# Risk-Sensitive Capital Requirements and Pro-Cyclicality in Lending

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# List of Abbreviations

| | |
|---|---|
| Art. | Article |
| BCBS | Basel Committee of Banking Supervision |
| *cf.* | compare |
| Ch. | Chapter |
| EC | European Community |
| EEC | European Economic Community |
| *e.g.* | for example |
| EMU | Economic and Monetary Union of the European Union |
| Eq. | Equation |
| EU | European Union |
| f | and the following page |
| Fig. | Figure |
| ff | and following pages |
| G-10 | Group of Ten (Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom and the United States) |
| G-20 | Group of Twenty (Argentina, Australia, Brazil, Canada, China, France, Germany, India, Indonesia, Italy, Japan, Mexico, Russia, Saudi Arabia, South Africa, South Korea, Turkey, the United Kingdom, the United States, and the European Union) |
| GDP | Gross Domestic Product |
| IAS | International Accounting Standards |
| *ibid.* | *ibidem* |
| *i.e.* | that is |
| IRB | Internal-Ratings-Based |
| IS | Investment/Saving equilibrium |

| | |
|---|---|
| Iss. | Issue |
| LM | Liquidity preference and Money supply |
| n/a | not applicable; not available |
| No. | number |
| OECD | Organization for Economic Co-operation and Development |
| p. | page |
| PCA | Prompt Corrective Action |
| PD | probability of default |
| PIT | point-in-time |
| pp. | pages |
| Res. | Result |
| S&P | Standard & Poor's |
| StdA | Standardized Approach |
| TTC | through-the-cycle |
| UK, U.K. | United Kingdom of Great Britain and Northern Ireland |
| US | United States |
| USA | United States of America |
| VaR | value-at-risk |
| Vol. | Volume |
| w/o | without |
| w/r/t | with respect to |

# List of Frequently Used Symbols

| | |
|---|---|
| $b_{d_x,t}$ | the bank's default barrier at $t$, $t = 1, 2$, in terms of non-standardized portfolio returns |
| $b_{d_z,t}$ | the bank's default barrier at $t$, $t = 1, 2$, in terms of standardized portfolio returns |
| $b_{s_z}$ | the bank's solvency barrier in terms of standardized portfolio returns (one-period model) |
| $c$ | Cooke ratio |
| $c_i$ | risk weight for loan $i$ |
| $C_j$ | feasibility set characterizing the contingent pay-offs of the bank to its depositor ("Case $j$") |
| $\mathrm{Corr}(\cdot,\cdot)$ | correlation operator |
| $D$ | contracted deposit volume (bank debt) |
| $\tilde{D}$ | deposit repayment |
| $e$ | Euler's number |
| $\mathrm{E}(\cdot)$ | expectations operator |
| $i$ | index number for corporate borrowers/sector, $i = 1, 2$ |
| $j$ | index number for the Case $C_j$ |
| $j$ | index number for corporate borrower $j$ belonging to any of both sectors $i$ |
| $k(\cdot)$ | constraint function under regulation with fixed risk weights $c_i$ |
| $L$ | contracted total loan volume |
| $L_i$ | contracted volume of the loan granted to borrower $i$ |
| $\tilde{L}$ | total loan repayments |
| lim | limit operator |
| max | maximization operator |
| $n$ | number of borrowers, firms/projects from one sector |
| $p_i$ | success probability of firm/project $i$ |
| $\tilde{p}_i^j$ | success probability of firm/project $j$ from sector $i$ |
| $\mathrm{P}(\cdot)$ | probability operator |

| | |
|---|---|
| $q$ | common success probability of both projects, Bernoulli model |
| $q_j$ | the probability of bank solvency/full deposit redemption given Case $j$, Bernoulli model |
| $R_D$ | contracted gross interest rate on deposits $D$ |
| $R_f$ | gross interest rate on the risk-free asset |
| $R_i$ | contracted gross interest rate on loan/sector-specific loan type $i$ |
| $s_i$ | standard deviation of returns from sector $i$ |
| $s_{1,2}$ | covariance of returns from Sectors 1 and 2 |
| $S$ | superscript indicating an equilibrium under regulation by fixed risk weights $c_i$ |
| $t$ | index of time, $t = 0, 1, 2$ (two-period model) |
| $U(\cdot)$ | the household's preference function |
| $u_H(\cdot)$ | the household's von Neumann-Morgenstern utility index |
| $v(\cdot, \cdot)$ | constraint function under regulation by VaR, Bernoulli model |
| $V$ | superscript indicating an equilibrium under regulation by VaR |
| $\mathrm{V}(\cdot)$ | variance operator |
| $w(\cdot)$ | constraint function under regulation by VaR, normal model |
| $W_B$ $(W_{B,0})$ | the bank's initial equity (in the two-period model) |
| $\tilde{W}_B$ | the bank's equity at the end of the period (one-period models) |
| $\tilde{W}_{B,t}$ | the bank's equity at $t$, $t = 1, 2$ (two-period model) |
| $W_H$ $(W_{H,0})$ | the household's initial wealth (in the two-period model) |
| $\tilde{W}_H$ | the household's wealth at the end of the period (one-period models) |
| $\tilde{W}_{H,t}$ | the household's wealth at $t$, $t = 1, 2$ (two-period model) |
| $\tilde{x}$ | normally distributed gross return on the bank's total loan portfolio |
| $\tilde{x}_i$ | normally distributed gross return on the bank's sector-$i$ loan portfolio |
| $\tilde{x}_t$ | normally distributed gross return on the bank's total loan portfolio realized at $t$, $t = 1, 2$ |
| $\tilde{X}_i$ | Bernoulli random number indicating firm/project $i$'s success or failure |
| $\tilde{X}_i^j$ | Bernoulli random number indicating firm/project $j$'s success or failure |
| $z_t$ | standardized normally distributed gross return on the bank's total loan portfolio realized at $t$, $t = 1, 2$ |

| | |
|---|---|
| $\alpha_i$ | gross return on project $i$ in case of success |
| $\gamma$ | the household's absolute risk aversion |
| $\mu_j$ | expected marginal gross return on deposits under Case $j$ |
| $\mu_i$ | expected gross return on loans from sector $i$ |
| $\rho$ | correlation between sectors |
| $\sigma_j$ | volatility of marginal gross return on deposits under Case $j$ |
| $\sigma_i$ | volatility of gross return on loans from sector $i$ |
| $\tau$ | regulatory parameter under regulation with value-at-risk |
| $\varphi(\cdot)$ | density function of the standard normal distribution |
| $\Phi(\cdot)$ | cumulative distribution function of the standard normal distribution |
| | |
| $\ell$ | loan-allocation rate/portfolio-allocation rate |
| $\mathcal{N}(\mu, \sigma)$ | normal distribution with parameters $\mu$ and $\sigma$ |
| $\mathbb{R}$ | the real numbers |
| $*$ | superscript indicating laissez-faire equilibrium |

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# Part I

# Introduction

# Chapter 1

# The Issue of Pro-Cyclicality

Risk-sensitive capital requirements aim at enforcing appropriate minimum amounts of capital to absorb losses resulting from credit defaults and other risk sources, notably market and operational risk. These requirements are considered to be important as they are supposed to strengthen financial stability. Nevertheless, practitioners and scholars have worried about negative side effects arising from capital-based regulation. One such topic concerns the reinforcement of cyclicality in lending by regulation. These concerns are based upon the notion that regulatory rules, which become tighter in downturns, restrict the bank's total lending beyond what banks would do for reasons of risk management. As a consequence, valuable projects are not financed during recessions, which may in turn enhance the cyclical troughs in their depth and length. A situation commonly known as “credit crunch” or “credit rationing” arises.

In this thesis, we address the question of if and how bank capital regulation may enhance cyclical patterns in lending. To do this, a bank model is set up, and different types of capital requirements are considered and compared to the outcomes if the bank remains unregulated. The mechanisms at work that fuel pro-cyclical patterns in lending can be also distinguished according to the characteristics of capital requirements. We add to the literature insofar as the literature about the pro-cylicality of capital requirements has not been concerned with a bank that simultaneously takes its leverage, asset risks, probability of bankruptcy, size, and costs of debt finance into account. Likewise, there is no work of this kind considering that the deposit volume and the deposit interest rate are based on decisions made by a risk-averse household and a risk-neutral bank. The deposit interest rate and the deposit volume reflect the bank's risk-taking and, if regulation is present, the regulatory constraint. The impact of capital requirements are analyzed by considering the sensitivity of the lending volume to changes of fundamental economic variables, called “shocks”, under a VaR approach, under approaches with fixed risk weights, and under a laissez-faire economy.

The simplest form of capital regulation is to require banks to adhere to a fixed ratio of equity to assets or deposits. During recessions, credit losses will increase and diminish the banks’ equity capital positions. Consequently, the volume of the loan portfolio must shrink in order to attain the regulatory capital ratio once again. As a consequence, lost loan positions are no longer completely filled. The overall potential credit capacity of the financial system shrinks.

Under Basel I, capital rules were already more complex than a simple capital ratio: banks had to meet a weighted capital ratio, *i.e.* a specific weight was assigned to each asset position. All weighted asset positions must meet a given relation to regulatory capital. These weights partly reflected notions of credit risk as some governmental authorities were given a risk weight of zero and building loans had preferential weights compared to other loans.[1] Therefore, losses could not only result in decreasing loan portfolio volumes, but also in loan portfolio shifts, here notably to the disadvantage of plain commercial and retail loans, and in favor of sovereigns (Haubrich/Wachtel, 1993, p. 3f; Berger/Udell, 1994, p. 586; Furfine, 2001, p. 34). It has been often attempted to trace back the 1990/91 Credit Crunch to the introduction of the capital requirements based on the Basel I Accord.

The Basel II rules aimed at determining minimum bank capital in accordance to the risk taken on the asset side. As recessions are characterized by decreasing prospects for firms and increasing losses that firms must bear, the firms’ credit-worthiness shrinks. With increasing credit risk in turn, banks must hold more capital for a given volume of loans. If, moreover, banks must bear losses themselves, a massive downturn in bank credit supply will take place according to the critics and sceptics of risk-sensitive capital requirements. Thus, some people view risk-sensitive capital requirements as worsening cyclical downturns in credit supply compared to what risk-insensitive capital requirements may cause.

However, all these criticisms often neglect the fact that lending by its very nature may be a cyclical business. In the next sections of this chapter, we present the literature that is concerned with the issue of if and how capital regulation may result in pro-cyclical patterns in lending.[2]

[1] Dewatripont/Tirole (1994, Ch. 3), provide an account of the implemented rules based on the Basel I Accord, particularly for the EU. In Germany, these rules were implemented as the ‘Grundsatz I’ of which §13 listed all eligible risk weights.

[2] For different reviews of literature on banking regulation, in particular capital requirements, we

To start with, Section 1.2.2 presents theories of how lending and business cycles may interact in the absence of regulation. Section 1.2 reviews the introduction of the Basel I Accord and the 1990/91 Credit Crunch. The empirical literature that asks if and to what extent the introduction of Basel I was responsible for the downturn in granting corporate loans, is discussed. Section 1.2.2 presents theoretical arguments in favor of pro-cyclical effects through capital requirements. Section 1.3 is devoted to the Basel II Accord. Its basic mechanisms are presented (Section 1.3.1). The introduction of further amendments, commonly known as Basel III are discussed. Section 1.3.2 reviews the literature that is concerned with the cyclicality of the required minimum capital. Section 1.3.3 discusses the role of capital buffers, *i.e.* the excess capital that is held by commercial banks beyond what is required by regulators. Section 1.3.4 highlights the impact that the type of the loan rating system has on the cyclicality of capital requirements. Section 1.4 points out that there are also other regulatory norms that may fuel pro-cyclicality. Finally, Section 1.5 discusses the impact capital adequacy rules may have on the risk-taking of banks.

The next chapter, Chapter 2, outlines the framework of our analysis. It refers to the types of shocks considered, our notion of pro-cyclicality, and to the basic framework.

The following two parts, Part II and III, are the core of this thesis. Two distinct versions of the basic model described in Chapter 2 are analyzed under different regulatory regimes. The impacts of shocks under these regimes are assessed using comparative static analyses which are performed numerically, except for few instances of equilibria. In Part II, the bank grants loans to two firms which run different, potentially correlated projects whose outcomes are Bernoulli distributed. The analysis is restricted to one period. Credit risk is either regulated by fixed risk weights or by a VaR approach. Part II consists of two chapters.

In Chapter 3, the model set-up is explained and some theoretical implications are derived. In Chapter 4, the question of pro-cyclicality resulting from capital adequacy rules is numerically discussed. Part II also addresses issues of risk-taking.

In Part III, the bank faces many firms whose different Bernoulli-distributed projects are weakly correlated and whose projects can be lumped together into two distinct sectors. In the aggregate, the bank faces normally distributed loan-portfolio returns on which the bank's and the household's decisions are based (Ch. 5). Chapter 6 is devoted to the analysis of the one-period model. In Chapter 7, the analysis is extended to two periods. Both fixed risk weights and a VaR approach are considered

refer to Bhattacharya/Boot/Thakor (1998), Santos (2000), Stolz (2002), Van Hoose (2007,2008), and Borio/Zhu (2008).

for the one-period analysis as regulatory capital requirements, whereas the analysis of the two-period model is restricted to a comparison of the laissez-faire equilibrium to the equilibrium under the VaR approach. The final part, Part IV, contains conclusions. Derivations of theoretical results can be found in the Appendix.

A main result is that the effect of a given shock depends on the type of the shock. Equity shocks, *i.e.* changes in bank equity as a result of gains or losses, lead to pro-cyclical effects on lending. Expectation shocks, *i.e.* changes of distribution parameters or of borrowers' productivity, are dampened by regulation. Thus, regulation may exacerbate downturns after losses have been realized, whereas regulation constrains bank lending if more favorable outcomes are expected, thus hampering economic recovery. Moreover, there is evidence of counter-cyclical effects through regulation on the level of single loan volumes. These observations can be made regardless of the risk sensitivity of the respective capital adequacy rule. Interestingly, a regulated bank may grant higher loan volumes to less risky firms if risk-sensitive capital requirements are binding than it would do under fixed requirements or under a laissez-faire regime.

The two-period model supports these findings. In particular, there are no signs that a financial accelerator based on a propagation mechanism via the bank's equity exists. That is, the sensitivities of the expected total loan volume in the second period with respect to expected equity which prevails after the first period are always the same size as they are in the one-period model with respect to the bank's initial equity.

Furthermore, this thesis shows that enhanced risk-taking may occur under capital regulation in the absence of other regulatory requirements, such as deposit insurance. Rather, a flat capital requirement alone induces the bank to take more risk than it does under any other capital adequacy regime considered in this thesis (*i.e.* different risk weights, VaR approach) and than under laissez-faire.

## 1.1 The Pro-Cyclicality of Lending

As the business cycle is characterized by up- and downturns in aggregate output, it is of interest of how firms' access to finance and firms' financing demands are affected. The standard IS/LM model is frequently used in first place to analyze of how the goods market is affected by financing conditions, notably by the interest rate and by money supply. The standard IS/LM model does not address monetary transmission via the bank loans, however.

Bernanke/Binder (1988) were the first who studied the role of commercial lending in the IS/LM-framework. The conventional LM curve, that only recognizes money and bonds as financial assets, is extended by (commercial) loans. As a consequence, (government) bonds and (commercial) loans are no longer perfect substitutes. Bernanke/Blinder derive loan supply from a generic bank balance sheet and the loan demand relation is set ad hoc. The standard IS/LM-model thus becomes a limit case of their model.

If loans and bonds are perfect substitutes, the curve representing the equilibria on the goods and credit market reduces to the familiar IS-curve. If money and bonds are perfect substitutes, the economy slips into the well-known liquidity trap (Bernanke/Blinder 1988, p. 436). In between, there is now a role for the credit market to affect the goods market and total equilibrium. In particular, a positive credit supply shock raises bond interest rates and aggregate output. Furthermore, reserve requirements may take more expansionary effects than in the standard IS/LM model.

Bernanke/Gertler/Gilchrist (1999) refine the notion of how credit and credit availability may cause macroeconomic up- and downturns. They incorporate credit market imperfections into a dynamic, general equilibrium New Keynesian model. Firms may borrow funds, but at a rate that is higher than their internal cost of capital. The spread, termed external finance premium, is due to agency problems and depends inversely on the firm's net worth, whereas the latter is as such pro-cyclical. These two ingredients form the financial accelerator (Bernanke/Gertler/Gilchrist, 1996, p. 4). Contrasting this model with conventional New Keynesian frameworks serves to compare the impacts that the firms' net worths have on transmitting shocks. Numerical studies illustrate that the thus implied financial accelerator propagates and amplifies shocks. This view, however, stands in contrast to the work of Bernanke/Blinder (1988) and Blum/Hellwig (1995)[3] insofar as the latter consider amplifications via the banks' ability of granting loans. However, the existence of the financial accelerator in the sense that lending amplifies macroeconomic shocks cannot be confirmed for Germany (Eickmeier/Hofmann/Worms, 2006).

One alternative explanation of the cyclicality of lending is the so-called "institutional memory hypothesis" which has been put forward by Berger/Udell (2004). This hypothesis says that the loan officers' abilities to judge loans according to their default risks deteriorate after the last bust as time has passed. As a result, credit standards are eased with upswings, notably by decreasing premia. Berger/Udell

[3]Their work is discussed in Section 1.2.2, p. 10.

(2004) analyze data on US banks to support this thesis. They find a statistically and economically significant relation between loan growth and the time elapsed since the last busts. As they also control for other demand and supply factors, they regard this significant relation as sufficiently supportive for their hypothesis. However, as also Berger/Udell (2004) note, the institutional memory hypothesis could also simultaneously show up with other phenomena. Furthermore, it could be criticized as a truism. In this vein, without recurring on this hypothesis, Ayuso/Pérez/Saurína (2004, p. 261) note that banks "might tend to behave during a boom as if it were to last for ever."

## 1.2 The Basel I Accord and Pro-Cyclicality

### 1.2.1 The Introduction of Basel I and the 1990/91 Credit Crunch

At the beginning of the nineties the Basel-I-rules were introduced and some further regulatory changes came into force in the USA. More specifically, those requirements were partially introduced in 1990 and they finally went into full effect in 1992[4] Moreover, a minimum capital ratio based on the total, non-weighted asset volume was required from 1990 on (*cf.* Berger/Udell, 1994, p. 585, and Berger/Herring/Szegö, 1995, p. 403).

During this period, the aggregate loan volume remained at a constant level in the USA whereas the volume of government securities held by commercial banks increased (Haubrich/Wachtel, 1993, p. 3f; Berger/Udell, 1994, p. 586). In turn, the total of commercial and industrial loans outstanding strongly went down from 1990 on, and the bank capital held increased (Haubrich/Wachtel, 1993, p. 3f; Berger/Udell, 1994, p. 586; Furfine, 2001, p. 34). In particular, Berger/Herring/Szegö (1995, p. 402f) report that US bank capital ratios raised from 6.21% at the end of 1989 to 8.01% at the end of 1993. It was widely attempted to regard the new regulations as cause for the observed changes in the banks' portfolios.

However, not all academics were convinced by the view that regulation caused a credit crunch or even worsened the macro-economic decline. Yet, the obvious decline

[4] *Cf.* Avery/Berger (1991, p. 862-864) concerning the prompt-corrective action plan, and Berger/Udell, 1994, p. 585 concerning the introduction of Basel I in the USA. According to Rochet (1992, p. 1137) and Freixas/Rochet (1997, p. 239), the capital requirements based on the Basel I Accord went into effect in 1993 for all commercial banks in the EEC.

in commercial and industrial loans on the one hand and the rise of government securities on banks' balance sheets on the other, fueled concerns that capital requirements could worsen recessions. In other words, the fear of "pro-cyclical" effects on lending by capital regulation was born. Furthermore, the observed portfolio shifts in favor of default-risk free securities added to the debate about the relation between risk-taking and minimum capital requirements which will be addressed later in this chapter.

The hypothesis that the introduction of the Basel I capital requirements caused the credit crunch in the US is supported by Haubrich/Wachtel (1993), Brinkmann/Horvitz (1995), Shrieves/Dahl (1995), and Jackson *et al.* (1999), amongst others. In particular, Jackson *et al.* (1999) claim that Basel I possibly contributed to constrained lending in Japan, too, whereas they confirm for the remaining G-10 countries that the introduction of Basel I seems to have induced weakly capitalized banks to increase capital ratios only. Similarly, Wagster (1999) finds evidence of constrained lending through Basel I for the US and Canada at the beginning of the nineties, but does not find it for the UK, Germany, and Japan.

Aggarwal/Jacques (1998) attribute the increasing bank capital ratios in the USA during the years 1991 to 1993 to the PCA plan. This view is also held by Furfine (2001, p. 51). Although he admits that an increase in risk-based capital requirements could have added to the credit crunch, this increase could not fully explain it. Berger/Herring/Szegö (1995) loosely ascribe the increase of US bank capital ratios to the introduction of the new capital requirements, the PCA plan, and of the risk-based deposit insurance premia. Peek/Rosengreen (1995) concentrate their study on New England and find evidence that banks were strongly constrained by declining capital, calling this phenomenon "capital crunch". As data on loans is too scarce, they cannot confirm the existence of a credit crunch, *i.e.* decline in bank loan supply.

The view that the introduction of risk-based capital requirements and other regulatory measures during 1990 to 1992 caused the credit crunch is not unanimously shared amongst academics, however. Berger/Udell (1994, p. 625) give evidence that alternative supply side mechanisms "are somewhat more consistent with the data" but their "quantitative effects are not substantial." But they also believe that non-risk related credit crunch hypotheses, as regulatory pressure, could have played a role. Bernanke/Lown (1991) consider a weak loan demand as main cause for the slowdown in lending. However, they do not deny that weak bank capital bases have impaired banks' lending potential. A full account of the various hypotheses considered to have caused the decline in loans at the beginning of the 1990s in the

US is provided by Berger/Udell (1994, pp. 586-588).

### 1.2.2 Theoretical Research on the Pro-Cyclicality of Basel I

Blum/Hellwig (1995) were the first[5] to deal with the issue of pro-cyclicality of capital requirements. The implications of a flat regulatory capital-to-asset ratio are analyzed within an IS/LM-framework. To be able to illustrate its potential propagating effects, they introduce a stylized banking sector. Blum/Hellwig (1995) thus distinguish in contrast to standard IS/LM analysis between commercial loans, government securities, and demand deposits where the latter is determined by the money demand. The banking sector comprises equity and demand deposits on the liability side, and commercial loans and bonds on the asset side. Credit risk and other types of risks that prevail in banking are neglected.

Given a real shock to aggregate demand, regulation causes a raise in the aggregate demand multiplier that is higher than the raise in the aggregate demand multiplier in the otherwise same economy where the banking sector is not regulated. Put differently, the sensitivities of equilibrium prices and output relative to shocks increase as well. Thus, regulation enforces pro-cyclicality. But their model cannot address the question of how much risk the banking sector takes after a shock has occurred and to what extent shocks have an impact on the risk positions of firms and depositors.

Thakor (1996) adds to the view that regulatory measures can cause a decline in loans by proving that increased capital requirements lead to increasing credit rationing in the sense that the borrower's probability of being denied for credit goes up. The argument builds on asymmetric information and the bank's cost of screening. Furthermore, Thakor (1996) asks if monetary policies lose their effectiveness under a regime of minimum capital requirements. He conjectures that monetary policy may have opposite effects on aggregate lending depending on how these impulses affect the term structure of interest rates.

Bliss/Kaufman (2003) show by considering a simplified bank balance sheet that the injection of an additional amount of reserves beyond what is initially required leads to a lower increment in earning assets if capital requirements are binding than it is the case without capital requirements. As a consequence, a considerably large amount of total reserves is held in excess given binding capital requirements. Thus, these excess reserves, that would allow for an appropriate increase in deposits, and

[5] *Cf.* Freixas/Rochet, 1997, p. 274.

hence in earning assets, can only be used to expand the volume of earning assets if new equity capital is raised. But in recessions, this is unlikely to occur as it becomes too costly. So, capital requirements reinforce cyclical downturns regardless of their risk-sensitiveness.

## 1.3 The Basel II Accord and Pro-Cyclicality

### 1.3.1 The Basel II Accord

The Basel II Accord (BCBS 2004), which has taken effect in Germany on the 1st of January 2008 (Deutsche Bundesbank, 2009, p. 55), offers the Standardized Approach and the "Internal-Ratings-Based Approach" (IRB Approach) for determining the required minimum capital for credit risk.[6]

In principle, the Standardized Approach assigns a risk weight to any claim according to the economic sector to which the debtor belongs and according to the debtor's credit risk. The credit risk appraisal should be reflected in a credit grade given by an external credit assessment institution (BCBS, 2004, Art. 52). Risk weights usually comprise several credit grades given a class of debtors. There are risk weights for claims on sovereigns and central banks (BCBS, 2004, Art. 53, 55), banks (*ibid.*, Art. 63), and corporates (Art. 66). Interestingly, claims secured by residential property are assigned a weight of 35% (Art. 72f), whereas a weight of 100% is required concerning commercial real estate because of the turbulence in the 1980ies and 1990ies (Art. 74). The Standardized Approach can be seen as a refinement of the Basel I Accord that merely distinguished claims by their sector-specific origin or by the collateral pledged.[7] Consequently, risk sensitivity of capital requirements has increased from the Basel I to the Basel II Accord.

Risk sensitivity increases even further if the Basel I rules are compared to the IRB Approach. The risk weights of the IRB Approach are derived from a credit risk model where the returns of claims follow a one-factor model with normally distributed factor loadings and where the claims are fully granular. Gordy (2003) characterizes

[6]We do not consider minimum capital requirements for securitization which follow different rules (*cf.* BCBS, 2004, pp. 113-136). In the European Union, the directives 2006/48/EC and 2006/49/EC build the legal basis for national law-setting. In Germany, the law concerning the new capital requirements based on the Basel II Accord ("Solvabilitätsverordnung") came into force on the 1st of January 2007 allowing for a one-year transition period. In this thesis, we will exclusively refer to the document launched in June 2004 by the Basel Committee on Banking Supervision (BCBS, 2004).

[7]For Germany, we refer to §13 Grundsatz I, listing all then eligible risk weights.

the assumptions that the portfolio be fully granular and that there be at most a single systematic risk factor as necessary and, with a few regularity conditions, as sufficient to obtain the portfolio-invariant additivity of risk weights, laying thus the theoretical foundations for the IRB Approach. Portfolio-invariance means that the capital charge of each asset item does not depend on the characteristics of the associated portfolio.[8] Given the underlying model, the IRB risk weights yield the 0.1%-quantile of a notional portfolio's credit loss distribution (BCBS, 2004, Art. 272). Hence, required minimum capital in the IRB approach can be understood as the capital level that absorbs all losses with 99.9% probability.

Thus, many theoretical papers, which model the portfolios' returns differently from what the IRB Approach assumes, represent this approach by requesting the bank to keep a given confidence level based on a VaR approach for economic capital. Models using VaR to represent the Basel II regime include Bühler/Koziol (2005), Dangl/Lehar (2004), Daníelsson/Shin/Zigrand (2004), Estrella (2004), and Daníelsson/Zigrand (2008). We follow this modeling approach in our analysis, too.

As the increased risk sensitivity in capital requirements is obvious, much research has been devoted to the cyclicality of minimum required capital compared to the Basel I regime. This literature will be reviewed and discussed in Section 1.3.2. Section 1.3.3 reviews the work concerned with capital buffers, *i.e.* the amount of capital held above what is required by regulation. Section 1.3.4 is devoted to the role of loan rating systems which has attracted a lot of attention since banks must estimate the probability of default (PD) in the IRB Approach and may also estimate other inputs after approval (BCBS, 2004, Art. 245-248).[9] Because these estimates are sensitive to the credit cycle in one way or the other, these estimates may be an additional source of cyclicality. Section 1.3.5 presents further literature that is concerned with pro-cyclicality due to risk-sensitive capital requirements.

As a response to the recent subprime crisis that in effect infected many other financial markets, amendments to the Basel II framework have been discussed in depth since 2009. These amendments have been known as Basel III and comprise a bundle of measures (BCBS 2010). These new rules are concerned with the quality and size of the bank's equity capital, the total leverage, and liquidity. The new rules will be phased in from 1st January 2013 on and shall take full effect from 2019 on. In times

[8]Outlines of the specific model and a derivation of the thus implied risk weight can also be found in Vasicek (1991), Schönbucher (2000), and Hartmann-Wendels (2003, pp. 117-120).

[9]These two options are called "foundation approach" and "advanced approach". The foundation approach requires that banks only estimate the default probabilities, whereas under the advanced approach banks have to estimate all the IRB-model's input parameters.

of stress, banks will be allowed to draw on the “conservation buffer” which will be introduced from 2016 on. The conservation buffer shall thus address the problem that banks reach the regulatory minimum amount of equity capital especially in adverse economic situations in which new equity cannot be raised. This option to absorb losses, however, will be linked to tighter regulatory constraints on the distribution of earnings (BCBS, 2010). Another lesson from the recent financial crisis has led to stricter liquidity rules.[10]

Most of importance in respect to this thesis is the attempt to establish an anti-cyclical buffer (*cf.* BCBS 2010). The buffer will account for 0 to 2.5 percentage points of the capital ratio. It is supposed to curtail credit growth in expansive economic phases. So the build-up of excessive risk shall be contained. Consequently, this anti-cyclical buffer will not change results concerning cyclicality compared to the current Basel II framework when downturns are considered. As the results of this thesis suggest that capital requirements already lead to lower loan volumes and in particular to lower credit growth than it is the case in a regime of laissez-faire, we shall not expect different qualitative results concerning upturns, either.

### 1.3.2 Pro-Cyclicality of Required Minimum Capital

The empirical research on the 1990/91 credit crunch has strengthened the view that bank capital and bank capital requirements are key drivers of total lending. Consequently, many scholars feared that an increased cyclicality in capital requirements by the Basel II Accord could amplify fluctuations in lending and thus in aggregate output.

For example, Bikker/Metzemakers (2007) find, based on bank data over OECD countries, that regulatory capital over risk weighted assets is pro-cyclical whereas the ratio of capital over risky assets is mostly not. Particularly, Bikker/Metzemakers (2007) and Bouvatier/Lepetit (2008) detect a tendency that riskier (and also smaller) banks maintain relatively low ratios. This relation implies that the regulatory capital of those banks may rather infringe regulatory thresholds if these thresholds depend on risk.

The works of Carpenter/Whitesell/Zakrajšek (2001), Ervin/Wilde (2001), Segoviano/Lowe (2002), Goodhart/Hofmann/Segoviano (2004), Kashyap/Stein (2004), Illing/Paulin (2005), and Gordy/Howells (2006) determine what amount of capital

[10] For an account of the crisis *cf.* Calomiris (2008) and Hellwig (2008), amongst others. With respect to the perceived phenomenon of (wholesale) bank-runs, we refer to Gorton (2008, 2009).

would have been required in the recent past for notional loan portfolios, that they deem representative, if Basel II had been already in place.

Their findings can be summarized as follows:

Segoviano/Lowe (2002) and Goodhart/Hofmann/Segoviano (2004) find strong amplifications in required minimum capital for Mexican banks in the 1990ies. Their simulations of the loan portfolios, that are based on Mexican, Norwegian, and US-American data each, show that required minimum capital is more cyclical under the IRB Approach than under the Standardized Approach (Goodhart/Hofmann/Segoviano, 2004). They observe this enhanced cyclicality even though they feed the IRB formulæ with Moody's data which are regarded as through-the-cycle estimates.

Illing/Paulin (2005) confirm for Canadian banks for the years 1984 to 2003 that required capital would have become more volatile under Basel II. Kashyap/Stein (2004) detect that Basel II rules might have raised required minimum capital by 2% to 160% compared to Basel I. Ervin/Wilde (2001) determine that bank capital would have been required to increase by 3% under the Basel I and by 20% under Basel II for US-loan portfolios during 1990. The increase in required capital from Basel I to Basel II is based on credit migrations.

Also Hofmann's (2005) simulation study suggests that the internal IRB approach enhances the cyclicality of lending compared to the Basel-I-framework, but that both regulatory regimes dampen this cyclicality compared to the economic capital approach pursued by the otherwise unregulated bank. The set-up is kept simple: the bank's equity is exogenously fixed, debt-finance not considered, and the loan portfolio is homogeneous. The study can only account for changes in the total loan volume that are based on cyclical changes of capital requirements. Credit risk is based on a one-factor model. Cyclical swings in credit risk are exclusively given by increasing and decreasing default barriers of the obligors in the bank's loan portfolio. The fluctuation in the default barrier is in turn given by a stylized, sine-shaped business-cycle model and by the evolution of total lending.

To sum up, the studies of Ervin/Wilde (2001), Segoviano/Lowe (2002), Goodhart/Hofmann/Segoviano (2004), Kashyap/Stein (2004), Hofmann (2005), and Illing/Paulin (2005) give evidence that the switch from Basel I to Basel II implies an increase in the amount and in the cyclicality of required capital.

In contrast, Carpenter/Whitesell/Zakrajšek (2001) do not find any substantial increased cyclicality in bank capital requirements under the Standardized Approach of Basel II compared to those of the preceding risk weighting rules. Their analysis is based on a study of a representative loan portfolio during the years from 1970 to 2000. Gordy/Howells (2006) put the studies suggesting increasing cyclicality through Basel II into perspective by illustrating how different re-investment rules after loan defaults affect the cyclicality of capital. Their study is based on a dynamic simulation setting. In particular, they can show that re-investment rules may have a larger impact on the cyclicality of required capital than capital adequacy rules do.

All these studies consider the cyclicality of bank capital exclusively as a matter of the cyclicality of required capital. As required capital results from the composition of the bank's loan portfolio, the lending policy is crucial for the total result. This strand of literature emanates from historic, average loan portfolios and from exogenously fixed portfolio strategies.

The appeal of the literature that is concerned with the cyclicality of minimum capital requirements is two-fold: first, a considerable number of these studies have a strong relation to real-world bank portfolios. Second, this literature considers the evolution of capital through time, often for the full length of a business cycle.

As shortcomings, three important points can be listed. First, these studies neglect potential banks' internal capital targets. Second, they neglect potential endogenous reactions to a new set of capital rules which seems to be important in light of the theoretical literature on risk-taking as well as in light of the empirical studies on the 1990/91 credit crunch. In contrast, the bank reacts differently given the various regulatory regimes and must consider the re-financing costs associated with its risk-taking in this thesis. Third, as in Section 1.3.3 discussed, banks hold capital buffers on top of what is required. Therefore, their behavior and their potential influence on capital cyclicality shall be discussed.

Estrella (2004) is also concerned with the cyclicality of bank capital but some of these drawbacks are addressed as he analyzes the bank's optimal capital choice with and without regulation in a dynamic, infinite-horizon model.

The bank's choice of its level of equity is associated with costs. These costs are the cost of holding capital, of failure and of capital adjustment. Both bank loans and deposits are given exogenously.

If adjustment costs are neglected, the optimally chosen level of capital is identical to the economic capital calculated by the VaR approach at an endogenously determined confidence level. This characterization allows to decide whether a confidence level set by regulation is binding or not.

If adjustment costs are introduced and if losses follow a stylized business cycle, the optimal level of capital of the unregulated bank is negatively correlated with the VaR-restricted capital level if separately determined period-by-period by the regulator. The reason is that the latter neglects adjustment costs such that capital is sooner raised after a shock in loan losses than the bank would optimally do without regulation.

Furthermore, the VaR constraint is positively related to external capital flows. Hence, regulation is expected to be binding under adverse economic situations, giving rise to the concern that regulation amplifies recessions.

Since there is no endogenous trade-off between raising new external capital and granting loans, the issue of pro-cyclicality of loan volumes cannot be directly addressed. Arguments can only be indirectly run via capital decisions and restrictions. The model cannot address the question of risk-taking under regulation, either, because the bank is only concerned with optimally adjusting its equity capital. Though adjustment costs are considered in this study, too, these costs are symmetrical for both up- and downswings so that equity can be raised during recessions at unchanged conditions.

Yet, it is questionable whether we should be concerned with cyclicality in capital at all: bank capital cannot be adjusted easily, especially not in times of increasing numbers of loan defaults. Rather, bank capital is fixed, only changing with losses and retained earnings. What must be adjusted, given the credit risk perceived and the capital given, is the bank's total loan volume. Hence, the lending volume should be the variable of first interest.

### 1.3.3 Pro-Cyclicality and Capital Buffers

In reality, banks hold capital beyond what regulation requires. This excess capital is usually referred to as capital buffer.

Banks hold capital buffers to reduce the cost of infringing capital requirements. The cost of infringing capital requirements is associated with the loss of the bank's charter or franchise value, which becomes the higher, the higher future expected returns are. According to Milne/Whalley (2001), increased recapitalization costs or a higher auditing frequency can further raise the bank's internal target of capital. Alfon/Argimon/Bascuñana-Ambrós (2004, p. 20) confirm that UK-based banks and thrifts hold capital buffers in order to avoid costly recapitalization when needed most. Jackson/Perraudin/Saporta (2002) explain capital buffers empirically by the improved access to the swap market in terms of contract conditions. Lindquist (2004) finds that Norwegian banks merely hold capital buffers to signal solvency to the market and to the regulator. Moreover, regulators suppose banks to hold capital buffers as well (BCBS, 2004, Art. 757f).

The existence of capital buffers raises the question of whether research on pro-cyclicality of minimum capital requirements and lending may become pointless. In particular, it could be claimed that it becomes dispensable to compare capital and lending under the Basel I regime with those under the Basel II regime. This view is held by Bikker/Metzemakers (2007) who do not fear pro-cyclical effects on lending with the introduction of Basel II despite the pro-cyclicality of capital ratio (*cf.* p. 13 of his thesis) since they can show that most banks hold significant capital buffers enabling them to smooth fluctuations in realized and expected losses.

The notion that capital buffers might absorb some of the cyclicality inherent in minimum capital requirements is sustained by the empirical magnitudes that Ervin/Wilde (2001) and Furfine (2001) report. According to Ervin/Wilde the IRB weights proposed at that time resulted in half of what had been required under the Basel I Accord whereas Furfine (2001, p. 52) points out, based on Carey's (1998) data, that roughly two thirds of bank loan portfolio are rated BBB or BB and thus are not subject to changes in capital requirements if the Standardized Approach is used. Jackson/Perraudin/Saporta (2002) estimate that most banks of their sample[11] aim at a survival probability higher than 99.9% whereas the Basel I framework only implies survival probabilities ranging from 99.0% to 99.9%. In particular, these banks should have met *ceteris paribus* the minimum capital requirements after the adoption of the Basel II rules.

Peura/Jokivuolle (2004) calculate the capital charges according to Basel I and Basel II for high-quality and average-quality loan portfolios.[12] Furthermore, they add a capital buffer such that the regulatory capital amount is not infringed by a probability of 99%. This probability is referred to as "the confidence level applied to regulatory adequacy" (*ibid.*, p. 1809). This strategy of holding a buffer is compared to holding economic capital such that solvency is kept by a probability of 99.95% which is in line with the empirical findings of Jackson/Perraudin/Saporta (2002). In sum, the simulation study of Peura/Jokivuolle (2004) results in capital buffers that lead to a less cyclical total capital amount than it is the case if the bank seeks to keep

[11]Their data is based on a Federal Reserve Board survey and by Deutsche Bank's annual report. The data comprises the distributions of credit quality in banks' loan portfolios.

[12]The percentages of credit grades are along the lines of Gordy's (1998) data on US loan portfolios.

a given solvency probability. These results hold for both loan portfolio qualities and for both regulatory regimes, *i.e.* Basel I and II. Interestingly, capital buffers become the higher relative to required capital, the higher the portfolio's loan quality is as the potential for down-side movements grows.

Gambacorta/Mistrulli (2004) give empirical evidence of the notion that capital buffers smooth the cyclicality in lending as they show for Italian banks that the higher capital buffers are, the less pro-cyclical lending becomes after GDP-shocks have realized.

Jacques (2005) and Heid (2007) illustrate the role of capital buffers concerning the pro-cyclicality of risk-based capital requirements in a framework that is based on the model used by Blum/Hellwig (1995). Jacques (2005) and Heid (2007) emphasize that pro-cyclicality of risk-based capital requirements can only be mitigated if, at least, capital buffers strictly increase in aggregate output and thus move in opposite direction to what has been observed under the Basel I regime.[13] The results of Repullo/Suarez (2008) show that such a behavior of capital buffers under Basel II may prevail.

Both, in Jacques' (2005) and Heid's (2007) models, the single, portfolio-wide risk weight depends on aggregate output. Jacques (2005) shows that ratings migrations unambiguously can add to increased cyclicality in lending despite of capital buffers held. Heid (2007) contributes to the literature with two findings: first, capital buffers may absorb the cyclicality of required capital, thus mitigating the overall effect on the capital ratio. Second, the capital buffer under risk-sensitive capital requirements decreases in downturns.

In Heid's (2007) model, the representative bank maximizes its expected profits given its funding constraint while keeping a given probability not to infringe the required regulatory equity amount. The latter constraint results in the capital buffer. Its dependence on aggregate output is given by the assumptions that underpin the macroeconomic framework used. A shortcoming of the study of Jacques (2005) is, however, that the capital buffer and its relation to the business cycle in terms of aggregate output is exogenous.

Kajuth (2008) demonstrates in a micro-economic model of a banking sector that the degree of pro-cyclicality may well depend on the degree of capitalization and

[13]A lot of empirical evidence on the counter-cyclicality of capital buffers with respect to GDP has been gathered, including Ayuso/Pérez/Saurína (2004) for Spanish banks, Bikker/Metzemakers (2007) for OECD-countries based banks, Jokipii/Milne (2006) for most types of EU-25 based banks, Lindquist (2004) for Norwegian banks, and Stolz/Wedow (2005) for German savings and cooperative banks.

on the behavior of the interbank lending rate toward aggregate risk. The degree of capitalization reflects the distribution of capital buffers and, in particular, also entails a subset of banks that face binding capital requirements. Similarly to Jacques (2005) and Heid (2007), capital requirements are inversely related to aggregate output. Given intermediate levels of capitalization, and a low elasticity of the interbank rate with respect to aggregate output, regulation strongly affects lending in a pro-cyclical way. For high elasticities the opposite is true.

Zhu (2008) illustrates within a dynamic bank model that capital buffers absorb some of the cyclicality of minimum capital requirements. But the calibrated model also shows that banks that are not effectively constrained by capital requirements may even exhibit the strongest cyclical swings in lending as they can freely engage in lending during upswings. The lending cycle of effectively constrained banks, though holding a buffer, are less pronounced.

As in Zhu's (2008) analysis, banks, that are constrained by regulation, hold a capital buffer on what is required, it remains unclear to what extent regulation may actually affect lending in a pro-cyclical way in his model. In general, it is not clear where the benchmark must be set so that a comparison of a constrained and an unconstrained banks becomes meaningful in reality. The work of Zhu (2008) is close to this thesis insofar as the effects of capital requirements on the lending volume dependent on the state of the economy are analyzed. In contrast to our approach, depositors are assumed to be risk-neutral and there is one single bank asset only of which the volume strictly decreases in the total loan volume.

Over and above, we take the view that holding capital in excess of what regulation requires does not extinct the problem of potential pro-cyclical effects on lending for two reasons: first, if capital buffers are held to keep the probability of being closed by regulatory authorities small, and if these buffers are either absolutely fixed or are a fixed portion of required capital, pro-cyclicality remains an important issue to investigate. Second, as Repullo/Suarez (2008, p. 35) point out, extrapolating the behavior of the capital buffers under the Basel I to the Basel II regime may be misleading as the new requirements are supposed to have a different impact on bank behavior in general than the Basel I requirements just had. Thus, simply extrapolation past behavior does not withstand the Lucas critique according to Repullo/Suarez (2008).

In particular, they show within a two-period framework that capital buffers become pro-cyclical under risk-sensitive capital requirements. In contrast, capital buffers are counter-cyclical if a Basel-I-style framework is in place.

The latter result is in line with empirical findings. Different empirical studies show in deed that the capital buffers held by banks have been inversely linked to GDP and have been thus counter-cyclical.[14] Yet, it is still questionable to what extent total capital actually absorbs the cyclicality of required capital. Alfon/Argimon/Bascuñana-Ambrós (2004, p. 29) observe for UK-based banks and thrift institutions that at most half of the changes in capital requirements have been absorbed by the buffers held in the period from 1997 to 2002 under the Basel I regime.

### 1.3.4 Pro-Cyclicality and Loan-Rating Systems

Pro-cyclicality in required capital or lending might not be due to the risk sensitivity of the Basel II Accord alone, but also due to the mode of how risk is measured. Concerning risk measurement techniques, two main classifications have emerged: through-the-cycle (TTC) and point-in-time (PIT) estimates. TTC estimates are based on data that comprise at least the full length of an economic cycle, as its name suggests. PIT estimates, however, rest upon data of short time periods. PIT estimates may serve as basis for the calculations of TTC. Consequently, PIT are more prone to changes with the economic cycle going ahead whereas TTC estimates are rather constant unless the reference entity goes into bankruptcy or faces any other severe idiosyncratic shock (*e.g.* Amato/Furfine, 2004).

Additionally to estimated default probabilities, estimated loss given default rates may result in a further source of cyclicality as the study of Altman/Resti/Sironi (2002) suggests. They detect a negative relation between recovery rates and default probabilities and confirm by simulations that considering this correlation in the IRB approach enhances pro-cyclicality.

All in all, it seems intuitive that IRB-based capital requirements based on PIT estimates will fluctuate stronger than if the IRB formulæ are fed by TTC estimates. In principle, this notion carries over to the Standardized Approach, where risk weights only change in a discrete manner. This intuition has been confirmed by empirical studies and simulations.

Concerning the Spanish mortgage market, Saurína/Trucharte (2007) conclude that TTC-ratings could much alleviate pro-cyclicality compared to PIT-ratings used in the IRB formulæ. In the same vein, Illing/Paulin (2005) confirm that the volatility of

[14] *Cf.* Ayuso/Pérez/Saurína (2004) for Spanish banks, Bikker/Metzemakers (2004) for OECD countries, Jokipii/Milne (2006) for pan-European samples, Lindquist (2004) for Norway, and Heid/Porath/Stolz (2004) as well as Stolz/Wedow (2005) for German savings and cooperative banks.

required capital is higher under the use of PIT estimates, derived from bond credit spreads, than under the use of TTC ratings. For low-rated assets this problem may worsen since the respective volatility increases with decreasing loan quality. Likewise, Kashyap/Stein (2004) detect the highest and the lowest increments in capital requirements under the KMV model which is supposed to deliver PIT estimates, but not under the credit ratings provided by S&P, which are regarded as TTC estimates. Also Goodhart/Hofmann/Segoviano (2004), who judge the IRB foundations-approach as highly pro-cyclical according to their simulation results, emphasize that capital requirements could become even more pro-cyclical in reality as banks regard their internal ratings as PIT whereas their simulation study and their calculations of default probabilities are based on Moody's TTC estimates.

Catarineu-Rabell/Jackson/Tsomocos (2005) ask within a general-equilibrium set up which rating system banks optimally choose if they can select between a pro-cyclical, counter-cyclical, and a cycle-neutral risk weight for corporate loans. More specifically, a pro-cyclical risk weight strictly decreases as the expected recovery rate increases whereas the opposite holds for a counter-cyclical risk weight. This assumption can be thought of as if expected recovery is assigned to applicable probability-of-default bands in different ways. The cycle-neutral risk weight remains constant throughout the cycle. Capital requirements are assumed to be binding at the end-date.

If banks are free to choose, they opt for a counter-cyclical approach as it maximizes expected profits. If regulators prevent banks from doing so, banks will most likely use a pro-cyclical rating approach. This results in excessive lending in booms at the expense of sharp contractions as soon as defaults start to rise in recessions. Hence, pro-cyclical estimates of default probabilities result in pro-cyclicality of lending under risk-based capital requirements. Catarineu-Rabell/Jackson/Tsomocos conclude that TTC ratings are the first-best solution in terms of maximizing household and corporate welfare and minimizing default.

Pederzoli/Toricelli/Tsomocos (2010) find by numerical instances of a general equilibrium model that firms and the household are better-off in terms of expected utility if PIT estimates are used and if the recession probability exceeds one half. More specifically, using PIT ratings leads to a stronger reduction in corporate loan rates and a weaker reduction in deposit interest rates compared to a TTC rating system. If an expansion is more likely, the opposite is true. Hence, also in this model PIT estimates affect the economy in a pro-cyclical way. Whether the bank prefers PIT or TTC estimates when a recession is expected depends on its role on the interbank loan market. The net lender prefers TTC while the net borrower PIT. Their results do not necessarily contradict those of Catarineu-Rabell/Jackson/Tsomocos (2005) as the study of Pederzoli/Toricelli/Tsomocos (2010) builds upon a richer environment concerning the banking sector.

To summarize, there is empirical as well as theoretical evidence that the estimation technique used to determine the risk weights under Basel II influences the cyclicality in required capital and in the lending volume. In particular, PIT ratings should be deemed to add to the pro-cyclicality of lending compared to the use of TTC rating systems. But it is not the aim of this thesis to examine the impact of loan rating systems on total lending and other decisions. Rather, the effects of capital-adequacy rules as such compared to a regime of laissez-faire are of main interest.

### 1.3.5 Further Theoretical Research on the Pro-Cyclicality of Basel II

Bühler/Koziol (2005) address the question of pro-cyclicality in lending by considering an economy with three agents: a firm, an investor (household), and a bank. All these agents have constant absolute risk-aversion and have unlimited liability. The firm's return on its real investment, the return on the loan, and on the deposit redemption are exogenously given and normally distributed. Specifically, the returns of the bank's asset ("loan") and liability side ("deposit") are linked by an exogenous correlation. The bank is regulated by a VaR approach. They detect counter-cyclical effects of regulation if there are shocks which affect the firm's return risk or the firm's productivity rate. Pro-cyclical effects occur if the firm's equity changes. Hence, whether regulation amplifies shocks in a pro-cyclical manner or not depends on the sort of shock that has affected the borrower's risk profile.

Also Zhu's (2008) study that is based on a dynamic bank model questions the notion that capital requirements have an inherent tendency to enforce the cyclicality in lending. The calibrated model shows that banks that are not effectively constrained by capital requirements exhibit stronger cyclical swings in lending during upswings than those banks that are constrained by regulation. Constrained banks, however, hold a capital buffer beyond what is required so that some of the cyclicality may be absorbed by total capital.[15]

Repullo/Suarez (2008) analyze the effects of capital requirements within a two-period framework. The bank is risk-neutral and managed in the interest of the

[15] *Cf.* the discussion under Section 1.3.3.

shareholders. Deposits are treated as fully insured debt. They draw comparisons between the Basel I, the Basel II, and a laissez-faire regime. Loan interest rates, capital ratios, and capital buffers are the most volatile under the Basel II framework. Their volatilities increase with the volatility of the exogenously given credit risk. A shortcoming of the model is that the absolute magnitude of the loan volume is exogenously fixed.

Tanaka (2002) incorporates credit risk in a stylized way into an IS/LM framework and shows that after an increase in the overall default probability aggregate output contracts stronger under Basel II than under Basel I. However, if default probabilities are high, the loan supply is less sensitive to changes in the loan interest rate under Basel II than it is the case under Basel I.

Daníelsson/Shin/Zigrand (2004) analyze the persistence of shocks within a simulated multi-period, sequence equilibrium model. Under VaR-based capital requirements, shocks are more persistent than in the laissez-faire equilibrium. Moreover, regulation exacerbates price fluctuations.

## 1.4 Further Regulations and Pro-Cyclicality

This thesis deals with the pro-cyclical impact of capital requirements. However, there are also some other rules and regulations that are supposed to amplify credit cycles.

Accounting and loan-loss provisioning rules have been blamed for enhancing pro-cyclicality in one way or another.

Bouvatier/Lepetit (2008) find that non-discretionary loan-loss provisioning affects lending in a pro-cyclical manner. As explanation, they claim banks to behave myopically. That is, they make little provisions in upswings and face increasing provisions due to increasing losses in downswings. Furthermore, there is little evidence of counter-cyclical provisioning behavior which could mitigate the problem. In particular, poorly capitalized banks do not tend to build provisions in good times.

In this vein, the study of Bikker/Metzemakers (2005) shows that GDP growth and provisioning are negatively related to each other. Small banks as well as large banks are found to base provisioning on past losses which is considered as pro-cyclical. Also Lindquist (2004) confirms that there is little propensity of banks to increase provisions in good times. Lindquist (2004) gives evidence that unspecified loan loss provisions and capital buffers seem to be substitutes. However, Bikker/Metzemakers

(2005) and Handorf/Zhu (2006) can also confirm patterns of income-smoothing via provisioning for some banks.

Another issue is fair-value accounting which has been held responsible for aggravating the downturn in fixed-income markets. Price drops in some markets have been accused of melting down banks' capital positions as the values of held-for-sale securities had steadily declined. Even before the recent financial crisis that has been started in 2007, Taylor/Goodhart (2004) pointed to the issue of pro-cyclicality through the introduction of fair-value accounting standards as laid down in IAS 39. Issues, that may lead to a "double squeeze" (Taylor/Goodhart 2004, p. 16) entail the accounting of regulatory capital and the determination of fair values of non-marketable assets, *i.e.* in particular loans. Enria *et al.* (2004) assert in their simulation study that losses could be doubled in adverse stress scenarios under full fair-value accounting compared to the accounting principles having been in place then (*cf.* Enria *et al.*, 2004, Table 5, p. 23).

In a nutshell, the current practice of provisioning does not seem to be able to balance credit cycles via equity smoothing whereas there are indications for increasing cyclicality through fair-value-accounting.

## 1.5 Risk-Taking, Systemic Risk, and Capital Requirements

### 1.5.1 Basel I and Risk-Taking

Beyond increased pro-cyclicality in terms of higher fluctuations in bank capital, lending, and prices or interest rates, respectively, there have been concerns that capital requirement do not curtail, as intended, but increase risk-taking, both on the individual as well as on the sector-wide level. Regulators are concerned with risk-taking as banking regulation is actually in place to reduce the risks the financial sector takes. Enforcing financial stability and depositors protections are actually the main rationales for regulating banks.[16]

In particular, flat capital requirements, as given by the Basel I Accord, have been often blamed for setting wrong incentive such that banks take as much risk as possible given a fixed amount of regulatory capital. The increased risk sensitivity of the Basel II Accord was intended to remove these deficiencies. Yet, the mistrust

[16] *Cf.* Dewatripont/Tirole (1994, p. 31f), Freixas/Rochet (1997, p. 264), and Santos (2000).

in Basel II concerning the issue of increased risk-taking has not disappeared.

A required minimum of capital establishes a minimum risk-participation of equity holders and elevates the bank's default barrier. Thus, a required minimum of capital serves as a rationale why capital regulation is in place. In particular, this was why the introduction of the Basel I capital requirements were welcome after decades of eroding bank capital ratios. Basel I was supposed to curtail risk-taking and to strengthen capital ratios according to Jacques/Nigro (1997). In particular, many US-American banks had faced large losses in asset values and thus declining capital ratios at that time after the savings and loans crisis in the 1980ies.

Calling for a minimum amount of capital is considered to be an important regulatory goal since the bank equity holders' limited liability results in risk-loving behavior (Sinn, 1982; Gollier/Koehl/Rochet, 1997; Goodhart, 1996, p. 12f). As a consequence, low-capitalized banks take excessive risks, known as "gambling for resurrection" (Rochet, 1992, Calem/Rob, 1998, and Blum, 1999). In light of the bank failures in 2008, a maximum-leverage rule, irrespective of risk, is gaining popularity among G-20 leaders (G-20 Leaders' Statement, 2009).

#### 1.5.1.1 Empirical Evidence and Criticism

Shrieves/Dahl (1992), Jacques/Nigro (1997), Aggarwal/Jacques (1998), Ediz/Michael/Perraudin (1998), and Rime (2001) are among the scholars who confirmed a positive relation between capital and the risks assumed.

More specifically, Jacques/Nigro (1997), Rime (2001), and Altunbas *et al.* (2007) find evidence that capital requirements effectively prevented banks from increased risk-taking. Ediz/Michael/Perraudin (1998, p. 20) and Rime (2001) claim that capital requirements induced banks to raise their capital ratios, irrespective of internal capital targets. In contrast to the other articles cited above, Shrieves/Dahl (1992) analyze pre-Basel I bank data. They report a strong and positive relation between capital and risk-taking among US banks in the years 1984 to 1986 and conclude that market discipline works well. Following this Shrieves/Dahl (1992), Heid/Porath/Stolz (2004) find evidence that German savings banks try to keep a given level of buffer capital in accordance with their risks. They consider the period from 1993 to 2000.

These views on the relation between risk-taking and the amount of capital held have been criticized for being too static and for disregarding new techniques in credit finance.

Jackson *et al.* (1999) already report exponentially increasing volumes of securitization in the USA and Europe, recognized as a method to reduce required capital. Over the years, this way of economizing on capital for given levels of risk, that has become known as “regulatory capital arbitrage”, has attracted increasing attention and caused concerns (Jones, 2000; Franke/Krahnen, 2005). Nowadays, securitization has been made responsible for decreasing lending standards and thus for having laid the grounds amongst others that finally cumulated to the sub-prime crisis (Dell’Ariccia/Igan/Laeven, 2008). Moreover, the contagious effects of securitization have been revealed by this crisis (Gorton, 2008).

#### 1.5.1.2 Theoretical Criticism

The possibility that capital requirements can imply the choice of rather risky portfolios has brought about a large strand of literature on its own.

One topic has been the interplay between capital requirements and deposit insurance and their complementary effects.

Bond/Crocker (1993) show that optimal insurance premia depend on the bank’s capitalization, that serves as an indicator of its probability of default, and that the optimal insurance plan does not fully cover losses in the bankruptcy state. The latter implies that there is a role for monitoring bank capital by depositors, preventing moral hazard. Similarly, Buser/Chen/Kane (1981) prove that banks can raise their value under flat deposit insurance premia by increasing their leverages without bounds.[17] Further research on the optimal mix of deposit insurance premia and capital requirements such that the implicit subsidies given by tax payers are minimized has been performed by Freixas/Gabillon (1999).

These findings serve as a rationale to explain why the deposit insurer seeks to tax insured institutions implicitly, be it by capital requirements or by threatening institutions to withdraw charters.

Despite of this kind of complementarity between deposit insurance and capital requirements there have been concerns that there are still bad incentives for banks, in particular regardless of the amount of equity capital required, such that banks hold riskier portfolios under regulation than they would do without. In this vein, Koehn/Santomero (1980) demonstrate that a stricter, but flat capital-to-asset ratio

[17] Amongst others, Bhattacharya/Boot/Thakor (1998, pp. 755-757) and Santos (2000, p. 16) review the implications of flat deposit insurance premia on the banks’ risk-taking behavior as well as the role of risk-based capital requirements to risk-based deposit insurance.

has ambiguous effects on the bank's probability of default if deposit insurance premia are constant. Similar conclusions are drawn by Gennotte/Pyle (1991).

But these negative effects of fixed rate deposit insurance contracts can be mitigated by introducing risk-sensitive capital requirements because they impose an upper bound on the probability of bank solvency, as Kim/Santomero (1988) show. Furlong/Keeley (1989) show that also flat capital requirements reduce the banks' risk-taking if the option value of deposit insurance, that implies a subsidy to the bank, is properly accounted for. More precisely, they show that the marginal value of the deposit insurance option declines with increasing asset risk if the capital ratio rises.[18]

But flat deposit insurance need not to be the sole source of increased risk-taking. Even if the bank's liabilities bear risks that are correlated with the bank assets' returns, neither a constraint on leverage nor on portfolio composition alone constrain the bank's probability of ruin, as Kahane (1977) shows within a mean-variance framework.

Rochet (1992) shows that choosing risk weights such that they are proportional to systematic risks helps to decrease the bank's probability of failure, if banks behave as portfolio managers and maximize expected utility. Limited liability requires an additional minimum level of capital in order that the bank is prevented from choosing inefficient portfolios. He criticizes earlier work on risk-taking for two reasons: for the assumption of complete markets being used to price the option value (*cf.* Kareken/Wallace, 1978; Dothan/Williams, 1980), which is incompatible with the existence of banks, or for neglecting limited liability as done by the portfolio models.

Calem/Rob (1999) consider the problem of risk-taking within an infinite-horizon framework. They consider a deposit insurance scheme that charges the bank an extra fee if its capital falls short the regulatory thresholds and that charges a flat premium else. If this deposit insurance scheme is in place, there is a U-shaped relation between capital and risk-taking. Low-capital as well as high-capital banks take excessive risks. The premium surcharges or tighter capital requirements aggravate the problem of risk-taking. Risk-sensitive capital requirements[19] are not sufficient to reduce

[18] Furlong/Keeley (1989) and Keeley/Furlong (1990) criticize in this respect the mean-variance framework as used by Koehn/Santomero (1980) and Kim/Santomero (1988) amongst others. In particular, they criticize that thus the option value implied by deposit insurance has been neglected so far. Merton (1977, 1978) was the first who characterized deposit insurance costs by a put option on common stock.

[19] Risk-based capital requirements linearly increase in the amount invested into the risky assets as soon as a fixed proportion of risky investments is surpassed. This rule is meant to mimic the

risk in their framework, but uninsured deposits (with depositors being risk-neutral) discipline the bank, *i.e.* reduce risk-taking for low capital levels compared to the case of insured deposits.

Blum (1999) was the first to examine inter-temporal effects analytically within a two-period framework. Assuming deposits are insured at a fixed-rate, a bank takes more risk if the capital requirement is binding in the first period since bank capital thus becomes more worthy in the second period. Contrary to Calem/Rob (1999) and others, Blum (2002, p. 1429) challenges the view that subordinated debt prevents banks from excessive risk-taking. Rather, he claims, the higher the contracted interest rate on that debt, the more valuable the 'option to go bankrupt' becomes, and the bank takes even more risk if it does not commit to a level of risk *ex ante*.

### 1.5.2 Basel II and Risk-Taking

#### 1.5.2.1 The Three Pillars of Basel II

As a consequence of these rather disappointing results, the academic literate has begun to pay more attention to risk-sensitive capital requirements. But as many studies suggest that even risk-sensitive capital requirements alone do not prevent banks from increased risk-taking, the other two pillars of Basel II besides capital requirements, market discipline and auditing (supervision), have gained in importance. Uninsured bank debt is considered as to foster market discipline.

In particular, Calem/Rob (1999) and Decamps/Rochet/Roger (2004) call for uninsured debt whereas Milne/Whalley (2001) call for continuous auditing to prevent low-capitalized banks from gambling for resurrection. Also Dangl/Lehar (2004) show within their continuous-time bank model that VaR-based capital requirements reduce the auditing frequency such that the IRB approach should be regarded as superior to the Standardized Approach or the Basel I Accord. Benink/Wihlborg (2002) ask for mandatory subordinated debt to strengthen market discipline and disclosure of risks. Blum (2002) questions this proposal. He argues that once the conditions for issuing subordinated debt are contracted, the bank with limited liability considers the associated costs as sunk and may even take more excessive risk. Only *ex ante* credible commitments can go against this problem.

In contrast, Jokivuolle/Vesala (2007) still find positive effects of Basel II without considering the other pillars. They show that risk-based capital requirements

Basel-I requirements.

alleviate the problem of risk-taking and cross-subsidization of loan rates in a competitive loan market where asymmetric information between lenders and borrowers is present.

#### 1.5.2.2 Systemic Risk

Aside from risk-taking on the single-bank level, the impact on risk-taking within the whole banking system has attracted increased attention. This research question has been motivated by the idea that an altered interdependence between banks could arise because of capital requirements such that more banks are prone to other banks' insolvencies or to adverse market outcomes while their own risk appears to be reduced. Thus, the inherent fragility of banking (Diamond/Dybvig, 1983) and bank net works (Allen/Gale, 2000) in the absence of regulation could not be addressed by capital adequacy rules.[20] Even worse, regulation would jeopardize its aim of enhancing financial stability.

Eichberger/Summer (2005) study the trade-off between the risk exposure of single banks versus the risk-exposure on the interbank market and how it is affected by capital requirements. Although capital requirements unambiguously reduce risks on the single-bank level in their model (there are no adverse portfolio allocations possible as banks can grant loans to one single firm only), risks from loans granted in the interbank market may increase for the following two reasons: first, banks operating under a binding capital requirement will increase lending on the interbank market since it is considered as riskless by both banks and regulators *ex ante*. Consequently, interbank loan rates are lower under regulation than without regulation. Therefore, initially well-capitalized banks, for which regulation is not binding, increase lending to their local firms by increased borrowing on the interbank market since there are no other borrowing opportunities for banks. Equity and deposits are exogenously fixed. Thus, shocks in the real sector are more likely to damage remote banks under regulation than without regulation.

In contrast, there are also arguments especially for risk-sensitive capital requirements to reduce systemic risk. Acharya (2001) shows that banks with limited liability tend to take correlated investments, thus increasing systemic risk. To counterbalance this effect, capital requirements should account "sufficiently" for correlations (*ibid.*, Proposition 7, p. 29). Consequently, a fixed-weight regime for capital requirements

[20] Deposit insurance which is nowadays in place in many countries (*cf.* BCBS, 1998) probably prevented banks from runs by retail depositors by and large during the recent crisis. However, the banking system has been subject to a run by wholesale investors, as Gorton (2009) points out.

cannot mitigate this behavior and hence cannot reduce systemic risk. Yet, it is still to be proven if the IRB-formulæ sufficiently account for real-world correlations.

Daníelsson/Zigrand (2008) call for capital requirements, too, since in general equilibrium excessive risk-taking occurs at the expense of other market participants' risk positions. At the heart of this mechanism lies the notion that everyone considers its own impact as negligible. VaR-based capital requirements can mitigate systemic risk, but the possibility that markets cannot clear increases, as a sufficient number of agents may lose their ability to absorb risks, *i.e.* to trade. Moreover, liquidity decreases, asset price volatility may rise, and price co-movements may appear even if asset prices are stochastically uncorrelated.

# Chapter 2

# Framework of the Analysis

## 2.1 Exogenous Shocks and the Business Cycle

A bank or an investor in general may be affected by different types of shocks. As pointed out by Allen/Saunders (2003, p. 2) any profit or loss observed can be caused by an *ex ante* shift in the return distribution or it is simply due to a realization based on a fixed loss distribution. We will refer to the latter as *realized shocks* and to the former as *expectation shocks*. We will treat all kind of shocks as exogenous. This classification will be useful for characterizing the results obtained in this thesis. Also the results obtained by Bühler/Koziol (2005) can be distinguished according to this classification of the underlying shock. They present how realized losses, *i.e.* decreases in capital, imply pro-cyclical effects whereas shocks affecting risk and productivity imply counter-cyclical reactions.

### 2.1.1 Equity Shocks

In the one-period models, equity shocks are understood as exogenous changes in the bank's initial equity: initial equity is considered to be lower than in the base case if a negative shock has occurred the period before, which has resulted in losses that had to be borne by the bank, hence reducing equity. Conversely, initial equity that is higher than in the base case, is associated with profits made in a notional previous period. Beyond that, different levels of bank equity may be linked to the state of the business cycle as pars pro toto: high levels of equity represent a boom state whereas low levels of equity reflect a bust. By this sort of co-movement of bank equity with the state of the business cycle, changes in bank equity will be considered as pro-cyclical. Consequently, it makes sense to refer to co-movements of other magnitudes with bank equity as pro-cyclical too. A formal definition will be given below. The level of bank equity is thus a backward-looking variable that characterizes the state of the cycle and the soundness of the bank.

### 2.1.2 Expectation Shocks

Expectation shocks comprise changes of default probabilities, expected returns on projects, project volatilities, and of correlations. Productivity shifts will also be considered as expectation shocks. All these variables contain the agents' expectations and knowledge about the ability of single firms to honor their debt obligations. They form the credit worthiness of the whole economy and thus affect the riskiness of loans and deposits. Shifts in these variables reflect changes in the agents' minds about future prospects. Lower expected returns and lower productivity are associated with an economic downturn. The reverse holds true with respect to volatilities and correlations as increasing values of volatilities and correlations are linked with rising uncertainty about future outcomes. Moreover, higher correlations hamper diversification. To summarize, expectation shocks refer to the future state of the cycle and affect the economy's fundamentals.

### 2.1.3 Interference of Different Shocks

Of course, in reality, we may be confronted with several different shocks at once. But it is unclear to what extent a given negative shock is due to shifts in distributions, or to what extent it is just bad luck. Economic subjects may update their beliefs after a negative or positive shock has realized. A behavior such as this complicates things further, as expectation shocks may follow realized shocks (*i.e.* equity shocks) whatever the reasons for the latter might have been.

However, considering different shocks at once does not help understand the impacts that regulation might have on the economy since different mechanisms will come into play at once. Furthermore, it is *ex ante* not clear, how different types of shocks should be mixed in a meaningful way. In reality, even countervailing shocks may come into force simultaneously. Yet, shocks may also arise independently.

For these reasons, we will confine ourselves to analyze regulatory effects for isolated shocks within the one-period frameworks considered.

Within a two-period model, the interference of expectation and realized shocks occurs as follows: in the first period, the economy faces a shock concerning expected magnitudes. Consequently, the distribution of the bank's equity in the middle of the time elapsed is affected and so is the distribution of final outcomes. Hence, expectation shocks in $t = 0$ imply on average realized shocks in $t = 1$.

## 2.2 Different Types of Cyclical Behavior

Having started with different types of shocks, we would like to classify the reactions of endogenous variables toward these exogenous shocks. This classification shall serve to distinguish some of the various impacts regulation may have on the behavior of endogenous variables. Endogenous variables may encompass loan and deposit volumes, deposit interest rates and portfolio effects. Classifying those impacts also helps to clarify our notion of pro-cyclicality of regulation, *cf.* Bühler/Koziol/Sygusch (2008), compared to other concepts that can be found in the literature. Above all, we do not ask if (regulatory) capital is pro-cyclically affected by regulation or not, such as some authors do (*cf.* Estrella, 2004; Goodhart/Hofmann/Segoviano, 2004; Gordy/Howells, 2006, and Kashyap/Stein, 2004). Rather, we will ask how changes in the bank's initial equity will affect endogenous variables with and without regulation. That is, changes in the bank's equity are treated as exogenous shocks amongst others whose effects ought to be analyzed.

Let $\theta$ be an exogenous parameter whose change represents either a realized or an expectation shock. Consider the parameter values $\theta_1$ and $\theta_2$, $\theta_1 < \theta_2$, belonging to any given interval $[\underline{\theta}, \overline{\theta}]$, $\underline{\theta} < \overline{\theta}$, on which regulation is binding. Let $\Theta$ be an endogenous variable where $\Theta_1^*$ denotes the optimal equilibrium outcome without regulation at parameter value $\theta_1$ and $\Theta_1^r$ that under a given regulatory regime at $\theta_1$. Likewise, we define $\Theta_2^*$ as well as $\Theta_2^r$. Consider the differences

$$
\begin{aligned}
\Delta\theta &= \theta_2 - \theta_1 \ , \\
\Delta\Theta^* &= \Theta_2^* - \Theta_2^* \ , \\
\Delta\Theta^r &= \Theta_2^r - \Theta_1^r \ .
\end{aligned}
$$

Regulation may refer to any concept of capital requirement.

Then we call regulation *pro-cyclical* on $[\underline{\theta}, \overline{\theta}]$ if

$$
\frac{\Delta\Theta^r}{\Delta\theta} > \frac{\Delta\Theta^*}{\Delta\theta} \geq 0 \quad \text{or} \quad -\frac{\Delta\Theta^r}{\Delta\theta} > -\frac{\Delta\Theta^*}{\Delta\theta} \geq 0 \tag{2.2.1}
$$

holds for all $\theta_1$, $\theta_2 \in [\underline{\theta}, \overline{\theta}]$, $\theta_1 < \theta_2$.

This definition captures all those situations where endogenous variables under regulation are affected more strongly by a shock than those without regulation. Moreover, this notion of pro-cyclicality is always linked to amplifications that go into the same direction under both regimes. Thus, by pro-cyclicality we understand either contractions or expansions that are enforced by regulation.

We say that regulation is *not pro-cyclical* on $[\underline{\theta}, \overline{\theta}]$ if

$$\left|\frac{\Delta\Theta^r}{\Delta\theta}\right| \leq \left|\frac{\Delta\Theta^*}{\Delta\theta}\right| \quad \text{and} \quad \left[\operatorname{sgn}\left(\frac{\Delta\Theta^r}{\Delta\theta}\right) = \operatorname{sgn}\left(\frac{\Delta\Theta^*}{\Delta\theta}\right) \text{ or } \frac{\Delta\Theta^r}{\Delta\theta} = 0\right] \tag{2.2.2}$$

holds for all $\theta_1$, $\theta_2 \in [\underline{\theta}, \overline{\theta}]$, $\theta_1 < \theta_2$.

This definition spans those situations where the endogenous variable with and without regulation is affected in the same direction but the effect for the case without regulation is stronger. Thus, if regulation is not pro-cyclical, regulation dampens endogenous cyclical effects.

Regulation is said to be *counter-cyclical* on $[\underline{\theta}, \overline{\theta}]$ if

$$\frac{\Delta\Theta^r}{\Delta\theta} < 0 < \frac{\Delta\Theta^*}{\Delta\theta} \quad \text{or} \quad \frac{\Delta\Theta^r}{\Delta\theta} > 0 > \frac{\Delta\Theta^*}{\Delta\theta} \tag{2.2.3}$$

holds for all $\theta_1$, $\theta_2 \in [\underline{\theta}, \overline{\theta}]$, $\theta_1 < \theta_2$.

Counter-cyclicality includes all cases where the endogenous variable under regulation reacts in the opposite direction compared to the case where the bank is unregulated.

Finally we call regulation *on average pro-cyclical* on $[\underline{\theta}, \overline{\theta}]$ if

$$\operatorname{sgn}\left(\frac{\Delta\Theta^r}{\Delta\theta}\right),\ \operatorname{sgn}\left(\frac{\Delta\Theta^*}{\Delta\theta}\right) \ \in\ \{0,1\} \quad \forall\ \theta_1, \theta_2 \in [\underline{\theta}, \overline{\theta}] \quad \text{and}$$

$$\frac{\Delta\Theta^r}{\Delta\theta} \ >\ \frac{\Delta\Theta^*}{\Delta\theta} \ \geq\ 0$$

or if

$$\operatorname{sgn}\left(\frac{\Delta\Theta^r}{\Delta\theta}\right),\ \operatorname{sgn}\left(\frac{\Delta\Theta^*}{\Delta\theta}\right) \ \in\ \{-1,0\} \quad \forall\ \theta_1, \theta_2 \in [\underline{\theta}, \overline{\theta}] \quad \text{and}$$

$$-\frac{\Delta\Theta^r}{\Delta\theta} \ >\ -\frac{\Delta\Theta^*}{\Delta\theta} \ \geq\ 0 \tag{2.2.4}$$

holds for all $\theta_1 \in [\underline{\theta}, \overline{\theta})$ and for $\theta_2 = \overline{\theta}$.

This definition is weaker than Definition (2.2.1) because pro-cyclicality on average may allow for subsets of $[\underline{\theta}, \overline{\theta}]$ on which the unregulated endogenous variable reacts more strongly on a given shock than the regulated one does. The latter may emerge by differences in convexity or concavity, respectively. [1]

## 2.3 The Model's Structure

### 2.3.1 The Basic Set-Up

We aim at assessing the consequences that specific shocks as classified above have on the amount of lending when minimum capital requirements are binding compared to the case of a laissez-faire economy. Contrary to the majority of the literature, we do not only consider our analysis on the relation between the borrowers and the bank, but track the financial linkages further to the bank's creditors, the households.[2]

Therefore, we consider a three-sector economy comprising of firms, a bank, and a household. The firms operate on a perfectly competitive goods market. The firms' risks are related to output and/or price fluctuations as the demand of their specific product may change. These mechanisms are not modeled explicitly. Particularly, the model is not closed by equating the firms' supply of goods with the household's demand. Instead, our framework concentrates on the financial linkages to the banks and finally to the household.

The household can invest its initial endowments into a riskless asset yielding an exogenous, riskless interest rate and into bank deposits. Deposits are not insured. Thus, the credit risk of the bank's loan portfolio translates into the pay-offs of the deposits. Early withdrawal of deposits is excluded. Therefore, the deposits considered are in fact a standard one-period debt contract on which the bank may default in some states of the world. The household is risk-averse and maximizes its utility over final wealth.

The bank's exclusive access to firms and its exclusive role as a financial intermediary between firms and the household is given by assumption. The bank has monopoly power on the loan market and the deposit market. The bank can only grant corporate loans and issue risky deposits. Thus, this model only allows an analysis of shifts between corporate loans, but not between corporate loans and risk-free

[1] Results 15 and 26 will present conditions where regulation has on average pro-cyclical effects. Examples in Section 6.6, illustrated Figures 6.9 and 6.10 provide evidence of the reasonability of this definition.

[2] Of course, the literature that is concerned with the fragility of banking and deposit convertibility concentrates on the relationship between the bank and its depositors. However, the focus is there on liquidity problems and other risks, such as credit risk, are neglected.

sovereign bonds as it was perceived in the 1990/91 Credit Crunch in the US (Haubrich/Wachtel, 1993, p. 3f; Berger/Udell 1994, p. 586; Furfine, 2001, p. 34). The firms apply for credit in a situation of perfect competition amongst each other such that their expected profits are driven to zero in equilibrium.

The risk-neutral bank maximizes expected final wealth by simultaneously choosing the loan interest rates, the composition of the loan portfolio and the deposit interest rate. The deposit interest rate does not only depend on the deposit volume but also on the composition of the loan portfolio since the latter is a key determinant for the default risk of the deposit. We assume that the bank can reliably commit to every arbitrary loan portfolio composition which ultimately shapes the state-dependent pay-offs anticipated by the household. There is no asymmetric information between firms, banks, and households.

Loan and deposit contracts are fixed at the beginning of the period and are paid off at the end of the period when uncertainty is resolved. Both the firms and the bank may default on their obligations and have limited liability. If the loan redemptions do not cover the promised repayment amount of the deposits, the bank defaults and the depositor obtains the total loan redemptions. Also the risk-free sovereign bond lasts for one period each. There are no other contracts available.

Below, the assumptions of the bank's risk-neutral behavior, the restrictions on bank liabilities, in particular equity finance, and the bank's market power are further discussed.

### 2.3.2 Fixed Bank's Equity Capital and Uninsured Deposits

We assume the following financial restrictions: bank capital is exogenously fixed in the one-period model or at the beginning of the first period in the two-period model whereas the volume of deposits (bank debt) is endogenously determined. Bank capital endogenously results from past gains or losses only in the two-period version at the start of the second period. Since deposits are uninsured and since the household is risk-averse, the bank faces an endogenous trade-off between deposit volume and deposit costs where the latter depend on the risk of the bank's loan portfolio, *i.e.* the loan portfolio composition. Beyond the bank's loan policy, deposits may also become more costly for the bank and deposit supply may decrease after expectation shocks.

The first assumption reflects that it is hardly possible for banks to raise new capital in economic downturns due to negative signalling effects or in very short time. This concern is also shared by the BCBS (BCBS, 2004, Art. 757, c). Instead, banks will have to adjust their loan portfolios concerning volume and risk which stands in contrast to the microeconomic approach of Estrella (2004) and the simulation studies of Goodhart/Hofmann/Segoviano (2004), Kashyap/Stein (2004), and Gordy/Howells (2006). Repullo/Suarez (2008) take an intermediate view in their model. There, banks can only raise new equity capital if they enter the market or grant new loans.

In our model framework, deposits are uninsured such that there are feedback effects from the risk-averse depositor to the bank if overall risk increases or the bank chooses specifically risky portfolios so as to counterbalance a constrained business volume which may be the case under binding capital requirements. This feature is novel to the literature on banks which can be distinguished concerning credit risk of deposits/bank debt or the role of depositors along the following lines: first, if the bank issues uninsured debt, the counter-party is risk-neutral with infinite elasticity of the deposit supply (Dermine, 1986, p. 107; Calem/Rob, 1999, p. 320 and pp. 342-346; Zhu, 2008, p. 173). Second, independent of the final risk-characteristics of the pay-offs from the deposit contract, portfolio choice models treat the risk transmission from the asset to the liability side as exogenous, by assuming assets and liabilities are correlated by an exogenous parameter (Hart/Jaffee, 1974; Kahane, 1977; Kim/Santomero, 1988, and Bühler/Koziol, 2005). This modeling technique can be also found with continuous-time models, as in Pennacchi (2005). Third, some papers assume exogenously given deposit cost functions (*e.g.* Blum, 1999) or exogenously given deposit-supply functions (Klein, 1971, Monti, 1972, and Dermine, 1986, p. 102). Lastly, the vast majority of papers which treat deposits as insured do not model the deposit supply side at all (Estrella, 2004; Eichberger/Summer, 2005; and all simulation-based studies cited so far).

Since there are no informational asymmetries between the bank and the household, the bank credibly commits to its loan allocation and hence to the level of risk which in turn determines the deposit interest rate and the feasible deposit volume. So the bank must incur the costs of its level of risk-taking and cannot shift these costs to other agents as a deposit insurance corporation such that there are market-disciplining effects.

Furthermore, bank equity will play the role of a buffer in case of bankruptcy. Since the household is fully informed, it can perfectly assess this role of bank equity in case of the bank's failure with respect to residual pay-offs. Bank equity thus serves as a device to share risks between the bank (or its managing owners) and the debtor (here the household). This function has been formerly analyzed by Gertler/Hubbard (1993) for firms in general that issue uninsured debt.

### 2.3.3 The Bank's Risk-Neutrality

The interaction between the household and the bank implies that the risk-neutral bank does not pick the loan with the highest expected return but diversifies credit risk in a non-trivial way.

In contrast, diversification of risk has often been achieved in past models by introducing risk-aversion at the bank level so that the bank maximizes expected utility. An important strand of this kind of literature are the so-called portfolio models. This approach has been pursued by Hart/Jaffee (1974), Kahane (1977), Koehn/Santomero (1980), Kim/Santomero (1988), and Bühler/Koziol (2005), amongst others.

Santomero (1984, p. 582) states that models that work under the assumption of risk-aversion implicitly build on the notion that the bank is run by managers who cannot diversify their human capital or that the bank is owned by investors who have limited access to other investment opportunities. Conversely, he claims that the assumption of risk-neutrality can be aligned with banks being owned by investors who have broad access to further investment opportunities such that diversification is no longer aimed at the bank's portfolio level, but at the investors' asset universe. However, the assumption of risk-neutrality has also been simply viewed as an approximation (Baltensperger, 1980, p. 25). So far, many models with a risk-neutral banking firm have evolved, such as Blum (1999), Calem/Rob (1999), Dermine (1986), Eichberger/Summer (2005), Estrella (2004) Klein (1971), Monti (1972), Repullo/Suarez (2008), Suarez (1994), Thakor (1996), and Zhu (2008), to name a few. In contrast, risk-averse banks and other risk-averse financial agents can also be found in general equilibrium approaches. In Catarineu-Rabell/Jackson/Tsomocos (2005), firms and banks have $\mu$-$\sigma$ preference functions as objective function. Similarly, banks in Daníelsson/Zigrand (2008) and traders in Daníelsson/Shin/Zigrand (2004) maximize expected utility.

### 2.3.4 Market Power in Banking

In our model, the bank has monopoly power on both the loan and deposit market, thus standing in this respect in the tradition of the neo-classical models of Klein

(1971), Monti (1972), Dothan/Williams (1980), Dermine (1986), and Blum (1999) who all consider a double-sided monopoly as well.[3] But by the endogenous risk-transfer and the endogenous interaction between a risk-neutral bank and a risk-averse depositor, we considerably depart from their frameworks. Let us review some reasons and evidence of market power in banking.

The bank's monopoly power on the market for loans can be explained as a result of relationship banking: once firms are bound to a given bank, the bank can extract the monopoly rent on loans (Sharpe, 1990, p. 1069; Repullo/Suarez, 2008).

Though relationship banking is deemed a convincing argument for monopoly power on the loan market, monopoly power on the deposit market is seen as rather controversial as depositors are considered to have access to a sufficiently high number of alternative banks. If deposits are fully insured, vagueness about the bank's asset quality should be negligible even in a world of asymmetric information and, at least concerning term deposits, perfect competition should emerge. A counter-argument frequently put forward is about locally fragmented markets such that alternatives to depositors are too scarce for sustaining a competitive environment (*e.g.* Saunders/Schumacher, 2000). In this vein, there are also authors who assume monopoly power of banks exclusively on the deposit market (Kareken/Wallace, 1978; Rochet, 1992, p. 1143; and Suarez, 1994). Recent technological advances, such as internet-based banking, may weaken this argument, but may not render it obsolete as long as there is demand for individual consultations.

Greenbaum/Thakor (2007, p. 108) explain market power of banks on the deposit side by two sources that are related to the bank's size. First, economies of scale arise by diversification: the bigger the bank, the more loans can be made and the more granular the loan portfolio becomes. Thus risks can be reduced with increasing size. Risk reduction, in turn, makes the depositors better-off, even if they are risk-neutral as they face a concave pay-off from uninsured debt. As a result, the offered deposit interest rate is the lower, the bigger the bank is. With increasing size, banks may reduce deposit interest rates stronger than the reduction of risk justifies. Single depositors will accept, as long as they have no similar diversification possibilities. Second, credit analyses associated with the loans made are done once by the bank and are borne by many depositors. Otherwise, if there is no bank, the same costly screening efforts will be performed directly by the depositors (then simply lenders). As coordination generally fails, these efforts will partly be done more than once,

[3] As the bank is actually on the demand side of the deposit market, the bank's behavior should be rather termed as monopsonistic (*cf.* Mas-Colell/Winston/Green, 1995, p. 500; Hendersen/Quandt, 1958, p. 164).

raising the costs per deposit.

These economies-of-scale argument work when the number of depositors per bank is large and if there is no deposit insurance. As result, the bank is a natural monopoly.

Thus, the question whether there are monopolistic structures in banking or not is an issue of institutional settings, of arrangements in banking regulation, and in the end of empirical evidences. Baltensperger (1980, p. 18) argues that “probably [...] specific circumstances” may justify whether or not the assumption of monopolistic behavior can be justified for modeling banks.

Evidence is given by Keeley (1990) who identifies bank charter values with market power on loan and deposit markets. However, declining capital ratios in the USA are seen as if bank charters lose their value or, equivalently, market power decreases. Similarly, Neven/Röller (1999) confirm a cartel-like conduct of banks concerning granting mortgages, but at a decreasing rate over time. Both Keeley (1990) and Neven/Röller (1999) associate the declining market power with deregulations. Further evidence of anti-competitive behavior is found by Molyneux/Lloyd-Williams/Thornton (1994) for Germany and de Bandt/Davis (2000) for small banks in Germany and France. Gischer/Stiele (2009) find that the locally segmented market of savings banks in Germany is characterized by monopolistic competition.

To summarize, this model framework allows us to study a bank’s simultaneous choice of the risk allocation concerning potentially correlated investments and on deposit costs which are captured by the interest rate on deposits. Furthermore, the size of the bank, *i.e.* the single loan volumes and the deposit volume endogenously emerge.[4] In particular, credit risk is endogenously transferred to the household, resulting in a feed-back effect for the bank when it takes deposit supply as given for maximizing its own expected wealth.

This model framework then serves to compare these decisions under various regimes. The laissez-faire equilibrium will be compared with the equilibria that occur under the Standardized Approach and a VaR approach to determine economic capital at a given confidence level. The latter approach is meant to represent the IRB Approach as done by Bühler/Koziol (2005), Dangl/Lehar (2004), Daníelsson/Shin/Zigrand

[4]Contrary to Baltensperger’s (1980, p. 18, 21) criticism on bank models with monopoly power concerning deposits, the assumption of monopoly power is not necessary to obtain an endogenous upper bound on the bank’s size in our framework. A competitive market structure between bank and depositor can be modeled by maximizing the household’s expected utility or preference function given a zero-expected-profit condition imposed on the bank. That is, the bank acts on behalf of the depositor as otherwise the depositor would choose an alternative bank.

(2004), Estrella (2004), and Daníelsson/Zigrand (2008).

# Part II

# Bernoulli Distributed Loan Redemptions

# Chapter 3

# Theoretical Analysis

## 3.1 Introduction

In this chapter, a specific model that is based on the framework outlined in Chapter 2 is set up and discussed. There are two firms who demand credit, a household that supplies deposits, and a bank as intermediary between the firms and the household. The analysis is restricted to one period.

The agents are introduced in the next section, Section 3.2. The firms are presented in Section 3.2.1. The household's decision is outlined in Section 3.2.3, the bank's in Section 3.2.4. Section 3.2.2 lays the grounds for the household's and the bank's objective function as it highlights the state- and portfolio-dependent pay-offs from the deposit contract.

Section 3.3 deals with the equilibria with and without regulation. Section 3.3.1 examines the equilibrium without regulation in general and, moreover, highlights some specific equilibria that can be stated explicitly. Likewise, equilibria under the Standardized Approach (Section 3.3.2), and under a VaR-based approach (Section 3.3.3) are discussed. Specific equilibria arising under binding regulation are compared with specific equilibria arising under the laissez-faire economy presented in Section 3.3.1. First conclusions on the cyclical impact of regulation are drawn. Section 3.4 concludes. In particular, Table 3.9 provides an overview over the results on regulatory impacts through the Standardized Approach, and Table 3.10 summarizes the findings concerning the VaR-based regulatory approach.

The whole Part II is based on Bühler/Koziol/Sygusch (2008), but considerably extends that study. Results, that can be also found in Bühler/Koziol/Sygusch (2008), are indicated as such. Part II adds to Bühler/Koziol/Sygusch (2008) by the theoretical analysis of the model, *i.e.* the non-numerical analysis of equilibria and the thus obtained results on cyclical impacts by regulation. Furthermore, the numerical analysis performed in Chapter 4 exceeds the study presented by Bühler/Koziol/Sygusch (2008).

## 3.2 The Model

### 3.2.1 The Firms

We assume that there are two firms $i$, $i = 1, 2$, where each can run a single risky project.[1] The outcome of each project is Bernoulli-distributed, *i.e.* a project can either be successful, $\tilde{X}_i = 1$, with probability $p_i$, or it can fail, $\tilde{X}_i = 0$ with probability $1 - p_i$. Firms exhibit a linear production technology where each unit of capital used transforms into $\alpha_i > 1$ units of output. Firms may scale their projects arbitrarily high. Firms are risk-neutral and for simplicity they are exclusively financed by (bank) debt $L_i$. As a result, their production functions are given as follows:

$$\alpha_i \cdot \tilde{X}_i \cdot L_i, \quad i = 1, 2 \ .$$

At the end of the period each loan is repaid conditional on the firm's success. Due to the firms' limited liability, the bank obtains the following payment from each loan contract:

$$\min \left\{ \ L_i \cdot R_i, \ \alpha_i \cdot \tilde{X}_i \cdot L_i \right\}, \quad i = 1, 2 \ . \tag{3.2.1}$$

where $R_i$ denotes the gross interest rate on the nominal debt volume $L_i$. $R_i$ and $L_i$ are endogenous. By risk-neutrality, both firms invest as long as their expected profits are non-negative. Therefore, they maximize:

$$\begin{gathered} \max_{L_i \geq 0} \ \mathrm{E}\left[ \ \max\left\{ \ \alpha_i \cdot \tilde{X}_i \cdot L_i \ - \ L_i \cdot R_i, \ 0 \ \right\} \right] \\ \Leftrightarrow \\ \max_{L_i \geq 0} \ p_i \cdot \max\left\{\alpha_i - R_i, \ 0\right\} \cdot L_i \ . \end{gathered} \tag{3.2.2}$$

Let us assume that both firms accept any loan volume as long as their expected wealth is equal to or greater than zero and that loan volumes are non-negative. Loan demand by firm $i$ can then be characterized by the following loan demand

[1] A short version of this section can be found in Bühler/Koziol/Sygusch (2008, p. 138f).

correspondence,

$$L_i^d(R_i) \begin{cases} \geq 0 & \text{if} \quad R_i \leq \alpha_i \\ = 0 & \text{if} \quad R_i > \alpha_i \end{cases} , \ i = 1, 2 \ . \tag{3.2.3}$$

Consequently, the bank, which supplies loans as a monopolist, does not forgo any profits if it sets each loan interest rate $R_i$ equal to the marginal productivity $\alpha_i$ of firm $i$, because the monopolistic bank is still free to scale the loan volume arbitrarily up and down. Thus, it always chooses[2]

$$R_i^* = \alpha_i, \quad i = 1, 2 \ . \tag{3.2.4}$$

The resulting gross interest rate $R_i^*$ is independent of the bank's refinancing conditions and hence of the deposit market, the household, and the regulation. We make use of this result from here on without explicitly referring to it.

Firms are not only price-takers on the market for corporate loans, but implicitly also on the goods market as their productivity is fixed and transforms the financing conditions linearly into goods. Hence, the firms operate under perfect competition. Instead of prices equaling marginal cost, the dimensionless marginal productivity $\alpha_i$ equals a dimensionless cost rate, expressed by the gross loan interest rate $R_i$. Inputs are capital in form of loans and thus denominated in dollars.

Unless otherwise stated, we will assume firm $i = 2$ as the riskier firm in terms of a higher default probability $1 - p_2$, a higher default variance, and a higher expected (gross) return $p_2\alpha_2$,

$$(1 - p_1) < (1 - p_2), \quad p_1(1 - p_1) < p_2(1 - p_2), \quad p_1\alpha_1 < p_2\alpha_2 \ , \tag{3.2.5}$$

implying that:

$$\alpha_1 < \alpha_2, \ \mathrm{V}(\alpha_1\tilde{X}_1) < \mathrm{V}(\alpha_2\tilde{X}_2) \ \Leftrightarrow \ p_1(1 - p_1)\alpha_1^2 < p_2(1 - p_2)\alpha_2^2 \quad \text{and} \quad p_1 > \frac{1}{2} \ .$$

Undertaking risky projects shall dominate the risk-free investment yielding the gross rate $R_f$:

$$p_1\alpha_1 > R_f \geq 1 \ . \tag{3.2.6}$$

Thus, the risk-neutral firms would not invest their capital into the riskless bonds even if they could.

[2]Also in Repullo/Suarez (2008), the bank charges the firms the total return in case of success as interest on loans.

Hence, the productivity parameters are bounded from below, $\alpha_i > 1$ by (3.2.6). In what follows,

$$\alpha_i \leq 2 \quad \forall i = 1, 2 \ , \tag{3.2.7}$$

is additionally assumed as upper bound such that returns from production and hence on loans, do not become too big. Assumption (3.2.7) helps to directly derive Results 3, 4, 9, 11, and 12. To obtain Result 7, Assumption (3.2.7) is further restricted to $\alpha_i \leq \frac{4}{3}$, which is only sufficient, however.

Furthermore, the Assumptions (3.2.6) and (3.2.7) result in

$$p_1 > p_2 > \frac{1}{2} \tag{3.2.8}$$

which is not restrictive, as the probability $1-p_i$ represents firm $i$'s default probability which is thought to be far lower than a half in this context. Equality, *i.e.* $p_1 = p_2$, as far as considered, will be deliberately noted.

On the level of individual firms, we have cared so far about how returns of each project are distributed. In order to decide the allocation of funds among these two firms, it is necessary to consider the joint distribution of the projects' returns. As the marginals are given by $p_i$, $i = 1, 2$, the joint distribution is restricted to

$$\begin{array}{rcl} \mathrm{P}(\tilde{X}_1 = 1, \tilde{X}_2 = 1) \ + \ \mathrm{P}(\tilde{X}_1 = 1, \tilde{X}_2 = 0) & = & p_1 \\ \mathrm{P}(\tilde{X}_1 = 0, \tilde{X}_2 = 1) \ + \ \mathrm{P}(\tilde{X}_1 = 0, \tilde{X}_2 = 0) & = & 1 - p_1 \\ \mathrm{P}(\tilde{X}_1 = 1, \tilde{X}_2 = 1) \ + \ \mathrm{P}(\tilde{X}_1 = 0, \tilde{X}_2 = 1) & = & p_2 \\ \mathrm{P}(\tilde{X}_1 = 1, \tilde{X}_2 = 0) \ + \ \mathrm{P}(\tilde{X}_1 = 0, \tilde{X}_2 = 0) & = & 1 - p_2 \end{array} .$$

By introducing the notation

$$q \ \equiv \ \mathrm{P}\left(\tilde{X}_1 = 1, \tilde{X}_2 = 1\right) \ , \tag{3.2.9}$$

we can rewrite the above system of equations to yield

$$\begin{array}{rcl} \mathrm{P}(\tilde{X}_1 = 1, \tilde{X}_2 = 0) & = & p_1 - q \\ \mathrm{P}(\tilde{X}_1 = 0, \tilde{X}_2 = 1) & = & p_2 - q \\ \mathrm{P}(\tilde{X}_1 = 0, \tilde{X}_2 = 0) & = & 1 - p_1 - p_2 + q \end{array} . \tag{3.2.10}$$

To ensure that the Probabilities (3.2.10) that form the joint distribution are all positive, the probability $q$ must be bounded by (*cf.* Joe, 2001, p. 210)

$$p_1 + p_2 - 1 \ < \ q \ < \ \min\{p_1, p_2\} \ . \tag{3.2.11}$$

Note that $p_1 + p_2 - 1 > 0$ holds due to (3.2.6) and (3.2.7). Furthermore, Condition (3.2.11) guarantees that the correlation of both projects, $\tilde{X}_1$ and $\tilde{X}_2$, given by

$$\mathrm{Corr}(\tilde{X}_1, \tilde{X}_2) = \frac{q - p_1 p_2}{\sqrt{p_1(1-p_1)p_2(1-p_2)}} \,, \tag{3.2.12}$$

is bounded on $(-1, 1)$. A correlation of one between both projects is associated with $p_1 = q$ and $p_2 = q$, whereas both projects are perfectly negatively correlated if and only if $q = 0$, $p_1 = \frac{1}{2}$, and $p_2 = \frac{1}{2}$ hold. For $p_1 \neq p_2$, the actual bounds can be quite apart from $-1$ and 1. Because of $p_1 > p_2 > \frac{1}{2}$ these bounds are as follows:

$$-1 \;<\; -\frac{1-p_1}{p_2} \;<\; \mathrm{Corr}(\tilde{X}_1, \tilde{X}_2) \;<\; \frac{p_2}{p_1} \;<\; 1. \tag{3.2.13}$$

That is, the upper bound is determined by the ratio of the firms' success probabilities whereas the lower bound is equal to the ratio of Firm 1's default probability over the other firm's success probability. If success probabilities are rather close-knit together and close to one, the lower bound is close to zero and the feasible range of correlations is located asymmetrically around zero: if Firm 1 undertakes its project successfully with $p_1 = 0.99$ and Firm 2 with $p_2 = 0.98$, their default correlation ranges at most on $(-\frac{1}{98}, \frac{98}{99})$.[3]

### 3.2.2 The Repayment of Deposits

In this section we analyze the repayments of the uninsured deposits at the end of the period. As soon as these repayments are fully characterized, the objective functions of the household and the bank can be analytically presented.

There are two distinct determinants that characterize the default risk of the deposits. First, and basically, the uninsured deposits bear risks since there are risks on the bank's asset side, *i.e.* by the Bernoulli-distributed loan redemptions discussed in the last section. Second, the riskiness of the deposits is shaped by the bank's portfolio compositions: the relation between the amounts of promised loan redemptions to each other and their respective relations to the amount of promised deposit redemption characterize under which states of the world deposits can be redeemed as promised and under which states the depositor is left with a stake in the residual

[3] The restrictions on parameter values increase with the number of projects, *i.e.* marginal Bernoulli distributions considered. Chaganty/Joe (2006) set out explicit conditions on parameters for three- (*ibid.*, pp. 198-200) and four-dimensional Bernoulli distributions (*ibid.*, pp. 202-203). In Section 5.2, we will deal with a multivariate Bernoulli distribution that is set up by a mixture model following the general pattern proposed by Joe (2001, pp. 210).

bank portfolio redemption. Deposits are always senior to equity.

Let $D$ denote the deposit volume and $R_D$ the respective gross interest rate, *i.e.* $D \cdot R_D$ denotes the promised repayment of the deposit volume inclusive of the promised interest. Each promised loan redemption is given by $L_i R_i$, $i = 1, 2$. According to (3.2.4), the bank's optimal choice of the loan interest rate is not affected by any other decisions to be taken. Therefore, we proceed with $R_i = R_i^* = \alpha_i$, $i = 1, 2$.

Because there are no information asymmetries, it is known to the household as the potential depositor how much equity $W_B \geq 0$ the bank initially possesses and that there are only the household's deposits $D \geq 0$ and the bank's initial equity $W_B$ at the bank's disposal to grant two loans to two distinct firms. We explicitly allow the bank to intermediate between depositors and borrowers even if it does not initially have any equity.[4]

Thus, each loan volume granted can be directly linked to the bank's liabilities. So the balance sheet identity implies

$$\begin{aligned} L_1 &\equiv \ell \cdot (D + W_B) \ , \\ L_2 &\equiv (1 - \ell) \cdot (D + W_B) \ , \\ L &\equiv L_1 + L_2 \equiv D + W_B \ , \end{aligned} \tag{3.2.14}$$

where $\ell$ represents the percentage amount of available total capital $D + W_B$ that is devoted as loan to the Firm 1. Henceforth $\ell$ will be called the loan-allocation rate. $L$ is referred to as the total loan volume.

We exclude the possibilities of short-sells, *i.e.* that the lender-borrower relationships become reversed. Therefore, the loan-allocation rate is restricted to the unit interval,

$$\ell \in [0, 1] \ . \tag{3.2.15}$$

The probability that deposits are paid back as promised depends crucially on the relations amongst the promised deposit repayment, $DR_D$, and promised loan repayments, $L_i \alpha_i$, $i = 1, 2$. Two risky loans and a single debt instrument on one balance sheet imply that we must consider four cases for analyzing the state-dependent pay-offs that arise from the deposit contract. Furthermore, we rule out $\alpha_1 R_1 + L_2 \alpha_2 < DR_D$ since the bank will then go bankrupt with certainty, resulting in an expected final wealth of zero for the bank. Hence, this choice is clearly dominated by any other policy. The objective function of the bank owners is presented in

[4] It is not until the two-period framework in Chapter 7 that, at beginning of the second period, zero equity is exclusively associated with bankruptcy.

Section 3.2.4, p. 63.

Making use of (3.2.4) and (3.2.14), these four cases can be formulated by the following sets $C_j$, $j = 1, \ldots, 4$, of which each set is defined contingent on promised repayments:

**Case 1 : neither loan suffices to fully redeem deposits .**

$$C_1 \;=\; \{\ (D, \ell, R_D) :\ DR_D \geq \alpha_1 \ell \cdot (D + W_B),\ DR_D \geq \alpha_2 (1-\ell) \cdot (D + W_B)\ \}\ , \tag{3.2.16}$$

**Case 2 : only loan 2 suffices to fully redeem deposits.**

$$C_2 \;=\; \{\ (D, \ell, R_D) :\ \alpha_1 \ell \cdot (D + W_B) \leq DR_D \leq \alpha_2 (1-\ell) \cdot (D + W_B)\ \}\ , \tag{3.2.17}$$

**Case 3 : only loan 1 suffices to fully redeem deposits.**

$$C_3 \;=\; \{\ (D, \ell, R_D) :\ \alpha_2 (1-\ell) \cdot (D + W_B) \leq DR_D \leq \alpha_1 \ell \cdot (D + W_B)\ \}\ , \tag{3.2.18}$$

**Case 4 : either loan suffices to fully redeem deposits.**

$$C_4 \;=\; \{\ (D, \ell, R_D) :\ DR_D \leq \alpha_1 \ell \cdot (D + W_B),\ DR_D \leq \alpha_2 (1-\ell) \cdot (D + W_B)\ \}\ . \tag{3.2.19}$$

We will refer to these sets henceforth as "Cases" with a capital letter. These Cases can be explained as follows: Case 1 represents those structures of the bank's balance sheet that require both loans to be fully redeemed in order to be able to pay-off the depositor as promised. Case 2 represents those structures where the bank can fully pay-off its depositor if exclusively Loan 2 is fully redeemed, but cannot do so, if exclusively Loan 1 is fully paid off. Case 3 is the mirror image to Case 2 and Case 4 subsumes those structures where any single loan redemption is sufficient to fully redeem deposits.[5]

None of the Cases (3.2.16) to (3.2.19) can be excluded from equilibrium *ex ante.*

[5]It can be shown by complete induction that there are $2^{n+1} - n - 2$ such Cases to be considered if there are $n$ loans on the bank's balance sheet matched to a single debt instrument. This still holds true if one out of the $n$ loans is the safe asset as the number of such cases is only determined by the number of relations between promised redemptions that allow for a full redemption of deposits. Particularly, if the bank potentially held three assets, we would have to consider eleven distinct repayment schedules for deposits. In particular, holding a volume of the risk-free asset equal to or in excess of the deposit volume does not increase the bank's expected final wealth compared to autarky without deposit finance.

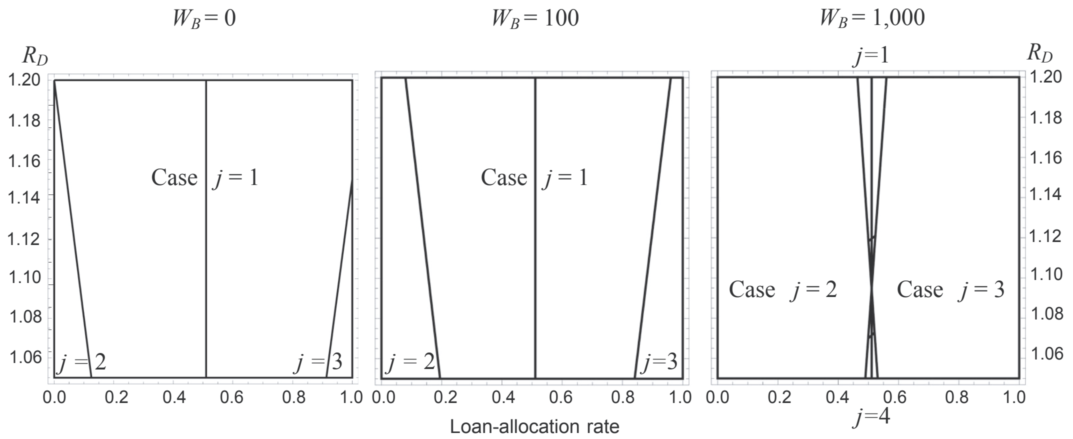


Figure 3.1: **The Case constraints given a fixed deposit volume**

*This figure illustrates the feasible $\ell$-$R_D$ combinations given a fixed deposit volume, $D = 1157.59$, and three different levels of the bank's initial equity $W_B$. Feasible $\ell$-$R_D$ tuples are separated according to their associated Cases $j$, $j = 1, \ldots, 4$. The vertical line represents $\ell \equiv \frac{\alpha_2}{\alpha_1+\alpha_2}$. Parameter values are given according to Table 4.1. The chosen deposit volume of $D = 1157.59$ equals its respective laissez-faire equilibrium value given that $W_B = 100$.*

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Depending on the parameters, each of these Cases may turn out to be optimal for the bank. Note that not every loan-allocation rate $\ell \in [0, 1]$ is feasible given a Case $j$. Under Case 2, the loan-allocation rate $\ell$ may only range between

$$\ell \ \in \ [\ 0, \ \frac{\alpha_2}{\alpha_1 + \alpha_2}\ ] \tag{3.2.20}$$

and under Case 3 between

$$\ell \ \in \ [\ \frac{\alpha_2}{\alpha_1 + \alpha_2}, \ 1]\ . \tag{3.2.21}$$

These bounds hold independent of specific values considered for the variables $D$ and $R_D$ as long as the total capital/loan volume is strictly positive, $D + W_B > 0$.

Figure 3.1 displays feasible $\ell$-$R_D$ combinations for a fixed amount of deposits and different levels of the bank's initial equity. The $\ell$-$R_D$ tuples are separated according to their associated Cases.

Now, the state-dependent pay-offs arising from the deposit contract will be examined such that expected pay-offs and the variance of pay-offs can be determined given a Case $j$ where $\tilde{D}_j$ denotes the stochastic pay-off given Case $j$ and where

$$\tilde{D} \tag{3.2.22}$$

denotes the stochastic pay-off from the deposit contract without referring to any specific case.

According to the definition of

**Case 1 (neither loan suffices to fully redeem deposits)**

the realizations of the firms' success variables $(\tilde{X}_1, \tilde{X}_2)$ result in the following pay-off structure:

$$\tilde{D}_1 = \begin{cases} \min\{DR_D, \alpha_1 L_1 + \alpha_2 L_2\} = DR_D, & \text{if} \quad \tilde{X}_1 = 1,\ \tilde{X}_2 = 1 \\ \min\{DR_D, \alpha_1 L_1\} = \alpha_1 L_1, & \text{if} \quad \tilde{X}_1 = 1,\ \tilde{X}_2 = 0 \\ \min\{DR_D, \alpha_2 L_2\} = \alpha_2 L_2, & \text{if} \quad \tilde{X}_1 = 0,\ \tilde{X}_2 = 1 \\ \min\{DR_D, 0\} = 0, & \text{if} \quad \tilde{X}_1 = 0,\ \tilde{X}_2 = 0 \end{cases} \tag{3.2.23}$$

The expected repayment of deposits is

$$\begin{aligned} \mathrm{E}(\tilde{D}_1) &= q \cdot DR_D \ + \ [\ (p_1 - q)\alpha_1 \ell + (p_2 - q)\alpha_2(1-\ell)\ ] \cdot (D + W_B) \\ &= \mu_1 \cdot (D + W_B) \ - \ q \cdot W_B R_D, \end{aligned} \tag{3.2.24}$$

where $\mu_1$ is defined as

$$\mu_1 \ = \ qR_D + (p_1 - q)\alpha_1 \ell \ + \ (p_2 - q)\alpha_2(1-\ell), \tag{3.2.25}$$

and represents the *marginal* expected gross return on deposits.

The variance of the repayments, $\tilde{D}_1$, can be stated as

$$\begin{aligned} \mathrm{V}(\tilde{D}_1) \ = \ & \sigma_1^2 \cdot (D + W_B)^2 \ + \ 2 \cdot qR_D \cdot (\mu_1 - R_D) \cdot D \cdot W_B \ + \\ & + \ qR_D \cdot [2\mu_1 - (1+q)R_D] \cdot W_B^2 \end{aligned} \tag{3.2.26}$$

where

$$\sigma_1 \ = \ \sqrt{qR_D^2 + (p_1 - q)(\alpha_1 \ell)^2 + (p_2 - q)\left(\alpha_2(1-\ell)\right)^2 - \mu_1^2} \tag{3.2.27}$$

is the volatility associated with the gross return obtained on deposits given in Case 1.

