Essays in Labor and Family Economics

Inauguraldissertation
zur Erlangung des akademischen Grades
eines Doktors der Wirtschaftswissenschaften
der Universität Mannheim

vorgelegt von
Hanno Foerster

im Frühjahrs-/Sommersemester 2019
Abteilungssprecher: Prof. Dr. Jochen Streb
Referent: Prof. Dr. Michèle Tertilt
Korreferent: Prof. Dr. Gerard J. van den Berg

Tag der Verteidigung: 01. August 2019
Acknowledgements

I am extremely grateful to my advisers Gerard van den Berg and Michèle Tertilt for their outstanding guidance and support. I benefited incredibly from their mentorship and their thoughtful, constructive feedback on my research.

Furthermore I would like to thank Hans-Martin von Gaudecker, Katja Kaufmann and many Professors at the University of Mannheim who have shared their advice- Antoine Camous, Sebastian Findeisen, Andreas Gulyas, Anne Hannusch, Matthias Meier, Andreas Peichl, Anna Raute and Minchul Yum.

My fellow students at the CDSE enriched this thesis significantly through many helpful discussions and made my graduate studies a lot of fun. I especially want to thank Albrecht Bohne, Tobias Etzel, Niklas Garnadt, Anna Hammerschmid, Lukas Henkel, Ruben Hipp, Mario Meier, Jan Nimczik, Tim Obermeier, Alexander Paul and Laura Pohlan.

Financial support from ERC grant SH1-313719 and funding by the German Research Foundation (DFG) through CRC TR 224 is gratefully acknowledged. I am also grateful to Aarhus University and Statistics Denmark for providing the data used in the first chapter of this dissertation.

I am very thankful to my parents, my sister and my friends for their support and encouragement. Most importantly, I want to thank Eva for her love and invaluable support.
## Contents

Acknowledgements iii  
Preface 1  

1 The Impact of Post-Marital Maintenance on Dynamic Decisions and Welfare of Couples 5  
1.1 Introduction ........................................ 5  
1.2 Institutional Background .......................... 10  
1.2.1 Child Support .................................... 10  
1.2.2 Alimony .......................................... 11  
1.2.3 Maintenance Payments .......................... 12  
1.3 Data and Descriptive Statistics .................. 12  
1.3.1 Maintenance Payments: Data vs. Imputations .... 14  
1.3.2 Evidence from Event Studies: Work Hours around Divorce 16  
1.4 Model ............................................ 18  
1.5 Estimation ........................................ 26  
1.5.1 Pre-set Parameters .............................. 26  
1.5.2 Directly Estimated Parameters .................. 27  
1.5.3 Method of Simulated Moments Estimation ....... 29  
1.6 Underlying Frictions and First Best Allocation .... 31  
1.6.1 First Best Scenario .............................. 32  
1.6.2 Characterization of the First Best Allocation and Underlying Frictions 34  
1.7 Policy Simulations ................................ 36  
1.7.1 The Impact of Child Support on Time Use and Consumption 36  
1.7.2 The Impact of Alimony on Time Use and Consumption 39
1.7.3 The Impact of Child Support and Alimony on Divorce Rates . . . . . . 42
1.8 Welfare Analysis ................................................................. 43
   1.8.1 Welfare Comparisons and Optimal Policy .......................... 43
   1.8.2 Comparison to First Best ............................................. 45
1.9 Conclusion ................................................................. 46

2 A Structural Analysis of Vacancy Referrals with Imperfect Monitoring and Sickness Absence 49
   2.1 Introduction ................................................................. 49
   2.2 Institutional Background .............................................. 53
      2.2.1 UI Benefits .......................................................... 53
      2.2.2 Vacancy Referrals and Sanctions ................................ 53
      2.2.3 Sick Leave ......................................................... 55
   2.3 Data ................................................................. 55
   2.4 Model ................................................................. 57
   2.5 Estimation ................................................................. 61
   2.6 Estimation Results ...................................................... 66
   2.7 Policy Simulations ...................................................... 72
   2.8 Conclusion ................................................................. 78

3 The Equilibrium Effects of Vacancy Referrals 81
   3.1 Introduction ................................................................. 81
   3.2 Institutional Background .............................................. 84
   3.3 Model ................................................................. 85
   3.4 Data ................................................................. 90
   3.5 Estimation ................................................................. 95
      3.5.1 Pre-set and Directly Estimated Parameters .................... 95
      3.5.2 Generalized Method of Moments Estimation .................... 96
      3.5.3 Parameter Estimates and Model Fit ............................ 96
   3.6 Policy Simulations ...................................................... 99
      3.6.1 The Impact of VRs on Aggregate Labor Market Outcomes .... 99
      3.6.2 Measuring the Equilibrium Effects of VRs .................... 101
   3.7 Conclusion ................................................................. 102
# Appendix to Chapter 1

A.1 Maintenance Payments, Details and Functional Forms ................. 113
A.2 Computational Details ............................................. 115
A.3 Timing of Events ..................................................... 116
A.4 Child Custody ......................................................... 116
A.5 Model Fit ............................................................... 122
A.6 Figures ................................................................. 123

# Appendix to Chapter 2

B.1 Derivations ........................................................... 125
   B.1.1 Value of Employment ........................................... 125
   B.1.2 Terminally Sanctioned Unemployed ............................. 126
   B.1.3 Derivation of the System of Reservation Wage Equations ..... 127
   B.1.4 Likelihood Contributions ....................................... 129
B.2 Figures ............................................................... 130
B.3 Tables ................................................................. 132

# Appendix to Chapter 3

C. Appendix to Chapter 3

CV ................................................................. 139
List of Figures

1.1 Child support rules ................................................................. 11
1.2 Alimony rules ................................................................. 11
1.3 Maintenance payments, data and imputations ............................... 15
1.4 Maintenance payments by payer’s labor income, data and imputations 15
1.5 Maintenance payments by no. children, data and imputations ............. 15
1.6 Weekly work hours around divorce ........................................... 17
1.7 Model fit ................................................................. 31
1.8 Event study: relative consumption ........................................... 39
1.9 Event study: relative consumption ........................................... 42
1.10 Welfare comparisons: changing child support (b) ......................... 44
1.11 Welfare comparisons: changing alimony (τ) ................................. 44
1.12 Welfare comparison: status quo, optimal maintenance policy and first best 46

2.1 Model fit: accepted wages ...................................................... 68
2.2 Implied reservation wages, h: health restrictions, a: apprenticeship ....... 71
2.3 Increasing sanction enforcement, reservation wages ......................... 73
2.4 Sending more VRs, reservation wages ....................................... 75

3.1 No. of VRs and applications, worker- and firm-side ......................... 94
3.2 Model fit, no. of applications .................................................. 98
3.3 Counterfactual policy scenarios .............................................. 100
3.4 Worker match rates, given VR/ no VR ..................................... 101

A.1 Timing of events for married couples ....................................... 116
A.2 Women’s weekly work around divorce, by number of children ........... 123
A.3 Men’s weekly work around divorce, by number of children ................ 124
List of Tables

1.1 Summary statistics, Danish register data ........................................... 13
1.2 Summary statistics (age re-weighted), Danish time use survey ................. 14
1.3 Pre-set parameters .............................................................................. 27
1.4 Distribution of initial no. of children .................................................. 28
1.5 Fertility process .................................................................................. 28
1.6 MSM parameter estimates ................................................................... 30
1.7 Simulated outcomes: removing frictions from the model ....................... 35
1.8 The effect of changing child support ($b$) on married couples’ time use .... 37
1.9 The effect of changing child support ($b$) on divorced couples’ time use .... 37
1.10 The effect of changing child support ($b$) on couples’ relative consumption .. 38
1.11 The effect of changing alimony ($\tau$) on married couples’ time use ........... 40
1.12 The effect of changing alimony ($\tau$) on divorced couples’ time use ........... 41
1.13 The effect of changing alimony ($\tau$) on couples’ relative consumption ........ 42
1.14 The effect of changing child support ($b$) on divorce rates .................... 43
1.15 The effect of changing alimony ($\tau$) on divorce rates ............................ 43
1.16 Mean outcomes: status quo, optimal maintenance policy and first best ...... 45

2.1 Parameter estimates, basic specification. ............................................. 68
2.2 Parameter estimates, full specification .................................................. 69
2.3 Model fit .............................................................................................. 70
2.4 Changing sanction enforcement, simulation results ............................... 73
2.5 Changing the VR rate, simulation results .............................................. 74
2.6 Eliminating VR induced sick reporting ............................................... 77
2.7 Eliminating VR induced sick reporting, $p_{doc}$ top 20% ......................... 78
Preface

This dissertation studies questions in labor and family economics. It consists of three self-contained chapters, that, as a common underlying theme, study how policy and institutions influence economic decisions made by individuals and families. In all three chapters I develop economic models that I use in combination with micro-data from different sources to address questions of high policy relevance.

The topics I study in this dissertation, within the fields of labor and family economics, fall into two broad areas. The first area is the study of how aspects of divorce law shape married couples’ and divorced individuals’ decision-making and impact on their wellbeing. The second area is the analysis of labor markets and the evaluation of active labor market policies that are designed to help unemployed workers find jobs. Chapter 1 relates to the former, chapters 2 and 3 to the latter of these research areas.

Chapter 1 is titled “The Impact of Post-Marital Maintenance on Dynamic Decisions and Welfare of Couples”. This chapter studies post-marital maintenance payments (child support and alimony) between divorced ex-spouses. In many countries divorce law mandates such payments to insure the lower earner in married couples against financial losses upon divorce. I study how maintenance payments affect couples’ intertemporal decisions and welfare. To this end I develop a dynamic model of family labor supply, housework, savings and divorce and estimate it using Danish register data. The model captures the policy trade off between providing insurance to the lower earner and enabling couples to specialize efficiently, on the one hand, and maintaining labor supply incentives for divorcees, on the other hand. I use the estimated model to analyze counterfactual policy scenarios in which child support and alimony payments are changed. The welfare maximizing maintenance policy is to triple child support payments and reduce alimony by 12.5% relative to the Danish status quo. Switching to the welfare maximizing policy makes men worse off, but comparisons to a first best scenario.
reveal that Pareto improvements are feasible, highlighting the limitations of maintenance policies.

Chapter 2 is titled “A Structural Analysis of Vacancy Referrals with Imperfect Monitoring and Sickness Absence” and is coauthored with Gerard van den Berg and Arne Uhlendorff. In many OECD countries unemployment insurance agencies assign job vacancy referrals (VRs) to unemployment benefit recipients. Refusals to apply for VRs are sanctioned with temporary benefit reductions. In this chapter we study the impact of VRs and sanctions on unemployed workers’ job search behavior, accounting for the possibility that workers may strategically report sick to avoid sanctions. We develop a structural model of unemployed workers’ job search behavior that incorporates VRs and sanctions. Our model takes into account that unemployed workers may rationally seek to get a sick note to circumvent a sanction. We account for rich observed and unobserved heterogeneity. We estimate our model using German administrative data from social security records that are linked to caseworker records on VRs, sickness absences and sanctions. The estimated model is used to simulate various counterfactual policy scenarios. We find that increasing sanction enforcement reduces moral hazard, while increasing the VR rate leads to more moral hazard by increasing the option value of search. According to our estimates 9.6% of sick reports among unemployed workers are induced by VRs. We find substantial heterogeneity in the effects of eliminating VR induced sick reporting. Effects are modest for around 80% of the population. For the remaining 20% of workers shutting down VR induced sick reporting reduces the mean unemployment duration by 0.8 months.

Chapter 3 is titled “The Equilibrium Effects of Vacancy Referrals” and is coauthored with Gerard van den Berg. A main purpose of VRs is bringing together unemployed workers and firms who otherwise would not have matched. Existing quasi-experimental evidence shows that VRs positively impact the job finding probabilities of VR recipients. The impact on the economywide employment rate however depends on the magnitude of equilibrium effects. This chapter studies the equilibrium effects of VRs and the channels through which they operate. We develop a search and matching model that accounts for three channels through which equilibrium effects arise: crowding out in the hiring process, disincentive effects on workers own job search effort and changes in firms’ vacancy posting behavior. We estimate our model using German data on unemployed workers’ job search behavior and firm hiring decisions. Based on the estimated model we find the average individual level effect (net of
equilibrium effects) of a VR on job finding probabilities is 3.1 p.p. In contrast, a reform that increases the VR rate from 58% (the status quo) to 80% increases job finding rates only by 2.8 p.p. A simple randomized experiment would overstate the effect of the reform by 14%. The main channel accountable for the discrepancy is crowding out in the hiring process.
Chapter 1

The Impact of Post-Marital Maintenance on Dynamic Decisions and Welfare of Couples

1.1 Introduction

Marital breakdown often has severe financial consequences for the lower earner in divorcing couples. The U.S. poverty rate among women who got divorced in 2009 was 21.5%, compared to 10.5% for divorced men and 9.6% for married people (Elliott and Simmons, 2011). For this reason most societies have divorce laws that mandate post-marital maintenance payments, such as alimony and child support, to insure the lower earner in couples against losing access to their partner’s income upon divorce.

Over the past decade fierce political debates about reducing post-marital maintenance payments have emerged in several countries, including the U.S., Germany, the U.K. and France. These debates were typically dominated by two economic arguments: Those in favor of reducing maintenance payments emphasized that divorcees who receive high maintenance payments have little incentive to work and become economically self-sufficient. Those in favor of high maintenance payments argued that people who invest less in their careers after getting married, e.g., because they spend time on child-care and housework, should be insured against the drop in financial resources upon divorce. How relevant is each of these arguments quantitatively? And how should post-marital maintenance policies be designed to balance
both arguments?

In this paper I provide the first study of how maintenance payments should be designed to balance an important policy trade off. In particular, I ask how child support and alimony payments should be designed to provide insurance to the lower earner in couples and enable couples to specialize efficiently, on the one hand, and maintain labor supply incentives for divorcees, on the other hand.

A number of empirical studies documents that alimony and child support payments influence the behavior of married and divorced couples. Many studies document that increases in child support lead to a reduction in divorced father’s labor supply (Holzer et al. (2005); Cancian et al. (2013)). There is also evidence that introducing alimony for existing couples leads to a decrease in women’s and an increase in men’s labor supply (Rangel (2006); Chiappori et al. (2016); Goussé and Leturcq (2018)).\(^1\) The empirical evidence strongly suggests that maintenance payments influence couples’ behavior. Nonetheless, I argue that to draw conclusions about how maintenance policies affect couples’ welfare, a joint economic framework of couples’ consumption, labor supply and time allocation and (endogenous) divorce is needed.

To examine the consequences of post-marital maintenance policies for couples’ welfare, I develop a dynamic structural model of married and divorced couples’ decision-making. In my model divorced ex-spouses are linked by maintenance payments, which depend on both ex-spouses’ labor earnings, their number of children and who the children stay with after divorce.

Decision-making of divorced couples is modeled as non-cooperative (dynamic) game. In deciding about their labor supply, each ex-spouse takes into account how own choices influence her/his ex-spouse’s choices and how the stream of maintenance payments is affected. Accounting for the strategic interdependence in ex-spouses’ labor supply decisions, which arises because of maintenance payments, is a novel feature relative to the previous literature.

