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Master thesis:

Analysis of the dynamic effects of fiscal policy using a factor augmented VAR approach

by

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1 Introduction

During the 30 years preceding the world economic crisis starting in 2007, most economists agreed on the thesis that governments should limit their interventions in the economy to the enforcement of a legal framework, to providing certain public goods and let the markets do "the rest". The world economic crisis in 2007-09 triggered a new discussion among economists, and governments all over the world implemented fierce Fiscal Policy measures to overcome the crisis.

This revived the topic and put it back into the focus of economic research, as the main questions remain unresolved: it is not clear whether government interventions have an effect on the economy, and which one. As an example, it is not clear whether an increase in government spending has a positive effect, a negative effect or no effect at all on GDP. Research on the matter is done by theoretical modelling and empirical studies.

Regarding the theoretical models, these can be divided into two broad categories: neoclassical models and (neo-)Keynesian models. In the neoclassical world, private households react to increased government spending by cutting private consumption, because they anticipate increasing taxes in the future. Moreover, because government spending usually means "deficit spending", demand for capital increases, driving up the interest rate and thus crowding out private investment. On the other hand, in Keynesian models increasing government spending increases aggregate demand, leaving aggregate supply untouched: prices are assumed to be sticky in the short run. The usual way to reconcile these contradictory findings is by noting that Keynesian models better explain the economy in the short run whereas neoclassical models are better suited for the long run.

Two main approaches are traditionally used to address the topic empirically: the narrative approach and the SVAR approach. The first one, applied for example by Ramey (2011), tries to identify exogenous events triggering spikes in government spending or revenue. Most commonly this approach uses some sorts of war dummies. The SVAR approach on the other handside tries to achieve identification by imposing structural restrictions in vector autoregressive models. An example for this approach is Blanchard & Perotti (2002).

The main problem with both approaches is anticipated Fiscal Policy. Generally speaking it takes at least various months for a government to implement new measures, which is very much alike for taxes and for spending. Moreover, intentions to raise or lower taxes or spending are – at least in democratic countries – publicly known, making it possible for the general public to adjust their behaviour, whereas the empirical researcher "sees" the new measures only when these appear in the data, which is often years later.

This study tries to tackle this and other issues by implementing a structural Factor Augmented VAR. It extends the SVAR approach, overcoming various limitations. To start with, "conventional" time series econometrics faces a peculiar data structure: there are hundreds of variables made available for analysis by statistical and other institutions, but the number of observations is very limited and the sample size cannot easily be inceased either. Badly enough, standard VARs impose tight limitations on the researcher regarding the number of series, as the number of parameters to be estimated increases quadratically.

The Factor Augmented VAR – or FAVAR – allows the inclusion of literally hundreds of variables. The motivation is, on one handside, that economic agents keep track of a much richer information set than is typically found in VARs and, on the other handside the fact that the choice of a variable to reflect certain economic concepts is arbitrary. Another huge benefit is that the effects on various variables can be analyzed in one model, which is impossible for simple VAR models.

Interestingly, the author could not find a study, which applied Factor Augmented VARs to analyze the effects of Fiscal Policy. That is the aim of this study. The methodology is as follows: Identification is done via block lower triangular exclusion restrictions – this is similar to Bernanke, Boivin & Eliasz (2005) – and seems most appropriate for the case here. After estimation of the reduced form model and the structural equation, an Impulse Response Analysis was performed to assess the effects of Fiscal Policy. The analysis was done for five key variables: GDP, Average Hours, Consumer Price Index, Industrial Production and Interest Rate Spread. For each of those, the effects of a spending shock as well as of a tax shock was analyzed.

Literature on Dynamic Factor Models in general is available in abundance, but on Factor Augmented VARs availability is rather sparse. The method of Dynamic Factor Models was introduced by Geweke (1977). Influential work includes, for example Stock & Watson (2002). The method of Factor Augmented VARs (as well as the term "FAVAR") was introduced and first applied by Bernanke et al. (2005). Their study focuses on the effects of monetary policy. A good overview and introduction to the topic is given by Stock & Watson (2005).

The remainder of this paper is organized as follows: Section 2 introduces the Dynamic Factor Model and the Factor Augmented VAR. Moreover it gives an overview regarding estimation of these models. The application to the Fiscal Policy topic is presented in Section 3. The next section (Section 4) contains a brief description of the data used in this study. Section 5 presents the empirical findings, and Section 6 concludes.

2 The theory of Dynamic Factor Model and Factor Augmented VARs

The presentation of this section is closely related to Stock & Watson (2005). In most parts I adopt the same notation.

2.1 The Dynamic Factor Model

The principal idea behind a Dynamic Factor Model is to assume that all observable series, or variables, are driven by a relatively small number of unobserved series (factors), plus some idiosyncratic disturbance, which might be due to measurement errors or because of differences in the concepts of variables. Regarding the latter, consider for example GDP and GNP, which move rather parallel, but measure slightly different concepts of economic activity.

Formally speaking, the observed series X_{it}, $i = 1, \ldots, n$, $t = 1, \ldots, T$ are modelled as a function of the unobserved dynamic factors f_t, plus a serially correlated error term, i.e.

$$X_{it} = \tilde{\lambda}_i(L) f_t + u_{it} \quad (1)$$

$$u_{it} = \delta_i(L) u_{i,t-1} + v_{it} \quad (2)$$

with $E(f_t v_{is}) = 0 \; \forall i, t, s$.

Expanding (1) with $1 - \delta_i(L)L$ and plugging it into (2) yields a representation with serially uncorrelated error term:

$$X_{it} = \lambda_i(L) f_t + \delta_i(L) X_{i,t-1} + v_{it} \quad (3)$$

with $\lambda_i(L) = (1 - \delta_i(L)L)\tilde{\lambda}_i(L)$.

The dynamic factors are usually assumed to follow a vector autoregressive process. Writing (3) in matrix notation – i.e. defining $X_t = \begin{pmatrix} X_{1,t} & \ldots & X_{n,t} \end{pmatrix}'$ and setting

$$\lambda(L) = \begin{pmatrix} \lambda_1(L) \\ \vdots \\ \lambda_n(L) \end{pmatrix}, \quad D(L) = \begin{pmatrix} \delta_1(L) & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \delta_n(L) \end{pmatrix} \text{ and } v_t = \begin{pmatrix} v_{1t} \\ \vdots \\ v_{nt} \end{pmatrix} \quad (4)$$

one then obtains

$$X_t = \lambda(L) f_t + D(L) X_{t-1} + v_t \quad (5)$$

$$f_t = \Gamma(L) f_{t-1} + \eta_t \quad (6)$$

with $E(\eta_t v_{is}) = 0 \; \forall i, t, s$.

2.2 The static form of the Dynamic Factor Model

Estimation of the system (14) and (15) requires two-sided smoothing and cannot be applied for structural analysis as is done in this paper. To circumvent this, a static representation can be derived under certain assumptions, which leads to estimates suitable for the purpose here.

Assuming $\lambda(L)$ being of degree $p-1$ and defining $F_t = \begin{pmatrix} f_t' & \dots & f_{t-p+1}' \end{pmatrix}'$, equations (14) and (15) can be rewritten in static form:

$$X_t = \Lambda F_t + D(L)X_{t-1} + v_t \tag{7}$$

$$F_t = \phi(L)F_{t-1} + G\eta_t \tag{8}$$

where F_t is of degree $r \times 1$ with $q \leq r \leq pq$.

