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Income Differences and Input-Output Structure

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Abstract

We consider a multi-sector general equilibrium model with IO linkages, sector-specific productivities and tax rates. Using tools from network theory, we investigate how the IO structure interacts with productivities and taxes in the determination of aggregate income. We show that aggregate income is a simple function of the first and second moments of the distribution of the IO multipliers, sectoral productivities and sectoral tax rates. We then estimate the parameters of the model to fit their joint empirical distribution. Poor countries have more extreme distributions of IO multipliers than rich economies: there are a few high-multiplier sectors, while most sectors have very low multipliers; by contrast, rich countries have more sectors with intermediate multipliers. Moreover, the correlations of these with productivities and tax rates are positive in poor countries, while being negative in rich ones. The estimated model predicts cross-country income differences extremely well, also out-of-sample. Finally, we perform a number of counterfactuals and compute optimal tax rates.

KEY WORDS: input-output structure, networks, productivity, cross-country income differences

JEL CLASSIFICATION: O11, O14, O47, C67, D85

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1 Introduction

One of the fundamental debates in economics is about how important differences in factor endowments – such as physical or human capital stocks – are relative to aggregate productivity differences in terms of explaining cross-country differences in income per capita. The standard approach to address this question is to specify an aggregate production function for value added (e.g., Caselli, 2005). Given data on aggregate income and factor endowments and the imposed mapping between endowments and income, one can back out productivity differences as a residual that explains differences between predicted and actual income. However, this standard approach ignores that GDP aggregates value added of many economic activities which are connected to each other through input-output linkages.[1] By contrast, a literature in development economics initiated by Hirschman (1958) has long emphasized that economic structure is of first-order importance to understand cross-country income differences.[2]

Consider, for example, a productivity increase in the Transport sector. This reduces the price of transport services and thereby increases productivity in sectors that use transport services as an input (e.g., Mining). Increased productivity in Mining in turn increases productivity of the Steel sector by reducing the price of iron ore, which in turn increases the productivity of the Transport Equipment sector. In a second-round effect, the productivity increase in Transport Equipment improves productivity of the Transport sector and so on. Thus, input-output (IO) linkages between sectors lead to multiplier effects. The IO multiplier of a given sector summarizes all these intermediate effects and measures by how much aggregate income will change if productivity of this sector changes by one percent. The size of the sectoral multiplier effect depends to a large extent on the number of sectors to which a given sector supplies and the intensity with which its output is used as an input by the other sectors.[3] We document that there are large differences in IO multipliers across sectors – e.g., most infrastructure sectors, such as Transport and Energy, have high multipliers because they are used as inputs by many other sectors,[4] while a sector such as Textiles – which does not provide inputs to many sectors – has a low multiplier. As a consequence, low productivities in different sectors will have very distinct effects on aggregate income, depending on the size of the sectoral IO multiplier.

[1] An important exception that highlights sectoral TFP differences is the recent work on dual economies. This literature finds that productivity gaps between rich and poor countries are much more pronounced in agriculture than in manufacturing or service sectors and this fact together with the much larger value added or employment share of agriculture in poor countries can explain an important fraction of cross-country income differences.

[2] More recent contributions highlighting the role of economic structure for aggregate income are Ciccone (2002) and Jones (2011 a,b).

[3] The intensity of input use is measured by the IO coefficient, which states the cents spent on that input per dollar of output produced. There are also higher-order effects, which depend on the number and the IO coefficients of the sectors to which the sectors that use the initial sector's output as an input supply.

[4] The view that infrastructure sectors are of crucial importance for aggregate outcomes has also been endorsed by the World Bank. In 2010, the World Bank positioned support for infrastructure as a strategic priority in creating growth opportunities and targeting the poor and vulnerable. Infrastructure projects have become the single largest business line for the World Bank Group, with $26 billion in commitments and investments in 2011 (World Bank Group Infrastructure Update FY 2012-2015).

In this paper, we address the question how differences in economic structure across countries – as captured by IO linkages between sectors – affect cross-country differences in aggregate income per capita. To this end, we combine data from the World Input-Output Database (Timmer, 2012) and the Global Trade Analysis project (GTAP Version 6), in order to construct a unique dataset of IO tables, sectoral total factor productivities and sectoral tax rates for a large cross section of countries in the year 2005.[5] With this data in hand, we investigate how the IO structure interacts with sectoral TFP differences and taxes to determine aggregate per capita income. First, we document that in all countries there is a relatively small set of sectors which have very large IO multipliers and whose performance thus crucially affects aggregate outcomes. Moreover, despite this regularity, we also find that there do exist substantial differences in the network characteristics of IO linkages between poor and rich countries. In particular, low-income countries typically have a very small number of average and high-multiplier sectors, while high-income countries have a more dense input-output network. To visualize these differences, in Figure 1 we plot a graphical representation of the IO matrices of two countries: Uganda (a very poor country with a per capita GDP of 964 PPP dollars in 2005) and the U.S. (a major industrialized economy with a per capita GDP of around 42,500 PPP dollars in 2005). The columns of the IO matrix are the producing sectors, while the rows are the sectors whose output is used as an input. Thus, a dot in the matrix indicates that the column sector uses some of the row sector's output as an input and a blank space indicates that there is no significant connection between the two sectors.[6]

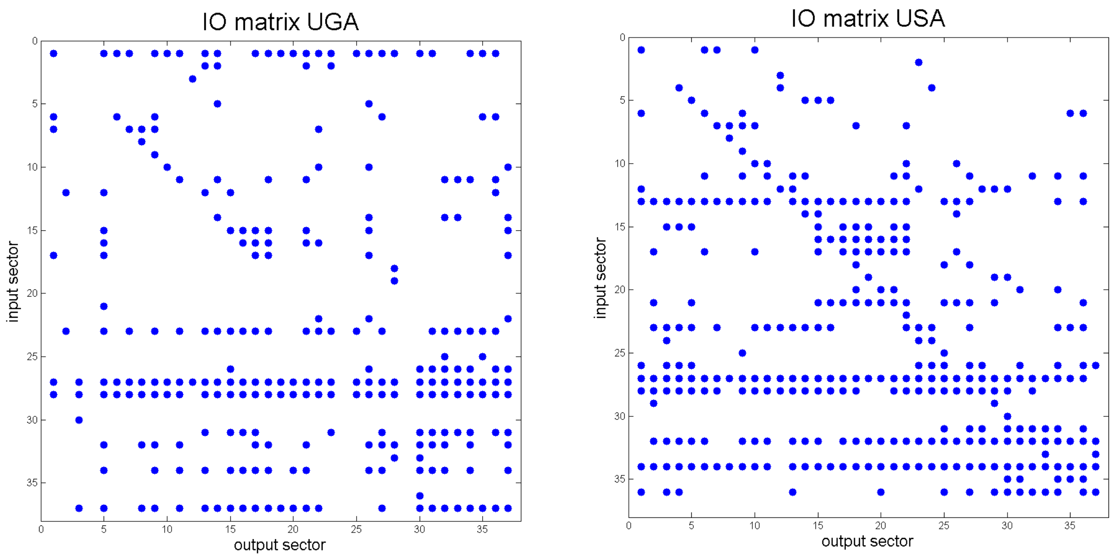

Figure 1: IO-matrices by country: Uganda (left), USA (right)

By comparing the matrices it is apparent that in Uganda there are only four sectors that supply to

[5]Data on sectoral TFPs and tax rates are available for 39 countries and data on IO tables for 70 countries.

[6]Data are from GTAP version 6, see the data appendix for details. The figure plots IO coefficients defined as cents of industry j output (row j) used per dollar of output of industry i (column i). To make the figure more readable, we only plot linkages with at least 2 cents per dollar of output.

most other sectors.[7] These are Agriculture (row 1), Electricity (row 23), Wholesale and Retail Trade (row 27), and Transport (row 28). These sectors are the high-IO-multiplier sectors, where a change in sectoral productivity has a relatively large effect on aggregate output. Most other sectors are quite isolated in Uganda, in the sense that their output is not used as an input by many sectors. In contrast, the U.S. has a much larger number of sectors that supply to many others: Chemicals (row 13), Electricity (row 23), Construction (row 26), (Wholesale and Retail) Trade (row 27), Transport (row 28), Financial Services (row 32), and Business Services (row 34), among others. This difference in IO structure between rich and poor countries has important implications for aggregate income differences: in Uganda changes in the productivity of a few crucial sectors have large effects on aggregate income, while productivity in most sectors does not matter much for aggregate outcomes, because these sectors are isolated. By contrast, in the U.S. productivity levels of many more sectors have a significant impact on GDP because the IO network is much denser. To some extent this is good news for low-income countries: in those countries policies that focus on a few crucial sectors can have a large effect on aggregate income, while this is not true for middle-income and rich countries.

Having described the salient features of cross-country differences in IO structure, we model IO structures using tools from network theory. We analytically solve a multi-sector general equilibrium model with IO linkages, sector-specific productivities and tax rates. We then estimate this model using a statistical approach that employs the moments of the distributions instead of actual values. The crucial advantage of this strategy is that it allows us to derive a simple closed-form expression for aggregate per capita income that conveniently summarizes the interactions between IO structure, productivities and tax rates, without having to deal with the complicated input-output matrices directly: aggregate income is a simple function of the first and second moments of the distribution of IO multipliers, sectoral productivities and sectoral tax rates.[8] Higher average IO multipliers and average sectoral productivities have a positive effect on income per capita, while higher average tax rates reduce it. Moreover, a positive correlation between sectoral IO multipliers and productivities increases income, while a positive correlation between IO multipliers and tax rates has the opposite effect. This is intuitive: high sectoral productivitities have a larger positive impact if they occur in high-multiplier sectors, while high tax rates in high-multiplier sectors are very distortionary. We estimate the parameters of the model to fit the joint

[7]See Table A-3 in the Supplementary Appendix for the complete list of sectors.

[8]In the light of Hulten's (1978) results, one may be concerned that using a full structural general equilibrium model and exploiting the information contained in the entire IO matrix adds little compared to using only production data and computing aggregate TFP as a weighted average (where the adequate 'Domar' weights correspond to the shares of sectoral gross output in GDP) of sectoral productivities. Absent distortions, Domar weights equal sectoral IO multipliers and summarize all the direct and indirect effect of IO linkages. However, as Basu and Fernald (2002) show, in the presence of distortions the connection between sectoral productivities and aggregate TFP is substantially more complicated and effectively depends on the distribution of sectoral distortions and intermediate input use. Moreover, such a reduced-form approach does not allow to assess which features of the IO structure matter for aggregate outcomes. Finally, only with a structural approach such as the one adopted in this paper one can compute counterfactual outcomes due to changes in IO structure, productivities or tax rates.

empirical distribution of IO multipliers, productivities and tax rates for the countries in our sample, allowing them to vary with income per capita in order to account for cross-country differences in these characteristics. We find that low-income countries have more extreme distributions of IO multipliers: while most sectors have very low multipliers, there are a few very high-multiplier sectors. In contrast, rich countries have relatively more sectors with intermediate multipliers. Moreover, while sectoral IO multipliers and productivities are positively correlated in low-income countries, they are negatively correlated in high-income ones. Similarly, IO multipliers and tax rates are positively correlated in poor countries and negatively correlated in rich ones.

With the parameter estimates in hand, we use our closed-form expression for income per capita as a function of IO structure to predict income differences across countries. In contrast to standard development accounting, where the model is exactly identified, this provides an over-identification test because parameter estimates have been obtained using data on IO multipliers, productivities and taxes only. We find that our model predicts cross-country income differences extremely well both within the sample of countries that we have used to estimate the parameter values and also out of sample, i.e., in the full Penn World Tables sample (around 150 countries). Our model predicts up to 97% of the cross-country variation in relative income per capita, which is extremely large compared to standard development accounting. Moreover, our model with IO linkages does much better in terms of predicting income differences than a model that just averages estimated sectoral productivities and ignores IO structure. In fact, such a model actually over-predicts cross-country income differences. The reason is that the large sectoral TFP differences that we observe in the data are mitigated by the IO structure, since very low productivity sectors tend to be isolated in low- and middle-income countries. Thus, if we measure aggregate productivity levels as an average of sectoral productivities, income levels of middle- and low-income countries would be significantly lower than they actually are.

Moreover, we perform a number of counterfactuals. First, we impose the IO structure of the U.S. on all countries, which forces them to use the relatively dense U.S. IO network. We find that the U.S. IO structure would significantly reduce income of low- and middle-income countries. For a country at 40% of the U.S. income level (e.g., Mexico) per capita income would decline by around 40% and income reductions would amount to up to 80 % for the world's poorest economies (e.g., Congo). The intuition for this result is that poor countries tend to have higher-than-average relative productivity levels (relative to those of the U.S. in the same sector) in precisely those sectors that have higher IO multipliers[9], while their typical sector is quite isolated from the rest of the economy. This implies that they do relatively well given their really low productivity levels in many sectors. Consequently, if we impose the much

[9] An important exception is agriculture, which, in low-income countries, has a high IO multiplier and a below-average productivity level.

denser IO structure of the U.S. on poor countries – which would make their typical sector much more connected to the rest of the economy – they would be significantly poorer.

Second, we impose that sectoral IO multipliers and productivities are uncorrelated. This scenario would again hurt low-income countries, which would lose up to 50% of their per capita income, because they have above average productivity levels in high-multiplier sectors. By contrast, high-income countries would gain up to 50% in terms of income per capita, since they tend to have below-average productivity levels in high-multiplier sectors.

Third, reducing distortions from taxes on gross output would have more modest effects. If low-income countries did not have above-average tax rates in high-multiplier sectors, they would gain up to 4% of per capita income, while imposing the tax structure of a country at the U.S. income level on them (with a relatively low variance of tax rates and lower tax rates in high-multiplier sectors) would increase their income by up to 6%.

We also study optimal taxation and the welfare gains from moving from the current tax rates to an optimal tax system that keeps tax revenue constant. Our results suggest that when the government is concerned with maximizing GDP per capita subject to a given level of tax revenue, the actual distribution of tax rates in rich countries is close to optimum. In poor countries, on the other hand, the mean of the distribution is too low and the variance is too high relative to the optimal values. Furthermore, for a given value of tax variance, a negative correlation of taxes with IO multipliers is optimal. Thus, the actual negative correlation in rich countries and the positive one in poor countries contributes to a larger income gap between countries than the one that would prevail if poor countries had an optimal tax system in place. Some of the poorest countries in the world could gain up to 30 % in terms of income per capita by moving to an optimal tax system.

Finally, we perform a number of robustness checks. First, we show that allowing for an elasticity of substitution between intermediates different from unity – which effectively makes IO coefficients endogenous to equilibrium prices – is hard to reconcile with the data. Moreover, we extend our baseline model and incorporate cross-country differences in final demand structure and imported intermediate inputs; we also differentiate between skilled and unskilled labor inputs. We find that our results are robust to any of these extensions.

1.1 Literature

We now turn to a discussion of the related literature.

Our work is related to the literature on development accounting (level accounting), which aims at quantifying the importance of cross-country variation in factor endowments – such as physical, human or natural capital – relative to aggregate productivity differences in explaining disparities in income per capita across countries. This literature typically finds that both are roughly equally important in accounting for cross-country income differences (see, e.g., Klenow and Rodriguez-Clare, 1997; Hall and Jones, 1999; Caselli, 2005). The approach of development accounting is to specify an aggregate production function for value added (typically Cobb-Douglas) and to back out productivity differences as residual variation that reconciles the observed income differences with those predicted by the model given observed variation in factor endowments. Thus, this approach naturally abstracts from any cross-country differences in the underlying economic structure across countries. We contribute to this literature by showing how aggregate value added production functions can be derived in the presence of input-output linkages that differ across countries. Moreover, we show that incorporating cross-country variation in input-output structure is of first-order importance in explaining cross-country income differences. In complementary work, Grobovsek (2013) incorporates intermediate goods into a two-sector model with intermediates and finds that poor countries have much lower productivities in intermediate compared to final production, which can potentially explain a substantial portion of cross-country income differences.

The importance of intermediate linkages and IO multipliers for aggregate income differences has been highlighted by Fleming (1955), Hirschmann (1958), and, more recently, by Ciccone (2002) and Jones (2011 a,b). The last two authors emphasize that if the intermediate share in gross output is sizable, there exist large multiplier effects: small firm (or industry-level) productivity differences or distortions that lead to misallocation of resources across sectors or plants can add up to large aggregate effects. These authors make this point in a purely theoretical context. While our setup in principle allows for a mechanism whereby intermediate linkages amplify small sectoral productivity differences, we find that there is little empirical evidence for this channel: cross-country sectoral productivity differences estimated from the data are even larger than aggregate ones, and the sparse IO structure of low-income countries helps to mitigate the impact of very low productivity levels in some sectors on aggregate outcomes.

Our finding that sectoral productivity differences between rich and poor countries are larger than aggregate ones is instead similar to those of the literature on dual economies and sectoral productivity gaps in agriculture (Caselli, 2005; Chanda and Dalgaard, 2008; Restuccia, Yang, and Zhu, 2008; Vollrath, 2009; Gollin et al., 2014). Also closely related to our work – which focuses on changes in the IO structure as countries' income level increases – is a literature on structural transformation. It emphasizes sectoral productivity gaps and transitions from agriculture to manufacturing and services as a reason for cross-country income differences (see, e.g., Duarte and Restuccia, 2010 for a recent contribution). However, this literature abstracts from intermediate linkages between industries.

In terms of modeling approach, our paper adopts the framework of the multi-sector real business cycle model with IO linkages of Long and Plosser (1983); in addition we model the input-output structure as a network, quite similarly to the setup of Acemoglu et. al. (2012). In contrast to these studies, which deal with the relationship between sectoral productivity shocks and aggregate fluctuations, we are interested in the question how sectoral productivity *levels* interact with the IO structure to determine aggregate income *levels*. Moreover, while the aforementioned papers are mostly theoretical, we provide a comprehensive empirical study of the impact of cross-country differences in IO structure on income.

Other recent related contributions are Oberfield (2014) and Carvalho and Voigtländer (2014), who develop an abstract theory of endogenous input-output network formation, and Boehm (2014), who focuses on the role of contract enforcement on aggregate productivity differences in a quantitative structural model with IO linkages. Differently from these papers, we do not try to model the IO structure as arising endogenously and we take sectoral productivity differences as exogenous. Instead, we aim at understanding how given differences in IO structure and sectoral productivities translate into aggregate income differences.

