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Firm size, economic risks, and the cross-section of international stock returns

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ABSTRACT

Recent empirical evidence from developed markets indicates a negative relation between value premium and firm size. We find that the value premium in small stocks is consistently priced in the cross-section of international returns, whereas the value premium in big stocks is not. Based on US data, we show that the small-stock value premium is associated with business cycle news and reflects changes in macroeconomic, especially credit market related risks. Our results hold true for regional and global equity markets and remain valid after controlling for firm characteristics and prominent profitability and investment factors.

1. Introduction

Numerous papers document inconsistency of the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) with several regularities of asset pricing data. Perhaps most notably, the CAPM fails to explain the well documented value and size premiums in equity returns, i.e. higher returns on stocks with relatively high book-to-market equity ratios and higher returns on stocks with relatively low market equity, respectively. In response to this deficit, Fama and French (1993) develop a three-factor model with factors mimicking the returns on the aggregate stock market, firm size, and book-to-market equity:

$$E\left(R^{i,e}\right) = \lambda_0 + \lambda_m \beta^i_m + \lambda_{smb} \beta^i_{smb} + \lambda_{hml} \beta^i_{hml} + e^i. \quad (1)$$

In cross-sectional regression (1), $E\left(R^{i,e}\right)$ is the expected return on asset i in excess of the risk-free rate, β^i_m is the sensitivity of asset i to the market excess return, β^i_{smb} is the sensitivity of asset i to the aggregate size premium, β^i_{hml} is the sensitivity of asset i to the aggregate value premium, and lambdas are the associated factor risk premiums. In this model, the aggregate size premium *SMB* is measured by the difference between the returns on diversified portfolios of small and big stocks (small-minus-big), and the aggregate value premium *HML* is measured by the difference between the returns on diversified portfolios of high and low book-to-market stocks (high-minus-low).

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While the model in Eq. (1) captures patterns in the post-1962 US average returns better than the CAPM, its overall performance leaves substantial room for improvements. For example, when confronted with high average returns on international micro-caps, the specification in Eq. (1) generates a significant pricing error. Relatedly, the three-factor model fails to rationalize the documented size effect in value premium, i.e. the negative relation between the value premium and firm size. Fama and French (2012) find these shortcomings for global equity markets and for local equity returns at a regional level. Cakici and Tan (2014) verify this empirical evidence for developed capital markets at a country level.

Against this background the contribution of our paper is twofold. First, we show that the value premium in small stocks tends to be associated with macroeconomic news. The analogous evidence for the value premium in big stocks turns out much weaker in our sample. We document this difference between small-stock and big-stock value premiums by evaluating a simple empirical approximation of Merton's (1973) intertemporal capital asset pricing model (ICAPM). Our analysis suggests that the profitability of small value firms is related to macroeconomic, especially credit market related risks. If small firms are badly collateralized and have limited excess to external financing (Gertler & Gilchrist, 1994) and high book-to-market firms are highly leveraged and suffer from cash flow problems (Fama & French, 1995), then the credit market situation should have a direct impact on the profitability of value firms with low market equity.

Secondly, we show that the exposure to the small-stock value factor is priced in average international stock returns. By contrast, we find no feedback from the big-stock value factor on stock performance. We gain this insight from a modified version of the three-factor Fama–French model which decomposes the aggregate value factor in the small-stock value premium and the big-stock value premium:

$$E\left(R^{i,e}\right) = \lambda_0 + \lambda_m \beta_m^i + \lambda_{smb} \beta_{smb}^i + \lambda_{hmls} \beta_{hmls}^i + \lambda_{hmlb} \beta_{hmlb}^i + e^i. \tag{2}$$

In cross-sectional regression (2), β_{hmls}^i is the sensitivity of asset *i* to the small-stock value premium and β_{hmlb}^i is the sensitivity of asset *i* to the big-stock value premium. We measure the small-stock value premium *HMLS* by the difference between the returns on diversified portfolios of small stocks with high and low book-to-market ratios (high-minus-low small), and the big-stock value premium *HMLB* by the difference between the returns on diversified portfolios of big stocks with high and low book-to-market ratios (high-minus-low big).

From a technical perspective, the representation in Eq. (2) emerges as a natural response to the observed patterns in the data. Recent studies document a negative relation between value premium and firm size. Fama and French (2012) find larger value premiums for small market capitalization stocks and smaller value premiums for big market capitalization stocks in North America, Europe, and Asia Pacific. Cakici and Tan (2014) derive similar conclusions for country-specific portfolios of stocks in 23 developed international equity markets.

In economic terms, *HMLB* and *HMLS* could be motivated by differences in the return-generating mechanisms for large capitalization and small capitalization firms. For instance, Hou and Van Dijk (2012) find that small firms experience large negative profitability shocks after the early 1980s, while big firms experience large positive cash-flow shocks. Alternatively, Eun, Huang, and Lai (2008) argue that returns on large-cap firms are driven by common factors, whereas returns on small-cap firms primarily response to idiosyncratic factors. While the current intensification in comovement of large-caps mitigates their benefits for cross-border diversification, small and locally oriented stocks become increasingly important as a vehicle in international portfolio diversification.

In contrast to the evidence for the UK in Gregory, Tharyan, and Christidis (2013), we find significant differences in risk prices associated with small-stock and big-stock value factors. Our results show that *HMLS* captures cross-sectional variation in returns and commands a significant premium in the US, regional and global stock returns. By contrast, there is no premium for *HMLB* risk exposures. Interestingly, the pricing error is typically insignificant in Eq. (2) as opposed to Eq. (1). Lewellen, Nagel, and Shanken (2010) warn against a false treatment of the slopes in cross-sectional regressions such as (1) and (2). When we follow their recommendation and impose a risk-free rate restriction, we find that the specification in Eq. (2) can double the adjusted $\overline{R^2}$ measure of the original three-factor model.

We guard against the possibility that the model in Eq. (2) is misspecified since it does not contain the prominent profitability and investment factors (see among others Hou, Xue, & Zhang, 2015). In particular, we employ US and international data and evaluate a recently proposed five-factor model of Fama and French (2015):

$$E\left(R^{i,e}\right) = \lambda_0 + \lambda_m \beta_m^i + \lambda_{smb} \beta_{smb}^i + \lambda_{rmw} \beta_{rmw}^i + \lambda_{cma} \beta_{cma}^i + \lambda_{hml} \beta_{hml}^i + e^i \tag{3}$$

and its modified version with small-stock and big-stock value factors:

$$E\left(R^{i,e}\right) = \lambda_0 + \lambda_m \beta_m^i + \lambda_{smb} \beta_{smb}^i + \lambda_{rmw} \beta_{rmw}^i + \lambda_{cma} \beta_{cma}^i + \lambda_{hmls} \beta_{hmls}^i + \lambda_{hmlb} \beta_{hmlb}^i + e^i. \tag{4}$$

In Eqs. (3) and (4), β_{rmw}^i denotes the sensitivity of asset *i* to the aggregate profitability and β_{cma}^i measures the sensitivity of asset *i* to the aggregate investment. In these representations, *RMW* is the difference between the returns on diversified portfolios of stocks with robust and weak profitability (robust-minus-weak), and *CMA* is the difference between the returns on diversified portfolios of the stocks of low and high investment firms (conservative-minus-aggressive). All results remain valid after controlling for firm characteristics and prominent momentum, profitability and investment factors, and hold true for regional and global equity markets. Our tests indicate that the small-stock value premium mimics credit market related risks and is priced consistently in the cross-section of returns in line with the basic prediction of the intertemporal asset pricing theory.

In general, our analysis is related to papers which highlight the importance of small stocks for asset pricing.[1] More specifically, we build on the business cycle literature which shows that small and value firms suffer stronger from aggregate economic shocks. For example, theoretical work by Bernanke and Gertler (1989), Gertler and Gilchrist (1994), and Kiyotaki and Moore (1997) implies that credit market conditions can have very different effects on firms of different size. Because small firms do not have much collateral, they will be affected more adversely by lower liquidity and higher discount rates following an economic shock. In this vein, Gomes, Yaron, and Zhang (2006) and Zhang (2005) argue that size and value premiums are inherently conditional on the business cycle conditions and likely countercyclical.

There are also several empirical studies which argue that vulnerability to macroeconomic sources of risk and variations in credit market conditions can rationalize premiums in equity markets. Perez-Quiros and Timmermann (2000) and Covas and Den Haan (2011) show that expected returns and financial characteristics of small firms exhibit much higher sensitivity to the business cycle and variables that measure credit market conditions. Duarte and Kapadia (2016) argue that increases in the firm's market value should signify economic contractions accompanied by increases in discount rates. They propose a new variable measured as the change in the market equity weight of the largest firms in the aggregate stock market and name it *GVD* (Goliath versus David). *GVD* is therefore per construction similar to *SMB*. [2] While *GVD* performs well in forecasting equity market returns, treasury bond returns, and business cycle variables, the ability of this variable to rationalize the negative relation between value and size appears very limited.[3]

The remainder is organized is follows. Section 2 describes the data set. Section 3 highlights the link between economic news and the small-stock value premium. Section 4 explores the cross-sectional implications of the size effect in value premium by evaluating specifications in Eqs. (1)–(4)for international equity markets and Section 5 concludes.

2. Data

Our analysis is based on a standard US data set covering the period from July 1963 to June 2016, and a relatively new international data set running from July 1990 to June 2016. The latter is of shorter size, but it has the benefit of broad coverage of micro-cap stocks. We consider global markets and three developed regional markets which include North America, Europe, and Asia Pacific.[4] These data are freely available on the website of Ken French.

2.1. Explanatory factors

Table 1 shows summary statistics for explanatory factor returns. Average market excess returns (*Mkt*) vary from 0.42% in the global markets to 0.70% in the region of Asia Pacific in monthly terms. The size premium is in general unstable. It switches signs and is typically statistically insignificant. We consider two versions of the size factor (*SMB*): $SMB_{(1)}$ is computed out of six value-weighted portfolios formed on size and book-to-market as the average return on three small portfolios minus the average return on three big portfolios. This factor is conventionally employed in the three-factor Fama–French model. $SMB_{(2)}$ is computed based on six value-weighted portfolios formed on size and book-to-market, six value-weighted portfolios formed on size and operating profitability, and six value-weighted portfolios formed on size and investment. It is calculated as the average return on the nine small stock portfolio minus the average return on the nine big stock portfolios. This factor is used in the five-factor Fama–French model. The profitability (*RMW*) and investment (*CMA*) factors are throughout positive and significant. The only exception is Asia Pacific where *RMW* is measured rather imprecisely. The value factor (*HML*) is positive, significantly different from zero, except for North America, and economically between 0.22% and 0.60% per month.