Married spouses are influenced by maintenance policies as their outside options (their values of divorce) are affected by maintenance payments. In modeling decision-making in marriage I build on the limited commitment framework (see Kocherlakota (1996); Ligon et al.

---

\(^1\)Looking at other divorce law changes, empirical studies find effects of introducing unilateral divorce on divorce rates (e.g., Friedberg (1998) and Wolfer (2006)) and labor supply of married and divorced couples (e.g., Gray (1998); Stevenson (2007) and Stevenson (2008)). The magnitude of the effects often depend on the asset division regime, e.g., Voena (2015).
that has previously been used to model intertemporal household decision-making, e.g., by Mazzocco (2007), Voena (2015) and Fernández and Wong (2016). Married spouses experience “love shocks”, which account for non-economic reasons for staying married. If one spouse prefers divorce to staying married (e.g., because of a bad love shock draw) this may lead to a shift in bargaining power from the spouse who prefers staying married to the spouse who wants to divorce. Changes in maintenance policies impact each spouses’ value of divorce and thus may trigger shifts in bargaining power or lead to divorce. The model includes savings in a risk-free asset and “learning by doing” human capital accumulation, i.e., by working during marriage model agents can increase their future expected wages and thus self-insure against losing resources upon divorce. By this mechanism maintenance payments weaken the individual incentives to supply labor and thus increase the possibilities for intra-household specialization according to comparative advantage. Maintenance payments thus facilitate efficient household specialization, while lowering maintenance payments promotes two-earner households.

The model is estimated using rich longitudinal data from Danish administrative records. Besides marital histories, labor supply and wages, the data include information on post-marital maintenance payments between ex-spouses, the number of children a couple has together, the children’s age and who the children stay with, if a couple divorces. The estimated model matches the targeted data moments and replicates event-studies that document the evolution of work hours around divorce.

To assess how maintenance policies affect couples’ decisions and welfare, I use the estimated model as a policy lab to conduct counterfactual experiments. Based on such policy experiments I show that the (ex-ante) welfare maximizing policy is characterized by increased (tripled) child support payments and slightly lower alimony payments (12.5% lower), relative to the Danish status quo policy. Increasing child support induces married couples to specialize more, leads to smoother consumption paths around divorce and to a moderate reduction in labor supply among divorced women. Increasing alimony payments in contrast fails to provide insurance: Alimony payments lead to a strong reduction in labor supply among divorced men and women. Because of the strong labor supply reduction, increasing alimony payments leads to larger consumption drops upon divorce for women (i.e., women’s consumption around divorce is reduced by a larger amount than for men).

---

2See Chiappori and Mazzocco (2017) for a detailed description of limited commitment framework applied to household decision-making.

3See Doepke and Tertilt (2016) for an analysis of the impact of divorce risk on savings.
divorce becomes less smooth). I thus show that alimony payments may have the opposite of the effect that is intended by policymakers.

To study how close maintenance policies can bring couples to efficiency, I compare the welfare maximizing policy to a first best scenario, in which frictions (limited commitment and non-cooperation in divorce) are removed from the model. The first best allocation is characterized by full consumption insurance and a higher degree of specialization among married couples, relative to the status quo and the welfare maximizing policy. In terms of women’s and men’s ex-ante wellbeing, I find that the first best allocation is a Pareto improvement relative to the status quo, while under the welfare maximizing maintenance policy women fare better, while men fare worse than under the status quo.

The contribution of this paper is threefold. First, I develop and estimate a model that incorporates a novel trade off that is relevant for studying maintenance policies. In my model maintenance payments provide insurance to the lower earner in couples and facilitate efficient intra-household specialization, but distort divorcees’ labor supply incentives. This paper provides the first study of how maintenance payments should be designed in light of this trade off. I thereby add to a small literature that studies alimony and child support payments (see, e.g., Weiss and Willis (1985); Weiss and Willis (1993); Del Boca and Flinn (1995); Flinn (2000)). Previous studies in this literature have used static models of divorced couples decision-making to study how compliance with maintenance policies and cooperation between ex-spouse can be encouraged by policymakers. Considering maintenance payments in a dynamic environment allows me to study how married couples, who face a risk of divorcing later in life, are affected by maintenance policies and analyze how maintenance payments interact with channels by which married spouses can self-insure, like human capital accumulation and savings.

Second, my research contributes to a literature that estimates dynamic economic models to study the impact of divorce law changes on household decisions and welfare. A large part of this literature is focused on studying switches from mutual-consent to unilateral divorce and asset division upon divorce (e.g., Chiappori et al. (2002); Voena (2015); Bayot and Voena (2015); Fernández and Wong (2016) and Reynoso (2018)). Less attention has been paid to policies like child support and alimony payments, that make spouses financially

---

4 For an overview of this literature see Del Boca (2003).
5 See Abraham and Laczo (2015) for a theoretical analysis of optimal asset division upon divorce.
interdependent beyond divorce. A notable exception is a study by Brown et al. (2015), who study the impact of child support on child investments and fertility. My paper adds to this literature by examining child support and alimony payments in a framework that fully accounts for the strategic interdependence that such policies induce between ex-spouses’ labor supply and savings decisions. Accounting for the strategic link between ex-spouses and by considering both extensive and intensive margin adjustments of women’s and men’s labor supply allows me to give a complete account of the labor supply disincentives incurred by maintenance policies.⁶

As a third contribution, this paper examines a first best scenario that serves as benchmark of what can be attained by maintenance policies (and divorce law changes more generally). I identify two key frictions that maintenance policies can help mitigate, limited commitment and non-cooperation in divorce. Removing these frictions yields the first best scenario. The first friction, limited commitment, has received a lot of attention in the previous literature (see Mazzocco (2007); Voena (2015); Fernández and Wong (2016); Lise and Yamada (2018)). The second friction, non-cooperation in divorce, featured in most models of divorcees decision-making, but few have studied the welfare loss that non-cooperation in divorce entails and to what extent this loss can be overcome by policy.⁷ Using a decomposition I show that non-cooperation in divorce plays a larger role than the limited commitment friction. By providing this analysis I extend the work of previous studies that have examined welfare consequences of divorce law changes (e.g., Brown et al. (2015); Voena (2015); Fernández and Wong (2016)). Contrasting the welfare maximizing maintenance policy to the first best allocation, allows me to study in what respects the welfare maximizing maintenance policy falls short relative to the first best allocation. In particular, I find that the first best scenario is a Pareto improvement over the welfare maximizing maintenance policy, indicating that there is scope for improvements in couples’ welfare beyond what is attained by the welfare maximizing maintenance policy.

The remainder of this paper is organized as follows. The following section describes the institutional background. Section 3.4 describes the data and presents empirical evidence from

---

⁶Previous studies in the literature focus exclusively on the extensive margin of female labor supply and take it as given that men always work full time.

⁷A notable exception is Flinn (2000), who analyzes a framework in which divorced couples endogenously choose between cooperation and non-cooperation and studies to what extent policymakers can encourage cooperation between ex-spouses.
event-studies. Section 1.4 develops my model and section 1.5 describes the estimation. In section 1.6 I discuss the key frictions in my model and characterize the first best scenario. Section 1.7 shows results from policy simulations. In section 1.8 I draw welfare comparisons, solve for the welfare maximizing policy and contrast it with the first best allocation. Section 2.8 concludes.

1.2 Institutional Background

In most OECD countries divorce law formulates rules by which it is determined what amount of maintenance payments needs to be payed within divorced couples. These rules typically formulate how maintenance payments are to be computed based on both ex-spouse’s labor incomes, the ex-couple’s number of children and the childrens’ age.\footnote{See de Vaus et al. (2017) and Skinner et al. (2007) for comparisons of maintenance payments in the OECD.} The precise rules differ across countries and countries also differ in whether the rules are applied rigidly or serve as broad guidelines. For some countries, like, e.g., the U.S., is known that compliance with maintenance rules is low.\footnote{Low compliance rates were found, e.g., for the US (see Weiss and Willis (1985), Del Boca and Flinn (1995) and Case et al. (2000)).} I use Denmark as an example to study the impact of maintenance payments for three interrelated reasons: First, In Denmark rigid rules are applied to determine the amount of maintenance that is to be payed, second, maintenance payments are strongly enforced by the Danish government,\footnote{See Skinner et al. (2007) for an overview of which countries apply rigid rules versus broad guidelines.} and third, Danish administrative records that contain information on maintenance payments allow me to study to what extent the institutional rules are reflected in actual payments. In the following I describe the rules that are used to determine the size and duration of child support and alimony payments in Denmark.\footnote{Qualitatively the following descriptions apply to a wide range of countries. All functional forms and quantities inserted for policy parameters are specific to Denmark.}

1.2.1 Child Support

Child support is to be payed from the non-custodial to the custodial parent for each child under the age of 18 a divorced couple has together. The payments are computed based on the child support payer’s labor income and the number of children. Consider divorced ex-spouses \(f\) and \(m\). Suppose \(s \in \{f, m\}\) holds custody of \(n_s\) children and the other ex-spouse
Notes: Each figure is plotted for the 2004 value of the respective policy parameter (i.e., for $B = 9420$ and $\tau = 0.2$).

$s \in \{f, m\} \setminus s$ has monthly labor earnings $I_s$. Then the non-custodial parent $\tilde{s}$ is mandated to make monthly child support payments

$$cs(n_s, I_{\tilde{s}}, B) = B \cdot a(n_s, I_{\tilde{s}})$$

to the custodial parent $s$, where $B$ is a basic money amount and $a(n_s, I_{\tilde{s}}) \geq 1$ is a factor that is increasing in the child support payer’s labor earnings $I_{\tilde{s}}$ and the number of children $n_s$. The functional form of $a(n_s, I_{\tilde{s}})$ and values for $B$ for 1999-2010 are provided in appendix A.1. Figure 1.1 provides a graphical illustration of the dependence of child support payments on $n_s$ and $I_{\tilde{s}}$. Child support payments for a given child need to made as long as the child is under the age of 18.

1.2.2 Alimony

Alimony payments are to be payed from the higher earning to the lower earning ex-spouse within a divorced couple. These payments are mandated independently of whether the divorced couple has children. Suppose $s \in \{f, m\}$ is the higher-earning and $\tilde{s} \in \{f, m\} \setminus s$ is the lower-earning ex-spouse in terms of monthly labor earnings, i.e., $I_s > I_{\tilde{s}}$. As a simple rule of thumb alimony payments equal a fraction $\tau$ of the monthly labor income difference, i.e.,

$$\tau \cdot (I_s - I_{\tilde{s}}).$$
There are several exceptions to the rule of thumb taking the form of caps on alimony payments. These caps ensure that:

1. If the receiver’s labor income is below $C_1$, alimony payments equal $\tau \cdot (I_s - C_1)$.
2. The maintenance payer’s labor earnings net of maintenance payments are not less than $C_2$.
3. The maintenance receiver’s labor earnings plus maintenance payments do not exceed $C_3$.

For the formal functional form of alimony payments, $alim(I_s, I_{\tilde{s}}, \tau)$, including the three caps see appendix A.1. Figure 1.2 gives a graphical example for the functional dependence of alimony on $I_s$ and $I_{\tilde{s}}$. Alimony payments may last for up to ten years, but end if the receiving ex-spouse remarries or cohabits with a new partner.

### 1.2.3 Maintenance Payments

Maintenance payments equal the sum of child support and alimony, subject to a cap on the total amount of maintenance payments that ensures that the maintenance payer does not have to pay more than a third of her/his income. Denote by $M_f$ the overall maintenance payments that are made from ex-husband to ex-wife (if $M_f > 0$) or from ex-wife to ex-husband (if $M_f < 0$) by the ex-wife and by $M_m$ the payments made or received by the ex-husband ($M_m = -M_f$ denotes the same payments from the ex-husbands perspective). The overall maintenance payments equal

$$M_f(n_f, n_m, I_f, I_m) = -M_m(n_f, n_m, I_m, I_f) =$$

$$\min \left\{ \frac{1}{3} I_m, cs(n_f, I_m) + alim(I_m, I_f) \right\} - \min \left\{ \frac{1}{3} I_f, cs(n_m, I_f) + alim(I_f, I_m) \right\}.$$

In my dynamic model I account for post-marital maintenance payments by adding $M_f$ and $M_m$ to the budget set of the ex-wife and ex-husband respectively.

### 1.3 Data and Descriptive Statistics

I use Danish register data covering 33 years from 1980 to 2013. The data include all Danish individuals who have been married at some point during the covered period. For each year I observe each individual’s annual labor income, labor force status and hours worked. Hours
worked are employer-recorded in five bins of weekly hours (<10, 10-19, 20-29, 30-37 and ≥ 38). Moreover I observe each individual’s marital history (starting from 1980) and number of children as recorded in the Danish birth register. For divorced individuals I additionally observe the amount of maintenance payments they make to or receive from their ex-spouse and with which parent divorced couples’ children continue to live after divorce. I restrict the sample to couples where both spouses are in their first marriage, aged between 25 and 58 and where at least one spouse is working in at least one sampled year. Furthermore I exclude couples where one spouse has a child from a previous relationship. The final sample includes 279,197 couples (558,394 individuals) and 4,912,474 couple-year observations. Table A.10 presents summary statistics for the final sample.

Table 1.1: Summary statistics, Danish register data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>38.70</td>
<td>7.68</td>
</tr>
<tr>
<td>Employed female</td>
<td>0.88</td>
<td>0.32</td>
</tr>
<tr>
<td>Employed male</td>
<td>0.93</td>
<td>0.26</td>
</tr>
<tr>
<td>Weekly hours worked female (cond. on working)</td>
<td>33.80</td>
<td>7.67</td>
</tr>
<tr>
<td>Weekly hours worked male (cond. on working)</td>
<td>34.36</td>
<td>8.22</td>
</tr>
<tr>
<td>Annual earnings female (DKK 1000s)</td>
<td>219</td>
<td>147</td>
</tr>
<tr>
<td>Annual earnings male (DKK 1000s)</td>
<td>299</td>
<td>241</td>
</tr>
<tr>
<td>No. of children (married)</td>
<td>1.40</td>
<td>0.98</td>
</tr>
<tr>
<td>% divorced after 5 years</td>
<td>6.91</td>
<td>25.38</td>
</tr>
<tr>
<td>% divorced after 10 years</td>
<td>15.28</td>
<td>35.98</td>
</tr>
<tr>
<td>% divorced after 15 years</td>
<td>21.57</td>
<td>41.13</td>
</tr>
<tr>
<td>% divorced after 20 years</td>
<td>25.26</td>
<td>43.44</td>
</tr>
<tr>
<td>% divorced after 25 years</td>
<td>28.29</td>
<td>45.04</td>
</tr>
</tbody>
</table>

Notes: Summary statistics from Danish register data. Pooled sample of 4,912,474 couple-year observations.

For the estimation of the structural model I further make use of information on housework

\(^{12}\)See Lund and Vejlin (2015) for a detailed description of the measurement of hours worked in Danish register data.

\(^{13}\)By using information from the Danish birth register I can distinguish the biological children that a couple has together from children living with the couple that are not biological children of the couple (e.g., children that one of the spouses has with someone else).

\(^{14}\)Maintenance payments are recorded by tax authorities. The data source is the maintenance payer’s tax declaration.

