Essays on Financial Frictions in Macroeconomics

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Eidesstattliche Erklärung

Hiermit erkläre ich, dass ich die vorliegende Dissertation selbstständig angefertigt und die benutzten Hilfsmittel vollständig und deutlich angegeben habe.

Mannheim, 30. August 2017 Johannes Pöschl

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Chapter 1

General Introduction

This dissertation consists of two non-consecutive chapters which do not follow a common theme. Instead of searching for unifying aspects, it is more useful to highlight their differences. The first chapter focuses on a positive question, namely *How can we rationalize the debt maturity structure of the corporate non-financial sector both in the aggregate and at the firm level?* and only briefly touches upon efficiency considerations. The second chapter poses a normative question, namely *How should a macro-prudential regulator set capital requirements for depository institutions in the presence of an unstable shadow banking sector?*, where the question of economic efficiency is of course central. The first chapter is about the capital structure dynamics of non-financial firms, whereas in the second chapter, there is no capital structure of non-financial firms. Instead the second chapter includes a detailed model of the financial sector with different types of institutions, whereas the financial sector in the first chapter is a very stylized homogeneous financial market.

Nevertheless, commonalities do exist. Both chapters assume that agents only have a very limited set of financial contracts at their disposal and do not try to rationalize what determines the set of these contracts. These financial contracts are debt and equity contracts. In both chapters, it is the debt contracts which create externalities. I abstract however from the many agency problems associated with equity financing discussed in the corporate finance literature, e.g. in Jensen and Meckling (1976), Grossman and Hart (1980), Shleifer and Vishny (1986) and Tirole (2006).

In the first chapter, an externality arises at the firm level, since the owners of a firm do not care about the value of outstanding debt. In the second chapter, an externality arises in general equilibrium, because price-taking banks fail to internalize the negative effects of their own borrowing decisions on borrowing constraints of other banks and hence the financial stability of the banking system as a whole. In both cases, these externalities create a motive for the government to regulate lending. While this is not explicitly addressed in the first chapter, the second chapter evaluates various regulatory policies. In particular, we find that counter-cyclical capital requirements on retail banks are an effective policy tool to reduce financial instability.

Regarding the first chapter, I document that the share of long-term debt in total debt of US non-financial firms is pro-cyclical. Furthermore, this pro-cyclicality is more pronounced for smaller firms: the long-term debt share of small firms has a higher standard deviation and correlation with output than the long-term debt share of large firms. I construct a quantitative model in which firms optimally choose investment, leverage, debt maturity, dividends, and default. Firms face idiosyncratic and aggregate risk. When they choose their debt maturity, firms trade off default premia and roll-over costs. As a result, financially constrained firms endogenously prefer to issue short-term debt, because they face high default premia on long-term debt. Financially unconstrained firms issue long-term debt, because it has lower roll-over costs. The model, which is parameterized to match cross-sectional moments, can match stylized facts about the level and dynamics of the maturity structure of debt both in the aggregate and along the firm size distribution. Regarding the effects of outstanding debt on investment, it is not short-term debt, but long-term debt which leads to substantial under-investment due to a debt overhang effect.

The second chapter is joint work with Xue Zhang. We study the macroeconomic effects of regulating depository institutions in an economy with a shadow banking sector. Systemic bank runs on the shadow banking sector occur occasionally and can cause large recessions. Importantly, the probability of such runs is closely tied to economic fundamentals. A higher bank capital requirement reduces the frequency of systemic bank runs by increasing the fire sale price of capital. For this effect, it is crucial that the regulators relax capital requirements during a bank run. A static capital requirement will instead increase the frequency of banking crises. The cost of higher capital requirements is a reduction of the aggregate capital stock due to less financial intermediation. We calibrate our model to match stylized facts of the U.S. banking system and banking crises in the developed countries after World War II. Our numerical results indicate that bank capital requirements for depository institutions are effective in reducing systemic bank runs. By imposing a bank capital requirement of 15 percent, the frequency of bank runs decreases from 2.7 to 0.8 runs per 100 years. Meanwhile, the capital stock decreases by about 5 percent. Despite the fact that bank capital regulation can effectively eliminate bank runs, it is not desirable: The welfare cost of such bank capital requirements outweighs the benefit of fewer bank runs.

Both chapters highlight the dramatic effects that financial frictions relating in particular to debt contracts can have on real economic outcomes like output and investment. They also stress the importance of the non-linear nature of these transmission channels, in the first chapter in the form of the default decisions of firms and in the second chapter in the form of large, systemic bank runs. These non-linearities require the use of sophisticated numerical methods to characterize the results of these models, which is a challenge in itself. The results in this dissertation square well with the existing empirical and theoretical literature, but they also show new channels that lead to a role for the regulation of the capital structure of both non-financial and financial firms. These are two areas of research which, while very active, still leave many research questions to be explored. I hope that this dissertation will become part of a fruitful conversation in economic research in these areas.

Chapter 2

Corporate Debt Maturity and Investment over the Business Cycle

2.1 Introduction

Excessive reliance on short-term debt by firms has arguably contributed to a large extent to the fall in investment during the 2007-2009 financial crisis. Short-term debt exposes firms to the risk of an unexpected decrease in credit availability, which in turn forces them to cut down investment to repay outstanding debt. Duchin, Ozbas, and Sensoy (2010) report that firms with high short-term debt restricted investment more during the financial crisis, while firms with high long-term debt did not significantly reduce investment. Almeida, Campello, Laranjeira, and Weisbrenner (2012) report that firms with a high fraction of debt maturing when the crisis hit decreased investment substantially more firms without such a debt position.

Despite these observations, the macroeconomic literature has so far not considered which factors determine the maturity structure of corporate debt. Typically, macroeconomic models with financial frictions treat all debt as short-term.[1] Yet for an average publicly traded U.S. firm between 1984 and 2012, only 36.8 percent of outstanding debt matured within the next year. In the aggregate, only 15.54 percent of corporate debt matured within the next year for the same time period. This discrepancy between the standard model assumption on short-term debt and the actual maturity structure observed in the data is economically important for how corporate debt affects corporate investment over the business cycle: Theories where firms rely exclusively on short-term debt emphasize liquidity constraints, such that firms reduce investment in a recession due to a high cost of refinancing.[2] Theories where firms will mostly use long-term debt emphasize other frictions like the debt overhang problem first introduced by Myers (1977). According to this theory, firms reduce investment due to the failure of shareholders to internalize the benefits of investment to holders of outstanding debt.

In this chapter, I study the determinants of the maturity structure of debt, both in the cross-section and over the business cycle. Importantly, I also consider the cyclical dynamics of the maturity structure for firms of different size. I document that in the aggregate, the share of long-term debt in total debt is pro-cyclical. This correlation varies substantially by firm size: For the largest 10 percent of firms as measured by assets, the share

[1] Examples are Jermann and Quadrini (2012), Khan and Thomas (2013), Gourio (2013) and Gilchrist, Sim, and Zakrajsek (2014).

[2] Seminal examples are Gomes (2001) and Hennessy and Whited (2007).

of long-term debt in total debt is *counter*-cyclical, while it is pro-cyclical for all other firms. In addition, large firms tend to use a larger share of long-term debt in general. Because the firm size distribution is very right-skewed, the behavior of large firms dominates the aggregate effect, such that it is easy to overlook the pro-cyclical debt maturity dynamics for the vast majority of small firms.

To propose an explanation for these debt maturity dynamics, I construct a quantitative dynamic model that allows for a rich capital structure of firms, firms can issue short-term debt, long-term debt and equity. They invest in productive capital, which is illiquid due to investment adjustment costs. Firms face both idiosyncratic and aggregate productivity risk. They also face aggregate consumption risk which affects asset prices in two ways: First, consumption risk creates a risk premium for assets whose returns co-move with consumption. This is true for both equity and debt in the model, since cash flows to equity are pro-cyclical and default rates are counter-cyclical, lowering the payouts to creditors in recessions. Second, it creates a time-varying term structure of risk free interest rates: short-term interest rates are lower than long-term interest rates in a recession and higher in an expansion. Due to limited liability, firms can default on outstanding debt. Because default risk is endogenously pro-cyclical, bond prices reflect both expected losses from default and a risk premium.

The key ingredients of the model are these endogenous default premia, exogenous debt and equity issuance costs and a tax benefit of debt. Issuance costs reflect costs like underwriting fees, which the corporate finance literature considers to be important for corporate capital structure decisions, see e.g. Gomes (2001), Hennessy and Whited (2007), Titman and Tsyplakov (2007) and Eisfeldt and Muir (2016). When equity issuance costs are higher than debt issuance costs, as they are in my parametrization of the model, they create an incentive for low productivity firms to issue debt to avoid having to issue equity. The tax deductibility of interest expenses creates an incentive to issue debt for high productivity firms, which is also widely considered to be an important determinant of the corporate capital structure, see for example Modigliani and Miller (1963), Fischer et al. (1989) and the empirical evidence in Heider and Ljungqvist (2015).

The main trade-off between short-term debt and long-term is the following: On the one hand, financially unconstrained firms that want to maintain a high leverage ratio for the tax benefit of debt want to keep the expected cost of rolling over debt low. They can do so by issuing long-term debt, since long-term debt only has to be rolled over infrequently. On the other hand, financially constrained firms that issue debt because they lack internal funds want cheap external liquidity and hence want to keep the default premium on newly issued debt low. They can do so by issuing short-term debt, since the default premium on short-term debt is endogenously lower than the default premium on long-term debt. There are two reasons for this: First, long-term debt prices default risk over a longer time horizon and second, long-term debt creates an ex post incentive misalignment between the firm owners and the long-term creditors that increases the probability of default. A pecking order theory arises: Very high productivity firms will issue long-term debt. Medium productivity firms will rely on internal funds, as long as they have sufficient internal liquidity. Low productivity firms will use short-term debt, since they lack internal liquidity. Very low productivity firms will issue equity, since they are effectively excluded from credit markets due toe prohibitively high default premia. Since productivity is positively correlated with firm size, my model can match the stylized fact that small firms use a larger share of long-term debt. Importantly, the model generates a inverse u-shaped relationship between the long-term debt share and firm size, since very large firms and very small firms will not use short-term debt.

The model can also match the stylized fact that the corporate debt maturity structure is pro-cyclical: If aggregate productivity decreases, the productivity distribution shifts to the left, such that firms will use more equity and short-term debt and less long-term debt. It can match the stylized fact that the debt maturity structure of small firms is more pro-cyclical than the debt maturity structure of large firms, because small firms tend to be in the region of the productivity distribution where firms issue short-term debt.

To understand the incentive misalignment between shareholders and long-term creditors better, consider the case of a firm that has some outstanding long-term debt and a positive default probability in the next period. This firm has to decide how much to invest today. For simplicity, assume that the investment is financed with internal funds. If the firm will default in some states of the world in the next period, an investment today hence constitutes an intertemporal transfer from the owners of the firm to the creditors, because the firm owners carry the entire cost of the investment today, while the benefit of the investment in default states in the next period accrues to the creditors. The firm has therefore an incentive to underinvest, if it does not care about the value of debt. For short-term debt, this incentive to underinvest does not arise, because the effect of the investment on the value of short-term debt is internalized by the firm through the effect on current bond issuance revenue. But for long-term debt, this effect is not internalized, because newly issued long-term debt constitutes only a fraction of all outstanding long-term debt. Knowing that the firm will not act in their interest ex post, long-term creditors will hence demand a higher default premium ex ante.

To test the quantitative importance of this theory for aggregate and firm-level debt maturity structure dynamics, I calibrate the model to match several cross-sectional moments, among them the average share of long-term debt in the cross-section and the default rate. The calibrated model captures that firms endogenously use mostly long-term debt and that larger firms use a larger share of long-term debt. While I choose the model parameters to match cross-sectional moments, it can also match aggregate correlations, notably the pro-cyclicality of investment, the long-term debt share and long-term debt issuance and the counter-cyclicality of equity issuance, leverage and the default rate.

I show that issuance costs of debt and equity are important to match the level and dynamics of the debt maturity structure: Absent of equity issuance costs, there is no motive to issue short-term debt, because firms are never financially constrained: The cost of a unit of external equity is always one. Firms will then exclusively use long-term debt, independently of their size. Absent of debt issuance costs firms find it optimal to avoid using long-term debt, also independently of their size. As a consequence, a model without debt and equity issuance costs cannot explain the size heterogeneity in the level and dynamics of the debt maturity structure. This results complements Crouzet (2015), who shows in a similar quantitative model without debt issuance costs that the optimality of short-term debt is a result of the incentive problem between the firm and the creditors. Finally, consumption risk helps to improve the model fit for the maturity dynamics substantially, because it leads to a counter-cyclical slope of the term structure of risk-free interest rates over the business cycle.

I conclude by discussing the channels through which outstanding corporate debt affects investment in the model. The model nests several channels discussed in the literature, namely the debt overhang channel and the liquidity constraints channel. These channels lead to under-investment relative to an unlevered firm. The debt overhang channel is the incentive to under-invest in the presence of long-term debt discussed above. The liquidity constraints channel arises because access to external funds is costly, such that firms will under-invest due to a higher cost of capital. I show that the debt overhang channel is quantitatively the more important channel in my model, and that it is primarily small firms which under-invest. Under-investment is quantitatively large: On average, the 25 percent smallest firms would choose an investment-capital ratio that is 37.5 percent higher if they had no debt. The debt overhang channel contributes 94 percent of this under-investment. Coming back to the motivation, it is exactly not short-term debt, but long-term debt, which causes under-investment in this model. Therefore, it is not a priori clear that a regulator should encourage firms to use more long-term debt, unless the different channels through which outstanding debt affects investment are well understood.

2.2 Review of the Literature

My paper primarily is related to the literature on the cyclicality of the capital structure of non-financial firms. The closest paper to mine is Jungherr and Schott (2016). In independent and simultaneous work, they focus on the maturity dynamics of aggregate liabilities, using aggregate data from the financial accounts of the United States. They find the maturity dynamics of aggregate liabilities to be *counter*-cyclical. There are two possible reasons why their data leads them to a different conclusion: First, their measure of short-term liabilities includes trade credit. This is important, since it is well known that during and after the financial crisis, there has been a collapse of trade and hence trade credit (Chor and Manova (2012)). This would show up as an increase in the maturity structure of debt in their data. In contrast, trade credit is a separate credit category in Compustat, which is the data I use. Second, their measure of short-term debt includes all loans except mortgages and excludes all bonds. In contrast, loans and bonds in Compustat are classified according to their actual maturity. This is also potentially an issue, because the fraction of loans in total debt financing is known to be pro-cyclical (de Fiore and Uhlig (2011)), which would show up as an increase in the maturity structure of debt during recessions in their data. In terms of the theoretical analysis, my focus is more on firm heterogeneity over the business cycle, whereas they focus on aggregate dynamics.

Various other characteristics of the business cycle dynamics of the capital structure of non-financial firms have been studied in the literature, for example the choice of debt vs equity in Covas and Haan (2011) and Jermann and Quadrini (2012), of loans vs bonds in de Fiore and Uhlig (2011), Crouzet (2016) and Xiao (2017) or the choice between unsecured vs secured debt in Azariadis, Kaas, and Wen (2016).

My paper is also related to the literature on how financial frictions amplify business cycle fluctuations and hence affect the real economy. Khan and Thomas (2013) develop a heterogeneous firm model in which firms issue secured short-term debt if they lack internal funds for investment. They do not consider default decisions. They propose a model which still focuses on short-term debt, but includes endogenous default in Khan, Senga, and Thomas (2014). Furthermore, the debt issuance motive and hence leverage of firms in their model is negatively related to firm size.[3] By allowing firms to choose between different types of debt for different debt issuance motives, I can endogenously achieve a positive cross-sectional relation between firm size and leverage. Gilchrist, Sim, and Zakrajsek (2014) study uncertainty shocks in a heterogeneous agent model with financial frictions and defaultable, short-term debt. Gomes, Jermann, and Schmid (2016) develop a heterogeneous agent model with nominal long-term debt, in which inflation risk affects investment and default through a debt overhang channel. None of these papers considers however a debt maturity choice.

There is also a literature in corporate finance which studies the maturity structure of corporate debt, typically abstracting from aggregate dynamics. The closest paper to mine in this literature is Crouzet (2015),which discusses the determinants of the debt maturity structure in a stylized model with frictionless investment, no equity issuance and no aggregate uncertainty. He and Milbradt (2014) discuss the dynamics of debt maturity in a continuous time model. They solve their model in closed form and provide a theoretical discussion of the existence of various equilibria. My focus is different: I use a quantitative model to study the dynamics of debt maturity

[3] Specifically, this is true for their baseline firm setup. They improve their cross-sectional fit by adding a second type of firms to the model which instead of optimally choosing how much debt to issue use a simple rule that relates debt issuance to the square of the capital stock of the firm.

in a setting with rich cross-sectional heterogeneity of firms. Importantly, I discuss the role of aggregate uncertainty and investment for the maturity structure of debt. The focus on investment distinguishes my paper also from Chen, Xu, and Yang (2016), who investigate maturity choice in a He and Milbradt (2014)-type model with illiquid bond markets and endogenous default. They show that a liquidity-default spiral may lead firms to shorten their maturity structure during recessions, despite the existence of rollover risk. They do however not discuss the dynamics of maturity choice: conditional on the aggregate state variable, debt maturity is static in their model.

There is furthermore a large literature in corporate finance and asset pricing that studies the role of macroeconomic risk for the investment and financing decisions of firms in dynamic models, starting with Gomes (2001). My paper builds on the model of Kuehn and Schmid (2014), study the implications of endogenous investment for credit spreads in a model with only long-term debt. Hackbarth, Miao, and Morellec (2006) study leverage dynamics with long-term debt and aggregate uncertainty in a continuous-time framework and show that leverage is counter-cyclical. Bolton, Chen, and Wang (2013) study cash holdings in a single-factor model and find that firms issue external funds in times of low funding costs to build up precautionary cash buffers. The financing costs in their model are exogenous, since they do not consider risky debt. In a similar framework, Eisfeldt and Muir (2016) use an estimated model to provide evidence of a separate financial factor for the build up of precautionary cash buffers through the issuance of external funds. Warusawitharana and Whited (2016) have a similar focus as Bolton et al. (2013) and Eisfeldt and Muir (2016), but provide a behavioral foundation for their financial shock in the form of an equity misvaluation shock. I contribute to this literature by studying the effects of aggregate risk on the debt maturity structure of firms.

Finally, my paper is related to the literature which studies how the maturity structure of debt affects investment, which starts with the seminal paper by Myers (1977). Moyen (2007) discusses the role of different maturity structures for the quantitative importance of debt overhang. There are some differences between her framework and mine, most notably that in her model, firms hold either only short-term debt or long-term debt and there is no endogenous debt maturity choice. Also, the long-term debt contract in her model is different. Diamond and He (2014) also discuss the effect of different maturity structures on debt overhang in a very different class of models for an exogenously given debt maturity structure.

I proceed as follows: In section 2.3, I show the main facts about the maturity structure of debt of U.S. firms. In section 2.4, I outline the model of the decision problem of an equity-value maximizing firm and the bond pricing equations. Section 2.5 illustrates the determinants of the maturity structure of debt. Section 2.6 discusses my calibration strategy. In section 2.7, I present the numerical results for aggregate debt maturity dynamics. Section 2.8 discusses under-investment. Section 2.9 concludes.

2.3 Stylized Facts about the Corporate Debt Maturity Structure

In this section, I describe the main empirical facts that I want to explain with the model. All data are from Compustat, for the time period from the first quarter of 1984 to the last quarter of 2012. Long-term debt is defined as all debt with a residual maturity of at least one year. Debt includes notes, bonds, loans, credit lines and bankers acceptances. A detailed description of the data can be found in Section 2.6.1.

2.3.1 Heterogeneity in the Level of the Debt Maturity Structure

First, consider the row "Aggregate" of Table 2.1. The first column shows the share of long-term debt in total debt in the aggregate for the Compustat sample. On average, 84 percent of the outstanding debt of US non-financial firms has a residual maturity of at least one year. This is in stark contrast to the macroeconomic literature with financial frictions, which often assumes that the entire stock of corporate debt matures within the next quarter.

Second, consider the remaining rows of the first column of Table 2.1. These show the distribution of the long-term debt share conditional on the firm size distribution. A robust feature of the data is that larger firms tend to have a higher share of long-term debt: The average share of long-term debt of the smallest quartile of firm by assets is average only 41.9 percent, whereas the largest one percent of firms holds on average 82.3 percent of debt debt as long-term debt. The second column shows that the

	(1) Mean, LT Share	(2) SD, LT Share	(3) FracAssets	(4) FracDebt
0% to 25%	0.419	0.368	0.006	0.006
25% to 50%	0.562	0.352	0.037	0.026
50% to 75%	0.706	0.327	0.151	0.131
75% to 90%	0.799	0.274	0.287	0.297
90% to 95%	0.822	0.252	0.195	0.207
95% to 99%	0.825	0.239	0.243	0.251
99% to 100%	0.823	0.232	0.081	0.082
0% to 90%	0.608	0.364	0.481	0.460
All Firms	0.632	0.359	1.000	1.000
Aggregate	0.845	0.012		
Observations				552987

Table 2.1: Summary statistics of the long-term debt share, by firm size.

standard deviation of the maturity structure within a given size quantile is monotonically decreasing in size, which is also a robust feature of the data. Not only do larger firms have a larger share of long-term debt, but their maturity structure is also less volatile.

In the last two columns, I show the fraction of assets and debt that firms within a given size quantile account for, that is, the marginal distributions of assets and debt in Compustat. These make it clear that while the distributions of debt and assets are very right-skewed, the smallest 90 percent account nonetheless for a substantial fraction of 48 percent of assets and 46 percent of debt in the data.

2.3.2 Aggregate and Firm-Level Dynamics of the Debt Maturity Structure

In terms of dynamics, the maturity structure of debt of non-financial US firms varies substantially over the business cycle at the aggregate level. In Figure 2.1, I plot the cyclical component of the share of long-term debt in total debt for all firms in the Compustat sample, where I detrend the series using a simple linear-quadratic trend. It is evident that the share of long-term debt decreases during recessions and increases during expansions. For example, the long-term debt share during the financial crisis decreased

	(1) GDP(t-1)	(2) GDP(t)	(3) GDP(t+1)
0% to 25%	0.288**	0.335***	0.283**
25% to 50%	0.265**	0.408***	0.449***
50% to 75%	0.621***	0.722***	0.760***
75% to 90%	0.398***	0.547***	0.633***
90% to 95%	0.135	0.227*	0.286**
95% to 99%	−0.324***	−0.229*	−0.155
99% to 100%	−0.140	−0.0633	−0.006 32
0% to 90%	0.467***	0.613***	0.690***
All Firms	0.227*	0.396***	0.502***
Observations			116

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2.2: Correlations of the detrended long-term debt share with detrended log real corporate sales, by firm size.

from 88.75 percent in the first quarter of 2007 to 84.97 percent in the last quarter of 2008.

In Table 2.2, I report the correlation coefficients between real corporate log sales and the share of long-term debt in total debt. Aggregate sales is my preferred measure of output, since it is the closest equivalent to output measured in my model and since it reflects cyclical fluctuations in the corporate sector better than real GDP. I present the correlations computed using real GDP or real corporate log profits in Appendix 2.A. In the aggregate, the debt maturity structure is pro-cylical, with a correlation of 0.396 for all firms, which increases to 0.613 for the smallest 90 percent of firms. This means that the fraction of due payments on outstanding debt increases exactly when internal funds are most valuable for firms, a seemingly puzzling observation.

The maturity structure also varies widely at the firm level. In Figure 2.2, I plot the time series for the long-term debt share of the smallest 50 percent, the firms in the 50-75, 75-90 and 90-100 percent size quantiles. I follow Covas and Haan (2011) in choosing these cutoffs. The time series are detrended using a linear quadratic trend. In the figure, they are furthermore smoothed using a moving average filter with two lags. In Table 2.2, the series are not smoothed. Figure 2.2 shows the long-term debt share of small firms is pro-cyclical, while the long-term debt share of large firms is counter-cyclical.

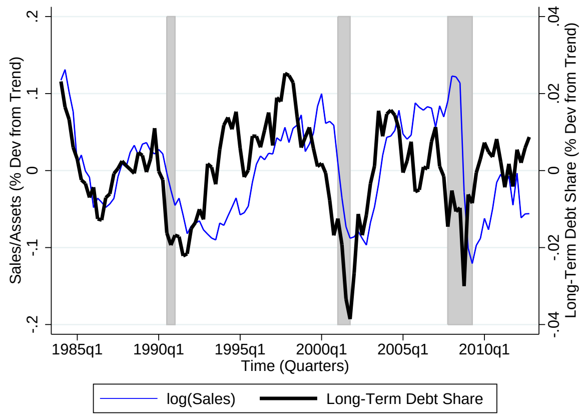

Figure 2.1: Figure 2.1 shows the cyclical component of the aggregate share of long-term debt in total debt for the bottom 90 % non-financial firms by size in the Compustat Quarterly database. The thin line is real aggregate sales. Both series are detrended with a linear-quadratic trend. The shaded areas indicate the NBER recession episodes.

This is confirmed by the correlations I report in Table 2.2: The long-term debt share of the firms up to the 90 percent size quantile is pro-cyclical, while for the largest 10 percent of firms, the long-term debt share is countercyclical. There is also an inverse u-shaped relationship between firm size and the correlation of the long-term debt share with aggregate output: For very small and very large firms, this correlation is lower than for medium-sized firms. These differences are large: For the 50 percent to 75 percent quantile, the contemporaneous correlation between the long-term debt share and real output is 0.7222 and is significantly different from zero at the 0.1 percent level. For the 95 to 99 percent size quantile, the contemporaneous correlation is -0.229 and is significantly different from zero at the 1 percent level.

Explaining these patterns of the debt maturity structure is interesting for two reasons: First, they provide additional evidence about which financial frictions determine the capital structure decisions of non-financial firms both at the micro-economic and macro-economic level. If firms predominantly rely on long-term debt, rollover costs are more important to the firms, if they use mostly short-term debt, default premia and default incentives are more important. Second, the maturity structure itself is a factor that determines through which channels outstanding debt affects the investment decisions of firms. Specifically, an important question that

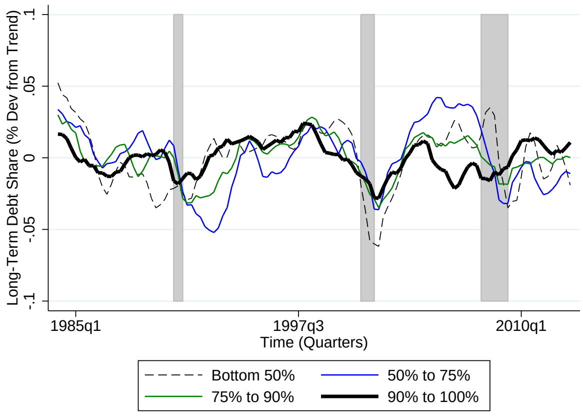

Figure 2.2: Figure 2.2 shows the cyclical component of the aggregate share of long-term debt for non-financial firms in the Compustat Quarterly database for different size bins. All series are detrended with a linear-quadratic trend. For better visibility, all series for this figure are smoothed using a moving average filter with two lags. The shaded areas indicate the NBER recession episodes.

arises is whether rollover costs or debt overhang are more important for the investment behavior of financially constrained firms. Answering this question crucially depends on understanding the optimal maturity choices of firms, because firms take the effect of their current debt maturity choices on future investment decisions into account.

2.4 Model

The model consists of many firms i, a competitive bond market for short-term debt and long-term debt, which specifies the menu of bond prices which is idiosyncratic to each firm and a representative household which owns all firms and holds all debt in the economy. Time is discrete: $t = 0, 1, \ldots, \infty$. The unit of time is a quarter.

Each firm i uses capital $K_{i,t}$ to produce output, subject to aggregate and idiosyncratic productivity risk. At each time t, firms decide how much to invest in capital, $K_{i,t+1}$, how much short-term debt $B^S_{i,t+1}$ and long-term debt $B^L_{i,t+1}$ to issue, how much dividend $D_{i,t}$ to pay and whether to default.

There are two types of debt contracts, short-term debt and long-term debt which I model as bonds. Their prices are determined on a competitive market, which prices default risk at the firm level. More precisely, creditors take into account that firms might default at any point in time in the future and they form expectations over the uncertain recovery value of a bond in default. Aggregate consumption risk yields a stochastic discount factor which generates an additional risk premium for both equity and risky debt.

In section 2.4.1, I present the firm problem. In section 2.4.2, I lay out how the bond prices are determined. I derive the stochastic discount factor from an exogenous consumption process and household preferences in section 2.4.3.

2.4.1 Firm Problem

Objective Function

The objective function of the firm is the present value of equity payouts. The dividend payout of firm i in period t is denoted by $D_{i,t}$. $EIC(D)$ is an equity issuance cost which is zero when dividends are non-negative and positive when dividends are negative. Future cash flows are discounted with a stochastic discount factor $\Lambda(C_t, C_s)$, where C_t denotes aggregate consumption at time t. The present value of dividends at time t is given by

$$\mathbb{E}_t\left[\sum_{s=t}^{T}\Lambda(C_t, C_s)\left(D_{i,s} - EIC(D_{i,s})\right)\right]. \tag{2.4.1}$$

T denotes the period in which it is optimal for the firm to default. In what follows, I will write the model in recursive form, using the notation that $X_t = X$ and $X_{t+1} = X'$ for any variable X.

Technology

The profit function of the firm is given by

$$\Pi(K_i, \tilde{A}_i, Z) = \tilde{A}_i Z K_i^{\alpha} - \psi, \tag{2.4.2}$$

where $\alpha < 1$ describes returns to scale at the firm level. $\tilde{A}_i$ is the idiosyncratic productivity of the firm, which evolves according to

$$\ln \tilde{A}_i' = \rho \ln \tilde{A}_i + \sigma^A \varepsilon_i^A, \tag{2.4.3}$$
$$\varepsilon_i^A \sim \text{i.i.d.} N(0,1).$$

The idiosyncratic productivity shocks ε_i^A are uncorrelated over time and across firms. Z is the aggregate productivity, common to all firms in the economy, which also follows a first-order autoregressive process:

$$\ln Z' = \rho \ln Z + \sigma^Z \varepsilon^Z, \qquad (2.4.4)$$
$$\varepsilon^Z \sim \text{i.i.d.} N(0,1).$$

In addition, firms have to pay a fixed cost of operation ψ. This cost arises only if the firm continues to operate, independently of whether the firm produces or not. It can be interpreted for example as a maintenance cost or as the sum of administrative expenses. In the corporate finance literature, such a fixed production cost is used, for example, in Gomes (2001).

Since idiosyncratic and aggregate productivity have the same persistence, they can be collapsed into the state variable $A_i = \tilde{A}_i Z$, which evolves according to

$$\ln A_i' = \rho \ln A_i + \sigma^A \varepsilon_i^A + \sigma^Z \varepsilon^Z. \qquad (2.4.5)$$

The conditional density function of A_i' is denoted by $f(A_i'|A_i)$.

Investment

Capital follows the standard law of motion:

$$K_i' = (1-\delta)K_i + I_i, \qquad (2.4.6)$$

where δ is the depreciation rate and I_i is investment. When installing new capital or selling old capital, the firm has to incur a quadratic capital adjustment cost with functional form

$$AC(K_i, K_i') = \frac{\theta}{2}\left(\frac{K_i'}{K_i} - 1 + \delta\right)^2 K_i. \qquad (2.4.7)$$

With these capital adjustment costs, I capture in a simple way that capital is illiquid. This form of capital adjustment costs is common in the investment literature, see for example Hayashi (1982). It is widely used in the corporate finance literature, for example in Bolton, Chen, and Wang (2013) and Eisfeldt and Muir (2016). Furthermore, Bloom (2009) reports that at the firm level, quadratic capital adjustment costs yield a good description of firm level investment behavior, even if the capital adjustment costs are non-convex at the plant level.

