Decision Making in the European Union: Externalities and Incomplete Information

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Chapter 1

General Introduction

Whenever a group of individuals has to reach an agreement about a common cause, some method of collective decision making will be applied. In this spirit, the theory of collective decisions is relevant for almost every aspect of our lives. Examples include a family who wants to decide where to go for holidays or a group of friends who has to choose how to spend the night.

Likewise, every branch of economics knows its own collective decision problem: Industrial organization is concerned with decisions on the future course of a firm made by the board of directors, labor economics considers recruitment decisions which are often taken by a commission and public choice theory analyzes the provision of public goods.

In politics, there exist numerous applications of the theory of collective decisions. To name a few, consider the agreement of a coalitional government on future taxes or the decision of a committee on the reform of a certain law. International organizations like the European Union have to build consensus about various politics, ranging from assessing excessive deficits under the Stability and Growth Pact to the development of a Common Foreign and Security Policy. Moreover, the Governing Council of the European Central Bank decides on the common monetary policy for all members of the European Monetary Union.

Bearing in mind the broad relevancy of collective decisions, it is not astonishing that examples of their analysis reach as far back as the fourth century before Christ. Classical writings include Aristotle in ancient Greece and Kautilya in ancient India who explored collective decision making in their books respectively entitled *Politics* and *Economics* (see Arrow et al. [2002]). Moreover, there exists a long tradition of analyzing collective decisions by means of mathematical methods. The rigorous analysis of collective decision making was established in the eighteenth century by contributions of Borda [1781] and Condorcet [1785] who were the first to theoretically investigate the performance of different decision procedures.

During the further evolution of this theory, emphasis has been put on the (strategic) interaction of individuals in collective decision making. Since no individual lives in isolation, its actions, decisions and thoughts influence others and thus in turn may affect the commonly desired decision. If the well-being of an individual is directly affected by the actions of others, a so-called externality is present. Therefore, external effects are inherently omnipresent in collective decision making. For example, the decision of the Governing Council of the European Central Bank on the common monetary policy may affect members as well as non-members of the European Monetary Union in various ways. Furthermore, decisions on environmental standards taken by the member states of the European Union have positive external effects on neighboring countries.

Once rational individuals perceive the possibility of manipulating the decision in the direction of their own interest, they will not hesitate to exploit this opportunity. The importance of strategic aspects in collective decision making was already noted by Arrow [1951] who pointed out that "Once a machinery for making social choices from individual tastes is established, individuals will find it profitable, from a rational point of view, to misrepresent their tastes by their actions".

Taking strategic behavior and private information (tastes in Arrow's sense) into account, this leads to the theory of mechanism design. This theory analyzes collective decisions when individual preferences are not publicly observable. As a result, individuals must be relied upon to reveal this information. Again, it was Arrow [1951] who observed that "Even in a case where it is possible to construct a procedure showing how to aggregate individual tastes into a consistent social preference pattern, there still remains

the problem of devising rules of the game so that individuals will actually express their true tastes even when they are acting rationally".

In terms of mechanism design, any decision process can be seen as a game form and the mechanism in turn describes the rules of this game. Specific equilibrium concepts are borrowed from non-cooperative game theory in order to predict the behavior of rational individuals. If an equilibrium of the game induced by the mechanism exists, it will be possible to assign an outcome to the mechanism. Which allocations remain achievable depends on the informational environment. On this note, mechanism design theory studies the extent to which the information revelation problem constraints the way collective decisions can respond to individual preferences. Important early contributions to this theory include (among many others) Clarke [1971], Groves [1973], Gibbard [1973], Satterthwaite [1975], Arrow [1979], D'Aspremont and Gérard-Varet [1979] as well as Myerson and Satterthwaite [1983].

This dissertation analyzes three different collective decision problems when external effects are present and information is incomplete. In Chapter 2, I focus on collective decisions where individually desired outcomes are correlated but not identical. Each individual holds private information that mainly concerns his own desired policy but also affects all other individuals. The analysis is general, but has numerous applications including, for example, the common monetary policy of the European Union or its environmental policy. This chapter originates from collaboration with Hans Peter Grüner and will be published in the European Economic Review. Chapter 3 analyzes fiscal stabilization decisions under the European Stability and Growth Pact when member states experience negative spillovers of debt issuance due to a common monetary policy. Private information on behalf of interest groups about costs or benefits of delayed stabilization leads to a war of attrition that may be mitigated by a fine for deficits. In Chapter 4, the influence of different decision procedures on the entry decision into a monetary union in the presence of asymmetric shocks and externalities in decision making is analyzed.

1.1 Collective Decisions with Interdependent Valuations

This chapter studies collective decisions with private information about desired policies, when the individually preferred decision of an individual does not only depend on his own private information but also on the private information of the others. In contrast to the existing literature, we neither restrict attention to political outcomes under individual utility maximization nor focus on efficient aggregation of perfectly coordinated interests, but instead vary the extent to which individual interests influence each other. The question is how existing mechanisms perform when individual preferences are correlated to a certain extent.

Considering direct mechanisms in which participation is not voluntary and monetary transfers are ruled out a priori, we focus on two specific mechanisms, the median and the mean mechanism. The median mechanism implements the median announcement, whereas the mean mechanism implements the average of all announcements. The main difference between these two mechanisms is how the implemented decision is influenced by changes in the announcements of private information. Under the median mechanism changes in extreme positions are disregarded, since the final decision solely depends on the median. Under the mean mechanism extreme positions influence the decision, since here all available information is taken into account.

In this setting, we establish the existence of symmetric Bayesian Nash equilibria of the corresponding games. In equilibrium, under the median mechanism individuals understate their private information, whereas under the mean mechanism they overstate it. As a consequence, the performance of the mechanisms depends upon the extent to which individual interests influence each other. With weak interdependencies, the median mechanism outperforms the mean mechanism, whereas with strong interdependencies it is better to use the average as decision mechanism.

Applications of this setting include the decision about the provision of public goods and decision processes in international organizations, teams or firms. The example that inspired this work is the decision process in the *European Central Bank*. At least part of the information of members of the *Central Bank Council* may be seen as their private information, for example experience or expectations.

1.2 The European Stability and Growth Pact as a Device for adequate Stabilization

A common monetary policy as conducted by the European Central Bank affects other national economic policies, for example fiscal or labor market policies. In this chapter, I focus on the first of these relationships, i.e. the interaction between one monetary authority and many independent national fiscal policies, and analyze fiscal stabilization decisions under the European Stability and Growth Pact. Three main questions are considered: rationales for the pact, reasons for signing it and possible improvements.

Focus of the analysis is the adoption of a welfare increasing fiscal reform. Private information about the costs of delayed stabilization leads to a war of attrition between interest groups in the member countries of a monetary union. Since the group that concedes first has to bear the larger part of the necessary cutbacks, it is individually rational for each group not to give in. I extend the basic set-up to the case of a monetary union in order to model negative spillovers of debt issuance due to a common monetary policy. Moreover, I introduce a fine for deficits to mimic some features of the *European Stability and Growth Pact*. Allowing for private benefits of delayed stabilization is another important feature of the model.

The first result of this chapter is that deficit sanctions can indeed enforce national commitment to stabilization. In the presence of private benefits of delay however, the effect is not unambiguously positive. However, if the pact is carefully designed with respect to the height of the fine, its introduction will lead to increases in welfare. On the other hand, the pact does not implement the ex post efficient decision, i.e. delay of stabilization when this is desirable. Therefore, I adapt an expected externality mechanism to this setting. When there exists private information, this

mechanism yields the best achievable outcome, but is unfortunately not able to ensure voluntary participation ex post.

1.3 To join or not to join? Monetary Unification & Decision Procedures

In tradition with the literature on *Optimum Currency Areas* this chapter analyzes the costs and benefits of joining a monetary union. Here, the trade-off concerns the costs of giving up independent monetary policy making and the benefits of gaining influence on union decisions. Assuming that the candidate economy is influenced by union wide shocks as well as by country-specific ones, accession reduces the possibility to react independently on country-specific shocks. Supposing further that also decisions taken inside the union influence the economic situation of the candidate, entry offers the opportunity to influence decisions taken inside the union. On the other hand, before accession potential members have to react on union decisions in order to support their national economies what may involve substantial costs.

In addition, the influence of three different decision procedures applied in a federal union on the entry decision of a potential member is analyzed. The decision procedures differ in the degree of influence they assign to union members in joint decision taking. The first determines the union wide decision as to maximize welfare. According to the second mechanism the median of desired policies of the members is implemented. Finally, I consider a procedure called *rotation*. Here, an exogenously fixed probability is introduced with which one member may determine the union wide decision alone. The performance of the procedures depends on the degree of interdependencies between economies as well as on the specification of national preferences.

The result of this chapter is that concentrating on the perspective of a potential member unification performs relatively poor compared to separation. This holds irrespective of the decision procedure. The only possibility to make entry profitable for a potential member is to grant it a merely

exclusive decision right in common policies. However and despite the fact that transaction costs are not explicitly modelled, there exist combinations of interdependencies and preferences that make entry desirable.

Chapter 2

Collective Decisions with Interdependent Valuations

2.1 Introduction

2.1.1 Decisions with public values

Many collective decision problems have in common that there is some agreement between the individuals who are supposed to take decisions as well as some disagreement. This chapter models such situations in an environment of asymmetric information by including interdependencies between individual preferences. This means that the individually preferred decision of a group member does not only depend on his own private information but also on the other members' private information. The questions are how the decision mechanism should be designed and how existing mechanisms perform when individual preferences are correlated.

We consider a specific class of collective decision problems where each of these has the following properties: In order to take a common decision, all agents obtain private information (a signal) about their most desired policy. However, no individual is perfectly informed about what would be the privately optimal policy. This imperfection is due to spillover effects

between the desired policies. The information of all individuals could be used to calculate the private bliss points whereby each individuals' private information yields more information about its own bliss point than any other individual's private information. Decision problems are characterized by one single parameter which measures the extent to which private information affects all individuals.

In this setting we analyze a specific class of mechanisms. Participation is not voluntary, therefore we can ignore any individual rationality constraints. Moreover, our mechanisms do not condition monetary transfers on the agents' announcements, in fact all monetary transfers are ruled out a priori. Instead, the mechanisms map individual announcements of the private information into the collective decision.

We concentrate on two mechanisms, i.e. the median and the mean mechanism. The median mechanism implements the median announcement, whereas the mean mechanism implements the average of all announcements. The main difference between these two mechanisms is how they deal with the announcements of private information. Under the median mechanism changes in extreme positions are disregarded, since the median alone determines the final decision. On the contrary, the nature of the mean mechanism is to take all available information into account. Therefore, under the mean mechanism extreme positions influence the decision.

The main result of this chapter is the identification of two symmetric Bayesian Nash equilibria of the respective games. The performance of the mechanisms depends upon the extent to which spillover effects affect the economy. With weak interdependencies, the median mechanism dominates the mean mechanism, whereas with strong interdependencies it is optimal to use the average as decision mechanism.

2.1.2 Some applications

One can think of a number of different applications of our framework: Any setting in which the individually preferred decision does not only depend on the agents' own information but as well on the signals of the others fits well into this framework.

2.1 Introduction

One example is the decision process in a common central bank like the European Central Bank (ECB). Here, national central banks may care about a policy that accommodates macroeconomic shocks in their own country while taking a collective decision about common monetary policy. However, due to demand spillover effects, shocks in one country may affect the desired policy in other participating countries. Most of the information used by national central bankers within the Central Bank Council is common knowledge. Our model relates to a situation where at least part of the information of council members is their private information. Hard data is exchanged in the European System of Central Banks before decisions are taken in the Governing Council. This data concerns recent economic indicators, new figures on growth, employment etc. Nevertheless, there is some soft information available to central bank governors which cannot be communicated. This information arises partly from the national central bank governors' experience in dealing with national data. Actually, this is one justification why national central bankers should play a role in the decision process. Private information may also concern expectations about the way in which other major macroeconomic players such as governments or trade unions will react to certain central bank decisions. If national central bankers are experts in this sense then there may be scope for manipulation of information. If interdependencies are strong, the other central bankers' information will be important for the nationally desired policy.

Our setting can also be applied to other international decisions such as decisions about environmental policy. Basically, the nation states are interested in achieving less pollution in their own country. On the other hand, they have to take a common decision about certain environmental standards. In addition, it may be the case that national governments possess private information about the national amount of emissions, the costs to reduce emissions or the economic consequences of a reduction. However, the environmental situation in one country is co-determined by the emissions in the neighboring countries. The nearer the location of countries, the more important becomes private information obtained in any single country. Take as a recent example the negotiations about a

common European standard of reduction in CO_2 emissions in preparation for the $Kyoto\ Protocol$.

Another important application of our framework is the collective decision on the provision of a public good. Imagine a situation where information about the desirability of a certain public good comes in two forms: (i) a measure of individual desirability of this particular good and (ii) a quality component that affects the desirability of all individuals in the same way. Suppose that each agent receives a combined signal which is related to his private component and to the quality of the good. In this case preferences about the provision of the public good are correlated but not identical.

Collective decision problems having the features described above can be found also in many areas besides politics. Consider the following example taken from industrial organization: a decision about the future orientation of a firm has to be made. This decision has to be taken by the different heads of department. First of all, these heads are interested in the performance of their own department. Beside this, they possess specific knowledge about the conditions, needs or prospects of it. However, their opinion about the future development of the firm is influenced by the conditions obtaining in other departments as well.

2.1.3 Relation to literature

The mechanism design literature offers solutions to related problems, but none to our specific setting. If there are no spillovers and side-payments are allowed it always will be possible to obtain (Bayesian) incentive compatibility using an expected externality mechanism.³ If informational and

¹Another, possibly more complex and more appropriate, way to model quality components in public good decisions would be to consider two signals, one on individual preferences and the second on quality.

²In our model the desired policy is linear in all agents' signals. This particular specification arises in the public goods example when the desired quantity of the public good is a convex combination of the signal and the quality component and when the quality component is linearly related to all agents' private signals.

³See Mas-Colell, Whinston and Green [1995], Arrow [1979] or D'Aspremont and Gérard-Varet [1979].

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allocational externalities are considered but monetary transfers are allowed, the problem of efficient design has been analyzed in auction environments.⁴ Instead, we study the case where spillovers are present and monetary transfers are excluded a priori. Moreover, we do not consider or look for optimal mechanisms but concentrate on the performance of two specific mechanisms.

Concerning the analysis of collective decisions, the setting of this chapter builds an intermediate case between two frameworks commonly used in the literature: On one hand, political outcomes under individual utility maximization are analyzed, i.e. the case of zero spillovers (see Vaubel and Willet [1991] and the references therein). On the other hand, the literature deals with efficient aggregation of perfectly co-ordinated interests, i.e. 100% spillovers (see Piketty [1999] and the references therein). Our research focuses on the intermediate case: what kind of political outcomes under different information aggregation mechanisms are to be expected if individual interests are correlated to a certain extent? Thus, we do not analyze political outcomes for a fixed degree of spillovers, but instead vary the extent to which individual preferences influence each other.

Closest to our analysis is a recent paper by Rausser et al. [2003]. These authors focus on so-called aggregation games under incomplete information. Like in our setting, members of a group receive private signals and have to take a collective decision. In contrast to our work, Rausser et al. differentiate between two individual characteristics: one is privately known, the other is publicly observed. Each individual's payoff depends on his observable characteristic, the aggregate of individuals' type realizations and the outcome of the game. Thus, attention is restricted to what we call the mean mechanism. In addition, a lower bound for admissible reports is introduced. Rausser et al. establish the existence of a pure strategy equilibrium in which individuals' equilibrium strategies are strictly monotone in their types. In equilibrium, the incentives to misreport are twofold: high types overstate their private information, whereas low types understate it. This kind of behavior is present in our analysis of the mean mechanism as

⁴See for example Fieseler, Kittsteiner and Moldovanu [2003] or Jehiel and Moldovanu [2001].

well, since types above zero over-report and types below zero under-report. The types at which announcement behavior changes are different, i.e. zero in our analysis, some positive type in Rausser et al. This is due to the different assumptions on the type space: whereas Rausser et al. focus on positive types, we allow for negative ones as well.

Related to our work is a recent paper by Casella [2002]. In a similar informational environment but with private values she proposes a voting scheme for deliberations taken by committees that meet regularly. At each meeting, committee members are allowed to store their vote for future use. Although the scheme cannot achieve the first-best with more than two voters, making votes storable typically leads to ex ante welfare gains. Her paper differs from ours in that we do not consider developments over time. Instead, we study a one-shot game excluding also any reputation effects.

Our model differs from a common value setting where individuals receive imperfect signals about a desirable course of action in the following respect. An example of such a setting is the Condorcet jury problem where all members of a jury receive a signal about the true state of the world. All individuals would agree about the correct decision once they are perfectly informed. In the common value setting individual information is correlated with the true state of the world. In our model, all signals are related to the desired decision in a deterministic manner. Hence, in our set-up individuals would still disagree about the collective decision when perfectly informed. Moreover, perfect information would obtain if all private signals were known.

The existing literature on decision making in a central banking context focuses either on monetary stabilization policies comparing alternative types of appointees (i.e. having different mandates) in the *ECB Council* (e.g. Von Hagen and Süppel [1994]), on the implications of different policy objectives of the common central bank (e.g. Gros and Hefeker [2002] and Grüner [1999]) or on equilibrium incentive contracts in a multi-principal agency framework (e.g. Dixit and Jensen [2000]). Instead, we ask how the decision mechanism of the *ECB* should be designed if national central bankers possess private information and their preferences are partly

2.2 The Model

correlated. Our result translates to this context in the following way: the larger the common component between member states, for example the more national macroeconomic shocks affect all members of the union, the more weight should be given to extreme positions in the *ECB Council*.

The remainder of this chapter is organized as follows: Section 2 introduces the model presenting the first-best solution and the two mechanisms we aim to study. The respective equilibria under those mechanisms are calculated in Section 3 where results about the truthful revelation properties of the equilibria are given as well. The next section compares the performance of the mean and the median mechanism for different degrees of interdependencies and develops a concrete example. The robustness of our results with respect to the specification of individual utility is analyzed in Section 5 and Section 6 concludes. All omitted proofs can be found in Appendix A.

2.2 The Model

We consider an economy which is populated by n individuals where n is an odd number. A decision $x \in \mathbb{R}$ has to be taken. Each individual receives private information θ_i . The parameters θ_i are distributed independently over the same support $\Theta \subseteq \mathbb{R}$. Let $f(\theta_i)$ denote the respective density. We assume that the expected value of all signals equals zero, i.e. $E[\theta_i] = 0 \,\forall i$. The vector of private information is θ . Individual preferences over outcomes are characterized by the following von-Neumann-Morgenstern utility function

$$u_i(x,\theta) = -\left(x - \theta_i^*\right)^2$$

where θ_i^* describes the individually preferred decision. Individual utility is maximized at $x = \theta_i^*$ (i.e. when the individually preferred decision is actually implemented). The highest attainable utility is zero. The larger the difference between the implemented policy x and the individually preferred decision θ_i^* , the smaller is individual utility.

The individually preferred policy θ_i^* is a convex combination of i's type and the average of all others' types, i.e.

$$\theta_i^* = (1 - \alpha) \,\theta_i + \frac{\alpha}{n - 1} \sum_{j \neq i} \theta_j.$$

The parameter $\alpha \in [0, \bar{\alpha}]$ measures the extent to which interdependencies align the preferences of all individuals. For the upper bound value $\bar{\alpha} := \frac{n-1}{n}$ all individuals share a common utility function with a maximum at $x = \theta_{Mean}$ where $\theta_{Mean} = \sum_{i=1}^{n} \theta_i / n$. For the lower bound $\alpha = 0$ each individual has his signal θ_i as a private bliss point. For example, α might measure the importance of a common quality component of a public good, the degree of spillover effects between the members of the *European Union*, closeness of geographical location or the degree to which firm departments are interlinked.

2.2.1 Welfare

First, we describe the decision that maximizes welfare when there are no informational asymmetries. Welfare is defined as the sum of individual utilities, i.e. we take a utilitarian perspective. The sum of all utilities is maximized if x^* maximizes

$$\sum_{i=1}^{n} u_i(x,\theta) = \sum_{i=1}^{n} -(x - \theta_i^*)^2.$$

Optimality requires

$$-\sum_{i=1}^{n} 2(x^* - \theta_i^*) = 0.$$
 (1)

Thus, it follows immediately that the decision x^* is independent of the degree of interdependency α and is determined by the average of all private signals.

Lemma 1 The sum of all utilities is maximized at $x^* = \theta_{Mean}$.

2.2 The Model

Proof of Lemma 1:

Reorganizing (1) and substituting for θ_i^* yields the result.

The intuition for this result is that by summing up the utilities of all individuals any spillover effects are automatically taken into account. Lemma 1 implies that the mean mechanism would yield the first-best if truth telling could be implemented for all degrees of interdependency. However, if informational asymmetries are present, this first-best solution remains no longer attainable.

2.2.2 The mechanisms

We consider the following two direct mechanisms excluding any monetary transfers and ignoring participation constraints: All individuals are asked for an announcement $\hat{\theta}_i \in \mathbb{R}$ of their private signal. The vector of announcements is $\hat{\theta}$. Depending on these announcements the collective decision x is taken.

The first mechanism we study is the median mechanism. Let

$$x_{Median} = x_{Median}(\hat{\theta}) := \hat{\theta}_{Median}$$

where $\hat{\theta}_{Median}$ is the median of all announcements.⁵ The median mechanism implements this announcement and thus replicates majority decisions for the case of zero spillovers. Another feature of this mechanism is that changes in extreme positions are disregarded, since the final decision solely depends on the median announcement.

The second mechanism we analyze is the mean mechanism. This mechanism is a mathematical representation of a decision procedure where changes of all agents' statements about the desired outcome are taken into account. It is one example of a mechanism where the intensity of an individual's statement always affects the collective decision. Let

$$x_{Mean} = x_{Mean}(\hat{\theta}) := \frac{1}{n} \sum_{i=1}^{n} \hat{\theta}_i.$$

⁵Let θ_{Median} be the signal of the median individual. In general $\theta_{Median} = \hat{\theta}_{Median}$ is not true.

This mechanism asks the individuals for their private information and implements the average of all announcements. Consequently, the mean mechanism uses all available information, changes in extreme positions are taken into account and thus extreme positions influence the common decision. This mechanism would implement the welfare maximizing solution if agents were to tell the truth. Our analysis of the mean mechanism therefore describes the outcome when people behave rational in a set-up where truth telling would maximize welfare.

2.3 The Results

In this section, we present two Bayesian Nash equilibria of the games introduced above, one of the median and one of the mean mechanism. These equilibria imply truthful revelation for certain degrees of interdependencies.

2.3.1 Equilibria

It is useful to define the desired policy of type θ_i conditional on being the median individual. It is given by

$$x(\theta_i) := (1 - \alpha) \,\theta_i + \alpha \, \left(\frac{1}{2} E \, \left[\theta_k \mid \theta_k < \theta_i \right] + \frac{1}{2} E \, \left[\theta_k \mid \theta_k > \theta_i \right] \right).$$

This is the expected value of player i's ideal point θ_i^* conditional on player i being the median individual. We show that the profile of announcements $\widehat{\theta} = (\widehat{\theta}_1, \dots, \widehat{\theta}_n)$ where $\widehat{\theta}_i = x(\theta_i) \ \forall i = 1, \dots, n$ constitutes a symmetric Bayesian Nash equilibrium under the median mechanism. There is one and only one such equilibrium.⁶

Let $U(\hat{\theta}_i, \theta_i)$ denote the expected utility of player i when he is of type θ_i , announces $\hat{\theta}_i$ and all players $j \neq i$ announce according to $x(\theta_j)$. In a first

⁶There exists a continuum of other symmetric equilibria in which all individuals announce the same type irrespective of their signal, i.e. $\hat{\theta}_i(\theta_i) = \widetilde{\theta} \ \forall i$. If all others announce according to $\hat{\theta}_j(\theta_j) = \widetilde{\theta}$, it will be a best reply for i to announce $\widetilde{\theta}$ as well, since on no account its announcement will change the implemented decision under the median mechanism.

2.3 The Results

step, we show that playing $x(\theta_i)$ is locally optimal when all other players play $x(\theta_i)$.

Lemma 2 For given θ_i , the function $U(\hat{\theta}_i, \theta_i)$ has a local maximum at $x(\theta_i)$.

Proof of Lemma 2:

Consider a marginal deviation from this announcement $x(\theta_i)$. Either player i is pivotal or he is not pivotal when he announces $x(\theta_i)$. If player i is not pivotal then a marginal deviation will not alter the outcome x_{Median} . If player i is pivotal, then his announcement will be the median announcement and therefore his type θ_i will be the median of all types. A marginal deviation does not pay because it alters the implemented decision x_{Median} . However, $x(\theta_i)$ is already chosen optimally conditional on player i being the median. Hence, any marginal deviation from $x(\theta_i)$ reduces $U(\hat{\theta}_i, \theta_i)$.

Lemma 3 For given θ_i , the function $U(\hat{\theta}_i, \theta_i)$ is single peaked at $x(\theta_i)$.

Proof of Lemma 3:

Consider any other announcement $\hat{\theta}'_i < x(\theta_i)$ of player i. Either player i is pivotal at $\hat{\theta}'_i$ or he is not pivotal. If player i is not pivotal, then a marginal deviation will not alter the outcome x_{Median} . If his announcement is pivotal, then one half of the other players $j \neq i$ will make an announcement below $\hat{\theta}'_i$. This in turn means that one half of the other players has a type below $x^{-1}(\hat{\theta}'_i)$, since all $j \neq i$ announce according to $x(\theta_j)$. Player i's conditional desired policy is then given by the expected

⁷Note that $x^{-1}(\cdot)$ exists because the expected signal of the others conditional on being the median type will be strictly increasing in the type if the distribution has full support. Hence, $x(\cdot)$ is strictly increasing in θ_i . Moreover, $x(\cdot)$ is differentiable in θ_i .

ideal point of player i conditional on one half of the other players having types smaller than $x^{-1}\left(\hat{\theta}_{i}'\right)$, i.e.

$$x'(\theta_i) = (1 - \alpha) \theta_i + \alpha \left(\frac{1}{2} E\left[\theta_k | \theta_k < x^{-1} \left(\hat{\theta}_i'\right)\right] + \frac{1}{2} E\left[\theta_k | \theta_k > x^{-1} \left(\hat{\theta}_i'\right)\right] \right). \tag{2}$$

On the other hand, according to the definition of $x(\theta_i)$ it must be true that

$$\hat{\theta}_{i}' = (1 - \alpha) x^{-1} \left(\hat{\theta}_{i}' \right) + \alpha \left(\frac{1}{2} E \left[\theta_{k} | \theta_{k} < x^{-1} \left(\hat{\theta}_{i}' \right) \right] + \frac{1}{2} E \left[\theta_{k} | \theta_{k} > x^{-1} \left(\hat{\theta}_{i}' \right) \right] \right).$$

$$(3)$$

Finally, it follows from

$$\hat{\theta}_i' < x(\theta_i) \Leftrightarrow x^{-1} \left(\hat{\theta}_i' \right) < \theta_i$$

and from equations (2) and (3) that

$$\hat{\theta}_i' < x'(\theta_i).$$

Thus, a marginal increase in the announcement $\hat{\theta}'_i$ increases individual i's expected utility, since it shifts the implemented decision in the direction of its ideal point.

The same line of argumentation holds for announcements $\hat{\theta}'_i > x(\theta_i)$. The single peakedness of $U\left(\hat{\theta}_i, \theta_i\right)$ at $x\left(\theta_i\right)$ follows from this and from the local optimality of $x(\theta_i)$.

Proposition 1 There exists a unique strictly monotonous symmetric equilibrium under the median mechanism. The equilibrium strategy is

$$x(\theta_i) = (1 - \alpha) \theta_i + \alpha \left(\frac{1}{2} E \left[\theta_k \mid \theta_k < \theta_i \right] + \frac{1}{2} E \left[\theta_k \mid \theta_k > \theta_i \right] \right) \forall i = 1, \dots, n.$$

Proof of Proposition 1:

Existence and uniqueness follow immediately from Lemmata 2 and 3.

In the special case of a uniform distribution the equilibrium strategy turns out to be linear. 2.3 The Results

Corollary 1 Assume that all θ_i are distributed uniformly over [-1, 1]. Then, the equilibrium strategy under the median mechanism is given by $\hat{\theta}_i(\theta_i) = \left(1 - \frac{1}{2}\alpha\right)\theta_i \ \forall i$.

Under the median mechanism individuals understate their private information. With increasing degrees of interdependency (larger α) the private signal becomes less valuable. Since individuals know that the median of all announcements will be implemented, they try to profit from the others' information. Their announcement is closer to zero than their true signal. This is due to the fact that individuals with extreme signals take into account that – if they are the median – the expected value of the other agents' signals will be closer to zero than their own signal.

While the median mechanism has an equilibrium in linear strategies only for a certain distribution of types, the mean mechanism always has a linear equilibrium. This equilibrium is unique up to a constant. According to this strategy individuals overstate their private information.

Proposition 2 For all constants c_1, \ldots, c_n with $\sum_{i=1}^n c_i = 0$ the following strategies constitute a Bayesian Nash equilibrium: $\hat{\theta}_i(\theta_i) = n(1-\alpha)\theta_i + c_i$ $\forall i$. There exists no other equilibrium.

