategies. In section 2.4 the strategies are quantified for several cases. Numerical examples are explained in section 2.5. Section 2.6 concludes. The appendix includes some proofs and solutions. Figures are presented in the last section.

2.2 The Model

The asset value of the bank generates cash-flow x which is assumed to follow a geometric Brownian motion, following Merton (1974) and Leland (1994). The banker makes decisions on investment and the regime of operation. There are two choices of portfolios. Each one represents a regime of operation and the bank cannot operate under a combination of two regimes. The representative banker can switch from the current portfolio to another at each moment. When the bank monitors its creditors, it receives a higher mean of cash-flow and a lower risk. This regime is called Good. Consequently, I call the other regime Bad in which the bank stops monitoring. In this case the risk

[2] See Pham (2005a) for a survey on the aspects of stochastic control problems.

increases while the bank loses the mean value.[3] Thus, the cash-flow process is denoted by

$$\begin{aligned} dx &= \begin{cases} \mu_G x dt + \sigma_G x d\omega & \text{the Good regime is operating} \\ \mu_B x dt + \sigma_B x d\omega & \text{the Bad regime is operating} \end{cases} \\ x(0) &= x_0 > 0. \end{aligned} \tag{2.1}$$

where we have $\sigma_G < \sigma_B$ for risk levels, and for drifts $\mu_G > \mu_B$. ω is a white noise variable. Assume that all agents are risk neutral with an instantaneous discount rate r. Thus, the deposit rate, d, is equal to the risk free rate r. Interpreting the Good regime as operation under delegated monitoring, note that this monitoring is assumed to be costless or to have a variable component which has already been subtracted from the original drift of the cash-flow. In other words, μ_G could be interpreted as $\mu - m$ where mx is the proportion of cash-flow lost in monitoring.

When the banker closes the bank at default, the bank's assets are liquidated for a value of αx, where α is given exogenously.[4] Because we are in a risk neutral world, the expected net present value of the cash-flow (conditional on the information available at time t) has to coincide with the current value of the unlevered bank,

$$W_t = \mathbb{E}_{x_t}\left[\int_t^{+\infty} e^{-r\tau} x_\tau d\tau\right] = \frac{x_t}{r-\mu}, \tag{2.2}$$

where $\mathbb{E}_x$ is the expectation operator over variable x. In order to have a positive cash-flow for all asset values above zero, I need an arbitrage free model,[5] i.e. $\mu < r$. But for a real bank holding deposits (levered bank) equation (2.2) no longer holds because of bankruptcy risk. In this model long-term deposits are fully insured, with the face value normalized to one. Thus, the equity-holders are residual claimants. There is an instantaneous switching cost, k, which is paid by the equity-holders at switching moments.

In the following subsection, I concentrate on the case where the bank has only one type of regime. This will introduce incentives of the bank for switching between two regimes. Throughout, I assume a simple world where the bank operates in the absence of outsider intervention.

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[3]I consider a general case rather than only risk shifting which is supposed in the most of asset substitution literature.

[4]Note that α is not necessarily less than 1, since the state variable is the cash-flow. This is unlike to Leland (1994) in which the state variable is the asset value of the firm and there is a fractional bankruptcy cost or cost of liquidation.

[5]The integral never converges for $\mu > r$.

2.2.1 No Asset Substitution

I explain the simple stopping problem for the bank in this section. Suppose the bank chooses one type of regime at time $t = 0$, and operates for its entire lifetime without asset substitution and any change in the portfolio. Thus, the bank has the only option to stop when operation is no longer beneficial. That means as soon as the equity-holders' wealth becomes negative, the banker stops operating and liquidates the assets.

In the absence of outsider intervention, the banker on behalf of equity-holders abandons the operation as the cash-flow drops below threshold x_C. Although a firm ex-ante maximizes the value of its asset portfolio, a levered bank ex-post (when deposits are in place) maximizes the equity value. In this framework the earnings of the bank from the cash-flow, before deposit payment and extra benefit, is determined by:

$$W(x) = \mathbb{E}_x \left[\int_0^{\tau_C} x_t e^{-rt} dt + e^{-r\tau_C} \alpha x_C \right] \tag{2.3}$$

where stopping time τ_C is a random variable, defined as the first instant where x_t falls below x_C, given $x_0 = x$. Then W is found typically by solving the ordinary differential equation (ODE)[6]

$$rW = (1/2)\sigma^2 x^2 W_{xx} + \mu x W_x + x. \tag{2.4}$$

The general solution is:

$$W(x) = \frac{x}{r - \mu} + K_1 x^{\gamma_1} + K_2 x^{\gamma_2}, \tag{2.5}$$

where $\gamma_1 > 1, \gamma_2 < 0$ are the roots of:

$$(1/2)\sigma^2 \gamma(\gamma - 1) + \mu\gamma - r = 0, \tag{2.6}$$

and thus equal to:

$$\gamma = \frac{-(\mu - (1/2)\sigma^2) \pm \sqrt{(\mu - (1/2)\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}. \tag{2.7}$$

The coefficients K_1, K_2 are determined by the boundary conditions:

$$W(x_C) = \alpha x_C, \tag{2.8}$$

[6]Refer to Dixit and Pindyck (1994) "Equivalent Risk-neutral Valuation", P. 121-125.

and

$$x \to +\infty, \quad W(x) \overset{asymptotically}{\longrightarrow} \frac{x}{r-\mu}. \tag{2.9}$$

The latter condition attributes to the case that the high cash-flow prevents bankruptcy. Therefore, since bankruptcy is unlikely, the earnings of the bank converges to the asset value of an unlevered bank with the same cash-flow.

From (2.9) we have $K_1 = 0$, and W is determined by

$$W(x) = \frac{x}{r-\mu} + (\alpha - \frac{1}{r-\mu})x_C(\frac{x}{x_C})^{\gamma_2}. \tag{2.10}$$

The second term in (2.10) indicates the option value associated with the irreversible closure at x_C. As condition (2.9) shows, this option value converges to zero for a high value of cash-flow.

With a similar approach, I can find the closed forms for other contingent claims, i.e. the market value of deposits and the market value of equity:

- The **market value of the uninsured deposits**: In contrast to the insured contract held by the depositors, that is always worth 1, the coupon flow d provided by the bank is not insured. The market value of the uninsured deposits $D(x)$ is the present value of coupon flow r. Whenever the cash-flow is below the coupon flow, the banker has to inject money into the bank in order to survive the situation. Since this claim is exposed to the default risk, the insurer bears the difference between the insured value and the market value of the coupon flow, i.e. $1 - D(x)$. This is the current value of possible future expenditures necessary to guarantee the full face value to depositors in case of bank closure.[7] The claim $D(x)$ satisfies the ODE below:

$$rD = (1/2)\sigma^2 x^2 D_{xx} + \mu x D_x + r. \tag{2.11}$$

  Therefore, $D(x)$ has a power function closed form with coefficients found from boundary conditions. The first boundary condition is $D(x_C) = \alpha x_C = W(x_C)$, also called "absolute priority rule". According to this rule, the equity-holders receive nothing from the asset value at the closure time.[8] Since a high amount of cash-flow rules out default risk, the market value of deposits converges to the

[7]For simplicity I assume that the bank pays the insurance premium equal $1 - D(x_0)$ at time $t = 0$. Yet, the insurance premium could follow a more complicated process, e.g. a regular payment.

[8]If $\alpha x_C > 1$, then $\alpha x_C - 1$ is given to equity-holders. But in this case there is no uncertainty for deposits. As I see later, a lower closure threshold still increases the market value of equity. Thus, I can assume $\alpha x_C < 1$, and absolute priority rule holds.

principal. Hence, another boundary condition is $x \to +\infty$, $D(x) \to d/r = 1$. Then, $D(x)$ is given by

$$D(x) = 1 + (\alpha x_C - 1)(\frac{x}{x_C})^{\gamma_2}. \tag{2.12}$$

- The **market value of the equity**: Being a residual claim, the value of equity is determined by[9]

$$\begin{aligned} E(x) &= W(x) - D(x) \\ &= \frac{x}{r-\mu} - 1 + (1 - \frac{x_C}{r-\mu})(\frac{x}{x_C})^{\gamma_2}. \end{aligned} \tag{2.13}$$

As in Leland (1994), when there is no protection for the debt,[10] bankruptcy occurs only if the firm cannot meet the required instantaneous deposit payment by issuing additional equity, i.e. when the equity value falls to zero. Of course, given the absolute priority rule, the equity value is zero at closure. Maximizing the social value of the bank, W, gives $x_C = 0$. However, the limited liability of equity prevents x_C from being arbitrarily small.[11] Thus, maximizing nonnegative $E(x)$ for all values of $x > x_C$ sets the closure threshold as

$$0 < x_c = \frac{\gamma_2(r-\mu)}{\gamma_2 - 1} < 1 \qquad (\gamma_2 < 0), \tag{2.14}$$

which is the result of the "smooth-pasting" condition

$$dE/dx_C|_{x=x_c} = 0. \tag{2.15}$$

From equations (2.7) and (2.14), the closure threshold depends on risk free interest rate r and the process of cash-flow such that

$$\frac{\partial \gamma_2}{\partial \sigma} > 0, \; \frac{\partial \gamma_2}{\partial \mu} < 0 \; \Rightarrow \; \gamma_{B2} > \gamma_{G2}, \tag{2.16}$$

and

$$\frac{\partial x_c}{\partial \sigma} = -\frac{r-\mu}{(\gamma_2 - 1)^2}\frac{\partial \gamma_2}{\partial \sigma} < 0, \tag{2.17}$$

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[9] The value of equity is also found directly from $E(x) = \mathbb{E}_x[\int_0^{\tau_C}(x_t - r)e^{-rt}dt]$. The second order differential equation $rE = (1/2)\sigma^2 x^2 E_{xx} + \mu x E_x + x - r$ and appropriate boundary conditions give the same closed forms. I will follow this approach in the next section.

[10] The debt (deposit) is insured from the depositor's point of view. However, there is no constraint on the bank to meet the required instantaneous deposit payment.

[11] Still, a lower closure threshold raises the equity value as $dE/dx_C \leq 0$ for $x_C > 0$.

where γ_{i2} is parameter γ_2 associated with regime $i \in \{G, B\}$. The closure threshold is not monotonic in the drift as the derivative indicates

$$\frac{\partial x_c}{\partial \mu} = -\frac{r-\mu}{(\gamma_2 - 1)^2}\frac{\partial \gamma_2}{\partial \mu} - \frac{\gamma_2}{\gamma_2 - 1} \lesseqgtr 0. \tag{2.18}$$

Therefore, the closure threshold depends on the regime of operation and the parameters.

2.2.2 Comparison of the Two Regimes

Having learned about the stopping strategies of a bank under a single regime, now the two regimes (Good and Bad) can be compared. As (2.13) shows, the equity value is a convex function of the cash-flow. For a sufficiently high value of the cash-flow under each regime $i \in \{G, B\}$, $E_i(x)$ asymptotically converges to $\frac{x}{r-\mu_i} - 1$ which equals the asset value of an unlevered bank with the same cash-flow net of the deposits principal of the levered bank. Because $\mu_G > \mu_B$, the equity value under regime Good is higher than the equity value under regime Bad for the high amount of cash-flow. If $\mu_G = \mu_B$ the closure of the bank under the Bad regime occurs below the closure under the Good regime. Figure 2.1 shows the case in which higher risk is always preferred, since it increases the value of equity, $E_B(x) \succ E_G(x)$.

As $\mu_G > \mu_B$, and x_C is not monotonic with respect to drift μ, the difference between two drifts yields different results. When $d\mu = \mu_G - \mu_B$ is high compared to $d\sigma = \sigma_G - \sigma_B$, and the drift coefficient of the Good portfolio, μ_G, tends to the risk free interest rate r, the bank with the Good portfolio closes at a lower threshold than the bank with Bad portfolio. Further, for a sufficiently high value of the cash-flow the expected value of equity under the Good regime is higher than the expected value of equity under the Bad regime. In such a case, the bank definitely prefers the Good portfolio with the higher equity value to the Bad portfolio, shown in figure 2.2. Alternatively, $\mu_G > \mu_B$ leads to $E_G(x) > E_B(x)$ for the high cash-flow while non-monotonic closure may result in $x_G > x_B$. Then the equity value functions of the two regimes cross as figure 2.3 shows. Therefore, depending on the cash-flow level the bank may prefer a different regime. With the intuitions from the figures, the following propositions discuss all cases. The proofs are in the appendix.

Proposition 2.1 *Assume that the bank has two possible portfolio choices:* (μ_G, σ_G) *and* (μ_B, σ_B)*, where* $\mu_G > \mu_B$*,* $\sigma_G < \sigma_B$*. Then for all* x *in the operation area of the bank,* $E_G(x) > E_B(x)$ *if and only if* $x_G < x_B$*.*

Proof See appendix.

Proposition 2.2 *Assume two regime choices for the bank such that* $\mu_G = \mu_B = \mu$*. Then* $x_G > x_B$ *and* $E_B(x) > E_G(x)$*, for all* x *in the operation area of the bank.*

Proof See appendix.

If drifts of two regimes are equal, equity value functions converge asymptotically to the same line for high values of cash-flow as figure 2.2 shows. The next corollary regarding figure 2.3 follows directly from the propositions above.

Corollary 2.1 *Necessary and sufficient conditions in order to have crossing equity value functions are $\mu_G > \mu_B$ and $x_G > x_B$.*

Proof For sufficiently high value of cash-flow we have

$$x \to +\infty, \quad E(x) \longrightarrow \frac{x}{r-\mu} - 1.$$

Hence $\exists M \in \mathbb{R}$ such that $\forall x > M$, $E_G(x) > E_B(x)$ iff $\mu_G > \mu_B$. From proposition 2.1, $\exists\ m \in \mathbb{R}$ such that $\forall x < m$, $E_G(x) < E_B(x)$ iff $x_G > x_B$, as $x_G = x_B$ only yields $E_G(x) > E_B(x)$ because of a higher drift. Since equity functions are continuous, $\exists\ x_s \in \mathbb{R}, m \leq x_s \leq M$ such that $E_G(x_s) = E_B(x_s)$. Because of convexity of the two equity functions the cross is unique. □

Remark 2.1 Note that I do not consider any potential preference on strategies or closure. For instance, Decamps et al. (2004) assume that the Good regime dominates the closure decision which is always preferred to the Bad regime. To implement this assumption, I need that $\frac{1}{r-\mu_G} > \alpha > \frac{1}{r-\mu_B}$. But in this paper I suppose that for all positive cash-flows the expected value of the bank, operating perpetually under either the Good or the Bad portfolio, is preferred to closure. However, the trade off between a higher drift and a higher risk is the important feature.

Taking the Good regime, the closure point moves by any change in the drift and risk level of cash-flow. The crossing holds iff the closure point decreases by lowering the drift and increasing the risk level. It means I need conditions under which the total differential of x_C is negative, i.e. $dx_C < 0$. Since $d\mu < 0$ and $d\sigma > 0$, from (2.17), (2.18) and

$$dx_C = \frac{\partial x_C}{\partial \mu}\, d\mu + \frac{\partial x_C}{\partial \sigma}\, d\sigma, \tag{2.19}$$

we see that $dx_C < 0$ iff

$$\frac{d\sigma}{d\mu} < \frac{\frac{\partial \gamma_2}{\partial \mu}}{\frac{\partial \gamma_2}{\partial \sigma}} + \frac{\gamma_2(\gamma_2 - 1)}{(r-\mu)\frac{\partial \gamma_2}{\partial \sigma}}. \tag{2.20}$$

Excluding trivial cases in which the bank chooses only one regime with certainty, there are opportunities for advantages of each regime. For a high value of the cash-flow the banker chooses the Good regime and monitors creditors. Then, bankruptcy is less likely. On the other hand, when the cash flow is too low, only survival is important for the bank. The high risk guaranties non-zero equity value and makes operation possible for lower cash-flow. That means the Bad regime is more attractive for a bank in distress.

2.3 Switching Strategies in a Crossing Case

The last section showed that the two regimes might have advantages and disadvantages for different values of the cash-flow. This result provides the intuition for switching from one regime of operation to another as the cash-flow varies. In this section, assume that the parameters satisfy inequality (2.20) which yield crossing equity functions as shown in figure 2.3.

The bank has three choices at each moment, i.e. the Good portfolio, the Bad portfolio or closure. Unlike closure, the two regimes can be reversibly abandoned at a cost. This cost may be arbitrarily high, thereby ruling out switching. Denote the bank's three possible actions by {Stick,Switch,Stop}, for sticking to the current regime, switching to another regime, and stopping the operation, respectively.

The general optimal switching model applies in this case. Following conjectures of Dangl and Lehar (2004), in a model including a lump-sum linear cost of switching, "control limits policies"indicate the optimal decision. The intuition from our previous results makes control limits policies applicable. The bank prefers the Good portfolio for higher cash-flow but the Bad portfolio for lower cash-flows. Since there is a lump-sum cost, control limits policy leads to an interval $[S_G, S_B]$ exerting the minimal control. As long as the cash-flow is in the interior of the interval, the bank sticks to the current regime. If cash-flow x falls below S_G, the bank with regime Good must switch to the Bad regime, and if x rises up S_B the bank with regime Bad must switch to the Good regime.

Although this policy seems intuitively reasonable, such a switching-stopping model does not fit to the former entering/exiting models leading to the control limit policies. Dangl and Lehar (2004) refer to such models which are basically different from their switching-stopping model. Thus, the model needs a direct solution without any prediction. On one hand, to solve the problem directly, I need equity value functions to find the optimal switching-stopping strategies. On the other hand, I can find the equity value function only by proper boundary conditions resulting from the optimal strategies of the bank. The viscosity solutions argument is a proper approach which considers the two optimization problems (maximizing the objective function and finding the optimal boundaries) simultaneously. Therefore, I state the switching model (between the two risky regimes) as the viscosity solutions arguments of Pham (2005b). His arguments mathematically fit to my model under the constraint that the bank only has a switching problem.

2.3.1 The Switching Model

I verify switching strategies using the framework of Pham (2005b) in this section. In order to exclude the closure problem, suppose there is enough support for the bank in distress. When there is welfare cost of closure, governments consider bail out policies. In here, we can assume the government recapitalizes the bank by public funds and the bank operates forever with cash-flows above $X_{SC} > 0$.[12] Having only a switching problem, this model satisfies assumptions H1-H4 of Pham (2005b) such that,

- **H1.** Lipschitz condition holds for the linear drift and variance of state variable x.
- **H2.** Variances are positive under each regime.
- **H3.** The out-flow function, $x - r$, is Lipschitz continuous.
- **H4.** The switching cost is positive, and sticking to the current regime is costless.

My value function is the equity value denoted by $\Psi_i(x)$ for regime i. Define the differential operator $\Delta_i F(x)$ for any value function $F(x)$ under regime i as

$$\Delta_i F(x) = \mu_i x F'(x) + \frac{\sigma_i^2}{2} x^2 F''(x). \tag{2.21}$$

Theorem 1.3.1 of Pham (2005b) proves the existence of the viscosity solution to an ordinary differential equation. I state it for this model in the next proposition.[13]

Proposition 2.3 *Assuming constant drift and risk of the state variable x and the linear profit function $P(x) = x - r$, for each regime i, the value function Ψ_i is a continuous viscosity solution on (X_{SC}, ∞) to the variational inequality:*

$$min\{r\Psi_i(x) - \Delta_i\Psi_i(x) - P(x), \Psi_i(x) - (\Psi_{j\neq i}(x) - k)\} = 0, \quad x > X_{SC}. \tag{2.22}$$

This means that for both regimes, we have supersolution and subsolution properties:

- *Viscosity supersolution property: for any $\bar{x} > X_{SC}$ and $F \in C^2(X_{SC}, \infty)$ s.t. $\bar{x}$ is a local minimum of $\Psi_i - F, \Psi_i(\bar{x}) = F(\bar{x})$, we have*

$$min\{rF(\bar{x}) - \Delta_i F(\bar{x}) - P(\bar{x}), \Psi_i(\bar{x}) - (\Psi_{j\neq i}(\bar{x}) - k)\} \geq 0, \tag{2.23}$$

- *Viscosity subsolution property: for any $\bar{x} > X_{SC}$ and $F \in C^2(X_{SC}, \infty)$ s.t. $\bar{x}$ is a local maximum of $\Psi_i - F, \Psi_i(\bar{x}) = F(\bar{x})$, we have*

$$min\{rF(\bar{x}) - \Delta_i F(\bar{x}) - P(\bar{x}), \Psi_i(\bar{x}) - (\Psi_{j\neq i}(\bar{x}) - k)\} \leq 0. \tag{2.24}$$

Based on (2.22) there exists a continuation area for each regime. Whenever the cash-flow is in the interior of the operation area, the bank sticks to the current regime. Then the equity value is a solution to an ODE determined by the first term in (2.22) being equal

[12] We can later call X_{SC} the closure threshold.
[13] See Pham (2005b), Appendix for the proof of a general case.

zero. The continuation area is connected to the switching region where it is optimal to change the regime. Switching area is a closed set by definition: as soon as the cash-flow reaches the boundaries of the operation area and falls in the switching region, the second term in (2.22) will be equal to zero. This equality is the value matching condition net the switching cost. Moreover, Lemma 1.4.1 of Pham (2005b) adds the smoothness of the value functions in their continuation region. Having the value functions from the last proposition, I need the optimal boundary conditions to find the switching/continuation regions. The optimality condition resulting from theorem 1.4.1 of Pham (2005b) is the so called smooth-fit property[14] over the boundaries of the switching regions. Note that there is no explicit solution in Pham (2005b). I denote the switching point from regime Good to regime Bad, S_G and the switching point from Bad to Good, S_B. We do not know yet if they exist uniquely. However, under assumptions H1 to H4 proposition 2.4 gives the smooth-fit property conditions for any switching point.

Proposition 2.4 *For $i \in \{G, B\}$, the value function Ψ_i is continuously differentiable on (X_{SC}, ∞). Moreover, at S_G and S_B we have*

$$\Psi'_G(S_G) = \Psi'_B(S_G), \tag{2.25}$$

$$\Psi'_G(S_B) = \Psi'_B(S_B). \tag{2.26}$$

I divide the whole range of the cash-flow for regime i, (X_{SC}, ∞), to SW_i and C_i, as the switching region for regime i and its continuation region, respectively. The two subsets intersect in S_i. In this work viscosity solution arguments cannot directly prove that the continuation region and the switching region of a regime $i \in \{G, B\}$ only connect as cases (a) and (b) of figure 2.4. Therefore, the case (c) is also possible since it satisfies all results of Pham (2005b), although it might not intuitively be reasonable for this model.

2.3.2 The Optimal Stopping-Switching Model

In order to find the overall strategies of a bank I need to add its optimal closure decision to the switching problem, since in the absence of outsider intervention different actions of the bank are not independent. The closure decision influences the switching actions and vice versa. I characterize the optimal switching strategies of each regime including the closure decision in this section. The mathematical solution of the combined stopping-switching model is far beyond this work.[15] Consequently, I develop the model taking into account the economical intuitions in this section. The first intuitive conjecture is that lemma 4.1 of Vath and Pham (2007) holds for the stopping-switching

[14] Or the smooth-pasting condition.

[15] Vath and Pham (2007) find explicit solutions for a special setting of switching model which is different from this model.

model similar to the stopping model.[16]

Conjecture 1. The value function, optimal equity $\hat{\Psi}_i$, $i \in \{G, B\}$, is smooth C^2 on continuation region C_i and satisfies

$$r\hat{\Psi}_i(x) - \Delta_i\hat{\Psi}_i(x) - P(x) \quad = \quad 0.$$

Lemma 2.1 *The closure threshold is above zero, i.e.* $\exists X_{SC} > 0, \hat{\Psi}_i(X_{SC}) = 0$ *for* $i \in \{G, B\}$ *which is the optimal regime for the low cash-flows.*

Proof If the bank never closes above zero zero cash-flow and operates under regime i arbitrarily close to 0, then $0 \in C_i$. From conjecture 1, $\hat{\Psi}_i$ has a general form

$$\Psi_i(x) \quad = \quad \frac{x}{r - \mu_i} - 1 + K_{i1}x^{\gamma_{i1}} + K_{i2}x^{\gamma_{i2}}.$$

When x converges to 0, the first and third terms converge to zero as well. Then if $K_{i2} \neq 0$ the forth term converges to infinity. And for $K_{i2} = 0$, $\hat{\Psi}_i$ converges to -1. By contradiction, $0 \in C_i$ and closure threshold X_{SC} is above zero. $\square$

Conjecture 2. For all x, $\hat{\Psi}_i$ is monotonically increasing in x, $i \in \{G, B\}$.

Proposition 2.5 *As* $\mu_G > \mu_B$*, the switching region of regime Bad is a non-empty set.*

Proof In general, the bank can switch or close down when the current regime is no longer beneficial. No matter which regime or strategy is optimal, the boundary condition must hold: when the state variable cash-flow rises sufficiently, each claim asymptotically converges to the claim on an unlevered bank with the same cash-flow. It means that

$$x \to +\infty, \quad \hat{\Psi}_i(x) \overset{asymptotically}{\longrightarrow} \frac{x}{r - \mu_i} - 1, \quad i \in \{G, B\}.$$

Therefore, the higher drift of the Good regime trivially increases the social value and the equity value of the bank with high cash-flow. Accordingly, the better choice for high cash-flow is regime Good. Operating under the Bad regime a bank which has sufficiently high cash-flow reduces the risk and switches to regime Good. Of course with a higher cost of switching the switching point from Bad to Good increases. However, for any $0 < k < \infty$ there exists $S_B < \infty$ such that $\hat{\Psi}_B(x) = \hat{\Psi}_G(x) - k$ (from the boundary condition). It indicates that the switching region of the Bad regime includes (S_B, ∞), i.e. $SW_B \supseteq (S_B, \infty)$ and $SW_B \neq \oslash$. $\square$

[16] In this section, I state some economical conjectures where the mathematical proof is missing.

Lemma 2.2 *The optimal closure threshold of the bank is x_G or $X_{SC} \leq x_B$.*

Proof If the bank follows operating in regime Good and never switches it has to close optimally at x_G. Where $x_B < x_G$, having the switching opportunity the bank is able to continue operation with the lower cash-flows under regime Bad. Since the switching action is optimal only if it increases the equity value, for all x in the operation area we have lower bounds $E_i(x) \leq \hat{\Psi}_i(x),\ i \in \{G, B\}$. Thus, the zero of $\hat{\Psi}$ occurs below the zero of $E_i,\ i \in \{G, B\}$ or at an equal level of the cash-flow. It means that for the switching case, if the bank switches to regime Bad or stays with this regime for the low cash-flow it stops operation at a smaller threshold than the no-substitution case. For the same reason, having switching opportunity the bank may decides to operate under regime Good if it can close at a lower cash-flow. Concluding from both switching cases, for the closure threshold of the stopping-switching model, X_{SC}, we have $X_{SC} \leq x_B < x_G$. □

Lemma 2.3 *The switching regions, SW_G and SW_B do not intersect.*

Proof If SW_G and SW_B intersect, the switch is not stable. Because if in each switch the bank loses an amount of cash-flow and enters to the switching area of another regime, it has to switch back. This is not an optimal policy as the bank loses the cash-flow continuously for the cost of switching back and forth. Thus, only the two continuation regions, C_G and C_B can intersect. □

Remark 2.2 Suppose there exist two switching points for regime Bad, m_B and M_B where $m_B < M_B$ and continuation area $C_B = (m_B, M_B)$. Define the switching region of the Good regime as an interval $SW_G = (m_G, M_G)$. From lemma 2.3 we must have that $SW_G \subseteq C_B$, shown in figure 2.5. Then, if the cash-flow decreases from a high value $M > M_B$ to a lower level $m < m_B$ we see that two times switching is more costly than operating along M to m only under regime Good. With multiple switches the bank faces multiple costs of switching

$$
\begin{aligned}
\hat{\Psi}_G(m) &< \hat{\Psi}_G(m_G) = \hat{\Psi}_B(m_G) - k \\
&< \hat{\Psi}_B(M_B) - k = \hat{\Psi}_G(M_B) - 2k \\
&< \hat{\Psi}_G(M) - 2k.
\end{aligned}
$$

Without switch along M to m, the equity value decreases smoothly. Any other case of two or more switching points for each regime decreases the equity value with the same intuition as the case in figure 2.5, unless a lower closure threshold can be achieved. Albeit, if the bank could close at a lower level of cash-flow under regime Good, there would be no incentive to switch to the Bad regime in the meantime. As the incentive for choosing regime Bad is a lower closure, we can exclude all multiple switchings which are too costly. Doing so, there is at most one switching point for each regime and the switching regions are convex sets. That means the optimal continuation area of each regime is a convex set too as shown in parts (a) and (b) of figure 2.4.