The number of empirical studies investigating cross-country differences in IO structure is quite limited. In the most comprehensive study up to that date, Chenery, Robinson, and Syrquin (1986) find that the intermediate input share of manufacturing increases with industrialization and – consistent with our evidence – that input-output matrices become less sparse as countries industrialize. Most closely related to our paper is the contemporaneous work by Bartelme and Gorodnichenko (2014). They also collect data on IO tables for many countries and investigate the relationship between IO linkages and aggregate income. In reduced form regressions of aggregate input-output multipliers on income per worker, they find a positive correlation between the two variables. Moreover, they investigate how distortions affect IO linkages and income levels. Differently from the present paper, they do not use data on sectoral productivities and tax rates and they do not use network theory to represent IO tables. As a consequence, they do not investigate how differences in the distribution of multipliers and their correlations with productivities and tax rates impact on aggregate income, which is the focus of our work. Furthermore, they do not address the question of optimal taxation given the IO structure, while we do.

The outline of the paper is as follows. In the next section we describe our dataset and present some descriptive statistics. In the following section, we lay out our theoretical model and derive an expression for aggregate GDP in terms of the IO structure, sectoral productivities and tax rates. Subsequently, we turn to the estimation and model fit and we present a number of counterfactual results. Then we turn to optimal taxation followed by a number of robustness checks. The final section presents our conclusion.

2 Dataset and descriptive analysis

2.1 Data

IO tables measure the flow of intermediate products between different plants or establishments, both within and between sectors. The ji'th entry of the IO table is the value of output from establishments in industry j that is purchased by different establishments in industry i for use in production.[10] Dividing the flow of industry j to industry i by gross output of industry i, one obtains the IO coefficient γ_{ji}, which states the cents of industry j output used in the production of each dollar of industry i output.

To construct a dataset of input-output tables for a range of high- and low-income countries and to compute sectoral total factor productivities, tax rates and countries' aggregate income and factor endowments, we combine information from three datasets: the World Input-Output Database (WIOD, Timmer, 2012), the Global Trade Analysis Project (GTAP version 6, Dimaranan, 2006), and the Penn World Tables, Version 7.1 (PWT 7.1, Heston et al., 2012).[11]

The first dataset, WIOD, contains IO data for 39 countries classified into 35 sectors in the year 2005. The list of countries and sectors is provided in the Supplementary Appendix Tables A-1 to A-3. WIOD data also provides all the information necessary to compute gross-output-based sectoral total factor productivity: real gross output, real sectoral capital and labor inputs, Purchasing Power Parity (PPP) price indices for sectoral gross output and sectoral factor payments to labor and capital. Moreover, WIOD provides information on sectoral net tax rates (taxes minus subsidies) on gross output. The second dataset, GTAP version 6, contains data for 70 countries and 37 sectors in the year 2004. We use GTAP data to obtain more information about IO tables of low-income countries and we construct IO coefficients for all 70 countries. Finally, the third dataset, PWT 7.1, includes data on income per capita in PPP, aggregate physical capital stocks and labor endowments for 155 countries in the year 2005. In our analysis, PWT data is used to make out-of-sample predictions with our model.

2.2 IO structure

To begin with, we provide some descriptive analysis of the input-output structure of the set of countries in our data. To this end, we consider the sample of countries from the GTAP database. First, we sum IO multipliers of all sectors to compute the aggregate IO multiplier. While a sectoral multiplier indicates the change in aggregate income caused by a one percent change in productivity of one specific sector, the aggregate IO multiplier tells us by how much aggregate income changes due to a one percent change in

[10] Intermediate output must be traded between establishments in order to be recorded in the IO table, while flows that occur within a given plant are not measured.

[11] In the main text we only provide a rough description of the datasets. Details can be found in the Supplementary Appendix.

productivity of all sectors. Figure 2 (left panel) plots aggregate IO multipliers for each country against GDP per capita (relative to the U.S.).

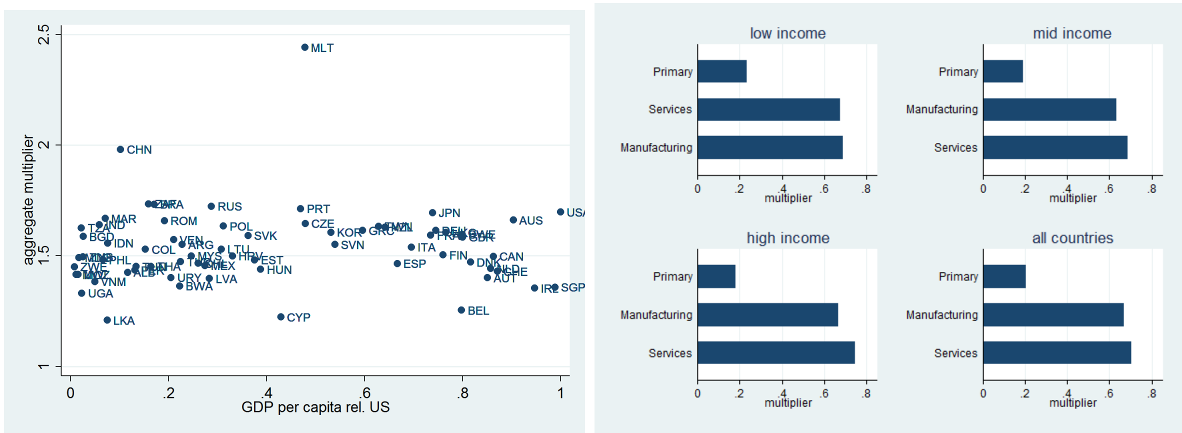

Figure 2: Aggregate IO-multipliers by country (left), sectoral IO-multipliers by income level (right)

We observe that aggregate multipliers for the GTAP sample average around 1.6 and are uncorrelated with the level of income. This implies that a one percent increase in productivity of all sectors raises per capita income by around 1.6 percent on average.[12]

Next, we compute separately the aggregate IO multipliers for the three major sector categories: primary sectors (which include Agriculture, Coal, Oil and Gas Extraction and Mining), manufacturing and services. Figure 2 (right panel) plots these multipliers by income level. Here, we divide countries into low income (less than 10,000 PPP Dollars of per capita income), middle income (10,000-20,000 PPP Dollars of per capita income) and high income (more than 20,000 PPP Dollars of per capita income).

We find that multipliers are largest in services (around 0.65 on average), slightly lower in manufacturing (around 0.62) and smallest in primary sectors (around 0.2). As before, the level of income does not play an important role in this result: the comparison is similar for countries at all levels of income per capita.[13] We conclude that at the aggregate-economy level or for major sectoral aggregates there are no systematic differences in IO structure across countries.

Let us now look at differences in IO structure at a more disaggregate level (37 sectors). To this end, we compute sectoral IO multipliers separately for each sector and country. Figure 3 presents kernel density plots of the distribution of (log) sectoral multipliers for different levels of income per capita.

The following two facts stand out. First, for any given country the distribution of sectoral multipliers is *highly skewed*: while most sectors have low multipliers, a few sectors have multipliers way above the average. A typical low-multiplier sector (at the 10th percentile) has a multiplier of around 0.02 and the

[12] Aggregate multipliers for the WIOD sample are somewhat larger (with a mean of around 1.8) and also uncorrelated with the level of per capita income. A simple regression of the aggregate multipliers from the GTAP sample on those from the WIOD data for the countries for which we can measure both gives a slope coefficient of around 0.8 and the relationship is strongly statistically significant

[13] Very similar results are obtained for the WIOD sample. The only difference is that primary sectors are somewhat more important in low-income countries compared to others.

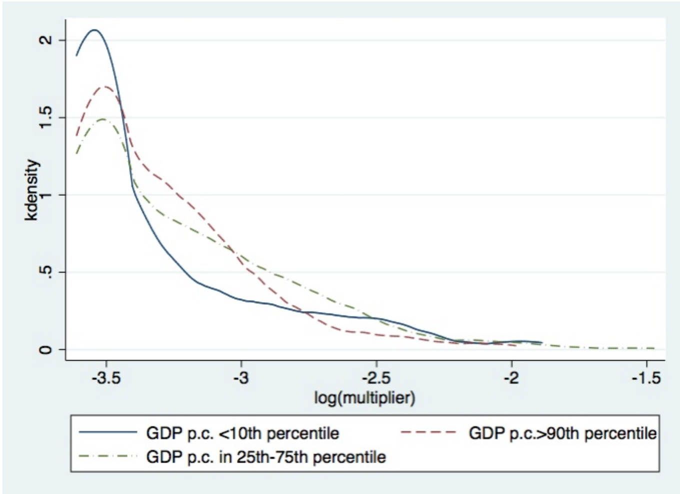

Figure 3: Distribution of sectoral log multipliers (GTAP sample)

median sector has a multiplier of around 0.03. By contrast, a typical high-multiplier sector (at the 90th percentile) has a multiplier of around 0.065, while a sector at the 99th percentile has a multiplier of around 0.134.

Second, the distribution of multipliers in low-income countries is *more skewed towards the extremes* than it is in high-income countries. In poor countries, almost all sectors have very low multipliers and a few sectors have very high multipliers. Differently, in rich countries the distribution of sectoral multipliers has significantly more mass in the center.

Finally, we investigate which sectors tend to have the largest multipliers. We thus rank sectors according to the size of their multiplier for each country. Figure 4 plots sectoral multipliers for a few selected countries, which are representative for the whole sample: a very poor African economy (Uganda (UGA)), a large emerging economy (India (IND)) and a large high-income country (United States (USA)). It is apparent that the distribution of multipliers of Uganda is such that the bulk of sectors have low multipliers, with the exception of Agriculture, Electricity, Trade and Inland Transport. By contrast, a typical sector in the U.S. has a larger multiplier, while the distribution of multipliers of India lies between the one of Uganda and the one of the U.S.[14]

In the lower panels of the same figure we plot sectoral multipliers averaged across countries by income level. Note that while the distributions of multipliers now look quite similar for different levels of income, this is an aggregation bias, which averages out much of the heterogeneity at the country level. From this Figure we see that in low-income countries the sectors with the highest multipliers are Trade,

[14]One might be concerned that the IO structure in poor countries is mismeasured due to the importance of the informal sector and that the size of linkages is understated (manufacturing census and survey data does not include this part of the economy). However, the fact that estimated average multipliers do not differ with GDP per capita and that agriculture has strong IO linkages in developing countries, even though most agricultural establishments are in the informal sector, mitigates this concern. In addition, the distribution of output is heavily skewed towards the largest firms (which operate in the formal economy) and even more so in developing countries (Alfaro et al., 2008), so that the mismeasurement in terms of output and intermediate input demand is small.

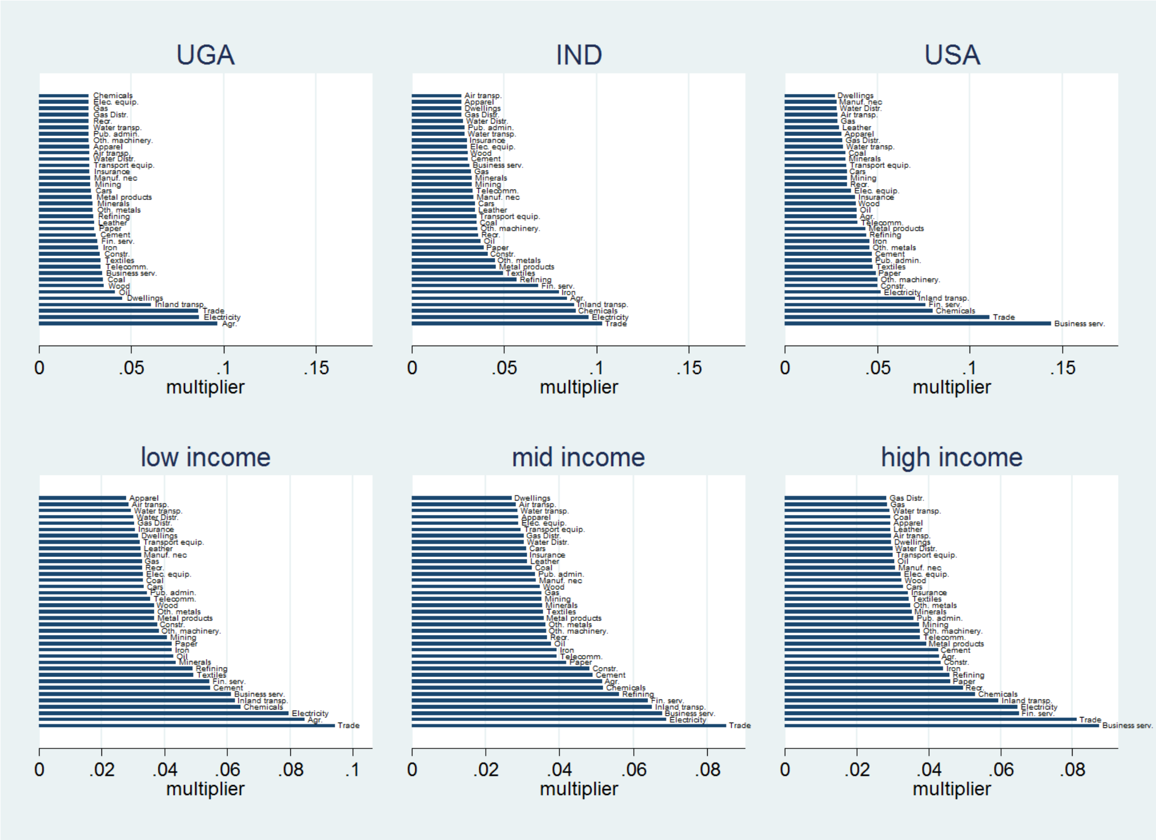

Figure 4: Sectoral IO-multipliers by country (top panel)/ income level (bottom panel)

Electricity, Agriculture, Chemicals, and Inland Transport. Turning to the set of middle- and high-income countries, the most important sectors in terms of multipliers are Trade, Electricity, Business Services, Inland Transport and Financial Services.

Thus, overall the sectors with the highest multipliers are mostly service sectors. Agriculture is one notable exception for countries with an income level below 10,000 PPP dollars, where agricultural products are an input to many sectors. Moreover, in low-income countries Chemicals and Petroleum Refining tend to have a large multiplier, too. In general though, typical manufacturing sectors have intermediate multipliers (around 0.04). Finally, the sectors with the lowest multipliers are also mostly services: Apparel, Air Transport, Water Transport, Gas Distribution and Dwellings (Owner-occupied houses). Given the large number of sectors with low multipliers, the specific sectors differ more across income groups. The figures for individual countries confirm the overall picture.

2.3 Productivities and taxes

We now provide some descriptive evidence on sectoral total factor productivity (TFPs) relative to the U.S., tax rates as well as their correlations with sectoral multipliers. Here, we use the countries in the WIOD sample, because this information is available only for this dataset. In Table 1 we provide means and standard deviations of relative productivities and tax rates by income level, as well as the correlation between multipliers and productivities or tax rates. To compute the correlations, we consider deviations from country means, so they are to be interpreted as within-country correlations. Moreover, in Figure 5 we plot correlations between multipliers and log productivities and tax rates for two selected countries, which are representative for countries at similar income levels: India (IND) and Germany (DEU).

Table 1: Descriptive statistics for TFPs and tax rates

Sample	N	avg. TFP	std. TFP	avg. tax rate	std. tax rate	corr. TFP, mult. (within)	corr. tax, mult. (within)
low income	236	0.445	0.430	0.047	0.041	0.251	0.019
mid income	340	0.619	0.593	0.046	0.036	0.06	-0.092
high income	745	1.109	0.801	0.049	0.034	-0.156	-0.124
all	1,321	0.891	0.765	0.049	0.036	-0.101	-0.034

The following empirical regularities arise. First, average sectoral productivities are highly positively correlated with income per capita, while average tax rates are not correlated with income per capita. Second, in low-income countries productivity levels of high-multiplier sectors are above their average productivity relative to the U.S., while in richer countries productivities in these sectors tend to be below average. This is demonstrated by the examples in Figure 5. For instance, India has productivity levels above its average in the high-multiplier sectors – Chemicals, Inland Transport, Refining and Electricity – while its productivity levels in the low-multiplier sectors such as Car Retailing, Telecommunications and Business Services are below average. An exception is India's high-multiplier sector Agriculture, where the productivity level is very low. This confirms the general view that poor countries tend to have particularly low productivity levels in this sector. In contrast, rich European economies, such as Germany – which according to our data is absolutely more productive than the U.S. in manufacturing sectors – tend to have below average productivity levels in high-multiplier sectors such as Financial Services, Business Services and Transport. Qualitatively the same pattern of correlation is observed between sectoral multipliers and taxes. Low-income countries have above average tax rates in high-multiplier sectors, while high-income countries have below average tax rates in these sectors. India, for example, taxes gross output in high-multiplier sectors such as Inland Transport, Chemicals and Refining relatively heavily compared to its average sector, while Germany taxes the high-multiplier sectors such as Financial or Business Services at below average rates.