2.3 VAR form of the Dynamic Factor Model

Putting (7) and (8) together yields the VAR form:

$$\begin{pmatrix} F_t \\ X_t \end{pmatrix} = \begin{pmatrix} \phi(L) & 0 \\ \Lambda\phi(L) & D(L) \end{pmatrix} \begin{pmatrix} F_{t-1} \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{F_t} \\ \varepsilon_{X_t} \end{pmatrix} \tag{9}$$

with

$$\begin{pmatrix} \varepsilon_{F_t} \\ \varepsilon_{X_t} \end{pmatrix} = \begin{pmatrix} I \\ \Lambda \end{pmatrix} G\eta_t + \begin{pmatrix} 0 \\ v_t \end{pmatrix} \tag{10}$$

2.4 MA form of the Dynamic Factor Model

For various purposes, especially for structural analysis, it is useful to denote the Factor Model in terms of its moving average representation, which is derived here. To start, denote the static factors as a (linear) function of its error terms:

$$\begin{aligned} F_t &= \phi(L)F_{t-1} + G\eta_t \\ [I - \phi(L)L]F_t &= G\eta_t \\ F_t &= [I - \phi(L)L]^{-1}G\eta_t \, . \end{aligned}$$

Repeating the same for X_t and plugging in the result from above one obtains

$$\begin{aligned} X_t &= \Lambda F_t + D(L)X_{t-1} + v_t \\ &= \Lambda[I - \phi(L)L]^{-1}G\eta_t + D(L)X_{t-1} + v_t \\ [I - D(L)L]X_t &= \Lambda[I - \phi(L)L]^{-1}G\eta_t + v_t \\ X_t &= \underbrace{[I - D(L)L]^{-1}\Lambda[I - \phi(L)L]^{-1}G}_{=:B(L)}\eta_t + \underbrace{[I - D(L)L]^{-1}v_t}_{u_t} \\ &= B(L)\eta_t + u_t \end{aligned} \tag{11}$$

This moving average representation can also be used to derive "factor augmented" impulse response functions, which are similar to those obtained by Cholesky type "standard" structural VARs, as identification is also achieved via normalizations and not via economic modelling.

2.5 Structural Analysis

Let ζ_t denote the q structural shocks to the dynamic factors. They are assumed to be related to the "reduced form" structural shocks in a linear form, i.e.

$$\zeta_t = H\,\eta_t \tag{12}$$

where H is an invertible $q \times q$ matrix. It is assumed that

$$E\zeta_t\zeta_t' = I \tag{13}$$

i.e. $H\Sigma_\eta H' = I$.

To distinguish reduced-form and structural parameters, denote structural parameters with "$\star$", e.g. $\lambda^\star(L)$ is the structural form lag polynomial of the Dynamic Factor Model. Defining $f_t^\star = Hf_t$, $\lambda^\star(L) = \lambda(L)H^{-1}$ and $\Gamma^\star(L) = H\Gamma(L)H^{-1}$ one can rewrite the system (14) - (15) in structural form:

$$X_t = \lambda^\star(L)f_t^\star + D(L)X_{t-1} + v_t \tag{14}$$

$$f_t^\star = \Gamma^\star(L)f_{t-1}^\star + \zeta_t \tag{15}$$

One can then obtain the moving average representation as follows:

$$X_t = B^\star(L)\zeta_t + u_t \tag{16}$$

where $B^\star(L)$ is $(n \times q)$ with $B^\star(L) = B(L)H^{-1}$.

Now, H can be identified via restricting $B^\star(L)$ ("timing restrictions") and/or $\lambda^\star(L)$ ("factor loading restrictions"), which means identification can range from exact identification to heavy overidentification.

Timing restrictions can be further divided into contemporaneous timing restrictions and long run restrictions. Timing restrictions imply that the structural shock of a variable does not have an influence on other variables within a period. For example, the assumption that a change in GDP in a certain quarter does not affect taxes within the same quarter would be such a restriction. Timing restrictions are the identification method of choice here and will be discussed in the following subsections.
For a discussion of factor loading restrictions and other identification schemes see for example Stock & Watson (2005), sections 3.4 and 3.5.

2.5.1 Contemporaneous timing restrictions: Cholesky factorization

From the MA-representation (16) it follows that

$$\varepsilon_{X_t} \;=\; B_0^\star \zeta_t + v_t \tag{17}$$

where $B_0^\star$ is the "zero-lag-term" in $B^\star(L)$.

Suppose for $B_0^\star$ a lower triangular form, i.e. let

$$B_0^\star \;=\; \begin{pmatrix} B_{0;q}^\star \\ B_{0;-q}^\star \end{pmatrix} \tag{18}$$

with

$$B_{0;q}^\star = \begin{pmatrix} x & 0 & \dots & 0 \\ x & x & \ddots & \vdots \\ \vdots & \ddots & x & 0 \\ x & \dots & x & x \end{pmatrix} \qquad \text{and} \qquad B_{0;-q}^\star = \begin{pmatrix} x & \dots & x \\ \vdots & \ddots & \vdots \\ x & \dots & x \end{pmatrix} \tag{19}$$

where $B_{0;q}^\star$ is $(q \times q)$, $B_{0;-q}^\star$ is $(n - q \times q)$ and the elements marked with "x" are unrestricted elements different from zero. It is easy to see that this structure imposes $q(q-1)/2$ restrictions and that – together with the normalization (13) – H is exactly identified.

Analogously to $B_{0;q}^\star$ define the first q rows of B_0 as $B_{0;q}$. Obviously $B_{0;q}^\star \zeta_t \;=\; B_{0;q}\eta_t$ and thus $B_{0;q}^\star {B_{0;q}^\star}' \;=\; B_{0;q}\, \Sigma_\eta\, {B_{0;q}}'$. $B_{0;q}^\star$ is lower triangular by assumption, thus

$$B_{0;q}^\star \;=\; \text{Chol}(B_{0;q}\, \Sigma_\eta\, {B_{0;q}}') \tag{20}$$

With $B^\star(L) = B(L)H^{-1} = [I - D(L)L]^{-1}\Lambda[I - \phi(L)L]^{-1}GH^{-1}$ it follows from the recursion formula of the moving average representation of VAR processes that $B_0^\star = \Lambda G H^{-1}$ and thus

$$H \;=\; \left[\text{Chol}(B_{0;q}\, \Sigma_\eta\, {B_{0;q}}')\right]^{-1} \Lambda_q G\,, \tag{21}$$

where Λ_q contains the first q rows of Λ.

2.5.2 Contemporaneous timing restrictions: Partial identification via block lower triangular exclusion restrictions

In this case the ordering of the variables is not "complete" as in the case of the Cholesky factorization, but instead the variables are divided into subgroups. The identifying assumption then is that the variables within one group do not contemporaneously influence variables of another group.

One example for this is the method used by Bernanke et al. (2005) – which was actually the "invention" of FAVAR models. They divide their set of variables into three subgroups: fast moving ones, the interest rate, and slow moving ones. Denote by ζ_t^F, ζ_t^S and ζ_t^R the structural shocks of fast moving and slow moving variables and of the interest rate, respectively. They assume that ζ_t^S can affect all variables, ζ_t^R can affect only the interest rate and fast moving variables and ζ_t^F can only affect fast moving variables, within a period respectively. $B_0^\star$ thus has a block lower triangular form:

$$B_0^\star = \begin{pmatrix} B_{0,SS}^\star & 0 & 0 \\ B_{0,RS}^\star & B_{0,RR}^\star & 0 \\ B_{0,FS}^\star & B_{0,FR}^\star & B_{0,FF}^\star \end{pmatrix}. \tag{22}$$

The dimension of $B_{0,SS}^\star$ is $(n_S \times q_s)$, of $B_{0,RS}^\star$ $(1 \times q_S)$, $B_{0,RR}^\star$ is a scalar, $B_{0,FS}^\star$ is $(n_F \times q_S)$, $B_{0,FR}^\star$ is $(n_F \times 1)$ and $B_{0,FF}^\star$ is $(n_F \times q_F)$, with $n = n_S + n_F + 1$.