Proposition 2.6 *Assume optimal S_G and S_B exist uniquely. Then switching points are different and $S_G < S_B$.*

Proof The higher drift of the Good regime increases the social value and the equity value of the bank when the cash-flow rises sufficiently such that these claims asymptotically converge to the claims on an unlevered bank with the same cash-flow. Hence, above some certain level of the cash-flow, a bank operating under regime Good sticks to it. If the bank was operating under regime Bad would switch to regime Good above this level. Thus, the threshold level is an upper boundary of the operation area of the Bad regime. Under the assumption of uniqueness of switching points, C_B is a convex set with lower boundary X_{SC}. It follows that the upper boundary is S_B and $C_B = (X_{SC}, S_B)$. Further, the continuation area of the Good regime is convex since S_G is unique. Having monotonically increasing equity values, $S_G > X_{SC}$ since by definition $\hat{\Psi}_B(X_{SC}) = 0$ and $\hat{\Psi}'_B(X_{SC}) = 0$ while

$$\hat{\Psi}_B(S_G) = \hat{\Psi}_G(S_G) + k > 0 \qquad (k > 0).$$

From lemma 2.3, the operation area of regime Good is a super set of $SW_B = [S_B, \infty)$. Therefore, the bank should stick to operation under regime Good above S_G and $C_G = (S_G, \infty)$, where $S_G \leq S_B$.

If switching from Good to Bad and from Bad to Good occur at the same value of cash-flow, i.e. $S_G = S_B = S$, then we have

$$\Psi_G(S) = \Psi_B(S) - k, \quad \Psi_B(S) = \Psi_G(S) - k, \tag{2.27}$$

$$\acute{\Psi_G}(S) = \acute{\Psi_B}(S). \tag{2.28}$$

From (2.27), we have

$$\begin{aligned} \Psi_G(S) &= \Psi_B(S) - k \\ &= \Psi_G(S) - k - k \\ &= \Psi_G(S) - 2k. \end{aligned}$$

which holds only if $k = 0$. By contradiction to $k > 0$, the first assumption is not satisfied, i.e. $S_G \neq S_B$. □

Accordingly, a bank with regime Good gambles for resurrection when in distress and increases the risk loosing the expected return. Then higher risk will help to close the bank in a lower level of cash-flow. Figure 2.7 shows the optimal strategies of the bank. When the cash-flow falls below S_G, the bank operating in the Good regime switches to the Bad regime. Having the Bad regime, when the cash-flow drops bellow X_{SC}, the bank will close. But if the cash-flow increases and raises above S_B, the bank switches from Bad to Good in order to benefit from larger return.

Conjecture 3.
a) An increase in $k > 0$ decreases value functions $\hat{\Psi}_i,\ i \in \{G, B\}$.
b) When switching is costless $k = 0$, the equity Ψ_i^o reaches maximum value, i.e. $\forall x, \Psi_i^o(x) > \hat{\Psi}_i(x),\ i \in \{G, B\}$.

The intuition is that the equity values are american option like claims. The higher the strike price, the lower the option value. That means the equity value decreases when switching cost increases. Therefore, when $k = 0$ the equity values are maximal.

Remark 2.3 When switching cost is high the bank does not switch unless in excessive need. Therefore, I expect the higher k, the bank switches at a lower level to Bad regime, i.e at a lower S_G. Similarly for the other way of switching, a bank with regime Bad postpones switching to a larger value of cash-flow. It means that the higher k, the larger S_B. Interval (S_G, S_B) expands by increasing k while equity values decrease. However, S_G is prevented from being arbitrarily small since $S_G > X_{SC} > 0$. Hence, there exists k^* such that any $k > k^*$ rules out switching from Good to Bad. The banker stops operation under regime Good instead of expensive switch. Then we have $SW_B = (S_B, +\infty)$ and $SW_G = \oslash$. This gives that $C_B = (X_{SC}, S_B)$ and $C_G = (x_G, +\infty)$ as figure 2.6 shows.

Remark 2.4 In the limit when $k = 0$, it follows that $S_G = S_B = S$. Having the costless switching opportunity, the argument of remark 2 is no longer valid. At any level of cash-flow, the bank can instantly switch without any cost to increase the equity value or survive distress.

2.4 Quantification of the Optimal Strategies

Applying a system of optimality conditions, I model the equity value of the Bad regime for the cash-flow bounded in an interval $[X_{SC}, S_B]$ and the equity value for the Good regime in the interval $[S_G, +\infty)$. As the solution of the second order ODE from (2.22), we have the general closed forms of the equity value functions:

$$\Psi_G(x) = \frac{x}{r - \mu_G} - 1 + K_1 x^{\gamma_{G1}} + K_2 x^{\gamma_{G2}}, \tag{2.29}$$

$$\Psi_B(x) = \frac{x}{r - \mu_B} - 1 + K_3 x^{\gamma_{B1}} + K_4 x^{\gamma_{B2}}. \tag{2.30}$$

Add the boundary conditions for all barriers:

$$\Psi_G(S_G) = \Psi_B(S_G) - k, \tag{2.31}$$
$$\Psi_B(S_B) = \Psi_G(S_B) - k, \tag{2.32}$$
$$x \to +\infty \Rightarrow \Psi_G(x) \overset{asymptotically.}{\longrightarrow} \frac{x}{r-\mu_G} - 1, \tag{2.33}$$
$$\Psi_B(X_{SC}) = 0. \tag{2.34}$$

Then $K_1 = 0$ from (2.33), and I find the rest of coefficients versus S_G, S_B and X_{SC} from the system of equations below:

$$\begin{cases} \frac{X_{SC}}{r-\mu_B} - 1 + K_3 X_{SC}^{\gamma_{B1}} + K_4 X_{SC}^{\gamma_{B2}} = 0 \\ \frac{S_G}{r-\mu_G} - 1 + K_2 S_G^{\gamma_{G2}} = \frac{S_G}{r-\mu_B} - 1 + K_3 S_G^{\gamma_{B1}} + K_4 S_G^{\gamma_{B2}} - k \\ \frac{S_B}{r-\mu_B} - 1 + K_3 S_B^{\gamma_{B1}} + K_4 S_B^{\gamma_{B2}} = \frac{S_B}{r-\mu_G} - 1 + K_2 S_B^{\gamma_{G2}} - k \end{cases} \tag{2.35}$$

Since the coefficients are complicated functions of barriers, I present them in detail in the appendix. Given the coefficients in the closed forms of the equity value functions, I use the optimality conditions (2.25), (2.26) and the smooth-fit property bellow in order to determine S_G, S_B and X_{SC},

$$\acute{\Psi}_B(X_{SC}) = 0. \tag{2.36}$$

The following system of non-linear equations determines all barriers simultaneously:

$$\frac{1}{r-\mu_G} + K_2\gamma_{G2}S_G^{\gamma_{G2}-1} = \frac{1}{r-\mu_B} + K_3\gamma_{B1}S_G^{\gamma_{B1}-1} + K_4\gamma_{B2}S_G^{\gamma_{B2}-1} \tag{2.37}$$
$$\frac{1}{r-\mu_B} + K_3\gamma_{B1}S_B^{\gamma_{B1}-1} + K_4\gamma_{B2}S_B^{\gamma_{B2}-1} = \frac{1}{r-\mu_G} + K_2\gamma_{G2}S_B^{\gamma_{G2}-1} \tag{2.38}$$
$$\frac{1}{r-\mu_B} + K_3\gamma_{B1}X_{SC}^{\gamma_{B1}-1} + K_4\gamma_{B2}X_{SC}^{\gamma_{B2}-1} = 0 \tag{2.39}$$

Substituting for K_2, K_3 and K_4 in (2.37)-(2.39), I have a set of three nonlinear equations to solve for the three unknown variables. Since the equations also contain cross multiplications of the unknown variables, an analytical solution is not possible.[17] Such models can only be solved numerically. However, the next proposition confirms the expected result.

[17] The closed form of the equity value points out that $X_{SC} > 0$. This property and negative out-flow for $x < r$, make it impossible to apply the approach by Vath and Pham (2007) to get explicit solution.

Proposition 2.7 *In system of equations (2.37)-(2.39) there exist X_{SC}, S_G and S_B such that:*
(1)$X_{SC} > 0$,
(2)$S_G \neq 0$ *and* $S_G \neq X_{SC}$,
(3)$S_B < \infty$.

Proof See appendix.

2.4.1 Two Alternative Cases

Before finding numerical evidences for the general stopping-switching model, I explain two cases in which the general model no longer fits. Remarks 2.3 and 2.4 discuss two special cases for the switching cost k. Equations (2.37)-(2.39) can not give solutions for $k > k^*$ or where $k = 0$. Therefore, in this subsection I adjust the framework of the model for each of the two boundary cases.[18]

2.4.1.1 Costless Switching

For $k = 0$ that $S_G = S_B = S$, the closed forms of the equity value functions are the same as (2.29)-(2.30). But the boundary conditions need to change, as we have only three boundary conditions for four unknown K_j's,

$$\hat{\Psi}_G(S) = \hat{\Psi}_B(S), \tag{2.40}$$

$$x \to +\infty \Rightarrow \hat{\Psi}_G(x) \overset{asymptotically.}{\longrightarrow} \frac{x}{r - \mu_G} - 1, \tag{2.41}$$

$$\hat{\Psi}_B(x_{SC}) = 0. \tag{2.42}$$

From (2.41), again $K_1 = 0$ in (2.29). Since only (2.40) and (2.42) are not enough for determining K_2, K_3 and K_4, I have to add smooth-pasting condition (2.28).

Given coefficients, the optimality condition (2.35) determines the closure threshold. However, another optimality condition is necessary in order to find the switching point. To achieve an optimal closure and switching strategy together, I use the following optimality condition:[19]

$$\frac{\partial \hat{\Psi}_G}{\partial S} - \frac{\partial \hat{\Psi}_G}{\partial X_{SC}}|_{x=X_{SC}} \frac{\partial X_{SC}}{\partial S} = 0, \tag{2.43}$$

where

$$\frac{\partial X_{SC}}{\partial S} = \frac{\frac{\partial(\hat{\Psi}'_B(X_{SC}))}{\partial S}}{\frac{\partial(\hat{\Psi}'_B(X_{SC}))}{\partial X_{SC}}}. \tag{2.44}$$

[18] I exclude the rigorous details of calculation as nonlinear systems of equations do not give any explicit solution.

[19] See Leland (1998)

Since the smooth fit property is satisfied at the switching point, optimizing equity value for the Good regime yields optimality of the equity value under the Bad regime. Condition (2.43) takes into account the total differential of the value function and condition (2.44) considers the optimality of closure with respect to the switching strategy. The system of non-linear equations (2.28) and (2.43), substituting for (2.44), indicates the critical points.

2.4.1.2 Too Costly Switching

Suppose $k > k^*$ and the bank in the Bad regime never switches. The associated equity value function is given by (2.13) and (2.14),

$$\hat{\Psi}_G(x) = E_G(x) = \frac{x}{r-\mu_G} - 1 + (1 - \frac{x_G}{r-\mu_G})(\frac{x}{x_G})^{\gamma_{G2}} \tag{2.45}$$

$$x_G = \frac{\gamma_{G2}(r-\mu_G)}{\gamma_{G2}-1}. \tag{2.46}$$

The closed form of $\hat{\Psi}_B$ is (2.30) with coefficients indicated by (2.32) and (2.34). Then we have optimality conditions (2.26) and (2.35).

2.5 Numerical Examples

This section presents three examples of the switching model combined with closure for the general and special cases. First, we should look at the crossing behavior of two equity value functions, assuming no switch. The parameters in real values are in Table 1.

Table 1. Parameters

Parameter	r	σ_G	σ_B	μ_G	μ_B
Value	0.1	0.08	0.2	0.03	0.02

Table 2 shows the closure points under the two different regimes with incentives for switching since closures satisfy $x_G > x_B$. Figure 2.8 sketches the equity functions. Then I build the proper system of equations for the general case and alternatives substituting for parameters. Note that nonlinear equations basically have multiple solutions and I have to choose the feasible solution.

Table 2. A Crossing Case

γ_{G1}	γ_{G2}	γ_{B1}	γ_{B2}	x_G	x_B	$E_B(x_G)$
2.79714	-11.1721	2.23607	-2.23607	0.0642492	0.0552786	0.023886

2.5.1 General Case: Two Switches

Since the Bad regime always switches to Good, $S_B < +\infty$ and the higher k the larger S_B. However, from (2.22) a necessary and sufficient condition for switching from Good to Bad is $0 < k \leq \hat{\Psi}_B(S_G) - \hat{\Psi}_G(S_G)$ such that S_G itself is dependent on k. The solution in table 3 shows that $S_G > x_G$ does not necessarily hold since switching opportunity increases the equity values, $\hat{\Psi}_G > E_G$ and $\hat{\Psi}_B > E_B$. Even $k > E_B(x_G)$ gives two switching points.

Table 3. Combined Switching and Closure

k	10^{-15}	0.01	0.02	0.05	0.1	0.15
X_{SC}	0.052354	0.052755	0.052746	0.053188	0.053715	0.054141
S_G	0.103470	0.083786	0.078051	0.068565	0.060124	0.054774
S_B	0.103471	0.116189	0.120477	0.133016	0.158241	0.190163

The interval (S_G, S_B) expands by increasing k. When the switching cost is large, the bank waits till it is necessary to change the regime of operation. Hence, it switches to the Bad regime at a more stressful level. With $k = 0.15$ the switching point S_G is very close to the closure threshold. My try for k larger than or equal to 0.156 ended up S_G being less than X_{SC} which is infeasible.[20] The bank operating under Bad regime also switches to Good when it can cover the cost and this increases S_B. Figure 2.9 shows the case of $k = 0.02$. Applying low values of k I find two switching points converge such that for $k = 10^{-15}$ they are extremely close.

2.5.2 A Costless Switch

When $k = 0$, costless switches happen at the same level of cash-flow, $S_G = S_B = S$. The bank in regime Bad switches to Good if the cash-flow rises S, while the bank operating under regime Good switches to Bad as soon as the cash-flow falls bellow S. Then, if the cash-flow decreases further and reaches X_{SC} the bank with Bad regime stops operating. Under above parameters, as figure 2.10 shows the two critical points are $S = 0.1034702$ and $X_{SC} = 0.0523536$. One could expect these values from the general case above. The closure threshold obtains the smallest value, comparing to table 3. The switching point S is inside intervals $[S_G, S_B]_k$ for all $k > 0$.

[20] Unfortunately, the analysis cannot explain this boundary for k.

2.5.3 Too Costly Switch

Table 4 gives risk reduction results of our example under high switching costs.

Table 4. Too Costly Switching and Closure

k	0.16	0.2	0.5	1	100
X_{SC}	0.0542028	0.0544057	0.0549685	0.0551454	0.0552782
S_B	0.197326	0.228221	0.510823	1.01398	101.305

Figure 2.11 sketches case $k = 0.16$. As k grows the Bad regime is still beneficial since $X_{SC} < x_G$. We see in table 4 that the higher the switching cost the larger the closure threshold and switching point. The critical points are also larger than the general case with two switches. By increasing the cost switching from Good to Bad becomes unprofitable. However, risk reduction is still valuable but at a larger level of cash-flow.

2.6 Conclusion

This paper develops a continuous time model to verify banks' risk-taking behavior. Two regimes of operation are available to a representative bank at each moment of time. The difference is in both return and risk levels of the portfolio chosen under each regime. It is assumed that the bank is already operating in the market. The question is how the operation should continue further in time. Investigating the gambling for resurrection rationale shows that when the cash-flow decreases below a certain level the bank takes more risk (regime Bad). If the cash-flow raises above a larger threshold the bank switches to the less risky regime which generates a higher expected return (regime Good). The cash-flow thresholds are named switching points. Optimal switching strategies for regime Good and Bad are presented. Insolvency is defined as if the cash-flow falls below the deposit payment at each moment. The deposits are fully insured. In the severe case of insolvency that the bank's equity values zero, the bank goes bankrupt and stops operation.

This research extends the literature on risk-taking behavior in the sense that it includes the change in the return in addition to the risk changes. Hence, advantages of a regime creates incentives for the bank to stick on it or switch to another regime. In this regard, the paper promotes the switching model of Dangl and Lehar (2004) who only study the risky asset substitution problem. In my model, the switchings behavior is a result of the trade off between the return and risk. Optimally there exists at most one switching point for each regime. However, the switching cost affects optimal strategies of switching and closure in case of bankruptcy. A high switching cost rules out risk taking by lowering switching criteria and increasing closure threshold. It influences risk reduction incentives by pushing the switching point upward. Costless switching ends up in one switching point, above which the bank operates under Good regime. Below this criteria, the bank operates under regime Bad, unless the cash-flow drops at the closure threshold.

The stopping-switching problem which optimizes the bank's switching and closure strategies does not have a closed form solution. The extreme cases where the switching cost is too high or the switching is costless cannot be solved as the boundary cases of the general setup. Thus, I combine the switching model of Pham (2005b), the basic stopping model, and economical intuitions in order to optimize strategies of a bank for each case. Founding a verification theorem for each case is as severe as inventing a stochastic control model which could solve the general switching-stopping problem. This should be done as further stochastic control studies.

The entrance problem, i.e. under which regime a bank starts operation at $t = 0$ is left for further research. The initial regime depends on the initial capital. Having the relationship between the cash-flow and the capital, the results of switching-stopping model can explain this problem only partially: we need at least positive capital which requires $x_0 > X_{SC}$. If $x_0 \in SW_i, i \in \{G, B\}$ the bank optimally starts under regime j.

In case of bankruptcy, the "lender of last resort" (LOLR) has to bear all deposits. Thus, the high risk of insolvency is not in favor of the supervisory agency who has to play as the LOLR. This research excludes outsiders' intervention. Yet, the setup can be extended to include the regulator's role as the social planner who maximizes the total surplus of the bank operation.

Appendix

Proof of Proposition 2.1. Where the equity value in regime Good is always higher than in regime Bad, it is trivial that the closure threshold is lower. For the other way, if $x_G < x_B$, while $\mu_G > \mu_B$, then for the first derivatives $\acute{E}(.)$ we have

$$\begin{aligned}
&\forall\ x > x_B > x_G,\\
x\acute{E}_G(x) - x\acute{E}_B(x) &= x(\frac{1}{r-\mu_G} - \frac{1}{r-\mu_B}) - \gamma_{G2}(\frac{x_G}{r-\mu_G} - 1)(\frac{x}{x_G})^{\gamma_{G2}} \\
&\quad +\gamma_{B2}(\frac{x_B}{r-\mu_B} - 1)(\frac{x}{x_B})^{\gamma_{B2}} \\
&> x_B(\frac{1}{r-\mu_G} - \frac{1}{r-\mu_B}) - \gamma_{G2}(\frac{x_G}{r-\mu_G} - 1)(\frac{x}{x_B})^{\gamma_{G2}} \\
&\quad +\gamma_{B2}(\frac{x_B}{r-\mu_B} - 1)(\frac{x}{x_B})^{\gamma_{B2}} \\
&> ((\frac{x_G}{r-\mu_G} - \frac{x_B}{r-\mu_B}) \\
&\quad -\gamma_{G2}(\frac{x_G}{r-\mu_G} - 1) + \gamma_{B2}(\frac{x_B}{r-\mu_B} - 1))(\frac{x}{x_B})^{\gamma_{B2}} \\
&= (\underbrace{(\frac{x_G}{r-\mu_G})(1-\gamma_{G2}) + \gamma_{G2}}_{0} - \\
&\quad \underbrace{(\frac{x_B}{r-\mu_B})(1-\gamma_{B2}) + \gamma_{B2}}_{0})(\frac{x}{x_B})^{\gamma_{B2}} \\
&= 0.
\end{aligned}$$

Therefore, $\acute{E}_G(x) > \acute{E}_B(x), \forall x > x_B$; and since $E_G(x_B) > E_B(x_B) = 0$, for all possible cash flows x we have $E_G(x) > E_B(x)$. □

Proof of Proposition 2.2. If $\mu_G = \mu_B = \mu$, then $d\mu = 0$ in (2.19). It follows from (2.17) that $x_G > x_B$. Therefore, $\forall x > x_G$,

$$\begin{aligned}
E_G(x) - E_B(x) &= (1 - \frac{\pi}{r} - \frac{x_G}{r-\mu})(\frac{x}{x_G})^{\gamma_{G2}} - (1 - \frac{\pi}{r} - \frac{x_B}{r-\mu})(\frac{x}{x_B})^{\gamma_{B2}} \\
&< (1 - \frac{\pi}{r} - \frac{x_B}{r-\mu})((\frac{x}{x_G})^{\gamma_{G2}} - (\frac{x}{x_B})^{\gamma_{B2}}) \\
&< (1 - \frac{\pi}{r} - \frac{x_B}{r-\mu})((\frac{x}{x_G})^{\gamma_{G2}} - (\frac{x}{x_B})^{\gamma_{G2}}) \\
&< (1 - \frac{\pi}{r} - \frac{x_B}{r-\mu})((\frac{x}{x_G})^{\gamma_{G2}} - (\frac{x}{x_G})^{\gamma_{G2}}) \\
&= 0. \ \square
\end{aligned}$$

Solutions to system of equations (2.35):

$$K_2 = (r^2(S_B^{\gamma B2}S_G^{\gamma B1} - S_B^{\gamma B1}S_G^{\gamma B2} - k(S_B^{\gamma B2} + S_G^{\gamma B2})X_{SC}^{\gamma B1} + kS_B^{\gamma B1}X_{SC}^{\gamma B2} + kS_G^{\gamma B1}X_{SC}^{\gamma B2}) + S_B(S_G^{\gamma B2}X_{SC}^{\gamma B1} - S_G^{\gamma B1}X_{SC}^{\gamma B2})(\mu_G - \mu_B) + k(-S_G^{\gamma B2}X_{SC}^{\gamma B1} + S_G^{\gamma B1}X_{SC}^{\gamma B2})\mu_G\mu_B + S_B^{\gamma B2}(S_G^{\gamma B1}\mu_G(X_{SC} + \mu_B) - X_{SC}^{\gamma B1}(S_G(\mu_G - \mu_B) + k\mu_G\mu_B)) + S_B^{\gamma B1}(-S_G^{\gamma B2}\mu_G(X_{SC} + \mu_B) + X_{SC}^{\gamma B2}(S_G(\mu_G - \mu_B) + k\mu_G\mu_B)) + r(k(S_G^{\gamma B2}X_{SC}^{\gamma B1} - S_G^{\gamma B1}X_{SC}^{\gamma B2})(\mu_G + \mu_B) + S_B^{\gamma B2}(kX_{SC}^{\gamma B1}(\mu_G + \mu_B) - S_G^{\gamma B1}(X_{SC} + \mu_G + \mu_B)) + S_B^{\gamma B1}(-kX_{SC}^{\gamma B2}(\mu_G + \mu_B) + S_G^{\gamma B2}(X_{SC} + \mu_G + \mu_B))))/((S_B^{\gamma B2}S_G^{\gamma G2}X_{SC}^{\gamma B1} - S_B^{\gamma B1}S_G^{\gamma G2}X_{SC}^{\gamma B2} + S_B^{\gamma G2}(-S_G^{\gamma B2}X_{SC}^{\gamma B1} + S_G^{\gamma B1}X_{SC}^{\gamma B2}))(r - \mu_G)(r - \mu_B))$$

$$K_3 = (X_{SC}^{\gamma B2}(k(S_B^{\gamma G2} + S_G^{\gamma G2})(r - \mu_G)(r - \mu_B) - (-S_B^{\gamma G2}S_G + S_BS_G^{\gamma G2})(\mu_G - \mu_B)) + (S_B^{\gamma B2}S_G^{\gamma G2} - S_B^{\gamma G2}S_G^{\gamma B2})(r - \mu_G)(r - X_{SC} - \mu_B))/((S_B^{\gamma B2}S_G^{\gamma G2}X_{SC}^{\gamma B1} - S_B^{\gamma B1}S_G^{\gamma G2}X_{SC}^{\gamma B2} + S_B^{\gamma G2}(-S_G^{\gamma B2}X_{SC}^{\gamma B1} + S_G^{\gamma B1}X_{SC}^{\gamma B2}))(r - \mu_G)(r - \mu_B))$$

$$K_4 = (X_{SC}^{\gamma B1}(-k(S_B^{\gamma G2} + S_G^{\gamma G2})(r - \mu_G)(r - \mu_B) + (-S_B^{\gamma G2}S_G + S_BS_G^{\gamma G2})(\mu_G - \mu_B)) - (S_B^{\gamma B1}S_G^{\gamma G2} - S_B^{\gamma G2}S_G^{\gamma B1})(r - \mu_G)(r - X_{SC} - \mu_B))/((S_B^{\gamma B2}S_G^{\gamma G2}X_{SC}^{\gamma B1} - S_B^{\gamma B1}S_G^{\gamma G2}X_{SC}^{\gamma B2} + S_B^{\gamma G2}(-S_G^{\gamma B2}X_{SC}^{\gamma B1} + S_G^{\gamma B1}X_{SC}^{\gamma B2}))(r - \mu_G)(r - \mu_B))$$

Proof of Proposition 2.7. Proof by contradiction for each case:
(1) By replacing coefficients K_2, K_3 and K_4 in (2.39) and ordering powers of X_{SC} in the equation I have

$$
\begin{aligned}
&(X_{SC}^{\gamma_{B1}+1}(r-\mu_G)(1-\gamma_{B1})(S_B^{\gamma_{G2}}S_G^{\gamma_{B2}}-S_B^{\gamma_{B2}}S_G^{\gamma_{G2}})+\\
&X_{SC}^{\gamma_{B1}}\gamma_{B1}(r-\mu_G)(r-\mu_B)(S_B^{\gamma_{G2}}S_G^{\gamma_{B2}}-S_B^{\gamma_{B2}}S_G^{\gamma_{G2}})+\\
&X_{SC}^{\gamma_{B1}+\gamma_{B2}}(\gamma_{B1}-\gamma_{B2})(k(r-\mu_G)(r-\mu_B)(S_G^{\gamma_{G2}}+S_B^{\gamma_{G2}})+(\mu_G-\mu_B)\\
&(S_BS_G^{\gamma_{G2}}-S_GS_B^{\gamma_{G2}}))+X_{SC}^{\gamma_{B2}+1}(r-\mu_G)(-1+\gamma_{B2})(S_B^{\gamma_{G2}}S_G^{\gamma_{B1}}-S_B^{\gamma_{B1}}S_G^{\gamma_{G2}})+\\
&X_{SC}^{\gamma_{B2}}\gamma_{B2}(r-\mu_G)(r-\mu_B)(S_B^{\gamma_{B1}}S_G^{\gamma_{G2}}-S_B^{\gamma_{G2}}S_G^{\gamma_{B1}}))/\\
&((X_{SC}^{\gamma_{B1}+1}(S_B^{\gamma_{B2}}S_G^{\gamma_{G2}}-S_B^{\gamma_{G2}}S_G^{\gamma_{B2}})+\\
&X_{SC}^{\gamma_{B2}+1}(S_B^{\gamma_{G2}}S_G^{\gamma_{B1}}-S_B^{\gamma_{B1}}S_G^{\gamma_{G2}}))(r-\mu_G)(r-\mu_B))=0
\end{aligned}
\tag{2.47}
$$

Simplifying this equation term by term with respect to the denominator, we see that the left hand side of the equation gives the following limit

$$
\lim_{X_{SC}\to 0}\left(\frac{1-\gamma_{B2}}{r-\mu_B}-\frac{\gamma_{B2}}{X_{SC}+\frac{X_{SC}^{\gamma_{B1}-\gamma_{B2}+1}}{\frac{(S_B^{\gamma_{G2}}S_G^{\gamma_{B1}}-S_B^{\gamma_{B1}}S_G^{\gamma_{G2}})}{(S_B^{\gamma_{B2}}S_G^{\gamma_{G2}}-S_B^{\gamma_{G2}}S_G^{\gamma_{B2}})}}}\right).
$$

When $X_{SC}\to 0$, the fact that the limit must go to 0 gives $\gamma_{B2}=0$ and $\gamma_{B2}-1=0$ which contradict.

