# Coordinated Planning in Revenue Management

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# Abstract

Revenue management has been applied in service industries for more than thirty years. Since then, revenue management has been transferred to other industries like manufacturing or e-fulfillment. Short-term revenue management decisions are taken based on other, longer-term decisions such as decisions about actual capacity, segment-based prices or the price fences in place. While optimization approaches have been developed for each of these planning tasks in isolation, existing approaches typically do not consider interactions between planning tasks. This thesis considers coordinated planning in revenue management, that is the interaction of revenue management decisions with other planning tasks.

First, we provide an overview of both the literature on coordinated decision making in the context of revenue management in different industries, and the literature on existing frameworks, which aim to structure the planning tasks around revenue management. We find that the planning tasks relevant to revenue management differ across the industries considered. Moreover, planning tasks are relevant on different hierarchical levels in different industries. We discuss an approach for an industry-independent framework.

Based on the relevant planning tasks identified, we investigate the long-term performance of revenue management and therefore the integration of revenue management and customer relationship management. We present a stochastic dynamic programming approach, where the firm's allocation decision impacts future customer demands by influencing the repurchase probabilities of customers, depending on whether their request has been accepted or rejected. We show that a protection level policy is not necessarily optimal in a two-period setting. In a numerical study, we find that the value of looking ahead in time is low on average but may be substantial in some scenarios. However, the benefit from regular demand updates is considerably higher than the additional value of looking ahead in time on average.

Lastly, we investigate the interaction of revenue management and fencing. We account for the trade-off between price-driven demand leakage on the one hand and costs for fencing on the other hand. We show that fencing decisions have an impact on the optimal capacity allocation, but that this is not the case vice versa as the fencing decision does not depend on the allocation decision. Taking both decisions sequentially is therefore optimal. We extend our approach in order to account for additional stock-out-based demand substitution. Then, both decisions depend on each other and firms should take both decisions simultaneously.

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# Chapter 1

# Introduction

## 1.1 Motivation

Revenue management has been applied in service industries (such as airlines, car rental and hotels) for more than thirty years, but it has widened its scope since then. On the one hand, revenue management does not focus solely on capacity allocation, on the other hand, the concept of revenue management has already been - partly due to advances in technology and the internet - transferred to non-traditional industries such as e-fulfillment or manufacturing (Ng et al., 2008). At its core, revenue management deals with service differentiation when a firm faces demand for its limited capacity from heterogeneous customers. Heterogeneity stems from, for example, different levels of willingness-to-pay or the different strategic importance of customers to a firm. Typical revenue management approaches include the allocation of capacity to several customer classes, overbooking, or dynamic pricing. Having been applied for decades, the basic techniques are well known, and several extensions of the basic models exist in literature.

Revenue management aims for short-term demand fulfillment. Short-term revenue management decisions are taken based on other longer-term decisions such as decisions about actual capacity, segment-based prices or the price fences in place. These longer-term decisions may affect revenue management decisions. While optimization approaches have been developed for each of these planning tasks in isolation, the existing approaches typically do not consider interactions between planning tasks. Revenue management decisions are, however, "highly interdependent with decisions made in other key areas of (airline) planning" (McGill and van Ryzin, 1999). Cizaire (2011) also stresses the importance of accounting for these interdependencies both between pricing and revenue management in particular, and with other planning tasks. From a different perspective, Weber et al. (2003) also find that each decision is treated separately in revenue management support systems. Optimizing each decision separately and hence taking decisions sequentially is not necessarily optimal from either a mathematical point of view or from a system perspective. In order to consider the complex decision hierarchy as a whole, as well as the interdependencies involved, all system components should be taken into account (Weber et al., 2003).

As well as the interaction of revenue management with pricing decisions, several other planning tasks, which interact with short-term revenue management decisions, have been identified in the literature. Consider the following examples:

- Next to segmenting customers according to demographic variables (such as age, income etc.) airlines started to introduce frequent flyer programs twenty years ago[1]. Since then, airlines have collected a great deal of information about their customers, but the revenue management systems in place rarely consider the customer specific information but manage availability according to the segment-based product prices (Noone et al., 2003). In general, this purely short-term orientation is not necessarily optimal in the long run. Considering customer relationships is crucial for the long-term performance of revenue management (see, e.g., Ovchinnikov et al., 2014).

- According to the Domestic Airline Fares Consumer Report 2012, large U.S. carriers (such as American Airlines or United) offer seats at a variety of prices within a booking period. In general, the airlines aim to maximize their profits with these complicated fare structures. In order to retain customer segments and to mitigate price-driven demand leakage, the firms design price fences (e.g. fare restrictions). The efficiency of the price fences in place is therefore crucial for the success of revenue management (see also Hanks et al., 2002; Kimes, 2002; Zhang and Bell, 2012).

- Airline schedule planning itself already consists of several decisions[2] which cannot be made simultaneously due to the resulting problem complexity. The scheduling problems are thus solved sequentially (Barnhart and Cohn, 2004). In particular, schedule planning is based on very simplistic assumptions regarding the revenues of an airline and therefore barely mirror the interaction of revenue management and the actual scheduling decisions. The schedules resulting from pure cost-minimizing models may differ significantly from the schedules resulting from an integrated model, which is not based on average fares but better reflects the actual complexity of the pricing systems in place.

In this thesis, we consider coordinated planning in revenue management, that is we investigate the interaction of revenue management decisions with other planning tasks. The research goals of this project are formulated in Section 1.2, and Section 1.3 provides the outline of the thesis.

[1]Lufthansa started the Miles-and-More program in 1993 (Lufthansa, 2013).

[2]According to Barnhart and Cohn (2004) airline schedule planning can be split into schedule design, fleet assignment, aircraft maintenance routing, and crew scheduling.

## 1.2 Research Goals and Methodology

As shown in the examples above, revenue management decisions are related to other also longer-term planning tasks. This thesis is concerned with decisions related to revenue management, and their interaction with revenue management. Revenue management considers, for example, short-term capacity allocation or (dynamic) pricing decisions (e.g. Talluri and van Ryzin, 2004b). These short-term revenue management decisions are related to other planning tasks both short-term and medium-term. Research has focused primarily on revenue management techniques, in particular their refinement, or on modeling customer behavior more realistically. Thus, the basic underlying models are well established. However, the interaction of short-term revenue management decisions with planning tasks in other decision hierarchies has not been a focus in literature so far. There are, to the best of our knowledge, only a few research contributions which investigate coordinated decision making across decision hierarchies in a revenue management setting. Although the number of contributions in the field of revenue management has been growing quickly, an extensive framework, which contains and structures the various planning tasks related to revenue management across different industries, is still lacking. Current frameworks focus on traditional applications for standardized services. It is thus the goal of this thesis to contribute to this broader view of revenue management. In general, the thesis aims to:

- contribute to the body of quantitative models in the field of revenue management,
- generalize the industry-dependent interrelations of planning tasks on different decision hierarchies in revenue management, and
- contribute to a better understanding of revenue management decision making and its interrelation with other planning tasks.

In order to achieve these objectives, this thesis centers on particular research questions. In a first step, we want to achieve a systematic and better understanding of planning tasks that are interrelated with short-term revenue management decisions.

RQ1: *Which planning tasks have an impact on short-term revenue management decisions across different industries?*

Based on an overview of other relevant planning tasks in the context of revenue management, we investigate the interrelationships of short-term revenue management decisions with two other planning tasks in terms of quantitative models. This allows for an analysis of the firm's trade-offs when taking two decisions simultaneously, compared to sequential decision making.

RQ2: *How can other planning hierarchies and tasks be captured in quantitative revenue management models in order to support coordinated decision making?*

RQ3: *How do short-term revenue management decisions change when interrelated planning tasks are considered additionally?*

To answer RQ1, we provide an overview of both the quantitative literature on coordinated decision making in the context of revenue management in different industries, as well as the literature on existing frameworks, which aim to structure the planning tasks around revenue management. We also contrast the literature on coordinated decision making in the context of revenue management with planning frameworks for the different industries from supply chain management literature. Based on this literature study, we discuss how far the relevant interrelated planning tasks in the different industries can be generalized and captured in an industry-independent framework for revenue management.

To answer RQ2 and RQ3, we present two distinct quantitative revenue management models where the firm accounts for additional longer-term planning tasks. In particular, we investigate the interaction of revenue management with customer relationship management, as well as with fencing decisions.

## 1.3 Outline of the Thesis

This thesis is organized in five chapters. Following this introductory chapter, Chapter 2 gives an overview of literature which combines other planning tasks on different planning hierarchies with short-term revenue management decisions. Based on this overview, we develop a generalized framework of planning tasks across different industries related to short-term revenue management decisions.

In the following two chapters we investigate the interaction of revenue management decisions with medium-term planning tasks. Chapter 3 deals with the long-term performance of revenue management and therefore the integration of revenue management and customer relationship management. Previous empirical research has investigated the way customers adapt their behavior depending on product availability in previous periods. This kind of endogenous customer behavior is not yet accounted for in quantitative revenue management research. We thus present a multi-periodic quantity-based stochastic dynamic programming approach which incorporates endogenous customer behavior over time. In a first step, we present the model formulation with general effects on customer demand. Customer behavior is class-wise aggregated and depends on the firm's actual allocation decision, as this decision directly impacts product availability for the respective customer classes. Within the subsequent numerical study, we specify the general demand model. We compare the optimal allocation strategy with several myopic allocation heuristics. Depending on customer reactions towards acceptance or rejection, different allocation strategies are shown to be optimal over time. The value of applying the optimal dynamic allocation strategy relative to simplifying heuristics also depends on these customer characteristics.

Chapter 4 investigates the interaction of revenue management and fencing. Price fences aim to tailor a firm's products to the preferences of its customer segments. In particular, they prevent customers with a high willingness-to-pay from buying lower priced products. We investigate a single period setting with two customer classes where part of the low-value demand arises from customers with high willingness-to-pay. The amount of price-driven demand leakage depends both on the price difference and on the effort a firm puts into designing and controlling the price fences. This effort comes at a cost. We therefore investigate the trade-off between price-driven demand leakage on the one hand and costs for fencing on the other hand. We build on a model proposed by Zhang and Bell (2012) for a newsvendor setting without capacity restrictions. We find that fencing decisions have an impact on the optimal capacity allocation between the two customer classes, but that this is not the case vice versa as the fencing decision does not depend on the allocation decision. Taking both decisions sequentially is therefore optimal. We extend our approach in order to account for additional stock-out-based demand substitution. Both decisions depend on each other and firms should take both decisions simultaneously.

Chapter 5 summarizes our findings and provides directions for further research.

# Chapter 2

# Decisions Related to Revenue Management - An Overview and Framework

## 2.1 Introduction

The concept of revenue management originates from service industries such as airlines, hotels or car rental services (see, e.g., Philips, 2005). Facing (at least in the short term) a fixed capacity (or supply), revenue management makes use of customer heterogeneity in order to maximize the firm's (expected) profit from this capacity. Customers differ with regard to their willingness-to-pay, their profitability or their preferences. By applying revenue management techniques, a firm actively manages demand either by protecting capacity for higher-value customers or through differentiated pricing (Talluri and van Ryzin, 2004b).

Nowadays, revenue management is also applied in other industries like such as manufacturing, retail or e-fulfillment (Talluri and van Ryzin, 2004b). While the main idea, to make use of customer heterogeneity, is the same, the approaches differ, for example with regard to the fixed capacity considered or from where the benefit through revenue management actually stems. For airlines (as for hotels or the car rental business), costs are sunk as variable costs for the actual service are considerably less than the fixed costs for the firm's capacity. In these cases, maximizing profit is equivalent to maximizing revenue. This is not the case in other industries. In manufacturing, customers differ with regard to their strategic importance. Here, a firm typically focuses on long-term customer relationships rather than on short-term sales. Moreover, the costs of fulfilling an order are much more complex to determine, as they additionally depend on the operational production schedule as well as how the firm fulfills demand. For example, customer requests can be served from stock or from future available production in make-to-stock settings. In brick-and-mortar retail, the revenue management approaches applied are price-based. E-fulfillment settings differ from the traditional revenue management applications in another way. Here, revenue management is not only about revenues but also about delivery costs. Applying revenue management results in cost savings by managing demand for delivery time windows based on heterogeneous delivery preferences instead of directly increasing revenues (Agatz, 2009).

Accepting or rejecting customer orders is at the core of short-term revenue management. Revenue management is therefore typically categorized as an operational planning task within the sales department of a firm in common supply chain management planning frameworks (see, e.g., Fleischmann et al., 2008). However, revenue management decisions depend on other short-term and longer-term planning tasks. Decisions with a longer-term planning horizon are, for example, those about available capacity, customer segmentation or product prices. Revenue management decisions either depend on or interact with other planning tasks (Klein and Steinhardt, 2008). In different industries, different planning tasks are particularly relevant and affect the firm's revenue management decisions. The integration of longer-term planning tasks with short-term revenue management decisions has already been identified as a research gap by several authors (see, e.g., McGill and van Ryzin, 1999; Pinchuk, 2002; Barnhart et al., 2003; Sherali et al., 2005).

Several authors have examined the interaction of revenue management with other planning tasks. Examples are the papers by Sen and Zhang (1999), Zhang et al. (2010) and Kocabiykoglu et al. (2011). Sen and Zhang (1999) consider coordinated decisions both about the firm's capacity and on a protection level. Zhang et al. (2010) investigate coordinated decisions on fencing and on order quantities for two customer segments. Kocabiykoglu et al. (2011) investigate coordinated pricing and capacity allocation decisions. Several other papers have focused on the interaction of revenue management with planning tasks in specific industries. Barnhart and Cohn (2004) and Barnhart et al. (2009) consider the interaction of airline scheduling decisions with revenue management. Approaches for make-to-stock manufacturing settings consider optimal capacity allocation and replenishment decisions (see, e.g., Kleijn and Dekker, 1999; Melchiors et al., 2000; de Véricourt et al., 2002; Deshpande et al., 2003; Arslan et al., 2007). However, a literature overview focusing on the interaction of revenue management with other planning tasks across different industries does not yet exist, to the best of our knowledge. In existing overviews, the focus is instead on specific revenue management techniques or industries, rather than on the interaction of different planning tasks.

Frameworks, which structure a firm's planning tasks and consider revenue management, are available in two streams of literature: supply chain planning literature and revenue management literature. Current supply chain planning frameworks aim to give an overview of the planning tasks in different planning hierarchies. The supply chain planning matrix by Fleischmann et al. (2008) is one of the state-of-the-art supply chain planning frameworks, but it focuses strongly on a manufacturing context. Neither the traditional or non-traditional industries applying revenue management, or the respective relevant planning tasks are therefore satisfactorily represented within this framework. In particular, the longer-term planning tasks affecting the firm's short-term revenue management decisions differ among different industries. Comparable frameworks exist for other supply chain settings such as retail or e-fulfillment. In the field of revenue management research, several authors present frameworks accounting for decisions that are strongly related to short-term revenue management. As well as their relatively narrow perspective on revenue management, they particularly neglect longer-term and operations-related decisions in most of the cases. Tactical and strategic planning tasks are considered in few cases, and most of the frameworks focus strongly on airline applications.

We build on the major drawbacks discussed above. We aim to investigate the currently available literature on a firm's decisions related to revenue management and compare these decisions across different industries. We focus on the interaction of different planning tasks with revenue management decisions. In order to structure the relevant planning tasks across industries, we give an overview of current revenue management and supply chain planning frameworks. This allows a comparison of current frameworks to currently available research in order to identify research gaps. We also aim to investigate how far it is possible to create a framework for revenue management and related planning tasks that is applicable across different industries.

This chapter thus results in the following contributions:

- We give an overview of other planning tasks considered in revenue management literature. We consider both traditional and non-traditional revenue management applications and industries and focus on the interaction of short-term revenue management decisions with other planning tasks, both short-term and longer-term.

- We give an overview of existing frameworks for planning tasks related to revenue management. They have either a narrow focus on revenue management or are industry-specific in many cases. Therefore, we compare these frameworks to frameworks from traditional supply chain management.

- Based on the comparison and consolidation of planning tasks related to revenue management in different industries, as well as industry-specific supply chain planning frameworks, we propose a general framework applicable across different industries.

Section 2.2 provides an introduction to revenue management as well as an overview of alternative formulations of Littlewood's standard model. The different model formulations serve as a modeling basis in the subsequent chapters of this thesis. We therefore intentionally take a broader perspective in order to provide a conceptual basis for our research. In Section 2.3 we provide an overview of the available literature, combining short-term revenue management decisions with other planning tasks. Additionally, Sections 2.4 and 2.5 provide overviews of already existing frameworks from both revenue management and supply chain planning literature. All frameworks presented account for revenue management as one of the firm's planning tasks. Sections 2.3 - 2.5 build on Speck (2014). Based on the overviews in Sections 2.3 - 2.5, we discuss and propose a general revenue management framework, which structures all the observed integrated decisions across different industries in Section 2.6. In our discussion, we first summarize the main findings from the literature overview. Then, we contrast the revenue management frameworks with these findings from the literature overview. This allows to evaluate their strenghts and weaknesses. Afterwards, we also compare the supply chain planning frameworks with the literature overview in order to identify potential other planning tasks which might be relevant for revenue management decisions. Based on the previous comparisons, we propose a generalized framework for revenue management and related planning tasks. Finally, Section 2.7 summarizes our findings.

## 2.2 Revenue Management

According to Talluri and van Ryzin (2004b), revenue management refers to "demand management decisions as well as the methodologies and systems to make them". Revenue management can be thought of as a complement to supply chain management (Talluri and van Ryzin, 2004b). Essentially, revenue management is based on service differentiation. Thus, a firm offers its capacity to different customer classes under different terms at different prices in order to maximize the (expected) profit from its given limited supply (Talluri and van Ryzin, 2004b; Philips, 2005). In different streams of literature, revenue management is alternatively denoted as yield management, demand management or demand fulfillment.

The Airline Deregulation Act of 1978 can be considered as the origin of revenue management applications to date (Talluri and van Ryzin, 2004b). From that time, the major U.S. American airlines started to offer part of their capacity at very low prices, based on the fact that the marginal costs incurred for a seat are almost equal to zero. However, the airlines had to ensure that customers with high willingness-to-pay did not request the low-priced seats. Therefore, they introduced a combination of fare restrictions and capacity allocations (Talluri and van Ryzin, 2004b; Philips, 2005; Klein and Steinhardt, 2008). Traditionally, the main applications of revenue management are in the airline and hotel industry, but the general idea has been transferred to several other industries. Examples include applications in make-to-stock and make-to-order manufacturing, retail, and e-fulfillment. These non-traditional applications differ to some extent from traditional airline applications.

Several authors provide indications regarding the impact of introducing revenue management on a firm's profit. Talluri and van Ryzin (2004b) estimate that revenues increase by 4 - 5% by applying revenue management. Klein and Steinhardt (2008) report on benefits around 2 - 5% for traditional passenger airlines (see also Hanks et al., 1992; Smith et al., 1992). Friend and Walker (2001) note that retailers may achieve a 5 - 15% increase in gross margins from assortment and pricing optimization. Based on two consultancy studies, Philips (2005) shows the high leverage of pricing in improving a firm's profitability compared to improvements affecting the firm's variable or fixed costs. Pricing is often the "area that can be improved the most with the least investment" (Philips, 2005).

The following business conditions are considered the main drivers for the applicability of revenue management (for extensive overviews see e.g. Kimes, 1989; Talluri and van Ryzin, 2004b; Philips, 2005; Klein and Steinhardt, 2008):

- *Customer heterogeneity*
  Due to customer heterogeneity, a firm can segment its customers and offer relatively standardized products at different prices to the customer classes. If customers do not exhibit different willingness-to-pay for (basically) the same product or service, the firm cannot benefit from price or service differentiation and thus revenue management.

- *Demand uncertainty*
  With increasing demand uncertainty, the benefit of sophisticated demand management decisions increases as the risk to make bad decisions increases.

- *Production inflexibility*
  When facing relatively fixed and inflexible capacities, different customer segments with different willingness-to-pay compete for this capacity. A firm can then benefit from deciding about the availability of the capacity for the different customer segments. This is not the case if capacities are flexible. In this case, a firm can easily increase its capacity (or supply) in order to meet, for example, higher than expected demand. The fact that services cannot be stored adds additional complexity and inflexibility.

Talluri and van Ryzin (2004b) additionally list other conditions such as the *availability of data systems* or the *appropriate management culture*. These conditions are supportive and focus less on the products offered or the customer interface.

Literature differs with regard to revenue management techniques depending on a firm's decision variables. A firm either decides on allocations for the different customer segments or dynamically adjusts prices. Which of these techniques is applied essentially depends on the extent to which the firm is able to use it in order to manage demand. The application of the different techniques varies across and within industries (Talluri and van Ryzin, 2004b). The different approaches can be clustered into two streams according to Talluri and van Ryzin (2004b): Quantity-based and price-based revenue management. Quantity-based revenue management comprises single-resource and network capacity control as well as overbooking, while dynamic pricing and auctions are referred to as price-based revenue management. Figure 2.1 summarizes the different approaches. The different revenue management techniques are briefly introduced below as they form the theoretical basis of the chapters in the remainder of this thesis.

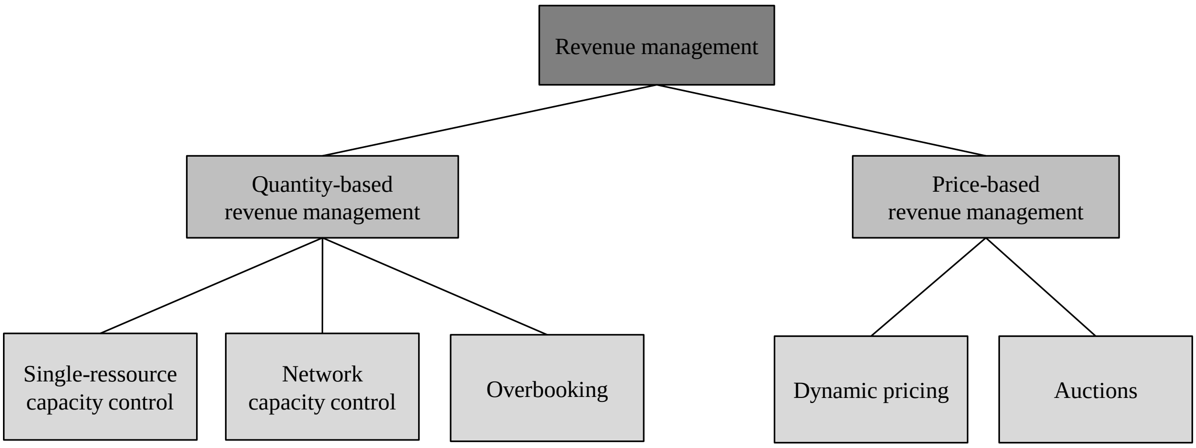


Figure 2.1: Classification of revenue management techniques (according to Talluri and van Ryzin, 2004b)

### 2.2.1 Quantity-Based Revenue Management

By making use of quantity-based approaches, the firm differentiates the availability of the product (or service) across different customer classes (see, e.g., Talluri and van Ryzin, 2004b; Klein and Steinhardt, 2008). The price for each product is predetermined and constant within the booking horizon. The firm allocates total available capacity to the different products and thus to the customer classes. If customers always attend, the allocation planning problems can be further sub-divided into single-resource and network revenue management problems. The approaches differ with regard to the number of resources considered as well as their relationship to each other. If customers exhibit a significant probability of not attending, the firm typically overbooks its capacity in order to increase its utilization.

#### 2.2.1.1 Single-Resource Capacity Control

Single-resource capacity control aims to manage demand for a single resource by optimally allocating this resource to different customer classes in order to maximize the firm's profit. In reality, products consist of multiple resources in most of the cases, however complex problems for multiple resources are frequently solved as a collection of single-resource problems in practice due to their lower complexity (Talluri and van Ryzin, 2004b).

The availability of capacity to different customer classes is managed by different types of controls. In literature, booking limits, protection levels and bid prices are distinguished (see, e.g., Talluri and van Ryzin, 2004b). Booking limits restrict the maximum amount of capacity available for a particular customer class at a certain time. Protection levels specify the amount of capacity protected for a particular customer class. Both types of controls can be partitioned or nested. Under partitioned allocations, the amount of capacity protected is dedicated to a particular customer class only.[1] Under nested allocations, lower-value products are also available to higher-value customer classes in case they are not entirely sold to the lower-value customers. Bid prices are threshold prices such that a request is accepted if its revenue exceeds the bid price and rejected otherwise.

As for the single-resource capacity control problem, the earliest model is due to Littlewood (1972). The models in Chapter 3 and 4 build on this model. We therefore present different model formulations and much of the notation used in these two chapters, in the remainder of this section. First, we introduce Littlewood's model for nested allocations as a stochastic dynamic programming model, an equivalent closed-form static optimization model, and an approach based on marginal analysis. Secondly, we introduce Littlewood's model for partitioned allocations. Here, we focus on the closed-form static model formulation.

Littlewood's model constitutes a well-known approach to managing a single resource with given capacity $C$ (Littlewood, 1972; Belobaba, 1987b; Curry, 1990) for a single booking period. Two customer classes $i$ ($i = 1, 2$) are considered. Customer demand $D_i$ is stochastic with pdf $f_i$ and cdf $F_i$. Customer orders arrive in a low-before-high order, that is the lower-value Class 2 demand arrives prior to the higher-value Class 1 demand (see e.g. Talluri and van Ryzin, 2004b). Protecting a particular amount of capacity for the higher-value customers ensures that the capacity is not depleted due to high low-value demand. The demands of the two customer classes are assumed to be independent random variables. The prices $p_i$ are exogenously given and $p_1 > p_2$ holds by assumption. Thus, Class 1 customers are strictly more valuable to the firm than the lower class customers. Customers demand single units of capacity and orders cannot be canceled.

Littlewood's model can be formulated in terms of a stochastic dynamic programming model as shown, for example, by Talluri and van Ryzin (2004b) for the general case of $n$ customer classes. The demand realization $d_i$ of a particular customer class $i$ is assumed to be known at the beginning of each stage, while the available remaining capacity $\tilde{C}$ denotes the state in each stage. Thus, the stages correspond to the different customer classes. In each stage, the firm decides about how many customer requests to accept (denoted as $acc_i$) from the number of requests $d_i$. Given the remaining capacity after deciding on $acc_i$, the demand of the next more profitable customer class $i-1$ is realized. Based on this general setting for $n > 2$, the stochastic dynamic programming approach for the case of two customer classes can be derived as a special case. Let $V_i$ denote the value function at the beginning of Stage $i$. Once Class 2 demand has realized, the firm decides how many of the requests to accept out of $d_2$ requests. The firm's optimal decision follows from

$$\max_{0 \le acc_2 \le d_2} p_2 \cdot acc_2 + V_1(C - acc_2). \tag{2.2.1}$$

$C - acc_2$ is the remaining capacity for the higher-value customer class. The value function

[1] Partitioned allocations are alternatively denoted as distinct allocations (see, e.g., Belobaba, 1987a).

entering Stage 2, $V_2(C)$, is then the expected value of this optimization with respect to $D_2$. Thus, the Bellman equation reads as

$$V_2(C) = E[\max_{0 \leq acc_2 \leq d_2} p_2 \cdot acc_2 + V_1(C - acc_2)]. \tag{2.2.2}$$

The optimal policy is obtained by backward induction starting with the lower-value Class 2 demand as there is only a single stage in the two-class model. The optimal policy is to protect a particular amount of the total available capacity for the higher-value customer class. Thus, a booking limit policy is optimal. In the remainder, $y$ denotes the protection level for the higher-value customers. A maximum amount of $C - y$ units of capacity is then available for the less profitable Class 2 customers. $C - y$ denotes the booking limit. Once the booking limit is depleted, all further Class 2 requests are rejected. The firm's optimal allocation policy can be implemented at the beginning of the planning horizon and prior to any demand realizations as the protection level is independent of actual demand realizations. The optimal protection level is nested.

As a static protection level policy is optimal, Littlewood's model can equivalently be formulated in terms of a static optimization problem where the firm decides about $y$ given both demand distributions. According to the stochastic dynamic programming approach, a nested protection level is the optimal allocation policy. In this case, the firm's optimization problem reads as (see, e.g., Williamson, 1992; Philips, 2005; Kocabiykoglu et al., 2011)

$$\max_{0 \leq y \leq C} E[\pi(y)] = \max_{0 \leq y \leq C} p_1 \cdot E[\min{(C - \min{(C - y, D_2)}, D_1)}] + p_2 \cdot E[\min{(C - y, D_2)}]. \tag{2.2.3}$$

Differentiating the expected profit with regard to $y$ yields

$$\begin{aligned}
\frac{dE[\pi(y)]}{dy} &= p_1 \cdot E[I_{q_1 > y, q_2 > C - y}] - p_2 \cdot E[I_{q_2 > C - y}] \\
&= p_1 \cdot \mathbb{P}(q_1 > y, q_2 > C - y) - p_2 \cdot \mathbb{P}(q_2 > C - y) \\
&= p_1 \cdot \mathbb{P}(q_1 > y) \cdot \mathbb{P}(q_2 > C - y) - p_2 \cdot \mathbb{P}(q_2 > C - y) \\
&= \mathbb{P}(q_2 > C - y) \cdot (p_1 \cdot \mathbb{P}(q_1 > y) - p_2) \\
&= (1 - F_2(C - y)) \cdot (p_1 \cdot (1 - F_1(y)) - p_2).
\end{aligned} \tag{2.2.4}$$

The second order derivative reads as

$$\frac{d^2E[\pi(y)]}{d^2y} = f_2(C - y) \cdot (p_1 \cdot \mathbb{P}(q_1 > y) - p_2) - p_1 \cdot (1 - F_2(C - y)) \cdot f_1(y). \tag{2.2.5}$$

As $\frac{d^2E[\pi(y)]}{d^2y} < 0$ does not necessarily hold, the expected profit function for a nested protection level is not necessarily strictly concave in $y$, however it is strictly concave under certain conditions. Let $h_i = \frac{f_i}{1 - F_i}$ denote the hazard rate of the demand distribution of class

$i$. The expected profit function is strictly concave in $y$, if $h_2(C - y) < h_1(y)$ holds $\forall y$. If customer demands follow a Poisson distribution this condition holds if $E[D_1] < E[D_2]$.

Although the expected profit function is not strictly concave in general, the optimal nested protection level is unique as the expected profit function is unimodal in $y$. The following holds:

$$\frac{dE[\pi(y)]}{dy} \begin{cases} > 0, & \text{if } y < \tilde{y}, \\ = 0, & \text{if } y = \tilde{y}, \\ < 0, & \text{if } y > \tilde{y}. \end{cases} \tag{2.2.6}$$

For $y < \tilde{y}$, marginally increasing the protection level also marginally increases the expected profit. For $y > \tilde{y}$, any further increase in the protection level results in a marginally decreasing expected profit. Thus, the protection level resulting from the first order condition is the unique optimal solution which maximizes the firm's expected profit. A unique optimal nested protection level exists. The first order condition is necessary and sufficient.

Setting the first order derivative equal to zero yields

$$(1 - F_2(C - y)) \cdot (p_1 \cdot (1 - F_1(y)) - p_2) = 0 \tag{2.2.7}$$

as the first order optimality condition for the nested protection level. Solving the first order optimality condition for $y$ results in

$$F_1(y^{LW}_{nested}) = 1 - \frac{p_2}{p_1} \Leftrightarrow y^{LW}_{nested} = F_1^{-1}(1 - \frac{p_2}{p_1}) \tag{2.2.8}$$

which is known as Littlewood's rule and gives a closed-form expression for the optimal nested protection level $y^{LW}_{nested}$ in a single booking period. The firm's optimal allocation policy is to protect $\min(y^{LW}_{nested}, C)$ units of capacity for the higher-value customer class.

Alternatively, the optimal allocation policy can be obtained through a marginal analysis (see, e.g., Talluri and van Ryzin, 2004b; Klein and Steinhardt, 2008). Facing a Class 2 request assuming that the firm has still $\hat{y}$ units of capacity left, the firm has to decide whether to accept or reject this request. The underlying trade-off is illustrated in Figure 2.2. If the firm rejects the request, this particular unit of capacity is still available for Class 1 later in the booking period. In this case, the firm's marginal expected revenue equals $p_1 \cdot \mathbb{P}(D_1 \geq \hat{y}) + 0 \cdot \mathbb{P}(D_1 < \hat{y}) = p_1 \cdot (1 - F_1(\hat{y}))$. If the firm accepts the Class 2 request, this particular unit of capacity is not available for Class 1 but the firm definitely earns $p_2$. The optimal protection level, $y^{LW}_{nested}$, results from equating the expected marginal revenues from both alternatives.

Littlewood's model can also be formulated for partitioned allocations. For partitioned

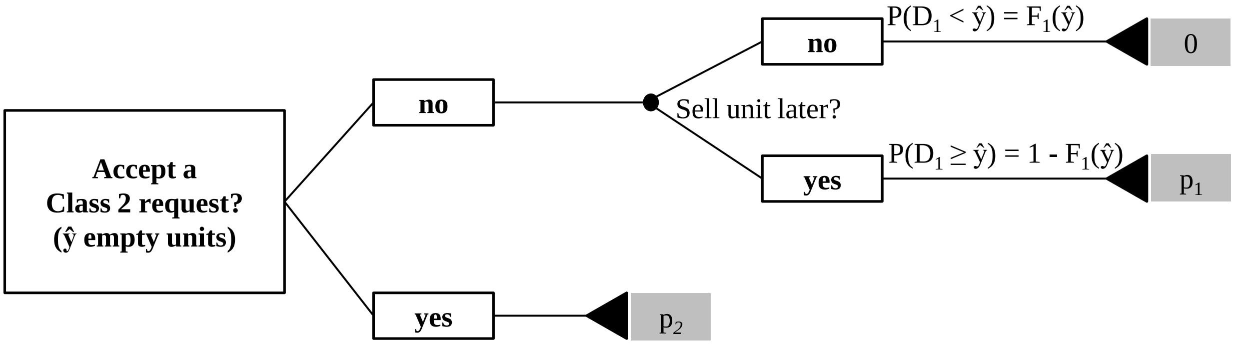


Figure 2.2: Littlewood's model: marginal analysis (adapted from Klein and Steinhardt, 2008)

allocations, the firm's static optimization problem reads as

$$\max_{0 \leq y \leq C} E[\pi(y)] = \max_{0 \leq y \leq C} p_1 \cdot E[\min(y, D_1)] + p_2 \cdot E[\min(C - y, D_2)]. \tag{2.2.9}$$

The objective function differs from the objective function under a nested protection level with regard to the Class 1 revenue. Under partitioned allocations, no capacity that is initially reserved for Class 2 customers is available to Class 1 customers.

Differentiating $E[\pi(y)]$ with regard to $y$ yields

$$\begin{aligned} \frac{dE[\pi(y)]}{dy} &= p_1 \cdot E[I_{q_1 > y}] - p_2 \cdot E[I_{q_2 > C-y}] \\ &= p_1 \cdot \mathbb{P}(q_1 > y) - p_2 \cdot \mathbb{P}(q_2 > C - y) \end{aligned} \tag{2.2.10}$$

as the first order derivative. The second order derivative results in

$$\frac{d^2 E[\pi(y)]}{d^2 y} = -p_1 \cdot f_1(y) - p_2 \cdot f_2(C - y). \tag{2.2.11}$$

As $\frac{d^2 E[\pi(y)]}{d^2 y} < 0$ holds independent of the choice of $y$, the expected profit function is strictly concave in $y$. Thus, a unique optimal partitioned protection level exists. The first order optimality condition is therefore necessary and sufficient for optimality.

Setting the first order derivative equal to zero yields

$$p_1 \cdot (1 - F_1(y)) = p_2 \cdot (1 - F_2(C - y)) \tag{2.2.12}$$

as the first order optimality condition for the partitioned protection level. The optimal protection level $y_{part}^{LW}$ is the solution to this optimality condition. In contrast to the nested protection level, $y_{part}^{LW}$ cannot be represented as an explicit solution. The firm's optimal allocation policy is to protect $\min(y_{part}^{LW}, C)$ units of capacity for the higher-value customer class.

When comparing both protection levels, it is intuitive that a nested protection level is the optimal allocation policy in the sense that it always performs at least as well as a partitioned protection level. Comparing the optimality conditions for both the nested and the partitioned protection level means that the partitioned protection level is always greater than the nested protection level for given customer demand distributions. This is due to the higher underage costs in case of a partitioned protection level. While Class 1 customers may be served from unsold units of the booking limit under nested allocations, this is not the case under partitioned allocations. Unsold units of capacity that were previously protected for the lower-value customer class are therefore lost for the firm. Despite its sub-optimality, the partitioned protection level is frequently applied in practice (Talluri and van Ryzin, 2004b). It is also frequently used in literature as an approximation due to the properties of the underlying optimization problem discussed above (see, e.g., Weatherford, 1997). The strict concavity property also simplifies the analysis for extensions of the standard model.

Despite its simplifying assumptions, Littlewood's model has been widely applied in revenue management research (see, e.g., Pfeifer (1989), Brumelle et al. (1990), Belobaba and Weatherford (1996), Weatherford (1997), Kocabiykoglu et al. (2011)). Several extensions of Littlewood's model exist in the literature. Belobaba (1987b) provides an extension to multiple customer classes resulting in the well-known expected-marginal-seat-revenue heuristic. Weatherford (1997) presents an approach to coordinated pricing and allocation decisions based on Littlewood's model. Building on the work of Weatherford (1997), Kocabiykoglu et al. (2011) further investigate the coordination of pricing and capacity allocation.

#### 2.2.1.2 Network Capacity Control

Network capacity control approaches aim to manage demand for multiple resources (Talluri and van Ryzin, 2004b). In the airline example, network revenue management aims to manage multi-leg flights (see, e.g., Klein and Steinhardt, 2008). Thus, a product[2] consists of a bundle of resources from a particular origin to a specific destination. If resources are available for different products, the resources are interdependent as the lack of availability of a particular resource limits sales of other products (Talluri and van Ryzin, 2004b).

Accounting for these interdependencies creates additional value for the firm as opposed to neglecting them. Boyd and Bilegan (2003) report 2% additional revenues when applying network revenue management techniques instead of single-resource techniques. However, according to Talluri and van Ryzin (2004b), implementing network revenue management poses significant implementation and methodological challenges for firms in different areas. Exact optimization in particular is significantly more complex and impossible for practical purposes compared to single-resource capacity control, where several exact optimization methods exist. Approximations such as deterministic and randomised linear programming have been proposed (for overviews see, e.g., Talluri and van Ryzin, 2004b; Klein and Steinhardt, 2008)

[2]Denoted as an origin-destination itinerary fare class combination (according to Talluri and van Ryzin, 2004b).

in order to solve network revenue management problems efficiently.

#### 2.2.1.3 Overbooking

If there is a considerable probability that customers will not show up, it makes sense for a firm to overbook its capacity in order to increase the resulting load factor and not lose revenue (e.g. Kasilingam, 1997b). While capacity allocation is essentially aimed at achieving the optimal demand mix, overbooking is concerned with increasing a firm's capacity utilization (Talluri and van Ryzin, 2004b). Overbooking also comes at costs that might arise from denying passengers what they have paid for: on the one hand costs due to passenger compensation, and on the other hand the potential loss of goodwill (see, e.g., Wangenheim and Bayón, 2007). Approaches to the area of overbooking can be clustered into static and dynamic models. Static models can be grouped into service-based and cost-based approaches, depending on which data is available (for details see Talluri and van Ryzin, 2004b; Philips, 2005; Klein and Steinhardt, 2008).

Both Talluri and van Ryzin (2004b) as well as Klein and Steinhardt (2008) provide extensive overviews on overbooking approaches under different assumptions as well as several extensions of the basic models. An early (airline) overbooking approach is presented by Rothenstein (1971). Specific approaches for overbooking in air cargo can be found in Kasilingam (1997a) and Kasilingam (1997b). Liberman and Yechiali (1978) extend early airline overbooking approaches and apply them to hotel settings.

### 2.2.2 Price-Based Revenue Management

When applying price-based revenue management approaches, firms vary prices dynamically over time and the total remaining capacity is typically available to all customers. Price-based approaches include dynamic pricing as well as auctions.

#### 2.2.2.1 Dynamic Pricing

In contrast to quantity-based methods, demand is managed by varying prices over time when applying price-based revenue management. Firms in certain industries apply price-based revenue management techniques as they are easier to manage in response to changing market conditions. For dynamic pricing applications, customer demand is explicitly modeled as a price-dependent process (Talluri and van Ryzin, 2004b). Examples include markdown pricing, discount pricing and price promotions (Talluri and van Ryzin, 2004b). For a review, we refer to Elmaghraby and Keskinocak (2003). Dynamic pricing has been successfully applied both in settings with a fixed capacity (e.g., when stocks cannot be replenished) and in settings where capacity is replenished frequently. Their review builds on that by Bitran and Caldentey (2002), who focus primarily on fixed capacities.

#### 2.2.2.2 Auctions

While firms decide about the price for a particular product/service at a particular point in time when applying dynamic pricing techniques, auctions work the other way around (Talluri and van Ryzin, 2004b). Customers offer a price they are willing to pay and a firm decides which bid(s) to accept. In contrast to dynamic pricing, where a firm must somehow estimate the customers' demand, auctions allow each customer's individual willingness-to-pay to be uncovered virtually directly. Several types of auctions exist (for an overview see, e.g., Talluri and van Ryzin, 2004b; Webster, 2009), which are typically applied in different markets. As auctions constitute competitive games, where multiple players compete with each other, they are typically modeled by means of game-theoretic approaches (Webster, 2009).

## 2.3 Coordination of Revenue Management with Related Planning Tasks - A Review

In this section, we review literature on models which investigates the interaction of revenue management decisions with other short-term or longer-term planning tasks. We focus on four industries, where revenue management is currently applied: service industries, manufacturing, retail, and e-fulfillment. These four industries mirror both traditional and non-traditional applications of revenue management. Moreover, this choice also mirrors the different revenue management approaches: quantity-based revenue management is prevalent in the first two industries while price-based revenue management is prevalent in retail settings. In e-fulfillment settings, both approaches are applied.

We categorize planning tasks as operations-related or marketing-related, depending on whether they determine the firms available capacity or whether they are located at the customer interface directly. Doing so mirrors the general objective in supply chain management: to match supply with demand (Chopra and Meindl, 2010). In the literature overview, we consider papers which investigate the interrelationship of revenue management with some other planning task. In most of the papers considered, a firm takes two (interrelated) decisions: A (quantity- or price-based) revenue management decision in combination with another operations- or marketing-related decision. We also consider papers where one of these decisions is not an explicite decision variable but is considered through approximations. This is, for example, the case in papers which consider interrelated decisions with very different time horizons. Which of the two decisions is approximated depends on the focus of the respective paper.

In addition to industry-specific planning tasks, forecasting and customer relationship management are two additional industry-independent areas of planning. Forecasts are a crucial prerequisite for many managerial decisions including revenue management decisions. This is true independent of the industry considered. Customer relationship management aims to establish long-term customer relationships. This view has been prevalent in many industries, too.

### 2.3.1 Quantity-Based Revenue Management for Service Industries

Quantity-based revenue management is prevalent in traditional service industries such as airlines or hotels (see Section 2.2). Operations-related decisions considered in literature in combination with revenue management include flight schedules, fleet assignment, available capacity, and dynamic capacity management. Marketing-related planning tasks refer to decisions on customer segmentation and fencing, prices, and the overbooking level. Approaches differ with regard to their focus on specific applications. Traditionally, most of the literature refer to airlines. Hotel or car-rental applications are less frequent.

#### 2.3.1.1 Operations-Related Planning Tasks

**Scheduling and Fleet Assignment**

In their schedule design, airlines decide which markets to serve with what frequency and how to schedule flights to meet these frequencies (Barnhart and Cohn, 2004). Schedule design starts around 12 months prior to the schedule's operation and typically consists of two steps. First, the appropriate frequencies is determined for each market. Timetables are then developed (Lohatepanont and Barnhart, 2004). In the next step, fleet assignment aims for the profit-maximizing assignment of a limited number of aircraft to the serviced flight legs subject to various operational constraints (Jacobs et al., 2012). Fleet assignments determine the available capacity on a particular flight as the basis for short-term revenue management.

In their overview, Barnhart and Cohn (2004) additionally consider aircraft maintenance routing and crew scheduling (see also Lohatepanont and Barnhart, 2004). These planning tasks differ with regard to their time horizons and are interdependent. However, according to Barnhart and Cohn (2004), an integrated model is intractable, therefore the planning decisions are taken sequentially. Lohatepanont (2001) and Lohatepanont and Barnhart (2004) investigate simultaneous decisions about schedule design and fleet assignment. Lohatepanont and Barnhart (2004) find significant benefits compared to a sequential approach. In these approaches, revenue management decisions are not explicitly considered but revenues are estimated.

Schedule design and fleet assignment decisions are typically taken based on average fares and average unconstrained demands per leg (Barnhart and Cohn, 2004; Jacobs et al., 2012). Average fares and demands depend on the schedule offered and vice versa (Barnhart and Cohn, 2004). Moreover, revenue management and its effects on both the fares and customer demands are neglected (Barnhart and Cohn, 2004; Barnhart et al., 2009). Barnhart et al.

(2009) allow for a wide class of revenue functions. Again, the firm only makes assignment decisions while revenue management decisions are approximated by revenue functions.

Farkas (1996) investigates the impact of network effects and revenue management on customer spill and the resulting costs in both a sequential and a simultaneous approach for coordinated revenue management and fleet assignment. The fleet assignments in the simultaneous approach differ substantially from the fleet assignments in the sequential approach.

Kniker (1998) and Barnhart et al. (2002) build on the simultaneous approach of Farkas (1996). They additionally account for passenger recapture. Kniker (1998) shows that optimal sequential fleet assignment decisions are also optimal in the simultaneous approach under certain conditions. Dumas and Soumis (2008) present a stochastic extension of the approaches by Kniker (1998) and Barnhart et al. (2002). Jacobs et al. (2008) integrate linear approximations of the total revenue into the fleet assignment problem. Jacobs et al. (1999) and Smith (2004) propose a different approach. Jacobs et al. (1999) embed a bid price model into a fleet assignment model. Sandhu and Klabjan (2006) investigate an extension of the bid-price approaches of Jacobs et al. (1999) and Smith (2004) by accounting for passenger and cargo demand. The benefit from taking both decisions simultaneously is substantial.

**(Medium-Term) Capacity Planning**

The approaches to medium-term capacity planning are similar to the fleet assignment approaches discussed above, but they do not explicitly involve airline settings. Carroll and Grimes (1995) consider the system-wise integration of longer-term capacity planning and short-term revenue management. They describe the revenue management system at the car rental company Hertz and its links to other information and decision support systems. They stress the importance of accounting for potential interdependencies. Sen and Zhang (1999) analytically investigate simultaneous decisions on the protection level and the initial capacity based on Littlewood's model. Taking both decisions simultaneously yields a significantly higher expected profit compared to a sequential approach.

**Dynamic Capacity Management**

Where fleet assignment approaches assign capacity to legs on a medium-term basis, dynamic capacity management approaches consider very short-term capacity reassignments. Supply can be better matched with demand by making use of short-term demand information. Examples include same-day aircraft swaps. These decisions may be even more short-term than actual allocation decisions. According to Peterson (1986), forecasting and the introduction of aircraft families are crucial for short-term capacity reassignments.

Berge and Hopperstad (1993) investigate dynamic reassignments by means of simulation. Forecasts are updated based on realized bookings. The benefit of dynamically reassigning capacities is significant, at 1% - 5% of additional profit. Revenue management is considered in the forecasting step as, for example, boardings that are denied depend on revenue management. Pilla et al. (2008) and Pilla et al. (2012) investigate a two-stage approach and approximate the expected revenues by means of multivariate adaptive regression but do not explicitly model revenue management.

Sherali et al. (2005) consider a deterministic mixed-integer programming model for dynamic reassignments within an aircraft family. They do not explicitly consider different customer classes but an aggregate customer acceptance decision.

Frank et al. (2006) simulate fleet assignment decisions and booking control decisions as well as their interdependencies. Booking control decisions are taken by means of standard quantity-based single-leg methods. Both the resulting revenues and the resulting load factors increase the closer reassignments are allowed prior to departure.

In contrast to the previous approaches, de Boer (2003) integrates the effects of dynamic capacity management into a revenue management model. She proposes modifications of the closed-form expressions for the optimal booking limits based on Littlewood's model. She develops a heuristic, which accounts for future capacity changes. The heuristic outperforms traditional expected-marginal-seat-revenue heuristics by up to 1%. Similarly, Bish et al. (2011) determine booking limits under the possibility of swapping aircrafts. Based on two flights, which can potentially be swapped, they derive a heuristic booking limit.

#### 2.3.1.2 Marketing-Related Planning Tasks

Literature on the interaction of revenue management with marketing-related planning tasks in service industries can be categorized into three streams: coordination of customer segmentation and fencing with allocation planning, coordinated pricing and allocation planning, and coordinated overbooking and allocation planning. These decisions are typically taken by the marketing department because they hold the necessary knowledge about the firm's customers. Moreover, these decisions are highly interdependent (see, e.g., Cizaire, 2011). Typically, customer segments are determined on a longer-term basis compared to prices and fences, which are typically made based on the customer segmentation in place. Pricing and fencing decisions are made on a longer-term basis compared to the quantity-based revenue management decisions.

**Customer Segmentation and Fencing**

De Boer (2003) investigates the effects of imperfect customer segmentation in a setting where customers always request the lowest available fare. A nested booking limit policy is not necessarily optimal in contrast to a setting with perfect segmentation. If a nested policy is not optimal, prices are either too low or the products are not sufficiently restricted. Ignoring imperfect segmentation results in significant revenue loss.

Based on the firm's customer segmentation, fences are imposed in order to mitigate price-driven demand leakage (Philips, 2005; Zhang and Bell, 2012). Belobaba and Weatherford (1996) determine optimal booking limits under exogenous customer buy-up behavior. Fiig et al. (2010) observe that fares in practice are becoming less restrictive, therefore they propose to consider demand leakage directly when determining the booking limits.

Zhang and Bell (2007) explicitly consider price-driven demand leakage as well as the costs to mitigate it. They derive the optimal prices as well as the order quantities analytically in a two-class newsvendor setting. A sequential approach which determines the order quantities prior to the prices performs close to optimal. Zhang et al. (2010) extend this approach by accounting for optimal fences under lost sales. Kim and Bell (2011) extend this approach by explicitly accounting for asymmetrical demand leakage. In both approaches, optimal demand leakage and order quantities are independent.

Raza (2014) determines the firm's optimal price differentiation strategy (in terms of threshold values for the segments willingness-to-pay), the optimal prices and capacity allocations when facing price-driven demand leakage. Similarly to Zhang and Bell (2007), he finds that a sequential approach yields good results.

**Pricing**

The literature stream on coordinated pricing and capacity allocation dates back to the work of Weatherford (1997). The need to consider the effects of prices on customer demands and thus on booking limits has been widely recognized (see also de Boer, 2003). All approaches consider quantity-based revenue management and in most cases do not focus on a specific industry but are motivated by service-related problems. We therefore consider these approaches in this section, although they could also be applied in other industries.

Weatherford (1997) analytically investigates joint pricing and capacity allocation based on Littlewood's model and extensions. He finds optimality gaps of around 3% - 5% compared to sequential decisions. Feng and Xiao (2006) investigate a stochastic dynamic programming approach where the firm decides on which customer classes to open at which price. The optimal policy is a threshold policy based on minimum acceptable prices. Kocabiykoglu et al. (2011) build on the approach of Weatherford (1997). They investigate the impact of the properties of demand models on the benefit from taking both decisions simultaneously.

De Boer (2003) investigates simultaneous pricing and capacity allocations in networks in both a single-period and a multi-period approach. She provides the optimality conditions as well as conditions for concavity for a single period. Numerically, she determines substantial benefits of up to 21.4% compared to the optimal benchmark pricing policy.

Kuyumcu and Popescu (2006) show that the simultaneous pricing and allocation problem boils down to a pure pricing problem if demand is deterministic and standard regularity assumptions on demand hold. Cizaire (2011) considers a multi-period joint pricing and allocation problem for multiple products. She confirms the results of Kuyumcu and Popescu (2006) with regard to deterministic demand. For stochastic demand, her approach yields a benefit of up to 6% compared to the deterministic case and up to 7% compared to a sequential approach. For multi-class problems, she develops several heuristics, which lead to better results than the optimal policy for deterministic demand.

