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The Political Economy of Immigration and Population Ageing

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Abstract

I investigate the effects of population ageing on immigration policies. Voters' attitude towards immigrants depends on how the net gains from immigration are divided up in the society by the fiscal policy. In the theoretical literature this aspect is treated as exogenous to the political process because of technical constraints. This generates inconsistent predictions about the policy outcome. I adopt a new equilibrium concept for voting models to analyse the endogenous relationship between immigration and fiscal policies and solve this apparent inconsistency. I show that the elderly and the poor have a common interest in limiting immigration and in increasing public spending. This exacerbates the effects of population ageing on public finances and results in a high tax burden on working age individuals and further worsens the age profile of the population. Moreover, I show that if the share of elderly population is sufficiently large, then a society is unambiguously harmed by the tightening in the immigration policy caused by the demographic change. The implications of the model are consistent with the patterns observed in UK attitudinal data and in line with the findings of the empirical literature about migration.

JEL classification: D72, C71, J610, H550.
Keywords: Immigration, Ageing, Policy, Voting.

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1 Introduction

What are the effects of population ageing on immigration policies? Do ageing societies tend to impose excessive restrictions on the inflow of foreign workers and if so, why? Should we expect an adjustment in immigration and spending policies to mitigate the impact of population ageing on public finances? This paper attempts to answer these questions using a theoretical model. In particular, I investigate why rapidly ageing countries - that arguably need more legal immigration - are imposing increasing restrictions to the inflow of immigrant workers and how this choice affects the tax burden faced by the working population. I also analyse the effects of these policy changes on the welfare of current and future generations. The importance of these questions is related to the vast fiscal effects of population ageing and immigration. The increase in longevity implies rising costs for the public sector, in particular the ones of public pensions and health care. The fall in the fertility rates causes an insufficient growth in the tax base. Both result in a pressure on public finances and tax rates. Several scholars and policy makers suggest that legal immigration can help in mitigating the effects of this problem, but this can happen only if there is political support for an increasingly open immigration policy. This analysis is therefore crucial to assess the fiscal soundness of ageing societies in the long run. Immigration also have demographic, social and cultural implications. Hence the study of immigration policies is also important to understand the evolution of the structure of our society in a broader sense.

1.1 Methods

In keeping with previous literature (Razin and Sadka, 1999), I analyse a political economy model with overlapping generations, in which voters differ in their income and in their age. In contrast with previous literature, however, I depart from a unidimensional policy space. Specifically, in each period the society chooses a two-dimensional policy consisiting of an immigration quota and of the provision of an imperfect public good. The elderly receive an exogenous public pension that is financed by the tax revenues. The government budget is balanced, hence the political choice determines the tax rate on labour income. The bi-dimensionality of the policy allows one to model endogenously both the immigration policy and how the net fiscal benefits from immigration are divided up in the society. In detail, if immigrants generate a fiscal surplus, voters can employ it to increase public spending and/or to reduce taxes. The first choice mostly benefits the elderly and the low-income individuals, while the second favours the high earners. This implies that the way in which the net gains are divided up by the fiscal system is crucial to correctly assess the attitude towards immigration of different groups of voters. An endogenous analysis of both the immigration and the fiscal policy requires a bi-dimensional policy. Thus the standard tools in the Political Economy literature - based on unidimensionality - cannot be used to answer this question. In order to address this problem, I adopt a dynamic version of the model of electoral competition and of the concept of coalitional equilibrium proposed in Dotti (2015). In such theoretical framework simple ordinal preference restrictions are sufficient to deliver existence of equilibrium and sharp comparative static results on the policy outcome. This is a consequence of a key restriction on the political process. Specifically, single politicians cannot commit to any platform other than their ideal policies, but they can form coalitions to enhance their ability to commit through internal agreements. Coalitions must be stable in equilibrium, in the sense that no subcoalition has a strict incentive to deviate and propose a different policy platform. I adopt this notion of equilibrium to study an overlapping generations model of immigration and public spending. This allows me to analyse how shocks on the longevity and on the fertility of the population affect immigration policy, public spending and the tax rate faced by the working population.

1.2 Summary of Results

I show that the elderly and the low income individuals have a common interest in reducing immigration and increasing public spending. Population ageing causes both an increase in the political power of these groups and a pressure on the government budget due to the rising cost of pensions. These two channels underpin the main results of this paper, which are as follows.

First, I show that, if the share of elderly is sufficiently large, a rise in the longevity and/or a fall in the natural growth rate of the population cause a tightening in the immigration policy and an increase in public spending. The reduced inflow of immigrant workers implies a reduction in the tax base. This, together with the rise in public spending in public goods and pensions, causes a sharp rise in the tax rate. Hence the political process tends to exacerbate the effects of population ageing on public finances.

Second, the effects of demographic shocks tend to worsen with time. In detail, a reduction in the immigration quota in the current period implies a change in the future age profile of the population because immigrants are mostly young and have weakly higher fertility rates relative to the natives. This causes further population ageing in the following periods and reinforces the effects.

Third, if the share of retired population is sufficiently large, then the tightening in the immigration policy generates a welfare loss for the society as a whole and harms the future generations.

These results suggest that ageing countries, that arguably need more immigration, tend to reduce it instead. This causes vast and persistent welfare and demographic effects and can affect the fiscal sustainability of the public sector in these countries.

1.3 Related Literature

Population ageing has been significant since the mid-twentieth century and it is expected to have dramatic demographic consequences in the next decades (see Figure 1). On one hand there are strong theoretical and empirical arguments in support of legal immigration as an instrument to ensure the financial soundness of a rapidly ageing society (Razin and Sadka, 1999 and Dustmann and Frattini, 2014). On the other hand the recent political debate in many countries is dominated by the discussion about how to limit the inflow of foreigners by introducing increasingly restrictive immigration policies. In many European countries this political agenda has leaded to a substantial tightening of immigration restrictions from 1994 (Boeri and Brucker, 2005) as shown in Figure 2. About the USA, Ortega and Peri (2009) provide evidence of an increase in the restrictiveness of immigration policy in the period 1994-2005. These trends in the implemented policies are consistent with a widespread and increasing aversion to immigration in those countries. Attitudinal data show that in the UK the share of citizens that would like immigration into their country to decrease has risen from 72.8% to a staggering 79.1% during the last 10 years (British Social Attitude Survey, 2003-2013). Moreover, the elderly are consistently more averse to immigration relative to the young. In the UK 85.7% of the individuals aged 60 or over would like less immigration while 71.2% of the individuals under 40 years old share the same opinion (British Social Attitude Survey, 2013). In the USA, the corresponding values are 47.3% and 39.2% (General Social Survey, 2014). These statistics suggest that population ageing may play an important role in the collective choice about immigration policies.

The empirical studies of the determinants of immigration policy are mostly based on attitudinal data and provide two main consistent facts that are relevant for this paper. The first fact is that age, education and income have a significant impact on the disapproval of further immigration and that in particular the elderly tend to have stronger preferences against further immigration in comparison with the young. Dustmann and Preston (2007), Facchini and Mayda (2007) and Card *et al.* (2011), using respectively data from the British Social Attitude Survey, the International Social Survey Programme and the European Social Survey, all support this finding. The latter paper also provides evidence that this result is mainly due to the perceived effect of immigration on the composition of the community in which the respendents live (or "compositional amenities") and to its economic effects. The second important fact is that economic hostility to immigration is driven by concern about effects on public finances at least as much as by effects on labour market outcomes (Dustmann and Preston, 2006, 2007; Boeri, 2010). Consistently with this finding, Milner and Tingley (2009) show that public finance aspects play a major role in shaping the immigration policy in the US. This is somewhat surprising given that there is not convincing empirical evidence about negative net effects of legal immigration on public finances (Preston, 2014), and that on the contrary some studies suggest that legal immigrants may be net contributors to the fiscal system in several countries (Dustmann *et al.* 2010, Dustmann and Frattini 2014). Lastly, the empirical literature about public spending provides an important result for this analysis. That is, population ageing affects fiscal policies in two key ways. On one hand there are direct effects - largely exogenous to the political process - due to changes in the cost of pensions, health care and education (Banks and Emmerson, 2003). On the other hand there is evidence that indirect political effects play an important role in shaping spending policies (Persson and Tabellini, 1999; Galasso and Profeta, 2004). Accounting for these two aspect is crucial to understand how demographic shocks affect the tax rates.

These three empirical findings justify some of the modelling choices of this paper. In particular: (i) the choice of an overlapping generation model with a crucial role for the elderly in shaping the equilibrium policy, (ii) the main role played by the political determination of tax rates and public spending in shaping the attitudes of different individuals towards immigration, (iii) the explicit account for the "compositional amenities" in the preferences of native individuals, (iv) the inclusion of both exogenous and endogenous effects of ageing on the size of public spending.

The theoretical literature has analyzed the effects of population ageing on three political outcomes that are crucial for this paper, namely: (i) the immigration policies, (ii) the public spending policies, and how these two affect (iii) the tax policy (Razin and Sadka 1999, 2000, Razin *et al.* 2002). The use of unidimensional models to study this problem (that is largely prevalent in the literature) has constrained the analysis to a unique endogenous outcome variable. The implication is that fiscal and immigration policies have been studied separately. This resulted in two complementary streams of literature whose key trade-offs are going to be relevant in the model proposed in this paper.

The first analyzes the political effects of ageing on public spending and intergenerational redistribution. Persson and Tabellini (1999) show that in a simple overlapping generation model the extent of intergenerational redistribution towards the elderly is increasing in the share of elderly population, and Tabellini (1990), Lindert (1996) and Perotti (1996) provide a partial empirical support to this hypothesis. Razin *et al.* (2002) propose a second channel: a larger share of elderly implies a higher tax burden on the median voter, because it corresponds to a lower share of taxpayers relative to the share of net benefit receivers. These two channels imply opposite effects of ageing on the level of public spending in equilibrium: the pro-tax coalition becomes larger but each taxpayer is relative less supportive of public spending.

The second stream of literature analyzes the determinants of immigration policy. If on one hand some papers focus on immigration policies related to the *quality* of immigrants, such as skill requirements (Benhabib, 1996 and Ortega, 2005), on the other hand the prevalent approach - of which this paper is an example - analyses policies that restrict the *number* of immigrants such as immigration quotas (see Preston, 2014 for a survey). These papers (Kemnitz, 2003; Krieger, 2003; Ben-Gad, 2012) emphasize the importance of intergenerational aspects such as the pension system and the investment in education in explaining the determinants of the political choice about immigration policies.

A crucial finding in this literature is that the unidimensionality assumption has important consequences on the predictive power of these models. In particular, it generates inconsistent predictions about the comparative statics of the outcome variable depending on the specific restrictions that are imposed in order to satisfy the required condition. An example of these paradoxical effects is described in Facchini and Mayda (2008, 2009) and Haupt and Peters (1998). They study a simple economy characterized by a linear income tax and assume that revenues are lump-sum rebated to all citizen. In this setting one may choose to meet the requirement of unidimensionality by imposing the exogeneity of either (i) the level of public spending in benefits or of (ii) the income tax rate. These two assumptions corresponds respectively to the classes of (i) "Tax adjustment models" (TAM, e.g. Scholten and Thum, 1996) and "Benefit adjustment models" (BAM, e.g Razin and Sadka, 1999, 2000) and imply opposite predictions about the relationship between pre-tax income, age and attitude towards immigration (Figure 1-2-3-4). Specifically, the first model implies that the elderly and the low income individuals are more hostile to immigration than the young and high income, while the opposite is true in the second model. The intuition that underpins these two apparently contradictory results lies in the consequence of an increase in the legal inflow of immigrants. Consider for instance the case in which immigrants are net contributors to the fiscal system. If publicly provided benefits are set exogenously, then the effect of an increase in immigration is a fall in the tax rate. Conversely, if the exogenous variable is the tax rate, then the effect is a rise in public spending per capita. As a result, in the former case immigration benefits mostly the young and high income voters, while in the latter the elderly and the low income individuals enjoy the largest share of the gains. In a recent paper Preston (2014) clarifies that the source of this inconsistency lies in how the social gains generated by immigration are divided up among different groups. This division is an output of the political process, but existing models treat it as an input. The issue is even more relevant for the purposes of this paper because I aim not only to understand the patterns of immigration policy, but more generally to address how a democratic society responds to population ageing in terms of immigration and fiscal policy, and the overall consequences on the public finances. These questions can be addressed only in a framework that allows immigration, spending and tax policy to be endogenously determined.

The theoretical literature has recognized the crucial importance of multidimensionality of the policy space in order to study the determinants of immigration policies, but all the existing studies are based on unidimensional models because of technical reasons. The early papers by Plott (1967), Tullock (1967) and Devis *et al.* (1972) have established rather restrictive conditions for the existence of a *Condorcet Winner* - a platform that is preferred to any alternative by a majority of voters - if the policy space is multidimensional. Grandmont (1978) has elegantly generalized these conditions with the concept of *Intermediate Relations.* The use of Grandmont's result in Political Economy applications is restricted to simple problems of redistribution (e.g. Borge and Rattsø, 2004) because of the extreme constraints that it imposes on preferences' heterogeneity. These requirements are way too restrictive for applications in which different subgroups of the voting population (such as the working age and the retired individuals in this paper) have sufficiently heterogeneous preferences over the set of available policies[1].

Alternatives to unidimensional voting models are popular in the literature, but they are not generally useful to answer questions about the comparative statics of the equilibrium policy outcomes because they do not deliver sharp analytical predictions about the policy response to a shock to the voters' distribution. This can be due either to a large multiplicity of equilibria,

[1]In the supplementary material I provide an example of why the Grandmont conditions usually fail to apply in this framework, and in particular to the model that I present in section 3 of this paper.

like in the Citizen-Candidate models (Besley and Coate, 1997) and in the Party Unanimity Nash Equilibrium (Roemer, 1999), or to the lack of sufficiently robust analytical comparative statics results, like in Probabilistic Voting models (Lindbeck et al. 1987, Banks et al. 2003). A more detailed analysis of the advantages and disadvantages of different theoretical framework in the study of comparative statics in models of electoral competition is provided in Dotti (2015). An attempt to model collective choices over immigration policies and welfare spending allowing for a multidimensional policy space is in Razin *et al.* (2011, 2014). They characterize the type of political coalitions that may prevail among skilled, unskilled and elderly voters in an overlapping generation models that shares several features with the one proposed in this paper. Neverthless, their approach is unsuitable to answer the questions of this paper, because of two reasons. First, they assume exogenous tax rates. Thus, the implications in terms of preferences for immigration are the same as the ones of Benefit adjustment models. Secondly, the assumptions they impose to tackle the multidimensionality of the policy space severely limit the possibility of deriving comparative statics results about the equilibrium policy outcome. This paper is based on another stream of literature (Levy 2004, 2005) which exploits the role of coalitions and political parties in ensuring stability in a multidimensional deterministic voting model. I adopt a dynamic version of the model of electoral competition proposed by Dotti (2015). Such framework, under appropriate preferences restrictions, delivers sharp predictions about the equilibrium policy outcome, and it is therefore suitable to answer the questions of the paper.

1.4 Organization of the paper

The paper is organized as follows. In the next section I introduce the main model and an equilibrium concept that allows me to answer the questions. Section 3 presents the main results of the paper, which are stated in Theorems 7-8. In section 4 I propose four extensions of the basic framework. In section 5 I analyze the welfare implications of the main predictions of the paper. Section 6 provides an analysis of the determinants of the attitude towards immigration in the UK based on the British Social Attitude Survey and show that they are consistent with the one implied by the model proposed in this paper. Lastly, in section 7 I discuss some limitations of this work and future directions of research.

2 A Political Model of Immigration and Spending Policy

This section is constituted by two parts. In the first I describe the features of the political process. In the second I present the economic model of immigration and public spending and I formally define the notion of equilibrium. These two theoretical tools are then used to derive the main results of this paper, which are stated in section 3.

2.1 The Political Process

I define a political process that translates individual preferences into a policy outcome x_t in each period t. The elements of the vector x_t represent the relevant policy outcomes, namely the immigration quota (M_t) and the uniform provision of an imperfect public good (Y_t). I adopt a dynamic version of the political model of electoral competition introduced in a companion working paper (Dotti 2015). It is a general tool with a potentially large range of applicability, some of which are mentioned in the concluding section of Dotti (2015). The closest example in the literature is in Levy (2004, 2005). A formal definition of the equilibrium concept is provided in section 2.3 (Definition 1), while a detailed description of the political process and its properties in the static case is available in Dotti (2015).

The political process is based on the assumption that voters can form coalitions in order to enhance their capacity to influence the policy outcome. Each individual can be the member of only one coalition, thus a *coalition structure* is defined as a partition of the set of voters. As in Levy (2005), a coalition can only offer credible policies, that is, policies in the Pareto set of its members. Thus, when a voter runs as an individual candidate, he can only offer his ideal policy, as in the "citizen-candidate" model. On the other hand, when heterogeneous individuals join together in a coalition, their Pareto set is larger than the set of their ideal policies. This assumption captures the idea that within a coalition individuals can commit to policies that represents a compromise among the members, and that these internal agreements are credible for the voting population provided that not all the members have an incentive to renegotiate the terms of the deal. Individuals play a two stage game: in the first stage they form coalitions in support of a certain proposed policy platform (or no policy) and in the second stage a voting game is played over the set of policies that are proposed by at least one coalition in the previous stage. Coalitions are required to be stable in equilibrium, in the sense that each coalition must possess at least one policy vector in its Pareto set such that - if the policy is proposed - there is no subcoalition that have a strict incentive to deviate and propose a different platform (named a *deviator* in this case)[2]. If the deviation occurs the policy initially proposed by the coalition may become unfeasible. Therefore the profitability of a deviation depends on the behavior of the remaining part of the coalition that did not participate in the deviation. I assume that this subgroup responds to the deviation by proposing a policy (if any) that is capable of reducing the final payoff of some (or of all) the deviating players and therefore to prevent the deviation, and no policy if such platform does not exist. It can be shown that the main results of this section are robust to different assumptions about such behaviour (see Dotti 2015). Moreover, I assume that the profitability of a deviation is determined by the final outcome of the voting process[3]. Specifically, voters fully anticipate not only the effects of their

[2] One can also allow for mergers between coalitions with no effects on the results in Theorems 3-4-5.

[3] Alternatively one can assume that the equilibrium choices of other coalitions do not affect the behavior of potential deviators (in such case the stability is purely *internal* to the coalition), with no effects on the comparative statics results, see Dotti (2015).

strategies in the current period, but also the effects on the equilibrium in the following periods. The latter effects are derived assuming rational expectations that satisfy the Markov property. This means that expectations about future equilibrium outcomes depend uniquely on the state of the economy in the current period. Details are provided in section 2.3. I assume a tie-breaking rule for the case in which, given the other platforms that are offered in equilibrium, all members of a given coalition are indifferent between offering a platform and running at all. Specifically, I impose that in equilibrium a coalition facing such a situation does not propose any platform. The same restriction is assumed in Levy (2005) and it is justified if one considers some small costs of running for elections which are not explicitly assumed in the model. If there is at least one policy in the Pareto set of a certain coalition that does not face any deviator, then this policy is *feasible* and the coalition is *stable*. A *stable coalition structure* is a partition of the set of voters in which all coalitions that are part of such partition are stable in the sense described above. Before observing the coalition structure each coalition (including one-member coalitions) propose either a feasible policy platform or no policy. Then the coalition structure and the proposed platforms are observed by all the players. Voters (the whole population) vote one of the available policy platforms and the election's outcome is a weak *Condorcet Winner*, which I name a *winning policy*. If no policy is offered or no weak *Condorcet Winner* exists, a default policy is implemented which is worse for all players than any other outcome[4]. A set of platforms (named a *policy profile*), a stable coalition structure and a winning policy given expectations about future policy outcomes constitute a *Markov-Perfect coalitional equilibrium* of the game if one of the coalition is a (weak) *Condorcet Winner* of the voting game at the second stage (see Dotti 2015). Notice that, differently from Levy (2005), I do not assume sincere voting: the existence of a *Condorcet Winner* at the second stage of the voting game implies a result that is robust to a fully sophisticated voting behavior and to a number of different voting protocols.

The main difficulty in applying the concept of coalitional equilibrium to the analysis in this paper is related to the dynamic nature of the problem. Specifically, voters' expectations about the effects of current policy choices on future outcomes may affect the equilibrium behaviour. Moreover, because of this dynamic aspect, multiple equilibria are, in principle, possible. However, under appropriate restrictions on voters' expectations, the analysis in each period t becomes equivalent to the one of a static problem, as I am going to clarify in section 2.3. In the next section I present the economic model of immigration that I adopt in this paper, and in section 2.3 I will provide sufficient conditions on voters' expectations such model satisfy, in each period t, the condition for a coalitional equilibrium.

[4]The comparative statics results apply even if the default policy is the platform implemented in the previous period, see Appendix B.4.

2.2 The Economic Environment

In this section I introduce an economic model of immigration and public spending in the spirit of the ones in the literature, in particular of Razin and Sadka (1999). Differently from the latter, I allow for the endogeneity of both the spending variable (an imperfect Public Good) and the immigration policy (in the form of a quota in each period t).

2.2.1 Demographic Structure

Consider an overlapping generation model with three generations in each period t: the children (ch), the working age population (y) and the elderly (o). In each period only the native individuals of working age and the elderly (which include both the native and immigrants of the previous period) have voting rights (highlighted in capital letters in Fig. 7).

time	$t-1$			t			$t+1$	
born $t-3$	OLD	(o)	$\rightarrow$	$\times$				
born $t-2$	NATIVE (n) Immigrant (m)	(y)	$\rightarrow$	OLD	(o)	$\rightarrow$	$\times$	
born $t-1$	Children	(ch)	$\rightarrow$	NATIVE (n) Immigrant (m)	(y)	$\rightarrow$	OLD	(o)
born t				Children	(ch)	$\rightarrow$	NATIVE (n) Immigrant (m)	(y)
born $t+1$							Children	(ch)

Fig. 7. Structure of Overlapping Generations

Each period has length normalized to 1 and it is characterized by a native working age population of size n_t and a number of immigrants m_t in their working age. Natives and immigrants have potentially different exogenous expected fertility rates denoted by σ_t^n and σ_t^m respectively. An elderly individual at time t has life expectancy $l_{t-1} \leq 1$. At the end of each period immigrants and their children are fully assimilated to the native population in terms of costs and fertility behavior. The size of each part of the population is summarized in Fig. 8. Denote with o_t the size of the elderly population, i.e. $o_t = l_{t-1}(n_{t-1} + m_{t-1})$. Notice that o_t is an increasing function of longevity. This assumption captures in a simple way the implications of a more realistic continuous time model[5].

[5] In a continuous time model the number of elderly in each moment in time t is given by $\int_{s=1}^{1+l} n_{t-s}(s) + m_{t-s}(s)ds$ which is also linearly increasing in the longevity l and in the size of the oldest generation of elderly $n_{t-1-l}(1+l) + m_{t-1-l}(1+l)$.

$t-1$			t			$t+1$	
$l_{t-2}(n_{t-2}+m_{t-2})$	(o)	$\rightarrow$	$\times$				
$n_{t-1}+m_{t-1}$	(y)	$\rightarrow$	$l_{t-1}(n_{t-1}+m_{t-1})$	(o)	$\rightarrow$	$\times$	
$\sigma^n_{t-1}n_{t-1}+\sigma^m_{t+1}m_{t-1}$	(ch)	$\rightarrow$	n_t+m_t	(y)	$\rightarrow$	$l_t(n_t+m_t)$	(o)
		born	$\sigma^n_t n_t+\sigma^m_t m_t$	(ch)	$\rightarrow$	$n_{t+1}+m_{t+1}$	(y)
					born	$\sigma^n_{t+1}n_{t+1}+\sigma^m_{t+1}m_{t+1}$	(ch)

Fig. 8. Size of each generation

Thus, the total number of individuals that possess voting rights at time t is $N_t = n_t + o_t$. Also notice that the way in which I define the size of different groups in the population implies a number of voters that is not necessarily a natural number, while in reality that must be the case. Given that the object of this study are policies that are typically decided at country level, and that the effects of this approximation tend to disappear as the number of individuals grows large, these assumptions are reasonable and commonly used in the literature (e.g. Razin and Sadka, 1999).