(2) Similar to part (1), I replace the coefficients in (2.38) and order it versus the powers of S_G.[21] The limit of resulting equation when $S_G \to 0$ is equal zero if

$$\gamma_{B2} = \gamma_{B1} \quad \text{or} \quad (\frac{S_B}{X_{SC}})^{\gamma_{B1}} = -k\frac{(r-\mu_G)(r-\mu_B)}{(r-\mu_G)(r-\mu_B) - X_{SC}(r-\mu_G)}$$

$$\text{and:} \quad r = \mu_G \text{ or } r = \mu_B \text{ or } k = 0 \text{ or } \gamma_{B2} = 0 \text{ or } S_B = \infty \text{ or } X_{SC} = 0$$

$$\text{and:} \quad \mu_G = \mu_B.$$

These necessary conditions are inconsistent and not satisfied. Therefore, $S_G \neq 0$.

Assume $S_G = X_{SC} = S$, rewrite (2.38) and (2.39) replacing for the coefficients and S. We must have equation (2.38) plus equation (2.47) equals zero. This gives:

$$S = \frac{(1-k)\gamma_{G2}(r-\mu_G)}{\gamma_{G2} - 1}$$

First of all we find $S < 0$ for $k > 1$. Next, $k = 1$ yields $S = 0$ which is impossible. Then, for $k < 1$ the two smooth-pasting conditions (2.38) and (2.39) no longer hold. The contradiction rejects the hypothesis.

(3) By S_B converging to ∞, the equation (2.37), substituting for coefficients K_2, K_3 and K_4, holds if $\mu_G = \mu_B$ or $\gamma_{B1} = 1$. Since the latter cannot be true, equality of the drifts is the necessary condition. Under this condition, (2.47) indicates that X_{SC} is exactly the closure threshold in no-switch case for the Bad regime. Following this result, S_G from (2.38) is

$$S_G = x_B \left(\frac{k\gamma_{G2}(1-\gamma_{B2})}{\gamma_{G2} - \gamma_{B2}}\right)^{\gamma_{B2}}$$

Such S_G may cause $S_G < x_B$. Despite the result is consistent with proposition 2.2, the assumption of unequal drifts rejects the hypothesis of $S_G \to \infty$. $\square$

[21] Since the resulting equation is more rigorous than helpful, I do not mention it here.

Figures

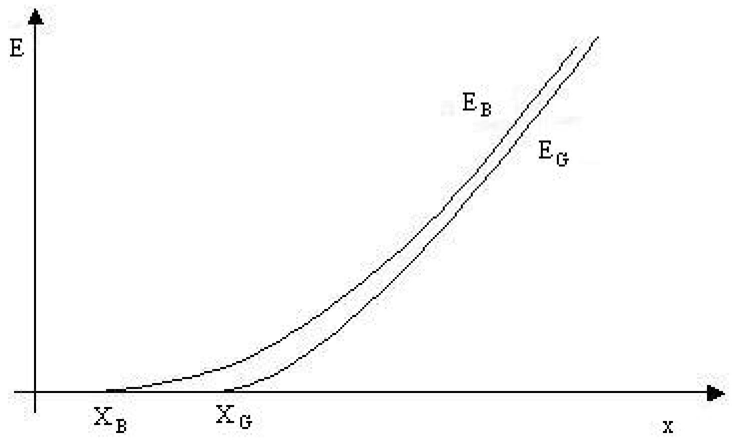

Figure 2.1: The equity value vs. the cash-flow, where $\mu_G = \mu_B$.

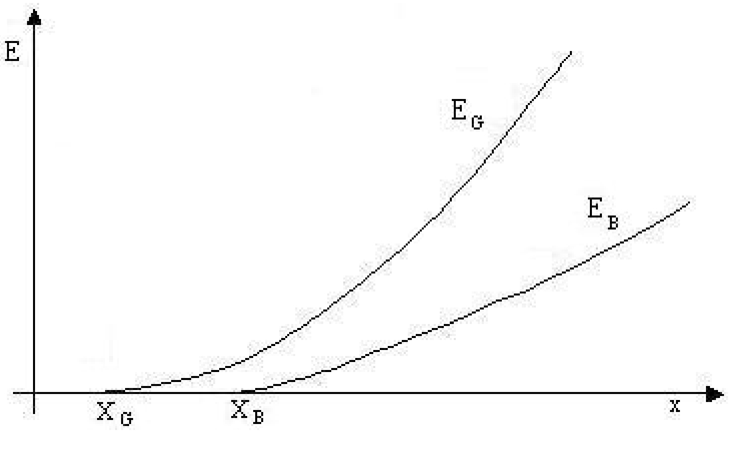

Figure 2.2: The equity value vs the cash-flow where $\mu_G > \mu_B$ and $x_G < x_B$.

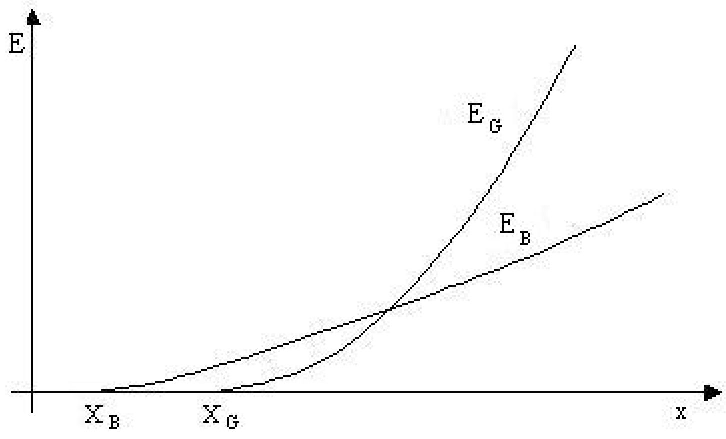

Figure 2.3: Value of equity vs the cash-flow where $\mu_G > \mu_B$ and $x_G > x_B$.

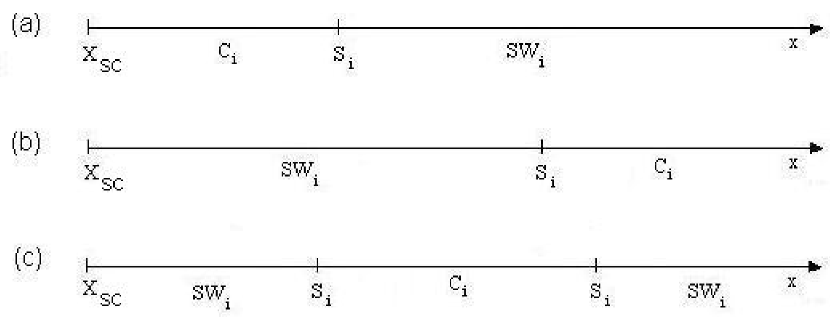

Figure 2.4: Examples of continuation and switching areas for a regime $i \in \{G, B\}$. The bank operates under regime i as long as $x \in C_i$ and it will switch to regime $j \neq i$ as soon as $x \in SW_i$.

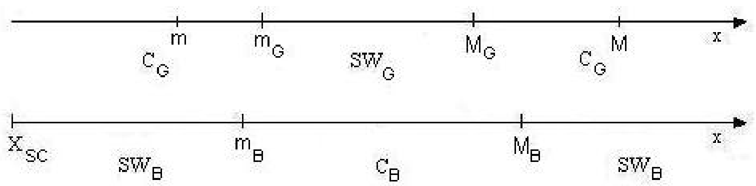

Figure 2.5: An example of more than one switching point for each regime.

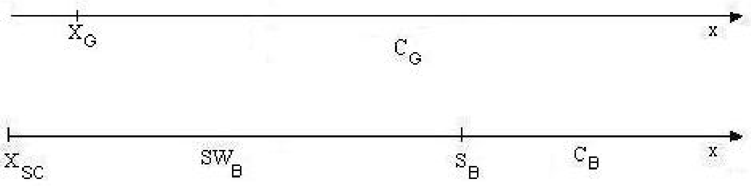

Figure 2.6: Only one switching point: from regime Bad to regime Good

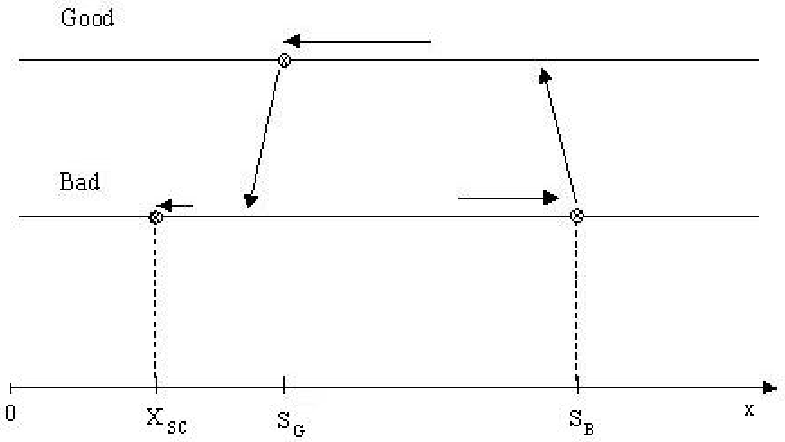

Figure 2.7: Optimal switching decisions and the optimal operation regions.

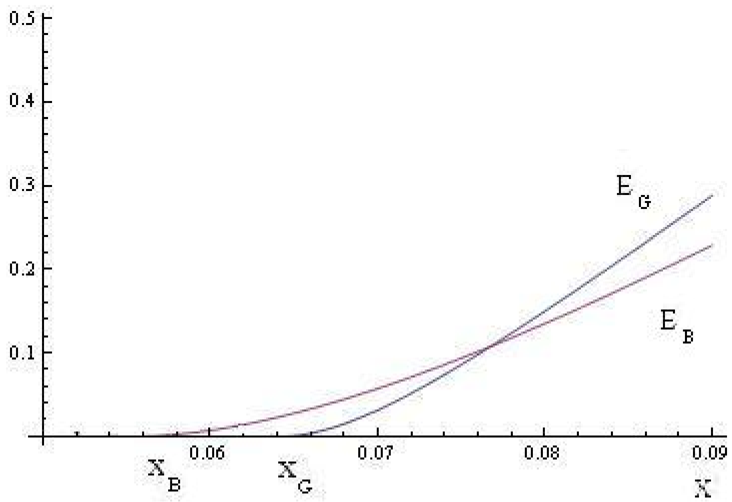

Figure 2.8: Trivial cases.

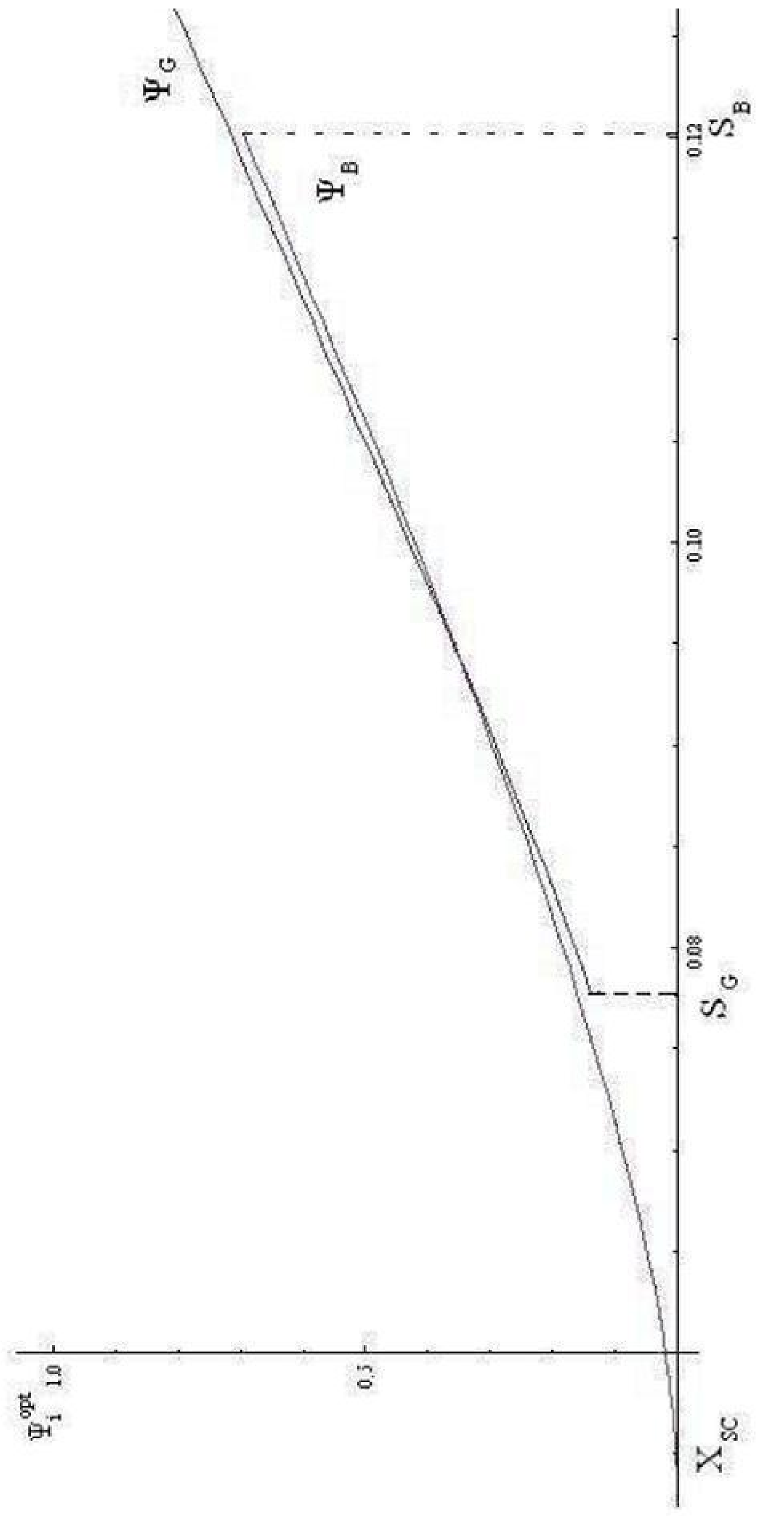

Figure 2.9: A general case in optimal switching-stopping, $k = 0.02$.

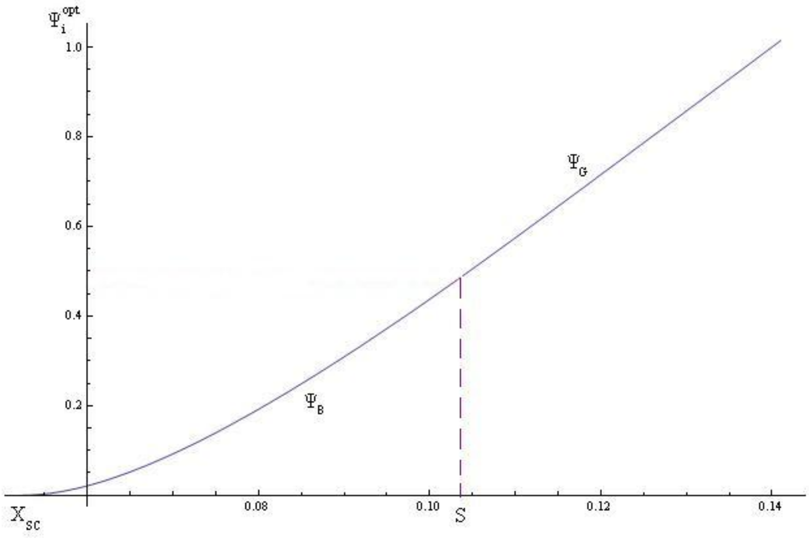

Figure 2.10: Costless switching.

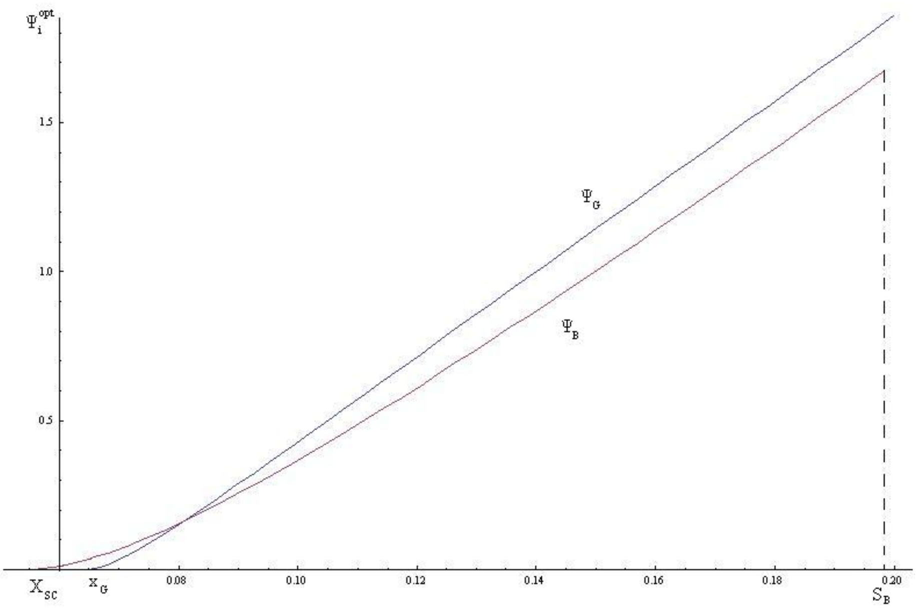

Figure 2.11: Too costly switching, $k = 0.16$.

Chapter 3

Switching Models for Banking: Is Gambling for Resurrection Valid?

3.1 Introduction

"Gambling for resurrection" has been covered in the financial as well as political economics literature. The conventional wisdom is that an agent under distress gambles and takes higher risk in order to survive. Under standard assumption that a bank can choose one of two different regimes of operation, i.e. one portfolio with a higher return and another portfolio with a higher risk, the relationship between a bank's capital and risk level is monotonic. The contribution of this paper is to show that the monotonicity does not always hold true.

In the first part, this paper examines the endogenous choice of risk-return regime for risk-neutral bankers who maximize the equity value. The bank's wealth is normalized with respect to the deposit value, and there is hence a one to one relationship between the net wealth and the capital (equity). With limited liability and fully insured deposits the bank increases its risk, loosing part of the expected return of the investment. However, it can then operate under a safer regime when having enough wealth. In a discrete time static model the "cutoff values" below which a bank takes more risk are found in terms of the net wealth. The risk-taking strategy is mainly influenced by the cost of effort to reduce the risk. If the bank could operate without an extra cost to monitor its creditors, it would increase the risk only below a unique level of the initial wealth. However, if the bank has to pay for having a better chance to succeed in its operation, the optimal risk-taking strategy is obtained by multiple cutoff values. In fact, a bank may go bankrupt at failure with a higher level of the initial wealth under the less risky portfolio because of the effort cost. This can lead to the multiple cutoff value policy.

The second part of this paper extends the setup to a dynamic model to investigate the intertemporal risk-taking behavior. The two periods of the model are connected to each other through the dividend policy. At the beginning of each period the bank chooses a risk-return regime of operation by maximizing the net present value (NPV). The main finding in this part is that beside gambling for resurrection a bank may reduce its risk by switching from the risky regime to the safer regime if the low risk portfolio is sufficiently advantageous. The optimal risk-return choice varies depending on the capital level and the dividend payment. The risk reduction or gambling for resurrection strategy is impacted also by the interest rate which determines the deposit payment. Due to the assumption that failure brings bankruptcy, the uniqueness of a cutoff value is robust with respect to the effort cost.

Risk-taking behavior has been widely discussed in the literature, mostly as a base for regulation studies. Decamps et al. (2004) verify Basel (II) regulatory policies in a continuous-time model but without dynamic regime switching. I follow them to specify effort cost to the safer regime. The discrete-time model of my paper which includes endogenous regime switching contributes to this avenue of studies. A good reference of switching models is the continuous-time model of Dangl and Lehar (2004). They assume standard gambling for resurrection with two switching points which are identical in the absence of switching cost. Nevertheless, a continuous time model does not necessarily have closed form solutions. This obstacle makes it impossible to verify the bank's endogenous portfolio choice analytically in a general continuous time model. Therefore, in this chapter I structure the bank's problem in a discrete-time model.

The model presented in this chapter is generalized comparing to Dangl and Lehar (2004) in the sense that both return and risk level of portfolio change from one regime to another. Further, I define no deficiency assumption on any regime to the extent that even the riskier regime may have positive net present value (NPV). This is different from the assumption in Decamps et al. (2004) which gives priority to liquidation rather than operating under the riskier regime. Also in credit rationing, Stiglitz and Weiss (1981) assign the riskier projects for being inefficient. In contrary, in this paper I discuss advantages and disadvantages of each regime, free from deficiency assumptions.

The focus of studies in some other literature is on the risk of creditors or firms. In that view, the bank has to take a monitoring position to avoid risky creditors. Stiglitz and Weiss (1981) relate the risk-taking behavior of creditors to the interest rate and analyze credit rationing. In their paper, as demand and supply of loans are functions of the interest rate, it plays the role of screening device for the bank. High interest rates attract riskier borrowers and decrease the bank's expected profit. The bank is reluctant to take risk and monitors its creditors through the interest rate at which they are willing to borrow. In the lender-borrower relationship, Berlin and Mester (1992) define a bank's role in preventing a firm's gambling for resurrection. The bank may receive a noisy signal indicating the success/failure of the firm in order to then allow/restrict renegotiation of loan covenants.

My work is in the class of studies concerned with regulating the banks' risk-taking and deal directly with the risk incentives. Diamond and Dybvig (1983) study bank deposit contracts and risk-taking incentives of bank-managers which lead to speculative bank-runs. Mailath and Mester (1994) solve the bank-regulatory game in a two period model in which the bank accesses to one risk-free and one risky asset. They explain regulatory forbearance and how the regulatory agency cannot commit ex ante to a tough closure policy. The bank takes higher risk and the regulator wants to impose closure before the net present value of the bank's assets become negative. However, from a social welfare perspective, it is almost always optimal to let an under-capitalized bank continue to operate. This generates bad incentives for the bankers from an ex ante point of view to take risk. Acharya and Yorulmazer (2007) observe a herding behavior among many banks to increase the risk as a result of the managers' moral hazard, and the regulator's problem regarding closure policy. Cordella and Yeyati (2003) analyze the moral hazard problem within a multi-period model but assume independent risk-taking strategy in each period. The existing risk-taking studies can be summerized as either an agent chooses between a safe asset and a risky investment in a one period model or if a dynamic model is prsented, each period is independent and unaffected by other periods. This motivates my work to challenge the classical idea of gambling for resurrection, allowing fully endogenous risk-taking behavior.

The following section sets up a one period model to determine the switching cutoff values and the associated policies. Section 3.3 develops the setup to a two period model. In section 3.4 the optimal dividend policy is investigated. Section 3.5 presents some numerical examples to illustrate the result. Section 3.6 concludes. The appendix includes the proof of a remark.

3.2 The One-Period Model with Discrete Return

Assume a risk neutral world. The initial status of the bank is W_0 which consists of initial equity, $A_0 \geq 0$, and deposit principal normalized to one, i.e. $W_0 = A_0 + 1$. Deposits are fully insured and shareholders have limited liability. Operation of the bank has a constant returns to scale technology with rate of return (RR) z_i under regime $i \in \{0, 1\}$. At the end of period, stochastic variable z_i returns R_i in case of success which occurs with probability P_i under regime i. In case of failure, the rate or return of regime i is r_i,

$$z_i = \begin{cases} R_i > 0 & \text{with probability } P_i, \\ r_i > -1 & \text{with probability } 1 - P_i. \end{cases} \tag{3.1}$$

Following Stiglitz and Weiss (1981), assume that regime 1 has higher expected return and more concentrated distribution than regime 0. This means that $\mu_1 > \mu_0$ where $\mu_i = P_i R_i + (1 - P_i) r_i$ but $R_1 < R_0$ and $r_0 \leq r_1$. It must be then $P_1 > P_0$. Further, I assume that regime 1 bears monetary effort cost $e \geq 0$. The cost can be interpreted as expenses of monitoring creditors (delegated monitoring as in Diamond (1984)). By definition, none of the two regimes is essentially inefficient or less preferable. The equity value at time $t = 1$ is,

$$W_1 = max(0, (W_0 - ie)(1 + z_i) - C). \tag{3.2}$$

In order to create incentives for the bank to monitor its creditors, the added value of high effort regime should exceed its cost, i.e. for $W_0 \geq 1$,

$$W_0(1 + \mu_1) - W_0 > e(1 + \mu_1) \quad \Leftrightarrow \quad e < \frac{\mu_1}{1 + \mu_1}. \tag{3.3}$$

The risk free interest rate in the market is $0 < r_f < 1$, where $r_f < R_i$ for $i \in \{0, 1\}$. Generally the bank faces two optimization problems. First it must decide on how much capital to invest in a risky regime, and second it must decide which regime to take. Due to the risk neutrality assumption, the bank invests either all the wealth or nothing. We can thus translate the bank's first problem to an entrance decision. The bank decides to enter the market and starts operation if the expected profit of investing in a risky regime is more than the expected value of saving the initial capital. Define discount rate $\beta = \frac{1}{1+r_f}$, the bank take any risky regime i if,

$$A_0 \leq \beta E(W_1, i). \tag{3.4}$$

For the bank to start operation, a necessary condition is that the bank must be solvent if the operation succeeds. Otherwise, the bank would never choose that regime of operation. This requires for any regime i,

$$W_0 \geq \frac{C}{1 + R_i} + ie, \ i \in \{0, 1\}. \tag{3.5}$$

Since $\frac{C}{1+R_0} < \frac{C}{1+R_1} + e$, the high effort regime may cause insolvency where the low effort operation is solvent.

Yet, if the NPV is negative the bank has incentive to operate under a risky regime because of a higher return (comparing to the risk free RR) in case of success. Therefore, solvency and profitability of a risky regime i is the necessary and sufficient condition for the bank to take deposit and start operation under that regime,

$$A_0 \leq \beta P_i((A_0 + 1 - ie)(1 + R_i) - C). \tag{3.6}$$

To solve the inequality for A_0, the sign of $\beta P_i(1+R_i)-1$ is important. This term can be interpreted as the NPV of success return for investing one unit in risky regime i, although the effort cost of regime 1 must be considered too. Many papers, e.g. Mailath and Mester (1994), assume that a risky regime has negative NPV. With their assumption the result is gambling for resurrection: if $\beta P_i(1+R_i) < 1$, the bank chooses risky investment i where the capital is below a threshold,

$$A_0 \leq \frac{\beta P_i(C-(1-ie)(1+R_i))}{\beta P_i(1+R_i)-1}. \tag{3.7}$$

which is meaningful (positive) iff its numerator is negative. Alternatively, suppose success of any risky regime is profitable,

$$\beta P_i(1+R_i) > 1, \quad i \in \{0,1\}. \tag{3.8}$$

Contrary to the classical idea, the bank operates under a risky regime iff,

$$A_0 \geq \frac{\beta P_i(C-(1-ie)(1+R_i))}{\beta P_i(1+R_i)-1}. \tag{3.9}$$

The unique cut-off value policy obtains for the entrance decision if the numerator is positive. Otherwise, the bank starts operating under a risky regime for any positive level of initial capital.[1] When (3.9) holds, the bank enters the market and takes deposits. Hence, for the initial wealth we have,

$$W_0 \geq G_i = \frac{\beta P_i(C+ie(1+R_i)-1)}{\beta P_i(1+R_i)-1}. \tag{3.10}$$

Now focus on the bank's second problem: optimization of risk-return regime. Regime i is risky for some level of initial wealth if its failure brings out insolvency, i.e. $W_0 < \frac{C}{1+r_i} + ie$. Without loss of generality suppose $\frac{C}{1+r_0} < \frac{C}{1+r_1} + e$. Again the high effort regime though succeeds with a higher probability causes insolvency in case of failure for initial wealth $\frac{C}{1+r_0} < W_0 < \frac{C}{1+r_1} + e$ where the bank under the low effort regime is solvent.[2] Correspondingly, the regime choice is different in each possible case. The bank optimally chooses regime i_0 at the beginning of the period to maximize the expected value of its equity,

$$E(W_1, i_0) \geq E(W_1, i), \quad i_0 \neq i \in \{0,1\}. \tag{3.11}$$

The following proposition describes the bank's optimal strategy.