Proof of Proposition 2:

Take any profile $\hat{\theta}_{-i} = (\hat{\theta}_1(\theta_1), \dots, \hat{\theta}_{i-1}(\theta_{i-1}), \hat{\theta}_{i+1}(\theta_{i+1}), \dots, \hat{\theta}_n(\theta_n))$ as given. Individual i maximizes its expected utility if $\hat{\theta}_i$ maximizes

$$E\left[-\left(x_{Mean}-\theta_i^*\right)^2\right] = E\left[-\left(\frac{\sum_{j=1}^n \hat{\theta}_j}{n}-\theta_i^*\right)^2\right].$$

Substituting for θ_i^* yields

$$\max_{\hat{\theta}_i} E \left[-\left(\frac{\sum_{j=1}^n \hat{\theta}_j}{n} - (1 - \alpha) \theta_i - \frac{\alpha}{n-1} \sum_{\substack{j=1 \ j \neq i}}^n \theta_j \right)^2 \right]$$

or

$$\max_{\hat{\theta}_i} E \left[-\left(\frac{\hat{\theta}_i}{n} + \frac{1}{n} \sum_{\substack{j=1\\j \neq i}}^n \hat{\theta}_j - (1 - \alpha) \theta_i - \frac{\alpha}{n-1} \sum_{\substack{j=1\\j \neq i}}^n \theta_j \right)^2 \right]. \tag{4}$$

Let

$$a := \frac{1}{n} \sum_{\substack{j=1\\j \neq i}}^{n} \hat{\theta}_j - (1 - \alpha) \theta_i - \frac{\alpha}{n-1} \sum_{\substack{j=1\\j \neq i}}^{n} \theta_j.$$

Then, (4) becomes

$$\max_{\hat{\theta}_i} E \left[-\left(\frac{\hat{\theta}_i}{n} + a\right)^2 \right]$$

or

$$\max_{\hat{\theta}_i} E\left[-\frac{\hat{\theta}_i^2}{n^2} - \frac{2a}{n}\hat{\theta}_i - a^2\right].$$

Optimality requires

$$-\frac{2}{n^2}\hat{\theta}_i - \frac{2}{n}E\left[a\right] = 0$$

and thus,

$$\hat{\theta}_i = -nE\left[a\right].$$

Using the fact that all θ_j have an expected value of zero and that expectations are taken over all $j \neq i$, we get

$$E[a] = \frac{1}{n}E\left[\sum_{\substack{j=1\\j\neq i}}^{n} \hat{\theta}_{j}\right] - (1 - \alpha)\theta_{i}$$

and thus,

$$\hat{\theta}_i = n (1 - \alpha) \theta_i - E \left[\sum_{\substack{j=1 \ j \neq i}}^n \hat{\theta}_j \right]. \tag{5}$$

Hence, the best reply with respect to any profile $\hat{\theta}_{-i}$ has to be linear with slope $n(1-\alpha)$. The fact that $\sum_{i=1}^{n} c_i = 0$ follows from (5) and the assumption that $E[\theta_i] = 0 \ \forall i$.

In what follows we restrict attention to announcement strategies of the form $\hat{\theta}_i(\theta_i) = n(1-\alpha)\theta_i \,\forall i$, since the constant c_i has no effect on expected utility.

Under the mean mechanism individuals exaggerate their private signal in order to cancel out the average taking implied by this mechanism. With increasing degrees of interdependency this behavior becomes counterproductive and announcements approach the true signal.

Using the above calculated equilibrium strategies, it follows that

Corollary 2 The median mechanism has a symmetric Bayesian Nash equilibrium in which agents announce their type truthfully if $\alpha = 0$.

Corollary 3 The mean mechanism has a symmetric Bayesian Nash equilibrium in which agents announce their type truthfully if $\alpha = \bar{\alpha} = \frac{n-1}{n}$.

2.4 Comparison of the mean and the median mechanism

This section compares the performance of the median and the mean mechanism for different degrees of interdependencies. First, we present results that obtain independent of the underlying distribution of information parameters and for any number of agents. In the second part, we develop a concrete example with uniform distribution and a small number of individuals.

2.4.1 General results

We start the comparison by analyzing the properties of the two mechanisms at the corner values of α , i.e. at $\alpha = 0$ and at $\alpha = \bar{\alpha}$.

Proposition 3 For $\alpha = 0$ and $n \geq 3$ the sum of expected utilities is larger under the median than under the mean mechanism.

Proof of Proposition 3:

For $\alpha = 0$ it holds that $\theta_i^* = \theta_i$. Since the whole setting is symmetric, it suffices to show that individual expected utility is larger under the median than under the mean mechanism, i.e. that

$$E\left[-(x_{Median}-\theta_i)^2\right] > E\left[-(x_{Mean}-\theta_i)^2\right].$$

First, consider the mean mechanism. In equilibrium individuals announce according to $\widehat{\theta}_i(\theta_i) = n\theta_i$ and thus expected utility for individual i is given by

$$E\left[-\left(x_{Mean} - \theta_{i}\right)^{2}\right] = E\left[-\left(\frac{\sum_{i=1}^{n} n \theta_{i}}{n} - \theta_{i}\right)^{2}\right]$$

$$= E\left[-\left(\sum_{i=1}^{n} \theta_{i} - \theta_{i}\right)^{2}\right]$$

$$= E\left[-\left(\sum_{i=1}^{n} \theta_{i}\right)^{2}\right]$$

$$= E\left[-\left(\sum_{j=1}^{n} \theta_{j}\right)^{2}\right]$$

$$= -(n-1) E\left[\theta_{i}^{2}\right].$$

Under the median mechanism individuals announce their type truthfully, i.e. $\widehat{\theta}_i(\theta_i) = \theta_i$. Let θ_{Median} denote the signal of the median individual. Consider a situation in which n individuals decide and individual i is not included in the collective decision, i.e. a situation in which θ_{Median} and θ_i are not correlated. Expected utility in this situation represents an upper bound to losses (a lower bound to individual expected utility), since participating in the decision process can only improve upon the situation. If θ_{Median} and θ_i are independent, individual expected utility is given by

$$E\left[-\left(x_{Median} - \theta_{i}\right)^{2}\right] = E\left[-\left(\theta_{Median} - \theta_{i}\right)^{2}\right]$$

$$= -E\left[\theta_{Median}^{2}\right] - E\left[\theta_{i}^{2}\right]$$

$$> -2E\left[\theta_{i}^{2}\right].$$

The last inequality follows, because $E\left[\theta_{Median}^2\right]$ can never exceed the variance of any single information parameter θ_i . This is true because the product of probabilities can never exceed 1:

$$E\left[\theta_{Median}^{2}\right]$$

$$= \int_{\Theta} \theta_{i}^{2} p\left[\theta_{1} \leq \theta_{i}\right] \dots p\left[\theta_{\frac{n-1}{2}} \leq \theta_{i}\right] p\left[\theta_{\frac{n-1}{2}+1} \geq \theta_{i}\right] \dots p\left[\theta_{n} \geq \theta_{i}\right] f(\theta_{i}) d\theta_{i}$$

$$< \int_{\Theta} \theta_{i}^{2} f(\theta_{i}) d\theta_{i}$$

$$= E\left[\theta_{i}^{2}\right].$$

It remains to show that the lower bound on expected utility under the median exceeds expected utility under the mean mechanism for $n \geq 3$:

$$-2E\left[\theta_i^2\right] > -\left(n-1\right)E\left[\theta_i^2\right]$$

 \Leftrightarrow

$$(n-3)E\left[\theta_i^2\right] > 0$$

which is true for all n > 3. For n = 3 the lower bound on expected utility under the median equals expected utility under the mean mechanism. Since the lower bound is strict, this completes the proof. \blacksquare

The median mechanism induces agents to tell the truth in the private values case. This leads to a relatively high welfare level when median and mean are correlated. The mean mechanism instead induces agents to exaggerate their private information. This explains the superiority of the median mechanism. In general, the median mechanism does not implement the first-best, since the first-best will only be implemented if the private information of the median individual by instance is equal to the average of all types.

Proposition 4 For $\alpha = \bar{\alpha}$ it holds that (i) the mean mechanism yields the first-best and (ii) the median mechanism does not yield the first-best.

Proof of Proposition 4:

(i) From Proposition 2 it follows that under the mean mechanism agents announce their type truthfully if $\alpha = \bar{\alpha}$. Thus $x_{Mean} = \frac{1}{n} \sum_{i=1}^{n} \theta_i$ and it holds that $\theta_i^* = \frac{1}{n} \sum_{i=1}^{n} \theta_i$. We obtain for the sum of expected utilities

$$E\left[\sum_{i=1}^{n} -(x_{Mean} - \theta_i^*)^2\right] = 0.$$

(ii) It remains to show that

$$0 > E\left[\sum_{i=1}^{n} - \left(x_{Median} - \theta_i^*\right)^2\right].$$

Suppose that the equilibrium under the median mechanism from Proposition 1 maximizes expected utility. Let $s(\theta)$ be the respective equilibrium profile. Then

$$x(s(\theta)) = \theta_{Mean} = \frac{1}{n} \sum_{i=1}^{n} \theta_i \ \forall \ \theta.$$

However, from Proposition 1 we know that for some θ some partial derivatives of $x(s(\theta))$ are zero. This is so because the median is unaffected by changes in extreme positions. Hence, $x(s(\theta))$ cannot equal θ_{Mean} for all θ .

The intuition for this result is as follows: for $\alpha = \bar{\alpha}$ individuals announce their type truthfully under the mean mechanism and we know already that the first-best solution is the average of all types. This yields part

(i). However, the median mechanism does neither imply truth telling for $\alpha = \bar{\alpha}$ nor does it implement the average of all private signals. Therefore, it does not yield the first-best.

Now, we turn to the behavior of the sum of expected utilities under both mechanisms for intermediate values of α .

Lemma 4 Consider the median mechanism. The sum of expected utilities is continuous in α .

Lemma 5 Consider the mean mechanism. The sum of expected utilities is (i) continuous in α , (ii) strictly increasing in α and (iii) attains its maximum at $\alpha = \bar{\alpha}$.

Lemma 6 Consider the mean mechanism. For n > 1 the sum of expected utilities is (i) continuous in n, (ii) strictly decreasing in n and (iii) attains its maximum at $n = \frac{1}{1-\alpha}$.

All the above yields our main result concerning the comparison between the median and the mean mechanism:

Proposition 5 There exists an α_1 below which the sum of expected utilities is larger under the median than under the mean mechanism and there exists an α_2 above which the sum of expected utilities is larger under the mean than under the median mechanism.

Proof of Proposition 5:

The result follows directly from Propositions 3 and 4 and Lemmata 4 and 5. \blacksquare

If individual preferences are strongly correlated, then making all agents participate in the decision will be better than restricting entry into the decision process. If there is only a small common component then it will be better to use the median mechanism. The intuition is that for weak interdependencies the equilibrium strategy under the median mechanism implies announcement behavior close to truth telling whereas the equilibrium strategy under the mean mechanism leads to strong exaggeration of

private information. Therefore, average taking is outperformed by ignoring some of the information available. Since the degree to which α influences untruthful announcement behavior is stronger under the mean mechanism, this intuition holds for a wide range of interdependencies, only for very high degrees it is reversed.

2.4.2 Example with uniform distribution

Assume that all θ_i are distributed uniformly over [-1,1] and that n=3.8 Then, the comparison between the median and the mean mechanism can be extended and we obtain the result that there exists a unique point of intersection for the respective sums of expected utility under the two mechanisms.

Corollary 1 tells us that for uniform distribution the equilibrium strategy under the median mechanism is given by $\hat{\theta}_i(\theta_i) = \left(1 - \frac{1}{2}\alpha\right)\theta_i \ \forall i$ and thus $x_{Median} = (1 - \frac{1}{2}\alpha)\theta_{Median}$. Then, expected utility for individual 2 is given by

$$E\left[-\left(x_{Median} - \theta_{2}^{*}\right)^{2}\right]$$

$$= -\frac{1}{4} \int_{-1}^{1} \int_{-1}^{\theta_{2}} \int_{\theta_{2}}^{1} \left(\left(1 - \frac{1}{2}\alpha\right)\theta_{2} - (1 - \alpha)\theta_{2} - \frac{\alpha}{2}\left(\theta_{1} + \theta_{3}\right)\right)^{2} d\theta_{3} d\theta_{1} d\theta_{2}$$

$$-\frac{1}{4} \int_{-1}^{1} \int_{-1}^{\theta_{2}} \int_{\theta_{1}}^{\theta_{2}} \left(\left(1 - \frac{1}{2}\alpha\right)\theta_{3} - (1 - \alpha)\theta_{2} - \frac{\alpha}{2}\left(\theta_{1} + \theta_{3}\right)\right)^{2} d\theta_{3} d\theta_{1} d\theta_{2}$$

$$-\frac{1}{4} \int_{-1}^{1} \int_{\theta_{2}}^{1} \int_{\theta_{1}}^{1} \left(\left(1 - \frac{1}{2}\alpha\right)\theta_{1} - (1 - \alpha)\theta_{2} - \frac{\alpha}{2}\left(\theta_{1} + \theta_{3}\right)\right)^{2} d\theta_{3} d\theta_{1} d\theta_{2}$$

$$= -\frac{11}{20}\alpha^{2} + \frac{11}{15}\alpha - \frac{4}{15}.$$

Since our whole setting is symmetric, we obtain for the sum of expected utilities under the median mechanism

⁸We conducted a similar analysis for the case of n = 5 and n = 7, but for expositional purposes only the detailed results for n = 3 are given.

$$\sum_{i=1}^{3} E\left[-\left(x_{Median} - \theta_2^*\right)^2\right] = -\frac{33}{20}\alpha^2 + \frac{11}{5}\alpha - \frac{4}{5}.$$
 (6)

Differentiating with respect to α yields

$$\frac{d\left(E\left[\sum_{i=1}^{3} - (x_{Median} - \theta_i^*)^2\right]\right)}{d\alpha} = -\frac{33}{10}\alpha + \frac{11}{5}$$
 (7)

which is larger than zero $\forall \alpha \in [0, \bar{\alpha})$. Thus, the sum of expected utilities is strictly increasing under the median mechanism and yields a maximum for $\alpha = \frac{2}{3}$.

Under the mean mechanism agents announce $\hat{\theta}_i(\theta_i) = 3(1-\alpha)\theta_i$ for n=3. Thus, $x_{Mean} = (1-\alpha)\sum_{i=1}^{3}\theta_i$. The sum of expected utilities is then given by

$$\sum_{i=1}^{3} E\left[-\left(x_{Mean} - \theta_{i}^{*}\right)^{2}\right]$$

$$= -\frac{3}{8} \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} \left((1-\alpha)\sum_{i=1}^{3} \theta_{i} - (1-\alpha)\theta_{1} - \frac{\alpha}{2}(\theta_{2} + \theta_{3})\right)^{2} d\theta_{1} d\theta_{2} d\theta_{3}$$

$$= -\frac{9}{2}\alpha^{2} + 6\alpha - 2. \tag{8}$$

Differentiating with respect to α yields

$$\frac{d\left(E\left[\sum_{i=1}^{n} -(x_{Mean} - \theta_{i}^{*})^{2}\right]\right)}{d\alpha} = -9\alpha + 6.$$
(9)

From Proposition 5 we know that there exists an α_1 below which the sum of expected utilities is larger under the median than under the mean mechanism and that there exists an α_2 above which the sum of expected utilities is larger under the mean than under the median mechanism. For uniform distribution of the information parameters the two points coincide, i.e. $\alpha_1 = \alpha_2 =: \alpha^*$.

The existence of a unique point of intersection can be established by comparing the slopes of the sum of expected utility under the two mechanisms, since both are strictly positive for the relevant range of α . Under the median mechanism the slope is given by equation (7) and under the mean mechanism by equation (9), respectively. For all $\alpha \in [0, \bar{\alpha})$ it is true that

$$-\frac{33}{10}\alpha + \frac{11}{5} < -9\alpha + 6$$

and therefore α^* exists.

The sum of expected utilities under the median mechanism is given by equation (6) and under the mean mechanism by equation (8). The point of intersection α^* is then determined by

$$-\frac{33}{20}\alpha^{*2} + \frac{11}{5}\alpha^* - \frac{4}{5} = -\frac{9}{2}\alpha^{*2} + 6\alpha^* - 2$$

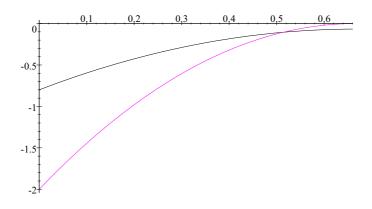
which yields

$$\alpha_{1,2}^* = \frac{2}{3} \pm \frac{2}{57} \sqrt{19}.$$

Since $\alpha_1^* > \bar{\alpha} = \frac{2}{3}$, α_2^* is the unique point of intersection in the relevant interval.

The following pictures show the sum of expected utilities under the mean and the median mechanism for n=3, 5 and 7 and uniform distribution of information parameters. In each of these cases, we obtain a unique point of intersection α^* .

Figure 1: n = 3



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Figure 2: n = 5

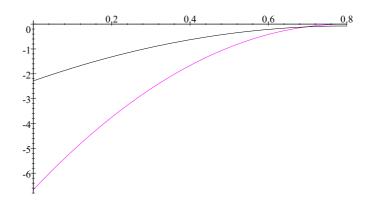
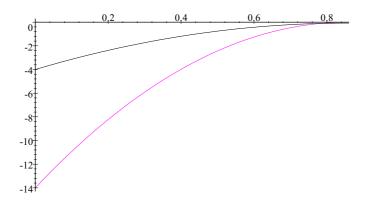


Figure 3: n = 7



2.5 Robustness

So far, our results have been derived for a particular class of quadratic utility functions. These utility functions describe a situation where individuals are risk averse with respect to the size of the deviation of the decision x from their ideal point θ_i^* . Our result on the welfare comparison of the two mechanisms presented in Proposition 5 also holds under a linear specification of utility which corresponds to the case of risk neutrality.

Consider the following von-Neumann-Morgenstern utility function

$$u(x, \theta_i^*) = -|x - \theta_i^*|.$$

In contrast to Lemma 1 we get

Lemma 7 Consider any vector θ of private signals. Benthamian welfare is maximized if and only if $x^* = median(\theta_1^*, \dots, \theta_n^*)$.

Proof of Lemma 7:

Consider any deviation from the median of all ideal policies, i.e. from $median(\theta_1^*, \ldots, \theta_n^*)$. With linear utility, this deviation costs more than half of the individuals one unit while less than half of the individuals gain.

Figure 4 presents the basic intuition underlying this result for the case of three individuals. This extends to any number n of individuals.

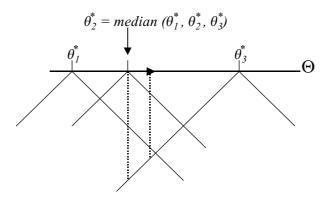


Figure 4: Utility under Risk Neutrality

It is useful to consider the implications of the changed utility specification for an individually desired policy \tilde{x} when individual i does not know its ideal point with certainty. Since all θ_i are distributed independently with density $f(\theta_i)$, this defines a probability distribution for each θ_i^* over Θ . Let $\psi(\theta_i^*)$ denote the density of this distribution and $median(\psi(\theta_i^*))$ the respective median.

Lemma 8 Consider an individual i with an ideal point θ_i^* distributed according to the density $\psi(\theta_i^*)$. Ex ante individual i's preferred decision \widetilde{x} is the median of the distribution of his own ideal points, i.e. median $(\psi(\theta_i^*))$.

⁹Under quadratic specification of utility this policy \tilde{x} corresponds to the mean of the distribution of the individually desired policy θ_i^* .

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Proof of Lemma 8:

Any deviation from this choice triggers a marginal loss of 1 with probability of more than 1/2 and a marginal gain of one unit with a probability below 1/2.

Proposition 1 no longer holds under the linear specification of utility. In order to transfer the result, we have to introduce some more notation: Let $\phi(\theta_i)$ denote the density of the distribution of the average of the signals of all agents $j \neq i$ conditional on θ_i being the median. Let $median(\phi(\theta_i))$ denote the median of this distribution.

Proposition 6 Consider a linear utility function of the form $u(x, \theta_i^*) = -|x - \theta_i^*|$. There exists a unique symmetric strictly monotonous equilibrium under the median mechanism. The equilibrium strategy is

$$\hat{\theta}_i(\theta_i) = (1 - \alpha) \theta_i + \alpha \ median (\phi(\theta_i)) \ \forall i, i = 1, \dots, n.$$

Proof of Proposition 6:

Following the same line of argumentation that leads to Proposition 2, note first that the announcement $\hat{\theta}_i(\theta_i)$ is a local maximum of player i's expected utility given θ_i and all other $j \neq i$ announcing according to $\hat{\theta}_j(\theta_j)$. Consider a marginal deviation from this announcement. If player i is not pivotal, then a marginal deviation will not alter the outcome x_{Median} . If player i is pivotal, then we know from Lemma 8 that he will choose the median of the distribution of his own ideal points as the common decision. Conditional on being the median, this becomes $(1 - \alpha) \theta_i + \alpha \mod(\phi(\theta_i))$. Thus, marginal deviations from $\hat{\theta}_i(\theta_i)$ reduce expected utility of player i.

Moreover, player i's expected utility is single peaked at $\hat{\theta}_i(\theta_i)$ given θ_i and all other $j \neq i$ announcing according to $\hat{\theta}_j(\theta_j)$. This is due to the monotonicity of the median of $\phi(\theta_i)$ in the signal of player i and the consequential existence of $\hat{\theta}_i^{-1}(\cdot)$. This completes the proof.

It can be easily seen that the equilibrium of Proposition 2 still exists under the linear specification of the utility function if the distribution of information parameters is symmetric. The intuition is that any change in θ_i does neither affect the median of the distribution of the average of the signals of the other individuals nor does it affect the median of the distribution of the average of the announcements of the other individuals. Therefore, according to Lemma 8, agent i's favorite decision is shifted by $n(1-\alpha)\Delta\theta_i$.

Another straightforward consequence of Lemma 7 is that the median mechanism maximizes ex ante expected utility in the private values case. In this setting, the median mechanism (again) induces truth telling. This maximizes welfare with linear utility. Moreover, for symmetric distributions of types, the median mechanism still yields higher ex ante expected utility than the mean mechanism in the private values case. Proposition 4 (i) on the mean mechanism instead holds and it remains true that the mean yields more expected utility than the median mechanism in the case where $\alpha = \overline{\alpha}$. Continuity again implies that there exist upper and lower bounds on α such as in Proposition 5.

2.6 Conclusion

The problem analyzed in this chapter is one of collective decision taking. If the individuals who are supposed to take a common decision are asymmetrically informed and if there are interdependencies between the individually desired policies, how should a decision mechanism be designed that maximizes the sum of expected utilities? Our analysis of this problem concentrates on two specific mechanisms, i.e. the median and the mean mechanism, and obtains the following: Under the median mechanism individuals understate their private information, whereas under the mean mechanism they overstate it. As a result, the median performs better than the mean mechanism in terms of expected utility maximization for weak interdependencies. For high degrees of interdependency is this mechanism outperformed by the mean mechanism.

Returning to the provision of a public good, we can conclude that the median mechanism will perform better than taking the average if the idiosyncratic component of individual preferences is strong. Disregarding 2.6 Conclusion 37

extreme positions – as does the median mechanism – seems to be favorable in this case. Only if the quality component becomes very important, it would be better to change the decision mechanism and use the average of announcements.

Starting from our research there are three directions to proceed. First, in this chapter we abstracted from individual rationality considerations, since in many collective decisions participation is not voluntary. However, if we take participation constraints into account, the traditional solution prescribes an outside option to be implemented if an individual opts out. This in turn leads to changed interim individual behavior and to different equilibrium outcomes – with the status quo maintaining in many instances. But in our setting, due to interdependent valuations, even individuals not participating in the mechanism would be affected by the common decision. This would imply endogenizing the participation constraint (following Jehiel et al. [1996]).

Relating our results to decisions taken by majority rule, it is not obvious how a majoritarian decision in a committee should be modeled. In the private values case the median mechanism is certainly a good approximation of the actual decision procedure. However, in a setting with interdependencies there may be scope for pre-vote communication. When communication plays a role, the median mechanism need not be the correct representation of a majoritarian decision. However, the result of this chapter indicates, that voting without communication can dominate the mean mechanism. This leads to the second extension, the analysis of pre-vote communication in a modified two-stage game. The question is if an improvement upon the equilibria of the original game is possible when people are allowed to communicate before they have to vote. It is well known that equilibrium behavior may be affected if agents have the opportunity to exchange information prior to playing some game (see Crawford and Sobel [1982]).

Finally, another question we did not address is the design of an optimal mechanism for the class of collective decision problems studied. This would mean to find a mechanism that implements the first-best for all degrees of spillovers, not only for the maximum amount.

Chapter 3

The European Stability and Growth Pact as a Device for adequate Stabilization

3.1 Introduction

Since European monetary unification has taken place, attention has shifted from the overall desirability of a monetary union in Europe to the influence that a common monetary policy as conducted by the *European Central Bank (ECB)* has on other national economic policies (e.g. Hughes Hallet et al. [2001]). Examples are the interplay between one monetary authority and many independent fiscal authorities or many independent labor market policies.

This chapter concentrates on the first of these relationships and tries to answer three main questions: First, I try to find reasons for such arrangements as the European Stability and Growth Pact (SGP). It is not obvious in the first place, why independent countries voluntarily confine the freedom of their national fiscal policies. Moreover, I want to answer why the SGP was signed, despite the fact that future situations could arise

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in which the fulfillment of the pact would be hard to achieve – and this was foreseeable at the point in time of signing. Finally and as a contribution to the ongoing debate about the reform of the *European SGP*, I introduce a direct mechanism in order to analyze whether it may achieve better outcomes than the pact.

Details of the features of the SGP are given in Section 3.2.2, but let me here briefly introduce its rationale. According to Eichengreen and Wyplosz [1998] the most important reason for the European SGP is to protect the common central bank from pressures for an inflationary debt bail-out. These pressures could arise because in a monetary union the costs of debt (in terms of higher inflation) are borne by all residents of the EMU zone rather than solely by the citizens of the responsible country. Thus, governments have an incentive to run riskier policies (higher deficits). Moreover, if the common monetary policy is geared to union averages, the inflationary response of the common central bank to an increase in debt in one of many countries will tend to be smaller.

If the union central bank is politically independent, negative spillovers created by a common monetary policy will constitute no problem. In the case of the ECB this fact is not only discussed in-depth in public (e.g. The Economist [2003]), but some features embodied in the Treaties of the $European\ Union$ and in the ECB's own statutes [1992] raise some doubts. First of all, the Maastricht Treaty contains a no-bail-out rule that prohibits the ECB from purchasing public debt directly from the issuer. Secondly, the main goal of monetary policy as conducted by the ECB is to maintain price stability. This, however, does not rule out the possibility to include other economic factors in conducting monetary policy whenever price stability is not in danger. Finally, the $European\ SGP$ may be interpreted as an instrument of further protection for the common central bank, since it confines the possibility of member countries to issue debt. As such there exist at least three provisions to safeguard the common central bank, despite it being entirely independent on paper.

¹⁰Other reasons could be to offset the political bias towards excessive deficits in participating countries or to encourage policy co-ordination among member states.

In this chapter I do not ask the question why budget deficits arise, but shift attention to the adoption of a welfare increasing fiscal reform. Private information about the costs of delayed stabilization leads to a war of attrition between interest groups in the member countries of a monetary union. The period preceding the reform is seen as a period of conflict among those parts of the population. Each group has an incentive to wait, because the group that concedes first is assumed to bear the larger part of the reform costs. By introducing private information an explanation is provided as to why a reform that is unanimously regarded as beneficial might be delayed.

I extend the basic model to the case of a monetary union in order to allow for negative spillovers due to a common monetary policy. Moreover, I introduce a fine for deficits to resemble some features of the *European SGP*. Additionally and in contrast to other models, I allow for private benefits of delayed stabilization as well. This renders situations possible in which it is not efficient to stabilize immediately.

The first result of this chapter is that a deficit ceiling like the one given in the SGP can enforce national commitment to stabilization. This will hold even if the rules of the pact are weakened with respect to the payment of fine. But due to the possibility of private benefits of delay the effect immediate stabilization has on welfare is not unambiguous. The SGP might induce interest groups to stabilize in situations when this is not desirable. I show that the introduction of a well designed fine unambiguously increases welfare as compared to the war of attrition. This also may explain why the member countries of EMU signed the pact in the first place. On the other hand, there always exist types that would like to abandon the pact ex post. This may yield an explanation for the current difficulties concerning the enforcement of the SGP. Moreover, it is impossible to achieve ex post efficiency with such arrangements as the SGP. Therefore, I analyze another mechanism (expected externality mechanism) that overcomes this drawback. Unfortunately, this mechanism is also not able to achieve vo-

 $^{^{11}}$ For an interesting account of the interaction between interest groups and central banks see Maier [2002].

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luntary participation ex post and would thus experience similar difficulties as the SGP with respect to its enforcement.

Furthermore, my analysis may contribute to the current discussion about how to reform the European SGP, since it introduces new actors (interest groups) and private information on their behalf into the process of stabilization decisions. Attention is drawn to possible weaknesses concerning the design of the current pact: the introduction of a fine will lead to welfare gains only if its height is chosen appropriately. Moreover, the possibility of renegotiations weakens the commitment effect of the pact and thus an argument in favor of stringent enforcement is provided. Finally, I show that a better result in terms of efficiency may be achieved using a decentralized form of agreement, i.e. a direct revelation mechanism that defines transfers directly between the interest groups.

3.1.1 Relation to literature

The literature mainly focuses on the justification of the above given rationale for the $European\ SGP$, namely potential spillovers from fiscal policies onto monetary policy. For example, Dixit and Lambertini [2001] show in a Barro-Gordon [1983a] type model that freedom of national fiscal policies can undermine the ECB's monetary commitment whenever there exist conflicting interests among the authorities. The value of precommitment in monetary policy is completely negated if fiscal policies are discretionary, since the reaction functions of the fiscal authorities impose a constraint on the monetary policy rule. The authors conclude that this result may possibly justify fiscal constraints like the SGP.

Others instead argue that coordinated fiscal policies may induce the *ECB* to follow a more expansionary monetary policy (see e.g. Beetsma and Bovenberg [1998]). This opposite result is due to the assumed fiscal leadership: many non-cooperating fiscal players strengthen the strategic position of the common central bank which favors lower inflation than the fiscal players do. Coordination of fiscal policies eliminates this effect.

Using a dynamic games approach, Engwerda et al. [2002] also analyze the interaction of fiscal stabilization policies in a monetary union. In their

set-up a pact reduces the degree of fiscal activism and this in turn leads to suboptimal macroeconomic policies. Moreover, the authors study a fiscal transfer system operating through the EU budget (i.e. union wide unemployment benefits). Such a scheme reduces the need for national fiscal policy to stabilize output and thus leads to an improved performance of the pact.

Another strand of literature explains why budget deficits arise. A model developed by Alesina and Tabellini [1990] is extended to monetary unions. Budget deficits emerge because of political uncertainty. Beetsma and Uhlig [1999] show that in a monetary union a pact will be preferred to autonomy, since it confines the possibilities for free-riding. Including moral hazard, Beetsma and Jensen [2003] come to the same result, i.e. that a pact eliminates the exacerbation of debt accumulation that may arise in monetary unions. This paper also provides an explanation for the non-automatic alleviation of sanctions and the detailed assessment procedure of the *European SGP*.

The formal set-up of this chapter builds on a war of attrition model developed by Alesina and Drazen [1991]. These authors were the first to analyze the possible delay of welfare increasing reforms due to private information. Their model has been applied extensively, for example by Casella and Eichengreen [1996] or in a European context by Carré [2000]. Carré introduces an exogenous deadline for stabilization to mimic the Maastricht Criteria for participation in the European Monetary Union. Here, I follow Persson and Tabellini [2000] who present a two-period version of the (continuous time) war of attrition of Alesina and Drazen.

