Asset Price Booms and Busts, and Expectations: Theory and Empirical Evidence

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Preface

The prices of assets, such as stocks or houses, display large persistent variations, which are sometimes referred to as bubbles, or, more neutrally, as booms and busts. The source of these swings in asset prices, which are far larger than can be explained by fundamentals, have long fascinated as well as puzzled economists (see for instance Bagehot (1873), and Kindleberger (1978)). It is fair to say, that despite considerable interest in the topic, a consensus on what explains this phenomenon has not yet been reached (see Cochrane (2016)).

Explanations of asset price dynamics range from models in which variations are fully efficient (e.g. Campbell and Cochrane (1999)) to models where they are caused by irrational investors driven by psychological biases (Barberis, Shleifer, and Vishny (1998)). This ambiguity was also reflected in the 2013 Nobel prize awards to Eugene Fama, Lars Peter Hansen, and Robert Shiller, the intellectual fathers of, respectively, 'efficient markets' and 'behavioral finance'. A different approach, which is pursued in this dissertation, considers a world where agents are fully rational, but possess only imperfect knowledge of the structure of the economy in which they live. As Adam and Marcet (2011) have shown, if agents do not know the true mapping from fundamentals to asset prices (which is precisely the source of the puzzle for economists), it is quite possible that their beliefs about asset prices deviate from so-called Rational Expectations.

This dissertation adds to the literature on asset price booms and busts and expectations in three self-contained chapters, each of which coincides with a specific paper. The first chapter is based on the paper 'Stock Price Booms and Expected Capital Gains' which is joint work with Klaus Adam and Albert Marcet. The paper has been published as Adam, Beutel, and Marcet (2014) and is forthcoming in the American Economic Review (AER). The latest version is Adam, Marcet, and Beutel (2017). The paper incorporates and builds on prior work by Adam and Marcet (2010). Parts of the paper, especially the numerical solution strategy, also build on my Master thesis (Beutel (2011)). The second chapter is based on the paper 'Can a Financial Transaction Tax Prevent Stock Price Booms?' which is joint work with Klaus Adam, Albert Marcet, and Sebastian Merkel. The paper has been published in the Journal of Monetary Economics as Adam, Beutel, Marcet, and Merkel (2015). Parts of this paper have also appeared as part of the Master thesis of Sebastian Merkel (Merkel (2014)). The third and most recent chapter is based on the paper 'Smart Money? On Stock Market Expectations of Professional Investors', which is single-authored. The paper is available as Beutel (2016).

The first chapter shows that allowing for deviations from Rational Expectations explains the otherwise puzzling booms and busts in asset prices. In this model, agents use the Kalman filter to form beliefs about future price growth based on the data they observe. As a consequence, past positive (negative) surprises in price growth lead to upward (downward) revisions in their beliefs about future price growth. Upward revisions of beliefs, lead to higher stock prices, which in turn can lead to further positive surprises in price growth. Hence, a self-reinforcing belief-driven asset price boom has emerged.

We show that under a plausible calibration, this model can quantitatively replicate the postwar history of U.S. stock prices and the expectations of U.S. households. The empirical success of the model is based on a crucial difference to previous models of adaptive learning. Previous models have focussed on imperfect knowledge about *exogenous* processes for fundamentals, whereas in our model, agents have imperfect knowledge about the *endogenous* process for prices. Only in the latter case, a feedback loop between beliefs and realizations emerges, which is able to generate the empirically observed magnitudes of asset price volatility (and persistence).

Additional empirical support for this model comes from survey data on stock return expectations, which also motivates deviations from Rational Expectations. Using several surveys of household expectations on the stock market, we show that return expectations co-move positively with the price-dividend ratio. Beliefs in our model, which are the main driver of the results, replicate the empirically observed dynamics of households' expectations. In contrast, Rational Expectations models are inconsistent with a positive correlation between the price-dividend ratio and return expectations, which is shown formally in the chapter as well. While subjective beliefs about endogenous variables substantially enhance the empirical plausibility of our asset pricing model, they also render the model substantially more difficult to solve than under Rational Expectations. While a Rational Expectations version of our model can be solved analytically, this is not the case for the model with subjective beliefs, except in the special case of vanishing noise. (All details in the chapter.) The chapter is therefore based on a numerical solution approach called time iteration with the endogenous grid-point method, which allows us to derive quantitative implications of the general model with subjective beliefs.

The second chapter investigates whether a financial transaction tax could be used to reduce the likelihood of asset price boom-bust cycles. To this end it builds on the model developed in chapter one, extending it to include heterogeneous agents and a financial transaction tax. Mitigating asset price boom-busts could be of interest, since, under the maintained model, booms and busts are in fact inefficient bubbles. Moreover, some authors have associated asset price booms in the stock and housing market with financial crises (e.g. Reinhart and Rogoff (2011)). The European Commission has advanced a proposal for the introduction of a European Financial Transaction Tax, which among other things should contribute to "providing disincentives for transactions which do not enhance the efficiency of financial markets" (European Commission (2013)). However, contrary to conventional wisdom, our results indicate that introducing a financial transaction tax would in fact increase the likelihood of asset price bubbles.

The intuition for this surprising finding is that the tax introduces inaction regions into agents' stock demand functions. In consequence, changes in stock supply lead to larger price changes, which in turn make it more likely that the economy enters a belief-driven boom. The interaction of the tax with the self-reinforcing boom mechanism is crucial for our results. In other words, the presence of a quantitatively credible source of asset price volatility, such as belief-driven booms and busts, can be essential for assessing the impact of a financial transaction tax. An additional contribution of the model is the introduction of heterogeneous agents, who differ in their speed of belief updating. The resulting heterogeneous belief dynamics, generate trade in equilibrium and thereby allow us to capture additional empirical patterns of the data. This makes our model the first to capture the empirical patterns of turnover, beliefs and stock prices jointly.

The third chapter tries to move our understanding of asset price booms and busts and expectations one step further by considering the differences in expectations between different types of agents - namely between households and professional investors. The first chapter presented an explanation for asset price booms and busts based on subjective beliefs of households which is consistent with several important empirical facts. The second chapter showed that this can have important policy implications. Is the puzzle thus solved?

The models of chapter one and two focus exclusively on agents with extrapolative beliefs, in the sense that recent positive surprises lead to upward revisions of return beliefs. It was shown that these beliefs are consistent with survey evidence on the expectations of average households. However, such beliefs lead to very large systematic expectational errors and are at odds with much of standard finance theory. Thus, it is questionable that these beliefs are also entertained by investment professionals, for instance at banks or hedge funds. What happens in financial markets if professional investors and households hold such different beliefs? Conjectures range from the well-known idea that professional investors' arbitrage should restore efficiency, to the possibility that speculation by well-informed investors could aggravate asset price booms and busts.

The chapter is not trying to give an answer to these questions, but rather tries to find out how "smart" investment professionals' expectations are in the first place. Using several recently developed econometric tests and a unique collection of three data sets on investment professionals' expectations, I am able to document several new findings. Expectations of professionals are indeed different from those of average households. Looking at the correlation of expectations and the price-dividend ratio, investment professionals' expectations do not appear to be of the extrapolative type, such that there might be an important role for this type of agents in models of financial markets. Therefore, I go on to investigate which model of expectations could be used to characterize professionals' expectations. The hypothesis that all professionals have Rational Expectations can be rejected, even when allowing for general, possibly asymmetric loss functions which are unknown to the econometrician. Zooming into the micro data, I find that Rational Expectations can only be rejected for around one third of the respondents. For those where forecast optimality is rejected, I find that the rejection cannot be explained by simple canonical models of information rigidities.

Chapter 1

Stock Price Booms and Expected Capital Gains

1.1 Abstract[1]

The booms and busts in U.S. stock prices over the post-war period can to a large extent be explained by fluctuations in investors' subjective capital gains expectations. Survey measures of these expectations display excessive optimism at market peaks and excessive pessimism at market troughs. Formally incorporating subjective price beliefs into an otherwise standard asset pricing model with utility maximizing investors, we show how subjective belief dynamics can temporarily de-link stock prices from their fundamental value and give rise to asset price booms that ultimately result in a price bust. The model successfully replicates (1) the volatility of stock prices and (2) the positive correlation between the price dividend ratio and expected returns observed in survey data. We show that models imposing objective or 'rational' price expectations cannot simultaneously account for both facts. Our findings imply that large parts of U.S. stock price fluctuations are not due to standard fundamental forces, instead result from self-reinforcing belief dynamics triggered by these fundamentals.

1.2 Introduction

Following the recent boom and bust cycles in a number of asset markets around the globe, there exists renewed interest in understanding better the forces contributing to the emergence of such drastic asset price movements. This paper argues that movements in investor optimism and pessimism, as measured by the movements in investors' subjective expectations about future capital gains, are a crucial ingredient for understanding these fluctuations.

[1]This chapter is based on (verbally quoted from) the paper Adam, Beutel, and Marcet (2014).

We present an asset pricing model that incorporates endogenous belief dynamics about expected capital gains. The model gives rise to sustained stock price booms and busts and is consistent with the behavior of investors' capital gains expectations, as measured by survey data. The model suggests that more than half of the variance of the price dividend ratio in U.S. post-WWII data is due to movements in subjective expectations.

The standard approach in the consumption-based asset pricing literature consists of assuming that stock price fluctuations are fully efficient. Campbell and Cochrane (1999) and Bansal and Yaron (2004), for example, present models in which stock price fluctuations reflect the interaction of investor preferences and stochastic driving forces in a setting with optimizing investors who hold rational expectations.

The empirical evidence we present casts considerable doubt on the prevailing view that stock price fluctuations are efficient. Specifically, we show that the RE hypothesis gives rise to an important counterfactual prediction for the behavior of investors' expectations. This counterfactual prediction is a model-independent implication of the RE hypothesis, but - as we explain below - key for understanding stock price volatility and its efficiency properties.

As previously noted by Fama and French (1988), the empirical behavior of asset prices implies that rational return expectations correlate *negatively* with the price dividend (PD) ratio.[2] Somewhat counter-intuitively, the RE hypothesis thus predicts that investors have been particularly pessimistic about future stock returns in the early part of the year 2000, when the tech stock boom and the PD ratio of the S&P500 reached its all-time maximum. As we document, the available survey evidence implies precisely the opposite: all quantitative survey measures of investors' return (or capital gain) expectations available for the U.S. economy, unambiguously and unanimously correlate *positively* with the PD ratio; and perhaps not surprisingly, return expectations reached a temporary maximum rather than a minimum in the early part of the year 2000, i.e., precisely at the peak of the tech stock boom, a fact previously shown in Vissing-Jorgensen (2004). Using a formal test we confirm that the survey data is at odds with the RE hypothesis at any conventional significance level because survey expectations and RE covary differently with the PD ratio.

The positive comovement of stock prices and survey expectations suggests that price fluctuations are amplified by overly optimistic beliefs at market peaks and by overly pessimistic beliefs at market troughs. Furthermore, it suggests that investors' capital gains expectations are influenced - at least partly - by the capital gains observed in the past, in line with evidence presented by Malmendier and Nagel (2011). Indeed, a simple adaptive updating equation captures the time series behavior of the survey data and its correlation with the PD ratio very well.

[2] The RE hypothesis implies also a negative correlation between the PD ratio and expected capital gains. Since most variation in returns is due to variation in capital gains, we tend to use both terms interchangeably.

Taken together, these observations motivate the construction of an asset pricing model in which investors hold subjective beliefs about the capital gains from stock investments.[3] We incorporate such beliefs into a Lucas (1978) asset pricing model, assuming that agents are uncertain about the capital gains process but invest optimally given their beliefs and update beliefs according to Bayes' law.

With this modification, the Lucas model with standard time separable preferences and standard stochastic driving processes becomes quantitatively consistent with the observed volatility of stock prices and the positive correlation between the PD ratio and subjective return expectations. Considering the same model under RE, produces - amongst other things - too little price volatility and the wrong sign for the correlation between the PD ratio and expected returns.

The strong improvement in the model's empirical performance arises because agents' attempts to improve their knowledge about price behavior can temporarily de-link asset prices from their fundamental (RE) value and give rise to belief-driven boom and bust cycles in stock prices. This occurs because with imperfect information about the price process, optimal behavior dictates that agents use past capital gains observations to learn about the stochastic process governing the behavior of capital gains; this generates a feedback between capital gain expectations and realized capital gains.

Suppose, in line with the empirical evidence, that agents become more optimistic about future capital gains whenever they are positively surprised by past capital gains.[4] A positive surprise then increases asset prices further, whenever increased optimism leads to an increase in investors' asset demand. If this effect is sufficiently strong, then positive surprises trigger further positive surprises and thus further price increases. As we show analytically, stock prices in our model do increase with capital gain optimism whenever the substitution effect of increased optimism dominates the wealth effect of such belief changes. Asset prices in the model then display sustained price booms, similar to those observed in the data.

After a sequence of sustained increases, countervailing forces come into play that endogenously dampen the upward price momentum, eventually halt it and cause a reversal. Specifically, in a situation where increased optimism about capital gains has led to a stock price boom, stock prices make up for a larger share of agents' total wealth.[5] As we show analytically, this eventually causes the wealth effect to become as strong as (or even stronger than) the substitution effect.[6] Increases in optimism then cease to cause further increases in stock

[3]As is explained in Adam and Marcet (2011), the presence of subjective price beliefs reflects a lack of common knowledge about agents' beliefs and preferences.

[4]Such positive surprises may be triggered by fundamental shocks, e.g., a high value for realized dividend growth.

[5]This occurs because stock prices are high, but also because agents discount other income streams, e.g., wage income, at a higher rate.

[6]With CRRA utility, this happens whenever the coefficient of relative risk aversion is

demand and thus stock prices, so that investors' capital gains expectations turn out to be too optimistic relative to the realized outcomes. This induces downward revision in beliefs, which gives rise to negative price momentum and an asset price bust.

The previous arguments show how belief dynamics can temporarily delink asset prices from their fundamental value. Clearly, these price dynamics are inefficient as they are not justified by innovations to preferences or other fundamentals.

We obtain these results even though we depart from the standard paradigm in a minimal way only. Specifically, we assume that investors are internally rational (IR) in the sense of Adam and Marcet (2011). This implies that all investors hold an internally consistent system of beliefs about variables that are exogenous to their decision problem and choose investment and consumption optimally. Although agents' beliefs do not fully capture the actual behavior of prices in equilibrium, in line with the survey evidence, agents' beliefs are broadly plausible given the behavior of equilibrium prices and the behavior of prices in the data. In particular, agents believe the average growth rate of stock prices to slowly drift over time, which is consistent with the presence of prolonged periods of price booms followed by price busts.

The current paper shows how the framework of internal rationality allows studying learning about market behavior in a model of intertemporal decision making, while avoiding some of the pitfalls of the adaptive learning literature, where agents' belief updating equations and choices are often not derived from individual maximization. We thus show how explicit microfoundations can guide modelling choices in settings featuring subjective beliefs about market outcomes, as is the case in settings imposing RE.

The remainder of the paper is structured as follows. Section 1.4 documents that there is a strong positive correlation between the PD ratio and survey measures of investors' return and capital gain expectations and that this is incompatible with the RE hypothesis. It then documents that from a purely statistical standpoint approximately two thirds of the variation in the PD ratio of S&P500 can potentially be accounted for by variations in expected capital gains. Section 2.5 presents our asset pricing model with subjective beliefs. For benchmark purposes, section 1.6 determines the RE equilibrium. Section 1.7 introduces a specific model for subjective price beliefs; it does so by relaxing agents' prior beliefs about price behavior relative to the RE equilibrium beliefs. This section also derives the resulting Bayesian updating equations characterizing belief dynamics over time, involving learning about the permanent component of stock price growth. After imposing market clearing in section 1.8, we present closed form solutions for the PD ratio in section 1.9 in the special case of vanishing uncertainty. We then explain how the interaction between belief updating dynamics and price outcomes can endogenously generate boom and bust dynamics in asset prices. Section 1.10 considers the model

larger than one.

with empirically plausible amounts of uncertainty and documents its ability to replicate the time series behavior of the postwar US PD ratio and of the survey data. Section 1.11 documents that the model under learning replicates important asset pricing moments much better than under RE. A conclusion briefly summarizes and presents an outlook on future research avenues. Technical material and proofs can be found in the appendix.

1.3 Related Literature

The literature on adaptive learning previously studied the role of deviations from RE in asset pricing models. Work by Bullard and Duffy (2001) and Brock and Hommes (1998), for example, explores learning about price forecasting and shows that learning dynamics can converge to complicated attractors that increase asset return volatility, if the RE equilibrium is unstable under learning dynamics.[7] Lansing (2010) shows how near-rational bubbles can arise under learning dynamics when agents forecast a composite variable involving future price and dividends. Branch and Evans (2011) present a model where agents learn about risk and return and show how it gives rise to bubbles and crashes. Boswijk, Hommes and Manzan (2007) estimate a model with fundamentalist and chartist traders whose relative shares evolve according to an evolutionary performance criterion, showing that the model can generate a run-up in asset prices and subsequent mean-reversion to fundamental values. DeLong et al. (1990) show how the pricing effects of positive feedback trading survives or even get amplified by the introduction of rational speculators. Timmermann (1993, 1996) explores learning about dividend behavior but finds overall limited pricing implications. Cogley and Sargent (2008) have studied a model of robustness, where agents learn about fundamentals and behave according to max-min utility.

We contribute to this literature in three ways. First, we compare the implications of our model more closely to the data, both in terms of matching the time series of asset prices and survey data, as well as in terms of matching asset pricing moments.

Second, we specify proper microfoundations for agents' infinite horizon decision problem with subjective beliefs and derive agents' optimal consumption plans and belief updating equations from this problem. The subjective consumption plans are then used to price the stock market. Earlier work on infinite horizon models in the adaptive learning literature typically falls short of specifying proper optimization problems. As explained in section 2 in Adam and Marcet (2011), this leads to arbitrariness in the modeling of agents' behavior, which can affect model predictions and the resulting conclusions. Important progress has been made in recent work by Eusepi and Preston (2011, 2013), who derive choices from properly formulated optimization problems featuring

[7] Stability under learning dynamics is defined in Marcet and Sargent (1989).

subjective beliefs. Here we go a step further by jointly deriving the optimal decisions and the belief updating rules from the utility maximization problem, instead of making appeal to the anticipated utility framework in Kreps (1998), which implies that future belief revisions are abstracted from when deriving decisions.

Third, we are able to derive our main results using a closed-form solution. This provides clearer insights into the economic mechanisms driving the asset pricing results. We also discuss issues of existence and uniqueness of optimal plans in models with subjective beliefs and conditions under which the optimal plan has a recursive representation. Furthermore, we explain why rational agents can hold separate subjective beliefs about prices and fundamentals.

Fuster, Herbert and Laibson (2011) present an asset pricing model where fundamentals exhibit momentum in the short-run and partial mean reversion in the long-run and where agents underparameterize the fundamental process, thereby missing the long-run mean reversion. They show how such a model can give rise to pro-cyclical excess optimism as in the present paper. Fundamentals in our model display neither momentum nor mean reversion, excess optimism and pessimism arise instead endogenously from the interaction between price outcomes and expectations.

Hassan and Mertens (2011) present a stock market model where investors deviate from fully rational behavior, as agents make small common errors in formulating expectations. They show how the market amplifies these errors and how this can have large welfare consequences by shifting investment away from domestic production opportunities into foreign safe bonds. The present model does not consider effects on output and welfare, instead derives empirical implications for stock price volatility and the behavior of expectations in a setting with fully optimal behavior and Bayesian updating, given imperfect knowledge of the economy.

Adam, Marcet and Nicolini (2016) quantitatively evaluate the ability of models of learning to explain asset price volatility. To be able to formally estimate the model using the method of simulated moments, they rely on a number of short-cuts. In particular, they assume dividends to be a negligible part of total income, so that consumption equals exogenous labor income. As a result, the stochastic discount factor is exogenous. While being analytically convenient, this prevents the emergence of the wealth effects referred to in the introduction, requiring asset price booms to be stopped by exogenously imposing an upper bound on agents' beliefs.[8] Clearly, this prevents a discussion of asset price booms and their end. They also do not discuss survey evidence.

The experimental and behavioral literature provides further evidence supporting the presence of subjective price beliefs. Asparouhova, Bossaerts, Roy and Zame (2011), for example, implement the Lucas asset pricing model in the

[8]The performance of the model in terms of quantitatively replicating asset pricing moments is, however, robust to the precise value chosen for this upper bound, because the bound is binding only rarely along the equilibrium path.

experimental laboratory and document that there is excess volatility in prices that is unaccounted for by the rational expectations equilibrium and that likely arises from participants' expectations about future prices. Furthermore, the type of learning employed in the present model is in line with evidence presented in Malmendier and Nagel (2011) who show that experienced returns affect beliefs about future asset returns.[9]

1.4 Stock Prices & Expectations: Facts

This section explains how two important and widely accepted asset pricing facts imply a counterfactual behavior for the behavior of stock price expectations, whenever one imposes that agents hold rational price expectations. We present the evidence informally in section 1.4.1 and derive a formal statistical test in section 1.4.2. The test shows that the RE hypothesis is inconsistent with the behavior of the survey data due to the way survey expectations covary with the PD ratio. Section 1.4.3 illustrates how simple adaptive prediction of prices, in line with Malmendier and Nagel (2011, 2016), quantitatively captures the relationship between survey expectations and the PD ratio. It also shows how, in a purely statistical sense, variations in expected capital gains can potentially account for up to two thirds of the variation of the U.S. PD ratio over the postwar period.

1.4.1 Survey Expectations and the PD Ratio

This section explains how the presence of boom and bust dynamics in stock prices, together with the unpredictability of dividend growth, imply that rational stock return forecasts should correlate *negatively* with the PD ratio. It then documents that survey measures of investors' return expectations correlate instead *positively* with the PD ratio.

The discrepancy in terms of correlations with the PD ratio is in line with recent independent findings by Greenwood and Shleifer (2014a). The positive co-movement between survey return expectations and the PD ratio has also been noted before by Vissing-Jorgensen (2004) and Bacchetta, Mertens, and Wincoop (2009). While generally insightful, one must emphasize that - in econometric terms - the presence of such a discrepancy is only suggestive. In particular, if investors possess private information that is not observed by the econometrician or if survey expectations are measured with error, as one can reasonably expect to be the case, then the correlation between fully rational return forecasts and the PD ratio will differ from the correlation between realized returns and the PD ratio. Simply comparing correlations is thus not

[9] Nagel and Greenwood (2009) show that - in line with this hypothesis - young mutual fund managers displayed trend chasing behavior over the tech stock boom and bust around the year 2000.

sufficient to reject the hypothesis that survey expectations are rational. Furthermore, formal tests must always take into account the *joint* distribution of the correlation estimates in order to make statistically valid statements. While the informal discussion below abstracts from these aspects, the next section takes them fully into account.

As is well known, stock prices experience substantial price booms and price busts. Figure 1.1 illustrates this behavior for the post-WWII period for the United States, using the quarterly price dividend ratio (PD) of the S&P 500 index.[10] The PD ratio displays persistent run-ups and reversals, with the largest one occurring around the year 2000. This shows that price growth can persistently outstrip dividend growth over a number of periods, but that the situation eventually reverses. In fact, the quarterly autocorrelation of the PD ratio equals 0.98. Similar run-ups and reversals can be documented for other mature stock markets, e.g., for the European or Japanese markets.

Figure 1.1: Quarterly PD Ratio of the S&P 500

Equally well-known is the fact that the growth rate of dividends is largely unpredictable, e.g., Campbell (2003). It is especially hard to predict using the PD ratio. The R^2 values of an in-sample predictive regression of cumulative dividend growth 1, 5 or 10 years ahead on a constant and the log PD ratio

[10] Quarterly dividend payments have been deseasonalized in a standard way by averaging them across the current and preceding 3 quarters. See appendix 2.12.1 for details about the data used in this section.

are rather small and amount to 0.03, 0.04, and 0.07, respectively, for the U.S. post-war data.[11]

Taken together the previous two facts imply that under RE one would expect that the PD ratio *negatively* predicts future stock market returns. To see this, let the asset return R_{t+1} be defined as

$$R_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\frac{P_{t+1}}{D_{t+1}} + 1}{\frac{P_t}{D_t}} \frac{D_{t+1}}{D_t},$$

where P denotes the stock price and D dividends. Given a high value of P_t/D_t, we have - due to the mean reverting behavior of the PD ratio - that $P_{t+1}/D_{t+1} < P_t/D_t$ on average. Since D_{t+1}/D_t is unpredictable, it follows that a high PD ratio negatively predicts future returns.[12] A symmetric argument holds if P_t/D_t is low.

In the setup just described, expectations about future stock returns should covary negatively with the PD ratio if investors hold RE. In particular, rational expectations about stock returns should be very low at the height of the tech stock boom in the year 2000 when the PD ratio reached its historical peak.

Survey evidence on investors' return expectations displays instead a strong *positive* correlation between investors' expected returns and the PD ratio. Figure 1.2 depicts this for our preferred survey, the UBS Gallup Survey, which is based on a representative sample of approximately 1.000 U.S. investors that own at least 10.000 US$ in financial wealth.[13] Figure 1.2 graphs the US PD ratio (the black line) together with measures of the cross-sectional average of investors' one-year ahead expected real return.[14] Return expectations are expressed in terms of quarterly real growth rates and the figure depicts two expectations measures: investors' expectations about the one year ahead stock market return, as well as their expectations about the one year ahead returns on their own stock portfolio. These measures behave very similarly over the period for which both series are available, but the latter is reported for a longer time period, so that we focus on it as our baseline. Figure 1.2 reveals that there is a strong positive correlation between the PD ratio and expected returns. The correlation between the expected own portfolio returns and the PD ratio is +0.70 and even higher for the expected stock returns (+0.82).

[11] We use logPD as a regressor, in line with Campbell (2003). The R^2 values are unchanged when using the level of the PD ratio instead.

[12] There may exist, of course, other predictors of future returns which correlate negatively with the PD ratio and that overturn the negative relationship between PD ratio and expected stock returns emerging from the forces described above. We take these formally into account in our statistical test in section 1.4.2.

[13] About 40% of respondents own more than 100.000 US$ in financial wealth. As documented below, this subgroup does not behave differently.

[14] To be consistent with the asset pricing model presented in later sections we report expectations of real returns. The nominal return expectations from the survey have been transformed into real returns using inflation forecasts from the Survey of Professional Forecasters. Results are robust to using other approaches, see the subsequent discussion.

Moreover, investors' return expectations were highest at the beginning of the year 2000, which is precisely the year the PD ratio reached its peak during the tech stock boom. At that time, investors expected annualized real returns of around 13% from stock investments. Conversely, investors were most pessimistic in the year 2003 when the PD ratio reached its bottom, expecting then annualized real returns of below 4%.

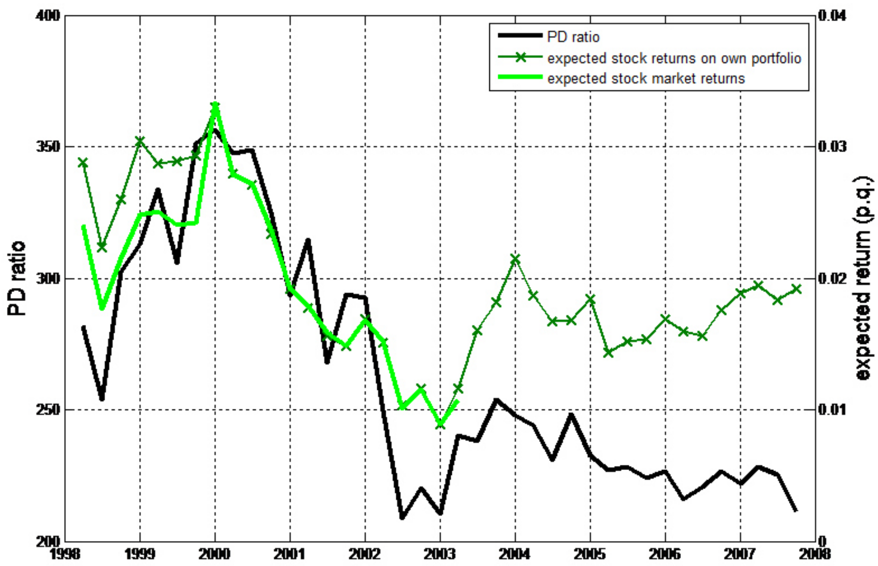

Figure 1.2: PD ratio and investors' expected returns (UBS Gallup Survey)

Table 1.1 shows that the strong positive correlation evident from figure 1.2 is robust to a number of alternative approaches for extracting expectations from the UBS survey, such as using the median instead of the mean expectation, when using inflation expectations from the Michigan survey to obtain real return expectations, when considering plain nominal returns instead of real returns, or when restricting attention to investors with more than 100.000 US$ in financial wealth. The numbers reported in brackets in table 1.1 (and in subsequent tables) are autocorrelation robust p-values for the hypothesis that the correlation is smaller or equal to zero.[15] The p-values for this hypothesis are all below the 5% significance level and in many cases below the 1% level.

[15]The sampling width is four quarters, as is standard for quarterly data, and the test allows for contempraneous correlation, as well as for cross-correlations at leads und lags. The p-values are computed using the result in Roy (1989).

UBS Gallup	Nominal Return Exp.		Real Ret. Exp. (SPF)		Real Ret. Exp. (Michigan)	
	Average	**Median**	**Average**	**Median**	**Average**	**Median**
Own portfolio, >100k US$	0.80 (0.01)	0.78 (0.01)	0.79 (0.01)	0.77 (0.01)	0.84 (0.01)	0.83 (0.01)
Own portfolio, all investors	0.80 (0.01)	0.76 (0.02)	0.79 (0.01)	0.75 (0.02)	0.84 (0.01)	0.80 (0.01)
Stock market, >100k US$	0.90 (0.03)	0.89 (0.04)	0.90 (0.03)	0.88 (0.03)	0.91 (0.03)	0.88 (0.03)
Stock market, all investors	0.90 (0.03)	0.87 (0.04)	0.90 (0.03)	0.87 (0.04)	0.91 (0.03)	0.88 (0.03)

Table 1.1: Correlation between PD ratio and 1-year ahead expected return measures (UBS Gallup Survey, robust p-values in parentheses)

Shiller Survey	**Nominal Capital Gain Exp.**		**Real Capital Gain. Exp. (SPF)**		**Real Capital Gain Exp. (Michigan)**	
Horizon	**Average**	**Median**	**Average**	**Median**	**Average**	**Median**
1 month	0.46 (0.01)	0.48 (0.01)	0.45 (0.01)	0.47 (0.01)	0.46 (0.01)	0.49 (0.01)
3 months	0.57 (0.01)	0.64 (0.00)	0.54 (0.01)	0.61 (0.00)	0.56 (0.01)	0.62 (0.01)
6 months	0.58 (0.01)	0.75 (0.01)	0.54 (0.02)	0.70 (0.01)	0.56 (0.02)	0.71 (0.01)
1 year	0.43 (0.03)	0.69 (0.01)	0.38 (0.05)	0.62 (0.01)	0.42 (0.04)	0.64 (0.02)
10 years	0.74 (0.01)	0.75 (0.01)	0.66 (0.02)	0.71 (0.01)	0.71 (0.02)	0.75 (0.01)

Table 1.2: Correlation between PD ratio and expected stock price growth (Shiller's Individual Investors' Survey, robust p-values in parentheses)

A positive and statistically significant correlation is equally obtained when considering other survey data. Table 1.2 reports the correlations between the PD ratio and the stock price growth expectations from Bob Shiller's Individual Investors' Survey.[16] The table shows that price growth expectations are also strongly positively correlated with the PD ratio, suggesting that the variation in expected returns observed in the UBS survey is due to variations in expected

[16] Shiller's price growth data refers to the Dow Jones Index. The table thus reports the correlation of the survey measure with the PD ratio of the Dow Jones.

capital gains. Table 1.2 also shows that correlations seem to become stronger for longer prediction horizons.

Table 1.3 reports the correlations for the stock return expectations reported in the Chief Financial Officer (CFO) survey which surveys chief financial officers from large U.S. corporations. Again, one finds a strong positive correlation; it is significant at the 1% level in all cases.

Table 1.4 reports the correlations between the PD ratio and the realized real returns (or capital gains) in the data, using the same sample periods as are available for the surveys considered in tables 1 to 3, respectively. The point estimate for the correlation is negative in all cases, although the correlations fall short of being significant the 5% level due to the short sample length for which the survey data is available. Nevertheless, table 1.4 suggests that investors' expectations are most likely incompatible with RE. The next section investigates this issue more formally.

CFO Survey	**Nominal Return Exp.**		**Real Return Exp. (SPF)**		**Real Return Exp. (Michigan)**	
	Average	**Median**	**Average**	**Median**	**Average**	**Median**
1 year	0.71 (0.00)	0.75 (0.00)	0.62 (0.00)	0.69 (0.00)	0.67 (0.00)	0.72 (0.00)

Table 1.3: Correlation between PD ratio and 1-year ahead expected stock return measures (CFO Survey, robust p-values in parentheses)

Variables	Time Period	Stock Index	Correlation
PD, 1 year-ahead real return	UBS Gallup sample (stock market exp.)	S&P 500	−0.66 (0.08)
PD, 1 year-ahead real price growth	Shiller 1 year sample	Dow Jones	−0.42 (0.06)
PD, 10 year-ahead real price growth	Shiller 10 year sample	Dow Jones	−0.88 (0.16)
PD, 1 year-ahead real return	CFO sample	S&P 500	−0.46 (0.06)

Table 1.4: Correlation between PD and actual real returns/capital gains (robust p-value in parentheses)

1.4.2 Survey Expectations versus Rational Expectations

Using a formal econometric test, this section shows that the RE assumption is indeed incompatible with the behavior of survey expectations. As suggested by the informal arguments in the previous section, the failure is due to the fact that RE and survey expectations covary differently with the PD ratio.

The test approach presented below is immune to the presence of measurement error in surveys, allows for unobserved information on the side of investors and properly takes into account the joint distribution of estimates.

Let $E_t^{\mathcal{P}}$ denote agents' subjective (and potentially less-than-fully-rational) expectations operator based on information up to time t, and $R_{t,t+N}$ the cumulative stock returns between period t and $t+N$. Furthermore, let $\widetilde{E_t^{\mathcal{P}}} R_{t,t+N}$ denote the (potentially noisy) measurement of expected returns, as obtained - for example - from survey data.[17] Since we shall consider both the rationality of real return expectations and the rationality of excess return expectations, we let $\widetilde{E_t^{\mathcal{P}}} R_{t,t+N}$ denote both, the expectations of real returns and the expectations of real excess returns. We construct excess return expectations following Bacchetta et al. (2009), i.e., assume that the N period ahead interest risk-free interest rate is part of agents' information set and subtract it from the expected stock return.[18]

Given the observed (excess) return expectations, one can write the *regression* equation[19]

$$\widetilde{E_t^{\mathcal{P}}} R_{t,t+N} = a^N + c^N \frac{P_t}{D_t} + u_t^N, \tag{1.1}$$

where the regression residual u_t^N captures the variation in agents' expectations that cannot be linearly attributed to the price-dividend ratio. It summarizes all other information that agents believe to be useful in predicting $R_{t,t+N}$, as well as potential measurement error from survey data.[20,21] We then have the orthogonality condition

$$E\left(x_t u_t^N\right) = 0 \tag{1.2}$$

for $x_t' = (1, P_t/D_t)$ where the operator E denotes the objective expectation for the *true* data generating process, independently of how agents' expectations are formed. Finally, we let $\widehat{c}^N$ denote the OLS estimator of c^N in equation (1.1).

In the special case with rational expectations ($E_t^{\mathcal{P}} = E_t$) equation (1.1) implies

$$R_{t,t+N} = a^N + c^N \frac{P_t}{D_t} + u_t^N + \varepsilon_t^N \tag{1.3}$$

where ε_t^N is equal to the sum of the prediction error $R_{t,t+N} - E_t R_{t,t+N}$ from the *true* data-generating process minus the measurement error contained in

[17] As is standard, we assume the measurement error to be uncorrelated with regressors, i.e., the PD ratio.

[18] As in Bacchetta et al. (2009), we use the constant maturity interest rates available from the FRED database at the St. Louis Federal Reserve Bank.

[19] This regression is well-defined, as long as agents' measured expectations $\widetilde{E_t^{\mathcal{P}}} R_{t,t+N}$ and the PD ratio P_t/D_t are stationary and have bounded second moments.

[20] The residual u_t^N is likely to be correlated with current and past observables (other than the PD ratio) and thus serially correlated.

[21] Since the Shiller survey reports expectations about capital gains instead of returns, we interpret the variable $R_{t,t+N}$ as the real (excess) growth rate of stock prices between periods t and $t+N$ when using the Shiller survey.

survey data. Importantly, ε_t^N is orthogonal to all past observations dated t or earlier and satisfies

$$E\left[x_t\left(u_t^N+\varepsilon_t^N\right)\right]=0, \tag{1.4}$$

so that an estimate of c^N that is consistent with the RE assumption can be derived by estimating (1.3) with OLS. We let $\widehat{\widehat{c}}^N$ denote this estimate.

The correlations reported in tables 1-4 imply - by construction - that $\widehat{c}^N > 0$ and $\widehat{\widehat{c}}^N < 0$. Yet, the regression estimates are useful here because under the RE hypothesis $\widehat{c}^N$and $\widehat{\widehat{c}}^N$ are consistent estimates of the *same* parameter c^N. This allows to formally test the RE hypothesis, i.e., $H_0 : \widehat{c}^N = \widehat{\widehat{c}}^N$.[22,23] Clearly, if the asset price and survey data were generated by a rational expectations model, say the models of Campbell and Cochrane (1999) or Bansal and Yaron (2004), this test would be accepted.

Survey measure	$\widehat{c}\cdot 10^3$	$\widehat{\widehat{c}}\cdot 10^3$	p-value $H_0 : \widehat{c}=\widehat{\widehat{c}}$	$\widehat{c}\cdot 10^3$	$\widehat{\widehat{c}}\cdot 10^3$	p-value $H_0 : \widehat{c}=\widehat{\widehat{c}}$
	S&P 500, real returns					
	Survey Average			Survey Median		
UBS*, >100k, 1 yr, SPF	0.56	-2.93	0.0000	0.44	-2.93	0.0000
UBS*, >100k, 1 yr, Michigan	0.55	-2.93	0.0000	0.43	-2.93	0.0000
UBS*, all, 1 yr, SPF	0.54	-2.93	0.0000	0.45	-2.93	0.0000
UBS*, all, 1 yr, Michigan	0.53	-2.93	0.0000	0.44	-2.93	0.0000
CFO, 1 yr, SPF	0.20	-1.88	0.0004	0.24	-1.74	0.0366
CFO, 1 yr, Michigan	0.26	-1.88	0.0002	0.32	-1.74	0.0252
	Dow Jones, real price growth					
	Survey Average			Survey Median		
Shiller, 1 yr, SPF	0.23	-1.48	0.0000	0.23	-1.48	0.0000
Shiller, 1 yr, Michigan	0.28	-1.48	0.0000	0.29	-1.48	0.0000
Shiller, 10 yrs, SPF	4.11	-6.48	0.0000	5.49	-6.48	0.0000
Shiller, 10 yrs, Michigan	3.51	-6.48	0.0000	4.89	-6.48	0.0000

*stock market return expectations

Table 1.5a: Forecast rationality test (returns)

[22] Under the RE hypothesis, the correlations in tables 1-3 are not equal to the corresponding correlations reported in table 1.4, albeit both should have the same sign. Constructing a formal test for the sign of the correlations being equal is a fairly non-trivial task.

[23] To obtain p-values for $H_0 : \widehat{c}^N = \widehat{\widehat{c}}^N$, we stack up equations (1.1) and (1.3), create a SUR system of equations to find the *joint* distribution of $\widehat{c}^N$ and $\widehat{\widehat{c}}^N$ and build a t-test for H_0. We use serial-correlation and heteroskedasticity robust asymptotic covariance matrix of the estimators, using 4 lags, results are robust to increasing the lag length to up to 12 lags. For each considered survey we use data on actual (excess) returns (or price growth) for the same time period for which survey data is available when computing the p-values. Further details of the test are described in appendix 1.13.2.