[1] Cordella and Yeyati (2003) have similar result, albeit with respect to rate of returns and interest rate.

[2] Without the assumption regarding the thresholds, proposition 3.1 has to be repeated for a case $1+\mu_1 > P_0(1+R_0)$, but the result does not change.

Proposition 3.1 ***(I)*** *Suppose for expected returns $P_1(1+R_1) > P_0(1+R_0)$.*
(a) *If $P_1(1+R_1) > 1+\mu_0$, the regime choice is characterized by a unique cutoff, i.e. for each set of parameters there exists only one cutoff value, in terms of net wealth, above which the bank takes less risk. Figure 3.1 shows possible cases.*
(b) *If $P_1(1+R_1) < 1+\mu_0$, multiple cutoff values characterize the optimal regime choices in two out of five feasible orders of thresholds, $S_1, S_2, S_3, \frac{C}{1+r_0}$ and $\frac{C}{1+r_1}+e$, as shown by figure 3.2. In each multiple-cutoff strategy three cutoff values in terms of net wealth exist such that the bank's regime choice differs from lower to higher than each. In other cases, unique cutoff value gives the optimal regime choices.*
(II) *If $P_0(1+R_0) > P_1(1+R_1)$, then unique cutoff value policy is optimal.*

Proof (I.a) As failure of a regime may cause bankruptcy, find initial wealth levels associated to each likely failure case. Three cases are possible: the bank is solvent for any return of each regime, it is insolvent only at failure of regime 1, it is insolvent at a failure. Compare expected profits under two regime choices.

1. Suppose the bank is solvent for all returns, i.e. $W_0 \geq \frac{C}{1+r_1} + e$. From (11) regime 1 makes the bank better off iff

   $$(W_0 - e)(1+\mu_1) - C \geq W_0(1+\mu_0) - C.$$

   Consequently, the bank chooses regime $i_0 = 1$ iff

   $$W_0 \geq S_1 = e(1+\mu_1)/(\mu_1 - \mu_0), \tag{3.12}$$

   and $i_0 = 0$ otherwise.

2. When $\frac{C}{1+r_0} \leq W_0 < \frac{C}{1+r_1} + e$, failure of regime 1 makes the bank insolvent because of effort cost but at return r_0 the bank is still solvent. It chooses regime $i_0 = 1$ iff

   $$\begin{aligned} &P_1((W_0 - e)(1+R_1) - C) \geq W_0(1+\mu_0) - C \quad \Leftrightarrow \\ &W_0 \geq S_2 = \frac{(P_1 - 1)C + eP_1(1+R_1)}{P_1(1+R_1) - 1 - \mu_0} \end{aligned} \tag{3.13}$$

3. Neither r_0, nor r_1 yield solvency, i.e. $W_0 < \frac{C}{1+r_0}$. The bank prefers the high effort regime iff

   $$\begin{aligned} &P_1((W_0 - e)(1+R_1) - C) \geq P_0(W_0(1+R_0) - C) \quad \Leftrightarrow \\ &W_0 \geq S_3 = \frac{C(P_1 - P_0) + eP_1(1+R_1)}{P_1 - P_0 + P_1R_1 - P_0R_0}. \end{aligned} \tag{3.14}$$

In spite of having three thresholds, notice that $S_1 > \frac{C}{1+r_1} + e$ contradicts $S_2 < \frac{C}{1+r_1} + e$ and vice versa,

$$S_2 < \frac{C}{1+r_1} + e \quad \Leftrightarrow$$
$$\frac{((P_1-1)(1+r_1) - [P_1(1+R_1) - 1 - \mu_0])C}{(P_1(1+R_1)-1-\mu_0)(1+r_1)} <$$
$$\frac{e[(P_1(1+R_1)-1-\mu_0) - P_1(1+R_1)]}{P_1(1+R_1)-1-\mu_0}$$
$$\Leftrightarrow \quad \frac{C}{1+r_1} > \frac{e(1+\mu_0)}{\mu_1 - \mu_0}.$$

Therefore, where $S_1 > \frac{C}{1+r_1} + e$, S_2 is not feasible as $S_2 > \frac{C}{1+r_1} + e$. Then optimal regime is 0 for all $\frac{C}{1+r_0} < W_0 < \frac{C}{1+r_1} + e$. Yet, $S_2 > \frac{C}{1+r_0}$ contradicts $S_3 < \frac{C}{1+r_0}$ and vice versa.

$$\frac{C}{1+r_0} < S_2 \quad \Leftrightarrow$$
$$\frac{C(P_1(1+R_1)-1-\mu_0-(P_1-1)(1+r_0))}{(P_1(1+R_1)-1-\mu_0)(1+r_0)} < \frac{eP_1(1+R_1)}{P_1(1+R_1)-1-\mu_0} \tag{3.15}$$

$$\Leftrightarrow \quad \frac{C(P_1(1+R_1)-P_0(1+R_0)-(P_1-P_0)(1+r_0))}{(1+r_0)(P_1(1+R_1)-1-\mu_0)} < \frac{eP_1(1+R_1)}{P_1(1+R_1)-1-\mu_0} \tag{3.16}$$

$$\Leftrightarrow \quad \frac{C(P_1(1+R_1)-P_0(1+R_0)-(P_1-P_0)(1+r_0))}{(1+r_0)(P_1(1+R_1)-P_0(1+R_0))} < \frac{eP_1(1+R_1)}{P_1(1+R_1)-P_0(1+R_0)} \tag{3.17}$$

$$\Leftrightarrow \quad S_3 > \frac{C}{1+r_0},$$

Thus, the optimal regime is 0 for all $W_0 \leq \frac{C}{1+r_0}$. It means that S_1 is the unique cutoff value. In a similar way, we end up having S_2 or S_3 as a unique cutoff value iff one of them is feasible, ruling out feasibility of two others.

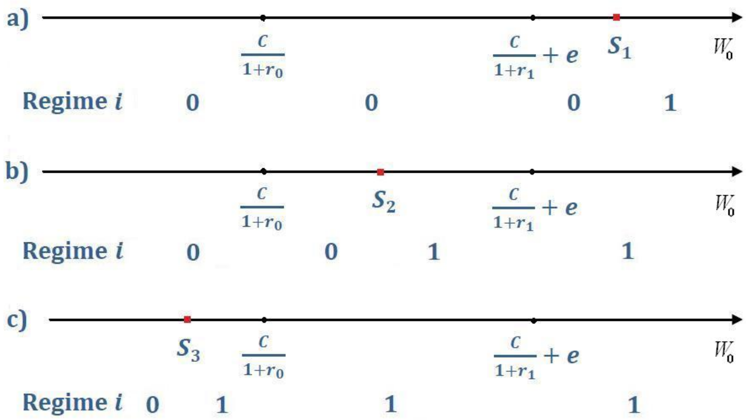

Figure 3.1: Switching strategy in a one period model with discrete return, expected gross success-return of regime 1 exceeds expected return of regime 0.

(I.b) Under assumption $P_1(1 + R_1) < 1 + \mu_0$, the inequality (3.13) changes. In other words, now regime 1 is the optimal choice for $W_0 \leq S_2$, and regime 0 is optimal otherwise. In contrast to part (I.a), inequalities (3.16) and (3.17) reverse. Here $S_2 > \frac{C}{1+r_0}$ requires $S_3 < \frac{C}{1+r_0}$, and $S_2 < \frac{C}{1+r_1} + e$ directs to $\frac{C}{1+r_1} < S_1$. If S_1 is feasible, then feasibility of S_2 makes S_3 feasible, case (a) in figure 3.2. However, S_2 might be lower than $\frac{C}{1+r_0}$. Then for $\frac{C}{1+r_0} \leq W_0 < \frac{C}{1+r_1} + e$ we have $i_0 = 0$. But it means that S_3 is above $\frac{C}{1+r_0}$ and the optimal regime is 0 where $W_0 < \frac{C}{1+r_0}$. Case (b) shows the unique cutoff value S_1. With feasible S_3, feasibility of S_2 rather than case (a) can also lead to case (c) where S_1 is above $\frac{C}{1+r_1}$ and still infeasible. Hence, three cutoff values are S_3, S_2 and $\frac{C}{1+r_1} + e$. In case (d), S_1 is infeasible but $\frac{C}{1+r_1} < S_1$. This demands $S_2 < \frac{C}{1+r_1} + e$, but we have infeasibility of $S_2 < \frac{C}{1+r_0} < S_3$. Therefore, $\frac{C}{1+r_1}$ is the unique cutoff value. The last possible case is (e) in figure 3.2. There, feasible S_3 appears with $S_2 > \frac{C}{1+r_1} + e$ which brings out S_1 infeasible. Hence, S_3 is the unique cutoff value, as below S_2 between two boundaries $\frac{C}{1+r_0}$ and $\frac{C}{1+r_1} + e$ the optimal strategy is regime 1.

(II) In a similar approach as (I.a), we observe the bank is reluctant to exert effort. Since $S_3 < 0$, below $\frac{C}{1+r_0}$ the optimal choice is regime 0. Still, if $P_1(1 + R_1) > 1 + \mu_0$, then $S_2 > \frac{C}{1+r_0}$ because $S_3 < 0 < \frac{C}{1+r_0}$. In this case, either S_1 or S_2 is the unique cutoff value (cases (a) and (b) in figure 3.1). But $P_1(1 + R_1) < 1 + \mu_0$ causes $S_2 < \frac{C}{1+r_0}$. Therefore, either S_1 is feasible and the unique cutoff value or it is only above $\frac{C}{1+r_1}$, making $\frac{C}{1+r_1} + e$ the unique cutoff value (cases (b) and (d) in figure 3.2).$\square$

Remark 3.1 In part (I.b) of proposition 3.1, the expected return of the high effort regime equals its return in the likely case of success and that is lower than the expected return of the risky regime. It occurs when the return of regime 0 in case of failure is not too small and this motivates for taking higher risk of regime 0. Thus, if failure

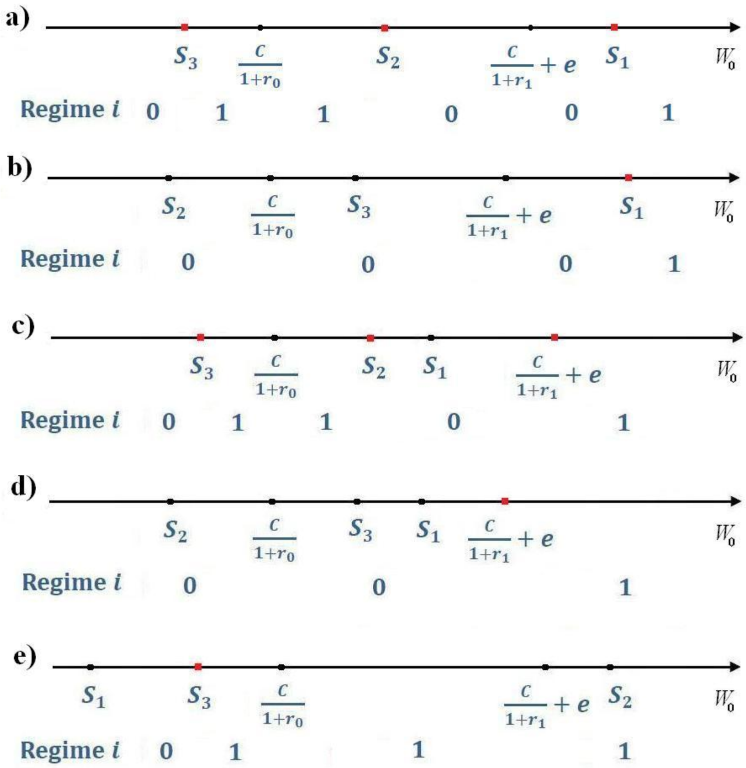

Figure 3.2: Switching strategy in a one period model with discrete return, expected return of regime 0 surpass expected gross success-return of regime 1.

makes the bank insolvent under regime 1 but solvent for regime 0, as above S_2, the bank prefers regime 0. However, effort cost is small enough such that the bank exerts effort to gain more through higher probability of success. This can be seen in the interval between feasible S_2 and S_3. Insolvency at failure of either regime 0 or 1 makes regime 1 more interesting since its failure has a lower probability. Under assumption $P_1(1+R_1) < P_0(1+R_0)$ in part (II), in expectation the bank is more profitable if succeeds in regime 0 than in regime 1. Hence, it is reluctant to choose the safer regime. This brings out a unique cutoff policy, with a large cutoff value comparing to all other cases.

Remark 3.2 In the benchmark case without effort cost, i.e. $e = 0$, the optimal regime is $i_0 = 1$ because of its higher expected return, as long as $W_0 \geq \frac{C}{1+r_0}$. Note that now the order changes for boundaries $\frac{C}{1+r_1} < \frac{C}{1+r_0}$. Within the same method as proposition 3.1, cutoff values are found based upon the similar assumptions. Nevertheless, the optimal regime choice is given by a unique cutoff value for each case. The proof is included in the appendix.

Comparing the results of remark 3.2 and proposition 3.1, we see that effort cost plays an important role for the bank's choice of the regime of operation. The classical gambling for resurrection obtains as long as there is no effort cost. This result is for instance seen in Dangl and Lehar (2004) if changing from one regime to another, i.e. regime-switch, costs nothing. While the switching cost yields separate thresholds for the risk reduction and gambling for resurrection, the effort cost for a portfolio with higher expected return induces non-monotone regime choices. Decamps et al. (2004) consider an effort cost in their continuous time model, but they assume negative NPV of the riskier regime which makes it worse than no operation. Therefore, they obtain a standard gambling for resurrection cutoff policy. The model in this paper shows that effort cost on the one hand and no deficiency (no negative NPV) of risky regimes on the other hand are the source of a multiplicity of cutoff values.

Remark 3.3 *Robustness Check of Proposition 3.1 for Non-monetary Effort Cost:* Alternatively, there might be non-monetary effort cost for regime 1, which does not affect the return of the portfolio but inflicts an additional monitoring cost on the bank. Consequently, the bank has to pay e at the end of the period and the equity value is

$$W_1 = (1 + z_{i_0})W_0 - i_0 e - C. \tag{3.18}$$

Then the solvency value in terms of net wealth is

$$W_0 \geq \frac{i_0 e + C}{1 + z_{i_0}}. \tag{3.19}$$

Assume that $\frac{C}{1+r_0} < \frac{e+C}{1+r_1}$. With the same value of e, regime choice cutoff values turn out to be smaller compared to the original model with monetary cost, since the effort cost is paid out once returns are realized. For high level of net wealth above $\frac{e+C}{1+r_1}$, between two boundaries and below $\frac{C}{1+r_0}$, the bank brings effort iff W_0 is, respectively in each interval, above the following cutoff values,

$$\hat{S}_1 = \frac{e}{\mu_1 - \mu_0}, \tag{3.20}$$

$$\hat{S}_2 = \frac{(P_1 - 1)C + eP_1}{P_1(1 + R_1) - (1 + \mu_0)} \tag{3.21}$$

$$\hat{S}_3 = \frac{C(P_1 - P_0) + eP_1}{P_1(1 + R_1) - P_0(1 + R_0)}. \tag{3.22}$$

Nonetheless, proposition 3.1 including some multi-cutoff strategies is satisfied.

We can interpret regime strategies in the one period model as a short-run decision in a dynamic model. However, the short-run decision can be different from the long-run decision. If at the beginning of each period the bank optimizes not only the profit of the end of period but also a stream of future profits the decision for risk taking may

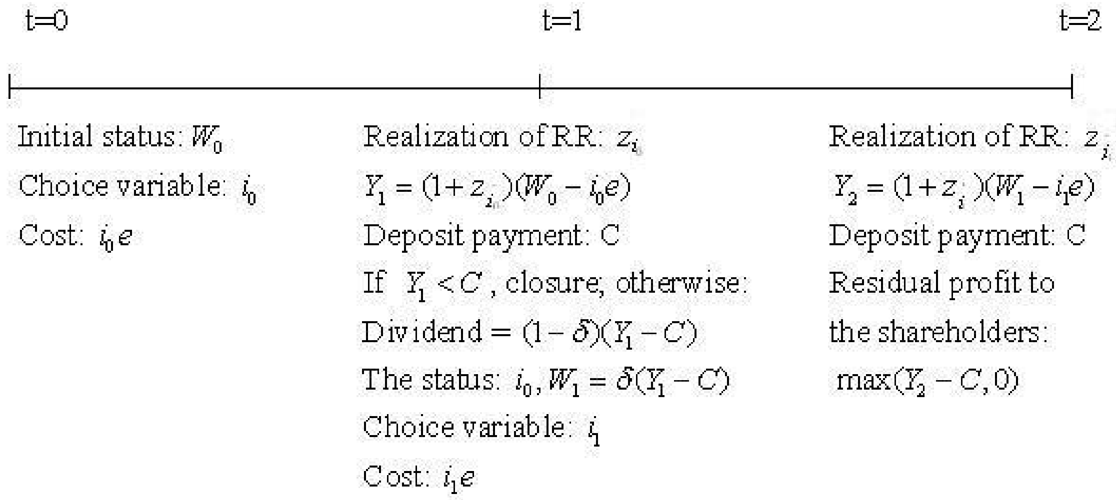

Figure 3.3: The Time-line of two-period model

change. The intuition is that the bank faces an intertemporal decision on its profit. For instance, the profit of one period operation adds on the bank's wealth which determines the regime of operation for the next period. Since the result of one period influences next periods, the bank has to take into account the consequences of its today's decision on the future. To capture the intertemporal effects, in the following section I analyze the optimal behavior of banks in a dynamic setup of two periods.

3.3 The Two-Period Model with Discrete Return

Suppose there are three dates, $t = 0, 1$ and 2. At the beginning of period one, $t = o$, the bank receives deposits normalized to 1 that it has to pay back in equal payments C at $t = 1, 2$. Having initial equity, the initial wealth W_0 exceeds principal.[3] At the end of the first period the bank has to pay dividend out of positive profit. The dividend is assume to be an exogenously given fraction of the first period outcome less deposit payment, $(1 - \delta)(Y_1 - C) > 0$. The remaining wealth, $\delta(Y_1 - C) > 0$, covers effort cost and generates outcome in the next period.[4] At $t = 2$ residual profits (after deposit payment) are paid to shareholders. Therefore, the bank on behalf of the shareholders aims to optimize the dividend of the first period added to the final profit. The discount rate is $0 < \beta < 1$. The rate of return (RR) is z_i given by equation (3.1). Figure 3.3 sketches the timing of the model.

[3]Alternatively, deposits could be defined as being rolled over each period. This would however not affect the results.

[4]Note that $0 < \delta < 1$ represent the reinvestment ratio. This notation makes the further calculations simpler.

Again I assume that there is monetary effort cost e for regime 1. The bank may switch at $t = 1$ to a different regime of operation in the second period, i.e. $i_0 \neq i_1$. Different form Dangl and Lehar (2004), I abstract from switching costs in the present model in which the low risk induces effort costs. The second period operation and regime choice are known from the one period model. The first period decisions are affected by the second period, as the bank has to consider its net present value of two periods operation. The best strategy at time $t = 0$ is the solution to the following optimization problem:

$$max_{i_0} \qquad E((1-\delta)\, max(Y_1 - C, 0) + \beta\, max(Y_2 - C, 0)), \tag{3.23}$$

where

$$Y_1 = (1 + z_{i_0})(W_0 - i_0 e), \tag{3.24}$$

$$W_1 = max(0, \delta(Y_1 - C)), \tag{3.25}$$

$$Y_2 = \begin{cases} (1 + z_{i_1})(W_1 - i_1 e) & \text{for } W_1 > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{3.26}$$

The wealth of the bank must cover the deposit payment, dividend and the cost of operation under an appropriate regime. After the realization of the return at the end of each period the bank is solvent if its associated level of wealth is larger than the deposit payment. However, at $t = 1$ after paying for deposit and dividend the bank may not have enough money for continuation under either regime. Then the bank closes and pays the remaining wealth to the shareholders. Hence, when making its decision regarding i_0 and i_1 the bank must consider costs and solvency. That means for some range of W_0 even success in the first period is not enough to continue operation. For such values, the model reduces to the one period model already discussed in the last section. Therefore, a second period will only be relevant if $Y_2 - C > 0$. Indeed, first it is required $Y_1 - C > 0$ which is verified in the last section. The sufficient condition

$$\begin{aligned} &Y_2 - C > 0 \Leftrightarrow \\ &(\delta((1 + z_{i_0})(W_0 - i_0 e) - C) - i_1 e)(1 + z_{i_1}) - C > 0 \end{aligned} \tag{3.27}$$

extends to several cases under each some outcome cannot be solvent. The reverse cases, where $Y_2 < C$, can be described by

$$z_{i_1} < \frac{C}{\delta((1 + z_{i_0})(W_0 - i_0 e) - C) - i_1 e} - 1. \tag{3.28}$$

This determines the relation between the returns of the first period and the second period which does not bring solvency for the bank at the end of two periods operation.

3.3.1 No Risk of Insolvency

For now focus on the very special case where initial wealth is sufficiently high such that all returns of the first and the second period are solvent. From (3.25) it means that for all values of z_{i_0} and z_{i_1} where $i_0, i_1 \in \{0, 1\}$, initial wealth should exceed

$$W_0 > T(z_{i_0}, z_{i_1}) = \frac{C(1+\delta(1+z_{i_1}))}{\delta(1+z_{i_0})(1+z_{i_1})} + \frac{e(i_1 + i_0\delta(1+z_{i_0}))}{\delta(1+z_{i_0})}. \tag{3.29}$$

Proposition 3.2 *If initial wealth of the bank satisfies (3.29), the unique cutoff value policy holds if $S_1 > T(z_{i_0}, z_{i_1})$. Otherwise the bank never chooses regime 0. Therefore, risk-return choice of each period is independent of another period.*

Proof To find switching strategies, the model is solved by backward induction. The solution to the second period is the same as the one period model with W_1 as initial wealth. Consequently from (3.12), the bank operates under regime $i_1 = 1$ iff $W_1 \geq S_1$, i.e.

$$W_1 \geq \frac{e(1+\mu_1)}{\mu_1 - \mu_0}, \tag{3.30}$$

and regime 0 otherwise. To solve the first period optimization problem, assume the bank operates under a given regime i_1 in the second period. Plug (3.24)-(3.26) in (3.23). Since $Y_2 > C$, the objective function follows

$$\begin{aligned} max_{i_0} \quad & E[(1-\delta)((1+z_{i_0})(W_0 - i_0 e) - C) + \\ & \beta((1+z_{i_1})(\delta((1+z_{i_0})(W_0 - i_0 e) - C) - i_1 e) - C)] \end{aligned} \tag{3.31}$$

which is, using equation (3.25), equivalent to

$$\begin{aligned} max_{i_0} \quad & (1-\delta)[(1+\mu_{i_0})(W_0 - i_0 e) - C] \\ & +\beta[(1+\mu_{i_1})(\delta((1+\mu_{i_0})(W_0 - i_0 e) - C) - i_1 e) - C] \end{aligned}$$

Rearranging yields

$$\begin{aligned} max_{i_0} \quad & (1+\mu_{i_0})[1-\delta+\beta\delta(1+\mu_{i_1})]W_0 \\ & -((1+\mu_{i_0})[1-\delta+\beta\delta(1+\mu_{i_1})]i_0 + \beta(1+\mu_{i_1})i_1)e \\ & -(1-\delta+\beta+\beta\delta(1+\mu_{i_1}))C. \end{aligned} \tag{3.32}$$

The optimal regime choice of $t = 0$ maximizes the net present value of two periods for any given regime in the second period. Therefore, for each i_1 the bank is better off by choosing $i_0 = 1$ if the expected value of two periods under $i_0 = 1$ is better than or equal to the expected value under $i_0 = 0$. This condition can be simplified to

$$(1 + \mu_0)[1 - \delta + \beta\delta(1 + \mu_{i_1})]W_0 \leq \\ (1 + \mu_1)[1 - \delta + \beta\delta(1 + \mu_{i_1})]W_0 - (1 + \mu_1)[1 - \delta + \beta\delta(1 + \mu_{i_1})]e,$$

which gives

$$W_0 \geq \frac{e(1 + \mu_1)}{\mu_1 - \mu_0}. \tag{3.33}$$

The threshold is identical to S_1 which implies that the first period decision is independent of the second period.

Hence, I can consider each period in isolation. This is a result of the assumption that the dividend ratio is given exogenously. Since the bank is solvent for all returns, the future outcome does not affect the current situation. However, the feasibility condition requires the threshold to be greater than $T(z_{i_0}, z_{i_1})$ for $i_0, i_1 \in \{0, 1\}$. If not, the bank chooses only the regime with a higher expected return. □

In addition to the independence of the regime choices in the two periods for the special case above, the cutoff value is only affected by average returns. No return makes the bank insolvent and the risk is irrelevant. The variances and return intervals thus do not appear in the regime choice decisions for this situation. The bank chooses the low effort regime only if it cannot afford the effort cost associated with the high mean return. The effect of variances in regime strategies are examined in the next subsection that involves some risk of insolvency.

3.3.2 Operating under Risk of Bankruptcy

When the outcome of the first period is low such that the bank needs higher outcome in the second period, condition (3.29) is crucial. For some cases, failure may cause insolvency, but even success return may not be sufficient for one more period operation. The general setting is explored in the next section as part of numerical example, since equation (3.29) extends to too many conditions which cannot be solved in a general case. To obtain analytical solutions and gain intuition, I have to limit the setting to a simple benchmark. Now, assume the extreme case in which the bank loses total wealth and goes bankrupt if it fails. It means that RR z_i from (3.1) returns $r_i = -1, i = 0, 1$ in case of failure. Then similar to proposition 3.2, we compare the expected returns of operation under two alternative regimes. The bank asserts effort iff the success of regime 1 is more profitable than success of regime 0,

$$P_1[(1 + R_1)(W_1 - e) - C] \geq P_0[(1 + R_0)W_1 - C].$$

That gives the unique cutoff point of the second period (hence in terms of W_1), below which the bank chooses riskier regime,

$$W_1 \geq S_3 = \frac{C(P_1 - P_0) + eP_1(1 + R_1)}{P_1(1 + R_1) - P_0(1 + R_0)}. \tag{3.34}$$

In the next step, I apply backward induction to solve for the bank's regime choices in the first period. Proposition 3.3 describes switching and cutoff strategies under the risk of bankruptcy, i.e. (3.29) does not hold or (3.27) is violated.