This chapter is also related to the recent (extensive) discussion about a reform of the European SGP (e.g. Buti et al. [2003], European Commission [2002] or Zimmermann [2003]). The reform proposals range from avoiding procyclical public spending over redefining the medium-term budgetary target to creating a European Council of Experts to tackle the partisan application of the rules. This chapter develops a new argument in favor of a stringent and carefully designed pact, since the introduction of a fine only leads to welfare gains if its height is appropriate. It also yields insights

about how the current SGP could be improved upon in the presence of private information using a more decentralized form of agreement.

The remainder of this chapter is organized as follows: Section 2 applies a war of attrition model to the context of a monetary union. In order to mimic some features of the European SGP two extensions including a fine for deficits are introduced and compared with respect to the resulting probability of delayed stabilization. Welfare, voluntary participation and efficiency are considered in Section 3. In the next section, I adapt an expected externality mechanism and compare its features to the other scenarios. Extensions to the basic model are analyzed in Section 5 and Section 6 concludes. All omitted proofs can be found in Appendix B.

3.2 The Model

In this section, I adapt a simplified version of Alesina and Drazen's [1991] war of attrition to the context of a monetary union. I consider a twostage (respectively three-stage) game of private information and solve it by backward induction. First, the interest groups in each country decide whether fiscal stabilization occurs. After that, the common central bank determines the monetary policy for the entire union. Introducing private information on behalf of the interest groups yields an explanation as to why welfare improving stabilizations are delayed. I then extend the basic model by adding another stage in which the governments of the participating countries implement a fine for deficits. Two different scenarios with fines are considered, one without and one with renegotiations, and compared to the basic set-up. Introducing a fine should resemble some features of the European SGP. The main result of this section is the identification of symmetric Bayesian Nash equilibria of the respective games. In addition, it can be shown that the introduction of a fine for deficits will lead to stabilization occurring for a wider range of information parameters – even if renegotiations are allowed.

3.2.1 War of attrition

In order to explain why budget deficits are not eliminated at once Alesina and Drazen [1991] introduce private information about the costs of delaying stabilization. Here, following Persson and Tabellini [2000], I present a one-shot version of a war of attrition and extend it to a situation in which two countries belong to a monetary union.¹²

There are two stages of the game and two countries c=A,B that form a monetary union. Monetary policy is conducted at the union level, whereas fiscal policy is decided upon at national level. Countries are assumed to be identical both in their economic and political structure. The society of each country consists of two powerful interest groups (or two parties in a coalition government) i=1,2. These groups have to decide whether stabilization should occur at once (s) or is being delayed (n). Let v_i^c denote the action of group i in country $c, v_i^c \in \{s, n\}$. Each group has an incentive to let the other bear the brunt of the necessary adjustment. This incentive is measured by one single parameter α which may be interpreted as a measure of polarization of the respective society. The question is how probable the delay of stabilization is under these circumstances.

Utility of group i in country c is denoted by $u_i^c\left(v_i^c,v_j^c\right)$ and consists of two parts. The first describes utility due to the stabilization decision and depends on both groups' actions. In addition, the groups dislike inflation $\pi\left(\cdot\right)$ which is determined in the second period (see below) and depends on the equilibrium decisions about stabilization in both countries. It is here where the spillovers due to the common monetary policy come into play. This utility cost arises independently of which decision about stabilization is taken in the own country.

If both groups are in favor of stabilization, then it will take place and no deficit arises. Utility of this stabilization decision is normalized to zero and thus total utility of group i in country c is given by $u_i^c(s,s) := -\pi(\cdot)$.

¹²To some readers it may sound queer to speak of a war of attrition in a one-shot game, since there hardly can be any attrition. I am aware of this, but still think that the notion has some appeal in the current context.

If one of the groups concedes, then stabilization will take place as well, but there exists an advantage of not giving in. This advantage is measured by the parameter $\alpha > 0$ and utility is $u_i^c(s,n) := -\alpha - \pi(\cdot)$, $u_i^c(n,s) := \alpha - \pi(\cdot)$, respectively. If one group gives in, that group will have to bear the main burden of the necessary cutbacks.

Finally, if neither group stabilizes, then stabilization will be delayed. Utility in this case is given by $u_i^c(n,n) := \theta_i^c d - \pi(\cdot)$, where d > 0 measures the size of the fiscal problem.¹³ The costs of debt policy enter additively in the utility function. They can be interpreted as either a suboptimal spending allocation over time, or other costs associated with debt issuance: perhaps part of the deficit is financed through a distortionary inflation tax or a high debt causes general macroeconomic instability because of an unsustainable budgetary position.

A crucial assumption is that the $\theta_i^{c'}s$ are private information to group i. They are distributed independently on $[\underline{\theta}, \overline{\theta}] \subseteq \mathbb{R}$ according to some commonly known distribution function $F(\cdot)$ ($f(\cdot)$ denotes the corresponding density function). I extend the usual framework by allowing for private benefits of delayed stabilization as well, i.e. $\overline{\theta}$ may be larger than zero. Consider for example a situation in which the economy is hit by an external shock that makes debt issuance eligible. To which degree or how the groups may profit from debt issuance is their private information. In order to allow for private costs and private benefits of delayed stabilization I assume that the support includes zero, i.e. $\underline{\theta} \leq 0$ and $\overline{\theta} \geq 0$. In addition, assume $\overline{\theta} > \frac{\pi(\cdot)}{d}$. This guarantees that there exist realizations of private information such that delaying stabilization is profitable. Note that private benefits of delay do not conflict with individual costs of giving in: interest groups may privately benefit from debt issuance, but nevertheless political reasons may render giving in costly.

¹³Persson and Tabellini [2000] present a two-period framework: If both groups are against stabilization, debt will be issued in *period* 1, but will have to be repaid by the end of *period* 2. Since the decision about stabilization is only taken once at the beginning of *period* 1, it depends on the sum of utility over both periods. Assuming an interest rate equal to zero, debt cancels out. In order to keep notation as simple as possible, I concentrate on overall utility.

Since the whole setting is symmetric, the following analysis is conducted for group i in country c, but applies also to group j and the respective groups in country -c. Summarizing the above, utility of group i in country c when this war of attrition is played is given by

$$u_{i}^{c}\left(v_{i}^{c},v_{j}^{c}\right):=\left\{ \begin{array}{ll} -\pi\left(\cdot\right), & \text{if } v_{i}^{c}=s,v_{j}^{c}=s\\ -\alpha-\pi\left(\cdot\right), & \text{if } v_{i}^{c}=s,v_{j}^{c}=n\\ \alpha-\pi\left(\cdot\right), & \text{if } v_{i}^{c}=n,v_{j}^{c}=s\\ \theta_{i}^{c}d-\pi\left(\cdot\right), & \text{if } v_{i}^{c}=n,v_{j}^{c}=n \end{array} \right..$$

The modelling of the monetary union closely follows Beetsma and Jensen [2003]. A common central bank (CB) sets monetary policy for the entire union. For simplicity, assume that the CB controls the inflation rate directly. The CB's objective function is given by

$$U_{CB} := -\lambda \pi \left(\cdot\right)^{2} + \left(1 - \lambda\right) \overline{d} * \pi \left(\cdot\right) \tag{10}$$

where \overline{d} denotes the average debt level in the union. The inflation rate depends on the decision about stabilization and this decision is determined in equilibrium (see below). The CB attaches weight $0 \le \lambda \le 1$ to the goal of price stability and weight $(1 - \lambda)$ to the budgetary situation of the union members. As such its objective function is a convex combination of the two conflicting goals. The case of $\lambda = 1$ corresponds to a CB that is able to commit to zero inflation. In contrast, $\lambda < 1$ describes a situation in which the CB is vulnerable to political pressures from member countries of the monetary union and consequently not able to entirely ignore the countries' budgetary positions.

The timing is as follows. First, the interest groups play the war of attrition about stabilization. After that, the *CB* selects the inflation rate. Using backward induction, this game is solved for a subgame perfect Bayesian Nash equilibrium.¹⁴

 $^{^{14}}$ Forming beliefs is not relevant in the context presented here, since the equilibrium decision of the CB depends on the decision taken about stabilization not on the types of the interest groups.

The CB takes the decision of the groups about stabilization as given and maximizes (10) over $\pi(\cdot)$. This yields

$$\pi^*\left(\cdot\right) = \beta \overline{d} \tag{11}$$

with $\beta := \frac{1-\lambda}{2\lambda}$. Note that β is the inverse of the relative independence of the CB, i.e. the more weight the CB attaches to price stability, the smaller is β . In equilibrium, the inflation rate is proportional to the average debt level in the union and inflation is decreasing in the independence parameter λ (increasing in β). If the CB puts no weight on the budgetary positions of the member states ($\lambda = 1$), equilibrium inflation will be zero. Thus, if the CB is entirely independent, there will be no negative spillovers due to the common monetary policy.

Let us now turn to the individual decision of the groups about stabilization. Let $p_i^c(\theta_i^c)$ denote the probability that group i in country c (with private information θ_i^c) wants to stabilize. It turns out, that there exists a critical value of private information such that the groups will stabilize if and only if their private information lies below this critical value. In equilibrium, this critical value is defined implicitly.

Proposition 7 Assume $-\frac{\alpha}{d} + \frac{\beta}{2} \geq \underline{\theta}$. Under the war of attrition exists a unique symmetric Bayesian Nash Equilibrium. The equilibrium strategy of group i in country c is given by

$$p_{i}^{c}\left(\theta_{i}^{c}\right) = \begin{cases} 0, & \text{if } \theta_{i}^{c} > T^{WA} \\ 1, & \text{if } \theta_{i}^{c} \leq T^{WA} \end{cases}$$

where

$$T^{WA} := \frac{-\alpha}{(1 - F(T^{WA})) d} + \frac{\beta}{2}.$$

Proof of Proposition 7:

(i) Derivation of $p_i^c(\theta_i^c)$:

In equilibrium the group's utility depends not only on the decision whether to stabilize in its own country, but also on the decision taken in the other country. This is due to the spillovers of debt policy that emerge in a monetary union where the central bank is subject to political pressures. The interest groups anticipate equilibrium behavior of the CB in the second stage of the game, i.e. utility now depends on $\pi^*(\cdot)$. Let $v^c := (v_i^c, v_j^c)$ and equilibrium utility of group i in country c be denoted by $u_i^c(v^c, v^{-c})$.

Group i in country c wants to stabilize whenever expected utility when stabilizing is larger than expected utility when not stabilizing, i.e. when the expected net gain of group i in country c from stabilization is positive:

$$E[u_i^c \left(v^c, v^{-c}\right) | v_i^c = s] - E[u_i^c \left(v^c, v^{-c}\right) | v_i^c = d] \ge 0$$

$$\Leftrightarrow$$

$$-\alpha + \frac{\beta}{2} \left(1 - p_j^c \left(\theta_j^c \right) \right) d - \left(1 - p_j^c \left(\theta_j^c \right) \right) \theta_i^c d \ge 0. \tag{12}$$

Thus, group i wants to stabilize whenever θ_i^c is below some critical value T^{WA} defined by

$$T^{WA} := \frac{-\alpha}{\left(1 - p_i^c(\theta_i^c)\right)d} + \frac{\beta}{2}.$$

In any symmetric equilibrium group j faces an identical decision problem. Thus, it also wants stabilization whenever $\theta_j^c \leq T^{WA}$. Then, it must be the case that $p_j^c\left(\theta_j^c\right) \equiv \Pr(\theta_j^c \leq T^{WA}) = F(T^{WA})$, where $F\left(\cdot\right)$ denotes the cumulative distribution function of θ_j^c . Using this, the equilibrium value of T^{WA} is implicitly defined by

$$T^{WA} = \frac{-\alpha}{(1 - F(T^{WA}))d} + \frac{\beta}{2}.$$

(ii) Existence:

Since T^{WA} is strictly increasing and $\frac{-\alpha}{(1-F(T^{WA}))d} + \frac{\beta}{2}$ is strictly decreasing in T^{WA} a unique solution exists if and only if

$$\begin{array}{ccc} \underline{\theta} & \leq & \frac{-\alpha}{(1 - F(\underline{\theta}))d} + \frac{\beta}{2} \\ & \Leftrightarrow & \\ \underline{\theta} & \leq & -\frac{\alpha}{d} + \frac{\beta}{2}. \end{array}$$

The following picture shows the equilibrium probability of stabilization $p_i^c(\cdot)$ as a function of private information θ_i^c . For $\theta_i^c \leq T^{WA}$ this probability is equal to one, for $\theta_i^c > T^{WA}$ it is zero. The intuition underlying this result is that it will be individually rational to stabilize if the private costs of delayed stabilization are high.

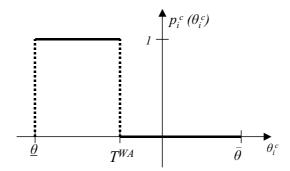


Figure 5: Equilibrium Strategy under War of Attrition

Some remarks concerning this equilibrium seem to be appropriate at this point: First note, that equilibrium strategies are independent of the stabilization decision in the other country. This is due to the fact that utility is additively separable with respect to this decision.

Secondly and like in any war of attrition, asymmetric equilibria in pure strategies may exist. Consider a situation in which group i does not concede with certainty. Then, it is a best reply for group j to concede if the private costs of delay exceed the costs of giving in and vice versa.

Furthermore, there exists a straightforward interpretation of the assumption that guarantees existence of the cut-off equilibrium presented in Proposition 7. Suppose, one group knows with certainty that the other group in the respective country will not concede. A symmetric (responsive) equilibrium exists if and only if it is individually profitable for the lowest possible type to stabilize, i.e. if the costs of conceding are below the costs of delay: $\alpha \leq -\underline{\theta}d + \frac{\beta}{2}d$. Responsive in this context means that the equilibrium strategies should react to changes in type realizations. ¹⁵ If the respective assumption is not satisfied, then equilibrium strategies will

¹⁵Note, that no *full* responsive equilibrium exists in the spirit that equilibrium stabilization probabilities are different for all different types, i.e. $p_i^c(\theta_i^c) \neq p_i^c\left(\widehat{\theta}_i^c\right) \forall \theta_i^c \neq \widehat{\theta}_i^c$.

be irresponsive, i.e. in equilibrium $p_i^c(\theta_i^c) = 0$ for all $\theta_i^c \in [\underline{\theta}, \overline{\theta}]$. Thus, in this case there exists a unique equilibrium in pure strategies in which both groups do not concede.

Let us now turn to the question how the other model parameters influence the stabilization decision. Group i wants to stabilize whenever equation (12) holds. Thus, it will be more advantageous to stabilize if the costs of giving in are small (α small, low polarization of society). The more weight the CB attaches to budgetary positions (high β), the more advantageous is it to stabilize, since stabilization decreases the inflation rate the CB chooses in equilibrium. Whenever there exist private costs associated with debt, stabilization will be more advantageous if the other group's probability of stabilization is low (p_j^c small) and if the fiscal problem is large (large d).¹⁶ These observations are formalized in the following lemma.

Lemma 9 T^{WA} is strictly increasing in β , d and strictly decreasing in α .

3.2.2 SGP without renegotiations

In this section, I first give an overview of the provisions of the SGP in the $European\ Union$. After that, the war of attrition model of the previous section is extended to include some of these features and solved for the respective equilibrium.

Features of the European Stability and Growth Pact

The European Stability and Growth Pact (adopted in 1997) clarifies the provisions of the Excessive Deficit Procedure (Art. 104). It calls for fiscal positions to be balanced or in surplus and urges surveillance of medium-term fiscal positions with the goal of providing an early warning signal if the 3% reference value for budget deficits is at risk. It also clarifies the conditions under which participants will be allowed to exceed the deficit

¹⁶These comparative static results are in accordance with the results obtained by Alesina and Drazen [1991] and by Persson and Tabellini [2000] as far as they are transferable.

ceiling without being determined to have an excessive deficit. This happens under exceptional circumstances or in the case of a severe economic downturn (defined as a fall in real GDP by at least 2% on a yearly basis).

Countries which are found to have an excessive deficit are forced to make a non-interest bearing deposit that will be transformed into a fine if the fiscal excess is not eliminated within two years. The minimum deposit or potential fine is 0.2% of GDP. It increases with each percentage point that the actual deficit exceeds the reference value until a maximum of 0.5% of GDP is reached.

Sanctions are not applied automatically but require a qualified majority in the Council of Economics and Finance Ministers. ¹⁷ Before sanctions are actually imposed, a long and detailed assessment of the country's situation is made. This procedures involves regular monitoring and reporting by the European Commission and recommendations and decisions by the Council of Economics and Finance Ministers. For simplicity, I abstract here from those definitional and procedural details and consider a fine proportional to the size of the fiscal problem. Furthermore, this fine has to be paid whenever a deficit arises.

The European SGP also foresees redistribution of fines among participating countries: countries which do not experience an excessive deficit get an equal share of the fines paid by countries where an excessive deficit has been assessed. This construction should induce countries which fulfill the SGP to vote in favor of determination of an excessive deficit in other countries. Unfortunately, in reality this pecuniary incentive seems to be too weak in order to achieve any punishment of member countries.¹⁸

 $^{^{17}}$ For an experimental study of voting over these sanctions see Irlenbusch et al. [2003]. They conclude that the institutional rules of the SGP do not perform well. Excluding the fiscal sinners from the procedure improves the outcome considerably.

¹⁸However, in reality (unlike in the model I present here) the punishment of excessive deficits is a repeated game. The repetition may lead to a "no punishment" equilibrium. Focussing on optimal fiscal strategies in a multi-period game, Vranceanu and Warin [2001] show that no excess deficits may prevail even without any sanctions.

Model set-up including a fine for deficits

In this section, I add a third stage to the game. Before the war of attrition about stabilization is played, the governments of the two countries decide to punish deficits union wide with a fine ϕ proportional to the size of the deficit. I assume $\phi \geq 0$. It would be possible to allow for negative values of ϕ as well (i.e. rewards for deficits), but this does not seem to be a realistic assumption. In contrast to reality, I allow for fines that exceed the initial size of the fiscal problem, i.e. $\phi > 1$. The fine has to be paid whenever stabilization is not achieved, i.e. whenever both groups in one country are against stabilization. Each group pays half of the fine. In the other three possible cases no fine has to be paid, since stabilization occurs immediately. Moreover, the country in question gets a redistribution of the fine paid by the other union member whenever stabilization could not be achieved abroad. The redistributed amount is divided equally between the two interest groups. Depending on being for or against stabilization, utility of group i in country c is now given by

$$u_{i}^{c}\left(v^{c},v^{-c}\right) := \begin{cases} \frac{\phi}{2}q^{-c}\left(\theta^{-c}\right)d - \pi\left(\cdot\right), & \text{if } v_{i}^{c} = s, v_{j}^{c} = s \\ \frac{\phi}{2}q^{-c}\left(\theta^{-c}\right)d - \alpha - \pi\left(\cdot\right), & \text{if } v_{i}^{c} = s, v_{j}^{c} = n \\ \frac{\phi}{2}q^{-c}\left(\theta^{-c}\right)d + \alpha - \pi\left(\cdot\right), & \text{if } v_{i}^{c} = n, v_{j}^{c} = s \\ -\frac{\phi}{2}d + \theta_{i}^{c}d - \pi\left(\cdot\right), & \text{if } v_{i}^{c} = n, v_{j}^{c} = n \end{cases}$$

where $\theta^{-c} := (\theta_i^{-c}, \theta_j^{-c})$ and $q^{-c}(\theta^{-c})$ denotes the probability of stabilization delay in country -c, i.e. $q^{-c}(\theta^{-c}) := (1 - p_i^{-c}(\theta_i^{-c})) (1 - p_j^{-c}(\theta_j^{-c}))$.

Here, the timing is as follows: First, the governments of country A and country B decide about the height of the fine ϕ . After this decision, the interest groups fight a war of attrition about stabilization. Finally, the CB selects the inflation rate. Again, the game is solved by backward induction.

Since the sole influence of the introduction of a fine is on the groups' decision about stabilization and this is taken as given by the CB, its intro-

¹⁹This may be seen as a pure theoretical possibility, but leads to interesting effects concerning welfare due to the fact that with unrestricted (positive) fines it is possible to reach all equilibrium cut-off values above T^{WA} .

duction has no influence on the decision problem of the *CB*. Thus, $\pi^*(\cdot)$ is still given by equation (11).

Again, there exists a critical value of private information such that the groups will want to stabilize if and only if their private information lies below this critical value. The interpretation of the assumption guaranteeing existence is similar to that given in Section 3.2.1. However, since the assumption is not as strict as before, existence of the symmetric responsive equilibrium becomes more likely.

Proposition 8 Assume $-\frac{\alpha}{d} + \frac{\beta}{2} + \phi \ge \underline{\theta}$. Under the SGP without renegotiations exists a unique symmetric Bayesian Nash Equilibrium. The equilibrium strategy of group i in country c is given by

$$p_i^c(\theta_i^c) = \begin{cases} 0, & \text{if } \theta_i^c > T^P \\ 1, & \text{if } \theta_i^c \le T^P \end{cases}$$

where

$$T^{P} := \frac{-\alpha}{(1 - F(T^{P})) d} + \frac{\beta}{2} + \frac{\phi}{2} \left(1 + \left(1 - F(T^{P}) \right)^{2} \right).$$

Proof of Proposition 8:

(i) Derivation of $p_i^c(\theta_i^c)$:

Following the same argumentation as before, group i in country c wants to stabilize whenever the expected net gain from stabilization is positive:

$$E[u_{i}^{c}\left(v_{i}^{c}, v_{j}^{c}\right) | v_{i}^{c} = s] - E[u_{i}^{c}\left(v_{i}^{c}, v_{j}^{c}\right) | v_{i}^{c} = d]$$

$$= -\alpha + \frac{\beta}{2} \left(1 - p_{j}^{c}\left(\theta_{j}^{c}\right)\right) d - \left(1 - p_{j}^{c}\left(\theta_{j}^{c}\right)\right) \theta_{i}^{c} d$$

$$+ \frac{\phi}{2} d\left(1 - p_{j}^{c}\left(\theta_{j}^{c}\right)\right) \left[1 + q^{-c}\left(\theta^{-c}\right)\right]. \tag{13}$$

Thus, group i in country c wants to stabilize whenever θ_i^A is below some critical value T^P defined by

$$T^{P} := \frac{-\alpha}{\left(1 - p_{i}^{c}\left(\theta_{i}^{c}\right)\right) d} + \frac{\beta}{2} + \frac{\phi}{2}\left(1 + q^{-c}\left(\theta^{-c}\right)\right).$$

In any symmetric equilibrium group j in country c and both groups in country -c face an identical decision problem. Thus, they also want stabilization whenever their private information parameter lies below this value, i.e. whenever $\theta_j^c \leq T^P$, $\theta_i^{-c} \leq T^P$ and $\theta_j^{-c} \leq T^P$. Thus, it must be the case that $p_j^c \left(\theta_j^c\right) \equiv \Pr(\theta_j^c \leq T^P) = F(T^P)$ and $p_i^{-c} \left(\theta_i^{-c}\right) \equiv \Pr(\theta_i^{-c} \leq T^P) = F(T^P)$, respectively. Using this, the equilibrium value of T^P is implicitly defined by

$$T^{P} = \frac{-\alpha}{(1 - F(T^{P})) d} + \frac{\beta}{2} + \frac{\phi}{2} \left(1 + \left(1 - F(T^{P}) \right)^{2} \right).$$

(ii) Existence:

Since T^P is strictly increasing and $\frac{-\alpha}{(1-F(T^P))d} + \frac{\beta}{2} + \frac{\phi}{2} \left(1 + \left(1 - F(T^P)\right)^2\right)$ is strictly decreasing in T^P a unique solution exists if and only if

$$\frac{\theta}{\theta} \leq \frac{-\alpha}{(1 - F(\underline{\theta})) d} + \frac{\beta}{2} + \frac{\phi}{2} \left(1 + (1 - F(\underline{\theta}))^2 \right)
\Leftrightarrow
\underline{\theta} \leq -\frac{\alpha}{d} + \frac{\beta}{2} + \phi.$$

Again, group i wants to stabilize whenever equation (13) is positive. The first three terms are the same as in equation (12) and thus the influence of α and β is identical. Whenever there exist private costs associated with debt, stabilization will be more advantageous if the other group's probability of stabilization is low $(p_j^c \text{ small})$ and if the fiscal problem is large (d high). Moreover, stabilization is more advantageous the higher the fine ϕ and the smaller the other country's probability of stabilization $(p_i^{-c} \text{ and } p_j^{-c} \text{ small})$. Thus, the introduction of a fine for deficits renders stabilization individually more profitable.

Lemma 10 T^P is strictly increasing in ϕ , β , d and strictly decreasing in α .

The higher is the fine, the wider is the range of private information for which stabilization is achieved. For ϕ going to infinity, T^P converges to $\overline{\theta}$.

However, since there may exist private benefits of delayed stabilization, it might happen that a fine that is too high leads to stabilization when this is not desirable. Therefore, the effect *more* stabilization has on welfare is not unambiguously positive.

Unfortunately, at this stage it is not possible to solve the game completely, i.e. to determine the equilibrium decision of the governments about the height of the fine. This is due to the implicit definition of the cutoff value for private information that determines the equilibrium decision about stabilization. I return to this issue below in order to examine the welfare effects of the SGP.

3.2.3 SGP with renegotiations

In the set-up described in the previous section, the fine ϕ will be collected under any circumstances. However, think of a situation in which in both countries the groups cannot reach an agreement about stabilization. Then, in both countries stabilization is delayed and both have to pay the fine, but there is no country to profit from redistribution. Thus, the money will be burned – literally speaking. Therefore, in this section I present a set-up that seems to be more realistic (taking into account the never-ending discussion about the design and even about the overall usefulness of the European SGP): when stabilization cannot be achieved in both participating countries, then no country has to pay any fine. I call this set-up SGP with renegotiations.

The only difference to the previous section is utility when all groups are against stabilization, i.e. $v_i^c = v_j^c = n$, c = A, B. For group i in country c utility is now given by

$$u_{i}^{c}(n,n):=-rac{\phi}{2}\left[1-q^{-c}\left(heta^{-c}
ight)\right]d+ heta_{i}^{c}d-\pi\left(\cdot
ight).$$

Country c will not have to pay the fine, if in country -c stabilization cannot be achieved either $(q^{-c}(\theta^{-c}) = 1)$.

As in the two set-ups considered before, there exists a cut-off value of private information such that the groups want to stabilize if and only if

their private information lies below this critical value. The assumption guaranteeing existence has the same interpretation as before. Existence of the equilibrium is more likely than under the war of attrition, but not as likely as without the possibility of renegotiations.

Proposition 9 Assume $-\frac{\alpha}{d} + \frac{\beta + \phi}{2} \ge \underline{\theta}$. Under the SGP with renegotiations exists a unique symmetric Bayesian Nash Equilibrium. The equilibrium strategy of group i in country c is given by

$$p_i^c(\theta_i^c) = \begin{cases} 0, & \text{if } \theta_i^c > T^{SGP} \\ 1, & \text{if } \theta_i^c \le T^{SGP} \end{cases}$$

where

$$T^{SGP} := \frac{-\alpha}{(1 - F(T^{SGP})) d} + \frac{\beta + \phi}{2}.$$

Proof of Proposition 9:

(i) Derivation of $p_i^c(\theta_i^c)$:

The expected net gain of group i in c from stabilization is given by

$$E[u_i^c(v_i^c, v_j^c) | v_i^c = s] - E[u_i^c(v_i^c, v_j^c) | v_i^c = d]$$

$$= -\alpha + \frac{\beta}{2} (1 - p_j^c(\theta_j^c)) d - (1 - p_j^c(\theta_j^c)) \theta_i^c d + \frac{\phi}{2} (1 - p_j^c(\theta_j^c)) d. (14)$$

Now, group i in country c wants to stabilize whenever θ_i^c is below some critical value T^{SGP} defined by

$$T^{SGP} := \frac{-\alpha}{\left(1 - p_j^c\left(\theta_j^c\right)\right)d} + \frac{\beta + \phi}{2}.$$

Using the same argumentation as above, the equilibrium value of T^{SGP} is implicitly defined by

$$T^{SGP} = \frac{-\alpha}{(1 - F(T^{SGP})) d} + \frac{\beta + \phi}{2}.$$

(ii) Existence:

Since T^{SGP} is strictly increasing and $\frac{-\alpha}{(1-F(T^{SGP}))d} + \frac{\beta+\phi}{2}$ is strictly decreasing in T^{SGP} a unique solution exists if and only if

$$\begin{array}{rcl} \underline{\theta} & \leq & \frac{-\alpha}{(1-F(\underline{\theta}))\,d} + \frac{\beta+\phi}{2} \\ & \Leftrightarrow & \\ \underline{\theta} & \leq & -\frac{\alpha}{d} + \frac{\beta+\phi}{2}. \end{array}$$

Here, group i wants to stabilize whenever equation (14) is positive. The influence of all parameters is the same as in Section 3.2.2, but the influence of the fine ϕ is not as strong as before. This is due to the possibility of renegotiations.

Lemma 11 T^{SGP} is strictly increasing in ϕ , β , d, strictly decreasing in α .

Again, the higher the fine, the wider is the range of private information for which stabilization is achieved. The effects on welfare are as ambiguous as before. And as aforementioned, it is not possible to solve this game completely due to the implicit definition of the equilibrium cut-off value of private information.

3.2.4 Comparison

In this section, I compare the three different (implicitly defined) cut-off values T^{WA} , T^P and T^{SGP} with respect to the resulting probability of delayed stabilization. Assuming $-\frac{\alpha}{d} + \frac{\beta}{2} \geq \underline{\theta}$ all exist and are unique as shown above. It turns out that for positive fines the SGP without renegotiations leads to the best result concerning the probability of stabilization, but even if renegotiations are allowed stabilization will be achieved for a wider range of information parameters than under the war of attrition.

Lemma 12 For positive fines, stabilization is more likely under the SGP without renegotiations than under the SGP with renegotiations, i.e.

$$\forall \phi > 0: T^P > T^{SGP}$$
.

Thus, the first result is that the possibility of renegotiations induces the groups to delay stabilization more often. But the next result shows that renegotiations do not entirely destroy the positive effect the introduction of a (positive) fine has on individual behavior in equilibrium.

Corollary 4 For positive fines, stabilization is more likely under the SGP with renegotiations than under the war of attrition, i.e.

$$\forall \phi \geq 0 : T^{SGP} \geq T^{WA}.$$

As an immediate consequence we get

Corollary 5 For positive fines, stabilization is more likely under the SGP without renegotiations than under the war of attrition, i.e.

$$\forall \phi \ge 0: T^P \ge T^{WA}.$$

Thus, any form of SGP leads to stabilization occurring in more instances than under the war of attrition. Therefore, one may interpret the SGP as a commitment device for stabilization. The fine will induce the interest groups to stabilize more often, even if renegotiations are possible.²⁰

As aforementioned, the effect more likely stabilization has on welfare is not unambiguous, since there may exist private benefits of delayed stabilization. If the fine induces the interest groups to stabilize too often under such circumstances, then the SGP will have a negative effect on the welfare of participating countries.