With this scheme ζ_t^R is identified, as well as the spaces spanned by ζ_t^S and ζ_t. To see this formally, one can rewrite (12) to get:

$$\begin{pmatrix} \zeta_t^S \\ \zeta_t^R \\ \zeta_t^F \end{pmatrix} = \begin{pmatrix} H_S' \\ H_R' \\ H_F' \end{pmatrix} \eta_t \tag{23}$$

Then (17) becomes, using the restrictions in (22) and the notation from (23)

$$\begin{aligned} \varepsilon_{Xt}^S &= B_{0,SS}^\star H_S' \eta_t + v_t^S \\ \varepsilon_{Xt}^R &= B_{0,RS}^\star H_S' \eta_t + B_{0,RR}^\star H_R' \eta_t + v_t^R \\ \varepsilon_{Xt}^F &= B_{0,FS}^\star H_S' \eta_t + B_{0,FR}^\star H_R' \eta_t + B_{0,FF}^\star H_F' \eta_t + v_t^F , \end{aligned} \tag{24}$$

where ε_{Xt}^S, ε_{Xt}^R and ε_{Xt}^F are the innovation vectors of the slow variables, the interest rate and the fast variables and of dimension $(n_S \times 1)$, (1×1) and $(n_F \times 1)$ respectively.

Now, it is easily seen that $\text{rank}(B_{0,SS}^\star H_S') = q_S$, because $B_{0,SS}^\star$ is of dimension $(n_S \times q_S)$, H_S' is of dimension $(q_S \times q)$ and assuming that $n_S \geq q_S$. This means that the projection of ε_{Xt}^S onto η_t spans a space of dimension q_S. It is the same space as the one spanned by ζ_t^S.

In the same manner, because the space spanned by ζ_t^S is identified and $B_{0,RR}^\star H_R'$ is $(1 \times q)$, ζ_t^R is identified (up to scale) as $\zeta_t^R = \text{Proj}(\varepsilon_{Xt}^R|\eta_t) - \text{Proj}(\varepsilon_{Xt}^R|\zeta_t^S)$. Finally, the space spanned by ζ_t^F is the space of η_t that is orthogonal to ζ_t^S and ζ_t^R.

2.5.3 Partial identification via block lower triangular exclusion restrictions: a more general approach

A more general approach would be to define groups of similar variables, e.g. put stock market variables into one group, exchange rates into another, and so on. Assume a number of q groups with n_i variables within each of them, such that $\sum_{i=1}^{q} = n$. In this case, $B_0^\star$ has a similar form to the one discussed above, i.e. is block lower triangular:

$$B_0^\star = \begin{pmatrix} B_{0,11}^\star & 0 & \dots & 0 \\ B_{0,12}^\star & B_{0,22}^\star & \ddots & 0 \\ \vdots & \vdots & \ddots & 0 \\ B_{0,1q}^\star & B_{0,2q}^\star & \dots & B_{0,qq}^\star \end{pmatrix}. \quad (25)$$

The dimension of $B_{0,ij}^\star$ in (25) is $(n_i \times 1)$, i.e. a vector. With the same argument as above (partitioning H and ε_{Xt}, ...) one can algebraically show identification up to scale of H.

Note however that it is implicitly assumed that each group of variables corresponds to one structural shock. For a clean econometric identification thus a theoretical model would be needed that explained the macroeconomy using a number of m variables. This model could then be used to

- collect various time series for each theoretical variable, thus creating the m groups of variables. Application of the factor model could be justified with measurement errors and slightly different economic concepts, i.e. the standard arguments of Dynamic Factor Models,
- ex ante imposing m as the number of factors, i.e. setting $q = m$ and not to be in need of estimating this and
- using relationships of the variables in the model to identify H – for example applying the Wold causal ordering to the m groups of variables as in (25).

2.6 Estimation

2.6.1 General notes

There are a few different methods available to estimate Dynamic Factor Models. One is to construct the Gaussian likelihood by using the Kalman filter, knowing that (14) and (15) together form a linear state space model. This method however is computationally demanding, which is why the present paper focuses on estimation using Principal Component Analysis.

Regarding the structural models, estimation is basically a two-step procedure: First, the reduced form Factor Model, G and $\{\eta_t\}$ are estimated using Principal Component Analysis and secondly, the identifying restrictions of the structural part are used to estimate H and $\{\zeta_t\}$ and thus the structural impulse response function $B^\star(L)$ can be constructed.

2.6.2 VAR representation of the Factor Model

Estimation is a multi-step procedure:

(1) Estimation of the number of static factors
This is conveniently done using the information criteria developed by Bai & Ng (2002). These criteria can be computed using the sample covariance matrix either of X_t or of the prefiltered variables $\tilde{X}_t \;=\; (I - \hat{D}(L)L)X_t$.

The authors suggest the use of IC_{p1}, defined as

$$IC_{p1}(k) \;=\; ln\left(V(k, \hat{F}^k)\right) + k\left(\frac{N+T}{NT}\right) ln\left(\frac{NT}{N+T}\right) \tag{26}$$

and/or the use of IC_{p2}, defined as

$$IC_{p2}(k) \;=\; ln\left(V(k, \hat{F}^k)\right) + k\left(\frac{N+T}{NT}\right) ln\left(C^2_{NT}\right)\;, \tag{27}$$

where

$$V(k, \hat{F}^k) \;=\; \min_{\Lambda} \frac{1}{NT} \sum_{i=1}^{n} \sum_{t=1}^{T} \left(X_{it} - \lambda_i^k \hat{F}_t^k\right)^2 \tag{28}$$

is the sum of squared residuals divided by N and T, of the factor model using k factors and $C_{NT} = min\{\sqrt{N}, \sqrt{T}\}$.

(2) Estimation of the static factors
As mentioned earlier, regarding estimation of the factors the focus of this paper is on Principal Component Analysis. However, the situation here is more complicated than in the "standard" case of Dynamic Factor Models, given that the X_t do not only depend on the factors, but also have an autoregressive component (see equation 7).

The minimization problem to obtain estimators for F_t and Λ is given by

$$\min_{F_1,\ldots,F_T,\Lambda.D(L)} \quad T^{-1} \sum_{t=1}^{T} [(I - D(L)L)X_t - \Lambda F_t]'[(I - D(L)L)X_t - \Lambda F_t] \tag{29}$$

Estimation is done iteratively: starting with an initial estimate of $D(L)$, $\{F_t\}$ can be obtained by Principal Component Analysis (the first r principal components of $\tilde{X}_t = (I - D(L)L)X_t$. Using this, individual regressions of X_{it} on F_t and $(X_{i,t-1}, \ldots, X_{i,t-a_i+1})$ can be used, with a_i being the order of $\delta_i(L)$. These steps do not increase the sum of squares of the minimization problem, thus the procedure converges.

(3) Estimation of the restricted VAR coefficients
By defining or estimating the number of lags in (8), F_t can be regressed onto its lags, thus obtaining an estimate for $\phi(L)$. Using $\hat{\Lambda}$ and $\hat{D}(L)$ from the previous step and $\hat{\phi}(L)$ from this, the VAR coefficient matrix in (9) is obtained.

(4) Estimation of the number of dynamic factors
The Dynamic Factor Model (the static form and the VAR form) provide various overidentifying restrictions. For a discussion of these see for example Stock & Watson (2005), section 2.3.