2.2.2 Individual Preferences

An individual i of working age (y) at time t has preferences that are represented by a utility function whose arguments are consumption of private goods C_s and the imperfect Public Good Y_s, and the share of immigrants in the total population of working age M_s in the form:

$$U_t^{i,y}\left(C_t^{i,y}, C_{t+1}^{i,o}, M_t, M_{t+1}, Y_t, Y_{t+1}\right) = C_t^{i,y} + b(Y_t) - c(M_t) + \beta l_t\left[C_{t+1}^{i,o} + d(Y_{t+1}) - \hat{c}(M_{t+1})\right]$$

where β is a parameter capturing how an individual discounts future utility. For retired individuals $U_t^{i,o}$ is constructed in a similar way, except that it only includes consumption and share of immigrants in the current period of life:

$$U_t^{i,o}\left(C_t^{i,o}, M_t, Y_t\right) = l_{t-1}\left[C_t^{i,o} + d(Y_t) - \hat{c}(M_t)\right]$$

The functions d, c and $\hat{c}$ are restricted to take only weakly positive values. Moreover, b and d are strictly concave while c and $\hat{c}$ are strictly convex.

2.2.3 Production

Individual productivity is given by ϵ_t^i and has average $\bar{\epsilon}_t$. The distribution of ϵ_t^i is perfectly observed by all agents and it does not change over time. I denote its continuous c.d.f. with Q, its p.d.f. with q and I assume $q(0) > 0$. Immigrants have the same expected productivity as the natives. Individuals are endowed with 1 unit of time and their labour supply is perfectly inelastic. I assume a linear production function $F_t(L_t) = \xi_t L_t$ in which the total supply of effective labour is given by $L_t = (m_t + n_t)\bar{\epsilon}_t$. Perfect competition on the labour market implies a wage rate per unit of effective labour $w_t = \xi_t$. Therefore individual pre-tax income is given by:

$$y_t^i = w_t \epsilon_t^i$$

and has average $\bar{y}_t$. The assumption of inelastic labour supply simplifies the results and it is not crucial for driving the pay-offs of the model (in the supplementary online material I show that the results are identical if all individuals have the same tax elasticity of labour supply). The assumption of a linear production function rules out the effects of changes in the aggregate labour supply on wages and it is common in the literature (e.g. Razin and Sadka, 2000). It is justified if one considers that in a more complex economy these effects tend to be offset by the adjustment in the stock of capital of the economy - not explicitly assumed in this analysis - that occurs in the relatively long time framework of a generation. This adjustment is particularly strong if firms have access to international capital markets (see Ben-Gad, 2012). In the additional online material I show that the main results of this paper are mostly unaffected in the case of a strictly concave production function.

2.2.4 Public Finances

The public sector raises revenues through a linear tax τ_t on labour income and spend them in the publicly provided good Y_t and in pensions for the elderly. In section 4.3 I introduce an extension of the model in which the government also provides public education. The government faces an exogenous amount of forgone tax revenues $\lambda_t = \lambda(w_t)$ per immigrant. This assumption captures the idea that the certain skills may be country-specific and therefore the immigrant may earn less than native individuals with similar productivity levels. Alternatively one can assume that immigrants and natives have different average productivities $\bar{\epsilon}_t^m$, $\bar{\epsilon}_t^n$, and λ_t to be a function $\lambda(w_t, \bar{\epsilon}_t^m, \bar{\epsilon}_t^n)$ that captures the net forgone government revenue due to the difference in income[6].

I assume a Pay-As-You-Go pension system (in section 4.1 I present an extension in which I allow for a partially funded system). The state pension paid to an individual i at time t is denoted by p_{t-1}^i and has average $\bar{p}_{t-1}$. It is promised to a working age individual at time $t-1$ and it is predetermined at time t. It is a constant flow, such that the total transfer is $l_{t-1}p_{t-1}^i$ (the flow amount times the time the pension is going to be paid for). It is a function of the relative income of the pensioner in the previous period $y_{t-1}^i/\bar{y}_{t-1}$ and of the growth rate of working age population. At time $t-1$, when the promise is made, m_t is not yet determined, because it is a function of the immigration policy at time t. Thus the promised pension is a function of an

[6]Notice that these two assumptions have consequences on the post-tax income of the immigrants

exogenously fixed amount of immigrants $\hat{m}_t$ (can be equal to zero). This assumption allows voters to ease the burden of pension on the working age population by choosing an immigration quota larger than $\hat{m}_t$. The assumptions on the pension system ensure that a certain positive amount of pensions is provided even if the pivotal voter typically prefers no pensions at all. Although not explicitly modeled in this paper, the assumption of an exogenous positive provision of public pensions in an overlapping generation model is justified in a game theoretical framework like the one in Rangel and Zeckhauser (2001). Lastly, two parameters $\alpha \geq 0$, $\gamma \geq 0$ determine the features of public pension system. In detail, the pension system can be either Beveridgean (if $\gamma = 0$), Bismarckian (if $\alpha = 0$) or a combination of the two. The state pension p^i_{t-1} is given by the formula:

$$p^i_{t-1} = \left(\alpha + \gamma \frac{y^i_{t-1}}{\bar{y}_{t-1}}\right) \frac{n_t + \hat{m}_t}{n_{t-1} + m_{t-1}} = \left(\alpha + \gamma \frac{y^i_{t-1}}{\bar{y}_{t-1}}\right) \frac{\bar{\sigma}_{t-1}}{(1 - \widehat{M}_t)}$$

where $\bar{\sigma}_{t-1} = \frac{n_t}{(m_{t-1}+n_{t-1})}$ is the natural growth factor of the working population between period $t-1$ and t and $\widehat{M}_t = \frac{\hat{m}_t}{n_t + \widehat{m}_t}$ is the share of immigrants implied by the default level of immigration $\widehat{m}_t$. Notice that if native and immigrants have different birth rates, i.e. $\sigma^n_t \neq \sigma^m_t$, then the natural growth rate of the population $\bar{\sigma}_t$ is itself endogenous in the immigration policy, and in particular: $\bar{\sigma}_t = \frac{\sigma^n_t n_t + \sigma^m_t m_t}{n_t + m_t} = \sigma^m_t M_t + \sigma^n_t (1 - M_t)$. Lastly, notice that the total cost of the pension system per taxpayer is decreasing in the number of immigrant workers that are allowed to enter the country in period t, while p^i_t is increasing in M_{t-1} if $\sigma^m_{t-1} > \sigma^n_{t-1}$.

I assume that the government budget is balanced in every period. The choice of not allowing for public debt simplifies the analysis and does not affect the trade-offs of the model. The government budget constraint ensure that the total public spending in public goods, pensions and the costs of immigration do not exceed the total tax revenue, and has form:

$$Y_t(m_t + n_t) + l_{t-1}\bar{p}_{t-1}(m_{t-1} + n_{t-1}) + \lambda_t m_t \leq \tau_t(m_t + n_t)\bar{y}_t$$

Assume that the governmental budget constraint is satisfied with equality (it must be true at any equilibrium of the voting game[7]). Using the formula for the pensions the governmental budget constraint can be rewritten as follows:

$$\tau_t = \tau(M_t, Y_t, \bar{y}_t) = \bar{y}_t^{-1} \left(\lambda_t M_t + (\alpha + \gamma) l_{t-1} \frac{(1 - M_t)}{(1 - \widehat{M}_t)} + Y_t\right)$$

Notice that this formula implies that working age voters can ease the tax burden on their income by voting for a more open immigration policy. The intuitition is that, if the number of immigrants increases, then the expenditure in pensions is going to be shared among a larger number of taxpayers. This results in lower income taxes. I can use this formula to state the feasibility

[7] In the case in which the pivotal voter is retired or has zero income one has to rule out Pareto inferior outcomes to ensure this result.

condition of the policy space:

$$0 \leq \tau_t(M_t, Y_t, \bar{y}_t) \leq k$$

for some $k < 1$. This restriction ensures that the implied tax rate on income will not exceed 1 or becomes negative. Notice that this restriction is crucial for the results in the next section to apply: if the tax rate hits the upper bound then the model and its predictions become similar to the ones of a standard Benefit Adjustment Model (See Appendix B.5). It is easy to show that the consumption of private goods of a young individual is given by her post-tax income such that $C_t^{i,y} = (1 - \tau_t) y_t^i$. lastly, the consumption of old people at time t depends only on the amount of pensions provided by the government, i.e. $C_t^{i,o} = p_{t-1}^i$.

2.2.5 Policy Space

I assume that voters face a two-dimensional policy space in each period t. Namely, a policy platform consist of an immigration policy M_t, and of a level of public spending in the imperfect public good Y_t. Moreover, I assume that both the immigration policy M_t and the spending policy Y_t lie between zero and an upper bound, i.e. $0 \leq M_t \leq \overline{M}$ and $0 \leq Y_t \leq \overline{Y}$. A typical platform is given by a two dimensional vector $x_t = (x_{1t}, x_{2t})$ with $x_{1t} = M_t$ and $x_{2t} = -Y_t$.

2.2.6 Voters' Objective Function

Substituting the formulas for C_t^y and C_{t+1}^o into the utility function of a young voter one gets the indirect utility function $\nu_t^{i,y} = \nu^y(M_t, Y_t, M_{t+1}, Y_{t+1}; y_t^i)$:

$$\nu_t^{i,y} = (1 - \tau_t) y_t^i + b(Y_t) - c(M_t) + \beta l_t \left[\left(\alpha + \gamma \frac{y_t^i}{\bar{y}_t} \right) \frac{\bar{\sigma}_t}{(1 - \widehat{M}_{t+1})} + d(Y_{t+1}) - c(M_{t+1}) \right]$$

The next step is to state the objective function of the elderly. Using the formula for $C_t^{i,o}$ into the utility function of an elderly voter I get $\nu_t^{i,o} = \nu^o(M_t, Y_t; y_{t-1}^i)$:

$$\nu_t^{i,o} = l_{t-1} \left[\left(\alpha + \gamma \frac{y_{t-1}^i}{\bar{y}_{t-1}} \right) \frac{\bar{\sigma}_{t-1}}{(1 - \widehat{M}_t)} + d(Y_t) - \hat{c}(M_t) \right]$$

The formula above delivers the main intuition that underpins the results in this paper. Notice that retired individuals internalize (indirectly) the positive effects of immigration through the level of public spending in the imperfect Public Good. The key difference with traditional models is that the tax rate on income is also an endogenous variable. Thus, the elderly always prefer, given a certain level of public spending, a policy that finances it with high taxes on the income of native workers rather than with a larger number of immigrants. This result follows from the fact that in this model the elderly dislike immigration as much as the young but, differently from the latter, they do not internalize the negative effects of high taxes on the working age population. Moreover, notice that the same preferences represented by $\nu_t^{i,o}$ are also represented by the function $\nu_t^o = d(Y_t) - \hat{c}(M_t)$ for all the elderly at time t. This objective function implies that the attitude of the elderly towards immigration is always more hostile than the one of any working age individual. This is true even if immigrants are net contributors in financing the public spending of which the elderly are net beneficiaries. This implication of the model is consistent with the empirical findings outlined in section 1 and it is crucial in order to understand the comparative statics of the equilibrium outcomes of the model that I will present in the next sections of this paper. Define $\theta_t^{i,y}$ as the ratio of i's income to mean income at time t:

$$\theta_t^{i,y} = \frac{\epsilon_t^i}{\bar{\epsilon}_t} = \frac{y_t^i}{\bar{y}_t}$$

The preferences of each young native individual i are uniquely identified by the parameter $\theta_t^{i,y} \in \Theta_t^y$ with $\Theta_t^y = [\underline{\theta}_t^y, \bar{\theta}_t^y]$. Notice that the function $\nu_t^{i,y}$ can be written as function of one exogenous parameter $\theta_t^{i,y}$ and of the choice variables $(M_t, Y_t, M_{t+1}, Y_{t+1})$ at time t and $t+1$, plus the parameters $\varphi_t = (\{\alpha, \beta, \gamma, \sigma_{t+s-1}^n, \sigma_{t+s-1}^m, l_{t+s-1}\}_{s=0}^{\infty})$. Moreover, the definition of $\theta_t^{i,y}$ implies that the cumulative distribution of $\theta_t^{i,y}$ is the same as the one of ϵ_t^i. The value of $y_{t-1}^i / \bar{y}_{t-1}$ does not affect the preferences of an elderly individual j over x_t, therefore all the elderly have the same preferences. This means that we can set a unique parameter $\theta_t^{j,o} = \theta_t^o \in \Theta_t^o$ which identifies the preferences of each elderly individual j at time t such that $\Theta_t^o = \{\theta_t^o\}$. I assign to all the elderly a parameter $\theta_t^o = -1$. I can now define the parameter set:

$$\Theta_t = \{\Theta_t^y \cup \Theta_t^o\}$$

which is a totally ordered set. In order to show that the preferences described in this section satisfy the conditions for the existence of a coalitional equilibrium I define a new objective function that includes both $\nu_t^{i,y}$ and $\nu_t^{i,o}$and has the following form:

$$\nu_t^i = \nu(x_t, x_{t+1}; \theta_t^i, \varphi_t) = \begin{cases} \nu_t^{i,y} & if \quad age = y \\ \kappa \nu_t^o & if \quad age = o \end{cases}$$

with $x_{1t} = M_t$ and $x_{2t} = -Y_t$ and for an arbitrarily large $\kappa > 0$. Notice that κ represents a strictly increasing transformation of the original objective function of the elderly therefore $\kappa \nu_t^o$ implies the same preferences as ν_t^o.

2.3 Markov-Perfect Coalitional Equilibrium

The equilibrium concept is a dynamic version of the coalitional equilibrium in Dotti (2015), that is described in Dotti (2015). I assume rational expectations on and off equilibrium. This imples that, given the history up to the the current period, the expectations are the same for all the voters. I also assume that such expectations only depend on the state of the economy in that period. Notice that, under this assumption, the state of the economy at the beginning of period $t+1$ is fully summarized by the share of elderly to young natives $g_{t+1} = o_{t+1}/n_{t+1}$, which is therefore the unique endogenous state in the dynamic process. Denote with $h_t = \{x_s, g_s\}_{s=0}^{t}$ the full history of policy choices and states observed by all agents up to time t, with $h_t \in H_t$. Denote with $x_{t+s}^{**}(g_t, \varphi_t, h_{t-1})$ the expectation at time t about the equilibrium policy at time $t+s$ given the state of the economy at time t and the history up to time $t-1$[8], and with $x_t^*(g_t, \varphi_t, h_{t-1})$ the policy actually implemented at time t. I assume that:

$$x_{t+s}^{**}(g_t, \varphi_t, h_{t-1}) = x_{t+s}^{**}(g_t, \varphi_t, h'_{t-1})$$

for all histories $h_{t-1}, h'_{t-1} \in H_{t-1}$ and all $s \geq 0$. Moreover, the assumption of rational expectations implies that $x_{t+s}^{**}(g_{t+s}, \varphi_{t+s}, h_{t-1+s}) = x_{t+s}^{*}(g_{t+s}, \varphi_{t+s}, h_{t-1+s})$ for all $s \geq 0$. These two assumptions imply that the dynamic system satisfies the Markov property. That is, there is no equilibrium in which different histories correspond to different equilibrium choices given an identical economic environment is. I also assume that $x_{t+s}^{**}(g_{t+s}, \varphi_{t+s}, h_{t-1+s})$ is twice differentiable with respect to g_{t+s}. This condition will prove to be satisfied in any Markov-Perfect coalitional equilibrium under appropriate restrictions (see Lemma 6). Notice that expectations are assumed to be a function of the state of the economy at time t, which is fully summarized by the state of the economy g_t. Also notice that g_{t+1} is perfectly known at the end of time t because there is no uncertainty about the distribution of future productivity[9]. Given these assumption, I define a Markov-Perfect coalitional equilibrium as follows.

Definition 1. *A Markov-Perfect coalitional equilibrium at time t is (i) a partition $\mathbb{P}_t$ of the set of voters at time t, (ii) a policy profile A_t, (iii) a winning policy x_t^*, and (iv) a set of expectations about future policies $\{x_{t+s}^{**}(g_t, \varphi_t, h_{t-1})\}_{s=0}^{\infty}$, such that (i) $(\mathbb{P}_t, A_t, x_t^*)$ is a coalitional equilibrium of the voting game given state g_t and given expectations about current and future policies $\{x_{t+s}^{**}(g_t, \varphi_t, h_{t-1})\}_{s=0}^{\infty}$ (ii) expectations are rational, i.e. $x_{t+s}^{**}(g_{t+s}, \varphi_{t+s}, h_{t-1+s}) = x_{t+s}^{*}(g_{t+s}, \varphi_{t+s}, h_{t-1+s})$ for all i* and for all $s \geq 0$, *and (iii) satisfy the Markov Property, i.e. $x_{t+s}^{**}(g_t, \varphi_t, h_{t-1}) = x_{t+s}^{**}(g_t, \varphi_t, h'_{t-1})$*, for all $s \geq 0$, all i and all $\varphi_t \in \Phi_t$. For ease of notation,

[8] The value of x_t^{**} is also a function of the distribution of productivity Q, but this is omitted in the formula.

[9] The result is the same if one allows for uncertainty and the size of the population is very large, because the law of large numbers implies that the identity of the median voter in the next period is known with probability equal to 1.

I am going to denote a coalitional equilibrium with $(\mathbb{P}_t, A_t, x_t^*\{x_{t+s}^{**}\}_{s=0}^{\infty}; g_t)$, in which I have suppressed the arguments of x_t^* and of each x_{t+s}^{**}. Using this notion of equilibrium, I can state the following Lemma:

Lemma 1. *In a Markov-Perfect coalitional equilibrium - if it exists - (i) each individual's ideal policy x_t^i and (ii) the equilibrium policy x_t^* at time t are invariant - conditional on g_t - to the history up to time $t-1$, i.e. $x_t^i(g_t, \varphi_t, h_{t-1}) = x_t^i(g_t, \varphi, h'_{t-1})$ and $x_t^*(g_t, \varphi_t, h_{t-1}) = x_t^*(g_t, \varphi_t, h'_{t-1})$ $\forall t$ and $\forall h_{t-1}, h'_{t-1} \in H_{t-1}$.*

Proof. See Appendix A.1.1.

These results imply that the history up to time $t-1$ is irrelevant for all aspects of the model conditional on g_t. This, from now on I am going to suppress the argument h_{t-1+s} from the formulas of x_{t+s}^*, x_{t+s}^{**} and x_{t+s}^i. Summarizing, g_t is the unique endogenous state variable of this dynamic system and a coalitional equilibrium in this model (if it exists) is a temporary equilibrium that depends only on the value of the state variable g_t at time t and is independent of the previous history conditional on g_t. Notice that g_t is the ratio of elderly relative to native individuals of working age, and therefore it represents the crucial variable in order to determine the identity of the pivotal voter. Lemma 1 allows one to disregard the effects of current policy choices (other than the effects on g_{t+1}) on future equilibrium outcomes when calculating the optimality conditions for each voter. This implies that future equilibrium policy outcomes affect the individual objective functions at time t only through their effects on g_{t+1}. The consequence is that, given expectations $x_{t+1}^{**}(g_t, \varphi_t)$, I can write a working age voter's objective function $V_t^i = V(x_t; \theta_t^i, \varphi_t, g_t)$ as follows:

$$V_t^i = V(x_t; \theta_t^i, \varphi_t, g_t) = \nu(x_t, x_{t+1}^{**}(g_t, \varphi_t); \theta_t^i, \varphi_t)$$

where $x_{t+1}^{**}(g_t, \varphi_t)$ represents the expected equilibrium policies at time $t+1$ which are a function solely of g_t and φ_t. Similarly, one can define the corresponding objective functions of young and old voters, $V_t^{i,y} = V^y(M_t, Y_t; \theta_t^i, \varphi_t, g_t)$ and $V_t^{i,o} = V^o(M_t, Y_t; \theta_t^i, \varphi_t, g_t)$ respectively. Notice that these two objective function implies that an interior solution for the optimal policy of individual i with a partially open immigration policy $x_{2t} = M_t > 0$ may exist even if immigrants "contribute less than what they take out" in the current period, or more precisely if - at a given policy $x_t = (M_t, -Y_t)$ - a marginal increase in the number of immigrants at constant Y_t implies, *ceteris paribus*, a rise in the income tax rate τ_t. This is true because if immigrants have higher fertility rates in comparison with the natives ($\sigma_t^m > \sigma_t^n$), then a native individual of working age will have a future benefit from immigration. Specifically, higher immigration today implies a lower dependency ratio tomorrow and, as a consequence, a more generous state pension system. This implies that this model is not affected by the dichotomy between "skilled migration" and "unskilled migration" in the patterns of attitude towards immigration and income that is typical of traditional models such as Facchini and Mayda (2008). In the model proposed in this paper the attitude towards immigration may improve with income even if the immigrants are a net burden for the society in the short run, because preferences accounts for the future positive effect of immigration. Moreover, these future benefits are increasing with income if the Bismarckian component of the pension system is positive ($\gamma > 0$). Using the previously defined V_t^i function I can state the following result:

Lemma 2. *The function $V(x_t; \theta_t^i, \varphi_t, g_t)$ satisfies SM and SSCP in $(x_t; \theta_t^i)$ for all $\theta_t^i \in \Theta_t$ and all $\varphi_t \in \Phi_t$ for any given state g_t.*

Proof. See Appendix A.1.2.

This Lemma is crucial in order to establish existence of a coalitional equilibrium, and therefore to derive all the results in the next section of this paper. The intuition about how this result can be proved relies on the effect of the Markov assumption. Recall that the expectations about the policy outcome in any future period $t+s$ are assumed to depend uniquely on the state of the economy at the beginning of such period (g_{t+s}), and that $g_{t+s} = \frac{o_{t+s}}{n_{t+s}} = \frac{l_{t+s}}{M_{t+s}(\sigma_{t+s}^m - \sigma_{t+s}^n) + \sigma_{t+s}^n}$. Thus, the only choice at time t that can affect the value of g_{t+1} - and therefore all future expectations - is the one about the immigration policy M_t. As a consequence, conditional on M_t and given parameters φ_t, the expectations about the policy implemented in the future periods are unaffected by changes in Y_t or θ_t^i. This makes the cross-partial derivatives of V with respect to each policy dimension $x_{k,t}, x_{j,t}$, $k \neq j$ and with respect to $x_{k,t}, \theta_t^i$ for all k realtively easy to calculate. Thus, the sufficient conditions for *SM* and *SSCP* - that are based on the sign of such cross derivatives - can be shown to hold.

2.3.1 Conditions for a Markov-Perfect Coalitional Equilibrium

Following the static analysis in Dotti (2015), denote with $\wedge$ and $\vee$ the meet and joint operators over a lattice (see Dotti 2015). Recall that (i) $\theta_t^i \in \Theta_t$ is the parameter that identifies the preference of a voter i , that (ii) the parameter space Θ_t is a totally ordered set, and that (iii) $\varphi_t \in \Phi_t$ is a vector of parameters that do not differ across voters. I state the conditions for a Markov-Perfect coalitional equilibrium to exist and satisfy some desirable properties.

1. The policy space X_t must be a subset of the the d-dimensional real space R^d with typical element x_t, such that the partially ordered set $(X_t, \leq)$ is a convex and complete sublattice of R^d.

2. Each individual i must be endowed with a reflexive, complete and transitive preference ordering $\succeq^i$ represented by an objective function $V : X_t \times \Theta_t \times \Phi_t \to R$ that is jointly

continuous in x_t and $\theta_t - concave$ [10].

3. Individual preferences are such that the function V satisfies, given the state g_t:

   (a) *Supermodularity (SM)* in x_t: $V(x_t' \vee x_t''; \theta_t, \varphi_t, g_t) - V(x_t'; \theta_t, \varphi_t, g_t) \geq V(x_t''; \theta_t, \varphi_t, g_t) - V(x_t' \wedge x_t''; \theta_t, \varphi_t, g_t)$ for all $\theta_t \in \Theta_t$, for all $\varphi_t \in \Phi_t$ and for all $x_t', x_t'' \in X_t$.

   (b) *Strict Single Crossing Property (SSCP)* in (x_t, θ_t): $V(x_t'; \overline{\theta}_t, \varphi_t, g_t) - V(x_t''; \overline{\theta}_t, \varphi_t, g_t) > V(x_t'; \underline{\theta}_t, \varphi_t, g_t) - V(x_t''; \underline{\theta}_t, \varphi_t, g_t)$ for all $x_t', x_t'' \in X_t$ such that $x_t' \geq x_t''$ and $x_t' \neq x_t''$, for all $\varphi_t \in \Phi_t$ and for all $\overline{\theta}_t, \underline{\theta}_t \in \Theta_t$ such that $\overline{\theta}_t > \underline{\theta}_t$.