Following Fama and French (2012), we construct *HMLS* and *HMLB* based on six elementary portfolios formed on size and book-to-market equity. These six basic portfolios are denoted by SG, SN, SV, BG, BN and BV, where S and B indicate small or big and G, N, and V indicate growth (low book-to-market equity), neutral, or value (high book-to-market equity). The small-value and big-value factors are computed as $HMLS = SV\text{-}SG$ and $HMLB = BV\text{-}BG$, respectively. Small-stock value premium ranges from 0.44% to 0.88%, while the big-stock value premium lies in an interval between −0.00% and 0.33%. The last column in the table provides the return differentials between *HMLS* and *HMLB*. In global and regional markets, the value premium is higher among small stocks and lower among big stocks. For instance, the difference in value premiums of small and big stocks is 0.32% in the USA with a t-statistic of 2.80, and 0.43% in global markets with a t-statistic of 3.28. The size effect in value premium appears economically relatively less important for Europe.

[1] For instance, Chan, Chen, and Hsieh (1985) and Keim and Stambaugh (1986) report that small stocks exhibit the greatest sensitivity to overall changes in expected risk premium. That is, when expected risk premiums on all assets change, the risk premiums on small stocks change the most.

[2] In an earlier draft of this paper, we studied the cross-sectional implications of a decomposion of the *SMB* factor for firms with high and low book-to-market equity, but found statistically weaker and economically generally inconclusive evidence.

[3] While Duarte and Kapadia (2016) do not study the cross-sectional implications of *GVD* for risk premiums in asset markets, Table 9 in their paper is indicative of this observation.

[4] Since there is no clear relation between value premium and firm size in Japan, we do not include Japan in our data set. Cakici, Fabozzi, and Tan (2013) find no robust size effect in value and momentum premiums in emerging market stock returns.

Table 1
Summary Statistics of Explanatory Factors. This table shows means, standard deviations, and *t*-statistics for explanatory factor returns in % per month in global, regional, and US stock markets. *Mkt*, *RMW*, *CMA* and *HML* are the standard market, operating profitability, investment and value factors. $SMB_{(1)}$ is the size factor in the three-factor Fama–French model, $SMB_{(2)}$ is the size factor in the five-factor Fama–French model. *HMLS* is the small-stock value factor and *HMLB* is the big-stock value factor. These factors are constructed as *HMLS* = *SV*-*SG* and *HMLB* = *BV*-*BG* out of six portfolios formed on 2 × 3 sorts on size and book-to-market equity: *SG*, *SN*, *SV*, *BG*, *BN* and *BV*, where *S* and *B* indicate small and big, respectively; and *G*, *N*, and *V* indicate growth (low book-to-market), neutral, and value (high book-to-market), respectively. Column Diff. denotes the return differentials between *HMLS* and *HMLB*. The sample period is July 1963 to June 2016 for the USA and July 1990 to June 2016 otherwise.

	Mkt	$SMB_{(1)}$	$SMB_{(2)}$	*RMW*	*CMA*	*HML*	*HMLS*	*HMLB*	Diff.
Global									
Mean	0.42	0.03	0.11	0.36	0.26	0.30	0.52	0.09	0.43
Std.	4.37	2.07	1.99	1.46	1.91	2.30	2.64	2.51	2.32
t-Stat.	1.70	0.25	1.01	4.31	2.40	2.34	3.48	0.62	3.28
North America									
Mean	0.62	0.09	0.17	0.34	0.31	0.22	0.44	−0.00	0.45
Std.	4.29	3.06	2.79	2.45	2.69	3.24	4.05	3.04	3.05
t-Stat.	2.57	0.54	1.09	2.45	2.02	1.21	1.94	−0.01	2.58
Europe									
Mean	0.45	−0.02	0.05	0.43	0.22	0.31	0.46	0.15	0.31
Std.	5.01	2.26	2.22	1.51	1.87	2.41	2.61	3.01	2.91
t-Stat.	1.57	−0.13	0.42	3.99	2.05	2.25	3.13	0.90	1.88
Asia Pacific									
Mean	0.70	−0.21	−0.08	0.23	0.40	0.60	0.88	0.33	0.55
Std.	6.05	3.05	2.98	2.88	2.64	3.04	3.13	4.07	3.99
t-Stat.	2.06	−1.23	−0.46	1.39	2.67	3.51	4.95	1.42	2.44
USA									
Mean	0.50	0.22	0.25	0.24	0.31	0.35	0.51	0.19	0.32
Std.	4.44	3.09	3.04	2.24	2.01	2.80	3.20	3.08	2.84
t-Stat.	2.84	1.78	2.09	2.75	3.88	3.14	4.00	1.57	2.80

2.2. Test asset returns

Table 2 gives monthly average returns and standard deviations for 25 portfolios formed on size and book-to-market equity for global, regional, and the US stock markets. The portfolios are organized in a squared matrix with low growth (*G*) stocks at the left, value (*V*) stocks at the right, small (*S*) stocks at the top, and big (*B*) stocks at the bottom. Column Diff. gives the differences in extreme value and extreme growth portfolios for each size category.

Four features of the data are worth noting. First, in each size group, there is a standard value effect in every market we examine: Value portfolios have higher average returns than growth portfolios. The only exception are extreme value and extreme growth portfolios in the highest size category in North America. Second, we see some evidence of a standard size effect: Small portfolios tend to have higher average returns than big portfolios. This pattern is particularly strongly pronounced for extreme value portfolios. Third, we can also see a reversed size effect in returns: For extreme growth stocks, small portfolios have lower average returns than big portfolios. Finally and most importantly, we observe a clearly pronounced size effect in value premium, i.e. the value premium is declining with size.

3. Economic risks and small-stock value premium

In this section, we take a closer look at the macroeconomic determinants of the size effect in value premium. We do so by analyzing the economic links between *HMLS* and *HMLB*, on the one hand, and sources of systematic economic risk, on the other hand. Our inspiration for this exercise comes from several recent contributions which argue that factor mimicking portfolios constructed on the basis of firm characteristics can be related to macroeconomic fundamentals. Known examples include Liew and Vassalou (2000) and Vassalou (2003) who show that size and value factors capture shocks to economy-wide growth prospects; Vassalou and Xing (2004) who relate the momentum factor to default risk; and Hahn and Lee (2006) and Petkova (2006) who find that the level and the slope of the yield curve can be interpreted as alternative proxies for size and value premiums in the cross-section of stock returns.

3.1. Theoretical framework of the ICAPM

The ICAPM of Merton (1973) predicts that the risk premium should be determined by asset return sensitivities to innovations in state variables which govern changes in the investment opportunity set. Assets with high loadings on good news about future expected returns on the market portfolio should have higher expected returns. These assets command a positive risk premium because they reduce the ability of investors to hedge against worsening investment opportunities.

Following Campbell (1996) we express the return generating process as

Table 2
Descriptive Statistics of International Portfolios. The table shows average excess returns and standard deviations in % per month on 25 portfolios formed on 5 × 5 sorts on size and book-to-market equity for global, regional, and US stock markets. *G* and *V* indicate growth (low book-to-market) and value (high book-to-market), respectively; *S* and *B* indicate small (low market equity) and big (high market equity), respectively. Column Diff. gives differences in extreme value and extreme growth portfolios. The sample period is July 1963 to June 2016 for the USA and July 1990 to June 2016 otherwise.

	G	2	3	4	*V*	Diff.	*G*	2	3	4	*V*
	Mean						Std.				
Global											
S	0.11	0.36	0.60	0.64	0.92	0.81	5.78	5.32	4.98	4.56	4.34
2	0.13	0.38	0.47	0.57	0.63	0.51	5.72	5.11	4.66	4.42	4.45
3	0.23	0.39	0.46	0.51	0.62	0.40	5.65	5.11	4.70	4.51	4.63
4	0.39	0.41	0.46	0.54	0.56	0.17	5.52	4.65	4.59	4.47	4.76
B	0.36	0.40	0.45	0.47	0.41	0.05	4.51	4.36	4.41	4.47	5.27
North America											
S	0.40	0.57	0.90	0.83	1.13	0.73	8.04	6.93	6.22	5.42	5.39
2	0.26	0.57	0.78	0.74	0.82	0.57	7.38	6.57	5.57	4.97	5.24
3	0.74	0.63	0.74	0.72	0.88	0.14	7.02	5.80	5.14	4.72	4.96
4	0.78	0.64	0.79	0.75	0.82	0.04	6.55	5.18	4.66	4.72	4.82
B	0.59	0.57	0.59	0.64	0.49	−0.10	4.63	4.18	4.26	4.24	5.32
Europe											
S	−0.12	0.26	0.40	0.48	0.67	0.79	5.61	5.29	5.04	4.91	4.91
2	0.21	0.37	0.45	0.63	0.69	0.48	5.73	5.33	5.09	5.15	5.38
3	0.25	0.51	0.49	0.48	0.64	0.39	5.80	5.27	5.20	5.30	5.72
4	0.42	0.46	0.47	0.54	0.56	0.14	5.48	5.00	5.15	5.44	5.92
B	0.34	0.47	0.50	0.59	0.44	0.10	4.94	4.83	5.30	5.62	6.48
Asia Pacific											
S	0.40	0.34	0.77	1.03	1.48	1.08	8.33	7.73	7.34	7.11	7.29
2	−0.08	0.17	0.37	0.57	0.92	1.00	7.17	7.52	6.63	7.01	7.56
3	0.10	0.32	0.67	0.73	0.70	0.60	7.15	6.99	6.72	6.94	7.44
4	0.58	0.77	0.59	0.81	1.02	0.44	6.60	6.28	6.24	6.69	7.84
B	0.64	0.73	0.71	0.75	0.92	0.28	6.44	6.20	6.25	6.30	8.07
USA											
S	0.59	1.15	1.13	1.36	1.45	0.86	8.89	7.23	6.54	6.06	6.68
2	0.83	1.11	1.24	1.28	1.34	0.51	7.27	5.78	5.42	5.24	5.86
3	0.86	1.15	1.10	1.23	1.39	0.53	6.59	5.58	4.79	5.10	5.42
4	0.96	0.95	1.05	1.21	1.16	0.21	6.06	5.13	5.14	4.88	5.99
B	0.82	0.87	0.89	0.84	1.02	0.20	4.97	4.49	4.22	5.21	5.45

$$R_t^{i,e} = \beta_0^i + \beta_m^i Mkt_t + \sum_{k=1}^{K} \beta_{\tilde{s}^k}^i \tilde{s}_t^k + \varepsilon_t^i, \tag{5}$$

where $\tilde{s}^k$ represents the innovation to the state variable k and $\beta_{\tilde{s}^k}^i$ represents the sensitivity of asset i to the risk factor k, $k = 1, \ldots, K$. In equilibrium, the unconditional average excess return on stock i is then given by

$$E\left(R^{i,e}\right) = \beta_0^i + \lambda_m \beta_m^i + \sum_{k=1}^{K} \lambda_{\tilde{s}^k} \beta_{\tilde{s}^k}^i + e^i, \tag{6}$$

where $\lambda_{\tilde{s}^k}$ is the price of risk associated with innovation in the state variable k for $k = 1, \ldots, K$. This representation captures the idea that the set of pricing factors which includes the market portfolio and innovations in the relevant state variables should capture differences in returns across assets. Moreover, it is only the unexpected component of the state variable which should command a premium in equilibrium.