3 Theoretical framework

3.1 Model

In this section we present our theoretical framework, which will be used in the remainder of our analysis. Consider a static multi-sector economy with taxes. n competitive sectors each produce a distinct good

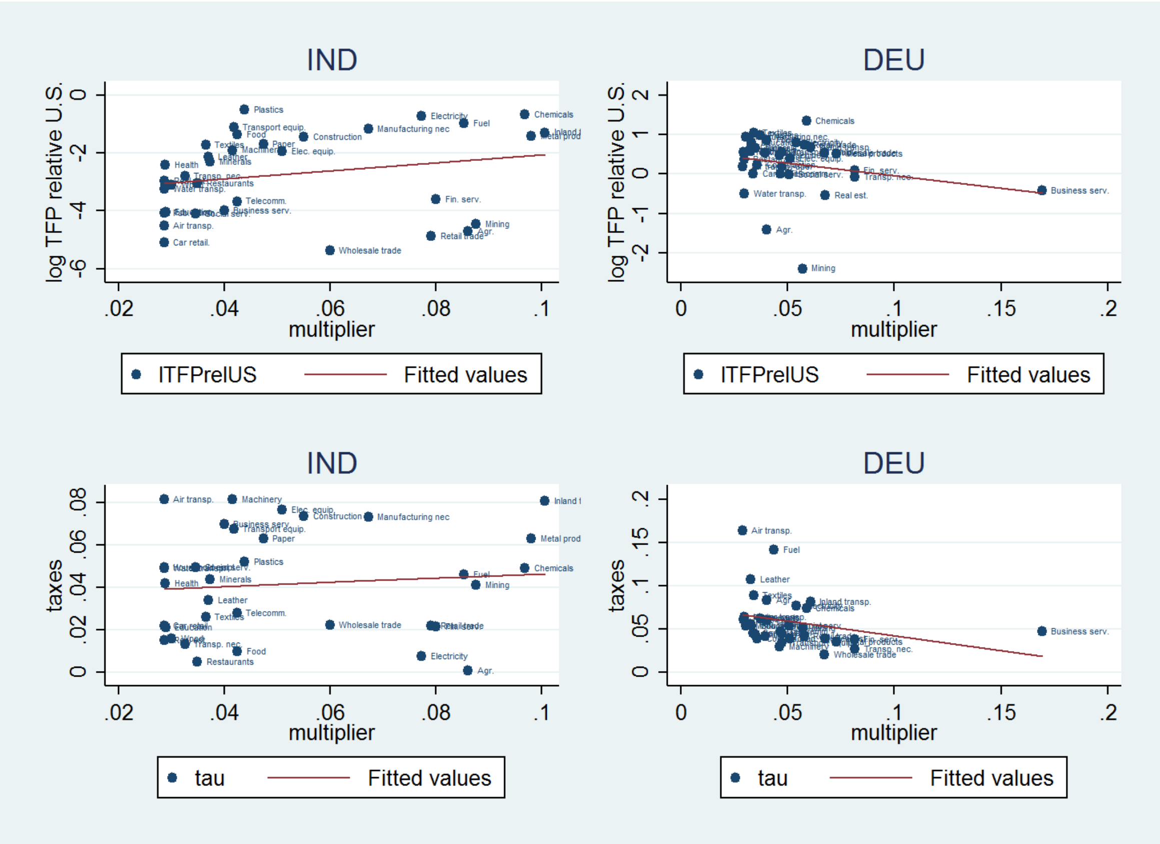

Figure 5: Correlation between IO-multipliers and productivity/taxes

that can be used either for final consumption or as an input for production. The technology of sector $i \in 1:n$ is Cobb-Douglas with constant returns to scale. Namely, the output of sector i, denoted by q_i, is

$$q_i = \Lambda_i \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{1-\gamma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}}, \tag{1}$$

where Λ_i is the exogenous total factor productivity of sector i, k_i and l_i are the quantities of capital and labor used by sector i and d_{ji} is the quantity of good j used in production of good i (intermediate goods produced by sector j).[15] The exponent $\gamma_{ji} \in [0, 1)$ represents the share of good j in the production technology of firms in sector i, and $\gamma_i = \sum_{j=1}^{n} \gamma_{ji} \in (0, 1)$ is the total share of intermediate goods in gross output of sector i. Parameters α, $1 - \alpha \in (0, 1)$ are shares of capital and labor in the remainder of the inputs (value added).

Given the Cobb-Douglas technology in (1) and competitive factor markets, γ_{ji}'s also correspond to the entries of the IO matrix, measuring the value of spending on input j per dollar of production of good i.[16] We denote this IO matrix by $\mathbf{\Gamma}$. Then the entries of the j'th row of matrix $\mathbf{\Gamma}$ represent the values of

[15] In section 6 and Appendix A we consider the case of an open economy, where sectors' production technology employs both domestic and imported intermediate goods.

[16] Strictly speaking, the entries in the IO matrix (IO coefficients) have to be adjusted for taxes. To see this, consider sector i's first-order condition with respect to output of sector j, which is given by: $(1 - \tau_i)\gamma_{ji} p_i q_i = p_j d_{ji}$. Thus, the *empirical* IO coefficients (demand of sector i for sector j per dollar of sector i's output) in *basic* prices $\tilde{p}_i \equiv p_i(1 - \tau_i)$ (excluding transport costs and taxes) are given by $IO^b_{ji} = \frac{\tilde{p}_j d_{ji}}{\tilde{p}_i q_i} = \frac{p_j(1-\tau_j)d_{ji}}{p_i(1-\tau_i)q_i} = \gamma_{ji}(1 - \tau_j)$. Consequently, the IO coefficient of the using sector i in basic prices depends negatively on the tax rate in the supplying sector j. Next, consider the empirical

spending on a given input j per dollar of production of each sector in the economy. On the other hand, the elements of the i'th column of matrix $\mathbf{\Gamma}$ are the values of spending on inputs from each sector in the economy per dollar of production of a given good i.[17]

Resources in the economy are allocated with distortions. In this paper distortions are regarded as sector-specific proportional taxes on gross output. For the main part of the analysis we consider taxes as exogenous reductions in firms' revenue, and the total revenue from taxation is spent on government expenditures. In the last section we endogenize taxes by addressing the problem of optimal taxation. Throughout the paper taxes in sector i are denoted by τ_i. We assume that $\tau_i \leq 1$ and interpret negative taxes as subsidies.

Output of sector i can be used either for final consumption, y_i, or as an intermediate good:

$$y_i + \sum_{j=1}^{n} d_{ij} = q_i \qquad i = 1 : n \tag{2}$$

Final consumption goods are aggregated into a single final good through another Cobb-Douglas production function:

$$Y = y_1^{\frac{1}{n}} \cdot ... \cdot y_n^{\frac{1}{n}}. \tag{3}$$

This aggregate final good is used in two ways, as households' consumption, C, and government consumption, G, that is, $Y = C + G$. Note that the symmetry in exponents of the final good production function implies symmetry in consumption demand for all goods. This assumption is useful as it allows us to focus on the effects of the IO structure and the interaction between the structure and sectors' productivities and tax rates in an otherwise symmetric framework. It is, however, straightforward to introduce asymmetry in consumption demand by defining the vector of demand shares $\boldsymbol{\beta} = (\beta_1, .., \beta_n)$, where $\beta_i \neq \beta_j$ for $i \neq j$ and $\sum_{i=1}^{n} \beta_i = 1$. The corresponding final good production function is then $Y = y_1^{\beta_1} \cdot ... \cdot y_n^{\beta_n}$. This more general framework is analyzed in section 6, where we consider extensions of our benchmark model.

Finally, the total supply of capital and labor in this economy are assumed to be exogenous and fixed

IO coefficients in *user* prices (including taxes and transport costs): $IO_{ji}^{u} = \frac{p_j d_{ji}}{p_i q_i} = \gamma_{ji}(1 - \tau_i)$. Thus, the IO coefficient of sector i in user prices depends negatively on its own tax rate τ_i. To the extent that taxes correspond to actual observable tax rates on gross output, we can adjust empirical IO coefficients for them. Differently, if taxes represent unmeasured distortions or wedges, we are unable to correct IO coefficients for their impact. However, in footnote 24 below we explain that unmeasured distortions do not systematically bias IO multipliers as long as IO coefficients are measured in user prices and production functions are Cobb-Douglas.

[17]According to our notation, the sum of elements in the i'th column of matrix $\mathbf{\Gamma}$ is equal to γ_i, the total intermediate share of sector i.

at the levels of K and 1, respectively:

$$\sum_{i=1}^{n} k_i = K, \tag{4}$$
$$\sum_{i=1}^{n} l_i = 1. \tag{5}$$

To complete the description of the model, we provide a formal definition of a competitive equilibrium with distortions.

Definition A competitive equilibrium is a collection of quantities q_i, k_i, l_i, y_i, d_{ij}, Y, C, G and prices p_i, p, w, and r for $i \in 1:n$ such that

1. y_i solves the profit maximization problem of a representative firm in a perfectly competitive final good's market:

$$\max_{\{y_i\}} p y_1^{\frac{1}{n}} \cdot ... \cdot y_n^{\frac{1}{n}} - \sum_{i=1}^{n} p_i y_i,$$

taking $\{p_i\}, p$ as given.

2. $\{d_{ij}\}$, k_i, l_i solve the profit maximization problem of a representative firm in the perfectly competitive sector i for $i \in 1:n$:

$$\max_{\{d_{ji}\},k_i,l_i} (1-\tau_i) p_i \Lambda_i \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{1-\gamma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot ... \cdot d_{ni}^{\gamma_{ni}} - \sum_{j=1}^{n} p_j d_{ji} - r k_i - w l_i,$$

taking $\{p_i\}$ as given (τ_i and Λ_i are exogenous).

3. Households' budget constraint determines C: $C = w + rK$.

4. Government's budget constraint determines G: $G = \sum_{i=1}^{n} \tau_i p_i q_i$.

5. Markets clear:

   (a) r clears the capital market: $\sum_{i=1}^{n} k_i = K$,

   (b) w clears the labor market: $\sum_{i=1}^{n} l_i = 1$,

   (c) p_i clears the sector i's market: $y_i + \sum_{j=1}^{n} d_{ij} = q_i$,

   (d) p clears the final good's market: $Y = C + G$.

6. Production function for q_i is $q_i = \Lambda_i \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{1-\gamma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot ... \cdot d_{ni}^{\gamma_{ni}}$.

7. Production function for Y, is $Y = y_1^{\frac{1}{n}} \cdot ... \cdot y_n^{\frac{1}{n}}$.

Note that households' and government consumption are simply determined by the budget constraints, so that there is no decision for the households or government to be made. Moreover, total production of the aggregate final good, Y, which is equal to $\sum_{i=1}^{n} p_i y_i$ due to the Cobb-Douglas technology in a competitive final good's market, represents GDP (total value added) per capita.

3.2 Equilibrium

The following proposition characterizes the equilibrium value of the logarithm of GDP per capita, which we later refer to equivalently as aggregate output or aggregate income or value added of the economy.

Proposition 1. *There exists a unique competitive equilibrium. In this equilibrium, the logarithm of GDP per capita, $y = \log(Y)$, is given by*

$$\begin{aligned} y &= \sum_{i=1}^{n}\mu_i\lambda_i + \sum_{i=1}^{n}\mu_i \log(1-\tau_i) + \sum_{i=1}^{n}\sum_{j\ s.t.\ \gamma_{ji}\neq 0}\mu_i\gamma_{ji}\log\gamma_{ji} + \sum_{i=1}^{n}\mu_i(1-\gamma_i)log(1-\gamma_i) - \log n + \\ &+ \log\left(1+\sum_{i=1}^{n}\tau_i\bar{\mu}_i\right) + \alpha\log K, \end{aligned} \tag{6}$$

where

$$\begin{aligned} \boldsymbol{\mu} &= \{\mu_i\}_i = \frac{1}{n}[\boldsymbol{I}-\boldsymbol{\Gamma}]^{-1}\mathbf{1}, && n\times 1 \textit{ vector of multipliers} \\ \boldsymbol{\lambda} &= \{\lambda_i\}_i = \{\log\Lambda_i\}_i, && n\times 1 \textit{ vector of sectoral log-productivity coefficients} \\ \boldsymbol{\tau} &= \{\tau_i\}_i, && n\times 1 \textit{ vector of sector-specific taxes} \\ \bar{\boldsymbol{\mu}} &= \{\bar{\mu}_i\}_i = \frac{1}{n}[\boldsymbol{I}-\bar{\boldsymbol{\Gamma}}]^{-1}\mathbf{1}, && n\times 1 \textit{ vector of multipliers corresponding to } \bar{\boldsymbol{\Gamma}} \\ \bar{\boldsymbol{\Gamma}} &= \{\bar{\gamma_{ji}}\}_{ji} = \{\frac{\tau_i}{n} + (1-\tau_i)\gamma_{ji}\}_{ji}, && n\times n \textit{ input-output matrix adjusted for taxes} \end{aligned}$$

Proof. The proof of Proposition 1 is provided in the Appendix.

Thus, due to the Cobb-Douglas structure of our economy, aggregate per capita GDP can be represented as a log linear function of terms that represent aggregate productivity and summarize the aggregate impact of sectoral productivities and taxes via the IO structure, and the capital stock per worker weighted by the capital share in GDP, α.

Two important outcomes are suggested by the proposition. First, aggregate output is an increasing function of sectoral productivities and it is a decreasing function of sector-specific taxes, at least in the vicinity of small positive $\{\tau_i\}_i$.[18] That is, larger sectoral productivities increase and larger taxes decrease

[18] Note that the partial derivative of y with respect to τ_i is equal to:

$$\frac{\partial y}{\partial \tau_i} = \frac{-\mu_i}{1-\tau_i} + \frac{\bar{\mu}_i}{1+\sum_{i=1}^{n}\tau_i\bar{\mu}_i} = \frac{-\mu_i\left(1+\sum_{i=1}^{n}\tau_i\bar{\mu}_i\right)+\bar{\mu}_i-\tau_i\bar{\mu}_i}{(1-\tau_i)\left(1+\sum_{i=1}^{n}\tau_i\bar{\mu}_i\right)} \approx \frac{-\mu_i\sum_{i=1}^{n}\tau_i\bar{\mu}_i-\tau_i\bar{\mu}_i}{(1-\tau_i)\left(1+\sum_{i=1}^{n}\tau_i\bar{\mu}_i\right)},$$

where the last equality employs the approximation $\bar{\mu}_i \approx \mu_i$ at low $\{\tau_i\}_i$.

aggregate output. Observe that the positive component of the effect of taxes, associated with the term $\log\left(1+\sum_{i=1}^{n}\tau_i\bar{\mu}_i\right)$ in (6), accounts for the fact that larger taxes do not only reduce firms' revenues but also contribute to government expenditures and thereby increase GDP. Second, and more importantly, the impact of each sector's productivity and tax on aggregate output is proportional to the value of the sectoral IO multiplier μ_i, and hence, the larger the multiplier, the stronger the effect. This means that the positive effect of higher sectoral productivity and the negative effect of a higher tax on aggregate output are stronger in sectors with larger multipliers.[19]

The vector of sectoral multipliers, in turn, is determined by the features of the IO matrix through the Leontief inverse, $[\boldsymbol{I}-\boldsymbol{\Gamma}]^{-1}$.[20] The interpretation and properties of this matrix as well as a simpler representation of the vector of multipliers are discussed in the next section.

3.3 Intersectoral network. Multipliers as sectors' centrality

The input-output matrix $\boldsymbol{\Gamma}$, where a typical element γ_{ji} captures the value of spending on input j per dollar of production of good i, can be equivalently represented by a directed weighted network on n nodes. Nodes of this network are sectors and directed links indicate the flow of intermediate goods between sectors. Specifically, the link from sector j to sector i with weight γ_{ji} is present if sector j is an input supplier to sector i.

For each sector in the network we define the *weighted in-* and *out-degree*. The weighted in-degree of a sector is the share of intermediate inputs in its production. It is equal to the sum of elements in the corresponding *column* of matrix $\boldsymbol{\Gamma}$; that is, $d_i^{in}=\gamma_i=\sum_{j=1}^{n}\gamma_{ji}$. The weighted out-degree of a sector is the share of its output in the input supply of the entire economy. It is equal to the sum of elements in the corresponding *row* of matrix $\boldsymbol{\Gamma}$; that is, $d_j^{out}=\sum_{i=1}^{n}\gamma_{ji}$. Note that if weights of all links that are present in the network are identical, the in-degree of a given sector is proportional to the number of sectors that supply to it and its out-degree is proportional to the number of sectors to which it is a supplier.

The interdependence of sectors' production technologies through the network of intersectoral trade, helps to obtain some insights into the meaning of the Leontief inverse matrix $[\boldsymbol{I}-\boldsymbol{\Gamma}]^{-1}$ and the vector of sectoral multipliers $\boldsymbol{\mu}$.[21] A typical element l_{ji} of the Leontief inverse can be interpreted as the percentage increase in the output of sector i following a one-percent increase in productivity of sector j. This result takes into account all – direct and indirect – effects at work, such as for example, the effect of raising productivity in sector A that makes sector B more efficient and via this raises the output in sector C.

[19]The value of sectoral multipliers is positive due to a simple approximation result (8) in the next section.

[20]See Burress (1994).

[21]Observe that in this model the Leontief inverse matrix is well-defined since CRS technology of each sector implies that $\gamma_i<1$ for any $i\in 1:n$. According to the Frobenius theory of non-negative matrices, this then suggests that the maximal eigenvalue of $\boldsymbol{\Gamma}$ is bounded above by 1, and this, in turn, implies the existence of $[\boldsymbol{I}-\boldsymbol{\Gamma}]^{-1}$.

Then multiplying the Leontief inverse matrix by the vector of weights $\frac{1}{n}\mathbf{1}$ adds up the effects of sector j on all the other sectors in the economy, weighting each by its share $\frac{1}{n}$ in GDP. Thus, a typical element of the resulting vector of IO multipliers reveals how a one-percent increase in productivity of sector j affects the overall value added in the economy.

In particular, for a simple one-sector economy, the multiplier is given by $\frac{1}{1-\gamma}$, where γ is a share of the intermediate input in the production of that sector. Moreover, $\frac{1}{1-\gamma}$ is also the value of the *aggregate* multiplier in an n-sector economy where only one sector's output is used (in the proportion γ) as an input in the production of all other sectors.[22] Thus, if the share of intermediate inputs in gross output of each sector is, for example, $\frac{1}{2}$ ($\gamma = \frac{1}{2}$), then a one-percent increase in TFP of each sector increases the value added by $\frac{1}{1-\gamma} = 2$ percent. In more extreme cases, the aggregate multiplier and hence, the effect of sectoral productivity increases on aggregate value added becomes infinitely large when $\gamma \to 1$ and it is close to 1 when $\gamma \to 0$. This is consistent with the intuition in Jones (2011b).

One important observation is that the vector of multipliers is closely related to the *Bonacich centrality* vector corresponding to the intersectoral network of the economy.[23] This means that sectors that are more "central" in the network of intersectoral trade have larger multipliers and hence, play a more important role in determining aggregate output.