In order to illustrate this point consider Figure 6. It shows the decisions that are implemented in equilibrium under the war of attrition and the SGP with renegotiations depending on private information. Stabilization will be achieved in a country, if one of the interest groups in this country has private information below T^{WA} (below T^{SGP} respectively). But due to the possibility of private benefits of delay, it is not always efficient to

 $^{^{20}}$ If we allow for negative fines, the order of the cut-off values will be reversed, i.e. $\forall \phi \leq 0 : T^{WA} \geq T^{SGP} \geq T^P$. Thus, in the case of rewards for deficits the war of attrition induces the interest groups to stabilize for the widest range of information parameters and any reward worsens the outcome.

stabilize. Thus, there exists a trade-off concerning more likely stabilization: with positive fines, the SGP induces the interest groups to stabilize in more instances. This is efficient as long as the sum of private information is below the sum of negative spillovers due to the common monetary policy (light grey area). On the other hand, the SGP also leads to more stabilization when this is not efficient, i.e. when the sum of private benefits of delay exceeds the negative spillovers (dark grey area).

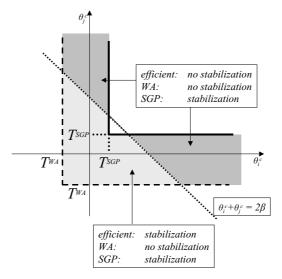


Figure 6: Equilibrium Stabilization Decisions

3.3 Solution of the game

In order to obtain explicit results for the cut-off values, I from now on restrict attention to private information uniformly distributed over $\left[\underline{\theta}, \overline{\theta}\right]$. First, I calculate the respective values. Furthermore, it is now possible to solve the game, i.e. to determine the height of the fine chosen by governments in the first stage. Implementing this fine unambiguously increases welfare. Finally, I analyze whether voluntary participation can be guaranteed and if the pact leads to an efficient outcome. Unfortunately, ex post voluntary participation and ex post efficiency both cannot be achieved.

²¹Unfortunately, for the SGP without renegotiations it is still not possible to obtain an explicit result. Therefore, I concentrate on the war of attrition and the SGP with renegotiations (on T^{WA} and T^{SGP}). This restriction seems to be reasonable considering the more realistic assumptions underlying the derivation of T^{SGP} as compared to those underlying T^P .

3.3.1 Equilibrium cut-off values

With a uniform distribution of private information I obtain as the equilibrium cut-off value under the war of attrition²²

$$T_{WA}^* = \frac{\frac{\beta}{2} + \overline{\theta}}{2} - \sqrt{\left(\frac{\frac{\beta}{2} - \overline{\theta}}{2}\right)^2 + \frac{\alpha}{d}\left(\overline{\theta} - \underline{\theta}\right)}.$$

Under the SGP with renegotiations the equilibrium value is given by

$$T_{SGP}^* = \frac{\frac{\beta + \phi}{2} + \overline{\theta}}{2} - \sqrt{\left(\frac{\frac{\beta + \phi}{2} - \overline{\theta}}{2}\right)^2 + \frac{\alpha}{d}\left(\overline{\theta} - \underline{\theta}\right)}.$$

The comparative static results obtained in Sections 3.2.1 and 3.2.3 and the comparison of the two cut-off values obtained in Section 3.2.4 are still valid. Thus, for positive fines it holds that $T_{SGP}^* \geq T_{WA}^*$. In addition, it is possible to obtain the following comparative static results: The smaller possible costs or the larger possible benefits of delayed stabilization, the more likely stabilization becomes. This holds for the war of attrition as well as for the SGP.

Lemma 13 T_{WA}^* and T_{SGP}^* are both increasing in $\overline{\theta}$ and in $\underline{\theta}$.

3.3.2 Optimal fine

By setting a union wide fine in the first stage of the game, the governments of the two member countries of the monetary union want to maximize expected welfare in the union – anticipating equilibrium behavior of the interest groups and the CB in the following stages of the game.

Let $x_{SGP}^c(\cdot)$ denote the equilibrium decision about stabilization under the SGP in country c, i.e.

$$x_{SGP}^{c}(\theta^{c}) := \begin{cases} stabilize \ not \ (n), & if \ \theta_{i}^{c} > T_{SGP}^{*} \land \theta_{j}^{c} > T_{SGP}^{*} \\ stabilize \ (s), & otherwise \end{cases}$$

²²This value will be unique if we assume $-\frac{\alpha}{d} + \frac{\beta}{2} \ge \underline{\theta}$ and take account of the fact that it should lie inside the support. The same applies to the cut-off value under the SGP.

Further, let $w_i^c\left(x_{SGP}^c(\theta^c), x_{SGP}^{-c}(\theta^{-c}), \theta_i^c\right)$ denote utility of group i in country c in equilibrium. For $x_{SGP}^{-c}(\theta^{-c}) = s$ this is given by

$$\begin{split} w_{i}^{c}\left(x_{SGP}^{c}(\theta^{c}), s, \theta_{i}^{c}\right) := \\ \begin{cases} & -\pi^{*}\left(\cdot\right), \; \text{if } x_{SGP}^{c}(\theta^{c}) = s, \; \theta_{i}^{c} \leq T_{SGP}^{*} \wedge \theta_{j}^{c} \leq T_{SGP}^{*} \\ & -\alpha - \pi^{*}\left(\cdot\right), \; \text{if } x_{SGP}^{c}(\theta^{c}) = s, \; \theta_{i}^{c} \leq T_{SGP}^{*} \wedge \theta_{j}^{c} > T_{SGP}^{*} \\ & \alpha - \pi^{*}\left(\cdot\right), \; \text{if } x_{SGP}^{c}(\theta^{c}) = s, \; \theta_{i}^{c} > T_{SGP}^{*} \wedge \theta_{j}^{c} \leq T_{SGP}^{*} \\ & -\frac{\phi}{2}d + \theta_{i}^{c}d - \pi^{*}\left(\cdot\right), \; \text{if } x_{SGP}^{c}(\theta^{c}) = d, \; \theta_{i}^{c} > T_{SGP}^{*} \wedge \theta_{j}^{c} > T_{SGP}^{*} \end{split}$$

and for $x_{SGP}^{-c}(\theta^{-c}) = n$ by

$$\begin{split} w_{i}^{c}\left(x_{SGP}^{c}(\theta^{c}), n, \theta_{i}^{c}\right) := \\ \begin{cases} \frac{\phi}{2}d - \pi^{*}\left(\cdot\right), & \text{if } x_{SGP}^{c}(\theta^{c}) = s, \; \theta_{i}^{c} \leq T_{SGP}^{*} \wedge \theta_{j}^{c} \leq T_{SGP}^{*} \\ \frac{\phi}{2}d - \alpha - \pi^{*}\left(\cdot\right), & \text{if } x_{SGP}^{c}(\theta^{c}) = s, \; \theta_{i}^{c} \leq T_{SGP}^{*} \wedge \theta_{j}^{c} > T_{SGP}^{*} \\ \frac{\phi}{2}d + \alpha - \pi^{*}\left(\cdot\right), & \text{if } x_{SGP}^{c}(\theta^{c}) = s, \; \theta_{i}^{c} > T_{SGP}^{*} \wedge \theta_{j}^{c} \leq T_{SGP}^{*} \\ \theta_{i}^{c}d - \pi^{*}\left(\cdot\right), & \text{if } x_{SGP}^{c}(\theta^{c}) = d, \; \theta_{i}^{c} > T_{SGP}^{*} \wedge \theta_{j}^{c} > T_{SGP}^{*} \end{split}$$

As a welfare measure on union level I consider the sum over the utilities of the involved interest groups:

$$EW_{SGP}(\theta) := E_{\theta} \left[\sum_{\substack{i=1,2\\c=A,B}} w_i^c \left(x_{SGP}^c(\theta^c), x_{SGP}^{-c}(\theta^{-c}), \theta_i^c \right) \right]. \tag{15}$$

Lemma 14 There exists a unique fine ϕ^* that maximizes expected welfare. This fine is given by

$$\phi^* = \frac{1}{3} \left(5\beta - 2\overline{\theta} \right) + \frac{3}{2} \frac{\alpha}{d} \frac{\overline{\theta} - \underline{\theta}}{\overline{\theta} - \beta}.$$

Thus, in equilibrium under the SGP the governments implement ϕ^* and stabilization is achieved in a country when one of the respective interest groups has private costs below $T^*_{SGP}(\phi^*) = -\frac{1}{3}\overline{\theta} + \frac{4}{3}\beta$.

The following comparative static results with respect to the welfare maximizing fine can be obtained. The higher the deficit, the higher possible

private benefits of delay or the smaller possible private costs, the smaller is the optimal fine. This is due to the fact that increases in d, $\overline{\theta}$ and $\underline{\theta}$ induce the interest groups to stabilize for a wider range of private information, i.e. lead to an increase in T^*_{SGP} . Therefore, to obtain the same level of expected welfare only a smaller fine is necessary. The higher polarization of society is, the larger is the optimal fine: an increase in α leads to a decrease in T^*_{SGP} and thus a higher fine is needed to offset this effect.

The influence of changes in β is more complex: the more weight the CB attaches to the budgetary positions of the member states, the wider is the range for which stabilization occurs, i.e. T_{SGP}^* is increasing in β . Following the same logic as before, the optimal fine could decrease in β . But ϕ^* is increasing in β . This is due to the fact that β also represents the externality an interest group imposes onto the others. The effect an increase in β has on the equilibrium cut-off value goes in the right direction (more stabilization), but is not enough to internalize the externality created. Therefore, stronger incentives (higher fines) are required to induce the right behavior. This is formalized in the following lemma.

Lemma 15 The welfare maximizing fine ϕ^* is strictly decreasing in d, $\overline{\theta}$ and $\underline{\theta}$ and strictly increasing in α and β .

3.3.3 Welfare

One important question concerning such arrangements as the European SGP is whether they improve the situation of the participating countries. Therefore, in this section I compare expected welfare under the basic war of attrition to a situation in which the SGP is in place.

For $\phi = 0$, the respective equilibrium cut-off values are identical under the war of attrition and under the SGP. Hence, expected welfare is the same under both arrangements. Thus, it is always possible to replicate the outcome of the war of attrition by an SGP with zero fine. Since ϕ^* is chosen as to maximize expected welfare, introducing a $\phi^* \neq 0$ can only enhance the outcome under the SGP. Intuitively, the reason is that under the SGPit is individually rational for the interest groups to achieve stabilization in more instances, i.e. for a wider range of information parameters. And this is good for society as a whole, since in equilibrium this positive effect of *more* stabilization exceeds its negative effect of inducing the interest groups to stabilize too often.

Corollary 6 The SGP yields at least as much expected welfare as the war of attrition when ϕ^* is implemented.

Imposing a union wide fine for deficits need not be welfare improving. In general, the effects depend on the parameter constellation. For example, if possible private benefits of delay are small compared to the negative spillovers created by the common monetary policy (i.e. $\overline{\theta} \leq \frac{3}{2}\beta$), then the SGP will always increase welfare. For other constellations there may exist fines such that the interest groups are induced to stabilize too often.²³

Lemma 16 Let
$$\widehat{\frac{\alpha}{d}} := \frac{(\overline{\theta} - \beta)(2\overline{\theta} - 3\beta)}{\overline{\theta} - \theta}$$
 and $\widetilde{\frac{\alpha}{d}} := \frac{2}{9} \frac{(\overline{\theta} - \beta)(2\overline{\theta} - 5\beta)}{\overline{\theta} - \theta}$.

(i) Consider $\overline{\theta} \leq \frac{3}{2}\beta$.

For positive fines, the SGP yields more expected welfare than the war of attrition.

- (ii) Consider $\frac{3}{2}\beta < \overline{\theta} \leq \frac{5}{2}\beta$.
- a) For $\frac{\alpha}{d} \geq \frac{\widehat{\alpha}}{d}$ and positive fines the SGP yields more expected welfare than the war of attrition.
- b) For $\frac{\alpha}{d} < \frac{\widehat{\alpha}}{d}$ there exists $\overline{\phi}$ larger than ϕ^* such that the war of attrition yields more expected welfare than the SGP for all ϕ larger than $\overline{\phi}$.
- (iii) Consider $\frac{5}{2}\beta < \overline{\theta}$.
- a) For $\frac{\alpha}{d} \geq \frac{\widehat{\alpha}}{d}$ and positive fines the SGP yields more expected welfare than the war of attrition.
- b) For $\frac{\widetilde{\alpha}}{d} \leq \frac{\alpha}{d} < \frac{\widehat{\alpha}}{d}$ there exists $\overline{\phi}$ larger than ϕ^* such that the war of attrition yields more expected welfare than the SGP for all ϕ larger than $\overline{\phi}$.
- c) For $\frac{\alpha}{d} < \frac{\widetilde{\alpha}}{d}$ and positive fines the war of attrition yields more expected welfare than the SGP.

 $^{^{23}}$ In Lemma 16 attention is restricted to positive fines. Similar results may be obtained considering rewards for deficits.

These results underline the importance of a well designed SGP with respect to the height of the fine. A pact will only lead to increases in expected welfare as compared to the war of attrition if the fine is chosen adequately.

3.3.4 Voluntary participation

In any democratic environment it is important to take into account whether the affected parts of the population would like to participate. The following analysis presumes that the governments of the respective countries (who actually signed the SGP and thus imposed the fine) will only be able to make that commitment and to enforce it if all interest groups in their country are in favor of it.²⁴ In order to analyze if voluntary participation of the interest groups can be achieved, their expected utility has to be compared with a situation in which there is no SGP, i.e. a situation in which the basic war of attrition is played. This comparison is relevant ex ante, i.e. at a point in time when neither group knows their private costs or benefits of delayed stabilization, and interim, i.e. when both groups know their own type, but not that of the other groups. Ex post the relevant benchmark is utility when withdrawing from the pact after stabilization decisions have been taken.

Ex ante

It will be individually rational for each interest group to participate in the SGP before learning the own type if expected utility under the SGP exceeds that under the war of attrition, i.e. if for group i in country c

$$E_{\theta}\left[w_{i}^{c}\left(x_{SGP}^{c}\left(\theta^{c}\right), x_{SGP}^{-c}\left(\theta^{-c}\right), \theta_{i}^{c}\right)\right] \geq E_{\theta}\left[w_{i}^{c}\left(x_{WA}^{c}\left(\theta^{c}\right), x_{WA}^{-c}\left(\theta^{-c}\right), \theta_{i}^{c}\right)\right].$$

²⁴Note the difference to usual participation constraints: these solely refer to those agents holding private information. In my setting, not the privately informed interest groups decide whether to participate but their governments.

Ex ante voluntary participation can be achieved in equilibrium, i.e. when ϕ^* is implemented. Since the whole setting is symmetric, this follows immediately from Corollary 6.

Corollary 7 The ex ante participation constraint of all interest groups is fulfilled under the SGP with renegotiations when ϕ^* is implemented.

Interim

Now, each group knows its own type, but stabilization decisions have not been taken yet. Then, it is interim individually rational for group i in country c to participate if for all θ_i^c

$$E_{\theta_{-i}}\left[w_{i}^{c}\left(x_{SGP}^{c}\left(\theta^{c}\right), x_{SGP}^{-c}\left(\theta^{-c}\right), \theta_{i}^{c}\right) \mid \theta_{i}^{c}\right] \\ \geq E_{\theta_{-i}}\left[w_{i}^{c}\left(x_{WA}^{c}\left(\theta^{c}\right), x_{WA}^{-c}\left(\theta^{-c}\right), \theta_{i}^{c}\right) \mid \theta_{i}^{c}\right].$$

In order to analyze these constraints, split the support into three parts: consider $\theta_i^c \leq T_{WA}^*$, $T_{WA}^* < \theta_i^c \leq T_{SGP}^*$ and $\theta_i^c > T_{SGP}^*$, since these types have the same expectations about which stabilization decision will be taken.

The interim participation constraints for types $\theta_i^c \leq T_{WA}^*$ are always fulfilled, since the existence of one such group leads to stabilization taking place in country c. Thus, these groups never have to pay any fine, but will profit from redistribution when stabilization is delayed in the other country.

Whether the interim participation constraints for higher types can be fulfilled depends on the overall constellation of parameters. In general, interim voluntary participation is (ceteris paribus) easier to assure the smaller are the fine and the deficit, the smaller are spillovers from debt policy (the more weight the *CB* attaches to price stability) and the higher is the polarization of society.

In order to illustrate this further, consider first the interim participation constraint of types $T_{WA}^* < \theta_i^c \leq T_{SGP}^*$. This is given by

$$\frac{1}{\left(\overline{\theta} - \underline{\theta}\right)^{3}} \left[\int_{T_{SGP}^{*}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} (-\alpha) d\theta_{i}^{-c} d\theta_{j}^{-c} d\theta_{j}^{c} + \int_{\underline{\theta}}^{\overline{\theta}} \int_{T_{SGP}^{*}}^{\overline{\theta}} \int_{T_{SGP}^{*}}^{\overline{\theta}} \left(-\frac{1}{2}\beta d + \frac{1}{2}\phi d \right) d\theta_{i}^{-c} d\theta_{j}^{-c} d\theta_{j}^{c} \right]$$

 \geq

$$\frac{1}{(\overline{\theta} - \underline{\theta})^{3}} \left[\int_{\underline{\theta}}^{T_{WA}^{*}} \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} (\alpha) d\theta_{i}^{-c} d\theta_{j}^{-c} d\theta_{j}^{c} + \int_{\underline{\theta}}^{\overline{\theta}} \int_{T_{WA}^{*}}^{\overline{\theta}} \int_{T_{WA}^{*}}^{\overline{\theta}} \left(-\frac{1}{2}\beta d \right) d\theta_{i}^{-c} d\theta_{j}^{-c} d\theta_{j}^{c} + \int_{T_{WA}^{*}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta_{i}^{c} d - \frac{1}{2}\beta d \right) d\theta_{i}^{-c} d\theta_{j}^{-c} d\theta_{j}^{c} \right].$$

Rearranging and solving for θ_i^c yields

$$\theta_{i}^{c} \leq \frac{\beta}{2} \left[1 + \frac{\left(\overline{\theta} - T_{WA}^{*}\right)^{2} - \left(\overline{\theta} - T_{SGP}^{*}\right)^{2}}{\left(\overline{\theta} - T_{WA}^{*}\right)\left(\overline{\theta} - \underline{\theta}\right)} \right] + \frac{\phi}{2} \left[\frac{\left(\overline{\theta} - T_{SGP}^{*}\right)^{2}}{\left(\overline{\theta} - T_{WA}^{*}\right)\left(\overline{\theta} - \underline{\theta}\right)} \right] + \frac{\alpha}{d} \left[\frac{-\left(\left(\overline{\theta} - T_{SGP}^{*}\right) + \left(T_{WA}^{*} - \underline{\theta}\right)\right)}{\left(\overline{\theta} - T_{WA}^{*}\right)} \right].$$

This will be fulfilled for all $T^*_{WA} < \theta^c_i \leq T^*_{SGP}$ if and only if it can be fulfilled for $\theta^c_i = T^*_{SGP}$.

Analogously, the interim participation constraint of types $\theta_i^c > T_{SGP}^*$ can be derived as

$$\frac{1}{(\overline{\theta} - \underline{\theta})^{3}} \left[\int_{\underline{\theta}}^{T_{SGP}^{*}} \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} (\alpha) d\theta_{i}^{-c} d\theta_{j}^{-c} d\theta_{j}^{c} + \int_{\underline{\theta}}^{\overline{\theta}} \int_{T_{SGP}^{*}}^{\overline{\theta}} \int_{T_{SGP}^{*}}^{\overline{\theta}} \left(-\frac{1}{2}\beta d + \frac{1}{2}\phi d \right) d\theta_{i}^{-c} d\theta_{j}^{-c} d\theta_{j}^{c} + \int_{T_{SGP}^{*}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta_{i}^{c} d - \frac{1}{2}\beta d - \frac{1}{2}\phi d \right) d\theta_{i}^{-c} d\theta_{j}^{-c} d\theta_{j}^{c} \right]$$

 \geq

$$\frac{1}{(\overline{\theta} - \underline{\theta})^{3}} \left[\int_{\underline{\theta}}^{T_{WA}^{*}} \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} (\alpha) d\theta_{i}^{-c} d\theta_{j}^{-c} d\theta_{j}^{c} + \int_{\underline{\theta}}^{\overline{\theta}} \int_{T_{WA}^{*}}^{\overline{\theta}} \int_{T_{WA}^{*}}^{\overline{\theta}} \left(-\frac{1}{2}\beta d \right) d\theta_{i}^{-c} d\theta_{j}^{-c} d\theta_{j}^{c} + \int_{T_{WA}^{*}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta_{i}^{c} d - \frac{1}{2}\beta d \right) d\theta_{i}^{-c} d\theta_{j}^{-c} d\theta_{j}^{c} \right].$$

This simplifies to

$$\theta_{i}^{c} \leq \frac{\beta}{2} \left[1 + \frac{\left(\overline{\theta} - T_{WA}^{*}\right)^{2} - \left(\overline{\theta} - T_{SGP}^{*}\right)^{2}}{\left(T_{SGP}^{*} - T_{WA}^{*}\right)\left(\overline{\theta} - \underline{\theta}\right)} \right] - \frac{\phi}{2} \left[\frac{\left(\overline{\theta} - T_{SGP}^{*}\right)\left(T_{SGP}^{*} - \underline{\theta}\right)}{\left(T_{SGP}^{*} - T_{WA}^{*}\right)\left(\overline{\theta} - \underline{\theta}\right)} \right] + \frac{\alpha}{d}$$

and will be fulfilled for all $\theta_i^c > T_{SGP}^*$ if and only if it holds for $\theta_i^c = \overline{\theta}$. In equilibrium,

$$\phi^* = \frac{1}{3} \left(5\beta - 2\overline{\theta} \right) + \frac{3\alpha \left(\overline{\theta} - \underline{\theta} \right)}{2d \left(\overline{\theta} - \beta \right)}$$

$$T_{SGP}^* \left(\phi^* \right) = -\frac{1}{3} \overline{\theta} + \frac{4}{3} \beta.$$

Inserting this, both constraints simplify to functions of $\overline{\theta}, \underline{\theta}, \beta$ and $\underline{\alpha}$. In general, the interim participation constraint can never be fulfilled for all types θ_i^c if

$$\frac{\beta}{2} \left[1 + \frac{\left(\overline{\theta} - T_{WA}^*\right)^2 - \left(\overline{\theta} - T_{SGP}^* \left(\phi^*\right)\right)^2}{\left(T_{SGP}^* \left(\phi^*\right) - T_{WA}^*\right) \left(\overline{\theta} - \underline{\theta}\right)} \right] \\
- \frac{\phi^*}{2} \left[\frac{\left(\overline{\theta} - T_{SGP}^* \left(\phi^*\right)\right) \left(T_{SGP}^* \left(\phi^*\right) - \underline{\theta}\right)}{\left(T_{SGP}^* \left(\phi^*\right) - T_{WA}^*\right) \left(\overline{\theta} - \underline{\theta}\right)} \right] + \frac{\alpha}{d} \\
< \frac{\beta}{2} \left[5 - \frac{3 \left[\left(\overline{\theta} - T_{WA}^*\right)^2 - \left(\overline{\theta} - T_{SGP}^* \left(\phi^*\right)\right)^2\right]}{\left(\overline{\theta} - T_{WA}^*\right) \left(\overline{\theta} - \underline{\theta}\right)} \right] \\
- \frac{\phi^*}{2} \left[\frac{3 \left(\overline{\theta} - T_{SGP}^* \left(\phi^*\right)\right)^2}{\left(\overline{\theta} - T_{WA}^*\right) \left(\overline{\theta} - \underline{\theta}\right)} \right] + \frac{\alpha}{d} \left[\frac{3 \left(\left(\overline{\theta} - T_{SGP}^* \left(\phi^*\right)\right) + \left(T_{WA}^* - \underline{\theta}\right)\right)}{\left(\overline{\theta} - T_{WA}^*\right)} \right].$$

Since it is not possible to solve the constraints in an adequate manner for all parameter constellations, I now consider two concrete examples. Let first $\left[\underline{\theta}, \overline{\theta}\right] = [-1, 1]$ and $\beta = \frac{1}{2}$. These parameters imply a further constraint on $\frac{\alpha}{d}$ (i.e. $\frac{\alpha}{d} \leq \frac{5}{4}$). Then it can be shown, that the interim participation constraint for types above T^*_{SGP} is not fulfilled for $\theta^c_i = \overline{\theta} = 1$ for all admissible $\frac{\alpha}{d}$ values. But if we change β to $\frac{1}{4}$, then all interim participation constraints (including those for types $T^*_{WA} < \theta^c_i \leq T^*_{SGP}$) will be fulfilled for $\frac{\alpha}{d} \leq \frac{1}{16}$. Thus, whether all groups would like to participate in the SGP after learning their types depends on the range of private information, the height of the deficit, the spillovers due to monetary policy and the degree of polarization.

Ex post

At this point in time, all private information is common knowledge and stabilization decisions are taken (and since I consider a one-shot game not reversible). Therefore and in contrast to the ex ante and interim considerations, the relevant benchmark for voluntary participation is no longer the outcome of the war of attrition, but the pay-off when withdrawing from the pact ex post. This view emanates from the assumption that governments will only be able to enforce the pact if all interest groups are in favor of it. It also presumes that governments have the possibility to unilaterally abandon the SGP ex post (or that consensus among participating countries is necessary for enforcing it, as it seems to be the case in the European $Monetary\ Union$ — despite the fact that only a qualified majority is required for assessing an excessive deficit).

Not surprisingly, the result is that there always exist realizations of types for which groups are worse off when having to pay the fine. Suppose types are such that stabilization is delayed in one country, but achieved in the other. Under the *SGP with renegotiations* this is the only constellation in which fines have to be paid and redistribution occurs. Then, the groups in the non-stabilization country will experience the spillovers from debt policy and their private benefits of delay but will have to pay the fine if the pact is executed. If the pact is abandoned, the fine will not be enforced but stabilization decisions are taken: private benefits and spillovers remain unchanged. Groups which experience high private benefits of delay thus prefer to abandon the pact.

On the other hand, there exists no realization of types such that abandoning of the pact would be preferred by all interest groups (i.e. by both countries), since one country always profits from redistribution.²⁵ This reasoning is summarized in the following corollary.

Corollary 8 (i) Consider $\theta_i^c > T_{SGP}^*$ for i = 1, 2 and $\theta_i^{-c} \leq T_{SGP}^*$ for at least one i in -c. Ex post both groups in country c will be better off if the SGP is not executed. (ii) There exists no realization of types such that abandoning of the pact will be preferred by all groups, i.e. by both countries.

Treated from a purely theoretical point of view, the outcome (unilateral abandoning of the pact under certain circumstances) would be foreseen by

 $^{^{25}}$ This result would be different under the SGP without renegotiations: if type realizations are such that stabilization will be delayed in both countries, then all groups would be willing to abandon the pact – since they still would have to pay the fine.

rational agents ex ante and thus, the SGP would lose all its commitment power. In practice, these results may yield an explanation as to why the members of the $European\ Monetary\ Union$ ex ante signed the SGP, but now (ex post) try to water down its arrangements. Moreover, an explanation is provided for the conflicting views of participating countries concerning the enforcement of the fine.

3.3.5 Efficiency

Although an (with respect to the height of the fine) carefully designed SGP leads to more expected welfare than a situation without further commitment, that arrangement experiences one important drawback: the SGP does not induce the interest groups to stabilize in all situations where stabilization would be desirable ex post. On the one hand, there exist combinations of information parameters for which the sum of private costs is above the spillovers, but both groups are against stabilization. On the other hand, there may as well arise situations in which there exist private benefits of delay, but the SGP induces the interest groups to stabilize (see Figure 6).

Ex post efficiency in this context means that stabilization should not occur if the private benefits of stabilization exceed the negative spillovers from debt policy in either country. Let $\theta := (\theta^A, \theta^B)$. The ex post efficient decision is defined as

$$x^*(\theta) := \left(x_A^*(\theta^A), x_B^*(\theta^B)\right) \tag{16}$$

where

$$x_c^*(\theta^c) = \begin{cases} stabilize\ (s), & if\ \sum_{i=1}^2 \theta_i^c \le 2\beta \\ stabilize\ not\ (n), & otherwise \end{cases}.$$

Lemma 17 The SGP does not implement the ex post efficient decision.

Thus, it is not possible to achieve ex post efficiency with an arrangement such as the simplified version of the *European SGP* presented here. The next section introduces a mechanism that overcomes this drawback.

3.4 Direct mechanism

In this section I shift attention to a revelation mechanism that could be used to achieve adequate fiscal stabilization in a monetary union. I concentrate on direct mechanisms, i.e. the participants are asked for their private information and depending on their announcements a decision is taken and transfers are paid. Thus and in contrast to the SGP, there is no involvement of the respective governments. However, there has to exist an authority to implement the mechanism.

I construct an expected externality or D'Aspremont-Gérard-Varet (AGV) mechanism²⁶ for the context presented above. Such mechanisms implement the ex post efficient decision, are incentive compatible and ensure budget balancedness. To the subject of voluntary participation I return in Section 3.4.3. The given mechanism internalizes the spillovers from debt policy by appropriately designed transfers.²⁷ Implementing this mechanism is unambiguously welfare increasing, but ex post voluntary participation cannot be achieved either.

3.4.1 Expected externality mechanism

Here, I adapt the concept of an expected externality mechanism to the context of negative spillovers due to a common monetary policy. The groups' announcements about their private information are denoted by $\tilde{\theta}_i^c$. Utility of group i in country c is given by

$$U_i^c\left(\widetilde{\theta}, \theta_i^c\right) := w_i^c\left(x^*(\widetilde{\theta}), \theta_i^c\right) + Z_i^c\left(\widetilde{\theta}\right)$$

 $^{^{26}\}mathrm{See}$ Mas-Colell, Whinston and Green [1995], Arrow [1979] or D'Aspremont and Gérard-Varet [1979].