To derive testable implications of the asset pricing Eq. (6), we need to specify the dynamics of the state variables. We do so by adopting a vector autoregressive (VAR) approach of Campbell (1996). We include the market excess return as the first element of a state vector s_t. The remaining elements of s_t are variables which are relevant for forecasting returns. For simplicity, we assume that all variables in s_t are zero-mean, and the vector s_t follows a first-order autoregressive process:

$$s_t = As_{t-1} + \tilde{s}_t, \tag{7}$$

where A is a companion matrix of constant parameters and $\tilde{s}_t$ is a vector of innovations in the state variables used as pricing factors in Eq. (6).

3.2. Empirical ICAPM approximations

In specifying the VAR, we use a combination of macroeconomic aggregates and risk mimicking portfolio returns to proxy for time-varying investment opportunities. We employ the aggregate dividend yield (*dy*), the default spread (*def*), the term spread (*term*), the unanticipated inflation (*ui*) and the short-term interest rate (*rf*) as our macroeconomic state variables. The motivation for these variables stems from a long list of prior studies. For example, the aggregate dividend yield in combination with the short-term rate reveals a strong forecasting ability for the conditional distribution of future returns (Ang & Bekaert, 2007). The term and default spreads are two of the most widely used proxies for expectations about future interest rates and time-varying risk premiums (Keim & Stambaugh, 1986). The unexpected inflation is strongly related to time-varying risk aversion and future returns and could thus also be useful in capturing the hedging concerns (Chen, Roll, & Ross, 1986; Brandt & Wang, 2003). Beyond these macroeconomic aggregates, we include the returns on the factor mimicking portfolios *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* into the VAR. We then estimate Eq. (7) to extract the news series of each state variable. The latter are used as pricing factors in the cross-sectional regressions summarized in Eq. (6).

We start with a simple specification based on a subset of macroeconomic factors:

$$E\left(R^{i,e}\right) = \lambda_0 + \lambda_m\beta^i_m + \lambda_{\widetilde{dy}}\beta^i_{\widetilde{dy}} + \lambda_{\widetilde{def}}\beta^i_{\widetilde{def}} + \lambda_{\widetilde{term}}\beta^i_{\widetilde{term}} + \lambda_{\widetilde{ui}}\beta^i_{\widetilde{ui}} + \lambda_{\widetilde{rf}}\beta^i_{\widetilde{rf}} + e^i. \tag{8}$$

Additionally, we estimate two models which include the returns on the factor mimicking portfolios and innovations in macroeconomic predictors jointly. We consider a specification which is based on Eq. (2)

$$E\left(R^{i,e}\right) = \lambda_0 + \lambda_m\beta^i_m + \lambda_{smb}\beta^i_{smb} + \lambda_{hmls}\beta^i_{hmls} + \lambda_{hmlb}\beta^i_{hmlb} + \lambda_{\widetilde{dy}}\beta^i_{\widetilde{dy}} + \lambda_{\widetilde{def}}\beta^i_{\widetilde{def}} + \lambda_{\widetilde{term}}\beta^i_{\widetilde{term}} + \lambda_{\widetilde{ui}}\beta^i_{\widetilde{ui}} + \lambda_{\widetilde{rf}}\beta^i_{\widetilde{rf}} + e^i. \tag{9}$$

and one which is based on Eq. (4)

$$\begin{aligned} E\left(R^{i,e}\right) = {} & \lambda_0 + \lambda_m\beta^i_m + \lambda_{smb}\beta^i_{smb} + \lambda_{rmw}\beta^i_{rmw} + \lambda_{cma}\beta^i_{cma} + \lambda_{hmls}\beta^i_{hmls} + \lambda_{hmlb}\beta^i_{hmlb} + \lambda_{\widetilde{dy}}\beta^i_{\widetilde{dy}} + \lambda_{\widetilde{def}}\beta^i_{\widetilde{def}} + \lambda_{\widetilde{term}}\beta^i_{\widetilde{term}} \\ & + \lambda_{\widetilde{ui}}\beta^i_{\widetilde{ui}} + \lambda_{\widetilde{rf}}\beta^i_{\widetilde{rf}} + e^i. \end{aligned} \tag{10}$$

Our main hypothesis is that if there is a relation between small-stock value factor and risks associated with business cycle fluctuations, then *HMLS* should lose its explanatory power once we control for assets' sensitivities to macroeconomic risks. We investigate this hypothesis empirically in Section 3.4.

We note that specifications in Eqs. (9) and (10) are different from the empirical ICAPM approximation studied in Petkova (2006) since we explicitly take heed of challenges in pricing small stocks and focus on small-stock and big-stock book-to-market factors. Furthermore, we include the actual returns on factor mimicking portfolios as pricing factors in our specifications because these factors (and not their innovations) form the basis for the empirical models we focus on. In specifications (9) and (10), we employ innovation series obtained from a VAR which includes the respective factor mimicking portfolios. Our estimates do not change qualitatively when we exclude portfolio returns from the state vector, or when we estimate Eqs. (8)–(10) with innovations in market excess return and innovations in *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB*.

3.3. VAR estimation

We estimate a first order VAR model based on US data over the period from July 1963 to June 2016. We measure the aggregate market dividend yield as in Campbell (1991) by a trailing 12-month dividend yield on the S&P500 index computed from the online data base of Robert Shiller. Similar to Petkova (2006), we proxy the default spread by the difference between the yield on the Moody's long-term corporate BAA bond and the yield on a 10-year government constant maturity bond, and the short-term rate by the 1-month T-bill yield. The data on bond yields are from the FRED database of the Federal Reserve Bank of St. Louis and the T-bill series is from the online data library of Ken French. The unanticipated inflation is measured as a difference between the actual CPI rate and inflation expectations as in Chen et al. (1986). As a proxy for expected inflation we use the bi-annual 12-month inflation forecasts from the Livingston Survey interpolated to monthly frequency. To match the timing horizon, in each month we compute portfolio returns with a one-year holding period. The data on returns are from the webpage of Ken French.

Table 3 shows results of contemporaneous time-series regressions of innovations in macroeconomic state variables on the factor mimicking portfolios. The upper panel gives results from the time-series regression of the following form:

$$\widetilde{s} = c_0 + c_1Mkt + c_2SMB + c_3HMLS + c_4HMLB + \varepsilon, \tag{11}$$

while the bottom panel summarizes the results from a modified specification:

$$\widetilde{s} = c_0 + c_1Mkt + c_2SMB + c_3RMW + c_4CMA + c_5HMLS + c_6HMLB + \varepsilon. \tag{12}$$

In Eqs. (11) and (12), we suppress the time indices for simplicity of representation.

The innovations to state variable $\widetilde{s}$ are obtained as residuals from a first-order VAR with *Mkt*, *dy*, *def*, *term*, *ui*, *rf*, and the respective factor mimicking portfolios. The VAR in the upper panel includes *SMB*, *HMLS* and *HMLB* in addition to the macroeconomic variables, whereas the VAR in the bottom panel includes *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* in addition to the macroeconomic variables. The first column of Table 3 shows the dependent variable, and the remaining columns give coefficient estimates on the independent variables listed in the column header. Heteroskedasticity and autocorrelation adjusted *t*-statistics (Newey & West, 1987) are provided in parentheses below coefficient estimates.

Table 3

Time-Series Regressions of Innovations in ICAPM State Variables on Factors. This table shows the results of contemporaneous time-series regressions of innovations in dividend yield ($\widetilde{dy}$), default spread ($\widetilde{def}$), term spread ($\widetilde{term}$), unanticipated inflation ($\widetilde{ui}$), and the short-term rate ($\widetilde{rf}$) on the factors in the three-factor and five-factor Fama–French models with *HML* decomposed in *HMLS* and *HMLB*. The innovations to the state variables are computed in a VAR system. In parentheses below coefficient estimates are Newey and West (1987) corrected *t*-statistics. The adjusted $\overline{R^2}$ measures are in percent. The sample period is July 1963 to June 2016.

	$\widetilde{s} = c_0 + c_1 Mkt + c_2 SMB + c_3 HMLS + c_4 HMLB + \varepsilon$						
	c_1	c_2	c_3	c_4			$\overline{R^2}$
$\widetilde{dy}$	−19.61	−3.94	−5.48	4.34			48.57
	(−16.26)	(−2.79)	(−2.84)	(2.44)			
$\widetilde{def}$	0.34	−0.24	0.92	−0.69			1.53
	(1.12)	(−1.01)	(2.52)	(−1.83)			
$\widetilde{term}$	1.01	0.46	1.41	−0.36			2.51
	(2.84)	(1.32)	(2.54)	(−0.79)			
$\widetilde{ui}$	−0.71	−0.43	−0.25	0.44			0.66
	(−1.73)	(−0.94)	(−0.42)	(0.59)			
$\widetilde{rf}$	0.01	0.10	−0.63	0.07			1.41
	(0.08)	(0.51)	(−2.64)	(0.30)			
	$\widetilde{s} = c_0 + c_1 Mkt + c_2 SMB + c_3 RMW + c_4 CMA + c_5 HMLS + c_6 HMLB + \varepsilon$						
	c_1	c_2	c_3	c_4	c_5	c_6	$\overline{R^2}$
$\widetilde{dy}$	−20.09	−4.98	−7.82	−3.58	−2.14	3.20	50.18
	(−16.77)	(−3.17)	(−3.64)	(−1.17)	(−0.93)	(1.79)	
$\widetilde{def}$	0.44	−0.31	0.42	0.70	0.54	−0.57	1.73
	(1.49)	(−1.24)	(0.99)	(1.27)	(1.14)	(−1.42)	
$\widetilde{term}$	0.96	0.18	−1.50	−0.08	1.87	−0.84	3.21
	(2.56)	(0.50)	(−2.83)	(−0.11)	(2.96)	(−1.70)	
$\widetilde{ui}$	−0.81	−0.67	−1.55	−0.77	0.43	0.19	1.21
	(−2.08)	(−1.48)	(−2.15)	(−0.69)	(0.59)	(0.25)	
$\widetilde{rf}$	−0.04	0.05	−0.59	−0.35	−0.34	−0.07	1.71
	(−0.26)	(0.23)	(−1.57)	(−0.78)	(−0.96)	(−0.26)	

Our estimates indicates that fluctuations in the market excess return and *RMW* are related to innovations in the aggregate dividend yield, term spread and unanticipated inflation. The return on *SMB* covaries negatively with dividend yield innovations. We find a strong feedback from *HMLS* on several economic aggregates, and especially on the unpredicted component of the term spread. By contrast, the relation between *HMLB* and macroeconomic news series is weak. We obtain qualitatively similar results when we use innovations in *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* as regressors.

Importantly, Table 3 highlights a significant relation of the profitability of small value firms to macroeconomic fundamentals in general and risks associated with business cycle fluctuations in particular. This evidence is weaker and often absent in case of big value firms. If low market equity firms are young and have bad access to external capital markets (Gertler & Gilchrist, 1994) and high book-to-market firms are highly leveraged (Fama & French, 1995), then it is reasonable to assume that small value firms will be more sensitive to the state of the business cycle and especially the credit market situation. These estimates appear to support a risk-based interpretation of size effect in value premium.