Regarding condition 1, I assume that X_t is the same in all periods. The assumptions on the policy space stated in section 2.2.5 ensure that this condition is satisfied. Condition 2 simply requires that the objective function satisfies some basic properties. Lastly, condition 3 is equivalent to state that all voters can be ordered along a single preference dimension over a multidimensional choice set. These assumptions on individual preferences are common in many fields of Economic Theory. Notice that condition 3 is stated in a very general form, but in the case of a twice differentiable objective function one can simply adopt the sufficient conditions in Milgrom and Shannon (1994) in order to verify that the function satisfies *SM* and *SSCP*. Namely, one needs to check that the following conditions hold. (i) $\frac{\partial^2 V}{\partial x_{i,t} \partial x_{j,t}} \geq 0$ $\forall x_t \in X_t$, $\forall i \neq j$, and (ii) $\frac{\partial^2 V}{\partial x_{i,t} \partial \theta_t} > 0$ $\forall x_t \in X_t$, $\forall \theta_t \in \Theta_t$, $\forall i$. These sufficient conditions are usually easier to verify in comparison with the one implied by the definitions of *SM* and *SSCP*. Because of that, in the next sections I am going to make frequent use of these sufficient conditions.

2.3.2 Monotone Comparative Statics

Denote the set of ideal policies of voter i in period t given state g_t (and for a given expectations $\{x_{t+s}^{**}(g_t, \varphi_t)\}_{s=0}^{\infty}$) with $I_t(i_t) \equiv \{x_t | x_t \in \arg\max_{y \in X_t} V(y; \theta_t^i, \varphi_t, g_t)\}$,[11], and define the set of *equilibrium policies* as the union of all the policies that are winning policies in some coalitional equilibrium of the game for given expectations $\{x_{t+s}^{**}(g_t, \varphi_t)\}_{s=0}^{\infty}$. Notice that because Θ_t is a totally ordered set, one can identify a median element θ_t^v. The individual characterized by this value of the parameter is the *median voter* denoted by the index v_t[12]. In this setting, conditional on g_t and given expectations, the political process at time t is identical to the one

[10] For any function f defined on the convex subset X_t of R^d , we say that f is concave in direction $v \neq 0$ if, for all x, the map from the scalar s to $f(x + sv)$ is concave. (The domain of this map is taken to be the largest interval such that $x + sv$ lies in X_t.) We say that f is $i - concave$ if it is concave in direction v for any $v > 0$ with $v_i = 0$. See Quah (2007).

[11] Notice that the completeness of X_t implies compactness in the order-interval topology. On bounded sets in R^d, the order-interval topology coincides with the Eucilidean topology (Birkhoff 1967). Hence $I_t(i_t) \neq \varnothing$ for all i.

[12] In the case of a discrete even number of voters I assume that the ties are broken in favor of the individual with the lower index. Different assumptions would not affect the results in the next paragraphs.

of the static model described in Dotti (2015). Thus, the results stated in such paper hold in the framework proposed here with minor modifications. Specifically, if the three conditions stated in the previous section are satisfied, then the following theorems hold for any value of the state g_t.

Theorem 3. *(Median Voter Theorem). If conditions 1-2-3 are satisfied, then (i) A Markov-Perfect coalitional equilibrium of the voting game exists; (ii) in any equilibrium the set of winning policies is a subset of the set of ideal points of the median voter v_t; (iii) if the median voter has a unique ideal policy, then the set of equilibrium policies is a singleton.*

Proof. See Appendix A.1.3.

Theorem 4. *(Monotone Comparative Statics). If conditions 1-2-3 are satisfied, then the set of equilibrium policies of the voting game is (i) a sublattice of X_t which is (ii) monotonic nondecreasing in θ_t^v.*

Proof. See Appendix A.1.3.

Lastly, consider a totally ordered subset $\Phi' \subseteq \Phi$ and suppose that the objective function $V(x_t, \theta_t, \varphi_t)$ satisfies the *Single Crossing Property (SCP)* in (x_t, φ_t), namely $V(x_t', \theta_t, \overline{\varphi}_t) - V(x_t'', \theta_t, \overline{\varphi}_t) \geq V(x_t', \theta_t, \underline{\varphi_t}) - V(x_t'', \theta_t, \underline{\varphi_t})$ for all $x_t' \geq x_t''$, and for all $\overline{\varphi}_t, \underline{\varphi_t} \in \Phi'$ such that $\overline{\varphi}_t \geq \underline{\varphi_t}$. Then I can state the following result:

Theorem 5. *(Monotone Comparative Statics 2). If conditions 1-2-3 are satisfied, then the set of equilibrium policies of the voting game is monotonic nondecreasing in φ_t.*

Proof. See Appendix A.1.3.

The interpretation of this generalized Median Voter Theorem is identical to the one provided for the static case described in Dotti (2015). Notice that, while $I_t(v_t)$ depends on voters' expectations, the identity of the median voter v_t is independent of expectations, thus the median voter is the same in all colaitional equilibria of the voting game. The results in this sections provide a tool to analyze the effects of a shock on the distribution of voters or on a preference parameter on the policy outcome that emerges in a political equilibrium. One only has to verify that an economic model satisfies the conditions stated in this section and then use Theorems 4-5 to formulate the predictions about the comparative statics of the platform that is implemented in equilibrium. Following this approach, I derive the main results of this paper, which are stated in the next section.

3 Results

In this section I present the main results of the paper, namely the existence and characterization of the voting equilibrium, the analytical comparative statics results, the dynamics of the equilibrium outcome and the simulation of the other long-run implications of the model. Notice that all the results described in this section - except for the cases in which the opposite is explicitly stated - are also valid for the extended version of the model with endogenous public education presented in section 4.3. The proofs in Appendix A include both the basic model and the extended one (the objects that refer to the extended model are denoted with a tilda in the proofs).

3.1 Equilibrium Existence and Characterization

Using the results in the previous sections we get that (i) the Policy Space $(X_t, \leq)$ is a convex and complete sublattice of R^2; (ii) the parameter set Θ_t is a totally ordered set; (iii) the objective function $V(x_t; \theta_t^i, \varphi_t, g_t)$ satisfies *Supermodularity* in x_t and the *Strict Single Crossing Property* in $(x_t; \theta_t^i)$. Therefore all the conditions for the existence of a Markov-Perfect coalitional equilibrium are satisfied provided that $V(x_t; \theta_t^i, \varphi_t, g_t) = \nu(x_t, x_{t+1}^{**}(g_t, \varphi_t); \theta_t^i, \varphi_t)$ is such that $x_{t+1}^{**}(g_t, \varphi_t)$ are rational expectations and V is concave in x. Moreover, if the objective function of each working age individual is strictly concave, then - given expectations - the ideal policy of the median voter is unique. Notice that, because the indirect utility function ν is continuous, twice differentiable and strictly concave in x_t in each period t, and because of the assumptions on Q and on expectations previously stated, there exists a threshold on $\hat{\sigma}$ such that if $|\sigma_t^m - \sigma_t^n| \leq \hat{\sigma}$, then V is also continuous and strictly concave in x_t. Thus, I can state the following result.

Lemma 6. *If $|\sigma_{t+s}^m - \sigma_{t+s}^n| \leq \hat{\sigma}$ for some $\hat{\sigma} > 0$ and all $s \geq 0$, then (i) a Markov-Perfect coalitional equilibrium for the voting game exists. Moreover, (ii) in any Markov-Perfect coalitional equilibrium at time t the equilibrium policy is the unique ideal point of the median voter $x_t^v = x_t^* \in I_t(v_t)$. (iii) The parameter θ_t^v that identifies the median voter is weakly decreasing in g_t. If $\sigma_t^m - \sigma_t^n$ is arbitrarily small, then (iv) there is a unique equilibrium policy that is chosen in any Markov-Perfect Coalitional Equilibrium in period t.*

Proof. See Appendix A.1.4.

Notice that the condition on $\sigma_{t+s}^m - \sigma_{t+s}^n$ is sufficient but not necessary for results (i), (ii) and (iii) in Lemma 6. In the rest of the paper, I am going to assume that ν and φ_t are such that continuity and strict concavity are satisfied for any x_{t+1}^{**} that implies rational expectations[13]. Notice that Lemma 6 does not postulate the *uniqueness* of the Markov-Perfect coalitional equilibrium in points (i), (ii), (iii). The reason is that, even if the equilibrium is unique conditional on g_t and

[13]Notice that the requirement of joint continuity and concavity in x_t are necessary for a coalitional equilibrium, but not for the Citizen-Candidate version of the equilibrium in which individuals only run as single candidates. Thus, one does not have to impose these two restrictions if such simpler model of electoral competition is adopted.

on expectations $\{x^{**}_{t+s}(g_t, \varphi_t)\}^{\infty}_{s=0}$, there may be different rational expectations $\{x^{**}_{t+s}(g_t, \varphi_t)\}^{\infty}_{s=0}$ that may support different policies in equilibrium. Neverthless, the comparative statics results in the next sections are valid in any equilibrium, thus the analysis is not affected by arbitrary equilibrium selection rules[14]. Having established existence of an equilibrium and (conditional) uniqueness of the policy outcome, I can use the result of the Monotone Comparative Statics of the equilibrium outcome in order to study the effects of shocks on the voters' distribution on the equilibrium policy outcome.

3.2 Main Result: Comparative Statics

In this section I analyse the short-run effects of shocks on the parameters that are related to population ageing on the equilibrium policy outcome. That is, how the equilibrium policy vector changes as a consequence of a shock - in the period in which the shock is observed - relative to the equilibrium level in absence of any shock. One has to account for four aspects: (i) how the direct preferences over policies of each native individual of working age are affected by the shock ("preference effect"), (ii) how the indirect preferences changes because of the effects of the shocks on the governmental budget constraint ("budget effect") and (iii) how the identity of the pivotal voter changes as a consequence of the changes in the demographic composition of the population induced by the shock ("political effect"). Lastly, one has to account for the ability of a fully rational agent to anticipate that if $\sigma^m_t \neq \sigma^n_t$, then the choice of the immigration policy at time t affects the demographic structure of the voting population in the following periods and can therefore change the political equilibrium in the future. One may think that voters are unlikely to really anticipate this (iv) "sophisticated effect", therefore whenever this aspect is relevant in this section I will distinguish between the predictions that emerge with "naive" agents - i.e. if voters expectations do not account for future political effects of current policies - and the ones implied by fully "sophisticated" agents. The approach used is the following. First I verify if there is any effects of type (i), (ii) and (iv). In detail, if V^i_t satisfies the condition of Theorem 5 for a given value of g_t, then the theorem can be used to establish the sign of these effects. Then I study the effects of type (iii). If g_t is affected by the shock, then Lemma 6 (iii) implies a change in the parameter that identifies the pivotal voter and therefore Theorem 4 can be used to formulate the predictions. The results about the tax rate τ_t stated in this section refer to the case in which immigrants provide, on average, a contribution to public finances sufficient to ensure that τ_t is weakly decreasing in M_t. This is true whenever the average cost per pensioner is sufficiently large, namely if $l_{t-1}\bar{p}_{t-1} \geq \lambda_t$. The results about M_t and Y_t are valid even if the latter condition does not hold.

[14]The Markov assumption implies that each function $x^{**}_{t+s}(g_t, \varphi_t)$ is uniquely affected by g_t and φ_t, thus the comparative statics results are valid in *any* equilibrium of the game, provided that no changes in the functions $x^{**}_{t+s}(g_t, \varphi_t)$ occur.

3.2.1 Unanticipated Rise in the Longevity of the Retired Population

I analyse the effects of a marginal increase in l_{t-1} keeping other parameters constant. That is, the longevity of the current elderly increases, keeping the longevity of other generations and birth rates unchanged. Recall that $x_t = (M_t, -Y_t)$. For effects of type (i)-(ii)-(iv) one can verify that V_t^i satisfies the *SCP* by studying the cross derivatives of $V_t^{i,y}$ with respect to each policy dimension and l_{t-1}. Denote with $V_{M_t}^{i,y}$ ($V_{Y_t}^{i,y}$) the partial derivative of $V_t^{i,y}$ with respect to the policy dimesion M_t (Y_t) and with $V_{M_t l_{t-1}}^{i,y}$ ($V_{Y_t l_{t-1}}^{i,y}$) the cross derivative of $V_t^{i,y}$ with respect to M_t (Y_t) and a parameter l_{t-1}. In this case we have:

$$V_{M_t l_{t-1}}^{i,y} = \frac{\theta_t^i (\alpha + \gamma)}{(1 - \widehat{M}_t)} \geq 0$$

$$V_{Y_t l_{t-1}}^{i,y} = 0$$

Consider a vector of parameters $\tilde{\varphi}_t \in \Phi_t$. Define a subset $\Phi_{j,t} \subseteq \Phi_t$ as follows: $\Phi_{j,t} := \{\varphi_t \in \Phi_t | \varphi_{t,k} = \tilde{\varphi}_{t,k} \forall k \neq j\}$, where j is the position of the longevity of the elderly l_{t-1} at time t in vector φ_t. Notice that Φ_j is a totally ordered set. Moreover, the signs of the cross derivatives imply that $V(x_t; \theta_t^v, \varphi_t, g_t)$ satisfies *SM* and *SCP* in $(x_t; \varphi_t)$, it also satisfies *SM* and *SSCP* in $(z_t; \varphi_t)$ where $z_t = (x_{1t}, -x_{2t})$. The conditions of Theorem 5 are satisfied, therefore at constant g_t the effect is a weak rise in M_t and no changes in Y_t. Moreover, $V_{M_t}^{i,y}$ is solely affected through the budget constraint hence the effect is of type (ii) ("budget effect"). For effects of type (iii) notice that $g_t = \frac{l_{t-1}}{\bar{\sigma}_{t-1}}$ is increasing in l_{t-1}. Lemma 6 (iii) implies that θ_t^v is decreasing in g_t. Hence Theorem 4 implies a weak increase in the public spending variable Y_t and a weakly more restrictive immigration policy M_t. The total effect of an increase in l_{t-1} is therefore weakly positive on the public spending variables Y_t and ambiguous on the immigration policy M_t. There are cases in which one effects dominates and therefore the comparative statics result for the immigration policy is also sharp. In particular, I can state the following results:

Theorem 7. *The effect of an increase in the life expectancy l_{t-1} is weakly positive on the spending policy and ambiguous on the immigration policy. Moreover, there exists a threshold $\hat{g}_t \in [0, 1]$ such that if $g_t \geq \hat{g}_t$ then the effect on immigration policy is unambiguously (weakly) negative and the effect on the tax rate is strictly positive.*

Proof. See Appendix A.2.1.

In order to get an intuition of what drives this results, consider the following cases. If $g_t = 1$

(i.e. there are as many working age individuals as elderly), then the pivotal voter has $\theta_t^v = 0$, which implies that $V_{M_t l_{t-1}}^{v,y} = 0$. Thus, there is no "budget effect" and the "political effect" weakly dominates. On the other hand, consider the case in which the variance of the income distribution is arbitrarily close to zero (e.g. $y_t^i = y_t$ for all i). In this case, as long as $g_t \neq 1$, the θ_t^v of the pivotal voter is unaffected by changes in the share of elderly, which implies that the "political effect" is zero and that the "budget effect" weakly dominates. Theorem 7 is a consequence of the negative relationship between age and attitude towards immigration and the positive one between age and attitude towards public spending implied by the the model. This result suggests the existence of a link between the size of the two effects and two characteristics on the voting population: the share of elderly and the degree of income inequality. Moreover, it implies that the sign of the effect of an increase in longevity on the equilibrium level of the immigration quota is the one implied by the Tax Adjustment Model if the share of elderly is large enough and there is sufficient income inequality, and the one implied by the Benefit Adjustment Model in societies characterized by opposite features (see section 1.3).

3.2.2 Unanticipated Fall in the Natural Growth Rate of the Woking Age Population

The natural growth rate of the native population is $\frac{n_t}{n_{t-1}+m_{t-1}} - 1 = \bar{\sigma}_{t-1} - 1$. In this model the effect of an unanticipated fall in such rate has same sign as the one of a decrease in the lagged birth rate of the natives σ_{t-1}^n. This is true because one can show that $\bar{\sigma}_{t-1} = \sigma_{t-1}^m M_{t-1} + \sigma_{t-1}^n (1 - M_{t-1})$, which implies that $\bar{\sigma}_{t-1}$ is predetermined at time t. This kind of shock corresponds for instance to the case in which the birth rate actually experienced during the period $t-1$ is smaller than the one expected at the beginning of that period. Therefore I analyse the effects of a shock on σ_{t-1}^n. I can state the following:

Theorem 8. *The effect of a decrease in the growth rate of the working age population is a weak decrease in the openness of the immigration policy and a weak increase in spending in the imperfect Public Good and in the tax rate.*

Proof. See Appendix A.2.2.

Notice that conditional on g_t the shock has no effect on the equilibrium policy outcome (i.e. there are no effects of type (i), (ii), (iv)). The reason is that the pension system adjusts its size to changes in the birth rate for the reasons described in section 2. Nevertheless a fall in σ_{t-1}^n implies a rise in g_t, which corresponds to a "political effect". Using Theorem 4 one gets the result stated above.

3.2.3 Rise in Life Expectancy of the Working Age Population

I analyse the effects of a shock on the life expectancy of the current working age population l_t, keeping all the other elements of vector φ_t unchanged. First of all notice that g_t is unaffected by changes in l_t, which means that there is no "political effect". The results of this paragraph are summarized in Theorem 9.

Theorem 9. *The effect of an increase in the life expectancy l_t is ambiguous on the immigration policy. If voters are "naive" then the effect is weakly positive. If the birth rate of the native is the same as the one of the immigrants, then there is no effect.*

Proof. See Appendix A.2.3.

In order to understand this result it is useful to analyze the cross derivative of $V_t^{i,y}$ with respect to the immigration policy M_t and the parameter l_t.

$$V^{i,y}_{M_t l_t} = \underbrace{\frac{\beta(\alpha+\gamma\theta_t^i)}{(1-\widehat{M}_{t+1})}(\sigma_t^m-\sigma_t^n)}_{preferences \quad effect} - \underbrace{\frac{d}{dl_t}\left\{\frac{\beta l_t^2}{\bar{\sigma}_t{}^2}\left[d'(Y^{**}_{t+1})\frac{dY^{**}_{t+1}}{d\theta^v_{t+1}} - \hat{c}'(M^{**}_{t+1})\frac{dM^{**}_{t+1}}{d\theta^v_{t+1}}\right]\frac{d\theta^v_{t+1}}{dg_{t+1}}(\sigma_t^m-\sigma_t^n)\right\}}_{sophisticated \quad effect}$$

First of all, notice that if $\sigma_t^m = \sigma_t^n$, then the cross derivatives are equal to zero and g_{t+1} is unaffected by changes in l_t, therefore a shock on l_t has no effects on the equilibrium outcome. If $\sigma_t^m \geq \sigma_t^n$ the sign of $V^{i,y}_{M_t l_t}$ is ambiguous. The reason is that two different effects enter the formula. On one hand a rise in the life expectancy makes consumption after retirement more attractive. This increases the desirability of better future pensions and therefore implies a more favorable attitude towards immigration ("preferences effect"). On the other hand more immigration today reduces the value of g_{t+1}. This changes the expected equilibrium policy in the next period in a way that harms a retired individual ("sophisticated effect"). In particular, a decrease in g_{t+1} causes a weak rise in M^{**}_{t+1} and a weak fall in Y^{**}_{t+1}, because of the future political effect. Which of the two effects dominates depends on many aspects, including the income distribution at time $t+1$ and the values of g_t and g_{t+1}. In particular notice that if the variance of the income distribution of the working age population tends to zero, then $\frac{d\theta^v_{t+1}}{dg_{t+1}} = 0$ and therefore the "preferences effect" dominates. Finally, if agents are "naive" then there is no "sophisticated effect" and therefore an increase in l_t has a weakly positive effect on the openness of the immigration policy.

3.2.4 Fall in the Birth Rate of the Native Population

I analyse the effects of a fall in σ_t^n keeping all other parameters constant. The results are summarized in the following theorem.

Theorem 10. *The effect of a decrease in the birth rate of the native population σ_t^n is ambiguous on the immigration policy and on the tax rate. If voters are "naive", then the effect is weakly positive on the immigration policy and weakly negative on the tax rate.*

Proof. See Appendix A.2.4.

Similarly to the previous case, the presence of a "sophisticated effect" and of a "preferences effect" that can have opposite sign implies that the sign of the comparative statics is ambiguous. If voters are "naive", then the preferences effect implies a weakly less restrictive immigration policy M_t. Moreover, if the immigrants are net contributors to the fiscal system, this also implies a weak fall in the tax rate τ_t. The intuition is that a fall in the birth rate of the natives implies a stronger positive impact of immigration of future pensions and no fiscal effects in the short run. Theorem 10 implies that a fall in the birth rate can have positive effects on public finances and cause a fall in the tax rate because of an increasingly liberal immigration policy in the short run. If this result may seem paradoxical, section 3.4 clarifies that this effect is true only in the current period, while in the long run a fall in the birth rate may have strong negative effects on public finances and tax rates.

3.2.5 Shocks on the Income Distribution of the Working Age Population

Given the state g_t, a shock on the income distribution of the working age population affects the equilibrium outcome if and only if it implies a change in the pivotal voter θ_t^v. If this is the case, it represents a shock of type (iii), if it is not, it has no effects. For instance, a shock that results in a median preserving spread of the distribution of θ_t does not imply any change in the identity of the median voter and therefore it does not affect the policy outcome. Thus, I can state the following result.

Theorem 11. *An increase in the median to mean income ratio implies in equilibrium (i) a weak increase in the openness of the immigration policy M_t and (ii) a weak decrease in the public spending in the imperfect Public Good Y_t. Moreover, (iii) if the immigrants are net contributors to the fiscal system then it also implies a weak fall in the tax rate τ_t.*

Proof. Results (i), (ii), follows directly from Theorem 4. Result (iii) follows directly from the governmental budget constraint and results (i), (ii). Q.E.D.

Corollary 12. *The equilibrium levels of Y_t and M_t respond in opposite directions to shocks to the voters' distribution.*

Proof. Straightforward from Theorem 11.

Notice that this result implies a positive correlation between the tightness of the immigration policy and the spending in the imperfect public good. This suggests that the concerns about the relationship between an open immigration policy and cuts to public benefits, which are documented in all attitudinal studies, may have some ground in the observed policy outcomes even if immigrants are net contributors to the tax system.

3.3 Steady-State Equilibrium

I define a long-run equilibrium of the overlapping generation model as a sequence of Markov-Perfect coalitional equilibria from time t onwards. Within this class, I define a steady-state as follows.

Definition 2. *A steady-state at time t is a sequence of Markov-Perfect coalitional equilibria $\left\{(\mathbb{P}_{t+s}, A_{t+s}, x^*_{t+r}, \{x^{**}_{t+s+r}\}_{r=0}^{\infty}; g_{t+s})\right\}_{s=0}^{\infty}$ such that, in each time $t+s$, and in absence of shocks on the parameters φ_{t+s}, (i) the policy platform implemented in equilibrium is the same in each period $t+s$, i.e. $x^*_{t+s} = x^{**}_{t+s} = x^{ss}$ for all $s \geq 0$, and (ii) the state of the economy is constant $g_{t+s} = g^{ss}$ for all $s \geq 0$.*

In the definition above, the superscript ss denote the steady-state value of a state or a control variable. In other words, in a steady state the equilibrium policy and the natural growth rate of the population are constant over time. Recall that g is the only state that evolves endogenously in the dynamic system and that at a Markov-Perfect coalitional equilibrium in each period $t+s$ - conditional on g_{t+s} and on expectations $\{x^{**}_{t+s+r}\}_{r=0}^{\infty}$ - the equilibrium policy x^*_t may not be unique. Neverthless, if the conditions of Lemma 6 are satisfied, then the set of equilibrium policies is a singleton, and in order to show that the economy is at a steady state one has to show that $g_s = g^{ss}$ for all $s > t$. Conditional uniqueness also implies that if $g_{t+s} = g_{t+s+1}$ in period $t+s$ and if the parameters are such that $(l_{t+s}, \sigma^m_{t+s}, \sigma^m_{t+s}) = (l, \sigma^m, \sigma^m)$ for all $s > 1$, i.e. $\varphi_{t+s} = \varphi$ for all $s > 1$, then $g_{t+s} = g_t = g^{ss}$ for all $s > t$. In such case, if $g_{t+1} = g_{t+2}$, then the economy is at a steady state.

Lemma 13. *If there exists a Markov-Perfect Coalitional Equilibrium in each period $t+s$, for all $s \geq 0$, then (i) an equilibrium for the OLG model at time t exists . Moreover, if $\varphi_{t+s} = \varphi$ for all $s > 0$, then (ii) there is an equilibrium that always converges to a steady-state. Lastly, if*

$\sigma^m_t = \sigma^n_t = \sigma_t$, then (iii) the political equilibrium at time t is independent of the previous political choices and the economy converges immediately to the steady state after a shock.

Proof. See Appendix A.2.5.

Notice that this statement does not necessarily imply that the steady state is unique, except for case (iii).