Debt Financing

The firm can issue short-term debt $B_{S,i}$ and long-term debt, $B_{L,i}$. Short-term debt takes the form of a one-period contract. Long-term debt takes the form of a contract with stochastic maturity μ.[4] This formulation is a common way to model long-term debt without introducing too many state variables in the model. It is for example used in the corporate finance literature in Hackbarth, Miao, and Morellec (2006) and Kuehn and Schmid (2014), but also in the literature on sovereign debt, for example in Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012) or Chatterjee and Eyigungor (2012). The level of long-term debt therefore evolves according to

$$B'_{L,i} = (1-\mu)B_{L,i} + J_{L,i}, \tag{2.4.8}$$

where $J_{L,i}$ denotes long-term debt issuance. Long-term debt cannot be repaid early: $J_{L,i} \geq 0$. Leary and Roberts (2005) report that firms adjust their leverage only slowly towards a target leverage, which is consistent with a transaction cost for repurchases or, similarly, a repurchase constraint. Both short-term debt and long-term debt pay a coupon c.

Debt is risky, because firms can default. Issuance occurs at state-contingent prices Q_S and Q_L for short-term debt and long-term debt respectively. I will explain how these bond prices are determined in equilibrium in section 2.4.2.

There is a linear issuance cost ξ for debt. Debt issuance costs are equal for short-term debt issuance and long-term debt issuance. The functional form for debt issuance costs is given by

$$DIC(B'_{S,i}, J_{L,i}) = \xi\left(|B'_{S,i}| + |J_{L,i}|\right). \tag{2.4.9}$$

These issuance costs can be interpreted as flotation fees for new bond issues or bank fees for new loans. Such costs can arise in addition to the endogenous default premium. Typically, the literature considers either a combination of fixed and linear debt issuance costs or one of the two. An

[4] Each unit of debt is infinitely divisible, such that a fraction μ will come due every period, while the remaining fraction $1-\mu$ is rolled over into the next period.

example for the former is Kuehn and Schmid (2014). A model which uses a purely linear issuance cost is Titman and Tsyplakov (2007).

Corporate Income Tax

There is a proportional corporate income tax τ. Consistent with the U.S. tax code, taxable income is calculated as income less operating costs, depreciation and interest expense. This implies there is a tax benefit for investment as well as debt issuance. As a consequence, from the perspective of the shareholder, debt issuance is cheaper than equity issuance, because a fraction of interest expense is implicitly rebated by the government. There is therefore an incentive for the firm to increase leverage up to the point where the marginal cost of debt in the form of issuance costs and the change in the default premium equal the marginal tax benefit of debt. This trade-off between the tax shield and the default premium is an important determinant of leverage, see for example Kraus and Litzenberger (1973) or Fischer, Heinkel, and Zechner (1989). Effectively, it lowers the required return on equity relative to the required return on debt, making the creditors of the firm more patient than it's shareholders. In this model, it is the reason why large, financially unconstrained firms issue debt.

Dividends and Equity Financing

For convenience, I use the total amount of outstanding debt, $B_i = B_{S,i} + B_{L,i}$ and the fraction of long-term debt, $M_i = B_{L,i}/B_i$, as state variables. I collect the endogenous state variables of firm i in the tuple $\mathcal{S}_i = (K_i, B_i, M_i)$. K_i is the capital stock of the firm, B_i is the total amount of outstanding debt, and M_i is the fraction of outstanding debt that was issued in the form of long-term debt.

Dividends are given residually by the budget constraint of the firm,

$$
\begin{aligned}
D_i = (1-\tau)\,&\underbrace{\left[\Pi(K_i, A_i) - \delta K_i - \psi - cB_i\right]}_{\text{Taxable Income}} \\
&- \underbrace{((1-M_i) + \mu M_i)B_i}_{\text{Principal Repayment}} + \underbrace{K_i - K_i'}_{\text{Gross Investment}} - AC(K_i, K_i') \\
&+ \underbrace{Q_S(1-M_i')B_i'}_{\text{Revenue from ST Debt Issuance}} + \underbrace{Q_L(M_i'B_i' - (1-\mu)M_iB_i)}_{\text{Revenue from LT Debt Issuance}} \\
&- \underbrace{DIC((1-M_i)B_i, M_i'B_i' - (1-\mu)M_iB_i)}_{\text{Debt Issuance Cost}}. \qquad (2.4.10)
\end{aligned}
$$

Dividends can be negative. In this case, the firm issues seasoned equity and has to pay an equity issuance cost.[5] These costs are meant to capture monetary costs, such as underwriting fees, but also non-monetary costs like managerial effort and signaling costs conveyed through the issues.[6] The equity issuance cost consists of a fixed component ϕ_0 and a linear component ϕ_1, such that average cost of issuing equity is decreasing in the size of the issue. The functional form is

$$EIC(D_i) = (\phi_0 + \phi_1 |D_i|)\, \mathbb{1}_{(D_i<0)}. \qquad (2.4.11)$$

This functional form is consistent for large firm with the empirical evidence in Altinkilic and Hansen (2000) and the structural estimation results in Hennessy and Whited (2007).[7] It is for example used in Gomes (2001), Cooper and Ejarque (2003), Gomes and Schmid (2010) and Kuehn and Schmid (2014).

Firm Problem and Default

If the firm decides not to default, its problem is then to maximize the present value of dividends by choosing the capital stock K_i', debt B_i', the fraction of long-term debt M_i', and dividends D_i. The value function of a

[5] Seasoned equity meaning that it is an issue by an already publicly traded firm.

[6] An example of a model in which equity issuance is associated with higher signalling costs than debt issuance is Myers and Majluf (1984).

[7] Although the main point of Altinkilic and Hansen (2000) is exactly that this formulation is not appropriate for smaller firms.

continuing firm i can be summarized as

$$V^C(\mathcal{S}_i, A_i, C) = \max_{K_i', B_i', M_i', D_i} \{D_i - EIC(D_i)$$
$$+\mathbb{E}\left[\Lambda(C, C') \int_{-\infty}^{\infty} V(\mathcal{S}_i', A_i', C') f(A_i'|A_i) dA_i' | C\right]\right\}, \quad (2.4.12)$$

subject to the budget constraint 2.4.10 and the constraints $K_i' \geq 0$, $B_i' \geq (1-\mu)M_iB_i$ and $(1-\mu)M_iB_i/B_i' \leq M_i' \leq 1$. The last two constraints arise due to the assumption that long-term debt cannot be repurchased.

Default occurs if the firm does not repay its debt, either for strategic reasons or because the firm cannot raise sufficient funds to repay outstanding liabilities. Since shareholders can simply walk away if the value of owning the firm becomes negative, the value of the firm to shareholders is bounded below by 0. The total value of equity is then

$$V(\mathcal{S}_i, A_i, C) = \max\left\{V^C(\mathcal{S}_i, A_i, C), 0\right\}. \quad (2.4.13)$$

2.4.2 Bond Markets

Payouts to Creditors

The bond market is competitive. Bonds are discounted with the same discount factor as equity. Both bonds pay a fixed coupon. Coupons are calculated according to

$$c = c_S = c_L = \frac{1}{\beta} - 1. \quad (2.4.14)$$

That is, coupons are chosen such that the values of risk-free bond prices in the absence of aggregate risk are both equal to 1.

If the firm does not default, the payment to the short-term creditors is $1 + c$. The payment to the long-term creditors is $\mu + c$. The outstanding fraction $(1-\mu)$ of long-term debt is valued by creditors at the end-of period bond price Q_L', such that the value of owning one unit of a long-term bond that is not in default is given by $\mu + c + (1-\mu)Q_L'$.

If the firm decides to default on its outstanding debt, the firm is liquidated after production has taken place.[8] There is a cross-default clause: a

[8] Corbae and D'Erasmo (2017) report that liquidations according to Chapter 7 of the US bankruptcy code account for about 20 percent of all US defaults. The rest are reorganizations following Chapter 11 of the US bankruptcy code. While allowing for

default on short-term debt triggers a default on long-term debt and vice versa. In addition, there is a pari passu clause: bond holders have equal claims on the liquidation value of the firm, independent of the maturity of their bond. The liquidation value consists of the profits plus the depreciated capital stock. Consistent with the U.S. tax code, it is not possible to deduct interest expense from taxable income in default.

A complication of my model with quadratic capital adjustment costs is that capital is illiquid. I interpret the capital adjustment cost as a primitive of the model that also has to be paid if the firm is liquidated. Therefore, it is not optimal to uninstall the entire capital stock of the firm, because below some optimal disinvestment $I^* < 0$, the marginal adjustment cost from disinvesting an additional unit of the capital stock outweighs the marginal benefit. This optimal level of disinvestment is the solution to

$$\max_{I \leq 0} = -I - \frac{\theta}{2} \left(\frac{I}{K} \right)^2 K.$$

The solution is given by $I^* = -\frac{K}{\theta}$. With this disinvestment, the adjustment cost is given by $\frac{K}{2\theta}$, such that creditors receive $I^* - \frac{K}{2\theta}$ from the liquidation of the capital stock in the current period. The recovery value per unit of the bond is therefore

$$R(\mathcal{S}_i, A_i) = \chi \max \left(\frac{(1-\tau)(\Pi(K_i, A_i) - \psi - \delta K_i) - I^* - \frac{K}{2\theta}}{B_i}, 0 \right). \tag{2.4.15}$$

Note that the model has an endogenous liquidation loss on capital due to the capital adjustment cost. Hennessy and Whited (2007) and Kuehn and Schmid (2014) assume instead that a fixed fraction of the firm value or the firm's assets is lost in liquidation.

Bond Pricing Equations

It is useful to define a default threshold set for productivity. The default threshold set is implicitly defined by

$$a^*(\mathcal{S}_i, C) = \left\{ A \in \mathcal{A} : V^C(\mathcal{S}_i, A, C) = 0 \right\}. \tag{2.4.16}$$

reorganizations would change the recovery value in default, it would not change the model mechanism substantially otherwise.

Suppose that the continuation value function V^C is strictly increasing in idiosyncratic productivity. Then, for each $(\mathcal{S}_i, C)$, the default threshold is unique and equation 2.4.16 defines a function for the default threshold productivity: For $A \leq a^*(\mathcal{S}_i, C)$, the firm will default, for $A > a^*(\mathcal{S}_i, C)$, the firm will continue.

The bond price functions are then

$$\begin{aligned} q_S(\mathcal{S}_i', A_i, C) = \mathbb{E}\Bigg[\Lambda(C, C')\Bigg(\int_{a^*(\mathcal{S}_i', C')}^{\infty}(1+c)f(A_i'|A_i)dA_i' + \\ \int_{-\infty}^{a^*(\mathcal{S}_i', C')} R(\mathcal{S}_i', A_i')f(A_i'|A_i)dA_i'\Bigg)\Big|C\Bigg], \end{aligned} \tag{2.4.17}$$

and

$$\begin{aligned} q_L(\mathcal{S}_i', A_i, C) &= \mathbb{E}\Bigg[\Lambda(C, C')\Bigg(\int_{a^*(\mathcal{S}_i', C')}^{\infty}(\mu + c + (1-\mu)Q_L')f(A_i'|A_i)dA_i' \\ &\quad + \int_{-\infty}^{a^*(\mathcal{S}_i', C')} R(\mathcal{S}_i', A_i')f(A_i'|A_i)dA_i'\Bigg)\Big|C\Bigg], \\ Q_L' &= q_L(\mathcal{S}_i'', A_i', C'). \end{aligned} \tag{2.4.18}$$

That is, bond prices reflect the future default probabilities and the value of the firm in default. Future cash flows are discounted at the stochastic discount factor. Notably, while the short-term bond price only reflects the next period default probability, the long-term bond price captures the entire future path of default probabilities through its recursive dependence on Q_L'. In what follows, I will call the next-period default probability short-run default risk and the default probability after the next period long-run default risk.

2.4.3 Stochastic Discount Factor

Equity and debt payouts are discounted with the stochastic discount factor

$$\Lambda(C, C') = \beta\left(\frac{C'}{C}\right)^{-\sigma}. \tag{2.4.19}$$

This discount factor is derived from a household whose consumption process co-moves with the aggregate productivity process: Household preferences are time-separable with discount factor β. The period felicity function has a constant relative risk aversion σ. The utility function therefore takes the recursive form

$$U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma} + \beta \mathbb{E}_{C'} \left[U(C') | C \right] \tag{2.4.20}$$

The consumption process is driven by aggregate productivity and a process $\tilde{C}$ that is uncorrelated with aggregate productivity. $\tilde{C}$ represents cyclical movements in aggregate consumption that are unrelated to productivity, for example due to other economic shocks. It also follows a first order autoregressive process:

$$\ln \tilde{C}' = \rho \ln \tilde{C} + \sigma^C \varepsilon^C. \tag{2.4.21}$$

Aggregate consumption depends on productivity Z with weight λ_1 and on $\tilde{C}$ with weight λ_2:

$$\ln C = \lambda_0 + \lambda_1 \ln Z + \lambda_2 \ln \tilde{C}. \tag{2.4.22}$$

Combining equations 2.4.19 and 2.4.22 yields the stochastic discount factor $\Lambda(C, C')$. This discount factor leads to a risk premium in the model, which has been established to be an important component of bond yields.[9] While the risk premium in the model is exogenous, default premia do reflect the endogenous default decisions of firms in the model, and are therefore endogenously determined as well.

Aggregate consumption is exogenous, which by itself is not central for the main results. It implies that the term structure of risk free interest rates is exogenous. Note however that I allow for correlation between aggregate productivity and aggregate consumption. The exogeneity of the term structure of risk-free interest rates is plausible, since I only model the markets for risky non-financial corporate debt and equity. The markets for government debt or household debt, for example, are outside the model. According to the Financial Accounts of the US, corporate business debt constituted only 17.9 percent of all outstanding debt in 2015. The market value of equity of non-financial domestic corporations constituted about 27.2 percent of the net wealth of the US for the same year. In addition, the assumption of an exogenous aggregate consumption or an exogenous stochastic discount factor is common in the asset pricing and corporate finance

[9] See for example Chen (2010) or Bhamra, Kuehn, and Strebulaev (2010)

literature. It is for example used in Campbell and Cochrane (1999).

2.4.4 Equilibrium

The recursive competitive equilibrium for this economy is given by a set of policy functions $h : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \rightarrow \mathbb{S}$ for capital, debt, and the share of long-term debt; a default policy function $d : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \rightarrow \{0, 1\}$, value functions $V_C : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \rightarrow \mathbb{R}$ and $V : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \rightarrow \mathbb{R}$; and bond price functions $q_S : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \rightarrow \mathbb{R}$ and $q_L : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \rightarrow \mathbb{R}$, such that for every firm i:

- for any $(\mathcal{S}_i, A_i, C) \in \mathbb{S} \times \mathbb{A} \times \mathbb{C}$, given q_S and q_L, $h(\mathcal{S}_i, A_i, C)$, maximizes the continuation problem 2.4.12, with the solution to the firm problem given by $V_C(\mathcal{S}_i, A_i, C)$.

- for any $(\mathcal{S}_i, A_i, C) \in \mathbb{S} \times \mathbb{A} \times \mathbb{C}$, given q_S and q_L, the firm chooses a default policy d such that

$$d(\mathcal{S}_i, A_i, C) = \begin{cases} 0 & \text{if } V^C(\mathcal{S}_i, A_i, C) > 0 \\ 1 & \text{if } V^C(\mathcal{S}_i, A_i, C) \leq 0 \end{cases} .$$

  The value function V is given by

$$V(\mathcal{S}_i, A_i, C) = V^C(\mathcal{S}_i, A_i, C)\,(1 - d(\mathcal{S}_i, A_i, C))$$

- for any $(\mathcal{S}_i', A_i, C) \in \mathbb{S} \times \mathbb{A} \times \mathbb{C}$, given h, d, V_C and V, $q_S(\mathcal{S}_i', A_i, C)$ and $q_L(\mathcal{S}_i', A_i, C)$ are the solutions to the bond pricing equations 2.4.17 and 2.4.18.

The first equilibrium condition states that the firm makes optimal investment and debt issuance decisions, taking the bond prices as given, the second states that the firm makes an optimal default decision, taking the bond prices as given, and the third condition states that bond price schedules incorporate the true default probability of the firm, taking firm policies as given.

2.5 The Determinants of Debt Maturity

In this section, I will outline the determinants of the maturity structure of a single firm and how it varies with productivity. First, I will explain the different channels that determine the maturity structure of the firm. Then, I will discuss how aggregate consumption shocks affect the optimal maturity choice.

I will denote $Q_L = q_L(\mathcal{S}_i', A_i, C)$, $Q_S = q_S(\mathcal{S}_i', A_i, C)$, and, with some abuse of notation, $V = V(\mathcal{S}_i, A_i, C)$ and $V^C = V^C(\mathcal{S}_i, A_i, C)$ to increase readability.

Throughout this section, I assume that the value function V is once differentiable in K, B, M and A and the bond price functions Q_S and Q_L are differentiable in K', B', M' and A. I further assume that the short-term and long-term bond prices are weakly increasing in A, i.e. $\frac{\partial Q_S}{\partial A} \geq 0$ and $\frac{\partial Q_L}{\partial A} \geq 0$. I do not make these assumptions when I solve the model numerically later on.

2.5.1 The General Case

The optimal maturity choice is given by the first order condition with respect to M' in the continuation problem of the firm presented in equation 2.4.12. I denote as λ_D the contemporaneous shadow cost of internal funds. λ^D is positive if the firm has to issue costly equity to avoid bankruptcy.[10] Further, I denote as $\lambda_{M,0}$ and $\lambda_{M,1}$ the multipliers for the constraints $M' \geq (1-\mu)MB/B'$ and $M' \leq 1$, respectively.[11]

The first order condition for M' in the Lagrangian to problem 2.4.12 is

$$\begin{aligned}\frac{\partial V^C}{\partial M'} = &\Bigg[(Q_L - Q_S)B' + \\ &\frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1+\lambda_D) + \\ &\mathbb{E}\left[\Lambda(C,C')\frac{\partial V'}{\partial M'}|A,C\right] = \lambda_{M,1} - \lambda_{M,0}. \qquad (2.5.1)\end{aligned}$$

[10]The non-convex issuance costs for debt and equity require the introduction of this additional multiplier. With respect to equity issuance, the firm problem can be split into three sub-problems: In problem (1) the firm pays dividends. Then, $\lambda_D = 0$. In problem (2), the firm does not pay dividends, but also does not issue equity. Then, $D_i = 0$ becomes one of the equilibrium conditions of the model and λ_D is found residually from the first-order condition for investment. Finally, in problem (3), the firm issues equity. In that case, $\lambda_D = \phi_1$. The value function V^C is the envelope of these three subproblems.

[11]The lower bound on M' arises from the assumption of no debt repurchases.

Differentiating equation 2.4.10, the envelope condition yields

$$\begin{aligned}\frac{\partial V}{\partial M} = \frac{\partial D}{\partial M} &= (\mu + (1-\mu)Q_L - 1 + (1-\mu)\xi)B(1+\lambda_D)\\ &= (1-\mu)(Q_L - 1 + \xi)B(1+\lambda_D).\end{aligned} \tag{2.5.2}$$

if the firm is in a no default state. Otherwise, the $\partial V/\partial M = 0$, that is, the value of equity in default is insensitive to the state of the firm. Combining 2.5.1 and 2.5.2, we get

$$\begin{aligned}\frac{\partial V^C}{\partial M'} = &\Bigg[(Q_L - Q_S)B' + \\ &\frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}(M'B' - (1-\mu)MB)\Bigg](1+\lambda_D) + \\ &\mathbb{E}\left[\Lambda(C,C')(1-\mu)(Q_L' - 1 + \xi)B'(1+\lambda_D')\mathbb{1}_{(V'>0)}|A,C\right] = \lambda_{M,1} - \lambda_{M,0}.\end{aligned} \tag{2.5.3}$$

The interpretation of this first order condition is that the benefit and the cost of marginally increasing the share of long-term debt must be equal. This decision concerns only the *maturity structure* of debt in the next period, but not the *leverage*. The total amount of debt issuance and hence the leverage choice is given by the first-order condition for total debt, B'.

The choice of M' is a portfolio choice how to allocate borrowing among different types of debt for a given amount of total debt B'. The cost of issuing marginally more long-term debt is here the opportunity cost of issuing marginally less short-term debt.[12]

2.5.2 Optimal Maturity Choice without Consumption Risk

First, I will focus on the main trade-off between short-term debt and long-term debt in a setup without consumption risk. That is, $\lambda_1 = \lambda_2 = 0$. In this case, the discount factor $\Lambda(1,1)$ equals β and the risk-free bond prices for short-term debt and long-term debt are both equal to 1.

The first order condition contains many different terms, so I will consider three different types of firms: In the first case, I discuss the case of a firm which never defaults. This is the case of a low leverage, high productivity firm. For such a firm, neither short-term debt nor long-term debt are

[12] Of course, B' and M' are in the end jointly determined.

risky. In the second case, the firm may default only after the next period. The firm hence has no short-run default risk, but some long-run default risk. Then, short-term debt is risk-free while long-term debt is risky. In the third case, the firm also has some short-run default risk, such that both short-term debt and long-term debt are risky.

In this way, I can introduce the channels that affect maturity choice one by one. I will first focus on the main trade-off between rollover costs and default risk and then add other channels.

Case I: No Default Risk

In the case of no default risk and no consumption risk, bond prices do not include a default premium: $Q_S = Q_L = 1$. In addition, bond prices are insensitive to changes in the maturity structure of the firm: $\partial Q_S/\partial M' = \partial Q_L/\partial M' = 0$. The first order condition 2.5.3 reduces to

$$\frac{\partial V^C}{\partial M'} = \beta(1-\mu)\xi B' \mathbb{E}\left[(1+\lambda'_D)|\mathcal{Y}\right] = \lambda_{M,1} > 0.$$

This optimality condition states that the benefit of increasing the long-term debt share is that the firm has to pay less rollover costs, ξ, in the next period if it uses relatively more long-term debt. Consequently, a firm that can issue debt without risk wants to set the long-term debt share as high as possible: $M' = 1$, which implies a positive multiplier $\lambda_{M,1} > 0$.

Case II: Risk-Free Short-Term Debt, Risky Long-Term Debt

If the firm has no short-run default risk, but some long-run default risk, the short-term bond price does not include a default premium, while the long-term bond price does: $Q_S = 1, Q_L < 1$. The short-term bond price remains insensitive to the maturity structure of the firm: $\partial Q_S/\partial M' = 0$.

The first-order condition for the long-term debt share becomes[13]

$$\frac{\partial V^C}{\partial M'} = \left[\underbrace{(Q_L - 1)\, B'}_{\text{Change, Marginal Revenue}} + \underbrace{\frac{\partial Q_L}{\partial M'} (M'B' - (1-\mu)MB)}_{\text{Change, Intramarginal LT Revenue}} \right] (1+\lambda_D) +$$

$$\beta \mathbb{E} \left[(1-\mu)(\underbrace{1 - Q_L'}_{\substack{\text{Change, Endogenous} \\ \text{Rollover Cost}}} + \underbrace{\xi}_{\substack{\text{Change, Exogenous} \\ \text{Rollover Cost}}}) B'(1+\lambda_D') | A_i \right]$$

$$= 0.$$

There are four important terms: The first two terms describe the change in the revenue from issuing new bonds if the firm decides to issue long-term bonds instead of short-term bonds. These are the costs of issuing a higher share of long-term debt. The last two terms describe the change in future rollover costs if the firm issues marginally more long-term debt. These are the benefits of issuing a higher share of long-term debt. Relative to case I in Section 2.5.2, the first three terms are new, whereas the exogenous rollover cost also arises in a situation with risk-free short-term *and* long-term debt.

The first term is the *change in the marginal revenue of debt issuance*: If the firm issues marginally more debt as risky long-term debt instead of risk-free short-term debt, it has to incur a default premium, captured by the term $(Q_L - 1)$. If this default premium is high, the firm prefers to issue short-term debt by setting a low M'.

The second term captures how a marginally larger long-term debt share affects the *intramarginal revenue from long-term debt issuance*. This effect arises, since a higher share of long-term debt today adversely affects firm policies in the future: The derivative $\frac{\partial Q_L}{\partial M'}$ is given by differentiating equation 2.4.18 with respect to M':

$$\frac{\partial Q_L}{\partial M'} = \beta(1-\mu)\mathbb{E}\left[\frac{\partial Q_L'}{\partial K''}\frac{\partial K''}{\partial M'} + \frac{\partial Q_L'}{\partial B''}\frac{\partial B''}{\partial M'} + \frac{\partial Q_L'}{\partial M''}\frac{\partial M''}{\partial M'} | A_i\right].$$

Since the envelope theorem does not apply to the bond price, the effects of current choices on future choices enter the current bond price.[14] It is not

[13] From this section onward, I focus on the case of an interior solution, so $\lambda_{M,1} = \lambda_{M,0} = 0$.

[14] The reason for why the envelope theorem does not apply to the bond prices is that firms do maximize over the market value of equity, but not over the market value of

possible to find analytic expressions for $\frac{\partial K'}{\partial M}$, $\frac{\partial B'}{\partial M}$ and $\frac{\partial M'}{\partial M}$. In the numerical solution to my model, the policy function for the next period capital stock K'' is decreasing in M', while the policy function for the next period level of debt B'' is increasing in M'. The former is the debt overhang, the latter the debt dilution channel. Both of these channels are discussed in detail in Jungherr and Schott (2017). This is because the firm acts only in the interest of the shareholder and does therefore not internalize the effect of its decisions of the value of debt in default states. Since the benefits of investment and the costs of debt issuance arise in the future, the larger the share of firm value that accrues to long-term debt, the lower will be investment and the higher debt issuance. As a consequence, a higher share of long-term debt will in general increase long-run default risk, by adversely affecting future firm policies, which drives down the price of long-term debt today. In this case, $\frac{\partial Q_L}{\partial M'} < 0$.

The third term is the *endogenous rollover cost*. If the firm issues short-term debt, it has to repay the entire amount at the face value in the next period. If the firm instead issues long-term debt, it can roll over a fraction $1 - \mu$ of debt at the market value. The market value of long-term debt is below its face value because of long-run default risk. Therefore, being able to roll over long-term debt at the market value leads to lower rollover costs for the firm.

In addition, since the market value of long-term debt is low whenever cash flows to equity are low, long-term debt creates a hedging benefit to shareholders. However, this hedging benefit is quantitatively not that important since in the region of the state space where firms issue long-term debt, the equity value is almost linear, since the firm is far away from being liquidity constrained. Therefore shareholders are almost risk-neutral and do not value the hedging benefit highly.

In this case, the firm trades off roll-over costs of short-term debt against the long-term default premium and the negative incentive effect of long-term debt issuance. This is the main trade-off I consider and therefore deserves to be discussed in more detail. Consider the case of a firm with low productivity and a low capital stock. This firm issues debt due to a high value of internal funds, that is, since λ_D is high. It will have a low probability of long-term survival, and hence face a high long-term default premium on long-term debt. This is captured by the term $Q_L - 1 <$

debt.

0. Furthermore, by issuing long-term debt, such a firm would decrease the incentive for future investment, since a part of that investment would essentially be an intertemporal transfer of current shareholder funds to future bondholder funds. This is captured by the term $\frac{\partial Q_L}{\partial M'} < 0$. If these effects outweigh the rollover costs, such a liquidity-constrained firm will choose to mostly issue short-term debt.

Now consider a firm with a high capital stock and a high productivity. The motive for such a firm to issue debt is not a high value of λ_D, but the tax benefit of debt. Such a firm has a low long-term default probability, and hence $Q_L - 1$ and $\frac{\partial Q_L}{\partial M'}$ will be close to 0. Therefore, such a firm will mostly be concerned about the rollover costs of debt and will issue long-term debt, as described in case I.

In this model, firms endogenously use different types of debt for different motives: Liquidity constrained firms use short-term debt, while firms which care mostly about the tax benefit of debt use long-term debt. These two motives will later on give rise to the cyclical dynamics of debt maturity: Intuitively, the fraction of firms which issues short-term debt due to liquidity constraints increases in a recession, while the fraction of firms which issues long-term debt due to the tax benefit decreases. The motive to issue debt due to a liquidity shortfall is counter-cyclical, while the tax benefit of debt net of the default premium is pro-cyclical. As a consequence, the aggregate long-term debt share in the model will be pro-cyclical.

Case III: Risky Short-Term Debt and Long-Term Debt

If short-term debt is also risky, the first order condition for M' is

$$\begin{aligned}\frac{\partial V^C}{\partial M'} = \Bigg[& \underbrace{(Q_L - Q_S)B'}_{\text{Change, Marginal Revenue}} + \\ & \underbrace{\frac{\partial Q_S}{\partial M'}(1-M')B'}_{\text{Change, Intramarginal ST Revenue}} + \underbrace{\frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)}_{\text{Change, Intramarginal LT Revenue}}\Bigg](1+\lambda_D) + \\ & \beta\mathbb{E}\left[(1-\mu)(1-Q_L'+\xi)B'(1+\lambda_D')\mathbb{1}_{(V'>0)}|A_i\right] = 0.\end{aligned}$$

In this case, there are two new terms relative to the case in which only long-term debt is risky: First, the short-term bond price and the long-term bond price also incorporate a premium for short-run default risk. The long-term bond price can be written as

$$Q_L = Q_S + (1-\mu)\beta\mathbb{E}\left[(Q'_L - 1)\,\mathbb{1}_{(V'>0)}|A_i\right].$$

Hence, $Q_L - Q_S = (1-\mu)\beta\mathbb{E}\left[(Q'_L - 1)\,\mathbb{1}_{(A'>a^*)}|A_i\right]$. Note that lengthening the maturity structure of debt does not change the default premium that the firm has to pay for short-run default risk, since both long-term debt and short-term debt price short-run default risk. The premium long-run default risk is the only change in the marginal revenue that arises when the firm increases the long-term debt share.

Second, if short-term debt is risky, the short-term bond price is also sensitive to the maturity structure of the firm. What matters for the short-term bond price is how a change in the maturity structure of debt affects short-run default risk of the firm: The derivatives of the short-term and long-term bond prices in the case of risky short-term debt and long-term debt are given by:

$$\frac{\partial Q_S}{\partial M'} = \underbrace{[1 + c - R(K, B, a^*, 1)]}_{\text{Change in Repayment}} \underbrace{\frac{\frac{\partial V'}{\partial M'}}{\frac{\partial V'}{\partial A'}} f(a^*|A)}_{\substack{\text{Change in Next Period}\\ \text{Default Probability}}} > 0,$$

$$\begin{aligned}\frac{\partial Q_L}{\partial M'} = &\underbrace{[\mu + c + (1-\mu)Q'_L - R(K, B, a^*, 1)]}_{\text{Change in Repayment}} \underbrace{\frac{\frac{\partial V'}{\partial M'}}{\frac{\partial V'}{\partial A'}} f(a^*|A)}_{\substack{\text{Change in Next Period}\\ \text{Default Probability}}} \\ &+ \beta(1-\mu)\underbrace{\int_{a^*}^{\infty}\left(\frac{\partial Q'_L}{\partial K''}\frac{\partial K''}{\partial M'} + \frac{\partial Q'_L}{\partial B''}\frac{\partial B''}{\partial M'} + \frac{\partial Q'_L}{\partial M''}\frac{\partial M''}{\partial M'}\right) f(A'|A)dA}_{\text{Change in Long-Term Repayment and Default Probability}}.\end{aligned}$$

Interestingly, increasing the long-term debt share can have opposite effects on the bond prices: A higher long-term debt share increases the price of short-term debt, because it reduces the short-run default risk. However, a higher long-term share also reduces investment in the next period and increases debt issuance in the next period, such that long-run default risk increases. In the quantitative version of my model, the latter effect dominates for the long-term bond price, such that lengthening the maturity structure increases the short-term bond price and decreases the long-term bond price.