 $^{^{27}}$ In contrast to the fiscal transfer system introduced in Engwerda et al. [2002], here transfers are paid directly by the interest groups. Nevertheless, they may be interpreted as a union wide scheme of unemployment benefits as well.

where

$$w_i^c\left(\boldsymbol{x}^*(\widetilde{\boldsymbol{\theta}}), \boldsymbol{\theta}_i^c\right) := \left\{ \begin{array}{ccc} 0, & if \ \sum_{i=1}^2 \widetilde{\boldsymbol{\theta}}_i^c \leq 2\beta \wedge \sum_{i=1}^2 \widetilde{\boldsymbol{\theta}}_i^{-c} \leq 2\beta \\ -\frac{\beta}{2}d, & if \ \sum_{i=1}^2 \widetilde{\boldsymbol{\theta}}_i^c \leq 2\beta \wedge \sum_{i=1}^2 \widetilde{\boldsymbol{\theta}}_i^{-c} > 2\beta \\ \boldsymbol{\theta}_i^c d - \frac{\beta}{2}d, & if \ \sum_{i=1}^2 \widetilde{\boldsymbol{\theta}}_i^c > 2\beta \wedge \sum_{i=1}^2 \widetilde{\boldsymbol{\theta}}_i^{-c} \leq 2\beta \\ \boldsymbol{\theta}_i^c d - \beta d, & otherwise \end{array} \right.$$

and $Z_i^c(\cdot)$ is a transfer to group *i* in country *c*. Note that the decision that is implemented depends on the announcements of the interest groups about their private information, but utility depends on the true value of θ_i^c .

The transfers are designed in such a way as to internalize the expected externality of a change in group i's announcement on the other groups and moreover, in order to guarantee a balanced budget:

$$Z_{i}^{c}\left(\widetilde{\theta}\right) := E_{\theta_{-i}}\left[\sum_{j\neq i} w_{j}\left(x^{*}(\widetilde{\theta}_{i}^{c}, \theta_{-i}), \theta_{j}\right)\right]$$
$$-\frac{1}{3}\sum_{j\neq i} E_{\theta_{-j}}\left[\sum_{-j} w_{-j}\left(x^{*}(\widetilde{\theta}_{j}, \theta_{-j}), \theta_{-j}\right)\right].$$

The first term on the right hand side represents the expected benefits of groups' $j \neq i$ when group i announces its type to be $\widetilde{\theta}_i^c$ and groups $j \neq i$ tell the truth. Therefore, it is a function of only group i's announcement, not of the announcements of the other groups. It thus represents the expected change in utility of groups $j \neq i$ due to a change in i's announcement. The second term is the share group i has to pay of the transfers to the other groups and therefore guarantees a balanced budget. With a uniform distribution of types on $[\underline{\theta}, \overline{\theta}]$ I obtain

$$Z_{i}^{c}\left(\widetilde{\theta}\right) = \frac{d}{\overline{\theta} - \underline{\theta}} \left(\frac{1}{2}\widetilde{\theta}_{i}^{c}\left(\beta - \widetilde{\theta}_{i}^{c}\right) - \frac{1}{3}\left(\frac{1}{2}\widetilde{\theta}_{j}^{c}\left(\beta - \widetilde{\theta}_{j}^{c}\right) + \sum_{i=1}^{2} \frac{1}{2}\widetilde{\theta}_{i}^{-c}\left(\beta - \widetilde{\theta}_{i}^{-c}\right)\right)\right). \tag{17}$$

As aforementioned, the transfers are designed to fulfill two tasks: first, the effect the announcement of group i has on the implemented decision should be internalized. Second, a balanced budget should be achieved.

Consider expected transfers to group i as a function of its announcement $\widetilde{\theta}_i$ (Figure 7). The sole effect a change in $\widetilde{\theta}_i$ has is its influence on the decision. Expected transfers of group i are strictly concave in $\widetilde{\theta}_i$ and are maximized at $\widetilde{\theta}_i = \frac{\beta}{2}$. The actual magnitude depends on the support, the size of the deficit and on the degree of spillovers. If group i announces to be of type $\frac{\beta}{2}$, the probability to change the decision is minimized. But a small deviation to either side increases the probability to change the decision that is implemented. Therefore, expected transfers strictly decrease to either side of $\frac{\beta}{2}$. In principle, at $\frac{\beta}{2}$ the other groups, which either have benefits of delay or prefer to stabilize, compensate group i for minimally influencing the decision with its announcement.

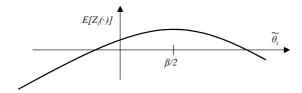


Figure 7: Expected Transfers as Function of the Announcement

In order to analyze the incentives underlying announcement behavior in equilibrium consider expected utility of implementing the ex post efficient decision. This not only depends on the own announcement, but also on the true type. We can distinguish three different cases depicted in Figure 8. First, consider $\theta_i = \frac{\beta}{2}$: expected utility of implementing the ex post efficient decision is constant in the announcement. This is due to the fact that expected utility is the same under stabilization and under delayed stabilization. For types below $\frac{\beta}{2}$, expected utility of implementing the expost efficient decision is a strictly decreasing linear function of the announcement: the higher the probability of delaying stabilization the worse for types below $\frac{\beta}{2}$, since they suffer from delay. The slope depends on the true type and is steeper the larger are private costs. For types above $\frac{\beta}{2}$ the opposite applies: the higher the probability of delay, the better for types that have private benefits. All functions intersect at $2\beta - \overline{\theta}$. This is due to the fact that for this announcement expected utility is independent of the true type.

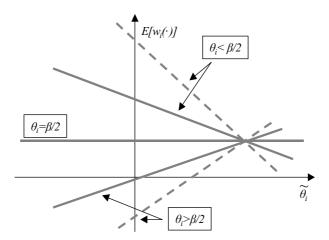


Figure 8: Expected Utility of Implementing ex post efficient Decision

Now, we can analyze the incentives underlying announcement behavior: Consider group i that is of type $\theta_i < \frac{\beta}{2}$. Has this group an incentive to announce some $\widetilde{\theta}_i \neq \theta_i$? There are two possible deviations: first, consider a deviation to some $\widetilde{\theta}_i = \widehat{\theta}_i < \theta_i$. The marginal expected gain of the increased probability of stabilization due to this change in the announcement is smaller than the marginal decrease in expected transfers. The reason is that the absolute value of the slope of expected transfers is larger than the slope of expected utility of implementing the expost efficient decision. The order of magnitude of the respective slopes changes exactly at the true type. Therefore, deviations to some $\widetilde{\theta}_i = \widehat{\theta}_i > \theta_i$ $\left(\widehat{\theta}_i < \frac{\beta}{2}\right)$ are not profitable either:

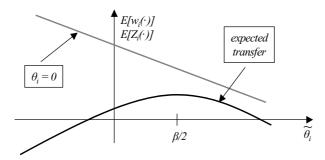


Figure 9: Expected Transfers and Expected Utility of Type Zero

The above given reasoning is formalized in the following corollary which is a direct application of the AGV-Theorem.²⁸

²⁸See for example Mas-Colell, Whinston and Green [1995], page 885.

Corollary 9 The social choice function $g(\cdot) = (x^*(\cdot), Z(\cdot))$ where $x^*(\cdot)$ is given by equation (16) and $Z(\cdot) := (Z_i^A(\cdot), Z_j^A(\cdot), Z_i^B(\cdot), Z_j^B(\cdot))$ by equation (17) is truthfully implementable in Bayesian Nash equilibrium and leads to a balanced budget.

Thus, there exists a mechanism that is able to deal with private information on behalf of the interest groups efficiently. Moreover, this mechanism needs no outside financing. However, there has to exist an authority to enforce the implementation of this mechanism.²⁹ In addition, it has to be possible to identify the respective interest groups to actually apply it.

3.4.2 Welfare

The AGV mechanism implements the ex post efficient decision for all realizations of private information. In order to achieve this a scheme of direct transfers between the interest groups is introduced. This scheme is budget balanced, therefore its effect is purely redistributional. Thus, the AGV mechanism maximizes expected welfare and consequently outperforms the war of attrition and the SGP in terms of expected welfare.

Corollary 10 The AGV mechanism maximizes ex ante expected welfare.

3.4.3 Voluntary participation

In this section, I again consider three different points in time (i.e. ex ante, interim and ex post) in order to evaluate whether it is individually rational to participate in the AGV mechanism. Since the analysis here concerns a direct revelation mechanism, no other player except the interest groups is involved, i.e. no government decides about participation (like it was the case under the SGP).

²⁹In the European Union this task could be assigned to the Commission for example.

Ex ante

As an immediate consequence of Corollary 10 and since the whole setting is symmetric, it is individually rational for all interest groups to participate in the AGV mechanism ex ante.

Corollary 11 The ex ante participation constraint of all interest groups is fulfilled under the AGV mechanism.

Interim

Whether the interim participation constraints under the AGV mechanism can be fulfilled for all types depends on the overall parameter constellation as well. Therefore, I first derive some general results and then give two concrete examples. These suggest that it depends on the parameter constellation as well which interim constraints (AGV or SGP) are more demanding.

Under the AGV mechanism, interim expected utility of a type θ^c_i is a strictly convex function of the own type. It obtains its global minimum at $\theta^c_i = 2\beta - \overline{\theta}$ and is given by

$$\begin{split} E_{\theta_{-i}}\left[w_{i}\left(\boldsymbol{x}^{*}\left(\boldsymbol{\theta}\right),\boldsymbol{\theta}_{i}^{c}\right)+Z_{i}^{c}\left(\boldsymbol{\theta}\right)\mid\boldsymbol{\theta}_{i}^{c}\right]\\ &=\frac{1}{\left(\overline{\boldsymbol{\theta}}-\underline{\boldsymbol{\theta}}\right)^{3}}\left[\int_{\underline{\boldsymbol{\theta}}}^{\overline{\boldsymbol{\theta}}}\int_{2\beta-\theta_{j}^{-c}}^{\overline{\boldsymbol{\theta}}}\int_{\underline{\boldsymbol{\theta}}}^{2\beta-\theta_{i}^{c}}\left(-\frac{1}{2}\beta\boldsymbol{d}\right)d\theta_{j}^{c}d\theta_{i}^{-c}d\theta_{j}^{-c}\\ &+\int_{\underline{\boldsymbol{\theta}}}^{\overline{\boldsymbol{\theta}}}\int_{\underline{\boldsymbol{\theta}}}^{2\beta-\theta_{j}^{-c}}\int_{2\beta-\theta_{i}^{c}}^{\overline{\boldsymbol{\theta}}}\left(\boldsymbol{\theta}_{i}^{c}d-\frac{1}{2}\beta\boldsymbol{d}\right)d\theta_{j}^{c}d\theta_{i}^{-c}d\theta_{j}^{-c}\\ &+\int_{\underline{\boldsymbol{\theta}}}^{\overline{\boldsymbol{\theta}}}\int_{2\beta-\theta_{j}^{-c}}^{\overline{\boldsymbol{\theta}}}\int_{2\beta-\theta_{i}^{c}}^{\overline{\boldsymbol{\theta}}}\left(\boldsymbol{\theta}_{i}^{c}d-\beta\boldsymbol{d}\right)d\theta_{j}^{c}d\theta_{i}^{-c}d\theta_{j}^{-c}\\ &+\int_{\underline{\boldsymbol{\theta}}}^{\overline{\boldsymbol{\theta}}}\int_{\underline{\boldsymbol{\theta}}}^{\overline{\boldsymbol{\theta}}}\left[Z_{i}^{c}\left(\boldsymbol{\boldsymbol{\theta}}\right)\right]d\theta_{j}^{c}d\theta_{i}^{-c}d\theta_{j}^{-c}\right]. \end{split}$$

Under the war of attrition, consider interim expected utility for two different parts of the support: for types $\theta_i^c \leq T_{WA}^*$ it is constant in the type, strictly negative and given by

$$E_{\theta_{-i}} \left[w_i^c \left(x_{WA}^c \left(\theta^c \right), x_{WA}^{-c} \left(\theta^{-c} \right), \theta_i^c \right) \mid \theta_i^c \leq T_{WA}^* \right]$$

$$= \frac{1}{\left(\overline{\theta} - \underline{\theta} \right)^3} \left[\int_{T_{WA}^*}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left(-\alpha \right) d\theta_i^{-c} d\theta_j^{-c} d\theta_j^c$$

$$+ \int_{\underline{\theta}}^{\overline{\theta}} \int_{T_{WA}^*}^{\overline{\theta}} \int_{T_{WA}^*}^{\overline{\theta}} \left(-\frac{1}{2}\beta d \right) d\theta_i^{-c} d\theta_j^{-c} d\theta_j^c \right].$$

Thus, a sufficient (but not necessary) condition for the interim participation of types below T_{WA}^* for all parameter constellations is

$$E_{\theta_{-i}}\left[w_{i}\left(x^{*}\left(\theta\right),\theta_{i}^{c}\right)+Z_{i}^{c}\left(\theta\right)\mid\theta_{i}^{c}=2\beta-\overline{\theta}\right]\geq0$$

$$\Leftrightarrow$$

$$\beta\geq\frac{2}{3}\overline{\theta}+\frac{1}{3}\underline{\theta}.$$

For types $\theta_i^c > T_{WA}^*$ interim expected utility under the war of attrition is strictly increasing in the own type and given by

$$\begin{split} E_{\theta_{-i}}\left[w_{i}^{c}\left(x_{WA}^{c}\left(\theta^{c}\right),x_{WA}^{-c}\left(\theta^{-c}\right),\theta_{i}^{c}\right)\mid\theta_{i}^{c}>T_{WA}^{*}\right]\\ &=\frac{1}{\left(\overline{\theta}-\underline{\theta}\right)^{3}}\left[\int_{\underline{\theta}}^{T_{WA}^{*}}\int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{\theta}}^{\overline{\theta}}\left(\alpha\right)d\theta_{i}^{-c}d\theta_{j}^{-c}d\theta_{j}^{c}\\ &+\int_{\underline{\theta}}^{\overline{\theta}}\int_{T_{WA}^{*}}^{\overline{\theta}}\int_{T_{WA}^{*}}^{\overline{\theta}}\left(-\frac{1}{2}\beta d\right)d\theta_{i}^{-c}d\theta_{j}^{-c}d\theta_{j}^{c}\\ &+\int_{T_{WA}^{*}}^{\overline{\theta}}\int_{\underline{\theta}}^{\overline{\theta}}\left(\theta_{i}^{c}d-\frac{1}{2}\beta d\right)d\theta_{i}^{-c}d\theta_{j}^{-c}d\theta_{j}^{c}\right]. \end{split}$$

In order to determine for which type $\theta_i^c > T_{WA}^*$ this constraint is most demanding, consider the difference in interim expected utility under the AGV mechanism and under the war of attrition. This is strictly convex in the own type and obtains its global minimum at $\theta_i^c = 2\beta - T_{WA}^*$. Thus,

all interim participation constraints for types $\theta_i^c > T_{WA}^*$ are fulfilled if and only if the difference at $\theta_i^c = 2\beta - T_{WA}^*$ is positive. This cannot be achieved for all parameter constellations.

Let first $[\underline{\theta}, \overline{\theta}] = [-1, 1]$ and $\beta = \frac{1}{2}$. Then, the interim participation constraint for types below T_{WA}^* is always fulfilled. That for types above T_{WA}^* holds for very small $\frac{\alpha}{d}$ values. In contrast to the SGP, here interim voluntary participation is achievable. For $\beta = \frac{1}{4}$, there also exist values of $\frac{\alpha}{d}$ for which all interim participation constraints hold.

Ex post

Unfortunately and despite all its advantages, the AGV mechanism is not able to ensure that all types of interest groups would like to participate ex post. For example, this is the case for type realizations that guarantee all groups high private benefits of delay (such that the mechanism prescribes delay of stabilization in both countries). If one group experiences higher benefits than the others, it will not be individually rational for this group to participate in the mechanism ex post, since it would have to compensate the others through transfers (which do not accrue when withdrawing from the mechanism).

Lemma 18 The ex post participation constraint is not fulfilled under the AGV mechanism.

Thus, this mechanism would experience the similar difficulties concerning its enforcement expost as the SGP in the $European\ Union$ does.

3.5 Extensions

In this section, I present three extensions to the set-up considered so far. The first one deals with a situation in which the countries that sign the SGP do not belong to a monetary union. Apart from any free-riding issues that may arise in a monetary union between one monetary authority and many independent fiscal authorities, there may exist a rationale for an SGP, since

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again the war of attrition among interest groups may be attenuated by a fine for deficits. Thus, the fine may be used as a device for stabilization among independent countries as well.

Secondly, I consider the other extreme, monetary and fiscal union. Concentrating primarily on an AGV mechanism, I here refer to a situation in which there exists not only a common monetary policy, but also a common fiscal policy, i.e. either stabilization occurs in both countries or it is delayed in both. Thus, the so-called constrained ex post efficient decision is not to stabilize when the sum of private benefits of all four interest groups exceeds the spillovers due to the common monetary policy. This decision cannot be implemented using a union wide fine either. In contrast to Section 3.4.2, here the AGV mechanism is outperformed by the SGP in terms of expected welfare, since under the SGP asymmetric decisions remain available.

Finally, I extend the analysis to n countries. The results obtained for the 2-country case concerning the comparison of the cut-off values still hold, i.e. the SGP without renegotiations leads the interest groups to stabilize for the widest range of private information. Moreover, an increase in the number of union members reduces the effect home deficit has on equilibrium inflation and thus leads to less stabilization. On the other hand, enlarging the number of participating countries reduces the negative impact of renegotiations.

3.5.1 Independent countries

In this section, the countries that sign the SGP do not belong to a monetary union. Apart from any free-riding issues, there may exist a rationale for an SGP, since again the fine induces the interest groups to stabilize for a wider range of information parameters.³⁰

³⁰I do not explicitly consider welfare and efficiency, but presume that the effects are similar to those obtained for the monetary union case.

War of attrition

Since in this setting there are no negative spillovers due to a common monetary policy, expected utility of group i in country c when the war of attrition is played is simply given by

$$u_{i}^{c}\left(v_{i}^{c},v_{j}^{c}\right) = \left\{ egin{array}{ll} 0, & ext{if } v_{i}^{c} = s, v_{j}^{c} = s \\ -lpha, & ext{if } v_{i}^{c} = s, v_{j}^{c} = n \\ lpha, & ext{if } v_{i}^{c} = n, v_{j}^{c} = s \\ heta_{i}d, & ext{if } v_{i}^{c} = n, v_{j}^{c} = n \end{array}
ight..$$

The same argumentation as in Section 3.2.1 leads to a unique symmetric Bayesian Nash Equilibrium³¹ of

$$p_{i}^{c}\left(\theta_{i}^{c}\right) = \begin{cases} 0, & \text{if } \theta_{i}^{c} > \widetilde{T}^{WA} \\ 1, & \text{if } \theta_{i}^{c} \leq \widetilde{T}^{WA} \end{cases}$$

where

$$\widetilde{T}^{WA} := \frac{-\alpha}{\left(1 - F(\widetilde{T}^{WA})\right)d}.$$

The comparative static results obtained in Section 3.2.1 concerning d and α remain unchanged and so does their interpretation.

SGP without renegotiations

Here again, the governments of both countries decide to punish deficits with a fine ϕ . Depending on being for or against stabilization, expected utility of group i in country c is now given by

$$u_{i}^{c}\left(v_{i}^{c},v_{j}^{c}\right) = \begin{cases} \frac{\phi}{2}q^{-c}\left(\theta^{-c}\right)d, & \text{if } v_{i}^{c} = s, v_{j}^{c} = s\\ \frac{\phi}{2}q^{-c}\left(\theta^{-c}\right)d - \alpha, & \text{if } v_{i}^{c} = s, v_{j}^{c} = n\\ \frac{\phi}{2}q^{-c}\left(\theta^{-c}\right)d + \alpha, & \text{if } v_{i}^{c} = n, v_{j}^{c} = s\\ -\frac{1}{2}\phi d + \theta_{i}^{c}d, & \text{if } v_{i}^{c} = n, v_{j}^{c} = n \end{cases}.$$

³¹Existence of all three equilibria of the respective games can be guaranteed by appropriately resizing the assumptions on $\underline{\theta}$.

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Again, there exists a critical value of private information such that the groups will want to stabilize if and only if their private information lies below this critical value. The same line of argumentation as in Section 3.2.2 leads to a unique symmetric Bayesian Nash Equilibrium of

$$p_i^c(\theta_i^c) = \begin{cases} 0, & \text{if } \theta_i^c > \widetilde{T}^P \\ 1, & \text{if } \theta_i^c \le \widetilde{T}^P \end{cases}$$

where

$$\widetilde{T}^P := \frac{-\alpha}{\left(1 - F(\widetilde{T}^P)\right)d} + \frac{1}{2}\left(1 + \left(1 - F(\widetilde{T}^P)\right)^2\right)\phi.$$

The influence of α and d on the individual decision about stabilization is like before. The fine induces the interest groups to stabilize in more instances.

SGP with renegotiations

Allowing for renegotiations changes expected utility only when all groups are against stabilization and is in this case now given by

$$u_i^c(n,n) = -\frac{\phi}{2} (1 - q^{-c} (\theta^{-c})) d + \theta_i^c d.$$

As in the set-ups considered before, there exists a cut-off value for private information such that the groups will stabilize if and only if their private information lies below this critical value. Here, I obtain the following unique symmetric Bayesian Nash Equilibrium:

$$p_i^c(\theta_i^c) = \begin{cases} 0, & \text{if } \theta_i^c > \widetilde{T}^{SGP} \\ 1, & \text{if } \theta_i^c \le \widetilde{T}^{SGP} \end{cases}$$

where

$$\widetilde{T}^{SGP} := \frac{-\alpha}{\left(1 - F(\widetilde{T}^{SGP})\right)d} + \frac{\phi}{2}.$$

The influence of d, α and ϕ is the same as above, only the effect of ϕ is again not as strong as before due to the possibility of renegotiations.

Comparison

For positive fines, the SGP without renegotiations again leads to stabilization occurring for the widest range of private information parameters, but the SGP with renegotiations outperforms the war of attrition as well.³² Thus, any form of SGP leads to stabilization occurring in more instances than under the war of attrition. Therefore, one may implement a pact as a commitment device also among independent countries.

3.5.2 Fiscal union

In this second extension, I return to the case of a monetary union, but do not allow for country-specific stabilization decisions (under the direct mechanism) anymore. This means that either stabilization occurs in both countries or in none, the possibility of stabilization in one country and delay in the other is ruled out a priori. Thus, the ex post efficient decision under this restriction is not to stabilize in both countries whenever the sum of private benefits of all four groups exceeds the negative spillovers due to the common monetary policy. I assume that this modification has no influence on the respective war of attrition (SGP) game where decentralized stabilization decisions remain available.³³ One interpretation of the setting in which only the authority that implements the mechanism underlies the restriction of one remaining stabilization instrument is that of a *European Council of Experts*: not only monetary policy is delegated to the *ECB* but fiscal policy is determined at union level as well. Attention is restricted

³²This can be shown using the same techniques as in Section 3.2.4.

 $^{^{33}}$ A possibility to overcome this asymmetry in fiscal flexibility is the following: interest groups i (from both countries) jointly fight a war of attrition against interest groups j (again from both countries). In this war of attrition, only one stabilization decision for the union as a whole remains available. I presume that this would lead to an improved performance of the AGV mechanism.

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to efficiency, welfare and voluntary participation. The exclusion of asymmetric decisions leads to an improved performance of the SGP vis-à-vis an (adapted) AGV mechanism in terms of expected welfare. This is due to the decreased flexibility in fiscal policy making.

Efficiency

Ex post efficiency under the restriction of only one stabilization decision means that stabilization should not occur if the private benefits of delay exceed the negative spillovers in the whole union. Thus, the constrained ex post efficient decision is defined as

$$x_U^*(\theta) := \begin{cases} stabilize(s), & if \sum_{i=1}^2 \theta_i^A + \sum_{i=1}^2 \theta_i^B \le 4\beta \\ stabilize(not(n), & otherwise \end{cases} .$$
 (18)

Lemma 19 The SGP does not implement the constrained ex post efficient decision.

Although the *SGP* leads to more likely stabilization than a situation without further commitment, that arrangement again experiences the drawback that it does not induce the interest groups to stabilize in all situations where stabilization would be desirable ex post. The interpretation is identical to the case without a fiscal union.

Expected externality mechanism

Here, I construct an expected externality mechanism under the constraint that only one stabilization decision may be taken for the whole union. The approach is similar to that in Section 3.4.1. Utility of group i in country c is given by

$$U_i^c\left(\widetilde{\theta}, \theta_i^c\right) := w_i^c\left(x_U^*(\widetilde{\theta}), \theta_i^c\right) + z_i^c\left(\widetilde{\theta}\right)$$

where

$$w_i^c\left(x_U^*(\widetilde{\theta}), \theta_i^c\right) := \begin{cases} 0, & if \sum_{i=1}^2 \widetilde{\theta}_i^c + \sum_{i=1}^2 \widetilde{\theta}_i^{-c} \le 4\beta \\ \theta_i^c d - \beta d, & otherwise \end{cases}$$

and $z_i^c(\cdot)$ is a transfer to group i in country c. Again, the decision that is implemented depends on the announcements of the interest groups about their private information, but utility depends on the true value of θ_i^c .

Following the same approach as in Section 3.4.1 transfers are designed to fulfill two tasks: first, the effect the announcement of group i has on the expected utility of the other groups should be internalized and second, a balanced budget should be achieved. With a uniform distribution I obtain

$$z_{i}^{c}\left(\widetilde{\theta}\right) = \frac{\left(\beta - \frac{1}{2}\widetilde{\theta}_{i}^{c}\right)\widetilde{\theta}_{i}^{c} - \frac{1}{3}\left(\left(\beta - \frac{1}{2}\widetilde{\theta}_{j}^{c}\right)\widetilde{\theta}_{j}^{c} + \sum_{i=1}^{2}\left(\beta - \frac{1}{2}\widetilde{\theta}_{i}^{-c}\right)\widetilde{\theta}_{i}^{-c}\right)}{\overline{\theta} - \underline{\theta}}d. \tag{19}$$

The interpretation given in Section 3.4.1 can be readily adapted to these transfers and the same argumentation with respect to the incentives concerning announcement behavior applies. Again, the AGV mechanism overcomes the efficiency problems induced by private information.

Corollary 12 The social choice function $h_U(\cdot) = (x_U^*(\cdot), z(\cdot))$ where $x_U^*(\cdot)$ is given by equation (18) and $z(\cdot) := (z_i^A(\cdot), z_j^A(\cdot), z_i^B(\cdot), z_j^B(\cdot))$ by equation (19) is truthfully implementable in Bayesian Nash equilibrium and leads to a balanced budget.

Welfare

Despite implementing the constrained ex post efficient decision for a fiscal union the above presented mechanism is outperformed by the SGP in terms of expected welfare. This is due to the fact that there may arise situations in which it would be desirable to implement an asymmetric decision, i.e. to stabilize in only one country and delay stabilization in the other. Under the SGP there exists one fiscal policy for each country, whereas in a fiscal union only one policy is available for the union as a whole, i.e. one stabilization decision which covers both countries.

Lemma 20 When ϕ^* is implemented the SGP yields more expected welfare than the AGV mechanism.

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Participation constraints

In addition to Lemma 20 it can be shown that there exist parameter constellations for which this adapted AGV mechanism is outperformed in terms of expected utility by the war of attrition as well.³⁴ Thus, the missing stabilization instrument may lead to severe problems concerning voluntary participation – even ex ante. Consequently, the interim participation constraints cannot be fulfilled in these cases either.

Not surprisingly, it is not possible to induce all interest groups to participate voluntarily ex post. There exist always types that lose due to the mechanism. Consider for example such a realization of types that stabilization is delayed. Under the (adapted) AGV mechanism each interest group receives its private benefits of delay and has to endure the negative spillovers created by the common monetary policy. As soon as the type realizations differ, some transfers have to be paid. That group with the highest realized value of θ_i^c has to compensate the others. Since without the mechanism no such payments would accrue, this high-type group prefers to withdraw from the mechanism.

3.5.3 More countries

In this section I extend the analysis³⁵ to a union consisting of n countries, $c = 1, \ldots, n$. Since the whole setting is symmetric, the following analysis is conducted for country h. The results obtained for the 2-country case in Section 3.2.4 still hold, i.e. the SGP without renegotiations leads the interest groups to stabilize for the widest range of information parameters. In addition, an increase in the number of union members reduces the effect the home deficit has on equilibrium inflation and thus leads to less stabilization at home. On the other hand, it turns out that enlarging the

³⁴For example, the ex ante participation constraint is never fulfilled for $\beta \leq \frac{3}{8}\overline{\theta} + \frac{5}{8}\underline{\theta}$ and $\frac{\alpha}{d} < \frac{\widehat{\alpha}}{d}$ as defined in Lemma 16.

³⁵Attention is restricted to the implicit definition of the cut-off values with general distribution functions, because even with a uniform distribution of types it is not possible to solve them explicitly (except under the war of attrition).

number of participating countries reduces the negative impact of allowing for renegotiations.

Again, the analysis begins at the last stage of the game. The CB's objective function is given by

$$U_{CB} = -\lambda \pi^{2} (\cdot) + (1 - \lambda) \overline{d} * \pi (\cdot)$$

where \overline{d} now denotes the average debt level in the *n*-member union. Its maximization over π yields

$$\pi^*\left(\cdot\right) = \beta \overline{d}$$

with β defined as before. Again, equilibrium inflation is proportional to the average debt level in the union and is increasing in β .

War of attrition

The basic war of attrition is not changed by enlarging the number of union members since there is no connection between behavior at home and abroad except for the spillovers of debt policy. The only difference is the changed decision of the CB about the equilibrium inflation rate.

The same argumentation as in Section 3.2.1 leads to a unique symmetric Bayesian Nash Equilibrium 36 of

$$p_i^h\left(\theta_i^h\right) = \left\{ \begin{array}{l} 0, & \text{if } \theta_i^h > t^{WA} \\ 1, & \text{if } \theta_i^h \le t^{WA} \end{array} \right.$$

where

$$t^{WA} := \frac{-\alpha}{(1 - F(t^{WA})) d} + \frac{\beta}{n}.$$

The comparative static results concerning β , d and α remain unchanged. The new insight is that enlarging the number of union members leads to less stabilization. This is due to the fact that the effect of home deficit on equilibrium inflation is smaller the more countries participate, since the inflationary response of the CB is smaller.

Lemma 21 t^{WA} is strictly decreasing in n.

 $^{^{36}}$ Existence of all equilibria again can be guaranteed by appropriately resizing the assumptions on $\underline{\theta}$.

