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CONTRACT DESIGN WITH LIMITED COMMITMENT

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Abstract. *We consider the problem of a principal who wishes to contract with a privately informed agent and is not able to commit to not renegotiating any mechanism. That is, we allow the principal, after observing the outcome of a mechanism to renegotiate the resulting contract without cost by proposing a new mechanism any number of times. We provide a general characterization of renegotiation-proof states of such a renegotiation. The proposed solution concept provides an effective and easy-to-use tool to analyze contracting problems with limited commitment. We apply the solution concept to a setting with a continuous type space, private values and non-linear contracts. We find that the optimal contracts for the principal are pooling and satisfy a "no-distortion-at-the-bottom" property.*

JEL classification: C72, C73, C78, D82

Keywords: Principal-Agent models, renegotiation, commitment, Coase-conjecture

1. Introduction

Motivation and results. Consider the problem of a principal (she) who is endowed with all the bargaining power and wishes to contract with a privately informed agent (he). As a consequence of the revelation principle, we can usually dispense with the details of the particular procedure that the principal may use to close the contract and focus on direct revelation mechanisms (Myerson, 1979). This approach is valid only if the principal honors the rules of the proposed mechanism and the agent trusts that this is the case. By playing the mechanism, however, the agent reveals information and the contracts resulting from optimal mechanisms are typically inefficient. Both parties therefore can benefit if the resulting contract is renegotiated, that is, if the principal proposes a new mechanism after observing the outcome of the original mechanism. In this case, the agent may decide whether to participate in the new mechanism or whether

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to stick to the original contract. If he decides to participate in the new mechanism, the contract resulting of this new mechanism can still also be subject to renegotiation. If the principal cannot commit to not renegotiating any contract and there is no deadline that puts an end to the renegotiation, backward induction cannot be applied and the revelation principle is hard to restore.[1]

We follow Gretschko and Wambach (2016) and characterize the set of renegotiation-proof states of such a (re-)negotiation.[2] A state of the negotiation is a tuple consisting of the current signed contract of the agent and the belief of the principal that was formed by observing the previous choices of the agent.[3] A state is said to be renegotiation-proof if the principal will not propose a new mechanism once such a state has been reached. Renegotiation-proof states are not identified one-by-one but simultaneously as a set. The key insight is that whether a state is renegotiation-proof or not will depend on whether it can be improved by other renegotiation-proof states. A state is said to be improved by another set of states if there is a mechanism which leads from the initial state to the other states and which makes (all types of) the agent and the principal (for the given belief) better off. In essence, renegotiation-proof states cannot be improved by other renegotiation-proof states while states that are not renegotiation proof can be improved by renegotiation-proof states.

The characterization of the set of renegotiation-proof states therefore is based on two simple properties. First, for every renegotiation-proof state there are no other renegotiation-proof states that can be reached by proposing a new mechanism and would make the principal strictly better off (*internal consistency*). Second, in any state of the negotiation it is feasible to reach renegotiation-proof states by proposing a new mechanism (*external consistency*).[4] Both properties reflect the sequential rationality of the

[1]Thus, Bester and Strausz (2001) cannot be directly applied.

[2]We extend the definitions in Gretschko and Wambach (2016) by considering arbitrary type spaces and not merely discrete ones.

[3]We focus on states rather than contracts as whether the principal would like to renegotiate the currently signed contract crucially depends on her belief.

[4]Without frictions, every history of the negotiation can be represented as a single-stage mechanism. Thus, for the definition of renegotiation-proofness it is sufficient to consider only a single-stage mechanism. Put differently, every equilibrium of a subgame starting at some history can be represented by a single-stage mechanism that is incentive compatible for the agent. That is, if after some history an equilibrium eventually induces a type conditioned lottery over contracts and posterior beliefs, then there is an incentive compatible single-stage mechanism that induces the same type dependent lotteries and generates at least as high payoff to the principal.

principal. Suppose the negotiation reaches a renegotiation-proof state and the principal proposes a new mechanism. External consistency ensures that by proposing a new mechanism she will renegotiate to reach renegotiation-proof states. Internal consistency implies that the resulting states do not make the principal better off than the original state would have. [5]

One of the main advantages of a general characterization of renegotiation-proof states is that it provides an effective and easy-to-use tool to analyze specific instances of the general problem. We apply the solution concept to a setting with a continuous type space, private values and non-linear contracts.[6] This setting encompasses many applications in which it is natural for the principal to suffer from the inability to rule out renegotiation like selling when price and quality matter, procurement, or franchising. As the principal example of the setting we use that of a seller selling a good to a privately informed buyer. The contracts consist of two dimensions: price and quality.

With full commitment, it is optimal for the principal to offer a continuum of contracts, the types fully separate with only the highest type receiving an efficient contract. This is the well-known "no-distortion-at-the-top" result. Clearly this is not sustainable if the principal is not able to commit. If there is full separation of types with inefficient contracts, the principal must know the type of the agent and could propose a new, strictly better, mechanism offering efficient quality to each type.

We show that without commitment the set of optimal renegotiation-proof contracts for the principal has the following features.[7] Firstly, the principal offers a *countably* infinite number of contracts. Secondly, each contract is signed by a *pool* of types of the agent that is of positive measure. Thirdly, the lowest type in each pool receives an efficient contract, every other type in each pool receives an *inefficient* contract. Thus, the result differs markedly from the full-commitment benchmark.

To prove the result, we start by showing that renegotiation-proof states must be either efficient and separating or pooling so that one of the types in the pool receives his efficient

[5]The proposed solution concept is closely related to the approach introduced by Vartiainen (2013) to analyze auctions without commitment. We comment on the relationship below and in in Section 6.

[6]A setting similar to Mussa and Rosen (1978).

[7]To improve readability, we will sometimes speak of renegotiation-proof contracts rather then renegotiation-proof states. A renegotiation-proof contract is then a contract that with an appropriate belief of the principal can constitute a renegotiation-proof state in the sense described above.

quality.[8] Pooling states where one of the agent types receives his efficient quality can be renegotiation-proof if the only other renegotiation-proof states that can be reached, starting from the pooling state, are efficient and separating. This is due to the fact that such states do not make the principal strictly better off in comparison to the pooling states. Having defined the general structure of renegotiation-proof states, we show that, optimally, the principal offers a mechanism that leads to pooling contracts only. In each pool, the lowest type will receive his efficient quality. That is, there is "no distortion at the bottom". This is due to the fact that efficient and separating contracts would imply a high information rent to the agent which can be reduced by offering pooling contracts instead. Thus, efficient and separating contracts cannot be optimal. Moreover, if the principal needs to provide efficient quality to one type in each pool, it is optimal to do so for the lowest type. This allows the principal to reduce the information rent of the higher types in the other pools.

Our solution concept is closely related to the solution concept that Vartiainen (2013) introduced to analyze auctions without commitment. Vartiainen specifies conditions that are imposed on the mechanism selection strategy of the principal by sequential rationality. He thereby identifies the selection strategies that the principal will not be tempted to change.[9] In particular, he requires that a mechanism selection strategy is consistent and optimal. The former condition implies that employing the mechanism selection strategy ex-ante should not contradict employing it ex-post. That is, if a mechanism is chosen after some history, it should make the principal weakly better off than any of the subsequent mechanisms prescribed by her mechanism selection strategy. The later condition implies that the principal should choose a mechanism that maximizes her payoff among the mechanisms which are consistent with the selection strategy. Observe that if the principal will not change her selection strategy after observing the play of the agent, given that the agent plays truthfully, it is optimal for the agent to play the mechanism

[8]A state is efficient and separating if the the principal has a singleton belief about the type of agent and the contract for this type is efficient. A state is pooling if the belief of the principal admits a positive measure of types. To improve readability, we sometimes speak of pooling contracts and efficient and separating contracts instead of states.

[9]As Vartiainen (2013) considers a situation with a total lack of commitment, his approach needs to be adapted to our setting in which we focus on renegotiation. With a total lack of commitment the agent does not have the option to retain the contract which was generated by a previous mechanism instead of playing the new mechanism.

truthfully.[10] We demonstrate the equivalence between the solution concept introduced by Vartiainen (2013) and our solution concept. On one hand, every consistent, optimal and history independent selection strategy of the principal results in a renegotiation-proof set of states. On the other hand, for any given set of renegotiation-proof states there exists an consistent, optimal and history independent selection strategy that generates this set of states.

The main advantage of abstracting from selection strategies and focus on renegotiation-proof states only is that optimal and consistent selection strategies may be rather complex. Thus, to demonstrate the cutting power of our solution concept, we analyze the famous Coase (1972) conjecture and rederive the "gap - no gap" result in a simple way. That is, we analyze the problem of a seller selling a durable good to a buyer who has private information about his valuation where price is the only relevant dimension. We show that if there is a gap between the cost of the seller and the lowest value of the buyer – the "gap" case – the seller can at most charge a price equal to the lowest valuation of the buyer. If the cost of the seller, however, is above the lowest valuation – the "no-gap" case – the seller can charge the monopoly price (Ausubel and Deneckere, 1989).[11]

Vartiainen (2013) also analyzes the Coase conjecture and writes that in principle it is possible to construct mechanism selection strategies that are consistent and optimal for the no-gap case. However, those selection strategies would be complex. Thus, he refrains from construction and makes the additional assumption that mechanism selection strategies need to be stationary. In this case, even in the no-gap case, the seller sells at a price equal to his costs. As we are able to rederive the "gap – no gap" result in a simple way, we demonstrate that shifting the focus from mechanism selection strategies to states simplifies the application of the solution concept.

Related literature. Gretschko and Wambach (2016) show how renegotiation-proof states arise as a perfect Bayesian equilibrium of the mechanism design game. Furthermore, they apply the solution concept to different models with a discrete type space. They find that

[10]Playing the mechanism truthfully means choosing the message that gives the agent the best possible payoff given her type (possibly mixing when indifferent).

[11]Liu et al. (2017) provide an interesting extension of (Ausubel and Deneckere, 1989) to auctions. They show that without commitment the monopoly solution is not achievable. This result is corroborated by Vartiainen (2013) who demonstrates that an English auction without reserve price is the only mechanism that is implementable without commitment.

with private values only efficient contracts are renegotiation-proof. With common values, however, inefficient contracts can be renegotiation-proof. This finding is corroborated by Strulovici (2017) who shows that if in a specific infinite-horizon bargaining protocol friction disappears, efficient and fully separating contracts arise in any Perfect Bayesian Equilibrium if values are private and the type space is discrete.[12] This is different to the results provided in this manuscript, given that with a continuous type space, inefficient contracts can be renegotiation-proof even in the case of private values. In the private value case, for any pooling contract renegotiation towards efficient separating contracts is feasible. It is also strictly profitable to the principal if types are discrete, thus pooling cannot not be a renegotiation proof state. With continuous types, however, such a renegotiation does not lead to an additional profit for the principal. More precisely, after any history the principal can use Vickrey-Clarke-Groves mechanisms where the agent's payment is the cost of provision to implement the efficient solution. With continuous types, this payment schedule is unique up to an additive constant. Hence, the principal makes a zero profit along the efficient path if the current contract is efficient for one of the types of agent. Thus, the principal can commit to some inefficient contracts, as renegotiating to efficient contracts would generate no additional profit for her. For discrete types, the maximal profit from implementing efficient contracts starting from an inefficient one is always positive. Thus, renegotiating to efficient contracts makes the principal always better off. We do not take a stance on whether the discrete or the continuous type model is the more relevant one. However, we like to point out that without commitment one has to be careful about which model to use.

Asheim and Nilssen (1997) consider a monopolistic insurance market with a finite type-space. They use assumptions regarding the characterization of renegotiation-proof states which resemble our characterization. That is, they rely on properties similar to internal and external consistency to characterize renegotiation-proof states. As in our case, this approach proves to be very useful in deriving clear results for an otherwise very complex problem.

[12] In a similar set-up, Maestri (2017) uses a refinement that in any subgame the principal induces the continuation equilibrium that maximizes her payoffs. As in Strulovici (2017), when frictions disappear, only efficient contracts arise in equilibrium.

Neeman and Pavlov (2013) argue that for outcomes of a mechanism to be renegotiation-proof under any renegotiation procedure there must be no Pareto improvements after the mechanism has been played. That is, they take the view that if the mechanism designer is agnostic about the specific renegotiation game that is played after the mechanism, the outcome of the mechanism must be ex-post efficient to survive renegotiation under any renegotiation procedure. The conceptual problem with this approach is that it permits all Pareto-improving outcomes to be blocking, even if those outcomes are themselves subject to renegotiation. In our approach, we require states to be renegotiation-proof only with respect to states that are themselves renegotiation-proof. In contrast to Neeman and Pavlov (2013), our approach allows for inefficient results.

Bester and Strausz (2001), Hörner and Samuelson (2011), Skreta (2006), and Skreta (2015) limit renegotiation to finite procedures. This approach allows for interesting equilibrium analysis but still leaves the principal with a considerable amount of commitment power. In our frictionless setting, limiting the renegotiation to n opportunities would allow the principal to implement the full commitment outcome. She could simply pass on $n-1$ opportunities and then propose the optimal contracts.