Proposition 3.3 *When $r_i = -1$, $i = 0, 1$, for an exogenous $\delta \in (0, 1)$, the unique-cutoff policy optimizes risk-return regime of the first period. There exist a nonempty switching area, in terms of net wealth.*

Proof The bank chooses i_0 by maximizing objective function (3.23) which gives

$$\begin{aligned} max_{i_0} \quad & E[(1-\delta)\, max((1+z_{i_0})(W_0 - i_0 e) - C, 0) + \\ & \beta\, max((1+z_{i_1})(\delta((1+z_{i_0})(W_0 - i_0 e) - C) - i_1 e) - C, 0)] \end{aligned} \tag{3.35}$$

The optimal choice is affected by i_1 since not all returns have positive value for the bank. Although, i_1 is known by the threshold in (3.34) at $t = 1$, the bank needs to realize it at $t = 0$. The operation continues for the second period only after success at the first period. Hence, substitute W_1 from (3.25) and (3.26) in (3.34), we have $i_1 = 1$ iff $W_0 \geq Q_{i_0}$ such that

$$Q_{i_0} = \frac{C(P_1 - P_0 + \delta(P_1(1 + R_1) - P_0(1 + R_0))) + eP_1(1 + R_1)}{\delta(P_1(1 + R_1) - P_0(1 + R_0))(1 + R_{i_0})} + i_0 e. \tag{3.36}$$

Since $Q_0 < Q_1$, the bank at $t = 0$ finds its optimal choice of the first period,
(I) $i_1 = 1$ iff $W_0 \geq Q_1$ where

$$Q_1 = \frac{C(P_1 - P_0 + \delta(P_1(1 + R_1) - P_0(1 + R_0))) + eP_1(1 + R_1)}{\delta(P_1(1 + R_1) - P_0(1 + R_0))(1 + R_1)} + e, \tag{3.37}$$

(II) $i_1 = 0$ iff $W_0 < Q_0$ where

$$Q_0 = \frac{C(P_1 - P_0 + \delta(P_1(1 + R_1) - P_0(1 + R_0))) + eP_1(1 + R_1)}{\delta(P_1(1 + R_1) - P_0(1 + R_0))(1 + R_0)}, \tag{3.38}$$

(III) $i_1 \neq i_0$ iff $Q_0 \leq W_0 < Q_1$, i.e. $[Q_0, Q_1]$ is a nonempty subset of switching area.

Now, I analyze the choice of the first period risk-return regime, i_0, in each of the three intervals. First assume $W_0 \geq Q_1$, then high effort regime is optimal iff

$$\begin{aligned}&P_1[(1-\delta)((1+R_1)(W_0-e)-C)+\beta P_1((1+R_1)(\delta((1+R_1)\\&(W_0-e)-C)-e)-C)] \;\geq\; P_0[(1-\delta)((1+R_0)W_0-C)+\\&\beta P_1((1+R1)(\delta((1+R_0)W_0-C)-e)-C)].\end{aligned}$$

Threshold U_1 obtains such that $i_0 = 1$ iff $W_0 \geq U_1$,

$$\begin{aligned}U_1 \;=\;& (C(P_1-P_0)(1-\delta+\delta\beta P_1(1+R_1)+\beta P_1)+\\& eP_1(1+R_1)(1-\delta+\delta\beta P_1(1+R_1)+\beta(P_1-P_0)))/\\& ((P_1(1+R_1)-P_0(1+R_0))(1-\delta+\delta\beta P_1(1+R_1))). \qquad (3.39)\end{aligned}$$

Next, if $Q_0 \leq W_0 < Q_1$ the bank switches from the regime it has at $t = 0$ to another regime at $t = 1$. Thus, there are two options of regime combination: $(i_0 = 1, i_1 = 0)$ and $(i_0 = 0, i_1 = 1)$. The bank is better off by the former regime combination iff

$$\begin{aligned}&P_1[(1-\delta)((1+R_1)(W_0-e)-C)+\beta P_0((1+R_0)\delta\\&((1+R_1)(W_0-e)-C)-C)] \;\geq\; P_0[(1-\delta)((1+R_0)W_0-C)+\\&\beta P1((1+R1)(\delta((1+R_0)W_0-C)-e)-C)].\end{aligned}$$

This gives threshold U_2 for W_0, below which the bank asserts no effort and takes higher risk,

$$\begin{aligned}U_2 \;=\;& (C((P_1-P_0)(1-\delta)+\delta\beta P_1P_0(R_0-R_1))+eP_1(1+R_1)(1-\delta\\& -\beta P_0+\delta\beta P_0(1+R_0)))/((P_1(1+R_1)-P_0(1+R_0))(1-\delta)). \qquad (3.40)\end{aligned}$$

For low initial wealth $W_0 < Q_0$, the bank chooses $i_0 = 1$ at $t = 0$, though it undertakes higher risk in the second period, iff

$$\begin{aligned}&P_1[(1-\delta)((1+R_1)(W_0-e)-C)+\beta P_0((1+R_0)(\delta((1+R_1)\\&(W_0-e)-C))-C)] \;\geq\; P_0[(1-\delta)((1+R_0)W_0-C)+\beta P_0\\&((1+R_0)(\delta((1+R_0)W_0-C))-C)].\end{aligned}$$

This demands $W_0 \geq U_3$ with threshold

$$\begin{aligned}U_3 \;=\;& (C(P_1-P_0)(1-\delta+\delta\beta P_0(1+R_0)+\beta P_0)+\\& eP_1(1+R_1)(1-\delta+\delta\beta P_0(1+R_0)))/\\& ((P_1(1+R_1)-P_0(1+R_0))(1-\delta+\delta\beta P_0(1+R_0))). \qquad (3.41)\end{aligned}$$

Nevertheless, only feasible thresholds are cutoff values that requires them to satisfy $U_3 \leq Q_0$, $Q_0 \leq U_2 < Q_1$ and $Q_1 \leq U_1$. Note that $Q_0 \neq Q_1$, and all the thresholds cannot be equal. Q_0 and Q_1 are both continuous, decreasing and convex in δ,

$$\frac{dQ_1}{d\delta} = \frac{-(C(P_1 - P_0) + eP_1(1 + R_1))}{\delta^2(P_1(1 + R_1) - P_0(1 + R_0))(1 + R_1)} < 0, \tag{3.42}$$

$$\frac{dQ_0}{d\delta} = \frac{-(C(P_1 - P_0) + eP_1(1 + R_1))}{\delta^2(P_1(1 + R_1) - P_0(1 + R_0))(1 + R_0)} < 0, \tag{3.43}$$

$$\frac{d^2Q_1}{d\delta^2} > 0, \ \frac{d^2Q_0}{d\delta^2} > 0. \tag{3.44}$$

Having equal denominators in U_1 and U_3, high probability of success and expected return in regime 1 brings $U_1 > U_3$. Moreover, these two monotone thresholds have monotone first derivatives with respect to δ. Also U_2 is increasing and convex in δ,

$$\begin{aligned} U_1 - U_3 \ = \ & (C\beta[(P_1 - P_0)(1 - \delta) + \delta\beta P_0 P_1(R_1 - R_0)] + \\ & eP_1(1 + R_1)(P_1 - P_0))/((P_1(1 + R_1) - P_0(1 + R_0)) \\ & (1 - \delta + \delta\beta P_0(1 + R_0))(1 - \delta + \delta\beta P_1(1 + R_1))) > 0, \end{aligned} \tag{3.45}$$

$$\frac{dU_1}{d\delta} = \frac{\beta P_1(P_1 - P_0)(C + e(1 + R_1))(1 - \beta P_1(1 + R_1))}{(P_1(1 + R_1) - P_0(1 + R_0))(1 - \delta + \beta P_1(1 + R_1))^2}, \tag{3.46}$$

$$\frac{dU_2}{d\delta} = \frac{\beta C P_0 P_1(R_0 - R_1) + \beta e P_0 P_1 R_0(1 + R_1)}{(1 - \delta)^2(P_1(1 + R_1) - P_0(1 + R_0))} \tag{3.47}$$

$$\frac{dU_3}{d\delta} = \frac{\beta P_0(P_1 - P_0)C(1 - \beta P_0(1 + R_0))}{(P_1(1 + R_1) - P_0(1 + R_0))(1 - \delta + \beta P_0(1 + R_0))^2}, \tag{3.48}$$

$$\frac{d^2U_2}{d\delta^2} > 0 \tag{3.49}$$

Hence, as functions of δ, each of U_1, U_2 and U_3 intersect Q_0 and Q_1 only once.

I verify that the intersection of U_1 and Q_1, denoted by δ_1, is identical to the intersection of U_2 and Q_1. It is a root of the equation

$$\begin{aligned} & C[\delta P_0(R_0 - R_1)(1 - \delta + \delta\beta P_1(1 + R_1)) - (1 - \delta)(P_1 - P_0)] = e(1 + R_1) \\ & [(1 - \delta)(P_1 - \delta P_0 + P_0(1 + R_0)) + \delta\beta P_0 P_1(1 + R_1)(1 - \delta(1 + R_0))]. \end{aligned} \tag{3.50}$$

This is equivalent to $U_2 = Q_1$, as well. When $U_1 > Q_1$, the LHS in equation (3.50) is larger than its RHS. This implies $U_2 > Q_1$, and vice versa. Thus, feasibility of U_1 demands infeasibility of U_2 and the other way around.

Similarly, I find that $U_2 = Q_0$ occurs at δ_0 which is the solution to the following equation,

$$C[\delta P_0(R_0 - R_1)(1 - \delta + \delta\beta P_0(1 + R_0)) - (1 - \delta)(P_1 - P_0)] =$$
$$eP_1(1 + R_1)(\delta(1 + R_0) - 1)(1 - \delta + \delta\beta P_0(1 + R_0)). \tag{3.51}$$

This equation imposes $U_3 = Q_0$ as well. If the LHS is larger than the RHS in this equation then $U_3 < Q_0$ which gives $Q_0 > U_2$. Conversely, feasibility of U_2 makes U_3 infeasible. Compare the two equations, coefficients of δ in (3.50) are larger than those in (3.51). Therefore, $\delta_1 > \delta_0$, which completes the sufficient conditions for feasibility of only one of the thresholds per given δ. It follows that given δ, only one of thresholds $U_j, j = 1, 2, 3$ is the unique cutoff value.□

Remark 3.4 The model with non-monetary cost is analogous to this case.

3.4 Numerical Examples

In order to illustrate the results, I present three benchmark sets of parameters for the two-period model. The first example is in line with the setting presented in proposition 3.3. Next, I investigate two more general cases in which the bank can be solvent at failure of a regime.

3.4.1 Bankruptcy at Failure

Figure 3.4 shows regime choices and switching strategies for different initial wealth levels and investment policy without effort cost e, as it does not influence the generality of the result. In this case we have $r_0 = r_1 = -1$.

When the effort cost is equal to 0 the bank takes regime 1 for a lower level of initial wealth since the expected return is higher at each level of wealth. In addition, lower dividend ratio, i.e. higher δ, creates more incentive to undertake risk. To summarize, there are two trade-offs. The intertemporal one is associated to the dividend and reinvestment decision. Though the reinvestment ratio is exogenous, the bank needs to decide about postponing either higher risk or higher return. Another trade-off is between higher probability of success or higher success return. The optimal combination of regime choices for two periods is a result of two trade-offs.

When the dividend ratio is low, failure does not bring a large loss to the bank whose wealth is also small. Since the profit of success is low, the bank behaves indifferent between failure and the low dividend. Thus, it gambles for resurrection first. If the bank succeeds it has sufficient wealth and plays safe in the second period. In this area we observe risk reduction from period one to two.

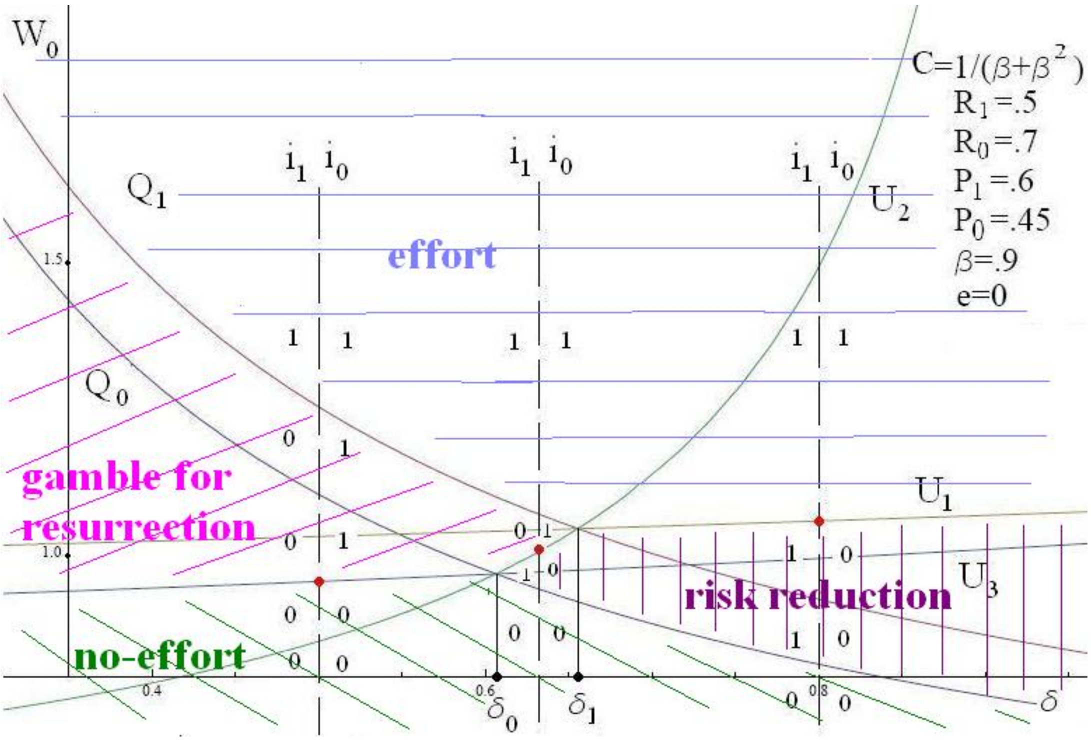

Figure 3.4: Inter-temporal Switching strategies in a two-period model without effort cost, i.e. $e = 0$, showing four possible regime-combinations in the optimal region w.r.t initial wealth W_0 and reinvestment ratio δ

For the wealth level between Q_0 and Q_1, the bank can afford the effort cost and takes the safer regime for high dividend. In that case, it has to play risky in the second period because reinvestment is low such that the bank does not have sufficient wealth to start the second period under the safer regime. This area is included for the gambling for resurrection strategy from the first period to the second.

In addition, note that the switching area is actually a super-set of $[Q_0, Q_1]$, depending on δ. Given the dividend ratio, if U_2 is feasible ($\delta_0 < \delta < \delta_1$), both two-way switching strategies are taken in this range of wealth.

3.4.2 Solvency at Failure

Suppose $r_i > -1,\ i \in \{0, 1\}$. The thresholds of proposition 3.1 give possible cutoff points for the second period. Consider the following examples:

Example 1. Take parameter set:

$$\begin{array}{ccccccccc} R_1 & r_1 & P_1 & R_0 & r_0 & P_0 & \beta & \delta & e \\ 0.75 & -0.1 & 0.65 & 0.9 & -0.3 & 0.45 & 0.9 & 0.99 & 0.3 \end{array} \tag{3.52}$$

Since the deposit is fully insured, the payment equals $C = \frac{1}{\beta+\beta^2}$. Note that (3.8) is not true for regime 0 but is satisfied under regime 1. Therefore, the bank may take regime 0 for $W_0 \le G_0 = 3.31$. But regime 1 is worthier than no operation for all positive wealth because $G_1 = -14.7693$. Checking the assumptions of proposition 3.1 and S_js, we find case (b) of figure 3.2 with unique cutoff value $S_1 = 2.05$ for the second period. In order to have $W_1 > S_1$ the bank needs for each regime and its outcome of the first period, z_{i_0},

$$W_0 > Q(z_{i_0}) = \frac{C + S_1/\delta}{1 + z_{i_0}} + i_0 e. \tag{3.53}$$

We see that $Q(R_0) < Q(R_1) < Q(r_1) < Q(r_0)$. Next, I verify whether each combination of outcomes of two periods is solvent. It means that I compute 16 thresholds from (3.29) for all outcomes of two periods. Locate them on intervals made by $Q(z_{i_0})$s for the initial wealth. Find optimal regime in each interval bounded to the described thresholds by comparing expected profits of solvent outcomes. At the end the unique cutoff point is 2.24, where $Q(R_1) < 2.24 < Q(r_1)$. For all $1 < W_0 < 2.24$ the bank operates under riskier regime while if it was operating for only one period it would take safer regime already above $S_1 < 2.24$. In other words, having the opportunity to operate for one more period the bank takes the safer regime at a larger capital comparing to the case of one period operation.

Example 2. Assume parameters

$$\begin{array}{ccccccccc} R_1 & r_1 & P_1 & R_0 & r_0 & P_0 & \beta & \delta & e \\ 0.65 & 0.1 & 0.65 & 0.75 & 0 & 0.3 & 0.57 & 0.99 & 0.2 \end{array} \tag{3.54}$$

Now, for one period operation (e.g. second period) we have case (a) of figure 3.2 with three consistent cutoff points $S_3 = 1.106$, $S_2 = 1.158$ and $S_1 = 1.253$. Consequently, there are four thresholds $Q(z_{i_0}), i_0 \in \{r_0, r_1, R_0, R_1\}$ for each case. Also take into account 16 thresholds from equation (3.29). The numerical solution determines the unique cutoff policy as many thresholds are infeasible. The cutoff value is 1.76 below which the bank chooses regime $i_0 = 0$.

A risky investment is worthy in regime 0 (based on (3.10)) for $W_0 \le G_0 = 1.154$, and in regime 1 if $W_0 \le G_1 = 1.193$ where (3.8) does not hold true. Only two thresholds $T(R_0, R_0) = 1.007$ and $T(R_0, R_1) = 1.145$ are below G_is. Still, G_is are bellow all $Q(z_{i_0})$s. Hence, the bank operates under the risky regime in the second period. The bank can survive two periods only if it chooses the risky regime at $t = 0$ and succeeds. Thus, operating for only one period under regime 1 produces a higher expected profit than operating under regime 0 in the first period and hoping to succeed and continue for the second period. Since in this example $S_2 < G_1 < G_0 < S_1$, the bank decides about regimes based on S_js and operates for only one period below G_is. To conclude, the bank operates for one period under regime 0 where $S_2 \le W_0 < G_0$. But it takes the safer regime for $S_3 \leq W_0 < S_2$ and the riskier regime below S_3.

Comparison of the two examples shows that when the risk free interest rate is high (discount factor β is small) the deposit payment is large. Therefore, in operation, the bank loses capital. As far as possible, it operates one period and takes risk non-monotonically in terms of the capital level. Otherwise, if the deposit payment is low it can be solvent at failure and has less tendency towards risk. Then, it can operate for two periods and follows the unique cutoff policy at the first period.

3.5 Endogenous Reinvestment

To complete the optimization problem of the bank, I include its dividend policy and find optimal δ. This decision is made at $t = 1$ simultaneous with the regime choice decision. The bank must be solvent by then and the first period outcome is realized such that $Y_1 > C$. The objective function is still (3.31). For simplicity, we assume $r_0 = r_1 = -1$. For every i_1, the optimal δ obtains from

$$max_\delta \ (1-\delta)(Y_1 - C) + \beta P(z_{i_1})((1+z_{i_1})(\delta(Y_1 - C) - i_1 e) - C). \tag{3.55}$$

Rearrange it for δ^*,

$$\begin{aligned} \delta^* = \quad argmax \quad & \delta(\beta P(z_{i_1})(1+z_{i_1}) - 1)(Y_1 - C) + (Y_1 - C) \\ & - I_{\delta>0}\left[\beta P(z_{i_1})(i_1 e(1+z_{i_1}) + C)\right]. \end{aligned} \tag{3.56}$$

Since this equation is a linear function of δ, the reinvestment ratio depends on the sign of its coefficient in (3.50). Notice that this is the same problem as the bank has at $t = 0$ when it decides to enter the game. The bank reinvests all of its capital in a risky regime iff (3.8) holds true, $\beta P_i(1 + R_i) - 1 > 0$. Therefore, if a bank operated for one period, from section 3.2, the solvent bank would pay no dividend and $\delta^* = 1$.

If the bank reinvest, its capital at $t = 1$ is $W_1 = Y_1 - C$. In order to have a profitable investment in regime i, the NPV should be positive,

$$W_1 \geq \frac{\beta P_i(ie(1+R_i) + C)}{\beta P_i(1+R_i) - 1}. \tag{3.57}$$

Therefore, for operating under regime 1 the bank needs

$$W_1 \geq G_0 = \frac{\beta P_0 C}{\beta P_0(1+R_0) - 1}, \tag{3.58}$$

and for operating under regime 0 it should be that,

$$W_1 \geq G_1 = \frac{\beta P_1(e(1+R_1)+C)}{\beta P_1(1+R_1)-1}. \tag{3.59}$$

Still the bank chooses between regime 0 and 1 at $t = 1$ based on cutoff policy. Feasibility of cutoff value S_1 requires $S_1 > G_0$ and $S_1 > G_1$. Both conditions are satisfied where

$$\frac{C(P_1 - P_0 - \beta P_0 P_1(R_0 - R_1))}{\beta P_0(1+R_0)-1} > eP_1(1+R_1). \tag{3.60}$$

Knowing all about the second period, the bank finds its first period investment strategies by backward induction.

Endogenously optimization of the reinvestment reduces to a bang-bang policy of reinvesting all or nothing, as agents are risk neutral. When the bank reinvests all the outcome of the first period, the problem is similar to the regime choice optimization in the two period model. The optimal strategy can again be characterized by unique cutoff policy as assumptions of proposition 3.3 hold. This makes the endogenous reinvestment model another robustness check to the findings in the previous sections.

3.6 Conclusion

This work questions gambling for resurrection and verifies existence of a non-monotonic relationship between the capital level of a bank and its portfolio risk. The standard rationale of banks taking risk under distress is violated in a static model which compares two different regimes of operation. This is beyond plenty of studies which focus only on selecting risky or risk-free asset, e.g Mailath and Mester (1994). Indeed, the cost of effort to reduce the risk from one risky regime to the less risky one plays the main role to rule out standard cutoff policy. The risk is less in one portfolio since the bank exerts effort to monitor creditors. The riskier projects however produce a larger return if succeed while the probability of failure and associated losses are larger. I observe two types of risk-return strategies. The first type is in line with the standard rationale: the risk neutral banker chooses the riskier asset when the capital decreases, in order to benefit from a higher return in case of success and hope to survive the distress. However, in the second type, the risk-taking decision depends on the initial level at which the capital begins to decay. First, when the capital decreases from a high level, a bank with less risk faces bankruptcy in case of failure, because the monitoring effort is paids out of the capital. Therefore, the bank stops monitoring and the risk increases. Risky mortgages are examples of such behavior in banks. Further the capital decreases, the bank operating under a risky regime also goes bankrupt in case of failure. Thus, the bank changes its portfolio to the one with less risk of failure. Nevertheless, with very low capital at which neither monitoring nor risky projects can survive in case of failure, the bank goes on operating under risky regime.

In the dynamic model, contrary to Dangl and Lehar (2004), the risk-taking and the intertemporal switching strategies are endogenous (not forced by a regulator) and influence each other. Under their hypothesis of gambling for resurrection, the cutoff value of the risk-taking policy is identical to the switching point, when the switching cost is omitted. Therefore, no switch means the bank chooses a certain fixed regime. In this paper, if a switching area exist it includes cutoff values. The switching area of each regime is a continuous set with at most one point of intersection with the switching area of the other regime. Depending on the dividend ratio, each switching area may narrow or widen and one may disappear. When both exist they intersect on a unique cutoff value. Low dividend raises gambling for resurrection but high dividend causes the bank to reduce risk in the first period. Yet the impact is reversed after paying dividend out since there is no outside investor and the bank is poor. Monitoring cost increases risk-taking incentive gently but does not have a structural effect.

The findings regarding non-monotonic risk-taking policies contribute to the banking regulation literature. Banks finance their investments in large parts by deposits. Imperfect transferability of banks' assets make banks' liquidation costly. To make profit the bank needs to spend on monitoring the creditors. Yet, with limited liability and insufficient capital the bank shirks in order to increase the equity value. From a regulatory point of view, the closure policy with a sufficiently high capital ratio requirement would eliminate the risk-taking incentives. However, a strict regulatory closure policy is not socially optimal. The regulator should not only protect the depositors but also optimize the social value of the bank. This way, the supervisory agency ends up in large scale forbearance in case of a crisis. Instead, my results propose more accurate screening of risks in the banks, in the first place. The possible methods could be the more market based approaches, for instance partial private insurance and risk-based taxing.

Appendix

Proof of remark 3.2: Suppose $1 + \mu_1 > P_0(1 + R_0)$. If $\frac{C}{1+r_1} \leq W_0 < \frac{C}{1+r_0}$, the bank takes regime 1 above threshold S_1^0 defined below,

$$P_1((1 + R_1)W_0 - C) + (1 + P_1)((1 + r_1)W_0 - C) \geq$$
$$P_0((1 + R_0)W_0 - C) \tag{3.61}$$
$$W_0 \geq S_1^0 = \frac{C(1 - P_0)}{1 + \mu_1 - P_0(1 + R_0)}. \tag{3.62}$$

The LHS and RHS of (3.61) are the expected returns under regime 1 and 0, respectively. If $W_0 < \frac{C}{1+r_1}$ the LHS of inequality (3.61) reduces to only $P_1((1+R_1)W_0-C)$. Assuming $P_1(1 + R_1) - P_0(1 + R_0) > 0$, the optimal regime is 0 below a threshold,

$$S_2^0 = \frac{C(P_1 - P_0)}{P_1(1 + R_1) - P_0(1 + R_0)}. \tag{3.63}$$

To check the feasibility of the threshold, compare them to boundaries We see that $S_1^0 < \frac{C}{1+r_0}$ follows from assumption $\mu_0 < \mu_1$. Yet for another boundary we have

$$S_1^0 > \frac{C}{1 + r_1} \Leftrightarrow \mu_1 - \mu_0 < (r_1 - r_0)(1 - P_0), \tag{3.64}$$

which equals

$$S_2^0 \geq \frac{C}{1 + r_1}, \quad \text{if} \quad P_1(1 + R_1) > P_0(1 + R_0) \tag{3.65}$$
$$S_2^0 < \frac{C}{1 + r_1}, \quad \text{if} \quad P_1(1 + R_1) < P_0(1 + R_0). \tag{3.66}$$

In (3.66) however, $S_2^0 < 0$. It means that $i_0 = 0$ for $W_0 < \frac{C}{1+r_1}$ and S_1^0 is feasible and the unique cutoff value. But when $P_1(1 + R_1) > P_0(1 + R_0)$, either S_2^0 or S_1^0 is feasible. Accordingly, we end up in unique-cutoff policy.

If conditions $1 + \mu_1 < P_0(1 + R_0)$ is violated, then $S_1^0 < 0$ and infeasible. The same holds for S_2^0 if $P_1(1 + R_1) < P_0(1 + R_0)$. It follows that the only cutoff value is $\frac{C}{1+r_1}$. Yet, for $P_1(1 + R_1) > P_0(1 + R_0)$, see that $S_2^0 \not< \frac{C}{1+r_1}$. Hence the bank takes regime 0 below unique cutoff value $\frac{C}{1+r_1}$. $\square$

Chapter 4

The Theories of Bank Regulation and Systemic Failures

4.1 Introduction

This paper surveys the recent literature on bank regulation, in particular for regulating systemic risk. Traditionally, there has been micro-prudential banking regulation focusing on individual banks and the risk they hold. As an example, the survey of Bhattacharya et al. (1998) covers the literature in the economics of bank regulation prior to Basel II. Subsequently, until around, the focus of the literature was on the optimal combination and implementation of the Basel II accords. The 2007-2009 financial crisis, however, has highlighted the interdependencies in the banking sector and in the financial industry as a whole. As a consequence, systemic risk issues have been in the focus of the recent theories on banking regulation and studies concentrated on macro-prudential regulation strategies.

These latest experiences provide the motivation to review how the banking regulation theories have been progressing. Hence, the contribution of this survey is to connect the previous bank regulation literature that has focused on a single entity with the most recent ideas on taking systemic risk into account. The focus is to show how the latter is complementary to the former and in which directions both strains of the academic discussions should progress.

First in a basic setup, I discuss bank failures. The fundamental problems that potentially lead to bank default in the expense of depositors are addressed. These are the shareholders' risk-taking and managerial moral hazard. To protect depositors the supervisory authority has to regulate banks on behalf of depositors. However, the authority faces time-inconsistency problem in solving this. The regulator who wants ex-ante to reduce the risk-taking incentives by threatening to liquidate assets in case of default, might have to forego the liquidation ex-post because of the high social cost. Solutions to this time-inconsistency problem are presented in the first part of this article. Hereby, I mostly concentrate on the latest studies in which risk-based approaches, e.g. Basel II, are considered.