According to

**Case 2 (only Loan 2 suffices to fully redeem deposits)**

the stochastic repayment, $\tilde{D}_2$, differs from that obtained under Case 1, $\tilde{D}_1$, only if

$\tilde{X}_1 = 0$ and $\tilde{X}_2 = 1$ emerge. This event results in

$$\tilde{D}_2 \;=\; \min\{DR_D; \alpha_2 L_2\} \;=\; DR_D \quad \text{if} \quad \tilde{X}_1 = 0,\; \tilde{X}_2 = 1\;. \tag{3.2.28}$$

The expected value of $\tilde{D}_2$ is given by

$$\begin{aligned} \mathrm{E}(\tilde{D}_2) \;&=\; p_2 \cdot DR_D \;+\; (p_1 - q)\alpha_1 \ell \cdot (D + W_B) \\ &=\; \mu_2 \cdot (D + W_B) \;-\; p_2 \cdot W_B R_D \end{aligned} \tag{3.2.29}$$

where $\mu_2$ is

$$\mu_2 \;=\; p_2 R_D \;+\; (p_1 - q)\alpha_1 \ell\;. \tag{3.2.30}$$

$\tilde{D}_2$ has as variance

$$\begin{aligned} \mathrm{V}(\tilde{D}_2) \;&=\; \sigma_2^2 \cdot (D + W_B)^2 \;+\; 2 \cdot p_2 R_D \cdot (\mu_2 - R_D) \cdot D \cdot W_B \;+ \\ &\quad+\; p_2 R_D \cdot [\; 2\mu_2 - (1 + p_2) R_D \;] \cdot W_B^2 \end{aligned} \tag{3.2.31}$$

where the volatility of gross deposit returns, $\sigma_2$, is

$$\sigma_2 \;=\; \sqrt{p_2 R_D^2 + (p_1 - q)(\alpha_1 \ell)^2 - \mu_2^2}\;. \tag{3.2.32}$$

The deposit repayments given

**Case 3 (only Loan 1 suffices to fully redeem deposits)**

are symmetric to Case 2 and we obtain

$$\begin{aligned} \mathrm{E}(\tilde{D}_3) \;&=\; p_1 \cdot DR_D \;+\; (p_2 - q)\alpha_2 (1 - \ell) \cdot (D + W_B) \\ &=\; \mu_3 \cdot (D + W_B) \;-\; p_1 \cdot W_B R_D \end{aligned} \tag{3.2.33}$$

as expected pay-off and

$$\begin{aligned} \mathrm{V}(\tilde{D}_3) \;&=\; \sigma_3^2 \cdot (D + W_B)^2 \;+\; 2 \cdot p_1 R_D \cdot (\mu_3 - R_D) \cdot D \cdot W_B \;+ \\ &\quad+\; p_1 R_D \cdot [\; 2\mu_3 - (1 + p_1) R_D \;] \cdot W_B^2 \end{aligned} \tag{3.2.34}$$

as the respective variance. The expected gross return on deposits and the associated volatility are

$$\mu_3 \;=\; p_1 R_D \;+\; (p_2 - q)\alpha_2 (1 - \ell), \tag{3.2.35}$$

$$\sigma_3 \;=\; \sqrt{p_1 R_D^2 \;+\; (p_2 - q)\alpha_2^2 (1 - \ell)^2 - \mu_3^2}\;. \tag{3.2.36}$$

Under

**Case 4 (either loan suffices to fully redeem deposits)**

the depositor receives the promised redemption $DR_D$ as long as at least one firm is successful. Hence, the expected pay-off and its variance are:

$$\begin{aligned} \mathrm{E}(\tilde{D}_4) &= (p_1 + p_2 - q) \cdot DR_D = \mu_4 \cdot D, && (3.2.37) \\ \mathrm{V}(\tilde{D}_4) &= \sigma_4^2 \cdot D^2, && (3.2.38) \end{aligned}$$

where

$$\begin{aligned} \mu_4 &= (p_1 + p_2 - q)R_D, && (3.2.39) \\ \sigma_4 &= \sqrt{(p_1 + p_2 - q)R_D^2 - \mu_4^2}\ . && (3.2.40) \end{aligned}$$

The expected repayments of deposits and its associated variance can be displayed in one formula across all four Cases if we introduce a new symbol for the Case-dependent probability that deposits are fully repaid:

$$q_j \;=\; \mathrm{P}(\tilde{D}_j = D \cdot R_D) \;=\; \begin{cases} q, & \text{in Case } j = 1, \\ p_2, & \text{in Case } j = 2, \\ p_1, & \text{in Case } j = 3, \\ p_1 + p_2 - q, & \text{in Case } j = 4. \end{cases} \qquad (3.2.41)$$

Their interpretation is two-fold: from the depositor's point of view, $q_j$ is primarily the probability that deposits are redeemed as contracted. From the bank's point of view, $q_j$ is primarily its survival probability. $1 - q_j$ states the bank's probability of bankruptcy. The probability $q_j$ crucially depends on the composition of the loan portfolio and the relation of promised loan pay-offs to promised deposit. This issue will be addressed in detail in this chapter from Section 3.2.4 on.

The expected repayment of deposits thus becomes

$$\mathrm{E}(\tilde{D}_j) \;=\; \mu_j \cdot D \;+\; (\mu_j - q_j R_D) \cdot W_B\ ,\ j = 1, \ldots, 4\ , \qquad (3.2.42)$$

and the variance of the repayments can be stated as

$$\begin{aligned} \mathrm{V}(\tilde{D}_j) \;=\; & \sigma_j^2 \cdot (D + W_B)^2 \;+\; 2 \cdot q_j R_D \cdot (\mu_j - R_D) \cdot D \cdot W_B \;+ \\ & +\; q_j R_D \cdot [2\mu_j - (1 + q_j)R_D] \cdot W_B^2\ ,\ j = 1, \ldots, 4\ . \end{aligned} \qquad (3.2.43)$$

### 3.2.3 The Household

The household allocates its initial wealth, $W_H$ between risky bank deposits, promising $D \cdot R_D$, and risk-free assets yielding the exogenous gross interest rate $R_f \geq 1$. The household is not allowed to short-sell any assets or to borrow. It maximizes its final wealth, $\tilde{W}_H$, according to a $\mu$-$\sigma$-preference function. This maximization problem can be considered as a simplified version of a household's one-period savings-consumption decision where consumption is not explicitly modeled.

As there is no asymmetric information, the depositor knows the possible portfolio compositions that are feasible for the bank and values the offered deposit contracts according to their risks and possible returns. Likewise, the bank cannot cheat and commits to each contract.[6]

Final wealth equals under any of the four Cases

$$\tilde{W}_H = \tilde{D} + (W_H - D)R_f \; . \tag{3.2.44}$$

As the distribution of $\tilde{D}$ changes along changing triples $(D, \ell, R_D)$, so does the distribution of final wealth $\tilde{W}_H$. In particular, if the variables $\ell$ and $R_D$ are fixed, the distributions of $\tilde{D}$ and $\tilde{W}_H$ solely depend on $D$.

The $\mu$-$\sigma$-preferences over final wealth, $\tilde{W}_H$, are

$$U(\tilde{W}_H) = \mathrm{E}(\tilde{D}) + (W_H - D) \cdot R_f - \frac{1}{2} \cdot \gamma \cdot \mathrm{V}(\tilde{D}) \; . \tag{3.2.45}$$

The parameter $\gamma > 0$ reflects the household's degree of risk-aversion. Given Case $j$, the utility $U(\tilde{W}_H)$ is characterized by the expected deposit pay-off $\mathrm{E}(\tilde{D}_j)$ and the associated variance $\mathrm{V}(\tilde{D}_j)$, as given by (3.2.42) and (3.2.43).

[6]Dermine (1986), Calem/Rob (1999), and Zhu (2008) also study bank models with uninsured deposits, but, contrary to our model, depositors are risk-neutral. In Calem/Rob (1999, p. 344), the depositor takes the bank's initial equity and the bank's portfolio-allocation rate into account. But Calem/Rob need not consider different deposit repayment schedules as they simply consider redemptions from the bank's loan portfolio on the return level and split them into safe returns and risky returns modeled by a uniform distribution. Also Dermine (1986) allows the bank to hold a risky commercial loan and a riskless bond, but he does not consider the case that the bond redemption could exceed the promised deposit redemption. In Zhu (2008, p. 178), the bank has only one asset that can be affected by shocks and the depositor internalizes the bank-specific shock only (Zhu, 2008, p. 178, Eq. (5)). Hence, all three papers have in common, that agents deal with a single source of risk. Calem/Rob (1999, p. 346) further question the depositors' abilities of observing the allocation rate and note that related empirical evidence is lacking. We note that, in reality, the commitment problem may be a problem of both asymmetric information and timing, *i.e.* the bank will reveal its asset choice at most when its financing is fixed (Bhattacharya/Boot/Thakor, 1998, p. 756).

Given a fixed loan-allocation rate $\ell$, a fixed deposit interest rate $R_D$, and a fixed Case $j$, the household chooses that amount of deposits $D$ that maximizes its utility as stated in (3.2.45) at which the optimal deposit volume must be from $[0, W_H]$.

Therefore, the household's optimization problem is given by

$$\begin{aligned} \max_{D} \quad & U(\tilde{W}_H), \\ \text{s.t.} \quad & \ell, R_D, j \text{ fixed }, \\ & 0 \leq D \leq W_H \ . \end{aligned} \tag{3.2.46}$$

The unconstrained optimization problem does not account for the lower and the upper bound on $D$, $0 \leq D \leq W_H$, and will be referred to by (3.2.46'). As $U(\tilde{W}_H)$ is strictly concave in $D$ given a specific Case $j$, the solution to Problem (3.2.46') is unique within each Case $j$ and denoted by $D^u(\ell, R_D; j)$. $D^u(\ell, R_D; j)$ is the unconstrained deposit-supply function. In general, magnitudes referring to Problem (3.2.46') or to its solution $D^u(\ell, R_D; j)$ will be super-indexed by $u$. $U(\tilde{W}_H)$ is strictly concave in $D$ within each Case $j$, as

$$\frac{\partial^2 U(\tilde{W}_H)}{\partial D^2} \;=\; -\,\gamma\sigma_j^2, \quad j = 1, \ldots, 4 \ , j \text{ fixed} \ ,$$

holds.

Solutions to Problem (3.2.46') are derived for given values of $\ell$ and of $R_D$, and for a given Case $j$ by taking, for each of the four Cases separately, the first partial derivative of $U(\tilde{W}_H)$ with respect to $D$ for fixed $\ell$ and $R_D$ and solving the associated first-order condition. That is, the functions $D^u(\ell, R_D; j)$ are Case-wise maximizers to Problem (3.2.46').

We dispense with the constraints associated with Cases 1 to 4 for the following reason: as the bank is a monopolist on both the loan and the deposit market, it will choose its risk-return profile such that its objective (*i.e.* expected wealth) is maximized. This objective is reached by choosing the appropriate loan-allocation rate $\ell$ and deposit interest rate $R_D$ within a given Case $j$ such that the solution is feasible in the sense that it meets one of the respective constraints as given by (3.2.16) to (3.2.19). Doing so for each of the four Cases results in Case-wise optimal choices which will be denoted by $(\ell^*(j), R_D^*(j))$. The choices $(\ell^*(j), R_D^*(j)), D^s(\ell^*(j), R_D^*(j); j))$ must meet the definition of the respective set $C_j$, as defined by (3.2.16) to (3.2.19), respectively. The triple resulting in the highest expected wealth is the final solution. For details, we refer to Section 3.3

and Subsection 3.3.1, respectively.

As a consequence, we can analyze the household's decision problem given each of the four Cases $j$ for any fixed pair of $\ell$ and $R_D$, *i.e.* we need not care if a given solution to the household's Problem (3.2.46) under one Case given $\ell$ and $R_D$ dominates a solution under another Case given the same values of $\ell$ and $R_D$. Instead, that deposit volume and its associated Case will prevail in equilibrium that leads to the highest bank's expected wealth given a fixed pair of $\ell$ and $R_D$.

Therefore, we can separately derive the explicit solutions to Problem (3.2.46') and thus for Problem (3.2.46). The solution to Problem (3.2.46) is denoted by $D^s(\ell, R_D; j)$ and will be referred to as the constrained solution.

Result 1 states the formulæ for the household's deposit-supply function and provides some of its basic properties.[7]

**Result 1.** *The unconstrained deposit supply $D^u(\cdot)$ that maximizes Problem* (3.2.46) *is unique under each Case $j$, $j = 1, \ldots, 4$, and given by*

$$D^u(\ell, R_D; j) \;=\; \frac{\mu_j - R_f}{\gamma \sigma_j^2} + \frac{q_j R_D (R_D - \mu_j) - \sigma_j^2}{\sigma_j^2} \cdot W_B,\ \ j = 1, \ldots, 4\ . \tag{3.2.47}$$

*Under Case* 4 *or if the bank grants only one loan in Cases* 2 *or* 3*, the unconstrained supply function simplifies to:*

$$D^u(\ell, R_D; j) \;=\; \frac{\mu_j - R_f}{\gamma \sigma_j^2},\ \text{ for either } j = 4,\ \text{ or } j = 2, \ell = 0,\ \text{ or } j = 3, \ell = 1\ . \tag{3.2.48}$$

*The constrained deposit-supply function reads*

$$D^s(\ell, R_D; j) = \min\left\{\max\left\{D^u(\ell, R_D; j), 0\right\}; W_H\right\},\ \ j = 1, \ldots, 4\ . \tag{3.2.49}$$

*In each Case $j$, $j = 1, \ldots, 4$, the functions $D^u(\ell, R_D; j)$ and $D^s(\ell, R_D; j)$ are continuous and $D^u(\ell, R_D; j)$ is differentiable both in its variables, $\ell$ and $R_D$, and in all its parameters.*

As the deposit-supply functions $D^s(\ell, R_D; j)$ are optimal decisions given a fixed Case $j$, the optimal deposit supply can migrate from one Case to another through changes in the variables $\ell$ and $R_D$ or through changes in parameter values.

In Case 4, the deposit-supply function is independent of the bank's equity, $W_B$, for the following reason: the depositor receives the promised payment $D \cdot R_D$ if at least

[7] A version of this result can be found in Bühler/Koziol/Sygusch (2008, p. 142).

one loan is redeemed and nothing if both firms fail. Thus, the contingent repayments of the deposits are independent of the bank's initial equity $W_B$ under Case 4 and so is the deposit-supply function $D^u(\ell, R_D; 4)$. For the same reason the productivity parameters (optimal gross rates on the firms' debts) $\alpha_1$ and $\alpha_2$ do not enter the deposit-supply function under Case 4. We can argue analogously if the bank grants a single loan only given Case 2 and 3, respectively.

Therefore, the deposit-supply function depends on the initial equity $W_B$ under the Cases where $W_B$ is crucial for the residual loan portfolio value if the bank defaults. To be more precise, the dependence on $W_B$ emerges whenever two loans are effectively granted under Cases 1, 2, and 3.

The first term of (3.2.47), apart from the composition of $\mu_j$ and $\sigma_j$, represents the optimal allocation of funds if an investor with $\mu$-$\sigma$-preferences can choose between a risky asset and a risk-free asset whose pay-offs are linked linearly to the amount of funds invested.

If, furthermore, the pay-off from the risky asset depended linearly on funds $W_B$ invested by another party, say a bank, this very investor would supply funds equal to

$$\frac{\mu_j - R_f}{\gamma\sigma^2} \; - \; W_B \;, \tag{3.2.50}$$

as long as the investor has still the opportunity to store funds into the risk-free asset. The funds invested by the other party/the bank lower *ceteris paribus* the supply of funds by the $\mu$-$\sigma$ investor as any gains by the own funds invested is enhanced by $W_B$.

Finally, the term

$$\frac{q_j R_D(R_D - \mu_j)}{\sigma_j^2} \cdot W_B$$

reflects the non-linearity in the marginal gains and the marginal risk of the deposit contract.

Next, Result 2 will characterize the household's deposit-supply function dependent on the deposit interest rate $R_D$. Furthermore, Result 2 will be the basis on which the bank's Optimization Problem (3.3.1) will be analyzed, as it provides an upper bound to $R_D$.

Therefore, let us define the following critical interest rates on deposits: $\underline{R}_D(\ell; j)$ denotes the critical deposit interest rate where the unconstrained deposit-supply function $D^u(\ell, R_D; j)$ becomes zero (given a fixed loan-allocation rate $\ell$ and given a fixed Case $j$):

$$\underline{R}_D(\ell;j) \in \{\ R_D|\ D^u(\ell,R_D;j)=0\ \}\,, \tag{3.2.51}$$

$\overline{R}^u_D(\ell;j)$ is the deposit interest rate that maximizes the unconstrained deposit-supply function $D^u(\ell,R_D;j)$ (given a fixed loan-allocation rate $\ell$ and given a fixed Case $j$):

$$\overline{R}^u_D(\ell;j) \in \left\{\ R_D|\ D^u(\ell,\overline{R}^u_D(\ell;j);j) \geq D^u(\ell,R_D;j) \quad \forall\ R_D \geq \underline{R}_D(\ell;j)\ \right\}\ . \tag{3.2.52}$$

$\overline{R}_D(\ell;j)$ represents the lowest deposit interest rate that maximizes the (constrained) deposit-supply function $D^s(\ell,R_D;j)$:

$$\overline{R}_D(\ell;j) := \min\left\{\ \overline{R}^u_D(\ell;j),\ \{\ R_D|R_D=\inf_{R_D}\{D^s(\ell,R_D;j)=W_H\}\ \}\ \right\}.$$

Thus, $\underline{R}_D(\ell;j) \leq \overline{R}_D(\ell;j) \leq \overline{R}^u_D(\ell;j)$ holds true by definition if $\overline{R}^u_D(\ell;j)$ exists where equality for the latter relation means that the household's deposit supply is not restricted by its initial wealth. The former relation is trivial.

The unconstrained deposit-supply function shows the following behavior in $R_D$. The existence of $\underline{R}_D(\ell;j)$ and $\overline{R}_D(\ell;j)$ has been also put forward by Bühler/Koziol/Sygusch (2008, p. 143). Here, we also provide a proof that can be found in Appendix A.1.1.

**Result 2.** *For each Case $j$ and for $\ell$ fixed, exactly one interest rate $\underline{R}_D(\ell;j)$ exists for which the unconstrained deposit-supply function becomes zero, and exactly one interest rate $\overline{R}^u_D(\ell;j)$ exists that maximizes the unconstrained deposit-supply function with respect to $R_D$. The unconstrained deposit-supply function approaches asymptotically zero from above if $R_D$ goes to infinity given any of the four Cases. Given Case $j$, the unconstrained deposit-supply function increases strictly monotonically in $R_D$ on $[\underline{R}_D(\ell;j),\overline{R}^u_D(\ell;j)]$ and decreases strictly monotonically on $[\overline{R}^u_D(\ell;j),\infty)$.*

Briefly, the unconstrained deposit-supply function is hump-shaped in $R_D$ on $[\underline{R}_D(\ell;j),\infty)$ given Case $j$, that is deposit supply is backward bending.

Clearly, Result 2 can be extended to the case of the constrained deposit-supply function which is zero for all $R_D \leq \underline{R}_D(\ell;j)$ and potentially flat for $R_D$ on some interval if the initial wealth $W_H$ is lower than the maximum value of the unconstrained supply function.

The economic intuition behind Result 2 can be understood by the pay-off structure of the standard risky debt contract and by the investor's risk-aversion level: The higher the promised interest rate $R_D$ grows, the higher the promised pay-off $D \cdot R_D$ becomes. Thus, it is natural to increase the supply of funds $D$ in order to further increase the final promised pay-off. However, deposits are risky and the household is risk-averse. Therefore, there is a point at which the benefits of further boosting $D$ do not outweigh the associated costs in terms of a rising variance. Consequently, the household will lower $D$ if the deposit interest rate $R_D$ is further increased. Specifically, the promised pay-off $D \cdot R_D$ may still rise while the household deposits less and less funds. Ultimately, if $R_D$ approaches infinity, deposit supply falls to zero.

Finally, let us characterize the critical interest rate $\underline{R}_D(\ell; j)$. We define the following Case-dependent magnitudes

$$\begin{array}{rcl} R_j^\mu & = & \mu_j - q_j R_D \\ R_j^\sigma & = & \sigma_j^2 + \mu_j^2 - q_j R_D^2 \end{array} , \quad j = 1, \ldots, 4, \quad \ell,\ j \text{ fixed}, \tag{3.2.53}$$

where $R_j^\mu$ denotes the marginal expected gross return on deposits exclusively the state of full deposit redemption and $R_j^\sigma$ is the associated second, non-centered moment. As a consequence,

$$R_j^\sigma - (R_j^\mu)^2 \ \geq \ 0 \tag{3.2.54}$$

is a variance and thus positive. $R_j^\mu$ and $R_j^\sigma$ are independent of $R_D$. In Case $j = 4$, both, $R_4^\mu$ and $R_4^\sigma$, become zero, as the bank defaults if and only if both firms default.

**Result 3.** *The interest rate $\underline{R}_D(\ell; j)$ for which the unconstrained deposit supply function becomes zero is given by*

$$\underline{R}_D(\ell; j) \ = \ \frac{R_f - R_j^\mu + \gamma W_B \cdot \left[R_j^\sigma - (R_j^\mu)^2\right]}{q_j \cdot (1 + \gamma W_B \cdot R_j^\mu)} \ , \quad \ell \text{ fixed},\ j = 1, \ldots, 4\ . \tag{3.2.55}$$

*It decreases strictly in $W_B$ in Cases 2 and 3. In Case 4, the interest rate $\underline{R}_D(\ell; 4)$ is independent of $W_B$ due to $R_4^\mu = R_4^\sigma = 0$. $\underline{R}_D(\ell; 4)$ always exceeds $R_f$ and simplifies to*

$$\underline{R}_D(4) \ = \ \frac{R_f}{q_4}\ .$$

*If $W_B = 0$,*

$$\underline{R}_D(\ell; j) \ = \ \frac{R_f - R_j^\mu}{q_j} \ > \ R_f\ , \quad \ell \text{ fixed},\ j = 2, 3,$$

*holds in Cases* 2 *and* 3. *The condition*

$$\alpha_i \; < \; \frac{1-q}{p_i - q} \cdot R_f, \; i = 1, 2 \; , \tag{3.2.56}$$

*is sufficient to obtain* $\underline{R}_D(\ell; 1) > R_f$ *in Case* 1 *if* $W_B = 0$.

*The critical interest rate* $\underline{R}_D(\ell; j)$ *remains positive, if* $W_B$ *goes to infinity,*

$$\lim_{W_B \to \infty} \underline{R}_D(\ell; j) \; = \; \frac{R_j^\sigma - (R_j^\mu)^2}{q_j \cdot R_j^\mu} \; \geq \; 0, \quad \ell \text{ fixed}, \; j = 1, 2, 3 \; .$$

The relation $\underline{R}_D(\ell; j) > R_f$ holds in $W_B = 0$ since the household is only willing to bear the credit risk of the deposits if they yield a higher expected return than the risk-free rate $R_f$.[8] The higher is the bank's initial equity $W_B$, the lower the credit spread on deposits becomes, as the buffer function of the bank's equity gains in importance. In particular, $\underline{R}_D(\ell; j)$ strictly decreases in $W_B$ in the Cases $j = 2, 3$. In Case 1, this relation generally remains opaque, however. In Case 4 or if loan-allocation rates are either zero (Case 2) or one (Case 3), contingent payments from the deposit contract are not affected by $W_B$, and so is $\underline{R}_D(\ell; j)$ (*cf.* Result 1 and Formulæ (3.2.24), (3.2.29), (3.2.33), and (3.2.37)).

The critical interest rate $\underline{R}_D(\ell; j)$, $j = 1, 2, 3$, is negatively related to $W_B$ in such a way that it may even fall below $R_f$ for sufficiently high amounts of bank capital. The limit of $\underline{R}_D(\ell; j)$ if $W_B$ goes to infinity marks the bottom line of which the positive sign is guaranteed by (3.2.54).

The following result illustrates that the buffer function of the bank's initial equity $W_B$ may stipulate the deposit supply.

**Result 4.** *Let* $\mu_j \geq R_f$ *and furthermore, let* $\ell \neq 0$ *in Case* 2, *and* $\ell \neq 1$ *in Case* 3. *Then,*

$$\frac{\partial D^u(\ell, R_D; j)}{\partial W_B} \; > \; 0 \tag{3.2.57}$$

*holds for the Cases* $j = 2$ *and* 3.

Let us close this section by discussing the household's deposit supply in Case 4.

**Remark 1.** *The characteristics of deposit supply in Case* 4 *can be stated in closed*

[8] We note that the additional (and only sufficient) bounds in Case 1 are rather of the same order of magnitude as the bounds assumed in (3.2.7). The base case scenario, as given in Table 4.1, yields as bounds according to (3.2.56) 1.70625 for $\alpha_1$, and 4.55 for $\alpha_2$. Assumption (3.2.56) is not further considered in what follows.

*form. The unconstrained deposit supply, $D^u(\ell, R_D; 4)$, becomes zero at*

$$\underline{R}_D(4) \;=\; \frac{1}{q_4} \cdot R_f \ ,$$

*and attains its unique maximum at*

$$\overline{R}^u_D(4) \;=\; \frac{2}{q_4} \cdot R_f \ ,$$

*as $D^u(\ell, R_D; 4)$ is strictly increasing for all $R_D < \frac{2}{q_4} R_f$ and strictly decreasing otherwise. If deposit supply is constrained by $W_H$, the critical rate $\overline{R}_D(\ell; j)$ where $D^u(\ell, R_D; 4)$ just attains $W_H$ is*

$$\overline{R}_D(4) \;=\; \frac{q_4 \;-\; \sqrt{q_4^2 - 4\gamma q_4(1-q_4)W_H R_f}}{2\gamma q_4(1-q_4)W_H}$$

*with*

$$\lim_{W_H \downarrow 0} \overline{R}_D(4) \;=\; \underline{R}_D(4) \ .$$

### 3.2.4 The Bank's Expected Pay-off

The bank is a risk-neutral intermediary between the firms and the household and has limited liability.[9] It exerts monopoly power on both markets: the one for loans and the one for deposits. As a result, the bank charges

$$R_i^* = \alpha_i$$

as gross interest rate on loans while being able to scale the loan volumes arbitrarily and it takes the household's (constrained) deposit-supply function as given when it maximizes its expected final wealth. Deposit supply differs according to the four Cases defined in (3.2.16) to (3.2.19) as the bank's probabilities of solvency differ as well. Given a Case $j$, the bank does creditably commit to this Case. Thus the bank does not arbitrarily combine deposit supply schedules with its own repayment schedules.

According to

**Case 1 (neither loan suffices to fully redeem deposits)**,

defined by (3.2.16), the bank remains only solvent if both loans are fully repaid which

[9] The following notes and formulæ in this section can be found on pp. 144f in Bühler/Koziol/Sygusch (2008).

happens with probability $q$. If only one single firm succeeds, the bank must cede the associated loan redemption fully to the depositor. With probability $1 - p_1 - p_2 + q$, there is no payment flow. Thus, using the loan-allocation rate defined by (3.2.14), the bank's objective function takes the following form under Case 1:

$$\begin{aligned} \mathrm{E}\left[\tilde{W}_B(\ell, R_D; 1)\right] &= q \cdot [\ \alpha_1 \ell + \alpha_2(1-\ell) - R_D\ ] \cdot D^s(\ell, R_D; 1) \\ &\quad + q \cdot [\ \alpha_1 \ell + \alpha_2(1-\ell)\ ] \cdot W_B\ . \end{aligned} \tag{3.2.58}$$

According to

**Case 2 (only Loan 2 suffices to fully redeem deposits)**,

defined by (3.2.17), the bank keeps solvent if both loans are fully redeemed or if the loan $L_2$ is fully paid back:

$$\begin{aligned} \mathrm{E}\left[\tilde{W}_B(\ell, R_D; 2)\right] &= [\ q\alpha_1 \ell + p_2\alpha_2(1-\ell) - p_2 R_D\ ] \cdot D^s(\ell, R_D; 2) \\ &\quad + [\ q\alpha_1 \ell + p_2\alpha_2(1-\ell)\ ] \cdot W_B\ . \end{aligned} \tag{3.2.59}$$

Symmetrical to Case 2,

**Case 3 (only Loan 1 suffices to fully redeem deposits)**

results in the following objective function:

$$\begin{aligned} \mathrm{E}\left[\tilde{W}_B(\ell, R_D; 3)\right] &= [\ p_1\alpha_1 \ell + q\alpha_2(1-\ell) - p_1 R_D\ ] \cdot D^s(\ell, R_D; 3) \\ &\quad + [\ p_1\alpha_1 \ell + q\alpha_2(1-\ell)\ ] \cdot W_B\ . \end{aligned} \tag{3.2.60}$$

Finally, the bank keeps solvent as long as at least a single loan is fully paid back if it lends according to

**Case 4 (either loan suffices to fully redeem deposits):**

$$\begin{aligned} \mathrm{E}\left[\tilde{W}_B(\ell, R_D; 4)\right] &= [\ p_1\alpha_1 \ell + p_2\alpha_2(1-\ell) - (p_1 + p_2 - q) R_D\ ] \cdot D^s(R_D; 4) \\ &\quad + [\ p_1\alpha_1 \ell + p_2\alpha_2(1-\ell)\ ] \cdot W_B\ . \end{aligned} \tag{3.2.61}$$

Case 4 is the only case which results in an objective function that is linear in the loan-allocation rate $\ell$. Thus, if Case 4 is feasible, the bank always chooses an allocation rate at the corner of the feasibility set $C_4$ given by (3.2.19). This idea can be further developed both with and without regulation.

Results 9 and 11 will characterize Case 4 as laissez-faire equilibrium. Results 19 and 21 refer to Case 4 as equilibrium under regulation by fixed risk weights, and finally Results 24 and 28 given a VaR approach. Case 4 can arise as equilibrium both under equal and under different projects the two firms undertake.

## 3.3 Equilibrium and Sensitivities

### 3.3.1 The Equilibrium without Regulation

#### 3.3.1.1 General Characterization

Given each Case $j$, the bank maximizes its expected final wealth, $\mathrm{E}[\tilde{W}_B(\ell, R_D; j)]$, with respect to the loan-allocation rate $\ell$ and the interest rate on deposits $R_D$.[10] The tuple $(\ell^*(j), R_D^*(j))$ denotes the allocation rate and the interest rate that maximize $\mathrm{E}\left[\tilde{W}_B(\ell, R_D; j)\right]$ under Case $j$. The triple $(\ell^*, R_D^*; j^*)$ denotes the loan-allocation rate, the interest rate on deposits, and the associated Case that maximize $\mathrm{E}\left[\tilde{W}_B(\ell, R_D; j)\right]$ considering all the four Cases, *i.e.* $(\ell^*, R_D^*; j^*)$ is the solution to the following maximization problem:[11]

$$\begin{array}{rl} \max\limits_{\ell,\ R_D} & \mathrm{E}\left[\tilde{W}_B(\ell, R_D; j)\right] \\ \text{s.t.} & \ell \in [0,1] \\ & \exists j:\ (D^s(\ell, R_D; j),\ \ell,\ R_D)\ \in\ C_j\ j = 1, \ldots, 4\ . \end{array} \tag{3.3.1}$$

We will refer to $(\ell^*, R_D^*; j^*)$ as the equilibrium without regulation. Equilibrium results are used later on as a benchmark to judge potential pro-cyclical impacts resulting from regulation.

**Result 5.** *An equilibrium $(\ell^*, R_D^*; j^*)$ always exists. The optimal interest rate on deposits given Case $j$, $R_D^*(\ell; j)$, satisfies $R_D^*(\ell; j) \in \left(\underline{R}_D(\ell; j), \overline{R}_D(\ell; j)\right]$, and is unique given a fixed loan-allocation rate $\ell$. Hence the optimal deposit volume $D^* \equiv D^s(\ell^*, R_D^*; j^*)$ is always strictly positive.*

The existence of an optimizing triple $(\ell^*, R_D^*; j^*)$ holds for two reasons: the sets $C_j$ defining the Cases $j = 1, \ldots, 4$ are compact sets by definition. As a consequence, we obtain compact sets with respect to the loan-allocation rate $\ell$ given fixed deposit

[10] Observe that the optimal interest rates on loans are simply $R_i = \alpha_i$, $i = 1, 2$, independent of the Case $j$ considered, *cf.* (3.2.4).

[11] Bühler/Koziol/Sygusch (2008, p. 145, Maximization Problem (16)).

interest rates $R_D$. Thus, bounds on $\ell$ are established that are tighter than the short-sell restrictions, as outlined for Cases 2 and 3 by (3.2.20) and (3.2.21), respectively. The borders arising out of Cases 1 and 4 cannot be analytically stated. Concerning the deposit rate $R_D$, Result 2 helps by providing definite bounds. Furthermore, we can even show the uniqueness of $R_D^*(\ell; j)$. The proof is provided in Appendix A.2.1.1.[12]

Optimal solutions $(\ell^*, R_D^*; j^*)$ typically consist of solutions in the interior concerning the loan-allocation rate, $\ell$, as any optimum is a compromise on risky lending between risk-neutral bank owners and risk-averse depositors. If the risk-neutral bank granted only its own capital $W_B$ as loans, it would always choose to grant a loan only to the company yielding the highest expected return (by (3.2.5), the second firm). The same holds true if the bank issues risk-free deposits where risks are assumed by an exogenous agency (deposit insurer) that charges the bank a fee which is independent of the risks taken by the bank. As deposits are uninsured and as depositors are risk-averse, there is an incentive, even for the bank acting as a monopolist on the market for deposits, to diversify risks.

By the parameters considered in Chapter 4, it will turn out that the unregulated bank mainly grants loans such that Case 1 prevails. Unregulated equilibria according to Cases 3 and 4 do not arise in the examples presented in this section. Thus, we find the typical risk-reducing effect through uninsured debt as long as both parties are fully informed about each other. So, in our framework, uninsured deposits can fully unfold their disciplinary effects. This does not imply, however, that there is no scope for choosing extreme loan-allocation rates, such as $\ell^* = 0$ or $\ell^* = 1$ given some parameter values. Figure 3.2 shows feasible $\ell$-$R_D$ combinations under the base case scenario.

If $\ell^* = 0$ holds, the equilibrium is characterized by Case 2. Likewise, $\ell^* = 1$ is always associated with Case 3.

In Case 1, loan-allocation rates equal to $\ell^* = 0$ or $\ell^* = 1$ would result in $R_D^* > \alpha_2$, and $R_D^* > \alpha_1$, respectively, implying (due to Result 2) a negative contribution of deposit-taking to the final wealth expected by the bank. This contradicts optimality, as the bank can improve by changing to Case 2, or 3, respectively. In Case 4, choices $\ell^* = 0$ and $\ell^* = 1$ contradict $D^* > 0$, which also prevails as a maximum under this Case. Furthermore, choosing $\ell^* = 0$ does not generally dominate $\ell^* = 1$ under Case 3 even if $p_2\alpha_2 > p_1\alpha_1$ is assumed, because the bank promises different interest rates

[12]Existence and bounds on $R_D^*$ have been also formulated by Bühler/Koziol/Sygusch (2008, p. 145), without providing the proof, however.

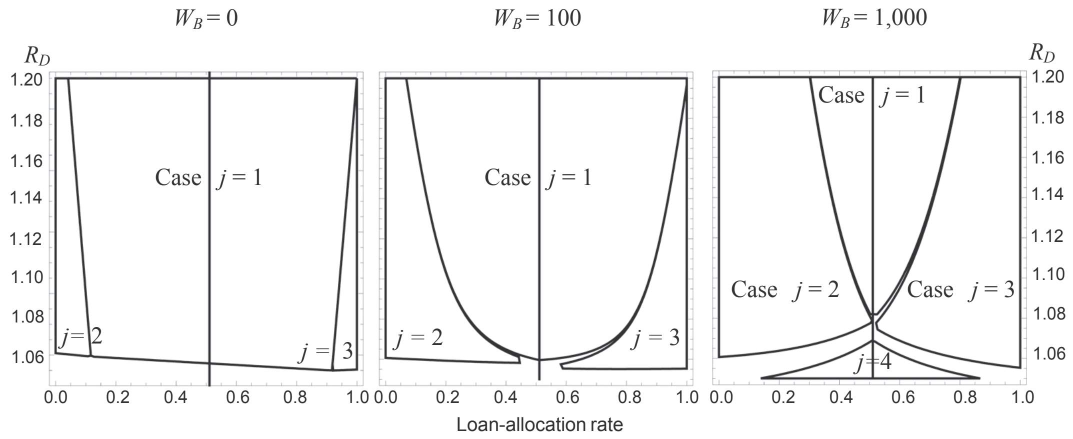


Figure 3.2: **The Case constraints given the household's deposit-supply function**

*This figure illustrates the feasible $\ell$-$R_D$ combinations given the household's deposit supply, $D^s(\ell, R_D, j)$ for three different levels of the bank's initial equity $W_B$. Feasible $\ell$-$R_D$ tuples are clustered by their associated Cases $j$, $j = 1, 2, 3, 4$. The vertical line represents $\ell \equiv \frac{\alpha_2}{\alpha_1+\alpha_2}$. Parameter values are given according to Table 4.1. Cf. Figure 3.1 where the deposit volume is exogenously fixed to $D = 1157.59$.*

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$R_D^*$ to the depositor under both Cases. The solution $\ell^* = 0$ is discussed in depth in the next section.

Moreover, Result 5 shows that it is always worthwhile for the bank to grant loans, even if it has no initial equity. So the bank benefits from its role as a financial intermediary that is assumed to have exclusive access to investment in the firms' projects.

In the next sections, we will analyze some specific optimal choices of the bank which can be potential equilibria. In general, however, there is no explicit characterization of these choices possible for Cases 1 to 3. For Case 1, we only consider the case of two equally distributed projects, for Case 2 the corner solution in $\ell^* = 0$.

#### 3.3.1.2 Characterization if Case 1 Prevails

Contrary to Assumption (3.2.5), let us consider equal success probabilities $p_1 = p_2 = p$ and equal productivity rates $\alpha_1 = \alpha_2 = \alpha$. Hence, both loans have equal expected return and variance. The optimum given Case 1 can be stated analytically. However, the comparative static properties under Case 1 remain opaque. The following result summarizes some properties of this potential equilibrium:

**Result 6.** *Let $p_1 = p_2 = p$ and $\alpha_1 = \alpha_2 = \alpha$. Suppose Case 1 is optimal. Then the* *bank allocates its funds equally as loans to both firms,*

$$\ell^* = \frac{1}{2} \ ,$$

*and promises*

$$R_D^* \ = \ \frac{q\alpha\cdot\big[2(1-q)R_f \ - \ (3-2p)(p-q)\alpha \ + \ (1-p)(p-q)(1-2p+2q)\alpha^2\gamma W_B\big] \ + \ \alpha\cdot\sqrt{Q_1^*(p,q,\alpha)}}{2\cdot q\cdot\big\{(1-q)R_f \ + \ [2q-p(1+q)]\alpha \ + \ \frac{1}{2}(p-q)(1-2p+3q-2pq)\alpha^2\gamma W_B\big\}}$$

*as the gross interest rate to the household where*

$$\begin{aligned} Q_1^*(p,q,\alpha) \ &= \ q\cdot[p(1-p)+q-p^2]\cdot \\ &\cdot\big\{2(1-q)R_f^2 - 4(p-q)\alpha R_f + (2p-q)(p-q)\alpha^2 + \ldots \\ &+ \tfrac{1}{2}(p-q)(1-2p+q)\alpha^2\gamma W_B\left[4\left(R_f-(p-q)\alpha\right)+(1-2p+2q)(p-q)\alpha^2\gamma W_B\right]\big\} \,. \end{aligned}$$

*This maximum choice is unique given Case* 1.

Furthermore, one can specify the optimal deposit volume and the bank's optimal expected wealth but their analytical representations are not very meaningful. By the equality of both loan distributions there is no disaccord among the risk-neutral bank and the risk-averse depositor about the optimal loan-allocation rate $\ell^*$ in Case 1. But the interest rate $R_D^*$, that the bank promises to its depositor, is still lower than that preferred by the household as long as $R_D^* < \overline{R}_D^u(\ell;1)$ holds, *cf.* Result 5. The derivation of this result can be found in Appendix A.2.1.2.

The sign of the derivative of $R_D^*$ with respect to $W_B$ is ambiguous and therefore it is unclear whether the marginal change of the total loan volume towards a marginal change in the bank capital is higher or lower than one and even whether this marginal rate of change is positive or not. But there is a sufficient condition on $\alpha$ such that the slope of $D^*$ is strictly positive with respect to $W_B$:

**Result 7.** *Let $p_1 = p_2 = p$ and $\alpha_1 = \alpha_2 = \alpha$. Assume Case* 1 *is optimal and $\alpha \leq \frac{4}{3}$. Then,*

$$\frac{dD^*}{dW_B} \ > \ 0$$

*holds.*

The additional restriction on $\alpha$, given by $\alpha \leq \frac{4}{3}$, instead of the general Assumption (3.2.7), is only sufficient to obtain the result. Hence, there are optimal choices of the bank under Case 1 with $\alpha > \frac{4}{3}$ and $\frac{dD^*}{dW_B} > 0$. The proof to this result is outlined in Appendix A.2.1.3.

#### 3.3.1.3 Characterization if Case 2 with $\ell^* = 0$ Prevails

In this section, we characterize Case 2 when the optimal loan-allocation rate $\ell^*$ is chosen to be zero. The following result outlines the sufficient conditions.

**Result 8.** *Suppose that the household is not constrained by its initial wealth, $W_H$, and that, given Case 2, the expected final wealth of the bank, $\mathrm{E}[\tilde{W}_B(\ell, R_D; 2)]$, has a unique maximum in $\ell$ on $\mathbb{R}$. Furthermore, assume*

$$(2 - p_2)(q\alpha_1 - p_2\alpha_2)R_f + p_2(p_2\alpha_2 - p_1\alpha_1)R_f + p_2^2(p_1 - q)\alpha_1\alpha_2 \;\leq\; 0 \;, \qquad (3.3.2)$$

*and*

$$\begin{aligned}&(p_2^2\alpha_2^2 - R_f^2) \cdot [(q - p_1p_2)\alpha_1 R_f - (1 - p_2)p_2\alpha_2 R_f + (p_1 - q)\alpha_1 p_2\alpha_2] \;- \\ &\quad - \;2p_2(1 - p_2)\alpha_2^2\gamma W_B R_f \cdot \\ &\qquad \cdot \{(2 - p_2)(p_2\alpha_2 - q\alpha_1)R_f - p_2(p_2\alpha_2 - p_1\alpha_1)R_f - p_2^2(p_1 - q)\alpha_1\alpha_2\} \;\leq\; 0\end{aligned} \qquad (3.3.3)$$

*hold, then*

$$\ell^* \;=\; 0 \quad \textit{and} \quad R_D^* \;=\; \frac{2\alpha_2 R_f}{p_2\alpha_2 + R_f} \;>\; R_f$$

*are optimal given Case 2. The sensitivities of the total loan volume, $L^*$, and the deposit volume, $D^*$, to shocks in bank equity $W_B$ simplify to*

$$\frac{dL^*}{dW_B} \;=\; 1 \quad \textit{and} \quad \frac{dD^*}{dW_B} \;=\; 0 \;. \qquad (3.3.4)$$

Condition (3.3.3) guarantees that $\ell^* = 0$ is the optimal choice given that there is a unique maximum of $\mathrm{E}[\tilde{W}_B(\ell, R_D; 2)]$ for fixed $R_D$. Condition (3.3.2) determines exclusively the sign of the second derivative $\frac{\partial^2 \mathrm{E}[\cdot]}{\partial W_B \partial \ell}$ such that the optimal loan-allocation rate $\ell$ shrinks with increasing bank capital if short-sells of bank assets were allowed. Thus, the constrained loan-allocation rate, *i.e.* the loan-allocation rate $\ell$ restricted to the unit interval, remains zero for increasing $W_B$.

As $\ell^*$ is kept fixed at zero, the optimal deposit interest rate $R_D^*$ is also constant with respect to $W_B$, such that only the direct effect of a marginal change in $W_B$ affects the total loan volume $L^*$. Note that in Equation (3.3.4) the direct partial derivative of deposits, $D^*$, with respect to bank capital, $W_B$, vanishes according to (3.2.48). Appendix A.2.1.4 provides the derivation of these results.

The importance of the Conditions (3.3.2) and (3.3.3) can also be grasped from a rather economic perspective: the left-hand side of each of the conditions becomes smaller, as the gap between both loans' marginal expected gross returns grows. This gap is measured in terms of $p_2\alpha_2 - p_1\alpha_1$ and $p_2\alpha_2 - q\alpha_1$, respectively. In particular, the negative of the first two terms in (3.3.2)

$$\underbrace{(2-p_2)}_{>p_2}\underbrace{(p_2\alpha_2 - q\alpha_1)}_{>(p_2\alpha_2 - p_1\alpha_1)} R_f - p_2(p_2\alpha_2 - p_1\alpha_1)R_f$$

is positive due to (3.2.5). Thus, both the Conditions (3.3.2) and (3.3.3) are negatively affected.

Condition (3.3.2) excludes

$$\frac{\partial^2 \mathrm{E}[\tilde{W}_B(\ell, R_D; 2)]}{\partial\ell\,\partial W_B} \;>\; 0\;.$$

But as soon as the opposite to (3.3.2) holds true and if furthermore[13]

$$\frac{\partial^2 \mathrm{E}[\tilde{W}_B(\ell, R_D; 2)]}{\partial\ell\,\partial R_D} \;<\; 0$$

is assumed, the sensitivities of the loan-allocation rate $\ell^*$ and the deposit interest rate $R_D^*$ show opposite reactions to shocks in $W_B$,

$$\frac{d\ell^*}{dW_B} \;>\; 0 \quad \text{and} \quad \frac{dR_D^*}{dW_B} \;<\; 0\;.$$

Consequently, the effect on the total loan volume remains ambiguous as the first-order conditions imply that the optimal deposit supply is increasing in both the loan-allocation rate as well as in the deposit interest rate.

**Remark 2.** *Given that $\ell = 0$ remains optimal for the constraint maximization problem for changes in the following parameters, the optimal total loan volume reacts as follows:*

$$\frac{dL^*}{dp_2} \;=\; \frac{p_2(p_2\alpha_2^2 - R_f^2) + (1-p_2)R_f^2}{4p_2^2(1-p_2)^2\alpha_2^2 R_f\gamma} \;>\; 0 \tag{3.3.5}$$

$$\frac{dL^*}{d\alpha_2} \;=\; \frac{R_f}{2p_2(1-p_2)\alpha_2^3\gamma} \;>\; 0 \tag{3.3.6}$$

$$\frac{dL^*}{d\gamma} \;=\; -\,\frac{p_2^2\alpha_2^2 - R_f^2}{4p_2(1-p_2)\alpha_2^2 R_f\gamma} \;<\; 0 \tag{3.3.7}$$

$$\frac{dL^*}{dR_f} \;=\; -\,\frac{p_2^2\alpha_2^2 + R_f^2}{4p_2(1-p_2)\alpha_2^2 R_f^2\gamma} \;<\; 0 \tag{3.3.8}$$

[13]The opposite does not occur if parameter values are chosen that are close to those given in Table 4.1. In particular, success probabilities are supposed to be close to one.

Table 3.1: **Parameter values satisfying Conditions** (3.3.2) **and** (3.3.3)

*The upper panel reports parameter values that fulfill Conditions* (3.3.2) *and* (3.3.3) *stated in Result 8. The lower panel reports the bank's optimal choices in each Case* $j = 1, \ldots, 4$*. The equilibrium is characterized by Case* 2*.*

- *Panel A: Parametrization*

| $p_1$ | $p_2$ | $q$ | $\alpha_1$ | $\alpha_2$ | $W_B$ | $\gamma$ | $W_H$ | $R_f$ |
|---|---|---|---|---|---|---|---|---|
| 0.99 | 0.98 | 0.975 | 1.1 | 1.2 | 100 | 0.05 | 3000 | 1.05 |

- *Panel B: Optimal Choices*

| $\ell^*(j)$ | $R_D^*(j)$ | $j$ | $D^*(\cdot,\cdot;j)$ | $\mathrm{E}[\tilde{W}_B(\cdot,\cdot;j)]$ | $U(\tilde{W}_H)$ | Eq./Res. |
|---|---|---|---|---|---|---|
| 0.513879 | 1.10128 | 1 | 112.628 | 117.187 | 3153.18 | n/a |
| 0 | 1.13208 | 2 | 47.3214 | 120.75 | 3151.41 | Res. 8 |
| 0.521739 | 1.09082 | 3 | 111.028 | 118.084 | 3152.1 | n/a |
| 0.39307 | 1.07471 | 4 | 67.3131 | 119.058 | 3150.65 | (3.3.11) |

These formulæ can be obtained by calculation given the (assumed) optimal deposit volume $D^u(0, R_D^*, 2)$. The signs are intuitive: either if the risk reduces by an increasing success probability $p_2$ or if the expected gross return on the second firm loan, $\alpha_2$, increases, the total loan volume increases too. The total loan volume decreases if the household's risk-aversion $\gamma$ increases. The same effect holds true if the return on the risk-free sovereign bond, $R_f$, rises.

Table 3.1 shows a parameter set that fulfills Conditions (3.3.2) and (3.3.3). Thus, the tuple $(\ell^*, R_D^*) = (0, \frac{2\alpha_2}{p_2\alpha_2 + R_f})$ is the bank's optimal choice given Case 2. What is more, this bank's choice under Case 2 dominates the choices according to all other Cases. An equilibrium such as this may also prevail if both firms' projects exhibit equal returns and risks as the example provided in Table 3.3 illustrates.

It is worth noting that the bank chooses in each Case the lowest possible loan-allocation rate, *i.e.* it devotes the lowest possible amount of funds as loan to Firm 1. Specifically, $\ell^*(3) = \frac{\alpha_2}{\alpha_1 + \alpha_2}$ holds in Case 3, *cf.* (3.2.21). The solution given Case 4 can also be analytically represented and is given by (3.3.11) in the next result, Result 9.

#### 3.3.1.4 Characterization if Case 4 Prevails

If Case 4 is optimal for the bank, the equilibrium can be stated analytically. The following result describes this equilibrium:

**Result 9.** *Suppose that the household is not constrained by its initial wealth, $W_H$. Then the maximum $(\ell^*, R_D^*, 4)$ given Case 4 is unique and $\alpha_1 L_1^* = D^* R_D^*$ holds. Furthermore, if*

$$0 \;<\; W_B \;\leq\; \frac{\left[p_2^2\alpha_2(q_4\alpha_1-R_f)-(p_2^2-q^2)\alpha_1R_f\right]\alpha_1R_f+p_2^2\alpha_2(q_4\alpha_1-R_f)\left[(\alpha_1+\alpha_2)R_f-q_4\alpha_1\alpha_2\right]}{4p_2^2(1-q_4)q_4R_f\alpha_1^2\alpha_2^2\gamma} \tag{3.3.9}$$

*holds, the optimal loan-allocation rate and the optimal deposit interest rate are given by*

$$\begin{aligned} \ell^* &= \frac{\alpha_2}{\alpha_1+\alpha_2}, \\ R_D^* &= \frac{(\alpha_1+\alpha_2)R_f+q_4\alpha_1\alpha_2+\sqrt{\left[(\alpha_1+\alpha_2)R_f-q_4\alpha_1\alpha_2\right]^2+4q_4(1-q_4)\alpha_1^2\alpha_2^2\gamma W_B R_f}}{2q_4\left[\alpha_1+\alpha_2-(1-q_4)\alpha_1\alpha_2\gamma W_B\right]}, \qquad (3.3.10) \\ &\quad \textit{with} \\ R_D^* &> \frac{1}{q_4}R_f \overset{\text{Remark 1}}{=} \underline{R}_D(4). \end{aligned}$$

*Consequently, the promised loan redemptions are equal, i.e. $\alpha_1 L_1^* = \alpha_2 L_2^*$. If Condition (3.3.9) is violated, $\alpha_1 L_1^* < \alpha_2 L_2^*$ prevails implying*

$$\begin{aligned} \ell^* &= \frac{2p_2\alpha_2R_f\left\{p_2\alpha_2q_4\alpha_1-[p_2\alpha_2+(p_2-q)\alpha_1]R_f\right\}}{(p_2\alpha_2q_4\alpha_1)^2-[p_2\alpha_2+(p_2-q)\alpha_1]^2R_f^2+4q_4(1-q_4)p_2^2\alpha_1^2\alpha_2^2\gamma W_B R_f} \;\in\; \left(0, \frac{\alpha_2}{\alpha_1+\alpha_2}\right), \\ R_D^* &= \frac{2p_2\alpha_1\alpha_2R_f}{p_2\alpha_2q_4\alpha_1+[p_2\alpha_2+(p_2-q)\alpha_1]R_f} > \frac{1}{q_4}R_f \overset{\text{Remark 1}}{=} \underline{R}_D(4), \qquad (3.3.11) \\ D^* &= \frac{\left\{p_2\alpha_2q_4\alpha_1-[p_2\alpha_2+(p_2-q)\alpha_1]R_f\right\}\cdot\left\{p_2\alpha_2q_4\alpha_1+[p_2\alpha_2+(p_2-q)\alpha_1]R_f\right\}}{4(1-q_4)q_4p_2^2\alpha_1^2\alpha_2^2\gamma R_f}. \end{aligned}$$

*For $W_B = 0$, the loan-allocation rate $\ell^*(4)$ is indeterminate. $R_D^*(4) = \frac{1}{q_4}R_f$ and $D^*(4) = 0$ holds. Then Case 4 is strictly dominated by choices according to other Cases and can thus never arise in equilibrium.*

*The solution characterized by (3.3.10) and by (3.3.11) is continuous in $W_B$.*

The relation $\alpha_1 L_1^* = D^* R_D^*$ is a direct consequence of the assumption $p_1\alpha_1 < p_2\alpha_2$ as the bank forgoes more profits for each marginal unit of funds it grants to Firm 1 more than required by the feasibility set $C_4$. Technically, the equality $\alpha_1 L_1^* = D^* R_D^*$ determines the optimal interest rate on deposits the bank chooses. As this choice depends on the loan-allocation rate, either the optimal loan-allocation rate maximizes the bank's expected wealth freely while fulfilling the bounds given by the set $C_4$, *i.e.* $\alpha_2 L_2^* > D^* R_D^*$, or the optimal loan-allocation rate must be chosen such that $\alpha_2 L_2^* = D^* R_D^*$ holds as well. If $W_B$ strictly exceeds the threshold (3.3.9), the solution presented under (3.3.11) arises, in particular, $\alpha_2 L_2^* > D^* R_D^*$ holds. The derivations of the formulæ are outlined in Appendix A.2.1.5.

**The relation between $W_B$ and the occurrence of Case 4 in equilibrium** Note that Case 4 can only be optimal for sufficiently high bank capital, $W_B$, or with sufficiently high productivity parameters $\alpha_i$, $i = 1, 2$: consider $W_B = 0$ and $\alpha_i \leq 2$. Then the definition of Case 4 according to (3.2.19) implies

$$\alpha_1 \ell \cdot D^s(\cdot, 4) \;\geq\; D^s(\cdot, 4) \cdot R_D \quad \text{and} \quad \alpha_2 (1-\ell) \cdot D^s(\cdot, 4) \;\geq\; D^s(\cdot, 4) \cdot R_D \;.$$

For $D^s(\cdot, 4) > 0$, we obtain $R_D \leq \min\{\alpha_1 \ell, \alpha_2(1-\ell)\}$. By $\alpha_i \leq 2$ and $\ell \in [0, 1]$, $\min\{\alpha_1 \ell, \alpha_2(1-\ell)\} \leq 1$ holds. Hence, $R_D \leq 1$, and especially $R_D \leq R_f$. This is a contradiction to $D^s(\cdot, 4) > 0$ as the critical deposit interest rate, at which deposit supply becomes zero, is $\underline{R}_D(4) = \frac{1}{q_4} R_f > R_f$. Thus, only $D^s(\cdot, 4) = 0$ is feasible, implying zero expected wealth of the bank. But the bank can simply improve by choosing a loan-allocation rate compatible with any other Case $j = 1, 2, 3$. Particularly the choice discussed in Result 8 is feasible under $W_B = 0$ and thus is a potential candidate for the Maximization Problem (3.3.1). Alternatively, one can argue by the optimal deposit interest rate $R_D^*(4)$ in (3.3.10) if $W_B$ lies beneath the threshold (3.3.9). At $W_B = 0$, $R_D^*(4)$ becomes $\frac{1}{q_4} R_f \equiv \underline{R}_D(4)$ and we obtain $D^s(\ell, \underline{R}_D(4); 4) = 0$.

Table 3.2 illustrates the relation between the bank's initial capital $W_B$, the different optimal choices within Case 4, and the actual optimal choices by the bank across all Cases, *i.e.* the equilibria. It suggests a positive relation between the bank's initial capital $W_B$ and the occurrence of Case 4 in equilibrium. The intuition for this relation is as follows: the higher $W_B$, the easier for the bank to find loan allocations that fulfill the Case constraints of Case 4. Specifically, for $W_B$ above (3.3.9), the deposit supply and the deposit interest rates are fixed in equilibrium such that allocating increasing funds to Firm 2 with increasing equity is not associated with any additional costs. In particular, the household would ask for a higher risk premium if the bank granted more funds as loan to Firm 2 according to the constraints determining Case 2.

The parameter values considered in Table 3.2 are those of the base case, as given in Table 4.1 except for the values of $W_B$. The threshold for $W_B$ according to (3.3.9)

Table 3.2: **Optimal choices given Case 4 vs. equilibria**

*The upper panel reports the optimal choices given Case 4 for different levels of bank capital $W_B$. The threshold for $W_B$ according to (3.3.9) amounts to 2040.14 The choices given Case 4 are contrasted with the actual optimal choices by the bank, i.e. the equilibria, shown in the lower panel. Except for $W_B$, parameter values are those of the base case, as given in Table 4.1.*

- *Panel A: Case 4*

| $W_B$ | $\ell^*(4)$ | $R_D^*(4)$ | $D^*(\cdot,\cdot;4)$ | $\mathrm{E}[\tilde{W}_B(\cdot,\cdot;4)]$ | $U(\tilde{W}_H)$ | Eq./Res. |
|---|---|---|---|---|---|---|
| 0 | $[0,1]$ | 1.0521 | 0 | 0 | 3150 | Res. 9 |
| 100 | 0.510638 | 1.05434 | 125.717 | 130.826 | 3150.14 | (3.3.10) |
| 1000 | 0.510638 | 1.07437 | 1205.49 | 1278.3 | 3163.39 | (3.3.10) |
| 2040.14 | 0.510638 | 1.09735 | 2348.54 | 2543.68 | 3203.03 | (3.3.10), (3.3.11) |
| 4000 | 0.352999 | 1.09735 | 2348.54 | 4872 | 3203.03 | (3.3.11) |
| 10000 | 0.181481 | 1.09735 | 2348.54 | 12000 | 3203.03 | (3.3.11) |

- *Panel B: Equilibrium*

| $W_B$ | $\ell^*$ | $R_D^*$ | $j^*$ | $D^*$ | $\mathrm{E}[\tilde{W}_B^*]$ | $U(\tilde{W}_H^*)$ | Eq./Res. |
|---|---|---|---|---|---|---|---|
| 0 | 0.585096 | 1.10905 | 1 | 1276.13 | 77.7021 | 3182.87 | n/a |
| 100 | 0.571138 | 1.10802 | 1 | 1303.59 | 197.22 | 3185.06 | n/a |
| 1000 | 0.574687 | 1.08839 | 3 | 1545.95 | 1282.5 | 3180.5 | n/a |
| 2040.14 | 0.510638 | 1.09735 | 4 | 2348.54 | 2543.68 | 3203.03 | (3.3.10), (3.3.11) |
| 4000 | 0.352999 | 1.09735 | 4 | 2348.54 | 4872 | 3203.03 | (3.3.11) |
| 10000 | 0.181481 | 1.09735 | 4 | 2348.54 | 12000 | 3203.03 | (3.3.11) |

amounts to 2040.14.

We note that it is by incidence that Case 4 prevails in equilibrium at this threshold. For example, Case 2 arises in equilibrium in the scenario considered in Table 3.1, although the bank's equity $W_B$ clearly exceeds the threshold (3.3.9) which equals 58.7378 in that parametrization. Instead, increasing $W_B$ rather supports Case 2 as equilibrium, since Condition (3.3.2) guarantees that Condition (3.3.3) to obtain Result 8 is negatively affected by $W_B$.

Though the deposit-supply function $D^s(\ell, R_D; 4)$ is independent of $W_B$ in Case 4 according to Result 1, the household will not be generally indifferent between different levels of $W_B$ if Case 4 occurs in equilibrium: if and only if $W_B$ is below the threshold (3.3.9), the household appreciates increasing $W_B$, *i.e.* the buffer function that $W_B$ fulfills if the bank defaults. This property can be readily obtained from Result 9:

**Result 10.** *Assume Case* 4 *to be optimal. Then the optimal deposit volume strictly increases in the bank's initial equity* $W_B$, *i.e.*

$$\frac{dD^*}{dW_B} > 0$$

*if Condition* (3.3.9) *holds and is independent of* $W_B$ *otherwise.*

The proof is given in Appendix A.2.1.6. This result shows that there can be a positive relation between the bank's initial equity, $W_B$, and the optimal deposit volume in equilibrium even if the out-of-equilibrium deposit supply does not depend on $W_B$.

Considering once again equal success probabilities $p_1 = p_2 = p$ and equal productivity rates $\alpha_1 = \alpha_2 = \alpha$, Case 4 changes its nature: the bank may also choose $\ell = \frac{1}{2} \equiv \frac{\alpha_2}{\alpha_1+\alpha_2}$ as maximum given Case 4:

**Result 11.** *Suppose that the household is not constrained by its initial wealth,* $W_H$, *and that* $p_1 = p_2 = p$ *and* $\alpha_1 = \alpha_2 = \alpha$ *hold. If*

$$0 \ < \ W_B \ \leq \ \frac{(p\alpha - R_f) \cdot [(2p+q)R_f - q_4 p\alpha]}{4p^2\alpha^2(1-q_4)\gamma R_f} \tag{3.3.12}$$

*holds, the maximum* $(\ell^*, R_D^*, 4)$ *given Case* 4 *is unique and characterized by the solution*

$$\begin{aligned} \ell^* &= \tfrac{1}{2} \\ R_D^* &= \tfrac{q_4\alpha + 2R_f + \sqrt{\left[2R_f - q_4\alpha\right]^2 + 4q_4(1-q_4)\alpha^2\gamma W_B R_f}}{2q_4[2-(1-q_4)\alpha\gamma W_B]} \\ & \quad \textit{with} \\ R_D^* &\geq \tfrac{1}{q_4} R_f \overset{Remark\ 1}{=} \underline{R}_D(4) \ , \end{aligned} \tag{3.3.13}$$

*resulting in the following signs for the sensitivities of the deposit interest rate,*

$$\frac{\partial R_D^*}{\partial W_B} > 0, \ \frac{\partial R_D^*}{\partial \gamma} > 0, \ \frac{\partial R_D^*}{\partial R_f} > 0, \ \frac{\partial R_D^*}{\partial q_4} < 0 \ . \tag{3.3.14}$$

*If Condition* (3.3.12) *is violated,* $\alpha L_1^* \neq \alpha L_2^*$ *may prevail, implying*

$$\begin{aligned}
\ell^* &\in \left[ \frac{2pR_f(p\alpha - R_f)}{q_4(p^2\alpha^2 - R_f^2) + 4p^2\alpha^2(1-q_4)\gamma W_B R_f} \; , \; \frac{(p\alpha - R_f)\big[q_4 p\alpha - qR_f\big] + 4p^2\alpha^2(1-q_4)\gamma W_B R_f}{q_4(p^2\alpha^2 - R_f^2) + 4p^2\alpha^2(1-q_4)\gamma W_B R_f} \right] \\
& \quad \textit{with} \\
\ell^* &\in (0,1) \\
R_D^* &= \frac{2p\alpha R_f}{q_4(p\alpha + R_f)} > \frac{1}{q_4} R_f \overset{\textit{Remark } 1}{=} \underline{R}_D(4) \; , \\
D^* &= \frac{q_4(p^2\alpha^2 - R_f^2)}{4p^2\alpha^2(1-q_4)\gamma R_f} \; ,
\end{aligned} \tag{3.3.15}$$

*resulting in the following signs for the sensitivities of the deposit interest rate,*

$$\frac{\partial R_D^*}{\partial W_B} = 0, \; \frac{\partial R_D^*}{\partial \gamma} = 0, \; \frac{\partial R_D^*}{\partial R_f} > 0, \; \frac{\partial R_D^*}{\partial q} > 0, \; \frac{\partial R_D^*}{\partial p} < 0, \; \frac{\partial R_D^*}{\partial \alpha} > 0 \; , \tag{3.3.16}$$

*and in the following signs for the sensitivities of the deposit volume,*

$$\frac{\partial D^*}{\partial W_B} = 0, \; \frac{\partial D^*}{\partial \gamma} < 0, \; \frac{\partial D^*}{\partial q} < 0, \; \frac{\partial D^*}{\partial p} > 0, \; \frac{\partial D^*}{\partial \alpha} > 0 \; , \tag{3.3.17}$$

*For* $W_B = 0$*, Case* 4 *can never arise in equilibrium, cf. Result 9. The solution characterized by* (3.3.13) *and* (3.3.15) *is continuous in* $W_B$*.*

The derivation of the main formulæ are shown in Appendix A.2.1.7.

Unlike in the case of two different firms' projects with respect to their risk-return characteristics, two distinct, but otherwise equal projects let the bank either choose the tuple $(\ell, R_D)$ such that both Case constraints are binding or such that none of the constraints are binding. Thus, if none of the Case constraints are binding, the bank can offer the interest rate on deposits that maximizes its expected final wealth as in the unconstrained problem. To make this choice feasible, the bank is required to have a sufficiently high amount of initial equity $W_B$, characterized by (3.3.12).[14]

Since the return and the risks of the deposits are not affected as long as the bank's choice does not infringe the two Case constraints, the bank can freely allocate its funds to both firms within Case 4, as given by (3.3.15).

This rationale, however, does not apply under Case 4 as soon as the firms' projects are different. Then the risk-neutral bank always grants as much funds as possible as

[14]The positive sign of the threshold (3.3.12) can be easily seen by considering the following sequence of lower bounds, $(2p+q)R_f - q_4 p\alpha \leq (1+q)R_f - q_4 p\alpha = (1+q)R_f - (2p-q)p\alpha > (1+q)R_f - p\alpha > 1 + q - 2p > 0$, where Properties (3.2.5), (3.2.41), (3.2.8), and (3.2.6) as well as (3.2.7) are sequentially applied.

loan to Firm 2 if the expected return on the second firm's loan exceeds that on the first firm's. Hence, the bank's choice is always effectively restricted by the constraint $\alpha_1 \ell \left[D^u(R_D;4) + W_B\right] \geq D^u(R_D;4)R_D$ irrespective of its initial endowment with $W_B$.

This structural difference can also be read off by the formulæ in Result 11 compared to those in Result 9 as follows: all formulæ shown by Result 11 can be directly derived from those in Result 9 except for the upper bound on the optimal loan-allocation rates in (3.3.15).

If the bank is constrained by its initial wealth $W_B$ according to the threshold (3.3.12), both Case constraints are effectively binding, as it is the case under the general parametrization analyzed in Result 9. Then the shadow costs of each binding constraint reflects the gain of financial intermediation. That is, the shadow costs are strictly positive if and only if

$$\frac{p\alpha - q_4 R_D^*}{q_4 R_D^* - R_f} \overset{(3.2.41)}{\equiv} \frac{p\alpha - (2p-q)R_D^*}{(2p-q)R_D^* - R_f} > \frac{p\alpha}{R_f} \tag{3.3.18}$$

holds. The left-hand side of the inequality shows the expected marginal gross profit on deposits for the bank over the spread between the expected equilibrium return on deposits $q_4 R_D^*$ and the risk-free interest rate $R_f$, which is the yield of the household's alternative investment. The right-hand side shows the bank's expected gross return on its loan portfolio over the risk-free rate. Thus, the ratio of (expected) gross returns on the inequality's right-hand side represents the ratio of (expected) gross returns in autarky whereas the ratio on the inequality's left-hand side embodies the ratio of expected excess returns if the bank and the household bargain for deposits under the conditions of Case 4.

The sensitivity of the optimal deposit interest rate $R_D^S$ according to (3.3.13) with respect to $q_4$ results in

$$\frac{\partial R_D^*}{\partial p} < 0, \quad \text{and} \quad \frac{\partial R_D^*}{\partial q} > 0,$$

as $q_4 = 2p - q$. These two sensitivities can be explained as follows: the deposit interest rate strictly decreases in $p$ as a higher success probability $p$ means reduced risks, *i.e.* a higher probability of full redemption of the deposits. In contrast, increasing $q$ is associated with an increasing correlation among both projects, *cf.* (3.2.12). As under Case 4 deposits can be fully paid back as long as a single loan is fully redeemed, a growing correlation among both projects endangers full deposit redemption. However, the total effect of $q_4$ on the total loan/deposit volume is ambiguous as the direct impact is positive if $R_D > \underline{R}_D(4)$,

$$\frac{\partial D^s(R_D;4)}{\partial q_4} \; > \; 0 \; ,$$

thus (partially) off-setting the indirect effect via $R_D^*$.

In contrast, if the Equilibrium (3.3.15) prevails, the total effect on the total loan/deposit volume can be directly calculated: more risk in terms of $q$ let the optimal deposit volume shrink. Furthermore, with a higher success probability $p$, the bank can collect more funds from the risk-averse household. Calculating the effect of $\alpha$ on $D^*$ yields

$$\frac{\partial D^*}{\partial \alpha} \; = \; \frac{q_4 R_f}{2p^2(1-q_4)\alpha^3\gamma} \; > \; 0 \; .$$

Finally, the sensitivity with respect to $p$ is given by

$$\frac{\partial D^*}{\partial p} \; = \; \frac{p(p^2\alpha^2 - R_f^2) + (2p-q)(1-2p+q)R_f^2}{2p^3\alpha^2(1-q_4)^2\gamma R_f} \; > \; 0 \; .$$

Concerning the Solution (3.3.13), the sensitivities with respect to $W_B$, $\gamma$, and $R_f$ can be directly read off the Formula (3.3.13): the numerator strictly increases in these variables, while the denominator strictly decreases in $W_B$ and $\gamma$ and is independent of $R_f$.

The sensitivities of the total loan/deposit volume are given by Result 10. Hence, if and only if Condition (3.3.12) holds, the optimal deposit volume strictly increases in $W_B$, otherwise it remains independent of $W_B$. Thus, the sensitivity of the total loan volume under Case 4 can be summarized by

$$\frac{\Delta L(\ell^*, R_D^*, 4)}{\Delta W_B} \; \geq \; 1 \tag{3.3.19}$$

where the threshold given by (3.3.9) and (3.3.12), respectively, induces a kink at which the slope of $L(\ell^*, R_D^*, 4)$ migrates from numbers strictly greater than one to one.

#### 3.3.1.5 Final Remarks

It can be shown that a positive, partial cushion effect is present in equilibrium independent of the Case that may prevail. That is, a marginal increase translates in equilibrium into a marginal, partial increase of the deposit volume:

**Result 12.** *Suppose that the household is not constrained by its initial wealth, $W_H$, and that $p_1 = p_2 = p$ and $\alpha_1 = \alpha_2 = \alpha$ hold. Then a marginal change in bank capital translates positively into a marginal change in deposits, i.e.*

$$\frac{\partial D^s(\ell^*, R_D^*; j^*)}{\partial W_B} \geq 0$$

*where equality holds if and only if either $\ell^* = 0$, or $\ell^* = 1$, or Case 4 with $W_B$ exceeding the thresholds given by (3.3.9) and by (3.3.12), respectively, prevail in equilibrium.*

Note that we have not presumed any specific Case to arise as the optimal one in Result 12. Again, for given deposit interest rates and given deposit volumes, corner solutions with either $\ell^* = 0$ or $\ell^* = 1$ would be optimal. Concerning Case 1, Result 12 is weaker than Result 7 as the former does a statement with respect to the partial effect of the bank's initial equity while the latter does to the total effect. So Result 12 can dispense with any additional assumptions (on $\alpha_i$).

Even if the project returns have equal distributions, one fails to generally characterize the optimal solutions in Case 2 and 3, respectively. Likewise, it remains opaque under which conditions a specific Case turns out to be the optimal one.

Specifically, the equilibrium may not be unique: first, Case 2 and 3 always lead to the same expected final wealths of both the bank and the household so that agents are always indifferent between Case 2 and 3. That is, Cases 2 and 3 become identical except for the (optimal) loan-allocation rate. Table 3.3 provides an example where the optimal choice in Case 2 can be characterized according to Result 8.

Second, multiple loan allocations may even be feasible within Case 4, namely if $W_B$ exceeds the threshold (3.3.12). Third, the bank is indifferent between allocating its funds between all four Cases if the firms' projects are perfectly correlated, *i.e.* if $p = q$ holds.

Finally, we note that a change in the optimal Case $j^*$ may result in a discontinuity in the optimal deposit volume $D^s(\ell^*, R_D^*; j^*)$ because a change in the Case means that the bank picks out another deposit-supply function $D^s(\ell, R_D; j)$ to maximize

Table 3.3: **Example of equal projects in the Bernoulli model w/o regulation**
*The upper panel shows an example set of parameter values where the success probabilities and the productivity parameters of both projects are equal. The lower panel reports the bank's optimal choices in each Case $j = 1, \ldots, 4$. The equilibrium is characterized by Cases 2 and 3, respectively.*

- *Panel A: Parametrization*

| $p$ | $q$ | $\alpha$ | $W_B$ | $\gamma$ | $W_H$ | $R_f$ |
|---|---|---|---|---|---|---|
| 0.92 | 0.9 | 1.15 | 10 | 0.001 | 3000 | 1.05 |

- *Panel B: Optimal Choices*

| $\ell^*(j)$ | $R_D^*(j)$ | $j$ | $D^*(\cdot,\cdot;j)$ | $\mathrm{E}[\tilde{W}_B(\cdot,\cdot;j)]$ | $U(\tilde{W}_H)$ | Eq./Res. |
|---|---|---|---|---|---|---|
| $\frac{1}{2}$ | 1.14547 | (Case) 1 | 48.4323 | 10.5472 | 3150.33 | Res. 6 |
| 0 | 1.14564 | (Case) 2 | 41.2513 | 10.7456 | 3150.08 | Res. 8 |
| 1 | 1.14564 | (Case) 3 | 41.2513 | 10.7456 | 3150.08 | n/a |
| $\frac{1}{2}$ | 1.11782 | (Case) 4 | 10.5929 | 10.6568 | 3150.00 | (3.3.13) |

its expected wealth associated with that Case. But, choosing another Case with its associated deposit supply function formally corresponds to picking out another local maximum, *cf.* (3.2.46). Thus, even marginal changes in the optimal loan-allocation rate $\ell^*$ and in the deposit interest rate $R_D^*$ may result in a break in the optimal deposit volume. As those marginal changes arise due to variations in exogenous parameters, a shock, as discussed in Sections 2.1 and 2.2, may result in jumps in the total loan/deposit volume. Examples are provided by Figures 4.1, 4.11, 4.19, and 4.20.

### 3.3.2 The Equilibrium with Regulation by Fixed Risk Weights

#### 3.3.2.1 General Characterization

In this paragraph, we analyze the bank's choice under regulation schemes where risk weights are assigned to each asset the bank holds on its banking book. This approach has already been put forward under the Basel I Accord and has been refined under the Basel II Accord, known as the Standardized Approach. The risk weights are supposed to account for the credit risk each asset bears. In our model,

Table 3.4: **Risk weights for corporates in the Standardized Approach**

*This table shows the risk weights for claims on corporates that have been set forth by BCBS (2004, Art. 66). The symbol $c_S$ represents the credit grade of the country of incorporation.*

| Credit grade | AAA to AA- | A+ to A- | BBB+ to BB- | lower than BB- | Unrated |
|---|---|---|---|---|---|
| Risk weight | 0.2 | 0.5 | 1 | 1.5 | $\max\{1, c_S\}$ |

we consider the loans as claims on corporates in the sense of Basel II. Thus we use the risk weights $c_i$ proposed in Article 66 of the Basel II Accord (BCBS 2004). They are presented in Table 3.4 where $c_S$ represents the risk weight of the country of incorporation.

We do not consider any other risks. According to Article 40, banks are required to maintain a capital ratio that must not fall below

$$c \;=\; 8\% \tag{3.3.20}$$

where that ratio is defined as regulatory capital, here simply $W_B$, over risk weighted assets, here given by $c_1 L_1 + c_2 L_2$, that is the following requirement

$$c \cdot (c_1 \cdot L_1 + c_2 \cdot L_2) \;\leq\; W_B$$

must be met. The regulatory parameter $c$ is also known as the Cooke ratio. This regulatory constraint can be re-written as a constraint on the total deposit volume:

$$\begin{aligned} D^s(\ell, R_D; j) &\leq k(\ell) \cdot W_B \;, \\ \text{where} \quad k(\ell) &= \frac{1 - c \cdot [c_1 \ell + c_2(1-\ell)]}{c \cdot [c_1 \ell + c_2(1-\ell)]} \;, \\ \text{with} \quad k(\ell) &\in [\frac{22}{3}, 61.5] \;. \end{aligned} \tag{3.3.21}$$

Due to (3.2.5), Loan 2 has a higher probability of default and a higher volatility on returns than Loan 1. Thus,

$$c_1 \;\leq\; c_2 \tag{3.3.22}$$

if we assume credit ratings decrease in default probabilities. If both loans are equally weighted, the choice of the loan-allocation rate $\ell$ does not affect the allowed maximum volume of deposits and loans. If both loans are weighted differently, $c_1 < c_2$, a shift in the loan allocation in favor of the less risky loan, namely Loan 1, causes the deposit-to-equity-ratio $k(\ell)$ to increase by

$$\begin{aligned} k'(\ell) &= \frac{c_2 - c_1}{c \cdot [c_1 \ell + c_2(1-\ell)]^2} \\ \text{with } k'(\ell) &\in (\frac{5}{3}, 406.25] , \quad \text{if } c_2 > c_1 , \end{aligned} \tag{3.3.23}$$

having presumed the risk weights to take values according to Table 3.4. The higher the loan-allocation rate $\ell$, the higher is the effect, *i.e.* the marginal benefit from increasing the loan-allocation rate $\ell$ increases:

$$k''(\ell) = \frac{2(c_2 - c_1)^2}{c\,[c_1 \ell + c_2(1-\ell)]^3} > 0 , \quad \text{if } c_1 \neq c_2 . \tag{3.3.24}$$

Thus, the bank exhibits increasing returns to scale in terms of a higher business volume in exchange for choosing less risky portfolio compositions. The costs are a loss of expected return on its portfolio.

The bank's decision problem under regulation by (exogenous) risk weighting of assets can be formulated as follows (Bühler/Koziol/Sygusch, 2008, p. 147, Maximization Problem (19)):

$$\begin{aligned} \max_{\ell,\ R_D} \quad & \mathrm{E}\left[\tilde{W}_B(\ell, R_D; j)\right] \\ \text{s.t.} \quad & \ell \in [0,1] \\ & \exists j : \ (D^s(\ell, R_D; j),\ \ell,\ R_D) \in C_j \\ & D^s(\ell, R_D; j) \leq k(\ell) \cdot W_B . \end{aligned} \tag{3.3.25}$$

We will refer to $(\ell^S, R_D^S; j^S)$ as the equilibrium, or the bank's optimal choice under the Standardized Approach. Regulation is said to be binding if the regulatory constraint holds with equality, *i.e.* $D^s(\ell, R_D; j) = k(\ell) \cdot W_B$.

**Result 13.** *An equilibrium $(\ell^S, R_D^S; j^S)$ always exists.*[15]

Clearly, the existence result as shown by Result 5 carries over to the case of regulation: the loan-allocation rate $\ell$ is restricted to $[0,1]$, the maximum interest rate on deposits $R_D$ is bounded from above by Result 2, and the definition of each Case $j$ is based on a compact set $C_j$ in $\ell$-$R_D$ space given a specified deposit-supply function. Additionally, the regulatory constraint imposes an additional definite bound on the set of feasible $\ell$-$R_D$ combinations.

[15] Bühler/Koziol/Sygusch (2008, p. 147).

If the household is not constrained by its initial wealth $W_H$ and if a local maximum $(\ell^S, R_D^S; j^S)$ to Problem (13) exists, the maximum $(\ell^S, R_D^S; j^S)$ satisfies the Kuhn-Tucker conditions and a multiplier $\lambda \geq 0$ exists such that[16]

$$\begin{aligned}
\frac{\partial \mathrm{E}[\tilde{W}_B(\ell^S, R_D^S; j^S)]}{\partial \ell} &= \lambda \cdot \left[\frac{\partial D^u(\ell^S, R_D^S; j^S)}{\partial \ell} - k'(\ell^S) \cdot W_B\right] \\
\frac{\partial \mathrm{E}[\tilde{W}_B(\ell^S, R_D^S; j^S)]}{\partial R_D} &= \lambda \cdot \frac{\partial D^u(\ell^S, R_D^S; j^S)}{\partial R_D} \\
\lambda \cdot \left[D^s(\ell^S, R_D^S; j^S) \ - \ k(\ell^S) \cdot W_B\right] &= 0
\end{aligned}$$

holds. For this exposition, we have assumed that neither $\ell^S$ hits the borders of the unit interval nor $(\ell^S, R_D^S; j^S)$ lies at the boundary of $C_{j^S}$,

If $\lambda^S > 0$ holds, regulation is binding and $\lambda^S > 0$ is the shadow cost of forgoing profit opportunities by reducing the business volume to $\left[k(\ell^S) + 1\right] \cdot W_B$.

To assess the potential pro-cyclical impact regulation has on lending, we consider the sensitivities of the equilibrium deposit (loan) amount to exogenous parameters. Changes in those parameters are considered as shocks. Such a shock is a loss (or gain) in initial equity by realized credit losses (or by loan redemptions exclusively as promised) in a notional preceding period. If regulation is binding and if the optimal Case $j^S$ does not vary, the effect of a change in initial equity on the deposit volume is given by

$$\frac{dD^s(\ell^S, R_D^S; j^S)}{dW_B} = k(\ell^S) \ + \ k'(\ell^S) \cdot \frac{d\ell^S}{dW_B} \cdot W_B \ . \tag{3.3.26}$$

One can identify two sources affecting this sensitivity. The first or direct effect equals the bank's deposit-to-equity ratio, reflecting the direct marginal impact of initial equity on the optimal deposit volume. The second, or indirect effect, is due to a modification in the loan-allocation rate $\ell^S$. This effect becomes more pronounced as the bank becomes more capitalized and it may affect the sensitivity positively or negatively. If it is optimal for the bank to grant only a single loan, $\frac{d\ell^S}{d\theta} = 0$ prevails and the latter effect vanishes. It also vanishes if loans are equally weighted, since $k'(\ell^S) = 0$ for $c_1 = c_2$ holds. Then, comparable to Basel I, the bank's optimal choice of the loan-allocation rate does not affect the feasible maximum loan and deposit volume and the sensitivity of lending to equity-shocks is given by

$$\frac{dL^S}{dW_B} = \frac{1}{cc_1} \in [\frac{25}{3}, 62.5] \ .$$

[16]For an exposition we refer to Mas-Collel/Winston/Green, (1995, p. 959, Theorem M.K.2), or Takayama (1994, p. 93f).

If we consider shocks with respect to firms' productivity, to firms' probabilities of success, or to the common success probability $q$, the direct effect is canceled out such that the complete sensitivity is given by

$$\frac{dD^s(\ell^S, R_D^S; j^S)}{d\theta} = k'(\ell^S) \cdot \frac{d\ell^S}{d\theta} \cdot W_B \tag{3.3.27}$$

where $\theta$ is one of the parameter $p_i$, $q$, or $\alpha_i$, $i = 1, 2$. The same reasoning applies to shocks that simultaneously affect several of these parameters. The numerical studies concerning these parameters in the next chapter, *cf.* Sections 4.3.1 and 4.4.1, are based on such common multipliers. The deposit (loan) volume is only affected by the second order effect. The total loan and deposit volumes remain unaffected if loans are equally weighted, due to $k'(\ell^S) = 0$, or as long as it is optimal for the bank to grant only a single loan, implying $\frac{d\ell^S}{d\theta} = 0$.

To conclude, these second-order effects imposed by a risk-sensitive regulation enhance the sensitivity of total lending toward shocks compared to regulation with fixed and equal risk weights. Figures 4.1 in connection with 4.2, 4.11, and 4.19 provide examples. On the level of single loans, the opposite may be true, as shown by Figure 4.23 because, under fixed risk weights, loan-allocation rates move while the total loan volume remains fixed.

But risk-sensitive regulation need not necessarily add to pro-cyclicality in the sense that lending volumes react more strongly on shocks than lending volumes granted by an unregulated bank as the examples above show. The same applies to a variation of this model with loan-portfolio returns approximated by the normal distribution. The associated numerical results will be discussed in Section 6.6.

To grasp some further insights into the impact regulation has on the bank's optimal choices, we are going to outline some equilibria that can be stated in closed form. In general, there is no explicit characterization possible for Cases 1 to 3. For Case 1 we consider again the case of two equally distributed projects, for Case 2 the corner solution in $\ell^S = 0$.

#### 3.3.2.2 Characterization if Case 1 Prevails

Such as in Paragraph 3.3.1.2, p. 67, we analyze the equilibrium under Case 1 if both firms' projects are equal concerning their expected return and their variance:

**Result 14.** *Let $p_1 = p_2 = p$, $\alpha_1 = \alpha_2 = \alpha$, and $c_1 = c_2$. Suppose that regulation is binding, the household is not constrained by its initial wealth and that Case 1 is* *optimal. Then the bank allocates its funds equally as loans to both firms, i.e.* $\ell^S = \frac{1}{2}$*, and promises*

$$R_D^S = \frac{cc_1 q + (2 - cc_1)(p-q)q\alpha\gamma W_B - \sqrt{Q_1^S(c_1,\alpha)}}{2(1-cc_1)(1-q)q\gamma W_B}$$

*as gross interest rate on deposits to the household where*

$$\begin{aligned} Q_1^S(c_1,\alpha) &= c^2 c_1^2 q^2 \left[1 + (p-q)\alpha\gamma W_B\right]^2 \\ &\quad + 4cc_1(1-cc_1)q\left[(p-q)\alpha - (1-q)R_f\right]\gamma W_B \\ &\quad - 2(1-cc_1)q(p-q)(1-2p+q)\alpha^2\gamma^2 W_B^2 \ . \end{aligned}$$

*The total loan volume* $L^S$ *is given by* $L^S = \frac{1}{cc_1} \cdot W_B$*. The choice* $(\ell^S, R_D^S)$ *is unique given Case* 1*. The signs of the sensitivities of the deposit interest rate* $R_D^S$ *are given as follows,*

$$\frac{\partial R_D^S}{\partial W_B} > 0, \ \frac{\partial R_D^S}{\partial \gamma} > 0, \ \frac{\partial R_D^S}{\partial R_f} > 0, \ \frac{\partial R_D^S}{\partial c} < 0, \ \frac{\partial R_D^S}{\partial c_1} \equiv \frac{\partial R_D^S}{\partial c_2} < 0 \ . \tag{3.3.28}$$

The sensitivities of the optimal deposit interest rate $R_D^S$ can be explained as follows: increasing equity $W_B$ results in a higher interest rate, despite of the risk-reducing effect of the bank equity's buffer function, because $R_D$ is the only variable left to increase the deposit volume with a growing regulatory constraint $k(\ell^S) \cdot W_B$ as the optimal loan-allocation rate $\ell^S$ does not change in equilibrium. Likewise, a higher risk weight $c_i$ or a higher Cooke ratio $c$ reduces $R_D^S$ because the bank can only reduce its total loan volume, $L^S = \frac{1}{cc_1} \cdot W_B$, by lowering the deposit supply via the interest rate promised.

$R_D^S$ increases with $R_f$ as the household can substitute bank deposits for sovereign bonds. Were $R_D^S$ too low relative to $R_f$, the bank might not be able to fully exploit the regulatory constraint.

The derivation of the result is shown in Appendix A.2.2.1.

Finally, we can draw the following comparison between regulation and non-regulation if Case 1 is optimal for equally distributed firm projects:

**Result 15.** *Suppose that the assumptions of Results 6 and 14 hold. Then regulation makes the total loan volume,* $L^S$*, and the deposit interest rate* $R_D^S$ *pro-cyclical concerning changes in* $c$ *and* $c_i$*. Except for* $W_B$*, the regulated total loan volume is not pro-cyclical concerning shocks in any other parameter. Furthermore, regulation results in a smaller interest rate on deposits,* $R_D^S < R_D^*$*. Assume additionally*

*$\alpha \leq \frac{4}{3}$. Then regulation also makes the total loan volume, $L^S$, on average pro-cyclical concerning shocks in $W_B$.*

Pro-cyclicality concerning the regulatory parameters $c$ and $c_1$ is trivial. Furthermore, it is obvious from $L^S = \frac{1}{cc_1} \cdot W_B$ that reactions of the total loan volume to shocks other than shocks in $W_B$, $c$, and $c_i$ are dampened by regulation. The optimal total loan volume does not depend on the optimal loan-allocation rate because both risk weights are equal, $c_1 = c_2$. Consequently, the total loan volume does not depend on any further parameters via the loan-allocation rate. Note that $\ell^S = \frac{1}{2}$ holds also under regulation due to $c_1 = c_2$.

Shocks in the bank's initial equity $W_B$ are pronounced by regulation and affect the total loan volume more heavily. This mechanism is mainly due to the assumption that the bank may not hold any capital buffers in excess of the regulatorily necessary capital. Thus, binding regulation always implies that the total optimal loan volume goes along the upper bound $\left[k(\ell^S) + 1\right] \cdot W_B$ which is fixed in the case considered by Result 15 to $\frac{1}{cc_1} W_B$. But there will be some equity threshold $\overline{W}_B$ beyond which regulation is no longer binding. Likewise, the regulation prevents the bank from doing intermediation when $W_B = 0$. Thus, the total loan volume evolves linearly and strictly increases in $W_B$ for all $W_B \in [0, \overline{W}_B]$. At $W_B = \overline{W}_B$, regulation ceases to bind and thus from $W_B = \overline{W}_B$ on, the total loan volume coincides with the unregulated one. Now consider the unregulated bank in Case 1. It can collect deposits and grant loans even if it does not have any equity, *i.e.*

$$D^u(\frac{1}{2}, R_D^*; 1)\Big|_{W_B=0} = \frac{(p-q)\alpha + qR_D^* - R_f}{\gamma\sigma_1^2} > 0 \, .$$

In $W_B = \overline{W}_B$, the unregulated bank grants a total loan volume as high as $\frac{1}{cc_1} \cdot \overline{W}_B$ by definition. In between, $L^* > L^S$ holds, and $L^*$ is strictly increasing if $\alpha \leq \frac{4}{3}$ is assumed (*cf.* Result 7 and the associated Appendix A.2.1.3). Hence, the positive slope of $L^S$ in $W_B$ exceeds on $[0, \overline{W}_B]$ on average the positive slope of $L^*$, *i.e.*

$$\frac{1}{c \cdot c_1} = \frac{L^S(\overline{W}_B) - L^S(W_B)}{\overline{W}_B - W_B} > \frac{L^*(\overline{W}_B) - L^*(W_B)}{\overline{W}_B - W_B} \overset{\text{Result 7}}{>} 1, \tag{3.3.29}$$

for all $W_B \in [0, \overline{W}_B)$. Thus, regulation affects the total loan volume on average pro-cyclically according to Definition (2.2.4).

The effect on the deposit interest rate, $R_D^S < R_D^*$, has already been shown in Appendix A.2.2.1. It is simply due to the fact that the deposit interest rate $R_D$ is the only control variable left to the bank to meet the regulatory constraint $\frac{1-cc_1}{cc_1} \cdot W_B$

as it is optimal for the bank to choose $\ell = \frac{1}{2}$ with or without a binding regulatory constraint. Furthermore, this result clearly reflects the monopolistic power the bank is assumed to have on the deposit market.

Note that if both firms' projects can be distinguished according to their risk and return, the loan-allocation rate without regulation $\ell^*$ and the loan-allocation rate under a binding regulatory constraint $\ell^S$ will differ from each other. Hence, in general, it remains an open question of how regulation will affect the risk-neutral bank owners' trade-off between the loan-allocation rate and the deposit interest rate.

The numerical analyses in Chapter 4 suggest that regulation affects both the deposit interest rate and the loan-allocation rate negatively from the household's point of view, *i.e.* $R_D^S < R_D^*$ and $\ell^S < \ell^*$ prevail.

#### 3.3.2.3 Characterization if Case 2 with $\ell^S = 0$ Prevails

One such equilibria could be that the bank optimally grants all its funds as loans to the riskier firm:

**Result 16.** *Suppose that the household is not constrained by its initial wealth, $W_H$, and that the expected final wealth of the bank, $\mathrm{E}[\tilde{W}_B(\ell, R_D; j)]$, is maximized in $\ell^S = 0$ on $\mathbb{R}$ and that regulation is binding. Then*

$$R_D^S = \frac{cc_2p_2 - \sqrt{c^2c_2^2p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f}}{2(1-cc_2)p_2(1-p_2)\gamma W_B} > \frac{1}{p_2}R_f$$

*is the optimal interest rate on deposits resulting in the sensitivities*

$$\frac{\partial R_D^S}{\partial W_B} > 0, \ \frac{\partial R_D^S}{\partial \gamma} > 0, \ \frac{\partial R_D^S}{\partial R_f} > 0, \ \frac{\partial R_D^S}{\partial p_2} < 0, \ \frac{\partial R_D^S}{\partial c} < 0, \ \frac{\partial R_D^S}{\partial c_2} < 0 \ . \tag{3.3.30}$$

*The total loan volume $L^S$ is given by $L^S = \frac{1}{cc_2}W_B$.*

The sensitivity of the optimal deposit interest rate $R_D^S$ with respect to $p_2$ can be explained as follows: an increasing success probability $p_2$ means reduced risks if anything else is held fixed. Thus, an increasing success probability $p_2$ results in a lower interest rate. With respect to the remaining sensitivities, we refer to the discussion following Result 14.

The total loan volume $L^S$ strictly increases in $W_B$, and strictly decreases in the regulatory parameters $c$ and $c_2$. Under the assumption that there are parameter values such that the unregulated as well as the regulated bank chooses $\ell = 0$ as optimal loan-allocation rate one immediately obtains from the Results 8 and 16:

**Result 17.** *Assume that the conditions of Result 8 and 16 hold. Then regulation makes the total loan volume, $L^S$, pro-cyclical concerning shocks in $W_B$, and concerning changes in $c$ and $c_2$. But the regulated total loan volume is not pro-cyclical concerning shocks in any other parameter. Regulation makes the deposit interest rate pro-cyclical concerning shocks in $W_B$, $\gamma$, $R_f$, $c$, and $c_2$. The regulated deposit interest rate is not pro-cyclical concerning shocks in any other parameter.*

Concerning the total loan volume, this result is trivial. The same applies to the comparison of the sensitivities of the deposit interest rates with respect to the bank's initial equity $W_B$, the gross return $\alpha_2$, and the household's risk-aversion parameter $\gamma$. Concerning the sensitivity to $R_f$ we refer to the Appendix A.2.2.3.

#### 3.3.2.4 Characterization if Case 4 Prevails

Next, we examine the characteristics of the equilibrium if it is optimal for the bank to structure its balance sheet such that either promised loan redemption suffices to fully pay-off the depositor (Case 4). First, we provide bounds on the productivity parameter (equilibrium loan interest rate) $\alpha_i$ beyond which regulation effectively becomes binding.

**Result 18.** *Let $W_B > 0$. If*

$$\alpha_2 \;>\; \frac{2 \cdot \left\{1 - c \cdot \left[c_1 \ell^S + c_2(1 - \ell^S)\right]\right\}}{q_4} \cdot R_f \;\geq\; \frac{1.76}{q_4} \cdot R_f \qquad (3.3.31)$$

*holds true, the Regulatory Constraint* (3.3.26) *becomes binding in Case* 4.

If gross returns on loans, $\alpha_i$, are too low, regulation does not become binding in Case 4. Rather, the bank's optimal choice is limited by the Case constraints such that the highest feasible deposit volume under regulation is not met. Increasing $\alpha_i$ implies a higher leeway on the bank's choice opportunities, in particular a higher leeway on the deposit volume. Consequently, if $\alpha_i$ is sufficiently high, optimal choices will hit the Regulatory Constraint (3.3.26).

This mechanism may prevent the bank from choosing Case 4 under regulation if $\alpha_i$ lies below (3.3.31). Since other Cases will allow for higher deposit volumes, specifically for $D^S(\ell, R_D; j) = k(\ell) \cdot W_B > D^S(4)$, $j = 1, 2, 3$, it is likely that the bank can determine the loan-allocation rate $\ell$ and the deposit interest rate $R_D$ such that it overall profits from granting more loans and issuing more deposits than it is feasible in Case 4.

The formal exposition of the argument can be found in Appendix A.2.2.4. Table 3.5 illustrates of how the optimal choice in Case 4 under regulation and of how the equilibrium outcome under regulation are affected by $\alpha_i$. To present this effect more pronounced, an equal parametrization for both projects is considered. $\alpha_i = 1.95301$, $i = 1, 2$, is just that value for $\alpha_i$ at which both the Case constraints as well as the regulatory constraint hold with equality. Thus, this point marks the transition between the regime with both Case constraints only binding to that one in which the regulatory constraint is exclusively binding.

The following result characterizes the equilibrium under Case 4 if regulation is binding:

**Result 19.** *Suppose Case 4 be optimal, that the household is not constrained by its initial wealth, that the optimal loan-allocation rate is from the interior of the unit interval, and that regulation is binding. If*

$$\frac{p_2\alpha_2}{p_1\alpha_1} \geq \frac{c_2}{c_1} > 1 \ , \quad \textit{where} \quad \frac{p_2\alpha_2}{p_1\alpha_1} \overset{(3.2.5)}{>} 1 \ , \tag{3.3.32}$$

*holds, the equilibrium is characterized by the following loan and deposit redemption volumes*

$$\alpha_1 L_1^S = D^S R_D^S \quad \wedge \quad \alpha_2 L_2^S > D^S R_D^S \ , \tag{3.3.33}$$

*where*

$$\begin{aligned} \ell^S &= \frac{\frac{c_2}{c_2-c_1} \cdot (q_4 R_D^S - R_f) - \frac{(1-cc_2)}{c(c_2-c_1)} \cdot q_4(1-q_4)\gamma W_B (R_D^S)^2}{q_4 R_D^S - R_f + q_4(1-q_4)\gamma W_B (R_D^S)^2} \ , \\ R_D^S &= \frac{cc_2 q_4 \alpha_1 + c(c_2-c_1)R_f - \sqrt{Q_4^S(c_1, c_2, \alpha_1)}}{2\left[c(c_2-c_1)q_4 + (1-cc_2)q_4(1-q_4)\alpha_1\gamma W_B\right]} \ , \end{aligned} \tag{3.3.34}$$

*with*

$$Q_4^S(c_1, c_2, \alpha_1) = c^2\left[c_2 q_4 \alpha_1 - (c_2 - c_1)R_f\right]^2 - 4cc_2(1-cc_2)(1-q_4)q_4\alpha_1^2\gamma W_B R_f \ .$$

*If*

$$\frac{p_2\alpha_2}{p_1\alpha_1} \geq \frac{c_2}{c_1} = 1 \ , \tag{3.3.35}$$

Table 3.5: **Example illustrating Result 18**

*The upper panel reports parameter values used in this example. The intermediate panel reports the bank's optimal choices in Case 4 for different productivity parameter values under regulation, the lower panel shows the equilibrium outcomes.*

- *Panel A: Parametrization*

| $p_1 = p_2$ | $q$ | $W_B$ | $\gamma$ | $W_H$ | $R_f$ | $c$ | $c_1 = c_2$ |
|---|---|---|---|---|---|---|---|
| 0.992 | 0.985 | 100 | 0.008 | 3000 | 1.05 | 0.08 | 1 |

- *Panel B: Optimal solutions given Case 4*

| $\alpha_1 = \alpha_2$ | $\ell^S(4)$ | $R_D^S(4)$ | $D^S(\cdot,\cdot;4)$ | $\mathrm{E}[\tilde{W}_B^S(\cdot,\cdot;4)]$ | $U(\tilde{W}_H)$ | Eq./Res. |
|---|---|---|---|---|---|---|
| 1.1 | 0.5 | 1.05202 | 109.557 | 113.528 | 3150.05 | (3.3.13) |
| $\frac{1.76R_f}{2p-q} = 1.84985$ | 0.5 | 1.0573 | 698.717 | 727.672 | 3152.18 | (3.3.13) |
| $\frac{2(1-c\cdot c_1)R_f}{2p-q} = 1.93393$ | 0.5 | 1.06036 | 1035.33 | 1081.36 | 3154.82 | (3.3.13) |
| 1.953005 | 0.5 | 1.06142 | 1150 | 1202.32 | 3155.95 | (3.3.13) |
| $\frac{2(1-c\cdot 0.5)R_f}{2p-q} = 2.01802$ | [ 0.483892, 0.516108 ] | 1.06142 | 1150 | 1282.93 | 3155.95 | Res. 21 |
| $\frac{2(1-c\cdot 0.2)R_f}{2p-q} = 2.06847$ | [ 0.47209, 0.52791 ] | 1.06142 | 1150 | 1345.49 | 3155.95 | Res. 21 |

- *Panel C: Equilibria*

| $\alpha_1 = \alpha_2$ | $\ell^S$ | $R_D^S$ | $j^S$ | $D^S$ | $\mathrm{E}[\tilde{W}_B^S]$ | $U(\tilde{W}_H^S)$ | Eq./Res. |
|---|---|---|---|---|---|---|---|
| 1.1 | 0.5 | 1.07669 | 1 | 538.837 | 120.723 | 3156.38 | Res. 6 |
| $\frac{1.76R_f}{2p-q} = 1.84985$ | 0.4707<br>0.5293 | 1.06427 | 2<br>3 | 1150 | 1072.08 | 3157.75 | n/a |
| $\frac{2(1-c\cdot c_1)R_f}{2p-q} = 1.93393$ | 0.49464 | 1.06232 | 2 | 1150 | 1177.82 | 3156.78 | n/a |
| 1.953005 | 0.5 | 1.06142 | 4 | 1150 | 1202.32 | 3155.95 | (3.3.13) |
| $\frac{2(1-c\cdot 0.5)R_f}{2p-q} = 2.01802$ | [ 0.483892, 0.516108 ] | 1.06142 | 4 | 1150 | 1282.93 | 3155.95 | Res. 21 |
| $\frac{2(1-c\cdot 0.2)R_f}{2p-q} = 2.06847$ | [ 0.47209, 0.52791 ] | 1.06142 | 4 | 1150 | 1345.49 | 3155.95 | Res. 21 |

*holds, the equilibrium satisfies* (3.3.33), *where*

$$\ell^S = \frac{cc_1q_4\alpha_1 - \sqrt{Q_4^S(c_1,c_1,\alpha_1)}}{2(1-q_4)q_4\alpha_1^2\gamma W_B}\ , \tag{3.3.36}$$

*with*

$$Q_4^S(c_1,c_1,\alpha_1) = c^2c_1^2q_4^2\alpha_1^2 - 4cc_1(1-cc_1)q_4(1-q_4)\alpha_1^2\gamma W_B R_f\ .$$

*The deposit interest rate can be directly derived from* (3.3.34).

*If and only if*

$$1 \overset{(3.2.5)}{>} \frac{p_1\alpha_1}{p_2\alpha_2} \geq \frac{c_1}{c_2} + \frac{c_2-c_1}{c_2}\cdot\frac{R_f}{p_1\alpha_1}\cdot\frac{c(c_2-c_1)R_f - cc_1q_4\alpha_2 + \sqrt{Q_4^S(c_2,c_1,\alpha_2)}}{3cc_1(c_2-c_1)R_f + cc_1q_4\alpha_2 - 4(1-cc_1)(1-q_4)\alpha_2\gamma W_B R_f - \sqrt{Q_4^S(c_2,c_1,\alpha_2)}}, \tag{3.3.37}$$

*where*

$$Q_4^S(c_2,c_1,\alpha_2) = c^2\left[c_1q_4\alpha_2 + (c_2-c_1)R_f\right]^2 - 4cc_1(1-cc_1)(1-q_4)q_4\alpha_2^2\gamma W_B R_f$$

*holds, the equilibrium is characterized by the following loan and deposit redemption volumes*

$$\alpha_1 L_1^S > D^S R_D^S \quad \textit{and} \quad \alpha_2 L_2^S = D^S R_D^S \tag{3.3.38}$$

*where*

$$\ell^S = \frac{(\alpha_2 - R_D^S)(q_4R_D^S - R_f) + q_4(1-q_4)\alpha_2\left(R_D^S\right)^2\gamma W_B}{\left[(q_4R_D^S - R_f) + q_4(1-q_4)\left(R_D^S\right)^2\gamma W_B\right]\alpha_2}\ ,$$

$$R_D^S = \frac{c(c_2-c_1)R_f - cc_1q_4\alpha_2 + \sqrt{Q_4^S(c_2,c_1,\alpha_2)}}{2q_4\left[c(c_2-c_1) - (1-cc_1)(1-q_4)\alpha_2\gamma W_B\right]}\ . \tag{3.3.39}$$

The asymmetric allocation of funds to both loans according to (3.3.33) and (3.3.38), respectively, is due to regulation. Without regulation, it may happen that both Inequalities (3.2.19) that define the feasibility set $C_4$ hold with equality, and (3.3.10) prevails. This is not the case when regulation is supposed to be binding, unless at that point where the Regulatory Constraint (3.3.21) just starts to hold with equality. Else, either the tuple $(\ell^*, R_D^*; 4)$ according to (3.3.10) induces the deposit volume violating the Regulatory Bound (3.3.21) and thus renders the solution unfeasible under regulation, or the bank does not fully exploit the total loan/deposit volume that is feasible under regulation, contradicting the assumption of binding regulation. Table 3.5, Panel B, provides an example with respect to different values of the productivity parameter and two equally distributed projects. At $\alpha = 1.953005$, both the regulatory constraint and the Case constraints hold with equality. Below, only the Case constraints are fully exploited, above only the regulatory constraint.

Under binding regulation, the optimal loan-allocation rate $\ell^S(4)$ is mainly driven by the relation of both risk weights $c_1$ and $c_2$ to the expected gross returns on both loans, $p_1\alpha_1$ and $p_2\alpha_2$. As deposit supply does not depend on the loan-allocation rate under Case 4, the Regulatory Constraint (3.3.21) is only affected by the loan-allocation rate via the risk weighting function $c_1\ell + c_2(1-\ell)$. Thus, the risk-neutral bank has only an incentive to grant a higher loan volume to Firm 1 than to Firm 2, if there is a leeway on Constraint (3.3.21), *i.e.* $c_1 < c_2$, that is big enough to compensate for granting more to the less profitable firm. In this model, a compensation in favor of the bank is a lower deposit interest rate to be charged. As the deposit interest rate depends on the risks associated with deposits and on the household's risk aversion, any condition that results in $\alpha_1 L_1^S > \alpha_2 L_2^S$ should depend on these parameters. Condition (3.3.37) does. It is necessary and sufficient to obtain $\alpha_1 L_1^S > \alpha_2 L_2^S$.

In contrast, Condition (3.3.32) does neither account for the household's risk aversion nor for the credit risk of loans and deposits. It results in a loan-allocation rate and in a deposit interest rate such that $\alpha_1 L_1^S < \alpha_2 L_2^S$ prevails. Hence, a condition solely depending on the risk weights $c_i$ and on the expected returns $p_i\alpha_i$ is too weak to sustain a shift in the portfolio composition in favor of the less risky, but also less profitable Loan 1.

Details to the derivation of Result 19 can be found in Appendix A.2.2.5. Note furthermore, that by the sufficiency of Condition (3.3.32), Solution (3.3.34) can also prevail as equilibrium for $\frac{p_2\alpha_2}{p_1\alpha_1} < \frac{c_2}{c_1}$.

Result 19 allows the following conclusion:

**Result 20.** *Suppose as equilibria the choices characterized by Results* 9 *and* 19*. Then regulation makes the total loan volume, $L^S$, and the deposit interest rate $R_D^S$ pro-cyclical concerning changes in $c$ and $c_i$ and regulation results in a smaller interest rate on deposits, $R_D^S < R_D^*$.*

*Suppose that either Assumption* (3.3.32) *or* (3.3.35) *holds. Then the regulated deposit interest rate is not pro-cyclical concerning shocks in $\alpha_2$.*

*In contrast, if Assumption* (3.3.37) *holds, the regulated deposit interest rate is not pro-cyclical concerning shocks in $\alpha_1$. Furthermore, the interest rate, and thus the deposit volume, strictly increase in $W_B$ as long as regulation is binding,*

$$\frac{dR_D^S}{dW_B} > 0, \quad \textit{and} \quad \frac{dD^S}{dW_B} > 0 .$$

As the deposit-supply function depends only on $R_D$ under Case 4, as the total loan/deposit volume is assumed to be effectively constrained by regulation, *i.e.* $D^S < D^*$, and, finally, as deposit supply strictly increases in $R_D$ under both regimes in equilibrium, the relation $R_D^S < R_D^*$ holds. The sign of the sensitivity $\frac{dR_D^S}{dW_B}$ is derived in Appendix A.2.2.6. The sign of the sensitivity of the total deposit volume directly results from the sign of the deposit rate's sensitivity. Concerning the fact that the deposit-supply function increases in $R_D$ in equilibrium, we refer to Result 5 and Result 10.

Finally, Case 4 could also prevail in equilibrium if both firms are equal. The equilibrium can then be characterized as follows:

**Result 21.** *Suppose that the household is not constrained by its initial wealth, $W_H$, and that $p_1 = p_2 = p$, $\alpha_1 = \alpha_2 = \alpha$, and $c_1 = c_2$ hold, and that regulation is binding. Then the bank allocates its funds, given Case 4, within the range*

$$\ell^S \in \left[\frac{cc_1q_4\alpha - \sqrt{Q_4^S(c_1,c_1,\alpha)}}{2q_4(1-q_4)\alpha^2\gamma W_B},\ 1 + \frac{2cc_1(1-cc_1)R_f}{cc_1q_4\alpha - \sqrt{Q_4^S(c_1,c_1,\alpha)}} - \frac{cc_1}{(1-q_4)\alpha\gamma W_B}\right]$$

*and promises*

$$R_D^S = \frac{cc_1q_4\alpha - \sqrt{Q_4^S(c_1,c_1,\alpha)}}{2(1-cc_1)q_4(1-q_4)\alpha\gamma W_B} \tag{3.3.40}$$

*as gross interest rate to the household, where*

$$Q_4^S(c_1,c_1,\alpha) = c^2c_1^2q_4^2\alpha^2 - 4(1-cc_1)cc_1q_4(1-q_4)\alpha^2\gamma W_B R_f\ .$$

*The total loan volume $L^S$ is given by $L^S = \frac{1}{cc_1} \cdot W_B$. The signs of the sensitivities of the deposit interest rate $R_D^S$ are given as follows,*

$$\frac{\partial R_D^S}{\partial W_B} > 0,\ \frac{\partial R_D^S}{\partial \gamma} > 0,\ \frac{\partial R_D^S}{\partial R_f} > 0,\ \frac{\partial R_D^S}{\partial q_4} < 0,\ \frac{\partial R_D^S}{\partial c} < 0,\ \frac{\partial R_D^S}{\partial c_1} \equiv \frac{\partial R_D^S}{\partial c_2} < 0\ . \tag{3.3.41}$$

The sensitivity of the optimal deposit interest rate $R_D^S$ with respect to $q_4$ is discussed on p. 77 following Result 11. With respect to the remaining sensitivities, we refer to the explanations following Result 14.

The equality of the projects' returns and risk results in a range of feasible loan-allocation rates. The lower bound for this range can also be directly derived from (3.3.36) by setting $p_1 = p_2$ and $\alpha_1 = \alpha$. The upper bound of this range stems from the appropriate Case constraint. The optimal deposit interest rate can also be directly traced back to Formulæ (3.3.34) and (3.3.39), respectively. Note that the assumption of a binding regulatory constraint requires the productivity parameters at least to exceed (3.3.31) as stated in Result 18.

The derivation of the solution is discussed in Appendix A.2.2.7.

Result 15, that refers to Case 1, applies as well, in particular $R_D^S < R_D^*$ holds as outlined in Appendix A.2.2.7. The reference equilibrium is that characterized in Result 11.

But the arguments concerning pro-cyclicality on average of the total loan/deposit volumes with respect to shocks in $W_B$ must be slightly altered. First, the upper bound on $\alpha$ assumed in Result 15 and derived in Result 7 can be dispensed with. Instead, $\alpha$ must exceed (3.3.31) in order to make the regulatory constraint be binding.