Jacobs et al. (2010) evaluate the fit between an airline's pricing strategy and its short-term allocation policy. The proposed metric captures the trade-off between the allocated capacity and the optimal price: if allocations for a customer class are too low, the price is too high. The firm might then have to reject profitable customer requests.

In contrast to the previous approaches, Pinder (2004) investigates optimal prices and capacity allocations in a two-class project management setting. Weber et al. (2003) consider joint pricing and capacity allocations from a system perspective by analyzing the links between a firm's pricing system and its revenue management system. The authors propose real-integrated systems in order to ensure good decisions.

**Overbooking Level**

Subramanian et al. (1999) and Gosavi and Das (2002) consider simultaneous decisions about overbooking levels and booking limits. Gosavi and Das (2002) build on the single-leg standard model and allow for overbooking. Costs for bumping passengers due to overbooking and customer cancellations are considered. Their approach outperforms the widely applied standard nested expected-marginal-seat-revenue heuristics.

Ringbom and Shy (2002) investigate a two-class problem with an adjustable curtain between higher-value and lower-value passengers. The curtain can be adjusted until shortly before departure. Boarding can be denied to lower-value passengers while higher-value passengers are guaranteed boarding. The authors determine the firm's optimal allocation policy.

### 2.3.2 Quantity-Based Revenue Management in Manufacturing

Quantity-based revenue management is also applied in manufacturing. Harris and Pinder (1995), Rehkopf and Spengler (2005) and Spengler et al. (2006) apply revenue management in make-to-order manufacturing. Quante et al. (2009) consider revenue management in a make-to-stock setting. Operations-related decisions consider decisions on production or replenishment quantities and decisions on the production schedule. Marketing-related decisions specifically related to a manufacturing context involve customer segmentation and pricing.

#### 2.3.2.1 Operations-Related Planning Tasks

Decisions about available capacity are taken at different hierarchical levels. By determining production schedules, firms also decide on their available capacity on a short-term basis.

**Production / Replenishment Quantities**

The stream on coordinated revenue management and production / replenishment decisions is a large stream of literature, which is denoted as inventory rationing in literature (Quante et al., 2009).

Ha (1997a) extends the lost-sales setting of Ha (1997b) and investigates a two-class make-to-stock setting where the firm decides on the production quantity and on the demand to be fulfilled from each customer class in each period. If demand is not fulfilled from available stock, it is backordered. Customers differ with regard to their costs for waiting. The optimal policy is characterized by two parameters: a production base-stock level and a rationing level. De Véricourt et al. (2002) extend the approach to $N$ customer classes.

Carr and Duenyas (2000) consider a two-class setting where the firm decides on order acceptance, production quantities and on the fraction of capacity to reserve for customers with long-term contracts. Their orders must be fulfilled and are produced on a make-to-stock basis. Short-term Class 2 orders are more profitable than Class 1 orders and are produced on a make-to-order basis. The benefit of considering both the order acceptance and production decisions simultaneously is substantial. Iravani et al. (2012) build on the approach of Carr and Duenyas (2000), however they assume full backordering of Class 1 requests instead of lost sales. Simultaneous decisions result in an average profit increase of 8% compared to a simple base-stock policy.

Benjaafar and El Hafsi (2006) consider the optimal production and inventory control of an assemble-to-order production system under lost sales. The firm decides on the production of components for the single end-product and whether customer orders are fulfilled from on-hand inventory. The optimal policy is a base-stock production policy, as in the approach by Ha (1997a). The optimal allocation is a rationing policy with different rationing levels for the different customer classes. Highest-value Class 1 orders are always fulfilled if enough on-hand inventory is available. A stationary base-stock level policy performs fairly well compared to the optimal dynamic policy. ElHafsi (2009) builds on the approach of Benjaafar and El Hafsi (2006) by accounting for multiple non-unitary demand classes. Cheng et al. (2011) additionally consider failure-prone machines. Benjaafar et al. (2010) build on the work of Ha (1997a) and Benjaafar and El Hafsi (2006) by accounting for both backorders and lost sales. The two customer classes differ with regard to backordering and rejection costs. An order can be fulfilled from stock, can be backordered or rejected.

Defregger and Kuhn (2007) investigate a make-to-order setting with a limited inventory capacity. In each period, the firm decides on whether to accept or reject a potential incoming order and on replenishments. The proposed heuristic prefers orders with a high profitability in relation to the necessary capacity to fulfill it. The heuristic outperforms a first-come-first-served policy by 2% - 4%. The optimal policy yields an additional average reward of 0% - 1% compared to the heuristic.

**Production Scheduling**

Compared to most of the service settings, a firm is relatively flexible with regard to its available capacity in the short term in manufacturing settings. This is particularly relevant in make-to-order settings, where the firm still has scheduling flexibility. Typically, a firm considers the order's profitability, its capacity consumption and the operational feasibility to schedule it in the current production schedule, such that the request can be fulfilled at a certain point in time (see, e.g., Huang et al., 2011).

Two literature streams can be distinguished, which both deal with scheduling and order acceptance decisions: in the first stream, the firm decides on order scheduling and due date quoting to customers. In order to make a decision about order acceptance and to quote a reliable due date, the firm must account for the importance of the customer as well as the utilization of the production system. Scheduling and due date quoting are thus interrelated. A second stream investigates coordinated order acceptance and order scheduling decisions.

Overviews of simultaneous order scheduling and due date quoting decisions are provided by Keskinocak and Tayur (2004) and Slotnick (2011). Kolisch (2001) provides an overview of order acceptance, due date quoting and scheduling in make-to-order manufacturing.

Duenyas (1995) was among the first to consider simultaneous decisions on order scheduling and due date quoting. Customers differ with regard to their willingness-to-pay and their lead time preferences. Due to equal processing times and equal tardiness costs across all orders, scheduling according to the earliest due date is optimal.

Keskinocak et al. (2001) consider joint scheduling and due date quoting in a two-class setting. One of the customer classes prefers a faster lead time and exhibits a higher willingness-to-pay. They consider both immediate and delayed quotation and compare different heuristics. Immediate quotation is closest to revenue management as the acceptance decision is solely based on expectations about the profitability of future orders. Plambeck (2002) considers a similar problem, however, all customer orders are accepted. As in Keskinocak et al. (2001), customers differ with regard to their willingness-to-pay and the urgency of their orders. Urgent customers are served immediately, but they pay a premium for the fast service. Lead times quoted to the lower-value customers are proportional to queue length, including an adjustment which accounts for the probability of an urgent order.

In contrast to Duenyas (1995), in the approach of Jalora (2006) the firm incurs holding costs. The firm trades-off capacity utilization and holding costs. According to Jalora (2006), an order is accepted if its opportunity cost of scheduling is smaller than its profit. The order is produced in the period with the minimum opportunity cost of scheduling. The expected profit increases by up to 34% compared to a first-come-first-served policy.

Guhlich et al. (2014) consider simultaneous decisions on order acceptance and due date quoting while simultaneously considering the production schedule. In contrast to the previous approaches, both the intermediate materials and the assembly capacity are limited. Decisions are taken online, based on a bid-price approach.

Maglaras (2006) investigates a dynamic pricing approach. The firm simultaneously decides on prices and the production sequence. It is optimal to schedule orders according to a greedy policy which minimizes holding costs. The scheduling decision is shown to be independent of the dynamic pricing strategy and vice versa.

#### 2.3.2.2 Marketing-Related Planning Tasks

**Customer Segmentation**

Meyr (2008) investigates the interaction of customer segmentation and capacity allocation based on real-world data. Customer segments are determined based on customer profitability. Capacity allocations are determined for a varying number of customer segments following the approach of Meyr (2009). The expected profit increases with the number of customer segments while the marginal value of additional customer segments decreases. The model already yields good results for a moderate number of segments. Meyr (2008) therefore concludes that a moderate number of customer segments is sufficient. Moreover, he finds that the appropriate segmentation model is more important than the appropriate allocation model.

**Pricing**

Harris and Pinder (1995) consider simultaneous pricing and capacity allocations in an assemble-to-order context. In contrast to Weatherford (1997), they investigate partitioned allocations and account for holding costs incurred for the allocated quantities. Becher (2009) formulates a joint pricing and capacity allocation model by means of fuzzy controllers.

### 2.3.3 Price-Based Revenue Management in Retail

Typically, retail companies manage demand through dynamic pricing (see, e.g., Talluri and van Ryzin, 2004b). The main interrelated planning tasks are comparable to the relevant decisions in the service context. Operations-related planning tasks involve decisions on initial inventories and on capacity rationing. Decisions on the product offer or the assortment and the selling format are considered in literature as marketing-related decisions.

#### 2.3.3.1 Operations-Related Planning Tasks

A firm's "capacity" in a retail environment typically consists of the available amount of products. Two procurement modes can be distinguished: either a firm decides on an initial inventory at the beginning of the planning horizon or they replenish within the planning horizon (Elmaghraby and Keskinocak, 2003). The first literature stream mirrors the notion of revenue management for seasonal products due to fixed initial available capacity. The second stream on revenue management when products are frequently replenished is similar to inventory rationing models in a manufacturing context. For the literature overview, we focus on decisions about initial inventories in the remainder of this section. Elmaghraby and Keskinocak (2003) also provide an overview of models considering replenishments within the planning horizon.

**Initial Inventories**

Smith and Achabal (1998) investigate a markdown setting where a firm decides on the initial inventory level and the prices. Customer demand is a function of price, time and the inventory level. Optimal prices and markdowns are higher if demand is sensitive to the inventory level. In this case, the firm should allow leftovers instead of offering high discounts at the end of the season (see also Elmaghraby and Keskinocak, 2003). Urban and Baker (1997) investigate a single markdown possibility in a newsvendor setting. Neglecting the impact of inventory levels and prices on demand results in a substantial profit loss.

Cachon and Swinney (2009) investigate a firm's decisions regarding initial inventory and one price change during the selling season facing myopic, strategic and discount customers. Both the optimal initial inventory and the optimal price discount are lower due to strategic customer behavior. In this case, a fixed price schedule is never optimal and the optimal price discount takes place later in the planning horizon.

Subrahmanyan and Shoemaker (1996) analyse a finite selling season without reorder possibilities where the product loses its value at the end of the season. The season is subdivided into two periods. First, the firm decides on both the initial inventory and price. After observing demand, the firm decides on the price in the second period. Learning is valuable when facing demand uncertainty. An additional reorder possibility increases the expected profit and decreases the optimal initial inventory.

Mantrala and Rao (2001) developed an integrated IT system. It first determines an optimal price path (from a set of feasible prices) for a given initial inventory level, then it determines the optimal initial inventory by comparing these solutions. While optimal dynamic pricing yields the highest expected profits, a fixed price strategy outperforms automatic markdowns without consideration of demand patterns. Dynamic pricing is able to adjust for a suboptimal initial inventory (see also Elmaghraby and Keskinocak, 2003).

Several papers focus on the impact of the initial inventory on the optimal prices and thus the resulting expected profit. Lin and Yao (2003) determine optimal markdown policies. The optimal policy only accepts orders with the highest price until a certain time threshold. A higher initial inventory results in higher and earlier markdowns. Bell and Zhang (2005) investigate a newsvendor setting with dynamic pricing. Their approach is similar to the approach by Subrahmanyan and Shoemaker (1996). Both the initial inventory and price are determined first, and the price can be adapted after observing demand. Small deviations from the optimal initial inventory do not have a major impact on the expected profit.

**Capacity Rationing**

The literature stream on capacity rationing is related to the firm's decision on initial inventories. However, literature on capacity rationing in retail applications considers strategic customer behavior. It may then be optimal to deliberately understock products in order to create an incentive for customers to buy earlier in the planning horizon at higher prices.

Liu and van Ryzin (2008) consider a firm which commits to a price path and decides on the optimal initial inventory facing deterministic demand in a first step. The firm trades off potential lost-sales and the additional revenue from more customers buying earlier at higher prices. Rationing and dynamic pricing are not beneficial if customers are risk-neutral.

Gallego et al. (2008) consider a similar setting. Customers cannot observe the inventory level and are assumed to be risk-neutral. The firm's optimal strategy is then not to ration inventory and to charge a single fixed price. This changes if demand is stochastic or if not all customers are acting strategically. Then, dynamic pricing becomes valuable. In a multi-period setting it is beneficial to convincingly announce that inventory is scarce.

Liu and van Ryzin (2011) build on the work of Liu and van Ryzin (2008) by assuming that customers do not have rational expectations but only learn about the firm's decisions over time by observing past capacities. The firm's optimal decisions result in either a capacity rationing equilibrium or in a low-price-only equilibrium depending on customer expectations. Rationing is only optimal if changing customer expectations is not costly.

#### 2.3.3.2 Marketing-Related Planning Tasks

Due to the focus on dynamic pricing strategies, customer segmentation is not prevalent in retail revenue management applications. However, other longer-term decisions about the product offer, assortment planning and the selling format play a role.

**Product Offer, Assortment Planning**

In the approach of Gallego and van Ryzin (1997), a firm decides on which products to offer at what prices facing deterministic demands. Demand interdependencies between the end-products are explicitly considered. A first heuristic determines the optimal prices for a given assortment. A second heuristic fulfills orders on a first-come-first-served basis. Maglaras and Meissner (2006) build on the work of Gallego and van Ryzin (1997). They show that the model can equivalently be formulated as a quantity-based problem. Based on this result, the authors develop further heuristics for solving the joint assortment and pricing problem.

Dong et al. (2009) consider horizontal product differentiation according to product quality. Consumers choose a product based on price, availability and their quality preferences. The retailer decides on which product versions to order and on the respective order quantities prior to the start of the selling season and independent of the subsequent dynamic pricing problem. The value of dynamic pricing depends on the degree of inventory scarcity. The optimal prices are affected by the total inventory level, the inventory level of the individual product and its quality. Small deviations from the optimal inventories do not have a major impact on the expected profit. Akcay et al. (2010) consider a similar setting. Suh and Aydin (2011) build on the work of Akcay et al. (2010) and Dong et al. (2009). They consider two customer classes and assume that the initial inventories are given. The difference in the optimal prices between the products decreases with increasing inventory levels.

**Selling Format**

Retailers may provide different information about product availability. Availability information can affect strategic customer behavior and thus the benefit from dynamic pricing.

Yin et al. (2009) investigate two inventory display formats as a means to mitigate strategic customer behavior and its effects on the firm's dynamic pricing strategy. Either the firm displays all available units or only one unit. Displaying only one unit may increase the perceived shortage risk. When changing display formats, the inventory level does not necessarily change, as appropriate price changes are typically sufficient. If the retailer's per-unit cost are relatively high, profits increase by up to 20% when displaying only one unit.

Su and Zhang (2009) investigate product availability information in a newsvendor model when customers incur costs for stock-outs at the retailer. The firm determines an observable price and an unobservable stocking quantity. Su and Zhang (2009) study two possible selling formats: Either the firm commits to a stocking quantity ex ante or it provides availability guarantees to customers. Both selling formats may increase a firm's profit.

Allon and Bassamboo (2011) investigate the impact of a firm's ability to communicate unverifiable information about product availability. A single retailer providing information on its own cannot create credibility with the customers, therefore, the firm will never be able to affect customer behavior. This result also holds for endogenous prices.

### 2.3.4 Revenue Management in E-fulfillment

E-fulfillment considers order fulfillment at internet retailers (see, e.g., Agatz et al., 2008). In contrast to brick-and-mortar retail stores, an internet retailer is responsible for the delivery of the products. Managing the delivery operations properly has been shown to have a significant effect on customer retention (see, e.g., Boyer and Hult, 2005). Customers making use of convenient home delivery have significant revenue potential (Goebel et al., 2012).

Agatz (2009) compares revenue management in an e-fulfillment setting with traditional single-leg airline revenue management.Three main differences are important for the remainder of this section. First, the firm's product to the customer consists of the physical products ordered and its delivery. While the physical products can be replenished on a regular basis, it is typically the delivery capacity which is limited, at least in the short run. Second, the limited delivery capacity can also be managed on an operational basis by means of, for example, routing decisions. By managing delivery demand, a firm can still influence its costs. In service settings, all costs are typically sunk. A third difference to traditional revenue management are the revenue sources. They consist of both the margins of the physical products and the delivery fees. Delivery fees serve as incentives for the customers. Firms can thus either manage delivery demand by managing the availability of time slots for deliveries or by pricing the time slots accordingly.

Operations-related planning tasks include decisions on delivery routing and scheduling as well as time slot management. Decisions on delivery service pricing and product bundling are considered as marketing-related planning tasks.

#### 2.3.4.1 Operations-Related Planning Tasks

Agatz (2009) provides an overview of the typical order fulfillment process in e-fulfillment. After the order intake, the internet retailer plans the routes, picks the necessary products for the order and finally delivers the order. Delivery capacity is the limited resource. Similar to manufacturing, available capacity can also be influenced on a short-term basis.

**Delivery Routing and Scheduling**

Campbell and Savelsbergh (2005) study internet retailer order acceptance decisions, decisions about the delivery time and the corresponding delivery routing. Customer preferences with regard to delivery times are known. In a first step, they determine whether it is possible to integrate the order into a delivery schedule and the resulting additional costs. It is then possible to decide whether the order should be accepted or rejected. Considering both the profitability of an order and the feasibility of scheduling yields considerable benefits. Offering only some delivery time slots to customers and/or differentiating prices would even further increase profits. In a similar approach, Espinoza et al. (2008) considers demand management for on-demand air transportation services.

Azi et al. (2012) consider whether dynamically arriving delivery requests can be included in delivery tours, and if they should be accepted or rejected. Future requests are considered when deciding on the acceptance of an order which proves to be valuable for the firm.

Ehmke and Campbell (2014) study different approaches to the acceptance problem, which differ with regard to the information included about the routes. The authors explicitly consider stochastic travel times in order to account for congestion. Intuitively, they find that dynamic acceptance rules perform best, as they benefit most from travel time information.

Cleophas and Ehmke (2014) present an iterative approach: first, the necessary delivery capacity is estimated by means of expected future requests and used as part of a routing problem. Customer requests are then accepted or rejected such that the total value of the accepted requests is maximized under the delivery capacity constraint. Finally, optimal delivery tours are generated for the accepted requests. The benefit of combining both decisions depends greatly on the forecast quality and the firm's ability to segment its customers.

**Time Slot Management**

Campbell and Savelsbergh (2006) build on the work of Campbell and Savelsbergh (2005). A firm decides on which order to accept for which time slot and the incentives the firm should provide for customers in order to choose a particular time slot. The probabilities of choosing a specific time slot are known but depend on delivery fees. Either the firm provides incentives for different time slots of the same length or it provides incentives for the customers to accept a longer time slot. Campbell and Savelsbergh (2006) report on higher profits due to lower delivery costs and more customers finally placing an order.

Confessore et al. (2008) build on the work of Campbell and Savelsbergh (2006). The authors develop an algorithm for quickly calculating the transportation costs associated with a certain time window (including the routing of the order). This information is provided to customers in order to induce them to choose a longer time window. Thus, the sum of the provider's transportation cost and the customer's cost associated with a longer time window is minimized. Longer delivery time windows can still result in lower total costs.

Agatz (2009) investigates dynamic slotting. The firm decides on which delivery time slots, out of all available time slots, should be offered to a particular customer. In contrast to Campbell and Savelsbergh (2005), customer preferences for different delivery time slots are not known ex ante. The two decisions are taken sequentially, with the set of all available time slots as an input to the dynamic slotting problem. The presented heuristics all outperform a first-come-first-served approach.

#### 2.3.4.2 Marketing-Related Planning Tasks

Marketing-related planning tasks refer to delivery service pricing and product bundling, which is a valuable technique used by many e-commerce retailers.

**Delivery Service Pricing**

Asdemir et al. (2009) investigate pricing of delivery options. The firm trades-off its capacity utilization and heterogeneous customer preferences. The approach is reminiscent of the approaches of Confessore et al. (2008) or Campbell and Savelsbergh (2006). The probability of choosing a particular delivery option depends on its utility and on its price. Considering two customer classes, the different delivery options and the influence of the available capacity may result in lower prices for more profitable customers.

**Product Bundling**

Netessine et al. (2006) investigate an e-commerce retailer's dynamic decisions about which product to offer in a bundle and how to price the bundle in order to maximize profit. Either emergency replenishments are possible or a stock-out results in lost sales. In the first setting, the optimal price for the bundle depends on the availability of the additional, but not of the original product. Under lost sales, the bundling decision depends on customer preferences rather than availability considerations as long as sufficient inventory is available. Simultaneous bundling and dynamic pricing is stated as a valuable extension.

Gürler et al. (2009) consider a two-product setting with a fixed initial inventory. In contrast to Netessine et al. (2006), the firm decides on the proportion of inventory to use for bundling and on the respective prices. Products which are bundled at the beginning of the season are not available for individual product requests. In this case, the inventory decision is similar to an assortment decision. If individual product prices are low, all alternatives are offered and a high bundle price is optimal. Otherwise, the optimal bundle price is such that only bundles are sold.

### 2.3.5 Industry-Independent Planning Tasks

#### 2.3.5.1 Forecasting

According to, for example, McGill and van Ryzin (1999) and Chiang et al. (2007), forecasting is crucial for revenue management decisions as their quality directly depends on the forecast accuracy. The need to obtain forecasts is industry-independent, however, forecasts differ in what they actually have to predict. In a hotel setting, demand for types of rooms at certain points in time has to be estimated. In a cargo revenue management setting, demand is multi-dimensional: forecasts are needed for the time, size and weight of a request. In addition, forecasts may depend on the type of revenue management applied. Booking curves (i.e. demand as a function of time), no-show probabilities or cancellation curves have to be estimated for quantity-based revenue management. For price-based approaches, estimating the demand functions and cross-price elasticities is crucial (Talluri and van Ryzin, 2004b).

Several authors have quantified the impact of accurate forecasts on the performance of revenue management. Lee (1990) finds that a 10% increase in forecast accuracy increases revenues by 0.5% - 3% on high demand flights. Pölt (1998) estimates that a 20% reduction of the forecast error yields a 1% increase in revenue.

Forecasting is generally considered an integral part of revenue management (see, e.g., Klein and Steinhardt, 2008), however, forecasting is not really regarded as a separate planning decision (see, e.g., Ng and Yip, 2011). Accounting for forecasting in revenue management models typically allows the investigation of the performance of revenue management for given forecasts obtained in different ways. Weatherford and Kimes (2003) provide an overview of issues regarding forecasting for revenue management in a hotel setting. For example, a firm should determine the level of aggregation, the relevant time period for forecasts and how to handle outliers.

Weatherford et al. (2001) investigate aggregated versus disaggregated forecasting approaches. They find that disaggregating forecasts by a combination of rate class and length of stay dominates more aggregated forecasts. Disaggregating by rate class dominates disaggregating by length of stay. Weatherford and Pölt (2002) investigate the general issue of unconstraining bookings to demand in order to improve forecast accuracy in an airline setting. More sophisticated methods should be used in order to obtain good forecasts. Weatherford and Kimes (2003) compare different forecasting methods based on real-world data. Standard forecasting approaches such as exponential smoothing or moving averages provide the most robust forecasts. Neuling et al. (2004) present passenger name records as an opportunity to improve forecast accuracy. Ng and Yip (2011) support the findings by

Weatherford et al. (2001). They propose forecast demand per customer segment.

#### 2.3.5.2 Customer Relationship Management

Customer relationship management focuses on the long-term value of customers to the firm (see, e.g., Berger and Nasr, 1998). The importance of focusing on long-term customer relationships in order to maximize long-term profits has been stressed for several years and independent of industries (Noone et al., 2003). The goals of both concepts may contradict each other: maximizing the short-term profit from a particular flight does not necessarily maximize the firm's long-term profits. While several papers investigate the integration of both concepts qualitatively, only few papers present models for integrated revenue management and customer relationship management decisions.

Typical customer relationship management decisions are decisions on spending for customer acquisition and retention or, more generally, on the effort a firm puts into managing its customer base (Reinartz and Venkatesan, 2008). The firm trades off the benefits of customer relationship management with the costs of its effort.

Von Martens (2009) and von Martens and Hilbert (2011) model the firms' decisions on a protection level, based on Littlewood's model and under additional consideration of a customer's long-term value. Customer profitability is defined as the weighted average of the customer's willingness-to-pay and long-term value, but the customer's long-term value is independent of the allocation decision when considering a lost-for-good setting. Buhl et al. (2011) investigate a deterministic model where the repurchase probabilities depend on the allocation decision.

Pfeifer and Ovchinnikov (2011) argue that customer relationships cannot be considered independent due to the typically fixed capacity in revenue management settings. Customer lifetime value is not then the maximum amount a firm should be willing to spend on the acquisition and retention of a customer. Ovchinnikov et al. (2014) model firms' decisions regarding acquisition and retention spending in a stochastic dynamic programming approach facing limited capacity. They show analytically that the incremental value of a particular customer is less than the customer's lifetime value. Thus, customer lifetime value is inadequate if capacity is limited. Spending is typically constant in models based on customer lifetime value but changes dynamically when accounting for customers' real incremental values.

## 2.4 Planning Frameworks for Revenue Management - A Review

Several frameworks have already been proposed which aim to structure revenue management decisions, and their interrelationships with other short-term and longer-term planning tasks.

Within this section, we provide an overview of existing frameworks in chronological order.

Vinod (2004) discusses the major components of revenue management applications in the hotel industry. The components are illustrated in Figure 2.3. Hotels segment their customers

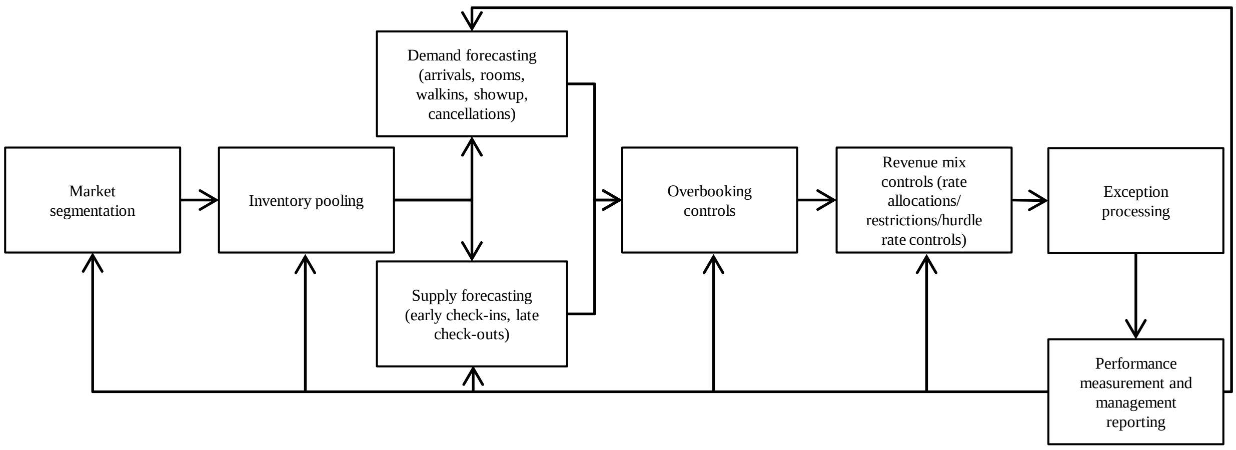


Figure 2.3: Hotel revenue management application (adapted from Vinod, 2004)

with regard to certain attributes in order to ensure a revenue-maximizing mix of customer segments and offered rates. They then pool their available rooms into several rate classes so that the respective rates are relatively homogeneous while the rate classes themselves are heterogeneous. Based on forecasts, overbooking and traditional revenue management controls are applied in order to maximize revenue from the available rooms. Exceptions are processed individually. Measuring the performance of the booking system yields feedback for the different revenue management sub-systems.

Philips (2005) presents a framework for pricing-related decisions and an overview of planning tasks related to quantity-based revenue management. Philips (2005) differs regarding pricing-related decisions between operational activities such as the analysis of different pricing alternatives, their execution and finally their evaluation. Prerequisites such as market segmentation are performed less frequently. The framework is shown in Figure 2.4.

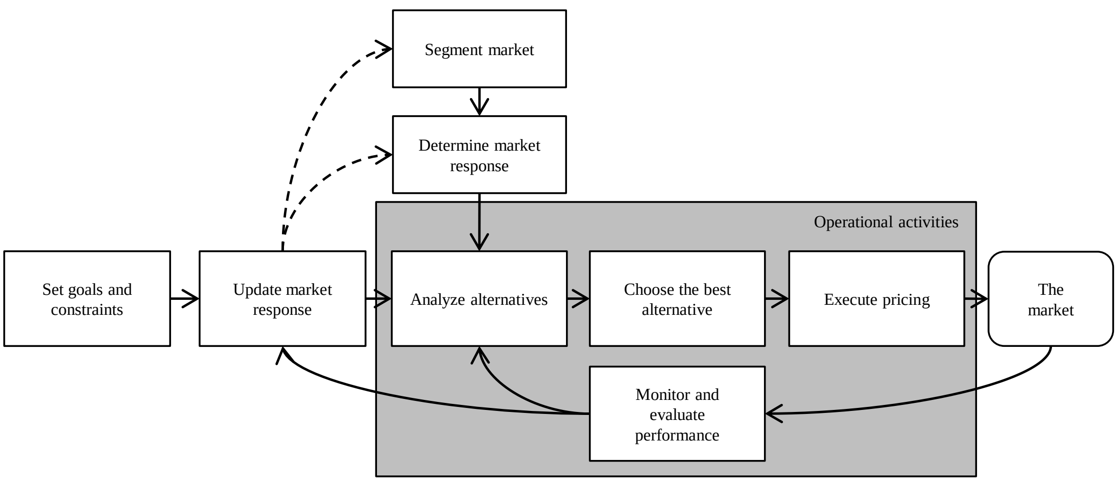


Figure 2.4: The pricing and revenue optimization process (adapted from Philips, 2005)

As for quantity-based revenue management, Philips (2005) differentiates between strategic, tactical and real time planning tasks. Decisions about market segmentation and segment-based prices are taken on an annual or quarterly basis. Decisions about the capacity allocations are taken on a weekly or daily basis while the actual order acceptance decisions are taken in real time. Table 2.1 summarizes the decision hierarchies.

| **Level** | **Description** | **Frequency** |
|---|---|---|
| Strategic | Segment market and differentiate prices | Quarterly or annually |
| Tactical | Calculate and update booking limits | Daily or weekly |
| Operational | Determine which booking to accept / reject | Real time |

Table 2.1: The three levels of revenue management decisions (according to Philips, 2005)

For standardized services, Klein and Steinhardt (2008) also differentiate between strategic, tactical and operational planning tasks. The framework is illustrated in Figure 2.5.

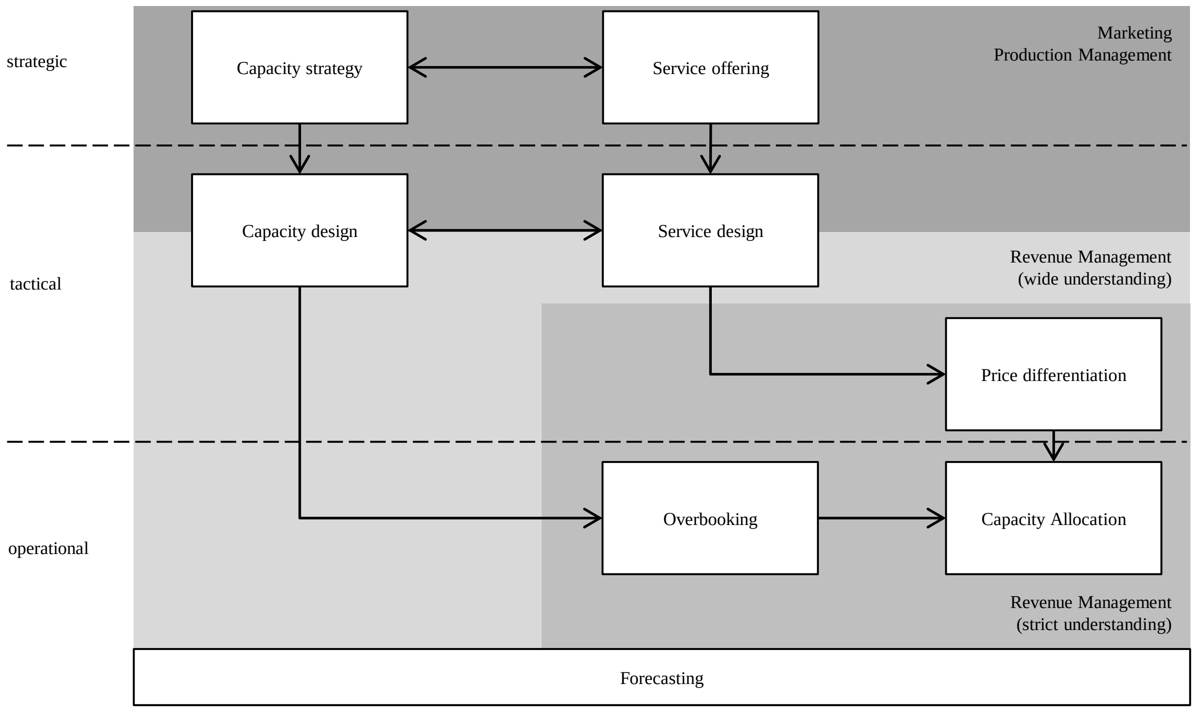


Figure 2.5: Planning tasks in revenue management (adapted from Klein and Steinhardt, 2008)

On a strategic level, the interdependent decisions on the firm's capacity and its service offering are mainly taken in the marketing and production departments. The service offering refers to the depth and width of the services offered. Within the capacity strategy the firm decides on the total available capacity and its configuration (e.g. the types and number of aircrafts). Depending on the financial effort for adapting capacity, this decision can be strategic (e.g. airlines) or tactical (e.g. car rental).

The tactical capacity design and service design decisions correspond to the planning tasks on the strategic level. Capacity design decisions determine the actual usage of the available capacity, for example by fleet assignment. Service design decisions further differentiate and develop the service offering. One example is schedule design. Price differentiation decisions assign prices to the services offered to the different customer segments. Price fences are meant to prevent demand leakage between the customer segments.

The main operational planning tasks are overbooking and capacity allocation. These short-term revenue management decisions are taken in the marketing or sales department. As all of the planning tasks are based on forecasts, forecasting is considered a supporting tool.

Ng et al. (2008) consider revenue management as an end-to-end solution from the firm's supply to customer demand. The authors propose a revenue management system which comprises four decision sets and various components, as shown in Figure 2.6. The value set

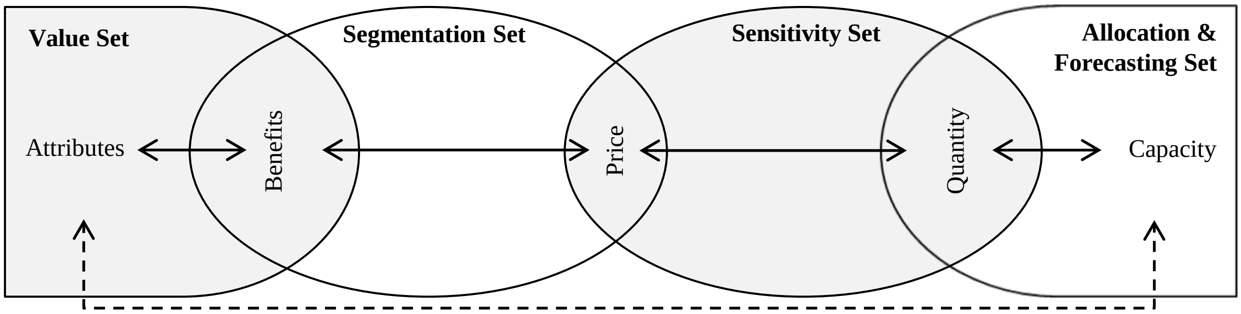


Figure 2.6: Decision sets in revenue management (adapted from Ng et al., 2008)

covers the interrelation of the benefits required from a product and the respective product attributes in order to provide these benefits. Within the segmentation set, the firm matches the benefits required with the prices for different customer segments. The required benefits support customer segmentation (see also Ng, 2006). Customer demand properties differ across the customer segments (referred to as sensitivity set). Firms decide about the allocation of their total capacity to the different customer segments based on demand forecasts, which are considered within the allocation and forecasting set.

The framework proposed by von Martens (2009) considers traditional transaction-based revenue management but also its adaption in order to consider aspects from customer relationship management. The framework is illustrated in Figure 2.7. The analysis of the firm's environment, the decision of the firm's objective and the development of its strategy are considered to be strategic planning tasks. Among other planning tasks, service design and pricing decisions are considered on a tactical basis. von Martens (2009) focuses on revenue management decisions, and in particular (relationship-based) booking control, and forecasting with regard to operational planning tasks.

Cizaire (2011) focuses on the interaction between airline pricing and seat capacity allocation. The corresponding framework, including other interrelated decisions, is shown in Figure 2.8. While the first three planning tasks determine the airline's operations and thus the available seat capacity on each origin-destination combination, pricing and revenue man-

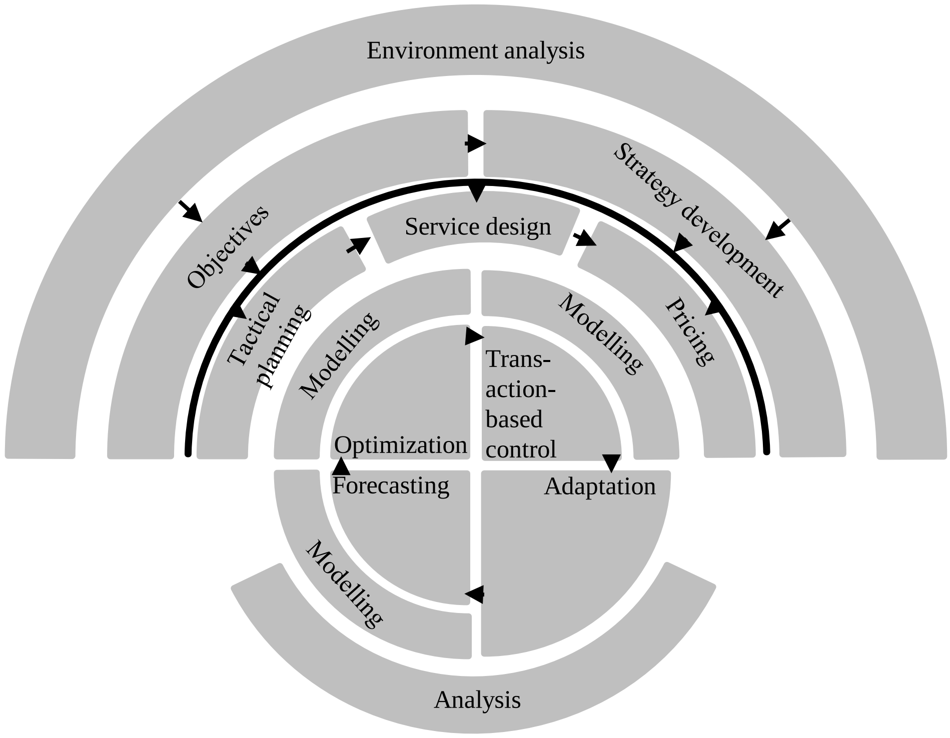


Figure 2.7: Framework for (relationship-based) capacity control (adapted from von Martens, 2009)

agement aim to maximize revenues from this capacity (Cizaire, 2011). The hierarchical order represents the different time horizons of the decisions.

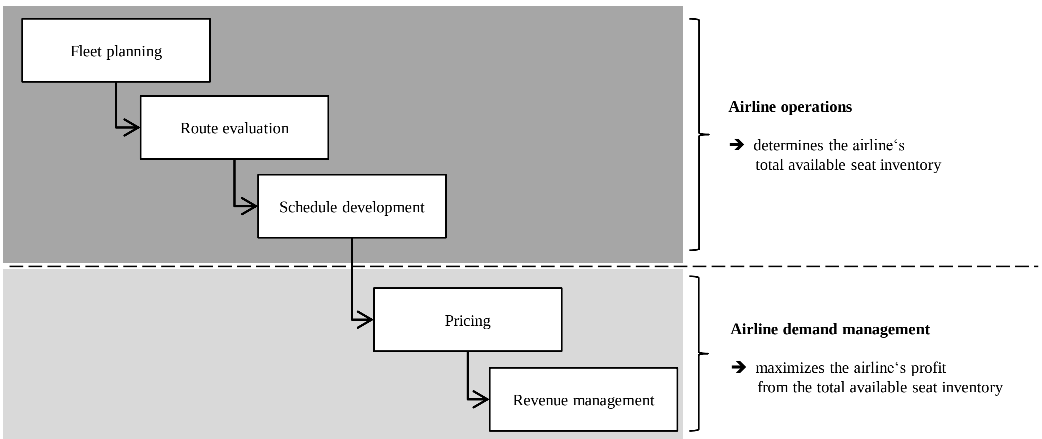


Figure 2.8: Sequential airline planning (adapted from Cizaire, 2011)

## 2.5 Supply Chain Planning Frameworks

In order to discuss the applicability of a common framework for revenue management and its related planning tasks across all considered industries, we give an additional overview of general supply chain planning frameworks for these industries. As discussed in the literature overview in Section 2.3, also operations-related decisions are interrelated with revenue management decisions. In contrast to the previous revenue management frameworks, supply chain planning frameworks focus particularly on operations-related decisions. Therefore, we additionally consider these frameworks.

### 2.5.1 The Supply Chain Planning Matrix

The supply chain planning matrix derived by Fleischmann et al. (2008) structures planning tasks in a manufacturing business-to-business setting. The planning tasks along the good's flow are structured according to their time horizon and the supply chain processes. The supply chain planning matrix is illustrated in Figure 2.9.

Long-term planning tasks refer to the product program and strategic sales planning, the physical distribution structure, plant location and the production system and the materials program and supplier selection. Mid-term planning tasks are on, for example, mid-term sales planning, distribution planning and capacity planning. Short-term planning tasks consider, for example, short-term sales planning, warehouse replenishment, transport planning and lot-sizing. These planning tasks arise in almost every manufacturing context to some extent, while their importance differs for particular supply chains (Fleischmann et al., 2008).

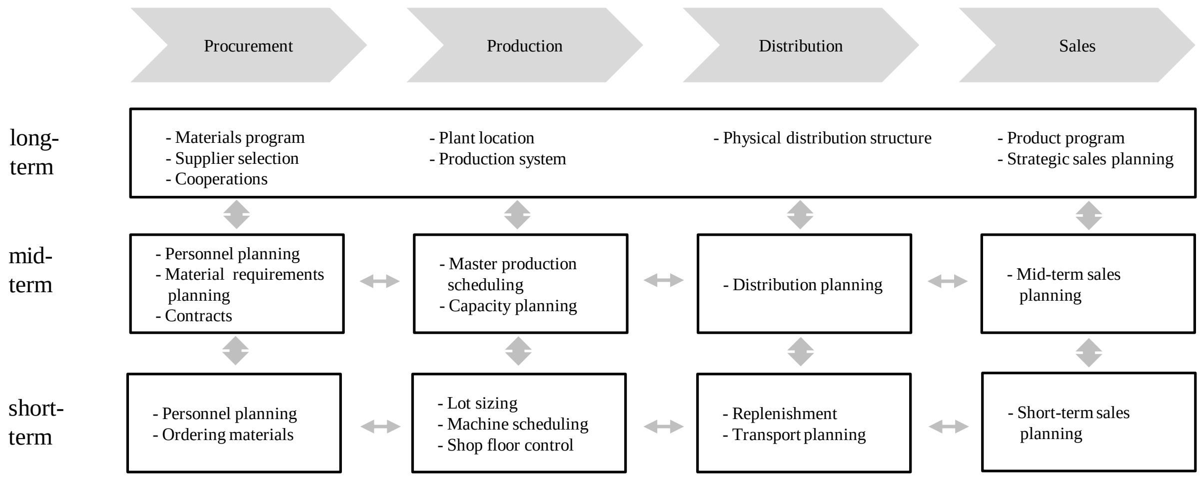


Figure 2.9: The supply chain planning matrix (Fleischmann et al., 2008)

Demand management is considered in terms of short-term sales planning. Demand fulfillment refers to demand management in make-to-stock settings (Fleischmann and Meyr, 2004) and involves planning tasks at and downstream of the decoupling point[3] (Quante et al., 2009). Customer orders are fulfilled from stock, which is still available to promise. If enough stock is available, an order is accepted, otherwise, it is rejected or delivered later (Fleischmann et al., 2008). Demand fulfillment and revenue management are similar (Quante et al., 2009). While replenishments are considered in demand fulfillment, they are not con-

[3]See Hoekstra and Romme (1992) for a definition of the decoupling point.

sidered in service applications as capacity cannot be stored. Quante et al. (2009) provide a framework for structuring different kinds of demand and supply management approaches according to the underlying decision variables.

### 2.5.2 The Retail Demand and Supply Chain Planning Framework

Both Hübner and Kuhn (2012) and Hübner et al. (2013) stress the importance of structuring retail planning tasks and of considering their interactions. Hübner et al. (2013) provide an overview of planning tasks in a (grocery) retail supply chain. They match available literature on planning tasks in a retail supply chain with the supply chain planning matrix by Fleischmann et al. (2008). The resulting framework is illustrated in Figure 2.10. The

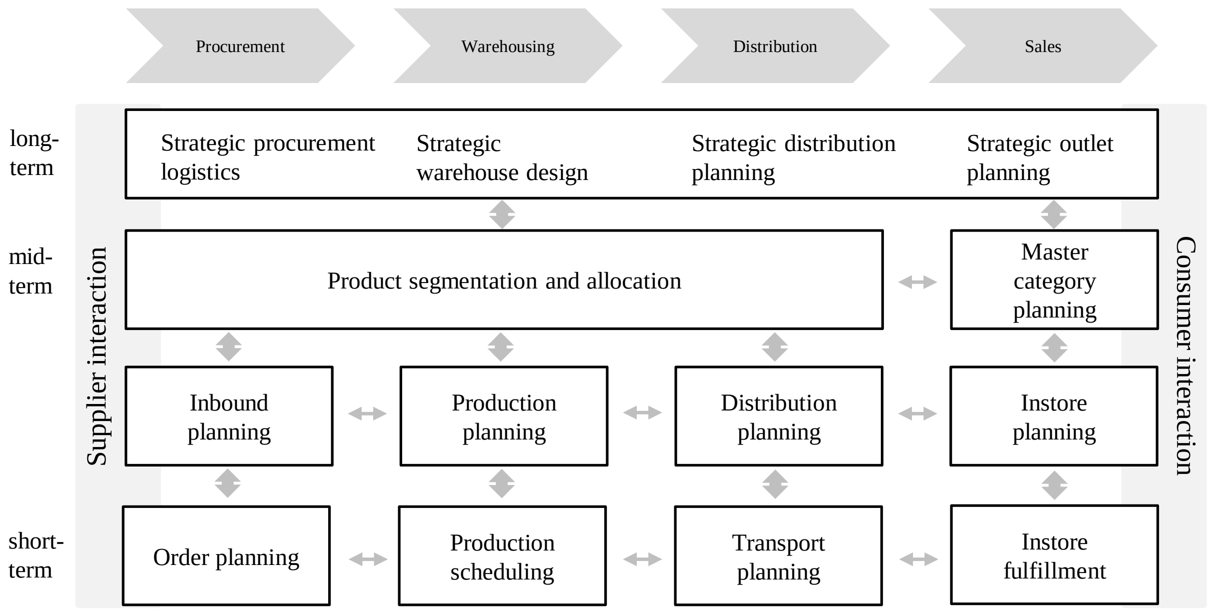


Figure 2.10: The retail demand and supply chain planning framework (adapted from Hübner et al., 2013)

framework differs from the supply chain planning matrix with regard to the horizontal axis. While Fleischmann et al. (2008) considers the production of goods, Hübner et al. (2013) consider decisions related to warehousing as no goods are produced. The framework's focus is on brick-and-mortar retail rather than e-fulfillment settings. Outlet and instore planning tasks play an important role as sales decisions. Compared to the manufacturing context, retail operations have a stronger focus on goods distribution and in-store logistics. The customer interface is also more prevalent in retail supply chains due to the direct contact with consumers and the more downstream location of the decoupling point. Forecasting and sales planning are thus more important in retail (Hübner et al., 2013).

Short-term sales planning entails forecasts and adjusts inventory levels and prices (Hübner et al., 2013). Forecasts are specified on a daily to weekly basis. For perishable products, the short-term alignment of inventory levels and prices impacts consideration of mark-down pricing in order to sell them before the expiry date (see, for example, Federgruen and Heching, 1999).

### 2.5.3 The E-fulfillment Planning Framework

E-fulfillment refers to the internet retail business (Agatz, 2009). Agatz et al. (2008) address issues in e-fulfillment settings by providing an overview of the relevant planning tasks compared to other industries. They also build their framework and literature review on the supply chain planning matrix. The supply chain processes correspond to the processes in the above retail framework except for the fact that the goods' distribution in retail equals the delivery of goods in e-fulfillment. That the stages are to a large extent identical is rather intuitive, as e-fulfillment is a particular type of retail business. On the vertical axis, planning tasks are structured according to their planning horizon. The resulting framework is illustrated in Figure 2.11.

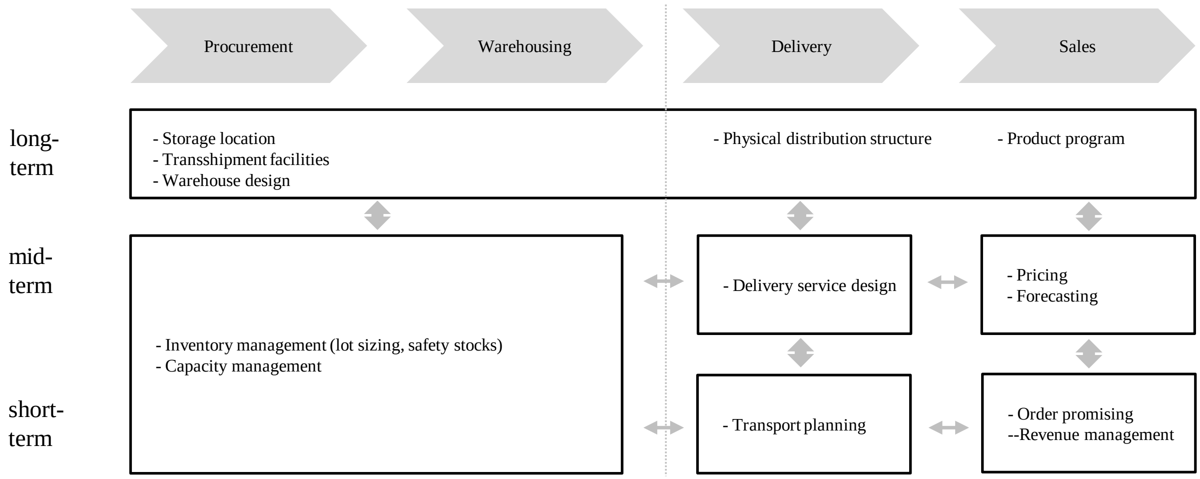


Figure 2.11: The e-fulfillment planning framework (adapted from Agatz et al., 2008)

E-fulfillment differs from brick-and-mortar retailing with regard to the actual product being offered. While stores offer only goods, delivery is part of the actual product offering in e-fulfillment. Delivery is an important determinant of customer satisfaction (Boyer and Hult, 2005). Due to the importance of delivery services, marketing and operations planning tasks are even more strongly interlinked than in retail (Agatz et al., 2008). Agatz et al. (2008) discuss the relevant planning tasks in two categories: sales and delivery planning, and warehousing and procurement.

Compared to the previous frameworks for manufacturing and retail, Agatz et al. (2008) explicitly consider revenue management. Revenue management in e-fulfillment differs from traditional revenue management as (delivery) costs also play an important role (Campbell and Savelsbergh, 2005). Delivery costs and thus the profitability of an order depend on order characteristics such as the location and the preferred delivery time window. This is another reason for considering the sales and delivery stages in a single category. The impact of demand management on the delivery costs is typically large compared to the low (grocery) retail margins (Agatz, 2009). Revenue management in e-fulfillment therefore focuses on managing demand for delivery time windows in order to maximize the expected profit from selling and delivering the requested goods.