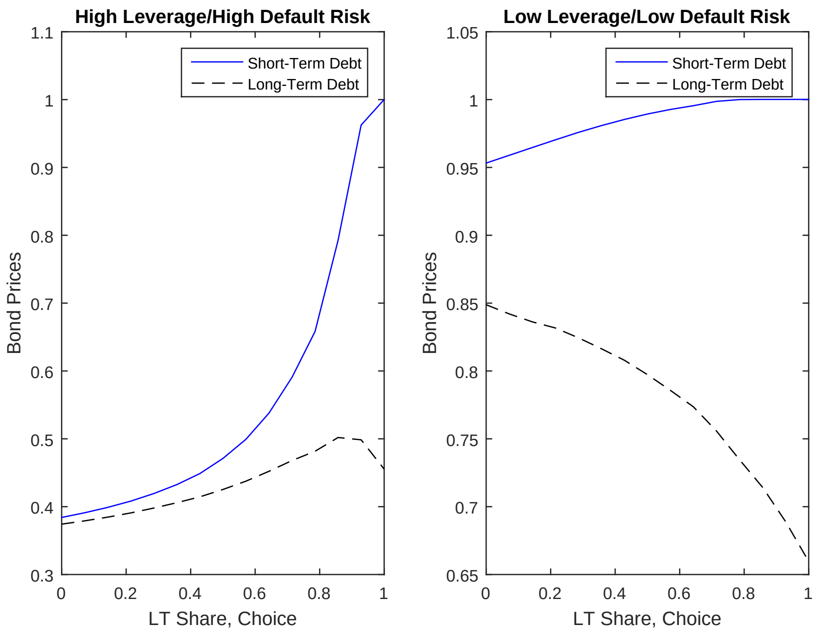

Figure 2.3: Bond prices as a function of the long-term debt share choice M'. The left panel depicts a situation with high debt and high next period default risk, the right panel depicts a situation with low debt and low next period default risk. Capital, productivity and the aggregate state are the same in the left and the right panel.

In Figure 2.3, I depict the bond price as a function of the long-term share for two different levels of debt. All other variables are chosen to be the same. The parameters are from my baseline calibration. In the left panel, the level of debt chosen is high and as a consequence, the next period default probability is high. The long-term bond price and the short-term bond price both increase for the most part in response to an increase in the long-term debt share. This is because such an increase lowers the next period default probability, which is here the dominant effect.

In the right panel, the level of debt is low and therefore the next period default probability is low. The long-term bond price decreases mostly if the long-term debt share increases. This is because a higher long-term debt share in this case leads to a higher long-term probability of default. In contrast to that, the short-term bond price increases, as in the left panel, monotonically with a higher long-term debt share.

In summary, if short-term debt and long-term debt are risky, the trade-off is fundamentally the same as in the case of risk-free short-term debt and risky long-term debt. Relative to the case with risk-free short-term debt, there is an additional benefit of issuing long-term debt, since a higher long-term debt share reduces the short-run default probability. However, in the quantitative model, firms will still prefer to issue short-term debt if they are liquidity constrained.

2.5.3 The Effect of Consumption Risk on the Optimal Maturity Structure

In the presence of consumption risk, the difference in bond prices can be decomposed into two terms:

$$Q_L - Q_S = \underbrace{Q_L^{RF} - Q_S^{RF}}_{\text{Risk-Free Term Spread}} + \underbrace{(Q_L - Q_L^{RF}) - (Q_S - Q_S^{RF})}_{\text{Long-Term Default Premium}}.$$

The new first term is the difference in risk-free bond prices. The second term is the long-term default premium. Bonds yield a fixed stream of income which is $1+c$ for short-term bonds and $\mu+c$ per period for long-term bonds. In the presence of aggregate risk, a marginal unit of consumption more in a recession is more valuable than a marginal unit of consumption in an expansion. Hence, conditional on being in a recession, the fact that consumption is mean-reverting implies that consumption in the next period will be higher than in the current period and hence that risk-free bond prices in a recession are lower than in an expansion. Further, the positive autocorrelation of the shocks implies that the risk-free long-term bond price in the recession is lower than the risk-free short-term bond price, since the short-term bond will repay more in periods closer to the present, where consumption is lower.

With this decomposition, the first order condition for the long-term debt share can be rewritten as

$$\begin{aligned}\frac{\partial V^C}{\partial M'} = \Bigg[&\left((Q_L^{RF} - Q_S^{RF}) + (Q_L - Q_L^{RF} - Q_S + Q_S^{RF})\right) B' + \\ &\frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1+\lambda_D) + \\ &\mathbb{E}\left[\Lambda(C,C')\frac{\partial V'}{\partial M'}|A_i\right] = 0.\end{aligned}$$

Consumption risk is important for two reasons. First, the risk-free short-term bond price is higher than the risk-free long-term bond price in a recession and lower in expansions, as described above. In other words, the term structure of risk-free bond *yields* is downward sloping in expansions and upward sloping in recessions. Without the shadow cost of internal funds, λ_D, this would not matter, since firms would then discount future cash-flows at the same discount factor as creditors. However, a firm that places a high value of internal funds today versus tomorrow, i.e. with $\lambda_D > \lambda'_D$, will be myopic and hence prefer short-term debt relative to long-term debt *more* in a recession.

Second, as outlined in Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010), in the presence of aggregate risk, the bond yield contains a risk-premium in addition to the risk-neutral default premium if the default probability is higher in recessions. This is because in that case, cash-flows from the firms to creditors are low exactly when creditors value cash flows highly.

In summary, consumption risk introduces a new channel for the determination of the maturity structure through the time variation in the term structure of risk-free rates and emphasizes the importance of the default channel relative to the rollover channel by increasing default premia. The fact that the short-term bond price is higher than the long-term bond price in recessions makes short-term debt even more attractive if the firm issues debt due to liquidity constraints in a recession. This channel amplifies the counter-cyclicality of short-term debt issuance. In addition, the higher and more cyclical default premia reduce the net tax benefit of long-term debt, particularly in recessions. This effect should amplify the pro-cyclicality of long-term debt issuance.

2.6 Mapping the Model to the Data

In this section, I will describe the data and the selection of parameters. I divide the set of parameters into three subsets: I take the first set from the literature. I estimate the second set of parameters directly from aggregate data. The third set of parameters is chosen to match cross-sectional data moments in simulations of the model. I solve the model using value function iteration. The interested reader will find a detailed description of the solution algorithm in Appendix 2.C.

2.6.1 Data

Cross-Sectional Data

Firm data are from the merged CRSP Compustat Quarterly North America database. I focus on US firms. The observation unit is a firm-quarter. I include data from the first quarter of 1984 to the last quarter of 2012.[15] I exclude regulated firms (SIC code 4900-4999), financial firms (SIC code 6000-6999) and non-profit firms (SIC code 9000-9999) from my sample, since the model is not appropriate for such firms. Furthermore, I exclude those observations which do not report total assets or those which report either negative assets or a negative net capital stock.

I calculate all flow variables from the cash flow statements of firms. Investment is capital expenditures minus sales of property, plant and equipment. Short-term debt issuance is defined as change in current debt. Long-term debt issuance is long-term debt issuance minus long-term debt reduction. Equity issuance is sale of common and preferred stock minus purchase of common and preferred stock minus dividends. All flows are normalized by lagged total assets. I calculate the share of long-term debt to total debt as long-term debt divided by long-term debt plus current debt. I follow Whited (1992) to calculate market leverage: It is defined as the book value of short-term debt plus the market value of long-term debt divided by the sum of the book market of short-term debt, the market value of long-term debt and the market value of equity. To calculate the market value of long-term debt, I use the method by Bernanke et al. (1988). The market value of common stock is defined as the share price times the number of shares. The market value of preferred stock is defined by the current dividend for preferred stock divided by the current federal funds rate.

The data for the default rate is taken from Ou et al. (2011). I use the default rate for the largest sample, namely all firms from 1920 to 2011, which corresponds to 1.1 percent.

Aggregate Data

I use the productivity time series from Fernald (2014) to compute productivity shocks and real personal consumption expenditure from the Bureau of

[15] Data are in principle available since the first quarter of 1976, but before 1984, the sample composition in Compustat changed markedly from quarter to quarter.

Name	Value	Role	Reference
Parameters from the Literature			
β	$1.04^{-1/4}$	Discount Factor	4% Annual Risk Free Rate
σ	2	Utility Curvature	Campbell and Cochrane (1999)
μ	0.05	Long-Term Debt Repayment Rate	5 Year Maturity
τ	0.14	Corporate Income Tax Rate	Graham (2000)
χ	0.8	Recovery Rate	Kuehn and Schmid (2014)
θ	4	Capital Adjustment Cost	Warusawitharana and Whited (2016)
α	0.35	Prod Function Curvature	Moyen (2007)
ρ_A	0.95	Persistence, Idiosyncratic Productivity	Katagiri (2014)
σ_A	0.1	Volatility, Idiosyncratic Productivity	Katagiri (2014)
Calibrated Parameters			
ξ	0.0025	Linear Debt Issuance Cost	Average Long-Term Debt Share
ϕ_0	0.1	Fixed Equity Issuance Cost	Size, Equity Issuance
ϕ_1	0.04	Linear Equity Issuance Cost	Frequency, Equity Issuance
ψ	1.8	Fixed Production Cost	Annual Default Rate
Estimated Parameters			
σ^Z	0.0071	Volatility, Aggregate Productivity	
σ^C	0.0058	Volatility, Consumption	
λ_1	0.113	Productivity Coefficient in Consumption	
λ_2	1	Consumption Coefficient	

Table 2.3: Parameter Choices. This table shows all model parameters, grouped into three categories: The first category shows parameters chosen from the literature, the second category shows parameters from the literature, the third category shows parameters taken from a production function estimation using a dynamic panel data estimator.

Economic Analysis (BEA) to compute the stochastic progress for consumption. For real GDP, I either use real GDP from the BEA or aggregate real sales from Compustat. To compute real sales, I deflate nominal sales with a four quarter moving average of the consumer price index. All series are detrended with a quadratic trend, as are the other time series I aggregate from Compustat data.

2.6.2 Parameter Choices

Parameters from the Literature

For the preferences of the representative household, I use a time preference rate, $\frac{1}{\beta} - 1$, of 4 percent per year and a utility curvature coefficient σ of 2, which is the value used in Campbell and Cochrane (1999). In the baseline calibration, I set the maturity of long-term debt to 5 years. This implies a quarterly repayment rate μ of 5 percent. Following Graham (2000), I set the corporate income tax rate τ to 14 percent. This is substantially lower

	(1) ln(Z)	(2) ln(C)
L.ln(Z)	0.9662***	-0.0263
	(34.14)	(-0.32)
ln(Z)		0.1128
		(1.44)
L.ln(C)		0.9172***
		(29.37)
Observations	115	115

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2.4: Estimation of aggregate processes.

than the true marginal US corporate income tax rate, but corresponds to about the actual average tax rate of firms in the US. I set the recovery rate in default to 0.8 as in Kuehn and Schmid (2014). I set the autocorrelation of the productivity shock to 0.95 and the standard deviation of idiosyncratic productivity to 0.1, which is similar to Katagiri (2014). I use $\alpha = 0.35$ for the production function curvature. I set the capital adjustment cost θ to 4. Warusawitharana and Whited (2016) estimate an adjustment cost between 4 and 6 for large firms in Compustat. Bloom (2009) estimates an adjustment cost of 4.8 on Compustat data in his purely quadratic adjustment cost specification.

Estimated Parameters

There are three parameters that govern aggregate uncertainty in the model: The volatility σ^Z of the aggregate productivity shock, the volatility σ^C of the consumption shock and the coefficient of productivity in consumption, λ_1. I set, without loss of generality, $\lambda_2 = 1$. The persistence of both aggregate shocks is equal to the persistence of the idiosyncratic shocks to keep the state space tractable. I estimate the followings regression on detrended productivity and consumption data:

$$\begin{aligned} \ln Z_t &= \rho^Z \ln Z_{t-1} + \eta_t^Z \\ \ln C_t &= \lambda_1 \rho^Z \ln Z_{t-1} + \lambda_1 (\ln Z_t - \rho^Z \ln Z_{t-1}) + \rho^C \ln C_{t-1} + \eta_t^C \end{aligned}$$

The results are in Table 2.4. The standard deviation of η_t^Z is 0.0071, the standard deviation of η_t^C 0.0058. Hence, I set $\lambda_1 = 0.113$, $\sigma^Z = 0.0071$, and $\sigma^C = 0.0058$. The regression further shows that setting $\rho^Z = \rho^C = 0.95$ is well within the range of plausible parameters.

Parameters Set to Match Cross-Sectional Moments

I choose the debt issuance cost parameter ξ, the equity issuance cost parameters ϕ_0 and ϕ_1 and the fixed production cost ψ to match a set of cross-sectional moments. I choose to match the average share of long-term debt, the average size and frequency of equity issuance, the cumulative one year default rate and the cross-sectional standard deviation of the investment capital ratio.

The average share of long-term debt is informative about the linear debt issuance cost ξ: the higher is the debt issuance cost ξ, the less attractive is short-term debt relative to long-term debt for the purpose of the tax benefit, and the higher is the share of long-term debt. The size and frequency of equity issuance are informative about the equity issuance costs ϕ_0 and ϕ_1: A higher ϕ_0 leads to a larger conditional size of equity issuance and a lower frequency of equity issuance. A higher ϕ_1 leads to a smaller conditional size of equity issuance, and a lower frequency of equity issuance. The default rate helps to identify the fixed cost ψ: A higher value for ψ implies a higher default rate.

Table 2.3 shows the parameters resulting from the moment matching exercise. The fixed cost ψ corresponds to 10.54% of the steady state capital stock of the model. Kuehn and Schmid (2014) use a linear production cost that corresponds to 4% of the lagged capital stock.

	Model	Data
Long-Term Debt Share, Mean	76.894	63.200
Equity Issuance, Size	7.549	12.200
Equity Issuance, Frequency	24.068	10.100
Def Rate	0.776	1.146

Table 2.5: Model fit.

Table 2.5 reports targeted moments from a numerical simulation. The model matches the average long-term debt share well. The default rate in the model is close to the default rate in the data of about 1.1 percent. The model can also match the size and frequency of equity issuance. Overall, while the match between the model and the data is by no means perfect, it delivers plausible numbers for all targeted moments.

2.6.3 Simulation Procedure

I simulate a panel of 5000 firms for 2000 quarters. I use a burn-in period of 1000 quarters. Defaulted firms are replaced with new firms which draw a new productivity from the unconditional productivity distribution. Changing these values does not affect the results. These firms start out with zero debt and a very small capital stock.

In the data, short-term debt is defined as debt with a maturity of less than 1 year. This definition includes long-term debt with a residual maturity of less than 1 year. In the model, debt with a maturity of less than 1 year is given by

$$(1 - M_{i,t})B_{i,t} + (1 - (1 - \mu)^4)M_{i,t}B_{i,t}. \tag{2.6.1}$$

Therefore, the share of long-term debt in total debt at the firm level is given by

$$\frac{B_{i,t} - (1 - M_{i,t})B_{i,t} - (1 - (1 - \mu)^4)M_{i,t}B_{i,t}}{B_{i,t}} = (1 - \mu)^4 M_{i,t}. \tag{2.6.2}$$

Market leverage at the firm level is calculated as the market value of debt divided by the market value of debt plus the ex dividend value of equity:

$$\frac{Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}}{V_{i,t} - D_{i,t} - EIC(D_{i,t}) + Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}}. \tag{2.6.3}$$

Book leverage is given by

$$\frac{B_{S,i,t+1} + B_{L,i,t+1}}{K_{i,t+1}}. \tag{2.6.4}$$

Finally, the Market-to-Book ratio is

$$\frac{V_{i,t} - D_{i,t} - EIC(D_{i,t}) + Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}}{K_{i,t+1}}. \tag{2.6.5}$$

2.7 Results

2.7.1 Aggregate Results

In this section, I report the implications of the model for the dynamics of the aggregate maturity structure. The aim of this section is to show that the model, which is parameterized to match firm-level moments, can also match aggregate dynamics. First, I will report aggregate means and correlations. Second, I consider how well the model fits the cross-sectional distributions of leverage and the debt maturity structure in the data and third how well it replicates the dynamics of the debt maturity structure across the size distribution of firms. Explaining the heterogeneity in the level and dynamics of the maturity structure for firms of different size classes is a key contribution of the paper relative to the existing literature.

Aggregate First Moments

	Data	Model
Long-Term Debt Share, Mean	0.845	0.763
Long-Term Debt Share, StDev	0.012	0.014
Book Leverage, Mean	0.433	0.525
Book Leverage, StDev	0.033	0.032
Market Leverage, Mean	0.228	0.199
Market Leverage, StDev	0.040	0.011
Investment/Capital, Mean	0.032	0.037
Investment/Capital, StDev	0.005	0.003

Table 2.6: Aggregate summary statistics

In Table 2.6, I report aggregate summary statistics. I report moments for the aggregate long-term debt share, which is the key variable the model should explain, as well as book and market leverage, which together with the long-term debt share fully characterize the capital structure of the firms in the model. Since investment is a key reason why firms want to issue debt in this model, I also report moments for aggregate investment.

The mean and standard deviation of the aggregate long-term debt share are well matched by the model. As in the data, the aggregate fraction of debt maturing within the next year is relatively low. The standard deviation of the long-term debt share in the model is similar to the data.

Overall, despite the fact that that the model only has two contracts with different debt maturity, compared to the limitless contracting options in the data, it explains the aggregate maturity structure in the data well.

Aggregate book leverage in the model is much higher than in the data, but the volatility is similar. Book leverage is of separate interest, since it is much easier to measure in the data than market leverage. This is because it does not depend on prices and hence expectations. It is however market leverage which is relevant for corporate decisions, since market leverage reflects expectations about the fraction of future cash flows accruing to the creditors of the firm. The model can also match the aggregate market leverage, but the volatility of market leverage is too low. This is because market leverage is calculated using stock prices in the data, and the model does not generate sufficient stock price volatility.

The model can also match the mean and standard deviation of the aggregate investment to capital ratio well. The mean of the investment to capital ratio is by construction equal to the deprecation rate, which corresponds surprisingly well to the average aggregate investment to capital ratio. The volatility of investment in the data is higher than in the model. Arguably, the model misses many important drivers of aggregate investment like uncertainty shocks (see for example Bloom (2009) and Bachmann and Bayer (2014)) or investment-specific technology shocks (see for example Fisher (2006) and Justiniano et al. (2011)).

Aggregate Correlations

	Data	Model
Long-Term Debt Share	0.386	0.324
Book Leverage	−0.099	−0.400
Market Leverage	−0.277	−0.551
Short-Term Debt Issuance/Capital	0.179	−0.295
Long Term Debt Issuance/Capital	0.510	0.222
Debt Issuance/Capital	0.383	−0.256
Equity Issuance/Capital	−0.390	−0.510
Investment/Capital	0.390	0.670

Table 2.7: Aggregate correlations with log(output).

In Table 2.7, I report the correlations of the aggregate time series of the model with my preferred measure of output, aggregate sales. This is the better measure of corporate cash flows than real GDP and is also strongly pro-cyclical and hence a good proxy variable for the business cycle.

The model matches the signs of the correlations of the long-term debt share, book leverage and market leverage with output. In terms of flows, it matches the signs of investment, long-term debt issuance, equity issuance and the default rate with output.

Importantly, the long-term debt share in the model is pro-cyclical. The model matches the correlation in the data quantitatively well.

Both book leverage and market leverage in the model are counter-cyclical. Market leverage has a more negative correlation with output than book leverage, because the market value of equity is more volatile than the market value of debt. This is also true in the data, although the model overstates the cyclicality of market leverage in the data. A negative correlation between market leverage and output implies that the fraction of cash flows accruing to creditors increases in a recession, which increases debt overhang problems in recessions and leads to a counter-cyclical default rate.

Short-term debt issuance is counter-cyclical in the baseline model, in contrast to the data. The main motive for short-term debt issuance are financial constraints in the form of a positive shadow cost of internal funds λ_D. As financial constraints are more severe in recessions, short-term debt issuance is higher during recessions. My measure of short-term debt issuance in the model is also not perfect, since I only observe the change in current debt, which also includes maturing long-term debt. Since I do not observe long-term debt maturing and repurchased separately, this problem cannot be easily remedied.

In line with the data, long-term debt issuance is pro-cyclical, since long-term debt is issued by unconstrained firms due to the tax benefit of debt. In a recession, default premia increase, while the tax benefit is constant. As a consequence, firms will issue less long-term debt.

Total debt issuance is counter-cyclical in the baseline model, also in contrast with the data. The reason for this is that counter-cyclical short-term debt issuance is too high relative to pro-cyclical long-term debt issuance. A model in which the maturity of long-term debt μ is also a choice variable could potentially better match both the dynamics of the long-term debt share and debt issuance.

Equity issuance is counter-cyclical in the baseline model as well as in the data. Financially constrained firms which cannot or do not want to issue debt will issue equity instead. This result is in line with the results in Jermann and Quadrini (2012), who also report counter-cyclical equity issuance. I interpret equity issuance strictly as liquidity injections due to a lack of internal funds, and measure it in the data accordingly. Other motives for equity issuance outlined in Fama and French (2005) exist, but those are not captured by this model. This is of relevance, since Covas and Haan (2011) document that other measures, which include for example stock compensation for employees or equity swaps during mergers are actually pro-cyclical.

Finally, the investment-capital ratio in the model is pro-cyclical, as it is in the data. The relatively weak pro-cyclicality is surprising. This is due to the fact that I scale investment by the last period capital stock, which is also pro-cyclical. This mutes the pro-cyclicality of the investment to capital ratio relative to the level of investment.

Overall, while the model predicts a too high counter-cyclicality of short-term debt issuance and hence total debt issuance, it replicates aggregate dynamics in the data well. In particular, it matches the main fact that the aggregate maturity structure of debt is pro-cyclical.

2.7.2 The Cross-Section over the Cycle

	Data	Model
0% to 25%	0.333	0.541
25% to 50%	0.473	0.204
50% to 75%	0.658	0.123
75% to 90%	0.519	0.227
90% to 95%	0.269	0.100
95% to 99%	−0.214	0.053
99% to 100%	−0.086	0.027

Table 2.8: Correlations between the long-term debt share and log(output) across the firm size distribution.

To investigate how firms adjust their issuance in response to macroeconomic shocks, I group the firms from the simulated panel into 7 size quantiles and compute the correlation of the long-term debt share with output within each of these size quantiles.

As in the data, the cyclicality of the long-term debt shares varies substantially by firm size. Both in the model and in the data, it is the medium sized firms for which the long-term debt share is the most sensitive to cyclical fluctuations. Small firms in the model are constrained and use predominantly short-term debt even during expansions, while medium sized firms are unconstrained during expansions, but constrained during recessions. The larger the firms, the less likely it is that they are financially constrained during a recession, and the lower is their need to issue short-term debt.

These results support the hypothesis that the main driver of short-term debt issuance and hence a short maturity structure are liquidity constraints. In addition, the model also lends support to the theory that the fraction of liquidity constrained firms increases during a recession.

Note also that for the largest firms, the maturity structure is basically acylical both in the model and in the data. The puzzling firms, from the perspective of the model, are the firms in the 95 to 99 percent size quantile for which the maturity structure of debt is *counter*-cyclical. Jungherr and Schott (2016) construct a model in which the share of long-term debt is counter-cylical due to debt dilution: As firms in their model issue more debt, they shorten the maturity structure of debt because newly issued debt constitutes a very large share of newly issued debt, which aligns incentives between firm owners and creditors. Creditors prefer short-term debt because long-term debt creates debt dilution issues. The distinction between their model and my model is that my model has a clear pecking order in which short-term and long-term debt are issued for different reasons, whereas in their model, short-term debt and long-term debt are to some extent substitutes.

One possible way to reconcile these results is if fixed costs constitutes a large share of debt issuance costs, such that it is beneficial for large firms to also issue short-term debt due to the tax benefit. However, in such a model, large firms would choose a short debt maturity structure, which runs counter to the observation that the debt maturity structure is strictly increasing in firm size in the data.

	Long-Term Debt Share		Market Leverage	
	Data	Model	Data	Model
Mean	63.210	76.896	22.862	24.472
StDev	35.914	8.186	26.292	16.830
Correlation with Size	0.414	0.635	0.068	−0.647
Correlation with Market To Book	−0.190	0.059	−0.236	−0.295
Correlation with Book Leverage	−0.030	−0.508	0.413	0.903

Table 2.9: Cross-sectional summary statistics.

2.7.3 Untargeted Cross-Sectional Moments

In Columns 1 and 2 of Table 2.9, I show moments of the cross-sectional distribution of the long-term debt share of the model. I report the mean and standard deviation, as well as correlations with firm size and the market to book ratio, which are important determinants of leverage in the corporate finance literature, as well as the correlation with book leverage. Size is an important proxy variable for financial constraints, while the market to book ratio of the firm measures its growth opportunities and is highly correlated with its productivity. Note that I only target the mean for the long-term debt share. All numbers report percentages.

The model can accurately capture the mean for the long-term share, which is a targeted moment. The standard deviation of the long-term debt share is too low relative to the data. One reason is that in the data, there is a non-negligible fraction of firms which uses exclusively short-term debt.

The model can also correctly account for the positive correlation between the share of long-term debt and firm size, which is crucial to match aggregate moments. The correlation with the market to book ratio is low, as in the data. However, the correlation with the market to book ratio is too weak in the model. The correlation with book leverage is too high. One issue is that the motive to use leverage in the model is too weak for large firms, such that small firms will simultaneously have a high book leverage and a short debt maturity structure, which leads to this strong negative correlation.

In Columns 3 and 4 of Table 2.9, I show the distribution of market leverage. The model can match the average market leverage as well as the standard deviation of the market leverage well. However, the model predicts a strong negative correlation between market leverage and firm size, while this correlation is weakly pro-cyclical in the data. The reason is that due to the fixed production cost, it is mostly small firms which issue debt because they are liquidity constrained. Firms in which growth options constitute a large fraction of the firm value use less leverage, as shown by the negative correlation between the market-to-book ratio and market leverage. As expected, the correlation between book leverage and market leverage is positive both in the model and the data.

2.7.4 Robustness

In this section, I want to further illustrate the importance of issuance costs for the main results. In addition, I investigate the role of consumption risk in the model.

	(1) Data	(2) Baseline	(3) No DIC	(4) No EIC	(5) $\sigma = 0$
Long-Term Debt Share, Mean	0.845	0.763	0.620	0.681	0.561
Long-Term Debt Share, StDev	0.012	0.014	0.058	0.050	0.009
Book Leverage, Mean	0.433	0.525	0.547	1.169	0.515
Book Leverage, StDev	0.033	0.032	0.024	0.191	0.010
Market Leverage, Mean	0.228	0.199	0.221	0.438	0.213
Market Leverage, StDev	0.040	0.011	0.013	0.068	0.004
Investment/Capital, Mean	0.032	0.037	0.037	0.037	0.037
Investment/Capital, StDev	0.005	0.003	0.003	0.004	0.001

Table 2.10: Aggregate moments, alternative models. "No DIC" refers to the model without debt issuance costs, "No EIC" to the model without equity issuance costs and $\sigma = 0$ to the model with a risk neutral representative household.

Table 2.10 shows aggregate moments for alternative model specifications. In the first alternative model in column (3), I eliminate debt issuance costs. Specifically, I set ξ to zero. In this case, the exogenous component of the debt rollover costs in the first order condition 2.5.3 is zero. Hence, this model substantially reduces rollover costs. In the absence of debt issuance costs, firms prefer to mostly issue short-term debt to attain the tax benefit of debt. Short-term debt avoids the problem of the incentive misalignment between shareholders and creditors with respect to future investment and debt issuance decisions. As a consequence, short-term debt can be issued at lower default risk premia and hence at lower costs. The model without issuance costs is at odds with the data, because it predicts a very low long-term debt share.

The second alternative model in column (4) has no equity issuance costs. I set ϕ_0 and ϕ_1 equal to zero. Without equity issuance costs, firms can issue equity at all time at the price of one. This shuts down the variation in the shadow cost of internal funds λ^D over time. As a consequence, the motive to issue short-term debt decreases in this model. Firms also issue a substantially higher amount of debt than in the baseline model: Leverage is about twice as high as in the baseline model.

For the third alternative model version in column (5), I consider the case of a risk-neutral representative household. I set the risk aversion parameter σ in this model to zero. Without risk-aversion, the household's consumption risk is irrelevant for firm decisions. In particular, there is no term structure of risk-free interest rates and default risk is priced at the risk-neutral default probability. In terms of aggregate first moments, eliminating the risk aversion of the representative household substantially reduces the long-term debt share and the volatility of all aggregate variables.

	(1) Data	(2) Baseline	(3) No DIC	(4) No EIC	(5) $\sigma = 0$
Long-Term Debt Share	0.386	0.324	0.346	0.475	0.463
Book Leverage	-0.099	-0.400	-0.531	-0.578	-0.129
Market Leverage	-0.277	-0.551	-0.635	-0.598	-0.615
Short-Term Debt Issuance/Capital	0.179	-0.295	-0.308	-0.441	-0.145
Long Term Debt Issuance/Capital	0.510	0.222	0.231	-0.123	0.708
Debt Issuance/Capital	0.383	-0.256	-0.297	-0.436	-0.039
Equity Issuance/Capital	-0.390	-0.510	-0.403	-0.248	-0.747
Investment/Capital	0.390	0.670	0.648	0.566	0.875

Table 2.11: Aggregate correlations, alternative models. "No DIC" refers to the model without debt issuance costs, "No EIC" to the model without equity issuance costs and $\sigma = 0$ to the model with a risk neutral representative household.

Table 2.11 shows aggregate correlations for alternative model specifications. The model without debt issuance costs overall matches the signs of the correlations in the data well. However, in contrast to the data, the correlation of short-term debt issuance with output and total debt issuance with output are negative. In addition, even with a small issuance cost, the correlation of leverage with output is much lower. In addition, the correlation of equity issuance with output is lower. The small debt issuance cost however does not substantially affect the correlation between output and investment.

The model without equity issuance costs predicts a much lower correlation between total debt issuance and output. In particular, even the correlation between long-term debt issuance and output becomes negative. A potential explanation is that debt dilution is much more prominent in this model, since leverage is almost twice as high. The incentive to dilute debt is stronger in recessions. The correlation between equity issuance and output is much more negative in this model. Finally, market leverage is strongly counter-cyclical and the long-term debt share in this model is very pro-cyclical. This shows that variation in the shadow cost of internal funds λ^D is an important driver of aggregate debt maturity dynamics in this model.

The model with a risk-neutral representative household generates in general much stronger correlations, since the aggregate productivity shock is now the only shock which drives aggregate dynamics. The counter-cyclicality of equity issuance increases dramatically, while the counter-cyclicality of short-term debt issuance decreases. The pro-cyclicality of long-term debt issuance increases. This shows that the term structure implied by the stochastic discount factor of the representative investor matters both for the optimal long-term leverage and the choice between equity and short-term debt when the firm is liquidity constrained.

2.8 Debt Maturity, Leverage and Investment

2.8.1 Debt Overhang or Liquidity Constraints?

I now discuss how debt overhang affects investment and debt issuance decisions in the model, and how the maturity structure of debt affects the severity of debt overhang. In the model, high leverage can lead to under-investment of a firm relative to an otherwise identical unlevered firm. I hence define under-investment I^- as the difference between the investment decision of an unlevered and a levered firm which are otherwise identical. Let $I^*(K, B, M, A, C)$ denote the optimal investment policy of the levered firm. Then, under-investment I^- can be computed as

$$I^-(K, B, M, A, C) = I^*(K, 0, M, A, C) - I^*(K, B, M, A, C). \qquad (2.8.1)$$

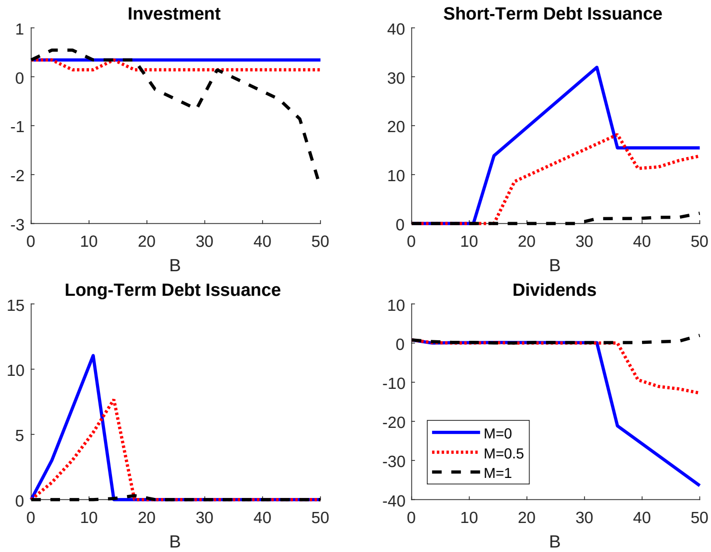

Figure 2.4: Policy functions for capital and debt as a function of debt state variable for three different values of the long-term debt share state variable. The capital stock and idiosyncratic productivity are held constant at the mean level., Aggregate consumption is set at the lower level.