Hahn and Lee (2006) and Petkova (2006) also provide evidence that size and value premiums proxy for the risks associated with business cycle fluctuations. However, these studies investigate the performance of a standard three-factor Fama–French model and do not take heed of pricing small stocks. Unlike Hahn and Lee (2006) and Petkova (2006), we focus on differences between small-stock and big-stock book-to-market equity factors and investigate alternative ICAPM specifications. Our approach is also different from Duarte and Kapadia (2016) who construct a version of size factor which they label *GVD* (Goliath versus David). They show that *GVD* is strongly related to changes in aggregate business conditions. Duarte and Kapadia (2016) study the forecasting ability of *GVD* and hence look at the time series properties of *GVD*, whereas we investigate the cross-sectional implications of the factors.

We have explored the sensitivity of these results in several dimensions. We experimented with alternative measures of default and term spreads based on the difference between the log yield on Moody's BAA and AAA corporate bonds, and the difference between the log yield on the 10-year US government bonds and 3-month Treasury bills. We worked with other measures of the dividend yield based on the ratio of dividends to prices and past dividends to current prices. In addition, we used inflation expectation series from the Michigan Survey which are available since the late 1970s and inflation expectations from the Survey of Professional Forecasters which are available since the early 1980s. We have also considered further candidate state variables such as the expected inflation, monthly and annual industrial production growth, or consumption. We employed innovations in these series obtained from a VAR system and the first differences in these series as our pricing factors following Chen et al. (1986). None of these changes had a significant impact on our conclusions.

3.4. Cross-sectional evidence

In what follows, we run a horse race of asset pricing tests. Our testing procedure relies on a two-stage regression methodology of Black, Jensen, and Scholes (1972) and Cochrane (2005) which emerges as one of the most popular approaches for estimating and testing linear asset pricing models. The first stage runs an unconditional time-series regression to obtain the estimates of factor loadings. These regressions take the following form:

$$R^{i,e} = \beta_0^i + \beta_m^i Mkt + \beta_{smb}^i SMB + \beta_{hmls}^i HMLS + \beta_{hmlb}^i HMLB + \varepsilon^i, \tag{13}$$

$$R^{i,e} = \beta_0^i + \beta_m^i Mkt + \beta_{smb}^i SMB + \beta_{rmw}^i RMW + \beta_{cma}^i CMA + \beta_{hmls}^i HMLS + \beta_{hmlb}^i HMLB + \varepsilon^i, \tag{14}$$

$$R^{i,e} = \beta_0^i + \beta_m^i Mkt + \beta_{\widetilde{dy}}^i \widetilde{dy} + \beta_{\widetilde{def}}^i \widetilde{def} + \beta_{\widetilde{term}}^i \widetilde{term} + \beta_{\widetilde{ui}}^i \widetilde{ui} + \beta_{\widetilde{rf}}^i \widetilde{rf} + \varepsilon^i, \tag{15}$$

$$R^{i,e} = \beta_0^i + \beta_m^i Mkt + \beta_{smb}^i SMB + \beta_{hmls}^i HMLS + \beta_{hmlb}^i HMLB + \beta_{\widetilde{dy}}^i \widetilde{dy} + \beta_{\widetilde{def}}^i \widetilde{def} + \beta_{\widetilde{term}}^i \widetilde{term} + \beta_{\widetilde{ui}}^i \widetilde{ui} + \beta_{\widetilde{rf}}^i \widetilde{rf} + \varepsilon^i, \tag{16}$$

or

$$R^{i,e} = \beta_0^i + \beta_m^i Mkt + \beta_{smb}^i SMB + \beta_{rmw}^i RMW + \beta_{cma}^i CMA + \beta_{hmls}^i HMLS + \beta_{hmlb}^i HMLB + \beta_{\widetilde{dy}}^i \widetilde{dy} + \beta_{\widetilde{def}}^i \widetilde{def} + \beta_{\widetilde{term}}^i \widetilde{term} + \beta_{\widetilde{ui}}^i \widetilde{ui} + \beta_{\widetilde{rf}}^i \widetilde{rf} + \varepsilon^i. \tag{17}$$

In the second stage, the factor risk premiums are estimated from a cross-sectional regression of average excess returns on the estimated betas. These regressions are summarized in Eqs. (2), (4), (8), (9), and (10) above.

Table 4 provides an overview of second-stage regressions. It shows risk premium estimates across alternative model specifications. We employ a standard set of 25 US portfolios sorted by size and book-to-market equity as our test assets. The sample period runs from July 1963 to June 2016. In parentheses below each coefficient are Shanken (1992) corrected t-statistics. The last three columns give adjusted $\overline{R}^2$ statistics and the mean squared and mean absolute pricing errors (*MSPE* and *MAPE*) in % p.a.

The top panel of Table 4 summarizes the results for the model in Eq. (2) which contains *Mkt*, *SMB*, *HMLS* and *HMLB* as pricing factors. The estimate of the intercept of 14.35% p.a. with a t-statistic of 7.74 indicates a significant pricing error. However, a high $\overline{R}^2$ of 70.85% suggest that the specification fits the data reasonably well. We find a significant premium on the *SMB* factor but a negative and statistically insignificant market risk premium. Interestingly, our results suggest that the loadings on *HMLS* represent an important determinant of average returns, while there is no feedback from *HMLB* sensitivities on average stock performance. Consistent with the evidence from time-series regressions in Table 3, these estimates highlight differences in the impact of small-stock and big stock-stock value factors on average stock returns.

Table 4

Baseline Risk Premium Estimates. The table shows the estimated factor risk prices in % p.a. from cross-sectional regressions of portfolio excess returns on a constant and their betas. The test assets are 25 US size and book-to-market equity sorted portfolios. The tested models include (i) a model with *Mkt*, *SMB*, *HMLS* and *HMLB* factors; (ii) a model with *Mkt*, *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* factors; (iii) a model with innovations in macro state variables including the dividend yield ($\widetilde{dy}$), default spread ($\widetilde{def}$), term spread ($\widetilde{term}$), unanticipated inflation ($\widetilde{ui}$), and the short-term rate ($\widetilde{rf}$); (iv) a model with *Mkt*, *SMB*, *HMLS* and *HMLB* factors and innovations in macro state variables; (v) a model with *Mkt*, *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* factors and innovations in macro state variables. Shanken (1992) corrected t-statistics are in parentheses below coefficient estimates. $\overline{R}^2$ is the cross-sectional adjusted R^2. *MSPE* and *MAPE* are the mean squared and mean absolute pricing errors in % p.a.

λ_0	λ_m	λ_{smb}	λ_{rmw}	λ_{cma}	λ_{hmls}	λ_{hmlb}	$\lambda_{\widetilde{dy}}$	$\lambda_{\widetilde{def}}$	$\lambda_{\widetilde{term}}$	$\lambda_{\widetilde{ui}}$	$\lambda_{\widetilde{rf}}$	$\overline{R}^2$	*MSPE*	*MAPE*
Model with *Mkt*, *SMB*, *HMLS* and *HMLB* Factors														
14.35	−1.08	2.74			2.87	0.71						70.85	1.54	0.99
(7.74)	(−0.50)	(3.31)			(3.76)	(0.86)								
Model with *Mkt*, *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* Factors														
15.29	−1.08	3.01	0.84	0.20	2.67	0.33						67.44	1.54	0.97
(8.40)	(−0.47)	(3.55)	(1.01)	(0.28)	(2.91)	(0.37)								
	Model with Macro State Variables													
13.42	4.89						−3.64	−0.41	0.46	0.39	−0.17	77.39	1.07	0.80
(1.81)	(0.43)						(−0.82)	(−2.02)	(1.08)	(1.14)	(−2.65)			
	Model with *Mkt*, *SMB*, *HMLS* and *HMLB* and Macro State Variables													
12.01	1.04	1.66			2.01	3.06	0.21	−0.23	0.64	0.12	−0.19	85.65	0.57	0.57
(3.57)	(0.24)	(1.03)			(1.22)	(1.92)	(0.13)	(−2.08)	(3.14)	(0.51)	(−4.43)			
	Model with *Mkt*, *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* and Macro State Variables													
11.87	2.20	2.38	−4.35	0.73	3.24	2.52	−0.10	−0.05	0.89	0.37	−0.21	86.43	0.45	0.50
(2.68)	(0.36)	(1.18)	(−2.22)	(0.33)	(1.45)	(1.09)	(−0.05)	(−0.23)	(2.93)	(1.17)	(−3.71)			

In the second row of Table 4 we estimate a specification in Eq. (14) which decomposes the *HML* factor in the five-factor Fama–French model in *HMLS* and *HMLB* varieties. In general, introducing the profitability and investment factors has no significant impact on the model performance. The specifications in the first two rows of the table generate fairly similar pricing errors and measures of fit. Our conclusions with respect to the relative importance of *HMLS* and *HMLB* remain unaffected. We find again a significant premium for the small-stock value factor and an insignificant premium for the big-stock value factor.

Next, we investigate the possibility that the explanatory ability of *HMLS* is associated with risks related to the business cycle. To do so, we first estimate a simple ICAPM specification in Eq. (15). This empirical representation includes the market excess return and innovations in macroeconomic state variables. We find that our ICAPM version produces a slightly better fit and somewhat lower pricing errors compared to the specifications which are based solely on the factor mimicking portfolios. For instance, the ICAPM model generates an $\overline{R^2}$ of 77.39% against 70.85% and 67.44% for the Fama–French models with decomposed *HML* factor. Also in terms of the *MSPE*s and *MAPE*s, the ICAPM specification appears to better fit the data. Most of the explanatory power of the model can be attributed to loadings on the innovations in the default spread and short-term rate which are usually employed in the literature as measures of credit market conditions. The estimate of $\lambda_{\widetilde{def}}$ is equal to -0.41% p.a. with a t-statistic of -2.02; and the estimate of $\lambda_{\widetilde{rf}}$ is equal to -0.17% p.a. with a t-statistic of -2.65. The prices of risk related to other macroeconomic factors turn out insignificant in our sample.

We then consider specifications which include both financial and macroeconomic risk factors. Our hypothesis is that if *HMLS* proxies for risks related to the business cycle, then β_{hmls}'s should lose their explanatory power once we control for sensitivities to economic risks. The results of specifications in Eqs. (16) and (17) are given in the last two rows of Table 4. Several points stand out. First, we can see that these more complex specifications perform to some extent better overall. These models produce very high $\overline{R^2}$ statistics which exceed 85% and very low pricing errors which are about twice as low as the models based on factor mimicking portfolios. Secondly, this improvement in the general model fit is partly due to a higher significance of the macro state variables and partly due to increased significance of the financial factors. For example, the specifications in Eqs. (16) and (17) attach a significant premium to the innovations in the term spread and the operating profitability factor. Third, *SMB* and *HMLS* become insignificant while *HMLB* becomes marginally significant after we control for assets' risk exposures to macroeconomic news. This last point hence also suggests that the ability of *HMLS* exposures to capture average return differentials (see e.g. the first two rows of the table) may reflect the ability of the small-stock value premium to capture macroeconomic, especially credit market related risks. We therefore cannot reject our hypothesis of a significant link between small-stock value premium and macroeconomic news. An alternative specification with innovations in (instead of actual) portfolio returns leads to qualitatively similar evidence.