The second part of the article explores the topics of regulating systemic risk. Widespread bank-failures, named systemic failure, bring externalities into the financial system. The externalities consist of bank-runs contagion and massive bank-failures such that no private institute is able to compensate for the losses. In order to prevent this, governments have to take the systemic risks into account for regulation. In banks, systemic risk may originate from either the liability or the asset side of the balance-sheet. Bank-runs are examples of exogenous shocks which appear in the liability side. Moral hazard and risky investments generate risk in the asset side. In this paper, the components of both idiosyncratic (exogenous) shocks on banks and the contribution of banks (endogenous shocks) to systemic crises are studied. To overcome the time-inconsistency problem in each of these cases, ex-ante optimal macro-prudential regulation policies are required.

My article covers this topic following the literature which specifically refers to the experience of the recent crisis, its origins and consequences. For this reason, after presenting the systemic risk regulatory proposals; e.g. granting healthy banks, systemic risk sensitive capital adequacy and taxing, I review the statistics about the resolution policies applied in the past crisis events.

In order to show how this survey relates to the existing literature, I first give an overview of previous survey studies. In an integrated model, Bhattacharya et al. (1998) analyze different deposit insurance related moral hazards and regulatory policies. Reasons for the existence of banks are discussed on the asset side as well as the liability side of the balance sheet. On one hand, they explain the delegated monitoring idea of Diamond (1984) that banks monitor the creditors on behalf of depositors at a lower cost than non-intermediated bilateral contracting between investors and entrepreneurs. On the other hand, the argument of Diamond and Dybvig (1983) is presented that the intermediaries contribute to improve risk sharing and provide liquidity better than non-intermediated case where investors would have to wait for the payoffs from the long term investments. Based on Diamond and Dybvig (1983), the governments' deposit insurance can prevent panic bank-runs. However, the insured banks have incentives for moral hazard in the sense to keep lower liquid reserve and to seek riskier portfolios. The regulatory policies are needed to attack these risk-taking activities.

In 1988, the Basel I accord introduced the capital requirements to rule out incentives for risk taking. However, studies such as Gennotte and Pyle (1991) and Boot and Greenbaum (1993) show particular situations in which stringent capital constraints do not reduce the risk in banking sector. Besanko and Kanatas (1996) emphasize that when the inside and outside equities are extremely distinguishable the higher capital requirement can reduce the bank's incentive to monitor the borrowers and increase risk. Further studies on fair priced and risk sensitive deposit insurance by for instance Chan et al. (1992) illustrate that the moral hazard can not be prevented if the regulator can not observe the bank's risk.

To confront private information problems, partial deposit insurance is suggested as a regulatory instrument which brings forth market discipline as the actuaries have to measure the bank's risk and also uninsured depositors monitor the bank. Peters (1994) points out that the informed uninsured depositors, with their own endowments at risk, will monitor and discipline banks better than governmental regulators do.

Risk-based capital adequacy is another cure to the moral hazard issue. This idea provided supports for the Federal Deposit Insurance Corporation (FDIC) Improvement Act of 1991 in the US and also evolved in the Basel I guidelines. FDIC prompt corrective action mandates progressive penalties against banks that exhibit progressively deteriorating capital ratios. Bank closure is considered as a threat to reduce incentives for risk in this law. Dahl and Spivey (1995) investigate banks' efforts for recovery under the closure threat forced by the FDIC. They find that the determination of failure for an undercapitalized bank is better defined in terms of the banks' capacity for recovery than the likelihood for further decay since the bank can recapitalize quickly by equity infusion. Empirical studies assessing the cost and benefit of the FDIC prompt correction, such as Jones and King (1995), suggest that the risk-based capital standards should improve to better recognize the credit risk of troubled banks.

Assessing regulatory closure policy shows less efficiency as expected, though it is not socially optimal either. For instance, Boot and Thakor (1993) argue that the regulator cares about its reputation and does not exert closure when it is needed and this again raises the risk taking by banks. Instead of a tough closure policy, Fries et al. (1997) propose optimal reorganization of the bank and closure rule beside fair pricing of deposit guarantees. The other branch of studies focusing on moral hazard issues, e.g. Leland (1994) and Leland and Toft (1996b), works on the capital structure to prevent asset substitution.

The 1988 Basel I capital framework evolved overtime. The Basel committee issued an amendment to refine the framework to address risks, e.g. market risks, other than credit risk. Accordingly, banks were allowed, subject to strict quantitative and qualitative standards, to use internal value-at-risk models as a basis for measuring their market risk capital requirements. In June 1999 the committee issued a proposal for a new capital adequacy framework. After nearly six years of challenging works, the Basel II capital framework was released in June 2004. It consists of three pillars: minimum capital adequacy expanded standards of the 1988 Accord; supervisory review; and regulatory closure to strengthen market discipline.

Following the works on how to refine regulatory strategies which resulted in the Basel II standards, most of literature prior to the recent crisis has focused on how to mix the three pillars of Basel II and improve their implementation. Prescott (2004) verifies that the banks have incentives not to reveal the true level of risk. Stochastic audit is found to be more effective comparing to the periodical banks' reports of their risks. In a model of optimal bank closure with stochastic audit Bhattacharya et al. (2006) find an optimal combination of capital requirement, closure rule and frequent audit which can eliminate risk-taking incentive for banks. Decamps et al. (2004) and Dangl and Lehar (2004) take a similar approach towards the gambling for resurrection problem for banks in distress. Further, there are researches on details of the Basel II implementation. For instance, Repullo and Suarez (2004) focus on loan pricing and demonstrate that the banks which adopt the internal rating based on the Basel II attract low risk firms by reduction in their loan rates.

Considering banks as liquidity-creators, the bank fragility issue relates also to the capital market risks and the market-driven fragility. In this regard, Boot and Thakor (2008) review the existing literature on the interbank relationship as well as the integration of banks and markets. In the recent years of crisis some empirical researches examined the effectiveness of Basel II. Also there have been studies about the regulatory policies in emerging markets. The view on banks as institutions that are closely related to each other and to the entire economy directed recent studies to focus on the analysis of systemic banking regulation.

This new strain in the literature motivates a new survey to collect their findings and explain different regulatory proposals in an analytical framework. The optimal regulatory strategies depend on the background problems, whether it is an exogenous shock or one of the moral hazard issues. I describe the possible regulatory confrontations related to each category of problems.

The paper follows in the next five sections. Section 4.2 outlines the basic setup and bank failure problems. In section 4.3, I review different policies that the regulatory authority can apply to individual banks. Section 4.4 discusses the effects of systemic failures. Subsequently, section 4.5 presents the regulatory proposals addressing systemic risk. Section 4.6 includes the statistics of the recent crises. At last, section 4.7 summarizes and concludes.

4.2 The Basic Model: Failures in Banks

Market failures can provide the intuition for the existence of a supervisory authority. This section consists of the basic setup that allows for a stringent analysis of the particular problems causing market failures in the banking sector. The details concerning possible regulatory actions are investigated in the next section.

In a risk neutral world, assume a representative bank receives 1 unit of deposit at date 0. Having equity E the bank's total wealth $A = 1 + E$ can be invested in a liquid risk free asset which returns $r > 0$, per unit of investment at date 1. The manager working for shareholders, can alternatively give loans to risky credits. To monitor creditors and have less risk, the bank has to bear cost $e \geq 0$ drawn out of the wealth at $t = 1$. This regime[1] of operation, denoted by $i = 1$, generates $R > r$ per unit of the investment with probability P_1 or zero otherwise, at time $t = 1$. However, because of the effort cost, the shareholders or the manager may decide to shirk[2] ($i = 0$) which increases the risk, reducing the probability of success to $P_0 < P_1$. The shareholders can not observe the manager's decision until the return is realized at date 1.[3] If they ask the manager to stop monitoring she will do, but they can not force the manager to monitor. This happens since the manager receives some non-pecuniary benefit $Q > 0$ if the shirking regime succeeds and generates $R - q,\ q \geq 0$, per unit of the investment at the end of the period.

The depositors are paid a fixed amount$D$ at date 1. Risky regime i brings the total[4] expected profit at the end of period,

$$\Pi_i \ = \ P_i \ max(0, A(R - (1 - i)q) - ie - D). \tag{4.1}$$

The return is higher under more risky regime when it succeeds, i.e. $R - q \geq R - e$ which requires $e \geq q$. But shirking is inefficient in the sense that its total return is less than monitoring regime: $R - q + Q < R$.

For the expected profit of risky operation to be positive, the capital should exceed some thresholds,

$$E \ \geq \ \frac{D - R + q + i(e - q)}{R - (1 - i)q}. \tag{4.2}$$

If the bank could define the deposit payment endogenously, the manager would propose D such that the bank would be solvent in case of success. This means from (4.2) for the deposit payment

$$D \ \leq \ (1 + E)(R - (1 - i)q) - ie. \tag{4.3}$$

Since the bank should motivate depositors, they have to pay them at least the same as the risk free return, i.e. $D \geq r$. Then it follows

[1] The decision or plan of operating with a specific portfolio is known as the regime of operation.

[2] To stop monitoring the creditors.

[3] The depositors have no information at all.

[4] In this risk neutral setup, the investment decision is optimally to invest all in a risky asset or nothing. This has been justified in my second paper.

$$(1 + E)(R - (1 - i)q) - ie \quad \geq \quad r. \tag{4.4}$$

One main problem of high risk (low probability of success) is a negative net present value (NPV). This occurs when there is no monitoring on the creditors, where for each unit of investment $P_0(R - q) - r < 0$. Given deposit payment D, without loss of generality assume $\frac{D+e}{R} > \frac{D}{R-q}$.[5] Then, the shareholders rather prefers less risk iff $\Pi_1 \geq \Pi_0$, which requires for the total capital level,

$$E \quad \geq \quad \frac{(P_1 - P_0)(D - R) + P_1 e - P_0 q}{(P_1 - P_0)R + P_0 q} = \hat{E}. \tag{4.5}$$

4.2.1 The Moral Hazard

The choice of regime depends on how the shareholders compensate the manager to work for their interest. This part of work outlines the possible managerial contract that the shareholders can offer and motivate the manager to operate in their favorite regime, though it may be risky.

When the manager is paid a certain salary s independent of her performance, the expected profit of the shareholders in (4.1) changes substituting $A - s$ for A. Then the shareholders take the less risky regime for large capital levels, i.e. $E \geq \hat{E} + s$. If the capital was not sufficiently high the shareholders would prefer more risk. Where $E < \hat{E} + s$, they would only make a fixed payment s to the manager at the beginning of the period as less as her outside option utility in order to have her in the firm. Then the manager would work for her private benefit and shirk which would be also in the interest of the shareholders.

However, if the shareholders want to have less risk the manager's salary should depend on the performance which influences the success and failure of the bank. To motivate for the less risky regime the shareholders offer an incentive compatible (IC) contract to the manager paying a salary s only when the bank's operation succeeds and the manager's expected profit under regime $i = 1$ is higher than under regime $i = 0$:

$$P_1 s \geq P_0(s + Q) \qquad \Longleftrightarrow \qquad s \geq \frac{QP_0}{P_1 - P_0}. \tag{4.6}$$

Note that the shareholders make such a contract iff for their expected profit $\Pi_1(s) \geq \Pi_0(s)$ where

$$\Pi_i(s) \quad = \quad P_i \; max(0, A(R - (1 - i)q) - ie - D - s). \tag{4.7}$$

[5]Otherwise, the calculation changes but not the result.

For this we have a new capital requirement,

$$E \geq \frac{(P_1 - P_0)(D - R + s) + P_1 e - P_0 q}{(P_1 - P_0)R + P_0 q} = E^*. \quad (4.8)$$

Another option for the shareholders is to define the salary as a share of the profit[6], $s_i = \alpha \Pi_i$ where Pi_i is defined in (4.1). If the shareholders' expected return $(1-\alpha)\Pi_i$ is higher under regime $i = 1$, the IC constraint changes to $s_1 > s_0 + P_0 Q$, or

$$\alpha(P_1[AR - e - D]) \geq \alpha(P_0[A(R-q) - D]) + P_0 Q. \quad (4.9)$$

Since α is independent from the regime choice, the shareholders decision is redundant to the one presented in (4.5). Therefore, if $E > \hat{E}$, fraction α must satisfy

$$\alpha \geq \frac{QP_0}{[A(P_1 - P_0)R + P_0 q] - eP_1 - D(P_1 - P_0)} = \hat{\alpha}. \quad (4.10)$$

4.2.2 Deposit Insurance and the Moral Hazard

In case of insolvency, even if all the outcome of its operation goes to the depositors, it is less than the promised deposit payment. Thus, the excessive risk of default is at the expense of depositors in the absence of any guarantee. This subsection examines whether protecting depositors is an effective strategy to prevent failure in the banking sector.

A defaulted bank has to go bankrupt and the shareholders and the manager receive nothing. A fair priced deposit insurance can protect the depositors. The insurer can be a private company or the state. However, whenever the high amount of deposit payment is not manageable by the private insurer the state has to intervene. Since a large scale default can influence the entire economy, the state has to bear this responsibility.

For having a faired price insurance, the premium equals the expected value of the worst default case, i.e.

$$m = D(1 - P_0). \quad (4.11)$$

which should be also subtracted from the profit function in (4.1). The state could then levy a tax on the bank equal to m. This prevents subsidization of the bank by taxpayers, in case of a default. However, the insurance premium or tax would shift the capital requirement for less risk-taking to $\hat{E} + m$. Regarding managerial contract, the fixed IC salary s does not change since the IC constraint (4.6) remains the same.

[6]See for instance Acharya and Yorulmazer (2007) that I explain in section 4.4.

If the manager is paid a share of the profit, the shareholders' decision is as above but the IC constraint (4.9) changes. In other words, $A - m$ appears instead of A in both sides of (4.9). This raises the minimum required managerial share because now we need

$$\alpha \geq \frac{QP_0}{[(A-m)(P_1-P_0)R + P_0 q] - eP_1 - D(P_1-P_0)} = \alpha^*. \qquad (4.12)$$

where $\alpha^* > \hat{\alpha}$. This shows that the insurance or tax system increases the shareholders' risk-taking incentives by shifting the minimum capital threshold upward. With an analogous argument, the insurance does not reduce the managerial moral hazard.

4.2.3 Introducing the Regulatory Actions

According to the result of the last subsection, there is a need for a regulatory agency which not only provides resolutions in case of failure but also has power to force ex-ante risk reduction policies.

For low capital levels the shareholders prefer the riskier regime of operation and let the manager fulfil their interest. As a straight forward result of the shareholders' risk-taking being related to the capital level, the regulator may offer a capital adequacy rule to prevent the risk-taking. It follows that the regulator closes the bank where the capital is below the required level A^* or $\hat{A}$ depending on the managerial contract.

My simple setup does not include any bankruptcy cost. In the real world, any bank failure influences its creditors and depositors. The creditors will not receive further investment. And for the depositors, they can not follow their plan to use the payment. For instance, a company may stop its development since it has not received the deposit payment, or received it later only through the insurance payment. These social costs make the closure policy not to be ex-post optimal. Mailath and Mester (1994) describe how the closure policy cannot be imposed. Yet, given the incentive compatible contract the bank with less risky loans may default too. Thus, the banking system demands for optimizing the resolution policies rather than only closure. Freixas (1999) considers partially insured deposits and examines the liquidity provision policy where closure is not ex-post optimal. Freixas and Rochet (2010) concentrate on introduction of a systemic tax that requires a regulatory authority with the power to replace the manager and shareholders.

The following sections describe how an optimal resolution policy depends on whether it is an individual bank failure or a systemic failure. The next section surveys the resolution policies for individual bank defaults. Explaining regulatory strategies focused on a single bank makes a proper background for extending the model further to examine systemic crises. More systemic risk regulatory policies are presented in section 4.5.

4.3 The Resolution of Individual Bank Failures

This section addresses the possible intervention policies of a strong regulatory/supervisory authority to resolve costly bankruptcies. The main problem that the regulator faces when taking action against a risky or failed bank is known as time-inconsistency. This problem and the alternatives to deal with it are explained in the next subsections. In each case, the basic set-up described above may change slightly to fit the requirements. For instance, the time horizon and risk aversion/neutrality may differ. We see how policy implications may change from one situation and set of assumptions to the other.

4.3.1 The Time-Inconsistency Problem

A strong regulator should have the power to shut down the operation of a bank which is taking excessive risk, as this ex-post reaction can influence ex-ante the investment of the bank. In order for the regulator to have the opportunity of supervisory visit to the bank, I must consider a time horizon more than one period. Mailath and Mester (1994) assume that the regulator has two options at date 1 and the bank has two periods of operation if the regulator, visiting at date 1, lets it operate for one more period. This subsection analyzes the model of Mailath and Mester (1994) which looks into the effectiveness of the regulatory closure policy.

I exclude the friction between the manager of the bank and the shareholders. Suppose, the bank decides only between the liquid asset with certain return r (safe) and regime of operation $i = 1$ (risky). The bank receives 1 unit of deposit at the beginning of each period and invests all. The risky assets mature at the end of the second period. For less complication assume the risky loans are free of the cost of monitoring effort, i.e. $e = 0$. The inefficiency of the risky asset is defined as having negative NPV, $P_1 R < r$. There is a fixed cost of closure C, borne by the regulator who repays fully the depositors of the failed bank.

Note that the regulator is redundant if the bank takes no risky investment for the two periods (i_1, i_2) =(safe,safe). If the bank loses in the second period it has to pay everything even out of the profit of the first period. Therefore, it prefers[7] (risky,safe) strictly to (risky,risky) iff the NPV is larger for (risky,safe) than for (risky,risky),

$$P_1[(R-1)+(r-1)] > 2p_1^2(R-1). \tag{4.13}$$

First, suppose the bank takes the strategy of switching from one regime to the other at the beginning of the second period. The regulatory policy is to close the bank if it chooses safe for the first period because the bank would otherwise choose risky for the

[7] (risky,safe) and (safe,risky) are equivalent.

second period. Hence, the optimal solution is for the bank to play (risky,safe) and for the regulator to leave it open.

Next, consider the bank always plays risky in the second period, if it is open. If the regulator closes the bank the expected cost for the regulator will be $(1 - P_1) + C$ that is the closure cost plus the deposit payment. If the bank remains open the regulator's expected cost is $2(1 - P_1)^2 + 2(1 - P_1)P_1(2 - R) + C(1 - P_1^2)$. The first term is the expected deposit payment when the bank loses in both periods. The second term is the expected cost if the bank succeeds in one of the two periods. And the last term is the cost of closure in case the bank loses in at leas one of the two periods. Comparing the regulator's expected costs of policies, we see that the bank will be closed iff the expected cost of closure is less than leaving the bank open,

$$C < \frac{(1 - P_1)(1 - 2P_1(R - 1))}{P_1^2}. \tag{4.14}$$

When (14) does no hold, the bank plays (risky,risky) because the regulator will play open. Otherwise if (4.14) is satisfied, the bank chooses between taking risk facing liquidation or staying under certainty. But then the regulator knows that the bank would play risky in the second period. In a similar method we can find that the bank plays risky at the first period and the regulator closes it iff

$$C < \frac{(1 - P_1)(2 - r)}{P_1}. \tag{4.15}$$

Therefore, the cost of closure is the important variable to the regulator. Since closure may be less costly in the future, the regulator can not commit ex-ante to be severe. When the deposit value of a bank is very high the social cost of closure becomes large that may lead to non-liquidation and bailout. This policy generates moral-hazard incentives. This "too big to fail (TBTF)" problem is more discussed in the next section as it is not only an issue of a single bank, but also may affect the banking system.

4.3.2 Optimal Liquidity Provision

When bank closure and liquidation of assets are not ex-post optimal, a central bank may find it essential to provide liquidity to the bank. This idea has been addressed as the Lender of Last Resort (LOLR). The question in this part of the work is that how the liquidity provision can optimally solve the time-inconsistency problem when the cost of bankruptcy is large.

Despite all arguments against LOLR that it will cause the central bank to face the consequent moral hazard problem and increasing risk, Freixas (1999) claims that the result depends on the degree to which a bank's deposits are insured. Freixas (1999) investigates two possible sources of risk, i.e exogenous and endogenous, and whether the regulatory policy should change from one case to the other. He sorts out an efficient implementation of liquidity provision based on a cost benefit analysis. The main differences to the setup from section 4.2 are that β percent of deposits are uninsured and the bank is investing only in the risky asset (i=1).

A negative exogenous liquidity shock which causes failure of risky loans brings financial distress. The time horizon is one period, at the end of which the central bank reacts in case of a default. The promised payment to insured and uninsured deposits are $(1-\beta)(1+r_D)$ and $\beta(1+r_L)$, respectively. The expected value of the bank under regulatory bailout or liquidation (in case of insolvency) is denoted by V_L and V_C. Since the liquidation value is non-zero, the fair priced insurance premium changes to[8]

$$m = (1-P_1)max[(1-\beta)(1+r_D) - V_L, 0] \tag{4.16}$$

and the subsidy by bailout sums up to

$$S = \beta(1+r_L) - max[V_L - (1-\beta)(1+r_D), 0], \tag{4.17}$$

assumed to be positive. Let Δ be the difference between costs of continuation and liquidation. The regulatory decision depends on Δ which is decreasing in closure cost C. When the central bank has no commitment for closure, since C is increasing in bank's wealth, $A = E + 1$, for some range of parameters the TBTF problem holds in the sense that if a bank with asset A is bailed out, all larger banks would be optimally rescued.

Assume that the central bank makes commitment to a specific regulatory resolution policy. Let $\theta > 0$ be the probability that the central bank rescues the bank. The optimal regulatory policy is determined by maximizing total surplus of the bank's actions subject to the incentive compatibility condition which requires a higher bank profit under bailout for any given β. Freixas (1999) assumes that C is increasing in β. Thus, he finds that either to bailout or to use a mixed strategy (between liquidation and bailout with $\theta > 0$) is optimal depending on the amount of uninsured debt, β. The mixed strategy is interpreted as "constructive ambiguity", which had previously only been justified in macroeconomics level.

In the second part, Freixas (1999) takes into account the moral hazard problem where the risk level is chosen endogenously. The result about the optimal regulatory is similarly dependent on β as long as the monitoring effort cost is not considered. First I describe his general setup with endogenous risk taking, then explain how the monitoring assumption influences the optimal regulatory actions.

The bank has a continuum of risk levels and chooses the probability of success at a cost $\varphi(P)$, assumed to be strictly increasing, convex and twice differentiable. The difference

[8] Compare it to (4.11).

to my setup is that in his model the differentiability of $\varphi(P)$ is necessary for optimizing the risk level.[9] Indeed, in his paper there is no agency problem between the shareholders and the manager. The probability of gross return $x = x(\beta)$ is $P = P(x)$. The profit of the bank for total return equals,

$$\Pi = P[x(\beta) - \beta(1 + r_L) - (1 - \beta)(1 + r_D)] - m - \varphi(P). \tag{4.18}$$

The first best $\hat{P}$ can be obtained from $x(\beta) = \frac{d\varphi(P)}{dP}$. However, since rate of return to uninsured deposits r_L and m should be already adjusted rationally for P, the first order condition (maximizing Π) yields higher risk, i.e. smaller probability of success than $\hat{P}$. Rewrite profit function to observe the relationship between parameters θ and P,

$$\Pi = \Pi_0 + (1 - P)\theta S, \tag{4.19}$$

where Π_0 is the expected profit the bank would gain in the absence of any subsidy (if no bailout). Concavity of Π (resulting from convexity of $\varphi(P)$) and the derivative of the first order condition show that P is decreasing in θ. It means that liquidation is more frequent as the bailout policy would increase the bank's riskiness[10]. In addition, welfare analysis shows that taking more risk decreases social surpluses of bailout policy. Yet, the optimal policy for the central bank is either a systemic bailout or a mixed strategy.

In the case that the effort level of the bank determines the risk, the probability of success P and its cost φ are thus functions of effort level e (a bounded value). The optimal policy is similar to other cases; however, Freixas (1999) verifies that moral hazard effect appears differently. Though, the social cost of bankruptcy implies that it is optimal to rescue a bank with small level of uninsured debt; a larger amount of uninsured debt generates a closer monitoring of the bank by its creditors. Hence, the moral hazard effect works counter-balanced. A mixed strategy stimulates banks to keep more uninsured debt and tighter monitoring.[11] This is the case also where the LOLR is able to commit to bailout with some positive probability. Nevertheless, the bailout policy increases the bank's riskiness, decreases monitoring effort and the marginal benefits of rescuing banks.

The work of Cordella and Yeyati (2003) on the moral hazard problem focuses on the value effect of bail-out policy where the central bank announces and commits ex-ante to rescue banks in times of exogenous macroeconomic shocks. The probability of success not only depends on the risk choice of the bank, but also is affected by a state dependent term η, which is unobservable by the central bank. In a dynamic multi-period setup,

[9] In section 2 of this paper I consider a monitoring effort cost to bring a higher probability of success. Thus, in my model $\varphi(P)$ takes only two values: e for P_1 but 0 for P_0.

[10] Sufficient conditions are considered to avoid corner solutions.

[11] The result is in line with the rationale that subordinated debt helps to have a better banking discipline.

the risky investment returns x with probability $P(x, \eta) = \eta P(x)$ where $0 \leq \eta \leq 1$ is i.i.d and $P(x)$ is decreasing.[12]

With full deposit insurance, in the absence of bail-out policy, it is shown that quite intuitively the bank never chooses lower risk than socially optimal. The central bank follows a constructive ambiguity approach. The shareholders may recapitalize the bank in case of failure by raising capital in the capital market, even if the central bank does not bail out. In the non-recapitalization scenario, the probability of bailout θ becomes a negative function of η. Then a state-independent bail-out policy, $\theta(\eta) = \theta$, increases risk-taking of the bank as we could expect. However, regardless of the bank's decision on capitalization, the central bank minimizes the risk. The optimal risk-minimizing bailout policy is obtained by a threshold $\hat{\eta}$ below which the central bank rescues the bank with certainty and lets it fail otherwise. Under this strategy, the bank always takes risk more than optimal level. An alternative optimizing approach is to maximizing the central bank's objective, which considers the possible efficiency cost of bailout. This approach brings about similar regulatory policy with a threshold at least as large as the risk-minimizing threshold. In other words, the central bank is never less generous than the risk-minimizing policy.

Accordingly, constructive ambiguity is beneficial to rule out the moral hazard problem arising from the bank's endogenous risk taking. However, on occasion of macroeconomic shocks, systemical intervention of the central bank contingent on the exogenous conditions is desirable as it creates risk-reducing value effects.

4.3.3 Takeover as an Incentives For Risk Reduction

Beside closure and bailout policies, the supervisory agency may allow for takeover of the failed bank a healthy financial institute. This policy has been promoted as an incentive program. In a dynamic model, Perotti and Suarez (2002) argues that a solvent bank can buy a failed institution and benefit from the increase in its charter value.

In the setup presented by Perotti and Suarez (2002), a new branch of the bank enters the market on a random basis determined by the regulator. The regulator decides also how to resolve the failures. If both branches fail, she will employ two new banker and lets them to compete in a duopoly. But if only one bank fails, she should optimize whether to allow for takeover by the other branch.

For each bank, the return to a prudent lending is certain. There is an opportunity for speculative lending which generates extra return but leaves the bank exposed to exogenous solvency shocks. Monopoly is more profitable for a bank due to the absence

[12] To simplify the model the bank only chooses Markov strategies in risk-taking. This simplification makes a closed form solution possible but reduces the problem to a specific case in which risk-taking in each period is independent of and has no impact on other periods.

of competition but the rent comes at a cost. The stochastic entry of the new branch turns monopoly to duopoly. Therefore, the lending structure of each branch impacts the other bank. A bank may speculate in a monopoly but in duopoly it can be allowed to buy the failed branch if it is solvent. Thus, in a duopoly the bank has less incentive for speculative lending because of the reward for being solvent. By takeover the survived bank is temporarily a monopolist. The higher rent in this case makes a new branch willing to enter the market.

The supervisory agency as a social planer optimizes the entrance and takeover policy, minimizing the social losses in case of failures. It leads to allowing takeover and implementing an optimum mixture of prudence and competition through an adequate level of new entry rate. This way banks convert from speculative lending into strategic decisions in order to remain solvent.