\(^{15}\)This case would be complicated to study as there would be child support payments to be made or received for the children from previous relationships as well.
hours. These data are obtained from the *Danish Time Use Survey*, which was conducted in 2001 among a 2,105 households representative sample of the Danish population.\(^{16}\) Table 1.2 presents summary statistics computed by re-weighting the data to match the age distribution of my main sample. A limitation of the *Danish Time Use Survey* is that married couples cannot be distinguished from cohabiting ones and divorced individuals cannot be distinguished from singles. I therefore pool these groups when making use of the time use data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>St. dev.</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housework hours female (married/cohabiting)</td>
<td>18.82</td>
<td>9.93</td>
<td>1271</td>
</tr>
<tr>
<td>Housework hours female (divorced/single)</td>
<td>19.92</td>
<td>8.94</td>
<td>156</td>
</tr>
<tr>
<td>Housework hours male (married/cohabiting)</td>
<td>10.83</td>
<td>8.08</td>
<td>1227</td>
</tr>
<tr>
<td>Housework hours male (divorced/single)</td>
<td>12.48</td>
<td>7.62</td>
<td>169</td>
</tr>
</tbody>
</table>

*Notes:* Summary statistics from the Danish Time Use survey 2001. Cross-section of 2,105 households. The data are re-weighted to match the age distribution in the Danish register data. Housework hours are total weekly hours spent on household chores and child care.

### 1.3.1 Maintenance Payments: Data vs. Imputations

Previous work based on U.S. data generally found low compliance with maintenance policies and was therefore mainly focused on understanding how compliance behavior may respond to policy changes (Weiss and Willis (1985); Weiss and Willis (1993); Del Boca and Flinn (1995); Flinn (2000)).\(^{17}\) In Denmark in contrast maintenance policies are strongly enforced by the government, which allows me to take compliance as given, when studying the impact of policy changes.\(^{18}\)

To confirm in the data to what extent actually implemented maintenance payments correspond to the institutional rules I compute annual imputed maintenance payments for each divorced couple in my sample based on the Danish institutional rules described in section 1.2 and check to what extent the imputations conform with maintenance payments recorded in the administrative data.

\(^{16}\)For a detailed description of the data see Browning and Gørtz (2012).

\(^{17}\)For a survey of these studies see Del Boca (2003).

\(^{18}\)In Denmark, if the ex-spouse mandated to pay maintenance refuses to make the payments a public agency helps to collect the outstanding payments. In case of non-compliance this agency can withhold tax refunds (see Rossin-Slater and Wiist (2018)).
**Figure 1.3:** Maintenance payments, data and imputations

**Figure 1.4:** Maintenance payments by payer’s labor income, data and imputations

**Figure 1.5:** Maintenance payments by no. children, data and imputations

**Notes:** The figures are based on observations, covering all divorced couples in my sample. Figure 1.3 and 1.4 display binned scatter-plots, where each dot corresponds to a percentile of the underlying distribution.

Figures 1.3 - 1.5 show how well the imputations match the observed data regarding several aspects. Figure 1.3 plots average imputed maintenance payments against observed maintenance payments in a binned scatter plot. The plot exhibits some small deviations, but by and large is clustered around the 45 degree line, confirming that on average the imputations of maintenance payments are close to the payments observed in the data. Figure 1.4 shows how maintenance payments evolve with the maintenance payer’s labor income in the observed data and for my imputations of maintenance payments respectively. Both the maintenance imputations and the maintenance data exhibit a positive gradient in the
payer’s labor income that is steepest between 300,000 and 500,000 DKK and somewhat flatter outside this income range. This gradient however is somewhat steeper in the imputations than in the data. Figure 1.5 shows imputed and actual annual maintenance payments by number of children. My imputations capture that maintenance payments are increasing in the number of children divorced couples have and the magnitude of the increase is similar in my imputations and in the data. The level of maintenance payments however is higher in the imputations than in the data for couples with 1,2 and 3 children, while being somewhat lower for couples with 0 children. Overall, the displayed relationships show that the institutional rules about maintenance payments are reflected in the actual payments, although the precise amounts may deviate to some extent.

1.3.2 Evidence from Event Studies: Work Hours around Divorce

To understand the relevance of post-marital maintenance payments it is important to know to what extent (and in what direction) divorcing spouses adjust their labor supply upon divorce. This subsection presents empirical evidence on the order of magnitude by which women and men adjust their labor supply before and after getting divorced. I conduct event study regressions that exploit variation in the timing of divorce to separate labor supply changes that are associated with divorce from general marriage duration and time trends.\footnote{In similar analyses Fisher and Low (2015) and Fisher and Low (2016) consider the evolution of divorcing spouses’ labor income (as well as other sources of income) after divorce.}

As outcome variable I consider work hours, as recorded in the Danish register data. This measure of work hours corresponds to weekly work hours and distinguishes between 5 work hours bins (\(<10, 10-19, 20-29, 30-37\) and \(\geq38\)). I code work hours to be equal to 0 in case of non-participation, 38 in case of full-time and equal to the mid-point of the respective bin, if work hours fall into one of the bins. Following the specification used in Kleven et al. (2018) I include calendar year fixed effects as well as fixed effects that control for the time that elapsed since a couple got married for the first time. Denote by \(h_{ict}\) the weekly work hours of individual \(i\) in calendar year \(c \in \{1980, 1981, ..., 2013\}\) in \(t\) year after first getting married. I run the following regression separately for women and men

\[
h_{it} = a_{c(i,t)} + b_t + \sum_{r=-3}^6 \kappa_r \cdot D_{it+r} + \nu_{it}, \tag{1.1}
\]
where $D_{it}$ is a dummy indicating whether individual $i$ gets divorced after having been married for $t$ years. $b_i$ are fixed effects that control for $t$, the time that elapsed since $i$ got married for the first time. $a_{c(i,t)}$ are calendar time fixed effects, where $c(i,t)$ denotes the calendar year in which $t$ years have elapsed since $i$ got married for the first time. I consider an event time window of 3 years before and 6 years after divorce. Panel A and B in Figure 1.6 plot the coefficient estimates separately for women and men. Panel C and D in Figure 1.6 show coefficient estimates from separate regressions by number of children (and for women/men).\(^{20}\)

**Figure 1.6:** Weekly work hours around divorce

Panel A: women

![Figure Panel A: Women](image)

Panel B: men

![Figure Panel B: Men](image)

Panel C: women, by number of kids

![Figure Panel C: Women with Kids](image)

Panel D: men, by number of kids

![Figure Panel D: Men with Kids](image)

**Notes:** Each figure contains coefficient estimates of 1.1, for women (panel A), men (panel B) and separately by number of children (panel C and D). Included are all individuals in my sample, that are observed for at least 3 periods prior and 6 periods after getting divorced.

\(^{20}\)For a better overview panel B and C in Figure 1.6 do not include confidence intervals. The respective graphs along with 95% confidence intervals are displayed in separate figures, A.2 and A.3.
The graphs show that both men and women reduce their labor supply upon divorce. Following divorce both men and women reduce their weekly work hours by 0.75 hours. For men this is complemented by a 0.5 work hours reduction in the three years preceding divorce. These findings have interesting implications, in the context of maintenance payments. First, if divorcing spouses reduce their work hours (and thus their earnings) the mandated amount of maintenance payments is affected. In particular for the person paying child support and/or alimony, a reduction in own earnings reduces the amount of mandated payments. For the person receiving alimony, in contrast, a reduction in own earnings increases the received alimony payments. At the same time maintenance payments directly improve the financial situation of the maintenance receiver, i.e., the consumption effect of a reduction in the receiver’s earnings is mitigated by maintenance payments.

1.4 Model

This section describes a dynamic structural model of labor supply, home production, savings and divorce that incorporates the following main features of married and divorced couples’ decision-making: 1. divorced ex-spouses are linked by maintenance payments and interact non-cooperatively, 2. married couples make decisions cooperatively subject to limited commitment, i.e., bargaining power and divorce rates respond to changes in married spouses’ outside options, 3. agents are forward looking and working improves their future wages, i.e., working during marriage mitigates financial losses upon divorce.

In the model a female individual $f$ and a male individual $m$ interact in each time period either as married couple or as divorced ex-spouses. The model is set in discrete time, $m$ and $f$ are married in period 1 and decide in each time period $t \in \{1, 2, ..., T\}$ about work hours $h_f, h_m$, housework hours $q_f, q_m$, (private) consumption $c_f, c_m$, savings in a joint asset $A_t$ and (if married) whether to stay married or get divorced. Work hours are discrete, i.e., each spouses working hours are chosen from finite sets $\mathcal{H}_f$ and $\mathcal{H}_m$. In period $T$ spouses retire and live as retirees until period $T + R$.

---

21In a similar analyses for the U.S. Johnson and Skinner (1986) and Mazzocco et al. (2014) find that women increase and men decrease work hours around divorce. Johnson and Skinner (1986) find effects in the years preceding divorce for women. Effects preceding divorce could be due to anticipation of divorce or because of events that cause persistent changes in labor supply as well as persistent changes in the divorce probability.
At the outset of the model, in period $t = 1$, couples are heterogeneous in their initial number of children, $n_1$ and initial assets $A_1$. During marriage a new child is born in each time period $t < T$ with probability $p(t, n_t)$, which is a function of $t$ and $n_t$, the number of children already present in the household.\(^{22}\)

As I model couples who are just married at the outset of the model, household formation is taken as given. The model hence is useful for studying the impact of policy changes on the population of already married couples, but does not address how household formation is affected by post-marital maintenance payments.

**Preferences**

Model agents $s \in \{f, m\}$ derive utility from private consumption $c_s$, from a household good $Q$ and from leisure time $\ell_s$. The household good represents a couple’s children well-being as well as goods and services produced within the household, like home made meals and cleaning up. $Q$ is produced from time inputs $q_f, q_m$ and is a public good within married couples, but becomes private when a couple divorces.

Intra-period utility is additively separable in consumption, leisure, the household good and a taste shock that affects an individual’s utility of being married relative to being divorced. The intra-period utility function of married spouses $s \in \{f, m\}$ is given by\(^{23}\)

$$
u_s^{mar}(c_s, \ell_s, Q, \xi_s) = \frac{c_s^{1+\eta_s}}{1+\eta_s} + \frac{\ell_s^{1+\gamma_s}}{1+\gamma_s} + \frac{\lambda(n) Q^{1+\kappa}}{1+\kappa} + \xi_s,$$

where $n$ denotes the couple’s number of children and $\lambda(n) = B \cdot (1 + b \cdot n)$, i.e., the relevance of the household good depends on the number of children present in the household. In order to account for persistence in the taste for marriage $\xi_s$ is assumed to follow a random walk with shocks correlated across $s$. Specifying $\xi_s$ to be individual specific rather than specific to the couple, allows for greater flexibility in marital status dynamics.\(^{24}\)

---

\(^{22}\)Not modeling an endogenous fertility process is in line with the previous literature that evaluates divorce law changes using formal economic models (e.g., Fernández and Wong (2016), Voena (2015), Bayot and Voena (2015), Reynoso (2018)). See Adda et al. (2017) for dynamic structural model of career choices and fertility and Doepke and Kindermann (2016) for a household bargaining model with endogenous fertility.

\(^{23}\)Time subscripts are omitted for convenience. $Q$ is a public good within married households and hence has no $s$ subscript.

\(^{24}\)Imposing marriage specific quality shocks, i.e., $\xi_f = \xi_m$ within each married couple, rules out situations where the spouse who benefits most in economic terms from the marriage wants to divorce while the spouse who benefits least in economic terms wants to maintain the marriage.
The intra-period utility function of divorced ex-spouses is given by

\[ u_{\text{div}}(c_s, h_s, Q_s) = \frac{c_s^{1+\eta_s}}{1+\eta_s} + \psi_s \frac{\ell_s^{1+\gamma_s}}{1+\gamma_s} + \lambda(n_s)\frac{Q_s^{1+\kappa}}{1+\kappa}, \]

where the subscript on $Q_s$ accounts for the fact that the household good $Q$ is not public within divorced couples and $n_s$ denotes the number of children living with spouse $s$ after divorce.

**Home Production**

Each spouse $s \in \{f, m\}$ has a time budget $H_s$, which is allocated between work, home production and leisure time, i.e., $H_s = h_s + q_s + \ell_s$. The technology by which the household good $Q$ is produced takes female and male home production time $q_f, q_m$ as inputs and has a constant elasticity of substitution form

\[ Q = F_Q(q_f, q_m) = \left(aq_f^\sigma + (1-a)q_m^\sigma\right)^{1/\sigma}, \]

where $\sigma$ controls the degree of substitutability between $q_f$ and $q_m$ and the factor $a \in [0, 1]$ captures productivity differences between the male and the female time input. The parameters $\sigma$ and $a$ jointly determine to what extent male and female non-work time are substitutes or complements in the process of producing the household good. Importantly married couples produce the household good jointly, while in divorced ex-couples each ex-spouse produces a separate household good, i.e., during marriage $Q = F_Q(q_f, q_m)$ and in divorce $Q_f = F_Q(q_f, 0)$ and $Q_m = F_Q(q_m, 0)$.

**Economies of Scale and Expenditures for Children**

I account for economies of scale in married couples’ consumption and expenditures for children by specifying the household expenditure function (cf. Voena (2015))

\[ F_x(c_f, c_m, n) = e(n)(c_f^\rho + c_m^\rho)^{\frac{1}{\rho}}. \]

For $\rho \geq 1$ and given expenditures $x_t = F_x(c_f, c_m, n)$ this functional form allows married couples to enjoy economies of scale from joint consumption, while there are no economies of scale if
only one spouse consumes. \( e(n) \geq 1 \) is an equivalence scale that accounts for expenditures for children, where \( e(0) = 1 \) and \( e(n) \) is strictly increasing in \( n \). A married couple with \( n \) children and private consumption levels \( c_f, c_m \) hence has expenditures \( x^\text{mar}_n = F_x(c_f, c_m, n) \). The individual expenditures of divorcees \( f, m \) with consumption levels \( c_f, c_m \) are \( x^\text{div}_f = F_x(c_f, 0, n_f) \) and \( x^\text{div}_m = F_x(0, c_m, n_m) \), meaning there are no economies of scale from joint consumption and each divorcee has expenditures only for children that continue to live with her/him.

**Wages**

For each spouse \( s \in \{f, m\} \) the wage process depends on human capital \( K_{ft}, K_{mt} \) and an i.i.d. random component \( \epsilon_{st} \)

\[
\ln(w_{st}) = \phi_{0s} + \phi_{1s}K_{st} + \epsilon_{st},
\]

\( \epsilon_{st} \) iid \( \sim \mathcal{N}(0, \sigma_{se}) \).

Human capital \( K_{st} \) is discrete with values \( \{0, 1, 2, \ldots, K_{\text{max}}\} \) and is accumulated through learning by doing.\(^{25}\) In particular from period \( t \) to \( t + 1 \), the stock of human capital \( K_{st} \) increases by one unit with probability \( p_K(h_{st}) \), which is strictly increasing in period \( t \) working hours. As functional form for \( p_K \) I impose \( p_K(h_{st}) = 1 - \exp(-\alpha_s h_{st}) \), where \( \alpha_s \) controls how responsive the human capital process is to work hours. At the same time \( K_{st} \) constantly depreciates with (exogenous) probability \( p_\delta \). This leads to the following law of motion for human capital:

\[
K_{st} = \begin{cases} 
\min\{K_{st-1} + 1, K_{\text{max}}\} & \text{with prob. } p_K(h_{t-1})(1 - p_\delta) \\
K_{st-1} & \text{with prob. } p_K(h_{t-1})p_\delta + (1 - p_K(h_{t-1}))(1 - p_\delta) \\
\max\{K_{st-1} - 1, 0\} & \text{with prob. } (1 - p_K(h_{t-1}))p_\delta.
\end{cases}
\]

Allowing for learning by doing adds an important dynamic component to the model. By working during marriage model agents can increase their individual expected future wages and thereby can self-insure against losing access to their spouses income upon divorce.