Under-investment can arise for two reasons: First, a high level of debt maturing in the current period reduces the amount of internal funds available to the firm, such that the firm has to access some costly external funding. Due to the higher cost of capital compared to the case with internal funding, the firm will choose a lower level of investment. Second, if the firm has a high level of long-term debt outstanding, it will under-invest due to the debt overhang effect.

In Figure 2.4, I plot the policy functions as a function of outstanding debt for three different values of the maturity of outstanding debt. The solid line is for a firm with only short-term debt outstanding, the dotted line for a firm with an intermediate maturity structure and the dashed line for a firm with only long-term debt outstanding. The other state variables are held constant at the mean level.

For firm with only short-term debt, a higher level of debt reduces investment due to a higher cost of capital if the firm has to issue external funds. Since the firm in the figure is always constrained, the level of investment is insensitive to further increases in the level of debt. For a longer maturity structure of debt, the reduction in investment is more dramatic. The reason for this is that a higher capital stock is costly to the shareholders today. However, a large part of its future benefit accrues to the creditors instead of the shareholders, either through a lower default risk or through a higher recovery value in default. As a consequence, investment constitutes a costly transfer from shareholders to creditors. With only short-term debt, this transfer is priced into the bond price of newly issued debt. Since with short-term debt, all future outstanding debt is issued in the current period, the value of this transfer is fully internalized by the shareholder. With long-term debt, however, the effect on the value of outstanding debt carried over from the last period is not internalized by the shareholder today, and as such the firm will use a lower capital stock than an unlevered, otherwise identical firm.

2.8.2 Measuring Under-Investment

	Actual Investment Rate	Under-investment, Total	Underinvestment, Due to Lack of Internal Funds	Underinvestment, Due to Debt Overhang
0% to 25%	0.047	0.018	0.001	0.017
25% to 50%	0.037	0.006	-0.002	0.007
50% to 75%	0.037	0.007	-0.000	0.007
75% to 90%	0.038	0.001	0.000	0.000
90% to 95%	0.038	-0.001	0.000	-0.001
95% to 99%	0.038	-0.000	0.000	-0.001
99% to 100%	0.039	-0.000	0.000	-0.001
All Firms	0.040	0.008	-0.000	0.008

Table 2.12: Underinvestment, total and decomposition.

To investigate whether under-investment is quantitatively important in the model and whether this under-investment arises due to costly external funding or debt overhang, I conduct the following decomposition: Consider a firm with states (K, B, M, A, C) and another firm that has the same amount of debt coming due in the current period as the original firm, but no further debt. This latter firm is described by the states $(K, (\mu M + (1 - M))B, 0, A, C)$. The latter firm has the same net worth as the original firm, but it does not have a debt overhang problem, since it has no outstanding long-term debt that is rolled over into the next period. Then, the part of under-investment that is due to costly external funds is given by

$$I^{-,CF}(K, B, M, A, C) = I^{*}(K, 0, M, A, C) - I^{*}(K, (\mu M + (1 - M))B, 0, A, C). \tag{2.8.2}$$

The part of under-investment due to debt overhang is then given by

$$I^{-,DO}(K,B,M,A,C) = I^{-}(K,B,M,A,C) - I^{-,CF}(K,B,M,A,C). \tag{2.8.3}$$

I report the average under-investment and the decomposition for different firm size classes in Table 2.12. I normalize under-investment by the capital stock of the respective firm. Overall, the average firm under-invests about 0.8 percent relative to the capital stock compared to a firm without any debt. Most of that under-investment is driven by the debt overhang effect. In addition, smaller firms under-invest more: the smallest 25 percent of firms by size under-invest about 1.8 percent relative to the capital stock, which corresponds to an investment rate which is 38 percent too low, while the largest 25 percent of firms basically do not under-invest. Most of the under-investment is driven by the debt overhang effect, while the lack of internal funds plays a negligible role.

In summary, in contrast to many macroeconomic models with only short-term debt, debt overhang is the most important driver of under-investment in this model. This often overlooked channel can have a quantitatively large impact on investment behavior, in particular for small firms.

2.9 Conclusion

I study the determinants of aggregate and firm-level corporate debt maturity dynamics in a quantitative model with rich cross-sectional heterogeneity. In the model, firms prefer to issue long-term debt if they are financially unconstrained, because debt issuance costs imply that the tax advantage of long-term debt is much bigger than the tax advantage of short-term debt. Maturity dynamics are driven by liquidity constrained firms issuing short-term debt to cover liquidity shortfalls.

The model can match levels and dynamics of the debt maturity structure both at the firm level and in the aggregate, and is consistent with other established facts about the dynamics of corporate financing and investment decisions.

The question of regulation arises naturally, given the incentive misalignment between shareholders and long-term creditors present in the model. Can and should regulatory authorities develop rules such that the preferences of bondholders are better reflected in the decisions of firms? The results in this paper suggest that such rules can lead to higher investment rates.

Some interesting extensions of the model are left for future research. For example, I abstract from cash holdings and credit lines, which are additional sources of funds firms can use to reduce the incidence of liquidity shortfalls. Further, there are no labor market frictions in the model, which might be another important reason to issue short-term debt through a working capital requirement as in Jermann and Quadrini (2012).

2.A Other Measures of Corporate Cash Flows

In this section, I describe how using other measures of corporate cash flows affects the main empirical facts.

	(1) GDP(t-1)	(2) GDP(t)	(3) GDP(t+1)
0% to 25%	0.176	0.202*	0.195*
25% to 50%	−0.0398	0.0518	0.101
50% to 75%	0.569***	0.651***	0.707***
75% to 90%	0.127	0.233*	0.311***
90% to 95%	−0.256**	−0.205*	−0.157
95% to 99%	−0.488***	−0.444***	−0.373***
99% to 100%	−0.302**	−0.234*	−0.170
0% to 90%	0.158	0.256**	0.328***
All Firms	−0.114	0.0001	0.101
Observations	116	116	116

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2.13: Correlations of the Detrended Long-Term Debt Share with Detrended Logged real GDP, by Firm Size.

In Table 2.13, I report the correlation between the long-term debt share and real GDP. The upside of using real GDP is that it is conventionally used in the business cycle literature to compute the cyclicality of other variables. The downside is that it is only imperfectly related to corporate cash flows. This is problematic from a theoretical and empirical perspective: The mapping to the model counterpart, namely real aggregate corporate cash flow, is imperfect. For example, real GDP might include shocks to other sectors of the economy which are uncorrelated with cash flow in the corporate sector. For example, a sharp decrease in added value from the financial sector or the public sector would not be contemporaneously reflected in corporate cash flow.

Indeed, using real GDP instead of real aggregate sales changes the correlations substantially: The correlation between the long-term debt share of all firms and real GDP is zero, and the correlation between the long-term debt share and the bottom 90 percent of firms by size is only one third as high compared to the correlation when using sales.

In Table 2.14, I report the correlations of the long-term debt share with aggregate log real corporate profits. Using profits instead of sales is a

	(1) GDP(t-1)	(2) GDP(t)	(3) GDP(t+1)
0% to 25%	0.248**	0.274**	0.137
25% to 50%	0.311***	0.415***	0.344***
50% to 75%	0.589***	0.716***	0.705***
75% to 90%	0.405***	0.548***	0.559***
90% to 95%	0.231*	0.300**	0.297**
95% to 99%	−0.162	−0.0648	0.0110
99% to 100%	−0.176	−0.0592	0.0191
0% to 90%	0.530***	0.666***	0.659***
All Firms	0.322***	0.494***	0.539***
Observations	116	116	116

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2.14: Correlations of the Detrended Long-Term Debt Share with Detrended Log Real Corporate Profits, by Firm Size.

better measure of corporate cash flows if there is significant cyclicality in the operating costs of firms, such that fluctuations in revenues do not present a full picture of the funds firms have available. However, the correlations are broadly similar to those in Table 2.2. The biggest difference is that with profits as measure of corporate cash flows, the maturity structure of the largest five percent is practically acyclical.

2.B Derivation of the Derivatives in the Main Text

As in section 2.5, I assume that the bond prices are differentiable once in K', B', M' and A and value function is differentiable once in K, B, M and A. Further, I assume that the short-term and the long-term bond price are non-decreasing in A.

2.B.1 Value Function

The first order condition for the long-term debt share is given by

$$
\begin{aligned}
\frac{\partial V}{\partial M'} = & \Bigg[(Q_L - Q_S)B' + \\
& \frac{\partial Q_S}{\partial M'}(1 - M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1 - \mu)MB\right)\Bigg](1 + \lambda_D) + \\
& \mathbb{E}\left[\Lambda(C, C')\frac{\partial V'}{\partial M'}|\mathcal{Y}\right] = \lambda_{M,1} - \lambda_{M,0}
\end{aligned}
$$

The envelope condition yields

$$
\begin{aligned}
\frac{\partial V}{\partial M} &= (\mu + (1 - \mu)Q_L - 1 + (1 - \mu)\xi)B(1 + \lambda_D) \\
&= (1 - \mu)(Q_L - 1 + \xi)B(1 + \lambda_D)
\end{aligned}
$$

if the firm is in a no default state and

$$
\frac{\partial V}{\partial M} = 0
$$

if the firm is in a default state.

The derivative of the value function with respect to A is given by

$$
\begin{aligned}
\frac{\partial V}{\partial A_i} = & (1 - \tau)K^\alpha + \frac{\partial Q_S}{\partial A}(1 - M')B' + \frac{\partial Q_L}{\partial A}(M'B' - (1 - \mu)MB) + \\
& \mathbb{E}\left[\Lambda(C, C')\int_{-\infty}^{\infty}\frac{\partial V'}{\partial A_i'}\frac{\partial A_i'}{\partial A_i}f(A_i'|A_i)dA_i'|C\right]
\end{aligned}
$$

Using $\frac{\partial A_i'}{\partial A_i} = \rho\frac{A_i'}{A_i}$ yields

$$
\begin{aligned}
\frac{\partial V}{\partial A_i} = & (1 - \tau)K^\alpha + \frac{\partial Q_S}{\partial A}(1 - M')B' + \frac{\partial Q_L}{\partial A}(M'B' - (1 - \mu)MB) + \\
& \mathbb{E}\left[\Lambda(C, C')\int_{-\infty}^{\infty}\frac{\partial V'}{\partial A_i'}\rho\frac{A_i'}{A_i}f(A_i'|A_i)dA_i'|C\right]
\end{aligned}
$$

in non-default states and

$$
\frac{\partial V}{\partial A_i} = 0
$$

in default states.

Expanding the recursion, one can see that this value function derivative depends on the entire future path of the derivatives of the production function and the bond prices with respect to how the sequence of productivity changes with a current change in the productivity. The production function derivative is positive. As shown below, the short-term bond price derivative is nonnegative. It not possible to analytically determine the sign of these bond prices. If these derivatives are nonnegative, which I assume and which is the case in simulations, the derivative of the value function with respect to idiosyncratic productivity is positive, i.e. $\frac{\partial V}{\partial A_i} > 0$, if the firm is in a non-default state.

2.B.2 Short-Term Bond Price

The sign of the short-term bond price derivative depends on how the default cutoff varies with the share of long-term debt. The default cutoff function $a^*(K_i, B_i, M_i, C)$, which exists if the value function derivative with respect to idiosyncratic is positive, i.e. $\frac{\partial V}{\partial A_i} > 0$, is implicitly defined by the equation

$$V(K_i, B_i, M_i, a^*(K_i, B_i, M_i, C), C) = 0$$

Using the implicit function theorem, the derivative for a^* is given by:

$$\frac{\partial a^*}{\partial M_i} = -\frac{\frac{\partial V}{\partial M_i}}{\frac{\partial V}{\partial A_i}}$$

Since $\frac{\partial V}{\partial M_i} > 0$ and $\frac{\partial V}{\partial A_i} > 0$, $\frac{\partial a^*}{\partial M_i} < 0$, i.e. the default threshold in the next period is decreasing in M.

With this information and using Leibniz rule, the short-term bond price derivative with respect to the long-term debt share can be computed as

$$\frac{\partial Q_S}{\partial M_i'} = \mathbb{E}\left[\Lambda(C, C')\left(R(K_i', B_i', a^*, C') - (1+c)\right) f(a^*|A) \frac{\partial a^*}{\partial M_i'} | C\right]$$

Since $1 + c \geq R(K, B, a^*, C)$, i.e. the creditor cannot recover more than his claim per unit of the bond in default, and $\frac{\partial a^*}{\partial M'} \leq 0$, i.e. the default cutoff for the next period is lower if the long-term share in the next period is higher, this derivative is positive.

2.B.3 Long-Term Bond Price

For the long-term bond price, the derivative with respect to the long-term debt share is given by

$$\frac{\partial Q_L}{\partial M'} = \mathbb{E}\left[\Lambda(C,C')\left[[R(K_i',B_i',a^*,C') - (\mu + c + (1-\mu)Q_L')] f(a^*|A)\frac{\partial a^*}{\partial M_i'}\right.\right.$$
$$\left.\left. +(1-\mu)\int_{a^*}^{\infty}\left(\frac{\partial Q_L'}{\partial K''}\frac{\partial K''}{\partial M'} + \frac{\partial Q_L'}{\partial B''}\frac{\partial B''}{\partial M'} + \frac{\partial Q_L'}{\partial M''}\frac{\partial M''}{\partial M'}\right) f(A'|A)dA\right]|C\right]$$

It is not possible to determine the sign of this derivative, for two reasons: First, it is not necessarily the case that $(\mu + c + (1-\mu)Q_L') > R(K_i', B_i', a^*, C')$, since the continuation bond price $Q_L' = q_L(\mathcal{S}_i', a^*, C')$, which represents a part of the claim of the creditor to the firm, might be low.

Second, the future bond price also changes with future firm decisions, which depend on the policy for the long-term debt share today. Since the policy functions are unknown, it is not possible to determine these derivatives.

In general, the long-term bond price can therefore decrease in the long-term debt share. This can be the case in two situations: First, if defaulting would actually lead to a higher payoff for creditors. Second, if a higher long-term share increases default risk after the next period through adversely affecting the firm policies in the next period.

2.C Numerical Algorithm

My solution algorithm is a value function iteration algorithm based on Hatchondo et al. (2016). It works as follows.

1. Start with a guess for the expected value function and bond prices. The equilibrium for the infinite horizon model might not be unique. I therefore follow Hatchondo and Martinez (2009) and approximate the infinite horizon value functions by finite horizon value functions for the first period. Therefore, the initial guesses are the terminal value function and the terminal bond prices.

2. Compute the policy functions and value function. I approximate the value function between grid points using linear interpolation. For

capital and debt, I use grids with 15 points, respectively. For the share of long-term debt, I use 5 grid points. For the idiosyncratic productivity shock, I use 9 and for the aggregate productivity shock 5 grid points. I use a choice grid with 250 points for capital, 250 points for debt and 100 points for the long-term debt share.

3. Update the expected value function and bond prices. For the calculation of expectations, I approximate the expected value function $\mathbb{E}\left[V(\mathcal{S}_i', A_i, C)\right]$, $q_S(\mathcal{S}_i', A_i, C)$, and $q_L(\mathcal{S}_i', A_i, C)$ using linear interpolation on $(\mathcal{S}_i', A_i)$. I treat A_i as continuous, using Gauss-Legendre quadrature to calculate the expectation over A_i'. I compute these expectations on a Grid with 30 points for K_i', 30 for B_i' and 5 for M_i' and 9 points for A_i. I use 25 quadrature points to compute the expectations approximating the integrals piecewise in the default and no default regions, using the exact default cutoffs for A_i. I use three Gauss-Hermite quadrature nodes for ϵ^Z and ε^C. By approximating the expectations, I only have to calculate expectations only once outside the maximization step instead of many times within the maximization step.

4. Repeat until the updating errors in the expected value function and bond prices are smaller than 1e-4.

The long-term bond price function requires some smoothing to converge. I use a smoothing weight of 0.001 on the bond price and no smoothing on the value function, once the bond price error is sufficiently small.

Chapter 3

Bank Capital Regulation and Endogenous Banking Crises

with Xue Zhang

3.1 Introduction

The 2007-09 global financial crisis revealed the fragility of the financial sector as well as the weakness of the existing financial regulatory system. Stringent bank regulations have been introduced globally subsequent to the financial crisis: In 2010, Basel III was released by the Basel Committee on Banking Supervision to strengthen the regulation, supervision and risk management of the banking sector of its member countries.[1] In particular, the committee recommends to increase the minimum capital requirement during normal times to 10.5%. This includes a capital conservation buffer of 2.5%, which can be lifted during periods of financial distress.

Nonetheless, there is little consensus among policymakers and researchers regarding the optimal level of bank capital regulation. On the one hand, advocates of regulation argue that higher capital requirements make the banking system more resilient to banking panics. On the other hand, opponents of regulation weigh in that higher capital requirements reduce desirable financial intermediation. This leads to our research question: How should a regulator faced with this trade-off design the regulatory policy?

We study the macroeconomic effects of regulating the depository institutions in a general equilibrium framework with two banking sectors, namely a retail and a shadow banking sector. Taking shadow banking into consideration is crucial for two reasons. First, it has grown tremendously over the last decades into an essential part of the modern financial system.[2] Second, it was the collapse of the shadow banking sector that led to the financial turmoil which eventually turned into a global financial crisis. We define shadow banks as financial institutions that (i) are outside the regulatory framework of banks, (ii) borrow from other financial institutions using money market instruments, and (iii) are more efficient than retail banks in capital investments. Examples of shadow banks by our definition include finance companies, stand-alone broker-dealers, asset-backed security originators, and non-bank affiliated structured investment vehicles.

We embed the banking and financial crises model developed by Gertler,

[1] In 2010, the Dodd-Frank Wall Street Reform and Consumer Protection Act was signed into federal law in the United States. In 2011, the European Commission adopted a legislative package called Capital Requirements Directive (CRD) IV, which reflects the Basel II and Basel III rules on capital measurement and capital standards.

[2] According to the Global Shadow Banking Monitoring Report 2016 by the Financial Stability Board, in 2015, shadow banking accounts for 13% of the total financial system, and the shadow banking to GDP ratio is around 70%.

Kiyotaki, and Prestipino (2016) into an otherwise standard real business cycle model with endogenous capital accumulation. The Gertler, Kiyotaki, and Prestipino (2016) framework features financial crises in the form of systemic bank runs where retail banks run on shadow banks. This framework links bank capital structure to the outbreak of financial crises, therefore enabling us to analyze how regulators can enhance financial stability through bank capital regulation. Adding endogenous capital accumulation is important for evaluating the steady state effect of bank capital regulation. Since there are business cycle fluctuations in the model, we can compare it's dynamics to business cycle dynamics in the data, which allows us to compa.

Our model captures the following trade-off of bank capital requirements: On the positive side, under a higher capital requirement, retail banks can use their capital buffers to absorb the liquidated assets of shadow banks during banking crises. Therefore, the liquidation price of capital is higher, which in turn reduces the liquidation loss of the retail banks due to a run. As the likelihood of a shadow bank run is positively related to this liquidation loss, the frequency of bank runs is reduced. As a result, the financial stability of the economy is enhanced through higher capital requirements. We show that for this effect to work, it is crucial that capital regulation is dynamic: In particular, the regulator should relax capital requirements during a bank run, allowing retail banks to draw down their capital buffers. A constant capital requirement will instead *increase* the frequency of bank runs and is therefore not an effective macro-prudential policy instrument. On the negative side, a higher capital requirement pushes up the cost of financing for both retail and shadow banks, resulting in less financial intermediation, a lower aggregate capital stock, and eventually lower output of the economy.

In addition, with higher capital requirements there is a shift of retail banks away from direct lending towards lending to shadow banks on the wholesale funding markets.[3] This is because wholesale lending has a lower weight in the leverage ratio of the retail banks. Consequently, the relative share of financial intermediation conducted in the shadow banking sector

[3] Shin (2009) defines wholesale funding as *nonretail funding that does not fall under either covered bonds or securitized notes.* He discusses the case of Northern Rock, which used short and medium-term notes issued to institutional investors as wholesale funding instruments. Another example for a wholesale funding market is the tri-party repo market discussed in Martin et al. (2014).

will increase as the capital requirement on retail banks increases.

We calibrate our model to match stylized facts about the U.S. banking system and the length and frequency of financial crises in OECD countries after World War II. The calibrated model generates financial crises of plausible magnitudes and business cycle co-movements between real and financial variables similar to the U.S. data. The calibrated model suggests that bank runs are costly in welfare terms. More specifically, assuming a log utility function, households are willing to pay 0.2 percent in permanent consumption equivalent units to avoid bank runs, even though bank runs are rare events. Risk-neutral retail and shadow banks are willing to pay 1.1 and 8.5 percent in consumption equivalent units to avoid bank runs respectively.

Using the calibrated model, we investigate the effects of different capital requirements on financial stability and welfare. Imposing a bank capital requirement of 15 percent leads to a decrease in the bank run frequency from 2.7 to 0.8 runs per 100 years. However, the steady state capital stock decreases by approximately 5 percent under this capital requirement policy. Regarding welfare, households lose with a higher bank capital requirement due to the negative steady state effect, whereas retail banks and shadow banks benefit from it. If the retail and shadow banks are small in consumption terms relative to households, as they are in our calibration, higher capital requirements for depository institutions tend to reduce welfare despite effectively eliminating banking crises.

The regulator can address two inefficiencies by imposing capital requirements:

First, due to a moral hazard problem, both retail and shadow banks face endogenous leverage constraints. In particular, banks with zero net worth cannot borrow. If a drop in the capital price reduces the value of the shadow banks' assets below the value of their liabilities, implying a negative net worth, shadow banks will default and their assets will be liquidated, further depressing the capital price. A systemic shadow bank run occurs. Such bank runs happen in equilibrium and are anticipated. We focus on the case that retail banks run on shadow banks and abstract from retail bank runs.[4] The regulator can improve upon the competitive equilibrium

[4] During the 07-09 financial crisis, many shadow banks experienced a run from their creditors and experienced insolvency as a consequence, the most well-known cases being the collapse of Bear Stearns Companies and the failure of Lehman Brothers. The traditional banking sector, on the other hand, was largely shielded from the financial cata-

by imposing borrowing limits, because private agents do not internalize the effects of their leverage decisions on the likelihood of bank runs.

Second, as price-taking agents also do not internalize the effects of their decisions on asset prices, there is a pecuniary externality: Agents overborrow in expansions, tightening future leverage constraints in recessions. By tightening lending during expansions, and relaxing it during recessions, the policymaker can reduce cyclical fluctiations. This externality has been discussed for example by Lorenzoni (2008) and Dávila and Korinek (2017) and in quantitative work by Bianchi (2011).

Capital requirements also have steady state effects on output in our model. This is a key distinction from Gertler et al. (2016), where the capital stock is constant. Two assumptions are crucial for these effects. First, there are no equity markets between households and banks. Therefore, banks can only respond to a higher capital requirement by contracting the asset side of their balance sheet. This way, the steady state capital stock decreases. The second assumption is the ranking of agents' investment efficiencies. Households are the least skilled investors and shadow banks are the most skilled investors. The investment skills of agents are captured by a capital holding cost, which corresponds to screening and monitoring expenses in capital investment. Under a higher bank capital requirement, capital reallocates from retail banks to households and shadow banks. As a result, financial intermediation decreases.

3.2 Literature Review

This paper is related to three strands of literature. The first literature studies the role of financial frictions as a driving force of financial crises. Early models include Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). In these papers, it is the non-financial firms rather than the financial intermediaries which are subject to financial frictions. After the 07-09 financial crisis, the emphasis of this literature has shifted to linking financial frictions in the banking sector to the outbreak of the worst global financial crisis since the Great Depression.[5] Gertler, Kiyotaki, and Prestipino (2016) develop a canonical macroeconomic framework of financial

strophe by government interventions.

[5] E.g. Gertler and Kiyotaki, 2011; Gertler and Karadi, 2011; Brunnermeier and Sannikov, 2014; Boissay, Collard, and Smets, 2016.

crisis in the form of bank runs. They extend Gertler and Kiyotaki (2015) by including shadow banking sector, which played an important part in the onset of the 07-09 financial crisis. In this paper, we build on their framework by introducing capital accumulation and retail bank capital regulation to analyse the welfare and financial stabilization effect of bank capital regulation.

The second related literature studies micro- and macro-prudential optimal bank capital regulation in macroeconomic models which consider different trade-offs for the regulation policy.

The micro-prudential literature focuses on an agency problem at the bank level. The positive effect of bank capital regulation captured by this literature is less risk-taking by banks (see Nguyen (2014) and Begenau (2016)) and limiting moral hazard problems of banks (Van den Heuvel (2008)). Regarding the cost of bank capital regulation,Van den Heuvel (2008) argues that a tighter capital regulation reduces banks' ability to create liquidity. Similarly, Nguyen (2014) emphasizes that capital regulations lead to less bank lending, which in turn causes lower growth of the economy. Begenau (2016) proposes the opposite effect in bank lending as in Van den Heuvel (2008) and Nguyen (2014). She argues that the safe and liquid bank deposit is a desirable asset for households, and as banks face a higher capital requirement, the supply of bank debt decreases, resulting in a lower financing cost for the banks; therefore, bank lending increases as a result of bank capital regulation rather than decreases.

The macro-prudential literature instead focuses on market externalities.Dávila and Korinek (2017) provide general conditions under which price dependent borrowing constraints lead to inefficiencies and characterize the optimal policy. Our model is different, because it has two nested borrowers and we discuss regulating the intermediate borrower, whereas they are concerned about regulating the ultimate borrower. Other important papers in that literature model banking crises as occasionally binding borrowing constraints, e.g. Lorenzoni (2008), Bianchi (2011) and Bianchi and Mendoza (2015).

Our paper departs from the existing literature on optimal bank capital regulation in two ways: First, by modelling endogenous banking crises directly, we consider the impact of bank capital regulation on the stability of the financial system, rather than on the risk-taking behavior of financial institutions. Second, we explicitly model shadow banking as a part of the financial sector and study the implications of retail bank capital regulation in this setup, which has not been done in these studies. Third, relative to the existing macro-prudential literature, we consider bank runs as an additional and severe externality which can motivate regulation.

Recently, there is a growing literature that studies the role of shadow banking system in evaluating the bank regulation policy, such as Plantin (2015) and Huang (2015). The way they model shadow banking corresponds to the off-balance sheet shadow banking activities conducted by traditional banks. In contrast, we consider shadow banking to be an independent banking sector that conducts financial intermediation outside the regulatory framework of banks. Our notion of shadow banks corresponds to the external shadow banking sector in contrast to the off-balance-sheet shadow banking activities carried out by traditional banks. We consider the internal shadow banking as part of the traditional banking as bank capital regulations (such as Basel III) are, or at least supposed to be, implemented on a fully consolidated basis. In contrast to internal shadow banking, external shadow banking is a result of gains from specialization and vertical integration rather than a result of regulatory arbitrage as is the internal shadow banking (Adrian and Ashcraft (2012)).

The closest paper to ours is Begenau and Landvoigt (2017), which also studies retail bank capital requirements in an economy with an unregulated external shadow banking sector and endogenous capital accumulation. The key difference to their framework is the flow of funds in our economy. In our model, households have direct access to capital markets and there is a wholesale funding market that links the retail and the shadow banking sector. In their model, households hold both debt and equity of retail and shadow banks but have no access to the capital market. There is also no interbank market between the two banking sectors. Consequently, they find that the spillover effects of regulating retail bank capital on shadow bank decisions are small. They also model bank runs, but the probability of a bank run is determined exogenously and independently of the liquidation loss in their model.

We proceed as follows: In Section 3.3, we explain the model, which we calibrate in Section 3.4. We discuss the main mechanism in Section 3.5 and present the welfare cost of bank runs as well as the welfare effects of bank capital regulation in Section 3.6. Section 3.7 concludes.

3.3 Model

We build on the infinite horizon general equilibrium model with endogenous bank runs by Gertler, Kiyotaki, and Prestipino (2016). A key feature of this model is that the financial intermediation sector is subdivided into a retail and a shadow banking sector. In their model, aggregate capital supply is fixed and banks are unregulated. To study the welfare implications of bank capital regulation, we extend the model to include capital accumulation and retail bank capital regulation. Also, bank runs in our model are persistent and can last for potentially many periods.

We consider a closed economy populated with three types of agents: households, retail banks, and shadow banks. There is a unit mass of agents of each type. Agents differ in their utility function, their investment efficiency, and their planning horizon. In addition, retail banks and shadow banks face an endogenous leverage constraint due to a moral hazard problem. Retail banks face an additional type capital requirement. Figure 3.1 shows an overview of the flow of funds in the no-run equilibrium in the model: Households, retail banks and shadow banks hold capital, which they purchase from capital producers and lend to final goods producers. Households also lend on the deposit market to retail banks, which in turn lend on the wholesale funding market to shadow banks. During a bank run, it is the wholesale funding market that will break down, because the retail banks will not roll over their lending to the shadow banks.

3.3.1 Households

Households maximize utility from consumption. Their utility function is given by

$$\mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \ln(c_s^H) \right], \tag{3.3.1}$$

where $\mathbb{E}_t$ denotes the expectation conditional on time t information and β is the discount factor of the household. c_t^H denotes household consumption in period t. We follow the convention that lower case letters for variables denote individual variables, while upper case letters denote aggregate variables.

Households consume, invest in capital k_{t+1}^H and make deposits d_{t+1}^H at banks. They supply labor inelastically and receive W_t as labor income.

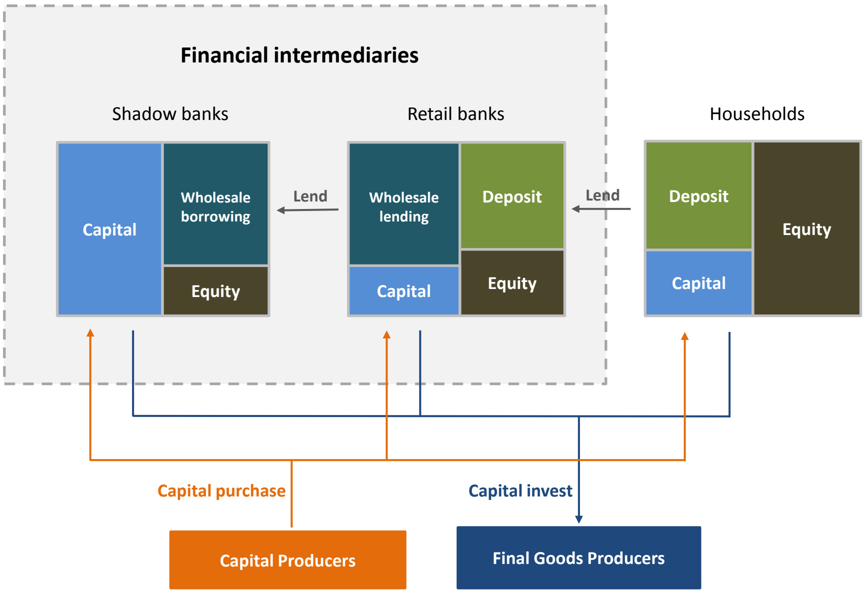

Figure 3.1: Model overview.

In addition, they own the capital producers and receive the profits Π_t^Q.[6] Deposits yield a safe gross return R_{t+1}^D in the subsequent period. Capital can be sold and purchased at price Q_t and yields an uncertain net return r_{t+1}^K in the subsequent period. It depreciates at rate δ. The remaining fraction of the capital stock in the next period is valued at the next period capital price, Q_{t+1}. The net worth of the household at the beginning of period t is given by

$$n_t^H = \left[r_t^K + (1-\delta)Q_t\right] k_t^H + R_t^D d_t^H + W_t + \Pi_t^Q. \tag{3.3.2}$$

In reality, households delegate credit supply to banks, because banks have a cost advantage at monitoring non-financial firms.[7] To capture this efficiency advantage of banks in a simplified way, we follow Gertler, Kiyotaki, and Prestipino (2016) and introduce a quadratic holding cost for new

[6]Profits are 0 in steady state, but may arise outside of the steady state due to a quadratic capital adjustment cost.