## 2.6 A Framework for Revenue Management

In this section, we aim to derive a generalized framework for revenue management and interrelated operations-related and marketing-related planning tasks. First, we summarize the main findings from the literature overview in Section 2.3. Second, we discuss the properties of the revenue management frameworks available in literature and reviewed in Section 2.4. Third, we compare the planning tasks considered in the supply chain planning frameworks with the planning tasks considered in literature in order to identify potential research gaps. Finally, we propose a generalized framework for revenue management and related planning tasks.

### 2.6.1 Main Findings from the Literature Overview

In this section, we summarize the main findings from the literature overview in Section 2.3 and compare the planning tasks considered in literature across the different industries. Table 2.2 shows the respective planning tasks on different hierarchical planning levels identified within the literature overview, for all considered industries.

Comparing the planning tasks considered across the four industries demonstrates that other planning tasks are relevant in different contexts. From an abstract point of view, operations-related decisions determine the firm's available capacity and marketing-related decisions manage a firm's interface with its customers. However, taking a closer look at the respective relevant decisions, they differ across the industries considered. While scheduling decisions are relevant in the manufacturing and e-fulfillment context, they are not relevant in a traditional retail context. Due to the relevance of short-term scheduling decisions, manufacturing and e-fulfillment settings are similar (Agatz, 2009).

Moreover, the same planning tasks are relevant on different hierarchical planning levels in different industries. As an example, again consider scheduling decisions. While scheduling decisions are taken on a short-term basis in both manufacturing and e-fulfillment, scheduling decisions are considered on a strategic level in traditional service industries. Another planning task which arises in different hierarchical levels is the decision about prices. Pricing decisions are taken on a medium-term basis, based on the firm's customer segmentation, whenever quantity-based revenue management is prevalent. However, as soon as price-based revenue management is prevalent, they are taken on a short-term basis.

In order to benefit from customer heterogeneity by means of quantity-based revenue management, firms typically segment their customer base into several customer classes. However, customer segmentation and its interaction with short-term revenue management decisions has only been explicitly investigated by Meyr (2008). Approaches investigating how firms should determine their fences are related but take the respective fencing decisions for a given number of customer classes. An integrated approach considering simultaneous decisions on the number of customer segments and the respective allocations does not yet exist. The firm does not necessarily benefit from additional customer classes when accounting for additional costs, for example, for managing the resulting complexity or through additional demand substitution between the customer classes.

Another area where research is rather limited is research on the interaction of revenue management with product bundling. Product bundling is particularly promising in the field of online retailing (Netessine et al., 2006) and is thus regularly applied. Little research exists in this field, however. This is probably due to the focus on limited delivery capacity instead of the available product quantities when additionally accounting for revenue management. Although decisions on replenishments can be considered similar to a traditional retail context, product bundling (and potentially other planning tasks) can be taken on an even shorter-term basis by internet retailers as opposed to traditional retailers. Through the internet, these decisions can be taken online in real-time and thus dynamically for each customer request. Click histories of customers are also available which supports product bundling decisions.

### 2.6.2 Comparison of Revenue Management Frameworks with the Literature Overview

In the following analysis and discussion of the revenue management frameworks, we focus on several aspects: whether the frameworks refer to specific industries, whether different departments as decision makers are represented, whether the frameworks account for different time horizons of different planning tasks, and the extent to which the frameworks cover the planning tasks discussed in the literature overview in Section 2.3 and illustrated in Table 2.2.

All revenue management frameworks reviewed more or less refer to standardized services and thus traditional applications of revenue management. Only one of the frameworks of Philips (2005) considers dynamic pricing explicitly. All other frameworks focus on quantity-based revenue management. However, none of the frameworks accounts for short-term operations-related decisions, such as scheduling decisions, and thus do not apply to manufacturing or e-fulfillment.

Decision hierarchies are considered in almost all frameworks. While Klein and Steinhardt (2008) account for them explicitly, the other frameworks consider the hierarchical levels implicitly through the sequence of planning tasks.

| **Industry** | **Service** | | **Manufacturing** | | **Retail** | | **E-fulfillment** | |
|---|---|---|---|---|---|---|---|---|
| **Time horizon** | Operations | Marketing | Operations | Marketing | Operations | Marketing | Operations | Marketing |
| **Long-term** | Schedule design | | | | | | | |
| **Medium-term** | Fleet assignment | Customer segmentation | Production quantity | Customer segmentation | Initial inventory | Product offer | | Delivery service pricing |
| | Capacity planning | Fencing | Replenishment quantity | Pricing | Capacity rationing | Assortment planning | | Product bundling |
| | | Pricing | | | | Selling format | | |
| **Short-term** | Dynamic capacity management | Revenue management | Scheduling | Revenue management | | Revenue management | Delivery routing | Revenue management |
| | | Overbooking level | | | | | Scheduling | |

Table 2.2: Planning tasks across all industries

Except for the framework of Klein and Steinhardt (2008), the frameworks do not explicitly distinguish between operations- and marketing-related decisions. While Vinod (2004), Philips (2005) and Ng et al. (2008) do not consider operations-related decisions at all, von Martens (2009) and Cizaire (2011) do so. However, they do not consider the firm's related decisions in different hierarchical levels.

Most of the frameworks take a rather narrow perspective on short-term revenue management decisions in combination with, at most, medium-term marketing-related decisions. Exceptions are the frameworks by Klein and Steinhardt (2008), von Martens (2009) and Cizaire (2011). They also consider a long-term perspective, but their scope is different. von Martens (2009) does not discuss actual long-term planning tasks while Klein and Steinhardt (2008) and Cizaire (2011) account for them explicitly. These frameworks cover all decision hierarchies but to a different extent. The framework by Klein and Steinhardt (2008) considers the longer-term decisions in more detail.

Customer relationship management as a marketing-related area of planning is not considered explicitly in any of the frameworks. It is only implicitly considered in the framework of von Martens (2009) through the adaption of transaction-based revenue management decisions.

To summarize, there are several approaches in the literature which aim to provide a framework for revenue management and related planning tasks. However, none of the frameworks completely mirrors the notion of McGill and van Ryzin (1999) and integrates the current state of literature as provided in Section 2.3. In particular, most of the frameworks neither consider operations-related planning tasks nor do they consider marketing-related planning tasks such as customer relationship management. Table 2.3 gives an overview of the presented approaches and summarizes the previous discussion and evaluation of the existing frameworks.

| **Framework** | **Industry** | **Departments** | **Hierarchy** | **Literature coverage** |
|---|---|---|---|---|
| Vinod (2004) | hotels | no | (yes) | low |
| Philips (2005) | services | no | (yes) | low |
| Klein and Steinhardt (2008) | services | yes | yes | medium |
| Ng et al. (2008) | services | no | no | low |
| von Martens (2009) | services | no | (yes) | medium |
| Cizaire (2011) | airlines | (yes) | (yes) | medium |

Table 2.3: Planning frameworks in literature: Overview of characteristics

### 2.6.3 Comparison of Supply Chain Planning Frameworks with the Literature Overview

Comparing the above supply chain planning frameworks with the summary of the literature overview in Table 2.2 allows a comparison of planning tasks considered in literature with actual planning tasks, and thus potential research gaps in revenue management literature. As discussed above, supply chain planning frameworks have a broad focus on several supply chain processes and particularly consider relevant operations-related planning tasks. We first discuss general differences in the results from the literature overview and the supply chain planning frameworks. We then discuss similarities and differences for each industry in more detail.

In contrast to the supply chain planning frameworks, we do not explicitly further split the operations-related decisions according to different departments of the firm as we wanted to keep the literature overview general.

Long-term planning tasks are relevant in general, as shown in the frameworks. However, the interaction of long-term planning tasks with short-term revenue management has only rarely been considered. Somehow, this is intuitive as long-term decisions only affect short-term decisions indirectly by their effects on medium-term decisions. One exception is research into services as shown in Table 2.2. Here, interactions of long-term and short-term planning tasks are considered. In this case, however, research does not really consider two planning tasks but only considers approximations for short-term revenue management decisions and their impact on customer demands when taking long-term decisions. In general, research on simultaneous decisions focuses on medium- and short-term planning tasks.

In the following, we compare the findings from the literature overview with the supply chain planning frameworks for each industry in more detail. Comparing the planning tasks for manufacturing in Table 2.2 with the supply chain planning matrix in Figure 2.9 demonstrates that both procurement and distribution decisions have not yet been a focus in combination with revenue management. While procurement decisions are partly considered through inventory models, distribution decisions have not yet been addressed at all. One reason for this might be their different focus: while supply chain planning frameworks identify the relevant planning tasks in general, research on the interaction of revenue management with other planning tasks is restricted by only considering limited capacities. Otherwise, revenue management is not applicable. Typically, either the end-product, intermediate product(s) or the production capacity is limited in a manufacturing context, depending on whether a make-to-stock, assemble-to-order or make-to-order setting is considered. Procurement and distribution decisions hardly affect these possibly scarce capacities and might therefore not be in the focus when considering decisions related to revenue management. All other medium- and short-term planning tasks considered in the supply chain planning matrix correspond to planning tasks discussed in the literature overview.

In retail settings, warehousing operations and the distribution of goods are not considered in the literature overview in the retail context. Again, this is intuitive. The limited resource is typically the available amount of products. This is determined on a medium-term basis. Marketing-related decisions affect the product offer. By deciding on, for example, the selling format, the retailer may limit the available capacity of a particular product artificially. The planning tasks investigated in interaction with revenue management match the general supply chain planning tasks discussed by Hübner et al. (2013). In contrast to the supply chain planning frameworks for a manufacturing and e-fulfillment context, Hübner et al. (2013) consider the interaction of a firm's sales decisions with customer behavior.

In e-fulfillment, the focus of both the available frameworks and the available literature is on distribution decisions. Their importance as part of the actual product offer is the main difference from a traditional retail context. Therefore, for example, actual product procurement decisions are not considered. They also arise in the context of e-fulfillment but are very similar to retail settings. Revenue management in e-fulfillment is two-fold: demand for delivery time windows can be managed by either pricing them appropriately or by managing their availability for specific customers. In contrast, only one particular type of revenue management is prevalent in the other industries considered.

While forecasting is considered in the supply chain planning frameworks as part of the sales planning, customer relationship management is not considered in most of the supply chain planning frameworks. Only the retail framework potentially accounts for customer relationship management, due to the interaction of sales planning tasks with customer behavior.

To summarize, the relevant supply chain planning tasks have also been considered in literature in terms of their interaction with short-term revenue management decisions. More planning tasks are considered in the above supply chain planning frameworks compared to the planning tasks in the literature overview. There are two reasons for this. First, the literature overview focuses on limited resources only, meaning planning tasks which are not related to these resources are not considered. Second, long-term planning tasks are not directly interlinked to short-term revenue management decisions, and therefore, they are only rarely considered in terms of interaction with revenue management.

### 2.6.4 A Generalized Framework for Revenue Management

In this section we propose a generalized framework for revenue management. The framework builds on the findings from the previous sections and hence on the literature overview as well as the revenue management and supply chain planning frameworks reviewed before.

The different industries differ regarding the type of revenue management applied. While quantity-based revenue management is prevalent in a service and manufacturing context, price-based revenue management is prevalent in a retail context. This difference affects the relevant related planning tasks. While pricing and customer segmentation is relevant as a basis for quantity-based revenue management, pricing is the short-term revenue management decision itself in industries where price-based revenue management is prevalent. When comparing the discussed frameworks for revenue management, this may be a reason why they either consider quantity-based or price-based revenue management and not both types of revenue management simultaneously.

As shown and already discussed above in Section 2.3, operations-related decisions determine a firm's available capacity. We have seen in the literature overview that the actual decisions about a firm's available capacity are taken by different departments in different industries. This is because the potentially limited capacities across industries differ. For example, in a manufacturing context production quantities or production schedules are determined by the production department while delivery routes are determined by the distribution planning department in an e-fulfillment context. Decisions about available capacity are taken on different hierarchical levels in different industries. Depending on the industry, the respective capacities are fixed either on a medium-term or on a short-term basis. In an airline context, aircraft are typically assigned to particular flights on a medium-term basis while short-term production schedules have to be considered in a manufacturing context in order to determine the available capacity. Due to the strong focus in literature on revenue management applications for standardized services, the frameworks for revenue management also focus on these applications and thus a single industry application. However, the above observation also explains to some extent why the discussed revenue management frameworks focus on one industry application only.

In general, not every planning task is relevant in every industry context. This is related to the previously discussed differences of operations-related planning tasks across industries. As discussed, when comparing the results from the literature overview with the supply chain planning frameworks, planning tasks from some planning areas are also irrelevant with regard to revenue management. Which planning tasks are irrelevant depends on the industry considered. While distribution decisions are related to revenue management in e-fulfillment, they are more or less irrelevant for revenue management in a traditional retail context. This is because the firm's limited capacity actually results from planning tasks in different areas and thus differs across industries.

In order to cover research on the interaction of revenue management with other planning tasks, the industry-independent planning tasks regarding forecasting and customer relationship management should be included. While forecasting is considered in existing frameworks, customer relationship management is not. This is probably due to the typically short-term orientation of revenue management.

Due to the different relevant planning tasks in different industries, the development of a general framework for revenue management faces the following trade-off: while all industries should be considered with the respective relevant planning tasks, a general framework should at the same time offer a high degree of generalizability. Being too industry-specific results in a low generalizability due to the differences in the relevant planning tasks. Being too general, however, possibly results in relatively abstract categories of planning tasks.

As discussed above, the supply chain planning frameworks further split the operations-related decisions in order to assign them to different departments of the firm. The industries differ with regard to which department actually decides on the limited capacity as the basis for revenue management. Considering a firm's departments thus explicitly stresses the differences of the industries. However, this contradicts the idea of a general framework across different industries. Omitting a firm's departments is in line with the framework of Klein and Steinhardt (2008).

Forecasting is necessary at every hierarchical level. For example, forecasts are necessary as a basis for determining a firm's total available capacity on a long-term basis or for determining fleet assignments in order to match supply with demand on single legs. Customer relationship management also entails longer-term and short-term features.

Most of the previously discussed issues complicate the applicability of a single framework covering both price-based and quantity-based revenue management and the related planning tasks across all industries. We therefore distinguish between quantity-based and price-based applications in the following.

First, we consider quantity-based revenue management applications. Comparing the medium-term marketing-related planning tasks illustrated in Table 2.2 across the different quantity-based applications demonstrates that they are rather similar. Decisions about customer segmentation, assortment planning or the selling format determine which products are offered in which way. Thus, they can all be summarized as decisions about a firm's service offering, in general. On a short-term basis, (quantity-based) revenue management decisions are taken.

Medium-term operations-related decisions determine a firm's capacity and are denoted as medium-term capacity planning. Comparing the medium-term operations-related planning tasks in Table 2.2 across the quantity-based applications demonstrates that they are also rather similar. Deciding on the production capacity or replenishment quantities in a manufacturing context is comparable to the fleet assignments in the airline context: all of these decisions determine the firm's available capacity on a medium-term basis.

On a short-term basis, capacity may be more or less flexible, depending on the particular industry. While dynamic capacity management allows for short-term adjustments of the available capacity in the service context, scheduling decisions in the manufacturing or e-fulfillment context work similarly. In general terms, these decisions can be summarized under short-term capacity planning as they determine a firm's available capacity on a short-term basis. Short-term capacity planning has not yet been considered within a revenue management planning framework so far.

While the different planning tasks can be summarized into the rather general categories above, they differ in the actual modeling approaches as do, for example, the relevant costs across the different industries.

Considering the rather general categories of planning tasks allows for a generic revenue management framework for quantity-based applications as illustrated in Figure 2.12.

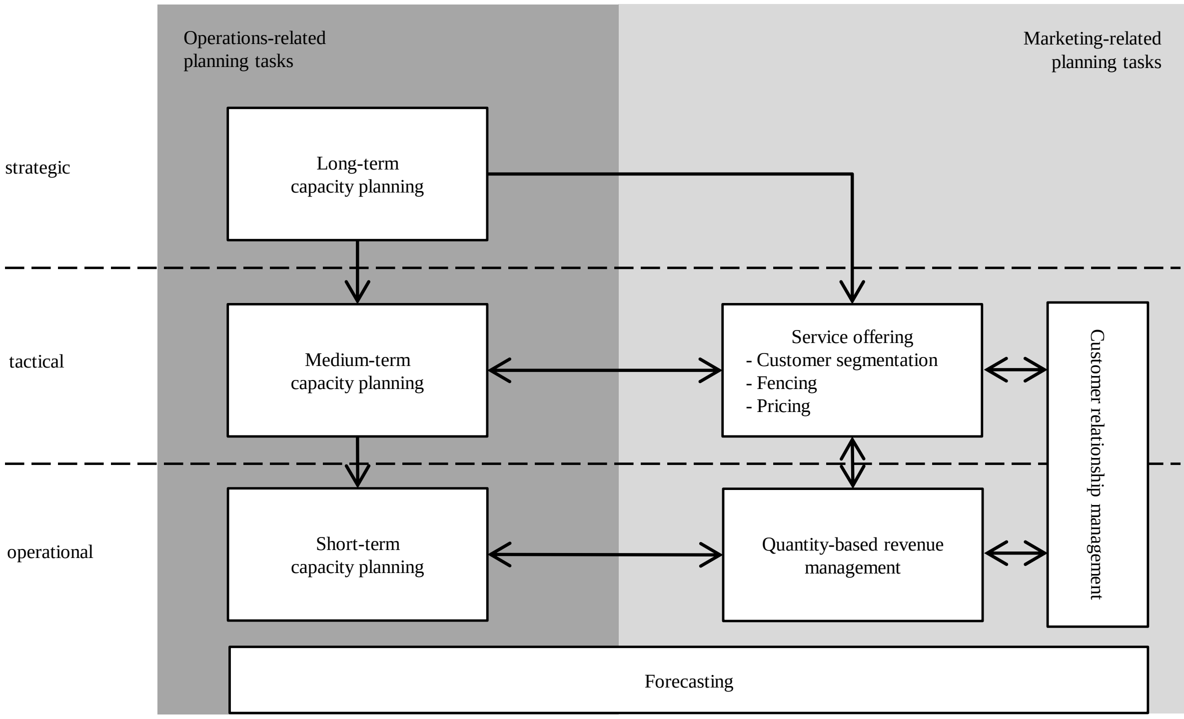


Figure 2.12: A framework for quantity-based revenue management applications

Comparing the above framework to the frameworks discussed in Section 2.4 yields that it is similar to the framework of Klein and Steinhardt (2008) in many facets. Our framework thus builds on the framework of Klein and Steinhardt (2008). However, in contrast to the framework of Klein and Steinhardt (2008), the above framework does not focus on airline applications or services in general. It represents the findings from the literature overview on planning tasks that interact with revenue management: both short-term capacity decisions (such as scheduling decisions) and customer relationship management are considered. These planning tasks are not considered in the framework of Klein and Steinhardt (2008). However, these two extensions mirror two current trends in revenue management research: revenue management is transferred to non-traditional applications and the focus on long-term customer relationships from marketing research also starts to affect revenue management.

In contrast to the supply chain planning frameworks, the operations-related planning tasks are not further split into, for example, procurement or distribution decisions. As discussed above, the various quantity-based revenue management applications differ with regard to which type of operations-related decisions affect the firm's capacity. Summarizing the different areas of operations-related decisions allows a general framework to be provided.

As an analogy to the framework for quantity-based revenue management applications, it is also possible to provide a framework for price-based revenue management on this level of abstraction. We derive the framework for price-based revenue management applications from the literature overview regarding retail and (partially) e-fulfillment applications. The resulting framework is illustrated in Figure 2.13.

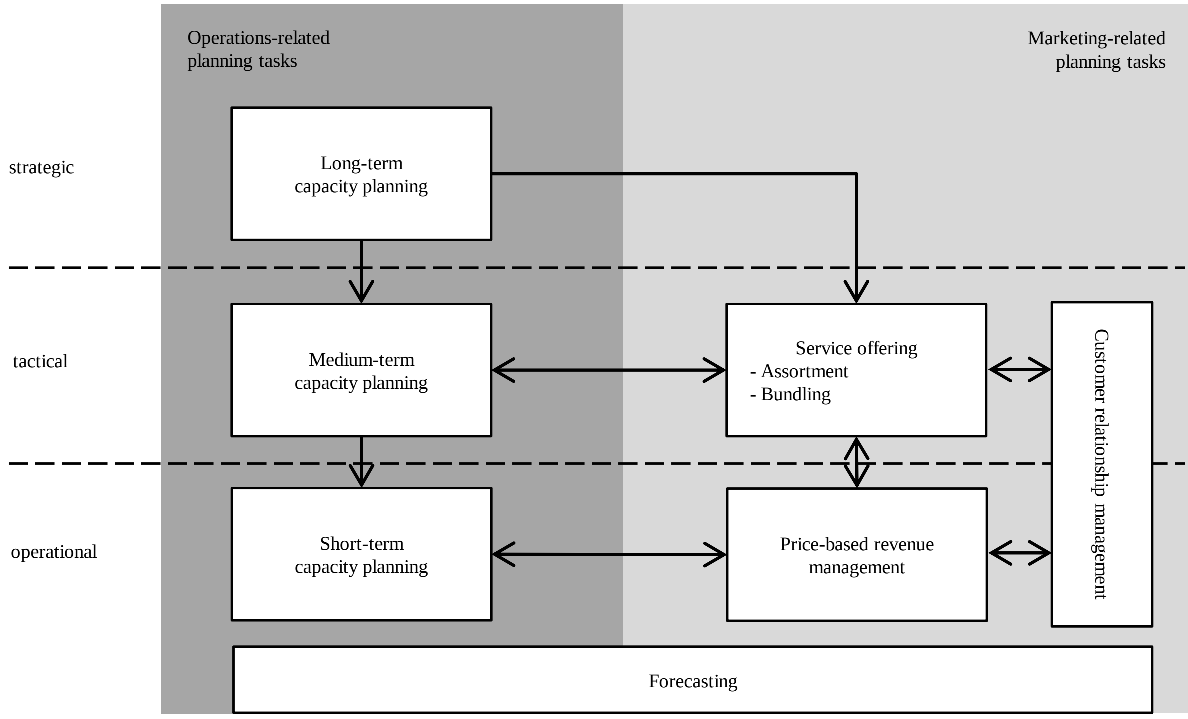


Figure 2.13: A framework for price-based revenue management applications

The framework in Figure 2.13 differs from the framework for quantity-based revenue management applications mainly with regard to the relevant marketing-related medium-term planning tasks. In contrast to the proposed framework for quantity-based revenue management, pricing is now an operational demand management decision itself.

## 2.7 Conclusion

After a short overview of basic revenue management concepts, we have investigated the literature on coordinated decision-making in revenue management in this chapter. Revenue management decisions focus on the operational level and are typically based on longer-term decisions such as decisions on a firm's capacity, the customer segmentation in place or a firm's prices. In this chapter, we focus on the interaction of revenue management decisions with these other planning tasks.

In order to consider both quantity-based and price-based revenue management and both traditional and non-traditional applications of revenue management, we focus on four industries: quantity-based revenue management in service applications and manufacturing, and price-based revenue management in retail and e-fulfillment. We review existing planning frameworks both for revenue management in general and for the industries discussed. Based on the existing frameworks and the literature overview, we propose a general framework covering the observed relevant planning tasks.

Research that includes an additional planning task which is related to revenue management is rather rare compared to research on pure revenue management decisions. More research also exists in traditional applications such as services and retail, compared to the non-traditional applications. To the best of our knowledge, an overview focusing on the interplay of revenue management decisions with other planning tasks does not yet exist. Existing reviews either focus on particular revenue management applications or, for example, demand modeling for revenue management.

From the literature overview, we find that planning tasks related to revenue management differ across the considered industries. While capacities are determined on a medium-term basis in the service context, the firm is more flexible on a short-term basis in manufacturing and e-fulfillment settings. Marketing-related decisions such as pricing differ regarding their time horizon depending on whether quantity-based or price-based revenue management is prevalent. While pricing is a typical medium-term decision in quantity-based applications, prices serve as the actual demand management lever on the operational level in price-based revenue management applications. Comparing the planning tasks investigated with supply chain planning frameworks for the considered industries demonstrates that the planning tasks identified in the frameworks have also been considered with regard to their interaction with revenue management.

In addition, we review existing revenue management frameworks. In contrast to the supply chain planning frameworks, they focus only on revenue management and directly related planning tasks. The revenue management frameworks discussed focus strongly on quantity-based approaches for standardized services and neither consider dynamic pricing nor apply to manufacturing or e-fulfillment. Operations-related (in particular short-term operations-related decisions) and longer-term planning tasks are not considered in most cases. Decisions from the field of customer relationship management have not yet been considered in these revenue management frameworks, however, literature has already addressed the interaction of revenue management with customer relationship management.

Based on the findings from the literature overview and both the revenue management and supply chain planning frameworks, we discuss and provide a general framework for revenue management and related planning tasks across different industries. In general, the trade-off is between generalizability and the details considered from the different industries. The marketing-related decisions differ significantly depending on whether quantity-based or price-based revenue management is applied. Within the framework, we only consider operations-related decisions in general on different hierarchical levels instead of accounting for different operations departments. We distinguish between quantity-based and price-based applications.

The resulting general framework builds on the framework of Klein and Steinhardt (2008) but differs from existing approaches in two main issues: we account for current research into the interaction of revenue management with customer relationship management and we consider short-term operations-related decisions. Customer relationship management affects other marketing-related decisions such as segmentation or pricing, and thus also revenue management decisions. Other criteria next to short-term profitability should be considered. Existing frameworks have not yet explicitly considered customer relationship management. The same is true for short-term operations-related planning tasks. They are particularly relevant in manufacturing and e-fulfillment applications, which have not yet been a focus.

In this chapter, we consider four different revenue management applications. More applications do exist (see, e.g., the overview by Talluri and van Ryzin, 2004b). Within our framework, the decision categories are very general. The identified decision categories do not necessarily have to apply to industries not considered here. Whether the frameworks also apply to other industries should be investigated as a new direction of research. This is true in particular for non-traditional revenue management applications to which revenue management ideas are transferred and adapted.

# Chapter 3

# Revenue Management and Customer Relationship Management: Coordinating Short-term and Long-term Performance

## 3.1 Introduction

Revenue management is concerned with demand management decisions on a transactional basis (Talluri and van Ryzin, 2004b) while, for example, customer relationship management proposes to focus on long-term customer relationships instead of maximizing short-term profits. Empirical research suggests that customers adapt their purchase behavior according to the service they have experienced from a firm in the past. Consequently, short-term allocation decisions in a revenue management setting might also affect future customer behavior. Our work deals with these intertemporal demand interdependencies and investigates how a firm should allocate capacity to customer classes when customers adapt their repurchase probabilities depending on whether their order has been accepted or denied.

Revenue management originates from, and gains great success in, service industries, such as the airline, hotel and car rental businesses (see Section 2.2) but has also been recognized as advantageous in other industries, including manufacturing and retailing. Customer relationship management, which originates from marketing research, proposes a focus on long-term customer relationships. Differential resource allocation to customers with different economic values for the firm in order to maximize the customers' values (Reinartz and Venkatesan, 2008) plays a key role in both approaches. Both concepts differ with regard to their objectives. While revenue management yields optimal short-term allocation decisions, which maximize revenues within a booking horizon, customer relationship management aims to establish profitable long-term customer relationships. As the long-term orientation has gained significant importance within the marketing and management literature, some earlier papers in the field of revenue management have already identified the need for the integration of both concepts (Belobaba, 2002; Esse, 2003; Noone et al., 2003; Shoemaker, 2003).

Another aspect that is relevant regarding the integration of a long-term perspective into revenue management decisions, is the customers' perceived fairness of revenue management techniques. Empirical research investigates how far customer satisfaction and thus loyalty is affected by applying revenue management (i.e., in particular by denying customer requests). Customer loyalty has proved to be a key driver for profitable long-term customer relationships and consequently constitutes an important objective in customer relationship management. In general, results indicate that short-term allocation decisions might affect (due to being perceived as unfair) customer satisfaction and loyalty and therefore also customer relationships in the long run (see, e.g., Suzuki, 2004; Wangenheim and Bayón, 2007). These findings imply that customer demand should be considered interdependent over time. Future customer demand thus also depends on current allocation decisions. Both revenue management and customer relationship management therefore interact. In the field of inventory management, several authors have already studied models where the effects of poor customer service on customer demand in subsequent periods are incorporated (e.g., Adelman and Mersereau, 2013; Olsen and Parker, 2008). These papers are based on the findings of empirical research regarding customer reactions towards physical stock-outs. Stock-outs cause customers (amongst others) to switch brand or store (see, e.g., Campo et al., 2000). Accordingly, stock-outs might have severe impacts in the long-run (e.g., Anderson et al., 2006). In general, there are few analytical contributions, including intertemporal demand effects caused by customer service. The papers particularly focus on inventory-related problems and not on issues related to revenue management.

In this chapter, we focus on the interaction between the short-term and the long-term performance of revenue management, assuming that revenue management decisions affect future customer demand. We therefore investigate how intertemporal demand effects, caused by accepting or denying customer requests, can be captured in a standard revenue management model in order to support optimal decision making from a long-term perspective. Our approach is intentionally generic and not tailored to a specific setting or application of revenue management (e.g., airline or hotel revenue management). Our approach is relevant to several industries. Industries which typically apply revenue management have also started to introduce customer relationship management programs aimed at long-term customer relationships. These firms thus manage capacity through revenue management in the short run and at the same time actively manage relationships with their customers independent of revenue management. We investigate how short-term revenue management decisions should change due to the intertemporal demand effects considered. We are particularly interested in the value of considering the effects with regard to the expected profit and how the actual allocation decisions of the firm change.

In order to do so, we present a quantity-based two-class stochastic dynamic programming model which accounts for the intertemporal effects of a firm's allocation decisions about customer demand. In particular, we build on the well-known static stochastic two-class revenue management model by Littlewood (1972) and extend it for multiple booking horizons in order to account for intertemporal demand effects. The model demonstrates optimal capacity allocations in the long run by considering two trade-offs: the trade-off between the heterogeneous customer classes within a booking period and the trade-off between the demands of the customer classes across subsequent booking periods. We analytically derive a heuristic for a partitioned protection level, which accounts for part of the intertemporal demand effects. In addition, we compare the optimal allocation policy with both the derived heuristic and other allocation heuristics based on Littlewood's model within a detailed numerical study. We find that the derived heuristic performs close to optimal in most of the scenarios. However, even heuristics which merely update customer demand perform fairly well. In general, updating customer demand forecasts is particularly relevant when facing intertemporal dependent demands.

To summarize, this chapter makes the following contributions:

- We give an overview of both empirical and analytical research on perceived customer service and its long-term effects.
- Based on the results of previous research, we present a two-class stochastic dynamic programming model which maximizes the expected profit within a planning horizon consisting of multiple booking periods. Subsequent booking periods are linked by intertemporal effects on customer demand caused by accepting and rejecting customer requests. The model presented differs from existing approaches in two ways:
    - While the existing approaches at the interface of revenue management and customer relationship management determine allocations based on the customer's value (determined as a weighted average of the customer's willingness-to-pay and the customer lifetime value), the firm makes its decisions based on short-term contributions (i.e. the customer's willingness-to-pay) as in the standard model. However, it anticipates the effects of allocation decisions on customer demands in subsequent periods.
    - The approaches of anticipating customers' reactions towards service in the field of inventory management track relationships with single customers. In contrast, we focus on aggregate effects on the level of customer classes.
- We analytically derive an allocation heuristic which accounts for the intertemporal effects on the demand of high-value customers.
- Within a numerical study, we compare the optimal allocation policy to several allocation heuristics. We evaluate the value of anticipating the intertemporal demand effects, and investigate the impact of several parameters on the allocation decisions.

We find that applying the optimal allocation policy on average yields an additional 3.85% of expected profit compared to applying a static protection level throughout the entire planning horizon and 0.57% of expected revenue compared to applying Littlewood's rule reactively in every booking period. Thus, updating demand forecasts (and the protection level) is particularly important. Our heuristic approach performs close to optimal in almost all scenarios. As for the impact of different parameters, we show that the intertemporal demand effects of higher-value Class 1 customers are particularly important and have the strongest influence on both the difference in the allocation decisions and the profit gaps. The protection level increases over time (with the capacity as the natural boundary) under all considered dynamic allocation policies, as soon as Class 1 customers react positively towards their requests being accepted. The allocation policies differ regarding how quickly they adapt the protection level. Thus, a considerable share of the total profit gap results from the first several booking periods, in which the allocation decisions differ the most.

The remainder of this chapter is organized as follows. We review the related literature in Section 3.2. In Section 3.3 we present a two-class stochastic dynamic programming model which incorporates intertemporal effects of allocation decisions on customer demand based on the stochastic dynamic formulation of Littlewood's model. For a planning horizon of two booking periods, we discuss the underlying trade-offs and derive a closed-form expression for a heuristic partitioned protection level under simplifying assumptions in Section 3.4. In Section 3.5 we present a particular approach for modeling demand interdependencies, discuss several myopic allocation heuristics and present the results of the numerical study. Section 3.6 summarizes our findings.

## 3.2 Literature

This chapter builds on several streams of literature related to revenue management and long-term customer behavior in general. General overviews of revenue management are given in the books of Talluri and van Ryzin (2004b) and Philips (2005) (see also Section 2.2). In the following, we investigate five streams of literature which are particularly relevant for our modeling approach. First, we give a short overview of empirical research on customer perceptions of revenue management. Second, we review both empirical and quantitative literature on customer perceptions of denied service. Third, we introduce basic findings from marketing research which theoretically link the perception of short-term service failures to long-term customer behavior. Fourth, we review a stream of literature, which focuses on revenue management with endogenous customer behavior. Fifth, and closest to our work, we investigate research into the interface of revenue management and customer relationship management.

### 3.2.1 Customer Perceptions of Revenue Management

Empirical research into revenue management mainly investigates customer perceptions of different revenue management techniques. The focus is on the perceived fairness of allocation decisions (i.e., inventory control and overbooking) and pricing decisions (e.g., Kimes, 2002; Kimes and Wirtz, 2003; Choi and Mattila, 2004, 2006; Wirtz and Kimes, 2007). Wirtz et al. (2003) give an overview of potential conflicts arising from applying revenue management in customer oriented firms. The authors argue that revenue management techniques might influence customer satisfaction negatively and thus harm the firm's success in the long run. Accordingly, a firm should, for example, give special treatment to loyal customers.

Wangenheim and Bayón (2007) and Suzuki (2004) both investigate the behavioral consequences of overbooking and the customer treatment (denial, downgrade or upgrade) offered by an airline. Within a longitudinal analysis, in contrast to Suzuki (2004), Wangenheim and Bayón (2007) find support for significantly negative effects of downgrading and denied boarding, resulting in reduced future transactions and revenues from these customers. When comparing the effects across customer segments, the authors find significantly stronger negative effects for high-value customers while Suzuki (2004) only finds weakly significant effects for leisure travelers. According to Wangenheim and Bayón (2007) the results clearly show the need to consider the consequences of the allocation strategy applied. Regarding seat allocation, Lindenmeier and Tscheulin (2008) study the impact of inventory control and denied boarding on customer satisfaction. Denied boarding is shown to have a negative impact on customer satisfaction for both leisure and business travelers (Hwang and Wen, 2009). The negative effect is significantly stronger for business travelers, supporting the findings from Wangenheim and Bayón (2007).

We build on these results by integrating the empirically observed effects of customer acceptance and rejection into a quantitative model. We allow for effects in either direction and explicitly account for the heterogeneity of effects across customer classes.

### 3.2.2 Customer Perceptions of Denied Service

Empirical findings with regard to the effects of denied service due to revenue management are supported by findings in other fields of research. Considering physical stock-outs under the assumption that denied service due to revenue management is comparable to physical stock-outs, research also finds that customers (depending on product and consumer characteristics and situational factors) may react negatively in terms of switching store or product (see, e.g., Campo et al., 2000). Anderson et al. (2006) find that stock-outs have both short-term and long-term effects. While order cancellation rates increase significantly in the short-term, the authors also find significantly lower repurchase probabilities (and thus decreasing customer loyalty) and revenues in the long run.

Several authors have already incorporated the effects of poor service in terms of physical stock-outs on customer demand in subsequent periods in quantitative models (see, e.g., Adelman and Mersereau, 2013; Olsen and Parker, 2008; Liu et al., 2007; Gaur and Park, 2007; Gans, 2002; Hall and Porteus, 2000). These papers mainly consider inventory management problems in a competitive setting, where competing firms are explicitly modeled (except from Adelman and Mersereau (2013)). As a consequence of bad service, customers defect to a competitor within these settings.

Conceptually, our approach integrates the empirically observed effects of poor customer service under the reasonable assumption that customers also perceive denied requests as poor service. Regarding the modeling approach, as per Adelman and Mersereau (2013), we do not model competition (and thus customer switching behavior) explicitly. On the other hand, unlike Adelman and Mersereau (2013), we do not model individual but aggregate (i.e., class-wise) customer behavior. Nevertheless, with the assumption that rejecting a customer due to revenue management techniques has an effect on future customer demand, our approach is consistent with the approaches in this stream of research.

### 3.2.3 Basic Theories on the Effects of Customer Perceptions on Customer Loyalty and Firm Profitability

The third stream of literature conceptually links s firm's decisions to long-term customer behavior and thus the firm's profitability. In principle, all a firm's decisions can potentially have an effect on customer behavior. Customer satisfaction plays a key role in marketing research. Research has investigated its antecedents and its effects on customer behavior for decades (see, e.g., Homburg and Giering, 2001; Homburg and Stock-Homburg, 2008).

By taking particular management decisions, the firm affects customer mindsets, triggers customer (dis)satisfaction and thus customer behavior. Customer behavior refers both to actual buying decisions in the short run and customer loyalty in the long run. The link between customer satisfaction and customer loyalty has often been investigated in literature (for an overview see, e.g., Homburg and Bucerius, 2006). As a central result, research in this field finds a positive relationship between customer satisfaction and customer loyalty, both based on customer intentions and observed customer behavior (see, e.g., Bolton and Lemon, 1999; Mittal and Kamakura, 2001).

According to Gupta et al. (2004) and Gupta and Lehmann (2008), customer retention is crucial for financial success in the firm's markets and thus finally its value in terms of shareholder value (see also Kalwani and Narayandas, 1995; Kumar, 1999). Figure 3.1 summarizes the firm's resulting value chain (see Homburg and Bucerius (2006) for a comparable illustration). Marketing research, and particularly customer relationship management, both investigate the links between the different steps in Figure 3.1 and the entire value chain. Revenue management however is less long-term oriented and traditionally investigates only the link between the firm's decisions and the resulting market success.

Accounting for the effects of short-term allocation decisions on customer behavior in sub-

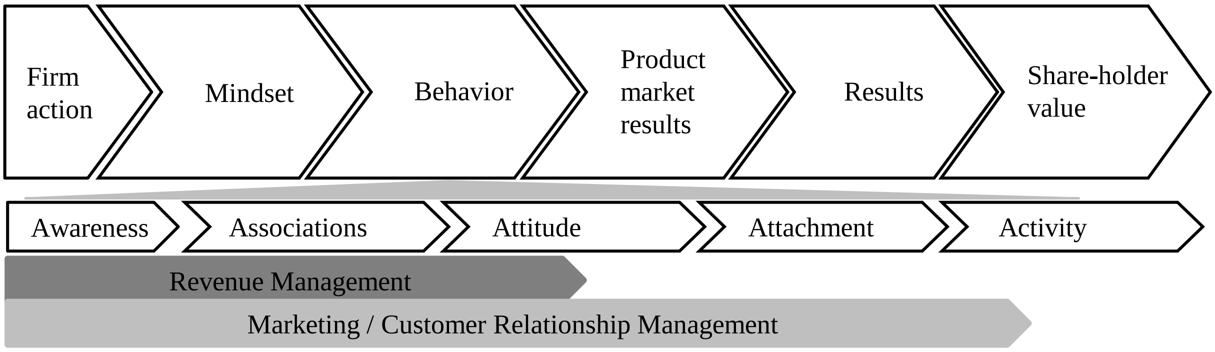


Figure 3.1: The value chain (adapted from Gupta and Lehmann, 2008)

sequent booking periods, reflects the value chain discussed above. According to the value chain of Gupta and Lehmann (2008), firms should consider the effect of their decisions on customer behavior, which affects the long-term profitability in terms of the share-holder value. Maximizing the expected profit over several booking periods accounting for the decisions' effects, mirrors this underlying notion.

In particular, we build on the link between customer satisfaction resulting from the firm's capacity allocation decisions, and customer loyalty investigated in this stream of research. Within the above value chain, customer satisfaction is formed by the customer's attitude towards the firm's action and affects the customer's activity, which determines whether a customer is, for example, a loyal customer or not. The findings in this stream support the assumption that customer demands in subsequent booking periods are interdependent.

### 3.2.4 Revenue Management with Endogenous Customer Behavior

We also contribute to the stream of research on revenue management facing endogenous customer behavior. In contrast to typical consumer modeling in operations management, customer demand is not exogenously given but depends on the firm's decisions when modeled as endogenous (e.g., Elmaghraby and Keskinocak, 2003; Shen and Su, 2007). Most of this research addresses endogenous behavior within one booking period, for example, in terms of strategic customer behavior or the integration of customer choice models into revenue management models (for an overview see Shen and Su, 2007).

Customers behave strategically if they optimally adapt their behavior in response to a firm's pricing or allocation decisions (Talluri and van Ryzin, 2004b). Su (2007) investigates a dynamic pricing problem, where a firm sells a finite inventory to customers who exhibit heterogeneous willingness-to-pay and different degrees of impatience. Both aspects affect the firm's optimal policy. For example, markdown pricing is particularly beneficial if high-value customers are proportionally less patient. Yin et al. (2009) investigate the impact of inventory display formats on the performance of optimal markdown pricing. They find that a firm may benefit from only displaying one unit of the product instead of all units available due to a higher perceived shortage risk from the customer's point of view. Jerath et al.

(2010) investigate last-minute selling and opaque selling in a competitive environment when facing strategic customer behavior. Direct last-minute selling is beneficial when customers have high valuations and when there is little service differentiation in the market. Otherwise, opaque selling dominates.

Liu and van Ryzin (2011) investigate a multi-period setting, where each period consists of two sub-periods: a full-price and a markdown-price period. Customers learn about the firm's capacities over time and adapt their purchase decisions over time according to their expectations about product availability. Depending on a threshold value with regard to the customer expectations, either rationing is optimal or the firm sells the entire capacity at a low price.

Li et al. (2014) empirically investigate the existence of strategic customer behavior in the airline market. For two data sets, they find that 5.2% - 19.2% of the customers behave strategically. These customers request capacity either at the beginning of the booking period or at the end. By comparing markets, they find that strategic customer behavior does not necessarily hurt revenues.

By integrating customer reactions towards their order being accepted or rejected, we borrow the notion of the richer modeling of customer behavior from this stream of research. In particular, customers adapt their purchasing behavior over time depending on the firm's allocation decision.

### 3.2.5 Interface of Revenue Management and Customer Relationship Management

This chapter combines both the short-term (transactional) view of revenue management and the long-term (relational) view of customer relationship management. While both concepts are based on customer heterogeneity, they differ with regard to their objectives. Revenue management decides (based on the customers' willingness-to-pay) about customer acceptance or rejection in the short term in order to maximize the expected profit from a particular booking period. In contrast, customer relationship management models typically decide on customer acquisition and retention by means of long-term oriented customer values such as, for example, customer lifetime value. Based on qualitative propositions regarding the integration of both concepts (see, e.g., Belobaba, 2002; Esse, 2003; Noone et al., 2003; Shoemaker, 2003), von Martens and Hilbert (2011) present, to the best of our knowledge, a first analytical approach involving an integrated quantity-based revenue management model where they incorporate a long-term perspective by substituting the segment price (or willingness-to-pay respectively) by a convex combination of the segment price and the customer lifetime value. The effects of customer acceptance and/or rejection on customer repurchase probabilities are not accounted for in their approach as the authors model a lost-for-good setting. Rejecting a particular customer's order thus not only causes the loss of the current revenue but also the loss of this customer's total long-term value. As an extension, Buhl et al. (2011)

present a deterministic linear approach where the authors explicitly incorporated deterministic repurchase probabilities depending on whether individual customers had been denied capacity.

In a different approach, Ovchinnikov et al. (2014) formulates a stochastic dynamic programming model. His approach closely follows typical approaches in customer relationship management, as the firm decides about customer retention and acquisition spending facing a fixed service capacity. However, the actual operational acceptance and rejection decisions, and their influence on customer retention (as in the approach of von Martens and Hilbert (2011)) are not considered.

By explicitly accounting for the long-term expected profit over several booking periods, our approach is in line with previous approaches focusing on the interface of revenue management and customer relationship management. Compared to the approach of von Martens and Hilbert (2011), allocation decisions in our approach are taken based on segment-based prices (or willingness-to-pay respectively) as in standard revenue management models and not by means of the customer lifetime value. While Buhl et al. (2011) follow a deterministic linear approach, we investigate a stochastic dynamic setting. We allow for general types of effects on customer demand caused by order acceptance and/or rejection.

## 3.3 A Stochastic Dynamic Programming Model for Long-Term Optimal Allocation Decisions

The model presented in the remainder of this section is based on the well-known static, stochastic two-class capacity allocation model of Littlewood (1972), which was introduced in Section 2.2.1.1. In particular, it builds on the stochastic dynamic programming formulation of Littlewood's model. We extend this basic model to a multi-period setting, incorporating effects of the firm's allocation decision on customer demands in subsequent booking periods. Consequently, customer demands are interdependent across booking periods. We intentionally choose the simplest available and most well understood stochastic allocation model as a starting point in order to gain insights into how accounting for intertemporal demand effects affects the optimal allocation policy in a generic setting.

### 3.3.1 Assumptions

In order to consider the effects of customer order acceptance and rejection on future customer demand within the above setting, we model several subsequent booking periods ($t = 1, ..., T$) within the firm's finite planning horizon $T$. Each of the single booking periods builds on Littlewood's model. We make the following assumptions.

**Assumption 1:** *Assumptions of Littlewood's model hold in each booking period*
We assume that the assumptions underlying Littlewood's model (see Section 2.2.1.1) hold in each booking period within the multi-period setting. In particular, we consider two customer classes, with Class 2 demand arriving prior to Class 1 demand. In addition, by this assumption, customer demand distributions within each booking period are assumed to be independent. Although the assumptions of Littlewood's model constitute a simplified view of customer demand only, they are referred to and often applied in the literature.

**Assumption 2:** *Capacity is exogenous and constant throughout the planning horizon*
Capacity is assumed to remain constant over the entire planning horizon $T$. When considering, for example, an airline setting, flight schedules are planned on a strategic or tactical level and can hardly be adjusted in the relevant time period in the setting considered in this chapter (e.g., several subsequent booking periods). As the presented multi-period setting can be interpreted as a sequence of booking periods for single-leg flights in an airline context, this assumption seems to be realistic for moderate values of $T$ not exceeding the length of a pre-determined flight schedule.

**Assumption 3:** *Prices are exogenous and constant throughout the planning horizon*
In order to investigate the impact of customer acceptance and rejection (and thus of allocation decisions), we assume that a firm does not make use of pricing in order to manage customer demand but manages demand only by means of capacity allocation. In particular, prices are assumed to be static and therefore constant over the entire planning horizon.

**Assumption 4:** *Customer acceptance and rejection affect future customer demand*
Based on the findings discussed in the literature review, we assume that a customer's future purchase behavior depends on whether their current capacity request is accepted or rejected. In the context of capacity allocation, a firm's actual allocation policy affects the number of customer orders of both classes that can be accepted or have to be rejected within a booking period. Thus, the firm's allocation policy potentially influences the future customer demand of both customer classes. Customers might adapt their own repurchase behavior and/or influence other customers (re)purchase behavior through word-of-mouth. Again, this assumption is in line with the findings reported in the literature review in Section 3.2. Accounting for customer reactions to acceptance and rejection separately is additionally backed by prospect theory, which argues that customers value perceived losses and gains differently (see, e.g., Kahneman and Tversky (1979)). In analogy to Littlewood's model, $D_i^t$ (i.e., the random demand of customer class $i$ in booking horizon $t$) follows a cdf $F_i^t$. We assume that the demand distribution $F_i^{t+1}$ in booking horizon $t+1$ depends on the previous demand distribution $F_i^t$, customer demand $d_i^t$ in $t$ and the number of accepted requests $acc_i^t$

of Class $i$ in $t$. Therefore,

$$F_i^{t+1} = g_i(F_i^t, d_i^t, acc_i^t) \quad (\forall i, t = 1, ..., T-1) \tag{3.3.1}$$

reflects the interdependence of customer demands in subsequent booking periods for continuous and differentiable functions $g_i$. As both the actual customer demand and the accepted requests in booking period $t$ are considered, this approach also accounts for the rejected requests in booking period $t$. Note that this definition is very generic and allows for many different specific approaches of modeling the intertemporal demand effects. We make additional simplifying assumptions on $g_i$ when deriving a heuristic protection level analytically. Within the numerical study, we present a particular approach for modeling the above interrelation of demand distributions in subsequent booking periods.

### 3.3.2 Stochastic Dynamic Programming Formulation

The firm's goal is to maximize the total expected profit over the planning horizon $T$ ($\in N$) consisting of $T$ booking periods ($t = 1, ..., T$). In each of the booking periods, the firm faces an allocation problem as considered in Littlewood's model. However, compared to Littlewood's model (as presented in Section 2.2.1.1), the firm faces not only the trade-off between the different customer classes within a single booking period but also an additional trade-off over subsequent booking periods. The additional trade-off arises from the fact that customer demands are interrelated over time in the multi-period setting considered. Thus, the firm must also account for the effect of its allocation decisions on customer demands in subsequent booking periods. The firm's allocation decision in a particular booking period $t$ should consequently not be myopic (i.e., optimal within a particular booking period) but should maximize the total expected profit until the end of the planning horizon $T$ in order to anticipate the intertemporal demand effects.

In order to determine the optimal allocation policy in this setting, we formulate the firm's optimization problem in terms of a stochastic dynamic programming model. Note that it would also be possible to formulate the optimization problem analogous to the second approach presented in Section 2.2.1.1 by assuming a protection level policy. Doing so directly specifies the type of allocation policy, and a closed-form model representation is possible. Due to the trade-offs mentioned above, it is however not clear whether a protection level policy is optimal at all. We discuss the potential effects and implications of a firm's allocation decision within Section 3.4.

Compared to the stochastic dynamic formulation of Littlewood's model for a single-period setting (see Section 2.2.1.1), single booking periods (rather than the different customer classes) correspond to the stages of the model. Customer demands (and therefore the allocation decisions) are interdependent over time, due to the trade-offs faced by the firm. The stochastic dynamic programming model can be solved for the optimal allocation policy by backward induction, starting from the end of the planning horizon.

**Parameterization of the State Space**

Let $S^t$ denote the state space in booking period $t$ comprising all admissible states $s^t$ in $t$. In general, the state space is specified such that the parameters are known in each booking period $t$ (as they correspond to the model's stages) and such that the information is sufficient for the firm to make its decisions (Howard, 1971). The demand distributions of both customer classes are exogenously given in the single-period model formulation. Specifying the state space by the remaining capacity for a particular customer class is therefore sufficient in the single-period multi-class setting. In addition, Talluri and van Ryzin (2004b) show for Littlewood's single-period two-class problem that a model formulation where the firm makes its decision based on a particular demand realization $d_2$ is equivalent to the model formulation where the firm does not explicitly account for particular Class 2 demand realization but only the Class 2 demand distribution.