Second, we must consider that, without regulation, the optimal deposit volume also goes to zero in Case 4 if the bank's initial equity, $W_B$, goes to zero. Hence, Case 4 can only occur from a given threshold $\widehat{W}_B > 0$ on in the laissez-faire equilibrium.[17] Third, there is the Bound (3.3.12) concerning $W_B$, beyond which the unregulated deposit volume is independent of $W_B$, and below which the unregulated deposit volume strictly increases in $W_B$, as summarized by Formula (3.3.19) concerning the total loan volume. Let this bound be $\widehat{\widehat{W}}_B$. Fourth, the deposit volume strictly increases in $W_B$ under binding regulation by a constant rate of $\frac{1-cc_1}{cc_1}$. Finally, binding regulation implies by definition $D^S < D^*$. Let $\overline{W}_B$ be the threshold at which regulation ceases to bind.

Merging these observations leads to the following result:

There is a range of values of $W_B$, $W_B \in [0, \widehat{W}_B]$, where the optimal loan/deposit volumes are strictly increasing with regulation. The slope of the unregulated deposit volume $D^*$, which is not necessarily positive (*cf.* Results 7 and 12), is on average lower than the slope of the regulated volume $D^S$. This is due to the fact that $D^* > D^S = 0$ holds in $W_B = 0$ and that $D^S$ catches up to $D^*$ by a constant, positive slope of $\frac{1-cc_1}{cc_1}$ as both loan volumes are equally weighted. Without regulation, we will observe any of the Cases 1 to 3, but not Case 4.

If regulation is still binding for $W_B \in [\widehat{W}_B, \overline{W}_B]$, the regulated total loan/deposit volume further catches up to the unregulated volumes by its constant slope, whereas $D^*$ is strictly increasing in $W_B$ as we have assumed Case 4 to hold in the laissez-faire equilibrium. Hence, regulation is supposed to influence total lending in a pro-cyclical manner on average. The same reasoning applies to $W_B < \widehat{\widehat{W}}_B$ if $\widehat{W}_B \leq \widehat{\widehat{W}}_B < \overline{W}_B$

[17] *Cf.* Result 9 and the subsequent discussion on p. 73.

holds and thus the unregulated total loan volume is unaffected by changes in $W_B$.

If we further consider the different dependencies of $R_D^S$ according to (3.3.40) and of $R_D^*$ according to (3.3.13) and (3.3.15), respectively, concerning the same parameters, the effects of regulation can be summarized as follows:

**Result 22.** *Suppose that the assumptions of Results 11, 18, and 21 hold. Then regulation makes the total loan volume, $L^S$, and the deposit interest rate $R_D^S$ pro-cyclical concerning changes in $c$ and $c_i$. Furthermore, the regulated total loan volume is pro-cyclical on average concerning shocks in $W_B$, but $L^S$ is not pro-cyclical concerning shocks in any other parameter. The regulated deposit interest rate is not pro-cyclical concerning shocks in $\alpha$. Above all, regulation results in a smaller interest rate on deposits, $R_D^S < R_D^*$. Let* (3.3.15) *be the equilibrium without regulation. Then the regulated deposit interest rate $R_D^S$ is pro-cyclical concerning changes in $W_B$ and $\gamma$.*

### 3.3.3 The Equilibrium with Regulation by a Value-at-Risk Approach

#### 3.3.3.1 General Characterization

As already pointed out in Section 1.3.1, the assignment of risk weights according to the internal-ratings-based approach can be interpreted as the calculation of credit VaR measures per loss given default and per exposure at default for each single loan. Instead of applying the thus derived risk weights, we determine the VaR of the whole portfolio based on the default probabilities and default correlations given by this model.

Unexpected losses to the bank's loan portfolio are defined as all realizations of the loan portfolio's value below the expected redemption $\mathrm{E}[\tilde{L}]$, *i.e.* unexpected losses are given by

$$\mathrm{E}[\tilde{L}] \; - \; \tilde{L} \; > \; 0 \; , \tag{3.3.42}$$

where $\tilde{L} \; = \; \alpha_1 \cdot \tilde{X}_1 \cdot L_1 \; + \; \alpha_2 \cdot \tilde{X}_2 \cdot L_2$.

A VaR constraint requires that unexpected losses not exceed a given threshold by a probability of $\bar{p}$. This threshold is linked to the bank's initial equity $W_B$ by a multiplier $\tau > 0$. Thus, the bank is required to meet

$$\mathrm{P}\left(\mathrm{E}(\tilde{L}) - \tilde{L} \geq \tau W_B\right) \; \leq \; \bar{p} \; . \tag{3.3.43}$$

The lower the $\tau$, the more often unexpected losses exceed the threshold $\tau W_B$ for given loan volumes. Hence, a lower $\tau$ requires the bank to decrease its loan volumes if the level of confidence $\bar{p}$ is fixed. Thus, $\tau$ quantifies the strictness of regulation. Furthermore, the value of $\tau$ can be regarded as the required resilience of equity to unexpected losses. If $\tau = 1$, the bank's initial equity should fully absorb all (un)expected losses up to a given probability $\bar{p}$. With $\tau < 1$, the initial equity can withstand losses such that a strictly positive final equity value remains within probability $\bar{p}$. For $\tau > 1$, total wipe-outs of the initial equity are tolerated within the probability $\bar{p}$.

Similar to the parameter $\tau$, the strictness of the Regulatory Constraint (3.3.43) increases with decreasing initial equity $W_B$: the lower $W_B$ is, the larger the domain of loan volumes becomes for which the inequality $\tilde{L} \leq \mathrm{E}(\tilde{L}) - \tau W_B$ continues to hold and accordingly $\mathrm{P}\left(\mathrm{E}(\tilde{L}) - \tilde{L} \geq \tau W_B\right)$ for given values of $\ell$ and $R_D$ rises. In order to reach the confidence level $\bar{p}$ again, the domain of previously feasible loan/deposit-volumes must shrink. Thus the initial equity $W_B$ has a pro-cyclical effect on the total loan volume $L$, given fixed $\ell$ and $R_D$.

The question is how the VaR Constraint (3.3.43) can be aligned with the internal-ratings-based approach in Basel II.

The latter requires that fractional losses conditional on a realization of the macro-factor not exceed a fixed threshold by $1-\bar{p} = 0.999$. The calculation of this threshold loss rate is given in BCBS (2004, Art. 272).[18] As this quantile is measured in terms of loss rates it is multiplied by the exposure at default, yielding an absolute measure for the VaR at the 0.1%-level. Subtracting expected losses results in unexpected losses. More precisely, the IRB formula thus determines unexpected losses at the 0.1%-level that must be fully covered by the bank's (regulatory) capital.[19]

Since unexpected losses are defined according to (3.3.42), $\tau = 1$ is most suitable if (3.3.43) is to reflect the IRB approach. In Chapter 4, the numerical studies make use of $\tau = 4$ to exaggerate the sensitivity of capital requirements, especially with respect to $W_B$. We will come back to $\tau = 1$ in Chapters 6 and 7.

Depending on the ordering of the two loans and on the confidence level $\bar{p}$, several constraints arise from the credit VaR Condition (3.3.43). They are shown in detail in Table 3.6. Their derivation can be found in Appendix A.2.3.1.

[18] Schönbucher (2000) and Vasicek (1991) derive the associated credit loss distribution function whereas Hartmann-Wendels (2003, pp. 117-120) illustrates the meaning of the IRB risk weights. Gordy (2003) characterizes the necessary and sufficient assumptions that underpin the basic IRB formula. Concerning this interpretation, see also Gordy/Howells (2006, p. 397).

[19] *Cf.* Article 272 in connection with Articles 40 and 44, BCBS (2004).

Table 3.6: **Constraints by regulation through VaR**

*This table shows the constraints on the total loan volume in relation to the bank's initial equity $W_B$ if the bank must fulfill a VaR constraint according to (3.3.43). The constraint depends on the prescribed level of confidence $\bar{p}$ and on the relation between both promised loan redemptions $\alpha_i L_i$, $i = 1, 2$.*

- *Panel A:* $\alpha_1 L_1 \; < \; \alpha_2 L_2$ (excluding Case 3)

| $\bar{p}$ | feasibility constraint |
|---|---|
| $[1 - p_1 - p_2 + q,\ 1 - p_2)$ | $[p_2\alpha_2(1-\ell) - (1-p_1)\alpha_1\ell] \cdot L \; < \; \tau W_B$ |
| $[1 - p_2,\ 1 - q)$ | $[p_2\alpha_2(1-\ell) - (1-p_1)\alpha_1\ell] \cdot L \; \leq \; \tau W_B$<br>and<br>$[p_1\alpha_1\ell - (1-p_2)\alpha_2(1-\ell)] \cdot L \; < \; \tau W_B$ |

- *Panel B:* $\alpha_1 L_1 \; = \; \alpha_2 L_2$

| $\bar{p}$ | feasibility constraint |
|---|---|
| $[1 - p_1 - p_2 + q,\ 1 - p_1)$ | $(p_1 + p_2 - 1) \cdot \alpha_1 L_1 \; < \; \tau W_B$ |
| $[1 - p_1, 1 - q)$ | $(p_1 + p_2 - 1) \cdot \alpha_1 L_1 \; \leq \; \tau W_B$ |

- *Panel C:* $\alpha_1 L_1 \; > \; \alpha_2 L_2$ (excluding Case 2)

| $\bar{p}$ | feasibility constraint |
|---|---|
| $[1 - p_1 - p_2 + q,\ 1 - p_1)$ | $[p_1\alpha_1\ell - (1-p_2)\alpha_2(1-\ell)] \cdot L \; < \; \tau W_B$ |
| $[1 - p_1,\ 1 - q)$ | $[p_1\alpha_1\ell - (1-p_2)\alpha_2(1-\ell)] \cdot L \; \leq \; \tau W_B$<br>and<br>$[p_2\alpha_2(1-\ell) - (1-p_1)\alpha_1\ell] \cdot L \; < \; \tau W_B$ |

The level $\bar{p} = 1 - q$ results in a condition that is always fulfilled if $L_1, L_2, W_B \geq 0$ holds with at least one strictly positive value. Hence, condition $\bar{p} = 1 - q$ does not have an impact on the bank's behavior.

The bank's decision problem under regulation by a VaR constraint can be formulated as follows:[20]

$$\begin{aligned}
\max_{\ell,\ R_D} \quad & \mathrm{E}\left[\tilde{W}_B(\ell, R_D; j)\right] \\
\text{s.t.} \quad & \ell \in [0,1] \qquad\qquad (3.3.44) \\
& \exists j :\ (D^s(\ell, R_D; j),\ \ell,\ R_D) \; \in \; C_j \\
& \mathrm{P}\left[\ \tilde{L} \leq (p_1\alpha_1\ell + p_2\alpha_2(1-\ell)) \cdot (D^s(\ell, R_D; j) + W_B) \; - \; \tau \cdot W_B\ \right] \; \leq \; \bar{p}\ .
\end{aligned}$$

[20] Bühler/Koziol/Sygusch (2008, p. 149).

We will refer to $(\ell^V, R_D^V; j^V)$ as the equilibrium, or the bank's optimal choice under the VaR approach. Regulation is said to be binding if the maximum of the unregulated problem, $(\ell^*, R_D^*; j^*)$ does not fulfill the given level of confidence $\bar{p}$.

Two remarks concerning the regulatory constraint are in order.

First, strictly speaking, a well-defined solution to Problem (3.3.44) cannot be guaranteed as the credit-VaR constraint may entail open feasibility sets for some confidence levels $\bar{p}$ as indicated by Table 3.6. The openness results from the cumulative distribution function that is continuous from the right. Thus, for practical reasons, the maximization problem is altered by adding a slack variable $\epsilon > 0$ such that the strict inequalities, as shown in Table 3.6 become weak. Then the feasibility constraints are compact sets and the existence result in Result 5 carries over to the case of regulation by a VaR constraint, analogous to the case of regulation by fixed risk weights, *cf.* Result 13.

Second, the bank can choose between two constraints to reach a confidence level of $\bar{p} \in [1 - p_2, 1 - q)$, as shown in Panel A of Table 3.6 for the loan allocation $\alpha_1 L_1 < \alpha_2 L_2$. Symmetrically, the bank has two such choices if it wishes to allocate funds such that $\alpha_1 L_1 > \alpha_2 L_2$ prevails if the confidence level is set to $\bar{p} \in [1-p_2, 1-q)$, *cf.* Panel C. Since the inequalities in Table 3.6 show a tight link between initial equity and the feasible total loan/deposit volume, binding regulation always reduces the level of the total loan/deposit volume. As a consequence the bank will choose, if possible, the constraint for a given level of confidence $\bar{p}$ that mitigates this volume reduction most strongly.

Note that for the constraints shown in Table 3.6, Panel A, given $\bar{p} \in [1 - p_2, 1 - q)$,

$$p_2\alpha_2(1-\ell) - (1-p_1)\alpha_1\ell \;>\; p_1\alpha_1\ell - (1-p_2)\alpha_2(1-\ell) \qquad \text{iff} \quad \ell < \frac{\alpha_2}{\alpha_1+\alpha_2}$$

holds. Likewise, the following ordering holds for constraints given in Table 3.6, Panel C:

$$p_1\alpha_1\ell - (1-p_2)\alpha_2(1-\ell) \;>\; p_2\alpha_2(1-\ell) - (1-p_1)\alpha_1\ell \qquad \text{iff} \quad \ell > \frac{\alpha_2}{\alpha_1+\alpha_2}$$

given $\bar{p} \in [1 - p_1, 1 - q)$. Because the bank always opts for the weakest constraint given a compulsory range of feasible loan-allocation rates, the constraints imposed by a VaR regulation according to (3.3.43) can be summarized by

$$[v(\ell,\bar{p}) + \epsilon(\ell,\bar{p})] \cdot [D^s(\ell, R_D; j) + W_B] \;\leq\; \tau \cdot W_B \tag{3.3.45}$$

where $v(\ell, \bar{p})$ is given by

$$v(\ell,\bar{p}) = \begin{cases} \begin{cases} p_2\alpha_2(1-\ell) - (1-p_1)\alpha_1\ell, & \bar{p} \in [1-p_1-p_2+q, 1-p_2) \\ p_1\alpha_1\ell - (1-p_2)\alpha_2(1-\ell), & \bar{p} \in [1-p_2, 1-q) \end{cases} , & \ell < \frac{\alpha_2}{\alpha_1+\alpha_2} \\ \\ \begin{cases} (p_1+p_2-1)\alpha_1\ell \end{cases} , \quad \bar{p} \in [1-p_1-p_2+q, 1-q) \ , & \ell = \frac{\alpha_2}{\alpha_1+\alpha_2} \\ \\ \begin{cases} p_1\alpha_1\ell - (1-p_2)\alpha_2(1-\ell), & \bar{p} \in [1-p_1-p_2+q, 1-p_1) \\ p_2\alpha_2(1-\ell) - (1-p_1)\alpha_1\ell, & \bar{p} \in [1-p_1, 1-q) \end{cases} , & \ell > \frac{\alpha_2}{\alpha_1+\alpha_2} \end{cases} \tag{3.3.46}$$

and $\epsilon(\ell, \bar{p})$ defined as

$$\epsilon(\ell,\bar{p}) \quad = \quad \begin{cases} 0 & \text{if} \quad \bar{p} \in [1-p_1, 1-q) \text{ and } \ell = \frac{\alpha_2}{\alpha_1+\alpha_2} \ , \\ \epsilon > 0 & \text{else} \quad . \end{cases}$$

The function $\epsilon(\ell, \bar{p})$ takes a positive, fixed number $\epsilon > 0$ if the regulatory constraint would otherwise result in a strict inequality and the feasibility set of the bank's optimizing problem would not be compact. This slack variable becomes dispensable only in one single case, as shown in the formula above.

The function $v(\ell, \bar{p})$ is continuous in $\ell$ for all $\ell \in [0, 1]$ for fixed $\bar{p}$. Thus, if it is binding, the Regulatory Constraint (3.3.45) imposes a continuous bound in addition to the other ones of Problem (3.3.1) without regulation.

The function $v(\ell, \bar{p})$ is only piecewise differentiable in the loan-allocation rate $\ell$ for fixed $\bar{p}$, *i.e.* it is only differentiable in $\ell$ for fixed $\bar{p}$ within the ranges $(0, \frac{\alpha_1}{\alpha_1+\alpha_2})$ and $(\frac{\alpha_1}{\alpha_1+\alpha_2}, 1)$, respectively. So, differentiability can be violated as one may have to consider multiple regulatory constraints, which may be the case for two reasons: first, in Cases 1 and 4, feasible loan-allocation rates range on intervals around $\frac{\alpha_2}{\alpha_1+\alpha_2}$. Second, it may be worthwhile for the bank to follow a tighter level of confidence than it is obliged to do (we provide an example in Chapter 4). Then the feasible loan-allocation rates may be on either side of $\frac{\alpha_2}{\alpha_1+\alpha_2}$ such that different constraints must be considered.

If, for given $\bar{p}$, the optimal loan-allocation rates never leave the spaces characterized by Cases 2 or 3, $v(\ell, \bar{p})$ is differentiable in $\ell$. Differentiability in $\ell$ also holds for $\bar{p} = 1 - p_1$ if the bank does not wish to comply with a stricter level of confidence.

With these definitions, the Maximization Problem (3.3.44) reads

$$\begin{array}{ll} \max\limits_{\ell,\ R_D} & \mathrm{E}\left[\tilde{W}_B(\ell, R_D; j)\right] \\ \text{s.t.} & \ell \in [0,1] \\ & \exists j:\ (D^s(\ell, R_D; j),\ \ell,\ R_D)\ \in\ C_j \\ & [v(\ell,\bar{p}) + \epsilon(\ell,\bar{p})] \cdot [D^s(\ell, R_D; j) + W_B]\ \leq\ \tau \cdot W_B\ . \end{array} \tag{3.3.47}$$

If the bank is regulated by a VaR constraint, the main features of this economy are summarized by the following result:

**Result 23.** *An equilibrium $(\ell^V, R_D^V; j^V)$ always exists. If the bank initially has no equity, $W_B = 0$, if $p_1 + p_2 > 1$ holds, and if the level of confidence $\bar{p}$ is strictly below $1-p_1$, the bank cannot issue any deposits, $D^s(\ell^V, R_D^V; j^V) = 0$. If $\bar{p} \geq 1-p_1$, strictly positive deposit volumes are also feasible under $W_B = 0$.*

The feasibility of positive loan and deposit volumes without any initial equity is surprising at first glance. However, its feasibility can be explained by the Bernoulli distribution that enables the bank to arbitrarily concentrate credit risk: if the bank grants the majority of its funds to one of the two firms, the expected repayments from the whole loan portfolio minus the regulatory fraction of initial equity, $\mathrm{E}[\tilde{L}] - \tau \cdot W_B$, are dominated by the promised loan redemption volume by that very borrower. Depending on the firm that has been granted the larger loan volume, either a level of confidence of $1 - p_1$ or $1 - p_2$ is attained. In terms of the loan-allocation rate $\ell$, a level of confidence of $1 - p_1$ is achieved with rather high values of $\ell$ which may be associated with Case 3 in equilibrium. Likewise, for $W_B = 0$, $\bar{p} = 1 - p_2$ implies low values of $\ell$ that may result in Case 2 in equilibrium. Technically, the bank has to choose $(\ell^V, R_D^V; j^V)$ such that

$$\lambda \cdot \{[v(\ell,\bar{p}) + \epsilon(\ell,\bar{p})] \cdot D^s(\ell, R_D; j)\} \ = \ 0$$

holds. Let regulation be binding; that is, that the regulatory constraint holds with equality. Hence, by $D^s(\ell, R_D; j) \geq 0$, $D^s(\ell^V, R_D^V; j^V) = 0$ or $v(\ell,\bar{p}) + \epsilon(\ell) = 0$ must be fulfilled. As the former leads to $\mathrm{E}[\tilde{W}_B(\cdot)] = 0$ because of $D^s(\ell^V, R_D^V; j^V) = 0$ and $W_B = 0$, the bank chooses $\ell^V$ such that $v(\ell^V, \bar{p}) + \epsilon(\ell^V) = 0$ holds, implying positive expected wealth, $\mathrm{E}[\tilde{W}_B(\cdot)] > 0$.

If $W_B = 0$ and $\bar{p} = 1 - p_2$, the bank allocates its total funds to loans according to

$$\ell^V \ = \ \frac{(1-p_2)\alpha_2 - \epsilon}{(1-p_2)\alpha_2 + p_1\alpha_1} \in (0, \frac{\alpha_2}{\alpha_1 + \alpha_2}) \text{ and } j^V = 2\ , \tag{3.3.48}$$

and, if $W_B = 0$ and $\bar{p} = 1 - p_1$, according to [21]

$$\ell^V = \frac{p_2\alpha_2 + \epsilon}{(1-p_1)\alpha_1 + p_2\alpha_2} \in (\frac{\alpha_2}{\alpha_1+\alpha_2}, 1) \text{ and } j^V = 3 \ . \tag{3.3.49}$$

Given $\bar{p} = 1 - p_2$, $\ell^V = 0$ implies that regulation is not binding, as $v(0, 1-p_2) < 0$ holds. Symmetrically, $\ell^V = 1$ results in non-binding regulation at a confidence level of $\bar{p} = 1 - p_1$ as $v(1, 1-p_1) < 0$ holds. In contrast, neither $\ell = 0$ nor $\ell = 1$ can reach a level of confidence of $\bar{p} = 1 - p_1 - p_2 + q$.

If $\bar{p} = 1 - p_1 - p_2 + q$, the regulatory constraint function $v(\ell, \bar{p})$ is always strictly positive because $p_1 + p_2 > 1$ holds due to (3.2.5). Hence, the feasible deposit volume $D^V$ must reduce to zero if $W_B = 0$. If the bank is still meant to comply with the regulatory constraint for $D^V = 0$, the tightest constraint must be less than $\tau$. For $\bar{p} = 1 - p_1 - p_2 + q$, the regulatory constraint function $v(\ell, \bar{p})$ attains its minimum in $\ell = \frac{\alpha_2}{\alpha_1+\alpha_2}$. Thus, the following equality should hold:

$$(p_1 + p_2 - 1) \cdot \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2} \ < \ \tau \ . \tag{3.3.50}$$

This guarantees that the bank always has the choice of complying with a confidence level of $\bar{p} = 1 - p_1 - p_2 + q$. The parameters of the base case, as shown in Table 4.1, comply with Restriction (3.3.50), while the parameters used in the example shown in Table 3.7 do not.

#### 3.3.3.2 Characterization of Case 1 if $l^V = \frac{\alpha_2}{\alpha_1+\alpha_2}$ Prevails

If $\bar{p} = 1 - p_1 - p_2 + q$ is required, the regulatory constraint function attains its minimum in $\ell = \frac{\alpha_2}{\alpha_1+\alpha_2}$. The bank will allocate its funds to both loans according to this rate if initial equity is sufficiently scarce. As a consequence, it will favor the less risky loan. In this paragraph, we take a closer look at Case 1.

**Result 24.** *Let $\bar{p} \in [1 - p_1 - p_2 + q, 1 - p_1)$. Suppose that regulation is binding, that the household is not constrained by its initial wealth, and that Case 1 with*

[21] The optimal interest rates $R_D^V$ can be represented analytically, too, but their expressions are very lengthy and signs of sensitivities and order of magnitudes remain opaque. This is even still a problem if the first-order conditions are solved for $R_D$ for fixed $\ell$ as $\frac{\partial D^s(\cdot)}{\partial \ell}$ and $\frac{\partial D^s(\cdot)}{\partial R_D} \cdot \frac{\partial R_D^V}{\partial \ell}$ will typically exhibit opposite signs making thus the effect of $\frac{\partial \ell^V}{\partial \theta}$ unclear.

$\ell^V = \frac{\alpha_2}{\alpha_1+\alpha_2}$ *prevails. Then the deposit interest rate is*

$$R_D^V(1) = \frac{q\left[\epsilon+(p_1+p_2-1)\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]+q(p_1+p_2-2q)\left[2\tau-\epsilon-(p_1+p_2-1)\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\gamma W_B - \sqrt{Q_1^V(\ell^V,\bar{p})}}{2q(1-q)\left[\tau-\epsilon-(p_1+p_2-1)\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\gamma W_B} \tag{3.3.51}$$

*where*

$$\begin{aligned} Q_1^V(\ell^V,\bar{p}) &= q^2\left[\epsilon+(p_1+p_2-1)\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]^2 + \\ &+ 2q\left[\epsilon+(p_1+p_2-1)\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\cdot \end{aligned}$$
$$\cdot\left\{q(p_1+p_2-2q)\left[\epsilon+\frac{(p_1+p_2-1)\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}-2\left[(1-q)R_f-\frac{(p_1+p_2-2q)\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\left[\tau-\epsilon-\frac{(p_1+p_2-1)\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\right\}\gamma W_B$$
$$+(p_1+p_2-2q)\left\{(p_1+p_2-2q)q\left[\epsilon+\frac{(p_1+p_2-1)\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]^2-4(1-p_1-p_2+q)\tau\left[\tau-\epsilon-\frac{(p_1+p_2-1)\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\right\}\frac{\alpha_1^2\alpha_2^2\gamma^2W_B^2}{(\alpha_1+\alpha_2)^2}\,.$$

*The associated deposit volume amounts to*

$$D^V = \frac{(\alpha_1+\alpha_2)(\tau-\epsilon)-(p_1+p_2-1)\alpha_1\alpha_2}{(\alpha_1+\alpha_2)\epsilon+(p_1+p_2-1)\alpha_1\alpha_2}\cdot W_B\,. \tag{3.3.52}$$

*The signs for the sensitivities of the deposit interest rate $R_D^V$ are given as follows,*

$$\frac{\partial R_D^V}{\partial W_B} > 0,\ \frac{\partial R_D^V}{\partial \gamma} > 0,\ \frac{\partial R_D^V}{\partial R_f} > 0,\ \frac{\partial R_D^V}{\partial \tau} > 0\,. \tag{3.3.53}$$

We note that the equilibrium examined above is not the only possible characterization of an equilibrium under Case 1 if VaR regulation at this confidence level is binding. We provide examples in Chapter 4: Figure 4.4 shows that $\ell^V$ deviates from $\frac{\alpha_2}{\alpha_1+\alpha_2}$ to catch up with $\ell^*$. As a result, the total loan volume exhibits a kink and its pro-cyclical behavior slackens (Fig. 4.2).

Beyond Case 1, other Cases may prevail under this level of confidence, also if $W_B > 0$ holds. So, the numerical studies in Chapter Chapter 4 provide examples for Case 2 to occur. Figure 4.22 illustrates that $\ell^V$ may drift to zero, which causes in turn a migration from Case 1 to Case 2 and jumps in total lending, as Figure 4.20 shows.

The reason for $R_D^V$ increasing in $W_B$ is as follows: because the loan-allocation rate $\ell^V$ and the deposit-supply function itself are independent of $W_B$, the interest rate $R_D^V$ is the only variable the bank can use to steer the deposit volume with respect to changes in $W_B$. Furthermore the overall deposit volume $D^V$ strictly increases in $W_B$, as does the deposit-supply function in $R_D^V$ according to Result 5. A similar mechanism is at work in the equilibria presented by Result 14 and 21. In particular, this argument applies to those equilibria under VaR regulation where both firms' projects are equal and where $\ell^V = \frac{1}{2}$ (*cf.* Results 25 and 28).

The derivations of the formulæ are presented in Appendix A.2.3.2.

According to (3.3.52) the total loan volume increases linearly with a slope exceeding one,

$$\frac{\partial L^V}{\partial W_B} = \frac{(\alpha_1+\alpha_2)\cdot\tau}{(\alpha_1+\alpha_2)\epsilon+(p_1+p_2-1)\alpha_1\alpha_2} > 1 .$$

Since the loan-allocation rate does not depend on the bank's initial equity, the relation between the total loan/deposit volume and $W_B$ simplifies to a simple linear relation as in the case under regulation with fixed risk weights and equality in the firms' projects, see Result 15. But it remains unclear if shocks in $W_B$ affect the total loan/deposit volume in a pro-cyclical manner as the monotonicity behavior of the loan/deposit volume in the laissez-faire equilibrium remains opaque under equally general conditions.

Concerning shocks in the success probabilities $p_i$ and the gross interest rates on loans, $\alpha_i$, there is a potential for counter-cyclical effects as the total loan volume under regulation reacts upon those shocks by

$$\frac{\partial L^V}{\partial p_i} = -\frac{\alpha_1\alpha_2(\alpha_1+\alpha_2)\cdot\tau\cdot W_B}{\left[(\alpha_1+\alpha_2)\epsilon+(p_1+p_2-1)\alpha_1\alpha_2\right]^2} < 0 , \quad i=1,2 ,$$
$$\frac{\partial L^V}{\partial \alpha_i} = -\frac{(p_1+p_2-1)\cdot\alpha_{3-i}^2\cdot\tau}{\left[(\alpha_1+\alpha_2)\epsilon+(p_1+p_2-1)\alpha_1\alpha_2\right]^2} < 0 , \quad i=1,2 .$$

The same signs prevail if common shocks to $p_i$ and $\alpha_i$, respectively, are considered,

$$\frac{\partial L^V}{\partial m_p} = (p_1+p_2)\cdot\frac{\partial L^V}{\partial p_i} , \quad \text{and} \quad \frac{\partial L^V}{\partial m_\alpha} = (\alpha_1+\alpha_2)\cdot\frac{\alpha_i}{\alpha_{3-i}}\cdot\frac{\partial L^V}{\partial \alpha_i} , \quad i=1,2 ,$$

where $m_p$ is the common multiplier for $p_i$ and $m_\alpha$ the common multiplier concerning $\alpha_i$, $i\in\{1,2\}$, respectively.

That higher success probabilities and higher gross returns on the firms' projects (*i.e.* higher loan interest rates) lessen the optimal total loan volume seems surprising at first glance but is actually tautological: the higher $p_i$ or the higher $\alpha_i$ become, the tighter the regulatory constraint becomes, implying in turn this property. These effects can thus be explained via the sensitivities of

$$v\left(\frac{\alpha_2}{\alpha_1+\alpha_2}, 1-p_1-p_2+q\right) = (p_1+p_2-1)\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2} .$$

A rise in $p_i$ or in $\alpha_i$ always translates into a rise of $v(\cdot)$, thus making the regulatory constraint stricter and making the maximum feasible loan/deposit volume smaller. This simple relation is particularly due to the simple formula for the optimal loan-allocation rate that strictly increases in each $\alpha_i$ and does not depend on any other parameter.

Furthermore, note that we have implicitly assumed for the above argument that the level of confidence $\bar{p}$ is kept structurally fixed for changes in $p_i$, in the sense that $\bar{p}$ still complies with a confidence level from $[1 - p_1 - p_2 + q, 1 - p_1)$ after changes in the probabilities $p_i$. That is, movements in $p_i$ are assumed to be tight enough such that the level of confidence $\bar{p}$ does not leave its initial range.

The total loan volume, $L^V$, increases in $\tau$ as an increasing $\tau$ means a relaxing regulatory constraint: the higher is this multiplier, the higher unexpected losses can become that are tolerated at a fixed level of confidence $\bar{p}$ according to (3.3.45), *i.e.*

$$\frac{\partial L^V}{\partial \tau} \; = \; \frac{(\alpha_1 + \alpha_2) \cdot W_B}{[(\alpha_1 + \alpha_2)\epsilon + (p_1 + p_2 - 1)\alpha_1\alpha_2]^2} \; > \; 0 \; .$$

By the binding regulation and by the fact that loans are granted such that their promised redemptions are equal to each other, the household's trade-off between risk and return loses its importance. Thus, the total loan/deposit volume does neither depend on its absolute risk aversion $\gamma$ nor on the gross return on the risk-free asset $R_f$. We note that this argument does not hold in general: especially if the bank's initial equity exceeds a given threshold under Case 1, the equilibrium loan-allocation rate may depend on other parameters than the productivity parameters as outlined in Result 24. In this respect we refer to the examples, as shown by Figures 4.4 and 4.22.

As shown by Result 24, there is a complementary effect between the risky deposits and the risk-free asset which is reflected by

$$\frac{\partial R_D^V(1)}{\partial R_f} \; > \; 0 \; . \tag{3.3.54}$$

The interest rate offered by the bank strictly increases in the rate on the risk-free asset since for the respective sensitivity of the total deposit volume

$$\underbrace{\frac{\partial D^V}{\partial R_f}}_{=0} \; \equiv \; \underbrace{\frac{\partial D^u(\ell^V, R_D^V(j); j)}{\partial R_f}}_{<0} + \underbrace{\frac{\partial D^u(\ell^V, R_D^V(j); j)}{\partial R_D}}_{>0} \cdot \frac{\partial R_D^V(j)}{\partial R_f} \; = \; 0 \; , \quad j \in \{1, 4\} \; ,$$

holds true. The positive sign of the partial derivative of $D^u$ with respect to $R_D$ is due to the assumed optimality of the choice, as shown by Result 24, *cf.* in this respect Result 5.

To gain further insights into the sensitivities of total loan volumes under the VaR regulation, we consider the case of identically distributed project returns. This approach allows us to draw comparisons with the sensitivities of the respective unregulated total loan volumes and we can judge whether regulation affects lending in a pro-cyclical way or not. Result 24 becomes:

**Result 25.** *Let* $p_1 = p_2 = p$ *and* $\alpha_1 = \alpha_2 = \alpha$. *Assume that* $2(\tau + \epsilon) > (2p - 1)\alpha$ *holds. Suppose Case* 1 *with* $\ell^V = \frac{1}{2}$ *is optimal, that regulation is binding, and that the household is not constrained by its initial wealth. Furthermore, let* $\bar{p} \in [1 - 2p + q, 1 - p)$. *Then the bank promises*

$$R_D^V \;=\; \frac{q\cdot[(2p-1)\alpha+2\epsilon]\cdot[1-(p-q)\alpha\gamma W_B] \;+\; 4q(p-q)\alpha\gamma\cdot\tau W_B \;-\; \sqrt{Q_1^V(p,q,\alpha)}}{2q(1-q)[2(\tau-\epsilon)-(2p-1)\alpha]\gamma W_B}$$

*as gross interest rate on deposits to the household where*

$$\begin{aligned} Q_1^V(p,q,\alpha) \;&=\; q^2\cdot\{(2p-1)\alpha+2\epsilon+(p-q)\alpha[2(2\tau-\epsilon)-(2p-1)\alpha]\gamma W_B\}^2 \\ &\quad -\; 4q(1-q)\gamma W_B[2(\tau-\epsilon)-(2p-1)\alpha]\cdot\Big\{[(2p-1)\alpha+2\epsilon]\big[R_f-(p-q)\alpha\big]+(p-q)(1-2p+2q)\alpha^2\gamma\tau W_B\Big\}\;. \end{aligned}$$

*Furthermore, if*

$$\tau \;\geq\; \frac{[(2p-1)\alpha + 2\epsilon]\left[(p-q)^2\alpha^2 - (p-q)(2-q)\alpha R_f + (1-q)R_f^2\right]}{(p-q)(2p-q)\alpha^2 - 4(p-q)\alpha R_f + 2(1-q)R_f^2}$$

*holds, the interest rate on deposits strictly increases in the bank's initial equity,*

$$\frac{\partial R_D^V}{\partial W_B} \;>\; 0\;.$$

*The optimal deposit volume is given by*

$$D^V \;=\; \frac{[2(\tau-\epsilon) - (2p-1)\alpha]\, W_B}{(2p-1)\alpha + 2\epsilon}\;.$$

If both firms are equal, some basic comparisons can be drawn between the outcome when the bank is unregulated and when it is regulated by a VaR approach:

**Result 26.** *Suppose that the assumptions of Result* 6 *and* 25 *hold. Then regulation makes the total loan volume,* $L$*, pro-cyclical concerning changes in* $\tau$*, i.e. the regulated total loan volume,* $L^V$*, strictly increases. But the regulated total loan volume is neither pro-cyclical concerning shocks in* $\gamma$ *nor shocks in* $R_f$*. Furthermore, regulation results in a lower interest rate on deposits,* $R_D^V < R_D^*$. *Assume furthermore* $\alpha \leq \frac{4}{3}$. *Then regulation makes the total loan volume,* $L$*, on average* *pro-cyclical concerning shocks in $W_B$.*

The proof to this result is analogous to the proof to Result 15. The effects of regulation on the total loan/deposit-volume are straightforward since the respective sensitivities under regulation are given by

$$\begin{aligned} \frac{\partial L^V}{\partial \tau} &= \frac{2}{[(2p-1)\alpha + \tau]} > 0 \\ \frac{\partial L^V}{\partial \gamma} &= 0 = \frac{\partial L^V}{\partial R_f} \qquad . \end{aligned}$$

Except for these parameters, no further conclusions on pro- or counter-cyclical effects by credit-VaR can be drawn as the sensitivities of the total loan volume to any other shocks are unknown in the case of the unregulated bank. Given the assumptions of Result 25, the remaining sensitivities of the regulated total loan volume are as follows:

$$\begin{aligned} \frac{\partial L^V}{\partial p} &= -\frac{4\alpha\tau}{[(2p-1)\alpha + \tau]^2} < 0 \,, \\ \frac{\partial L^V}{\partial \alpha} &= -\frac{2(2p-1)\tau}{[(2p-1)\alpha + \tau]^2} < 0 \,. \end{aligned}$$

The reasons for these signs are exactly the same reasons that have been put forward in connection with Result 24 as Result 25 represents just a special case of the former. Again, Case 1 does not need to be the optimal Case under binding VaR regulation when both firms are equal as the following example illustrates:

#### 3.3.3.3 Characterization if Case 4 Prevails

As under regulation with fixed risk weights, Case 4 allows for equilibria with an asymmetric allocation of promised loan redemptions. These solutions are supported by appropriate levels of confidence:

**Result 27.** *Suppose that regulation is binding,that the household is not constrained by its initial wealth, that $W_B > 0$, and that Case 4 is optimal. Let $\bar{p} \in [1-p_1, 1-p_2)$. Then*

$$\alpha_1 L_1^V > D^V R_D^V \,, \quad \alpha_2 L_2^V = D^V R_D^V \,,$$

$$
\begin{aligned}
\ell^V &= \frac{(\alpha_2 - R_D^V)(q_4 R_D^V - R_f) + q_4(1-q_4)\gamma W_B \alpha_2 \left(R_D^V\right)^2}{\left[q_4 R_D^V - R_f + q_4(1-q_4)\left(R_D^V\right)^2 \gamma W_B\right]\alpha_2} \,, \\
R_D^V &= \frac{\left[(1-p_1)\alpha_1 + p_2\alpha_2\right] R_f + q_4\left[(1-p_1)\alpha_1 - \epsilon\right]\alpha_2 + \sqrt{Q_4^V(1-p_1)}}{2q_4\left\{(1-p_1)\alpha_1 + p_2\alpha_2 - (1-q_4)\alpha_2\left[(1-p_1)\alpha_1 + \tau - \epsilon\right]\gamma W_B\right\}} \,,
\end{aligned}
\tag{3.3.55}
$$

*where*

$$
\begin{aligned}
Q_4^V(1-p_1) &= \left\{\left[(1-p_1)\alpha_1 + p_2\alpha_2\right] R_f - q_4\alpha_2\left[(1-p_1)\alpha_1 - \epsilon\right]\right\}^2 \\
&\quad + 4q_4(1-q_4)\alpha_2^2 \cdot \left[(1-p_1)\alpha_1 - \epsilon\right] \cdot \left[(1-p_1)\alpha_1 + \tau - \epsilon\right]\gamma W_B R_f
\end{aligned}
$$

*is feasible in equilibrium.*

*If* $\bar{p} \in [1-p_2, 1-q)$*, then*

$$
\alpha_1 L_1^V = D^V R_D^V \,, \quad \alpha_2 L_2^V > D^V R_D^V \,,
$$

$$
\begin{aligned}
\ell^V &= \frac{(q_4 R_D^V - R_f) R_D^V}{\left[q_4 R_D^V - R_f + q_4(1-q_4)\left(R_D^V\right)^2 \gamma W_B\right]\alpha_1} \,, \\
R_D^V &= \frac{\left[p_1\alpha_1 + (1-p_2)\alpha_2\right] R_f + q_4\left[(1-p_2)\alpha_2 - \epsilon\right]\alpha_1 + \sqrt{Q_4^V(1-p_2)}}{2q_4\left\{p_1\alpha_1 + (1-p_2)\alpha_2 - (1-q_4)\alpha_1\left[(1-p_2)\alpha_2 + \tau - \epsilon\right]\gamma W_B\right\}} \,,
\end{aligned}
\tag{3.3.56}
$$

*where*

$$
\begin{aligned}
Q_4^V(1-p_2) &= \left\{\left[p_1\alpha_1 + (1-p_2)\alpha_2\right] R_f - q_4\alpha_1\left[(1-p_2)\alpha_2 - \epsilon\right]\right\}^2 \\
&\quad + 4q_4(1-q_4)\alpha_1^2 \cdot \left[(1-p_2)\alpha_2 - \epsilon\right] \cdot \left[(1-p_2)\alpha_2 + \tau - \epsilon\right]\gamma W_B R_f
\end{aligned}
$$

*is feasible in equilibrium. If* $W_B = 0$ *neither* (3.3.55) *nor* (3.3.56) *can occur as equilibrium.*

Technically, those formulæ can be obtained by first solving the Case constraint for the loan-allocation rate that is assumed to bind. Second, the VaR constraint is solved for $R_D$. Here, two solutions arise whereas one cannot be optimal as it exceeds the other by far, *cf.* Result 5. If $W_B = 0$ holds, the optimal deposit interest rates presented in Result 27 imply that the bank can no longer accept deposits: the deposit volume goes to zero as $R_D^V = \frac{1}{q_4} R_f$ holds for $W_B = 0$ and the loan-allocation rate is indeterminate. Then potential equilibria may be characterized by (3.3.48) or (3.3.49), respectively.

If both equilibria are compared with the laissez-faire Equilibrium (3.3.11), regulation affects the total loan/deposit volume and the deposit interest rate in a pro-cyclical way concerning shocks in $W_B$. The reason is that the respective unregulated outcomes do not depend on $W_B$, while the regulated outcomes do, in particular at both levels of confidence.

Both presented potential equilibria indicate (such as the extreme loan-allocation rates given by (3.3.48) and (3.3.49) if $W_B = 0$) that intermediate levels of confidence, *i.e.* $\bar{p} \in [1 - p_1, 1 - q)$, support loan allocations that tend to one single loan. This property arises since a given level of confidence is strongly related to the failure probability of a given loan due to the Bernoulli distribution.

Let us provide some deeper intuition as to why such loan-allocation rates may be optimal for the bank owners when regulation is binding. Consider first a level of confidence of $\bar{p} = 1 - p_2$. If the bank owners allocate their funds such that $\alpha_2 L_2 < \alpha_1 L_1$ holds, the VaR Constraint (3.3.45) requires the bank to comply with

$$\mathrm{E}(\tilde{L}) - \tau \cdot W_B \overset{!}{<} \alpha_1 L_1 \ .$$

But by turning the loan allocation once around, *i.e.* choosing $\alpha_1 L_1 < \alpha_2 L_2$, the bank still complies with the $(1 - p_2)$-level as it fulfills now

$$\mathrm{E}(\tilde{L}) - \tau \cdot W_B \overset{!}{<} \alpha_2 L_2 \ .$$

By doing so, the bank owners can raise their expected return on their loan portfolio, thus raising their overall expected wealth if the total promised deposit redemption $D^V \cdot R_D^V$ does not increase too much.

Consider now a level of confidence of $\bar{p} = 1 - p_1$. If the bank owners allocate their funds such that $\alpha_1 L_1 < \alpha_2 L_2$ holds, the VaR Constraint (3.3.45) requires that the bank complies with

$$\mathrm{E}(\tilde{L}) - \tau \cdot W_B \overset{!}{<} \alpha_1 L_1 \ ,$$

*i.e.* to the $(1 - p_1 - p_2 + q)$-level because of $1 - p_1 - p_2 + q < 1 - p_1 < 1 - p_2$. Again, by reversing the loan allocation, *i.e.* choosing $\alpha_2 L_2 < \alpha_1 L_1$, the bank still complies with the $(1 - p_1)$-level while lifting up the constraint to the higher loan redemption volume; that is, say to

$$\mathrm{E}(\tilde{L}) - \tau \cdot W_B \overset{!}{<} \alpha_1 L_1 \ .$$

Thus, by doing so, the bank owners can raise their business volume and thus mitigate the negative size effect on their business induced by regulation. Furthermore, deposit interest rates will not rise, as long as the loan portfolio's risk and the risk of deposits has declined.

Both arguments are also reflected by the regulatory function $v(\ell, \bar{p})$ given by (3.3.46): $v(\ell, 1-p_2)$ has its unique maximum in $\ell = \frac{\alpha_2}{\alpha_1+\alpha_2}$ suggesting that the bank chooses $\ell^V < \frac{\alpha_2}{\alpha_1+\alpha_2}$ to mitigate the reduction in the loan/deposit volume while not forgoing profits as it would be the case if it chose $\ell^V > \frac{\alpha_2}{\alpha_1+\alpha_2}$. This notion is illustrated by the example given in Table 3.7.

This example illustrates that the regulatory constraint affects the loan-allocation rate depending on the required level of confidence. If $\bar{p} \in [1-p_1, 1-p_2)$, the bank chooses loan-allocation rates above $\frac{\alpha_2}{\alpha_1+\alpha_2}$. At this level of confidence, Case 2 is not feasible under this parametrization: Case 2 requires the bank to comply with

$$[p_2\alpha_2(1-\ell) - (1-p_1)\alpha_1\ell + \epsilon] \cdot [D^s(\ell, R_D; 2) + W_B] \;\leq\; \tau W_B \ .$$

As $v(\ell, 1-p_2)$ reaches its minimum in $\ell = \frac{\alpha_2}{\alpha_1+\alpha_2}$, the bank can minimize the regulatory constraint just at this point given fixed values of the deposit interest rate. But the parametrization, as shown in Table 3.7 results in

$$v(\frac{\alpha_2}{\alpha_1+\alpha_2}, 1-p_2) \;=\; v(0.50099, 0.2) \;=\; 0.380111 \;>\; 0.1 \;=\; \tau \ .$$

Hence, there is no feasible solution under Case 2 unless regulation is dispensed with for $D^V = 0$ under Case 2. Yet, there is always a non-trivial solution under Case 3 for $\bar{p} = 1-p_1$ because $v(1, 1-p_1) = -(1-p_1)\alpha_1 < 0$ holds. Then the bank is in fact not restricted in its choice under regulation given Case 3, as shown in Table 3.7, Panels B and D. But in fact the choice compatible with Case 4 is the equilibrium. It is characterized by (3.3.55). In the examples discussed in the following chapter, Chapter 4, Case 4 does not emerge. Without regulation, the equilibrium is given by the bank's optimal choice given Case 2. Above all, both the bank and the household are in fact better off without regulation.

Likewise, the bank chooses loan-allocation rates below $\frac{\alpha_2}{\alpha_1+\alpha_2}$ if a confidence level of $\bar{p} \in [1-p_2, 1-q)$ must be met. Here, the solution given Case 2 is the equilibrium under regulation. By $v(0, 1-p_2) = -(1-p_2)\alpha_2 < 0$ the bank can freely choose its deposit interest rate and deposit volume. By incidence, the regulated equilibrium coincides with the laissez-faire equilibrium.

It is striking that the deposit interest rates under Case 1 both with and without regulation exceed the respective average gross return on loans, $\alpha_1\ell + \alpha_2(1-\ell)$. This is the case as any other solutions with $R_D(1)$ below $\alpha_1\ell + \alpha_2(1-\ell)$ are not feasible under Case 1: the deposits volumes become too low so that the Case constraints are

Table 3.7: **Example illustrating Result 27**

*The upper panel reports parameter values used in this example. Panel B reports the bank's optimal choices under regulation with VaR at a confidence level of $\bar{p} \in [1-p_1, 1-p_2)$, Panel C the bank's optimal choices under VaR at a confidence level of $\bar{p} \in [1-p_2, 1-q)$. The lower panel shows the bank's optimal choices without regulation.*

- *Panel A: Parametrization*

| $p_1$ | $p_2$ | $q$ | $\alpha_1$ | $\alpha_2$ | $W_B$ | $\gamma$ | $W_H$ | $R_f$ | $\tau$ | $\epsilon$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.802 | 0.8 | 0.79 | 1.25 | 1.255 | 50 | 0.001 | 3000 | 1 | 0.1 | 0.0001 |

- *Panel B: Optimal solutions under regulation with VaR at a confidence level of $\bar{p} \in [1-p_1, 1-p_2)$*

| $\ell^V(j)$ | $R_D^V(j)$ | $j$ | $D^V(\cdot,\cdot;j)$ | $\mathrm{E}[\tilde{W}_B^V(\cdot,\cdot;j)]$ | $U(\tilde{W}_H)$ | Eq./Res. |
|---|---|---|---|---|---|---|
| 0.783236 | 1.29961 | 1 | 157.782 | 45.0104 | 3003.98 | n/a |
| n/a | n/a | 2 | n/a | n/a | n/a | n/a |
| 1 | 1.25341 | 3 | 20.9745 | 50.6369 | 3000.05 | n/a |
| 0.743814 | 1.23663 | 4 | 17.7565 | 50.6649 | 3000.04 | (3.3.55) |

- *Panel C: Optimal solutions under regulation with VaR at a confidence level of $\bar{p} \in [1-p_2, 1-q)$*

| $\ell^V(j)$ | $R_D^V(j)$ | $j$ | $D^V(\cdot,\cdot;j)$ | $\mathrm{E}[\tilde{W}_B^V(\cdot,\cdot;j)]$ | $U(\tilde{W}_H)$ | Eq./Res. |
|---|---|---|---|---|---|---|
| 0.219244 | 1.30158 | 1 | 157.307 | 45.2417 | 3003.92 | n/a |
| 0 | 1.25746 | 2 | 23.5749 | 50.7423 | 3000.07 | Res. 8 |
| 1 | 1.25341 | 3 | 20.9745 | 50.6369 | 3000.05 | n/a |
| 0.258458 | 1.23667 | 4 | 17.8734 | 50.7139 | 3000.04 | (3.3.56) |

- *Panel D: Optimal solutions without regulation*

| $\ell^*(j)$ | $R_D^*(j)$ | $j$ | $D^*(\cdot,\cdot;j)$ | $\mathrm{E}[\tilde{W}_B^*]$ | $U(\tilde{W}_H^*)$ | Eq./Res. |
|---|---|---|---|---|---|---|
| 0.50099 | 1.26343 | 1 | 49.9257 | 49.8315 | 3000.99 | n/a |
| 0 | 1.25746 | 2 | 23.5749 | 50.7423 | 3000.07 | Res. 8 |
| 1 | 1.25341 | 3 | 20.9741 | 50.6369 | 3000.05 | n/a |
| 0.304016 | 1.23798 | 4 | 22.4032 | 50.7188 | 3000.06 | (3.3.11) |

violated. Irrespective of a specific parametrization chosen and the bank's behavior under the other Cases, this choice can never be an equilibrium as it will be always dominated by the choice $\ell = 0$ under autarky, resulting in $p_2\alpha_2 W_B$, here in numbers 50.6 ($\ell = 0$ is due to (3.2.5)). This choice is in turn always dominated by the choice $(\ell(j), R_D(j); j) = (0, R_D(0; 2); 2)$, here resulting in 50.6369. Note that without regulation, $(0, R_D(0; 2); 2)$ is always feasible.

To sum up, Case 3 may rather arise in equilibrium if $\bar{p} \in [1 - p_1, 1 - p_2)$ is required, while Case 2 may rather occur if $\bar{p} \in [1 - p_2, 1 - q)$ is set. So, in equilibrium, changes in the Case are very likely to occur for intermediate levels of confidence. Specifically, Case 2 will rather dominate Cases 1 and 4 for $\bar{p} = [1 - p_2, 1 - q)$ as the higher expected return on the second loan is an incentive to the risk-neutral bank owners to shift the loan-allocation rate far enough to reach Case 2. By the lower expected return on the first loan, the incentive to deviate from either Case 1 or 4 in favor of Case 3 is not that strong for $\bar{p} = [1 - p_1, 1 - p_2)$. Figure 4.3 further illustrates these incentives induced by the different levels of confidence.

If both firms's project returns and risks are equal, Case 4 may prevail as equilibrium if a confidence level of $\bar{p} = 1 - 2p + q$ is set. It can be characterized as follows:

**Result 28.** *Let $p_1 = p_2 = p$ and $\alpha_1 = \alpha_2 = \alpha$. Suppose Case 4 is optimal, that regulation is binding, and that the household is not constrained by its initial wealth. Furthermore, let $\bar{p} \in [1 - 2p + q, 1 - p)$. Then the bank chooses $\ell^V = \frac{1}{2}$ as loan-allocation rate and promises*

$$R_D^V = \frac{(2p - q)\left[(2p - 1)\alpha + 2\epsilon\right] - \sqrt{Q_4^V(p, q, \alpha)}}{2(2p - q)(1 - 2p + q)\left[2(\tau - \epsilon) - (2p - 1)\alpha\right]\gamma W_B}$$

*as gross interest rate on deposits to the household where*

$$\begin{aligned} Q_4^V(p, q, \alpha) &= (2p - q)^2\left[(2p - 1)\alpha + 2\epsilon\right]^2 \\ &- 4(2p - q)(1 - 2p + q)\left[2(\tau - \epsilon) - (2p - 1)\alpha\right] \cdot \left[(2p - 1)\alpha + 2\epsilon\right]\gamma W_B R_f \ . \end{aligned}$$

*The optimal deposit volume is given by*

$$D^V = \frac{\left[2(\tau - \epsilon) - (2p - 1)\alpha\right] W_B}{(2p - 1)\alpha + 2\epsilon} \ , \tag{3.3.57}$$

*resulting in the following signs for the sensitivities*

$$\frac{\partial D^V}{\partial W_B} > 0, \ \frac{\partial D^V}{\partial \gamma} = 0, \ \frac{\partial D^V}{\partial R_f} = 0, \ \frac{\partial D^V}{\partial p} < 0, \ \frac{\partial D^V}{\partial q} = 0, \ \frac{\partial D^V}{\partial \alpha} < 0, \ \frac{\partial D^V}{\partial \tau} > 0 \ , \tag{3.3.58}$$

*in turn implying the signs for the sensitivities of the deposit interest rate* $R_D^V$,

$$\frac{\partial R_D^V}{\partial W_B} > 0,\ \frac{\partial R_D^V}{\partial \gamma} > 0,\ \frac{\partial R_D^V}{\partial R_f} > 0,\ \frac{\partial R_D^V}{\partial p} < 0,\ \frac{\partial R_D^V}{\partial q} > 0,\ \frac{\partial R_D^V}{\partial \alpha} < 0,\ \frac{\partial R_D^V}{\partial \tau} > 0\ . \tag{3.3.59}$$

By $p_1 = p_2$ and $\alpha_1 = \alpha_2$, the bank's objective function, $\mathrm{E}\left[\tilde{W}_B(\ell, R_D; 4)\right]$, is independent of the loan-allocation rate $\ell$. Assume furthermore, that the Case constraints are not violated, *i.e.* that the bank will choose $(\ell^V(4), R_D^V(4))$ from the interior. Then the Lagrangian depends on $\ell$ only via the regulatory constraint function $v(\ell, 1-2p+q)$, which is

$$v(\ell, 1-2p+q) = \begin{cases} (p-\ell)\alpha & \text{if} \quad \ell \leq \frac{1}{2}\ , \\ p\alpha - \alpha(1-\ell) & \text{if} \quad \ell > \frac{1}{2}\ . \end{cases}$$

Consequently, the bank first chooses $\ell$ such that the regulatory burden is minimized, resulting in $\ell^V = \frac{1}{2}$. Obviously, the choice is compatible with Case 4 for any $D^V \geq 0$. $R_D^V$ results from the regulatory constraint if it holds with equality, as assumed.

Concerning the principle of the derivation of the interest rate's sensitivities, we refer to Result 21. The deposit interest rate strictly decreases in $\alpha$ as with $\ell^V = \frac{1}{2}$, $R_D$ is the only variable left to the bank to decrease the total loan/deposit volume while the regulatory function $v(\frac{1}{2}, \bar{p})$ strictly increases. The deposit-supply function $D^s(R_D; 4)$ does not depend on $\alpha$ in Case 4. The remaining sensitivities can be explained along the lines of the discussion on p. 77 following Result 11.

Comparing the equilibrium given by Result 28 with the laissez-faire equilibrium characterized by Result 11 allows for the following conclusions concerning cyclical impacts:

**Result 29.** *Suppose that the assumptions of Results 11, and 28 hold. Then regulation makes the total loan volume,* $L^V$, *and the deposit interest rate* $R_D^V$ *pro-cyclical concerning changes in* $\tau$ *and regulation results in a smaller interest rate on deposits,* $R_D^V < R_D^*$. *But* $L^V$ *is not pro-cyclical concerning shocks in* $\gamma$, $R_f$, *and* $q$. *Let* (3.3.13) *be the reference equilibrium without regulation. Then the regulated total loan volume is pro-cyclical on average concerning shocks in* $W_B$.

*If* (3.3.15) *is the reference equilibrium without regulation, the regulated total loan volume is pro-cyclical concerning shocks in* $W_B$, *but is counter-cyclical concerning shocks in* $p$ *and* $\alpha$. *The regulated deposit interest rate* $R_D^V$ *is pro-cyclical concerning changes in* $W_B$ *and* $\gamma$, *but is counter-cyclical concerning changes in* $\alpha$.

Table 3.8: **Example of equal projects under VaR-based regulation, Bernoulli model**

*The upper panel shows an example set of parameter values with equal success probabilities and equal productivity parameters for both projects. The lower panel reports the bank's optimal choices in each Case $j = 1, \ldots, 4$. The equilibrium under VaR regulation at a confidence level of $\bar{p} = 1 - p_1 - p_2 + q$ is characterized by Case 4. Without regulation, the equilibrium is given under Cases 2 and 3, respectively, cf. Table 3.3.*

- *Panel A: Parametrization*

| $p$ | $q$ | $\alpha$ | $W_B$ | $\gamma$ | $W_H$ | $R_f$ | $\tau$ | $\epsilon$ |
|---|---|---|---|---|---|---|---|---|
| 0.92 | 0.9 | 1.15 | 10 | 0.001 | 3000 | 1.05 | 0.99 | 0.0001 |

- *Panel B: Optimal Choices*

| $\ell^V(j)$ | $R_D^V(j)$ | $j$ | $D^V(\cdot,\cdot;j)$ | $\mathrm{E}[\tilde{W}_B(\cdot,\cdot;j)]$ | $U(\tilde{W}_H)$ | Eq./Res. |
|---|---|---|---|---|---|---|
| $\frac{1}{2}$ | 1.14195 | 1 | 10.4927 | 10.426 | 3150.23 | Res. 25 |
| 0.498618 | 1.12965 | 2 | 10.4255 | 10.541 | 3150.12 | n/a |
| 0.501382 | 1.12965 | 3 | 10.4255 | 10.541 | 3150.12 | n/a |
| $\frac{1}{2}$ | 1.11781 | 4 | 10.4927 | 10.6562 | 3150 | Res. 28 |

The cyclical impacts can be read-off by the signs of the sensitivities stated in Results 11, and 28.

Let us revisit the example shown in Table 3.3. Table 3.8, Panel B, reports the bank's optimal choices if it must comply with a confidence level of $\bar{p} = 1 - p_1 - p_2 + q$. The regulatory parameter $\tau$ is set equal to 0.99 such that regulation just becomes binding under Case 4. The equilibrium under regulation is characterized by Case 4. Both, under Case 1 and 4, the regulated bank can issue the same volume of deposits. But the risks the household must bear under these two Cases are different in the sense that the probability of full deposit redemption differs. As a consequence, the deposit interest rate is lower under Case 4 than under Case 1, resulting in total in a higher expected final wealth for the bank under Case 4.

If $\tau = 1$, regulation is effectively binding in Cases 1 to 3, too, while in Case 4, regulation just becomes binding such that the bank's choice is identical to its choice without regulation, *cf.* Table 3.3. Then the equilibrium under regulation is characterized by the bank's choice according to Case 4 as well.

## 3.4 Summary

In this chapter, a three-sector model of financial intermediation was introduced and analyzed. Some specific equilibria, distinguished by Case and by whether both firms' projects are different in return and risk or not, have been characterized. Explicit expressions for the loan-allocation rate, the deposit interest rate, and the total loan/deposit volume have been obtained. These characterizations led to first insights concerning the issue of whether risk-sensitive capital adequacy rules may enhance or even create cyclical patterns in lending. Results 15, 17, 20, 22, 26, and 29 give first answers. They have been summarized in Table 3.9 concerning an approach of fixed risk weights and in Table 3.10 concerning a VaR approach.

Shocks in the bank's initial capital, reflecting past gains or losses, seem to affect lending in a pro-cyclical manner. This observation also applies to the deposit interest rate. Shocks in the depositor's risk-aversion, however, have little or no impact on regulated outcomes. Changes in the success probabilities $p_i$ and $q$ or marginal gross returns on loans, $\alpha$, have mostly no pro-cyclical effect. In contrast, we observe counter-cyclical effects concerning shocks in $p_i$ and $\alpha_i$ under the VaR regulation. The specific instance is characterized by Case 4 and by projects that have the same success probability and gross return. Clearly, changes in the regulatory parameters affect the optimal loan volumes and deposit interest rates under a binding regulation so that there are pro-cyclical effects in this respect by definition.

The results so far address few specific equilibria only so that there are many open questions left. In particular, in most equilibria considered and compared, the loan-allocation rate remains the same, no matter if it is the laissez-faire or the regulated equilibrium. Hence, we have not considered any effects concerning the allocation of risks. These effects, in turn, could have strong impacts on the total loan/deposit volume and on deposit interest rates and thus alter their cyclical behavior.

Furthermore, the equilibria considered have not been fully characterized concerning all potential pro-cyclical effects. For many shocks the issue of pro-cyclicality remains unsolved, even if the signs of the respective sensitivities with and without regulation are known. But then, their absolute magnitudes remain opaque or some further strong and specific assumptions are required.

Table 3.9: **Summary of cyclical effects through regulation by fixed risk weights**

*This table summarizes the cyclical effects that have been theoretically characterized in Section 3.3.2. The different notions of cyclicality are based upon the definitions (2.2.1) to (2.2.4). The last row addresses some specific volume effects.*

| | **Case** 1 | | **Case** 2 | | **Case** 4 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| equilibrium under regulation | $p_1 = p_2$, $\alpha_1 = \alpha_2$, $c_1 = c_2$ Result 14 | | $c_1 < c_2$ $\ell^S = 0$ Result 16 | | $p_1 \neq p_2$, $\alpha_1 \neq \alpha_2$ (3.3.34), (3.3.36) Result 19 | | (3.3.39) Result 19 | | $p_1 = p_2$, $\alpha_1 = \alpha_2$ $c_1 = c_2$ Result 21 | | | |
| reference equilibrium w/o regulation | $p_1 = p_2$ $\alpha_1 = \alpha_2$ Result 6 | | $\ell^* = 0$ Result 8 | | $p_1 \neq p_2$, $\alpha_1 \neq \alpha_2$ (3.3.10), (3.3.11) Result 9 | | | | $p_1 = p_2$, $\alpha_1 = \alpha_2$ (3.3.13) Result 11 | | (3.3.15) Result 11 | |
| cyclical effects stated in | Result 15 | | Result 17 | | Result 20 | | | | Result 22 | | | |
| sensitivities | $L^S$ | $R_D^S$ | $L^S$ | $R_D^S$ | $L^S$ | $R_D^S$ | $L^S$ | $R_D^S$ | $L^S$ | $R_D^S$ | $L^S$ | $R_D^S$ |
| $W_B$ | pro-cyclical on average if $\alpha \leq \frac{4}{3}$ | | pro-cyclical | pro-cyclical | | | | | pro-cyclical on average | | pro-cyclical on average | pro-cyclical |
| $\gamma$ | | | | | | | | | | | | |
| $R_f$ | | | | | n/a | n/a | n/a | n/a | | n/a | | |
| $p_1$ | not pro-cyclical | n/a | not pro-cyclical | not pro-cyclical | | | | | not pro-cyclical | | not pro-cyclical | n/a |
| $p_2$ | | | | n/a | | | | | | | | |
| $q$ | | | | | | | | | | | | |
| $\alpha_1$ | | | | not pro-cyclical | | | | not pro-cyclical | | not pro-cyclical | | not pro-cyclical |
| $\alpha_2$ | | | | | | not pro-cyclical | | n/a | | | | |
| $c$ | | | pro-cyclical | pro-cyclical | | | | | | | | |
| $c_1$ | pro-cyclical | pro-cyclical | not pro-cyclical | not pro-cyclical | pro-cyclical | pro-cyclical | pro-cyclical | pro-cyclical | pro-cyclical | pro-cyclical | pro-cyclical | pro-cyclical |
| $c_2$ | | | pro-cyclical | pro-cyclical | | | | | | | | |
| absolute values | $L^S = \frac{1}{cc_1} \cdot W_B$, $R_D^S < R_D^*$ | | $L^S = \frac{1}{cc_2} \cdot W_B$, $R_D^S < R_D^*$ | | $R_D^S < R_D^*$ | | | | $L^S = \frac{1}{cc_1} \cdot W_B$ , $R_D^S < R_D^*$ | | | |

Table 3.10: **Summary of cyclical effects by VaR-based regulation**

*This table summarizes the cyclical effects that have been theoretically characterized in Section 3.3.3. The different notions of cyclicality are based upon the Definitions* (2.2.1) *to* (2.2.4). *The last row addresses some specific volume effects.*

| | **Case** 1 | | | | | | | | | **Case** 4 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| equilibrium under regulation | $p_1 \neq p_2$, $\alpha_1 \neq \alpha_2$ <br> $\bar{p} = 1 - p_1 - p_2 + q$ <br> Result 24 | | | $p_1 = p_2$, $\alpha_1 = \alpha_2$ <br> $\bar{p} = 1 - 2p + q$ <br> Result 25 | | $p_1 \neq p_2$, $\alpha_1 \neq \alpha_2$ <br> $\bar{p} = 1 - p_1$ \| $\bar{p} = 1 - p_2$ <br> Res. 27: (3.3.55), (3.3.56) | | | | $p_1 = p_2$, $\alpha_1 = \alpha_2$ <br> $\bar{p} = 1 - 2p + q$ <br> Result 28 | | | |
| reference equilibrium w/o regulation | $p_1 \neq p_2$ <br> $\alpha_1 \neq \alpha_2$ <br> n/a | | | $p_1 = p_2$ <br> $\alpha_1 = \alpha_2$ <br> Result 6 | | $p_1 \neq p_2$, $\alpha_1 \neq \alpha_2$ <br> (3.3.10) <br> Result 9 | | (3.3.11) <br> Result 9 | | $p_1 = p_2$, $\alpha_1 = \alpha_2$ <br> (3.3.13) <br> Result 11 | | (3.3.15) <br> Result 11 | |
| cyclical effects discussed by/on | pp. 102 - 104 | | | Result 26 | | n/a | | p. 107 following Res. 27 | | Result 29 | | | |
| sensitivities | $L^V$ | $R_D^V$ | $\ell^V$ | $L^V$ | $R_D^V$ | $L^V$ | $R_D^V$ | $L^V$ | $R_D^V$ | $L^V$ | $R_D^V$ | $L^V$ | $R_D^V$ |
| $W_B$ | pro-cyclical on average | n/a | not pro-cyclical | pro-cyclical on average | n/a | n/a | n/a | pro-cyclical | | pro-cyclical on average | n/a | pro-cyclical | pro-cyclical |
| $\gamma$ | not pro-cyclical | | | not pro-cyclical | | | | n/a | n/a | not pro-cyclical | | not pro-cyclical | |
| $R_f$ | | | | | | | | | | | | | n/a |
| $p_i$ | n/a | | | n/a | | | | | | n/a | | counter-cyclical | |
| $q$ | not pro-cyclical | | | not pro-cyclical | | | | | | not pro-cyclical | | not pro-cyclical | |
| $\alpha_i$ | n/a | | n/a | | | | | | | n/a | | counter-cyclical | counter-cyclical |
| $\tau$ | pro-cyclical | pro-cyclical | pro-cyclical | pro-cyclical | pro-cyclical | pro-cyclical | | pro-cyclical | | pro-cyclical | pro-cyclical | pro-cyclical | pro-cyclical |
| absolute values | $L^V = \frac{\tau}{(p_1+p_2-1)\alpha_1\ell^V+\epsilon} \cdot W_B$, <br> $\ell^V = \frac{\alpha_2}{\alpha_1+\alpha_2}$ | | | $L^V = \frac{2\tau}{(2p-1)\alpha+2\epsilon} \cdot W_B$, <br> $\ell^V = \ell^* = \frac{1}{2}$, $R_D^V < R_D^*$ | | | | | | $L^V = \frac{2\tau}{(2p-1)\alpha+2\epsilon} \cdot W_B$, <br> $\ell^V = \ell^* = \frac{1}{2}$, $R_D^S < R_D^*$ | | | |

# Chapter 4

# Numerical Analysis of Regulatory Impacts

## 4.1 Introduction

So far general results concerning the existence of an equilibrium allocation and some results concerning some specific equilibrium outcomes were obtained. There was only partial evidence of pro-cyclicality and other potential effects induced by regulation. Results 15, 17, 20, 22, 26, and 29 provide first notions on how regulation can affect loan and deposit volumes depending on the state of the cycle. Particularly, changes in the bank's initial equity to which loan volumes are ultimately pegged seem to affect total loan volumes in a qualitatively different manner than changes in expected prospects. However, as Result 17 is characterized by a single loan granted in equilibrium and as Results 15, 22, 26, and 29 are based on the assumption of both firms undertaking equal projects, the typical properties of risk-based capital regulation cannot come into effect. For this reason, we will now study the model numerically in order to account for differences in the expected returns and in the return volatilities of the firms' projects. Thus there is a serious role for risk-sensitive capital regulation and equilibrium outcomes will depend to a greater extend on the differing prospects of the firms' projects, on the differentiated treatment of these projects by regulation, and on the household's risk attitude who is the ultimate investor in this model.

Furthermore, the numerical analysis may provide deeper insights into regulatory effects as it can explicitly account for migrations from one Case to another due to regulation. The results shown so far have always compared an outcome under regulation to a given laissez-faire outcome without questioning the relevance of the

Table 4.1: **Parameter values of the base case, Bernoulli model**

*The upper panel reports the parameter values used for this numerical analysis of the model set forth in Chapter 3. The lower panel reports the thus implied values of the expected gross returns on single loan redemptions and the variances of these returns. Furthermore, the correlation of both loans' returns are shown.*

- *Panel A:*

| $p_1$ | $p_2$ | $q$ | $\alpha_1$ | $\alpha_2$ | $W_B$ | $\gamma$ | $W_H$ | $R_f$ | $\tau$ | $\epsilon$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.995 | 0.99 | 0.987 | 1.15 | 1.2 | 100 | 0.008 | 3000 | 1.05 | 4 | 0.000001 |

- *Panel B:*

| $\mathrm{E}[\tilde{L}_1/L_1]$ | $\mathrm{E}[\tilde{L}_2/L_2]$ | $\mathrm{V}[\tilde{L}_1/L_1]$ | $\mathrm{V}[\tilde{L}_2/L_2]$ | $\mathrm{Corr}[\tilde{L}_1/L_1, \tilde{L}_2/L_2]$ |
|---|---|---|---|---|
| $=$ 1.14425 | $=$ 1.188 | $\approx$ 0.0811137 | $\approx$ 0.119398 | $\approx$ 0.277856 |

Cases thus considered. As a by-product, it becomes more meaningful in this vein to point out other regulatory effects, notably those on the loan allocation. This may complement the discussions initiated by Results 18 and 20, or by the equilibria under the VaR approach characterized by (3.3.48) and (3.3.49), respectively.

The base case scenario is given by the parameter values outlined in Table 4.1. Parameter values are chosen such that Conditions (3.2.5), (3.2.6), and (3.2.7) are fulfilled. The expected return, variance of returns, and the correlation of the loans granted to both firms are identical to those of the corresponding projects as implied by the clearing of the loan market (3.2.4).[1]

As the default probabilities of the loans amount to 0.5% and 1.0%, respectively, credit grades of BBB and BB+ apply if these probabilities are considered as historical one-year default probabilities.[2] Such credit grades translate into a

[1]Bühler/Koziol/Sygusch (2008) study the model outlined in Chapter 3 by the same parameter values, but only for three regimes, the laissez-faire equilibrium, the Standardized Approach with both risk weights equaling one, $c_i = 1$, and the VaR approach with confidence level $\bar{p} = 1.0\%$. By their numerical analysis (*ibid.*, pp. 150-158), they can draw the same main conclusions.

[2]*Cf.* Crouhy/Galai/Mark (2000, p. 68), Gordy/Lütkebohmert (2007, p. 15), Hull (2003, p. 626). In periods of stress, such as the 1990/91-recession in the US, one-year default probabilities of BB-rated loans can even rise up to 3.5% whereas those of BBB-rated loans keep around 0.6%, as do Ervin/Wilde (2001 p. S30) report. Following Carey (1998), Furfine (2001, p. 52) reports that roughly one third of total loans can be asserted each to the rating classes BBB and BB, respectively, in typical bank loan portfolios, and Illing/Paulin (2005, p. 169) state that, concerning Canadian banks, "a little more than two-thirds of bank exposures [are] rated investement grade (*i.e.*, BBB- or higher)" , thus underpinning the empirical relevance of the credit risk structure assumed by the

risk weight of 1.0 according to Table 3.4. But we will also consider a risk weight of $c_1 = 0.5$ to account for the lower risk of Firm 1. The correlation, $\text{Corr}[\tilde{L}_1/L_1, \tilde{L}_2/L_2] \approx 0.28$, is, interpreted as a loan-return correlation, relatively high. The Basel Committee assumes the average asset-return correlation to take values from $[0.12, 0.24]$ in the formulæ for the IRB-risk weights (BCBS, 2004, Art. 272). Moreover, asset-return correlations always translate to loan-return correlations that are much lower, unless they approach one, as shown by Gersbach/Lipponer (2003, p. 365f). In all comparative static analyses considered in the next sections, the household's and subsequently the bank's decisions are not constrained by the household's initial wealth $W_H$.

# 4.2 Equity Shocks

## 4.2.1 Total Loan Volumes and Pro-Cyclical Effects

Figure 4.1 shows the results of a comparative static analysis with respect to changes in the bank's initial equity $W_B$. Both forms of regulation amplify the sensitivity of the total loan volume to changes in $W_B$. Thus, according to Definition (2.2.1), regulation is pro-cyclical. Let us separate the single building blocks that add up to the total effect.

First, in $W_B = 0$, the bank issues a strictly positive amount of deposits and provides loans in the absence of regulation according to Result 5, while it is constrained under the VaR approach and its lending activities are scaled down to zero under the Standardized Approach. Second, when $W_B$ is high enough, both Regulatory Constraints (3.3.21) and (3.3.45) ease and are no longer binding and the total loan volume is the same under all regimes. Third, under a binding regulatory constraint, the total regulated loan volume is always lower than the non-regulated total loan volume. Fourth, the total loan volumes strictly increase in $W_B$ under all three regimes. The fact that the unregulated loan volume is the least sensitive toward changes in $W_B$, finally guarantees the pro-cyclical effect. In numbers, the equal weighting under the Standardized Approach leads to a slope of 12.5, whereas the total loan volume $L^V$ increases by 2.95 on average under the VaR approach with a confidence level of $1 - p_1 = 0.5\%$ and by only 1.28 without regulation.

Recall that the total loan/deposit volume is positive in $W_B = 0$ under the VaR

parameter values given in Table 4.1. Similar numbers can be found in Gordy (1998) which are based on data from internal Federal Reserve Board surveys.

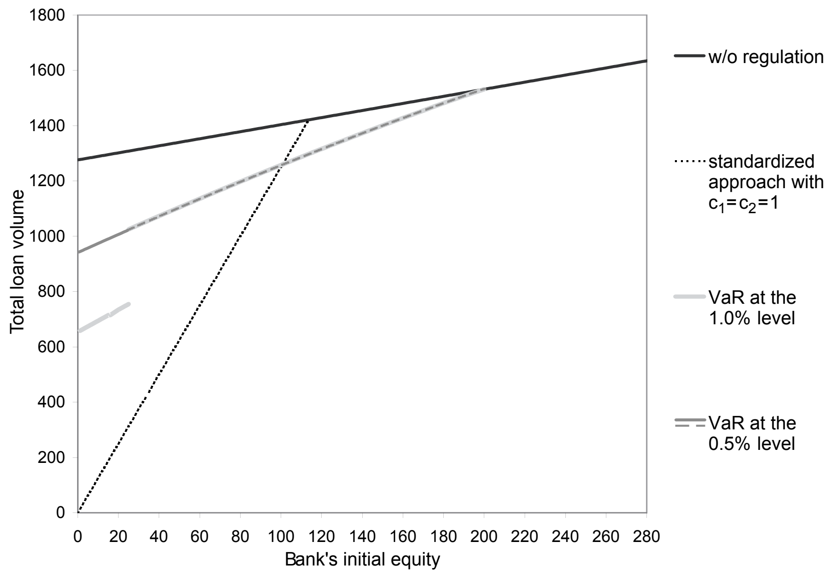


Figure 4.1: **Total loan volume as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium total loan volume $L = L_1 + L_2$ on the bank's initial equity $W_B$. The total loan volume is shown for four different regimes: the laissez-faire equilibrium, the Standardized Approach with both risk weights equaling one, $c_i = 1$, the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.5\%$. Cf. Bühler/Koziol/Sygusch (2008, Fig. 1, p. 152).*

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approach to regulation, since the bank can choose the loan-allocation rate $\ell$ such that the regulatory constraint function $v(\ell, 1-p_2)$ becomes negative while still attaining a level of confidence of $1 - p_2 = 1\%$. Particularly, the bank chooses the optimal loan-allocation rate and the optimal Case according to (3.3.48), resulting in $\ell^V \approx 1.0378\%$ and $j^V = 2$. From $W_B \approx 25$ on, it becomes more profitable for the bank to choose a loan portfolio that satisfies the stronger confidence level of $1 - p_1 = 0.5\%$, although it is not obliged to do so. Then the regulatory constraint function becomes

$$v(\ell, 0.005) \;=\; p_2\alpha_2(1-\ell) - (1-p_1)\alpha_1\ell$$

while the bank chooses loan-allocation rates above $\frac{\alpha_2}{\alpha_1+\alpha_2}$ as presented in Figure 4.3. The change in the confidence level, and hence in the structure of the regulatory constraint function $v(\ell, \bar{p})$, results in a jump of the total loan volume/deposit volume at $W_B \approx 25$. Despite the rather extreme loan-allocation rates chosen around $W_B = 25$, the loan allocation is compatible with Case 1 from $W_B \approx 20$ on.

Because the equilibrium deposit volume under the VaR approach will always be

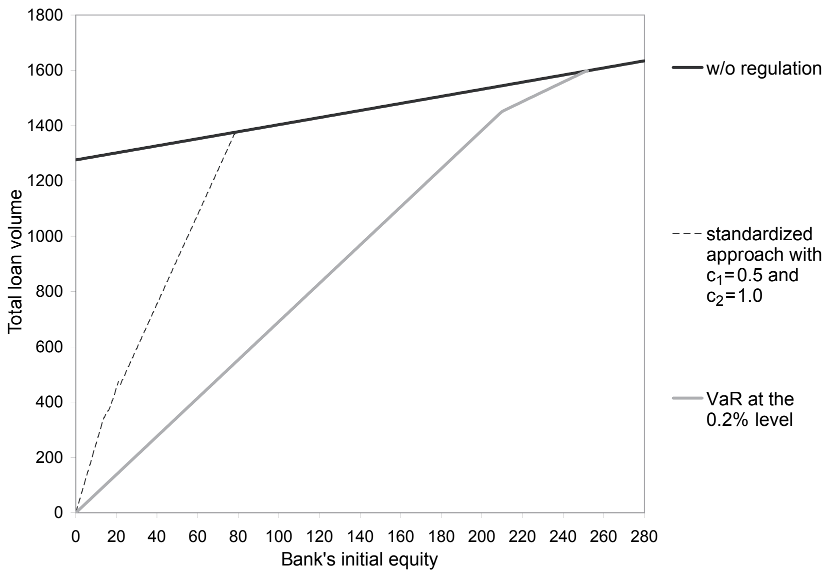


Figure 4.2: **Total loan volume as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium total loan volume $L = L_1 + L_2$ on the bank's initial equity $W_B$. The total loan volume is shown for three different regimes: the laissez-faire equilibrium, the Standardized Approach with different risk weights, where $c_1 = 0.5$ and $c_2 = 1$, and the VaR approach with confidence level $\bar{p} = 0.2\%$.*

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positive in $W_B = 0$ for intermediate levels of confidence, here $\bar{p} \in \{1 - p_1, 1 - p_2\}$ are chosen, and because zero equity always implies a zero amount of risky loans under the Standardized Approach, the VaR approach tends to be less pro-cyclical than the latter approach. However, according to Result 23, $D^V = 0$ holds for the tightest confidence levels; that is, for $\bar{p} < 1 - p_1$. Then, both curves representing the equilibrium volumes of total loans under either VaR regimes of regulation start at zero and their slopes depend on the specific values that are attached to given levels of risk.

The reactions of the loan volumes under the VaR approaches would be further dampened by lower values of $\tau$. Moreover, the domain on which regulation is binding would be enlarged proportionately.

Figure 4.2 compares the total loan volume $L^V$ for $\bar{p} = 1 - p_1 - p_2 + q = 0.2\%$ to the unregulated volume and the volume under the Standardized Approach with differing risk weights $c_i$. In this example, $L^V$ is less pro-cyclical than $L^S$, regardless of whether the loans are weighted equally (*cf.* Fig. 4.1), or not. As the risk weight $c_1$ has been reduced from 1 to 0.5 (whereas $c_2$ remains fixed to 1), the regulatory constraint function $k(\ell)$ increases (for negligible indirect effects via $\ell^S$). Thus the slope of the

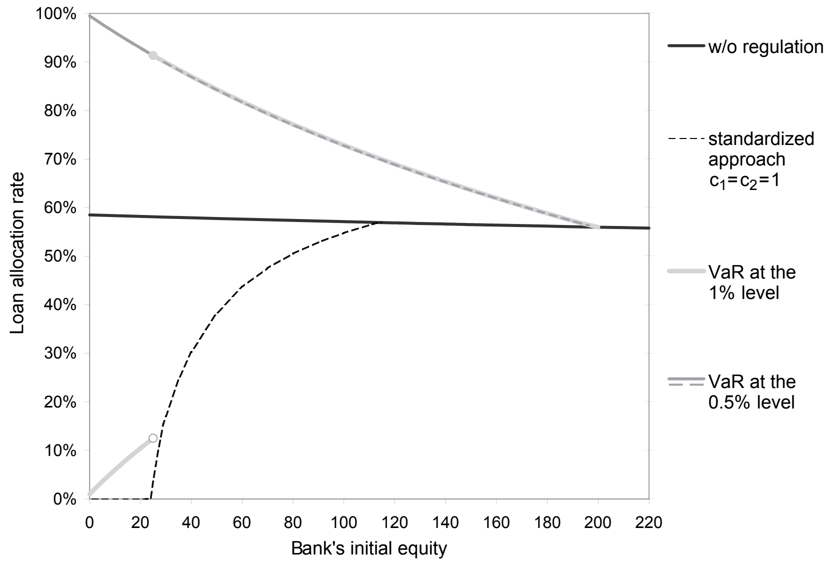


Figure 4.3: **Loan-allocation rate as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium loan-allocation rate $\ell$ on the bank's initial equity $W_B$. The loan-allocation rate is shown for four different regimes: the laissez-faire equilibrium, the Standardized Approach with both risk weights equaling one, $c_i = 1$, the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.5\%$. Cf. Bühler/Koziol/Sygusch (2008, Fig. 2, p. 153).*

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feasible total loan volume becomes steeper with respect to $W_B$. This reduction in the risk weights affects the total loan volume $L^S$ in a pro-cyclical manner compared to both the unregulated case and the case of equal risk weights.

The loan allocation slightly affects the total loan volume, as shown by Figure 4.2 for the Standardized Approach with different risk weights. It is a only piecewise straight line. The different slopes of the total loan volume in the Standardized Approach, as shown in Figure 4.2 can be traced back to the equilibrium loan-allocation rates which are depicted as a function of the bank's equity in Figure 4.4: the total loan volume $L^S$ increases by a slope of 25 for $W_B \leq 13$ because the loan-allocation rate remains at one, $\ell^S = 1.00$. From $W_B = 23$ on, $L^S$ strictly increases at an almost constant rate of roughly 16.15, whereas the total loan volume fluctuates in between: first, $\ell^S$ falls from one to 0.892 for $W_B \in [13, 21]$, and second, $\ell^S$ drops from 0.892 to 0.817 at $W_B = 22$ as the loan allocation migrates from Case 3 to 1.

Concerning the Standardized Approach, we can conclude that regulation seems to be more pro-cyclical as the eligible risk weights decrease.

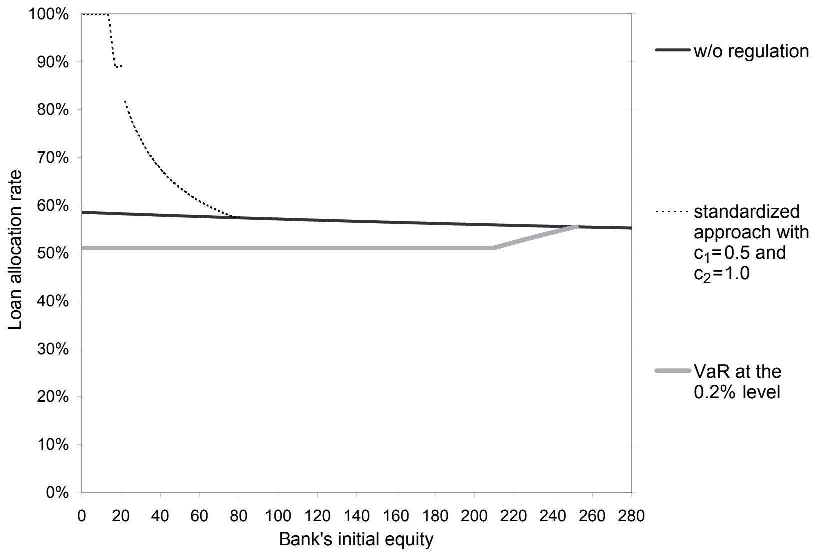


Figure 4.4: **Loan-allocation rate as a function of the bank's initial equity**
*This figure illustrates the dependence of the equilibrium loan-allocation rate $\ell$ on the bank's initial equity $W_B$. The loan-allocation rate is shown for three different regimes: the laissez-faire equilibrium, the Standardized Approach with different risk weights, where $c_1 = 0.5$ and $c_2 = 1$, and the VaR approach with confidence level $\bar{p} = 0.2\%$.*

### 4.2.2 Tighter Confidence Levels and Total Volumes

If the level of confidence is tightened to $1 - p_1 = 0.5\%$, the bank does exactly comply with this level as long as regulation is effectively binding. In contrast, the bank voluntarily switches to this stricter confidence level at $W_B \approx 25$ when the regulatory confidence level is set equal to $1 - p_2 = 1\%$, as illustrated in Figures 4.1 and 4.3.

There are two off-setting effects that give rise to this switch from the weak to the tight level of confidence: granting a higher loan volume to Firm 2 than to Firm 1 is favored if the bank must comply with the weak level, *i.e.* with $\bar{p} = 1 - p_2$. As a result, there are higher expected gains on the loan portfolio which must be rewarded by a higher deposit interest rate to the household, however. If the bank voluntarily decides to meet the tighter confidence level $\bar{p} = 1 - p_1$, expected gains on the loan portfolio will be lower as a preference for Loan 1 will relax the regulatory constraint. As this portfolio composition is less risky, the deposit interest rate is also lower. Furthermore, the bank may even collect a higher deposit volume.

In this example, the bank chooses its optimal loan-allocation rate and its optimal deposit interest rate according to Case 2 if $\bar{p} = 1 - p_2$ is required, and according to Case 3 if $\bar{p} = 1 - p_1$ must be met, as long as $W_B \leq 16$ holds. In particular, the bank allocates its funds according to (3.3.48) and (3.3.49), respectively at $W_B = 0$. The reason is that the regulatory function vanishes at $W_B = 0$, *i.e.* $v(\ell^V, \bar{p}) + \epsilon = 0$, and the bank can freely scale the deposit volume $D^u(\ell^V, R_D; j^V)$ via $R_D$.

For $W_B \leq 25$, the two offsetting effects are in numbers as follows: the difference in the expected gross return on deposits due, $q_j R_D^{V,1-p_2} - q_j R_D^{V,1-p_1}$, ranges from 2.0% up to 2.5%, while the difference in expected gross returns on the loan portfolio ranges between 3.9% and 4.2%. At $W_B = 25$, the marginal expected return on intermediation is still by 1.9 percentage points larger for the bank if it mets $\bar{p} = 1 - p_2$ than if it complies with a confidence level equal to $\bar{p} = 1 - p_1$. But under the tighter confidence level the bank can collect by far more deposits (the difference is equal to approximately 270 dollars), despite of the lower interest rate on deposits to be promised, such that the bank is in total slightly better off by following $\bar{p} = 1 - p_1$.

To sum up, the bank may voluntarily obey to tighter confidence levels than it is obliged to do and a tighter VaR-based regulation does not always result in lower total loan/deposit volumes. Figure 4.1 illustrates this issue by the jump in the total loan volume at $W_B = 25$ if $\bar{p} = 1 - p_2$ is required.

### 4.2.3 Single Loans and Pro-Cyclical Effects

Single loan volumes $L_i$ may react to changes in the bank's initial equity differently than the respective total loan volume $L = L_1 + L_2$. Different ways of regulation also cause differences in sensitivities.

Under the Standardized Approach with equal risk weights the loan volume of Firm 1 remains zero for $W_B \leq 24$, as Figures 4.3 and 4.5 show. At the point where $L_1^S$ turns positive, the equilibrium supply of Loan 2 exhibits a kink (*cf.* Fig. 4.6). At approximately $W_B \approx 30$, the equilibrium migrates from Case 2 to Case 1 which is reflected by the second kink in the loan supply to Firm 2 because the fraction of total loans that can be devoted to Firm 2 must be reduced considerably under Case 1 compared to Case 2.

The low total loan volumes $L^S$ for $W_B$ close to zero under the Standardized Approach with equal risk weights are accompanied by loan-allocation rates $\ell^S$ equal or close to zero. Obviously, the bank thus seeks to set off low volumes with high returns. Figure 4.3 shows that this behavior basically holds for higher equity values $W_B$ as well, albeit to a lesser extent.

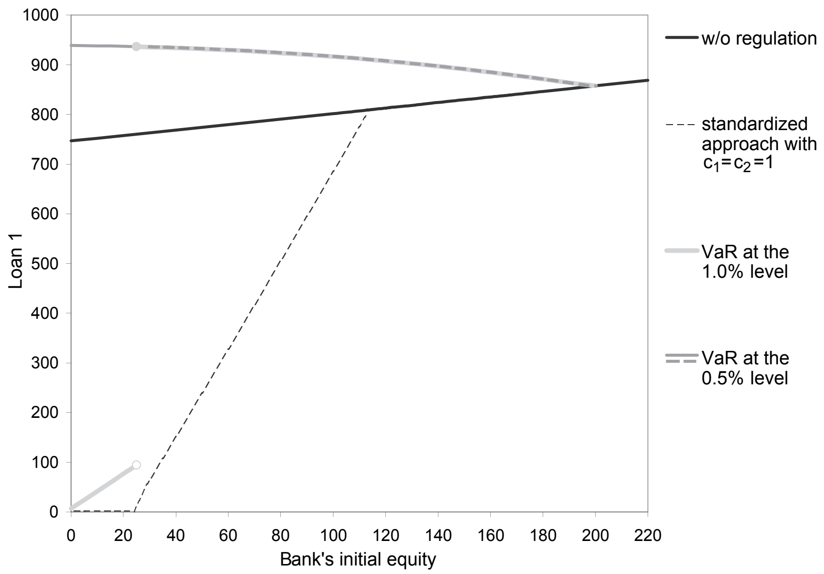


Figure 4.5: **Loan to Firm 1 as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium loan volume granted to Firm 1 on the bank's initial equity $W_B$. The loan volume $L_1$ is shown for four different regimes: the laissez-faire equilibrium, the Standardized Approach with both risk weights equaling one, $c_i = 1$, the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.5\%$.*

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Under the VaR approach, single loan volumes can be larger than in the case of an unregulated bank though the total loan volume is smaller, *cf.* Figures 4.5 to 4.7. The reason is twofold: first, the bank chooses rather extreme loan-allocation rates for intermediate levels of confidence and low values of equity. Figure 4.3 illustrates that point. Particularly, a confidence level of $\bar{p} = 1 - p_2$ favors Firm 2 whereas $\bar{p} = 1 - p_1$ makes the bank prefer to grant credit for Project 1. If, additionally, the regulated total loan volumes are closer to the unregulated total loan volumes than they are to zero, the volume of a single loan under the VaR regulation may be higher than the volume of its unregulated counterpart. With both the regulated and the unregulated total loan volume close to each other, a single loan volume may exceed its unregulated counterpart even if the level of confidence is set to $\bar{p} = 1 - p_1 - p_2 + q$. Figure 4.7 illustrates that point for $L_2^V$ and $L_2^*$ if $\bar{p} = 1 - p_1 - p_2 + q = 0.2\%$. Figures 4.5 to 4.7 show the reactions of single loan volumes toward equity shocks.

Therefore a single firm may even benefit from risk-based capital requirements even though it undertakes more risky projects than its competitors. This effect is also found in the following two numerical examples analyzed in Sections 4.3.1 and 4.4.1.

As regulated loan-allocation rates do not move parallel to their unregulated

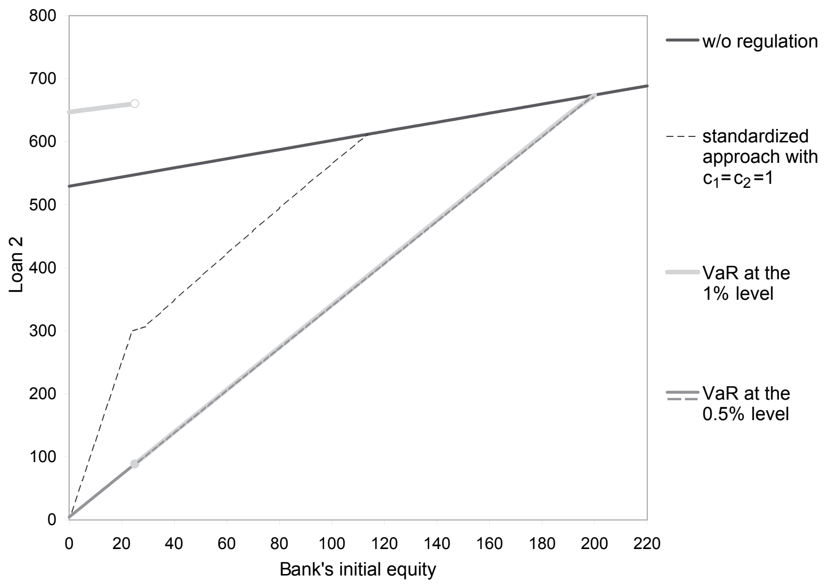


Figure 4.6: **Loan to Firm 2 as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium loan volume granted to Firm 1 on the bank's initial equity $W_B$. The loan volume $L_1$ is shown for four different regimes: the laissez-faire equilibrium, the Standardized Approach with both risk weights equaling one, $c_i = 1$, the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.5\%$. Cf. Bühler/Koziol/Sygusch (2008, Fig. 3, p. 154).*

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counterparts, single loan volumes granted under regulation do not always show pro-cyclical patterns even if total loan volumes do. Particularly, $L_1^S$ is not pro-cyclical for $W_B \leq 24$ when the risk weights are equal, *i.e.* $c_1 = c_2 = 1$, implying $\ell^S = 0$ (Fig. 4.5). Non-pro-cyclical behavior can also be observed for $L_2^S$ if risk weights distinguish between credit risk, $c_1 = 0.5, c_2 = 1$, and the bank's initial equity is small, *i.e.* $W_B \leq 13$, resulting in $\ell^S = 1$. (*cf.* Fig. 4.7). Conversely, $L_2^S$ evolves pro-cyclically for $c_1 = c_2 = 1$ and $L_1^S$ does so for $c_1 < c_2$ except for the areas where migrations from one Case to another occur.

Under the VaR approach things are even more diverse: the loan granted to Firm 1, $L_1^V$, is counter-cyclical for $\bar{p} = 1 - p_1 = 0.5\%$ and pro-cyclical for $\bar{p} = 1 - p_2 = 1.0\%$ and $W_B < 25$. For $\bar{p} = 1 - p_1 = 0.5\%$, $L_2^V$ is pro-cyclical for all $W_B$ and not pro-cyclical if $\bar{p} = 1 - p_2 = 1.0\%$ and $W_B < 25$.[3]

Also the loan volume granted to Firm 2, $L_2^V$, under $\bar{p} = 1 - p_1 - p_2 + q = 0.2\%$ shows pro- and non-pro-cyclical behavior, albeit $L_2^V$ is very flat from $W_B = 210$ on.

To conclude, there seems to be a tendency that single loan volumes are pro-cyclical

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[3]The respective slopes are approximately $\frac{\Delta L_2^V}{\Delta W_B} \in (0.48, 0.58)$ vs. $\frac{\Delta L_2^*}{\Delta W_B} \approx 0.73$.

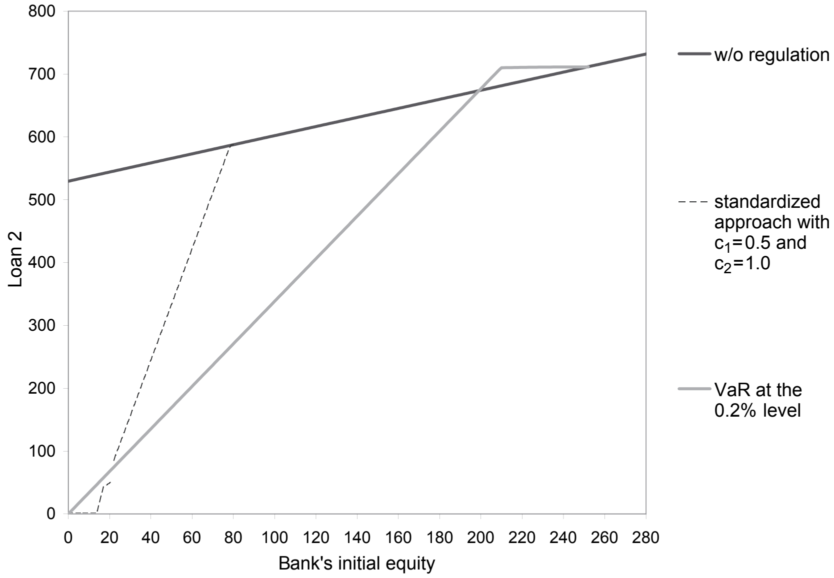


Figure 4.7: **Loan to Firm 2 as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium loan volume granted to Firm 1 on the bank's initial equity $W_B$. The loan volume $L_1$ is shown for three different regimes: the laissez-faire equilibrium, the Standardized Approach with different risk weights, where $c_1 = 0.5$ and $c_2 = 1$, and the VaR approach with confidence level $\bar{p} = 0.2\%$.*

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if their volumes under regulation are lower than their unregulated equivalents.

### 4.2.4 Return Volatilities of Total Loans and Deposits

Without regulation, the risk-neutral bank faces a risk-return trade-off that is solely caused by the risk-averse household's deposit supply. The equilibrium loan-allocation rate is characterized by this difference in risk attitudes. It is shifted in favor of the high-return loan from that loan-allocation rate that would be chosen by the household. This disagreement vanishes if the firms' projects are identically distributed. In this section, we discuss how this conflict of interest is resolved under the different regimes.

Figures 4.3 and 4.4 show the dependence of the loan-allocation rate $\ell$ on the bank's initial equity $W_B$ under different regimes. Without regulation, the decrease in $\ell^*$ with increasing $W_B$ is based on two reasons: on the one hand, increasing bank capital generally means that the monopolistic impact of the risk-neutral bank-owners increases as there is a maximum deposit supply characterized by $\overline{R}_D(\ell; j)$. As a consequence, $\ell^*$ moves in favor of the bank owners. The higher the value of $W_B$,

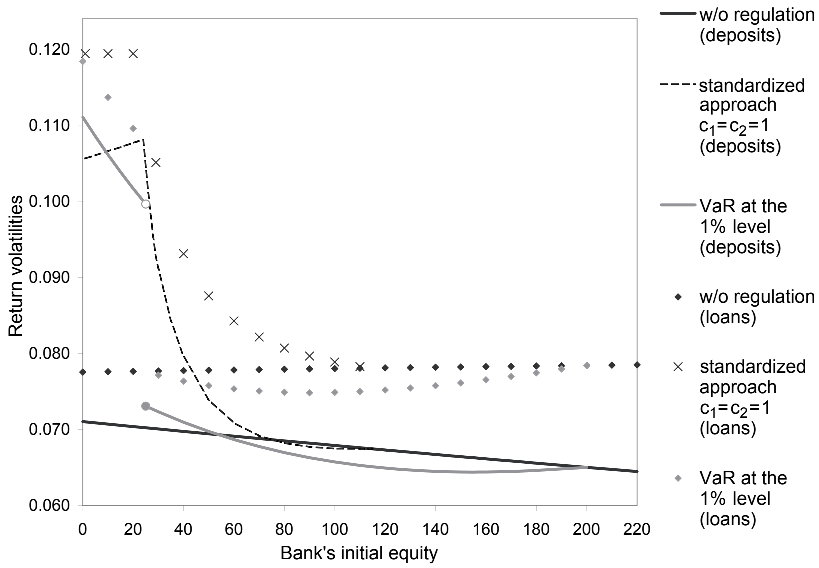


Figure 4.8: **Return volatilities as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium return volatilities on the bank's initial equity $W_B$. Solid and dashed lines represent volatilities of returns on deposits, $\sigma_D$. Diamonds and crosses represent volatilities on total loans, $\sigma$. Return volatilities are shown for three different regimes: the laissez-faire equilibrium, the Standardized Approach with both risk weights equaling $c_i = 1$, and the VaR approach with confidence level $\bar{p} = 1\%$. Source: Bühler/Koziol/Sygusch (2008, Fig. 4, p. 155).*

the greater the share of equity to total funds and the higher the fraction $1 - \ell^*$ the bank grants as a loan to the second, the high-risk (and high-return) firm. On the other hand, higher bank capital means a larger cushion for depositors. The higher the value of $W_B$, the higher the recovery rate of deposits so that deposit supply increases other things being equal, as it has been pointed out for some specific equilibria in Chapter 3. Hence, the bank may lower the loan-allocation rate $\ell^*$ and thus increase the expected returns and volatilities without forgoing an increase in deposit volume or without increasing deposit interest rates. Figure 4.2 illustrates this point in connection with Figures 4.3 and 4.10. Concerning the volatility of return on total loans, we refer to Figure 4.8.

Figure 4.3 shows that regulation by identical risk weights has adverse effects on the optimal portfolio. The low-risk loan is always less weighted than it is without regulation. This choice by the bank owners can be explained as follows: under the Standardized Approach with equal risk weights, the total loan volume that the bank can grant is fixed in proportion to $W_B$. Hence, with decreasing equity, the total loan volume shrinks by a rate of 12.5. The bank offsets the low volume with

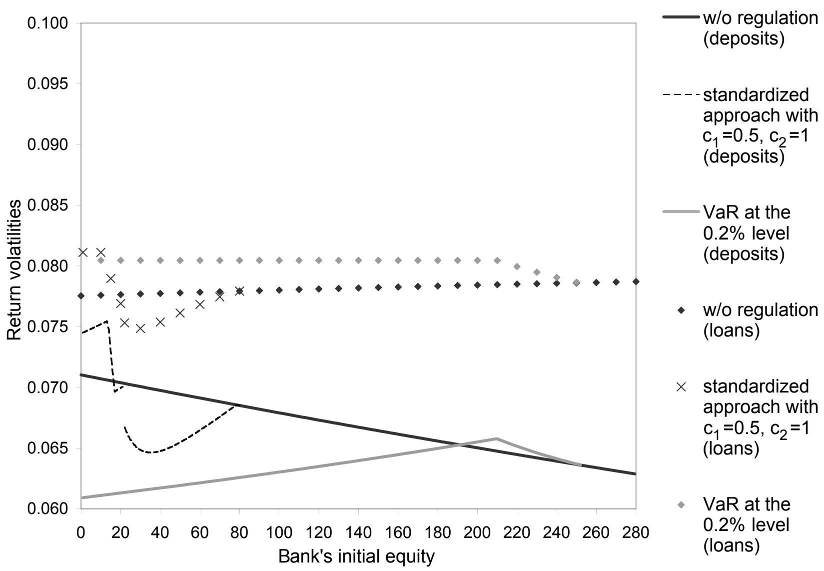


Figure 4.9: **Return volatilities as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium return volatilities on the bank's initial equity* $W_B$*. Solid and dashed lines represent the volatilities of returns on deposits. Diamonds and crosses represent the volatilities of returns on total loans. Return volatilities are shown for three different regimes: the laissez-faire equilibrium, the Standardized Approach with different risk weights, where* $c_1 = 0.5$ *and* $c_2 = 1$*, and the VaR approach with confidence level* $\bar{p} = 0.2\%$*.*

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higher expected returns by widening the fraction of total capital granted as a loan to the second firm.

If risk weights sufficiently account for differences in the credit risk of loans, the regulated bank may prefer the low-risk loan over the high-risk loan. Unlike when it is dealing with equal risk weights, the bank chooses a loan-allocation rate of one, $\ell^S = 1.00$, if $W_B \leq 13$ holds given $c_1 = 0.5$ and $c_2 = 1$. For $W_B \geq 14$ until regulation ceases to bind, $\ell^S > \ell^*$ still holds (*cf.* Figures 4.3 and 4.4). But as a consequence, regulation mainly corrodes incentives for diversification. Let us measure the depositor's risk by the return volatility $\sigma_D$ of deposits, defined as

$$\sigma_D \equiv \sqrt{\mathrm{V}(\frac{\tilde{D}_j}{D})} \,, \tag{4.2.1}$$

where $j = 1, \ldots, 4$ denotes one of the Cases and $\tilde{D}_j$ is the stochastic, Case-dependent and state-dependent redemption of deposits as partially outlined in (3.2.23) and (3.2.23).

Figures 4.8 and 4.9 show the return volatilities of the bank's loan portfolios and the return volatilities of the associated deposits under different regimes. The former are depicted by diamonds for the unregulated case and for the VaR approach and by crosses for the Standardized Approach. The latter are represented by solid and dashed lines, respectively.

Clearly, the bank absorbs the return volatility from its loan portfolio such that the returns on deposits are always less volatile. Two mechanisms are at work: the pay-off structure of the deposit contract truncates extreme (though favorable) outcomes and thus lessens volatility, whereas the residual volatility is absorbed by the bank's initial equity $W_B$. These two effects can be particularly well studied in the case without regulation: The gap between the volatilities of returns on deposits (black solid line) and the volatilities of returns on total loans (black diamonds) at $W_B = 0$ reflects the cut in volatility by the upper bound the deposit contract imposes, while the growing gap between those volatilities represents the cushion effect. The risk-absorbing role banks play is emphasized by Bitz (2006, p. 4, pp. 14-21). Greenbaum/Thakor (2007, p. 108) understand this risk transformation function, which is enhanced by the property that single loan volumes often exceed single deposit volumes, even as a driver for monopoly power in banking.

Figure 4.9 shows that depositors are not better protected for all levels of bank equity by the Standardized Approach than they are in case of an unregulated bank. In fact, the depositor is worse off in these terms under the Standardized Approach whenever the return volatility on the regulated loan portfolio is higher than the return volatility on the unregulated loan portfolio. One observes a similar result if risk weights are fixed, *cf.* Figure 4.8: depositors face a lower return volatility on their deposits under regulation compared to the unregulated case if the regulated loan portfolio's return volatility approaches that of the unregulated loan portfolio. Lowering the risk weight $c_1$ from 1 to 0.5 reduces the overall return volatilities for both the loan portfolio and for deposits, as the comparison of Figure 4.8 and Figure 4.9 reveals.

This also applies to the volatilities of the returns on deposits under the VaR approach: a stricter level of confidence decreases the deposit return volatility. However, this is not true for the volatilities of portfolio returns. Furthermore, depositors may bear more risk in terms of volatilities under the VaR approach than they do under the unregulated regime if the high-risk loan $L_2^V$ outweighs its unregulated counterpart $L_2^*$ in volume. By and large, the deposit return volatilities under $\bar{p} = 1 - p_1 = 0.5\%$ are between the return volatilities under the next weaker, $1 - p_2$, and the next stricter, $1 - p_1 - p_2 + q$, level of confidence.

Figures 4.8 and 4.9 show, in essence, that the stricter confidence levels are under a

VaR approach or the more risk weights distinguish between high- and low risk loans, the better the protection for the depositors. Despite this property, the depositors may still bear more risk in terms of return volatilities than they do in a world without regulation. This holds regardless of whether regulation is risk-sensitive or not. In contrast, the volatility of deposit redemptions, $V(\tilde{D})$, is always reduced through regulation which is due to the volume reductions.

### 4.2.5 Deposit Interest Rates

Figure 4.10 shows that the deposit interest rate $R_D^*$ offered by the unregulated bank strictly decreases in $W_B$, reflecting the fact that as $W_B$ increase, it becomes easier for the bank to collect deposits from the risk-averse household.

The behavior of the deposit interest rate under the various regulatory regimes can be essentially traced back to two roots: increasing interest rates are present if the bank primarily seeks to collect more deposits as the maximum feasible total loan/deposit volume increases with $W_B$. We refer to this phenomenon as the volume aspect. In this context, the bank suffers from the positive relation between deposit volume and deposit interest rates in equilibrium, as stated in Result 2.

The risk aspect is understood to be the property that the deposit interest rate decreases with increasing equity, as the default risk of deposits thus decreases.

In particular, the volume aspect outweighs the risk aspect if the risk changes only slightly or if it remains unchanged, as is the case under the Standardized Approach for low values of bank equity. Interest rates strictly decrease with $W_B$ under regulation if the bank considerably reduces risks with increasing equity. This is particularly true under the VaR approaches for low values of equity. Otherwise, the volume effect dominates.

Although the deposit interest rate seems to be the main instrument for the bank to reduce deposit volumes to the volumes that are allowed by regulation, regulation may induce the bank to choose optimal loan portfolios that are so risky that the bank must compensate the risk-averse depositor for the higher risk with higher interest rates. Thus, the regulated bank promises higher deposit interest rates than it would do without regulation. Such an increase in deposit interest rates can be observed under the VaR approach for $\bar{p} = 1 - p_2 = 1\%$ and $W_B < 25$, as illustrated in Figure 4.10.