3.4 Dynamics

The analysis of the dynamics of the OLG model is a complex exercise because of the number of different short-run effects described in the previous sections. There are anyways interesting results that can be stated about the long-run effects of shocks in this framework. In particular, I present two analytical results: (i) the long-run effects of an unanticipated permanent shock on the longevity of the elderly l_{t-1} and/or on the natural growth rate of the native population $\bar{\sigma}_{t-1}$ on the sequence of political equilibria from the period after the shock until the economy converges to a new steady state (keeping other parameters constant); (ii) the long-run effects of an unanticipated permanent shock in the life expectancy (l_t) and/or on the expected birth rate of the native population (σ^n_t) in the case in which immigration does not cause changes in the age profile of the society (i.e. $\sigma^m_t - \sigma^n_t \leq \eta$ for arbitrarily small η). For the other cases which I cannot address analytically I propose a simulation in section 3.5 which show that the results are not qualitatively different from the one presented in the following paragraphs.

3.4.1 Long-Run Effects of a Permanent Shock on the Longevity of the Retired Population and on the Natural Growth Rate of the Working Age Population

The sign of the long-run effects of a positive shock on the longevity l_{t-1} or on the natural growth rate of working age population $\bar{\sigma}_{t-1} - 1$ at time t depend on the ambiguous short-run effects on the immigration policy stated in Theorems 7-8. In order to address the effects at period $t+1$ and the following ones it is sufficient to notice that, given that the shock is permanent (i.e. $l_{t+s} = l_t$ or $\sigma^n_{t+s} = \sigma^n_t$ for all $s \geq 0$), the collective choice problem at time $t+1$ is identical to the one at time t except for the value of g. Thus, there is an equilibrium in which expectations are time-independent. In such equilibrium, all the changes in the policy choices at time $t+1$ must be due to the evolution of the endogenous state g_{t+1}. The results in Lemmas 7b-8b apply to this class of equilibrium. In particular notice that g_{t+1} is strictly decreasing in M_t, and therefore if $M_t \geq M_{t-1}$ ($M_t \leq M_{t-1}$) then $g_{t+1} \leq g_t$ ($g_{t+1} \geq g_t$). Lemma 6 (iii) ensures that the parameter that identifies the pivotal voter changes accordingly $\theta^v_{t+1} \geq \theta^v_t$ ($\theta^v_{t+1} \leq \theta^v_t$). Therefore I can state the following results.

Theorem 7b. *The long-run effect of an increase in l_{t-1} on the immigration policy has same sign as the short-run effect and a weakly larger magnitude. If $g_t \geq \hat{g}_t$ then the effect on immigration policy is (weakly) negative and the effect on the public spending and the tax rate is strictly positive.*

Proof. See Appendix A.2.5.

Similarly one can analyse the long-run effects of a fall in the natural growth rate of the native population of working age (or equivalently of σ_{t-1}^n, see section 3.2.2). The result is the following.

Theorem 8b. *The long-run effect of a decrease in the natural growth rate of the native population is a weak decrease in the openness of the immigration policy and a weak increase in spending in the imperfect Public Good and in the tax rate. All the effects have weakly larger magnitude relative to the short-run effects.*

Proof. See Appendix A.2.5.

These results imply that the effects of population ageing are persistent and tend to increase in magnitude in the periods after the shock. The reason is that - if immigrants have higher fertility rate relative to the natives - then a change in the size of the immigration flow affects the distribution of voters in the following periods. In particular, a more restrictive immigration policy in the current period implies further population ageing in the future and therefore an increase in magnitude of the initial effects.

3.4.2 Long-Run Effects of a Permanent Rise in Life Expectancy

I analyse the long run effects of changes in l_t. This shock generates a number of effects that affect the temporary equilibrium as described in the previous sections. Moreover, the specific path of policies depends on the timing of the different shocks (for instance shocks on l_t and l_{t-1} may occur simultaneously). I study the case in which σ_t^m is arbitrarily close to σ_t^n and the shock is permanent, i.e. l_{t+s} is equal to the new value of l_t for all $s > t$. This case is simple to analyze because the long run effects at time t and $t+1$ after the shock correspond, respectively, to the temporary effects of a rise in l_t and l_{t-1} described in the previous sections. Moreover, given that $\sigma_{t+s}^m - \sigma_{t+s}^n$ is arbitrarily small, then the "preference effect" and the "sophisticated effect" can be disregarded and the economy converges to the new steady state one period after the shock[15]. Under the proposed restrictions I can state a sharper result:

[15]Notice that under such restriction, the Markov-Perfect coalitional equilibrium is unique in every period (see Lemma 6), hence - in contrast with the previous paragraph - there is no need to select a class of equilibria.

Theorem 14. *The long-run effects of an increase in the life expectancy is a weak rise in public spending and, if $g_t \geq \hat{g}_t$, a weak decrease in the openness of the immigration policy.*

Proof. In the case of $\sigma_t^m - \sigma_t^n \leq \eta$ the sign of the long-run effect corresponds to the short-run effect of an increase in l_{t-1}. Q.E.D.

3.4.3 Long-Run Effects of a Permanent Fall in the Birth Rate of the Natives

I study the long-run effects of a marginal fall in σ_t^n, in the case in which $\sigma_t^m - \sigma_t^n$ is arbitrarily small and the shock is permanent, i.e. the rise of σ_t^n implies that σ_{t+s}^n will be equal to the new value of σ_t^n for all $s > t$.

Theorem 15. *The long-run effects of a marginal decrease in the birth rate of the native population is a weak rise in public spending. The effect on the openness of the immigration policy is ambiguous at time t and weakly negative in the following periods. If voters are "naive" the effect on the openness of the immigration policy is weakly positive at time t and weakly negative in the following periods.*

Proof. In the case of $\sigma_t^m - \sigma_t^n \leq \eta$ the long-run effect corresponds to the short-run effect of an decrease in σ_t followed by a decrease in σ_{t-1}. Q.E.D.

The results in this section suggest that if immigrants are not too different from the natives in terms of fertility rates, then the long run effects of population ageing follow the patterns of the corresponding short-run effects.

3.5 Simulation

Some interesting cases cannot be fully described analytically, in particular the long-run effects of permanent shocks on the parameters in the case in which the birth rate of immigrant is different from the one of the natives. In order to study these cases I run a simulation of the model whose results are extensively presented in the supplementary material of this paper. This exercise shows that the effects due to the sophistication of voters may be substantial in terms of levels of the equilibrium policy, but they do not generally imply qualitatively different predictions about the shape of the curves describing the policy response to shocks on the parameters. I find that for several different parametrizations - if the difference in the birth rate of immigrants and natives

$(\sigma_t^m - \sigma_t^n)$ is not too large[16] - then the predictions of Theorems 14 and 15 are valid even if voters are "sophisticated" (Figures 9-10-11-12) . Figures 13 and 14 show the response of the immigration policy M_t and of the spending policy Y_t to a permanent rise in the life expectancy of the retired population, both for the case of "naive" voters (dashed lines) and of sophisticated voters (solid lines). Although the shape of the two lines is very similar, the equilibrium level is different. Sophisticated individuals fully internalize the effect of current immigration on the composition of the society in the following period. In particular, they anticipate that more immigration in the current period would imply a higher share of young individuals in the next period, and therefore an equilibrium policy that is less favorable to them when they will be retired. Therefore the equilibrium with "sophisticated" voters features a more restrictive immigration policy and a higher public spending in comparison with the case of "naive" voters.

The simulation exercise can also help to understand the factors that determine the speed of convergence to the steady state after a shock. The crucial aspect is that the speed is decreasing in the size of the "sophisticated effect", specifically in the value of $\sigma_t^m - \sigma_t^n$. Figures 15-16 show the path of convergence of the immigration policy M_t after a positive (solid line) and a negative (dashed line) shock on the endogenous state g_t, in the case of high difference (Fig. 15, $\sigma_t^m - \sigma_t^n = 1$) and low difference (Fig. 16, $\sigma_t^m - \sigma_t^n = 0.2$) in the birth rates of immigrants and natives. This exercise suggests that the key results in the previous sections still apply even to the cases in which the long run effects of shocks in the model cannot be characterized analytically. Thus, I can conclude that - in a society characterized by a very large share of retired individuals - population ageing leads to a policy that is closer to the needs of the elderly. In particular, high public spending and increasingly restrictive immigration policies are going to be implemented. These policy changes imply an increasing tax burden on the individuals of working age and may affect the fiscal sustainability of public spending in the long run.

4 Extensions

In this section I propose three extensions in which I introduce alternative forms of public intervention in social spending and a different legal status of the immigrants. I describe if and how the equilibrium political choices differ from the one presented in section 3. Specifically, I analyse the implication of the model if (i) the pension system is partially funded, if (ii) immigrants do not acquire voting rights and if (iii) the government provides public education. On one hand the main comparative statics results of this paper remain generally valid. On the other hand these exercises deliver some understanding of how different rules in the public sector may affect the attitude towards immigration of the voting population. Lastly, in section 4.4, I describe (iv) an extension of the model in which the labour market is segmented. In particular, I study the case

[16]For large value of $\sigma_t^m - \sigma_t^n$ the steady-state may not be unique and a shock may cause a transition to a different equilibrium path. Moreover, the conditions in Lemma 6 may not be satisfied.

in which the elderly demand specific services, such as home care, and only immigrants possess the skills to provide such services. In this case the results may differ substantially from the ones of the baseline model.

4.1 Partially Funded Pension System

The assumption of a pure Pay-As-You-Go pension system is a very stylized description of how the social security for the elderly is organized in most developed countries. In particular partially funded pension schemes are becoming increasingly common. There is empirical evidence of an increasing size of the funded part of the pension relative to the "state pension" in European countries (Galasso and Profeta, 2004). The theoretical analysis proposed by Rangel and Zeckhauser (2001) suggests that this phenomenon may also be related to the increase in the number of elderly relative to the working age population. In the model proposed in this paper I did not explicitly account for savings. One simple possibility is to model the funded part of the pension system as a form of compulsory savings. Under this assumption each individual has to save an amount $\psi(\rho)s(y_t^i)$ when young and she will receive $(1+r)\psi(\alpha+\gamma)s(y_t^i)$ when retired, where r is the exogenous interest rate and ψ is a strictly decreasing function. The total pension received by i at time $t+1$ becomes:

$$p_t^i = \left(\alpha + \gamma \frac{y_t^i}{\bar{y}_t}\right) \frac{\bar{\sigma}_t}{(1-\widehat{M}_{t+1})} + (1+r)\psi(\alpha+\gamma)s(y_t^i)$$

This formulation implies that if the state-pension component falls (e.g. if γ decreases) then the funded pension part rises. Notice that - because the utility function is linear in consumption - the size of the compulsory saving does not affect voter preferences over policies. Thus, the effect of a marginal transition towards a fully funded pension system simply corresponds to the effect of a fall in the Beveridgean part of the state pension α or of a fall in the Bismarckian part γ (or both). Hence I can state the following result:

Theorem 16. *The effect of a marginal decrease in the size of the public pension system in the short run is an increase in the restrictiveness of the immigration policy. In the long run, the effect is an increase in restrictions to immigration and an increase in public spending in the imperfect Public Good. The total effect on the tax rate is ambiguous.*

Proof. See Appendix B.1.

The intuition that underpins this result is simple. If the share of the Pay-As-You-Go component of the pension system decreases in favor of a fully funded scheme, then the fiscal gains from immigration for a worker decrease because the total size of public pension expenditures to be shared among the working age population is smaller. Moreover, the future gains from immigration also decrease, because public finance aspects have a lower impact on the overall pension enjoyed by a retired individual. Therefore all voters become more averse to immigration and ask for a more restrictive policy. In the long run, if the immigrants have higher fertility rates relative to the natives, this political choice causes an increase in the share of elderly individuals, with consequences that are similar to the ones described in section 3.4.1 for the case of increasing life expectancy. Namely, a further tightening in the immigration policy and an increase in the endogenous part of public spending prevail in equilibrium. An important aspect of this analysis is that if the size of the state pension system becomes too small (e.g. small $\alpha + \gamma$) then the total gains from immigration for a working age individuals may become negative, which implies an equilibrium in which the most restrictive immigration policy is implemented.

4.2 Voting Rights: *Ius Soli* vs. *Ius Sanguinis*

In the previous sections I have assumed that the children of immigrants that are born in the guesting country are awarded the voting right when they become adults (*Ius Soli*). Moreover, in the model voting rights can be also acquired after a sufficiently long period of legal residency. These assumptions are consistent with the legal procedures to obtain citizenship - and consequently voting rights - in several countries such as the US, Canada and France. In many other countries - such as the UK, Japan, Germany and Italy - the legal requirements are often quite different and they do not typically imply an automatic award of the citizenship based of the place of birth only. The most common case is that at least one of the parents must possess the citizenship in order for the children to obtain the same status (*Ius Sanguinis*). It is out of the scope of this paper to formulate assumptions that precisely describe the law of different countries. Nevertheless, in order to understand the possible effects of different legal requirements, it is useful to analyze the consequences of the opposite assumption in comparison with the one in section 3.2 of this paper. Namely, in this section I assume a pure form of *Ius Sanguinis,* in which neither the immigrants nor their children ever obtain the nationality. This assumption is clearly extreme and only serves as a term of comparison.

The main implications of the model stated in Theorems 7-8 are unaffected by this modification, except that for one aspect. Specifically, immigrants and their children do not become members of the voting population at any point in time. Therefore the choice of the immigration policy does not affect the future composition of the voting population. This implies that there is no "sophisticated effect" in this case, and therefore some of the results in section 4 are sharper. Namely for any σ_t^n, σ_t^m such that $\sigma_t^m \geq \sigma_t^n$ one gets:

Theorem 17. *(i) the short-run effect of a rise in l_t in unambiguously (weakly) positive on the immigration policy M_t and weakly negative on the tax rate τ_t; the long-run effects of (ii) an* *increase in the life expectancy and of (iii) a decrease in the birth rate of the native population is a weak rise in public spending and, if $g_t \geq \hat{g}$, a weak increase in the openness of the immigration policy at time t followed by a weak fall in the following periods.*

Proof. The relevant variable for determining the pivotal voter is in this case $\tilde{g}_t = \frac{l_{t-1}\tilde{n}_{t-1}}{\tilde{n}_t} = \frac{l_{t-1}}{\sigma^n_{t-1}}$ where $\tilde{n}_{t-1} \leq n_{t-1}$ is the number of young individuals that possess voting rights at time $t-1$ and it is smaller or equal to the number of individuals that are born in the country. Notice that $\tilde{g}_t$ is independent of M_{t-1}. The rest of the analysis is unaffected. All the proofs are identical to the ones for Theorems 9-10 except that no "sophisticated effect" occurs.

Theorem 17 suggests that the analytical predictions of the model are not strongly affected by the cross-country differences in the law that regulates the acquisition of the citizenship, and that - on the contrary - some results tend to become sharper and less sensitive to changes in parameter values if an extreme version of the *Ius Sanguinis* is assumed.

4.3 Endogenous Public Education

I analyse an extension of the model in which the income of an individual depends not only on the wage rate and on her productivity, but also on the amount of education she received when she was a child. I assume that education is uniformly provided by the government and has decreasing returns given by the strictly concave function f. Individual fertility of natives is given in this alternative setting by the random variable k^i_t, that is i.i.d. and with $E[k^i_t] = \sigma^n_t$. I get that the income of an individual i at time t can be written as follows:

$$y^i_t = f(e_{t-1})w_t\epsilon^i_t$$

and the total supply of effective labour at time t becomes $L_t = f(e_{t-1})\bar{\epsilon}_t(n_t + m_t)$. The budget constraint accounts for the public spending in education, such that the formula for the tax rate on labour income becomes:

$$\tau_t = \tau(e_t, M_t, Y_t, \bar{y}_t) = \bar{y}_t^{-1}\left[\bar{\sigma}_t e_t + \lambda_t M_t + (\alpha+\gamma)l_{t-1}\frac{(1-M_t)}{(1-\widehat{M}_t)} + Y_t\right]$$

Moreover, I assume that working age individuals (retired individuals) care about the utility of their children (grandchildren) such that the utility function of an individual of generation a can be written as follows:

$$\widetilde{U}^{i,a}_t = U^{i,a}_t\left(\left\{C^i_s, M_s, Y_s\right\}^{t+1}_{s=t}\right) + \delta^a E\left[k^i_t U^{j,y}_t\left(\left\{C^j_s, M_s, Y_s\right\}^{t+2}_{s=t+1}\right)\right]$$

Lastly, I assume that the number of voters is large, such that the uncertainty about the size of the future generation does not affect the result. Notice that given the assumptions about k^i_t

the preferences shown above can also represent individuals that care about the *next generation* rather than about their children and grandchildren. The structure of the overlapping generations model in the same as in the baseline model, except for the presence of an additional endogenous state e_{t-1} which affects the average income at time t. A coalitional equilibrium exists under the assumptions stated in Lemma 6 and most results of this augmented model about the comparative statics of shock on life expectancy are the same as the ones described in the previous section. (See Appendix A.2). The interesting aspect of this analysis is the counterintuitive effect of population ageing on public investment in education (per child). Such effects are stated in the following theorem.

Theorem 18. *The effects of an increase in the longevity of the retired population l_{t-1} and /or of a decrease in the growth rate of native population σ^n_{t-1} is a weak increase in the public spending in education per child e_t.*

Proof. See Appendix B.1.

The intuition that underpins this result is that if an elderly individual cares about her grandchildren (i.e. $\delta^o > 0$), then she will always support any policy that increases the spending in education through a rise in the taxes on the working age population, because she is not affected by this rise in the tax rate. The consequence of Theorem 18 is that the next generation may enjoy a better education and a higher pre-tax income as a consequence of population ageing. Notice that the overall welfare effect of the policy adjustment is not necessarily positive for these individuals. The negative side for future generations may come from the results in Theorems 7 and 8, which hold also in the augmented model (see Appendix A). In particular, in period t a more restrictive immigration policy is implemented. Thus, the future generations may have to face an society with a larger share of elderly which implies, *ceteris paribus*, higher tax rates on labour income and more public spending. Such policy can be harmful for the most productive individuals of the next generation. The second result is the following.

Theorem 19. *If voters are "naive" and $\frac{l_t p^v_{t+1}}{e_t} \geq \frac{\theta^v_t}{\beta}$, then the effects of a decrease in the birth rate of the native population σ^n_t is a weak fall in the public spending in education per child e_t and a weak increase in the openness of the immigration policy. Else both effect have an ambiguous sign.*

Proof. See Appendix A.2.4.

Theorem 19 suggests that the cost of public education may play a role in shaping the effects shocks on fertility rates on the immigration policy. On one hand immigration tends to reduce the pressure of the pensions system on public finances, on the other hand it causes an increase in the total costs of public education. If the latter effect is sufficiently strong, the predictions are going to be different from the one implied by the baseline model. This is particularly relevant if one considers that several countries are implementing reforms in order to reduce the Pay-As-You-Go share of the pensions received by the elderly in favor of a fully funded system (see section 4.1). Nevertheless, the public expenditures for the elderly represents a large share of the governmental budget in most western countries and, more importantly, they consistently exceed the ones in education and childcare (OECD 2015, 2015b). Notice that the assumption $\frac{l_t p_{t+1}^v}{e_t} \geq \frac{\theta_t^v}{\beta}$ is satisfied if β is close to 1 and the median cost of a pensioner is weakly larger than the cost of educating a child. Thus, OECD data suggest that such assumption is consistent with the facts about public spending in most OECD countries.

4.4 Services for the Elderly ("Elderly Goods")

In this section I present the results of an extension of the model in which the labour market is segmented. In particular, I study the case in which immigrants possess the skills to provide those services that are needed only by the elderly, such as home care, while the natives workers do not. This may be the case if immigrants are selected by the firms in the receiving country on the basis of their qualifications and previous work experience. In the next line I describe informally the characteristics that differentiate this setting from the baseline model. A detailed description of the economic environment is provided in Appendix B.2. Suppose that the elderly consume a different private good denoted by O_t. This good is produced with the same technology as the consumption good C_t and the imperfect public good Y_t, but only the immigrant workers are capable of producing it. Immigrants can also be employed in the production of the other goods. For simplicity I assume that there is no difference in the average tax payments of immigrants and natives, i.e. $\lambda_t = 0$, that the default immigration is $\widehat{M_t} = 0$ and I analyse the case in which $\sigma_t^m - \sigma_t^n$ is arbitrarily small. There are two possibilities. If at the equilibrium there are enough immigrant workers to satisfy the demand for "elderly goods" at a sufficiently low price, then the segmentation of the labour market is irrelevant and the results are identical to the baseline model. The perfect substitutability in production and the perfect competition ensure that all prices are are unaffected by immigration choices. The implications change dramatically if in the proximity of an equilibrium there are not enough immigrant workers to satisfy the demand for the "elderly good" at the constant price[17]. I can state the following result.

Theorem 20. *If $g_t \leq 1$ then at the equilibrium, if it exists, the immigration policy is $M_t = 0$, else a positive level of immigration is possible.*

Proof. Appendix B.2.

This result implies that, as long as the majority of voters is of working age, the society always chooses the most restrictive immigration policy. Moreover, a shock on the longevity or the fertility

[17]Notice that multiplicity of equilibria is possible in this case.

of the native population does not affect the immigration policy in equilibrium. The channel that underpins this result is the effect of immigration on equilibrium prices. Specifically, immigrants are endogenously hired in the sector that produces the "elderly good" O_t, but they consume only the other two goods C_t and Y_t. As a result, immigration in equilibrium implies a rise in the relative prices faced by the young natives, offsetting the fiscal benefits generated by immigrants and making working age voters extremely hostile to immigration. The conclusion one can derive from this section is that some implications of the analysis presented in section 3 of this paper are true for this extended case only if in the proximity of the equilibrium the immigration policy is not too restrictive. If the number of immigrants is too low to satisfy the demand of services for the elderly, then some predictions in section 3 of the paper are no longer valid. The result in this case is somewhat paradoxical: a society that is in great need of immigrants to satisfy the demand of services for the elderly tend to be very averse to any positive level of immigration of specialized workers. Additional details and results about this extension of the model are available in the supplementary online material.

5 Welfare Analysis

In the previous section I have proved that a rise in the longevity or a fall in the birth rate of the native population generates a political pressure towards more restrictive immigration policy. This does not necessarily imply that this change is desirable on the point of view of the society as a whole. In this section I present a welfare analysis which shows that, if a society has certain demographic characteristics, a marginal increase in the restrictions to immigration is unambiguously harmful for the society. I define a measure of the wellbeing of the society in the form of a *Social Welfare Function* (SWF). The idea that is exploited in this section is the following. If at an equilibrium policy the marginal effect of an increase in a policy dimension $x_{j,t}$ on the SWF is greater than than the one of the median voter (and at the equilibrium $x^*_{j,t} < \bar{x}_{j,t}$), then there exists a policy with $x'_{j,t} > x^*_{j,t}$ which is welfare improving. This implies in turn that if, as a consequence of a shock, a certain policy dimension j is such that $x^*_{j,t-1} > x^*_{j,t}$, then $x_{j,t}$ has moved in the "wrong direction" on a social welfare point of view and that the society would benefit, *ceteris paribus*, from a marginal change in the direction of $x^*_{j,t-1}$. In other words, the society is harmed by the change in policy at the margin. Consider a SWF that is a weighted average of the utility of each individuals of the working age generation (y), of the retired generation (o) at time t and the expected future utility of the children (ch), where $\mu^a_t(\theta^i_s)$ represents the Pareto weight assigned to an individual i of generation a at time t. Notice that I am not ruling out either the possibility that the SWF attributes zero weight to the immigrants or the possibility that some or all the immigrants have positive weight[18]. The SWF has form:

$$SWF(x_t, ; \varphi_t, g_t) = \int_0^{\bar{\theta}_t} \mu^y_t(\theta^i_t) V^y(x_t; \theta^i_t, \varphi_t, g_t) q(\theta^i_t) d\theta^i_t +$$

[18]One has to specify the objective function of an immigrant in this case.

$$+\int_{0}^{\bar{\theta}_{t-1}} \mu_t^o(\theta_{t-1}^i)V^o(x_t;\theta_{t-1}^i,\varphi_t,g_t)q(\theta_{t-1}^i)d\theta_{t-1}^i+\int_{0}^{\bar{\theta}_{t+1}} \mu_{t+1}^y(\theta_{t+1}^i)E_t\left[V^y(x_{t+1}^{**};\theta_{t+1}^i,\varphi_{t+1},g_{t+1})\right]q(\theta_{t+1}^i)d\theta_{t+1}^i$$

Most welfare implications of this analysis depend on the Pareto weights assigned to each individual in the *SWF*. For instance, some results that can be obtained using a specific *SWF* (e.g. Utilitarian or Rawlsian) are presented in the supplementary material. Nevertheless an interesting general result can be stated under relative weak restrictions on the *SWF*. Specifically, I analyse the welfare effects of changes in the immigration policy keeping the other policy dimension constant at the equilibrium level. This analysis is also consistent with the extended model presented in section 4.3.