As for the parameterization of the state space in the multi-period setting, we build on the model formulation of the single-period setting. The remaining capacity entering a particular booking period $t$ always corresponds to the available capacity $C$, which differs from the single-period multi-class approach discussed by Talluri and van Ryzin (2004b). The capacity is independent of the firm's decision and not necessary in order to specify the state space. In contrast to the single-period setting, both customer demand distributions $F_i^t$ (and thus the random customer demands $D_i^t$) are endogenous and required by the firm to take the allocation decision. Without knowing the demand distributions, the firm cannot quantify the impact of alternative allocation decisions. In order to closely stick to the model formulation from Section 2.2.1.1, we additionally account for the demand realization $d_2^t$ of the low-value customer class as part of the state space. From this it follows that demand uncertainty with regard to Class 2 has already been resolved prior to the firm's decision in each stage. As in the single-period setting, this is not necessary, but is more intuitive from our point of view. Consequently, a particular state $s^t$ in $t$ is parameterized by

$$s^t = (F_1^t, F_2^t, d_2^t) \quad (\forall t) \tag{3.3.2}$$

in the remainder of this chapter.

Under the chosen parameterization of the state space, the optimal allocation decision in booking period $t$ is determined for each possible demand realization $d_2^t$ separately. It thus explicitly accounts for the possibility that the firm's decision depends on Class 2 demand realization. In general, this is the case, if a booking limit policy is not necessarily optimal. The chosen parameterization is also more intuitive as it explicitly mirrors the notion that individual customers react towards whether their request has been accepted or denied.

Alternatively, the state space could be parameterized solely by the demand distributions $F_1^t$ and $F_2^t$ of both customer classes. If the firm still makes its decision for each Class 2 demand realization, the two parameterizations are equivalent. In this case, they only differ with regard to consideration of the probabilities for the demand realizations $d_2^t$. While they are considered in booking period $t-1$ in terms of the expected revenue to go in the chosen parameterization, the probabilities are directly considered in booking period $t$ under the alternative parameterization.

The computational effort for solving the model is consequently also independent of the parameterization of the state space in this case as all possible Class 2 demand realizations must be considered anyway. As common for stochastic dynamic models, this optimization problem becomes computationally expensive very quickly, depending on the number of booking periods considered. While the number of states equals the number of Class 2 demand realizations in the first period, the size of the state space has already significantly increased in the second booking period. The model has to be evaluated for all combinations of demand distributions and possible Class 2 demand realizations. The size of the state space (i.e., the potential demand distributions that have to be considered in each stage) affects the computation time, thus the longer the planning horizon, the larger the state space and the longer the necessary computation time.

**Decisions**

Based on a particular state $s^t = (F_1^t, F_2^t, d_2^t)$, the firm decides how many Class 2 requests (out of $d_2^t$ requests) to accept in booking period $t$. Thus, the firm only takes a single allocation decision in each booking period. This decision is the same as in the single-period setting. The firm's decision variable is denoted by $acc_2^t$ in the remainder of the chapter and denotes the requests from Class 2 customers accepted in booking period $t$. The set of admissible decisions in a booking period $t$ depends on the current state $s^t$ and is denoted by $A^t(s^t) = 0, 1, ..., min(d_2^t, C)$ $(s^t \in S^t,\ t < T)$. The firm can either accept $d_2^t$ (if $d_2^t < C$) or $C$ (if $d_2^t \geq C$) customer requests at most. The remaining capacity $C - acc_2^t$ is subsequently available for Class 1 customers on a first-come-first-served basis. Acceptance of Class 1 demand is not a decision for the firm. Given a Class 1 demand realization $d_1^t$ and the remaining capacity $C - acc_2^t$, it is always beneficial for the firm to accept as many Class 1 requests as possible (i.e., $acc_1^t = min(d_1^t, C - acc_2^t)$). Therefore, the number of accepted Class 1 requests results directly from the firm's allocation decision $acc_2^t$ in combination with Class 1 demand realizations $d_1^t$.

**Transitions**

In the single-period multi-class setting, the firm's allocation decisions affect the available capacity for each customer class. In the multi-period setting considered, the firm's decision additionally affects customer demands in subsequent booking periods. In particular, the demand distributions are interrelated over time. Based on the fact that the firm only takes a single allocation decision $acc_2^t$ in each booking period $t$, the transition of the demand distributions over time in Equation (3.3.1) can be further specified and stated as

$$F_i^{t+1} = g_i(F_i^t, d_i^t, acc_2^t) \quad (\forall i, t = 1, ..., T-1). \tag{3.3.3}$$

Compared to Equation (3.3.1), Equation (3.3.3) mirrors the fact that the firm only decides how many Class 2 requests to accept in each booking horizon $t$.

Given the above definitions of both the state space and the intertemporal demand effects, the transition occurs as follows. Assume that the firm faces a particular state $s^t = (F_1^t, F_2^t, d_2^t)$ in booking horizon $t$. By taking a particular allocation decision $acc_2^t$, the firm will end up in state $s^{t+1} = (F_1^{t+1}, F_2^{t+1}, d_2^{t+1}) = (g_1(F_1^t, d_1^t, acc_2^t), g_2(F_2^t, d_2^t, acc_2^t), d_2^{t+1})$ in booking period $t+1$, where $d_2^{t+1}$ is drawn from $F_2^{t+1}$.

To summarize, the firm decides about $acc_2^t$ for given $d_2^t$. The decision $acc_2^t$ also determines the number of rejected Class 2 requests. The number of rejected Class 2 customers results from $\max(0, d_2^t - acc_2^t)$. The resulting demand distribution $F_2^{t+1}$ in booking horizon $t+1$ thus follows from $acc_2^t$. Given the remaining capacity $C - acc_2^t$, Class 1 demand realizes based on $F_1^t$. Its realization $d_1^t$ determines the number of accepted and rejected Class 1 requests. As Class 1 customer requests are accepted on a first-come-first-served basis, $\min(d_1^t, C - acc_2^t)$ Class 1 requests are accepted while $\max(0, d_1^t - (C - acc_2^t))$ requests are rejected. Depending on $d_1^t$, different Class 1 demand distributions $F_1^{t+1}$ result in booking period $t+1$. Figure 3.2 illustrates the transition process.

**Profit**

The expected revenue $r^t(s^t, acc_2^t)$ in a particular booking period $t$ given the current state $s^t$ depending on the allocation decision $acc_2^t$ can be expressed as

$$r^t(s^t, acc_2^t) = p_2 \cdot acc_2^t + p_1 \cdot E_{D_1^t(s^t)}[\min(D_1^t(s^t), C - acc_2^t)]. \tag{3.3.4}$$

By accepting $acc_2^t$ Class 2 requests, the firm collects a revenue of $p_2 \cdot acc_2^t$ from Class 2. The second term in the above expected revenue formula determines the expected revenue from Class 1, given the remaining capacity $C - acc_2^t$.

In order to maximize the total expected profit over the entire planning horizon, the firm makes an allocation decision in each booking period $t$. Let $V^t(s^t)$ denote the total expected profit to go in a particular state $s^t$, from booking period t until the end of the planning horizon. The corresponding Bellman equation, which summarizes the above model ingredients, can be expressed as

$$V^t(s^t) = \max_{acc_2^t \in A^t(s^t)} [r^t(s^t, acc_2^t) + E_{D_2^{t+1}(s^{t+1}), D_1^t(s^t)}[V^{t+1}(s^{t+1})]] \quad (\forall t) \tag{3.3.5}$$

with boundary conditions $V^T = r^T(s^T, acc_2^T)$ $(\forall s^T)$. The optimal decision for a particular state $s^t$ in each booking period maximizes the sum of the expected current and the expected

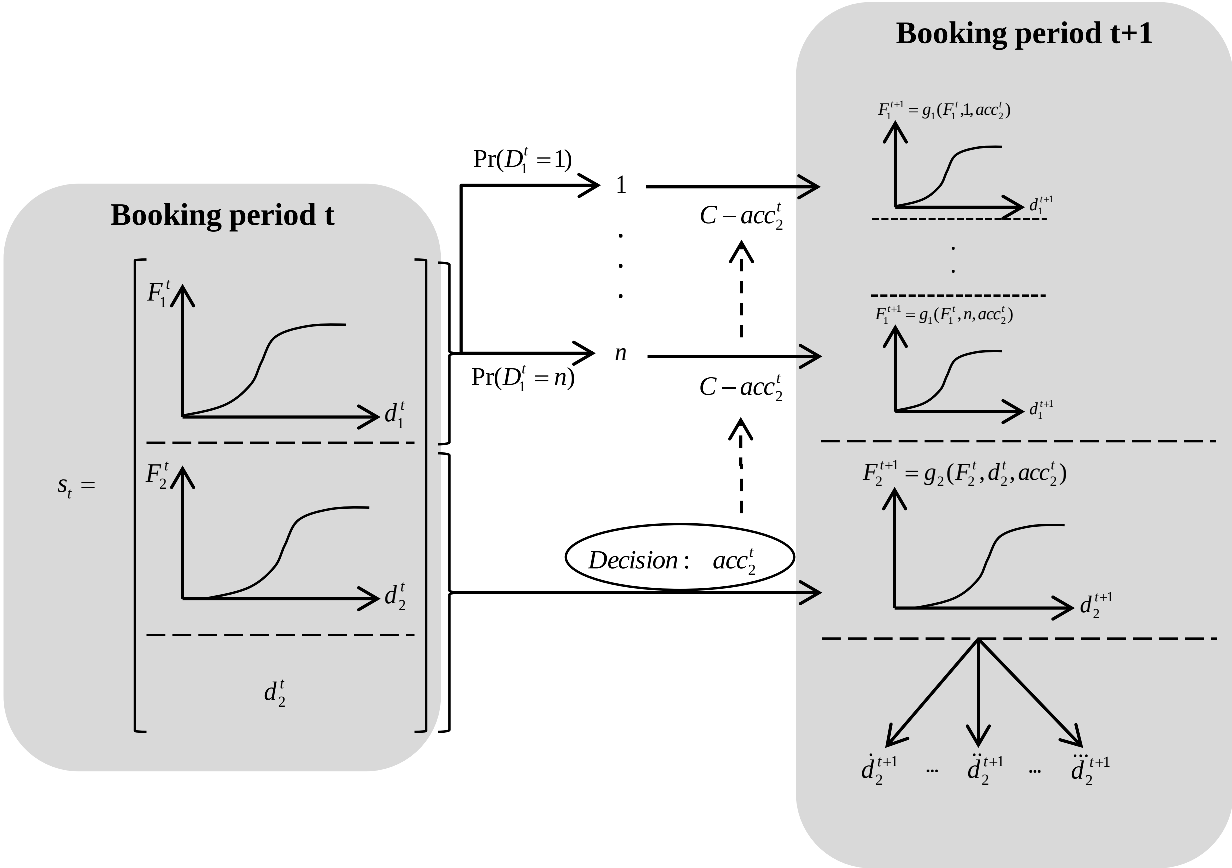


Figure 3.2: Transition process

future revenue.

Thus, for each admissible state $s^t$ in booking period $t$, the firm chooses the allocation decision $acc_2^t$ which maximizes the sum of the current expected revenue $r^t(s^t, acc_2^t)$ and the expected revenue to go until the end of the planning horizon. The above model can be solved by backward induction starting from booking period $T$.

### 3.3.3 Optimal Allocation Policy in Booking Period T

Intuitively, as the planning horizon ends in booking period $T$, the allocation decision in $T$ has no influence on customer demands in further booking periods and therefore corresponds to the single-period setting. Consequently, Littlewood's rule yields the optimal allocation decision in booking period $T$ in each admissible state $s^T$. The optimal allocation decision $acc_2^T$ at the end of the planning horizon therefore satisfies

$$F_1^T(C - acc_2^T) = 1 - \frac{p_2}{p_1} \quad (\forall s^T) \tag{3.3.6}$$

in case of a nested protection level.

## 3.4 Two-Period Setting: Analysis

In order to derive insight into properties of the stochastic dynamic programming model and the allocation policy, we restrict our analysis to two booking periods (i.e., $T = 2$) in the remainder of this section. We aim to investigate the impact of considering the effect of a firm's allocation decisions on both the current booking period and on a future booking period. The firm takes an allocation decision in each of the two booking periods, however, the allocation decision in the second booking period is known beforehand. As discussed in Section 3.3.3, the firm applies Littlewood's rule in each admissible state $s^2$. The resulting state $s^2$ depends on the firm's decision in the first booking period. Therefore, the allocation decision in the second booking period directly results from the allocation decision in the first period. Thus, the allocation decision in the first booking period is the only allocation decision to consider in terms of model analysis in the two-period setting. A protection level policy is not necessarily optimal as the allocation decision might depend on the current state as we will discuss below. First, we investigate the trade-offs faced by the firm and provide a marginal analysis analogous to the marginal approach for Littlewood's model (see Section 2.2.1.1) in Section 3.4.1. Second, we analytically derive a heuristic approach for a partitioned protection level in Section 3.4.2.

### 3.4.1 Marginal Analysis of the Allocation Decision: Trade-offs and Implications

Analogous to the marginal approach for Littlewood's rule (see Figure 2.2), the two-period setting can be investigated by marginal analysis. In line with the single-period setting, we consider a Class 2 request assuming a remaining capacity of $\hat{y}$ units. The firm can either accept or reject this particular request. The firm decides by comparing the marginal expected revenues from both alternatives. In addition to the marginal revenues $p_1$ and $p_2$ in the first booking period, the marginal effects on the expected revenue in the second booking period have to be considered. In the following, $\Delta_i^+$ ($\Delta_i^-$) denotes the non-negative (non-positive) expected marginal impact on the revenue from Class $i$ in the second booking period caused by accepting (rejecting) a request from customer Class $i$ in the first booking period.

If the firm rejects the Class 2 request, the marginal revenue depends on whether the unit can be sold to Class 1 at the higher price $p_1$ or not. If it is not sold, the firm does not obtain any revenue from this particular unit of capacity. If it is sold later in the booking period, the firm obtains a revenue of $p_1$ from customer Class 1. In this case, the firm benefits from an additional marginal effect $\Delta_1^+$ on the Class 1 revenue in the subsequent booking period. Independent of whether the unit can be sold or not, rejecting the Class 2 request has an expected marginal effect $\Delta_2^-$ on the Class 2 revenue in the subsequent booking period.

If the firm accepts the Class 2 request, they obtain a revenue of $p_2$ as in the standard model. In addition, accepting the request has an expected marginal effect $\Delta_2^+$ on the Class

2 revenue in the subsequent booking period. However, this unit of capacity could have potentially been sold to Class 1. This is the case with probability $\mathbb{P}(D_1 > \hat{y})$. The firm then additionally incurs an expected marginal effect $\Delta_1^-$ on the Class 1 revenue in $T = 2$ as an additional Class 1 request must be rejected. Figure 3.3 illustrates the resulting decision tree.

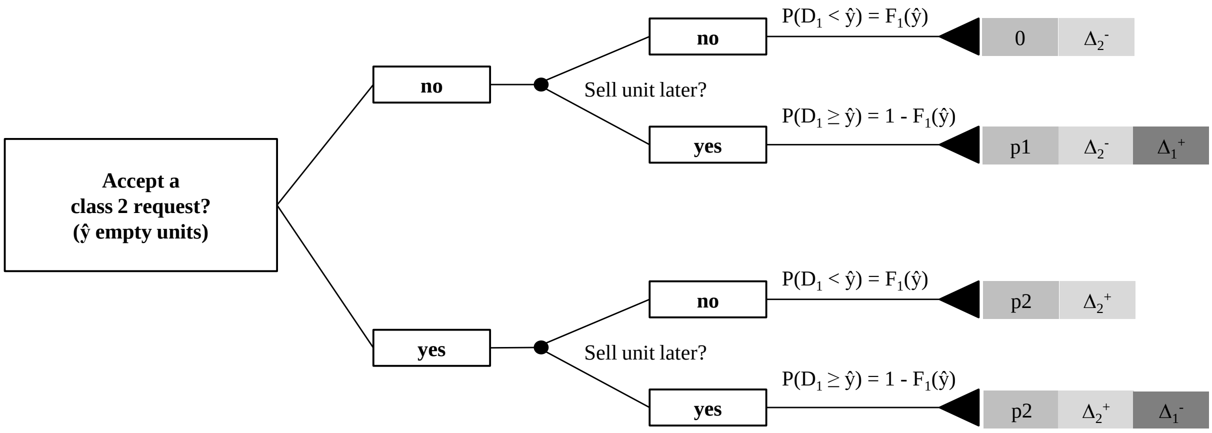


Figure 3.3: Marginal analysis and trade-offs (T = 2)

The decision tree in Figure 3.3 differs from Figure 2.2 in two aspects: the pay-offs in general, and the marginal pay-off in case of accepting a Class 2 request.

1. In the single-period setting, the firm's allocation decision is only based on $p_1$ and $p_2$. Thus, only pay-offs in the current booking period are relevant. This is not the case in the investigated two-period setting. Here, the additionally induced expected marginal pay-offs caused by the decision's marginal impact on customer demand in $T = 2$ are also relevant.

2. In the single-period setting, accepting a Class 2 request yields a revenue of $p_2$ in the current booking period. This is also the case in the considered two-period setting. However, accepting a Class 2 request also induces potentially positive effects on Class 2 demand in $T = 2$. In addition, it also plays a role whether this particular unit of capacity could have been sold to Class 1 instead. If so, accepting the Class 2 request might affect Class 1 demand in $T = 2$ if an additional Class 1 request must be rejected in the first booking period because this unit has been sold to Class 2. This is irrelevant when only one booking period is considered.

The above decision tree demonstrates the following optimality condition:

$$p_2 + \Delta_2^+ + (1 - F_1^1(\hat{y})) \cdot \Delta_1^- = (1 - F_1^1(\hat{y})) \cdot p_1 + \Delta_2^- + (1 - F_1^1(\hat{y})) \cdot \Delta_1^+. \qquad (3.4.1)$$

The firm is indifferent between the expected marginal revenue in case of accepting the request and the expected marginal revenue in case of rejecting the request. Resolving this condition for $\hat{y}$ yields

$$F_1^1(\hat{y}) = 1 - \frac{p_2 + \Delta_2^+ - \Delta_2^-}{p_1 + \Delta_1^+ - \Delta_1^-}. \tag{3.4.2}$$

Thus, the allocation decision depends on all marginal pay-offs (both in the current and the future booking period). For $\Delta_i^+ = -\Delta_i^- = \Delta_i$, this expression further simplifies to

$$F_1^1(\hat{y}) = 1 - \frac{p_2 + 2 \cdot \Delta_2}{p_1 + 2 \cdot \Delta_1}. \tag{3.4.3}$$

If the expected marginal effects $\Delta_i^+$ and $\Delta_i^-$ are constant (and thus independent of the firm's allocation decision), the above conditions in Equations (3.4.2) and (3.4.3) yield an optimal booking limit policy, analogous to Littlewood's standard model. This follows from the fact that only the left-hand side depends on $\hat{y}$. The resulting protection level may deviate from Littlewood's rule in absolute terms depending on the relationship between the prices and the marginal demand effects.

Next, we investigate the impact of the different parameters on the optimal protection level resulting in this case. Let $cf = 1 - \frac{p_2 + 2 \cdot \Delta_2}{p_1 + 2 \cdot \Delta_1}$ denote the above critical fractile for ease of notation. As $F_1^1$ increases in its argument, $\frac{dcf}{dp_2} < 0$ and $\frac{dcf}{d\Delta_2} < 0$, the resulting protection level decreases in $p_2$ and $\Delta_2$. As $F_1^1$ increases in its argument, $\frac{dcf}{dp_1} > 0$ and $\frac{dcf}{d\Delta_1} > 0$, the resulting protection level increases in $p_1$ and $\Delta_1$. Thus, the more profitable Class 1 customers are for the firm both in the current and future booking period, the more capacity should be reserved for them. In turn, the more profitable Class 2 customers are for the firm, the higher their booking limit should be. The impact of $p_1$ and $p_2$ are equivalent to Littlewood's standard model. The impact of the marginal effects $\Delta_1$ and $\Delta_2$ is intuitive.

The above representation of the critical fractile for constant expected marginal effects also allows for comparing the resulting protection level to Littlewood's rule. Littlewood's rule for nested allocations is given in Equation (2.2.8). Relating both critical fractile solutions demonstrates that the firm should reserve more capacity for Class 1 customers compared to Littlewood's rule if

$$1 - \frac{p_2 + 2 \cdot \Delta_2}{p_1 + 2 \cdot \Delta_1} > 1 - \frac{p_2}{p_1} \Leftrightarrow \frac{\Delta_1}{\Delta_2} > \frac{p_1}{p_2}. \tag{3.4.4}$$

The firm should thus reserve more capacity for Class 1 customers in the first booking period if the future revenue ratio (i.e., $\frac{\Delta_1}{\Delta_2}$) of an additionally reserved unit of capacity is greater than the current revenue ratio (i.e., $\frac{p_1}{p_2}$). Accordingly, the resulting optimal protection level is less than the protection level resulting from Littlewood's rule, if the future revenue ratio is less than the current revenue ratio.

In the above analysis, the marginal intertemporal effects $\Delta_i^+$ and $\Delta_i^-$ $(i = 1, 2)$ are assumed to be constant. Then, a booking limit policy is optimal. However, it is not necessarily the case that the marginal effects are constant and independent of the firm's allocation decision. In general, they are not constant but depend on the number of accepted and rejected requests in the first booking period and therefore on the firm's allocation decision $\hat{y}$ in the first booking period. This is what we consider next.

To this end, we further specify the marginal effects. In the remainder, $\Delta_{D_i}$ denotes the marginal effect on the demand of customer Class $i$ in $T = 2$. This allows for explicit expressions to be given for the marginal effects on the firm's allocation policy on the expected profit in $T = 2$ when these effects depend on the firm's allocation policy in $t = 1$. For ease of analysis, we assume that the firm's allocation decision only affects the expected demands in the second booking period. Thus, demand uncertainty is not affected and the resulting demand distributions in $T = 2$ result from shifting the demand distributions from the first booking period.

First, we consider the marginal effect $\Delta_2$ on the Class 2 revenue in $T = 2$. Again, we consider $\hat{y}$ units of available capacity in $t = 1$. The allocation decision in $t = 1$ only has a marginal effect on the Class 2 revenue in $T = 2$, if the firm can gain additional revenue by its allocation decision. Accordingly, the firm's allocation decision affects Class 2 revenue in $T = 2$ only if the resulting Class 2 demand in $T = 2$ is less than the available booking limit in $T = 2$. This yields

$$\Delta_2 = p_2 \cdot \Delta_{D_2} \cdot \mathbb{P}(D_2^1 + \Delta_{D_2} \cdot (C - \hat{y}) \leq C - [y^{1,LW} + \Delta_{D_1} \cdot \min(\hat{y}, D_1^1)]) \tag{3.4.5}$$

as an expression for the marginal effect on the Class 2 revenue in $T = 2$. The firm gains $p_2$ for each additional unit of Class 2 demand $\Delta_{D_2}$, if the resulting Class 2 demand is less than or equal to the available booking limit in the second booking period. Class 2 demand in $T = 2$ equals $D_2^2 = D_2^1 + \Delta_{D_2} \cdot (C - \hat{y})$ as $C - \hat{y}$ Class 2 requests have already been accepted. Littlewood's rule is optimal in the second booking period. Due to the simplifying assumptions, the protection level in $T = 2$ follows from shifting the Littlewood protection level $y^{1,LW}$ by the effects on Class 1 demand, i.e. $y^{2,LW} = y^{1,LW} + \Delta_{D_1} \cdot \min(\hat{y}, D_1^1)$. As a protection level policy is optimal in $T = 2$, the firm can at most sell $C - y^{2,LW}$ units of capacity to Class 2. If Class 2 demand is greater than the available capacity anyway, accepting an additional Class 2 request in the first booking period will not have an effect on the firm's revenue in the second booking period.

As for the marginal effect $\Delta_1$, the logic is the same. Again, the firm's allocation decision will only have a marginal impact on the firm's revenue, if the firm can fulfill additional Class 1 requests in $T = 2$. Therefore,

$$\begin{aligned}
\Delta_1 =& p_1 \cdot \Delta_{D_1} - p_2 \cdot \Delta_{D_1} \cdot \mathbb{P}(D_2^1 + \Delta_{D_2} \cdot (C - \hat{y}) > C - [y^{1,LW} + \Delta_{D_1} \cdot \min(\hat{y}, D_1^1)]) \\
=& (p_1 - p_2 \cdot \mathbb{P}(D_2^1 + \Delta_{D_2} \cdot (C - \hat{y}) > C - [y^{1,LW} + \Delta_{D_1} \cdot \min(\hat{y}, D_1^1)])) \cdot \Delta_{D_1} \\
=& (p_1 - p_2) \cdot \Delta_{D_1} \\
& + p_2 \cdot \Delta_{D_1} \cdot \mathbb{P}(D_2^1 + \Delta_{D_2} \cdot (C - \hat{y}) \leq C - [y^{1,LW} + \Delta_{D_1} \cdot \min(\hat{y}, D_1^1)])
\end{aligned} \tag{3.4.6}$$

yields an expression for the marginal effect of accepting an additional Class 1 request in $t = 1$ on the firm's revenue in $T = 2$. As demand distributions and thus the optimal protection level are shifted depending on the firm's decision, the firm will gain additional expected revenue of $p_1 \cdot \Delta_{D_1}$. However, the firm loses Class 2 revenue if the resulting Class 2 demand in $T = 2$ is greater than the resulting booking limit.

The above expressions for the marginal impacts on the firm's revenue in $T = 2$ themselves depend on the allocation decision in the first booking period. Both sides of the above optimality condition in Equation (3.4.2) therefore depend on the actual allocation decision unless $\Delta_{D_1} = \Delta_{D_2} = 0$. In this case, Equation (3.4.3) demonstrates Littlewood's rule, as the current allocation decision does not affect future demands. While $F_1^1(\hat{y})$ is increasing in $\hat{y}$, it is not even clear if the right hand-side of the optimality condition is monotonic in $\hat{y}$ and if a booking limit policy is generally optimal in the considered setting.

In order to investigate the monotonicity properties, we first consider the properties of the probability term involved in the expressions for $\Delta_1$ and $\Delta_2$. Rearranging terms yields

$$\begin{aligned} &\mathbb{P}(D_2^1 + \Delta_{D_2} \cdot (C - \hat{y}) \leq C - [y^{1,LW} + \Delta_{D_1} \cdot \min(\hat{y}, D_1^1)]) \\ &= \mathbb{P}(D_2^1 \leq C - y^{1,LW} - \Delta_{D_2} \cdot (C - \hat{y}) - \Delta_{D_1} \cdot \min(\hat{y}, D_1^1)). \end{aligned} \tag{3.4.7}$$

The above probability is increasing in $\hat{y}$ if $D_1^1 \leq \hat{y}$. If $D_1^1 > \hat{y}$, it depends on the relation of $\Delta_{D_1}$ and $\Delta_{D_2}$ whether the above probability is increasing or decreasing in $\hat{y}$. If $\Delta_{D_2} > \Delta_{D_1}$, the above probability is increasing in $\hat{y}$, while it is decreasing in $\hat{y}$ if $\Delta_{D_2} < \Delta_{D_1}$. For $\Delta_{D_2} = \Delta_{D_1}$, the above probability is independent of $\hat{y}$.

Substituting $\Delta_1$ and $\Delta_2$ in Equation 3.4.3, under the assumption that both the negative and positive marginal impact of the allocation decision on the revenue in $T = 2$ are the same, results in

$$F_1^1(\hat{y}) = 1 - \frac{p_2 + 2 \cdot p_2 \cdot \Delta_{D_2} \cdot \mathbb{P}(D_2^1 + \Delta_{D_2} \cdot (C - \hat{y}) \leq C - [y^{1,LW} + \Delta_{D_1} \cdot \min(\hat{y}, D_1^1)])}{p_1 + 2 \cdot \Delta_{D_1} \cdot [p_1 - p_2 \cdot \mathbb{P}(D_2^1 + \Delta_{D_2} \cdot (C - \hat{y}) > C - [y^{1,LW} + \Delta_{D_1} \cdot \min(\hat{y}, D_1^1)])]} \tag{3.4.8}$$

as the optimality condition for the firm's allocation decision. The impact of $\hat{y}$ on the right hand-side (i.e., whether it is increasing or decreasing in $\hat{y}$) depends on the relationship of its impact on the numerator and the denominator. No general statement can be made regarding general monotonicity properties as both the numerator and the denominator increase in $\hat{y}$ based on the properties of the probability terms involved. Therefore, the overall impact of the firm's allocation decision on the right-hand side in Equation (3.4.8) is not clear and thus it is not clear either whether a booking limit policy is optimal, in general.

In contrast to Littlewood's standard model, the firm accounts for both current revenues and the impact of current decisions on future revenues. In order to discuss and explain the drivers of a non-monotonic allocation policy, we consider the firm's trade-off with regard to each individual Class 2 request in the first booking period. At the beginning of the booking period, the firm will typically accept each Class 2 request. With an increasing number of Class 2 requests, the expected marginal revenue from accepting a Class 2 request however is decreasing while the expected marginal revenue from rejecting the request is increasing. Thus, the firm starts to reject Class 2 requests at some level $\hat{d}_2^1$. As more Class 2 requests arrive, the expected marginal revenue from rejecting these requests again increases with every further Class 2 request. At some particular Class 2 request $\tilde{d}_2^1$, the expected marginal revenue from accepting this requests is again greater than or equal to the expected marginal revenue from rejecting it. Thus, the firm will accept $\tilde{d}_2^1$ again. When applying a booking limit policy, the firm would not accept any further Class 2 requests after having rejected the first Class 2 request. Figure 3.4 shows the resulting number of accepted requests for $\hat{d}_2^1 = 15$ and $\tilde{d}_2^1 = 21$.

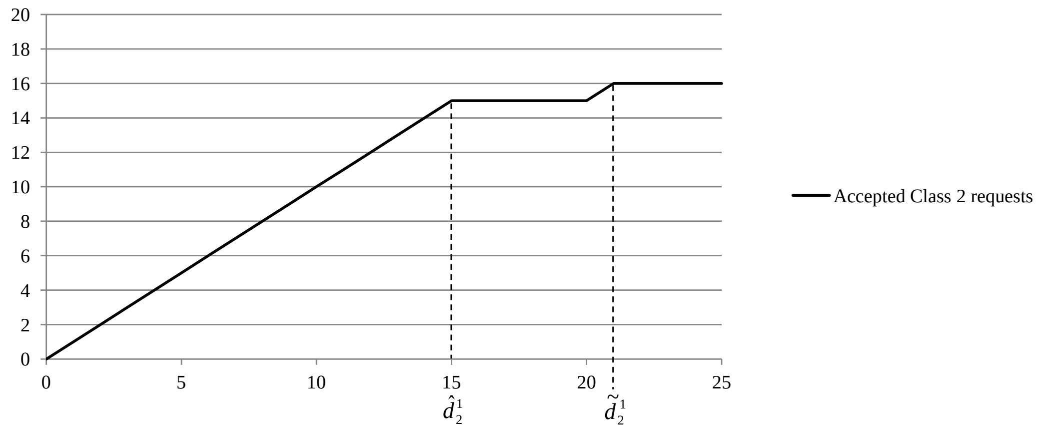


Figure 3.4: Accepted Class 2 requests under a non-monotone allocation policy

While the theoretical result that a booking limit policy is not necessarily optimal when accounting for potential effects on future demand is derived under the assumption that the demand distributions in $T = 2$ result from shifting the demand distributions in $t = 1$, this assumption does not hold in our numerical study in Section 3.5.

In order to provide some further insight, we consider two simplified cases in the following. First, we assume $\Delta_{D_1} = 0$. Then, the firm's allocation decision does not affect Class 1 revenue in $T = 2$. In this case, Equation (3.4.8) simplifies to

$$F_1^1(\hat{y}) = 1 - \frac{p_2 + 2 \cdot p_2 \cdot \Delta_{D_2} \cdot \mathbb{P}(D_2^1 + \Delta_{D_2} \cdot (C - \hat{y}) \leq C - y^{1,LW})}{p_1}. \tag{3.4.9}$$

The right hand-side of the above condition is decreasing in $\hat{y}$ as the fraction is increasing in $\hat{y}$ while the left hand-side is increasing in $\hat{y}$. Therefore, a booking limit policy is optimal in this case. As in the standard setting, the optimal protection level increases in $p_1$ and decreases in $p_2$. Under the assumption that accepting an additional Class 2 request results in a non-negative marginal effect, the numerator is greater than $p_2$. The resulting protection level is therefore less than or equal to Littlewood's rule.

Next, we assume $\Delta_{D_2} = 0$. The optimality condition in Equation (3.4.8) then simplifies to

$$F_1^1(\hat{y}) = 1 - \frac{p_2}{p_1 + 2 \cdot ((p_1 - p_2) \cdot \Delta_{D_1} + p_2 \cdot \Delta_{D_1} \cdot \mathbb{P}(D_2^1 \leq C - [y^{1,LW} + \Delta_{D_1} \cdot \min(\hat{y}, D_1^1)]))}. \tag{3.4.10}$$

The fraction on the right hand-side of the above optimality condition is decreasing in $\hat{y}$ while the left hand-side is increasing in $\hat{y}$. Again, a protection level policy is optimal. As the denominator is larger than $p_1$, the resulting protection level is greater than or equal to Littlewood's rule.

To summarize, the firm faces two trade-offs in the setting considered. As in the standard model, the firm trades-off the two customer classes within a single booking period. The second trade-off arises from the impact of the firm's decision on the customer demands in subsequent booking periods. A critical fractile solution can be derived by marginal analysis. If the expected marginal revenue effects are constant, a booking limit policy is optimal as in the standard model. However, analyzing the critical fractile solution is rather complex if the marginal revenue effects depend on the firm's allocation decision. Then, both sides of the optimality condition depend on the firm's allocation decision. No general statement can be made with regard to the monotonicity properties. In general, therefore, a booking limit policy may not necessarily be optimal in the considered setting. However, a booking limit policy is optimal if the firm's decision only affects the demand of one of the customer classes. Then, the firm will protect more (less) capacity than according to Littlewood's rule if only Class 1 (Class 2) demand is affected.

### 3.4.2 Partitioned Allocation Heuristic

As discussed in Section 3.3, the stochastic dynamic model formulation is rather complex and computationally expensive. As the marginal effects of the firm's allocation decision on the profit in $T = 2$ themselves depend on the allocation decision, deriving a simple booking limit as the optimal allocation decision as in Littlewood's model is also not necessarily optimal as discussed in Section 3.4.1.

In the following, we aim to derive a closed-form allocation heuristic, which is easy to implement on the one hand but still accounts for the effects on customer demand in the subsequent booking periods on the other hand. In order to derive such a closed-form solution for a protection level, we analyse the above two-period setting under several additional simplifications, which allow for the derivation of a closed-form solution. We evaluate the quality of the resulting heuristic numerically in Section 3.5.

We derive the heuristic decision rule based on the stochastic dynamic approach for the two-period setting. For two booking periods, the total expected profit given a particular state $s^1$ reads as

$$V^1(s^1) = \max_{0 \leq acc_2^1 \leq min(d_2^1, C)} [r^1(s^1, acc_2^1) + E_{D_2^2(s^2), D_1^1(s^1)}[V^2(s^2)]], \tag{3.4.11}$$

where the firm decides how many Class 2 customers to accept in the first booking period (i.e., $acc_2^1$).

In order to derive a heuristic allocation policy, we consider the following simplifications.

**Simplification 1:** *The firm uses a booking limit policy in each booking period t*
This simplification excludes non-monotonic allocation policies as discussed in the previous section and ensures that the resulting allocation policy can be easily implemented. The firm decides on a protection level $y^t$ in each booking period.

**Simplification 2:** *Class 2 demand exceeds any booking limit in each booking period t*
This simplification states that all available units of capacity can always be sold to Class 2. Class 1 requests will never be fulfilled from the booking limit available for Class 2 customers, thus the resulting protection level is a partitioned protection level. We therefore denote the firm's partitioned protection level in the first booking period as $y^{1,part}$ and the heuristic itself as *PARTITIONED*. Accordingly, $p_2$ can be considered to correspond to the salvage value in a newsvendor setting.

**Simplification 3:** *Additive linear demand effects shift the underlying demand distributions*
This simplification controls the degree to which the allocation decision in $t = 1$ affects customer demands in $T = 2$. We only account for marginal effects on the expected demand in $T = 2$ and ignore potential effects on demand uncertainty in $T = 2$. Thus, the demand distributions in $T = 2$ result from shifting the initial demand distributions, i.e. $E[D_i^2(d_i^1, y^{1,part})] = E[D_i^1] + \tilde{g}_i(d_i^1, y^{1,part})\ \forall i = 1, 2, \forall d_i^1$. The effects are assumed to be linear additive, resulting in constant marginal effects of the allocation decision on customer demands. The optimal protection level in $T = 2$ according to Littlewood's rule (see Section 3.3.3) results from shifting the Littlewood protection level $y^{1,LW}$ in the first booking period by the demand effects and thus reads as

$$y^{2,LW}(d_1^1, y^{1,part}) = y^{1,LW} + \tilde{g}_1(d_1^1, y^{1,part}), \quad \forall d_1^1. \tag{3.4.12}$$

As the firm's allocation decision is not based on a particular Class 2 demand realization but is a static protection level according to the first simplification, we reformulate Equation (3.4.11). The firm maximizes its total expected profit $E[\pi^{total}(y^{1,part})]$ by deciding on $y^{1,part}$.

Under the above simplifications, the firm's optimization problem can be stated as

$$\max_{0\leq y^{1,part}\leq C} E[\pi^{total}(y^{1,part})] = \max_{0\leq y^{1,part}\leq C} E[\pi^1(y^{1,part})] + E[\pi^2(y^{1,part})], \tag{3.4.13}$$

with

$$\begin{aligned} E[\pi^1(y^{1,part})] &= p_2 \cdot (C - y^{1,part}) + p_1 \cdot E[\min(D_1^1, y^{1,part})] \\ &= p_2 \cdot (C - y^{1,part}) + p_1 \cdot E[D_1^1] - p_1 \cdot E[\max(0, D_1^1 - y^{1,part})] \end{aligned} \tag{3.4.14}$$

and

$$\begin{aligned} E[\pi^2(y^{1,part})] =& p_2 \cdot E[C - y^{2,LW}(d_1^1, y^{1,part})] \\ & + p_1 \cdot E[\min(D_1^2, y^{2,LW}(d_1^1, y^{1,part}))] \\ =& p_2 \cdot E[C - y^{2,LW}(d_1^1, y^{1,part})] \\ & + p_1 \cdot E[D_1^2] - p_1 \cdot E[\max(0, D_1^2 - y^{2,LW}(d_1^1, y^{1,part}))] \\ =& p_2 \cdot E[C - y^{2,LW}(d_1^1, y^{1,part})] + p_1 \cdot E[D_1^1 + g_1(d_1^1, y^{1,part})] \\ & - p_1 \cdot E[\max(0, D_1^1 + \tilde{g}_1(d_1^1, y^{1,part}) - y^{1,LW} - \tilde{g}_1(d_1^1, y^{1,part}))] \\ =& p_2 \cdot E[C - y^{2,LW}(d_1^1, y^{1,part})] + p_1 \cdot E[D_1^1 + \tilde{g}_1(d_1^1, y^{1,part})] \\ & - p_1 \cdot E[\max(0, D_1^1 - y^{1,LW})] \end{aligned} \tag{3.4.15}$$

The expected Class 1 demand in $T = 2$ depends on the firms allocation decision $y^{1,part}$. For any given pair of a protection level and a Class 1 demand realization $d_1^1$ in the first booking period, the effects on the protection level and on the expected Class 1 demand in the second booking period are the same. Therefore, $E[\max(0, D_1^2 - y^{2,LW}(d_1^1, y^{1,part}))] = E[\max(0, D_1^1 - y^{1,LW})]$ in Equation (3.4.15).

Differentiating the total expected profit with regard to $y^{1,part}$ yields

$$\begin{aligned}\frac{dE[\pi^{total}(y^{1,part})]}{dy^{1,part}} =& - p_2 + p_1 \cdot E[I_{D_1^1 > y^{1,part}}] \\ & + p_2 \cdot E[-\frac{dy^{2,LW}(d_1^1, y^{1,part})}{dy^{1,part}}] + p_1 \cdot E[\frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}}] \\ =& - p_2 + p_1 \cdot \mathbb{P}(D_1^1 > y^{1,part}) \\ & + (p_1 - p_2) \cdot E[\frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}}] \\ =& - p_2 + p_1 \cdot \mathbb{P}(D_1^1 > y^{1,part}) \\ & + (p_1 - p_2) \cdot \frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}} \cdot \mathbb{P}(D_1^1 > y^{1,part}) \\ =& \mathbb{P}(D_1^1 > y^{1,part}) \cdot [p_1 + (p_1 - p_2) \cdot \frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}}] - p_2 \\ =& (1 - F_1^1(y^{1,part})) \cdot [p_1 + (p_1 - p_2) \cdot \frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}}] - p_2\end{aligned} \tag{3.4.16}$$

as the first order derivative. In the above first order derivative, $\frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}}$ denotes the marginal effect of the firm's allocation decision on the expected Class 1 demand in $T = 2$. We make use of $E[\frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}}] = \frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}} \cdot \mathbb{P}(D_1^1 > y^{1,part})$ in order to rearrange terms. Doing so reflects the third simplification and the fact that the firm's allocation decision only has a marginal effect on Class 1 demand in $T = 2$ if additional Class 1 demand can be fulfilled by marginally increasing $y^{1,part}$. This is only the case if the Class 1 demand is greater than the protection level.

Setting the first order derivative in Equation (3.4.16) equal to zero and rearranging terms yields

$$F_1^1(y^{1,part}) = 1 - \frac{p_2}{p_1 + (p_1 - p_2) \cdot \frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}}} \tag{3.4.17}$$

as the optimality condition for $y^{1,part}$. The heuristic protection level in $t = 1$ based on the simplifying assumptions made, directly follows from this optimality condition. According to the third simplification, $\frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}}$ is constant.

The analogy of the optimality equation to Littlewood's rule (see Section 2.2.1.1) allows a comparison of the resulting protection level $y^{1,part}$ with $y^{1,LW}$. The relationship of both protection levels depends on the marginal intertemporal demand effects $\frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}}$ as $p_1 -$

$p_2 > 0$. The following cases result:

$$y^{1,part} \begin{cases} < y^{1,LW}, -\frac{p_1}{p_1-p_2} < \frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}} < 0, \\ = y^{1,LW}, \frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}} = 0, \\ > y^{1,LW}, \frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}} > 0. \end{cases} \tag{3.4.18}$$

Compared to Littlewood's rule, the firm should thus reserve more capacity for Class 1 customers if reserving an additional unit of capacity for Class 1 customers has a non-negative marginal effect on Class 1 demand in $T = 2$ (i.e. if $\frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}} \geq 0$). The term $(p_1 - p_2) \cdot (1 + \frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}})$ can be interpreted as the underage costs in the above critical fractile solution in Equation (3.4.17). $p_1 - p_2$ expresses the direct marginal impact in the first booking period while $(p_1 - p_2) \cdot \frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}}$ is the marginal impact of the firm's decision in $T = 2$ under the above simplifications. The marginal effects on Class 2 demand are not considered in the optimality equation for the heuristic protection level $y^{1,part}$. They cannot have a marginal impact in the second booking period by the second simplification.

The second order derivative with regard to $y^{1,part}$ equals

$$\begin{aligned} \frac{d^2 E[\pi^{total}(y^{1,part})]}{d^2 y^{1,part}} = & - f_1^1(y^{1,part}) \cdot (p_1 + (p_1 - p_2) \cdot \frac{d\tilde{g}_1(d_1^1, y^{1,part})}{dy^{1,part}}) \\ & + (1 - F_1^1(y^{1,part})) \cdot (p_1 - p_2) \cdot \frac{d^2\tilde{g}_1(d_1^1, y^{1,part})}{d^2 y^{1,part}}. \end{aligned} \tag{3.4.19}$$

The properties of the simplified expected profit function depend on the properties of the marginal effect of $y^{1,part}$ on the Class 1 demand in $T = 2$. For non-negative marginal effects as discussed above, the first part of the second derivative is non-positive. If the demand effects are linear in $y^{1,part}$ (i.e., $\frac{d^2\tilde{g}_1(d_1^1, y^{1,part})}{d^2 y^{1,part}} = 0$) as assumed above, the total expected profit is concave in $y^{1,part}$ under the stated simplifications. Thus, $y^{1,part}$ is the unique optimal protection level in this case.

## 3.5 Numerical Study

In the following numerical study, we evaluate the performance and compare the decisions taken according to the different allocation policies. In particular, we compare the optimal allocation policy resulting from the stochastic dynamic programming model, the *PARTITIONED* heuristic derived above, and other allocation heuristics discussed in the remainder of this section. First, we present and discuss a particular approach to modeling intertemporal demand effects $g_i$ (see Section 3.3), in order to investigate the general model above numerically. Several other multi-period allocation heuristics (in addition to *PARTITIONED*) are then presented. These heuristics are myopic and based on Littlewood's model and therefore serve as benchmarks within the numerical study. To summarize, the numerical study aims to:

- compare short-term oriented allocation heuristics with the optimal allocation policy resulting from the stochastic dynamic programming approach with regard to (a) the resulting performance and (b) the resulting allocation decisions,
- investigate the influence of different parameters on the resulting performance (e.g., under which conditions our approach outperforms the other allocation heuristics and when, on the contrary, a simpler heuristic will suffice) and on the different allocation decisions,
- investigate the dynamics with regard to allocation decisions over time.

The numerical analysis is divided into two parts which differ with regard to the length of the planning horizon. In the first part (Part A), we compare all considered allocation strategies for a planning horizon of $T = 4$. In this part, we particularly focus on the first two objectives of the numerical study: we compare the performance and the resulting allocation decisions before undertaking a sensitivity analysis to show the parameters' impact on performance and allocation decisions. In order to investigate the dynamics of allocation decisions over time, we extend the planning horizon to $T = 10$ in the second part (Part B) of the numerical study. As the computation time increases significantly due to the longer planning horizon, we exclude the optimal allocation policy from this part. In Part B we focus on the impact of the planning horizon on the performance gaps by comparing the results for $T = 10$ with the results in Part A and the dynamics in the allocation decisions over time.

### 3.5.1 Study Design

#### 3.5.1.1 Modeling Customer Demand

Within the numerical study, we model customer demand by means of a Poisson distribution with expected demand $E[D_i^t] = \lambda_i^t$ $(\forall i, t)$. It is common to assume Poisson distributions in revenue management research (see, e.g., Talluri and van Ryzin (2004b) and Shen and Su (2007)).

In order to investigate the allocation strategies numerically, the demand transition function $g_i$ must be specified. As the Poisson distribution is fully parameterized by $\lambda_i^t$, $g_i$ in this case describes the transition from $\lambda_i^t$ to $\lambda_i^{t+1}$. In the remainder of the numerical study, we specify the transition by:

$$\lambda_i^{t+1} = \lambda_i^t + a_i \cdot acc_i^t - r_i \cdot (d_i^t - acc_i^t) \quad (\forall s^t, i, t = 1, ..., T-1). \tag{3.5.1}$$

While $acc_i^t$ denotes the number of accepted customers, $d_i^t - acc_i^t$ denotes the number of rejected customers in customer class $i$ given a demand realization $d_i^t$. According to this specification, the expected demand in booking period $t+1$ results from the expected demand in $t$ and the customers' reactions to acceptance ($a_i$) and rejection ($r_i$). The firm cannot accept more requests than available, i.e. $acc_i^t \leq d_i^t$. Within this representation, the intertemporal demand effects are assumed to be linear in the number of accepted and rejected customers.

The above representation can be interpreted as an approximation of a model which considers individual customers. Instead of considering individual customers with individual reactions towards their requests being accepted or rejected, we aggregate the customers class-wise by assuming average reactions towards acceptance and rejection.

To this end, consider individual customers and individual repurchase probabilities as follows. Let $prob_{ij}^t$ denote the purchase probability of a customer $j$ from customer class $i$ in booking period $t$. Furthermore, assume a finite customer population $P_i$ in either customer class $i$. Then, $E[D_i^t] = \sum_{j=1}^{P_i} prob_{ij}^t$ is the expected demand of customer class $i$ in booking period $t$. Assuming a particular demand realization $d_i^t$, the expected demand in the subsequent booking period can be expressed as

$$E[D_i^{t+1}] = \sum_{j=1}^{acc_i^t} \min(prob_{ij}^t + \Delta_{ij}^+, 1) + \sum_{j=acc_i^t+1}^{d_i^t} \max(prob_{ij}^t - \Delta_{ij}^-, 0) + \sum_{j=d_i^t+1}^{P_i} prob_{ij}^t, \tag{3.5.2}$$

where $\Delta_{ij}$ denotes the (non-negative) changes in repurchase probability in case of acceptance ($\Delta_{ij}^+$) or rejection ($\Delta_{ij}^-$) of a customer $j$. The first part of the above equation denotes the total expected demand from the previously accepted customers in booking period $t+1$. As for the repurchase probability, $prob_{ij}^t + \Delta_{ij}^+ \leq 1$ ($\forall j = 1..acc_i^t$) must hold. The second part is the expected demand in $t+1$ of all customers being rejected in booking period $t$. As the repurchase probability is non-negative, $prob_{ij}^t - \Delta_{ij}^- \geq 0$ ($\forall j = acc_i^t + 1..d_i^t$) must hold. The third part accounts for all customers who do not request a unit of capacity in booking period $t$. For these customers, the repurchase probability does not change.

Assuming that all customers within Class $i$ are homogeneous allows for consideration of average repurchase probabilities $prob_i^t$ and average changes in repurchase probabilities $\Delta_i$. Equation (3.5.2) can then be rearranged as follows:

$$\begin{aligned} E[D_i^{t+1}] =& min(prob_i^t + \Delta_i^+, 1) \cdot acc_i^t + \max(0, prob_i^t - \Delta_i^-) \cdot (d_i^t - acc_i^t) \\ &+ prob_i^t \cdot (P_i - d_i^t), \end{aligned} \tag{3.5.3}$$

Rearranging terms under the further assumptions that $prob_i^t + \Delta_i^+ < 1$ and $prob_i^t - \Delta_i^- > 0$ yields

$$\begin{aligned} E[D_i^{t+1}] =& (prob_i^t + \Delta_i^+) \cdot acc_i^t + (prob_i^t - \Delta_i^-) \cdot (d_i^t - acc_i^t) + prob_i^t \cdot (P_i - d_i^t) \\ =& prob_i^t \cdot P_i + \Delta_i^+ \cdot acc_i^t - \Delta_i^- \cdot (d_i^t - acc_i^t). \end{aligned} \tag{3.5.4}$$

After replacing $\Delta_i^+ = a_i$ and $\Delta_i^- = r_i$, the above representation for the demand transition follows from this term. Thus, $a_i$ and $r_i$ can be interpreted as average marginal changes in repurchase probabilities. However, the above modeling of the demand transition process is only an approximation as it only holds for $prob_i^t + \Delta_i^+ < 1$ and $prob_i^t - \Delta_i^- > 0$ as discussed above. In addition, the approximation models aggregate demand rather than individual demand due to the assumption of homogeneity in customers within a customer class.

#### 3.5.1.2 Allocation Heuristics

**Partitioned Allocation Heuristic**

Accounting for the above specification of the intertemporal demand effects, the *PARTITIONED* heuristic (see Section 3.4.2) can be specified. For the considered protection level policy, the transition of Class 1 demand follows as

$$\begin{aligned} E[D_1^2] &= E[D_1^1] + a_1 \cdot \min(D_1^1, y^{1,part}) - r_1 \cdot \max(0, D_1^1 - y^{1,part}) \\ &= E[D_1^1] + \begin{cases} a_1 \cdot D_1^1, & \text{if } D_1^1 \leq y^{1,part}, \\ a_1 \cdot y^{1,part} - r_1 \cdot (D_1^1 - y^{1,part}), & \text{else .} \end{cases} \end{aligned} \tag{3.5.5}$$

Thus, the marginal impact of the firm's decision on the protection level $y^{1,part}$ results in

$$\frac{dE[D_1^2]}{dy^{1,part}} = \begin{cases} 0, & \text{if } D_1^1 < y^{1,part}, \\ a_1 + r_1, & \text{if } D_1^1 > y^{1,part}. \end{cases} \tag{3.5.6}$$

If $D_1^1 \leq y^{1,part}$, the firm's allocation decision has no marginal impact on $D_1^2$. If $D_1^1 > y^{1,part}$, the decision's marginal impact equals to $a_1 + r_1$. Accordingly,

$$F_1^t(y^{t,part}) = 1 - \frac{p_2}{p_1 + (p_1 - p_2) \cdot (a_1 + r_1)} \quad (\forall t, \forall s^t) \tag{3.5.7}$$

results as the optimality equation for $y^{1,part}$ after replacing $E[\frac{dD_1^2(s^2)}{dy^{1,part}} | D_1^1(s^1) > y^{1,part}] = a_1 + r_1$ in Equation (3.4.17).