To see what centrality means in terms of simple network characteristics, such as sectors' out-degree, consider the following useful approximation for the vector of multipliers. Since none of $\mathbf{\Gamma}$'s eigenvalues lie outside the unit circle (cf. footnote 21), the Leontief inverse and hence the vector of multipliers can be expressed in terms of a convergent power series:

$$\boldsymbol{\mu} = \frac{1}{n}[\boldsymbol{I} - \mathbf{\Gamma}]^{-1}\mathbf{1} = \frac{1}{n}\left(\sum_{k=0}^{+\infty} \mathbf{\Gamma}^k\right)\mathbf{1}.$$

As long as the elements of $\mathbf{\Gamma}$ are sufficiently small, this power series is well approximated by the sum of the first terms. Namely, consider the norm of $\mathbf{\Gamma}$, $\|\mathbf{\Gamma}\|_\infty = \max_{i,j\in 1:n} \gamma_{ji}$, and assume that it is sufficiently small. Then

$$\frac{1}{n}\left(\sum_{k=0}^{+\infty} \mathbf{\Gamma}^k\right)\mathbf{1} \approx \frac{1}{n}(\boldsymbol{I} + \mathbf{\Gamma})\mathbf{1} = \frac{1}{n}\mathbf{1} + \frac{1}{n}\mathbf{\Gamma}\mathbf{1}.$$

Consider that $\mathbf{\Gamma}\mathbf{1} = \boldsymbol{d^{out}}$, where $\boldsymbol{d^{out}}$ is the vector of sectors' out-degrees, $\boldsymbol{d^{out}} = \left(d_1^{out}, .., d_n^{out}\right)'$. This leads to the following simple representation of the vector of multipliers:

$$\boldsymbol{\mu} \approx \frac{1}{n}\mathbf{1} + \frac{1}{n}\boldsymbol{d^{out}}, \tag{7}$$

[22] Recall that aggregate multiplier is equal to the sum of all sectoral multipliers and represents the effect on aggregate income of a one percent increase in the productivity of each sector.

[23] Analogous observation is made in Acemoglu et al. (2012), with respect to the *influence vector*. For the definition and other applications of the Bonacich centrality notion in economics see Bonacich, 1987; Jackson, 2008; and Ballester et al., 2006.

so that for any sector i,

$$\mu_i \approx \frac{1}{n} + \frac{1}{n} d_i^{out}, \qquad i = 1 : n. \tag{8}$$

Thus, larger multipliers correspond to sectors with larger out-degree, the simplest measure of sector's centrality in the network.[24] In view of the statement in the previous section, this implies that sectors with the largest out-degree have the most pronounced impact on aggregate value added of the economy. Hence, the changes in productivity and taxes in such "central" sectors affect aggregate output most.

For the sample of countries in our GTAP data, both rich and poor, the approximation of sectoral multipliers by sectors' out-degree (times and plus $1/n$) turns out to be quite good, as demonstrated by Figure 6.

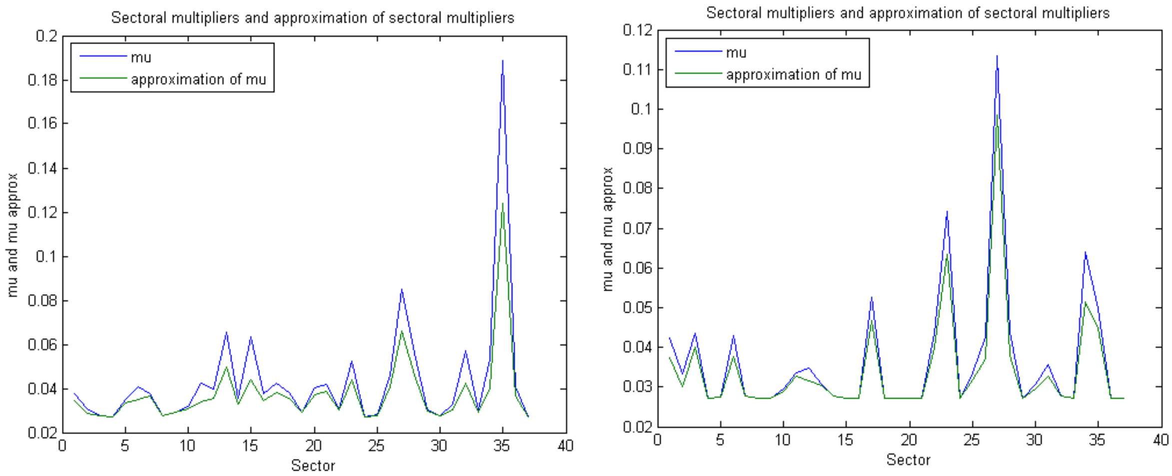

Figure 6: Sectoral multipliers in Germany (left) and Botswana (right). GTAP sample.

In what follows we will consider that the in-degree of all sectors is the same, $\gamma_i = \gamma$ for all i. While clearly a simplification, this assumption turns out to be broadly consistent with the empirical distribution of sectoral in-degrees of countries from our GTAP sample. In fact, the distribution of in-degrees in all countries is strongly peaked around the mean value, which suggests that on the demand side sectors are rather homogeneous, i.e., they use intermediate goods in approximately equal proportions.[25] This is in sharp contrast with the observed distribution of sectoral out-degrees that puts most weight on small

[24] Returning to the discussion of the impact of unmeasured distortions on IO multipliers in footnote 16, observe that the *empirical* out-degree of sector j in *basic* prices is given by $d_j^{out,b} = (1-\tau_j)\sum_{i=1}^{N} \gamma_{ji}$. It follows that the empirical out-degree measured in basic prices, $d_j^{out,b}$, and hence, the empirical IO multiplier of sector j in basic prices is lower when sector j itself is taxed more (τ_j larger). Differently, the empirical out-degree of sector j in *user* prices is given by: $d_j^{out,u} = \sum_{i=1}^{N}(1-\tau_i)\gamma_{ji}$. Hence, the empirical IO multiplier of sector j in user prices is reduced when the *using* sectors i are taxed more (when the τ_is are larger). By contrast, the IO multiplier of sector j measured in user prices does not depend on its own tax rate τ_j. Therefore – at least in the Cobb-Douglas model – a tax or subsidy on the revenue of a sector does not affect its empirical IO multiplier measured in user prices.

Observe that the WIOD IO data are measured in basic prices, while the GTAP IO data are measured in user prices. Hence, empirical IO multipliers in the WIOD data might be larger for sectors that receive implicit subsidies and lower for sectors with high distortions. Instead, this does not apply to the GTAP data. We note that in the GTAP data the problem of unmeasured distortions is likely to be more severe because this data includes many more low-income countries with potentially high unmeasured implicit tax rates.

[25] Note that essentially the same assumption of constant in-degree ($\gamma_i = 1$) is employed in Acemoglu et al., 2012, and in Carvalho et al., 2010.

values of out-degrees but also assigns a non-negligible weight to the out-degrees that are way above the average, displaying a fat tail. That is, on the supply side sectors are rather heterogenous: relatively few sectors supply their product to a large number of sectors in the economy, while many sectors supply to just a few. Figure A-1 in the Supplementary Appendix provides an illustration of empirical distributions of in- and out-degree for different levels of income per capita.

Note that the fat-tail nature of out-degree distribution is also inherent to the distribution of sectoral multipliers. Moreover, according to both distributions, the proportion of sectors with very low and very high out-degree and multiplier is larger in low-income countries. This similarity between the distribution of sectoral out-degrees and multipliers is consistent with the derived relationship (8) between d_i^{out} and μ_i for each sector.

3.4 Expected aggregate output

To estimate the model we use a statistical approach that allows us to represent aggregate income as a simple function of the first and second moments of the distribution of the IO multipliers, sectoral productivities and sectoral tax rates. The distribution of multipliers, or sectors' centralities, captures the properties of the intersectoral network in each country, while the correlation between the distribution of multipliers and productivities and between multipliers and distortions captures the interaction of the input-output structure with sectoral productivities and distortions.

In the next section, we show that the *joint* distribution of sectoral multipliers, productivities (relative to the U.S.) and taxes $(\mu_i, \Lambda_i^{rel}, \tau_i)$ is close to *log-Normal*, so that the joint distribution of log's of the corresponding variables, $(\log(\mu_i), \log(\Lambda_i^{rel}), \log(\tau_i))$ is *Normal*.[26] Here i refers to the sector and $\Lambda_i^{rel} = \frac{\Lambda_i}{\Lambda_i^{US}}$. In particular, the fact that the distribution of μ_i is log-Normal means that while the largest probability is assigned to relatively low values of a multiplier, a non-negligible weight is assigned to high values, too. That is, the distribution is positively skewed, or possesses a fat right tail. Empirically, we find that this tail is fatter and hence, the variance and the mean of μ_i are larger in countries with lower income.[27]

Given the log-Normal distribution of $(\mu_i, \Lambda_i^{rel}, \tau_i)$, the expected value of the aggregate output in each country can be evaluated using the expression for y in (6). We first impose a few simplifying assumptions. First, we consider that for each sector i of a given country, the triple $(\mu_i, \Lambda_i^{rel}, \tau_i)$ is drawn from the *same* trivariate log-Normal distribution, as estimated for this country. Second, we assume that all variables on the right-hand side of (6), apart from μ_i, Λ_i^{rel} and τ_i, are not random. Moreover, all non-zero elements

[26]To be precise, the distribution of $(\log(\mu_i), \log(\Lambda_i^{rel}), \log(\tau_i))$ is a *truncated* trivariate Normal, where $\log(\mu_i)$ is censored from below at a certain $a > 0$. This is taken into account in our empirical analysis. However, the difference from a usual, non-truncated Normal distribution turns out to be inessential. Therefore, for simplicity of exposition, in this section we refer to the distribution of $(\log(\mu_i), \log(\Lambda_i^{rel}), \log(\tau_i))$ as Normal and to the distribution of $(\mu_i, \Lambda_i^{rel}, \tau_i)$ as log-Normal.

[27]See the distribution parameter estimates in the next section.

of the input-output matrix $\mathbf{\Gamma}$ are the same, that is, $\gamma_{ji} = \widehat{\gamma}$ for any i and j whenever $\gamma_{ji} > 0$, and the in-degree $\gamma_i = \gamma$ for all i.[28] Third, to simplify the analysis of the benchmark model, we omit the positive term with taxes, $\log\left(1 + \sum_{i=1}^{n} \tau_i \bar{\mu}_i\right)$, on the right-hand side of (6). It is easy to show that such modification means treating distortions as pure waste, rather than taxes contributing to government budget. In section 6, we implement the "full" model, including the omitted term, and show that the difference between treating distortions as a pure waste or taxes is not empirically relevant. Furthermore, we regard the values of $\{\tau_i\}_i$ as sufficiently small, which allows approximating $\log(1 - \tau_i)$ with $-\tau_i$. Finally, in order to express sectoral log-productivity coefficients λ_i in terms of the relative productivity Λ_i^{rel}, we use the approximation $\lambda_i = \log(\Lambda_i) \approx \Lambda_i^{rel} + \big(\log(\Lambda_i^{US}) - 1\big)$, which, strictly speaking, is only good when Λ_i is sufficiently close to Λ_i^{US}.

Under these assumptions, the expression for the aggregate output y in (6) simplifies and can be written as:

$$y = \sum_{i=1}^{n} \mu_i \Lambda_i^{rel} - \sum_{i=1}^{n} \mu_i \tau_i + \sum_{i=1}^{n} \mu_i \gamma \log(\widehat{\gamma}) + \log(1-\gamma) - \log n + \alpha \log(K) - (1+\gamma) + \sum_{i=1}^{n} \mu_i \log(\Lambda_i^{US}). \quad (9)$$

The expected aggregate output, $E(y)$, is then equal to :

$$\begin{aligned} E(y) &= n\Big(E(\mu)E(\Lambda^{rel}) + cov(\mu, \Lambda^{rel}) - E(\mu)E(\tau) - cov(\mu, \tau)\Big) + (1+\gamma)(\gamma \log(\widehat{\gamma}) - 1) + \\ &+ \log(1-\gamma) - \log n + \alpha \log(K) + E(\mu) \sum_{i=1}^{n} \log\left(\Lambda_i^{US}\right). \end{aligned} \quad (10)$$

From this expression, we see that higher expected multipliers $E(\mu)$ lead to larger expected income $E(y)$ for the same fixed levels of $E(\Lambda^{rel})$, $E(\tau)$ and covariances, as soon as $E(\Lambda^{rel}) > E(\tau)$, which holds empirically for most countries. Moreover, since aggregate value added depends positively on the covariance term $cov(\mu, \Lambda^{rel})$, higher relative productivities have a larger impact if they occur in sectors with higher multipliers. Similarly, higher tax rates reduce aggregate income by more if they are imposed on sectors with higher multipliers, as indicated by $cov(\mu, \tau)$.

The expression for expected aggregate income in (10) can be written in terms of the parameters of the normally distributed $(\log(\mu), \log(\Lambda^{rel}), \log(\tau))$, by means of the relationships between Normal and

[28] These conditions on γ_{ji} and γ allow us to express $\sum_{j \text{ s.t. } \gamma_{ji} \neq 0} \mu_i \gamma_{ji} \log \gamma_{ji}$ as $\mu_i \gamma \log(\widehat{\gamma})$ since the number of non-zero elements in each column of $\mathbf{\Gamma}$ is equal to $\frac{\gamma}{\widehat{\gamma}}$, and $\sum_{i=1}^{n} \mu_i (1-\gamma_i) log(1-\gamma_i) = log(1-\gamma)$ since $\sum_{i=1}^{n} \mu_i(1-\gamma_i) = \mathbf{1}'[\boldsymbol{I} - \mathbf{\Gamma}] \cdot \frac{1}{n}[\boldsymbol{I} - \mathbf{\Gamma}]^{-1}\mathbf{1} = \frac{1}{n}\mathbf{1}'\mathbf{1} = 1$. Moreover, $\sum_{i=1}^{n} \mu_i \approx 1 + \gamma$ because from (8) it follows that $\sum_{i=1}^{n} \mu_i \approx 1 + \frac{\sum_{i=1}^{n} d_i^{out}}{n} = 1 + \frac{\sum_{i=1}^{n} d_i^{in}}{n}$ and $d_i^{in} = \gamma_i = \gamma$ for all i.

log-Normal distributions:[29]

$$E(y) = n\left(e^{m_\mu+m_\Lambda+1/2(\sigma_\mu^2+\sigma_\Lambda^2)+\sigma_{\mu,\Lambda}} - e^{m_\mu+m_\tau+1/2(\sigma_\mu^2+\sigma_\tau^2)+\sigma_{\mu,\tau}}\right) + (1+\gamma)(\gamma\log(\widehat{\gamma})-1) + \\ + \log(1-\gamma) - \log n + \alpha\log(K) + e^{m_\mu+1/2\sigma_\mu^2}\sum_{i=1}^{n}\log\left(\Lambda_i^{US}\right), \quad (13)$$

where m_μ, m_Λ, m_τ are the means and σ_μ^2, σ_Λ^2, σ_τ^2 and $\sigma_{\mu,\Lambda}$ and $\sigma_{\mu,\tau}$ are the elements of the variance-covariance matrix of the Normal distribution.

This is the ultimate expression that we use in the empirical analysis of the benchmark model in section 4.

4 Empirical analysis

In this section we estimate the parameters of the Normal distribution of $(\log(\mu), \log(\Lambda^{rel}), \log(\tau))$ for the sample of countries for which we have data. We allow parameter estimates to be functions of GDP per capita in order to model the systematic underlying differences in IO structure, productivity and tax rates that we have discussed in section 2. With the parameter estimates in hand we then use equation (13) to evaluate the predicted aggregate income in these countries (relative to the one of the U.S.)[30] and compare our baseline model with three simple alternatives which do not encompass some of the three elements present in our model: productivity differences, taxes or country-specific IO structure. Finally, we conduct a series of counterfactual exercises where we investigate how differences in IO structure, distribution of taxes and estimated correlation patterns between log multipliers and log productivities or taxes matter for cross-country income differences.

4.1 Structural estimation

The vector of log multipliers, log relative productivities and log tax rates $\mathbf{Z} = (\log(\mu), \log(\Lambda^{rel}), \log(\tau))$ is drawn from a (truncated) trivariate Normal distribution.[31]

[29] These relationships are:

$$E(\mu) = e^{m_\mu+1/2\sigma_\mu^2}, \qquad E(\Lambda^{rel}) = e^{m_\Lambda+1/2\sigma_\Lambda^2}, \qquad E(\tau) = e^{m_\tau+1/2\sigma_\tau^2}, \quad (11)$$

$$cov(\mu,\Lambda) = e^{m_\mu+m_\Lambda+1/2(\sigma_\mu^2+\sigma_\Lambda^2)}\cdot\left(e^{\sigma_{\mu,\Lambda}}-1\right), cov(\mu,\tau) = e^{m_\mu+m_\tau+1/2(\sigma_\mu^2+\sigma_\tau^2)}\cdot\left(e^{\sigma_{\mu,\tau}}-1\right) \quad (12)$$

[30] In order to predict relative rather than absolute output, we use equation (13) differenced with the value of predicted aggregate income for the U.S.

[31] The formula for the truncated trivariate Normal, where $\log(\mu)$ is censored from below at a is given by $f(\mathbf{Z}|log(\mu)\geq a) = \frac{1}{\sqrt{(2\Pi)^3|\mathbf{\Sigma}|}}exp[-1/2(\mathbf{Z}-\mathbf{m})'\mathbf{\Sigma}^{-1}(\mathbf{Z}-\mathbf{m})]/(1-F(a))$, where $F(a) = \int_{-\infty}^{a}\frac{1}{\sigma_\mu\sqrt{(2\Pi)}}exp[-1/2(\log(\mu)-m_\mu)^2/\sigma_\mu^2]d\log(\mu)$ is the cumulative marginal distribution of $\log(\mu)$ and where

$$\mathbf{m} = \begin{pmatrix} m_\mu \\ m_\Lambda \\ m_\tau \end{pmatrix}, \mathbf{\Sigma} = \begin{pmatrix} \sigma_\mu^2 & \rho_{\mu\Lambda}\sigma_\mu\sigma_\Lambda & \rho_{\mu\tau}\sigma_\mu\sigma_\tau \\ \rho_{\mu\Lambda}\sigma_\mu\sigma_\Lambda & \sigma_\Lambda^2 & \rho_{\Lambda\tau}\sigma_\Lambda\sigma_\tau \\ \rho_{\mu\tau}\sigma_\mu\sigma_\tau & \rho_{\Lambda\tau}\sigma_\Lambda\sigma_\tau & \sigma_\tau^2 \end{pmatrix} \quad (14)$$

The vector of parameters to be estimated using Maximum Likelihood estimation is $\mathbf{\Theta} = (\mathbf{m}, \mathbf{\Sigma})$, where $\mathbf{m}$ is the vector of means and $\mathbf{\Sigma}$ denotes the variance-covariance matrix. In order to allow for structure, productivity and taxes to differ across countries we model both $\mathbf{m}$ and $\mathbf{\Sigma}$ as linear functions of log(GDP per capita).