\(^{25}\)By making these assumptions I can include human capital for both spouses, while keeping the dimension of the state space manageable. In my estimations I impose \( K_{\text{max}} = 4 \).
Problem of Divorced Couples

Divorced couples are linked by maintenance payments and interact non-cooperatively. Each ex-spouse makes choices to maximize her/his own discounted lifetime utility, taking into account how decisions affect the stream of maintenance payments that flows from one ex-spouse to the other. As both ex-spouses’ decisions jointly impact the amount of maintenance payments, the interaction of divorced couples becomes strategic.

In each time period each ex-spouse chooses her/his time allocation between work hours, home production hours and leisure time as well as consumption and savings in a risk free asset $A_{st+1}$, subject to the budget constraint

$$x_{st}^{div} = w_{st} h_{st} + \Xi_t M_{st} + (1 + r) A_{st} - A_{st+1},$$

where $r$ denotes the risk free interest rate and maintenance payments are denoted by $M_{ft} = -M_{mt} = M_f(n_{ft}, n_{mt}, w_{ft} h_{ft}, w_{mt} h_{mt})$. Note that $f$’s work hours decision hence impacts $m$’s decision problem through the maintenance payments $M_m$ in $m$’s budget constraint (vice versa $m$’s work hours decision also affect $f$’s budget constraint). Period $t$ maintenance payments depend on the each ex-spouse’s period $t$ labor income and the number of children living with each ex-spouse. The functional form of $M_f$ is as described in section 1.2, i.e., corresponds exactly to the Danish institutional setting. To account for the duration for which maintenance payments are made I introduce an indicator variable $\Xi_t$ that equals 1 as long as maintenance payments are ongoing. In each period maintenance payments are discontinued ($\Xi_t = 0$) with probability $1 - p_M$, implying an average duration of maintenance payments of \( \frac{1}{1-p_M} \) time periods. Once discontinued maintenance payments remain at zero (i.e., if $\Xi_t = 0$ then $\Xi_{t+1} = 0$).

In order to determine allocations in this setting I restrict my attention to Markov-Perfect equilibria. To rule out multiplicity of equilibria which often occurs in simultaneous-move games I impose sequential (stackelberg type) decision-making within time periods. In particular I assume that within each time period $m$ chooses first and $f$ responds optimally to

\footnote{Flinn (2000) analyzes a framework in which the interaction mode between divorcees is endogenous.}
m’s choices.\(^{27,28}\)

Denote the period \(t\) decisions of spouse \(s\) by \(\iota_s = (c_{st}, h_{st}, q_{st}, \ell_{st}, A_{st+1})\). In the second stage of time period \(t\), \(f\) solves the following decision problem. Given \(m\)’s first stage choices \(\iota_{mt}\) and given the vector of period \(t\) state variables \(\Omega_t^{\text{div}} = (A_{ft}, A_{mt}, n_{ft}, n_{mt}, K_{ft}, K_{mt}, \epsilon_{ft}, \epsilon_{mt}, \Xi_t)\), \(f\) solves\(^{29}\)

\[
\tilde{\iota}_{ft} = \arg\max_{\iota_{ft}} u_{ft}^{\text{div}}(c_{ft}, \ell_{ft}, Q_{ft}) + \beta \mathbb{E}_t[V_{ft+1}(\Omega_{t+1}^{\text{div}})] \tag{1.3}
\]

s.t. \[x_{ft}^{\text{div}} = w_{ft}h_{ft} + \Xi_t M_f(n_{ft}, n_{mt}, w_{ft}h_{ft}, w_{mt}h_{mt}) + (1 + r)A_{ft} - A_{ft+1}\]
\[Q_{ft} = F_Q(q_{ft}, 0)\]
\[H_f = h_{ft} + q_{ft} + \ell_{ft}.\]

In the first stage, \(m\) makes his decision taking into account how it influences his female ex-spouse’s second stage response \(\tilde{\iota}_{ft}\), i.e., \(m\) solves

\[
\iota_{mt}^* = \arg\max_{\iota_{mt}} u_{mt}^{\text{div}}(c_{mt}, \ell_{mt}, Q_{mt}) + \beta \mathbb{E}_t[V_{mt+1}^{\text{div}}(\tilde{\Omega}_{t+1}^{\text{div}})] \tag{1.4}
\]

s.t. \[x_{mt}^{\text{div}} = w_{mt}h_{mt} + \Xi_t M_m(n_{ft}, n_{mt}, w_{ft}h_{ft}, w_{mt}h_{mt}) + (1 + r)A_{mt} - A_{mt+1}\]
\[Q_{mt} = F_Q(0, q_{mt})\]
\[H_m = h_{mt} + q_{mt} + \ell_{mt},\]

where \(\tilde{h}_{ft}\) denotes \(f\)’s optimal work hours response and \(\tilde{\Omega}_{t+1}^{\text{div}}\) is the vector of state variables given \(f\)’s optimal second stage response. Given \(m\)’s optimal choices \(\iota_{mt}^*\) and \(f\)’s optimal responses

\[
\iota_{ft}^* = \tilde{\iota}_{ft}(\iota_{mt}^*),
\]

\(^{27}\)(Weiss and Willis, 1993) model decision-making of divorced couples as (static) stackelberg game. Kaplan (2012) imposes sequential decision-making to ensure uniqueness of a Markov-Perfect equilibrium in a similar dynamic two-player setting, where youths interact with their parents. His paper provides a discussion of multiplicity of Markov-Perfect equilibria in dynamic two-player settings.

\(^{28}\)Changing the timing of the game such that \(f\) moves first tends to produce unrealistically low levels of male labor supply.

\(^{29}\)\(f\)’s optimal choices depend functionally on \(m\)’s first stage choices (e.g., for labor supply \(\tilde{h}_{ft} = \tilde{h}_{ft}(\iota_{mt})\)). For convenience I suppress the functional dependence in my notation.
the value of divorce for ex-spouse $s \in \{f, m\}$ is given by

$$V_{st}^{\text{div}}(\Omega_{t}^{\text{div}}) = u_s^{\text{div}}(c_{st}^*, \ell_{st}^*, Q_{st}^*) + \beta E_t[V_{st+1}^{\text{div}}(\Omega_{t+1}^{*\text{div}})]$$

(1.5)

where $c_{st}^*, h_{st}^*, Q_{st}^*$ denote the respective components of $r_{st}^*$ and $\Omega_{t+1}^{*\text{div}}$ is the vector of state variables given optimal period $t$ choices of $f$ and $m$. Given the period $T$ value of divorce $V_{sT}^{\text{div}}$ (the value of entering retirement as divorcée) for $s \in \{f, m\}$ the decision problems (1.3) and (1.4) and equation (1.5) recursively define the value of divorce $V_{st}^{\text{div}}$ for every period $t \in \{1, \ldots, T-1\}$ for $s \in \{f, m\}$.

**Division of Assets upon Divorce and Child Custody**

If a couple divorces in period $t$ savings in the joint asset $A_t$ are divided among the divorcing spouses. I assume that property is divided equally, such that each spouse receives $\frac{A_t}{2}$. Equal property division is a close approximation to the property division regime that is in place in Denmark, where assets accumulated during marriage are divided equally, but assets held prior to marriage are exempt from property division.

Upon divorce it is furthermore decided which spouse receives physical custody of the divorcing couples children. I assume all children either stay with their mother, $n_{ft} = n_t$, with exogenous probability $p_{\text{cust}_f}$, or with their father, $n_{mt} = n_t$, with probability $1 - p_{\text{cust}_f}$. In case of multiple children I do not account for cases where some children stay with their mother, while others stay with their father, as this would increase the dimensionality of the state space and increase the computational complexity of the model solution drastically. In my sample I observe that in 93% of all divorcing couples all children stay with one parent, while in 7% of all cases some children stay with each parent.

**Problem of Married Couples**

Married couples make decisions cooperatively subject to limited commitment. In limited commitment models of the family the outside options of both spouses impact the distribution of bargaining power between husband and wife and the propensity of the couple to divorce. As policy changes to post-marital maintenance payments affect each spouse’s outside option, the limited commitment framework allows maintenance payments to impact the intra-household
distribution of bargaining power and divorce rates.

In each time period married couples choose work hours, home production hours, (private) consumption for each spouse and savings in the joint asset $A_{t+1}$. Define the vector of period $t$ state variables of a married couple by $\Omega_{t}^\text{mar} = (\mu_t, A_t, n_t, K_{ft}, K_{mt}, \epsilon_{ft}, \epsilon_{mt}, \xi_{ft}, \xi_{mt})$ and denote a married couple’s choice variables by $\iota_t = (c_{ft}, c_{mt}, h_{ft}, h_{mt}, q_{ft}, q_{mt}, \ell_{ft}, \ell_{mt}, A_{t+1}, D_t)$, where $D_t = 1$ indicates the couple’s decision to get divorced in $t$. Conditional on the decision to stay married ($D_t = 0$) and for given female bargaining power $\mu_t$ the couple solves the constrained maximization problem

\[
\begin{align*}
\iota_t^* & = \arg\max_{\iota_t} \mu_t \left( u_{ft}^\text{mar}(c_{ft}, \ell_{ft}, Q_{ft}, \xi_{ft}) + \beta E_t[V_{ft+1}] \right) \\
 & \quad + (1 - \mu_t) \left( u_{mt}^\text{mar}(c_{mt}, \ell_{mt}, Q_{mt}, \xi_{mt}) + \beta E_t[V_{mt+1}] \right) \\
\text{s.t.} & \quad x_{t}^\text{mar} = w_f h_{ft} + w_m h_{mt} + (1 + r)A_t - A_{t+1} \\
& \quad Q_t = F_Q(q_{ft}, q_{mt}) \\
& \quad H_f = h_{ft} + q_{ft} + \ell_{ft} \\
& \quad H_m = h_{mt} + q_{mt} + \ell_{mt}
\end{align*}
\]

and the value of marriage for spouse $s$ is

\[
V_{st}^\text{mar}(\Omega_{t}^\text{mar}) = u_s(c_{st}^*, \ell_{st}^*, Q_s^*, \xi_{st}) + \beta E_t[V_{st+1}],
\]

where $c_{st}^*, q_{st}^*, \ell_{st}^*$ are the respective components of $\iota_t^*$ and $Q_s^*$ is the quantity of the home good that is produced at $q_{ft}^*, q_{mt}^*$.

The $t+1$ continuation value $V_{st+1}$ depends on whether the couple stays married $D_{t+1} = 0$ or gets divorced $D_{t+1} = 1$ in $t + 1$ and is given by

\[
V_{st+1} = D_{t+1} V_{st+1}^\text{div}(\Omega_{t+1}^\text{div}) + (1 - D_{t+1}) V_{st+1}^\text{mar}(\Omega_{t+1}^\text{mar}).
\]

In the limited commitment framework intra-household bargaining power may shift if one spouses participation constraint is violated. If at given female bargaining power $\mu_t$ both spouses participation constraints are satisfied, i.e.,

\[
V_{st}^\text{mar}(\Omega_{t}^\text{mar}) \geq V_{st}^\text{div}(\Omega_{t}^\text{div}) \quad \text{for} \ s \in \{f, m\},
\]
then it is individually rational for both spouses to stay married. In this case the couple stays married and makes decisions according to (1.6). If however the participation constraint (1.8) is violated for one spouse but not the other, bargaining power is increased (if $f$’s participation constraint is violated) or decreased (if $m$’s participation constraint is violated) until the spouse whose participation constraint is binding is just indifferent between staying married and getting divorced. Divorce occurs if no value of $\mu_t$ exists such that both spouses’ participation constraints are satisfied simultaneously.

Policy changes to post-marital maintenance payments typically increase the value of one spouse’s outside option while decreasing the value of the other spouse’s outside option. Under limited commitment this may trigger changes in intra-household bargaining power. Furthermore divorce rates may respond to such policy changes, if divorce becomes too attractive relative to staying married for (at least) one spouse and if reallocating bargaining power cannot restore the incentives to stay married for both spouses.

1.5 Estimation

To obtain estimated values for the structural parameters of my model I proceed in three steps. First, a small subset of the model parameters is set externally to match values from the previous literature. Next, several model parameters are estimated directly from the data without making use of the structural model. The remaining parameters are estimated by the method of simulated moments (MSM), (see Pakes and Pollard (1989); McFadden (1989)), i.e., I use numerical optimization techniques to find model parameters such that a set of simulated model moments match the corresponding moments from the data as close as possible. The next subsections describe each of the three steps of obtaining estimates of my model parameters in more detail.

1.5.1 Pre-set Parameters

I pre-set several model parameters to match values from the literature. These parameters and the values that I fix them at are summarized in Table 1.3. I set the relative risk-aversion $\eta$ to 1.5 and the annual discount factor to 0.98 in line with Attanasio et al. (2008). As annual interest rate I take the average yearly deposit rate across my sample period, which is
published by the Danish Central Bank. Following Voena (2015) I fix the economies of scale parameter \( \rho \) at 1.4023, which is the value implied by the McClements scale.

I set a time period to correspond to three years to keep the computational complexity manageable and in line with previous studies (see Voena (2015); Reynoso (2018)). I solve the model for \( T = 10 \) and \( TR = 4 \), i.e., for individuals whose working life lasts for 30 years after getting married and who spent live for 12 years as retirees after their working life has ended. For both spouses, \( f \) and \( m \) the domain of weekly working hours is restricted to four values: non-participation (0 hours) two levels of part-time work (20 and 30 hours) and full time work (38 hours). To arrive at annual work hours I impose that one year consists of 49 working weeks. I fix the overall weekly time budget at 50 hours \((H_f = H_m = 50)\), such that if a person works full time there is a residual of 12 hours to be allocated between weekly housework and leisure. Finally I fix the initial bargaining weight at \( \mu_0 = 0.5 \), i.e., bargaining power is assumed to be equal at the outset of the model.

Table 1.3: Pre-set parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implied annual discount factor:</td>
<td>0.98</td>
<td>Attanasio et al. (2008)</td>
</tr>
<tr>
<td>Risk aversion (( \eta )):</td>
<td>1.5</td>
<td>Attanasio et al. (2008)</td>
</tr>
<tr>
<td>Implied annual interest rate:</td>
<td>0.046</td>
<td>Abildgren (2005)</td>
</tr>
<tr>
<td>Economies of scale (( \rho )):</td>
<td>1.4023</td>
<td>implied by McClements scale</td>
</tr>
<tr>
<td>Number of time periods (( T )):</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Duration of retirement (( TR )):</td>
<td>4 time periods</td>
<td></td>
</tr>
<tr>
<td>Implied weekly work hours domain:</td>
<td>{0, 20, 30, 38}</td>
<td></td>
</tr>
</tbody>
</table>

Notes: For ease of interpretation the table presents the implied annual discount factor and interest rate and the implied weekly work hours domain, rather than the corresponding numbers for a model time period (which corresponds to three years).