Two of these can be used to estimate the number of dynamic factors. The method presented here builds upon restriction 3 which states that $E(\varepsilon_{Xt}\varepsilon'_{Xt}) = \Lambda G \Sigma_\eta G' \Lambda' + \Sigma_v$, i.e. ε_{Xt} – the innovations of X_t – follow a factor model. Then, estimation of ε_{Xt} proceeds by using the previous estimation results: $\hat{\varepsilon}_{Xt}$ are obtained as the residuals of system (9). The estimate $\hat{q}$ for q is obtained using the Bai & Ng (2002) information criteria on the covariance matrix of $\hat{\varepsilon}_{Xt}$.

(5) Estimation of the space spanned by dynamic shocks
The method used to obtain the dynamic factor error terms η_t is derived from the assumption of uncorrelated innovations and with the aim of maximizing the trace of $\Sigma_X = E(XX')$, with the usual ordering, i.e. the first factor contributes the biggest part of the total variance, and so on.
Formally, rewrite $B(L)$ from the moving average representation (11) as $B(L) = A(L)G$ and normalize $\Sigma_\eta = I$. Then it follows:

$$\begin{aligned} \mathrm{tr}\{E[(B(L)\eta_t)(B(L)\eta_t)']\} &= \mathrm{tr}\{E[(A(L)G\eta_t)(A(L)G\eta_t)']\} \\ &= \mathrm{tr}\{\ \sum_{j=0}^{\infty} A_j G \Sigma_\eta G' A'_j\ \} = \mathrm{tr}\{\ G'\ (\ \sum_{j=0}^{\infty} A'_j A_j\ )\ G\ \} \end{aligned}$$

and further

$$\mathrm{tr}\,\{\Sigma_X\} = \mathrm{tr}\{\ G'\ (\ \sum_{j=0}^{\infty} A'_j A_j\ )\ G\ \} + \mathrm{tr}\{\Sigma_u\} \tag{30}$$

which means that the trace is maximized by constructing G as the q eigenvectors of $\sum_{j=0}^{\infty} A'_j A_j$ that correspond to the highest eigenvalues. The estimator $\hat{G}$ is then computed as the sample version of this matrix of eigenvectors using $A(L)$ as defined above and replacing population moments with the corresponding sample analogs.

2.6.3 Structural models with Cholesky type restrictions

Estimation of the structural part in this case is straightforward, by replacing the population matrices in (21) with their estimated counterparts. These estimates can be obtained as described in the previous section.

2.6.4 Structural models with block lower triangular restrictions

Estimation in this case is along the scheme of identification (section 2.5.2):

(1) Estimate, as discussed above, the innovations ε_{Xt} and η_t, i.e. obtain $\hat{\varepsilon}_{Xt}$ and $\hat{\eta}_t$

(2) Estimate the number of structural shocks in the slow variables q_S. This can be done as the estimation of q discussed above, but using only the slow variables.

(3) Estimate $B^\star_{0,SS}$ and H_S using reduced rank regression: regress $\hat{\varepsilon}_{Xt^S}$ on $\hat{\eta}_t$, restricting the rank of the coefficient matrix to q_S. Then, $\hat{\zeta}^S_t$ can be calculated from the results. Regarding reduced rank regression, Section 2.7 gives a brief introduction.

(4) Estimate ζ_t^R and $\hat{H}_R$ with the formulas from section 2.5.2:

- A linear projection can be represented by a projection matrix – which depends on the space onto which is projected – times the element that is projected. More formally: $\text{Proj}(\varepsilon_{Xt}^R|\eta_t) = P_\eta \varepsilon_{Xt}^R$ and $\text{Proj}(\varepsilon_{Xt}^R|\zeta_t^S) = P_{\zeta_t^S} \varepsilon_{Xt}^R$, where P_η and $P_{\zeta_t^S}$ are the projection matrices.
- As "ordinary least squares" has the interpretation of linear projections, these can be estimated using OLS. The projection matrices then become $P_{\hat{\eta}} = \eta_t'(\eta_t\eta_t')^{-1}\eta_t$ and $P_{\zeta_t^S} = \zeta_t^{S\prime}(\zeta_t^S\zeta_t^{S\prime})^{-1}\zeta_t^S$. The projections then essentially become "fitted values".
- Replace population moments with estimated ones: $\hat{\zeta}_t^R = \text{Proj}(\hat{\varepsilon}_{Xt}^R|\hat{\eta}_t) - \text{Proj}(\hat{\varepsilon}_{Xt}^R|\hat{\zeta}_t^S)$.
- By regressing $\hat{\zeta}_t^R$ on $\hat{\eta}_t$ one obtains $\hat{H}_R$.
- The corresponding parts of $B_0^\star$ can be estimated using linear regression.

(5) The space of η_t that is orthogonal to ζ_t^S and ζ_t^R can be estimated using OLS: regress η_t onto ζ_t^S and ζ_t^R and take the residuals as an estimate of the space spanned by ζ_t^F (using the normalization $\Sigma_\zeta^F = I$). H_F' and $B_{0,Fc}'$, $c \in \{S, R, F\}$ can then be obtained as above.

2.7 Digression on Reduced Rank Regression

This method is about estimating a multivariate, linear regression model, where the coefficient matrix has not full rank. It was first introduced by Anderson (1951). Probably the best presentation of the topic, with references to economic modelling, is given in Reinsel & Velu (1998). The digression here is closely related to the latter.

As mentioned above, the starting point is a multivariate linear regression model. Let

$$Y_k = CX_k + \epsilon_k, \qquad k = 1, \dots, T \tag{31}$$

where Y_k is a $m \times 1$ dimensional vector ("dependent variables") and X_k is a vector of dimension $n \times 1$ ("independent variables"). The ϵ_k are modelled to be independent for different k.

The key point for reduced rank regression then is to assume that the $m \times n$ dimensional coefficient matrix C is not of full rank, i.e.

$$\text{rank}(C) = r \leq \min\{m, n\} \tag{32}$$

In this case, C can be written as $C = AB$, with A being $(m \times r)$, B being $(r \times n)$ and both matrices having full rank r:

$$rank(A) = rank(B) = r \tag{33}$$

Thus, (31) can be written as

$$Y_k = ABX_k + \epsilon_k, \qquad k = 1, \dots, T \tag{34}$$

The estimators for A and B are presented in Theorem 2.2 in Reinsel & Velu (1998), page 28:

Theorem 2.2 Suppose the $(m+n)$-dimensional random vector $(Y', X')'$ has mean vector 0 and covariance matrix with $\Sigma_{yx} = \Sigma'_{xy} = \text{Cov}(Y, X)$ and $\Sigma_{xx} = \text{Cov}(X)$ nonsingular. Then for any positive-definite matrix Γ, an $m \times r$ matrix A and an $r \times n$ matrix B, for $r \leq \min(m, n)$, which minimize

$$\text{tr}\{E[\Gamma^{1/2}(Y - ABX)(Y - ABX)'\Gamma^{1/2}]\} \tag{35}$$

are given by

$$A^{(r)} = \Gamma^{-1/2}[V_1, \ldots, V_r] = \Gamma^{-1/2}V \ , \qquad B^{(r)} = V'\Gamma^{1/2}\Sigma_{yx}\Sigma_{xx}^{-1} \tag{36}$$

where $V = [V1, ..., V_r]$ and V_j is the (normalized) eigenvector that corresponds to the j$^{\text{th}}$ largest eigenvalue λ_j^2 of the matrix $\Gamma^{1/2}\Sigma_{yx}\Sigma_{xx}^{-1}\Sigma_{xy}\Gamma^{1/2}$ $(j = 1, 2, \ldots, r)$.

A and B are not determined up to scale, but only the space spanned by these: for any nonsingular matrix $H_{r\times r}$, $C = AHH^{-1}B$ is another possible decomposition. Thus, to get a unique solution, usually normalization conditions are imposed. Regarding (36), the implicit normalization is $V'V = I_r$, which reflects on A and B in the following manner:

$$B\Sigma_{xx}B' = \Lambda^2 \qquad \text{and} \qquad A'\Gamma A = I_r \tag{37}$$

with $\Lambda^2 = diag(\lambda_1^2, ..., \lambda_r^2)$.