First, we estimate the statistical model on the WIOD sample (35 sectors, 39 countries). We find that m_μ is decreasing in log(GDP per capita), while σ_μ is not a significant function of per capita GDP for this sample. We thus restrict the second parameter to be constant in the reported estimates. The point estimates and standard errors of all parameters are presented in Table 2. m_μ is decreasing in log(GDP per capita) with a slope of around -0.08. The log of σ_μ^2 is around -0.642. Hence, in the WIOD sample poor countries have a distribution of log multipliers with a slightly higher average than rich countries but with the same dispersion, implying that the distribution of the level of multipliers has a larger mean and a larger variance in poor countries (see formulas in footnote 30). Average log productivity, m_Λ is strongly increasing in log GDP per capita (with a slope of around 1.3), while the standard deviation of log productivity, σ_Λ is a decreasing function of the same variable. This implies that rich countries have higher average log productivity levels and less variation across sectors than poor countries. Similarly, average log taxes, m_τ, are slightly increasing in log(GDP per capita) (with a slope of 0.09), whereas the variability of tax rates, as described by $\log(\sigma_\tau^2)$, is decreasing with income. Finally, note that the correlation between log multipliers and log productivity, $\rho_{\mu\Lambda}$, is a decreasing function of log(GDP per capita). Similarly, the correlation between log multipliers and log distortions, $\rho_{\mu\tau}$ is also decreasing in per capita income. These correlations imply that poor countries have above average productivity levels and taxes in sectors with higher multipliers, while rich countries have productivities and taxes which are lower than their average levels in these sectors. Figure A-2 in the Supplementary Appendix provides density plots of the empirical and estimated distributions of log multipliers, log productivity and log distortions. It is apparent that the estimated distributions fit the empirical ones quite well. Finally, Figure 7 plots the parameter estimates of the correlation coefficients $\rho_{\mu\Lambda}$ and $\rho_{\mu\tau}$ as functions of log(GDP per capita).

To obtain more information on the IO structure of low-income countries, we now re-estimate our statistical model on the GTAP sample (37 sectors, 70 countries). For these countries, we only have information on IO multipliers but not on productivity levels and taxes. Therefore, we estimate a univariate Normal distribution for m_μ and σ_μ. Table 3 reports the results. We find that m_μ is now an insignificant function of income and we therefore report the estimate for constant m_μ. By contrast, for the larger sample the standard deviation of log multipliers, σ_μ, is now significantly smaller for rich than for poor countries. This implies that both the mean and the standard deviation of the corresponding

.

Table 2: Maximum likelihood

	WIOD sample	
	Coef.	Std. Err.
$\mathbf{m}_{\mu}$:		
constant	0.648	0.431
log(gdp per capita)	-0.081*	0.047
$\log(\sigma_{\mu}^{2})$:		
constant	-0.642***	0.145
$\mathbf{m}_{\mathbf{\Lambda}}$:		
constant	-13.327***	1.287
log(gdp per capita)	1.287***	0.142
$\log(\sigma_{\mathbf{\Lambda}}^{2})$:		
constant	4.102***	0.735
log(gdp per capita)	-0.375***	0.074
$\mathbf{m}_{\tau}$:		
constant	-3.847***	0.464
log(gdp per capita)	0.090***	0.046
$\log(\sigma_{\tau}^{2})$:		
constant	1.870***	0.617
log(gdp per capita)	-0.284***	0.062
z-transformed $\rho_{\mu\mathbf{\Lambda}}$:		
constant	3.440***	0.813
log(gdp per capita)	-0.352***	0.083
z-transformed $\rho_{\mu\tau}$:		
constant	1.010*	0.607
log(gdp per capita)	-0.126**	0.061
Log likelihood	-107.885	
Observations	1281	

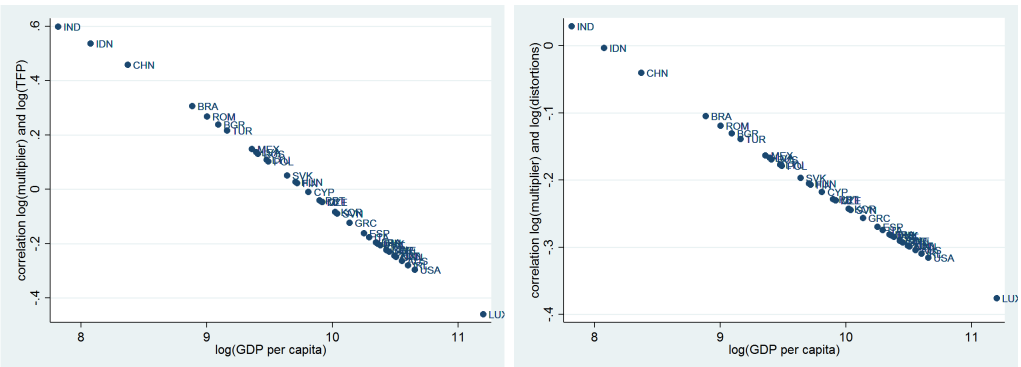

Figure 7: Estimated correlation between multiplier and productivity (left), distortions (right)

distributions of multipliers are larger in poor countries than in rich ones: in poor countries the average sector has a larger multiplier and there is more mass in the right tail of the distribution. We summarize these empirical findings below.

Table 3: Maximum Likelihood

	GTAP sample	
	Coef.	Std. Err.
$\mathbf{m}_\mu$:		
constant	-3.274***	0.072
$\mathbf{log}({\sigma_\mu}^2)$:		
constant	0.328***	0.029
log(GDP per capita)	-0.008***	0.003
Log likelihood	10,069.322	
Observations	2,553	

Summary of estimation results:

1. *The estimated distribution of IO multipliers has a larger variance and more mass in the right tail in poor countries compared to rich ones.*
2. *The estimated distribution of productivities has a lower mean and a larger variance in poor countries compared to rich ones.*
3. *The estimated distribution of tax rates has a lower mean and a larger variance in poor countries compared to rich ones.*
4. *IO multipliers and productivities correlate positively in poor countries and negatively in rich ones.*
5. *IO multipliers and tax rates correlate positively in poor countries and negatively in rich ones.*

4.2 Predicting cross-country income differences

With the parameter estimates $\hat{\mathbf{\Theta}}$ in hand, we now use equation (13) (differenced relative to the U.S.) to predict income per capita relative to the U.S.[32] We compare our baseline model which features country-specific IO linkages, sectoral productivity differences and taxes with three simple alternatives. The first one, which we label the 'naive model', has no IO structure, no productivity differences and no taxes, so that $y = E(y) = \alpha log(K)$. The second model, in contrast, has sectoral productivity differences but no IO linkages. It is easy to show that under the assumption that sectoral productivities follow a log-Normal distribution, predicted log income in this model is given by $E(y) = e^{m_\Lambda + 1/2\sigma_\Lambda^2} + \alpha \log(K) +$

[32]The expression for $E(y)$ for the truncated distribution of $(\mu_i, \Lambda_i^{rel}, \tau_i)$ is somewhat more complicated and less intuitive. However, the results for aggregate income using a truncated normal distribution for μ are very similar to the estimation of (13) and we therefore use the formulas for the non-truncated distribution. The details can be provided by the authors.

$\frac{1}{n}\sum_{i=1}^{n}(\log(\Lambda_i^{US}))-1$.[33] The third alternative model features sectoral productivity differences, taxes and IO linkages but keeps the IO structure constant for all countries (by restricting the mean and the variance of the distribution of multipliers to be independent of per capita GDP in the estimation). In addition to the estimated parameter values $\hat{\mathbf{\Theta}}$, we need to calibrate a few other parameters. As standard, we set $(1-\alpha)$, the labor income share in GDP, equal to 2/3. Moreover, we set γ, the share of intermediates in gross output, equal to 0.5, which corresponds to the average level in the WIOD dataset. Finally, we set n equal to 35, which corresponds to the number of sectors in the WIOD dataset.

To evaluate model fit, we provide the following tests: first, we regress income per capita relative to the U.S. predicted by the model on actual data for GDP per capita relative to the U.S. If the model fits perfectly, the estimate for the intercept should be zero, while the regression slope and the R-squared should equal unity. Second, as a graphical measure for the goodness of fit, we also plot predicted income per capita relative to the U.S. against actual relative income. Note that these statistics provide over-identification tests for our model since there is no intrinsic reason for the model to fit data on relative per capita income: we have not tried to match income data in order to estimate the parameters of the distribution of IO multipliers, productivities or taxes. Instead, we have just allowed the joint distribution of these parameters to vary with the level of income per capita.

The results for the first test are reported in Table 4. In column (1), we report statistics for the 'naive' model. In column (2), we report results for the model with productivity differences but no IO structure. In column (3) we report results for the baseline model (13), where we take the parameter estimates as obtained from the WIOD data (using parameters for the distribution of multipliers from Table 2 above). In column (4), we force the distribution of multipliers to be the same across countries by restricting both m_μ and σ_μ^2 to be constant. Finally, in column (5) we report results for the baseline model when the distribution of multipliers is estimated from the GTAP dataset (using parameters for the distribution of multipliers from Table 3).

We now present the results of this exercise. The 'naive' model fails in predicting relative income across countries (column (1)). As is well known, a model without productivity differences predicts too little variation in income per capita across countries. Still, in the WIOD sample, which consists mostly of high-income countries, it does relatively well: the intercept is 0.426, the slope coefficient is 0.735 and the R-squared is 0.888. The simple model with productivity differences but no IO linkages (column (2)) performs better but it generates too much variation in income compared to the data, implying that aggregate productivity differences estimated from sectoral data are larger than what is necessary to generate the observed differences in income: the intercept is -0.155, the slope coefficient is 0.929 and the

[33] $Y=\prod_{i=1}^{n}\Lambda_i^{1/n}(K)^\alpha$, hence $y=\frac{1}{n}\sum_{i=1}^{n}\lambda_i+\alpha\log(K)$. Using our approximation for productivity relative to the U.S., taking expectations and assuming that Λ_i follows a log-Normal distribution, we obtain the above formula.

R-squared is 0.865. We now move to the first specification with IO structure. In column (3) we report results for the baseline model with the IO structure estimated from WIOD data. This model indeed performs better than the one without IO structure: the intercept is no longer statistically different from zero, the slope coefficient equals 0.922 and the R-squared is 0.853. A visual comparison of actual vs. predicted relative income in Figure 8 confirms the substantially better fit of the model with IO linkages compared to the one without IO structure, which underpredicts relative income levels of most countries. Next, we test if cross-country differences in IO structure are part of the explanation of improved fit. In column (4) we restrict the coefficients of m_μ and σ_μ^2 to be the same for all countries but we continue to allow for cross-country differences in the correlation between productivity and IO structure as well as in the correlation between taxes and IO structure. We find that this model fits the data much worse than the one with income-varying IO structure: the intercept is 0.225, the slope coefficient drops to 0.843 and the R-squared to 0.738, thus indicating that cross-country differences in IO structure are important for predicting differences in income across countries. Finally, in column (5) we use the estimated IO structure from the GTAP sample in our baseline IO model. The GTAP data is more informative about cross-country differences in IO structure than the WIOD data because it includes a much larger sample of low- and middle-income countries, which allows estimating differences in structure across countries much more precisely. The above estimates from the GTAP data indicate that poorer countries have a distribution of multipliers with a significantly fatter right tail compared to rich countries. Using these estimates, we find that the size of the intercept drops to 0.007 and is not statistically different from zero, while the slope coefficient is equal to 0.901 and the R-squared is 0.834 Thus, this specification outperforms both the model without IO structure and the one with constant IO structure in terms of predicting income differences and performs comparably to the one where the IO structure is estimated from the WIOD data.[34]

Observe that there are three main factors that determine the improved fit of the baseline model with IO structure compared to the model without IO structure: first, the difference in the IO structure between high and low-income countries, where poor countries in the sample have only a few highly connected sectors and many sectors that are relatively isolated, while rich countries have more intermediately connected sectors; second, the fact that in contrast to rich countries poor economies have higher than average productivity levels in high-multiplier sectors; third, the fact that poor countries have relatively higher taxes in high-multiplier sectors. We will investigate the impact of each of these factors separately

[34]We have also checked the fit of a model that computes aggregate TFP as an average of sectoral TFPs using Domar weights (gross output/GDP). This is the theoretically consistent aggregation with IO linkages when no distortions are present. The fit of this model is quite poor. The intercept is 0.831, the slope coefficient 0.635 and the R-squared for this model is only 0.0036. Thus, our model with IO structure performs much better than simply computing predicted GDP by aggregating sectoral data. There are several reasons for this: first, sectoral TFPs are quite noisy and the Domar weights exacerbate outliers; second, in the presence of distortions, Domar weights do not correctly aggregate sectoral productivities, while our model does.

in the next section, but we first turn to the model fit in two alternative samples.

Table 4: Model Fit: World IO sample

	Naive model	No IO structure	WIOD IO structure	Constant IO structure	GTAP IO structure
constant	0.426***	-0.155***	-0.014	0.225**	0.007
	(0.064)	(0.030)	(0.021)	(0.038)	(0.023)
slope	0.735***	0.929***	0.922***	0.843***	0.907***
	(0.117)	(0.066)	(0.057)	(0.080)	(0.061)
Observations	39	39	39	39	39
R-squared	0.557	0.865	0.853	0.738	0.834

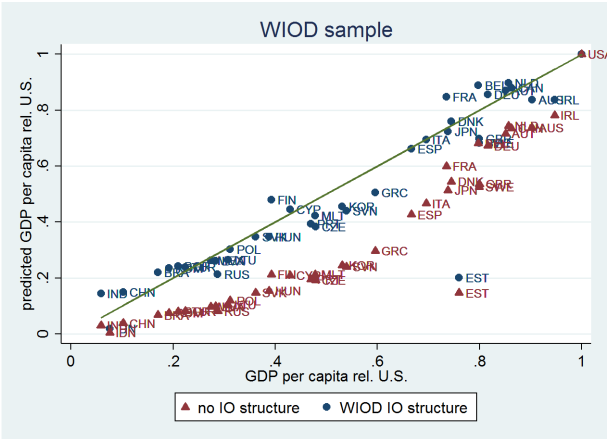

Figure 8: Predicted income per capita: baseline model with estimated IO structure

Using our model together with the parameter estimates obtained from the WIOD and GTAP data, we predict relative income for the sample of GTAP countries (70 countries) and the sample of countries in the Penn World Tables for which we have the necessary information on capital stocks (155 countries). The latter sample is usually employed for development accounting exercises. In Table 5, columns (1)-(4), we present results for the GTAP sample. In column (1) we report results for the 'naive' model, which does relatively poorly in predicting relative income for this sample: the intercept is 0.363, the slope coefficient is 0.781 and the R-squared is 0.888. In column (2), by contrast, we present results for the model with productivity differences but no IO structure. Again, this model performs much better than the 'naive' one: the intercept drops to -0.067, the slope coefficient rises to 0.807 and the R-squared improves to 0.901. Now we turn to the baseline model with IO structure. In column (3) we report the results for the baseline model where we take the parameter estimates for the distribution of multipliers from the GTAP sample. This model does much better than the 'naive' one and also better than the model without IO structure in terms of fitting the regression of predicted on actual income: the intercept is 0.127, the slope coefficient is 0.812 and the R-squared is 0.977. The increased goodness of fit can also be seen from Figure 9, left panel, where we plot predicted income against actual income for the baseline model and the model without IO structure. While the second considerably underpredicts income for most countries, the model with IO structure is extremely close to the 45 degree line. Only for the poorest countries it overpredicts their relative income somewhat. Finally, in column (4) we report results for the baseline model where the estimates of the IO structure are derived from the WIOD sample: we now get the intercept of 0.084, the slope coefficient of 0.808 and the R-squared of 0.962. Thus, this model performs slightly worse than the one where we have used the GTAP IO structure because it predicts somewhat smaller differences in IO structure across countries.

Finally, we discuss the results for the Penn World Tables sample (columns (5)-(8)). Here the performance of the 'naive' model is again quite poor and it strongly overpredicts income for poor countries, indicating that productivity differences matter for explaining aggregate income differences. In column (5) the intercept is 0.342 and the slope coefficient is 0.823 with an R-squared of 0.831. In column (6) we report results for the model without IO structure, which has a negative intercept (-0.037), an even smaller slope coefficient of 0.759 and an R-squared of 0.894. This model is again outperformed by our baseline model with the GTAP IO structure: the slope coefficient for this model is 0.775 and the R-squared increases substantially to 0.966. Thus, the model performs quite well in predicting relative income across countries, even in a sample that is much larger than the one from which we have estimated the parameters of the model. The good fit can also be seen clearly from the right panel of Figure 9, where most data points are extremely close to the 45 degree line. Again, the model overpredicts relative income levels somewhat for very low-income countries. Finally, in column (8), we report results for the baseline model when estimating the IO structure from the WIOD sample. This model does slightly worse than the previous one, but still performs better than the model without IO structure: the slope coefficient is 0.763, and the R-squared is 0.953. We conclude that including an IO structure into the model helps to significantly improve model fit. To wrap up, we now present a summary of our findings.

Summary of model fit:

1. *The baseline model with estimated IO structure performs substantially better in terms of predicting cross-country income differences than a model without technology differences (which underestimates income differences) and a model with technology differences but without IO structure (which overestimates income differences).*

2. *The baseline model with IO structure estimated from GTAP data performs slightly better than the same model with IO structure estimated from WIOD data.*

3. *The above results hold for three different samples of countries: the WIOD dataset (39 countries), the GTAP dataset (70 countries) and the Penn World Tables dataset (155 countries).*

Now that we have shown that the baseline model with IO structure performs very well in terms of predicting relative income levels across countries, we turn to several counterfactual exercises in order to understand better how the interplay between IO structure, sectoral productivity levels and taxes determines income differences across countries.