[7]Diamond (1984) develops a model in which a monitoring advantage of banks arises through diversification.

capital. This capital holding cost takes the form

$$\frac{\eta^H}{2}\left(\frac{k^H_{t+1}}{K_t}\right)^2 K_t.$$

The capital holding cost represents the cost of screening and monitoring investment projects for the investors.

Following Gertler, Kiyotaki, and Prestipino (2016), we assume that retail banks purchase capital management services from specialized companies.[8] Retail banks pay a linear fee f^R_t to these companies for each unit of capital managed. The capital management firms, in turn, incur a quadratic capital management cost. The profit of these capital management firms is given by:

$$f^R_t \tilde{K}^R_{t+1} - \frac{\eta^R}{2}\left(\frac{\tilde{K}^R_{t+1}}{K_t}\right)^2 K_t$$

where f^R_t is the capital management fee per unit of capital. These firms are owned by the households, who receive their profits. The capital management firms operate in a competitive market, which means the equilibrium fee f^R_t is taken as given by them and retail banks, and is determined in equilibrium such that capital management firms are willing to manage the capital of the retail banks, i.e. such that $\tilde{K}^R_{t+1} = K^R_{t+1}$.

This assumption is important for technical reasons. It ensures that the decision problem of the retail banks is linear in their net worth. Therefore it is sufficient to characterize the decision problem of a representative retail bank.

The optimization problem of the household can be summarized as

$$\max_{\left\{c^H_t, k^H_{t+1}, d^H_{t+1}, \tilde{K}^R_{t+1}\right\}_{t=0}^{\infty}} \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \ln\left(c^H_t\right)\right], \tag{3.3.3}$$

s.t.

$$c^H_t = n^H_t - Q_t k^H_{t+1} - d^H_{t+1} - \frac{\eta^H}{2}\left(\frac{k^H_{t+1}}{K_t}\right)^2 K_t + \left(f^R_t - \frac{\eta^R}{2}\frac{\tilde{K}^R_{t+1}}{K_t}\right)\tilde{K}^R_{t+1},$$

$$k^H_{t+1}, d^H_{t+1}, c^H_t \geq 0,$$

with n^H_t given by Equation 3.3.2.

[8] Such companies are for example appraisal management companies, which determine the value of a property, or credit bureaus, which determine the credit worthiness of a household.

3.3.2 Banks

There is a unit measure of both retail banks (R-banks) and shadow banks (S-banks) in the economy. J-banks, J $\in$ {R,S} can take deposits d_{t+1}^J from households and borrow or lend on the wholesale funding market b_{t+1}^J. In addition, they purchase capital k_{t+1}^J, which is invested into consumption goods production.[9] Gertler, Kiyotaki, and Prestipino (2016) derive conditions such that in the unregulated equilibrium, it will be optimal for R-banks to use deposits and to lend on the wholesale funding market, while S-banks will not use deposits and borrow on the wholesale funding market. These conditions are identical in our model.

Banks differ in their ability to invest in capital. In particular, retail banks pay a linear fee $f_t^R k_{t+1}^R$ for the capital management services provided by specialized capital management firms as described in the previous section. There is no such fee and no capital holding cost for the shadow banks.[10]

Banks have linear consumption utility. With probability σ^J, banks of type J receive an exit shock. In the case of such a shock, the banks liquidate their assets, consume their net worth and exit the economy. To keep the measure of banks constant over time, new banks with mass σ^J enter the economy with an exogenous endowment $\upsilon^J K_t/\sigma^J$.[11]

Both types of banks can divert a fraction of their assets after they have made their borrowing and lending decisions. How much they can divert depends both on the type of assets and the financing of the assets. Capital investment is easier to divert than wholesale lending, and assets financed through wholesale funding are harder to divert compared to those financed by deposits or equity.[12] In particular, a fraction ψ, $0 < \psi < 1$, of equity

[9]In practice, banks' lending to the non-financial sector is largely in the form of debt rather than equity. In the context of our model, banks' investment in the non-financial sector takes the form of equity investment rather than debt. This is a common assumption in the literature with financial intermediation for simplicity - otherwise another layer of liability of the non-financial sector has to be added. Under the current assumption, default on bank loans can be related to bankruptcy of the non-financial firms.

[10]Adrian and Ashcraft (2012) discuss reasons for the existence of shadow bank credit intermediation in addition to retail bank credit intermediation. They argue that securitization allowed shadow banks to reduce informational frictions in credit markets, thereby offering loans to high-risk creditors which yield a superior return.

[11]We scale the endowment of newly entering banks by the capital stock to ensure that the arguably stylized assumptions on entry do not affect the comparative statics through changes in the relative size of the endowment.

[12]Diversion entails the liquidation of the banks' assets and a subsequent default on

or deposit financed capital investment can be diverted. A smaller fraction $\gamma\psi$, $0 < \gamma < 1$, of equity or deposit financed wholesale loans is divertible. A fraction $\omega\psi$, $0 < \omega < 1$, of wholesale funding financed capital investment can be diverted. ω captures the monitoring intensity of the creditors of wholesale lending. Adrian and Ashcraft (2012) argue that due to deposit insurance, depositors have a lower incentive to monitor investments of the borrower than wholesale lenders who lend against securitized assets. For the former, the implicit government guarantee is enough to ensure depositors that their lending is risk-free, whereas for the latter, the riskiness of their lending depends on the diversification of the borrower. For $\gamma < 1$, the intuition lies in the higher standardization of wholesale lending compared to other lending activities. The collateral underlying for example a repo contract, which is a typical wholesale lending instrument, is often a high quality government bond, whose market value is easy to verify for creditors. The collateral underlying a loan can for example be real estate, for which only a rough estimate of the market value exists. Hence, the potential for diversion is much higher for loans compared to wholesale lending.

If banks divert assets, they will not repay their liabilities. Their creditors, either the households or wholesale lenders, will force the banks to exit the economy if they observe diversion. Because diversion occurs at the end of the period before next period uncertainty realizes, an incentive constraint on the banks can ensure that diversion will never occur in equilibrium. This incentive constraint states that the benefit of diversion must be smaller or equal to the continuation value of the bank.

Figure 3.2 displays the timing of intra-period decisions of banks. The intra-period problem of a J-bank consists of three stages: survival, borrowing and investment decisions and diversion. After the productivity uncertainty of the final goods producers, Z_t, has realized, banks receive the exit shock. If they exit, they consume their net worth, otherwise they make their investment and borrowing decisions. New banks enter the economy and also make these intertemporal decisions. Finally, after banks have decided how much to invest and how much to borrow, they can decide whether or not to divert their assets. If they divert, they consume the gain from diversion, default on their next period debt and exit the economy. Other-

creditors. One way to justify why equity financed assets cannot be diverted fully is that diversion creates a loss to the diverting bank which is equal to $1 - \psi$ times the diverted assets. Since banks utility is linear in consumption, such a cost may either be a pecuniary cost in the form of a penalty or a stigmatic utility cost.

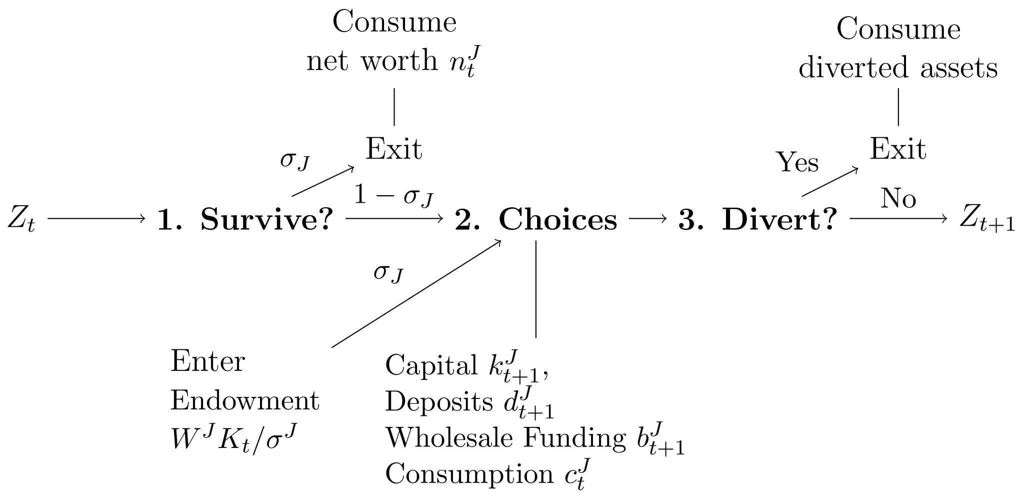

Figure 3.2: Sequence of decisions for a bank of type J.

wise, they transition to the next period and the same intra-period problem repeats.

Shadow Banks

If shadow banks do not exit, they consume c_t^S, borrow funds on the wholesale funding market b_{t+1}^S and invest in capital k_{t+1}^S. If they do exit, they consume their net worth n_t^S. The utility function of shadow banks is linear in consumption:

$$\mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \left\{ \left[\beta(1-\sigma^S)\right]^{s-t} \left[\sigma^S n_s^S + (1-\sigma^S)c_s^S\right] \right\} \right\}, \tag{3.3.4}$$

where β is the time preference rate, σ^S is the exit shock, n^S is the net worth of the shadow bank and c^S is consumption in the case that the shadow bank does not exit. The net worth of an incumbent shadow bank in period t is given by

$$n_t^S = R_t^K k_t^S - R_t^B b_t^S. \tag{3.3.5}$$

A new shadow bank is endowed with an exogenous amount of resources when entering the economy, which equals their net worth in period t:

$$\tilde{n}_t^S = \frac{\upsilon^S K_t}{\sigma^S}. \tag{3.3.6}$$

The balance sheet constraint of shadow banks requires that assets equal liabilities plus equity:

$$Q_t k_{t+1}^S = b_{t+1}^S + \underbrace{n_t^S - c_t^S}_{\text{Equity}}. \tag{3.3.7}$$

Since shadow banks borrow exclusively from retail banks and lend only to final goods producers, their payoff from diversion is given by

$$\psi\left[(n_t^S - c_t^S) + \omega b_{t+1}^S\right].$$

An incentive constraint ensures that diversion never occurs in equilibrium. It states that

$$\psi\left[(n_t^S - c_t^S) + \omega b_{t+1}^S\right] \leq \beta \mathbb{E}_t\left[V_{t+1}^S\right], \tag{3.3.8}$$

where $\mathbb{E}_t\left[V_{t+1}^S\right]$ is the continuation value of the shadow bank defined below. This constraint states that the value from continuing to operate the shadow bank must be at least as high as the value of diverting assets.

Continuing shadow banks reinvest their entire net worth, i.e. $c_t^S = 0$. This is an optimal choice, whenever

$$Q_t < \beta \mathbb{E}_t\left[\frac{V_{t+1}^S}{n_{t+1}^S} R_{t+1}^K\right].$$

This equation says that even if the shadow bank invests his entire net worth in capital, the benefit of investment still exceeds the cost of investment. We verify that this condition holds in our numerical solution.

The incentive constraint is always binding. In that case, the problem of a shadow bank reduces to

$$V_t^S = \max_{\left\{k_{s+1}^S, b_{s+1}^S\right\}_{s=t}^{\infty}} \mathbb{E}_t\left[\sum_{s=t}^{\infty}(\beta(1-\sigma^S))^{s-t}\sigma^S n_s^S\right], \tag{3.3.9}$$

s.t.

$$b_{t+1}^S = Q_t k_{t+1}^S - n_t^S, \tag{Balance Sheet Constraint}$$

$$\psi\left(n_t^S + \omega b_{t+1}^S\right) \leq \beta \mathbb{E}_t\left[\sigma^S n_{t+1}^S + (1-\sigma^S)\psi\left(n_{t+1}^S + \omega b_{t+2}^S\right)\right], \tag{IC}$$

$$k_{t+1}^S, b_{t+1}^S \geq 0,$$

with net worth given by 3.3.5 for incumbent banks and by 3.3.6 for new shadow banks. Finally, we define leverage as the market value of assets over equity, i.e.

$$\phi_t^S \equiv \frac{Q_t k_{t+1}^S}{n_t^S}.$$

Retail Banks

Retail banks consume c_t^R, invest in capital k_{t+1}^R, take deposits from households d_{t+1}^R, and lend money to shadow banks on the wholesale funding market b_{t+1}^R. Their utility function is given by

$$\mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \left\{ \left[\beta(1-\sigma^R)\right]^{s-t} \left[\sigma^R n_s^R + (1-\sigma^R) c_s^R\right] \right\} \right\}. \tag{3.3.10}$$

In period t, incumbent retail banks obtain a gross return on capital, $R_t^K k_t^R$, and a gross return from lending to shadow banks, $R_t^B b_t^R$. They return $R_t^D d_t^R$ to households for their deposits. The retail bank's net worth in period t is given by

$$n_t^R = R_t^K k_t^R + R_t^B b_t^R - R_t^D d_t^R. \tag{3.3.11}$$

The net worth of newly entering retail banks is given by

$$\tilde{n}_t^R = \frac{\upsilon^R K_t}{\sigma^R}. \tag{3.3.12}$$

The balance sheet of retail banks states that assets equal liabilities plus equity:

$$(Q_t + f_t^R) k_{t+1}^R + b_{t+1}^R = d_{t+1}^R + n_t^R - c_t^R. \tag{3.3.13}$$

We again focus on the case of zero consumption of continuing banks, which is optimal whenever

$$Q_t + f_t^R < \beta \mathbb{E}_t \left[\frac{V_{t+1}^R}{n_{t+1}^R} R_{t+1}^K \right].$$

Since retail banks lend on the wholesale funding markets and refinance themselves exclusively through deposits and equity, their payoff from diversion is

$$\psi \left[(Q_t + f_t^R) k_{t+1}^R + \gamma b_{t+1}^R \right].$$

Their incentive constraint states that

$$\psi\left[(Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R\right] \leq \beta \mathbb{E}_t\left[V_{t+1}^R\right]. \tag{3.3.14}$$

Further, define the leverage ratio of retail banks as

$$\phi_t^R \equiv \frac{(Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R}{n_t^R}. \tag{3.3.15}$$

This leverage ratio describes how retail banks can lever up their net worth by using deposits. This leverage ratio excludes a fraction $(1-\gamma)b_{t+1}^R$ of wholesale loans, because this fraction is non-divertable and can therefore be completely financed with deposits. The fraction of equity financing used by retail banks is given by $\frac{1}{\phi_t^R}$. With this formulation, the incentive constraint for retail bank pins down the leverage ratio:

$$\psi \phi_t^R n_t^R \leq \beta \mathbb{E}_t\left[V_{t+1}^R\right]. \tag{3.3.16}$$

In this sense, the incentive constraint can be interpreted as a market imposed leverage constraint.

3.3.3 Capital Regulation

The regulator can impose a capital requirement on depository institutions, which stipulates that the bank's equity cannot be less than a fraction of its assets. Importantly, we assume that whether a bank is regulated depends on whether the bank uses deposits or not and not on the type of bank per se. We make this assumption to avoid situations where households will shift deposits from regulated retail banks to unregulated shadow banks. This assumption can be justified, because the ability to issue deposits requires participation in a deposit insurance scheme, like the FDIC in the U.S., which is usually attached to stringent oversight requirements. In equilibrium, the regulated banks will be the retail banks. We assume that the regulator weighs assets in the same way as the market. That is, a fraction $1-\gamma$ of wholesale loans can be financed completely with deposits and does not count towards assets in the capital requirement. Accordingly, the capital requirement can be formulated as

$$\frac{1}{\phi_t^R} \geq \frac{1}{\bar{\phi}_t}, \tag{3.3.17}$$

where $\bar{\phi}_t$ is the maximum leverage ratio the regulator allows.

Recall that there are two classes of assets on the retail bank's balance sheet: capital holdings, $\left(Q_t + f_t^R\right) k_{t+1}^R$ and wholesale lending, b_{t+1}^R. The interpretation of 3.3.17 is that the retail bank can finance at most $1 - \frac{1}{\bar{\phi}_t}$ share of its capital holdings and at most $1 - \frac{\gamma}{\bar{\phi}_t}$ share of its wholesale lending with deposits. In other words, with $\gamma < 1$, at least a share $\frac{1}{\bar{\phi}_t}$ of capital holding, but only a share $\frac{\gamma}{\bar{\phi}_t}$ of wholesale lending has to be financed by retail banks' own equity.

The problem of the retail bank in the presence of a regulatory capital requirement can be summarized as

$$V_t^R = \max_{\left\{k_{s+1}^R, b_{s+1}^R, d_{s+1}^R\right\}_{s=t}^{\infty}} \mathbb{E}_t \left\{ \sigma^R \sum_{s=t}^{\infty} \left[\beta(1-\sigma^R)\right]^{s-t} n_s^R \right\}, \tag{3.3.18}$$

$$\begin{aligned}
&\text{s.t.} \\
&(Q_t + f_t^R)k_{t+1}^R + b_{t+1}^R = d_{t+1}^R + n_t^R, && \text{(Balance Sheet Constraint)} \\
&(Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R \le \bar{\phi}_t n_t^R, && \text{(Regulatory Capital Requirement)} \\
&\psi\left[(Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R\right] \le \\
&\beta \mathbb{E}_t \left\{\sigma^R n_{t+1}^R + (1-\sigma^R)\psi\left[(Q_{t+1} + f_{t+1}^R)k_{t+2}^R + \gamma b_{t+2}^R\right]\right\}, && \text{(IC)} \\
&k_{t+1}^R, d_{t+1}^R, b_{t+1}^R \ge 0,
\end{aligned}$$

with n_t^R given by Equation 3.3.11 for incumbent retail banks and 3.3.12 for new retail banks. We refer to the economy in which the regulatory bank capital requirement is so low that retail bank leverage is always determined by the incentive constraint of the retail banker as the baseline economy.

3.3.4 Production

Final Goods Producers

Final goods are produced using a Cobb-Douglas production technology that takes labor and capital as input:

$$Y_t = Z_t F(K_t, L_t) = Z_t K_t^{\alpha} L_t^{1-\alpha}. \tag{3.3.19}$$

The price of the final good is normalized to one. Productivity Z_t follows an AR(1) process:

$$\ln(Z_t) = (1-\rho^Z)\mu^Z + \rho^Z \ln(Z_{t-1}) + \epsilon_t, \tag{3.3.20}$$

where $|\rho^Z| < 1$ and $\epsilon_t \sim N(0, \sigma^Z)$.

The final goods producers hire labor from households and borrow capital from households, retail banks and shadow banks to produce final goods. They take the prices of the production inputs and output as given and make zero profits.[13] They maximize profits taking the aggregate wage W_t and the rental rate of capital r_t^K as given:

$$\max_{L_t, K_t} \Pi_t = Z_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t - r_t^K K_t.$$

The first order conditions of the final goods producers' problem determine the wage and rental rate of capital in equilibrium:

$$W_t = (1-\alpha) Z_t K_t^{\alpha} L_t^{-\alpha}, \tag{3.3.21}$$

$$r_t^K = \alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha}. \tag{3.3.22}$$

Capital Producers

Capital producers use a technology which transforms one unit of final goods into one unit of capital goods:

$$Y_t^K = F^K(I_t) = I_t, \tag{3.3.23}$$

where Y_t^K is the amount of capital produced in period t and I_t is the amount of consumption goods used for the production. Adjustment to the production of capital goods is costly. The capital adjustment cost takes the form

$$\frac{\theta}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t, \tag{3.3.24}$$

where δ is the depreciation rate of capital. This form of capital adjustment cost implies that whenever the investment rate differs from the depreciation rate, a positive proportional adjustment cost is incurred. Therefore, the relative price of capital goods is endogenous. Importantly, the adjustment cost is scaled by the aggregate capital stock K_t, which the capital producers take as given.

Due to the capital adjustment cost, the profit function of the capital producer is concave. Therefore the capital producers may earn a non-zero

[13]Since the final goods producers make zero profits, it does not matter who owns the them.

profit. We assume that the capital producers are owned by the households and any profits or losses are transferred to the households each period.

The capital producers' problem can be summarized as:

$$\max_{I_t} \Pi_t^Q = Q_t I_t - I_t - \frac{\theta}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{3.3.25}$$

The first order condition of the capital producer yields an expression for the capital price:

$$Q_t = 1 + \theta\left(\frac{I_t}{K_t} - \delta\right). \tag{3.3.26}$$

3.3.5 Aggregation and Equilibrium

Aggregation

Since the policy functions of an individual banker are linear in net worth, we will characterize the equilibrium in terms of a representative household and the aggregate decisions of the banking sectors. The aggregate net worth of the retail and shadow banking sector is given by the sum of the net worth of incumbent and newly entering banks:

$$\begin{aligned} N_t^R &= \left(R_t^K K_t^R + R_t^B B_t - R_t^D D_t\right)(1-\sigma^R) + \upsilon^R K_t, \\ N_t^S &= \left(R_t^K K_t^S - R_t^B B_t\right)(1-\sigma^S) + \upsilon^S K_t. \end{aligned}$$

Aggregate output is given by production net of the capital holding costs:

$$Y_t = Z_t K_t^\alpha + \upsilon^R K_t + \upsilon^S K_t - \frac{\eta^H}{2}\left(\frac{K_{t+1}^H}{K_t}\right)^2 K_t - \frac{\eta^R}{2}\left(\frac{K_{t+1}^R}{K_t}\right)^2 K_t. \tag{3.3.27}$$

We define as aggregate gross investment $\tilde{I}_t$ as the total expenditure necessary to change the capital stock from K_t to K_{t+1}. Therefore, our measure of aggregate investment includes the capital adjustment costs: Define I_t as net investment excluding capital adjustment costs, that is

$$I_t = K_{t+1} - (1-\delta)K_t.$$

Then, gross investment is given by

$$\tilde{I}_t = I_t + \frac{\theta}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{3.3.28}$$

A similar distinction between gross investment and net investment is for example used in Christiano et al. (2005).

Since there is a representative household, the individual consumption and aggregate consumption are equal, $c_t^H = C_t^H$. Household consumption can be inferred from the aggregate resource constraint:

$$C_t^H = Y_t - \tilde{I}_t - \sigma^R \frac{N_t^R - \upsilon^R K_t}{1-\sigma^R} - \sigma^S \frac{N_t^S - \upsilon^S K_t}{1-\sigma^S} \tag{3.3.29}$$

No-Run Equilibrium

In the absence of bank runs, we use a standard sequential equilibrium definition. Crucially, in the no-run equilibrium, each retail bank expects all other retail banks to roll over shadow bank debt, such that a bank run will never arise in this equilibrium. Taking the bank capital regulation policy $\bar{\phi}_t$ as given, the no-run equilibrium is a sequence of prices

$$\left\{Q_t, r_t^K, R_t^D, R_t^B, W_t, f_t^R\right\}_{t=0}^{\infty}$$

and allocations for

- households, $\left\{C_t^H, K_{t+1}^H, D_{t+1}^H\right\}_{t=0}^{\infty}$,
- retail banks, $\left\{C_t^R, K_{t+1}^R, D_{t+1}^R, B_{t+1}^R\right\}_{t=0}^{\infty}$,
- shadow banks, $\left\{C_t^S, K_{t+1}^S, B_{t+1}^S\right\}_{t=0}^{\infty}$,
- final goods producers, $\{K_t, L_t\}_{t=0}^{\infty}$, and
- capital producers, $\{I_t\}_{t=0}^{\infty}$,

that solve the respective optimization problems of all agents as defined above, clear the markets for

- capital $K_t = K_t^H + K_t^R + K_t^S$,
- labor $L_t = 1$,
- investment goods $I_t = K_{t+1}^H + K_{t+1}^R + K_{t+1}^S - (1-\delta)K_t$,

- deposits $D^R_{t+1} = D^H_{t+1}$,
- wholesale funding $B^S_{t+1} = B^R_{t+1}$,
- and capital management services $K^R_{t+1} = \tilde{K}^R_{t+1}$,

and satisfy the aggregate resource constraint 3.3.29. In a no-run equilibrium, the bank run condition 3.3.31, which is discussed in detail in the next subsection, does not hold.

Shadow Bank Run Equilibrium

As in the model of Gertler, Kiyotaki, and Prestipino (2016), retail banks can run on shadow banks. We consider only runs on the shadow banking sector as a whole. If such a run happens, the assets of the shadow banks are liquidated at the liquidation price Q^*_t. The retail banks recover the assets of the shadow banks instead of their lending. Incumbent shadow banks exit once their banks are liquidated. Define x_t as the recovery rate of retail banks:

$$x_t = \xi_t \frac{\left[r^K_t + (1-\delta)Q^*_t\right] K^S_t}{(1 + r^B_t)B_t} \tag{3.3.30}$$

where ξ_t is a liquidation value shock following an iid log normal distribution with mean 0 and variance σ^ξ. The liquidation value shock helps to quantitatively pin down the frequency of bank runs. One interpretation for this liquidation cost shock is that it represents a market illiquidity discount in the collateral market due to search frictions.[14] If the liquidity premium is high, retail banks incur an additional loss on the recovery value, because they receive illiquid capital instead of a liquid repayment of their wholesale lending.

Runs are persistent and continue into the next period with probability $1 - \pi$. New shadow banks start re-entering the economy at rate σ^S only once the run has ended.

Bank runs can be self-fulfilling. In that case, the market price of capital deteriorates in anticipation of a bank run. This weakens balance sheets of shadow banks so much that they cannot repay their liabilities. As a consequence, it is optimal for the retail banks to run on shadow banks,

[14] For a microfoundation for time-varying liquidity discounts in collateral markets, see for example He and Milbradt (2014). In their model, collateral of defaulted bonds is sold on a search market as in Duffie et al. (2005). Liquidity is determined endogenously by the default decision of the bond issuer.

because if they do not run, their claim on the shadow banks becomes worthless once the shadow banks are liquidated. However, a run will occur only if the assets of the shadow banks, valued at the liquidation price of capital, are insufficient to cover the liabilities of shadow banks, that is, if

$$x_t \leq 1. \tag{3.3.31}$$

We assume that once this condition is fulfilled, a bank run will be triggered. The liquidation condition in 3.3.31 states that, if a bank run happens, i.e. shadow banks' assets are liquidated, the retail banks suffer a loss on their wholesale lending. If all retail banks coordinate not to run, then even if the bank run condition holds, bank runs would never take place. Our assumption eliminates this possibility by implying that retail banks can never successfully coordinate. Hence in our model, bank runs do not arise randomly as a consequence of sunspot shocks, but are closely tied to the fundamentals of the shadow banking sector. Gorton (1988) presents evidence that historically, bank runs in the United States were indeed related to an increased fundamental riskiness of deposits, that is, the bank run frequency was historically higher during times when expected losses on deposits were high. Further, the large number of retail banks in an economy and the high competition in the retail banking business reduces the ability of banks to coordinate absent, for example, a credible lender of last resort (Rochet and Vives (2004)).

Since the recovery rate is strictly increasing in ξ_t, the probability of a bank run happening in t can be written as a state-contingent cutoff $\bar{\xi}_t$, which is defined by

$$\bar{\xi}_t = \frac{(1 + r_t^B)B_t}{\left[r_t^K + (1-\delta)Q_t^*\right] K_t^S}.$$

The probability of a bank run in t conditional on Z_t is given by

$$p_t \equiv F(\bar{\xi}_t).$$

This probability is endogenous. It depends, in particular, on the liquidation price of capital during a bank run: A lower liquidation price of capital makes a bank run both more severe and more likely. While our liquidation shock assumption is slightly different from the sunspot shock which determines the bank run probability in Gertler, Kiyotaki, and Prestipino (2016), it also leads to a bank run probability that is decreasing in x_t.

3.4 Calibration

We now outline our calibration strategy and provide evidence for the fit of the model by comparing untargeted moments from the model to the data. Of particular interest is the ability of the model to produce financial crises that are similar to financial crises observed in developed economies. We also check whether the model can generate realistic business cycle dynamics for real and financial variables.

3.4.1 Calibration Strategy

We calibrate the model using US data from 1990 to 2007 for the model economy. Since financial crises are relatively rare events and there are not enough observations for a single country, we use financial crises data of OECD countries after WWII for the calibration of bank run parameters. The length of one period in the model is a quarter. We divide the parameters into three groups. Parameters in the first group are taken from the literature. The second group of parameters is set to match steady state properties of the model to the data. The third group is calibrated to match dynamic properties of the model.

Parameters in Panel (a) of Table 3.1 are set following the literature. The capital share of final good production and the quarterly depreciation rate of capital are set to be 0.36 and 0.025, respectively, following Christiano et al. (2005). The banks endowments υ^R and υ^S to yield a planning horizon of shadow banks of about two years and retail banks of about five years, similar to Gertler et al. (2016).[15] These are the same targets for the banks endowment as in Gertler and Karadi (2011). We set the discount rate of households β to target an annual steady state return on deposits of 4 percent.

We set the adjustment cost parameter θ to match an elasticity of the capital price to the investment-to-capital ratio of 0.25, which is the target of Bernanke et al. (1999). This implies a parameter of $\theta = 10$. [16] There

[15] The planning horizons are simply $1/\sigma^R$ and $1/\sigma^S$.

[16] The elasticity of the capital price to investment is given by $\frac{\partial Q_t}{\partial I_t}\frac{I_t}{Q_t} = \theta\frac{1}{K_t}\frac{I_t}{1+\theta\left(\frac{I_t}{K_t}-\delta\right)}$.

(a) Parameters from the literature				
α	δ	υ^R, υ^S	β	θ
0.36	0.025	0.001	0.9902	10
Christiano et al. (2005)		Gertler et al. (2016)	$R^D = 1.04$	$\frac{\partial \ln(Q_t)}{\partial \ln(I_t)} = 0.25$

(b) Parameters set to match steady state properties				
γ	η^H	η^R	σ^R	σ^S
0.6676 $R^{K,R} - R^B = 0.4\%$	0.0286 $R^K - R^{K,R} = 1.2\%$	0.0071 $R^{K,R} - R^D = 1.2\%$	0.0521 $K^R/K = 0.4$	0.1273 $K^S/K = 0.4$
ψ	ω			
0.2154 $\phi^R = 10$	0.5130 $\phi^S = 20$			

(c) Parameters set to match dynamic properties			
ρ^Z	σ^Z	σ^ξ	π
0.9 $\rho(Y_t, Y_{t-1}) = 0.9$	0.01 $\sigma(Y_t) = 0.025$	0.0188 Crisis freq. of 0.68% per quarter	12/13 Runs last 3.25 yrs on avg

Table 3.1: Calibration.

is considerable variation in the choice of this parameter in the literature. This parameter is very important for capital price dynamics, which in turn are crucial for the possibility of bank runs, so it deserves some discussion. Christiano and Fisher (2003) estimate an elasticity of the capital price to the investment to capital ratio of 0.76, targeting asset price comovement with real GDP. Gertler et al. (2007) target an elasticity of the capital price to the investment-capital ratio of 2. In our model, these elasticities would correspond to adjustment cost parameters of 30 and 80, respectively. Such high adjustment costs would however lead to counterfactually smooth investment rates.

More recent papers in the financial accelerator literature often use investment adjustment costs instead of capital adjustment costs. Using investment adjustment costs instead of capital adjustment costs would introduce another endogenous state variable, which would complicate the global solution of our non-linear model substantially. However, since these models typically target the steady state elasticity of the price of capital with respect to investment, we can compare their choice to our target. For example, Gertler and Kiyotaki (2011) target an elasticity of investment to

Evaluated in Steady State, this expression reduces to $\theta\delta$.

the price of capital of 1.5, which would correspond to a parameter value of 60.

The business cycle literature generally uses much lower parameter values for the quadratic capital adjustment costs. This allows them to match the high volatility of investment relative to output in the data. Begenau and Landvoigt (2017) use a parameter value of 2, Christensen and Dib (2008) of 0.5. Basu and Bundick (2017) also estimate a value of around 2 for their quadratic capital adjustment cost. By using a value of 10, we use a high value relative to the business cycle literature, but a very conservative value relative to the financial accelerator literature.