In contrast, deposit interest rates under regulation are always lower than their

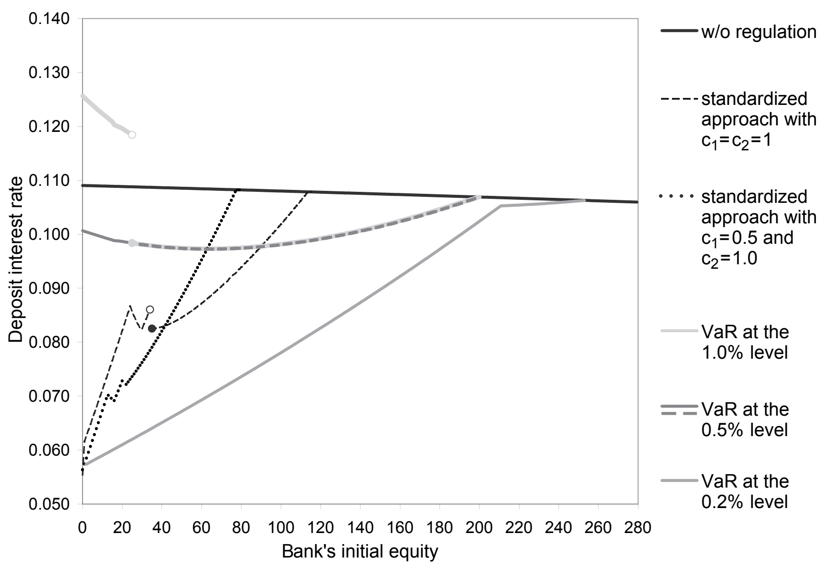


Figure 4.10: **Deposit interest rate as a function of the bank's initial equity**
*This figure illustrates the dependence of the equilibrium deposit interest rate $R_D$ on the bank's initial equity $W_B$. Deposit interest rates are shown for all six different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with the confidence levels $\bar{p} = 1\%$, $\bar{p} = 0.5\%$ and $\bar{p} = 0.2\%$.*

unregulated counterparts for equally distributed firm projects because the minimum variance portfolio is chosen regardless of the regime; *i.e.* $\ell = \frac{1}{2}$ holds, as shown by Results 15, 22, 26, and 29.

Concerning cyclicality, we can observe the following: if the risk aspect seems to dominate the choice of the deposit interest rate, deposit interest rates may become pro-cyclical since they decrease under both regimes. If the volume aspect seems to dominate the choice of the deposit interest rate, deposit interest rates become counter-cyclical since they increase, while $R_D^*$ strictly decreases.

## 4.3 Credit Risk Shocks

### 4.3.1 Total Volumes and Cyclical Dampening

Shocks in credit risk are modeled by parallel shifts in the single and in the common success probabilities. That is, the parameters $p_1$, $p_2$, and $q$ are scaled by a common multiplier $m$. The lower Bound (3.2.11) on the joint default probability $q$ implies $1 - p_1 - p_2 + q > 0$, and thus, if all the probabilities $p_1$, $p_2$, and $q$ are jointly scaled

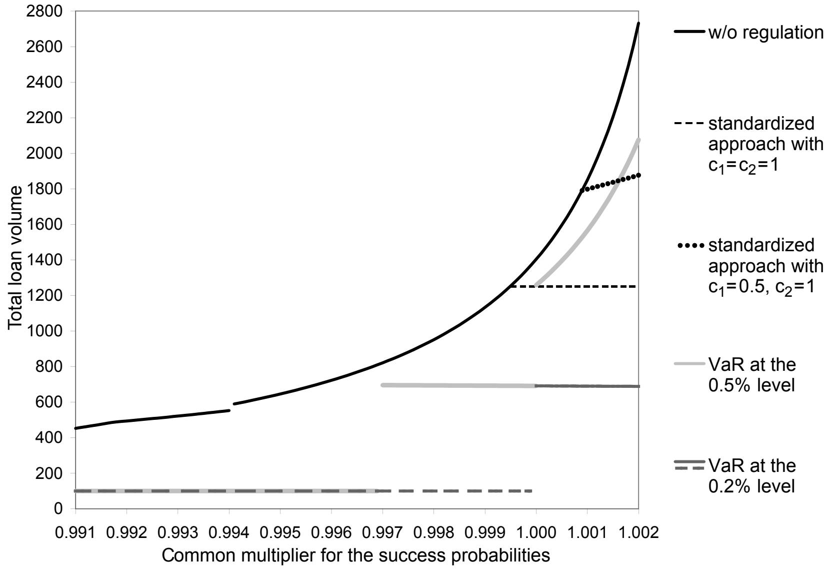


Figure 4.11: **Total loan volume as a function of shifts in success probabilities**
*This figure illustrates the dependence of the equilibrium total loan volume* $L$ *on shifts in each project's success probability* $p_i$ *and in their common success probability* $q$. *Shifts in success probabilities are modeled using a common multiple* $m$, $0.991 \leq m \leq 1.002$. *Total loan volumes are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with the confidence levels* $\bar{p} = 0.5\%$ *and* $\bar{p} = 0.2\%$.

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by $m$, results in the following upper bound for $m$ whereas Condition (3.2.6) in the following lower bound such that $m$ is restricted to

$$\frac{R_f}{p_1 \alpha_1} < m < \frac{1}{p_1 + p_2 - q} \overset{\text{Def. (3.2.41)}}{\equiv} \frac{1}{q_4} .$$

Economically, the lower bound is set such that the expected gross return on the first firm's project exceeds the risk-free rate. The upper bound guarantees that the deposit redemption remains risky under Case 4. Furthermore, the upper bound implies $mp_i < 1$, as $p_i < q_4$, $i = 1, 2$.

As a result, the correlation $\text{Corr}(\tilde{X}_1, \tilde{X}_2)$ strictly decreases in $m$,

$$\frac{\partial \text{Corr}(\tilde{X}_1, \tilde{X}_2)}{\partial m} = -\frac{p_1(p_2 - q)(1 - mp_2) + p_2(p_1 - q)(1 - mp_1)}{2\sqrt{p_1 p_2} \cdot \sqrt{(1 - mp_1)(1 - mp_2)}^3} < 0 . \quad (4.3.1)$$

Given the bounds imposed on $m$ from above, the correlation between the projects

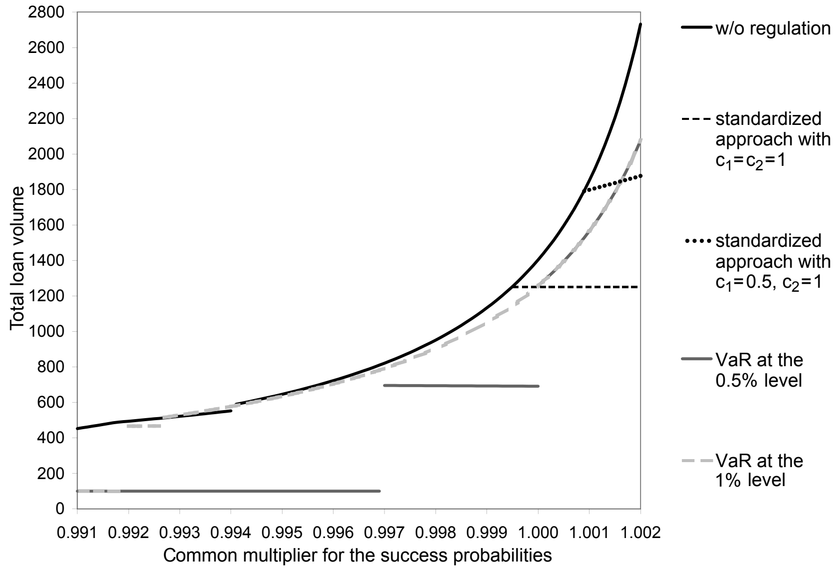


Figure 4.12: **Total loan volume as a function of shifts in success probabilities**

*This figure illustrates the dependence of the equilibrium total loan volume $L$ on shifts in each project's success probability $p_i$ and in their common success probability $q$. Shifts in success probabilities are modeled using a common multiple $m$, $0.991 \leq m \leq 1.002$. Total loan volumes are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with the confidence levels $\bar{p} = 1\%$ and $\bar{p} = 0.5\%$. Cf. Bühler/Koziol/Sygusch (2008, Fig. 6, p. 157).*

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thus varies between

$$\frac{q(p_1+p_2-q)-p_1p_2}{\sqrt{p_1p_2(p_1-q)(p_2-q)}} \;<\; \mathrm{Corr}(\tilde{X}_1,\tilde{X}_2) \;<\; \frac{q\alpha_1-p_2R_f}{\sqrt{p_2(\alpha_1-R_f)(p_1\alpha_1-p_2R_f)}}\ .$$

Given the parameter values from Table 4.1, we let the multiplier $m$ take values from $[0.9500, 1.0020]$ in steps of 0.0001. The upper bound is slightly below $\frac{1}{p_1+p_2-q}$. The lower bound was chosen because the regulation under fixed risk weights becomes binding for values far above 0.9500 (*cf.* Fig. 4.11), and because the VaR approach with a confidence level of $\bar{p} = 1\%$ does not result in strictly positive deposit volumes for $m < 0.9920$ (*cf.* Fig. 4.12).[4]

Using the parameter values from Table 4.1, the correlation lies in the interval $(-0.004936, 0.9383)$. The parameter choice $m \in [0.9500, 1.0020]$ reduces this range to $[-0.004122, 0.9039]$ (bounds rounded to four leading digits each).

Because lower values for the multiplier $m$ imply higher probabilities of default, low values of $m$ can be associated with a less favorable state of the economy. Moreover,

[4]The total loan volume $L^V$ reduces to the bank's initial equity $W_B$, in this example $W_B = 100$.

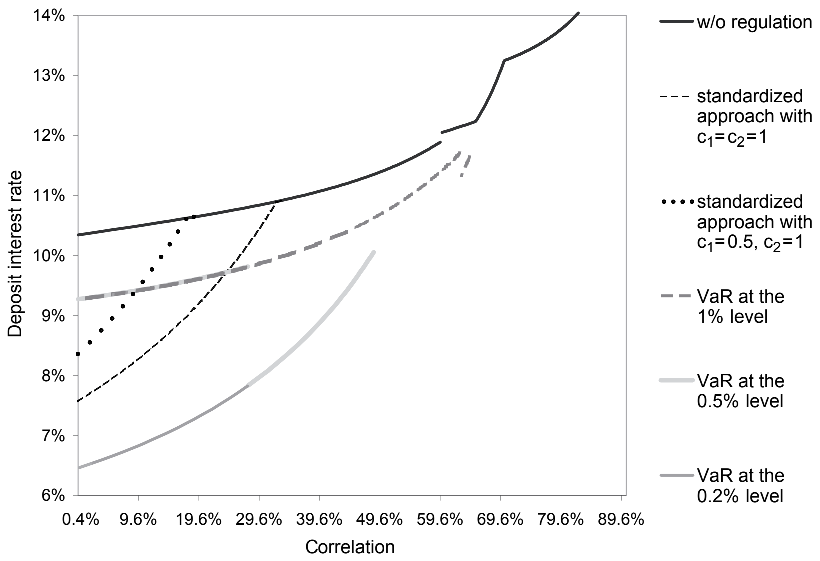


Figure 4.13: **Deposit interest rate as a function of the correlation**

*This figure illustrates the dependence of the equilibrium deposit interest rate $R_D$ on the project returns' correlation. Changes in the correlation are based on joint shifts in each project's success probability $p_i$ and in their common success probability $q$. These shifts are modeled using a common multiple $m$, $0.991 \leq m \leq 1.002$. Deposit interest rates are shown for all six different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with the confidence levels $\bar{p} = 1\%$, $\bar{p} = 0.5\%$ and $\bar{p} = 0.2\%$.*

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a bad economic situation in terms of higher default probabilities is linked to a higher (default) correlation, since $\mathrm{Corr}(\tilde{X}_1, \tilde{X}_2)$ strictly decreases in $m$ according to (4.3.1).[5] The regulatory formula for the average asset-return correlation establishes the qualitative relation (*cf.* BCBS, 2004, Art. 272) which has been confirmed by empirical studies such as those by Lopez (2004) and Vondra/Weiser (2006). Beyond empirical evidence and the purely mathematical relation between default probabilities and default correlations, covariations in asset prices may even arise in downturns when asset prices are otherwise stochastically independent. This may be due to agents' wealth constraints or fire sales, in particular due to risk-sensitive capital regulation as Daníelsson/Zigrand (2008) show in their general equilibrium model.

Figures 4.11 and 4.12 show the total loan volumes under the various regimes dependent on shifts in the success probabilities $p_1$, $p_2$, and $q$ by the multiplier $m$,

[5] Erlenmaier/Gersbach (2001) use a Merton-type framework and show in their Proposition 1, p. 6, that default correlations increase for increasing default probabilities as long as they remain below 0.5. Gersbach/Lipponer (2003) take a broader view on the relation between shocks, default probabilities, and default correlations.

with $m = 1$ referring to the base case. The higher these success probabilities are, the higher the total loan volume without regulation becomes. This is intuitive: the lower the default risk, the higher the propensity of the risk-averse household to supply deposits for given loan-allocation rates and for given deposit interest rates. Moreover, as risks are reduced, the bank can even lower deposit interest rates in equilibrium, as illustrated in Figure 4.13. The abscissa is drawn in terms of correlations and thus the graphs must be reversed in terms of the multiplier $m$.

Especially, deposit interest rates increase under all regimes with the loans' correlation, hence with aggregate risk. This monotonic relation holds except for the point at which the equilibrium changes from one Case to another, and the point at which the structure of the level of confidence effectively attained by the bank changes.

The change from Case 1 to Case 2 happens at a correlation of 59.89% ($m = 0.9940$, *cf.* also Fig. 4.11 and 4.14) in the laissez-faire equilibrium. The steep ascent in $R_D^*$ thereafter reflects a reduction in the loan-allocation rate from $\ell^* = 25.59\%$ to zero, resulting in higher risks for the depositor beyond the shifts in the correlation/the probabilities.

If a confidence level of $\bar{p} = 0.5\%$ is required, the bank will decide to comply with $\bar{p} = 1 - mp_1 - mp_2 + mq$ from a correlation of 27.79% on, *i.e.* below $m = 1.0000$. As a consequence, the bank effectively attains confidence levels between 0.2% and 0.4994% for correlations ranging from 28.76% to 48.56% (corresponding to multipliers from $m = 0.9999$ down to 0.9970). The change is also reflected in a jump in the total loan volume granted, as shown in Figure 4.11 with respect to $m$ and in Figure 4.14 with respect to the correlation. From $m = 1.0000$ on (that is, for correlations equal to and below 27.79%) the bank attains confidence levels varying from 0.3010% to 0.5%, corresponding to the structural level of $1 - mp_1$. The associated higher deposit interest rate results in a sudden increase of the total loan volume (*cf.* Fig. 4.11 and 4.14).

If the bank must comply with $\bar{p} = 1.0\%$, there is another jump from $m = 0.9926$ to 0.9927. This jump is solely characterized by a considerable increase in the loan-allocation rate from 27.06% to 33.84%. At this point, there is neither a Case migration nor a structural change in the level of confidence that is effectively met. But a structural change such as this also occurs under this regulatory regime, namely at $m = 0.9960$, corresponding to a correlation of 53.01%. Below this critical value of $m$, the bank effectively complies with a structural level of $1 - mp_1 - mp_2 + mq$, corresponding to confidence levels ranging from 0.61% to 1.00%. Above this critical

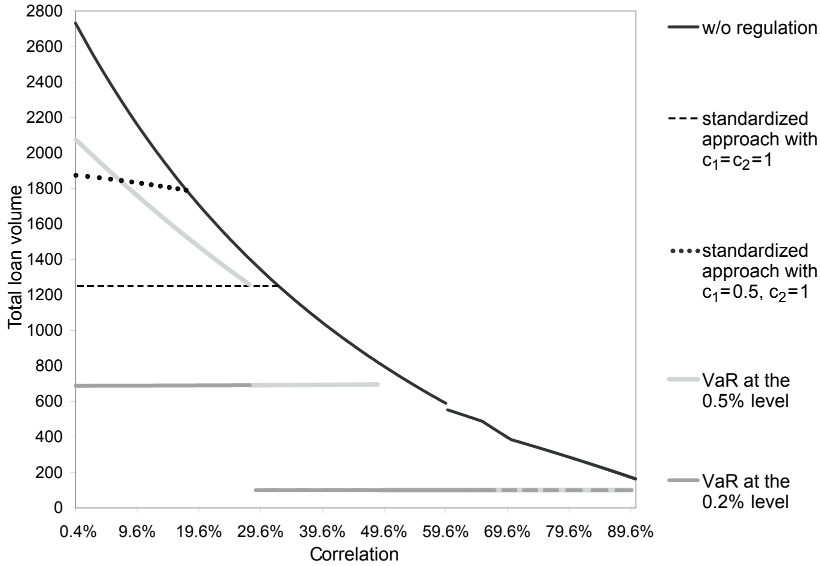


Figure 4.14: **Total loan volume as a function of the correlation**

*This figure illustrates the dependence of the equilibrium total loan volume $L$ on the project returns' correlation. Changes in the correlation are based on joint shifts in each project's success probability $p_i$ and in their common success probability $q$. These shifts are modeled using a common multiple $m$, $0.991 \leq m \leq 1.002$. Deposit interest rates are shown for all six different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with the confidence levels $\bar{p} = 1\%$, $\bar{p} = 0.5\%$ and $\bar{p} = 0.2\%$.*

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value of $m$, the bank voluntarily complies with a level equal to $1-mp_1$, corresponding to values from 0.30% to 0.90%.

Deposit interest rates are smaller under regulation because the deposit volume is lower and risks are reduced. However, they react more strongly on changes in correlation and success probabilities. Thus, regulation affects interest rates in a pro-cyclical manner even if it does not do so with the total loan/deposit volumes.

As the total loan volume is strictly increasing in $m$ and as the total loan volume that is feasible under regulation is always lower than the unregulated total loan volume, there is a $m$ above which regulation is binding. As regulation remains binding from each of these critical values for $m$ on, the regulated total loan volumes are on average not pro-cyclical. Inversely, the regulated bank behaves as if it were unregulated in severe economic contractions (equivalent to higher correlations or lower values for $m$). Hence, regulation rather deters the bank from participating in the fruits of an upswing. These issues are illustrated in Figures 4.11, 4.12, and 4.14.

There is, however, one exception in our example. Under the VaR approach with

Table 4.2: **Effects on the total loan volume by VaR-based regulation, Bernoulli model**

*This table reports equilibria under a VaR-based regulation with a required confidence level of* 1% *and the corresponding equilibria in the laissez-faire economy.* $m$ *is the joint multiplier referring to the success probabilities* $p_i$ *and to the common success probability* $q$*. Otherwise, all parameters are given as in Table 4.1. We note that the total loan/deposit volume under regulation exceeds its unregulated counterpart for* $0.9927 \leq m \leq 0.9940$.

| | | **w/o regulation** resulting in $\bar{p}^* = 1 - mp_2$ for $0.9862 \leq m \leq 0.9967$ | | | | | | **VaR regulation at** 1% resulting in $\bar{p}^V = 1 - mp_1 - mp_2 + mq$ for $0.9920 \leq m \leq 0.9959$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $m$ | $\text{Corr}(\tilde{X}_1, \tilde{X}_2)$ | $\ell^*$ | $R_D^*$ | $j^*$ | $D^*$ | $\text{E}[\tilde{W}_B^*]$ | $\sigma^*$ | $\ell^V$ | $R_D^V$ | $j^V$ | $D^V$ | $\text{E}[\tilde{W}_B^V]$ | $\sigma^V$ |
| 0.9920 | 64.99% | 25.41% | 1.1222 | 2 | 394.2 | 141.4 | 0.01772 | 26.89% | 1.1178 | 1 | 366.4 | 141.2 | 0.01746 |
| 0.9922 | 64.54% | 25.22% | 1.1220 | 2 | 396.8 | 141.9 | 0.01750 | 26.95% | 1.1157 | 1 | 366.7 | 141.6 | 0.01718 |
| 0.9924 | 64.08% | 25.03% | 1.1219 | 2 | 404.8 | 142.3 | 0.01728 | 27.00% | 1.1147 | 1 | 366.9 | 142.0 | 0.01690 |
| 0.9926 | 63.61% | 24.84% | 1.1217 | 2 | 410.3 | 142.8 | 0.01706 | 27.06% | 1.1137 | 1 | 367.2 | 142.4 | 0.01661 |
| 0.9927 | 63.36% | 24.75% | 1.1216 | 2 | 413.1 | 143.1 | 0.01695 | 33.84% | 1.1174 | 1 | 415.8 | 142.7 | 0.01583 |
| 0.9930 | 62.62% | 24.47% | 1.1214 | 2 | 421.7 | 143.9 | 0.01662 | 35.47% | 1.1167 | 1 | 428.8 | 143.5 | 0.01527 |
| 0.9932 | 62.10% | 24.28% | 1.1212 | 2 | 427.6 | 144.4 | 0.01640 | 36.55% | 1.1162 | 1 | 437.8 | 144.0 | 0.01490 |
| 0.9934 | 61.57% | 24.09% | 1.1210 | 2 | 433.6 | 144.9 | 0.01619 | 37.63% | 1.1156 | 1 | 447.2 | 144.6 | 0.01454 |
| 0.9936 | 61.03% | 23.90% | 1.1209 | 2 | 439.8 | 145.5 | 0.01597 | 38.71% | 1.1151 | 1 | 456.8 | 145.1 | 0.01418 |
| 0.9938 | 60.47% | 23.71% | 1.1207 | 2 | 446.2 | 146.1 | 0.01575 | 39.79% | 1.1146 | 1 | 466.8 | 145.7 | 0.01382 |
| 0.9940 | 59.89% | 23.52% | 1.1205 | 2 | 452.7 | 146.7 | 0.01553 | 40.86% | 1.1141 | 1 | 477.1 | 146.4 | 0.01347 |
| 0.9941 | 59.60% | 33.65% | 1.1189 | 1 | 489.6 | 147.0 | 0.01420 | 41.40% | 1.1134 | 1 | 482.4 | 146.7 | 0.01330 |
| 0.9944 | 58.68% | 34.97% | 1.1183 | 1 | 507.3 | 148.0 | 0.01368 | 43.01% | 1.1131 | 1 | 498.8 | 147.7 | 0.01278 |

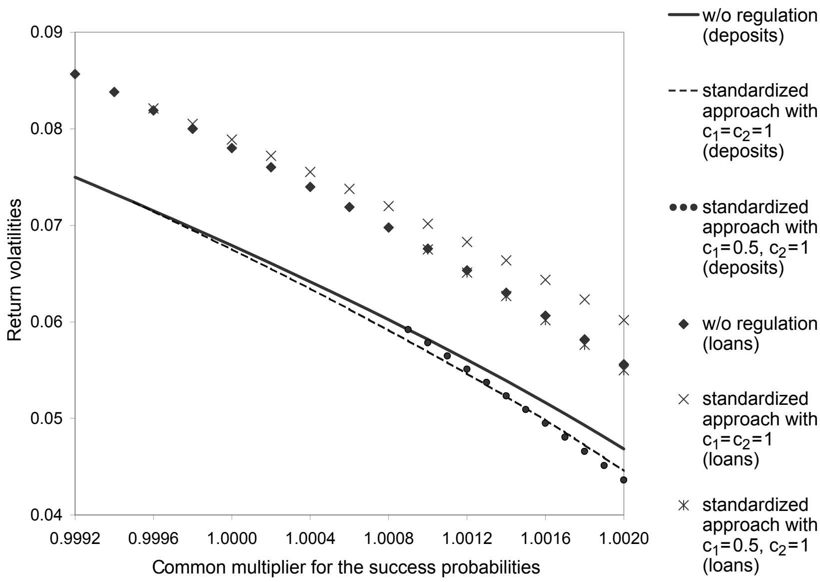


Figure 4.15: **Return volatilities as a function of shifts in success probabilities**

*This figure illustrates the dependence of the equilibrium return volatilities on shifts in each project's success probability $p_i$ and in their common success probability $q$. Shifts in success probabilities are modeled using a common multiple $m$, $0.991 \leq m \leq 1.002$. Solid and dashed lines and dots, respectively, represent volatilities of returns on deposits, $\sigma_D$. Diamonds and crosses represent volatilities on total loans, $\sigma$. Return volatilities are shown for three different regimes: the laissez-faire equilibrium, and the Standardized Approach with both equal and different risk weights.*

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$\bar{p} = 1\%$, we observe that the *total* loan volume $L^V$ exceeds the unregulated total loan volume $L^*$. This is surprising as a binding Regulatory Constraint 3.3.45 suggests that the regulated total loan volume is always lower than its unregulated counterpart. But the required level of confidence is essential, too.

In this example, the bank keeps a level of confidence of $1 - mp_2$ for $0.9927 \leq m \leq 0.9940$ (corresponding to correlations ranging between 63.36% and 59.89%) without regulation. This structural level of confidence is associated with a low fraction of total funds that the bank grants as a loan to Firm 1 (*cf.* Fig. 4.16). The result is associated with Case 2. But if the bank is required to meet a confidence level of 1%, it chooses a structural level of confidence equal to $1 - mp_1 - mp_2 + mq$ for $0.9920 \leq m \leq 0.9959$, corresponding to correlation values between 64.99% and 53.41%.

The resulting reduction in risk by the VaR-approach is that essential for some values of the multiplier $m$, that the regulated bank issues more deposits than if it is unregulated. Table 4.2 illustrates this phenomenon in numbers. It can also

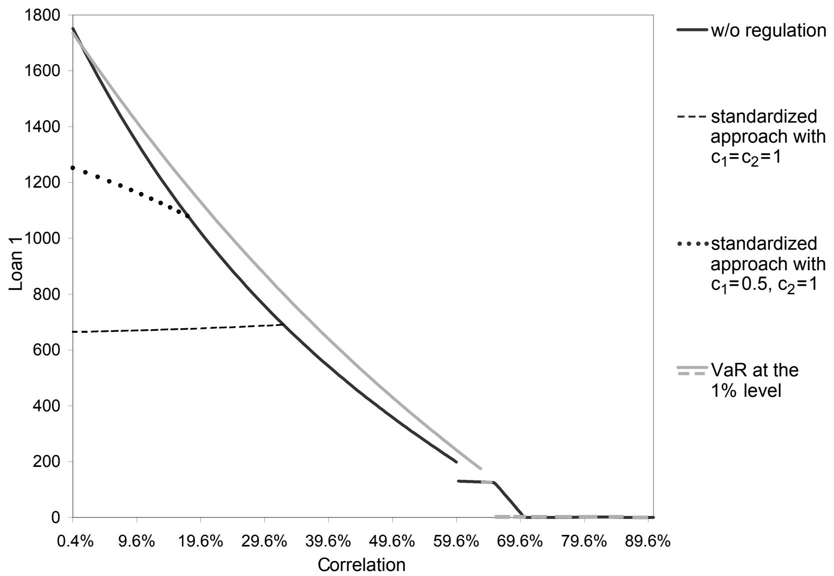


Figure 4.16: **Loan to Firm 1 as a function of the correlation**

*This figure illustrates the dependence of the equilibrium loan volume granted to Firm* 1 *on the project returns' correlation. Changes in the correlation are based on joint shifts in each project's success probability* $p_i$ *and in their common success probability* $q$. *These shifts are modeled using a common multiple* $m$, $0.991 \leq m \leq 1.002$. *The loan volume* $L_1$ *is shown for four different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR with the confidence level* $\bar{p} = 1\%$.

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be seen in Figure 4.17. The regulated total loan volume exceeds its unregulated counterpart where the absolute difference between the volatilities of gross deposit returns is up to a multiple of 10 higher that it is the case close to this area. In numbers, this gap ranges between 0.0012 and 0.00206 where the unregulated total loan volume is higher and only between 0.00026 and 0.00090 where it is not.

From $m = 0.9941$ on, the unregulated total loan volume $L^*$ is again larger than $L^V$ at the 1%-level. As long as $L^V > L^*$ holds here, $L^V$ is also more sensitive toward changes in $W_B$. Hence, the VaR approach makes total lending pro-cyclical on this domain.

Otherwise the VaR approach at the 1% confidence level is not pro-cyclical either, but it shows the highest sensitivity except for the jump discontinuities with the other VaR regimes (*cf.* Fig. 4.12). In contrast, the Standardized Approach with equal risk weights has zero sensitivity to $W_B$. The latter is obvious as by $c_1 = c_2$ the total loan volume $L^S$ is given by $\frac{1}{c \cdot c_1} \cdot W_B$ and hence $L^S$ is independent of changes in risk unless $c_1$ moves. Under the Standardized Approach with $c_1 = 0.5$ and $c_2 = 1$, the total loan volume strictly increases in $m$ albeit more slightly than $L^*$.

### 4.3.2 Single Loans and the Role of Constant Risk Weights

Figure 4.16 reveals that the bank, regulated with $c_1 = 0.5$, pushes lending to Firm 1 when correlation decreases; that is, when success probabilities rise. Here again, the lower risk weight makes the low-risk, but also low-return loan more attractive to the bank as one dollar lent requires only 0.04 cents of equity, *i.e.* half the equity than it is the case for $c_1 = 1$. Thus, the total loan volume can be increased by increasing the loan-allocation rate as the monotonicity and convexity of the regulatory constraint function $k(\ell)$ suggests, *cf.* (3.3.23) and (3.3.24). Higher loan volumes, in turn, lead to higher expected wealth for the bank. In this example, the regulated bank increases the total loan volume by choosing a loan-allocation rate $\ell^S$ that even exceeds $\ell^*$.

Figure 4.15 shows that the loan-allocation rate $\ell^S$ chosen under $c_1 = 0.5$ reduces the volatility of returns on the loan portfolio compared to the case with equal risk weights and compared to the unregulated case. Comparing both instances of the Standardized Approach shows that risk-reducing effect on the deposit return volatility is fairly small and ambiguous in this example.

Although the loan-allocation rate $\ell^S$ is higher than $\ell^*$ for $c_1 = 0.5$, the volume lent to Firm 1 is not pro-cyclical according to Definition (2.2.2), neither with respect to the correlation nor to the shift parameter $m$. Given $c_1 = c_2 = 1$, the volume granted to Firm 1, $L_1^S$, is counter-cyclical.

If $c_1 = 0.5$ and $c_2 = 1.0$, the high-risk firm, Firm 2, is granted less when expectations become more optimistic. Concerning equal risk weights, we observe the opposite as economies in equity are no longer present. That is, the better the prospects, the higher the volume of $L_2^S$ becomes. The unregulated bank also increases $L_2^*$ with expectations turning better. As a result, $L_2^S$ is not pro-cyclical if $c_1 = c_2 = 1$ holds, but is counter-cyclical for $c_1 = 0.5$ with $c_2 = 1$.

Although risk weights that reflect the differences in credit risk, in this example by $c_1 = 0.5$ and $c_2 = 1$, result in loan portfolio compositions that are less risky than in the unregulated case, equal risk weights also show a positive effect. If the success probabilities decrease, the loan volume granted to the less risky loan increases, albeit weakly. This stabilizing effect by regulation is, however, qualified by the fact that regulation is no longer binding from a point on at which the probabilities are sufficiently low.

### 4.3.3 Flexible Risk Weights

If we allow the risk weights $c_i$ to move with shifts in the default probabilities, results will be mixed as risk weights will be both equal and different. Consider the different eligible values of the risk weights according to the Standardized Approach, *cf.* Table 3.4. Let $c_i = 0.5$ if $1 - mp_i \leq 0.50\%$ and $c_i = 1$ if $1 - mp_i \leq 2.00\%$ is fulfilled, and $c_i = 1.5$ else.[6] Then the risk weights $c_i$ take the following values dependent on $m$,

$$c_1 = \begin{cases} 1.5 & \text{if} \quad 0.9176 < m \leq 0.9849 \\ 1.0 & \text{if} \quad 0.9850 \leq m \leq 1.0000 \\ 0.5 & \text{if} \quad 1.0001 \leq m < 1.0021 \end{cases}, \quad c_2 = \begin{cases} 1.5 & \text{if} \quad 0.9176 < m \leq 0.9898 \\ 1.0 & \text{if} \quad 0.9899 \leq m < 1.0021 \end{cases}.$$

Comparing eligible total loan volumes to those chosen by the unregulated bank, leads to the conclusion that regulation is binding on $m \in [0.9995, 1.0000]$ and on $[1.0009, 1.0020]$. Moreover, on $[0.9995, 1.0000]$, both risk weights are equal to one, whereas on $[1.0009, 1.0020]$, $c_1 = 0.5$ and $c_2 = 1$ holds. In between, that is on $[1.0001, 1.0008]$, $c_1 = 0.5$ and $c_2 = 1$ holds true as well and regulation is not binding.

Thus, the results from the two different instances of the Standardized Aprroach carry over to those two domains on which regulation is binding: the total lending volume under binding regulation is not pro-cyclical. The volume lent to Firm 1 is counter-cyclical on $[0.9995, 1.0000]$ and not pro-cyclical on $[1.0009, 1.0020]$. From a global perspective, total lending becomes erratic. Slight changes in the success probabilities make the total loan volume migrate within three different regimes. The switch at $m = 1.0000$ causes a jump in the total loan volume from $1,250$ to $1,438.1$. As long as the regulatory constraint is not binding, the bank can widen its total loan volume up to $1,739.1$. Thereafter, regulation is binding again and the growth of the total loan volume becomes weaker, and the total loan volume ranges from $1,789.8$ to $1,839.8$ for $1.0009 \leq m \leq 1.0020$.

### 4.3.4 Effects under the Value-at-Risk Approach

Apart from the jumps, the total loan volumes $L^V$ are not pro-cyclical compared to $L^*$. Depending on the shifts in the success probabilities and the level of confidence, regulation may even affect lending slightly counter-cyclically.

[6] The lowest feasible default probability in this example amounts to $1-mp_1 > 1-1.0021 \cdot 0.995 = 0.0029105$, still justifying a risk weight equal to 0.5.

First, let us consider the VaR approach with a level of confidence of 1%, as shown in Figure 4.12. Because the success probabilities move with $m$, different structural levels are eligible: below $m = 0.9920$, the regulated bank is not allowed to grant any loans financed by debt. Hence, the regulated total loan volume equals the bank's initial equity, here 100. By the bank owners' risk-neutrality, all capital is granted as loan to Firm 2. For $0.9920 \leq m \leq 0.9950$, the bank can only comply with a confidence level of 1% by allocating its loans such that $\bar{p} = 1 - mp_1 - mp_2 + mq$ is fulfilled. From $m = 0.9950$ on, the bank could already switch to a level of $\bar{p} = 1 - mp_1$, which is not optimal, however, such that $\bar{p} = 1 - mp_1 - mp_2 + mq$ is maintained till $m = 0.9959$. From $m = 0.9960$ on, the bank finally chooses $\bar{p} = 1 - mp_1$. Although the bank owners could choose loan portfolios whose redemptions conform to $\bar{p} = 1 - mp_2$ from $m = 1.0000$ on, they do not switch the structural levels again. As a result, loan-allocation rates above $\frac{\alpha_2}{\alpha_1+\alpha_2}$ are chosen.

Because the bank chooses $\ell^V \leq \frac{\alpha_2}{\alpha_1+\alpha_2}$ for $\bar{p} = 1 - mp_1 - mp_2 + mq$, $0.9920 \leq m \leq 0.9959$ and $\ell^V > \frac{\alpha_2}{\alpha_1+\alpha_2}$ otherwise, the total deposit volume is tied to equity and expected returns according to

$$D^V = \left( \frac{\tau}{m \cdot p_2\alpha_2(1-\ell^V) - (1 - m \cdot p_1)\alpha_1\ell^V + \epsilon} - 1 \right) \cdot W_B$$

as stated in (3.3.46). $D^V$ is directly and indirectly, via $\ell^V$, affected by $m$. The direct effect,

$$\frac{\partial D^V}{\partial m} = -\frac{\tau W_B \cdot \left[p_2\alpha_2(1-\ell^V) + p_1\alpha_1\ell^V\right]}{\left[m \cdot p_2\alpha_2(1-\ell^V) - (1 - m \cdot p_1)\alpha_1\ell^V + \epsilon\right]^2} < 0 \tag{4.3.2}$$

is always negative whereas the indirect effect,

$$\frac{\partial D^V}{\partial \ell} \cdot \frac{d\ell^V}{dm} = -\frac{\tau W_B \cdot [-m \cdot p_2\alpha_2 - (1 - mp_1)\alpha_1]}{\left[m \cdot p_2\alpha_2(1-\ell^V) - (1 - m \cdot p_1)\alpha_1\ell^V + \epsilon\right]^2} \cdot \frac{d\ell^V}{dm} \tag{4.3.3}$$

is ambiguous as the behavior of the equilibrium loan-allocation rate is in general unclear. $D^V$ strictly increases in $\ell$ because of $p_2\alpha_2 > p_1\alpha_1$.

For $\bar{p} = 1\%$, the loan-allocation rate $\ell^V$ strictly increases and the indirect effect outweighs the direct effect resulting in a strictly increasing total loan volume $L^V$ as soon as the bank is allowed to issue debt $D^V > 0$ (which is the case from $m = 0.9920$ onwards). This increasing behavior is shown by Figure 4.12.

Issuing a strictly positive amount of deposits and compliance with a confidence level of 0.5% is feasible from $m = 0.9970$ on (*cf.* Fig. 4.11 and 4.12), to one of 0.2%

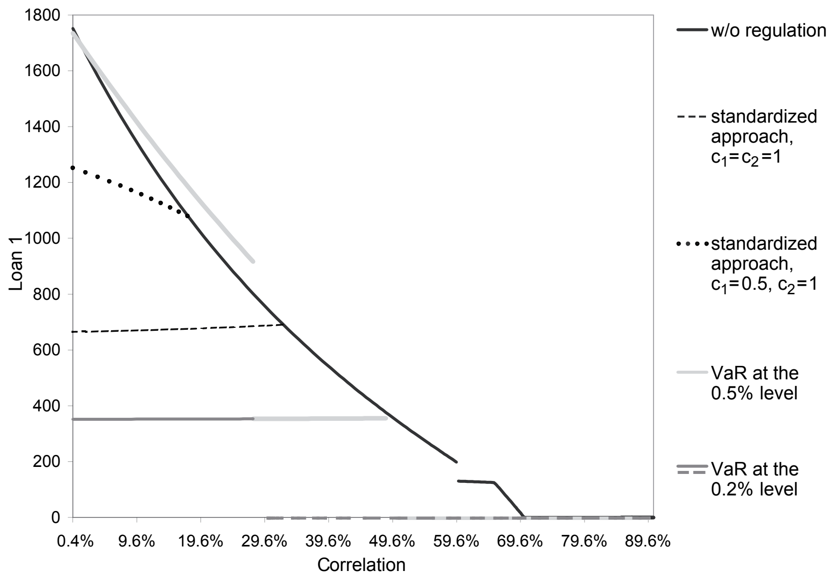


Figure 4.17: **Loan to Firm 1 as a function of the correlation**

*This figure illustrates the dependence of the equilibrium loan volume granted to Firm 1 on the project returns' correlation. Changes in the correlation are based on joint shifts in each project's success probability $p_i$ and in their common success probability $q$. These shifts are modeled using a common multiple $m$, $0.991 \leq m \leq 1.002$. The loan volume $L_1$ is shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with the confidence levels $\bar{p} = 0.5\%$ and $\bar{p} = 0.2\%$.*

from $m = 1.0000$ on (*cf.* Fig. 4.11). Under the 0.5% regime the bank can relax its decisions to a level of $1 - mp_1$ from $m = 1.0000$ onwards which it effectively does. Below it follows a structural level equal to $1 - mp_1 - mp_2 + mq$, corresponding to values between 0.2100% and 0.4994%. The migration from one structural level of confidence to the other causes a jump at $m = 1.0000$ (correlation equal to 27.79%). The bank's behavior implies that the confidence levels of $\bar{p} = 1\%$ and $\bar{p} = 0.5\%$ yield the same results for $m = 1.0000$ onwards (for correlations below the critical value of 27.79%). Especially, regulation does not have a pro-cyclical impact, neither on the total nor on the single loan volumes on this domain.

But for $m \leq 0.9999$, the VaR approach that requires a level of confidence of 0.5%, affects total lending counter-cyclically: the optimal loan-allocation rate is equal to $\frac{\alpha_2}{\alpha_1+\alpha_2}$ and thus only the direct effect is present. In absolute terms, the total loan volume $L^V$ shrinks from 595.68 to 591.47. Compared to the slope of $L^*$, this counter-cyclical effect is small: an absolute change in probabilities by 0.30% translates into a relative cutback in the lending volume of $-0.7\%$. These sensitivities have already been discussed in connection with Result 24.

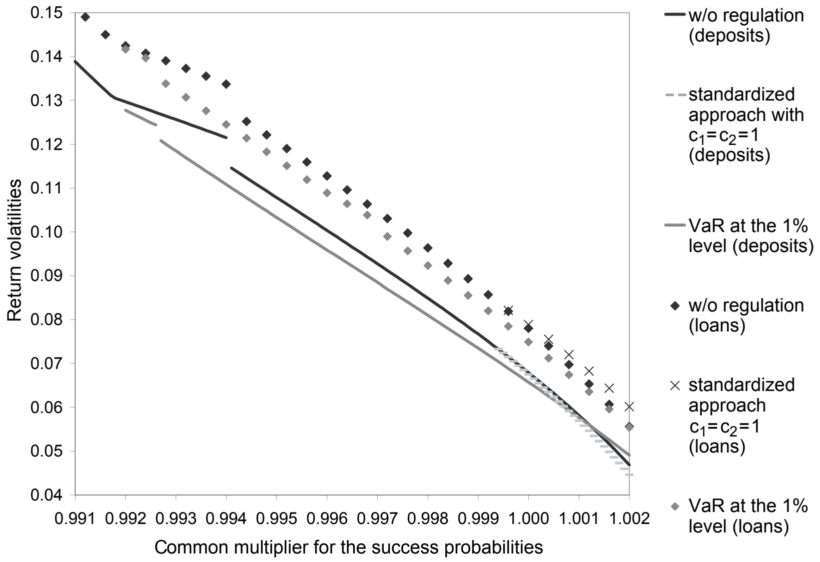


Figure 4.18: **Return volatilities as a function of shifts in success probabilities**

*This figure illustrates the dependence of the equilibrium return volatilities on shifts in each project's success probability* $p_i$ *and in their common success probability* $q$*. Shifts in success probabilities are modeled using a common multiple* $m$*,* $0.991 \leq m \leq 1.002$*. Solid and dashed lines represent volatilities of returns on deposits,* $\sigma_D$*. Diamonds and crosses represent volatilities on total loans,* $\sigma$*. Return volatilities are shown for three different regimes: the laissez-faire equilibrium, the Standardized Approach with both risk weights equaling one,* $c_i = 1$*, and the VaR approach with confidence level* $\bar{p} = 1\%$*.*

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Under $\bar{p} = 0.2\%$, the bank always chooses $\ell^V = \frac{\alpha_2}{\alpha_1+\alpha_2}$ such that only the direct effect is present, which is negative. Here, an absolute decrease of 0.20% in default probabilities causes a relative change in the total loan volume by roughly $-0.4\%$.

As outlined by (4.3.3), the bank owners choose, given any of the confidence levels 0.2%, 0.5%, or 1.0% and given the value of the success probabilities, those structural confidence levels that result in $v(\bar{p}, \ell^V) = mp_2\alpha_2(1-\ell^V) - (1-mp_1)\alpha_1\ell^V$. Thus the total feasible deposit volume strictly increases in $\ell$ for $\ell \neq \frac{\alpha_2}{\alpha_1+\alpha_2}$, setting incentives to choose higher loan-allocation rates than the bank owners do without regulation. Indeed, this is true for $\bar{p} = 1.0\%$, but only partly for $\bar{p} = 0.5\%$ and never for $\bar{p} = 0.2\%$: from $m = 0.9984$ on, the unregulated bank owners offer a loan-allocation rate higher than $\frac{\alpha_1}{\alpha_1+\alpha_2}$.

Mostly, the household benefits from these higher loan-allocation rates in terms of deposit return volatilities if $\bar{p} = 1\%$. The return volatility of the bank's loan portfolio reduces for $\bar{p} = 1\%$, too, as Figure 4.18 shows.

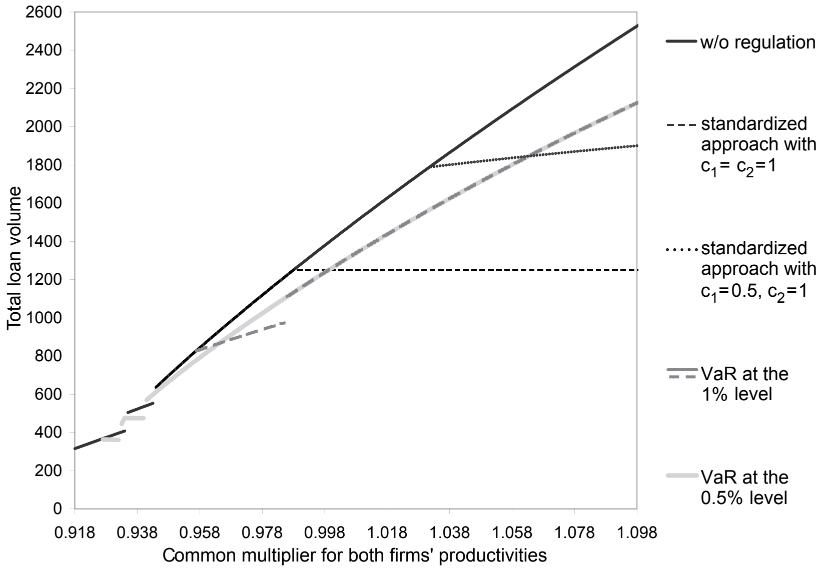


Figure 4.19: **Total loan volume as a function of shifts in productivity**
*This figure illustrates the dependence of the equilibrium total loan volumes $L$ on shifts in each project's productivity $\alpha_i$. Shifts in productivity are modeled by a common multiple $m$. Total loan volumes are shown for four different regimes: the laissez-faire equilibrium, and the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1\%$ and $\bar{p} = 0.5\%$. Cf. Bühler/Koziol/Sygusch (2008, Fig. 5, p. 156).*

## 4.4 Productivity Shocks

### 4.4.1 Total Volumes and Cyclical Dampening

Finally, let us consider changes in productivity modeled by jointly scaling the parameters $\alpha_i$. Higher productivity is associated with more favorable economic conditions: total output increases given the inputs and given risks: the economy as a whole is better-off.

The productivity multiplier $m$ is bounded from below by the condition $R_f < p_1\alpha_1 m$, *i.e.* by Condition (3.2.6), and from above by $\alpha_2 m \leq 2$, *i.e.* by Condition (3.2.7), hence altogether by

$$\frac{R_f}{p_1\alpha_1} < m \leq \frac{2}{\alpha_2} .$$

The lower bound for the multiplier is identical to the lower bound of the probability multiplier as both multipliers affect the projects' expected gross returns identically. The upper bound is due to (3.2.7), an assumption that is not necessary for the model to work, but that has been helpful to derive some general results. In this example, the household is already constrained by its initial wealth $W_H$ from $m = 1.156$ on whereas the upper bound allows for values of the multiplier up to 1.666. If $W_H$ had been set equal to 4000, the household would have been constrained from 1.266 on. In between, that is for $1.156 \leq m \leq 1.266$, the unregulated economy would qualitatively behave as it does for $0.958 \leq m \leq 1.156$. In particular, both the Standardized Approach and the VaR-approach at all three levels of confidence are binding.

Though the multiplier concerning the gross returns $\alpha_i$ shifts the projects' expected returns as the multiplier concerning the probabilities $p_i$ and $q$ does, a fundamental difference is that the latter multiplier has been assumed to affect the projects' correlation. Another difference shows up in the sensitivity of the total loan/deposit volume in equilibrium: it exhibits a higher sensitivity to changes in the probabilities $p_i$ and $q$ than to changes in the gross returns $\alpha_i$. Furthermore, the equilibrium total loan volumes in the unregulated economy and under the VaR-requirements at 0.5% and at 1%, are concave in the multiplier for both gross returns, but convex in the multiplier for the single and the joint success probabilities within each domain between the jumps.

Low values of the multiplier $m$ mean low revenues for the firms in case their projects succeed and thus lower expected loan redemptions. In the end, lower revenues to both firms imply lower residual values to the household if the bank defaults. In short, lower values of this multiplier $m$ reduce the household's willingness to save its funds by means of bank deposits. It will rather raise the weight of the risk-free asset in its portfolio. If $m$ rises, the opposite is true. Figure 4.19 and 4.20, respectively, show that the unregulated total loan/deposit volume strictly increases with increasing productivity.

As both forms of regulations result in fixing a maximum lending amount and as the unregulated bank strictly increases its total loan/deposit volume in the productivity multiplier $m$, regulation becomes binding from a given threshold value of $m$ on. Consequently, regulation does not have a pro-cyclical impact on total lending. Comparing pairwise Figure 4.19 to Figure 4.12 and Figure 4.20 to Figure 4.11 shows that the various regulatory regimes show similar patterns concerning cyclical effects in both scenarios, in the one with joint changes in the probabilities $p_i$ and $q$ and in the one with joint changes in the gross returns $\alpha_i$. Furthermore, there are some similarities concerning the pairwise overlapping behavior of total loan volumes resulting from different required confidence levels. This behavior will be discussed in Subsection 4.4.3.

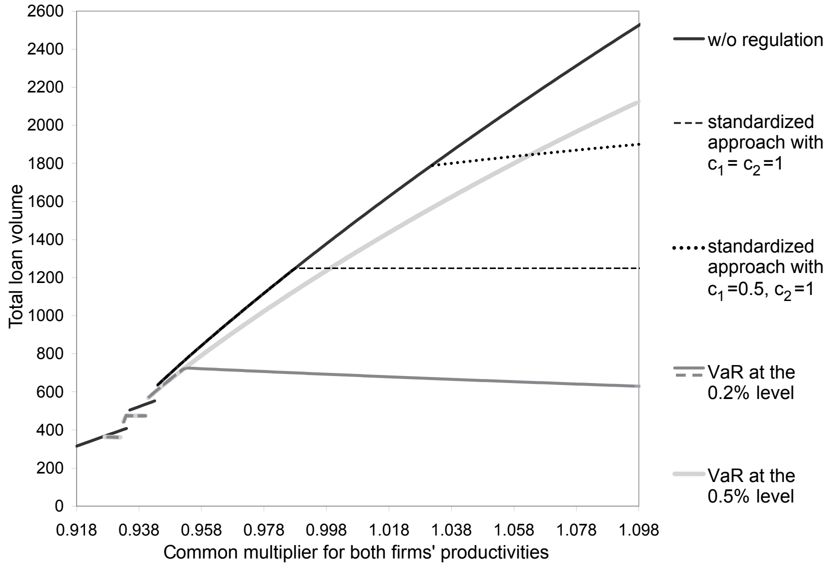


Figure 4.20: **Total loan volume as a function of shifts in productivity**

*This figure illustrates the dependence of the equilibrium total loan volumes $L$ on shifts in each project's productivity $\alpha_i$. Shifts in productivity are modeled by a common multiple $m$. Total loan volumes are shown for four different regimes: the laissez-faire equilibrium, and the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 0.5\%$ and $\bar{p} = 0.2\%$.*

## 4.4.2 Effects under the Standardized Approach

If both loans are equally weighted under the Standardized Approach there is no reaction of the total loan volume to changes in the productivity as long as the regulation is binding. It is fixed to $\frac{1}{c \cdot c_1} \cdot W_B$. Thus, the regulated total loan volume is not pro-cyclical. Furthermore, both loans and thus state-contingent deposit redemptions become so attractive with increasing productivity that the bank shifts its funds in favor for the riskier Loan 2. Despite the increasing risk associated with the loan portfolio under binding regulation, the deposit interest rate $R_D^S$ is lower than the interest promised by the unregulated bank, $R_D^*$. Moreover, $R_D^S$ strictly decreases in the multiplier $m$, while $R_D^*$ strictly increases, thus sustaining the growing deposit volume in the case without regulation. As a result, the deposit interest rate is counter-cyclically affected by regulation.

Figure 4.21 (4.22, respectively) shows that the volume granted as a loan to Firm 1, $L_1^S$, strictly decreases in its expected gross return, while Figure 4.23 (4.22, respectively) shows that the volume granted to Firm 2 strictly increases in the productivity multiplier. From $m = 1.195$ on, the volume granted to Firm 2 exceeds

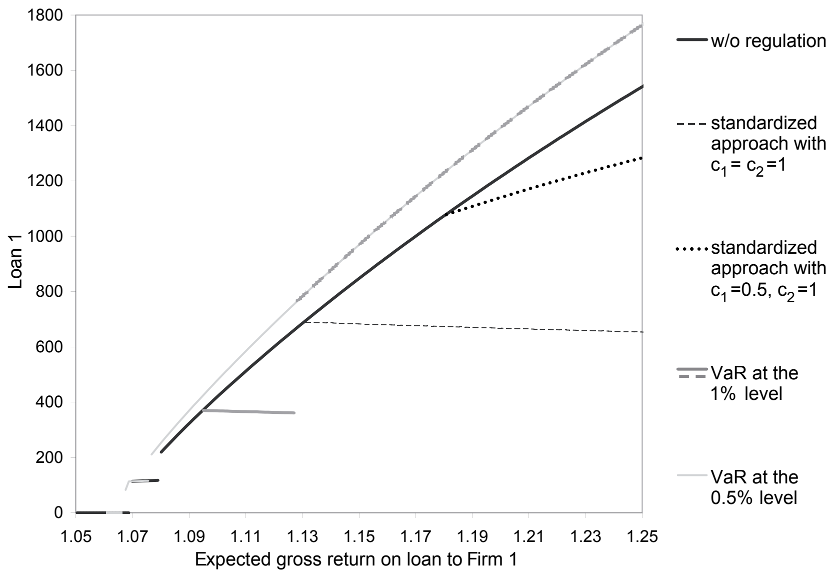


Figure 4.21: **Loan to Firm 1 as a function of its expected gross return**

*This figure illustrates the dependence of the equilibrium loan volumes granted to Firm 1 on its expected gross return. Changes in its expected gross return $p_1\alpha_1 m$ are caused by $m$ where $m$ jointly scales each project's productivity $\alpha_i$. The loan volumes $L_1$ are shown for four different regimes: the laissez-faire equilibrium, and the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1\%$ and $\bar{p} = 0.5\%$.*

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that granted to Firm 1. From $m = 1.458$ on, the bank's choice corresponds to Case 2 while it has been compatible to Case 1 below. Both these thresholds are not shown in the figures considered.

Although the loan granted to the high-risk firms grows under a binding regulation, its volume is not pro-cyclically affected, whereas the loan to the low-risk firm shows a counter-cyclical pattern. Thus the regulated bank will build up more risk in a boom period compared to an unregulated bank, which also translates into a higher return volatility of loan redemptions, as shown by Figure 4.26. Regulation by equal risk weights only dampens the return volatility of the deposit pay-off.

This example illustrates that risk-insensitive regulation may induce banks to relax their lending standards in good times, resulting in an excessive build-up of risks that may materialize some periods later and thus worsen a downturn. Therefore, regulation may affect lending and the size of the real economy in a pro-cyclical manner.

This type of counter-cyclical risk-taking has also been empirically documented. Ayuso/Pérez/Saurína (2004) have analyzed this phenomenon with Spanish banks.

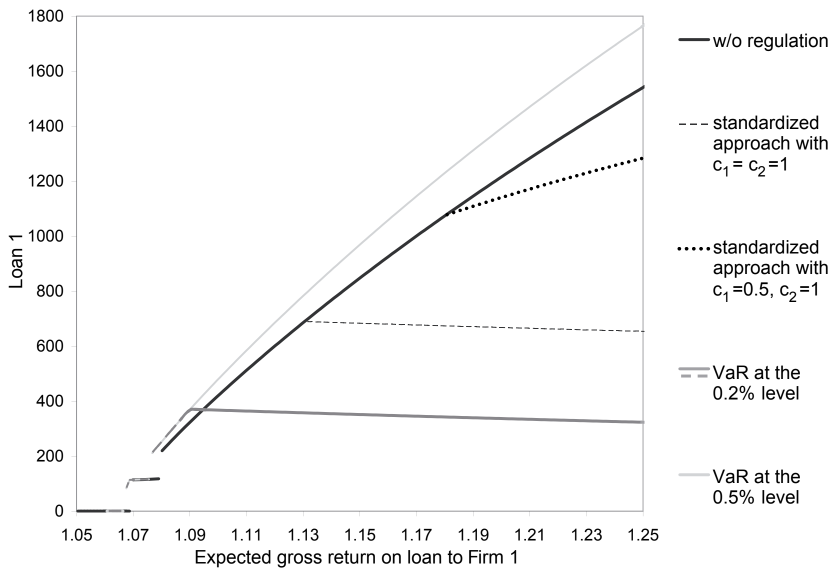


Figure 4.22: **Loan to Firm 1 as a function of its expected gross return**
*This figure illustrates the dependence of the equilibrium loan volumes granted to Firm 1 on its expected gross return. Changes in its expected gross return $p_1\alpha_1 m$ are caused by $m$ where $m$ jointly scales each project's productivity $\alpha_i$. The loan volumes $L_1$ are shown for four different regimes: the laissez-faire equilibrium, and the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 0.5\%$ and $\bar{p} = 0.2\%$.*

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They find that the accumulation of risks is reflected by counter-cyclical capital buffers, a relation that has been reported for many other countries and jurisdictions as well.[7] As a matter of fact, these studies were still carried out under the dated Basel I Accord where lower default probabilities could not affect the volume of regulatory capital needed.[8] Crude risk weighting, as it is proposed by the Standardized Approach, however, will show a comparative sluggishness toward changes in credit risk.

The phenomenon of counter-cyclical risk-taking could also be aligned with the notion that loan officers could be oblivious. This hypothesis, the so-called "institutional memory hypothesis", was confirmed by Berger/Udell (2004) for United-States based banks during the eighties and nineties.

If a risk weight of $c_1 = 0.5$ is assigned to Loan 1 whereas Loan 2 is still weighted by $c_2 = 1$, the bank can increase its total loan volume by increasing the volume granted

[7] *Cf.* Bikker/Metzemakers (2007) for OECD countries, Jokipii/Milne (2006) for pan-European samples, Lindquist (2004) for Norway, and Stolz/Wedow (2005) for German savings and cooperative banks.

[8] By the new Accord, however, through-the-cycle or long-run credit ratings are requested by supervisory authorities, *cf.* BCBS (2004, Art. 447).

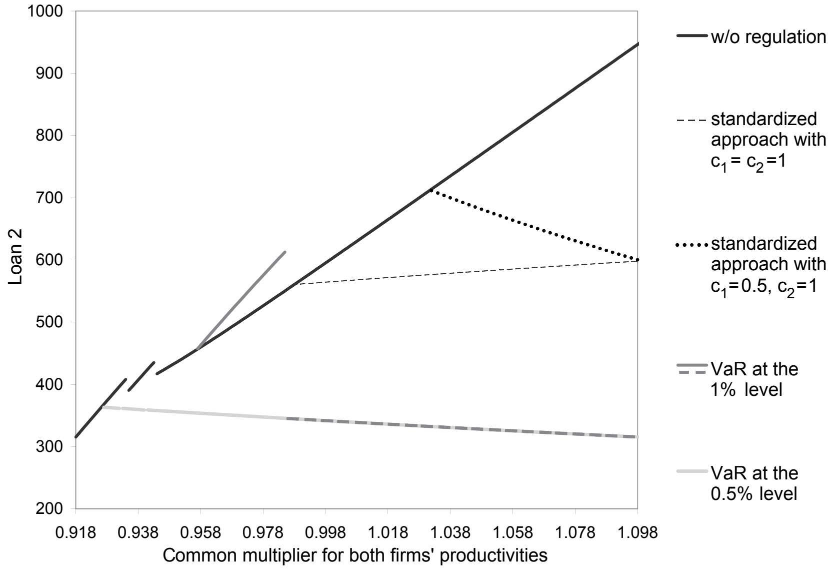


Figure 4.23: **Loan to Firm 2 as a function of shifts in productivity**

*This figure illustrates the dependence of the equilibrium loan volumes granted to Firm 2 on shifts in each project's productivity $\alpha_i$. Shifts in productivity are modeled by a common multiple $m$. The loan volumes $L_2$ are shown for four different regimes: the laissez-faire equilibrium, and the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1\%$ and $\bar{p} = 0.5\%$.*

---

as a loan to Firm 1. As a consequence, the total loan volume strictly increases in the firms' productivity and we observe the opposite to the case of equal risk weights concerning single loan volumes under a binding regulation. That is, the low-risk firm benefits from increasing productivity, but not as much as it does if the bank is unregulated. The volume granted as a loan to the riskier firm, Firm 2, is counter-cyclical under binding regulation. Figure 4.26 shows that regulation lowers the return volatility on the bank's loan portfolio if $c_1 = 0.5$ and $c_2 = 1$ holds, in contrast to the case of equal risk weights.

### 4.4.3 Effects under the Value-at-Risk Approach

Under the VaR approach the picture is much more diverse than under the Standardized Approach. Apart from some jumps, the total loan volume is not pro-cyclically affected by regulation if the confidence levels $\bar{p}$ are set equal to 0.5% or 1.0%. For $\bar{p} = 0.2\%$, the total loan volume is counter-cyclically affected by binding regulation (*cf.* Fig. 4.20). Single-loan volumes can be both pro- or counter-cyclical, depending on the level of confidence but also on the level of productivity.

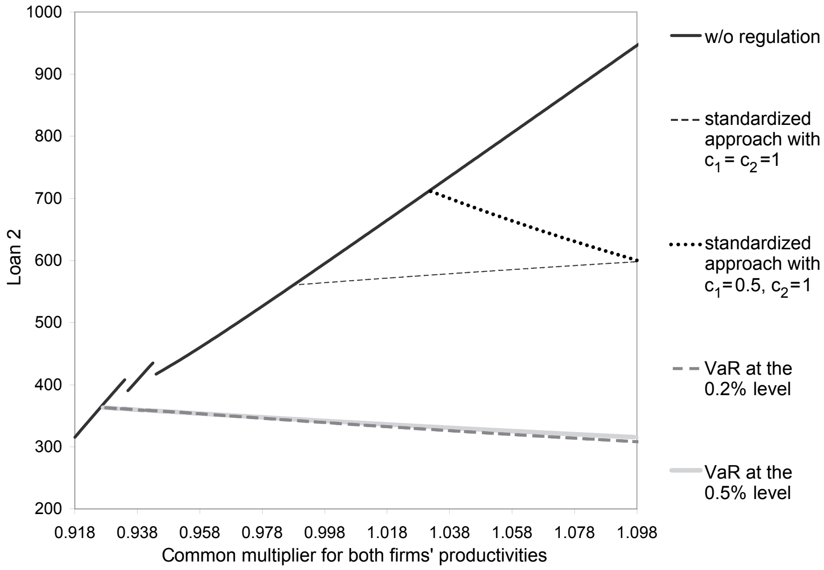


Figure 4.24: **Loan to Firm 2 as a function of shifts in productivity**

*This figure illustrates the dependence of the equilibrium loan volumes granted to Firm 2 on shifts in each project's productivity $\alpha_i$. Shifts in productivity are modeled by a common multiple $m$. The loan volumes $L_2$ are shown for four different regimes: the laissez-faire equilibrium, and the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 0.5\%$ and $\bar{p} = 0.2\%$.*

---

Furthermore, single loan volumes can outweigh their unregulated counterparts in volume, as illustrated in Figures 4.21 to 4.23.

In what follows, the effects under different confidence levels are discussed separately, starting with $\bar{p} = 1\%$ and proceeding to the other required levels of confidence in descending order.

If $\bar{p} = 1\%$ is required, regulation becomes binding from $m = 0.957$ on. Up to $m = 0.985$, it is optimal simply to retain $1 - p_2$ as level of confidence. From $m = 0.986$ on, however, the bank switches voluntarily to $\bar{p} = 1 - p_1 = 0.5\%$, resulting in a loan-allocation rate $\ell^V$ that exceeds $\ell^*$ such that the volume granted to Firm 1 under regulation is higher than the volume granted without regulation, as Figure 4.21 shows. In the figure, this jump is identified with an expected gross return on the first firm's project of approximately 1.13. The volume of $L_1^V$ exceeds that of $L_1^*$ from that threshold on, and $L_1^V$ reacts in a pro-cyclical manner to further increases in productivity. The effects that the change in the level of confidence brings about not only cause jumps in $\ell^V$ and $L_1^V$, but in the total loan volume $L^V$ also, as illustrated in Figure 4.19. Concerning the loan to Firm 2, $L_2^V$, we observe the opposite: it exceeds its unregulated counterpart in volume for $0.957 < m \leq 0.985$ and is pro-cyclical on this domain. For $m \geq 0.986$, it is even counter-cyclical. In this area, its volume equals that granted under $\bar{p} = 0.5\%$, as Figure 4.23 shows.

If the confidence level $\bar{p}$ equals 1%, the bank is free to meet all structural confidence levels from $\bar{p} = 1 - p_1 - p_2 + q$ to $\bar{p} = 1 - p_2$, as $p_1 > p_2 = 99\%$ holds. If it complies with $\bar{p} = 1 - p_2$, but not with $\bar{p} = 1 - p_1$, the regulatory constraint function $v(\bar{p}, \ell)$ becomes, according to (3.3.46)

$$v(1 - p_2, \ell) \;=\; p_1 m\alpha_1\ell - (1 - p_2)m\alpha_2(1 - \ell) \;,$$

in connection with

$$\ell^V \;<\; \frac{\alpha_2}{\alpha_1 + \alpha_2} \;.$$

In this example, the bank, required to reach 1%, effectively reaches the $\bar{p} = (1-p_2)$-level for $0.957 \leq m \leq 0.985$. On this domain, the regulated bank chooses loan-allocation rates ranging from 44.74% to 37.10%. In particular,

$$\ell \;>\; \frac{(1 - p_2)\alpha_2}{(1 - p_2)\alpha_2 + p_1\alpha_1} \;\approx\; 1.038\%$$

is fulfilled by the parameter values given in Table 4.1, such that $v(1 - p_2, \ell) > 0$ holds in equilibrium.

But for $0.986 \leq m \leq 1.666$, the bank voluntarily chooses to meet the stricter level of $\bar{p} = 0.5\%$. The corresponding regulatory constraint is therefore

$$v(1 - p_1, \ell) \;=\; p_2 m\alpha_2(1 - \ell) - (1 - p_1)m\alpha_1\ell \;,$$

with

$$\ell^V \;>\; \frac{\alpha_2}{\alpha_1 + \alpha_2} \;,$$

as the domain of appropriate loan-allocation rates. This confidence level results in loan-allocation rates varying from 68.92% to 94.61%. Therefore,

$$\ell \;<\; \frac{p_2\alpha_2}{p_2\alpha_2 + (1 - p_1)\alpha_1} \;\approx\; 99.52\%,$$

holds in this example, implying $v(1-p_1, \ell) > 0$ in equilibrium. Thus, the regulatory constraint function always remains positive if the bank is required to attain $\bar{p} = 1\%$.

The voluntary change in the confidence level shows that an absolute rise in both projects' gross returns can be sufficient to make the risk-neutral bank shift its loan portfolio in favor of the low-risk loan, in this case Loan 1.

Joint increases in productivity always have a negative direct effect on the eligible total loan/deposit volume as long as $v(\bar{p}, \ell) > 0$ holds, since

$$\frac{\partial D^V}{\partial m} = -\frac{\tau \cdot W_B \cdot \frac{\partial v(\bar{p}, \ell^V)}{\partial m}}{\left[v(\bar{p}, \ell^V) + \epsilon\right]^2} \tag{4.4.1}$$

is negative if

$$\frac{\partial v(\bar{p}, \ell^V)}{\partial m} = \frac{v(\bar{p}, \ell^V)}{m} > 0$$

holds. The latter is the case here, as shown above. Thus, the indirect effect (4.3.3) must be positive again and outweigh the direct effect as the total loan/deposit volume strictly increases in the joint productivity multiplier. Hence,

$$\frac{\partial D^V}{\partial \ell} \cdot \frac{d\ell^V}{dm} > 0$$

must hold true, whereas we note that both single derivatives are negative for $0.957 \leq m \leq 0.985$, and positive for $0.986 \leq m \leq 1.666$.

Concerning the volume of total lending under the levels of confidence equal to 1% and equal to 0.5%, there is a remarkable outcome for $0.957 \leq m \leq 0.985$. On this domain, the total loan volume $L^V$ under the tighter confidence level $\bar{p} = 0.5\%$ exceeds the total loan volume under $\bar{p} = 1\%$. In fact, the bank can attain the less strict level with a lower loan-allocation rate $\ell$, *i.e.* with a riskier loan portfolio. loan-allocation rates are 9.87 to 31.49 percentage points lower than where a confidence level equal to $\bar{p} = 0.5\%$ is required. The higher risk is rewarded to the household by deposit rates that are raised by 65 to 91 base points. This compensation does not suffice to assure that the household supplies at least as much deposits as it supplies under the confidence level $\bar{p} = 0.5\%$. Despite the lower total loan/deposit volume and the higher deposit interest rates, the monopolistic bank can still increase its expected final wealth by 0.09 to 1.35 dollars if it attains $\bar{p} = 1\%$. For this reason, the bank does not voluntarily stick to the tighter level of 0.5% to raise a higher deposit volume. Likewise, if regulation requires 0.5%, the bank can never achieve the gains it could obtain under the less strict level of 1%.

The VaR approach with confidence level $\bar{p} = 0.5\%$ becomes binding at $m = 0.927$. Up to $m = 0.952$, it is even optimal for the bank to comply with $\bar{p} = 1 - p_1 - p_2 + q = 0.2\%$. If the bank effectively attained the $\bar{p} = (1 - p_1)$ level, it would have to choose loan-allocation rates above $\frac{\alpha_2}{\alpha_1 + \alpha_2}$, *cf.* (3.3.46). In contrast, compliance with the stricter level of $\bar{p} = 1 - p_1 - p_2 + q = 0.2\%$ allows for loan-allocation rates below

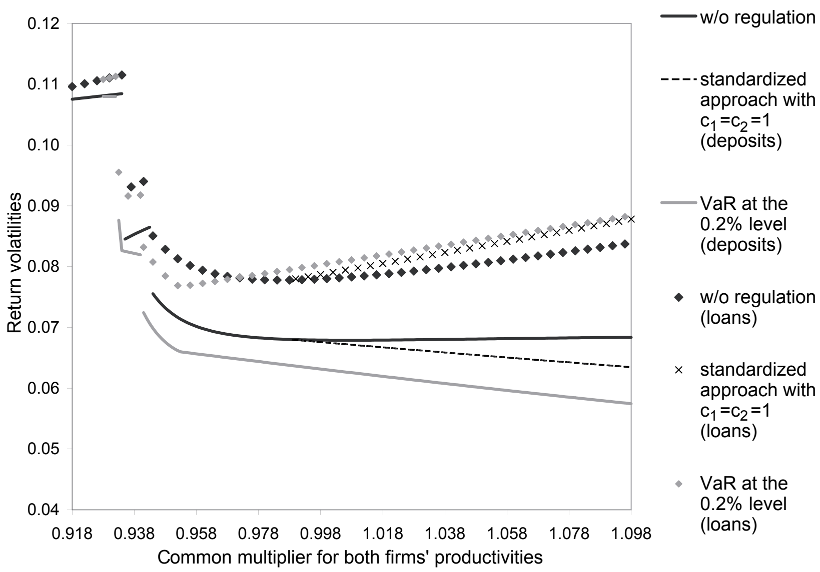


Figure 4.25: **Return volatilities as a function of shifts in productivity**

*This figure illustrates the dependence of the equilibrium return volatilities on shifts in each project's productivity $\alpha_i$. Shifts in productivity are modeled using a common multiple $m$. Solid and dashed lines represent volatilities of returns on deposits, $\sigma_D$. Diamonds and crosses represent volatilities on total loans, $\sigma$. Return volatilities are shown for three different regimes: the laissez-faire equilibrium, the Standardized Approach with both risk weights equaling one, $c_i = 1$, and the VaR approach with confidence level $\bar{p} = 0.2\%$.*

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$\frac{\alpha_2}{\alpha_1+\alpha_2}$, *cf.* (3.3.46), thus increasing marginal returns on the loan portfolio. The unregulated bank attains a level of confidence equal to 1% on this domain and chooses loan-allocation rates between zero and 41.76%, *i.e.* also lower than $\frac{\alpha_2}{\alpha_1+\alpha_2}$.

From $m = 0.953$ on, the bank chooses $\ell^V$ and $R_D^V$ such that $\bar{p} = 0.5\%$ is reached. From $m = 0.941$ on, the total loan volume is not pro-cyclical, but the volume granted to Firm 1 is pro-cyclical. Loan 2 behaves counter-cyclically.

For multipliers of $0.927 \leq m \leq 0.952$, the requirement $\bar{p} = 0.2\%$ leads to the same results as described for $\bar{p} = 0.5\%$. From $m = 0.953$ on, the bank will remain with $\ell^V = \frac{\alpha_2}{\alpha_1+\alpha_2}$ as this loan-allocation rate allows for the highest feasible total loan/deposit volume under $\bar{p} = 1 - p_1 - p_2 + q = 0.2\%$ (*cf.* Result 24). Consequently, only the direct effect is present, and is negative since $v(1 - p_1 - p_2 + q, \frac{\alpha_2}{\alpha_1+\alpha_2}) > 0$ holds, *cf.* (4.4.1). As a consequence, the total loan volume and the single loan volumes strictly decrease in the multiplier for $\alpha_i$ in equilibrium and thus become counter-cyclical under $\bar{p} = 0.2\%$.

The effects of the VaR regulation on return volatilities are mixed: with all confidence

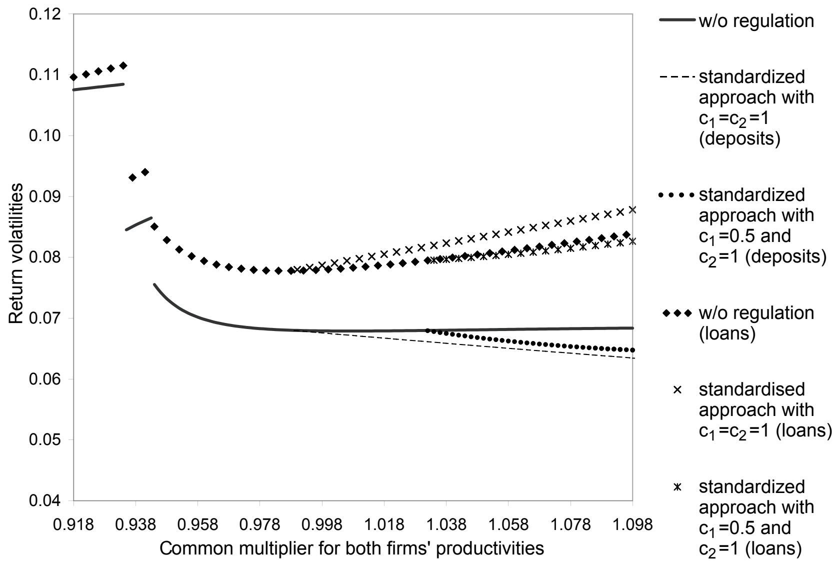


Figure 4.26: **Return volatilities as a function of shifts in productivity**

*This figure illustrates the dependence of the equilibrium return volatilities on shifts in each project's productivity $\alpha_i$. Shifts in productivity are modeled using a common multiple $m$. Solid and dashed lines and dots represent the volatilities of returns on deposits. Diamonds and crosses represent the volatilities of returns on total loans. Return volatilities are shown for three different regimes: the laissez-faire equilibrium, and the Standardized Approach with both equal and different risk weights.*

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levels, the return volatility on the regulated loan portfolio is dampened for rather low values of the productivity multiplier $m$, but is enhanced for high $m$. If $\bar{p} = 1\%$ is required, the return volatilities of the regulated loan portfolio also exceed its unregulated counterparts for $0.957 \leq m \leq 0.985$, *i.e.* on the domain, where the bank still only attains a confidence level of $\bar{p} = 1\%$, and does not yet reach the tighter level of $\bar{p} = 0.5\%$.

If the bank must satisfy $\bar{p} = 1\%$ or $\bar{p} = 0.5\%$, the deposit return volatility is dampened given relatively low values for the firms' productivity, but it becomes enhanced for rather high multipliers as well. It is only under a required confidence level of $\bar{p} = 0.2\%$ that the deposit return volatility is reduced for all levels of productivity. The effects of the VaR-based regulation at $\bar{p} = 0.2\%$ on return volatilities are shown in Figure 4.25.

Comparing loan volumes granted under the VaR approaches to those granted under the Standardized Approaches does not yield a clear-cut picture concerning procyclicality either.

Results can be best summarized by stating that volumes and volatilities react more strongly to shocks under regulation the more leeway regulation concedes to banks. Under the most rigid forms of regulation, such as equal risk weights or the tightest eligible confidence level, volumes are the most impervious to shocks. Rigidity, however, does not mean that risks are effectively reduced.

## 4.5 Summary

The theoretical and numerical analyses suggest that the potential of pro-cyclical effects caused by capital requirements on lending depends mainly on three factors: the kind and tightness of regulation, the type of shock considered, and the loan volume considered (*i.e.* total vs. single volumes).

Total loan volumes are affected pro-cyclically when there are shocks in the bank's initial equity because capital requirements, both the Standardized Approach as well as the VaR approach, are directly linked to the bank's level of equity. Although the proportionality factor may change in the loan-allocation rate, it is more or less fixed to a given value, or at least to a given range of values.

There is no pro-cyclical effect, however, if we consider joint moves in the firm projects' success probabilities or in the firm projects' productivity. Under the tightest level of confidence enabling positive total loan volumes, the regulated total loan volume is counter-cyclical concerning rising productivity. In particular, a joint rise in success probabilities or in productivity always have a direct negative impact on the feasible total loan/deposit volume under the VaR approach.[9]

Concerning the single loan volumes, results are much more diverse. Under all forms of capital adequacy rules considered, single loan volumes may react pro-cyclically, non-pro-cyclically, or even counter-cyclically to any kind of shocks. Under binding VaR-requirements either single loan volume may also outweigh its unregulated counterpart. With the parameter values considered, this phenomenon cannot be confirmed under the Standardized Approach as it reduces the feasible total loan volume too strongly. Similarly, Repullo/Suarez (2004) show within a one-period bank model that Basel II is more favorable for borrowers than Basel I in terms of loan interest rates.

Besides the issues of cyclicality, this study gives some insights into how the bank's risk-taking can be affected by regulation.

[9] Note that a joint rise in productivity leads to a reduction in the feasible total loan/deposit volume under the VaR approach if and only if $v(\bar{p}, \ell) > 0$ holds.

First, let us consider the loan-allocation rate. Equal risk weights under the Standardized Approach result in lower loan-allocations rates than if the bank is unregulated. Thus, the high-risk loan becomes relatively more important to the bank: lower business volumes are compensated by high-yield, and highly risky investments.

If the risk weights differentiate according to the default risk of the loans, the bank will choose higher loan-allocation rates under regulation than without regulation.

Under the VaR approach, the relation between $\ell^V$ and $\ell^*$ depends on the confidence level held which can be different from, *i.e.* tighter than, the confidence level required. With $\bar{p} = 1 - p_2$, the regulated bank always chooses lower, with $\bar{p} = 1 - p_1$ always higher loan-allocation rates than without regulation. Things are mixed under $\bar{p} = 1 - p_1 - p_2 + q$ as the bank mostly chooses the intermediate level of $\frac{\alpha_2}{\alpha_1+\alpha_2}$, hence the equilibrium outlined by Result 24.

The regulated bank devotes a higher fraction to the more risky, *i.e.* more profitable loan, for the following two distinct reasons: under the Standardized Approach with fixed risk weights, the bank has to reduce its loan volumes and hence must forgo profit opportunities under a binding regulatory constraint. Contrary to risk weights accounting for differences in the default probability, there is no possibility to weaken the regulatory constraint on total volumes. The forgone profit opportunities by this volume reduction are therefore offset by increasing the marginal return, and hence the credit risk.

Under the VaR approach, this adverse selection occurs if the bank must comply with $\bar{p} = 1 - p_2$, always preferring Loan 2. Thus, firms featuring more risk in their production plans might even face situations where they have a comparative advantage among their less risky competitors on the market for bank loans.

Volatilities of returns on the loan portfolio can become both higher or lower through regulation, irrespective of the form of capital rule considered. The same applies to the volatilities of returns on deposits. Thus, risks are rather contained by volume, not by structure.

Yet, particularly those results concerning risk-taking under the VaR approach may be driven by the Bernoulli distribution. If the redemptions of the whole loan portfolio are modeled by a continuous distribution, the jumps in loan volumes and loan-allocation rates should vanish for two reasons: first, Case migrations can no longer occur and second, changes in confidence levels under a VaR approach are no longer achieved by jumps in the loan-allocation rate. In the following part we therefore present a variation of the model that allows for a normal approximation of aggregate loan redemptions.

# Part III

# Normally Distributed Loan Redemptions

# Chapter 5

# A Basis for the Normal Distribution

## 5.1 Introduction

We extend the analysis of Chapter 3 and 4 to more than two firms. More precisely, there are two groups of firms, each containing $n > 1$ firms. As in the Bernoulli framework, each firm undertakes one single risky project that can either be successful or fail. Each project's return depends on the returns of some other projects. We assume that the number of projects which any single project outcome can be correlated with may be arbitrarily high, but fixed. Thus, the depth of the dependence structure does not expand with the number of projects and sector-wise returns can be approximated by the normal distribution. The set-up on the firm level builds up on a random probability model as it can be found in its general form in Joe (2001). The normal approximation simplifies the analysis compared to that performed in the preceding part, Part II, as now one can dispense with the distinction of different deposit redemption cases.

The following sections will present the underlying dependence structure that allows for an approximation by the normal distribution. Chapter 6 discusses the bank's and the household's objectives in an analogous setting, as can be found in Chapter 3 and as sketched in Chapter 2. Section 6.6 analyzes the impact through regulation on the bank's lending behavior in a similar way as performed in Chapter 4. Chapter 5 and 6 are based upon Bühler/Koziol/Sygusch (2007), but considerably extend their analysis, in particular concerning the numerical study of regulatory impacts on lending and the exposition of the underlying dependence structure.

## 5.2 Firms and Sectors

There are, analogous to Section 3.2.1, two types of projects $i$ whose outcomes are Bernoulli-distributed, and which can be differentiated according to their probability of success and two types of firms that can be differentiated according to their technology. Assumptions (3.2.5), (3.2.6), and (3.2.7) are supposed to hold as well, unless we explicitly refer to the case of equality. A given technology and a given project can either be perfect complements that fit together for production, or completely useless. As each firm is assumed to have a fixed technology, they have no choice about the project they undertake. Consequently, there is *ex ante* a group of firms running project $i = 1$ (with technology $i = 1$) and another group of firms running project $i = 2$ (with technology $i = 2$). Each of both groups is referred to as a sector. Both sectors are assumed to be equal in size such that each sector contains $n$ firms. Firms are indexed by $j$. $\tilde{X}_i^{(j)}$ denotes the random success of a single project $i$ undertaken by firm $j$, $j = 1, \ldots, n$. The outcome of each project is Bernoulli-distributed, *i.e.* a project is either successful, $\tilde{X}_i^{(j)} = 1$, or it fails, $\tilde{X}_i^{(j)} = 0$.

In addition to Chapter 3, the success of undertaking a single project does not only depend on the performance of one other firm, but potentially on the performance of many other firms. Thus, each project, and in turn the redemption of each single loan, depends on a number of risk factors. These risk factors affect the actual outcome of each project by influencing the outcome of the corresponding success probability $\tilde{p}_i^{(j)}$ that characterizes the risk-return profile of project $i$ undertaken by firm $j$. Therefore, firms can be differentiated by their idiosyncratic risk within each sector $i$. Thus success probabilities are random variables and each Bernoulli-distribution can formally be stated as

$$\tilde{X}_i^{(j)} \sim \mathrm{B}(\tilde{p}_i^{(j)}), \quad \mathrm{P}(\tilde{X}_i^{(j)} = 1) = \tilde{p}_i^{(j)}, \quad i = 1, 2, \; j = 1, \ldots, n \; . \tag{5.2.1}$$

This set-up is known as Bernoulli-mixture model (Joe, 2001).

More specifically, consider firm $j$, $j \neq 1, n$, from sector $i$. Let its success probability $\tilde{p}_i^{(j)}$ be exposed to firm $j$'s idiosyncratic factor $\tilde{\xi}_i^{(j)}$ and to the idiosyncratic risk of its follower within the same sector, *i.e.* to $\tilde{\xi}_i^{(j+1)}$. This common risk factor represents common demand shocks to which the respective firms may be exposed to as they produce similar products or operate in the same geographical region. Likewise, the firm shares risks with its predecessor, reflected by $\tilde{\xi}_i^{(j-1)}$. Hence, firm $j$ from sector $i$ shares the risk loading $\tilde{\xi}_i^{(j)}$ with both its predecessor and its follower. Moreover, the firm considered also faces some risks stemming from the other sector which can be interpreted by supply chain relations. Specifically, we assume that it is exposed to the idiosyncratic risk of firm $j$ from the other sector, labeled $3-i$, *i.e.* to the risk factor $\tilde{\xi}^{(j)}_{3-i}$. The assumption that projects cannot be correlated with any other project too far away points out that even firms producing similar products do not necessarily suffer from the same shock the same time. To represent the preceding notions, we set up the following model for each probability $p_i^{(j)}$,

$$\tilde{p}_i^{(j)} \equiv \begin{cases} 1 - b_i^{(1)} \cdot \tilde{\xi}_i^{(1)} - c_i^{(2)} \cdot \tilde{\xi}_i^{(2)} - d_i^{(1)} \cdot \tilde{\xi}_{3-i}^{(1)} \,, & j = 1 \\ 1 - a_i^{(j-1)} \cdot \tilde{\xi}_i^{(j-1)} - b_i^{(j)} \cdot \tilde{\xi}_i^{(j)} - c_i^{(j+1)} \cdot \tilde{\xi}_i^{(j+1)} - d_i^{(j)} \cdot \tilde{\xi}_{3-i}^{(j)} \,, & j = 2, \ldots, n-1 \\ 1 - a_i^{(n-1)} \cdot \tilde{\xi}_i^{(n-1)} - b_i^{(n)} \cdot \tilde{\xi}_i^{(n)} - d_i^{(n)} \cdot \tilde{\xi}_{3-i}^{(n)} \,, & j = n \end{cases} \tag{5.2.2}$$

for $i = 1, 2$ where $a_i^{(j)}, b_i^{(j)}, c_i^{(j)}, d_i^{(j)}$ are fixed, strictly positive numbers and where $\tilde{\xi}_i^{(j)}$ are mutually independent, uniformly distributed random variables with $\tilde{\xi}_i^{(j)} \sim \mathrm{U}(0, 2t)$ for all $i = 1, 2$, $j = 1, \ldots, n$. Hence, the success probabilities only take values between zero and one if furthermore

$$2 \cdot t \cdot (a_i^{(j)} + b_i^{(j)} + c_i^{(j)} + d_i^{(j)}) < 1 \tag{5.2.3}$$

is fulfilled. Their support is thus given by[1]

$$\mathrm{supp}(\tilde{p}_i^{(j)}) = \begin{cases} \left[1 - 2 \cdot (b_i^{(1)} + c_i^{(2)} + d_i^{(1)}) \cdot t, \ 1\right] \,, & j = 1 \\ \\ \left[1 - 2 \cdot (a_i^{(j-1)} + b_i^{(j)} + c_i^{(j+1)} + d_i^{(j)}) \cdot t, \ 1\right] \,, & j = 2, \ldots, n-1 \\ \\ \left[1 - 2 \cdot (a_i^{(n-1)} + b_i^{(n)} + d_i^{(n)}) \cdot t, \ 1\right] \,, & j = n \end{cases} ,$$

hence, $\mathrm{supp}(\tilde{p}_i^{(j)}) \subsetneq (0, 1]$.

For simplicity, we assume that the parameters $a_i^{(j)}, b_i^{(j)}, c_i^{(j)}, d_i^{(j)}$ are only sector-specific, not firm-specific. The expected probability of success, $\mathrm{E}[\tilde{p}_i^{(j)}] = \bar{p}_i$ , simplifies to

$$\bar{p}_i^{(j)} = \begin{cases} 1 - (b_i + c_i + d_i) \cdot t \,, & j = 1 \\ 1 - (a_i + b_i + c_i + d_i) \cdot t \,, & j = 2, \ldots, n-1 \,, \quad i = 1, 2 \,. \\ 1 - (a_i + b_i + d_i) \cdot t \,, & j = n \end{cases}$$

As a consequence, the success of firm $j$ from sector $i$ has as expectation value and

[1] Under the base case outlined in Table 5.1, the support is $[0.99, 1]$ for $\tilde{p}_1^{(j)}$ and $[0.98, 1]$ for $\tilde{p}_2^{(j)}$, respectively, with $j = 2, \ldots, n-1$.

variance

$$\mathrm{E}[\tilde{X}_i^{(j)}] \;=\; \bar{p}_i^{(j)}, \quad \text{and} \quad \mathrm{V}[\tilde{X}_i^{(j)}] \;=\; \bar{p}_i^{(j)} \cdot (1-\bar{p}_i^{(j)}) \ , \quad i=1,2, \ j=1,\ldots,n \ , \quad (5.2.4)$$

respectively, analogous to any generic Bernoulli random variable. Details are given by (B.1.3) to (B.1.5) in Appendix B.1. By

$$\bar{p}_i \;\equiv\; 1-(a_i+b_i+c_i+d_i)\cdot t \qquad (5.2.5)$$

we denote to the expected success probabilities for firms $j = 2,\ldots,n-1$. This probability is linked to the generic firm $j$ from sector $i$ whose success will characterize the limit properties of aggregate loan redemptions.

The factor model (5.2.2) implies that within each sector a given firm's outcome is correlated with the outcomes of its next two followers and its last two predecessors, that is

$$\mathrm{Corr}(\tilde{X}_i^{(j)}, \tilde{X}_i^{(j\pm1)}) \;=\; \frac{1}{3}\cdot\frac{(a_i+c_i)b_i t^2}{\bar{p}_i(1-\bar{p}_i)} \ , \ 1 \le j\pm1 \le n \ , \qquad (5.2.6)$$

$$\mathrm{Corr}(\tilde{X}_i^{(j)}, \tilde{X}_i^{(j\pm2)}) \;=\; \frac{1}{3}\cdot\frac{a_i c_i t^2}{\bar{p}_i(1-\bar{p}_i)} \ , \ 1 \le j\pm2 \le n \ . \qquad (5.2.7)$$

Each firm's undertaken project is also correlated with the projects' outcomes of the three neighboring firms from the other sector, hence

$$\mathrm{Corr}(\tilde{X}_i^{(j)}, \tilde{X}_{(3-i)}^{(j-1)}) \;=\; \frac{1}{3}\cdot\frac{(a_i d_{3-i}+c_{3-i}d_i)t^2}{\sqrt{\bar{p}_1(1-\bar{p}_1)\bar{p}_2(1-\bar{p}_2)}} \ , \ 2\le j\le n \ , \qquad (5.2.8)$$

$$\mathrm{Corr}(\tilde{X}_i^{(j)}, \tilde{X}_{(3-i)}^{(j)}) \;=\; \frac{1}{3}\cdot\frac{(b_i d_{3-i}+b_{3-i}d_i)t^2}{\sqrt{\bar{p}_1(1-\bar{p}_1)\bar{p}_2(1-\bar{p}_2)}} \ , \ 1\le j\le n \ , \qquad (5.2.9)$$

$$\mathrm{Corr}(\tilde{X}_i^{(j)}, \tilde{X}_{(3-i)}^{(j+1)}) \;=\; \frac{1}{3}\cdot\frac{(c_i d_{3-i}+a_{3-i}d_i)t^2}{\sqrt{\bar{p}_1(1-\bar{p}_1)\bar{p}_2(1-\bar{p}_2)}} \ , \ 1\le j\le n-1 \ . \qquad (5.2.10)$$

The derivation of these formulæ is shown in Appendix B.1 which follows the expositions by Joe (2001, pp. 211, 219-220). Note that model parameters cannot be chosen arbitrarily, but mixture models have the advantage that consistent, non-trivial correlation structures can be set up for a large number of random variables in a convenient way.[2] Furthermore, mixture models are by far less parameter-consuming than it is the case with contingency tables.[3]

[2]Chaganty/Joe (2006) characterize restrictions on correlations matrices for binary random variables and point out that there are positive-definite correlation-matrices that are not compatible with any set of univariate success probabilities.

[3]The number of parameters increases only linearly in the number $n$ of random variables

The intra- and inter-industrial linkages arising from this model are illustrated by Figure 5.1. Each horizontal and each vertical arrow represents a common risk factor with the respective neighbor. These dependencies result in the correlations given by (5.2.6) and (5.2.9). Diagonal arrows and bowed arrows represent correlations that are based on common risk weights via common neighbors, *cf.* (5.2.7), (5.2.8), and (5.2.10).

Given the preceding model, let us re-write each firm's objective in a complete analogy to Section 3.2.1. Particularly, let $L_i^{(j)}$ be the loan volume granted to firm $j$ from sector $i$ and $R_i^{(j)}$ be the gr oss interest rate on this loan. Technology is linear and sector specific such that each firm $j$ from sector $i$ can multiply each dollar invested by $\alpha_i > 1$ in case of success. The risky redemption from the loan contract is thus given by

$$\tilde{L}_i^{(j)} = \min\left\{L_i^{(j)} \cdot R_i^{(j)},\ \alpha_i \cdot \tilde{X}_i^{(j)} \cdot L_i^{(j)}\right\}\ ,$$

and expected final wealth by

$$\mathrm{E}\left[\max\left\{\alpha_i \cdot \tilde{X}_i^{(j)} \cdot L_i^{(j)} - L_i^{(j)} \cdot R_i^{(j)},\ 0\right\}\right] \ =$$

$$\mathrm{E}\left[\tilde{p}_i^{(j)} \cdot \max\left\{\alpha_i - R_i^{(j)}, 0\right\} \cdot L_i^{(j)} + (1 - \tilde{p}_i^{(j)}) \cdot 0\right] = \bar{p}_i^{(j)} \cdot \max\left\{\alpha_i - R_i^{(j)}, 0\right\} \cdot L_i^{(j)},$$

being equal to (3.2.2) except for the expectations formed about the success probability. Because firms are risk-neutral and because firms are not endowed with any equity initially, they will demand debt finance as given by (3.2.3). Consequently, the bank optimally sets the loan interest rates according to $R_i^* = \alpha_i$ for any loan granted to a firm from sector $i$ and the average loan volume in sector $i$ amounts to

$$L_i^* \ = \ \frac{1}{n} \cdot L_i, \quad i = 1, 2,\ j = 1, \ldots, n, \tag{5.2.11}$$

given that the bank lends an aggregate loan volume of $L_i$ to the whole sector $i$. Again, the aggregate loan volume will have to be determined by the refinancing opportunities that the household offers to the bank.

considered if mixture models are used whereas the approach by contingency tables requires $2^n - 1$ parameters (*cf.* Joe, 2001, p. 211).

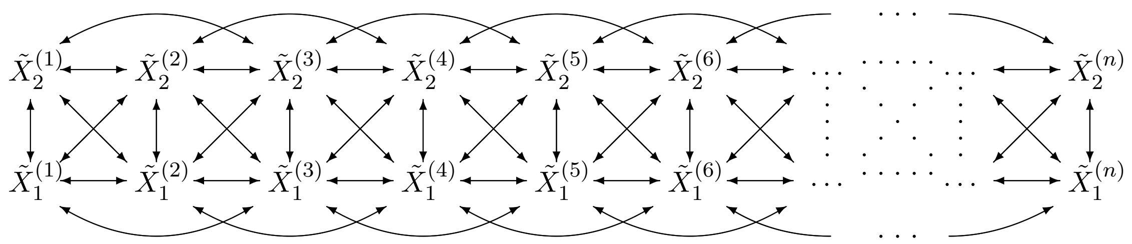


Figure 5.1: **Correlation structure between projects**

*This figure illustrates the dependence structure between project successes if the success probabilities are given by* (5.2.2)*. Each arrow represents a positive correlation between the respective projects.*

## 5.3 The $m$-Dependence Structure and Its Limit Distribution

The aim of this section is to characterize this dependence structure further and to present a framework that enables the derivation of the limit distributions for the sums of sector-wise loan repayments $\sum_{j=1}^{n} \tilde{L}_i^{(j)}$ and for the common repayments.

Beyond the classical central limit theorem that particularly presumes mutual independence of the random variables $\tilde{X}_j$ considered within the sequence $\{\tilde{X}_j\}_{i=j,\ldots,n}$, Serfling (1968, pp. 1158f) presents the following conditions,

$$\lim_{n\to\infty} \mathrm{E}\left[\left(\frac{1}{\sqrt{n}} \cdot \sum_{j=a+1}^{a+n} \left(\tilde{X}_j - \mathrm{E}\left[\tilde{X}_j\right]\right)\right)^2\right] = s^2,\ s^2 > 0, \quad \text{uniformly for all } a \geq 0, \tag{5.3.1}$$

$$\mathrm{E}\left|\tilde{X}_j\right|^{2+\delta} \leq M,\ M < \infty, \quad \text{for some } \delta > 0\ , \tag{5.3.2}$$

that are necessary so that the following normalized sum of the random variables $\tilde{X}_j$, $j = 1, \ldots, n$,

$$\frac{1}{\sqrt{n} \cdot s} \cdot \sum_{j=1}^{n} \left(\tilde{X}_j - \mathrm{E}\left[\tilde{X}_j\right]\right)\ ,$$

is asymptotically normal with mean zero and variance equal to one, *i.e.*

$$\lim_{n\to\infty} \mathrm{P}\left[\frac{\sum_{j=1}^{n} \tilde{X}_j - \sum_{j=1}^{n} \mathrm{E}[\tilde{X}_j]}{\sqrt{n} \cdot s} \leq z\right] = \Phi(z)\ ,$$

where $\Phi(z)$ is the cumulative distribution function of the standard normal distribution. If the random variables are independent, Assumptions (5.3.1) and (5.3.2) become sufficient and the classical central limit theorem holds.

For dependent random variables, Assumption (5.3.1) may fail in general. (5.3.1) requires the variance of the sum of $\tilde{X}_j$, $j = 1, \ldots, n$, not to increase by an order higher than $n$. However, $\mathrm{V}(\sum_{j=1}^{n} \tilde{X}_j)$ grows by $n^2$ for general covariance patterns. So it is necessary to establish a covariance structure among the random variables that excludes any dependencies between two sets of random variables as soon as these two sets get too far apart. A notion such as this can be rendered more precisely by the so-called $m$-dependence (Hoeffding/Robbins, 1948, p. 773; Billingsley, 1995, p. 364; and Serfling, 1968, p. 1162):

**Definition 1.** *A sequence $\{\tilde{X}_i\} = \{\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_n\}$ is said to be $m$-dependent if the two sequences $\{\tilde{X}_{a-r}, \tilde{X}_{a-r+1}, \ldots, \tilde{X}_a\}$ and $\{\tilde{X}_b, \tilde{X}_{b+1}, \ldots, \tilde{X}_{b+s}\}$ are independent sets of random variables if $b - a > m$ for all $r, s \geq 0$ and $m$ a fixed constant.*

Clearly, 0-dependence is equivalent to independence whereas the higher is the value for $m$, the deeper becomes the dependence structure. Note, however, that $m$ is a fixed constant such that the depth of the dependence structure does not increase in $n$. Consequently, the number of covariance terms in the variance of the sum over $\tilde{X}_i$, $i = 1, \ldots, n$, increases only linearly in $n$ such that Assumption (5.3.1) is fulfilled.

Now, consider sector $i$ as defined by the factor model (5.2.2) in Section 5.2 and take $\tilde{X}_i^{(3)}$ as a starting point. Then $\tilde{X}_i^{(3)}$ depends on $\tilde{X}_i^{(j)}$ up to $j = 5$ and down to $j = 1$ (*cf.* Fig. 5.1). Thus the two sequences $\{\tilde{X}_i^{(1)}, \tilde{X}_i^{(2)}, \tilde{X}_i^{(3)}\}$ and $\{\tilde{X}_i^{(6)}, \tilde{X}_i^{(7)}, \ldots, \tilde{X}_i^{(6+s)}\}$, $s \geq 0$, and so forth are independent sets of random variables. By Definition 1, the sequence $\tilde{X}_i^{(j)}$ is 2-dependent. Similarly, the bivariate sequence $\{\ (\tilde{X}_1^{(j)}, \tilde{X}_2^{(j)})\ \}$ is 2-dependent as well. This 2-dependence structure can be illustrated by replacing the projects' returns $\tilde{X}_i^{(j)}$ in Figure 5.1 by the factor loadings $\tilde{\xi}_i^{(j)}$ according to (5.2.2).

Given $m$-dependent sequences, central limit theorems can be applied again, in particular given the Assumptions (5.3.1) and (5.3.2) (Serfling, 1968, p. 1159). His theorem resorts to the seminal work of Hoeffding/Robbins (1948). Furthermore, we can take advantage of the assumption that the random variables are identically distributed, *i.e.* of (5.2.1) and (5.2.4). As a direct consequence, we can relax Condition (5.3.1) in the univariate case (Hoeffding/Robbins, 1948, pp. 774, 776). Moreover, we present the theorem applicable to the bivariate case which can, in turn, be readily stated for the univariate case (*ibid.* pp. 776, 776-777 and *cf.* Billingsley, 1995, p. 364, who extends the univariate case to so-called $\alpha$-mixing distributions):

**Theorem 1 (Hoeffding/Robbins, 1948).** *Let* $\{(\tilde{X}_1^{(j)}, \tilde{X}_2^{(j)})\} = \{(\tilde{X}_1^{(1)}, \tilde{X}_2^{(1)}), (\tilde{X}_1^{(2)}, \tilde{X}_2^{(2)}), \ldots, (\tilde{X}_1^{(n)}, \tilde{X}_2^{(n)})\}$ *be an* $m$*-dependent sequence of random vectors which are identically distributed and whose expectations are normalized to zero,* $\left(\mathrm{E}(\tilde{X}_1^{(j)}), \mathrm{E}(\tilde{X}_2^{(j)})\right) = (0,0)$*, and let* $\mathrm{E}|\tilde{X}_1^{(j)}|^3 < \infty$ *and* $\mathrm{E}|\tilde{X}_2^{(j)}|^3 < \infty$ *hold. Then as* $n \to \infty$*, the random vector* $\frac{1}{\sqrt{n}}\left(\sum_{j=1}^n \tilde{X}_1^{(j)}, \sum_{j=1}^n \tilde{X}_2^{(j)}\right)$ *has a limiting normal distribution with mean* $(0,0)$ *and covariance matrix*

$$\begin{pmatrix} s_1^2 & s_{1,2} \\ s_{1,2} & s_2^2 \end{pmatrix}$$

*where*

$$s_1^2 = \mathrm{E}(\tilde{X}_1^{(1)}\tilde{X}_1^{(1)}) + 2 \cdot \sum_{j=1}^{m} \mathrm{E}(\tilde{X}_1^{(1)}\tilde{X}_1^{(j+1)}) \,, \tag{5.3.3}$$

$$s_{1,2} = \mathrm{E}(\tilde{X}_1^{(1)}\tilde{X}_2^{(1)}) + \sum_{j=1}^{m} \left[\mathrm{E}(\tilde{X}_1^{(1)}\tilde{X}_2^{(j+1)}) + \mathrm{E}(\tilde{X}_1^{(j+1)}\tilde{X}_2^{(1)})\right] \,, \tag{5.3.4}$$

$$s_2^2 = \mathrm{E}(\tilde{X}_2^{(1)}\tilde{X}_2^{(1)}) + 2 \cdot \sum_{j=1}^{m} \mathrm{E}(\tilde{X}_2^{(1)}\tilde{X}_2^{(j+1)}) \,. \tag{5.3.5}$$

Based on this theorem, we will approximate the distribution of aggregate loan redemptions, both sector-wise and in total, in the following section.

## 5.4 The Limit Distributions of the Aggregate Loans

For sufficiently large, but still finite $n$ we may neglect the first and the $n$-th firm belonging to each sector. Thus, aggregate loan redemptions $\tilde{L}_i$,

$$\tilde{L}_i \equiv \sum_{j=1}^{n} \tilde{L}_i^{(j)} \equiv \sum_{j=1}^{n} \alpha_i \cdot \tilde{X}_i^{(j)} \cdot L_i^{(j)} \,, \; i = 1, 2 \,, \tag{5.4.1}$$

consist of identically distributed loan redemptions on the micro-level. Treating each loan redemptions $\tilde{L}_i^{(j)}$ as being equal to each other within each sector $i$, *i.e.* $\tilde{L}_i^{(j)} = \tilde{L}_i^{(k)}$ for all $j \neq k$, $i$ fixed, facilitates computations of aggregate statistical moments, and, in addition, makes Theorem 1 applicable. Therefore, we can approximate the distribution of the non-centered sums

$$\frac{1}{\sqrt{n}} \cdot \sum_{j=1}^{n} \tilde{X}_i^{(j)}$$

by the normal distribution

$$\mathcal{N}(\ \sqrt{n}\bar{p}_i\alpha_i,\ s_i^2\ )$$

where $\sqrt{n}\bar{p}_i\alpha_i = \frac{1}{\sqrt{n}} \cdot \mathrm{E}[\sum_{j=1}^{n} \tilde{X}_i^{(j)}]$, and where $s_i$ is given according to (5.3.3) and (5.3.5), respectively. Furthermore, the tuple of both sums is approximately bivariate normal,

$$\frac{1}{\sqrt{n}} \cdot \left( \sum_{j=1}^{n} \tilde{X}_1^{(j)}, \sum_{j=1}^{n} \tilde{X}_2^{(j)} \right)^{\top} \overset{d}{\sim} \mathcal{N}(\ \sqrt{n} \cdot \begin{pmatrix} \bar{p}_1\alpha_1 \\ \bar{p}_2\alpha_2 \end{pmatrix}, \begin{pmatrix} s_1^2 & s_{1,2} \\ s_{1,2} & s_2^2 \end{pmatrix}\ )\ .$$

To obtain the limit distributions for both loan portfolios, we scale the sums shown above by $\sqrt{n} \cdot L_i$. Thus, we obtain the aggregate loan redemptions as defined by (5.4.1) with the following approximate limiting distribution:

$$\tilde{L}_i \equiv \sum_{j=1}^{n} \tilde{L}_i^{(j)} \equiv \sum_{j=1}^{n} \tilde{X}_i^{(j)} \cdot L_i^{(j)} \overset{d}{\sim} \mathcal{N}(\ \bar{p}_i\alpha_i \cdot L_i,\ \frac{1}{n} \cdot s_i^2 \cdot L_i^2\ )\ . \tag{5.4.2}$$

Analogously for the bivariate case,

$$\left(\tilde{L}_1, \tilde{L}_2\right)^{\top} \overset{d}{\sim} \mathcal{N}( \begin{pmatrix} \bar{p}_1\alpha_1 \cdot L_1 \\ \bar{p}_2\alpha_2 \cdot L_2 \end{pmatrix}, \frac{1}{n} \cdot \begin{pmatrix} s_1^2 \cdot L_1^2 & s_{1,2} \cdot L_1 \cdot L_2 \\ s_{1,2} \cdot L_1 \cdot L_2 & s_2^2 \cdot L_2^2 \end{pmatrix}\ )\ , \tag{5.4.3}$$

approximately holds. Concerning the last two derivations, we have made use of the Property (5.2.11) that is valid for the optimal loan volumes granted by the bank.

Thus, each dollar invested into loan portfolio $i$ yields a gross return $\tilde{x}_i$ that is approximately normally distributed with mean $\mu_i$ and variance $\sigma_i^2 \equiv \frac{s_i^2}{n}$,

$$\tilde{x}_i \sim \mathcal{N}(\mu_i, \sigma_i^2)\ , \tag{5.4.4}$$

where $\mu_i$ and $\sigma_i$ are approximately given by

$$\begin{aligned} \mu_i &= \bar{p}_i \cdot \alpha_i\ , \\ \sigma_i \equiv \frac{s_i}{\sqrt{n}} &= \frac{1}{\sqrt{n}} \cdot \sqrt{\bar{p}_i \cdot (1-\bar{p}_i) + \frac{2}{3}(a_i c_i + a_i b_i + b_i c_i)t^2} \cdot \alpha_i\ ,\ i = 1, 2\ , \end{aligned} \tag{5.4.5}$$

Table 5.1: **Parameter values of the base case, Bernoulli mixture model**

*This table reports the parameter values of the base case for the factor model given by* (5.2.2). *The Bernoulli distribution characterized by the success probabilities* $\tilde{p}_i^{(j)}$ *takes values for its expectation and its variance that equal those of the Bernoulli distribution used in Part II under the parameter values given in Table 4.1.*

| $a_1$ | $b_1$ | $c_1$ | $d_1$ | $a_2$ | $b_2$ | $c_2$ | $d_2$ | $t$ |
|---|---|---|---|---|---|---|---|---|
| 0.02 | 0.05 | 0.02 | 0.01 | 0.04 | 0.10 | 0.04 | 0.02 | 0.05 |

and where $\bar{p}_i$ is given according to (5.2.5). Let the gross return on the sum of both loan portfolios be given by

$$\tilde{x} \;=\; \ell \cdot \tilde{x}_1 \;+\; (1-\ell) \cdot \tilde{x}_2 \;, \tag{5.4.6}$$

where $\ell$ is defined in complete analogy to (3.2.14) and, in conjunction with the model considered in this part, referred to as the portfolio-allocation rate.

Consequently, the return $\tilde{x}$ is normally distributed, $\tilde{x} \sim \mathcal{N}(\mu, \sigma^2)$, with

$$\begin{aligned} \mu \;&=\; \ell \cdot \mu_1 \;+\; (1-\ell) \cdot \mu_2 \;, \\ \sigma^2 \;&=\; \ell^2 \cdot \sigma_1^2 \;+\; 2 \cdot \ell \cdot (1-\ell) \cdot \sigma_{1,2} \;+\; (1-\ell)^2 \cdot \sigma_2^2 \;. \end{aligned} \tag{5.4.7}$$

where $\sigma_{1,2}$ is the covariance that is given by

$$\sigma_{1,2} \;\equiv\; \frac{s_{1,2}}{n} \;=\; \frac{1}{3n} \cdot [(a_2+b_2+c_2) \cdot d_1 + (a_1+b_1+c_1) \cdot d_2] \cdot t^2 \cdot \alpha_1 \alpha_2 \;. \tag{5.4.8}$$

Thus, aggregate loan redemptions $\sum_{i=j}^{n} \tilde{L}_i^{(j)}$ and aggregate gross returns $\tilde{x}_i$, respectively, are approximately correlated by

$$\rho \;=\; \frac{\frac{1}{3} \cdot [(a_1+b_1+c_1) \cdot d_2 + (a_2+b_2+c_2) \cdot d_1] \cdot t^2}{\prod_{i=1}^{2} \sqrt{\bar{p}_i(1-\bar{p}_i) + \frac{2}{3}(a_i b_i + a_i c_i + b_i c_i) t^2}} \;. \tag{5.4.9}$$

Concerning these formulæ we refer to Appendix B.2 where their derivations are demonstrated and the respective formulæ considering the aggregate loan repayments $\tilde{L}_i$ are given by (B.2.3) to (B.2.6).

For illustrative purposes, we consider our Bernoulli mixture model for a given set of parameter values, as shown in Table 5.1 and compare the limit distribution with a

Table 5.2: **Correlation values of the base case, Bernoulli mixture model**

*This table reports the values for the project return correlations under the base case. Both inter- and intra-sectoral correlations are shown. Numbers are rounded to three leading digits.*

| | | | |
|---|---|---|---|
| $\mathrm{Corr}(\tilde{X}_1^{(j)}, \tilde{X}_1^{(j+1)})$ | $\mathrm{Corr}(\tilde{X}_1^{(j)}, \tilde{X}_1^{(j+2)})$ | $\mathrm{Corr}(\tilde{X}_2^{(j)}, \tilde{X}_2^{(j+1)})$ | $\mathrm{Corr}(\tilde{X}_2^{(j)}, \tilde{X}_2^{(j+2)})$ |
| 0.000335 | 0.0000670 | 0.000340 | 0.000680 |
| $\mathrm{Corr}(\tilde{X}_1^{(j)}, \tilde{X}_2^{(j-1)})$ | $\mathrm{Corr}(\tilde{X}_1^{(j)}, \tilde{X}_2^{(j)})$ | $\mathrm{Corr}(\tilde{X}_1^{(j)}, \tilde{X}_2^{(j+1)})$ | |
| 0.0000479 | 0.000120 | 0.0000479 | |

simulated distribution.

