Discussion Paper No. 15-020

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Willi Semmler[†a,b] and Christian R. Proaño[a,c]

[a]The New School of Social Research, New York
[b]Zentrum für Europäische Wirtschaftsforschung (ZEW), Mannheim
[c]Macroeconomic Policy Institute (IMK), Düsseldorf

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Abstract

The recent financial and sovereign debt crises around the world have sparked a growing literature on models and empirical estimates of defaultable debt. Frequently households and firms come under default threat, local governments can default, and recently sovereign default threats were eminent for Greece and Spain 2012-13. Moreover, Argentina experienced an actual default in 2001. What causes sovereign default risk, and what are the escape routes from default risk? Previous studies such as Arellano (2008), Roch and Uhlig (2013) and Arellano et al. (2014) have provided theoretical models to explore the main dynamics of sovereign defaults. These models can be characterized as threshold models in which there is a convergence toward a good no-default equilibrium below the threshold and a default equilibrium above the threshold. However, in these models aggregate output is exogenous, so that important macroeconomic feedback effects are not taken into account. In this paper, we 1) propose alternative model variants suitable for certain types of countries in the EU where aggregate output is endogenously determined and where financial stress plays a key role, 2) show how these model variants can be solved through the Nonlinear Model Predictive Control numerical technique, and 3) present some empirical evidence on the nonlinear dynamics of output, sovereign debt and financial stress in some euro area and other industrialized countries.

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Keywords: Sovereign default risk, financial stress, macroeconomic dynamics, euro crisis
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†Corresponding author. E-mail: semmlerw@newschool.edu.

1 Introduction

As it is well known, the sharp increase in sovereign default risk premia of several euro area countries (in particular Greece, Portugal, Spain and Ireland) since 2008 deteriorated significantly the debt-refinancing conditions of the central governments. These developments were magnified by two main forces which led to the outbreak of a full-fledged sovereign debt crisis that threatened the macroeconomic stability of the euro area, and the European unification project as a whole. First, the surge of the yields of sovereign bonds led also to an increase in the borrowing costs of the corporate and the household sector in the euro area, affecting in a direct manner the private sector's consumption and investment decisions. And second, once the break-up of the euro area began to be considered as an actual possibility by the financial markets, this risk was also incorporated in the sovereign default risk premia of several euro area countries (see for instance Coimbra, 2014 and Semmler and Gervorkyan, 2014), leading to even higher sovereign yields. All these developments let aggregate output and income decrease even further, generating less sovereign revenues and increasing primary fiscal deficits.

The historical experience of previous sovereign debt crises suggests that successful strategies implemented in previous sovereign debt crises around the world such as debt relief, a change of the debt maturity structure, the announcement of credit guarantees have not only reduced the sovereign debt servicing burdens, but have managed to lower the credit costs for private borrowers and the overall financial stress as well, increasing so nominal production and income.[1] For instance, Reinhart and Trebesch (2014) show that debt relief accounts historically for roughly 20 percent of recovery and macroeconomic improvement. They attribute improved ratings and reduced debt service to this improvement. If successful these measures are likely to increase economic activity, aggregate income and tax revenues, and to reduce sovereign deficit and fiscal debt levels – all this lowering sovereign default risk and aiding to escape a default threat environment. Yet, the debt reduction may mean that there is a balance sheet effect on the side of the creditors, for example in the banks' asset position. This could trigger, as the EU crisis has shown, some threat of banking insolvency, possibly entailing further financial stress, as discussed by Brunnermeier and Oehmke (2012) and Coimbra (2014).[2]

At the theoretical level, the occurrence of euro area sovereign crisis and the related surge in perceived sovereign default risk of many EMU countries triggered a new generation of theoretical studies which investigate how such a high default risk regime can emerge in the first place, as well as how policies should be designed to reverse eminent defaults in the second place.[3] In particular, Arellano (2008), Roch and Uhlig (2013) and Arellano et al. (2014) analyze consumption-based models on sovereign

[1]For an early study on the escape routes see Tobin (1963, 1987).

[2]In contrast, historical experience with partial and full default episodes has shown that interest rates and credit costs are likely to rise rapidly immediately after the default, leading to a reduction of aggregate consumption and investment. This effect depends however on the reaction of output: With a falling output, credit spreads can surge, deteriorating the macroeconomic performance even more, see Arellano (2008).

[3]While in early models of fiscal sustainability (see e.g. Bohn, 1998) the transmission mechanisms from sovereign debt to economic activity and vice-versa were mostly linear, recent studies such as Mittnik and Semmler (2012), Baum et al. (2012), De Grauwe (2012), Blanchard and Leigh (2013), Blanchard et al. (2013), among others, have highlighted the fact that the multiplier might be state dependent and fiscal consolidation could be strongly contractionary when implemented in the midst of a recession.

debt and default risk with optimal choice of sovereign bonds, where aggregate income fluctuations are exogenously determined. The main escape routes suggested by such theoretical frameworks are consumption reduction (see also Uhlig, 2014), or bailouts with "fair bond pricing" (see Gavin et al. 2013).

Yet, as we will argue, the insight from these types of models needs to be complemented with a study of further macroeconomic feedback channels in order to investigate what important routes are available to escape from a regime of high sovereign default risk. In this paper we thus propose a broader macroeconomic model which also includes economic production and capital investment. We make production and income fluctuations endogenous, include capital stock as state variable and investment as a decision variable. Since there can be a pass-through of sovereign risk to the private sector, and the reverse, the main route is likely to be to reduce default threat through reducing financial market stress and credit spreads, using exceptional monetary policy tools. Because the sovereign risk is passed-through to the private sector borrowing cost, the transition out of this debt crisis regime to a better regime also works through those channels, namely through a reduction of financial stress and interest cost, not only for sovereign bonds but also for private borrowing.[4][5]

Accordingly, we propose here a small scale macro model which, in contrast to the above mentioned previous literature, includes economic production and capital investment. We make production and income fluctuations endogenous, including capital stock as state variable and investment as a decision variable. Because the sovereign risk is allowed to be passed-through to the private sector borrowing cost, the transiting out of this regime to a better regime, also works through those channels, namely through a reduction of financial stress and interest cost, not only for sovereign bonds but also for private borrowing. In this context exceptional monetary policy becomes particularly important, since there are often strong adverse macroeconomic feedback loops.

In contrast to the standard approach where eventually destabilizing feedback loops tend to be smoothed out due to the infinite horizon underlying the agent's economic decisions, or are only captured in some adverse shocks,[6] we propose here an alternative framework with a finite planning horizon that allows for endogenous destabilizing feedback loops. To solve this model variant we use the Nonlinear Model Predictive Control algorithm proposed by Grüne and Pannek (2011), see also Grüne et al. (2013) and the appendix of this paper. We build a regime change model variant with high sovereign debt, high financial stress and strong macro feedback loops. On the other hand, what maybe typical is a regime with moderate sovereign debt, but low financial stress and low credit spreads and also more favorable macroeconomic feedback loops.

The remainder is organized as follows. In section 2 we set up a model along the lines of Arellano

[4]See also Lorenzoni and Werning (2013) who distinguish between a good equilibrium with no default and a bad equilibrium associated with a slowly emerging debt crisis.

[5]Note that in the subsequent discussion we leave aside the issue of generating sustainable sovereign debt through fiscal policy such as fiscal consolidations, through an increase in taxes or a cut in public spending, strategies which have not been particularly successful in a regime of economic and financial stress as discussed by Buiter (2010), see also Mittnik and Semmler (2012).

[6]Note that there are also now models of regime change available that move away from those smoothness properties, see Farmer et al. (2009).

(2008) and Arellano et al. (2014) and modify it to analyze default risk and non-default risk countries. In section 3 we study empirically the nonlinear relationship between output growth, sovereign debt and financial stress in the main euro area countries and other industrialized economies. Finally, we draw some concluding remarks from this study in section 4. The appendix outlines the numerical procedure (called Nonlinear Model Predictive Control or NMPC) used for the solution of our model.

2 Theoretical Analysis

As mentioned, recent studies such as Arellano (2008), Roch and Uhlig (2013), Lorenzoni and Werning (2013) and Arellano et al. (2014) have elaborated on the dynamics of sovereign default risk. We here briefly discuss their standard default risk model and then introduce our extension with production and investment.