Evans and Reiche (2015) assume that after an initial mechanism is played, the principal can offer a new mechanism and the agent may choose whether to retain the outcome of the original mechanism or to participate in the new mechanism. They assume that there is no friction in-between the mechanism proposals, as do we. After the new mechanism is played, the renegotiation is over and there is no scope for further offers from the principal. In this setting, the optimal mechanism from the point of view of the principal is easy to implement if she proposes the null mechanism in the first round and the optimal mechanism in the second round. What makes the analysis of Evans and Reiche (2015) interesting is the fact that they allow a third party whose goals are not aligned with the principal to propose the initial mechanism. This third party must then take into account that the outcome of the mechanism may be subject to renegotiation.

Organization of the manuscript. The manuscript proceeds as follows. In Section 2, we introduce the general model and the commitment problem of the principal. In Section 3, we derive the solution concept and the optimization problem of the principal.

In Section 4, we apply the solution concept to a setting with a continuous type space, private values and non-linear contracts. In Section 5, we demonstrate how our solution concept can be used to retrieve the Coase conjecture and the "gap – no gap" result. In Section 6, we compare our solution concept to the solution concept used by Vartiainen (2013). Section 7 concludes.

2. The Setup

Preferences. A principal (she) and an agent (he) want to implement a contract w from a metric space of contracts W. If a contract w is implemented, the utility of the principal amounts to $v(w)$ where $v : W \to \mathbb{R}$ is a von Neumann-Morgenstern utility function. The utility of the agent is given by $u(w, \theta)$ where $u : W \times \Theta \to \mathbb{R}$ is a von Neumann-Morgenstern utility function and depends on the agent's type θ. The agent's type is private information to the agent and is drawn from a metric space Θ. Endow Θ with the Borel σ-algebra and denote by $\Delta(\Theta)$ the set of all probability measures on the Borel σ-algebra. Let $d(\cdot, \cdot)$ be a metric on Θ. We will denote by θ_ϵ the ϵ-neighborhood of θ. That is, the set of all $\theta' \in \Theta$ such that $d(\theta, \theta') < \epsilon$. The principal's prior about the type of the agent is characterized by $\mu_0(\cdot) \in \Delta(\Theta)$. The prior is common knowledge between the agent and the principal. For $\mu \in \Delta(\Theta)$, we will denote by $\text{supp}(\mu)$ the support of μ. That is, θ is in $\text{supp}(\mu)$ whenever $\mu(\theta_\epsilon) > 0$ for all $\epsilon > 0$. If no contract is implemented, both parties receive the outside option contract denoted by $w_0 \in W$.

Mechanisms. To elicit information from the agent and implement a contract the principal uses a mechanism. A mechanism is a tuple $M = (\mathcal{Z}, w(\cdot))$ consisting of a metric space of messages $\mathcal{Z}$ and a function $w : \mathcal{Z} \to W$. A mechanism works as follows. The agent chooses a message $z \in \mathcal{Z}$. When the message z is sent, $w(z)$ generates a contract.[13] Denote by $\mathcal{M}$ the set of all mechanisms. Denote by 1_w the mechanism that generates w for sure. That is, $w(z) \equiv w$ for all $z \in \mathcal{Z}$.

Example 1. To fix ideas, consider the following specification of the model that we will analyze in Section 4. The principal is a seller that sells a good to the agent, the buyer. A contract $(q, p) \in W = \mathbb{R}^2_+$ specifies the quality q and the price p of the good. The seller

[13]Here it is assumed that communication is direct. For an analysis of contracting with renegotiation and mediated communication, see for example Pollrich (2017) or Strausz (2012).

incurs a cost of $c(q)$ when producing a good of quality q . Her pay-off from a contract (q, p) is $p - c(q)$. A buyer of type $\theta \in \Theta = [\underline{\theta}, \bar{\theta}]$ enjoys a utility of $u(q, \theta)$ when consuming a good of quality q. His utility from a contract (p, q) is $u(q, \theta) - p$. Higher types of buyer enjoy a higher utility and a higher marginal utility from consuming the good. One class of mechanism for this problem are direct mechanisms with $\mathcal{Z} = \Theta$, in which the buyer directly reports his type and receives a contract according to his type. That is, $M = (\Theta, (q(\theta), p(\theta)))$.

The problem. The problem for the principal is that she cannot commit to not renegotiating $w(z)$ after the mechanism has been played. That is, after M is played and the principal observes z, she will update her belief about the type of the agent to $\mu(\cdot : z)$. After observing z the principal may propose a new mechanism $M' = (\mathcal{Z}', w'(\cdot))$. The agent can then decide whether he wants to play the new mechanism or whether he wants to hold on to the initially generated contract $w(z)$. In other words, $w(z)$ is the new outside-option contract of the agent. If the agent decides to play M', the principal again observes the message, updates her belief, and may again renegotiate the new contract by proposing a new mechanism. Overall, the principal is not able to commit to not renegotiating any contract produced by any mechanism. Whenever a mechanism is played, the principal may propose a new mechanism and the agent may decide to either hold on to his current contract or to participate in the new mechanism. Thus, we are concerned with the question of what mechanisms will not be renegotiated at the ex-post stage.

To be more precise, consider the following negotiation. At each stage $t = 1, 2, ..$ the principal proposes a mechanism $M_t = (\mathcal{Z}_t, w_t(\cdot)) \in \mathcal{M}$. The agent chooses a message $z_t \in \mathcal{Z}_t$ or decides to hold on to the contract w_{t-1} that was generated by the mechanism M_{t-1} in stage $t-1$. The contract w_{t-1} is implemented if in stage t the principal offers $M_t = 1_{w_{t-1}}$. Denote a history before the principal moves in round t by

$$h_t^p = \{(M_1, z_1, w_1), (M_2, z_2, w_2), \ldots, (M_{t-1}, z_{t-1}, w_{t-1})\} .$$

A history realized before the agent moves is

$$h_t^a = \{(M_1), (M_2, z_1, w_1), \ldots, (M_t, z_{t-1}, w_{t-1})\} .$$

In round 1 there is no relevant history for the principal, so $h_1^p = \emptyset$. Let $\mathcal{H}_t^a$ be the set of all histories for the agent at round t and let $\mathcal{H}_t^p$ be the set of all histories for the principal.[14]

Strategies and beliefs. Before we discuss the solution concept it is useful to define strategies and beliefs of the principal and the agent.

A mechanism selection strategy σ^p of the principal prescribes in each round t the mechanism M_t that the principal will choose conditional on the history h_t^p. That is, σ^p is a sequence of maps

$$\sigma_t^p : \mathcal{H}_t^p \to \mathcal{M}.$$

Endow $\mathcal{Z}_t$ with the Borel σ-algebra and denote by $\Delta(\mathcal{Z}_t)$ the set of all probability measures on the Borel $\sigma-$algebra. A behavior strategy σ_t^a of an agent of type θ prescribes in each round t a probability distribution over messages in $\mathcal{Z}_t$ conditional of the history h_t^a. That is, σ^a is a sequence of maps

$$\sigma_t^a : \mathcal{H}_t^a \times \Theta \to \Delta\left(\mathcal{Z}_t \cup \{w_{t-1}\}\right).$$

The belief system of the principal is a sequence $\{\mu_0, \mu_1, \ldots\}$ where $\mu_{t-1} \in \Delta(\Theta)$ are the beliefs held after a history h_t^p.[15]

States and outcome function. There are two additional concepts that are useful to define before we turn to the solution concept. We define the state of the negotiation as the chosen contract of the agent and the resulting belief of the principal. That is, we call $C_t = (w_{t-1}(z), \mu_{t-1}(\cdot : z))$ the state of the negotiation after mechanism $M_{t-1} = (\mathcal{Z}_{t-1}, w_{t-1}(\cdot))$ and define $\Gamma = W \times \Delta(\Theta)$ as the set of all possible states. In particular, every history h_t leads to a state that was generated by the sequence of mechanisms and choices of the agent that lead to this history. That is, for a given h_t, $C_{h_t} = (w_{t-1}, \mu_{t-1})$. For a given mechanism $M_t = (\mathcal{Z}_t, w_t(\cdot))$ and a given history h_t we will denote by $f(M_t, \sigma^a(h_t^a)) \subset \Gamma$ the set of states generated by this mechanism. That is, a state $C = (w, \mu)$ is in $f(M_t, \sigma_t^a(h_t^a))$ if – given the strategy σ^a of the agent – there exists a message $z \in \mathcal{Z}$ that is chosen by some

[14] We will drop the superscript from h^p and h^a whenever we refer to both or whenever it is unambiguous whose history is used.

[15] We slightly abuse notation as we suppress that different histories in period t might lead to a different posterior.

type of the agent with positive probability such that $w_t(z) = w$ and $\mu_t(\cdot : z) = \mu$. Each mechanism selection strategy of the principal together with the strategy of the agent induces a set of possible states $\Omega = \bigcup_{h_t \in \mathcal{H}} f(\sigma_t^p(h_t), \sigma_t^a(h_t^a)) \subset \Gamma$.

3. Solution Concept: Renegotiation Proof States

In this section we closely follow Gretschko and Wambach (2016) and identify renegotiation-proof states. That is, states $C_t = (w_{t-1}, \mu_{t-1})$, such that the principal will not renegotiate w_{t-1}. We extend the definitions in Gretschko and Wambach (2016) by considering arbitrary type spaces and not merely discrete ones. In Section 6, we demonstrate that renegotiation-proof states arise from mechanism selection strategies that are consistent and optimal in the sense of Vartiainen (2013). We focus on renegotiation-proof states $C_t = (w_{t-1}, \mu_{t-1})$ rather then renegotiation-proof contracts w_{t-1}, as whether the principal will want to renegotiate will crucially depend on her belief.

We start by observing that every subgame after some history h_t can be represented by a single stage mechanism that the principal will not renegotiate. That is, if after history h_t the subsequent play of the agent and the principal eventually induce a type conditioned lottery over states and posterior beliefs, then there is a mechanism and a strategy of the agent that induces the same type dependent lotteries over states and posterior beliefs. In any subgame after h_t the strategy of the principal and the strategy of the agent induces a distribution over potential states in $\bigcup_{\{h_s \in \mathcal{H}: s \geq t, h_t \subset h_s\}} f(\sigma^p(h_s), \sigma^a(h_s))$. Instead of playing out the game according to those histories, the principal can offer a mechanism $M = (\mathcal{Z}, w(\cdot))$ such that $w[\mathcal{Z}]$ includes all contracts that are part of the states in $\bigcup_{\{h_s \in \mathcal{H}: s \geq t, h_t \subset h_s\}} f(\sigma^p(h_s), \sigma^a(h_s))$ and the agent mixes between messages $z \in \mathcal{Z}$ in a way that induces the same probability distribution over states and posterior beliefs as playing out the histories would. Thus, we will focus on single-stage mechanisms after each history. The problem then boils down to identify states after a single-stage mechanism that the principal would not like to renegotiate given the strategy of the agent. We will call such states renegotiation-proof states, denote the set of renegotiation-proof states by Ω and derive its properties in the following.

Agent's incentives. Suppose a history h_t that the current state of negotiation is $C_t = (w_{t-1}, \mu_{t-1})$ and the principal offers a mechanism $M_t = (\mathcal{Z}_t, (w_t(\cdot))$ such that if the agent chooses $z \in \mathcal{Z}_t$ optimally every choice leads to a renegotiation-proof state. That is, the negotiation will be over after the agent chooses a message z and the contract $w_t(z)$ is generated. Then the agent indeed should choose optimally in the single-stage mechanism. The optimal choice of the agent can be characterized as follows.

Lemma 1 (*Agent's strategy*)**.** *In a single-stage mechanism an agent with type θ will choose $z \in \mathcal{Z}_t$ (possibly mixing between messages) such that $u(w_t(z), \theta) \geq u(w_t(z'), \theta)$ for all $z' \in \mathcal{Z}_t$. In particular, $u(w_t(z), \theta) > u(w_t(z'), \theta)$ for all $z' \in \mathcal{Z}_t / \{z\}$ implies that $\sigma^a(h_t, \theta)[A] = 1$ for all $\sigma^a(h_t, \theta)$-measurable $A \subset \mathcal{Z}_t$ such that $z \in A$.*

Denote by $\nu^{M_t} \in \Delta(\mathcal{Z}_t)$ the measure on $\mathcal{Z}_t$ that is induced by the strategy $\sigma^a(h^t, \theta)$ of the agent and the belief of the principal μ_{t-1} after history h_t. That is, for all A in the Borel σ-algebra on $\mathcal{Z}_t$ it must hold $\nu^{M_t}(A) = \int_\Theta \sigma^a(h_t, \theta)[A] \, \mathrm{d}\mu_{t-1}$.

Optimal play of the agent is then understood in the following sense. The agent will choose the message z from the message space that gives him the highest payoff. Whenever more than one message gives the agent the highest possible payoffs, the agent may mix between those messages. Note that the same message may be optimal for more than one type of the agent. Thus, optimal play of the agent does not need to be fully revealing. In fact, we will show below that the optimal mechanism from the point of view of the principal is not fully revealing the type of the agent.