The parametrization in Table 5.1 results in $\bar{p}_1 = 0.995$ and $\bar{p}_2 = 0.99$ and thus each single loan redemption has the same distribution as considered in Table 4.1. However, correlations (*cf.* Formulæ (5.2.6) to (5.2.10)) are lower, in fact close to zero, as illustrated in Table 5.2. On the other hand, loan-return correlations are always well below their associated asset-return correlations unless the latter approaches one as emphasized by Gersbach/Lipponer (2003, p. 365f).[4]

Let us assume that each sector consists of $n = 40,000$ firms. For simplicity, we normalize every single loan volume $L_i$ to one, such that the aggregate loan volume $L_i$ equals the number $n$ of borrowers. Running the simulation 200 times leads to the distributions with moments, as shown in Table 5.3. These simulated values are compared to the theoretical values given by (B.2.3) to (B.2.5) in Appendix B.2 for parameter values equal to those from Table 5.1.

By this model we are able to approximate the aggregate loan redemptions by the normal distribution, though intra- and inter-sectoral correlation patterns between the firms' projects are present. Inter-sectoral correlations are incurred by the sector-wise variance as in (5.3.3) and (5.3.5), respectively. Intra-sectoral correlations on the firms' level result in the aggregate inter-sectoral correlation as given by (5.3.4). Particularly, this model can represent a positive correlation between sector-wise loan redemptions on an aggregate level without losing the central limit property of normality.

Concerning the equilibrium analysis in this model framework, a smooth limit distribution has the well-known advantage that we have not to analyze each loan individually. If we accounted separately for every single of the $2 \cdot n$ loans,

[4]Compare also the values from Table 5.2 with the value of the correlation parameter given in Table 4.1, Panel B and the associated discussion of these values considered for the Bernoulli-model on p. 119.

Table 5.3: **Comparison between simulated and limiting distribution**

*This table reports the values of moments of the random sums* $\sum \tilde{X}_1^{(j)}$, $i = 1, 2$, *i.e. of the sector-wise loan redemptions. Each sum/sector consists of* $n = 40{,}000$ *firms as defined by* (5.2.1)*. In the simulation each sum was drawn* $N = 200$ *times. The moments of the limiting normal distribution are given by* (B.2.3) *to* (B.2.5) *in Appendix B.2. If necessary, numbers are rounded to six leading digits.*

| statistical moment | simulated distribution | limiting distribution |
|---|---|---|
| $\mathrm{E}\left[\sum \tilde{X}_1^{(j)}\right] \equiv \mu_1 \cdot L_1$ | 45,769.9 | 45,770 |
| $\mathrm{V}\left[\sum \tilde{X}_1^{(j)}\right] \equiv \sigma_1^2 \cdot L_1^2$ | 253.125 | 263.389 |
| Jarque-Bera test statistic | 1.75630 | 0 |
| $\mathrm{E}\left[\sum \tilde{X}_2^{(j)}\right] \equiv \mu_2 \cdot L_2$ | 47,522.2 | 47,520 |
| $\mathrm{V}\left[\sum \tilde{X}_2^{(j)}\right] \equiv \sigma_2^2 \cdot L_2^2$ | 566.898 | 570.470 |
| Jarque-Bera test statistic | 0.118472 | 0 |
| $\mathrm{Cov}\left[\sum \tilde{X}_1^{(j)}, \sum \tilde{X}_2^{(j)}\right] \equiv \sigma_{1,2} \cdot L_1 \cdot L_2$ | 0.160122 | 0.1656 |
| $\mathrm{E}\left[\frac{1}{2} \cdot \left(\sum \tilde{X}_1^{(j)} + \sum \tilde{X}_2^{(j)}\right)\right] \equiv \mu \cdot (L_1 + L_2)$ | 46,646.0 | 46,645 |
| $\mathrm{V}\left[\frac{1}{2} \cdot \left(\sum \tilde{X}_1^{(j)} + \sum \tilde{X}_2^{(j)}\right)\right]$ | 205.086 | 208.548 |
| Jarque-Bera test statistic | 1.84903 | 0 |

$2^{2n+1} - 2 \cdot n - 2$ Cases would have to be considered as outlined in Section 3.2.2, in the footnote on p. 51. Finally, the application of the VaR approach is facilitated.

A drawback of this approach is that the gross returns $\tilde{x}$ and, thus, the aggregate loan portfolio redemptions $\tilde{L}$, may be negative with probability $\Phi(-\frac{\mu}{\sigma})$. This probability decreases in the square root of the number of firms $n$ because of (5.4.5). If the whole loan portfolio only consists of loans from one sector, $\tilde{L} = \tilde{L}_i$, this probability reduces to $\Phi(-\frac{\mu_i}{\sigma_i})$, *i.e.*

$$\begin{aligned} \Phi(-\frac{\mu_i}{\sigma_i}) &\overset{(5.4.5)}{=} \Phi(-\frac{\bar{p}_i \alpha_i \cdot \sqrt{n}}{\sqrt{\bar{p}_i \cdot (1 - \bar{p}_i) + \frac{2}{3}(a_i c_i + a_i b_i + b_i c_i) t^2 \cdot \alpha_i}}) \\ &\overset{\text{Table 5.1}}{\approx} \Phi(-\frac{\bar{p}_i}{\sqrt{\bar{p}_i \cdot (1 - \bar{p}_i)}} \cdot \sqrt{n}) \ . \end{aligned}$$

For large $n$, this probability becomes negligible. Furthermore, Properties (3.2.5) and (3.2.6) transfer via (5.4.5) to

$$\mu_2 \ > \ \mu_1 \ > \ R_f \ \geq \ 1 \ . \tag{5.4.10}$$

Fulfilling additionally the inequality $a_1b_1 + a_1c_1 + b_1c_1 \ < \ a_2b_2 + a_2c_2 + b_2c_2$ is thus a sufficient condition to obtain

$$\sigma_1 \ < \ \sigma_2 \ . \tag{5.4.11}$$

Furthermore, following (3.2.7), we set

$$\mu_i \ < \ 2 \quad \text{and by (5.2.3)} \quad \sigma_i \ < \ 0.1 \ , \tag{5.4.12}$$

where the latter is obtained if we additionally assume that loan portfolio $i$ consists of at least 144 borrowers, *i.e.* $n \geq 144$.[5] As 0.2 is rather typical for asset-return volatilities, this bound on $\sigma_i$, which represents the volatility of returns on loan portfolio $i$, does not seem to be too restrictive.

As a consequence, we obtain by (5.4.10) and (5.4.12)

$$\frac{\mu}{\sigma} \ \geq \ \frac{\min\{\mu_1, \mu_2\}}{\max\{\sigma_1, \sigma_2\}} \ > \ \frac{1}{0.1} \ = \ 10 \tag{5.4.13}$$

for the ratio of expected gross returns over their respective standard deviations, resulting in the following approximations concerning the density function $\varphi(x)$ and the cumulative distribution function $\Phi(x)$ of the standard normal distribution

$$\begin{aligned} \varphi(\tfrac{\mu}{\sigma}) &\approx 0, \quad \text{since} \quad \varphi(\tfrac{\mu}{\sigma}) < \varphi(10), \\ \Phi(\tfrac{\mu}{\sigma}) &\approx 1, \quad \text{since} \quad \Phi(\tfrac{\mu}{\sigma}) > \Phi(10), \end{aligned} \tag{5.4.14}$$

and likewise $\Phi(-\frac{\mu}{\sigma}) \approx 0$. Whenever (5.4.14) is used in deriving results, we will explicitly refer to it. If we do not use this approximation and the positive chance of negative redemptions of the loan portfolio remains, we assume that these losses are borne by a governmental authority that is not explicitly modeled.

The one-factor model on which the IRB approach is based can be considered as a Bernoulli-mixture model with firm default as Bernoulli-distributed random variable and with the macro factor as the mixing distribution (Giesecke 2004, p. 26). In contrast to our model, the derivation of the distribution of aggregate defaults (loan redemptions) rests on a given realization of a single macro factor.[6] Thus they reflect a centralized dependence structure whereas our model pursues a decentralized dependence structure.

[5]Condition (5.2.3) allows to bound $t$ by $\frac{1}{2} \cdot \frac{1}{a_i+b_i+c_i+d_i}$ resulting in $(a_ic_i + a_ib_i + b_ic_i)t^2 < \frac{1}{4} \cdot \frac{a_ic_i+a_ib_i+b_ic_i}{(a_i+b_i+c_i+d_i)^2} < \frac{1}{4} \cdot \frac{1}{2}$. Let $n \geq 144$, we finally obtain $\sigma_i \leq \frac{1}{12} \cdot \sqrt{\frac{1}{3}} \cdot 2 \approx 0.0962$.

[6] *Cf.* Giesecke (2004, pp. 21-32), Lütkebohmert (2009), Schönbucher (2000), and BCBS (2004, Art. 272). Gordy (2003) provides a rigorous characterization of these single-factor models.

# Chapter 6

# The One-Period Model

## 6.1 Introduction

By the last chapter, Chapter 5, the grounds have been laid for approximating weakly dependent, Bernoulli-distributed loan redemptions by the normal distribution. Intra- and inter-sectoral dependencies have been considered on the firm level. In total, aggregate loan redemptions related to each of both portfolios are normally distributed and both aggregate loan redemptions preserve the inter-sectoral dependency.

The model that is considered in this chapter is analogous to that analyzed in Part II. The two single firms, however, are now replaced by portfolios of which the returns are normally distributed and which may be correlated, in line with the discussions in Chapter 5. The risk-averse household supplies deposits and the banks serves as an intermediary between firms and the household.

The bank's objective is outlined in the next section, Section 6.2, followed by an analysis of the household's decision in Section 6.3. In Section 6.4, the equilibrium absent of regulation is discussed, in Section 6.5 the equilibrium if the bank is regulated by a VaR-based approach. In contrast to Part II, the Standardized Approach is no longer considered as it does not yield any further insights in this framework considered. The properties of the regulatory constraint as such have been already discussed in Section 3.3.2.

As a general analysis does not yield further insights into the potential impacts regulation may have, the analysis is purely based on numerical examples as done in Section 6.6. The laissez-faire equilibrium is compared to the outcomes under a VaR-based regulation and partly also to the outcomes arising under the Standardized

Approach. Section 6.7 concludes.

## 6.2 The Bank's Objective

The bank is risk-neutral and maximizes its expected final wealth. It acts as a financial intermediary between households and firms. All contracts start at the beginning of the period considered and mature at the end of the period.

The bank owners can grant loans to firms from two sectors as discussed in Chapter 5. We have already pointed out by (5.2.11) how the bank optimally chooses the loan interest rates $R_i$ and the loan volumes $L_i$ on the individual firm level. The whole model outlined in the preceding section presumes an equal number of firms in each sector. However, the bank is still free to assign different total loan volumes $L_i$ to each sector. We assume that this decision on the aggregate level is based on the bank's view of the aggregate distribution, in the sense that it considers the limiting distribution for each sector and for both sectors together. Thus, we can build on the normal framework, and the return of the bank's loan portfolio will follow (5.4.6). The decision variable $\ell$ is referred to as the portfolio-allocation rate. Again, there are no short selling opportunities to the bank, $\ell \in [0, 1]$. The bank refinances itself by fixed, exogenously given equity $W_B$ and deposits $D$ supplied by the household. The deposit contract is a standard debt contract, with the following pay-off function

$$\max\{x \cdot (D + W_B),\ D \cdot R_D\} \ ,$$

where $D$ is the amount deposited at the bank at the beginning of the period, $R_D$ the contracted gross interest rate on deposits and $x$ the realized gross return on the loan portfolio. The return on the loan portfolio is given by (5.4.6) and hence $\tilde{x}$ is normally distributed according to

$$\tilde{x} \ \sim \ \mathcal{N}(\mu, \sigma^2) \ ,$$

where the expected return $\mu$ and the associated volatility $\sigma$ are given by (5.4.7), including (5.4.8). In particular, both moments depend on the portfolio-allocation rate $\ell$ and so does the bank's expected final wealth $\mathrm{E}[\tilde{W}_B(\ell, R_D)]$.

Because of the bank's limited liability its expected final wealth $\mathrm{E}[\tilde{W}_B(\ell, R_D)]$ simply contains the expected difference of in- and outflows in those states where the bank is able to pay back deposits and interest due as promised

$$\mathrm{E}\left[\tilde{W}_B(\ell, R_D)\right] \;=\; \int\limits_{\frac{DR_D}{D+W_B}}^{\infty} [\; x \cdot (D+W_B) - D \cdot R_D \;] \cdot \frac{1}{\sigma} \cdot \varphi(\frac{x-\mu}{\sigma})\; dx \;,$$

simplifying to

$$\begin{aligned} \mathrm{E}\left[\tilde{W}_B(\ell, R_D)\right] \;&=\; \sigma \cdot (D+W_B) \cdot \varphi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \\ &\quad + \;[\mu \cdot (D+W_B) - D \cdot R_D] \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \;, \end{aligned} \tag{6.2.1}$$

where

$$\Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \;\equiv\; \mathrm{P}[\; \tilde{x} \;\geq\; \frac{DR_D}{D+W_B} \;] \tag{6.2.2}$$

is the probability of deposits being fully redeemed. It can be aligned with the probabilities $q_j$ given by (3.2.41) in the Bernoulli model. Henceforth, let

$$b_{s_z} \;=\; \frac{\mu - \frac{DR_D}{D+W_B}}{\sigma} \tag{6.2.3}$$

be the solvency barrier for the bank after having standardized the returns on the loan portfolio by their expected return and their volatility. In contrast to the bank's success probability $q_j$, $j = 1, \ldots, 4$, that accounts for the lumped character of loan redemptions and of full deposit redemption in the Bernoulli model, $\Phi(b_{s_z})$ depends on the loan portfolio risk characterized by $\ell$, the bank's debt volume $D$, initial equity $W_B$, and hence leverage.

The difference $\mu \cdot (D+W_B) - D \cdot R_D$ would be the bank's expected final wealth if it had unlimited liability. Intermediation is valuable to the bank as long as expected wealth given by (6.2.1) exceeds the pure equity investment, *i.e.* $\max\{\mu_1, \mu_2\} \cdot W_B$. If initial equity is zero, $W_B = 0$, intermediation is even valuable for deposit interest rates $R_D$ up to the expected return $\mu$ on the loan portfolio because of

$$\sigma \cdot D \cdot \varphi(\frac{\mu - R_D}{\sigma}) \;>\; 0 \;.$$

The additional costs associated with the increasing risk of bankruptcy are given by

$$\frac{\partial \Phi(b_{s_z})}{\partial D} \;=\; -\,\frac{1}{\sigma} \cdot \frac{W_B \cdot R_D}{(D+W_B)^2} \cdot \varphi(b_{s_z}) \;<\; 0 \;, \tag{6.2.4}$$

*i.e.* by a decreasing probability of solvency.

The marginal return on the bank's expected final wealth by each dollar of deposits $D$ raised is independent of theses costs as it is equal to

$$\begin{aligned}\frac{\partial \mathrm{E}[\tilde{W}_B(\cdot)]}{\partial D} &= \sigma \cdot \varphi(b_{s_z}) \ + \ (\mu - R_D) \cdot \Phi(b_{s_z}) \\ &\quad + \sigma \cdot (D + W_B) \cdot \varphi(b_{s_z}) \cdot \frac{\mu - \frac{DR_D}{D+W_B}}{\sigma^2} \cdot \underbrace{\frac{W_B R_D}{(D+W_B)^2}}_{\equiv -\sigma \cdot \frac{\partial b_{s_z}}{\partial D}} \\ &\quad + \ [\mu(D + W_B) - DR_D] \cdot \varphi(b_{s_z}) \cdot \frac{\partial b_{s_z}}{\partial D} \\ &= \sigma \cdot \varphi(b_{s_z}) \ + \ (\mu - R_D) \cdot \Phi(b_{s_z}) \ . \end{aligned} \tag{6.2.5}$$

Thus the increasing risk of bankruptcy is not directly internalized by the risk-neutral bank, but indirectly, via the deposits supply, by the risk-averse household. The same argument as with respect to deposit supply applies Analogous to the portfolio-allocation rate which can take effects in both directions.

## 6.3 The Household

As described in Section 3.2.3, the household can allocate its initial wealth, $W_H$, between risky bank deposits, promising a pay-off of $D \cdot R_D$, and risk-free assets yielding the exogenous gross return $R_f$. The household maximizes its expected utility over final wealth $\tilde{W}_H$ resulting in its optimal supply of deposits. The household is assumed to be risk-averse with a constant coefficient of absolute risk aversion $\gamma > 0$. Its von Neumann-Morgenstern utility function over outcomes $y$ is given by

$$u_H(y) \ = \ -e^{-\gamma \cdot y} \ .$$

Outcomes are given in dollars and thus, risk-aversion is measured in one over dollars as utility is cardinal.

Since deposits are uninsured and are arranged by a standard debt contract, their redemption depends on the performance of the bank's loan portfolio and is thus risky, featuring the following pay-off profile,

$$\tilde{D} \ = \ \max\left\{\min\left\{D \cdot R_D, \ \tilde{x} \cdot (D + W_B)\right\}, \ 0\right\} \ ,$$

where $\tilde{x}$ is the stochastic, normally distributed gross return on the bank's aggregate loan portfolio according to (5.4.6).

The household judges deposit contracts according to their risk-return profiles. The latter depend on the bank's leverages, the exogenous risk-return profiles of each sector-specific loan portfolio and the bank's endogenous decisions about the portfolio-allocation rate $\ell$ and the deposit interest rate $R_D$. As there is no asymmetric information, these inputs are perfectly observable to the household and the bank credibly commits to each pair $(\ell, R_D)$. The household takes $(\ell, R_D)$ as given when maximizing its expected utility for $D$.

The household's final wealth amounts to

$$\tilde{W}_H \;=\; \max\left\{\min\left\{D \cdot R_D,\; \tilde{x} \cdot (D + W_B)\right\},\; 0\right\} \;+ (W_H - D) \cdot R_f \;,$$

being concave in the deposit volume $D$ for given portfolio returns $\tilde{x}$. Then the household's expected utility over final wealth from its portfolio decision is given by

$$\begin{aligned}
\mathrm{E}\left[u_H(\tilde{W}_H)\right] \;=\; & \int_{-\infty}^{0} -e^{-\gamma\cdot(W_H - D)\cdot R_f} \cdot \frac{1}{\sigma} \cdot \varphi(\frac{x-\mu}{\sigma})\; dx \\
& + \int_{0}^{\frac{DR_D}{D+W_B}} -e^{-\gamma\left[x(D+W_B)+(W_H-D)R_f\right]} \cdot \frac{1}{\sigma} \cdot \varphi(\frac{x-\mu}{\sigma})\; dx \\
& + \int_{\frac{DR_D}{D+W_B}}^{\infty} -e^{-\gamma\left[DR_D+(W_H-D)R_f\right]} \cdot \frac{1}{\sigma} \cdot \varphi(\frac{x-\mu}{\sigma})\; dx \;.
\end{aligned}$$

The first integral represents all states where the bank completely loses its loans. Consequently, there are zero pay-offs from the deposit contract and only the risk-free asset is redeemed with its interest bearing.

The second integral represents all states where the bank still goes bankrupt since it cannot meet its obligations related to the debt contract, but where firms pay back a positive amount on an aggregate level. Thus, the residual value from the loan portfolio is transferred to the household in addition to the pay-off from the risk-free asset.

The last integral summarizes all states where the bank can meet its obligations to the household and the household is paid-off as promised.

Evaluating the integrals yields

$$
\begin{aligned}
\mathrm{E}\left[u_H(\tilde{W}_H)\right] \;=\; & -e^{-\gamma(W_H-D)R_f} \cdot \Phi(-\frac{\mu}{\sigma}) \\
& -e^{-\gamma\left[\mu(D+W_B)+(W_H-D)R_f-\frac{1}{2}\gamma\sigma^2(D+W_B)^2\right]} \\
& \cdot \left[\Phi(\frac{\mu-\gamma\sigma^2(D+W_B)}{\sigma}) - \Phi(\frac{\mu-\frac{DR_D}{D+W_B}-\gamma\sigma^2(D+W_B)}{\sigma})\right] \\
& -e^{-\gamma\left[DR_D+(W_H-D)R_f\right]} \cdot \Phi(\frac{\mu-\frac{DR_D}{D+W_B}}{\sigma}) \; .
\end{aligned}
\tag{6.3.1}
$$

The last and the first line are each given by the household's utility of the respective pay-offs weighted by the probability of the states considered. The intermediate lines represent those states where the household is partially paid-off. On this domain, expected utility depends on the well-known $\mu$-$\sigma$ representation of risky pay-offs that exhibit normal returns and are judged according to CARA utility, adjusted by the appropriate probability.

Since the household is only allowed to seize the bank's assets in case of the bank's bankruptcy, this utility representation is multiplied by its respective probability. The probability of these intermediate states are adjusted by the portfolio's volatility $\sigma$ and the household's coefficient of absolute risk-aversion $\gamma$.

The household's supply of deposits arises from the following maximization problem:

$$
\max_D \; \mathrm{E}\left[u_H(\tilde{W}_H)\right] \quad \text{s.t.} \quad 0 \;\leq\; D \;\leq\; W_H \; . \tag{6.3.2}
$$

As in Section 3.2.3, the unconstrained optimization problem does not account for the lower and the upper bound $0 \leq D \leq W_H$ and will be referred to by (6.3.2'). Unconstrained magnitudes will be super-indexed by $u$. The solution to the Unconstrained Problem (6.3.2') is called $D^u(\ell, R_D)$.

**Result 30.** *The household's objective function* $\mathrm{E}[u_H(\tilde{W}_H)]$ *is strictly concave in $D$ for all $D \in \mathbb{R}$. The unconstrained deposit supply $D^u(\ell, R_D)$ that maximizes Problem* (6.3.2') *always exists and is unique.*

Expected utility over final wealth is a strictly concave function of final wealth $\tilde{W}_H$. In turn, final wealth is concave in the deposit volume $D$ for given portfolio returns. The expectations operator preserves concavity. Thus, $\mathrm{E}[u_H(\tilde{W}_H)]$ is strictly concave in $D$ for all $D \in \mathbb{R}$.

To establish the existence of a maximum $D^u$, we consider the behavior of expected

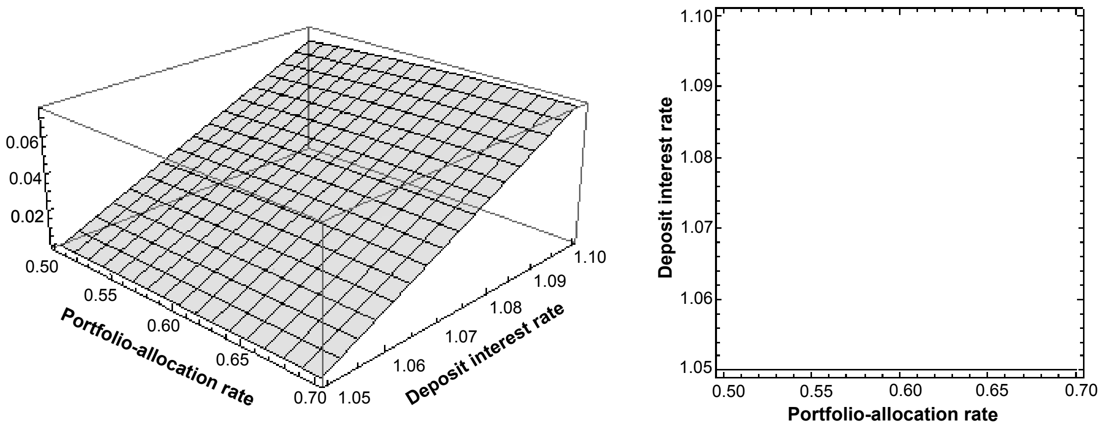


Figure 6.1: **Illustration of Condition** (6.3.4)

*The figure illustrates the functional expression dependent on $\ell$ and $R_D$ that is assumed to be greater than zero according to Condition* (6.3.4)*. It is shown for the parameter values from Table 6.1 given the relation $D \equiv \hat{D}$. The graph on the left-hand side shows this functional expression dependent on $\ell$ and $R_D$. The graph on the right-hand side indicates that this functional expression approaches zero (and even becomes negative) if $R_D$ is close to $R_f$. Otherwise, the condition is satisfied for the parameters as given in Table 6.1.*

utility for $|D|$ approaching infinity. One can show that the expected utility function diverges to minus infinity for both $D \longrightarrow -\infty$ and $D \longrightarrow \infty$. In between, expected utility is continuous. Thus, at least one $D$ exists that maximizes the household's expected utility. Strict concavity of the expected utility function establishes uniqueness. For details we refer to Appendix C.1.

Since $D^u(\ell, R_D)$ may be negative or exceed $W_H$, we define the constrained deposit-supply function $D^s(\ell, R_D)$ analogous to (3.2.49):

$$D^s(\ell, R_D) \; = \; \min\left\{\max\left\{D^u(\ell, R_D), 0\right\}, W_H\right\} \; . \tag{6.3.3}$$

The deposit-supply function can only be obtained numerically. For details of calculations we refer to Section 6.6. Still, some basic general properties of the deposit-supply function can be derived. First, let us assume the following relation for the return on the household's portfolio:

$$\frac{R_D - R_f}{\sigma} - \frac{\varphi(\hat{b}_{s_z})}{\Phi(\hat{b}_{s_z})} \; > \; 0, \tag{6.3.4}$$

where

$$\hat{b}_{s_z} \equiv \frac{\mu - \frac{\hat{D}R_D}{\hat{D}+W_B}}{\sigma} \quad \text{with} \quad \hat{D} \equiv \frac{\mu - R_f}{\gamma\sigma^2} - W_B \; .$$

Technically, Condition (6.3.4) assures that the household's first-order condition is positive at $D = \hat{D}$. Figure 6.2 illustrates this condition for the base case. Thus, $D^u$ exceeds $\hat{D}$, as shown in Appendix C.2. Economically, the household recognizes the buffer function that the bank's initial equity has concerning the redemption of deposits. Conversely, if the household supplied $D$ dollars to the bank and got back (a contractually fixed fraction of) $\tilde{x} \cdot (D + W_B)$ in any state of the world, the amount of deposits supplied would decrease with every dollar of equity initially provided in a ratio of one to one, as given by (6.3.5), *i.e.* deposit supply would be equal to $\hat{D}$ (adjusted by the appropriate parameter).

Unconstrained deposit supply is a smooth function in the portfolio-allocation rate $\ell$, the deposit interest rate $R_D$, and in all its parameters. It is affected by the trade-off between the expected return on the loan portfolio and the associated volatility. Likewise, a higher return $R_f$ on the risk-free asset lowers deposit supply given the deposit contract characterized by $\ell$ and $R_D$. The following result summarizes these properties:

**Result 31.** *The deposit-supply function is continuous and the unconstrained deposit-supply function is differentiable in the portfolio-allocation rate $\ell$, in the deposit interest rate $R_D$, and in all other parameters. The unconstrained deposit-supply function strictly decreases in $R_f$.*

*If Approximation* (5.4.14) *is assumed and if Condition* (6.3.4) *is satisfied, the unconstrained deposit-supply function exceeds*

$$D^u(\ell, R_D) \; > \; \frac{\mu - R_f}{\gamma\sigma^2} \; - \; W_B \; . \tag{6.3.5}$$

*Moreover, the unconstrained deposit-supply function strictly increases in $\mu$ and strictly decreases in $\sigma$. Let $\ell_{max}$ be the portfolio-allocation rate where $D^u(\ell, R_D)$ attains its maximum for given values of $R_D$ and let $\ell_{\sigma_{min}}$ be the portfolio-allocation rate that minimizes $\sigma$. Then, if $\ell_{\sigma_{min}} \in (0,1)$ holds, the following relations apply,*

$$\ell_{max} \begin{cases} > \; \ell_{\sigma_{min}} & \text{if} \quad \mu_1 \; > \; \mu_2 \\ = \; \ell_{\sigma_{min}} & \text{if} \quad \mu_1 \; = \; \mu_2 \\ < \; \ell_{\sigma_{min}} & \text{if} \quad \mu_1 \; < \; \mu_2 \end{cases} . \tag{6.3.6}$$

Existence, given by Result 30, in conjunction with the implicit function theorem leads to differentiability of the unconstrained solution (Mas-Colell/Winston/Green, 1995, p. 941f and Heuser, 1992, p. 295f). The constrained deposit supply $D^s(\ell, R_D)$, given by (6.3.3), is differentiable in $\ell$, $R_D$, and in all other parameters, except for those parameter values where $D^s(\ell, R_D)$ equals zero or $W_H$.

The behavior of the deposit-supply function in $\mu$ and $\sigma$ is derived in Appendix C.2. Since the deposit-supply function depends on $\ell$ solely via $\mu$ and $\sigma$, we can state for the unconstrained deposit supply

$$\frac{\partial D^u}{\partial \ell} = \underbrace{\frac{\partial D^u}{\partial \mu}}_{>0 \text{ Res. } 31} \cdot \frac{\partial \mu}{\partial \ell} + \underbrace{\frac{\partial D^u}{\partial \sigma}}_{<0 \text{ Res. } 31} \cdot \frac{\partial \sigma}{\partial \ell} .$$

By assuming $\ell_{\sigma_{min}} \in (0,1)$, $\frac{\partial \sigma}{\partial \ell}$ changes signs on $(0,1)$ once in the variance minimum portfolio-allocation rate. Suppose $\mu_1 > \mu_2$. Then $\frac{\partial D^d}{\partial \ell} > 0$ holds for all $\ell \leq \ell_{\sigma_{min}}$. Thus, the household demands more deposits as $\ell$ becomes higher. Above all, $\ell_{max} > \ell_{\sigma_{min}}$ holds. Symmetrically, we can argue for $\mu_1 < \mu_2$ by starting the reasoning from $\ell \geq \ell_{\sigma_{min}}$.

Second, to characterize the deposit-supply function $D^s(\ell, R_D)$ with respect to the deposit interest rate, it is again helpful to introduce the three critical deposit interest rates $\underline{R}_D(\ell)$, $\overline{R}_D(\ell)$, and $\overline{R}_D^u(\ell)$ for every given portfolio-allocation rate $\ell$, *cf.* Definitions (3.2.51) and (3.2.52).

$\underline{R}_D(\ell)$ denotes the critical deposit interest rate where the unconstrained deposit-supply function $D^u(\ell, R_D)$ becomes zero:

$$\underline{R}_D(\ell) \in \{ R_D | R_D = \sup_{R_D} \{ D^u(\ell, R_D) = 0 \} \} . \tag{6.3.7}$$

$\overline{R}_D^u(\ell)$ is the deposit interest rate that maximizes the unconstrained deposit-supply function $D^u(\ell, R_D)$:

$$\overline{R}_D^u(\ell) \in \left\{ R_D | D^u(\ell, \overline{R}_D^u(\ell)) \geq D^u(\ell, R_D) \quad \forall \; R_D \geq \underline{R}_D(\ell) \right\} . \tag{6.3.8}$$

$\overline{R}_D(\ell)$ represents the minimum deposit interest rate that maximizes the (constrained) deposit-supply function $D^s(\ell, R_D)$:

$$\overline{R}_D(\ell) = \min \left\{ \overline{R}_D^u(\ell), \; \{ R_D | R_D = \inf_{R_D} \{ D^s(\ell, R_D) = W_H \} \} \right\} , \tag{6.3.9}$$

leading to the relation $\underline{R}_D(\ell) \leq \overline{R}_D(\ell) \leq \overline{R}_D^u(\ell)$ by definition if these maxima exist.

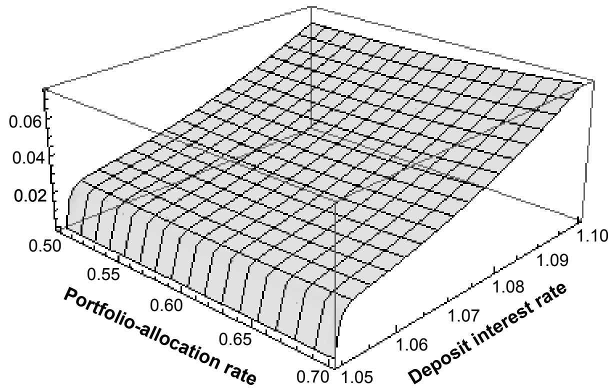


Figure 6.2: **Illustration of Condition** (6.3.10)

*This figure illustrates the dependence of the functional expression on $\ell$ and $R_D$ that is assumed to be lower than one according to Condition* (6.3.10). *The functional expression is shown for the parameter values from Table 6.1 given the deposit-supply function $D^u(\ell, R_D)$ which solves the Maximization Problem* (6.3.2'). *The condition is satisfied for the parameters as given in Table 6.1.*

Moreover, we assume

$$\frac{\varphi(b^u_{s_z})}{\Phi(b^u_{s_z})} \cdot \frac{D^u \cdot W_B \cdot R_f}{\sigma(D^u + W_B)^2} \; < \; 1 \; , \tag{6.3.10}$$

where

$$b^u_{s_z} \; \equiv \; \frac{\mu - \frac{D^u \cdot R_D}{D^u + W_B}}{\sigma} \; ,$$

to establish the following result which is in fact a re-formulation of Result 2 considering that we need not differentiate between different Cases as necessary in the model discussed in Part II.

**Result 32.** *If Condition* (6.3.10) *holds, exactly one interest rate $\underline{R}_D(\ell)$ exists for which the unconstrained deposit-supply function becomes zero, and exactly one interest rate $\overline{R}^u_D(\ell)$ exists that maximizes the unconstrained deposit-supply function with respect to $R_D$. Furthermore, the constrained deposit-supply function $D^s(\ell, R_D)$ increases strictly monotonically in $R_D$ for $R_D \in [\underline{R}_D(\ell), \overline{R}^u_D(\ell)]$, and decreases strictly monotonically on $[\overline{R}^u_D(\ell), \infty)$.*

The economic intuition for this result is the same as for Result 2: the risk-averse investor with CARA utility will not increase its deposit supply beyond a given threshold interest rate $R_D = \overline{R}_D(\ell)$, as the contingent pay-offs from the deposit contract have already attained a given level of global satiation due to relatively high values of $R_D$. Moreover, the bank's probability of solvency decreases for the deposit volume $D$, as illustrated by (6.2.4).[1] Technically, this proof follows the same approach as that for Result 2 within the Bernoulli framework. Details are presented in Appendix C.3. In fact, $\overline{R}_D^u(\ell)$ is implicitly given by

$$\overline{R}_D^u(\ell) \;=\; \frac{(1+\gamma\cdot\overline{D}^u\cdot R_f)\cdot\Phi(\overline{b}_{s_z}^u)}{\varphi(\overline{b}_{s_z}^u)\cdot\frac{\overline{D}^u\cdot W_B}{\sigma\cdot(\overline{D}^u+W_B)^2}+\gamma\cdot\overline{D}^u\cdot\Phi(\overline{b}_{s_z}^u)} \overset{(6.3.5)}{<} R_f \;+\; \frac{\sigma^2}{\mu-R_f-\gamma\sigma^2\cdot W_B}\;,$$

with

$$\overline{b}_{s_z}^u \;\equiv\; \frac{\mu-\frac{\overline{D}^u\cdot\overline{R}_D^u(\ell)}{\overline{D}^u+W_B}}{\sigma}\;,$$

where $\overline{D}^u \equiv D^u(\overline{R}_D^u(\ell),\ell)$ and where $\ell$ takes any arbitrary, but fixed value from $[0,1]$.

The critical interest rate $\underline{R}_D(\ell)$ barely exceeds $R_f$,

$$\underline{R}_D(\ell) \;=\; \frac{1}{\Phi(\frac{\mu}{\sigma})}\cdot R_f \;=\; R_f+\varepsilon,\ \varepsilon>0\;,\quad \text{by}\ \ (5.4.14)\quad \underline{R}_D(\ell)\;\approx\;R_f\;,$$

*i.e.* the success probability of full deposit redemption approaches one as $D$ goes to zero. The economic reason is that an investment equal to zero dollars by the household yields zero dollars with certainty at the end of the period. If final payments deviated from zero, one party would always be granted an arbitrage opportunity at the expense of the counter-party. Technically, this holds true because of (5.4.14).

## 6.4 Equilibrium without Regulation

Without facing any regulation, the bank chooses the portfolio-allocation rate $\ell$ and the deposit interest rate $R_D$ such that its Objective (6.2.1) given the household's deposit-supply function $D^s(\ell, R_D)$ is maximized:

$$\begin{aligned} \max_{\ell,\ R_D}\quad & \mathrm{E}\left[\tilde{W}_B(\ell,R_D)\right] \\ \text{s.t.}\quad & \ell\in[0,1] \\ & D \;\equiv\; D^s(\ell,R_D)\;. \end{aligned} \tag{6.4.1}$$

[1]In the model with Bernoulli distributed loan redemptions, the bank's probability of solvency solely depends on the Case $j$. The Case, in turn, is determined by the relation between $D$, $R_D$, and $\ell$.

The pair $(\ell^*, R_D^*)$ denotes the maximizer to this problem and will be referred to as the equilibrium without regulation. The basic properties, already shown and discussed within the framework with the Bernoulli distribution, are valid too:

**Result 33.** *An equilibrium $(\ell^*, R_D^*)$ always exists. The deposit interest rate, $R_D^*$, satisfies $R_D^* \in (\underline{R}_D(\ell), \overline{R}_D(\ell)]$. Hence, the optimal deposit volume $D^* \equiv D^s(\ell^*, R_D^*)$ is always strictly positive.*

Expected final wealth $\mathrm{E}[\tilde{W}_B(\ell, R_D)]$ is continuous in $\ell$ and $R_D$. The portfolio-allocation rate $\ell$ is restricted to the compact set $[0,1]$ by definition. The domain of $R_D$ can be restricted to a compact set as well without excluding any optima. Since

$$\frac{\partial \mathrm{E}[\tilde{W}_B(\cdot)]}{\partial R_D} \;=\; -\,D \cdot \Phi(b_{s_z}) \;<\; 0$$

holds for fixed $D$, the bank does not increase deposit interest rates beyond $\overline{R}_D(\ell)$: for $R_D$ higher than $\overline{R}_D(\ell)$, expected wealth decreases by both higher interest rates $R_D$ and lower deposit volumes $D$, where the latter is due to Result 32. As for all $R_D < \underline{R}_D(\ell)$ the deposit volume remains zero, deposit interest rates below $\underline{R}_D(\ell)$ do not alter the value of the bank's expected final wealth and can thus be skipped. Consequently, $R_D^*$ must be restricted to $[\underline{R}_D(\ell), \overline{R}_D(\ell)]$ and a solution to the Maximization Problem (6.4.1) always exists. Because of

$$\begin{aligned}
\left.\frac{d\mathrm{E}[\tilde{W}_B(\cdot)]}{dR_D}\right|_{R_D=\underline{R}_D} &= \left.\left(\frac{\partial \mathrm{E}[\tilde{W}_B(\cdot)]}{\partial D}\cdot\frac{\partial D^u(\cdot)}{\partial R_D}\right)\right|_{R_D=\underline{R}_D} + \left.\frac{\partial \mathrm{E}[\tilde{W}_B(\cdot)]}{\partial R_D}\right|_{R_D=\underline{R}_D} \\
&\overset{(6.2.5)}{=} -\,D^u(\ell, \underline{R}_D)\cdot\Phi(\underline{b}_{s_z}) \\
&\quad + \{\sigma\cdot\varphi(b_{s_z}) \;+\; (\mu - R_D)\cdot\Phi(b_{s_z})\}\cdot\left.\frac{\partial D^u(\cdot)}{\partial R_D}\right|_{R_D=\underline{R}_D} \\
&= \left\{\sigma\cdot\varphi(\underline{b}_{s_z}) + (\mu - \underline{R}_D)\cdot\Phi(\underline{b}_{s_z})\right\}\cdot\left.\frac{\partial D^u(\cdot)}{\partial R_D}\right|_{R_D=\underline{R}_D} \;>\; 0\,,
\end{aligned} \tag{6.4.2}$$

it is not optimal for the bank to forgo debt finance, *i.e.* $D = 0$ is never optimal. Consequently, $R_D^*$ lies in $(\underline{R}_D(\ell), \overline{R}_D(\ell)]$ and $D^*$ is strictly positive. Note that $R_D^* = \overline{R}_D(\ell)$ can be an equilibrium if $\overline{R}_D(\ell) < \overline{R}_D^u(\ell)$ holds; that is, whenever the household is constrained by its initial wealth in equilibrium, $D^s(\ell^*, \overline{R}_D(\ell)) \equiv W_H$.

If the household is not constrained by its initial wealth, choosing $R_D^* = \overline{R}_D(\ell)$ cannot be optimal for the bank because of

$$\begin{aligned}\left.\frac{\partial \mathrm{E}[\tilde{W}_B(\cdot)]}{\partial R_D}\right|_{R_D=\overline{R}_D(\ell)} &= -\ D^u(\ell, R_D) \cdot \Phi(b_{s_z}) \\ &\quad + \ \{\sigma \cdot \varphi(b_{s_z}) \ + \ (\mu - R_D) \cdot \Phi(b_{s_z})\} \cdot \left.\frac{\partial D^u(\cdot)}{\partial R_D}\right|_{R_D=\overline{R}_D(\ell)} \\ &= -\ D^u(\ell, \overline{R}_D(\ell)) \cdot \Phi(\bar{b}_{s_z}) \ < \ 0 \ .\end{aligned}$$

The risk-averse household affects the bank's choice of the optimal portfolio-allocation rate such that the risk neutral bank management does not allocate all its funds to the loan portfolio with the higher expected return. Still, the relation between both loan portfolios' expected returns affects the bank's decision stronger than it does with the household's:

**Result 34.** *Assume that $\ell^* \in (0,1)$, $\ell_{\sigma_{min}} \in (0,1)$, and that the household is not constrained by its initial wealth. Then the following relations hold for the optimal portfolio-allocation rate:*

$$\ell^* \begin{cases} > \ \ell_{max} \ > \ \ell_{\sigma_{min}} & \text{if} \quad \mu_1 \ > \ \mu_2 \\ = \ \ell_{max} \ = \ \ell_{\sigma_{min}} & \text{if} \quad \mu_1 \ = \ \mu_2 \\ < \ \ell_{max} \ < \ \ell_{\sigma_{min}} & \text{if} \quad \mu_1 \ < \ \mu_2 \end{cases} . \tag{6.4.3}$$

The derivation is as follows. By the conditions set out in the result, no further constraints must be considered when analyzing the first-order constraints. The first-order condition to (6.5.3) with respect to $R_D$ is given by

$$\begin{aligned}\frac{\partial \mathrm{E}[\tilde{W}_B(\cdot)]}{\partial R_D} &= -\ D^u(\ell, R_D) \cdot \Phi(b_{s_z}) \\ &\quad + \ \{\sigma \cdot \varphi(b_{s_z}) \ + \ (\mu - R_D) \cdot \Phi(b_{s_z})\} \cdot \frac{\partial D^u(\cdot)}{\partial R_D} \ \stackrel{!}{=} \ 0 \ .\end{aligned}$$

Thus,

$$\frac{\partial \mathrm{E}[\tilde{W}_B(\cdot)]}{\partial D} = \sigma \cdot \varphi(b_{s_z}) \ + \ (\mu - R_D) \cdot \Phi(b_{s_z}) \ > \ 0 \tag{6.4.4}$$

holds. The first-order condition with respect to $\ell$ reads

$$\frac{\partial \mathrm{E}[\tilde{W}_B(\cdot)]}{\partial \ell} = [\ D^u(\ell, R_D) + W_B\ ] \cdot \underbrace{\left\{\varphi(b_{s_z}) \cdot \frac{\partial \sigma}{\partial \ell} + (\mu_1 - \mu_2) \cdot \Phi(b_{s_z})\right\}}_{\equiv \mathrm{E}[\tilde{W}_B]_\ell} + \underbrace{\{\sigma \cdot \varphi(b_{s_z}) + (\mu - R_D) \cdot \Phi(b_{s_z})\}}_{\equiv \mathrm{E}[\tilde{W}_B]_D} \cdot \frac{\partial D^u(\cdot)}{\partial \ell} \overset{!}{=} 0\ .$$

Since $D^u(\ell, R_D) + W_B > 0$ and $\mathrm{E}[\tilde{W}_B]_D > 0$ hold, $\mathrm{E}[\tilde{W}_B]_\ell$ and $\frac{\partial D^u(\cdot)}{\partial \ell}$ are either of opposite sign or zero.

If $\mu_1 = \mu_2$ the first-order condition simplifies such that $\mathrm{E}[\tilde{W}_B]_\ell$ and $\frac{\partial D^u}{\partial \ell}$ are reduced to multiples of $\frac{\partial \sigma}{\partial \ell}$. One solution is then $\ell^* = \ell_{\sigma_{min}}$. Consequently, the household's deposit supply is maximized with respect to $\ell$ due to Result 31. As $\mathrm{E}[\tilde{W}_B]_D > 0$ holds, this is in turn in the interest of the bank. Hence, the optimal portfolio-allocation rate is given by $\ell^* = \ell_{\sigma_{min}}$ and is unique. Consequently,

$$\sigma \cdot \varphi(b_{s_z}) + (\mu - R_D) \cdot \Phi(b_{s_z}) > [D^u(\ell, R_D) + W_B] \cdot \varphi(b_{s_z}) \tag{6.4.5}$$

must hold, as $\frac{\partial^2 \mathrm{E}[\tilde{W}_B]}{\ell^2} < 0$ is unambiguously linked to a unique maximum in $\ell$.

Consider now $\mu_1 > \mu_2$: there is no incentive for the bank to put a weight on the first sector less that is lower than the variance-minimum loan-allocation rate. If it did, it could improve in two ways by raising the portfolio-allocation rate: first, a higher $\ell$ would directly increase the bank's expected final wealth. Second, a higher $\ell$ would raise deposits $D$ according to Result 31 and thus increase the bank's expected final wealth as well.

The first (direct) effect is formally due to the assumption $\mathrm{sgn}(\mathrm{E}[\tilde{W}_B]_\ell) = \mathrm{sgn}(\mu_1 - \mu_2)$. By this property, $\mathrm{E}[\tilde{W}_B]_\ell > 0$ holds and thus $\frac{\partial D^u}{\partial \ell} < 0$. By the latter we can conclude $\ell^* > \ell_{max}$.

For $\mu_1 < \mu_2$ we can argue analogously.

## 6.5 Equilibrium with Regulation by a Value-at-Risk Approach

The bank faces the same sort of VaR regulation as described in Section 3.3.3.1: regulation requires the bank that unexpected losses,

$$\left[\mathrm{E}[\tilde{W}_B] \;-\; \tilde{L}\right]^+ \;,$$

must not exceed a given threshold (a fraction/multiple $\tau$ of initial bank equity $W_B$) by more than a given probability $\bar{p}$:

$$\mathrm{P}\left(\mathrm{E}(\tilde{L}) - \tilde{L} \geq \tau W_B\right) \;\leq\; \bar{p}\;. \tag{6.5.1}$$

Since the gross return on the whole loan portfolio is normally distributed, the VaR requirement results in the following upper bound on the deposit volume[2]

$$D \;\leq\; w(\ell)\cdot W_B \quad \text{where} \quad w(\ell) \;=\; \max\{\,-\frac{\tau}{\Phi^{-1}(\bar{p})\cdot\sigma} \;-\; 1\;,\;0\,\}\;, \quad \bar{p} \;<\; \frac{1}{2}. \tag{6.5.2}$$

The regulatory constraint becomes weaker as the bank gains more equity or the value of the parameter $\tau$ increases. The former applies to $\bar{p} > \Phi(-\frac{\tau}{\sigma})$ and $\bar{p} < \frac{1}{2}$, for the latter to hold, only $\bar{p} < \frac{1}{2}$ must be fulfilled. The regulatory constraint strictly increases in $\bar{p}$ for either $\bar{p} \in (0, \frac{1}{2})$ or $\bar{p} \in (\frac{1}{2}, 1)$.

As volatility is the only parameter characterizing the distribution of the loan redemptions that goes into (6.5.2), the VaR constraint seems to foster decisions that reduce risk. The leeway for the deposit volume is the highest under regulation if the bank chooses the variance minimum portfolio-allocation rate. Increasing portfolio-allocation rates that are below $\ell_{\sigma_{min}}$ imply a higher feasible deposit volume, and *vice versa* for portfolio-allocation rates that are above $\ell_{\sigma_{min}}$,

$$\frac{\partial w}{\partial \ell} \;=\; \frac{\tau}{\Phi^{-1}(\bar{p})\cdot\sigma} \;\cdot\; \frac{\partial\sigma}{\partial\ell} \begin{cases} >\;0 & \text{if} \;\; \ell \;<\; \ell_{\sigma_{min}} \\ =\;0 & \text{if} \;\; \ell \;=\; \ell_{\sigma_{min}} \\ <\;0 & \text{if} \;\; \ell \;>\; \ell_{\sigma_{min}} \end{cases} .$$

However, under the conditions of Result 34, the equilibrium portfolio-allocation rate cannot be further or differently characterized than done under the equilibrium without regulation.

[2] *Cf.* Dangl/Lehar (2004, p. 108) for a similar VaR based formulation of capital requirements considering normally distributed portfolio returns.

Sufficiently high volatilities $\sigma_1$ and $\sigma_2$ may prevent the regulated bank from issuing deposits. More precisely, if $\sigma \geq -\frac{1}{\Phi^{-1}(\bar{p})}$ holds for any feasible portfolio-allocation rate $\ell$, the bank is only able to allocate its equity $W_B$ to that loan portfolio which yields the higher expected return $\mu_i$.

The bank's maximization problem under regulation is given by

$$\begin{aligned} \max_{\ell,\ R_D} \quad & \mathrm{E}\left[\tilde{W}_B(\ell, R_D)\right] \\ \text{s.t.} \quad & \ell \in [0,1] \\ & D \ \equiv \ D^s(\ell, R_D) \\ & D \ \leq \ w(\ell) \cdot W_B \ . \end{aligned} \tag{6.5.3}$$

The tuple $(\ell^V, R_D^V)$ denotes the equilibrium under the VaR regulation. Regulation is said to be binding, if the laissez-faire equilibrium $(\ell^*, R_D^*)$ does not fulfill the given level of confidence $\bar{p}$. The properties of the equilibrium without regulation presented in Results 33 and 34 carry over to the equilibrium under regulation. Apart from lower total loan and deposit volumes, no further properties of both equilibria can be shown in general.

**Result 35.** *An equilibrium $(\ell^V, R_D^V)$ always exists. Assume that $\ell^V \in (0,1)$, $\ell_{\sigma_{min}} \in (0,1)$, and that the household is not constrained by its initial wealth. Then the optimal portfolio-allocation rate $\ell^V$ also satisfies the relations in* (6.4.3).

The argument for the existence of $(\ell^V, R_D^V)$ runs as in the unregulated case. The additional regulatory Condition (6.5.2) restricts the space of the choice variables $(\ell, R_D)$ further. The argument for the behavior of the optimal portfolio-allocation rate $\ell^V$ can be shown analogously as follows. Consider the Lagrangian[3]

$$\mathcal{L}(\ell, R_D, \lambda) \ = \ \mathrm{E}\left[\tilde{W}_B(\ell, R_D)\right] - \lambda \cdot \left[D^u(\ell, R_D) - w(\ell) \cdot W_B\right] \ .$$

The first-order condition with respect to $R_D$ becomes[4]

$$\begin{aligned} \frac{\partial \mathcal{L}(\cdot)}{\partial R_D} \ = \ & - \ D^u(\ell, R_D) \cdot \Phi(b_{s_z}) \\ & + \ \{\sigma \cdot \varphi(b_{s_z}) + (\mu - R_D) \cdot \Phi(b_{s_z}) - \lambda\} \cdot \frac{\partial D^u(\cdot)}{\partial R_D} \stackrel{!}{=} 0 \ , \end{aligned}$$

[3]Note that assuming $\ell^V \in (0,1)$ is equivalent to assuming $\sigma < \frac{1}{-\ \Phi-1(\bar{p})}$ as a violation of the latter inequality implies either $\ell^V = 0$ or $\ell^V = 1$, dependent on what loan portfolio yields the higher return, and zero deposits. Moreover, $w(\ell)$ becomes differentiable.

[4]*Cf.* the derivation of Formula (6.4.2).

resulting in

$$\sigma \cdot \varphi(b_{s_z}) + (\mu - R_D) \cdot \Phi(b_{s_z}) - \lambda \; > \; 0 \; .$$

Thus, the marginal impact of deposits is also strictly positive under regulation and can be more concisely written as $\mathcal{L}_D \equiv \mathrm{E}[\tilde{W}_B]_D - \lambda > 0$. The first-order condition with respect to $\ell$ reads[5]

$$\begin{aligned} \frac{\partial \mathcal{L}(\cdot)}{\partial \ell} &= [D^u(\ell, R_D) + W_B] \cdot \underbrace{\left\{ \varphi(b_{s_z}) \cdot \frac{\partial \sigma}{\partial \ell} + \frac{\lambda}{\sigma} \cdot \frac{\partial \sigma}{\partial \ell} + (\mu_1 - \mu_2) \cdot \Phi(b_{s_z}) \right\}}_{\mathcal{L}_\ell} \\ &\quad + \; \{\sigma \cdot \varphi(b_{s_z}) + (\mu - R_D) \cdot \Phi(b_{s_z}) - \lambda\} \cdot \frac{\partial D^u(\cdot)}{\partial \ell} \stackrel{!}{=} 0 \; . \end{aligned}$$

Due to $\mathcal{L}_D > 0$ and $D^u(\ell, R_D) + W_B \; > \; 0$, the derivative $\frac{\partial D^u(\cdot)}{\partial \ell}$ and the remainder $\mathcal{L}_\ell$ are either of opposite sign or zero.

For $\mu_1 = \mu_2$ we can argue exactly as we did for Result 34. Moreover, choosing $\ell^V = \ell_{\sigma_{min}}$ maximizes the eligible deposit volume according to (6.5.2).

Again, for $\mu_1 > \mu_2$, there is no incentive for the bank to put a weight on the first sector that is lower than the variance-minimum loan-allocation rate. If it did, it could now improve even in three ways by raising the portfolio-allocation rate: besides the former two reasons that have already been mentioned in connection with Result 34, increasing $\ell$ toward $\ell_{\sigma_{min}}$ also increases deposit volume that is eligible by regulation. Thus, $\ell^V > \ell_{max} > \ell_{\sigma_{min}}$ is obtained. We can argue symmetrically for $\mu_1 < \mu_2$.

If loan redemptions from both loan portfolios are equal in their expected returns, $\mu_1 = \mu_2$, as well as in their volatilities, $\sigma_1 = \sigma_2$, and if regulation is binding, the equilibrium deposit volume $D^V$ equals

$$D^V \;\equiv\; D^u(\frac{1}{2}, R_D^V) \;=\; \left( - \frac{\tau \cdot \sqrt{2}}{\Phi^{-1}(\bar{p}) \cdot \sigma_1 \cdot \sqrt{1+\rho}} - 1 \right) \cdot W_B, \quad \Phi(-\frac{\tau}{\sigma}) < \bar{p} < \frac{1}{2} \; .$$

In particular, the equilibrium total loan/deposit volume strictly decreases in the single portfolios' volatilities $\sigma_1$ and in their correlation $\rho$ and remains unaffected by changes in expected returns. If the bank is regulated by fixed risk weights, as studied in Section 3.3.2 for Bernoulli-distributed loan redemptions, the total loan volume under regulation is simply given by $L^S = \frac{1}{c \cdot c_1} \cdot W_B$. As long as risk is measured by the volatility $\sigma_i$, the VaR approach affects the total loan volume in a pro-cyclical way compared to the total loan volume granted under a fixed-risk weight regime.

[5]The expression $\lambda \cdot \frac{\tau}{-\Phi^{-1}(\bar{p})\sigma^2} \cdot W_B \cdot \frac{\partial \sigma}{\partial \ell}$ has been substituted by $[D^u(\cdot) + W_B] \cdot \frac{\lambda}{\sigma} \cdot \frac{\partial \sigma}{\partial \ell}$ because the Regulatory Constraint (6.5.2) holds with equality in equilibrium.

Table 6.1: **Parameter values of the base case, normal model**

*This table reports the base case parameter values used for the numerical analysis of the model set forth in this chapter. The parameter values of the bank's initial wealth, the household's coefficient of absolute risk aversion, and the gross return rate on the risk-free asset are identical to those given by Table 4.1. The parameter values of the expected gross returns and the return volatilities, and the inter-sectoral loan-return correlation result from the mixture model set forth in Chapter 5 with the parametrization according to Table 5.1. Expectations are precisely quoted whereas volatilities and the correlation are rounded to six leading digits.*

| $W_B$ | $\gamma$ | $R_f$ | $\tau$ | $\mu_1$ | $\mu_2$ | $\sigma_1$ | $\sigma_2$ | $\rho$ |
|---|---|---|---|---|---|---|---|---|
| 100 | 0.008 | 1.05 | 1 | $= 1.14425$ | $= 1.188$ | $\approx 0.0811463$ | $\approx 0.119495$ | $\approx 0.000426955$ |

However, once it is recognized that both the portfolio volatility $\sigma_i$ as well as the risk weight $c_i$ depend positively on the default probability $1-p_i$, $1-p_i \in (0, \frac{1}{2})$, it is still unclear whether $L^S$ or $L^V$ reacts stronger to changes in $1-p_i$. Arguing strictly by the Standardized Approach on the one hand and the internal-ratings-based approach on the other, $L^S$ is piecewise insensitive with up to three jumps according to Table 3.4 whereas $L^V$ changes smoothly.

## 6.6 Numerical Analysis of Regulatory Impacts

Let us analyze this model with the parameter values as given in Table 5.1 which result in the same values of the redemption distributions on the individual firm loan level as in Chapter 4. Because of the inner-sectoral correlations between firms, sector-wise volatilities $\sigma_i$ depart slightly from the values shown in Table 4.1. The Bernoulli mixture model set forth in Chapter 5 given the parametrization according to Table 5.1 leads, however, to a cross-sectoral correlation that is much lower than the correlation in the Bernoulli model. Table 6.1 shows these values. For simplicity, comparative statics are performed as if the household's initial wealth $W_H$ is arbitrarily high, such that only unconstrained equilibria are considered under the base case. The regulatory multiplier $\tau$ is set equal to one, reflecting the notion of the IRB approach to determine the overall loan portfolio's VaR. For a discussion on the meaning of $\tau$, we refer to p. 96.

The deposit-supply function is computed as follows: first the household's expected utility is numerically maximized for given tuples $(\ell, R_D)$. Second, by linearly interpolating the triples $(\ell, R_D, D)$, we can set up the household's deposit supply

Table 6.2: **Alternative parameter values, normal model**

*The upper and the lower panel report an alternative parametrization each. $\Delta\ell$ and $\Delta R_D$ represent the step-size of the grid in the $\ell - R_D$ space on which the household's expected utility is maximized with respect to deposit supply $D$ .*

- *Panel A:*

| $\mu_1$ | $\mu_2$ | $\sigma_1$ | $\sigma_2$ | $\rho$ | $W_B$ | $\gamma$ | $W_H$ | $R_f$ | $\tau$ | $\bar{p}$ | $\Delta\ell$ | $\Delta R_D$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.06 | 1.08 | 0.02 | 0.03 | 0 | 102 | 0.4 | 4000 | 1 | 1 | 0.1% | 0.001 | 0.00001 |

- *Panel B:*

| $\mu_1$ | $\mu_2$ | $\sigma_1$ | $\sigma_2$ | $\rho$ | $W_B$ | $\gamma$ | $W_H$ | $R_f$ | $\tau$ | $\bar{p}$ | $\Delta\ell$ | $\Delta R_D$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.08 | 1.12 | 0.06 | 0.1 | 0.3 | 100 | 0.2 | 4000 | 1 | $\frac{1}{3}$ | 1.0% | 0.01 | 0.0001 |

function $D^s(\ell, R_D)$ which is cut off at 0. We can then numerically determine the bank's maximum choice both with and without regulation. The step lengths and the domains of $\ell$ and $R_D$ may vary with the (regulatory) regime considered when equilibria were computed under the base case parametrization given by Table 6.1.

Beyond that base case we also consider two further parameter constellations to support our exemplary findings (Bühler/Koziol/Sygusch, 2007). The first set of parameters is given by Table 6.2, Panel A. These values are chosen such that at $W_B = 102$ regulation ceases to bind (if we consider a step size of one concerning $W_B$). As regulation, only a VaR approach is considered. The level of confidence is set equal to $\bar{p} = 0.1\%$ and the scaling parameter $\tau$ is equal to one. The bank's assets are riskier in the scenario characterized by the parameter values shown in Table 6.2, Panel B. The VaR approach considered is tighter with respect to the scaling parameter $\tau$ but weaker in terms of the confidence level $\bar{p}$. The coefficient of absolute risk-aversion $\gamma$ has been set to a lower level to allow for a propensity to supply deposits that is high enough in order to make the regulatory constraint binding.

### 6.6.1 Equity Shocks

Figure 6.3 shows the (equilibrium) total loan volumes with and without regulation as a function of the bank's initial capital $W_B$. The total loan volume increases

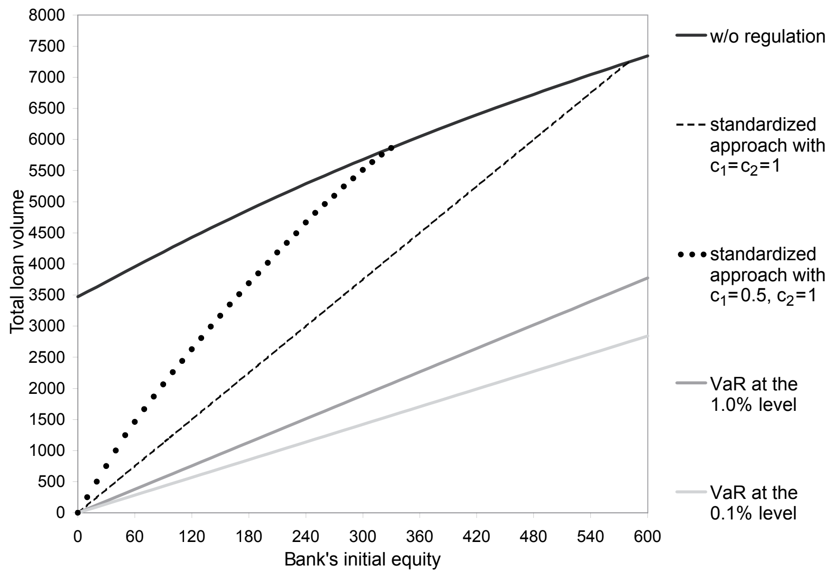


Figure 6.3: **Total loan volume as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium total loan volume $L = L_1 + L_2$ on the bank's initial equity $W_B$. The total loan volume is shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.1\%$.*

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monotonically in the bank's initial equity under all regimes. Without regulation, the bank demands and the households supplies deposits even if the bank owners do not initially bring in any equity. This is not feasible under the capital requirements considered. In contrast to the preceding Bernoulli-distribution framework, the VaR approach requires a strictly positive amount of initial equity for debt-financed lending due to the Regulatory Constraint (6.5.2) that arises out of the normality of aggregate loan redemptions. Hence, regulation is binding at least for values of initial equity around zero. As all regulatory regimes link the feasible total loan volume to the bank's initial equity, the former will strictly increase in $W_B$, at least on average, or if countervailing indirect effects via the loan-allocation rate or the deposit interest rate can be neglected. If there is some point $W_B$ where regulation ceases to bind, the regulated total loan volume will increase more steeply on average than the unregulated volume in order to catch up with the latter. As a consequence, regulation has a pro-cyclical effect on average as already confirmed within the model in Chapter 4.

This pro-cyclicality on average can be observed in connection with the Standardized Approach. The slopes, both under equal risk weights and under different risk weights, exceed the slope of the unregulated total loan volume. The latter exceeds,

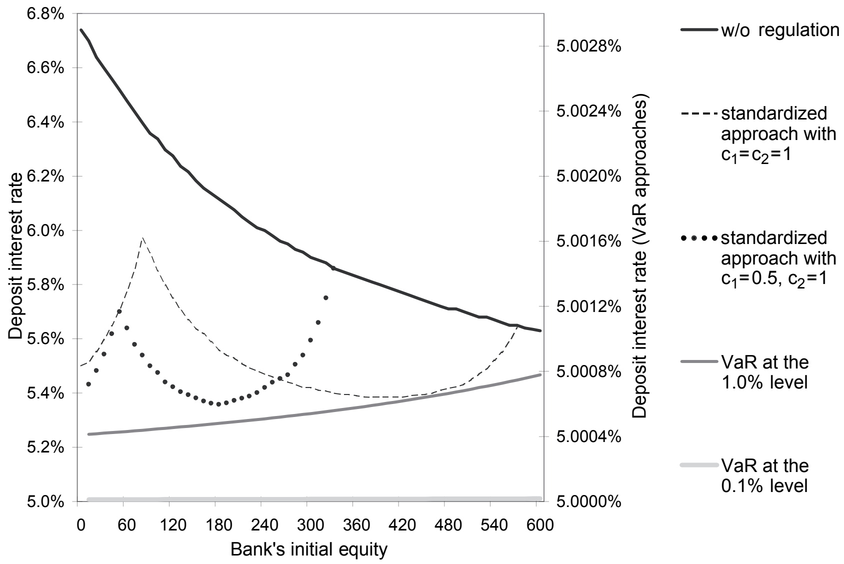


Figure 6.4: **Deposit interest rate as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium deposit interest rate $R_D$ on the bank's initial equity $W_B$. Deposit interest rates are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.1\%$.*
*Left-hand scale: deposit interest rates granted without regulation and under the Standardized Approach.*
*Right-hand scale: deposit interest rates granted under the VaR approach.*
*We note that the deposit interest rate under the VaR approach with $\bar{p} = 0.1\%$ barely exceeds the risk-free rate $R_f - 1 = 5.0\%$, i.e. ranges between $5.0000124\%$ and $5.0000174\%$ for $W_B \in [0, 600]$.*

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however, the slopes of the total loan volumes under both VaR approaches which is here due to the low regulatory parameter $\tau$, $\tau = 1$.

Figure 6.4 shows the deposit interest rates under the regimes considered. As in the Bernoulli framework, the deposit interest rate does not only incorporate the bank's credit risk, but serves also as the instrument to fix the total loan and deposit volume. Consequently, the deposit interest rates under regulation are lower than their unregulated counterparts. Furthermore, deposit interest rates strictly increase as a function of the bank's capital under the VaR approaches.

This behavior can also be observed under the Standardized Approach with fixed risk weights for lower values of initial equity. But thereafter, the risk-reducing effect dominates and the deposit interest rates shrink as a result of increased diversification: the kink in the deposit interest rate under the regime with equal risk weights is associated with the change to allocating strictly positive amounts of capital as loans to firms from the first sector. This kink in deposit interest rates is

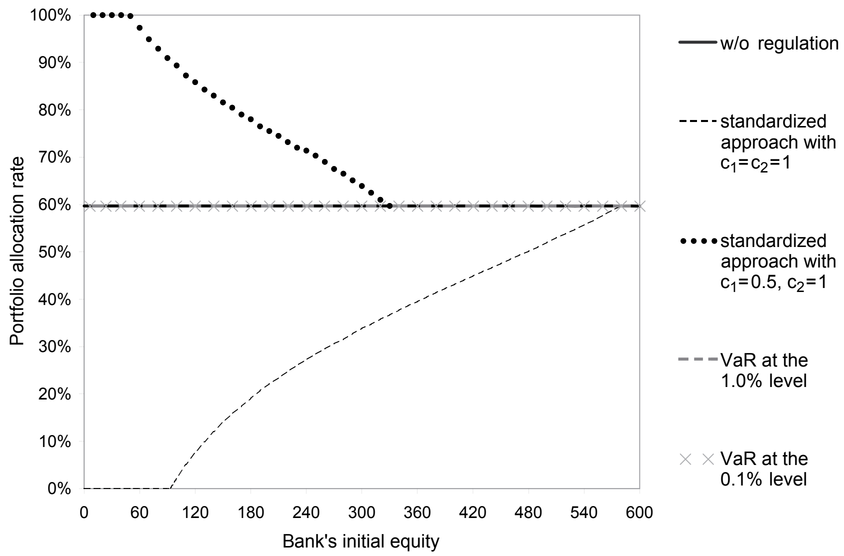


Figure 6.5: **Portfolio-allocation rate as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium portfolio-allocation rate $\ell$ on the bank's initial equity $W_B$. Portfolio-allocation rates are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.1\%$.*

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reflected by the raise of the portfolio-allocation rate from zero to strictly positive levels from $W_B \approx 93$ on in Figure 6.5. For $c_1 = 0.5$, $c_2 = 1$, this kinks already occurs at $W_B \approx 50$.

The effects of higher risk-taking if risk weights are equal also arise here. Employing different risk weights, $c_1 < c_2$, results in high allocation rates for loans to the first sector if the bank's initial wealth is low (Fig. 6.5). By doing so, the bank can attract deposits relatively cheaply from the household. In particular, the return volatilities on the loan redemptions, $\sigma$, and the return volatilities on deposits, $\sigma_D$, are lower than if loans from both sectors are equally weighted (*cf.* Fig. 6.6), whereas $\sigma_D$ is analogously defined to (4.2.1) as

$$\sigma_D \equiv \sqrt{\mathrm{V}(\tfrac{\tilde{D}}{D})} \,. \tag{6.6.1}$$

The risk-reducing effect of unequal risk weights is also reflected by the probability of full deposit redemption which mostly exceeds that under equal risk weighting (*cf.* Fig. 6.7).

Figure 6.6 also exemplifies the cushion effect of the bank's initial equity which has

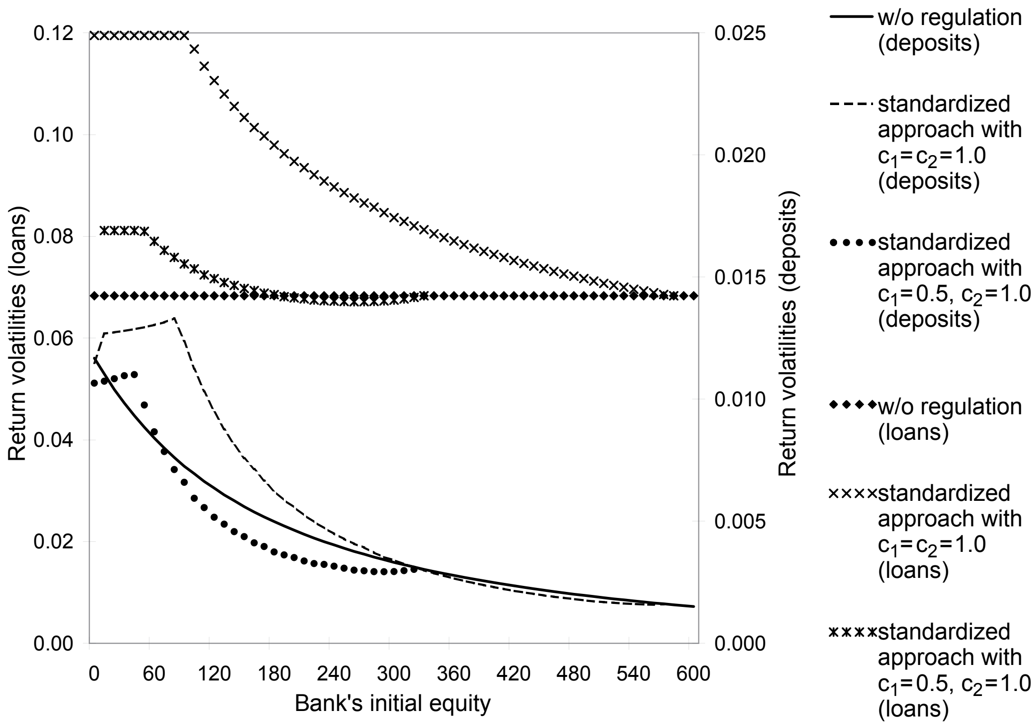


Figure 6.6: **Return volatilities as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium return volatilities on the bank's initial equity $W_B$. Solid and dashed lines and dots, respectively, represent the volatilities of returns on deposits, $\sigma_D$. Diamonds and crosses represent the volatilities of returns on total loans, $\sigma$. Return volatilities are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.1\%$.*

been already confirmed for the Bernoulli model in Part II.[6] This cushion effect can be observed for all regimes by the gap between volatilities of returns on the whole loan portfolio and the volatilities of deposit returns. Without regulation, this gap strictly increases with increasing $W_B$ and is even positive for $W_B = 0$ since the maximum pay-off from deposits is fixed and bounded. Note that this initial gap also arises if repayments from the loan portfolio are bounded from above as it is the case in the framework with a Bernoulli distribution.

Figure 6.8 illustrates that the bank may prefer granting loans to firms of the first sector compared to the non-regulated case, if it can weight the two types of loans differently. Compared to the regime with equal risk weights, different risk weights

[6] Result 4 shows the cushion effect to exist in Cases 2 and 3 out-of-equilibrium whereas Result 10 refers to the total cushion effect under Case 4 in equilibrium and Result 12 to the partial cushion effect in equilibrium if both projects are equal. Figures 4.8 and 4.9 illustrate the return volatilities within the Bernoulli model for its base case parametrization.

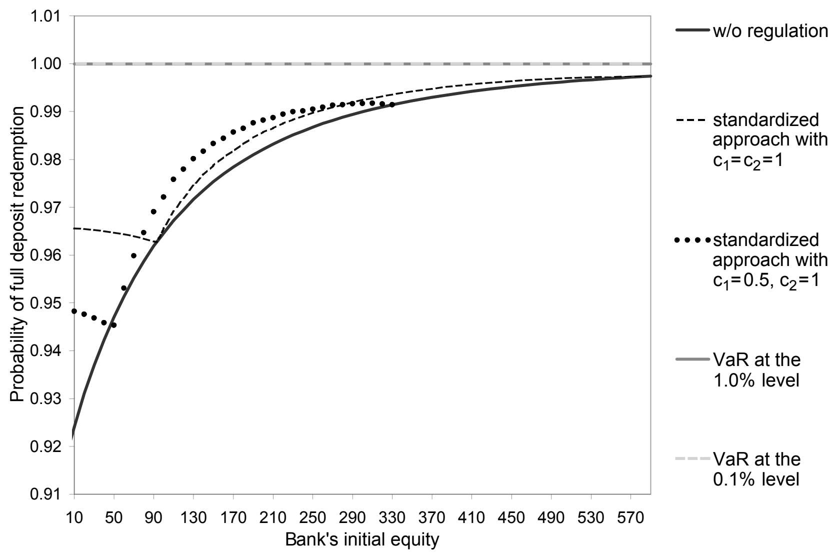


Figure 6.7: **Probability of full deposit redemption**

*This figure illustrates the dependence of the equilibrium portfolio-allocation rate $\ell$ on the bank's initial equity $W_B$. Portfolio-allocation rates are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.1\%$.*

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may result in a higher sensitivity of the total loan volume to shocks in $W_B$ as the (average) risk weight of the loan portfolio is lower than the (average) risk weight under the regime with flat weights equal to one, *cf.* Figures 6.3 and 6.8. Considering the total loan volume, the sensitivity ranges between 16.38 and 25 with $c_1 = 0.5$ and $c_2 = 1$ and is thus up to twice as much as the sensitivity with weights equal to $c_1 = c_2 = 1$. This examples thus illustrates that more risk-sensitive capital requirements may result in pro-cyclicality. The VaR approach with $\tau = 1$ leads to the opposite conclusion for the range of values of $W_B$ considered. That is, formally

$$\underbrace{w(\ell^V)}_{\text{direct effect}} + \underbrace{w'(\ell^V) \cdot \frac{d\ell^V}{dW_B} \cdot W_B}_{\text{indirect effect}} < \underbrace{k(\ell^S)}_{\text{direct effect}} + \underbrace{k'(\ell^S) \cdot \frac{d\ell^S}{dW_B} \cdot W_B}_{\text{indirect effect}}$$

holds. For most of the values of $W_B$ considered, the indirect effect is larger under the Standardized Approach, regardless of the risk weighting, than under any of both VaR approaches (*cf.* Fig. 6.5). The same applies to the direct effects, too. In particular, $w(\ell^V) < k(\ell^S)$ is fulfilled if $\tau$ does not exceed

$$\tau < \left[-\Phi^{-1}(\bar{p})\right] \cdot \sigma|_{\ell=\ell^V} \cdot \frac{1}{c \cdot [c_1\ell^S + c_2(1-\ell^S)]} ,$$

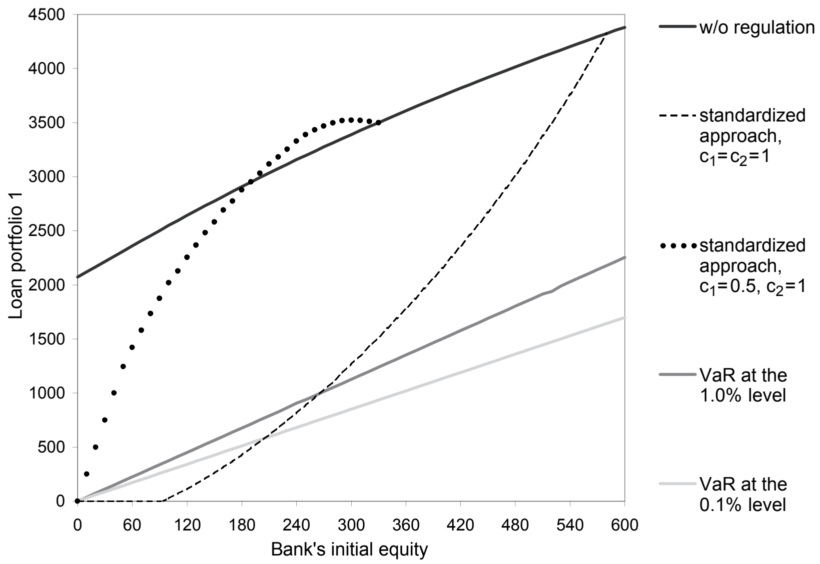


Figure 6.8: **Loan volume to Sector 1 as a function of the bank's initial equity**

*This figure illustrates the dependence of the equilibrium loan volume granted to Sector 1 on the bank's initial equity $W_B$. Loan volumes $L_1$ are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.1\%$.*

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where

$$\begin{aligned}
[-\Phi^{-1}(\bar{p})] &\geq 2.32635 \text{ if } \bar{p} \leq 0.01 \ , \\
\sigma &\in \left[ \sqrt{\tfrac{(1-\rho^2)\sigma_1^2\sigma_2^2}{\sigma_1^2-2\rho\sigma_1\sigma_2+\sigma_2^2}} \ , \max\{\sigma_1, \sigma_2\} \right], \text{ and} \\
\tfrac{1}{c\cdot[c_1\ell^S+c_2(1-\ell^S)]} &\in [8\tfrac{1}{3}, 62.5] \text{ if } 0.2 \leq c_i \leq 1.5 \ .
\end{aligned}$$

Hence, the lowest upper bound for $\tau$ such that the direct effect is smaller under the VaR approach than under the Standardized Approach is given by

$$\tau \ < \ 19.3862 \cdot \sqrt{\frac{(1-\rho^2)\sigma_1^2\sigma_2^2}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}} \ ,$$

amounting to 1.3016 in the base case as given by Table 6.1, to 0.322607 for the parameters as given by Table 6.2, Panel A, and to 1.1096 if Panel B is considered. Given the parametrization according to Table 6.2, Panel A, the VaR-regulated total loan volume actually increases on average by 18.260 if it is considered as a function of $W_B$ (*cf.* Fig. 6.9), but only by roughly 1.4 if the parameters from Panel B are considered (*cf.* Fig. 6.10).

Within the presented framework, all forms of regulation enhance the depositor's protection in terms of the probability of full redemption of deposits (including interest, *cf.* Definition (6.2.2)), as shown by Figure 6.7. The portfolio-allocation rates in Figure 6.5 indicate that volatilities of returns on the loan portfolio and also on deposits are higher under both forms with fixed risk weights $c_i$ than if the bank is unregulated. This also holds true for the regime with different risk weights as the scope for diversification is not fully exploited either. Only the VaR approaches effectively dampen the return volatilities because of the low deposit volumes and the low deposit interest rates on the one hand and the negligible impact on the portfolio-allocation rate on the other. Thus, a reduction in leverage already leads to a reduction of the probability of full deposit redemption.

Next, let us consider the parameter values, as shown in Table 6.2, Panel A and B. The respective VaR approaches result in similar patterns concerning total loan volumes. Figure 6.9 illustrates that the unregulated total loan volume may be more sensitive to changes in the bank's initial equity than the regulated volume for low equity values. The curve of total loan volumes is clearly concave such that it is emphasized once more that pro-cyclicality of regulation with respect to changes in initial equity may only hold on average, but not at every point. The total loan volumes emerging from the parameter values from Table 6.2, Panel B, further confirm this point, *cf.* Figure 6.10.

To sum up, these numerical studies suggest that regulation enhances pro-cyclicality in bank lending in general if regulation is binding and a shock has occurred that has impaired the bank's initial equity. In particular, this notion applies where regulation has just started to bind: let $\bar{W}_B$ be the level of equity below which regulation is binding. Once the bank falls below the critical value $\bar{W}_B$, we obtain $L^r < L^*$ by definition and thus we should observe $\frac{\Delta L^r}{\Delta W_B} > \frac{\Delta L^*}{\Delta W_B} > 0$ for some $W_B$ still sufficiently close to $\bar{W}_B$.

However, examples have also shown that the sensitivity of total lending may even be higher without regulation for low values of initial equity $W_B$. The lower is $W_B$, the more pronounced the cushion effect becomes that equity signals to the risk-averse depositors. Thus, the total loan/deposit volume that can be raised becomes more sensitive to changes in $W_B$, the closer the values considered for $W_B$ are to zero. In contrast, the sensitivities of total loan volumes under either sort of regulation are dominated by the almost linear relation between total lending and initial equity.

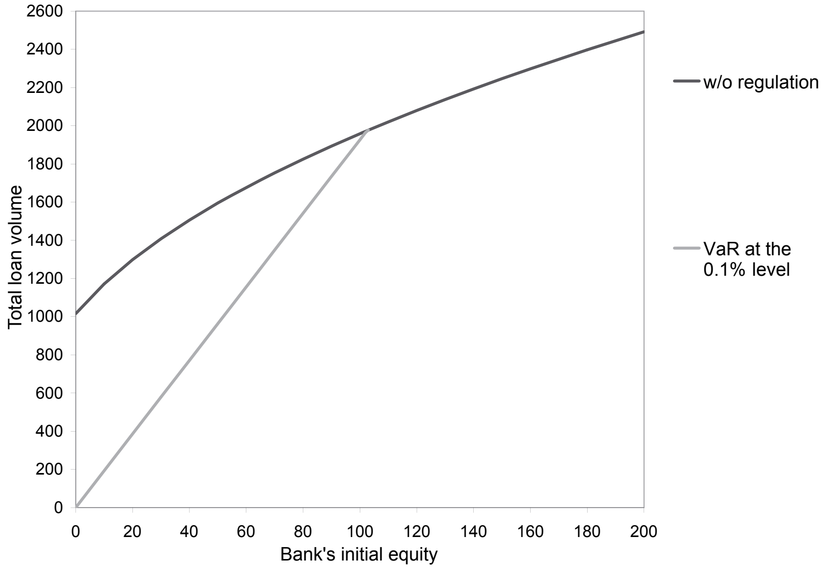


Figure 6.9: **Total loan volume as a function of $W_B$, Table 6.2, Panel A**

*This figure illustrates the dependence of the equilibrium total loan volume $L$ on the bank's initial equity $W_B$. Results are based on the parametrization given by Table 6.2, Panel A. Total loan volumes $L$ are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level $\bar{p} = 0.1\%$.*

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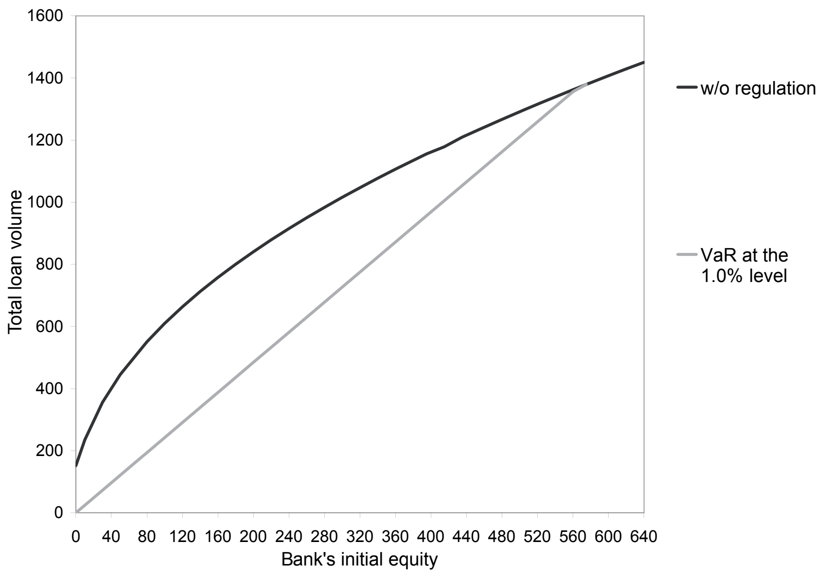


Figure 6.10: **Total loan volume as a function of $W_B$, Table 6.2, Panel B**

*This figure illustrates the dependence of the equilibrium total loan volume $L$ on the bank's initial equity $W_B$. Results are based on the parametrization given by Table 6.2, Panel B. Total loan volumes $L$ are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level $\bar{p} = 1.0\%$.*

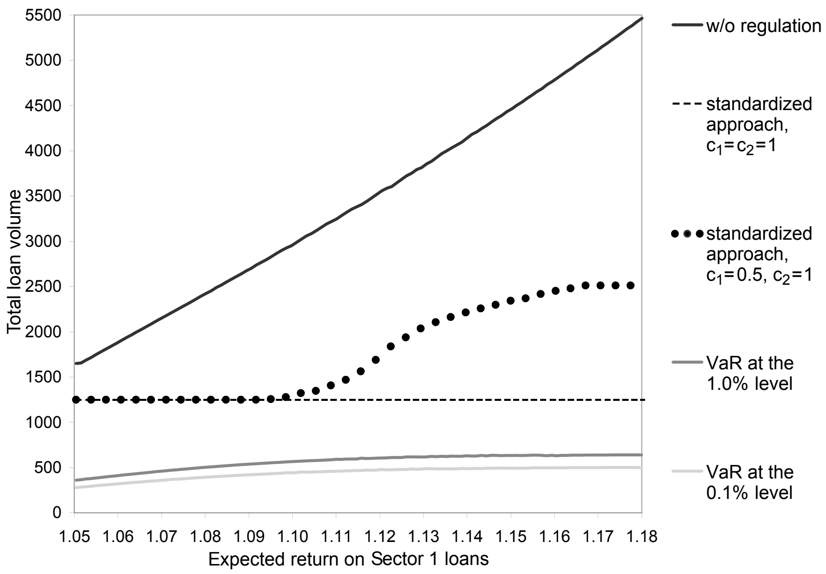


Figure 6.11: **Total loan volume as a function of $\mu_1$**

*This figure illustrates the dependence of the equilibrium total loan volume $L$ on the expected gross return of first-sector loans, $\mu_1$. Total loan volumes $L$ are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and with $\bar{p} = 0.1\%$.*

## 6.6.2 Expected Return Shocks

Figure 6.11 shows the comparative static results for expected return shocks concerning redemptions from the first loan portfolio. The higher the expected return, the more loans are granted to that sector (not shown). Even the total loan volume strictly increases under all regimes. The latter need not be the case, as Figure 6.12 highlights.

As regulation puts an upper bound on the feasible total loan/deposit volume by the Standardized Approach according to (3.3.21) and the VaR approach according to (6.5.2), respectively, and as the unregulated total loan volume strictly increases, regulation has in general a dampening effect on the loan volume sensitivities to expected return shocks. Under risk-adjusted portfolio weights with $c_1 = 0.5$ and $c_1 = 1$, however, sensitivities may be higher which is mainly due to changes from one extreme portfolio allocation to the other. In the base case, if expected returns on the first portfolio are considerably low ($\mu_1 \leq 1.09390$), the bank only grants loans to firms of Sector 2. Likewise, if $\mu_1 \geq 1.16485$, solely firms of the first sector can borrow funds. In between, there is a relatively sharp increase in the portfolio-allocation rate and thus in the aggregate loan volume to Sector 1 firms. Because

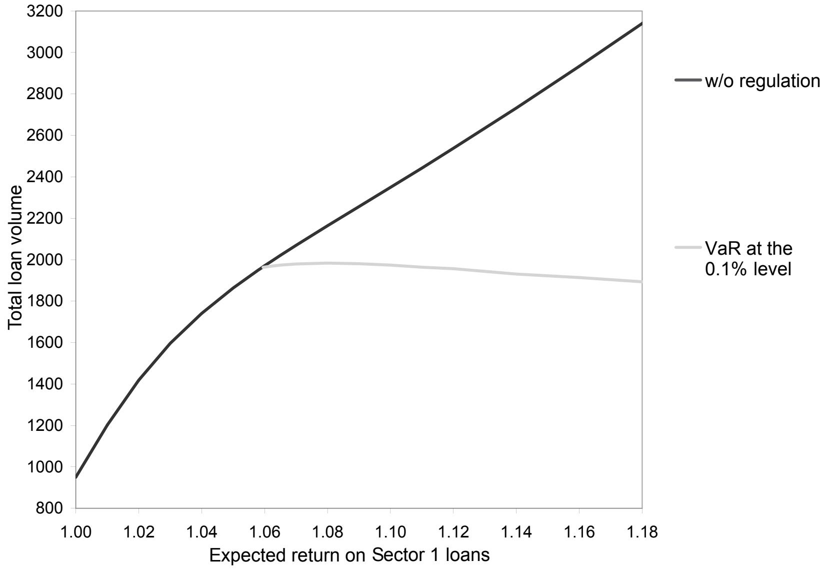


Figure 6.12: **Total loan volume as a function of $\mu_1$, Table 6.2, Panel A**

*This figure illustrates the dependence of the equilibrium total loan volume L on the expected gross return of first-sector loans, $\mu_1$. Results are based on the parametrization given by Table 6.2, Panel A. Total loan volumes L are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level $\bar{p} = 0.1\%$.*

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of augmented expectations, the total loan volume increases strongly as well. The sensitivity of the regulated total loan volume may outreach the sensitivity of the unregulated total loan volume as indicated by Figure 6.11. The same applies to $L_1$ whereas the aggregate loan volume granted to Sector 2 is sloped downwards and thus behaves counter-cyclically.

With parameter values according to Table 6.2, Panel A, the portfolio-allocation rate strictly increases under both regimes and does so even further for $\mu_1 > \mu_2$ after they have crossed the variance-minimum portfolio-allocation rate $\ell_{\sigma_{min}}$, where $\ell_{\sigma_{min}}$ is chosen if $\mu_1$ attains $\mu_2 = 1.08$ (*cf.* Result 35)[7] Hence, the maximum feasible total loan volume under the VaR approach reaches its global maximum value over all expected returns, as the variance is the lowest at this point (*cf.* the VaR Constraint (6.5.2)). As $\sigma$ strictly increases again, if $\mu_1$ increases beyond $\mu_2$, a decreasing total loan volume is obtained under VaR-based regulation due to the Constraint Function (6.5.2).

[7]The comparative static analysis in $\mu_1$ under the base case has been performed until $\mu_1$ reaches $\mu_2$. This example is meant to show that the VaR-regulated total loan volume becomes counter-cyclical if $\mu_1$ increases beyond $\mu_2$. This phenomenon could be observed under the base case as well.

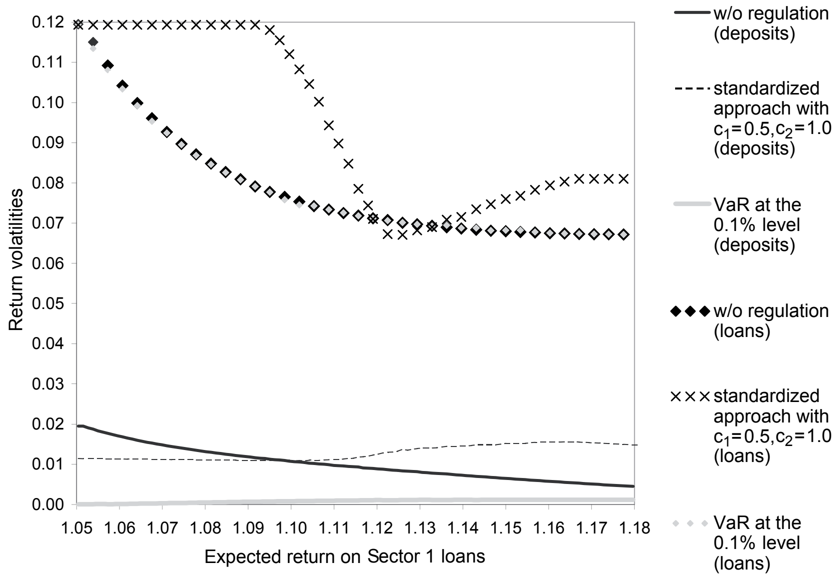


Figure 6.13: **Return volatilities as a function of $\mu_1$**

*This figure illustrates the dependence of the equilibrium return volatilities on the expected gross return of first-sector loans, $\mu_1$. Solid and dashed lines represent volatilities of returns on deposits, $\sigma_D$. Diamonds and crosses represent volatilities on total loans, $\sigma$. Return volatilities are shown for three different regimes: the laissez-faire equilibrium, the Standardized Approach with different risk weights, where $c_1 = 0.5$ and $c_2 = 1$, and the VaR approach with confidence level $\bar{p} = 0.1\%$.*

Figure 6.13 illustrates the issue of risk-taking beyond the choice of portfolio-allocation rates. Evidently, the extreme allocations under the Standardized Approach with different risk weights translate into higher return volatilities concerning both the loan portfolio and the deposits. As a consequence, risk-sensitive weights do not mitigate volatilities compared to the regime with fixed risk weights except for a rather narrow range of intermediate values.[8] At $\mu \approx 1.125$, the regulated bank chooses the variance-minimum loan-allocation rate though $\mu_1 < \mu_2 = 1.188$ still holds. With increasing $\mu_1$, the bank prefers raising expected returns at the cost of increasing risk and the return volatility starts to rise until $\sigma = \sigma_1$ is reached by $\ell^S = 100\%$ from roughly $\mu_1 = 1.16714$ on. In contrast, the unregulated loan-allocation rate $\ell^*$ attains the variance-minimum at $\mu_1 = \mu_2$ (not shown). The latter applies to the VaR-regime as well.

The probability of full deposit redemption is highest under the VaR approaches, namely throughout higher than 99.98%. Under the Standardized Approach,

[8] If $c_1 = c_2 = 1$ holds, the loan-portfolio return volatility $\sigma$ is equal to 0.11949 for $\mu_1 < 1.13738$. It strictly decreases for $\mu_1 > 1.13738$, and achieves its minimum equal to 0.06745 in $\mu_1 = 1.18773$. The deposit return volatility is similar to that under the regime with different risk weights.

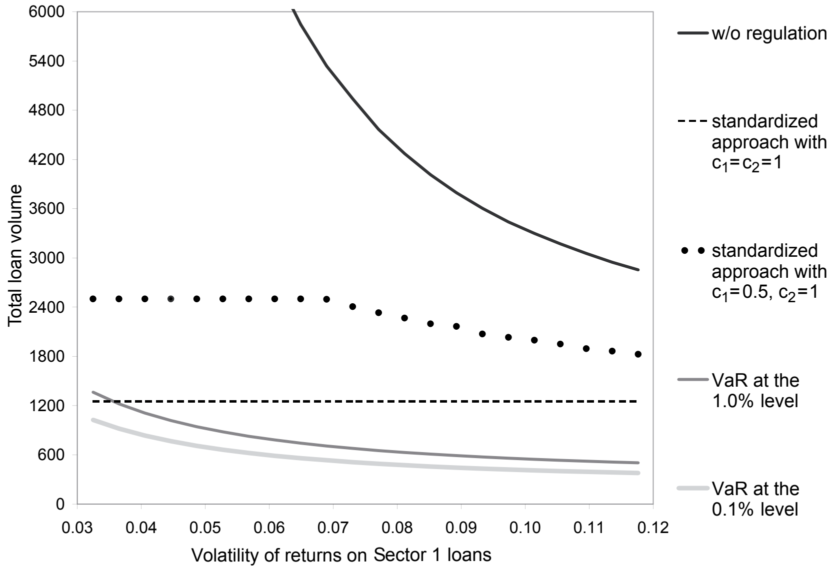


Figure 6.14: **Total loan volume as a function of $\sigma_1$**

*This figure illustrates the dependence of the equilibrium total loan volumes L on the volatility of returns on first-sector loans, $\sigma_1$. Total loan volumes are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and with $\bar{p} = 0.1\%$. The built-in chart highlights the complete $\sigma_1$-$L^*$ chart for all $\sigma_1$ considered.*

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depositors also enjoy higher success probabilities despite increased portfolio risks due to the reduced leverage. For the Standardized Approach with different risk weights, results are mixed.

To sum up, lending under the VaR approach is not pro-cyclical for $\mu_1 \leq \mu_2$ and for $W_H$ being sufficiently high, and even counter-cyclical for $\mu_1 > \mu_2$. If loans are equally weighted, *i.e.* $c_1 = c_2$, there is no pro-cyclical effect on loan volumes either. Crude distinctions of risks by fixed risk weights, in this example by setting $c_1 = 0.5$ and $c_2 = 1$, may accelerate the sensitivity in total lending as well as in sector-wise lending.

### 6.6.3 Return Volatility Shocks

Figure 6.14 shows the comparative static results for volatility shocks. The unregulated total loan volume $L^*$ strictly decreases in $\sigma_1$ reflecting the risk-aversion of the depositors. The total loan volumes under the VaR approach strictly decrease as well, whereas two effects can be identified. First, as without regulation, the equilibrium loan-allocation rate strictly decreases in $\sigma_1$ as loans to Sector 1 become

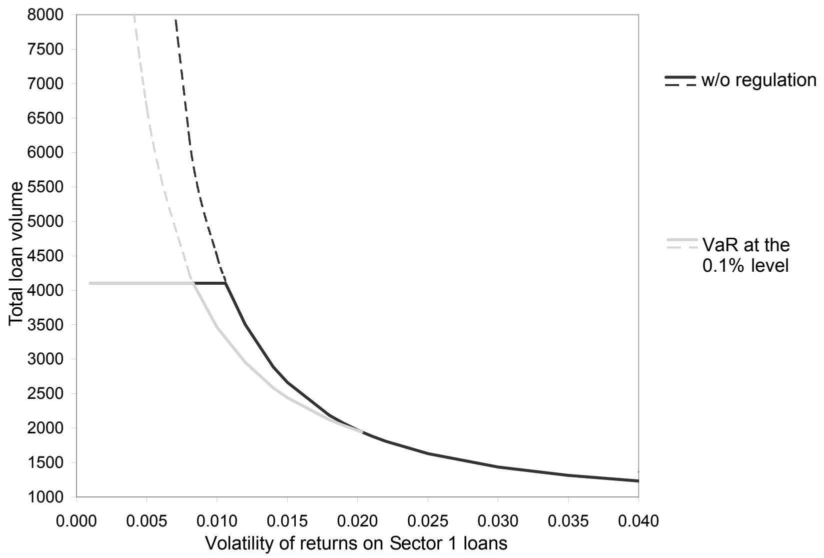


Figure 6.15: **Total loan volume as a function of $\sigma_1$, Table 6.2, Panel A**

*This figure illustrates the dependence of the equilibrium total loan volume $L$ on the volatility of returns on first-sector loans, $\sigma_1$. Results are based on the parametrization given by Table 6.2, Panel A. Total loan volumes $L$ are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level $\bar{p} = 0.1\%$.*

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less favorable. As the regulatory constraint function $w(\ell)$ strictly increases in the loan-allocation rate $\ell$ for $\ell \in [0, \ell_{\sigma_{min}}]$, we obtain a negative effect in total, *i.e.* the total loan volume shrinks. Second, the regulatory constraint function $w(\ell)$ also strictly declines in $\sigma_1$, notably because of $\rho > 0$, which enhances the former effect. The drop in the unregulated total loan volume is stronger than those in the total loan volumes resulting from the VaR approach. As a consequence, regulation is non pro-cyclical concerning total lending. Figure 6.15, which is produced using the alternative parametrization from Table 6.2, Panel A, gives a more detailed view on total loan volumes under return volatility shocks and thus illustrates the dampening effect of VaR regulation. If the expected economic situation worsens, the risk-averse depositor becomes reluctant to lend to the bank such that the total loan/deposit volume shrinks to a point where regulation ceases to bind from a fixed value of $\sigma_1$ on, in this example from $\sigma_1 \approx 0.202$ on.

In the base case scenario we do not explicitly consider the case where the household is constrained by its initial wealth $W_H$, but in the scenario shown in Figure 6.15 we do. Wealth constraints may lead to pro-cyclicality as follows:

For $\sigma_1 \leq 0.0083$, the bank would like to attract more deposits, even under regulation,

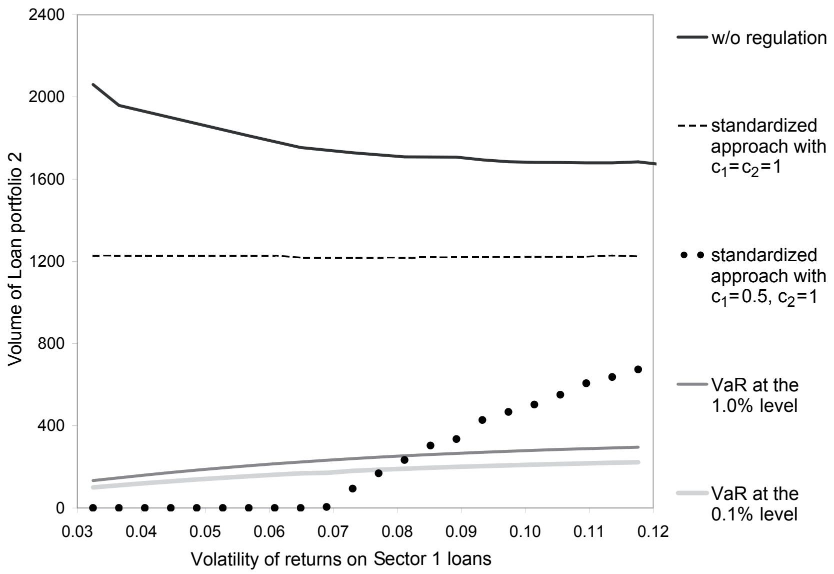


Figure 6.16: **Loan volume to Sector 2 as a function of $\sigma_1$**

*This figure illustrates the dependence of the equilibrium loan volumes granted to Sector 2 on the volatility of returns on first-sector loans, $\sigma_1$. Loan volumes $L_2$ are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and with $\bar{p} = 0.1\%$.*

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than the household is able to provide. Hence, in spite of binding regulation, there is no effect on the total loan volume and regulation is not pro-cyclical according to Definition (2.2.2). For $\sigma_1$ in $[0.0083, 0.0106]$, the unregulated bank is constrained by the household's budget whereas the regulated bank is only restricted by the Regulatory Requirement (6.5.2). Consequently, we obtain $L^* = W_B + W_H$ without regulation, but a risk-sensitive total loan volume under regulation. Hence, the VaR-based regulation is pro-cyclical. If the bank were regulated by equal risk weights, we would not observe a pro-cyclical effect either. It is not until $[0.0106, 0.0202]$ that the dampening effect through regulation begins to hold.

Though the binding regulation for $\sigma_1 \leq 0.0083$ is not reflected by total loan volumes, it is reflected by the different the loan-allocation rates and the different deposit interest rates, where $\ell^V > \ell^S$ and $R_D^V < R_D^S$ holds. If the household is not constrained by its initial wealth $W_H$ under either regimes, both loan-allocation rates are equal and approach the variance-minimum loan-allocation rate $\ell_{\sigma_{min}}$ with decreasing $\sigma_1$.

In principle, we should always expect one of the three effects of regulation on total lending as described above if $\sigma_1$ varies for the following reasons:

First, as the depositor's budget is bounded (as short selling is excluded), the level

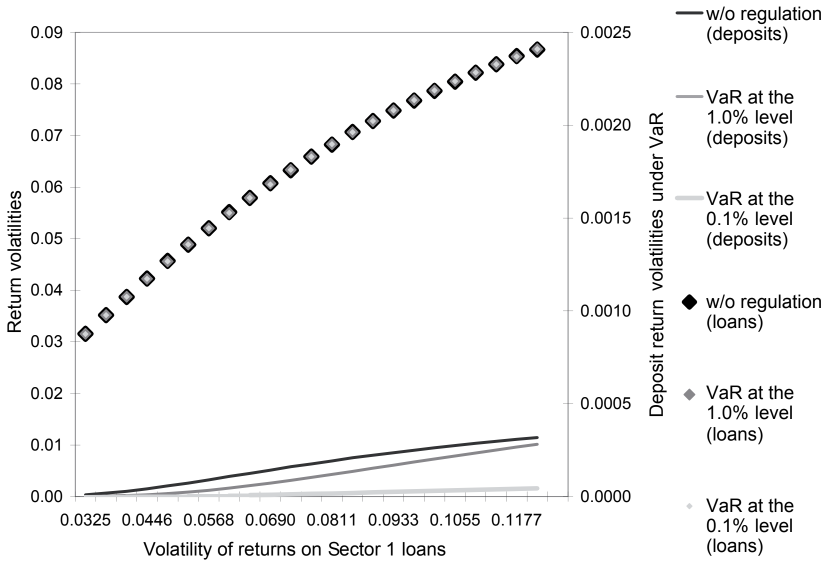


Figure 6.17: **Return volatilities as a function of $\sigma_1$**

*This figure illustrates the dependence of the equilibrium return volatilities on the volatility of returns on first-sector loans, $\sigma_1$. Solid and dashed lines represent volatilities of returns on deposits, $\sigma_D$. Diamonds and crosses represent volatilities on total loans, $\sigma$. Return volatilities are shown for three different regimes: the laissez-faire equilibrium and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.1\%$.*

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of shocked expectations can always be such that deposit supply is restricted by the initial wealth independent of whether the bank is regulated or not.[9]

Second, the regulated equilibrium deposit volume $D^V$ will be already below $W_H$ for lower values of $\sigma_1$ than the unregulated volume $D^*$ since $D^V \leq D^*$ holds by definition. By the assumption of strictly decreasing equilibrium deposit volumes, this property creates a pro-cyclical effect of the VaR regulation according to Definition (2.2.1).

Third, an expectation shock can take values such that the household's initial wealth $W_H$ does not constrain equilibrium total loan/deposit volumes anymore. If furthermore curvature does not change too much[10], the curve representing the unregulated total loan/deposit volume is steeper than that under regulation and

[9]For this reasoning, the assumption that the unconstrained equilibrium deposit volume strictly decreases in $\sigma_i$, $i = 1, 2$ is crucial. The strictly monotonic decline must hold for both regulated and unregulated volumes. Note that Result 31 does not account for the endogenous effects on $\ell$ and $R_D$ in equilibrium , however.

[10]For the importance of curvature on cycle-related effects, we refer to the example shown in Figure 6.10 concerning equity shocks.

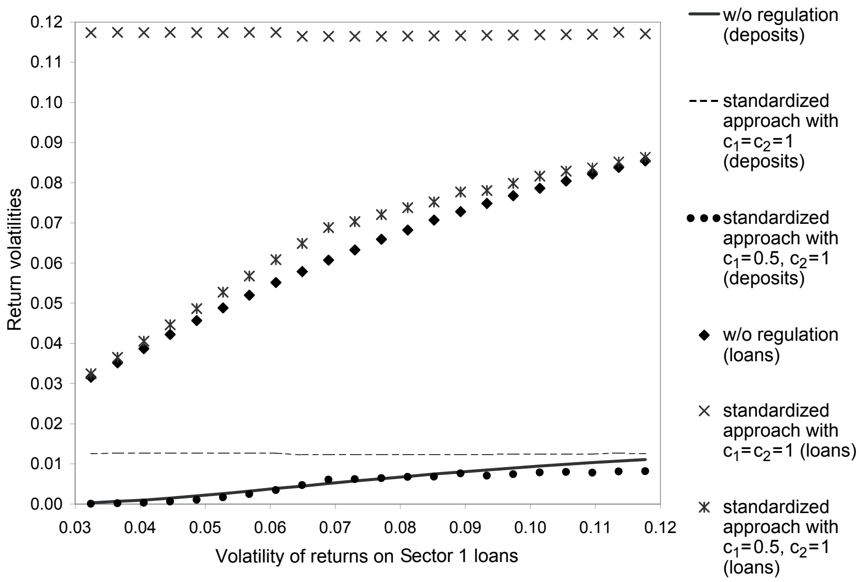


Figure 6.18: **Return volatilities as a function of $\sigma_1$**

*This figure illustrates the dependence of the equilibrium return volatilities on the volatility of returns on first-sector loans, $\sigma_1$. Solid and dashed lines represent volatilities of returns on deposits, $\sigma_D$. Diamonds and crosses represent volatilities on total loans, $\sigma$. Return volatilities are shown for three different regimes: the laissez-faire equilibrium and the Standardized Approach with both equal and different risk weights.*

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thus there is no longer any pro-cyclical impact on lending.[11]

Apart from the total loan volumes under the VaR approach, Figure 6.14 depicts the total loan volumes under the Standardized Approach. In case of $c_1 < c_2$ , the total loan volume decreases in $\sigma_1$ as well and is not pro-cyclical either. For $\sigma_1 < 0.069$, the total loan volume does not react to changes in $\sigma_1$ and the loan-allocation rate $\ell^S$ remains close to 100%. Consequently, the total loan volume is not pro-cyclically affected. If $\sigma_1 > 0.069$, the regulated bank reduces its loan-allocation rate and total lending shrinks under regulation. Then it is conceivable that it can be pro-cyclical, though it is not in this example.

The stronger decline in the loan-allocation rate $\ell^S$ compared to the decrease in the unregulated loan-allocation rate, $\ell^*$, results in a stronger deflation of $L_1^S$ compared to $L_1^*$. Thus, on the level of sectors, we observe a pro-cyclical effect. This sharper decline in $\ell^S$, in turn causes the loan volume granted to firms from the second sector

[11]As the analysis in Section 4.3.1 suggests, these arguments also hold for shocks in correlations. Within the normal framework, we refer to the next section, Section 6.6.4. The preceding arguments hold for shocks in expected returns as well (recall Figures 6.11 and 6.12 for the normal framework and Figures 4.19 and 4.20 concerning the Bernoulli framework).