Finally, we guard against the possibility that the high explanatory power of the asset pricing models can arise from the commonality effects in portfolios sorted on characteristics. Following the recommendation of Lewellen et al. (2010) we expand the set of our test assets and evaluate the empirical specifications in Eqs. (13)–(17) on 10 industry and 25 size- and book-to-market sorted portfolios as well as on 10 industry and 25 size- and momentum sorted portfolios. Confronted with this broader set of test asset returns, we generally observe lower $\overline{R^2}$ statistics and higher pricing errors. However, our main conclusions with respect to the relative importance of *HMLS* and *HMLB*, and the relation of *HMLS* to the business cycle risks, remain unaffected.[5] In addition, we extend the models in Eqs. (13)–(17) with portfolio characteristics. For instance, Jagannathan and Wang (1998) argue that taking account of additional return predictors provides a valid test for a model specification. We find that portfolio characteristics turn out often insignificant and typically do not add substantial explanatory power. Our estimates generally reinforce the view that macroeconomic risk factors contain most of the information summarized in the *HMLS* factor. After controlling for exposures to innovations in the relevant state variables, the small-stock value factor becomes essentially superfluous in explaining the cross-section of average stock returns. We thus conclude that the profitability of small value firms is related to business cycle news.

4. Cross-sectional implications of the size effect in value premium: International evidence

Our analysis hitherto suggests that small-stock value premium is associated with risks related to business cycle fluctuations and reflects changes in macroeconomic, especially credit market related risks. This result does not apply to the value premium specific to big stocks. The result that small-stock variety of the *HML* factor is priced in the cross-section of US returns thus turns out in line with the basic insight of the ICAPM. In this section, we ask whether this result holds generally true and applies to global and regional returns.

4.1. Risk loadings

Tables 5 and 6 show the betas in the modified versions of the three- and five-factor Fama–French models with *HMLS* and *HMLB* factors. The betas are obtained from multiple time-series regressions summarized in Eqs. (13) and (14) above. To save space, we report only the estimates of the factor loadings for 25 global portfolios sorted by size and book-to-market. The

[5] We omit these results for brevity of representation. These and several additional robustness checks are readily available from the authors upon request.

Table 5

Betas in the Four-Factor Model. The table shows the betas on the *Mkt*, *SMB*, *HMLS* and *HMLB* factors for 25 global portfolios formed on 5 × 5 sorts on size and book-to-market equity in the left half of the table and their *t*-statistics in the right half of the table. The betas are computed in multiple time-series regressions for each portfolio excess return separately. *G* and *V* indicate growth (low book-to-market) and value (high book-to-market), respectively; *S* and *B* indicate small (low market equity) and big (high market equity), respectively. Column Diff. gives differences in extreme value and extreme growth portfolios. The sample period is July 1990 to June 2016.

	G	2	3	4	*V*	Diff.	*G*	2	3	4	*V*
			β_m						*t*-stat. (β_m)		
S	0.96	0.96	0.94	0.93	0.95	−0.01	38.50	38.91	45.74	48.52	61.87
2	1.01	1.00	0.96	0.99	1.02	0.01	51.18	61.60	68.30	57.87	79.47
3	1.07	1.03	1.04	1.00	1.07	0.00	53.35	60.25	48.65	56.69	56.70
4	1.08	1.04	1.05	1.02	1.05	−0.02	46.52	47.56	54.22	44.81	46.39
B	0.99	0.98	0.97	0.95	1.03	0.04	63.45	51.03	68.46	65.59	56.99
			β_{smb}						*t*-Stat. (β_{smb})		
S	1.19	1.12	1.08	1.05	1.12	−0.07	19.85	18.89	18.79	24.94	32.75
2	0.98	0.82	0.90	0.75	0.93	−0.05	32.84	16.87	26.74	14.14	38.06
3	0.68	0.73	0.55	0.48	0.59	−0.09	14.05	19.36	9.04	9.26	14.44
4	0.49	0.40	0.29	0.28	0.28	−0.21	12.09	6.72	6.16	4.79	5.18
B	−0.20	−0.25	−0.13	−0.21	−0.35	−0.15	−4.94	−6.67	−5.23	−6.56	−8.62
			β_{hmls}						*t*-Stat. (β_{hmls})		
S	−0.53	−0.31	−0.14	0.15	0.54	1.07	−7.07	−4.70	−2.21	3.94	17.05
2	−0.57	−0.28	0.01	0.28	0.58	1.15	−13.97	−6.75	0.17	5.73	19.67
3	−0.49	−0.22	0.12	0.24	0.52	1.01	−9.40	−5.15	2.37	4.75	16.62
4	−0.43	0.07	0.18	0.29	0.31	0.74	−6.69	1.08	3.97	6.20	7.79
B	−0.00	0.04	−0.00	−0.01	−0.17	−0.17	−0.05	1.07	−0.12	−0.17	−2.87
			β_{hmlb}						*t*-Stat. (β_{hmlb})		
S	0.06	0.11	0.12	0.12	−0.02	−0.08	1.00	2.28	3.31	3.80	−0.63
2	0.07	0.14	0.09	0.14	0.08	0.01	2.01	4.07	3.26	4.39	3.37
3	−0.10	0.04	0.10	0.23	0.17	0.27	−2.94	1.04	2.50	5.56	4.85
4	−0.17	−0.01	0.10	0.17	0.35	0.53	−3.12	−0.28	2.51	3.91	9.89
B	−0.56	−0.13	0.18	0.39	0.73	1.29	−12.09	−2.64	3.99	10.11	13.64

results are very similar for each regional market we consider, i.e. North America, Europe and Asia Pacific. The representation style in Tables 5 and 6 is analogous to Table 2. The portfolios are organized in a squared matrix with growth stocks at the left, value stocks at the right, small stocks at the top, and large stocks at the bottom. Column Diff. gives differences in extreme value and extreme growth portfolios in each size category.

The patterns in the beta estimates for the market and *SMB* risk factors are qualitatively similar for both models under examination. We can see that value stocks often have lower market and *SMB* betas compared to growth stocks, while the evidence for the *RMW* and *CMA* factors in Table 6 is mixed. For some size categories, the operating profitability and investment betas are higher for value versus growth stocks, whereas the opposite is true for other size categories. It is important to note that patterns in the market, *SMB*, *RMW* and *CMA* betas do not capture the negative relation between size and value we seek to explain.

A different picture emerges once we look at the *HMLS* and *HMLB* betas. Both beta types are typically higher for value than for growth stocks, and this could explain the standard value effect in returns. However, there is an important difference in small-stock and big-stock *HML* beta varieties. While the spreads in *HMLB* loadings increase almost monotonically from low market capitalization to high market capitalization stocks as indicated in column Diff., the spreads in *HMLS* loadings tend to decline with size with the only exception of stocks in the second size group. Hence, patterns in *HMLS* betas tend to resemble patterns in average returns. We take this as evidence that the *HMLS* factor can have a potential to capture the negative relation between firm size and value we observe in the data.

In sum, Tables 5 and 6 indicate one key shortcoming of standard risk factors: They fail to rationalize a negative relation between value premium and firm size. In stark contrast, we find that sensitivities to the small-stock book-to-market factor resemble patterns in average returns. In the following section we ask whether splitting the standard *HML* factor in its small-stock and big-stock varieties is helpful in explaining cross-sectional differences in returns on international stocks.

4.2. Prices of risk

We first compare the performance of the original three-factor Fama–French model with an augmented specification which includes the *Mkt*, *SMB*, *HMLS* and *HMLB* factors. Table 7 presents cross-sectional risk premium estimates in % p.a. for specifications in Eqs. (1) and (2) along with Shanken (1992) corrected *t*-statistics in parentheses. As a measure of "goodness-of-fit" we employ the adjusted $\overline{R}^2$ from a cross-sectional regression of average returns on the betas and a constant. As test assets we use 25 portfolios formed on size and book-to-market equity constructed for global equity markets and the regions of North America, Europe and Asia Pacific.

Table 6
Betas in the Six-Factor Model. The table shows the betas on the *Mkt*, *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* factors for 25 global portfolios formed on 5 × 5 sorts on size and book-to-market equity in the left half of the table and their *t*-statistics in the right half of the table. The betas are computed in multiple time-series regressions for each portfolio excess return separately. *G* and *V* indicate growth (low book-to-market) and value (high book-to-market), respectively; *S* and *B* indicate small (low market equity) and big (high market equity), respectively. Column Diff. gives differences in extreme value and extreme growth portfolios. The sample period is July 1990 to June 2016.

	G	2	3	4	V	Diff.	G	2	3	4	V
			β_m						t-Stat. (β_m)		
S	0.94	0.93	0.92	0.92	0.92	−0.01	36.47	32.87	36.99	42.17	58.23
2	1.02	0.98	0.97	1.00	1.02	−0.00	52.64	53.28	65.55	54.77	78.05
3	1.05	1.04	1.06	1.04	1.10	0.05	39.58	59.62	53.05	54.73	49.01
4	1.06	1.05	1.08	1.07	1.09	0.03	44.63	45.13	51.69	45.78	36.52
B	0.97	1.01	0.99	0.96	0.99	0.03	59.88	55.73	68.73	56.22	56.07
			β_{smb}						t-Stat. (β_{smb})		
S	1.17	1.11	1.08	1.05	1.11	−0.05	20.05	17.77	17.63	21.51	35.09
2	0.99	0.81	0.92	0.78	0.92	−0.07	34.36	17.89	27.12	16.26	38.27
3	0.66	0.75	0.58	0.53	0.62	−0.04	13.86	21.69	11.18	11.67	16.37
4	0.45	0.42	0.33	0.35	0.33	−0.12	14.86	7.38	8.56	6.96	7.12
B	−0.21	−0.23	−0.11	−0.19	−0.40	−0.18	−7.86	−8.87	−4.97	−5.91	−10.55
			β_{rmw}						t-Stat. (β_{rmw})		
S	−0.21	−0.08	−0.06	−0.06	−0.13	0.08	−2.64	−0.96	−0.78	−1.14	−2.78
2	−0.16	−0.13	−0.03	0.02	−0.17	−0.02	−3.49	−2.15	−0.52	0.46	−5.93
3	−0.25	−0.02	0.10	0.13	0.00	0.25	−3.21	−0.31	1.62	2.65	0.03
4	−0.31	0.01	0.10	0.25	0.12	0.43	−4.82	0.09	1.65	4.83	1.67
B	−0.07	0.09	0.15	0.11	−0.19	−0.12	−1.43	1.77	3.79	2.16	−3.62
			β_{cma}						t-Stat. (β_{cma})		
S	−0.14	−0.25	−0.22	−0.16	−0.29	−0.15	−1.61	−2.77	−3.12	−2.49	−5.94
2	0.15	−0.15	0.01	0.05	0.03	−0.12	2.71	−2.07	0.25	0.98	0.92
3	−0.01	0.07	0.11	0.24	0.28	0.29	−0.09	1.42	1.96	3.06	3.82
4	0.03	0.03	0.24	0.29	0.27	0.24	0.44	0.55	3.36	5.23	3.23
B	−0.22	0.26	0.03	0.02	−0.27	−0.05	−3.74	4.86	0.75	0.39	−4.99
			β_{hmls}						t-Stat. (β_{hmls})		
S	−0.55	−0.32	−0.17	0.11	0.57	1.12	−6.04	−3.54	−2.00	1.88	14.44
2	−0.70	−0.28	−0.09	0.18	0.51	1.21	−14.85	−5.27	−1.91	2.94	17.76
3	−0.50	−0.32	−0.01	0.06	0.34	0.84	−6.55	−5.91	−0.13	0.92	6.54
4	−0.42	0.02	0.03	0.09	0.14	0.56	−5.01	0.25	0.47	1.55	2.55
B	0.13	−0.05	−0.04	−0.02	0.03	−0.10	2.65	−1.34	−1.07	−0.46	0.68
			β_{hmlb}						t-Stat. (β_{hmlb})		
S	−0.01	0.07	0.08	0.08	−0.06	−0.04	−0.21	1.54	1.92	2.45	−1.65
2	−0.04	0.10	0.03	0.09	−0.01	0.03	−1.10	2.60	1.11	2.85	−0.31
3	−0.18	−0.02	0.07	0.18	0.09	0.27	−5.51	−0.39	1.51	4.52	2.62
4	−0.25	−0.04	0.06	0.14	0.31	0.56	−4.86	−0.99	1.36	3.87	8.88
B	−0.53	−0.15	0.20	0.41	0.77	1.29	−12.15	−3.09	4.87	10.72	16.58