Next, we define which states can be induced by a mechanism in which the agent chooses optimally. That is, we define conditions on a set of states $\{C(z) = (w(z), \mu(\cdot, z)) : z \in \mathcal{Z}\}$ such that there exists a mechanism $M = (\mathcal{Z}, w(\cdot))$ that, if played optimally starting at h_t, generates this set of states. This set of states will then be called feasible. Those conditions encompass all possibilities of optimal play of the agent. Thus, they are independent from h_t other than through the induced state C_{h_t}. This implies that the definition of feasibility is with respect to C_t rather then h_t. Those conditions are necessary for states to be renegotiation-proof.

Definition 1 (*Feasibility*)**.** Let $\mathcal{Z}$ be a metric space. We call a set of states

$$\{C(z) = (w(z), \mu(\cdot, z)) : z \in \mathcal{Z}\}$$

feasible for the agent starting from $C_{h_t} = (w_{t-1}, \mu_{t-1})$ if the following conditions are satisfied.

(i) *(Individual rationality)* For all $\theta \in \text{supp}(\mu)$ there exists a $z \in \mathcal{Z}$ such that $u(w(z), \theta) \geq u(w_{t-1}, \theta)$.

(ii) (*Incentive compatibility*) For any $\theta \in \text{supp}(\mu)$, if there exists a z and a z' in $\mathcal{Z}$ such that $u(w(z), \theta) > u(w(z'), \theta)$ then $\theta \notin \text{supp}(\mu_{t-1}(\cdot : z'))$.

(iii) (*Bayesian consistency*) For the probability measure $\nu \in \Delta(\mathcal{Z})$ generated from a optimal strategy of the agent and the initial belief μ_{t-1} it holds that

$$\int_{\mathcal{Z}} \mu(\theta_\epsilon : z) d\nu = \mu_{t-1}(\theta_\epsilon).$$

We define by $IC(C) : \Gamma \to 2^\Gamma$ the mapping from some C to all feasible states starting from C. That is, $\{C(z) : z \in \mathcal{Z}\}$ is an element of $IC(C)$ if $\{C(z) : z \in \mathcal{Z}\}$ satisfies conditions (i) to (iii).

For $\{C(z) : z \in \mathcal{Z}\}$ to be states that can be generated by a mechanism starting from C_{h_t} it is necessary that the agent is weakly better off compared to the initial situation in state C_{h_t} (requirement (i)). The principal takes the optimal behavior into account when updating her belief (requirement (ii)). From the ex-ante point of view of the principal optimal play of the agent and the initial belief μ_{t-1} induce a probability distribution ν over the set of messages $\mathcal{Z}$. This should be consistent with Bayesian updating (requirement (iii)).

The conditions follow endogenously from the fact that the agent will choose an improvement if he beliefs that the contract is terminal. That is, feasible states were derived under the assumption that the agent does not expect further negotiations and thus chooses (possibly mixing) the optimal contract from his point of view. Optimal play by the agent, however, will only be induced if the principal will not renegotiate the proposed mechanism for any feasible state. Thus, we turn our attention to states in which the principal would not renegotiate.

Principal's strategy. As argued above, for our solution concept, it is sufficient to focus on strategies of the principal that prescribe her to offer a single-stage mechanism that results only in renegotiation-proof states if played optimally by the agent, and end the negotiation afterwards. Let Ω define the set of renegotiation-proof states that we yet have to define. Thus, we focus on strategies of the principal such that if $C_{h_t} \notin \Omega$, then $\sigma^p(h_t) = (Z, w(\cdot))$ such that $\{C(z) = (w(z), \mu_{t-1}(\cdot : z)) : z \in \mathcal{Z}\} \in IC(C_{h_t})$ and $\{C(z) = (w(z), \mu_{t-1}(\cdot : z)) : z \in \mathcal{Z}\} \subset \Omega$. If $C_{h_t} \in \Omega$, then $\sigma^p(h_t) = 1_{w_{t-1}}$.

We now turn our attention to the definition of Ω, the set of renegotiation-proof states. To characterize renegotiation-proof states, it is convenient to introduce some notation on which feasible states make the principal weakly better off when proposing a new mechanism in history h_t with state C_{h_t}. That is, we define conditions on a given set of feasible states

$$\{C(z) = (w(z), \mu_{t-1}(\cdot, z)) : z \in \mathcal{Z}\} \in IC(C_{h_t})$$

such that there exists a mechanism $M = (\mathcal{Z}, w(\cdot))$ that, if played optimally by the agent, generates this set of states and makes the principal weakly better off.

Definition 2. Let $\mathcal{Z}$ be a metric space. A feasible set of states

$$\{C(z) = (w(z), \mu(\cdot, z)) : z \in \mathcal{Z}\} \in IC(C_{h_t})$$

makes the principal weakly better off starting from $C_{h_t} = (w_{t-1}, \mu_{t-1})$ if the following condition is satisfied.

$$v(w_{t-1}) \leq \int_{\mathcal{Z}} v(w(z))\mathrm{d}\nu. \tag{1}$$

With $\nu \in \Delta(\mathcal{Z})$ denoting the probability measure generated from a optimal strategy of the agent and the initial belief μ_{t-1} (Lemma 1).

We define by $X(C) : \Gamma \to 2^\Gamma$ the mapping from some C to all feasible states that make the principal better of starting from C. That is, $\{C(z) : z \in \mathcal{Z}\}$ is an element of $X(C)$ if $\{C(z) : z \in \mathcal{Z}\} \in IC(C)$ and satisfies inequality (1).

We are now in the position to define renegotiation-proof states. Renegotiation-proof states are not identified one-by-one but simultaneously as a set. The key insight is that whether a state is renegotiation-proof or not will crucially depend on whether it can be improved by other feasible renegotiation-proof states. That is, renegotiation-proof states cannot be improved by other renegotiation-proof states. States that are not renegotiation-proof, can be improved by feasible *renegotiation-proof* states. Thus, the set of renegotiation-proof states should have two properties. First, independent of the current state of the negotiation it should be feasible to reach renegotiation-proof states. This is to ensure that our solution concept is well defined. Second, whenever the negotiation has reached a renegotiation-proof state, the principal should not be better off by renegotiating to another renegotiation-proof state. This is to ensure that the principal will end the negotiation after reaching a renegotiation proof state. The following definition formalizes these conditions.

Definition 3 (Renegotiation proofness)**.** $\Omega \subset \Gamma$ is a set of *renegotiation-proof states* if the following holds true.

(i) *(External consistency)* If C is not in Ω, there exists $\{C(z) : z \in \mathcal{Z}\} \in X(C)$ such that $\{C(z) : z \in \mathcal{Z}\} \subset \Omega$. That is, there is $\{C(z) : z \in \mathcal{Z}\} \in IC(C)$ that makes the principal better off and is renegotiation proof.

(ii) *(Internal consistency)* For all $C = (w, \mu)$ in Ω and for all $\{C(z) : z \in \mathcal{Z}\} \in IC(C)$ such that $\{C(z) : z \in \mathcal{Z}\} \subset \Omega$ it holds that $v(w) \geq \int_{\mathcal{Z}} v(w(z)) \mathrm{d}\nu$. That is, for all $\{C(z) : z \in \mathcal{Z}\} \in X(C)$ such that $\{C(z) : z \in \mathcal{Z}\} \subset \Omega$ it holds that $v(w) = \int_{\mathcal{Z}} v(w(z)) \mathrm{d}\nu$.

It is crucial to understand that the restrictions we place on the set of renegotiation-proof states reflect sequential rationality. Thus, the solution concept is consistent with standard solution concepts. Indeed, in a slightly simpler setting, Gretschko and Wambach (2016) show that for every set of renegotiation-proof states there exists a perfect Bayesian equilibrium of the mechanism design game that implements the optimal state in the set. Thus, renegotiation-proofness can be seen as a refinement of perfect Bayesian equilibrium.

To see how renegotiation-proofness implies sequential rationality, suppose the principal – instead of implementing the contract in a renegotiation-proof state – deviates and proposes a new mechanism M. By external consistency, the states resulting from the new mechanism can be further improved by renegotiation-proof states. Thus, the principal benefits from proposing a new mechanism M' that leads to renegotiation-proof states. It follows from internal consistency that proposing M could not have been profitable in the first place.

The same argument applies to any finite deviation by the principal. This is due to the fact that starting from some state C, any finite sequence of proposed mechanisms can be interpreted as just one feasible mechanism. Following the same argument as above, deviation to this mechanism cannot be profitable. In terms of infinite long deviations, Gretschko and Wambach (2016) show that, with the appropriate assumptions regarding the payoffs of infinite terminal histories, every infinite deviation can be improved by a finite one. In particular, this implies incentive compatibility of the mechanisms for the agent as if the principal follows the proposed selection strategy after any history the mechanisms will not be redesigned ex-post and the agent can play the mechanism optimally. Four remarks are in order.

Remark 1. Our solution concept is equivalent to the solution concept introduced by Vartiainen (2013). We demonstrate this equivalence in Section 6. The main advantage of the introduced concept is that it allows us to dispense with the details of the particular mechanism selection strategy of the principal and directly characterize renegotiation-proof states. Much like the revelation principle, this provides a tool to analyze contracting problems with limited commitment which is both effective and easy to use. We contrast the cutting power of our concept as compared with Vartiainen (2013) in Section 5 where we analyze the Coase conjecture and rederive the "gap-no-gap" result.

Remark 2. The key idea is that states that are not renegotiation proof but can be improved by renegotiation-proof states shall not block the principal's mechanism choice. That is, the principal might not renegotiate a contract even if there is an other mechanism that would make the principal strictly better off. This implies that negotiation can stop even if there is room for Pareto improvement. In particular, our solution concept does not ad-hoc rule out inefficiencies. In fact, as we will show below, the optimal mechanism from the point of view of the principal will be inefficient.

Remark 3. Our concept is related to the concept of von Neumann-Morgenstern stability (von Neumann and Morgenstern (1944)). There are two main differences. The first one is technical: von Neumann and Morgenstern (1944) define their concept as a dominance relation between elements of a set. In our context, the dominance relation is defined between elements of a set (the states) and sets of elements of a set (sets of states). The second is that in our concept internal consistency implies that if one of the players (the principal) is not made strictly better off by proposing a new mechanism that leads to renegotiation-proof states, she can decide not to propose a new mechanism. Von Neumann and Morgenstern's concept in our context would require that as long as one of the parties (i.e. principal and agent) can be made better off without making the other party strictly worse off the principal should propose a new mechanism. However, we consider the case where the principal has all the bargaining power. Thus, if she is indifferent between proposing a new mechanism and sticking to the current contract, there is no explicit need for the principal to propose a new mechanism. She might be better off by not proposing a new mechanism. Indeed, as we will show below, the optimal renegotiation-proof mechanism will result in inefficiencies. Thus, the mechanism could be improved further from the point of view of the agent, but not the principal.

Remark 4. The definition of the set of renegotiation-proof states is related to the concept of weakly renegotiation proof equilibrium as proposed by Farrell and Maskin (1989) An equilibrium of an infinitely repeated game is called weakly renegotiation proof if equilibrium payoffs of different subgames cannot be strictly Pareto ranked. Following a similar logic, internal consistency ensures that payoffs of different feasible states that are in Ω cannot make the principal strictly better off without leaving the agent strictly worse off.

We are now in the position to state the principal's optimization problem.

Lemma 2 (Principal's problem)**.** *Denote by $C_0 = (w_0, \mu_0)$ the initial contract-belief pair and by Ω sets of renegotiation-proof states. The principal's optimization problem can be* *written as*

$$\max_{\Omega,\{C(z)\}} \quad \int_{\mathcal{Z}} v(w(z))\mathrm{d}\nu$$
$$s.t. \quad \{C(z) = (w(z), \mu_0(\cdot : z)) : z \in \mathcal{Z}\} \in X(C_0)$$
$$\{C(z) = (w(z), \mu_0(\cdot : z)) : z \in \mathcal{Z}\} \subset \Omega.$$

Some useful results. One of the main advantages of our solution concept is that we can abstract from the mechanism selection strategy of the principal and simply construct sets Ω of renegotiation-proof states. That is, we need to construct sets Ω that are internally and externally consistent. Before we turn our attention to specific applications of the solution concept, we will state the following two results that facilitate the construction of renegotiation-proof Ω.

Lemma 3. *If $X(C) = \{C\}$, then C is in any renegotiation-proof Ω which satisfies the conditions of Definition 3.*

Proof. Follows directly from external consistency. If $\{C\}$ is the only element of $X(C)$, then C must be in any Ω. □

In particular, Lemma 3 implies that any Pareto-efficient allocation must be in Ω.