Test outcomes are reported in table 1.5a using stock returns and table 1.5b using excess returns. Both tables report the point estimates $\widehat{c}$ and $\widehat{\widehat{c}}$, as well as the p-values for $H_0 : \widehat{c}^N = \widehat{\widehat{c}}$, using the survey data sources considered in the previous section.[24] The point estimates satisfy in all but two cases $\widehat{c} > 0$ and always satisfy $\widehat{\widehat{c}} < 0$. The difference between the two estimates is statistically significant at the 1% level in all cases, except for the survey median from the CFO survey. Given the relatively short sample lengths, this is a remarkable outcome. Tables 5a and 5b thus provide overwhelming evidence against the notion that survey expectations are rational.

Survey measure	$\widehat{c} \cdot 10^3$	$\widehat{\widehat{c}} \cdot 10^3$	p-value $H_0 : \widehat{c} = \widehat{\widehat{c}}$	$\widehat{c} \cdot 10^3$	$\widehat{\widehat{c}} \cdot 10^3$	p-value $H_0 : \widehat{c} = \widehat{\widehat{c}}$
	S&P 500, real excess returns					
	Survey Average			Survey Median		
UBS*, >100k, 1 yr, SPF	0.25	-3.02	0.0000	0.14	-3.02	0.0000
UBS*, >100k, 1 yr, Michigan	0.24	-3.02	0.0000	0.14	-3.02	0.0000
UBS*, all, 1 yr, SPF	0.23	-3.02	0.0000	0.15	-3.02	0.0000
UBS*, all, 1 yr, Michigan	0.23	-3.02	0.0000	0.14	-3.02	0.0000
CFO, 1 yr, SPF	0.04	-1.97	0.0006	0.12	-1.66	0.0801
CFO, 1 yr, Michigan	0.04	-1.97	0.0005	0.12	-1.66	0.0796
	Dow Jones, real excess price growth					
	Survey Average			Survey Median		
Shiller, 1 yr, SPF	-0.04	-1.68	0.0001	-0.04	-1.68	0.0000
Shiller, 1 yr, Michigan	-0.05	-1.68	0.0001	-0.05	-1.68	0.0000
Shiller, 10 yrs, SPF	2.24	-7.98	0.0000	3.62	-7.98	0.0000
Shiller, 10 yrs, Michigan	2.08	-7.98	0.0000	3.46	-7.98	0.0000

Table 1.5b: Forecast rationality test (excess returns)

1.4.3 How Models of Learning May Help

This section illustrates that a simple 'adaptive' approach to forecasting stock prices is a promising alternative to explain the joint behavior of survey expectations and stock price data.

Figure 1.2 shows that the peaks and troughs of the PD ratio are located very closely to the peaks and troughs of investors' return expectations. This suggests that agents become optimistic about future capital gains whenever they have observed capital gains in the past. Such behavior can be captured by models where agents expectations are influenced by past experience prompting us to assume for a moment that agents' subjective conditional capital gain expectations $\widetilde{E}_t \left[P_{t+1}/P_t \right]$ evolve according to the following adaptive prediction

[24] Tests for 'own portfolio' expectations are not shown because we do not observe agents' returns on their own portfolio.

model

$$\widetilde{E}_t\left[P_{t+1}/P_t\right] = \widetilde{E}_{t-1}\left[P_t/P_{t-1}\right] + g\left(\frac{P_t}{P_{t-1}} - \widetilde{E}_{t-1}\left[P_t/P_{t-1}\right]\right), \qquad (1.5)$$

where $g > 0$ indicates how strongly capital gain expectations are updated in the direction of the forecast error. While equation (1.5) may appear ad-hoc, we show in section 1.7 how a very similar equation can be derived from Bayesian belief updating in a setting where agents estimate the persistent component of price growth from the data.

One can use equation (1.5) and feed into it the historical price growth data of the S&P 500 over the postwar period. Together with an assumption about capital gain expectations at the start of the sample this will deliver a time series of implied capital gain expectations $\widetilde{E}_t\left[P_{t+1}/P_t\right]$ that can be compared to the expectations from the UBS survey.[25] Figure 1.3 reports the outcome of this procedure when assuming initial beliefs in Q1:1946 to be equal to -1.11% per quarter and $g = 0.02515$, which minimizes the sum of squared deviations from the survey evidence.[26] Figure 1.3 shows that the adaptive model captures the behavior of UBS expectations extremely well: the correlation between the two series is equal to $+0.89$.

A similarly strong positive relationship between the PD ratio and the capital gains expectations implied by equation (1.51) exists over the entire postwar period, as figure 1.4 documents. The figure plots the joint distribution of the capital gains expectations (as implied by equation (1.51)) and the PD ratio in the data. When regressing the PD ratio on a constant and the expectations of the adaptive prediction model, one obtains an R^2 coefficient of 0.55; using also the square of the expectations, the R^2 rises further to 0.67. Variations in expected capital gains can thus account - in a purely statistical sense - for up to two thirds of the variability in the postwar PD ratio.[27]

The previous findings suggest that an asset pricing model consistent with equation (1.5), which additionally predicts a positive relationship between the PD ratio and subjective expectations about future capital gains, has a good chance of replicating the observed positive co-movement between price growth expectations and the PD ratio. The next sections spell out the microfoundations of such a model. As we show, the model can simultaneously replicate the behavior of stock prices and stock price expectations.

[25] We transform the UBS survey measures of return expectations into a measure of price growth expectations using the identity $R_{t+1} = \frac{P_{t+1}}{P_t} + \frac{D_{t+1}}{P_t} = \frac{P_{t+1}}{P_t} + \beta^D \frac{D_t}{P_t}$ where β^D denotes the expected quarterly growth rate of dividends that we set equal to the sample average of dividend growth over Q1:1946-Q1:2012, i.e, $\beta^D = 1.0048$. Results regarding implied price growth are very robust towards changing β^D to alternative empirically plausible values.

[26] The figure reports growth expectations in terms of quarterly real growth rates.

[27] Interestingly, the relationship between implied price growth expectations and the PD ratio seems to have shifted upwards after the year 2000, as indicated by the squared icons in figure 1.4. We will come back to this observation in section 1.10.

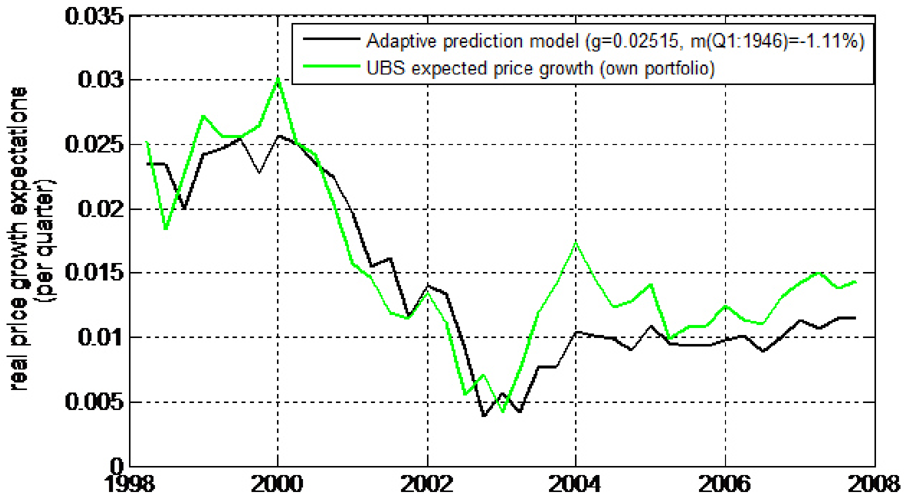

Figure 1.3: UBS survey expectations versus adaptive prediction model

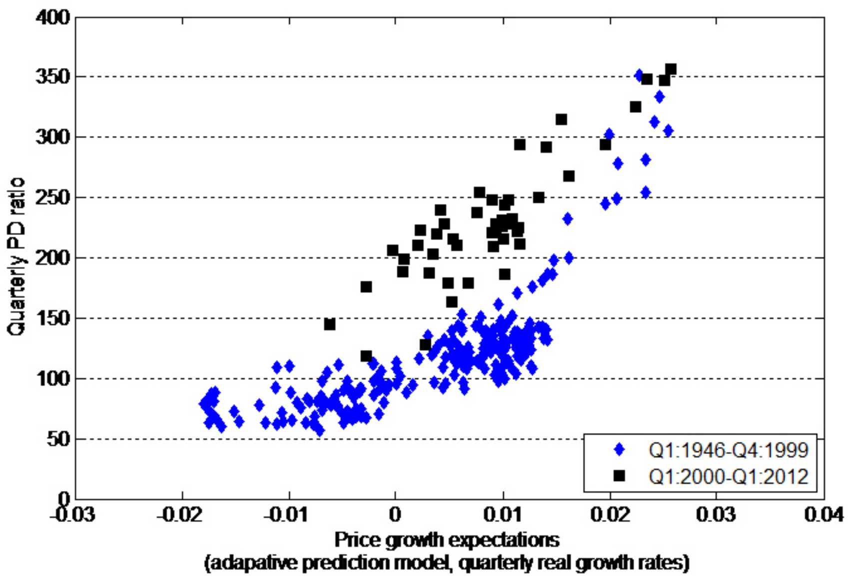

Figure 1.4: PD ratio S&P 500 vs. adaptive price growth predictions

1.5 A Simple Asset Pricing Model

Consider an endowment economy populated by a unit mass of infinitely lived agents $i \in [0,1]$ with time-separable preferences. Agents trade one unit of a stock in a competitive stock market. They earn each period an exogenous non-dividend income $W_t > 0$ that we refer to as 'wages' for simplicity. Stocks deliver the dividend $D_t > 0$. Dividend and wage incomes take the form of perishable consumption goods.

The Investment Problem. Investor i solves

$$\max_{\left\{C_t^i \geq 0, S_t^i \in \mathcal{S}\right\}_{t=0}^{\infty}} E_0^{\mathcal{P}^i} \sum_{t=0}^{\infty} \delta^t \, u\left(C_t^i\right) \tag{1.6}$$

$$\text{s.t.} \qquad S_t^i P_t + C_t^i = S_{t-1}^i \left(P_t + D_t\right) + W_t \quad \text{for all } t \geq 0$$

where $S_{-1}^i = 1$ and C^i denotes consumption, u the instantaneous utility of the consumer, assumed to be continuous, differentiable, increasing and strictly concave, S^i the agent's stockholdings, chosen from some compact, non-empty and convex set $\mathcal{S}$ such that $1 \in \mathcal{S}$, $P \geq 0$ the (ex-dividend) price of the stock, $D \geq 0$ an exogenous dividend, $W \geq 0$ the exogenous wage income, and $\mathcal{P}^i$ the agent's subjective probability measure, which may or may not satisfy the rational expectations hypothesis. Further details of $\mathcal{P}^i$ will be specified below.

Dividend and Wage Income. As standard in the literature, we assume that dividends grow at a constant rate and that dividend growth innovations are unpredictable

$$\ln D_t = \ln \beta^D + \ln D_{t-1} + \ln \varepsilon_t^D, \tag{1.7}$$

where $\beta^D \geq 1$ denotes gross mean dividend growth, $\ln \varepsilon_t^D$ an i.i.d. growth innovation described further below.

We also specify an exogenous wage income process W_t , which is chosen such that the resulting aggregate consumption process $C_t = W_t + D_t$ is empirically plausible. First, in line with Campbell and Cochrane (1999), we set the standard deviation of consumption growth to be 1/7 of the standard deviation of dividend growth. Second, again following these authors, we set the correlation between consumption and dividend growth equal to 0.2. Third, we choose a wage process such that the average consumption-dividend ratio in the model ($E\left[C_t/D_t\right]$) equals the average ratio of personal consumption expenditure to net dividend income, which equals approximately 22 in U.S. postwar data.[28] All this can be parsimoniously achieved using the following wage income process

$$\ln W_t = \ln \rho + \ln D_t + \ln \varepsilon_t^W,$$

where

$$\begin{pmatrix} \ln \varepsilon_t^D \\ \ln \varepsilon_t^W \end{pmatrix} \sim iiN\left(-\frac{1}{2}\begin{pmatrix} \sigma_D^2 \\ \sigma_W^2 \end{pmatrix}, \begin{pmatrix} \sigma_D^2 & \sigma_{DW} \\ \sigma_{DW} & \sigma_W^2 \end{pmatrix}\right) \tag{1.8}$$

[28] See appendix 1.13.3 for details.

and $E\varepsilon_t^D = E\varepsilon_t^W = 1$. Given the variance of dividend growth σ_D^2, which can be estimated from dividend data, one can use σ_{DW} and σ_W^2 to impose the desired volatility of consumption growth and the desired correlation with dividend growth. Furthermore, one can choose $\rho = 22$ to obtain the targeted average consumption-dividend ratio. Appendix 1.13.3 explains how this is achieved.

The Underlying Probability Space. Agents hold a set of subjective probability beliefs about all payoff-relevant variables that are beyond their control. In addition to fundamental variables such as dividends and wage income, agents also perceive competitive stock prices to be beyond their control. Therefore, the belief system also specifies probabilities about prices. Formally, letting Ω denote the space of possible realizations for infinite sequences, a typical element $\omega \in \Omega$ is given by $\omega = \{P_t, D_t, W_t\}_{t=0}^{\infty}$. As usual, Ω^t then denotes the set of all (nonnegative) price, dividend and wage histories from period zero up to period t and ω^t its typical element. The underlying probability space for agents' beliefs is then given by $(\Omega, \mathcal{B}, \mathcal{P}^i)$ with $\mathcal{B}$ denoting the corresponding σ-Algebra of Borel subsets of Ω, and $\mathcal{P}^i$ a probability measure over $(\Omega, \mathcal{B})$.

The agents' plans will be contingent on the history ω^t, i.e., the agent chooses state-contingent consumption and stockholding functions

$$C_t^i \; : \; \Omega^t \to \mathcal{R}^+ \tag{1.9}$$

$$S_t^i \; : \; \Omega^t \to \mathcal{S} \tag{1.10}$$

The fact that C^i and S^i depend on price realizations is a consequence of optimal choice under uncertainty, given that agents consider prices to be exogenous random variables.

The previous setup is general enough to accommodate situations where agents learn about the stochastic processes governing the evolution of prices, dividends, and wages. For example, $\mathcal{P}^i$ may arise from a stochastic process describing the evolution of these variables that contains unknown parameters about which agents hold prior beliefs. The presence of unknown parameters then implies that agents update their beliefs using the observed realizations of prices, dividends and wages. A particular example of this kind will be presented in section 1.7 when we discuss learning about stock price behavior.

The probability space defined above is more general than that specified in a RE analysis of the model, where Ω contains usually only the variables that are exogenous to the model (in this case D_t and W_t), but not variables that are endogenous to the model and exogenous to the agent only (in this case P_t). Under the RE hypothesis, agents are assumed to know the pricing function $P_t((D, W)^t)$ mapping histories of dividends and wages into a market price. In that case prices carry redundant information and can be excluded from the probability space without loss of generality. The more general formulation we entertain here allows us to consider agents who do not know exactly which price materializes given a particular history of dividends and wages; our agents do have a view about the distribution of P_t conditional on $(D, W)^t$, but in their minds this is a proper distribution, not a point mass as in the RE case. Much akin to academic economists, investors in our model have not converged on a single asset pricing model that associates one market price with a given history of exogenous fundamentals.

Parametric Utility Function. To obtain closed-form solutions, we consider in the remaining part of the paper the utility function

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \qquad \text{with } \gamma > 1, \tag{1.11}$$

and also consider agents who hold rational expectations about dividends and wages ($\mathcal{P}^i$ incorporates knowledge of the process (2.6)), so as to be able to isolate the pricing effects arising from subjective capital gains beliefs. We furthermore assume that

$$\delta\beta^{RE} < 1, \tag{1.12}$$

where $\beta^{RE} \equiv (\beta^D)^{1-\gamma} e^{\gamma(\gamma-1)\sigma_D^2/2}$, which insures existence of an equilibrium under rational price expectations. Since solving the optimization problem (2.2) for general (potentially non-rational) price beliefs is non-standard, appendix 1.13.4 discusses conditions guaranteeing existence of an optimum, sufficiency of first order conditions and the existence of a recursive solution. These conditions are all satisfied for the preference specification (1.11) and the subjective price beliefs introduced in the remaining part of the paper and guarantee that the optimal solution to (2.2) takes the form

$$S_t^i = S^i\left(S_{t-1}^i, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t^i\right). \tag{1.13}$$

where m_t^i is a sufficient statistic characterizing the subjective distributions about future values of $\left(\frac{Dt+j}{D_{t+j-1}}, \frac{P_{t+j}}{D_t+j}, \frac{W_{t+j}}{D_{t+j}}\right)$ for $j > 0$.

1.6 Rational Expectations (RE) Equilibrium

As a point of reference, we determine the equilibrium stock price implied by the RE hypothesis. Appendix 1.13.5 derives the following result:

Proposition 1 *If agents hold rational expectations and if price expectations satisfy the usual transversality condition (stated explicitly in appendix 1.13.5), then RE equilibrium price is given by*

$$\frac{P_t^{RE}}{D_t} = (1 + \rho\varepsilon_t^W)^\gamma b \frac{\delta\beta^{RE}}{1 - \delta\beta^{RE}} \tag{1.14}$$

where $b \equiv E[(1+\rho\varepsilon_t^W)^{-\gamma}\left(\varepsilon_t^D\right)^{1-\gamma}] e^{\gamma(1-\gamma)\frac{\sigma_D^2}{2}}$ *and* $\beta^{RE} \equiv (\beta^D)^{1-\gamma} e^{\gamma(\gamma-1)\sigma_D^2/2}$.

The PD ratio is an iid process under RE, thus fails to match the persistence of the PD ratio observed in the data. Moreover, since the volatility of ε_t^W tends to be small, it fails to match the large variability of stock prices. Furthermore, the RE equilibrium implies a *negative* correlation between the PD ratio and expected returns, contrary to what is evidenced by survey data. To see this note that (1.14) implies

$$\ln P_{t+1}^{RE} - \ln P_t^{RE} = \ln \beta^D + \ln \varepsilon_{t+1}^P, \tag{1.15}$$

where $\varepsilon_{t+1}^P \equiv \varepsilon_{t+1}^D (1 + \rho\varepsilon_{t+1}^W)/(1 + \rho\varepsilon_t^W)$, so that one-step-ahead price growth expectations covary negatively with the current price dividend ratio.[29] Since the dividend component of returns also covaries negatively with the current price, the same holds true for expected returns.

In the interest of deriving analytical solutions, we consider below the limiting case with vanishing uncertainty $(\sigma_D^2, \sigma_W^2 \to 0)$. The RE solution then simplifies to the perfect foresight outcome

$$\frac{P_t^{RE}}{D_t} = \frac{\delta\beta^{RE}}{1 - \delta\beta^{RE}}, \tag{1.16}$$

which has prices and dividends growing at the common rate β^D.

1.7 Learning about Capital Gains and Internal Rationality

Price growth in the RE equilibrium displays only short-lived deviations from dividend growth, with any such deviation being undone in the subsequent period, see equation (1.15). Price growth in the data, however, can persistently outstrip dividend growth, thereby giving rise to a persistent increase in the PD ratio and an asset price boom; conversely it can fall persistently short of dividend growth and give rise to a price bust, see figure 1.1. This behavior of actual asset prices suggests that it is of interest to relax the RE beliefs about price behavior. Indeed, in view of the behavior of actual asset prices in the data, agents may entertain a more general model of price behavior, incorporating the possibility that the growth rate of prices persistently exceeds/falls short of the growth rate of dividends. To the extent that the equilibrium asset prices implied by these beliefs display such data-like behavior, agents' beliefs will be generically validated.

Generalized Price Beliefs. In line with the discussion in the previous paragraph, we assume agents perceive prices evolving according to the process

$$\ln P_{t+1} - \ln P_t = \ln \beta_{t+1} + \ln \varepsilon_{t+1}, \tag{1.17}$$

[29] The PD ratio under RE is proportional to $1 + \rho\varepsilon_t^W$, see equation (1.14), while ε_{t+1}^P depends inversely on $1 + \rho\varepsilon_t^W$.

where ε_{t+1} denotes a transitory shock to price growth and β_{t+1} a persistent price growth component that drifts slowly over time according to

$$\ln \beta_{t+1} \;= \ln \beta_t + \ln \nu_{t+1} \; . \tag{1.18}$$

This setup can capture periods with sustained increases in the PD ratio ($\beta_{t+1} > \beta^D$) or sustained decreases ($\beta_{t+1} < \beta^D$).[30] In the limiting case where the variance of the innovation $\ln \nu_{t+1}$ becomes small, the persistent price growth component behaves almost like a constant, as is the case in the RE solution.

For simplicity, we assume that agents perceive the innovations $\ln \varepsilon_{t+1}$ and $\ln v_{t+1}$ to be jointly normally distributed according to

$$\begin{pmatrix} \ln \varepsilon_{t+1} \\ \ln \nu_{t+1} \end{pmatrix} \sim iiN \left(\begin{pmatrix} -\frac{\sigma_\varepsilon^2}{2} \\ -\frac{\sigma_v^2}{2} \end{pmatrix}, \begin{pmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_\nu^2 \end{pmatrix} \right). \tag{1.19}$$

Since agents observe the change of the asset price, but do not separately observe the persistent and transitory elements driving it, the previous setup defines a filtering problem in which agents need to decompose observed price growth into the persistent and transitory subcomponents, so as to forecast optimally.

To emphasize the importance of learning about price behavior rather than learning about the behavior of dividends or the wage income process, which was the focus of much of an earlier literature on learning in asset markets, e.g., Timmermann (1993, 1996), we continue to assume that agents know the processes (2.6), i.e., hold rational dividend and wage expectations.

Internal Rationality of Price Beliefs. Among academics there appears to exist a widespread belief that rational behavior and knowledge of the fundamental processes (dividends and wages in our case) jointly *dictate* a certain process for prices and thus the price beliefs agents can rationally entertain.[31] If this were true, then rational behavior would imply rational expectations, so that postulating subjective price beliefs as those specified in equation (2.8) would be inconsistent with the assumption of optimal behavior on the part of agents.

This view is correct in some special cases, for example when agents are risk neutral and do not face trading constraints. If fails to be true, however, more generally. Therefore, agents in our model are 'internally rational': their behavior is optimal given an internally consistent system of subjective beliefs about variables that are beyond their control, including prices.

[30] We deliberately do not incorporate any mean-reversion into price growth beliefs as we seek to determine model-endogenous forces that lead to a reversal of asset price booms and busts, rather than having these features emerge because they are hard-wired into beliefs. Incoporating such mean reversion in prices would not be difficult though. Furthermore, as we discus below, return expectations display some degree of mean reversion even with the present specification.

[31] We often received this reaction during seminar presentations.

To illustrate this point, consider first risk neutral agents with rational dividend expectations and ignore limits to stock holdings. Forward-iteration on the agents' own optimality condition (1.42) then delivers the present value relationship

$$P_t = E_t\left[\sum_{i=1}^{T} \delta^i D_{t+i}\right] + \delta^T E_t^{\mathcal{P}^i}\left[P_{t+T}\right],$$

which is independent of the agents' own choices. Provided agents' price beliefs satisfy a standard transversality condition ($\lim_{T\to\infty} \delta^T E_t^{\mathcal{P}^i}\left[P_{t+T}\right] = 0$ for all i), then each rational agent would conclude that there must be a degenerate joint distribution for prices and dividends given by

$$P_t = E_t\left[\sum_{i=1}^{\infty} \delta^i D_{t+i}\right] \text{ a.s.} \tag{1.20}$$

Since the r.h.s of the previous equation is fully determined by dividend expectations, the beliefs about the dividend process deliver the price process compatible with optimal behavior. In such a setting, it would be plainly inconsistent with optimal behavior to assume the subjective price beliefs (2.8)-(2.9).[32]

Next, consider a concave utility function $u(\cdot)$ satisfying standard Inada conditions. Forward iteration on (1.42) and assuming an appropriate transversality condition then delivers

$$P_t\; u'(C_t^i) = E_t^{\mathcal{P}^i}\left[\sum_{j=1}^{\infty} \delta^j\; D_{t+j}\; u'(C_{t+j}^i)\right] \text{ a.s.} \tag{1.21}$$

Unlike in equation (1.20), the previous equation depends on the agent's current and future consumption. Equation (1.21) thus falls short of mapping beliefs about the dividend process into a price outcome. Indeed, given *any* equilibrium price P_t, the agent will choose her consumption plans such that (1.21) holds, i.e., such that the price equals the discounted sum of dividends, discounting with her on internally rational consumption plan.[33] Equation (1.21) thus fails to deliver any restriction on what optimizing agents can possibly believe about the price process.

With the considered non-linear utility function, we can thus simultaneously assume that agents maximize utility, hold the subjective price beliefs (2.8)-(2.9) and rational beliefs about dividends and wages.

Learning about the Capital Gains Process. The beliefs (2.8) give rise to an optimal filtering problem. To obtain a parsimonious description of

[32]See Adam and Marcet (2011) for a discussion of how in the presence of trading constraints, this conclusion breaks down, even with risk-neutral consumption preferences.

[33]This follows directly from the fact that consumption plans must satisfy (1.42) at all contingencies.

the evolution of beliefs we specify conjugate prior beliefs about the unobserved persistent component $\ln \beta_t$ at $t = 0$. Specifically, agent i's prior is

$$\ln \beta_0 \sim N(\ln m_0^i, \sigma^2), \tag{1.22}$$

where prior uncertainty σ^2 is assumed to be equal to its Kalman filter steady state value, i.e.,

$$\sigma^2 \equiv \frac{-\sigma_\nu^2 + \sqrt{(\sigma_\nu^2)^2 + 4\sigma_\nu^2\sigma_\varepsilon^2}}{2}, \tag{1.23}$$

and the prior is also assumed independent of all other random variables at all times. Equations (2.8), (2.9) and (1.22), and knowledge of the dividend and wage income processes (2.6) then jointly specify agents' probability beliefs $\mathcal{P}^i$.

The optimal Bayesian filter then implies that the posterior beliefs following some history ω^t are given by[34]

$$\ln \beta_t | \omega^t \sim N(\ln m_t^i, \sigma^2), \tag{1.24}$$

with

$$\ln m_t^i = \ln m_{t-1}^i - \frac{\sigma_v^2}{2} + g\left(\ln P_t - \ln P_{t-1} + \frac{\sigma_\varepsilon^2 + \sigma_v^2}{2} - \ln m_{t-1}^i\right) \tag{1.25}$$

$$g = \frac{\sigma^2}{\sigma_\varepsilon^2}. \tag{1.26}$$

Agents' beliefs can thus be parsimoniously summarized by a single state variable (m_t^i) describing agents' degree of optimism about future capital gains. These beliefs evolve recursively according to equation (2.11) and imply that

$$E_t^{\mathcal{P}^i}\left[\frac{P_{t+1}}{P_t}\right] = e^{\ln m_t^i}\, e^{\sigma^2/2}, \tag{1.27}$$

which is - up to the presence of a log and exponential transformation and some variance correction terms - identical to the adaptive prediction model considered in section 1.4.3.

Nesting PF Equilibrium Expectations. The subjective price beliefs (2.8),(2.9) and (1.22) generate perfect foresight equilibrium price expectations in the special case in which prior beliefs are centered at the growth rate of dividends, i.e.,

$$\ln m_0^i = \ln \beta^D,$$

and when considering the limiting case with vanishing uncertainty, where $(\sigma_\varepsilon^2, \sigma_\nu^2, \sigma_D^2, \sigma_W^2) \to 0$. Agents' prior beliefs at $t = 0$ about price growth in

[34]See theorem 3.1 in West and Harrison (1999). Choosing a value for σ^2 different from the steady state value (1.23) would only add a deterministically evolving variance component σ_t^2 to posterior beliefs with the property $\lim_{t\to\infty} \sigma_t^2 = \sigma^2$, i.e., it would converge to the steady state value.

$t \geq 1$ then increasingly concentrates at the perfect foresight outcome $\ln \beta^D$, see equations (2.8) and (2.9). With price and dividend expectations being at their PF value, the perfect foresight price $PD_0 = \delta\beta^{RE}/(1-\delta\beta^{RE})$ becomes the equilibrium outcome at $t = 0$ in the limit. Importantly, it continues to be possible to study learning dynamics in the limit with vanishing risk: keeping the limiting ratio $\sigma_\nu^2/\sigma_\varepsilon^2$ finite and bounded from zero as uncertainty vanishes, the Kalman gain parameter g defined in (2.12), remains well-specified in the limit and satisfies $\lim \frac{\sigma_\nu^2}{\sigma_\varepsilon^2} = \lim \frac{g^2}{1-g}$. We will exploit this fact in section 1.9 when presenting analytical results.

1.8 Dynamics under Learning

This section explains how equilibrium prices are determined under the subjective beliefs introduced in the previous section and how they evolve over time.

Agents' stock demand is given by equation (1.13). Stock demand depends on the belief m_t^i, which characterizes agents' capital gains expectations. These beliefs evolve according to (2.11). As a benchmark, we shall now assume that all agents hold identical beliefs ($m_t^i = m_t$ for all i). While agents may initially hold heterogenous prior beliefs m_0^i, heterogeneity would asymptotically vanish because all agents observe the same price history. The asset dynamics derived under the assumption of identical beliefs thus describe the long-run outcome of the model.

Using this assumption and imposing market clearing in periods t and $t-1$ in equation (1.13) shows that the equilibrium price in any period $t \geq 0$ solves

$$1 = S\left(1, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t\right), \tag{1.28}$$

which exploits the fact that the total supply of stocks is equal to one.

The beliefs m_t and the price dividend ratio P_t/D_t are now simultaneously determined via equations (2.11) and (2.16). Unfortunately, this simultaneity could give rise to multiple market clearing price and belief pairs, due to a complementarity between realized capital gains and expected future capital gains.[35] While this multiplicity may be a potentially interesting avenue to explain asset price booms and busts, analyzing price dynamics within such a setting would require introducing non-standard features, such as an equilibrium selection device for periods in which there are multiple solutions to (2.11) and (2.16). Instead, we resort to a standard approach of using only lagged information for updating beliefs.

[35] Intuitively, a higher PD ratio implies higher realized capital gains and thus higher expectations of future gains via equation (2.11). Higher expected future gains may in turn induce a higher willingness to pay for the asset, thereby justifying the higher initial PD ratio.

Appendix 1.13.6 shows that the simultaneity can be overcome by slightly modifying the information structure. The modification is relatively straightforward and consists of assuming that agents observe at any time t information about the lagged temporary price growth component ε_{t-1} entering equation (2.8). The appendix then shows that Bayesian updating implies that

$$\ln m_t = \ln m_{t-1} + g\left(\ln P_{t-1} - \ln P_{t-2} - \ln m_{t-1}\right) + g \ln \varepsilon_t^1, \tag{1.29}$$

where updating now occurs using only lagged price growth (even though agents do observe current prices) and where $\ln \varepsilon_t^1 \sim iiN(\frac{-\sigma_\varepsilon^2}{2}, \sigma_\varepsilon^2)$ is a time t innovation to agent's information set (unpredictable using information available to agents up to period $t-1$), which reflects the information about the transitory price growth component ε_{t-1} received in period t.

With this slight modification, agents' beliefs m_t are now pre-determined at time t, so that the economy evolves according to a uniquely determined recursive process: equation (2.16) determines the market clearing price for period t given the beliefs m_t and equation (1.29) determines how time t beliefs are updated following the observation of the new market clearing price.[36]

1.9 Equilibrium: Analytic Findings

This section derives a closed form solution for the equilibrium asset price for the special case where all agents hold the same subjective beliefs $\mathcal{P}$ and where these beliefs imply no (or vanishing) uncertainty about future prices, dividends and wages. While the absence of uncertainty is unrealistic from an empirical standpoint, it allows deriving key insights into how the equilibrium price depends on agents' beliefs, as well as on how prices and beliefs evolve over time.[37] The empirically more relevant case with uncertainty will be considered in section 1.10 using numerical solutions.

The next section provides a closed form expression for the equilibrium PD ratio as a function of agents' subjective expectations about future stock market returns. Section 1.9.2 then discusses the pricing implications of this result for the subjective capital gains beliefs introduced in section 1.7. Finally, section 1.9.3 shows how the interaction between asset price behavior and subjective belief revisions can temporarily de-link asset prices from their fundamental value, i.e., give rise to a self-feeding boom and bust in asset prices along which subjective expected returns rise and fall.

[36]There could still be an indeterminacy arising from the fact that $S(\cdot)$ is non-linear, so that equation (2.16) may not have a unique solution, but we have encountered such problems neither in our analytical solution nor when numerically solving the model.

[37]In the absence of uncertainty one can evaluate more easily the expectations of nonlinear functions of future variables showing up in agents' FOCs.

1.9.1 Main Result

The following proposition summarizes our main finding:[38]

Proposition 2 *Suppose* $u(C) = C^{1-\gamma}/(1-\gamma)$, *agents' beliefs* $\mathcal{P}$ *imply no uncertainty about future prices, dividends and wages, and*

$$\lim_{T\to\infty} E_t^{\mathcal{P}} R_T > 1 \text{ and } \lim_{T\to\infty} E_t^{\mathcal{P}} \left(\sum_{j=1}^{T} \left(\prod_{i=1}^{j} \frac{1}{R_{t+i}} \right) W_{t+j} \right) < \infty, \tag{1.30}$$

then the equilibrium PD ratio in period t is given by

$$\begin{aligned} \frac{P_t}{D_t} &= \left(1 + \frac{W_t}{D_t}\right) \sum_{j=1}^{\infty} \left(\left(\delta^{\frac{1}{\gamma}}\right)^j \left(E_t^{\mathcal{P}} \prod_{i=1}^{j} \frac{1}{R_{t+i}} \right)^{\frac{\gamma-1}{\gamma}} \right) \\ &\quad - \frac{1}{D_t} E_t^{\mathcal{P}} \left(\sum_{j=1}^{\infty} \left(\prod_{i=1}^{j} \frac{1}{R_{t+i}} \right) W_{t+j} \right) \end{aligned} \tag{1.31}$$

Conditions (1.30) insure that the infinite sums in the pricing equation (1.31) converge.[39] Under the additional assumption that agents hold rational wage and dividend expectations, equation (1.31) simplifies further to

$$\begin{aligned} \frac{P_t}{D_t} &= (1+\rho) \sum_{j=1}^{\infty} \left(\left(\delta^{\frac{1}{\gamma}}\right)^j \left(E_t^{\mathcal{P}} \prod_{i=1}^{j} \frac{1}{R_{t+i}} \right)^{\frac{\gamma-1}{\gamma}} \right) \\ &\quad - \rho \left(\sum_{j=1}^{\infty} \left(\beta^D\right)^j \left(E_t^{\mathcal{P}} \prod_{i=1}^{j} \frac{1}{R_{t+i}} \right) \right). \end{aligned} \tag{1.32}$$

We now discuss the implications of equation (1.32), focusing on the empirically relevant case where $\rho > 0$ and $\gamma > 1$.

Consider first the upper term on the r.h.s. of equation (1.32), which is decreasing in the expected asset returns. This emerges because for $\gamma > 1$ the wealth effect of a change in return expectations then dominates the substitution effect, so that expected asset demand and therefore the asset price has a tendency to decrease as return expectations increase. The negative wealth effect thereby increases in strength if the ratio of wage to dividend income (ρ) increases. This is the case because higher return expectations also reduce the present value of wage income.

Next, consider the lower term on the r.h.s. of equation (1.32), including the negative sign pre-multiplying it. This term depends positively on the

[38] The proof can be found in appendix 1.13.7.

[39] These are satisfied, for example, for the expectations associated with the perfect foresight RE solution. Equation (1.31) then implies that the PD ratio equals the perfect foresight PD ratio (1.16), as is easily verified. Conditions (1.30) are equally satisfied for the subjective beliefs defined in section 1.7, when considering the case with vanishing uncertainty $(\sigma_\varepsilon^2, \sigma_\nu^2, \sigma_D^2, \sigma_W^2) \to 0$.

expected returns and captures a substitution effect that is associated with increased return expectations. This substitution effect only exists if $\rho > 0$, i.e., only in the presence of non-dividend income, and it is increasing in ρ. It implies that increased return expectations are associated with increased stock demand and thus with a higher PD ratio in equilibrium. It is this term that allows the model to match the positive correlation between expected returns and the PD ratio.

This substitution effect is present even in the limiting case with log consumption utility ($\gamma \to 1$). The upper term on the r.h.s. of equation (1.32) then vanishes because the substitution and wealth effects associated with changes in expected returns cancel each other, but the lower term still induces a positive relationship between prices and return expectations. The substitution effect is also present for $\gamma > 1$ and can then dominate the negative wealth effect arising from the upper term on the r.h.s. of (1.32). Consider, for example, the opposite limit with $\gamma \to \infty$. Equation (1.32) then delivers

$$\frac{P_t}{D_t} = \sum_{j=1}^{\infty} \left(1 + \rho \sum_{j=1}^{\infty} \left(1 - \left(\beta^D\right)^j\right)\right) \left(E_t^{\mathcal{P}} \prod\nolimits_{i=1}^{j} \frac{1}{R_{t+i}}\right).$$

Since $\beta^D > 1$, there is a positive relationship between prices and expected asset returns, whenever ρ is sufficiently large. The two limiting results ($\gamma \to 1$ and $\gamma \to \infty$) thus suggest that for sufficiently large ρ the model can generate a positive relationship between return expectations and the PD ratio, in line with the evidence obtained from survey data.

1.9.2 PD Ratio and Expected Capital Gains

We now consider the implications of equation (1.32) for the subjective capital gains beliefs introduced in section 1.7.[40] Equation (1.32) implies a non-linear relationship between the PD ratio and the subjective capital gain expectations m_t, but one cannot obtain a closed-form solution for the PD ratio as a function of the capital gains expectations.[41] Figure 1.5 depicts the relationship between

[40] Appendix 1.13.8 proves that condition (1.30) is then satisfied for all beliefs $m_t > 0$.

[41] More precisely, with vanishing uncertainty the beliefs from section 1.7 imply

$$E_t^{\mathcal{P}}\left[P_{t+i}\right] = \left(m_t\right)^i P_t,$$

which together with perfect foresight about dividends allows expressing agents' expectations of future inverse returns as a function of m_t and the current PD ratio:

$$E_t^{\mathcal{P}} \frac{1}{R_{t+i}} = \frac{E_t^{\mathcal{P}} P_{t+i-1}}{E_t^{\mathcal{P}} P_{t+i} + E_t^{\mathcal{P}} D_{t+i}} = \frac{\left(m_t\right)^{i-1} \frac{P_t}{D_t}}{\left(m_t\right)^i \frac{P_t}{D_t} + \left(\beta^D\right)^i}.$$

Substituting this into (1.32) one can solve numerically for P_t/D_t as a function of m_t.

the PD ratio and m_t using the parameterization employed in our quantitative application in section 1.10, but abstracting from future uncertainty.[42]

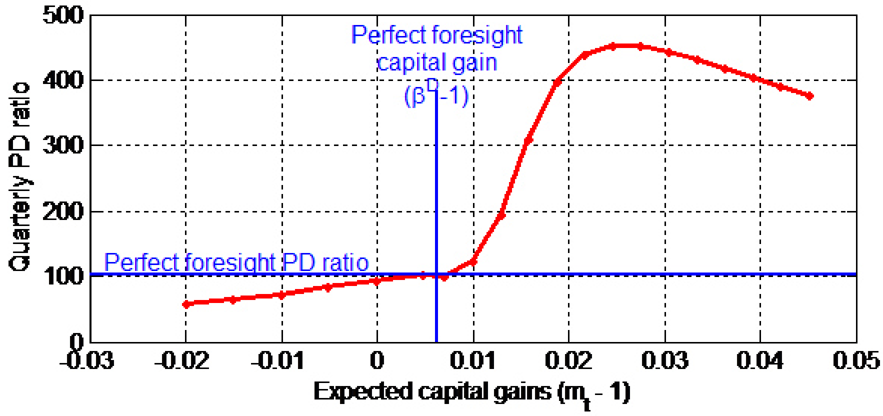

Figure 1.5: PD ratio and expected capital gains (vanishing noise)

Figure 1.5 shows that there is a range of price growth beliefs in the neighborhood of the perfect foresight value ($m_t = \beta^D$) over which the PD ratio depends positively on expected price growth, similar to the positive relationship between expected returns and the PD ratio derived analytically in the previous section. Over this range, the substitution effect dominates the wealth effect because our calibration implies that dividend income finances only a small share of total consumption (approximately 4.3%). As a result, stock market wealth is only a small share of the total present value of household wealth (the same 4.3%) when beliefs assume their perfect foresight value ($m_t = \beta^D$).