Parameters in Panel (b) of Table 3.1 are set to match steady state properties of the economy. We use the same targets for these parameters as Gertler, Kiyotaki, and Prestipino (2016). To find data equivalents for the steady state values, Gertler, Kiyotaki, and Prestipino (2016) assume that the U.S. economy was in steady state in the years before the financial crisis of 2007-2009. We use leverage ratios of 10 and 20 for retail banks and shadow banks, respectively, to calibrate the diversion parameters ψ and ω. We choose the remaining diversion parameter γ to match an average annualized spread between the return on retail lending and the return on wholesale funding of 0.4 percent. We set the exit shock probabilities σ^R and σ^S such that the share of assets intermediated by retail banks and shadow banks in steady state are respectively 40 percent. These values correspond to the respective share of intermediated assets in the data between 2003 and 2007.[17] For the capital holding cost parameters η^H and η^R, we target spreads between the return on shadow bank lending and retail bank lending and between the return on retail bank lending and the deposit rate of 1.2 percent in annualized terms, respectively.

Parameters in Panel (c) of Table 3.1 are calibrated to match dynamic properties of the model. We choose ρ^Z and σ^Z to match the conditional volatility and the autocorrelation of detrended GDP for the United States. Two key parameters for the welfare cost of bank runs are the volatility of the liquidation cost shock σ^ξ and the persistence of the run π. We choose the persistence of financial crises such that the average length of a financial crisis is 3.25 years. We calibrate the volatility of the liquidation cost σ^ξ

[17] According to Gertler, Kiyotaki, and Prestipino (2016), assets intermediated by retail banks comprise equity of non-financial firms, bonds, commercial paper, household and non-financial firm loans, mortgages and consumer credit. For shadow banks, intermediated assets comprise equity of non-financial firms, mortgages and consumer credit.

to match an annual frequency of bank runs of 2.7 percent or one bank run every 36.76 years.

We use data of historical banking crises between 1970-2011 from Table A1 in Laeven and Valencia (2012), where the authors provide a comprehensive database on systemic banking crises during 1970-2011. They classify a time period as a systemic financial crisis if it exhibits *"[s]ignificant signs of financial distress in the banking system (as indicated by significant bank runs, losses in the banking system, and/or bank liquidations)."* and *"[s]ignificant banking policy intervention measures in response to significant losses in the banking system."* (Laeven and Valencia (2012), p. 228) This definition corresponds well to a crisis in our model, which is characterized by a shadow bank run, liquidation of the shadow banking sector and losses on both capital holdings and wholesale lending for the retail banking sector.

We calculate the frequency of a financial crises per country and quarter by dividing the number of financial crises which occurred during the period 1970-2011 in the OECD countries by the number of countries (i.e. 35) and the length of the period in quarters. We sum up the length of all financial crises which happened in the OECD countries during this time[18] and divide the number by the number of financial crises to get the average length of financial crises.

3.4.2 Numerical Solution and Simulation Procedure

We solve the model nonlinearly with a polynomial projection method on a sparse (Smolyak) grid, using the toolbox of Judd et al. (2014). Details of the solution algorithm are in Appendix 3.D.

The model has three shocks: The productivity shock ϵ_t, the liquidation cost shock ξ_t and the re-entry-shock π_t. We simulate $N = 5000$ economies for $T = 1000$ periods. We discard the first 200 periods to eliminate the effects of initial conditions. One issue in simulating the model is that the net worth N_t^R and N_t^S and the price of capital Q_t are simultaneously determined by a nonlinear equation. Therefore, we guess an initial path for $\left\{Q_t^d\right\}_{t=1}^T$, use this path to update the optimal policies and compute $\left\{N_t^{R,d+1}\right\}_{t=1}^T$ and $\left\{N_t^{S,d+1}\right\}_{t=1}^T$ and use these new sequences of net worth

[18] The banking crises started in 2008 in many countries do not have specific ending date in this table. In this case we set a uniform ending date of 2012.

to compute the updated sequence $\left\{Q_t^{d+1}\right\}_{t=1}^T$. We iterate on the simulation until the distance between $\left\{Q_t^d\right\}_{t=1}^T$ and $\left\{Q_t^{d+1}\right\}_{t=1}^T$ becomes small.

3.4.3 Untargeted Moments

Financial Crises

In this section, we show that the model is able to generate financial crises which are quantitatively similar to the crises we observe in the data. Schularick and Taylor (2012) use an event study approach to measure the cumulative change in real and financial aggregate variables caused by a financial crisis relative to the pre-crisis trend. Their empirical definition of a financial crisis follows Laeven and Valencia (2012). The dataset they use covers 14 economies spanning the years 1870 to 2008. For the comparison, we use their post-WW2 results, which are comparable to our calibrated model using the post-WW2 US and OECD data.

	Schularick and Taylor (2012)			Model		
	0 yr	0-1yr	0-2yr	0 yr	0-1yr	0-2yr
real GDP	-2.02%	-4.46%	-6.3%	-1.89%	-1.46%	-1.16%
real Investment	-3.45%	-12.45%	-19.9%	-8.32%	-6.15%	-4.69%
Bank Assets	-1.89%	-6.98%	-7.7%	-3.69%	-2.89%	-2.21%

Table 3.2: Untargeted financial crisis moments.

Table 3.2 reports the results. Using the same data and method as in Schularick and Taylor (2012), we calculate the average cumulative percentage change in real GDP, real investment, and bank assets after a systemic banking crisis.[19] We distinguish three time intervals: the first year, the first two years, and the first three years after the start of the crisis.

Our measure of bank assets in the model is given by $(Q_t + f_t^R)K_{t+1}^R + Q_tK_{t+1}^S$. Real GDP is Y_t as defined in equation 3.3.27 and real investment is $\tilde{I}_t$ as defined in equation 3.3.28. For instance, in the case of the first three years after the systemic crisis, given that a bank run starts in period t, we

[19] In Table 2 in Schularick and Taylor (2012), the authors report the cumulative percentage change 0-5 years after the start of the banking crises. According to our model calibration however, a systemic bank run lasts on average 3 years. Therefore we recalculate the Table 2 results for the 0-2 (instead of 0-5) year cumulative effects of banking crises using their data and method to make it comparable to the simulation results of our model. We consolidate both banking sectors for the comparison.

compute the log change in the variables of interest between period t and period $t+11$. Using the same method as Schularick and Taylor (2012) on our simulated data, we estimate the average change of the real and financial variables for each period between t and $t+11$ by the panel regression:

$$d.log(X_{i,t}) = \alpha_i + \sum_{s=0}^{11} \beta_{s+1} run_{i,t-s} + \varepsilon_{i,t}$$

where $X_{i,t} \in \left\{ Y_{i,t}, \tilde{I}_{i,t}, (Q_{i,t} + f^R_{i,t})K^R_{i,t+1} + Q_{i,t}K^S_{i,t+1} \right\}$ and $run_{i,t-s}$ is a dummy variable which takes the value of 1 if a run happens in period $t-s$. α_i is the unconditional growth rate of X_i in country i, β_{s+1} is the growth rate s periods after the start of a financial crisis. We then add up the coefficients of the dummy variables to get the cumulative effect of banking crises on the variables of interest 0-2 years after the crises happened.

As shown in Table 3.2, the immediate effect of systemic banking crises on real GDP from the model matches quite closely with that in the data. In general, our model economy reacts stronger in the first year after the crisis and recovers faster afterwards from the recession compared to the data, in which a banking crisis has a more persistent negative effect on the economy.

Business Cycle Statistics

The model should also be consistent with the business cycles in the U.S. to lend additional confidence to the ability of the model to account for fluctuations. The data are from the NIPA and the Flow of Funds between 1986Q1 and 2010Q4. For the wholesale funding rate we only have data from 2001Q1 onwards. We stop in 2010, because afterwards there has been a secular decline in wholesale funding of the shadow banking sector. We describe the data in Appendix 3.A. A notable deviation from the business cycle literature is that instead of the Hodrick-Prescott (HP) filter, we detrend the data using the routine proposed by Hamilton (2017), which avoids the spurious correlations that can arise with HP-filtered time series.

Columns (1) and (2) of Table 3.3 report unconditional standard deviations of variables relative to output for the data and the model.[20] We calibrate the model such that output volatility roughly matches that in the data. Consumption is less volatile than output both in the model and the

[20]In the first row we report the standard deviation of output instead.

	$\sigma(X)/\sigma(Y)$		$\rho(X,Y)$		$\rho(X_t, X_{t-1})$	
	(1) Data	(2) Model	(3) Data	(4) Model	(5) Data	(6) Model
Output (Y_t)	0.027	0.029	1.000	1.000	0.898	0.931
Consumption (C_t^H)	0.929	0.527	0.916	0.864	0.887	0.961
Investment ($\tilde{I}_t$)	4.368	1.632	0.943	0.966	0.886	0.905
Deposits (D_{t+1})	2.449	1.657	0.759	0.895	0.891	0.946
Wholesale Lending (B_{t+1})	10.379	9.606	0.187	0.625	0.853	0.963
Deposit Rate (R_{t+1}^D)	0.648	0.104	0.431	-0.494	0.900	0.898
Wholesale Funding Rate (R_{t+1}^B)	0.532	0.136	0.249	-0.665	0.599	0.889
Return on Equity (R_t^K/Q_{t-1})	6.385	1.089	0.379	0.184	0.829	0.445

Table 3.3: Untargeted business cycle moments.

data. Investment is more volatile. The volatility of deposits and the volatility of wholesale lending match the data quite well. In terms of interest rates, the volatility of the deposit interest rate, the wholesale funding rate and the return on equity is a bit low relative to the data, which is not surprising given that we use a simple log utility function.

Columns (3) and (4) report the contemporaneous correlations of all variables with output. As in the data, consumption, investment and deposits are strongly pro-cyclical. The most problematic statistic is the correlation between wholesale lending and output, which is positive in the model, but only weakly positive in the data. This seems puzzling, since one of the stylized facts of the financial crises is a contraction in wholesale lending. There is a clear trend break in the data around the year 2002 for wholesale lending, which is not properly picked up by either the HP-filter or the Hamilton filter. This trend break may be responsible for our counter-intuitive observation of acyclical wholesale lending. The deposit rate is fairly acyclical both in the model and in the data, and the return on equity is weakly pro-cyclical.

Finally, columns (5) and (6) show that the model can roughly match the autocorrelations in the data, with the exception of the return on capital being too weakly autocorrelated in the model.

3.5 Discussion

In this section, we discuss how a regulator can improve financial stability through retail bank capital regulation. We will also discuss the steady state effects of bank capital regulation, in which capital requirements reduce financial intermediation. This leads to the trade-off between the frequency of bank runs and financial intermediation that is central to our results. We determine the quantitative importance of this trade-off in section 3.6.

3.5.1 How Can Bank Capital Regulation Increase Welfare?

There are two key inefficiencies in the model which retail bank capital requirements can address: First, retail banks do not internalize the effect of their leverage and asset allocation decisions on the probability of bank runs. We call this the run externality. Naturally, this externality arises only in models with endogenous bank runs. Second, there is a feedback loop between the incentive constraints of the banks and the price of capital, which is also not internalized by the retail banks. This feedback loop increases the frequency and severity of both bank runs and business cycle fluctuations. We call this the capital price externality. Bank capital requirements can address these inefficiencies in two ways: First, bank capital requirements force retail banks to reduce leverage. Since they cannot issue equity to households, they will instead intermediate fewer funds to the shadow banking sector. Second, if retail banks build up higher capital buffers during normal times, they can absorb the liquidated capital of the shadow banking sector during a bank run more easily and therefore stabilize the liquidation price of capital.

The Run Externality

At first glance, it might appear to be odd that retail banks do not internalize the impact of their leverage choice on bank runs. After all it is the retail banks which initiate the run. We consider only systemic bank runs, in which the whole retail banking sector runs on the whole shadow banking sector. The probability of such a systemic bank run depends only on aggregate equilibrium prices and quantities and not on bank-specific variables. Hence, from the perspective of an individual retail bank, the probability of bank runs is exogenous. As we show below, while retail banks charge a premium on wholesale lending for the expected loss in a bank run, they do not internalize that by increasing leverage and thereby increasing lending to the shadow banking sector, they increase the probability of a bank run in the next period.

The equilibrium choices k^R_{t+1}, b^R_{t+1} and d^R_{t+1} of the retail bank are determined by either the incentive constraint 3.3.14 or the capital requirement 3.3.17, the balance sheet constraint 3.3.13 and one first order condition. Define $\Omega^R_{t+1} = V^R_{t+1}/n^R_{t+1}$. This expression corresponds to the average, and, due to the linearity of the banks problem in net worth, marginal value of an additional unit of net worth. Substituting the incentive constraint and the balance sheet constraint into equation 3.3.18 for k^R_{t+1} and d^R_{t+1} and differentiating with respect to b^R_{t+1}, we get that

$$\mathbb{E}_t \left[(1 - p_{t+1})\, \Omega^R_{t+1} \left(\frac{R^K_{t+1}}{Q_t + f^R_t} - R^D_{t+1} \right) + \int_0^{\bar{\xi}_{t+1}} \Omega^{R*}_{t+1} \left(\frac{R^{K*}_{t+1}}{Q_t + f^R_t} - R^D_{t+1} \right) dF(\xi) \right]$$
$$= \frac{1}{\gamma} \mathbb{E}_t \left[(1 - p_{t+1})\, \Omega^R_{t+1} (R^B_{t+1} - R^D_{t+1}) + \int_0^{\bar{\xi}_{t+1}} \Omega^{R*}_{t+1} (x_{t+1} R^B_{t+1} - R^D_{t+1}) dF(\xi) \right]. \tag{3.5.1}$$

We derive this condition in Appendix 3.C.2. It essentially states that retail banks must be indifferent between lending on the wholesale market and holding capital. The return on wholesale lending is lower than the return on capital holdings, because the retail bank can use more leverage to finance wholesale lending, i.e. since $\gamma < 1$. Both the ex post return on capital holdings and the ex post return on wholesale lending are lower if a run occurs in the next period: The return on capital holdings is lower, because those capital holdings are valued at the liquidation price if a bank run occurs, since

$$R^K_{t+1} = r^K_{t+1} + (1 - \delta) Q_{t+1} > r^K_{t+1} + (1 - \delta) Q^*_{t+1} = R^{K*}_{t+1}.$$

The return on wholesale lending is lower, because retail banks will not recover the full amount when a bank run occurs, since $x_{t+1} < 1$ by definition of the bank run equilibrium. Rearranging the first order condition 3.5.1 yields the following expression for the ex ante (safe) return on retail bank lending:

$$R^B_{t+1} = \gamma \mathbb{E}_t \left[(1 - p_{t+1})\, \Omega^R_{t+1} \frac{R^K_{t+1}}{Q_t + f^R_t} + \int_0^{\bar{\xi}_{t+1}} \Omega^{R*}_{t+1} \frac{R^{K*}_{t+1}}{Q_t + f^R_t} dF(\xi) \right] + (1-\gamma) R^D_{t+1} \mathbb{E}_t \left[(1 - p_{t+1})\, \Omega^R_{t+1} + \int_0^{\bar{\xi}_{t+1}} \Omega^{R*}_{t+1} dF(\xi) \right] \times \underbrace{\left(\mathbb{E}_t \left[(1 - p_{t+1})\, \Omega^R_{t+1} + \int_0^{\bar{\xi}_{t+1}} \Omega^{R*}_{t+1} x_{t+1} dF(\xi) \right] \right)^{-1}}_{\text{Liquidation Loss Premium}} \quad (3.5.2)$$

Consider first the case in which bank runs are unanticipated, i.e. $\mathbb{E}_t [p_{t+1}] = 0$. In this case, the ex-post return on wholesale lending is safe. Then, equation 3.5.2 reduces to

$$R^B_{t+1} = \gamma \frac{\mathbb{E}_t \left[\Omega^R_{t+1} \frac{R^K_{t+1}}{Q_t + f^R_t} \right]}{\mathbb{E}_t \left[\Omega^R_{t+1} \right]} + (1-\gamma) R^D_{t+1}.$$

The fraction γ of wholesale lending is financed in the same way as capital holdings and must yield a return equal to the expected return on capital. The fraction $1-\gamma$ of wholesale lending is non-divertable and can therefore be fully financed with deposits. Since retail banks are competitive, this fraction must therefore yield a return which corresponds to the return on deposits. The full equation 3.5.2 essentially includes an additional premium for the liquidation loss in the denominator plus an adjustment to the marginal value of an additional unit of equity, Ω^R_{t+1}, which varies between bank run and no-run states. Importantly, however, retail banks do not incorporate how their decisions change the probability of a bank run, that is, in their eyes $\mathbb{E}_t \left[\partial p_{t+1} / \partial \phi^R_t \right] = 0$. Therefore, when leveraging up and expanding their balance sheet, retail banks invest too much in wholesale lending from a social welfare perspective.

Effects of a Static Capital Requirement on the Bank Run Probability

Figure 3.3 shows that a constant retail bank capital requirement can substantially increase the probability of a bank run in the next period. It is therefore not an appropriate policy to address the bank run externality. In this figure, we plot the next period bank run probability $\mathbb{E}_t[p_{t+1}]$ and

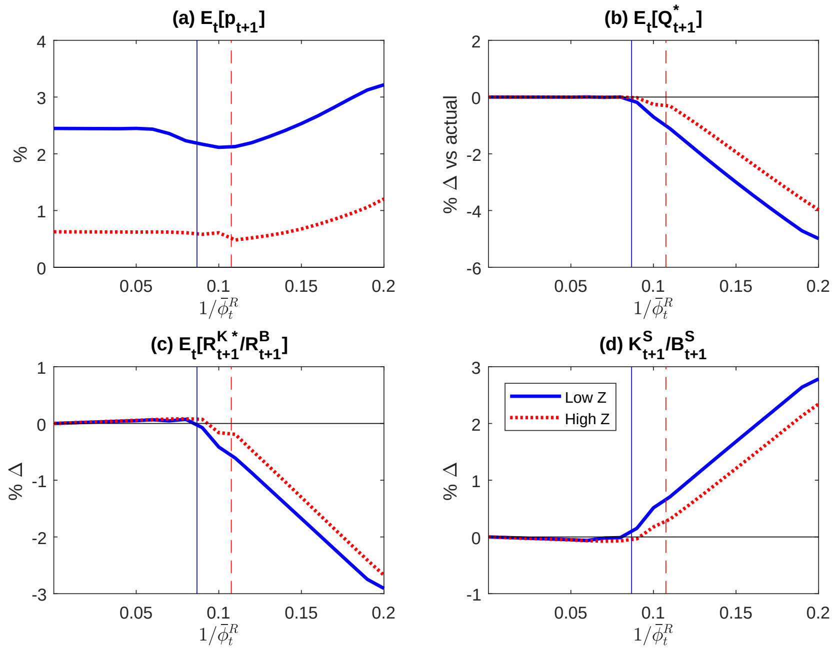

Figure 3.3: General equilibrium effects of a constant retail bank capital requirement on the bank run probability.

the two components of the bank run condition 3.3.30 as a function of $\bar{\phi}^R_t$. We set K^H_t, K^R_t, K^S_t, B_t, D_t, R^D_t and R^B_t at the steady state level and report results for two values of Z_t, one unconditional standard deviation above and below the unconditional mean, respectively. We fix a level of $\bar{\phi}^R_t$, recompute the general equilibrium given this fixed level of $\bar{\phi}^R_t$ and then compute the statistics of interest.

First, in Panel (a), we report the bank run probability as a function of $\bar{\phi}^R_t$. The solid line depicts the bank run probability if productivity is one standard deviation below the mean, the dotted line if productivity is one standard deviation above the mean. We mark the policies the retail banks choose in the absence of regulation by the thin vertical solid and dashed lines, respectively. Note that the market-imposed leverage ratio of retail banks ϕ^R_t is *counter*-cyclical, because during an expansion, retail banks will increase wholesale lending relative to capital holdings. Since wholesale lending enters leverage only with weight γ, this shift in the composition of assets reduces the leverage of retail banks.

As we can see in Panel (a) of Figure 3.3, imposing a higher capital requirement can increase the future probability of a bank run. By increasing the capital requirement from 0 to about 20 percent, the probability of a bank run increases from 2.5 percent to around 3 percent per quarter in a recession, and from 0.6 percent to about 1.1 percent in an expansion. For low values of $1/\bar{\phi}_t^R$, there is no effect, since the capital requirement is never binding. There is a small effect even before the capital requirement starts binding in the current period, since it will already bind in some states of the world in the next period.

To see through which channels the capital requirements impact the run probability, we decompose the bank run condition given by equation 3.3.30 into two components. The first component, R_{t+1}^{K*}/R_{t+1}^{B}, is the spread between asset and liability returns for shadow banks during a bank run. We show in Panel (b) that increasing the capital requirement reduces this spread, meaning that it is less likely that shadow banks can repay their liabilities. This effect is mostly due to a lower liquidation price of capital. The second component, K_{t+1}^{S}/B_{t+1}, is the assets-to-debt ratio of the shadow banks. This ratio is inversely related to the tightness of the incentive constraint of shadow banks. The higher the leverage of shadow banks, the is lower the ratio of assets to liabilities. A higher leverage ratio of retail banks hence tightens the incentive constraint of shadow banks. We can see in Panel (c) that the reduction in the first component less than one for one offset by an increase in the second component. Because of this under-adjustment, the probability of a bank run increases as the capital ratio of retail banks decreases. The reason for the strong increase in the bank run probability is that the regulator imposes a *less* counter-cyclical capital requirement than the market. This forces retail banks to delever *more* during a bank run compared to the case of no regulation, which in turn lowers the future liquidation price of capital and hence increases the bank run probability. Therefore, the regulator should optimally relax the capital requirement as much as possible ex post during a bank run to reduce the probability of a bank run ex ante.

Whether the bank run probability increases or decreases depends theoretically on whether K_{t+1}^{S}/B_{t+1}^{S} increases more or less than R_{t+1}^{K*}/R_{t+1}^{B} increases. This in turn depends on how much the incentive constraint of the shadow bank tightens in response to an reduction in the future net worth of the shadow bank. However note that the capital to debt ratio is determined one period in advance by the incentive constraint. Hence, it takes the expectation of creditors over no-run and run states into account, while the profitability during a run is determined in the period of the run.

Therefore, there will likely be an under-adjustment of the capital to debt ratio ex ante such that the profitability effect dominates ex post.

Effects of a Dynamic Capital Requirement on the Bank Run Probability

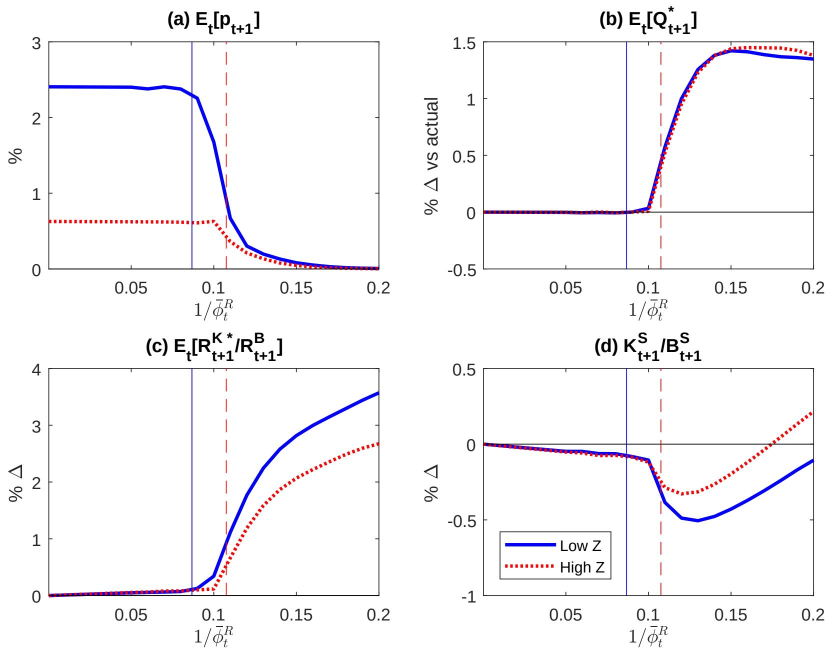

Figure 3.4: General equilibrium effects of a run-contingent retail bank capital requirement on the bank run probability.

We show the effects of a policy that imposes a capital requirement only during the no-run equilibrium in Figure 3.4. This policy is successful in eliminating bank runs: Panel (a) shows that imposing a capital requirement of 20 percent can eliminate bank runs both in recession and in expansion states. Panel (b) shows that the lower bank run probability is due to a higher spread between the returns on assets and liabilities of shadow banks during a bank run. This effect is primarily driven by a higher liquidation price of capital: As the capital requirement is relaxed during a run, retail banks can increase leverage and will hence expand their capital holdings, the only asset they have access to in the case of a bank run. This higher investment demand will increase the liquidation price of capital. From Panel (c), we can see that this policy increases shadow bank leverage initially, because a lower bank run probability relaxes the incentive constraint of shadow banks by raising their continuation value, allowing them to use more leverage and hence a lower ratio of assets to liabilities. However, leverage also decreases for capital requirements higher than 12 percent. This reversal occurs because for a small enough bank run probability, a reduction in the bank run probability only relaxes the incentive constraint little. In addition, a higher capital requirement still tightens the incentive constraint of shadow banks during normal times.

In summary, agents fail to internalize the effects of their decision on the probability of a bank run. In particular, the probability of a bank run is very sensitive to the capital ratio of retail banks, especially if the economy is in a recession. Therefore, retail bank capital requirements can be an effective way of reducing shadow bank runs, but only if the regulator relaxes them during a bank run.

The Capital Price Externality

Due to the incentive constraints 3.3.8 and 3.3.14, the extent to which banks can leverage their equity depends on the aggregate capital price. A low capital price lowers the maximum leverage ratio of shadow banks, since the value of diverting assets today decreases less than the continuation value. Therefore, banks have to deleverage, which lowers their desired level of investment. This lower level of investment in turn leads to a lower capital price. The model therefore exhibits a feedback loop, the classical financial accelerator effect, which operates through the endogenous capital price and the incentive constraints.[21]

An inefficiency arises, because banks do not internalize the effects of their current borrowing decisions on the future aggregate price of capital. By borrowing less during times of high capital prices, banks could reduce the co-movement between the tightness of the incentive constraint and the price of capital and thereby reduce the strength of this feedback loop.[22] This feedback loop is especially prominent in the economy with bank runs, because bank runs are more likely to occur and more severe if the capital price is more volatile.

A regulatory policy that is designed to restrict deposit lending during

[21] The financial accelerator effect was first introduced in the business cycle literature by Bernanke et al. (1999).

[22] This mechanism is well known in the literature, see for example Lorenzoni (2008) for theoretical work and Bianchi (2011) for numerical work that studies this pecuniary externality.

good economic conditions and ease deposit lending during bad economic conditions can increase welfare by mitigating the feedback loop and reducing the frequency and severity of bank runs. In contrast, a policy that restricts lending in all states of the world equally may actually reduce welfare, because such a policy acts as a tighter borrowing constraint and makes the feedback loop more severe.

To see this, compare the incentive constraint 3.3.14 of the retail banks to the case of a constant capital requirement 3.3.17. We get:

$$\psi\left[(Q_t+f_t^R)k_{t+1}^R+\gamma b_{t+1}^R\right]\leq\beta\mathbb{E}_t\left[V_{t+1}^R\right]$$
$$\psi\left[(Q_t+f_t^R)k_{t+1}^R+\gamma b_{t+1}^R\right]\leq\bar{\phi}n_t^R$$

While the left hand sides are identical, the right hand sides differ. The incentive constraint has the continuation value of the retail bank on the right-hand side, while the capital requirement has the current net worth times some constant on the right-hand side. In particular, rewriting the incentive constraint slightly, we get

$$\psi((Q_t+f_t^R)k_{t+1}^R+\gamma b_{t+1}^R)\leq\beta\mathbb{E}_t\left[\Omega_{t+1}^R\frac{n_{t+1}^R}{n_t^R}\right]n_t^R.$$

Using that both the marginal value of net worth Ω_{t+1}^R and the net worth growth rate n_{t+1}^R/n_t^R are independent from n_t^R, which we show in Appendix 3.C.2, we can state that the derivative of the right hand side of the incentive constraint with respect to n_t^R is given by $\beta\mathbb{E}_t\left[\Omega_{t+1}^R\frac{n_{t+1}^R}{n_t^R}\right]$, which in our calibration is *counter*-cyclical. The derivative of the right hand side of the capital requirement with respect to n_t^R is $\bar{\phi}$, which is constant. Hence, the market imposed leverage partially offsets fluctuations in net worth of the retail bank, which reduces the pro-cyclicality of the retail bank balance sheet. The constant regulatory capital requirement does not do this.

Effects of a Static Capital Requirement on the Capital Price

In Figure 3.5, we illustrate that a higher constant retail bank capital requirement $1/\bar{\phi}_t^R$ reduces both the expected future capital price and the expected future liquidation price of capital. We follow the same procedure to compute these statistics as in Figure 3.3. The expected future capital price $\mathbb{E}_t\left[Q_{t+1}\right]$ in Panel (a) incorporates the probability that a bank run may occur in the next period.

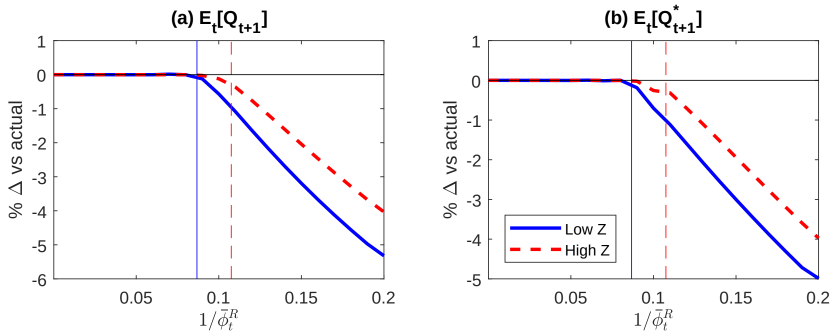

Figure 3.5: General equilibrium effects of a constant capital requirement on the future capital price.

The thick, solid line in the left panel of Figure 3.5 is the expected future capital price as a function of $\bar{\phi}_t^R$ if productivity Z_t today is one unconditional standard deviation below the mean. The thin, solid line denotes the level of ϕ_t^R that retail banks actually choose in equilibrium. We report the expected capital price relative to the expected capital price under the actually chosen policy, which in a recession implies a capital ratio of around 9 percent. Imposing a capital requirement $1/\bar{\phi}_t^R$ of 20 percent decreases the future capital price by more than 5 percent. The unconditional quarterly standard deviation of the capital price in the simulated model is about 0.6 percent, so this is a large change. The reason for this effect is that the regulator tightens the financial constraint of the retail banking sector, which forces them to deleverage, reducing both lending to final goods producers and shadow banks. Retail banks and shadow banks are forced to contract their balance sheets, which reduces investment and the price of capital.

The thick, dashed line in the left panel of Figure 3.5 is the same function, except when productivity is one standard deviation above the mean. In this case, retail banks choose a higher capital ratio, which is shown by the thin, dashed line. Increasing the capital ratio has a weaker effect on the future price of capital. Imposing a capital requirement of 20 percent reduces the capital price by only 4 percent. Looking at the right panel, we see that imposing a constant capital requirement has a similar effect on the future liquidation price of capital.

Effects of a Dynamic Capital Requirement on the Capital Price

In Figure 3.6, we show the effects of a capital requirement that is imposed during the no-run equilibrium, but relaxed to zero during a bank run. This

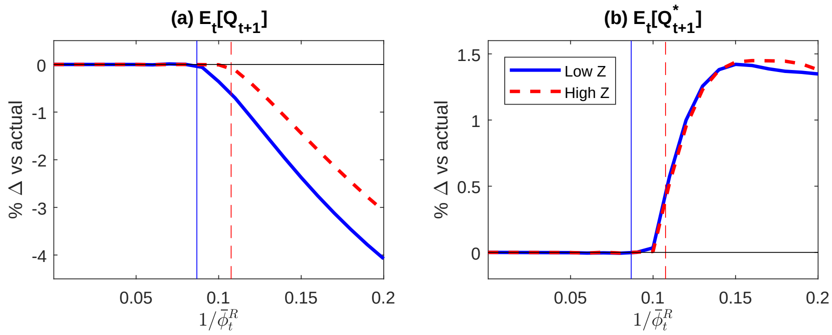

Figure 3.6: General equilibrium effects of a run-contingent retail bank capital requirement on the future capital price.

policy has a weaker negative effect on the expected future capital price, because it has a strong positive effect on the future liquidation price of capital. In addition, this policy reduces the frequency of bank runs, which in turn lowers the probability that assets are valued at the liquidation price of capital, increasing the capital price even further. Contrasting Panels (a) of Figures 3.5 and 3.6, we can see that by setting a capital requirement of 20 percent that is relaxed during a bank run, the regulator still reduces the capital price, but by only 4 instead of more than 5 percent if he does not relax the capital requirement during a run. Panel (b) shows that a higher capital requirement now strongly increases the liquidation price of capital, both in recessions and expansions: Imposing a capital requirement of 20 percent increases the liquidation price of capital by 1.5 percent relative to the model without regulation. This is because by restricting lending today, the regulator increases the capital buffer of retail banks during a bank run, which increases their ability to increase leverage and therefore their capital holdings during a bank run. As a consequence, the liquidation price is higher compared to the baseline model.