Lemma 4. *Let Ω satisfy the conditions of Definition 3. For any $C = (w, \mu)$ if there exists $\{C' = (w', \mu)\}$ in $X(C)$ such that $v(w') > v(w)$, then C is not in Ω.*

Proof. Suppose to the contrary there exist $C \in \Omega$ and a $C' = (w', \mu)$ such that $\{C'\} \in X(C)$ and $v(w') > v(\mathrm{w})$. In this case, internal consistency implies that C' is not in Ω. External consistency implies that there exists $\{C(z) : z \in \mathcal{Z}\} \in X(C')$ with $\{C(z) : z \in \mathcal{Z}\} \subset \Omega$. This implies that $\int_{\mathcal{Z}} v(w(z))\mathrm{d}\nu \geq v(w')$. As $\{C(z) : z \in \mathcal{Z}\}$ is feasible starting from C', $\{C(z) : z \in \mathcal{Z}\}$ is also feasible starting from C. Together with $\int_{\mathcal{Z}} v(w(z))\mathrm{d}\nu \geq v(w') > v(\mathrm{w})$ this violates internal consistency and it follows that neither $C \notin \Omega$ nor Ω is a set of renegotiation-proof states. □

Lemma 4 has an intuitive interpretation. For any potential state, if there exists a single contract that would be accepted by the agent independent of his type and makes the principal strictly better off, then the initial state cannot be renegotiation-proof. If this were the case, the principal could simply offer a mechanism in which this contract could be implemented for any message from the agent. Such a mechanism would be played truthfully by the agent since it does not reveal any additional information and would make the principal strictly better off. Thus, the contract resulting from the original state would be renegotiated.

4. Design of non-linear contracts with limited commitment

We now turn to the main application of our solution concept. We will proceed as follows. Firstly, we will set up the model. Secondly, we will introduce applications of the model. Thirdly, we will state the main result, briefly describe the main intuition and give an outline of the proof. Fourthly, we will introduce a simple example to illustrate the main result. Finally, we will provide a proof of the main result.

Set up. Consider a principal who wants to implement a two-dimensional contract $w = (p, q)$ with $q \in \mathbb{R}_+$ and $p \in \mathbb{R}$. If a contract (p, q) is implemented, the utility of the principal is given by

$$v(w) = p - c(q).$$

Denote by $c_q(\cdot)$ the derivative of $c(\cdot)$ with respect to q and by $c_{\mathrm{qq}}(\cdot)$ the second derivative of $c(\cdot)$ with respect to q. Assume that $c_q(\cdot) > 0$ and $c_{qq}(\cdot) > 0$.

The utility of the agent is given by

$$\bar{u}(w, \theta) = u(q, \theta) - p.$$

The type θ of the agent is taken from $\Theta = \left[\underline{\theta}, \bar{\theta}\right]$. Denote by u_q the derivative of u with respect to q and by u_{qq} the second derivative of u with respect to q. Similarly, denote by u_θ the derivative of u with respect to θ and by u_{q_θ} the cross-derivative of u with respect to q and θ. Assume that $u_q > 0$ and $u_{qq} \leq 0$ and that u satisfies single crossing. That is, $u_\theta > 0$ and $u_{q\theta} > 0$, a larger type receives larger utility and larger marginal utility from a given q. The principal's prior about the agent's type is given by $\mu_0 \in \Delta(\Theta)$. Assume that $\text{supp}(\mu_0) = \Theta$.[16] The initial contract w_0 is $(0, 0)$.

Applications. The initial model can be interpreted to fit, but is not limited to, the following applications.

[16]In particular, full support of μ_0 on $\Theta = \left[\underline{\theta}, \bar{\theta}\right]$ rules out discrete distributions.

(1) *Selling when price and quality matter.* The principal is a seller that sells a good to the agent, the buyer. The contract (q, p) specifies the quality q and the price p of the good. The seller incurs a cost of $c(q)$ when producing a good of quality q. A buyer of type θ enjoys a utility of $u(q, \theta)$ when consuming a good of quality q. Higher types of buyer enjoy a higher utility and a higher marginal utility from consuming the good.

(2) *Procurement.* The principal is a buyer who procures a good from the agent, the seller. The contract (p, q) specifies the quantity q and the price $-p$ of the good. The buyer derives a utility of $-c(q)$ when procuring a quantity q of the good. A seller of type θ incurs a cost of $-u(q, \theta)$ when producing a quantity q of the good. Higher types of seller enjoy a lower cost of production and a lower marginal cost of production.

(3) *Franchising.* The principal is a manufacturer who produces a quantity q of a good at cost $c(q)$ and sells the good to a retailer, the agent, at price p. The retailer faces a demand of $D(t, \theta)$ for the good with t denoting the resale price and θ the demand shock that is private knowledge to the retailer. Higher θ imply a higher demand and a higher marginal demand. The profit for the retailer from selling the good is $u(q, \theta) = tD(t, \theta)$.

(4) *Labor contracts.* The principal is a potential employer and the agent is a worker. The contract (p, q) specifies the effort q of the agent and his wage $-p$. The principal derives a utility of $-c(q)$ from the effort of the agent. An agent with productivity θ incurs a cost of effort of $-u(q, \theta)$. Higher types of agent enjoy a lower cost of effort and a lower marginal cost of effort.

In what follows we will adopt the "*selling when price and quality matter*" interpretation of the model and call q the quality of the good and p the price.

Useful properties of the model. Before we turn our attention to the main result, we first state some useful definitions for and properties of the considered model. Denote by $q^*(\theta)$ the efficient quality for a given type θ. The efficient quality is implicitly given by

$$-v_q(q^*(\theta)) = u_q(q^*(\theta), \theta). \tag{2}$$

Given the assumptions we made about v and u, $q^*(\theta)$ is unique and satisfies

$$q^*_\theta(\theta) = \frac{u_{q\theta}(q^*(\theta), \theta)}{v_{qq}(q^*(\theta)) - u_{qq}(q^*(\theta), \theta)} > 0.$$

Definition 4. Define μ^θ as the probability measure that puts probability 1 on type θ. That is, for all measurable sets A, $\mu^\theta(A) = 1$ whenever $\theta \in A$ and $\mu^\theta(A) = 0$ otherwise. We will call an state

(1) $C = ((p, q), \mu^\theta)$ *a separating* state. Efficient and separating states are denoted by $C = ((p, q^*(\theta), \mu^\theta)$,

(2) $C = ((p, q), \mu)$ *a pooling* state if it holds $\mu_0(\text{supp}(\mu)) > 0$. With μ_0 being the initial belief of the principal.

Lemma 5. *If the principal can commit to any mechanism, there exists a direct, individually rational and incentive compatible mechanism that implements the efficient quality. That is, there exists a price function $p(\theta)$ such that for all types θ,*

$$u(q^*(\theta), \theta) - p(\theta) \geq u(q^*(\hat{\theta}), \theta) - p(\hat{\theta})$$

for all $\hat{\theta} \in \Theta$. This holds true for all $p(\theta)$ such that

$$p_\theta(\theta) = u_q(q^*(\theta), \theta)q^*(\theta) > 0. \tag{3}$$

Proof. See Fudenberg and Tirole (1990) Theorem 7.3 for a proof. □

Lemma 6. *If every type obtains his efficient quality and the prices satisfy (3), then the principal is indifferent between all contracts; she obtains the same profit from all types. That is, $v(q^*(\theta)) - p(\theta) = k$ for some constant k.*

Proof. This is a consequence of $p_\theta(\theta) - v_q(q^*(\theta))q^*_\theta(\theta) = (u_q q^*(\theta), \theta) - v_q(q^*(\theta)))q^*_\theta(\theta) = 0$. Due to equation (2) and equation (3). □

The main result. Our main result is that the optimal set of states $\{C(z)\}_{z\in\mathbb{N}}$ that will not be renegotiated by the seller has the following properties. Firstly, all states that are achieved with positive probability are pooling states. Secondly, for each of these pooling states the lowest type choosing this contract obtains his efficient quality; all other types receive an inefficient quality.

Theorem 1. *The profit maximizing set of renegotiation-proof states* $\{C(z)\}_{z\in\mathbb{N}}$ *is countably infinite and has the following properties*

(1) (Pooling) *All* $C(z) = ((p,q),\mu)$ *are pooling states.*

(2) (No distortion at the bottom) $C(z) = ((p_z, q^*(\theta_z)),\mu)$ with $\theta_z = \min(\text{supp}(\mu))$.

The maximization problem of the principal becomes:[17]

$$\begin{aligned} \max_{\{\theta_z\}_{z\in\mathbb{N}}} \quad & \sum_{z\in\mathbb{N}}(p_z - v(q^*(\theta_z))\mu_0([\theta_z,\theta_{z+1}]) \\ \text{s.t.} \quad & \theta_{z+1} > \theta_z,\ \theta_0 = \underline{\theta},\ \text{and}\ \theta_z < \bar{\theta} \\ & u(q^*(\theta_{z+1}),\theta_{z+1}) - p_{z+1} = u(q^*(\theta_z),\theta_{z+1}) - p_z. \end{aligned} \tag{4}$$

One remark is in order with respect to the solution of the maximization problem of the principal.

Remark 5. To solve the maximization problem of the principal we can use the incentive constraints $(q^*(\theta_{z+1}),\theta_{z+1}) - p_{z+1} = u(q^*(\theta_z),\theta_{z+1}) - p_z$ to back out the optimal prices. That is, setting $p_0 = \underline{\theta}$ and solving recursively yields $p_k = \sum_{t=1}^{k} u(q^*(\theta_t),\theta_t) - u(q^*(\theta_{t-1}),\theta_t) + \underline{\theta}$. The optimization problem of the principal can be rewritten to give

$$\begin{aligned} \max_{\{\theta_z\}_{z\in\mathbb{N}}} \quad & \sum_{z\in\mathbb{N}}\left[\left(\sum_{t=1}^{z} u(q^*(\theta_t),\theta_t) - u(q^*(\theta_{t-1}),\theta_t) + \underline{\theta}\right) - v(q^*(\theta_z))\mu_0([\theta_z,\theta_{z+1}])\right] \\ \text{s.t.} \quad & \theta_{z+1} > \theta_z,\ \theta_0 = \underline{\theta},\ \text{and}\ \theta_z < \bar{\theta} \end{aligned} \tag{5}$$

This problem can then be approximated numerically for finite z. We demonstrate this approach in the example below.

We will now provide an intuition for the result and an outline of the proof. Moreover, we will provide a simple example that illustrates the results and contrasts them with models with full commitment.

Intuition and outline for the proof of the main result. To gain some intuition for our main result, observe that by Lemma 3 efficient and separating states must be in any set of renegotiation-proof states. Thus, only states that cannot be strictly improved by a set of efficient and separating states can be in any set of renegotiation-proof states. The

[17]A solution to this problem exists, as a solution to the auxiliary problem with $\theta_{z+1} \geq \theta_z$ and $\theta_z \leq \bar{\theta}$ exists and Lemma 11 shows that it necessarily is on the interior.

only states that cannot be improved by a set of efficient and separating states are pooling states with a connected support in which one of the types in the pool receives his efficient quality. This is due to the fact that the type that received his efficient quality in the pooling state, also receives his efficient quality in a feasible set of efficient and separating states. Thus, by Lemma 6, if the principal were to propose a new mechanism that led to efficient and separating states, she would make the same profit as with the pooling state. The profit maximizing states among such pooling states is then a countably infinite set of pooling states with the lowest type in the pool receiving his efficient quality. This stems from the fact that separation of types is only possible with efficient states. However, efficient and separating states imply a higher information rent than, say, two pooling states would. Moreover, if the principal offers the efficient quality to the lowest type in the pool, she reduces the information rent of the higher types that are not in the pool. The proof of Theorem 1 proceeds by a series of lemmata that reflect the intuition given above. The outline is instructive for the understanding of the structure of renegotiation-proof sets.

Firstly, we show that for any state C there exists a set of efficient and separating states that are feasible and make the principal weakly better off (Lemma 7). Secondly, we demonstrate that the following states *cannot* be part of any renegotiation-proof Ω:

(1) Inefficient and separating states, that is, $C = ((p, q), \mu^{\theta})$ such that $q \neq q^{*}(\theta)$ (Lemma 8).

(2) States with a belief of the principal whose support has a gap, that is, $C = ((p, q), \mu)$ such that $\text{supp}(\mu)$ is not connected (Lemma 9).

(3) States with a quality that is efficient for a type that is not in the support of the belief of the principal, that is, $C = ((p, q), \mu)$ such that $q = q^{*}(\theta)$ but $\theta \notin \text{supp}(\mu)$ (Lemma 10).

Thus, every set of renegotiation-proof states must consist of a combination of efficient and separating states and pooling states with a connected support such that one of the types in the pool receives his efficient quality.

Thirdly, we prove that the profit maximizing set of states $\{C(z)\}$ among the states with a connected support of $\mu(z)$ and an efficient quality for one type in $\text{supp}(\mu(z))$

takes the form described in Theorem 1. That is, the set is countably infinite, $\mu(z)$ is not degenerate for every $C(z)$, and the lowest type in support of $\mu(z)$ receives his efficient quality (Lemma 11). The proof of Lemma 11 proceeds along five steps.