1.5.2 Directly Estimated Parameters

A subgroup of parameters are estimated directly from the data. These parameters are 1. the parameters governing the fertility process 2. the parameters governing compliance with maintenance policies and the duration of maintenance payments and 3. parameters related
to child custody.

**Fertility process** The parameters of the fertility process are the initial (period 1) distribution of children

\[ p_{n_1}(n) = P(n_1 = n) \quad \text{for} \quad n \in \{0, 1, 2, 3\} \]

and the probabilities of giving birth to an additional child as a function of the model time period \( t \) and the number of children already present in the household\(^{30}\)

\[ p_n(t, n_t) = P(\text{birth}|t, n_t) \quad \text{for} \quad n_t \in \{0, 1, 2\}, \; 1 \leq t < T. \]

I estimate \( p_{n_1}(n) \) and \( p_n(t, n_t) \) by computing the corresponding sample means and Markov transition probabilities from the Danish birth register data. The estimates for \( p_{n_1} \) are reported in Table 1.4. The matrix of estimated Markov transition probabilities is presented in Table 1.5. Note that for \( t \geq 4 \) (i.e., after 12 years of marriage) birth probabilities generally are practically equal to 0.

**Table 1.4:** Distribution of initial no. of children

<table>
<thead>
<tr>
<th>( n )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{n_1}(n) )</td>
<td>0.34</td>
<td>0.37</td>
<td>0.25</td>
<td>0.04</td>
</tr>
</tbody>
</table>

*Notes:* Source: Danish birth register.

**Table 1.5:** Fertility process

<table>
<thead>
<tr>
<th>( n = 0 )</th>
<th>( n = 1 )</th>
<th>( n = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_n(t = 1, n_1 = n) )</td>
<td>0.25</td>
<td>0.23</td>
</tr>
<tr>
<td>( p_n(t = 2, n_2 = n) )</td>
<td>0.08</td>
<td>0.19</td>
</tr>
<tr>
<td>( p_n(t = 3, n_3 = n) )</td>
<td>0.02</td>
<td>0.06</td>
</tr>
<tr>
<td>( p_n(t = 4, n_4 = n) )</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>( p_n(t \geq 5, n_5 = n) )</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

*Notes:* Source: Danish birth register.

\(^{30}\)Note that I allow couples to have at most 3 children, i.e., \( p_n(t, 3) = 0 \) for all \( t \).
1.5.3 Method of Simulated Moments Estimation

The remaining model parameters that are estimated using the method of simulated moments are (for $s \in \{f, m\}$) the parameters governing preferences for leisure $\gamma_s$, $\psi_s$ and preferences for the home good $B$, $b$, $\kappa$, the parameters governing home production $a$, $\sigma$, the love shock parameters $\mu_\xi$, $\sigma_\xi$, $r_\xi$ and the parameters governing the wage processes $\phi_{0s}$, $\phi_{1s}$, $\sigma_{\epsilon, s}$, $\alpha_s$, $p_{\delta, s}$.

I denote the vector of structural model parameters estimated by MSM by $\theta$. For a given $\theta$ I solve the structural model by backwards recursion, simulate data for 20,000 hypothetical couples and compute the vector of simulated moments $m(\theta)$. MSM-estimates $\tilde{\theta}$ are obtained by minimizing the distance between simulated model moments and their empirical counterparts $\tilde{m}$

$$\min_{\theta} (m(\theta) - \tilde{m})'\tilde{W}(m(\theta) - \tilde{m}).$$

The empirical moments I target are conditional averages of working hours, housework hours and wages, where I condition on marital status (married/ divorced) and number of children.\(^{31}\) I also target the fraction of ever divorced couples by time that elapsed since couples got married. Overall I target 53 empirical moments.

As weighting matrix $\tilde{W}$ I use the diagonal matrix with the inversed variances of the empirical moments as diagonal entries.\(^{32}\) The MSM parameter estimates are presented in Table 1.6 together with asymptotic standard errors (see, e.g., Newey and McFadden (1994)). For an assessment of the model fit Figure 1.7 contrasts average outcomes computed from model simulations with the respective empirical moments computed from my data. In particular Panel A-C of Figure 1.7 show average work hours, housework hours and wages (computed separately by marital status, but averaged over number of children). Panel D shows the fraction of ever divorced couples by the time that elapsed, since they first got married. Overall the model matches the considered data moments well, even though the model simulations deviate slightly from the data for married men’s wages and work hours (my model slightly under-predicts these moments) and divorced women’s housework hours (which are slightly over-predicted by my model). To give the full picture of how well my model fits all 53 targeted empirical moments Table A.10 contrasts all targeted empirical moments.

\(^{31}\)As the data from the Danish Time Use Survey feature few observations on people with two or more children I compute joint moments for this group, i.e., target average housework hours separately for three groups: people with no children, people with one child and people with two or more children.

\(^{32}\)Altonji and Segal (1996) show that using the efficient weighting matrix leads to undesirable finite sample properties.
with their counterparts from model simulations at the estimated parameters. Relative to Figure 1.7, Table A.10 also shows how well my model captures heterogeneity in the observed outcomes across couples with different numbers of children. Even though the model is a bit sparse on couples with no kids, the model generally captures heterogeneity by number of children well. E.g., for couples with children the model does a good job at capturing the variation work hours and housework hours across number of children.

**Table 1.6: MSM parameter estimates**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leisure preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>-2.2</td>
<td>0.0112</td>
</tr>
<tr>
<td>$\psi_f$</td>
<td>0.08</td>
<td>0.0032</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>-2.2</td>
<td>0.0140</td>
</tr>
<tr>
<td>$\psi_m$</td>
<td>1.20</td>
<td>0.0027</td>
</tr>
<tr>
<td>Home good preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_f$</td>
<td>0.0017</td>
<td>0.26 $\cdot 10^{-3}$</td>
</tr>
<tr>
<td>$B_m$</td>
<td>0.0010</td>
<td>0.48 $\cdot 10^{-3}$</td>
</tr>
<tr>
<td>$b$</td>
<td>0.25</td>
<td>0.0077</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>-1.19</td>
<td>0.028</td>
</tr>
<tr>
<td>Home good production</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>0.51</td>
<td>0.064</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.15</td>
<td>0.0082</td>
</tr>
<tr>
<td>Marriage preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_\xi$</td>
<td>0.0094</td>
<td>0.20 $\cdot 10^{-3}$</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>0.12</td>
<td>0.0093</td>
</tr>
<tr>
<td>Wage processes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_{0f}$</td>
<td>4.05</td>
<td>0.07</td>
</tr>
<tr>
<td>$\phi_{1f}$</td>
<td>0.4</td>
<td>0.043</td>
</tr>
<tr>
<td>$\alpha_f$</td>
<td>0.69 $\cdot 10^{-4}$</td>
<td>0.19 $\cdot 10^{-4}$</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon_f}$</td>
<td>0.22</td>
<td>0.02</td>
</tr>
<tr>
<td>$\delta_f$</td>
<td>0.11</td>
<td>0.01</td>
</tr>
<tr>
<td>$\phi_{0m}$</td>
<td>4.31</td>
<td>0.08</td>
</tr>
<tr>
<td>$\phi_{1m}$</td>
<td>0.42</td>
<td>0.030</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>0.72 $\cdot 10^{-4}$</td>
<td>0.16 $\cdot 10^{-4}$</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon_m}$</td>
<td>0.21</td>
<td>0.06</td>
</tr>
<tr>
<td>$\delta_m$</td>
<td>0.14</td>
<td>0.02 $\cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

**Notes:** Model parameters estimated by MSM and asymptotic standard errors. The estimates are obtained by fitting average work hours, housework hours and wages by marital status and number of children as well as the fraction of ever divorced couples by the time that elapsed since couples got married. For an assessment of the model fit see Table A.10.
Figure 1.7: Model fit

Panel A: Weekly work hours

Panel B: Weekly housework hours

Panel C: Wages

Panel D: Divorce

Notes: The figures display mean data moments (solid lines) and simulated model moments (dotted lines) by marital status and separately for women/men. Data moments on work hours, housework hours and divorce are computed from Danish register data. Data moments on housework are computed based on the Danish Time Use Survey. Model moments are computed based on simulations for $N = 20,000$ couples. For the fit regarding all 53 data moments see Table A.10.

1.6 Underlying Frictions and First Best Allocation

Before analyzing counterfactual policy scenarios and asking what the welfare maximizing maintenance policy is, it is worthwhile to consider what the frictions in my model are that
can potentially be mitigated by maintenance policies. A first friction, which has been studied a lot in the previous literature, is limited commitment (see Mazzocco (2007); Voena (2015); Fernández and Wong (2016); Lise and Yamada (2018)). Since married spouses cannot commit to staying married, it needs to be ensured that each spouse is better off married than divorced (i.e., participation constraints need to be satisfied) in each time period and in each state. Ensuring that these participation constraints are satisfied is what keeps married spouses from fully insuring each other and introduces scope for re-bargaining, when participation constraints are violated.

The second friction is non-cooperation in divorce. Because of non-cooperation in divorce there is no mutual insurance between divorcees, i.e., there is an inefficient lack of insurance against income losses upon divorce. Maintenance payments can help to rectify this lack of insurance. One consequence of lacking insurance against income losses upon divorce are strong incentives for married individuals to work and accumulate human capital to self-insure. These individual incentives to supply a lot of labor reduce the possibilities for intra-household specialization, as specialization requires one spouse to work little and mainly engage in home production. By reducing the individual need for self-insurance, maintenance policies may (partially) strengthen the overall incentives for intra-household specialization and thus help married households to realize specialization gains.

1.6.1 First Best Scenario

This subsection characterizes a first best scenario in which both frictions, limited commitment and non-cooperation in divorce are removed from the model. In this first best version of my model spouses/ex-spouses cooperate under full commitment for the entire time horizon independent of whether they are married or got divorced. Couples thus fully realize gains from mutual insurance and household specialization. The first best scenario I consider yields an ex-ante Pareto-efficient allocation and is characterized by the following features: 1. within a couple labor income is fully shared between spouses/ex-spouses for the entire time horizon of the model, 2. married as well as divorced couples bargain at equal bargaining weights over labor supply, housework hours and consumption given the couples joint labor income, 3. couples get divorced if and only if divorce is Pareto efficient. Divorcees do not experience

33My definition of “first best” does not allow for insurance across households, i.e., does not correspond to the complete markets definition of “first best”. 

32
love shocks $\xi$, do not enjoy economies of scale from joint consumption, do not engage in joint home production and the produced home goods are consumed privately.\textsuperscript{34}

Formally, the first best allocation is the solution to the following dynamic problem. Denote the vector of choice variables $\xi_t = (c_{ft}, c_{mt}, h_{ft}, h_{mt}, q_{ft}, q_{mt}, \ell_{ft}, \ell_{mt}, A_{t+1}, D_t)$. For divorced couples the first best allocation solves

$$
\iota_t^{fb,div} = \arg\max_{\iota_t} \mu_t \left( u_t^{div}(c_{ft}, \ell_{ft}, Q_{ft}) + \beta E_t[V_{ft+1}^{fb,div}] \right) + (1 - \mu_t) \left( u_t^{div}(c_{mt}, \ell_{mt}, Q_{mt}) + \beta E_t[V_{mt+1}^{fb,div}] \right)
$$

s.t. $x_{ft}^{div} + x_{mt}^{div} = w_{ft}h_{ft} + w_{mt}h_{mt} + (1 + r)A_t - A_{t+1}$

$$
\begin{align*}
Q_{ft} &= F_Q(q_{ft}, 0) \\
Q_{mt} &= F_Q(0, q_{mt}) \\
H_f &= h_f + \ell_f + q_f \\
H_m &= h_m + \ell_m + q_m,
\end{align*}
\]}

where the continuation values are defined by

$$
V_{st}^{fb,div} = u_s^{div}(c_{st}^{fb,div}, \ell_{st}^{fb,div}, Q_{st}^{fb,div}) + \beta E_t[V_{st+1}^{fb,div}]. \quad (1.9)
$$

For married couples the first best allocation solves

$$
\iota_t^{fb,mar} = \arg\max_{\iota_t} \mu_t \left( u_t^{mar}(c_{ft}, \ell_{ft}, Q_{ft}, \xi_{ft}) + \beta E_t[V_{ft+1}^{fb}] \right) + (1 - \mu_t) \left( u_t^{mar}(c_{mt}, \ell_{mt}, Q_{mt}, \xi_{mt}) + \beta E_t[V_{mt+1}^{fb}] \right)
$$

s.t. $x_{ft}^{mar} = w_{ft}h_{ft} + w_{mt}h_{mt} + (1 + r)A_t - A_{t+1}$

$$
\begin{align*}
Q_t &= F_Q(q_{ft}, q_{mt}) \\
H_f &= h_f + \ell_f + q_f \\
H_m &= h_m + \ell_m + q_m
\end{align*}
$$

\textsuperscript{34}Letting married and divorced couples bargain at fixed but not necessarily equal bargaining weights defines a class of first best (i.e., ex-ante Pareto-efficient) allocations. Recall that initial bargaining weights are assumed to be equal (i.e., $\mu_0 = 1 - \mu_0 = 0.5$, see section 1.5), so the first best allocation under equal bargaining weights is a reasonable benchmark for comparison.
where the continuation values are defined by
\[
V_{st}^{fb} = (1 - D_t)V_{st}^{fb,mar} + D_t V_{st}^{fb,div}
\]
\[
V_{st}^{fb,mar} = u_s^{mar}(c_{st}^{fb,mar}, x_{st}^{fb,mar}, Q_t^{fb,mar}, \xi_{st}) + \beta \mathbb{E}_{t}[V_{st+1}^{fb}]
\]
and where \( D_t = 1 \) is an indicator variable that indicates divorce. Finally married couples get divorced if divorce is Pareto efficient, i.e., if (and only if)\(^{35}\)
\[
\mu_t V_{ft}^{fb,div} + (1 - \mu_t) V_{mt}^{fb,div} > \mu_t V_{ft}^{fb,mar} + (1 - \mu_t) V_{mt}^{fb,mar}.
\]

### 1.6.2 Characterization of the First Best Allocation and Underlying Frictions

To characterize the first best scenario, I solve for the first best allocation under equal bargaining weights at the estimated model parameters and draw comparisons to the allocation obtained under the status quo policy (i.e., under \( B = 9420, \tau = 0.2 \)). In order to study the magnitude of each of the underlying frictions I additionally solve and simulate a version of my model in which only non-cooperation in divorce is removed from the model, while the other friction, limited commitment, is left in place.\(^{36}\)

As a second notable difference the first best allocation exhibits a higher degree of household specialization than the status quo allocation, reflecting that the frictions in the model prevent married couples from specializing efficiently. Compared to the status quo, under the first best scenario married women’s housework hours are higher by 2.3% and work hours are lower by 1.3%, while married men’s housework hours are lower by 1.9% and work hours are higher by 0.6%. Among divorced couples, in the first best scenario women work more hours in the household (by 15.1%) and less in the labor market (by 10.8%), while divorced men work less in the household (by 23.8%) and supply more work hours (by 12.0%) under first best, relative to the status quo.