In order to derive the ML estimator, which is asymptotically efficient under the following assumptions, define $\mathbf{Y} = [Y_1, \ldots, Y_T]$ and $\mathbf{X} = [X_1, \ldots, X_T]$, with T being the number of observations. Moreover, assume that $\epsilon_k \overset{iid}{\sim} N(\mathbf{0}, \Sigma_{\epsilon\epsilon})$.

When population moments are replaced by sample moments, i.e. $\hat{\Sigma}_{yx} = T^{-1}\mathbf{YX'}$, $\hat{\Sigma}_{xx} = T^{-1}\mathbf{XX'}$ and

$$\hat{A}^{(r)} = \Gamma^{-1/2}[\hat{V}_1, \ldots, \hat{V}_r] \ , \qquad \hat{B}^{(r)} = \hat{V}'\Gamma^{1/2}\hat{\Sigma}_{yx}\hat{\Sigma}_{xx}^{-1}, \tag{38}$$

the log-likelihood is maximized by setting $\Gamma = \tilde{\Sigma}_{\epsilon\epsilon}^{-1}$, where

$$\tilde{\Sigma}_{\epsilon\epsilon} = T^{-1}(\mathbf{Y} - \tilde{C}\mathbf{X})(\mathbf{Y} - \tilde{C}\mathbf{X})' \tag{39}$$

is obtained from the full rank regression model by ordinary least squares, i.e. $\tilde{C} = \mathbf{YX'}(\mathbf{XX'})^{-1}$.

3 Application of structural Factor Augmented VARs to Fiscal Policy

3.1 SVAR approach of Blanchard & Perotti (2002)

This section is basically a summary of Blanchard & Perotti (2002).

Their baseline VAR model is composed of taxes, government spending and GDP, respectively. Government spending is defined as total purchases of goods and services and as tax variable they use total receipts minus transfers minus interest payments. Each of these variables has quarterly frequency and enters the model in log real, per capita terms. Denote the variables as T_t, G_t and X_t and let $Y_t = (\; T_t \quad G_t \quad X_t \;)'$. Then, their model is given by

$$Y_t \;=\; A(L)\, Y_{t-1} + U_t \tag{40}$$

where $U_t = (\; t_t \quad g_t \quad x_t \;)'$ are the error terms of the three variables and $A(L)$ is a lag polynomial of order 4 .

The relationship between the reduced form error terms and the structural errors is assumed to be the following:

$$\begin{aligned} t_t &= a_1 x_t \;+\; a_2 e_t^g \;+\; e_t^t \\ g_t &= b_1 x_t \;+\; b_2 e_t^t \;+\; e_t^g \\ x_t &= c_1 t_t \;+\; c_2 g_t \;+\; e_t^x \end{aligned} \tag{41}$$

where $e_t^i, i \in \{t, g, x\}$ are the structural shocks.

The identification strategy builds upon the assumption that by using quarterly data, contemporaneous reactions of taxes and/or government spending to GDP shocks can be ruled out, as there are neither automatic responses nor discretionary ones that are transmitted fast enough. The remaining structural parameters are estimated using elasticities and an instrumental variables approach.

3.2 The Fiscal Policy structural FAVAR

The identification strategy applied in this study is similar to the one used by Bernanke et al. (2005). As outlined in the previous section, the two variables of interest are government spending and taxes. The remaining variables are divided into two groups: fast-moving variables and slow moving variables.

Denote by ζ_t^F, ζ_t^S, ζ_t^I and ζ_t^G the structural shocks of fast moving and slow moving variables, of taxes ("income") and of government spending, respectively. Identification is achieved by assuming that ζ_t^S can affect all variables within the period, ζ_t^G and ζ_t^I can affect all but the slow ones within a period and ζ_t^F can only affect fast moving variables within a period.

This pattern is very intuitive and it is observed in the "real world". Take the stock market ("fast") and GDP ("slow") as an example. While a sudden change in GDP almost immediately affects the stock market, it is very unlikely that the day-to-day up and downs of the stock market will affect GDP within a period.

Regarding the ordering of ζ_t^G and ζ_t^I, this is ambiguous: Does a government first react to a crisis by adjusting taxes or by increasing or reducing government spending? The author could not think of any definitve argument that would resolve this, thus two versions were analyzed: one with taxes ordered first and one with spending ordered first.

Thus, $B_0^\star$ has a block triangular form:

$$B_0^\star = \begin{pmatrix} B_{0,SS}^\star & 0 & 0 & 0 \\ B_{0,iS}^\star & B_{0,ii}^\star & 0 & 0 \\ B_{0,jS}^\star & B_{0,ji}^\star & B_{0,jj}^\star & 0 \\ B_{0,FS}^\star & B_{0,Fi}^\star & B_{0,Fj}^\star & B_{0,FF}^\star \end{pmatrix}. \tag{42}$$

where $(i,j) \in \{(G,I),(I,G)\}$. Extending the argument in Section 2.5.2, (23) then becomes

$$\begin{aligned} \varepsilon_{Xt}^S &= B_{0,SS}^\star H_S' \eta_t + v_t^S \\ \varepsilon_{Xt}^i &= B_{0,iS}^\star H_S' \eta_t + B_{0,ii}^\star H_i' \eta_t + v_t^i \\ \varepsilon_{Xt}^j &= B_{0,jS}^\star H_S' \eta_t + B_{0,ji}^\star H_i' \eta_t + B_{0,jj}^\star H_j' \eta_t + v_t^j \\ \varepsilon_{Xt}^F &= B_{0,FS}^\star H_S' \eta_t + B_{0,Fi}^\star H_i' \eta_t + B_{0,Fj}^\star H_j' \eta_t + B_{0,FF}^\star H_F' \eta_t + v_t^F \ . \end{aligned} \tag{43}$$

By the same argument as in Section 2.5.2, the space spanned by ζ_t^S is identified, making it possible to identify ζ_t^i and ζ_t^j up to scale, according to

$$\zeta_t^i = \text{Proj}(\varepsilon_{Xt}^i | \eta_t) - \text{Proj}(\varepsilon_{Xt}^i | \zeta_t^S) \tag{44}$$

and

$$\zeta_t^j = \text{Proj}(\varepsilon_{Xt}^j | \eta_t) - \text{Proj}(\varepsilon_{Xt}^j | \zeta_t^S, \zeta_t^i) \ . \tag{45}$$

Thus the structural shocks are identified and can be used for Impulse Response Analysis.

4 The Data

4.1 General information

As explained earlier in this paper, this study tries to remain as close as possible to the paper of Blanchard & Perotti (2002). This means that taxes and government spending are defined in the same way as in their paper.

The data is for the United States and was obtained from the Federal Reserve Bank of St. Louis (2015). It spans the horizon from 1985Q1 to 2014Q4 and belongs to one of 12 groups, as outlined in Table 1.

Table 1: Groups of variables used

Group	# of variables	Fast vs. Slow
Average Hourly Earnings	3	S
Employment and Labor Market	34	S
Exchange Rates	9	F
Housing	9	F
Interest Rates	13	F
Money and Credit Quantity Aggregates	17	F
National Accounts	5	S
National Income and Product Accounts	5	S
Policy Variables	2	–
Prices	11	S
Production and Business Activity	18	S
Stock Market	3	F
Total	127	–

The mapping of variables to either being "slow" or "fast" is done via the groups. Information on which groups are denominated "slow" and which ones "fast" can also be gathered from Table 1. In Appendix A the complete list of variables with further details can be found.