(1) We show that in the set of optimal states higher types receive higher quality.

(2) We demonstrate that the principal optimally makes a higher profit from the higher agent types.

(3) We argue that efficient and separating states are not optimal for the principal. This is due to the fact that if the principal offers the efficient quality to a connected subset of types while satisfying the incentive compatibility constraints, she will not obtain additional rent from higher types. Those types receive a high information rent. In this case, the principal can reduce the information rent by offering pooling contracts.

(4) We show that the lowest type in a pooling and connected state will receive his optimal quality in the profit-maximizing set of states. If in the pooling state the quality is optimal for some intermediate type, the quality for the lower types is distorted in the wrong direction. That is, they receive too much quality. In this case, the seller prefers to give some types a lower quality which reduces the information rent to higher types. The best way to achieve this is to give the lowest type in the pool the efficient quality, such that the quality for all the types is distorted in the right direction.

(5) We demonstrate that the set of profit-maximizing states must be countably infinite. To see this, let us suppose to the contrary that the number of states is finite. In this case, there exists an state that contains all types above some θ. It follows that the principal is better off if she splits this pool into two pools: one where she obtains the same profit as before and that contains the lower types of agent and a second one where she obtains a higher profit and that is chosen by the higher types of agent.

Fourthly, we demonstrate that the resulting set is internally and externally consistent and thus renegotiation-proof (Lemma 12).

Overall, we construct the optimal set of renegotiation-proof contracts from the point of view of the principal starting from (w_0, μ_0). Of course, there exist other sets of renegotiation-proof contracts that are internally and externally consistent and could be implemented by a principal with a consistent mechanism selection strategy. For example, if the initial contract w_0 is efficient for some type $\theta \in [\underline{\theta}, \bar{\theta}]$ the set including all efficient and separating states and the initial state (w_0, μ_0) can be supported as renegotiation proof. Even though it would not be the optimal set of contracts. Moreover, Lemma 12 implies any set of pooling states such that one of types in the support of μ_z receives his efficient quality can be sustained as renegotiation proof. Even the set that divides the optimal pools from Theorem 1 even further. Again, such a set of renegotiation-proof states would not be optimal from the point of view of the principal at (w_0, μ_0).

The construction also illustrates how internal and external consistency work together to achieve recursively renegotiation-proof states. Suppose the principal decides to split one of the pools $[\theta_z, \theta_{z+1})$ further by renegotiating and offering for example one additional contract that splits the pool in $[\theta_z, \theta)$ and $(\theta, \theta_{z+1}]$. Such a split can be profitable given our construction. However, from each of the new pools the only feasible renegotiation-proof states are efficient and separating states for all of the types in the pools. Those states would have been also feasible in the first place. However, they make the principal exactly indifferent between pooling the types in $[\theta_z, \theta_{z+1})$ or offering efficient contracts.

Before we turn our attention to the proof of Theorem 1, we shall illustrate our results by means of an example.

Illustration of the results by means of an example. Let $v(p, q) = p - 1/2q^2$ and $u(p, q, \theta) = \theta q - p$. Let $\Theta = [1, 2]$ and μ_0 the uniform measure on $[1, 2]$. In this case the efficient quality for each type is given by $q^*(\theta) = \theta$. We compare four different scenarios.

(i) First best. If the principal is able to observe the agent's type, the principal can extract all the surplus from the agent. She will offer contract $(p, q) = (\theta^2, \theta)$ to an agent of type θ and thereby achieve the first-best allocation from her point of view. The overall profit of the principal is then given by

$$\int_1^2 \frac{1}{2}\theta^2 \, \mathrm{d}\theta = \frac{7}{6}.$$

(ii) Efficient contracting. If the principal cannot observe the agent's type, she is still able to implement the efficient quality levels. Efficient quality levels are achieved by offering a menu of contracts $(p, q) = (0.5(1 + q^2), q)$ for $q \in [1, 2]$. In this case, for each type θ of the agent it is optimal to choose $(0.5(1 + \theta^2), \theta)$ so that every type of agent obtains his efficient quality. Moreover, the agent with the type $\theta = 1$ obtains a rent of 0, every other type of agent receives a positive rent. The principal obtains the same profit from every type of agent which amounts to

$$\frac{1}{2}\left(1 + \theta^2\right) - \frac{1}{2}\theta^2 = \frac{1}{2}.$$

(iii) Second best – with commitment. If the principal cannot observe the agent's type and can fully commit, she can implement distorting contracts to maximize her own profit. The optimal contract for each type θ is then given by $(p, q) = (\theta^2 - 1, 2(\theta - 1))$. In this case, the agent of type $\theta = 2$ obtains a contract with his efficient quality. Every other type of agent $\theta \in [1, 2)$ obtains a contract with a quality that is lower than his efficient one. The result is often called "*no distortion at the top*". The second-best state can be achieved by offering a menu of contracts $(p(q), q) = (q^2/4 + q, q)$. The profit of the principal is given by

$$\int_1^2 \theta^2 - 1 - 2(\theta - 1)^2 \, \mathrm{d}\theta = \frac{2}{3}.$$

(iv) Third best – without commitment. Theorem 1 provides us with a structure of the set of renegotiation-proof states. Any increasing sequence θ_z with contracts $(p_z, q^*(\theta_z))$ such that $u(q^*(\theta_{z+1}), \theta_{z+1}) - p_{z+1} = u(q^*(\theta_z), \theta_{z+1}) - p_z$ constitutes a set of renegotiation-proof states. Thus, we merely have to solve for the optimal such sequence. Using the incentive compatibility constraints in equation (4), that is, $\theta_{z+1}^2 - p_{z+1} = \theta_{z+1}\theta_z - p_z$, and setting $p_0 = 1$, the program in Theorem 1 can be rewritten to give

$$\begin{aligned} \max \quad & 1 + \sum_{z \in \mathbb{N}} \left((2 - \theta_{z+1})\, \theta_{z+1} - \frac{1}{2}\theta_z^2 \right) (\theta_{z+1} - \theta_z) \\ \text{s.t.} \quad & \theta_{z+1} > \theta_z, \ \theta_0 = 1, \ \text{and } \theta_z < 2 \end{aligned}.$$

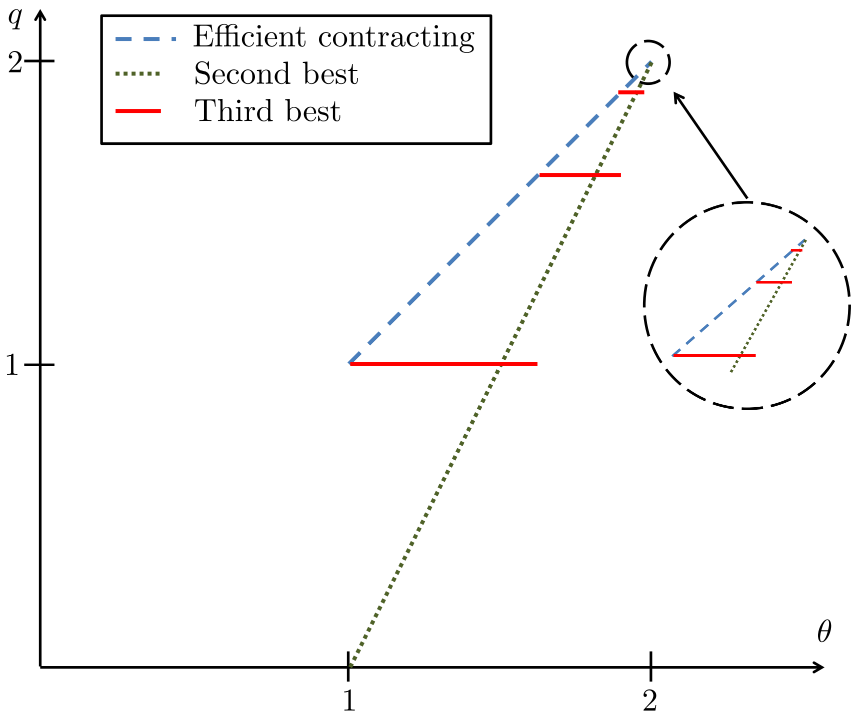

FIGURE 1. The comparison of the efficient quality levels (blue line) with the quality levels with the second-best solution with commitment (green line) and the third-best solution without commitment (red line).

We approximated this program for $z \in \{0, \ldots, 6\}$ numerically.[18] With six contracts the optimal pooling of types is given by

$$\begin{aligned}
[\theta_0, \theta_1] &= [1, 1.63], & (p_0, q_0) &= (1, 1), & u(p_0, q_0, \theta_0) &= 0\\
[\theta_1, \theta_2] &= [1.63, 1.87], & (p_1, q_1) &= (2.03, 1.63), & u(p_1, q_1, \theta_1) &= 0.63\\
[\theta_2, \theta_3] &= [1.87, 1.95], & (p_2, q_2) &= (2.48, 1.87), & u(p_2, q_2, \theta_2) &= 1.012\\
[\theta_3, \theta_4] &= [1.95, 1.98], & (p_3, q_3) &= (2.63, 1.98), & u(p_3, q_3, \theta_3) &= 1.17\\
[\theta_4, \theta_5] &= [1.98, 1.99], & (p_4, q_4) &= (2.69, 1.98), & u(p_4, q_4, \theta_4) &= 1.23\\
[\theta_5, \theta_6] &= [1.99, 2], & (p_5, q_5) &= (2.71, 1.99), & u(p_5, q_5, \theta_6) &= 1.25.
\end{aligned}$$

The profit of the principal is given by

$$0.5773.$$

The overview of the results is depicted in Figure 1.

To get an idea for how the optimization changes we additionally summarized the results of the simulations for $z \in \{0, 1\}, \ldots, z \in \{0, 1, 2, 3, 4, 5\}$ in Table 1. As will be

[18] Adding z=7 and thereby a seventh contract increased the profit of the principal only in the order of 10^{-5}.

$\max z$	θ_1	θ_1	θ_3	θ_4	Profit
2	1.67				0.5740
3	1.64	1.88			0.5771
4	1.63	1.87	1.96		0.5773
5	1.63	1.87	1.95	1.98	0.5773

TABLE 1. Development of pooling of types for numerical solutions with maximum number of pools from 2 to 5.

corroborated by Lemma 11 the principal ideally splits pools at the top of the type space if she gains an additional degree of freedom. This is consequence of the fact that splitting pools at the bottom of the type space would distort incentives for higher types in an unfavorable way.

Proof of the main result. We will now turn our attention to the proof of Theorem 1.

Lemma 7. *For any state $C = ((p, q), \mu)$ there exists a set of efficient and separating states $\{C(z)\}$ which is feasible and makes the principal weakly better off. That is, $\{C(z)\} \in X(C)$.*

Proof. The proof is relegated to the appendix. □

Lemma 8. *If $\theta \neq \theta'$, a separating $C = ((p, q^*(\theta), \mu^{\theta'})$ cannot be an element of any set of renegotiation-proof states Ω.*

Proof. Starting from $C = ((p, q^*(\theta), \mu_{\theta'})$, there exists a feasible state $\bar{C} = ((\bar{p}, q^*(\theta')), \mu_{\theta'})$ that makes the principal strictly better off. Lemma 4 implies the result. □

In what follows, we will call states such that $\text{supp}(\mu)$ is non-degenerated, pooling states.

Lemma 9. *If $\text{supp}(\mu)$ is not connected, $C = ((p, q), \mu)$ cannot be an element of any set of renegotiation-proof states Ω.*

Proof. That $\text{supp}(\mu)$ is not connected implies that there exist θ' and θ'' in $\text{supp}(\mu)$ with $\theta'' > \theta'$ such that $\mu((\theta', \theta'')) = 0$. Suppose that $q = q^*(\hat{\theta})$ for some $\hat{\theta} \leq \theta'$.[19] We will show that there exists a feasible set of efficient and separating states that make the principal strictly better off. Consider the following menu of contracts $\{(p(\theta), q^*(\theta)) : \theta \in \text{supp}(\mu)\}$ with $p_\theta = u_q q^*_\theta$. For $\theta \leq \theta'$ set $p(\hat{\theta}) = p$. For $\theta \geq \theta''$ set $p(\theta'') = p(\theta') + u(q^*(\theta''), \theta'') -$

[19]The case $\hat{\theta} > \theta'$ works analogously.

$u(q^*(\theta'), \theta'')$. From Lemma 5 it follows that each type of agent is better off with the contract with his efficient quality. That is, an agent of type θ is better off with $(p(\theta), q^*(\theta))$. The seller is strictly better off, since she makes the same profit from all types $\theta \leq \theta'$ and strictly more profit from types $\theta \geq \theta''$. This is a consequence of the efficiency of $q^*(\theta'')$ and

$$[p(\theta'') - v(q^*(\theta''))] - [p(\theta') - v(q^*(\theta'))] =$$

$$[u(q^*(\theta''), \theta'') - v(q^*(\theta''))] - [u(q^*(\theta'), \theta'') - v(q^*(\theta'))] > 0.$$

Thus, we constructed a feasible set of efficient and separating states that make the principal strictly better off. As, due to Lemma 7, all efficient and separating states are in every set of renegotiation-proof states, the initial state C could not have been part of any set of renegotiation-proof states. □

Lemma 7, Lemma 8 and Lemma 9 taken together illustrate why similar models with discrete type spaces lead to efficient states: with discrete type spaces the support of μ cannot be connected. Thus, renegotiation-proof states need to be separating and this is only possible with efficient states. In the following we will call states such that $\text{supp}(\mu)$ is connected, connected states.