2.1 The infinite horizon standard model on sovereign default risk

The standard theoretical framework can be summarized in a generic way as follows:

$$V\,(b,y) = \max_{c_t, b_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t (U(c_t) - \chi d) \tag{1}$$

subject to

$$c_t + (1-\theta)b_t = y_t + q(b_{t+1}, y_t)(b_{t+1} - \theta b_t) \tag{2}$$

where $V\,(b,y)$ is the expected present discounted value of the agent's utility out of consumption c_t (either private or by the government), with y_t being aggregate income (or government revenues), b_t outstanding (private or government) debt, and χd the cost of default when a default event d occurs. Agents maximize thus the expected present value of intertemporal utility – with $U(\,\cdot\,)$ being the period utility function – by choosing c_t and b_{t+1} subject to a period budget constraint given by eq.(2), where θ denotes the fraction of bonds repaid in a given period. Aggregate income y_t is assumed to be given by an exogenous stochastic process, and $q(b_{t+1}, y_t)$ is the bond price given the choice of issued bonds b_{t+1} and the income stream y_t.[7]

Since for the bond yield we have R_t=$1/q(b_{t+1}, y_t)$, and $r_t = R_t - 1$, we can write for $\theta = 0$ a state equation in continuous time such as[8]

$$db_t = (r_t b_t - (1 + r_t)(y_t - c_t))dt. \tag{3}$$

According to eq.(3) the increase in sovereign debt from t to $t + 1$ is driven by the bond price – or the bond yield respectively –, the current sovereign debt level and the current surplus, the latter given by the exogenous income fluctuations and the endogenously chosen consumption stream.[9] As

[7] Arellano (2008) and for most part of their analysis also Roch and Uhlig (2013) assume $\theta = 0$.

[8] While in discrete time b_{t+l} is a choice variable to be decided in time period t, in a continuous time variant of this model one can write this expression as in eq.(3).

[9] As mentioned, in the context of sovereign debt the exogenous income fluctuation is interpreted in Roch and Uhlig (2013) as tax revenues, and c as public consumption.

it should be clear, as income is driven by an exogenous process macroeconomic feedback loops from output to sovereign debt – for instance, through their impact on income and credit spreads – are not endogenized in such frameworks.

The above literature studies a high stress regime with high debt, and low income stream resulting in low bond prices and high yields, and a low stress regime with low debt, low yields and higher bond prices. An extreme case is given when the sovereign chooses to default, with an appropriate effect on consumption and welfare. In this case it is assumed that the country actually chooses defaults when the welfare of non-defaulting $V(b,k)$ becomes lower than that the welfare associated with a default $V^{def}(k)$, thus when $V(b,k) < V^{def}(k)$. This means that welfare is smaller with borrowing than for default and autarky. This is an interesting case to be considered – as done e.g. in Arellano et al. (2014) –, though such an event has not yet occurred in the euro-area (this issue will be further discussed below).

What is a limitation of those models is that since output and income are stochastic, macroeconomic feedback loops of stabilizing or destabilizing nature which affect growth rates, the default risk, and the macroeconomic performance as a whole are by and large neglected. Nonetheless, such macroeconomic feedback loops between sovereign debt, bonds yields and output can bring the economy into a downward spiral of low growth, high sovereign risk and increasing sovereign debt. Yet the reverse, more stabilizing spiral, may also hold. Therefore, it is key to acknowledge that these macroeconomic feedback effects may be state-dependent, and that their overall effect may switch from destabilizing to stabilizing.

2.2 A model variant 1: Weak macroeconomic feedback effects

Building on the just discussed modeling approach we first present a model variant based on a finite decision horizon and with weak macro feedbacks. We do so to characterize short-run behavior in a more realistic manner, as the infinite horizon framework implies a pronounced smoothness in the evolution of the choice variables by construction, as discussed by Grüne, Semmler and Stieler (2013). We also introduce production and investment. Accordingly, the eqs. (1) and (2) in our modified framework read[10]

$$V\left(k,d\right) = \max_{c_t,i_t} E \int_0^T e^{-\rho t}(\ln(c_t) - \chi(\mu_t - \mu^*)^2)dt \tag{4}$$

subject to

$$\begin{aligned} dk_t &= (i_t - \delta k_t)dt & (5) \\ db_t &= (r(\cdot)b_t - (1 + r(\cdot))(y_t - c_t - i_t - \varphi(i_t)))dt & (6) \end{aligned}$$

In eq.(4) there are preferences over log utility, now penalized by some excess with $\mu_t = b_t/k_t$ and μ^* being steady state debt-to-capital ratio.[11] The decision variables are consumption, c_t and investment

[10]For details of how such type of short decision horizon model can approximate models with longer time horizons well on the basis of much less information, see Grüne et al. (2013).

[11]Roch and Uhlig (2013) allow for a one-time cost of default. We stretch this default cost out over time, making it depending on the excess leveraging. A similar approach as ours was been proposed by Blanchard (1983).

i_t.[12] Eq.(5) represents the evolution of capital stock, which increases due to investment but declines due to the capital depreciation δk_t. Eq.(6) represents the dynamics of sovereign debt, with $y = Af(k) = Ak^{\alpha}$ being the production function, and $A > 0$. The interest payment on debt $r(\cdot)b_t$ is state dependent, with the interest rate r_t being an increasing function of the issued bonds and a decreasing function in the capital stock, in a similar way as in Arellano (2008).[13] Moreover, debt decreases with the surplus $y_t - c_t - i_t - \varphi(i_t)$ – the excess of income over spending, and rises with a deficit. Moreover, $\varphi(i_t)$ is the adjustment cost for investment, which is presumed to be quadratic. We thus introduce production and a capital stock dynamics, as well as the evolution of sovereign debt by defining debt now as state variable.

Note that the model has now two decision variables and two state variables,[14] and that we make a distinction between the discount rate ρ and the interest rate r_t, the latter being affected by the credit spread. In more advanced settings, see for example Brunnermeier and Sannikov (2014), the capital stock is shared by households and financial intermediaries, but in our setting it remains relatively passive in this respect.

As previously mentioned, a key mechanism in the dynamics of sovereign default risk discussed by Arellano (2008) and Lorenzoni and Werning (2013) is the endogenous reaction of the bond yield to the perceived sovereign risk. In the following we also make the yield on bonds a nonlinear function of the sovereign leveraging. Yet it is reasonable to assume the function to be bounded, so we define it by a function such as:

$$r(s_t|\gamma, c^*) = [1 + \exp(-\gamma(s_t - c^*))]^{-1}, \ \gamma > 0, \ c^* > 0 \tag{7}$$

This function makes now the sovereign credit cost depending, in a nonlinear way, on a state variable s_t (here μ_t, the debt-to-capital ratio), a threshold variable, c^* (here equal to μ^*) and a slope parameter, γ.[15] Note that this logistic function is roughly the nonlinear function that has been estimated empirically in De Grauwe and Ji (2012) in their analysis of EU debt and bond yield data (see also Corsetti et al. 2012).[16]

When there are state dependent risk premia and credit cost, but the macro feedback loops are weak, interest rates can still stay low. Credit risk and financial stress do not build up and there are

[12] Since we mainly want to focus on endogenous credit spreads arising from sovereign risk, with pass-through of public default risk to private borrowing cost, we interpret consumption and investment spending as private as well as public spending, and income y_t as GDP, a fraction of it be used as tax revenue. Accordingly, neither public spending nor tax revenues will be explicitly modeled, but they are simply endogenized through movements of the aggregates.

[13] There the bond yield is a function of the size of bond issuing and the stochastic income shock.

[14] We could have formulated the second state equation in terms of net worth and leveraging, the latter as a decision variable as in Brunnermeier and Sannikov (2014). We prefer leveraging here as a state variable where then debt can only sluggishly be redeemed and issued again.

[15] The above represents the logistic function often used in regime change models such as the STAR and STVAR models such as Schleer and Semmler (2013).

[16] In Arellano (2008) and Roch and Uhlig (2013) the sovereign risk and credit spreads are driven by exogenous shocks in income and bond issuing. Arellano (2008) justifies an increasing interest rate (discount rate) with the high negative correlation of risk perception of lenders with the output shock. In more standard DSGE models the rise of risk premia and its persistence on a high level is often simply modeled through large shocks with some strong persistent, see also Gilchrist et al. (2011).

little adverse macro feedback loops. Figure 1 illustrates this case, with initially low stress and with borrowing cost below some threshold. For these initial conditions we can observe the capital stock to converge to the steady state.