Table 5: Model Fit: Alternative Samples

	GTAP sample				PWT sample			
	Naive model	No IO structure	GTAP IO structure	WIOD IO structure	Naive model	No IO structure	GTAP IO structure	WIOD IO structure
constant	0.363***	-0.067***	0.127***	0.084***	0.342***	-0.037***	0.146***	0.110***
	(0.022)	(0.014)	(0.093)	(0.009)	(0.012)	(0.007)	(0.004)	(0.005)
slope	0.781***	0.807***	0.812***	0.808***	0.823***	0.759***	0.775***	0.763***
	(0.039)	(0.046)	(0.019)	(0.025)	(0.034)	(0.041)	(0.016)	(0.024)
Observations	70	70	70	70	155	155	155	155
R-squared	0.888	0.903	0.977	0.962	0.831	0.894	0.966	0.953

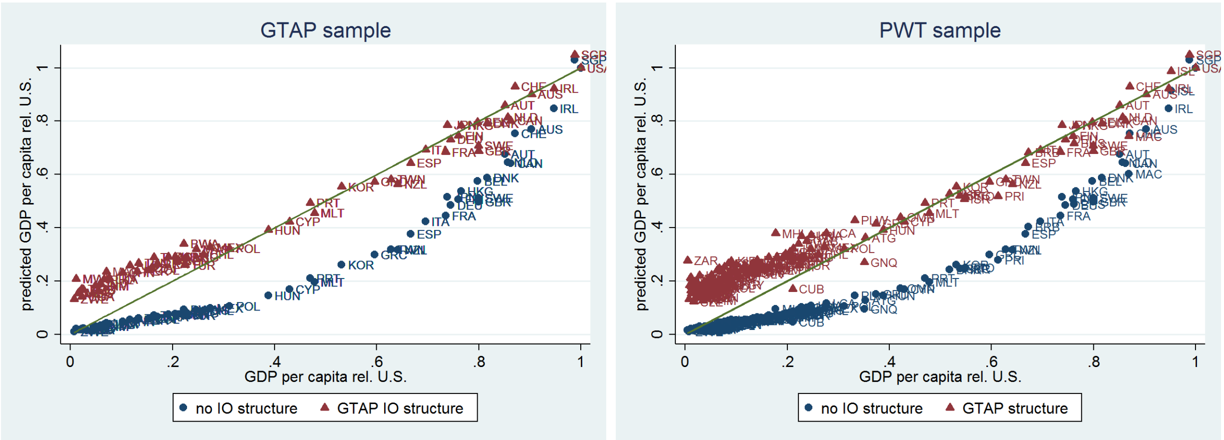

Figure 9: Predicted income per capita: baseline model with estimated IO structure

4.3 Counterfactual experiments

We first investigate how differences in IO structure – as summarized by the distribution of multipliers – matter for cross-country income differences. Thus, in our first counterfactual exercise we set the distribution of multipliers equal to the U.S. one for all countries by fixing m_μ and σ_μ^2 at the levels for the U.S.[35] As mentioned above (see footnote 24), as long as multipliers are measured in user prices and sectoral production functions are Cobb-Douglas, this is a valid experiment. Indeed, under these assumptions IO multipliers are neither systematically related to unmeasured sector-specific distortions nor to productivity levels. Consequently, it is possible to separate sectoral efficiencies from the IO structure.[36] The result of this experiment can be grasped from Figure 10, upper left panel, which plots the counterfactual change in income per capita (in percent of the initial level of income per capita)

[35]The experiment holds m_μ fixed and reduces σ_μ for virtually all countries, since according to Table 3 σ_μ is a decreasing function of GDP per capita. For a log-normal distribution such a change shifts mass away from the lower and upper tails towards the center of the distribution.

[36]Moreover, note that productivity levels are unaffected by changes in the distribution of IO multipliers even when technologies are not factor-neutral. To see this, note that labor-augmenting or intermediate-augmenting rather than Hicks-neutral technologies would imply:

against GDP per capita relative to the U.S. It can be seen that virtually all countries would lose in terms of income if they had the U.S. IO structure. These losses are decreasing in income per capita and range from negligible for countries with income levels close to the U.S. one, to 80 percent of per capita income for very poor countries such as Congo (ZAR) or Zimbabwe (ZWE).

The reason why most countries lose in this counterfactual experiment is the form of the distribution of multipliers in the U.S.: high-income countries have a distribution of multipliers with less mass in the right tail than poor countries but much more mass in the middle range of the distribution. This implies that a typical sector in the U.S. is intermediately connected. Given the distribution of productivities in low-income countries, which has a low mean, high variance and positive correlation with multipliers, they perform much worse with their new IO structure: now their typical sector – which is much less productive than in the U.S. – has a higher multiplier and thus is more of a drag on aggregate performance. Moreover, they can no longer benefit much from the fact that their super-star, high-multiplier sectors are relatively productive because the relative importance of these sectors for the economy has been reduced. To put it differently, recall that in low-income economies, a few sectors, such as Energy, Transport and Trade, provide inputs for most other sectors, while the typical sector provides inputs to only a few sectors. Thus, it suffices to have comparatively high productivity levels in those crucial sectors in order to obtain a relatively satisfactory aggregate outcome. By contrast, in the industrialized countries most sectors provide inputs for several other sectors (the IO network is quite dense), but there are hardly any sectors that provide inputs to most other sectors. Thus, with such a dense IO structure fixing inefficiencies in a few selected sectors is no longer enough to achieve a relatively good aggregate performance.

In the second counterfactual exercise, we set the correlation between log multipliers and log productivities, $\rho_{\mu\Lambda}$, to zero. We can see from the upper right panel of Figure 10 that poor countries (up to around 40 percent of the U.S. level of income per capita) would lose substantially (up to 50 percent) in terms of their initial income, while rich countries would gain up to 60 percent. Why is this the case? From our estimates, poor countries have a positive correlation between log multipliers and log productivities, while rich countries have a negative one (see Figure 7). This implies that poor countries are doing relatively well despite their low average productivity levels, because they perform significantly better than average precisely in those sectors that have a large impact on aggregate performance. The opposite is true in rich countries, where the same correlation tends to be negative. Eliminating this link

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$$q_i = \left[k_i^{\alpha}(\Lambda_i l_i)^{1-\alpha}\right]^{1-\gamma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}}, \tag{15}$$
$$q_i = \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{1-\gamma_i} (\Lambda_i^{\gamma_i}) d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}}$$

In this case, a change in the γ_{ji}s (reflecting a change in the distribution of multipliers) would also affect measured productivity $\Lambda_i^{(1-\alpha)(1-\gamma_i)}$ or $\Lambda_i^{\gamma_i}$. While this is true in general, our counterfactual exercise remains correct even in this case due to the assumption that the intermediate share $\gamma_i = \sum_{j=1}^{N} \gamma_{ji} = 0.5$ and thus is constant across sectors. Therefore, any change in IO structure that is implied by a change in the parameters m_μ and σ_μ leave productivities unaffected.

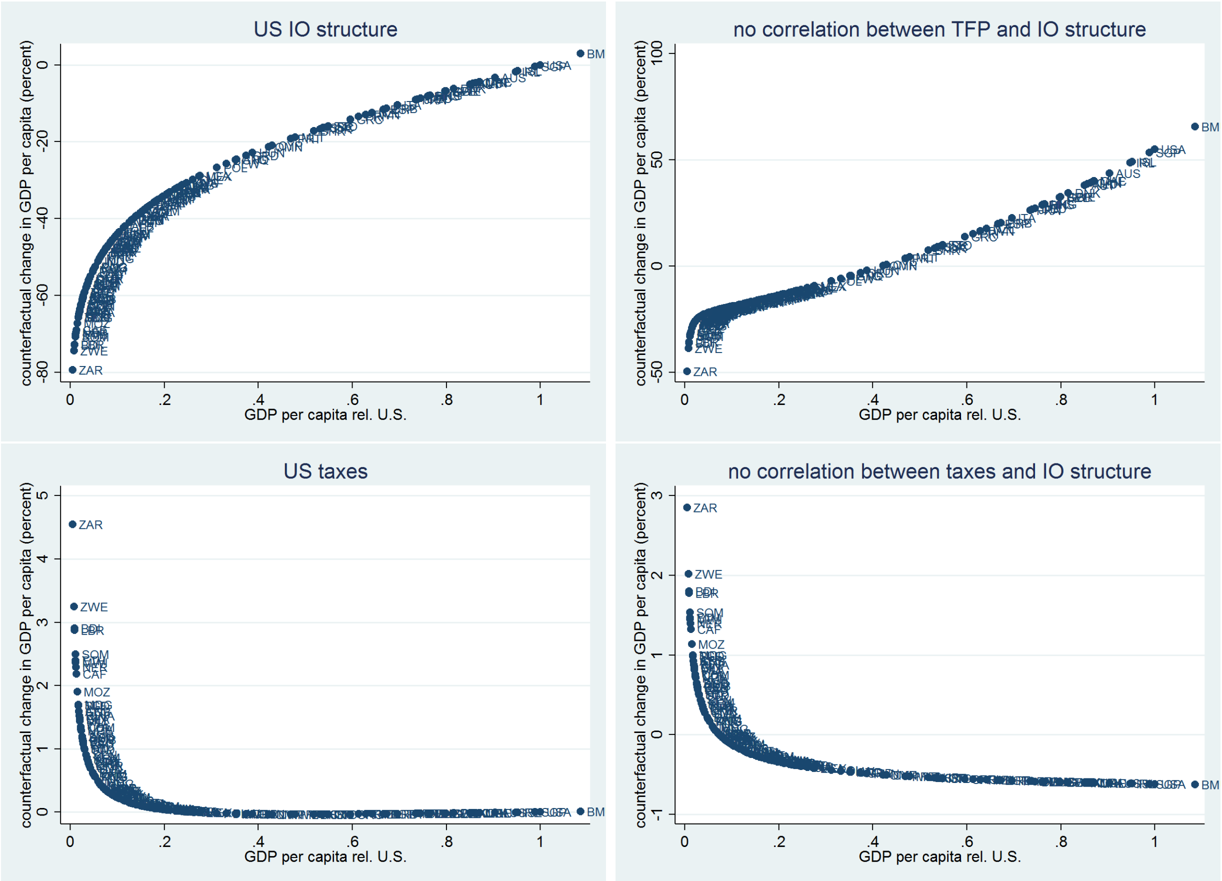

Figure 10: Counterfactuals

worsens aggregate outcomes in poor countries and improves those in rich countries further.

Next, we turn to a counterfactual where we set the distribution of log taxes as well as their correlation with log multipliers equal to the one of a country at the U.S. income level.[37] As can be seen from Table 2, average tax rates in rich countries are somewhat higher than in low-income countries, but they have a much lower variance across sectors and are thus less distortionary. Moreover, the correlation between multipliers and tax rates is negative for countries at the U.S. income level. The lower left panel of Figure 10 plots changes in income per capita (in percent) against GDP per capita relative to the U.S. One can see that setting distortions equal to the U.S. level provides negligible gains for most countries. Only countries with less than 20 percent of the U.S. income level gain significantly, with a maximum of around 5 percentage points for Congo (ZAR). Thus, income gains from reducing tax distortions to the U.S. level are modest for most countries.[38]

In the final experiment we set the correlation between log multipliers and log taxes to zero for all countries. The lower right panel of Figure 10 plots the resulting changes in per capita income (in

[37]We set the distribution of taxes equal to the estimated one for a country at the U.S. level of per capita GDP and not to the actual values of the U.S., which does not charge any taxes on gross output.

[38]Observe that this does not imply that distortions that imply misallocation of resources across sectors are small. Our data uses information on actual tax rates on gross output and the bulk of these distortions are captured by low sectoral productivity levels rather than by high tax rates.

percent) against GDP relative to the U.S. level. Again, income changes resulting from this experiment are relatively small. Poor countries – which empirically exhibit a positive correlation between multipliers and distortions– experience small increases in income (up to 3 percentage points for Congo (ZAR)), while rich countries – which empirically have a negative correlation between multipliers and tax rates – lose around one percentage point of income per capita.

Summary of counterfactual experiments:

1. *Imposing the dense IO structure of the U.S. on poor economies would reduce their income levels by up to 80 percent because a typical sector, which has a lower productivity than the high-multiplier sectors in these economies, would become more connected.*

2. *If poor economies did not have above-average productivity levels in high-multiplier sectors, their income levels would be reduced by up to 40 percent.*

3. *Imposing the distribution of tax rates of a country with the U.S. income level on poor economies would lead to moderate income gains of up to 5 percent.*

4. *If poor economies did not have above-average tax levels in high-multiplier sectors, their income levels would increase by up to 3 percent.*

5 Optimal taxation

The theoretical model employed so far considers tax rates as exogenously given and wasteful. In this section, we introduce an active role for the government and address the problem of optimal taxation. To do that, in principle we should specify the objective function of the government or social planner which is to be maximized by the choice of tax rates. However, given that in this model the market allocation is Pareto-optimal, an unconstrained planner would choose zero taxation. We thus analyze the problem of optimal taxation for exogenously specified government expenditures. The appealing feature of analyzing such "semi-optimal" taxation schemes is that they are much less dependent on the specific welfare function. Indeed, as long as welfare increases with individual consumption C, any welfare function would generate the same outcome for exogenously fixed government consumption G. In short, we will designate this analysis as GDP per capita maximization with exogenous G.

Importantly, optimal taxes will be obtained via a statistical approach, in line with the rest of the paper. As before, it is assumed that sectoral IO multipliers, taxes and productivities are drawn from a trivariate log-Normal distribution. The optimization task is then to maximize the expected value of GDP through an appropriate choice of the mean, variance and covariance of the tax distribution, keeping the other parameters fixed. All expected values are computed via a Monte Carlo method, and the fixed parameter values are kept at their empirical levels.

5.1 Optimal taxes: setup

To derive characteristics of optimal tax scheme, we use the equilibrium expression for log GDP. We consider the optimization problem in which this expression is maximized subject to a given level of government consumption. To solve that problem, we follow a statistical approach and instead of considering actual values of taxes $\{\tau_i\}$, we focus on the first and second moments of the distribution of taxes that generate the highest predicted aggregate output $E(y)$ for a given level of expected tax revenues/government consumption as computed from the data.[39] The expected values of aggregate output, and tax revenues/government consumption are computed via a Monte Carlo optimization method under the assumption that sectoral IO multipliers, taxes and productivities follow a trivariate log-Normal distribution. All parameters of this distribution, apart from those that relate to the distribution of taxes, are fixed at the levels of their empirical estimates. Then by varying the mean, variance and covariance of the tax distribution,[40] or more precisely, the mean, variance and covariance of the corresponding Normal distribution of the logarithm of taxes, we derive the features of the optimal tax scheme. The results of this numerical analysis can be briefly summarized as follows.

5.2 Optimal taxes: results

We assume that for each country, government consumption is fixed at the level generated by the estimated distributions. We find that the optimal tax distribution is degenerate with variance $\sigma_\tau^2 \to 0$. The correlation between taxes and IO multipliers is not relevant in the limit. Empirically, the optimal mean tax rate in rich countries is close to the estimated ones (around 5%), while in poor countries the optimal mean tax is substantially higher than the estimated ones (for some poor countries the optimal mean tax rate can be twice as large as the estimated mean tax rate).

In fact, the estimated distribution of tax rates in rich countries turns out to be close to optimum, featuring low variance and optimal mean. In poor countries, instead, the variance is high and the estimated mean tax rate is substantially lower than the optimal one. Moreover, there is a large positive correlation between tax rates and sectoral IO multipliers in poor countries, which ensures that high-multiplier sectors are taxed more. The latter is precisely the reason why a given level of tax revenues in

[39] An analytical solution in terms of actual values of tax rates (that maximize y subject to a given level of tax revenues) appears feasible only under some strong simplifying assumptions, which eventually lead to trivial or corner values of tax rates. We therefore resort to the statistical approach, which is also consistent with our approach in the prior empirical analysis.

[40] By covariance we mean the covariance between the distribution of taxes and IO multipliers, as the covariance between taxes and productivities does not affect the calculated values.

poor countries can be reached with a lower mean tax rate than prescribed in optimum. Indeed, under the optimal tax scheme all sectors should be taxed evenly, and then raising the same amount of tax revenues requires a higher mean tax. Still, we find that the distortion loss associated with high (optimal) mean tax is small compared to the loss associated with taxing high-multiplier sectors more. The left panel of Figure 11 plots welfare gains (in terms of percentage gains in GDP) of moving to a uniform tax rate that generates the same revenue as the current tax system against GDP per capita. The welfare gains are basically zero for all high-income countries but they can rise to up to 30% of GDP for some of the poorest countries in the world. This is consistent with our previous counterfactuals, which showed that poor countries would gain most from moving to the U.S. parameters of the tax distribution. However, in that experiment gains were much smaller since taxes were considered as wasteful.

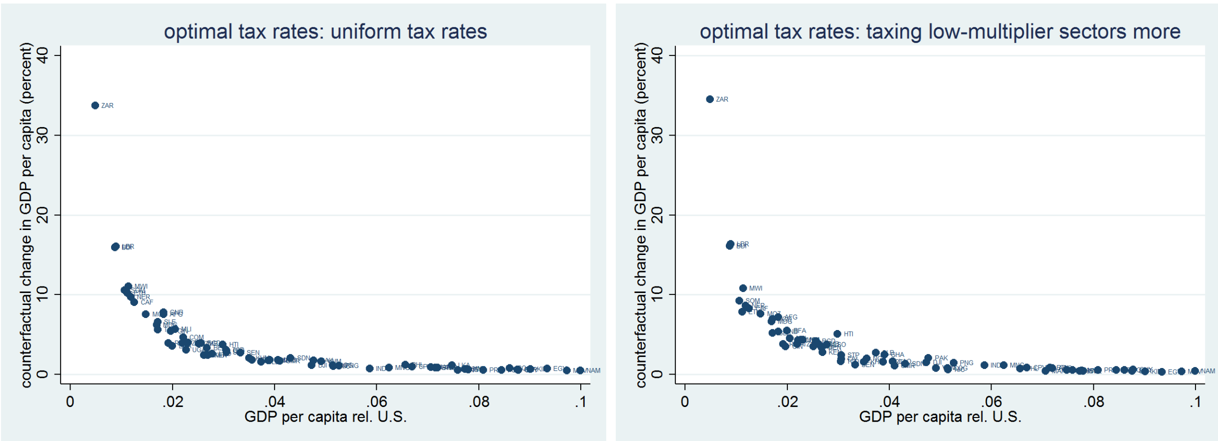

Figure 11: Optimal taxation

We also perform a more unusual experiment. Indeed, as there might be reasons why tax rates cannot be uniform, we want to explore the role of the covariance between taxes and IO multipliers for a given variation in tax rates. We set the variance of the tax rate distribution to be equal to the estimated value in each country and examine the role of choosing the optimal correlation between the distribution of tax rates and sectoral IO multipliers and the mean tax rate that keeps tax revenue constant. We find that the optimal tax distribution has negative correlation with sectoral IO multipliers, so that consistently with the findings of our empirical analysis, more central sectors should be taxed less. The right panel of Figure 11 plots the percentage gains in GDP per capita of moving to the optimal correlation between taxes and multipliers that keeps tax revenue constant. Again, welfare gains are substantial for very poor countries. Moreover, moving to a negative correlation between taxes and multipliers and increasing average tax rates would imply gains which are almost as large as those of moving to a uniform tax rate.