5.1 Welfare Effects of a Marginal Opening in the Immigration Policy

Assume that $c'(M_t) < \infty$ for all $x_t \in X_t$ and that at the equilibrium $0 < M_t < \overline{M}_t$, i.e. the solution is internal for the immigration policy. Then I can state the following result.

Theorem 21. *For any Social Welfare Function $SWF(x_t;\varphi_t,g_t)$ that assigns a strictly positive weight to each native individual of working age, there exist a threshold $\breve{g}_t \in [0,1]$ such that if $g_t \geq \breve{g}_t$ then a marginal tightening in the immigration policy caused by a change in the equilibrium outcome reduces the Social Welfare.*

Proof. See Appendix B.3.

The intuition that underpins this result is that - as g_t tends to 1 - the parameter θ_t^v that identifies the pivotal voter get close to 0. On one hand, the benefits for the individuals of working age from a marginal opening of the immigration policy increase rapidly as M_t approaches 0. On the other hand, the cost of immigration becomes increasingly small at low levels of M_t. If $\theta_t^v = 0$, then $M_t = 0$, which implies that the marginal social gains from immigration are very large relative to the marginal social costs. Also notice that the converse of the statement in Theorem 21 is not always true. Specifically, a threshold $\breve{g}_t \in [0,1]$ such that if $g_t \leq \breve{g}_t$ then the society would benefit from a marginally more restrictive immigration policy may not exists for all the *SWF*s with the features stated above. Nevertheless, such threshold $\breve{g}_t$ exists for Utilitarian and Rawlasian *SWF*. The result in Theorem 21 suggests that societies characterized by high income inequality and/or by a high share of elderly on the total population (which have a g_t close to 1 or larger) are likely to adopt excessively restrictive immigration policies. Moreover, it implies that a tightening in the immigration law - for instance the one caused by population ageing - reduces the Social Welfare. In other words, the policy adjustment of the immigration quota is harmful for the society. This result is suggestive in the light of the increasingly and rather controversial restrictions to immigrations that have been progressively introduced in countries characterized by a rapidly ageing population and by a high degree of income inequality, such as the UK and the

USA, or in countries that feature by a very large elderly population, such as Japan or Italy. In the supplementary material I propose a welfare analysis about the effects of a change in the public spending in the imperfect Public Good and in education. These results are less general because they rely on more restrictive assumptions about the SWF (e.g. Utilitarianism). Nevertheless, they suggest that the allocation of public spending may be too generous for the imperfect Public Good and perhaps insufficient for education in society characterized by high income inequality and by a large share of elderly.

6 Empirical Evidence

In this section I investigate the determinants of the attitudes towards immigration and public spending of adult residents in Great Britain using data from the British Social Attitude Survey, and in particular from the rounds of data 2009 - 2011 - 2013 that includes a specific section about immigration. The dataset accounts for a total of 6639 observations. The explanatory variables are the age of the respondent, the income decile of the household and the highest educational qualification attained by the respondent, on a scale from 1 (postgraduate degree) to 8 (no qualification). Observations of individuals with foreign qualifications have been omitted. Dummy variables capture whether the household includes children, and if the respondent is a woman, if she lives in rural areas, if she is born abroad and if she is not part of any religion. Characteristics related to the employment status and type are captured by dummies. In particular, I include the effects of being employed in a manual job, unemployed or retired.

6.1 Determinants of Attitude towards Immigration

The outcome variable LETIN captures the attitude towards further immigration in the country. The question is "Do you think the number of immigrants to Britain nowadays should be increased a lot, increased a little, remain the same as it is, reduced a little or reduced a lot?" and the respondents must choose a value on a discrete scale from 1 ("increased a lot") to 5 ("reduced a lot"). The variable LETIN measures therefore the degree of aversion towards further immigration. I use an ordered Logit model because of the discrete and ordered nature of the outcome variable. Table 1 presents the results of this analysis. In line with what observed in the literature and with what is implied by the model proposed in this paper, the age of the respondent exhibit a significant positive relationship with the hostility towards immigration. Moreover, the parameter on household income is negative and significant in all the specifications. This means that high income individuals tend to be less averse to immigration relative to the low income, and this is consistent with the implications of model. Similarly, low level of education tend to be associated with a stronger aversion to immigrants. Lastly, the presence of children in the household, the location in a urban area and the birth of the respondent outside of the UK are all significantly related to a more positive attitude towards immigrants.

Table 1. Determinants of Attitudes Towards Immigration

	Ordered Logit - Dependent Variable: LETIN				
	(1)	(2)	(3)	(4)	(5)
VARIABLES	Robust SE	Cluster SE	Robust SE	Cluster SE	Cluster SE
Age	0.00949***	0.00949***	0.0117***	0.0117***	
	(0.00196)	(0.00202)	(0.00269)	(0.00310)	
Income	-0.0258**	-0.0258***	-0.0235*	-0.0235***	-0.0241***
	(0.0114)	(0.00917)	(0.0120)	(0.00655)	(0.00884)
LowEdu	0.248***	0.248***	0.238***	0.238***	0.257***
	(0.0177)	(0.0206)	(0.0185)	(0.0220)	(0.0195)
BornAbroad	-0.790***	-0.790***	-0.768***	-0.768***	-0.806***
	(0.154)	(0.162)	(0.157)	(0.169)	(0.164)
Children	-0.267***	-0.267***	-0.247***	-0.247***	-0.225***
	(0.0650)	(0.0822)	(0.0668)	(0.0888)	(0.0785)
Rural	0.172***	0.172***	0.167***	0.167***	0.182***
	(0.0323)	(0.0624)	(0.0324)	(0.0620)	(0.0615)
Sex			0.0778	0.0778	
			(0.0625)	(0.0744)	
Unemployed			-0.0793	-0.0793	
			(0.143)	(0.110)	
ManualWork			0.139*	0.139	
			(0.0720)	(0.0893)	
Retired			-0.0813	-0.0813	
			(0.112)	(0.0943)	
Religion			0.0779	0.0779	
			(0.0635)	(0.0516)	
year	-0.0635**	-0.0635**	-0.0615**	-0.0615**	-0.0668**
	(0.0298)	(0.0314)	(0.0298)	(0.0308)	(0.0323)
Age65					0.209**
					(0.0857)
Observations	4,421	4,421	4,421	4,421	4,421

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

6.2 Determinants of Attitude towards Public Spending

The outcome variable TaxSpend is a measure of the attitude towards public spending financed through taxation. This variable capture a fundamental trade-off that drives the results in section 3. Namely, it measures the degree of aversion to higher taxes in exchange of more social spending. The question is "Suppose the government had to choose between the three options on this card: reduce taxes and spend less on health, education and social benefits, Keep taxes and spending on these services at the same level as now, Increase taxes and spend more on health, education and social benefits. Which do you think it should choose?" and the respondents must choose a value on a discrete scale from 1 ("spend less") to 3 ("spend more"). I use an ordered Logit model for the same reasons explained in the previous section. Table 2 shows the results of this analysis. The relationship between the outcome variable and the age and the income of the respondent are both significant and the signs are consistent with the implication of the model.

Table 2. Determinants of Attitude towards Public Spending

Ordered Logit - Dependent Variable: TaxSpend

VARIABLES	(1) Robust SE	(2) Cluster SE	(3) Robust SE	(4) Cluster SE	(5) Cluster SE
Age	0.0102***	0.0102***	0.0124***	0.0124***	
	(0.00161)	(0.00178)	(0.00270)	(0.00282)	
Income	-0.0320***	-0.0320***	-0.0422***	-0.0422***	-0.0385***
	(0.00983)	(0.00996)	(0.0126)	(0.0123)	(0.0129)
LowEdu	-0.0451***	-0.0451***	-0.0669***	-0.0669***	-0.0473***
	(0.0134)	(0.00997)	(0.0175)	(0.0145)	(0.0150)
BornAbroad			-0.0424	-0.0424	-0.0760
			(0.111)	(0.0702)	(0.0733)
Children			-0.0765	-0.0765	-0.0262
			(0.0677)	(0.0828)	(0.0816)
Rural			-0.0103	-0.0103	0.00376
			(0.0323)	(0.0310)	(0.0281)
Sex	0.180***	0.180***	0.188***	0.188***	0.161***
	(0.0504)	(0.0476)	(0.0637)	(0.0492)	(0.0490)
Unemployed	0.251**	0.251**	0.202	0.202	0.144
	(0.116)	(0.125)	(0.142)	(0.138)	(0.127)
ManualWork			0.202***	0.202***	0.185***
			(0.0691)	(0.0506)	(0.0483)
Retired			-0.173	-0.173*	0.230**
			(0.106)	(0.0946)	(0.117)
Religion	0.218***	0.218***	0.235***	0.235***	0.167***
	(0.0517)	(0.0486)	(0.0645)	(0.0598)	(0.0594)
year	0.00639	0.00639	0.00614	0.00614	0.000110
	(0.0298)	(0.0314)	(0.0300)	(0.0310)	(0.0310)
Age65					-0.136
					(0.112)
Observations	6,639	6,639	4,421	4,421	4,421

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

In line with with the previous literature, unemployment is also related with a more favorable attitude towards public spending. It is somewhat surprising that low levels of education are associated with a stronger aversion to taxes and public spending. This may be due to factors that are not considered in the theoretical analysis and that are likely to vary across different education level, such as knowledge of the structure of the fiscal system, awareness of the demographic and economic structure of the country and degree of altruism.

6.3 Discussion

The analysis in this section provides a strong support for two crucial implications of the model regarding voters' preferences in Britain. Namely, the analysis of the attitudinal data in the BSA suggests that older age tend to be associated with stronger aversion towards immigration and with a higher propensity to increase the size of public intervention in public spending policies, even if this implies higher taxes. Moreover, the analysis implies that (conditional and unconditional on the level of education), higher levels of income tends to corresponds to a more positive attitude towards immigrants and to a stronger propensity to cut taxes and public spending. It may be worth to underline that this analysis does not make any claim about a causal relationship between the variables of interest. The results in section 6.1 are consistent with other similar studies in the literature that use alternative dataset and analyse other countries or group of countries. For instance Dustmann and Preston (2007), Facchini and Mayda (2007) and Card *et al.* (2011), using respectively data from the British Social Attitude Survey, the International Social Survey Programme and the European Social Survey, all support these findings. Thus, one can conclude that there is substantial empirical evidence in support of the patterns of attitudes induced by age and income that are implied by the model proposed in this paper, even if no causal relationship can be claimed. A more general question concern the empirical support to the main predictions of the paper, which concern the comparative statics of the policy outcome. Specifically, it would be critical for this stream of literature to assess in future research if population ageing tend to be associated to more restrictive immigration policies and, if so, to what extent this is due to a causal link between these two variables. The answer to this question is not strightforward. First of all, population ageing is a demographic phenomenon that produces effects on a very long time span and it is likely to be associated to a number of other economic and political transformations. Thus, it is not an easy task to disentangle its effect on specific policies, such as immigration, from other endogenous processes that may induce correlation between the the variables of interest. Moreover, immigration policies are not easy to measure. The "tightness" of an immigration policy is a multidimensional concept, in the sense that such policies can be restricted in various ways, targeting different kinds of immigrants, etc. Moreover, its relationship with the number of immigrants that legally enter a country in a given period of time may be highly endogenous. For instance, on one hand it is reasonable to expect that a country with a more restrictive immigration policy allows, *ceteris paribus.* a smaller number of immigrants to enter the country relative to one with a more liberal set of rules. On the other hand, a country that is subject to a more intense *immigration pressure*, for instance because it is more attractive for potential immigrants, may tend to experience a larger inflow of immigrants even if its immigration policy is more restrictive in comparison with a less attractive country. Similarly, an increase in the immigration pressure due to exogenous factors may translate into a more restrictive immigration law and to a larger inflow of foreigners in the country. In other words, immigration choices and policy choices are two interdependent endogenous processes, and this must be accounted for if one aims to study the latter in isolation from the former. Lastly, immigration policies are often formulated in terms of *qualitative* requirements, which may not be easy to translate into an objective measure of "tightness". For instance, the immigration law often assign different status to potential immigrants that possess different education levels, or that come from specific countries. Attempts to measure the "tightness" of immigration policies have been made by Boeri and Brucker (2005) for 15 European countries countries and by Ortega and Peri (2009) for 14 OECD countries. Their measures consist of a number of indexes constructed under different definitions of "tightness" of an immigration policy. The limitations in the use of these data are not neglegible. Specifically, the low number of observations, the limited extent of time variation that can be exploited and the robustness of the findings to different concepts of "tightness" are important issues. Thus, this literature did not provide so far enough evidence in support or against the predictions of the model proposed in this paper. This remains an open and challenging question for future research.

7 Concluding Remarks

This paper investigates the interaction between two crucial demographic, economic and social processes in our society: ageing and immigration. The aim is to analyse how these two processes shape policy choices in democratic countries, and how such policy choices may affect the demographic profile of the society. In particular, I study the effects on immigration policies of two major demographic changes that have caused population ageing in western societies, namely increasing life expectancy and decreasing birth rates. The main finding concerns the fiscal consequences of population ageing. That is, if the share of elderly population is large enough, population ageing increases the political pressure to restrict the inflow of immigrant workers into the country and to rise public spending. This result implies that the negative effects of population ageing on public finances - due to increasing costs for public pensions - may be exacerbated by the endogenous political effects on immigration and public spending policies. Direct and indirect effects of the ageing phenomenon may affect the overall fiscal soundness of the public sector in the long run. The second result looks at the demographic consequences of ageing. In particular, I show that the effects of a demographic shock on the age profile of the population tend to worsen with time because of the endogenous political effects on the immigration policies. Specifically, I find that an ageing society tends to support increasingly restrictive immigration policies. This translates into a reduced number of immigrants and - in some cases - into further population ageing in the future. The third finding is about social welfare. I show that the changes in the immigration policy induced by population ageing tend to harm the society, in particular the young individuals and future generations. One element that emerges from this analysis is that the way in which costs and benefits generated by immigration are divided up in the society is crucial to determine the attitudes towards immigration of different demographic groups. This implies that an analysis of the political processes that lead to the division of these net gains is essential in order to assess the political effects of ageing on immigration policies. Thus, the study of the latter cannot abstract from how fiscal policies are determined.

There are anyway some limitations in this analysis that one has to consider. First, in this study the endogenous adjustment of wages has no effect on the equilibrium policy choices. This is due to the assumption that the individual labour supply is perfectly inelastic both at the extensive and at the intensive margin. This modelling choice is justified by theoretical (Ben-Gad, 2004) and empirical considerations (Dustmann and Preston, 2006, 2007; Boeri, 2010) and can be relaxed to some extent (see additional material). Nevertheless this aspect is likely to play a role in shaping immigration policies. Thus, this is a topic that calls for further research. Secondly, I do not fully investigate the effects of the heterogeneity in the productivity of immigrants. This aspect is likely to be relevant given that such heterogeneity may be - at least to some extent - endogenous in the political process. For instance, simple theoretical models suggest that countries with a generous welfare system may attract relatively low skilled immigrants (Borjas, 1999), and that the attitude towards different types of immigration may vary with the composition of skills of the native population (Benhabib, 1996). Even if the empirical literature provide limited support for two these channels (see Preston, 2014), they represents important elements to enrich the study of the determinants of immigration policy. Lastly, a deeper analysis of the determinants of the aversion to immigration due to concerns related to the effects on the "compositional amenities" of the society is needed in order to better understand what other factors shape immigration policies. This aspect has been shown to play a major role in attitudinal studies (Card *et al.*, 2011) and it is an active field of research in other disciplines (see Brettell and Hollifield, 2007), but it has not been sufficiently analyzed with the tools of economic theory. A more general remark should be made about the model of political interaction and the equilibrium concept adopted in this paper. This framework represents a tool that does not only serves for the purposes of this analysis, but it is sufficiently general to be used in many other applications in Political Economy. In a companion paper (Dotti, 2015) I show how this theoretical framework can be useful to analyze problems of redistribution in the spirit of Meltzer and Richard (1981) and to reconcile the controversial predictions about the relationship between income inequality and size of the government - that are typical of that literature - with the empirical evidence. In this paper, I show that such framework can be extended to study simple dynamic problems in an OLG model. There are many other questions in Political Economy for which the multidimensionality of the policy space represents a major obstacle in the analysis, and that therefore represent a promising field of application for the voting model presented in this paper. Examples of these potential new applications are described in Dotti (2015).

Lastly, I emphasize that this analysis delivers an essentially pessimistic message about the evolution of our society in the immediate future and its consequences for the young generations. If population ageing means an increasing power for the elderly to shape public policies according to their needs, the main victims of this process are going to be the young, both the ones born in rich countries and the ones native of poorer regions. On one hand the former will have to support the fiscal burden of an increasingly large and long-living elderly population through high tax rates on their income. On the other hand the latter are going to be prevented from searching for better employment opportunities by the excessively restrictive immigration policies that are going to be implemented in the high income countries.

Appendix

A Proofs: Main Results

Appendix A includes the proofs to the main results of the paper. Specifically, in appendix A.1 I prove the Lemmas related to the existence of a Markov-Perfect coalitional equilibrium. In appendix A.2 I provide proofs of the main comparative statics results.

A.1 Existence of Equilibrium

A.1.1 Markov Property of Ideal Policies

Lemma 1. *In a Markov-Perfect coalitional equilibrium - if it exists - (i) each individual's ideal policy x_t^i and (ii) the equilibrium policy x_t^* at time t are invariant - conditional on g_t - to the history up to time $t-1$, i.e. $x_t^i(g_t, \varphi_t, h_{t-1}) = x_t^i(g_t, \varphi_t, h'_{t-1})$ and $x_t^*(g_t, \varphi_t, h_{t-1}) = x_t^*(g_t, \varphi_t, h'_{t-1})$ $\forall t$ and $\forall h_{t-1}, h'_{t-1} \in H_{t-1}$.*

Proof. Part (ii) is simply a consequence of rational beliefs and of the Markov property. Part (i) follows the F.O.C.s for each individual i at time t:

$$\widetilde{V}_{M_t}^{i,y} = -c'(M_t) + \theta_t^i (\alpha + \gamma) \frac{l_{t-1}}{(1 - \widehat{M}_t)} - \theta_t^i \lambda_t + \left(\frac{\beta l_t}{(1 - \widehat{M}_{t+1})} (\alpha/\theta_t^i + \gamma) - e_t \right) (\sigma_t^m - \sigma_t^n) \theta_t^i +$$

$$+\beta l_t \sum_{j=1}^{3} \frac{\partial V_{t+1}^{i,o}}{\partial x_j} \frac{dx_{j,t+1}^{**}}{dM_t}$$

$$\widetilde{V}_{Y_t}^{i,y} = -\theta^i + b'(Y_t) + \beta l_t \sum_{j=1}^{3} \frac{\partial V_{t+1}^{i,o}}{\partial x_j} \frac{dx_{j,t+1}^{**}}{dY_t}$$

$$\widetilde{V}_{e_t}^{i,y} = -E(\sigma_t)\theta_t^i + \delta \sigma_t^n f'(e_t) E(\bar{\omega}_{t+1}) + \beta l_t \sum_{j=1}^{3} \frac{\partial V_{t+1}^{i,o}}{\partial x_j} \frac{dx_{j,t+1}^{**}}{de_t}$$

as x_{t+1}^{**} only depends upon $g_{t+1} = \frac{l_t}{\bar{\sigma}_t(M_t)}$, (and in the case of endogenous education, the pivotal voter is unaffected by e_t) then the above reduces to:

$$\widetilde{V}_{M_t}^{i,y} = -c'(M_t) + \theta_t^i (\alpha + \gamma) \frac{l_{t-1}}{(1 - \widehat{M}_t)} - \theta_t^i \lambda_t + \left(\frac{\beta l_t}{(1 - \widehat{M}_{t+1})} (\alpha/\theta_t^i + \gamma) - e_t \right) (\sigma_t^m - \sigma_t^n) \theta_t^i +$$

$$+\beta l_t \sum_{j=1}^{3} \frac{\partial V_{t+1}^{i,o}}{\partial x_j} \frac{dx_{j,t+1}^{**}}{dg_{t+1}} \frac{l_t(\sigma_t^m - \sigma_t^n)}{\bar{\sigma}_t^2}$$

$$\widetilde{V}_{Y_t}^{i,y} = -\theta^i + b'(Y_t)$$

$$\widetilde{V}_{e_t}^{i,y} = -E(\sigma_t)\theta_t^i + \delta\sigma_t^n f'(e_t)E(\bar{\omega}_{t+1})$$

Given that the F.O.C.s are invariant to h_s for all $s < t$ and that the optimum is unique, then the individual ideal policy must be invariant to h_s for all $s < t$ as well. Q.E.D.

A.1.2 *SM* and *SSCP*

Lemma 2. *The function $V(x_t;\theta_t^i,\varphi_t,g_t)$ satisfies SM and SSCP in $(x_t;\theta_t^i)$ for all $\theta_t^i \in \Theta$ and for all $\varphi_t \in \Phi_t$ for any given state g_t.*

Proof. Given the definition of V_t^i ($\widetilde{V}_t^i$ for the full model):

$$V_t^i = V(x_t;\theta_t^i,\varphi_t,g_t) = \begin{cases} V^{i,y} & if \quad young \quad then \quad \theta = \theta_t^i \\ \kappa V^o & if \quad old \quad then \quad \theta = -1 \end{cases}$$

With $x_{1t} = M_t$, $x_{2t} = -Y_t$, $x_{3t} = -e_t$ and for an arbitrarily large $\kappa > 0$. Notice that κ represents a strictly increasing transformation of the original objective function of the elderly therefore κV^o implies the same preferences as V^o. First I need to show that each component $V_t^{i,y}$, V_t^o ($\widetilde{V}_t^{i,y}$, $\widetilde{V}_t^o$) satisfies the required properties and then I will show that it also holds for the overall function V_t^i ($\widetilde{V}_t^i$). Recall the objective function of a young individual in the baseline model is:

$$V_t^{i,y} = (1-\tau_t)\omega_t^i + b(Y_t) - c(M_t) + \beta l_t \left(\frac{(\alpha + \gamma\theta_t^{i,y})\bar{\sigma}_t}{(1-\widehat{M}_{t+1})} + d(Y_{t+1}^{**}) - c(M_{t+1}^{**}) \right)$$

and in the full model is:

$$\widetilde{V}_t^{i,y} = (1-\tau_t)f(e_{t-1})\omega_t^i + b(Y_t) - c(M_t) +$$
$$\beta l_t \left((\alpha + \gamma\theta_t^{i,y}) \frac{\bar{\sigma}_t}{(1-\widehat{M}_{t+1})} + d(Y_{t+1}^{**}) - c(M_{t+1}^{**}) \right) + \delta^y \sigma_t^n f(e_t)\bar{\omega}_{t+1}$$

Below I derive the conditions for the full model with endogenous education. Given these conditions, the ones for the baseline model are straightforward. Given that the function $\widetilde{V}_t^{i,y}$ is twice differentiable under the assumption stated in section 2.3, sufficient conditions for SM and $SSCP$ are simply related to the sign of the cross derivatives and in particular: $\widetilde{V}_{e_tM_t}^{i,y}, \widetilde{V}_{e_tY_t}^{i,y} \leq 0$, $\widetilde{V}_{M_tY_t}^{i,y} \geq 0$ for all $x \in X$ and all $\theta_i^t \in \Theta$ and $\widetilde{V}_{e_t\theta_t^i}^{i,y}, \widetilde{V}_{Y_t\theta_t^i}^{i,y} < 0$, $\widetilde{V}_{M_t\theta_t^i}^{i,y} > 0$ for all $x \in X$ and all $\theta_i^t \in \Theta$. The first derivatives are:

$$\widetilde{V}_{M_t}^{i,y} = -c'(M_t) + \theta_t^i(\alpha+\gamma)\frac{l_{t-1}}{(1-\widehat{M}_t)} - \theta_t^i\lambda_t + \left(\frac{\beta l_t(\alpha/\theta_t^i + \gamma)}{(1-\widehat{M}_{t+1})}(\alpha/\theta_t^i+\gamma) - e_t \right)(\sigma_t^m - \sigma_t^n)\theta_t^i +$$

$$+\beta l_t \left(d'(Y^{**}_{t+1}) \frac{\partial Y^{**}_{t+1}}{\partial M_t} - c'(M^{**}_{t+1}) \frac{\partial M^{**}_{t+1}}{\partial M_t} \right)$$

$$\widetilde{V}^{i,y}_{Y_t} = -\theta^i + b'(Y_t)$$

$$\widetilde{V}^{i,y}_{e_t} = -\bar{\sigma}_t \theta^i_t + \delta \sigma^n_t f'(e_t) \bar{\omega}_{t+1}$$