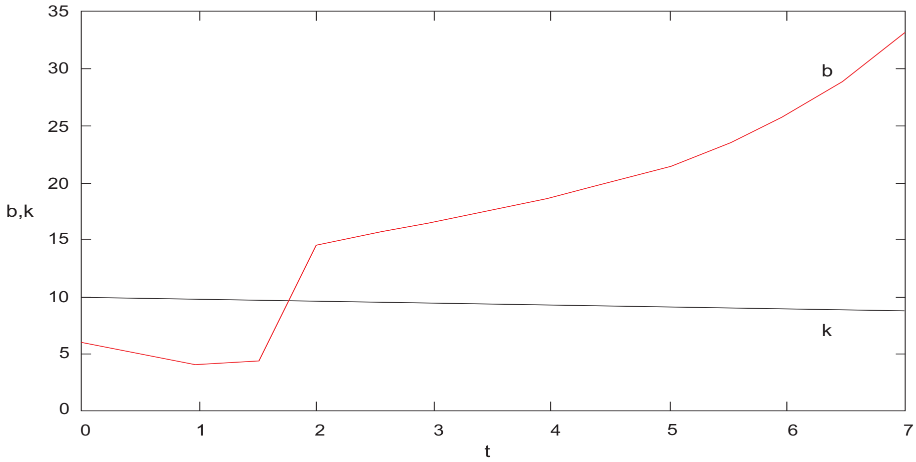

Figure 1: Dynamics for capital stock k and debt b under borrowing costs rising nonlinearly in high leveraging regime, though macro feedback loops might be weak. Initial conditions $k(0) = 10$, $b(0) = 6$. Debt dynamics eventually unstable, though GDP growth is by and large unaffected.

Note that in this scenario variant we have set here the macro feedback loops to aggregate demand to be weak. We still obtain the sovereign debt to be unstable eventually (see the upper trajectory in Figure 1), but output and capital accumulation stabilize. The blac trajectory in Figure 1 represents the path of the capital stock (denoted by k) with low initial debt and weaker macro feed back loops, with a parallel evolution of output (not depicted here). But even at lower initial condition, the debt dynamics becomes unstable. What is modeled here is what has been called the feedback of financial market stress to aggregate demand and output.[17] Note that leveraging and credit cost above a certain threshold[18] may become unsustainable, due to our transition of eq. (7) to a higher stress regime.

2.3 Model variant 2: Strong macro feedback loops

We consider now a slightly modified variant of the above case. As mentioned above adverse macro feed back effects arising from sovereign leveraging can affect credit cost for consumption and investment as well as banking vulnerability. There are not only endogenous risk premia, rise of interest rates and prices of assets declining but the macro feedback loops are likely to trigger a serious decline in aggregate demand and output[19] and thus banks' operating income and market valuation – with

[17]This is what a recent IMF study defines as follows: "The risk channel amplifies the transmission of shocks to aggregate demand, unless monetary policy manages to offset the spillover from sovereign default risk to private funding costs". Corsetti et al. (2012).

[18]Those thresholds can be empirically estimated, see Schleer and Semmler (2013).

[19]See Blanchard and Leigh (2013) and Corsetti et al. (2012). They show of how empirically for example sovereign debt and banking risk also increases private borrowing cost and thus make aggregate demand falling. They employ, as

the consequence of a further reduction of credit supplies by banks. So the real side starts to have effects on the financial sector and the reverse. We thus focus here on economic mechanisms that entail endogenous feedback loops of the financial stress to macroeconomic activity, generating non-linearities, possibly giving rise to greater instability. This is likely to occur if the central bank is not attempting – or not being able – to pursue a monetary policy to reduce financial market stress and to bring down credit cost through credit policies.

Though optimal payouts and investment might be targeted, actual operating income of banks, are likely to decline due to macroeconomic feedback loops. So overall we may experience that actual gross operating income y_t^a in eq.(8) may be only a fraction of y_t and negatively dependent on $r_t(|\gamma, c^*)$, namely:

$$
\begin{aligned}
y_t^a &= (1 - r(s_t|\gamma, c^*))y_t && (8)\\
db_t &= r(s_t|\gamma, c^*)b_t - (1 + r(s_t|\gamma, c^*))(y_t^a - c_t - i_t - \varphi(i_t))dt && (9)
\end{aligned}
$$

Note that in eq.(8) we have defined actual operating income to be driven by aggregate activity in the regime of high financial stress $(1 - r(s_t|\gamma, c^*))(i + c)$, where actual consumption and investment, responding to financial stress, $(1-r(s_t|\gamma, c^*))$, are determining actual income. So the optimally chosen decision in each time period of the state variables are actually not realized, but the actual outcome depends on the degree of financial stress and the macro feedbacks triggered by this. The numerical results, obtained again through the application of the NMPC algorithm, are shown in Figure 2.

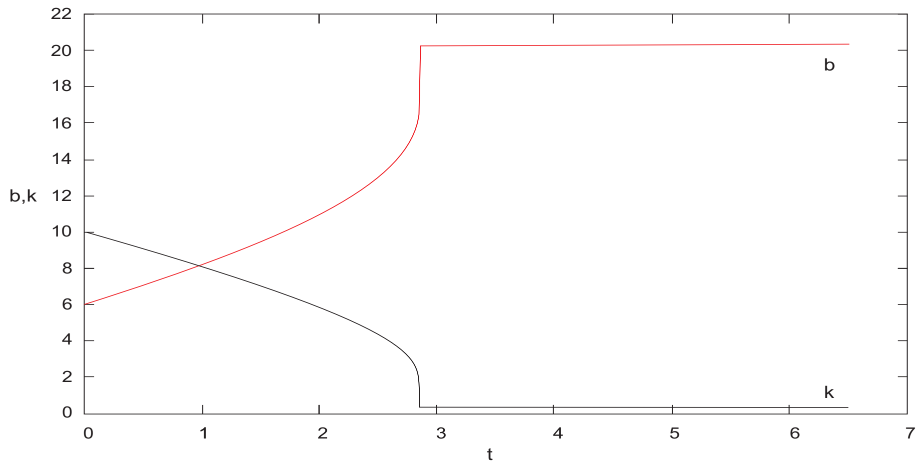

Figure 2: Dynamic paths of capital stock k, and debt b for moderate debt, but high credit spread and default risk. Initial conditions $k(0) = 10$, $b(0) = 6$, steady state debt to capital ratio of $\mu^* = 0.3$, with debt dynamics quickly unstable.

In Figure 2 we assume that the initial debt-to-capital stock ratio is roughly 0.6 which corresponds

we do here, the spillover effects of risk spreads to aggregate demand, but one can also think of another channel through which macro economic contractions are triggered. A reduction of loan supplies by banks will set in when asset prices and net worth of banks fall, i.e. they will reduce their loan supply to households and firms, see Gerali et al. (2010) and De Grauwe and Machiarelli (2013).

approximately to a sovereign debt to income ratio of 1.2. The function described by eq.(7), representing the steeply rising credit spread, makes credit cost rising with higher debt ratio and higher financial stress. Note also if in this case we were to look at the asset side of the economy, asset prices are likely to fall or do not grow any more and capital gains could become negative and the income y would fall and surpluses would shrink, the debt service rise with higher interest rates and debt sustainability becomes threatened.

Since in our model we have also investment and capital accumulation we can trace better the macro feedback mechanisms that models based solely on consumption tend to overlook. The economic intuition of the upper and lower unstable trajectories is that stronger macroeconomic feedback loops with negative impact on demand, declining output and income and credit cost rise, may be due to the following:[20]

- There is the wealth effect reducing aggregate demand – when asset prices fall and there is a depreciation of assets, aggregate demand would fall and with lower collateral value of assets (in our case of bonds) banks would reduce loans or increase credit cost[21]
- The share of households that are income and credit constrained, in the sense of Galí et al. (2007), and households that are higher leveraged and are under financial stress,[22] are significantly rising in a contraction period of the business cycle, and thus demand falls (see e.g. Mittnik and Semmler 2012, 2013).
- As the financial market forces trigger banking and financial stress,[23] the central bank may have no instruments available – or is not willing – to force the interest rate down further and/or to reduce risk premia and credit cost, which again may adversely affect demand and output
- A fraction of private households start strongly deleveraging that reduces income and liquidity of other households and firms (see e.g. Eggertsson and Krugman, 2012). This might be accompanied by a Fisher debt deflation process, causing higher real debt and declining demand because of expected price fall (the Tobin effect).[24]
- Finally, there could occur even a worse feedback: a weak financial sector, holding risky sovereign bonds or other debt instruments, may come under severe stress, because debt may go into default and banks reduce lending to the real economy, or worse, may even default.[25]

[20]A systematic study of macroeconomic feedback effect, know from the history of macroeconomics, partly stabilizing partly destabilizing, are extensively elaborated on in Chiarella, Flaschel and Semmler (2014). Note that we leave here aside the macro feedback effects of nominal type. Those can be build in as well in such a dynamic macro model, as it is e.g. done in Charpe et al. (2013) and Semmler and Ernst (2014).