Finally, in unreported simulations we have considered an alternative welfare function where government expenditure enters households' utility in a Cobb-Douglas fashion. The results were very similar to those of the model that takes government expenditure as given.

6 Robustness checks

In this section, we report the results of a number of robustness checks in order to show that our findings do not hinge on the specific restrictions imposed on the baseline model. We consider the following extensions. First, we discuss an extension of our setup to CES production functions. Second, we estimate parameters using the approximated distribution of multipliers, where we employ the representation of multipliers in terms of sectoral out-degrees. Third, we allow for skilled and unskilled labor as separate production factors. Fourth, we generalize the final demand structure and introduce expenditure shares that differ across countries and sectors. Fifth, we generalize the model by introducing imported intermediate inputs. Finally, we consider taxes as government revenue instead of treating them as wasteful. We then show that none of these generalizations changes the basic conclusions of the baseline model. The formulas for aggregate income implied by these more general models as well as detailed derivations can be found in the Supplementary Appendix.

6.1 CES production function

For our counterfactual exercises to be valid, it is important that the structural model is correctly specified. One specific concern may be that sectoral production functions are not Cobb-Douglas, so that the elasticity of substitution between different intermediates is different from unity. If this were the case, IO coeffients would no longer be sector-country-specific constants but would instead be endogenous to equilibrium prices. While it has been observed that for the U.S. the IO matrix has been remarkably stable over the last decades despite large shifts in relative prices (Acemoglu et al., 2012) – an indication of a unit elasticity – we still consider here a more general CES sectoral production function that allows for an elasticity of substitution between intermediate inputs that is different from one. The sectoral production functions are now given by:

$$q_i = \Lambda_i \left(k_i^{\alpha} l_i^{1-\alpha} \right)^{1-\gamma_i} M_i^{\gamma_i}, \tag{16}$$

where $M_i \equiv \left(\sum_{j=1}^{N} \gamma_{ji} d_{ji}^{\frac{(\sigma-1)}{\sigma}} \right)^{\frac{\sigma}{(\sigma-1)}}$. The rest of the model is specified as in section 3.1.

With CES production functions the equilibrium cannot be solved analytically, so one has to rely on numerical solutions. However, it is straighforward to show how IO multipliers are related to sectoral productivities and distortions. The relative expenditure of sector i on inputs produced by sector j relative to sector k is given by:

$$\frac{p_j d_{ji}}{p_k d_{ki}} = \left(\frac{p_j}{p_k} \right)^{1-\sigma} \left(\frac{\gamma_{ji}}{\gamma_{ki}} \right) \tag{17}$$

Thus, if $\sigma > 1$ ($\sigma < 1$), each sector i spends relatively more on the inputs provided by the sector that charges a lower (higher) price. That sector then has a higher (lower) multiplier, as the multiplier is proportional to sector's outdegree $d_j^{out} = \sum_{i=1}^{N} p_j d_{ji}$. Moreover, since prices are inversely proportional to productivity and directly proportional to tax rates, sectors with higher productivity charge lower prices, while sectors with higher tax rates charge higher prices. This then implies that when $\sigma > 1$, sectoral multipliers and productivities (tax rates) are positively (negatively) correlated in *all* countries, while when $\sigma < 1$, the opposite is true. We confirm these results in unreported simulations. These predictions are not consistent with our empirical finding that multipliers and productivities are positively correlated in low-income countries, while they are negatively correlated in high-income countries. They are also not compatible with the fact that tax rates and multipliers are positively correlated in poor countries and negatively in rich ones. Consequently, unless the elasticity of substitution differs systematically across countries, the data on IO tables, sectoral productivities and tax rates are difficult to reconcile with CES production functions.

6.2 Approximation of multipliers

Next, we provide results when estimating the distribution of log multipliers from a first- and second-order approximation in terms of IO network characteristics, instead of using the actual empirical distribution of log multipliers. Following the discussion in section 3.3, the first-order approximation of multipliers is $\boldsymbol{\mu} \approx \frac{1}{n} + \frac{1}{n}\boldsymbol{\Gamma}\mathbf{1}$, while the second-order approximation is $\boldsymbol{\mu} \approx \frac{1}{n} + \frac{1}{n}\boldsymbol{\Gamma}\mathbf{1} + \frac{1}{n}\boldsymbol{\Gamma}^2\mathbf{1}$. The first-order approximation abstracts from higher-order interconnectedness and only considers direct outward linkages (weighted out-degrees), while the second-order approximation also considers second-order interconnectedness (the weighted out-degree of sectors to which each sector delivers).

Empirically, there is little difference between the actual distribution of multipliers and the estimated distributions based on the first and second-order approximation.[41]. Columns (1) and (2) of Table 6 report model fit results for the first- and second-order approximation. It is apparent that the difference in performance between the models is relatively small. The intercepts are now -0.106 and -0.133 and the slope coefficients are 0.989 and 0.974, respectively, for the first- and second-order approximation, compared to -0.014 and 0.922 for the baseline model with IO structure estimated on the WIOD sample. Observe also that the first-order approximation, which is most consistent with the formula for aggregate income (13), performs slightly better than the second-order approximation, indicating that modeling second-order interconnectedness does not help to improve our understanding how differences in countries' IO structure affect aggregate income.

[41]See Figure A-3 in the Supplementary Appendix

6.3 Cross-country differences in demand structure

So far we have abstracted from cross-country differences in the final demand structure, which also matter for the values of sectoral multipliers because sectors with larger final expenditure shares will have a larger impact on GDP. In the next robustness check, we thus consider a more general demand structure. More specifically, we model the production function for the aggregate final good as $Y = y_1^{\beta_1} \cdot ... \cdot y_n^{\beta_n}$, where β_i is allowed to be country-sector-specific. The advantage of this specification is that it picks up differences in the final demand structure that may have an impact on aggregate income. The drawback is that now multipliers become functions of both differences in the IO structure and differences in final demand. Thus, this specification does not allow one to differentiate between the two channels. The vector of sectoral multipliers is now defined as $\boldsymbol{\mu} = \{\mu_i\}_i = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta}$, where $\boldsymbol{\beta} = (\beta_1, .., \beta_n)'$. Its interpretation, however, is identical to the one before: each sectoral multiplier μ_i reveals how a change in productivity (or taxes) of sector i affects the overall value added in the economy.

The results for this model can be found in column (3) of Table 6. The intercept is now -0.12 and the slope coefficient is 0.901, which is somewhat worse than the performance of our baseline model. This indicates that – within the context of our model – modeling differences in the final demand structure does not help to understand differences in aggregate income. The reason seems to be that modeling differences in final demand structure introduces additional noise in the multiplier data, which makes it harder to estimate the systematic features of inter-industry linkages.

6.4 Skilled labor

Next, we split aggregate labor endowments into skilled and unskilled labor. Namely, let the technology of each sector $i \in 1 : n$ in every country be described by the following Cobb-Douglas function:

$$q_i = \Lambda_i \left(k_i^{\alpha} u_i^{\delta} s_i^{1-\alpha-\delta}\right)^{1-\gamma_i-\sigma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot ... \cdot d_{ni}^{\gamma_{ni}}, \tag{18}$$

where s_i and u_i denote the amounts of skilled and unskilled labor used by sector i, $\gamma_i = \sum_{j=1}^{n} \gamma_{ji}$ is the share of intermediate goods in the total input use of sector i and α, δ, $1 - \alpha - \delta \in (0, 1)$ are the respective shares of capital, unskilled and skilled labor in the remainder of the inputs. The total supply of skilled and unskilled labor in the economy is fixed at the exogenous levels of S and U, respectively. We define skilled labor as the number of hours worked by workers with at least some tertiary education and we define unskilled labor as the number of hours worked by workers with less than tertiary education. Information on skilled and unskilled labor inputs is from WIOD. We calibrate $\delta = 1/6$ to fit the college skill premium of the U.S. Results are provided in column (4) of Table 6. The intercept is -0.055 and the slope coefficient is 0.943, which is very close to the baseline model. We conclude that the results are not sensitive to the definition of labor endowments.

6.5 Imported intermediates

So far we have abstracted from international trade and we have assumed that all goods have to be produced domestically. Here, we instead allow for both domestically produced and imported intermediates. We thus assume that sectoral production functions are given by:

$$q_i = \Lambda_i \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{1-\gamma_i-\sigma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}} \cdot f_{1i}^{\sigma_{1i}} f_{2i}^{\sigma_{2i}} \cdot \ldots \cdot f_{ni}^{\sigma_{ni}}, \tag{19}$$

where d_{ji} are domestically produced intermediate inputs and f_{ji} are imported intermediate inputs. Domestic and imported intermediate inputs are assumed to be imperfectly substitutable, and γ_{ji}, σ_{ji} denote the shares of each domestic and imported intermediate, respectively, in the value of sectoral gross output. We change the construction of the IO tables accordingly by separating domestically produced from imported intermediates and then re-estimating the distribution of IO multipliers.

The results for model fit with this specification are given in column (5) of Table 6. The intercept is now -0.153 and the slope coefficient is 1.004. The intuition for why results remain very similar comes from the fact that most high-multiplier sectors tend to be services, which are effectively non-traded. Therefore, allowing for trade does not change the statistical distribution of multipliers and the implied predicted income in every country very much. We thus conclude that our results are quite robust to allowing for trade in intermediates.

6.6 Taxes as government revenue

As a final robustness check, we rebate tax revenues collected by the government lump sum to households instead of considering them as wasted. The results for this model are presented in column (6) of Table 6. We find that the results are basically equivalent to those in the baseline model: the intercept is 0.017 and not statistically different from zero, while the slope coefficient is 0.920. Thus, rebating government revenue to households does not make a difference. We therefore conclude that our baseline model is pretty robust to a number of extensions and alternative assumptions.

7 Conclusions

In this paper we have studied the role of input-output structure of the economy and its interaction with sectoral productivities and tax distortions in explaining income differences across countries. In contrast to the typical approach in the literature on development accounting, dual economies and structural transformation, we model input-output linkages between sectors and the difference in these linkages

Table 6: Robustness: World IO sample

	1st order approximation	2nd order approximation	Expenditure shares	Skilled labor	Imported intermediates	No waste
constant	-0.106***	-0.133**	-0.128***	-0.055***	-0.153***	0.017
	(0.022)	(0.059)	(0.027)	(0.002)	(-0.026)	(0.021)
slope	0.989***	0.974***	0.901***	0.943***	1.004***	0.920***
	(0.056)	(0.059)	(0.064)	(0.055)	(0.063)	(0.057)
Observations	39	39	39	39	39	39
R-squared	0.883	0.884	0.865	0.845	0.884	0.852

across countries explicitly. Moreover, our approach is to a large extent empirical, which complements the predominantly theoretical analysis of previous studies on cross-country differences in IO structure.

We first develop and analytically solve a multi-sector general equilibrium model with IO linkages, sector-specific productivities and taxes. We then estimate this model using a statistical approach that allows us to derive a simple closed-form dependence of aggregate per capita income on the first and second moments of the joint distribution of IO multipliers, sectoral productivities and tax rates. We estimate the parameters of this distribution to fit the corresponding empirical distribution of IO multipliers, productivities and tax rates for the countries in our sample, allowing them to vary with income per capita. The estimates imply important cross-country differences in countries' IO structure as well as in the interaction between IO structure and sectoral productivities and taxes. First, in low-income countries the distribution of sectoral IO multipliers is more extreme: while most sectors have very low multipliers, the multipliers of a small number of sectors are very high compared to the average. In contrast, the distribution of sectoral multipliers in rich countries allocates a relatively large weight to intermediate values of multipliers. Moreover, while in poor countries sectoral IO multipliers and productivities are positively correlated, in rich countries this correlation is negative. Similarly, the correlation of IO multipliers and tax rates is positive in poor countries but negative in rich.

These cross-country differences in the distribution of IO multipliers and their interaction with productivities and taxes lead to the difference in predicted income. We find that our (over-identified) model predicts cross-country income differences extremely well both within and out of sample. In fact, the generated predictions are much more accurate than those of a model that measures aggregate productivity as an average of the estimated sectoral productivities and ignores IO structure. Such a model overpredicts the variation in per capita income. The reason is that the empirically large sectoral TFP differences are actually mitigated by the IO structure. In paticular, as very low-productivity sectors in poor countries tend to be badly connected, they are not that relevant for the aggregate economy.

Our counterfactual experiments suggest that if we impose the much denser IO structure of the U.S. on poor countries and thereby increase the overall significance of their worst-productivity sectors, the per capita income of these countries could decline by as much as 80%. That is, given the very low productivity levels of many sectors in poor countries, having these sectors largely isolated effectively benefits these economies. Similarly, eliminating the correlation of sectoral multipliers and productivities would hurt poor countries but benefit the rich ones, due to the fact that the correlation of multipliers and productivities is positive in poor countries and negative in rich ones. At last, reducing distortions from taxes on gross output would improve the aggregate economic performance of poor countries; however, the associated per capita income changes would be relatively small.

Finally, we study the problem of optimal taxation and analyze the welfare gains from moving to an optimal tax system in all countries, while keeping tax revenues constant. Our findings suggest that when the government aims at maximizing GDP per capita for a given level of tax revenue, the actual distribution of tax rates is close to optimum in rich countries, but in poor countries, the mean of the distribution is too low and the variance is too high relative to the optimum. We also find that for a given value of tax variance, a negative correlation of taxes with IO multipliers is optimal, which once again suggests a relative advantage of the tax scheme implemented in rich countries. We find that some of the poorest countries in the world could gain up to 30% in terms of income per capita by moving to an optimal tax system, while benefits for the rich countries would be negligible.

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Appendix: Proofs for the benchmark model and its extensions

Proposition 1 and formulas for aggregate output in the main text are particular cases of Proposition 2 that applies in a generic setting – with imported intermediates, division of labor into skilled and unskilled labor inputs and unequal demand shares. A brief description of this economy, as well as Proposition 2 and its proof are provided below.

- The technology of each of n competitive sectors is Cobb-Douglas with constant returns to scale. Namely, the output of sector i, denoted by q_i, is

$$q_i = \Lambda_i \left(k_i^{\alpha} u_i^{\delta} s_i^{1-\alpha-\delta} \right)^{1-\gamma_i-\sigma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}} \cdot f_{1i}^{\sigma_{1i}} f_{2i}^{\sigma_{2i}} \cdot \ldots \cdot f_{ni}^{\sigma_{ni}},$$

  where s_i and u_i are the amounts of skilled and unskilled labor, d_{ji} is the quantity of the domestic good j and f_{ji} is the quantity of the imported good j used by sector i. $\gamma_i = \sum_{j=1}^{n} \gamma_{ji}$ and $\sigma_i = \sum_{j=1}^{n} \sigma_{ji}$ are the respective shares of domestic and imported intermediate goods in the total input use of sector i and α, δ, $1-\alpha-\delta$ are the respective shares of capital, unskilled and skilled labor in the remainder of the inputs.

- A good produced by sector i can be used for final consumption, y_i, or as an intermediate good:

$$y_i + \sum_{j=1}^{n} d_{ij} = q_i \qquad i = 1:n$$

- Final consumption goods are aggregated into a single final good through another Cobb-Douglas production function:

$$Y = y_1^{\beta_1} \cdot \ldots \cdot y_n^{\beta_n},$$

  where $\beta_i \geq 0$ for all i and $\sum_{i=1}^{n} \beta_i = 1$.

- This aggregate final good can itself be used in one of three ways, as households' consumption, government expenditures or as export to the rest of the world:

$$Y = C + G + X.$$

- Exports pay for the imported intermediate goods, and we impose a balanced trade condition:

$$X = \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{p}_j f_{ji},$$

  where $\overline{p}_j$ is the exogenous world price of the imported intermediate goods.

- Households finance their consumption through income:

$$C = w_U U + w_S S + rK.$$

- Government finances its expenditures through tax revenues:

$$G = \sum_{i=1}^{n} \tau_i p_i q_i.$$

- The total supply of physical capital, unskilled and skilled labor are fixed at the exogenous levels of K, U and S, respectively:

$$\begin{aligned} \sum_{i=1}^{n} k_i &= K, \\ \sum_{i=1}^{n} u_i &= U, \\ \sum_{i=1}^{n} s_i &= S. \end{aligned}$$

For this "generic" economy, the competitive equilibrium with distortions is defined by analogy with the definition in section 3.1. The solution is described by Proposition 2.

Proposition 2. *There exists a unique competitive equilibrium. In this equilibrium, the logarithm of GDP per capita, $y = \log(Y/(U+S))$, is given by*

$$\begin{aligned} y \;=\; & \frac{1}{\sum_{i=1}^{n} \mu_i (1-\gamma_i-\sigma_i)} \Bigg[\sum_{i=1}^{n} \mu_i \lambda_i + \sum_{i=1}^{n} \mu_i \log(1-\tau_i) + \sum_{i=1}^{n} \sum_{j \ s.t. \gamma_{ij} \neq 0} \mu_i \gamma_{ji} \log \gamma_{ji} + \sum_{i=1}^{n} \sum_{j \ s.t. \sigma_{ij} \neq 0} \mu_i \sigma_{ji} \log \sigma_{ji} - \\ & - \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i \sigma_{ji} \log \bar{p_j} + \sum_{i=1}^{n} \beta_i log \beta_i + \sum_{i=1}^{n} \mu_i (1-\gamma_i-\sigma_i) log(1-\gamma_i-\sigma_i) \Bigg] + \log \left(1 + \sum_{i=1}^{n} \tau_i \bar{\mu_i} + \sum_{i=1}^{n} (1-\tau_i) \sigma_i \bar{\mu_i} \right) + \\ & + \alpha \log K + \delta log U + (1-\alpha-\delta) log S - \log(U+S). \end{aligned} \tag{20}$$

where

$$\begin{aligned} \boldsymbol{\mu} &= \{\mu_i\}_i = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1} \boldsymbol{\beta}, && n \times 1 \textit{ vector of multipliers} \\ \boldsymbol{\lambda} &= \{\lambda_i\}_i = \{\log \Lambda_i\}_i, && n \times 1 \textit{ vector of sectoral log-productivity coefficients} \\ \bar{\boldsymbol{\mu}} &= \{\bar{\mu_i}\}_i = [\boldsymbol{I} - \bar{\boldsymbol{\Gamma}}]^{-1} \boldsymbol{\beta}, && n \times 1 \textit{ vector of multipliers corresponding to } \bar{\boldsymbol{\Gamma}} \\ \bar{\boldsymbol{\Gamma}} &= \{\bar{\gamma_{ji}}\}_{ji} = \{\beta_j \tau_i + \beta_j (1-\tau_i) \sigma_i + (1-\tau_i) \gamma_{ji}\}_{ji}, && n \times n \textit{ input-output matrix adjusted for taxes and trade} \end{aligned}$$

Proof. Part I: Calculation of $\log w_U$.