Figure 1.5 also reveals that there exists a capital gains belief beyond which the PD ratio starts to decrease. Mathematically, this occurs because if $m_t \to \infty$, expected returns also increase without bound[43], so that $E_t^{\mathcal{P}} \prod_{i=1}^{j} \frac{1}{R_{t+i}} \to 0$. From equation (1.32) one then obtains $\frac{P_t}{D_t} \to 0$.

The economic intuition for the existence of a maximum PD ratio is as follows: for higher m_t the present value of wage income is declining, as in-

[42] The parameterization assumes a moderate degree of risk aversion $\gamma = 2$, a quarterly discount factor of $\delta = 0.995$, quarterly real dividend growth equal to the average postwar growth rate of real dividends $\beta^D = 1.0048$, and $\rho = 22$ to match the average dividend-consumption ratio in the U.S. over 1946-2011, see section 1.10 for further details.

[43] This follows from $E_t^{\mathcal{P}} R_{t+i+1} = E_t^{\mathcal{P}} \frac{P_{t+i+1}+D_{t+i+1}}{P_{t+i}} > E_t^{\mathcal{P}} \frac{P_{t+i+1}}{P_{t+i}} = m_t$.

creased price growth optimism implies higher expected returns[44] and therefore a lower discount factor. This can be seen by noting that the FOC (1.42) can alternatively be written as

$$1 = \delta E_t^{\mathcal{P}} \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} \right],$$

which implies that increased return expectations $E_t^{\mathcal{P}} R_{t+1}$ imply a lower discount factor $\delta E_t^{\mathcal{P}} \left[(C_{t+1}/C_t)^{-\gamma} \right]$.[45] With increased optimism, the present value of wage income thus falls. At the same time, stock market wealth initially increases strongly. Indeed, at the maximum PD ratio, stock market wealth amounts to approximately 4.5 times the value it assumes in the perfect foresight solution, see figure 1.5. This relative wealth shift has the same effect as a decrease in the wage to non-wage income ratio ρ. As argued in section 1.9.1, for sufficiently small values of ρ the income effect starts to dominate the substitution effect, so that prices start to react negatively to increased return optimism.

1.9.3 Endogenous Boom and Bust Dynamics

We now explain how the interplay between price realizations and belief updating can temporarily de-link asset prices from their fundamental values. This process emerges endogenously and takes the form of a sustained asset price boom along which expected returns rise and that ultimately results in a price bust along which expected returns fall. This feature allows the model to generate volatile asset prices and to capture the positive correlation between expected returns and the PD ratio.

Consider figure 1.5 and a situation in which agents become optimistic, in the sense that their capital gains expectations m_t increase slightly above the perfect foresight value $m_{t-1} = \beta^D$ entertained in the previous period.[46] Figure 1.5 shows that this increase in expectations leads to an increase in the PD ratio, i.e., $P_t/D_t > P_{t-1}/D_{t-1}$. Moreover, due to the relatively steep slope of the PD function, realized capital gains will strongly exceed the initial increase in expected capital gains. The belief updating equation (1.29) then implies further upward revisions in price growth expectations and thus further capital gains, leading to a sustained asset price boom in which the PD ratio and return expectations jointly move upward.

The price boom comes to an end when expected price growth reaches a

[44] This is hown in appendix 1.13.9, which depicts the relationship between expected capital gains and expected returns at various forecast horizons.

[45] This holds true under the maintained assumption of no or vanishing uncertainty.

[46] In the model with uncertainty, such upward revisions can be triggered by fundamentals, e.g., by an exceptionally high dividend growth realization in the previous period, which is associated with an exceptionally high price growth realization.

level close to where the PD function in figure 1.5 reaches its maximum.[47] At this point, stock prices grow at most at the rate of dividends (β^D), but agents hold considerably more optimistic expectations about future capital gains ($m_t > \beta^D$). Investors' high expectations will thus be disappointed, which subsequently leads to a reversal.

The previous dynamics are also present in a stochastic model considered in the next sections. They introduce low frequency movements in the PD ratio, allowing the model to replicate boom and bust dynamics and thereby to empirically plausible amounts of asset price volatility, despite assuming standard consumption preferences. These dynamics also generate a positive correlation between the PD ratio and expected returns.[48]

1.10 Historical PD Ratio and Survey Evidence

This section considers the asset pricing model with subjective beliefs and uncertainty; it shows that the model can successfully replicate the low-frequency movements in the postwar U.S. PD ratio, as well as the available survey evidence.

Solving the non-linear asset pricing model with subjective beliefs is computationally costly, which prevents us from pursuing formal estimation or moment matching. We thus resort to calibration.

Table 1.6 reports the calibrated parameters and the calibration targets.[49] The mean and standard deviation of dividend growth (β^D and σ_D) are chosen to match the corresponding empirical moments of the U.S. dividend process. The ratio of non-dividend to dividend income (ρ) is chosen to match the average dividend-consumption ratio in the U.S. for 1946-2011.[50] The standard deviation of wage innovations (σ_W) and the covariance between wage and dividend innovations (σ_{DW}) are chosen, in line with Campbell and Cochrane (1999), such that the correlation between consumption and dividend growth is 0.2 and the standard deviation of consumption growth is one seventh of the the standard deviation of dividend growth.[51] The perceived uncertainty in stock price growth (σ_ε) is set equal to the empirical standard deviation of stock price growth.[52]

[47] In the model with noise, fundamental shocks, e.g., a low dividend growth realization, can cause the process to end well before reaching this point.

[48] While the arguments above only show that expected capital gains correlate positively with the PD ratio, Appendix 1.13.9 shows that expected capital gains and expected returns comove positively, so that expected returns also comove positively with the PD ratio.

[49] The targets are chosen to match features of the fundamental processes emphasized in the asset pricing literature.

[50] See appendix 1.13.3 for further details.

[51] For details on how this can be achieved, see appendix 1.13.3.

[52] Since the gain parameter g will be small, the contribution of σ_ν^2 in (2.8) is negligable.

Parameter	Value	Calibration Target
β^D	1.0048	average quarterly real dividend growth
σ_D	0.0192	std. deviation quarterly real dividend growth
ρ	22	average consumption-dividend ratio
σ_{DW}	$-3.74 \cdot 10^{-4}$	*jointly chosen* s.t. $corr_t(C_{t+1}/C_t, D_{t+1}/D_t) = 0.2$
σ_W	0.0197	and $std_t(C_{t+1}/C_t) = \frac{1}{7} std_t(D_{t+1}/D_t)$
σ_ε	0.0816	std. deviation of quarterly real stock price growth

Table 1.6: Model calibration

This leaves us with four remaining parameters: the belief updating parameter g, the initial price growth belief $m_{Q1:1946}$, the time discount factor δ and the risk aversion parameter γ. We choose $g = 0.02515$ and $m_{Q1:1946} = -1.11\%$, in line with the values employed in constructing figure 1.3, which allowed matching the UBS survey expectations. We then assume risk aversion of $\gamma = 2$ and choose the quarterly discount factor δ, so as to obtain a good match between the model-implied and the empirical PD ratio over the postwar period. It turns out that $\delta = 0.995$ achieves a good fit.

Figure 1.6 depicts the equilibrium PD ratio obtained from numerically solving the asset pricing model with uncertainty, together with the equilibrium PD ratio in the absence of uncertainty analyzed in the previous section.[53] While the presence of price, dividend, and wage risk lowers the equilibrium PD ratio compared to a setting without risk, the functional form of the relationship remains qualitatively unchanged. The findings obtained in the previous section thus continue to apply in the presence of quantitatively realistic amounts of uncertainty.

[53]The numerical solution is obtained by numerically determining the stock demand function (1.13) solving the FOC (1.42) under the subjectively perceived dividend, wage and price dynamics, where agents understand that their beliefs evolve according to (1.29). The PD ratio as a function of m_t depicted in figure 1.6 is determined from the market clearing condition (2.16) assuming $W_t/D_t = \rho$, to be comparable with the value this variable assumes in the vanishing risk limit. We verified that in the limiting case without uncertainty, our numerical solution algorithm recovers the analytical solution derived in proposition 2. Furthermore, in the case with uncertainty, we insure the accuracy of the numerical solution by verifying that the Euler equation errors are in the order of 10^{-5} over the relevant area of the state space. Insuring this requires a considerable amount of adjustment by hand of the grid points and grid size used for spanning the model's state space. This prevents us from formally estimating the model, as the model cannot be solved with sufficient accuracy using an automated procedure. Further details of the solution approach are described in appendix 1.13.10. The MatLab code used for solving the model is available upon request.

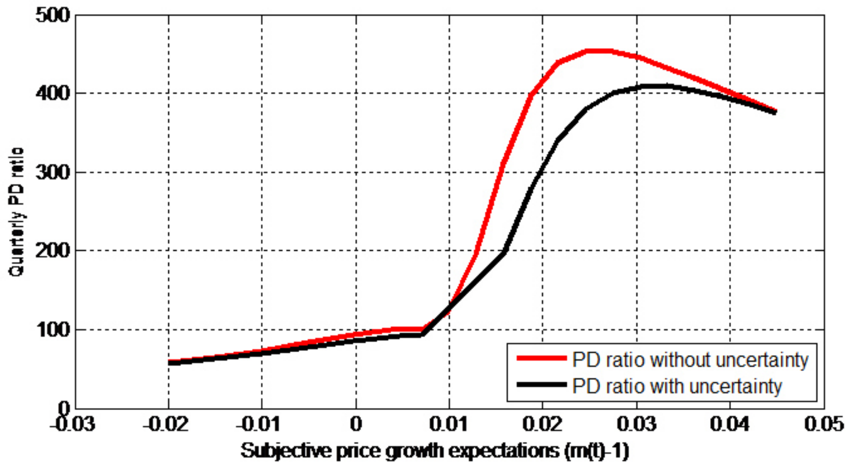

Figure 1.6: The effects of uncertainty on the equilibrium PD ratio

We now evaluate the ability of the model to replicate the postwar time series of the PD ratio. We do so by first feeding the historical capital gains into our model-based belief updating equation (1.29), so as to obtain a model-implied value for expected capital gains.[54] The resulting series is shown in figure 1.7. It displays a strong rise and fall of price growth expectations around the year 2000, as well as relatively low capital gains expectations from the mid 1970's to the mid 1980's. In a second step, we use the model-implied equilibrium PD function to derive a model-implied time series for the PD ratio associated with the model-based beliefs. Figure 1.8 depicts this model-implied PD series and compares it with historical PD series.[55]

[54] We thereby shut down all other sources of information about price growth, i.e., set $\ln \varepsilon_t^1 = 0$ for all t in equation (1.29).

[55] When computing the model-implied PD ratio, we set the non-dividend to dividend income ratio equal to its steady state value ($W_t/D_t = \rho$), so as to obtain only pricing effects due to variation in subjective capital gains expectations. The effects of fundamental shocks to wages and dividends will be considered in section 1.11.

Figure 1.7: Price growth expectations implied by Bayesian updating and historical price growth information

Figure 1.8 reveals that the model captures a lot of the low-frequency variation in the historically observed PD ratio. It captures particularly well the variations before the year 2000, including the strong run-up in the PD ratio from the mid 1990's to year 2000. The model also predicts a strong decline of the PD ratio after the year 2000, but overpredicts the decline relative to the data. For the period up to and including the year 2000, the variance of the gap between the model predicted PD and the PD in the data amounts to just 20.1% of the overall variance of the PD in the data. In this sense, the subjective belief model is capable of capturing approximately 79.9% of the variation of the PD ratio in the data. Since the fit deteriorates some time after the year 2000, it explains - using the same measure - about 52.5% of the variance for the full sample. We find this to be a remarkable result.

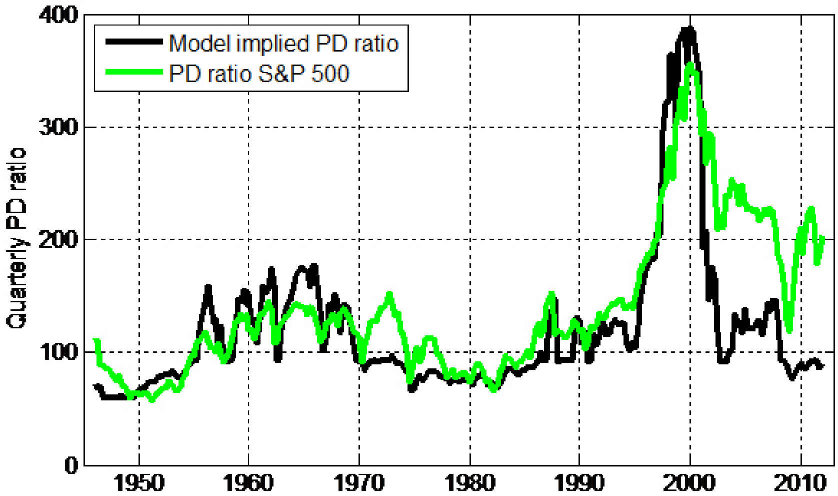

Figure 1.8: PD ratio - model vs. data

Figure 1.9 depicts the model-implied price growth expectations and those implied by the UBS survey.[56] While the model fits the survey data overall well, the model predicts after the year 2003 considerably lower capital gains expectations, which partly explains why the model underpredicts the PD ratio in figure 1.8 towards the end of the sample period. Yet, the expectations gap in figure 1.9 narrows considerably after the year 2004, while this fails to be the case in figure 1.8. Underprediction of expected price growth thus explains only partly the deterioration of the fit of the PD ratio towards the end of the sample period .

The gap after the year 2000 emerging in figure 1.8 is hardly surprising, given the empirical evidence presented in figure 1.4, which shows that the relationship between the PD ratio and the expectations implied by equation (1.5) has shifted upward in the data following the year 2000. While we can only speculate about potential reasons causing this shift, the exceptionally low real interest rates implemented by the Federal Reserve following the reversal of the tech stock boom and following the collapse of the subsequent housing boom may partly contribute to the observed discrepancy. Formally incorporating the effects of monetary policy decisions - while of interest - is beyond the scope of the present paper.

[56] See footnote 25 for how to compute price growth expectations from the UBS survey.

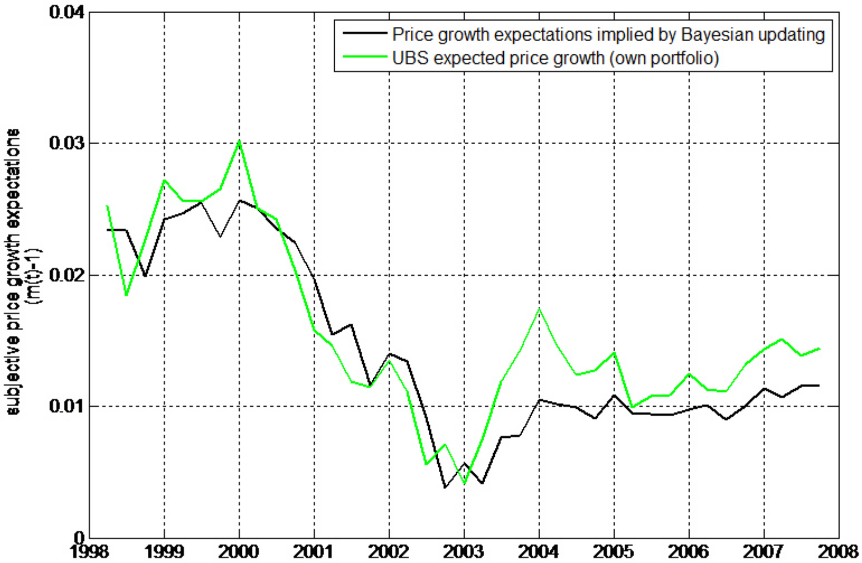

Figure 1.9: Price growth expectations: UBS survey vs. Bayesian updating model

1.11 Model Simulations

The previous section evaluated to what extent subjective belief updating dynamics alone can explain the behavior of the PD ratio in the data, but it ignored the role of the fundamental dividend and wage processes as ultimate drivers of asset price and belief dynamics. This section evaluates the ability of the model to replicate key asset pricing moments, using model simulations with dividend and wage shocks as fundamental drivers.

To do so, we compare the asset pricing moments in the data to those obtained from simulating the model, considering both the model with subjective beliefs as well as the RE model. We use the parameters from table 1.6 to simulate the model and formally evaluate the model fit by reporting t-statistics for a number of asset pricing moments.[57]

Table 1.7 reports the data moments (column 2 of the table), the moments of the subjective beliefs models and the implied t-statistic (columns 3 and 4), as well as the moments and t-statistics of the RE version of the model (columns 5 and 6).[58] The first eight asset pricing moments listed in the table are those

[57]The t-statistic is based on an estimate of the standard deviation of the data moment as a measure of uncertainty, where we estimate the standard deviation of the moment in the data using standard procedures. This delivers an asymptotically valid t-test given the parameter values.

[58]All variables are reported in terms of quarterly real values.

considered in Adam, Marcet, and Nicolini (2016); we augment these by the correlation between the PD ratio and expected stock returns, as implied by the UBS survey data.

The model with subjective beliefs turns out to be able to quantitatively account for many asset pricing moments, even though parameters have not been chosen to maximize the fit.[59] The RE version of the model performs rather poorly. Besides generating insufficient asset price volatility, it wrongly predicts a negative sign for the correlation between the PD ratio and investors' expected returns.

Table 1.7 reveals that the subjective belief model quantitatively replicates 7 of the 9 considered moments at the 1% significance level, while the RE version matches only 3 of the 9 moments. The learning model replicates particularly well the mean of the PD ratio (denoted by E[PD] in the table), the high autocorrelation of the PD ratio (Corr[PD_t,PD_{t-1}]), the regression coefficient obtained from regressing 5 year ahead excess returns on the current PD ratio (c), as well as the R^2 of that regression (R^2).[60] The model similarly matches the mean of the stock returns (E[r^s]) and the positive correlation between the PD ratio and expected returns (Corr[PD_t,$\mathrm{E}_t^{\mathcal{P}}\mathrm{R}_{t+1}$]). It generates a somewhat too high value for the standard deviation of the PD ratio (Std[PD]) and - as a result - predicts a too high value for the standard deviation of stock returns (Std[r^s]). The learning model also misses the equity premium, although it produces about half of the premium observed in the data. This is a considerable success, given the low degree of risk aversion assumed ($\gamma = 2$).

	U.S. Data	**Subj. Beliefs**		**RE**	
	Moment	Moment	t-stat	Moment	t-stat
E[PD]	139.7	122.2	0.70	105.5	1.37
Std[PD]	65.3	97.3	-2.17	3.94	4.15*
Corr[$\mathbf{PD}_t$, $\mathbf{PD_{t-1}}$]	0.98	0.98	0.54	-0.0058	>100*
Std[$\mathbf{r}^s$]	8.01	9.44	-3.57*	4.23	9.50*
c	-0.0041	-0.0049	0.67	-0.0126	7.08*
R^2	0.24	0.18	0.47	0.12	0.93
E[r^s]	1.89	1.93	-0.09	1.50	0.84
E[r^b]	0.13	0.97	-5.10*	1.50	-8.28*
UBS Survey Data:					
Corr[$\mathbf{PD}_t$,$\mathbf{E}_t^{\mathcal{P}}\mathbf{R}_{t+1}$]	0.79	0.85	-0.79	-0.99	24.86*

* indicates rejection at the 1% level

Table 1.7: Asset pricing moments

Since none of these moments have been targeted when calibrating the

[59]While this would be desirable, numerically solving the model with high accuracy is rather time-consuming.

[60]The regression also includes a constant, which is statistically insignificant and whose value is not reported.

model, the ability of the subjective belief model to quantitatively replicate the data moments is surprisingly good. This is especially true when compared to the performance of the model under RE. Comparing the last column in table 1.7 to column 4 in the same table shows that the t-statistics all increase (in absolute terms) when imposing RE, with some increases being quite dramatic.

The RE version of the model produces insufficient asset price volatility, i.e., too low values for the standard deviation of the PD ratio and of stock returns. It also produces a tiny equity premium only and gets the sign of the correlation between the PD ratio and expected stock returns wrong. This highlights the strong quantitative improvement in the empirical performance obtained by incorporating subjective belief dynamics. It also highlights that - according to our model - asset price volatility is to a large extent due to subjective belief dynamics.

1.12 Conclusions

We present a model with rationally investing agents that gives rise to market failures in the sense that the equilibrium stock price deviates from its fundamental value. These deviations take the form of asset price boom and bust cycles that are fueled by the belief-updating dynamics of investors who behave optimally given their imperfect knowledge of the world. Investors update beliefs about market behavior using observed market outcomes and Bayes' law, causing their subjective expectations about future capital gains to comove positively with the price-dividend ratio, consistent with the evidence available from investor surveys. As we argue, this feature cannot be replicated within asset pricing models that impose rational price expectations.

We relax slightly the RE assumption but maintain full rationality of investors. The fact that a fairly small deviation from a standard asset pricing model significantly improves the empirical fit of the model strongly suggests that issues of learning are important when accounting for stock price fluctuations. Indeed, our empirical analysis shows that more than half of the observed variation of the S&P500 PD ratio over the post-war period can be accounted for by variations in subjective beliefs.

If asset price dynamics are to a large extent influenced by investors' subjective belief dynamics, i.e., by subjective optimism and pessimism, then the asset price fluctuations observed in the data are to considerable extent inefficient. Due to a number of simplifying assumptions, this did not yield adverse welfare implications within the present setup.[61] For more realistic models incorporating investor heterogeneity, endogenous output or endogenous stock supply, such fluctuations can give rise to significant distortions that affect welfare. Exploring these within a setting that gives rise to quantitatively credible amounts of asset price fluctuations appears to be an interesting avenue for further re-

[61] This is true if one evaluates welfare using ex-post realized consumption.

search. Such research will in turn lead to further important questions, such as whether policy can and should intervene with the objective to stabilize asset prices.

1.13 Appendix

1.13.1 Data Sources

Stock price data: our stock price data is for the United States and has been downloaded from 'The Global Financial Database' [62]. The period covered is Q1:1946-Q1:2012. The nominal stock price series is the 'SP 500 Composite Price Index (w/GFD extension)' (Global Fin code '_SPXD'). The daily series has been transformed into quarterly data by taking the index value of the last day of the considered quarter. To obtain real values, nominal variables have been deflated using the 'USA BLS Consumer Price Index' (Global Fin code 'CPUSAM'). The monthly price series has been transformed into a quarterly series by taking the index value of the last month of the considered quarter. Nominal dividends have been computed as follows

$$D_t = \left(\frac{I^D(t)/I^D(t-1)}{I^{ND}(t)/I^{ND}(t-1)} - 1 \right) I^{ND}(t)$$

where I^{ND} denotes the 'SP 500 Composite Price Index (w/GFD extension)' described above and I^D is the 'SP 500 Total Return Index (w/GFD extension)' (Global Fin code '_SPXTRD '). We first computed monthly dividends and then quarterly dividends by adding up the monthly series. Following Campbell (2003), dividends have been deseasonalized by taking averages of the actual dividend payments over the current and preceding three quarters.

Stock market survey data: The UBS survey is the UBS Index of Investor Optimism, which is available (against a fee) from the Roper Center at the University of Connecticut.[63]

The quantitative question on stock market expectations has been surveyed over the period Q2:1998-Q4:2007 with 702 responses per month on average and has thereafter been suspended. For each quarter we have data from three monthly surveys, except for the first four quarters and the last quarter of the survey period where we have only one monthly survey per quarter. The Shiller survey data covers individual investors over the period Q1:1999Q1-Q4:2012 and has been kindly made available to us by Robert Shiller at Yale University. On average 73 responses per quarter have been recorded for the question on stock price growth. Since the Shiller data refers to the Dow Jones, we used the PD ratio for the Dow Jones, which is available at http://www.djaverages.com/, to compute correlations. The CFO survey is collected by Duke University and CFO magazine and collects responses from U.S. based CFOs over the period Q3:2000-Q4:2012 with on average 390 responses per quarter, available at http://www.cfosurvey.org/ .

Inflation expectations data: The Survey of Professional Forecasters

[62] http://www.globalfinancialdata.com

[63] http://www.ropercenter.uconn.edu/ data_access/data/datasets/ubs_investor.html

(SPF) is available from the Federal Reserve Bank of Philadelphia.[64] The Michigan Surveys of Consumers are collected by Thomson Reuters/University of Michigan.[65]

1.13.2 Details of the t-Test in Section 1.4.2

Under the RE hypothesis, equations (1.1) and (1.3) both hold for the same parameters a^N, c^N, given any horizon N. These two equations define a standard SUR model. Dependent variables are $E_t^{\mathcal{P}} R_{t+N}^N$ and R_{t+N}^N, where the latter is the N-period rate of return and $E_t^{\mathcal{P}} R_{t+N}^N$ is the observed survey expectation at time t, explanatory variables in both equations are $x_t = (1, \frac{P_t}{D_t})$, satisfying the orthogonality conditions (1.2)-(1.4). For expositional clarity we relabel the true parameters in equation (1.3) as $\left(\bar{a}^N, \bar{c}^N\right)$. The aim is to design efficient estimators of the true parameters $\beta_0^N \equiv (a^N, c^N, \bar{a}^N, \bar{c}^N)$ and to test the hypothesis $H_0 : c^N = \bar{c}^N$.

As is standard in SUR models, without any additional assumption on the distribution of $u, \varepsilon, P/D$, the OLS estimator equation by equation β_T defined by

$$\beta_T \equiv \begin{bmatrix} a_T^N \\ c_T^N \\ \bar{a}_T^N \\ \bar{c}_T^N \end{bmatrix} = \left(\sum_{t=1}^{T} x_t x_t' \otimes I_2 \right)^{-1} \sum_{t=1}^{T} x_t \begin{bmatrix} E_t^{\mathcal{P}} R_{t+N}^N \\ R_{t+N}^N \end{bmatrix},$$

where I_2 is a 2×2 identity matrix, is consistent and efficient among the set of estimators using only orthogonality conditions (1.2)-(1.4).

To simplify on notation we now drop the superscripts N in the remaining part of this appendix. As is well known, with stationarity, strong ergodicity and bounded second moments, the estimator is consistent and its asymptotic distribution as $T \to \infty$ is given by

$$\sqrt{T} \left(\beta_T - \beta_0\right) \to N \left(0, \ \left[E(x_t x_t') \otimes I_2\right]^{-1} \ S_w \ \left[E(x_t x_t') \otimes I_2\right]^{-1}\right), \qquad (1.33)$$

where

$$\begin{aligned} S_w &= \Gamma_0 + \sum_{k=1}^{\infty} \Gamma_k + \Gamma_k' \\ \Gamma_k &= E\left(\begin{bmatrix} u_t \\ u\varepsilon_t \end{bmatrix} \left[u_{t-k}, u\varepsilon_{t-k}\right] \otimes x_t x_{t-k}' \right), \end{aligned}$$

where $u\varepsilon_t \equiv u_t + \varepsilon_{t+N}$. To build the test-statistic, we now only need to find an estimator for var-cov matrix in (1.33).

We can estimate $E\left(x_t x_t'\right)$ by $\frac{1}{T} \sum_{t=1}^{T} x_t x_t'$. To estimate the Γ_k terms, we exploit the special form of the error $u\varepsilon_t$. In particular, partition each Γ_k into

[64] http://www.phil.frb.org/research-and-data/real-time-center/survey-of-professional-forecasters/

[65] http://www.sca.isr.umich.edu/

four 2×2 matrices, with $\Gamma_{ij,k}$ denoting the $(i,j)-th$ element of this partition. Then, letting $\widehat{u}_t$ and $\widehat{u\varepsilon}_t$ denote the calculated errors of each equation, we use standard estimators

$$\Gamma_{11,k,T} = \frac{1}{T-k}\sum_{t=1}^{T-k} \widehat{u}_t \; \widehat{u}_{t-k} \; x_t x'_{t-k}$$

$$\Gamma_{12,k,T} = \frac{1}{T-k}\sum_{t=1}^{T-k} \widehat{u}_t \; \widehat{u\varepsilon}_{t-k} \; x_t x'_{t-k}$$

Since u_t is not a forecasting error, there is no reason why $\Gamma_{11,k}$ should be zero for any k. We deal with this by using Newey-West weights to truncate the infinite sum in S_w.

Since ε_{t+N} is a forecast error using information up to t we have

$$\Gamma_{21,k} = E([u_t + \varepsilon_{t+N}]\, u_{t-k} x_t x'_{t-k}) = \Gamma_{11,k} \text{ for all } k \geq 0,$$

so estimated $\Gamma_{11,k,T}$ is an estimate of $\Gamma_{21,k,T}$. Furthermore, we have

$$\begin{aligned} \Gamma_{22,k} &= E(u\varepsilon_t \; u\varepsilon_{t-k} \; x_t x'_{t-k}) \\ &= \Gamma_{12,k} + E(\varepsilon_{t+N}\varepsilon_{t+N-k} \; x_t x'_{t-k}) \qquad \text{for all } k \end{aligned}$$

where the second equality follows from $E(\varepsilon_{t+N} \; u_{t-k} \; x_t x'_{t-k}) = 0$. Moreover, since ε_{t+N} is orthogonal to $\varepsilon_{t+N-k} x_t x'_{t-k}$ for $k \geq N$ we have $\Gamma_{22,k} = \Gamma_{12,k}$ for $k \geq N$. Therefore, we can use the relationship

$$\begin{aligned} \Gamma_{22,k,T} &= \Gamma_{21,k,T} + \frac{1}{T-k}\sum_{t=1}^{T-k} (\widehat{u\varepsilon}_t - \widehat{u}_t)\;(\widehat{u\varepsilon}_{t-k} - \widehat{u}_{t-k})\; x_t x'_{t-k} \qquad \text{for } k < N \\ &= \Gamma_{21,k,T} \qquad \text{for } k \geq N \end{aligned}$$

which allows using the estimated $\Gamma_{21,k,T}$ as our estimate for $\Gamma_{22,k}$.

1.13.3 Parameterization of the Wage Process

To calibrate ρ we compute the average dividend-consumption share in the U.S. from 1946-2011, using the 'Net Corporate Dividends' and the 'Personal Consumption Expenditures' series from the Bureau of Economic Analysis. This delivers an average ratio of $\rho = 22$. Following Campbell and Cochrane (1999) we then choose the standard deviation of one-step-ahead consumption growth innovations to be 1/7 of that of one-step-ahead dividend growth innovations, i.e.,

$$\sqrt{\frac{var_t(\ln C_{t+1} - \ln C_t)}{var_t(\ln D_{t+1} - \ln D_t)}} = \frac{1}{7},$$

and the correlation between one-step-ahead consumption and dividend growth to be equal to 0.2, i.e.

$$\frac{cov_t(\ln C_{t+1} - \ln C_t, \ln D_{t+1} - \ln D_t)}{\sqrt{var_t(\ln C_{t+1} - \ln C_t) var_t(\ln D_{t+1} - \ln D_t)}} = 0.2$$

To achieve this we need to compute the required variance and covariances. We have

$$\begin{aligned} var_t(\ln D_{t+1} - \ln D_t) &= \sigma_D^2 \\ var_t(\ln C_{t+1} - \ln C_t) &= var_t\left(\ln(D_{t+1} + W_{t+1}) - \ln(D_t + W_t)\right) \\ &= var_t\left(\ln\left(D_{t+1} + \rho D_{t+1}\varepsilon_{t+1}^W\right)\right) \\ &= var_t(\ln D_{t+1} + \ln(1 + \rho\varepsilon_{t+1}^W)) \\ &= var_t(\ln D_{t+1}) + 2cov_t(\ln D_{t+1}, \ln(1 + \rho\varepsilon_{t+1}^W)) + var_t(\ln(1 + \rho\varepsilon_{t+1}^W)) \\ &= \sigma_D^2 + 2cov_t(\ln \varepsilon_{t+1}^D, \ln(1 + \rho\varepsilon_{t+1}^W)) + var_t(\ln(1 + \rho\varepsilon_{t+1}^W)) \qquad (1.34) \end{aligned}$$

and

$$\begin{aligned} cov_t(\ln C_{t+1} - \ln C_t, \ln D_{t+1} - \ln D_t) &= cov_t(\ln C_{t+1}, \ln \varepsilon_{t+1}^D) \\ &= cov_t(\ln(D_{t+1} + W_{t+1}), \ln \varepsilon_{t+1}^D) \\ &= cov_t(\ln D_{t+1} + \ln(1 + \rho\varepsilon_{t+1}^W), \ln \varepsilon_{t+1}^D) \\ &= cov_t(\ln \varepsilon_{t+1}^D + \ln(1 + \rho\varepsilon_{t+1}^W), \ln \varepsilon_{t+1}^D) \\ &= \sigma_D^2 + cov_t(\ln(1 + \rho\varepsilon_{t+1}^W), \ln \varepsilon_{t+1}^D) \qquad (1.35) \end{aligned}$$

Linearly approximating $\ln(1 + \rho\varepsilon_{t+1}^W)$ around the unconditional mean $\varepsilon^W = 1$ delivers

$$\ln(1 + \rho\varepsilon_{t+1}^W) \approx c + \frac{\rho}{1+\rho} \ln \varepsilon_{t+1}^W + O(2)$$

where c is a constant and $O(2)$ a second order approximation error. Using this approximation we have

$$var_t(\ln C_{t+1} - \ln C_t) \approx \sigma_D^2 + 2\frac{\rho}{1+\rho}\sigma_{DW} + \left(\frac{\rho}{1+\rho}\right)^2 \sigma_W^2 \qquad (1.36)$$

So that

$$\sqrt{\frac{var_t(\ln C_{t+1} - \ln C_t)}{var_t(\ln D_{t+1} - \ln D_t)}} \approx \sqrt{1 + 2\frac{\rho}{1+\rho}\frac{\sigma_{DW}}{\sigma_D^2} + \left(\frac{\rho}{1+\rho}\right)^2 \frac{\sigma_W^2}{\sigma_D^2}} = \frac{1}{7} \qquad (1.37)$$

Using the approximation we also have

$$\begin{aligned} \frac{cov_t(\ln C_{t+1} - \ln C_t, \ln D_{t+1} - \ln D_t)}{\sqrt{var_t(\ln C_{t+1} - \ln C_t) var_t(\ln D_{t+1} - \ln D_t)}} &\approx \\ \frac{\sigma_D^2 + \frac{\rho}{1+\rho}\sigma_{WD}}{\sqrt{\left(\sigma_D^2 + 2\frac{\rho}{1+\rho}\sigma_{WD} + \left(\frac{\rho}{1+\rho}\right)^2 \sigma_W^2\right)\sigma_D^2}} &= 0.2 \qquad (1.38) \end{aligned}$$

Using (1.37) to substitute the root in the denominator in (1.38) we get

$$\frac{\sigma_D^2 + \frac{\rho}{1+\rho}\sigma_{WD}}{\frac{1}{7}\sigma_D^2} = 0.2 \Longleftrightarrow \sigma_{WD} = -\frac{68}{70}\frac{1+\rho}{\rho}\sigma_D^2 \qquad (1.39)$$

Using (1.37) we then get

$$\begin{aligned}\sigma_W^2 &= -\frac{48}{49}\left(\frac{1+\rho}{\rho}\right)^2\sigma_D^2 - 2\frac{1+\rho}{\rho}\sigma_{WD}\\ &= \frac{236}{245}\left(\frac{1+\rho}{\rho}\right)^2\sigma_D^2. \qquad (1.40)\end{aligned}$$

1.13.4 Existence of Optimum, Sufficiency of FOCs, Recursive Solution

Existence of Optimum & Sufficiency of FOCs. The choice set in (2.2) is compact and non-empty. The following condition then insures existence of optimal plans:

Condition 1. The utility function $u(\cdot)$ is bounded above and for all $i \in [0,1]$

$$E_0^{\mathcal{P}^i}\sum_{t=0}^{\infty}\delta^t\, u\left(W_t + D_t\right) > -\infty. \qquad (1.41)$$

The expression on the left-hand side of condition (1.41) is the utility associated with never trading stocks ($S_t^i = 1$ for all t). Since this policy is always feasible, condition (1.41) guarantees that the objective function in (2.2) is also bounded from below, even if the flow utility function $u(\cdot)$ is itself unbounded below. The optimization problem (2.2) thus maximizes a bounded continuous utility function over a compact set, which guarantees existence of a maximum.

Under the assumptions made in the main text (utility function given by (1.11), knowledge of (2.6) and $\delta\beta^{RE} < 1$), condition 1 holds, as can be seen from the following derivation:

$$
\begin{aligned}
& E_0^{\mathcal{P}^i} \sum_{t=0}^{\infty} \delta^t \; u\left(W_t + D_t\right) \\
= \; & E_0 \sum_{t=0}^{\infty} \delta^t \; u\left(W_t + D_t\right) \\
= \; & E_0 \sum_{t=0}^{\infty} \delta^t \left((1+\rho\varepsilon_t^W) D_t\right)^{1-\gamma} \\
= \; & \left((1+\rho\varepsilon_0^W) D_0\right)^{1-\gamma} + E_0 \sum_{t=1}^{\infty} \delta^t \left((1+\rho\varepsilon_t^W) D_t\right)^{1-\gamma} \\
= \; & \left((1+\rho\varepsilon_0^W) D_0\right)^{1-\gamma} + E_0 \sum_{t=1}^{\infty} \left(\delta \left(\beta^D\right)^{1-\gamma}\right)^t \left((1+\rho\varepsilon_t^W)\varepsilon_t^D \prod_{k=1}^{t-1} \varepsilon_t^D\right)^{1-\gamma} \\
= \; & \left((1+\rho\varepsilon_0^W) D_0\right)^{1-\gamma} + E\left[\left((1+\rho\varepsilon^W)\varepsilon^D\right)^{1-\gamma}\right] \cdot E_0 \sum_{t=1}^{\infty} \left(\delta \left(\beta^D\right)^{1-\gamma}\right)^t \left(\prod_{k=1}^{t-1} \varepsilon_t^D\right)^{1-\gamma} \\
= \; & \left((1+\rho\varepsilon_0^W) D_0\right)^{1-\gamma} + E\left[\left((1+\rho\varepsilon^W)\varepsilon^D\right)^{1-\gamma}\right] \cdot \sum_{t=1}^{\infty} \left(\delta \left(\beta^D\right)^{1-\gamma}\right)^t \left(e^{\frac{\sigma_D^2}{2}\gamma(\gamma-1)}\right)^{t-1} \\
= \; & \left((1+\rho\varepsilon_0^W) D_0\right)^{1-\gamma} + \frac{E\left[\left((1+\rho\varepsilon^W)\varepsilon^D\right)^{1-\gamma}\right]}{e^{\frac{\sigma_D^2}{2}\gamma(\gamma-1)}} \cdot \sum_{t=1}^{\infty} \left(\delta\beta^{RE}\right)^t
\end{aligned}
$$

Since (2.2) is a strictly concave maximization problem the maximum is unique. With the utility function being differentiable, the first order conditions

$$u'(C_t^i) = \delta E_t^{\mathcal{P}^i}\left[u'(C_{t+1}^i)\frac{P_{t+1}+D_{t+1}}{P_t}\right] \tag{1.42}$$

plus a standard transversality condition are necessary and sufficient for the optimum.

Recursive Solution. We have a recursive solution whenever the optimal stockholding policy can be written as a time-invariant function $S_t^i = S^i(x_t)$ of some state variables x_t. We seek a recursive solution where x_t contains appropriately rescaled variables that do not grow to infinity. With this in mind, we impose the following condition:

Condition 2 The flow utility function $u(\cdot)$ is homogeneous of degree $\eta \geq 0$. Furthermore, the beliefs $\mathcal{P}^i$ imply that $\theta_t \equiv \left(\frac{D_t}{D_{t-1}}, \frac{P_t}{D_t}, \frac{W_t}{D_t}\right)$ has a state space representation, i.e., the conditional distribution $\mathcal{P}^i(\theta_{t+1}|\omega^t)$ can be written as

$$
\begin{aligned}
\mathcal{P}^i(\theta_{t+1}|\omega^t) &= \mathcal{F}^i(m_t^i) && (1.43)\\
m_t^i &= \mathcal{R}^i(m_{t-1}^i, \theta_t) && (1.44)
\end{aligned}
$$

for some finite-dimensional state vector $\ln m_t^i$ and some time-invariant functions $\mathcal{F}^i$ and $\mathcal{R}^i$.