[21]This has been called the "pass through of sovereign risk" to the private sector.

[22]The share of those households matter, since there is empirical evidence that the drop in demand will be larger for households with larger debt, that are forced to deleverage more, see Eggertsson and Krugman (2012).

[23]This is documented by the ZEW financial condition index as presented in Schleer and Semmler (2013).

[24]Though we have not build in a price dynamics in our model the described the reader might find the above results quite intuitive, for a detailed discussion of such and further macroeconomic feedback effects can be found in Charpe et al. (2013).

[25]This is what Brunnermeier and Oehmke (2012) call "diabolic loop", see also Bolton et al. (2011), the latter present data on the sovereign debt holdings of banks.

We expect thus, starting with a leverage ratio roughly above normal, that the above feedback mechanisms are likely to lead to lower asset value, higher risk premia, higher credit costs, less credit supply – and thus higher financial market stress – and lower investment and consumption demand, lower output, leading to a contraction in the utilization of the capital stock, and falling capital stock, with increasing credit spreads and so on. With the latter the excess sovereign debt rises due to high yields, low bond prices and insolvency risk is likely to rise.[26]

Given those above sketched adverse macro feedback loops, it is likely that a regime switch to a high stress regime occurs where the vulnerability of financial intermediaries increase and a faster deterioration of demand and output, as well as capital stock, can be triggered which has then again feedback effects from the real to the financial side. This is happening the more, the more central banks fail to successfully undertake an unconventional intervention into the money and asset markets. Conventional monetary policy will not help in this case but credit and financial market policies are needed instead.[27]

2.4 A model variant 3: Escape routes from sovereign default risk

Looking at the transition function given by eq.(7) escape routes from sovereign default risk might emerge, if credit costs are brought down and credit channels are unblocked by central banks credit and financial market policies. Such a low credit cost, low stress regime, is characterized by low interest rates on borrowing, lower leveraging and little credit spreads with an appropriate pass through to households' and firms' borrowing conditions. This can be seen as equivalent to the case of the central bank successfully pursuing a low – or near zero – interest rate policy which keeps the economy in a low financial stress regime, where also the credit costs are low and rather constant, and in addition we have $y_t^a = y_t$.[28]

This more benign scenario is characterized by

$$db_t \quad = \quad (rb_t - (1+r)(y_t - c_t - i_t - \varphi(i_t)))dt \tag{10}$$

with the agent's maximization problem still being given by eq.(4). As in the previous model variants, we have here again two decision variables and two state variables.

We again have made a distinction between the discount rate and interest rate, the latter impacted by leveraging. Assuming here $r = 0.03$, the outcome of both the financial stress and the macro feedbacks are captured in the lower and upper trajectories of Figure 3.

As it can be clearly observed, when interest rate and bond yields are at a low level and do not react endogenously to adverse macroeconomic conditions, a positive primary surplus resulting from the excess of income over consumption and investment will reduce the sovereign leveraging over time.

[26] This could equivalently create a downward spiral in net worth, if the model is written in terms of net worth, as in Stein (2012) and Brunnermeier and Sannikov (2014).

[27] See Correia et al. (2013) who make a distinction between conventional monetary policy, reducing the interest rate and credit policies, reducing the spread.

[28] A country in the Euro-Area such as Germany seems to have enjoyed such conditions, after 2010.

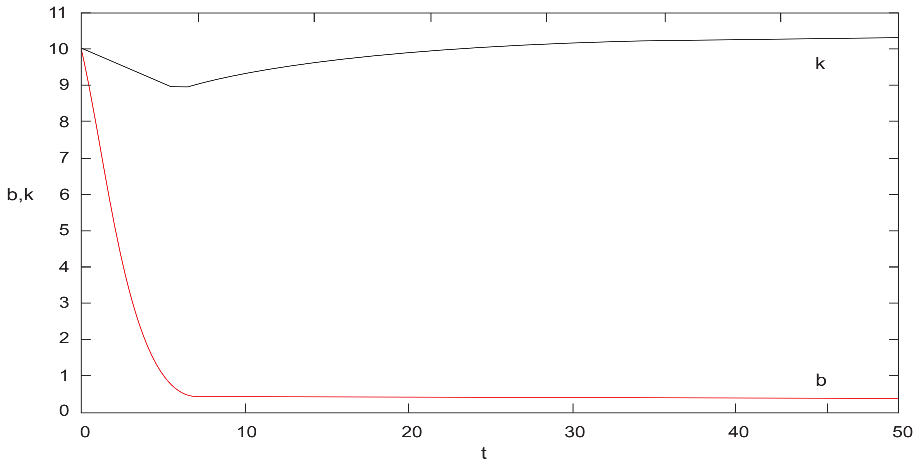

Figure 3: Stable capital stock k and debt b. Dynamics under low interest rates, and no adverse macro feedback loops, both starting from the same initial conditions as in the previous cases.

In this scenario – where adverse macroeconomic feedback loops are not present – sovereign debt can be stabilized by appropriate debt reduction policies and credit policies by the central bank, in a case when output is determined endogenously, as it is the case in the present framework. Those macro channels are however not available in model variants of sect. 2.1. We will get back to the above three cases of sect. 2.2-2.4. In particular, we want to know if we could observe those cases in Euro-Area countries in recent times.

3 Empirical Analysis

Next we study such country differences with respect to their vulnerability to default traps. Can we detect our above mentioned three cases in the data? We explore a nonlinear linkage between sovereign debt, financial stress (as a proxy for sovereign default risk) and output growth, on the basis of quarterly data for 13 advanced countries. Though the financial stress index (FSI) we are using[29] contains also components other than bond yields, the index is constructed so that significant sovereign default risk should show up in the FSI. Our methodology is to investigate by means of nonlinear single-equation and panel estimation techniques if and for what countries or country groups, sovereign debt and financial stress affect economic activity in our sample of 13 industrialized countries.

3.1 Data Description

In our econometric analysis we use quarterly seasonally adjusted data on the net sovereign debt of the general government, GDP at constant prices, the financial stress index (FSI), and the real interest rate

[29]The financial stress index (FSI) – a composite indicator comprising information on the *banking-sector beta*, stock market returns, time-varying stock market return volatility, sovereign debt spreads, and an exchange market pressure index – stems from the IMF data set discussed in Danninger et al. (2009) and Cardarelli et al. (2009).

on long-term government bonds from 1981:1 to 2013:2 for the following 13 countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Germany (DEU), Spain (ESP), Finland (FIN), France (FRA), the United Kingdom (GBR), Greece (GRC), Italy (ITA), Japan (JPN), the Netherlands (NLD), Portugal (PRT), and the United States (USA). The data on real GDP and the interest rate on long-term government bonds stems from the OECD Economic Outlook database and Eurostat (for details on this data set, see Proaño et al. (2014)).

Figure 4 illustrates the evolution of the net sovereign debt-to-GDP ratio, the financial stress and the output growth for 13 advanced economies over the period 1981Q1 to 2013Q2.