A special case are the "variables of interest" or "policy variables". These are composed of taxes and government spending. The expenditure variable ("spending") is defined as total purchases of goods and services, the revenue variable ("taxes") as total receipts minus transfers minus interest payments. All the variables cover the general government, i.e. federal, state and local governments and social security. Both – taxes and spending – are transformed to real, log, per capita terms. A total of seven variables was retrieved to construct these variables of interest, which can also be found in Appendix A. Regarding GDP, the same transformations as for taxes and spending were used, which is why this variable is with the policy variables group in the appendix, but counted as part of the "National Accounts" group in Table 1.

4.2 Seasonality, Stationarity and Standardization

Seasonality: Most of the data was already seasonally adjusted when retrieved. In cases where not, seasonal adjustment by a simple dummy variables regression approach was done. None of the series exhibited a significant seasonal pattern however and thus no adjustment was necessary.

Stationarity: All series were manually inspected in order to decide which transformation was suitably applied as a first step (mostly first differences or first differences of log). The series were then transformed according to the derived transformations and an ADF test was computed for each series to test whether the series is stationary or not. In case a non-stationary series was detected, the first difference was taken and the ADF test was applied again. In total (keeping the transformations from the first step in mind) twice differencing was allowed as maximum. Nonetheless, all series were stationary according to the ADF test after maximum twice differencing.

Standardization: Posterior to the transformations and checks mentioned, all series were standardized to have sample mean zero and unit sample standard deviation. This is a common step in Factor Analysis in order to "permit" each variable an influence on the factor estimates.

4.3 Properties of the policy variables

A plot of log real per capita government expenditures can be found in Figure 1. As expected, the series is quite persistent, because government spending is not per se linked to economic activity. It shows a general upward trend over time and only recently (2010 and after) there seems to be a downward trend.

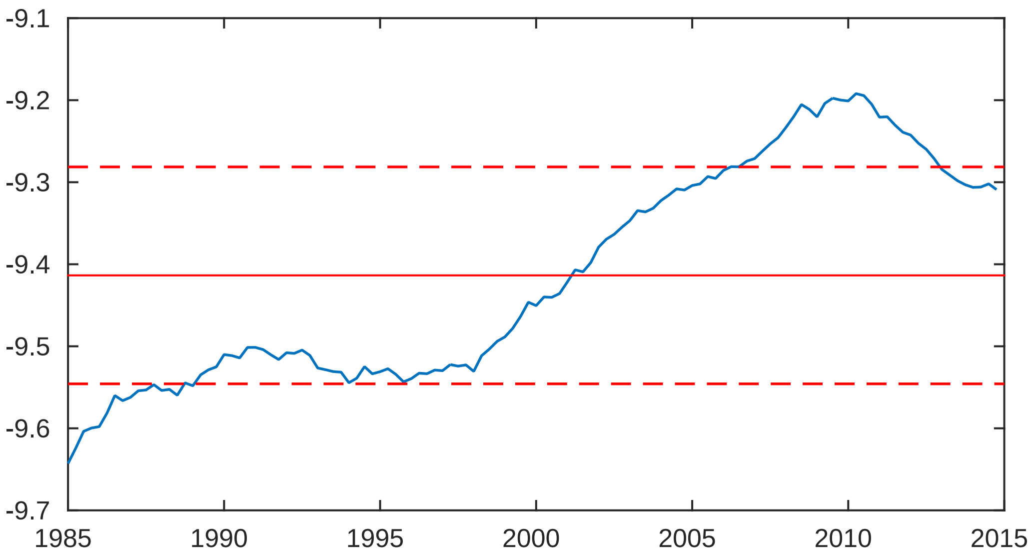


Figure 1: Real Government Expenditures, per capita, in logarithms, 1985Q1 – 2014Q4

The picture is a bit different for government revenue – or taxes, depicted in Figure 2. This series is also persistent, but not that highly as spending. This reflects the fact that most taxes are proportional ones and thus vary with economic activity. In booms, taxes go up, whereas government revenue decreases in crises. For example, the world economic crisis 2007-09 is clearly reflected by the series. As of trends, there seems to be no clear trending behaviour in taxes.

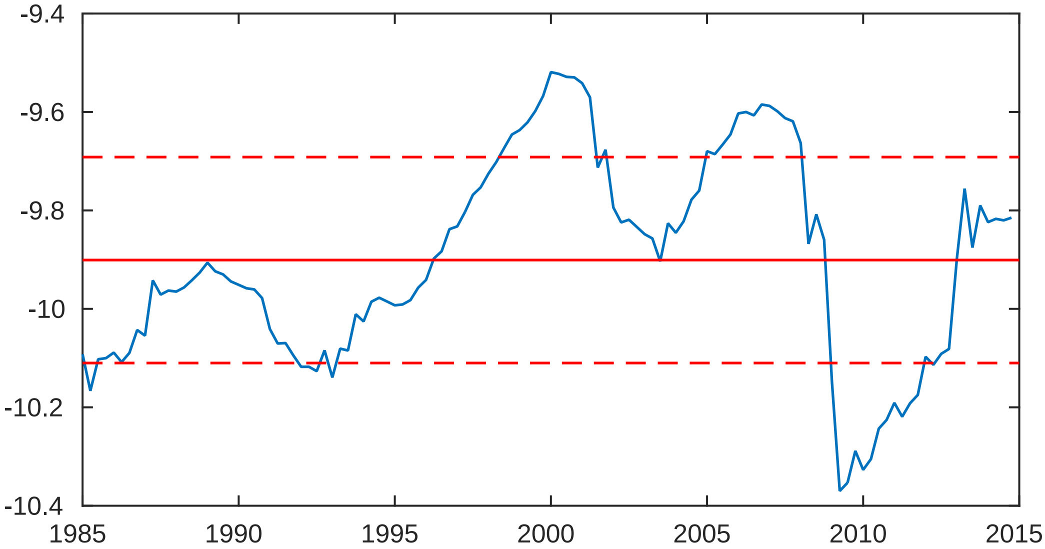


Figure 2: Log real per capita government revenues, 1985Q1 – 2014Q4

5 Empirical Results

5.1 Estimation results for the Factor Augmented VAR

The number of static and dynamic factors was estimated as $\max\{IC_{p1}, IC_{p2}\}$, applied to the covariance matrix of X_t and to the covariance matrix of ε_{Xt}, respectively.

At first, a smaller dataset than the one mentioned in Section 4 was used. The problem was that only $\hat{r} = 3$ static factors were chosen. This number did not leave space for structural estimation, as the number of dynamic factors $\hat{q}$ has to be smaller or equal to $\hat{r}$ and moreover it has to be the case that $\hat{r} \geq \hat{r}_S \geq \hat{q}_S$. The information criteria seemed to work well though, because manually changing these values to higher ones led to numerical problems as singular matrices, for example.

To fix this, the number of variables was extended from initially 76 series to the mentioned 127 series. This then produced more convincing estimates regarding the number of factors. The estimates can be found in Table 2. As can be seen, the number of static factors was estimated as 5 and the number of dynamic factors as 3. Interestingly, in an estimation using only the slow variables, both the static and the dynamic factors were estimated to be 4, which, again, does not make much sense. To proceed, the number of dynamic factors in the slow variables was restricted to be two less than the overall number of dynamic factors – i.e. $\hat{q}_S = \hat{q} - 2$ – leaving one structural (dynamic) shock per policy variable. In contrast to the study of Stock & Watson (2005), the information criteria were nowhere near to being flat but exhibited a clear minimum.