At a first look, the benchmark three-factor model of Fama and French (1993) seems to do a reasonable job in fitting the data. The adjusted $\overline{R}^2$ statistics vary from 39.95% for North America to 71.57% for global markets. The explanatory ability of the model is largely due to the *HML* factor, while the *SMB* factor lacks power in picking return differentials across stocks. Yet, a closer look at the first column of the upper panel reveals that the model generates an economically large and statistically significant pricing error in each market under consideration. In annual percentage points, the pricing error is close to 40% for Asia Pacific and slightly below 20% for North America and global equity markets. In addition, the market risk premium is estimated to be negative in each case.

Furthermore, the estimates in Table 7 suggest that there are substantial benefits from decomposing *HML* along the size dimension. First, judged by the average pricing error in the first column of the table, i.e. the intercept estimates, the specification in Eq. (2) outperforms its three-factor counterpart for regional and global returns. For instance, the estimates of the constant term drop from 18.84% to 7.13% per annum for North America, and from 38.88% to 29.12% for Asia Pacific. In case of Europe, the intercept declines from 14.24% to less than 1% and even becomes insignificant. For global and regional markets, a model with *HMLS* and *HMLB* factors results in lower mean absolute and mean squared pricing error and higher $\overline{R}^2$ statistics. For example, the *MSPE* declines from 6.32% to 4.93% and the adjusted $\overline{R}^2$ statistic increases from 52.16% to 60.82% for Asia Pacific. In addition, the model with the decomposed *HML* factor tends to generate market risk premiums which correspond closer to the actual factor means. Importantly, in economic but also in statistical terms, the small-stock value factor emerges as the main driver of the model performance and the key to more accurate asset pricing, greater explanatory power, and generally lower pricing errors. In line with the baseline evidence for the USA in Table 4, we find significant risk premium estimates for *HMLS*, but no feedback of *HMLB* exposures on average stock returns.

Table 7
Cross-Sectional Implications of the Size Effect in the Three-Factor Model. The table shows the estimated factor risk prices in % p.a. from cross-sectional regressions of excess returns on 25 global or 25 regional portfolios sorted on size and book-to-market equity on a constant and their betas. The tested models include (i) a standard three-factor Fama–French model with *Mkt*, *SMB*, and *HML* factors; and (ii) a modified model with *Mkt*, *SMB*, *HMLS* and *HMLB* factors. Shanken (1992) corrected *t*-statistics are in parentheses below coefficient estimates. $\overline{R^2}$ is the cross-sectional adjusted R^2. *MSPE* and *MAPE* are the mean squared and mean absolute pricing errors in % p.a. The sample period is July 1990 to June 2016.

Three-Factor Fama–French Model

	λ_0	λ_m	λ_{smb}	λ_{hml}	$\overline{R^2}$	*MSPE*	*MAPE*
Global	18.46	−13.29	0.44	3.06	71.57	0.98	0.69
	(5.35)	(−2.90)	(0.30)	(1.92)			
North America	18.84	−11.01	0.87	2.46	39.95	2.24	1.17
	(4.77)	(−2.23)	(0.41)	(1.10)			
Europe	14.24	−8.58	−0.77	3.30	60.05	1.43	0.80
	(2.80)	(−1.41)	(−0.50)	(1.94)			
Asia Pacific	38.88	−31.33	−0.54	7.36	52.16	6.32	2.08
	(5.10)	(−3.61)	(−0.25)	(3.35)			

Model with *Mkt*, *SMB*, *HMLS* and *HMLB* Factors

	λ_0	λ_m	λ_{smb}	λ_{hmls}	λ_{hmlb}	$\overline{R^2}$	*MSPE*	*MAPE*
Global	12.44	−7.33	0.62	5.28	0.57	73.13	0.89	0.69
	(1.77)	(−0.95)	(0.44)	(2.87)	(0.33)			
North America	7.13	0.77	1.18	4.97	−1.27	51.76	1.72	1.02
	(1.24)	(0.12)	(0.56)	(1.77)	(−0.59)			
Europe	0.76	4.73	−0.45	4.87	−0.79	67.59	1.11	0.77
	(0.09)	(0.54)	(−0.29)	(2.64)	(0.37)			
Asia Pacific	29.12	−21.00	−0.90	10.96	2.40	60.82	4.93	1.88
	(3.83)	(−2.43)	(−0.41)	(4.72)	(0.82)			

Table 8
Cross-Sectional Implications of the Size Effect in the Five-Factor Model. The table shows the estimated factor risk prices in % p.a. from cross-sectional regressions of excess returns on 25 global or 25 regional portfolios sorted on size and book-to-market equity on a constant and their betas. The tested models include (i) a standard five-factor Fama–French model with *Mkt*, *SMB*, *RMW*, *CMA*, and *HML* factors; and (ii) a modified model with *Mkt*, *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* factors. Shanken (1992) corrected *t*-statistics are in parentheses below coefficient estimates. $\overline{R^2}$ is the cross-sectional adjusted R^2. *MSPE* and *MAPE* are the mean squared and mean absolute pricing errors in % p.a. The sample period is July 1990 to June 2016.

Five-Factor Fama–French Model

	λ_0	λ_m	λ_{smb}	λ_{rmw}	λ_{cma}	λ_{hml}	$\overline{R^2}$	*MSPE*	*MAPE*
Global	19.10	−13.89	1.02	1.70	2.74	3.19	69.26	0.96	0.69
	(3.78)	(−2.35)	(0.74)	(0.83)	(1.25)	(2.02)			
North America	13.07	−5.33	1.77	2.48	6.31	2.00	39.17	2.06	1.07
	(2.48)	(−0.88)	(0.92)	(0.90)	(2.07)	(0.89)			
Europe	3.99	1.61	−0.21	1.51	3.55	2.81	63.66	1.18	0.77
	(0.57)	(0.20)	(−0.14)	(0.61)	(1.57)	(1.67)			
Asia Pacific	27.03	−19.46	0.44	2.09	12.89	8.20	57.49	5.08	1.79
	(3.36)	(−2.14)	(0.21)	(0.74)	(3.72)	(3.77)			

Model with *Mkt*, *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* Factors

	λ_0	λ_m	λ_{smb}	λ_{rmw}	λ_{cma}	λ_{hmls}	λ_{hmlb}	$\overline{R^2}$	*MSPE*	*MAPE*
Global	5.58	−0.39	1.18	1.71	−0.99	5.69	0.61	76.69	0.69	0.61
	(0.75)	(−0.05)	(0.86)	(0.83)	(−0.47)	(3.11)	(0.35)			
North America	5.19	2.81	1.50	0.87	3.40	5.12	−1.30	47.31	1.69	1.00
	(0.86)	(0.42)	(0.78)	(0.29)	(1.12)	(1.83)	(−0.60)			
Europe	0.28	5.23	−0.15	0.36	2.34	4.65	0.88	64.48	1.09	0.77
	(0.03)	(0.60)	(−0.10)	(0.15)	(0.96)	(2.56)	(0.42)			
Asia Pacific	18.23	−9.58	−0.66	−1.09	12.19	11.34	2.97	73.30	3.03	1.53
	(2.19)	(−1.03)	(−0.32)	(−0.36)	(3.50)	(4.91)	(1.00)			

In Table 8, we compare the performance of the original five-factor Fama–French model with an augmented specification which includes the *Mkt*, *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* factors. These specifications are introduced in Eqs. (3) and (4) above. Faced with our test assets, the five-factor model in Table 8 performs to some extent better than the three-factor model in Table 7. However, the *RMW* factor is never priced, and the *CMA* factor is significant only in the regions of North America and Asia Pacific. Similar to our earlier conclusions, we find that splitting the *HML* factor along the size dimension tends to improve the general model fit. Compared to the upper panel, the bottom panel of Table 8 shows lower pricing errors, higher $\overline{R^2}$ statistics, and smaller and often insignificant intercept term estimates. The *HMLS* betas are priced significantly in global and regional markets, while the risk premium for the *HMLB* factor switches sign and cannot be distinguished from zero in statistical terms. Thus, we find generally similar evidence with respect to the *HMLS* and *HMLB* factors once we control for the prominent investment and profitability factors.

We address the concern of Lewellen et al. (2010) regarding a false treatment of the slopes in second-stage regressions as free parameters. This is done by imposing a theoretical restriction ex ante and enforcing a zero-beta constraint. In Tables 9 and 10, we evaluate a restricted version of the three-factor and five-factor Fama–French models and their modified specifications with *HMLS* and *HMLB* factors. As we deal with excess returns, we exclude a constant from the Eqs. (1)–(4).

Table 9
Three-Factor Model with Restricted Zero-Beta Rate. The table shows the estimated factor risk prices in % p.a. from cross-sectional regressions of excess returns on 25 global or 25 regional portfolios sorted on size and book-to-market equity on their betas. The tested models include (i) a standard three-factor Fama–French model with *Mkt*, *SMB*, and *HML* factors; and (ii) a modified model with *Mkt*, *SMB*, *HMLS* and *HMLB* factors. Shanken (1992) corrected *t*-statistics are in parentheses below coefficient estimates. $\overline{R^2}$ is the cross-sectional adjusted R^2. *MSPE* and *MAPE* are the mean squared and mean absolute pricing errors in % p.a. The sample period is July 1990 to June 2016.