As it can be observed in Figure 4 while there is a significant heterogeneity between the net sovereign debt-to-GDP ratios across the considered countries (with Italy (ITA) and Japan (JPN) having the highest debt-to-GDP ratio at the end of the sample), a common feature among all EMU countries is the sizable deterioration of the fiscal balance (as measured by the sovereign debt-to-GDP ratio) resulting from the sharp drop in output growth associated to the 2007-08 global economic recession and the subsequent euro area crisis. This countercyclical response of the sovereign debt-to-GDP ratio is of course not surprising due to its construction (with the GDP level in the denominator of the ratio). However, the question of causality between these two variables can of course not be reduced to this simple construction, but has to be addressed by means of econometric methods, as it will be done below, see also Proaño et al. (2014).[30]

In general, concerning the dynamics and interaction of the FSI and the GDP growth, Figure 4 illustrates clearly that while the output contraction during the recent financial crisis was rather similar across all countries considered, the FSI reaction was, on the other hand, quite differentiated, with countries such CAN, JPN, NLD and USA soaring with their FSI about three times higher than the remaining countries. But, maybe with the exception of the US, these were countries with little eminent default threat, see also the discussion in Proaño et al. (2014). These countries can be viewed as countries representing roughly the model variant 1 of sect. 2.2.

At first sight, the sovereign leveraging does not seem to have generally triggered a transition to high stress regimes with strong adverse macro feedback effects and eminent default threats. As to this nexus, the evolution of the sovereign debt-to-GDP ratios of Spain and the UK stand out, with an increase from a level of 20.4% in 2008Q3 to 63.5% in 2013Q2 of the former, and of 29.3% in 2008Q3 to 71.1% in 2012Q3 of the latter.[31] Yet, this dramatic increase in the debt-to-GDP ratio did not translate into a comparable reaction in the overall financial market stress – as the FSI in the UK increase to levels almost five times larger than in Spain at the height of the euro area crisis in 2009 –, nor to a similar reaction in GDP growth in the subsequent period. In this context an interesting case regards Italy, as one would expect its economic situation to be reflected by the model variants one or two discussed in the previous section. However, despite of the high level of the Italian debt-to-GDP

[30] In contrast to the present study, Proaño et al. (2014) focus on the role of the sovereign debt-to-GDP as a transition variable determining the switching between two regimes, as well as the joint effect of this variable with the financial stress index in determining the switching between for regimes.

[31] Note that we consider here net sovereign debt, that is, sovereign debt held only by the public, because gross sovereign debt – which includes intra-governmental debt – may not have a direct impact on the economy.

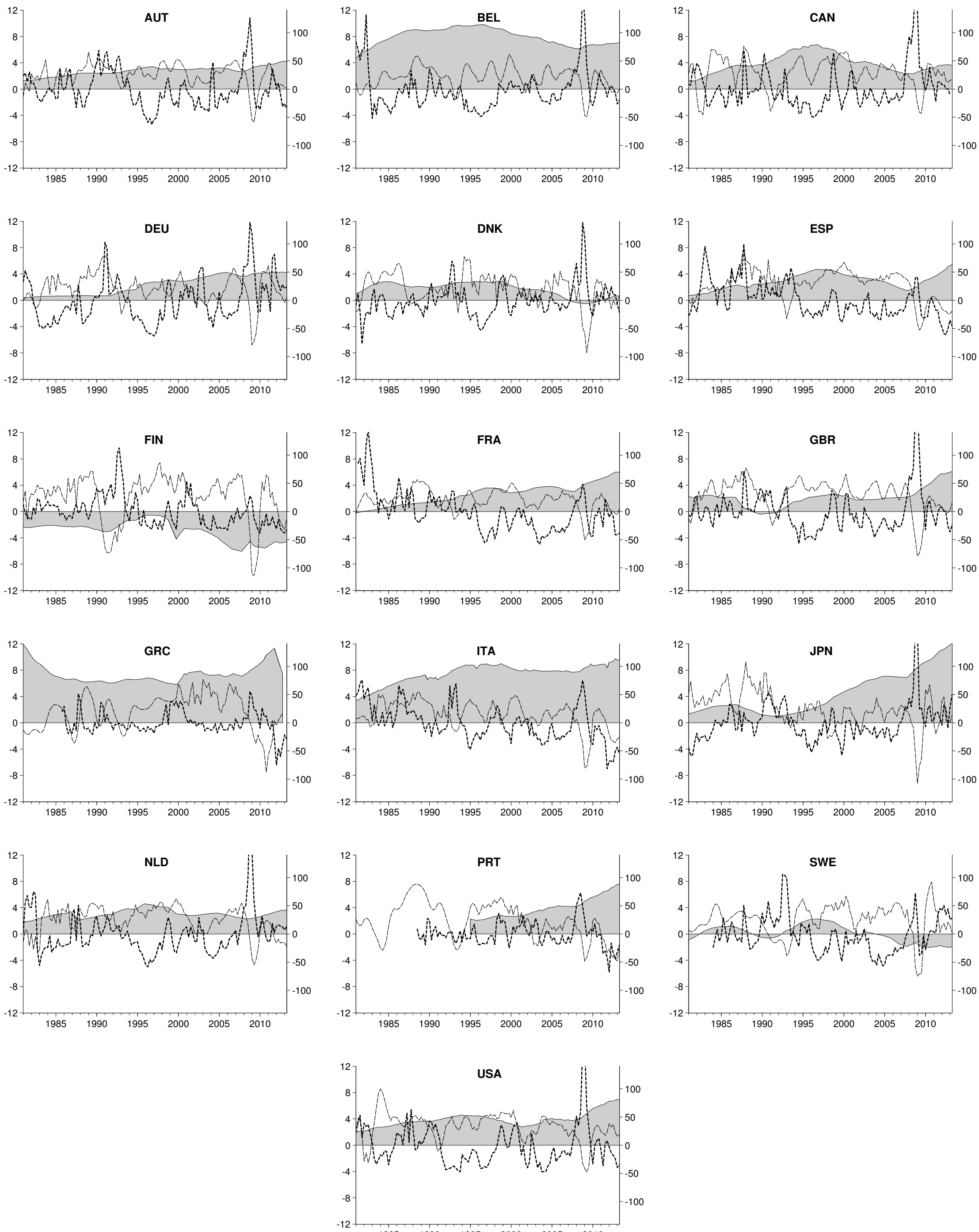

Figure 4: Net sovereign debt-to-GDP ratio (shaded area, left axis), year-to-year GDP growth rate (solid line, right axis) and financial stress index (dashed line, right axis) from 1981:1 to 2013:2. Seasonally adjusted quarterly data. Source: OECD Economic Outlook database, Eurostat and IMF.

ratio, the actual FSI in Italy did not increase significantly in the recent euro area crisis, therefore not fully activating the adverse macroeconomic feedback mechanism previously discussed. Further, Sweden and Germany seem to belong more to the model variant 1 or 3 above, despite the significantly high levels of the FSI in the recent euro area crisis. In contrast, in Portugal, a country that was close to sovereign default at some point, these adverse macroeconomic feedback mechanisms seem to have been particularly active despite of the relatively low levels of the FSI.

So, there seem to be various types of nonlinearities at work in the different countries of our sample. In order to gain some first insights into the possibly nonlinear linkage between output growth, sovereign debt and financial stress, in particular for sub-groups of countries, we run first a battery of linear country-specific and dynamic panel regressions, which can be jointly represented through the following general specification:

$$y_{it} = \mu_i + \mathbf{x}_{it}\boldsymbol{\beta} + \mathbf{w}_{it}\boldsymbol{\alpha} + \varepsilon_{it} \tag{11}$$

where $i = 1, \ldots, N$ is a country-index, $t = 1, \ldots, T$ is the time index, μ_i denotes a country-specific fixed effect, ε_{it} is an i.i.d. random disturbance with zero mean and a variance $\sigma^2_{\varepsilon i}$. y_{it} is the dependent variable (quarter-to-quarter real GDP growth), x_{it} is a vector of explanatory variables consisting of the the main regressors y_{it-1}, b_{it-1}, f_{it-1}, i.e. the lagged GDP growth rate, the government debt-to-GDP ratio and the financial stress index, respectively, as well as the ex-post real interest rate on long-term government bonds, as a control variable. Further, w_{it} is a vector of the main explanatory variables in their quadratic form, in its most general form given by

$$\mathbf{w}_{it} = (y^2_{it-1}, b^2_{it-1}, f^2_{it-1}).$$

Table 1 reports the estimation results of the most general specification where all quadratic terms are included in the set of regressors.[32] To start, it is interesting to note that the linear effect of debt-to-GDP ratio b_{t-1} on economic growth is statistically significant and negative only for a subset of countries, namely France, Greece and Portugal, while for the remaining countries is either statistically insignificant, or positive, as it is the case for Spain and Sweden. In contrast, the quadratic term b^2_{t-1} is negative and statistically significant only for Spain what, jointly with the first positive effect of b_{t-1}, seems to suggest that the – unexpectedly – positive effect of sovereign debt on GDP growth becomes smaller for larger values of b_{t-1}. By the same token, the positive and statistically significant coefficient of b^2_{t-1} in Greece suggests the negative effect of b_{t-1} on y_t decreases in absolute terms for larger values of b_{t-1}.