Lemma 10. *If $\tilde{\theta} \notin \text{supp}(\mu)$, $C = ((p, q^*(\tilde{\theta})), \mu)$ cannot be an element of any set of renegotiation-proof states.*

Proof. From Lemma 9 it follows that we need only to consider connected states $C = ((p, q^*(\theta)), \mu)$. Suppose $\tilde{\theta} < \min \text{supp}(\mu) = \theta'$.[20] We show that there exists a set of feasible and efficient states that make the principal strictly better off. Consider the following set of contracts: $(p(\theta), q^*(\theta))$ with $p_\theta = u_q q^*_\theta$ and $p(\theta') = p + \left[u(q^*(\theta'), \theta') - u(q^*(\tilde{\theta}), \theta')\right] > p$. It follows from Lemma 5 that each type of agent is better off with the contract with his efficient quality. That is, an agent of type θ is better off with $(p(\theta), q^*(\theta))$. The seller is strictly better off because she can offer to every agent type the efficient quality at a higher price. Thus, we constructed a set of efficient and separating states that are feasible and leave the principal better off. As, due to Lemma 7, all efficient and separating states are

[20] The case $\tilde{\theta} > \max \text{supp}(\mu)$ works analogously.

in every set of renegotiation-proof states, the initial state C could not have been part of any set of renegotiation-proof states. □

At this point we have shown that renegotiation-proof states must be either efficient and separating or pooling and connected such that one of the agent types in the support obtains his efficient quality. We now show that the profit-maximizing set of states among those states takes the form as described in Theorem 1.

Lemma 11. *The profit-maximizing set of states $\{C(z)\}_{z\in\mathbb{N}}$ among efficient and separating or pooling and connected states such that one of the agent types in the support obtains his efficient quality is countably infinite and has the following properties:*

(1) (Pooling) *For all $C(z) = ((p, q), \mu)$ it holds $\mu_0(\text{supp}(\mu)) > 0$*

(2) (No distortion at the bottom) $C(z) = ((p, q^*(\theta_z), \mu)$ with $\theta_z = \min(\text{supp}(\mu))$.

Proof. The proof is relegated to the appendix. □

It remains to be shown that a set of states as described in Lemma 11 is indeed renegotiation-proof.

Lemma 12. *Let $\{C(\theta_z) = (p_i, q^*(\theta_z), \mu_z)\}_{z\in\mathbb{N}} \in X(C_0)$ be a set of feasible states starting from C_0 such that $\mu_z([\theta_z, \theta_{z+1})) = 1$. There exists a set of renegotiation-proof states Ω such that $\{C(\theta_z)\}_{z\in\mathbb{N}} \subset \Omega$.*

Proof. Let $\Omega = \{C : C \text{ is efficient and separating}\} \cup \{C(\theta_z)\}_{z\in\mathbb{N}}$. Firstly, we consider external consistency. By Lemma 7, for any state C there exists a set of efficient and separating states which is feasible and makes the principal weakly better off. Thus, Ω is externally consistent.

Secondly, we consider internal consistency. Take any $C \in \Omega$. If C is efficient and separating, $X(C) = \{C\}$. If $C = C(\theta_z)$ for some z then the only feasible set of states in Ω that makes the principal weakly better off is a set of efficient and separating states such that θ_z obtains the same contract as before. From Lemma 6, it follows that the principal makes the same profit as before. Thus, internal consistency is not violated. □

5. The Coase conjecture

In this section we consider the Coase conjecture, which is a special instance of our set-up. The Coase conjecture argues that if a seller is not able to commit to not selling a durable good, she can at most charge a price equal to the lowest valuation of the buyer as long as the cost of the seller is strictly below the buyer's lowest valuation (gap case). Whenever the cost of the seller is equal or above the lowest valuation of the buyer (no-gap case), the seller is able to charge the monopoly price even without commitment (Ausubel and Deneckere (1989)). Our approach, using renegotiation-proof states, allows us to rederive this result in a simple manner.

Set up. Consider a monopolistic seller who is selling one object to a buyer. A contract is a tuple $w = (p, q)$ with $p \in \mathbb{R}$ specifying the price and $q \in \{0, 1\}$ specifying whether the good is exchanged ($q = 1$) or not ($q = 0$). The seller incurs a cost of c of producing the good and this cost is common knowledge between the seller and the buyer. Thus, the utility function of the seller is given by

$$v(w) = p - cq.$$

The buyer has a valuation of $\theta \in \Theta = \left[\underline{\theta}, \overline{\theta}\right]$ for consuming the good, which is private information to the buyer. Thus, the utility of the buyer is given by

$$u(w, \theta) = \theta q - p.$$

The seller's prior about the valuation of the buyer is given by $\mu_0 \in \Delta(\Theta)$. To simplify notation, assume that $\text{supp}(\mu_0) = \Theta$. The initial contract w_0 is $(0, 0)$. In the following we will study two cases; the gap case with $c < \underline{\theta}$ and the no-gap case with $c \geq \underline{\theta}$. In the first case, there is a gap between the cost of the seller and the lowest valuation of the buyer so it is common knowledge that there are gains from trade. In the second case, the buyer can have a valuation that makes trading the good inefficient.

The gap case. In what follows we will show that if $c < \underline{\theta}$, the unique set of renegotiation-proof states contains only states in which the good is traded. This implies that the seller will optimally charge a price equal to the lowest valuation and not propose a new mechanism after the buyer has accepted.

Proposition 1. *Let $c < \underline{\theta}$, the unique set of renegotiation-proof states Ω that satisfies the conditions of Definition 3 is $\Omega = \{C = ((p, q), \mu) : q = 1\}$.*

Proof. We show that for a given Ω that is internally and externally consistent, $((p, q), \mu)$ is in Ω if and only if $q = 1$.
"if": For any state $C = ((p, 1), \mu)$ there exists no set of states that would make both parties weakly better off. Thus, $X(C) = \{C\}$. It follows from Lemma 3 that C must be in Ω.
"only if": Let $C = ((p, 0), \mu)$. Consider the contract $(p + c + 1/2(\underline{\theta} - c), 1)$. The buyer strictly prefers this contract to $(p, 0)$ independent of his type. Moreover the seller is strictly better off than with contract $(p, 0)$. Thus, $\{((p + c + 1/2(\underline{\theta} - c), 1), \mu)\}$ is feasible and in $X(C)$. Lemma 4 gives the desired result. □

Proposition 1 implies that starting from a status quo in which the good is not traded, any final state cannot imply a price charged for the good that is above the lowest valuation that is in the support of the current belief of the principal. If this is not the case, some of the types of buyer would optimally choose not to buy the good, which would lead to $q = 0$ in some of the states. Such states, however, are not in the unique set of renegotiation-proof states.

That the set of renegotiation proof states $\Omega = \{C = ((p, q), \mu) : q = 1\}$ is unique implies the following optimization problem of the seller.

$$\begin{aligned} \max_{p} \quad & p - c \\ \text{s.t.} \quad & p \leq \underline{\theta} \end{aligned} .$$

The solution of this problem is the state $((\underline{\theta}, 1), \mu_0)$.

Corollary 1. *If $c < \underline{\theta}$ and the seller cannot commit to a mechanism, she can only charge a price of $p = \underline{\theta}$.*

This result is in agreement with the literature on the Coase conjecture: the monopolist competes herself down to the lowest valuation.[21]

The no-gap case. We now consider $c \geq \underline{\theta}$, the no-gap case. In this case, we construct an internally and externally consistent Ω such that in the optimal set of states within this Ω, the seller can charge the monopoly price. As the problem of the seller without commitment is a more constrained version of the problem of a seller with full commitment, charging the monopoly price must also be the optimal solution without commitment.

Proposition 2. *Let p^M denote the monopoly price. That is, $p^M = \arg\max_p (1-\mu_0(p))(p-c) > c$. An Ω as defined below is internally and externally consistent.*

$C = ((p,q), \mu)$ is in Ω if one of the following holds true

(1) $q = 1$ and for all θ in $\text{supp}(\mu)$ it holds $\theta \geq c$.

(2) $q = 0$ and for all θ in $\text{supp}(\mu)$ it holds $\theta \leq c$.

(3) $q = 0$, $\max(supp(\mu)) = p^M$, and there exists an $\epsilon > 0$ such that for all $\delta \leq \epsilon$ it holds $\mu\left[(c-\delta, c+\delta) \cap \text{supp}(\mu)\right] > 0$.

Proof. The proof is relegated to the appendix □

The set of renegotiation-proof states Ω consists of three types of sets. The first two are the efficient states. States in which the good is traded, and the valuations are above costs (1). States in which the good is not traded and valuations are below costs (2). The third consists of inefficient states in which the good is not traded, the highest valuation in the support of the belief of the principal is weakly below the monopoly price, and the belief puts positive mass on types of agents with valuations arbitrarily close to the cost (3). Observe that this defines a renegotiation proof set. Firstly, all efficient outcomes are in Ω. Secondly, with regard to states in (3), the only way to negotiate to other renegotiation-proof states would imply to offer a price equal to the cost. Thus, the set of renegotiation-proof states in Proposition 2 is constructed in such a way that the seller proposes a mechanism that leads to two states: either the object is traded at a price p^M and the seller believes that all buyers have a valuation above p^M, or the object is not traded and the seller beliefs that all buyers have a valuation below p^M. Whenever

[21] See for example Fudenberg et al. (1985), or more recently Strulovici (2017).

the object is not traded, the only other feasible renegotiation-proof states that make the principal weakly better off are that the good is traded at a price equal to c or not traded at all. Thus, the seller cannot profit from proposing a new mechanism whenever the object is not traded. As charging the monopoly price is the optimal mechanism under full commitment it must also be the optimal mechanism without commitment.

Corollary 2. *If $c \geq \theta$, the principal can charge the monopoly price p^M even without commitment.*

This result is in agreement with the non-cooperative bargaining literature. Ausubel and Deneckere (1989) show that in the no-gap case, in an infinite-horizon bargaining game, the seller can sustain the monopoly price in equilibrium if the frictions go to zero.

For a discrete type space, Vartiainen (2013) demonstrated how his solution concept can recreate the Coase conjecture in the gap case.[22] For the no-gap case he writes that, in principle, it is possible to construct mechanism selection strategies that the principal can commit to. However, those selection strategies would be complex. Thus, he refrains from constructing any such strategies and makes the additional assumption that mechanism selection strategies need to be stationary. In this case, even in the no-gap case, the seller sells at a price equal to her cost. In this section we demonstrated the simplicity and cutting power of our approach: to focus on states rather than selection strategies. We recreated the conceptual difference between the gap and no-gap case without resorting to complex selection strategies.

6. Comparison with Vartiainen (2013)

In this section we will shortly summarize the approach by Vartiainen (2013). Vartiainen argues that sequential rationality of the principal, and the agent's knowledge of this, requires that a mechanism selection strategy of the principal σ^p reflects consistency and optimization. To be able to define these conditions, we first need to develop some concepts.

We start by stating the payoff of the principal from using mechanism $M_t = (\mathcal{Z}_t, w_t(\cdot))$ after history h_t given that the agent plays this mechanism optimally in the sense of

[22] For a comparison of our solution concept with Vartiainen (2013) see Section 6.

Lemma 1. As above, denote by $\nu^{M_t} \in \Delta(\mathcal{Z}_t)$ the measure on $\mathcal{Z}_t$ that is induced by the strategy $\sigma^a(h^t, \theta)$ of the agent and the belief of the principal μ_t after history h_t. The payoff of the principal can then be written as

$$V(M_t) = \int_{\mathcal{Z}_t} v(w_t(z)) \mathrm{d}\nu^{M_t}.$$

Suppose at history h_t the principal can commit to following her mechanism selection strategy in the future. For $z \in \mathcal{Z}_t$ define h^z_{t+1} by $h^z_{t+1} = (h_t, (M_t, z, w_t(z)))$. We say the principal will not change a mechanism $M_t = (\mathcal{Z}_t, w_t(\cdot))$ today given that σ^p is followed in the future if for all $z \in \mathcal{Z}_t$ it holds

$$v(w_t(z)) \geq V(\sigma^p(h^z_{t+1})).$$

That is, after every possible realization of mechanism M_t, the mechanism selection strategy of the principal would prescribe to implementing a mechanism with a weakly lower expected surplus. Thus, if the principal follows her mechanism selection strategy, independent of the message of the agent, M_t would not be renegotiated ex post. Such a mechanism would be played optimally by the agent in the sense of Lemma 1 given the mechanism selection strategy of the principal.