Thirdly, the fraction of couples ever getting a divorce in the first best scenario is lower than under the status quo. In the first-best scenario divorced couples cooperate and married

\(^{35}\)It can be shown that under this condition no allocation in marriage or divorce exists that Pareto dominates \( c_{ft}^{fb,div}, c_{mt}^{fb,div}, h_{ft}^{fb,div}, h_{mt}^{fb,div} \).

\(^{36}\)For this version of the model the value of divorce is defined by (1.9) and the value of marriage by (1.7).
couples specialize efficiently, meaning that both the value of marriage and the value of divorce 
are higher than under the status quo policy. It thus depends on the relative magnitude of 
the changes in the value of marriage and the value of divorce, whether divorce becomes more 
or less attractive in the first scenario relative to the status quo. At the estimated structural 
parameters I find that 28.3% of couples ever get divorced, while only 26.8% divorce under 
the first best scenario.

Considering the allocation, where non-cooperation in divorce is removed from the model 
(column 2), such that limited commitment is the only friction, shows that the obtained 
allocation is generally very close to the first best allocation. This suggests that non-cooperation 
in divorce is the main friction that accounts for differences between the status quo and the 
first best scenario, while limited commitment plays a smaller role.

<table>
<thead>
<tr>
<th>Table 1.7: Simulated outcomes: removing frictions from the model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>---------------------------------------</td>
</tr>
<tr>
<td>Hours worked female (married)</td>
</tr>
<tr>
<td>Housework hours female (married)</td>
</tr>
<tr>
<td>Leisure female (married)</td>
</tr>
<tr>
<td>Hours worked male (married)</td>
</tr>
<tr>
<td>Housework hours male (married)</td>
</tr>
<tr>
<td>Leisure male (married)</td>
</tr>
<tr>
<td>Consumption ratio ( \frac{c_f}{c_m}, \text{married} )</td>
</tr>
<tr>
<td>Hours worked female (divorced)</td>
</tr>
<tr>
<td>Housework hours female (divorced)</td>
</tr>
<tr>
<td>Leisure female (divorced)</td>
</tr>
<tr>
<td>Hours worked male (divorced)</td>
</tr>
<tr>
<td>Housework hours male (divorced)</td>
</tr>
<tr>
<td>Leisure male (divorced)</td>
</tr>
<tr>
<td>Consumption ratio ( \frac{c_f}{c_m}, \text{divorced} )</td>
</tr>
<tr>
<td>% divorced in T</td>
</tr>
</tbody>
</table>

Notes: Mean outcomes by marital status, computed based on model simulations for \( N = 20,000 \) couples.
1.7 Policy Simulations

Given the structural parameter estimates, I use the model to explore the effects of policy changes on married and divorced couples’ behavior. I use the estimated model to simulate policy scenarios across which child support and alimony payments are varied. The following subsections study how married and divorced couples’ time use, consumption allocation and propensity to divorce adjusts as child support and alimony payments are changed.

1.7.1 The Impact of Child Support on Time Use and Consumption

This subsection considers policy scenarios in which the level of child support payments is varied. In particular I consider changes in the policy parameter $B$, which controls child support payments and corresponds to a parameter in the Danish real world institutions. The status quo policy parameters in Denmark are ($B = 9420, \tau = 0.2$). For convenience, I consider the normalized policy parameter $b = B/9420$ in the following. Conditional on the non-custodial parent’s income, the number of children, child support payments are homogeneous of degree one in $b$, i.e., as $b$ is multiplied by a factor $\alpha > 0$, mandated child support payments are multiplied by the same factor $\alpha$. In the considered counterfactual experiments I vary $b$ step-wise from no child support ($b = 0$) to quadrupled child support ($b = 4$) while the alimony policy is kept fixed at $\tau = 0.2$.

**Child support and couples’ time allocation** First, I look at how married couples’ time allocation changes as child support is increased. The results in Table 1.8 show that higher child support leads to a slightly higher degree of household specialization among married couples. Married women tend to supply less market work and more housework, while married men supply less housework and more market work. Quantitatively, as child support is increased from $b = 0$ to $b = 4$ housework hours among married women increase by 1.7% while their (market) work hours drop by 1.0% and leisure increases by 4.5%. At the same time married men’s average housework hours fall by 1.9% while their work hours increase by 0.6% and leisure decreases by 1.5%.
Table 1.8: The effect of changing child support ($b$) on married couples’ time use

<table>
<thead>
<tr>
<th>$b$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours worked female</td>
<td>30.2</td>
<td>30.1</td>
<td>30.0</td>
<td>29.9</td>
<td>29.9</td>
</tr>
<tr>
<td>Housework hours female</td>
<td>17.6</td>
<td>17.7</td>
<td>17.7</td>
<td>17.8</td>
<td>17.8</td>
</tr>
<tr>
<td>Leisure female</td>
<td>2.2</td>
<td>2.2</td>
<td>2.3</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>Hours worked male</td>
<td>32.8</td>
<td>32.9</td>
<td>33.0</td>
<td>33.0</td>
<td>33.0</td>
</tr>
<tr>
<td>Housework hours male</td>
<td>10.7</td>
<td>10.7</td>
<td>10.6</td>
<td>10.6</td>
<td>10.5</td>
</tr>
<tr>
<td>Leisure male</td>
<td>6.5</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
</tr>
</tbody>
</table>

Notes: Mean time uses of married couples for different child support policy regimes. Computed based on model simulations for $N = 20,000$ couples.

Table 1.9 shows the corresponding results for divorcees. Increasing child support, leads to a decrease in work hours of divorced women and, perhaps surprisingly, to an increase in work hours among divorced men. This is suggestive of a large income effect that dominates the substitution effect, which pushes towards higher male labor supply as child support is increased. Quantitatively, switching from $b = 0$ to $b = 4$ leads to a reduction in female work hours by 6.1% and to an increase in male work hours by 4.8%. At the same time female housework hours increase by 9.5% and male housework hours decrease by 0.9%. Average leisure time among divorced women increases by 6.3% while leisure time among divorced men decreases by 5.2%.

Table 1.9: The effect of changing child support ($b$) on divorced couples’ time use

<table>
<thead>
<tr>
<th>$b$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours worked female</td>
<td>29.4</td>
<td>28.6</td>
<td>27.9</td>
<td>27.6</td>
<td>27.6</td>
</tr>
<tr>
<td>Housework hours female</td>
<td>19.0</td>
<td>19.8</td>
<td>20.4</td>
<td>20.7</td>
<td>20.8</td>
</tr>
<tr>
<td>Leisure female</td>
<td>1.6</td>
<td>1.6</td>
<td>1.7</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>Hours worked male</td>
<td>31.1</td>
<td>31.7</td>
<td>32.1</td>
<td>32.4</td>
<td>32.6</td>
</tr>
<tr>
<td>Housework hours male</td>
<td>13.1</td>
<td>12.6</td>
<td>12.3</td>
<td>12.0</td>
<td>11.9</td>
</tr>
<tr>
<td>Leisure male</td>
<td>5.8</td>
<td>5.7</td>
<td>5.6</td>
<td>5.5</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Notes: Mean time uses of divorced couples for different child support policy regimes. Computed based on model simulations for $N = 20,000$ couples.

Child support and consumption insurance Next, I study the extent to which child support policies are successful in providing consumption insurance. Table 1.10 shows couples’
relative consumption by marital status, which provides a measure of how well individuals are insured against income losses upon divorce under each policy scenario. If child support payments work well as insurance device, the gap between relative consumption in marriage and divorce should narrow as child support is increased. The results in Table 1.10 show that child support policies indeed provide consumption insurance. Under all considered policy scenarios married couples relative consumption is close to 1, i.e., married men and women consume almost equally, while among divorcees women’s consumption is a lot lower than men’s. As child support is increased the relative consumption of divorced couples increases from 0.57 in the case of no child support ($b = 0$) to 0.78 in the $b = 4$ scenario. While child support is effective in mitigating the drop in relative consumption, full insurance, i.e., equal relative consumption in marriage and divorce, is not attained upon divorce even for high levels of child support.

**Table 1.10:** The effect of changing child support ($b$) on couples’ relative consumption

<table>
<thead>
<tr>
<th>$b$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_j^{mar}/c_m^{mar}$</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>$c_j^{div}/c_m^{div}$</td>
<td>0.57</td>
<td>0.62</td>
<td>0.67</td>
<td>0.73</td>
<td>0.78</td>
</tr>
</tbody>
</table>

**Notes:** Mean relative consumption by marital status for different child support policy regimes. Computed based on model simulations for $N = 20,000$ couples.

To address concerns that the patterns shown in Table 1.10 could mainly be driven by differences between couples who do get divorced and couples who do not get divorced, Figure 1.8 presents event study graphs, that show the drop in relative consumption upon divorce (at $t = 0$). This graph only includes individuals who do get divorced. The drop in relative consumption closely corresponds to the differences in relative consumption between married and divorced couples shown in Table 1.10. This shows that differences between couples who do get divorced and couples who do not get divorced are not a main driver of the results shown in Table 1.10.
1.7.2 The Impact of Alimony on Time Use and Consumption

I now turn to studying policy scenarios in which the level of alimony payments is changed and draw comparisons to the results for child support policies presented in the previous subsection. I consider changes in the policy parameter $\tau$, which controls alimony payments and corresponds to a real world parameter in the Danish institutions. Its status quo value is $\tau = 0.2$, which means that the higher-earning spouse needs to pay one fifth of the difference between the ex-spouses’ labor incomes, net of child support payments, to the lower-earning spouse.\footnote{If the caps on alimony or on overall maintenance payments are binding, the relationship between $\tau$ and the amount of alimony payments is more complicated. See section 1.2 for details.} For a given amount of both ex-spouses incomes and given that the caps on alimony payments are non-binding, alimony payments are homogeneous of degree one in $\tau$, i.e., as $\tau$ is multiplied by a factor $\alpha > 0$, alimony payments are multiplied by the same factor $\alpha$. I consider counterfactual policy experiments in which I vary $\tau$ across $\{0, 0.1, 0.2, 0.3, 0.4\}$, while child support is kept fixed (at $B = 1.2$). Alimony payments are thus increased step-wise from no alimony to doubled alimony, relative to the status quo in Denmark.\footnote{More precisely, alimony payments are doubled for divorced couples conditional on both ex-spouses labor incomes and child support payments and given that the caps on alimony payments and on overall maintenance payments are non-binding.}
**Alimony and couples’ time allocation** Table 1.11 shows how changes in alimony payments impact married couples’ time allocation. Among married women increasing the alimony policy parameter $\tau$, leads to a decrease in work hours and an increase in housework and leisure. In contrast there is virtually no impact on the time allocation of married men. Quantitatively, switching from $\tau = 0$ to $\tau = 0.4$ leads to a reduction in married women’s work hours by 2.1% and an increase in married women’s housework hours and leisure time by 3.2% and 1.6%, respectively.

Table 1.11: The effect of changing alimony ($\tau$) on married couples’ time use

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours worked female</td>
<td>30.3</td>
<td>30.2</td>
<td>30.1</td>
<td>30.0</td>
<td>30.0</td>
</tr>
<tr>
<td>Housework hours female</td>
<td>17.5</td>
<td>17.6</td>
<td>17.7</td>
<td>17.8</td>
<td>17.8</td>
</tr>
<tr>
<td>Leisure female</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>Hours worked male</td>
<td>32.8</td>
<td>32.9</td>
<td>32.9</td>
<td>33.0</td>
<td>33.0</td>
</tr>
<tr>
<td>Housework hours male</td>
<td>10.7</td>
<td>10.7</td>
<td>10.7</td>
<td>10.6</td>
<td>10.6</td>
</tr>
<tr>
<td>Leisure male</td>
<td>6.5</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
</tr>
</tbody>
</table>

*Notes:* Mean time uses of married couples for different alimony policy regimes. Computed based on model simulations for $N = 20,000$ couples.

Table 1.12 shows the corresponding results for divorced couples. In response to a switch from $\tau = 0$ to $\tau = 0.4$ the average work hours of divorced women drop by 36.1%. This is accompanied by both rising average housework hours (by 64%) and rising average leisure time (by 28.6%). For divorced men I find that average work hours fall by 5.9%, while housework hours and leisure time increase by 18.1% and 10.4% respectively among male divorcees.

Interestingly these results show that increasing alimony leads to much starker labor supply disincentives for both divorced women and divorced men than increasing child support. A plausible explanation is that alimony payments depend on the difference of ex-spouses’ incomes. As a consequence both alimony payer and receiver can manipulate alimony payments to their advantage by reducing work hours. Child support in contrast only depends on one ex-spouse’s (the non-custodial parent’s) income, while the child support receiver cannot manipulate child support payments by reducing work hours. Alimony payments thus have both an income and a substitution effect for both spouses, while child support have both effects for the paying spouse, but only an income effect for the child support receiver.
**Table 1.12:** The effect of changing alimony (τ) on divorced couples’ time use

<table>
<thead>
<tr>
<th>τ</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours worked female</td>
<td>31.0</td>
<td>29.9</td>
<td>28.6</td>
<td>27.4</td>
<td>27.3</td>
</tr>
<tr>
<td>Housework hours female</td>
<td>17.5</td>
<td>18.6</td>
<td>19.8</td>
<td>20.9</td>
<td>21.0</td>
</tr>
<tr>
<td>Leisure female</td>
<td>1.5</td>
<td>1.6</td>
<td>1.6</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>Hours worked male</td>
<td>33.1</td>
<td>32.4</td>
<td>31.7</td>
<td>31.1</td>
<td>30.5</td>
</tr>
<tr>
<td>Housework hours male</td>
<td>11.5</td>
<td>12.1</td>
<td>12.6</td>
<td>13.1</td>
<td>13.6</td>
</tr>
<tr>
<td>Leisure male</td>
<td>5.4</td>
<td>5.6</td>
<td>5.7</td>
<td>5.8</td>
<td>5.9</td>
</tr>
</tbody>
</table>

*Notes:* Mean time uses of divorced couples for different alimony policy regimes. Computed based on model simulations for \( N = 20,000 \) couples.

**Alimony and consumption insurance**  As a measure of how successful alimony payments are in providing consumption insurance Table 1.13 shows couples’ relative consumption by marital status for different values of the alimony policy parameter τ. Under all considered scenarios married men and women consume almost equally, while divorced women consume a lot less than divorced men. Increasing alimony payments does not narrow of the gap between divorced women’s and men’s consumption, but on the contrary, as alimony payments are increased relative consumption among divorcees decreases even further. The reason for the failure of alimony payments to provide consumption insurance, is that alimony payments entail strong labor supply disincentives for divorced women. These disincentives are stronger than those for men (see Table 1.12). Overall the drop in divorced women’s labor supply, causes a drop in labor income, which overrides any positive consumption effect of receiving alimony payments.

Figure 1.9 presents event study graphs, that show the drop in relative consumption upon divorce (at \( t = 0 \)). This graph only includes individuals who do get divorced, showing that the results from Table 1.13 are not driven by differences between couples who do get divorced and couples who do not get divorced.
Table 1.13: The effect of changing alimony (τ) on couples’ relative consumption

<table>
<thead>
<tr>
<th>τ</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^\text{mar}_f / c^\text{mar}_m$</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$c^\text{div}_f / c^\text{div}_m$</td>
<td>0.65</td>
<td>0.64</td>
<td>0.62</td>
<td>0.59</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Notes: Mean relative consumption by marital status for different alimony policy regimes. Computed based on model simulations for $N = 20,000$ couples.