Table 2: Estimated number of factors

$\hat{r}$	$\hat{q}$	$\hat{r}_S$	$\hat{q}_S$
5	3	4	4*

* (restricted to 1)

The initial estimate for $D(L)$ was then obtained via OLS estimation of the individual AR processes, ignoring the factors. Afterwards, estimation was done iteratively, as described in Section 2.6.2. As optimization target (in order to abort the sequence) the SSR was taken, additionally limiting the maximum number of iterations to 20, which was not exhausted, however.

After estimation of the model, Impulse Response Analysis was performed. For this, the system was "fed" with a unit shock (with a magnitude of one standard deviation). One standard deviation is, in the case of taxes 0.12 and in case of spending it is 0.18.

5.2 Issues

Estimation of the models posed quite a few challenging issues. A few were already mentioned above, namely estimation of the number of factors. Even after extending the dataset various problems persisted. To name a few:

- Estimation of the model for "government spending ordered first" went through, whereas structural estimation of "taxes ordered first" produced complex eigenvectors. This was remedied by extracting the real part and ignoring the complex part

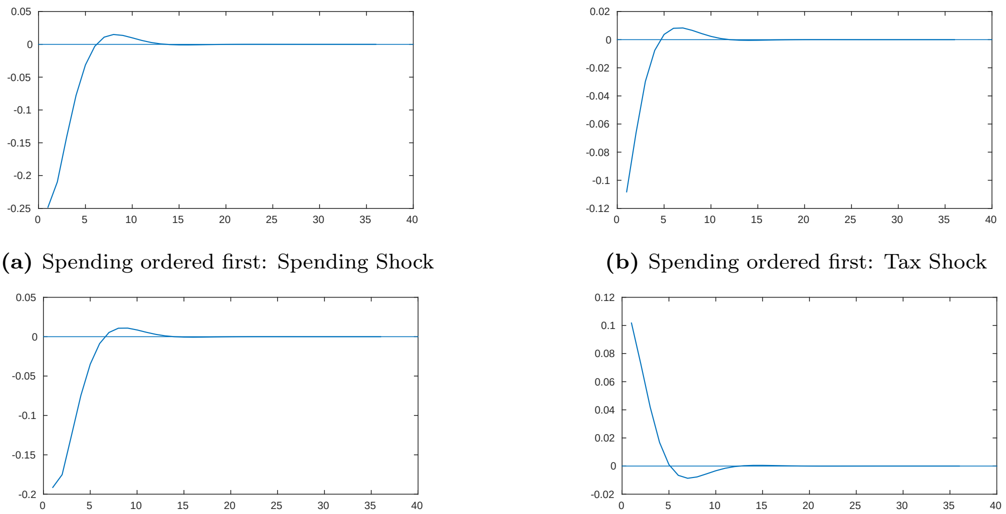
- To provide more concise information, it was planned to bootstrap confidence bands using Hall's percentile interval, doing 100 repetitions. However, this led to a bunch of problems: to complex results (in part triggered by the point mentioned above) and to nearly singular matrices.
- Surprisingly, reestimation of the number of factors from a simulated (bootstrapped) series led in many cases to much higher estimates than for the original data (for example $\hat{r} = 12$). This caused lots of (nearly) singular matrices in the next step and does not make sense, which is why for the bootstrapped series the numbers of factors were restricted to the ones estimated with the data a step earlier.

Because bootstrapping takes quite some time it was not possible to come up with a solution to all aspects within the time limit. The bootstrapped confidence bands thus had to be left out.

5.3 Impulse responses for selected variables

This section presents the estimated Impulse Response functions for selected variables, like GDP, hours worked, interest rate spread and others. The plots of the Impulse response functions for all other variables can be found in the electronic appendix.

Figure 3 depicts the reaction of GDP to shocks in spending and taxes. Interestingly the impact reaction is negative in three of the four cases, namely for spending shocks in both ordering scenarios as well as for the tax shock when spending is ordered first. Only when taxes are ordered first, a tax shock exhibits a positive reaction at impact. After the impact reaction, all Impulse Response Functions revert back to zero after roughly ten periods.



(a) Spending ordered first: Spending Shock

(b) Spending ordered first: Tax Shock

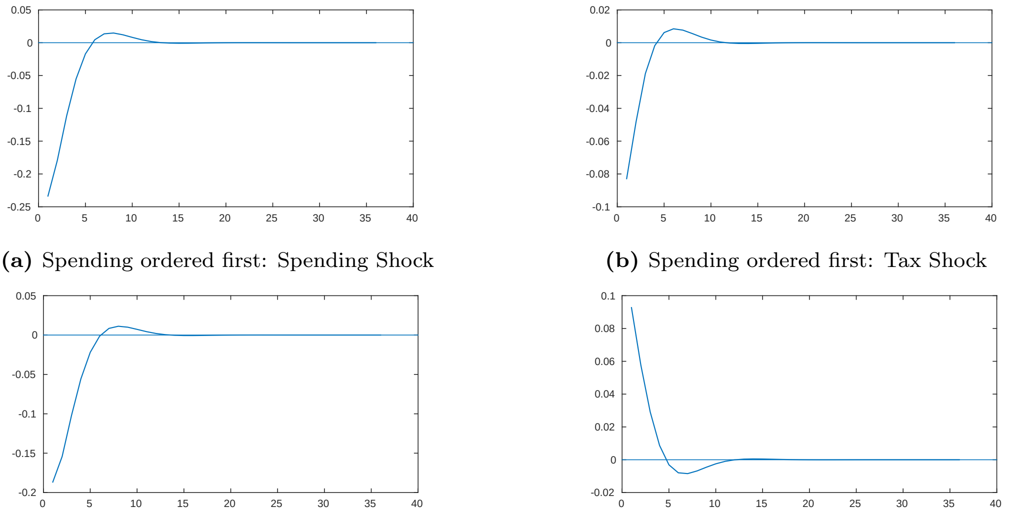
(c) Taxes ordered first: Spending Shock

(d) Taxes ordered first: Tax Shock

Figure 3: Impulse responses of GDP (Series ID: tGDP)

A strikingly similar picture to that of GDP is the one of "average weekly hours", depicted in Figure 4. Again, the responses for spending ordered first can be found in the top row, the responses for taxes ordered first in the bottom row.

For spending ordered first, both a spending shock and a tax shock lead to a very similar pattern in the response variable, differing only in magnitude. At impact, hours worked show a huge negative reaction, but revert back to zero rather quickly, after around 10 periods. Regarding taxes ordered first, the picture for a spending shock is the same as for spending ordered first, but a tax shock shows now the opposite effect: a big positive reaction, followed by a decline towards zero, which is reached again after approximately ten periods.



(a) Spending ordered first: Spending Shock

(b) Spending ordered first: Tax Shock

(c) Taxes ordered first: Spending Shock

(d) Taxes ordered first: Tax Shock

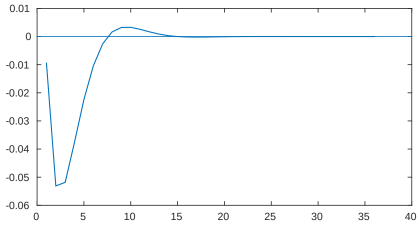
Figure 4: Impulse responses of Average Weekly Hours (Series ID: AWHMAN)

The Consumer Price Index (Figure 5) is quite different in its reaction to the policy shocks. After a small negative reaction on impact to a spending shock (spending ordered first), the CPI decreases further to reach its minimum a period later. From there it rises to converge to zero approximately after period ten. The shape of the curve is similar for a spending shock when taxes are ordered first. The only difference is the initial positive reaction.