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SGP without renegotiations

The model of Section 3.2.2 remains unaltered with respect to the set-up. Consider group i in country h and let $v := (v^1, \ldots, v^n)$. Depending on being for or against stabilization, expected utility is now given by

$$u_{i}^{h}\left(v\right) = \begin{cases} \frac{\frac{\phi}{2(n-1)} \sum_{c \neq h} q^{c}\left(\theta^{c}\right) d - \pi\left(\cdot\right), & \text{if } v_{i}^{h} = s, v_{j}^{h} = s \\ \frac{\phi}{2(n-1)} \sum_{c \neq h} q^{c}\left(\theta^{c}\right) d - \alpha - \pi\left(\cdot\right), & \text{if } v_{i}^{h} = s, v_{j}^{h} = n \\ \frac{\phi}{2(n-1)} \sum_{c \neq h} q^{c}\left(\theta^{c}\right) d + \alpha - \pi\left(\cdot\right), & \text{if } v_{i}^{h} = n, v_{j}^{h} = s \\ -\frac{\phi}{2} d + \theta_{i}^{h} d - \pi\left(\cdot\right), & \text{if } v_{i}^{h} = n, v_{j}^{h} = n \end{cases}$$

The same line of argumentation as in Section 3.2.2 leads to a unique symmetric Bayesian Nash Equilibrium of

$$p_i^h\left(\theta_i^h\right) = \begin{cases} 0, & \text{if } \theta_i^h > t^P\\ 1, & \text{if } \theta_i^h \le t^P \end{cases}$$

where

$$t^{P} := \frac{-\alpha}{(1 - F(t^{P})) d} + \frac{\beta}{n} + \frac{\phi}{2} \left(1 + \left(1 - F(t^{P}) \right)^{2} \right).$$

The comparative static results concerning β , d and α are qualitatively the same as before. The introduction of a fine for deficits again leads to stabilization occurring for a wider range of information parameters and enlarging the number of union members leads to less stabilization.

Lemma 22 t^P is strictly decreasing in n.

SGP with renegotiations

Allowing for renegotiations in the n-country set-up leads to changes in expected utility only in the case when all groups are against stabilization:

$$u_{i}^{h}(n,n):=-rac{\phi}{2}\left(1-\prod_{c
eq h}q^{c}\left(heta^{c}
ight)
ight)d+ heta_{i}^{h}d-\pi\left(\cdot
ight).$$

Here, the following unique symmetric Bayesian Nash Equilibrium obtains:

$$p_i^h\left(\theta_i^h\right) = \begin{cases} 0, & \text{if } \theta_i^h > t^{SGP} \\ 1, & \text{if } \theta_i^h \le t^{SGP} \end{cases}$$

where

$$t^{SGP} := \frac{-\alpha}{(1 - F(t^{SGP}))d} + \frac{\beta}{n} + \frac{\phi}{2} \left[1 + \left(1 - F(t^{SGP}) \right)^2 - \left(1 - F(t^{SGP}) \right)^{2(n-1)} \right].$$

Due to the last term on the right hand side (which enters negatively) it is not possible to obtain any clear-cut comparative static results on t^{SGP} . But since this term goes to zero as n goes to infinity, I presume that all results obtained for t^P apply to t^{SGP} as well.

Under the SGP with renegotiations enlarging the number of union members has another effect on the probability of stabilization. The more countries join the SGP, the stronger becomes the commitment effect even when allowing for renegotiations. The reason is that the only case in which renegotiations are possible is the one in which all countries delay stabilization. The probability that this happens declines when the number of participating countries increases. As this number goes to infinity the commitment when allowing for renegotiations becomes as strong as the commitment when there is no possibility for renegotiations at all. 37

The results on the number of countries may be applied to a future enlargement of the *European Monetary Union*. Admitting new members would have two opposing effects: on the one hand, the probability of delay would increase, since the interest groups anticipate that their behavior will exert less influence on the decision of the *CB*. On the other hand, the negative effect of allowing for renegotiations would be mitigated.

Comparison

Finally, let us turn to the comparison of the respective equilibrium cutoff values. Considering more countries does not change any of the results

 $^{^{37} \}text{For } n \rightarrow \infty : t^{SGP} \rightarrow t^P \text{, since for } n \rightarrow \infty : -\frac{\phi}{2} \left(1 - F(t^{SGP})\right)^{2(n-1)} \rightarrow 0.$

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concerning the probability of stabilization. Again, the *SGP without renego*tiations leads to stabilization occurring for the widest range of information parameters, but even if renegotiations are allowed, the result of the war of attrition will be improved upon. The argumentation again proceeds along the lines developed in Section 3.2.4.

3.6 Conclusion

This chapter introduces national interest groups and private information on their behalf into the discussion about the *European Stability and Growth Pact*. I extend a war of attrition model to the case of a monetary union and introduce a fine for deficits. Allowing for private benefits of delayed stabilization is another new feature of this model.

The first result of this chapter is that deficit sanctions indeed can enforce national commitment to stabilization. Even if we allow for renegotiations this result will hold. On the other hand, in the presence of private benefits of delay the effect immediate stabilization has on welfare is not unambiguously positive. If the pact is carefully designed with respect to the height of the fine, then it will unambiguously increase welfare. This yields an explanation as to why the member countries of EMU signed the pact. But there always exist types who prefer to abandon the pact ex post, what may explain the current difficulties concerning its enforcement. Finally, I show that it is impossible to achieve expost efficiency with the SGP. Therefore, I adapt an expected externality mechanism to my setting. Despite all its advantages this mechanism is also not able to achieve voluntary participation ex post. In addition, I presume that in reality there may exist political reasons not to implement such a decentralized system of transfers and there could arise problems in identifying the respective interest groups.

Starting from this research there are different directions to proceed. The first one would be to allow for the war of attrition (extended to a monetary union and by a fine for deficits) to happen in continuous time. One possibility could be to adapt the model presented by Carré [2000].

She already mentioned the extension of introducing uncertainty about the enforcement of a deadline into a war of attrition. Considering the current state of the art in the *European Union* this seems to be an approach that would well fit reality. It is conjecturable that there would arise interesting effects concerning the interaction of the height of the fine and the timing of stabilization.

Second, one could consider a reversal of timing, i.e. the respective wars of attrition are played before the decision about the fine is taken. Here, I presume that the optimal fine would be zero and that thus an SGP would make no difference. The same effect would arise if the CB acts first.

Allowing for more heterogeneity among countries is another promising direction for further research. In such a context, one could analyze the possible impact of enlargement on stabilization decisions (see for example Beetsma and Bovenberg [2003] for strategic debt accumulation or Hefeker [2001] for structural fiscal reform incentives in the presence of heterogeneity).

Finally, one could add another stage to the game in which the interest groups themselves are allowed to vote about the introduction of the fine and about its height. Since the interest groups decide about stabilization, why should they not determine the rest? In addition, one could introduce correlation between the information parameters of the respective interest groups in order to study the influence different voting rules have on the decision.

Chapter 4

To join or not to join? Monetary Unification & Decision Procedures

4.1 Introduction

This chapter deals with the decision of a country to join an already existing international organization or decision making body. I assume that the candidate economy is influenced by union wide shocks as well as by country-specific ones and that decisions taken inside the union exert influence on the economic situation of the potential member – even if it remains separate. On the other hand, the economy of the union remains unaffected by shocks and policy of the candidate. Thus, I model a relation with asymmetric spillover effects between the economies. This asymmetry tries to represent the position of a potential member vis-à-vis an already existing organization. The economic players are the central banks of the respective countries. Their preferences are characterized by a certain degree of conservatism. The question is how different decision procedures employed after unification perform compared to the situation in which the potential member remains separate. This depends on the one hand on the degree of interdependencies between the economies and on the other on the degree of conservatism of the respective central banks.

In a three-country model, all countries experience shocks and the respective central bankers have to decide about monetary policy which the central bankers can control directly. I consider two basic scenarios: separation and unification. Under the separation scenario, the original union members determine jointly their preferred decision first. The potential member observes their decision and determines its own desired policy taking into account the union decision. The candidate country remains separate, but experiences spillovers from shocks and policy of the union. On the other hand, it has no influence on the union decision.

In the basic set-up of unification the candidate country obtains full membership in a union consisting of three countries. Only one policy instrument remains available for the union as a whole. Here, I study three different decision procedures. These differ in the degree of influence they assign to union members in joint decision taking. The first one determines the union wide decision in order to maximize welfare, i.e. the sum of individual utilities. According to the second mechanism the median of desired policies of the three members will be implemented. Finally, I consider a procedure which I call *rotation*. It involves an exogenously fixed probability with which the new member may determine the union wide decision alone. The main differences between these procedures are how they affect the probability of being decisive and how they deal with extreme positions.

The basic trade-off concerns independence of policy making versus gains in influence. Assuming that the candidate economy is influenced by union wide shocks as well as by country-specific ones, accession offers the opportunity to influence decisions taken inside the union, but at the same time reduces the possibility to react independently on country-specific shocks. Assuming further that not only shocks but also decisions taken inside the union have a certain influence on the economic situation of the candidate, the candidate central banker has to react on union decisions in order to support the national economy. This may involve not negligible costs, particularly if the candidate – as a non-member – has no possibility to influence the decisions of the union.

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The main result of this chapter is that all unification scenarios perform poorly compared to the separation scenario from the perspective of the candidate. Entry will be never optimal if the union wide decision maximizes welfare. If the decision is taken according to the median, I am able to identify a region of conservatism and spillovers where it will be profitable for the candidate to join the union. Only if central bankers care a lot about unemployment and if interdependencies are strong between the economies, then it will pay to delegate the decision to the median. The median procedure ignores extreme positions in decision making and this will be profitable if preferences are close. Under rotation there exists a region of combinations of spillovers and conservatism for which no decision power could lead to entry. For the remaining combinations the new member would require a high probability of being decisive to make entry profitable. However, there exist combinations of interdependencies and conservatism that make entry profitable – although transaction costs are not explicitly modelled.

4.1.1 Relation to literature

This chapter tries to combine different aspects of monetary unification. In tradition with the literature on *Optimum Currency Areas* (*OCA*) originated by Mundell [1961] I analyze the costs and benefits of joining a monetary union. In *OCA* models, monetary unification leads to higher adjustment and lower transaction costs. I capture this by focusing on the trade-off between the costs of giving up independent monetary policy making (for an empirical assessment of these costs see e.g. Schuberth and Wehinger [1998]) and the benefits of gaining influence on union decisions. For review and discussion of the *OCA* literature see, for example De Grauwe [1992] or Masson and Taylor [1992]. Research has also focused on the relation of European monetary unification to *OCA* criteria (e.g. Bayoumi and Eichengreen [1997], Caporale [1993] or Eichengreen [1997]).

There exists an extensive literature focusing on different aspects of two institutional set-ups: flexible, discretionary exchange rate regimes versus monetary unions (see however Lane [2000] who compares the stabilization

properties of a currency union versus alternative exchange rate regimes and Martin [1995] who considers three institutional structures). The main focus is either on the implications for national fiscal policy after abandoning the exchange rate as a tool to cope with shocks (e.g. Kletzer [1998] or Swedish Government Report [2002]) or on the incentives for economic reform implied by monetary unification (e.g. Calmfors [2001] or Sibert and Sutherland [2000]). In contrast to my model, none of the above distinguishes between different decision procedures used inside a monetary union.

Concerning the comparison of different decision procedures in central banks, most of that literature focuses on how national interests in the ECB Council influence the common decision. For example Aksoy et al. [2002] analyze the impact of economic and institutional asymmetries on the effectiveness of monetary policy, Grüner [1999] considers the relationship between inflation and unemployment when the council members are either concerned with union wide aggregates or with domestic conditions, Hefeker [2003] assesses the economic implications of different decision making procedures (federal, purely national or a combination of both) and Von Hagen and Süppel [1994] compare monetary stabilization policies for alternative types of appointees (i.e. having different mandates) in the ECB Council. In my model, the presence of national interests is not assumed, but the different decision procedures can be interpreted as to foster national influences in different ways.

Related to this model are also papers that deal with the size of international unions. For example Kohler [2002] extends a shock stabilization game by non-cooperative coalition formation. Here, a limit to the stable coalition size arises due to a lack of demand for membership since the coalition formation process creates positive spillovers for the outsiders. In my model, decisions taken inside the union affect non-members in a negative way. Albertin [2001] analyzes the endogenous formation path of regional blocs. Formalizing both the incentives for member countries to enlarge and for non-members to join the bloc, there exists an upper bound to the size of the bloc (due to a binding supply-side constraint) and a trade-off between depth and width of integration. Different decision procedures play no role

4.2 The Model

in Albertin's analysis. In Alesina et al. [2001] the trade-off between the benefits of coordination and the loss of independent policy making endogenously determines the size, composition and scope of international unions. Decisions about common policies are taken by majority, i.e. the authors concentrate on the median of desired policies and again, do not consider the influence of different decision procedures. The equilibrium size is decreasing in the heterogeneity of member countries and in the scope of common policies. In contrast to this set-up, I focus on one specific common policy and do not allow for heterogeneous preferences. But the effect of heterogeneity in Alesina et al. is similar to the influence of interdependencies in my model: the "closer" economies are, the more profitable is membership.

The remainder of this chapter is organized as follows: Section 2 introduces the basic set-up of the model for the separation and unification scenarios. The three different decision mechanisms I aim to study under unification are presented and expected utility is calculated for each situation. The next section compares these results for different degrees of interdependencies and conservatism. Section 4 concludes. All proofs are relegated to Appendix C.2.

4.2 The Model

Consider a world consisting of three countries, i = 1, 2, 3. Countries 1 and 2 form a union, country 3 is a potential member of this union. The economic players are the respective central banks. Their loss functions are defined in inflation and unemployment similar to the one introduced by Barro and Gordon ([1983a] and [1983b]). Each country experiences a shock ε_i . These shocks are independently and uniformly distributed over [-1,1]. The vector of shocks is ε . Decisions $x_U \in \mathbb{R}$ and $x_C \in \mathbb{R}$ about monetary policy have to be taken by the union and the candidate central banker, respectively. For simplicity, I assume that the central bankers can control the inflation rate directly.

Consider two different scenarios. First, I deal with a situation in which country 3 remains separate and takes the decision of the union as given

when choosing its policy. I describe the decisions of the central bankers and calculate expected utility of the potential member. After that, the same analysis is conducted for full membership of country 3 considering three different decision procedures used after unification.

4.2.1 Separation

If country 3 remains outside the union, the set-up is modeled as a *Stackel-berg* game: First, the central banker of the union chooses his decision x_U for countries 1 and 2. After observing that decision, the central banker of country 3 sets his policy x_C – taking the decision x_U as given. The advantage of the union moving first should capture a situation in which the central banker of the candidate economy has to react on union decisions in order to support the national economy.³⁸

Consider countries 1 and 2: Preferences of each union member are characterized by the following von-Neumann-Morgenstern utility function in inflation and unemployment

$$u_i(x_U, \varepsilon_i) = -\left(\sigma x_U^2 + (1 - \sigma)(x_U - \varepsilon_i)^2\right), \ i = 1, 2.$$
 (20)

The parameter σ measures the degree of conservatism, i.e. the weight given to output and price stability, respectively. For $\sigma = 0$, unemployment is the exclusive goal, whereas for $\sigma = 1$ inflation is the sole target.

The decision x_U is jointly determined by the members of the union, i.e. by countries 1 and 2. The central banker of the union chooses x_U for the union as a whole to maximize the sum of utilities³⁹, i.e.

³⁸However, it is also technically impossible to consider the Cournot game among countries like in Canzoneri and Gray [1985] due to the one-sided specification of spillovers.

³⁹Note that this specification yields the same result as if the central banker would maximize the average of individual utilities, i.e. $\frac{u_U(x_U,\varepsilon_1,\varepsilon_2)}{2}$, or subject to the average of country-specific shocks, i.e. $\widetilde{u}_U(x_U,\varepsilon_1,\varepsilon_2) = -\left(\sigma x_U^2 + (1-\sigma)\left(x_U - \frac{\varepsilon_1 + \varepsilon_2}{2}\right)^2\right)$.

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$$u_{U}(x_{U}, \varepsilon_{1}, \varepsilon_{2}) = \sum_{i=1}^{2} u_{i}(x_{U}, \varepsilon_{i})$$

$$= -\left(\sigma x_{U}^{2} + (1 - \sigma)(x_{U} - \varepsilon_{1})^{2}\right)$$

$$-\left(\sigma x_{U}^{2} + (1 - \sigma)(x_{U} - \varepsilon_{2})^{2}\right).$$

This yields as a solution

$$x_U^* = \frac{1}{2} (1 - \sigma) (\varepsilon_1 + \varepsilon_2).$$

After observing the country-specific shocks, this decision will be implemented for the union. Depending on the degree of the preference parameter σ the union central banker tries to offset the average of shocks with his decision.

The basic structure of the above preferences applies to the candidate country as well, i.e.

$$u_3(x_C, x_U, \varepsilon) = -\left(\sigma x_C^2 + (1 - \sigma)\left((1 - \alpha)\left(x_C - \varepsilon_3\right) + \alpha\left(x_U - \frac{\varepsilon_1 + \varepsilon_2}{2}\right)\right)^2\right).$$

Again, $\sigma \in [0, 1]$ measures the preferences for output and price stability of the central banker. For simplicity, assume that the preferences of the union and candidate central banker are identical.

In contrast to the preferences of the union members, here the term relating to unemployment allows for spillover effects. The parameter $\alpha \in [0, 1]$ measures the extent to which shocks of countries 1 and 2 and the decision of the union central banker x_U affect the utility of country 3.⁴⁰ For $\alpha = 0$ there exist no spillovers, shocks and monetary policy of the union leave the candidate economy unaffected. On the other hand, $\alpha = 1$ describes a situation in which unemployment in country 3 depends solely on shocks

 $^{^{40}}$ In principle, the spillover parameter α could be any real number ($\alpha \in \mathbb{R}$). The restriction to $\alpha \in [0,1]$ means that I consider only positive spillovers (prosper-thy-neighbor). This a priori excludes any beggarthy-neighbor behavior like in Canzoneri and Gray [1985].

and decision of the union, whereas its own decision on monetary policy only affects inflation. $\alpha = \frac{2}{3}$ leads to equal weighting of candidate and union. For example, α might measure the degree of demand or productivity spillover effects between the members of the *European Union*.⁴¹

This specification of preferences captures a situation in which the economy of the potential member may be in a relatively weak position. Its economic situation is influenced by the shocks as well as by the policy obtaining inside the union, whereas the union itself does not experience any spillover effects from the candidate.⁴² In this sense, this three-country model may be interpreted as describing the situation of a potential member vis-à-vis a union consisting of many countries.

The central banker of the candidate country observes the union decision x_U^* and takes it as given (acts as Stackelberg-Follower). He chooses x_C to maximize

$$u_3(x_C, x_U^*, \varepsilon) = -\left(\sigma x_C^2 + (1 - \sigma)\left((1 - \alpha)\left(x_C - \varepsilon_3\right) + \alpha\left(x_U^* - \frac{\varepsilon_1 + \varepsilon_2}{2}\right)\right)^2\right).$$

As a solution we get

$$x_C^* = \frac{1}{2} (1 - \sigma) (\alpha - 1) \frac{\sigma \alpha (\varepsilon_1 + \varepsilon_2) + 2\varepsilon_3 (1 - \alpha)}{\sigma (\alpha^2 - 2\alpha) - (1 - \alpha)^2}.$$

Depending on the degree of conservatism and of spillover effects the decision is taken as to deal with the shocks. Consider $\sigma=0$: The sole target of monetary policy in this situation is to reduce unemployment. Since the candidate central banker has no influence on the decision of the union, his only possibility is to conquer the country-specific shock. Accordingly, he implements $x_C^* = \varepsilon_3$. For $\sigma = 1$ the decision is $x_C^* = 0$ what is due to the fact that the sole target of the central banker is price stability. The same decision is implemented for $\alpha = 1$. This time the reason is the lack

⁴¹For an empirical assessment of the asymmetry of shocks, see e.g. Clark and Shin [1998] or Frankel and Rose [1998].

 $^{^{42}}$ This specification captures one-sided interdependencies, but note that it is not identical to assuming correlation of shocks.

4.2 The Model

of influence on unemployment, since this solely depends on the situation obtaining inside the union. If the economy of the potential member is independent, i.e. $\alpha = 0$, then $x_C^* = (1 - \sigma) \varepsilon_3$. In this case, the central banker solely decides according to his preferences concerning inflation and unemployment.

Substituting for x_U^* and x_C^* , expected utility of country 3 depending on σ and α is given by

$$S(\alpha, \sigma) = E[u_{3}(x_{C}^{*}, x_{U}^{*}, \varepsilon)]$$

$$= \frac{1}{8} \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} u_{3}(x_{C}^{*}, x_{U}^{*}, \varepsilon) d\varepsilon_{1} d\varepsilon_{2} d\varepsilon_{3}$$

$$= \frac{1}{6} \sigma (1 - \sigma) \frac{\sigma^{2} \alpha^{2} + 2 (1 - \alpha)^{2}}{\sigma (\alpha^{2} - 2\alpha) - (1 - \alpha)^{2}}.$$
(21)

For $\sigma = 0$ and for $\sigma = 1$, expected utility is zero. This is due to the fact that in both cases there exists no conflict of interest for the central banker. Either the sole responsibility of monetary policy is to offset the country-specific shock or price stability is the sole target. For intermediate preferences $(\sigma \in (0,1))$ it is never possible to secure zero losses.

4.2.2 Unification

Consider now a situation in which country 3 obtains union membership. The basic structure of preferences remains unchanged, but only one policy instrument $x \in \mathbb{R}$ is now available to react to shocks. Accordingly, the utility function of country 3 changes to

$$u_3(x,\varepsilon) = -\left(\sigma x^2 + (1-\sigma)\left((1-\alpha)\left(x-\varepsilon_3\right) + \alpha\left(x-\frac{\varepsilon_1+\varepsilon_2}{2}\right)\right)^2\right), \quad (22)$$

whereas the preferences of countries 1 and 2 are still described by (20), i.e.

$$u_i(x,\varepsilon_i) = -\left(\sigma x^2 + (1-\sigma)\left(x-\varepsilon_i\right)^2\right), \ i = 1, 2.$$
 (23)

I consider three different decision procedures according to which the decision x for the union as a whole is determined.

Welfare Maximization

The first procedure takes the decision x_W as to maximize the sum of utilities, i.e.

$$u_W(x_W, \varepsilon) = \sum_{i=1}^3 u_i(x_W, \varepsilon)$$

$$= -\left(\sigma x_W^2 + (1 - \sigma) (x_W - \varepsilon_1)^2\right)$$

$$-\left(\sigma x_W^2 + (1 - \sigma) (x_W - \varepsilon_2)^2\right)$$

$$-\left(\sigma x_W^2 + (1 - \sigma) ((1 - \alpha) (x_W - \varepsilon_3) + \alpha (x_W - \frac{\varepsilon_1 + \varepsilon_2}{2}))^2\right).$$

This yields as joint decision

$$x_W^* = \frac{1}{6} (1 - \sigma) ((2 + \alpha) (\varepsilon_1 + \varepsilon_2) + 2\varepsilon_3 (1 - \alpha)).$$

If the exclusive target of all central bankers is price stability ($\sigma = 1$), then the joint decision will be $x_W^* = 0$. When there exist no spillover effects in country 3 ($\alpha = 0$), all three countries act according to identical preferences. Moreover, equal weight is assigned to each country's preferences and the joint decision amounts to $x_W^* = \frac{1}{3} (1 - \sigma) (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$. Depending on the degree of the preference parameter σ the union central banker tries to offset the average of shocks with his decision. If $\alpha = 1$, then $x_W^* = \frac{1}{2} (1 - \sigma) (\varepsilon_1 + \varepsilon_2)$: the shock in country 3 plays no role in determining the decision. Note that this decision is the same as the one taken in a union consisting only of countries 1 and 2.

Under this procedure expected utility of country 3 is given by

$$U_{W}(\alpha, \sigma)$$

$$= E \left[u_{3}(x_{W}^{*}, \varepsilon)\right]$$

$$= \frac{1}{8} \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} u_{3}(x_{W}^{*}, \varepsilon) d\varepsilon_{1} d\varepsilon_{2} d\varepsilon_{3}$$

$$= \frac{1}{18} (\sigma - 1) \left((4 + 5\sigma) \alpha^{2} - (4\sigma + 8) \alpha + 2\sigma + 4 \right). \tag{24}$$

4.2 The Model

This function is concave in α and convex in σ . Concavity in spillovers means that under welfare maximization it is good in terms of expected utility to experience intermediate interdependencies. In contrast, convexity in the conservatism parameter tells us, that here clear preferences in either direction are preferable. Again, for $\sigma = 1$, zero losses can be secured. But in contrast to expected utility under separation (21), this is not true for $\sigma = 0$. Despite the fact that output stability is the sole target of all central bankers, it will not be possible to conquer all shocks if only one policy instrument remains available.

Median

Under this procedure, the decision is determined by the median of the desired policies of the three countries. The central bankers of country 1 or 2 choose x_i^* in order to maximize (23):

$$u_i(x, \varepsilon_i) = -\left(\sigma x^2 + (1 - \sigma)(x - \varepsilon_i)^2\right), i = 1, 2.$$

The solution to this problem is given by

$$x_i^* = (1 - \sigma) \varepsilon_i, \ i = 1, 2.$$

If this desired policy of country 1 or 2 happens to be the median desired policy, it will be implemented as monetary policy for the whole union.

The central banker of the new member country chooses x_3^* in order to maximize (22):

$$u_3(x,\varepsilon) = -\left(\sigma x^2 + (1-\sigma)\left((1-\alpha)\left(x-\varepsilon_3\right) + \alpha\left(x-\frac{\varepsilon_1+\varepsilon_2}{2}\right)\right)^2\right).$$

This yields as a decision

$$x_3^* = \frac{1}{2} (1 - \sigma) \left(\alpha \left(\varepsilon_2 + \varepsilon_1 \right) + 2\varepsilon_3 \left(1 - \alpha \right) \right)$$

and x_3^* will be implemented if it constitutes the median policy of the union.

An important feature of this decision procedure is that the median policy depends on the degree of interdependencies α and on the realization of shocks. Therefore, it is endogenously determined which policy is "in the middle". Since this implicitly defines the probability of being pivotal, the decision power of the participating countries is endogenously determined as well.

Letting x_M^* denote the vector of preferred policies, i.e. $x_M^* = (x_1^*, x_2^*, x_3^*)$, expected utility⁴³ of country 3 is given by

$$U_{M}(\alpha, \sigma)$$

$$= E \left[u_{3}(x_{M}^{*}, \varepsilon) \right]$$

$$= \frac{1}{30} \frac{1 - \sigma}{\alpha - 2} \left[(\sigma - 16) \alpha^{3} + (2\sigma + 48) \alpha^{2} - (2\sigma + 48) \alpha + 4\sigma + 16 \right].$$
(25)

The basic behavior of this function is similar to expected utility (24) under the welfare maximizing procedure: $U_M(\alpha, \sigma)$ is concave in α and convex in σ , obtains zero expected utility for $\sigma = 1$.

Rotation

Finally, I consider the simplified mathematical representation of a so-called rotation procedure.⁴⁴ With probability $\rho \in [0,1]$ country 3 determines the union wide decision, with probability $\frac{1-\rho}{2}$ the desired policy of either country 1 or country 2 is implemented. For $\rho = 0$ country 3 has no influence on the common decision, whereas for $\rho = 1$ the new member acts as a dictator.

This specification of probabilities may be interpreted as giving the new member a permanent voting right, whereas the original members would have to share theirs. In comparison to the two decision procedures considered before, here the probability of actually determining the decision is

⁴³The detailed derivation of expected utility is given in Appendix C.1.

⁴⁴Rotation is one of the procedures discussed as reform option for an enlarged *ECB Council*. It involves that not every central bank governor would have a right to vote at each meeting. Other reform proposals include representation or executive decision. For a detailed analysis and discussion of reform options for decision procedures in an enlarged *European Monetary Union* see e.g. Baldwin et al. [2001].

4.2 The Model

not endogenously determined by the procedure and the preferences, but is exogenously fixed.

Expected utility of country 3 under this scenario can be decomposed into three parts, i.e. expected utility if the desired policy of country 1 is implemented, expected utility if the desired policy of country 2 is implemented and expected utility if its own desired policy is set into force, i.e.

$$U_{R}(\alpha, \sigma, \rho) = \frac{1 - \rho}{2} E\left[u_{3}(x_{1}^{*}, \varepsilon)\right] + \frac{1 - \rho}{2} E\left[u_{3}(x_{2}^{*}, \varepsilon)\right] + \rho E\left[u_{3}(x_{3}^{*}, \varepsilon)\right].$$

Since the decision problems of countries 1 and 2 are symmetric⁴⁵, this simplifies to

$$U_{R}(\alpha, \sigma, \rho)$$

$$= (1 - \rho) E \left[u_{3}(x_{1}^{*}, \varepsilon)\right] + \rho E \left[u_{3}(x_{3}^{*}, \varepsilon)\right].$$

Consider these terms separately. The desired decision of country 1 is

$$x_1^* = (1 - \sigma) \varepsilon_1.$$

If this is implemented, expected utility of country 3 is given by

$$E[u_{3}(x_{1}^{*},\varepsilon)] = \frac{1}{8} \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} u_{3}(x_{1}^{*},\varepsilon) d\varepsilon_{1} d\varepsilon_{2} d\varepsilon_{3}$$
$$= \frac{1}{6} (\sigma - 1) \left(3\alpha^{2} + (2\sigma - 6)\alpha - 2\sigma + 4 \right). \tag{26}$$

The desired policy of country 3 is

$$x_3^* = \frac{1}{2} (1 - \sigma) \left(\alpha \left(\varepsilon_2 + \varepsilon_1 \right) + 2\varepsilon_3 \left(1 - \alpha \right) \right)$$

and

$$E\left[u_{3}\left(x_{3}^{*},\varepsilon\right)\right] = \frac{1}{8} \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} u_{3}\left(x_{3}^{*},\varepsilon\right) d\varepsilon_{1} d\varepsilon_{2} d\varepsilon_{3}$$
$$= \frac{1}{6} \sigma\left(\sigma-1\right) \left(3\alpha^{2}-4\alpha+2\right). \tag{27}$$

⁴⁵Without loss of generality I consider expected utility of country 3 if the desired policy of country 1 is implemented.

Thus, overall expected utility under the rotation procedure is given by

$$U_R(\alpha, \sigma, \rho)$$

$$= \frac{1}{6} (\sigma - 1)$$

$$\left[3 \left(\rho \sigma - \rho + 1 \right) \alpha^2 - 2 \left(3 \rho \sigma - 3 \rho - \sigma + 3 \right) \alpha + 4 \rho \sigma - 4 \rho - 2 \sigma + 4 \right].$$

This function does not only depend on α and σ , but also on the probability ρ . It is increasing in ρ meaning that the higher the probability of being decisive for the new member, the higher is its expected utility. Again, $U_R(\alpha, \sigma, \rho)$ is concave in α and obtains zero expected utility for $\sigma = 1$.

4.3 Comparison

In this section, I compare expected utility of the potential member under the two different scenarios and the three decision mechanisms.⁴⁶ Would a potential member base its entry decision exclusively on these terms, there would be little scope for a union of three.