As discussed above, *PARTITIONED* anticipates the effect of the firm's allocation decision on Class 1 demand in the subsequent booking period. The heuristic is derived under some simplifications. To test their appropriateness, these simplifications do not hold within the setting of our numerical study. First, the firm will implement a booking limit policy following this approach but such a policy is not necessarily optimal as discussed above. Second, Class 2 demand does not always exceed the booking limit. Third, the demand distributions are not simply shifted within the numerical study. We shift the expected demand as shown in Equation (3.5.1), however, probabilities change as we model demand using a Poisson distribution. This allows for a comparison of the performance of the heuristic with the optimal allocation policy resulting from the stochastic dynamic programming approach and alternative short-term allocation heuristics discussed below.

**Myopic, Littlewood-Based Allocation Heuristics**

As the firm decides on a protection level in each period, applying Littlewood's rule in each admissible state $s^t$ in each booking period $t$ is an intuitive benchmark for the optimal allocation policy. Accordingly, the optimality condition results as

$$F_1^t(y^{t,LW}) = 1 - \frac{p_2}{p_1} \quad (\forall t, \forall s^t). \tag{3.5.8}$$

This heuristic mirrors a situation where the firm updates the demand forecasts regularly at the beginning of each booking period. The firm reacts to the updated demand forecast by setting the appropriate single-periodically optimal protection level but does not anticipate the effects of its decisions on future demands. We therefore refer to this heuristic as *REACT* in the remainder of the paper. The resulting total expected profit equals the sum of single-periodically optimal expected profits. When applying this heuristic, the firm does not look ahead to the future, they do not account for potential effects on future customer demand in the allocation decisions. Thus, comparing the performance of the optimal allocation policy with *REACT* demonstrates the value of looking ahead to the future.

As an additional benchmark, we consider a heuristic which completely neglects the intertemporal demand effects over time. When applying this heuristic, the firm protects $y^t = y^{1,LW} = const.$ $(\forall t, \forall s^t)$ units of capacity for the high-value customers in each booking period, independent of the particular state. We refer to this heuristic as *NEGLECT*. In general, this is the most short-sighted allocation heuristic, and it is dominated by *REACT*. However, comparing its performance with *REACT* allows quantification of the value of regularly updating customer demand.

#### 3.5.1.3 Experimental Design

Our numerical analysis is divided into two parts. We consider a planning horizon of $T = 4$ in Part A and $T = 10$ in Part B. While we compare all considered allocation strategies (i.e., the optimal allocation policy and all allocation heuristics) in Part A, Part B only considers the allocation heuristics. As it is intractable due to the size of the state space, we omit the optimal allocation policy in the second part. Accordingly, we compare the optimal allocation policy with the allocation heuristics with regard to the resulting performance and the actual allocation decisions, and investigate the impact of different parameters in Part A. While we test the same parameters in Part B, the latter part additionally allows investigation of the dynamics of the allocation decisions over time due to the longer planning horizon.

We consider a capacity of $C = 10$ units and a price for Class 2 customers of $p_2 = 100$. We choose the rather small value for $C$ in order to cope with the computationally demanding determination of the optimal allocation policy. In addition, we fix the ratio of the initial expected customer demands, i.e., $\frac{\lambda_2^1}{\lambda_1^1} = 1.5$. As for the remaining parameters of our model,

| Parameter description | Symbol | Parameter values |
|---|---|---|
| Capacity | C | 10 |
| (Expected) demand ratio | $\frac{\lambda_2^1}{\lambda_1^1}$ | 1.5 |
| (Expected) load factor in $t = 1$ | $\frac{\lambda_1^1+\lambda_2^1}{C}$ | 1.0, 1.5 |
| Price earned by selling to Class 1 | $p_1$ | 150, 250 |
| Price earned by selling to Class 2 | $p_2$ | 100 |
| Average effect on repurchase probability in case of accepting a request of Class $i$ | $a_i$ | 0.0, 0.2, 0.4 |
| Average effect on repurchase probability in case of rejecting a request of Class $i$ | $r_i$ | 0.0, 0.2, 0.4 |

Table 3.1: Experimental design for the numerical study

we run a full-factorial experimental design. By varying the load factor $\frac{\lambda_1^1+\lambda_2^1}{C} \in \{1, 1.5\}$, we vary the initial scarcity of the resource. The degree of customer heterogeneity is accounted for by varying $p_1 \in \{150, 250\}$. The marginal average demand effects $a_i$ and $r_i$, which account for customer heterogeneity in the long run, are $\forall i$ chosen as $a_i \in \{0, 0.2, 0.4\}$ and $r_i \in \{0, 0.2, 0.4\}$. Thus, we investigate scenarios where acceptance and/or denial result in no effects (i.e., $a_i = 0$ and/or $r_i = 0$) and scenarios where both decisions cause relatively high reactions (i.e., $a_i = 0.4$ and/or $r_i = 0.4$). According to Section 3.5.1.1, these parameters can be interpreted as average changes of repurchase probabilities. Thus we choose relatively small absolute values for these parameters. Note that the simplifying assumptions used to derive *PARTITIONED* do not hold in most of the considered scenarios and therefore the numerical results also give an indication of how well the heuristic performs under general conditions.

Table 3.1 summarizes the experimental design. There are $N = 2 \cdot 2 \cdot 3^4 = 324$ parameter combinations (denoted as scenarios in the remainder of this chapter) in total. For each scenario, we determine the respective optimal allocation policy by backward induction and the resulting total expected profit by forward recursion. All other allocation policies are directly evaluated by forward recursion as the respective protection levels can be easily determined by means of closed-form expressions. Thus, we do not simulate allocation policies but determine the allocation decisions and resulting expected profits exactly. Figure 3.5 shows the design of the code underlying the numerical study for the optimal allocation policy. First, we determine the expected profit resulting from Littlewood's rule for a set of possible combinations of demand distributions in booking period $T$. As Littlewood's rule is a static allocation policy which does not depend on demand realizations, the demand distributions themselves are sufficient for determining the firm's allocation decision in the last booking period. As a booking limit policy may not be optimal in the preceding booking periods $n = T - 1, ..., 1$, the optimal allocation decision is determined for each particular demand realization $d_2^n$ (and for each combination of demand distributions) by backward induction. In addition, we determine the expected number of customer requests accepted

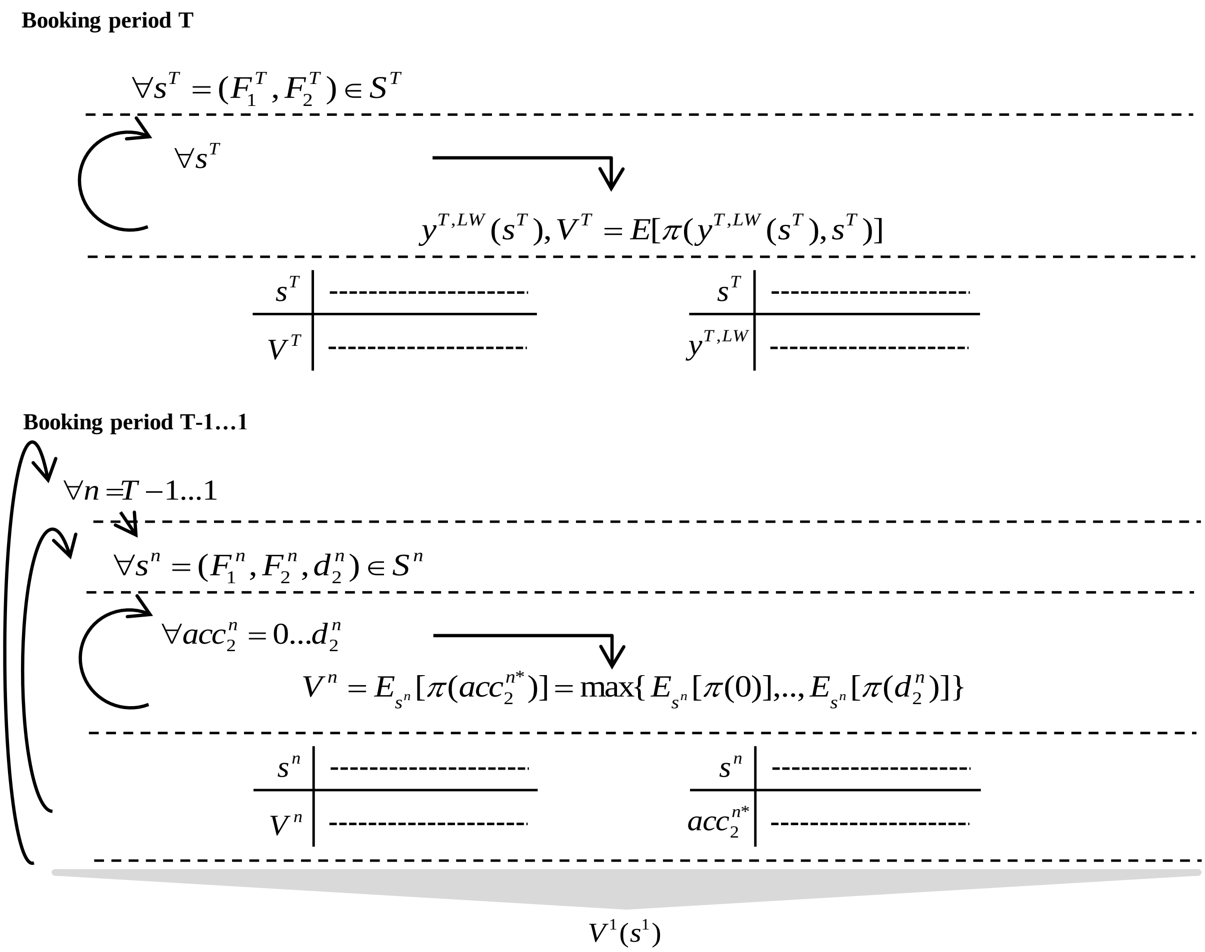


Figure 3.5: Optimization process and coding

and rejected in each of the periods.

All heuristics and the stochastic dynamic programming model have been coded in C++. The computational tests were executed on a personal computer with an Intel Core i7 3.20GHz processor and 32GB RAM, operated by the Microsoft Windows 7 Professional system.

## 3.5.2 Numerical Results

### 3.5.2.1 Part A: $T = 4$

In a first step, we investigate the performance and the resulting allocation decisions of the three heuristic allocation strategies and the optimal allocation policy for a planning horizon of four booking periods. First, we discuss aggregated results, then we investigate the impact of the different parameters on the performance and the decisions of the different allocation strategies in terms of a sensitivity analysis. Table 3.2 shows the spread of the optimality gaps over the 324 scenarios for the three considered allocation heuristics. Next to the respective minimum, average and maximum optimality gap, we indicate the 25%, 50% and 75% percentile optimality gap.

| Percentile | ***PARTITIONED*** | ***REACT*** | ***NEGLECT*** |
|---|---|---|---|
| min | 0.00% | 0.00% | 0.00% |
| 25% | 0.00% | 0.06% | 0.57% |
| 50% | 0.00% | 0.37% | 2.08% |
| 75% | 0.09% | 0.87% | 5.98% |
| max | 0.49% | 2.44% | 12.79% |
| average | 0.06% | 0.57% | 3.85% |

Table 3.2: Spread of optimality gaps (Part A)

From the results in Table 3.2, we note three things:

- *PARTITIONED* is on average very close (0.06%) to the optimal allocation policy indicating that the heuristic performs very well. The heuristic dominates the other two allocation heuristics based on Littlewood's rule. *PARTITIONED* yields the optimal allocation decisions in more than 50% of the scenarios. Its maximum optimality gap is 0.49%. Due to its very close to optimal performance and the fact that the *PARTITIONED* heuristic only accounts for Class 1 demand effects, it follows that accounting for Class 1 demand effects seems to be very important for the firm's allocation decisions. This is in line with the empirical results discussed in the literature overview and the notion of customer relationship management.

- *REACT* also performs close (0.57%) to optimal on average. Thus, reacting to the demand interactions still results in a good performance. The value of looking ahead in time is relatively low due to the small optimality gaps of *REACT* in most of the considered scenarios. Although very small, the optimality gap for *REACT* is around ten times as large as that of *PARTITIONED*. The benefit of applying revenue management itself is assumed to be only around 2-5% (see Chapter 2). *REACT* yields the optimal allocation decision in less than 25% of the scenarios with a maximum optimality gap of 2.44%. This indicates that scenarios exist where only reacting to demand interactions costs a significant share of revenue and where the value of looking ahead in time is high. We discuss the conditions under which this is the case later.

- *NEGLECT* performs significantly worse on average (3.85%) compared to the other two allocation heuristics. *NEGLECT* results in a optimality gap of more than 5.98% in 25% of the scenarios. Therefore, not even reacting to changes in customer demand has a significant impact on the firm's profit in many scenarios. This result clearly emphasises the importance of demand updating. The value of updating customer demands is considerable across almost all scenarios.

As the average value of looking ahead is rather small, we additionally relate it to the value of applying revenue management itself. In order to do so, we relate it to the average single-period value of applying Littlewood's rule instead of applying a first-come-first-served policy. For the considered scenarios, the average single-period value of applying revenue management results in 11.91%. Related to the value of applying revenue management, the above average optimality gap of 0.57% amounts to a relative revenue increase of about 4.79%. Thus, the further improvement of anticipating potential effects on future customer demands is considerable when related to the value of applying revenue management itself. Next to this argument, an additional 0.57% of a typical total revenue (of, e.g., an airline) means a considerable amount of money.

Table 3.3 shows the average, minimum and maximum computation times across all scenarios for the different allocation strategies. The indicated computation times comprise both determination of the optimal allocation decisions and their evaluation for the optimal allocation policy. For the remaining three allocation heuristics, the computation times indicated comprise only the forward evaluation of the resulting booking limit policies. On average,

| **Allocation strategy** | **Average** [sec] | **Minimum** [sec] | **Maximum** [sec] |
|---|---|---|---|
| Optimal allocation policy | 4660.61 | 0.72 | 20376.50 |
| *PARTITIONED* | 2391.06 | 0.58 | 10328.70 |
| *REACT* | 2388.40 | 0.50 | 10356.60 |
| *NEGLECT* | 2331.56 | 0.25 | 10133.60 |

Table 3.3: Computation times (Part A)

it requires 4660.61 seconds = 77.68 minutes to determine the optimal allocation policy by backward induction and to evaluate it afterwards for the parameters to be tested. The long computation times clearly indicate the complexity of the stochastic dynamic programming approach and the size of the underlying state space. The allocation heuristics need around half the time and still yield close to optimal solutions on average (see above). The computation times are that long although the available capacity only equals ten units. It would increase considerably for larger capacities (and thus customer demands) making the problem almost intractable for settings of practical relevance and sizes at least for the optimal allocation policy.

As discussed above, the state space considered is already rather large for the small capacity of 10 units and the short planning horizon of $T = 4$. To give an idea of the size of the state space, consider the following scenario: $a_1 = 0$, $r_1 = 0.2$, $a_2 = r_2 = 0$ with a load factor of 1.5. In this case, the maximum possible expected Class 1 demand equals $\hat{\lambda}_1^T = \lambda_1^1 + a_1 \cdot (T-1) \cdot C = 6$. However, $\frac{6}{0.2} + 1 = 31$ different possible values for the expected Class 1 demand may result. As both Class 2 demand effects equal zero, the expected Class 2 demand remains constant at the initial value of 9. However, we consider $\lfloor 9 + \sqrt{9} + 1 \rfloor = 13$ Class 2 demand realizations. The state space consists of $31 \cdot 13 = 403$ states and thus decisions to be taken and evaluated in the last booking period in the considered example. This is one of the scenarios for the smallest state space. Determining the size of the state spaces for all considered scenarios results in an average size of the state space of more than one million states for consideration per scenario.

In order to explain the different performance of the considered allocation heuristics, we analyse the actual resulting allocation decisions in the respective booking periods and the number of accepted requests in both customer classes over time.

Table 3.4 shows the average protection levels set by the firm under the different allocation strategies in the booking periods considered. On average the firm protects more capacity

| **Allocation strategy** | **t = 1** | **t = 2** | **t = 3** | **T = 4** |
|---|---|---|---|---|
| Optimal allocation policy | 5.36 | 5.71 | 6.34 | 6.82 |
| *PARTITIONED* | 5.06 | 5.24 | 6.05 | 6.77 |
| *REACT* | 4.50 | 5.17 | 5.90 | 6.56 |
| *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 |

Table 3.4: Average protection levels (Part A)

for Class 1 requests under the optimal allocation policy in comparison to the allocation heuristics. This also holds for the first booking period where the allocation decisions under each strategy are taken based on the same demand distribution parameters. Comparing the firm's average allocation decisions emphasizes the importance of focusing on Class 1 customers.

Average protection levels increase over time except for *NEGLECT*. On average, Class 1 demand increases due to increasing protection levels as more Class 1 requests can be fulfilled. The increasing average Class 1 demand again results in an increasing protection level. Thus, by allocating more capacity to the higher-value customer class, the firm succeeds in increasing customer loyalty. On average, the importance of Class 1 customers increases due to the additional intertemporal demand effects compared to the standard setting where customers only exhibit heterogeneous willingness-to-pay. Figure 3.6 demonstrates the average protection levels over time from Table 3.4. As shown in Figure 3.6, the gaps between the

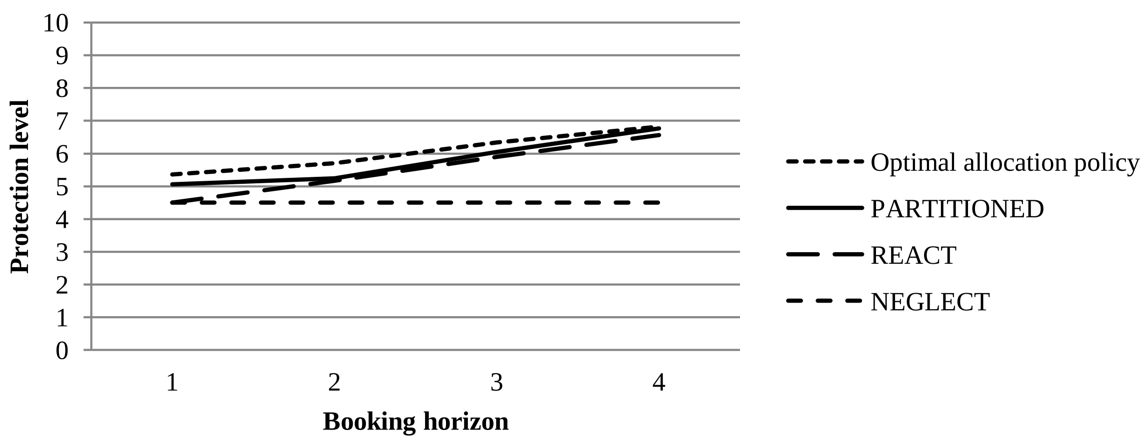


Figure 3.6: Average protection levels over time (Part A)

optimal protection level and the other allocation heuristics first increase and then decrease again. The optimality gaps are also mainly due to the differences in the protection levels within the first periods.

Due to the differences in the protection levels, the different allocation strategies offer different service for the customer classes. Typically, service levels for the different customer classes are presented in order to account for this fact. However, service levels are hard to interpret in our setting as expected demands change over time. Therefore, the service provided for a particular customer class does not necessarily correlate with the capacity dedicated to this customer class and thus the firm's allocation decision. In the following, we therefore analyse the number of accepted requests for each customer class. Table 3.5 shows the number of accepted requests for both customer classes in all booking periods and the sum of accepted requests over the entire planning horizon.

| **Customer class and allocation strategy** | | **t = 1** | **t = 2** | **t = 3** | **T = 4** | **sum** |
|---|---|---|---|---|---|---|
| Class 1 | Optimal allocation policy | 4.32 | 4.94 | 5.69 | 6.33 | 21.27 |
| | *PARTITIONED* | 4.20 | 4.82 | 5.58 | 6.27 | 20.86 |
| | *REACT* | 4.05 | 4.55 | 5.21 | 5.90 | 19.70 |
| | *NEGLECT* | 4.05 | 4.21 | 4.28 | 4.32 | 16.85 |
| Class 2 | Optimal allocation policy | 4.11 | 3.70 | 3.12 | 2.70 | 13.64 |
| | *PARTITIONED* | 4.42 | 3.92 | 3.25 | 2.72 | 14.30 |
| | *REACT* | 4.81 | 4.43 | 3.83 | 3.22 | 16.29 |
| | *NEGLECT* | 4.81 | 5.00 | 5.10 | 5.13 | 20.04 |

Table 3.5: Average accepted customer requests (Part A)

While the optimal allocation policy accepts more Class 1 requests in comparison with the other allocation heuristics, it also accepts fewer Class 2 requests. Thus, it is on average more favorable to focus on Class 1 customers compared to Class 2 customers. However, as we will see in Section 3.5.2.2, this is not the case in all considered scenarios. Depending on customer heterogeneity and the reactions of the customer classes towards acceptance and rejection, this can also be the other way around in some scenarios.

In a second step, we perform a sensitivity analysis in order to investigate the impact of particular parameters on the performance of the different allocation strategies and the actual resulting allocation policies. This allows for the identification of scenarios where it is particularly beneficial for the firm to allocate its capacity optimally instead of making use of allocation heuristics. Table 3.6 summarizes the average optimality gaps for different parameter values. The numbers give the average optimality gap if a single parameter is set constant at a particular value and all other parameters are varied.

First, we analyse the impact of the Class 1 demand effects. As shown in Table 3.6, the average optimality gap of *PARTITIONED* is almost invariant to changes in the parameters except for varying values of $a_1$ and $r_1$. In these cases, both the largest average optimality gaps and the largest spreads of average optimality gaps are observed. These parameters drive the profitability of serving Class 1 demand. The same holds for *REACT* and *NEGLECT*. Here, these parameters also have the greatest impact on the resulting average optimality

| Parameter | Value | *PARTITIONED* | *REACT* | *NEGLECT* |
|---|---|---|---|---|
| load factor | 1 | 0.06% | 0.53% | 3.31% |
| | 1.5 | 0.07% | 0.60% | 4.38% |
| p1 | 150 | 0.07% | 0.33% | 1.99% |
| | 250 | 0.05% | 0.81% | 5.71% |
| a1 | 0 | 0.03% | 0.20% | 0.33% |
| | 0.2 | 0.05% | 0.59% | 3.01% |
| | 0.4 | 0.11% | 0.92% | 8.21% |
| r1 | 0 | 0.02% | 0.17% | 3.81% |
| | 0.2 | 0.08% | 0.52% | 3.75% |
| | 0.4 | 0.09% | 1.00% | 3.99% |
| a2 | 0 | 0.08% | 0.59% | 3.54% |
| | 0.2 | 0.06% | 0.55% | 3.93% |
| | 0.4 | 0.06% | 0.56% | 4.09% |
| r2 | 0 | 0.06% | 0.59% | 3.96% |
| | 0.2 | 0.06% | 0.57% | 3.86% |
| | 0.4 | 0.07% | 0.54% | 3.72% |

Table 3.6: Average parameter-dependent optimality gaps (Part A)

gaps. Compared to *PARTITIONED*, the average optimality gaps are significantly larger with a larger spread for the myopic Littlewood-based allocation heuristics.

The optimality gaps partly significantly increase in the Class 1 demand effects. This holds in particular for the impact of $a_1$ as varying $a_1$ affects the performance of all three allocation heuristics. While both $a_1$ and $r_1$ affect the performance of *PARTITIONED* and *REACT*, the performance of *NEGLECT* is only significantly affected by $a_1$. Analyzing the optimality gaps for different values of $a_1$ yields the following results:

- *PARTITIONED* performance decreases but is still nearly optimal, with increasing Class 1 demand effects.
- Under high Class 1 demand effects, *REACT* loses revenue. This effect arises from its delayed reaction to the demand effects.
- Two aspects of *NEGLECT* are noteworthy: while *NEGLECT* performs very close to optimal for $a_1 = 0$, its performance is particularly bad for $a_1 = 0.4$.

Second, we analyse the impact of varying $p_1$ as the parameters affecting Class 1 profitability seem to be particularly important. While the performance of *PARTITIONED* is almost invariant to variations of $p_1$, varying $p_1$ has a considerably influence on the optimality gaps for *REACT* and *NEGLECT*. In particular, *REACT* performs significantly worse with increasing price $p_1$ due to the fact that the allocation strategy does not protect enough capacity for Class 1 demand on average in the first booking periods. Compared to *PARTITIONED*, *REACT* responds with some delay to the demand effects and thus loses revenue.

In contrast to the Class 1 demand effects, the Class 2 demand effects $a_2$ and $r_2$ do not have a significant impact on the performance of the firm's allocation strategies. The performance of both *PARTITIONED* and *REACT* in particular is almost invariant to changes in these parameters. Only the average performance of *NEGLECT* is affected by $a_2$.

Regarding the impact of the other parameters, only the impact of varying the load factor is noteworthy. However, varying the load factor only affects the average performance of *NEGLECT*. All other parameters have no noteworthy effects on the performance of the allocation heuristics.

In order to further investigate the optimality gaps, Table 3.7 compares the firm's average protection levels under the optimal allocation strategy and the allocation heuristics in all booking periods for all considered values of $a_1$ and $r_1$. The observed protection levels explain the optimality gaps which are also indicated in Table 3.7. The performance of the different allocation heuristics is mainly driven by $p_1$, $a_1$ and $r_1$ as discussed above. The other parameters do not affect the performance of the allocation strategies significantly. As $p_1$ is already part of Littlewood's standard model, we focus on investigating the impact of the intertemporal demand effects $a_1$ and $r_1$.

| Parameter | Value | Allocation strategy | t = 1 | t = 2 | t = 3 | T = 4 | Opt. gap |
|---|---|---|---|---|---|---|---|
| a1 | 0 | Optimal allocation policy | 4.85 | 4.53 | 4.32 | 4.16 | - |
| | | *PARTITIONED* | 4.83 | 4.38 | 4.26 | 4.15 | 0.03% |
| | | *REACT* | 4.50 | 4.36 | 4.22 | 4.09 | 0.20% |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 0.33% |
| | 0.2 | Optimal allocation policy | 5.45 | 5.73 | 6.40 | 7.07 | - |
| | | *PARTITIONED* | 5.08 | 5.24 | 6.07 | 6.98 | 0.05% |
| | | *REACT* | 4.50 | 5.18 | 5.90 | 6.71 | 0.59% |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 3.01% |
| | 0.4 | Optimal allocation policy | 5.78 | 6.89 | 8.32 | 9.21 | - |
| | | *PARTITIONED* | 5.25 | 6.11 | 7.83 | 9.17 | 0.11% |
| | | *REACT* | 4.50 | 5.99 | 7.58 | 8.89 | 0.92% |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 8.21% |
| r1 | 0 | Optimal allocation policy | 4.98 | 5.52 | 6.42 | 7.07 | - |
| | | *PARTITIONED* | 4.83 | 5.36 | 6.26 | 7.05 | 0.02% |
| | | *REACT* | 4.50 | 5.36 | 6.21 | 7.00 | 0.17% |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 3.81% |
| | 0.2 | Optimal allocation policy | 5.40 | 5.70 | 6.29 | 6.80 | - |
| | | *PARTITIONED* | 5.08 | 5.22 | 6.04 | 6.77 | 0.08% |
| | | *REACT* | 4.50 | 5.15 | 5.90 | 6.59 | 0.52% |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 3.75% |
| | 0.4 | Optimal allocation policy | 5.70 | 5.92 | 6.32 | 6.58 | - |
| | | *PARTITIONED* | 5.25 | 5.15 | 5.86 | 6.48 | 0.09% |
| | | *REACT* | 4.50 | 5.01 | 5.59 | 6.10 | 1.00% |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 3.99% |

Table 3.7: Average parameter-dependent protection levels and optimality gaps (Part A)

As shown above, the average optimality gaps increase with increasing Class 1 demand effects. As shown in Table 3.7, the average optimal protection levels are greater than the heuristic protection levels for $a_1 = 0.4$ or $r_1 = 0.4$. Under high Class 1 demand effects, the firm thus particularly benefits from protecting more capacity for Class 1 customers. This result is rather intuitive. Protecting more capacity for Class 1 customers results in higher future Class 1 demand as more customer requests can be accepted and fewer requests have to be rejected.

A closer look at $a_1 = 0.4$ shows (1) that the firm already protects almost the entire capacity for Class 1 customers after four booking periods, and (2) the above discussed delayed response of *REACT* compared to *PARTITIONED*. Thus, the main contribution to the optimality gap arises from the second and third booking period as the protection levels differ most there. This result is in line with the development of the total average protection levels over time shown in Figure 3.6 and discussed above. In contrast to *REACT*, *PARTITIONED* adapts the protection level faster and thus allows the acceptance of more Class 1 requests already in early booking periods.

As shown above in Table 3.7, *PARTITIONED* structurally underestimates the protection level compared to the optimal allocation policy although the heuristic protection level explicitly accounts for intertemporal Class 1 demand effects. This is due to the fact that *PARTITIONED* only accounts for two periods (i.e. the effects of a current allocation decision on the subsequent booking period) while the optimal allocation policy considers all remaining booking periods until the end of the planning horizon when taking an allocation decision. Thus, the optimal policy already anticipates the decision's effect on multiple future booking periods. Therefore, an even larger protection level results from the optimal allocation policy.

#### 3.5.2.2 Part B: $T = 10$

As shown above for $T = 4$, the various allocation heuristics differ with regard to the degree to which they respond to the demand effects. In Part B of the numerical study, we extend the planning horizon to $T = 10$ booking periods in order to further investigate the allocation decisions over time. We omit the optimal allocation policy in Part B due to being computationally intractable. The very close to optimal performance of *PARTITIONED* (see Section 3.5.2.1) gives additional support for omitting the optimal allocation policy.

First, we compare the allocation heuristics with regard to their performance and the dynamics in terms of the actual allocation decisions over time. The relative performance of *REACT* and *NEGLECT* in relation to *PARTITIONED* is denoted as a performance gap in the remainder of this section. We investigate the impact of the longer planning horizon on the relative performance of the allocation heuristics and the respective resulting protection levels. We analyse the impact of the different parameters on the firm's allocation decisions in the long run.

In order to investigate the impact of the longer planning horizon on the resulting performance gaps, we again determine the minimum, average and maximum, and the 25%, 50% and 75% percentile performance gap with regard to *PARTITIONED*. The results are shown in Table 3.8.

For a planning horizon of $T = 10$ booking periods, *REACT* performs on average 0.94% worse than *PARTITIONED*. The performance gap is greater than 1.35% in only 25% of the cases. The maximum performance gap amounts to 4.09%, which is considerable. However,

| **Percentile** | **React** | **Neglect** |
|---|---|---|
| min | -0.05% | 0.00% |
| 25% | 0.13% | 1.82% |
| 50% | 0.50% | 9.81% |
| 75% | 1.35% | 17.06% |
| max | 4.09% | 28.50% |
| average | 0.94% | 10.53% |

Table 3.8: Spread of performance gaps (Part B)

on average *REACT* still performs very well even with a longer planning horizon in most of the scenarios. In a very few scenarios, *REACT* performs even better than *PARTITIONED*.

*NEGLECT* performs on average 10.53% worse than *PARTITIONED*. The performance gap exceeds 9.81% in 50% of the scenarios and 17.06% in 25% of the scenarios. By applying *NEGLECT* the firm thus loses considerable revenue in most of the scenarios.

Compared to the results for $T = 4$ (shown in Table 3.2), the performance gaps (and thus both the value of looking ahead and the value of updating customer demand) increase over time. However, the performance gap of *NEGLECT* increases significantly more than the performance gap of *REACT*. Thus, with an increasing planning horizon, updating customer demands becomes increasingly important. This result is rather intuitive as the absolute demand effects increase over time while the allocation policy under *NEGLECT* does not change. Thus, the error made by the allocation policy increases over time.

Table 3.9 illustrates the performance gaps depending on the problem parameters.

| **Parameter** | **Value** | ***REACT*** | ***NEGLECT*** |
|---|---|---|---|
| load factor | 1 | 1.08% | 11.40% |
| | 1.5 | 0.79% | 9.66% |
| p1 | 150 | 0.81% | 7.22% |
| | 250 | 1.07% | 13.84% |
| a1 | 0 | 0.70% | 0.94% |
| | 0.2 | 1.50% | 12.74% |
| | 0.4 | 0.61% | 17.90% |
| r1 | 0 | 0.16% | 10.36% |
| | 0.2 | 0.79% | 10.36% |
| | 0.4 | 1.87% | 10.86% |
| a2 | 0 | 0.84% | 9.32% |
| | 0.2 | 0.95% | 11.01% |
| | 0.4 | 1.02% | 11.25% |
| r2 | 0 | 0.98% | 10.88% |
| | 0.2 | 0.94% | 10.58% |
| | 0.4 | 0.88% | 10.13% |

Table 3.9: Average parameter-dependent performance gaps (Part B)

The impact of the different parameters on the performance gaps is in principle the same as for $T = 4$. Again, the Class 1 demand effects have the strongest impact on the performance of the different allocation heuristics. In contrast to Part A, the impact of $a_1$ is non-monotone regarding the performance gap of *REACT* for the longer planning horizon. The largest average performance gap results for $a_1 = 0.2$. For even larger positive Class 1 demand effects, the performance gap decreases again. As the factors which drive the performance gaps are comparable to Part A, we will not discuss them further in detail for $T = 10$ but rather focus on the dynamics in the actual allocation decisions over time in the remainder of this section. Analyzing the allocation decisions explains the performance gaps and demonstrates insights about the long-term allocation strategy under the different allocation heuristics.

Table 3.10 shows the average protection levels set by the firm under the different allocation heuristics in all considered booking periods. On average the firm protects more capacity for

| **Allocation strategy** | **t = 1** | **t = 2** | **t = 3** | **t = 4** | **t = 5** | **t = 6** | **t = 7** | **t = 8** | **t = 9** | **T = 10** |
|---|---|---|---|---|---|---|---|---|---|---|
| *PARTITIONED* | 5.06 | 5.24 | 6.05 | 6.77 | 7.18 | 7.45 | 7.60 | 7.69 | 7.74 | 7.76 |
| *REACT* | 4.50 | 5.17 | 5.90 | 6.56 | 6.98 | 7.25 | 7.41 | 7.50 | 7.56 | 7.58 |
| *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |

Table 3.10: Average protection levels (Part B)

Class 1 requests under *PARTITIONED* in comparison to the other allocation heuristics. This also holds for the first booking period where the allocation decisions under each strategy are taken based on the same demand distribution parameters. The average protection levels in the first four booking periods are equivalent to the average protection levels in PART A. This would not necessarily be the case for the optimal allocation policy as the stochastic dynamic programming approach would anticipate the longer planning horizon.

Again, the average protection levels increase over time except for *NEGLECT*. The development of the average protection levels is illustrated in Figure 3.7. As for $T = 4$ in

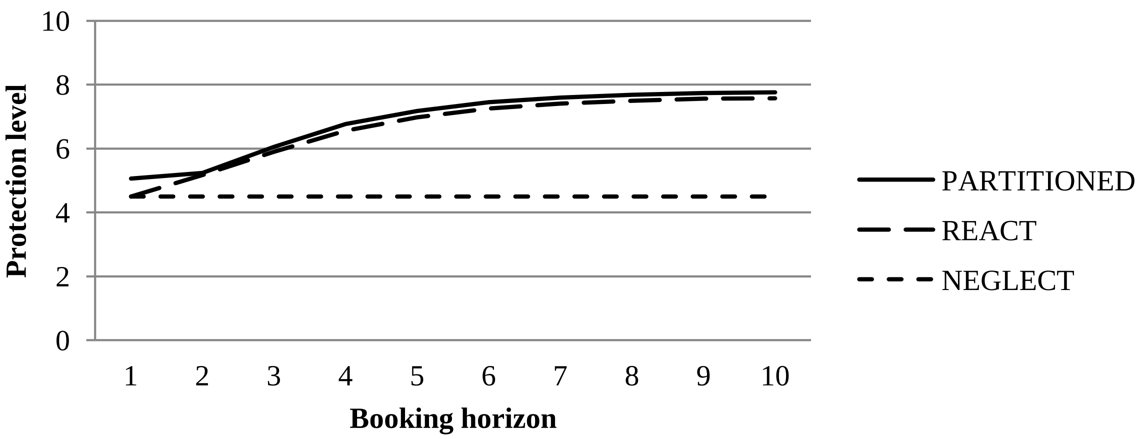


Figure 3.7: Average protection levels over time (Part B)

the previous section, the difference in the average protection levels resulting from *PARTITIONED* and *REACT* decreases over time. The main difference again stems from the first few booking periods. On average these two allocation strategies protect more than 75% of the available capacity for the higher-value customers at the end of the planning horizon. The average protection levels seem to converge towards the end of the planning horizon.

In order to further investigate the impact of the different parameters on the protection levels resulting from the allocation heuristics, Table 3.11 shows the average protection levels for the different parameter values in each booking period.

| Parameter | Value | Allocation strategy | t = 1 | t = 2 | t = 3 | t = 4 | t = 5 | t = 6 | t = 7 | t = 8 | t = 9 | T = 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| load factor | 1 | *PARTITIONED* | 4.11 | 4.12 | 4.90 | 5.76 | 6.41 | 6.82 | 7.08 | 7.26 | 7.37 | 7.44 |
| | | *REACT* | 3.50 | 4.07 | 4.75 | 5.53 | 6.15 | 6.57 | 6.85 | 7.04 | 7.15 | 7.21 |
| | | *NEGLECT* | 3.50 | 3.50 | 3.50 | 3.50 | 3.50 | 3.50 | 3.50 | 3.50 | 3.50 | 3.50 |
| | 1.5 | *PARTITIONED* | 6.00 | 6.36 | 7.21 | 7.77 | 7.96 | 8.07 | 8.12 | 8.12 | 8.11 | 8.09 |
| | | *REACT* | 5.50 | 6.27 | 7.05 | 7.59 | 7.82 | 7.92 | 7.96 | 7.97 | 7.96 | 7.94 |
| | | *NEGLECT* | 5.50 | 5.50 | 5.50 | 5.50 | 5.50 | 5.50 | 5.50 | 5.50 | 5.50 | 5.50 |
| p1 | 150 | *PARTITIONED* | 4.22 | 4.44 | 5.16 | 5.93 | 6.48 | 6.83 | 7.03 | 7.18 | 7.30 | 7.36 |
| | | *REACT* | 4.00 | 4.40 | 5.01 | 5.68 | 6.21 | 6.56 | 6.76 | 6.92 | 7.02 | 7.08 |
| | | *NEGLECT* | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 |
| | 250 | *PARTITIONED* | 5.89 | 6.04 | 6.94 | 7.61 | 7.89 | 8.07 | 8.18 | 8.20 | 8.19 | 8.17 |
| | | *REACT* | 5.00 | 5.95 | 6.79 | 7.44 | 7.76 | 7.93 | 8.05 | 8.09 | 8.09 | 8.08 |
| | | *NEGLECT* | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 |
| a1 | 0 | *PARTITIONED* | 4.83 | 4.38 | 4.26 | 4.15 | 4.04 | 3.94 | 3.85 | 3.75 | 3.67 | 3.58 |
| | | *REACT* | 4.50 | 4.36 | 4.22 | 4.09 | 3.96 | 3.84 | 3.72 | 3.61 | 3.51 | 3.41 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.2 | *PARTITIONED* | 5.08 | 5.24 | 6.07 | 6.98 | 7.72 | 8.44 | 8.96 | 9.32 | 9.56 | 9.71 |
| | | *REACT* | 4.50 | 5.18 | 5.90 | 6.71 | 7.41 | 8.04 | 8.55 | 8.92 | 9.17 | 9.33 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.4 | *PARTITIONED* | 5.25 | 6.11 | 7.83 | 9.17 | 9.79 | 9.97 | 10.00 | 10.00 | 10.00 | 10.00 |
| | | *REACT* | 4.50 | 5.99 | 7.58 | 8.89 | 9.58 | 9.86 | 9.95 | 9.98 | 9.99 | 10.00 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| r1 | 0 | *PARTITIONED* | 4.83 | 5.36 | 6.26 | 7.05 | 7.53 | 7.84 | 7.99 | 8.10 | 8.16 | 8.17 |
| | | *REACT* | 4.50 | 5.36 | 6.21 | 7.00 | 7.50 | 7.81 | 7.97 | 8.09 | 8.15 | 8.17 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.2 | *PARTITIONED* | 5.08 | 5.22 | 6.04 | 6.77 | 7.17 | 7.44 | 7.60 | 7.68 | 7.73 | 7.76 |
| | | *REACT* | 4.50 | 5.15 | 5.90 | 6.59 | 6.99 | 7.26 | 7.45 | 7.55 | 7.60 | 7.63 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.4 | *PARTITIONED* | 5.25 | 5.15 | 5.86 | 6.48 | 6.84 | 7.07 | 7.21 | 7.29 | 7.34 | 7.37 |
| | | *REACT* | 4.50 | 5.01 | 5.59 | 6.10 | 6.45 | 6.66 | 6.80 | 6.88 | 6.92 | 6.94 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| a2 | 0 | *PARTITIONED* | 5.06 | 5.24 | 6.07 | 6.80 | 7.23 | 7.50 | 7.67 | 7.77 | 7.83 | 7.85 |
| | | *REACT* | 4.50 | 5.17 | 5.92 | 6.61 | 7.06 | 7.34 | 7.51 | 7.63 | 7.69 | 7.72 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.2 | *PARTITIONED* | 5.06 | 5.24 | 6.05 | 6.76 | 7.17 | 7.44 | 7.58 | 7.67 | 7.72 | 7.74 |
| | | *REACT* | 4.50 | 5.17 | 5.90 | 6.55 | 6.97 | 7.22 | 7.38 | 7.47 | 7.52 | 7.53 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.4 | *PARTITIONED* | 5.06 | 5.24 | 6.04 | 6.74 | 7.15 | 7.41 | 7.55 | 7.64 | 7.68 | 7.70 |
| | | *REACT* | 4.50 | 5.17 | 5.88 | 6.52 | 6.92 | 7.18 | 7.33 | 7.42 | 7.46 | 7.48 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| r2 | 0 | *PARTITIONED* | 5.06 | 5.24 | 6.05 | 6.76 | 7.17 | 7.43 | 7.58 | 7.67 | 7.72 | 7.74 |
| | | *REACT* | 4.50 | 5.17 | 5.90 | 6.55 | 6.97 | 7.22 | 7.38 | 7.48 | 7.52 | 7.54 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.2 | *PARTITIONED* | 5.06 | 5.24 | 6.05 | 6.77 | 7.18 | 7.45 | 7.60 | 7.69 | 7.74 | 7.76 |
| | | *REACT* | 4.50 | 5.17 | 5.90 | 6.56 | 6.98 | 7.24 | 7.40 | 7.50 | 7.55 | 7.57 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.4 | *PARTITIONED* | 5.06 | 5.24 | 6.06 | 6.77 | 7.20 | 7.47 | 7.62 | 7.71 | 7.77 | 7.79 |
| | | *REACT* | 4.50 | 5.17 | 5.90 | 6.57 | 7.00 | 7.27 | 7.43 | 7.54 | 7.59 | 7.62 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |

Table 3.11: Average parameter-dependent protection levels (Part B)

As shown in Table 3.11, the average protection levels are constant (in case of *NEGLECT*) or increase (in case of *PARTITIONED* and *REACT*) over time for all parameter values except for $a_1 = 0$. Thus, $a_1$ affects the firm's allocation policy over time structurally.

First, we consider the impact of $a_1$ in more detail. For $a_1 = 0$, the average protection level decreases over time as other parameters such as Class 2 demand effects seem to affect the firm's allocation policy. As a result, the average protection level decreases over time. It seems to approach some value towards the end of the planning horizon as the average protection level decreases at a (at least) non-increasing rate.

As soon as $a_1 \geq 0.2$, the positive effects on Class 1 demand dominate the potential effects of all other parameters and thus steer the firm's allocation policy. As soon as Class 1 customers react positively towards their requests being accepted by the firm, it is beneficial for the firm to protect more capacity for the higher-value customers and thus to accept more

Class 1 requests. Due to the positive reaction towards acceptance, Class 1 demand further increases. As a result, the firm protects the total available capacity for the higher-value customer class at the end of the planning horizon for $a_1 = 0.4$ under both *PARTITIONED* and *REACT*. In general, the average protection level converges faster to the available capacity with increasing values of $a_1$. This result explains the non-monotone impact of $a_1$ on the performance gap discussed above. *PARTITIONED* and *REACT* differ with regard to how quickly they respond to the demand effects as discussed above. In line with the results for $T = 4$, all other parameters have no structural impact on the firm's average protection levels.

Secondly, for all other parameters except for $a_1$, the average protection levels increase over time and seem to approach some value towards the end of the planning horizon as the average protection levels increase at a decreasing rate. In contrast to $a_1 = 0.4$, this value is less than the total available capacity in all of these cases.

The results so far clearly indicate the need for the firm to specify the effects of their allocation decisions on Class 1 demand as these effects predominantly affect its allocation policy in the long run. Only as a second-order effect do the Class 2 demand effects affect the allocation decisions.

In order to gain insight into the factors driving the firm's allocation policy if $a_1 = 0$, we vary several other parameters and present the results. Figure 3.8 illustrates the impact of $r_1$ on the average protection level for $a_1 = 0$. The average protection levels for $r_1 = 0$ are not

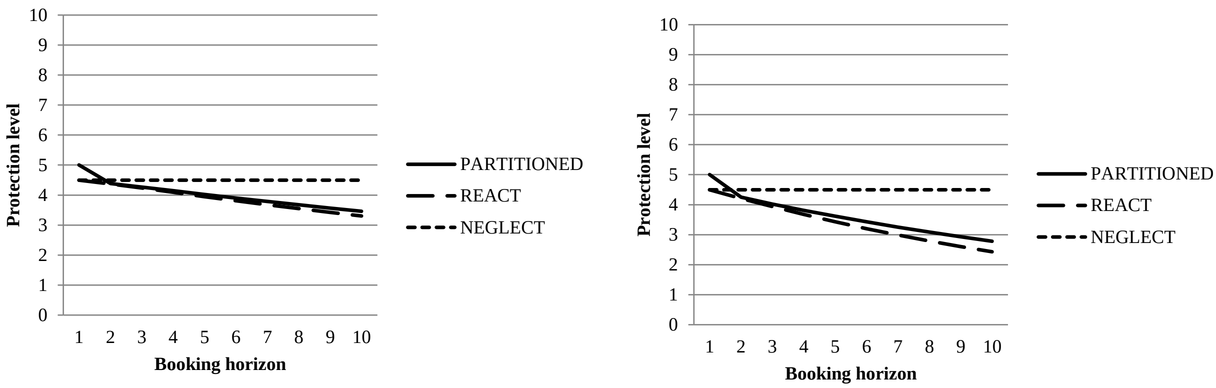


Figure 3.8: Allocation policy for $a_1 = 0$ and $r_1 = 0.2$ (left) and $r_1 = 0.4$ (right)

illustrated as Class 1 demand does not change throughout the planning horizon in this case. The average protection levels are therefore constant and identical under all three allocation heuristics. For $r_1 \geq 0.2$, the average protection levels decrease even further with increasing values of $r_1$. In both illustrated cases, *PARTITIONED* protects more capacity for Class 1 customers than *REACT* as the allocation heuristic anticipates the negative effect on Class 1 demand.

The (negative) effects on customer demand result in a decreasing Class 1 demand volume from the first booking period as (1) some Class 1 customers are always rejected, at least in some high demand scenarios; and (2) the Class 1 demand volume cannot be affected positively due to $a_1 = 0$. The firm should therefore protect less capacity in order to accept more Class 2 requests. In order to investigate the impact of the Class 2 demand effects in more detail, Table 3.12 illustrates the average protection levels for $a_1 = 0$, $r_1 \geq 0.2$ and varying values of $a_2$.

As shown in Table 3.12, the average protection levels decrease for all parameter combinations. The average protection levels decrease with an increasing $r_1$ and decrease for

| $r_1$ | $a_2$ | **Allocation strategy** | **t = 1** | **t = 2** | **t = 3** | **t = 4** | **t = 5** | **t = 6** | **t = 7** | **t = 8** | **t = 9** | **T = 10** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | 0 | *PARTITIONED* | 5.00 | 4.39 | 4.28 | 4.17 | 4.07 | 3.98 | 3.89 | 3.81 | 3.73 | 3.66 |
| | | *REACT* | 4.50 | 4.37 | 4.25 | 4.13 | 4.01 | 3.90 | 3.80 | 3.71 | 3.62 | 3.54 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.2 | *PARTITIONED* | 5.00 | 4.39 | 4.27 | 4.14 | 4.01 | 3.88 | 3.75 | 3.63 | 3.51 | 3.39 |
| | | *REACT* | 4.50 | 4.37 | 4.23 | 4.09 | 3.93 | 3.79 | 3.64 | 3.50 | 3.36 | 3.22 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.4 | *PARTITIONED* | 5.00 | 4.39 | 4.26 | 4.12 | 3.98 | 3.84 | 3.71 | 3.58 | 3.46 | 3.33 |
| | | *REACT* | 4.50 | 4.37 | 4.22 | 4.06 | 3.90 | 3.74 | 3.59 | 3.44 | 3.29 | 3.15 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| 0.4 | 0 | *PARTITIONED* | 5.00 | 4.25 | 4.04 | 3.86 | 3.70 | 3.56 | 3.44 | 3.32 | 3.21 | 3.12 |
| | | *REACT* | 4.50 | 4.21 | 3.96 | 3.74 | 3.55 | 3.38 | 3.22 | 3.08 | 2.96 | 2.84 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.2 | *PARTITIONED* | 5.00 | 4.25 | 4.02 | 3.80 | 3.59 | 3.39 | 3.20 | 3.02 | 2.84 | 2.67 |
| | | *REACT* | 4.50 | 4.21 | 3.93 | 3.66 | 3.41 | 3.16 | 2.92 | 2.70 | 2.48 | 2.29 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |
| | 0.4 | *PARTITIONED* | 5.00 | 4.25 | 4.00 | 3.77 | 3.54 | 3.33 | 3.12 | 2.93 | 2.74 | 2.57 |
| | | *REACT* | 4.50 | 4.21 | 3.91 | 3.62 | 3.34 | 3.07 | 2.82 | 2.58 | 2.37 | 2.17 |
| | | *NEGLECT* | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 | 4.50 |

Table 3.12: Average parameter-dependent protection levels for $a_1 = 0$ (Part B)

increasing values of $a_2$ for a given value of $r_1$. The effects on the lower-value demand are thus negative effect at the average protection level or, alternatively, result in increasing average booking limits. Thus, the firm will accept on average more Class 2 requests in these cases. Intuitively, the more capacity is available for the lower-value customers, the more positively they react towards their requests being accepted.

## 3.6 Conclusion

In this chapter we present an approach for capacity allocation when a firm faces interdependent customer demands over time. The chapter is motivated by empirical research on customer reactions towards revenue management on the one hand and analytical research contributions which integrate a long-term perspective into revenue management models on the other hand. While traditional revenue management takes a transactional perspective (i.e., focuses on short-term profit maximization), customer relationship management focuses on customer relationships and thus on a firm's long-term profitability. Both concepts interact: a firm's short-term allocation decisions have been shown to affect customer satisfaction and loyalty in the long run. This chapter addresses this interaction.

We present a stochastic dynamic programming model, which builds on Littlewood's rule, where the firm faces stochastic demand of two customer classes and decides how many requests to accept from the customer class with the lower willingness-to-pay in each booking period. However, we assume that the firm's allocation decision impacts future customer demands by influencing the repurchase probabilities of the customers, depending on whether their request has been accepted or rejected. The firm faces a two-fold trade-off in each booking period: on the one hand, the trade-off between the customer classes within a particular booking period and on the other hand the trade-off between customer demands in subsequent booking periods. With regard to the modeling of the demand interdependencies, our approach is intentionally generic allowing for multiple kinds of demand transitions over time.