Under Condition 2 problem (2.2) can then be re-expressed as

$$\max_{\{S_t^i \in \mathcal{S}\}_{t=0}^{\infty}} E_0^{\mathcal{P}^i} \sum_{t=0}^{\infty} \delta^t \, \mathcal{D}_t \, u\left(S_{t-1}^i\left(\frac{P_t}{D_t}+1\right) - S_t^i \frac{P_t}{D_t} + \frac{W_t}{D_t}\right), \tag{1.45}$$

given $S_{-1}^i = 1$, where $\mathcal{D}_t$ is a time-varying discount factor satisfying $\mathcal{D}_{-1} = 1$ and

$$\mathcal{D}_t = \mathcal{D}_{t-1}\left(\beta^D \varepsilon_t^D\right)^{\eta}.$$

The return function in (1.45) depends only on the exogenous variables contained in the vector θ_t. Since the beliefs $\mathcal{P}^i$ are assumed to be recursive in θ_t, standard arguments in dynamic programming guarantee that the optimal solution to (1.45) takes the form (1.13). This formulation of the recursive solution is useful, because scaling P_t and W_t by the level of dividends eliminates the trend in these variables, as desired. This will be useful when computing numerical approximations to $S^i(\cdot)$. The belief systems $\mathcal{P}^i$ introduced in section 1.7 will satisfy the requirements stated in condition 2.

1.13.5 Proof of Proposition 1

In equilibrium $S_t^i = 1$ for all $t \geq 0$, so that the budget constraint implies

$$C_t^i = D_t + W_t = (1+\rho\varepsilon_t^W)D_t.$$

Substituting into the agent's first order condition delivers

$$P_t = \delta E_t\left[\left(\frac{(1+\rho\varepsilon_{t+1}^W)D_{t+1}}{(1+\rho\varepsilon_t^W)D_t}\right)^{-\gamma}(P_{t+1}+D_{t+1})\right]. \tag{1.46}$$

Assuming that the following transversality condition holds

$$\lim_{j\to\infty} E_t\left[\delta^j \left\{\left(\frac{1+\rho\varepsilon_{t+j}^W}{1+\rho\varepsilon_t^W}\right)\frac{D_{t+j}}{D_t}\right\}^{-\gamma} P_{t+j}\right] = 0, \tag{1.47}$$

one can iterate forward on (1.46) to obtain

$$\frac{P_t}{D_t} = E_t\left[\sum_{j=1}^{\infty} \delta^j \left(\frac{1+\rho\varepsilon_{t+j}^W}{1+\rho\varepsilon_t^W}\right)^{-\gamma}\left(\frac{D_{t+j}}{D_t}\right)^{1-\gamma}\right],$$

Using $D_{t+j}/D_t = (\beta^D)^j \prod_{k=1}^{j} \epsilon^D_{t+k}$ one has

$$\begin{aligned}
\frac{P_t}{D_t} &= (1+\rho\varepsilon^W_t)^\gamma \sum_{j=1}^{\infty} \left(\delta(\beta^D)^{1-\gamma}\right)^j E_t \left[\left(1+\rho\varepsilon^W_{t+j}\right)^{-\gamma} \left(\prod_{k=1}^{j} \epsilon^D_{t+k}\right)^{1-\gamma} \right] \\
&= (1+\rho\varepsilon^W_t)^\gamma \sum_{j=1}^{\infty} \left(\delta(\beta^D)^{1-\gamma}\right)^j E_t \left[\left(1+\rho\varepsilon^W_{t+j}\right)^{-\gamma} (\epsilon^D_{t+j})^{1-\gamma} \right] E_t \left[\left(\prod_{k=1}^{j-1} \epsilon^D_{t+k}\right)^{1-\gamma} \right] \\
&= (1+\rho\varepsilon^W_t)^\gamma E_t \left[\left(1+\rho\varepsilon^W_{t+j}\right)^{-\gamma} (\epsilon^D_{t+j})^{1-\gamma} \right] e^{\gamma(1-\gamma)\sigma^2_D/2} \frac{\delta\beta^{RE}}{1-\delta\beta^{RE}},
\end{aligned}$$

as claimed in proposition 1.

1.13.6 Bayesian Foundations for Lagged Belief Updating

We now present a slightly modified information structure for which Bayesian updating gives rise to the lagged belief updating equation (1.29). Specifically, we generalize the perceived price process (2.8) by splitting the temporary return innovation $\ln \varepsilon_{t+1}$ into two independent subcomponents:

$$\ln P_{t+1} - \ln P_t = \ln \beta_{t+1} + \ln \varepsilon^1_{t+2} + \ln \varepsilon^2_{t+1}$$

with $\ln \varepsilon^1_{t+2} \sim iiN(-\frac{\sigma^2_{\varepsilon,1}}{2}, \sigma^2_{\varepsilon 1})$, $\ln \varepsilon^2_{t+1} \sim iiN(-\frac{\sigma^2_{\varepsilon 2}}{2}, \sigma^2_{\varepsilon 2})$ and

$$\sigma^2_\varepsilon = \sigma^2_{\varepsilon 1} + \sigma^2_{\varepsilon 2}.$$

We then assume that in any period t agents observe the prices, dividends and wages up to period t, as well as the innovations ε^1_t up to period t. Agents' time t information set thus consists of $I_t = \{P_t, D_t, W_t, \varepsilon^1_t, P_{t-1}, D_{t-1}, W_{t-1}, \varepsilon^1_{t-1}, ...\}$. By observing the innovations ε^1_t, agents learn - with a one period lag - something about the temporary components of price growth. The process for the persistent price growth component $\ln \beta_t$ remains as stated in equation (2.9), but we now denote the innovation variance by $\sigma^2_{\tilde{v}}$ instead of σ^2_v. As before, $\ln m_t$ denotes the posterior mean of $\ln \beta_t$ given the information available at time t. We prove below the following result:

Proposition 3 *Fix $\sigma^2_\varepsilon > 0$ and consider the limit $\sigma^2_{\varepsilon 2} \to 0$ with $\sigma^2_{\tilde{v}} = \sigma^2_{\varepsilon 2} g^2/(1-g)$. Bayesian updating then implies*

$$\ln m_t = \ln m_{t-1} + g\left(\ln P_{t-1} - \ln P_{t-2} - \ln m_{t-1}\right) - g \ln \varepsilon^1_t \tag{1.48}$$

The modified information structure thus implies that only lagged price growth rates enter the current state estimate, so that beliefs are predetermined, precisely as assumed in equation (1.29). Intuitively, this is so because lagged returns become infinitely more informative relative to current returns as $\sigma^2_{\varepsilon 2} \to 0$, which eliminates the simultaneity problem. For non-vanishing uncertainty $\sigma^2_{\varepsilon 2}$ the weight of the last observation actually remains positive but would still be lower than that given to the lagged return observation, see equation (1.51) in the proof below and the subsequent discussion for details.

We now sketch the proof of the previous proposition. Let us define the following augmented information set $\widetilde{I}_{t-1} = I_{t-1} \cup \{\varepsilon^1_t\}$. The posterior mean for β_t given $\widetilde{I}_{t-1}$, denoted $\ln m_{t|\widetilde{I}_{t-1}}$ is readily recursively determined via

$$\ln m_{t|\widetilde{I}_{t-1}} = \ln m_{t-1|\widetilde{I}_{t-2}} - \frac{\sigma^2_{\widetilde{v}}}{2} + \widetilde{g}\left(\ln P_{t-1} - \ln P_{t-2} - \ln \varepsilon^1_t + \frac{\sigma^2_{\widetilde{v}} + \sigma^2_{\varepsilon 2}}{2} - \ln m_{t-1|t-1}\right) \tag{1.49}$$

and the steady state posterior uncertainty and the Kalman gain by

$$\begin{aligned} \sigma^2 &= \frac{-\sigma^2_{\widetilde{\nu}} + \sqrt{\left(\sigma^2_{\widetilde{\nu}}\right)^2 + 4\sigma^2_{\widetilde{\nu}}\sigma^2_{\varepsilon 2}}}{2} \\ \widetilde{g} &= \frac{\sigma^2}{\sigma^2_{\varepsilon 2}} \end{aligned} \tag{1.50}$$

Standard updating formulas for normal distributions then imply that the posterior mean of $\ln \beta_t$ using information set I_t can be derived by updating the posterior mean based on $\widetilde{I}_{t-1}$ according to

$$\ln m_{t|I_t} = \ln m_{t|\widetilde{I}_{t-1}} + \frac{\sigma^2}{\sigma^2 + \sigma^2_{\varepsilon 1} + \sigma^2_{\varepsilon 2} + \sigma^2_{\widetilde{v}}}\left(\ln P_t - \ln P_{t-1} + \frac{\sigma^2_{\varepsilon 1} + \sigma^2_{\varepsilon 2} + \sigma^2_{\widetilde{v}}}{2} - \ln m_{t|\widetilde{I}_{t-1}}\right) \tag{1.51}$$

Since $\frac{\sigma^2}{\sigma^2 + \sigma^2_{\varepsilon 1} + \sigma^2_{\varepsilon 2} + \sigma^2_{\widetilde{v}}} < \frac{\sigma^2}{\sigma^2_{\varepsilon 2}} = \widetilde{g}$, the weight of the price observation dated t is reduced relative to the earlier observation dated $t-1$ because it is 'noisier'. Now consider the limit $\sigma^2_{\varepsilon 2} \to 0$ and along the limit choose $\sigma^2_{\varepsilon 1} = \sigma^2_{\varepsilon} - \sigma^2_{\varepsilon 2}$ and $\sigma^2_{\widetilde{v}} = \frac{g^2}{1-g}\sigma^2_{\varepsilon 2}$, as assumed in the proposition. Equation (1.51) then implies that $\ln m_{t|I_t} = \ln m_{t|\widetilde{I}_{t-1}}$, i.e., the weight of the last observation price converges to zero. Moreover, from $\sigma^2_{\widetilde{v}} = \frac{g^2}{1-g}\sigma^2_{\varepsilon 2}$ and (1.50) we get $\widetilde{g} = g$. Using these results, equation (1.49) can exactly be written as stated by equation (2.13) in the main text.

1.13.7 Proof of Proposition 2

The proof relies on the fact that in a situation without uncertainty the expectation of a non-linear function of 'random' variables is identical to the non-linear function of the expectation of these random variables, i.e., for some continuous non-linear function $f(\cdot,\cdot)$ and some random variables X_{t+j}, Y_{t+j} we have under the stated assumptions $E^{\mathcal{P}}_t f(X_{t+j}, Y_{t+j}) = f(E^{\mathcal{P}}_t X_{t+j}, E^{\mathcal{P}}_t Y_{t+j})$. Simplifying notation (and slightly abusing it) we let $X_{t+j} = E^{\mathcal{P}}_t X_{t+j}$ for all $j \geq 1$, so that X_{t+j} below denotes the subjective expectation conditional on information at time t of the variable X at time $t+j$. The first order conditions (1.42) can then be written as

$$\begin{aligned} 1 &= \left(\frac{C_{t+1+j}}{C_{t+j}}\right)^{-\gamma} \delta R_{t+1+j} \Longleftrightarrow \\ \frac{C_{t+1+j}}{P_{t+1+j}+D_{t+1+j}} &= \delta^{\frac{1}{\gamma}} \left(R_{t+1+j}\right)^{\frac{1-\gamma}{\gamma}} \frac{C_{t+j}}{P_{t+j}} \end{aligned} \tag{1.52}$$

for all $j \geq 0$. The budget constraint implies

$$S_{t-1}(P_t+D_t) = C_t - W_t + S_t P_t \Longrightarrow S_{t-1} = \frac{C_t - W_t}{P_t+D_t} + \frac{P_t}{P_t+D_t} S_t$$

Iterating forward on the latter equation gives

$$S_{t-1} = \frac{C_t-W_t}{P_t+D_t} + \frac{P_t}{P_t+D_t}\frac{C_{t+1}-W_{t+1}}{P_{t+1}+D_{t+1}} + \frac{P_t}{P_t+D_t}\frac{P_{t+1}}{P_{t+1}+D_{t+1}}\frac{C_{t+2}-W_{t+2}}{P_{t+2}+D_{t+2}} + \ldots$$

Repeatedly using equation (1.52) gives

$$\begin{aligned} S_{t-1} &= \frac{C_t-W_t}{P_t+D_t} + \frac{P_t}{P_t+D_t}\left(\delta^{\frac{1}{\gamma}}\left(R_{t+1}\right)^{\frac{1-\gamma}{\gamma}}\frac{C_t}{P_t} - \frac{W_{t+1}}{P_{t+1}+D_{t+1}}\right) \\ &\quad + \frac{P_t}{P_t+D_t}\frac{P_{t+1}}{P_{t+1}+D_{t+1}}\left(\delta^{\frac{1}{\gamma}}\left(R_{t+2}\right)^{\frac{1-\gamma}{\gamma}}\frac{C_{t+1}}{P_{t+1}} - \frac{W_{t+2}}{P_{t+2}+D_{t+2}}\right) + \ldots \\ &= \frac{C_t}{P_t+D_t} + \delta^{\frac{1}{\gamma}}\left(R_{t+1}\right)^{\frac{1-\gamma}{\gamma}}\frac{C_t}{P_t+D_t} \\ &\quad + \frac{P_t}{P_t+D_t}\delta^{\frac{1}{\gamma}}\left(R_{t+2}\right)^{\frac{1-\gamma}{\gamma}}\frac{C_{t+1}}{P_{t+1}+D_{t+1}} + \ldots \\ &\quad - \frac{W_t}{P_t+D_t} - \frac{W_{t+1}}{P_t+D_t}\frac{1}{R_{t+1}} - \frac{W_{t+2}}{P_t+D_t}\frac{1}{R_{t+1}R_{t+2}} - \ldots \\ &= \frac{C_t}{P_t+D_t} + \delta^{\frac{1}{\gamma}}\left(R_{t+1}\right)^{\frac{1-\gamma}{\gamma}}\frac{C_t}{P_t+D_t} \\ &\quad + \left(\delta^{\frac{1}{\gamma}}\right)^2\left(R_{t+2}R_{t+1}\right)^{\frac{1-\gamma}{\gamma}}\frac{C_t}{P_t+D_t} + \ldots \\ &\quad - \frac{1}{P_t+D_t}\left(\sum_{j=0}^{\infty} W_{t+j}\prod\nolimits_{i=1}^{j}\frac{1}{R_{t+i}}\right) \\ &= \frac{D_t}{P_t+D_t} + \frac{C_t}{P_t+D_t}\sum_{j=1}^{\infty}\left(\left(\delta^{\frac{1}{\gamma}}\right)^j\left(\prod\nolimits_{i=1}^{j}\frac{1}{R_{t+i}}\right)^{\frac{\gamma-1}{\gamma}}\right) \\ &\quad - \frac{1}{P_t+D_t}\left(\sum_{j=1}^{\infty}\left(\prod\nolimits_{i=1}^{j}\frac{1}{R_{t+i}}\right)W_{t+j}\right) \end{aligned} \tag{1.53}$$

Imposing on the previous equation $S_{t-1} = 1$ (the market clearing condition for period $t-1$ if $t > 1$, or the initial condition for period $t = 0$) and $C_t = D_t + W_t$ (the market clearing condition for period $t \geq 0$) one obtains the result stated in the proposition under the convention that $R_{t+i} = E_t^{\mathcal{P}} R_{t+i}$.

1.13.8 Verification of Conditions (1.30)

For the vanishing noise limit of the beliefs specified in section 1.7 we have

$$\begin{aligned} E_t^{\mathcal{P}}[P_{t+j}] &= (m_t)^j P_t \\ E_t^{\mathcal{P}}[D_{t+j}] &= \left(\beta^D\right)^j D_t \\ E_t^{\mathcal{P}}[W_{t+j}] &= \left(\beta^D\right)^j W_t. \end{aligned}$$

We first verify the inequality on the l.h.s. of equation (1.30). We have

$$\lim_{T\to\infty} E_t^{\mathcal{P}}[R_T] = m_t + \lim_{T\to\infty} \left(\frac{\beta^D}{m_t}\right)^{T-1} \beta^D \frac{D_t}{P_t},$$

so that for $m_t > 1$ the limit clearly satisfies $\lim_{T\to\infty} E_t^{\mathcal{P}}[R_T] > 1$ due to the first term on the r.h.s.; for $m_t < 1$ the second term on the r.h.s. increases without bound, due to $\beta^D > 1$, so that $\lim_{T\to\infty} E_t^{\mathcal{P}}[R_T] > 1$ also holds.

In a second step we verify that the inequality condition on the r.h.s. of equation (1.30) holds for all subjective beliefs $m_t > 0$. We have

$$\begin{aligned} \lim_{T\to\infty} E_t^{\mathcal{P}}\left(\sum_{j=1}^{T}\left(\prod\nolimits_{i=1}^{j}\frac{1}{R_{t+i}}\right)W_{t+j}\right) &= \lim_{T\to\infty} W_t E_t^{\mathcal{P}}\left(\sum_{j=1}^{T}\left(\beta^D\right)^j\left(\prod\nolimits_{i=1}^{j}\frac{1}{R_{t+i}}\right)\right) \\ &= \lim_{T\to\infty} W_t \sum_{j=1}^{T} X_j \qquad (1.54) \end{aligned}$$

where

$$X_j = \frac{\left(\beta^D\right)^j}{\prod_{i=1}^{j}(m_t + \left(\frac{\beta^D}{m_t}\right)^{i-1}\beta^D\frac{D_t}{P_t})} \geq 0 \qquad (1.55)$$

A sufficient condition for the infinite sum in (1.54) to converge is that the terms X_j are bounded by some exponentially decaying function. The denominator in (1.55) satisfies

$$\begin{aligned} &\prod\nolimits_{i=1}^{j}(m_t + \left(\frac{\beta^D}{m_t}\right)^{i-1}\beta^D\frac{D_t}{P_t}) \\ \geq\;& (m_t)^j + \left(\frac{\beta^D}{m_t}\right)^{j\left(\frac{j-1}{2}\right)}\beta^D\frac{D_t}{P_t}, \qquad (1.56) \end{aligned}$$

where the first term captures the the pure products in m_t, the second term the pure products in $\left(\frac{\beta^D}{m_t}\right)^{i-1}\beta^D\frac{D_t}{P_t}$, and all cross terms have been dropped. We then have

$$
\begin{aligned}
X_j &= \frac{\left(\beta^D\right)^j}{\prod_{i=1}^{j}(m_t + \left(\frac{\beta^D}{m_t}\right)^{i-1} \beta^D \frac{D_t}{P_t})} \\
&\leq \frac{\left(\beta^D\right)^j}{(m_t)^j + \left(\frac{\beta^D}{m_t}\right)^{j\left(\frac{j-1}{2}\right)} \beta^D \frac{D_t}{P_t}} \\
&= \frac{1}{\left(\frac{m_t}{\beta^D}\right)^j + \left(\frac{\beta^D}{m_t}\right)^{j\left(\frac{j-1}{2}\right)} \frac{1}{\left(\beta^D\right)^{j-1}} \frac{D_t}{P_t}},
\end{aligned}
$$

where all terms in the denominator are positive. For $m_t \geq \beta^D > 1$ we can use the first term in the denominator to exponentially bound X_j , as $X_j \leq \left(\frac{\beta^D}{m_t}\right)^j$; for $m_t < \beta^D$ we can use the second term:

$$
X_j \leq \frac{1}{\left(\frac{\beta^D}{m_t}\right)^{j\left(\frac{j-1}{2}\right)} \frac{1}{\left(\beta^D\right)^{j-1}} \frac{D_t}{P_t}} = \frac{1}{\left(\left(\frac{\beta^D}{m_t}\right)^{\frac{j}{2}} \frac{1}{\beta^D}\right)^{j-1} \frac{D_t}{P_t}}
$$

Since $m_t < \beta^D$ there must be a $J < \infty$ such that

$$
\left(\frac{\beta^D}{m_t}\right)^{\frac{j}{2}} \frac{1}{\beta^D} \geq \frac{\beta^D}{m_t} > 1
$$

for all $j \geq J$, so that the X_j are exponentially bounded for all $j \geq J$.

1.13.9 Capital Gains Expectations and Expected Returns: Further Details

Figure 1.10 depicts how expected returns at various horizons depend on agent's expected price growth expectations using the same parameterization as used in figure 1.5. It shows that expected returns covary positively with capital gains expectations for $m_t \geq \beta^D$, as has been claimed in the main text. The flatish part at around $m_t - 1 \approx 0.01$ arises because in that area the PD ratio increases strongly, so that the dividend yield falls. Only for pessimistic price growth expectations ($m_t < \beta^D$) and long horizons of expected returns we find a negative relationship. The latter emerges because with prices expected to fall, the dividend yield will rise and eventually result in high return expectations.

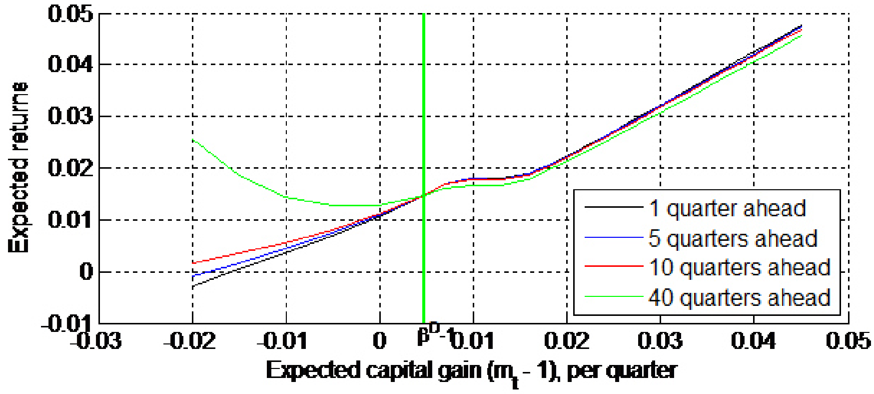

Figure 1.10: Expected return as a function of expected capital gain

1.13.10 Numerical Solution Algorithm

Algorithm: We solve for agents' state-contingent, time-invariant stockholdings (and consumption) policy (1.13) using time iteration in combination with the method of endogenous grid points. Time iteration is a computationally efficient, e.g., Aruoba et al. (2006), and convergent solution algorithm, see Rendahl (2015). The method of endogenous grid points, see Carroll (2006), economizes on a costly root finding step which speeds up computations further.

Evaluations of Expectations: Importantly, agents evaluate the expectations in the first order condition (1.42) according to their subjective beliefs about future price growth and their (objective) beliefs about the exogenous dividend and wage processes. Expectations are approximated via Hermite Gaussian quadrature using three interpolation nodes for the exogenous innovations.

Approximation of Optimal Policy Functions: The consumption/stockholding policy is approximated by piecewise linear splines, which preserves the non-linearities arising in particular in the PD dimension of the state space. Once the state-contingent consumption policy has been found, we use the market clearing condition for consumption goods to determine the market clearing PD ratio for each price-growth belief m_t.

Accuracy: Carefully choosing appropriate grids for each belief is crucial for the accuracy of the numerical solution. We achieve maximum (relative)

Euler errors on the order of 10^{-3} and median Euler errors on the order of 10^{-5} (average: 10^{-4}).

Using our analytical solution for the case with vanishing noise, we can assess the accuracy of our solution algorithm more directly. Setting the standard deviations of exogenous disturbances to 10^{-16} the algorithm almost perfectly recovers the equilibrium PD ratio of the analytical solution: the error for the numerically computed equilibrium PD ratio for any price growth belief m_t on our grid is within 0.5 % of the analytical solution.

Chapter 2

Can a Financial Transaction Tax Prevent Stock Price Booms?

2.1 Abstract[1]

We present a stock market model that quantitatively replicates the joint behavior of stock prices, trading volume and investor expectations. Stock prices in the model occasionally display belief-driven boom and bust cycles that delink asset prices from fundamentals and redistribute considerable amounts of wealth from less to more experienced investors. Although gains from trade arise only from subjective belief differences, introducing financial transactions taxes (FTTs) remains undesirable. While FTTs reduce the size and length of boom-bust cycles, they increase the likelihood of such cycles, thereby overall return volatility and wealth redistribution. Contingent FTTs, which are levied only above a certain price threshold, give rise to problems of equilibrium multiplicity and non-existence.

2.2 Introduction

Following the financial crisis, there has been a widespread desire among policymakers to introduce financial transaction taxes (FTTs). The European Commission, for example, proposed the introduction of FTTs in September 2011. Subsequently, France introduced in 2012 a 0.1% tax on stock market and related transactions and has recently increased the tax rate to 0.2%. Italy introduced a 0.1% tax on stock market transactions in 2013.[2]

One of the stated policy objectives of the European Commission is that FTTs should 'discourage financial transactions which do not contribute to the

[1]This chapter is based on (verbally quoted from) the paper Adam, Beutel, Marcet, and Merkel (2015).

[2]FTTs are already a widely used tax instrument in housing markets. Spain, for example, levies an 8% transaction tax on real estate transactions and Germany levies a 5% tax, both additionally levy capital gains taxes.

efficiency of financial markets'. The present paper seeks to analyze to what extent FTTs actually increase the efficiency of stock market transactions and stock market prices. In particular, it investigates whether FTTs can prevent boom and bust like dynamics in stock prices; over recent decades such price dynamics have become pervasive in a number of important stock markets and have contributed to the redistribution of wealth between different kinds of investors.[3] The effect that FTTs have on boom-bust like dynamics in stock markets should thus be of prime importance to policymakers.

To analyze this issue, we use a modeling framework that can generate stock price fluctuations roughly of the size observed in the data, including occasional large upswings and reversals in stock market prices. The model also quantitatively replicates important data moments characterizing the behavior of trading volume, as well as its comovement with stock prices and investor expectations. Credibly replicating the behavior of trading volume appears key for an analysis that seeks to understand the effects of taxing trading activity and is a distinguishing feature of the present analysis.[4]

Besides being quantitatively plausible, our modeling framework gives FTTs the best possible chance to generate positive welfare effects: first, we consider a framework where subjective belief components cause asset prices not to be fully efficient, so that there is - at least in principle - room for increasing the efficiency of financial market prices; second, within the presented framework, the gains from trade exist only in subjective terms, i.e., due to belief differences, so that taxing trading activity may appear desirable on a priori grounds, see Simsek (2013); third, we abstract from a number of adverse consequences likely to be associated with the introduction of FTTs, such as costly evasive behavior, which may involve redirecting orders to other exchanges, the adverse liquidity effects resulting from financial market fragmentation, or the costly creation of alternative financial instruments that are not subject to the tax.

Our main finding is that even within this very conducive setting, the introduction of FTTs fails to 'discourage transactions which do not contribute to the efficiency of financial markets'. Indeed, we find that the introduction of FTTs increases the likelihood that the stock market embarks on a significant boom and bust cycle in valuation, and thereby increases the overall amount of wealth redistribution. The reasons for this finding are subtle, as we explain below, but show that FTTs may actually not be a suitable policy instrument for increasing the efficiency of stock markets.

The modeling framework used in the present paper builds upon prior work by Adam, Beutel and Marcet (2014), which replicates stock price behavior within a representative agent framework with time separable preferences. The present analysis adds (1) by introducing investor heterogeneity and thereby equilibrium trade, (2) by showing that the resulting trading patterns are em-

[3]See Brunnermeier and Nagel (2004) for evidence on how the tech stock boom and bust around the year 2000 redistributed wealth between hedge fund and other investors.

[4]See section 2.3 for a discussion of the related literature.

pirically plausible, and (3) by studying the pricing and welfare effects of introducing FTTs.

While the equilibrium pricing patterns of the representative agent model in Adam, Beutel and Marcet (2014) prove rather robust to introducing agent heterogeneity, i.e., stock prices continue to be very volatile and to display occasional boom-bust cycles, the addition of agent heterogeneity helps in generating auto-correlated trading volume, trading volume that correlates positively with absolute price changes, and trading volume that correlates positively with investor disagreement, in line with what is found in the data.

The presented model is one where boom-bust dynamics arise from subjective price beliefs, but in a setting where investors take fully optimal investment decisions given their beliefs, following Adam and Marcet (2011). The introduction of subjective stock price beliefs is motivated by empirical evidence presented in Adam, Beutel and Marcet (2014), who show that the joint dynamics of realized capital gains and capital gain expectations, as observed from survey data, are strongly inconsistent with the rational expectations hypothesis. This implies - amongst other things - that rational asset price bubbles, e.g., those derived in classic work by Froot and Obstfeld (1991), are inconsistent with the joint dynamics of actual and expected capital gains in the data.

Following Adam, Beutel and Marcet (2014), we consider investors who hold subjective stock price beliefs of a kind such that Bayesian updating causes investors to extrapolate (to different degrees) past capital gains into the future. The degree of extrapolation is thereby calibrated to the one that we document to be present in survey data. In particular, we show that less experienced stock market investors extrapolate more compared to investors with longer investment experience.

Extrapolative behavior, which gives rise to investor optimism and pessimism, potentially supports a strong argument in favor of introducing FTTs. Specifically, in our setting, price booms emerge because investors become optimistic once they see past prices going up, causing them to bid up today's prices, thereby creating additional optimism in the next period and further price increases. FTTs can prevent investors from trading on their optimistic beliefs, i.e., prevent them from bidding up prices once optimism has increased, thereby preventing the positive feedback loop between price increases and increased optimism just described.

While intuitively plausible, this argument ignores an important additional consequence of FTTs. By preventing agents from trading, even arbitrarily small exogenous shocks to stock supply can have a disproportionately large effect on realized prices. Specifically, linear transaction taxes imply that investors, whose stockholdings are close to their subjectively optimal level, do not want to trade, unless there is a significant change in the stock price.[5] As a result, FTTs can increase price volatility in normal times. With realized prices

[5]This is so because the gains from trade are of second order close to the optimum, while the cost of the tax are of first order.

feeding into investors' beliefs, due to extrapolative behavior, this ultimately increases the likelihood that the stock market embarks on a large self-fueling boom and subsequent bust. The predicted effect of a 4% FTT is an increase by one third of the number of stock price boom episodes relative to the case without taxes.

Our quantitative analysis shows that FTTs manage to decrease the size and duration of stock price booms, including the volatility of prices during boom times. At the same time, FTTs increase price volatility during normal times.[6] The latter together with increased likelihood of (volatile) boom and bust episodes causes FTTs to increase overall stock price volatility.

Motivated by the observation that it is undesirable to levy FTTs in normal times, as they increase price volatility and thereby the likelihood of boom-bust cycles, we also consider the effects of state contingent taxes that are only levied once prices exceed a certain threshold. We show that such taxes give rise to non-continuous stock demand functions and thereby to problems of equilibrium multiplicity and non-existence. State-contingent transaction taxes appear problematic on these grounds.

The remainder of this paper is organized as follows. Section 2.3 discusses some of the related literature. Section 2.4 provides basic facts about the joint behavior of stock prices, trading volume and investor expectations that we seek to quantitatively match within our asset pricing framework. Section 2.5 introduces the asset pricing model. Section 2.6 shows that the model performs poorly in terms of replicating price and trading dynamics when investors hold rational price expectations. Section 2.7 evaluates the quantitative performance of the model with subjective price beliefs and in the absence of a transactions tax. In section 2.8 we show how stock price boom and bust dynamics redistribute wealth between different investor types. Section 2.9 presents the implications of introducing linear FTTs and section 2.10 considers the effects of state-contingent taxes. A conclusion briefly summarizes. Technical material and information about the employed data sources is summarized in an appendix.

2.3 Related Literature

The present paper is closely connected to an extensive literature on financial transaction taxes going at least back to the well-known proposal by Tobin (1978). We provide here a selective overview of the literature, making reference to work that is most closely related to the present paper.

In a comprehensive theoretical study, Dávila (2014) determines optimal linear transaction taxes for a setting where investors hold heterogeneous beliefs.

[6] Normal times are times that are not classified as boom times. Boom times begin when the quarterly price dividend ratio exceeds a certain level and end when the PD ratio falls below a certain lower level. In our numerical application, we set the first threshold to 250 and latter to 200. Results turn out to be are rather robust to the precise threshold values.

He shows that the optimal transaction tax of a social planner who maximizes social welfare under her own (possibly different) probability beliefs, depends on the cross-sectional covariance between investors' beliefs and equilibrium portfolio sensitivities.

Scheinkman and Xiong (2003) analyze how asset price bubbles and trading volume are affected by transactions taxes in a setting with risk neutral investors who face a short-sale constraint and who hold different beliefs because they assign different information content to publicly available signals. In their setting, transaction taxes strongly affect trading volume but may have only a limited effect on the size of asset price bubbles.

The present paper adds to these contributions by considering the effects of FTTs within a quantitatively credible setting that replicates important data moments describing the joint behavior of stock prices, trading volume and investor expectations. Furthermore, by incorporating learning from market prices, investors' belief distortions depend in important ways on market outcomes. This gives rise to feedback effects that are absent in models in which agents consider market prices to offer only redundant information.

In related work, Buss et al. (2013) consider the effects of FTTs and other policy instruments on stock market volatility in a production economy in which some stock market participants overinterpret the information content of public signals, as in Dumas et al. (2009). The present paper considers an endowment economy but evaluates model performance also with regard to the ability to match trading activity. Similar to our findings, Buss et al. (2013) show how financial transaction taxes increase the volatility of stock market returns.

With financial transaction taxes being almost equivalent to trading costs, the present paper also relates to the transaction costs literature. As in Constantinides (1986), transaction costs generate within the present setup partially flat demand curves, see also subsequent work by Aiyagari and Gertler (1991) and Heaton and Lucas (1996). Different from Constantinides (1986), the asset price effects of transaction costs fail to be of second order within the present setting because we consider agents that use price realizations to update beliefs about the price process. Guasoni and Muhle-Karbe (2013) and Vayanos and Wang (2012) provide recent surveys of the transaction cost literature.

Empirical evidence on the volatility effects of financial transaction taxes is provided in Umlauf (1993), Jones and Seguin (1997) and Hau (2006). These studies tend to find that market volatility increases with the introducing of a tax, see also McCulloch and Pacillo (2011) for a recent overview of the empirical literature. Coelho (2015) and Colliard and Hoffmann (2015) analyze the recent experiences with the introduction of FTTs in France and Italy, documenting how FTTs increase price volatility and reduce market depth.

The market microstructure literature also studies financial transaction taxes, focusing on the differential impact that such taxes have on the participation of noise traders, which create exogenous market volatility or mispricing, versus the participation of informed traders who evaluate prices according to fundamentals, see for example Jeanne and Rose (2002) or Hau (1998). The general conclusion of this theoretical literature is that if financial transaction taxes cause noise traders to participate less in the market, then market volatility can fall as a result.

2.4 Stock Prices, Price Expectations and Trading Volume: Empirical Evidence

This section documents key facts about the joint behavior of U.S. stock prices, investors' price expectations and stock market trading volume that we seek to quantitatively replicate with our asset pricing model. The next section presents empirical evidence about stock price behavior, the behavior of dividends and the behavior of average stock price expectations. Section 2.4.2 complements this with key facts about the behavior of trading volume and its relation with price behavior and the behavior of price expectations. It shows - amongst other things - that trading volume correlates positively with disagreement across investors about future prices. Finally, section 2.4.3 shows that disagreement between investors can be systematically related to investors' stock market experience.

2.4.1 Stock Prices, Dividends and Average Price Expectations

Table 2.1 presents key facts about the behavior of quarterly U.S. stock prices, dividends and stock return expectations as available from survey data.[7] The facts presented in table 2.1 are the main data moments guiding the analysis in Adam, Beutel and Marcet (2014) and we summarize them here for convenience.[8]

Table 2.1 shows that the average quarterly price dividend ratio ($E[PD]$) is around 140 and has a standard deviation ($std(PD)$) of approximately half its average value.[9] Stock prices are thus very volatile. The quarterly autocorrelation of the price dividend (PD) ratio ($corr(PD_t, PD_{t-1})$) is 0.98, showing that deviations of the PD ratio from its sample mean are very persistent over time. As a result, quarterly real stock returns are very volatile, with a standard deviation ($std(r^s)$) of around 8% per quarter. Real stock returns are thus much more volatile than real dividend growth, which has a standard deviation ($std(D_t/D_{t-1})$) of just 1.92%. The mean real stock return ($E[r^s]$)

[7] The data sources used in this and the subsequent sections are described in appendix 2.12.1.

[8] We include here all asset pricing facts considered in Adam, Beutel and Marcet (2014), except for those involving the bond market, as the present model does not feature a bond market.

[9] The quarterly PD ratio is defined as the price over quarterly dividend payments, see appendix 2.12.1 for further details.

is 1.89% per quarter and much higher than the average growth rate of real dividends ($E[D_t/D_{t-1} - 1]$), which equals 0.48% per quarter.

Table 2.1 also documents that the average investor's expected real returns in the UBS survey correlates strongly and positively with the PD ratio ($corr(PD_t, \overline{E}_t R_{t+1})$): the correlation equals 0.79.[10] Adam, Beutel and Marcet (2014) show that this fact is robust against using other survey data sources and against alternative ways to distill expectations from the survey data. They also show that this fact is inconsistent with investors holding rational price expectations, which is why we include the correlation between the PD ratio and expected returns in the set of data moments that we seek to match.

	U.S. Data 1949:Q1-2012:Q1
Stock prices:	
$E[PD]$	139.7
$std(PD)$	65.3
$corr(PD_t, PD_{t-1})$	0.98
$std(r^s)$	8.01%
$E[r^s]$	1.89%
Survey expectations:	
$corr(PD_t, \overline{E}_t R_{t+1})$	0.79
Dividends:	
$E[D_t/D_{t-1} - 1]$	0.48%
$std(D_t/D_{t-1})$	1.92%

Table 2.1: Quarterly stock prices, dividends and survey expectations

2.4.2 Trading Volume, Stock Prices and Disagreement

This section presents empirical facts about trading activity and its comovement with prices and price expectations. It shows that trading volume is highly persistent, that trading volume is largely uncorrelated with stock market valuation, instead correlates positively with *absolute* price changes. Furthermore, it documents - to our knowledge for the first time - that aggregate trading volume and disagreement about future aggregate stock market returns, as measured by survey data, are positively correlated.

The finance literature studies a range of empirical measures to capture trading activity, see Lo and Wang (2009) for an overview. To account for trading in individual shares, Lo and Wang argue that 'shares traded divided by shares outstanding is a natural measure of trading activity when viewed in

[10]The number reported in table 2.1 uses the mean of the expected returns of the own portfolio return expectations of all investors in the UBS survey. The survey data are avaiable from 1998:Q2 to 2007:Q2 and have been transformed into real values using the median of expected inflation reported in the survey of professional forecasters.

the context of standard portfolio theory and equilibrium asset-pricing models' (p.243). Clearly, for individual shares, this measure is identical to using the dollar volume of shares traded divided by the dollar volume of shares outstanding. Since this latter measure aggregates more naturally across different stocks and since we are interested in the aggregate stock market, we use the dollar volume of shares traded over the dollar volume of share outstanding as our preferred measure of trading volume.

We aggregate daily trading volume into a quarterly series by summing up the daily trading volumes over the quarter, following Lo and Wang (2009). While being standard, this procedure is likely going to lead to an overstatement of the model relevant trading volume, as many of the daily trades recorded in the data may be reversed with opposing trades within the same quarter. Indeed, with the advent of high frequency trading strategies, many of the recorded trades are likely to be undone within seconds, if not milliseconds. Dealing properly with this issue in the data is difficult, as it would require information about individual portfolios of all investors. We seek to account - at least partially - for the increasing share of high-frequency trades over time, therefore use detrended data on trading volume. Since detrending can affect the cyclical properties of the trading volume series, we report below only facts that turn out to be robust to a range of plausible detrending methods.

Figure 2.1 depicts the (undetrended) quarterly trading volume of the U.S. stock market, where data is available from January 1973. Trading volume displays a clear upward trend over time. In the early 1970's trade during a quarter amounted to around 5% of the market value of outstanding shares; at the end of the sample period this number reaches close to 50%; the data also shows temporary spikes in trading volume around the 1987, 2000 and 2008 stock market busts.