	Three-Factor Fama–French Model						
	λ_m	λ_{smb}	λ_{hml}		$\overline{R^2}$	*MSPE*	*MAPE*
Global	4.45	0.84	4.00		33.56	2.41	1.08
	(1.49)	(0.59)	(2.50)				
North America	7.00	1.32	3.42		5.47	3.70	1.39
	(2.38)	(0.62)	(1.52)				
Europe	5.32	−0.54	3.85		41.93	2.18	1.03
	(1.56)	(−0.35)	(2.25)				
Asia Pacific	7.05	−0.43	6.42		25.73	10.28	2.54
	(1.70)	(−0.20)	(2.94)				
	Model with *Mkt*, *SMB*, *HMLS* and *HMLB* Factors						
	λ_m	λ_{smb}	λ_{hmls}	λ_{hmlb}	$\overline{R^2}$	*MSPE*	*MAPE*
Global	4.83	0.96	5.30	0.72	67.05	1.14	0.84
	(1.62)	(0.67)	(2.88)	(0.41)			
North America	7.77	1.36	5.16	−1.48	51.35	1.82	1.07
	(2.64)	(0.64)	(1.83)	(−0.69)			
Europe	5.48	−0.43	4.87	0.78	69.11	1.11	0.77
	(1.61)	(−0.28)	(2.63)	(0.37)			
Asia Pacific	8.05	−0.99	9.93	1.62	48.44	6.81	2.18
	(1.95)	(−0.45)	(4.26)	(0.55)			

Table 10
Five-Factor Model with Restricted Zero-Beta Rate. The table shows the estimated factor risk prices in % p.a. from cross-sectional regressions of excess returns on 25 global or 25 regional portfolios sorted on size and book-to-market equity on their betas. The tested models include (i) a standard five-factor Fama–French model with *Mkt*, *SMB*, *RMW*, *CMA* and *HML* factors; and (ii) a modified model with *Mkt*, *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* factors. Shanken (1992) corrected *t*-statistics are in parentheses below coefficient estimates. $\overline{R^2}$ is the cross-sectional adjusted R^2. *MSPE* and *MAPE* are the mean squared and mean absolute pricing errors in % p.a. The sample period is July 1990 to June 2016.

	Five-Factor Fama–French Model								
	λ_m	λ_{smb}	λ_{rmw}	λ_{cma}	λ_{hml}		$\overline{R^2}$	*MSPE*	*MAPE*
Global	4.84	1.70	7.81	−3.34	2.78		55.83	1.46	0.91
	(1.63)	(1.22)	(3.82)	(−1.38)	(1.75)				
North America	7.44	2.07	1.35	9.41	2.04		31.65	2.43	1.21
	(2.54)	(1.08)	(0.45)	(2.87)	(0.91)				
Europe	5.54	−0.18	1.07	4.24	2.78		64.95	1.20	0.77
	(1.63)	(−0.12)	(0.39)	(1.68)	(1.66)				
Asia Pacific	7.25	0.36	1.16	9.41	8.19		48.93	6.43	1.95
	(1.75)	(0.17)	(0.41)	(2.73)	(3.76)				
	Model with *Mkt*, *SMB*, *RMW*, *CMA*, *HMLS* and *HMLB* Factors								
	λ_m	λ_{smb}	λ_{rmw}	λ_{cma}	λ_{hmls}	λ_{hmlb}	$\overline{R^2}$	*MSPE*	*MAPE*
Global	5.12	1.31	2.65	−2.65	5.59	0.57	77.22	0.71	0.63
	(1.72)	(0.95)	(1.23)	(−1.14)	(3.04)	(0.33)			
North America	7.96	1.54	0.26	3.78	5.25	−1.44	48.80	1.73	1.03
	(2.72)	(0.80)	(0.09)	(1.23)	(1.88)	(−0.67)			
Europe	5.50	−0.15	0.32	2.37	4.65	0.88	66.34	1.09	0.77
	(1.61)	(−0.10)	(0.12)	(0.89)	(2.55)	(0.42)			
Asia Pacific	8.59	−0.86	−2.11	9.90	10.87	3.15	69.92	3.60	1.63
	(2.08)	(−0.41)	(−0.70)	(2.82)	(4.68)	(1.06)			

Our results indicate that none of our conclusions are affected by this restriction. Moreover, the advantages of the modified specifications with *HMLS* and *HMLB* become pronounced even stronger. Splitting *HML* in two factors increases the overall model fit and lowers the pricing error. Our estimates support that the explanatory power of the modified specifications is mostly attributed to the small-stock value factor.

The findings in Section 4.2 turn out in line with our previous evidence. The results show that the augmented Fama–French models which decompose the *HML* factor into *HMLS* and *HMLB* factors do a better job in capturing cross-sectional patterns in returns compared to the original specifications. Furthermore, the small-stock value factor emerges as the key driver of this success. Our results hold true for regional and global markets.

4.3. Size effect in other factor mimicking portfolios

Our analysis so far has focused on cross-sectional implications of the size effect in value premium motivated by the evidence in Fama and French (2012) and Cakici and Tan (2014). We discussed the economic motivation for the decomposition of the aggregate value factor in *HMLS* and *HMLB* in Section 1. In this subsection, we ask whether there exists a size effect in other prominent factors such as momentum, operating profitability and investment. Subsequently, we investigate the impact of small-stock and big-stock varieties in *WML*, *RMW*, and *CMA* factors for the cross-section of international returns.***

We apply a similar methodology as in Section 2.1 to construct small-stock and big-stock factor varieties of *WML*, *RMW*, and *CMA*. For consistency with our previous analysis, we use the data over the period from November 1990 to June 2016 for regional and global markets, and from July 1963 to June 2016 for the USA. The small-stock and big-stock winner-minus-loser factors are constructed out of six portfolios formed on 2×3 sorts on size and momentum: *SL*, *SN*, *SW*, *BL*, *BN* and *BW*, where *S* and *B* indicate small or big, and *L*, *N*, and *W* indicate loser (low prior returns), neutral, and winner (high prior returns) portfolios as *WMLS* = *SW*-*SL* and *WMLB* = *BW*-*BL*. Analogously, the small-stock and big-stock operating profitability factors are constructed out of six portfolios formed on 2×3 sorts on size and operating profitability: *SW*, *SN*, *SR*, *BW*, *BN* and *BR*, where *S* and *B* indicate small or big, and *W*, *N*, and *R* indicate weak, neutral, or robust operating profitability portfolios as *RMWS* = *SR*-*SW* and *RMWB* = *BR*-*BW*. Accordingly, we build the small-stock and big-stock investment factors based on six portfolios formed on 2×3 sorts on size and operating profitability: *SC*, *SN*, *SA*, *BC*, *BN* and *BA*, where *S* and *B* indicate small or big, and *C*, *N*, and *A* indicate conservative, neutral, or aggressive investment portfolios as *CMAS* = *SC*-*SA* and *CMAB* = *BC*-*BA*. Table 11 gives descriptive statistics of these factors.

Table 11
Summary Statistics of Explanatory Factors. This table shows means, standard deviations, and *t*-statistics for explanatory factor returns in % per month in global, regional, and US stock markets. *WMLS* is the small-stock winner-minus-loser factor and *WMLB* is the big-stock winner-minus-loser factor. These factors are constructed as *WMLS* = *SW*-*SL* and *WMLB* = *BW*-*BL* out of six portfolios formed on 2×3 sorts on size and momentum: *SL*, *SN*, *SW*, *BL*, *BN* and *BW*, where *S* and *B* indicate small or big and *L*, *N*, and *W* indicate loser (low prior returns), neutral, and winner (high prior returns) portfolios. *RMWS* is the small-stock operating profitability factor and *RMWB* is the big-stock operating profitability factor. These factors are constructed as *RMWS* = *SR*-*SW* and *RMWB* = *BR*-*BW* out of six portfolios formed on 2×3 sorts on size and operating profitability: *SW*, *SN*, *SR*, *BW*, *BN* and *BR*, where *S* and *B* indicate small and big, and *W*, *N*, and *R* indicate weak, neutral, and robust operating profitability portfolios. *CMAS* is the small-stock investment factor and *CMAB* is the big-stock investment factor. These factors are constructed as *CMAS* = *SC*-*SA* and *CMAB* = *BC*-*BA* out of six portfolios formed on 2×3 sorts on size and operating profitability: *SC*, *SN*, *SA*, *BC*, *BN* and *BA*, where *S* and *B* indicate small and big, and *C*, *N*, and *A* indicate conservative, neutral, and aggressive investment portfolios. Column Diff. denotes the return differentials between small-stock and big-stock factors. The sample period is July 1963 to June 2016 for the USA and November 1990 to June 2016 otherwise.

	WMLS	*WMLB*	Diff.	*RMWS*	*RMWB*	Diff.	*CMAS*	*CMAB*	Diff.
Global									
Mean	0.83	0.44	0.39	0.37	0.34	0.03	−0.34	−0.17	-0.17
Std.	3.83	4.44	2.42	1.46	2.02	1.98	1.78	2.32	1.58
t-Stat.	3.80	1.73	2.86	4.48	2.98	0.27	−3.43	−1.33	−1.90
North America									
Mean	0.78	0.42	0.37	0.42	0.26	0.16	−0.46	−0.16	−0.30
Std.	4.99	5.18	2.80	2.84	2.65	2.52	2.56	3.20	2.15
t-Stat.	2.76	1.41	2.31	2.59	1.74	1.09	−3.18	−0.86	−2.50
Europe									
Mean	1.38	0.53	0.85	0.54	0.31	0.24	−0.32	−0.11	−0.21
Std.	3.75	4.76	2.83	1.23	2.44	2.41	1.85	2.28	1.83
t-Stat.	6.43	1.94	5.26	7.82	2.23	1.74	−3.08	−0.85	−2.06
Asia Pacific									
Mean	1.29	0.64	0.65	0.30	0.16	0.14	−0.51	−0.29	−0.22
Std.	4.31	5.70	4.24	2.70	4.24	4.16	2.21	4.06	3.85
t-Stat.	5.26	1.98	2.68	1.94	0.66	0.59	−4.08	−1.26	−1.01
USA									
Mean	0.92	0.46	0.46	0.31	0.17	0.14	−0.41	−0.21	−0.20
Std.	4.22	4.69	2.79	2.67	2.51	2.59	1.98	2.70	2.51
t-Stat.	5.47	2.47	4.14	2.96	1.76	1.35	−5.21	−1.96	−2.01

Similar to Fama and French (2012) and Cakici and Tan (2014) we find a negative relation between momentum and size, i.e. spreads in average returns decrease from low market equity stocks to high market equity stocks. This observation applies to both global and regional markets. On average, the momentum premium among small firms exceeds the momentum premium among big stocks by 0.39% per month in global markets. For comparison, the return differential between *WMLS* and *WMLB* is highest in Europe being equal to 0.85%. By contrast, we find no size effect in the operating profitability returns. Differences between *RMWS* and *RMWB* factor returns lie in the interval between 0.03% and 0.24% per month and are statistically indistinguishable from zero. Finally, there is a mixed evidence of size effect in investment sorted portfolio returns: Differences between *CMAS* and *CMAB* are always negative. However, their statistical significance is generally not robust.