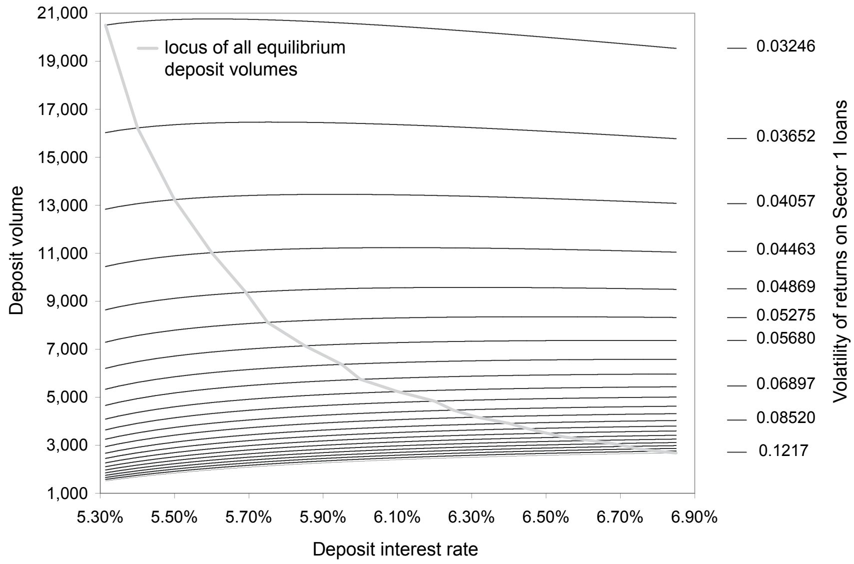


Figure 6.19: **Risk effects in equilibrium on deposits**

*This figure illustrates the influence of the volatility of returns on first-sector loans, $\sigma_1$, on the equilibrium deposit volume given optimally chosen loan-allocation rates. Deposit volumes are shown as functions of the deposit interest rate for different levels of $\sigma_1$. The curve on the top depicts $D^s(\ell^*, R_D)$ for $\sigma_1 \approx 0.03246$ whereas the curve on the bottom represents $D^s(\ell^*, R_D)$ for $\sigma_1 \approx 0.1217$. The grey curve is the locus of all equilibrium deposit volumes and deposit interest rates. This figure exclusively refers to the laissez-faire equilibrium.*

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to strictly increase. In contrast, the unregulated bank reduces also lending to firms belonging to Sector 2, reflecting its prudence due to the risk-averse depositor and its deposit-supply function to the rising loan return volatility $\sigma_1$. Thus, there is a counter-cyclical effect concerning lending to Sector 2. The second phenomenon is illustrated by Figure 6.16.

Deposit interest rates strictly increase in $\sigma_1$ under all regimes except for the Standardized Approach with both risk weights being equal to one (not shown). The equilibrium loan-allocation rates under these four regimes decrease in $\sigma_1$. Yet, the loan portfolio's return volatilities strictly increase (*cf.* Fig. 6.17 and 6.18), implying a strict partial decrease in deposit supply due to Result 31. Though lower loan-allocation rates result in higher expected portfolio returns $\mu$ and though deposit interest rates are raised, the total effect of all these changes on the deposit volume, and hence the total loan volume (*cf.* Fig. 6.15) is negative. Figure 6.19 illustrates this issue in the $R_D - D$ plane as the loan-allocation rate $\ell$ is fixed to $\ell^*$, *i.e.* to the rate that is optimally chosen by the unregulated bank. Thus, the deposit-supply function $D^s(\ell^*, R_D)$ exclusively depends on the deposit interest rate for fixed values of $\sigma_1$.

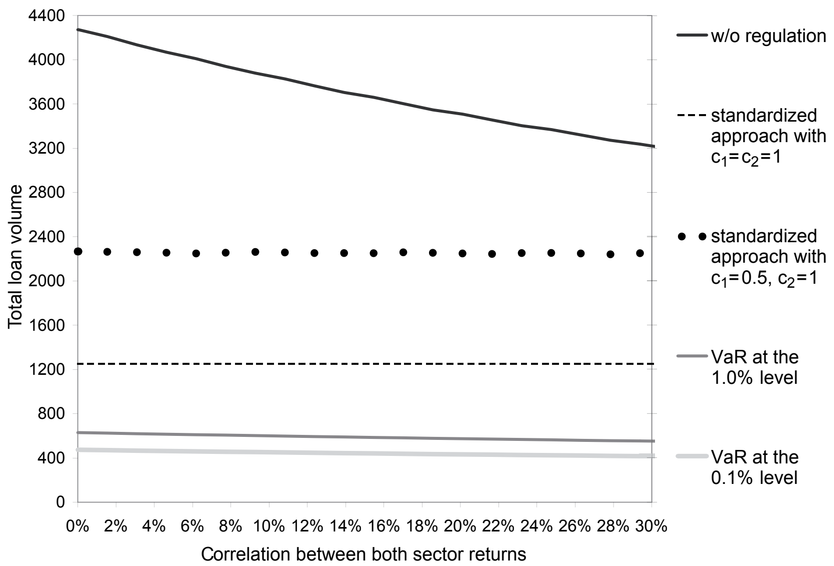


Figure 6.20: **Total loan volume as a function of the inter-sectoral correlation**
*This figure illustrates the dependence of the equilibrium total loan volumes $L$ on the inter-sectoral return correlation $\rho$. Total loan volumes $L$ are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.1\%$.*

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$D^s(\ell^*, R_D)$ is further depicted for different values of $\sigma_1$ to highlight the reduction in deposit volume by increasing $\sigma_1$.

Except for the Standardized Approach with both risk weights being equal, regulation raises the probability of full deposit redemption. This holds true even for the Standardized Approach with different risk weights where the volatility of the loan portfolio is greater than it is the case without regulation (*cf.* Fig. 6.18). But even under the regime with $c_1 = c_2 = 1$, this probability exceeds its unregulated counterpart from $\sigma_1 \approx 0.811$ on.

### 6.6.4 Correlation Shocks

Correlation shocks affect the loan return volatility $\sigma$ similarly as volatility shocks with respect to single sectors. Consequently, the effects on magnitudes that are dependent on both loan portfolios' returns are similar. In contrast to the volatility shocks considered, compensating the shock by shifting portfolio weights as seen in the last section (*cf.* Fig. 6.16) does not happen if the correlation remains at intermediate levels. Under the scenario considered in Figure 6.21, loan-allocation rates move strongly only when correlations exceed $\pm 0.7$. However, from an empirical point of view, these values are unrealistic as high for correlations on loan redemptions. Having the Merton-framework in mind, these values are also problematic from a theoretical point of view. As Gersbach/Lipponer (2003, p. 364f) show, default correlations are always positive and are bound by $\frac{2}{\pi} \cdot \arcsin(\rho_{\text{return}})$, where $\rho_{\text{return}}$ is the firms' asset correlation, if both firms are equal. Given their assumptions, a default correlation may only be above 0.7 if the asset-return correlation of two equal firms exceeds 0.891. Likewise, a default correlation of 0.31 would be associated with an asset correlation of 0.454. Hence, it seems reasonable to consider correlations to be from the interval $[0, 0.31]$ for the comparative static analysis in the base case.

As with increasing $\sigma_i$, an increase in $\rho$ result in a considerably higher $\sigma$, and in a partial decrease both in the unregulated total loan/deposit volume and in the VaR-constrained total loan/deposit volume because of Result 31 and because of the Regulatory Restriction (6.5.2), respectively. Figure 6.20 shows that these effects carry over to the equilibrium as well. As in the case of increasing volatilities, the unregulated total loan volume is more affected by increasing correlations than the VaR-regulated volume. Hence, VaR-based regulation does not enhance cyclicality under correlation shocks either.

The total loan volume does not change under the Standardized Approach, as the different risk weights do not account for symmetrical shifts in risks and the loan-allocation rate is not affected in noteworthy magnitudes. Hence, this kind of regulation does not affect lending in a pro-cyclical way.

Concerning pro-cyclicality, we observe patterns that are analogous to those in the case of volatility and expected return shocks. Figures 6.20 and 6.21 suggest that the regulated and the unregulated total loan volumes come closer to each other as the return correlation becomes higher while a decreasing $\rho$ leads to a stronger rise in the unregulated total loan volume than in the regulated total loan volumes. Furthermore, Figure 6.21 illustrates that regulation can be pro-cyclical with respect to the total loan volume if the depositor is wealth-constrained. As with shocks to $\sigma_1$, the regulated equilibrium deposit volume $D^V$ will be already below $W_H$ for lower values of $\rho$ than the unregulated volume $D^*$ since $D^V \leq D^*$ holds by definition. By the strictly decreasing equilibrium deposit volumes, this property creates a pro-cyclical effect of the VaR regulation.

Figure 6.22 shows that the volumes granted to firms from the first sector are reduced if the correlation between the firms of different sectors decreases. This

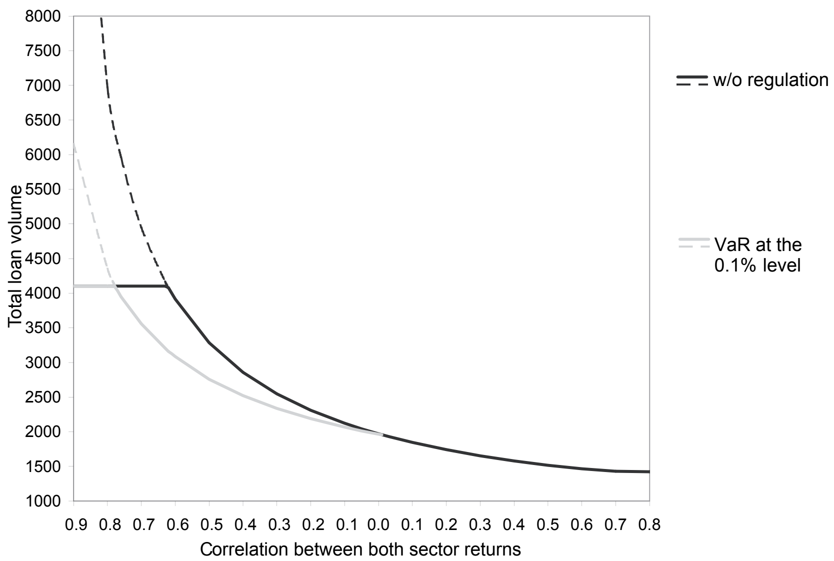


Figure 6.21: **Total loan volume as a function of $\rho$, Table 6.2, Panel A**

*This figure illustrates the dependence of the equilibrium total loan volume $L$ on the inter-sectoral return correlation $\rho$. Results are based on the parametrization given by Table 6.2, Panel A. Total loan volumes $L$ are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level $\bar{p} = 0.1\%$.*

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behavior results from decreasing total loan volumes (*cf.* Fig. 6.20) while the loan-allocation rates remain almost constant (not shown) within the range of values of return correlations considered. More specifically, $59.60\% < \ell^* < 59.80\%$ holds, and $\ell^V$ ranges under each of both VaR constraint between 59.68% and 59.79%. As Figure 6.22 indicates, loan-allocation rates strongly differ from their unregulated counterparts under the Standardized Approach. The magnitudes of the differences depends on whether the fixed risk weights distinguish the two types of loans: if $c_1 = c_2 = 1$, the banks prefers granting loans to the riskier sector implying $0 \leq \ell^S < 2.26\%$, while $\ell^S$ takes values from 88.59% to 89.70% if $c_1 = 0.5$ and $c_2 = 1$ holds. The associated risks are indicated by the return volatilities concerning total loans and deposits, respectively, as shown by Figures 6.24 and 6.25, which are discussed below. If the alternative parametrization from Table 6.2, Panel A, is considered, however, the unregulated loan-allocation rate $\ell^*$ strongly increases, namely from 62.8% to 65.1% for $0.0\% \leq \rho \leq 30\%$. Furthermore, the total loan volume is less sensitive to changes in $\rho$ for $0.0\% \leq \rho \leq 30\%$ than it is the case for $L^*$ under the base case. In numbers, we have $\frac{\Delta L^*}{\Delta \rho} = -1060.3$ under the parameter values from Table 6.2, Panel A, while $\frac{\Delta L^*}{\Delta \rho} = -3492.85$ holds in the base case as given by Table 6.1. Consequently, $L_1^*$ is also less sensitive to shocks in $\rho$ for $0.0\% \leq \rho \leq 30\%$

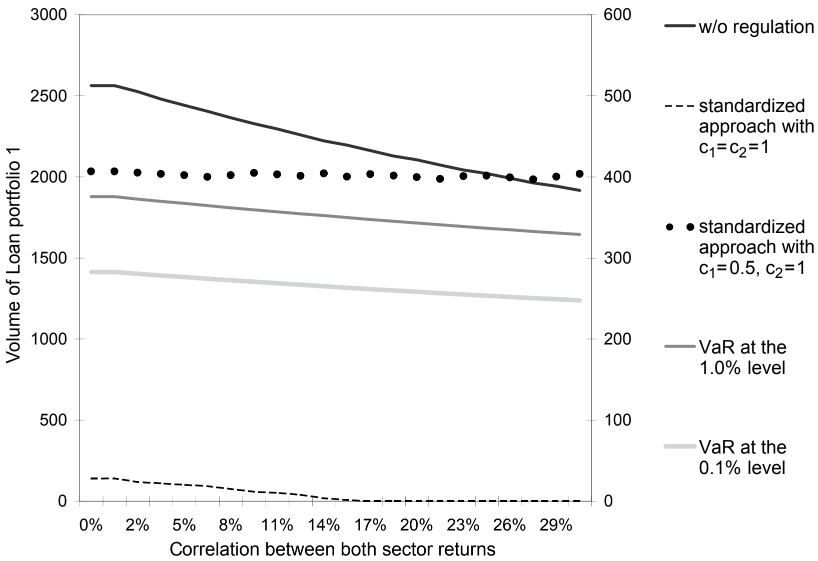


Figure 6.22: **Loan volume to Sector 1 as a function of $\rho$**

*This figure illustrates the dependence of the equilibrium loan volumes granted to Sector 1 on the inter-sectoral return correlation $\rho$. Loan volumes $L_1$ are shown for five different regimes: the laissez-faire equilibrium, the Standardized Approach with both equal and different risk weights, and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and with $\bar{p} = 0.1\%$.*
*Left-hand scale: w/o regulation and the Standardized Approach with different risk weights.*
*Right-hand scale: both VaR approaches.*

in the scenario given by Table 6.2 than in the base case.

Furthermore, if the household is wealth-constrained, pro- and counter-cyclical effects in lending concerning a single portfolio $L_i$, $i = 1, 2$, may arise, as shown in Figure 6.23. For $-90\% \leq \rho < -77\%$, $L_1^V$ and $L_1^*$ strictly increase where $L_1^V$ is steeper. Thus, the loan volume lend to firms from the first sector becomes pro-cyclical through regulation. For $-77\% < \rho < -62\%$, the regulated loan volume $L_1^V$ strictly decreases while $L_1^*$ further strictly increases. Thus, regulation has a counter-cyclical effect according to Definition (2.2.3). From $\rho = -63\%$ on, the loan volumes lend to firms from the first sector strictly decrease under both regimes, whereas the slope of the unregulated volume, $L_1^*$ with respect to changes in $\rho$ is larger. Thus, regulation is not pro-cyclical from $\rho = -63\%$ on.

On the domain where regulation has a counter-cyclical effect, *i.e.* for $\rho \in [-77\%, -63\%]$, the household is restricted by its initial wealth $W_H$ in the laissez-faire economy, while it is not if the bank is regulated. As a consequence, the regulated total loan volume is already downward-sloping for increasing $\rho$. The unregulated total loan volume, however, is fixed to $L^* = W_H + W_B$. As the loan-allocation rate

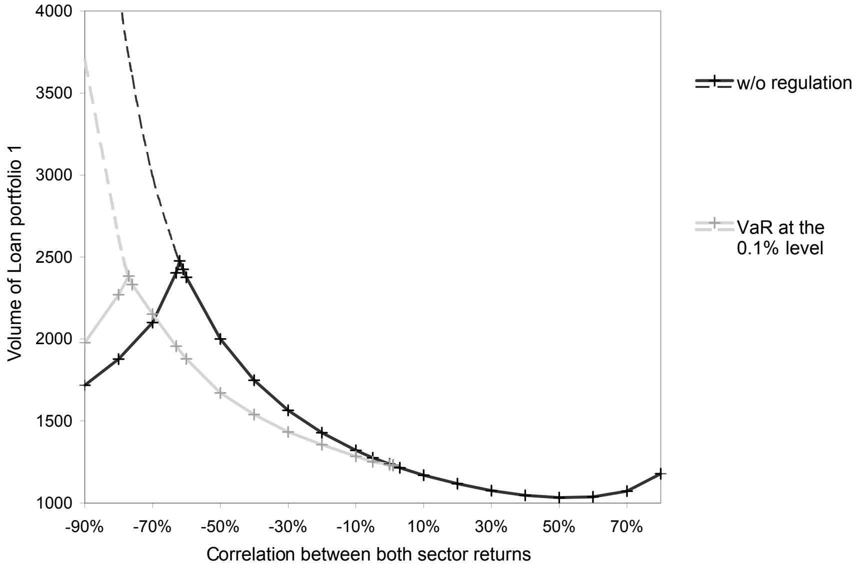


Figure 6.23: **Loan volume to Sector 1 as a function of $\rho$, Table 6.2, Panel A**
*This figure illustrates the dependence of the equilibrium loans volumes granted to Sector 1 on the inter-sectoral return correlation $\rho$. Results are based on the parametrization given by Table 6.2, Panel A. Loan volumes $L_2$ are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level $\bar{p} = 0.1\%$.*

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$\ell^*$ strictly increases in $\rho$ without regulation,[12] the loan volume $L_1^*$ strictly increases as well for $\rho \in [-77\%, -63\%]$.

Concerning risk-taking, we refer to Figures 6.24 and 6.25 which show the return volatilities of the loan portfolio and of the deposit for intermediate values of correlations in the base case. As in the other shock scenarios, the loan-allocation rates are not significantly altered by the VaR regulation. Thus, loan return volatilities are very close to their unregulated counterparts. Since the VaR regulation results in lower deposit volumes and deposit interest rates, deposit return volatilities are lower under regulation, too. Only under the Standardized Approach adverse effects occur. Since binding regulation always reduces the bank's leverage, the buffer effect of bank capital is also strengthened under the Standardized Approach such that deposit return volatilities scatter less than the associated loan return volatilities. Furthermore, the probabilities of full deposit redemption are also higher under both forms of regulation compared to the unregulated regime, whereas those under the VaR approach are the highest.

Analogous to volatility shocks, the increasing return volatilities $\sigma$ and $\sigma_D$,

[12] Also $\ell^V$ strictly increases with decreasing $\rho$.

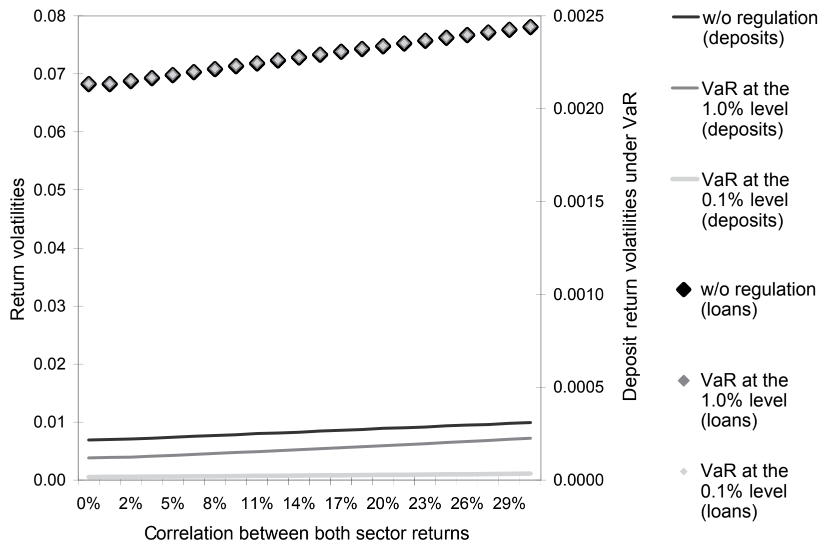


Figure 6.24: **Return volatilities as a function of $\rho$**

*This figure illustrates the dependence of the equilibrium return volatilities on the inter-sectoral loan-return correlation $\rho$. Solid and dashed line represent volatilities of returns on deposits, $\sigma_D$. Diamonds and crosses represent volatilities on total loans, $\sigma$. Return volatilities are shown for three different regimes: the laissez-faire equilibrium and the VaR approach with confidence levels $\bar{p} = 1.0\%$ and $\bar{p} = 0.1\%$.*

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respectively, lead in both scenarios considered in this analysis to higher deposit interest rates which has also been confirmed under the framework with the Bernoulli distribution (*cf.* Figure 4.13).

## 6.7 Summary

This numerical analysis has shown that, by and large, the results obtained in Chapter 4 carry over to this framework which considers normally distributed aggregate loan redemptions. The effects regulation can have depend on the kind of capital requirements and the exogenous shock as well as the endogenous variable considered.

Under past shocks that impaired the bank's equity once, a binding regulation affects total lending pro-cyclically. This again holds true for both the Standardized Approach and the VaR approach. Assigning different risk weights may partially lead to higher volumes granted to a single sector under regulation than without regulation, thus questioning a general pro-cyclical impact on lending on the disaggregate level (*cf.* Fig. 6.8). A similar effect has been observed under the

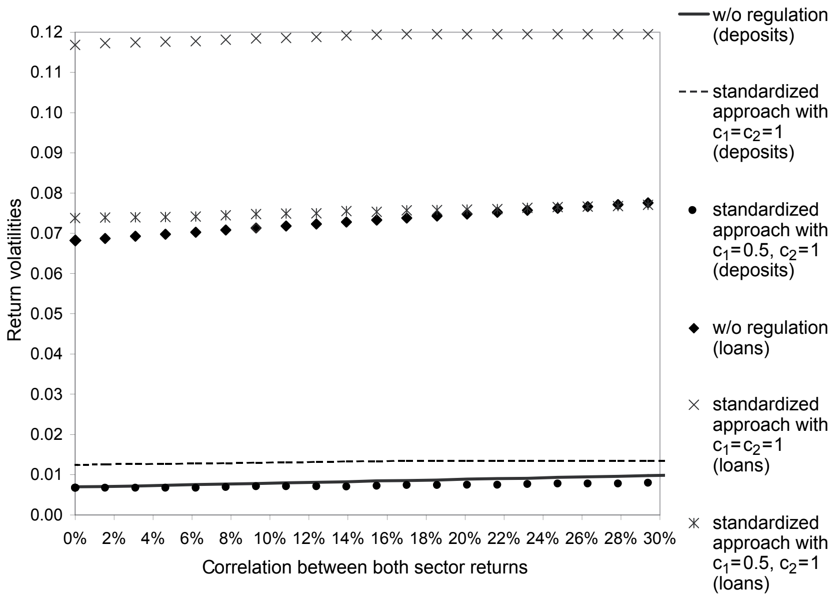


Figure 6.25: **Return volatilities as a function of $\rho$**

*This figure illustrates the dependence of the equilibrium return volatilities on the inter-sectoral loan-return correlation $\rho$. Solid and dashed lines represent volatilities of returns on deposits, $\sigma_D$. Diamonds and crosses represent volatilities on total loans, $\sigma$. Return volatilities are shown for three different regimes: the laissez-faire equilibrium and the Standardized Approach with both equal and different risk weights.*

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Bernoulli framework (*cf.* Fig. 4.5).

Furthermore, we provided an example emphasizing that the sensitivity of the unregulated total loan volume is crucial for sustaining pro-cyclical effects. The more concave the total loan/deposit volume is with respect to the bank's initial equity, the more likely it is that there is a domain on which regulation dampens the sensitivity of the total loan volume. The notion of pro-cyclicality on average thus becomes a suitable term.

If the agents' expectations are shifted, total lending mostly reduces sensitivities under binding regulation. But there are exceptions. Concerning shocks in the volatility $\sigma_1$, the sector-wide loan volume $L_1^S$ granted under the Standardized Approach with different risk weights deflates stronger than its unregulated counterpart. On the other side, we observe a counter-cyclical reaction in the loan volume granted to Sector 2, $L_2$: the regulated volume strictly increases on this domain, while the unregulated volume strictly decreases.

Furthermore, wealth-restrictions with the household may result in both pro- and counter-cyclical effects.

The VaR approach always reduces risks in terms of deposit return volatilities and the bank's probability of bankruptcy which is in line with Dangl/Lehar (2004) though their setting is different from ours, in particular in terms of the bank's binary portfolio choices that can be revealed only by auditing from time to time.

The adverse effects that were present under the VaR approach within the Bernoulli framework do not recur here as loan redemptions are no longer lumpy. Consequently, the regulatory constraint becomes smooth and extreme allocations in favor of either sector can no longer be observed. Concerning risk, the Standardized Approach offers mixed effects as in the Bernoulli-distribution based framework.

So far, the question of how a shock that has once occurred may further affect the economy could not be addressed. In particular, we wish to examine if expectation shocks have lasting effects. Consequently, we will extend this model to two periods in the following chapter.

# Chapter 7

# The Two-Period Model

## 7.1 Timeline and Decisions

In this chapter, we extend the model from Chapter 6 to two periods. Decisions are made at the beginning of each period, at $t = 0$ and at $t = 1$. All contracts last for one period each. Thus the household decides both at $t = 0$ and $t = 1$ on its portfolio composition and the bank does the same. We assume that the institutional framework from the one-period model holds for each period separately.

Both the bank and the household are perfectly informed about each other's sets of feasible decisions, as in the one-period models. They also have rational expectations regarding their own and the other's reaction in $t = 1$ to decisions made at $t = 0$. As a consequence, the household and the bank will base their decisions at $t = 0$ on the possible equilibria in $t = 1$. Likewise, the household and the bank will base their decisions at $t = 1$ on what happened then.

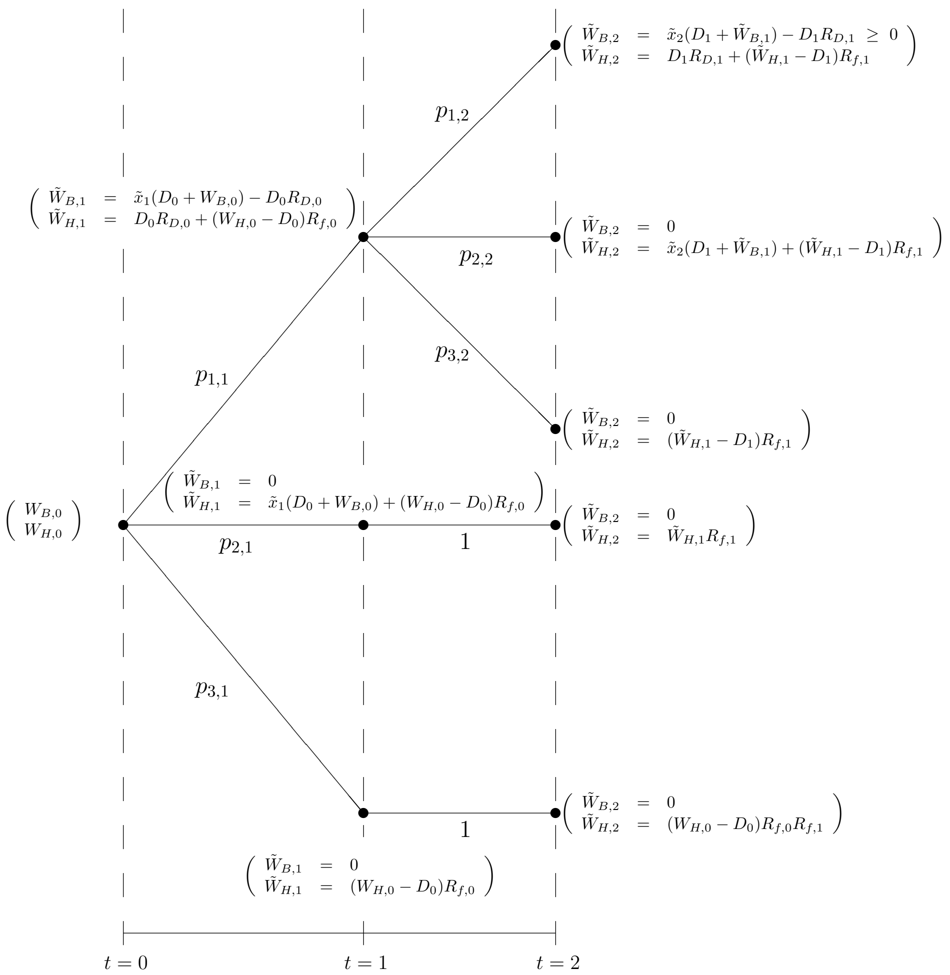
Therefore, the model will, as usual, be solved backwards. First, the decisions at $t = 1$ given any decisions and outcomes from the first period will be derived, and then the decisions and the equilibrium in $t = 0$, given all possible equilibria in $t = 1$, will be determined. Given the institutional framework, there are thus no incentives to deviate from the decisions thus derived and the solution is subgame-perfect. As the stages of this game are interpreted as a timeline, the equilibria can also be considered to be time-consistent: There is no point in time at which either the household or the bank can credibly threaten to deviate (*cf.* Mas-Colell/Winston/Green, 1995, p. 267ff).

We begin by analyzing the model in the second period. Expectations formed at $t = 1$ are indexed by 1, those formed at $t = 0$ by 0. The model's parameters referring to the returns on the bank's loan portfolio are indexed either by 1 or 2, depending on the period they refer to, or, equivalently, the point of time at which they are realized. Decision variables such as the loan-allocation rates, the deposit interest rates, and the deposit volumes are indexed by the point of time at which the decision was made; that is, either by $t = 0$ or $t = 1$. The time indices of the riskless rates refer, in analogy to the deposit interest rates, to the point in time at which the household starts to save its remaining funds, and not to the point in time at which the riskless rate is realized. Thus, $R_{f,t}$ denotes the riskless rate starting at $t$ and ending at $t + 1$. The same applies to the deposit interest rates $R_{D,t}$, whereas the loan-portfolio return $x_t$ indicates that it was realized in $t$ while the associated portfolio composition was chosen at $t - 1$. Likewise, the symbols $\mu_t$, $\sigma_t$, and $\rho_t$ denote the expected portfolio return, the portfolio's return volatility and the correlation of the sector returns associated with the returns to be realized in $t$. In particular, $\mu_1$, $\sigma_1$, and $\rho_1$ denote the initial distributional parameters referring to the first period. The parameters characterizing the return distributions of the respective subportfolios are indexed by $i, t$, $i = 1, 2$ and $t = 1, 2$. Hence, $\mu_{1,1}$ is the expected return on the loans granted to Sector 1, as expected at $t = 0$ for the first period. We assume that portfolio returns are independent across both periods. At the final date $t = 2$, the realizations of the portfolio returns $\tilde{x}_2$ result in the bank's final wealth $W_{B,2}$ and in the household's final wealth $W_{H,2}$.

Figure 7.1 outlines the wealth timeline. Each state is characterized by the bank's amount of equity and the household's wealth at each point in time $t$. Though there is a continuum of states at each point in time $t$ as gross returns on the bank's loan portfolio are normally distributed, these states can be summarized by a few nodes that reflect the main event sets. These main event sets are characterized by whether or not the bank can redeem the deposits as promised. Failure comprises both the set of states in which the household receives positive payments that are based on positive residual claims on the bank's assets and the set of states in which the household receives zero payments. If the bank fails after the first period, the household can only invest its funds in the riskless asset for the second period. Consequently, there are only two critical nodes in $t = 2$ following both nodes in $t = 1$ that symbolize the failure events. The paths linking the nodes are tagged by the conditional probabilities that can be assigned to each respective node, *i.e.* set of states, to which the path points.

Analogous to Definition (6.2.3), $\tilde{b}_{d_z,2}$ denotes the bank's solvency barrier at the end of the second period, and $b_{d_z,1}$ denotes the solvency barrier after the first period.

Figure 7.1: **Wealth timeline**




with

$$
\begin{aligned}
p_{1,t} &= \Phi(b_{d_z,t}) \\
p_{2,t} &= \Phi(\tfrac{\mu_t}{\sigma_t}) - \Phi(b_{d_z,t}) \\
p_{3,t} &= 1 - \Phi(\tfrac{\mu_t}{\sigma_t})
\end{aligned}
$$

$t = 1, 2$

Formally, these standardized barriers can be expressed as

$$\begin{aligned} b_{d_z,1} &= \frac{\mu_1 - \frac{D_0 R_{D,0}}{D_0 + W_{B,0}}}{\sigma_1} , \\ \tilde{b}_{d_z,2} &= \frac{\tilde{\mu}_2 - \frac{\tilde{D}_1 \tilde{R}_{D,1}}{\tilde{D}_1 + \tilde{W}_{B,1}}}{\tilde{\sigma}_2} . \end{aligned} \tag{7.1.1}$$

The solvency barrier at the end of the second period is a random variable given the available information at $t = 0$ and thus tagged by a tilde. In particular, all main components of $\tilde{b}_{d_z,2}$ are random variables as they depend on the realizations of the state variables $\tilde{W}_{B,1}$ and $\tilde{W}_{H,1}$. At $t = 1$ uncertainty concerning $\tilde{b}_{d_z,2}$ is resolved and we can simply write $b_{d_z,2}$. Likewise, any other realizations of random variables will be written without a tilde as well.

As the household is faced in $t = 1$ with the same portfolio decision as in the one-period model, the expected utility from (6.3.1) can be re-formulated as the household's objective function at $t = 1$. Hence we obtain

$$\begin{aligned} \mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,|(W_{B,1}, W_{H,1})\right] &= -e^{-\gamma(W_{H,1}-D_1)R_{f,1}} \cdot \Phi(-\frac{\mu_2}{\sigma_2}) \\ &\quad -e^{-\gamma\left[\mu_2(D_1+W_{B,1})+(W_{H,1}-D_1)R_{f,1}-\gamma\sigma_2^2(D_1+W_{B,1})^2\right]} \cdot \\ \cdot \Bigg[\Phi(\frac{\mu_2 - \gamma\sigma_2^2(D_1+W_{B,1})}{\sigma_2}) &- \Phi(\frac{\mu_2 - \frac{D_1 R_{D,1}}{D_1+W_{B,1}} - \gamma\sigma_2^2(D_1+W_{B,1})}{\sigma_2})\Bigg] \\ &\quad -e^{-\gamma\left[D_1 R_{D,1}+(W_{H,1}-D_1)R_{f,1}\right]} \cdot \Phi(b_{d_z,2}) , \end{aligned} \tag{7.1.2}$$

as expectations are formed conditional on realizations of $\tilde{W}_{B,1}$ and $\tilde{W}_{H,1}$ at $t = 1$.

As a consequence, the household's deposit supply in $t = 1$ has properties that are analogous to those in the one-period model, except for $W_{B,1} = 0$, since the bank is considered bankrupt in this state and is no longer allowed to operate. We assume

$$R_{D,1}\big|_{x_1 = \frac{D_0 R_{D,0}}{D_0 + W_{B,0}}} := 0 \tag{7.1.3}$$

resulting in

$$D_1^s(\ell_1, R_{D,1})\big|_{x_1 = \frac{D_0 R_{D,0}}{D_0 + W_{B,0}}} = 0$$

due to Result 32. This assumption prevents the household from depositing funds at the bank after the bank has already defaulted. In particular, Condition (7.1.3) ensures that

$$\mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,|W_{B,1}\right]\Big|_{x_1=b_{d_x,1}} \;=\; -\,e^{-\gamma W_{H,1}R_{f,1}}$$

holds, as derived in Appendix D.2 in connection with the proof of Result 36. Condition (7.1.3) leads to a jump in the second period's deposit-supply function at $W_{B,1}=0$. The supply function is continuous from below (and equal to zero) in terms of realized equity $W_{B,1}$ or, equivalently, in terms of portfolio realizations $x_1$.

A bankruptcy of the bank in $t=1$ results in the following expected losses for both the bank and the household: First, the bank can no longer generate expected wealth from intermediation in the second period as it will be closed. Second, after the bank's closure, the household can only invest its funds in the risk-free asset. Hence, the household also incurs a loss in expected wealth in the second period if the bank goes bankrupt, as it must forgo the opportunity to invest funds in risky deposits.

Analogous to the one-period model, (6.2.1), at $t=1$ the bank's expectation can be expressed as follows, conditional on $W_{B,1}$ and $W_{H,1}$,

$$\begin{aligned}\mathrm{E}_1\left[\tilde{W}_{B,2}(\ell_1,R_{D,1})\,|(W_{B,1},W_{H,1})\right] \;&=\; \sigma_2\cdot(D_1+W_{B,1})\cdot\varphi(b_{d_z,2})\\ &\quad+\;[\mu_2\cdot(D_1+W_{B,1})-D_1\cdot R_{D,1}]\cdot\Phi(b_{d_z,2})\end{aligned}$$

as its final wealth $\tilde{W}_{H,2}$. Thus, the bank's conditional decision in $t=1$ can be characterized as discussed in the one-period model if $W_{B,1}>0$ holds.

The equilibrium choice at $t=1$ will be denoted by $(\ell_1^*,R_{D,1}^*)$ if the bank is unregulated. The superscript $V$ indicates equilibrium choices under VaR-based regulation. Solving the model backwards, optimal choices made at the beginning of the second period are contingent on a continuum of potential decisions in $t=0$ and on the realized portfolio return $x_1$. All these parameters affect $(\ell_1^*,R_{D,1}^*)$ via the bank's equity $W_{B,1}$ and via the households's wealth $W_{H,1}$. If the household is not constrained in $t=1$, bank capital $W_{B,1}$ remains as the only link between the decisions made in the first and the second periods. The loan-allocation rate chosen at $t=0$, in turn, indirectly influences $W_{B,1}$ in two ways: first via $D_0^*$ and second via the distribution of the portfolio returns $\tilde{x}_1$. As variables in $t=1$ depend on $W_{B,1}$, which depends *inter alia* on the realized return $x_1$, all variables in $t=1$ are random variables in $t=0$. Thus, the household and the bank managers will base their expectations on the second period's equilibria when taking decisions at $t=0$. Formally, the connection between the second-period decision variables, here the portfolio-allocation rate, and the first-period variables is as follows:

$$\begin{aligned}\tilde{\ell}_1^* = \ell_1^*(\tilde{W}_{B,1}) &= \ell_1(\tilde{x}_1, D_0, R_{D,0}) \\ &= \ell_1\left(\tilde{x}_1, D_0^s(\ell_0, R_{D,0}), R_{D,0}\right) \\ &= \ell_1\left(\tilde{x}_1, D_0(\ell_0(\mu_{1,1}, \mu_{2,1}, \sigma_{1,1}, \sigma_{2,1}, \rho_1, \cdot), R_{D,0}(\cdot)), R_{D,0}(\cdot)\right) \ ,\end{aligned} \tag{7.1.4}$$

where $R_{D,0}(\cdot)$ potentially depends on the same parameters as $\ell_0(\cdot)$.

Given the optimal decisions in $t = 1$, $\left(\ell_1^*, R_{D,1}^*\right)$, the bank owners's expectations at $t = 0$ over final wealth in $t = 2$ are

$$\begin{aligned}&\mathrm{E}_0\left[\mathrm{E}_1\left[\tilde{W}_{B,2}(\tilde{\ell}_1^*, \tilde{R}_{D,1}^*)\,\Big|(\tilde{W}_{B,1}, \tilde{W}_{H,1})\right]\right] = \\ &\qquad \int\limits_{\frac{D_0 R_{D,0}}{D_0 + W_{B,0}}}^{\infty} \mathrm{E}_1\left[\tilde{W}_{B,2}(\ell_1^*, R_{D,1}^*)\,|(W_{B,1}, W_{H,1})\right] \cdot \frac{1}{\sigma_1} \cdot \varphi(\frac{x_1 - \mu_1}{\sigma_1})\ dx_1 \ .\end{aligned}$$

A further analytical evaluation of the integral is not possible. Even if both sector-wise expected returns $\mu_{1,i}$ are equal and the optimal loan-allocation rate $\ell_1^*$ becomes a constant, the interest rate on deposits will still depend on the bank's capital in $t = 1$ in a manner that makes it analytically intractable. Hence, characterizations of the decision in $t = 0$ can only be expressed numerically.

Assume that the household is not constrained by its wealth $W_{H,1}$ if it can decide to invest in bank deposits at $t = 1$; that is, $D_1^u < D_0^s \cdot R_{D,0} + (W_{H,0} - D_0^s) \cdot R_{f,0}$ holds. As a consequence, the expectations formed at $t = 1$ will be only conditional on $W_{B,1}$. Then the household's expected utility in $t = 0$ over final wealth $\tilde{W}_{H,2}$ in $t = 2$ is given by

$$
\begin{aligned}
&\mathrm{E}_0\left[\mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,\middle|\,\tilde{W}_{B,1}\right]\right] \;= \\
&= \int\limits_{\frac{D_0 R_{D,0}}{D_0+W_{B,0}}}^{\infty} -e^{-\gamma\left[D_0 R_{D,0}+(W_{H,0}-D_0)R_{f,0}-D_1\right]R_{f,1}} \cdot \Bigg\{ 1-\Phi(\frac{\mu_2}{\sigma_2}) \\
&\qquad + e^{-\gamma\left[\mu_2(D_1+W_{B,1})-\frac{1}{2}\gamma\sigma_2^2(D_1+W_{B,1})^2\right]} \cdot \\
&\quad \cdot \left[\Phi(\frac{\mu_1-\gamma\sigma_1^2(D_1+W_{B,1})}{\sigma_1}) - \Phi(\frac{\mu_1-\frac{D_1 R_{D,1}}{D_1+W_{B,1}}-\gamma\sigma_1^2(D_1+W_{B,1})}{\sigma_1})\right] \\
&\qquad\qquad + \; e^{-\gamma D_1 R_{D,1}} \cdot \Phi(b_{d_z,2}) \Bigg\} \cdot \frac{1}{\sigma_1} \cdot \varphi(\frac{x_1-\mu_1}{\sigma_1})\; dx_1 \\
&+ \int\limits_{0}^{\frac{D_0 R_{D,0}}{D_0+W_{B,0}}} -e^{-\gamma\left[x_1(D_0+W_{B,0})+(W_{H,0}-D_0)R_{f,0}\right]R_{f,1}} \cdot \frac{1}{\sigma_1} \cdot \varphi(\frac{x_1-\mu_1}{\sigma_1})\; dx_1 \\
&+ \int\limits_{-\infty}^{0} -e^{-\gamma(W_{H,0}-D_0)R_{f,0}R_{f,1}} \cdot \frac{1}{\sigma_1} \cdot \varphi(\frac{x_1-\mu_1}{\sigma_1})\; dx_1 \;.
\end{aligned}
$$

The first integral represents all states $W_{B,1}$ in $t = 1$ where bank debt is fully repaid to the household. The second integral describes all states in which positive aggregate loan redemptions are received by the bank but are too low to fully repay the obligations to the household. The third integral summarizes all states in which all borrowers fully default on their loans and the household is only paid off from the riskless asset.

As with the representation of the bank's expected equity value, the first integral cannot be simplified further if one solves for equilibrium. The reason is that the first integral represents the part of expected utility that has been formed conditional on $\tilde{W}_{B,1}$. In particular, $\tilde{D}_1$ is a random variable dependent on $\tilde{W}_{B,1}$. The same applies to $\tilde{R}_{D,1}$ and $\tilde{\ell}_1$, respectively. The latter variable affects $\mu_2$ and $\sigma_2$ such that $\mu_2$ in fact represents a conditional expectation and $\sigma_2$ a conditional volatility.

As this dependence is analytically unknown, a further analytical determination is not possible. As the bank will have defaulted in the other two sets of states, no random variables referring to the second period need be considered and both of the latter integrals can be further simplified. Expectations in $t = 0$ over final wealth in $t = 2$ can thus be re-formulated as

$$
\begin{aligned}
\mathrm{E}_0\left[\mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,\middle|\tilde{W}_{B,1}\right]\right] \;=\; & \int\limits_{\frac{D_0 R_{D,0}}{D_0+W_{B,0}}}^{\infty} \mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,|W_{B,1}\right]\cdot\frac{1}{\sigma_1}\cdot\varphi(\frac{x_1-\mu_1}{\sigma_1})\;dx_1 \\
& - \; e^{-\gamma\left[\mu_1(D_0+W_{B,0})+(W_{H,0}-D_0)R_{f,0}-\frac{1}{2}\gamma\sigma_1^2(D_0+W_{B,0})^2R_{f,1}\right]R_{f,1}}\cdot \\
\cdot\left[\Phi(\frac{\mu_1-\gamma\sigma_1^2(D_0+W_{B,0})R_{f,1}}{\sigma_1})-\Phi(\frac{\mu_1-\frac{D_0R_{D,0}}{D_0+W_{B,0}}-\gamma\sigma_1^2(D_0+W_{B,0})R_{f,1}}{\sigma_1})\right] & \\
& - \; e^{-\gamma(W_{H,0}-D_0)R_{f,0}R_{f,1}}\cdot\left[1-\Phi(\frac{\mu_1}{\sigma_1})\right]\;.
\end{aligned}
\tag{7.1.5}
$$

The integral in the first line will occasionally be denoted by $g(D_0)$ while $h(D_0)$ will occasionally refer to the remaining parts.

The conditional equilibrium in $t = 1$ features the same properties as the equilibrium in the one-period model examined in the preceding chapter, Chapter 6, if $W_{B,1}$ is changed parametrically. Consider the household's maximization problem in $t = 0$ given the equilibrium in $t = 1$:

$$
\begin{aligned}
\max_{D_0} \quad & \mathrm{E}_0\left[\mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,\middle|\tilde{W}_{B,1}\right]\right] \\
\text{s.t.} \quad & 0 \;\le\; D_0 \;\le\; W_{H,0} \\
& (\ell_1, R_{D,1}) \;=\; \left(\ell_1^*(\cdot), R_{D,1}^*(\cdot)\right)\;.
\end{aligned}
\tag{7.1.6}
$$

First of all, there is always at least one solution to Problem (7.1.6). This result is shown in Appendix D.1. In fact, the proof is obtained by rephrasing the proof of the existence of a solution stated in Result 30. To characterize sufficient conditions for which the household's deposit-supply function in $t = 0$ is unique, we define

$$
\begin{aligned}
\Delta_1^\top \;&=\; \left(1,\;\frac{\partial R_{D,1}^*}{\partial W_{B,1}},\;\frac{\partial \ell_1^*}{\partial W_{B,1}}\right)\;, \\
\Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}^\top \;&=\; \left(\frac{\partial \mathrm{E}_1\left[u_H(\cdot)|\cdot\right]}{\partial W_{B,1}},\;\frac{\partial \mathrm{E}_1\left[u_H(\cdot)|\cdot\right]}{\partial R_{D,1}},\;\frac{\partial \mathrm{E}_1\left[u_H(\cdot)|\cdot\right]}{\partial \ell_1}\right)
\end{aligned}
$$

if these derivatives exist. In particular, we use $\Delta_1\big|_{x_1=\frac{D_0R_{D,0}}{D_0+W_{B,0}}}$ and $\Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}\big|_{x_1=\frac{D_0R_{D,0}}{D_0+W_{B,0}}}$ to denote the limit values for which $x_1 = \frac{D_0R_{D,0}}{D_0+W_{B,0}}$ is approached from above.[1] The expression $\Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}$ always refers to the conditional

[1]Note that these terms would vanish if $x_1 = \frac{D_0R_{D,0}}{D_0+W_{B,0}}$ were approached from below because of

(laissez-faire) equilibrium $\left(\ell_1^*(\cdot), R_{D,1}^*(\cdot)\right)$ in $t = 1$. The definitions of $\Delta_1$ and $\Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}$ are only meaningful if $R_{D,0} > \underline{R}_{D,0}(\ell)$ holds where $\underline{R}_{D,0}(\ell)$ is given by Definition (6.3.7). As in the one-period model $\ell_t \in [0,1]$, $t \in \{0,1\}$, is assumed.

**Result 36.** *Assume that the conditions*

$$
\begin{aligned}
0 \;>\; & -2 \cdot \gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot g'(D_0) \;+\; h''(D_0) \\
& - \gamma^2 \cdot (R_{D,0} - R_{f,0})^2 \cdot R_{f,1}^2 \cdot g(D_0) \\
& + \frac{1}{\sigma_1} \cdot \frac{b_{d_z,1} \cdot R_{D,0} W_{B,0}}{\sigma_1 (D_0 + W_{B,0})^2} \cdot e^{-\gamma W_{H,1} R_{f,1}} \cdot \varphi(b_{d_z,1}) \\
& + \frac{(R_{D,0} W_{B,0})^2}{(D_0 + W_{B,0})^3} \;\cdot\; \Delta_1^\top \cdot \Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}\Big|_{x_1 = \frac{D_0 R_{D,0}}{D_0 + W_{B,0}}} \;\cdot\; \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1})
\end{aligned}
$$

*and*

$$
\frac{d\left[\Delta_1^\top \cdot \Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}\right]}{dW_{B,1}} \;<\; 0
$$

*hold, and the respective derivatives exist. In this case $D_0^s(\ell_0, R_{D,0})$ is unique.*

The derivation of Result 36 can be found in Appendix D.2.

The second condition states that the sensitivities with respect to the bank's initial equity $W_{B,1}$ become lower as equity increases. These declining sensitivities reflect the fact that the bank's equity $W_B$ loses its buffer function for the household at the margin as $W_B$ increases. This notion is also reflected by the numerical results discussed in Section 6.6 concerning equity shocks, *i.e.* changes in $W_B$. The numerical examples result in strictly concave curvatures of the total loan volume with respect to $W_B$ for the whole range of values chosen for $W_B$ within each of the parameter variations as given in Table 6.1 and Table 6.2, respectively (*cf.* Fig. 6.3, 6.9, and 6.10).

As the second condition is a derivative of first-order derivatives, it technically imposes regularity conditions on the curvature of the household's expected utility with respect to its deposit supply, deposits interest rates, portfolio-allocation rates, and bank equity in $t = 1$. Details are given in Appendix D.2, *cf.* especially Formulæ (D.2.10) to (D.2.15) and (D.2.16).

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Assumption (7.1.3).

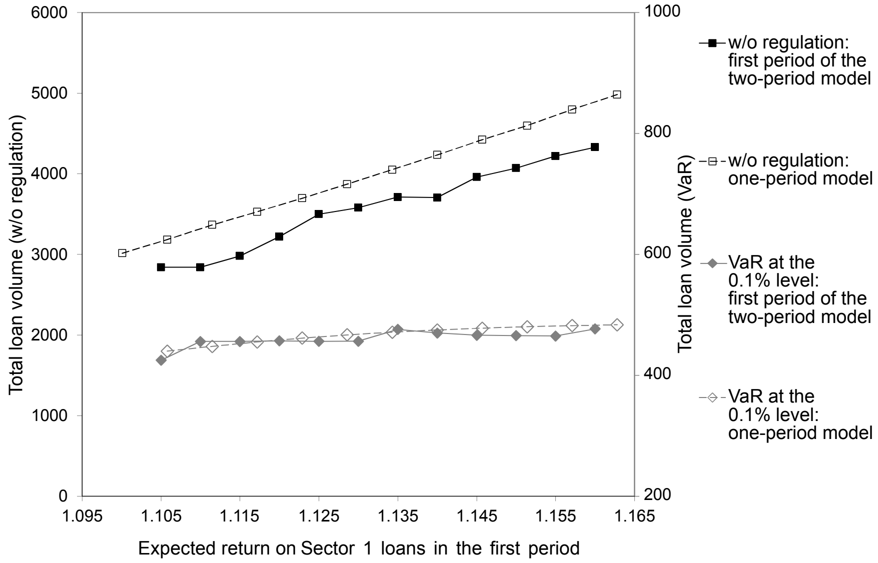


Figure 7.2: **Total loan volume $L_0$ $(L)$ as a function of $\mu_{1,1}$ $(\mu_1)$**

*This figure illustrates the dependence of the equilibrium total loan volumes on the expected gross return of first-sector loans, $\mu_{1,1}$ $(\mu_1)$. Solid lines represent total loan volumes granted in the first period of the two-period model, while dashed lines indicate total volumes in the one-period model. Total loan volumes are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level $\bar{p} = 0.1\%$.*

## 7.2 Numerical Analysis of Regulatory Impacts

The aim of this section is to determine whether expectation shocks have a lasting effect on lending and if they accelerate the cyclical nature of lending as soon as a VaR constraint comes into play. For this purpose, we examine expectation shocks in $t = 0$ while all other exogenous parameters are held fixed, in particular all those related to the second period. Thus, lending is potentially influenced in the following two ways: First, expectation shocks have a direct effect on the lending decision at $t = 0$. Second, the distribution of the bank's equity at $t = 1$, $\tilde{W}_{B,1}$, is affected. As a consequence, the outcomes of the lending decisions at $t = 1$ become shifted in a way that is consistent with the shifts in the distribution of $\tilde{W}_{B,1}$. So lending decisions at $t = 1$ are made as if a realized shock had occurred.

Consequently, the question of whether expectation shocks have a lasting effect is equivalent to asking the opposite, namely whether expectation shocks at $t = 0$ can be fully broken down into a first-period effect that is exclusively based on this original effect and into a second-period effect that is exclusively based on the realized effect. If this is true, results from the one-period analysis in Section 6.6 fully carry over to

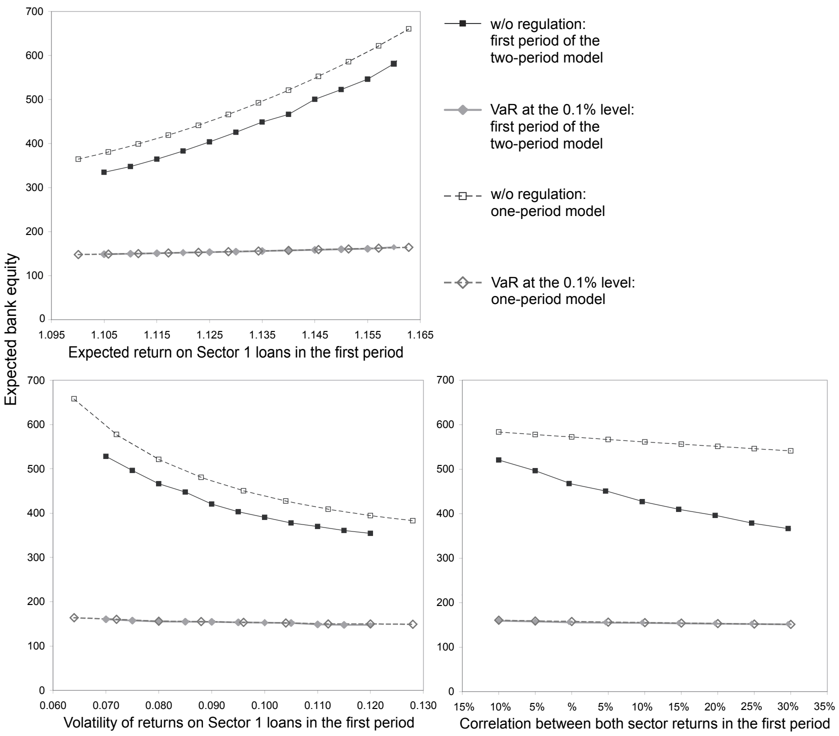


Figure 7.3: $\mathrm{E}_0(\tilde{W}_{B,1})$ **and** $\mathrm{E}(\tilde{W}_B)$ **as functions of expectation shocks**

*This figure illustrates the dependence of the bank's expected equity* $\mathrm{E}_0(\tilde{W}_{B,1})$ *on expectation shocks in* $t = 0$*; that is, when a single parameter of the distribution of gross returns on aggregate loans has varied in* $t = 0$ *(solid lines with filled-in boxes and diamonds, respectively). As a benchmark, the bank's expected equity* $\mathrm{E}(\tilde{W}_B)$ *is graphed for the one-period case (dashed lines with empty boxes and diamonds, respectively). The charts represent in counter-clockwise rotation: a shift in the expected gross return of first-sector loans,* $\mu_{1,1}$ $(\mu_1)$*; in the volatility of returns on first-sector loans,* $\sigma_{1,1}$ $(\sigma_1)$*; and in the inter-sectoral return correlation* $\rho_1$ $(\rho)$*. Expected equity is shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level* $\bar{p} = 0.1\%$*.*

---

the two-period framework. If not, the fact that agents include two periods in their optimization calculations will influence outcomes economically, thus giving rise to some kind of accelerator (*cf.* Bernanke/Gertler/Gilchrist, 1996, 1999). Furthermore, the bank's risk-taking behavior may change and in particular be adversely affected by regulation (Blum, 1999). In particular, Blum (1999) shows that regulation raises the value of second-period equity. Future equity can only be increased, however, if a higher risk level is chosen in the first period.

For the analysis, we use the parameter values from Table 6.1, rounded to two decimal places. Thus, the results concerning the single-period model shown in this chapter may deviate slightly from the results presented in the previous chapter.

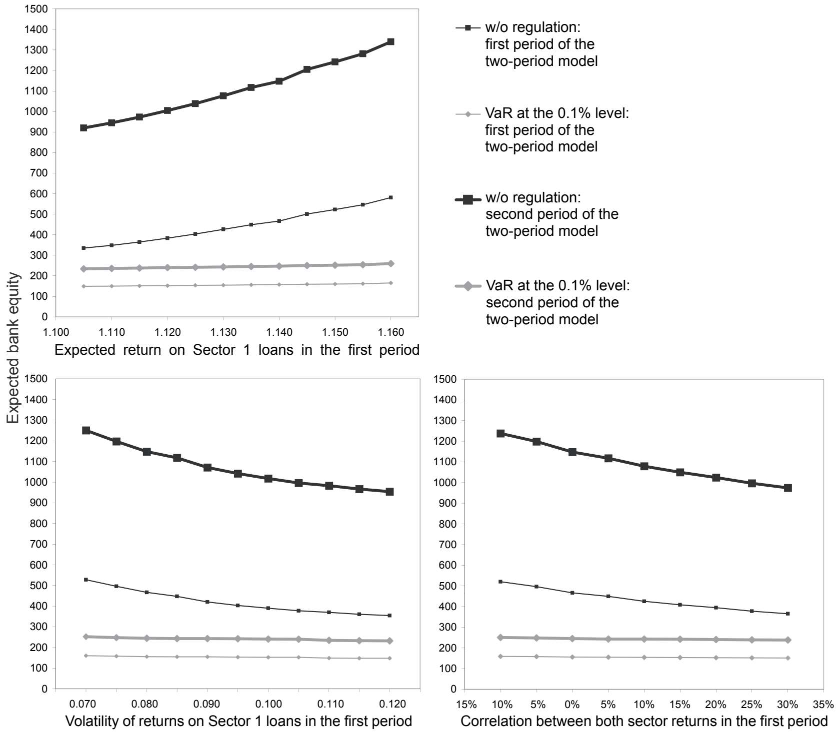


Figure 7.4: $\mathrm{E}_0(\tilde{W}_{B,1})$ **and** $\mathrm{E}_0(\tilde{W}_{B,2})$ **as functions of expectation shocks in** $t = 0$

*This figure illustrates the dependence of the bank's expected equity in* $t = 1$ *and* $t = 2$, $\mathrm{E}_0(\tilde{W}_{B,1})$ *and* $\mathrm{E}_0(\tilde{W}_{B,2})$*, respectively, on expectation shocks at* $t = 0$*. The charts represent in counter-clockwise rotation: a shift in the expected gross return of first-sector loans,* $\mu_{1,1}$*; in the volatility of returns on first-sector loans,* $\sigma_{1,1}$*; and in the inter-sectoral return correlation* $\rho_1$*. Expected equity is shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level* $\bar{p} = 0.1\%$*.*

### 7.2.1 Expected Return Shocks

We first vary the expected return on first-sector loans related to the first period, $\mu_{1,1}$. Figure 7.2 shows the total loan volumes granted by the unregulated and regulated bank at $t = 0$ and those granted in the one-period model. Qualitatively, an expected shock on $\mu_{1,1}$ has the same effects on the sensitivity of total loan volumes in the first period of the two-period model as in the one-period model (*cf.* Section 6.6.2). Without regulation, the gap between total loan volumes is striking. As, by construction, equilibrium magnitudes in $t = 0$ depend on expected equilibrium magnitudes related to the second period, this gap ought to be explained by the value that stems from the possibility of granting loans and generating positive expected profits in the second period.

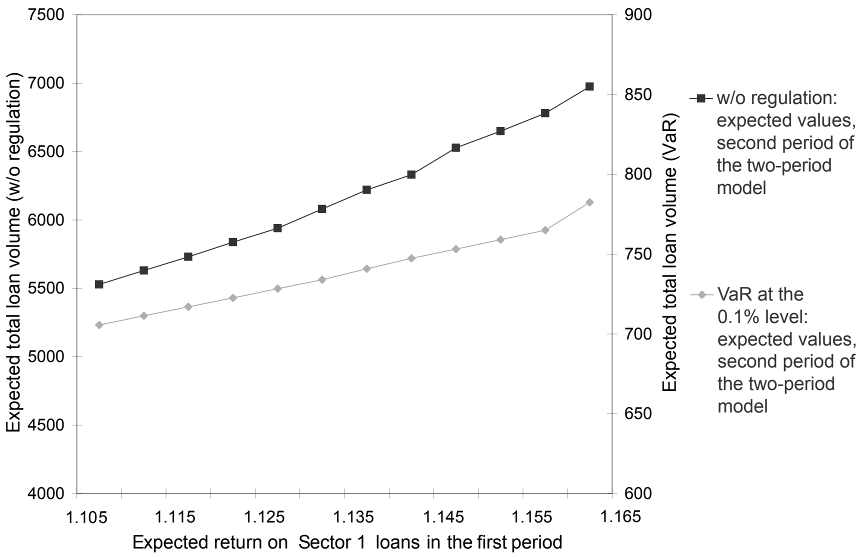


Figure 7.5: **Expected total loan volume $\mathrm{E}_0(\tilde{L}_1)$ as a function of $\mu_{1,1}$**

*This figure illustrates the dependence of the expected total loan volumes on the expected gross return of first-sector loans, $\mu_{1,1}$. Total loan volumes are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level $\bar{p} = 0.1\%$.*

---

In particular, lower total loan volumes result in lower expected equity in $t = 1$, as Figure 7.3 shows. But reduced business volume $L_0^*$ in the first period raises the success probability $\Phi(b_{d_z,1}^*)$ and hence, partially increases (assuming that second-period loan volumes are fixed) equity in $t = 2$ as expected at $t = 0$; that is, $\mathrm{E}_0[\tilde{W}_{B,2}^*]$. Therefore, the bank can raise its continuation value by further offering its intermediation service in the second period at the expense of expected profits after the first period. This continuation value can by quantified by the gap $\mathrm{E}_0[\tilde{W}_{B,2}^*] - \mathrm{E}_0[\tilde{W}_{B,1}^*]$, as shown by the upper chart in Figure 7.4.

Under regulation, we cannot confirm that this continuation effect has a significant adverse effect on risk-taking at $t = 0$. That is, the total loan volume $L_0^V$ granted in the first period of the two-period model is close to the total loan $L^V$ granted in the one-period model, as Figure 7.2 illustrates. Moreover, the gap between the respective expected levels of bank equity, $\mathrm{E}_0(\tilde{W}_{B,1}^V)$ and $\mathrm{E}(\tilde{W}_B^V)$, do not significantly differ.[2] As a result, the default probabilities $\Phi(b_{d_z,1}^V)$ and $\Phi(b_{s_z}^V)$ are close to each other, too.[3] Yet, the continuation value, given by $\mathrm{E}_0(\tilde{W}_{B,2}) - \mathrm{E}_0(\tilde{W}_{B,1})$, is clearly

[2]The relative deviation $\left(\mathrm{E}_0(\tilde{W}_{B,1}^V) - \mathrm{E}(\tilde{W}_B^V)\right)/\mathrm{E}_0(\tilde{W}_{B,1}^V)$ ranges between $-0.44\%$ and $0.86\%$ for $1.105 \le \mu_{1,1} \le 1.160$ and thus cannot be represented by the respective graphs, as shown in the upper chart of Figure 7.3.

[3]More precisely, the one-period default probability $\Phi(b_{d_z,1}^V)$, referring to the first period in the

positive, as illustrated by the respective graphs shown in the upper chart of Figure 7.4.

These findings stand in contrast to those obtained by Blum (1999), who shows that the regulated and thus volume-constrained bank chooses a higher risk level in the first period in order to raise its expected equity after the first period, as more equity allows the bank to increase its expected profits in the second period. A major difference between Blum's setting and ours is that in his model the risk-neutral bank is faced with an exogenous deposit cost function that is independent of risk, whereas in our setting the risk-averse household has a risk-disciplining effect in the two-period setting as well. Note that the VaR approach did not result in adverse effects concerning risk-taking in the one-period framework either.

Figure 7.5 shows the expected total loan volume $\mathrm{E}_0[L_1(\tilde{W}_{B,1})]$ granted in the second period. It strictly increases in $\mu_{1,1}$ under both the unregulated and the regulated regimes, although the increase is smaller under regulation. Thus, neither a pro- nor a counter-cyclical effect on lending exists on average in the second period.

It is still unclear to what extent second-period loan volumes are directly influenced by shocks in $\mu_{1,1}$ and to what extent they are influenced by the changes in the bank's equity $\tilde{W}_{B,1}$.

We try to answer this question by comparing the reaction of the total loan volume $L \equiv L(W_B)$ to shocks in $W_B$ in the one-period framework with the reaction of expected second-period total loan volumes $\mathrm{E}_0[L_1(\tilde{W}_{B,1})]$ to the bank's expected equity $\mathrm{E}_0[\tilde{W}_{B,1}]$, where both magnitudes result from the same expectation shock in $\mu_{1,1}$.

Concerning shocks in $\mu_{1,1}$, this comparison is drawn by means of the upper chart in Figure 7.6.

The curves representing $L^*$ and $\mathrm{E}_0[L_1^*(\tilde{W}_{B,1}^*)]$ are essentially parallel to each other if $L^*$ and $\mathrm{E}_0[L_1^*(\tilde{W}_{B,1}^*)]$ are plotted against $\mathrm{E}_0[\tilde{W}_{B,1}^*]$ on the abscissa. Thus, the shift in equity levels $W_{B,1}^*$ — implied by $\mu_{1,1}$ shocks in the two-period model — seems to mainly account for the sensitivity of $\mathrm{E}_0[L_1^*(\tilde{W}_{B,1}^*)]$ with respect to $\mathrm{E}_0[\tilde{W}_{B,1}^*]$. The parallel distance of $\mathrm{E}_0[L_1^*(\tilde{W}_{B,1}^*)]$ to $L^*(\mathrm{E}_0[\tilde{W}_{B,1}^*])$ can be explained by Jensen's inequality. In particular, Jensen's inequality implies $\mathrm{E}_0[L_1^*(\tilde{W}_{B,1}^*)] - L^*(\mathrm{E}_0[\tilde{W}_{B,1}^*]) < 0$ since $L^*(\cdot)$ and $L_1^*(\cdot)$ are strictly concave functions. As $\mathrm{E}_0[L_1^*(\tilde{W}_{B,1}^*)]$ and $L^*(\mathrm{E}_0[\tilde{W}_{B,1}^*])$ are shown in Figure 7.6 as dependent on $\mathrm{E}_0[\tilde{W}_{B,1}^*]$, whereas different

two-period model, strictly decreases from 0.00020% to 0.000023% given expected returns $\mu_{1,1} \in [1.105, 1.160]$. Choosing the same range for expected returns within the one-period model yields default probabilities from 0.00021% to 0.000023%.

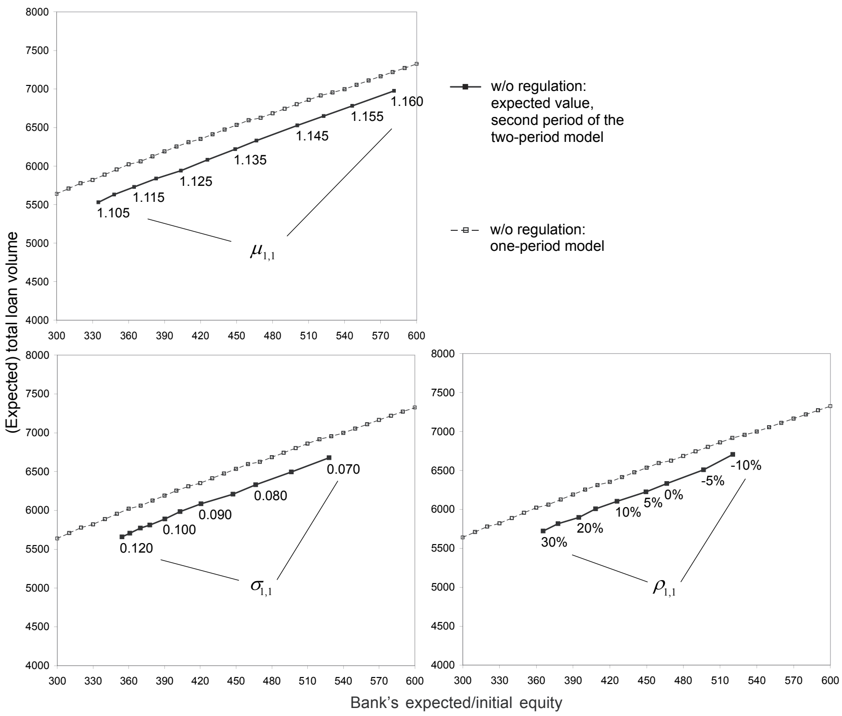


Figure 7.6: **(Expected) total loan volume as a function of the bank's (expected) equity w/o regulation**

*This figure illustrates the dependence of the expected total loan volume $\mathrm{E}_0(\tilde{L}_1)$ on the bank's expected equity $\mathrm{E}_0[\tilde{W}_{B,1}]$ when an expectation shock has occurred in $t = 0$ (solid lines with filled-in diamonds). The charts represent in counter-clockwise rotation: a shift in the expected gross return of first-sector loans, $\mu_{1,1}$; in the volatility of returns on first-sector loans, $\sigma_{1,1}$; and in the inter-sectoral return correlation $\rho_1$. As a benchmark, the total loan volume $L$, as a function of the bank's initial equity $W_B$, is graphed for the one-period case (dashed lines with empty diamonds).*

levels of $\mathrm{E}_0[\tilde{W}^*_{B,1}]$ are identified with different levels of $\mu_{1,1}$, the parallelism of $\mathrm{E}_0[L^*_1(\tilde{W}^*_{B,1})]$ and $L^*(\mathrm{E}_0[\tilde{W}^*_{B,1}])$ with respect to the $\mathrm{E}_0[\tilde{W}^*_{B,1}]$ axis emerges. To make the comparison meaningful, $\mathrm{E}_0[\tilde{W}^*_{B,1}]$ is identified with initial equity $W_B$ concerning the total loan volume $L^*(\cdot)$ in the one-period model. Note, however, that the functions $L^*(\cdot)$ and $L^*_1(\cdot)$ are not identical in general.

While the graphs in Figure 7.6 refer to the laisser-faire equilibrium, Figure 7.7 shows the (expected) total loan volumes under the VaR-based regulation at a confidence level of 0.1%.

Concerning $\mu_{1,1}$ shocks, the upper chart of Figure 7.7 illustrates that, under regulation, the sensitivity of the expected total loan volume granted at $t = 1$ with respect to $\mathrm{E}_0[\tilde{W}^V_{B,1}]$ is almost the same as in the one-period model with respect to

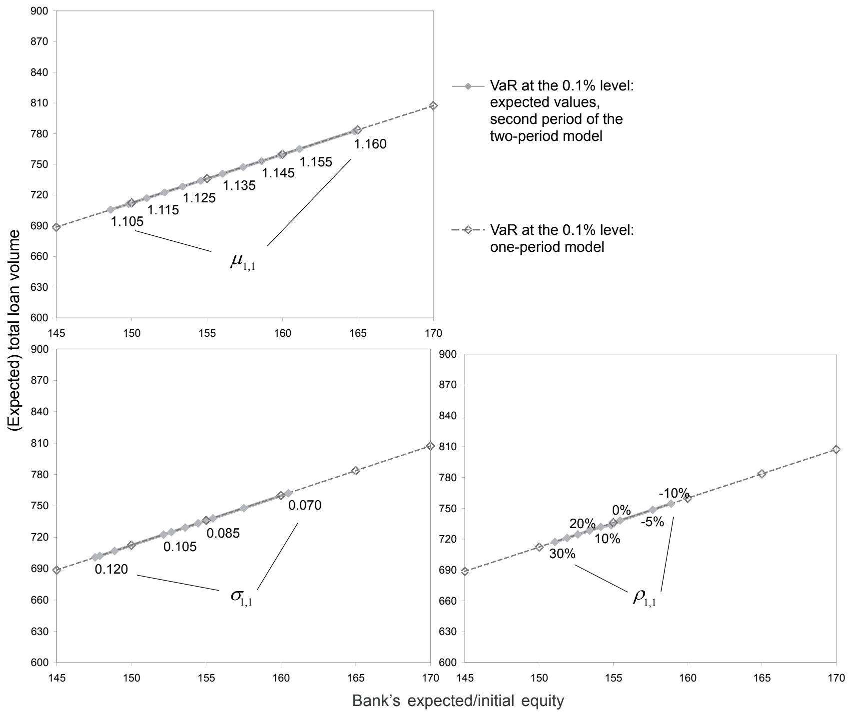


Figure 7.7: **(Expected) total loan volume as a function of the bank's (expected) equity under VaR**

*This figure illustrates the dependence of the expected total loan volume* $\mathrm{E}_0(\tilde{L}_1)$ *on the bank's expected equity* $\mathrm{E}_0[\tilde{W}_{B,1}]$ *when an expectation shock has occurred in* $t = 0$ *(solid lines with filled-in diamonds). The charts represent in counter-clockwise rotation: a shift in the expected gross return of first-sector loans,* $\mu_{1,1}$*; in the volatility of returns on first-sector loans,* $\sigma_{1,1}$*; and in the inter-sectoral return correlation* $\rho_1$*. As a benchmark, the total loan volume* $L$*, as a function of the bank's initial equity* $W_B$*, is graphed for the one-period case (dashed lines with empty diamonds).*

equally high levels of initial equity $W_B$. Thus, the sensitivity of $\mathrm{E}_0[\tilde{L}_1^V]$ can be mainly traced back to the shifts in $\tilde{W}_{B,1}^V$ and in $\mathrm{E}_0[\tilde{W}_{B,1}^V]$, respectively, that are in turn due to the expectation shock concerning $\mu_{1,1}$ in $t = 0$. The gap $\mathrm{E}_0[L_1^V(\tilde{W}_{B,1}^V)] - L^V(W_B)$ almost disappears under regulation, as total volumes are much lower than what they are without regulation.

Concerning $\mathrm{E}_0[\tilde{W}_{B,1}]$, the total expected loan volume granted at $t = 1$ is not pro-cyclically affected by regulation either: We obtain $\frac{\Delta \mathrm{E}_0[\tilde{L}_1^*]}{\Delta \mathrm{E}_0[\tilde{W}_{B,1}^*]} \approx 5.87$ without regulation versus $\frac{\Delta \mathrm{E}_0[\tilde{L}_1^V]}{\Delta \mathrm{E}_0[\tilde{W}_{B,1}^V]} \approx 4.75$ under regulation. We note that there are again regions around lower realized values of $\tilde{W}_{B,1}$ where the unregulated total loan volume reacts more strongly to shocks than the total loan volume under VaR regulation, due to the total loan volume's stronger concavity without regulation (*cf.* Fig. 6.3,

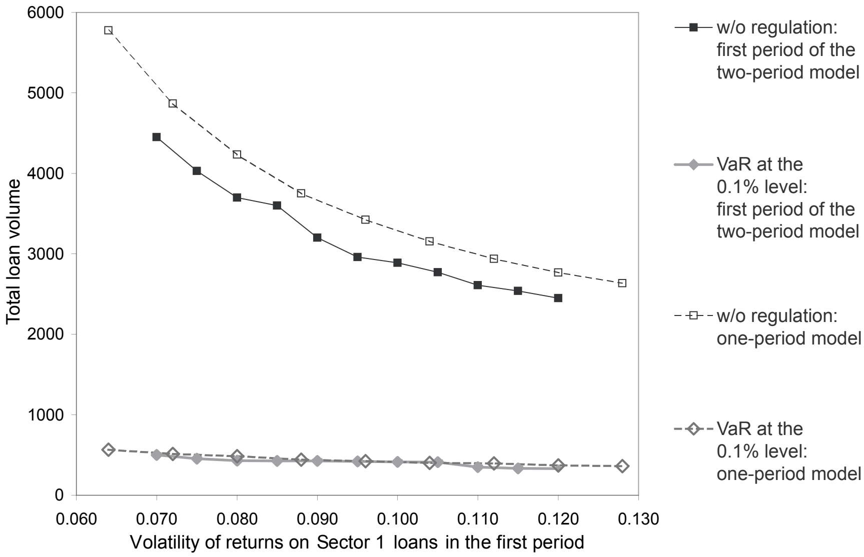


Figure 7.8: **Total loan volume $L_0$ $(L)$ as a function of $\sigma_{1,1}$ $(\sigma_1)$**

*This figure illustrates the dependence of the equilibrium total loan volumes on the volatility of returns on first-sector loans, $\sigma_{1,1}$ $(\sigma_1)$. Solid lines represent total loan volumes granted in the first period of the two-period model, while dashed lines indicate total volumes in the one-period model. Total loan volumes are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level $\bar{p} = 0.1\%$.*

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6.9, and 6.10 in the one-period framework).

### 7.2.2 Return Volatility Shocks

Figure 7.8 shows the comparative static results for volatility shocks in the first period. In this example, the volatility of returns on loans granted to Sector 1, $\sigma_{1,1}$, is varied at $t = 0$. As in the one-period model, total loan volumes decrease strictly in $\sigma_{1,1}$ with and without regulation. In particular, the regulatory constraint becomes tighter the higher $\sigma_{1,1}$ is; this would hold true even if the bank chose the variance-minimum loan-allocation rate. Regulation does not have a pro-cyclical impact as demonstrated in the one-period setting (*cf.* Fig. 6.14 and 6.15).

The total loan volume referring to the first period of the two-period problem is lower than the equivalent volume in the one-period set-up, partly resulting in default probabilities $\Phi(b_{d_z,1})$ that are lower than those obtained in the one-period model, $\Phi(b_{s_z})$. These effects observed under $\sigma_{1,1}$ shocks are thus similar to those observed with $\mu_{1,1}$ shocks. The continuation values in terms of $\mathrm{E}_0[\tilde{W}_{B,2}] - \mathrm{E}_0[\tilde{W}_{B,1}]$ can be identified by the graphs, as shown in the bottom-left chart of Figure 7.4. As with

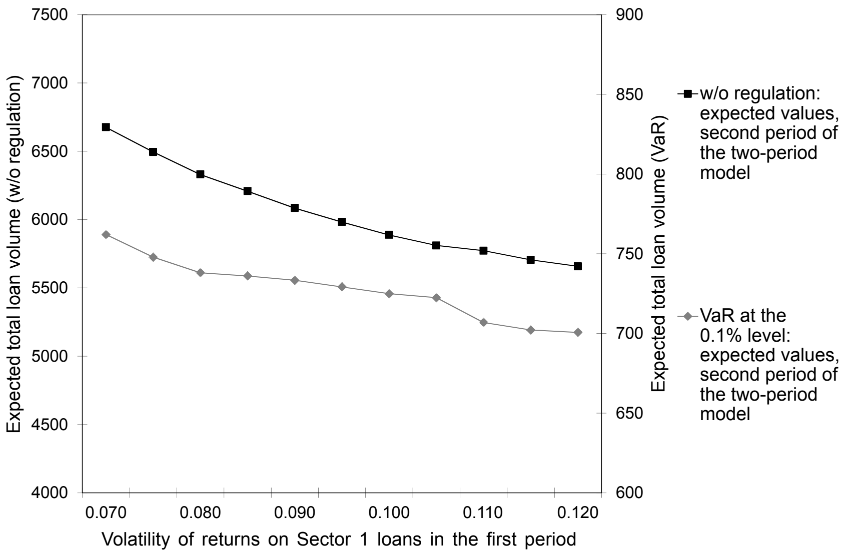


Figure 7.9: **Expected total loan volume** $\mathrm{E}_0(\tilde{L}_1)$ **as a function of** $\sigma_{1,1}$

*This figure illustrates the dependence of the expected total loan volumes on the volatility of returns on first-sector loans,* $\sigma_{1,1}$. *Solid lines represent total loan volumes granted in the first period of the two-period model, while dashed lines indicate total volumes in the one-period model. Total loan volumes are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level* $\bar{p} = 0.1\%$.

$\mu_{1,1}$ shocks, the effects on expected equity after the first period, $\mathrm{E}_0[\tilde{W}_{B,1}]$, compared to the bank's expected equity within the one-period setting, $\mathrm{E}[\tilde{W}_B]$, are negligible, as the bottom-left chart in Figure 7.3 illustrates.

As shown in Figure 7.9, expected total loan volumes strictly decrease in $\sigma_{1,1}$; the sensitivity is greater for the unregulated volumes than for the regulated volumes. Hence, there is neither a pro- nor a counter-cyclical effect. Considering the bottom-left charts in Figures 7.6 and 7.7 reveals that the shift in bank equity $W_{B,1}$ (implied by changes in $\sigma_{1,1}$) seems to be mainly responsible for the changes in expected loan volumes in the second period. Hence, there can be no accelerating mechanism with respect to total loan volume sensitivities, as is also the case with $\mu_{1,1}$ shocks. The positive sign of the gap $L^*(\mathrm{E}_0[\tilde{W}^*_{B,1}]) - \mathrm{E}_0[L^*_1(\tilde{W}^*_{B,1})]$ can be explained again by Jensen's inequality, since $L^*(\cdot)$ and $L^*_1(\cdot)$ are strictly concave, respectively.

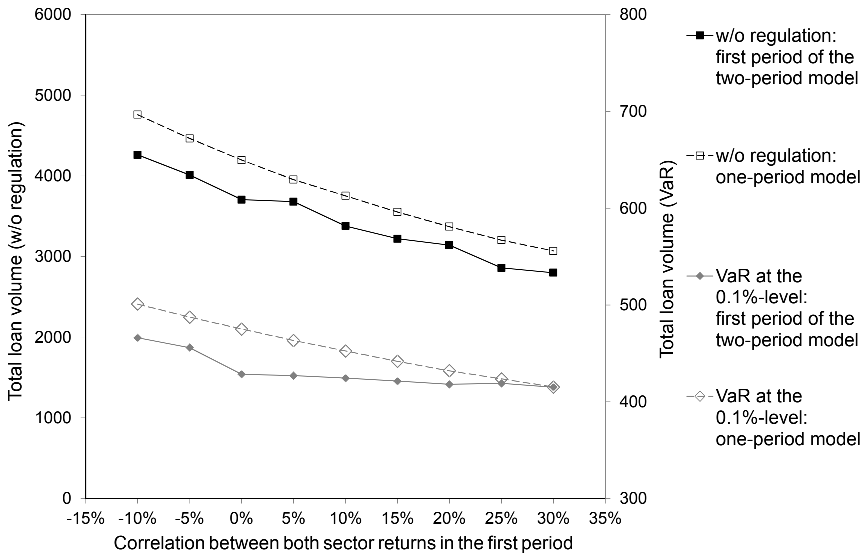


Figure 7.10: **Total loan volume $L_0$ $(L)$ as a function of $\rho_1$ $(\rho)$**

*This figure illustrates the dependence of the equilibrium total loan volumes on the inter-sectoral return correlation $\rho_1$ $(\rho)$. Total loan volumes are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level $\bar{p} = 0.1\%$.*

### 7.2.3 Return Correlation Shocks

Because correlation shocks concerning $\rho_1$ affect the portfolio return volatility $\sigma_1$ in the same manner as volatility shocks with respect to a single sector, we expect similar effects on total loan volumes.

Indeed, the total loan volumes with and without regulation strictly decrease in $\rho_1$ where contractions in unregulated volumes are stronger. Thus, we observe no procyclical impact caused by regulation, as in the one-period model and as it is the case with volatility shocks in the one- and in the two-period models.

Again, the decrease in the expected loan volumes $\mathrm{E}[L_1(\tilde{W}_{B,1})]$ is comparable to the effects of equity shocks in the one-period model, as illustrated in each bottom-right chart of Figures 7.6 and 7.7. We note that the same effects exist concerning risk-taking in the first period and concerning continuation values, as in both the preceding comparative-static analyses.

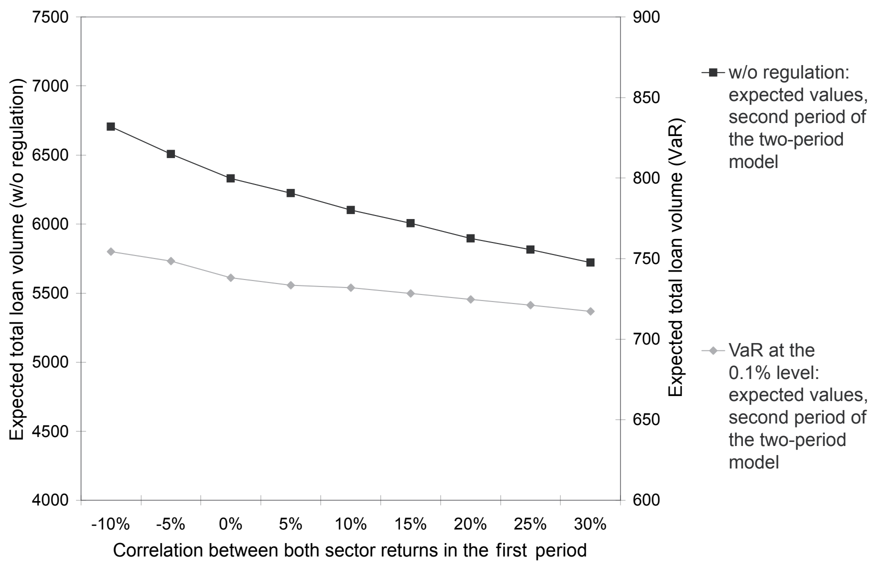


Figure 7.11: **Expected total loan volume** $\mathrm{E}_0(\tilde{L}_1)$ **as a function of** $\rho_1$

*This figure illustrates the dependence of the expected total loan volumes on the inter-sectoral return correlation* $\rho_1$*. Solid lines represent total loan volumes granted in the first period, while dashed lines indicate total volumes in the one-period model. Total loan volumes are shown for two different regimes: the laissez-faire equilibrium and the VaR approach with confidence level* $\bar{p} = 0.1\%$*.*

## 7.3 Summary

Examining two subsequent periods allows us to determine whether expectation shocks have lasting effects on loan volumes and their sensitivities to these shocks. By construction, expectation shocks occurring at $t = 0$ transmit to the second period via the bank's equity. Equity is thus an amplifier for expectation shocks, albeit only if *ex-post* realizations can be aligned with the *ex-ante* shifted expectations. In this respect, we may have identified a financial accelerator in the spirit of Bernanke/Gertler/Gilchrist (1996, 1999), though other propagation mechanisms, such as information asymmetries, are missing.

However, there is by no means an acceleration of sensitivities as such. Far from it: Total loan volumes react to expectation shocks as in the one-period setting when the very period is considered at whose beginning the shock occurred. Likewise, the expected or average reaction of total loan volumes in the second period to the shifts thus implied in the bank's equity are in the same order of magnitude as in the one-period model.

The most striking difference between the one-period and the two-period models is that total loan volumes in the first period may be well below their single-period counterparts. The most suitable explanation is that the bank recognizes its continuation value arising from further intermediation in another period when making its decisions at $t = 0$. The importance of continuation values to the bank's decision making and risk-taking has been analyzed by Blum (1999). Continuation values arise in our model due to the bank's monopoly power, which could be justified by the bank charters granted by regulatory authorities (Keeley, 1990).

# Part IV

# Final Remarks

# Final Remarks

The concern about pro-cyclical movements of aggregate lending volumes through capital adequacy rules has been debated since the nineties and has attracted increased attention during the consultation process for the amendments to capital rules which have become commonly known as the Basel II Accord. With this thesis, we add to the literature that is concerned with pro-cyclicality in the following manner: We analyze the sensitivity of the lending volume to changes of fundamental economic variables, called shocks, under a VaR approach, under approaches with fixed risk weights, and under a laissez-faire economy. We endogenize the bank's risk-allocation and size decision. In particular, the deposit volume and the deposit interest rate are based on decisions made by the household and the bank. The deposit interest rate and the deposit volume reflect the bank's risk-taking and the regulatory constraints. So far, the literature about pro-cylicality of capital requirements has not been concerned with a bank that simultaneously takes its leverage, asset risks, probability of bankruptcy, size, and costs of debt finance into account. Likewise, there is no work in this respect that analyzes the interaction between the bank and a risk-averse depositor while endogenizing for the bank's bankruptcy risk.

Our results concerning pro-cyclicality lead to the conclusion that the effect of a given shock should be distinguished according to the type of the shock. Equity shocks, *i.e.* changes in bank equity as result of gains or losses, lead to pro-cyclical effects on lending. Expectation shocks, *i.e.* changes of distribution parameters or of borrowers' productivity, are dampened by regulation. Regulation affects the total lending volume asymmetrically given each type of shock: Concerning equity shocks, regulation effectively constrains bank lending after realized losses, thus exacerbating downturns. This observation is in line with common fears. Concerning expectation shocks, regulation effectively constrains bank lending in anticipation of more favorable expectations on outcomes, thus hampering economic recovery. Except for pro- and non-pro-cyclical effects, we can even observe counter-cyclical effects through regulation on the level of single loan volumes. These insights can be found regardless of the degree of risk sensitivity of the respective capital adequacy rule. A bank may grant higher loan volumes to less risky firms under risk-sensitive capital requirements than it would do under fixed requirements or under a laissez-faire regime.[4]

The two-period model considered does not yield any further insights relative to the one-period model. In particular, there are no signs of a financial accelerator that could be based on a simple propagation mechanism via the bank's equity. If the sensitivities of the expected total loan volume in $t = 1$ toward expectation shocks at the beginning of the first period, $t = 0$, are considered as sensitivities with respect to expected equity in $t = 1$, these sensitivities are always of the same size as they are in the one-period model.

Furthermore, this work emphasizes that risk-taking may occur under capital regulation without any interplay with other regulatory measures, such as deposit insurance schemes. Rather, the degree of risk-taking can be directly aligned with the sort of capital rules considered. A flat capital requirement always induces the bank to take more risk than under any other regime considered (*i.e.* laissez-faire, different risk weights, VaR approach). The reduction in size is always compensated by taking more risk since this is the only possibility to raise the bank's expected final wealth. However, as capital requirements become risk-sensitive, our models yield mixed results. If loan redemptions are normally distributed, the VaR approach generally has no impact on risk-taking compared to the laissez-faire equilibrium. Thus, the "correct" alignment of capital rules with actual credit risk is of great importance, as analyzed by Kim/Santomero (1988) and Rochet (1992). But finding the "correct" risk weights or risk-determining models will be an unsolvable task in reality. Therefore, adverse side effects concerning risk-taking will always remain a problem to various extents. This problem is commonly known as model risk.

The mixed results of this thesis raise the question of the relevancy of the concerns about pro-cyclical effects through risk-sensitive capital requirements. The research done on the role of capital buffers suggest that these buffers may alleviate this problem to a considerable extent (Peura/Jokivuolle, 2004; Jacques, 2005; Heid, 2007, and Repullo/Suarez, 2008). Others, however, who consider pro-cyclicality as a major problem, propose rules of how pro-cyclicalty could be mitigated. In this vein, Kashyap/Stein (2004) and Gordy/Howells (2006) suggest counter-cyclical indexing of capital rules. Pennacchi (2005) advocates a regulatory regime comprising actuary fair deposit insurance premia and a fixed capital ratio instead of risk-sensitive capital

[4] *Cf.* Jokivuolle/Vesala (2007) for a slightly different result.

requirements.

This controversy concerning the importance of pro-cyclical impacts may for the most part exist because of the variety of the frameworks and variables analyzed. First of all, many authors, including Kashyap/Stein (2004) and Gordy/Howells (2006), exclusively consider the cyclicality of required capital, thereby assuming that capital is always completely taken up for lending, such that the existence of buffers is neglected. This assumption is justified as long as buffers can be seen as a rather constant endorsement on required capital. This view is also taken in our analysis. As a consequence, Basel II leads to increased cyclicality of capital compared to Basel I, in particular with expectation shocks, in their analyses. Consequently, the same cyclicality is also believed to be true concerning total loan volumes. But this result does not only differ from the conclusions drawn by the capital-buffer based views, but also from our reasoning as follows: In their view, swings in capital requirements are always identified with proportional swings in lending as the bank's reaction to changes of risk is not endogenized (apart from *ex-ante* fixed re-investment rules in simulation studies). In our analysis, though, the bank reacts differently given the various regulatory regimes and must account for the re-financing costs associated with its risk-taking.

Another point might be that the debate on pro-cyclicality often seems to be related to absolute lending levels, perhaps influenced by the experiences with the introduction of the Basel I Accord and the credit crunch.

But as the Basel II Accord was introduced in 2008 in the EU and as it has thus taken effect under the extra-ordinary circumstances of the ongoing financial and economic crisis, it will take years to fully assess and understand its potential impact on the business cycle. Furthermore, amendments, known as Basel III, will come into effect from 2013 on.

# Appendix A

# Proofs to Chapter 3

## A.1 The Household

### A.1.1 Proof of Result 2

**The Existence and Uniqueness of $\underline{R}_D(\ell; j)$**

To obtain $\underline{R}_D(\ell; j)$, the equation

$$D_j^s(\ell, R_D) \stackrel{!}{=} 0$$

must be solved for $R_D$ given $\ell$ and $j$. Multiplying this equality by $\gamma \cdot \sigma_j^2$ and simplifying reveals that this equation is linear in $R_D$. Thus, $R_D(\ell; j)$ is unique and is equal to (3.2.55) after having solved this equation under consideration of (3.2.53).

**The Existence and Uniqueness of $\overline{R}_D^u(\ell; j)$**

By the implicit function theorem the derivative of the unconstrained deposit-supply function with respect to $R_D$ is given by

$$\frac{dD_j^s}{dR_D} = -\frac{\frac{\partial^2 U_j(D)}{\partial D \partial R_D}}{\frac{\partial^2 U_j(D)}{\partial D^2}}.$$

Because of $\frac{\partial^2 U_j()}{\partial D^2} = -\gamma\sigma_j^2 < 0$, the sign of this derivative is exclusively determined by the sign of the second mixed derivative

$$\frac{\partial^2 U_j(D)}{\partial D \partial R_D} = q_j - \gamma \cdot \left[2q_j(R_D - \mu_j) \cdot D - q_j \cdot R_j^\mu \cdot W_B\right] \qquad \text{(A.1.1)}$$

which is positive if and only if

$$R_D \;<\; \frac{\frac{1}{\gamma} + 2\mu_j D + R_j^\mu \cdot W_B}{2D}$$

holds. Let

$$f_j(R_D) \;=\; \frac{\frac{1}{\gamma} + 2\mu_j D + R_j^\mu \cdot W_B}{2D}.$$

If $R_D$ approaches the zero $\underline{R}_D(\ell;j)$ from above, $f_j(R_D)$ becomes infinite,

$$\lim_{R_D \downarrow \underline{R}_D(\ell;j)} f_j(R_D) \;=\; +\infty \;. \tag{A.1.2}$$

Its derivative is given by

$$f_j'(R_D) \;=\; \frac{1}{2\gamma D^2} \cdot \left[ 2\gamma q_j D^2 - (1 + R_j^\mu \cdot W_B) \cdot \frac{\partial D_j^u}{\partial R_D} \right] \;. \tag{A.1.3}$$

Thus, the following relation between the signs of $\frac{\partial D_j^u}{\partial R_D}$ and $f_j'(R_D)$ can be established:

$$\begin{array}{c} \frac{\partial D_j^u}{\partial R_D} < 0 \;\Rightarrow\; f_j'(R_D) > 0 \\ f_j'(R_D) < 0 \;\Rightarrow\; \frac{\partial D_j^u}{\partial R_D} > 0 \end{array} \;. \tag{A.1.4}$$

At $R_D = \underline{R}_D(\ell;j)$, the deposit-supply function is strictly increasing due to

$$\left.\frac{\partial^2 U_j(D)}{\partial D \partial R_D}\right|_{R_D = \underline{R}_D(\ell;j)} \;=\; q_j + q_j \cdot R_j^\mu \cdot \gamma W_B \;>\; 0 \;. \tag{A.1.5}$$

As long as $f_j(R_D) > R_D$ holds for $R_D > \underline{R}_D(\ell;j)$, the deposit-supply function keeps increasing in $R_D$. Furthermore, $f_j(R_D)$ is decreasing in $R_D$ in the neighborhood of $\underline{R}_D(\ell;j)$ due to

$$\lim_{R_D \downarrow \underline{R}_D(\ell;j)} f_j'(R_D) \;=\; -\infty \;.$$

With increasing $R_D$, $f_j(R_D)$ could either run such that it crosses the identity $R_D \equiv R_D$ at least once, or run such that it never crosses the identity. Since the latter case is associated with $f_j(R_D) > R_D$ for all $R_D$, the deposit-supply function keeps increasing for all $R_D$. This contradicts the fact that

$$\lim_{R_D \to \infty} D^s(\ell, R_D; j) \;=\; 0^+ \tag{A.1.6}$$

holds, implying that a critical rate $R_D$ exists from which the deposit-supply function decreases in $R_D$. Hence, the function $f_j(R_D)$ crosses at least once the identity line. As soon as $R_D > f_j(R_D)$ holds, the deposit-supply function is decreasing.

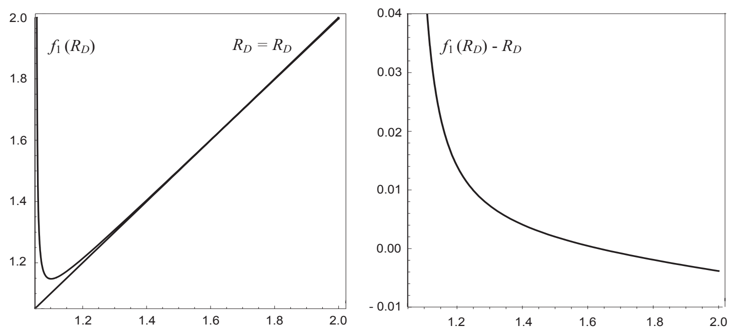


Figure A.1: **The function $f_1(R_D)$ and its relation to the identity $R_D \equiv R_D$**

*This figure illustrates the course of the function $f_1(R_D)$. The left-hand chart shows the graph of $f_1(R_D)$ with the parameter values given in Table 4.1 and given $\ell = 0.57114$. The right-hand chart shows the graph of $f_1(R_D) - R_D$ with the same parameter values to highlight the unique intersection point. The chosen loan-allocation rate equals its respective laissez-faire equilibrium value.*

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By (A.1.4), the function $f_j(R_D)$ starts increasing after this intersection. Hence, if $f_j(R_D)$ now crosses the identity another time, it crosses the identity from below. Therefore, $f_j'(R_D) > 1$ must hold in a potential second intersection point. Because of $f_j(R_D) = R_D$ in any intersection point, the deposit supply function has zero slope by the definition of $f_j(R_D)$ and $f_j'(R_D)$ becomes $f_j'(R_D) = q_j < 1$.

Thus, $f_j(R_D)$ keeps bounded by $R_D$ from the first intersection point on and the deposit-supply function decreases in $R_D$. So, the intersection is unique and, moreover, the deposit-supply function is maximized in that point with respect to $R_D$. Thus, the intersection point is equivalent to $\overline{R}_D^u(\ell)$ defined in (3.2.52).

Under Case 4, the behavior of the deposit-supply function concerning $R_D$ can also be shown by taking the derivative of the explicit function $D^u(\ell, R_D; 4)$ with respect to $R_D$.

Figure A.1 displays the graph of $f_1(R_D)$ given the parameter values from Table 4.1 to illustrate the idea of the proof. To highlight the unique intersection point, $f_1(R_D) - R_D$ is shown as well since this difference is only roughly one hundredth compared to the lengths of the intervals considered as ranges.

### A.1.2 Derivations concerning Result 3

Existence and uniqueness of $\underline{R}_D(\ell; j)$ is given by Result 2.

To obtain $\underline{R}_D(\ell; j)$, the equation

$$D^s(\ell, R_D; j) \stackrel{!}{=} 0$$

must be solved for $R_D$ given $\ell$ and $j$. Multiplying this equality by $\gamma \cdot \sigma_j^2$ and simplifying leads to (3.2.55) after having solved this equation under consideration of (3.2.53).

The derivative of $\underline{R}_D(\ell; j)$ is given by

$$\frac{\partial \underline{R}_D(\ell; j)}{\partial W_B} = \frac{\gamma \cdot \left[R_j^\sigma - R_f \cdot R_j^\mu\right]}{q_j \cdot (1 + \gamma W_B R_j^\mu)^2} .$$

Consider Case 2. The derivative becomes

$$\frac{\partial \underline{R}_D(2)}{\partial W_B} = \frac{\gamma \cdot (p_1 - q)\alpha_1 \ell \left[\alpha_1 \ell - R_f\right]}{p_2 \cdot (1 + \gamma W_B R_2^\mu)^2} .$$

By (3.2.20), we obtain

$$\frac{\partial \underline{R}_D(2)}{\partial W_B} \leq \frac{\gamma \cdot (p_1 - q)\alpha_1 \ell \left[(p_1 - q) \cdot \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2} - R_f\right]}{p_2 \cdot (1 + \gamma W_B R_2^\mu)^2} .$$

By (3.2.7), we have $\frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2} \leq 1$ and due to (3.2.6), we finally obtain

$$\frac{\partial \underline{R}_D(2)}{\partial W_B} \leq 0 .$$

Under Case 3 we can derive zero as an upper bound in an analogical way.

If $W_B = 0$, $\underline{R}_D(\ell; j)$ becomes

$$\underline{R}_D(\ell; j) = \frac{R_f - R_j^\mu}{q_j} .$$

In Case 2, we obtain:

$$\frac{R_f - R_2^\mu}{q_2} = \frac{R_f - (p_1 - q)\alpha_1 \ell}{p_2}.$$

This term strictly exceeds $R_f$ iff

$$(1 - p_2) R_f > (p_1 - q)\alpha_1 \ell$$

is fulfilled. By (3.2.6), $(1-p_2)R_f \geq 1-p_2$ holds, and by (3.2.7) as well as (3.2.20) we obtain $(p_1-q)\alpha_1\ell \leq (p_1-q)\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2} \leq p_1-q$ as higher bound on the right-hand side. Taking together this bound and the lower of the left-hand side, we obtain

$$(1-p_2)R_f \geq 1-p_2 \;>\; p_1-q \geq (p_1-q)\alpha_1\ell$$

and $1-p_2 > p_1-q$ is true because of (3.2.11). For Case 3 we can argue analogously.

Consider Case 1, we obtain:

$$\frac{R_f - R_1^\mu}{q_1} \;=\; \frac{R_f - (p_1-q)\alpha_1\ell - (p_2-q)\alpha_2(1-\ell)}{q}\,,$$

so that

$$(1-q)R_f \;>\; (p_1-q)\alpha_1\ell \;+\; (p_2-q)\alpha_2(1-\ell)$$

must be fulfilled. Taking the (sufficient) condition stated in the result, the right-hand side is bounded from above by

$$(p_1-q)\alpha_1\ell + (p_2-q)\alpha_2(1-\ell) \;<\; (1-q)R_f\ell + (1-q)R_f(1-\ell) \;=\; (1-q)R_f\,.$$

As this bound equals the left-hand side, we have shown the result.

### A.1.3 Proof of Result 4

*Proof.* Throughout this proof, we will consider the unconstrained deposit-supply function.

Let us start with Case 2. Deposit supply can be written also as

$$D^u(\ell, R_D; 2) = \frac{\mu_2 - R_f}{\gamma\sigma_2^2} + \frac{(p_1-q)\alpha_1\ell(\mu_2-\alpha_1\ell)}{\sigma_2^2}\cdot W_B\,.$$

Hence, the sign of

$$(p_1-q)\cdot\alpha_1\ell\cdot(\mu_2-\alpha_1\ell)$$

must be determined. By (3.2.20) we can establish as a lower bound

$$(p_1-q)\cdot\alpha_1\ell\cdot(\mu_2-\alpha_1\ell) \;\geq\; (p_1-q)\cdot\alpha_1\ell\cdot\left(\mu_2 - \frac{\alpha_1\cdot\alpha_1}{\alpha_1+\alpha_2}\right)\,.$$

By $\alpha_1, \alpha_2 \leq 2$ according to (3.2.7), we obtain $\frac{\alpha_1\cdot\alpha_1}{\alpha_1+\alpha_2} \leq 1$. By the assumption $\mu_j \geq R_f$ and by $R_f \geq 1$, the result obtains.

Given Case 3 the proof is analogous: We must determine the sign of

$$(p_2 - q) \cdot \alpha_2(1-\ell) \cdot [\mu_3 - \alpha_2(1-\ell)] \ .$$

By (3.2.21), $\mu_3 - \alpha_2(1-\ell)$ is bounded from below as follows:

$$\mu_3 - \alpha_2(1-\ell) \ \geq \ \mu_3 - \alpha_2 \cdot \left(1 - \frac{\alpha_2}{\alpha_1 + \alpha_2}\right) \geq \mu_2 - 1$$

whereas the last inequality is due to (3.2.7). Assumption $\mu_j \geq R_f$ leads to the result. □

## A.2 Results on Specific Instances of the Equilibrium

### A.2.1 The Equilibrium without Regulation

#### A.2.1.1 Proof of Result 5

*Proof.* The bank's objective function $\mathrm{E}[\tilde{W}_B(\ell, R_D; j)]$ is a composition of continuous functions. In particular, $D^s(\ell, R_D; j)$ is continuous (but not necessarily differentiable as it can be cut off by the household's initial wealth $W_H$ at some points, that is especially in $\overline{R}_D(\ell; j)$, $\overline{R}_D(\ell; j) < \overline{R}_D^u(\ell; j)$, for fixed loan-allocation rates). Furthermore, feasible loan-allocation rates $\ell$ are restricted to the compact sets $[0,1] \cap C_j$, $j = 1, \ldots, 4$ for fixed deposit rates $R_D$ by definition. Note that the sets $C_j$ are compact in $\ell$ for given $R_D$ as their Definitions (3.2.16) to (3.2.19) always contain their respective borders $\partial C_j$. In particular, the loan-allocation rate is restricted to (3.2.20) under Case 2 and (3.2.21) under Case 3, respectively. Thus, there is always at least one loan-allocation rate $\ell^* = \ell(R_D)$ maximizing the bank's objective function for given deposit interest rates $R_D$. This argument holds for each Case $j$ as well as globally across all four Cases.

Concerning $R_D$ we will argue case-by-case. Note that the bank's objective function is differentiable given each of the Cases $j = 1, \ldots, 4$. Let us start with

**Case 1:**

The partial derivative of the bank's objective function is given by

$$\frac{\partial \mathrm{E}[\tilde{W}_B(\ell, R_D; 1)]}{\partial R_D} \ = \ -q \cdot D^u(\ell, R_D; 1) \ + \ q \cdot [\alpha_1 \ell + \alpha_2(1-\ell) - R_D] \cdot \frac{\partial D^u(\ell, R_D; 1)}{\partial R_D}$$

whereas the household is assumed to be unconstrained. By Result 2 this derivative takes the following values in $\underline{R}_D(\ell; j)$, and $\overline{R}^u_D(\ell; j)$, respectively:

$$\begin{aligned}
\left.\frac{\partial \mathrm{E}[\tilde{W}_B(\ell,R_D;1)]}{\partial R_D}\right|_{R_D=\underline{R}_D(\ell;1)} &= q \cdot [\alpha_1\ell + \alpha_2(1-\ell) - \underline{R}_D(\ell;1)] \left.\frac{\partial D^u(\ell,R_D;1)}{\partial R_D}\right|_{R_D=\underline{R}_D(\ell;1)} \\
&> 0
\end{aligned}$$

$$\left.\frac{\partial \mathrm{E}[\tilde{W}_B(\ell,R_D;1)]}{\partial R_D}\right|_{R_D=\overline{R}^u_D(\ell;1)} = -q \cdot D^u(\ell, \overline{R}^u_D(\ell;1);1) \ < \ 0 \ .$$

Thus, the derivative $\frac{\partial \mathrm{E}[\tilde{W}_B(\ell,R_D;1)]}{\partial R_D}$ changes signs at least once on $(\underline{R}_D(\ell;1), \overline{R}^u_D(\ell;1))$ and hence there is at least one local maximum on $(\underline{R}_D(\ell;1), \overline{R}^u_D(\ell;1))$. Either if $\overline{R}^u_D(\ell;1) > \alpha_1\ell + \alpha_2(1-\ell)$ or $\overline{R}^u_D(\ell;1) \leq \alpha_1\ell + \alpha_2(1-\ell)$ holds, the expression

$$q \cdot [\alpha_1\ell + \alpha_2(1-\ell) - R_D] \cdot \frac{\partial D^u(\ell, R_D; 1)}{\partial R_D}$$

is always negative on $(\alpha_1\ell + \alpha_2(1-\ell), \overline{R}^u_D(\ell;1))$, and on $(\overline{R}^u_D(\ell;1), \alpha_1\ell + \alpha_2(1-\ell))$, respectively. Note that in the first case the marginal profit $[\alpha_1\ell + \alpha_2(1-\ell) - R_D]$ is negative, whereas the derivative of the deposit supply is positive due to Result 2 and that in the latter case signs are reversed.

Beyond $R_D > \alpha_1\ell + \alpha_2(1-\ell)$, the contribution of collecting deposits to the bank's expected final wealth is negative. Furthermore, the bank's expected final wealth is increasing at $R_D = \underline{R}_D(\ell;1)$. Thus, there is at least one maximizing interest rate $R_D$ on $(\underline{R}_D(\ell;1), \overline{R}^u_D(\ell;1))$. If deposit supply is effectively cut off by the household's expected wealth at a rate $\overline{R}_D(\ell;1) < \overline{R}^u_D(\ell;1)$, the optimal $R_D$ is bounded from above by $\overline{R}_D(\ell;1)$. In this case, the derivative $\frac{\partial \mathrm{E}[\tilde{W}_B(\ell,R_D;1)]}{\partial R_D}$ is negative for all those $R_D > \overline{R}_D(\ell;1)$ until the bank's expected final wealth turns negative, such as in the case of an unconstrained deposit supply.

Concerning uniqueness, consider again

$$\frac{\partial \mathrm{E}[\tilde{W}_B(\ell,R_D;1)]}{\partial R_D} \ = \ -q \cdot D^u(\ell, R_D; 1) \ + \ q \cdot [\alpha_1\ell + \alpha_2(1-\ell) - R_D] \cdot \frac{\partial D^u(\ell,R_D;1)}{\partial R_D} \ ,$$

using the implicit function theorem for representing the partial derivative $\frac{\partial D^u(\ell,R_D;1)}{\partial R_D}$, this partial derivative of the bank's objective function becomes

$$\begin{aligned}
&- \ q \cdot D^u(\ell, R_D; 1) + \\
&+ \ q \cdot [\alpha_1\ell + \alpha_2(1-\ell) - R_D] \cdot \frac{q \cdot \left\{ \ 1 - \gamma\left[2 \cdot (R_D - \mu_1) \cdot D^u(\ell,R_D;1) - R^\mu_1 \cdot W_B\right] \ \right\}}{\gamma\sigma_1^2}
\end{aligned}$$

which is positive if and only if

$$R_D \quad < \quad \frac{-\gamma\sigma_1^2 D^u(\ell,R_D;1)+[\alpha_1\ell+\alpha_2(1-\ell)-R_D]\cdot\left[1+\gamma W_B R_1^\mu+2\mu_1\gamma D^u(\ell,R_D;1)\right]}{2\gamma D^u(\ell,R_D;1)\cdot[\alpha_1\ell+\alpha_2(1-\ell)-R_D]}$$

holds. For simplicity, $D^u(\ell, R_D; 1)$ is rephrased by $D$. Let

$$\begin{aligned} g_1(R_D) \quad &= \quad \frac{-\sigma_1^2 D + [\alpha_1\ell + \alpha_2(1-\ell) - R_D] \cdot \left[\frac{1}{\gamma} + 2\mu_1 D + W_B \cdot R_1^\mu\right]}{2D \cdot [\alpha_1\ell + \alpha_2(1-\ell) - R_D]} \\ &\equiv \quad -\frac{\sigma_1^2}{2 \cdot [\alpha_1\ell + \alpha_2(1-\ell) - R_D]} + f_1(R_D) \end{aligned}$$

If $R_D$ approaches the zero $\underline{R}_D(\ell;1)$ from above, $g_1(R_D)$ becomes infinite,

$$\lim_{R_D \downarrow \underline{R}_D(\ell;1)} g_1(R_D) \;=\; -\frac{\sigma_1^2}{2 \cdot [\alpha_1\ell + \alpha_2(1-\ell) - \underline{R}_D(\ell;1)]} + \lim_{R_D \downarrow \underline{R}_D(\ell;1)} f_1(R_D) \;=\; +\infty \; . \tag{A.2.1}$$

Its derivative is given by

$$g_1'(R_D) \;=\; -\frac{\sigma_1^2}{[\alpha_1\ell + \alpha_2(1-\ell) - R_D]^2} \;+\; f_1'(R_D) \tag{A.2.2}$$

where $f_1'(R_D)$ is given according to (A.2.2). So, we obtain

$$\lim_{R_D \downarrow \underline{R}_D(\ell;1)} g_1'(R_D) \;=\; -\infty \; .$$

As the function $g_1(R_D)$ emerges from $f_1(R_D)$ by a variable shift, $g_1(R_D)$ can be discussed relative to the better-known function $f_1(R_D)$. On $(\ \underline{R}_D(\ell;1), \alpha_1\ell + \alpha_2(1-\ell)\ )$, the gap

$$f_1(R_D) - g_1(R_D) \;=\; \frac{\sigma_1^2}{2 \cdot [\alpha_1\ell + \alpha_2(1-\ell) - R_D]} > 0$$

is monotonically increasing until it becomes infinitely large as $R_D$ approaches $\alpha_1\ell + \alpha_2(1-\ell)$ from below. The fact that $g_1(R_D)$ is bounded by $f_1(R_D)$ which crosses the identity $R_D \equiv R_D$ exactly once on $(\ \underline{R}_D(\ell;1), \alpha_1\ell + \alpha_2(1-\ell)\ )$ and whose distance to $g_1(R_D)$ is increasing in $R_D$, implies that the function $g_1(R_D)$ crosses identity $R_D \equiv R_D$ exactly once, too. This unique intersection point is $R_D^*$ by the definition of $g_1(R_D)$.