Table 2.2 presents a number of facts about detrended trading volume. As a baseline, we use simple linear detrending, but the table also displays outcomes for other commonly used detrending methods. In particular, it considers linear-quadratic detrending, the outcomes obtained from HP-filtering with a smoothing parameter of 1600, as well as so-called moving average (MA) detrending, which normalizes trading volume by the average trading volume recorded in the preceding four quarters.

Table 2.2 shows that trading volume displays considerable autocorrelation across quarters. The autocorrelation is statistically significant at the 1% level for all detrending methods.[11] For higher frequencies, this is a well-known fact that has been documented in the finance literature, we show it here for the quarterly frequency at which we will evaluate our asset pricing model.

Table 2.2 also shows that there exists no statistically significant correlation

[11] We test the null hypothesis $H_0 : corr(\cdot,\cdot) = 0$ in this and subsequent tables using robust standard errors, following Roy and Cléroux (1993), which are implemented with a Newey-West estimator with 4 leads and lags.

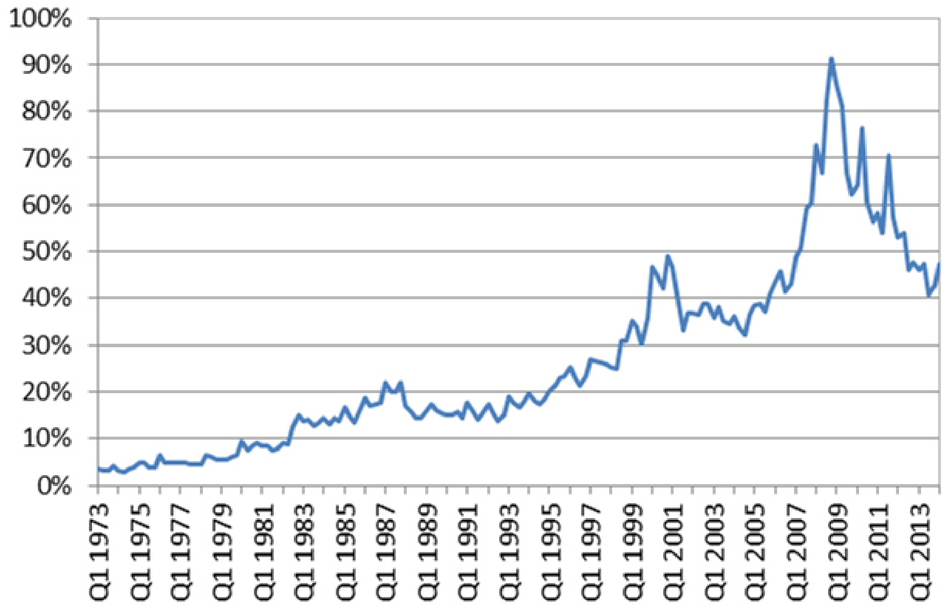

Figure 2.1: Quarterly trading volume (% of outstanding shares, undetrended)

between trading volume and the level of the PD ratio.[12] This illustrates that claims about the existence of a high positive correlation between the level of stock prices and trading volume, see for example Scheinkman and Xiong (2003) and the references cited therein, disappear once one removes the trend displayed by trading volume.[13]

The previous finding does not imply that trading volume and prices are unrelated. Indeed, as table 2.2 documents, trading volume correlates positively and in a statistically highly significant way with normalized absolute price changes. This finding holds again for all detrending methods. It is in line with patterns documented by Karpoff (1987) and shows that periods of high volume are associated with large relative price changes.

[12] Table 2.2 uses the undetrended PD ratio. Detrending the PD ratio leads to very similar conclusions. For example, using instead the linearly detrended or HP filtered PD ratio, the point estimates for the correlations with turnover range between -0.27 and 0.02, depending on the way turnover is detrended.

[13] Our findings also hold true if one uses data only up to the year 2006, which shows that results are not driven by the recent financial crisis.

	Detrending Method			
	Baseline (linear)	Linear-quadratic	HP filter	MA
$corr(TV_t, TV_{t-1})$	0.89***	0.88***	0.66***	0.43***
$corr(TV_t, PD_t)$	-0.07	0.01	-0.03	-0.06
$corr(TV_t, \|P_t/P_{t-1} - 1\|)$	0.34***	0.33***	0.33***	0.23***

*/**/*** indicates significance at the 10%/5%/1% significance level, respectively.

Table 2.2: Trading Volume and Price Behavior

The facts presented in table 2.2 are fairly standard in the light of the existing finance literature studying trading volume. We complement these facts below with additional empirical evidence on the relationship between trading volume and belief disagreement. Models in which investors disagree about the future prospects from investment have a long tradition in the finance literature, see Hong and Stein (2007) for a survey. We document in table 2.3 below that there exists a fairly robust positive correlation between aggregate trading volume and the amount of cross-sectional disagreement about future aggregate stock market returns.

Table 2.3 reports the correlation between trading volume and the cross-sectional standard deviations of real survey return expectations ($corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$), as obtained from various survey data sources.[14] The point estimate of the correlation is always positive and often statistically significant when using linear or linear-quadratic detrending or the HP filter. The evidence is less strong when detrending trading volume using the moving average approach, but is otherwise rather robust. Furthermore, to document that results are not driven by outliers in the surveys, table 2.3 also reports the correlation between detrended trading volume and the inter-quartile range (IQR) of the cross-section of survey expectations ($corr(TV_t, IQR(\widetilde{E}_t^i R_{t+1}))$).[15] Results turn out to be robust towards using this alternative dispersion measure.

Overall, the evidence in table 2.3 shows that trading volume and disagreement are positively correlated in the data.

[14]Since the Shiller survey asks for expected capital gains, the reported correlations for this survey pertain to the cross-sectional dispersion of capital gain expectations.

[15]For the CFO survey, we do not observe individual survey responses or the interquartile range, thus cannot perform this robustness check.

	Detrending Method			
	Baseline (linear)	Linear-quadratic	HP filter	MA
UBS-Gallup Survey (1-year horizon)				
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.41*	0.41**	0.43*	0.17
$corr(TV_t, IQR(\widetilde{E}_t^i R_{t+1}))$	0.36	0.50*	0.65**	0.41**
Shiller Survey (3-months horizon)				
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.37*	0.40*	0.43**	-0.06
$corr(TV_t, IQR(\widetilde{E}_t^i R_{t+1}))$	0.52**	0.54**	0.63***	0.19
Shiller Survey (6-months horizon)				
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.60***	0.60*	0.58***	0.03
$corr(TV_t, IQR(\widetilde{E}_t^i R_{t+1}))$	0.43*	0.46*	0.47**	0.09
Shiller Survey (1-year horizon)				
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.51**	0.52**	0.51***	0.18
$corr(TV_t, IQR(\widetilde{E}_t^i R_{t+1}))$	0.49**	0.55**	0.56***	0.23
CFO Survey (1-year horizon)				
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.70**	0.65**	0.64***	-0.02

$^{*}/^{**}/^{***}$ indicates significance at the 10%/5%/1% significance level, respectively.

Table 2.3: Trading volume and disagreement

2.4.3 Disagreement and Stock Market Experience

Given the evidence presented in the previous section, which shows that investor disagreement is systematically related to trading volume, this section explores potential sources of investor disagreement more closely. In particular, it shows that disagreement can be partly related to investor experience: the price expectations of investors with less stock market experience are more heavily influenced by recent stock market performance than those with more experience.

Adam, Beutel and Marcet (2014) show that the empirical time series behavior of the *average* price growth expectation in the UBS survey data ($\overline{E}_t[P_{t+1}/P_t]$) can be captured very well by an extrapolative updating equation of the form

$$\overline{E}_t[P_{t+1}/P_t] = \overline{E}_{t-1}[P_t/P_{t-1}] + g\left(\frac{P_t}{P_{t-1}} - \overline{E}_{t-1}[P_t/P_{t-1}]\right), \qquad (2.1)$$

which stipulates that the average investor extrapolates observed capital gains into the future. We document below that investors with different numbers of years of experience extrapolate to different degrees.

Figure 2.2 depicts the evolution of quarterly real price growth expectations held by investors with different years of stock market experience, as available from the UBS survey.[16,17] It shows that in the year 1999 and until the beginning of the year 2000, when prior stock market returns have been very high due to the preceding tech stock boom, it is the less experienced investors that tend to be most optimistic about future capital gains. Indeed, investors with 0-5 years of experience expect an average real capital gain of around 3.5% per quarter, i.e., a real gain of about 14% per year, while the most experienced group expects considerably lower capital gains (albeit still very high ones by historical standards). Following the subsequent stock market bust, belief dispersion across investor groups significantly narrows and reaches a low point during the stock market trough in the year 2003. Clearly, this happens because less experienced investors updated expectations more strongly during the market bust. Following the stock market recovery after the year 2003, belief dispersion widened again, with the least experienced investor group then holding once more the highest return expectations, while the two most experienced groups hold the lowest expectations.

Figure 2.2 suggests, in line with evidence presented in Malmendier and Nagel (2011), that the capital gain expectations of less experienced investors react more strongly to realized capital gains. We formally check this hypothesis by estimating the updating parameter g in equation (2.1) for each experience group separately, using the same approach as employed in Adam, Beutel and Marcet (2014). Table 2.4 reports the estimation outcome and shows that the updating parameter is monotonically decreasing with experience, with the updating parameter of the most inexperienced group of investors being approximately 75% higher than that of the most experienced investor group. The estimated updating gains are all statistically significantly different from zero at the 1% level.[18] Appendix 2.12.4 shows that the gains are significantly different from each other for sufficiently distant experience groups and that the gain of the most experienced investor group is different from those of all other

[16] We choose experience groups with equidistant group boundaries (except for the highest group) and in a way that groups are approximately of similar size. The reported results are robust to using different numbers of groups or different group boundaries, provided one does not consider too many groups, which causes results to become more noisy.

[17] The figure reports the 'own portfolio' return expectations from the UBS survey, as these are available for a longer time period. Results do not depend on this choice, though. We transform nominal return expectations into real expectations using the median inflation forecast from the Survey of Professional Forecasters. To be consistent with our asset pricing model, which models capital gain expectations, we transform real return expectations into a measure of real price growth expectations using the identity $R_{t+1} = \frac{P_{t+1}}{P_t} + \frac{D_{t+1}}{P_t} = \frac{P_{t+1}}{P_t} + \beta^D \frac{D_t}{P_t}$ where β^D denotes the expected gross quarterly real growth rate of dividends that we set equal to its sample average, i.e, $\beta^D = 1.0048$, see table 1. Results are very similar when using alternative plausible values for β^D. Also, since the UBS survey does not have a panel structure, the figure is based on a pseudo panel and reports at each point in time the median expectation of the considered experience group.

[18] Standard errors in table 2.4 and the p-values reported in appendix 2.12.4 are computed in a standard way, exploiting the fact that the procedure used for estimating the gain is a nonlinear least squares estimation.

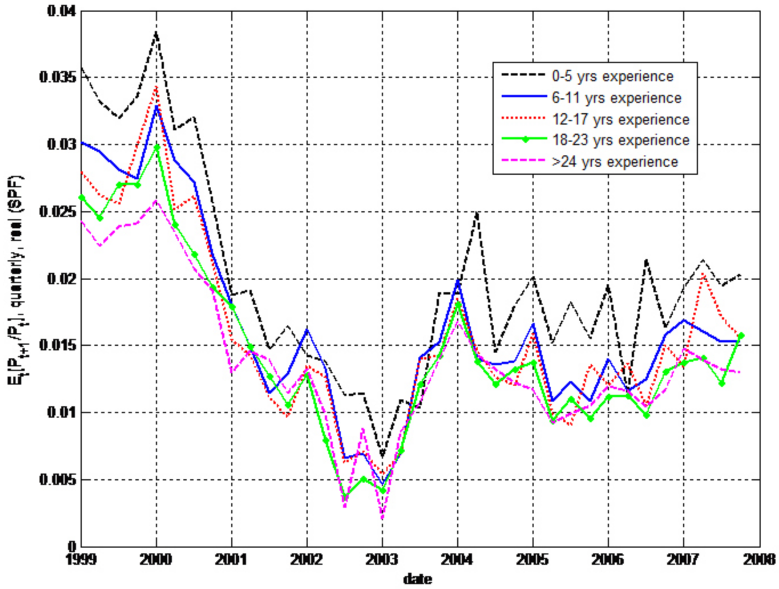

Figure 2.2: Price growth expectations by experience group (UBS survey, real, in quarterly growth rates)

groups at the 1% level.

Experience (yrs)	0-5	6-11	12-17	18-23	> 23
Estimated g^i	**0.0316**	**0.0286**	**0.0264**	**0.0230**	**0.0180**
(std. deviation)	(0.0028)	(0.0013)	(0.0017)	(0.0013)	(0.0090)

Table 2.4: Estimated updating parameters

2.5 The Asset Pricing Model

This section presents the asset pricing model that we use to replicate the empirical facts documented in the previous section. We consider a model with a unit mass of atomistic investors who trade on a competitive stock market, where trade may be subject to a linear transactions tax. At the beginning of each period, stocks pay a stochastic dividend D_t per unit and investors earn an exogenous wage income W_t. Income from both sources takes the form of perishable consumption goods.

There are $I \geq 1$ types of investors in the economy and a mass $\mu^i > 0$ of each type $i \in \{0, \ldots, I\}$, where $\sum_{i=1}^{I} \mu_i = 1$. Types differ with respect to the beliefs they entertain about the behavior of future stock prices and with regard to their accumulated stockholdings. For the special case without a financial transactions tax and when there is a single investor type, the setup reduces to the one studied in Adam, Beutel and Marcet (2014).

The Investment Problem. The representative investor of type $i \in \{1, ..., I\}$ solves

$$\max_{\left\{C_t^i \geq 0, S_t^i\right\}_{t=0}^{\infty}} E_0^{\mathcal{P}^i} \sum_{t=0}^{\infty} \delta^t \frac{\left(C_t^i\right)^{1-\gamma}}{1-\gamma} \tag{2.2}$$
$$\text{s.t.: } S_t^i P_t + C_t^i = S_{t-1}^i \left(P_t + D_t\right) + W_t - \tau \left|(S_t^i - S_{t-1}^i) P_t\right| + T_t^i$$
$$S_{-1}^i \text{ given,}$$

where C^i denotes consumption, $\gamma > 1$ the coefficient of relative risk aversion, S^i the agent's stockholdings, $P \geq 0$ the (ex-dividend) price of the stock, $\tau \geq 0$ a linear financial transactions tax, which is levied on the agents' trading volume $\left|(S_t^i - S_{t-1}^i) P_t\right|$ and $T^i \geq 0$ lump sum tax rebates.

Investors' choices are contingent on the history of variables that are exogenous to their decision problem, i.e., time t choices depend on $\{P_j, D_j, W_j, T_j\}_{j=0}^t$ and the initial condition S_{-1}^i. $\mathcal{P}^i$ denotes a subjective probability measure, which assigns probabilities to all possible infinite histories $\{P_t, D_t, W_t, T_t\}_{t=0}^{\infty}$. The agent's subjective probabilities may or may not coincide with the objective probabilities, i.e., agents may not know the true probabilities characterizing the behavior of the variables $\{P_t, D_t, W_t, T_t\}_{t=0}^{\infty}$, which are beyond their control, but agents are 'internally rational' in the sense of Adam and Marcet (2011), i.e., behave optimally given their beliefs about external variables.

We consider linear transaction taxes because they are most easily implemented in practice. In addition, non-linear transaction taxes would create incentives to either partition trades into smaller increments or bundle trades of several investors into larger packages, so as to economize on transaction costs. The resulting tax rate would effectively be linear again. To simplify the analysis, we also assume that transaction taxes paid by investors of type i are rebated in the same period in a lump sum fashion, i.e.,

$$T_t^i = \tau \left|(S_t^i - S_{t-1}^i) P_t\right|, \tag{2.3}$$

where S_t^i and S_{t-1}^i on the r.h.s. of the previous equation denote the choices of the representative investor of type i.[19] We thereby eliminate the income effects associated with raising transaction taxes.[20] Appendix 2.12.5 considers

[19] Agents' fully understand that what matters for tax rebates is the trading decision of the representative investor of type i and not their own decision.

[20] Alternative assumptions, e.g., a rebate that is identical across investors at each point in time, would make rebates dependent on the whole distribution of trades in equilibrium and thus on the distribution of investors' beliefs. This would add many additional state variables into investors' decision problem.

an alternative setup without tax rebates ($T_t^i \equiv 0$ for all t, i). It shows that the main quantitative findings are robust to assuming that taxes are not rebated to investors.

The exogenous wage and dividend processes take the form considered previously in Adam, Beutel and Marcet (2014), with dividends evolving according to

$$\ln D_t = \ln \beta^D + \ln D_{t-1} + \ln \varepsilon_t^D, \tag{2.4}$$

where $\beta^D \geq 1$ denotes the mean growth rate of dividends and, $\ln \varepsilon_t^D$ an i.i.d. growth innovation described further below. The wage income process W_t is chosen such that the resulting aggregate consumption process $C_t = W_t + D_t$ is empirically appealing.[21] In particular, we assume

$$\ln W_t = \ln \rho + \ln D_t + \ln \varepsilon_t^W, \tag{2.5}$$

where

$$\begin{pmatrix} \ln \varepsilon_t^D \\ \ln \varepsilon_t^W \end{pmatrix} \sim iiN \left(-\frac{1}{2} \begin{pmatrix} \sigma_D^2 \\ \sigma_W^2 \end{pmatrix}, \begin{pmatrix} \sigma_D^2 & \sigma_{DW} \\ \sigma_{DW} & \sigma_W^2 \end{pmatrix} \right) \tag{2.6}$$

which implies $E\varepsilon_t^D = E\varepsilon_t^W = 1$.

Substituting the constraint into the objective function and dividing the objective function by $D_0^{1-\gamma}$, the investor's problem can be written as

$$\max_{\{S_t^i\}_{t=0}^{\infty}} E_0^{\mathcal{P}^i} \sum_{t=0}^{\infty} \delta^t \left(\frac{D_t}{D_0}\right)^{1-\gamma} \frac{\left(S_{t-1}^i \left(\frac{P_t}{D_t} + 1\right) + \frac{W_t + T_t^i}{D_t} - \tau \left| \frac{(S_t^i - S_{t-1}^i) P_t}{D_t} \right| - S_t^i \frac{P_t}{D_t}\right)^{1-\gamma}}{1-\gamma}$$

$$s.t. : S_{-1}^i \text{ given} \tag{2.7}$$

Due to the linear transaction cost specification, the preceding optimization problem fails to be differentiable. We explain in section 2.5.1 how we deal with this difficulty.

Subjective Beliefs. To complete the description of the investment problem we now specify investors' subjective probability measure $\mathcal{P}^i$. We first assume that agents know the processes (2.4) and (2.5), i.e., hold rational dividend and wage expectations.[22] In a second step, we seek to specify subjective price beliefs in a way that allows us to capture the extrapolative nature of price expectations, as implied by survey data. In particular, following Adam, Beutel and Marcet (2014), we set up a belief system for prices that leads to expectation dynamics of the kind described by equation (2.1), which captures the empirical behavior of survey expectations. To this end, we endow agents

[21] For further details, we refer the reader to Adam, Beutel and Marcet (2014), section 4.

[22] This is motivated by the fact that wihtin the present setting with time separable preferences, (reasonable amounts of) extrapolation of wage and dividend beliefs would add very little to price volatility. This holds true for models with rational price expectations, as discussed in section 2 in Adam, Beutel and Marcet (2014), but also for models with subjective price beliefs, see for example section V.A in Adam, Marcet and Nicolini (2016).

with a belief system that allows for persistent deviations of the growth rate of prices from the growth rate of dividends. Specifically, we assume that agent i's perceived law of motion of prices is given by

$$\ln P_{t+1} - \ln P_t = \ln \beta^i_{t+1} + \ln \varepsilon^{1,i}_{t+2} + \ln \varepsilon^{2,i}_{t+1}, \tag{2.8}$$

where $\varepsilon^{1,i}_{t+2}, \varepsilon^{2,i}_{t+1}$ denote (not directly observable) transitory shocks to price growth and β^i_{t+1} a persistent price growth component that slowly drifts over time according to

$$\ln \beta^i_{t+1} = \ln \beta^i_t + \ln \nu^i_{t+1}, \tag{2.9}$$

and where the persistent component of price growth $\ln \beta^i_{t+1}$ is also unobserved. The setup just described can capture periods with sustained increases in the price dividend ratio ($\beta^i_{t+1} > \beta^D$), as well as periods with sustained decreases ($\beta^i_{t+1} < \beta^D$). The perceived innovations $\ln \varepsilon^{1,i}_{t+2}, \ln \varepsilon^{2,i}_{t+1}$ and $\ln v^i_{t+1}$ are assumed to be jointly normally distributed according to

$$\begin{pmatrix} \ln \varepsilon^{1,i}_{t+2} \\ \ln \varepsilon^{2,i}_{t+1} \\ \ln \nu^i_{t+1} \end{pmatrix} \sim iiN \left(\begin{pmatrix} -\frac{\sigma^2_{\varepsilon,1}}{2} \\ -\frac{\sigma^2_{\varepsilon,2}}{2} \\ -\frac{(\sigma^i_v)^2}{2} \end{pmatrix}, \begin{pmatrix} \sigma^2_{\varepsilon,1} & 0 & 0 \\ 0 & \sigma^2_{\varepsilon,2} & 0 \\ 0 & 0 & \sigma^{i\,2}_{\nu} \end{pmatrix} \right), \tag{2.10}$$

where the variances $\sigma^2_{\varepsilon,1}$, $\sigma^2_{\varepsilon,2}$ of the transitory components are identical for all agents. We allow the perceived variance of the innovation to the persistent component ($\sigma^{i\,2}_{\nu}$) to differ across investors, so as to be able to capture the different responsiveness of survey expectations to realized price growth rates, as documented in section 2.4.3.

The previous setup defines an optimal filtering problem for agents, in which they need to decompose observed price growth ($\ln P_{t+1} - \ln P_t$) into its persistent and transitory components ($\ln \beta^i_{t+1}$ and $\ln \varepsilon^{1,i}_{t+2} + \ln \varepsilon^{2,i}_{t+1}$, respectively). In the special case, that the two transitory shock components are both unobserved and can thus be combined to $\ln \varepsilon^i_t = \ln \varepsilon^{1,i}_{t+1} + \ln \varepsilon^{2,i}_t$ with variance $\sigma^2_\varepsilon = \sigma^2_{\varepsilon,1} + \sigma^2_{\varepsilon,2}$, Adam, Beutel and Marcet (2014) show, that under the assumption of a normal prior with variance equal to its Kalman filter steady state value, price growth beliefs can be summarized by a single state variable m^i_t that evolves according to

$$\begin{aligned} \ln m^i_t = \ln m^i_{t-1} - \frac{(\sigma^i_v)^2}{2} \\ + g^i \left(\ln P_t - \ln P_{t-1} + \frac{(\sigma^i_\varepsilon)^2 + (\sigma^i_v)^2}{2} - \ln m^i_{t-1} \right) \end{aligned} \tag{2.11}$$

$$g^i = \frac{(\sigma^i)^2}{\sigma^2_\varepsilon + (\sigma^i)^2}, \tag{2.12}$$

where

$$(\sigma^i)^2 \equiv \frac{-(\sigma^i_\nu)^2 + \sqrt{((\sigma^i_\nu)^2)^2 + 4(\sigma^i_\nu)^2 \sigma^2_\varepsilon}}{2}$$

is the Kalman filter steady state variance. The state variable $\ln m_t^i$ describes the mean of $\ln \beta_t^i$ conditional on the information available at time t, i.e., $\ln \beta_t^i$ is conditionally $N(\ln m_t^i, (\sigma^i)^2)$-distributed, which implies

$$E_t^{\mathcal{P}^i}\left[\frac{P_{t+1}}{P_t}\right] = m_t^i e^{(\sigma^i)^2/2}.$$

This previous result, together with equation (2.11) shows that optimal belief updating delivers - up to a log-exponential transformation - the updating equation (2.1) considered in the empirical section. Moreover, equation (2.12) shows that the optimal updating parameter g^i is a positive function of the variance $(\sigma_\nu^i)^2$, which allows us to replicate the empirically observed heterogeneity in the belief updating equations.

To avoid simultaneity between prices and price beliefs, which may give rise to multiple market clearing price and price belief pairs, we shall rely on a slightly modified information structure, where agents observe $\ln \varepsilon_t^{1,i}$ as part of their time t information set. Adam, Beutel and Marcet (2014) show how such a modified information structure gives rise to an updating equation of the form

$$\ln m_t^i = \ln m_{t-1}^i + g^i \left(\ln P_{t-1} - \ln P_{t-2} - \ln m_{t-1}^i\right) - g \ln \varepsilon_t^{1,i}, \qquad (2.13)$$

which has lagged price growth enter.[23]

To complete the description of the belief system, we need to specify investors' beliefs about the behavior of the lump sum tax rebate T_t^i. We shall assume that agents understand that the tax rebates do not depend on their own decision, instead on the choices of the representative investor of the same type i. Moreover, we assume that agents know the tax rebate function (2.3).[24]

Market Clearing. The stock market clearing condition is given by

$$\sum_{i=1}^{I} S_t^i \mu_i = 1 + u_t,$$

where the left-hand side denotes total stock demand by investors of all types and the right-hand side total stock supply. We incorporate a small exogenous stochastic component u_t into stock supply, which we assume to be white noise, uniformly distributed and to have support $[-\overline{u}, \overline{u}]$ for some $\overline{u} > 0$ sufficiently close to zero. Stock supply shocks u_t may thereby capture the issuance of new stocks or stock repurchases by firms.[25] We add these shocks because linear

[23] Price growth expectations are then given by $E_t^{\mathcal{P}^i}[P_{t+1}/P_t] = m_t^i$.

[24] This assumption considerably simplifies the analysis: since in equilibrium individual actions coincide with those of the representative investor of the same type, we do not need to incorporate any additional state variables that characterize the future evolution of lump sum taxes, when writing a recursive representation of the agents' decision problem.

[25] Alternatively, they may capture changes to asset float, as discussed in Ofek and Richardson (2003) and Hong, Scheinkman, and Xiong (2006). In any case, these shocks capture (exogenous) stock demand or supply that is not coming from the consumers described around equation (2.2).

financial transaction taxes lead to piecewise price-insensitive demand curves, which can give rise to equilibrium price indeterminacy in the absence of supply shocks. In our numerical applications, we make sure that $\overline{u}$ is sufficiently small such that it has no noticeable effects on the outcomes that emerge in the absence of a financial transaction tax. For the case with transaction taxes, the supply shock effectively only selects the equilibrium price whenever price-insenstitive demand curve may create the potential for price indeterminacy.

2.5.1 Solution Approach

This section explains how one can solve for the optimal solution of the non-differentiable problem (2.7). The approach we pursue consists of defining an alternative optimization problem with a differentiable transaction cost specification, so that a standard solution approach based on first order conditions can be applied. The alternative problem has the property that all choices that are feasible in the original problem are also feasible in the alternative problem. Therefore, if the optimal solution to the differentiable problem is a feasible choice in (2.7), then it must also solve (2.7).

The alternative problem we consider is

$$\max_{\{S_t^i\}_{t=0}^{\infty}} E_0^{\mathcal{P}^i} \sum_{t=0}^{\infty} \delta^t \left(\frac{D_t}{D_0}\right)^{1-\gamma} \frac{\left(S_{t-1}^i\left(\frac{P_t}{D_t}+1\right)+\frac{W_t+T_t^i}{D_t}-\tau_t^i\frac{(S_t^i-S_{t-1}^i)P_t}{D_t}-S_t^i\frac{P_t}{D_t}\right)^{1-\gamma}}{1-\gamma}$$
$$s.t.: S_{-1}^i \text{ given} \tag{2.14}$$

where $\tau_t^i \in [-\tau, \tau]$ denotes a state-contingent but fully linear transaction tax/subsidy and where T_t^i is given by (2.3). Problem (2.14) is differentiable and can be solved in a standard way using first-order conditions. Moreover, since $\tau_t^i \in [-\tau, \tau]$, all stockholding plans that are feasible in the original problem (2.7) continue to be feasible in the alternative problem (2.14).

Suppose that the state-contingent transactions cost function τ_t^i and the associated optimal stockholding plan $\{S_t^{i,opt}\}_{t=0}^{\infty}$ solving (2.14) jointly satisfy for all $t \geq 0$ the following property

$$\begin{array}{ll} \tau_t^i = \tau & \text{at contingencies where } S_t^{i,opt} > S_{t-1}^{i,opt} \\ \tau_t^i = -\tau & \text{at contingencies where } S_t^{i,opt} < S_{t-1}^{i,opt} \\ \tau_t^i \in [-\tau, \tau] & \text{at contingencies where } S_t^{i,opt} = S_{t-1}^{i,opt}, \end{array} \tag{2.15}$$

then $\{S_t^{i,opt}\}_{t=0}^{\infty}$ is also feasible in the original problem (2.7) and thus the solution to (2.7). The task of solving the original non-differentiable problem (2.7) is thus equivalent to finding a state contingent tax function τ_t^i such that condition (2.15) holds for the optimal solution of the alternative differentiable problem (2.14).

For a given $\{\tau_t^i\}_{t=0}^{\infty}$ the solution to (2.14) is characterized by the first order condition

$$\left(\frac{C_t}{D_t}\right)^{-\gamma}(1+\tau_t^i)\frac{P_t}{D_t}=\delta E_t^P\left(\frac{C_{t+1}}{D_{t+1}}\right)^{-\gamma}\left(\frac{D_{t+1}}{D_t}\right)^{1-\gamma}\left(\frac{P_{t+1}}{D_{t+1}}(1+\tau_{t+1}^i)+1\right) \tag{2.16}$$

As noted above, investor i's subjective beliefs can be summarized by the recursively evolving state variable m_t^i. Provided the state contingency of the tax function can be expressed in the form $\tau_t^i=\tau^i(S_{t-1}^i,\frac{P_t}{D_t},\frac{W_t}{D_t},m_t^i)$, where the arguments in the function should be interpreted as the choices and beliefs of the representative agent of type i, the optimal stock holding policy then also has a recursive representation of the form $S_t=S^i(S_{t-1}^i,\frac{P_t}{D_t},\frac{W_t}{D_t},m_t^i)$, by the same arguments as put forward in Adam, Beutel and Marcet (2014).[26,27]

Our numerical solution routines, which are described in appendix 2.12.2 simultaneously solve for the functions $\tau^i(\cdot)$ and $S^i(\cdot)$ that jointly satisfy equations (2.15) and (2.16). Numerically solving for the optimal solution is computationally costly. Despite extensive reliance on parallelization, the numerical computation of the solution and the evaluation of the Euler errors takes around 30 hours of computing time.

2.6 Outcomes under Objective Price Beliefs

Before presenting the model outcome under subjective price beliefs, this section briefly discusses the model predictions for the case where agents hold rational price expectations. With objective price beliefs and with investors holding identical initial stock endowments, differences between investor types disappear. The model then reduces to a representative agent rational expectations model with time separable preferences. As shown in Adam, Beutel and Marcet (2014), the pricing implications of the model then display a well-known set of shortcomings. The standard deviation of the price dividend ratio, for instance, is one order of magnitude below that observed in the data and displays virtually no persistence over time. The model thus fails to replicate the large and protracted run-ups and reversals that can be observed for the PD ratio in U.S. data. The model also fails to replicate the positive correlation between the PD ratio and expected returns, as evidenced in survey data. Finally, with rational price expectations, the model does not give rise to trade in equilibrium, thus cannot be related to the documented facts on trading activity. As we show in the next section, model performance strongly improves, once one incorporates the kind of extrapolative behavior documented in survey data.

[26] The fact that the transaction costs are linear and that under the stated assumptions the tax rebate T^i is a function of the same state variables is key for this result.

[27] The fact that τ_t^i depends on S_{t-1}^i is just a convenient way to summarize dependence of the tax function on past values of P_t, D_t and W_t. It does not mean that the agent thinks that τ_t^i depends on its own choices, in fact, as should be clear from the first order condition (2.16), the agent takes τ_t^i as exogenously given.

2.7 Quantitative Model Performance

This section evaluates the quantitative performance of our asset pricing model in the absence of FTTs with subjective price beliefs given by equations (2.8) and (2.9). Performance is evaluated in terms of the ability to match the stylized facts presented in section 2.4. The effects of introducing FTTs will be studied in section 2.9.

We parameterize our model using the model parameters employed in Adam, Beutel and Marcet (2014), which are summarized in table 2.5. Table 2.5 also lists the value for the support of stock supply shocks, which is a new parameter and set such that the amount of trade caused by these shocks amounts to less than 0.3% of the average trading volume in a setting without FTTs. Since trading volume is only weakly affected for the considered range of FTTs, the same holds approximately true for the case with FTTs. Furthermore, we verify that in the absence of FTTs, stock supply shocks affect the model moments in almost non-noticable ways.

Motivated by the evidence in table 2.4, we consider a model with 5 agent types, each of which has mass 1/5, and assign to them the point estimates of the updating gains from table 2.4.[28]

Parameter	**Value**	**Calibration Target**
β^D	1.0048	average quarterly real dividend growth
σ_D	0.0192	std. deviation quarterly real dividend growth
ρ	22	average consumption-dividend ratio
σ_{DW}	$-3.74 \cdot 10^{-4}$	*jointly chosen* s.t. $corr_t(C_t/C_{t-1}, D_t/D_{t-1}) = 0.2$
σ_W	0.0197	and $std_t(C_t/C_{t-1}) = \frac{1}{7} std_t(D_t/D_{t-1})$
σ_ε	0.0816	std. deviation of quarterly real stock price growth
δ	0.995	average PD ratio
γ	2	- none -
$\overline{u}$	$1 \cdot 10^{-5}$	- none -

Table 2.5: Model calibration

Table 2.6 compares the model generated moments in the absence of FTTs to those in the data.[29] The model moments for our baseline calibration are reported in the third column, while the fourth column reports the associated t-ratios for each considered data moment.[30] Overall, our asset pricing model does a good job in replicating the pure stock price moments, i.e., the first

[28] Recall that we chose the experience groups in table 2.4 so as to have approximately the same number of investors in each group.

[29] All simulation results are based on 100.000 quarters of simulated data, where the first 10.000 quarters are considered as a burn-in and discarded when calculating model moments. Also, to make results from different simulations more comparable, we use fixed sequences for the exogenous driving processes (wages, dividends, stock supply shocks).

[30] The t-ratio is based on an estimate of the standard deviation of the data moment as a

five moments reported in the table. It matches particularly well the mean and autocorrelation of the PD ratio, as well as the mean of quarterly real stock returns. It produces, however, too much volatility for the PD ratio and for returns. The model also does a good job in capturing the observed high positive correlation between the PD ratio and average return expectations in the survey data ($corr(PD_t, \overline{E}_t R_{t+1})$).

Regarding the newly added moments, the model generates a high positive autocorrelation in trading volume ($corr(TV_t, TV_{t-1})$), albeit the model correlation is too high relative to the one found in the data. The model also manages to quantitatively capture the positive correlation between trading volume and (normalized) absolute price changes ($corr(TV_t, |P_t/P_{t-1} - 1|)$). When looking at the correlation between trading volume and the PD ratio ($corr(TV_t, PD_t)$), the model produces a fairly weak positive correlation, but one that is stronger than in the data. The model also generates a positive correlation between trading volume and cross sectional dispersion of return expectations ($corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$), but again overstates this correlation relative to the data.[31] The latter should not be surprising, given that in our simple model belief dispersion is the only reason why agents want to trade.

Since the baseline model produces an 'anti-puzzle' in the form of too much stock price volatility relative to the data, we also consider a model version in which we dampen the extrapolative component in belief updating. This is motivated by the fact that the updating gains in table 2.4 are themselves estimated with uncertainty. Specifically, we reduce the point estimates from table 2.4 by 2.5 times the estimated standard deviation of the point estimate[32], leaving all other parameters unchanged. The resulting model moments are reported in the second to last column in table 2.6 below, with the last column reporting the associated t-ratios. Price and return volatility are now in line with data, while all other moments remain largely unaffected.

Overall, we find that the model does a good job in quantitatively replicating the joint behavior of stock price, trading volume and price expectations.

measure of uncertainty. Since the data moments in table 2.6 are not truly data moments, but functions of such moments, we estimate the standard deviation using the so-called delta method, as described in Cox (1998) or in the online appendix to Adam, Marcet and Nicolini (2016).

[31] The data moment reported in table 2.6 is the one pertaining to the UBS survey, which has also been used to compute $corr(PD_t, \overline{E}_t R_{t+1})$ in the data.

[32] We use for each gain parameter the gain specific standard deviation reported in the last row of table 2.4.

	U.S. Data	**Baseline Model** (no tax)	t-ratio	**Reduced Gain** (no tax)	t-ratio
$E[PD]$	139.77	135.77	0.16	117.16	0.91
$std(PD)$	65.17	122.13	-3.84	92.96	-1.88
$corr(PD_t, PD_{t-1})$	0.98	0.98	0.84	0.98	-0.13
$std(r^s)$	8.00%	11.63%	-9.05	8.27%	-0.68
$E[r^s]$	1.89%	2.11%	-0.47	1.84%	0.11
$corr(PD_t, \overline{E}_t R_{t+1})$	0.79	0.84	-0.78	0.84	-0.73
$corr(TV_t, TV_{t-1})$	0.89	0.97	-4.29	0.97	-4.14
$corr(TV_t, PD_t)$	-0.07	0.37	-5.79	0.47	-7.09
$corr(TV_t, \|P_t/P_{t-1} - 1\|)$	0.34	0.25	1.12	0.28	0.78
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.41	0.95	-3.67	0.92	-3.50

Table 2.6: Quantitative match of the asset pricing model

2.8 Asset Price Booms and their Implications

This section illustrates that stock prices in the model occasionally embark on a self-sustaining asset price boom and bust cycle. Unlike in the representative agent model of Adam, Beutel and Marcet (2014), such cycles have large welfare implications for different agent types.

To illustrate the potential of the model to generate boom-bust cycles and to compute the welfare implications of such cycles, we conduct a simple controlled experiment using the baseline model from the previous section: we fix agents' initial stockholdings and initial beliefs at their ergodic sample means; we then shock the economy with n positive dividend growth shocks of a two standard deviation size. Such or larger positive dividend shocks occur with a probability of about 2.5% per quarter. We shut down all other shocks, including dividend growth shocks after period n. We begin the experiment with $n = 1$ and successively increase n until we obtain a stock price boom and bust cycle from period $n + 1$ onwards. Figure 2.3 depicts - for different values of n - the equilibrium outcomes for the PD ratio during the initial periods. While the PD ratio reacts very little to the positive news when $n = 1$ (stock prices, however, do react to the positive dividend news), the increase in price optimism starts to increase slightly the PD ratio for $n = 2$ and $n = 3$. For $n = 4$ one suddenly obtains a very large stock price boom and a subsequent price bust, see figure 2.4.[33]

The economic forces driving the boom and bust dynamics are explained in detail in Adam, Beutel and Marcet (2014). Here, we only note that the boom

[33]Increasing n further would lead to very similar boom-bust dynamics as for the case with $n = 4$.