We discuss the underlying trade-offs in a two-period setting by means of marginal analysis. As the protection level in the second booking period also depends on the firm's allocation decision in the first period, a protection level policy is not necessarily optimal. We show that a protection level policy results for specific cases where only one of the customer classes exhibits long-term demand effects. We analytically derive a closed-form heuristic for a protection level accounting for part of the demand interdependencies under simplifying assumptions based on the two-period setting. The heuristic accounts for the effects on Class 1 demand and neglects Class 2 demand effects.

We test the optimal allocation policy numerically and compare it against the derived heuristic and two other allocation heuristics based on Littlewood's rule. We compare the allocation policies with respect to their performance and the resulting allocation decisions. We find that the derived heuristic performs close to optimal in almost all of the scenarios, independent of the length of the planning horizon. The other allocation heuristics partially result in considerably worse performance, however the average optimality gap when applying $REACT$ is still relatively small. The value of looking ahead in time varies across the scenarios considered. While it is relatively low on average, it can be substantial in some scenarios. In particular, totally neglecting the demand interdependencies performs considerably worse in almost all of the considered scenarios. Thus, the value of updating customer demand on a regular basis is shown to be considerable. Comparing both optimality gaps demonstrates that the benefit from regular demand updates is considerably higher than the additional value of looking ahead in time on average.

The results indicate that it is particularly important for a firm to protect enough capacity for Class 1 customers as soon as these customers react positively towards their orders being accepted. As the derived heuristic focuses on Class 1 demand, this can be considered the main reason why it performs so well. If Class 1 customers react positively towards their request being accepted, we find both increasing protection levels and increasing Class 1 demand over time. Depending on the strength of the positive reaction, the firm even protects the total available capacity in some scenarios. In this case, it is beneficial to protect even more capacity than that resulting from applying Littlewood's rule in each booking period (as done by $REACT$). Only if Class 1 customers do not show a positive reaction, should the firm focus on Class 2 demand. Booking limits then increase with increasing Class 2 demand effects. In this case, Littlewood's rule protects slightly too much capacity for the higher-value customers. These numerical results support the discussion about the underlying trade-offs in Section 3.4.1.

New directions include, for example, the setup of the numerical study and the demand model. First, testing the allocation strategies for a larger total capacity could yield more insight about their performance. We test the allocation policies based on $C = 10$ units of capacity. This is due to the computational effort. Assuming such a small capacity also restricts the possible impact of different allocation policies, however, as they cannot differ much in absolute terms. Therefore, testing the performance for larger capacities could also have an impact on the value of looking ahead to the future as the policies can differ more strongly then.

The parameters used in the numerical study are hypothetical as we wanted to investigate several kinds of relationships between the parameters across the customer classes. However, it would be particularly interesting to investigate the strength of the effects (i.e., how strongly a repurchase probability is influenced by a request being accepted or denied) empirically. Thus, the performance of the different allocation strategies could be evaluated based on empirical data rather than theoretical parameters. In particular, it might also be interesting to see how customers react towards acceptance / denial in the long run in different industries. Our approach allows generally for all possible kinds and functional forms of intertemporal demand transitions, while we only account for a particular additive approach within the numerical study. Therefore, investigating other approaches for the aggregation from individual customer reactions to aggregated effects of customer classes and other modeling approaches for the demand transitions themselves will be valuable.

# Chapter 4

# Coordinating Fencing and Capacity Allocation

## 4.1 Introduction

Making use of customer heterogeneity, for example, by means of differential pricing is at the core of revenue management. In order to differentiate service among its customers, a firm typically applies customer segmentation techniques which aim to properly identify different customer segments and the factors which impact their demands. As such, a firm divides its heterogeneous customer base into subsets of relatively homogeneous customers with comparable preferences and/or willingness-to-pay (Wind, 1978; Rangan et al., 1992; Wedel, 2000). As discussed in Chapter 2, customer segmentation is not an operational planning task as it is seldom done on a day-to-day, routine basis (Talluri and van Ryzin, 2004b). However, the effectiveness of customer segmentation directly influences the effectiveness of short-term demand management, and its benefits for the firm. As such, this interaction provides one example of the interdependencies of different planning hierarchies affecting revenue management, as discussed in Chapter 2.

Firms such as airlines or hotels typically offer a service (e.g., a seat or a night in a particular type of room) at different prices. Customers select their desired product or service according to their preferences and their willingness-to-pay. The different prices are typically determined for particular fare products, linked to particular individual booking restrictions, which actually differentiate the offered products (Williamson, 1992). Commonly used restrictions which qualify for a cheaper fare in the airline context are, for example, a stay over a weekend or limited refunding policies. Such conditions are denoted as price fences. They aim to implement a profit maximizing differentiation scheme by tailoring the offered products (or services) to the actual customer segments. Price fences are therefore typically based on the differences in customer preferences across the customer segments (Botimer, 1996). In particular, fences are meant to prevent customers with a high willingness-to-pay from buying the lower-priced product. As soon as the firm differentiates prices among its customers, customers in high-priced segments are motivated to find a way to pay a lower price (Philips, 2005; Klein and Steinhardt, 2008). To summarize, price fences aim to preserve customer segmentation and limit demand leakage (or spill-over) between the market segments (Botimer, 1996; Zhang and Bell, 2012). Accordingly, the benefit of differential pricing (and thus the benefit of revenue management) depends on the quality of the firm's customer segmentation and on the effectiveness of the price fences in place.

Customer segmentation is not updated very frequently and should be stable over time in order to allow derivation of the appropriate strategies and concepts to manage the customer base. As fences are based on the characteristics of the customer segments, the decision about their specification is also not taken very frequently. Compared to the capacity allocation decision, the fencing decision is located on a higher hierarchical planning level. These decisions also affect short-term decisions in the context of revenue management as they influence the actual materialization of demand of the different customer classes.

Consider, for example, a setting where a firm succeeds in perfectly determining its fences. In this case, customer demand is perfectly separated and customers have no price-driven reason for deviating from the product they prefer at the predetermined prices. This is no longer the case if the firm fails to perfectly determine its fences. The actual demand of each customer class therefore depends on the firm's fences. As standard revenue management decisions are typically based on demand forecasts for the respective customer classes, the effectiveness of the fences in place affects a firm's revenue management decisions. For this reason, fences and capacity allocation should be considered jointly in order to maximize profits (Hanks et al., 2002). In addition to price-driven substitution, customers might also consider a different product if a stock-out occurs in a particular customer class. Additional stock-out-based demand substitution then occurs.

Although customer segmentation and price fences are a prerequisite for revenue management, only few papers have investigated their interrelationship with revenue management decisions. In particular, the interrelation has not yet been considered in terms of a quantitative revenue management approach. Within traditional revenue management models, customer segmentation is in most cases assumed to be perfect. Customer demand is also at most assumed to be price-dependent (i.e., dependent on the prices of all products offered) when assuming classical demand functions. Exceptions include studies on customer diversion or product substitution. Here, customer demand also depends on the capacity available for a particular customer class. Customer choice models additionally allow the integration of aspects other than prices, which have an influence on customer buying decisions (Talluri and van Ryzin, 2004a,b). For example, they allow for integration of the availability or the specification of the different fare products. Customer choice models allow for a detailed modeling of particular fences and their impact on customer demands. However, customer choice models consider the customer segments (and thus the underlying fences) in a particular setting as exogenous, while we explicitly consider the firm's decision on fences. In our approach, we choose an aggregated modeling of the fences' impact on customer demands and do not consider particular types of fences in order to consider them as a firm's decision. We do not consider individual customers and their choice probabilities but aim to derive structural properties of the firm's optimal decisions.

Within this chapter, we focus on the interaction of fencing decisions and short-term quantity-based revenue management decisions. We therefore first investigate how this interdependency can be captured in a standard revenue management model in order to support coordinated decision making. Based on this approach, we are interested in deriving the firm's optimal fencing and allocation decisions. Additionally, we aim to investigate the interdependency of both decisions. Due to price-driven demand substitution, lower-value demand increases. Thus, lower-value customers are likely to experience a stock-out of the lower-value product. We therefore additionally investigate a setting with both price-driven demand leakage and stock-out-based demand substitution. We are particularly interested in the impact of stock-out-based demand substitution on the firm's optimal fencing and capacity allocation decisions and on their interdependency. Finally, we aim to investigate the value of integrated decision making in comparison to independent fencing and allocation decisions.

In order to do so, we present a single-period static quantity-based two-class revenue management model which incorporates demand substitution effects. In particular, we build on the well-known model by Littlewood (1972) and additionally account for price-dependent customer demand, including price-driven demand leakage. A firm decides on the fences in place and on the protection level. To implement a particular fencing level, the firm incurs costs., and so it thus trades costs for fencing against the marginal impact of demand leakage on the expected revenue: strict fences may prevent demand leakage but result in higher costs. Based on this approach, we investigate the interaction of fencing and capacity allocation. We derive analytical expressions for the optimal fencing decision for both partitioned and nested allocations, assuming strictly convex fencing costs. We find that capacity allocation and fencing do not interact when accounting only for price-driven demand leakage. Classical hierarchical planning is therefore optimal in this case. In a second step, we extend the previous setting by additional stock-out-based demand substitution. We derive the optimality conditions for both the firm's fencing and allocation decisions. We find that the interaction of the two decisions depends on the stock-out-based substitution rates. If the stock-out-based substitution rates are the same for both customer classes, it is optimal to take both decisions sequentially. However, if the stock-out-based substitution rates differ between the customer classes, both decisions interact: the fences depend on the firm's protection level and vice versa. Within a numerical illustration, we compare the two-class setting with a single customer segment. Additionally, we compare the optimal coordinated decisions with hierarchical decision making in order to investigate the value of coordinated decision making.

To summarize, this chapter makes the following contributions:

- We give an overview of both qualitative and quantitative research on price differentiation and demand substitution in inventory management and revenue management. In the first part, we take a broader perspective and provide the conceptual basis for our approach. With regard to inventory management and revenue management, we focus on quantitative approaches that consider price-driven demand leakage or stock-out-based substitution.

- Based on previous approaches in standard newsvendor settings, we present a static single-period stochastic quantity-based revenue management model. We incorporate the fencing decisions by accounting for price-driven demand leakage and costs for fencing. Thus, we combine approaches from inventory management and revenue management. The firm decides on the fences in place and on the protection level. In contrast to Zhang et al. (2010), we investigate the firm's optimal decisions analytically. For price-driven demand leakage, we show that it is optimal to take both decisions sequentially in a hierarchical manner.
- Based on the previous approach, we additionally account for stock-out-based demand substitution. Again, the firm decides on the protection level and the fences in place. We analytically investigate the firm's optimal decisions and the underlying trade-offs.
- We provide numerical examples in order to illustrate the firm's optimal decisions and compare the two-class setting with a setting where the firm does not differentiate (i.e., serves a single customer segment).

The remainder of this chapter is organized as follows. After this introduction, Section 4.2 reviews the relevant literature on qualitative research on price fences, research on product line design and demand substitution in the fields of inventory and revenue management. Section 4.3 presents a monopolistic single-period stochastic quantity-based revenue management approach, based on Littlewood's model. The approach includes price-driven demand leakage and the firm's decision about fences. In Section 4.4, we extend this setting with additional stock-out-based substitution. We illustrate our results numerically in Section 4.5. Finally, Section 4.6 concludes and summarizes opportunities for further research.

## 4.2 A Review of Fencing in Revenue Management

As discussed in Section 4.1, we consider a setting where the firm decides both on fences and on the capacity allocation to different customer classes. The firm faces either only price-driven demand leakage or both price-driven demand leakage and additional stock-out-based demand substitution. Our approach builds on the fields of general price differentiation, product line design, inventory management and revenue management. First, we provide an overview of different concepts from price differentiation, customer segmentation and fencing. Here, we take a broader perspective and provide the conceptual basis for our approach with regard to, for example, the concept of fences and the classification of the different decisions in terms of taxonomies. In this first part, the focus is on qualitative basics. Subsequently, we provide an overview of research into product line design. Third, we give an overview of quantitative approaches from the field of inventory management. Fencing has rarely been considered in a revenue management model. We consider inventory management problems, due to existing approaches for modeling a firm's fencing decision in this field. Inventory management approaches thus serve as the basis for our modeling approach. They differ from our approach with regard to the firm's "capacity", however: while the capacity (i.e. the available resources) is the firm's decision in inventory models, it is given and fixed in revenue management settings. We consider both price-driven demand leakage and stock-out-based demand substitution. In the case of price-driven demand leakage, we discuss approaches which integrate a firm's fencing decision with inventory optimization. We discuss the literature on inventory management under stock-out-based demand substitution. Finally, we discuss quantitative revenue management approaches which explicitly incorporate (price-driven or stock-out-based) demand substitution. These approaches are closest to the setting investigated in this chapter. We discuss revenue management approaches with both traditional demand models and under customer choice.

### 4.2.1 Price Differentiation, Segmentation and Fencing

In this section, we provide an overview of price differentiation, customer segmentation and fences. These are the underlying concepts of our approaches presented in Sections 4.3 and 4.4. Additionally, we present taxonomies for structuring their interdependencies. The focus is on fundamentals rather than on recent research findings.

Firms differentiate their prices in order to maximize their revenues (see, e.g., Talluri and van Ryzin, 2004b). Traditionally, three types of price differentiation are distinguished in literature (see, e.g., Homburg and Krohmer (2006) and Klein and Steinhardt (2008)). Under first degree price differentiation, a firm charges the maximum willingness-to-pay to each customer. Typically, first degree price differentiation cannot be implemented, as customers do not reveal their willingness-to-pay directly. Under second degree price differentiation, customers are grouped into segments. Typically, the firm offers different products for the different segments and charges product-dependent prices. The customers choose their favorite product. Customer willingness-to-pay depends on the characteristics of the goods or service (e.g., the particular departure of a flight) or on situational aspects (e.g., different willingness-to-pay at different times). Thus, products are designed in a way such that their characteristics fit the identified customer segments (Botimer, 1996, 2000). For example, airlines offer a wide variety of fare products, differing in their attributes, for a single flight. Under third degree price differentiation, customers cannot choose their favorite product but qualify for a particular product by observable criteria (e.g., their age or gender). Here, the possibilities for fencing are limited.

We then focus on second degree price differentiation. A firm loses revenue if it does not succeed in ensuring that customers with a high willingness-to-pay buy the high-priced product. Rate restrictions are thus imposed on the different products (Talluri and van Ryzin, 2004b). These restrictions potentially limit the customer's flexibility and are therefore associated with lower prices (see, e.g., Botimer and Belobaba, 1999). According to Zhang and Bell (2012), such "a fence is a device that should preserve market segmentation and thus limit demand leakage between the firm's customer segments" (see also Alderighi, 2010). Following Philips (2005), price-driven demand leakage has the potential to eliminate the benefit of price differentiation. Imposing the appropriate and effective fences is thus crucial for the performance of revenue management (Hanks et al., 2002; Kimes, 2002) or service differentiation more generally. Accounting for the interdependency of fencing decisions and customer diversion is crucial, according to Botimer and Belobaba (1999).

Several kinds of fences can be distinguished. Fencing criteria such as the time of purchase or reservation, the length of a stay or the days of a stay are widespread. General overviews of different fences are given, for example, in Hanks et al. (2002), Talluri and van Ryzin (2004b), Klein and Steinhardt (2008) and Zhang and Bell (2012). Talluri and van Ryzin (2004b) focus particularly on the implementation of revenue management in different industries and give an overview of examples for the fare products, including corresponding restrictions for these industries. While Hanks et al. (2002) differentiate between physical and non-physical fences, Zhang and Bell (2012) distinguish between fences based on the purchase pattern (e.g., purchase time), on the product characteristics (e.g. time of usage) and on customer characteristics (e.g. age or spending).

Typically, products are differentiated by means of a combination of several fences (Talluri and van Ryzin, 2004b; Klein and Steinhardt, 2008). The resulting fare structures are thus rather complex (Botimer, 1996). Foran (2003) reports on the call center from British Airways, where more than half of the incoming calls were for servicing (i.e. explaining) the complexity of the offered fares. Due to the resulting complexity, Zhang et al. (2010) assume that fencing comes at a cost depending on the effort made in fencing.

As customer segmentation and fencing are interlinked, they should be considered jointly (Talluri and van Ryzin, 2004b). Zhang and Bell (2012) present a detailed overview and taxonomy of both market segmentation and price fencing methods. Customer segments are determined by descriptive methods (e.g., cluster analysis[1], discriminant analysis or finite mixture models[2]). They are characterized by particular values for the segmentation variables. These values are used to determine fences under the assumption that customers behave rationally according to their stated preferences and previous behavior. Figure 4.1 illustrates this two-step process. While Zhang and Bell (2012) focus on the development of

[1]For technical details see Ward (1963) and MacQueen (1967). For an overview of applications of customer segmentation see Punj and Stewart (1983) and Wedel and Kamakura (2000).

[2]For an overview of finite mixture models we refer to Wedel and Kamakura (2000) and McLachlan and Peel (2000). For applications of different finite mixture models we refer to Wedel et al. (1993), Wedel and

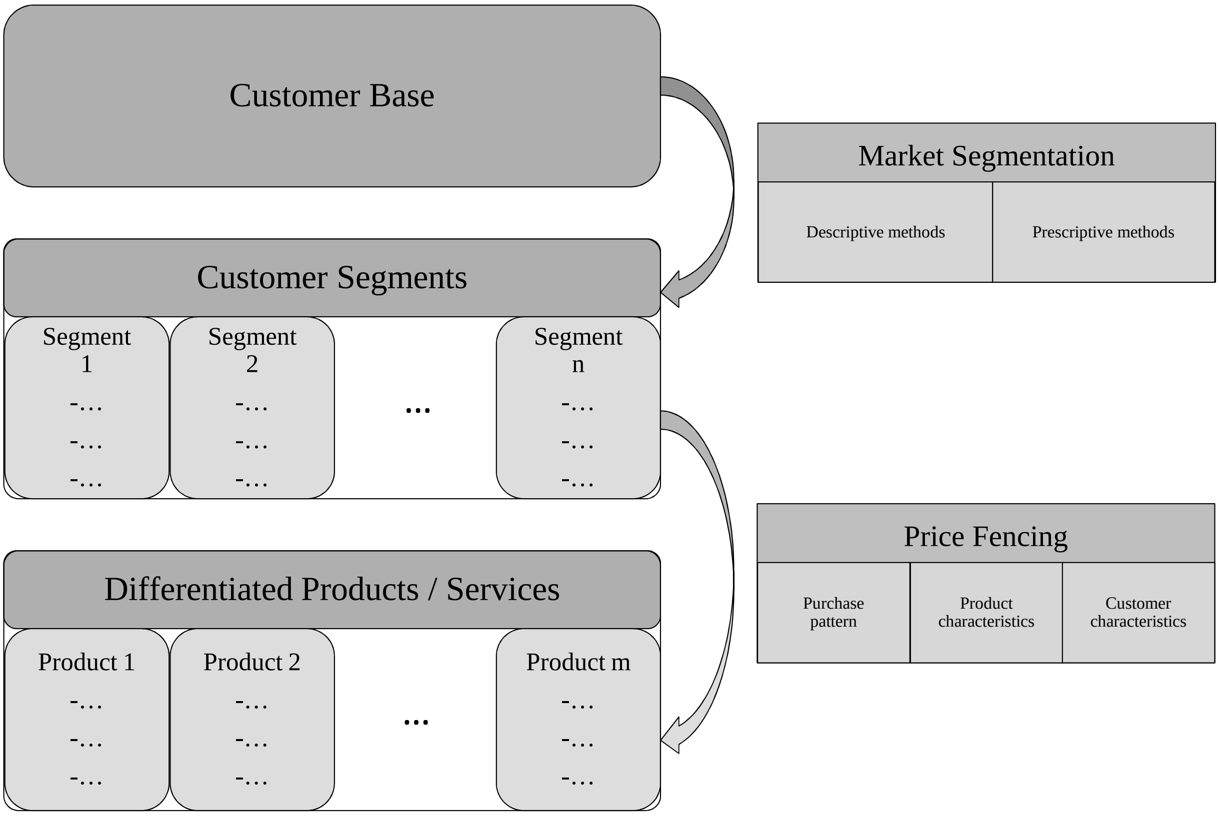


Figure 4.1: The process of market segmentation and price fencing (according to Zhang and Bell, 2012)

differentiated products based on customer segmentation, Cizaire (2011) focuses on how these decisions interact with capacity allocation. The process is illustrated in Figure 4.2. The pricing decisions mirror the taxonomy by Zhang and Bell (2012). Cizaire (2011) additionally accounts for the interaction of revenue management and pricing decisions.

In our approach, we build on the discussed qualitative basics of fences. We do not explicitly model particular types of fences (such as the time of booking) but present a stylized approach. Our approach builds on the general idea of fences, their definition and the effects of imperfect fences reported in literature. Imperfect fences result in price-driven demand leakage. We account for price-driven demand leakage in order to integrate the firm's fencing decision into a standard revenue management model.

### 4.2.2 Product Line Design

In this section, we provide an overview of product line design decisions. Product line design involves simultaneous decisions about how many products to offer, how to differentiate them and how to price them (Schön, 2010b). Product differentiation has analogies with fencing decisions: in both cases, key attributes are used in order to design different products.

DeSarbo (1995), Jedidi et al. (1996) and Jedidi et al. (1997).

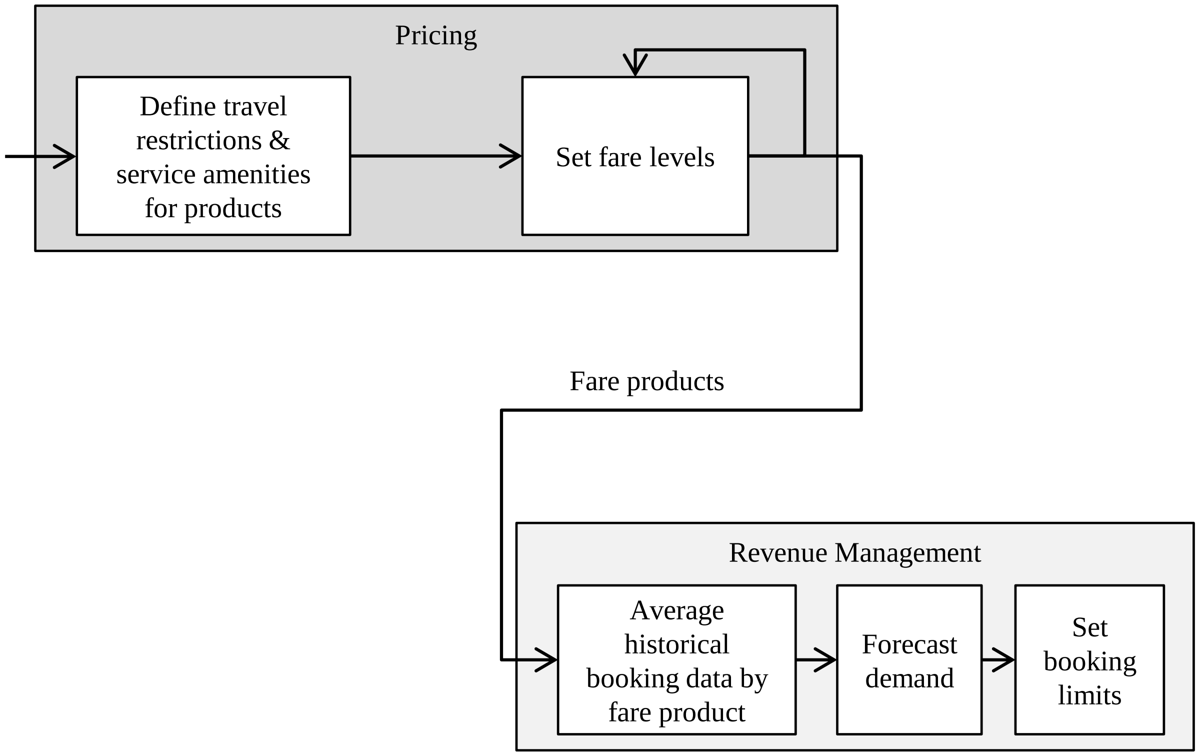


Figure 4.2: Pricing and revenue management (according to Cizaire, 2011)

However, product differentiation focuses on different variants of a (in most cases) physical product (e.g., different sizes, components etc. (McBride and Zufryden, 1988)) while fences are non-physical attributes of (in principle) the same physical product (see above).

On the one hand, the firm aims to increase product variety in order to meet the heterogeneous customer needs, resulting in a large variety of products and prices. On the other hand, costs and possible substitution effects must be balanced against the potential gains from product differentiation (Kekre and Srinivasan, 1990; Hopp and Xu, 2005; Schön, 2010a). Thus, by deciding which products to offer, a firm should maximize its profit by accounting for both revenues and costs implied by the decision on the product line design (see, e.g., McBride and Zufryden, 1988; Chen and Hausman, 2000). Customers may incur costs for evaluating the different product variants. These costs stem from the perceived complexity of the product portfolio (Kuksov and Villas-Boas, 2010).

Based on consumer preferences, early approaches in this field determine which product to offer to the market (Green and Krieger, 1985). Due to the heterogeneous needs of different market segments, Moorthy (1984), Green and Krieger (1985) and McBride and Zufryden (1988) extend these approaches by deciding on the offering of a subset of potential candidate products. They account for potential product substitution among the offered products, however, the firm does not decide on the product prices in their approaches. In a later approach, Chen and Hausman (2000), Schön (2010a) and Schön (2010b) extend these approaches according to the firm's additional decision on the product prices and choice-based customer demands. Due to the interdependencies in the demand effects and the large number of binary decisions, product line design decisions exhibit a high complexity and can typically only be tackled by heuristics for real-world instances (Schön, 2010a). Belloni et al. (2008) therefore present several practice-oriented heuristics for determining the product offering and pricing. They find that several of the heuristics find optimal or near-optimal solutions.

The stream of literature on product line design decisions is related to our approach through the notion of differentiation, however we do not investigate the choice to offer particular product variants but determine the firm's fencing decision for a given set of products. As in product line design approaches, demand substitution occurs, depending on the firm's fences. The firm does not decide on the product prices but on the allocation of the products to the different customer classes.

### 4.2.3 Substitution in Inventory Management

In this section, we discuss quantitative inventory management approaches, accounting for different types of demand substitution. Inventory management problems are similar to revenue management problems: however, while capacity is decided by the firm in inventory management models, it is given and fixed in revenue management. We borrow particular approaches for modeling different types of demand substitution in a revenue management setting from this field of literature. To this end, we consider models with stochastic demands for multiple products, which account for either price-driven or stock-out-based demand substitution. All approaches considered investigate newsvendor settings as this is closest to the revenue management setting considered in Sections 4.3 and 4.4.

If a firm faces price-dependent customer demand for multiple products, fences become particularly relevant in order to mitigate price-driven demand leakage. As a basis for this stream, Lau and Lau (1988) and Petruzzi and Dada (1999) investigated single-period, single-product joint pricing and inventory problems for a monopolistic newsvendor. Customer demand is price-dependent. For the multi-product setting, price-driven demand leakage assumes that the prices of the different products affect the customer perceptions of the different products and therefore their purchase behavior (Bitran and Caldentey, 2002).

Zhang and Bell (2007) investigate a monopolistic newsvendor facing demand from two customer segments. The firm decides both about the price to charge to each customer class and about the joint order quantity in order to satisfy aggregate customer demand. Price-driven demand leakage is modeled such that part of the demand for the higher-value product is lost to the lower-value product. Demand leakage is assumed to increase in the price difference between both products. Thus, the demand for a particular product depends on both product prices. The authors give analytical expressions for both the optimal prices and the optimal order quantity in the deterministic case and for the optimal order quantity in the stochastic case. They investigate the effects of demand uncertainty and demand leakage on the expected profit by means of a numerical example. They find that the optimal price difference decreases in demand leakage. The same holds for the profit gain relative to a setting without market segmentation.

Karakul and Chan (2008) and Zhang et al. (2010) investigate the problem when the firm aims to determine distinct order quantities for the two customer classes. Zhang et al. (2010) additionally consider the firm's fencing decision. Fences come into play by assuming that they affect demand leakage. The more effective the fences in place, the less price-driven demand leakage occurs. However, this comes at a cost. A firm therefore faces a trade-off between the resulting demand leakage and the costs for fencing. Zhang et al. (2010) (as Zhang and Bell, 2007) provide analytical results for given fences. Optimal fences are not investigated analytically but numerically. The paper differentiates between linear and strictly convex fencing costs. For strictly convex fencing costs, they find inner expected profit maxima numerically. The optimal leakage level is strictly positive and varies only slightly with demand uncertainty. Based on these numerical examples, Zhang et al. (2010) stress the fact that the characteristics of the fencing cost function are particularly important for deciding whether to segment at all. Kim and Bell (2011) extend this work by allowing customers not to buy at all. In their setting, they consider asymmetrical substitution, that is only a share of the spilled-over customers requests the lower-priced product.

Stock-out-based demand substitution occurs if customers substitute their requested product with another product due to a stock-out. Khouja (1999) provides an overview of several extensions of the single-period newsvendor problem. Well-known approaches are described in the papers by Parlar and Goyal (1984), Khouja et al. (1996) and Bassok et al. (1999), which all consider monopolistic newsvendors. Parlar and Goyal (1984) consider a two-product setting. They formulate the firm's decision problem under the assumption that both the salvage value and the penalties for lost sales are equal to zero. They show that the expected profit is strictly concave in the order quantities under certain conditions and provide an iterative procedure in order to determine the optimal order quantities. Khouja et al. (1996) revisit the two-product setting, however, they allow for positive salvage values and lost sales penalties. They provide lower and upper bounds on the optimal order quantities by Monte Carlo simulation. Bassok et al. (1999) consider the general case of $N$ products and $N$ demand classes under full downward-substitution. They investigate a two-stage decision problem: the firm determines the optimal order quantities for all products, which are then allocated to the different customer classes. Based on the properties of the optimal policy, they develop algorithms to efficiently determine the optimal allocations. Netessine and Rudi (2003) consider the competitive newsvendor problem under stock-out-based substitution.

In contrast to the above newsvendor settings, we consider a revenue management setting where the firm faces limited capacity and thus does not decide on order quantities but on capacity allocations. However, our modeling approach builds on the above modeling approaches in the field of inventory management. In particular, we build on the approach by Zhang et al. (2010) for modeling price-driven demand leakage and thus the impact of the firm's fencing decision on the expected profit. We also borrow the model of customer demand under stock-out-based demand substitution in our approach from the newsvendor problems under stock-out-based substitution discussed by, for example, Netessine and Rudi (2003).

### 4.2.4 Segmentation and Substitution in Revenue Management

In this section, we discuss quantitative literature from the field of quantity-based revenue management. While price-dependent demands, including price-based demand substitution, are accounted for, the firm's fencing decision is rarely explicitly addressed. First, we focus on the interrelation of customer segmentation and revenue management. We refer to this field, when comparing the two-class problem with a single-class problem within the numerical illustration in Section 4.5. Second, we consider models for joint pricing and capacity allocation as well as two distinct extensions. Although we do not account for prices as decision variables, these models are closest to the setting presented in Sections 4.3 and 4.4 as they investigate capacity allocation decisions and their interaction with longer-term pricing decisions.

Meyr (2008) investigates the interrelationship of decisions on the number of customer segments with the firm's allocation decision. The focus is on determining the appropriate number of customer segments. Meyr (2008) segments a given customer base by making use of cluster analysis, based on the customers' profitability, determines respective product allocations and simulates customer demand arrivals in order to evaluate the decisions in terms of expected profit. He finds that the marginal value of an additional customer segment decreases in the number of customer segments. He concludes that a moderate number of customer segments is sufficient in order to take advantage of service differentiation.

Several authors investigate settings where the firm decides on both prices and capacity allocations. Belobaba and Weatherford (1996) investigate customer substitution and its impact on revenue management decisions. They find that allowing for substitution is beneficial for the firm, however, the benefit decreases with increasing number of customer classes as the price difference to be exploited also decreases. Brumelle et al. (1990) extend the standard setting by allowing for stochastically dependent customer demands. They give analytical expressions for optimal booking policies and find that the optimal booking limits decrease, compared to Littlewood's rule, due to the dependent demands. Weatherford (1997) investigates a static single-resource revenue management setting where the firm decides on both the allocations and the prices. He considers additive-linear customer demands, where the expected demand depends on all prices and demand uncertainty adds to the expected demand (see also the demand models in Lau and Lau (1988); Petruzzi and Dada (1999); Zhang and Bell (2007); Zhang et al. (2010)). Kuyumcu and Popescu (2006) present a deterministic revenue management model building on the previous approach by Weatherford (1997). Kocabiykoglu et al. (2011) also build on the approach by Weatherford (1997). They compare sequential decision making with coordinated decision making. They find that coordinated decision making demonstrates significantly higher revenues. However, they also propose a hierarchical approach, which accounts for uncertainty in the pricing decision and mitigates the effects of poorly coordinated decisions.

Demand substitution is also considered in overbooking settings and multi-period settings. Karaesmen and van Ryzin (2004) investigate an overbooking setting with substitutable products. They show that accounting for substitution has a small, yet significant effect on the firm's expected profit. Bertsimas and de Boer (2005) study a multi-period multi-product (dynamic) pricing problem where the firm faces a limited supply of resources and has to decide how much of which product to offer at what price in each period. They present a stochastic dynamic approach allowing for general demand models.

Approaches accounting for demand substitution in the context of (quantity-based) revenue management are close to the setting we investigate in the remainder of this chapter. However, they do not explicitly account for price fences, their effects on customer demands or the effort by the firm to implement them. Instead of considering pricing as the firm's decision, we consider joint fencing and capacity allocation. Our approach is based on Littlewood's model (which is also the basis for the work by Belobaba and Weatherford (1996), Brumelle et al. (1990) and Weatherford (1997)). As we account for price fences based on approaches from the field of inventory management, we combine components from both inventory management and revenue management literature in our approach.

## 4.3 Price-Driven Demand Leakage

In order to investigate integrated fencing and capacity allocation, we consider a monopolistic setting where the firm faces stochastic price-dependent demand of two customer classes $i$ ($i = 1, 2$) for its finite and exogenous capacity $C$. As discussed in the previous section, the firm's fencing decision has not yet been explicitly accounted for in quantitative revenue management models. Thus, as a starting point, we choose the most basic model for capacity allocation under stochastic demand and therefore build on Littlewood's model (see Section 2.2.1.1). Despite the simplifying assumptions, Littlewood's model has been widely applied in literature (see, e.g., Pfeifer (1989), Brumelle et al. (1990), Belobaba and Weatherford (1996), Weatherford (1997), Kocabiykoglu et al. (2011)). The underlying assumptions of Littlewood's model (see Section 2.2.1.1) are assumed to hold also in our approach. However, we consider two decisions in our approach: the protection level and the firm's fences. While the protection level is a short-term decision on the operational level, fences are typically not updated on a short-term basis. Firms typically first decide on their fences and afterwards on the protection level based on the resulting customer demands.

The subsequent model formulation follows several steps. First, we characterize the underlying demand model which incorporates price-driven demand leakage, then we discuss the concept of a fencing cost function and characterize a particular formulation, based on previous literature. Thirdly, we present the firm's resulting optimization problem for a partitioned and a nested protection level. We then analyse the joint fencing and allocation problem for both types of protection levels. In order to compare our results with those of Zhang et al. (2010), we first investigate capacity allocation for given fences. This setting reflects a situation where the firm reacts to longer-term fencing decisions on the operational planning level. Afterwards, we analytically investigate both optimal fencing and allocation decisions.

### 4.3.1 Demand Model

In order to integrate the firm's fencing decision into our decision problem, we build on the demand model by Zhang et al. (2010). A share of the high-value Class 1 customers requests the product initially offered for the lower-value Class 2 customers. This share increases with increasing price difference $p_1 - p_2$ as it consequently becomes more attractive to request the lower-value product. Price-driven demand leakage is modeled by the leakage level $\gamma \geq 0$. The stochastic demand for product $i$ is denoted as $q_i(\gamma)$ $(\forall i)$. For the two customer classes, the stochastic demands are expressed as

$$q_1(\gamma) = D_1 - \gamma \cdot (p_1 - p_2) \tag{4.3.1}$$

and

$$q_2(\gamma) = D_2 + \gamma \cdot (p_1 - p_2) \tag{4.3.2}$$

where $D_i$ denotes the stochastic demand of customer class $i$ for the corresponding product and $\gamma \cdot (p_1 - p_2)$ characterizes the absolute price-driven demand leakage. $q_i(\gamma)$ thus is a random variable with cdf $F_i$ and pdf $f_i$. Both demands are assumed to be non-negative.

Representing the firm's fencing decision implicitly by the leakage level $\gamma$ is an aggregated modeling approach where we do not specify particular fences. The relevant product characteristics, which qualify as possible fences, stem from the actual customer segmentation, which is conducted prior to the fencing decision. In our approach, $\gamma$ represents the effort of designing the products, implementing the fences, and enforcing them.

Price-driven demand leakage is assumed to be deterministic. This is in line with the joint pricing and capacity allocation models discussed in the literature overview. They also assume deterministic effects of the firm's decision variables on customer demand. In our setting, this results in the fact that losing Class 1 demand increases Class 2 demand by a particular amount. Specifically, we assume that all leaked Class 1 customers request the lower-value product. Further implications and the impact of these assumptions on the optimal decisions are discussed below.

In contrast to Zhang et al. (2010), we do not focus on a particular functional form of the demand function. We assume exogenous prices, that is the firm does not decide on the product prices in our approach, and we therefore do not require a specific functional form of the demand function and consider demand as a general random variable. However, this modeling approach allows for many different explicit customer demand models. Different demand models exist in literature. For example, additive linear customer demand functions[3] (as assumed by Zhang et al., 2010) are amongst the most frequently applied demand functions in literature (see also, e.g., Talluri and van Ryzin, 2004b; Zhang and Bell, 2007; Klein and Steinhardt, 2008; Kim and Bell, 2011).

### 4.3.2 Fencing Costs

Setting up effective fences comes at a cost, which is related to the effort of designing, implementing and enforcing a complex system of fares (see, e.g. Foran, 2003). At the same time, more effective fences reduce price-driven demand leakage.

In line with Zhang et al. (2010), we model this relationship by assuming that the firm incurs costs of $K(\gamma)$, depending on its fencing decision. As discussed above, the firm's fencing decision is implicitly modeled by accounting for the leakage level as a decision variable.

We assume that $K(\gamma)$ is non-negative, continuously differentiable and monotonically decreasing in $\gamma$. While Zhang et al. (2010) propose both linear and strictly convex fencing cost functions, we focus on their representation for strictly convex fencing costs in the remainder of this chapter. Thus, the marginal costs for fencing (and thus for mitigating price-driven demand leakage) increase for stricter fences.

Zhang et al. (2010) represent strictly convex fencing costs by

$$K(\gamma) = \frac{K_0}{\gamma + K_1}, \tag{4.3.3}$$

with $K_0 > 0$ and $K_1 \geq 0$. The costs for a perfect fence amount to $\frac{K_0}{K_1}$. Moreover, $\frac{dK(\gamma)}{d\gamma} = -\frac{K_0}{(\gamma+K_1)^2} < 0$ and $\frac{d^2K(\gamma)}{d^2\gamma} = \frac{2 \cdot K_0}{(\gamma+K_1)^3} > 0$.

### 4.3.3 An Integrated Fencing and Capacity Allocation Model

We build our approach on Littlewood's standard model (see Section 2.2.1.1). The firm maximizes its expected revenue from selling the capacity $C$ to two customer classes minus the fencing costs by deciding on the leakage level $\gamma$ and the protection level $y$. Both decisions are static, they are taken once at the beginning of the planning horizon. We analyse the firm's decision problem for both a partitioned and a nested protection level.[4] The firm's

[3] Assuming additive linear demand functions results in $D_i = a_i - b_i \cdot p_i + \epsilon_i$ $(\forall i)$ where $\epsilon_i$ denotes the noise term.

[4] See Section 2.2.1.1 for details as well as a discussion of how both types of protection levels relate to each other.

optimization problem for a partitioned protection level can be stated as

$$\max_{0\le y\le C, 0\le\gamma} E[\pi(y,\gamma)] = p_2 \cdot E[\min(C-y, q_2(\gamma))] + p_1 \cdot E[\min(y, q_1(\gamma))] - K(\gamma). \quad (4.3.4)$$

For a nested protection level, the resulting expected profit can be stated as

$$\begin{aligned}\max_{0\le y\le C, 0\le\gamma} E[\pi(y,\gamma)] = {} & p_2 \cdot E[\min(C-y, q_2(\gamma))] \\ & + p_1 \cdot E[\min(C-\min(C-y, q_2(\gamma)), q_1(\gamma))] - K(\gamma). \end{aligned} \quad (4.3.5)$$

After having established the underlying assumptions and the firm's optimization problems, we now analyse the firm's optimal decisions when facing price-driven demand leakage. In line with the two different protection levels above, we first present the results for a partitioned protection level. The results for a nested protection level are subsequently presented.

### 4.3.4 Analysis: Partitioned Allocation Planning

In this section, we investigate the firm's optimization problem for a partitioned protection level stated in Equation (4.3.4). First, we consider the allocation decision for given fences. Second, we consider the simultaneous decisions about fences and the protection level.

#### 4.3.4.1 Allocation Planning under Exogenous Fences

Investigating the firm's allocation decision for given fences mirrors a situation where the firm takes the fencing decision prior to the allocation decision in a hierarchical manner. Investigating allocation planning for given fences is closest to the setting analytically investigated by Zhang et al. (2010). It allows for a comparison of our result for a limited capacity with the unconstrained setting investigated by Zhang et al. (2010).

For given fences, the firm's allocation planning problem reduces to Littlewood's standard model with demands depending on the firm's fences. The optimal partitioned protection level $y^{LW,part}$ thus follows from the optimality condition

$$p_2 \cdot \mathbb{P}(q_2(\gamma) > C - y) = p_1 \cdot \mathbb{P}(q_1(\gamma) > y). \quad (4.3.6)$$

The optimal allocation policy is to protect $\min(y^{LW}_{part}(\gamma), C)$ units of capacity, as the unconstrained univariate maximization problem is strictly concave in $y$ for given customer demands (see Section 2.2.1.1). Thus, a firm protects either the optimal protection level following from the previous optimality condition or the total available capacity.

As both customer demands depend on $\gamma$, the optimal protection level also depends on $\gamma$. The left hand-side of the optimality condition decreases in $\gamma$ while the right-hand side increases in $\gamma$. For stricter fences, the optimal protection level must thus increase, in order to ensure the optimality condition to hold. The inner optimal protection level decreases in

$\gamma$. This result is relatively intuitive as stricter fences decrease price-driven demand leakage, which leads to increasing Class 1 demand. Thus, firms should choose a higher protection level in the case of stricter fences.

Zhang et al. (2010) find that the optimal order quantity of the higher-value product increases with stricter fences. Our result is in line with these findings as the firm's optimal protection level reflects the available capacity for the higher-value customer class and also increases with stricter fences. As for the properties of the optimal solution, the inner optimal decisions are unique in both setting.

#### 4.3.4.2 Optimal Fences

In the remainder of this section, we consider both the fencing and the allocation decision simultaneously. First, we provide the first and second order derivatives with regard to the decision variables, in order to investigate the properties of the firm's decision problem. Second, we investigate the firm's optimal fencing and allocation decisions and their properties.

For partitioned allocations, the firm's optimization problem is stated in Equation (4.3.4). The first order derivative with regard to $y$ follows as

$$\begin{aligned}\frac{dE[\pi(y,\gamma)]}{dy} &= -p_2 \cdot E[I_{\{q_2(\gamma)>C-y\}}] + p_1 \cdot E[I_{\{q_1(\gamma)>y\}}] \\ &= -p_2 \cdot \mathbb{P}(q_2(\gamma) > C-y) + p_1 \cdot \mathbb{P}(q_1(\gamma) > y). \end{aligned} \tag{4.3.7}$$

The firm's allocation decision has a marginal impact on the firm's expected profit only if at least one of the customer demands exceeds the respective available capacity. On the one hand, the firm gains $p_1$ by reserving an additional unit of capacity for the higher-value customers. On the other hand, it loses $p_2$ as a previously serviced Class 2 customer is lost.

The first order derivative with regard to $\gamma$ results in

$$\begin{aligned}\frac{dE[\pi(y,\gamma)]}{d\gamma} &= p_2 \cdot E[\frac{dq_2(\gamma)}{d\gamma} \cdot I_{\{q_2(\gamma)\leq C-y\}}] + p_1 \cdot E[\frac{dq_1(\gamma)}{d\gamma} \cdot I_{\{q_1(\gamma)\leq y\}}] - \frac{dK(\gamma)}{(d\gamma} \\ &= p_2 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) \leq C-y) - p_1 \cdot (p_1-p_2) \cdot \mathbb{P}(q_1(\gamma) \leq y) \\ &\quad + \frac{K_0}{(\gamma+K_1)^2} \end{aligned} \tag{4.3.8}$$

This formula also has an intuitive interpretation. Fencing only affects the firm's revenue if at least one customer demand is lower than the respective available capacity. If $q_2(\gamma) \leq C-y$, part of the booking limit is still unsold. In this case, additional price-driven demand leakage increases the revenue gained from Class 2 customers. With regard to Class 1, fencing only impacts the firm's revenue, if $q_1(\gamma) \leq y$. If $q_1(\gamma) > y$, the firm would still be able to sell up to the protection level. As price-driven demand leakage is deterministic, the marginal impact of fencing on customer demands is also deterministic. The terms $\frac{dq_i(\gamma)}{d\gamma}$ can therefore be taken out of the expected value expressions in the above first order derivative, which allows for transformation of the first order derivative as shown above.

The second order derivatives with regard to $y$ and $\gamma$ are given by

$$\frac{d^2E[\pi(y,\gamma)]}{d^2y} = -\,p_2 \cdot f_2(C-y) - p_1 \cdot f_1(y), \tag{4.3.9}$$

$$\begin{aligned}\frac{d^2E[\pi(y,\gamma)]}{d^2\gamma} &= -\,p_2 \cdot (p_1-p_2)^2 \cdot f_2(C-y) - p_1 \cdot (p_1-p_2)^2 \cdot f_1(y) - \frac{2 \cdot K_0}{(\gamma+K_1)^3} \\ &= -\,(p_1-p_2)^2 \cdot [p_2 \cdot f_2(C-y) + p_1 \cdot f_1(y)] - \frac{2 \cdot K_0}{(\gamma+K_1)^3},\end{aligned} \tag{4.3.10}$$

and

$$\begin{aligned}\frac{d^2E[\pi(y,\gamma)]}{dyd\gamma} &= -\,p_2 \cdot f_2(C-y) \cdot \frac{dq_2}{d\gamma} + p_1 \cdot f_1(y) \cdot \frac{dq_1}{d\gamma} \\ &= -\,p_2 \cdot (p_1-p_2) \cdot f_2(C-y) - p_1 \cdot (p_1-p_2) \cdot f_1(y) \\ &= -\,(p_1-p_2) \cdot [p_2 \cdot f_2(C-y) + p_1 \cdot f_1(y)]\end{aligned} \tag{4.3.11}$$

The above second order derivatives provide some insight into the properties of the expected profit function. As $\frac{d^2E[\pi(y,\gamma)]}{d^2y} < 0$, the expected profit function is strictly concave in $y$ for given $\gamma$. Analogously, it is strictly concave in $\gamma$ for given $y$ as $\frac{d^2E[\pi(y,\gamma)]}{d^2\gamma} < 0$. $\frac{d^2E[\pi(y,\gamma)]}{dyd\gamma} < 0$ reflects the result from Section 4.3.4 that the protection level is decreasing in $\gamma$.

In order to investigate the properties of the firm's optimization problem, we investigate the determinant of the Hessian. The determinant of the Hessian, denoted as H in the remainder, results as

$$\begin{aligned} detH =& \frac{d^2E[\pi(y,\gamma)]}{d^2y} \cdot \frac{d^2E[\pi(y,\gamma)]}{d^2\gamma} - \left(\frac{d^2E[\pi(y,\gamma)]}{dyd\gamma}\right)^2 \\ =& (p_1 \cdot f_1(y) + p_2 \cdot f_2(C-y)) \cdot (p_1-p_2)^2 \cdot \frac{2 \cdot K_0}{(\gamma+K_1)^3}\end{aligned} \tag{4.3.12}$$

after substituting and rearranging the above second order derivatives. According to Section 4.3.1, $f_i(q_i(\gamma)) > 0$ holds by assumption. As $(p_1 \cdot f_1(y) + p_2 \cdot f_2(C-y)) > 0$, and $\frac{2 \cdot K_0}{(\gamma+K_1)^3} > 0$, the expected profit is jointly concave in $y$ and $\gamma$. Therefore, a unique optimal solution exists for the firm's unconstrained optimization problem.

However,the firm's optimization problem, stated in Equation (4.3.4), is constrained. In order to investigate the firm's optimal decisions, we investigate the Karush-Kuhn-Tucker conditions derived from the firm's constrained optimization problem stated in Equation (4.3.4). As the unconstrained objective function is jointly concave in $y$ and $\gamma$ and all constraints are convex functions in the firm's decision variables, the Karush-Kuhn-Tucker conditions are necessary and sufficient for optimality. The Lagrange function for the optimization problem follows as

$$L(y,\gamma,\lambda) = p_2 \cdot E[\min(C-y, q_2(\gamma))] + p_1 \cdot E[\min(y, q_1(\gamma))] - K(\gamma) - \lambda \cdot (y-C). \quad (4.3.13)$$

The last term represents the boundary condition $y \leq C$ stated in Equation (4.3.4). The Karush-Kuhn-Tucker conditions are the following:

$$\frac{dL(y,\gamma,\lambda)}{dy} = -p_2 \cdot \mathbb{P}(q_2(\gamma) > C-y) + p_1 \cdot \mathbb{P}(q_1(\gamma) > y) - \lambda \leq 0, \quad (4.3.14)$$

$$\begin{aligned}\frac{dL(y,\gamma,\lambda)}{d\gamma} =& p_2 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) \leq C-y) \\ & - p_1 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_1(\gamma) \leq y) + \frac{K_0}{(\gamma + K_1)^2} \leq 0, \end{aligned} \quad (4.3.15)$$

$$\frac{dL(y,\gamma,\lambda)}{d\lambda} = -(y-C) \geq 0, \quad (4.3.16)$$

$$y \cdot [-p_2 \cdot \mathbb{P}(q_2(\gamma) > C-y) + p_1 \cdot \mathbb{P}(q_1(\gamma) > y) - \lambda] = 0, \quad (4.3.17)$$

$$\gamma \cdot [p_2 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) \leq C-y) - p_1 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_1(\gamma) \leq y) + \frac{K_0}{(\gamma + K_1)^2}] = 0, \quad (4.3.18)$$

$$\lambda \cdot (-(y-C)) = 0, \quad (4.3.19)$$

$$y \geq 0, \quad (4.3.20)$$

$$\gamma \geq 0, \quad (4.3.21)$$

$$\lambda \geq 0. \quad (4.3.22)$$

Equations (4.3.14), (4.3.15) and (4.3.16) are the first order partial derivatives with regard to the firm's decision variables $y$, $\gamma$ and $\lambda$. Equations (4.3.17), (4.3.18) and (4.3.19) reflect the complementary slack with regard to the decision variables and $\lambda$. Equations (4.3.20), (4.3.21) and (4.3.22) represent the non-negativity constraints. According to the constraints regarding the decision variables, we have to investigate six cases. The cases result from combining $y = 0$, $0 < y < C$ and $y = C$ with $\gamma = 0$ and $\gamma > 0$. In the following, we investigate the firm's optimal decisions for these six cases.