We denote by $\mathcal{M}^{\sigma^p}(h_t)$ the set of all mechanisms that are truthfully playable at history h_t given that the principal follows her mechanism selection strategy σ^p afterwards. That is,

$$\mathcal{M}^{\sigma^p}(h_t) = \left\{ M = (\mathcal{Z}, w(z)) \in \mathcal{M} : v(w(z)) \geq V(\sigma^p(h^z_{t+1})) \text{ for all } z \in \mathcal{Z} \right\}.$$

We are now in the position to formally specify conditions that Vartiainen (2013) imposes on a mechanism selection strategy of the principal. The first condition requires consistency. That is, employing σ^p ex-ante should not contradict employing σ^p ex-post. The second condition implies optimality. Given σ^p and h_t, the principal should choose a mechanism that maximizes her payoff in the set $\mathcal{M}^{\sigma^p}(h_t)$. In particular, under the hypothesis that σ^p can be committed to in the future, the principal does not want to change σ^p for any message of the agent.

Definition 5. A mechanism selection strategy σ^p

(1) is *consistent* if $\sigma^p(h_t) \in \mathcal{M}^{\sigma^p}(h_t)$ for all $h_t \in \mathcal{H}$.

(2) is *optimal* if $V(\sigma^p(h_t)) \geq V(M)$ for all $M \in \mathcal{M}^{\sigma^p}(h_t)$ for all $h_t \in \mathcal{H}$.

A thorough discussion of this approach can be found in Vartiainen (2013). Vartianinen summarizes why Definition 5 implies that the principal can commit to the selection strategies that are consistent with it as follows. Note that optimality together with consistency reflects sequential rationality. This is due to the fact that a selection strategy σ^p with these properties maximizes the principal's payoff in $\mathcal{M}^{\sigma^p}$. That all states of $\sigma^p(h_t)$ are in $\mathcal{M}^{\sigma^p}(h_t)$ for every history guarantees that this act of optimization is consistent with foresight. That is, since σ^p is obeyed in the future, $\sigma^p \in \mathcal{M}^{\sigma^p}$ guarantees that σ^p will not be redesigned and thus can be committed to.

Vartiainen (2014) demonstrates that a mechanism selection rule that is consistent and optimal can be interpreted as a reduced form (weak) Perfect Bayesian Equilibrium of the mechanism design game as defined in Section 2. That is, the sub-games of this games can be viewed as single-stage mechanisms that result from a consistent mechanism selection strategy. In particular, consistency of the mechanism selection rule implies incentive compatibility of the mechanisms as if a mechanism M is in $\mathcal{M}^{\sigma^p}$ at any history it will not be redesigned ex-post and the agent can play the mechanism optimally.

We are now in a position to compare our solution concept with Vartiainen (2013).

Proposition 3. *The following holds true*

(1) Let σ^p be a history independent mechanism selection strategy that is consistent and optimal (Definition 5). In this case, $\bigcup_{C\in\Gamma} f(\sigma^p(C), C, \sigma^a)$ is internally and externally consistent (Definition 3).

(2) For every Ω that is internally and externally consistent such that for all $C \in \Gamma$ the solution to

$$\begin{aligned} \max_{\{C(z)\}} \quad & \int_{\mathcal{Z}} v(w(z))\mathrm{d}\nu \\ s.t. \quad & \{C(z)) = (w(z), \mu(\cdot : z)) : z \in \mathcal{Z}\} \in X(C) \\ & \{C(z)) = (w(z), \mu(\cdot : z)) : z \in \mathcal{Z}\} \subset \Omega. \end{aligned}$$

exists, there is a mechanism selection strategy of the principal σ^p *that is consistent and satisfies the one deviation property such that* $\bigcup_{C\in\Gamma} f(\sigma^p(C), C, \sigma^a) \subset \Omega$.[23]

Proof. The proof is relegated to the appendix. □

On one hand, a history independent, consistent and optimal mechanism selection strategy in the sense of Vartiainen (2013) generates for each subgame a set of states that is internally and externally consistent. On the other hand, whenever we have a set of renegotiation-proof states that is internally and externally consistent, the mechanism selection strategy of choosing a mechanism that leads to the principal-optimal renegotiaton-proof states is a consistent and optimal mechanism selection strategy. Thus, both solution concepts coincide.

7. Conclusion

The main contribution of this manuscript is to construct the optimal mechanism for the screening problem of a principal who is not able to commit to not renegotiating any contract resulting from the mechanism she proposes. In the case of private values with non-linear contracts, the optimal mechanism is inefficient given that it leads to a pooling of types. This is different from previous work on contract design without commitment that find that with private values the principal can only implement efficient contracts.[24] With continuous types, we observe a countably infinite number of pooling contracts which each satisfy a "no-distortion-at-the-bottom" property.

To arrive at the results we characterize the set of renegotiation-proof states by using internal and external consistency. The main advantage of this approach is that, in contrast to other definitions of renegotiation-proofness, we do not assume that renegotiation-proof states must be efficient. This is due to the fact that we do not consider states as blocking which themselves are subject to renegotiation.

We demonstrate the cutting power of our concept by also considering the Coase conjecture. We show that, if there is a gap between the costs of the seller and the lowest valuation of the buyer, the seller can only charge a price equal to the lowest valuation

[23]Whether for a given Ω the solution to the maximization problem exists, can be verified directly for each setting at hand. In our applications in Sections 4 and 5 this is straightforward.

[24]For example see Gretschko and Wambach (2016), Maestri (2017), Strulovici (2017), and Vartiainen (2013).

of the buyer. However, if there is no such gap, the seller is able to charge the monopoly price even without commitment.

Appendix

Proof of Lemma 7.

Proof. Three cases are relevant. Either there exists a type $\tilde{\theta}$ in $\text{supp}(\mu)$ such that $q = q^*(\tilde{\theta})$ or for $\theta' = \min(\text{supp}(\mu))$ it holds $q < q^*(\theta')$ or for $\theta'' = \max(\text{supp}(\mu))$ it holds $q > q^*(\theta'')$. We focus on the first case only, the other cases can be proven analogously.

Let $q = q^*(\tilde{\theta})$ for some $\tilde{\theta} \in \text{supp}(\mu)$. Consider the following set of efficient and separating states $\left\{C(\theta) = ((q^*(\theta), p(\theta)), \mu^\theta)\right\}$ such that $p(\tilde{\theta}) = p$ and $p_\theta(\theta) = u_q(q^*(\theta), \theta)q^*(\theta)$. An agent of type $\tilde{\theta}$ will receive the initial contract and an agent of type $\theta \neq \tilde{\theta}$ will receive a contract that makes him strictly better off. Thus, this set of states is individually rational for each type of the agent (condition (i) of Definition 1). Moreover, $p(\theta)$ satisfies equation 3. Thus, this set of states is incentive compatible (condition (ii)) and satisfies Bayesian consistency (condition (iii)). As a consequence of

$$p_\theta(\theta) - v_q(q^*(\theta))q^*_\theta(\theta) = (u_q q^*(\theta), \theta) - v_q(q^*(\theta)))q^*_\theta(\theta) = 0,$$

the principal is indifferent between all states in $\left\{C(\theta) = ((q^*(\theta), p(\theta)), \mu^\theta)\right\}$. In particular, she is then indifferent between the original state C and any state in

$$\left\{C(\theta) = ((q^*(\theta), p(\theta)), \mu^\theta)\right\}.$$

Hence, the proposed set of states satisfies individual rationality of the principal (condition (iv)). Overall, this implies that

$$\left\{C(\theta) = ((q^*(\theta), p(\theta)), \mu^\theta)\right\} \in X(C).$$

□

Proof of Lemma 11. The proof is divided into five steps. We show that

Step 1 *In the set of optimal* states*, higher types obtain higher quality*

Step 2 *In the set of optimal* states*, the seller achieves a weakly higher profit from higher types*

Step 3 *Efficient and separating* states *are not in the set of optimal* states

Step 4 *For every* state *in the set of optimal* states*, the lowest type in the support of the belief of the principal receives his efficient quality.*

Step 5 *The set of optimal* states *is countably infinite.*

Denote by $\{C(z)\}$ the set of feasible, profit-maximizing states for the seller starting from C^0 and by $\{w(z)\}$ the set of corresponding contracts.

Step 1: In the set of optimal states, higher types obtain higher quality.

Proof. The principal maximizes among efficient and separating or pooling and connected states such that one of the types receives his efficient quality. Thus, for $\theta_2 > \theta_1$ with q_2 and q_1 denoting the quality that type θ_2 respectively θ_1 receives, two cases are relevant. Firstly, both types obtain their efficient quality $q_2 = q^*(\theta_2)$ and $q_1 = q^*(\theta_1)$. In this case, as q^* is an increasing function, $q_2 > q_1$. Secondly, both types are in different pooling states.[25] In this case, as one of the types in each pool receives his efficient quality, the pools are connected and due to the fact that q^* is an increasing function, $q_2 > q_1$. Thirdly, both types are in the same pooling state. In this case, $q_2 = q_1$. Summing up, it follows that if $\theta_2 > \theta_1$, $q_2 \geq q_1$, that is, higher types receive a higher quality. □

Step 2: In the set of optimal states, the seller achieves a weakly higher profit from higher types.

Proof. We will show that for any type θ there exists an $\epsilon > 0$ such that for all types $\theta' \in (\theta, \theta + \epsilon)$ the seller realizes an equal or higher profit than with type θ. Let $C = ((p, q), \mu)$ such that $\theta \in \text{supp}(\mu)$. If $\theta < \max\{\text{supp}(\mu)\}$, that is, if C is a pooling and connected state and θ is not the largest type in the pool, there exists an $\epsilon > 0$ such that all types $\theta' \in (\theta, \theta + \epsilon)$ receive the same contract. Thus, the seller makes the same profit with all these types. Assume that $\theta = \max\{\text{supp}(\mu)\}$, that is, assume that θ is the largest type in the pool or that C is an efficient and separating state. All types $\theta' > \theta$ obtain a different contract than type θ. If there exists an $\epsilon > 0$ such that almost all types in

[25]Or one of the types receives his efficient quality and the other is in a pooling state.

$\theta' \in (\theta, \theta+\epsilon)$ receive their efficient contract, then the seller makes the same profit from all of these types (Lemma 6). If type θ also obtains his efficient quality, the seller makes the same profit from θ and any θ'. If type θ does not obtain his efficient quality, then the seller makes a strictly larger profit from any type θ'. Thus, assume that for all $\epsilon > 0$ the amount of types in $(\theta, \theta+\epsilon)$ that do not receive their efficient contract is of positive measure. In this case there exists an $\epsilon > 0$ such that all types in $(\theta, \theta+\epsilon)$ are in the same pooling state with a contract $\hat{w} = (\hat{p}, q^*(\hat{\theta}))$ for some $\hat{\theta}$. Call this state $\hat{C} = (\hat{w}, \hat{\mu})$. If the seller obtains the same profit or higher with $(\hat{p}, q^*(\hat{\theta}))$ than with (p, q), we are done. Thus, assume that the seller makes less profit with $(\hat{p}, q^*(\hat{\theta}))$. We will show that there exists a set of states that makes the seller strictly better off. In this case there exists a type θ' with $\theta < \theta' < \hat{\theta}$ and a contract $w' = (p', q^*(\theta'))$ such that $\bar{u}((p,q),\theta) = \bar{u}((p', q^*(\theta')),\theta)$, $\bar{u}((\hat{p}, q^*(\hat{\theta})),\hat{\theta}) > \bar{u}((p', q^*(\theta')),\hat{\theta})$, and $v((p', q^*(\theta')) > (\hat{p}, q^*(\hat{\theta}))$. That is, we can find a contract with an efficient quality for type $\theta' \in (\theta, \hat{\theta})$. Moreover, type θ is indifferent between his original contract and the new contract, type $\hat{\theta}$ strictly prefers his original contract, and the principal makes a higher profit from the new contract. Now consider the following set of states $\{C(z)\} \setminus \hat{C} \cup \{C', C''\}$ with $C' = (w', \mu')$ and $C'' = (\hat{w}, \mu'')$. By construction, there exists a type $\theta'' \in (\theta, \hat{\theta})$ such that all types between θ and θ'' prefer w' to w and all types between θ'' and $\max\{\text{supp}(\hat{\mu})\} = \tilde{\theta}$ prefer $\hat{w}$. Thus, set μ' such that it is Bayesian consistent and satisfies $supp(\mu') = (\theta, \theta'']$ and μ'' such that it is Bayesian consistent and satisfies $\text{supp}(\mu'') = (\theta'', \tilde{\theta})$. It follows that the constructed set of states, $\{C(z)\} \setminus \hat{C} \cup \{C', C''\}$, is feasible and leaves the principal better off. Thus, the original set of states, $\{C(z)\}$, could not have been optimal. □

Step 3: Efficient and separating states are not in the set of optimal states.