In summary, the retail bank capital ratio can affect future capital prices substantially, so a policy aimed at influencing this retail bank capital ratio can mitigate the financial accelerator effect. In particular, the regulator should tighten the capital requirement during expansions and relax it during recessions and bank runs to stabilize the capital price and reduce both the financial accelerator effect during normal business cycles and the probability of large bank runs.

3.5.2 The Steady State Effect of Retail Bank Capital Requirements

In this section, we explore the steady state implications of retail bank capital requirements. We characterize the non-stochastic steady state equilibrium absent of bank runs in Appendix 3.B. In the steady state, the capital adjustment cost is zero, therefore the price of capital Q equals 1. We denote steady state variables without time subscript.

In the following subsections, we conduct comparative statics analysis of the impact of varying bank capital regulation on retail banks, i.e. the consequence of changing the policy parameter $\frac{1}{\bar{\phi}_R}$ while keeping other parameters constant.

Capital Allocation and the Aggregate Capital Stock

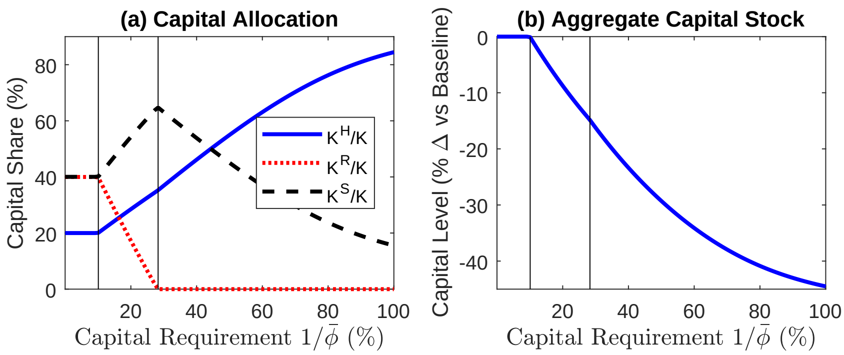

Figure 3.7: Steady state effect of a retail bank capital requirement on the capital allocation and the aggregate capital stock.

Other things being equal, under a tighter capital requirement, retail banks are faced with a higher financing cost as they have to use more costly capital and less relatively cheap deposit from the households. This higher financing cost is further passed on by the retail banks to the shadow banks. Therefore, the required return for capital investment from the banking sector increases. On the other hand, the required return for capital investment for households remains the same. As a result, the share of capital held by households increases. The first order condition regarding K^H_{t+1} in steady state is given by (B.5 in the appendix):

$$R^K = \frac{1}{\beta}\left(1 + \eta^H \frac{K^H}{K}\right).$$

Hence the return on capital, R^K, increases as the households hold a larger share of the capital stock. As investment becomes more costly, the aggregate capital stock and aggregate output decrease.

Figure 3.7 shows the comparative statics for the distribution of capital among households, retail banks and shadow banks in Panel (a) and the aggregate capital stock in Panel (b). We vary the minimum capital requirement between 0 and 100 percent. The solid line is the share of capital held by households, the dotted line is the share of retail bank capital holdings and the dashed line is the share of shadow bank capital holdings. For a retail bank capital requirement below 10 percent, indicated by the left vertical line, the capital requirement is not binding and the retail bank leverage ratio is determined by its incentive constraint.

For a retail bank capital requirement between 10 and 25 percent, the capital requirement is binding and retail banks will invest in both capital and wholesale lending. As the capital requirement increases in this range, retail banks substitute away from capital lending to wholesale lending. This is because an additional unit of capital lending requires $1/\bar{\phi}$ units of equity finance, while an additional unit of wholesale lending only requires $\gamma/\bar{\phi}$ units of equity finance. If the regulator tightens the retail bank capital requirement, wholesale lending will therefore become relatively more attractive for retail banks. Hence, direct capital holdings by retail banks decrease, and capital holdings by shadow banks increase in this range.

For a capital requirement of above 25 percent, indicated by the right vertical line, retail banks will only invest into wholesale lending. If the regulator increases the capital requirement in this range, retail banks can no longer substitute away from capital holdings and therefore can only reduce wholesale lending. Consequently, both retail and shadow banks will reduce their assets in this range.

In Panel (b), we plot the aggregate capital stock relative to the unregulated economy as a function of the capital requirement. The capital stock is very sensitive to the capital requirement: If retail banks were required to finance themselves using 100 percent equity, the capital stock would reduce by more than 40 percent. The reason for this strong effect is that banks in this economy cannot raise outside equity from households. Hence, a higher capital requirement forces retail banks to sharply cut the asset side of their balance sheet, which in turn forces the shadow banks to reduce their assets as well. If banks could issue equity to households, their required return on equity would not increase monotonically with a higher capital requirement, which would imply a lower bound on the aggregate capital stock.

Leverage and the Coverage Ratio of Shadow Banks

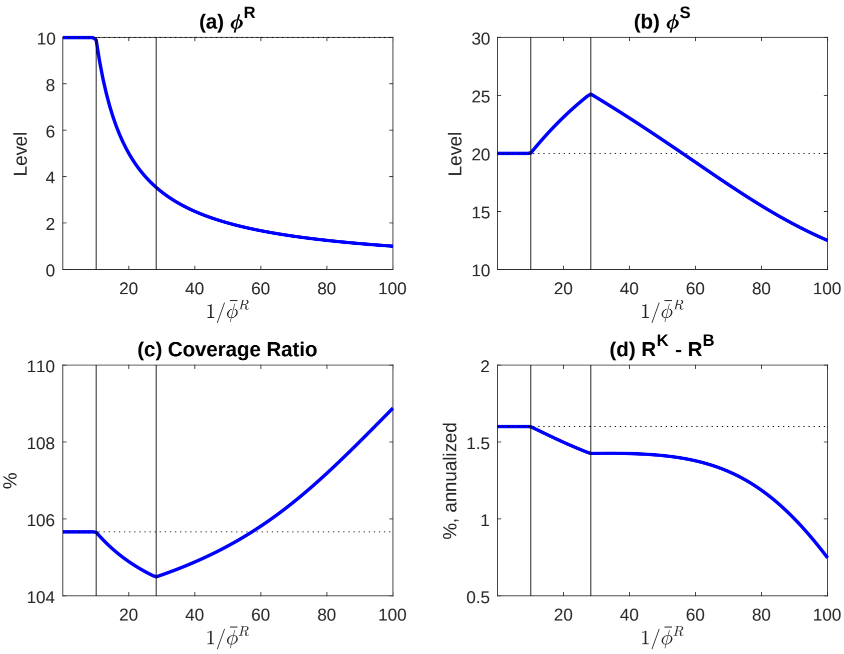

Figure 3.8: Steady state effect of a retail bank capital requirement on leverage and the coverage ratio.

In Figure 3.8, we report how leverage of retail and shadow banks and the coverage ratio of shadow banks change with the capital requirement. We define the steady state coverage ratio of shadow banks as the ratio of beginning of period assets over liabilities by shadow banks, i.e. as

$$\frac{R^K K^S}{R^B B^S} = \frac{R^K}{R^B} \frac{\phi^S}{\phi^S - 1}.$$

The coverage ratio is an interesting statistic, because it indicates how run-prone the shadow banking sector is. While it does not exactly correspond to the recovery rate of wholesale lending by the retail banks after a bank run, a given fall in the liquidation price of capital can ceteris paribus trigger a bank run more often if the coverage ratio is low.

Mechanically, an increase in the retail bank capital requirement lowers the leverage ratio of retail banks. For shadow banks, there are two cases.

As long as retail banks can substitute away from capital holdings towards wholesale lending, higher retail bank capital requirements increase the leverage ratio of shadow banks. When retail banks lend only on the wholesale market, regulating retail banks reduces shadow bank leverage.

The coverage ratio decreases as soon as the retail bank capital requirement becomes binding and increases as soon as retail banks exclusively lend on the wholesale funding market. This has a significant effect on the bank run probability: In the baseline model, the economy could sustain a drop in the liquidation price of capital of at most 5.5 percent without a bank run being triggered. With fully equity-financed retail banks, the economy could sustain a drop in the liquidation price of capital of more than 8.5 percent without a self-fulfilling bank run being triggered.[23]

The coverage ratio is decreasing in the leverage of shadow banks and increasing in the excess return $R^K - R^B$. Looking at the right upper and lower panel of Figure 3.8, we find that the increase in the coverage ratio is primarily driven by the lower leverage ratio of shadow banks, because $R^K - R^B$ is decreasing in the capital requirement throughout.

Consumption and Welfare

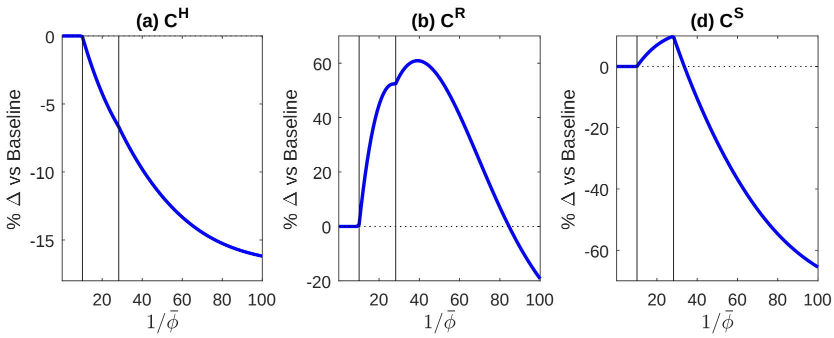

Figure 3.9: Steady state effect of a retail bank capital requirements on consumption.

Figure 3.9 shows how retail bank capital requirements affect consumption and hence welfare of agents in steady state. Households consume less in a regime with a higher capital requirement. For households this is because they can save less through deposits and make more inefficient direct

[23]The formula to calculate this threshold price is $Q^* = \left(\frac{R^B B}{K^S} - r^K\right)\frac{1}{1-\delta}$.

investment. Also, aggregate output decreases strongly due to the decrease in the aggregate capital stock, which lowers wages.

Retail banks and shadow banks, on the other hand, enjoy higher consumption under a higher capital requirement. This is because competitive retail banks do not internalize that by lending less to final goods producers they can increase the returns on their assets, $R^K/(Q + f^R)$ and R^B, relative to the return on their liabilities, R^D, and thereby increase their net worth. Shadow banks receive a consumption gain as long as they can increase leverage, and a decrease in consumption once retail banks can no longer substitute from direct lending to wholesale lending.

Overall, because in our calibration consumption of retail and shadow banks is very small relative to household consumption, higher retail bank capital requirements lead to a welfare loss. However, by reducing the coverage ratio of shadow banks, retail bank capital requirements can reduce the susceptibility of the economy to shadow bank runs.

3.6 Counterfactuals

We conduct two experiments in this section. First, we compute the welfare cost of bank runs from the perspective of households, retail banks and shadow banks. This experiment gives us an upper bound on the positive effect of a policy designed to reduce bank runs. Second, we consider different rules for capital requirements, both in an economy with and without shadow bank runs.

3.6.1 Welfare Computation

We compute welfare in consumption equivalent terms. We use the realized consumption sequences to compute welfare. For households, welfare computation is straightforward. For the banks, we include both incumbent and newly entering banks into our welfare measure. Since they have linear utility, the regulator can then simply consider the welfare of each of the banking sectors as a whole. The utility of banking sector J, $J \in \{R, S\}$, as a whole if net worth is constant over time is given by

$$U^J = \sigma^J \frac{N^J - \upsilon^J K}{1 - \sigma^J} + \beta(1 - \sigma^J)U^J, \tag{3.6.1}$$

	Baseline, No Runs	Baseline, With Runs	% Δ
C^H	0.824	0.822	0.203
C^R	0.576	0.570	1.055
C^S	0.205	0.189	8.467

Table 3.4: Agents' willingness to pay to avoid bank runs in the baseline model.

with consumption given by $C^J = \sigma^J \frac{N^J - \upsilon^J K}{1-\sigma^J}$. The consumption equivalent welfare of banking sector J is therefore

$$C^J_{\text{equiv}} = \left[1 - \beta(1-\sigma^J)\right] U^J. \tag{3.6.2}$$

3.6.2 The Welfare Cost of Bank Runs

Before investigating the welfare effects of bank capital requirements, we want to know how costly bank runs are in our calibrated model. For this purpose, we conduct the following experiment: We first simulate the model with the liquidation cost shock. We compute the permanent consumption equivalent of welfare for each type of agent in this economy. Next, we simulate a model without bank runs by setting the liquidation value shock to a large enough number. We then calculate the permanent consumption equivalent for each type of agent in this economy without bank runs. The difference between the two consumption equivalents is the welfare gain if bank runs are completely eliminated, expressed in permanent consumption equivalent units.

Table 3.4 shows how much households, retail banks and shadow banks are willing to pay to avoid bank runs. We report the results as percentage change in the consumption equivalent of welfare from eliminating bank runs. If welfare in consumption equivalent terms is given by C^J, the percentage change in welfare for agent J is

$$\frac{C^J_{\text{No Runs}} - C^J_{\text{Runs}}}{C^J_{\text{Runs}}}.$$

Shadow banks gain the most from eliminating bank runs and would be willing accept an 8.5 percent permanent decrease in consumption to avoid bank runs. Bank runs are also very costly for retail banks, who would pay 1.1 percent of their permanent consumption to eliminate bank runs. Households gain about 0.2 percent in quarterly consumption equivalent terms from the elimination of bank runs.

To compare our results to the literature, Chatterjee and Corbae (2007) estimate a consumption equivalent welfare gain from eliminating the likelihood of economic crises to be around 0.97 percent.[24] Their estimated contribution of a reduction in consumption volatility to this welfare gain is around 0.196 percent in consumption equivalent terms, which is comparable to the welfare gain of households in our model. Their depression state has a similar frequency and similar output effects to a financial crisis in our model: The unconditional probability of a depression state in their model is 9.75 percent, whereas the unconditional probability of a financial crisis state in our model is around 8 percent, which is also comparable. They assume however a constant relative risk aversion of 3, which is much higher than the value of 1 that we use.

Barro (2009) estimates a welfare gain of 4% in output equivalent terms for a representative household with log utility from eliminating consumption disasters like World War II. In his case, disasters however have an output cost of almost 30 percent on average, which is one order of magnitude larger than the output loss from a bank run in our model. Overall, we conclude that the welfare gain from eliminating bank runs is sizable for all agents in the economy.

3.6.3 Policy Experiments

We discuss two different rules for setting the capital requirement $\frac{1}{\bar{\phi}_t}$. First, we consider the simple case of a constant capital requirement:

$$\frac{1}{\bar{\phi}_t} = \frac{1}{\bar{\phi}}. \tag{3.6.3}$$

Second, we look at the case where the regulator can condition the capital requirement on whether or not the economy is in a run equilibrium. Denote as $\mathbb{1}_t^{\text{Run}}$ an indicator variable that is 1 if the economy experiences a run in period t and 0 otherwise. Then, we can write a capital requirement that

[24] Chatterjee and Corbae (2007) define economic crises as depressions of the same magnitude of the Great Depression in terms of increase in unemployment.

conditions on the no-run state as

$$\frac{1}{\bar{\phi}_t} = \frac{1}{\bar{\phi}}(1 - \mathbb{1}_t^{\text{Run}}). \tag{3.6.4}$$

Such a requirement has the advantage that the regulator can impose higher equity buffers of retail banks during normal times, which can be used to absorb the liquidated capital from shadow banks during a run, thereby pushing up the liquidation price of capital. In this sense, the more access to deposits retail banks have during a banking crisis, the higher the fire sale price of the capital will be ex post, and the less likely bank runs would happen ex ante. Therefore, the optimal capital requirement in face of a bank run is its lower bound, i.e. zero. In what follows we focus on this specific run-contingent capital requirement.

For each policy experiment, we are interested in two questions. i) How effective is the policy in reducing bank runs? ii) What is the welfare effect of the policy?

Constant Capital Requirements

First, we discuss the case of a constant capital requirement. Panel (a) of

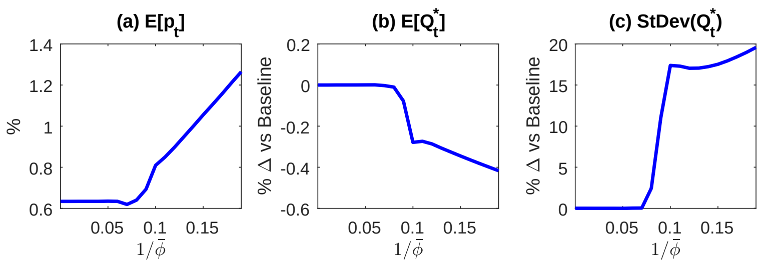

Figure 3.10: Probability of bank runs with a constant minimum capital requirement.

Figure 3.10 shows that the frequency of bank runs increases as the minimum capital requirement increases. The reason is that net worth of retail banks decreases during a bank run, which lowers their leverage capacity under a binding capital requirement and hence their ability to absorb the liquidated capital of the shadow banking sector. Therefore, this capital requirement has a negative effect on the liquidation price of capital, as shown in panel

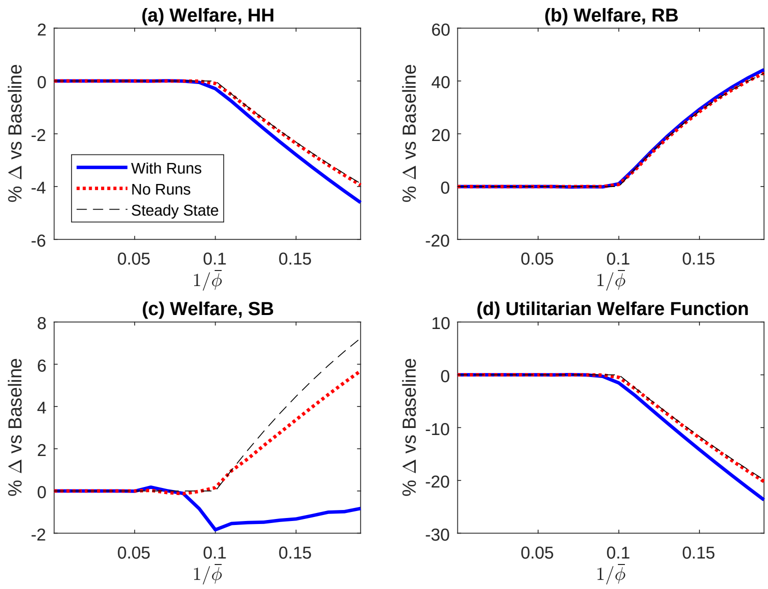

Figure 3.11: Welfare with a constant minimum capital requirement.

(b). Since the higher constant capital requirement implies that the capital price is more volatile, as shown in panel (c), bank runs become more likely.

In Figure 3.11, we show the welfare effects of a constant capital requirement. Welfare in panels (a) to (c) is measured in consumption-equivalent units. The utilitarian welfare function in panel (d) is expressed in utility. On the x-axis, we vary the capital requirement between 0 percent and 20 percent. On the y-axis, we show the percentage change in welfare relative to the capital requirement of 0 percent, which is never binding. We report the percentage change in welfare relative to the model without regulation for each type of agents as well as for the sum of utilities, which corresponds to a utilitarian welfare function. We show the results for three different versions of the model. The dashed line is the steady state equilibrium. The dotted line is the model without bank runs and the solid line is the model with bank runs. The purpose of including the steady state is to illustrate the strong steady state effect, and the purpose of including the No Runs case is to illustrate the isolated effect of the capital requirement on the capital price externality that is also present in the model without runs.

First, we can see that in steady state, a higher capital requirement reduces the welfare of households, but increases the welfare of retail and shadow banks. Overall, measured in consumption equivalent units, capital requirements are welfare reducing in steady state. Retail and shadow banks gain in welfare terms, because the return on their assets increases more than the return on their liabilities, which increases the net worth of incumbent banks.

Second, we can see that a constant capital requirement reduces welfare *more* in the dynamic model without bank runs relative to the steady state. This is because this constant capital requirement amplifies the effect of the pecuniary externality, as discussed in section 3.5.1.

Third, we can see that a constant capital requirement reduces welfare even more in the dynamic model with bank runs compared to the dynamic model without bank runs. The reason is that, as can be seen in Figure 3.10, in addition to amplifying the effect of the pecuniary externality, a higher capital requirement increases the frequency and severity of bank runs. To summarize the results: From a macro-prudential perspective, constant capital requirements not only distort the allocation of capital, which leads to a steady state welfare loss, but they also amplify the effects of the pecuniary externality during normal times and increase the frequency and severity of bank runs. While constant capital requirements may be beneficial at the microprudential level, our results indicate that macroprudential regulation of the retail banking sector should not use constant capital requirements.

Run-Contingent Capital Requirements

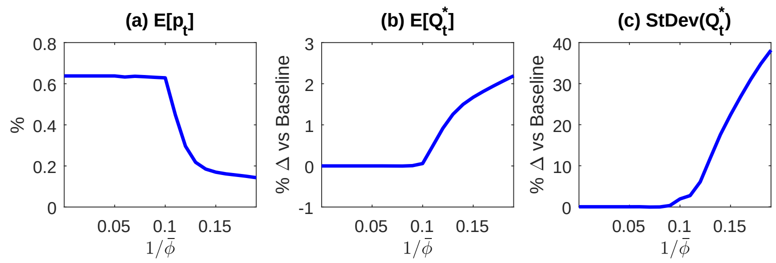

Figure 3.12: Probability of bank runs with a run-contingent capital requirement.

In Figures 3.12 and 3.13, we show the effects of a capital requirement that is only imposed if the economy is in the no-run equilibrium. First, we can see in Panel (a) in Figure 3.12 that by implementing a run-contingent capital requirement, the regulator can reduce the probability of bank runs substantially. As we show in Panels (b) and (c), the main channel through which the regulator achieves this effect is through a higher liquidation price of capital: Increasing the capital requirement to 20 percent increases the liquidation price of capital by more than 2 percent. While the run-contingent capital requirement also increases the volatility of the liquidation price of capital, this effect is more than offset by the higher mean of the liquidation price. Nevertheless, a policy which is designed to offset cyclical fluctuations in the price of capital may lead to superior welfare outcomes.

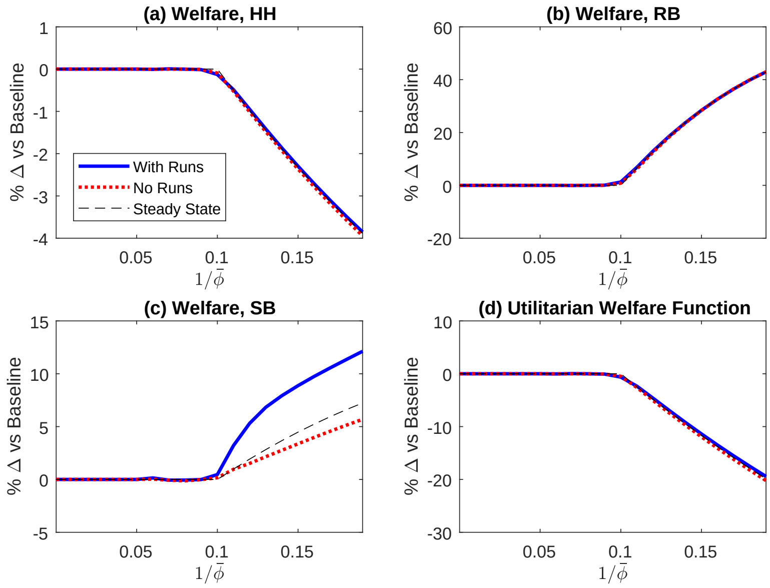

Figure 3.13: Welfare with a run-contingent capital requirement.

The welfare results for the steady state case and the case without bank runs are the same as in Figure 3.11. As in Figure 3.11, welfare in panels (a) to (c) is measured in consumption equivalent units. The welfare function in panel (d) is measured in utility. Focusing on the welfare results for the model with bank runs, we can see that a run-contingent capital requirement can undo the negative externality of capital requirements on the probability of bank runs. In fact, welfare of shadow banks increases more in the model with runs compared to the model without runs. However, the capital requirements are still overall welfare reducing, and more so in the dynamic model with runs than in the steady state. This is because the run-contingent capital requirement still increases the pro-cyclicality of the retail bank balance sheet constraint during normal times, which amplifies the pecuniary externality from the capital price and therefore the welfare cost of business cycles.

The intuition for the better performance of the run-contingent capital requirement relative to the constant capital requirement is as follows: A higher capital requirement increases the net worth of retail banks, which in turn increases the continuation value of the retail banks. This means that the incentive constraint and hence the market imposed borrowing constraint is relaxed. If the regulator now removes the capital requirement during a shadow bank run, the retail banks can increase leverage relative to the case without regulation. Hence, they can absorb the liquidated capital of the shadow banks more easily, which increases the liquidation price of capital. Finally, a higher liquidation price of capital reduces the ex post cost of realized bank runs and reduces the ex ante probability of bank runs. The success of this policy is illustrated by a relatively higher welfare gain from regulation in the model with bank runs for all agents compared to the steady state model. However, the steady state cost of bank capital regulation is still dominant, such that capital regulation overall lowers welfare. This high cost relies on the extreme assumption that banks can never raise outside equity from households, no even in the long run. Removing this constraint may yield a significantly less pessimistic cost of bank capital requirements.

3.7 Conclusion

We study the macroeconomic effects of retail bank capital regulation in a quantitative model with regulated retail banks and unregulated shadow banks. In our model, financial crises occur in the form of runs on shadow banks. There is a role for regulation in the model because banks do not internalize that their decisions affect the likelihood of financial crises, which leads to over-borrowing during normal times.

From the regulators' perspective, the trade-off that determines the optimal capital requirement is: On the one hand, higher capital requirements increase the ability of retail banks to absorb liquidation losses during a shadow bank run, thereby reducing the frequency and severity of bank runs.

For this effect, it is crucial that capital requirements are relaxed during a bank run. A higher constant capital requirement instead leads to more bank runs. On the other hand, tightening capital requirements reduces the steady state capital stock and output due to less financial intermediation.

We conclude that capital requirements on depository institutions are an effective policy instrument to reduce banking crises and increase financial stability. However, they can also create substantial costs for the economy, especially when capital accumulation is endogenous and equity issuance is very costly for banks. Therefore, the optimal capital requirement in a model with endogenous capital accumulation should be substantially lower than that in a model with exogenous capital.

An interesting extension of our model would be to include sticky prices and nominal debt. A bank run could then result in a Fisherian debt deflation spiral: The initial effects of the run depresses goods prices, which worsens the real debt burden of banks, which in turn depresses investment, and so on. Bank runs can then lead to episodes that cause the economy to be at the lower bound for the nominal policy interest rate. In this case, the possibility of bank runs will also affect how monetary policy should be conducted.

3.A Data

We measure output Y using real GDP, investment $\tilde{I}$ using real gross private domestic investment and household consumption C^H using personal consumption expenditures from the U.S. National Income and Product Accounts.

For the deposit rate R^D, we use the real effective federal funds rate, provided by the Federal Reserve Board, minus a four quarter moving average of the annualized inflation rate. We use the U.S. GDP deflator to construct the inflation rate. For the wholesale funding market rate R^B, we follow Gertler, Kiyotaki, and Prestipino (2016), who use the 90 day asset backed commercial paper rate, which is also provided by the Federal Reserve Board, minus a four quarter moving average of the annualized inflation rate. For the return on capital R_t^K/Q_{t-1}, we use the Wilshire 5000 index, a return index, which we also deflate with the GDP deflator.

For the construction of wholesale lending B and deposit lending D, we follow the procedure in Gertler, Kiyotaki, and Prestipino (2016). We take the data from the financial accounts of the U.S., provided by the Federal Reserve Board. We calculate deposits as the sum of the asset holdings of households and nonfinancial business of

1. checkable deposits and currency,
2. total time and savings deposits,
3. money market mutual fund shares, and
4. mutual fund shares.

The shadow banking sector comprises the following groups:

1. GSEs and federally related mortgage pools
2. Funding corporations
3. Finance companies
4. Security brokers and dealers
5. Issuers of asset-backed securities
6. Holding companies

We compute B as the net liability position of these groups in the following short-term asset classes:

1. Commercial Paper
2. Security repurchase agreements

We deflate the resulting time series using the GDP deflator. We detrend the time series for Y, $\tilde{I}$, C^H, D and B with the filter proposed in Hamilton (2017). We do not use the Hodrick-Prescott filter, since Hamilton (2017) reports that the Hodrick-Prescott filter can lead to spurious correlations and distorts the properties of the filtered series at the beginning and the end of the series. We plot the detrended time series in Figure 3.14. We detrend quantities in logs and interest rates in levels. The Hamilton filter leads to substantially different detrended time series compared to the Hodrick-Prescott filter. In particular, the Hamilton-filtered time series displays a much stronger recession in 2007-2009 in terms of output, investment and consumption.

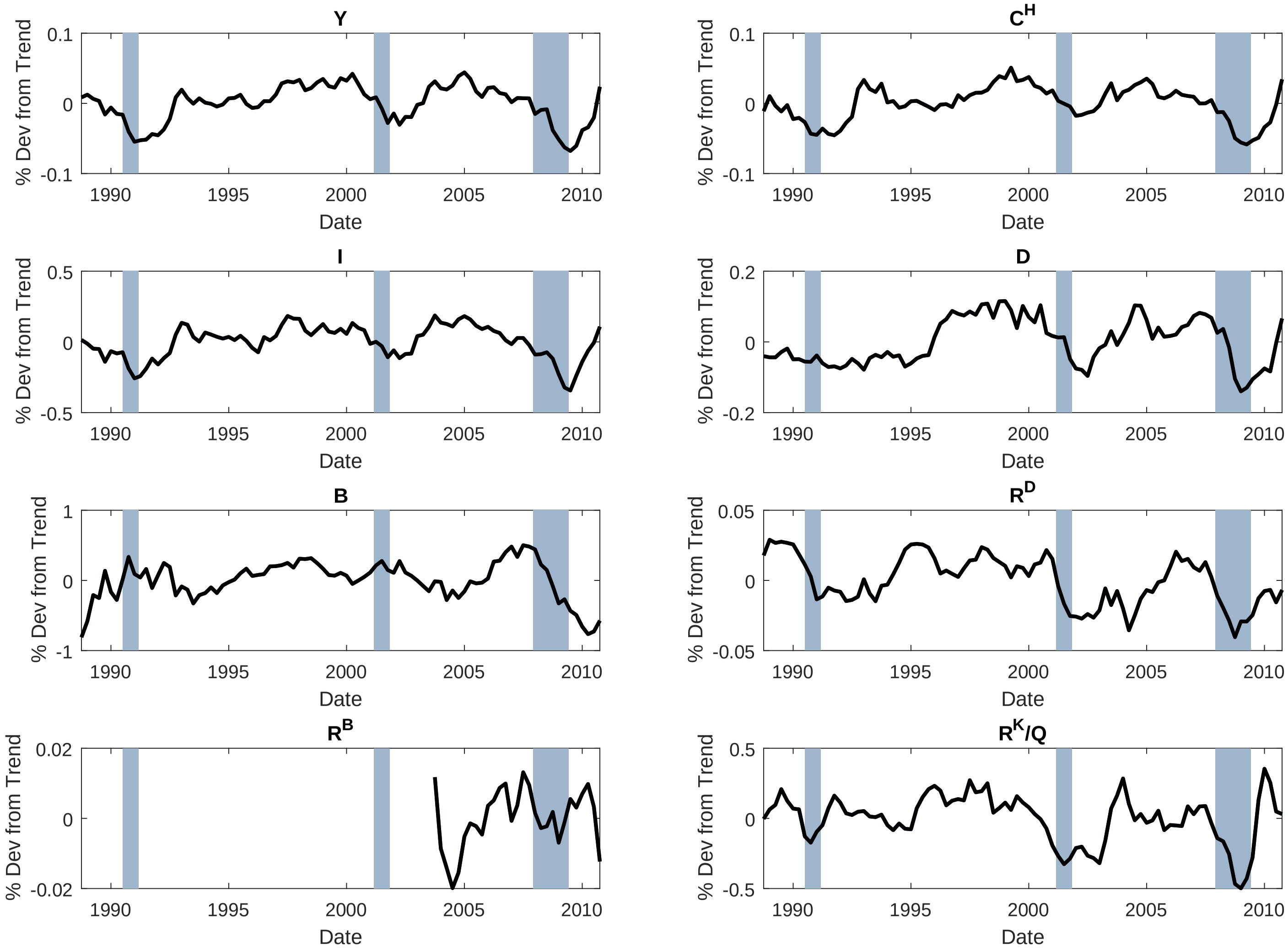

Figure 3.14: Detrended data.