I start the comparison with the following observation: For $\sigma=1$ and for all degrees of interdependency, separation and unification yield the same expected utility irrespective of the decision mechanism employed after unification. The reason is that if central bankers do not care about unemployment, then shocks and uncertainty will not influence their decisions. Thus, under each scenario the same decision is taken – irrespective of the decision procedure. This leads to zero losses since there exists no conflict of interest regarding inflation and unemployment for all central bankers.

The remaining part of this section deals with the effect different decision procedures will have on expected welfare of the candidate if central bankers care about unemployment.

⁴⁶I concentrate solely on the perspective of the potential member. I do not consider the net gains of monetary unification which could eventually lead to compensation payments. However, I return to this topic in the conclusion of this chapter.

4.3.1 Welfare Maximization

For this procedure I obtain a clear-cut result in favor of separation.

Proposition 10 If the decision after unification is taken as to maximize the sum of individual utilities, then there exists no combination of α and σ such that unification yields more expected utility than separation for the candidate.

If the decision about monetary policy after unification is taken jointly, this procedure will give the new member not enough influence on the common decision as to compensate for giving up its independence.

4.3.2 Median

Under this decision procedure the probability of the new member to be pivotal after unification increases. This leads to the following proposition.

Proposition 11 If the decision procedure after unification implements the median, then there exist combinations of α and σ such that unification yields more expected utility than separation for the candidate country.

Figure 10 shows implicitly the combinations of (α, σ) for which the difference in expected utility is zero. Entry is only profitable for a small range of combinations which is characterized by large spillovers and low weight on inflation: the economies have to be "close" and shocks have to be important. Under these circumstances, it pays to delegate the decision to the median, since the median procedure ignores extreme positions in decision making.

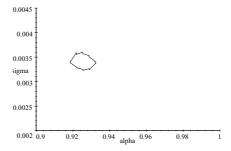


Figure 10: $(\alpha, \sigma) : S(\alpha, \sigma) = U_M(\alpha, \sigma)$

4.3.3 Rotation

Finally, considering the rotation procedure, I obtain a more optimistic result regarding the entry decision of a potential union member.

Proposition 12 (i) There exist combinations of α and σ for which separation yields more expected utility than unification $\forall \rho \in [0,1]$, (ii) for all other (α, σ) exists $\rho(\alpha, \sigma)$ such that separation and unification yield the same expected utility for country 3 and (iii) $\forall \rho > \rho(\alpha, \sigma)$ unification yields more expected utility for country 3 than separation $\forall (\alpha, \sigma)$.

The first part of this proposition tells us, that even if country 3 may determine the union wide decision after unification alone ($\rho = 1$), there will exist combinations of α and σ such that country 3 would prefer not to join that union.⁴⁷ Interpreting the boundary of such combinations as a function $\sigma(\alpha)$, this function is strictly decreasing in α . The smaller the spillovers, the larger has to be the weight given to inflation to make separation always preferable.

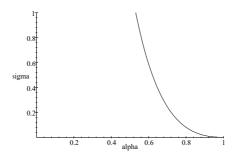


Figure 11: $\sigma(\alpha): S(\alpha, \sigma) = U_R(\alpha, \sigma, 1)$

The second part of Proposition 12 defines for all other combinations of α and σ a probability $\rho(\alpha, \sigma)$ of being pivotal which makes the candidate indifferent between no entry and entry. In principle, we can conclude that the potential member requires a high probability to be pivotal in order to

⁴⁷On first sight, one would expect that it is always possible for country 3 to secure itself zero losses when it acts as a dictator after unification. That this is not the case is due to the fact that after unification only one policy instrument remains available.

make entry profitable.⁴⁸ That the probability has to be relatively high can be seen by considering $\rho(\alpha, \sigma)$ at the corner values of the parameters α and σ . When there exist no interdependencies, i.e. $\alpha = 0$, the only way to make the candidate indifferent between entry and separation is to give him an exclusive decision right irrespective of the degree of central bank conservatism, i.e. $\rho(0,\sigma) = 1$. The same is true for $\sigma = 0$, central bankers who care solely about unemployment irrespective of the degree of interdependencies. If the candidate economy experiences the maximum amount of spillovers, then there will exist no probability to reach indifference, since $\rho(1,\sigma) = 1 + \sigma \geq 1 \,\forall \,\sigma$. Finally, if all central banks are conservative $(\sigma = 1)$, then $\rho(\alpha, 1)$ lies below 1 for small α :

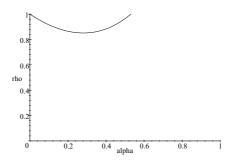


Figure 12: $\rho(\alpha, 1) : S(\alpha, \sigma) = U_R(\alpha, \sigma, \rho)$

4.3.4 Welfare Maximization – Median

Now, I sidestep a little from the main focus of my analysis and compare the relative performance of the welfare maximization and the median procedure. This comparison has no direct influence on entry decisions, but is related to it in the following way. The decision procedure employed in a union is of interest to any potential member, since the chosen procedure determines its influence on common decisions. Imagining that there are other factors that affect entry decisions, the relative performance of the different procedures becomes important. Since all countries in my model

⁴⁸This result resembles the one of Casella [1992]. In a general equilibrium model of monetary union she finds that a small economy will not enter the union unless its influence on union decisions is more than proportional to its size.

experience the same preferences for inflation and unemployment, this relative performance is independent of σ . It solely depends on the degree of interdependencies between the economies.

Proposition 13 (i) There exists α_1 such that for $\alpha < \alpha_1$ taking the decision as to maximize welfare yields more expected utility for country 3, (ii) there exists α_2 such that for $\alpha > \alpha_2$ the median mechanism yields more expected utility for country 3 and (iii) $\alpha_1 = \alpha_2 = \alpha^* = \frac{2}{7}$.

If the degree of interdependencies between the economies is small (small α), it will be profitable for the candidate to take account of all views when determining the union wide decision. This is due to the fact that under this procedure, extreme policies influence the final decision, whereas under the median procedure they are disregarded, since there the final decision solely depends on the median desired policy. If, on the other hand, the spillovers lie above some threshold (here $\alpha^* = \frac{2}{7}$), then the economies are sufficiently close in terms of preferences and extreme policies become rare. Moreover, since the probability of being decisive increases in α under the median procedure, this increase in power contributes to the result as well.⁴⁹

4.4 Conclusion

This chapter compares the influence of different decision procedures on unilateral entry decisions into existing monetary unions. If the candidate economy is influenced by union wide shocks as well as by country-specific ones and if decisions taken inside the union exert influence on the economic situation of the potential member, how will the decision procedures used after unification affect the decision of the potential member to join the union? My analysis concentrates on three specific decision procedures,

⁴⁹On first sight, these results seem to contradict those of Section 2.4, but in principle both are obtained by similar reasoning. The sole difference lies in the point of view: here, with weak interdependencies the *potential member* prefers to take (his) extreme position into account, whereas in Section 2.4 for society as a whole it is better to ignore extremes. A similar argument applies for the case of strong interdependencies.

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i.e. welfare maximization, median and rotation, and obtains the following: All unification scenarios perform relatively poor compared to separation irrespective of the decision procedure. The only chance to make entry profitable for a potential member is to grant it a merely exclusive decision right in common policies. But although transaction costs are not explicitly modelled, there exist combinations that make entry desirable.

Starting from this research there are three directions to proceed. The first extension concerns different parameter specifications. Considering $\alpha \in \mathbb{R}$ would allow for beggar-thy-neighbor behavior, like for example in Canzoneri and Gray [1985]. A similar effect may evolve if we consider the possibility of shocks and policy exerting different influence on the preferences of the potential member. Considering different parameter specifications, it would also be interesting to analyze the effect of country-specific preferences regarding inflation and unemployment.

A second extension would be to allow for two-sided spillovers. Not only the potential member is influenced by shocks and policy of the union but the same is true vice versa. Again, this would lead to different decisions of the respective central banks and to more strategic interaction.

Finally, a question I did not consider so far is the possibility or impossibility of compensation payments. Here, in a first step, one would have to assess if unification leads to any net gains, i.e. if the sum of expected utilities is increased after unification. This would create scope for transfers that could be offered to the potential member to make it enter the union. My intuition is that in this set-up there will be little opportunity for such side-payments. However, this may change if we allow for two-sided spillovers.

Appendix A

Chapter 2: Collective Decisions with Interdependent Valuations

Proof of Lemma 4:

Under the median mechanism the announcement strategies $\hat{\theta}_i(\theta_i)$ are continuous in the parameter α . Since the decision $x_{Median}(\hat{\theta})$ is continuous in the announcements, it is so in α . Finally, individual utility is continuous in the decision (and therefore in α) and in the individually desired policy θ_i^* , which is continuous in α as well. Thus the sum of expected utilities is continuous in α under the median mechanism.

Proof of Lemma 5:

Under the mean mechanism agents announce according to $\hat{\theta}_i(\theta_i)$ = $n(1-\alpha)\theta_i$. Thus, $x_{Mean} = (1-\alpha)\sum_{i=1}^n \theta_i$.

(i) CONTINUITY: The sum of expected utilities is given by

$$\sum_{i=1}^{n} E\left[-\left(x_{Mean} - \theta_{i}^{*}\right)^{2}\right]$$

$$= -nE\left[\left(\left(1 - \alpha\right) \sum_{i=1}^{n} \theta_{i} - \left(1 - \alpha\right) \theta_{i} - \frac{\alpha}{n-1} \sum_{j \neq i} \theta_{j}\right)^{2}\right]$$

$$= -nE\left[\left(\left(1 - \alpha\right) \sum_{j=1}^{n} \theta_{j} - \frac{\alpha}{n-1} \sum_{\substack{j=1\\j \neq i}}^{n} \theta_{j}\right)^{2}\right]$$

$$= -nE\left[\left(-\frac{\alpha n - n + 1}{n-1} \sum_{\substack{j=1\\j \neq i}}^{n} \theta_{j}\right)^{2}\right]$$

$$= -n\left(\frac{\alpha n - n + 1}{n-1}\right)^{2} E\left[\left(\sum_{\substack{j=1\\j \neq i}}^{n} \theta_{j}\right)^{2}\right]$$

which is continuous in α .

(ii) Monotonicity: Differentiating with respect to α yields

$$\frac{d\left(E\left[\sum_{i=1}^{n} - (x_{Mean} - \theta_{i}^{*})^{2}\right]\right)}{d\alpha} = -2n^{2} \frac{\alpha n - n + 1}{\left(n - 1\right)^{2}} E\left[\left(\sum_{\substack{j=1\\j \neq i}}^{n} \theta_{j}\right)^{2}\right]$$

which is larger than zero $\forall \alpha \in [0, \bar{\alpha}).$

(iii) Optimality:

$$\frac{d\left(E\left[\sum_{i=1}^{n}-(x_{Mean}-\theta_{i}^{*})^{2}\right]\right)}{d\alpha}=0\Longleftrightarrow\alpha=\frac{n-1}{n}=\bar{\alpha}.$$

Proof of Lemma 6:

(i) Continuity: The sum of expected utilities under the mean mechanism is given by

$$\sum_{i=1}^{n} E\left[-\left(x_{Mean} - \theta_{i}^{*}\right)^{2}\right] = -n\left(\frac{\alpha n - n + 1}{n - 1}\right)^{2} E\left[\left(\sum_{\substack{j=1\\j \neq i}}^{n} \theta_{j}\right)^{2}\right]$$
$$= -n\left(n - 1\right)\left(\frac{\alpha n - n + 1}{n - 1}\right)^{2} E\left[\theta_{i}^{2}\right]$$

which is continuous in n for n > 1.

(ii) Monotonicity: Differentiating with respect to n yields

$$\frac{d\left(E\left[\sum_{i=1}^{n} -(x_{Mean} - \theta_{i}^{*})^{2}\right]\right)}{dn} = -(\alpha n - n + 1)\frac{2\alpha n^{2} - 3\alpha n - 2n^{2} + 3n - 1}{(n-1)^{2}}E\left[\theta_{i}^{2}\right].$$

Since $\alpha n - n + 1 < 0$ and $2\alpha n^2 - 3\alpha n - 2n^2 + 3n - 1 < 0 \ \forall \ \alpha \in [0, \bar{\alpha})$, this is smaller than zero $\forall n > 1$.

(iii) Optimality:

$$\frac{d\left(E\left[\sum_{i=1}^{n} -(x_{Mean} - \theta_{i}^{*})^{2}\right]\right)}{dn} = 0 \Longleftrightarrow n = \frac{1}{1 - \alpha}.$$

Appendix B

Chapter 3: The European Stability and Growth Pact as a Device for adequate Stabilization

Proof of Lemma 9:

(i)

$$\frac{\partial T^{WA}\left(\cdot\right)}{\partial \beta} = \frac{-\alpha}{\left(1 - F\left(T^{WA}\right)\right)^{2} d} * f\left(T^{WA}\right) * \frac{\partial T^{WA}\left(\cdot\right)}{\partial \beta} + \frac{1}{2}$$

$$\frac{\partial T^{WA}\left(\cdot\right)}{\partial \beta} = \frac{1}{2\left[1 + \frac{\alpha}{\left(1 - F\left(T^{WA}\right)\right)^{2} d} * f\left(T^{WA}\right)\right]}$$

This is larger than zero, since α , d, $(1 - F(T^{WA}))^2$ and $f(T^{WA})$ are larger than zero.

(ii)

$$\frac{\partial T^{WA}\left(\cdot\right)}{\partial d} = \frac{\alpha}{\left(1 - F\left(T^{WA}\right)\right)^{2} d^{2}} \left(-f\left(T^{WA}\right) * \frac{\partial T^{WA}\left(\cdot\right)}{\partial d} d + \left(1 - F\left(T^{WA}\right)\right)\right)$$

$$\frac{\partial T^{WA}\left(\cdot\right)}{\partial d} \ = \ \frac{\alpha}{\left(1 - F\left(T^{WA}\right)\right)d^{2}\left[1 + \frac{\alpha}{\left(1 - F\left(T^{WA}\right)\right)^{2}d} * f\left(T^{WA}\right)\right]}$$

 \Rightarrow

$$\frac{\partial T^{WA}\left(\cdot\right)}{\partial d} > 0.$$

(iii)

$$\frac{\partial T^{WA}(\cdot)}{\partial \alpha} = \frac{-\left(1 - F\left(T^{WA}\right)\right) - \alpha * f\left(T^{WA}\right) * \frac{\partial T^{WA}(\cdot)}{\partial \alpha}}{\left(1 - F\left(T^{WA}\right)\right)^{2} d} \Leftrightarrow \frac{\partial T^{WA}(\cdot)}{\partial \alpha} = -\frac{1}{\left(1 - F\left(T^{WA}\right)\right) d\left[1 + \frac{\alpha}{d} \frac{f(T^{WA})}{\left(1 - F(T^{WA})\right)^{2}}\right]}$$

 \Rightarrow

$$\frac{\partial T^{WA}\left(\cdot\right)}{\partial\alpha}<0.$$

Proof of Lemma 10:

(i)

$$\frac{\partial T^{P}(\cdot)}{\partial \phi} = \left(1 - F(T^{P})\right) * f(T^{P}) * \frac{\partial T^{P}(\cdot)}{\partial \phi} * \phi + \frac{1}{2} \left(1 + \left(1 - F(T^{P})\right)^{2}\right) \\
- \frac{\alpha}{(1 - F(T^{P}))^{2} d} * f(T^{P}) * \frac{\partial T^{P}(\cdot)}{\partial \phi} \\
\Leftrightarrow \frac{\partial T^{P}(\cdot)}{\partial \phi} = \frac{1 + \left(1 - F(T^{P})\right)^{2}}{2\left(1 + \frac{\alpha}{(1 - F(T^{P}))^{2} d} * f(T^{P}) + (1 - F(T^{P})) * f(T^{P}) * \phi\right)}$$

$$\Rightarrow \frac{\partial T^{P}(\cdot)}{\partial \phi} > 0.$$

(ii)

$$\frac{\partial T^{P}(\cdot)}{\partial \beta} = \frac{-\alpha}{(1 - F(T^{P}))^{2} d} * f(T^{P}) * \frac{\partial T^{P}(\cdot)}{\partial \beta} + \frac{1}{2}$$

$$-\phi (1 - F(T^{P})) * f(T^{P}) * \frac{\partial T^{P}(\cdot)}{\partial \beta}$$

$$\Leftrightarrow \frac{\partial T^{P}(\cdot)}{\partial \beta} = \frac{1}{2 \left[1 + \frac{\alpha}{(1 - F(T^{P}))^{2} d} * f(T^{P}) + \phi (1 - F(T^{P})) * f(T^{P})\right]}$$

 \Rightarrow

$$\frac{\partial T^{P}\left(\cdot\right)}{\partial\beta}>0.$$

(iii)

$$\frac{\partial T^{P}\left(\cdot\right)}{\partial d} = \frac{\alpha}{\left(1 - F\left(T^{P}\right)\right) d^{2} \left[1 + \frac{\alpha}{\left(1 - F\left(T^{P}\right)\right)^{2} d} * f\left(T^{P}\right) + \phi\left(1 - F\left(T^{P}\right)\right) * f\left(T^{P}\right)\right]}$$

 \Rightarrow

$$\frac{\partial T^{P}\left(\cdot\right)}{\partial d} > 0.$$

(iv)

$$\frac{\partial T^{P}(\cdot)}{\partial \alpha} = \frac{-\left(1 - F\left(T^{P}\right)\right) - \alpha * f\left(T^{P}\right) * \frac{\partial T^{P}(\cdot)}{\partial \alpha}}{\left(1 - F\left(T^{P}\right)\right)^{2} d} \\
-\phi \left(1 - F\left(T^{P}\right)\right) * f\left(T^{P}\right) * \frac{\partial T^{P}(\cdot)}{\partial \alpha} \\
\Leftrightarrow \frac{\partial T^{P}(\cdot)}{\partial \alpha} = -\frac{1}{\left(1 - F\left(T^{P}\right)\right) d\left[1 + \frac{\alpha}{d} \frac{f(T^{P})}{\left(1 - F\left(T^{P}\right)\right)^{2}} + \phi\left(1 - F\left(T^{P}\right)\right) * f\left(T^{P}\right)\right]}$$

$$\Rightarrow \frac{\partial T^{P}(\cdot)}{\partial \alpha} < 0.$$

Proof of Lemma 11:

(i)

$$\frac{\partial T^{SGP}(\cdot)}{\partial \phi} = \frac{1}{2} - \frac{\alpha}{(1 - F(T^{PR}))^2 d} * f(T^{SGP}) * \frac{\partial T^{SGP}(\cdot)}{\partial \phi}
\Leftrightarrow \frac{\partial T^{SGP}(\cdot)}{\partial \phi} = \frac{1}{2\left(1 + \frac{\alpha}{(1 - F(T^{SGP}))^2 d} * f(T^{SGP})\right)}$$

 \Rightarrow

$$\frac{\partial T^{SGP}\left(\cdot\right)}{\partial\phi} > 0.$$

(ii)

$$\frac{\partial T^{SGP}\left(\cdot\right)}{\partial \beta} = \frac{-\alpha}{\left(1 - F\left(T^{SGP}\right)\right)^{2} d} * f\left(T^{SGP}\right) * \frac{\partial T^{SGP}\left(\cdot\right)}{\partial \beta} + \frac{1}{2}$$

$$\frac{\partial T^{SGP}\left(\cdot\right)}{\partial \beta} = \frac{1}{2\left[1 + \frac{\alpha}{d} \frac{f\left(T^{SGP}\right)}{\left(1 - F\left(T^{SGP}\right)\right)^{2}}\right]}$$

 \Rightarrow

$$\frac{\partial T^{SGP}\left(\cdot\right)}{\partial\beta} > 0.$$

(iii)

$$\frac{\partial T^{SGP}(\cdot)}{\partial d} = \frac{\alpha}{\left(1 - F\left(T^{SGP}\right)\right)^{2} d^{2}} \left(-f\left(T^{SGP}\right) * \frac{\partial T^{SGP}(\cdot)}{\partial d} * d + \left(1 - F\left(T^{SGP}\right)\right)\right)$$

$$\frac{\partial T^{SGP}\left(\cdot\right)}{\partial d} \;\; = \;\; \frac{\alpha}{\left(1 - F\left(T^{SGP}\right)\right) d^2 \left[1 + \frac{\alpha}{d} \frac{f\left(T^{SGP}\right)}{\left(1 - F\left(T^{SGP}\right)\right)^2}\right]}$$

 \Rightarrow

$$\frac{\partial T^{SGP}\left(\cdot\right)}{\partial d} > 0.$$

(iv)

$$\frac{\partial T^{SGP}\left(\cdot\right)}{\partial \alpha} = \frac{-\left(1 - F\left(T^{SGP}\right)\right) - \alpha * f\left(T^{SGP}\right) * \frac{\partial T^{SGP}\left(\cdot\right)}{\partial \alpha}}{\left(1 - F\left(T^{SGP}\right)\right)^{2} d} \Leftrightarrow \frac{\partial T^{SGP}\left(\cdot\right)}{\partial \alpha} = \frac{-1}{\left(1 - F\left(T^{SGP}\right)\right) d\left[1 + \frac{\alpha}{d} \frac{f\left(T^{SGP}\right)}{\left(1 - F\left(T^{SGP}\right)\right)^{2}}\right]}$$

 \Rightarrow

$$\frac{\partial T^{SGP}\left(\cdot\right)}{\partial\alpha}<0.$$

Proof of Lemma 12:

Assume that $T^{SGP} > T^P$.

It follows that

$$F\left(T^{SGP}\right) > F\left(T^{P}\right)$$

$$\Leftrightarrow$$

$$\frac{1}{1 - F\left(T^{SGP}\right)} > \frac{1}{1 - F\left(T^{P}\right)}$$

$$\Leftrightarrow$$

$$\frac{-\alpha}{(1 - F\left(T^{P}\right)) d} > \frac{-\alpha}{(1 - F\left(T^{SGP}\right)) d}$$

Since $\beta \geq 0, \phi \geq 0$ and $(1 - F(T^P))^2 \geq 0$, it follows

$$\begin{split} \frac{-\alpha}{\left(1-F\left(T^{P}\right)\right)d} + \frac{\beta}{2} + \frac{\phi}{2} + \frac{\phi}{2} \left(1-F(T^{P})\right)^{2} \\ \geq \frac{-\alpha}{\left(1-F\left(T^{SGP}\right)\right)d} + \frac{\beta}{2} + \frac{\phi}{2}. \end{split}$$

But this means that $T^P \geq T^{SGP}$, since $T^{SGP} := \frac{-\alpha}{(1-F(T^{SGP}))d} + \frac{\beta+\phi}{2}$ (see Proposition 9) and $T^P := \frac{-\alpha}{(1-F(T^P))d} + \frac{\beta}{2} + \frac{\phi}{2} \left(1 + \left(1 - F(T^P)\right)^2\right)$ (see Proposition 8). This contradicts the assumption that $T^{SGP} > T^P$. Hence, it must hold that $T^P \geq T^{SGP}$.

Proof of Corollary 4:

 $(i) \ \phi = 0 : T^{SGP} = T^{WA}$

(ii) T^{SGP} is strictly increasing in ϕ (see Lemma 11).

$$\Rightarrow \forall \phi > 0: T^{SGP} > T^{WA}$$
.

Proof of Lemma 13:

(*i*)

$$\frac{\partial T_{WA}^*\left(\cdot\right)}{\partial\overline{\theta}} = \frac{1}{2} - \frac{1}{2\sqrt{\left(\frac{\frac{\beta}{2} - \overline{\theta}}{2}\right)^2 + \frac{\alpha}{d}\left(\overline{\theta} - \underline{\theta}\right)}} \left(\frac{\overline{\theta} - \frac{\beta}{2}}{2} + \frac{\alpha}{d}\right).$$

It remains to show that

$$\sqrt{\left(\frac{\frac{\beta}{2} - \overline{\theta}}{2}\right)^2 + \frac{\alpha}{d}\left(\overline{\theta} - \underline{\theta}\right)} \ge \frac{\overline{\theta} - \frac{\beta}{2}}{2} + \frac{\alpha}{d}.$$

This is true whenever $\underline{\theta} \leq -\frac{\alpha}{d} + \frac{\beta}{2}$ holds.

(ii)

$$\frac{\partial T_{WA}^{*}\left(\cdot\right)}{\partial\underline{\theta}} = \frac{\alpha}{2d\sqrt{\left(\frac{\beta}{2} - \overline{\theta}}{2}\right)^{2} + \frac{\alpha}{d}\left(\overline{\theta} - \underline{\theta}\right)}} \ge 0.$$

(iii)

$$\frac{\partial T_{SGP}^{*}\left(\cdot\right)}{\partial\overline{\theta}}=\frac{1}{2}-\frac{1}{2\sqrt{\left(\frac{\beta+\phi}{2}-\overline{\theta}}\right)^{2}+\frac{\alpha}{d}\left(\overline{\theta}-\underline{\theta}\right)}}\left(\frac{\overline{\theta}-\frac{\beta+\phi}{2}}{2}+\frac{\alpha}{d}\right).$$

It remains to show that

$$\sqrt{\left(\frac{\frac{\beta+\phi}{2}-\overline{\theta}}{2}\right)^2+\frac{\alpha}{d}\left(\overline{\theta}-\underline{\theta}\right)}\geq \frac{\alpha}{d}+\frac{\overline{\theta}-\frac{\beta+\phi}{2}}{2}$$

This is true whenever $\underline{\theta} \leq -\frac{\alpha}{d} + \frac{\beta + \phi}{2}$ holds. (iv)

$$\frac{\partial T_{SGP}^{*}\left(\cdot\right)}{\partial\underline{\theta}} = \frac{\alpha}{2d\sqrt{\left(\frac{\underline{\beta}+\underline{\phi}}{2}-\overline{\theta}}\right)^{2} + \frac{\alpha}{d}\left(\overline{\theta}-\underline{\theta}\right)}} \geq 0.$$

Proof of Lemma 14:

The governments solve $\max_{\phi} EW_{SGP}(\theta)$ where $EW_{SGP}(\theta)$ is given by equation (15). With a uniform distribution we obtain

$$EW_{SGP}(\theta) = \frac{2d}{\left(\overline{\theta} - \theta\right)^2} \left(\overline{\theta} - T_{SGP}^*\right)^2 \left(\overline{\theta} + T_{SGP}^* - 2\beta\right).$$

Differentiating with respect to ϕ leads to

$$\frac{\partial EW_{SGP}(\theta)}{\partial \phi} = \frac{2d}{(\overline{\theta} - \underline{\theta})^2} \left\{ (\overline{\theta} - T_{SGP}^*)^2 * \frac{\partial T_{SGP}^*}{\partial \phi} - 2(\overline{\theta} - T_{SGP}^*) (\overline{\theta} + T_{SGP}^* - 2\beta) * \frac{\partial T_{SGP}^*}{\partial \phi} \right\}.$$

Setting this equal to zero yields as first order condition for the welfare maximizing fine $T^*_{SGP}\left(\phi^*\right) = -\frac{1}{3}\overline{\theta} + \frac{4}{3}\beta$, since $\frac{\partial T^*_{SGP}}{\partial \phi}$ is strictly larger than zero (see Lemma 11) and $T^*_{SGP}\left(\phi^*\right) = \overline{\theta}$ cannot be solved for $\phi \neq \infty$. Inserting T^*_{SGP} and solving for ϕ yields

$$\phi^* = \frac{1}{3} \left(5\beta - 2\overline{\theta} \right) + \frac{3}{2} \frac{\alpha}{d} \frac{\overline{\theta} - \underline{\theta}}{\overline{\theta} - \beta}.$$

Note that the second order condition is fulfilled as well, since the second partial derivative of expected welfare with respect to ϕ evaluated at ϕ^* is negative.

Proof of Lemma 15:

(i)

$$\frac{\partial \phi^*}{\partial d} = -\frac{3}{2} \frac{\alpha}{d^2} \frac{\overline{\theta} - \underline{\theta}}{\overline{\theta} - \beta} < 0$$

(ii)

$$\frac{\partial \phi^*}{\partial \overline{\theta}} = -\frac{2}{3} - \frac{3}{2} \frac{\alpha}{d} \frac{\beta - \underline{\theta}}{\left(\overline{\theta} - \beta\right)^2} < 0$$

(iii)

$$\frac{\partial \phi^*}{\partial \theta} = -\frac{3}{2} \frac{\alpha}{d} \frac{1}{\overline{\theta} - \beta} < 0$$

$$\frac{\partial \phi^*}{\partial \alpha} = \frac{3}{2} \frac{\overline{\theta} - \underline{\theta}}{d(\overline{\theta} - \beta)} > 0$$

(v)

$$\frac{\partial \phi^*}{\partial \beta} = \frac{5}{3} + \frac{3}{2} \frac{\alpha}{d} \frac{\overline{\theta} - \underline{\theta}}{(\overline{\theta} - \beta)^2} > 0$$

Proof of Lemma 16:

First, I consider expected welfare under the SGP and under the war of attrition for uniformly distributed types in general and then deal with different parameter constellations in turn.

Expected welfare under the SGP is continuous in ϕ and obtains its global maximum at ϕ^* (see Lemma 14). Additionally, $\lim_{\phi \to \infty} EW_{SGP} = -\infty$ and $\lim_{\phi \to \infty} EW_{SGP} = 0$. Moreover, it holds that

$$\phi^* > 0 \Leftrightarrow \frac{\alpha}{d} > \frac{2}{9} \frac{(\overline{\theta} - \beta)(2\overline{\theta} - 5\beta)}{\overline{\theta} - \theta} =: \frac{\widetilde{\alpha}}{d}$$

Expected welfare under the war of attrition is constant in ϕ and for a uniform distribution given by

$$EW_{WA} = \frac{2d}{\left(\overline{\theta} - \underline{\theta}\right)^2} \left(\overline{\theta} - T_{WA}^*\right)^2 \left(\overline{\theta} + T_{WA}^* - 2\beta\right).$$

It holds that

$$EW_{WA} > 0 \Leftrightarrow \frac{\alpha}{d} < \frac{(\overline{\theta} - \beta)(2\overline{\theta} - 3\beta)}{\overline{\theta} - \underline{\theta}} =: \frac{\widehat{\alpha}}{d}.$$

Furthermore, for $\phi = 0$ the war of attrition and the SGP yield the same expected welfare, since for $\phi = 0$ it holds that $T_{WA}^* = T_{SGP}^*$.

(i) Consider $\overline{\theta} \leq \frac{3}{2}\beta$.

$$\Rightarrow \frac{\widehat{\alpha}}{d} \le 0$$

$$\Rightarrow \forall \frac{\alpha}{d} : EW_{WA} \leq 0$$

$$\Rightarrow \forall \tilde{\phi} > 0 : EW_{SGP} > EW_{WA}.$$

Note that $\frac{\widetilde{\alpha}}{d} < 0$ for $\overline{\theta} \leq \frac{3}{2}\beta$ and thus $\phi^* > 0 \ \forall \frac{\alpha}{d}$.