4.4 Regulating Systemic Risk

Failure of a substantial part of the economy, meaning a large institute or many small ones, are considered as systemic failures. Mostly the regulation policies have so far focused on individual bank's risk. Therefore, insolvency of a bank is dealt with accurately in normal times. However, in addition there is a risk of systemic failures that lead to severe crisis. The recent crisis raised attention to the need for restructuring regulatory strategies in order to take account of systemic risks. This section states why it is necessary to regulate systemic financial crises and investigates the externalities involved in a systemic failure.

In 2008 the states let Lehman Brothers fail in order to limit moral hazard risk-taking. On the contrary to the government's interest, it led to a serious collapse of the financial system. Eventually, failure of this large financial institute spread to a significant part of the economy through direct and indirect interconnections to other institutes. Then, the second externality appeared. No private sector, including banks and insurance companies, could take over and compensate for the large scale failures of many banks and institutes.

Hence, the recent crisis shows traces of externalities in two main directions. The first externality is the spillover risk of one bank on other banks. The second is the collective failures of banks where healthy banks can not take them over. The former is discussed within a model of contagion and the latter as the too-many-to-fail problem, in the following subsections.

If the banking system is in danger to collapse all together, naturally the supervisory has to take precautionary reactions. After the failure of Lehman, the government could not let any other large financial institute fail, despite the fact that the bail-out policy strengthened moral-hazard. The costs and inconveniences on governments and supervisory authorities demonstrate needs for macro-prudential regulatory strategies that is the topic of the next section.

4.4.1 Contagion and Too Much Related Banks

To illustrate the first externality effect a bank failure has on the banking system, I refer to the case of transmitting bank-run, named contagion. Allen and Gale (2000) study the fragility of a banking system, where bank runs spread in the system. Their model is notable for my purpose since it separates the inter-bank structure from the risk-taking behavior. This approach helps to emphasize the spillover externality and avoids complexity caused by the risk optimization challenges. Allen and Gale (2000) consider the liquidity provider[13] role of banks which maximize their depositors' utility.

Assume there are four banks each operating in a different region, denoted by A, B, C and D. For simplicity, suppose each bank has no equity, i.e. $E = 0$, at date 0. The 1 unit of deposit (provided by the depositors of the same region) at time $t = 0$ is the only available source of wealth to each bank. Depositors demand d_1 and d_2 at dates 1 and 2, respectively.[14] However, each bank receives early demands with probability w_H or w_L in each region at date 1, where $0 < w_H < w_L < 1$. The manager works for the bank without moral hazard problem. The liquidity problem raises from the banks investment in an illiquid asset which takes two periods to mature. It means that each bank invests amount L in an asset which returns $R > 1$ per 1 unit at $t = 2$. Therefore, a bank may have to liquidate assets prematurely to pay to depositors. Liquidating one unit of investment produces $0 < \lambda < 1$ unit at $t = 1$.

Each bank decides about the inter-bank market, an investment portfolio and a deposit contract. Suppose a complete market in which every bank has deposits in each of other regions. Since all regions and consumers are equivalent, with out loss of generality assume in regions A and C there are early consumers with low probability, but in B and D with the high probability. In the complete market banks can easily transfer their excess supply of the liquid asset to the regions with excess demands at date 1. Suppose every bank has deposit $z = (w_H - \gamma)/2$ in each bank of three other regions, where $\gamma = \frac{w_H + w_L}{2}$. Now, the banks have to choose only the deposit contracts, d_1, d_2, and the risky investment L. Each bank maximizes the expected utility of consumers at time $t = 0$ in the following way,

$$\gamma u(d_1) + (1 - \gamma) u(d_2). \tag{4.20}$$

[13] The framework of Diamond and Dybvig (1983).

[14] The depositors could not expect the same level of utility a bank in their region brings them in autarky.

As the total of depositors' consumption in each period is a constant, it is optimal for the bank to hold the liquid asset (by itself or as deposit in other regions) for the early deposit demands, i.e. $\gamma d_1 \leq r(1-L)$. It gives the feasibility constraint of the second period, $(1-\gamma)d_2 \leq RL$. The objective function (4.20) increases as long as the consumption can be shifted from early deposit demand to the late demands using the liquid asset. The first order condition is obtained, $\acute{u}(d_1) \geq \acute{u}(d_2)$, where $\acute{u}(.)$ is the first derivative. This condition stops shifting deposit demand until $d_1 \leq d_2$ which is an incentive constraint for the depositors who wait longer. Otherwise, the depositors with late demand would be better off withdrawing at date 1. This optimization problem is the same as if a central planner optimizes risk sharing. Allen and Gale (2000) call this optimal allocation a first best allocation, which is also incentive efficient as seen above. There is no bank-run and no need for premature liquidation.

Consider a perturb state which occurs with probability zero, such that each bank in B, C, and D receives early deposit demands with probability γ, but they come to the bank in region A with probability $\gamma+\epsilon$, $\epsilon > 0$. If a bank is insolvent it may liquidate some of the illiquid asset to meet its commitment to early deposit demand. But it prefers to pay out of liquid assets at first, and next liquidates the deposits in other banks. If neither liquid asset, nor deposit liquidation helps, the bank will liquidate the illiquid assets at date 1. This is called liquidation "peking order[15]".

In order to prevent a run[16] in date 2, a bank with a fraction w of early deposits demands has to keep at least $(1-w)d_1/R$ units of the illiquid asset. Since the amount of illiquid asset is $1-L$, the highest amount that can be liquidated at $t=1$ is $(1-L-(1-w)d_1/R)$ which produces the buffer

$$b(w) \equiv \lambda(1-L-(1-w)d_1/R) \tag{4.21}$$

As long as the amount of illiquid asset a bank needs to liquidate is less than this buffer, the bank is insolvent but not bankrupt. In the perturb case, the assets of the bank in region A are valued at $r(1-L)+\lambda L+3zd_1$ at date 1. The last term comes from its deposits in three other regions, as in each one deposits are valued at d_1. The liabilities of bank A are valued at $(1+3z)q^A$, where q^j is the value of a bank's deposits in region j. Balancing assets and liabilities, q^A is found:

$$q^A = \frac{r(1-L)+\lambda L+3zd_1}{1+3z}. \tag{4.22}$$

The bank in region A is bankrupt whenever,

$$\epsilon d_1 \leq b(\gamma+\epsilon). \tag{4.23}$$

[15]It holds for small λ.
[16]The bank can not pay to the late deposit demands fully.

The loss to every other bank because of bankruptcy in A is $z(d_1 - q^A)$. Those banks will not be bankrupt iff the loss is less than their buffer,

$$z(d_1 - q^A) \quad \leq \quad b(\gamma). \tag{4.24}$$

Imagine that region A was only connected to B, and C only to D. Then, bankrun in one sector of the market would never transmit to another sector. Allen and Gale (2000) investigate the fragility of system where regions are incompletely connected where each bank has deposit only in one neighbor bank. Region A has deposits $2z = w_H - \gamma$ in B. Similarly B in C, C in D and region D has deposits $2z$ in region A. Under the assumption that two banks receive early deposit demands with high probability and two other with low ones, the first best allocation is still achieved. The reason is that the objective function is the same as (4.20) and the budget constraints for high liquidity shocks at dates 1 and 2 are, respectively,

$$\begin{aligned} w_H d_1 &= r(1-L) + (w_H - \gamma)d_1 \\ [(1-w_H) + (1-w_H)d_2] &= RL, \end{aligned}$$

which equal the previous constraints,

$$\gamma d_1 = r(1-L) \tag{4.25}$$

$$(1-\gamma)d_2 = RL. \tag{4.26}$$

Similarly for the regions with low liquidity shocks the same budget constraints hold. Thus, in this case the liquidity transfer makes the first best allocation possible. Nevertheless, in the perturb situation where (4.23) is satisfied for region A, the spillover to region D can be large enough that,

$$2z(d_1 - q^A) \quad > \quad b(\gamma). \tag{4.27}$$

Note that both inequalities (4.24) and (4.27) are possible. It means that, for a set of parameters the complete market can be safe from bank-run contagion but not the incomplete market. As bank in A is bankrupt, its assets are valued less than d_1. Therefore, the deposit of region D in A is not sufficient and it must liquidate more than the safe buffer. This in turn causes bankruptcy for the bank in D. In a similar way the losses transfer to the bank in region C and then to one in B. Accordingly, all banks connected by a chain of overlapping bank liabilities must go bankrupt.

As seen fragility of a system is different under complete and incomplete markets, but there is not a monotone relation. Actually, the level of inter-connection among banks determines how a contagion can spread.

If the outcome in Allen and Gale (2000) were risky, similar to my setup in section 4.2, the returns in different regions would not be perfectly correlated. In that case banks would gain from risk sharing and they would hold claim on each other. Ex-ante, the first best could be achieved. But, ex-post risk sharing would not be possible when the returns were known, as arrangements and bankruptcy rules would not work properly, in addition to the complicated analysis. This explains in some extend the more complexity of contagion in the real world, that it leads to difficulty of dealing with crisis.

This discussion of contagion and financial fragility concentrates on the liability structure of banks. The interbank relations has been addressed together with the TBTF problem. The source to the systemic risk in both issues can be the asset side of the banks' balance-sheets. Rochet and Tirole (1996) investigate the TBTF and suggest peer monitoring among commercial banks. In their work, TBTF occurs if the peer monitoring starts after the liquidity shock and it more depends on the size of interbank loans than size of the individual failed bank. Further work on interbank market is done in Freixas et al. (2000). They show that on one hand, interbank credit lines reduce the cost of holding reserves to cope with liquidity shocks. On the other hand, a contagion is inevitable in these connection lines. Insolvency of one bank affects the stability of the banking system because of a coordination failure, even if other banks are solvent. Moreover, the subsidy generated in the network of cross-liabilities allows the insolvent bank to continue its weak performance. If the central bank decides to liquidate this bank, it has to compensate for payments of the defaulting bank to the depending banks. Here two courses are available, inefficient liquidation of counterparts of an insolvent bank or bailout the defaulting bank. Therefore, Freixas et al. (2000) result in a moral hazard problem as TBTF.

4.4.2 Too Many To Fail

A soft regulator who ex-ante lowers monitoring capacity or ex-post rescues insolvent banks, not-being sufficiently generous, triggers banks to collude on disclosing their losses. Thus, many banks roll over their bad loans passively rather than to announce bankruptcy against defaults. Consequently, the regulator may need to repeat rescue or recapitalization in the future. Mitchell (1997) explains this issue and call it "too many to fail" (TMTF).

Acharya and Yorulmazer (2007) analyzes the herding behavior of banks leading to TMTF. Their work focuses on banks' inter-correlation of risk-taking and covers the three main regulatory actions: closure, bailout and take-over. This subsection studies the effectiveness of these policies in dealing with the TMTF.

The collective failure of many banks have been analyzed in the presence of regulatory actions which focus on individual banks. These regulatory systems have been limitedly effective as they could not prevent systemic failures. This subsection includes a similar setup that the regulatory policies exist but do not target systemic risks. We see how banks take advantage of it and initiate widespread failures. To complement the problem discussed in the last subsection, the focus is on the risk taking behavior of banks in this part. Further, the banks' herding on risk taking contributes to the second type of externality in a large crisis.

Consider two equal-sized banks A and B operating each for two periods. Each period has basically the setup of section 4.2. But it is adapted for letting the regulator into the model and a few simplifications. The regulator may intervene at time $t = 1$. The only source of fund for each bank is 1 unit of deposit per period. Deposits are debt contracts with maturity of one period. The banks benefit from full deposit insurance only in the first period which costs $ad_1,\ a > 0$, where d_1 is the deposit return at $t = 1$. Then d_2 denotes the deposit return at $t = 2$.

Moral hazard of a bank manager is defined as before. However, assume the probabilities of success do not change by moral hazard, i.e. $P_0 = P_1 = \bar{P}$, but the probability depends on the period. $\bar{P}_1$ and $\bar{P}_2$ stand for the probability of success independent of whom owns the bank's assets at $t = 1, 2$, respectively. Further, ignore the effort cost, i.e. $e = 0$. IC constraint (4.10) indicates that the banker needs a minimum share of $\alpha = \frac{Q}{q}$ not to commit moral hazard.

Define liquidation as selling the bank to outsiders who generates only $R - \delta$ in the success state. Acharya and Yorulmazer (2007) assume $\delta < q$ which means outsiders can manage the bank better than the moral hazard case but are not as productive as the bankers. This is in line with the literature that the moral hazard risk-taking is the most severe case in terms of social welfare as its outcome is the least.

The banks choose their interbank correlation, $\rho \in \{0, 1\}$ which refers to the correlation of their respective returns. Whereas $\rho = 0$, the two banks belong to two different industries, and $\rho = 1$ means that they choose the same industry. Having two banks A and B in the economy, 4 possible states at time $t = 1$ are given: SS, SF, FS, FF, while S and F recalling success and failure of bank A and B, respectively. Being in the same industry the joint probabilities of the 4 cases are, $\bar{P}_1, 0, 0, 1 - \bar{P}_1$. However, if the banks are independent (two different industries) then the joint probability of each state is given by multiplying the probabilities of the two outcomes.

To show that surviving bank will always buy the failed bank, take into account following assumptions: (i) without loss of generality, bank A has the bargaining power to offer to buy bank B, (ii) bank A will access to depositors of bank B after purchase, and (iii) deposit insurance is costly to the regulator when there is a bank failure. The surviving bank, A will always buy the failed bank in a price, $\psi = \bar{P}_2(R - \delta) - 1$ equal to what outsiders at most would pay in states SF. The surviving bank's expected profit from the investment in assets of the failed bank will be $\bar{P}_2 R - 1$. Therefore, it purchases the failed bank and receives the discount ($\bar{P}_2\delta$). This resolution policy is optimal also for the regulator. The misallocation cost is zero, comparing to the misallocation cost of selling to the outsiders, $\bar{P}_2\delta$. The fiscal cost for the regulator is $a(d_1 - \psi)$ in both cases. The bank's bailout policy includes no misallocation cost but fiscal cost ad_1.

In this unique subgame perfect equilibrium the regulator never intervenes in state SS. In state FF, if both banks are sold to outsiders, the regulator's objective function is

$$E(V_2^L) = 2[\bar{P}_2(R-\delta)-1] - a(2d_1 - 2\psi), \tag{4.28}$$

and if both are bailed out, it takes the value

$$E(V_2^B) = 2(\bar{P}_2R-1) - a(2d_1), \tag{4.29}$$

of course it is already assumed that the bank's manager has a minimum share of α in each bailed out bank. Bailing out one bank and liquidating the other one, the objective function takes a value between $E(V_2^L)$ and $E(V_2^B)$. As two banks are taken symmetrically, the regulator takes the same action towards either of them. Hence, both banks are liquidated if $E(V_2^L) \geq E(V_2^B)$ which gives $\delta \leq \delta^*$, where $\delta^* = \frac{a(\bar{P}_2R-1)}{\bar{P}_2(1+a)}$. Otherwise, the regulator bails them out and takes a share υ in each bank's equity $\upsilon < (1-\alpha)$.

Knowing the regulator's strategy, we find the banks' decision on the interbank correlation, which is their investment problem of date 0. Note that a bank's first period profit $\bar{P}_1R - d_1$ is independent of ρ. Therefore, its expected profit of two periods,

$$E(\pi_1) + E(\pi_2(\rho)) \tag{4.30}$$

is optimized with respect to ρ as $E(\pi_2(\rho))$ maximizes. When two banks invest in the same industry,[17]

$$E(\pi_2(1)) = \bar{P}_1E(\pi_2^{ss}) + (1-\bar{P}_1)E(\pi_2^{ff}). \tag{4.31}$$

But if they differentiate,

$$E(\pi_2(0)) = \bar{P}_1^2E(\pi_2^{ss}) + \bar{P}_1(1-\bar{P}_1)E(\pi_2^{sf}(0)) + (1-\bar{P}_1)^2E(\pi_2^{ff}), \tag{4.32}$$

where $E(\pi_2^{sf}(0)) = E(\pi_2^{ss}) + \bar{P}_2\delta$, as we discussed before that the surviving bank receives a discount $\bar{P}_2\delta$ by buying the failed bank in state SF. The choice of interbank correlation is determined by the tradeoff between this discount and the subsidy at being bailed out in state FF,

$$E(\pi_2(1)) - E(\pi_2(0)) = \bar{P}_1(1-\bar{P}_1)[E(\pi_2^{ff}(0)) - \bar{P}_1\delta]. \tag{4.33}$$

[17] The outcomes are only success for both or failure of both banks.

If the regulator liquidates two banks at $t = 1$, i.e. $\delta \leq \delta^*$, banks choose the highest level of correlation at $t = 0$. Otherwise, if banks are bailed out, $E(\pi_2^{ff}) = (1 - \upsilon)(\bar{P}_2 R - 1)$ exceeds subsidy[18] $\bar{P}_2\delta$ if and only if,

$$\upsilon \ < \ \upsilon^* = 1 - \frac{\bar{P}_2\delta}{(\bar{P}_2 R - 1)}. \tag{4.34}$$

Thus, if $\upsilon^* > 1 - \alpha$, for a bailout strategy of $\upsilon < 1 - \alpha$ banks takes $\rho = 1$. But if $\upsilon^* \leq 1 - \alpha$, they herd where the regulator takes very low share $\upsilon < \upsilon^*$. To make banks differentiate under a bailout policy the regulator has to take $\upsilon \in [\upsilon^*, 1 - \alpha]$.

However, the ex-ante optimal policy may differ from the regulator's ex-post policies. The losses in state FF inspire the regulator to implement closure policies that minimizes ex-ante the probability of this state. It means that the expected total output of the banking sector is maximized when banks invest in different industries.[19] In case $\delta \leq \delta^*$, obviously the ex-ante and ex-post policies are the same. In the more crucial case of $\delta > \delta^*$, the regulator needs to take a dilution $\upsilon > \upsilon^*$ to prevent herding. Where $\upsilon^* < 1 - \alpha$, the regulator can take $\upsilon = \upsilon^*$ to provide incentive for banks to deviate and still continue without moral hazard. Nevertheless, the most considerable case is when $\upsilon^* > 1 - \alpha$. Acharya and Yorulmazer (2007) find a set of parameters under which ex-ante it is optimal to liquidate both banks, as $\delta < q$ and the liquidation costs are smaller than the agency cost. But discussed above that ex-post it is optimal to bailout both banks, since the regulator is ex-post only maximizing the profits at state FF. The regulator ex-ante objects to reduce the likelihood of joint-failure. She may give up some of its profit and imposes a tougher liquidity policy in order to incentive less correlation between banks. Hence, state FF includes time inconsistency problem for large δ.

Acharya and Yorulmazer (2007) compare too-big-to-fail and too-many-to-fail, assuming two banks asymmetric in their sizes. Without loss of generality let bank A be the larger bank which takes deposit more than 1 unit. The result changes due to the assumption that the large bank has enough capital to buy the small bank but the small bank does not have enough fund to acquire the large bank's assets. Therefore, only in state SF the surviving large bank buys the small bank. In state FS, if $\delta > \delta^*$ the regulator bails out the failed large bank, since liquidating to outsiders is a misallocation. Otherwise, where $\delta \leq \delta^*$ the regulator liquidates any failed bank to outsiders. In their paper, Acharya and Yorulmazer (2007) show that state FF is similar to the symmetric case, unless for large δ. For $\delta \leq \delta^*$, the small bank is actually indifferent between high and low correlation, as it can not take over the assets of the other bank. But the big bank differentiates itself, as it gets always an extra benefit in state SF than state SS. In the contrary, for $\delta > \delta^*$,

[18] The subsidy of differentiating, likely surviving and buying the other bank in state SF.

[19] It can be verified by computing the total expected output generated by banks, net of liquidation and/or bailout policy, in a similar approach as above.

since the small bank has no opportunity to access the failed large bank's assets, only its bailout subsidy at state FF matters. The bail out subsidy for the large bank does not increase when the small bank fails too, whereas it does for the small bank if the big bank fails. This gives incentives to the small bank to herd with the big bank. Thus, the inter-bank correlation obtains by mixed strategies and there is no equilibrium in pure strategy. Accordingly, the TMTF mostly affect small banks. Empirical works of Jain and Gupta (1987) and Barron and Valev (2000) on US banks' lending behavior prior to the debt crisis of 1982-1984 support the results.

4.5 Macro-Prudential Regulation Policies

This section addresses prudential regulation policies dealing with the systemic risk. The focus is on the three sever cases of moral hazard: 1)TMTF 2)Too much related to fail and 3)TBTF. The first and the third cases have been briefly mentioned in the previous sections and this section concentrates on rather macro-prudential approaches towards these issues. The case of too much related too fail refers to a highly interconnected banking system liable to contagion and distributing the risk. Next subsections introduce policies against the distributed risk. Effectivity of each regulatory strategy is analyzed with respect to the source of moral hazard.

4.5.1 Dealing with TMTF

Acharya and Yorulmazer (2008) concentrate directly on the time-inconsistency problem as of TMTF. They show that granting surviving banks to take over the failed banks create incentives for taking less risk. With the setup of Acharya and Yorulmazer (2007) for n banks in an economy, this subsection analyzes the rewarding policy.

When too many banks are in default, the surviving banks may not have enough liquidity to acquire large amount of assets of all the failed ones. Therefore, the price of assets falls in the market such that outsiders of the banking sector can purchase some of the failed banks' assets. Even if the surviving bank want to issue equity to raise fund in order to be able to purchase all the failed banks' assets, they will need to compensate the outsiders as their competitors in the market for failed banks' assets. This will in turn reduces the price for assets of the surviving banks because they have to sell equity at a discount. Hence, they will still not access enough fund. The more failed banks, the lower the market-clearing price and the higher the total misallocation cost.[20]

[20] The total misallocation cost equals the number of failures times a constant misallocation cost $\bar{P}_2\delta$, where δ is the loss in return generated by outsiders.

Liquidating to outsiders is not ex-post optimal in a welfare analysis perspective. However, bailing out failed banks incurs a fiscal cost in the work of Acharya and Yorulmazer (2008). Thus, the regulator ex-post optimally bails out some of the failed banks as long as the marginal cost of bailout is less than the misallocation cost. Alternatively, suppose that the regulator provides sufficient liquidity to surviving banks to buy the same optimal number of failed banks. From the point of view of social welfare, the regulator has to pay the same amount of insurance cost and the total misallocation cost is not more than before, as surviving banks are the efficient users. Therefore, the ex-post social welfare cost with the alternative policy is as equal as the direct bailout policy.

Ex-ante the regulator wishes to avoid too many failures. The time inconsistency problem arises as she wants ex-ante to avoid herding among banks by threat of liquidating to outsiders but has to ex-post bailout the failed banks. To mitigate herding, the regulator takes dilution in the equity of the bailed out banks dependent on the severity of moral hazard.[21] The same result follows when the required liquidity is provided to surviving banks.

Yet, the endowment of outsiders influences the herding incentives. The less endowment the outsiders have, the price and the number of failed banks they together with the surviving banks can acquire decreases. The regulator has to provide liquidity for even smaller number of failures. This increases banks' incentives to differentiate as their surplus of takeing over failed banks raises. In turn, the regulator can take a smaller dilution to control herding over a larger range of α.

A surviving bank uses its first period profit, $R-d_1$, to purchase failed banks. When this resource, available to each surviving bank at date 1, exceeds the maximum price outsiders would pay for purchasing a failed bank, ψ, the bank can purchase larger amount of failed banks' assets. A surviving bank benefits more from the liquidity provision policy, as its purchase surplus outweighs the subsidy of bailout policy.

This way, the regulator encourages banks to differentiate through rather relax liquidation strategy. Comparing to ex-post optimal bailout policy, she can implement lower interbank correlation by a smaller stake in the bailed out banks. To summarize, the liquidity provision not only diminishes the likelihood of aggregate banking crisis but also dominates the bailout policy from an ex-ante standpoint.

4.5.2 Too Related To Fail and Capital Adequacy

The externality from one bank's investment to other banks', broadens prudential banking regulation studies towards a multiple-bank design. One extension approach is to take into account banks' correlation in the existing regulation strategies. This subsection

[21] As discussed in section 4.2, the choice of ϵ with respect to ϵ^* and α depends on the relation between δ and q.

describes two proposals which concern capital adequacy. As discussed in the previous sections, the ex-post optimal closure policies suffer from time-inconsistency problem. This directs us to ex-ante optimal designs. Capital requirement is then the core of such regulatory policies.

Acharya (2009) study the externalities of a bank failure on surviving banks. In a two banks setup similar to Acharya and Yorulmazer (2007), if bank B fails, a fraction $\varsigma < 1$ of its depositors migrate to bank A. Since the overall investment in the economy reduces, the return on the safe asset raises. This increases the cost of deposits on surviving banks, because the return to depositors equals the return to the safe asset in equilibrium (otherwise, there would be no investment or short-sell on it.).

Beside the "recessionary spill-over", there is a positive externality. Having more depositors, bank A can expand and acquire also the human capital of bank B. In fact, its cost of investment decreases to σ percent, and so does the investment in the risky asset. Thus, the total effect of two (negative and positive) externalities makes the difference between the profit in state SF and the profit in state SS, i.e. $E(\pi^{sf}) - E(\pi^{ss})$. This value which determines the bank's choice on interbank correlation, is by definition decreasing in ς but increasing in σ. For any σ, a threshold $\varsigma^*(\sigma)$ can be found below which the total externality is negative and banks have no incentive to differentiate. This situation can also hold for sufficiently high investment cost $\sigma^*(\varsigma)$, given ς. We end up in collective risk shifting, i.e. high ρ, for large σ and/or small ς, and low correlation otherwise.

As discussed in section 4.2, individual banks with low charter value (wealth) takes higher risk. Now the systemic risk shifting due to their correlation is extra to the individual failure risk. By definition, the loss of joint failure is larger than an individual bank failure. This provokes the need for regulatory actions against both systemic and individual risk-taking in Acharya (2009). Consequently, the regulator's closure policies (including liquidity provision) should exhibit less forbearance in the joint failure.

Regarding prudential treatments to penalize collective risk-taking and TMTF phenomena, ex-ante mechanisms such as capital requirement can be effectively improved. Since, the collective risk-shifting is based on externalities, a myopic capital adequacy regulation, independent of ρ, can at best mitigate individual risk-shifting.

Acharya (2009) shows that a capital adequacy regulation, increasing in the correlation of banks' portfolio and individual portfolio risk, moderates banks' systemic risk-shifting. The negative externality in state SF incentives banks to increase the probability of state SS by taking high correlation. However, the capital adequacy which depends on the endogenous negative externality induces the cost of capital in that case. Hence, the high cost of capital counteracts the negative externality. Accordingly, the proposal amends the myopic capital requirement strategy. It suggests that banks should hold more capital and take into account the general risk in economy in addition to their specific risk.

The next proposal contains rather practical view to the capital adequacy strategies. The main intuition is again about considering each bank's contribution to a systemic crisis. The systemic risk regulator can be compared to a senior manager who wants to prevent financial distress in a firm. She applies risk management technics to measure each division's contribution to the total risk of the firm. The equity is assumed a public good to the entire firm. Therefore, each unit must be charged according to the equity value used to support it. Acharya et al. (2009) imply similar approach for regulating crisis in the banking system.[22] As systemic risk is defined to occur endogenously, each bank's contribution is measured.

Current regulation policies should be adjusted to consider systemic risk in the banking system. Capital adequacy is thus as an intuitive regulatory instrument imposed to depend on each bank's measure of the systemic risk contribution. For instance, the Basel II capital requirement multiplied by this measured systemic factor is an improvement, consistent to the discussion above. The proposal is in fact an introduction to the Basel III regulatory accords. However, it can be enforced efficiently under circumstances that limit the cyclicality problem in the systemic risk measurement and the issue of fake decrease in leverage.

4.5.3 TBTF and Systemic Taxing

Since a big complex bank can not be liquidated, a natural prudential strategy is to tax its activities that bring negative externalities with the intuition to discourage the behavior leading to systemic risk. Further, the accumulated tax then could be used to fund the losses of the systemic crisis. However, from section 4.2 we know that for taxing being effective against risk-taking a proper design is necessary. Freixas and Rochet (2010) plan a systemic tax to deal with the extreme and rare event of large losses in a Systemically Important Financial Institution (SIFI).