Consider the profit maximization problems of a representative firm in the final goods market and in each sector. For a representative firm in the final goods market the FOCs allocate to each good a spending share that is proportional to the good's demand share β_i:

$$p_i y_i = \beta_i Y = \beta_i(C+G+X) = \beta_i(w_U U + w_S S + rK) + \beta_i \sum_{i=1}^{n} \tau_i p_i q_i + \beta_i \sum_{i=1}^{n}\sum_{j=1}^{n} \overline{p}_j m_{ji} \quad \forall i \in 1:n$$

where the price of the final good is normalized to 1, $p = 1$. For a firm in sector i the FOCs are:

$$(1-\tau_i)\alpha(1-\gamma_i-\sigma_i)\frac{p_i q_i}{r} = k_i \tag{21}$$

$$(1-\tau_i)\delta(1-\gamma_i-\sigma_i)\frac{p_i q_i}{w_U} = u_i \tag{22}$$

$$(1-\tau_i)(1-\alpha-\delta)(1-\gamma_i-\sigma_i)\frac{p_i q_i}{w_S} = s_i \tag{23}$$

$$(1-\tau_i)\gamma_{ji}\frac{p_i q_i}{p_j} = d_{ji} \quad j \in 1:n \tag{24}$$

$$(1-\tau_i)\sigma_{ji}\frac{p_i q_i}{\bar{p_j}} = f_{ji} \quad j \in 1:n \tag{25}$$

Substituting the left-hand side of these equations for the values of k_i, u_i, s_i, $\{d_{ji}\}$ and $\{f_{ji}\}$ in firm i's log-production technology and simplifying the obtained expression, we derive:

$$\begin{aligned}\delta \log w_U = \; & \frac{1}{1-\gamma_i-\sigma_i}\Big(\lambda_i + \log(1-\tau_i) + \log p_i - \sum_{j=1}^{n}\gamma_{ji}\log p_j + \sum_{j \text{ s.t. } \gamma_{ji}\neq 0}\gamma_{ji}\log\gamma_{ji} - \\ & -\sum_{j=1}^{n}\sigma_{ji}\log\bar{p_j} + \sum_{j \text{ s.t. } \sigma_{ji}\neq 0}\sigma_{ji}\log\sigma_{ji}\Big) - \alpha\log r - (1-\alpha-\delta)\log(w_S) + \\ & +\log(1-\gamma_i-\sigma_i) + \alpha\log(\alpha) + \delta\log\delta + (1-\alpha-\delta)\log(1-\alpha-\delta)\end{aligned} \tag{26}$$

Next, we use FOCs (21) – (25) and market clearing conditions for labor and capital to express r and w_S in terms of w_U:

$$w_U = \frac{1}{U}\delta\sum_{i=1}^{n}(1-\tau_i)(1-\gamma_i-\sigma_i)(p_i q_i) \tag{27}$$

$$w_S = \frac{1}{S}(1-\alpha-\delta)\sum_{i=1}^{n}(1-\tau_i)(1-\gamma_i-\sigma_i)(p_i q_i) = \frac{w_U U}{S}\frac{1-\alpha-\delta}{\delta} \tag{28}$$

$$r = \frac{1}{K}\alpha\sum_{i=1}^{n}(1-\tau_i)(1-\gamma_i-\sigma_i)(p_i q_i) = \frac{w_u U}{K}\frac{\alpha}{\delta} \tag{29}$$

Substituting these values of r and w_S in (26) we obtain:

$$\begin{aligned}\log w_U = \; & \frac{1}{1-\gamma_i-\sigma_i}\Big(\lambda_i + \log(1-\tau_i) + \log p_i - \sum_{j=1}^{n}\gamma_{ji}\log p_j + \sum_{j \text{ s.t. } \gamma_{ji}\neq 0}\gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^{n}\sigma_{ji}\log\bar{p_j} + \\ & + \sum_{j \text{ s.t. } \sigma_{ji}\neq 0}\sigma_{ji}\log\sigma_{ji}\Big) + \alpha\log K - (1-\delta)\log U + (1-\alpha-\delta)\log S + \log(1-\gamma_i-\sigma_i) + \log\delta\end{aligned}$$

Multiplying this equation by the ith element of the vector $\boldsymbol{\mu'Z} = \boldsymbol{\beta'1'}[\boldsymbol{I} - \boldsymbol{\Gamma'}]^{-1} \cdot \boldsymbol{Z}$, where $\boldsymbol{Z}$ is a diagonal matrix with $Z_{ii} = 1 - \gamma_i - \sigma_i$, and summing over all sectors i gives

$$\sum_{i=1}^{n} \mu_i(1-\gamma_i-\sigma_i)\log w_U = \sum_{i=1}^{n}\mu_i\lambda_i + \sum_{i=1}^{n}\mu_i\log(1-\tau_i) + \sum_{i=1}^{n}\beta_i\log p_i + \sum_{i=1}^{n}\sum_{j \text{ s.t. } \gamma_{ji}\neq 0}\mu_i\gamma_{ji}\log\gamma_{ji} -$$
$$-\sum_{j=1}^{n}\mu_i\sigma_{ji}\log\bar{p_j} + \sum_{j \text{ s.t. } \sigma_{ji}\neq 0}\mu_i\sigma_{ji}\log\sigma_{ji} + \sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) +$$
$$+\sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)\left(\alpha\log K - (1-\delta)\log U + (1-\alpha-\delta)\log S + \log\delta\right)$$

Next, we use the relation between the price of the final good p (normalized to 1) and prices of each sector goods, derived from a profit maximization of the final good firm that has Cobb-Douglas technology.[42] This relation implies that $\prod_{i=1}^{n}(p_i)^{\beta_i} = \prod_{i=1}^{n}(\beta_i)^{\beta_i}$, so that $\sum_{i=1}^{n}\beta_i\log p_i = \sum_{i=1}^{n}\beta_i\log\beta_i$, and the above equation becomes:

$$\begin{aligned}\log w_U \;=\; & \frac{1}{\sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)}\Big[\sum_{i=1}^{n}\mu_i\lambda_i + \sum_{i=1}^{n}\mu_i\log(1-\tau_i) + \sum_{i=1}^{n}\beta_i\log\beta_i + \sum_{i=1}^{n}\sum_{j \text{ s.t. } \gamma_{ji}\neq 0}\mu_i\gamma_{ji}\log\gamma_{ji} - \\ & - \sum_{j=1}^{n}\mu_i\sigma_{ji}\log\bar{p_j} + \sum_{j \text{ s.t. } \sigma_{ji}\neq 0}\mu_i\sigma_{ji}\log\sigma_{ji} + \sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i)\Big] + \\ & + \alpha\log K - (1-\delta)\log U + (1-\alpha-\delta)\log S + \log\delta \end{aligned} \tag{30}$$

Part II: Calculation of y.

Recall that our ultimate goal is to find $y = \log(Y/(U+S)) = \log(C+G+X) - \log(U+S)$. From the households' and government budget constraints and from the balanced trade condition, $C + G + X = w_U U + w_S S + rK + \sum_{i=1}^{n}\tau_i p_i q_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\overline{p}_j f_{ji}$, where in the last term, $\overline{p}_j f_{ji} = (1-\tau_i)\sigma_{ji}p_i q_i$ (cf. (25)). Below we show that $p_i q_i$ can be expressed as a product of $w_U U + w_S S + rK$ and another term that involves distortions and structural characteristics. Then using (28) and (29), we obtain the representation of $C + G + X$ as a product of w_U and another term determined by exogenous variables. This representation, together with (30), will then allow us to solve for y.

Consider the resource constraint for sector j, with both sides multiplied by p_j:

$$p_j y_j + \sum_{i=1}^{n} p_j d_{ji} = p_j q_j$$

[42] Profit maximization of the final good's firm implies that $\frac{\partial Y}{\partial y_i} = \frac{p_i}{p}$. On the other hand, since $Y = y_1^{\beta_1} \cdot ... \cdot y_n^{\beta_n}$, we have $\frac{\partial Y}{\partial y_i} = \beta_i\frac{Y}{y_i}$. Hence, $\beta_i\frac{Y}{y_i} = \frac{p_i}{p}$, or $y_i = \beta_i\frac{pY}{p_i}$. Substituting this in the production technology of the firm in final good market, we obtain:

$$Y = \prod_{i=1}^{n}\left(\beta_i\frac{pY}{p_i}\right)^{\beta_i} = pY\prod_{i=1}^{n}\left(\beta_i\frac{1}{p_i}\right)^{\beta_i}.$$

So, $p\prod_{i=1}^{n}\left(\beta_i\frac{1}{p_i}\right)^{\beta_i} = 1$. Now, since we used the normalization $p = 1$, it must be that $\prod_{i=1}^{n}(p_i)^{\beta_i} = \prod_{i=1}^{n}(\beta_i)^{\beta_i}$.

Using FOCs of the profit maximization problem of the final good's firm and a firm in sector i, this can be written as:

$$\beta_j Y + \sum_{i=1}^{n} (1-\tau_i)\gamma_{ji} p_i q_i = p_j q_j$$

or

$$\beta_j (w_U U + w_S S + rK) + \sum_{i=1}^{n} \left[\beta_j \tau_i + (1-\tau_i)\gamma_{ji}\right] p_i q_i + \beta_j \sum_{i=1}^{n}\sum_{j=1}^{n} (1-\tau_i)\sigma_{ji} p_i q_i = p_j q_j.$$

Using the fact that $\sum_{j=1}^{n} \sigma_{ji} = \sigma_i$ and combining terms, we obtain:

$$\beta_j (w_U U + w_S S + rK) + \sum_{i=1}^{n} \left[\beta_j \tau_i + \beta_j (1-\tau_i)\sigma_i + (1-\tau_i)\gamma_{ji}\right] p_i q_i = p_j q_j.$$

Denote by $a_j = p_j q_j$ and by $\bar{\gamma}_{ji} = \beta_j \tau_i + \beta_j (1-\tau_i)\sigma_i + (1-\tau_i)\gamma_{ji}$. Then the above equation in the matrix form is:

$$(w_U U + w_S S + rK)\,\boldsymbol{\beta} + \bar{\boldsymbol{\Gamma}}\boldsymbol{a} = \boldsymbol{a}$$

where $\boldsymbol{\beta} = (\beta_1, .., \beta_n)'$, $\bar{\boldsymbol{\Gamma}} = \{\bar{\gamma}_{ji}\}_{ji}$ and $\boldsymbol{a} = \{a_j\}_j$. Hence,

$$\boldsymbol{a} = (\boldsymbol{I} - \bar{\boldsymbol{\Gamma}})^{-1} (w_U U + w_S S + rK)\,\boldsymbol{\beta} = (w_U U + w_S S + rK)\bar{\boldsymbol{\mu}}$$

where $\bar{\boldsymbol{\mu}} = \left(\boldsymbol{I} - \bar{\boldsymbol{\Gamma}}\right)^{-1} \boldsymbol{\beta}$.[43] So, $a_i = p_i q_i = (w_U U + w_S S + rK)\bar{\mu}_i$ and therefore,

$$\begin{aligned} Y &= C + G + X = w_U U + w_S S + rK + \sum_{i=1}^{n} \tau_i p_i q_i + \sum_{i=1}^{n}\sum_{j=1}^{n} (1-\tau_i)\sigma_{ji} p_i q_i = \\ &= (w_U U + w_S S + rK)\left(1 + \sum_{i=1}^{n} \tau_i \bar{\mu}_i + \sum_{i=1}^{n} (1-\tau_i)\sigma_i \bar{\mu}_i\right) \end{aligned}$$

Using (28) and (29), this leads to

$$Y = \frac{w_U U}{\delta}\left(1 + \sum_{i=1}^{n} \tau_i \bar{\mu}_i + \sum_{i=1}^{n} (1-\tau_i)\sigma_i \bar{\mu}_i\right).$$

so that

$$y = \log Y - \log(U+S) = \log w_U + \log U + \log\left(1 + \sum_{i=1}^{n} \tau_i \bar{\mu}_i + \sum_{i=1}^{n} (1-\tau_i)\sigma_i \bar{\mu}_i\right) - \log\delta - \log(U+S).$$

[43] Notice that $\left(\boldsymbol{I} - \bar{\boldsymbol{\Gamma}}\right)^{-1}$ exists because the sum of elements in each column of $\bar{\boldsymbol{\Gamma}}$ is less than 1: $\sum_{j=1}^{n} (\beta_j \tau_i + \beta_j (1-\tau_i)\sigma_i + (1-\tau_i)\gamma_{ji}) = \tau_i + (1-\tau_i)\sigma_i + (1-\tau_i)\gamma_i = \tau_i + (1-\tau_i)(\sigma_i + \gamma_i) < 1$ for any $\sigma_i + \gamma_i < 1$ and any $\tau_i < 1$.

Finally, substituting $\log w_U$ with (30) yields our result:

$$\begin{aligned} y &= \frac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\Bigg[\sum_{i=1}^n \mu_i\lambda_i + \sum_{i=1}^n \mu_i \log(1-\tau_i) + \sum_{i=1}^n \sum_{j \text{ s.t.}\gamma_{ij}\neq 0} \mu_i\gamma_{ji}\log\gamma_{ji} + \sum_{i=1}^n \sum_{j \text{ s.t.}\sigma_{ij}\neq 0} \mu_i\sigma_{ji}\log\sigma_{ji} - \\ &- \sum_{i=1}^n\sum_{j=1}^n \mu_i\sigma_{ji}\log\bar{p_j} + \sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)log(1-\gamma_i-\sigma_i) + \sum_{i=1}^n \beta_i log\beta_i\Bigg] + \log\left(1+\sum_{i=1}^n \tau_i\bar{\mu_i} + \sum_{i=1}^n(1-\tau_i)\sigma_i\bar{\mu_i}\right) + \\ &+\alpha\log K + \delta logU + (1-\alpha-\delta)logS - \log(U+S). \end{aligned}$$

This completes the proof. □

Application of Proposition 2 to the case of the benchmark economy:

Proof. (Proposition 1) In case of our benchmark economy, we assume that: i) there is no distinction between skilled and unskilled labor, so that $\delta = 1-\alpha$ and the total supply of labor is normalized to 1; ii) demand shares for all final goods are the same, that is, $\beta_i = \frac{1}{n}$ for all i; iii) the economies are closed, so that no imported intermediate goods are used in sectors' production, that is, $\sigma_{ji} = 0$ for all $i, j \in 1:n$ and $\sigma_i = 0$ for all i. This simplifies the expression for y in Proposition 2 and produces:

$$\begin{aligned} y &= \frac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i)}\left(\sum_{i=1}^n \mu_i\lambda_i + \sum_{i=1}^n \mu_i\log(1-\tau_i) + \sum_{i=1}^n \sum_{j \text{ s.t. } \gamma_{ji}\neq 0} \mu_i\gamma_{ji}\log\gamma_{ji} + \sum_{i=1}^n \mu_i(1-\gamma_i)log(1-\gamma_i) - \log n\right) + \\ &+ \log\left(1+\sum_{i=1}^n \tau_i\bar{\mu_i}\right) + \alpha\log K, \end{aligned}$$

Now, observe that $\sum_{i=1}^n \mu_i(1-\gamma_i) = \mathbf{1}'[\boldsymbol{I}-\boldsymbol{\Gamma}]\cdot\frac{1}{n}[\boldsymbol{I}-\boldsymbol{\Gamma}]^{-1}\mathbf{1} = \frac{1}{n}\mathbf{1}'\mathbf{1} = 1$. Then the expression simplifies even further and leads to the result of Proposition 1:

$$\begin{aligned} y &= \sum_{i=1}^n \mu_i\lambda_i + \sum_{i=1}^n \mu_i\log(1-\tau_i) + \sum_{i=1}^n \sum_{j \text{ s.t. } \gamma_{ji}\neq 0} \mu_i\gamma_{ji}\log\gamma_{ji} + \sum_{i=1}^n \mu_i(1-\gamma_i)log(1-\gamma_i) - \log n + \\ &+ \log\left(1+\sum_{i=1}^n \tau_i\bar{\mu_i}\right) + \alpha\log K, \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{\mu} &= \{\mu_i\}_i = \frac{1}{n}[\boldsymbol{I}-\boldsymbol{\Gamma}]^{-1}\mathbf{1}, && n\times 1 \text{ vector of multipliers} \\ \boldsymbol{\lambda} &= \{\lambda_i\}_i = \{\log\Lambda_i\}_i, && n\times 1 \text{ vector of sectoral log-productivity coefficients} \\ \boldsymbol{\tau} &= \{\tau_i\}_i, && n\times 1 \text{ vector of sector-specific taxes} \\ \bar{\boldsymbol{\mu}} &= \{\bar{\mu_i}\}_i = \frac{1}{n}[\boldsymbol{I}-\bar{\boldsymbol{\Gamma}}]^{-1}\mathbf{1}, && n\times 1 \text{ vector of multipliers corresponding to } \bar{\boldsymbol{\Gamma}} \\ \bar{\boldsymbol{\Gamma}} &= \{\bar{\gamma_{ji}}\}_{ji} = \{\frac{\tau_i}{n} + (1-\tau_i)\gamma_{ji}\}_{ji}, && n\times n \text{ input-output matrix adjusted for taxes} \end{aligned}$$

□