Further, concerning the effect of financial stress on GDP growth, our estimates deliver a much more uniform picture for the analyzed countries. Indeed, we find that the linear coefficient is negative for all countries and statistically significant for 9 out of the 13 countries analyzed, and that the quadratic term f^2_{t-1} is also negative and statistically significant for Austria, Germany, Denmark, Italy and Japan. This seems to corroborate – at least in a preliminary manner – the nonlinear effect of leveraging and financial stress on growth as discussed in the previous section of this paper, for the model variant 1 and 3.[33]

[32] Additional regressions where the elements included in w_{it} were selectively included are available upon request.
[33] See also Brunnermeier and Sannikov (2014), for example.

Table 1: Country-specific least-squares regressions. Sample: 1981Q1–2013Q2

	c	y_{t-1}	b_{t-1}	f_{t-1}	y^2_{t-1}	b^2_{t-1}	f^2_{t-1}	$\bar{R}^2$	DW-Stats
AUT	-0.345	0.113	0.069	-0.047^{*}	0.123	-0.001	-0.016^{***}	0.227	1.836
	(0.805)	(0.104)	(0.048)	(0.025)	(0.089)	(0.001)	(0.006)		
BEL	0.052	0.618^{***}	0.001	-0.009	0.037	0.000	-0.001	0.479	1.594
	(1.508)	(0.096)	(0.033)	(0.018)	(0.085)	(0.000)	(0.003)		
CAN	-0.274	0.327^{***}	0.023	-0.039	0.076	-0.000	-0.003	0.367	1.891
	(0.521)	(0.096)	(0.020)	(0.030)	(0.065)	(0.000)	(0.003)		
DEU	1.529^{***}	-0.128	-0.042	-0.075^{**}	0.113^{**}	0.001	-0.021^{***}	0.251	1.997
	(0.455)	(0.084)	(0.028)	(0.032)	(0.044)	(0.001)	(0.005)		
DNK	0.157	-0.285^{**}	-0.003	-0.127^{***}	0.006	0.001	-0.012^{*}	0.218	2.053
	(0.210)	(0.115)	(0.024)	(0.046)	(0.054)	(0.001)	(0.007)		
ESP	-1.013^{*}	0.310^{***}	0.073^{***}	0.057	-0.207^{***}	-0.001^{**}	-0.006	0.239	1.913
	(0.528)	(0.105)	(0.028)	(0.036)	(0.052)	(0.000)	(0.007)		
FIN	1.754^{***}	0.083	0.049	-0.170^{***}	-0.053^{**}	0.000	0.018	0.239	1.985
	(0.560)	(0.091)	(0.026)	(0.065)	(0.024)	(0.000)	(0.011)		
FRA	1.392^{***}	0.403^{***}	-0.036^{***}	-0.077^{***}	-0.003	0.000	0.003	0.387	2.103
	(0.422)	(0.085)	(0.013)	(0.020)	(0.072)	(0.000)	(0.002)		
GBR	0.298	0.571^{***}	0.006	-0.055^{**}	-0.190^{**}	-0.000	0.005	0.385	1.912
	(0.199)	(0.134)	(0.008)	(0.024)	(0.086)	(0.000)	(0.004)		
GRC	17.747	0.012	-0.337^{**}	-0.152	-0.070	0.002^{**}	-0.028	0.249	2.031
	(6.085)	(0.164)	(0.130)	(0.132)	(0.057)	(0.001)	(0.045)		
ITA	1.914	0.270^{***}	-0.029	-0.007	-0.025	0.000	-0.024^{***}	0.255	2.011
	(1.433)	(0.089)	(0.034)	(0.026)	(0.044)	(0.000)	(0.006)		
JPN	0.127	-0.037	-0.010	-0.052	0.082	0.000	-0.011^{*}	0.073	2.043
	(0.533)	(0.124)	(0.013)	(0.047)	(0.063)	(0.000)	(0.007)		
NLD	0.735	-0.072	-0.020	-0.118^{***}	0.053	0.000	-0.003	0.201	2.016
	(1.576)	(0.091)	(0.081)	(0.034)	(0.061)	(0.001)	(0.003)		
PRT	2.955^{**}	0.032	-0.082^{*}	-0.079	-0.062	0.001	-0.016	0.271	2.062
	(1.175)	(0.131)	(0.044)	(0.061)	(0.078)	(0.000)	(0.016)		
SWE	0.819	0.195^{**}	0.012^{*}	-0.079^{**}	-0.044	-0.000	-0.006	0.296	1.945
	(0.238)	(0.093)	(0.007)	(0.033)	(0.042)	(0.000)	(0.006)		
USA	0.826	0.222^{*}	-0.003	-0.078^{***}	0.006	-0.000	0.000	0.270	2.017
	(0.905)	(0.126)	(0.031)	(0.025)	(0.080)	(0.000)	(0.003)		

Standard errors are in parenthesis. ***, ** and * denote the level of significance at 0.01%, 0.05% and 0.1%, respectively.

Next we attempt some more general results from panel regressions. On the basis of the above preliminary least squares regressions, we estimate dynamic panel regressions to exploit additionally the information along the cross-section dimension to obtain more robust results, as well as to identify potential differences between particular country sub-samples. As sub-samples we consider EMU and non-EMU countries, as well as Northern and Southern EMU countries. More specifically, we identify as "Northern" EMU countries Austria, Belgium, Germany, Finland, France and the Netherlands, and as "Southern" EMU countries Spain, Greece, Italy and Portugal.

Given the auto-regressive structure of the regression model given by eq.(11), we use the GMM estimator for dynamic panels proposed by Arellano and Bover (1995). Accordingly, in order to take care of the country-specific fixed effects in the dynamics panel structure, the variables are subject to a forward orthogonal deviation transformation, where each observation is subtracted by the mean of all future observations. This has the advantage of eliminating the fixed effects in a consistent manner while keeping the lags of the variables available as valid instruments. As dynamic instrumental variables, we use the first and second lags of the explanatory variables comprised in $\mathbf{x}_{it}$.

Table 2: Dynamic Panel Regression Results. Sample: 1981Q1 – 2013Q2

	y_{t-1}	b_{t-1}	f_{t-1}	r_{t-1}	b^2_{t-1}	f^2_{t-1}	J-Stat	J-Prob
All	0.239^{***}	-0.004^{*}	-0.075^{***}	0.004	-0.000	-0.004^{***}	522.291	0.000
	(0.021)	(0.002)	(0.006)	(0.006)	(0.000)	(0.001)		
EMU	0.327^{***}	-0.004^{*}	-0.052^{***}	0.007	-0.000	-0.006^{***}	477.800	0.000
	(0.019)	(0.002)	(0.007)	(0.006)	(0.000)	(0.001)		
Non-EMU	0.161^{***}	0.002	-0.088^{***}	0.005	-0.000	-0.002	386.598	0.369
	(0.020)	(0.002)	(0.005)	(0.005)	(0.000)	(0.001)		
North EMU	0.272^{***}	-0.002	-0.068^{***}	-0.001	-0.000	-0.002^{**}	426.566	0.043
	(0.019)	(0.001)	(0.006)	(0.004)	(0.000)	(0.001)		
South EMU	0.226^{***}	-0.002	-0.011^{**}	-0.024^{***}	-0.000^{***}	-0.016^{***}	321.283	0.501
	(0.009)	(0.002)	(0.005)	(0.003)	(0.000)	(0.001)		

Standard errors are in parenthesis. ***, ** and * denote the level of significance at 1%, 5% and 10%, respectively.

Table 2 reports our estimation results of these dynamic panel regressions. Our empirical findings can be summarized as follows: On the one hand, the coefficients of the lagged GDP growth rate are found to be statistically significant, of the expected sign and remarkably robust concerning their numerical value across all panels but the South EMU panel. Accordingly, GDP growth at date t depends positively on the GDP growth in the previous period. On the other hand, the coefficient of the debt-to-GDP ratio is found as negative and statistically significant only in the panel of all countries considered, as well as the panel of all EMU countries. However, the validity of the instrument sets and regression estimates is not corroborated by the J-statistics at standard significance levels, in contrast to the $J-$statistic associated with the dynamic panel regressions for the Non-EMU, North-EMU and South-EMU subgroups.