3.B Steady State

We focus on the case of the model under a binding capital requirement. Our approach is to first characterize the steady state allocation of capital for a given aggregate capital stock K. We then explain how the aggregate capital stock is determined. Given K, we can compute the gross return on capital and the wage as

$$R^K_{SS} = \alpha K^{\alpha-1}_{SS} + 1 - \delta \tag{3.B.1}$$

$$W_{SS} = (1-\alpha) Z K^{\alpha}_{SS} \tag{3.B.2}$$

Steady state interest rates are determined by the first order conditions with respect to D^H_{t+1}, and B^R_{t+1}:

$$R^D_{SS} = \frac{1}{\beta} \tag{3.B.3}$$

$$R^B_{SS} = \gamma \frac{R^K_{SS}}{1 + f^R_{SS}} + (1-\gamma) R^D_S S \tag{3.B.4}$$

Given R^K, K^H is determined by the euler equation of the household with respect to K^H

$$R^K_{SS} = \frac{1}{\beta}\left(1 + \eta^H \frac{K^H_{SS}}{K_{SS}}\right) \tag{3.B.5}$$

$$\frac{K^H}{K} = \frac{1}{\eta^H}\left(\beta R^K - 1\right) \tag{3.B.6}$$

We can now characterize the steady state allocation for the shadow banks: First, from the balance sheet constraint of shadow banks follows

$$B_{SS} = K^S_{SS} - N^S_{SS} \tag{3.B.7}$$

Plugging this into the law of motion for aggregate net worth, we can write net worth as

$$N^S = \frac{\upsilon^S K_{SS}}{1 - \left[\left(R^K - R^B\right)\frac{K^S}{N^S} + R^B\right]\left(1 - \sigma^S\right)}. \tag{3.B.8}$$

From the incentive constraint, we then get a quadratic condition for K^S/N^S:

$$\psi\left[\omega\frac{K^S}{N^S}+(1-\omega)\right] \tag{3.B.9}$$

$$=\beta\left[\sigma^S+(1-\sigma^S)\psi\left[\omega\frac{K^S}{N^S}+(1-\omega)\right]\right]\frac{1}{1-\sigma^S}\left(1-\upsilon^S\frac{K}{N^S}\right)$$

$$=\beta\left[\sigma^S+(1-\sigma^S)\psi\left[\omega\frac{K^S}{N^S}+(1-\omega)\right]\right]\left(\left(R^K-R^B\right)\frac{K^S}{N^S}+R^B\right) \tag{3.B.10}$$

We then can infer $K^S=K^S/N^SN^S$. The fraction of capital holdings of retail banks are given by the market clearing condition for capital goods:

$$\frac{K^R_{SS}}{K_{SS}}=1-\frac{K^H_{SS}}{K_{SS}}-\frac{K^S_{SS}}{K_{SS}}. \tag{3.B.11}$$

From the balance sheet of the retail banking sector, we get

$$D=\left(1+\eta^R\frac{K^R}{K}\right)K^R+B-N^R. \tag{3.B.12}$$

This allows us to substitute out D in the law of motion for aggregate net worth:

$$\begin{aligned}
N^R &= (R^K K^R+R^B B-R^D D)(1-\sigma^R)+\upsilon^R K\\
&= \big(R^K K^R+R^B B\\
&-R^D\left(\left(1+\eta^R\frac{K^R}{K}\right)K^R+B-N^R\right)\big)(1-\sigma^R)+\upsilon^R K\\
&= \left(\left(R^K-\left(1+\eta^R\frac{K^R}{K}\right)R^D\right)\frac{K^R}{N^R}\right.\\
&\left.+\left(R^B-R^D\right)\frac{B}{N^R}+R^D\right)N^R(1-\sigma^R)+\upsilon^R K
\end{aligned}$$

Hence,

$$N^R=\frac{\upsilon^R K}{1-(1-\sigma^R)\left(\left(R^K-\left(1+\eta^R\frac{K^R}{K}\right)R^D\right)\frac{K^R}{N^R}+\left(R^B-R^D\right)\frac{B}{N^R}+R^D\right)}. \tag{3.B.13}$$

Finally, from the capital requirement, we get

$$\left(1+\eta^R\frac{K^R}{K}\right)K^R+\gamma B$$
$$=\bar{\phi}\frac{\upsilon^R K}{1-(1-\sigma^R)\left(\left(R^K-\left(1+\eta^R\frac{K^R}{K}\right)R^D\right)\frac{K^R}{N^R}+(R^B-R^D)\frac{B}{N^R}+R^D\right)} \tag{3.B.14}$$

Substituting in the solutions for B from equation 3.B.7, N^R from equation 3.B.13 and K^R/K, from equation 3.B.11 this is a complicated nonlinear equation in K only.

Some additional variables of interest can be calculated residually. Total output is given by:

$$Y_{SS}=ZK_{SS}^{\alpha}+\upsilon^R K+\upsilon^S K-\eta^H\left(\frac{K^H}{K}\right)^2 K-\eta^R\left(\frac{K^R}{K}\right)^2 K. \tag{3.B.15}$$

Then, household consumption is characterized by the aggregate budget constraint:

$$C_{SS}^H=Y-\delta K-\sigma^R\frac{N^R-\upsilon^R K}{1-\sigma^R}-\sigma^S\frac{N^S-\upsilon^S K}{1-\sigma^S}. \tag{3.B.16}$$

3.C Equilibrium Conditions in the Dynamic Model

3.C.1 Households

The first-order conditions of the households' problem with respect to capital holding K_{t+1}^H and deposit D_{t+1}^H are given by:

$$FOC(K_{t+1}^H): \quad \frac{1}{C_t^H}(Q_t+\eta^H\frac{K_{t+1}^H}{K_t})=\beta\mathbb{E}_t\left(\frac{1}{C_{t+1}^H}R_{t+1}^K\right) \tag{3.C.1}$$
$$FOC(D_{t+1}^H): \quad \frac{1}{C_t^H}=\beta\mathbb{E}_t\left(\frac{1}{C_{t+1}^H}R_{t+1}^D\right) \tag{3.C.2}$$

The interpretation of these first-order conditions is standard. In the first expression, the left-hand side and the right-hand side are the marginal cost and marginal benefit of capital holding, respectively. The marginal cost of capital holding has two components. One is the price the households have to pay for purchasing the capital goods, and the second is the capital holding cost due to households' low investment skills.

In addition, the households decide how much capital to hold through the retail banking sector. The first order condition with respect to K_{t+1}^R yields a first order condition which pins down f_t^R:

$$f_t^R = \eta^R \left(\frac{K_{t+1}^R}{K_t} \right).$$

Aggregate consumption of the household sector can be inferred from the resource constraint of the economy. Therefore, we do not have to track the net worth of households as a state variable.

$$\begin{aligned} C_t^H = &Z_t K_t^\alpha + \upsilon^R K_t + \upsilon^S K_t - \frac{\eta^H}{2} \left(\frac{K_{t+1}^H}{K_t} \right)^2 K_t - \frac{\eta^R}{2} \left(\frac{K_{t+1}^R}{K_t} \right)^2 K_t \\ &- I_t - \frac{\theta}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t - \sigma^R \frac{N_t^R - \upsilon^R K_t}{1 - \sigma^R} - \sigma^S \frac{N_t^S - \upsilon^S K_t}{1 - \sigma^S} \end{aligned}$$

3.C.2 Banks

Shadow Banks

The incentive constraint of the shadow bank is given by

$$\psi(n_t^S + \omega b_{t+1}^S) = \beta \mathbb{E}_t \left[V_{t+1}^S \right]. \tag{3.C.3}$$

The balance sheet constraint of the shadow bank reads

$$Q_t k_{t+1}^S = n_t^S + b_{t+1}^S \tag{3.C.4}$$

The net worth of an incumbent shadow bank is

$$n_t^S = R_t^K k_t^S - R_t^B b_t^S \tag{3.C.5}$$

The value of the shadow bank before the realization of the exit shock is given by

$$\begin{aligned} V_t^S &= \sigma^S n_t^S + (1 - \sigma^S) \beta \mathbb{E}_t \left[V_{t+1}^S \right] \\ &= \sigma^S n_t^S + (1 - \sigma^S) \psi(n_t^S + \omega b_{t+1}^S), \end{aligned}$$

where the second line uses the binding incentive constraint to substitute out the continuation value. Plugging this expression into 3.C.3 yields the following characterization for the shadow banks choices for k^S_{t+1} and b^S_{t+1}:

$$\psi(n^S_t + \omega b^S_{t+1}) = \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi) n^S_{t+1} + \psi\omega(1-\sigma^S) b^S_{t+2} \right] \tag{3.C.6}$$

$$Q_t k^S_{t+1} = n^S_t + b^S_{t+1} \tag{3.C.7}$$

$$n^S_t = R^K_t k^S_t - R^B_t b^S_t \tag{3.C.8}$$

We now conjecture and verify that the policy functions for b^S_{t+1} and k^S_{t+1} are linear in net worth, such that it is sufficient to characterize the optimal choices of the shadow banking sector as a whole in equilibrium.

Theorem 3.C.1 (Linearity of Policy Functions)**.** *The policy functions for b^S_{t+1} and k^S_{t+1} which solve the problem of the shadow bank given by equations 3.C.6 to 3.C.8 are linear in net worth.*

Proof. Suppose that the policy functions are given by $b^S_{t+1} = A^S_b n^S_t$ and $k^S_{t+1} = A^S_k n^S_t$, respectively. Then, it follows from equation 3.C.8 that

$$\begin{aligned} n^S_{t+1} &= R^K_{t+1} k^S_{t+1} - R^B_{t+1} b^S_{t+1} \\ &= (R^K_{t+1} A^S_k - R^B_{t+1} A^S_b) n^S_t \\ &= A^S_n n^S_t. \end{aligned}$$

From equation 3.C.7 follows that

$$\begin{aligned} Q_t A^S_k n^S_t &= n^S_t + A^S_b n^S_t \\ A^S_k &= \frac{1 + A^S_b}{Q_t}. \end{aligned}$$

Finally, from 3.C.6 follows that

$$\begin{aligned} \psi(1+\omega A^S_b) n^S_t &= \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi(1+\omega A^S_b)) n^S_{t+1} \right] \\ &= \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi(1+\omega A^S_b)) A^S_n \right] n^S_t \\ &= \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi(1+\omega A^S_b)) (R^K_{t+1} \frac{1+A^S_b}{Q_t} - R^B_{t+1} A^S_b) \right] n^S_t \end{aligned}$$

This equation yields a solution for A^S_b that is independent of n^S_t.[25] Conse-

[25]Specifically, the solution is given by $A^S_b = -p + \sqrt{p^2 + q}$, with $p =$

quently, A_k^S and A_n^S are also independent of n_t^S. $\square$

Given the linearity of policy functions, it is sufficient to characterize the policies K_{t+1}^S and B_{t+1}^S of the aggregate shadow banking sector. These choices are the solutions to

$$\psi(N_t^S + \omega B_{t+1}^S) = \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi) \frac{N_{t+1}^S - \upsilon^S K_{t+1}^S}{1-\sigma^S} + \psi\omega(1-\sigma^S) B_{t+2}^S \right]$$
$$Q_t K_{t+1}^S = N_t^S + B_{t+1}^S$$
$$N_t^S = (R_t^K K_t^S - R_t^B B_t^S)(1-\sigma^S) + \upsilon^R K_t^S.$$

Retail Banks, No Regulation

We characterize the problem of a retail banks under a non-binding and a binding capital requirement. First, we consider the problem of a retail bank where the incentive constraint is binding. The incentive constraint is given by

$$\psi((Q_t + f_t^R) k_{t+1}^R + \gamma b_{t+1}^R) = \beta \mathbb{E}_t \left[V_{t+1}^R \right]. \tag{3.C.9}$$

The balance sheet constraint reads:

$$(Q_t + f_t^R) k_{t+1}^R + b_{t+1}^R = n_t^R + d_t^R. \tag{3.C.10}$$

Net worth is determined according to

$$n_t^R = R_t^K k_t^R + R_t^B b_t^R - R_t^D d_t^R. \tag{3.C.11}$$

$-\frac{1}{2}\frac{(1/\beta - \mathbb{E}_t[R_{t+1}^K/Q_t]) - \mathbb{E}_t[R_{t+1}^K/Q_t - R_{t+1}^B]}{\omega \mathbb{E}_t[R_{t+1}^K/Q_t - R_{t+1}^B]}$ and $q = \frac{1/\beta - \mathbb{E}_t[(\sigma^S + (1-\sigma^S)\psi) R_{t+1}^K/Q_t]}{\omega \mathbb{E}_t[R_{t+1}^K/Q_t - R_{t+1}^B]}$. When $\mathbb{E}_t\left[R_{t+1}^K/Q_t - R_{t+1}^B\right] > 0$ and $\mathbb{E}_t\left[R_{t+1}^B - 1/\beta\right] > 0$, this solution is unique.

These three equations pin down k^R_{t+1}, d^R_{t+1} and n^R_t. b^R_{t+1} is determined by a first order condition of the retail banks problem:

$$
\begin{aligned}
&\max_{\left\{k^R_{s+1},b^R_{s+1},d^R_{s+1}\right\}_{s=t}^{\infty}} \beta\mathbb{E}_t\left[V^R_{t+1}\right]\\
&\text{s.t.}\\
&V^R_t = \sigma^R n^R_t + (1-\sigma^R)\psi((Q_t + f^R_t)k^R_{t+1} + \gamma b^R_{t+1})\\
&n^R_t + d^R_{t+1} = (Q_t + f^R_t)k^R_{t+1} + b^R_{t+1}\\
&\psi((Q_t + f^R_t)k^R_{t+1} + \gamma b^R_{t+1}) = \beta\mathbb{E}_t\left[V^R_{t+1}\right]\\
&n^R_t = R^K_t k^R_t + R^B_t b^R_t - R^D_t d^R_t\\
&k^R_{t+1}, d^R_{t+1}, b^R_{t+1} \geq 0.
\end{aligned}
$$

We conjecture, as in the shadow banking problem, that the policy functions for k^R_{t+1}, b^R_{t+1} and d^R_{t+1} are linear in net worth n^R_t:

$$
\begin{aligned}
k^R_{t+1} &= A^R_k n^R_t\\
b^R_{t+1} &= A^R_b n^R_t\\
d^R_{t+1} &= A^R_d n^R_t
\end{aligned}
$$

Plugging in the conjectured policy functions, we can rewrite the maximization problem as

$$
\begin{aligned}
&\max_{\left\{k^R_{s+1},b^R_{s+1}\right\}_{s=t}^{\infty}} \beta\mathbb{E}_t\left[V^R_{t+1}\right]\\
&\text{s.t.}\\
&V^R_{t+1} = (\sigma^R + (1-\sigma^R)\psi((Q_{t+1} + f^R_{t+1})A^R_k + \gamma A^R_b))n^R_{t+1}\\
&\psi((Q_t + f^R_t)k^R_{t+1} + \gamma b^R_{t+1}) = \beta\mathbb{E}_t\left[V^R_{t+1}\right]\\
&n^R_{t+1} = R^K_{t+1}k^R_{t+1} + R^B_{t+1}b^R_{t+1} - R^D_{t+1}((Q_t + f^R_t)k^R_{t+1} + b^R_{t+1} - n^R_t)\\
&k^R_{t+1}, d^R_{t+1}, b^R_{t+1} \geq 0.
\end{aligned}
$$

Defining $\Omega^R_{t+1} \equiv V^R_{t+1}/n^R_{t+1} = (\sigma^R + (1-\sigma^R)\psi((Q_{t+1} + f^R_{t+1})A^R_k + \gamma A^R_b))$, the Lagrangian for this problem is given by

$$
\begin{aligned}
\mathcal{L} = &\beta\mathbb{E}_t\left[\Omega^R_{t+1}(R^K_{t+1}k^R_{t+1} + R^B_{t+1}b^R_{t+1} - R^D_{t+1}((Q_t + f^R_t)k^R_{t+1} + b^R_{t+1} - n^R_t))\right]\\
&+ \lambda\left[\psi((Q_t + f^R_t)k^R_{t+1} + \gamma b^R_{t+1})\right.\\
&\left.-\beta\mathbb{E}_t\left[\Omega^R_{t+1}(R^K_{t+1}k^R_{t+1} + R^B_{t+1}b^R_{t+1} - R^D_{t+1}((Q_t + f^R_t)k^R_{t+1} + b^R_{t+1} - n^R_t))\right]\right]
\end{aligned}
$$

This yields the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}^R} = \beta \mathbb{E}_t \left[\Omega_{t+1}^R (R_{t+1}^K - R_{t+1}^D (Q_t + f_t^R)) \right] (1-\lambda) + \lambda \psi (Q_t + f_t^R) = 0$$
$$\frac{\partial \mathcal{L}}{\partial b_{t+1}^R} = \beta \mathbb{E}_t \left[\Omega_{t+1}^R (R_{t+1}^B - R_{t+1}^D) \right] (1-\lambda) + \lambda \psi \gamma = 0$$

Combining these two equations and rearranging, we arrive at the condition

$$\mathbb{E}_t \left[\Omega_{t+1}^R \left(\frac{R_{t+1}^K}{Q_t + f_t^R} - R_{t+1}^D \right) \right] = \frac{1}{\gamma} \mathbb{E}_t \left[\Omega_{t+1}^R (R_{t+1}^B - R_{t+1}^D) \right].$$

This is basically a condition that ensures that the retail bank is indifferent between lending a marginal unit of funds to final goods producers or on the wholesale funding market.

Showing the linearity of policy functions works in the same way as in the shadow bank problem. Then, in equilibrium, it is sufficient to characterize the choices K_{t+1}^R, B_{t+1}^R and D_{t+1}^R of the retail banking sector as a whole. These choices are characterized by the following system of equations:

$$\psi((Q_t + f_t^R) K_{t+1}^R + \gamma B_{t+1}^R) = \beta \mathbb{E}_t \left[\Omega_{t+1}^R \frac{N_{t+1}^R - \upsilon^R K_{t+1}^R}{1 - \sigma^R} \right]$$
$$(Q_t + f_t^R) K_{t+1}^R + B_{t+1}^R = N_t^R + D_t^R$$
$$\mathbb{E}_t \left[\Omega_{t+1}^R \left(\frac{R_{t+1}^K}{Q_t + f_t^R} - R_{t+1}^D \right) \right] = \frac{F1}{\gamma} \mathbb{E}_t \left[\Omega_{t+1}^R (R_{t+1}^B - R_{t+1}^D) \right]$$
$$N_t^R = (R_t^K K_t^R + R_t^B B_t^R - R_t^D D_t^R)(1 - \sigma^R) + \upsilon^R K_t^R$$

Retail Banks, With Regulation

Under a binding regulatory capital requirement, the incentive constraint of the retail bank is replaced by the capital requirement:

$$\bar{\phi}_t n_t^R = (Q_t + f_t^R) k_t^R + \gamma b_{t+1}^R$$

Otherwise, the model is unchanged. In particular, the linearity of policy functions is preserved. Therefore, the choices of the aggregate retail banking sector are given by

$$
\begin{aligned}
\bar{\phi}_t N_t^R &= (Q_t + f_t^R) K_t^R + \gamma B_{t+1}^R \\
(Q_t + f_t^R) K_{t+1}^R + B_{t+1}^R &= N_t^R + D_t^R \\
\mathbb{E}_t \left[\Omega_{t+1}^R \left(\frac{R_{t+1}^K}{Q_t + f_t^R} - R_{t+1}^D \right) \right] &= \frac{1}{\gamma} \mathbb{E}_t \left[\Omega_{t+1}^R (R_{t+1}^B - R_{t+1}^D) \right] \\
N_t^R &= (R_t^K K_t^R + R_t^B B_t^R - R_t^D D_t^R)(1 - \sigma^R) + \upsilon^R K_t^R
\end{aligned}
$$

3.C.3 Production

From the problem of the capital producer follows

$$
Q_t = 1 + \theta \left(\frac{I_t}{K_t} - \delta \right).
$$

The first order conditions of the final goods producer yield

$$
\begin{aligned}
r_t^K &= \alpha Z_t K_t^{\alpha - 1} \\
W_t &= (1 - \alpha) Z_t K_t^{\alpha}.
\end{aligned}
$$

3.C.4 Full Statement of the Equilibrium Conditions

No Run Equilibrium

- Household:

$$
\begin{aligned}
&\frac{1}{C_t^H} \left(Q_t + \eta^H \frac{K_{t+1}^H}{K_t} \right) = \beta \mathbb{E}_t \left(\frac{1}{C_{t+1}^H} R_{t+1}^K \right) \\
&\frac{1}{C_t^H} = \beta \mathbb{E}_t \left(\frac{1}{C_{t+1}^H} R_{t+1}^D \right) \\
&f_t^R = \eta^R \frac{K_{t+1}^R}{K_t} \\
&C_t^H = Z_t K_t^{\alpha} + \upsilon^R K_t + \upsilon^S K_t - \frac{\eta^H}{2} \left(\frac{K_{t+1}^H}{K_t} \right)^2 K_t - \frac{\eta^R}{2} \left(\frac{K_{t+1}^R}{K_t} \right)^2 K_t \\
&\quad - I_t - \frac{\theta}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t - \sigma^R \frac{N_t^R - \upsilon^R K_t}{1 - \sigma^R} - \sigma^S \frac{N_t^S - \upsilon^S K_t}{1 - \sigma^S}
\end{aligned}
$$

- Shadow Bank:

$$\psi(N_t^S + \omega B_{t+1}^S) = \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi) \frac{N_{t+1}^S - \upsilon^S K_{t+1}^S}{1-\sigma^S} + \psi\omega(1-\sigma^S) B_{t+2}^S \right]$$
$$Q_t K_{t+1}^S = N_t^S + B_{t+1}^S$$
$$N_t^S = (R_t^K K_t^S - R_t^B B_t^S)(1-\sigma^S) + \upsilon^R K_t^S.$$

- Retail Bank:

$$\bar{\phi}_t N_t^R = (Q_t + f_t^R) K_{t+1}^R + \gamma B_{t+1}^R$$
$$(Q_t + f_t^R) K_{t+1}^R + B_{t+1}^R = N_t^R + D_t^R$$
$$\mathbb{E}_t \left[\Omega_{t+1}^R \left(\frac{R_{t+1}^K}{Q_t + f_t^R} - R_{t+1}^D \right) \right] = \frac{1}{\gamma} \mathbb{E}_t \left[\Omega_{t+1}^R (R_{t+1}^B - R_{t+1}^D) \right]$$
$$\Omega_t^R = \left(\sigma^R + (1-\sigma^R)\psi \left((Q_t + f_t^R) \frac{K_{t+1}^R}{\tilde{N}_t^R} + \frac{B_{t+1}^R}{\tilde{N}_t^R} \right) \right)$$
$$N_t^R = (R_t^K K_t^R + R_t^B B_t^R - R_t^D D_t^R)(1-\sigma^R) + \upsilon^R K_t^R$$
$$\tilde{N}_t^R = \frac{N_t^R - \upsilon^R K_t^R}{1-\sigma^R}$$

- Firms:

$$Q_t = 1 + \theta \left(\frac{I_t}{K_t} - \delta \right)$$
$$R_t^K = \alpha Z_t K_t^{\alpha-1} + (1-\delta) Q_t$$
$$W_t = (1-\alpha) Z_t K_t^{\alpha}$$

- Laws of Motion:

$$K_{t+1}^H + K_{t+1}^R + K_{t+1}^S = (1-\delta) K_t + I_t$$
$$\ln(Z_t) = (1-\rho^Z)\mu^Z + \rho_Z \ln(Z_{t-1}) + \epsilon_t$$

Run Equilibrium

- Household:

$$\frac{1}{C_t^{H,*}}\left(Q_t^* + \eta^H \frac{K_{t+1}^{H,*}}{K_t}\right) = \beta \mathbb{E}_t \left[(1-\pi)\frac{1}{C_{t+1}^H} R_{t+1}^K + \pi \frac{1}{C_{t+1}^{H,*}} R_{t+1}^{K,*}\right]$$

$$\frac{1}{C_t^{H,*}} = \beta \mathbb{E}_t \left[(1-\pi)\frac{1}{C_{t+1}^H} R_{t+1}^D + \pi \frac{1}{C_{t+1}^{H,*}} R_{t+1}^D\right]$$

$$f_t^{R,*} = \eta^R \frac{K_{t+1}^{R,*}}{K_t}$$

$$C_t^{H,*} = Z_t K_t^\alpha + \upsilon^R K_t - \frac{\eta^H}{2}\left(\frac{K_{t+1}^{H,*}}{K_t}\right)^2 K_t - \frac{\eta^R}{2}\left(\frac{K_{t+1}^{R,*}}{K_t}\right)^2 K_t$$

$$- I_t^* - \frac{\theta}{2}\left(\frac{I_t^*}{K_t} - \delta\right)^2 K_t - \sigma^R \frac{N_t^{R,*} - \upsilon^R K_t}{1-\sigma^R}$$

- Shadow Bank:

$$C_t^{S,*} = 0$$
$$N_t^{S,*} = 0$$
$$B_{t+1}^* = 0$$
$$K_{t+1}^{S,*} = 0$$

- Retail Bank:

$$\bar{\phi}_t N_t^{R,*} = (Q_t + f_t^R) K_{t+1}^{R,*}$$

$$(Q_t^* + f_t^{R,*}) K_{t+1}^{R,*} = N_t^{R,*} + D_{t+1}^{R,*}$$

$$\Omega_t^R = \left(\sigma^R + (1-\sigma^R)\psi\left((Q_t^* + f_t^{R,*})\frac{K_{t+1}^{R,*}}{\tilde{N}_t^{R,*}}\right)\right)$$

$$N_t^{R,*} = (R_t^{K,*} K_t^R + \xi_t R_t^{K,*} K_t^S - R_t^D D_t^R)(1-\sigma^R) + \upsilon^R K_t^R$$

$$\tilde{N}_t^{R,*} = \frac{N_t^{R,*} - \upsilon^R K_t}{1-\sigma^R}$$

- Firms:

$$Q_t^* = 1 + \theta\left(\frac{I_t^*}{K_t} - \delta\right)$$
$$R_t^{K,*} = \alpha Z_t K_t^{\alpha-1} + (1-\delta)Q_t^*$$
$$W_t = (1-\alpha)Z_t K_t^{\alpha}$$

- Laws of Motion:

$$K_{t+1}^{H,*} + K_{t+1}^{R,*} = (1-\delta)K_t + I_t^*$$
$$\ln(Z_t) = (1-\rho^Z)\mu^Z + \rho_Z \ln(Z_{t-1}) + \epsilon_t$$

3.D Computation

We solve the model nonlinearly using a time iteration algorithm. Solving the model nonlinearly is important, because bank runs can lead to large deviations from steady state, where perturbation algorithms are inaccurate.

The state space of the model is $\mathcal{S} = (N^R, N^S, K, Z)$ in the *"no bank run"* equilibrium and $\mathcal{S}^* = (N^{R,*}, K, Z)$ in the *"bank run"* equilibrium. We approximate the consumption policy functions $C^H(\mathcal{S}), V^R(\mathcal{S}), V^S(\mathcal{S}), C^{H,*}(\mathcal{S}^*)$ and $V^{R,*}(\mathcal{S}^*)$ and the capital prices $Q(\mathcal{S})$ and $Q^*(\mathcal{S}^*)$ using fourth order polynomials. We compute the polynomial coefficients by imposing that the polynomial approximations must be equal to the original functions on the grid. Specifically, denoting the polynomial coefficients by α and the polynomials by $\Pi(\mathcal{S})$, we impose for example for the consumption of households

$$\Pi(\mathcal{S}_i)\alpha_{C^H} = C^H(\mathcal{S}_i) \quad i = 1, \ldots, N. \tag{3.D.1}$$

for all N grid points. We use a Smolyak grid with order $\mu = 4$ for the endogenous states and $\mu = 3$ for the exogenous states. We compute the Smolyak grid and polynomials using the toolbox by Judd, Maliar, Maliar, and Valero (2014).

One slight complication of the model is that the future net worth values N_R and N_S, depends on $Q(\mathcal{S})$. This implies that, for example, the household net worth for a given function $Q(.)$ must be computed as a solution to the nonlinear function

$$N^{R,'} = \Big[(r^K + (1-\delta)Q(N^{R,'}, N^{S,'}, K', Z'))K^{R,'} + \\ R^{B,'}B' - R^{D,'}D'\Big](1-\sigma^R) + \upsilon^R K. \tag{3.D.2}$$

With this in mind, we will now outline our solution algorithm for the *"no bank run"* equilibrium. Suppose we are in iteration k and have initial guesses for the no-run consumption policy functions $C^H_{(k)}(\mathcal{S}), V^R_{(k)}(\mathcal{S})$, and $V^S_{(k)}(\mathcal{S})$ and the capital price function $Q_{(k)}(\mathcal{S})$ as well as values for the future net worth $N^{R,'}_{(k)}$ and $N^{S,'}_{(k)}$.

1. Given the value functions and the future net worth, compute the future value functions and capital prices as

$$Q'_{(k)} = .$$

2. Compute the new values for $(K^{H,'}, K^{R,'}, K^{S,'}, D', B', R^{D,'}, R^{B,'}, Q)$ for all grid points $i = 1, \ldots, N$ using the first order conditions 3.C.1, 3.C.2, 3.C.9, 3.C.10, 3.C.3, 3.3.26 and the leverage constraints 3.3.14 and 3.3.8. Compute the future net worth where necessary according to

$$\tilde{N}^{R,'}_{(k+1)} = \Big[(r^{K,'} + (1-\delta)Q_{(k)}(N^{R,'}_{(k)}, N^{S,'}_{(k)}, K', Z'))K^{H,'} \\ + R^{B,'}B' - R^{D,'}D'\Big](1-\sigma^R) + \upsilon^R K'. \tag{3.D.3}$$

   We compute expectations using Gauss-Hermite quadrature. Note that for each quadrature node Z', a different value of $\tilde{N}^{R,'}_{(k+1)}$ must be computed.

3. Using the new policies and prices, update the consumption function of the household using equation 3.3.29, and the value functions for the retail and shadow banks using equations 3.3.9 and 3.3.18.

4. Update the next period net worth values using 3.D.3, with some attenuation: $N^{R,'}_{(k+1)} = (1-\iota)N^{R,'}_{(k)} + \iota\tilde{N}^{R,'}_{(k+1)}$, with $\iota = 0.5$.

5. Repeat until the errors in the consumption, capital price and net worth values on the grid are small. We iterate until the maximum

error in consumption is smaller than 1e-5 and the maximum error in the net worth is smaller than 1e-5.

If bank runs are unanticipated, we can first solve for the *"no bank run"* equilibrium and then afterwards for the *"bank run"* equilibrium. Importantly, expectations during a bank run are taken over the future *"bank run"* and *"no bank run"* states. It is therefore necessary to keep track of two sets of net worth values, $N_{(k)}^{R,'}$ and $N_{(k)}^{R,',*}$. Otherwise, the algorithm works in the same way as for the *"no bank run"* equilibrium. For the anticipated run case, we use the unanticipated run case as initial guess and solve jointly for the *"no bank run"* and *"bank run"* policy functions.

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