However, this analysis is only true, if a solution such as this is neither restricted by the Case constraints nor by the depositor's wealth $W_H$.

The Case constraints may require the bank to choose $R_D > \alpha_1\ell + \alpha_2(1-\ell)$ given $\ell$

fixed. Because of $D^s(\ell, R_D; 1) > 0$, any such choice is dominated by autarky, *i.e.* by choosing $D^* = 0$ yielding $p_2\alpha_2 W_B$ because of (3.2.5) and the bank's risk neutrality. This choice, however, is in turn dominated by choosing $\ell = 0$ under Case 2 for all feasible parameter values as

$$(p_2\alpha_2 - p_2 R_D) \cdot \frac{p_2 R_D - R_f}{\gamma p_2(1-p_2)R_D^2} + p_2\alpha_2 W_B \;>\; p_2\alpha_2 W_B$$

holds for $\frac{1}{p_2}R_f < R_D < \alpha_2$. Due to (3.2.5) and (3.2.6), this double inequality may be fulfilled and in particular $R_D^* > \frac{1}{p_2}R_f \equiv \underline{R}_D(0;2)$ holds.

Consider the other possibility that the depositor is effectively constrained by its initial wealth: If $\overline{R}_D(\ell;1)$ is greater than the intersection point satisfying $R_D \equiv g(R_D)$, nothing changes. If $\overline{R}_D(\ell;1)$ is lower than that intersection point, the bank's objective function $\mathrm{E}[\tilde{W}_B(\ell, R_D;1)]$ is still strictly increasing in $R_D$ as $R_D < g(R_D)$ holds below that intersection. Thus, $R_D^* = \overline{R}_D(\ell;1)$ is optimal. As $\overline{R}_D(\ell;1) \equiv \overline{R}_D^u(\ell;1)$ if the depositor is not effectively constrained, we finally obtain that $R_D^*$ is uniquely from $(\ \underline{R}_D(\ell;1), \overline{R}_D(\ell;1)\ ]$.

Next, we consider

**Case 2:**

The proof for this Case is analogous to that for Case 1. First, let us consider the partial derivative of the bank's objective function

$$\frac{\partial \mathrm{E}[\tilde{W}_B(\ell,R_D;2)]}{\partial R_D} \;=\; -p_2 \cdot D^u(\ell, R_D; 2) \;+\; [q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2 R_D] \cdot \frac{\partial D^u(\ell,R_D;2)}{\partial R_D}$$

whereas the household is assumed to be unconstrained. By Result 2 this derivative takes the following values in $\underline{R}_D(\ell;2)$, and $\overline{R}_D^u(\ell;2)$, respectively:

$$\begin{aligned}
\left.\frac{\partial \mathrm{E}[\tilde{W}_B(\ell,R_D;2)]}{\partial R_D}\right|_{R_D=\underline{R}_D(\ell;2)} &= [q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2\underline{R}_D(\ell;2)] \cdot \left.\frac{\partial D^u(\cdot,\cdot;2)}{\partial R_D}\right|_{R_D=\underline{R}_D(\ell;2)} \\
&> 0 \\
\left.\frac{\partial \mathrm{E}[\tilde{W}_B(\ell,R_D;2)]}{\partial R_D}\right|_{R_D=\overline{R}_D^u(\ell;2)} &= -p_2 \cdot D_2^u(\ell, \overline{R}_D^u(\ell;2); 2) \;<\; 0\ .
\end{aligned}$$

Thus, the derivative $\frac{\partial \mathrm{E}[\tilde{W}_B(\ell,R_D;2)]}{\partial R_D}$ changes signs at least once on $(\underline{R}_D(\ell;2), \overline{R}_D^u(\ell;2))$ and hence there is at least one local maximum on $(\underline{R}_D(\ell;2), \overline{R}_D^u(\ell;2))$. Either if $\overline{R}_D^u(\ell;2) > \frac{q}{p_2}\alpha_1\ell + \alpha_2(1-\ell)$ or $\overline{R}_D^u(\ell;2) \le \frac{q}{p_2}\alpha_1\ell + \alpha_2(1-\ell)$ holds, the expression

$$[q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2\underline{R}_D(\ell;2)] \cdot \frac{\partial D^u(\ell, R_D; 2)}{\partial R_D}$$

is always negative on $(\frac{q}{p_2}\alpha_1\ell+\alpha_2(1-\ell), \overline{R}^u_D(\ell;2))$, and on $(\overline{R}^u_D(\ell;2), \frac{q}{p_2}\alpha_1\ell+\alpha_2(1-\ell))$, respectively. Note that in the first case the marginal profit $[q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2\underline{R}_D(\ell;2)]$ is negative, whereas the derivative of the deposit supply is positive due to Result 2 and that in the latter case signs are reversed.

Since beyond $R_D > \frac{q}{p_2}\alpha_1\ell + \alpha_2(1-\ell)$, the bank's expected final wealth is negative, $\mathrm{E}[\tilde{W}_B(\ell, R_D; 2)] < 0$, and since the bank's expected final wealth is increasing at $R_D = \underline{R}_D(\ell;2)$, there is at least one maximizing interest rate $R_D$ on $(\underline{R}_D(\ell;2), \overline{R}^u_D(\ell;2))$. If deposit supply is effectively cut off by the household's expected wealth at a rate $\overline{R}_D(\ell;2) < \overline{R}^u_D(\ell;2)$, the optimal $R_D$ is bounded from above by $\overline{R}_D(\ell;2)$. Then the derivative $\frac{\partial \mathrm{E}[\tilde{W}_B(\ell,R_D;2)]}{\partial R_D}$ is negative for all those $R_D > \overline{R}_D(\ell;2)$ until the bank's expected final wealth turns negative, such as in the case of an unconstrained deposit supply.

Concerning uniqueness, consider again

$$\frac{\partial \mathrm{E}[\tilde{W}_B(\ell,R_D;2)]}{\partial R_D} \;=\; -q\cdot D^u(\ell,R_D;2) \;+\; [q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2R_D]\cdot\frac{\partial D^u(\ell,R_D;2)}{\partial R_D}\;,$$

using the implicit function theorem for representing the partial derivative $\frac{\partial D^u(\ell,R_D;2)}{\partial R_D}$, this partial derivative of the bank's objective function becomes

$$\begin{aligned} &- \;\; p_2\cdot D^u(\ell,R_D;2) \,+ \\ &+ \;\; p_2\cdot[q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2R_D]\cdot\frac{p_2\cdot\left\{\;1-\gamma\left[2\cdot(R_D-\mu_2)\cdot D^u(\ell,R_D;2)-R_2^\mu\cdot W_B\right]\;\right\}}{\gamma\sigma_2^2}\end{aligned}$$

which is positive if and only if

$$R_D \;<\; \frac{-\gamma\sigma_2^2 D^u(\ell,R_D;2)+[q\alpha_1\ell+p_2\alpha_2(1-\ell)-p_2R_D]\cdot\left[1+\gamma W_B R_2^\mu+2\mu_2\gamma D^u(\ell,R_D;2)\right]}{2\gamma D^u(\ell,R_D;2)\cdot[q\alpha_1\ell+p_2\alpha_2(1-\ell)-p_2R_D]}$$

holds. For simplicity, $D^u(\ell,R_D;2)$ is rephrased by $D$. Let

$$\begin{aligned} g_2(R_D) \;&=\; \frac{-\sigma_2^2 D + [\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2R_D]\cdot\left[\frac{1}{\gamma} + 2\mu_2 D + W_B\cdot R_2^\mu\right]}{2D\cdot[\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2R_D]} \\ &\equiv\; -\frac{\sigma_2^2}{2\cdot[\alpha_1\ell+\alpha_2(1-\ell)-R_D]} + f_2(R_D)\end{aligned}$$

If $R_D$ approaches $\underline{R}_D(\ell;2)$ from above, $g_2(R_D)$ becomes infinite,

$$\begin{aligned} &\lim_{R_D\downarrow\underline{R}_D(\ell;2)} g_2(R_D) \;= \\ &= \;-\frac{\sigma_2^2}{2\cdot[q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2\underline{R}_D(\ell;2)]} \;+\; \lim_{R_D\downarrow\underline{R}_D(\ell;2)} f_2(R_D) \;=\; +\infty\;.\end{aligned}$$

Its derivative is given by

$$g_2'(R_D) \;=\; -\frac{\sigma_2^2}{\left[q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2R_D\right]^2} \;+\; f_2'(R_D) \tag{A.2.3}$$

where $f_2'(R_D)$ is given according to (A.2.3). So, we obtain

$$\lim_{R_D \downarrow \underline{R}_D(\ell;2)} g_2'(R_D) \;=\; -\infty \;.$$

As the function $g_2(R_D)$ emerges from $f_2(R_D)$ by a variable shift, $g_2(R_D)$ can be discussed relative to the better-known function $f_2(R_D)$. On $(\ \underline{R}_D(\ell;2), \frac{q}{p_2}\alpha_1\ell + \alpha_2(1-\ell)\ )$, the gap

$$f_2(R_D) - g_2(R_D) \;=\; \frac{\sigma_2^2}{2\cdot\left[q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2R_D\right]} > 0$$

is monotonically increasing until it becomes infinitely large as $R_D$ approaches $\frac{q}{p_2}\alpha_1\ell + \alpha_2(1-\ell)$ from below. The fact that $g_2(R_D)$ is bounded by $f_2(R_D)$ which crosses the identity $R_D \equiv R_D$ exactly once on $(\ \underline{R}_D(\ell;2), \frac{q}{p_2}\alpha_1\ell+\alpha_2(1-\ell)\ )$ and whose distance to $g_2(R_D)$ is increasing in $R_D$, implies that the function $g_2(R_D)$ crosses the identity $R_D \equiv R_D$ exactly once, too. This unique intersection point is $R_D^*$ by the definition of $g_2(R_D)$. Finally, we can argue such as in Case 1 when deposit supply is effectively bounded by the depositor's initial wealth $W_H$.

The proofs for the Cases 3 and 4 are analogous. Case-dependent expressions must be appropriately rephrased. The equilibrium given Case 4 can be determined analytically so that a proof by indirect arguments is not necessary. □

#### A.2.1.2 Proof of Result 6

*Proof.* Let us consider the following simplified version of the Maximization Problem (3.3.1) given Case 1,

$$\max_{\ell,\ R_D} \quad \mathrm{E}\left[\tilde{W}_B(\ell, R_D; 1)\right] \;,$$

and let us assume that the household is not constrained by its initial wealth $W_H$. The associated first-order conditions are:

$$q\cdot(\alpha - R_D)\cdot\frac{\partial D^u(\ell, R_D;1)}{\partial \ell} \;\stackrel{!}{=}\; 0 \tag{A.2.4}$$

$$q\cdot(\alpha - R_D)\cdot\frac{\partial D^u(\ell, R_D;1)}{\partial R_D} \;-\; q\cdot D^u(\ell, R_D;1) \;\stackrel{!}{=}\; 0\;. \tag{A.2.5}$$

By the first first-order constraint,

$$R_D^* = \alpha \quad \text{or} \quad \frac{\partial D^u(\ell, R_D; 1)}{\partial \ell} \;=\; 0$$

must hold. Consider first $R_D^* = \alpha$. Then the bank's expected wealth is only $q\alpha W_B$ as it forgoes any profit opportunities from intermediation. As

$$D^u(\ell, \alpha, 1) \;=\; \frac{p\alpha - R_f - (1 - 2\ell + 2\ell^2 - p)(p-q)\alpha^2\gamma W_B}{\{[1 - 2\ell(1-\ell)]\, p(1-p) + 2\ell(1-\ell)(q - p^2)\} \cdot \alpha^2 \cdot \gamma}$$

is strictly positive for example at $\ell = \frac{1}{2}$. And as it is still so for $R_D < \alpha$, the bank could lessen the deposit interest rate resulting in strictly positive expected wealth from doing intermediation. Thus, $R_D^* = \alpha$ contradicts optimality. Consequently, $\frac{\partial D^u(\ell, R_D;1)}{\partial \ell} = 0$ holds. By

$$\frac{\partial D^u(\ell, R_D; 1)}{\partial \ell} \;=\; \frac{\partial D^u(\ell, R_D; 1)}{\partial \sigma_1^2} \cdot \frac{\partial \sigma_1^2}{\partial \ell}\,,$$

$\ell = \frac{1}{2}$ solves the first first-order condition. Because of

$$\frac{\partial^2 D^u(\ell, R_D; 1)}{\partial \ell^2} \;=\; \frac{\partial^2 D^u(\ell, R_D; 1)}{\partial (\sigma_1^2)^2} \cdot \left(\frac{\partial \sigma_1^2}{\partial \ell}\right)^2 \;+\; \frac{\partial D^u(\ell, R_D; 1)}{\partial \sigma_1^2} \cdot \frac{\partial^2 \sigma_1^2}{\partial \ell^2}\,,$$

the solution $\ell = \frac{1}{2}$ is a local maximum as

$$\left.\frac{\partial^2 D^u(\ell, R_D; 1)}{\partial \ell^2}\right|_{\ell=\frac{1}{2}} \;=\; \underbrace{\frac{\partial D^u(\ell, R_D; 1)}{\partial \sigma_1^2}}_{<\,0} \cdot \underbrace{\frac{\partial^2 \sigma_1^2}{\partial \ell^2}}_{>\,0} \;<\; 0 \tag{A.2.6}$$

holds. As it is the only critical point independent of $R_D$, $\ell = \frac{1}{2}$ maximizes uniquely the objective $\mathrm{E}[\tilde{W}_B(\ell, R_D; 1)]$. By Result 5, the optimal $R_D^*$ is unique. In particular, due to

$$D^u(\tfrac{1}{2}, R_D; 1) \;=\; \frac{qR_D + (p-q)\alpha - R_f + (p-q)\left[qR_D + (p-q)\alpha - \alpha\frac{1}{2}\right]\alpha\gamma W_B}{\sigma_1^2 \cdot \gamma} \;>\; 0$$

the relation

$$\frac{\partial D^u(\frac{1}{2}, R_D; 1)}{\partial R_D} \;>\; 0$$

holds. For the positive sign of $[qR_D + (p-q)\alpha - \alpha\frac{1}{2}]$ we refer to Appendix A.2.1.8. $\square$

#### A.2.1.3 Proof of Result 7

*Proof.* The sensitivity of the optimal deposit volume with respect to $W_B$ is given by

$$\frac{dD^*}{dW_B} = \frac{\partial D^u(\cdot)}{\partial R_D} \cdot \frac{dR_D^*}{dW_B} + \frac{\partial D^u(\cdot)}{\partial W_B}$$

as

$$\left.\frac{\partial D^u(\cdot)}{\partial \ell}\right|_{\ell=\frac{1}{2}} = 0 \tag{A.2.7}$$

holds in optimum (*cf.* Appendix A.2.1.2). The sensitivities of the optimal loan-allocation rate $\ell^*$ and $R_D^*$ with respect to $W_B$ are given by

$$\begin{pmatrix} \frac{d\ell^*}{dW_B} \\ \frac{dR_D^*}{dW_B} \end{pmatrix} = -\frac{1}{\det\Big(H(\mathrm{E}[\cdot],\ell^*,R_D^*)\Big)} \cdot \begin{pmatrix} \frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D^2} & -\frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell \partial R_D} \\ -\frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell \partial R_D} & \frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell^2} \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell \partial W_B} \\ \frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D \partial W_B} \end{pmatrix}$$

where $H(f,x)$ denotes the Hessian of function $f$ evaluated at $x$. By

$$\left.\frac{\partial^2 \mathrm{E}[\tilde{W}_B(\ell,R_D;1)]}{\partial \ell \partial R_D}\right|_{\ell=\frac{1}{2}} = 0 \quad \text{and} \quad \left.\frac{\partial^2 \mathrm{E}[\tilde{W}_B(\ell,R_D;1)]}{\partial \ell \partial W_B}\right|_{\ell=\frac{1}{2}} = 0$$

the sensitivities simplify to

$$\begin{pmatrix} \frac{d\ell^*}{dW_B} \\ \frac{dR_D^*}{dW_B} \end{pmatrix} = -\frac{1}{\det\Big(H(\mathrm{E}[\cdot],\ell^*,R_D^*)\Big)} \cdot \begin{pmatrix} 0 \\ \frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell^2} \cdot \frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D \partial W_B} \end{pmatrix} .$$

As $(\ell^*, R_D^*)$ is an inner optimum to the maximization problem, the Hessian $H(\mathrm{E}[\cdot],\ell^*,R_D^*)$ is negative semidefinite (*cf.* Mas-Colell/Winston/Green, 1995, p. 955). The Hessian itself is simply given by

$$H(\mathrm{E}[\cdot],\ell^*,R_D^*) = \begin{pmatrix} \frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D^2} & 0 \\ 0 & \frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell^2} \end{pmatrix} ,$$

where

$$\left.\frac{\partial^2 \mathrm{E}[\tilde{W}_B(\ell,R_D;1)]}{\partial \ell^2}\right|_{\ell=\frac{1}{2}} < 0$$

can be derived from (A.2.4) and (A.2.6), respectively. By Result 6, the bank's objective function cannot be flat around $R_D^*$ either. Hence,

$$\left.\frac{\partial^2 \mathrm{E}[\tilde{W}_B(\ell,R_D;1)]}{\partial R_D^2}\right|_{\ell=\frac{1}{2},R_D=R_D^*} < 0$$

holds. Thus, $H(\mathrm{E}[\cdot], \ell^*, R_D^*)$ is even negative definite, and its determinant is strictly positive, $\det(H(\mathrm{E}[\cdot], \ell^*, R_D^*)) > 0$. So the sensitivities are well-defined. In total, the sign of $\frac{dR_D^*}{dW_B}$ arises out of the implicit function theorem as follows:

$$\frac{dR_D^*}{dW_B} = \underbrace{- \overbrace{\frac{1}{\det(\ H(\mathrm{E}[\cdot], \ell^*, R_D^*)\ )}}^{>\ 0} \cdot \overbrace{\frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell^2}}^{<\ 0}}_{>\ 0} \cdot \frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D \partial W_B} ,$$

which can be reduced to

$$\frac{dR_D^*}{dW_B} = - \frac{\frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D \partial W_B}}{\frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D^2}} ,$$

due to $\det(H(\mathrm{E}[\cdot], \ell^*, R_D^*)) = \frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D^2} \cdot \frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell^2}$.

The second mixed derivative of $\mathrm{E}[\cdot]$ with respect to $R_D$ and $W_B$ reads

$$\frac{\partial^2 \mathrm{E}[\tilde{W}_B(\ell, R_D; 1)]}{\partial R_D \partial W_B} = - q \cdot \frac{\partial D^u(\ell, R_D; 1)}{\partial W_B} + q \cdot (\alpha - R_D) \cdot \frac{\partial^2 D^u(\ell, R_D; 1)}{\partial R_D \partial W_B}$$

resulting in the total derivative of the deposit volume with respect to $W_B$ as follows:

$$\begin{aligned} \frac{dD^*}{dW_B} &= \frac{\partial D^u(\cdot)}{\partial R_D} \cdot \frac{dR_D^*}{dW_B} + \frac{\partial D^u(\cdot)}{\partial W_B} \\ &= - \frac{\partial D^u(\cdot)}{\partial R_D} \cdot \frac{- \frac{\partial D^u(\cdot)}{\partial W_B} + (\alpha - R_D) \cdot \frac{\partial^2 D^u(\cdot)}{\partial R_D \partial W_B}}{- 2 \cdot \frac{\partial D^u(\cdot)}{\partial R_D} + (\alpha - R_D) \cdot \frac{\partial^2 D^u(\cdot)}{\partial R_D^2}} + \frac{\partial D^u(\cdot)}{\partial W_B} . \end{aligned}$$

As $-\frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D^2} > 0$, we can multiply both sides of the equality by $-\frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D^2} > 0$ without changing either side's sign. As a consequence, the total derivative $\frac{dD^*}{dW_B}$ is positive if and only if

$$\frac{\partial D^u(\cdot)}{\partial R_D} \cdot \frac{\partial D^u(\cdot)}{\partial W_B} + (\alpha - R_D) \cdot \left[ \frac{\partial^2 D^u(\cdot)}{\partial R_D \partial W_B} \cdot \frac{\partial D^u(\cdot)}{\partial R_D} - \frac{\partial^2 D^u(\cdot)}{\partial R_D^2} \cdot \frac{\partial D^u(\cdot)}{\partial W_B} \right] > 0$$

holds. In the optimum $(\ell^*, R_D^*)$,

$$\frac{\partial D^u(\frac{1}{2}, R_D; 1)}{\partial R_D} > 0 \quad \text{and} \quad \frac{\partial D^u(\frac{1}{2}, R_D; 1)}{\partial W_B} > 0$$

are satisfied, *cf.* Appendices A.2.1.2 and A.2.1.8, respectively. Furthermore, the (unconstrained) deposit-supply function is strictly concave with respect to $R_D$: The second derivative of the deposit supply function with respect to $R_D$ reads

$$\left.\frac{\partial^2 D^u(\cdot)}{\partial R_D^2}\right|_{\ell=\frac{1}{2}} =$$

$$\frac{8q\Big\{-\alpha^3(p-q)^2[2q(p-q)+2p+q-1]+2(1-q)^2qR_D^2(3R_f-qR_D)-6\alpha(p-q)(1-q)qR_D\big[2R_f+(1-q)R_D\big]\Big\}}{\gamma\Big\{-(p-q)(1-2p+2q)\alpha^2+4q(p-q)\alpha R_D-2(1-q)qR_D^2\Big\}^3}$$

$$-\ \frac{8(p-q)q\alpha^2\Big\{-6q^3R_D+R_f-2pR_f+q^2\big[(9+6p)R_D+6R_f\big]+q\big[-3(1+2p)R_D+R_f-6pR_f\big]\Big\}}{\gamma\Big\{-(p-q)(1-2p+2q)\alpha^2+4q(p-q)\alpha R_D-2(1-q)qR_D^2\Big\}^3}$$

$$-\ \frac{4(p-q)^2q\alpha^4\cdot\{(p-q)[4(p-q)(1+q)-3]+(1-p)\}W_B}{\Big\{-(p-q)(1-2p+2q)\alpha^2+4q(p-q)\alpha R_D-2(1-q)qR_D^2\Big\}^3}$$

$$+\frac{4(p-q)q^2(1-q)\alpha R_D\cdot\Big\{6(1-2p+2q)[(1-q)R_D-(p-q)\alpha]\alpha-4(1-q)qR_D^2\Big\}W_B}{\Big\{-(p-q)(1-2p+2q)\alpha^2+4q(p-q)\alpha R_D-2(1-q)qR_D^2\Big\}^3}\ .$$

Numerical routines implemented in Mathematica show that the numerator is greater than zero on the domain specified by

$$\begin{aligned}\{(p,q,\alpha,R_D,R_f,\gamma,W_B)\quad :\quad & 2p-1\leq q\leq p\ ,\tfrac{1}{2}\leq p\leq 1,\ p\alpha>R_D>R_f\geq 1,\\ & \alpha\leq 2,\ \gamma\geq 0,\ W_B\geq 0\}\ .\end{aligned}$$

The denominator, however, becomes negative on the same domain. More specifically, the denominator takes all values from $[-0.0838413, 0)$. Thus, we obtain

$$\left.\frac{\partial^2 D^u(\ell,R_D;1)}{\partial R_D^2}\right|_{\ell=\frac{1}{2}} < 0$$

where all parameter values have not been more restricted than already done. Hence, the problem of determining the sign of $\frac{dD^*}{dW_B}$ can be broken down as follows:

$$\underbrace{\underbrace{\frac{\partial D^u(\cdot)}{\partial R_D}\cdot\frac{\partial D^u(\cdot)}{\partial W_B}}_{>0} + \underbrace{(\alpha-R_D)}_{>0}\cdot\left[\frac{\partial^2 D^u(\cdot)}{\partial R_D\partial W_B}\cdot\frac{\partial D^u(\cdot)}{\partial R_D}\right.}_{\textbf{sign?}}\ \left.-\ \underbrace{\frac{\partial^2 D^u(\cdot)}{\partial R_D^2}\cdot\frac{\partial D^u(\cdot)}{\partial W_B}}_{>0}\right] > 0$$

By $\frac{\partial D^u(\cdot)}{\partial R_D}>0$, it is now sufficient to show that

$$\frac{\partial D^u(\cdot)}{\partial W_B} + (\alpha-R_D)\cdot\frac{\partial^2 D^u(\cdot)}{\partial R_D\partial W_B} > 0$$

holds. But the sign of

$$\frac{\partial^2 D^u(\frac{1}{2}, R_D; 1)}{\partial R_D \partial W_B} = \frac{-2(p-q)q\alpha\left[(p-q)(1-2p+2q)\alpha^2-2(1-2p+2q)(1-q)\alpha R_D+2(1-q)qR_D^2\right]}{\left[(p-q)(1-2p+2q)\alpha^2-4(p-q)q\alpha R_D+2(1-q)qR_D^2\right]^2}$$

is ambiguous. At $\ell^* = \frac{1}{2}$, the expression $\frac{\partial D^u(\cdot)}{\partial W_B} + (\alpha - R_D) \cdot \frac{\partial^2 D^u(\cdot)}{\partial R_D \partial W_B}$ can be displayed as

$$- \frac{(p-q)\alpha\left\{(p-q)[(1+2q)(1-4(p-q))+4p(p-q)]\alpha^3+4q[2(p-q)(2p-3q)-(1-q)]\alpha^2 R_D\right\}}{\left[(p-q)(1-2p+2q)\alpha^2-4(p-q)q\alpha R_D+2(1-q)qR_D^2\right]^2}$$
$$- \frac{(p-q)\alpha\left\{2q[3(1-2p+q)+2q(1-q+5(p-q))]\alpha R_D^2-8(1-q)q^2R_D^3\right\}}{\left[(p-q)(1-2p+2q)\alpha^2-4(p-q)q\alpha R_D+2(1-q)qR_D^2\right]^2} .$$

The numerator except for the multiplier $(p-q)\alpha$ takes values from $[0, 0.238193]$ on the domain specified by

$$\left\{(p, q, \alpha, R_D, R_f, \gamma, W_B) \;:\; 2p-1 \leq q \leq p \,,\, \tfrac{1}{2} \leq p \leq 1,\; p\alpha > R_D > R_f \geq 1,\; \alpha \leq \tfrac{4}{3},\; \gamma \geq 0,\; W_B \geq 0\right\} .$$

Note that the firms' return parameter $\alpha$ has been further restricted as otherwise negative values become possible. Thus, $\alpha \leq \frac{4}{3}$ is an additional (compared to the assumptions made so far) and sufficient condition to ensure that

$$\frac{dD^*}{dW_B} > 0$$

holds. □

#### A.2.1.4 Proof of Result 8

*Proof.* Solving the first-order condition given Case 2

$$\left.\frac{\partial \mathrm{E}[\cdot]}{\partial R_D}\right|_{\ell=0} = -p_2 \cdot D^u(\cdot; 2) + [q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2 R_D] \cdot \frac{\partial D^u(\cdot; 2)}{\partial R_D} \stackrel{!}{=} 0 \quad \text{(A.2.8)}$$

for $R_D$ after having plugged-in $\ell = 0$ results readily in

$$R_D^* = \frac{2\alpha_2 R_f}{p_2\alpha_2 + R_f} \quad \text{(A.2.9)}$$

being the unique maximizer according to Result 5. Straight-forward calculations show that the optimal deposit volume becomes

$$D^u(0, R_D^*, 2) \;=\; \frac{(p_2\alpha_2 - R_f)\cdot(p_2\alpha_2 + R_f)}{4p_2(1-p_2)\alpha_2^2 R_f \gamma}$$

and the bank expects

$$\mathrm{E}\left[\tilde{W}_B(0, R_D^*, 4)\right] \;=\; \frac{(p_2\alpha_2 - R_f)^2}{4(1-p_2)\alpha_2 R_f \gamma} \;+\; p_2\alpha_2 W_B$$

as final wealth. A necessary and sufficient condition for $\ell^* = 0$ as optimum is that

$$\begin{aligned}
\frac{\partial \mathrm{E}[\cdot]}{\partial \ell}\bigg|_{\ell=0, R_D=R_D^*} &= (q\alpha_1 - p_2\alpha_2)\cdot(D^u(\cdot,2) + W_B) \;+ \\
&\quad + \;[q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2 R_D]\cdot \tfrac{\partial D^u(\cdot,2)}{\partial \ell} \\
&= (q\alpha_1 - p_2\alpha_2)\cdot(D^u(\cdot,2) + W_B) \;+\; p_2\cdot(\alpha_2 - R_D^*)\cdot \tfrac{\partial D^u(\cdot,2)}{\partial \ell} \\
&\overset{!}{\leq} 0
\end{aligned}$$

holds as we have assumed that there is a unique global maximizer with respect to the loan-allocation rate $\ell$ on the reals, $\mathbb{R}$. This condition can be stated in terms of the parameters of the model as

$$\begin{aligned}
&-(1-p_2)p_2\alpha_2 R_f \;+\; [(p_1 - q)p_2\alpha_2 + (q - p_1 p_2)R_f]\cdot\alpha_1 \;+ \\
&+\; 2p_2(1-p_2)\alpha_2^2\cdot\gamma W_B\cdot R_f\;\cdot \\
&\quad\cdot\;\{(2-p_2)(q\alpha_1 - p_2\alpha_2)R_f + p_2(p_2\alpha_2 - p_1\alpha_1)R_f + p_2^2(p_1-q)\alpha_1\alpha_2\}\;\overset{!}{\leq}\;0
\end{aligned}$$

such that we obtain Condition (3.3.3).

If this condition holds with equality, $\ell^* = 0$ is the maximizer of $\mathrm{E}[\tilde{W}_B(\ell,\cdot,2)]$ with respect to the whole real line $\mathbb{R}$. Here, we will refer to it as the unconstrained case. If, however, Condition (3.3.3) strictly holds, $\ell^* = 0$ is the corner solution to the maximization problem where the loan-allocation rate is restricted to $[0,1]$. We refer here to this case as the constrained case. First, let us consider

**The Unconstrained Problem**

which implies that the sensitivities of $\ell^*$ and $R_D^*$ can be calculated according to the implicit function theorem:

$$\begin{pmatrix} \frac{d\ell^*}{dW_B} \\ \frac{dR_D^*}{dW_B} \end{pmatrix} \;=\; -\;\frac{1}{\det\big(\;H(\mathrm{E}[\cdot],\ell^*,R_D^*)\;\big)}\cdot\begin{pmatrix} \frac{\partial^2\mathrm{E}[\cdot]}{\partial R_D^2} & -\frac{\partial^2\mathrm{E}[\cdot]}{\partial\ell\partial R_D} \\ -\frac{\partial^2\mathrm{E}[\cdot]}{\partial\ell\partial R_D} & \frac{\partial^2\mathrm{E}[\cdot]}{\partial\ell^2} \end{pmatrix}\cdot\begin{pmatrix} \frac{\partial^2\mathrm{E}[\cdot]}{\partial\ell\partial W_B} \\ \frac{\partial^2\mathrm{E}[\cdot]}{\partial R_D\partial W_B} \end{pmatrix}$$

where $H(f,x)$ denotes the Hessian of function $f$ evaluated at $x$. The notation $\mathrm{E}[\cdot]$ should be understood here as short-hand for the bank's expected final wealth given Case 2 and its respective derivatives as being evaluated at the optimum $(\ell^*, R_D^*)$. At this point, the second mixed derivative

$$\left.\frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D \partial W_B}\right|_{\ell=0, R_D=R_D^*} = -p_2 \cdot \frac{\partial D^u(\cdot,2)}{\partial W_B} + [q\alpha_1\ell + p_2\alpha_2(1-\ell) - p_2 R_D] \cdot \frac{\partial^2 D^u(\cdot,2)}{\partial R_D \partial W_B}$$

vanishes as both partial derivatives of the deposit-supply function vanish (*cf.* Formula (3.2.48) in Result 1) with respect to the derivative $\frac{\partial D^u(\cdot,2)}{\partial W_B}$) such that the sensitivities simplify to

$$\begin{pmatrix} \frac{d\ell^*}{dW_B} \\ \frac{dR_D^*}{dW_B} \end{pmatrix} = -\frac{1}{\det\left( H(\mathrm{E}[\cdot], \ell^*, R_D^*) \right)} \cdot \begin{pmatrix} \frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D^2} \cdot \frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell \partial W_B} \\ -\frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell \partial R_D} \cdot \frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell \partial W_B} \end{pmatrix} \tag{A.2.10}$$

As the maximum refers to the unconstrained problem (*i.e.* the bank is allowed to short sell) the determinant of the Hessian is positive (since the Hessian of $\mathrm{E}[\tilde{W}_B(\ell, R_D; 2)]$ is a $2 \times 2$-matrix and $(\ell^*, R_D^*) = (0, \frac{2\alpha_2 R_f}{p_2\alpha_2 + R_f})$ is assumed to be the maximizer of the problem). The relevant derivatives, all evaluated at $(\ell^*, R_D^*) = (0, \frac{2\alpha_2 R_f}{p_2\alpha_2 + R_f})$, are given as follows:

$$\begin{aligned}
\frac{\partial^2 \mathrm{E}[\cdot]}{\partial R_D^2} &= -\frac{(p_2\alpha_2 + R_f)^2}{8(1-p_2)\alpha_2^3 R_f^3 \gamma} < 0 \\
\frac{\partial^2 \mathrm{E}[\cdot]}{\partial W_B \partial \ell} &= \frac{(2-p_2)(q\alpha_1 - p_2\alpha_2)R_f + p_2(p_2\alpha_2 - p_1\alpha_1)R_f + p_2^2(p_1 - q)\alpha_1\alpha_2}{2(1-p_2)R_f} \\
\frac{\partial^2 \mathrm{E}[\cdot]}{\partial \ell \partial R_D} &= \frac{(p_2\alpha_2 + R_f)^2}{4p_2(1-p_2)\alpha_2^3 R_f \gamma} \cdot \frac{\partial^2 \mathrm{E}[\cdot]}{\partial W_B \partial \ell} + \\
&\quad + \frac{p_2(p_2\alpha_2 + R_f)^2 \cdot (p_1 - q)\alpha_1 \cdot \left[R_f - (2-p_2)\alpha_2 - 2(1-p_2)\alpha_2 \cdot \gamma W_B R_f\right]}{8(1-p_2)^2 \alpha_2^2 R_f^2 \gamma}
\end{aligned} \tag{A.2.11}$$

where the latter depiction has been chosen for convenience such that the following relations can be immediately read off:

$$\begin{aligned}
\left.\frac{\partial^2 \mathrm{E}\left[\tilde{W}_B(\cdot,2)\right]}{\partial W_B \partial \ell}\right|_{\ell=0, R_D=R_D^*} < 0 &\Rightarrow \left.\frac{\partial^2 \mathrm{E}\left[\tilde{W}_B(\cdot,2)\right]}{\partial \ell \partial R_D}\right|_{\ell=0, R_D=R_D^*} < 0 \\
\left.\frac{\partial^2 \mathrm{E}\left[\tilde{W}_B(\cdot,2)\right]}{\partial \ell \partial R_D}\right|_{\ell=0, R_D=R_D^*} > 0 &\Rightarrow \left.\frac{\partial^2 \mathrm{E}\left[\tilde{W}_B(\cdot,2)\right]}{\partial W_B \partial \ell}\right|_{\ell=0, R_D=R_D^*} > 0
\end{aligned} \tag{A.2.12}$$

since

$$\underbrace{R_f - \underbrace{(2-p_2)}_{>p_2}\alpha_2}_{<\ 0 \text{ by (3.2.5) and (3.2.6)}} - 2(1-p_2)\alpha_2 \cdot \gamma W_B R_f < 0$$

whereas all other terms, except for $\frac{\partial^2 \mathrm{E}\left[\tilde{W}_B(\cdot,2)\right]}{\partial W_B \partial \ell}$ itself, are always positive. But Condition (3.3.2)

$$(2-p_2)(q\alpha_1 - p_2\alpha_2)R_f + p_2(p_2\alpha_2 - p_1\alpha_1)R_f + p_2^2(p_1 - q)\alpha_1\alpha_2 \;\leq\; 0$$

is equivalent to

$$\frac{\partial^2 \mathrm{E}[\cdot]}{\partial W_B \partial \ell} \;<\; 0$$

such that, in line with (A.2.10), the unconstrained solution reacts to changes in $W_B$ according to

$$\frac{d\ell^*}{dW_B} \;<\; 0 \quad \text{and} \quad \frac{dR_D^*}{dW_B} \;>\; 0\ .$$

However, since, obviously, the optimal loan-allocation rate leaves the unit interval a change in $W_B$ implies that the optimal allocation rate actually remains at zero. Anything else contradicts feasibility in the constrained problem according to (3.3.1). Thus, the optimal value of $R_D^*$ does not change either such that we obtain

$$\frac{d\ell^*}{dW_B} \;=\; 0 \quad \text{and} \quad \frac{dR_D^*}{dW_B} \;=\; 0\ .$$

By (3.2.48), the optimal total loan volume, $L^*$ changes in deed one-to-one for every marginal change in $W_B$.

Suppose now

**The Constrained Problem**

in the sense that the optimal loan-allocation rate of the unconstrained problem is negative. Thus, Condition (3.3.3) strictly holds. Under Assumption (A.2.10), an increase in $W_B$ meant a further decrease in the optimal loan-allocation rate of the unconstrained problem. Even if the opposite holds and thus the optimal loan-allocation rate locally increases, the allocation rate will remain negative and hence, the optimal loan-allocation rate of the constrained problem, $\ell^*$, will be still kept fixed at zero. As in either case $\frac{\partial \mathrm{E}[\cdot]}{\partial \ell} < 0$ holds at $\ell = 0$ (by having assumed that the unconstrained optimum lie beneath zero), the implicit function theorem does not apply and marginal changes of $R_D^*$ in $W_B$ can be either implicitly deduced from the first-order Condition (A.2.8), or explicitly from the optimal Solution (A.2.9), both clearly implying again

$$\frac{d\ell^*}{dW_B} = 0 \quad \text{and} \quad \frac{dR_D^*}{dW_B} = 0 \; .$$

$\square$

#### A.2.1.5 Proof of Result 9

*Proof.* As we have assumed $D^u(\ell, R_D; 4) < W_H$ in equilibrium, we have to consider the following maximization problem for the bank under Case 4[1]

$$\begin{array}{rl} \max\limits_{\ell, R_D} & \mathrm{E}\left[\tilde{W}_B(\ell, R_D; 4)\right] \\ \text{s.t.} & D^u(\ell, R_D; 4) \cdot R_D \;\leq\; \alpha_1 \ell \left[D^u(\ell, R_D; 4) + W_B\right] \\ & D^u(\ell, R_D; 4) \cdot R_D \;\leq\; \alpha_2 (1-\ell) \left[D^u(\ell, R_D; 4) + W_B\right] \end{array}$$

implying the following first-order conditions

$$\begin{array}{rcl} \frac{\partial \mathcal{L}[.]}{\partial \ell} &=& -\frac{\left[q_4 R_D - R_f + q_4(1-q_4)\gamma W_B R_D^2\right]\cdot(p_2\alpha_2 - p_1\alpha_1 + \alpha_2\lambda_2 - \alpha_1\lambda_1)}{(1-q_4)q_4 R_D^2 \gamma} \stackrel{!}{=} 0 \\ \frac{\partial \mathcal{L}[.]}{\partial R_D} &=& \frac{[\alpha_1\ell(p_1+\lambda_1) + \alpha_2(1-\ell)(p_2+\lambda_2)](2R_f - q_4 R_D) - (q_4+\lambda_1+\lambda_2)R_f R_D}{(1-q_4)q_4 R_D^3 \gamma} \stackrel{!}{=} 0 \end{array}$$

$$\begin{array}{rcl} \lambda_1 \cdot \left\{D^u(\ell, R_D; 4)\cdot R_D - \alpha_1 \ell \left[D^u(\ell, R_D; 4) + W_B\right]\right\} &\stackrel{!}{=}& 0 \\ \lambda_2 \cdot \left\{D^u(\ell, R_D; 4)\cdot R_D - \alpha_2 (1-\ell) \left[D^u(\ell, R_D; 4) + W_B\right]\right\} &\stackrel{!}{=}& 0 \end{array}$$

Consider first $\lambda_1 = 0$ and $\lambda_2 = 0$; that is, none of the constraints constituting Case 4 according to (3.2.19) is binding. As the optimal interest rate will at least be equal to the zero of the deposit-supply function, *i.e.* $R_D^* > \frac{1}{q_4} R_f \equiv \underline{R}_D(4)$ (*cf.* (3.2.55)) given Case 4[2], we obtain

$$\frac{\partial \mathcal{L}[.]}{\partial \ell} \;\equiv\; \frac{\partial \mathrm{E}[.]}{\partial \ell} \;<\; 0$$

due to the assumption $p_1\alpha_1 < p_2\alpha_2$. Thus, the bank chooses the lowest feasible value for the loan-allocation rate $\ell$ given Case 4, implying

$$\alpha_1 L_1^* \;=\; D^u(\ell^*, R_D^*; 4) R_D^* \; .$$

This equality, in turn, contradicts the assumption that none of the constraints is binding. As a consequence $\lambda_1 \geq 0$.

[1] We can neglect the short-sell conditions imposed on $\ell$ as the Conditions (3.2.19) for Case 4 are always stricter.

[2] Note that the optimal interest rate strictly exceeds the deposit supply's zero in $R_D$ in equilibrium (*cf.* Result 5). That is, $R_D = \underline{R}_D(4)$ is never an equilibrium.

Due to the constraints imposed by Definition (3.2.19) of Case 4, the bank chooses its loan-allocation rate $\ell$ and $R_D$ such that

$$D^u(R_D^*) \cdot R_D^* \;=\; \alpha_1 L_1^* \;\leq\; \alpha_2 L_2^*$$

holds.

Let us proceed with $\alpha_1 L_1^* \;<\; \alpha_2 L_2^*$. Then we obtain $\lambda_2 = 0$ and the first-order conditions simplify to

$$\begin{aligned}
\frac{\partial \mathcal{L}[.]}{\partial \ell} \;&=\; -\,\frac{[q_4 R_D - R_f + q_4(1-q_4)\gamma W_B R_D^2] \cdot (p_2\alpha_2 - p_1\alpha_1 - \alpha_1\lambda_1)}{(1-q_4)q_4 R_D^2 \gamma} \;&\stackrel{!}{=}\; 0 \\
\frac{\partial \mathcal{L}[.]}{\partial R_D} &= \frac{[(p_1+\lambda_1)\alpha_1\ell + p_2\alpha_2(1-\ell)]\,(2 - q_4 R_D) - (q_4+\lambda_1)R_f R_D}{(1-q_4)q_4 R_D^3 \gamma} \;&\stackrel{!}{=}\; 0 \\
D^u(\ell, R_D; 4) \cdot R_D - \alpha_1\ell\,[D^u(R_D) + W_B] \;&&\stackrel{!}{=}\; 0
\end{aligned}$$

Solving

$$D^u(\ell, R_D; 4) \cdot R_D - \alpha_1\ell\,[D^u(R_D) + W_B] \;\stackrel{!}{=}\; 0$$

for $R_D$ leads to two solutions in $R_D$ whereas exactly one solution is feasible:

$$R_D^*(\ell) \;=\; \frac{q_4\alpha_1\ell + R_f + \sqrt{(q_4\alpha_1\ell + R_f)^2 - 4q_4 R_f \alpha_1\ell\,[1 - (1-q_4)\alpha_1\ell\gamma W_B]}}{2q_4\,[1 - (1-q_4)\alpha_1\ell\gamma W_B]}\;.$$

Solving

$$\frac{\partial \mathcal{L}[.]}{\partial R_D} \;\stackrel{!}{=}\; 0$$

for $\lambda_1$ uniquely leads to

$$\lambda_1^*(R_D^*(\ell), \ell) \;=\; \frac{[p_1\alpha_1\ell + p_2\alpha_2(1-\ell)]\,(2R_f - q_4 R_D) - q_4 R_D R_f}{R_D R_f - (2R_f - q_4 R_D)\alpha_1\ell}\;.$$

Thus, $p_2\alpha_2 - p_1\alpha_1 - \alpha_1\lambda_1^*(R_D^*(\ell), \ell) \;\stackrel{!}{=}\; 0$ is the necessary optimality condition for $\ell$ as the other factor of $\frac{\partial \mathcal{L}}{\partial \ell}$ is always strictly positive. This equation is quadratic in $\ell$ and has as solutions

$$\begin{aligned}
\ell_{(1)} \;&=\; \tfrac{1}{(1-q_4)\alpha_1\gamma W_B} \\
\ell_{(2)} \;&=\; \tfrac{2p_2\alpha_2 R_f\left[(p_2\alpha_2 - R_f)(p_1+p_2-q)\alpha_1 - (p_2\alpha_2 - p_1\alpha_1)R_f\right]}{(p_2^2\alpha_2^2 - R_f^2)q_4^2\alpha_1^2 - (p_2\alpha_2 - p_1\alpha_1)^2 R_f^2 - 2(p_2\alpha_2 - p_1\alpha_1)q_4\alpha_1 R_f^2 + 4q_4(1-q_4)p_2^2\alpha_1^2\alpha_2^2\gamma W_B R_f},
\end{aligned}$$

where the latter is the optimal loan-allocation rate and the former is unfeasible as $R_D^*(\ell)$ diverges at $\ell = \frac{1}{(1-q_4)\alpha_1\gamma W_B}$.

Plugging-in $\ell^* = \ell_{(2)}$ into $R_D^*$ yields $R_D^*$. Furthermore, the first Lagrange-Multiplier simplifies to $\lambda_1^* = \frac{p_2\alpha_2 - p_1\alpha_1}{\alpha_1}$ emphasizing the cost to the risk-neutral bank to grant the minimum feasible amount to Firm 1. The representation of $\ell_{(2)}$ emphasizes the differences in expected gross returns and between expected returns to the risk-free gross yield. For the representation in the result, (3.3.11), we re-arrange terms once again.

Having the optimum $(\ell^*, R_D^*)$, we can use the relation $\alpha_2(1-\ell^*)\left[D^u(R_D^*) + W_B\right] \geq D^u(R_D^*) \cdot R_D^*$ to derive the lower bound that $W_B$ must exceed such that this inequality strictly holds and thus the solution fulfills:

$$\frac{(p_2^2\alpha_2^2 - p_1^2\alpha_1^2)R_f^2 \; - \; q_4^2\alpha_1^2(p_2^2\alpha_2^2 - R_f^2) \; - \; 2q_4(p_2^2\alpha_2^2 - p_1\alpha_1 R_f)\alpha_1 R_f \; + \; 4p_2^2(1-q_4)q_4 R_f \alpha_1^2\alpha_2^2\gamma W_B}{4p_2^2(1-q_4)q_4\alpha_1^2\alpha_2\gamma R_f} >$$
$$> \frac{p_2\alpha_2(q_4\alpha_1 - R_f) - (p_2 - q)\alpha_1 R_f}{2p_2 q_4(1-q_4)\alpha_1\alpha_2\gamma} \; .$$

As only the left-hand side of this inequality depends on $W_B$ and as it is strictly increasing in $W_B$, solving for $W_B$ results in (3.3.9). Is $W_B$ equal to or lower than this bound, the inequality $\alpha_2(1-\ell^*)\left[D^u(R_D^*) + W_B\right] \; \geq \; D^u(R_D^*) \cdot R_D^*$ becomes an equality in equilibrium and thus the solution is characterized by

$$\alpha_1\ell^*\left[D^u(R_D^*) + W_B\right] = D^u(R_D^*) \cdot R_D^*$$

and

$$\alpha_2(1-\ell^*)\left[D^u(R_D^*) + W_B\right] = D^u(R_D^*) \cdot R_D^*$$

resulting in

$$\ell^* \;=\; \frac{\alpha_2}{\alpha_1 + \alpha_2}$$

and

$$R_D^* \;=\; \frac{(\alpha_1+\alpha_2)R_f + q_4\alpha_1\alpha_2 + \sqrt{\left[(\alpha_1+\alpha_2)R_f - q_4\alpha_1\alpha_2\right]^2 + 4q_4(1-q_4)\alpha_1^2\alpha_2^2\gamma W_B R_f}}{2q_4\left[\alpha_1+\alpha_2 - (1-q_4)\alpha_1\alpha_2\gamma W_B\right]} \; .$$

By $\ell^* = \frac{\alpha_2}{\alpha_1+\alpha_2}$ and $\lambda_1^* = \frac{p_2\alpha_2 - p_1\alpha_1}{\alpha_1} + \frac{\alpha_2}{\alpha_1}\lambda_2^*$ from $\frac{\partial\mathcal{L}[.]}{\partial\ell} \overset{!}{=} 0$ , the remaining first-order condition, $\frac{\partial\mathcal{L}[.]}{\partial R_D} \overset{!}{=} 0$, becomes

$$\begin{aligned}\alpha_1^2\alpha_2(p_1+p_2)(2R_f - q_4R_D^*) - q_4\alpha_1(\alpha_1+\alpha_2)R_D^*R_f \; - \\ - \; (p_2\alpha_2 - p_1\alpha_1)\left[(\alpha_1+\alpha_2)R_D^*R_f - \alpha_1\alpha_2(2R_f - q_4R_D^*)\right] \; \overset{!}{=} \; 0 \; ,\end{aligned}$$

yielding

$$\lambda_2^* \;=\; \frac{\left\{p_2\alpha_1\alpha_2 - \left[(p_2 - q)\alpha_1 + p_2\alpha_2\right]R_D^*\right\}R_f - p_2\alpha_1\alpha_2(q_4R_D^* - R_f)}{\alpha_1\alpha_2(q_4R_D^* - R_f) + \left[(\alpha_1+\alpha_2)R_D^* - \alpha_1\alpha_2\right]R_f} \; .$$

Finally, we derive the bounds on $\ell^*$ and on $R_D^*$. Let us show why

$$\ell_{(2)} \in (0, \frac{\alpha_2}{\alpha_1+\alpha_2}) \ ,$$

holds, *i.e.* why the bounds on the loan-allocation rate hold as given in (3.3.11). $\ell_{(2)}$ strictly decreases in $W_B$ with

$$\lim_{W_B\to\infty} \ell_{(2)} = 0^+ \ .$$

Second, $\ell_{(2)}$ becomes

$$\ell_{(2)} = \frac{\alpha_2}{\alpha_1+\alpha_2} \ ,$$

at the point where $W_B$ is equal to the Bound (3.3.9). Thus, the upper bound on $\ell_{(2)}$ is established.

$R_D$ from (3.3.10) strictly increases in $W_B$. For the border case $W_B = 0$, $R_D^* = \frac{1}{q_4}R_f > R_f$, implying $D^* = 0$ as $\underline{R}_D(4) = \frac{1}{q_4}R_f$. Assumption (3.2.7) is crucial here to obtain $R_D^* = \frac{1}{q_4}$ as it ensures $(\alpha_1+\alpha_2)R_f - q_4\alpha_1\alpha_2 > 0$ in connection with $R_f \geq 1$. If $(\alpha_1+\alpha_2)R_f - q_4\alpha_1\alpha_2$ were negative, $R_D^*$ would become $\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ at $W_B = 0$.

If $W_B$ equals the border given by (3.3.9), $R_D^*$ according to (3.3.10) becomes identical to the formula given in (3.3.11). Thus, the latter also exceeds $\underline{R}_D(4)$. □

#### A.2.1.6 Proof of Result 10

*Proof.* If Condition (3.3.9) is satisfied both inequalities defining Case 4 according to (3.2.19) hold with equality. In other words, an optimal and feasible solution under Case 4 is effectively constrained by $W_B$. Let us consider the following first-order conditions, starting with the partial derivative of the Lagrangian with respect to $R_D$:

$$\begin{aligned}
&- q_4\left[D^u(R_D)+W_B\right] + \left[p_1\alpha_1\ell + p_2\alpha_2(1-\ell) - q_4R_D\right]\cdot\tfrac{\partial D^u(R_D)}{\partial R_D} - \dots \\
&- \lambda_1\left[(R_D-\alpha_1\ell)\tfrac{\partial D^u}{\partial R_D} + D^u(R_D)\right] - \lambda_2\left[(R_D-\alpha_2(1-\ell))\tfrac{\partial D^u}{\partial R_D} + D^u(R_D)\right] \overset{!}{=} 0
\end{aligned}$$

$$\begin{aligned}
\lambda_1\cdot\left[D^u(R_D)R_D - \alpha_1\ell(D^u(R_D)+W_B)\right] &\overset{!}{=} 0 \\
\lambda_2\cdot\left[D^u(R_D)R_D - \alpha_2(1-\ell)(D^u(R_D)+W_B)\right] &\overset{!}{=} 0
\end{aligned}$$

As the bounds to Case 4 are binding, we obtain $R_D^* \geq \alpha_1 \ell^*$ and $R_D^* \geq \alpha_2(1-\ell^*)$ where equality holds if and only if $W_B = 0$. Furthermore, calculations show

$$\begin{aligned}
(R_D - \alpha_1 \ell)\tfrac{\partial D^u}{\partial R_D} + D^u(R_D) \quad &= \quad \tfrac{\left[R_D^* - \alpha_1 \ell^*\right] + \alpha_1 \ell^* (q_4 R_D^* - R_f)}{\gamma q_4 (1-q_4)(R_D^*)^3} \quad > \quad 0 \\
\left[R_D - \alpha_2 (1-\ell^*)\right] \tfrac{\partial D^u}{\partial R_D} + D^u(R_D) \quad &= \quad \tfrac{\left[R_D^* - \alpha_2(1-\ell^*)\right] + \alpha_2 (1-\ell^*)(q_4 R_D^* - R_f)}{\gamma q_4 (1-q_4)(R_D^*)^3} \quad > \quad 0
\end{aligned}$$

The positive signs are due to the binding conditions induced by (3.2.19) and by the fact that $R_D^* > \frac{1}{q_4} R_f \equiv \underline{R}_D(4)$ holds as otherwise we did not consider an optimal outcome (*cf.* Result 5). As $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ must hold due to the binding constraints, too, we obtain

$$-\lambda_1 \left[(R_D - \alpha_1 \ell)\frac{\partial D^u}{\partial R_D} + D^u(R_D)\right] - \lambda_2 \left[(R_D - \alpha_2(1-\ell))\,\frac{\partial D^u}{\partial R_D} + D^u(R_D)\right] \;\leq\; 0 \; .$$

Hence,

$$-q_4 \left[D^u(R_D) + W_B\right] + \left[p_1 \alpha_1 \ell + p_2 \alpha_2 (1-\ell) - q_4 R_D\right] \cdot \frac{\partial D^u(R_D)}{\partial R_D} \;\geq\; 0$$

holds, and in particular

$$\frac{\partial D^u(R_D)}{\partial R_D} \;>\; 0 \; . \tag{A.2.13}$$

With respect to $W_B$, the optimal deposit interest rate $R_D^*$ is of the form $f(W_B)$ where

$$f(W_B) \;=\; \frac{a + \sqrt{b + c \cdot W_B}}{d - e \cdot W_B}$$

with $a$, $b$, $c$, $d$, and $e$ strictly positive constants appropriately defined. Obviously, $f(W_B)$ ($R_D^*$, respectively) strictly increases in $W_B$. Altogether, we obtain

$$\frac{dD^u(R_D^*)}{dW_B} \;=\; \frac{\partial D^u(R_D)}{\partial R_D} \cdot \frac{\partial R_D^*}{\partial W_B} \;>\; 0 \; . \tag{A.2.14}$$

□

#### A.2.1.7 Proof of Result 11

*Proof.* As we have assumed $D^u(\ell, R_D; 4) < W_H$ in equilibrium, the maximization problem under Case 4 if both firms' projects are equal is given by

$$\begin{array}{rl} \max\limits_{\ell, R_D} & \mathrm{E}\left[\tilde{W}_B(\ell, R_D; 4)\right] \\ \text{s.t.} & D^u(\ell, R_D; 4) \cdot R_D \;\leq\; \alpha\ell\left[D^u(\ell, R_D; 4) + W_B\right] \\ & D^u(\ell, R_D; 4) \cdot R_D \;\leq\; \alpha(1-\ell)\left[D^u(\ell, R_D; 4) + W_B\right] \end{array}$$

implying the following first-order conditions

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \ell} &= -\frac{\left[(2p-q)R_D - R_f + (2p-q)(1-2p+q)\gamma W_B R_D^2\right]\cdot\alpha\cdot(\lambda_2-\lambda_1)}{(2p-q)(1-2p+q)R_D^2\gamma} \overset{!}{=} 0 \\ \frac{\partial \mathcal{L}}{\partial R_D} &= \frac{\left[p+\ell\lambda_1+(1-\ell)\lambda_2\right]\left[2R_f-(2p-q)R_D\right]\alpha - \left[(2p-q)+\lambda_1+\lambda_2\right]R_f R_D}{(2p-q)(1-2p+q)R_D^3\gamma} \overset{!}{=} 0 \end{aligned}$$

$$\begin{aligned} \lambda_1 \cdot \left\{D^u(\ell, R_D; 4)\cdot R_D - \alpha\ell\left[D^u(\ell, R_D; 4) + W_B\right]\right\} &\overset{!}{=} 0 \\ \lambda_2 \cdot \left\{D^u(\ell, R_D; 4)\cdot R_D - \alpha(1-\ell)\left[D^u(\ell, R_D; 4) + W_B\right]\right\} &\overset{!}{=} 0 \end{aligned}$$

By the first condition we obtain $\lambda_1^* = \lambda_2^*$ reflecting the equality of costs imposed by Case 4 if both projects are equal. Hence,

$$\frac{\partial \mathcal{L}}{\partial \ell} = 0$$

holds for all optimal choice under Case 4 regardless of the structures of the optimal loan-allocation rate and the optimal deposit interest rate.

Consider now $\lambda_1 = 0$ and $\lambda_2 = 0$; that is, none of the constraints constituting Case 4 according to (3.2.19) is binding. Then we obtain

$$\frac{\partial \mathcal{L}}{\partial R_D} = \frac{p\left[2R_f - (2p-q)R_D\right]\alpha - (2p-q)R_f R_D}{(2p-q)(1-2p+q)R_D^3\gamma} \overset{!}{=} 0$$

resulting in

$$R_D^* = \frac{2p\alpha R_f}{(2p-q)(p\alpha + R_f)}.$$

Plugging-in $R_D^*$ into the Case constraints yields the unique lower and the unique upper bound on the optimal loan-allocation rate $\ell^*$, respectively, given in (3.3.15). Analogous to the proof to Result 9 given in Appendix A.2.1.5 we obtain the threshold value for $W_B$ (3.3.12) above which both Case constraints never hold with equality in equilibrium.

Consequently, if $W_B$ is equal or below this threshold value, both Case constraints must hold with equality in equilibrium and by the projects' equality we obtain $\ell^* = \frac{1}{2}$. As both Case constraints are binding, the optimal interest rate $R_D^*$ solves $\alpha\frac{1}{2}\left[D^u(R_D^* + W_B\right] \stackrel{!}{=} D^u(R_D^*) \cdot R_D^*$.

The Lagrange multipliers $\lambda_1^* = \lambda_2^* = \lambda^*$ can be obtained from

$$(p + \lambda^*)\left[2R_f - (2p-q)R_D^*\right]\alpha - \left[(2p-q) + 2\lambda^*\right]R_f R_D^* \;\stackrel{!}{=}\; 0$$

after having simplified the numerator of $\frac{\partial \mathcal{L}[.]}{\partial R_D}$, yielding

$$\lambda^* \;=\; \frac{\left[p\alpha - (2p-q)R_D^*\right]R_f - \left[(2p-q)R_D^* - R_f\right]p\alpha}{\left[(2p-q)R_D^* - R_f\right]\alpha + (2R_D^* - \alpha)R_f}\;.$$

Finally, let us analyze the bounds of $\ell^*$ and $R_D^*$. The optimal deposit interests rates are special cases of those shown by Result 9. Thus the bounds and the behavior in $W_B$ are given in an analogous way.

Concerning $\ell$, let us consider the lower bound first,

$$\frac{2pR_f(p\alpha - R_f)}{q_4(p^2\alpha^2 - R_f^2) + 4p^2\alpha^2(1-q_4)\gamma W_B R_f}\;.$$

If $W_B$ equals the bound stated in (3.3.12), this term becomes $\frac{1}{2}$. As it strictly decreases in $W_B$, it is bounded from below by zero, *i.e.* by its limit value for $W_B \to \infty$. Even if $W_B = 0$ were allowed for this solution, this bound would remain lower than one:

$$\frac{2pR_f(p\alpha - R_f)}{q_4(p^2\alpha^2 - R_f^2)} \;=\; \frac{2pR_f}{q_4(p\alpha + R_f)} \;<\; 1 \;\stackrel{q_4 \equiv 2p-q}{\Leftrightarrow}\; qR_f \;<\; (2p-q)p\alpha\;.$$

The upper bound stated in (3.3.15),

$$\frac{(p\alpha - R_f)\left[q_4 p\alpha - qR_f\right] + 4p^2\alpha^2(1-q_4)\gamma W_B R_f}{q_4(p^2\alpha^2 - R_f^2) + 4p^2\alpha^2(1-q_4)\gamma W_B R_f}$$

becomes $\frac{1}{2}$ if $W_B$ is equal to the bound stated in (3.3.12), too. It strictly increases in $W_B$ and is bounded from above by its limit value for $W_B \to \infty$. This limit value is equal to one. Taking the derivative of this upper bound w/r/t $W_B$ shows that the strict monotonicity in $W_B$ is guaranteed by

$$q_4(p^2\alpha^2 - R_f^2) - (p\alpha - R_f)\left[q_4 p\alpha - qR_f\right] \;=\; (p\alpha - R_f)(q_4 + q)R_f \;=\; 2p(p\alpha - R_f)R_f \;>\; 0\,.$$

Consider Equilibrium (3.3.13). After some algebraic manipulation, the sensitivity of $R_D^*$, given by (3.3.14), w/r/t $q_4$ can be displayed as follows,

$$\frac{q_4\alpha\gamma W_B\sqrt{Q_4^*(p,q,\alpha)}+2R_f[2-(1-q_4)\alpha\gamma W_B]}{q_4[2-(1-q_4)\alpha\gamma W_B]\sqrt{Q_4^*(p,q,\alpha)}}\cdot R_D^* \;+\; \frac{(2-\alpha\gamma W_B)\alpha R_f}{q_4[2-(1-q_4)\alpha\gamma W_B]\sqrt{Q_4^*(p,q,\alpha)}}\ ,$$

where $Q_4^*(p,q,\alpha) = \left[2R_f - q_4\alpha\right]^2 \;+\; 4q_4(1-q_4)\alpha^2\gamma W_B R_f$. This expression for $\frac{\partial R_D^*}{\partial q_4}$ is positive since $2R_f \geq \alpha R_f$ due to (3.2.7) and since $2-(1-q_4)\alpha\gamma W_B \;>\; 2-\alpha\gamma W_B$ due to $q_4 > 0$ hold.

Consider Equilibrium (3.3.15). The sensitivities of $R_D^*$ w/r/t $q$, $p$, and $\alpha$ can be expressed as follows:

$$\begin{aligned}
\frac{\partial R_D^*}{\partial q} &= \underbrace{\frac{\partial R_D^*}{\partial q_4}}_{<0}\cdot\underbrace{\frac{\partial q_4}{\partial q}}_{=-1} \;>\; 0\ ,\\
\frac{\partial R_D^*}{\partial p} &= \frac{-2\alpha R_f(2p^2\alpha + qR_f)}{q_4^2(p\alpha+R_f)^2} \;<\; 0\ , \text{ and}\\
\frac{\partial R_D^*}{\partial \alpha} &= \frac{2pR_f^2}{q_4(p\alpha+R_f)^2} \;>\; 0\ .
\end{aligned}$$

□

#### A.2.1.8 Proof of Result 12

*Proof.* Suppose that the household is not constrained by its initial wealth $W_H$. The sign of the expression $q_jR_D(R_D-\mu_j)-\sigma_j^2$ determines the sign of the derivative $\frac{\partial D^u(\cdot)}{\partial W_B}$.

Let us consider Case 1 first. Since both projects are assumed to be equal, *i.e.* $p_1 = p_2 = p$ and $\alpha_1 = \alpha_2 = \alpha$, the expression $q_1R_D(R_D-\mu_1)-\sigma_1^2$ becomes

$$(p-q)\alpha\ell\left[\mu_1-\alpha\ell\right]+(p-q)\alpha(1-\ell)\left[\mu_1-\alpha(1-\ell)\right]\ .$$

As the optimal loan-allocation rate $\ell^*$ equals $\frac{1}{2}$, this expression further simplifies to

$$(p-q)\alpha\cdot\left[\mu_1-\alpha\frac{1}{2}\right]\ .$$

Due to $\ell^* = \frac{1}{2}$ according to Result 6 and $\alpha_i \leq 2$ according to (3.2.7), the relation $\alpha\ell^* \leq 1$ holds. Suppose now that $\mu_1 < \alpha\ell^* \leq 1$ hold. As a consequence, we obtain $\mu_1 < R_f$ as $R_f \geq 1$. Thus, $D^u(\ell^*, R_D^*) < 0$ and by excluding short-selling

$D^s(\ell^*, R_D^*) = 0$ contradicting $D^s(\ell^*, R_D^*) > 0$ stated by Result 5. Thus $\mu_1 > \alpha\ell^* \geq 1$ holds in $(\frac{1}{2}, R_D^*, 1)$ and hence

$$\frac{\partial D^s(\cdot)}{\partial W_B} \;>\; 0 \tag{A.2.15}$$

in equilibrium.

Concerning Case 2 and 3 and $\ell^* \neq 0$ or, $\ell^* \neq 1$, Result 4 applies. Only if the bank chooses a corner solution in the sense that either $\ell^* = 0$ (Case 2), or $\ell^* = 1$ (Case 3) holds, the dependence of the deposit volume from bank capital vanishes and thus the cushion effect vanishes, too. In essence, the same argumentation holds true if Case 4 prevails. These notions have been already captured under Result 1. □

## A.2.2 The Equilibrium with Regulation by Fixed Risk Weights

### A.2.2.1 Proof of Result 14

*Proof.* The first-order conditions to the maximization problem considered in Result 14 are:

$$\begin{aligned}
q \cdot (\alpha - R_D) \cdot \frac{\partial D^u(\ell, R_D; 1)}{\partial \ell} &\overset{!}{=} \lambda \cdot \left[\frac{\partial D^u(\ell, R_D; 1)}{\partial \ell} - k'(\ell) W_B\right] \\
q \cdot (\alpha - R_D) \cdot \frac{\partial D^u(\ell, R_D; 1)}{\partial R_D} \;-\; q \cdot D^u(\ell, R_D; 1) &\overset{!}{=} \lambda \cdot \frac{\partial D^u(\ell, R_D; 1)}{\partial R_D} \\
\lambda \cdot [D^u(\ell, R_D; 1) \;-\; k(\ell) \cdot W_B] &\overset{!}{=} 0
\end{aligned}$$

As both loans exhibit equal characteristics, especially concerning their credit risk, we have $c_1 = c_2$ and hence $k(\ell) = \frac{1-cc_1}{cc_1}$, *i.e.* $k(\ell)$ is independent of the loan-allocation rate $\ell$. Let us rewrite the first-order conditions as

$$\begin{aligned}
[q \cdot (\alpha - R_D) - \lambda] \cdot \frac{\partial D^u(\ell, R_D; 1)}{\partial \ell} &\overset{!}{=} 0 \\
[q \cdot (\alpha - R_D) - \lambda] \cdot \frac{\partial D^u(\ell, R_D; 1)}{\partial R_D} &\overset{!}{=} q \cdot D^u(\ell, R_D; 1) \\
\lambda \cdot [D^u(\ell, R_D; 1) \;-\; k(\ell) \cdot W_B] &\overset{!}{=} 0
\end{aligned}$$

Let us discuss all possible cases that are able to solve this non-linear system of equation.

First, let us consider $q\cdot(\alpha - R_D^S) - \lambda^S \;=\; 0$. This implies coercively $D^u(\ell^S, R_D^S, 1) = 0$ by the second equation. The third equation thus necessitates $\lambda^S = 0$ resulting recursively in $R_D^S = \alpha$. Then the bank's expected final wealth is only $q\alpha W_B$ as it forgoes any expected gains from intermediation although the regulatory constraint has not been (fully) exploited. Thus, a solution such as this contradicts optimality.

Second, we consider $q \cdot (\alpha - R_D^S) - \lambda^S \neq 0$. Thus we obtain $\frac{\partial D^u(\ell, R_D; 1)}{\partial \ell} = 0$ uniquely resulting in $\ell^S = \frac{1}{2}$. Furthermore, $q \cdot (\alpha - R_D^S) - \lambda^S \neq 0$ excludes $D^u(\ell, R_D; 1) = 0$ as the bank would forgo expected profits from intermediation without being forced to do so. By the short-sell constraint on deposits we obtain $D^u(\frac{1}{2}, R_D^S, 1) > 0$ which implies two subcases to consider.

First, we assume $q \cdot (\alpha - R_D^S) - \lambda^S < 0$ implying $\frac{\partial D^u(\ell, R_D; 1)}{\partial R_D} < 0$. By the first-order condition with respect to $R_D$ we obtain

$$\frac{\partial \mathrm{E}\left[\tilde{W}_B(\cdot)\right]}{\partial R_D} \;=\; q \cdot (\alpha - R_D) \cdot \frac{\partial D^u(\frac{1}{2}, R_D; 1)}{\partial R_D} \;-\; q \cdot D^u(\frac{1}{2}, R_D; 1) \;<\; 0$$

because of $\alpha - R_D > 0$ as anything else would contradict optimality. By Result 5, the optimal interest rate $R_D^*$ is unique for given loan-allocation rates. Here, we have just shown the first derivative of the unregulated bank's objective function. As this derivative is negative for $(\frac{1}{2}, R_D^S)$, uniqueness, as shown in the proof A.2.1.1, implies that $R_D^S > R_D^*$ for $\ell = \frac{1}{2}$. Because of the last first-order condition given by the regulatory constraint,

$$D^s(\frac{1}{2}, R_D^S, 1) \leq \frac{1 - cc_1}{cc_1} \cdot W_B$$

must hold. As we have assumed that regulation is binding, we have $D^s(\frac{1}{2}, R_D^S, 1) < D^*$. Thus, $R_D^S$ must have already surpassed the critical rates $\overline{R}_D(1)$, and $\overline{R}_D^u(1)$, respectively, where the latter is defined by (3.2.52). However, a deposit interest rate with $R_D^S > \overline{R}_D(1) > R_D^*$ contradicts optimality as the bank could choose a rate $R_D'$, $\overline{R}_D(1) > R_D^* > R_D'$, such that $D^s(\frac{1}{2}, R_D', 1) = \frac{1-cc_1}{cc_1} \cdot W_B$ holds, *i.e.* such that the same deposit volume as before can be attained, but that leads to a wider spread $\alpha - R_D$ that the bank can earn through intermediation. Note that the existence of such an interest rate $R_D'$ is due to the hump-shaped deposit-supply function characterized by Result 2. Thus, the subcase just considered contradicts optimality as well.

The second subcase and the only remaining case to consider is given by $q \cdot (\alpha - R_D^S) - \lambda^S > 0$. In analogy to the subcase just discussed we are now obtaining

$$\frac{\partial D^u(\frac{1}{2}, R_D; 1)}{\partial R_D} > 0$$

Consequently, a deposit interest rate $R_D^S$ with $R_D^* < R_D^S < \overline{R}_D(1)$ contradicts the assumption of a binding regulation. Thus, given $\frac{\partial D^u(\frac{1}{2}, R_D; 1)}{\partial R_D} > 0$ only an interest rate $R_D^S$ with $R_D^* > R_D^S$ may satisfy the regulatory constraint $\lambda \cdot \left[D^u(\frac{1}{2}, R_D; 1) - \frac{1-cc_1}{cc_1} \cdot W_B\right] = 0$. Furthermore, $R_D^S > \underline{R}_D(1)$ holds as otherwise the bank forgo expected from profits from intermediation without being forced to do so.

As $\underline{R}_D(1) < R_D^S < R_D^*$,

$$q \cdot (\alpha - R_D^S) \cdot \frac{\partial D^u(\frac{1}{2}, R_D^S, 1)}{\partial R_D^S} \; - \; q \cdot D^u(\frac{1}{2}, R_D^S, 1)$$

holds, we obtain $\lambda^S > 0$ and thus $D^S = \frac{1-cc_1}{cc_1} \cdot W_B$.

Note that the objective

$$\mathrm{E}\left[\tilde{W}_B(\ell, R_D; 1)\right] \; = \; q \cdot (\alpha - R_D) \cdot D^u(\ell, R_D; 1) \; + \; q\alpha W_B$$

cannot be heightened if corner solutions for $R_D$, such as $\underline{R}_D(1)$ or $\alpha$, would be chosen for any fixed loan-allocation rate $\ell$ that is compatible with Case 1. Furthermore, the following partial derivative becomes by the projects' equality

$$\frac{\partial D^u(\ell, R_D; 1)}{\partial \ell} \; = \; \frac{\partial D^u(\ell, R_D; 1)}{\partial \sigma^2} \cdot \frac{\partial \sigma^2}{\partial \ell}$$

implies $\ell^S = \frac{1}{2}$ as solution to the appropriate first-order condition of the bank for any given $R_D$. Furthermore, the second partial derivative of the bank's objective function with respect to $\ell$ becomes

$$\begin{aligned}
\left.\frac{\partial^2 \mathrm{E}[\tilde{W}_B(\ell, R_D; 1)]}{\partial \ell^2}\right|_{\ell = \frac{1}{2}} \; &= \; q \cdot (\alpha - R_D) \cdot \left[\frac{\partial^2 D^u(\cdot)}{\partial (\sigma_1^2)^2} \cdot \left(\frac{\partial \sigma_1^2}{\partial \ell}\right)^2 \; + \; \frac{\partial D^u(\cdot)}{\partial \sigma_1^2} \cdot \left.\frac{\partial^2 \sigma_1^2}{\partial \ell^2}\right|_{\ell = \frac{1}{2}}\right] \\
&= \; q \cdot (\alpha - R_D) \cdot \frac{\partial D^u(\cdot)}{\partial \sigma_1^2} \cdot \frac{\partial^2 \sigma_1^2}{\partial \ell^2} \; < \; 0
\end{aligned}$$

confirming that $\ell^S = \frac{1}{2}$ is a local maximizer given any $R_D$. As $\ell^S = \frac{1}{2}$ is the only critical point, it is also the only global maximizer with respect to the loan-allocation rate for any deposit interest rate $R_D$. Thus, we obtain as solution $\ell^S = \frac{1}{2}$ and

$$R_D^S \; = \; \frac{cc_1 + (2 - cc_1)(p - q)\alpha\gamma W_B - \sqrt{Q_1^S(c_1, \alpha)}}{2(1 - cc_1)(1 - q)\gamma W_B}$$

where $Q_1^S(c_1, \alpha)$ is as given by the result. The sensitivities of $R_D^S$ are derived by total differentiation of the optimal deposit volume. By $\ell^S = \frac{1}{2}$, we obtain $L^S = \frac{1}{cc_1} \cdot W_B$

and hence $D^S = \frac{1-cc_1}{cc_1} \cdot W_B$. Since

$$\frac{dD^S}{d\theta} \equiv \frac{\partial D^s(\frac{1}{2}, R_D^S; 1)}{\partial \theta} + \underbrace{\frac{\partial D^s(\frac{1}{2}, R_D^S; 1)}{\partial R_D}}_{>0 \text{ by Res.(5)}} \cdot \frac{\partial R_D^S}{\partial \theta}$$

holds for any parameter $\theta$, we subsequently obtain the signs for the deposit interest rate's sensitivities, as shown in the result. Specifically, for $\theta = \gamma$,

$$\frac{\partial D^s(\frac{1}{2}, R_D^S; 1)}{\partial \gamma} < 0$$

holds, resulting in

$$\frac{\partial R_D^S}{\partial \gamma} > 0$$

because of

$$\frac{dD^S}{d\gamma} = 0 \ .$$

Since $R_D^S$ is a function of $\gamma \cdot W_B$, and $\gamma, W_B > 0$,

$$\frac{\partial R_D^S}{\partial W_B} > 0$$

must be true, too. Analogous to $\gamma$, we can proceed with $R_f$. Finally,

$$\frac{dD^S}{dc} = -\frac{1}{c^2 c_1} \cdot W_B < 0$$

results in connection with

$$\frac{\partial D^s(\frac{1}{2}, R_D^S; 1)}{\partial c} = 0$$

in

$$\frac{\partial R_D^S}{\partial c} < 0 \ .$$

The same reasoning applies to $c_1$ and $c_2$, respectively, as they are assumed to be identical. □

#### A.2.2.2 Proof of Result 16

*Proof.* By assumption, $\ell^S = 0$ and hence $j^S = 2$ is optimal. Thus,

$$\begin{aligned} \frac{\partial \mathrm{E}[\tilde{W}_B(0, R_D; 2)]}{\partial R_D} &= \lambda \cdot \frac{\partial D^s(0, R_D; 2)}{\partial R_D} \\ \lambda \cdot [D^s(0, R_D; 2) - k(0) \cdot W_B] &= 0 \end{aligned}$$

must be solved for $\lambda$ and $R_D$ yielding

$$\begin{aligned}\lambda^S &= \frac{p_2 \cdot \left[\alpha_2 \cdot \sqrt{c^2 c_2^2 p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f} - cc_2 R_f\right]}{cc_2 \cdot \sqrt{c^2 c_2^2 p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f}} \\ R_D^S &= \frac{cc_2 p_2 - \sqrt{c^2 c_2^2 p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f}}{2(1-cc_2)p_2(1-p_2)\gamma W_B} \; .\end{aligned}$$

The sensitivity of $R_D^S$ with respect to $W_B$ is thus

$$\frac{\partial R_D^S}{\partial W_B} = \frac{1}{W_B} \cdot \left( \frac{cc_2}{\sqrt{c^2 c_2^2 p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f}} \cdot R_f \; - \; R_D^S \right) \; .$$

$\frac{\partial R_D^S}{\partial W_B}$ is thus positive if

$$\frac{cc_2}{\sqrt{c^2 c_2^2 p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f}} \cdot R_f \; > \; R_D^S$$

holds. Plugging-in the formula for $R_D^S$ and multiplying this inequality cross-wise, *i.e.* the left-hand side with the denominator of $R_D^S$ and the right-hand side by the square-root term yields:

$$\begin{gathered} 2cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f \\ > \\ cc_2 p_2 \cdot \sqrt{c^2 c_2^2 p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f} \; - \\ - \; c^2 c_2^2 p_2^2 \; + \; 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f \; . \end{gathered}$$

Simplifying and re-arranging terms yields

$$cc_2 p_2 - \sqrt{c^2 c_2^2 p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f} \; > \; 2(1-cc_2)(1-p_2)\gamma W_B R_f \; ,$$

which is true if and only if

$$R_D^S \; > \; R_f$$

holds.

Let us show that even $R_D^S > \frac{1}{p_2} \cdot R_f$ holds which is the case if and only if

$$cc_2 p_2 - \sqrt{c^2 c_2^2 p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f} \; > \; 2(1-cc_2)p_2(1-p_2)\gamma W_B \cdot \frac{1}{p_2} R_f$$

is true. Note that $cc_2 \leq 0.08 \cdot 1.5 = 0.12 < 1$. Re-ordering yields

$$cc_2p_2 - 2(1-cc_2)(1-p_2)\gamma W_B R_f \;>\; \sqrt{c^2c_2^2p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f}\;.$$

Assume that the term under the square-root is positive (as a result, the left-hand is then positive, too, which can be easily seen if the left-hand side is multiplied by $cc_2p_2 > 0$). Taking the square of both sides leads to

$$\begin{gathered} c^2c_2^2p_2^2 \;+\; 4(1-cc_2)^2(1-p_2)^2\gamma^2W_B^2R_f^2 \;-\; 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f \;> \\ >\; c^2c_2^2p_2^2 \;-\; 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f \\ \Leftrightarrow \\ 4(1-cc_2)^2(1-p_2)^2\gamma^2W_B^2R_f^2 \;>\; 0\;, \end{gathered}$$

which is always true.

Finally, let us show that the term under the square-root must be positive if regulation is binding in the case here considered. The term under the square root strictly decreases in $W_B$. Thus, the highest value $W_B$ can take such that the whole square root remains in the reals, is given by

$$c^2c_2^2p_2^2 \overset{!}{=} 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f\;.$$

Plugging-in into $R_D^S$ yields

$$R_D^S \;=\; \frac{2R_f}{p_2}\;, \quad \text{if} \quad W_B = \frac{cc_2p_2}{4(1-cc_2)(1-p_2)\gamma R_f}\;,$$

resulting in

$$D^S \;=\; \frac{p_2}{4(1-p_2)\gamma R_f}\;.$$

But this deposit volume is equal to

$$D^S \;=\; \frac{1-cc_2}{cc_2}\cdot W_B$$

iff

$$cc_2p_2 \;=\; 4(1-cc_2)(1-p_2)\gamma W_B R_f$$

holds, *i.e.* iff the assumption

$$c^2c_2^2p_2^2 \overset{!}{=} 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f$$

is fulfilled. In total,

$$\frac{\partial R_D^S}{\partial W_B} \;>\; 0$$

has been thus shown.

Combining $\frac{\partial R_D^S}{\partial W_B} > 0$ with the sensitivity of the optimal deposit volume with respect to $R_D^S$,

$$\frac{\partial D^s(0, R_D^S, 2)}{\partial R_D^S} \;=\; \frac{2R_f - p_2 R_D^S}{\gamma p_2(1-p_2)(R_D^S)^3}$$

results in the sensitivity of the total loan volume with respect to $W_B$. The other sensitivities of $R_D^S$ are as follows:

$$\begin{aligned}
\frac{\partial R_D^S}{\partial \alpha_2} &= 0 \\
\frac{\partial R_D^S}{\partial p_2} &= -\, cc_2 \cdot \left[\frac{p_2 R_D^S - R_f}{1-p_2} + \frac{1}{p_2} R_f\right] \\
\frac{\partial R_D^S}{\partial R_f} &= \frac{cc_2}{\sqrt{c^2 c_2^2 p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f}} \\
\frac{\partial R_D^S}{\partial c_2} &= -\, \frac{c \cdot (p_2 R_D^S - R_f)}{(1-cc_2) \cdot \sqrt{c^2 c_2^2 p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)\gamma W_B R_f}}
\end{aligned}$$

Alternatively, the signs of the sensitivities of the deposit interest rate $R_D^S$ can be determined according to

$$\frac{\partial D^s(0, R_D^S, 2)}{\partial R_D^S} \cdot \frac{\partial R_D^S}{\partial \theta} \;+\; \frac{\partial D^s(0, R_D^S, 2)}{\partial \theta} \;=\; \frac{\partial \left(\frac{1}{cc_2} W_B\right)}{\partial \theta},$$

is regulation is assumed to be binding and $\ell^S$ to be fixed to zero. □

#### A.2.2.3 Proof of Result 17

*Proof.* According to Result 8, the sensitivity of the deposit interest rate with respect to the risk-free rate is given by

$$\frac{\partial R_D^*}{\partial R_f} \;=\; \frac{2p_2\alpha_2^2}{(p_2\alpha_2 + R_f)^2}$$

and according to Result 16 that under regulation by

$$\frac{\partial R_D^S}{\partial R_f} \;=\; \frac{cc_2}{\sqrt{c^2 c_2^2 p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)R_f\gamma W_B}}$$

Suppose now that

$$\frac{\partial R_D^*}{\partial R_f} > \frac{\partial R_D^S}{\partial R_f}$$

holds. That is equivalent to

$$\begin{aligned}
2p_2\alpha_2^2 \cdot \sqrt{c^2c_2^2p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)R_f\gamma W_B} &> cc_2 \cdot (p_2\alpha_2 + R_f)^2 \\
&\Leftrightarrow \\
p_2\alpha_2^2 \cdot \sqrt{\ldots} - cc_2R_f^2 - 2cc_2p_2\alpha_2R_f &> p_2\alpha_2^2\left(cc_2p_2 - \sqrt{\ldots}\right) \\
&\Leftrightarrow \\
-p_2\alpha_2 \underbrace{\left[\underbrace{2R_f}_{\geq 2} cc_2 - \underbrace{\alpha_2}_{\leq 2} \cdot \sqrt{\ldots}\right]}_{\geq 2\cdot[cc_2-\sqrt{\ldots}]>0} - cc_2R_f^2 &> p_2\alpha_2^2 \underbrace{\left(cc_2p_2 - \sqrt{\ldots}\right)}_{>0},
\end{aligned}$$

resulting in a contradiction as especially

$$cc_2p_2 - \sqrt{c^2c_2^2p_2^2 - 4cc_2(1-cc_2)p_2(1-p_2)R_f\gamma W_B} > 0$$

holds since $R_D^S > 0$ is true. Thus, we obtain

$$0 < \frac{\partial R_D^*}{\partial R_f} < \frac{\partial R_D^S}{\partial R_f},$$

meaning that the deposit interest rate becomes pro-cyclical by regulation concerning changes in the risk-free rate according to Definition (2.2.1). □

#### A.2.2.4 Proof of Result 18

*Proof.* Case 4 requires

$$\alpha_1\ell^S(D^S + W_B) \geq D^SR_D^S \quad \text{and} \quad \alpha_2(1-\ell^S)(D^S + W_B) \geq D^SR_D^S$$

to hold. As regulation is supposed to be binding and by having assumed $D^S < W_H$ to hold in equilibrium, the deposit volume satisfies (3.3.21) and both Case constraints become

$$\alpha_1\ell^S \geq \left\{1 - c \cdot \left[c_1\ell^S + c_2(1-\ell^S)\right]\right\} \cdot R_D^S$$

and

$$\alpha_2(1-\ell^S) \geq \left\{1 - c \cdot \left[c_1\ell^S + c_2(1-\ell^S)\right]\right\} \cdot R_D^S$$

resulting for $R_D^S$ in

$$R_D^S \ \leq \ \min\left\{ \frac{\alpha_1 \ell^S}{1 - c \cdot [c_1 \ell^S + c_2(1-\ell^S)]}, \ \frac{\alpha_2(1-\ell^S)}{1 - c \cdot [c_1 \ell^S + c_2(1-\ell^S)]} \right\} .$$

Due to $\alpha_2 \geq \alpha_1$, $R_D^S$ can be at most

$$R_D^S \ \leq \ \min\left\{\ell^S, (1-\ell^S)\right\} \cdot \frac{\alpha_2}{1 - c \cdot [c_1 \ell^S + c_2(1-\ell^S)]} .$$

Because of $\ell \in [0,1]$, the highest upper bound to $\min\{\ell^S, (1-\ell^S)\}$ is a half, yielding

$$R_D^S \ \leq \ \frac{\alpha_2 \cdot \frac{1}{2}}{1 - c \cdot [c_1 \ell^S + c_2(1-\ell^S)]} .$$

This upper bound may not go below the zero of the deposit-supply function in Case 4, *i.e.* not below $\frac{1}{q_4} R_f$. If it did, $D^S = 0$, being a contradiction to (3.3.21) with $W_B > 0$. Hence, $\alpha_2$ must exceed the threshold

$$\alpha_2 \ > \ \frac{2 \cdot \left\{1 - c \cdot \left[c_1 \ell^S + c_2(1-\ell^S)\right]\right\}}{q_4} \cdot R_f \ \overset{c=0.08, c_i \leq 1.5}{\geq} \ \frac{1.76}{q_4} \cdot R_f$$

if regulation shall be binding in Case 4. □

#### A.2.2.5 Proof of Result 19

*Proof.* As we have assumed $D^u(\ell, R_D; 4) < W_H$ in equilibrium, we have to consider the following maximization problem for the bank under Case 4 under regulation with fixed risk weights

$$\begin{aligned}
\max_{\ell, R_D} \quad & \mathrm{E}\left[\tilde{W}_B(\ell, R_D; 4)\right] \\
\text{s.t.} \quad & D^u(\ell, R_D; 4) \cdot R_D \ \leq \ \alpha_1 \ell \left[D^u(\ell, R_D; 4) + W_B\right] \\
& D^u(\ell, R_D; 4) \cdot R_D \ \leq \ \alpha_2 (1-\ell) \left[D^u(\ell, R_D; 4) + W_B\right] \\
& c \cdot [c_1 \ell + c_2(1-\ell)] \left[D^u(\ell, R_D; 4) + W_B\right] \ \leq \ W_B
\end{aligned}$$

implying the following first-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \ell} &= -\frac{\left[q_4 R_D - R_f + q_4(1-q_4)\gamma W_B R_D^2\right]}{(1-q_4)q_4 R_D^2 \gamma} \cdot \\
&\quad \cdot \left[\alpha_2(p_2+\lambda_2) - \alpha_1(p_1+\lambda_1) + c(c_1-c_2)\lambda_3\right] &\stackrel{!}{=} 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R_D} &= -\frac{(q_4+\lambda_1+\lambda_2)R_f R_D + q_4 R_D \cdot \lambda_3 c[c_1\ell + c_2(1-\ell)]}{(1-q_4)q_4 R_D^3 \gamma} \\
&\quad + \frac{\{\alpha_1\ell(p_1+\lambda_1) + \alpha_2(1-\ell)(p_2+\lambda_2) - \lambda_3 c[c_1\ell+c_2(1-\ell)]\}(2R_f - q_4 R_D)}{(1-q_4)q_4 R_D^3\gamma} &\stackrel{!}{=} 0
\end{aligned}$$

$$\begin{aligned}
\lambda_1 \cdot \{D^u(\ell, R_D; 4)\cdot R_D - \alpha_1\ell\left[D^u(\ell,R_D;4) + W_B\right]\} &\stackrel{!}{=} 0 \\
\lambda_2 \cdot \{D^u(\ell, R_D; 4)\cdot R_D - \alpha_2(1-\ell)\left[D^u(\ell,R_D;4) + W_B\right]\} &\stackrel{!}{=} 0 \\
\lambda_3 \cdot \{c\cdot[c_1\ell + c_2(1-\ell)]\left[D^u(\ell,R_D;4) + W_B\right] - W_B\} &\stackrel{!}{=} 0
\end{aligned}$$

By assumption, regulation is binding. As both projects are different in expected returns and in variance, at least one Case constraint is binding, *cf.* Result 9 and Appendix A.2.1.5.

Let (3.3.33) prevail. Then, solving

$$\begin{aligned}
c\cdot[c_1\ell + c_2(1-\ell)]\left[D^u(R_D) + W_B\right] &\stackrel{!}{=} W_B \\
\alpha_1\ell\left[D^u(R_D) + W_B\right] &\stackrel{!}{=} D^u(R_D)\cdot R_D
\end{aligned}$$

for $\ell$ and $R_D$ leads to (3.3.34) after having excluded unfeasible outcomes. The shadow cost of regulation amounts to

$$\lambda_3^S = \frac{p_2\alpha_2\left[(\alpha_1 - R_D^S)R_f - (q_4 R_D^S - R_f)\alpha_1\right] - (p_2-q)\alpha_1 R_D^S R_f}{c\left[c_2\alpha_1(2R_f - q_4 R_D^S) - (c_2-c_1)R_D^S R_f\right]} \geq 0$$

whose positive sign results from the assumption that regulation is strictly binding. The numerator is positive if and only if

$$R_D^S \leq \frac{2p_2\alpha_1\alpha_2 R_f}{(p_2\alpha_2 - p_1\alpha_1)R_f + q_4\alpha_1(p_2\alpha_2 + R_f)} \stackrel{(3.3.11)}{\equiv} R_D^*$$

is satisfied, where the latter equivalence refers to the optimal choice in Case 4 without regulation that results in $\alpha_1 L_1^* < \alpha_2 L_2^*$. The denominator is strictly positive if and only if

$$R_D^S < \frac{2c_2\alpha_1 R_f}{(c_2-c_1)R_f + c_2 q_4\alpha_1}$$

is satisfied which is the case for $R_D^S < R_D^*$ as $R_D^* < \frac{2c_2\alpha_1 R_f}{(c_2-c_1)R_f + c_2q_4\alpha_1}$ holds. This inequality can be traced back to $0 < c_1 p_2\alpha_2 + c_2(p_2-q)\alpha_1$. Thus, binding regulation implies a reduction in the deposit rate.[3]

The shadow cost of granting a minimum to Firm 1 equals

$$\lambda_1^S = \frac{(c_1p_2\alpha_2 - c_2p_1\alpha_1)(2R_f - q_4R_D^S) + (c_2 - c_1)q_4R_D^SR_f}{c_2\alpha_1(2R_f - q_4R_D^S) - (c_2 - c_1)R_D^SR_f} \; .$$

Hence, $c_1p_2\alpha_2 - c_2p_1\alpha_1 \geq 0$ is sufficient to make the constraint $\alpha_1\ell\left[D^u(R_D) + W_B\right] \geq D^u(R_D)\cdot R_D$ be binding as $c_2 > c_1$ has been assumed. If $c_2 = c_1$, $c_1p_2\alpha_2 - c_2p_1\alpha_1 \geq 0$ is necessary, and solving the first-order conditions results in (3.3.36) for $\ell^S$ and (3.3.34) again for $R_D^S$. Note that (3.3.34) is not feasible for $\ell^S$ if $c_1 = c_2$ holds.

Likewise, if (3.3.38) prevails, the Solution (3.3.39) can be obtained and the associated Lagrange multipliers are given by

$$\lambda_3^S = \frac{p_1\alpha_1\left[(\alpha_2 - R_D^S)R_f - (q_4R_D^S - R_f)\alpha_2\right] - (p_1 - q)\alpha_2R_D^SR_f}{c\left[c_1\alpha_2(2R_f - q_4R_D^S) + (c_2 - c_1)R_D^SR_f\right]} \geq 0 \; ,$$

and

$$\lambda_2^S = \frac{(c_2p_1\alpha_1 - c_1p_2\alpha_2)(2R_f - q_4R_D^S) - (c_2 - c_1)q_4R_D^SR_f}{c_1\alpha_2(2R_f - q_4R_D^S) + (c_2 - c_1)R_D^SR_f}$$

which is positive if

$$c_2p_1\alpha_1 - c_1p_2\alpha_2 > \frac{(c_2 - c_1)q_4R_D^SR_f}{2R_f - q_4R_D^S}$$

holds. Lacking un upper bound to $R_D^S$ that is tight enough, we simply plug-in $R_D^S$ into the formula from above to obtain the following condition for $\lambda_2^S > 0$ to hold:

$$\frac{p_1\alpha_1}{p_2\alpha_2} \geq \frac{c_1}{c_2} + \frac{c_2-c_1}{c_2}\cdot\frac{R_f}{p_1\alpha_1}\cdot\frac{c(c_2-c_1)R_f - cc_1q_4\alpha_2 + \sqrt{Q_4^S(c_2,c_1,\alpha_2)}}{3cc_1(c_2-c_1)R_f + cc_1q_4\alpha_2 - 4(1-cc_1)(1-q_4)\alpha_2\gamma W_BR_f - \sqrt{Q_4^S(c_2,c_1,\alpha_2)}}$$

where

$$Q_4^S(c_2, c_1, \alpha_2) = c^2\left[c_1q_4\alpha_2 + (c_2 - c_1)R_f\right]^2 - 4cc_1(1 - cc_1)(1 - q_4)q_4\alpha_2^2\gamma W_BR_f \; .$$

□

[3] Instead of arguing for $R_D^S < R_D^*$ via the Lagrange-multiplier associated with the regulatory constraint we can make use of the fact that $D^S < D^*$ holds by the binding regulation and of the fact that the deposit-supply function depends on $R_D$ only under Case 4. Furthermore, we must consider that the slope of $D^u(R_D)$ is positive under both regimes (*cf.* Result 5, Result 2, and Remark 1.

#### A.2.2.6 Proof of Result 20

*Proof.* Let us derive the sign of the sensitivity of $R_D^S$ with respect to $W_B$. If Condition (3.3.37) holds, $R_D^S$ is given by (3.3.39) and is thus a function of $W_B$ of the following form:

$$R_D^S = \frac{a + \sqrt{b - c \cdot W_B}}{d - e \cdot W_B}$$

with $a, b, c, d, e > 0$ appropriately defined. Then we obtain

$$\begin{aligned}
\frac{\partial R_D^S}{\partial W_B} &= \frac{\frac{1}{2} \cdot \frac{1}{\sqrt{b - c \cdot W_B}} \cdot (-c) \cdot (d - ex) \; + \; \left(a + \sqrt{b - c \cdot W_B}\right) \cdot e}{(d - e \cdot W_B)^2} = \\
&= \frac{ae + e\sqrt{b - c \cdot W_B} - \frac{c}{2\sqrt{b - c \cdot W_B}} \cdot (d - ex)}{(d - e \cdot W_B)^2} = \\
&= \frac{2ae\sqrt{b - c \cdot W_B} + 2e(b - c \cdot W_B) - c \cdot (d - ex)}{2\sqrt{b - c \cdot W_B}(d - e \cdot W_B)^2} = \\
&= \frac{2ae\sqrt{b - c \cdot W_B} + 2eb - ce \cdot W_B - cd}{2\sqrt{b - c \cdot W_B}(d - e \cdot W_B)^2} = \\
&= \frac{2ae\sqrt{b - c \cdot W_B} + e(\overbrace{b - c \cdot W_B}^{>0}) + be - cd}{2\sqrt{b - c \cdot W_B}(d - e \cdot W_B)^2} .
\end{aligned}$$

This derivative is positive if $be > cd$ holds. The expression $be - cd$ can be simplified to yield

$$\begin{aligned}
& 2c^2(1 - cc_1)(1 - q_4)q_4\alpha_2\gamma W_B \cdot \\
\cdot \; & \left[c_1^2 q_4^2 \alpha_2^2 + (c_2 - c_1)^2 R_f^2 + 2c_1(c_2 - c_1)q_4\alpha_2 R_f - 4c_1(c_2 - c_1)\alpha_2 q_4\right] ,
\end{aligned}$$

of which the first factor is strictly positive, while the second factor is bounded from below by

$$\begin{aligned}
& \left[c_1^2 q_4^2 \alpha_2^2 + (c_2 - c_1)^2 R_f^2 + 2c_1(c_2 - c_1)q_4\alpha_2 R_f - 4c_1(c_2 - c_1)q_4\alpha_2\right] \geq \\
\geq \; & \left[c_1^2 q_4^2 \alpha_2^2 + (c_2 - c_1)^2 + 2c_1(c_2 - c_1)q_4\alpha_2 - 4c_1(c_2 - c_1)q_4\alpha_2\right] = \\
= \; & \left[c_1 q_4 \alpha_2 - (c_2 - c_1)\right]^2 \geq 0 ,
\end{aligned}$$

due to $R_f \geq 1$ according to (3.2.6) and due to $c_2 > c_1$ by Assumption (3.3.32). As a consequence, we obtain $\frac{\partial R_D^S}{\partial W_B} > 0$. □

#### A.2.2.7 Proof of Result 21

*Proof.* As we have assumed $D^u(\ell, R_D; 4) < W_H$ in equilibrium, we have to consider the following maximization problem for the bank under Case 4 under regulation with fixed risk weights if both firms' projects are equal:

$$\begin{aligned}
\max_{\ell, R_D} \quad & \mathrm{E}\left[\tilde{W}_B(\ell, R_D; 4)\right] \\
\text{s.t.} \quad & D^u(\ell, R_D; 4) \cdot R_D \;\leq\; \alpha\ell \left[D^u(\ell, R_D; 4) + W_B\right] \\
& D^u(\ell, R_D; 4) \cdot R_D \;\leq\; \alpha(1-\ell) \left[D^u(\ell, R_D; 4) + W_B\right] \\
& c \cdot c_1 \left[D^u(\ell, R_D; 4) + W_B\right] \;\leq\; W_B
\end{aligned}$$

implying the following first-order conditions

$$\frac{\partial \mathcal{L}}{\partial \ell} = -\frac{\left[(2p-q)R_D - R_f + (2p-q)(1-2p+q)\gamma W_B R_D^2\right] \cdot \alpha \cdot (\lambda_2 - \lambda_1)}{(1-2p+q)(2p-q)R_D^2\gamma} \stackrel{!}{=} 0$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R_D} = & -\frac{[(2p-q)-(\lambda_1+\lambda_2)]R_D R_f + cc_1\left[2R_f - (2p-q)R_D\right]\lambda_3}{(1-q_4)q_4 R_D^3\gamma} \\
& + \frac{\left[2R_f - (2p-q)R_D\right][p - \ell\lambda_1 - (1-\ell)\lambda_2]\alpha}{(1-q_4)q_4 R_D^3\gamma} \stackrel{!}{=} 0
\end{aligned}$$

$$\begin{aligned}
\lambda_1 \cdot \left\{D^u(\ell, R_D; 4) \cdot R_D - \alpha\ell \left[D^u(\ell, R_D; 4) + W_B\right]\right\} &\stackrel{!}{=} 0 \\
\lambda_2 \cdot \left\{D^u(\ell, R_D; 4) \cdot R_D - \alpha(1-\ell) \left[D^u(\ell, R_D; 4) + W_B\right]\right\} &\stackrel{!}{=} 0 \\
\lambda_3 \cdot \left\{cc_1 \left[D^u(\ell, R_D; 4) + W_B\right] - W_B\right\} &\stackrel{!}{=} 0
\end{aligned}$$

Regulation is binding by assumption. Suppose none of the Case constraints is binding, *i.e.*

$$\alpha L_1^S \;>\; D^S R_D^S \quad \text{and} \quad \alpha L_2^S \;>\; D^S R_D^S \, .$$

implying $\lambda_1^S = \lambda_2^S = 0$, and the remaining first-order conditions become:

$$\frac{\partial \mathcal{L}}{\partial R_D} = \frac{\left[2R_f - (2p-q)R_D\right]p\alpha - (2p-q)R_D R_f - cc_1\left[2R_f - (2p-q)R_D\right]\lambda_3}{(1-q_4)q_4 R_D^3\gamma} \stackrel{!}{=} 0$$

$$\lambda_3 \cdot \left\{cc_1 \left[D^u(\ell, R_D; 4) + W_B\right] - W_B\right\} \stackrel{!}{=} 0$$

Solving the regulatory constraint for $R_D$ leads to two solutions whereas one is the optimal one as given in Result 21. The Lagrange multiplier of the regulatory constraint becomes

$$\lambda_3^S = \frac{2p\alpha R_f - (2p-q)R_D^S(p\alpha + R_f)}{cc_1\left[2R_f - (2p-q)R_D^S\right]} \, .$$

The denominator is positive since $2R_f - (2p-q)R_D^S > 2R_f - (2p-q)\alpha \geq 2R_f - (2p-q)2 \geq 2 \cdot 1 - 1 \cdot 2 = 0$ holds. The numerator is positive if and only if

$$R_D^S \ < \ \frac{2p\alpha R_f}{(2p-q)(p\alpha + R_f)}$$

holds whereas the right-hand side is the promised deposit interest rate without regulation with the Case constraints not binding. Hence, by Result 20, we obtain $\lambda_3^S > 0$ and $R_D^S < R_D^*$.

Finally, the constraints given by the result yield the lower and upper bound for all loan-allocation rates that are feasible under this equilibrium.

Again, the sensitivities of $R_D^S$ can be derived by total differentiation of the optimal deposit volume. By $\ell^S = \frac{1}{2}$, we obtain $L^S = \frac{1}{cc_1} \cdot W_B$ and hence $D^S = \frac{1-cc_1}{cc_1} \cdot W_B$. Since

$$\frac{dD^S}{d\theta} \ \equiv \ \frac{\partial D^s(R_D^S;4)}{\partial \theta} \ + \ \underbrace{\frac{\partial D^s(R_D^S;4)}{\partial R_D}}_{>0 \text{ by Res.}(5)} \cdot \frac{\partial R_D^S}{\partial \theta}$$

holds for any parameter $\theta$, we subsequently obtain the signs for the deposit interest rate's sensitivities, as shown in the result. Specifically, for $\theta = \gamma$,

$$\frac{\partial D^s(R_D^S;4)}{\partial \gamma} \ < \ 0$$

holds, resulting in

$$\frac{\partial R_D^S}{\partial \gamma} \ > \ 0$$

because of

$$\frac{dD^S}{d\gamma} \ = \ 0 \ .$$

Since $R_D^S$ is a function of $\gamma \cdot W_B$, and $\gamma, W_B > 0$,

$$\frac{\partial R_D^S}{\partial W_B} \ > \ 0$$

must be true, too. Equivalently, we can argue by

$$\underbrace{\frac{dD^S}{dW_B}}_{>0} \ \equiv \ \underbrace{\frac{\partial D^s(R_D^S;4)}{\partial W_B}}_{\equiv 0} \ + \ \underbrace{\frac{\partial D^s(R_D^S;4)}{\partial R_D}}_{>0} \cdot \frac{\partial R_D^S}{\partial W_B}$$

in Case 4, *cf.* Res. 1.

Analogous to $\gamma$, we can proceed with $R_f$. Concerning the regulatory parameters $c$

and $c_i$, we refer to Appendix A.2.2.1. □

### A.2.3 The Equilibrium with Regulation by a Value-at-Risk Approach

#### A.2.3.1 The Regulatory Constraints

As $\tilde{L} \;=\; \alpha_1\tilde{X}_1L_1 + \alpha_2\tilde{X}_2L_2$ holds, the VaR constraint is in detail given by

$$\mathrm{P}\Big[\; \mathrm{E}[\alpha_1\tilde{X}_1L_1 + \alpha_2\tilde{X}_2L_2] - (\alpha_1\tilde{X}_1L_1 + \alpha_2\tilde{X}_2L_2) \geq \tau W_B \;\Big] \;\leq\; \bar{p}\;,$$

with $\mathrm{E}(\alpha_1\tilde{X}_1L_1 + \alpha_2\tilde{X}_2L_2) \;=\; p_1\alpha_1L_1 + p_2\alpha_2L_2$. To derive conditions that assure that a given level of confidence $\bar{p}$ is fulfilled, let us rearrange the probability to

$$\mathrm{P}\Big[\; \tilde{L} \;\leq\; p_1\alpha_1L_1 + p_2\alpha_2L_2 - \tau W_B \;\Big] \;\leq\; \bar{p}\;.$$

Thus, the bank complies with a fixed $\bar{p}$ if $\tilde{L}$ does not surpass the threshold $p_1\alpha_1L_1 + p_2\alpha_2L_2 - \tau W_B$ too often. This, in turn, depends on the relative magnitude of both loan redemptions to each other.

Let us first consider the case $\alpha_1L_1 < \alpha_2L_2$. By the joint distribution of $(\tilde{X}_1, \tilde{X}_2)$, we obtain the cumulative distribution function of the random variable $\tilde{L}$ as

$$\begin{array}{lcl} \mathrm{P}[0 \leq \tilde{L} < \alpha_1L_1] & = & 1 - p_1 - p_2 + q \\ \mathrm{P}[\tilde{L} \leq \alpha_1L_1] \;=\; \mathrm{P}[\tilde{L} < \alpha_2L_2] & = & 1 - p_2 \\ \mathrm{P}[\tilde{L} \leq \alpha_2L_2] \;=\; \mathrm{P}[\tilde{L} < \alpha_1L_1 + \alpha_2L_2] & = & 1 - q \\ \mathrm{P}[\tilde{L} \leq \alpha_1L_1 + \alpha_2L_2] & = & 1 \end{array} \;.$$

Let $\bar{p} = 1 - p_1 - p_2 + q$. Then, on the one hand, the VaR constraint becomes

$$\mathrm{P}\Big[\; \tilde{L} \;\leq\; p_1\alpha_1L_1 + p_2\alpha_2L_2 - \tau W_B \;\Big] \;=\; 1 - p_1 - p_2 + q\;,$$

as there is no event occurring with probability strictly lower than $1 - p_1 - p_2 + q$. On the other hand, we must consider $\mathrm{P}[0 \leq \tilde{L} < \alpha_1L_1] = 1 - p_1 - p_2 + q$. Because every cumulative distribution function is strictly increasing, the condition

$$p_1\alpha_1L_1 + p_2\alpha_2L_2 - \tau W_B \;<\; \alpha_1L_1$$

must be satisfied. With $L_1 \equiv \ell \cdot L \equiv \ell \cdot (D + W_B)$, we obtain the result shown in Table 3.6.

Now let $\bar{p} = 1 - p_2$: There are two ways to express this probability in terms of loan redemptions, namely by $\mathrm{P}[\tilde{L} \leq \alpha_1 L_1]$ and by $\mathrm{P}[\tilde{L} < \alpha_2 L_2]$, respectively. Hence, there are two possibilities to express the constraint the bank faces, either as a weak inequality,

$$p_1 \alpha_1 L_1 + p_2 \alpha_2 L_2 - \tau W_B \ \leq \ \alpha_1 L_1 \ ,$$

or as a strict inequality,

$$p_1 \alpha_1 L_1 + p_2 \alpha_2 L_2 - \tau W_B \ < \ \alpha_2 L_2 \ ,$$

where both are shown in Table 3.6, again in terms of the total loan volume $L$.

Note that for all $\bar{p} < 1 - p_2$, the constraint derived for $\bar{p} = 1 - p_1 - p_2 + q$ is binding as otherwise the level of confidence $\bar{p}$ would drop to $1 - p_2$ or to values over $1 - p_2$.

The arguments from above hold analogously for $\alpha_1 L_1 = \alpha_2 L_2$ and $\alpha_1 L_1 > \alpha_2 L_2$, respectively. Irrespective of the relative magnitude of both loan redemptions to each other, we can derive the constraint for $\bar{p} = 1 - q$, resulting in

$$p_1 \alpha_1 L_1 + p_2 \alpha_2 L_2 - \tau W_B \ < \ \alpha_1 L_1 + \alpha_2 L_2 \ ,$$

which is trivially fulfilled if $L_1, L_2, W_B \geq 0$ holds with at least one value being strictly positive (otherwise equality holds).