This supports the differentiated analysis between these country subgroups and thus the validity of the estimates of the corresponding dynamic panel regressions, despite their reduced cross-sectional dimension. Focusing thus on these last three dynamic panel regressions, our estimates suggest that the linear effect of financial stress on GDP growth is negative and statistically significant for the three considered subgroups, being the linear effect largest in Non-EMU countries and smallest in the South EMU countries. This effect is however overturned by the significantly larger coefficient of the quadratic terms f^2_{t-1} in the South-EMU panel, which represents the countries more associated to the model variant 2 of the above section. This corroborates the notion that financial market stress may affect in a more nonlinear manner the Southern as the Northern EMU countries.

As next we use country-specific and panel threshold regressions of EMU and non-EMU countries, as well as of Northern and Southern EMU countries. Specifically, we consider the following dynamic threshold regression specification with the financial stress index not only as the threshold variable, but also as a regime dependent regressor, i.e.

$$\begin{aligned} y_{i,t} &= \alpha_y y_{i,t-1} + \alpha_r r_{i,t-1} + \alpha_b b_{i,t-1} \\ &\quad + \beta^L_f f_{i,t-1} I(f_{i,t-1} \leq \gamma) + \beta^H_f f_{i,t-1} I(f_{i,t-1} > \gamma) + \delta I(f_{i,t-1} \leq \gamma) + \mu_i + \varepsilon_{i,t} \end{aligned} \tag{12}$$

where $i = 1, \dots, N$ denotes a country-index, $t = 1, \dots, T$ the time index, μ_i a country-specific fixed effect, ε_{it} an i.i.d. country-specific random disturbance with zero mean and a variance $\sigma^2_{\varepsilon i}$, and $I(\cdot)$ is an indicator function which takes on the value of one if the condition in its argument holds and zero otherwise. Hence, the coefficients β^L and β^H represent the effect of $b_{i,t-1}$ on the dependent variable $y_{i,t}$ for $f_{i,t-1} \leq \gamma^f$ and $f_{i,t-1} > \gamma^f$, respectively. δ is the difference between the intercept in regime H, i.e. μ_i, and the intercept in regime L, i.e. $\mu_i + \delta$, see Bick (2010).

As previously mentioned, we estimate the regression model described by eq.(12) using both single-equation and panel estimation techniques. Concerning the single-equation regressions (where of course the index i is given and the information comes only from the variation in the time index t), we follow Caner and Hansen (2004) and employ their IV estimator to account for the possible endogeneity of the variables comprised in x_{it} and z_{it}. Further, also following Caner and Hansen (2004) the threshold value which determines the switch of the indicator function $I(\cdot)$ is identified as the value which minimizes the residual sum of squares.

Concerning the panel dimension of our analysis, despite the fact that, as recently pointed out by Baum et al. (2014), a full distribution theory for such type of models has not been developed so far,[34] we estimate the dynamic threshold panel regression described by eq.(12) following Kremer et al. (2013), who extended the IV threshold estimation methodology developed in Caner and Hansen (2004) for the panel case using the data set of 13 countries spanning the same period (1981Q1 to 2013Q2) (see also Proaño et al. 2014).

Table 3 reports the estimation results for this specification, using the same country subgroups as in the previous regressions. As in the previous specification, despite the significant heterogeneity of the country-specific threshold values resulting from the single-equation regressions, the panel estimates of the different country subgroups are remarkably similar to each other with an average value of about 2.00. The parameter estimates for the lagged endogenous variable and for the interest rate are also broadly in line with the findings of the previous specifications.

The regime independent estimate of the effect of the debt-to-GDP ratio on growth also reveals some interesting insights in the context of our previous finding that, at a given state of the financial markets, high indebtedness impairs growth only in the stand-alone countries of our sample and not in EMU countries. Yet, the results obtained in the present specification suggest that, overall, the debt-to-GDP ratio, here assumed to affect growth linearly and thus independently from its actual level, impairs growth more in the EMU countries, in particular the Northern EMU countries, than in the stand-alone countries.

Concerning the regime-dependent regressor in this equation, namely the FSI, our estimates suggest that the negative effect of the FSI on GDP growth is higher in absolute terms in the high financial stress regime than in the low financial stress regime in most countries considered. Comparing EMU and non-EMU countries reveals that, during times of low financial stress, the FSI has a more pronounced growth effect in the former group than in the latter. During times of high financial stress, the estimated coefficients are similar in both groups. Further, within the EMU we find similar growth effects of financial stress for both the Northern and Southern EMU countries for the low-stress regime. Yet, for the high-stress regime the growth effect of the FSI is rather pronounced in the Southern EMU

[34]The distribution theory developed in Hansen (1999) for threshold panels applies only for the case of non-dynamic panels.

Table 3: Country-specific and panel dynamic threshold GMM estimation results with the financial stress index as the threshold variable and the financial stress index as a regime dependent regressor.

	γ	α_y	α_r	α_b	β_f^L	β_f^H	δ
	COUNTRY-SPECIFIC THRESHOLD REGRESSIONS						
	Northern EMU countries						
AUT	1.35	0.200*** (0.030)	−0.006** (0.002)	−0.063*** (0.012)	−0.051*** (0.011)	−0.310*** (0.011)	−0.921*** (0.042)
BEL	3.60	0.594*** (0.027)	0.001 (0.001)	−0.006*** (0.002)	0.000 (0.004)	0.011 (0.020)	0.269 (0.178)
DEU	-0.22	−0.092*** (0.031)	−0.012*** (0.002)	−0.164*** (0.023)	−0.120*** (0.031)	−0.254*** (0.017)	−0.543*** (0.108)
FRA	1.97	0.366*** (0.015)	−0.015*** (0.001)	−0.016* (0.008)	−0.077*** (0.008)	0.033** (0.016)	0.639*** (0.075)
NLD	1.12	−0.051 (0.036)	−0.001 (0.003)	−0.013 (0.010)	−0.108*** (0.023)	−0.216*** (0.013)	−0.667*** (0.063)
	Southern EMU countries						
ESP	0.22	0.129*** (0.043)	0.008*** (0.002)	−0.013 (0.012)	0.075*** (0.025)	0.046*** (0.016)	0.518*** (0.075)
ITA	-0.65	0.452*** (0.028)	−0.002* (0.001)	0.013** (0.007)	−0.128*** (0.013)	−0.086*** (0.012)	−0.452*** (0.043)
	non-EMU countries						
AUS	-0.43	−0.070* (0.036)	0.019*** (0.003)	−0.056*** (0.006)	−0.052* (0.026)	0.020 (0.013)	0.207*** (0.053)
CAN	-0.13	0.438*** (0.032)	0.006*** (0.002)	−0.048*** (0.007)	−0.059*** (0.021)	−0.110*** (0.016)	−0.383*** (0.072)
DNK	-0.30	−0.217*** (0.030)	0.016*** (0.004)	0.002 (0.011)	−0.155*** (0.038)	−0.272*** (0.033)	−0.561*** (0.086)
GBR	0.27	0.354*** (0.019)	0.001 (0.001)	0.015** (0.008)	0.049*** (0.015)	−0.069*** (0.010)	0.277*** (0.045)
JPN	2.89	0.109*** (0.028)	−0.007*** (0.001)	0.021 (0.018)	0.006 (0.012)	−0.051 (0.056)	0.774*** (0.221)
USA	1.58	0.271*** (0.024)	0.005** (0.002)	0.032*** (0.005)	−0.009 (0.009)	−0.018 (0.015)	0.452*** (0.061)
	PANEL THRESHOLD REGRESSIONS						
All	1.86	0.253*** (0.027)	0.003 (0.006)	−0.001 (0.001)	−0.029*** (0.010)	−0.072** (0.015)	0.138* (0.079)
EMU	1.84	0.310*** (0.026)	0.003 (0.008)	−0.004** (0.001)	−0.085** (0.033)	−0.070*** (0.009)	−0.226** (0.101)
non-EMU	2.06	0.131*** (0.032)	0.007 (0.008)	0.000 (0.001)	−0.045*** (0.012)	−0.079*** (0.016)	0.288*** (0.094)
North EMU	1.97	0.243*** (0.026)	0.006 (0.009)	−0.005*** (0.001)	−0.017* (0.010)	−0.048** (0.021)	0.270*** (0.097)
South EMU	2.45	0.277*** (0.008)	−0.005* (0.003)	−0.000 (0.001)	−0.026*** (0.006)	−0.203*** (0.016)	−0.794*** (0.074)

Standard errors are in parenthesis. ***, ** and * denote the level of significance at 1%, 5% and 10%, respectively.

countries. As expected, high financial stress seems to have had a strong effect on growth in these countries by triggering stronger adverse macro feedback effects.