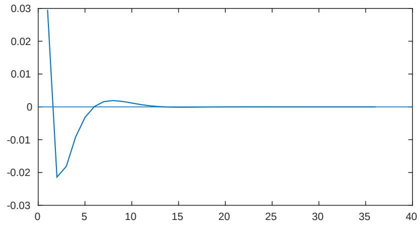
The same pattern can be observed for a tax shock, when spending is ordered first: a positive reaction at impact, followed by a strong negative reaction and a reversion to zero afterwards. A bit different is the situation for a tax shock (tax ordered first): it directly converges back to zero after a positive spike, without going into the negative.

Quite surprising is Figure 6. It shows the reaction of Industrial Production to the shocks, and is, except for the tax shock, when spending is ordered first, really different from the GDP figure.

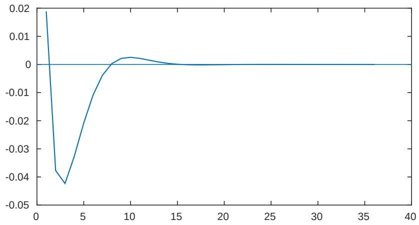
For spending shocks in both scenarios can be observed the same effects: after a positive reaction at impact, Industrial Production reverts back to a negative value, to converge to zero afterwards.



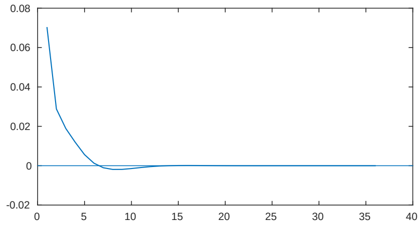
(a) Spending ordered first: Spending Shock



(b) Spending ordered first: Tax Shock



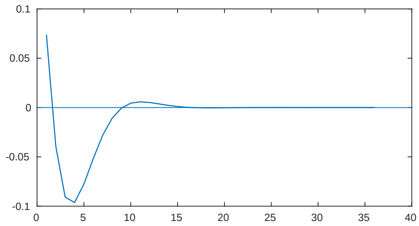
(c) Taxes ordered first: Spending Shock



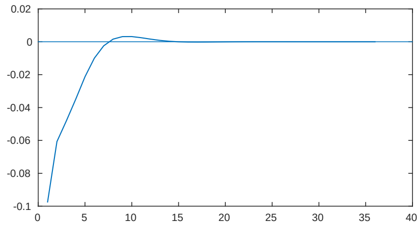
(d) Taxes ordered first: Tax Shock

Figure 5: Impulse responses of the Consumer Price Index (Series ID: CPALTT01USQ661S)

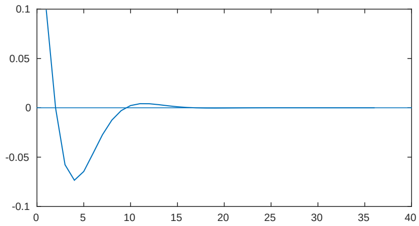
To the tax shock, in the case when spending is ordered first, Industrial Production reacts strongly negative at impact, then turning back to zero. When taxes are ordered first, a tax shock leads to a slight negative reaction on impact, followed by a huge positive peak and another turnaround towards zero thereafter.



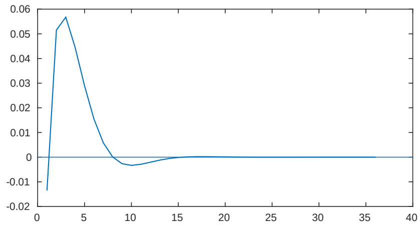
(a) Spending ordered first: Spending Shock



(b) Spending ordered first: Tax Shock



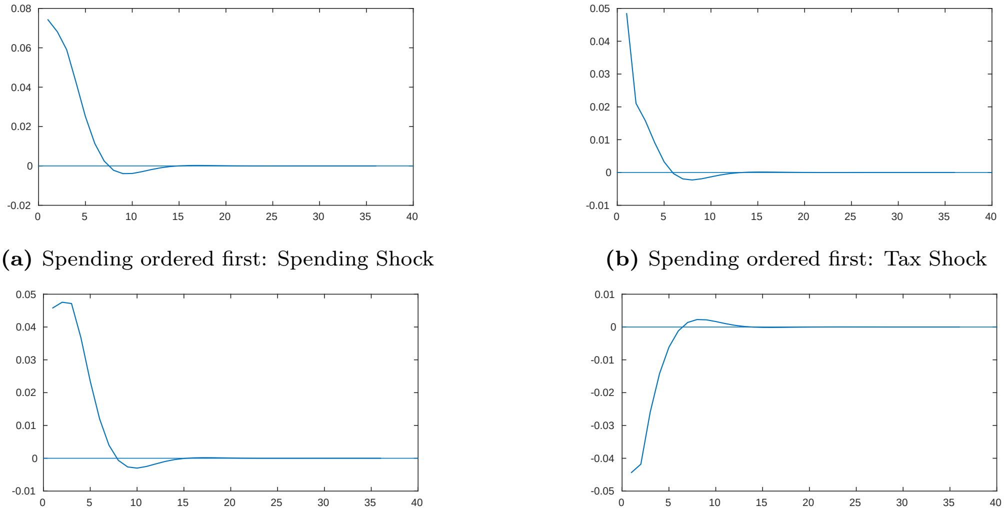
(c) Taxes ordered first: Spending Shock



(d) Taxes ordered first: Tax Shock

Figure 6: Impulse responses of Industrial Production (Series ID: IPB00004SQ)

The reaction of the interest rate spread (here the 3-Month Treasury Bill Minus Federal Funds Rate) is – for the cases depicted in panels 7a, 7b and 7c – positive at impact with a rather fast tendency towards zero after that. A tax shock, when ordered first, shows a negative reaction at impact and goes back to zero after approximately ten periods.



(a) Spending ordered first: Spending Shock

(b) Spending ordered first: Tax Shock

(c) Taxes ordered first: Spending Shock

(d) Taxes ordered first: Tax Shock

Figure 7: Impulse responses of the 3-Month Treasury Bill Minus Federal Funds Rate (Series ID: TB3SMFFM)

5.4 Discussion

The GDP figures are in line with neoclassical thinking, except for Panel 3d, which does not make sense, neither in a neoclassical world nor in a Keynesian one. This is only partly in line with Blanchard & Perotti (2002), as they find positive reactions of GDP to spending shocks. The figures for hours worked, in contrast, contradict the neoclassical view, as hours decrease after a spending shock, whereas the neoclassical paradigm predicts a decrease in private consumption and an increase in labor supply, which means, hours should go up.

Regarding Production, this should go up as a consequence of higher govenment spending, according to both the neoclassical as well as the Keynesian model. According to the study here it does not. Finally, an increasing interest rate spread as shown in the figures is in line with a neoclassical model (higher overall level also means a bigger absolute gap), whereas for Keynesian models it depends on the concrete formulation whether interest rates rise or fall.

6 Conclusion

Overall, the method shed some new light on different aspects. Depending on the variable, it sometimes confirmed the neoclassical view, sometimes it confirmed the Keynesian view, and sometimes it challenged both. Although the design of the study was kept close to Blanchard & Perotti (2002), it mainly did not confirm their findings.

Nonetheless, quite a few issues remain at hand. For starters, the method seems to be "data-hungry". Either it needs a lot more variables or more observations or both. Another aspect is whether the use of monthly data instead of quarterly improves the analysis. Most of the data in the present study would have been available with a monthly frequency, but the key variables – the fiscal policy variables – were only available with quarterly frequency.

A further question is, why so few factors were estimated, especially in comparison to Stock & Watson (2005), who mention around 10 static factors. A possible explanation is the lack of data, however, this needs to be addressed in future studies.

Needless to say that this area deserves further research. One possibility would be to start with a theoretical model and to use q groups to partition the dataset – as outlined in Section 2.5.3. Another promising branch would be to use long run restrictions to achieve identification.

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Andreas Költringer