**Case 1:** $0 < y < C$**,** $\gamma > 0$
First, we investigate inner optimal solutions to the joint fencing and allocation optimization problem. For inner optimal solutions, $\lambda = 0$ follows from Equation (4.3.19) and Equations (4.3.17) and (4.3.18) only hold if the first order partial derivatives of the expected profit function with regard to $y$ and $\gamma$ are equal to zero. It is therefore necessary and sufficient to investigate the first-order conditions following from Equations (4.3.14) and (4.3.15). The optimality equations for $y$ and $\gamma$ result as

$$p_2 \cdot \mathbb{P}(q_2(\gamma) > C - y) = p_1 \cdot \mathbb{P}(q_1(\gamma) > y), \tag{4.3.23}$$

and

$$p_2 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) \leq C - y) - p_1 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_1(\gamma) \leq y) = -\frac{K_0}{(\gamma + K_1)^2}. \tag{4.3.24}$$

Substituting Equation (4.3.23) in Equation (4.3.24) yields the optimality condition for the firm's fencing decision:

$$(p_1 - p_2)^2 = \frac{K_0}{(\gamma + K_1)^2}. \tag{4.3.25}$$

The optimal leakage level $\gamma^*_{part}$ results from this optimality equation:

$$\gamma^*_{part} = \frac{\sqrt{K_0}}{p_1 - p_2} - K_1. \tag{4.3.26}$$

Thus, the firm should define its fences such that $\gamma^*_{part}$ results as price-driven demand leakage. $\gamma^*_{part} > 0$ holds, iff $(p_1 - p_2)^2 < \frac{K_0}{K_1^2}$. The corresponding optimal protection level $y^{LW}_{part}(\gamma^*_{part})$ results from Equation (4.3.23) with $q_i(\gamma^*_{part})$ as the demand for product $i$ resulting from the firm's optimal fencing decision. Case 1 requires $y^{LW}_{part}(\gamma^*_{part}) < C$. This holds, iff $p_2 > p_1 \cdot \mathbb{P}(q_1(\gamma^*_{part}) > C)$.

Note that the inner optimal fencing decision $\gamma^*_{part}$ only depends on exogenous parameters and is therefore constant for a given problem. In particular, fences depend on the parameters of the cost function (i.e., $K_0$ and $K_1$) and on the revenue differential (i.e., $p_1 - p_2$). With increasing customer heterogeneity with regard to the different willingness-to-pay, the firm should define stricter fences in order to mitigate price-driven demand leakage as less leakage is then optimal. This result is intuitive. For given fences, price-driven demand leakage increases with increasing customer heterogeneity. In order to mitigate this effect, a firm should define stricter fences. The optimal level of price-driven demand leakage increases in $K_0$ and decreases in $K_1$.

When defining its fences such that the leakage level $\gamma^*_{part}$ results, the firm incurs fencing costs of

$$K(\gamma^*_{part}) = \sqrt{K_0} \cdot (p_1 - p_2). \tag{4.3.27}$$

$K(\gamma^*_{part})$ is strictly positive as $p_1 > p_2$ by assumption (see Section 2.2.1.1). The resulting costs for fencing increase with increasing price difference and thus customer heterogeneity as stricter fences are then optimal for the firm. Stricter fences come at a higher cost. As the value of capacity allocation generally increases with increasing price difference, the increasing fencing costs are a counter effect on the total expected profit.

**Case 2:** $0 < y < C$**,** $\gamma = 0$
For $0 < y < C$, $\lambda = 0$ follows from Equation (4.3.19).

$$p_2 \cdot \mathbb{P}(q_2(0) > C - y) = p_1 \cdot \mathbb{P}(q_1(0) > y). \tag{4.3.28}$$

follows as the optimality condition for $y$ from Equation (4.3.14). For $\gamma = 0$, Equation (4.3.15) reads as

$$p_2 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_2(0) \leq C - y) - p_1 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_1(0) \leq y) \leq -\frac{K_0}{K_1^2}. \tag{4.3.29}$$

Substituting Equation (4.3.28) in Equation (4.3.29) yields

$$(p_1 - p_2)^2 \geq \frac{K_0}{K_1^2} \tag{4.3.30}$$

as the condition for $\gamma = 0$ being the firm's optimal fencing decision. The firm's optimal protection level $y^{LW}_{part}(0)$ follows from Equation (4.3.28). Analogous to Case 1, this case only applies if $y^{LW}_{part}(0) < C$. This condition holds, if $p_2 > p_1 \cdot \mathbb{P}(q_1(0) > C)$.

**Case 3 and 4:** $y = 0$**,** $\gamma = 0$ **and** $y = 0$**,** $\gamma > 0$
For $y = 0$, $\lambda = 0$ follows from Equation (4.3.19). Consequently, Equation (4.3.14) simplifies to

$$-p_2 \cdot \mathbb{P}(q_2(\gamma) > C) + p_1 \cdot \mathbb{P}(q_1(\gamma) > 0) \leq 0. \tag{4.3.31}$$

As $\mathbb{P}(q_1(\gamma) > 0) = 1$, this further simplifies to

$$-p_2 \cdot \mathbb{P}(q_2(\gamma) > C) + p_1 \leq 0. \tag{4.3.32}$$

As $p_1 > p_2$, Equation (4.3.32) results in a contradiction. Therefore, reserving no capacity at all for the higher-value customers will never be an optimal decision for a firm, independent of the firm's fencing decision.

**Case 5:** $y = C$, $\gamma = 0$
For $y = C$ and $\gamma = 0$, Equation (4.3.14) simplifies to

$$-p_2 \cdot \mathbb{P}(q_2(0) > 0) + p_1 \cdot \mathbb{P}(q_1(0) > C) - \lambda = 0, \tag{4.3.33}$$

as Equation (4.3.17) only holds for $y = C$ if $\frac{dL(y,\gamma,\lambda)}{dy} = 0$. Equation (4.3.15) yields

$$p_2 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_2(0) \leq 0) - p_1 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_1(0) \leq C) + \frac{K_0}{K_1^2} \leq 0 \tag{4.3.34}$$

as the condition for $\gamma$. As $\mathbb{P}(q_2(0) > 0) = 1$, Equations (4.3.33) and (4.3.34) can be further simplified to

$$-p_2 + p_1 \cdot (1 - \mathbb{P}(q_1(0) \leq C)) - \lambda = 0, \tag{4.3.35}$$

and

$$-p_1 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_1(0) \leq C) + \frac{K_0}{K_1^2} \leq 0. \tag{4.3.36}$$

As $\lambda \geq 0$, Equation (4.3.35) holds iff $p_2 \leq p_1 \cdot \mathbb{P}(q_1(0) > C)$. Substituting Equation (4.3.35) in Equation (4.3.36) yields that Equation (4.3.36) holds iff $\frac{K_0}{K_1^2} \leq (p_1 - p_2)^2$. Then, $\gamma = 0$ and $y = C$ are the firm's optimal decisions.

**Case 6:** $y = C$, $\gamma > 0$
For $y = C$ and $\gamma > 0$, both Equation (4.3.14) and Equation (4.3.15) must hold with equality due to the slackness conditions in Equations (4.3.17) and (4.3.18). The analysis follows identical steps to those in Case 5 for $\gamma = 0$. Equation (4.3.14) results in

$$\lambda = p_1 \cdot (1 - \mathbb{P}(q_1(\gamma) \leq C)) - p_2. \tag{4.3.37}$$

Furthermore, Equation (4.3.18) yields

$$p_2 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) \leq 0) - p_1 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_1(\gamma) \leq C) + \frac{K_0}{(\gamma + K_1)^2} = 0. \tag{4.3.38}$$

Rearranging terms and simplifying this equation results in

$$p_1 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_1(\gamma) \leq C) = \frac{K_0}{(\gamma + K_1)^2}. \tag{4.3.39}$$

The left hand-side increases in $\gamma$ while the right hand-side decreases in $\gamma$. Thus, the unique optimal fencing decision $\gamma_C^*$ is the solution of this equation. Substituting $p_1 \cdot (1 - \mathbb{P}(q_1(\gamma) \leq$

$C)) - p_2 \geq 0$ in Equation (4.3.39) demonstrates that the optimal leakage level $\gamma_C^*$ is greater than or equal to $\gamma_{part}^*$. Thus, if a firm protects the total available capacity for the higher-value customers anyway, it is more beneficial to allow for higher leakage by setting less strict fences. By doing so, a firm saves costs due to the lower fencing effort. The marginal impact of setting less strict fences on Class 1 demand can be neglected due to the high Class 1 demand. Case 6 is complementary to Case 1. Therefore, $y = C$ and $\gamma_C^* > 0$ are the firm's optimal decisions if the conditions $p_2 \leq p_1 \cdot \mathbb{P}(q_1(\gamma_{part}^*) > C)$ and $\frac{K_0}{K_1^2} > (p_1 - p_2)^2$ hold.

To summarize, Table 4.1 provides an overview of the different optimal solutions in the cases considered. Cases 3 and 4 are not included in the overview as protecting $y = 0$ units of capacity has proved never to be optimal.

| Case | Optimal fencing | Optimal allocation | Conditions |
|---|---|---|---|
| 1 | $\gamma_{part}^* = \frac{\sqrt{K_0}}{(p_1-p_2)} - K_1$ | $y_{part}^{LW}(\gamma_{part}^*)$ | $\frac{K_0}{K_1^2} > (p_1 - p_2)^2$, $p_2 > p_1 \cdot \mathbb{P}(q_1(\gamma_{part}^*) > C)$ |
| 2 | 0 | $y_{part}^{LW}(0)$ | $\frac{K_0}{K_1^2} \leq (p_1 - p_2)^2$, $p_2 > p_1 \cdot \mathbb{P}(q_1(0) > C)$ |
| 5 | 0 | $C$ | $\frac{K_0}{K_1^2} \leq (p_1 - p_2)^2$, $p_2 \leq p_1 \cdot \mathbb{P}(q_1(0) > C)$ |
| 6 | $\gamma_C^*$: Equation (4.3.39) | $C$ | $\frac{K_0}{K_1^2} > (p_1 - p_2)^2$, $p_2 \leq p_1 \cdot \mathbb{P}(q_1(\gamma_{part}^*) > C)$ |

Table 4.1: The firm's optimal allocation and fencing decisions

The above conditions have some intuitive interpretations. If the marginal savings in fencing costs when deviating from perfect fences ($\frac{K_0}{K_1^2}$) are less than or equal to the marginal revenue loss due to price-driven demand leakage, reflected by $(p_1 - p_2)^2$, perfect fences are optimal for a firm. Otherwise, firms should allow for positive price-driven demand leakage by defining weaker fences. In this case the firm incurs higher price-driven demand leakage but also saves fencing costs.

The firm's allocation decision can still be characterized by Littlewood's rule for given fences as discussed in Section 4.3.4. Thus, the firm protects $\min(y_{part}^{LW}(\gamma^*), C)$ units of capacity for the higher-value customer class. It is optimal for a firm to take both decisions sequentially: while the firm first determines the optimal fences, the resulting customer demands $q_i(\gamma^*)$ serve as a basis for the optimal protection level. Taking both decisions simultaneously is of no additional value for the firm.

A firm's fencing decision is independent of the firm's available capacity and information on the demand distributions in Cases 1, 2 and 5. This is different in Case 6. In order to determine the optimal leakage level $\gamma_C^*$, the Class 1 demand distribution and the firm's capacity are necessary. However, Cases 5 and 6 generally have only a little practical relevance as protecting the total available capacity will rarely be optimal. Class 1 demand is typically only a share of the total available capacity, and therefore a firm would not protect the total capacity even without price-driven demand leakage. As the optimal protection level decreases with increasing price-driven demand leakage, the optimal protection level would be even lower.

As discussed above, Zhang and Bell (2012) consider an unconstrained setting but do not provide analytical results for their setting. Investigating their approach analytically allows a comparison of our result above with the unconstrained setting. The firm's expected profit in the unconstrained setting is

$$E[\pi] = p_2 \cdot E[q_2(\gamma)] + p_1 \cdot E[q_1(\gamma)] - K(\gamma). \tag{4.3.40}$$

The first order derivative with regard to $\gamma$ results as

$$\frac{dE[\pi]}{d\gamma} = p_2 \cdot (p_1 - p_2) - p_1 \cdot (p_1 - p_2) + \frac{K_0}{(\gamma + K_1)^2}. \tag{4.3.41}$$

From Equation (4.3.41), the first order condition yields

$$(p_1 - p_2)^2 = \frac{K_0}{(\gamma + K_1)^2}, \tag{4.3.42}$$

which is identical to the first order condition above in Equation (4.3.25) for the inner optimal solution in the constrained setting. Thus, the optimality condition in the unconstrained setting yields the identical optimal fencing decision resulting in the identical level of price-driven demand leakage $\gamma^*_{part}$, at least for the inner optimal solution. In contrast to our constrained setting, it is generally optimal to take the fencing decision independent of both the decision on the order quantities and information on the demand distributions.

The above results for the constrained setting are mainly driven by three assumptions: (1) price-driven demand leakage has a deterministic effect on the demands of both customer classes, (2) all leaked Class 1 customers request the lower-value product and (3) the customer demands themselves are independent of the allocation decision.

The first assumption simplifies the analysis. If this assumption is relaxed, and the fencing decision has a random effect on customer demands, the firm will not be able to ensure a leakage level with certainty by defining particular fences. Relaxing this assumption does not change the firm's optimal decisions qualitatively as long as price-driven demand leakage only depends on $\gamma$ and not additionally on demand realizations. This is because although being stochastic, leaked Class 1 demand (for any possible demand realization) is a fixed amount.

The second assumption means that losing a Class 1 customer due to price-driven demand leakage results in definitely gaining a Class 2 customer. Thus, a firm will decrease the protection level accordingly with increasing price-driven demand leakage. However, following the approach by Kim and Bell (2011), one could also assume that only a share of the leaked Class 1 customers requests the lower-value product. Even under this assumption, the properties of the expected profit function are preserved. Thus, both the structure and the properties of the optimal decisions remain unchanged.

The main result that both decisions can be taken sequentially, at least for practical purposes, is mainly driven by the third underlying assumption, the fact that customer demands are independent of the allocation decision. By accounting for additional stock-out-based substitution, we relax this assumption in Section 4.4 and discuss the implications below.

### 4.3.5 Analysis: Nested Allocation Planning

Analogous to the previous section, we split our analysis in two steps for the nested protection level. First, we consider the firm's optimal allocation decision for given fences. We then consider simultaneous decisions on fences and the protection level. In both parts, we compare our results to the results obtained for a partitioned protection level.

#### 4.3.5.1 Allocation Planning under Exogenous Fences

For given fences, a firm's allocation planning problem is equivalent to Littlewood's standard model as in the case of partitioned allocations. Setting the protection level according to Littlewood's rule is therefore optimal. However, customer demands depend on $\gamma$. The optimal protection level $y^{LW}_{nested}$ according to Littlewood's rule results from

$$\mathbb{P}(q_1(\gamma) \leq y) = 1 - \frac{p_1}{p_2}. \tag{4.3.43}$$

The firm's optimal allocation policy is to protect $y^* = \min(y^{LW}_{nested}(\gamma), C)$ as the unconstrained optimization problem is unimodular in $y$ as shown in Section 2.2.1.1.

The left hand-side of Equation (4.3.43) is increasing in $\gamma$ as $q_1(\gamma)$ is decreasing in $\gamma$, therefore the firm's inner optimal protection level $y^{LW}_{nested}(\gamma)$ decreases in $\gamma$. If $y^* = C$, then additional demand leakage does not affect the protection level. Thus, both the partitioned and the nested protection level are non-decreasing with stricter fences and corresponding decreasing price-driven demand leakage.

#### 4.3.5.2 Optimal Fences

In the following, we investigate the firm's two-dimensional optimization problem, as stated in Equation (4.3.5). The firm decides on both the fences and the nested protection level. As discussed in Section 2.2.1.1, the expected profit function is generally not concave but unimodular in $y$ for given demand distributions and thus for given fences. The joint optimization problem is therefore not jointly concave in $y$ and $\gamma$ either. Due to the properties of the expected profit function (in particular the missing concavity property), we focus only on the necessary first-order conditions and discuss the underlying trade-offs. Investigating the unconstrained optimization problem is supported by the result in Section 4.3.4 that both approaches yield the identical inner optimal solution and by the unimodularity property with regard to $y$.

The first order derivatives of the expected profit function in Equation (4.3.5) with regard to $y$ and $\gamma$ result in

$$\begin{aligned}
\frac{dE[\pi(y,\gamma)]}{dy} &= -p_2 \cdot E[I_{\{q_2(\gamma)>C-y\}}] + p_1 \cdot E[I_{\{q_1(\gamma)>y, q_2(\gamma)>C-y\}}] \\
&= -p_2 \cdot \mathbb{P}(q_2(\gamma) > C-y) + p_1 \cdot \mathbb{P}(q_1(\gamma) > y, q_2(\gamma) > C-y) \\
&= -p_2 \cdot \mathbb{P}(q_2(\gamma) > C-y) + p_1 \cdot \mathbb{P}(q_1(\gamma) > y) \cdot \mathbb{P}(q_2(\gamma) > C-y) \\
&= \mathbb{P}(q_2(\gamma) > C-y) \cdot [p_1 \cdot \mathbb{P}(q_1(\gamma) > y) - p_2],
\end{aligned} \tag{4.3.44}$$

and

$$\begin{aligned}
\frac{dE[\pi(y,\gamma)]}{d\gamma} =& p_2 \cdot \frac{dq_2(\gamma)}{d\gamma} \cdot E[I_{\{q_2(\gamma)\leq C-y\}}] + p_1 \cdot E[-\frac{dq_2(\gamma)}{d\gamma} \cdot I_{\{q_2(\gamma)\leq C-y, C-q_2(\gamma)<q_1(\gamma)\}} \\
&+ \frac{dq_1(\gamma)}{d\gamma} \cdot I_{\{q_2(\gamma)\leq C-y, C-q_2(\gamma)\geq q_1(\gamma)\}} + \frac{dq_1(\gamma)}{d\gamma} \cdot I_{\{q_2(\gamma)>C-y, q_1(\gamma)\leq y\}}] \\
&- \frac{dK(\gamma)}{d\gamma} \\
=& p_2 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) \leq C-y) \\
&+ p_1 \cdot [-(p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) \leq C-y, C-q_2(\gamma) < q_1(\gamma)) \\
&- (p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) \leq C-y, C-q_2(\gamma) \geq q_1(\gamma)) \\
&- (p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) > C-y, q_1(\gamma) \leq y)] + \frac{K_0}{(\gamma+K_1)^2} \\
=& -(p_1 - p_2)^2 \cdot \mathbb{P}(q_2(\gamma) \leq C-y) \\
&- p_1 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) > C-y, q_1(\gamma) \leq y) + \frac{K_0}{(\gamma+K_1)^2} \\
=& -(p_1 - p_2)^2 \cdot \mathbb{P}(q_2(\gamma) \leq C-y) \\
&- p_1 \cdot (p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) > C-y) \cdot \mathbb{P}(q_1(\gamma) \leq y) + \frac{K_0}{(\gamma+K_1)^2} \\
=& (p_1 - p_2) \cdot \mathbb{P}(q_2(\gamma) > C-y) \cdot [p_1 \cdot \mathbb{P}(q_1(\gamma) > y) - p_2] \\
&- (p_1 - p_2)^2 + \frac{K_0}{(\gamma+K_1)^2}.
\end{aligned} \tag{4.3.45}$$

Equation (4.3.44) is equivalent to the first order derivative in Littlewood's standard model except that customer demands depend on the firm's fences. Equation (4.3.45) reflects the marginal impact of the fencing decision on the firm's expected profit. The terms in Equation (4.3.45) have some interpretation. Price-driven demand leakage has a positive marginal impact on the revenue from Class 2 demand if this demand is less than the booking limit. Otherwise, additional price-driven demand leakage does not affect Class 2 revenues as the quantity up to the booking limit can be sold anyway. The second term represents the impact of price-driven demand leakage on the Class 1 revenue. The first two terms in brackets reflect the lost Class 1 sales if $q_2(\gamma) \leq C - y$, that is when more capacity than the protection level is available for Class 1 customers. In case of nesting, the firm loses $\frac{dq_1(\gamma)}{d\gamma} = -(p_1 - p_2)$ Class 1 demand. The third term in brackets reflects the lost Class 1 sales if $q_2(\gamma) > C - y$ and $q_1(\gamma) \leq y$. In this case, the firm loses Class 1 demand as $q_1(\gamma) \leq y$ but cannot benefit as it sells the booking limit to Class 2 anyway. Finally, the firm incurs marginal fencing costs.

The second order derivative with regard to $y$ reads

$$\begin{aligned}\frac{d^2E[\pi(y,\gamma)]}{d^2y} =& - p_2 \cdot f_2(C-y) \\ &+ p_1 \cdot [-f_1(y) \cdot \mathbb{P}(q_2(\gamma) > C-y) + \mathbb{P}(q_1(\gamma) > y) \cdot f_2(C-y)]. \end{aligned} \tag{4.3.46}$$

This term is not generally negative. As in Littlewood's standard model, the expected profit function is thus not generally concave in $y$ for given $\gamma$.

The second order derivative with regard to $\gamma$ results in

$$\begin{aligned}\frac{d^2E[\pi(y,\gamma)]}{d^2\gamma} =& (p_1-p_2)^2 \cdot [(p_1-p_2) \cdot f_2(C-y) - p_1 \cdot f_2(C-y) \cdot \mathbb{P}(q_1(\gamma) \leq y) \\ &- p_1 \cdot f_1(y) \cdot (1 - \mathbb{P}(q_2(\gamma) \leq C-y))] - \frac{2 \cdot K_0}{(\gamma+K_1)^3}. \end{aligned} \tag{4.3.47}$$

This term is also not generally negative. Therefore, the expected profit function for a nested protection level is not concave in $\gamma$ for given $y$, in general.

Setting Equation (4.3.44) equal to zero and solving for $y$ yields

$$p_1 \cdot \mathbb{P}(q_1(\gamma) > y) = p_2 \tag{4.3.48}$$

as the optimality condition for $y$. Substituting this optimality condition for $y$ into Equation (4.3.45) yields

$$(p_1 - p_2)^2 = \frac{K_0}{(\gamma + K_1)^2} \tag{4.3.49}$$

as the optimality condition for $\gamma$ and thus

$$\gamma^*_{nested} = \frac{\sqrt{K_0}}{p_1 - p_2} - K_1. \tag{4.3.50}$$

The resulting optimal demand leakage is identical to the derived optimal demand leakage for a partitioned protection level. Therefore, the resulting inner optimal fencing decision (and its properties) are the same under both types of protection levels. Again, $\gamma^*_{nested} > 0$ holds for $\frac{K_0}{K_1^2} > (p_1 - p_2)^2$. Thus, if a positive leakage level is optimal for the firm, the firm should define the same fences independent of whether they apply a partitioned or nested protection level. This holds as long as the resulting optimal protection level is less than the total available capacity, $y^{LW}_{nested}(\gamma^*_{nested}) < C$. Given the results for a partitioned protection level, this results is rather intuitive. If the value of the protection level itself (at least as long as the inner optimal protection level is less than the total available capacity) does not affect the firm's fencing decision, the type of allocation rule should not affect the firm's fencing decision either. As long as the inner optimal protection level does not exceed the total capacity, taking both decisions sequentially is also optimal for a nested protection level. As shown in Section 2.2.1.1, the expected profit function is unimodular in $y$ for given fences. The inner optimal protection level is therefore unique. Littlewood's rule follows from Equation (4.3.44) and yields $y_{nested}^{LW}(\gamma_{nested}^*)$ as the inner optimal protection level.

The resulting locally optimal decisions yield a local maximum of the expected profit function as both second order derivatives are negative for $\gamma^* = \gamma_{nested}^*$ and $y^* = y_{nested}^{LW}(\gamma_{nested}^*)$.

To summarize, our findings support the numerical results by Zhang et al. (2010). In our approach, we show that the inner optimal fencing decisions are identical under both protection levels, constant and thus independent of decisions about the protection level for both types of protection levels. Taking both decisions simultaneously is therefore of no additional value for the firm. The firm also takes the optimal decisions if it decides in a hierarchical manner. Our result therefore is in contrast to the claim that both decisions should generally be taken simultaneously by, for example, Hanks et al. (2002), and supports traditional hierarchical planning approaches as discussed in Chapter 2.

The main result, that the fencing decision can be taken independently of the allocation decision if the optimal protection level does not exceed the total capacity results from the definition of the customer demands, as discussed for the case of a partitioned protection level above. In order to account for customer demand depending on the firm's allocation decision, we consider additional stock-out-based substitution in the next section.

## 4.4 Additional Stock-Out-Based Substitution

In this section, we extend the previous setting. We still account for demand leakage due to the difference in product prices, however, we additionally consider stock-out-based demand substitution in case that the Class 2 product is unavailable. Compared to the previous setting, we thus consider two additional customer types by accounting for the fact that actual Class 2 customers might behave differently in case of a stock-out compared to leaked Class 1 customers. Both additional customer types re-substitute in cases where the booking limit quantity is already unavailable.

In the remainder of this section, we first present the underlying demand model with stock-out-based demand substitution. Stock-out-based demand substitution affects the demand model discussed in Section 4.3, which makes up the difference compared to the previous setting. The extended demand model is presented and discussed in Section 4.4.1. Afterwards, we give the extended models for a partitioned and nested protection level. Finally, we present the analyses again for both a partitioned and a nested protection level.

### 4.4.1 Demand Model

As described above, we account for additional stock-out based substitution in case the lower-value product is unavailable. These substitution effects impact Class 1 demand while Class 2 demand remains unchanged compared to the previous setting (see Section 4.3.1). The Class 1 demand model follows the approach for modeling stock-out-based substitution as presented by, for example, Netessine and Rudi (2003), Kök et al. (2009) or Jiang et al. (2011).

In line with Littlewood's model, we assume a low-before-high order arrival pattern. Due to price-driven demand leakage, requests for the lower-value product may stem from either initial Class 1 or Class 2 customers. Here, we assume that requests from initial Class 2 customers are also served first in the case of a stock-out. Stock-out-based demand substitution only occurs if the lower-value demand exceeds the booking limit. In case of a stock-out, the substitution rates may differ between initial Class 1 and Class 2 customers. The substitution rates are assumed to be exogenously given.

Following the previous notation and the above assumptions, Class 1 demand $q_1^s$ after substitution results from combining the demand model in Section 4.3.1 and the approach for modeling stock-out-based demand substitution as

$$q_1^s = \begin{cases} q_1(\gamma), & \text{for } q_2(\gamma) \leq C - y, \\ q_1(\gamma) + \alpha_1 \cdot (q_2(\gamma) - (C - y)), & \text{for } C - y < q_2(\gamma) \leq C - y + \gamma \cdot (p_1 - p_2), \\ q_1(\gamma) + \alpha_1 \cdot \gamma \cdot (p_1 - p_2) & \\ \qquad + \alpha_2 \cdot (D_2 - (C - y)), & \text{for } q_2(\gamma) > C - y + \gamma \cdot (p_1 - p_2), \end{cases} \tag{4.4.1}$$

where $\alpha_i$ $(i = 1, 2)$ denotes the share of actual demand of customer class $i$ for the lower-price product, which cannot be satisfied from the allocated quantity and is therefore (re-) substituted to the higher-value product. In contrast to the previous setting, effective Class 1 demand depends both on the firm's fencing decision and the allocation decision.

If $q_2(\gamma) \leq C - y$, demand for the lower-price product can be entirely satisfied by the available booking limit. Thus, no additional stock-out-based substitution occurs in this case. We denote this event by $NS$ in the remainder of this section. As soon as $q_2(\gamma)$ exceeds the booking limit, stock-out-based substitution occurs. This is the case in the second and third case in Equation (4.4.1).

If $C - y < q_2(\gamma) \leq C - y + \gamma \cdot (p_1 - p_2)$, the demand for the lower-value product exceeds the booking limit. Thus, not all the requests can be satisfied. As Class 2 requests occur prior to Class 1 requests due to the assumptions given in Section 2.2.1.1 (”low-before-high” order arrivals), we assume that Class 2 requests are served first. The booking limit is thus sufficient to satisfy demand from the actual Class 2 customers, but is not sufficient to satisfy all leaked Class 1 demand in this case. A share of $\alpha_1$ of the rejected leaked initial Class 1 customers again requests the higher-value product. This event is denoted as $S1$ in the remainder as only actual Class 1 customer re-substitute.

In the third case, i.e., if $q_2(\gamma) > C - y + \gamma \cdot (p_1 - p_2)$, or equivalently if $D_2 > C - y$, stock-out based substitution occurs by both the original Class 2 customers ($\alpha_2 \cdot (D_2 - (C - y))$) and the leaked Class 1 customers. In this case, the booking limit is too low even to satisfy the original Class 2 demand. Some customers from both customer classes are therefore not served. A share $\alpha_1$ of the leaked Class 1 demand ($\gamma \cdot (p_1 - p_2)$) re-substitutes for the Class 1 product. As customers from both customer classes engage in stock-out-based substitution, we denote this event as $S12$ in the remainder. Table 4.2 summarizes the possible events with regard to stock-out-based substitution discussed above.

| **Event** | **Product 2 demand after leakage** | **Product 1 demand after substitution** |
|---|---|---|
| NS | $q_2(\gamma) \leq C - y$ | $q_1(\gamma)$ |
| S1 | $C - y < q_2(\gamma) \leq C - y + \gamma \cdot (p_1 - p_2)$ | $q_1^{S1} = q_1(\gamma) + \alpha_1 \cdot (q_2(\gamma) - (C - y))$ |
| S12 | $q_2(\gamma) > C - y + \gamma \cdot (p_1 - p_2)$ | $q_1^{S12} = q_1(\gamma) + \alpha_1 \cdot \gamma \cdot (p_1 - p_2) + \alpha_2 \cdot (D_2 - (C - y))$ |

Table 4.2: Events with regard to stock-out-based demand substitution

### 4.4.2 Fencing Costs

In line with the previous section, we assume that a firm can influence price-driven demand leakage by managing its fences, thereby incurring costs of $K(\gamma)$. The stock-out-based substitution rates $\alpha_i$ are assumed to be exogenously given. In contrast to the fences (and therefore the resulting price-driven demand leakage), they are not a decision variable for the firm. Thus, we assume that the fencing costs are independent of the stock-out-based substitution rates $\alpha_i$. Given these assumptions, we again consider $K(\gamma) = \frac{K_0}{\gamma + K_1}$ as the relevant fencing cost function.

### 4.4.3 An Integrated Fencing and Capacity Allocation Model

Under additional stock-out-based demand substitution, Class 1 demand depends on both decisions of the firm: the fences and the protection level. A firm's optimization problems for partitioned and nested protection levels therefore result from replacing $q_1(\gamma)$ by $q_1^s(y, \gamma)$ in Equations (4.3.4) and (4.3.5), respectively. Thus

$$\max_{0 \leq y \leq C, 0 \leq \gamma} E[\pi(y, \gamma)] = p_2 \cdot E[\min(C - y, q_2(\gamma))] + p_1 \cdot E[\min(y, q_1^s(y, \gamma))] - K(\gamma) \quad (4.4.2)$$

for a partitioned protection level, and

$$\begin{aligned} \max_{0 \leq y \leq C, 0 \leq \gamma} E[\pi(y, \gamma)] &= p_2 \cdot E[\min(C - y, q_2(\gamma))] \\ &\quad + p_1 \cdot E[\min(C - \min(C - y, q_2(\gamma)), q_1^s(y, \gamma))] - K(\gamma). \end{aligned} \quad (4.4.3)$$

for a nested protection level.

Having stated the additional assumptions under stock-out-based substitution and the firm's optimization problem, we investigate a firm's optimal decisions in the remainder. Again, we first present the results for a partitioned protection level both for given fences and simultaneous decisions on the protection level and the fences. Subsequently, we consider the case of a nested protection level. For both types of protection levels, we investigate the necessary first-order optimality conditions and thus the trade-offs underlying the optimal inner solutions for the firm's decisions.

### 4.4.4 Analysis: Partitioned Allocation Planning

#### 4.4.4.1 Allocation Planning under Exogenous Fences

As in the previous section, we first consider the firm's allocation planning problem for given fences. We consider this setting in order to investigate the impact of both price-driven demand leakage and stock-out-based substitution on the optimal protection level.

Differentiating Equation (4.4.2) with regard to $y$ yields

$$\frac{dE[\pi(y,\gamma)]}{dy} = -\, p_2 \cdot E[I_{\{q_2(\gamma)>C-y\}}] + p_1 \cdot E[\frac{dq_1^s(y,\gamma)}{dy} \cdot I_{\{q_1^s(y,\gamma)\leq y\}} + I_{\{q_1^s(y,\gamma)>y\}}]. \tag{4.4.4}$$

The marginal effect of $y$ on Class 1 demand can be expressed as

$$\frac{dq_1^s}{dy} = \begin{cases} 0, & \text{if } q_2(\gamma) < C - y, \\ \alpha_1, & \text{if } C - y < q_2(\gamma) < C - y + \gamma \cdot (p_1 - p_2), \\ \alpha_2, & \text{if } q_2(\gamma) > C - y + \gamma \cdot (p_1 - p_2). \end{cases} \tag{4.4.5}$$

For $q_1^s(y,\gamma)$ as stated above in Section 4.4.1, Equation (4.4.4) results in

$$\begin{aligned} \frac{dE[\pi(y,\gamma)]}{dy} = &-\, p_2 \cdot \mathbb{P}(q_2(\gamma) > C - y) + p_1 \cdot [\alpha_1 \cdot \mathbb{P}(S1, q_1^{S1} \leq y) + \alpha_2 \cdot \mathbb{P}(S12, q_1^{S12} \leq y) \\ &+ \mathbb{P}(NS, q_1(\gamma) > y) + \mathbb{P}(S1, q_1^{S1} > y) + \mathbb{P}(S12, q_1^{S12} > y)] \end{aligned} \tag{4.4.6}$$

when accounting for all possible cases regarding stock-out-based substitution as illustrated in Table 4.2 and Equation (4.4.5), respectively. Both the marginal effect on Class 2 revenue and the marginal effect on Class 1 revenue depend on $\gamma$. However, only the marginal effect on Class 1 revenue depends on the substitution effects $\alpha_1$ and $\alpha_2$.

Equation (4.4.6) has some interpretation. On the one hand, increasing the protection level results in a marginal loss of $p_2$ if $q_2(\gamma) > C - y$. This is reflected by the first term in the above first order derivative and is equivalent to Littlewood's standard model. On the other hand, increasing the protection level results in a marginal revenue gain. This is reflected in the right-hand side of the above optimality condition. If $q_1^s(y,\gamma) \leq y$, increasing the protection level does not increase the firm's revenue by serving additional Class 1 requests directly. Increasing the protection level is identical to decreasing the booking limit. Therefore, additional stock-out-based substitution might occur in this case, which affects the firm's revenue indirectly. If $q_1^s(y,\gamma) > y$, then increasing the protection level results in additional direct revenue from Class 1. If the stock-out-based demand substitution rates are equal to zero (i.e., $\frac{dq_1^s(y,\gamma)}{dy} = 0$), Equation (4.4.6) is identical to Equation (4.3.7) in the previous setting without consideration of stock-out-based demand substitution.

The second order derivative results from differentiating Equation (4.4.6) with regard to $y$. As the resulting second order derivative is not generally negative, the expected profit function is not necessarily strictly concave in $y$. A unique optimal protection level is thus not guaranteed.

Setting the first order derivative equal to zero yields

$$\begin{aligned} p_2 \cdot \mathbb{P}(q_2(\gamma) > C - y) = &p_1 \cdot [\alpha_1 \cdot \mathbb{P}(S1, q_1^{S1} \leq y) + \alpha_2 \cdot \mathbb{P}(S12, q_1^{S12} \leq y) \\ &+ \mathbb{P}(NS, q_1(\gamma) > y) + \mathbb{P}(S1, q_1^{S1} > y) + \mathbb{P}(S12, q_1^{S12} > y)] \end{aligned} \tag{4.4.7}$$

as the first-order optimality condition for $y$. Both sides depend on $\gamma$. The left hand-side of Equation (4.4.7) is increasing in $\gamma$. $q_1^s(y,\gamma)$ is decreasing in $\gamma$ and $\frac{dq_1^s(y,\gamma)}{dy}$ is constant and independent of $\gamma$ in each of the cases discussed above. Thus, the right-hand side is decreasing in $\gamma$. If the firm sets weaker fences, the resulting price-driven demand leakage increases. Therefore, the left hand-side increases and the right-hand side decreases. Consequently, $y$ must decrease such that Equation (4.4.7) holds again. The local optimal protection level thus decreases with increasing (exogenous) demand leakage as in the previous setting where additional stock-out-based demand substitution is not considered. The stricter the firm's fences, the more capacity should be reserved for the higher-value customer class. This result holds locally, for any local optimum. Recall that the problem is not concave and may therefore have multiple local optima, in general.

Next, we investigate the impact of the substitution rates $\alpha_1$ and $\alpha_2$ on the firm's optimal allocation decision. In order to investigate their impact, we compare the settings with and without stock-out-based substitution. We aim to investigate whether the protection level under stock-out-based substitution is lower or greater than the protection level when no stock-out-based substitution occurs. Stock-out-based substitution affects Class 1 demand positively, and therefore $q_1^s(y,\gamma)$ is stochastically larger than $q_1(\gamma)$.

Assume that the firm sets a protection level $\tilde{y}$ when no stock-out based substitution occurs. In order to make a statement about the relationship of the optimal protection levels with and without stock-out-based demand substitution, we compare the first order derivatives in both cases. Specifically, assume that there exists a value $y$ such that the first order derivative with regard to the protection level at $y$ under stock-out-based substitution (Equation (4.4.4)) is equal to the first order derivative with regard to $y$ without stock-out-based substitution at

$\tilde{y}$ (Equation (4.3.7)). Formally, this implies that

$$p_1 \cdot E[\frac{dq_1^s(y,\gamma)}{dy} \cdot I_{\{q_1^s(y,\gamma)\leq y\}} + I_{\{q_1^s(y,\gamma)>y\}}] - p_2 \cdot E[I_{\{q_2(\gamma)>C-y\}}]$$
$$= p_1 \cdot E[I_{\{q_1(\gamma)>\tilde{y}\}}] - p_2 \cdot E[I_{\{q_2(\gamma)>C-\tilde{y}\}}] \tag{4.4.8}$$

holds. Rearranging terms results in

$$p_1 \cdot E[(I_{\{q_1^s(y,\gamma)>y\}} - I_{\{q_1(\gamma)>\tilde{y}\}}) + \frac{dq_1^s(y,\gamma)}{dy} \cdot (1 - I_{\{q_1^s(y,\gamma)>y\}})]$$
$$- p_2 \cdot E[I_{\{q_2(\gamma)>C-y\}} - I_{\{q_2(\gamma)>C-\tilde{y}\}}] = 0. \tag{4.4.9}$$

As $\frac{dq_1^s(y,\gamma)}{dy} \geq 0$ and $(1 - I_{\{q_1^s(y,\gamma)>y\}}) \geq 0$, $\frac{dq_1^s(y,\gamma)}{dy} \cdot (1 - I_{\{q_1^s(y,\gamma)>y\}}) \geq 0$ holds.

By means of Equation (4.4.9), we investigate how the protection level under stock-out-based demand substitution is related to $\tilde{y}$, that is whether the firm should choose the same, a smaller or a greater protection level under stock-out-based demand substitution.

First, assume $y \leq \tilde{y}$. Then, $I_{\{q_1^s(\tilde{y},\gamma)>\tilde{y}\}} - I_{\{q_1>\tilde{y}\}} > 0$ as $q_1^s(\tilde{y},\gamma)$ is stochastically larger than $q_1(\gamma)$. Moreover, $I_{\{q_2(\gamma)>C-\tilde{y}\}} - I_{\{q_2(\gamma)>C-\tilde{y}\}} \leq 0$. The left-hand side of the Equation (4.4.9) is thus non-negative. Therefore, $y \leq \tilde{y}$ yields a contradiction.

As we can exclude $y \leq \tilde{y}$, the protection level under stock-out-based demand substitution is greater than the optimal protection level without stock-out-based demand substitution, i.e. $y > \tilde{y}$. If the firm protects the total available capacity anyway, additional stock-out-based demand substitution does not affect the protection level. This result is rather intuitive as stock-out-based substitution increases Class 1 demand and does not come with additional penalty costs. Another argument for this result is that the firm's actual overage costs for protecting capacity for the higher-value customer class decrease due to increasing demand substitution.

The above result also holds for $p_1 \cdot E[I_{\{q_1(\gamma)>\tilde{y}\}}] - p_2 \cdot E[I_{\{q_2(\gamma)>C-\tilde{y}\}}] = 0$. Hence, also the optimal protection level under stock-out-based demand substitution is greater than the corresponding protection level without stock-out-based demand substitution.

#### 4.4.4.2 Optimal Fences

The firm maximizes its expected profit by deciding about $y$ and $\gamma$. The optimality equation for $y$ is given above by Equation (4.4.6). Differentiating the expected profit with regard to

$\gamma$ yields

$$
\begin{aligned}
\frac{dE[\pi(y,\gamma)]}{d\gamma} =& p_2 \cdot E[\frac{dq_2(\gamma)}{d\gamma} \cdot I_{\{q_2(\gamma)\le C-y\}}] + p_1 \cdot E[\frac{dq_1^s(y,\gamma)}{d\gamma} \cdot I_{\{q_1^s(y,\gamma)\le y\}}] - \frac{dK(\gamma)}{d\gamma} \\
=& p_2 \cdot (p_1 - p_2) \cdot \mathbb{P}(NS) - p_1 \cdot (p_1 - p_2) \cdot \mathbb{P}(NS, q_1(\gamma) \le y) \\
& - p_1 \cdot (p_1 - p_2) \cdot (1-\alpha_1) \cdot (\mathbb{P}(S1, q_1^{S1} \le y) + \mathbb{P}(S12, q_1^{S12} \le y)) \\
& + \frac{K_0}{(\gamma + K_1)^2} \\
=& (p_1 - p_2) \cdot [p_2 \cdot \mathbb{P}(NS) - p_1 \cdot \mathbb{P}(NS, q_1(\gamma) \le y) \\
& - p_1 \cdot (1-\alpha_1) \cdot (\mathbb{P}(S1, q_1^{S1} \le y) + \mathbb{P}(S12, q_1^{S12} \le y)] + \frac{K_0}{(\gamma + K_1)^2}.
\end{aligned}
\tag{4.4.10}
$$

This first order derivative has some intuitive interpretation. Additional price-driven demand leakage has a positive marginal effect on the revenue from the lower-value product if $q_2(\gamma) \le C - y$. Regarding the higher-value product, demand leakage only has a marginal impact on the revenue if $q_1^s(y,\gamma) \le y$. Otherwise, demand leakage does not have a marginal impact as the firm is still able to sell the protection level $y$. If no stock-out-based substitution occurs (i.e., $q_2(\gamma) \le C - y$), the (negative) marginal impact equals $\frac{dq_1(\gamma)}{d\gamma} = -(p_1 - p_2)$. As soon as stock-out-based substitution occurs, demand leakage has a marginal impact of $\frac{dq_1(\gamma)}{d\gamma} + \alpha_1 \cdot \frac{dq_2(\gamma)}{d\gamma} = -(p_1 - p_2) + \alpha_1 \cdot (p_1 - p_2) = -(p_1 - p_2) \cdot (1-\alpha_1) \le 0$ on the firm's revenue. In these cases, the impact is lower compared to a situation without stock-out-based substitution. On the one hand, demand leakage still decreases Class 1 demand directly. However, with stock-out-based substitution, this effect is lower in absolute terms because some of the leaked Class 1 customers again re-substitute for the higher-value product. Finally, demand leakage has a marginal impact on the fencing costs.

Making use of $\mathbb{P}(S1, q_1^{S1} \le y) = \mathbb{P}(S1) - \mathbb{P}(S1, q_1^{S1} > y)$ and $\mathbb{P}(S12, q_1^{S12} \le y) = \mathbb{P}(S12) - \mathbb{P}(S12, q_1^{S12} > y)$ allows for terms to be rearranged in Equation (4.4.10). After rearrangement, terms in Equation (4.4.10) correspond to terms in Equation (4.4.7). Substituting Equation (4.4.7) into Equation (4.4.10) after rearrangement finally yields

$$
-(p_1 - p_2) \cdot (p_1 \cdot (1 - (\alpha_1 - \alpha_2) \cdot \mathbb{P}(S12, q_1^{S12} \le y)) - p_2) = -\frac{K_0}{(\gamma + K_1)^2} \tag{4.4.11}
$$

as the optimality equation for $\gamma$.

Compared to the setting without stock-out-based substitution, Equation (4.4.11) differs from Equation (4.3.25) with regard to the term in brackets on the left-hand side. Both optimality conditions are equivalent, if the stock-out-based substitution rates are identical across the customer classes (i.e., if $\alpha_1 = \alpha_2$). Therefore, the same optimal fencing decisions result for $\alpha_1 = \alpha_2$ with and without consideration of additional stock-out-based demand substitution. In this case, the unconstrained decision about the firm's fences is independent of the allocation decision. Sequential planning is optimal in this case and the optimal protection level follows from Equation (4.4.6).

A firm's decision on its fences and the allocation decision are interdependent if $\alpha_1 \neq \alpha_2$ as the left-hand side of the above optimality equation also depends on $y$ in this case. Both decisions should then be taken simultaneously in order to maximize the total expected profit. Taking both decisions sequentially would neglect their interaction.

In order to investigate the interaction of the firm's decisions, we rearrange terms in Equation (4.4.11), yielding

$$\mathbb{P}(S12, q_1^{S12} \leq y) = [(p_1 - p_2)^2 - \frac{K_0}{(\gamma + K_1)^2}] \cdot \frac{1}{p_1 \cdot (p_1 - p_2) \cdot (\alpha_1 - \alpha_2)}. \quad (4.4.12)$$

First, we investigate how the firm's decisions and the substitution rates affect the left-hand side of the optimality equation. $\mathbb{P}(S12, q_1^{S12} \leq y)$ denotes the probability that the two events $S12$ and $q_1^{S12} \leq y$ occur simultaneously. $\mathbb{P}(S12)$ increases in $y$ and is independent of $\gamma$, $\alpha_1$ and $\alpha_2$. $\mathbb{P}(q_1^{S12} \leq y)$ is increasing in $y$ and $\gamma$ and decreasing in $\alpha_1$ and $\alpha_2$ (if $D_2 > C - y$). Thus, $\mathbb{P}(S12, q_1^{S12} \leq y) = \mathbb{P}(q_1^{S12} \leq y | S12) \cdot \mathbb{P}(S12)$ increases in $\gamma$ and decreases in $\alpha_1$ and $\alpha_2$. As the two events $S12$ and $q_1^{S12} \leq y$ are not independent, the overall effect of $y$ on the common probability $\mathbb{P}(S12, q_1^{S12} \leq y)$ is not clear. The right-hand side of Equation (4.4.12) is independent of $y$, increases in $\gamma$, decreases in $\alpha_1$ and increases in $\alpha_2$.

The left-hand side of the above optimality equation may either increase or decrease with increasing protection levels while the right-hand side is not affected. For an increasing protection level, the firm will have to adapt its fencing decision so that Equation (4.4.12) holds again. As both sides are increasing in $\gamma$, no general statement can be made about whether the firm should define stricter or weaker fences in response to an increasing protection level. Responding to a greater protection level by setting weaker fences results in lower costs for fencing, higher price-driven demand leakage and finally higher stock-out-based demand substitution (for given substitution rates). Due to the greater protection level, a greater share of the increasing demand after stock-out-based demand substitution can be fulfilled. In contrast to weaker fences, the firm can potentially also set stricter fences. Fencing costs then increase. However, total substitution (i.e. price-driven demand leakage and stock-out-based demand substitution) decreases. Thus, $q_1(\gamma)$ increases directly compared to weaker fences. Thus, it boils down to a trade-off between the fencing costs and the total substitution effects and their impact on fulfillment of Class 1 demand given a greater protection level, respectively. Several parameters are involved in this trade-off as shown in Equation (4.4.12).

In Equation (4.4.12), both sides depend on $\gamma$. In the following, we aim to identify bounds for the optimal leakage level under stock-out-based demand substitution. Due to the complex second order derivatives, we make use of the fact that the left-hand side denotes a probability ($\in [0, 1]$). This allows for deriving conditions for the optimal choice of $\gamma$, which depend on the problem parameters. Identifying these conditions is particularly interesting as these conditions allow for comparison with the optimal inner leakage level $\gamma^*_{part}$ without stock-out-based demand substitution. First, we investigate the conditions for the right-hand side to be greater than zero.

**Condition 1:** $[(p_1 - p_2)^2 - \frac{K_0}{(\gamma+K_1)^2}] \cdot \frac{1}{p_1 \cdot (p_1-p_2) \cdot (\alpha_1-\alpha_2)} \geq 0$
This condition holds, if either both terms are non-negative or if both terms are non-positive. Whether the second term is positive or negative depends on the relation of $\alpha_1$ and $\alpha_2$. Thus, $[(p_1-p_2)^2 - \frac{K_0}{(\gamma+K_1)^2}] \cdot \frac{1}{p_1 \cdot (p_1-p_2) \cdot (\alpha_1-\alpha_2)}$ is non-negative, if either $\alpha_1 > \alpha_2$ and $(p_1-p_2)^2 \geq \frac{K_0}{(\gamma+K_1)^2}$ or if $\alpha_1 < \alpha_2$ and $(p_1 - p_2)^2 \leq \frac{K_0}{(\gamma+K_1)^2}$. These conditions result in either

$$\alpha_1 > \alpha_2, \quad \gamma \geq \frac{\sqrt{K_0}}{p_1 - p_2} - K_1 = \gamma^*_{part}, \tag{4.4.13}$$

or

$$\alpha_1 < \alpha_2, \quad \gamma \leq \frac{\sqrt{K_0}}{p_1 - p_2} - K_1 = \gamma^*_{part}. \tag{4.4.14}$$

The locally optimal fencing decision thus results in a lower optimal price-driven demand leakage compared to the previous setting without stock-out-based demand substitution, if $\alpha_1 < \alpha_2$. The firm should define stricter fences in this case. For $\alpha_1 < \alpha_2$, relatively more actual Class 2 customers request the higher-value product. The share of Class 1 customers who re-substitute is relatively low, therefore, it is beneficial for the firm to directly pre-empt stock-out-based demand substitution from Class 1 by limiting price-driven demand leakage. If $\alpha_1 > \alpha_2$, the firm should define weaker fences compared to the previous setting. Higher price-driven demand leakage is optimal in this case. In line with the previous argument, the firm can tolerate higher price-driven demand leakage in this case as a higher share of the more profitable Class 1 customers re-substitutes for the higher-value product. The risk of actually losing Class 1 demand is then lower.

**Condition 2:** $[(p_1 - p_2)^2 - \frac{K_0}{(\gamma+K_1)^2}] \cdot \frac{1}{p_1 \cdot (p_1-p_2) \cdot (\alpha_1-\alpha_2)} \leq 1$
In addition to the above first condition, the right-hand side in Equation (4.4.12) must also be less than or equal to one. In the following, we investigate the two cases discussed above, i.e., $\alpha_1 > \alpha_2$ and $\gamma \geq \frac{\sqrt{K_0}}{p_1-p_2} - K_1 = \gamma^*_{part}$ and $\alpha_1 < \alpha_2$ and $\gamma \leq \frac{\sqrt{K_0}}{p_1-p_2} - K_1 = \gamma^*_{part}$. These conditions ensure that the right-hand side is non-negative and must therefore also hold.

First, we consider the conditions stated in Equation (4.4.13): $\alpha_1 > \alpha_2$ and $\gamma \geq \frac{\sqrt{K_0}}{p_1-p_2} - K_1 = \gamma^*_{part}$. Rearranging terms in $[(p_1 - p_2)^2 - \frac{K_0}{(\gamma+K_1)^2}] \cdot \frac{1}{p_1 \cdot (p_1-p_2) \cdot (\alpha_1-\alpha_2)} \leq 1$ under these conditions yields

$$(p_1 - p_2) \cdot [p_1 - p_2 - p_1 \cdot (\alpha_1 - \alpha_2)] \leq \frac{K_0}{(\gamma + K_1)^2}. \tag{4.4.15}$$

Depending on the relationship of the prices and the stock-out-based substitution rates, the left-hand side in Equation (4.4.15) is either positive or negative. This allows for identification of additional conditions.