Notice that the expectations M^{**}_{t+1} and Y^{**}_{t+1} are solely affected by M_t (through g_{t+1}) because of the Markov assumption. Calculate the cross derivatives of $\widetilde{V}^{i,y}_t$ with respect to each two policy dimensions:

$$\widetilde{V}^{i,y}_{e_t M_t} = -\theta^i_t (\sigma^m_t - \sigma^n_t) \leq 0$$

$$\widetilde{V}^{i,y}_{e_t Y_t} = V^{i,y}_{Y_t M_t} = 0$$

And with respect to each policy dimension and the parameter θ^i_t (recall that $x^{**}_t$is a function of solely g_{t+1} and it is therefore invariant to θ^i_t):

$$\widetilde{V}^{i,y}_{e_t \theta^i_t} = -E(\sigma_t) < 0$$

$$\widetilde{V}^{i,y}_{M_t \theta^i_t} = \frac{(\alpha + \gamma)\, l_{t-1}}{(1 - \widehat{M}_t)} - \lambda_t - \left(e_t - \frac{\beta l_t \gamma}{(1 - \widehat{M}_{t+1})} \right) (\sigma^m_t - \sigma^n_t) > 0$$

$$\widetilde{V}^{i,y}_{Y_t \theta^i_t} = -1 < 0$$

Notice that the FOCs with respect to M_t imply that an interior solution with a partially open migration policy $M_t > 0$ can exist even if immigrants "contribute less than what they take out" in the current period, or more precisely if at a given policy (e_t, Y_t, M_t) a marginal increase in the number of migrants at constant e_t, Y_t implies a rise in the income tax rate. This is true because a native individual of working age will have a future benefit from immigration $\frac{\beta l_t (\sigma^m_t - \sigma^n_t)}{(1 - \widehat{M}_{t+1})} (\alpha + \gamma \theta^i_t)$ which incorporates the fact that he will partially internalize the positive effect of immigration today on the governmental budget constraint in the following period through the adjustment in the pension system. This implies that this model is not affected by the dichotomy between "skilled migration" and "unskilled migration" in the patterns of attitude towards immigration and income that is typical of traditional models such as Facchini and Mayda (2008). In my model the attitude towards immigration may improve with income even if the immigrants are a net burden for the society in the short run, because if the Bismarkian component of the pension system is positive ($\gamma > 0$), then the future benefits of current immigration are increasing with income. The next step is to state the elderly's objective function and calculate its first derivatives. Using the formulas for C^o_{t+1} we get:

$$\widetilde{V}^o = l_{t-1} \left[d(Y_t) - c(M_t) \right] + \delta^o E(k^i_t) f(e_t) \bar{\omega}_{t+1}$$

First derivatives are:

$$\widetilde{V}^o_{e_t} = E(k^i_t) f'(e_t) E(\omega_{t+1}) > 0$$

$$\widetilde{V}^o_{M_t} = -l_{t-1}c'(M_t) < 0$$

$$\widetilde{V}^o_{Y_t} = l_{t-1}d'(Y_t) > 0$$

Cross derivatives:

$$\widetilde{V}^o_{e_t M_t} = \widetilde{V}^o_{e_t Y_t} = \widetilde{V}^o_{Y_t M_t} = 0$$

Notice that the preferences for (M_t, Y_t, e_t) are the same for all elderly individuals. Now I can show that the function $\widetilde{V}(x_t; \theta^i_t, \varphi_t, g_t)$ satisfies (i) *SM* and (ii) *SSCP* in $(x_t; \theta^i_t)$.
(i) *SM*. It follows from *SM* of $\widetilde{V}^{i,y}_t$ and $\widetilde{V}^o_t$. (ii) *SSCP*. I need to show that if $x'_t \geq x''_t$, $x'_t \neq x''_t$ and $\theta'_t > \theta''_t$ then

$$\widetilde{V}(x'_t; \theta''_t, \varphi_t, g_t) - \widetilde{V}(x''_t; \theta''_t, \varphi_t, g_t) > \widetilde{V}(x'_t, ; \theta'_t, \varphi_t, g_t) - \widetilde{V}(x''_t; \theta'_t, \varphi_t, g_t)$$

(ii) (a) $\theta'_t, \theta''_t \neq -1$. *SSCP* follows from *SSCP* of $V^{i,y}_t$and V^o_t. (ii) (b)$\theta' \neq -1$, $\theta''_t = -1$. Notice that $\widetilde{V}(x'_t; \theta''_t, \varphi_t, g_t) - \widetilde{V}(x''_t; \theta''_t, \varphi_t, g_t) > 0$ is always true under the assumption previously stated so it is sufficient to choose κ large enough such that *SSCP* holds trivially. (ii) (c) $\theta'_t, \theta''_t = -1$. Straightforward. Also notice that under the restriction the parameter set

$$\Theta_t := \left\{ \theta^i_t | \theta^i_t = \left\{ \begin{array}{ccc} \theta^{i,y}_t & if & age = y \\ -1 & if & age = o \end{array} \right\} \right\}$$

is a totally ordered set. Q.E.D.

A.1.3 Median Voter Theorem and Comparative Statics

Theorem 3. *(Median Voter Theorem). If conditions 1-2-3 are satisfied, then (i) A Markov-Perfect coalitional equilibrium of the voting game exists; (ii) in any equilibrium the set of winning policies is a subset of the set of ideal points of the median voter v_t; (iii) if the median voter has a unique ideal policy, then the set of equilibrium policies is a singleton.*

Proof. Consider an objective function $\nu(x_t, x^{**}_{t+1}(g_t, \varphi_t); \theta^i_t, \varphi_t)$ for some (common) expectations $x^{**}_{t+1}(g_t, \varphi_t)$. Notice that notion of coalitional equilibrium implies that rational expectations must exists at time t, because the political process implied by such equilibrium concept always delivers a policy outcome (it can be an equilibrium outcome in the form of a *Condorcet winner*, or, in case such outcome does not exist, a default policy x^0). Thus, rational expectations exist even if there is no Markov-Perfect coalitional equilibrium at time $t+1$. Moreover, given that in each period $t+s$ voters' indirect utility ν - conditional on g_{t+s} and x_{t+1+s} is unaffected by history up to time $t-1+s$, then there must be rational expectations $x^{**}_{t+1}(g_t, \varphi_t)$ that satisfy *MP*. Thus, choose a function x^{**}_{t+1} such that the expectations are rational and satisfy the *MP*. These two ensure that conditions (ii) and (iii) of the definition of Markov-Perfect coalitional equilibrium (Definition 1) are satisfied. The Markov assumption (*MP*) implies $x^{**}_{t+1}(g_t, \varphi_t) =$

$x^{**}_{t+1}(g_{t+1}(x_t,\varphi_t),\varphi_{t+1})$. Using such rational expectations, define the function $V(x_t;\theta^i_t,\varphi_t,g_t) = \nu(x_t,x^{**}_{t+1}(g_{t+1}(x_t,\varphi_t),\varphi);\theta^i_t,\varphi_t)$. This is the objective function that corresponds to the static case of coalitional equilibrium. Theorem 1 in Dotti (2015) states that, if condition 1-2-3 are satisfied, then a coalitional equilibrium exists in the form $(\mathbb{P}_t, A_t, x^*_t)$ (see Ch.1, Theorem 1). This implies that condition (i) of Definition 1 is also satisfied. Then a Markov-Perfect coalitional equilibrium exists. Results (ii) and (iii) in Theorem 3 follow directly from Theorem 1 in Dotti (2015) (see Dotti, 2015). Q.E.D.

Theorem 4. *(Monotone Comparative Statics). If conditions 1-2-3 are satisfied, then the set of equilibrium policies of the voting game is (i) a sublattice of X_t which is (ii) monotonic nondecreasing in θ^v_t.*

Proof. Theorem 3 implies that if conditions 1-2-3 are satified, then $(\mathbb{P}_t, A_t, x^*_t\{x^{**}_{t+s}\}^{\infty}_{s=0}; g_t)$ is a Markov-Perfect coalitional equilibrium and $(\mathbb{P}_t, A_t, x^*_t)$ is a coalitional equilibrium given the objective function $V(x_t;\theta^i_t,\varphi) = \nu(x_t,x^{**}_{t+1}(g_{t+1}(x_t,\varphi_t),\varphi);\theta^i_t,\varphi_t)$, in which x^{**}_{t+1} satisfy Rational Expectations and *MP*. Thus, the results in Theorem 2, in Dotti (2015) apply. Q.E.D.

Theorem 5. *(Monotone Comparative Statics 2). If conditions 1-2-3 are satisfied, then the set of equilibrium policies of the voting game is monotonic nondecreasing in φ.*

Proof. Theorem 3 implies that if conditions 1-2-3 are satified, then $(\mathbb{P}_t, A_t, x^*_t\{x^{**}_{t+s}\}^{\infty}_{s=0}; g_t)$ is a Markov-Perfect coalitional equilibrium and $(\mathbb{P}_t, A_t, x^*_t)$ is a coalitional equilibrium given the objective function $V(x_t;\theta^i_t,\varphi_t,g_t) = \nu(x_t,x^{**}_{t+1}(g_{t+1}(x_t,\varphi),\varphi);\theta^i_t,\varphi_t)$, in which x^{**}_{t+1} satisfy Rational Expectations and *MP*. Thus, the results in Theorem 3, in Dotti (2015) apply. Q.E.D..

A.1.4 Equilibrium Existence and Characterization

Lemma 6. *If $|\sigma^m_{t+s} - \sigma^n_{t+s}| \leq \hat{\sigma}$ for some $\hat{\sigma} > 0$ and all $s \geq 0$, then (i) a Markov-Perfect coalitional equilibrium for the voting game exists. Moreover, (ii) in any Markov-Perfect coalitional equilibrium at time t the equilibrium policy is the unique ideal point of the median voter $x^v_t = x^*_t \in I_t(v)$. (iii) The parameter θ^v_t that identifies the median voter is weakly decreasing in g_t. If $\sigma^m_t - \sigma^n_t$ is arbitrarily small, then (iv) there is a unique equilibrium policy that is chosen in any Markov-Perfect Coalitional Equilibrium in period t.*

Proof. (i) Consider expectations $\{x^{**}_{t+s}(g_t,\varphi_t)\}^{\infty}_{s=0}$ that are consistent with a MCE such that $x^{**}_{t+s}(g_{t+s},\varphi_{t+s})$ is unique and differentiable, and $\left|\frac{dx^{**}_{k,t+s}}{dx_{j,t+s}}\right| \leq c_{t+s}(k,j)$ for all k, j and all $s \geq 0$, in which $c_{t+s}(k,j)$ are numbers that are arbitrarily close to 0. I need to show that such expectations are rational for $\hat{\sigma}$ close enough to zero. Start with $x^{**}_t(g_t,\varphi_t)$. As stated, $x^{**}_{t+1}(g_t,\varphi_t)$ satisfies the Markov property, hence $x^{**}_{t+1}(g_{t+1}(x_t,\varphi_t),\varphi_{t+1}) = x^{**}_{t+1}(g_t,\varphi_t)$. Moreover, it is differentiable and consistent with a unique MCE. This means $x^{**}_{t+1}(g_{t+1},\varphi_{t+1}) = x^*_{t+1}(g_{t+1},\varphi_{t+1}) = x^v_{t+1}(g_{t+1},\varphi_{t+1})$, i.e. must be the unique ideal point of the median voter

v_{t+1}. Thus $\frac{dx^{**}_{k,t+1}}{dx_{j,t}} = \frac{\partial x^{**}_{k,t+1}}{\partial g_{t+1}} \frac{\partial g_{t+1}}{\partial x_{jt}} = \frac{\partial x^{**}_{k,t+1}}{\partial \theta^v_{t+1}} \frac{\partial \theta^v_{t+1}}{\partial x_{j,t}}$. Notice that $\frac{\partial \theta^v_{t+1}}{\partial x_{j,t}} = 0$ for all j except for the one such that $x_{j,t} = M_t$. In such case, $\frac{\partial \theta^v_{t+1}}{\partial M_t} = -\frac{l_t(\sigma^m_t - \sigma^n_t)}{[(\sigma^m_t - \sigma^n_t)M_t + \sigma^n_t]^2} \frac{1}{q(\theta^v_{t+1})}$, which is finite for all M_t and tends to 0 as $\sigma^m_t - \sigma^n_t \to 0$ (notice that if v_{t+1} is the median voter, then $q(\theta^v_{t+1}) > 0$, and continuity of q implies that his must be true in a neighborhood of θ^v_{t+1}). Moreover, $\frac{\partial x^{**}_{k,t+1}}{\partial \theta^v_{t+1}} = 0$ if k is in a corner solution of the maximization problem of the median voter, else $\frac{\partial x^{**}_{k,t+1}}{\partial \theta^v_{t+1}} = -\frac{V_{x_{k,t+1}\theta_{t+1}} + \sum_{j \neq k} V_{x_{k,t+1}x_{j,t+1}} V_{x_{j,t+1}\theta^v_{t+1}}}{V_{x_{k,t+1}x_{k,t+1}}}$. The numerator is finite (see A.1.2). About the denominator, it is finite if $\frac{dx^{**}_{k,t+2}}{dx_{j,t}}$ is finite for all k, j. But this is true because expectations are such that $\left| \frac{dx^{**}_{k,t+2}}{dx_{j,t+1}} \right| \leq c_{t+2}(k,j)$ for all k, j. One gets $\frac{dx^{**}_{k,t+1}}{dx_{j,t}}$ is the product of a finite factor times a factor that is continuous in $\sigma^m_t - \sigma^n_t$ and tends to zero as $\sigma^m_t - \sigma^n_t \to 0$. Hence, there exists $\hat{\sigma} > 0$ such that if $|\sigma^m_t - \sigma^n_t| \leq \hat{\sigma}$, then $\left| \frac{dx^{**}_{k,t+1}}{dx_{j,t}} \right| \leq c_{t+1}(k,j)$ for all k, j. Because $c_{t+1}(k,j)$ are arbitrarily close to zero, this implies that $V(x_t; \theta^i_t, \varphi_t, g_t)$ is strictly concave, thus $x^*_t(g_t, \varphi_t)$ is consistent with a MCE, unique and differentiable and $\frac{dx^*_{k,t}}{dx_{j,t}} \leq c_t(k,j)$ for all k, j. As rational expectations are assumed, then $x^{**}_t(g_t, \varphi)$ must also satisfy those properties. Similarly, one can show that x^{**}_{t+1} is consistent with MCE, unique, differentiable and satisfies $\frac{dx^{**}_{k,t+1}}{dx_{j,t+1}} \leq c_{t+1}(k,j)$ for all k, j given such expectations. Thus, recursively, one can show that this is true for all $x^{**}_{t+s}(g_{t+s}(x_{t+s-1}, \varphi_{t+s-1}), \varphi_{t+s})$ with $s \geq 0$ and, because of the Markov assumption, for $x^{**}_{t+s}(g_t, \varphi_t)$ for all $s \geq 0$. This means that $V(x_{t+s}, \theta^i_{t+s}, \varphi_{t+s}, g_{t+s})$ is continuous and strictly concave in x_{t+s} (it satisfies *SM* and *SSCP* because of Lemma 2) and that the expectations $\{x^{**}_{t+s}(g_t, \varphi_t)\}^{\infty}_{s=0}$ are rational and satisfy the Markov property. Summarizing, (i) Lemma 2 and the definitions of the policy space X_t and of the parameter space Θ_t, plus the result above imply that all the conditions for the existence of a coalitional equilibrium in Theorem 1 are satisfied. (ii) The strict concavity of the objective function of each working age individual and the convexity of X imply that the pivotal voter has a unique ideal policy, and therefore that is the only policy vector that can be implemented in any coalitional equilibrium of the voting game. (iii) If $g_t \leq 1$, then the median individual in the totally ordered set Θ_t solves $Q(\theta^v_t)n_t + l_{t-1}(m_{t-1} + n_{t-1}) = [1 - Q(\theta^v_t)]\, n_t$[19]. Rearranging and solving for θ^v_t one gets $\theta^v_t = Q^{-1}\left(\frac{1-g_t}{2}\right)$ which is weakly positive and weakly decreasing in g_t. If $g_t > 1$, then the parameter of the pivotal voter is fixed at $\theta^i_t = -1$. Lastly, for (iv), if $\sigma^m_t \to \sigma^n_t$ then $\frac{dx^{**}_{k,t+1}}{dx_{j,t}} \to 0$ for any rational expectations. Thus $V(x_t; \theta^i_t, \varphi_t, g_t)$ is strictly concave in x_t and in period t there is a unique policy vector x^*_t that is chosen in any MCE given g_t. Given that g_t is known at time t, then x^*_t must be unique. Q.E.D.

[19]The tie-breaking rule assumed in section 2.1.2 ensures that this formula is correct even if the number of voters is even.

A.2 Comparative statics

A.2.1 Unanticipated rise in the longevity of the retired population

Theorem 7. *The effects of an increase in the life expectancy l_{t-1} is weakly positive on the spending policy and ambiguous on the immigration policy. Moreover, there exists a threshold $\hat{g} \in [0,1]$ such that if $g_t \geq \hat{g}$ then the effect on immigration policy is unambiguously (weakly) negative and the effect on the tax rate is strictly positive.*

Proof. Calculate the cross derivatives of $V_t^{i,y}$ ($\widetilde{V}_t^{i,y}$) with respect to each policy dimension M_t, Y_t, e_t and the parameter l_{t-1}.

$$\widetilde{V}^{i,y}_{M_t l_{t-1}} = \frac{\theta_t^v (\alpha + \gamma)}{(1 - M_t^*)} > 0$$

$$\widetilde{V}^{i,y}_{Y_t l_{t-1}} = 0$$

$$\widetilde{V}^{i,y}_{e_t l_{t-1}} = 0$$

(i) Effects at fixed g_t. Consider a totally ordered subset $\Phi_t^j := \{\varphi_t \in \Phi_t | \varphi_{i,t} = \hat{\varphi}_{i,t} \forall i \neq j\}$ where j is the position of the longevity parameter in the vector φ_t, i.e. $\varphi_{j,t} = l_{t-1}$. Notice that $\widetilde{V}(x_t; \theta_t^i, \varphi_t, g_t)$ in Φ_t^j satisfies *SM* in (x_t) and *SSCP* in $(x_t; \varphi_t)$, it also satisfies *SM* in (z_t) and *SSCP* in $(z_t; \varphi_t)$ for $z_t = (x_{1t}, -x_{2t}, -x_{3t})$. Using Theorem 3, one gets $\triangle M_t \geq 0$, $\triangle Y_t = 0$, $\triangle e_t = 0$, $\triangle \tau_t \leq 0$. (ii) Recall that

$$g_t = \frac{l_{t-1}}{\bar{\sigma}_{t-1}}$$

which is increasing in l_{t-1}. Hence a rise in l_{t-1} corresponds to a change in the voter distribution such that the new median voter is lower than before. Hence $\triangle M_t \leq 0$, $\triangle Y_t \geq 0$, $\triangle e_t \geq 0$, $\triangle \tau_t \geq 0$. Total effect: ambiguous for M_t. But $\triangle e_t \geq 0$, $\triangle Y_t \geq 0$. Finally notice that if $g_t = 1$ then $\theta_t^v = 0$ and $\widetilde{V}^{i,y}_{M_t l_{t-1}} = 0$, which means that the "budget effect" is equal to zero and therefore the the political effect (weakly) dominates. Hence there exists a threshold $\hat{g} \in [0,1]$ (possibly $\hat{g} = 1$) such that if $g_t \geq \hat{g}$ then the effect on immigration policy is unambiguously (weakly) negative. Q.E.D.

A.2.2 Unanticipated fall in the natural growth rate of the native population

Theorem 8. *The effects of a decrease in the growth rate of the working age population is a weak decrease in the openness of the immigration policy and and a weak increase in spending in the imperfect Public Good and in the tax rate.*

Proof. Calculate the cross derivatives of $V_t^{i,y}$ ($\widetilde{V}_t^{i,y}$) with respect to each policy dimension M_t, Y_t, e_t and the parameter σ_{t-1}^n.

$$\widetilde{V}^{v,y}_{e_t \sigma^n_{t-1}} = \widetilde{V}^{v,y}_{Y_t \sigma^n_{t-1}} = \widetilde{V}^{v,y}_{M_t \sigma^n_{t-1}} = 0$$

Recall that, but the share of "old" voters decreases at each point in time:

$$g_t = \frac{l_{t-1}}{\bar{\sigma}_{t-1}}$$

which is decreasing in σ^n_{t-1}. Using Theorem 3, a fall in σ^n_{t-1} implies $\triangle M_t \leq 0$, $\triangle Y_t \geq 0$, $\triangle e_t \geq 0$, $\triangle \tau_t \geq 0$. Q.E.D.

A.2.3 Rise in the life expectancy of the working age population

Theorem 9. *The effects of an increase in the life expectancy l_t is ambiguous on the immigration policy. If voters are "naive" then the effect is weakly positive. If the birth rate of the native is the same as the one of the immigrants, then there is no effect.*

Proof. One needs to analyze the cross derivative of $\widetilde{V}^{i,y}_t$ with respect to M_t, Y_t, e_t and the parameter l_t. Define $\tilde{\pi}^{i,o}_{t+1} = (\alpha + \gamma\theta^i_t)\frac{\bar{\sigma}_t}{1-\widehat{M}_{t+1}} + d(Y_{t+1}) - c(M_{t+1})$ (this is only relevant for the case of endogenous public education).

$$\widetilde{V}^{i,y}_{M_t l_t} = \underbrace{\frac{\beta(\alpha + \gamma\theta^i_t)}{(1-\widehat{M}_{t+1})}(\sigma^m_t - \sigma^n_t)}_{preferences \quad effect} - \underbrace{\frac{\beta 2 l_t}{\bar{\sigma}_t{}^2}\left[\sum_{j=1}^{3} \frac{d\tilde{\pi}^{i,o}_{t+1}}{dx^{**}_{j,t+1}} \frac{\partial x^{**}_{j,t+1}}{\partial \theta^v_{t+1}}\right] \frac{d\theta^v_{t+1}}{dg_{t+1}}(\sigma^m_t - \sigma^n_t)}_{sophisticated \quad effect}$$

$$\underbrace{-\frac{\beta l_t^2}{\bar{\sigma}_t{}^2}\left[\sum_{j=1}^{3} \frac{d^2\tilde{V}^{i,o}_{t+1}}{dx^{**}_{j,t+1}} \frac{\partial x^{**}_{j,t+1}}{\partial \theta^v_{t+1}} \frac{dx^{**}_{j,t+1}}{dl_t} + \frac{d\tilde{\pi}^{i,o}_{t+1}}{dx^{**}_{j,t+1}} \frac{d}{dl_t}\left(\frac{\partial x^{2**}_{t+1}}{\partial \theta^v_{t+1}}\right)\right] \frac{d\theta^v_{t+1}}{dg_{t+1}}(\sigma^m_t - \sigma^n_t)}_{sophisticated \quad effect}$$

$$\widetilde{V}^{i,y}_{Y_t l_t} = 0$$

$$\widetilde{V}^{i,y}_{e_t l_t} = 0$$

First of all notice that if $\sigma^m_t = \sigma^n_t$, then the cross derivatives are equal to zero and g_{t+1} is unaffected by changes in l_t, therefore a shock on l_t has no effects on the equilibrium outcome.

If $\sigma_t^m \geq \sigma_t^n$ the sign of $\widetilde{V}^{i,y}_{M_t l_t}$ is ambiguous. The reason is that two different effects enter the formula. On one hand an increase in life expectancy increase the relative weight of consumption after retirement in the utility function of a working age individual, increasing the desirability of better future pensions and therefore of an increase in the number of immigrants at time t ("preferences effect"). On the other hand there is a "sophisticated effect" that concerns the effect of current political choices on future outcomes. If the "preferences" effect dominates, then using the same procedure as in C.5.1 I can show that $\widetilde{V}(x_t;\theta_t^i,\varphi_t,g_t)$ satisfies SM in (x_t) and $SSCP$ in $(x_t;\varphi_t)$ in Φ_t^j where $\varphi_{j,t}=l_t$, it also satisfies SM in (z_t) and $SSCP$ in $(z_t;\varphi_t)$, for $z_t=(x_{1t},-x_{2t},-x_{3t})$, which by Theorem 3 implies $\triangle M_t \geq 0$,$\triangle \tau_t \leq 0$ and no effect on the other variables. If the "sophisticated" effect dominates in a similar way one can show that $\triangle M_t \leq 0$, $\triangle \tau_t \geq 0$. If agents are "naive" then there is no "sophisticated effect" because $\frac{d\theta^v_{t+1}}{dg_{t+1}}=0$ and therefore an increase in l_t has a weakly positive effect on the openness of the immigration policy. Q.E.D.