In case of a SIFI failure, a public supervisory intervention is needed since no private insurance can cover the losses C, neither the shareholders want to recapitalize as their expected NPV is negative. In a multi-period setup, take into account the manager's moral hazard discussed in section 4.2. The SIFI generates a fixed positive cash flow μ in each period. But it may fail with a very small probability τ which increases by $d\tau$ because of the manager's moral hazard. Another main friction between the manager and shareholders is that she is more impatient as his discount factor ξ_M is smaller than shareholders' ξ.

Existence of a strong and independent systemic risk authority which has the power to restructure the bank and the ownership is necessary in Freixas and Rochet (2010), to

[22] A detailed discussion on the applied risk management methods is beyond this survey. Acharya et al. (2009) describe following methods: value-at-risk and expected shortfall, stress tests and aggregate risk scenarios, and pricing systemic risk.

establish ex-ante optimal regulation. The authority has to bear a cost for restructuring the bank Γ. As discussed in section 4.2, to avoid moral hazard, the manager's salary should depend on his performance and satisfy incentive compatibility constraint. Hence, she receives a bonus s at the end of each successful period that

$$(1-\tau)(s+\xi_M\omega) \leq (1-\tau-d\tau)(s+\xi_M\omega)+Q. \tag{4.35}$$

Still, she should be offered a one time payment G when she signs the contract. This fills the gap between his reserved utility U (his training cost) and the expected continuation payoff ω, i.e. $G = U - \omega$, where $\omega = (1-\tau)(s+\xi_M)$.

The systemic risk authority expropriates the shareholders after the crisis. The regulator sells the bank to new shareholders, naturally in a price equal to their expected benefit. The price is the expected value of the bank π net of the one-time offer to the new manager, i.e. $\Pi - G$ where $\Pi = \mu - T + (1-\tau)(-s+\Pi)$. The regulatory cost of restructuring reduces because of selling the bank. Hence, the systemic tax equal to the expected cost for the regulator in case of crisis, $T = \tau[C + \xi(\Gamma - (\Pi - G))]$.

The clever proposal of Freixas and Rochet (2010) is to consider the regulator offering a grace period to the new manager after the crisis. It means that if immediately after a restructuring the bank fails the manager will not be fired and the shareholders are not expropriated, but the bank is bailed out. For the manager not to take moral hazard in this period the minimum bonus of the period is $Q/(d\tau)$ which is larger than ω from (4.31). In return, her one time payment reduces because of bigger bonus in the grace period. After the grace period, everything is back to the contract mentioned above. Nevertheless, guaranteeing a grace period is socially beneficial iff the total cost of restructuring (immediately after previous restructuring) $\xi(U+\Gamma)$ is higher than the cost of loading the compensation of the manager $(\xi - \xi_M)\omega^*$, where ω^* is the manager's minimum continuation salary from (4.35). In other words, under such condition the one period grace contract is socially more beneficial.

To find the optimal contract, Freixas and Rochet (2010) control for the optimal probability of the bank being restructured. Furthermore, the question is whether the manager's payment contract is optimal with respect to her performance. The regulator optimizes the total social surplus of the bank. Though no managerial payment in case of crisis minimizes the managerial risk-taking incentives, a crisis implies restructuring the bank which is costly to the systemic risk authority. The trade-off brings the solution to the problem, as the sufficient requirements for having a grace period is explained above. Freixas and Rochet (2010) solve the recursive dynamic programming problem to justify the optimality of the contract with one grace period. The interesting point is that if there was no supervisory of the regulator, the new shareholders would refuse to compensate the new manager. The robustness of the result is also verified for larger τ.

4.5.4 Market-Based System and Other Alternatives

Above mentioned methods make the regulator responsible for measuring risks and implementation of resolution policies. Alternatively, an insurance against only systemic part of the risk would be a market-based complementary system. The insurer must compute the risk and in case of crisis can pay part of losses to the financial stability regulator, not directly to the institute.

As Acharya et al. (2009) discuss, to handle the crisis among insurance system, the insurers only provide coverage for a small percent of losses. The regulator has to still be the lender of last resort. However, the insurance companies would inspect the systemic risk of each bank carefully and regularly such that banks have less incentive to game than under fixed regulatory fees or capital requirements. This way the bank would limit its systemic risk and provide more transparency to decrease the insurance premia. The insurer's pricing provides also more information for the public and the financial stability regulator. Note that the insurance system can be combined and imposed together with the systemic-risk-based capital adequacy or taxing policies. Therefore, a public-private system would work more effectively both in examining the systemic risk and then in the rare crisis event.

4.6 Regulatory Policies in the Recent Crisis

After surveying the regulatory policies, it is time to investigate what have been so far done in the past crisis. This section presents the US regulatory data on bank and financial institutions failures. The sample starts from 1934 but the main focus is on the recent crises of 2007-2009 and its comparison to the past. The source of data is FDIC's Failures and Assistance Transactions database.[23] Unfortunately, detailed data on bailout are not available but there are data about other resolutions.

The resolution transactions are in three main categories: 1) assistance in which institution's charter value survives, 2) failure with termination of the charter value, and 3) payout, where the insurer pays the depositors directly and place assets in the liquidating receivership. Assistance transactions include transactions where a healthy institution acquires the entire bridge bank-type entity but certain other assets were moved to liquidating receivership, or open bank assistance transactions under a systemic risk determination. In a bridge bank transaction the FDIC itself acts temporarily as the acquirer. It provides uninterrupted service to bank customers while having sufficient time to market the institution. Reprivatization as management takeover with or without assistance at takeover, followed by a sale, is very rare in the data. The second category contains all types of "Purchase and Assumption" (P&A) agreements. In these resolution

[23]The data for year 2010 is up to August, 20.

transactions the healthy institution purchases some or all of assets of a failed institution and assumes some or all of the liabilities, including all insured deposits.[24]

Figures 4.1 to 4.4 show number of all transactions of the three categories in four time intervals. Figure 4.5 puts them all together in order to make comparison possible. The second category transactions are known as "failure, merger" in the figures. The trend of average total deposits in failure (category two and three) and assistance transactions provide information on volumes. In all years from 1934 to 1979, the total deposits under assistance sums up to about 6$ billion. Compare it with years after. In the 80s the average total deposit under assistance is much higher than under failures. It is increasing and the peak is 1.5$ billion in 1989. This is so while the number of assistance is always very small. It means that mostly large banks have been under assistance. Huge number of failures is seen in the 80s that is reversed in the 90s. The trend of systemic failure is decreasing in the 90s and so do the trend of average total deposits. However, a relatively larger volume of deposits were under failure transaction than assistance. Since 2000 there was not much problems in the banking system until 2008 and 2009. Though the number of failures and assistance is not as large as the 80s, the average total deposits is enormous. With low number of assistance transactions, up to about 6$ and 14$ billion are spent to assist total deposits per bank in 2008 and 2009, respectively. Note that the 7$ billion bailout to the financial system of the US is extra to these transactions. The important role of bank sale is observable. However, the systemic shocks were such extreme that they are mostly covered by huge cost for the government, i.e. the regulatory authority.

4.7 Conclusion

This paper surveys the development of banking regulation towards systemic risk regulation in the recent years. Regulating a single bank in normal times have been widely studied. Regulation strategies against a bank's risk-taking and resolution policies in case of a failure are well optimized. However, they have been limited to individual banks' problems.

Preventing or resolving a systemic crisis requires different policies. Ex-ante policies such as capital adequacy, taxing and/or deposit insurance should adjust for this purpose. Dependence of the adequate capital ratio not only on each bank's risk but also on banks' correlation would decrease banks' herding in risk-taking. Computing each bank's contribution to the systemic risk in a proper risk-management method, the capital adequacy or insurance premium should depend on this measurement too. Systemic taxing for a substantially important institute in an economy would diminish the risk if the regulator is strong enough to expropriate the ownership. Taxing and partial insurance can also

[24] A bridge bank transaction is also a type of P&A.

provide funding for the losses. Private insurance companies would also monitor banks' activities more closely and price premia more carefully. To protect the insurance system from transmitted loss, the partial insurance given to banks should only cover a fraction of their systemic risk.

Ex-post crisis resolutions should also be ex-ante optimal. Since at a crisis, asset liquidation is not ex-post optimal in majority of failures, forbearance policies should encourage risk-reduction. In other words, direct bailout would highly increase moral hazard and must be prohibited. Researches propose takeover of a failed bank by a healthy institute should be allowed and also granted. It means that, the regulator should provide liquidity to a survival of the crisis for purchasing failed institutes. The policy that empowers healthy banks involves the same social cost as a direct bailout, but has the great advantage that reduces collective risk-taking among banks.

Still, there is much space for further development of macro-prudential regulation. Further research could for instance consider the interbank relation. At failure of some banks, how could their connection to other parts of financial system be controlled to avoid transmission? How should this interconnection be ex-ante optimally regulated? Beside open questions regarding interbank relations, implementation of existing proposals is equally important. The practical way the supervisory authority should impose a policy or combination of policies depends on the economy and also legal systems. This provides broad area of research in both applied and theoretical topics.

Figures

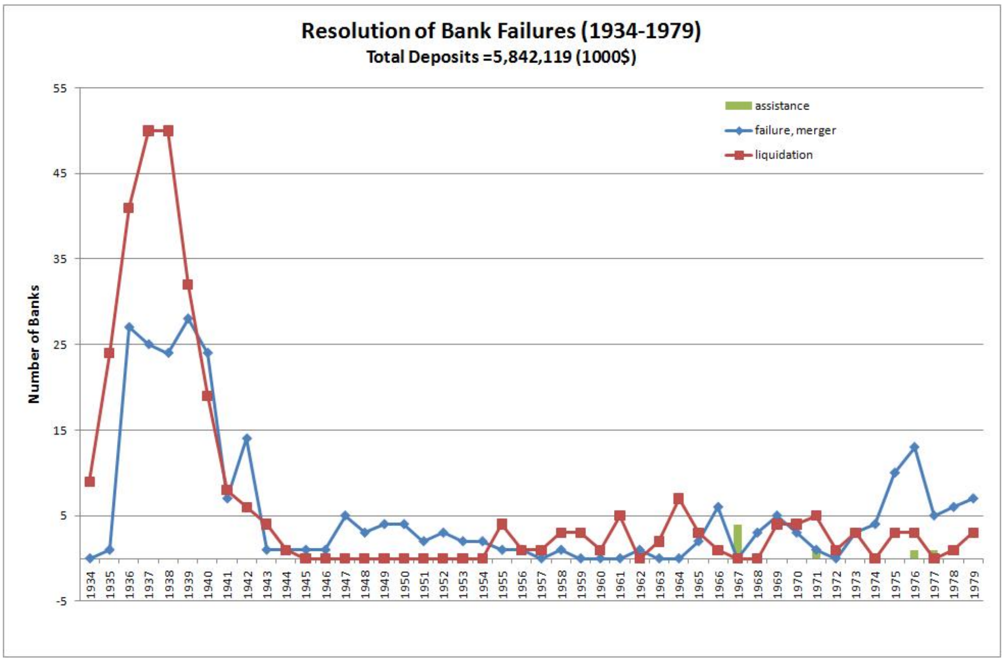

Figure 4.1. US Bank Resolutions 1934-1979.

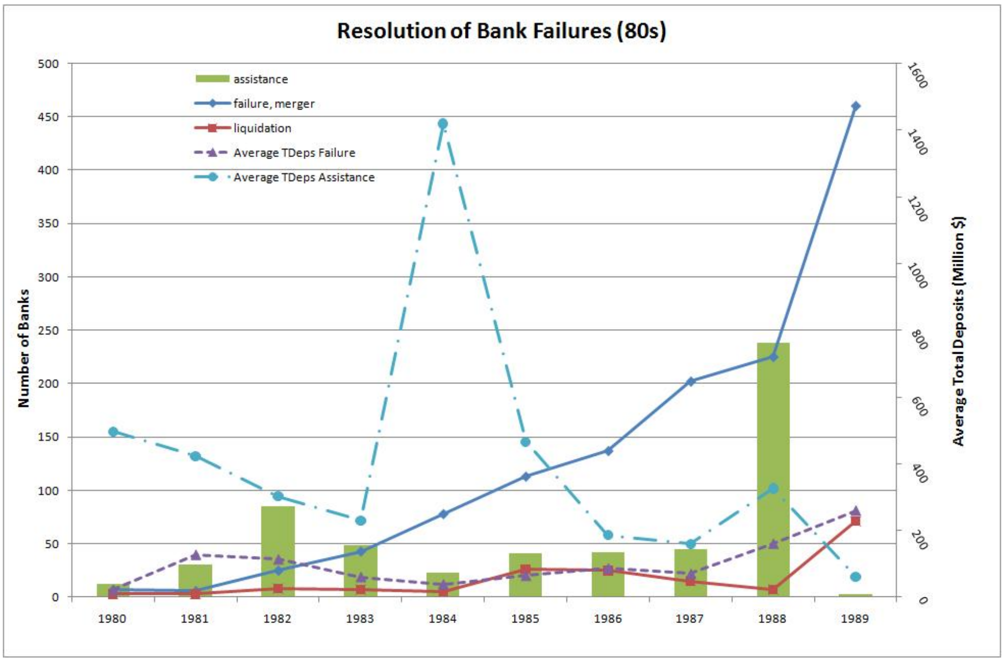

Figure 4.2. US Bank Resolutions in the 80s.

Assistance Transactions include: A/A transactions where assistance was provided to the acquirer who purchased the entire institution, or where assistance was provided under a systemic risk determination; and the institution's charter survived.

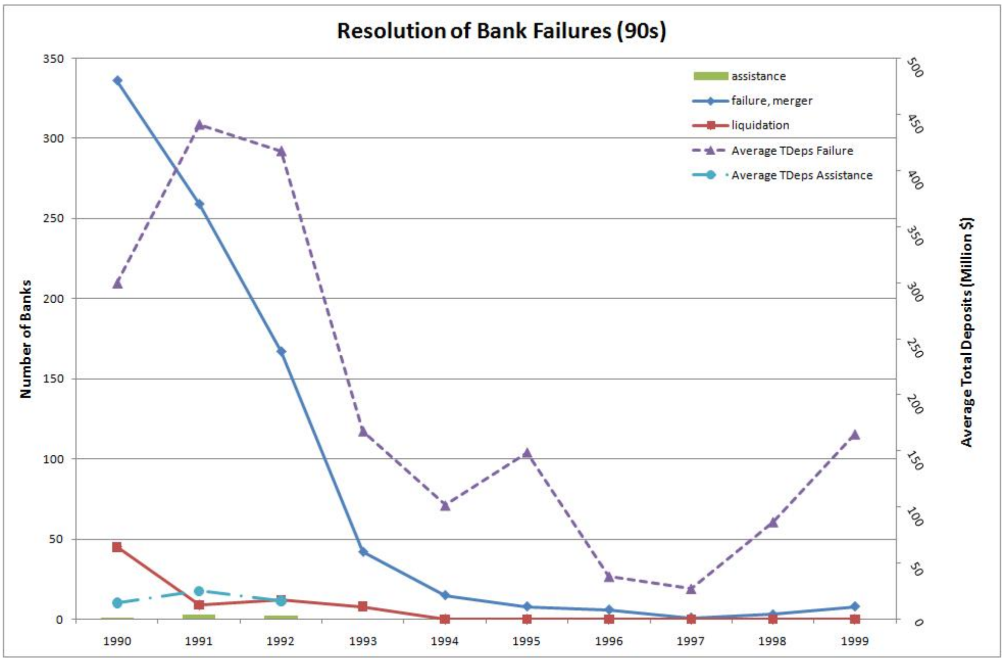

Figure 4.3. US Bank Resolutions in the 90s.

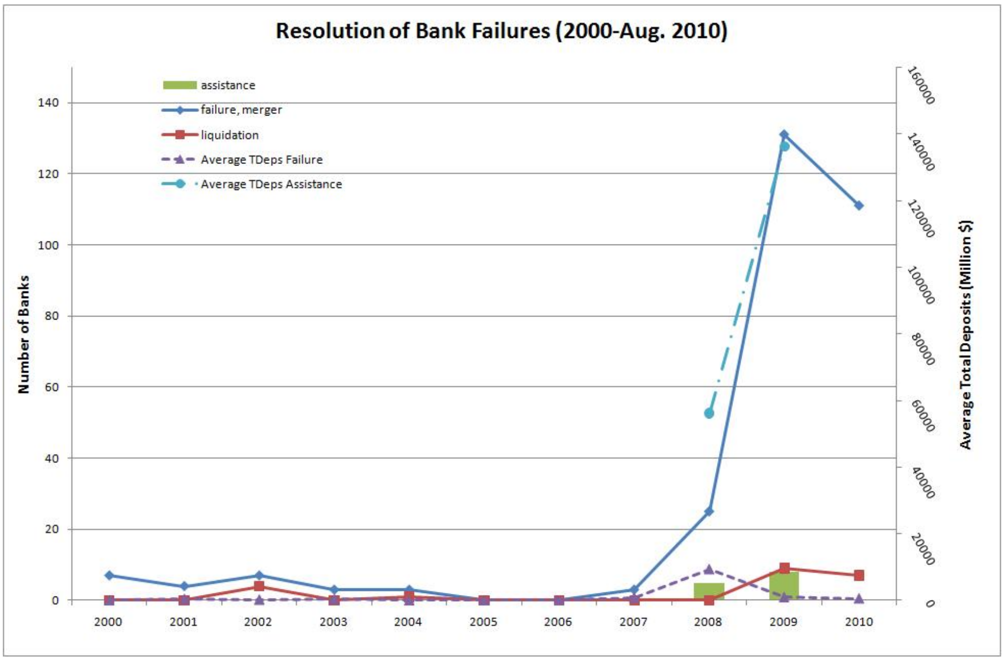

Figure 4.4. US Bank Resolutions since 2000.

Assistance Transactions include: A/A transactions where assistance was provided to the acquirer who purchased the entire institution, or where assistance was provided under a systemic risk determination; and the institution's charter survived.

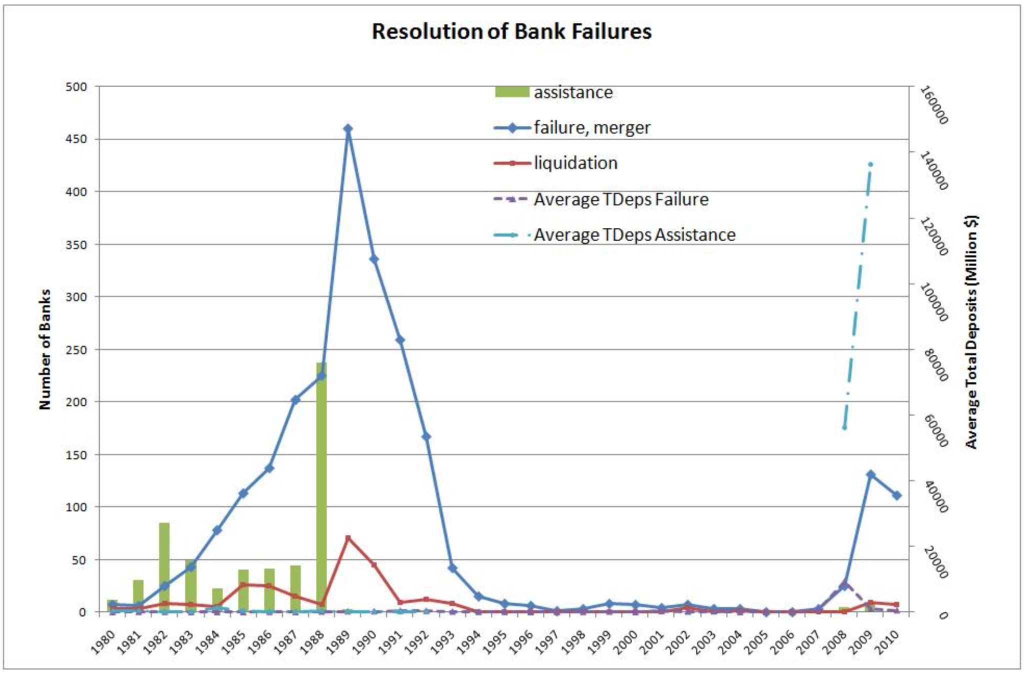

Figure 4.5. US Bank Resolutions 1980-August 2010

Bibliography

Acharya, V. and Yorulmazer, T. (2008). Cash in the market pricing and optimal resolution of bank failures. *The Review of Financial Studies*, 21:2705–2742.

Acharya, V. V. (2009). A theory of systemic risk and design of prudential bank regulation. *Journal of Financial Stability*, 5 (3):224–255.

Acharya, V. V., L.H. Pederson, T. P., and Richardson, M. (2009). *Regulating Systemic Risk*, chapter 13, pages 283–303. New York University Stern School of Business.

Acharya, V. V. and Yorulmazer, T. (2007). Too many to fail- an analysis of time-inconsistency in bank closure policies. *Journal of Financial Intermediation*, 16:1–31.

Allen, F. and Gale, D. (2000). Financial contagion. *Journal of Political Economy*, 108(1):1–33. Mimeo, The Wharton School, University of Pennsylvania.

Barron, J. and Valev, N. (2000). International lending by u.s. banks. *Journal of Money, Credit and Banking*, 32 (3):357–381.

Basel Committee (2001). Overview of the New Basel Capital Accord. Consultative document, BCBS, Basel, Switzerland.

Bayraktar, E. and Egami, M. (2007). On the one-dimensional optiomal switching problem. http://www-personal.umich.edu/ egami/switch.pdf.

Berlin, M. and Mester, L. (1992). Debt covenants and renegotiation. *Journal of Financial Intermediation*, 2:95–133.

Besanko, D. and Kanatas, G. (1996). Bank capital regulation: Time consistency, hedging, and incentive compatibility. *Journal of Financial Intermediations*, 5:160–83.

Bhattacharya, S., Boot, A., and Thakor, A. (1998). The economics of bank regulation. *Journal of Money, Credit and Banking*, 30:745–770.

Bhattacharya, S., Plank, M., Strobl, G., and Zechner, J. (2006). Bank capital regulation with random audits. *Journal of Economic Dynamics & Control*, 26:1301–1321.

Boot, A. and Greenbaum, S. I. (1993). Bank regulation, reputation, and rents: Theory and policy implications. In Mayer, C. and Vives, X., editors, *Capital Markets and Financial Intermediation*. Cambridge University Press.

Boot, A. and Thakor, A. (1993). Self-interested bank regulation. *The American Economic Review*, 83:206–212.

Boot, A. and Thakor, A. (2008). *The Accelerating Integration of Banks and Markets and its Implictions forRegulation*. The Oxford Handbook of Banking.

Brekke, K. and Oksendal, B. (1994). Optimal switching in an economic activity under uncertainty. *SIAM Journal of Control and Optimization*, 32:1021–1036.

Chan, Y., S.I., G., and Thakor, A. (1992). Is fairly priced deposit insurance possible? *The Journal of Finance*, 47:227–245.

Cordella, T. and Yeyati, E. L. (2003). Bank bailouts: Moral hazard vs. value effect. *Journal of Financial Intermediation*, 12:300–330.

Dahl, D. and Spivey, M. (1995). Prompt corrective action and bank efforts to recover from undercapitalization. *Journal of Banking and Finance*, 19:225–243.

Dangl, T. and Lehar, A. (2004). Value-at-risk vs. building block regulation in banking. *Journal of Financial Intermediation*, 13:96–131.

Decamps, J. and Djembissi, B. (2005). Revisiting the asset substitution effect in a contingent-claim analysissetting. Real Options, 9th Annual International Conference, Paris, June22-25.

Decamps, J., Roche, J., and Roger, B. (2004). The three pillars of basel ii: Optimizing the mix. *Journal of Financial Intermediation*, 13:132–155.

Diamond, D. (1984). Financial intermediation and delegated monitoring. *Review of Economic Studies*, 51:393–414.

Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91:401–19.

Dixit, A. and Pindyck, R. S. (1994). *Investment under Uncertainty*. Princeton University Press, Princeton, New Jersey.

Erricson, J. (1997). *Asset substitution, debt pricing, optimal leverage and maturity, creditrisk in corporate securities and derivatives: valuation and optimal capitalstructure choice*. PhD thesis, Stockholm School of Economics.

Fackler, P. (2004). Solving optimal switching models. Working paper, http://www.samsi.info/200506/fmse/workinggroup/ci/switch.pdf.

Freixas, X. (1999). Optimal bailout policy, conditional and constructive ambiguity. Discussion paper No. 237. LSE Financial Markets Group.

Freixas, X., Parigi, B., and Rochet, J. C. (2000). Systemic risk, interbank relation, and liquidity provision by the centralbank. *Journal of Money, Credit and Banking*, 32 (3):611–638.

Freixas, X. and Rochet, J. C. (2010). Taming sifis. In *3rd Conference on Financial Integration and Stability: Systemic Risk and Incentives; 23-24.09.2010, ZEW, University of Mannheim*.

Fries, S., Mella-Barral, P., and Perraudin, W. (1997). Optimal bank reorganization and the fair pricing of deposit guarantees. *Journal of Banking and Finance*, 21:441–468.

Gennotte, G. and Pyle, D. (1991). Capital controls and bank risk. *Journal of Banking and Finance*, 15:805–824.

Jain, A. and Gupta, S. (1987). Some evidence on herding behavior of u.s. banks. *Journal of Money, Credit and Banking*, 19 (1):78–89.

Jensen, M. and Meckling, W. (1976). Theory of the firm: Managerial behavior, agency costs, and ownership structure. *Journal of Financial Economics*, 4:305–360.

Jones, D. and King, K. (1995). The implementation of prompt corrective action: An assessment. *Journal of Banking and Finance*, 19:491–510.

Leland, H. (1994). Corporate debt value, bond covenants, and optimal capital structure. *The Journal of Finance*, 49:1213–1252.

Leland, H. and Toft, K. (1996a). Optimal capital structure, endogenous bankruptcy, and the term structureof credit spreads. *The Journal of Finance*, 51(3):987–1019.

Leland, H. and Toft, K. (1996b). Optimal capital structure, endogenous bankruptcy, and the term structureof credit spreads. *The Journal of Finance*, 51:987–1019.

Leland, H. E. (1998). Agency costs, risk management, and capital structure. Working Paper.

Mailath, G. and Mester, L. (1994). A positive analysis of bank closure. *Journal of Financial Intermediation*, 3:272–299.

Mitchell, J. (1997). Strategic credito passivity, regulation, and basel. University of Michigan Business School, The William Davidson Institute,working paper.

Perotti, E. and Suarez, J. (2002). Last bank standing: What do i gain if you fail? *European Economic Review*, 46:1599–1622.

Peters, S. (1994). Why are demand deposits sequential service constrained and insured? University of Connecticut, working paper, 1994.

Pham, H. (2005a). On some recent aspects of stochastic control and their applications. *Probability Surveys*, 2:506–549.

Pham, H. (2005b). On the smooth-fit property for one-dimentional optimal switching problem. forthcoming, Seminaire de Probabilites, Vol. XL.

Prescott, E. (2004). Auditing and bank capital regulation. *Federal Reserve Bank of Richmond Economic Quarterly*, 90/4.

Repullo, R. and Suarez, J. (2004). Loan pricing under basel capital requirements. *Journal of Financial Intermediation*, 13:496–521.

Rochet, J. and Tirole, J. (1996). Interbank lending and systemic risk. *Journal of Money, Credit and Banking*, 28(4):733–62.

Stiglitz, J. E. and Weiss, A. (1981). Credit rationing in markets with imperfect information. *The American Economic Review*, 71:393–410.

Vath, V. L. and Pham, H. (2007). Explicit solution to an optimal switching problem in the two-regime case. *SIAM, Journal on Control and Optimization*, 46:395–426.

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Eidesstattliche Erklärung

Hiermit erkläre ich, dass ich die Dissertation selbstständig angefertigt und mich anderer als der in ihr angegebenen Hilfsmittel nicht bedient habe, insbesondere, dass aus anderen Schriften Entlehnungen, soweit sie in der Dissertation nicht ausdrücklich als solche gekennzeichnet und mit Quellenangaben versehen sind, nicht stattgefunden haben.

Mannheim, 18. Januar 2011

Maryam Kazemi Manesh