If $\alpha_1 - \alpha_2 < 1 - \frac{p_2}{p_1}$, then the left-hand side is positive. In this case, the above condition in Equation (4.4.15) results in

$$\gamma^*_{part} < \gamma \leq \sqrt{\frac{K_0}{(p_1 - p_2) \cdot [p_1 - p_2 - p_1 \cdot (\alpha_1 - \alpha_2)]}} - K_1. \tag{4.4.16}$$

The right-hand side is greater than $\gamma^*_{part}$ as $(p_1 - p_2) \cdot [p_1 - p_2 - p_1 \cdot (\alpha_1 - \alpha_2)] < (p_1 - p_2)^2$. The firm's fencing decision is thus in the interval $\gamma^*_{part} \leq \gamma \leq \sqrt{\frac{K_0}{(p_1-p_2)\cdot[p_1-p_2-p_1\cdot(\alpha_1-\alpha_2)]}} - K_1$ for $\alpha_1 - \alpha_2 < 1 - \frac{p_2}{p_1}$.

If $\alpha_1 - \alpha_2 > 1 - \frac{p_2}{p_1}$, then the left-hand side is negative. In this case, the above condition results in

$$(\gamma + K_1)^2 \geq 0 > \frac{K_0}{(p_1 - p_2) \cdot [p_1 - p_2 - p_1 \cdot (\alpha_1 - \alpha_2)]}. \tag{4.4.17}$$

As the left-hand side is non-negative, this condition always holds. Therefore, the firm's fencing decision results in a leakage level greater than $\gamma^*_{part}$ for $\alpha_1 - \alpha_2 > 1 - \frac{p_2}{p_1}$. Thus, the firm defines weaker fences.

For $\alpha_1 < \alpha_2$ and $\gamma \leq \frac{\sqrt{K_0}}{p_1 - p_2} - K_1 = \gamma^*_{part}$, rearranging terms in the above inequality in Equation (4.4.15) under these conditions yields

$$(p_1 - p_2) \cdot [p_1 - p_2 + p_1 \cdot (\alpha_2 - \alpha_1)] \geq \frac{K_0}{(\gamma + K_1)^2}. \tag{4.4.18}$$

The left-hand side in Equation (4.4.18) is positive as $\alpha_1 < \alpha_2$. This condition results in

$$\gamma^*_{part} > \gamma \geq \sqrt{\frac{K_0}{(p_1 - p_2) \cdot [p_1 - p_2 + p_1 \cdot (\alpha_2 - \alpha_1)]}} - K_1. \tag{4.4.19}$$

A firm's fencing decision is therefore in the interval $\sqrt{\frac{K_0}{(p_1-p_2)\cdot[p_1-p_2+p_1\cdot(\alpha_2-\alpha_1)]}} - K_1 \leq \gamma \leq \gamma^*_{part}$ if $\alpha_1 < \alpha_2$. In this case, the price-driven demand leakage resulting from the firm's fencing decision is less than $\gamma^*_{part}$ and bounded below. Table 4.3 summarizes the conditions for the choice of the fencing decision derived above.

To summarize, it is again optimal to take both decisions sequentially if $\alpha_1 = \alpha_2$, otherwise, the firm's decisions interact. By analyzing the optimality condition, we find that no general statement can be made with regard to how the firm should adapt its fencing decision in order to respond to changes in the allocation decisions. The interaction of both decisions depends on the sensitivity with regard to the decision variables of the terms involved in the optimality condition. However, we find that the resulting optimal leakage level under additional stock-out-based substitution is greater (smaller) than $\gamma^*_{part}$ if $\alpha_1 > \alpha_2$ ($\alpha_1 < \alpha_2$).

| Bounds for the optimal leakage level | Conditions |
|---|---|
| $\sqrt{\frac{K_0}{(p_1-p_2)\cdot[p_1-p_2+p_1\cdot(\alpha_2-\alpha_1)]}} - K_1 \leq \gamma < \gamma^*_{part}$ | $\alpha_1 < \alpha_2$ |
| $\gamma^*_{part}$ | $\alpha_1 = \alpha_2$ |
| $\sqrt{\frac{K_0}{[(p_1-p_2)\cdot[p_1-p_2-p_1\cdot(\alpha_1-\alpha_2)]]}} - K_1 > \gamma > \gamma^*_{part}$ | $0 < \alpha_1 - \alpha_2 < 1 - \frac{p_2}{p_1}$ |
| $\gamma > \gamma^*_{part}$ | $0 < 1 - \frac{p_2}{p_1} < \alpha_1 - \alpha_2$ |

Table 4.3: Bounds for the optimal fencing decisions under stock-out-based demand substitution

### 4.4.5 Analysis: Nested Allocation Planning

In the remainder of this section, we investigate the firm's optimization problem as stated in Equation (4.4.3). In contrast to Section 4.3, we only investigate the necessary first-order conditions for the setting where the firm decides simultaneously about fences and the nested protection level. For nested allocations, the standard model has already been shown not to be concave in $y$ in Section 2.2.1.1. Thus, the extended model, including price-driven demand leakage and stock-out-based demand substitution, will not be concave in the firm's decisions either.

Differentiating the expected profit in Equation (4.4.3) with regard to $y$ yields

$$\begin{aligned}\frac{dE[\pi(y,\gamma)]}{dy} =& - p_2 \cdot E[I_{\{q_2(\gamma)>C-y\}}] \\ &+ p_1 \cdot E[I_{\{q_1^s(y,\gamma)>y, q_2(\gamma)>C-y\}} + \frac{dq_1^s(y,\gamma)}{dy} \cdot I_{\{q_1^s(y,\gamma)\leq y, q_2(\gamma)>C-y\}}] \\ =& - p_2 \cdot \mathbb{P}(q_2(\gamma) > C-y) \\ &+ p_1 \cdot (\mathbb{P}(S1, q_1^{S1} > y) + \mathbb{P}(S12, q_1^{S12} > y)) \\ &+ p_1 \cdot (\alpha_1 \cdot \mathbb{P}(S1, q_1^{S1} \leq y) + \alpha_2 \cdot \mathbb{P}(S12, q_1^{S12} \leq y)). \end{aligned} \tag{4.4.20}$$

Equation (4.4.20) has some intuitive interpretation. The protection level $y$ only has a marginal impact on the firm's total expected profit if stock-out-based substitution occurs (i.e., if $q_2(\gamma) > C - y$). On the one hand, the firm then loses $p_2$ when increasing $y$. On the other hand, the firm gains $p_1$ if the resulting demand for the higher-value product (including stock-out-based substitution) exceeds the protection level. Then the firm can sell an additional unit to Class 1 customers. If the resulting Class 1 demand after stock-out-based substitution is less than $y$, the firm gains $p_1 \cdot \alpha_i$ $(i = 1, 2)$ as increasing the protection level decreases the available booking limit for Class 2 and thus increases stock-out-based substitution which again increases demand for Product 1. As demand for Product 1 does not exceed $y$, the additional demand caused by stock-out-based substitution can be fulfilled at price $p_1$.

The first order derivative with regard to $\gamma$ results in

$$\begin{aligned}
\frac{dE[\pi(y,\gamma)]}{d\gamma} =& p_2 \cdot E[\frac{dq_2(\gamma)}{d\gamma} \cdot I_{\{q_2(\gamma)\leq C-y\}}] + p_1 \cdot E[\frac{dq_1^s(y,\gamma)}{d\gamma} \cdot I_{\{q_2(\gamma)>C-y, q_1^s(y,\gamma)\leq y\}} \\
&- \frac{dq_2(\gamma)}{d\gamma} \cdot I_{\{q_2(\gamma)\leq C-y, C-q_2(\gamma)<q_1(y,\gamma)\}} + \frac{dq_1^s(y,\gamma)}{d\gamma} \cdot I_{\{q_2(\gamma)\leq C-y, C-q_2(\gamma)\geq q_1(\gamma)\}}] \\
&- \frac{dK(\gamma)}{d\gamma} \\
=& p_2 \cdot (p_1 - p_2) \cdot \mathbb{P}(NS) \\
&+ p_1 \cdot ((\mathbb{P}(S1, q_1^{S1} \leq y) + \mathbb{P}(S12, q_1^{S12} \leq y)) \cdot (\alpha_1 - 1) \cdot (p_1 - p_2)) \\
&- (p_1 - p_2) \cdot \mathbb{P}(NS, C - q_2(\gamma) < q_1(\gamma)) \\
&- (p_1 - p_2) \cdot \mathbb{P}(NS, C - q_2(\gamma) \geq q_1(\gamma)) + \frac{K_0}{(\gamma + K_1)^2} \\
=& - (p_1 - p_2) \cdot ((p_1 - p_2) \cdot \mathbb{P}(NS) + p_1 \cdot (1 - \alpha_1) \cdot (\mathbb{P}(S1, q_1^{S1} \leq y) \\
&+ \mathbb{P}(S12, q_1^{S12} \leq y))) + \frac{K_0}{(\gamma + K_1)^2}.
\end{aligned} \tag{4.4.21}$$

Allowing marginally higher price-based demand leakage yields additional Class 2 revenue if Class 2 demand does not exceed the booking limit (i.e., if $q_2(\gamma) \leq C - y$). If stock-out-based substitution occurs (i.e., if $q_2(\gamma) > C - y$) and the resulting Class 1 demand does not exceed the protection level $y$, the firm on the one hand loses revenue due to marginally lower Class 1 demand ($p_1 \cdot \frac{dq_1(\gamma)}{d\gamma}$). On the other hand, the firm gains additional Class 1 demand as the marginally increasing Class 2 demand cannot be fulfilled and partially increases Class 1 demand (by increasing stock-out-based substitution), which can still be fulfilled. If no stock-out-based substitution occurs, the marginal effects differ, depending on whether the total demand exceeds capacity or not. If so, higher demand leakage causes lost revenues as the demand for Product 1 decreases but at the same time increases demand for Product 2. The last term represents the marginal effect on the substitution costs.

Rearranging terms in Equation (4.4.20), equating them to zero and substituting the resulting optimality condition for $y$ into the optimality condition for $\gamma$ following from Equation (4.4.21) yields

$$-(p_1 - p_2) \cdot (p_1 - p_2 - p_1 \cdot (\alpha_1 - \alpha_2) \cdot \mathbb{P}(S12, q_1^{S12} \leq y)) = -\frac{K_0}{(\gamma + K_1)^2} \tag{4.4.22}$$

as the optimality condition for $\gamma$. This is identical to the optimality condition for the leakage level under a partitioned protection level stated in Equation (4.4.11). The main results obtained for a partitioned protection level discussed above therefore also hold for a nested protection level: the decisions should be taken sequentially (simultaneously) if $\alpha_1 = \alpha_2$ ($\alpha_1 \neq \alpha_2$), and no statement can be made with regard to the impact of value of the protection level on the optimal price-driven demand leakage. Based on the firm's optimal decisions in the setting without stock-out-based demand substitution and the analogies of the decisions in both settings, this result is rather intuitive. If the value of the protection level does not affect the firm's fencing decision for $\alpha_1 \neq \alpha_2$, the type of protection level should not affect the firm's fences either.

## 4.5 Numerical Illustration

In this section, we provide numerical illustrations for the two settings considered in the previous sections: price-driven demand leakage both without and with additional stock-out-based demand substitution. By providing and discussing numerical examples, we aim to highlight the impact of different parameters on a firm's optimal decisions and thus on the expected profit. First, we present numerical examples for the setting which only accounts for price-driven demand leakage. Afterwards, we discuss numerical examples for the setting with additional stock-out-based demand substitution. In all numerical examples, we assume a partitioned protection level.

All numerical examples have been evaluated via MS Excel. They were calculated on a personal computer with an Intel Core Duo 2.40GHz processor and 2GB RAM, operated by the Microsoft Windows 7 Professional system.

### 4.5.1 Price-Driven Demand Leakage

In this section, we illustrate numerical examples of a firm's optimal fencing and allocation decisions for the setting discussed in Section 4.3. First, we define a basic parameter set as the reference case for the following numerical illustrations, then we illustrate the firm's optimal decisions, as derived in Section 4.3, for different parameter scenarios based on the reference case. We compare the investigated setting with two customer classes with a setting where the firm does not differentiate prices among its customers at all. This allows for an illustration of how different parameters affect the value of price differentiation itself.

In the following, we assume a capacity of 100 units. For ease of computation and due to its wide application in the field of revenue management, we model customer demands $D_i$ using Poisson distributions (in contrast to Zhang et al. (2010) who model customer demands with uniform distributions). We consider $\lambda_1 = 40$ and $\lambda_2 = 80$ as the parameters of the respective Poisson distributions. As for the exogenous prices, we consider $p_1 = 20$ and $p_2 = 15$. We assume strictly convex fencing costs as defined in Section 4.3.2. As for the cost parameters, we assume $K_0 = 1,000$ and $K_1 = 15$. The fencing cost function follows from Equation (4.3.3) as $K(\gamma) = \frac{1,000}{\gamma+15}$. Table 4.4 summarizes the chosen values for the parameters in the reference case.

| **Parameter** | $C$ | $\lambda_1$ | $\lambda_2$ | $p_1$ | $p_2$ | $K_0$ | $K_1$ |
|---|---|---|---|---|---|---|---|
| **Value** | 100 | 40 | 80 | 20 | 15 | 1,000 | 15 |

Table 4.4: The reference case under price-driven demand leakage

Based on these parameters, we first show how a firm should actually take the optimal decisions following the approach in Section 4.3. For the given parameters, $\frac{K_0}{K_1^2} = 4.44$ and $(p_1 - p_2)^2 = 25$. Thus, $\frac{K_0}{K_1^2} \leq (p_1 - p_2)^2$ and therefore the optimal leakage level in this case results in $\gamma^* = 0$. A firm should thus define its fences such that no price-driven demand leakage occurs at all. From Equation (4.3.3), the resulting fencing costs amount to $K(0) = \frac{1{,}000}{15} = 66.67$. Based on the demand distributions resulting from the optimal price-driven demand leakage according to the demand model in Section 4.3.1, the firm determines the optimal protection level. The firm should reserve $y^* = \min(100, y_{part}^{LW}(0)) = 36$ units of capacity for Class 1 customers. $y_{part}^{LW}(0) = 36$ results from Equation (4.3.7). The maximum expected profit based on the demand distributions and the optimal protection level results in $E[\pi(y^*, \gamma^*)] = E[\pi(36, 0)] = 1,592$. The reference case reflects Case 2 from the analysis in Section 4.3.

In order to illustrate the firm's optimal decisions $(\gamma^*, y^*)$ under price-driven demand leakage, derived in Section 4.3, we determine the optimal decisions for several scenarios. Each parameter setting reflects one of the cases (except for the Cases 3 and 4, which have been shown to be never optimal) resulting from the Karush-Kuhn-Tucker conditions in Section 4.3. Table 4.5 shows the investigated scenarios and the resulting optimal decisions. The resulting

| **Scenario** | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| $p_1$ | 20 | 20 | 20 | 20 |
| $p_2$ | 15 | 15 | **18** | **18** |
| $\lambda_1$ | 40 | **120** | 40 | **120** |
| $\lambda_2$ | 80 | 80 | 80 | 80 |
| $K_0$ | 1,000 | 1,000 | 1,000 | 1,000 |
| $K_1$ | 15 | 15 | 15 | 15 |
| $\frac{K_0}{K_1^2}$ | 4.44 | 4.44 | 4.44 | 4.44 |
| $(p_1 - p_2)^2$ | 25 | 25 | 4 | 25 |
| $y^*$ | 36 | 100 | 33 | 96 |
| $\gamma^*$ | 0 | 0 | 0.81 | 1.50 |
| $E[\pi(y^*, \gamma^*)]$ | 1,592.30 | 1,930.87 | 1,789.44 | 1935.38 |

Table 4.5: Optimal decisions for different scenarios

optimal decisions are illustrated in Figure 4.3. The first scenario reflects the defined reference case discussed above. Increasing Class 1 demand to $\lambda_1 = 120$ results in the second scenario. In this case, it is still optimal to select perfect fences as $4.44 \leq 25$. However the firm protects the total available capacity for Class 1 customers. Thus, the second scenario reflects Case 5 from the analysis in Section 4.3. Increasing $p_2$ to 18 based on the reference case demonstrates a scenario where an inner solution is optimal. This reflects Case 1 in Section 4.3. The firm decides on a strictly positive leakage level $\gamma^* = \gamma_{part}^* = 0.81$ and the corresponding protection level $y^* = y_{part}^{LW}(0.81) = 30 < C$. Finally, the fourth scenario shows a case where the firm again protects the total available capacity and a leakage level $\gamma_C^* > \gamma_{part}^* = 0.81$ is optimal. This parameter setting refers to Case 6 from the analysis in Section 4.3.

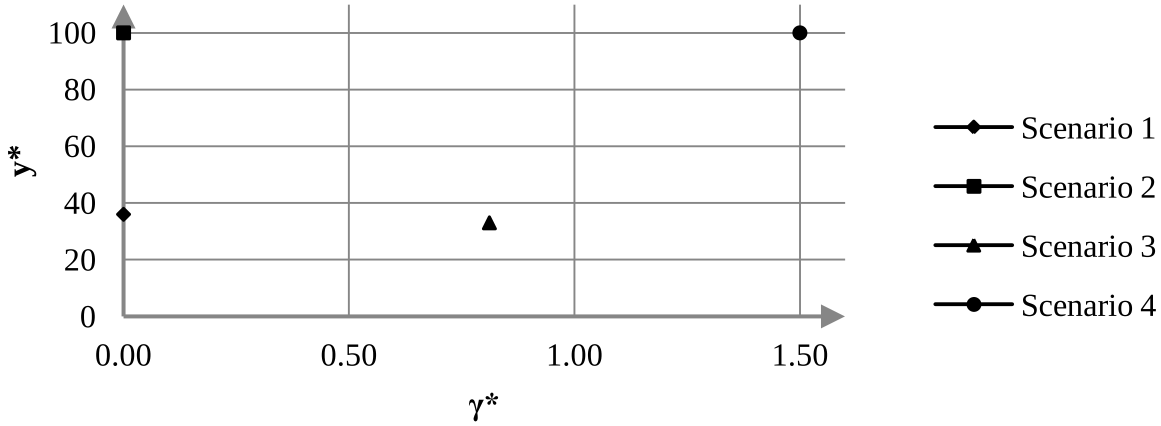


Figure 4.3: Optimal decisions for different parameter scenarios

In line with Zhang et al. (2010), we also compare the above two-segment setting with a setting where the firm does not differentiate at all. If the firm does not segment its customers, all customers receive the same service at the same price. We assume that the firm sets the price $p_{1+2} = p_2 = 15$ for all customers in this case. Alternatively, the firm could set some price $\tilde{p}$ such that $20 \geq \tilde{p} > 15$. However, expected demand from the lower-value class would decrease then as $\tilde{p}$ might be greater than the willingness-to-pay of at least some of the Class 2 customers who would request the lower-value product at $p_2 = 15$. We omit this by setting $p_{1+2} = p_2$. The total demand $D_{1+2}$ for the single customer segment thus equals the sum of the demands in the above discussed two-class settings. Therefore, $\lambda_{1+2} = 120$. When setting a single price for both products, fencing is not necessary,and so the firm does not incur costs for fencing. The expected profit at price $p_2$ equals $E[\pi_{1+2}] = p_2 \cdot E[min(C, D_{1+2})]$. For the parameters above, the firm's expected profit from a single customer segment equals $1,498$. Thus, price differentiation including fencing increases the expected profit by 6.28% for the reference setting discussed above.

In order to illustrate how the value of price differentiation depends on particular parameters, we vary customer heterogeneity in terms of price difference and a fencing cost parameter. In order to illustrate the impact of customer heterogeneity, we vary $p_2$. Table 4.6 shows the expected profit for the two-segment case, the firm's optimal allocation and fencing decisions, the expected profit in the single-segment setting, and the resulting value of price differentiation for varying prices $p_2$.

| $\mathbf{p_2}$ | **13** | **14** | **15** | **16** | **17** | **18** | **19** |
|---|---|---|---|---|---|---|---|
| $E[\pi(y^*, \gamma^*)]$ | 1,466.15 | 1,528.88 | 1,592.30 | 1,656.43 | 1,721.33 | 1,789.44 | 1,874.14 |
| $y^*$ | 38 | 37 | 36 | 35 | 34 | 33 | 16.62 |
| $\gamma^*$ | 0 | 0 | 0 | 0 | 0 | 0.81 | 18.73 |
| $E[\pi_{1+2}]$ | 1,298.40 | 1,398.28 | 1,498.15 | 1,598.03 | 1,697.91 | 1,797.78 | 1,897.66 |
| $(\frac{E[\pi(y^*,\gamma^*)]}{E[\pi_{1+2}]} - 1) \cdot 100\%$ | 12.92% | 9.34% | 6.28% | 3.65% | 1.38% | -0.46% | -1.24% |

Table 4.6: Value of price differentiation depending on customer heterogeneity

The value of price differentiation decreases with decreasing customer heterogeneity as shown in Table 4.6. It may even be more beneficial for a firm not to differentiate prices at all for low customer heterogeneity. In particular, for $p_2 \geq 18$, the expected profit under price differentiation is less than the expected profit for a single customer segment. With lower customer heterogeneity, the firm can weaken its fences in order to save fencing costs. Due to the low price differences, whether capacity is sold at $p_1$ or at $p_2$ does not make a big difference. Thus, the savings in fencing costs from not differentiating prices at all are greater than the additional expected benefit from price differentiation from some threshold price difference.

In order to show the impact of the fencing costs on the value of price differentiation, we vary the cost parameter $K_0$. This does not affect the expected profit when the firm does not differentiate its prices. Fencing is not necessary in this case. Thus, the expected profit from a single customer segment is constant and equals $E[\pi_{1+2}] = 1,498.15$. Table 4.7 shows the resulting expected profit for the two-segment case, the firm's optimal allocation and fencing decisions and the resulting value of price differentiation for varying values of $K_0$.

| $\mathbf{K_0}$ | **50** | **100** | **500** | **1,000** | **2,000** | **4,000** | **8,000** |
|---|---|---|---|---|---|---|---|
| $E[\pi(y^*, \gamma^*)]$ | 1,655.63 | 1,652.30 | 1,625.63 | 1,592.30 | 1,525.63 | 1,392.30 | 1,142.18 |
| $y^*$ | 36 | 36 | 36 | 36 | 36 | 36 | 22 |
| $\gamma^*$ | 0 | 0 | 0 | 0 | 0 | 0 | 2.89 |
| $E[\pi_{1+2}]$ | 1,498.15 | 1,498.15 | 1,498.15 | 1,498.15 | 1,498.15 | 1,498.15 | 1,498.15 |
| $(\frac{E[\pi(y^*,\gamma^*)]}{E[\pi_{1+2}]} - 1) \cdot 100\%$ | 10.51% | 10.29% | 8.51% | 6.28% | 1.83% | -7.07% | -23.76% |

Table 4.7: Value of price differentiation depending on $K_0$

The expected profit for two customer segments decreases with increasing $K_0$. As $E[\pi_{1+2}]$ is not affected, the value of price differentiation also decreases in $K_0$. For particularly high values of $K_0$ it is beneficial for the firm not to differentiate among its customers at all. Compared to the customer heterogeneity discussed above, the impact of the fencing costs on the benefit from price differentiation is larger for the examples illustrated. This is due to its direct impact in the objective function. Varying $K_0$ affects a firm's optimal decisions. While $\gamma^* = 0$ for $K_0 \leq 4,000$, it is optimal for the firm to allow price-driven demand leakage ($\gamma^* = 2.89$) for $K_0 = 8,000$.

The above results for varying values of $p_2$ and $K_0$ are in line with the results obtained by Zhang et al. (2010). He investigates the impact of demand uncertainty (under uniform demand distributions) on the firm's optimal decisions and also finds that not differentiating at all may be optimal in some parameter constellations.

### 4.5.2 Additional Stock-Out-Based Substitution

In this section, we present numerical examples for the setting discussed in Section 4.4. In this approach, we account for price-driven demand leakage and stock-out-based demand substitution. The basic setting from the previous section also serves as the reference case for the following numerical illustrations. In addition, we consider the stock-out-based substitution rates $\alpha_1 = 0.2$ and $\alpha_2 = 0.6$. Table 4.8 shows the resulting parameters in the reference case under additional stock-out-based demand substitution.

| **Parameter** | $C$ | $\lambda_1$ | $\lambda_2$ | $p_1$ | $p_2$ | $K_0$ | $K_1$ | $\alpha_1$ | $\alpha_2$ |
|---|---|---|---|---|---|---|---|---|---|
| **Value** | 100 | 40 | 80 | 20 | 15 | 1,000 | 15 | 0.2 | 0.6 |

Table 4.8: The reference case under additional stock-out-based substitution

First, we compare the results for the reference case with and without stock-out-based demand substitution. As above, we determine the optimal decisions according to the optimality conditions under stock-out-based substitution or equivalently by maximising the firm's expected profit. For the given parameters, the firm's optimal decisions are $\gamma^* = 0$ and $y^* = 75$. Thus, the firm should reserve a larger share of the total available capacity for Class 1 customers under stock-out-based demand substitution. As discussed above, this is intuitive as the resulting Class 1 demand under stock-out-based substitution stochastically dominates Class 1 demand under pure consideration of price-driven demand leakage. The resulting total expected profit is $E[\pi(75, 0)] = 1,706$. The firm's expected profit under stock-out-based demand substitution is larger than the expected profit under pure price-driven demand leakage. The expected profit from Class 2 equals $E[R_2(75, 0)] = 375$ and is thus lower compared to the previous setting. Although the expected underage decreases, the expected sales decrease due to the lower available booking limit. Class 1 revenue increases however, due to stock-out-based demand substitution and equals $E[R_1(75, 0)] = 1,398$. The firm benefits from additional Class 1 demand compared to the previous setting due to the higher protection level. The second effect overcompensates for the first effect. Table 4.9 summarizes a comparison of the firm's optimal decisions and the resulting expected profits with and without stock-out-based demand substitution.

| | $\gamma^*$ | $\mathbf{y}^*$ | $E[\pi(y^*, \gamma^*)]$ | $E[R_1(y^*, \gamma^*)]$ | $E[R_2(y^*, \gamma^*)]$ |
|---|---|---|---|---|---|
| Without stock-out-based substitution | 0 | 36 | 1,592 | 700 | 958 |
| With stock-out-based substitution | 0 | 75 | 1,706 | 1,398 | 375 |

Table 4.9: Comparison of the results for the reference case with and without stock-out-based substitution

As for the setting without stock-out-based demand substitution, we vary $p_2$ in order to show the impact of customer heterogeneity. Table 4.10 shows the resulting expected profit for the two-segment case, the firm's optimal allocation and fencing decisions, the expected profit in the single-segment setting and the resulting value of price differentiation for varying prices $p_2$.

Again, the value of price differentiation decreases with decreasing customer heterogeneity as shown in Table 4.6. Compared to the setting where only price-driven demand leakage is

| $\mathbf{p_2}$ | **13** | **14** | **15** | **16** | **17** | **18** | **19** |
|---|---|---|---|---|---|---|---|
| $E[\pi(y^*, \gamma^*)]$ | 1,677.57 | 1,686.02 | 1,705.90 | 1,734.55 | 1,770.96 | 1,823.02 | 1,898.16 |
| $y^*$ | 99 | 86 | 75 | 67 | 60 | 45 | 27 |
| $\gamma^*$ | 0 | 0 | 0 | 0 | 0 | 7.65 | 32.10 |
| $E[\pi_{1+2}]$ | 1,298.40 | 1,398.28 | 1,498.15 | 1,598.03 | 1,697.91 | 1,797.78 | 1,897.66 |
| $(\frac{E[\pi(y^*,\gamma^*)]}{E[\pi_{1+2}]} - 1) \cdot 100\%$ | 29.20% | 20.58% | 13.87% | 8.54% | 4.30% | 1.40% | 0.03% |

Table 4.10: Value of price differentiation depending on customer heterogeneity under stock-out-based demand substitution

considered, price differentiation under stock-out-based demand substitution outperforms a single customer segment in all the numerical examples considered. Under additional stock-out-based demand substitution, Class 1 demand is stochastically larger than without consideration of stock-out-based demand substitution. The firm protects a higher share of the total capacity for higher-value customer demands compared to the previous setting without stock-out-based demand substitution. This confirms the general result in Section 4.4, that the protection level under stock-out-based demand substitution is greater than the protection level without stock-out-based demand substitution. The firm therefore also benefits from stock-out-based demand substitution.

In order to compare the impact of the fencing cost parameters, we also again vary the cost parameter $K_0$. Varying $K_0$ does not affect the expected profit when the firm does not differentiate its prices. Thus, the expected profit in this case again equals $E[\pi_{1+2}] = 1,498.15$, as in the previous setting. Table 4.11 shows the resulting expected profit for the two-segment case, the firm's optimal allocation and fencing decisions and the resulting value of price differentiation for varying values of $K_0$.

| $\mathbf{K_0}$ | **50** | **100** | **500** | **1,000** | **2,000** | **4,000** | **8,000** |
|---|---|---|---|---|---|---|---|
| $E[\pi(y^*, \gamma^*)]$ | 1,769.23 | 1,765.90 | 1,739.23 | 1,705.90 | 1,639.23 | 1,513.62 | 1,326.81 |
| $y^*$ | 75 | 75 | 75 | 75 | 75 | 68 | 55 |
| $\gamma^*$ | 0 | 0 | 0 | 0 | 0 | 3.05 | 8 |
| $(\frac{E[\pi(y^*,\gamma^*)]}{E[\pi_{1+2}]} - 1) \cdot 100\%$ | 18.09% | 17.87% | 16.09% | 13.87% | 9.42% | 1.03% | -11.44% |

Table 4.11: Value of price differentiation depending on $K_0$ under stock-out-based demand substitution

The expected profit for two customer segments decreases with increasing values for $K_0$. As $E[\pi_{1+2}]$ is not affected, the value of price differentiation also decreases in $K_0$ as under pure price-driven demand leakage. The benefit through price differentiation under stock-out-based demand substitution is higher compared to the previous setting without stock-out-based demand substitution. Again, for high values of $K_0$, it is beneficial for the firm not to differentiate between its customers at all.

Based on one of the examples in Table 4.11, we also show the importance of taking both decisions simultaneously instead of taking them in a hierarchical manner. As outlined in Section 4.3, taking both decisions hierarchically results in $\gamma^* = \max(\gamma^*_{part}, 0)$. The optimal protection level under stock-out-based demand substitution follows from Equation (4.4.6).

Table 4.12 shows the value of simultaneous decision making for different values of $K_0$. $y^*$ and $\gamma^*$ denote the optimal simultaneous decisions while $\hat{y}^*$ and $\hat{\gamma}^*$ denote the optimal values of the decision variables when taking both decisions hierarchically. For example, consider the

| $\mathbf{K_0}$ | **50** | **100** | **500** | **1,000** | **2,000** | **4,000** | **8,000** |
|---|---|---|---|---|---|---|---|
| $E[\pi(y^*,\gamma^*)]$ | 1,769.23 | 1,765.90 | 1,739.23 | 1,705.90 | 1,639.23 | 1,513.62 | 1,326.81 |
| $y^*$ | 75 | 75 | 75 | 75 | 75 | 68 | 55 |
| $\gamma^*$ | 0 | 0 | 0 | 0 | 0 | 3.05 | 8 |
| $E[\pi(\hat{y}^*,\hat{\gamma}^*)]$ | 1,769.23 | 1,765.90 | 1,739.23 | 1,705.90 | 1,639.23 | 1,506.00 | 1,290.00 |
| $\hat{y}^*$ | 75 | 75 | 75 | 75 | 75 | 75 | 68 |
| $\hat{\gamma}^*$ | 0 | 0 | 0 | 0 | 0 | 0 | 2.89 |
| $(\frac{E[\pi(y^*,\gamma^*)]}{E[\pi(\hat{y}^*,\hat{\gamma}^*)]} - 1) \cdot 100\%$ | 0% | 0% | 0% | 0% | 0% | 0.51% | 2.85% |

Table 4.12: Value of simultaneous decision making

scenario with $K_0 = 4,000$. For the parameters given, $\hat{\gamma}^* = \max(\frac{\sqrt{4,000}}{20-15} - 15, 0) = 0$ follows as the optimal hierarchical leakage level. The optimal protection level for $\hat{\gamma}^* = 0$ results in $\hat{y}^* = 75$ and yields a total expected profit of 1,506. Thus, an additional 0.51% can be gained by simultaneously deciding on the fences and the protection level. The benefit from simultaneous decision making increases to 2.85% for $K_0 = 8,000$.

In order to illustrate the impact of customer heterogeneity with regard to the stock-out-based demand substitution rates, we vary $\alpha_1$ and $\alpha_2$. Table 4.13 shows the resulting expected profit for the two-segment case, the firm's optimal allocation and fencing decisions and the resulting value of price differentiation for the resulting combinations of $\alpha_1$ and $\alpha_2$.

The expected profit under stock-out-based demand substitution increases with increasing values of $\alpha_2$ for a given value of $\alpha_1$. The same holds for the benefit of price differentiation. This can be explained as follows. With increasing values of $\alpha_2$, the share of customers substituting for the higher-value product increases. The firm accounts for increasing stock-out-based demand substitution by further limiting the booking limit. In doing so, most of the customers requesting the lower-value product face a stock-out and re-substitute. The firm will therefore sell a greater share of its total available capacity at a higher price.

For a given value of $\alpha_2$, both the expected profit and the value of price differentiation are non-decreasing in $\alpha_1$. Perfect fences are optimal in all scenarios with $\alpha_1 + \alpha_2 \leq 0.8$. In these cases, the expected profit equals the expected profit without stock-out-based demand substitution. Due to the perfect fences, the firm does not incur price-driven demand leakage, therefore, stock-out-based demand substitution from leaked Class 1 customers does not occur either. Thus, the expected profit shown in the first column is completely independent of $\alpha_1$. As soon as $\alpha_1 + \alpha_2 > 0.8$, the expected profit increases in $\alpha_1$ for given $\alpha_2$.

The results in Table 4.13 also show that no general statement can be made with regard to the impact of the parameters on the firm's optimal decisions, as shown in the analysis in Section 4.4. Consider, for example, the two scenarios $\alpha_1 = 0.6$ and $\alpha_2 = 0.2$ and $\alpha_1 = 0.6$ and $\alpha_2 = 0.4$. With the increasing value of $\alpha_1$, the optimal protection level slightly decreases for $\alpha_2 = 0.2$ and increases for $\alpha_2 = 0.4$. However, the optimal leakage level increases for

| $\boldsymbol{\alpha_1}$ | **0** | | | | |
|---|---|---|---|---|---|
| $\boldsymbol{\alpha_2}$ | **0** | **0.2** | **0.4** | **0.6** | **0.8** |
| $y^*$ | 36 | 41 | 50 | 75 | 100 |
| $\gamma^*$ | 0 | 0 | 0 | 0 | 0 |
| $E[\pi(y^*, \gamma^*)]$ | 1,592.30 | 1,610.08 | 1,637.71 | 1,705.90 | 1,869.72 |
| $(\frac{E[\pi(y^*,\gamma^*)]}{E[\pi_{1+2}]} - 1) \cdot 100\%$ | 6.28% | 7.47% | 9.32% | 13.87% | 24.80% |
| $\boldsymbol{\alpha_1}$ | **0.2** | | | | |
| $\boldsymbol{\alpha_2}$ | **0** | **0.2** | **0.4** | **0.6** | **0.8** |
| $y^*$ | 36 | 41 | 50 | 75 | 100 |
| $\gamma^*$ | 0 | 0 | 0 | 0 | 8 |
| $E[\pi(y^*, \gamma^*)]$ | 1,592.30 | 1,610.08 | 1,637.71 | 1,705.90 | 1,887.26 |
| $(\frac{E[\pi(y^*,\gamma^*)]}{E[\pi_{1+2}]} - 1) \cdot 100\%$ | 6.28% | 7.47% | 9.32% | 13.87% | 25.97% |
| $\boldsymbol{\alpha_1}$ | **0.4** | | | | |
| $\boldsymbol{\alpha_2}$ | **0** | **0.2** | **0.4** | **0.6** | **0.8** |
| $y^*$ | 36 | 41 | 50 | 76 | 100 |
| $\gamma^*$ | 0 | 0 | 0 | 8 | 8 |
| $E[\pi(y^*, \gamma^*)]$ | 1,592.30 | 1,610.08 | 1,637.71 | 1,723.70 | 1,929.33 |
| $(\frac{E[\pi(y^*,\gamma^*)]}{E[\pi_{1+2}]} - 1) \cdot 100\%$ | 6.28% | 7.47% | 9.32% | 15.06% | 28.78% |
| $\boldsymbol{\alpha_1}$ | **0.6** | | | | |
| $\boldsymbol{\alpha_2}$ | **0** | **0.2** | **0.4** | **0.6** | **0.8** |
| $y^*$ | 36 | 41 | 49 | 100 | 100 |
| $\gamma^*$ | 0 | 0 | 8 | 8 | 8 |
| $E[\pi(y^*, \gamma^*)]$ | 1,592.30 | 1,610.08 | 1,657.96 | 1,816.34 | 1,948.27 |
| $(\frac{E[\pi(y^*,\gamma^*)]}{E[\pi_{1+2}]} - 1) \cdot 100\%$ | 6.28% | 7.47% | 10.67% | 21.24% | 30.05% |
| $\boldsymbol{\alpha_1}$ | **0.8** | | | | |
| $\boldsymbol{\alpha_2}$ | **0** | **0.2** | **0.4** | **0.6** | **0.8** |
| $y^*$ | 36 | 40 | 62 | 100 | 100 |
| $\gamma^*$ | 0 | 8 | 8 | 8 | 8 |
| $E[\pi(y^*, \gamma^*)]$ | 1,592.30 | 1,632.78 | 1,718.70 | 1,895.32 | 1,954.55 |
| $(\frac{E[\pi(y^*,\gamma^*)]}{E[\pi_{1+2}]} - 1) \cdot 100\%$ | 6.28% | 8.99% | 14.72% | 26.51% | 30.46% |

Table 4.13: Value of price differentiation depending on $\alpha_1$ and $\alpha_2$ under stock-out-based demand substitution

$\alpha_2 = 0.2$ and stays constant for $\alpha_2 = 0.4$.

To summarize, the numerical examples show the firm's optimal decisions and how several parameters affect these optimal decisions. For the chosen reference setting, both decisions can be taken sequentially without stock-out-based demand substitution. Under stock-out-based demand substitution, the expected profit can be maximized by using a non-linear solver in MS Excel. The considered parameters affect the expected profit in the same way in the two settings considered both with and without stock-out-based demand substitution. The expected profits in each setting increase with decreasing customer heterogeneity and decrease with increasing fencing costs. A firm benefits from additional stock-out-based demand substitution by (in most of the considered cases) protecting a higher share of capacity for the higher-value customer class. In this way, a firm ensures the sale of a high share of capacity at the higher price. Regarding the interface to customer segmentation, we provide numerical examples for price differentiation resulting in a lower expected profit compared to a single segment. Depending on the parameters, price differentiation is therefore not necessarily optimal at all.

## 4.6 Conclusion

In this chapter, we have investigated the interaction of a firm's fencing decision with the firm's capacity allocation decision. Typically, the fencing decision is taken prior to the allocation decision and updated less regularly. Thus, both decisions are taken in a hierarchical manner, that is allocations are determined for given fences. Fences aim to limit price-driven demand leakage. Without fences, customers with a high willingness-to-pay might have the motivation to request the lower-value product and the firm loses revenue.

Price-driven demand leakage and the firm's related decision about its fences have already been considered in the field of inventory management. In these approaches, the firm decides on the order quantities for the different customer classes in a newsvendor setting. Thus, capacity is flexible and the firm does not face an allocation problem. In revenue management research, price-driven demand leakage has not yet been considered explicitly. We combine both streams by integrating the customer demand model including price-driven demand leakage and thus the firm's fencing decision into a revenue management model.

We build our modeling approach on Littlewood's model. Our analysis is divided into two parts: first, we explicitly consider the impact of price-driven demand leakage on customer demand, building on the approach by Zhang et al. (2010). We investigate the firm's decision problem analytically and find that it is optimal for the firm (at least in practically relevant settings) to take both decisions sequentially. The optimal fencing decision then yields a leakage level which is constant for given problem parameters (prices and fencing cost parameters) and independent of the allocation decision. The optimal leakage level only depends on the customer heterogeneity with regard to the willingness-to-pay and the costs of fencing. For given fences, Littlewood's rule yields the optimal allocation decision for both partitioned and nested allocations.

The results are mainly driven by three assumptions: fencing has a deterministic effect on customer demands, all leaked Class 1 customers request the lower-value product and customer demands are independent of the allocation decision. The first assumption simplifies the analysis but does not affect the results structurally as long as price-driven demand leakage is a fixed amount and does not depend on demand realizations. When relaxing the second assumption, the properties of the optimization problem are preserved, that is the properties of the optimal decisions do not change. In order to account for the impact of the allocation decision on customer demands, we consider additional stock-out-based demand substitution in a second step. In particular, we modify the customer demands to include demand substitution in the case of a stock-out of the lower-value product.

Under stock-out-based demand substitution, the firm should c.p. protect more capacity for the higher-value customer class, compared to the setting without stock-out-based demand substitution. Class 1 demand under additional stock-out-based demand substitution is stochastically larger as some of the leaked Class 1 customers re-substitute for the higher-value product in case of a stock-out. The firm anticipates this substitution and therefore increases the protection level. Thus, the optimal protection level increases in the stock-out-based demand substitution rates.

When deciding on both its fences and the protection level simultaneously, a firm's decisions can be taken sequentially if the stock-out-based substitution rates are the same across the two customer classes. The unconstrained optimal fencing decision is identical to the fencing decision without stock-out-based substitution in this case. If stock-out-based substitution rates differ across the customer classes, both decisions interact and it is beneficial for the firm to take both decisions simultaneously. Compared to the setting without stock-out-based demand substitution, the firm should set weaker fences and thus allow more price-driven demand leakage if the Class 1 substitution rate is greater than the Class 2 substitution rate. If the relationship between the substitution rates is reversed, the firm should define stricter fences.

We illustrate our results numerically by means of several examples. Our analytical results are confirmed in the investigated examples. We show that price differentiation itself is not necessarily beneficial for a firm, depending on customer heterogeneity and the fencing costs. In particular, price differentiation is not profitable for low customer heterogeneity and high fencing costs. In this case a firm should serve a single customer class only.

For stock-out-based demand substitution, we illustrate the benefit of simultaneous optimization compared to taking hierarchical decisions on fences and the protection level. We show the impact of the substitution rates on the firm's optimal decisions and the value of price differentiation by comparing the resulting expected profits to the expected profit from a single customer segment. With stock-out-based demand substitution, the value of price differentiation is substantial. Comparing both the value of price differentiation and the value of simultaneous decision making yields that the value of price differentiation is significantly higher (at least for the examples considered) than the value of taking both decisions simultaneously. Thus, a large share of the overall benefit can already been gained by differentiating prices.

We consider a static decision problem where both decisions are taken once, at the beginning of the booking horizon. As discussed above, this is a simplification, which allows for investigating the setting in an analytically tractable way. In practical applications however, the allocation decision is typically taken dynamically while the fences are determined once for the entire booking horizon. Accounting for dynamic decisions therefore represents a practice-oriented model extension. Modeling dynamic allocation decisions affects the modeling approach. While fences are still a single decision variable for the entire booking horizon, the booking horizon must be divided into several time steps in order to account for dynamic allocation decisions. In each time step, the firm updates its allocation decision. Such a setting could be solved by, for example, a stochastic dynamic programming approach. Compared to our approach, complexity is increased, however, by taking dynamic decisions, the firm will also benefit.

# Chapter 5

# Conclusions and Further Research

Within this thesis, we have investigated the interaction of other planning tasks with short-term revenue management decisions. Revenue management decisions are applied on an operational level and aim to manage demand for a given fixed supply so that the firm maximizes its (expected) profit.

We have seen that short-term revenue management decisions are affected by many other, also longer-term, planning tasks. These decisions potentially impact the profitability of applying revenue management. In order to take optimal decisions, a firm needs to consider multiple planning tasks on different decision hierarchies simultaneously. This thesis has brought forward new quantitative approaches, which consider the integration of customer relationship management and fencing decisions with short-term capacity allocation decisions. In this concluding chapter, we summarize the main results of our research. We also discuss several potential areas for future research.

## 5.1 Results

In this section, we summarize and discuss the key results of this thesis, and therefore return to the research questions posed in Chapter 1.

RQ1: *Which planning tasks have an impact on short-term revenue management decisions across different industries?*
The literature overview in Chapter 2 has shown that several planning tasks, on different hierarchical levels, affect short-term revenue management decisions. When taking account of revenue management and another related planning task simultaneously, a firm's decisions change, compared to taking both decisions sequentially, in most of the cases. However, sequential heuristics also yield very good results in some cases.

The relevant related planning tasks (1) differ across the industries considered; and (2) the same related planning tasks may be on different hierarchical levels in the different industries. For example, scheduling (or routing) decisions are relevant in the manufacturing and e-fulfillment context and also affect the available capacity on a short-term basis. This is not the case for traditional airline or hotel applications. An example of the same planning task on different hierarchical levels is route planning. While route planning is a rather long-term planning task in the airline industry, it is typically performed on a very short-term basis after the order acceptance decision in an e-fulfillment context.

Based on these findings, we review and evaluate existing frameworks for revenue management and related planning tasks. The currently available frameworks are either tailored to a specific industry or have a rather narrow focus on largely marketing-related decisions. In most cases, operations-related planning tasks are not considered. This particularly holds for short-term operations-related planning tasks (such as scheduling decisions) which are prevalent in manufacturing or e-fulfillment. Planning tasks from the field of customer relationship management are only rarely considered.

We discuss whether a single framework can actually cover all relevant planning tasks across all industries. In general, the trade-off is between the extent to which industry-dependent characteristics are considered within a framework and its generalizability. Due to the significantly different role of marketing-related planning tasks in quantity-based and price-based revenue management applications, we separate these two fields. For both types of revenue management applications, we conclude that rather general frameworks are able to describe and to structure the relevant planning tasks on different hierarchical levels.

While operations-related decisions affect a firm's available capacity on different hierarchical levels, marketing-related decisions determine the service (or product) offer itself and revenue management decisions. In contrast to existing frameworks, we additionally consider the impact of the rather longer-term focus of customer relationship management on other marketing-related planning tasks.

RQ2: *How can other planning hierarchies and tasks be captured in quantitative revenue management models in order to support coordinated decision making?*
In Chapter 3 and 4 we investigated the interplay of a firm's capacity allocation decision with two other related planning tasks: customer relationship management and fencing. Both planning tasks typically have a longer-term character compared to short-term allocation decisions.

As for the integration of revenue management with customer relationship management, we do not explicitly model typical customer relationship management decisions but assume that customer demand is endogenous and depends on the firm's allocation decisions. Customers adapt their repurchase probabilities in subsequent booking periods, depending on whether their current request has been accepted or rejected. Following the approach to customer relationship management, a firm maximizes the total expected profit within the complete planning horizon instead of maximizing the respective expected profits within the single booking periods as in a pure revenue management approach.

In Chapter 4 we investigated the interrelationship of a firm's medium-term fencing decision with its short-term allocation decision. Again, we built our approach on Littlewood's two-class model. In a generalized approach, we do not explicitly model particular fences. We instead model the firm's fencing decision implicitly by accounting for the effect of the firm's fencing decision on price-driven demand leakage. By setting appropriate fences, the firm typically aims to prevent price-driven demand leakage. However, this comes at a cost for managing and enforcing a potentially complex system of fences. As a result, the firm trades off the marginal impact of stricter fences and the resulting marginal costs for setting stricter fences.

Building on the setting where we only consider price-driven demand leakage, we additionally present an extension where we account for additional stock-out-based demand substitution.

RQ3: *How do short-term revenue management decisions change when interrelated planning tasks are considered additionally?*
As for the integration of revenue management and customer relationship management, we show by marginal analysis for the two-period setting that a booking limit policy is not necessarily optimal any more. Thus, the resulting optimal allocation policy differs from standard allocation planning models for a single booking period. This is because a firm's current allocation decision affects future customer demand. After having rejected several requests, rejecting another request can be too expensive so that firms start to accept requests again.

Within the numerical study, we investigate the firm's allocation strategy over time. Here, we found that a static booking limit policy throughout the entire planning horizon performs considerably worse than all other allocation policies. While regularly updating the myopic booking limit yields good results, accounting for the effects of the firm's allocation policy on future Class 1 demand performs very close to optimal in almost all scenarios. We find that the protection level increases over time and approaches the total available capacity as soon as Class 1 customers react slightly positively towards their requests being accepted. Class 2 demand effects only affect the firm's allocation policy if this is not the case. Then, the protection level decreases slightly over time approaching some value.

As for the integration of revenue management and a firm's fencing decision, we derive a firm's optimal decisions under price-driven demand leakage analytically for partitioned allocations. We find that both decisions can be taken independently. Thus, sequential decision-making is optimal in this case. The firm can first determine its optimal fencing decision and then the optimal protection level based on the resulting demand. This is also true for nested allocations.

The main result under price-driven demand leakage, that a firm can take both decisions sequentially, does not necessarily hold when additional stock-out-based demand substitution is considered. Customer demands then also depend on the firm's allocation decision. As a result, both the allocation and fencing decision depend on each other if the stock-out-based substitution rates differ across customer classes. They should thus be taken simultaneously. If the stock-out-based substitution rates are the same, sequential decisions are again optimal. By investigating the firm's trade-offs from the first order optimality conditions, we show how different parameters affect the optimal decisions. Compared to the setting without stock-out-based demand substitution, a firm should set weaker (stricter) fences, if the Class 1 substitution rate is larger (smaller) than the Class 2 substitution rate.

By means of numerical examples we show that price differentiation itself is not necessarily beneficial for a firm, depending on the fencing costs and customer heterogeneity. For high fencing costs and a low degree of heterogeneity, the firm should not differentiate prices at all.

## 5.2 Further Research

Based on the literature overview in Chapter 2 and the little research identified in some areas of interaction with other decisions, we see interesting opportunities for further research.

The literature overview and the frameworks derived in Chapter 2 are based on four different revenue management applications or industries. Considering other additional industries would further strengthen the generalizability of our approach and potentially provide additional insights into relevant planning tasks.

The literature overview in Chapter 2, in combination with the more detailed overview in Chapter 4, has shown that there is only little research into the interface of customer segmentation and revenue management. In particular, none of the existing approaches deals with the question of the optimal segmentation of a firm's customer base in combination with revenue management. In the approach of Meyr (2008), increasing the number of segments results in a better segmentation. This result is in line with general results from cluster analyses. However, a larger number of customer segments may not necessarily be beneficial for the firm. In line with the underlying notion of Chapter 4, increasing the number of customer segments at the same time increases the complexity of managing the customer base, and the potential effects among the demands of the customer segments due to the decreasing differences of the segments. Accounting for such effects in a multi-class setting would result in a counter-effect to the benefits from service differentiation. When deciding on the optimal customer segmentation, the firm would thus trade off the benefits from an additional customer segment with the resulting additional costs.

In both Chapters 3 and 4, we investigate monopoly settings, however, it would be interesting to investigate the effects of competition on the firm's optimal decisions. Consider, for example, a firm's fencing decision. If demand leakage and substitution also depend on a competitor's decisions, the fencing strategy in the equilibrium (if it exists) might be different.

We have mainly investigated static decisions throughout the analytical parts of this thesis. Static decisions are taken only at the beginning of the planning horizon. This assumption simplifies derivation of the firm's optimal decisions but does not reflect real-world applications. Here, optimal decisions may be derived based on a static model but the decisions are typically adapted frequently (and thus applied dynamically) throughout the booking horizon, in order to account for the available information (e.g., the already realized demand) at different points in time. Thus, investigating either optimal dynamic decisions or the performance of static decisions when applied dynamically would be interesting.

Finally, we investigated theoretical models in Chapters 3 and 4 but did not investigate the underlying modeling assumptions empirically. The core idea behind them both is to integrate richer models of customer demand. It would be interesting to investigate customer demands empirically within the settings we studied in order to see how customers actually react, for example, towards their order being accepted or rejected. The same holds, for example, for the fencing costs assumed in Chapter 4.

Considering the above issues in this thesis, it is clear that revenue management interacts with a firm's other, also longer-term, decisions. This thesis has investigated these interactions, and can hopefully contribute to a broader view on revenue management.

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