Figure 1.9: Event study: relative consumption

Notes: The figure shows average relative consumption of couples around divorce for different alimony policy regimes. Computations are based on simulations for $N = 20,000$ couples. The figure includes couples that get divorced and are observed for 2 time periods before and 2 time periods after getting divorced (a time period corresponds to 3 years).

1.7.3 The Impact of Child Support and Alimony on Divorce Rates

In general divorce law changes can be expected to influence divorce rates, although ex-ante the direction of the effect that maintenance payments have on divorce rates is unclear.\textsuperscript{39} For the large majority of divorced couples in my sample the ex-wife is receiving maintenance payments and the ex-husband needs to make these payments, i.e., when maintenance payments are increased divorce is becoming more attractive for women and less attractive for men. Whether this leads to a change in divorce rates and in what direction divorce rates change depends

\textsuperscript{39}Chiappori et al. (2015) and Clark (2001) show that the Becker-Coase Theorem according to which divorce law changes do not impact divorce rates only holds under restrictive assumptions, if households consume both public and private goods.
among other things on the weight that individuals attach to their financial situation after divorce, when deciding whether to stay with their partner or get divorced. Tables 1.14 and 1.15 show the impact of changing child support and alimony respectively (i.e., changing $b$ and $\tau$) on the % of couples who ever get divorced.

**Table 1.14:** The effect of changing child support ($b$) on divorce rates

<table>
<thead>
<tr>
<th>$b$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>ever divorced (%)</td>
<td>28.4</td>
<td>28.3</td>
<td>28.1</td>
<td>27.8</td>
<td>27.5</td>
</tr>
</tbody>
</table>

*Notes:* Divorce rates for different child support policy regimes, computed based on model simulations for $N = 20,000$ couples.

**Table 1.15:** The effect of changing alimony ($\tau$) on divorce rates

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>ever divorced (%)</td>
<td>28.8</td>
<td>28.6</td>
<td>28.3</td>
<td>27.9</td>
<td>27.4</td>
</tr>
</tbody>
</table>

*Notes:* Divorce rates for different alimony policy regimes, computed based on model simulations for $N = 20,000$ couples.

### 1.8 Welfare Analysis

Given the underlying policy trade-off between providing insurance to the lower earner in married couples, enabling married couples to specialize efficiently and maintaining labor supply incentives it is interesting to ask what the “best” maintenance policy is. In this section I draw welfare comparisons between different policy regimes and solve for the welfare maximizing maintenance policy (i.e., the welfare maximizing combination of $b$ and $\tau$). Moreover I compare maintenance policies by how close they bring couples to the first best scenario characterized in section 1.6.

#### 1.8.1 Welfare Comparisons and Optimal Policy

To measure the welfare consequences of changes in post-marital maintenance policies I consider the ex-ante well-being of women and men. More precisely, I use the sum of time period 0
expected discounted utilities of women $E[V_{f0}^{mar}]$ and men $E[V_{m0}^{mar}]$ as welfare criterion (i.e.,
the utilitarian welfare criterion with equal weights)\textsuperscript{40}

$$W = E[V_{f0}^{mar}] + E[V_{m0}^{mar}] .$$

I first consider the welfare consequences of ceteris paribus changing child support (varying $b$) and alimony payments (varying $\tau$), while keeping the other policy fixed at the status quo. The results are displayed in Figures 1.10 and 1.11. Figure 1.10 shows that increasing child support by a factor of 2.5 relative to the status quo is welfare maximizing child support policy if alimony is kept fixed at the status quo level. For alimony in contrast a slight reduction (by 12.5%) relative to the status quo is welfare maximizing if child support is kept fixed at the status quo level.

![Figure 1.10: Welfare comparisons: changing child support ($b$)](image1.png)

![Figure 1.11: Welfare comparisons: changing alimony ($\tau$)](image2.png)

Notes: The figures show the utilitarian welfare criterion (at equal weights) for counterfactual policy scenarios. Figure 1.10 displays policy scenarios across which child support ($b$) is changed. Figure 1.11 displays scenarios across which alimony payments are changed ($\tau$ is changed). Each figure is based on model simulations for 20,000 couples.

To find the optimal maintenance policy, I search for the combination of ($b, \tau$) that maximizes $W$ and find that $b = 3, \tau = 0.175$ is the welfare maximizing child support/ alimony combination. A welfare maximizing reform would thus be to triple child support and slightly reduce alimony payments.\textsuperscript{41}

\textsuperscript{40}Note that the variables that expectations are taken over include $n_0$ the initial number of kids a couple has, i.e., welfare is evaluated for the average couple at the beginning of marriage. Recall that initial bargaining power in couples is assumed to be equal (i.e., $\mu_0 = 1 - \mu_0 = 0.5$). Considering the utilitarian criterion at equal welfare weights thus fixes the welfare weights at women’s and men’s initial bargaining weights.

\textsuperscript{41}More specifically, I compute the welfare criterion $W$ for each value of ($b, \tau$) in $\{0, 0.5, 1, ..., 4\} \times \{0, 0.05, 0.1, ..., 0.4\}$.
1.8.2 Comparison to First Best

To assess how close the optimal child support/ alimony combination can bring couples to the first best scenario, I compare allocations and couples’ welfare under the status quo policy \((b = 1, \tau = 0.2)\) to the optimal maintenance policy \((b = 3, \tau = 0.175)\) and the first best scenario. Table 1.16 presents outcomes for each of the three scenarios and Figure 1.12 compares women’s and men’s ex-ante welfare for each scenario. Comparing the columns of Table 1.16 from left to right shows that all considered outcomes are closer to first best under the optimal maintenance policy than under the status quo, i.e., the optimal maintenance policy induces couples to adjust their behavior towards the first best allocation.

Table 1.16: Mean outcomes: status quo, optimal maintenance policy and first best

<table>
<thead>
<tr>
<th>Variable</th>
<th>Status quo ((B^<em>, \tau^</em>))</th>
<th>First best</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours worked female (married)</td>
<td>30.1 30.0</td>
<td>29.7</td>
</tr>
<tr>
<td>Housework hours female (married)</td>
<td>17.7 17.7</td>
<td>18.1</td>
</tr>
<tr>
<td>Leisure female (married)</td>
<td>2.2   2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>Hours worked male (married)</td>
<td>32.9 33.0</td>
<td>33.1</td>
</tr>
<tr>
<td>Housework hours male (married)</td>
<td>10.7  10.6</td>
<td>10.5</td>
</tr>
<tr>
<td>Leisure male (married)</td>
<td>6.4   6.4</td>
<td>6.4</td>
</tr>
<tr>
<td>Consumption ratio ((c_f/c_m, married))</td>
<td>0.98  0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>Hours worked female (divorced)</td>
<td>28.6  27.9</td>
<td>25.5</td>
</tr>
<tr>
<td>Housework hours female (divorced)</td>
<td>19.8  20.4</td>
<td>22.8</td>
</tr>
<tr>
<td>Leisure female (divorced)</td>
<td>1.6   1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>Hours worked male (divorced)</td>
<td>31.7  32.1</td>
<td>35.5</td>
</tr>
<tr>
<td>Housework hours male (divorced)</td>
<td>12.6  12.3</td>
<td>9.6</td>
</tr>
<tr>
<td>Leisure male (divorced)</td>
<td>5.7   5.6</td>
<td>4.9</td>
</tr>
<tr>
<td>Consumption ratio ((c_f/c_m, divorced))</td>
<td>0.62  0.67</td>
<td>1.00</td>
</tr>
<tr>
<td>% divorced in (T)</td>
<td>28.3  28.0</td>
<td>26.8</td>
</tr>
</tbody>
</table>

Notes: Mean outcomes by marital status for status quo, optimal maintenance policy and first best scenario. Computed based on model simulations for \(N = 20,000\) couples.

\(\{0, 0.05, 0.1, ..., 0.4\}\). The reported welfare maximizing \((b, \tau)\) is the maximizer over this grid.
Figure 1.12: Welfare comparison: status quo, optimal maintenance policy and first best

Notes: The figure shows the mean expected discounted utility for women and men under the status quo policy, the optimal maintenance policy and the first best scenario. Computed based on model simulations for $N = 20,000$ couples.

Figure 1.12 shows that the first best allocation makes both women and men on average better off relative to the status quo, i.e., is a Pareto improvement over the status quo (on average). The optimal maintenance policy in contrast makes women better off, while men are worse off than under the status quo. This indicates that there is scope for improvement beyond the welfare maximizing maintenance policy according to my model and that allocations are feasible that make both women and men ex-ante better off.

1.9 Conclusion

This paper addresses the question how post-marital maintenance payments affects couples’ decision-making and how maintenance policies (child support and alimony policies) should be designed. I construct a dynamic economic model and estimate its structural parameters by method of simulated moments estimation, matching a range of empirical moments from rich Danish administrative data and time use data. The data include information on marriage and divorce, child custody, maintenance payments and housework hours. My model incorporates two driving forces that speak in favor of maintenance payments: providing insurance to the
lower earner in married couples and enabling married couples to specialize efficiently, as well as a mechanism that speaks against maintenance payments: maintenance payments lower the labor supply incentives of divorcees. The aim of policy is to balance this trade-off.

The model takes into account that divorced ex-spouses are linked by maintenance payments. Divorcees interact non-cooperatively. The strategic interaction that arises because ex-couples are linked through maintenance payments, is fully modeled. Married spouses make decisions cooperatively, subject to limited commitment. Another key model ingredient are “learning-by-doing” returns to work experience, which instill a conflict between individual incentives and what is optimal from the couples perspective. From the individual’s perspective it is optimal work a lot to accumulate returns to work experience and thereby self-insure against income losses upon divorce, while from the couples perspective it is optimal to specialize, i.e., to have one spouse work little and focus on housework. Maintenance payments reduce the need for self-insurance and thereby facilitate household specialization. Moreover, maintenance payments impact married spouses’ outside options and thereby may affect divorce rates and trigger shifts in intra-household bargaining power.

To assess how maintenance policies affect couples’ decisions and welfare, I use the estimated model as a policy lab to conduct counterfactual experiments. Based on such experiments I show that the (ex-ante) welfare maximizing policy is characterized by increased (tripled) child support payments and slightly lower alimony payments (12.5% lower), relative to the Danish status quo policy. Increasing child support induces married couples to specialize more, leads to smoother consumption paths around divorce and to a moderate reduction in labor supply among divorced women. Increasing alimony payments in contrast fails to provide insurance: Alimony payments lead to a strong reduction in labor supply among divorced men and women. Because of the strong labor supply reduction, increasing alimony payments leads to larger consumption drops upon divorce (i.e., consumption around divorce becomes less smooth).

To study how close maintenance policies can bring couples to efficiency, I compare the welfare maximizing policy to a first best scenario, in which frictions (limited commitment and non-cooperation in divorce) are removed from the model. The first best-scenario is characterized by full consumption insurance and a higher degree of specialization among married couples, relative to the welfare maximizing policy. In terms of women’s and men’s ex-ante wellbeing, the first best scenario is a Pareto improvement over the welfare maximizing

47
maintenance policy and the status quo policy, indicating that there is scope for improvement in couples well-being beyond what is attained by the welfare maximizing maintenance policy.
Chapter 2

A Structural Analysis of Vacancy Referrals with Imperfect Monitoring and Sickness Absence

2.1 Introduction

Unemployment insurance (UI) systems in OECD countries typically imply specific job search requirements for the receipt of unemployment benefits and offer some form of job search assistance. Unemployed job seekers who do not comply with the job search requirements usually risk a sanction. While the common goal of these eligibility rules and activation strategies is to reduce moral hazard and to increase the reemployment rate among UI benefit recipients, there exist large differences across countries in the institutions implemented to reach these goals (Immervoll and Knotz 2018b and Knotz 2018). These differences include variation in the strictness and the enforcement rate of sanctions, in the criteria for job offers a job seeker has to accept, and in the use of job vacancy referrals (VRs) to support and monitor the unemployed workers’ job search effort. Given the differences in the institutional setup across countries and the interdependencies of the different policy measures, it is difficult to use cross-country comparisons or reduced-form analyses to learn something about the relative effectiveness of counterfactual policy designs.

This paper develops and estimates a structural job search model that incorporates vacancy referrals and punitive sanctions, two main features of many UI systems. UI agencies usually
punish refusals to apply for referred job vacancies or to accept corresponding job offers by reducing unemployment insurance payments for a fixed time span. VRs complement individual job search effort of UI benefit recipients and the threat-effect of sanctions ensures that unemployed job-seekers cannot be too selective about applying for referred job vacancies. In the model, we additionally allow for the possibility to report sick after the receipt of a VR. In many UI systems the requirement to apply for a VR ceases in case of sickness. For a UI recipient this creates an incentive to call in sick strategically in response to receiving a VR that he deems unattractive. Our model further takes into account that job search effort is not perfectly observable and that the enforcement of sanctions is typically not structured by binding rules, but is - at least to some extent - subject to the discretion of caseworkers at the UI agency. As a direct consequence, the possibility of strategic sick reporting and the presence of imperfect sanction enforcement may hamper the effectiveness of VRs and sanctions in counteracting moral hazard. In evaluating policy changes related to VRs and sanctions it is thus important to take into account both these aspects.

We estimate our model using administrative data from social security records and from the public employment service in Germany. Our data covers the time period from 2000 to 2002. In particular, we use detailed information on unemployment and employment durations, benefit receipt, the arrival of vacancy referrals, imposed sanctions, sickness absence during unemployment and daily wages during employment. Additionally, the data feature a broad range of socioeconomic characteristics including education, family status and health restrictions. We use our model to simulate a range of counterfactual policy changes related to changing the VR arrival rate and the sanction enforcement rate. Our model is also suited to simulate changes in sanction duration (i.e. for how many time periods a sanction lasts) and sanction severeness (i.e. by how much UI benefits are reduced in case of a sanction). The results of these simulation exercises are helpful for understanding how effective different policy designs for sanctions and VRs are in reducing moral hazard and for evaluating their impact on accepted wages and unemployment durations.

We find that increasing sanction enforcement reduces moral hazard, i.e., incentivizes unemployed workers to reduce their reservation wages. By this mechanism increased sanction enforcement considerably increases job finding rates. According to our estimates under the status quo average sanction enforcement is low, at a 30.4% rate. Moving from the status quo to a 100% sanction enforcement rate reduces average unemployment duration by 0.6 months.
In contrast, increasing the VR rate leads to more moral hazard, i.e., leads unemployed workers to raise their reservation wages. Quantitatively, by this mechanism increasing the VR rate by a factor of 1.5 leads to a decrease in job take-up given VR receipt by 0.45 p.p. Nevertheless, a higher VR rate leads to higher job finding rates, as the higher frequency at which VRs arrive mechanically leads to higher job take-up, even at increased reservation wages. Overall an increase in the VR rate by a factor of 1.5 increases job take-up by 0.6 p.p., reflecting that a higher VR arrival rate mechanically leads to higher job take-up even at increased reservation wages.