In the context of the previous findings, these results imply that the FSI seems to be an important source of non-linearities in its own relationship to economic activity. Since we control for the growth effect of sovereign debt, the non-linear effects of the FSI on growth are independent of the sovereign debt-to-GDP ratio. This has important implications especially for the Southern EMU countries. Indeed, the estimation results of the last regression additionally suggest that much of the slowdown in economic growth over the recent years has been caused by high financial stress itself – and the possibly triggered macro feedback effects – and not directly by the increased level of sovereign debt.

So, as to our above three model variants of a transition to a high sovereign default risk countries it is not alone the increased level of sovereign leveraging, but rather the many feedback loops driving the financial stress, sovereign debt and output, moving the country closer to default risk. Among them are important financial variables and macro feedback effects that make the destabilizing forces endogenous. But we should also note that similar mechanisms and feedback loops might work to move the economy out of the default risk trap. Yet, only with a more fully specified macro model which reveals the feedback variables and loops one can observe a variety of escape routes and one can make appropriate policy suggestions.

4 Conclusions

This paper studied the macroeconomic consequences of a rise in the sovereign default risk and the overall financial market stress. To study these issues, we employed a theoretical model with two equilibria, one default risk equilibrium and one no-default equilibrium. We showed that such a framework is a useful modeling devise to understand default and non-default environments, and the switch between those two, as well as the role of macroeconomic amplification effects. As previously discussed, such model variants can also be solved when such amplifying macroeconomic feedback loops are operating in a finite decision horizon framework through the NMPC algorithm. We showed that this type of non-linear macroeconomic models are useful to study recent financial turmoil episodes, such as the US financial and the EU sovereign debt crises. Indeed, it thus seems worthwhile to study earlier historical experiences on debt reduction policies – for example the US Post-WW II strategy to reduce its sovereign debt, Mexico's debt reduction after 1994, the Latin American debt of the 1990s, Argentinean debt reduction after 2001, and the debt reduction for African countries a decade ago – more extensively from the viewpoint of macroeconomic framework such as the one outlined in this paper. Within such frameworks alternative economic policies can be studied which could allow the concerned economies to escape from the threat of sovereign default and move toward a more stable no-default macroeconomic environment.

As to the current regime, the Euro-Area finds itself in, a low growth regime with potentially rising financial stress again, monetary policy needs to be tailored toward this situation. In such regime, a general increase of the balance sheets of the Central Bank through Quantitative Easing may not be sufficient to get lending and economic activity again triggered. Specific credit policies are needed to aim at specific risk factors and specific bottlenecks. Liquidity mismatches resulting from such a regime will not disappear if interbank lending rates stay high in the near future. Moreover, the fragile situation of the balance sheets of households, firms and the government may not get solved by extensive purchases of treasure bonds, such as the OMT program of the ECB suggests. If the financial system is still uncapable to transmit the expansionary effects of such a general purchase of government bonds by the ECB to the private sector, more selective credit policies to affect specific credit spreads and funding cost and quantity measures to encourage credit flows to overcome credit bottlenecks might be needed. More research on specific risk factors and how to reduce their impact on economic activity appears to be advisable.

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Appendix: Numerical Procedure

For the numerical solution of the optimal control problem we do not apply here the dynamic programming (DP) approach as in Ernst and Semmler (2010). Though DP method has the advantage that a global solution to the optimal control problem can be found, by first computing an approximation to the optimal value V and then the optimal control, and its time path, is computed from V. For a detailed description of the specifics of the DP algorithm we are using we refer to Grüne and Semmler (2004). The main disadvantage of DP, however, is that its numerical effort typically grows exponentially with the dimension of the state variable. Hence, even for moderate state dimensions it may be impossible to compute a solution with reasonable accuracy.[35]

A remedy to this problem can be obtained by using nonlinear model predictive control (NMPC). Instead of computing the optimal value function for all possible initial states, NMPC only computes single (approximate) optimal trajectories. In order to describe the method, let us abstractly write the optimal decision problem as

$$\text{maximize} \quad \int_0^\infty e^{-\rho t}\ell(x(t),u(t))dt,$$

where $x(t)$ satisfies $\dot{x}(t) = f(x(t),u(t))$, $x(0) = x_0$ and the maximization takes place over a set of admissible control functions. By discretizing this problem in time, we obtain an approximate discrete time problem of the form

$$\text{maximize} \quad \sum_{i=0}^{\infty} \beta^i \ell(x_i,u_i), \tag{13}$$

where the maximization is now performed over a sequence u_i of control values and the sequence x_i satisfies $x_{i+1} = \Phi(h,x_i,u_i)$, Here $h > 0$ is the discretization time step, $\beta = e^{-\rho h}$ and Φ is a numerical scheme approximating the solution of $\dot{x}(t) = f(x(t),u(t))$ on the time interval $[ih,(i+1)h]$. For details and references in which the error of this discretization is analyzed we refer to Grüne and Semmler (2004).

The idea of NMPC now lies in replacing the maximization of the infinite horizon functional (13) by the iterative maximization of finite horizon functionals

$$\text{maximize} \quad \sum_{k=0}^{N} \beta^i \ell(x_{k,i},u_{k,i}), \tag{14}$$

for a truncated finite horizon $N \in \mathbb{N}$ with $x_{k+1,i} = \Phi(h,x_{k,i},u_{k,i})$ and the index i indicates the number of the iteration, cf. the algorithm below. Note that neither β nor ℓ nor Φ changes when passing from (13) to (14), only the optimization horizon is truncated.

Problems of type (14) can be efficiently solved numerically by converting them into a static nonlinear program and solving them by efficient NLP solvers, see. Grüne and Pannek (2012). In our simulations, we have used a discounted variant of the MATLAB routine nmpc.m available from

[35] Another global algorithm that works with gridding and computation of the value function and computation of the decision variables at each grid point, is used in Gavin et al. (2013), where a New Keynesian model is solved globally. They point out quite different solutions far from the steady state as compared to close to the steady state. Thus, they also show that nonlinearities matter. Yet for their algorithm it also holds that there is a curse of dimension.

www.nmpc-book.com, which uses MATLAB's fmincon NLP solver in order to solve the resulting static optimization problem.

Given an initial value x_0, an approximate solution of (1) can now be obtained by iteratively solving (2) as follows:

(1) for i=1,2,3,...

(2) solve (2) with initial value $x_{0,i} := x_i$ and denote the resulting optimal control sequence by $u^*_{k,i}$

(3) set $u_i := u^*_{0,i}$ and $x_{i+1} := \Phi(h, x_i, u_i)$

(4) end of for-loop

This algorithm yields an infinite trajectory x_i, $i = 1, 2, 3, \ldots$ whose control sequence u_i consists of all the first elements $u^*_{0,i}$ of the optimal control sequences for the finite horizon subproblems (2).

Under appropriate assumptions on the problem, it can be shown that the solution (x_i, u_i) (which depends on the choice of N in (2) converges to the optimal solution of (1) as $N \to \infty$. The main requirement in these assumptions is the existence of an optimal equilibrium for the infinite horizon problem (1). If this equilibrium is known, it can be used as an additional constraint in (2), in order to improve the convergence properties, see Angeli, Amrit and Rawlings (2012).

However, recent results have shown that without a priory knowledge of this equilibrium this convergence can also be ensured, see Grüne (2012), and this is the approach we use in the computations in this paper. It should be noted that the references just cited treat averaged instead of discounted infinite horizon problems. However, we conjecture that the main proofs carry over to the discounted case details of which will be addressed in future research. In any case, the solution generated by NMPC will always provide a lower bound for the true optimal solution. The procedure also allows for irregular impacts on the dynamics of the state variables and regime switches.[36]

[36]Note that in DSGE models regime switches are also perceived as something likely to occur which some literature starts to explore now, see Farmer et al. (2009).