A.2.4 Decrease in the birth rate of the natives

Theorem 10. *The effects of a decrease in the birth rate of the native population σ_t^n is a weak increase in the openness of the immigration policy and a fall in the tax rate. The effects of a decrease in the birth rate of the native population σ_t^n is ambiguous on the immigration policy. If voters are "naive", then the effect is weakly positive.*

Proof. Calculate the cross derivatives of $V_t^{i,y}$ ($\widetilde{V}_t^{i,y}$) with respect to each policy dimension M_t, Y_t, e_t and the parameter σ_t^n. $\tilde{v}^{i,o}_{t+1}$ is defined as in A.2.3.

$$\widetilde{V}^v_{M_t\sigma_t^n} = \underbrace{e_t\theta_t^v}_{b.e.} - \underbrace{\frac{\beta l_t(\alpha+\gamma\theta_t^v)}{(1-\widehat{M}_{t+1})}}_{preferences\quad effect} +$$

$$+\underbrace{\frac{\beta l_t}{\bar{\sigma}_t{}^2}\left[d'(Y^{**}_{t+1})\frac{\partial Y^{**}_{t+1}}{\partial\theta^v_{t+1}} - \hat{c}'(M^{**}_{t+1})\frac{\partial M^{**}_{t+1}}{\partial\theta^v_{t+1}}\right]\frac{d\theta^v_{t+1}}{dg_{t+1}}\left[1+\frac{2(1-M_t)(\sigma_t^m-\sigma_t^n)}{\bar{\sigma}_t}\right]}_{sophisticated\quad effect}+$$

$$+\underbrace{\frac{\beta l_t(\sigma_t^m-\sigma_t^n)}{\bar{\sigma}_t{}^2}\left\{\left[\sum_{j=1}^{3}\frac{d^2\tilde{\pi}^{i,o}_{t+1}}{dx^{**}_{j,t+1}}\left(\frac{\partial x^{**}_{j,t+1}}{\partial\theta^v_{t+1}}\right)^2\frac{dx^{**}_{j,t+1}}{d\sigma_t}+\frac{d\tilde{\pi}^{i,o}_{t+1}}{dx^{**}_{j,t+1}}\frac{\partial x^{2**}_{t+1}}{\partial(\theta^v_{t+1})^2}\right]\left(\frac{d\theta^v_{t+1}}{dg_{t+1}}\right)^2\right.}_{sophisticated\quad effect}+$$

$$+ \underbrace{\left[\sum_{j=1}^{3} \frac{d\tilde{\pi}_{t+1}^{i,o}}{dx_{j,t+1}^{**}} \frac{\partial x_{j,t+1}^{**}}{\partial \theta_{t+1}^{v}} \right] \frac{d^2 \theta_{t+1}^{v}}{d(g_{t+1})^2} \Bigg\}}_{sophisticated \quad effect}$$

Notice that in this case the effect of σ_t^n on future outcomes affect the pivotal voter. Also notice that $d'(Y_{t+1}^{**})\frac{\partial Y_{t+1}^{**}}{\partial \theta_{t+1}^{v}} - \hat{c}'(M_{t+1}^{**})\frac{\partial M_{t+1}^{**}}{\partial \theta_{t+1}^{v}} \leq 0$ because Theorem 11. Hence for $\sigma_t^m = \sigma_t^n$ the sophisticated effect is weakly positive hence the overall effect is ambiguous. If agents are naive then $\frac{d\theta_{t+1}^{v}}{dg_{t+1}} = 0$ and the overall sign is negative if and only if:

$$\frac{l_t p_{t+1}^{v}}{e_t} \geq \frac{\theta_t^{v}}{\beta}$$

i.e. the total transfer in pensions to the median voter at time $t+1$ is sufficiently large in comparison with his tax expenditure in education per pupil (notice that this is always true in the basic model with no public education).

$$\widetilde{V}_{Y_t \sigma_t}^{v} = 0$$

$$\widetilde{V}_{e_t \sigma_t^n}^{v} = -\theta_t^{v}(1 - M_t) + \delta f'(e_t)\bar{\omega}_{t+1} > 0$$

as long as $e_t > 0$ at the equilibrium (this condition is only relevant for the extended model). $\widetilde{V}(x_t; \theta_t^i, \varphi_t, g_t)$ satisfies *SM* in (z_t') and *SSCP* in $(z_t'; \varphi_t)$ in Φ^j $(\varphi_{j,t} = \sigma_t^n)$, where $z_t' = (-x_{1t}, -x_{2t}, -x_{3t})$, it also satisfies *SM* in (z_t'') and *SSCP* in $(z_t''; \varphi_t)$ where $z_t'' = (-x_{1t}, -x_{2t}, x_{3t})$. By Theorem 3 a fall in σ_t^n implies: $\triangle M_t \geq 0$, $\triangle Y_t = 0$, $\triangle e_t \leq 0$, $\triangle \tau_t \leq 0$. Q.E.D.

A.2.5 Steady-State Equilibrium

Lemma 13. *If there exists a Markov-Perfect Coalitional Equilibrium in each period $t+s$, for all $s \geq 0$, then (i) an equilibrium for the OLG model at time t exists . Moreover, if $\varphi_{t+s} = \varphi$ for all $s > 0$, then (ii) there is an equilibrium that always converges to a steady-state. Lastly, if $\sigma_t^m = \sigma_t^n = \sigma_t$, then (iii) the political equilibrium at time t is independent of the previous political choices and the economy converges immediately to the steady state after a shock.*

Proof. Fix the value of the parameters. (i) Notice that if a Markov-Perfect coalitional equilibrium exists in each period $t+s$ (Lemma 6), then an equilibrium of the OLG model also exists because it is simply a sequence of such temporary equilibria. Q.E.D. (ii) The equilibrium political choice at time t depends uniquely on the value of the state g_t. Notice that g_t depends on the parameters l_{t-1}, σ_t^m, σ_t^n and on the choice variable M_{t-1} but is is independent of anything else. This implies that the evolution of g depends uniquely on the evolution of M if l, σ^m, σ^n are constant over time. Notice that if $\sigma_t^m = \sigma_t^n = \sigma_t$ then the political equilibrium at time t is independent of the previous political choices because the state g_t is independent of history: $g_t = \frac{l_{t-1}}{\sigma_t} = g^*$, which implies in turn that the economy converges immediately to the steady state after a shock. Also notice that in this case the equilibrium is independent of the lagged value M_{t-1}, hence the steady-state is unique. If $\sigma_t^m > \sigma_t^n$ then this is no longer true and the convergence may take several periods. Finally notice that at constant parameters if $g_{t+s} = g_{t+s+1}$ for some $t+s$, then $g_{t+s+u} = g_{t+s}$ for all $s > 0$, i.e. $g_{t+s} = g_{t+s+1}$ is sufficient for a steady state. Suppose a steady state does not exists, i.e. $g_{t+s} \neq g_{t+s+1}$ for all $s \geq 0$. If $g_{t+1} > g_t$ $(<)$ then the pivotal voter $\theta_{t+1}^v \leq \theta_t^v$ $(\geq)$ which using Theorem 9 implies $M_{t+1}^* \leq M_t^*(\geq)$. This implies in turn that $g_{t+2} \geq g_{t+1}(\leq)$. If $g_{t+2} = g_{t+1}$ then we have reached a steady state. If instead $g_{t+2} > g_{t+1}(<)$ the process continues recursively. There are three possibilities. Either (1) the process stops because $g_{t+s} = g_{t+s+1}$ and a steady state is achieved, or (2) the process converges to some g^{ss}. Else, (3) suppose that $g_{t+s+1} - g_{t+s} > 0$ $(<)$ for all $s \geq 0$. if this is true, then the process implies $M_{t+s+1}^* < M_{t+s}^*(>)$ for all $s \geq 0$. Because the direction of this iterative process is monotonic (increasing or decreasing), if it does not converge to some M^{ss}, then it this implies that if M is unbounded it will diverge to $-\infty$ $(+\infty)$. But $M_t \in [\underline{M}, \overline{M}]$ by assumption, hence the process must stop at $M_{t+s}^* = \underline{M}$ $(\overline{M})$ for some $s \geq 0$. Notice that monotonicity under case (3) implies $g_{t+s+1} - g_{t+s} > 0$ $(<)$ and therefore $M_{t+s+1}^* < M_{t+s}^*(>)$, but this is impossible because $M_{t+s}^* = \underline{M}$ $(\overline{M})$. Hence, $M_{t+s+1}^* = \underline{M}$ $(\overline{M})$, which means $M_{t+s+1}^* = M_{t+s}^*$ and implies $g_{t+s+1} = g_{t+s}$. Hence the system has achieved a steady state, and this leads to a contradiction. Q.E.D. (iii) Straightforward from (ii) and Lemma 6. Q.E.D.

Theorem 7b. *The long-run effect of an increase in l_{t-1} on the immigration policy has same sign as the short-run effect and a weakly larger magnitude. If $g_t \geq \hat{g}$ then the effect on immigration policy is (weakly) negative and the effect on the public spending and the tax rate is strictly positive.*

Proof. If at time t the "Budget Effect" prevails, i.e. $M_t \geq M_{t-1}$, then $g_{t+1} \leq g_t$ and $\theta_{t+1}^v \geq \theta_t^v$ by Lemma 6. Using Theorem 2 one gets $M_{t+1} \geq M_t$ and $Y_{t+1} \leq Y_t$. Notice that this is implies recursively $\theta_{t+s+1}^v \geq \theta_{t+s}^v$ and therefore $M_{t+s+1} \geq M_{t+s}$ and $Y_{t+s+1} \leq Y_{t+s}$ for all $s > 0$. Hence I can conclude that at the new steady state $M^{ss} \geq M_t \geq M_{t-1}$ and $Y^{ss} \leq Y_t$ but $Y^{ss} \gtreqless Y_{t-1}$, which means that the long run effect of an increase in l_{t-1} is positive on the openness of the immigration policy and ambiguous on the public spending variable, which increases at the time in which the shock occurs and falls in the following periods. Similarly one can show that if at time t the "Political Effect" dominates, then at the new steady state $M^{ss} \leq M_t \leq M_{t-1}$ and $Y^{ss} \geq Y_t \geq Y_{t-1}$.

Theorem 8b. *The long-run effect of a decrease in the growth rate of the native population is a weak decrease in the openness of the immigration policy and a weak increase in spending in the imperfect Public Good and in the tax rate.All the effects have weakly larger magnitude relative to the short-run effects.*

Proof. Similar to the previous case.

8 B Proofs: Extensions and Welfare Analysis

Appendix B includes the proof to the results regarding extensions in section 4 of the paper and of the Welfare results in section 5. Moreover, it provides a formal description of the setup in the case of "Elderly goods" informally described in section 4.4.

B.1 Partially funded pension system

Theorem 16. *The effect of a marginal decrease in the size of the public pension system in the short run is an increase in the restrictiveness of the immigration policy. In the long run, the effect is an increase in restrictions to immigration and an increase in public spending in the imperfect Public Good. The total effect on the tax rate is ambiguous.*

Proof. It is sufficient to show that the objective function $V_t^{i,y}$ ($\widetilde{V_t}^{i,y}$) satisfies the *SCP* in α (γ). Calculate the cross derivatives of $V_t^{i,y}$ ($\widetilde{V_t}^{i,y}$) with respect to M_t,Y_t (e_t) and the parameter α (γ).

$$\widetilde{V}^{i,y}_{M_t\alpha} = \frac{\theta_t^i l_{t-1}}{(1-M_t^*)} + \frac{\beta l_t(\sigma_t^m - \sigma_t^n)}{(1-M_{t+1}^*)} \geq 0$$

and $\widetilde{V}^{i,y}_{Y_t\alpha} = 0$,$\widetilde{V}^{i,y}_{e_t\alpha} = 0$. Hence given a subset Φ_t^j defined as in C.5.1 with $\varphi_{j,t} = \alpha$ (γ), one can show that $\widetilde{V_t}^i$ satisfies the *SCP* with respect to $(x_t;\varphi_t)$ and to $(z_t;\varphi_t)$ with $z_t = (x_{1t,} - x_{2t} - x_{3t})$. Using Theorem 3 this implies that the short-run effect of a fall in α (γ) is $\triangle M_t \leq 0$. In the long run the effect of a weak fall in M_t is a rise in g_t, which implies in turn a "political effect" at time $t+1$ with $\triangle M_t \leq 0 \triangle Y_t \geq 0$, which implies recursively the same effect for all the periods after $t+1$ until the economy converges to a new steady state. Notice that the effect on the tax rate is ambiguous at time t because of a simultaneous reduction of the total cost of pension (as α falls) and of the workforce (because of the fall in M_t), while from time $t+1$ the tax rate increases until a new steady state is achieved, because of the fall in the workforce and the rise in public spending. Therefore the overall long-run effect is ambiguous. Q.E.D.

Theorem 18. *The effects of an increase in the longevity of the retired population l_{t-1} and /or of a decrease in the growth rate of native population σ_{t-1}^n is a weak increase in the public spending in education per child e_t.*

Proof. Straightforward from A.2.1 and A.2.2.

B.2 Services for the elderly ("elderly goods")

Suppose that the elderly consume a different private good, for instance home care, denoted by O_t while the young consume the private good C_t. The good O_t is produced with the same technology as the consumption good C_t and the imperfect public good Y_t, but only the immigrant workers are capable of producing it. For simplicity I assume that there is no cost of immigration, i.e. $\lambda_t = 0$, that the default immigration is $\widehat{M_t} = 0$ and I analyse the case in which $\sigma_t^m - \sigma_t^n$ is arbitrarily small. Also assume that the functions $a(Y)$ and $d(Y)$ are such that $-\frac{a''}{a'}Y \geq 1$ and $-\frac{d''}{d'}Y \geq 1$ for all Y in the policy space. There are two possibilities. If at the equilibrium there are enough immigrant workers, then the segmentation of the labour market is irrelevant and the results are identical to the baseline model. The perfect substitutability in production and the perfect competition ensure that all prices are are unaffected by immigration choices. The implications change dramatically if in the proximity of an equilibrium there are not enough immigrant workers to satisfy the demand at the constant price. In detail, the total demand of services for the elderly is given by:

$$O_t^{TD} = \frac{\bar{p}_{t-1} l_{t-1} n_t}{P_t^o} = \frac{(\alpha+\gamma) l_{t-1} n_t}{P_t^o}$$

Suppose that all the immigrants endogenously select themselves into the sector that produces O_t (this is the case if wages are higher in this sector), then the total supply is given by: $O_t^{TS} = \xi m_t \bar{\epsilon}_t$, and the equilibrium price of the elderly good P_t^O is $P_t^O = \frac{(\alpha+\gamma) l_{t-1} n_t}{\xi m_t \bar{\epsilon}_t}$. The zero profit condition implies that the total revenue in the elderly good sector must be equal to the total cost, thus one gets a different wage w_t^O in this sector, namely $w_t^O = \frac{(\alpha+\gamma) l_{t-1} n_t}{m_t \bar{\epsilon}_t}$, such that the total nominal income of the workers in the elderly good sector is $w_t^O \bar{\epsilon}_t m_t = (\alpha+\gamma) l_{t-1} n_t$. Notice that the perfect substitutability in production between the consumption good and the imperfect public good, together with the zero profit condition still imply $P_t^C = P_t^Y = P_t$ (else only one of the two would be produced and the result would still hold). Hence, in order to solves for the wage of the native workers, we can use the total demand of consumption and imperfect public good. Using the budget constraint one can show that $(C_t + Y_t)^{TD} = \frac{w_t^C \bar{\epsilon}_t n_t}{P_t}$. Because the total supply is $\xi n_t \bar{\epsilon}_t$ one can solve for the price $P_t = w_t^C / \xi$. The zero profit condition for the production of the consumption good holds for all prices P_t, namely $P_t \xi n_t \bar{\epsilon}_t - w_t^C \bar{\epsilon}_t n_t = 0$. Hence I can normalize $P_t = 1$ (this means that good C is the *numéraire*) and I get the wage $w_t^C = \xi$. A competitive equilibrium of this kind exists only if $w_t^O \geq w_t^C$. In this problem this condition is equivalent to: $\bar{p}_t \left(\frac{1-M_t}{M_t}\right) \geq \xi_t \bar{\epsilon}_t$. Notice that as long as positive pensions are paid, one can always find M_t small enough that such inequality is satisfied. I can now state the formulas for the consumption of young and old individuals.

$$C_t^{i,y} = (1-\tau_t)\xi \epsilon_t^i$$

and

$$C_t^{i,o} = \frac{[\alpha + \gamma(\epsilon_{t-1}^i / \bar{\epsilon}_{t-1})]\bar{\sigma}_{t-1}\xi}{(\alpha+\gamma) l_{t-1}} \frac{M_t}{1-M_t}$$

Finally notice that the government budget constraint is now different because the immigrants have different wages relative to the natives. In order to keep the problem tractable it is useful to define a new variable $\widetilde{Y}_t = \frac{Y_t}{(1-M_t)}$. In detail:

$$\tau_t = \frac{\widetilde{Y}_t}{[\xi\bar{\epsilon}_t + (\alpha+\gamma)l_{t-1}]} + \frac{(\alpha+\gamma)l_{t-1}}{\xi\bar{\epsilon}_t + (\alpha+\gamma)l_{t-1}}$$

The objective function of a young individual becomes:

$$V_t^{i,y} = (1-\tau_t)\xi\epsilon_t^i + a[\widetilde{Y}_t(1-M_t)] - c(M_t) + \beta l_t \left\{ \frac{[\alpha+\gamma(\theta_t^{i,y})]\bar{\sigma}_t\xi}{(\alpha+\gamma)} \frac{M_{t+1}^{**}}{1-M_{t+1}^{**}} + d[Y_{t+1}^{**}] - \hat{c}(M_{t+1}^{**}) \right\}$$

where $\theta_t^{i,y} = y_t^i/\bar{y}_t$. Notice that the assumption of $\sigma_t^m - \sigma_t^n$ arbitrarily small implies that M_{t+1}^{**} is unaffected by current policy choices. Thus, the first derivatives are:

$$V_{M_t}^{i,y} = -a'[\widetilde{Y}_t(1-M_t)]\widetilde{Y}_t - c'(M_t) < 0$$

$$V_{\widetilde{Y}_t}^{i,y} = a'[\widetilde{Y}_t(1-M_t)](1-M_t) - \frac{\xi\theta_t^{i,y}}{\xi + (\alpha+\gamma)l_{t-1}/\bar{\epsilon}_t}$$

Regarding the elderly, they have an objective function in the form:

$$V_t^{i,o} = \frac{[\alpha+\gamma(\epsilon_{t-1}^i/\bar{\epsilon}_{t-1})]\bar{\sigma}_{t-1}\xi}{(\alpha+\gamma)l_{t-1}} \frac{M_t}{1-M_t} - \hat{c}(M_t) + d[\widetilde{Y}_t(1-M_t)]$$

Notice that

$$V_{M_t}^{i,o} = \frac{[\alpha+\gamma(\epsilon_{t-1}^i/\bar{\epsilon}_{t-1})]\bar{\sigma}_{t-1}\xi}{(\alpha+\gamma)l_{t-1}(1-M_t)^2} - \hat{c}'(M_t) - d'[\widetilde{Y}_t(1-M_t)]\widetilde{Y}_t$$

and

$$V_{\widetilde{Y}_t}^{i,o} = d'[\widetilde{Y}_t(1-M_t)](1-M_t) > 0$$

One can notice that in this case the young individuals are more hostile to immigration and to public spending than the elderly. Using the same method presented in the paper, one can define a common objective function $V_t^i = V(x_t; \theta_t^i, \varphi_t, g_t)$ by setting $\theta_t^i = \theta_t^{i,y} = \epsilon_t^i/\bar{\epsilon}_t$ for the young individuals and $\theta_t^i = -\epsilon_{t-1}^i/\bar{\epsilon}_{t-1}$ for the elderly. Moreover I apply the increasing transformation $V_t^i = (1+\theta_t^i)V_t^{i,y}$ for all young individuals and $(\kappa - \theta_t^i)V_t^{i,o}$ with κ arbitrarily large (these transformation do not affect the preferences). Define $z_t = (-M_t, -\widetilde{Y}_t)$. I can show the following results.

Lemma 23. *(i) If l_{t-1} is small enough, the function V_t^i satisfies SM and SSCP in $(z_t; \theta_t^i)$. Therefore (ii) a coalitional equilibrium exists.*

Proof. (i) It is easy to show that $V_{M_t\theta_t^i}^{i,o} < 0$ and $V_{\widetilde{Y}_t\theta_t^i}^{i,y} > 0$ for all M_t, Y_t, θ_t^i. Because $V_{M_t}^{i,y} < 0$ and $V_{\widetilde{Y}_t}^{i,o} > 0$ for all M_t, Y_t, θ_t^i, then the *SSCP* is satisfied within the young and within the elderly respectively. Lastly, one need to show that $V(z_t'; \theta_t^i, \varphi_t, g_t) - V(z_t''; \theta_t^i, \varphi_t, g_t) > V(z_t'; \theta_t^j, \varphi_t, g_t) - V(z_t''; \theta_t^j, \varphi_t, g_t)$ for all $z_t' \geq z_t''$ and $z_t' \neq z_t''$and whenever i is a young individual and j is an elderly. Notice that for l_{t-1} arbitrarily small $V^{i,o}_{M_t} > 0$ for all M_t, Y_t, θ^i_t. Hence $V(z'_t; \theta^j_t, \varphi_t, g_t) - V(z''_t; \theta^j_t, \varphi_t, g_t)$ is strictly negative and because κ is arbitrarily large, the condition is satisified for all M_t, Y_t, θ^i_t. (ii) Straightforward from Theorem 3.

Theorem 20. *If $g_t \leq 1$ then at the equilibrium, if it exists, the immigration policy is $M_t = 0$, else a positive level of immigration is possible.*

Proof. If $g_t \leq 1$ and an equilibrium exists, then the pivotal voter is a young individual with $V^{i,y}_{M_t} < 0$. Hence her ideal policy is $M_t = 0$.

Further details and additional results for this extension are provided in the supplementary online material.