Proof. We will show that there is no interval $[\theta', \theta''] \subset \Theta$ with $\mu_0([\theta', \theta'']) > 0$ such that all types $\theta \in [\theta', \theta'']$ obtain their efficient quality $q^*(\theta)$. Denote the set of states that contain those types by $\{C(\theta)\} \subseteq \{C(z)\}$. We will find a different set of states that is feasible starting from C_0 and makes the principal strictly better off. Denote by p' the price of the contract that θ' obtains. Consider the following two contracts $w_1 = (p', q^*(\theta'))$ and $w_2 = (\hat{p}, q^*(\hat{\theta}))$ with $\hat{\theta} = (1/2)(\theta'' + \theta')$ and $\hat{p} = \bar{u}(q^*(\theta'), \hat{\theta}) - p'$. Now consider the following set of states $\{C(z)\} \setminus \{C(\theta)\} \cup \{C_1, C_2\}$ with $C_1 = (w_1, \mu_1)$ and $C_2 = (w_2, \mu_2)$.

By construction, all types not in $[\theta', \theta'']$ still prefer their old contract $w(z)$ to both w_1 and w_2. Types in $[\theta', \hat{\theta}]$ prefer w_1 to any other contract in $\{w(z)\} \setminus \{w(\theta)\}$. Types in $[\hat{\theta}, \theta'']$ prefer either w_2 or some contract in $\{w(z)\} \setminus \{w(\theta)\}$ that is also preferred by a higher type. Thus, there exist Bayesian consistent μ_1 and μ_2 such that the constructed set of states $\{C(z)\} \setminus \{C(\theta)\} \cup \{C_1, C_2\}$ is feasible starting from C_0 and makes the principal strictly better off.[26] □

Step 4: For every state in the set of optimal states, the lowest type in the support of the belief of the principal receives his efficient quality.

Proof. Suppose to the contrary that there exists a $C(\hat{z}) = (w(\hat{z}), \mu(\hat{z})) = ((p, q), \mu(\hat{z})) \in \{C(z)\}$ such that $\min \operatorname{supp}(\mu) = \theta$ but $q > q^*(\theta)$. Consider the following contract $(p', q^*(\theta))$ with $p' = \bar{u}(q^*(\theta), \theta) - \bar{u}(q, \theta) + p$ and the following set of states $\{C(z)\} \setminus \{C(z)\}_{z \geq \hat{z}} \cup C' \cup \left\{\hat{C}(z)\right\}_{z > \bar{z}}$ with $C' = ((p', q^*(\theta)), \mu')$ and $\hat{C}(z) = ((w(z), \hat{\mu}(z))$. That is, construct a set of states such that all contracts which are in the new set of states are the same as in $\{C(z)\}$ with the exception that (p, q) is swapped for $(p', q^*(\theta))$. By construction, all types below θ still prefer their old contract. Moreover, there exists a $\theta' > \theta$ such that all types in $[\theta, \theta') \subseteq \operatorname{supp}(\mu)$ prefer $(p', q^*(\theta))$ and all types in $[\theta', \max \operatorname{supp}(\mu)]$ prefer one of the contracts $w(z)$ with $z > \hat{z}$. By construction, the principal makes a strictly higher profit from types in $[\theta, \theta')$. Step 1 implies that the principal makes at least as much profit from types in $[\theta', \max \operatorname{supp}(\mu)]$ as if the set of states were $\{C(z)\}$. Thus, there exist Bayesian consistent μ' and $\{\hat{\mu}(z)\}$ such that the constructed set of states is feasible starting from C_0 and makes the principal strictly better off.[27] □

Step 5: The set of optimal states is countably infinite.

Proof. So far we have shown that the optimal set of states partitions $\left[\underline{\theta}, \overline{\theta}\right]$ in connected intervals of strictly positive measure. This implies that if the set of states is infinite it must be countable. Thus, we merely need to show that the set of optimal states is not

[26]The idea of this proof is straightforward. If the principal gives each type of agent in some interval his efficient quality and satisfies the incentive compatibility constraints, she does not earn any additional rent from higher types by increasing the information rent they earn. If the principal offers two pooling contracts instead, the information rent to the high types is reduced.

[27]The idea of the proof is the following. If a pooling state contains a contract where q is optimal for some intermediate type, the lower types are distorted in the wrong direction. They obtain too much quality. The seller thus prefers to give some of those types a contract with a lower quality.

finite. Suppose to the contrary that there exists a $\theta \in \Theta = [\underline{\theta}, \bar{\theta}]$ and $C = ((p, q^*(\theta)), \mu) \in \{C(z)\}$ such that $\text{supp}(\mu) = [\theta, \bar{\theta}]$. Now take any $\theta' \in (\theta, \bar{\theta})$, consider the contract $w' = ((p', q^*(\theta'))$ with $p' = \bar{u}(q^*(\theta'), \theta') - \bar{u}(q^*(\theta), \theta') + p$, and construct a new set of states $\{C(z)\} \setminus C \cup \{C_1, C_2\}$ with $C_1 = ((p, q^*(\theta)), \mu_1)$ and $C_2 = ((p', q^*(\theta')), \mu_2)$. Types in $[\theta, \theta')$ prefer $(p, q^*(\theta))$ and types in $[\theta', \bar{\theta}]$ prefer $(p', q^*(\theta'))$. The principal obtains a strictly higher profit from $(p', q^*(\theta'))$. Thus, there exist Bayesian consistent μ_1 and μ_2 such that the constructed set of states is feasible starting from C_0 and makes the principal strictly better off. □

Proof of Proposition 2.

Proof. We start by showing that Ω is externally consistent. Let $C = ((p, q), \mu)$ be not Ω. Consider the following two states $C^1 = ((p + (1-q)c, 1), \mu_1)$ and $C^2 = ((p - qc, 0), \mu_2)$ such that $\mu_1(\cdot) = \mu(\cdot : \theta \geq c)$ and $\mu_2(\cdot) = \mu(\cdot : \theta < c)$. That is, an state in which the good is traded at a price of $p + (1-q)c$ and an state in which the object is not traded and the price is $p - qc$. We check now whether the conditions of Definition 1 and Definition 2are satisfied and the proposed states are feasible and make the seller weakly better off.

(i) Individual rationality. State C^1 is individually rational for buyers with a valuation above c and state C^2 is individually rational for buyers with a valuation below c.

(ii) Incentive compatibility. State C^1 is strictly preferred by buyers with a valuation above c to state C^2. State C^2 is strictly preferred by buyers with a valuation below c. The seller does not believe that she is facing buyers with a valuation below c in state C^1 and buyers with a valuation above c in state C^2.

(iii) Bayesian consistency. The probability of reaching state C^1 is $\mu((\underline{\theta}, c])$ and of reaching C^2 is $\mu\left(\left[c, \overline{\theta}\right]\right)$. Thus, $\mu((\underline{\theta}, c])\,\mu_1 + \mu\left(\left[c, \overline{\theta}\right]\right)\mu_2 = \mu$.

(iv) Individual rationality of the seller. The seller is indifferent between C, C^1, and C^2.

Thus, $\{C^1, C^2\} \in X(C)$ is feasible, makes the seller weakly better off and by definition of Ω, C^1 and C^2 are in Ω.

Now that we have shown that Ω is externally consistent we turn our attention to internal consistency. We show that for all states $C \in \Omega$ there is no set of feasible states that is also in Ω and makes the seller strictly better off.

For every state with $q = 1$ and $\theta \geq c$ or $q = 0$ and $\theta \leq c$ for all θ in $\text{supp}(\mu)$ there exists no set of states which would make the seller strictly better off and the buyer not worse off. Thus, any such state cannot be improved.

We turn our attention to states $C = ((0, p), \mu)$ such that $\max(\text{supp}(\mu)) = p^M$ and there exists an $\epsilon > 0$ such that for all $\delta \leq \epsilon$ it holds $\mu[(c - \delta, c + \delta) \cap \text{supp}(\mu)] > 0$. Due to incentive compatibility of the buyer any feasible set of states starting from C consists of at most two states. One state where the object is exchanged ($q = 1$) and one where the object is not exchanged ($q = 0$). The set of states that merely consists of one state in Ω with $q = 0$ either does not make the seller better off or is not individually rational to the buyer starting from C. The set of states that merely consist of one state in Ω with $q = 1$ and a belief such that all $\theta \geq c$, either does not make the seller better off, due to $\mu[(c - \delta, c + \delta) \cap \text{supp}(\mu)] > 0$ or does not satisfy individual rationality of the buyer or does not satisfy Bayesian consistency. Thus, again due to $\mu[(c - \delta, c + \delta) \cap \text{supp}(\mu)] > 0$ the only set of feasible states that are in Ω is $\{C^1 = ((p + c, 1), \mu_1), C^1 = ((p, 0), \mu_2)\}$ with $\mu_1(\cdot) = \mu(\cdot : \theta \geq c)$ and $\mu_2(\cdot) = \mu(\cdot : \theta < c)$. However, the seller is indifferent is indifferent as to whether he does not sell at a price of p or sells at a price of $p + c$. Thus, starting from $C = ((0, p), \mu)$ as defined above, there exists no set of feasible states that is also in Ω and makes the seller better off. This implies that Ω is internally consistent. □

Proof of Proposition 3.

Proof. Ad (i): Let σ^p be a mechanism selection strategy that is consistent and optimal. Let $\Omega = \bigcup_{C \in \Gamma} f(\sigma^p(C), C, \sigma^a)$. External consistency follows, by definition, from the fact that for every C in Γ, $f(\sigma^p(C), C, \sigma^a)$ is in $X(C)$ and a subset of Ω. It remains to check that Ω is internally consistent. Let $C \in \Omega$ and $\{C(z) = (w(z), \mu(z)) : z \in \mathcal{Z}\} \in X(C)$ with $C(z) \in \Omega$ for all $z \in \mathcal{Z}$ and some metric space $\mathcal{Z}$. Denote by $M = (\mathcal{Z}, w(z))$ the mechanism that is induced by $\{C(z) : z \in \mathcal{Z}\}$. As $\{C(z) : z \in \mathcal{Z}\}$ is in $X(C)$, M exists and is well defined. Due to consistency of σ^p and the fact that $C(z)$ is in Ω it follows that $\sigma^p(C(z)) \leq v(w(z))$. Thus, M is in $\mathcal{M}^{\sigma^p}$. Due to optimality of σ it follows that $V(\sigma^p(C)) \geq V(M)$. Thus, again by consistency, $V(\sigma^p(C)) \leq v(w)$ and thereby $V(M) \leq v(w)$.

Ad (ii): Let Ω be externally and internally consistent and let the solution to

$$\begin{aligned} &\max_{\{C(z)\}} && \int_{\mathcal{Z}} v(w(z))\mathrm{d}\nu \\ (6) \qquad &\text{s.t.} && \{C(z)) = (w(z), \mu(\cdot : z) : z \in \mathcal{Z}\} \in X(C) \\ & && \{C(z)) = (w(z), \mu(\cdot : z) : z \in \mathcal{Z}\} \subset \Omega. \end{aligned}$$

exist. Define σ^p as follows. Choose for every $C = (w, \mu) \in \Omega$, $\sigma^p(C) = (\mathcal{Z}, w(z) \equiv w)$ and for $C \notin \Omega$, $\sigma^p(C) = (\mathcal{Z}, w(z))$ such that $\{C(z) : z \in \mathcal{Z}\} \in X(C)$, $\{C(z) : z \in \mathcal{Z}\} \subset \Omega$ and $\{C(z)\}$ is a solution to Problem 6. This is always possible due to external consistency and the fact that a solution exists. For any $C \in \Gamma$, $f(\sigma^p(C), C, \sigma^a)$ is a subset of Ω. Thus, by definition, for every $C' = (w', \mu') \in f(\sigma^p(C), C, \sigma^a)$, $v(w') \geq V(\sigma^p(C')) = v(w')$. It follows that $\sigma^p(C)$ is in $\mathcal{M}^{\sigma^p}(C)$ and thereby σ^p is consistent. For optimality take any $M = (\mathcal{Z}, w(z))$ in $\mathcal{M}^{\sigma^p}(C)$. Observe that $\{C(z) : z \in \mathcal{Z}\} = f(M, C, \sigma^a)$ is feasible and makes the principal weakly better off. First, suppose that $\{C(z) : z \in \mathcal{Z}\} \subset \Omega$ it follows from the definition of $\sigma^p(C)$ as the solution to Problem (6) that $V(\sigma^p(C)) \geq V(M)$. Thus, suppose that there exists $\mathcal{Z}' \subseteq \mathcal{Z}$ such that for all $z' \in \mathcal{Z}'$, $C(z') \notin \Omega$. Consider the following set of states $\mathcal{C} = \{C(z) : z \in \mathcal{Z} \setminus \mathcal{Z}'\} \cup \bigcup_{z' \in \mathcal{Z}'} f(\sigma(C(z'), C(z'), \sigma^a)$. By definition of σ^p, $\mathcal{C}$ is feasible starting from C and makes the principal weakly better off, $\mathcal{C} \subset \Omega$ and $V(\mathcal{C}) \geq V(M)$. It follows from the definition of σ^p as the solution of Problem (6) that $V(\sigma^p(C)) \geq V(\mathcal{C}) \geq V(M)$. □

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