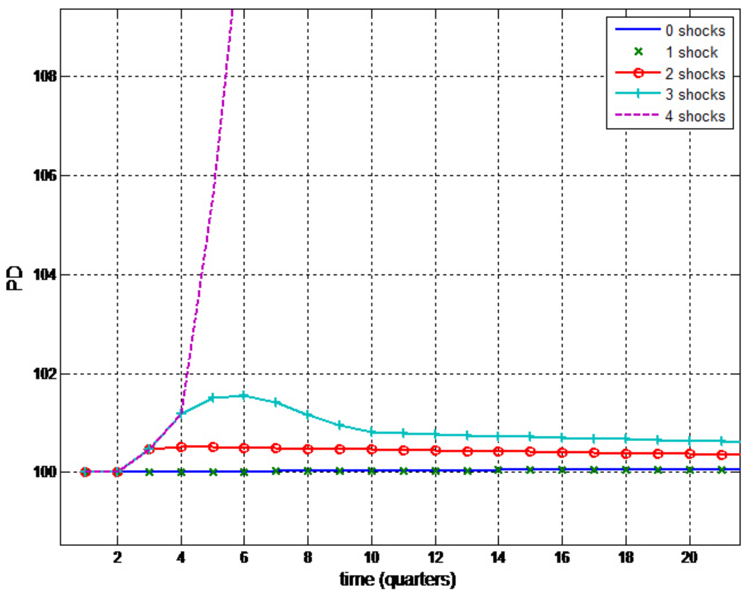

Figure 2.3: Response of the PD ratio to dividend growth shocks: initial periods

results from the fact that agents - having observed price increases - become optimistic about future price growth and eventually bid up stock prices by sufficient amounts, so that price increases and increasing optimism mutually reinforce each other. This effect is set in motion whenever a sufficient number of positive fundamental shocks, e.g., dividend growth shocks, occurs. The boom comes to an end, when agents' increased wealth leads them to eventually increase consumption demand, so that stock demand ceases to increase further with increased optimism. Prices then stagnate, which means that they fail to fulfill the high growth expectations of agents. Agents then revise growth beliefs downwards and set in motion a price bust. The bust causes a temporary undershooting of the PD ratio below its ergodic mean, but prices eventually return close to their ergodic mean absent further shocks, see figure 2.4.

Figure 2.5 depicts the PD ratio (top panel) together with agents' equilibrium trading decisions (middle panel) and return expectations (bottom panel) for the boom-bust episode triggered by four positive dividend growth shocks. To increase readability of the graph, we only report the trading patterns and return expectations of agents with the highest and lowest updating gain parameters.[34] In the UBS survey, high gains were estimated for agents with few years of stock market experience, while the most experienced group displayed a low updating gain. For this reason we refer to agents with a high (low) gain

[34] Agents types with intermediate updating gain values take intermediate decisions that are in between those shown in the figure.

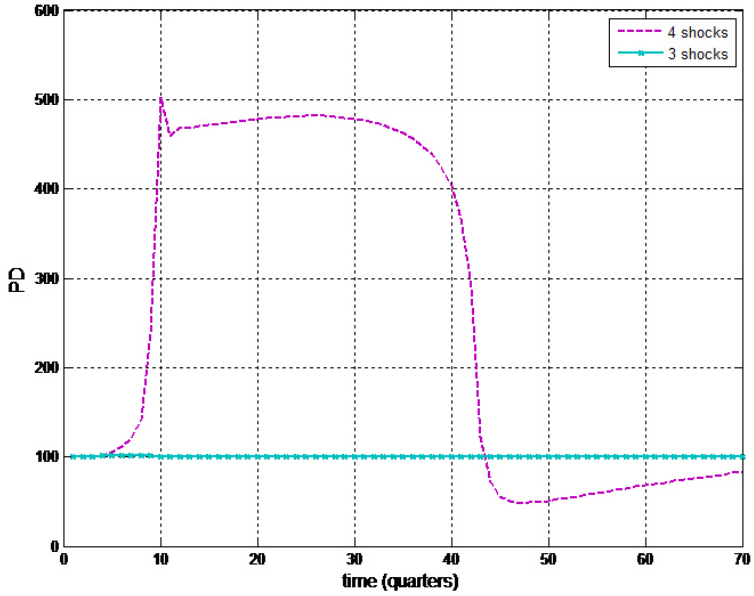

Figure 2.4: Response of the PD ratio: three versus four dividend growth shocks

as inexperienced (experienced) agents.

Figure 2.5 shows that in the initial phase of the stock price boom, inexperienced agents do rather well. They start buying stocks early on and well before prices approach their peak value. Experienced investors sell assets during the boom phase, i.e., much too early. Yet, once the PD ratio is high, inexperienced investors are much more optimistic about future returns than experienced investors, see the bottom panel. As a result, inexperienced investors continue buying stocks from low gain types at high prices (relative to dividends). Also, inexperienced investors continue buying during much of the price bust phase and only sell in significant amounts once the PD ratio started undershooting its long-run mean. Thus, even though inexperienced investors are doing well initially, this fails to be the case over the entire boom-bust cycle.

To gauge the welfare effects of a boom-bust episode, we compare the outcome in figure 2.5 to a situation in which the same shocks occur, but where agents hold their beliefs constant at the initial value, i.e., do not respond to the price movements triggered by the dividend growth shocks, so that there is no asset price boom. We can then compute the permanent proportional consumption variation that would make (ex-post realized) utility in the setting with constant beliefs and without an asset price boom identical to the (ex-post realized) utility in the setting with the asset price boom shown in figure 2.5. Outcomes are reported in table 2.7, which shows that asset price booms are extremely costly for inexperienced agents and extremely beneficial

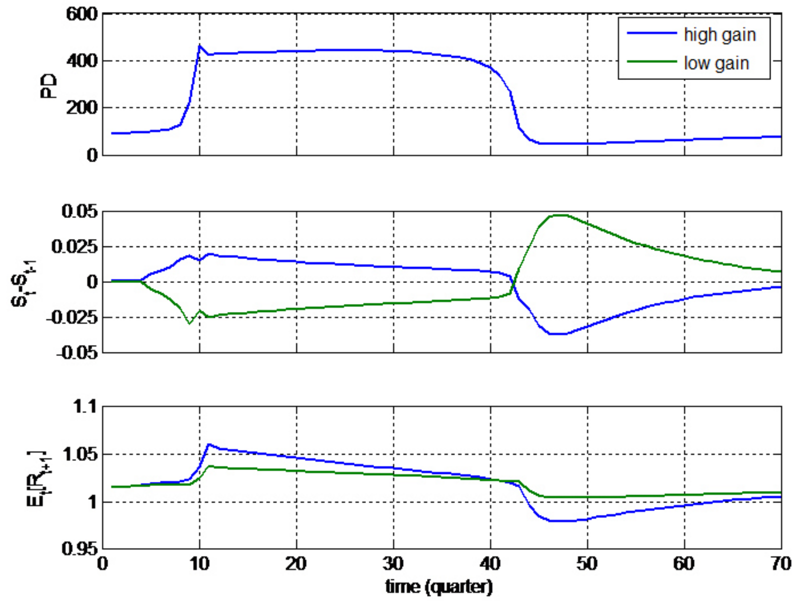

Figure 2.5: PD ratio, trading and return expectations over a boom-bust cylce (baseline model, no tax)

for experienced investors: the welfare equivalent consumption variations of a boom-bust episode amount to a permanent change in consumption of several percentage points.

Gain	0.0316	0.0286	0.0264	0.0230	0.0180
Permanent cons. variation	-7.01%	-3.51%	-1.27%	1.73%	5.24%

Table 2.7: Welfare cost of a stock price boom-bust episode

2.9 The Effects of Financial Transaction Taxes

We now consider the implications of introducing linear financial transaction taxes, focusing on the implication of FTTs for the behavior of asset pricing moments, the patterns of boom-bust dynamics and trading volume.

Table 2.8 reports how the asset pricing moments from the baseline model in table 2.6 are affected by various tax rates. The main effect of financial transaction taxes consists of increasing asset price volatility, as measured by the standard deviation of quarterly stock returns ($std(r^s)$) and the standard deviation of the PD ratio ($std(PD)$).[35] Except for the reduced correlation between trading volume and prices ($corr(TV_t, PD_t)$, $corr(TV_t, |P_t/P_{t-1} - 1|)$), the remaining asset pricing moments from table 2.6 prove to be rather robust towards the introduction of FTTs.

The last four rows in table 2.8 report a number of additional statistics about asset price boom-bust episodes and trading volume. These statistics allow to asses in greater detail why asset price volatility increases with the introduction of FTTs. The fourth to last row in table 2.8, for example, reports the number of asset price boom episodes per 100 years of simulated data, where we define the beginning of a boom as the first time in which the quarterly PD ratio exceeds a level of 250 and the end of a boom as the first time it falls below 200 thereafter.[36] The results in the table show that the number of stock price booms is monotonically increasing in the FTTs, with boom-bust episodes becoming about a third more likely relative to the case without transaction taxes when the tax rate reaches 4%.

The third and second to last rows in table 2.8 display, respectively, information about the length of the boom episodes and the average peak value of the PD reached during these episodes. It shows that booms tend to become shorter lived and somewhat less pronounced as the tax rate rises, but these effects are not very strong for tax rates up to 4%. As a result, the effect of an increased number of booms dominates and the standard deviation of the PD ratio increases with the tax rate. For a 10% tax rate, the decrease in the peak level of the PD during booms and the reduced length of stock price booms start to dominate, causing the standard deviation of the PD ratio to decreases, even if the standard deviation of returns still increases.

Somewhat surprisingly, the average trading volume (relative to the case without FTTs) tends to increase with the level of FTTs. This occurs because there is more trade during booms times, as belief disagreements are then larger, and because booms become more likely with the introduction of FTTs.

[35] For very high tax rates (10%) the volatility of the PD ratio starts to fall, while return volatility continues to increase. We discuss this issue further below.

[36] The reported numbers are very robust to choosing different thresholds because boom-bust episodes are periods in which prices display a clearly distinct behavior.

	No Tax	1% Tax	2% Tax	4% Tax	10% Tax
$E[PD]$	135.77	137.21	139.74	142.47	146.27
$std(PD)$	122.13	123.18	125.42	127.10	125.24
$corr(PD_t, PD_{t-1})$	0.98	0.98	0.98	0.98	0.98
$std(r^s)$	11.63%	11.85%	12.14%	12.55%	14.04%
$E[r^s]$	2.11%	2.14%	2.18%	2.24%	2.49%
$corr(PD_t, \overline{E}_t R_{t+1})$	0.84	0.85	0.86	0.87	0.89
$corr(TV_t, TV_{t-1})$	0.97	0.97	0.97	0.97	0.94
$corr(TV_t, PD_t)$	0.37	0.35	0.33	0.29	0.17
$corr(TV_t, \|P_t/P_{t-1} - 1\|)$	0.25	0.25	0.24	0.21	0.05
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.95	0.94	0.93	0.92	0.87
# of booms per 100 yrs*	1.82	1.95	2.12	2.40	3.06
average boom length (quarters)*	32.42	31.87	31.41	30.44	27.21
average boom peak (PD)*	491.03	485.82	480.31	469.95	443.86
$E[TV]$ relative to no tax	100.00%	99.64%	101.49%	102.52%	117.85%

*A boom starts in the first period in which the quarterly PD ratio exceeds a value of 250 and ends once it falls below 200.

Table 2.8: Effects of introducing financial transaction taxes

Table 2.9 reports the welfare implications associated with introducing different tax rates. Starting from the ergodic mean for stock holdings and beliefs in the no-tax economy, the table reports the welfare equivalent permanent consumption variation that would make different agent types in the economy with taxes as well-off in expected terms as in the economy without taxes.[37] Table 2.9 clearly shows that agents that extrapolate more, i.e., inexperienced investors in our survey sample, tend to lose, while more experienced investors tend to win in expected terms.[38] For all agent types, except the median type, whose utility is largely unaffected by the tax rate, the gains and losses monotonically increase with the tax rate. Wealth redistribution between investors thus increases with the tax rate.

[37] We use objective probabilities to compute agents' expected utility.

[38] The welfare effects in table 2.9 are smaller than those reported in table 2.7. The latter reports the effects of a single stock price boom episode relative to the counterfactual outcome without a boom. Since booms are (in expected terms) not likely to occur within the immediate future, when starting the simulation at the ergodic mean, the welfare effects in table 2.9 are not as large as those reported in table 2.7.

Gain	0.0316	0.0286	0.0264	0.0230	0.0180
1% Tax	-0.34%	-0.14%	-0.02%	0.06%	0.25%
2% Tax	-0.84%	-0.36%	-0.06%	0.16%	0.62%
4% Tax	-1.56%	-0.64%	-0.07%	0.28%	1.18%
10% Tax	-2.76%	-1.00%	0.10%	0.39%	2.15%

Table 2.9: Welfare implications of FTTs
(welfare equiv. permanent cons. variations)

Table 2.10 provides additional insights by reporting asset price moments conditional on being in a boom period, as defined above, and conditional on being in 'normal times', i.e., periods that are not identified as boom periods. Clearly, the PD ratio is considerably higher during boom times and so is the standard deviation of the PD ratio. Mean quarterly stock returns during boom periods are considerably higher than in normal times, but stock returns also display a considerably larger standard deviation. Furthermore, while the introduction of FTTs reduces the volatility of the PD ratio and returns during boom periods, FTTs increase both of these standard deviations during normal times. As we show below, it is precisely the increase in volatility during normal times coupled with extrapolative behavior which causes stock price booms to become more likely.

Table 2.10 shows that trading volume decreases with the size of the FTT during boom periods, but - somewhat paradoxically - increases during normal times. Upon closer inspection, we find that for tax rates up to 4% the increase in trading volume during normal times is purely driven by post-boom trading activity. As can be seen from figure 2.5, trading activity stays high long after the PD ratio returned to values below 200. Once one removes these post-boom periods from the normal times, trading volume is actually decreasing with the FTTs in normal times.[39]

[39] The situation is different for very high tax rates (10%). Trading activity then increases also during normal times, even when excluding post-boom periods. This occurs because the large increase in price volatility leads to an amount of belief disagreement and thus trade in normal times, which more than compensates the trade-reducing effect of the tax.

	No Tax	1% Tax	2% Tax	4% Tax	10% Tax
Boom times*					
$E[PD]$	424.63	419.79	415.04	406.56	384.06
$std(PD)$	44.93	44.13	44.27	42.89	42.01
$corr(PD_t, PD_{t-1})$	0.48	0.48	0.47	0.49	0.53
$std(r^s)$	23.06%	22.86%	22.52%	21.76%	19.34%
$E[r^s]$	3.68%	3.74%	3.67%	3.55%	3.15%
$E[TV]$ rel. to no tax	100.00%	96.13%	93.12%	88.27%	80.13%
Normal times+					
$E[PD]$	85.87	85.33	84.63	83.59	83.72
$std(PD)$	15.37	15.89	16.68	18.14	23.62
$corr(PD_t, PD_{t-1})$	0.84	0.84	0.85	0.85	0.86
$std(r^s)$	8.15%	8.36%	8.65%	9.31%	12.26%
$E[r^s]$	1.84%	1.84%	1.88%	1.95%	2.32%
$E[TV]$ rel. to no tax	100.00%	99.60%	101.43%	102.69%	129.90%

* A boom starts in the first period in which the quarterly PD ratio exceeds a value of 250 and ends once it falls below 200.

+ Normal times are all those periods not classified as boom periods.

Table 2.10: Conditional Asset Price Moments

To illustrate further how FTTs increase the likelihood of boom-bust cycles, we now perform a similar experiment as carried out in section 2.8 for the case without a tax. Specifically, we consider the model with a FTT of 4% and fix initial stockholdings and initial beliefs at their ergodic sample means. We then shock the economy with $n \geq 0$ positive dividend growth shocks of two standard deviations. Yet, this time we continue to let the small exogenous stock supply shocks operate at all times. These shocks are themselves not enough to generate stock price booms, but can do so in combination with dividend shocks.

Figure 2.6 depicts the probability that the economy embarks on a stock price boom as a function of the number of dividend growth shocks, integrating over possible realizations of the stock supply shocks.[40] For the case without a FTT, booms start to emerge once n increases above 4.[41] The situation differs for the case with a 4% FTT, where fewer fundamental shocks are required to start a boom episode. For $n \leq 1$, the economy never embarks on a stock price boom, but for $n = 2$ stock price booms emerge in more than 60% of the cases and for $n \geq 3$ virtually always. This shows that booms become more likely in

[40] As before, we define a boom as a stituation where the PD ratio subsequently increases above 250 at some point. We consider up to 12 quarters after the last dividend shock. Results prove very robust to choosing different thresholds and period limits. Probabilities are computed from averaging the outcome of 500 stochastic realizations.

[41] Since we now let stock supply shocks also operate in the case without a tax, this shows that the findings of figure 2.4 are robust to the introduction of the stock supply shock.

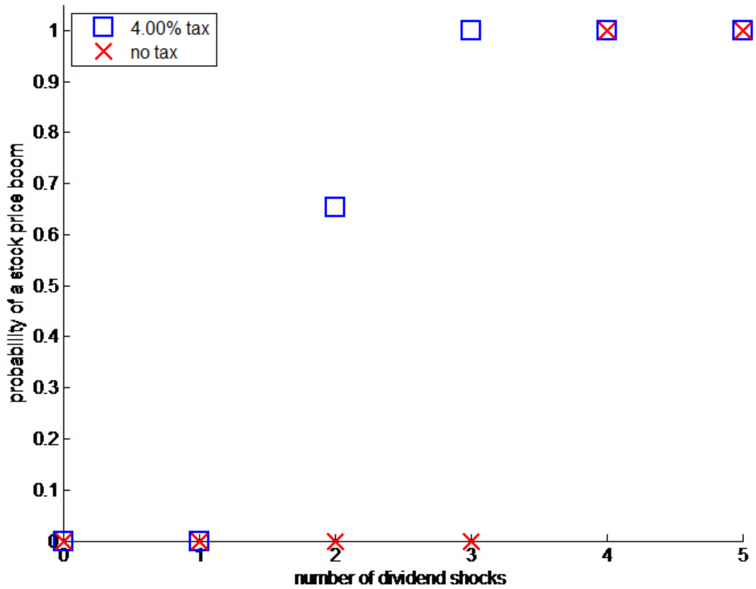

Figure 2.6: FTTs and the likelihood of stock price booms

a situation with FTTs, as fewer fundamental shocks are required to set it in motion.

Figure 2.7 illustrates the driving force giving rise to this outcome. The figure depicts the stock demand function for a 4% FTT.[42] It shows that around the level of prior stockholding (assumed to be equal to one), stock demand (shown on the vertical axis) is not sensitive to the stock price (shown on the horizontal axis). This price insensitivity of stock demand covers a considerable price range and is actually increasing with the tax rate.[43] Therefore, in the presence of FTTs, even very small exogenous variations in stock supply can lead to large movements in realized prices, explaining why prices become more volatile during 'normal times'. Since agents use realized price growth to update price expectations, FTTs increase the likelihood that stock prices embark on a belief-driven stock price boom.

2.10 State-Contingent Financial Transaction Taxes

Motivated by the results in the previous section, this section considers the effects of introducing state-contingent transaction taxes that are only levied

[42] The figure assumes $\tau = 4\%$ and the following values for the state variables: $W_t/D_t = \rho$, $m_t = \beta^D$, and $S_{-1} = 1$.

[43] Appendix 2.12.3 explains how one can accurately determine the inaction regions.

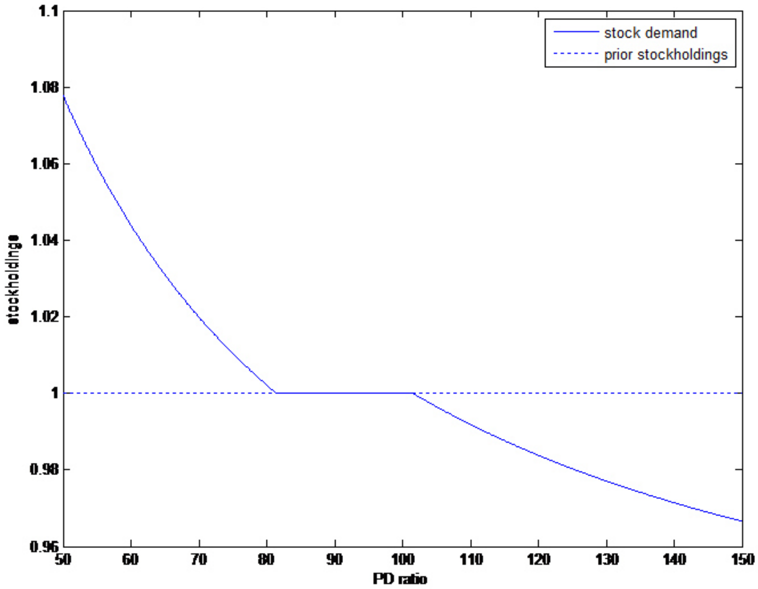

Figure 2.7: Stock demand function (4% tax)

once the PD ratio exceeds a certain (sufficiently high) threshold value $\overline{PD}$. The idea behind such a state-contingent tax is that it avoids the increase in price volatility during 'normal times', thereby avoiding that the stock market embarks with higher likelihood on a boom-bust cycle, while potentially limiting the duration and extent of stock price booms once they have taken hold.

Specifically, consider a setting with linear transaction taxes $\tau > 0$, which are levied only if $PD_t \geq \overline{PD}$, and zero taxes otherwise. We set the threshold value $\overline{PD}$ equal to 250, which is the value used to identify the beginning of a stock price boom episodes in previous sections. After solving for the optimal stock demand functions[44], it turns out that state-contingent taxes lead to problems of non-existence of equilibrium prices, as well as to the possibility of

[44]The solution strategy outlined in section 2.5.1 for the case with a non-state contingent tax can then still be applied because the tax function $\tau^i(S^i_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m^i_t)$ derived in section 2.5.1 can already depend on the PD ratio. Instead of satisfying equations (2.15) and (2.16), the tax function and the stock holding policy must now jointly satisfy the first order condition (2.16) and

$$\begin{array}{ll} \tau^i_t = \tau & \text{at contingencies where } S^{i,opt}_t > S^{i,opt}_{t-1} \text{ and } PD \geq \overline{PD} \\ \tau^i_t = -\tau & \text{at contingencies where } S^{i,opt}_t < S^{i,opt}_{t-1} \text{ and } PD \geq \overline{PD} \\ \tau^i_t \in [-\tau, \tau] & \text{at contingencies where } S^{i,opt}_t = S^{i,opt}_{t-1} \text{ and } PD \geq \overline{PD} \\ \tau^i_t = 0 & \text{otherwise,} \end{array}$$

so as to be feasible in the original problem with a non-differentiable tax function (above the PD threshold).

equilibrium multiplicities.

The non-existence problem is illustrated in figure 2.8, which depicts the excess stock demand (on the vertical axis) as a function of the price dividend ratio (horizontal axis). The figure depicts these functions for all agent types, as well as the aggregate excess demand function.[45]

Figure 2.8 shows that once the PD ratio exceeds its critical value $\overline{PD}$, agents want to buy or sell less stocks, i.e., the excess demand functions discontinuously jump to a value closer to the no trade line (the zero line). As a result, the aggregate excess demand function also has a jump at $PD = \overline{PD}$ and for the case depicted in figure 2.8, this leads to non-existence of an equilibrium price: the excess demand function is strictly positive for $PD < \overline{PD}$ but strictly negative for $PD \geq \overline{PD}$.

Obviously, the jump in the aggregate excess stock demand function does not necessarily have to be of the kind shown in figure 2.8. We also encountered cases in which there was an upward jump at the critical value $\overline{PD}$. This can happen whenever agents who seek to sell stocks respond more to the tax once its levied than agents who want to purchase stocks. Figure 2.9 depicts an example, where the aggregate excess demand jumps upwards at $PD = \overline{PD}$. As the figure illustrates, this can give rise to multiple market clearing equilibrium prices. Since realized prices feed into agents' price beliefs, price multiplicities have the potential to significantly increase price volatility.

While the non-existence problem could possibly be overcome by introducing taxes that are a continuous function of the PD ratio, the multiplicity issue is harder to address. One would have to design state-contingent taxes in such a way that aggregate stock excess demand functions are never upward sloping in the vicinity of the zero point. It is unclear which tax design would be able to achieve this outcome.

[45] To illustrate the effects in the most transparent way, we use the setting with a 10% transaction tax, but the effects are qualitatively the same for lower tax rates.

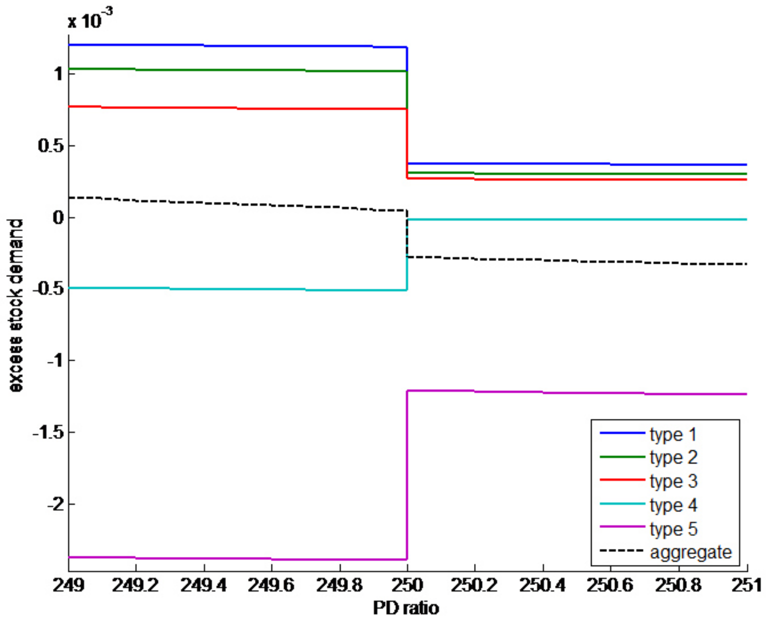

Figure 2.8: Non-existence of equilibrium with state-contingent FTTs

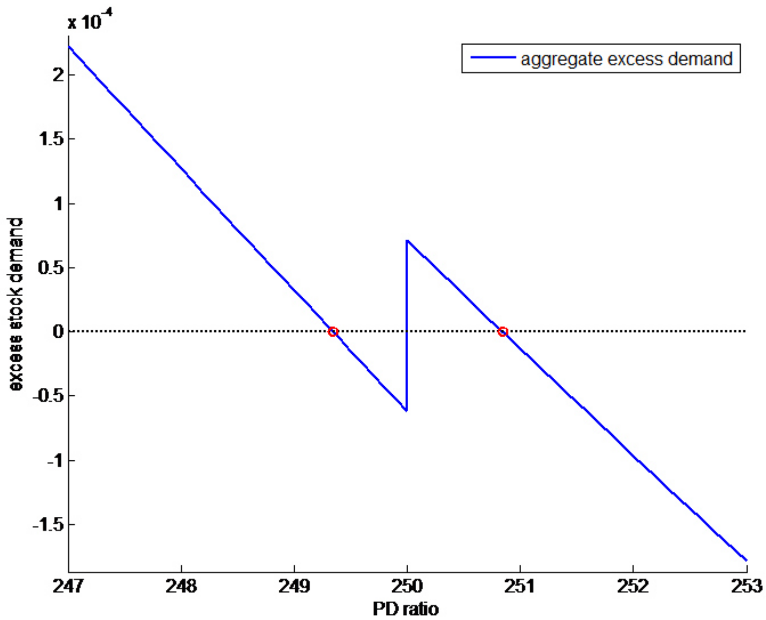

Figure 2.9: Multiple equilibrium prices with state-contingent transaction taxes

2.11 Conclusions

We present a quantitatively credible asset pricing model in which stock prices display occasional boom and bust cycles in valuation, which redistribute large amounts of wealth between different investor types. We show how the introduction of financial transactions taxes increases price volatility during ‘normal times’ and thereby the likelihood that the stock market embarks on a belief-driven boom and bust cycle. State-contingent transaction taxes, which seek to avoid the increase in price volatility during normal times, generate problems via equilibrium multiplicities and non-existence. Taken together, these findings cast serious doubts on whether financial transaction taxes can fruitfully contribute towards increasing the efficiency of stock market prices and transactions.

A key insight highlighted by the present framework is that the presence of extrapolation by investors makes it an important requirement that market interventions do not increase stock price volatility during normal times, so as to avoid creating additional boom-bust episodes. Throughout the analysis, we have taken the degree of extrapolation as given. Conceivably, market interventions can also have a direct effect on the degree to which investors extrapolate past capital gains. To the extent that FTTs reduce extrapolation, FTTs can generate additional benefits that are not captured within the present analysis and may overturn our results. Obviously, if FTTs give rise to more extrapolation, they generate additional costs and strengthen the point made in the present paper. Empirically investigating the effects of FTTs on the degree of investor extrapolation thus appears to be an interesting avenue for future research.

2.12 Appendix

2.12.1 Data sources

Stock price data: Our stock price data is for the United States and has been downloaded from 'The Global Financial Database'.[46] The period covered is Q1:1949-Q1:2012. The nominal stock price series is the "'SP 500 Composite Price Index (w/GFD extension)"' (Global Fin code "'_SPXD"'). The daily series has been transformed into quarterly data by taking the index value of the last day of the considered quarter. To obtain real values, nominal variables have been deflated using the 'USA BLS Consumer Price Index' (Global Fin code 'CPUSAM'). The monthly price series has been transformed into a quarterly series by taking the index value of the last month of the considered quarter. Nominal dividends have been computed as follows

$$D_t = \left(\frac{I^D(t)/I^D(t-1)}{I^{ND}(t)/I^{ND}(t-1)} - 1 \right) I^{ND}(t)$$

where I^{ND} denotes the "'SP 500 Composite Price Index (w/GFD extension)"' described above and I^D is the "'SP 500 Total Return Index (w/GFD extension)"' (Global Fin code "'_SPXTRD"'), which contains returns from price changes and dividend payouts. In the notation of our model, $I^D(t)$ is equal to P_t and $I^{ND}(t)/I^{ND}(t-1)$ equal to $(P_t + D_t)/P_{t-1}$.We first computed monthly dividends and then quarterly dividends by adding up the monthly series. Following Campbell (2003), dividends have been deseasonalized by taking averages of the actual dividend payments over the current and preceding three quarters.

Stock market survey data: The UBS survey is the UBS Index of Investor Optimism.[47] For all our calculations we use own portfolio return expectations from 1999:Q1 to 2007:Q2. We do not use data from 1998 due to missing values. The micro dataset of the UBS survey consists of 92823 record. Data-cleaning results in the removal of 18379 this records: Following Vissing-Jorgensen (2004), we ignore survey responses with stated expected returns larger than 95% in absolute value, which results in the elimination of 16380 observations. Furthermore, we ignore records, where the difference between the respondent's age and his stated stock market experience is less than 16 years, which eliminates 2378.[48]

The Shiller survey covers individual investors and has been kindly made available to us by Robert Shiller at Yale University. The survey spans the period 1999:Q1-2012:Q4. The CFO survey is collected by Duke University and CFO magazine and collects responses from about 450 CFOs. The data span the period 2000:Q3-2012:Q4.

[46] It is available at http://www.globalfinancialdata.com.

[47] See http://www.ropercenter.uconn.edu/data_access/data/datasets/ubs_investor.html.

[48] The two numbers do not add up to 18379, since some records satisfy both criteria for elimination.

Inflation expectations data: The Survey of Professional Forecasters (SPF) is available from the Federal Reserve Bank of Philadelphia.

Trading volume: We have daily data from Thomson Reuters Financial Datastream from 2nd January 1973 until 31st March 2014. We look at the series "'US-DS Market"' (TOTMKUS), an index of 1000 U.S. stocks traded on NYSE and Nasdaq.

We compute quarterly trading volume as follows: Starting from daily trading volume (DS: VA) and daily market value (DS: MV) we compute daily trading volume (VA/MV), i.e. the share of the market that is traded on each day. We then aggregate this up, following Lo and Wang (2009), by summing the shares over all trading days in the quarter, thus arriving at the share of the market that is traded in a particular quarter up to the last trading day of the quarter (end of March, June, September, December). Thus volume is measured over the same time period where expectations are measured. Moreover, end of quarter PDs are associated with the trading volume accumulated in the preceding 3 months.

2.12.2 Numerical solution approach

We now describe the solution strategy for determining the functions $S^i(\cdot)$ and $\tau^i(\cdot)$ and the associated lump sum rebate $T^i(\cdot)$. To simplify notation we drop all i superscripts. Also, instead of solving for the optimal stockholding function $S(\cdot)$, we solve in our numerical approach for the optimal consumption dividend ratio $C_t/D_t = CD(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$. There is a one-to-one mapping between the $S(\cdot)$ policy and the $CD(\cdot)$ policy due to the flow budget constraint, which implies

$$\frac{C_t}{D_t} = S_{t-1}\left(\frac{P_t}{D_t} + 1\right) + \frac{W_t + T_t}{D_t} - \tau_t(S_t - S_{t-1})\frac{P_t}{D_t} - S_t\frac{P_t}{D_t},$$

and due to the assumption that $\tau_t = \tau(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t^i)$ and $T_t/D_t = \tau\,|(S_t - S_{t-1})P_t/D_t|$.

We solve the first order condition by combining time iteration with an endogenous grid point method, thereby avoiding any root finding steps in the solution procedure. This considerably speeds up the numerical solution. We now describe this procedure in detail.

We start with a guess for the future consumption policy $CD^{(j)}(\cdot)$, the transactions tax function $\tau^{(j)}(\cdot)$ and the lump sum rebate relative to dividends $TD^{(j)}(\cdot) = T^j(\cdot)/D_t$, where the superscript (j) denotes the j-th guess in the time iteration procedure and where all functions depend on the arguments $(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$.

Given the guesses $CD^{(j)}(\cdot)$, $\tau^{(j)}(\cdot)$ and $TD^{(j)}(\cdot)$ and given an alternative grid of current values $(S_t, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$ - note this alternative grid contains S_t not S_{t-1} - we can compute the updated consumption policy $\widetilde{CD}^{(j+1)}(S_t, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$ and the updated marginal tax function $\widetilde{\tau}^{(j+1)}$, which are both defined over the alternative grid, by iterating on the FOC (2.16). In particular, equation (2.16) implies

$$\left(\widetilde{CD}^{(j+1)}\right)^{-\gamma}(1+\widetilde{\tau}_t^{(j+1)}) = \frac{\delta E_t^{\mathcal{P}}\left(CD^{(j)}\right)^{-\gamma}\left(\frac{D_{t+1}}{D_t}\right)^{1-\gamma}\left(\frac{P_{t+1}}{D_{t+1}}(1+\tau_{t+1}^{(j)})+1\right)}{P_t/D_t} \tag{2.17}$$

Given any point $(S_t, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$ on the alternative grid, we can compute the distribution over future (standard) grid points $(S_t, \frac{P_{t+1}}{D_{t+1}}, \frac{W_{t+1}}{D_{t+1}}, m_{t+1})$, using the perceived evolution over prices, dividends, wages and beliefs. Together with the guesses $CD^{(j)}(\cdot)$ and $\tau^{(j)}$, this allows evaluating the r.h.s. of (2.17) using a standard numerical integration method (we use deterministic integration based on quadrature points). For future reference, let $M(S_t, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$ denote the value of the r.h.s. of (2.17). The l.h.s. of equation (2.17) then implies that we have also determined the value of the product $(C_t/D_t)^{1-\gamma}(1-\tau_t)$, at every alternative grid point.

It now remains to compute the updated functions $CD^{(j+1)}$, $\tau_t^{(j+1)}$ and $TD^{(j+1)}$ which are defined over the standard grid $(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$. We do so by fixing an arbitrary alternative grid point $(S_t^*, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*)$ and by checking the range of possible situations $S_{t-1} \lesseqgtr S_t^*$.

We begin by conjecturing $S_{t-1} = S_t^*$. The flow budget constraint then determines the implied consumption dividend ratio, i.e.,

$$\frac{C_t}{D_t} = S_t^* + \left(\frac{W_t}{D_t}\right)^*. \tag{2.18}$$

We can then check whether the tax rate $\tau_t^{(j+1)}$ associated with (2.18), defined as

$$\left(\frac{C_t}{D_t}\right)^{-\gamma}(1+\tau_t^{(j+1)}) = M(S_t^*, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*), \tag{2.19}$$

satisfies $\tau_t^{(j+1)} \in [-\tau, +\tau]$. If so, then we have found the optimal consumption dividend ratio $CD^{(j+1)}$ and associated shadow tax rate $\tau_t^{(j+1)}$ at the standard grid point $\left(S_{t-1} = S_t^*, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*\right)$. The updated lump sum tax rebate over dividends at this gridpoint is simply $TD^{(j+1)} = 0$.

If the value of $\tau_t^{(j+1)}$ solving (2.19) satisfies $\tau_t^{(j+1)} > \tau$, then it must be that $S_t^* > S_{t-1}$.[49] We therefore set $\tau_t^{(j+1)} = \tau$ and determine the equilibrium consumption dividend ratio $CD^{(j+1)}$ from equation (2.17), which delivers

$$(CD^{(j+1)})^{-\gamma}(1+\tau) = M(S_t^*, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*).$$

[49]Reducing τ_t so that it satisfies $\tau_t \leq \tau$ requires that $(C_t/D_t)^{-\gamma}$ increases, see the l.h.s. of equation (2.17). From the flow budget constraint follows that this can only happen if S_{t-1} decreases below S_t^*, given the values for $(P_t/D_t)^*$ and $(W_t/D_t)^*$.

Finally, we use the budget constraint to compute the associated initial grid point S_{t-1}, which must solve

$$CD^{(j+1)} = S_{t-1}\left(\left(\frac{P_t}{D_t}\right)^* + 1\right) + \left(\frac{W_t}{D_t}\right)^* - S_t^*\left(\frac{P_t}{D_t}\right)^*, \qquad (2.20)$$

where we used the updated lump sum rebate function $TD^{(j+1)} = \tau \cdot (S_t^* - S_{t-1})\left(\frac{P_t}{D_t}\right)^*$. We have thus determined $CD^{(j+1)}$, $\tau^{(j+1)}$ and $TD^{(j+1)}$ at the grid point $\left(S_{t-1}, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*\right)$.

If the value of $\tau_t^{(j+1)}$ solving (2.19) satisfies $\tau_t^{(j+1)} < -\tau$, then we must assume $S_t^* < S_{t-1}$ and thus set $\tau_t^{(j+1)} = -\tau$. Using (2.17) we can determine the equilibrium consumption dividend ratio $CD^{(j+1)}$

$$u'(CD^{(j+1)})(1-\tau) = M(S_t^*, \left(\frac{P_t}{D_t}\right)^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*).$$

Again, we use the budget constraint to compute the associated grid point S_{t-1}, which must solve

$$CD^{(j+1)} = S_{t-1}\left(\left(\frac{P_t}{D_t}\right)^* + 1\right) + \left(\frac{W_t}{D_t}\right)^* - S_t^*\left(\frac{P_t}{D_t}\right)^*, \qquad (2.21)$$

where we use the updated lump sum rebate function $TD^{(j+1)} = -\tau \cdot (S_t^* - S_{t-1})\left(\frac{P_t}{D_t}\right)^*$

We perform the iterations described above until convergence of the functions $CD^{(j)}(\cdot)$, $\tau^{(j)}(\cdot)$ and $TD^{(j)}$.

2.12.3 Inaction Regions and Adaptive Grid Point Choice

A transaction tax leads to partially flat stock demand curves (inaction regions) and thereby introduces a high degree of nonlinearity - non-differentiabilities in the $\frac{P_t}{D_t}$-dimension - into the consumption policy function $CD^{(j)}(\cdot)$ and the associated shadow tax $\tau^{(j)}(\cdot)$. While linear interpolation between two grid points yields very accurate approximations of these functions for most $\frac{P_t}{D_t}$ values, this is generally not true close to the boundaries of the inaction regions, if these boundaries are not elements of our discretized state space.

Including the $\frac{P_t}{D_t}$ boundaries of the inaction region into the discretized state space poses two challenges: First, the exact locations of these boundaries are not known a priori, but depend on the optimal solution. Therefore, the $\frac{P_t}{D_t}$ grid is required to change in every iteration. We describe in the sequel how we use an adaptive grid point choice to ensure that our best guess for the inaction region boundaries is always part of the $\frac{P_t}{D_t}$ grid. Second, these boundaries are not independent of other states, but vary with $(S_{t-1}, \frac{W_t}{D_t}, m_t)$. Hence, the $\frac{P_t}{D_t}$ grid is not only required to change in every iteration of the algorithm, but also to be dependent on other state variables.[50] We clarify below how we interpolate our policy to states not contained in the discretized state space.