B.3 Welfare Analysis: Immigration Policy

Theorem 21. *For any Social Welfare Function $SWF(x_t; \varphi_t, g_t)$ that assigns a strictly positive weight to each native individual of working age, there exist a threshold $\breve{g}_t \in [0, 1]$ such that if $g_t \geq \breve{g}_t$ then a marginal tightening in the immigration policy caused by a change in the equilibrium outcome reduces the Social Welfare.*

Proof. Notice that the theorem above is stated for the baseline model without endogenous education. Here I show the proof for the full model with SWF denoted by $SWF(x_t; \varphi_t, g_t) = \widetilde{SWF}(M_t, Y_t, e_t; \varphi_t, g_t)$ for $x_t = (M_t, -Y_t, -e_t)$. The proof of the baseline model is straightforward. Define the overall weight of each generation as follows:

$$\int_0^{\bar{\theta}_t} \mu^y_t(\theta^i_t) q_t(\theta^i_t) d\theta_t{}^i = \mu^y$$

$$\int_0^{\bar{\theta}_{t-1}} \mu^o_t(\theta^i_{t-1}) q_{t-1}(\theta^i_{t-1}) d\theta_{t-1}{}^i = \mu^o$$

$$\int_0^{\bar{\theta}_{t+1}} \mu^y_{t+1}(\theta^i_{t+1}) q_{t+1}(\theta^i_{t+1}) d\theta_{t+1}{}^i = \mu^c$$

Normalize $\mu^y = 1$ and assume $\mu^y + \mu^o + \mu^c = \mu$ with $0 < \mu < \infty$. This can be done without loss of generality under the assumption that $\mu^y_t(\theta^i_t) > 0$ for each native individual of working age. Suppose the equilibrium policy x^*_t is such that $\underline{M}_t < M_t < \overline{M}_t$, which implies that a marginal opening in the immigration policy is feasible. If the difference between the marginal social benefit for the society from an increase in M_t and the marginal utility of M_t for the pivotal voter evaluated at the equilibrium policy vector is strictly positive, i.e.

$$\widetilde{WD}_{M_t}(M_t^*, Y_t^*, e_t^*; \varphi_t, g_t) = \widetilde{SWF}_{M_t}(M_t^*, Y_t^*, e_t^*; \varphi_t, g_t) - V_{M_t}^{v,y}(M_t^*, Y_t^*, e_t^*; \varphi_t, g_t) > 0$$

then a marginal increase in the openness of the immigration policy M_t is, ceteris paribus, beneficial for the society. Notice that if $\underline{M}_t < M_t < \overline{M}_t$, then $V_{M_t}^{v,y}(M_t^*, Y_t^*, e_t^*; \theta_t^v, \varphi_t, g_t) = 0$ from the F.O.C. The social benefit for the society from an increase in M_t is given by:

$$\widetilde{SWF}_{M_t} = \int_0^{\bar{\theta}_t} \mu_t^y(\theta_t^i)\tilde{V}_{M_t}^y(M_t^*, Y_t^*, e_t^*; \theta_t^i, \varphi_t, g_t)q_t(\theta_t^i)d\theta_t^i + \int_0^{\bar{\theta}_{t-1}} \mu_t^o(\theta_{t-1}^i)\tilde{V}_{M_t}^o(e_t, M_t, Y_t; \theta_t^i, \varphi_t, g_t)q_{t-1}(\theta_{t-1}^i)d\theta_{t-1}^i +$$

$$+ \int_0^{\bar{\theta}_{t+1}} \mu_{t+1}^y(\theta_t^i)E[\tilde{V}_{M_t}^y(M_{t+1}, Y_{t+1}, e_{t+1}; \theta_{t+1}^i, \varphi_{t+1}, g_t)]q_{t+1}(\theta_{t+1}^i)d\theta_{t+1}^i =$$

First of all notice that the linearity in consumption of the utility function implies $E[V_{M_t}^y(M_{t+1}, Y_{t+1}, e_{t+1}, ; \theta_{t+1}^i, \varphi_{t+1}, g_{t+1})] = V_{M_t}^y(M_{t+1}, Y_{t+1}, e_{t+1}; \bar{\theta}_{t+1}, \varphi_{t+1}, g_{t+1})]$ hence $\int_0^{\bar{\theta}_{t+1}} \mu_{t+1}^y(\theta_{t+1}^i)E[V_{M_t}^y(M_{t+1}, Y_{t+1}, e_{t+1}; \theta_{t+1}^i, \varphi_{t+1}, g_{t+1})]q_{t+1}(\theta_{t+1}^i)d\theta_{t+1}^i =$ $E[\mu_{t+1}^y(\theta_{t+1}^i)]V_{M_t}^y(M_{t+1}, Y_{t+1}, e_{t+1}; \bar{\theta}_{t+1}, \varphi_{t+1}, g_{t+1})$. Moreover, notice that a change in x_t only affects the future generation through a fall in g_{t+1}, which has no effects neither on the budget constraint at time $t+1$ nor on the preferences of an individual (it only affects the political equilibrium at time $t+1$). Therefore $V_{M_t}^y(M_{t+1}, Y_{t+1}, e_{t+1}, ; \theta_{t+1}, \varphi_{t+1}, g_{t+1})$ is independent of M_t and therefore *SSCP* implies: $V_{M_t}^y(M_{t+1}, Y_t, e_{t+1}; \bar{\theta}_{t+1}, \varphi_{t+1}, g_{t+1}) \geq$ $V_{M_t}^y(M_{t+1}, Y_{t+1}, e_{t+1}; \theta_{t+1}^v, \varphi_{t+1}, g_{t+1})$ as long as $\theta_{t+1}^v \leq \bar{\theta}_{t+1}$. I use the latter result and I substitute the formulas for $V_{M_t}^{i,y}$, $V_{M_t}^{i,o}$ into $\widetilde{WD}_{M_t}$, and I can write the following inequality:

$$\widetilde{WD}_{M_t} \geq \left[(\alpha+\gamma)\frac{l_{t-1}}{(1-M_t^*)} - \lambda_t + \left(\frac{\beta l_t \gamma}{(1-M_{t+1}^*)} - e_t\right)(\sigma_t^m - \sigma_t^n)\right]\left(\int_0^{\bar{\theta}_t} \theta_t^i \mu_t^y(\theta_t^i)q_t(\theta^i)d\theta_t{}^i - \theta_t^v\right) +$$

$$-c'(M_t)\int_0^{\bar{\theta}_{t-1}} \mu_t^o(\theta_{t-1}^i)g(\theta_{t-1}^i)d\theta_{t-1}^i =$$

Notice that:

$$V_{M_t}^{v,y} = -c'(M_t) + \theta_t^v(\alpha+\gamma)\frac{l_{t-1}}{(1-\widehat{M}_t)} - \theta_t^v\lambda_t + \left(\frac{\beta l_t}{(1-\widehat{M}_{t+1})}(\alpha/\theta_t^v + \gamma) - e_t\right)(\sigma_t^m - \sigma_t^n)\theta_t^v$$

also represent the FOC of the optimization problem of the pivotal individual. This implies that if at the equilibrium $\underline{M}_t < M_t$ then:

$$(\alpha+\gamma)\frac{l_{t-1}}{(1-\widehat{M}_t)} - \lambda_t + \left(\frac{\beta l_t \gamma}{(1-\widehat{M}_{t+1})} - e_t\right)(\sigma_t^m - \sigma_t^n) \geq \frac{1}{\theta_t^v}\left(c'(M_t) - \frac{\alpha\beta l_t(\sigma_t^m - \sigma_t^n)}{(1-\widehat{M}_{t+1})}\right)$$

Define the weighted average

$$E_{g_t}(\mu_t^y \theta^i) = \int_0^{\bar{\theta}_t} \lambda_t^y(\theta_t^i)\theta_t^i g_t(\theta_t^i) d\theta_t^i = h_{g_t} \int_0^{\bar{\theta}_t} \theta_t^i \dot{g}_t(\theta_t^i) d\theta_t^i = h_{g_t} E_{\dot{g}_t}(\theta_t)$$

for some p.d.f $\dot{g}_t$. Notice that $h_{g_t} E_{\dot{g}_t}(\theta_t) > 0$ under the assumption that $\mu_t^y(\theta_t^i) > 0$ for each native individual of working age. Therefore we can state the following inequality:

$$\widetilde{WD}_{M_t} \geq \left(c'(M_t) - \frac{\alpha\beta l_t(\sigma_t^m - \sigma_t^n)}{(1-\widehat{M}_{t+1})} \right) \frac{h_{g_t} E_{\dot{g}_t}(\theta_t) - \theta_t^v}{\theta_t^v} - c'(M_t)\mu^o$$

The F.O.C.s of the pivotal individual plus the assumption that immigrants are not net beneficiaries (in expectation) of the fiscal system imply $c'(M_t) - \frac{\alpha\beta l_t(\sigma_t^m - \sigma_t^n)}{(1-\widehat{M}_{t+1})} > 0$ for $\underline{M}_t < M_t < \overline{M}_t$. Finally notice that because of a previous assumption $c'(M_t) < \infty$ and that $\mu^o < 0$ imply:

$$\lim_{\theta_t^v \to 0^+} \left(c'(M_t) - \frac{\alpha\beta l_t(\sigma_t^m - \sigma_t^n)}{(1-\widehat{M}_{t+1})} \right) \frac{h_{g_t} E_{\breve{g}_t}(\theta) - \theta_t^v}{\theta_t^v} - c'(M_t)\mu^o = +\infty$$

Therefore, given a certain distribution of weights, either $\widetilde{WD}_{M_t}(M_t^*, Y_t^*, e_t^*; \varphi_t, g_t) > 0$ for all $\theta_t^v > 0$, else the Intermediate Value Theorem implies the existence of a threshold $0 < \breve{\theta}_t < \bar{\theta}$ such that $\widetilde{WD}_{M_t}(M_t^*, Y_t^*, e_t^*; \varphi_t, g_t) = 0$. This threshold is always meaningful because I have previously assumed that the distribution of θ_t is such that $q(0) > 0$ and therefore $\theta_t^j = 0$. Moreover, $\widetilde{WD}_{M_t}(M_t^*, Y_t^*, e_t^*; \varphi_t, g_t)$ is strictly decreasing in θ_t^v because $\widetilde{WD}_{M_t}$ is independent of θ_t^v and $V_{M_t}^{v,y}$ is strictly decreasing in θ_t^v because of the *SSCP*. Therefore if the wage distribution is such that $\theta_t^v < \breve{\theta}_t$ then $\widetilde{WD}_{M_t}(M_t^*, Y_t^*, e_t^*; \varphi_t, g_t) > 0$ which implies that it would be welfare improving to increase M_t. Lastly, because of Lemma 6 (iii), a threshold $\breve{g}_t \in [0,1]$ exists, such that if $g_t \geq \breve{g}_t$ iff $\theta_t^v < \breve{\theta}_t$, which implies the result stated. Q.E.D.

B.4 Alternative assumption about the default policy: Status quo

One may want to assume that the default platform is the policy implemented in the previous period (if feasible) In such case, $x_t^0 = x_{t-1}^*$. Following the same steps described in the proofs to Theorem 1 in Dotti (2015) one can show that there is no equilibrium in which a platform $x_t \in X_t$ such that $x_t \in M(v_t)$ is implemented. Neverthless, given that the default policy under this alternative assumption may not be the least preferred option for some players, then there may additional possible outcomes. Specifically, there may be (i) equilibria in which no coalition is active and the default policy is implemented and (ii) situations of instability, in which some coalitions are active only in order to prevent the victory of some other candidate. This may be possible because of the assumption that, if no Condorcet Winner exists in the final stage of the voting game, then x^0 is implemented. Suppose that is the case. The characterizations of all the equilibria given in Theorem 3 is no longer valid. Neverthless, the compartative statics results in Theorem 4-5 still apply. The reason is that in both cases (i) and (ii) the default option is implemented, i.e. $x_t^* = x^0$. Given that the default option is assumed to be the status quo, i.e. $x_t^0 = x_{t-1}^*$, this implies that the comparative statics is null, i.e. $x_t^* = x_{t-1}^*$ and therefore the outcome is nondecreasing in θ_t^v and φ_t as stated in the Theorems 4-5.

B.5 High tax rate

In section 3 we have restricted the policy space in such a way that for all $x_t \in X$ the tax rate is internal $0 < \tau_t < k < 1$. Suppose that this assumption fails and at an equilibrium $\tau_t = k$. In this case it is not straightforward to derive results in the full model. Nevetheless, some results can be obtained in the baseline model with $x_t = (M_t, -Y_t)$ under the assumption that $d'(Y_t) \leq b'(Y_t)$ for all $Y_t \in [0, \overline{Y}]$ and $\hat{c}'(M_t) \geq c'(M_t)$ for all $M_t \in [0, \overline{M}]$. If $\tau_t = k$ the policy space is unidimensional, thus the traditional Median Voter Theorem applies if voter preferences satisfy the *Spence-Mirrlees* condition. Consider the slope of the indifference curve of an working age individual i:

$$MRS_{M_t,Y_t}^{i,y} = -\frac{\left(\frac{\beta l_t}{(1-\widehat{M}_{t+1})}(\alpha/\theta_t^i + \gamma)\right)(\sigma_t^m - \sigma_t^n)\theta_t^i - c'(M_t)}{b'(Y_t)}$$

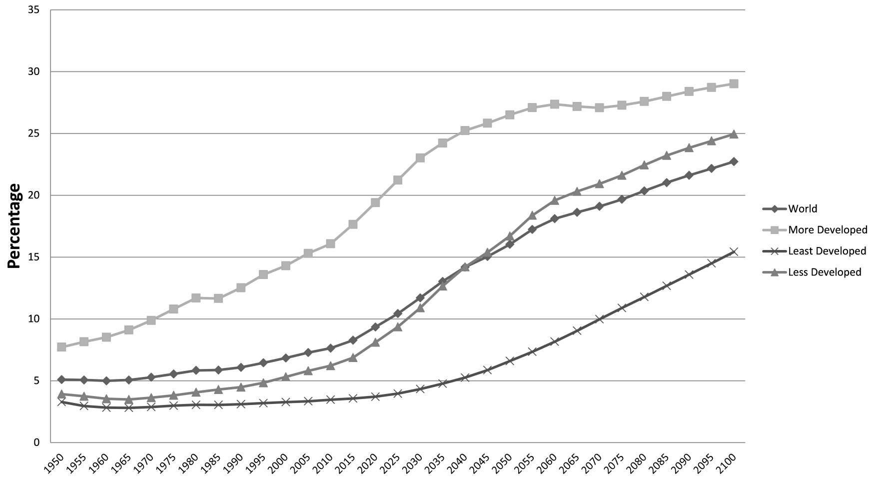
and its derivative with respect to θ_t^i:

$$\frac{\partial MRS_{M_t,Y_t}^{i,y}}{\partial \theta_t^i} = -\frac{\frac{\beta l_t \gamma}{(1-\widehat{M}_{t+1})}(\sigma_t^m - \sigma_t^n)}{b'(Y_t)} \leq 0$$

Moreover, notice that the MRS of any retired individual is given by $MRS_{M_t,Y_t}^{o} = \frac{\hat{c}'(M_t)}{d'(Y_t)}$, which implies that $MRS_{M_t,Y_t}^{o} \geq MRS_{M_t,Y_t}^{i,y}$ for all i. Thus, preferences satisfy the *Spence-Mirrlees* condition, and standard results can be applied to make predictions about the effects of changes in the pivotal voter on the equilibrium outcome. The results differ from the ones of most Benefit Adjustment Models. Specifically, an increase in the relative share of the elderly implies, *ceteris paribus*, an fall in public spending and a reduction of the immigration quota. In this framework I cannot derive analytical results about the effects of a rise in life expectancy, because this kind of shock typically involves not only a change in the pivotal voter but also in the position and slope of the budget constraint, such that the sign of the overall effect cannot be determined using the *Spence-Mirrlees* condition only.

Figures

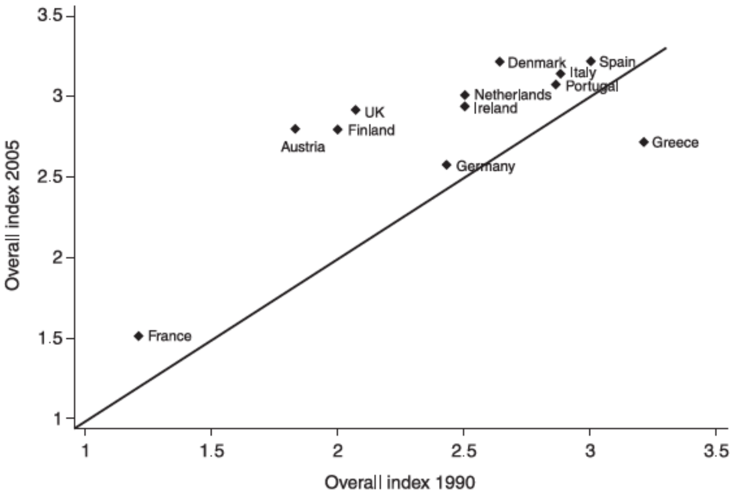
Figure 1: Share of Population of Age 65 or Older



Source: United Nations, 2015

Evolution of the share of population of age above 65 from 1950 to 2015 and the forecast for the next decades (source: United Nations, 2015).

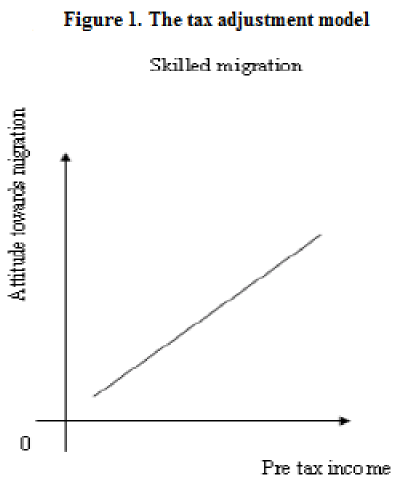
Figure 2: Trends in Migration Policies

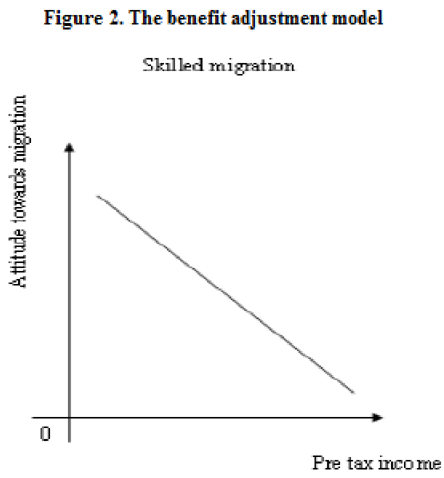


Source: Boeri and Brucker, 2005

Comparison of the value of the index of tightness of immigration policies proposed by Boeri and Brucker (2005) in 1990 and 2005 for 12 European countries.

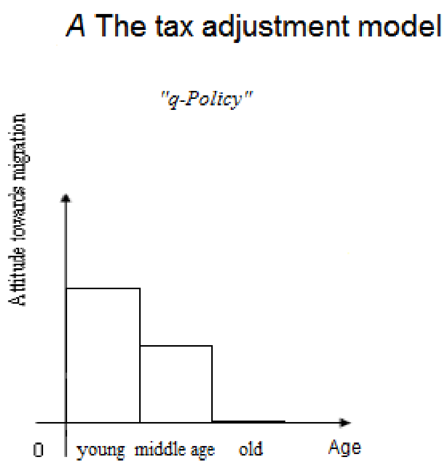
Figure 3-4: Effects of income on the attitudes towards immigration

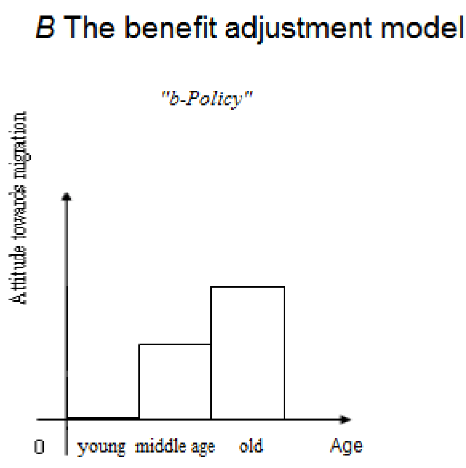




Relationship between income and attitude towards immigration (preferred number of immigrants) in a Tax Adjustment Model (Fig. 1) and in a Benefit Adjustment Model (Fig. 2). Based on Facchini and Mayda (2008).

Figure 5-6: Effects of age on the attitudes towards immigration





Attitude towards immigration (preferred number of immigrants) of different generations of voters in a Tax Adjustment Model (Fig. 3) and in a Benefit Adjustment Model (Fig. 4). Based on Haupt and Peters (1998).

Figure 9-10: Long-Run Effects of an Increase in Life Expectancy

Parameters: $\sigma^n = 1$, $\sigma^m = 1.5$, before shock $l = 0.6$, after shock: $l = 0.62$.

Figure 9

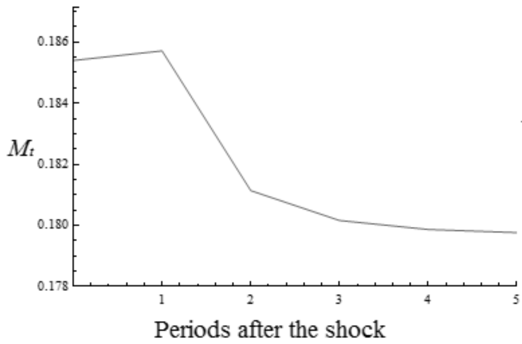
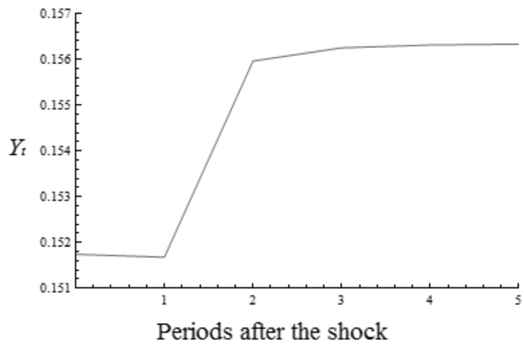


Figure 10



Effects of a positive shock on the life expectancy of the elderly on the immigration quota M_t (Fig. 9) and on public spending per worker Y_t (Fig. 10).

Figure 11-12: Long-Run Effects of a Decrease in the Birth Rate of the Natives

Parameters: $\sigma^n = 1.2$, $\sigma^m = 1.5$,$l = 0.6$, after shock: $\sigma^n = 1$.

Figure 11

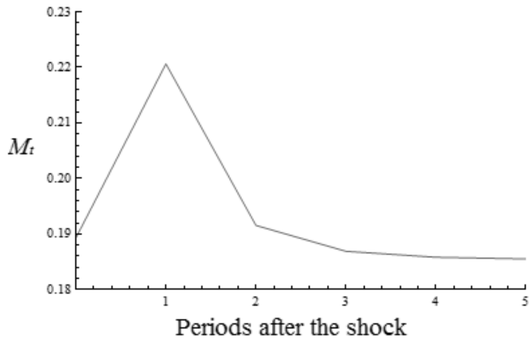
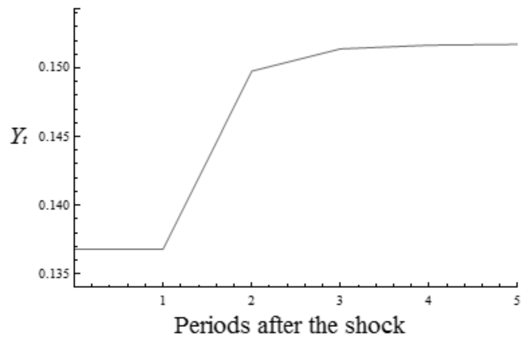


Figure 12



Effects of a negative shock on the birth rate of the native population on the immigration quota M_t (Fig. 11) and on public spending per worker Y_t (Fig. 12).

Figure 13-14: "Naive" vs. "Sophisticated" agents

Parameters: $\sigma^n = 1$, $\sigma^m = 1.5$, before shock $l = 0.6$, after shock: $l = 0.62$.

Figure 13

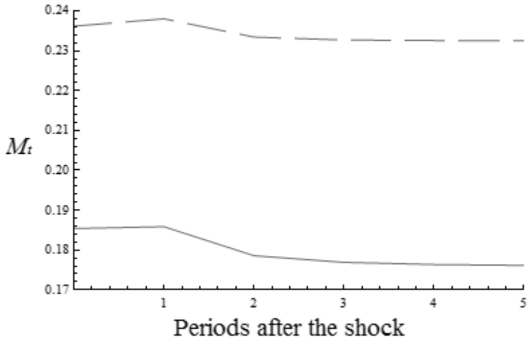
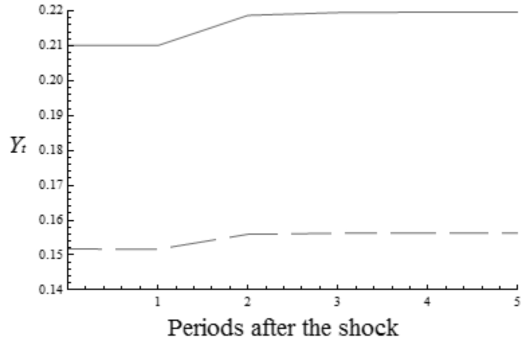


Figure 14



Effects of a positive shock on the life expectancy of the elderly on the immigration quota M_t (Fig. 13) and on public spending per worker Y_t (Fig. 14) for "naive" (dashed line) and "sophisticated" voters (solid line).

Figure 15-16: Convergence to the Steady-State

Parameters: $\sigma^n = 1$, $l = 0.6$.

Figure 15

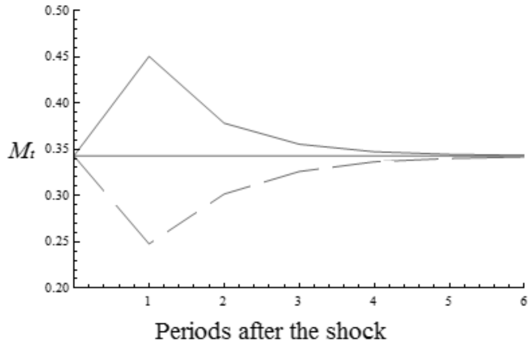
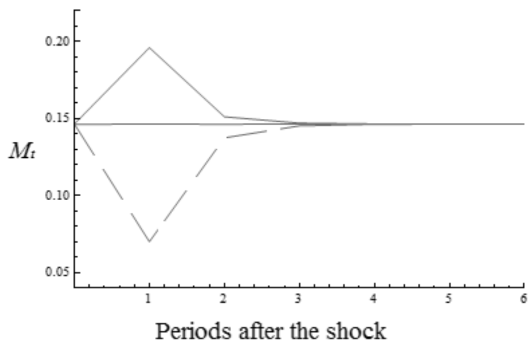


Figure 16



Effects of a temporary negative shock on g_t (solid lines) and of a temporary negative shock on g_t (dashed line) for $\sigma^m = 1.5$ (Fig.15) and $\sigma^m = 2$ (Fig.16).

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