#### A.2.3.2 Proof of Result 24

*Proof.* By assumption, regulation is binding and implies Case 1 with $\ell^V = \frac{\alpha_2}{\alpha_1+\alpha_2}$. Thus,

$$\left[v(\ell^V, \bar{p}) + \epsilon(\ell^V, \bar{p})\right] \cdot \left[D^u(\ell^V, R_D; 1) + W_B\right] \ \stackrel{!}{=} \ \tau \cdot W_B$$

must be fulfilled. As the household's initial wealth is not assumed to be binding either, we can consider the unconstrained deposit-supply function. By (3.3.46), we obtain $v(\ell^V, \bar{p}) = (p_1 + p_2 - q) \cdot \frac{\alpha_2}{\alpha_1+\alpha_2}$ and $\epsilon(\ell^V, \bar{p}) = \epsilon$. This equation yields two solutions for $R_D$ whereas one solution always exceeds the other. Thus, the optimal interest rate for the bank is unique, recall Result 5. The optimal interest rate is equal to

$$R_D^V(1) \ = \ \frac{q\left[\epsilon+(p_1+p_2-1)\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]+q(p_1+p_2-2q)\left[2\tau-\epsilon-(p_1+p_2-1)\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\gamma W_B \ - \ \sqrt{Q_1^V(\ell^V,\bar{p})}}{2q(1-q)\left[\tau-\epsilon-(p_1+p_2-1)\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\gamma W_B}$$

where

$$
\begin{aligned}
Q_1^V(\ell^V,\bar p) \;=\;& q^2\left[\epsilon+(p_1+p_2-1)\tfrac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]^2+\\
&+2q\left[\epsilon+(p_1+p_2-1)\tfrac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\cdot\\
\cdot&\left\{q(p_1+p_2-2q)\left[\epsilon+\tfrac{(p_1+p_2-1)\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\tfrac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}-2\left[(1-q)R_f-\tfrac{(p_1+p_2-2q)\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\left[\tau-\epsilon-\tfrac{(p_1+p_2-1)\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\right\}\gamma W_B\\
+&(p_1+p_2-2q)\left\{(p_1+p_2-2q)q\left[\epsilon+\tfrac{(p_1+p_2-1)\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]^2-4(1-p_1-p_2+q)\tau\left[\tau-\epsilon-\tfrac{(p_1+p_2-1)\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right]\right\}\tfrac{\alpha_1^2\alpha_2^2\gamma^2W_B^2}{(\alpha_1+\alpha_2)^2}
\end{aligned}
$$

The regulatory constraint results in Formula (3.3.52) due to $\ell^V=\frac{\alpha_2}{\alpha_1+\alpha_2}$. As $D^V$ goes to zero for $W_B$ going to zero and as $\ell^V$ is independent of $W_B$, the deposit interest rate must go to $\underline{R}_D(\frac{\alpha_2}{\alpha_1+\alpha_2};1)$. This limit

$$\lim_{W_B\downarrow 0} R_D^V(1)$$

can be derived by l'Hôpital's rule. Taking once the derivative of the numerator and the denominator with respect to $W_B$ and setting then $W_B=0$ yields

$$
\begin{aligned}
\lim_{W_B\downarrow 0} R_D^V(1) \;&=\; \frac{1}{q}R_f-\frac{p_1+p_2-2q}{q}\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\overset{(3.2.11)}{\in}\left(\frac{R_f-1}{q},\frac{1}{q}R_f\right)\,, \quad i.e.\\
\lim_{W_B\downarrow 0} R_D^V(1) \;&\overset{(3.2.55)}{=}\; \underline{R}_D\left(\frac{\alpha_2}{\alpha_1+\alpha_2};1\right).
\end{aligned}
$$

The signs for the sensitivities of $R_D^V$ w/r/t $W_B$, $\gamma$, $R_f$ and $\tau$ can be obtained by total differentiation of $D^V$. Consider $\gamma$ first.

$$\underbrace{\frac{dD^V}{d\gamma}}_{=0}\equiv\underbrace{\frac{\partial D^s(\ell^V,R_D^V;1)}{\partial\gamma}}_{<0}+\frac{\partial D^s(\ell^V,R_D^V;1)}{\partial\ell}\cdot\underbrace{\frac{d\ell^V}{d\gamma}}_{=0}+\underbrace{\frac{\partial D^s(\ell^V,R_D^V;1)}{\partial R_D}}_{>0\text{ by Res. 5}}\cdot\frac{dR_D^V}{d\gamma}\,,$$

resulting in

$$\frac{dR_D^V}{d\gamma}>0\,.$$

Analogously, the sign w/r/t $R_f$ obtains. As $R_D^V$ is a function of $\gamma\cdot W_B$,

$$\frac{dR_D^V}{dW_B}>0$$

holds as well. Concerning $\tau$,

$$\underbrace{\frac{dD^V}{d\tau}}_{>0}\equiv\underbrace{\frac{\partial D^s(\ell^V,R_D^V;1)}{\partial\tau}}_{=0}+\frac{\partial D^s(\ell^V,R_D^V;1)}{\partial\ell}\cdot\underbrace{\frac{d\ell^V}{d\tau}}_{=0}+\underbrace{\frac{\partial D^s(\ell^V,R_D^V;1)}{\partial R_D}}_{>0\text{ by Res. 5}}\cdot\frac{dR_D^V}{d\tau}\,,$$

holds, so that $\frac{dR_D^V}{d\tau} > 0$ must hold. □

#### A.2.3.3 Proof of Result 25

*Proof.* Since $\ell^V = \frac{1}{2}$ is assumed to be optimal, the conditions on Case 1, $C_1$, are not violated and furthermore it is assumed that a potential maximizer to Problem (3.3.44) does not lie at the border $\partial C_1$. Moreover, the household is assumed not to be constrained by its initial wealth $W_H$ and that $\bar{p} = 1 - 2p + q$ is the level of confidence to be maintained, altogether implying that the bank's Maximization Problem (3.3.44) can be simplified to

$$\begin{array}{rl} \max\limits_{\ell,\ R_D,\ j} & \mathrm{E}\left[\tilde{W}_B(\ell, R_D; j)\right] \\ \text{s.t.} & [v(\ell,\bar{p}) + \epsilon(\ell,\bar{p})] \cdot [D^u(\ell, R_D; j) + W_B] \ \leq\ \tau \cdot W_B \end{array}$$

where the constraint function, $v(\ell,\bar{p}) + \epsilon(\ell,\bar{p})$, is fixed to $v(\frac{1}{2}, 1-2p+q) + \epsilon(\frac{1}{2}, 1-2p+q) \equiv (p - \frac{1}{2})\alpha + \epsilon$ as the potential maximizer $\ell^V = \frac{1}{2}$ lies at the kink of $v(\ell,\cdot)$ in $\ell$.

Then a maximum $(\frac{1}{2}, R_D^V, 1)$ satisfies the Kuhn-Tucker conditions and there is a multiplier $\lambda \geq 0$ such that [4]

$$\begin{array}{rcl} \frac{\partial \mathrm{E}[\tilde{W}_B(\ell^V, R_D^V, 1)]}{\partial R_D} & = & \lambda \cdot \left[v(\frac{1}{2},\bar{p}) + \epsilon(\frac{1}{2},\bar{p})\right] \cdot \frac{\partial D^u(\ell^V, R_D^V, 1)}{\partial R_D} \\ 0 & = & \lambda \cdot \left\{ \left[v(\frac{1}{2},\bar{p}) + \epsilon(\frac{1}{2},\bar{p})\right] \cdot \left[D^u(\ell^V, R_D^V, 1) + W_B\right] \ - \ \tau \cdot W_B \right\} \end{array}$$

holds. Because regulation is assumed to be binding, we obtain $\lambda^V > 0$ implying

$$D^V \ = \ \frac{[2(\tau - \epsilon) - (2p-1)\alpha]\, W_B}{(2p-1)\alpha + 2\epsilon}$$

and the identity $D^u(\ell^V, R_D^V, 1) \equiv D^V$ leads to

$$R_D^V \ = \ \frac{q \cdot [(2p-1)\alpha + 2\epsilon] \cdot [1 - (p-q)\alpha\gamma W_B] \ + \ 4q(p-q)\alpha\gamma \cdot \tau W_B \ - \ \sqrt{q \cdot Q^V (1-2p+q)}}{2q(1-q)[2(\tau-\epsilon) - (2p-1)\alpha]\gamma W_B}$$

[4] As $\ell = \frac{1}{2}$ maximizes the bank's objective if both firms are equal when there is no regulation, too, it does not change results irrespective whether we take the derivative of the bank's objective function with respect to $\ell$ or not given that the regulatory constraint is fixed to $v(\frac{1}{2},\bar{p}) + \epsilon(\frac{1}{2},\bar{p})$.

as gross interest rate on deposits to the household where

$$\begin{aligned} Q^V(1-2p+q) \;=\; & q\left\{(2p-1)\alpha+2\epsilon+(p-q)\alpha\left[2(2\tau-\epsilon)-(2p-1)\alpha\right]\gamma W_B\right\}^2 \\ & -\; 4(1-q)\gamma W_B\left[2(\tau-\epsilon)-(2p-1)\alpha\right]\cdot \\ & \cdot\left\{\left[(2p-1)\alpha+2\epsilon\right]\left[R_f-(p-q)\alpha\right]+(p-q)(1-2p+2q)\alpha^2\gamma\tau W_B\right\}\;. \end{aligned}$$

The partial derivative of $R_D^V$ with respect to $W_B$ can be represented as

$$\begin{aligned} \frac{\partial R_D^V}{\partial W_B} \;=\; & \frac{[(2p-1)\alpha+2\epsilon]}{\sqrt{q\cdot Q^V(1-2p+q)}}\cdot \\ & \cdot\left\{\frac{q\cdot[(2p-1)\alpha+2\epsilon]+\left\{2q(p-q)\alpha\cdot\epsilon+4(\tau-\epsilon)\left[(p-q)\alpha-(1-q)R_f\right]+(2p-1)\left[2(1-q)R_f-(2-q)(p-q)\alpha\right]\alpha\right\}\cdot\gamma W_B}{2(1-q)[2(\tau-\epsilon)-(2p-1)\alpha]\gamma W_B^2}\right. \\ & \left.-\frac{\sqrt{q\cdot Q^V(1-2p+q)}}{2(1-q)[2(\tau-\epsilon)-(2p-1)\alpha]\gamma W_B^2}\right\} \end{aligned}$$

which is strictly positive if and only if

$$\begin{aligned} & \left\{q\cdot[(2p-1)\alpha+2\epsilon]\;+\right. \\ & \quad +\;\left\{2q(p-q)\alpha\cdot\epsilon+4(\tau-\epsilon)\left[(p-q)\alpha-(1-q)R_f\right]+\right. \\ & \quad \left.\left.+(2p-1)\left[2(1-q)R_f-(2-q)(p-q)\alpha\right]\alpha\right\}\gamma W_B\right\}^2 \;> \\ & >\;\sqrt{q\cdot Q^V(1-2p+q)}^{\,2} \end{aligned}$$

holds. Expanding both sides of the inequality and then subtracting the term of the right-hand side from the term of the left hand side leads to

$$\begin{aligned} & -4(1-q)\underbrace{\left[2(\tau-\epsilon)-(2p-1)\alpha\right]}_{>0}\gamma^2W_B^2\;\cdot \\ & \cdot\left\{\underbrace{-\left[2(\tau-\epsilon)-(2p-1)\alpha\right]\cdot\left\{(p-q)\alpha(\alpha-R_f)+\underbrace{\left[(1-q)R_f-(p-q)\alpha\right]R_f}_{>0}\right\}}_{<0}\right. \\ & \quad +(p-q)\alpha\cdot\left\{-(2p-1)(1-p+q)\alpha^2+2qR_f\epsilon\right. \\ & \quad \left.\left.+\left[(2-2p+q)(\tau-\epsilon)+q(2p-1)R_f-q\epsilon\right]\alpha\right\}\right\}\;>\;0 \end{aligned}$$

As the factor in the first line is negative, the second, embraced factor must be negative, too. As there are negative as well as positive expressions contained in the latter, the following condition on $\tau$,

$$\tau \;\geq\; \frac{[(2p-1)\alpha+2\epsilon]\left[(p-q)^2\alpha^2-(p-q)(2-q)\alpha R_f+(1-q)R_f^2\right]}{(p-q)(2p-q)\alpha^2-4(p-q)\alpha R_f+2(1-q)R_f^2}$$

makes the second factor negative, too, implying

$$\frac{\partial R_D^V}{\partial W_B} \; > \; 0 \; .$$

□

# Appendix B

# Derivations to Chapter 5

## B.1 Bernoulli Mixture Model

In this section, we present a fairly general formulation of a Bernoulli mixture model.[1] A Bernoulli mixture model is a model of Bernoulli-distributed random variables by which various patterns and degrees of dependence among the Bernoulli random variables can be accounted for. This is done by assuming the success probabilities to be random themselves. Finally, we will derive the first and all second moments for this model under the assumption that the success probabilities $\tilde{p}_i^{(j)}$ follow (5.2.2). For convenience, we dispense with the cases $j = 1$ and $j = n$ and treat the probabilities $\tilde{p}_i^{(j)}$ as if they were equally given across all firms $j$ within a given sector $i$. This is further justified by the fact that we aim at providing the moments of the limiting distribution of aggregate loan redemption in the next section, Section B.2.

The (cumulative) distribution functions of the probabilities $\tilde{p}_i^{(j)}$ are denoted by $F_i(p_i^{(j)})$ with support on $[0, 1]$, or a subarea thereof.

Realizations of the random variable $\tilde{p}_i^{(j)}$ are denoted by $p_i^{(j)}$. Given realizations $p_i^{(j)}$, the Bernoulli random variables $\tilde{X}_i^{(j)}$ are conditionally independent.

First, let us calculate the probability of a given outcome $\{\tilde{X}_i^{(j)}\}_{j=1,\dots,n} = \{x_i^{(j)}\}$, $x_i^{(j)} \in \{0, 1\}$, within a given sector $i$. This probability is given by (*cf.* Joe, 2001, pp. 211, 219)

$$\mathrm{P}\left[\tilde{X}_i^{(j)} = x_i^{(j)},\ j = 1, \dots, n\right] = \int_{\mathbf{p}_i \in [0,1]^n} \prod_{j=1}^{n} (p_i^{(j)})^{x_i^{(j)}} \cdot (1 - p_i^{(j)})^{1 - x_i^{(j)}} dF_i^{(1,\dots,n)}(\mathbf{p}_i), \tag{B.1.1}$$

[1] Expositions on general properties of (Bernoulli) exchangeable mixture models can be found in Joe (2001, pp. 211, 219-220) and in Frey/McNeil (2001, pp. 7-8).

where $\mathbf{p}_i = (p_i^{(j)})_{j=1,\ldots,n}$ and where $F_i^{(1,\ldots,n)}(\mathbf{p}_i)$ denotes the joint distribution function of $\tilde{\mathbf{p}}_i = (\tilde{p}_i^{(j)})_{j=1,\ldots,n}$. The same formula applies to any subset of $k_n$ random variables considered where $1 \leq k_n < n$.

Let us extend the model to calculate the probability of the joint outcome $\{(\tilde{X}_1^{(j)}, \tilde{X}_2^{(j)})_{j=1,\ldots,n}\} = \{(x_1^{(j)}, x_2^{(j)})_{j=1,\ldots,n}\}$ of both sectors. Let $F_{1,2}^{1,\ldots,n}(\mathbf{p}_1, \mathbf{p}_2)$ be the common distribution function of the bivariate tuple $(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2)$ of success probability vectors. Then the probability of a given drawing $\{(\tilde{X}_1^{(j)}, \tilde{X}_2^{(j)})_{j=1,\ldots,n}\} = \{(x_1^{(j)}, x_2^{(j)})_{j=1,\ldots,n}\}$ is computed according to

$$\begin{aligned} &\mathrm{P}\left[\tilde{X}_i^{(j)} = x_i^{(j)},\ j = 1, \ldots, n,\ i = 1, 2\right] = \\ &= \int\limits_{(\mathbf{p}_1, \mathbf{p}_2) \in [0,1]^{2n}} \prod_{i=1}^{2} \left( \prod_{j=1}^{n} (p_i^{(j)})^{x_i^{(j)}} \cdot (1 - p_i^{(j)})^{1 - x_i^{(j)}} \right) dF_{1,2}^{1,\ldots,n}(\mathbf{p}_1, \mathbf{p}_2) \end{aligned} \tag{B.1.2}$$

because of the conditional independence between all the random variables considered. Likewise, the probabilities of $k_n$ tuples, $1 \leq k_n < n$, can be calculated.

According to (B.1.1), the expected value of the random variable $\tilde{p}_i^{(j)}$ is equivalent to the success probability of the Bernoulli-distributed return $\tilde{X}_i^{(j)}$,

$$\mathrm{P}\left[\tilde{X}_i^{(j)} = 1\right] = \int_{p_i^{(j)}} p_i^{(j)} dF_i(p_i^{(j)}) = \mathrm{E}[\tilde{p}_i^{(j)}] = \bar{p}_i^{(j)} . \tag{B.1.3}$$

Then the expected value of $\tilde{X}_i^{(j)}$ is given by

$$\mathrm{E}[\tilde{X}_i^{(j)}] = \mathrm{E}[\bar{p}_i^{(j)} \cdot 1 + (1 - \bar{p}_i^{(j)}) \cdot 0] = \bar{p}_i^{(j)} \tag{B.1.4}$$

because of (B.1.3). Likewise, we obtain for the variance

$$\mathrm{V}[\tilde{X}_i^{(j)}] = \mathrm{E}\left[\left(\tilde{X}_i^{(j)}\right)^2\right] - \left(\bar{p}_i^{(j)}\right)^2 = \mathrm{E}\left[\bar{p}_i^{(j)} \cdot 1^2\right] - \left(\bar{p}_i^{(j)}\right)^2 = \bar{p}_i^{(j)} \cdot (1 - \bar{p}_i^{(j)}) . \tag{B.1.5}$$

As all Bernoulli random variables considered either take zero or one as values, all non-central moments can be presented by the respective joint success probabilities given by (B.1.1) and (B.1.2):[2]

$$
\begin{aligned}
\mathrm{E}[\tilde{X}_i^{(j)}] &= \mathrm{P}[\tilde{X}_i^{(j)} = 1] \ , \\
\mathrm{E}[\tilde{X}_i^{(j)} \cdot \tilde{X}_i^{(j+1)}] &= \mathrm{P}[\tilde{X}_i^{(j)} = \tilde{X}_i^{(j+1)} = 1] \ , \\
&\vdots \\
\mathrm{E}[\tilde{X}_i^{(1)} \cdot \tilde{X}_i^{(2)} \cdot \ldots \cdot \tilde{X}_i^{(n)}] &= \mathrm{P}[\tilde{X}_i^{(j)} = 1, j = 1, \ldots, n] \ , \\
\mathrm{E}[\tilde{X}_1^{(j)} \cdot \tilde{X}_2^{(j)}] &= \mathrm{P}[\tilde{X}_1^{(j)} = \tilde{X}_2^{(j)} = 1] \ , \\
\mathrm{E}[\tilde{X}_1^{(j)} \cdot \tilde{X}_2^{(j+1)}] &= \mathrm{P}[\tilde{X}_1^{(j)} = \tilde{X}_2^{(j+1)} = 1] \ , \\
&\vdots
\end{aligned}
\tag{B.1.6}
$$

Thus, covariance terms are given by

$$
\mathrm{Cov}(\tilde{X}_i^{(j)}, \tilde{X}_h^{(k)}) \ = \ \mathrm{P}[\tilde{X}_i^{(j)} = \tilde{X}_h^{(k)} = 1] \ - \ \bar{p}_i^{(j)} \cdot \bar{p}_h^{(k)} \ , \ i, h = 1, 2, \ j = 1, \ldots, n \ . \tag{B.1.7}
$$

Consider the factor model for the success probabilities according to (5.2.2),

$$
\tilde{p}_i^{(j)} \ = \ \begin{cases} 1 - b_i \cdot \tilde{\xi}_i^{(1)} - c_i \cdot \tilde{\xi}_i^{(2)} - d_i \cdot \tilde{\xi}_{3-i}^{(1)} \ , & j = 1 \\ 1 - a_i \cdot \tilde{\xi}_i^{(j-1)} - b_i \cdot \tilde{\xi}_i^{(j)} - c_i \cdot \tilde{\xi}_i^{(j+1)} - d_i \cdot \tilde{\xi}_{3-i}^{(j)} \ , & j = 2, \ldots, n-1 \\ 1 - a_i \cdot \tilde{\xi}_i^{(n-1)} - b_i \cdot \tilde{\xi}_i^{(n)} - d_i \cdot \tilde{\xi}_{3-i}^{(n)} \ , & j = n \end{cases}
$$

for $i = 1, 2$ where $a_i, b_i, c_i, d_i$ are fixed, strictly positive numbers and where $\tilde{\xi}_i^{(j)}$ are mutually independent, uniformly distributed random variables with $\tilde{\xi}_i^{(j)} \sim \mathrm{U}(0, 2t)$, $t > 0$, for all $i = 1, 2$, $j = 1, \ldots, n$. Furthermore, Condition (5.2.3) be fulfilled.

The univariate density function of each $\tilde{p}_i^{(j)}$, $j = 1, \ldots, n-1$, can be traced back to the mutually independent risk factors $\xi_i^{(j)}$ involved and, thus, is given by

$$
f_i(p_i^{(j)}) = f^{*4}(\xi_i^{(j-1)}, \xi_i^{(j)}, \xi_i^{(j+1)}, \xi_{3-i}^{(j)}) = \begin{cases} \frac{1}{(2t)^4} & \text{if} \quad 0 \le \xi_i^{(j-1)}, \xi_i^{(j)}, \xi_i^{(j+1)}, \xi_{3-i}^{(j)} \le 2t \\ 0 & \text{else} \end{cases} , \tag{B.1.8}
$$

for $2 \le j \le n-1$. Dependent on the lag between the Bernoulli random variables, the univariate density functions $f_i^{j;j+1}(p_i^j, p_i^{j+1})$, $f_i^{j;j+2}(p_i^j, p_i^{j+2})$, and the bivariate density functions $f_{1,2}^{j;j+1}(p_1^{(j)}, p_2^{(j-1)})$, $f_{1,2}^{j;j+1}(p_1^{(j)}, p_2^{(j)})$, and $f_{1,2}^{j;j+1}(p_1^{(j)}, p_2^{(j+1)})$ are of interest, too. They can be traced back to the associated factor loadings $\xi_i^{(j)}$ and $\xi_h^{(k)}$ analogous to $f_i(p_i^{(j)})$.

[2] All these $2^n - 1$ joint moments alternatively characterize the multivariate Bernoulli distribution, *cf.* Joe (2001, p. 211), Maydeu-Olivares/Joe (2005, p. 1010).

Following (B.1.3) and (B.1.4) we obtain for the expectations:

$$\mathrm{E}[\tilde{X}_i^{(j)}] \;=\; \mathrm{P}[\tilde{X}_i^{(j)} = 1] \;=\; \mathrm{E}[\tilde{p}_i^{(j)}] \;=\; 1 - (a_i + b_i + c_i + d_i) \cdot t \;,\; 2 \leq j \leq n-1 \;,$$

and in the same manner $\mathrm{V}(\tilde{X}_1^{(j)})$ via (B.1.5).

By (B.1.1) and (B.1.6), the second joint, non-central moment $\mathrm{E}[\tilde{X}_i^{(j)} X_i^{(j+1)}]$ is given by

$$\begin{aligned}
\mathrm{E}[\tilde{X}_i^{(j)} X_i^{(j+1)}] \;&= \\
&= \int\limits_{p_i^{(j)}, p_i^{(j+1)}} p_i^{(j)} \cdot p_i^{(j+1)} \cdot f_i(p_i^{(j)}, p_i^{(j+1)}) \; d(p_i^{(j)}, \tilde{p}_i^{(j+1)}) \\
&= \int\limits_{[0,2t]^6} p_i^{(j)} \cdot p_i^{(j+1)} \cdot f^{*6}(\xi_i^{(j-1)}, \xi_i^{(j)}, \xi_i^{(j+1)}, \xi_i^{(j+2)}, \xi_{3-i}^{(j)}, \xi_{3-i}^{(j+1)}) \; d(\xi_i^{(j-1)}, \ldots, \xi_{3-i}^{(j+1)}) \\
&= \; \bar{p}_i^2 \;+\; \frac{1}{3} \cdot (a_i + c_i) \cdot b_i \cdot t^2 \;,\; 2 \leq j \leq n-1 \;, \qquad \text{(B.1.9)}
\end{aligned}$$

resulting by (B.1.7) in

$$\mathrm{Cov}(\tilde{X}_i^{(j)}, \tilde{X}_i^{(j\pm1)}) \;=\; \frac{1}{3} \cdot (a_i + c_i) \cdot b_i \cdot t^2 \;,\; 2 \leq j \leq n-1 \;. \qquad \text{(B.1.10)}$$

By this 2-dependence structure, any second mixed moment of $X_i^{(j)}$ and $X_i^{(j+m)}$ with lag $|m| \geq 3$ becomes

$$\mathrm{E}(\tilde{X}_i^{(j)} X_i^{(j+m)}) = \mathrm{E}(\tilde{X}_i^{(j)}) \cdot \mathrm{E}(X_i^{(j+m)}) \;,\; 1 \leq j+m \leq n \;, \qquad \text{(B.1.11)}$$

within each sector, and any second mixed moment of $X_i^{(j)}$ and $X_{3-i}^{(j+m)}$ with lags of $|m| \geq 2$

$$\mathrm{E}(\tilde{X}_i^{(j)} X_i^{(j+m)}) = \mathrm{E}(\tilde{X}_i^{(j)}) \cdot \mathrm{E}(X_{3-i}^{(j+m)}) \;,\; 1 \leq j+m \leq n \;, \qquad \text{(B.1.12)}$$

across sectors. Thus, the other intra- and inter-sectoral covariance terms can be determined. The corresponding correlations are given by (5.2.6) to (5.2.10).

## B.2 Moments of Aggregate Loan Repayments

Let us determine the expectations, variances and covariances of the sector-wise aggregated loan portfolios

$$\tilde{L}_1 \;=\; \sum_{j=1}^{n} \tilde{X}_1^{(j)} \cdot \ell_1 \quad \text{and} \quad \tilde{L}_2 \;=\; \sum_{j=1}^{n} \tilde{X}_2^{(j)} \cdot \ell_2 \;,$$

where we treat all Bernoulli random variables, $\tilde{X}_i^{(j)}$, as homogeneous within each sector $i$. That is, we neglect the asymmetries arising at the borders of the sequence $\{\tilde{X}_i^{(1)}, \tilde{X}_i^{(2)}, \ldots, \tilde{X}_i^{(n-1)}, \tilde{X}_i^{(n)}\}$. Note that we are interested in the statistical moments of large portfolios, that is with many borrowers, and in the statistical moments of the associated limiting distribution such that this approach may be justified.

The expectations are thus simply given by

$$\mathrm{E}\left[\sum_{j=1}^{n} \tilde{X}_i^{(j)} \cdot L_i\right] \;=\; n \cdot \bar{p}_i \alpha_i \cdot \frac{L_i}{n} \;=\; \bar{p}_i \alpha_i \cdot L_i \;.$$

The intra-sectoral variance becomes

$$\begin{aligned}
\mathrm{V}\big(\sum_{j=1}^{n} \tilde{X}_i^{(j)} \cdot L_i\big) \;&=\; \sum_{j=1}^{n} \mathrm{V}(\tilde{X}_i^{(j)} \cdot L_i) \;+\; \sum_{j=1}^{n} \mathrm{Cov}\big(\tilde{X}_i^{(j)} \cdot L_i, \sum_{k=1,k\neq j}^{n} \tilde{X}_i^{(k)} \cdot L_i\big) \\
&= \sum_{j=1}^{n} \mathrm{V}(\tilde{X}_i^{(j)}) \cdot L_i^2 \;+\; \mathrm{Cov}(\tilde{X}_i^{(1)}, \tilde{X}_i^{(n)}) \cdot L_i^2 + \sum_{j=2}^{n} \mathrm{Cov}(\tilde{X}_i^{(j)}, \tilde{X}_i^{(j-1)}) \cdot L_i^2 \\
&+ \sum_{j=1}^{n-1} \mathrm{Cov}(\tilde{X}_i^{(j)}, \tilde{X}_i^{(j+1)}) \cdot L_i^2 + \mathrm{Cov}(\tilde{X}_i^{(n)}, \tilde{X}_i^{(1)}) \cdot L_i^2 \\
&+ \mathrm{Cov}(\tilde{X}_i^{(1)}, \tilde{X}_i^{(n-1)}) \;+\; \mathrm{Cov}(\tilde{X}_i^{(2)}, \tilde{X}_i^{(n)}) \;+\; \sum_{j=3}^{n} \mathrm{Cov}(\tilde{X}_i^{(j)}, \tilde{X}_i^{(j-1)}) \\
&+ \sum_{j=1}^{n-2} \mathrm{Cov}(\tilde{X}_i^{(j)}, \tilde{X}_i^{(j+2)}) \cdot L_i^2 \;+\; \mathrm{Cov}(\tilde{X}_i^{(n-1)}, \tilde{X}_i^{(1)}) \cdot L_i^2 \;+\; \mathrm{Cov}(\tilde{X}_i^{(n)}, \tilde{X}_i^{(2)}) \cdot L_i^2 \\
&= \; n \cdot \mathrm{V}(\tilde{X}_i^{(1)}) \cdot L_i^2 + 2 \cdot n \cdot \mathrm{Cov}(\tilde{X}_i^{(1)}, \tilde{X}_i^{(2)}) \cdot L_i^2 + 2 \cdot n \cdot \mathrm{Cov}(\tilde{X}_i^{(1)}, \tilde{X}_i^{(3)}) \cdot L_i^2
\end{aligned}$$

as all covariance terms with a distance of $|m| \geq 3$ reduce to zero according to (B.1.11). Hence, the variance of each aggregate loan repayment $\tilde{L}_i$ is finally given by

$$\mathrm{V}(\tilde{L}_i) \;=\; \frac{1}{n} \cdot \Big[\mathrm{V}(\tilde{X}_i^{(1)}) + 2 \cdot \mathrm{Cov}(\tilde{X}_i^{(1)}, \tilde{X}_i^{(2)}) + 2 \cdot \mathrm{Cov}(\tilde{X}_i^{(1)}, \tilde{X}_i^{(3)})\Big] \cdot L_i^2 \;. \qquad \text{(B.2.1)}$$

Due to Property (B.1.12), the covariance between the sums $\sum_{j=1}^n \tilde{X}_1^{(j)} \cdot \ell_1$ and $\sum_{j=1}^n \tilde{X}_2^{(j)} \cdot \ell_2$ is given by

$$\begin{aligned}
\mathrm{Cov}\Big(\sum_{j=1}^n \tilde{X}_1^{(j)} \cdot \ell_1, \sum_{j=1}^n \tilde{X}_2^{(j)} \cdot \ell_2\Big) &= \sum_{j=1}^n \mathrm{Cov}\Big(\tilde{X}_1^{(j)} \cdot \ell_1, \sum_{k=1}^n \tilde{X}_2^{(k)} \cdot \ell_2\Big) \\
&= \Big[\mathrm{Cov}(\tilde{X}_1^{(1)}, \tilde{X}_2^{(n)}) \;+\; \sum_{j=2}^n \mathrm{Cov}(\tilde{X}_1^{(j)}, \tilde{X}_2^{(j-1)}) \;+\; \sum_{j=1}^n \mathrm{Cov}(\tilde{X}_1^{(j)}, \tilde{X}_2^{(j)})\Big] \cdot \ell_1 \cdot \ell_2 \\
&\quad + \sum_{j=1}^{n-1} \mathrm{Cov}(\tilde{X}_1^{(j)}, \tilde{X}_2^{(j+1)}) \cdot \ell_1 \cdot \ell_2 \;+\; \mathrm{Cov}(\tilde{X}_1^{(n)}, \tilde{X}_2^{(1)}) \cdot \ell_1 \cdot \ell_2 \\
&= \Big[\mathrm{Cov}(\tilde{X}_1^{(2)}, \tilde{X}_2^{(1)}) \;+\; \mathrm{Cov}(\tilde{X}_1^{(1)}, \tilde{X}_2^{(1)}) \;+\; \mathrm{Cov}(\tilde{X}_1^{(1)}, \tilde{X}_2^{(2)})\Big] \cdot n \cdot \ell_1 \cdot \ell_2 \;,
\end{aligned}$$

hence

$$\mathrm{Cov}(\tilde{L}_1, \tilde{L}_2) \;=\; \frac{1}{n} \cdot \Big[\mathrm{Cov}(\tilde{X}_1^{(2)}, \tilde{X}_2^{(1)}) + \mathrm{Cov}(\tilde{X}_1^{(1)}, \tilde{X}_2^{(1)}) + \mathrm{Cov}(\tilde{X}_1^{(1)}, \tilde{X}_2^{(2)})\Big] \cdot L_1 \cdot L_2 \;. \qquad \text{(B.2.2)}$$

In light of the Bernoulli mixture model from Sections B.1 and 5.2, the moments of the aggregate loan repayments are given in terms of the factor loadings and the individual expected success probability $\bar{p}_i$ as follows:

$$\mathrm{E}(\tilde{L}_i) \;=\; \bar{p}_i \alpha_i \cdot L_i \;, \qquad \text{(B.2.3)}$$

$$\mathrm{V}(\tilde{L}_i) \;=\; \frac{1}{n} \cdot \Big[\bar{p}_i(1-\bar{p}_i) + \frac{2}{3}(a_i b_i + a_i c_i + b_i c_i) t^2\Big] \cdot \alpha_i^2 \cdot L_i^2 \;. \qquad \text{(B.2.4)}$$

The covariance becomes

$$\mathrm{Cov}(\tilde{L}_1, \tilde{L}_2) = \frac{1}{3 \cdot n} \cdot [(a_1 + b_1 + c_1) \cdot d_2 + (a_2 + b_2 + c_2) \cdot d_1] \cdot t^2 \cdot \alpha_1 \cdot \alpha_2 \cdot L_1 \cdot L_2 \;, \qquad \text{(B.2.5)}$$

resulting in

$$\mathrm{Corr}(\tilde{L}_1, \tilde{L}_2) \;=\; \frac{\frac{1}{3} \cdot [(a_1 + b_1 + c_1) \cdot d_2 + (a_2 + b_2 + c_2) \cdot d_1] \cdot t^2}{\prod_{i=1}^{2} \sqrt{\bar{p}_i(1-\bar{p}_i) + \frac{2}{3}(a_i b_i + a_i c_i + b_i c_i) t^2}} \qquad \text{(B.2.6)}$$

as inter-sectoral correlation.

# Appendix C

# Proofs to Chapter 6

## C.1 Proof of Result 30

*Proof.* To prove the existence of a solution to the household's unconstrained maximization problem it is crucial to show that

$$\lim_{D\to\pm\infty} \quad -e^{-\gamma\left[\mu(D+W_B)+(W_H-D)R_f-\gamma\sigma^2(D+W_B)^2\right]} \cdot \left[\Phi(\tfrac{\mu-\gamma\sigma^2(D+W_B)}{\sigma}) - \Phi(\tfrac{\mu-\frac{DR_D}{D+W_B}-\gamma\sigma^2(D+W_B)}{\sigma})\right] \quad = \quad -\infty$$

holds. The first term, $-e^{-\gamma\left[\mu(D+W_B)+(W_H-D)R_f-\gamma\sigma^2(D+W_B)^2\right]}$, diverges for $D \longrightarrow \pm\infty$ to $-\infty$ whereas the second term converges from above to zero. To determine the limit we use de l'Hôpital's rule. Therefore we consider the limit of

$$\lim_{D\to\pm\infty} \quad - \frac{\Phi(\frac{\mu-\gamma\sigma^2(D+W_B)}{\sigma}) - \Phi(\frac{\mu-\frac{DR_D}{D+W_B}-\gamma\sigma^2(D+W_B)}{\sigma})}{\left(e^{-\gamma\left[\mu(D+W_B)+(W_H-D)R_f-\gamma\sigma^2(D+W_B)^2\right]}\right)^{-1}} \ . \tag{C.1.1}$$

The derivative of the second term (the numerator) is

$$-\gamma\sigma\cdot\varphi(\tfrac{\mu-\gamma\sigma^2(D+W_B)}{\sigma}) \ + \ (\gamma\sigma + \tfrac{R_D W_B}{\sigma(D+W_B)^2})\cdot\varphi(\tfrac{\mu-\frac{DR_D}{D+W_B}-\gamma\sigma^2(D+W_B)}{\sigma}) \ > \ 0 \ .$$

Taking the derivative of the denominator with respect to $D$ and summarizing exponential factors yields:

$$-\left(e^{-\gamma\left[\mu(D+W_B)+(W_H-D)R_f-\gamma\sigma^2(D+W_B)^2\right]}\right)^{-1}\cdot \ \left[-\gamma(\mu-R_f)-2\gamma^2\sigma^2(D+W_B)\right] \ > \ 0 \ .$$

Let us define

$$
\begin{aligned}
E_1 &= e^{\left[-\frac{\mu^2}{2\sigma^2}-\gamma W_H R_f+\gamma D R_f+\frac{1}{2}\gamma^2\sigma^2(D+W_B)^2\right]} , \\
&\text{and} \\
E_2 &= e^{\left[-\gamma D R_D+\frac{1}{\sigma^2}\cdot\frac{D R_D}{D+W_B}\cdot(\mu-\frac{D R_D}{2(D+W_B)})\right]} .
\end{aligned}
$$

Their limiting behavior is given by

$$\lim_{D\to\pm\infty} E_1 = +\infty \ , \quad \lim_{D\to-\infty} E_2 = +\infty \ , \quad \text{and} \quad \lim_{D\to+\infty} E_2 = 0 \ . \tag{C.1.2}$$

Simplifying the ratio of these two derivatives by using the just defined terms $E_1$ and $E_2$ results in:

$$\frac{\gamma\sigma\cdot E_1\cdot\left\{\left[1+\frac{R_D}{\gamma\sigma^2(D+W_B)^2}\right]\cdot E_2-1\right\}}{\sqrt{2\pi}\cdot[\gamma(\mu-R_f)-2\gamma^2\sigma^2(D+W_B)]} .$$

Both the numerator and the denominator diverge for $D \longrightarrow \pm\infty$, but each with opposite sign. More precisely, we obtain for the numerator:

$$
\begin{aligned}
\lim_{D\to+\infty} \quad \gamma\sigma\cdot E_1\cdot\left\{\left[1+\frac{R_D}{\gamma\sigma^2(D+W_B)^2}\right]\cdot E_2-1\right\} &= -\infty \\
\lim_{D\to-\infty} \quad \gamma\sigma\cdot E_1\cdot\left\{\left[1+\frac{R_D}{\gamma\sigma^2(D+W_B)^2}\right]\cdot E_2-1\right\} &= +\infty
\end{aligned}
$$

In either case, the rule of de l'Hôpital can potentially be applied once more. The derivative of the denominator is simply $-2\sqrt{2\pi}\cdot\gamma^2\sigma^2$ such that the behavior of the numerator remains crucial.

The derivative with respect to $D$ of the numerator reads:

$$
\begin{aligned}
&\gamma\sigma\cdot E_1\cdot\left\{\left[1+\tfrac{R_D}{\gamma\sigma^2(D+W_B)^2}\right]\cdot E_2-1\right\}\cdot\left[\gamma R_f+\gamma^2\sigma^2\cdot(D+W_B)\right] \\
&+\gamma\sigma\cdot E_1\cdot\left\{\left[1-\tfrac{2R_D}{\gamma\sigma^2(D+W_B)^3}\right]\cdot E_2 \ + \ldots\right. \\
&\quad \left.\ldots+\left[1+\tfrac{R_D}{\gamma\sigma^2(D+W_B)^2}\right]\cdot\left[-\gamma R_D+\tfrac{1}{\sigma^2}\cdot\tfrac{R_D W_B}{(D+W_B)^2}\cdot\left(\mu-\tfrac{D R_D}{D+W_B}\right)\right]\cdot E_2-1\right\}
\end{aligned}
$$

Recall (C.1.2). Thus, for $D \longrightarrow +\infty$, the numerator becomes

$$
\begin{aligned}
&+\infty\cdot\{[1+0]\cdot 0-1\}\cdot[+\infty] \\
&+\infty\cdot\{[1-0]\cdot 0+[1+0]\cdot[-\gamma R_D+0\cdot(\mu-R_D)]\cdot 0-1\} \\
= \quad &\infty\cdot\{0-1\}\cdot\infty+\infty\cdot\{0+0-1\} \ = \ -\infty \ .
\end{aligned}
$$

Since the derivative of the denominator is equal to $-2\sqrt{2\pi}\cdot\gamma^2\sigma^2$, the whole ratio diverges for $D \longrightarrow +\infty$ to $+\infty$. By de l'Hôpital's rule this holds true for both preceding ratios and especially we obtain

$$\lim_{D \to +\infty} \quad -e^{-\gamma\left[\mu(D+W_B)+(W_H-D)R_f-\gamma\sigma^2(D+W_B)^2\right]} \cdot \left[\Phi(\tfrac{\mu-\gamma\sigma^2(D+W_B)}{\sigma}) - \Phi(\tfrac{\mu-\frac{DR_D}{D+W_B}-\gamma\sigma^2(D+W_B)}{\sigma})\right] \quad = \quad -\infty$$

because we have omitted the sign of Ratio (C.1.1).

Considering $D \longrightarrow -\infty$ yields:

$$\begin{aligned} & + \infty \cdot \{[1+0]\cdot\infty\} \cdot [-\infty] \\ & + \infty \cdot \big\{[1+0]\cdot\infty \ + \ [1+0]\cdot\big[-\gamma R_D + \tfrac{1}{\sigma^2}\cdot 0 \cdot (\mu - R_D)\big]\cdot\infty - 1\big\} \\ = \quad & \infty \cdot \{\infty\} \cdot [-\infty] \ + \ \infty \cdot \{[1-\gamma R_D]\cdot\infty\} \quad = \quad -\infty \ , \end{aligned}$$

as the first term, that is negative, outweighs the second term, that is positive, by an order of magnitude of $D$ (the first term grows at $E_1 \cdot E_2 \cdot D$ whereas the second term only by an order of magnitude of $E_1 \cdot E_2$). Thus we have,

$$\lim_{D \to -\infty} \gamma\sigma \cdot E_1 \cdot \left\{\left[1 + \frac{R_D}{\gamma\sigma^2(D+W_B)^2}\right]\cdot E_2 - 1\right\}[\gamma R_f + \gamma^2\sigma^2(D+W_B)] \quad = \quad -\infty \ ,$$

which in fact is irrespective of the sign of the second term. To summarize, we obtain the same result as for $D \longrightarrow +\infty$ and hence

$$\lim_{D \to \pm\infty} \quad -e^{-\gamma\left[\mu(D+W_B)+(W_H-D)R_f-\gamma\sigma^2(D+W_B)^2\right]} \cdot \left[\Phi(\tfrac{\mu-\gamma\sigma^2(D+W_B)}{\sigma}) - \Phi(\tfrac{\mu-\frac{DR_D}{D+W_B}-\gamma\sigma^2(D+W_B)}{\sigma})\right] \quad = \quad -\infty \ . \tag{C.1.3}$$

The remainder of the expected utility function (6.3.1) becomes for $D \longrightarrow -\infty$

$$\lim_{D \to -\infty} -e^{-\gamma(W_H-D)R_f} \cdot \Phi(-\frac{\mu}{\sigma}) \quad = \quad 0 \ ,$$

$$\lim_{D \to -\infty} -e^{-\gamma[DR_D+(W_H-D)R_f]} \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \quad = \quad -\infty \ ,$$

and for $D \longrightarrow +\infty$

$$\begin{aligned}\lim_{D \to +\infty} -e^{-\gamma(W_H - D)R_f} \cdot \Phi(-\frac{\mu}{\sigma}) &= -\infty \ , \\ \lim_{D \to +\infty} -e^{-\gamma[DR_D + (W_H - D)R_f]} \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) &= 0 \ ,\end{aligned}$$

where $R_D > R_f$ has been assumed to derive the limits of that part of expected utility that appreciates the state of full deposit redemption. Any other other relation between $R_D$ and $R_f$ does not change the overall result, however.

In the end, we have shown that

$$\lim_{D \to \pm\infty} \mathrm{E}[u_H(\tilde{W}_H)] = -\infty \qquad \text{(C.1.4)}$$

holds. Since $\mathrm{E}[u_H(\tilde{W}_H)]$ is continuous (and differentiable) for all real $D$, there is always at least one real number $D^u$ maximizing $\mathrm{E}[u_H(\tilde{W}_H)]$. By strict concavity of $\mathrm{E}[u_H(\tilde{W}_H)]$ with respect to $D$, $D^u$ is unique. □

## C.2 Proof of Result 31

*Proof.*

**The Lower Bound to the Unconstrained Deposit Supply Function**

Consider the partial derivative of expected utility with respect to $D$ which is given by:

$$
\begin{aligned}
\frac{\partial \mathrm{E}[u_H(\tilde{W}_H)]}{\partial D} \;=\; & - e^{-\gamma (W_H - D) R_f} \cdot \Phi(-\tfrac{\mu}{\sigma}) \cdot \gamma R_f \\[2ex]
& - e^{-\gamma \left[\mu(D+W_B) + (W_H - D)R_f - \frac{1}{2}\gamma\sigma^2 (D+W_B)^2\right]} \cdot \\
& \quad \cdot \left[ \Phi(\tfrac{\mu - \gamma\sigma^2 (D+W_B)}{\sigma}) \; - \; \Phi(\tfrac{\mu - \frac{DR_D}{D+W_B} - \gamma\sigma^2(D+W_B)}{\sigma}) \right] \cdot \\
& \quad \cdot (-\gamma) \cdot [\mu - R_f - \gamma\sigma^2 (D + W_B)] \\[2ex]
& - e^{-\gamma \left[\mu(D+W_B) + (W_H - D)R_f - \frac{1}{2}\gamma\sigma^2 (D+W_B)^2\right]} \cdot \\
& \quad \cdot \Big[ \varphi(\tfrac{\mu - \gamma\sigma^2(D+W_B)}{\sigma}) \cdot (-\gamma\sigma) \; - \\
& \qquad - \varphi(\tfrac{\mu - \frac{DR_D}{D+W_B} - \gamma\sigma^2 (D+W_B)}{\sigma}) \cdot \tfrac{1}{\sigma} \cdot \Big( -\tfrac{W_B R_D}{(D+W_B)^2} - \gamma\sigma^2 \Big) \Big] \\[2ex]
& - e^{-\gamma \left[ D R_D + (W_H - D) R_f \right]} \; \cdot \; \Phi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot (-\gamma) \cdot (R_D - R_f) \\[2ex]
& - e^{-\gamma \left[ D R_D + (W_H - D) R_f \right]} \; \cdot \; \varphi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot (-\tfrac{1}{\sigma}) \cdot \tfrac{W_B R_D}{(D+W_B)^2}
\end{aligned}
$$

Gathering all terms involving exponentials, in particular including those with the density function of the normal distribution $\varphi(\cdot)$, yields

$$
\begin{aligned}
\frac{\partial \mathrm{E}[u_H(\tilde{W}_H)]}{\partial D} \;=\; & - e^{-\gamma (W_H - D) R_f} \cdot \Phi(-\tfrac{\mu}{\sigma}) \cdot \gamma R_f \\[2ex]
& - e^{-\gamma \left[\mu(D+W_B) + (W_H - D)R_f - \frac{1}{2}\gamma\sigma^2 (D+W_B)^2\right]} \cdot \\
& \quad \cdot \left[ \Phi(\tfrac{\mu - \gamma\sigma^2 (D+W_B)}{\sigma}) \; - \; \Phi(\tfrac{\mu - \frac{DR_D}{D+W_B} - \gamma\sigma^2(D+W_B)}{\sigma}) \right] \cdot \\
& \quad \cdot (-\gamma) \cdot [\mu - R_f - \gamma\sigma^2 (D + W_B)] \\[2ex]
& - e^{-\gamma \left[ D R_D + (W_H - D) R_f \right]} \; \cdot \; \Phi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot (-\gamma) \cdot (R_D - R_f) \\[2ex]
& + e^{-\gamma (W_H - D) R_f} \; \cdot \; \varphi(\tfrac{\mu}{\sigma}) \cdot \gamma\sigma \\
& - e^{-\gamma \left[ D R_D + (W_H - D) R_f \right]} \; \cdot \; \varphi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot \gamma\sigma \; ,
\end{aligned}
$$

Dividing by $\gamma e^{-\gamma (W_H - D) R_f}$, the first-order condition becomes

$$
\begin{aligned}
- e^{-\gamma D R_D} \cdot \sigma \cdot \varphi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \; + \; \sigma \cdot \varphi(\tfrac{\mu}{\sigma}) & \\
+ \, e^{-\gamma \cdot \left[ \mu (D+W_B) - \frac{1}{2}\gamma\sigma^2 (D+W_B)^2 \right]} \cdot [R_f - \mu + \gamma\sigma^2 (D + W_B)] \cdot \Delta\Phi & \\
- R_f \cdot \Phi(-\tfrac{\mu}{\sigma}) \; + \; e^{-\gamma D R_D} \cdot (R_D - R_f) \cdot \Phi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \;\; = \;\; 0 \; , &
\end{aligned}
\tag{C.2.1}
$$

where

$$\Delta\Phi \;=\; \left[\Phi(\frac{\mu - \frac{DR_D}{D+W_B} - \gamma\sigma^2(D+W_B)}{\sigma}) - \Phi(\frac{\mu - \gamma\sigma^2(D+W_B)}{\sigma})\right] \;, \qquad \text{(C.2.2)}$$

hence, $-1 < \Delta\Phi < 0$ holds. By (5.4.14), the first-order condition may be approximated by

$$\begin{aligned} &- e^{-\gamma DR_D} \cdot \sigma \cdot \varphi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \\ &+ e^{-\gamma\cdot\left[\mu(D+W_B) - \frac{1}{2}\gamma\sigma^2(D+W_B)^2\right]} \cdot \left[R_f - \mu + \gamma\sigma^2(D+W_B)\right] \cdot \Delta\Phi \\ &+ e^{-\gamma DR_D} \cdot (R_D - R_f) \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \;=\; 0 \;. \end{aligned}$$

Let

$$\hat{D} \;=\; \frac{\mu - R_f}{\gamma\sigma^2} \;-\; W_B$$

be a solution to the household's problem. Hence, $R_f - \mu + \gamma\sigma^2(\hat{D} + W_B) \equiv 0$ holds. The approximated first-order condition becomes at $D = \hat{D}$

$$\begin{aligned} &- e^{-\gamma \hat{D} R_D} \cdot \sigma \cdot \varphi(\frac{\mu - \frac{\hat{D}R_D}{\hat{D}+W_B}}{\sigma}) \\ &e^{-\gamma \hat{D} R_D} \cdot (R_D - R_f) \cdot \Phi(\frac{\mu - \frac{\hat{D}R_D}{\hat{D}+W_B}}{\sigma}) \;>\; 0 \;, \end{aligned}$$

whereas the positive sign is due to Assumption (6.3.4). Thus, in conjunction with strict concavity and hence uniqueness, as established by Result 30, the household can improve unambiguously by increasing $D$. Hence, the optimal unconstrained deposit supply satisfies

$$D^u \;>\; \frac{\mu - R_f}{\gamma\sigma^2} \;-\; W_B \;.$$

### Behavior of the Deposit Supply Function in $R_f$

Consider the household's first-order condition

$$\begin{aligned} &- e^{-\gamma DR_D} \cdot \sigma \cdot \varphi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \;+\; \sigma \cdot \varphi(\tfrac{\mu}{\sigma}) \\ &+ e^{-\gamma\cdot\left[\mu(D+W_B) - \frac{1}{2}\gamma\sigma^2(D+W_B)^2\right]} \cdot [R_f - \mu + \gamma\sigma^2(D+W_B)] \cdot \Delta\Phi \qquad \text{(C.2.3)} \\ &- R_f \cdot \Phi(-\tfrac{\mu}{\sigma}) \;+\; e^{-\gamma DR_D} \cdot (R_D - R_f) \cdot \Phi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \;\stackrel{!}{=}\; 0 \;. \end{aligned}$$

Let $\mathrm{LHS}_{\mathrm{FOC}}$ be the left-hand side of this equation and by definition $\mathrm{LHS}_{\mathrm{FOC}}$ =

$\frac{\partial \mathrm{E}[\tilde{W}_H(\ell,R_D)]}{\partial D} \cdot \frac{1}{\gamma} \cdot e^{\gamma(W_H-D)R_f}$ holds.

Since we have $\frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial D^2} < 0$ by strict concavity, the signs of the partial derivatives $\frac{\partial D}{\partial \theta}$ are exclusively determined by the signs of $\frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial \theta \partial D}$ where $\theta$ is the respective parameter considered.

Let us define

$$\begin{aligned} A &= \left[\varphi(\tfrac{\mu}{\sigma}) - e^{-\gamma D R_D}\varphi(\tfrac{\mu-\frac{DR_D}{D+W_B}}{\sigma})\right] &< 0\,, \\ B &= e^{-\gamma\cdot\left[\mu(D+W_B)-\gamma\sigma^2(D+W_B)^2\right]}\cdot\Delta\Phi &< 0\,, \end{aligned}$$

where the sign of expression $A$ is due to Approximation (5.4.14), *i.e.* $\varphi(\frac{\mu}{\sigma}) \approx 0$.

Consider now the second mixed partial derivative of $\mathrm{LHS}_{\mathrm{FOC}}$ with respect to $R_f$, which is simply given by

$$\frac{\partial \mathrm{LHS}_{\mathrm{FOC}}}{\partial R_f} = B - \Phi(-\frac{\mu}{\sigma}) - e^{-\gamma D R_D}\cdot\Phi(\frac{\mu-\frac{DR_D}{D+W_B}}{\sigma}) < 0\,.$$

As

$$\frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial R_f \partial D} = \underbrace{\frac{\partial \mathrm{LHS}_{\mathrm{FOC}}}{\partial R_f}}_{<0}\cdot\frac{1}{\gamma}\cdot e^{\gamma(W_H-D)R_f} + \underbrace{\mathrm{LHS}_{\mathrm{FOC}}}_{\equiv 0}\cdot\frac{1}{\gamma}\cdot e^{\gamma(W_H-D)R_f}\cdot(-\gamma\cdot D)$$

holds,

$$\frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial R_f \partial D} < 0$$

obtains. Thus, the unconstrained deposit supply strictly decreases in $R_f$.

### Behavior of the Deposit Supply Function in $\mu$ and $\sigma$

To determine the sign of $\frac{\partial D}{\partial \mu}$ we must analyze $\frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial \mu \partial D}$ which is equal to $\frac{\partial \mathrm{LHS}_{\mathrm{FOC}}}{\partial \mu}$ except for constants. Thus this second mixed derivative is proportional to:

$$\begin{aligned} \frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial \mu \partial D} \sim\; & -\gamma\sigma\cdot(D+W_B)\cdot A \\ & + \frac{W_B\cdot R_D}{\sigma(D+W_B)}\cdot e^{-\gamma D R_D}\cdot\varphi(\tfrac{\mu-\frac{DR_D}{D+W_B}}{\sigma}) \\ & - \{1+[R_f-\mu+\gamma\sigma^2\cdot(D+W_B)]\cdot\gamma(D+W_B)\}\cdot B \end{aligned} \tag{C.2.4}$$

We know from above $A < 0$ and $B < 0$. The expression $[R_f - \mu + \gamma\sigma^2(D+W_B)]$ is positive because of (6.3.5). So we obtain

$$\frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial\mu\partial D} \;>\; 0\;, \quad \text{and thus} \quad \frac{\partial D^u(\ell, R_D)}{\partial\mu} \;>\; 0\;. \tag{C.2.5}$$

Next we consider the partial derivative with respect to $\sigma$ in an analogous way:

$$\begin{aligned}\tfrac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial\sigma\partial D} \;\sim\;& \{1+[R_f+\gamma\sigma^2(D+W_B)]\,\gamma(D+W_B)\}\cdot A\\ &+\tfrac{\gamma\sigma^2 DR_D(D+W_B)^2-R_D W_B[\mu(D+W_B)-DR_D]}{\sigma^2(D+W_B)^2}\cdot e^{-\gamma DR_D}\cdot\varphi(\tfrac{\mu-\frac{DR_D}{D+W_B}}{\sigma})\\ &+\;\{2+[R_f-\mu+\gamma\sigma^2(D+W_B)]\cdot\gamma(D+W_B)\}\cdot\gamma\sigma(D+W_B)\cdot B\end{aligned} \tag{C.2.6}$$

Approximating $\frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial\sigma\partial D}$ by $\varphi(\frac{\mu}{\sigma})\approx 0$ yields

$$\begin{aligned}&\tfrac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial\sigma\partial D} \;\approx\\ &\Big\{-1-[R_f+\gamma\sigma^2(D+W_B)]\,\gamma(D+W_B)+\gamma DR_D-\tfrac{[\mu(D+W_B)-DR_D]\cdot W_B\cdot R_D}{\sigma^2(D+W_B)^2}\Big\}\cdot\ldots\\ &\qquad\ldots\;\cdot e^{-\gamma DR_D}\cdot\varphi(\tfrac{\mu-\frac{DR_D}{D+W_B}}{\sigma})\\ &+\;\{2+[R_f-\mu+\gamma\sigma^2(D+W_B)]\cdot\gamma\cdot(D+W_B)\}\cdot\gamma\sigma\cdot(D+W_B)\cdot B\end{aligned} \tag{C.2.7}$$

The first term in curly braces is negative because

$$\begin{aligned}&-[R_f+\gamma\sigma^2(D+W_B)]\cdot\gamma\cdot(D+W_B)\;+\;\gamma DR_D\;<\\ &\quad<\;-[R_f+\mu-R_f]\cdot\gamma\cdot(D+W_B)\;+\;\gamma DR_D\;=\\ &\quad=\;-\gamma\cdot[\mu(D+W_B)-DR_D]\;<\;0\end{aligned}$$

holds by (6.3.5). The second term is negative for the same reasons as it is the case in the last line of Derivative (C.2.4). □

## C.3 Proof of Result 32

*Proof.* Here, the expression deposit supply always refers to the unconstrained deposit supply on the domain $[\underline{R}_D(\ell), \infty)$.

Since we have $\frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial D^2} < 0$ by strict concavity, the sign of $\frac{\partial D}{\partial R_D}$ is determined by the sign of $\frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial R_D \partial D}$. The latter reads:

$$\begin{aligned} \frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial R_D \partial D} \;=\; & -\frac{D \cdot W_B \cdot R_D}{\sigma(D+W_B)^2} \cdot e^{-\gamma D R_D} \cdot \varphi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \\ & + e^{-\gamma D R_D} \cdot \Phi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \\ & - \gamma D \cdot (R_D - R_f) \cdot e^{-\gamma D R_D} \cdot \Phi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \;. \end{aligned} \tag{C.3.1}$$

That is, $\frac{\partial^2 \mathrm{E}[u_H(\tilde{W}_H)]}{\partial R_D \partial D}$ is positive if and only if

$$R_D \;<\; \frac{(1+\gamma \cdot DR_f) \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma})}{\varphi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot \frac{D \cdot W_B}{\sigma(D+W_B)^2} + \gamma \cdot D \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma})} \tag{C.3.2}$$

holds. We define

$$f(R_D) \;=\; \frac{(1+\gamma \cdot DR_f) \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma})}{\varphi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot \frac{D \cdot W_B}{\sigma(D+W_B)^2} + \gamma \cdot D \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma})} \;.$$

To draw conclusions about the existence and uniqueness of $\overline{R}^u_D(\ell)$, we will analyze the behavior of $f(R_D)$ in $R_D$.

We note that $\frac{\partial D}{\partial R_D}$ exists on $(\underline{R}_D(\ell), \infty)$ by Result 31 and that

$$\lim_{R_D \downarrow \underline{R}_D(\ell)} f(R_D) \;=\; \infty \tag{C.3.3}$$

holds.

The derivative of the numerator of $f(R_D)$ with respect to $R_D$ is,

$$\frac{\partial\, \bullet}{\partial R_D} \;=\; \gamma R_f \cdot \frac{\partial D}{\partial R_D} \cdot \Phi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) + (1+\gamma D R_f) \cdot \varphi(\tfrac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot \Omega \tag{C.3.4}$$

where $\Omega$ is defined as

$$\Omega \;\equiv\; -\frac{D}{\sigma(D+W_B)} \;-\; \frac{R_D W_B}{\sigma(D+W_B)^2} \cdot \frac{\partial D}{\partial R_D} \;. \tag{C.3.5}$$

$\Omega < 0$ holds for $W_B = 0$ and $\Omega < 0$ holds for $W_B > 0$ if and only if

$$\frac{\partial D}{\partial R_D} \;>\; -\frac{D \cdot (D+W_B)}{R_D \cdot W_B}$$

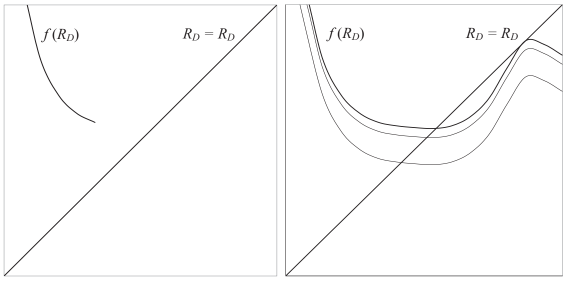


Figure C.1: **Feasible graphs for the function** $f(R_D)$

*This figure illustrates feasible graphs for the function $f(R_D)$ in relation to the identity $R_D \equiv R_D$. The left-hand chart summarizes the conclusions drawn by* (C.3.2), (C.3.7), *and* (C.3.3). *The right-hand chart shows generic graphs for $f(R_D)$ as implied by this proof.*

is satisfied. The derivative of the denominator of $f(R_D)$ with respect to $R_D$ is:

$$\begin{aligned}\frac{\partial \circ}{\partial R_D} \quad = \quad & - \varphi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot \frac{D \cdot W_B}{\sigma(D+W_B)^2} \cdot \frac{\mu - \frac{DR_D}{D+W_B}}{\sigma} \cdot \Omega \\ & + \gamma \cdot \frac{\partial D}{\partial R_D} \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \ + \ \gamma D \cdot \varphi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot \Omega \ . \qquad \text{(C.3.6)}\end{aligned}$$

After having expanded and simplified the numerator of $f'(R_D)$ we obtain

$$\begin{aligned}& \varphi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot \frac{D \cdot W_B}{\sigma(D+W_B)^2} \cdot \gamma \cdot R_f \cdot \frac{\partial D}{\partial R_D} \\ & + \varphi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma}) \cdot \frac{D \cdot W_B}{\sigma(D+W_B)^2} \cdot \frac{(1+\gamma DR_f) \cdot \left(\mu - \frac{DR_D}{D+W_B}\right)}{\sigma} \cdot \Omega \\ & - \gamma \cdot \Phi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma})^2 \cdot \frac{\partial D}{\partial R_D} \\ & + \varphi(\frac{\mu - \frac{DR_D}{D+W_B}}{\sigma})^2 \cdot \frac{DW_B}{\sigma(D+W_B)^2} \cdot (1+\gamma DR_f) \cdot \Omega\end{aligned}$$

Recall Condition (6.3.10), *i.e.*

$$\frac{\varphi(b^u_{s_z})}{\Phi(b^u_{s_z})} \cdot \frac{D^u \cdot W_B \cdot R_f}{\sigma(D^u + W_B)^2} \ < \ 1 \ ,$$

This condition allows the following two conclusions,

$$\begin{aligned}\frac{\partial D}{\partial R_D} \ > \ 0 \ \Rightarrow \ & f'(R_D) \ < \ 0 \ , \\ f'(R_D) \ > \ 0 \ \Rightarrow \ & \frac{\partial D}{\partial R_D} \ < \ 0 \ . \qquad \text{(C.3.7)}\end{aligned}$$

For $R_D \geq \underline{R}_D(\ell)$, $\frac{\partial D}{\partial R_D} > 0$ always holds, provided that $R_D$ is not "too big", because any agent, independent of his or her risk preferences, is ready to take some risk as soon as the promised return $R_D$ exceeds the zero $\underline{R}_D(\ell)$ which equals the expected return on deposits for an $\varepsilon \to 0$ invested into deposits.

Thus, in the neighborhood of $\underline{R}_D(\ell)$ with $R_D \geq \underline{R}_D(\ell)$ , $f(R_D)$ strictly decreases in $R_D$ and has a pole for $R_D$ approaching $\underline{R}_D(\ell)$ from below. These two properties base on (C.3.7) and (C.3.3), respectively. Furthermore, $\frac{\partial D}{\partial R_D} > 0$ implies via (C.3.2) the relation $R_D < f(R_D)$. The left-hand chart of Figure C.1 summarizes these observations.

Assume now that $f(R_D)$ increases while remaining *above* the identity. Then deposit supply decreases in $R_D$, $\frac{\partial D}{\partial R_D} < 0$, according to (C.3.7). This, in turn, leads to $R_D > f(R_D)$ because of Relation (C.3.2), meaning that $f(R_D)$ is in fact located *below* the identity: hence, a contradiction to our assumption. Consequently, $f(R_D)$ decreases until it crosses the identity, implying by (C.3.2) that $\frac{\partial D}{\partial R_D} < 0$ holds thereafter and thus $\overline{R}_D^u(\ell)$ exists.

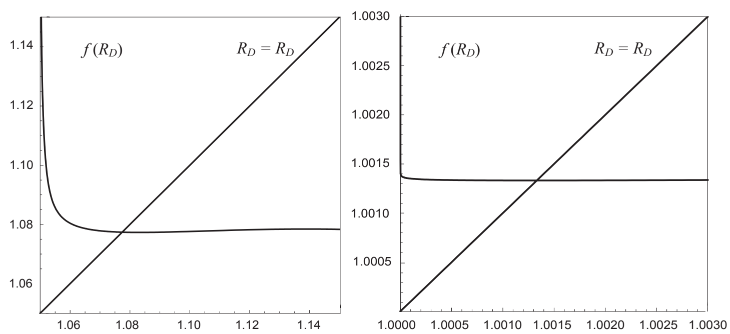


Figure C.2: **The function $f(R_D)$ and its relation to the identity $R_D \equiv R_D$**

*This figure illustrates the course of the function $f(R_D)$. The left-hand chart shows the graph of $f(R_D)$ with the parameter values given in Table 6.1 and given $\ell = 0.597$. The right-hand chart shows the graph of $f(R_D)$ with the parameter values given in Table 6.2, Panel A and given $\ell = 0.628$. Hence, the chosen loan-allocation rates equal their respective laissez-faire equilibrium values. Cf. Figure A.1.*

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Let us establish uniqueness. Assume first that $f(R_D)$ crosses the identity a second time from below. After this second intersection we have $R_D < f(R_D)$ implying $\frac{\partial D}{\partial R_D} > 0$ by (C.3.2). But we also have, at least in a neighborhood above that intersection, $f'(R_D) > 0$ leading to a contradiction because of (C.3.7). Thus, after this "intersection" we must obtain $f'(R_D) < 0$. That is, any potential second intersection with the identity is in fact a tangency point. However, this tangency point vanishes if the model is perturbed slightly. Thus, $\overline{R}_D^u(\ell)$ is generically unique.

The right-hand chart of Figure C.1 summarizes the idea of this proof.

Figure C.2 shows the relation between $R_D$ and $f(R_D)$ with the parameter values given in Tables 6.1 and 6.2, Panel A. The loan-allocation rates $\ell$ equal their laissez-faire equilibrium values, *i.e.* $\ell = 0.597$ and $\ell = 0.628$, respectively. $\square$

# Appendix D

# Proofs to Chapter 7

## D.1 Existence of the Deposit Supply Function in the First Period

Recall the household's expected utility in $t = 0$ over final wealth. First of all,

$$\int\limits_{b_{d_x,1}}^{\infty} \mathrm{E}_1 \left[ u_H(\tilde{W}_{H,2}) \,|W_{B,1} \right] \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1) \; dx_1 \;<\; 0$$

trivially holds for all $D_0 \in \mathbb{R}$ where

$$z_1 \;\equiv\; \frac{x_1 - \mu_1}{\sigma_1} \tag{D.1.1}$$

is the centralized return on the whole loan portfolio after the first period and where

$$b_{d_x,1} \;\equiv\; \frac{D_0 R_{D,0}}{D_0 + W_{B,0}} \tag{D.1.2}$$

is the bank's solvency barrier after the first period in terms of the non-centralized portfolio returns (*cf.* $b_{d_z,t+1}$).

Second, the limits for the last term if $D_0 \to \pm\infty$ are given by

$$\begin{aligned}
\lim_{D_0 \to -\infty} \; -e^{\gamma D_0 R_{f,0} R_{f,1}} \cdot \left[ 1 - \Phi(\frac{\mu_1}{\sigma_1}) \right] \;&=\; 0^- \;, \\
\lim_{D_0 \to +\infty} \; -e^{\gamma D_0 R_{f,0} R_{f,1}} \cdot \left[ 1 - \Phi(\frac{\mu_1}{\sigma_1}) \right] \;&=\; -\infty \;.
\end{aligned}$$

Let

$$\Delta\Phi_1 \;=\; \Phi\Big(\frac{\mu_2 - \gamma\sigma_2^2(D_1 + W_{B,1})}{\sigma_2}\Big) - \Phi\Big(\frac{\mu_2 - \frac{D_1 R_{D,1}}{D_1+W_{B,1}} - \gamma\sigma_2^2(D_1 + W_{B,1})}{\sigma_2}\Big) \tag{D.1.3}$$

and analogously

$$\Delta\Phi_0 \;=\; \Phi\Big(\frac{\mu_1 - \gamma\sigma_1^2(D_0 + W_{B,0})R_{f,1}}{\sigma_1}\Big) - \Phi\Big(\frac{\mu_1 - b_{d_x,1} - \gamma\sigma_1^2(D_0 + W_{B,0})R_{f,1}}{\sigma_1}\Big) \;. \tag{D.1.4}$$

Furthermore, for the last but second term one obtains

$$\lim_{D_0\to\pm\infty} \; -e^{-\gamma\left[\mu_1(D_0+W_{B,0})-D_0R_{f,0}-\frac{1}{2}\gamma\sigma_1^2(D_0+W_{B,0})^2R_{f,1}\right]R_{f,1}} \cdot \Delta\Phi_0 \;=\; -\infty \;,$$

for the derivation of this limit, we refer to Appendix C.1.

Altogether, we obtain

$$\lim_{D_0\to\pm\infty} \mathrm{E}_0\left[\mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,\middle|\,\tilde{W}_{B,1}\right]\right] \;=\; -\,\infty \;. \tag{D.1.5}$$

This property results in conjunction with differentiability in $D_0$ in at least one extremal point. That is,

$$\frac{\partial\, \mathrm{E}_0\left[\mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,\middle|\,\tilde{W}_{B,1}\right]\right]}{\partial D_0} \;\overset{!}{=}\; 0 \tag{D.1.6}$$

has at least one solution on $\mathbb{R}$ in $D_0$.

## D.2 Proof of Result 36

If the second period's choices, $D_1^s(\cdot)$, $R_{D,1}^*(\cdot)$, and $\ell_{D,1}^*(\cdot)$, were constants, the concavity in $D_0$ of the household's expected utility in $t = 0$ over final wealth $\tilde{W}_{H,2}$ would follow immediately. However, as $D_1^s(\cdot)$, $R_{D,1}^*(\cdot)$ and $\ell_{D,1}^*(\cdot)$ depend on $D_0$ in a non-trivial manner, the arguments presented so far, as done on p. 181 are not enough.

In the next paragraph, we will characterize the second derivative of $\mathrm{E}_0\left[\mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,\middle|\,\tilde{W}_{B,1}\right]\right]$ in each extremal point. By doing so, we obtain conditions for the second derivative's negative sign and hence uniqueness of $D_0$ can be established. To keep notations under integrals short, $\tilde{W}_{B,1}$ or realizations of it, $W_{B,1}$, will occasionally be skipped under the expectations operator and replaced by a dot.

**The first-order condition**

We define

$$\begin{aligned} g(D_0) \;&:=\; -\int_{b_{d_x,1}}^{\infty} e^{-\gamma(W_{H,1}-D_1)R_{f,1}} \cdot \Bigg\{ 1-\Phi(\frac{\mu_2}{\sigma_2}) \\ &\qquad + \; e^{-\gamma\left[\mu_2(D_1+W_{B,1})-\frac{1}{2}\gamma\sigma_2^2(D_1+W_{B,1})^2\right]} \cdot \Delta\Phi_1 \\ &\qquad + \; e^{-\gamma D_1 R_{D,1}} \cdot \Phi(b_{d_z,2}) \Bigg\} \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1)\; dx_1 \\ &\equiv \int_{b_{d_x,1}}^{\infty} \mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,|W_{B,1}\right] \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1)\; dx_1 \end{aligned} \tag{D.2.1}$$

and

$$\begin{aligned} h(D_0) \;&=\; -e^{-\gamma\left[\mu_1(D_0+W_{B,0})+(W_{H,0}-D_0)R_{f,0}-\frac{1}{2}\gamma\sigma_1^2(D_0+W_{B,0})^2R_{f,1}\right]R_{f,1}} \cdot \Delta\Phi_0 \\ &\quad -e^{-\gamma(W_{H,0}-D_0)R_{f,0}R_{f,1}} \cdot \left[1-\Phi(\frac{\mu_1}{\sigma_1})\right] \end{aligned} \tag{D.2.2}$$

such that

$$\mathrm{E}_0\left[\mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,\middle|\tilde{W}_{B,1}\right]\right] \;\equiv\; g(D_0) \;+\; h(D_0)$$

holds. Thus the household's first-order condition in $t=0$ can be represented by $g'(D_0)+h'(D_0)=0$ where

$$\begin{aligned} g'(D_0) \;=\; &\int_{b_{d_x,1}}^{\infty} \Bigg( \frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial W_{B,1}} \cdot (x_1 - R_{D,0}) \;+\; \frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial W_{H,1}} \cdot (R_{D,0}-R_{f,0}) \\ &+ \frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial R_{D,1}} \cdot \frac{\partial R^*_{D,1}}{\partial D_0} \;+\; \frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial \ell_1} \cdot \frac{\partial \ell^*_{D,1}}{\partial D_0} \Bigg) \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1)\; dx_1 \\ &- \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1=b_{d_x,1}} \cdot \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1}) \end{aligned}$$

and

$$h'(D_0) = \gamma R_{f,1}\left[\mu_1 - R_{f,0} - \gamma\sigma_1^2(D_0+W_{B,0})R_{f,1}\right]\cdot$$
$$\cdot\, e^{-\gamma\left[\mu_1(D_0+W_{B,0})+(W_{H,0}-D_0)R_{f,0}-\frac{1}{2}\gamma\sigma_1^2(D_0+W_{B,0})^2R_{f,1}\right]R_{f,1}}\cdot\Delta\Phi_0$$

$$+\gamma\sigma_1 R_{f,1}e^{-\gamma(W_{H,0}-D_0)R_{f,0}}\cdot\varphi(\frac{\mu_1}{\sigma_1})$$

$$-\left(\gamma\sigma_1 R_{f,1}+\frac{R_{D,0}W_{B,0}}{\sigma_1(D_0+W_{B,0})^2}\right)\cdot e^{-\gamma\left[D_0R_{D,0}+(W_{H,0}-D_0)R_{f,0}\right]R_{f,1}}\cdot\varphi(b_{d_z,1})$$

$$-\gamma R_{f,0}R_{f,1}\cdot e^{\gamma(W_{H,0}-D_0)R_{f,0}R_{f,1}}\cdot\left[1-\Phi(\frac{\mu_1}{\sigma_1})\right].$$

The derivative $g'(D_0)$ is obtained by the Leibniz rule and after some simplifying algebraic manipulations. Note that

$$\frac{\partial \mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,|W_{B,1}\right]}{\partial D_1} \equiv 0$$

holds given the equilibrium in $t=1$. Consequently, the dependencies on $D_0$ via $D_1^s(\cdot)$ vanish. The partial derivatives of expected utility $\mathrm{E}_1[u_H(\tilde{W}_{H,2})]$ have thus be understood as the direct dependencies on the respective variables and parameters, and never as dependencies via (equilibrium) deposit supply in $t=1$.

Because of

$$\frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial W_{H,1}} = -\gamma R_{f,1}\cdot\mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,|W_{B,1}\right] > 0 \qquad \text{(D.2.3)}$$

the derivative $g'(D_0)$ further simplifies to

$$g'(D_0) = \int\limits_{b_{d_x,1}}^{\infty}\left(\frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial W_{B,1}}\cdot(x_1-R_{D,0}) + \frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial R_{D,1}}\cdot\frac{\partial R_{D,1}^*}{\partial D_0}\right.$$
$$\left.+\frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial \ell_1}\cdot\frac{\partial \ell_{D,1}^*}{\partial D_0}\right)\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)\,dx_1$$

$$-\gamma\cdot R_{f,1}\cdot(R_{D,0}-R_{f,0})\cdot g(D_0)$$
$$-\mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1=b_{d_x,1}}\cdot\frac{1}{\sigma_1}\cdot\varphi(b_{d_z,1})\,. \qquad \text{(D.2.4)}$$

We note that

$$\lim_{x_1 \downarrow b_{d_x,1}} \frac{D_1 R_{D,1}}{D_1 + W_{B,1}} \;=\; 0 \tag{D.2.5}$$

holds which can be illustrated by de l' Hôpital's rule, yielding

$$\lim_{x_1 \downarrow b_{d_x,1}} \frac{\frac{\partial D_1}{\partial x_1} \cdot R_{D,1} \;+\; D_1 \cdot \frac{\partial R_{D,1}}{\partial x_1}}{\frac{\partial D_1}{\partial x_1} + (D_0 + W_{B,0})} \;=\; \frac{\frac{\partial D_1}{\partial x_1} \cdot 0 \;+\; 0 \cdot \frac{\partial R_{D,1}}{\partial x_1}}{\frac{\partial D_1}{\partial x_1} + (D_0 + W_{B,0})} \;=\; 0$$

due to the Boundary Condition (7.1.3) and by additionally assuming that the denominator will be unequal to zero generically. The partial derivatives do exist from above.

Hence, we obtain

$$\mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1 = b_{d_x,1}} \;=\; -\,e^{-\gamma W_{H,1} R_{f,1}}\;, \tag{D.2.6}$$

as

$$\lim_{x_1 \downarrow b_{d_x,1}} \Delta\Phi_1 \;=\; 0 \tag{D.2.7}$$

because of (D.2.5). Thus, assuming $R_{D,1} = 0$ at $x_1 = b_{d_x,1}$ instead of any other value $R_{D,1} \leq \underline{R}_{D,1}$ is not an arbitrary choice, but is sensible, as thus $\Delta\Phi_1 \equiv 0$ obtains which is necessary to transform the expectations $\mathrm{E}_1[u_H(\tilde{W}_{H,2})]\Big|_{x_1 = b_{d_x,1}}$ into a number independent of random drawings at the end of the second period. In particular, final wealth is not only known in $t = 1$ to the household if the bank has defaulted, but must also exactly equal the expression shown in (D.2.6).

At $D_0 = 0$ we obtain

$$\begin{aligned} h'(0) \;&=\; -\,\frac{R_{D,0}}{\sigma_1 W_{B,0}} \cdot e^{-\gamma W_{H,0} R_{f,0} R_{f,1}} \cdot \varphi(\frac{\mu_1}{\sigma_1}) \\ &\quad -\,\gamma R_{f,0} R_{f,1} \cdot e^{-\gamma W_{H,0} R_{f,0} R_{f,1}} \cdot \left[1 - \Phi(\frac{\mu_1}{\sigma_1})\right] \;<\; 0. \end{aligned}$$

Because the function $h(D_0)$ is a linear combination of strictly concave functions in $D_0$, $h(D_0)$ is strictly concave in $D_0$ which results in

$$h'(D_0) \;<\; 0 \quad \forall \quad D_0 \;\geq\; 0\,. \tag{D.2.8}$$

because of $h'(0) < 0$ by contradiction. Consequently,

$$g'(D_0) \;>\; 0 \tag{D.2.9}$$

holds in every extremal point.

**The second-order condition**

The second derivative of $g(D_0)$ is calculated by taking the following partial derivatives with respect to $D_0$:

$$\begin{aligned}
g''(D_0) \quad = \quad & \frac{d}{dD_0}\left[\int\limits_{b_{d_x,1}}^{\infty} \frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial W_{B,1}} \cdot (x_1 - R_{D,0}) \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1)dx_1\right] \\
& + \frac{d}{dD_0}\left[\int\limits_{b_{d_x,1}}^{\infty} \frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial R_{D,1}} \cdot \frac{\partial R^*_{D,1}}{\partial D_0} \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1)dx_1\right] \\
& + \frac{d}{dD_0}\left[\int\limits_{b_{d_x,1}}^{\infty} \frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial \ell_1} \cdot \frac{\partial \ell^*_1}{\partial D_0} \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1)dx_1\right] \\
& - \gamma \cdot R_{f,1} \cdot (R_{D,0} - R_{f,0}) \cdot g'(D_0) \\
& + \frac{d}{dD_0}\left[- \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1=b_{d_x,1}} \cdot \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1})\right] \ .
\end{aligned}$$

Due to the first-order condition we know $g'(D_0) > 0$ such that the last but one line is negative provided $R_{D,0} > R_{f,0}$.

Taking the remaining partial derivatives and ordering them results in

$$
\begin{aligned}
- \quad & \gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot g'(D_0) \\
+ \quad & \int_{b_{d_x,1}}^{\infty} \left[ \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1}^2} \cdot (x_1 - R_{D,0})^2 \; + \; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}^2} \cdot \left( \frac{\partial R_{D,1}^*}{\partial D_0} \right)^2 \; + \right. \\
& \qquad \left. + \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1^2} \cdot \left( \frac{\partial \ell_1^*}{\partial D_0} \right)^2 \right] \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1) dx_1 \\
+ \quad & \int_{b_{d_x,1}}^{\infty} \left[ \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1} \partial W_{H,1}} \cdot (x_1 - R_{D,0}) \; + \; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1} \partial W_{H,1}} \cdot \frac{\partial R_{D,1}^*}{\partial D_0} \; + \right. \\
& \qquad \left. + \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1 \partial W_{H,1}} \cdot \frac{\partial \ell_1^*}{\partial D_0} \right] \cdot (R_{D,0} - R_{f,0}) \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1) dx_1 \\
+ \quad & \int_{b_{d_x,1}}^{\infty} \left[ \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1} \partial D_1} (x_1 - R_{D,0}) + \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1} \partial D_1} \frac{\partial R_{D,1}^*}{\partial D_0} + \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1 \partial D_1} \frac{\partial \ell_1^*}{\partial D_0} \right] \cdot \\
& \qquad \cdot \left[ \frac{\partial D_1^s}{\partial D_0} \; + \; \frac{\partial D_1^s}{\partial R_{D,1}} \cdot \frac{\partial R_{D,1}^*}{\partial D_0} \; + \; \frac{\partial D_1^s}{\partial \ell_1} \cdot \frac{\partial \ell_1^*}{\partial D_0} \right] \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1) dx_1 \\
+ \quad & \int_{b_{d_x,1}}^{\infty} 2 \cdot \left\{ (x_1 - R_{D,0}) \left[ \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1} \partial \ell_1} \cdot \frac{\partial \ell_1^*}{\partial D_0} \; + \; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1} \partial R_{D,1}} \cdot \frac{\partial R_{D,1}^*}{\partial D_0} \right] \right. \\
& \qquad \left. + \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1} \partial \ell_1} \cdot \frac{\partial R_{D,1}^*}{\partial D_0} \cdot \frac{\partial \ell_1^*}{\partial D_0} \right\} \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1) dx_1 \\
+ \quad & \int_{b_{d_x,1}}^{\infty} (x_1 - R_{D,0})^2 \left[ \frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}} \frac{\partial^2 R_{D,1}^*}{\partial W_{B,1}^2} + \frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1} \frac{\partial^2 \ell_1^*}{\partial W_{B,1}^2} \right] \frac{1}{\sigma_1} \varphi(z_1) dx_1 \\
+ \quad & \frac{d}{dD_0} \left[ - \left. \mathrm{E}_1[u_H(\cdot)|\cdot] \right|_{x_1 = b_{d_x,1}} \cdot \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1}) \right] \\
- \quad & \frac{R_{D,0} W_{B,0}}{(D_0 + W_{B,0})^2} \left\{ \left. \left[ \frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}} \frac{\partial R_{D,1}^*}{\partial D_0} \right] \right|_{x_1 = b_{d_x,1}} + \left. \left[ \frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1} \frac{\partial \ell_1^*}{\partial D_0} \right] \right|_{x_1 = b_{d_x,1}} \right. \\
& \qquad \left. - \frac{R_{D,0} W_{B,0}}{D_0 + W_{B,0}} \cdot \left. \frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1}} \right|_{x_1 = b_{d_x,1}} \right\} \cdot \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1})
\end{aligned}
$$

By the relation

$$\frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial W_{H,1}} = -\,\gamma \cdot R_{f,1} \cdot \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]$$

we can re-arrange

$$\begin{aligned}
&\int\limits_{b_{d_x,1}}^{\infty} \left[ \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial W_{B,1} \partial W_{H,1}} \cdot (x_1 - R_{D,0}) \;+\; \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial R_{D,1} \partial W_{H,1}} \cdot \frac{\partial R^*_{D,1}}{\partial D_0} + \right. \\
&\qquad \left. + \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial \ell_1 \partial W_{H,1}} \cdot \frac{\partial \ell^*_1}{\partial D_0} \right] \cdot (R_{D,0} - R_{f,0}) \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1) dx_1 \\
\equiv \;\; & -\gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot \int\limits_{b_{d_x,1}}^{\infty} \left( \frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial W_{B,1}} \cdot (x_1 - R_{D,0}) + \right. \\
&\quad \left. + \frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial R_{D,1}} \cdot \frac{\partial R^*_{D,1}}{\partial D_0} \;+\; \frac{\partial \mathrm{E}_1[u_H(\tilde{W}_{H,2})|\cdot]}{\partial \ell_1} \cdot \frac{\partial \ell^*_{D,1}}{\partial D_0} \right) \;\cdot\; \frac{1}{\sigma_1} \cdot \varphi(z_1)\; dx_1
\end{aligned}$$

as we can change the order of derivatives with respect to $W_{H,1}$ and any of the variables $W_{B,1}$, $R_{D,1}$, and $\ell_1$. Using the formulæ for $g(D_0)$ and $g'(D_0)$, respectively, we can re-write the expression from above as

$$\begin{aligned}
&\int\limits_{b_{d_x,1}}^{\infty} \left[ \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial W_{B,1} \partial W_{H,1}} \cdot (x_1 - R_{D,0}) \;+\; \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial R_{D,1} \partial W_{H,1}} \cdot \frac{\partial R^*_{D,1}}{\partial D_0} + \right. \\
&\qquad \left. + \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial \ell_1 \partial W_{H,1}} \cdot \frac{\partial \ell^*_1}{\partial D_0} \right] \cdot (R_{D,0} - R_{f,0}) \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1) dx_1 \\
= \;\; & -\gamma^2 \cdot (R_{D,0} - R_{f,0})^2 \cdot R^2_{f,1} \cdot g(D_0) \\
& -\gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1 = b_{d_x,1}} \cdot \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1}) \\
& -\gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot g'(D_0)
\end{aligned}$$

Note that

$$\frac{d}{dD_0} \left[ \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1 = b_{d_x,1}} \right] \;\equiv\; \frac{\partial}{\partial D_0} \left[ -e^{-\gamma W_{H,1} R_{f,1}} \right]$$

holds because of (D.2.6), which is equal to

$$-\,\gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]_{x_1 = b_{d_x,1}} \;.$$

Hence, the total derivative

$$\frac{d}{dD_0}\left[-\,\mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1=b_{d_x,1}}\cdot\frac{1}{\sigma_1}\cdot\varphi(b_{d_z,1})\right]$$

and the integral

$$\int\limits_{b_{d_x,1}}^{\infty}\left[\frac{\partial^2\mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial W_{B,1}\partial W_{H,1}}\cdot(x_1-R_{D,0})\;+\;\frac{\partial^2\mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial R_{D,1}\partial W_{H,1}}\cdot\frac{\partial R^*_{D,1}}{\partial D_0}\;+\right.$$
$$\left.+\;\frac{\partial^2\mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial \ell_1\partial W_{H,1}}\cdot\frac{\partial \ell^*_1}{\partial D_0}\right]\cdot(R_{D,0}-R_{f,0})\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1$$

partially offset each other, that is we obtain

$$\begin{aligned}
&\int\limits_{b_{d_x,1}}^{\infty}\left[\frac{\partial^2\mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial W_{B,1}\partial W_{H,1}}\cdot(x_1-R_{D,0})\;+\;\frac{\partial^2\mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial R_{D,1}\partial W_{H,1}}\cdot\frac{\partial R^*_{D,1}}{\partial D_0}\;+\right.\\
&\qquad\left.+\;\frac{\partial^2\mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial \ell_1\partial W_{H,1}}\cdot\frac{\partial \ell^*_1}{\partial D_0}\right]\cdot(R_{D,0}-R_{f,0})\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1\\
&+\frac{d}{dD_0}\left[-\,\mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1=b_{d_x,1}}\cdot\frac{1}{\sigma_1}\cdot\varphi(b_{d_z,1})\right]\\
=\;&-\gamma^2\cdot(R_{D,0}-R_{f,0})^2\cdot R^2_{f,1}\cdot g(D_0)\\
&-\gamma\cdot(R_{D,0}-R_{f,0})\cdot R_{f,1}\cdot g'(D_0)\\
&-\mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1=b_{d_x,1}}\cdot\frac{1}{\sigma_1}\cdot\frac{\partial\varphi(b_{d_z,1})}{\partial D_0}\;,
\end{aligned}$$

where

$$\frac{\partial\varphi(b_{d_z,1})}{\partial D_0}\;=\;\varphi(b_{d_z,1})\cdot b_{d_z,1}\cdot\frac{R_{D,0}W_{B,0}}{\sigma_1(D_0+W_{B,0})^2}\;>\;0\;.$$

As a consequence, the second derivative can be further simplified to yield

$$
\begin{aligned}
&- \; 2\cdot\gamma\cdot(R_{D,0}-R_{f,0})\cdot R_{f,1}\cdot g'(D_0) \; - \; \gamma^2\cdot(R_{D,0}-R_{f,0})^2\cdot R_{f,1}^2\cdot g(D_0) \\
&+ \int\limits_{b_{d_x,1}}^{\infty}\left[\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1}^2}\cdot(x_1-R_{D,0})^2 \; + \; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}^2}\cdot\left(\frac{\partial R_{D,1}^*}{\partial D_0}\right)^2 \; + \right. \\
&\qquad \left. + \; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1^2}\cdot\left(\frac{\partial \ell_1^*}{\partial D_0}\right)^2\right]\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1 \\
&+ \int\limits_{b_{d_x,1}}^{\infty}\left[\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1}\partial D_1}(x_1-R_{D,0})+\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}\partial D_1}\frac{\partial R_{D,1}^*}{\partial D_0}+\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1\partial D_1}\frac{\partial \ell_1^*}{\partial D_0}\right]\cdot \\
&\qquad \cdot\left[\frac{\partial D_1^d}{\partial D_0} \; + \; \frac{\partial D_1^d}{\partial R_{D,1}}\cdot\frac{\partial R_{D,1}^*}{\partial D_0} \; + \; \frac{\partial D_1^d}{\partial \ell_1}\cdot\frac{\partial \ell_1^*}{\partial D_0}\right]\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1 \\
&+ \int\limits_{b_{d_x,1}}^{\infty} 2\cdot\frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial W_{B,1}\partial \ell_1}\cdot(x_1-R_{D,0})\cdot\frac{\partial \ell_1^*}{\partial D_0}\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1 \\
&+ \int\limits_{b_{d_x,1}}^{\infty} 2\cdot\frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial W_{B,1}\partial R_{D,1}}\cdot(x_1-R_{D,0})\cdot\frac{\partial R_{D,1}^*}{\partial D_0}\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1 \\
&+ \int\limits_{b_{d_x,1}}^{\infty} 2\cdot\frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial R_{D,1}\partial \ell_1}\cdot\frac{\partial R_{D,1}^*}{\partial D_0}\cdot\frac{\partial \ell_1^*}{\partial D_0}\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1 \\
&+ \int\limits_{b_{d_x,1}}^{\infty}(x_1-R_{D,0})^2\left[\frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}}\frac{\partial^2 R_{D,1}^*}{\partial W_{B,1}^2}+\frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1}\frac{\partial^2 \ell_1^*}{\partial W_{B,1}^2}\right]\frac{1}{\sigma_1}\varphi(z_1)dx_1 \\
&- \; \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1=b_{d_x,1}}\cdot\frac{1}{\sigma_1}\cdot\varphi(b_{d_z,1})\cdot\frac{b_{d_z,1}\cdot R_{D,0}W_{B,0}}{\sigma_1(D_0+W_{B,0})^2} \\
&+ \; \frac{(R_{D,0}W_{B,0})^2}{(D_0+W_{B,0})^3}\left\{\frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1}}\Big|_{x_1=b_{d_x,1}}+\left[\frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}}\frac{\partial R_{D,1}^*}{\partial W_{B,1}}\right]\Big|_{x_1=b_{d_x,1}}+\right. \\
&\qquad \left. + \left[\frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1}\frac{\partial \ell_1^*}{\partial W_{B,1}}\right]\Big|_{x_1=b_{d_x,1}}\right\}\cdot\frac{1}{\sigma_1}\cdot\varphi(b_{d_z,1})
\end{aligned}
$$

Because of the implicit function theorem and because the direct dependence on $D_0$ is exclusively based on the bank's and the household's wealth in $t = 1$, we obtain the relation

$$\frac{\partial D_1^s}{\partial D_0}+\frac{\partial D_1^s}{\partial R_{D,1}}\frac{\partial R_{D,1}^*}{\partial D_0}+\frac{\partial D_1^s}{\partial \ell_1}\frac{\partial \ell_1^*}{\partial D_0} \;=\; \frac{\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1}\partial D_1}(x_1-R_{D,0})+\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{H,1}\partial D_1}(R_{D,0}-R_{f,0})}{-\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial D_1^2}} - \frac{\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}\partial D_1}}{\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial D_1^2}}\cdot\frac{\partial R_{D,1}^*}{\partial D_0} - \frac{\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1\partial D_1}}{\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial D_1^2}}\cdot\frac{\partial \ell_1^*}{\partial D_0}\,.$$

But because of

$$\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{H,1}\partial D_1} \;=\; -\,\gamma\cdot R_{f,1}\cdot\frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial D_1} \;=\; 0$$

given the optimal choices in $t = 1$, we obtain

$$\begin{aligned}&\frac{\partial D_1^s}{\partial D_0} \;+\; \frac{\partial D_1^s}{\partial R_{D,1}}\cdot\frac{\partial R_{D,1}^*}{\partial D_0} \;+\; \frac{\partial D_1^s}{\partial \ell_1}\cdot\frac{\partial \ell_1^*}{\partial D_0}\\ =\; & -\frac{\frac{\partial^2 \mathrm{E}_1[u_H(\tilde W_{H,2})|\cdot]}{\partial W_{B,1}\partial D_1}(x_1-R_{D,0}) \;+\; \frac{\partial^2 \mathrm{E}_1[u_H(\tilde W_{H,2})|\cdot]}{\partial R_{D,1}\partial D_1}\frac{\partial R_{D,1}^*}{\partial D_0} \;+\; \frac{\partial^2 \mathrm{E}_1[u_H(\tilde W_{H,2})|\cdot]}{\partial \ell_1\partial D_1}\frac{\partial \ell_1^*}{\partial D_0}}{\frac{\partial^2 \mathrm{E}_1[u_H(\tilde W_{H,2})|W_{B,1}]}{\partial D_1^2}}\,,\end{aligned}$$

resulting in

$$\begin{aligned}&\int\limits_{b_{d_x,1}}^{\infty}\left[\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1}\partial D_1}(x_1-R_{D,0}) \;+\; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}\partial D_1}\frac{\partial R_{D,1}^*}{\partial D_0} \;+\; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1\partial D_1}\frac{\partial \ell_1^*}{\partial D_0}\right]\cdot\\ &\qquad\cdot\left[\frac{\partial D_1^d}{\partial D_0} \;+\; \frac{\partial D_1^d}{\partial R_{D,1}}\cdot\frac{\partial R_{D,1}^*}{\partial D_0} \;+\; \frac{\partial D_1^d}{\partial \ell_1}\cdot\frac{\partial \ell_1^*}{\partial D_0}\right]\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1\\ =\;&\int\limits_{b_{d_x,1}}^{\infty} -\frac{1}{\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial D_1^2}}\cdot\left[\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1}\partial D_1}\cdot(x_1-R_{D,0}) \;+\; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}\partial D_1}\cdot\frac{\partial R_{D,1}^*}{\partial D_0}\right.\\ &\qquad\left. +\;\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1\partial D_1}\cdot\frac{\partial \ell_1^*}{\partial D_0}\right]^2\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1 \;>\; 0\,.\end{aligned}$$

Thus, the second derivative can be further simplified to

$$
\begin{aligned}
&- \; 2\cdot\gamma\cdot(R_{D,0}-R_{f,0})\cdot R_{f,1}\cdot g'(D_0) \;-\; \gamma^2\cdot(R_{D,0}-R_{f,0})^2\cdot R_{f,1}^2\cdot g(D_0) \\
&+ \int\limits_{b_{d_x,1}}^{\infty} \left[\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1}^2}\cdot(x_1-R_{D,0})^2 \;+\; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}^2}\cdot\left(\frac{\partial R_{D,1}^*}{\partial D_0}\right)^2 \;+\right. \\
&\qquad \left. +\; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1^2}\cdot\left(\frac{\partial \ell_1^*}{\partial D_0}\right)^2\right]\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1 \\
&+ \int\limits_{b_{d_x,1}}^{\infty} -\frac{1}{\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial D_1^2}}\cdot\left[\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1}\partial D_1}\cdot(x_1-R_{D,0}) \;+\; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}\partial D_1}\cdot\frac{\partial R_{D,1}^*}{\partial D_0}\right. \\
&\qquad\qquad \left. +\; \frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1\partial D_1}\cdot\frac{\partial \ell_1^*}{\partial D_0}\right]^2\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1 \\
&+ \int\limits_{b_{d_x,1}}^{\infty} 2\cdot\frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial W_{B,1}\partial \ell_1}\cdot(x_1-R_{D,0})\cdot\frac{\partial \ell_1^*}{\partial D_0}\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1 \\
&+ \int\limits_{b_{d_x,1}}^{\infty} 2\cdot\frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial W_{B,1}\partial R_{D,1}}\cdot(x_1-R_{D,0})\cdot\frac{\partial R_{D,1}^*}{\partial D_0}\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1 \\
&+ \int\limits_{b_{d_x,1}}^{\infty} 2\cdot\frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial R_{D,1}\partial \ell_1}\cdot\frac{\partial R_{D,1}^*}{\partial D_0}\cdot\frac{\partial \ell_1^*}{\partial D_0}\cdot\frac{1}{\sigma_1}\cdot\varphi(z_1)dx_1 \\
&+ \int\limits_{b_{d_x,1}}^{\infty} (x_1-R_{D,0})^2\left[\frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}}\frac{\partial^2 R_{D,1}^*}{\partial W_{B,1}^2}+\frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1}\frac{\partial^2 \ell_1^*}{\partial W_{B,1}^2}\right]\frac{1}{\sigma_1}\varphi(z_1)dx_1 \\
&- \; \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1=b_{d_x,1}}\cdot\frac{1}{\sigma_1}\cdot\varphi(b_{d_z,1})\cdot\frac{b_{d_z,1}\cdot R_{D,0}W_{B,0}}{\sigma_1(D_0+W_{B,0})^2} \\
&+ \; \frac{(R_{D,0}W_{B,0})^2}{(D_0+W_{B,0})^3}\left\{\frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial W_{B,1}}\Bigg|_{x_1=b_{d_x,1}} + \left[\frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}}\frac{\partial R_{D,1}^*}{\partial W_{B,1}}\right]\Bigg|_{x_1=b_{d_x,1}} +\right. \\
&\qquad \left. +\left[\frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1}\frac{\partial \ell_1^*}{\partial W_{B,1}}\right]\Bigg|_{x_1=b_{d_x,1}}\right\}\cdot\frac{1}{\sigma_1}\cdot\varphi(b_{d_z,1})
\end{aligned}
$$

The remaining integrand is a quadratic form which can be illustrated by appropriately defining a $3\times 3$ matrix $\mathfrak{D}$. To do so, let us first consider the following $2\times 2$ matrices associated with second partial derivatives of the household's expected utility in $t=1$. All partial derivatives are taken with respect to parameters and variables, respectively, that are affected by the household's choice of deposits in $t = 0$. Only the associated determinants are of interest. Therefore we define:

$$\mathrm{D}_{W_{B,1},W_{B,1}} := \left|\begin{pmatrix} \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial W_{B,1}^2} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial W_{B,1}\partial D_1} \\ \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial W_{B,1}\partial D_1} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial D_1^2} \end{pmatrix}\right| \tag{D.2.10}$$

$$\mathrm{D}_{R_{D,1},R_{D,1}} := \left|\begin{pmatrix} \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial R_{D,1}^2} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial R_{D,1}\partial D_1} \\ \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial R_{D,1}\partial D_1} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial D_1^2} \end{pmatrix}\right| \tag{D.2.11}$$

$$\mathrm{D}_{\ell_1,\ell_1} := \left|\begin{pmatrix} \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial \ell_1^2} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial \ell_1\partial D_1} \\ \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial \ell_1\partial D_1} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})]}{\partial D_1^2} \end{pmatrix}\right| \tag{D.2.12}$$

$$\mathrm{D}_{W_{B,1},R_{D,1}} := \left|\begin{pmatrix} \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial W_{B,1}\partial D_1} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial W_{B,1}\partial R_{D,1}} \\ \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial D_1^2} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial R_{D,1}\partial D_1} \end{pmatrix}\right| \tag{D.2.13}$$

$$\mathrm{D}_{W_{B,1},\ell_1} := \left|\begin{pmatrix} \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial W_{B,1}\partial D_1} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial W_{B,1}\partial \ell_1} \\ \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial D_1^2} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial \ell_1\partial D_1} \end{pmatrix}\right| \tag{D.2.14}$$

$$\mathrm{D}_{R_{D,1},\ell_1} := \left|\begin{pmatrix} \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial R_{D,1}\partial D_1} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial R_{D,1}\partial \ell_1} \\ \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial D_1^2} & \frac{\partial^2 \mathrm{E}_1[u_H(\tilde{W}_{H,2})|W_{B,1}]}{\partial \ell_1\partial D_1} \end{pmatrix}\right| \tag{D.2.15}$$

Next, let us define the following symmetric $3 \times 3$ matrix $\mathfrak{D}$ whose entries are the determinants from above:

$$\mathfrak{D} = \begin{pmatrix} \mathrm{D}_{W_{B,1},W_{B,1}} & \mathrm{D}_{W_{B,1},R_{D,1}} & \mathrm{D}_{W_{B,1},\ell_1} \\ \mathrm{D}_{W_{B,1},R_{D,1}} & \mathrm{D}_{R_{D,1},R_{D,1}} & \mathrm{D}_{R_{D,1},\ell_1} \\ \mathrm{D}_{W_{B,1},\ell_1} & \mathrm{D}_{R_{D,1},\ell_1} & \mathrm{D}_{\ell_1,\ell_1} \end{pmatrix} \tag{D.2.16}$$

Furthermore let

$$\begin{aligned} \Delta_1^\top &= \left(1, \frac{\partial R_{D,1}^*}{\partial W_{B,1}}, \frac{\partial \ell_1^*}{\partial W_{B,1}}\right) \\ \Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}^\top &= \left(\frac{\partial \mathrm{E}_1\left[u_H(\cdot)|\cdot\right]}{\partial W_{B,1}}, \frac{\partial \mathrm{E}_1\left[u_H(\cdot)|\cdot\right]}{\partial R_{D,1}}, \frac{\partial \mathrm{E}_1\left[u_H(\cdot)|\cdot\right]}{\partial \ell_1}\right) . \end{aligned}$$

Thus we obtain:

$$\left.\frac{\partial^2 \mathrm{E}_0\left[\mathrm{E}_1\left[u_H(\tilde{W}_{H,2})\,\middle|\,\tilde{W}_{B,1}\right]\right]}{\partial D_0^2}\right|_{\frac{\partial \mathrm{E}_0[\mathrm{E}_1[u_H(\cdot)|\cdot]]}{\partial D_0}=0} =$$

$$
\begin{aligned}
& - \quad 2 \cdot \gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot g'(D_0) \; + \; h''(D_0) \\
& - \quad \gamma^2 \cdot (R_{D,0} - R_{f,0})^2 \cdot R_{f,1}^2 \cdot g(D_0) \\
& + \quad \int_{b_{d_x,1}}^{\infty} \frac{(x_1 - R_{D,0})^2}{\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial D_1^2}} \cdot \Delta_1^\top \cdot \mathfrak{D} \cdot \Delta_1 \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1) dx_1 \\
& + \quad \int_{b_{d_x,1}}^{\infty} (x_1 - R_{D,0})^2 \left[ \frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}} \frac{\partial^2 R_{D,1}^*}{\partial W_{B,1}^2} + \frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1} \frac{\partial^2 \ell_1^*}{\partial W_{B,1}^2} \right] \frac{1}{\sigma_1} \varphi(z_1) dx_1 \\
& - \quad \mathrm{E}_1[u_H(\tilde{W}_{H,2}) \,|W_{B,1}]\Big|_{x_1 = b_{d_x,1}} \cdot \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1}) \cdot \frac{b_{d_z,1} \cdot R_{D,0} W_{B,0}}{\sigma_1 (D_0 + W_{B,0})^2} \\
& + \quad \frac{(R_{D,0} W_{B,0})^2}{(D_0 + W_{B,0})^3} \; \cdot \; \Delta_1^\top \cdot \Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}\Big|_{x_1 = b_{d_x,1}} \; \cdot \; \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1}) \; .
\end{aligned}
$$

The expressions in the first line are negative given $D_0$ fulfills the first-order condition. However, the expressions in the second and fifth line are always positive and it is not clear if and how they offset each other. The integral in the third line is negative if the matrix $\mathfrak{D}$ is positive semi-definite. To be able to offset the impact of terms with negative signs, it is desirable to have $\mathfrak{D}$ positive definite.

Moreover, the definiteness of $\mathfrak{D}$ also influences the monotonicity of the vector product $\Delta_1^{\mathrm{T}} \cdot \Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}$ with respect to $x_1$. It decreases in $x_1$ if

$$
\begin{aligned}
\frac{d\left[\Delta_1^\top \cdot \Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}\right]}{dW_{B,1}} \quad = \quad & \frac{1}{\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial D_1^2}} \cdot \Delta_1^\top \cdot \mathfrak{D} \cdot \Delta_1 \; + \\
& + \; \left[ \frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}} \cdot \frac{\partial^2 R_{D,1}^*}{\partial W_{B,1}^2} \; + \; \frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1} \cdot \frac{\partial^2 \ell_1^*}{\partial W_{B,1}^2} \right]
\end{aligned}
$$

is negative.

Due to (D.2.4), the expression $-2 \cdot \gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot g'(D_0)$ can be rephrased by

$$
\begin{aligned}
& - \quad 2 \cdot \gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot \int\limits_{b_{d_x,1}}^{\infty} (x_1 - R_{D,0}) \cdot \Delta_1^{\top} \cdot \Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]} \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1)\ dx_1 \\
& + \quad 2 \cdot \gamma^2 \cdot (R_{D,0} - R_{f,0})^2 \cdot R_{f,1}^2 \cdot g(D_0) \\
& + \quad 2 \cdot \gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1 = b_{d_x,1}} \cdot \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1})
\end{aligned}
$$

resulting in

$$
\left.\frac{\partial^2 \mathrm{E}_0[u_H(\tilde{W}_{H,2})\,|W_{B,1}]}{\partial D_0^2}\right|_{\frac{\partial \mathrm{E}_0[u_H(\cdot)|\cdot]}{\partial D_0} = 0} =
$$

$$
\begin{aligned}
& + \quad \gamma^2 \cdot (R_{D,0} - R_{f,0})^2 \cdot R_{f,1}^2 \cdot g(D_0) \ + \ h''(D_0) \\
& + \quad \int\limits_{b_{d_x,1}}^{\infty} \frac{(x_1 - R_{D,0})^2}{\frac{\partial^2 \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial D_1^2}} \cdot \Delta_1^{\top} \cdot \mathfrak{D} \cdot \Delta_1 \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1) dx_1 \\
& - \quad 2 \cdot \gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot \int\limits_{b_{d_x,1}}^{\infty} (x_1 - R_{D,0}) \cdot \Delta_1^{\top} \cdot \Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]} \cdot \frac{1}{\sigma_1} \cdot \varphi(z_1)\ dx_1 \\
& + \quad \int\limits_{b_{d_x,1}}^{\infty} (x_1 - R_{D,0})^2 \left[ \frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial R_{D,1}} \frac{\partial^2 R_{D,1}^*}{\partial W_{B,1}^2} + \frac{\partial \mathrm{E}_1[u_H(\cdot)|\cdot]}{\partial \ell_1} \frac{\partial^2 \ell_1^*}{\partial W_{B,1}^2} \right] \frac{1}{\sigma_1} \varphi(z_1) dx_1 \\
& - \quad \left[ \frac{b_{d_z,1} R_{D,0} W_{B,0}}{\sigma_1 (D_0 + W_{B,0})^2} - 2\gamma (R_{D,0} - R_{f,0}) R_{f,1} \right] \cdot \mathrm{E}_1[u_H(\cdot)|\cdot]\big|_{x_1 = b_{d_x,1}} \cdot \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1})
\end{aligned}
$$

Thus, the positive term $-\gamma^2 \cdot (R_{D,0} - R_{f,0})^2 \cdot R_{f,1}^2 \cdot g(D_0)$ is offset by $-2 \cdot \gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot g'(D_0)$, which is negative, given that the first-order condition is fulfilled. In other words,

$$
- \quad 2 \cdot \gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot g'(D_0) \ + \ h''(D_0)
$$

$$
- \quad \gamma^2 \cdot (R_{D,0} - R_{f,0})^2 \cdot R_{f,1}^2 \cdot g(D_0)
$$

is negative.

Hence, in order to obtain a negative sign for the whole second derivative the following conditions must hold:

$$
\begin{aligned}
0 \;>\; & -\, 2 \cdot \gamma \cdot (R_{D,0} - R_{f,0}) \cdot R_{f,1} \cdot g'(D_0) \;+\; h''(D_0) \\
& -\, \gamma^2 \cdot (R_{D,0} - R_{f,0})^2 \cdot R_{f,1}^2 \cdot g(D_0) \\
& -\, \mathrm{E}_1[u_H(\tilde{W}_{H,2})\,|W_{B,1}]\Big|_{x_1 = b_{d_x,1}} \cdot \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1}) \cdot \frac{b_{d_z,1} \cdot R_{D,0} W_{B,0}}{\sigma_1 (D_0 + W_{B,0})^2} \\
& +\, \frac{(R_{D,0} W_{B,0})^2}{(D_0 + W_{B,0})^3} \;\cdot\; \Delta_1^\top \cdot \Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}\Big|_{x_1 = b_{d_x,1}} \;\cdot\; \frac{1}{\sigma_1} \cdot \varphi(b_{d_z,1}) \\
0 \;>\; & \frac{d\left[\Delta_1^\top \cdot \Delta_{\mathrm{E}_1[u_H(\cdot)|\cdot]}\right]}{dW_{B,1}}\,.
\end{aligned}
$$

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# Lebenslauf

| | |
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