Adaptive grid points: Since the non-differentiability problem only occurs in the $\frac{P_t}{D_t}$-dimension, we fix a vector $(S_{t-1}, \frac{W_t}{D_t}, m_t)$ in the sequel. First, we observe, that the interior of the inaction region in the $\frac{P_t}{D_t}$-dimension can be identified by the shadow tax function $\tau(\cdot)$: The optimal consumption (or, equivalently, stock holding) policy does not change in a neighborhood of the current value of $\frac{P_t}{D_t}$, if and only if $\tau\left(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t\right) \in (-\tau, \tau)$. Since in such cases $S_{t-1} = S_t$, the same relationship must hold for the function $\tilde{\tau}$ defined on the alternative "state space" $(S_t, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t)$. In our solution algorithm, we solve for this function $\tilde{\tau}$ by solving equation (2.19) under the assumption that consumption satisfies the no trade relationship (2.18) and set it to τ, whenever its value exceeds τ and to $-\tau$, whenever its value is less than $-\tau$. The boundaries of the inaction region are therefore given for those values of $\frac{P_t}{D_t}$, for which no trade consumption defined by (2.18) and $\tau_t^{(j+1)} \in \{-\tau, \tau\}$ solve equation (2.19). This yields two equations

$$\left(S_t^* + \left(\frac{W_t}{D_t}\right)^*\right)^{-\gamma} (1 \pm \tau) = M(S_t^*, \left(\frac{P}{D}\right)_{\pm}, \left(\frac{W_t}{D_t}\right)^*, m_t^*)$$

which we solve for the adapted grid points $\left(\frac{P}{D}\right)_{\pm}$ in each iteration of the above algorithm.[51] We make sure, that in our algorithm not only the functions $CD^{(j)}(\cdot)$, $\tau^{(j)}(\cdot)$ and $TD^{(j)}$, but also these adapted grid points converge. The present approach is similar to the approach proposed in Brumm and Grill (2014). The latter cover the discretized state space with simplices and look for 'just binding' constraints on each edge of these simplices. We only look at edges that are orthogonal to the $(S_{t-1}, \frac{W_t}{D_t}, m_t)$-hyperplane, which is computationally more efficient within the present setup.

Interpolation: We fix the set of initial grid points G_S, G_{WD}, G_{PD}, G_m for the state space. Our discretized state space is, however, not given by the product $G_S \times G_{WD} \times G_{PD} \times G_m$, but instead by

$$\begin{aligned} &G_S \times G_{WD} \times G_{PD} \times G_m \\ &\cup \{(S, WD, PD_+(S, WD, m), m) \mid (S, WD, m) \in G_S \times G_{WD} \times G_m\} \\ &\cup \{(S, WD, PD_-(S, WD, m), m) \mid (S, WD, m) \in G_S \times G_{WD} \times G_m\} \end{aligned}$$

The standard linear interpolation method on a Cartesian product of one-dimensional grids is therefore augmented as follows: for a given query point

[50] Including all inaction boundaries for any combination of $(S_{t-1}, \frac{W_t}{D_t}, m_t)$ into a common $\frac{P_t}{D_t}$ grid creates a computationally prohibitively large number of discretization points.

[51] Note, that $\left(\frac{P}{D}\right)_+$ and $\left(\frac{P}{D}\right)_-$ are functions of $((S_t^*, \left(\frac{W_t}{D_t}\right)^*, m_t^*)$, although this is suppressed in our notation.

(S_q, WD_q, PD_q, m_q), we first search for indices i, j, k, such that $S_q \in [S_i, S_{i+1}]$, $WD_q \in [WD_j, WD_{j+1}]$ and $m_q \in [m_k, m_{k+1}]$ and then linearly interpolate the policy in the PD-dimension for each combination $(S, WD, m) \in \{S_i, S_{i+1}\} \times \{WD_j, WD_{j+1}\} \times \{m_k, m_{k+1}\}$ using as a PD grid the intersection of the discretized state space with the line parallel to the PD-axis that crosses (S, WD, m). This yields eight interpolated policy values $CD_{u,v,w}$ with $(u, v, w) \in \{i, i+1\} \times \{j, j+1\} \times \{k, k+1\}$ of the function

$$(S, WD, m) \mapsto CD(S, WD, PD_q, m)$$

at the chosen closest (S, WD, m)-grid points. We then use ordinary three-dimensional linear interpolation to obtain the interpolated policy value for $CD(S_q, WD_q, PD_q, m_q)$, i.e.

$$\begin{aligned} & CD^{interp}(S_q, WD_q, PD_q, m_q) \\ & = \sum_{u=i,i+1} \sum_{v=j,j+1} \sum_{w=k,k+1} \frac{|S_q - S_u||WD_q - WD_v||m_q - m_w|}{(S_{i+1} - S_i)(WD_{j+1} - WD_j)(m_{k+1} - m_k)} CD_{u,v,w} \end{aligned}$$

We proceed analogously for linear extrapolation.

2.12.4 Testing for Equality of Gain Estimates in Table 2.4

Table A.2.1 reports the p-values for the null hypothesis $H_0 : g^i = g^j$ for $i \neq j$.

Experience Groups	6-11	12-17	18-23	>23
0-5	0.33	0.11	0.01	0.00
6-11	-	0.30	0.00	0.00
12-17	-	-	0.11	0.00
18-23	-	-	-	0.00

Table A.2.1: P-values for Equality of Gain Estimates

2.12.5 No Tax Rebates

Table A.2.2 reports the outcomes shown in table 2.8 in the main text for the case where tax revenue is not rebated to investors ($T_t^i = 0$ for all t,i). It shows that findings a robust to making this alternative assumption on tax rebates.

	No Tax	1% Tax	2% Tax	4% Tax	10% Tax
$E[PD]$	135.77	137.11	140.18	143.99	152.01
$std(PD)$	122.13	122.89	125.54	128.29	131.48
$corr(PD_t, PD_{t-1})$	0.98	0.98	0.98	0.98	0.98
$std(r^s)$	11.63%	11.72%	11.97%	12.26%	13.78%
$E[r^s]$	2.11%	2.12%	2.15%	2.19%	2.41%
$corr(PD_t, \overline{E}_t R_{t+1})$	0.84	0.85	0.86	0.87	0.89
$corr(TV_t, TV_{t-1})$	0.97	0.97	0.97	0.97	0.94
$corr(TV_t, PD_t)$	0.37	0.35	0.33	0.29	0.16
$corr(TV_t, \lvert P_t/P_{t-1} - 1\rvert)$	0.25	0.25	0.24	0.22	0.04
$corr(TV_t, std(\widetilde{E}_t^i R_{t+1}))$	0.95	0.94	0.93	0.91	0.87
# of booms per 100 yrs	1.82	1.92	2.08	2.32	2.88
average boom length (quarters)	32.42	31.97	31.44	30.72	28.24
average boom peak (PD)	491.01	487.57	484.54	478.82	468.96
$E[TV]$ relative to no tax	100.00%	97.33%	97.96%	96.69%	105.66%

Table A.2.2: Effects of introducing financial transaction taxes
(no tax rebate)

Chapter 3

Smart Money? Investment Professionals' Expectations about the Stock Market

3.1 Abstract[1]

Using a comprehensive data set based on three different surveys, I establish new empirical facts on investment professionals' expectations about future stock market returns. I show that professionals' expectations differ substantially from those of households. While household expectations may be well-described by an extrapolative model of expectations, investment professionals' expectations show a more sophisticated pattern. At the same time, professionals' expectations deviate from rational expectations, even when taking into account potential asymmetries in their loss functions. Micro level evidence confirms these findings, but also documents substantial heterogeneity in the cross-section of investment professionals. The deviations from rational expectations cannot be explained by simple models of information rigidities. Overall, the results point to an important role for including investment professionals' into models of financial markets although modelling their expectations and interactions with other agents may be challenging.

3.2 Introduction

It is well known to economists that "expectations matter" (Coibion and Gorodnichenko (2015)). But when does it matter most to take a closer look at how expectations are formed? The way we think about expectations is particularly consequential when deviations from rational expectations are large and such deviations produce strong economic effects. In this case, explanations

[1]This chapter is based on the paper Beutel (2016).

of economic phenomena and their implications may differ strongly from those derived under the assumption of rational expectations.

There is a growing body of evidence that this is the case in financial markets. Using direct evidence from surveys, Vissing-Jorgensen (2004), Bacchetta, Mertens, and Van Wincoop (2009), Greenwood and Shleifer (2014b), Greenwood and Shleifer (2014b), and Adam, Marcet, and Beutel (2017) document statistically and economically important deviations from rational expectations about the stock market.[2] Across several different surveys, respondents expect high returns when stock prices are high, which is diametrically opposed to models of rational expectations.[3] Very similar patterns, have been found for house price expectations, see Piazzesi and Schneider (2009), Case, Shiller, and Thompson (2012) and Gelain and Lansing (2014). On the theory side, it has been shown that taking into account the evidence on how expectations are formed, may explain otherwise puzzling phenomena such as the "Dot com" boom bust in the U.S. stock market (Adam, Marcet, and Beutel (2017)) or the U.S. housing boom prior to the financial crisis in 2008 (Gelain and Lansing (2014), Hoffmann (2016)).[4] Moreover, Winkler (2016) shows that incorporating extrapolative expectations into an otherwise standard DSGE macro model with financial frictions has strong implications for the effects of monetary policy, too. In summary, there is a growing amount of empirical evidence documenting deviations from rational expectations about stock prices (and house prices) and these deviations have strong implications in models of asset prices and even macroeconomic DSGE models.

However, while a consensus appears to have been reached on the extrapolative nature of the average households' stock return expectations, it should not be forgotten that households are not the only players on the stock market. In fact, the share of U.S. stocks owned directly by households decreased steadily over the last 60 years. While direct ownership by households amounted to 91.6% in 1950 (Friedman (1996)) it has decreased to less than 32% in 2007 (Lewellen (2011)). Thus, around two thirds of the toal stock market value is nowadays held by institutional investors, such as mutual funds, pension funds, banks, insurance companies, hedge funds and other types of investors.

Therefore, knowing more about the expectation formation of these professional investors may be crucial for understanding asset price dynamics. Of course, there is by no means a consensus on how professional investors affect stock prices. While the traditional view (Shleifer and Summers (1990)) is that

[2] Given that these surveys contain explicit and simple questions, it is unlikely that findings are due to respondents misunderstanding the questions. In anonymous surveys, it is also unlikely that answers are distorted by reputation considerations, or other incentives. Moreover, the evidence is robust across several surveys, differing in the questionnaire (framing), sampling period or index considered, and covers household respondents as well as CFOs.

[3] See Adam, Marcet, and Beutel (2017) for a formal statement of this point.

[4] Alternative explanations of asset price volatility include Campbell and Cochrane (1999), and Bansal and Yaron (2004). Given that these are rational expectations models, they stand in contrast to the empirical evidence on stock market expectations just described.

rational arbitrageurs bring prices closer to fundamentals, De Long, Shleifer, Summers, and Waldmann (1990) show that rational arbitrageurs may choose to "ride the bubble" and thereby aggravate booms and busts. At the same time, there is a debate about how much discretion upon their investments institutional investors actually exert, or in other words how sizeable the fraction of rational arbitrageurs is. While Lewellen (2011) argues that institutional investors mostly just mimic the market portfolio, other authors argue that institutional investors do show systematic investment patterns (Brunnermeier and Nagel (2004), Gompers and Metrick (2001)).[5] Thus, while there is a sizeable literature about the consequences of rational arbitrage and its limits (e.g. Shleifer and Vishny (1997), Abreu and Brunnermeier (2003)), little evidence exists about how smart or well-informed modern professional investors actually are.

In this paper, I therefore take a closer look at the rationality of investment professionals' expectations.[6] I do not want to take a stand on the impact of these investors on financial markets or beyond, but rather focus on establishing empirical facts on how they form expectations. These facts, could serve as a useful ingredient to models including both households and investment professionals.

My data consists of three different surveys of investment professionals' expectations (ZEW Financial Market Survey, Livingston Survey, Shiller Professional Investor Survey) and one survey about household expectations (Shiller Individual Investor Survey). To the best of my knowledge, this is the most comprehensive data set on investment professionals' stock market expectations used in the literature thus far. It allows me to draw conclusions which are robust across all data sources, and to establish new findings - for instance by directly comparing the expectations of households and investment professionals, by conducting forecast optimality tests at the level of the individual forecaster, or by testing for information rigidities in investment professionals' expectation formation process.

I establish that investment professionals' expectations differ markedly from those of households. While household expectations may be well described by

[5]Clearly, some professionals at institutions, e.g. mutual funds restricted to U.S. stocks, have little discretion over their investment shares into the aggregate stock market. However, it may be sufficient that some professional investors, for example hedge funds, have discretion over their investments. Another important group of stock market participants are firms, who decide on the quantity of equity issued in each period. Coibion and Gorodnichenko (2012) think of banks' expectations as proxies of the expectations of these firms in the context of inflation, which may also not be unreasonable for stock market expectations if one thinks of the consulting element included in banks' services. Finally, professionals' expectations may also matter by influencing households' expectations as in Carroll (2003).

[6]In the following, I will speak of investment professionals and households, where the latter refers to the average household as identified by representative surveys. The former refers to the group of people selected by the corresponding surveys, which focus on portfolio and fund managers, securities analysts or other specialists at banks, insurance companies, and other institutions.

an extrapolative model where expectations comove positively with the price-dividend ratio, this is not the case for investment professionals whose expectations are not significantly correlated with the price-dividend ratio. Thus, the expectation formation process of investment professionals has to be characterized seperately and there is a potential role for this type of investors in economic models.

To further characterize the expectation formation process of investment professionals, I test three leading models of expectation formation: Full Information Rational Expectations (FIRE), Sticky Information, and Noisy Information.

Using standard tests of FIRE, based on the orthogonality property of forecast errors with respect to information up to the time where the optimal forecast was made, I reject forecast optimality for aggregate expectations of investment professionals. A recent paper by Patton and Timmermann (2007b) shows that such rejections may in fact be due to asymmetric loss functions rather than non-optimality. Therefore, I also employ a quantile-based test proposed by these authors, which allows for asymmetric loss. However, forecast optimality at the aggregate level is still rejected.

I then zoom in to the micro level and conduct tests for each individual forecaster separately.[7] I find that, while there is a substantial fraction of individual respondents' forecasts for which I reject forecast optimality, for the majority of the respondents optimality cannot be rejected. Overall, while these findings support the rejection of FIRE at the aggregate level, they also point to substantial heterogeneity in the cross-section of investment professionals.

To further investigate the source of the rejection, I test whether investment professionals' expectations are consistent with the Sticky Information or Noisy Information model. Coibion and Gorodnichenko (2015) recently showed, that both of these models imply that aggregate forecast errors are predictable using aggregate forecast revisions. I reject this implication as well. This could suggest that frictions other than Sticky Information or Noisy Information may be responsible for the rejection of FIRE. For example, respondents simply may not know the true model of stock returns, such that they operate not only under incomplete information but potentially also (or alternatively) under a misspecified model of stock returns.

My paper is closely related to Coibion and Gorodnichenko (2015). However, while they test for Sticky Information and Noisy Information in expectations about inflation and several other macroeconomic variables, they do not consider expectations about the stock market. Aretz, Bartram, and Pope (2011) test forecast optimality of stock market expectations from the Livingston survey under asymmetric loss. While their results are broadly in line with my findings, they restrict themselves to Livingston survey data, and do not run

[7] These micro level tests can only be conducted for the ZEW Survey data, as it is the only of the three data sources considered for which sufficiently long time series from individual respondents are available.

tests separately for individual forecasters. Moreover, they do not consider the tests proposed by Coibion and Gorodnichenko (2015).

The rest of the paper is structured as follows. Section 2 reviews the literature on theories and tests of expectation formation more broadly. Section 3 introduces the datasets. Section 4 establishes differences between household and investment professionals' expectations. Section 5 explains the econometric approach and results for the tests of FIRE as well as of Sticky and Noisy Information. Section 6 concludes.

3.3 Theories and Tests of Expectation Formation in the Literature

In this section I review theories and empirical tests of expectation formation on the stock market, and more generally in macroeconomics. With respect to the empirical literature, I focus on tests using survey data on expectations. Survey data on expectations offer the possibility to test assumptions about expectations directly and independent of a specific economic model, whereas traditional tests of economic models are always joint tests of assumptions about expectations and the structure of the economic model (preferences, technology, market (in)completeness, exogenous processes, types of agents, etc.).

3.3.1 Expectation Formation on the Stock Market

Dominitz and Manski (2011) provide a useful framework for thinking about stock market expectations. They categorize agents forecasting stock returns into three types:[8]

1. Random-walk (RW) type: Believes that returns are distributed i.i.d. over time. Thus, the RW type uses the long-run distribution of returns to form beliefs about the future.

2. Persistence type (P) type: Believes that stock market performance is persistent, such that recent past returns are indicative of near future returns.

3. Mean-reversion (MR) type: Believes that stock returns mean-revert, such that a sequence of high (low) returns, may indicate low (high) returns in

[8]Other approaches which do not lend themselves readily into these categories are Barberis, Shleifer, and Vishny (1998) and Afik and Lahav (2015). The former present "a model of investor sentiment" based on psychological evidence, in which people see patterns in truly random sequences (and therefore start using misspecified models). The latter is an experimental setup capturing the idea, that in forecasting prices, people switch between extrapolating the recent past or applying historical patterns (e.g. boom-bust episodes).

the near future.[9]

On the empirical side, Dominitz and Manski (2011) show, that while aggregate expectations most closely resemble the persistence type, agents are really heterogeneous across types. Using data from the Michigan Survey of Consumers (from 2002-2004) they find that the shares of individuals of types (RW, P, MR) are (0.27, 0.41, 0.32) in a basic classification.

As mentioned in the introduction, a number of papers have also found stock market expectations to be of the persistence type. Graham and Harvey (2001) find that expectations of CFOs (mainly from the manufacturing sector) comove with past returns. Hurd, Van Rooij, and Winter (2011) look at a two year sample of dutch households' stock market expectations and find that expected capital gains are positively correlated with stock prices as well as stock ownership. Vissing-Jorgensen (2004) documents similar patterns for the UBS-Gallup survey, a representative survey of U.S. households. Adam, Marcet, and Beutel (2017) update the evidence for the UBS-Gallup survey, and additionally the CFO, and the Shiller individual investor survey. They show that the survey data formally rejects Full Information Rational Expectations and can be well-explained by Bayesian learning from past price growth. Adam, Beutel, Marcet, and Merkel (2015) show that the Bayesian learning model also applies to subgroups of investors with different years of experience in the stock market. Greenwood and Shleifer (2014b) present evidence from these three surveys as well, and additionally from the American Association of Individual Investor Sentiment Survey and Investors' Intelligence newsletter expectations, confirming the findings of the previous literature, that households tend to extrapolate past returns into the future.

3.3.2 Expectation Formation in Macroeconomics

In macroeconomics, microfounded theories of expectation formation lead to a different typology of models of expectation formation. Figure 3.1 provides an overview of existing theories.

In the Sticky Information model of Mankiw and Reis (2002), a randomly drawn fraction λ of the population updates its information set each period, while a fraction $1-\lambda$ continues to use the information from last period, and therefore operates with outdated plans. When agents update, they immediately acquire Full Information Rational Expectations (FIRE) beliefs for the given period. This has important implications for the effects of monetary policy on inflation and was sought to bring the model closer to the data.[10]

[9] The three types are not fully formalized in Dominitz and Manski (2011) which is reflected here. Nevertheless, the three types are useful as a framework for subsuming several of the existing approaches in the literature.

[10] Another well-known paper by Carroll (2003) can be thought of as providing microfoundations for Sticky Information. In their setup, households derive their expectations not from

In the Noisy Information model of Kydland and Prescott (1982), the level of technology reflects both a permanent and a transitory component but agents cannot separately identify these two components. In Woodford (2003), firms observe aggregate demand subject to idiosyncratic errors. Note that in Kydland and Prescott (1982), the Noisy Information is about an exogenous variable, whereas it is about an endogenous variable in Woodford (2003).[11]

As illustrated in Figure 3.1, both the Sticky and Noisy Information models are Rational Expectations models, given that while information is incomplete, agents' perceived model of the economy is correctly specified. By contrast, a different class of models arises, when agents do not know the true model of the economy. In such models, agents can have subjective beliefs about exogenous processes, as in the Bayesian RE model of Barberis, Shleifer, and Vishny (1998), and/or about endogenous processes, as in the Internally Rational Expectations Equilibrium of Adam and Marcet (2011).

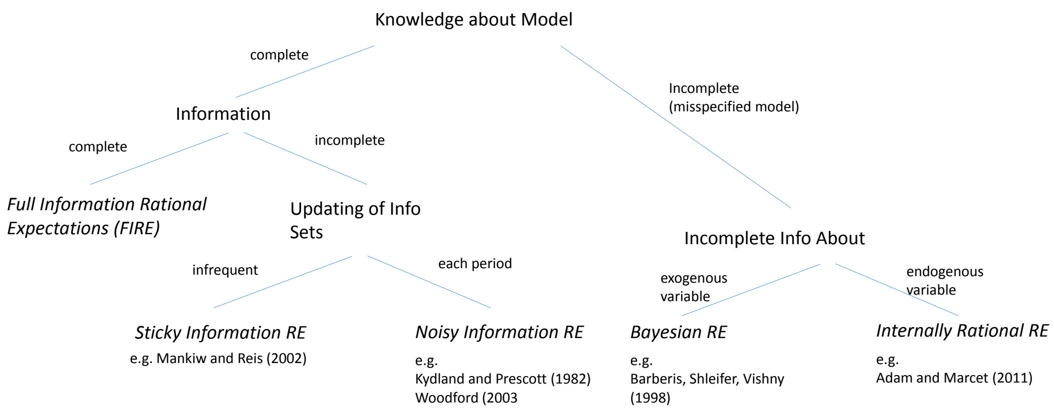

Figure 3.1: Overview of Theories of Expectation Formation

Mankiw, Reis, and Wolfers (2004) test empirically, which of the alternatives, Sticky Information, FIRE, or adaptive learning is capable of explaining survey data on inflation expectations. They reject FIRE after finding that median inflation forecasts are biased and corresponding forecast errors are

original data but from forecasts of professionals published in news media. Each period they encounter a newspaper article on the variables they have to forecast with probability λ, which leads to a mathematically equivalent formulation to Mankiw and Reis (2002). Carroll finds some empirical evidence for household expectations being influenced by those of professional forecasters, using the Michigan Consumer Survey and the Survey of Professional Forecasters. The updating rate λ may vary with the amount of newspaper articles published in a given period.

[11] Another influential paper motivating noisy information is Sims (2003) which emphasizes the role of limited information-processing capacities on behalf of the agents.

predictable by both the previous period's forecast error and recent macroeconomic data. Comparing data from the Michigan Survey of Consumers, Survey of Professional Forecasters (SPF), and Livingston Survey they also find that forecasts of consumers are less efficient than those of professionals.

Given that not only past observations of inflation, but also other macroeconomic variables affect forecasts, they conclude that a (naive) model of adaptive expectations is also inconsistent with the data. They show that a calibrated model of Sticky Information, in which agents update their beliefs infrequently, is able to generate patterns of disagreement similar to those observed in the surveys.

Andolfatto, Hendry, and Moran (2008) think that a Noisy Information model can explain deviations from FIRE found in survey data. They argue, that while in a Noisy Information model, agents' use of the Kalman Filter should lead to unbiased expectations and serially uncorrelated forecast errors in population, in short samples, significant bias and autocorrelation will be detected. Thus according to these authors, a Noisy Information rational expectations model is able to explain deviations from FIRE found in short samples.

Branch (2007) finds evidence for model-switching in expectation formation. They compare three different setups: the static Sticky Information model of Mankiw and Reis (2002), a Sticky Information model with time-varying updating frequency, and a model uncertainty framework, where each period, agents choose between the full information VAR, adaptive learning and a naive model. They suggest that the two dynamic models provide a better fit of survey data on inflation expectations than the static Sticky Information model.

In a suite of two papers, Coibion and Gorodnichenko (2012) and Coibion and Gorodnichenko (2015) present econometric tests which have a direct linkt to an underlying microfounded theory of expectation formation. They therefore offer clear advantages relative to more ad-hoc approaches such as Dominitz and Manski (2011).

Coibion and Gorodnichenko (2012) study the response of disagreement to macroeconomic shocks. They test whether the following classes of models are consistent with the data: Sticky Information, Noisy Information, or heterogeneous Noisy Information. They can reject the Sticky Information and heterogeneous Noisy Information models, as well as the alternative explanation of heterogeneity in loss aversion by Capistrán and Timmermann (2009) and therefore conclude that the basic Noisy Information model provides the best characterization of the data. Their analysis is "model-free" in the sense that their results do not hinge on a specific economic model, but instead are tested directly with survey data on expectations. On the other hand, the approach does rely on the identification of shocks, which requires a number of auxiliary assumptions.

A subsequent paper by Coibion and Gorodnichenko (2015) tests the above mentioned theories of expectation formation directly, without the need to identify shocks. Thus, the results of this paper hold independent of an economic model and require only a minimum of auxiliary assumptions. Therefore, my tests of the Sticky and Noisy Information models will build on Coibion and Gorodnichenko (2015).

3.4 Data

3.4.1 Survey Data on Investment Professionals' Expectations

I use three distinct data sets on stock market expectations, namely the ZEW Financial Market Survey, Livingston Survey, and Robert Shiller's Stock Market Confidence Indices Survey data.[12] The most important characteristics of these data sets are summarized in table 3.1.[13] To my knowledge, this is the most comprehensive collection of data on investment professionals stock market expectations analyzed so far.

I have chosen to use these three data sets, in order to cover several features which are desirable for the purpose of this paper. These are:

1. large cross-section dimension (reliable aggregate statistics)
2. long time series dimension (limit impact of sample period)
3. micro data available with **panel structure (**analysis at individual level)
4. expectations of **both** investment professionals and households available (differences in expectations of the two groups)

None of the three data sets in isolation satisfies all of these points, but taken together, the three data sets cover all of the desired features. While differences between the three surveys naturally imply some degree of heterogeneity of results, all main findings of my paper are robust across data sources.

[12]ZEW Financial Market Survey data provided by the Centre of European Economic Research (ZEW) at Mannheim. Livingston survey data provided by the Federal Reserve Bank of Philadelphia. Stock Market Confidence Indices data provided by International Center for Finance at Yale School of Management. I thank the respective institutions and their staff for making these data sets available to me.

[13]Corresponding realizations data has been obtained from Datastream (see Appendix for details).

	ZEW	**Livingston**	**Shiller Prof.**	**Shiller HH**
Respondent Type	Professionals	Professionals	Professionals	Households
Time Period	2003:02-2015:10	1952:6-2015:12	1989:7-2016:5	1999:2-2016:5
Frequency	Monthly	Semi-annually	Monthly	Monthly
Forecast of	DAX	S&P 500	Dow Jones	Dow Jones
Cross-section observations per wave (mean)	233	32	26	28
Panel	Yes	Yes	No	No
Horizon (months)	6	6, 12, (18, 24)	1, 3, 6, 12, 120	1, 3, 6, 12, 120
T (# of waves)	153	128	249	190
Total # of obs.	34993	4134	6397	5245

Table 3.1: Overview of Survey Data Sources

The longest available time series of stock market expectations is found in the Livingston survey, which is at the same time an (unbalanced) panel.[14] However, its cross-section is quite small, which may lead to considerable noise in cross-section aggregates. The latter is a crucial drawback given that the Coibion and Gorodnichenko (2015) tests are based purely on cross-section aggregates.

In contrast the ZEW Financial Market Survey has the largest cross-section. It is a monthly survey of investment professionals (mainly from banks) who are asked about their expected level of the German Stock Market Index DAX in 6 months and the uncertainty surrounding their expectation. As a (rotating) panel with around 250 valid quantitative responses per month, spanning the period from 2003:02 until 2015:10, it is clearly one of the most notable data sets on stock market expectations. At the same time it appears to be relatively little known in the international literature on stock market expectations (see the above literature review).

Finally, the only data source which allows to directly compare the expectations of investment professionals and households are the Shiller Surveys, which are conducted separately for these two groups of respondents. Similar to the Livingston survey the cross-section dimension of the Shiller surveys can be rather small.

Overall the ZEW Survey appears to offer the best quality of data, but it has to be augmented by the Livingston survey to be sure that results also hold

[14]Unfortunately, the Livingston survey data on stock market expectations is known to suffer from several problems before the Federal Reserve Bank (FRB) of Philadelphia took over its administration in 1990, such as unclear base-values and several changes of the index to be forecasted (see Aretz, Bartram, and Pope (2011)). Therefore, in line with Aretz, Bartram, and Pope (2011), I restrict myself to Livingston data from 1990 onwards.

over long time periods and also hold for U.S. data. The Shiller data is needed if one wants to have directly comparable data on investment professionals and households. An additional benefit of the Shiller and Livingston data is that they do not require the use of instruments in the estimation for tests of models of information rigidities. Therefore, including these data sources also provides a natural check of the results from the ZEW Survey, where instrumental variable estimation is necessary to implement the tests proposed by Coibion and Gorodnichenko (2015).

In choosing data sets, I have restricted myself to surveys including quantitative questions on stock market expectations (i.e. those where respondents had to give a number for expected index level or expected return) and surveys which are ongoing.[15] Table 3.2 shows the exact questions used in each survey.

3.4.2 Stock Market Realizations Data

To compute forecast errors and price-dividend ratios, I use the following stock market indices (datastream codes in parentheses): DAX-30 (DAXINDX; PI, RI), S&P 500 (S&PCOMP; PI, RI), and Dow Jones Industrial Average (DJINDUS; PI, RI). Details on how the price-dividend ratio is computed are given in the appendix. I also obtain from datastream the 3-month treasury bill rate (henceforth called the short term interest rate) for Germany (TRBD3MT) and the U.S. (DTB3).

3.4.3 Notation Used Throughout the Paper

Let $F_t(i)x_{t+h}$ denote agent i's h-period ahead forecast of variable x made at time t, and let $F_t x_{t+h} \equiv \frac{1}{N}\sum_{i=1}^{N} F_t(i)x_{t+h}$ denote the average forecast across all N agents in the cross-section.[16] In this paper, x_{t+h} will be the growth rate from t to $t+h$ of a stock market index.

[15]The first knock-out criterion excludes many alternative data sources (including another sub-question of the ZEW survey) on the grounds that it provides much coarser information than quantitative data, and on the grounds that quantification would require auxiliary assumptions which would reduce the decisiveness of the subsequent tests. The second knock-out criterion excludes the UBS-Gallup survey which was terminated in 2007 (somewhat unfortunately) rendering it the shortest time series of those mentioned here (it was also not conducted as a panel and included only households). Another well-known data set is the CFO survey, which started in 2001. The respondents were CFOs of U.S. companies (mainly from the manufacturing sector). One might a priori tend to think of these CFOs as investment professionals, however, it turns out that their aggregate expectations behave very much like those of households (see Adam, Marcet, and Beutel (2017)). Thus, the CFO survey expectations are difficult to classify neither as investment professionals nor as households and thus shed little light on the questions considered in this paper.

[16]Using the median instead of the mean to aggregate the cross-section of forecasts does not induce any major changes to my results.

For the tests of information rigidities in section 5.2 I use the more precise notation $F_t x_{t,t+h}$ instead of the simple notation $F_t x_{t+h}$ to make explicit that the growth rate is computed over the time period from t to $t+h$. While for all other sections, this would only complicate the notation given that we always have $s = q$ in $F_s x_{q,t+h}$, the more precise notation is necessary when computing forecast revisions in section 5.2, where we can have $s \neq q$.

<table>
<tr><th>ZEW</th><th>Livingston</th><th>Shiller Prof.</th><th>Shiller HH</th></tr>
<tr><td>Level of DAX
in 6 months:
Min (90%): __
Expectation: __
Max (90%): __</td><td>June 2016:
Please provide
your forecasts [...]:
STOCK PRICES
(S&P500)
Monthly Data,
End-of-Period
2016
29. Apr: 2065,30
30. Jun: __
30. Dec: __
2017
30. Jun: __
Annual,
End-of-Period
2015: 2043,94
2016: Same as
Dec 30 2016
2017: __</td><td colspan="2">How much of a change
in percentage terms do
you expect in the following
(use + before your number to
indicate an expected increase,
a − to indicate an expected
decrease, leave blanks where
you do not know):
Dow Jones
Industrial Average
In 1 month: __
In 3 months: __
In 6 months: __
In 1 year: __
In 10 years: __</td></tr>
</table>

Table 3.2: Exact Questions Asked in Each Survey

Notes: Exact questions are represented in very much the same format that has been used in the respective survey questionnaires. For example, in the June 2016 Livingston survey, realized values for April 2016 and end of 2015 are given to the forecaster, who is asked to provide her forecasts for the dates indicated. The ZEW Financial Market Survey and the Shiller Survey are confidential, whereas the Livingston survey data is publicly available. The data from the ZEW Financial Market Survey is not to be confused with the ZEW Indicator of Economic Sentiment, which is derived from a different question within the same survey (referring to the general macroeconomic outlook).[17]

[17] More information about each survey can be found on the following websites:
http://www.zew.de/WS99-1
https://www.philadelphiafed.org/research-and-data/real-time-center/livingston-survey
http://som.yale.edu/faculty-research/centers-initiatives/international-center-for-finance/data/stock-market-confidence-indices/stock-market-confidence-indices

Depending on the survey, the relevant index for computing the growth rate $x_{t,t+h}$ will be the DAX (ZEW survey), S&P 500 (Livingston survey), or the Dow Jones (Shiller survey). Notice that the DAX is a return index, such that in this case $x_{t,t+h}$ is a return (i.e. $x_{t,t+h} = \frac{P_{t+h}+D_{t+h}}{P_t}$, where P_t denotes the ex-dividend price of the index, and D_t denotes the dividend), whereas the S&P 500 and the Dow Jones are price indices, such that $x_{t,t+h}$ is the price growth rate (i.e. $x_{t,t+h} = \frac{P_{t+h}}{P_t}$). However, given that this is not important for the econometric treatment, I use the the same notation for returns and for price growth rates.

For the equations I use the general notation $F_t x_{t+h}$ to emphasize that they could be applied to forecasts about any variable. In the tables with the empirical results, I use more specific variable names, such as $F_t R_{t+h}$ to emphasize that this is an application to returns or respectively price growth rates on stock market indices (in both cases, I use $F_t R_{t+h}$).

3.5 Differences Between Household and Investment Professional Expectations

In this section, I show that while aggregate stock market expectations of households may be well described by an extrapolative model ("persistence" type of Dominitz and Manski (2011)), as found in several papers, this is not true for the expectations of investment professionals, which show a markedly different dynamic pattern. Therefore, a closer look at how investment professionals form expectations seems warranted.

The extrapolative nature of household expectations is documented in Adam, Marcet, and Beutel (2017) based on a variety of data sources on household expectations. The key stylized fact documented in this paper is that aggregate household expectations, $F_t R_{t+h}$, comove positively with the price-dividend ratio, PD_t, i.e. $\text{corr}(PD_t, F_t R_{t+h}) > 0$. The benchmark value for the correlation used in Adam, Marcet, and Beutel (2017) is 0.79, i.e. expectations comove strongly with the price-dividend ratio (see also Figure 2 in their paper). Thus household expectations are high when the price-dividend ratio is high and low during busts.

In contrast, when we look at the expectations of investment professionals in Figure 3.2 we see a very different pattern. Investment professional expectations peak during busts and are lower when the price-dividend ratio is high. Thus, investment professional expectations appear to be counter-cyclical, i.e. closer to Dominitz and Manski's "mean reversion" type.

As a consequence, I do not find robust evidence for a positive correlation of investment professional expectations and the price-dividend ratio. Table 3.3 shows the correlation, $\text{corr}(PD_t,\ F_t R_{t+h})$, for different data sources and forecast horizons. For households, I find significant positive correlations at all forecast horizons. In contrast, for investment professionals, I find that

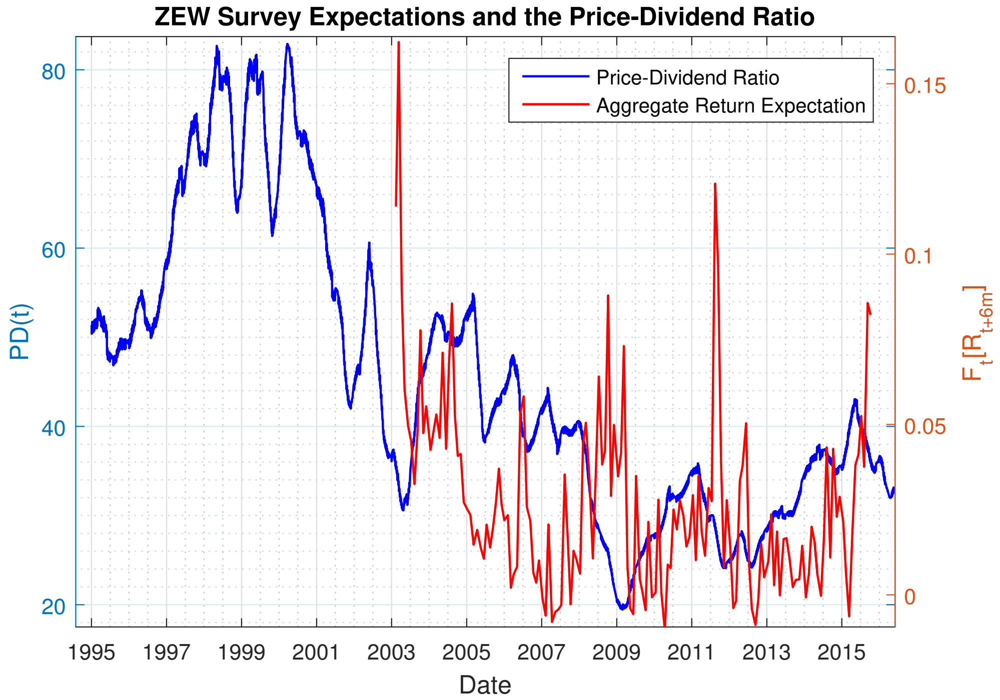

Figure 3.2: Investment Professionals' Expectations and Market Valuations
Notes: Monthly, aggregate net stock return expectations, $F_t x_{t+h}$, over the period 2003:02-2015:10 from the ZEW survey plotted on the right axis. Price-dividend ratio of the DAX-30 plotted on the left axis (in per annum units).

correlations are not significantly different from zero, except for one out of 15 specifications (The only exception occurs at the 10 year horizon, when the sample is restricted to coincide with the sampling period of household expectations).[18][19]

Thus, investment professional expectations do not seem to be well described by a simple model of extrapolative expectations as in Adam, Marcet, and Beutel (2017). For investment professionals, the pattern of expectations appears to be more complex. In the following section I will therefore investigate in more detail which model of expectation formation could potentially explain the observed beliefs of investment professionals.

[18]Notice that under the hypothesis that the expectations of both types of investors have the same correlation with the price-dividend ratio, a possible small sample bias (see Stambaugh (1999) or Campbell and Yogo (2006)) would affect the correlations of both types. Thus, even when not correcting for a possible small sample bias in correlations (as there is no standard way of doing so), the difference between the correlations shows that expectations of both types of investors covary differently with the price-dividend ratio.

[19]Findings are robust to using the median instead of the mean to aggregate the cross-section of expectations. Moreover, Adam, Beutel and Marcet (2016) document that using real instead of nominal return expectations have only neglible effects on such correlations. This is because fluctuations in asset price expectations of households are much larger than fluctuations in inflation or inflation expectations.

Survey (horizon)	**Households (1999-2016) (1)**	**Professionals (1999-2016) (2)**	**Professionals (sample start - 2016) (3)**
ZEW (6m)	-	-	0.126 (0.138)
Livingston (6m)	-	-0.278 (0.268)	-0.050 (0.251)
Livingston (12m)	-	-0.210 (0.275)	0.009 (0.260)
Shiller (1m)	**0.265***** (0.130)	0.098 (0.104)	0.067 (0.098)
Shiller (3m)	**0.400***** (0.134)	0.030 (0.112)	0.110 (0.106)
Shiller (6m)	**0.42***** (0.130)	-0.084 (0.124)	0.040 (0.116)
Shiller (12m)	**0.343***** (0.135)	-0.100 (0.128)	-0.010 (0.116)
Shiller (10y)	**0.707***** (0.183)	**0.274***** (0.123)	-0.025 (0.118)

Table 3.3: Comparison of Household and Investment Professional Expectations - Correlation with Price-Dividend Ratio

Notes: The table reports estimates of the contemporaneous correlation between the price-dividend ratio at time t, PD_t, and the aggregate forecast made at time t for different horizons h, $F_t R_{t+h}$, made by different groups of forecasters. Details on the computation of the price-dividend ratio, PD_t, are given in the appendix. Column (1) reports correlations for households. Column (2) reports correlations for professional investors, restricting the sample period to be identical with that of households. Column (3) reports correlations for professional investors using the full sample from each survey (see Table 3.1 for the respective sample periods). Significant correlations in bold. Serial correlation robust standard errors (Roy and Cléroux (1993)) in parentheses.