Based on this evidence, we ask whether the patterns related to the size effect in *WML* and *CMA* factors have important implications for the cross-section of stock returns. Table 12 summarizes our findings. It shows the estimated factor risk prices from cross-sectional regressions of the following form:

$$E\left(R^{i,e}\right) = \lambda_0 + \lambda_m\beta_m^i + \lambda_{smb}\beta_{smb}^i + \lambda_{hml}\beta_{hml}^i + \lambda_{wml}\beta_{wml}^i + e^i, \tag{18}$$

$$E\left(R^{i,e}\right) = \lambda_0 + \lambda_m\beta_m^i + \lambda_{smb}\beta_{smb}^i + \lambda_{hml}\beta_{hml}^i + \lambda_{wmls}\beta_{wmls}^i + \lambda_{wmlb}\beta_{wmlb}^i + e^i, \tag{19}$$

and

$$E\left(R^{i,e}\right) = \lambda_0 + \lambda_m\beta_m^i + \lambda_{smb}\beta_{smb}^i + \lambda_{hml}\beta_{hml}^i + \lambda_{rmw}\beta_{rmw}^i + \lambda_{cmas}\beta_{cmas}^i + \lambda_{cmab}\beta_{cmab}^i + e^i, \tag{20}$$

Table 12
Cross-Sectional Implications of the Size Effect in WML and CMA Factors. The table shows the estimated factor risk prices in % p.a. from cross-sectional regressions of excess returns on 25 global, regional, or US portfolios sorted on size and book-to-market equity on their betas. The tested models include (i) a standard four-factor Carhart model with *Mkt*, *SMB*, *HML* and *WML* factors; (ii) a modified model with *Mkt*, *SMB*, *HML*, *WMLS* and *WMLB* factors; and (iii) a modified model with *Mkt*, *SMB*, *HML*, *RMW*, *CMAS* and *CMAB* factors. Shanken (1992) corrected *t*-statistics are in parentheses below coefficient estimates. $\overline{R^2}$ is the cross-sectional adjusted R^2. *MSPE* and *MAPE* are the mean squared and mean absolute pricing errors in % p.a. The sample period is July 1963 to June 2016 for the USA and November 1990 to June 2016 otherwise.

	Four-Factor Carhart Model						
	λ_0	λ_m	λ_{smb}	λ_{hml}	λ_{wml}	$\overline{R^2}$	*MSPE*	*MAPE*
Global	17.93	−12.09	0.69	3.27	9.66	72.64	0.97	0.70
	(4.88)	(-2.56)	(0.49)	(2.05)	(1.35)			
North America	13.10	−3.69	1.09	3.19	32.15	70.32	1.10	0.84
	(2.77)	(-0.66)	(0.51)	(1.40)	(3.85)			
Europe	11.15	−4.78	−0.71	3.38	9.47	63.14	1.32	0.77
	(2.05)	(-0.74)	(-0.45)	(1.97)	(1.21)			
Asia Pacific	29.68	−20.29	−1.73	7.74	26.96	62.20	4.76	1.70
	(3.53)	(−2.17)	(−0.81)	(3.45)	(2.81)			
USA	5.43	1.55	2.05	4.87	33.47	72.09	1.46	0.95
	(1.19)	(0.30)	(1.35)	(3.49)	(3.52)			

	Model with *Mkt*, *SMB*, *HML*, *WMLS* and *WMLB* Factors								
	λ_0	λ_m	λ_{smb}	λ_{hml}	λ_{wmls}	λ_{wmlb}	$\overline{R^2}$	*MSPE*	*MAPE*
Global	17.28	−11.40	0.61	3.35	11.01	7.73	71.86	0.95	0.67
	(4.36)	(-2.31)	(0.43)	(2.10)	(1.31)	(1.04)			
North America	12.19	−2.83	1.13	3.15	29.72	35.60	69.33	1.08	0.82
	(2.32)	(-0.46)	(0.53)	(1.39)	(3.20)	(3.32)			
Europe	1.91	4.63	−0.32	3.48	24.78	6.10	73.78	0.89	0.65
	(0.26)	(0.56)	(-0.20)	(1.99)	(2.37)	(0.56)			
Asia Pacific	30.48	−21.04	−1.80	8.01	20.77	34.18	61.12	4.65	1.66
	(3.42)	(−2.15)	(−0.83)	(3.47)	(2.26)	(2.77)			
USA	6.47	0.24	2.47	4.90	31.11	20.82	76.69	1.16	0.84
	(1.44)	(0.05)	(1.66)	(3.51)	(3.53)	(1.90)			

	Model with *Mkt*, *SMB*, *HML*, *RMW*, *CMAS* and *CMAB* Factors									
	λ_0	λ_m	λ_{smb}	λ_{hml}	λ_{rmw}	λ_{cmas}	λ_{cmab}	$\overline{R^2}$	*MSPE*	*MAPE*
Global	11.05	−5.62	1.09	3.36	4.01	−3.56	3.51	76.70	0.69	0.60
	(1.99)	(−0.88)	(0.80)	(2.12)	(1.87)	(−1.74)	(1.17)			
North America	11.50	−3.46	1.83	1.94	4.61	−5.70	1.13	51.79	1.54	0.98
	(2.11)	(−0.56)	(0.95)	(0.87)	(1.75)	(−2.08)	(0.28)			
Europe	5.93	−0.34	−0.16	2.76	2.12	−1.94	−5.54	62.20	1.16	0.78
	(0.75)	(−0.04)	(−0.11)	(1.64)	(0.81)	(−0.73)	(−1.39)			
Asia Pacific	19.26	−10.43	−0.45	7.98	1.83	−13.63	−11.96	69.61	3.52	1.58
	(2.27)	(−1.11)	(−0.22)	(3.68)	(0.63)	(−4.00)	(−2.27)			
USA	4.94	1.38	3.20	3.89	6.45	−2.74	2.50	78.61	1.01	0.74
	(1.21)	(0.30)	(2.17)	(2.85)	(2.96)	(−1.28)	(0.86)			

where λ_{wmls} and λ_{wmlb} are the risk premiums associated with *WMLS* and *WMLB* factors, and λ_{cmas} and λ_{cmab} denote the risk premiums associated with *CMAS* and *CMAB* factors.

Several observations stand out. First, the *WML* factor in the conventional four-factor model of Carhart (1997) is priced in North America, Asia Pacific and the USA, but it is insignificant in Europe and global equity markets. A direct comparison of the upper panel of Table 12 with the upper panel of Table 7 reveals that *WML* can substantially improve the performance of a simple three-factor Fama–French model. In the middle panel of Table 12 we replace the *WML* factor with the *WMLS* and *WMLB* components. Except Europe, the benefits from decomposing the *WML* factor into its small-stock and big-stock components turn out largely negligible. There are no clear patterns with respect to the relative importance of the *WMLS* and *WMLB* factor varieties. For instance, λ_{wmls} and λ_{wmlb} are insignificant in global markets, but significantly positive in North America and Asia Pacific. In Europe and the USA, there is a significant premium for λ_{wmls} but an insignificant compensation for λ_{wmlb}.

Finally, the evidence is even more mixed when we evaluate the specification which splits *CMA* in *CMAS* and *CMAB*. The results of the empirical specification in Eq. (20) are given in the bottom panel of Table 12. This asset pricing model generates a slightly better fit than the standard five-factor Fama–French model (see for comparison Table 8), but the patterns in risk premiums on the *CMAS* and *CMAB* factors cannot tell a clear story about the determinants of the model's performance. The estimate of λ_{cmas} is throughout negative, but in two out of five cases insignificant. The estimate of λ_{cmab} switches signs and is significant only in Asia Pacific.

Overall, the results in this section suggest that the size effect in momentum and investment factors has no apparent impact on the cross-section of stocks returns. This result could be due to the fact that momentum and investment returns are per se weaker in international data. We therefore do not investigate the macroeconomic determinants of these small-firm and big-firm risk factors.

4.4. Further robustness checks

This section summarizes several robustness checks. First, we test the sensitivity of our results with respect to the choice of test assets and sample periods. The concern about the use of test assets that are formed on the basis of same characteristics as factor mimicking portfolios is not unusual in the literature. We follow the recommendation of Lewellen et al. (2010) and expand the set of test assets to industry portfolios to reduce the commonality effects. We also experiment with portfolios sorted on other characteristics, i.e. short- and long-term reversal, alternative value characteristics such as cash-flow-to-price, dividend-to-price, and earnings-to-price ratios. These data are available for the USA for a longer period starting in 1927. We confirm that the Fama–French models with decomposed *HML* factor produce a reliable explanation of patterns in average returns, and that the small-stock value factor is consistently rewarded with a significant premium.

Second, instead of assuming fix betas over the full sample, we allow the betas to vary over time. We obtain betas in rolling time-series regressions over a 6-year or 72-month period. Then we run a series of out-of-sample cross-sectional regressions of average returns over the next 6-year or 72-month period on these betas. In general, out-of-sample regressions produce a lower fit and higher pricing errors compared to the in-sample regressions, but our main results do not change qualitatively. In a further experiment, we run overlapping cross-sectional regressions relating time-varying betas to the next-month returns. This exercise generally leads to an increase in the magnitude of pricing errors, but it further reinforces the importance of small stocks in capturing commonalities in returns.

Third, to account for difficulties associated with pricing of micro-caps, we follow Cakici and Tan (2014) and Fama and French (2012), and re-run the asset pricing tests on a smaller set of portfolios based on 4x5 sorts by size and book-to-market equity. Indeed, we find that all models do to some extent better, but the specification with decomposed value factor continues to outperform the respective benchmark.

Next, as the residuals in the second-stage regressions can be correlated with each other, the standard textbook recommendation is to run a GLS instead of OLS (Cochrane, 2005). However, as the weighting matrix may be generally hard to estimate or invert, efficiency gains of GLS over OLS are associated with robustness losses. Kan, Robotti, and Shanken (2013) note that the OLS is more relevant in tests of a particular set of assets, whereas the GLS—despite the apparent difficulties in its interpretation—is more informative from an investment perspective. Our GLS estimates lead to similar inference.

Finally, betas are usually estimated from multiple regressions of asset returns on factors and are hence referred to as multivariate or multiple regression betas. Unless factors are uncorrelated, the first-stage regression might generate beta estimates which are unreliable. To guard against this possibility Jagannathan and Wang (1998) recommend to employ the so-called univariate or simple regression betas estimated for each factor separately. This beta computation method further supports our conclusions.

5. Conclusions

Our analysis is motivated by one old debate and one new question that are to date unsettled in the empirical asset pricing literature. The major debate to which we contribute is about financial characteristics and associated factor mimicking portfolios that provide the greatest explanatory power for returns in international stock markets. The new question which we address centres on how value returns vary with firm size.

We find two main insights. First, we show that small-stock book-to-market factor is rewarded with a pervasive and statistically significant premium in the cross-section of international stock returns, whereas the big-stock book-to-market factor is not. This result is important for practical applications such as calculations of the required rate of return and portfolio evaluations and has direct implications for asset pricing tests. To the extent that small stocks are increasingly used as a tool for international portfolio diversification, this result is particularly relevant for asset allocation decisions of fund managers and investment companies.

Second, our results suggest that the small-stock value premium related to the business cycle and reflects changes in macroeconomic, especially credit market related risks. This finding extends the literature that emphasizes the importance of macroeconomic fundamentals for our understanding of asset price changes. More generally, our analysis provides new evidence which supports the view that assets' exposures to systematic sources of risk are responsible for differences in expected returns across assets, i.e. the key insight in financial economics.

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