(ii) Consider
$$\frac{3}{2}\beta < \overline{\theta} \leq \frac{5}{2}\beta$$
.
 $\Rightarrow \frac{\widehat{\alpha}}{d} > 0$

a) For
$$\frac{\alpha}{d} \geq \frac{\widehat{\alpha}}{d} : EW_{WA} \leq 0$$
.

$$\Rightarrow \forall \phi > 0 : EW_{SGP} > EW_{WA}.$$

b) For
$$\frac{\alpha}{d} < \frac{\widehat{\alpha}}{d} : EW_{WA} > 0$$
.

$$\Rightarrow \exists \overline{\phi} > \phi^* : \forall \phi > \overline{\phi} : EW_{WA} > EW_{SGP}.$$

Note that $\frac{\widetilde{\alpha}}{d} \leq 0$ for $\overline{\theta} \leq \frac{5}{2}\beta$ and thus $\phi^* \geq 0 \ \forall \frac{\alpha}{d}$.

(iii) Consider
$$\frac{5}{2}\beta < \overline{\theta}$$
.

$$\Rightarrow \frac{\widetilde{\alpha}}{d} > 0 \text{ and } \frac{\widetilde{\alpha}}{d} > \frac{\widetilde{\alpha}}{d}$$

a) For
$$\frac{\alpha}{d} \geq \frac{\widehat{\alpha}}{d} : EW_{WA} \leq 0$$
 and $\phi^* \geq 0$

$$\Rightarrow \forall \phi > 0 : EW_{SGP} > EW_{WA}$$
.

b) For
$$\frac{\widetilde{\alpha}}{d} \leq \frac{\alpha}{d} < \frac{\widehat{\alpha}}{d} : EW_{WA} > 0$$
 and $\phi^* \geq 0$
 $\Rightarrow \exists \overline{\phi} > \phi^* : \forall \phi > \overline{\phi} : EW_{WA} > EW_{SGP}.$

$$\Rightarrow \exists \overline{\phi} > \phi^* : \forall \phi > \overline{\phi} : EW_{WA} > EW_{SGP}.$$

c) For
$$\frac{\alpha}{d} < \frac{\widetilde{\alpha}}{d} : EW_{WA} > 0$$
 and $\phi^* < 0$

$$\Rightarrow \forall \phi \geq 0 : EW_{SGP} \leq EW_{WA}$$
.

Proof of Lemma 17:

It suffices to find one specific vector of private information such that the equilibrium decision under the SGP is not ex post efficient. Let θ_1^A $\theta_1^B = \overline{\theta}$ and $\theta_2^A = \theta_2^B = -\frac{1}{3}\overline{\theta} + \frac{4}{3}\beta = T_{SGP}^*(\phi^*)$. Then, under the \overline{SGP} stabilization takes place in both countries, since in each country one group stabilizes. But, on the other hand, for c = A, B it holds that

$$\sum_{i=1,2} \theta_i^c > 2\beta.$$

Stabilization is not expost efficient (in either country).

Proof of Lemma 18:

It suffices to find one specific vector of information parameters θ for which it is not individually rational for one of the groups to participate in the AGV mechanism. Consider group 1 in country A and private information $\sum_{i=1}^{2} \theta_i^A > 2\beta$ and $\sum_{i=1}^{2} \theta_i^B > 2\beta$. Then, the AGV mechanism prescribes that stabilization does not occur in either country and utility of group 1 is given by

$$\begin{split} w_1^A\left(x^*(\theta),\theta_1^A\right) + Z_1^A\left(\theta\right) \\ &= \theta_1^A d - \beta d + \frac{\frac{1}{2}\theta_1^A\left(\beta - \theta_1^A\right) - \frac{1}{3}\left(\frac{1}{2}\theta_2^A\left(\beta - \theta_2^A\right) + \sum_{i=1,2}\frac{1}{2}\theta_i^B\left(\beta - \theta_i^B\right)\right)}{\overline{\theta} - \theta} d. \end{split}$$

If the mechanism is abandoned after all private information is common knowledge and stabilization decisions have been taken, then utility will be $\theta_1^A d - \beta d$.

It is not individually rational for group 1 to participate if

$$0 \ge \frac{\frac{1}{2}\theta_1^A \left(\beta - \theta_1^A\right) - \frac{1}{3}\left(\frac{1}{2}\theta_2^A \left(\beta - \theta_2^A\right) + \sum_{i=1,2} \frac{1}{2}\theta_i^B \left(\beta - \theta_i^B\right)\right)}{\overline{\theta} - \theta}d.$$

For $\theta_1^A = \theta_1^B = \beta + \varepsilon$ and $\theta_2^A = \theta_2^B = \beta$ with $\varepsilon > 0$ it holds that

$$\frac{1}{2}\theta_1^A\left(\beta-\theta_1^A\right) - \frac{1}{3}\left[\frac{1}{2}\theta_2^A\left(\beta-\theta_2^A\right) + \sum_{i=1,2} \frac{1}{2}\theta_i^B\left(\beta-\theta_i^B\right)\right] = -\frac{1}{3}\left(\beta+\varepsilon\right)\varepsilon.$$

This is smaller than zero, since β and ε are larger than zero.

Proof of Lemma 19:

Again, it suffices to find one specific vector of private information such that the equilibrium decision under the SGP is not constrained ex post efficient. Let $\theta_1^A = \theta_1^B = \overline{\theta}$ and $\theta_2^A = \theta_2^B = -\frac{1}{3}\overline{\theta} + \frac{4}{3}\beta = T_{SGP}^*(\phi^*)$. Then, under the SGP stabilization takes place in both countries, since in each country one of the groups stabilizes. But it also holds that

$$\sum_{c=A} \sum_{B} \sum_{i=1,2} \theta_i^c > 4\beta.$$

Stabilization is not ex post efficient. \blacksquare

Proof of Lemma 20:

In equilibrium, expected welfare under the SGP with a uniform distribution is given by

$$EW_{SGP}(\theta) = \frac{64}{27} \left(\overline{\theta} - \beta \right)^3 \frac{d}{\left(\overline{\theta} - \underline{\theta} \right)^2}.$$

Under the AGV mechanism expected welfare is

$$EW_{AGV}(\theta)$$

$$= \frac{1}{(\overline{\theta} - \underline{\theta})^4} \sum_{\substack{i=1,2\\c=A,B}} \left[\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \int_{-\theta_i^{-c} - \theta_j^{-c}}^{\overline{\theta}} (\theta_i^c d - \beta d) d\theta_i^c d\theta_j^c d\theta_i^{-c} d\theta_j^{-c} \right]$$

$$= (2\beta - \overline{\theta} - \underline{\theta}) (-8\beta + 3\overline{\theta} + 5\underline{\theta}) \frac{d}{\overline{\theta} - \underline{\theta}}.$$

Consider the difference in expected welfare as a function of β

$$D\left(\beta\right) := EW_{AGV} - EW_{SGP}.$$

This is a polynomial of third degree in β and as such has two possible extreme points β_1 and β_2 . It holds that $D\left(\beta_i\right)<0$ for i=1,2. Moreover, $D''\left(\beta_1\right)<0$ and $D''\left(\beta_2\right)>0$ for $\beta_1<\beta_2$ and $\beta_2>0$. Since $D\left(\cdot\right)$ is continuous in β and $\beta<\overline{\theta}$, $\lim_{\beta\to\overline{\theta}}D\left(\beta\right)=5\left(-\overline{\theta}+\underline{\theta}\right)d<0$ completes the proof.

Proof of Lemma 21:

$$\frac{\partial t^{WA}\left(\cdot\right)}{\partial n} = -\frac{\beta}{n^2 \left[1 + \frac{\alpha}{\left(1 - F\left(t^{WA}\right)\right)^2 d} * f\left(t^{WA}\right)\right]} < 0$$

Proof of Lemma 22:

$$\frac{\partial t^{P}\left(\cdot\right)}{\partial n}=-\frac{\beta}{n^{2}\left[1+\frac{\alpha}{\left(1-F\left(t^{P}\right)\right)^{2}d}*f\left(t^{P}\right)+\phi\left(1-F\left(t^{P}\right)\right)*f\left(t^{P}\right)\right]}<0$$

Appendix C

Chapter 4: Monetary Unification & Decision Procedures

C.1 Expected utility under unification with median decision

Expected utility of country 3 under the median mechanism (25) can be decomposed into three conditional expectations weighted by the respective probabilities:

$$U_{M}(\alpha,\sigma) = E\left[u_{3}(x_{M}^{*},\varepsilon)\right]$$

$$= prob\left(x_{1}^{*}is\ median\right) * E\left[u_{3}(x_{1}^{*},\varepsilon) \mid x_{1}^{*}is\ median\right]$$

$$+prob\left(x_{2}^{*}is\ median\right) * E\left[u_{3}(x_{2}^{*},\varepsilon) \mid x_{2}^{*}is\ median\right]$$

$$+prob\left(x_{3}^{*}is\ median\right) * E\left[u_{3}(x_{3}^{*},\varepsilon) \mid x_{3}^{*}is\ median\right].$$

The first and the second part describe expected utility of country 3 if country 1's preferred policy happens to be the median policy and if country 2's preferred policy happens to be the median policy, respectively. The third part deals with the situation in which country 3's preferred decision is median. I consider these cases now in turn.

- 1. Country 1's desired policy becomes median if either $x_1^* \geq x_2^*$ and $x_1^* \leq x_3^*$ or $x_2^* \geq x_1^*$ and $x_3^* \leq x_1^*$. These two scenarios are symmetric with respect to expected utility. Therefore, I consider the case $x_1^* \geq x_3^*$ and $x_1^* \leq x_2^*$ and multiply the resulting part of expected utility by 2.
 - (a) Country 1's preferred decision is given by $x_1^* = (1 \sigma) \varepsilon_1$ and country 2's by $x_2^* = (1 \sigma) \varepsilon_2$. Thus, $x_1^* \le x_2^*$ is equivalent to

$$\varepsilon_1 \leq \varepsilon_2$$
.

(b) Country 3's preferred decision is

$$x_3^* = \frac{1}{2} (1 - \sigma) \left(\alpha \left(\varepsilon_2 + \varepsilon_1 \right) + 2\varepsilon_3 \left(1 - \alpha \right) \right)$$

and therefore $x_1^* \ge x_3^*$ is the same as

$$\frac{2\varepsilon_1 - \alpha \left(\varepsilon_2 + \varepsilon_1\right)}{2\left(1 - \alpha\right)} \ge \varepsilon_3.$$

There exist additional constraints to be satisfied:

(c) $x_2^* \ge x_3^*$:

$$(1 - \sigma) \varepsilon_{2} \geq \frac{1}{2} (1 - \sigma) (\alpha (\varepsilon_{2} + \varepsilon_{1}) + 2\varepsilon_{3} (1 - \alpha))$$

$$\Leftrightarrow \frac{2\varepsilon_{2} - \alpha (\varepsilon_{2} + \varepsilon_{1})}{2 (1 - \alpha)} \geq \varepsilon_{3}$$

i. Which upper bound is binding for ε_3 ?

$$\frac{2\varepsilon_{2} - \alpha (\varepsilon_{2} + \varepsilon_{1})}{2(1 - \alpha)} \geq \frac{2\varepsilon_{1} - \alpha (\varepsilon_{2} + \varepsilon_{1})}{2(1 - \alpha)}$$

$$\Leftrightarrow$$

$$\varepsilon_{2} \geq \varepsilon_{1}$$

This is true for $x_1^* \leq x_2^*$. Thus, the binding upper bound for ε_3 is $\frac{2\varepsilon_1 - \alpha(\varepsilon_2 + \varepsilon_1)}{2(1-\alpha)}$.

(d) Additionally it must hold, that the upper bound on ε_3 lies above its lower bound, i.e.

$$\frac{2\varepsilon_{1} - \alpha (\varepsilon_{2} + \varepsilon_{1})}{2(1 - \alpha)} \geq -1$$

$$\Leftrightarrow$$

$$\varepsilon_{1} \geq \frac{-2(1 - \alpha) + \alpha \varepsilon_{2}}{2 - \alpha}$$

(e) Another question to be considered is whether the upper bound on ε_3 lies below 1 (otherwise there would exist no restrictions on ε_3), i.e.

$$\begin{array}{rcl}
1 & \geq & \frac{2\varepsilon_{1} - \alpha \left(\varepsilon_{2} + \varepsilon_{1}\right)}{2\left(1 - \alpha\right)} \\
\Leftrightarrow & \\
\frac{2\left(1 - \alpha\right) + \alpha\varepsilon_{2}}{2 - \alpha} & \geq & \varepsilon_{1}
\end{array}$$

i. Which upper bound is now binding for ε_1 ?

$$\frac{2(1-\alpha) + \alpha\varepsilon_2}{2-\alpha} \geq \varepsilon_2$$

$$\Leftrightarrow$$

$$1 > \varepsilon_2$$

This is true and thus the binding upper bound for ε_1 is ε_2 .

(f) What happens if

$$\frac{2\varepsilon_{1} - \alpha (\varepsilon_{2} + \varepsilon_{1})}{2(1 - \alpha)} \geq 1$$

$$\Leftrightarrow$$

$$\varepsilon_{1} \geq \frac{2(1 - \alpha) + \alpha \varepsilon_{2}}{2 - \alpha}$$

i. Which lower bound is now binding on ε_1 ?

$$\frac{2(1-\alpha) + \alpha\varepsilon_2}{2-\alpha} \ge \frac{-2(1-\alpha) + \alpha\varepsilon_2}{2-\alpha}$$

Thus, the binding lower bound on ε_1 is $\frac{2(1-\alpha)+\alpha\varepsilon_2}{2-\alpha}$.

ii. Lies this lower bound below the upper bound, i.e. ε_2 ?

$$\varepsilon_{2} \geq \frac{2(1-\alpha) + \alpha\varepsilon_{2}}{2-\alpha}$$

$$\Leftrightarrow$$

$$\varepsilon_{2} \geq 1$$

This is not true. Therefore, the case $\frac{2\varepsilon_1 - \alpha(\varepsilon_2 + \varepsilon_1)}{2(1-\alpha)} \ge 1$ need not be considered further.

Summarizing the above, the constellation $x_1^* \ge x_3^*$ and $x_1^* \le x_2^*$ is reflected in

$$1 \geq \varepsilon_2 \geq -1$$

$$\varepsilon_2 \geq \varepsilon_1 \geq \frac{\alpha \varepsilon_2 - 2(1 - \alpha)}{2 - \alpha}$$

$$\frac{2\varepsilon_1 - \alpha(\varepsilon_1 + \varepsilon_2)}{2(1 - \alpha)} \geq \varepsilon_3 \geq -1$$

If country 1's preferred decision happens to be the median, it will be implemented as common decision for the union consisting of three countries. Expected utility of country 3 conditional on x_1^* being median is then given by

$$\begin{aligned} & prob \left(x_{1}^{*} \, is \, median \right) * E \left[u_{3}(x_{1}^{*}, \varepsilon) \mid x_{1}^{*} \, is \, median \right] \\ &= \ 2 * \left[\frac{1}{8} \int_{-1}^{1} \int_{\frac{\alpha \varepsilon_{2} - 2(1 - \alpha)}{2 - \alpha}}^{\varepsilon_{2}} \int_{-1}^{\frac{2\varepsilon_{1} - \alpha(\varepsilon_{1} + \varepsilon_{2})}{2(1 - \alpha)}} u_{3}(x_{1}^{*}, \varepsilon) d\varepsilon_{3} d\varepsilon_{1} d\varepsilon_{2} \right] \\ &= \ \frac{2}{15} \frac{(\alpha - 1) \left(1 - \sigma\right)}{(\alpha - 2)^{3}} \\ &\left[-2\alpha^{4} + (\sigma + 12) \, \alpha^{3} - (5\sigma + 26) \, \alpha^{2} + (2\sigma + 24) \, \alpha - 8 \right]. \end{aligned}$$

2. The derivation of expected utility for the situation in which country 2's preferred decision is the median is analogous to the case of country 1. Therefore,

$$prob (x_{1}^{*} is median) * E [u_{3}(x_{1}^{*}, \varepsilon) | x_{1}^{*} is median]$$

$$+prob (x_{2}^{*} is median) * E [u_{3}(x_{2}^{*}, \varepsilon) | x_{2}^{*} is median]$$

$$= \frac{4}{15} \frac{(\alpha - 1) (1 - \sigma)}{(\alpha - 2)^{3}}$$

$$[-2\alpha^{4} + (\sigma + 12) \alpha^{3} - (5\sigma + 26) \alpha^{2} + (2\sigma + 24) \alpha - 8].$$

- 3. Country 3 becomes median if either $x_2^* \ge x_3^*$ and $x_1^* \le x_3^*$ or $x_3^* \ge x_2^*$ and $x_3^* \le x_1^*$. Again, these two scenarios are symmetric with respect to expected utility. Therefore, I consider the case of $x_2^* \ge x_3^*$ and $x_1^* \le x_3^*$ and multiply the resulting part of expected utility by 2.
 - (a) $x_2^* \ge x_3^*$ is equivalent to

$$\varepsilon_3 \leq \frac{2\varepsilon_2 - \alpha \left(\varepsilon_2 + \varepsilon_1\right)}{2\left(1 - \alpha\right)}.$$

(b) $x_1^* \le x_3^*$ is equivalent to

$$\frac{2\varepsilon_1 - \alpha \left(\varepsilon_2 + \varepsilon_1\right)}{2\left(1 - \alpha\right)} \le \varepsilon_3.$$

(c) Additionally, it must hold that $x_2^* \ge x_1^*$, i.e.

$$\varepsilon_2 \geq \varepsilon_1$$
.

(d) The additional decompositions of the integration boundaries result again from taking care of which bound is binding. The considerations are similar to those under (1.).

If country 3's preferred policy is median, it will be implemented accordingly.⁵⁰ Expected utility conditional on x_3^* being median is then given by

⁵⁰Note that the probability of country 3 to be pivotal under this procedure is given by the integral taken over 1. This probability is independent of σ and an increasing function of α : the larger the spillovers, the larger is the probability p of country 3 to be decisive. It is given by $p(\alpha) = -\frac{1}{3}\frac{\alpha+2}{\alpha-2}$.

$$prob\left(x_{3}^{*} is \, median\right) * E\left[u_{3}(x_{3}^{*}, \varepsilon) \mid x_{3}^{*} is \, median\right]$$

$$= \frac{1}{4} \left[\int_{-1}^{1-\alpha} \int_{\frac{\alpha\varepsilon_{2}-2(1-\alpha)}{2-\alpha}}^{\varepsilon_{2}} \int_{\frac{2\varepsilon_{1}-\alpha(\varepsilon_{1}+\varepsilon_{2})}{2(1-\alpha)}}^{\frac{2\varepsilon_{2}-\alpha(\varepsilon_{1}+\varepsilon_{2})}{2(1-\alpha)}} u_{3}(x_{3}^{*},\varepsilon) d\varepsilon_{3} d\varepsilon_{1} d\varepsilon_{2} \right.$$

$$+ \int_{1-\alpha}^{1} \int_{\frac{(2-\alpha)\varepsilon_{2}-2(1-\alpha)}{\alpha}}^{\varepsilon_{2}} \int_{\frac{2\varepsilon_{1}-\alpha(\varepsilon_{1}+\varepsilon_{2})}{2(1-\alpha)}}^{\frac{2\varepsilon_{1}-\alpha(\varepsilon_{1}+\varepsilon_{2})}{2(1-\alpha)}} u_{3}(x_{3}^{*},\varepsilon) d\varepsilon_{3} d\varepsilon_{1} d\varepsilon_{2}$$

$$+ \int_{1-\alpha}^{1-\alpha} \int_{-1}^{\frac{(2-\alpha)\varepsilon_{2}-2(1-\alpha)}{\alpha}} \int_{-1}^{1} u_{3}(x_{3}^{*},\varepsilon) d\varepsilon_{3} d\varepsilon_{1} d\varepsilon_{2}$$

$$+ \int_{1-\alpha}^{1} \int_{-1}^{\frac{\alpha\varepsilon_{2}-2(1-\alpha)}{2-\alpha}} \int_{-1}^{1} u_{3}(x_{3}^{*},\varepsilon) d\varepsilon_{3} d\varepsilon_{1} d\varepsilon_{2}$$

$$+ \int_{\frac{2-3\alpha}{2-\alpha}}^{1-\alpha} \int_{\frac{(2-\alpha)\varepsilon_{2}-2(1-\alpha)}{\alpha}}^{\frac{\alpha\varepsilon_{2}-2(1-\alpha)}{2-\alpha}} \int_{-1}^{\frac{2\varepsilon_{2}-\alpha(\varepsilon_{1}+\varepsilon_{2})}{2(1-\alpha)}} u_{3}(x_{3}^{*},\varepsilon) d\varepsilon_{3} d\varepsilon_{1} d\varepsilon_{2}$$

$$+ \int_{-1}^{1} \int_{-1}^{\frac{\alpha\varepsilon_{2}-2(1-\alpha)}{\alpha}} \int_{-1}^{\frac{2\varepsilon_{2}-\alpha(\varepsilon_{1}+\varepsilon_{2})}{2(1-\alpha)}} u_{3}(x_{3}^{*},\varepsilon) d\varepsilon_{3} d\varepsilon_{1} d\varepsilon_{2}$$

$$+ \int_{1-\alpha}^{1} \int_{-1}^{\frac{(2-\alpha)\varepsilon_{2}-2(1-\alpha)}{\alpha}} \int_{-1}^{1} u_{3}(x_{3}^{*},\varepsilon) d\varepsilon_{3} d\varepsilon_{1} d\varepsilon_{2}$$

$$+ \int_{1-\alpha}^{1} \int_{-1}^{\frac{(2-\alpha)\varepsilon_{2}-2(1-\alpha)}{\alpha}} \int_{-1}^{1} u_{3}(x_{3}^{*},\varepsilon) d\varepsilon_{3} d\varepsilon_{1} d\varepsilon_{2}$$

$$+ \int_{1-\alpha}^{1} \int_{-1}^{\frac{(2-\alpha)\varepsilon_{2}-2(1-\alpha)}{\alpha}} \int_{-1}^{1} u_{3}(x_{3}^{*},\varepsilon) d\varepsilon_{3} d\varepsilon_{1} d\varepsilon_{2}$$

Thus, overall expected utility of country 3 under the unification scenario with median decision is given by

 $= \frac{1}{30} \frac{(1-\sigma)\sigma}{(\alpha-2)^3} \left(\alpha^5 - 10\alpha^4 + 42\alpha^3 - 36\alpha^2 - 8\alpha + 16\right).$

$$U_{M}(\alpha, \sigma) = E[u_{3}(x_{M}^{*}, \varepsilon)]$$

$$= \frac{1 - \sigma}{30 (\alpha - 2)} [(\sigma - 16) \alpha^{3} + (2\sigma + 48) \alpha^{2} - (2\sigma + 48) \alpha + 4\sigma + 16].$$

C.2 Proofs

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Proof of Proposition 10:

Consider the difference between expected utility obtained by country 3 if independent and expected utility after accession when the decision for the union as a whole is taken jointly, i.e.

$$D_W(\alpha, \sigma) = S(\alpha, \sigma) - U_W(\alpha, \sigma)$$

$$= \frac{1}{18} \frac{(1 - \sigma)^2}{\sigma(\alpha^2 - 2\alpha) - (1 - \alpha)^2} f(\alpha, \sigma)$$

where $f(\alpha, \sigma)$ is a polynomial of fourth degree in α and second in σ . First note that $D_W(\alpha, \sigma)$ is continuous for all (α, σ) except for (1, 0) and $D_W(0, 0) > 0$. Solving $D_W(\alpha, \sigma) = 0$ yields as interior solutions

$$\sigma_{1,2} = \left(-\frac{5}{6}\alpha + \frac{2}{3} \pm \frac{1}{6}\sqrt{(25\alpha^2 - 40\alpha - 32)}\right) \frac{(\alpha - 1)^2}{\alpha}.$$

But for $\alpha \in [0,1]$ it holds that $25\alpha^2 - 40\alpha - 32 < 0$. Thus, $S(\alpha,\sigma) \neq U_W(\alpha,\sigma) \ \forall (\alpha,\sigma)$ and using the continuity of $D_W(\alpha,\sigma) : D_W(\alpha,\sigma) > 0$.

Proof of Proposition 11:

Consider the difference between expected utility obtained if independent and expected utility after accession when the decision for the union as a whole is taken by the median, i.e.

$$D_{M}(\alpha, \sigma) = S(\alpha, \sigma) - U_{M}(\alpha, \sigma)$$

$$= \frac{1}{30} \frac{(\sigma - 1)^{2}}{\left(\sigma (\alpha^{2} - 2\alpha) - (1 - \alpha)^{2}\right)(\alpha - 2)} g(\alpha, \sigma)$$

where $g(\alpha, \sigma)$ is a polynomial of fifth degree in α and of second degree in σ .

It holds that

$$D_M(0.93, 0.0035) < 0.$$

Since $D_M(\alpha, \sigma)$ is continuous for all (α, σ) except (1,0) there exists an environment around (0.93, 0.0035) in which unification yields more expected utility for country 3 than separation under the median mechanism.

Proof of Proposition 12:

Consider the difference between expected utility obtained if independent and expected utility after accession when the decision for the union as a whole is taken according to the rotation procedure, i.e.

$$D_{R}(\alpha, \sigma, \rho) = S(\alpha, \sigma) - U_{R}(\alpha, \sigma, \rho)$$
$$= \frac{1}{6} \frac{(1 - \sigma)^{2}}{\sigma (\alpha^{2} - 2\alpha) - (1 - \alpha)^{2}} k(\alpha, \sigma, \rho)$$

where $k(\alpha, \sigma, \rho)$ is a polynomial of fourth degree in α , of second in σ and linearly decreasing in ρ .

(i) Separation yields more expected utility if

$$D_R(\alpha, \sigma, \rho) > 0.$$

Note first that $D_R(\alpha, \sigma, \rho)$ is strictly decreasing in ρ , since $\forall \alpha$ and $\sigma \in [0, 1)$

$$\frac{\partial D_R(\alpha, \sigma, \rho)}{\partial \rho} = -\frac{1}{6} (\sigma - 1)^2 (3\alpha^2 - 6\alpha + 4) < 0.$$

Thus, it suffices to show that there exist (α, σ) such that $D_R(\alpha, \sigma, \rho)$ is positive for $\rho = 1$, i.e.

$$D_R(\alpha, \sigma, 1) = -\frac{1}{6}\sigma\alpha (\sigma - 1)^2 \frac{3\alpha^3 - 10\alpha^2 + (11 + \sigma)\alpha - 4}{\sigma (\alpha^2 - 2\alpha) - (1 - \alpha)^2} > 0.$$

Since $\sigma(\alpha^2 - 2\alpha) - (1 - \alpha)^2 < 0$, $D_R(\alpha, \sigma, 1)$ is positive if and only if $3\alpha^3 - 10\alpha^2 + (11 + \sigma)\alpha - 4$ is positive, i.e. if

$$\sigma > \frac{-3\alpha^3 + 10\alpha^2 - 11\alpha + 4}{\alpha}.$$

Thus, $D_R(\alpha, \sigma, \rho) > 0 \ \forall (\alpha, \sigma, \rho) \ \text{iff } \sigma > \frac{-3\alpha^3 + 10\alpha^2 - 11\alpha + 4}{\alpha}$.

C.2 Proofs

(ii) Solving $D_R(\alpha, \sigma, \rho) = 0$ for ρ yields

$$\rho(\alpha, \sigma) = -\frac{3\alpha^4 + (2\sigma - 12)\alpha^3 + (\sigma^2 - 5\sigma + 19)\alpha^2 + (4\sigma - 14)\alpha + 4}{(3\alpha^2 - 6\alpha + 4)(\sigma(\alpha^2 - 2\alpha) - (1 - \alpha)^2)}.$$

It holds that $\rho(\alpha, \sigma) > 0 \ \forall (\alpha, \sigma)$ and $\rho(\alpha, \sigma) < 1 \text{ for } \sigma < \frac{-3\alpha^3 + 10\alpha^2 - 11\alpha + 4}{\alpha}$. (iii) Unification yields more expected utility than separation $\forall (\alpha, \sigma)$ if $\rho > \rho(\alpha, \sigma)$ as defined above, since $D_R(\alpha, \sigma, \rho)$ is continuous in all three parameters and strictly decreasing in ρ .

Proof of Proposition 13:

(i) It holds $\forall \sigma \in [0,1)$ that

$$U_W(0,\sigma) = \frac{1}{9}(-1+\sigma)(2+\sigma)$$

> $\frac{1}{15}(-1+\sigma)(4+\sigma) = U_M(0,\sigma)$

and $\forall \sigma \in [0,1)$ it is true that

$$\frac{\partial U_W(\alpha, \sigma)}{\partial \alpha}|_{\alpha=0} = \frac{2}{9} (1 - \sigma) (\sigma + 2)$$

$$< \frac{2}{3} (1 - \sigma) = \frac{\partial U_M(\alpha, \sigma)}{\partial \alpha}|_{\alpha=0}.$$

Thus, there exists α_1 such that $\forall \alpha < \alpha_1$: $U_W(\alpha, \sigma) > U_M(\alpha, \sigma)$.

(ii) Consider the difference between expected utility of country 3 if the union wide decision is taken to maximize the sum of utilities and if it is taken according to the median, i.e.

$$D_{U}(\alpha, \sigma) = U_{W}(\alpha, \sigma) - U_{M}(\alpha, \sigma)$$
$$= \frac{2}{45} (\sigma - 1)^{2} (\alpha - 1)^{2} \frac{7\alpha - 2}{\alpha - 2}.$$

It holds that

$$D_U(1,\sigma) = \frac{\partial D_U(\alpha,\sigma)}{\partial \alpha}|_{\alpha=1} = 0$$

and $\forall \sigma \in [0,1)$

$$\frac{\partial D_U(\alpha, \sigma)}{\partial \alpha \partial \alpha}|_{\alpha=1} = -\frac{4}{9} (\sigma - 1)^2 < 0.$$

Taking (i) into account, this proves the existence of α_2 . (iii) Setting $D_U(\alpha, \sigma) = 0$ and solving for α^* yields as only interior solution $\alpha^* = \frac{2}{7}$.

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Hiermit erkläre ich, dass ich die Dissertation selbständig angefertigt und mich anderer als der in ihr angegebenen Hilfsmittel nicht bedient habe, insbesondere, dass aus anderen Schriften Entlehnungen, soweit sie in der Dissertation nicht ausdrücklich als solche gekennzeichnet und mit Quellenangaben versehen sind, nicht stattgefunden haben.

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