***Significant at the 1 percent level.

**Significant at the 5 percent level.

*Significant at the 10 percent level.

3.6 The Expectations Formation Process of Investment Professionals

This section presents tests of three of the most prevalent models of expectation formation, namely Full Information Rational Expectations (FIRE), Sticky Information, and Noisy Information.

3.6.1 Are Expectations of Investment Professionals Consistent with Full Information Rational Expectations?

Full Information Rational Expectations (FIRE) implies that agents' forecasts are optimal. An optimal forecast, $F_t^*(i)x_{t+h}$, is defined as minimizing the expected loss, $L(x_{t+h}, F_t(i)x_{t+h})$, associated with the forecast and the realization, conditional on the forecaster's information set at time t, I_t (see Patton and Timmermann (2007a)):

$$F_t^*(i)x_{t+h} \equiv \arg\min_{F_t(i)x_{t+h}} E[L(x_{t+h}, F_t(i)x_{t+h})|I_t], \tag{3.1}$$

where $I_t = \{(x_{t-k}, z_{t-k}) : k \geq 0\}$ naturally includes all past realizations of x, as well as additional predictor variables, z, up to the time where the forecast is made. In the following, I first derive testable implications of FIRE under mean-squared error (MSE) loss and test whether these are satisfied by investment professionals' expectations. Second, I derive testable implications of FIRE under more general, potentially asymmetric loss functions, which have been emphasized in the literature (Granger (1969), Patton and Timmermann (2007b)).

Mean-Squared-Error Loss

Under mean-squared-error (MSE) loss optimal forecasts have several well-known testable properties. Suppose the loss function is MSE:

$$L(x_{t+h}, F_t(i)x_{t+h}) \equiv (x_{t+h} - F_t(i)x_{t+h})^2 . \tag{3.2}$$

Then, the first-order condition of equation (3.1) reads:

$$0 = -2E[x_{t+h} - F_t^*(i)x_{t+h}|I_t]. \tag{3.3}$$

Thus, the optimal forecast under MSE loss is given by the conditional expectation of x_{t+h}:

$$F_t^*(i)x_{t+h} = E[x_{t+h}|I_t]. \tag{3.4}$$

Moreover, Patton and Timmermann (2007a) show that forecast errors, $e_{t+h} \equiv x_{t+h} - F_t^*(i)x_{t+h}$, resulting from optimal forecasts under MSE loss satisfy the following properties:

1. h-period forecast errors are serially uncorrelated at lags greater than or equal to h: $Cov(e_{t+h}, e_{t+h-k}) = 0$ for $k \geq h$
2. Forecast errors resulting from forecasts made at time t, are uncorrelated with information dated t or earlier: $Cov(e_{t+h}, \tilde{I}_t) = 0$, where $\tilde{I}_t \in I_t$.

These properties hold under the assumption that the process $\{x_t\}$ is covariance stationary. Notice that, while this requires the unconditional mean and variance of $\{x_t\}$ to be constant, the *conditional* mean and variance are allowed to be time-varying. Thus, the class of time series processes under which the above conditions hold is rather broad, including for instance ARMA processes with GARCH dynamics.

Thus, I test FIRE by estimating the following specifications. Uncorrelated forecast errors at lags greater than or equal to h imply $\beta = 0$ in the following equation:

$$x_{t,t+h} - F_t x_{t+h} = c + \beta(x_{t-h,t} - F_{t-h}x_t) + \varepsilon_{t,t+h}. \tag{3.5}$$

$F_t x_{t+h}$ is the cross-sectional average of all forecasts made in t. Thus, in line with most of the literature, I start with tests of forecast rationality at the aggregate level. If all individual forecasts are optimal, $\beta = 0$ also has to hold at the aggregate level. I discuss alternative specifications and conduct micro level tests in section 5.1.3. The equation is estimated for each survey data source separately. $x_{t,t+h}$ is the growth rate from t to $t+h$ of the index corresponding to each survey, i.e. the growth rates of the DAX (ZEW survey), the S&P 500 (Livingston survey) or the Dow Jones (Shiller survey), respectively.

In the next specification, I test whether forecast errors are uncorrelated with information dated t or earlier, i.e.whether $\beta = 0$ in the following equation:

$$x_{t,t+h} - F_t x_{t+h} = c + \beta z_t + \varepsilon_{t,t+h}. \tag{3.6}$$

Here, z_t could be any information contained in the information set at t. I focus on the price-dividend ratio, PD_t, and the short-term interest rate, i_t, as predictor variables. Both of these variables have been shown to forecast stock returns (Campbell and Thompson (2008), Cochrane (2008)). Campbell and Yogo (2006) show that predictability regressions using a highly persistent regressor such as the price-dividend ratio are subject to a small sample bias which leads to over-rejection of the null-hypothesis of no predictability.[20]

[20]The finite-sample bias, which has been introduced into the return predictability literature by Stambaugh (1999), basically results from a violation of "strict exogeneity" of the regressor, such that standard finite sample theory based on strict exogeneity is not applicable (Hayashi (2000)). The issue is that strict exogeneity would require the error term

Interestingly, they also show that no correction is required when using the short-term interest rate as regressor. Therefore, the short-term interest rate is a particularly useful variable in these kind of regressions.

Thus far, the short-term interest rate has been a reliable return predictor over the post-war sample (Campbell and Yogo (2006)). However, the short-term interest rate has recently become uninformative, given that it has been staying at zero almost constantly for several years (as some argue, perhaps partly as a result of central bank policies in the aftermath of the financial crisis.) Therefore, I have restricted the sample of the short-term interest rate to exclude the period of near zero interest rates. For the U.S., I use interest rate data only up to August 2008 and for Germany, where it took interest rates longer to reach zero, I use interest rate data up to June 2012.

In both of the previous regressions, I denote the error term by $\varepsilon_{t,t+h}$, since under the null-hypothesis of forecast optimality, it is the forecast error realized in period $t+h$ associated with an optimal forecast made in t. Thus, the first of the two properties of optimal forecasts under MSE loss shown above ($Cov(e_{t+h}, e_{t+h-k}) = 0$ for $k \geq h$), implies that the error terms of the regression will be serially correlated unless $h = 1$. This is the case for the ZEW survey, where $h = 6$ since the sampling frequency is monthly and forecasts are made 6 months ahead. Similarly, it is the case for the Shiller survey except for the 1-month forecast. For the Livingston survey, $h = 1$ given that the sampling frequency is at six month intervals and forecasts are made 6 months ahead (in other words, there is no overlap of forecast intervals). Therefore, to account for the serial correlation in the error terms, I use robust standard errors following Newey and West (1987) and Andrews and Monahan (1992).

For equation (3.5), orthogonality of regressor and error term also follows directly from property 1. For equation (3.6) orthogonality follows from the second property of optimal forecasts, which implies that $\text{Cov}(\varepsilon_{t,t+h}, z_t) = 0$. Therefore, both equations can be estimated by OLS.

Table 3.4 shows the results of the tests of FIRE under MSE loss. I restrict myself to forecasts at most at the 6 months horizon, given that Torous, Valkanov, and Yan (2004) as well as Boudoukh, Richardson, and Whitelaw (2008) document that standard estimates of coefficients and R^2 of return predictability at long horizons are unreliable.

The results in Table 3.4 show that across all specifications (surveys and horizons) the price-dividend ratio does not significantly predict investment professionals' forecast errors.[21] Correcting for potential small-sample bias of

to be orthogonal to past, current *and* future regressors. However, while optimal forecast errors are orthogonal to information dated t or earlier, they are not orthogonal to *future* regressors. Therefore, contrary to the argument in Bacchetta, Mertens, and Van Wincoop (2009), finite sample bias may also arise when predicting forecast errors, albeit the effect may be quantitatively smaller than in pure predictability regressions (of future realizations).

[21] Througout the paper, the number of observations, N, used in the regressions naturally varies by survey data source and by forecast horizon. Sometimes, N can vary additionally due to missing observations. For example, in the Shiller survey the number of observations

the coefficient on the price-dividend ratio is therefore unnecessary given that the bias goes in the direction of over-rejections of the null.

$\mathbf{R}_{t+h} - \mathbf{F}_t\mathbf{R}_{t+h}$	$\mathbf{PD}_t$ (1)	$\mathbf{i}_t$ (2)	$\mathbf{R}_t - \mathbf{F}_{t-h}\mathbf{R}_t$ (3)	
ZEW (6 m)	-0.002	**-0.038***	0.007	
	(0.002)	(0.022)	0.176	147
	R^2: 0.009	R^2: 0.114	R^2: 0.000	
Livingston (6 m)	-0.002	**0.034***	**0.469****	
	(0.001)	(0.018)	(0.179)	46
	R^2: 0.067	R^2: 0.201	R^2: 0.220	
Shiller (6 m)	-0.003	**0.028***	0.175	
	(0.002)	(0.011)	(0.148)	143
	R^2: 0.037	R^2: 0.174	R^2: 0.029	
Shiller (3 m)	-0.001	**0.012***	**0.264***	
	(0.001)	(0.006)	(0.136)	120
	R^2: 0.009	R^2: 0.088	R^2: 0.070	
Shiller (1 m)	0.000	**0.004***	0.13	
	(0.000)	(0.002)	(0.08)	133
	R^2: 0.001	R^2: 0.031	R^2: 0.015	

Table 3.4: Tests of Forecast Optimality under MSE Loss

Notes: The table reports estimates of the coefficient beta in equations (3.5) and (3.6) Columns 1-3 refer to univariate regressions of the forecast error on different potential predictor variables (including the lagged forecast error). N refers to the number of observations used in the regression with lagged forecast errors as regressors (i.e. column 3). Significant coefficients in bold. Newey West standard errors in parentheses.

***Significant at the 1 percent level.

**Significant at the 5 percent level.

*Significant at the 10 percent level.

The non-rejection of forecast optimality with respect to the price-dividend ratio is in line with the insignificant correlation of investment professionals' forecasts and the price-dividend ratio documented in the previous section. Given that the correlation between future *realized* returns and the

for the 3 months and 1 months horizon are lower than for the 6 months horizon due to several missing observations at the 3 months and 1 months horizon (which occur mainly between the beginning of the sample and the year 2002).

price-dividend ratio is either zero or negative, investment professionals' forecasts are in this sense more rational than those of households (for whom $\text{corr}(PD_t, F_t R_{t+h})$ is significantly positive). This may explain why for investment professionals FIRE cannot be rejected based on the price-dividend ratio, whereas for households Adam, Marcet, and Beutel (2017) reject FIRE.

In contrast, the short term interest rate significantly predicts forecast errors in all specifications. Coefficients are significant at the 10% level only, but the fraction of explained variance of the forecast errors (R^2) is substantial, especially for the 6 month horizon forecast errors. The low significance level of the short-term interest rate can at least partly be explained by the lower number of observations resulting from the above mentioned sample restriction due to the very low interest rate environment at the end of my sample. Lagged forecast errors are significant for some of the specifications.[22] Overall, the hypothesis of FIRE is rejected.

Unknown Loss

The previous section made it clear that the standard properties of optimal forecasts are derived under MSE loss. However, Patton and Timmermann (2007b) argue that it is often likely that agents use asymmetric loss functions. For example, stock market forecasters might dislike negative return surprises (negative forecast errors) more than positive surprises. In this case, the properties of optimal forecasts derived above do not hold (Patton and Timmermann (2007a)), unless conditional variances are constant (which clearly is not a realistic assumption for stock market returns). Thus, the above rejections of FIRE under MSE loss may be due either to a violation of FIRE or of the MSE loss assumption.

Since we do not know the loss function of our forecasters, we need properties of optimal forecasts under general loss functions of unknown form. Such are derived by Patton and Timmermann (2007b). Perhaps not surprisingly, it turns out that some restrictions on the loss function *and* the data generating process (DGP) are needed to obtain testable implications.

Specifically, let us assume that the loss function is a homogeneous function solely of the forecast error, $e_{t,t+h} \equiv x_{t+h} - F_t(i)x_{t+h}$ i.e.

$$L(x_{t+h}, F_t(i)x_{t+h}) = L(e_{t,t+h}) = g(a)L(e) \tag{3.7}$$

for some positive function g.

[22] Correcting for the short-sample bias in estimated autocorrelations (using the result in Sawa (1978), i.e. $\beta = \hat{\beta} + \frac{1+3\hat{\beta}}{T}$) does not introduce any significant changes to the estimated autocorrelations of forecast errors. Firstly, it can be seen from the formula, that for a positive estimated coefficient, $\hat{\beta}$, the corrected coefficient will always be even more positive such that rejcections of forecast optimality would remain. Secondly, the bias is small for the sample lengths considered here.

As a second assumption, let us assume that the DGP has dynamics in the conditional mean, $\mu_{t+h,t}$, and variance, $\sigma^2_{t+h,t}$, but no dynamics in higher moments of the conditional distribution. Specifically, let the DGP be given by

$$x_{t+h} = \mu_{t+h,t} + \sigma_{t+h,t}\eta_{t+h}, \ \eta_{t+h}|I_t \sim J_h(0,1), \tag{3.8}$$

where $J_h(0,1)$ is some distribution with mean 0 and unit variance which may depend on h, but does not depend on I_t.

Notice that again, the assumption on the DGP is rather mild and includes a broad range of sophisticated DGPs. The assumption on the loss function is also not very restrictive. It includes common loss functions such as mean absolute error (MAE), so called lin-lin loss (which is a linear function from the origin for positive and negative forecast errors, but slopes are different which introduces asymmetry) or asymmetric quadratic loss. On the other hand, it does exclude linear-exponential (Linex) loss.

Thus, it has to be noted that, under full generality with respect to the loss function and/or the DGP, testable properties of optimal forecasts cannot readily be derived (Patton and Timmermann (2007b) derive an "impossibility result" showing this explicitely.). This is were we approach the boundary of what can be said about the optimality of forecasts under very general circumstances. Another complication is that macroeconomists usually think of FIRE not in terms of point forecast optimality but in terms of agents' knowing the entire conditional distribution associated with the true DGP. However, here we also reach the boundaries of what is currently possible, both conceptually and practically given that the availability of survey data about density forecasts is limited up to now. Thus, these points remain challenging for both theoretical and empirical research.

Returning to what is currently feasible, let us suppose that the two assumptions specified in equations (3.7) and (3.8) hold. Then, Patton and Timmermann (2007b) show that the following binary variable is independent of any element of the time t information set I_t:.

$$V^*_{t+h,t} \equiv 1(x_{t+h} \leq F^*_t(i)x_{t+h}), \tag{3.9}$$

where $1(Q)$ equals 1 if Q is true and zero otherwise. Thus, this indicator variable is equal to 1 whenever the forecast error is negative and zero otherwise. To see why this indicator variable is independent of time t information, first note that the optimal forecast under (3.7) and (3.8) is given as

$$F^*_t(i)x_{t+h} = \mu_{t+h,t} + \sigma_{t+h,t}\theta^*_h, \tag{3.10}$$

where θ^*_h depends on the loss function and J_h, but not on time t. The proof of this result is given in Patton and Timmermann (2007b) and not reiterated here. What is important to note, however, is that due to the conditional variance component in the optimal forecast, forecast errors will be predictable by time t information. This highlights the difference to MSE loss, where the optimal forecast simply consists of the conditional mean $\mu_{t+h,t}$.

Moreover, given the optimal forecast under general loss functions, and letting $P[.|I_t]$ denote probabilities conditional on time t information, we have

$$\begin{aligned} P[V^*_{t+h,t} = 1|I_t] &= P[x_{t+h} \leq F^*_t(i)x_{t+h}|I_t] \\ &= P[\mu_{t+h,t} + \sigma_{t+h,t}\eta_{t+h} \leq \mu_{t+h,t} + \sigma_{t+h,t}\theta^*_h|I_t] \\ &= P[\eta_{t+h} \leq \theta^*_h|I_t] \\ &= q^*_h \end{aligned}$$

Thus, the optimal forecast $F^*_t(i)x_{t+h}$ is the q^*_h quantile of the distribution of x_{t+h} conditional on information I_t. Given that $V^*_{t+h,t}$ is a binary variable and q^*_h does not depend on t, $V^*_{t+h,t}$ is independent of any variable contained in the information set at t.

Thus forecast optimality under these more general conditions implies $\beta = 0$ in

$$V^*_{t+h,t} = c + \beta z_t + u_{t,t+h} \tag{3.11}$$

Our objective is to test whether the rejections of forecast optimality under MSE loss also hold under more general loss functions. Therefore, I use those variables for which optimality has been rejected under MSE loss, namely the short-term interest rate i_t and lagged forecast errors $x_{t-h,t} - F_{t-h}x_t$ as conditioning variables. Table 3.5 shows the results of these tests.[23]

Given the coarser information used in the quantile-based approach, it is not surprising that not all coefficients remain significant, as is the case for the short-term interest rate. However, across both conditioning variables we still get rejections for each of the three data sources and horizons considered. Thus, these results indicate that rejections of forecast optimality are robust to using more general, possibly asymmetric loss functions (at least as long as they are homogeneous functions solely of the forecast error).

[23]For the empirical results, I use $V^*_{t+h,t} \equiv 1(x_{t+h} > F^*_t(i)x_{t+h})$ instead of $V^*_{t+h,t} \equiv 1(x_{t+h} \leq F^*_t(i)x_{t+h})$. The only difference between the two variants is that all β coefficients change sign. In this way, coefficients in Table 3.5 have the same sign as those in Table 3 such that the two tables are easier to compare.

$1(\mathbf{R}_{t+h} - \mathbf{F}_t \mathbf{R}_{t+h} > 0)$	$\mathbf{i}_t$ (1)	$\mathbf{R}_t - \mathbf{F}_{t-h} \mathbf{R}_t$ (2)	**N**
ZEW (6 m)	**-0.135****	0.149	147
	(0.052)	0.449	
	R^2: 0.164	R^2: 0.002	
Livingston (6 m)	0.069	**1.589****	46
	(0.060)	(0.596)	
	R^2: 0.047	R^2: 0.128	
Shiller (6 m)	**0.072****	0.437	143
	(0.035)	(0.469)	
	R^2: 0.070	R^2: 0.010	
Shiller (3 m)	0.054	**1.32****	120
	(0.036)	(0.600)	
	R^2: 0.041	R^2: 0.038	
Shiller (1 m)	0.051	**2.355*****	133
	(0.033)	(0.839)	
	R^2: 0.028	R^2: 0.032	

Table 3.5: Tests of Forecast Optimality under Unknown Loss

Notes: The table reports estimates of the coefficient beta in equation (3.11). Columns 1-2 refer to univariate regressions of the indicator variable $1(R_{t+h} - F_t R_{t+h} > 0)$ on different potential predictor variables.The indicator variable $1(R_{t+h} - F_t R_{t+h} > 0)$ takes the values 1 if $R_{t+h} - F_t R_{t+h} > 0$ and 0 otherwise. N refers to the number of observations used in the regression with lagged forecast errors as regressors (i.e. column 2). Significant coefficients in bold. Newey West standard errors in parentheses.

***Significant at the 1 percent level.

**Significant at the 5 percent level.

*Significant at the 10 percent level.

Micro-Level Evidence

In line with most of the literature, the above tests have all been conducted at the level of aggregate expectations. This has (at least) two disadvantages: Firstly, non-rejection of forecast optimality can (in principle) occur, even when *all* forecasters in fact make sub-optimal forecasts. This is the case when heterogenous biases in individual forecasts have offsetting effects such that they average out in the aggregate forecast. Secondly, the other extreme can also occur: Even if only one (or only a few) of the forecasts is (sufficiently) suboptimal, while all other are optimal, the aggregate forecast would be suboptimal. Notice that the first problem remains even as the number of forecasters in the cross-section, n, goes to infinity, whereas the second problem vanishes asymptotically, as $n \to \infty$.[24,25]

To address these issues, I present evidence at the individual forecaster level, complementing the tests at the aggregate level. I do this by running the above tests of FIRE for each forecaster separately. To guard against small sample problems (and to make results comparable across forecasters), I do this for all forecasters with a certain minimum number of observations. Thus, I report findings for the ZEW survey in two setups with a minimum of 50 and 100 observations per forecaster, respectively. For the Shiller survey, tracking individuals over time is not possible, given that no identifiers for individuals are recorded. For the Livingston survey, given that there are less than 9 responses per forecaster on average, tests at the individual forecaster level are also not meaningful. (Grouping forecasters by affiliation is also severly limited by the fact that we have on average only 32 obervations per survey date for the Livingston survey.) Thus, I focus on zooming into the ZEW survey, in which we have $N_{\max} = 296$ forecasters with at least 50 forecast error observations, and $N'_{\max} = 138$ forecasters with at least 100 forecast error observations, such that tests at the individual level are meaningful.

Table 3.6 shows the results of running the specifications (3.5), (3.6), and (3.11) for each individual forecaster separately.

For different significance levels $a \in (0.01, 0.05, 0.1)$, I calculate the share of forecasters for which optimality is rejected, s_α. When running a large number

[24]These issues have also been noted in the literature. See Bonham and Cohen (2001) and Pesaran and Weale (2006) for a more formal discussion of these and related issues when aggregating or pooling forecasts.

[25]Potential alternatives for tests using micro-data could be to pool all individual observations or to use panel fixed effects. However, in the context of testing for forecast optimality, both of these approaches suffer from significant drawbacks. Therefore, I do not include them in this paper.

First, due to the common slope assumption, pooled and standard panel fixed effects regression also suffer from the two fundamental disadvantages just desribed for tests at the aggregate level. In addition, pooled regression and panel fixed effects come with additional problems specific to each of them.

A major additional problem with pooled regression is that a common intercept across all forecasters is assumed. When this is not the case, the estimated slope coefficient can be biased. Consider for example the case of regressing forecast errors on lagged forecast errors. Suppose that forecast optimality holds, but forecasters have heterogenous asymmetric loss. In this case, each forecaster's forecast errors are serially uncorrelated but each forecaster has a different unconditional bias. Pooling forecasts (and therefore not accounting for each forecaster's unconditional bias) would then detect forecast error autocorrelation and wrongfully conclude a rejection of forecast optimality.

The additional problem with panel regressions is that they can be particularly strongly affected by short-sample bias, which would lead to over-rejection of the H_0 (see Hjalmarsson (2008)).

of level α tests, we would of course expect a share of "chance" rejections due to estimation uncertainty. Therefore, I report "excess" rejection rates, defined as $s_\alpha - \alpha$. When these are positive, forecast optimality is rejected for more forecasters than could be explained by randomness in the estimated coefficients. The results in Table 3.6 show that excess rejection rates when using lagged forecast errors as conditioning variables are negative or close to zero in all specifications. In contrast, excess rejection rates are substantial when using the short term interest rate, i_t, as the conditioning variable. For example, for tests at level $a = 0.05$, I find an excess rejection rate of almost 30% for forecasters with at least 50 observations (Panel A), and a rate of 15% for those with at least 100 observations (Panel B). Differences between the two panels may at least partly be explained by the fact that forecasts in Panel B are more likely to include the period following the Lehman collapse in 2008, where returns and forecast errors have been difficult to predict by the short-term interest rate. An alternative explanation could be self-selection, where more successful forecasters stay longer in the sample.

"Excess" Rejection Rates, ZEW (6m)

Panel A.

minT= 50	MSE Loss		Unknown Loss	
	$\mathbf{i}_t$	$\mathbf{R}_t - \mathbf{F}_{t-h}\mathbf{R}_t$	$\mathbf{i}_t$	$\mathbf{R}_t - \mathbf{F}_{t-h}\mathbf{R}_t$
	(1)	(2)	(3)	(4)
$\alpha = 0.01$	0.132	0.023	0.163	0.014
$\alpha = 0.05$	0.295	0.011	0.353	-0.017
$\alpha = 0.1$	0.404	-0.015	0.466	-0.025
N	226	212	226	212

Panel B.

minT= 100	MSE Loss		Unknown Loss	
	$\mathbf{i}_t$	$\mathbf{R}_t - \mathbf{F}_{t-h}\mathbf{R}_t$	$\mathbf{i}_t$	$\mathbf{R}_t - \mathbf{F}_{t-h}\mathbf{R}_t$
	(1')	(2')	(3')	(4')
$\alpha = 0.01$	0.058	0.011	0.024	0.011
$\alpha = 0.05$	0.153	-0.008	0.221	-0.008
$\alpha = 0.1$	0.273	-0.047	0.392	-0.026
N	59	95	59	95

Table 3.6: Tests of Forecast Optimality at the Individual Forecaster Level

Notes: The table reports "excess" rejection rates from running equations (3.5), (3.6), (MSE Loss) and (3.11) (Unknown Loss) for each individual forecaster of the ZEW Financial Market Survey separately. Rejection rates at level a measure the share of forecasters, s_α, for which a given predictor variable is significant at level a. "Excess" rejection rates are defined as $s_\alpha - \alpha$. Panel A and B report "excess" rejection rates for all forecasters with more than 50 or respectively 100 (not necessarily adjacent) observations. The usual Newey West standard errors have been used in the significance tests.

Notice that, in contrast to the previous tables, N refers to the number of forecasters upon which the "excess" rejection rates are based. N reported in Panel A and B is the number of forecasters with at least 50 or, respectively, with at least 100 observations in the corresponding regression. Thus, for example, in Panel A, column (1), $N = 226$, is the number of forecasters, which have at least 50 matching observations for both the dependent variable ($R_{t+h} - F_t R_{t+h}$) and the predictor variable (i_t) in equation (3.6). This N is substantially lower than $N_{\max}$, since the short term interest rate i_t is restricted to the period up to June 2012, which excludes the last 40 survey dates, thereby reducing the number of forecasters which have at least 50 observations in both i_t and $R_{t+h} - F_t R_{t+h}$. Similarily, in column (2), N is based on the set of observations for which both $R_{t+h} - F_t R_{t+h}$ and $R_t - F_{t-h} R_t$ (with $h = 6$) are available. Every time a forecaster is skipping a survey (which happens frequently), two observations are lost for her regression at the individual level. Thus, again, we expect $N < N_{\max}$. Whether there are more or less responses in column (1) compared to column (2), or in column(1') compared to column(2') depends on the exact pattern of responses and non-responses and its interaction with the sample restriction on i_t. In Panel A, response behavior dominates N, whereas in Panel B, the sample restriction on i_t dominates N. (Given that the restriction limits the maximum possible number of observations per forecaster to 113, it also strongly limits the number of forecasters with more than 100 observations).

Overall, the conclusion is that, while we cannot reject forecast optimality for the majority of the respondents, a fraction of up to around 30 % of the respondents' forecasts are sub-optimal and therefore inconsistent with the FIRE hypothesis. Thus, one finding of our analysis thus far is, that there is a substantial amount of heterogeneity in the optimality of investment professionals' forecasts. The other finding is that FIRE is rejected at the aggregate level and this rejection is supported also by the micro level evidence. In the next section, we will therefore test two explanations for the deviations of FIRE, namely Sticky Information and Noisy Information.

3.6.2 Can Canonical Models of Information Rigidities (Sticky Information or Noisy Information) Explain the Deviations from FIRE?

A novel prediction of both the Sticky Information model and the Noisy Information model of expectations formation, proposed by Coibion and Gorodnichenko (2015), is that aggregate forecast errors should be predictable by revisions of aggregate forecasts, i.e. $\beta > 0$ in:

$$x_{t,t+h} - F_t x_{t,t+h} = c + \beta(F_t x_{t,t+h} - F_{t-1} x_{t,t+h}) + \nu_{t,t+h}. \tag{3.12}$$

While this is a special case of regressing forecast errors on information dated t or earlier, a key advantage of this specification is that the estimated parameter β maps directly into the underlying degree of information rigidity. Thus, in contrast to standard tests of FIRE, it has a direct economic interpretation.

In the Sticky Information model (i.e. provided $\beta > 0$) we have that the fraction of the population, λ, who do not update their information set in a given period is given by $\lambda = \frac{\beta}{1+\beta}$. In the Noisy Information model, the Kalman gain G, governs the degree of information rigidity $1 - G$, which is given by $1 - G = \frac{\beta}{1+\beta}$.

The derivation of equation (3.12) for Sticky Information is given in the appendix (for the derivation under Noisy Information see Coibion and Gorodnichenko (2015)). Notice that for the Sticky Information model, no assumption is needed, apart from the assumption that only a fraction $1 - \lambda$ of the population is updating each period. Thus the predicition holds independent of an economic model, and independent of the specific exogenous or endogenous stochastic processes. For the Noisy Information model, equation (3.12) requires the assumption that x_t follows an AR(1) process. More general AR(p) and VAR(p) processes are discussed in Coibion and Gorodnichenko (2015). However, tests of these are infeasible given my data set, as they would require forecast revisions at p different horizons.

As Coibion and Gorodnichenko (2015) point out, this prediction of the Sticky and the Noisy Information model only holds at the aggregate level. In the Sticky Information model, agents randomly update their information sets. If an agent does not update her information set, the forecast revision is zero, if she does update, she updates to FIRE such that the resulting forecast error is uncorrelated with her information set at time t (which includes her lagged revisions). In the Noisy Information model, agents use the Kalman filter, which implies that resulting forecast errors should also be unpredictable using their information set. Thus equation (3.12) can only be tested at the aggregate level.

The tricky part in estimating equation (3.12) is the construction of the forecast revisions $F_t x_{t,t+h} - F_{t-1} x_{t,t+h}$. Notice that these require forecasts made at different points in time about the same target date. In contrast, my survey data contains forecasts made at fixed horizons i.e. $F_t x_{t,t+h}, F_{t-1} x_{t-1,t+h-1}$. For the Livingston survey, I can use forecasts at the 6 months and 12 months horizon to construct 6 months horizon revisions $F_t x_{t,t+6} - F_{t-6} x_{t,t+6}$, by defining:

$$F_{t-6} x_{t,t+6} = \frac{F_{t-6} x_{t-6,t+6}}{F_{t-6} x_{t-6,t}}. \tag{3.13}$$

Thus, we can compute an implied lagged forecast of $x_{t,t+6}$ by dividing the lagged 12 months forecast by the lagged 6 months forecast. For the Shiller survey, I can use the same procedure to construct 6 months revisions, and additionally I can construct 3 months revisions from 3 and 6 months forecasts.

For the ZEW survey, I only have one forecast horizon (6 months). In this case, only a modified version of equation (3.12) can be estimated, namely:

$$x_{t,t+h} - F_t x_{t,t+h} = c + \beta(F_t x_{t,t+h} - F_{t-1} x_{t-1,t+h-1}) + \nu'_{t,t+h}. \text{(3.12')}$$

This means replacing forecast revisions by forecast changes at fixed horizon. Under the two models of information rigidities, the error term $\nu'_{t,t+h}$ then contains the FIRE error (orthogonal to information dated t or earlier) plus the following term:

$$\beta(F_{t-1} x_{t-1,t+h-1} - F_{t-1} x_{t,t+h}). \tag{3.14}$$

In this case, the error term $\nu'_{t,t+h}$ is correlated with the regressor, such that estimating (3.12') by OLS would result in biased coefficients. Therefore, I estimate this equation using an instrumentable variable (IV) approach. As usual, the instrument has to be correlated with the endogenous variable (here: changes in 6 months horizon stock return forecasts), and uncorrelated with the error term (difference between $t-1$ forecasts about returns from $t-1$ to $t+h-1$ and from t to $t+h$). I use the change in disagreement among forecasters between $t-1$ and t (measured as the change in the cross-sectional standard deviation of forecasts). It turns out that this variable is highly correlated with the endogenous variable and therefore a useful instrument. At the same time, this variable becomes known only in period t, such that exogeneity of the instrument is given.

Table 3.7 shows the results of the tests of information rigidities. Findings are not consistent with the two models of information rigidities. All estimated β coefficients are negative and only the coefficient for the ZEW survey is insignificant, likely due to the reduced precision introduced by the need to use instruments. In sum, the consistently negative estimates of β reject the possibility that the stock market expectations of investment professionals have been generated by these (simple) canonical models of information rigidities.

This is notable, given that these findings stand in contrast to those found by Coibion and Gorodnichenko (2015) for inflation expectations. This must not be a contradiction, but could reflect rational behavior of respondents when confronted with cost-benefit analysis of information acquisition and processing. In the context of a Sticky Information model, it may be perfectly rational to update information sets infrequently (e.g. on average every second quarter as the estimates of Coibion and Gorodnichenko (2015) suggest) given that the cost of updating inflation forecasts every quarter may be larger than its benefit. However, for the stock market, whose price index is far more volatile than the CPI, it may be very costly, not to update expectations at least once a month (ZEW) or respectively every quarter (Shiller 3m) or semi-annually (Shiller 6m and Livingston 6m).

The bottomline is that it is likely that agents use different models of expectation formation for different economic variables (e.g. inflation and stock returns). To my knowledge, this finding is not yet taken into account in economic models, although it could potentially lead to interesting new findings.[26]

$\mathbf{R}_{t+h} - \mathbf{F}_t\mathbf{R}_{t+h}$	$F_t\mathbf{R}_{t+h} - \mathbf{F}_{t-k}\mathbf{R}_{t,t+h}$	R^2	N	IV
ZEW (6m)	- 0.754 (0.720)	1^{st} stage: 0.510 2^{nd} stage: 0.007	152	Yes
Livingston (6m)	**-1.932***** (0.535)	0.116	46	No
Shiller (6m)	**- 1.107***** (0.349)	0.070	143	No
Shiller (3m)	**- 0.800**** (0.380)	0.061	118	No

Table 3.7: Tests of Information Rigidities

Notes: The table reports estimates of the coefficient beta in equation (3.12) and respectively (3.12'), where forecast errors are regressed on lagged forecast revisions. The last column indicates whether the equation had to be estimated by Instrumental Variables (IV) or could be estimated by OLS. Significant coefficients in bold. Newey West standard errors in parentheses.[27,28,29]
***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.

[26]Of course, the possibility remains that at higher frequencies (say daily or hourly) the sticky or noisy information model is a better approximation to the stock market expectation formation process than at monthly, quarterly or semi-annual frequency. Nevertheless, given that most (macro-)economic models are formulated at lower frequencies (e.g. quarterly), the failure of sticky and noisy info at these frequencies remains important.

[27]The number of observations for the ZEW survey is higher than in the previous tables, as forecast errors have to be lagged by 6 months to construct lagged forecast errors, whereas equation (3.12') requires only a 1 month lag.The number of observations for Shiller (3m) is slightly lower, because in order to construct the corresponding revisions, observations at both the 3 months and the 6 months horizon have to be used.

[28] Forecasts may also change because of changes in the composition of forecasters. However, including only forecasters who were present in the two surveys used to construct forecast revisions does not introduce any substantial changes to the results.

[29]Note that the persistence of forecast revisions is much lower than that of the price-dividend ratio. Therefore, in line with Coibion and Gorodnichenko (2015), I do not correct for a potential small sample bias.

3.7 Conclusion

This paper presented evidence from one of the most comprehensive data sets on investment professionals' stock market expectations, consisting of the ZEW survey, the Livingston survey and the Shiller survey, covering the period from 1990 to 2016 in the U.S. and Germany.

I find that professionals' expectations follow a markedly different pattern than those of households. While households' expectations comove positively with market valuation measured in terms of the price-dividend ratio, professionals' expectations peak during busts. Thus, while households' expectations may be well-described by an extrapolative model of expectations where agents expect high returns after a sequence of past high returns, professionals' expectations are more of a contrarian type. Therefore, establishing separate facts on investment professionals' expectations and including these types of agents into models of financial markets is potentially crucial for such issues as understanding financial bubbles and the impact of monetary policy on financial markets.

Therefore, this paper further characterizes the expectations of investment professionals. I find that Full Information Rational Expectations (FIRE) is rejected, even when taking account potentially asymmetric loss functions. I also add to the literature by showing that these tests are not just rejected at the aggregate level, but also for a substantial share of the individual forecasters. At the same time, these tests also reveal significant heterogeneity in the cross-section of forecasters.

Moreover, this paper is the first to apply the methodology of Coibion and Gorodnichenko (2015) for testing whether stock market expectations are consistent with simple models of information rigidities. In contrast to previous findings about inflation expectations, I show that simple Sticky or Noisy Information models cannot explain the deviations from FIRE in the context of stock market expectations. Thus, exploring the consequences of different expectation formation models for different economic variables might be an interesting avenue for future research.

With respect to further pinning down the expectation formation process of investment professionals, this paper has also highlights the importance of high quality survey data. One central aspect is that for tests at the individual forecaster level, it is important that forecasters stay in the survey long enough. This is the case for the ZEW survey. Moreover, for testing forecast optimality under asymmetric loss functions, it could be beneficial to have information about density forecasts (e.g. to infer conditional variances) included in the surveys. For testing models of information rigidities, it is important to have forecasts at several horizons available.

3.8 Appendix

3.8.1 Derivation of the Prediction of the Sticky Information Model

Following Coibion and Gorodnichenko (2015), suppose that each period only a fraction $1-\lambda$ of the population updates their information set (and consequently their forecast). When they update, they update to the full information Rational Expectation (FIRE). Let FIRE expectations be denoted $E_t x_{t+h}$ and let the average forecast across agents at time t of a random variable realized at time $t+h$ be denoted by $F_t x_{t+h}$. Then we have:

$$\begin{aligned} F_t x_{t+h} &= (1-\lambda) F_t^{updaters_t} x_{t+h} + \lambda F_t^{non-updaters_t} x_{t+h} \\ &= (1-\lambda) E_t x_{t+h} + \lambda((1-\lambda) E_{t-1} x_{t+h} + \lambda F_{t-1}^{non-updaters_{t-1}} x_{t+h}) \\ &= (1-\lambda) \sum_{k=0}^{\infty} \lambda^k E_{t-k} x_{t+h}, \end{aligned}$$

or equivalently,

$$F_t x_{t+h} = (1-\lambda) E_t x_{t+h} + \lambda F_{t-1} x_{t+h}, \tag{3.15}$$

such that the current average forecast $F_t x_{t+h}$ is a weighted average of the current rational expectation of x_{t+h} and last period's average forecast.

Let the full information rational expectation (FIRE) expectational error $x_{t+h} - E_t x_{t+h}$ be denoted $\nu_{t+h,t}$, i.e.

$$x_{t+h} - E_t x_{t+h} \equiv \nu_{t+h,t}. \tag{3.16}$$

Using definition (3.16) in (3.15) we obtain an equation of the form in the text:

$$x_{t+h} - F_t x_{t+h} = \frac{\lambda}{1-\lambda}(F_t x_{t+h} - F_{t-1} x_{t+h}) + \nu_{t+h,t},$$

where $v_{t+h,t}$ is the FIRE expectational error, which is (under MSE loss) unpredictable using information dated t or earlier.

3.8.2 Calculation of the Price-Dividend Ratio

By the definition of returns we have that dividends between $t-h$ and t are:

$$D(t-h,t) = R(t-h,t)P(t-h) - P(t). \tag{3.17}$$

Thus, given a price index $P(t)$ and a return index $IND_R(t)$, we compute $R(t-h,t) = IND_R(t)/IND_R(t-h)$ and $D(t-h,t)$ as in (3.17). For example, when t is at monthly frequency, we can compute quarterly dividends for $h=3$.

The price dividend ratio is then simply given as:

$$PD_t = \frac{P(t)}{(D(t-h,t) * 12/h)},$$

where the term $12/h$ normalizes PD_t to per annum units. I compute price-dividend ratios for each observation date in the sample (i.e. daily for the ZEW and Shiller surveys and monthly for Livingston).[30]

[30]Dividends could additionally be deseasoned as in Adam, Marcet, and Beutel (2017). This can make sense when fitting a theoretical model (which typically does not include seasonal effects) to empirical data. However, for the purpose of evaluting forecasts it is unnecessary, therefore I do not use deseasoning. In any case, the difference between the two methods is quantitatively minor since most of the variation in the price-dividend ratio is driven by the price component.

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