We further use our estimated model to study the consequences of VR induced sick reporting. We find that VR induced sick reporting accounts for a substantial share of overall sick reporting. According to our estimates, 9.6% of all observed sick reports are induced by VRs.\footnote{This number is in line with the empirical findings by van den Berg et al. (2019b).} To study the consequences of VR induced sick reporting for job search outcomes we consider a hypothetical scenario in which we eliminate VR induced sick reporting. Looking at averages across the whole worker population we find modest effects of shutting down VR induced sick reporting on job search outcomes. However, there is substantial heterogeneity in the population. For the 20% of workers with the highest propensity of VR induced sick reporting we find that eliminating VR induced sick reporting would reduce the mean unemployment duration by 0.8 months (i.e., by 7.7%) and modestly reduce average wages by 2.3%.

Our study contributes to the literature that uses structural models to evaluate active labor market policies (see e.g. Lise et al. 2015, Launov and Wälde 2016, Gautier et al. 2018 and Wunsch 2013). Fougère et al. (2009) estimate a partial equilibrium job search model to study the effect of job contacts through the public employment service (PES) on job seekers’ search effort. Their results suggest that an increase in the support through the PES has a negative impact on private search and that it reduces the time spent in unemployment. van den Berg and van der Klaauw (2018) estimate a job search model with formal and informal search to analyze the impact of monitoring on the use of the different search channels and employment outcomes. Their findings indicate that monitoring leads to a substitution of informal search – which cannot be observed by the PES – by perfectly observable formal search. Cockx et al. (2018) study effects of a system of monitoring and sanctions on search

\[1\text{This number is in line with the empirical findings by van den Berg et al. (2019b).}\]
effort in a non-stationary environment with imperfect monitoring. Their finding indicate that the intensity and the precision of the monitoring scheme is crucial for the effectiveness of the policy. Our paper is the first that provides a structural analysis of the interplay of vacancy referrals and sanctions. Moreover, we are the first who structurally investigate the role of strategic sick reporting in a monitoring system.

There exist a number of reduced-form analyses on job search monitoring and sanctions (see e.g. Lalive et al. 2005, van den Berg and van der Klaauw (2006), van den Berg et al. 2004, Boone et al. 2009 and Micklewright and Nagy 2010). These studies usually find positive effects of monitoring and imposed sanctions on reemployment rates. Some studies additionally indicate that these positive effects on employment probabilities go along with negative effects on initial wages (van den Berg and Vikström 2014 and Arni et al. 2013). There also exists some reduced-form evidence on the effects of VRs. van den Berg, Hofmann and Uhlendorff (2018) use the same data set as in this paper. Based on multi-spell duration models they show that the receipt of a VR increases the transition rate to employment and that these jobs go along with lower wages. They additionally find a positive impact of receiving a VR on the probability of reporting sick. In line with these results, Bollens and Cockx (2017) provide evidence that receiving a VR increases the transition rate to employment based on a sample of unemployed job seekers in Belgium. The reduced-form results reported in these studies cannot be used to study effects of alternative policy designs. The present paper provides a structural model which can be applied for counterfactual policy analysis.

The remainder of the paper is organized as follows: Section 2.2 describes the institutional background, i.e. the rules and institutions related to UI benefits, VRs and sanctions that German UI recipients face during our observation period. Section 2.3 describes the data. Section 2.4 develops the structural model. Section 2.5 derives the likelihood function and describes how we estimate our model. Section 2.6 presents estimation results. Section 2.7 presents the evaluation of counterfactual policies and section 2.8 concludes.
2.2 Institutional Background

In the following, we describe the institutional setting of the different policies relevant for our analysis. The description refers to our observation periods from 2000 to 2002.

2.2.1 UI Benefits

Unemployed who have worked at least twelve months within the last three years are eligible for UI benefits. The potential benefit duration depends on the age and the time spent in employment. It ranges from 6 months for individuals below the age 45 who have worked between 12 and 16 months in the last seven years to 32 months for unemployed job seekers who are older than 57 and have been employed for at least 64 months. The replacement rate corresponds to 67% for unemployed with at least one child and to 60% for individuals without children. After the expiration of the UI benefits unemployed are entitled to means-tested unemployment assistance with replacement ratios of 57% and 53%, respectively (Konle-Seidl, Eichhorst, and Grienberger-Zingerle, 2010).

2.2.2 Vacancy Referrals and Sanctions

With a vacancy referral, a caseworker asks an unemployed to apply for a specific job vacancy. A VR usually contains information about the occupation, the working hours and the starting date of the job, but not about the wage. The time lag between a VR and end of the hiring process depends on the sector and the occupation of the job vacancy. Qualitative evidence based on interviews with caseworkers indicates that this time lag is shorter for low skilled jobs than for high skilled positions, and that it usually does not exceed 2 weeks. Not applying to a referred job vacancy as well as not accepting a corresponding job offer can lead to a sanction. One condition for a sanction is that the job is “suitable”. This implies during our observation period between 2000 and 2002 that the job has to be within 2.5 hours of commuting distance and that – within the first 3 months after the start of the unemployment spell – the wage is above 80% of the previous wage. This wage threshold drops to 70% within months 4 and 6, and after 6 months of unemployment all jobs with a wage above the UI benefit level and within the 2.5 hours of commuting distance are defined to be suitable.

Not applying to a job after receiving a corresponding VR or refusing to accept a suitable
job offer is one of the main reasons for being sanctioned. In this case the unemployment benefit payments are cut completely for a period of 12 weeks. One strategy to prevent a job offer and the risk of being sanctioned might be to intentionally misbehave in the job interview. While it is more difficult to detect such behavior, it is still possible for the caseworker to impose a sanction in this case. In practice, this depends on the type of contact between the caseworker and the firm posting the job vacancy. Other reasons for “long” sanctions of 12 weeks are refusing to participate in or dropping out of active labor market policy measures. If an UI benefit recipient is not showing up at a scheduled meeting, this might lead to a short sanction of 2 weeks. All types of short and long sanctions imply a benefit cut by 100%.\(^2\)

After the imposition of a sanction, the unemployed job seeker is supposed to go on with his job search effort and to comply with the obligations for UI benefit recipients. If the unemployed does not follow the job search requirements, he or she risks an additional sanction. If the accumulated duration of sanctions within one unemployment spell is above 24 weeks, the unemployed loses all claims for UI benefits. Qualitative evidence based on caseworker interviews suggest that the monitoring of unemployed workers’ job search effort and the use of VRs do not change after a sanction. Sanctioned unemployed can apply for welfare benefits. These benefits are means-tested – i.e. they depend on the household income and savings – and not related to the previous wage.

Non-compliance with job search requirements is not always detected. First, during our observation period, between 400 and 1000 unemployed workers were allocated to one caseworker. This very high caseload implies that caseworkers cannot monitor the job search effort of all unemployed very closely. Second, even with an intense monitoring not every infringement can be fully observed. For example, detecting non-compliance after the receipt of a VR depends on the relationship between the caseworker and the human resources department of the employer offering the vacancy. Besides that, the caseworkers have some discretion whether or not to impose a sanction (Müller and Oschmiansky, 2006). After the detection of an violation of the job search requirements, the unemployed has the opportunity to explain and justify his behavior. At this stage, the caseworker has some degree of freedom to decide whether or not this justification is sufficient. If the caseworker evaluates the justification as insufficient, the benefits management department takes over and – in case of

\(^2\)Another reason for a long sanction are voluntary job quits. In this case, individuals do not receive any benefits in the first 12 weeks of their unemployment spell. In this paper we exclude individuals from our analysis who are facing this type of sanctions.
no objection – sends out a letter to the unemployed worker to inform him about the sanction. After that, the unemployed worker has the option to file an objection against the sanction.

2.2.3 Sick Leave

Following the guidelines for UI benefit recipients, unemployed job seekers have to hand in a sick note from a medical doctor to the PES if they are sick. While being sick, the unemployed continues receiving UI benefits and the UI entitlement duration continues to decline. This implies that there are no direct financial incentives to report sick during UI benefit receipt. However, during sickness absence, the unemployed does not have to comply with the job search requirements and therefore does not risk a sanction if he or she does not send out an application after the receipt of a VR. This implies that an unemployed has an incentive to report sick in case of real sickness and in case of a VR which refers to a job which is unattractive for the unemployed. There is no direct way for the PES to evaluate the sick note. Only after the sickness, the unemployed can be sent to the medical service of the PES. At this service, the doctors evaluate the general work-related health status. Moreover, the unemployed can freely chose their doctor and can change the physicians at any time. This allows them to search for a doctor who is cooperative and willing to hand out a sick note, and it is not possible for the PES to verify the sick note.

2.3 Data

Our analysis is based on administrative records from the German PES (Bundesagentur für Arbeit). The data contain daily information about employment and unemployment spells, participation in ALMPs, earnings and UI benefits. Moreover, we have information about basic sociodemographic characteristics including education, family status and health restrictions. As common for this type of data, we do not have information about self-employment, inactivity, and civil servants (Dundler, 2006).

Our sample consists of men entering unemployment in the year 2000 and who have been employed for at least 12 months before the entry into unemployment. We focus on West

---

3If an unemployed is sick with the same diagnosis for more than 6 weeks, the unemployed enters sickness benefits. This benefit scheme requires a specific medical certificate. This certificate can be verified by a doctor of the medical service of the health insurance. In this paper, we focus on short-term sickness.
Germany because in our observation period East and West German labor markets were substantially different. For example, public employment programs played an much more important role in the East German labor market. We select unemployed workers who are between 25 and 57 years old. The first age restriction is motivated by the educational system and the second one by the retirement schemes in Germany. In 2003, several labor market reforms have been introduced. Therefore, we right-censor our observation at December 31, 2002. Our final estimation sample consists of 69,788 individuals.

In our data we observe arrivals of VRs, imposed sanctions and periods of sickness absence. The main outcome variables are transitions from unemployment to work and accepted wages upon exit to employment. We have no information about working hours. Therefore, our wages correspond to daily gross wages.\(^4\) A transition from unemployment to employment is defined as a transition to regular jobs without receiving any benefits from the PES at the same time. Our model will be estimated in discrete time, and we discretize our duration data in monthly observations. Unemployment duration corresponds to the duration of benefit receipt. The institutional rules with respect to VRs and sanctions are the same for UI and UA benefits. Therefore, we do not distinguish between periods of these two types of benefits. Unemployment spells with transitions into inactivity, subsidized jobs or ALMP programs with training measure benefits (Unterhaltsgeld, UHG) are right-censored.

While we know the month in which an individual receives a VR, we have no information about the occupation or the sector of the firm with the open vacancy. We observe the intended length and the starting dates of sanctions. In our analysis we focus on long sanctions, and we exclude unemployment spells with a sanction due to voluntary job quits. For these types of sanctions, the sanction is imposed directly at the beginning of an unemployment spell. Besides that, we do not know the reason for long sanctions imposed at some point after the start of the unemployment spell. However, the majority of the observed sanctions are related to VRs. Following official statistics of the German PES, sanctions related to VRs were about 4 times as common as sanctions due to refusing or dropping out of a training measure (Bundesagentur für Arbeit, 2004). We define a sickness absence if in our data a sickness spell lasts at least 13 days. Given that the application period usually does not exceed

\(^4\)The wage information is right-censored at the social security contribution ceiling. This aspect should be of limited relevance for our analysis, since almost all observed post-unemployment wages are below this threshold. In 2002, the cap was at 4500 Euro per month in West Germany. Only 2.1% of our sample took up a job that paid more than 4000 Euro per month.
two weeks, this ensures that individuals who are sick following our definition can effectively avoid an application to an assigned VR.

It is important to note that we do not observe whether a sickness absence occurs due to a VR. Moreover, we do not know whether a job found after receiving a VR is the one which the unemployed has been referred to.

### 2.4 Model

Our structural model extends the standard sequential random search model (see e.g. Mortensen (1986)) by allowing for regular job offers as well as offers obtained through vacancy referrals (VRs). We furthermore account for sanctions and sickness absences. Turning down a job offer obtained through a VR may lead to a sanction, i.e., to a benefit reduction that lasts for several months. Sick notes can be handed in strategically after receiving a VR to avoid getting sanctioned. Our model generalizes the model that van den Berg et al. (2019b) use to interpret their reduced form results.

The model is set in discrete time. We consider an unemployed worker who is risk neutral and discounts the future at discount rate $\beta$. At the beginning of a time period a (non-sanctioned) unemployed worker collects unemployment benefits $b$. In any given period he may fall sick with probability $p_{sick}$. In this case no job offers can be accepted and the worker remains unemployed and does not receive a sanction. In case no sickness occurs, a regular job offer or a VR may arrive. In case a regular job offer arrives the unemployed worker observes the attached wage offer and decides whether to accept or reject. If a VR arrives the unemployed additionally needs to decide whether to hand in a sick note. If the unemployed worker turns down a job offer that he obtained through a VR, he may receive a sanction, i.e., a benefit reduction starting in the next time period.

**Law of motion of the state variables**

In a given period the decisions of the unemployed worker depend on two state variables. The first state variable, denoted by $s$, counts the remaining time periods of an ongoing sanction. If $s > 0$, the unemployed worker does not receive unemployment benefits for the next $s$ time periods. Upon arrival of a new sanction $s$ is increased by $K$, i.e., the unemployed worker is sanctioned by a benefit reduction for the subsequent $K$ time periods. The other state
variable \( ps \) counts the number of sanctions received in the past. If an unemployed worker’s accumulated past sanctions cross a threshold value \( ps \), a terminal sanction is imposed on him, i.e. he completely loses benefit eligibility.

**Sickness**

Sicknesses occur at rate \( p_{sick} \). If a sickness occurs the unemployed worker cannot accept any job offers, following the notion that sick individuals cannot apply for any jobs,\(^5\) in the given time period, but in case a VR arrives also cannot be sanctioned because he fails to apply.\(^6\) If a sickness occurs the unemployed worker thus moves on to the next period, without responding to regular job offers or VRs.

**Regular Job Offers**

Regular job offers arrive exogenously at rate \( p_{jo} \). A regular job offer is characterized by a random draw from the wage offer distribution \( F_{jo} \). Upon offer arrival the unemployed worker decides whether to accept or reject. If accepting he starts the job at the offered wage, while in case of rejection he continues his job search. Formally, the expected value of receiving a regular job offer is

\[
R_{jo}(s, ps) = \int \max \left\{ E(w, ps), U(\max\{s - 1, 0\}, ps) \right\} dF_{jo}(w),
\]

where \( U(s, ps) \) denotes the value of being unemployed in state \((s, ps)\) and \( E(w, ps) \) is the value of being employed at wage \( w \) and given past sanctions \( ps \).

**Vacancy Referrals and Sanctions**

VRs arrive randomly at rate \( p_{vr} \). A VR is characterized by a wage draw from the offer distribution \( F_{vr} \). We assume that the unemployed worker learns the wage offer attached to a referred vacancy immediately when he receives the VR. After observing the wage offer he decides whether to try to get a sick note to circumvent the VR or not. If he tries to get a sick note, he is successful in obtaining one with probability \( p_{doc} \). In this case the obligation

\(^5\)Recall that we focus on sicknesses that last for at least 2 weeks and have been certified by a doctor. In such cases a successful job application seems extremely unlikely.

\(^6\)The latter assumption is realistic, as sick benefit recipients should be able hand in a sick note to their caseworker with probability 1 and thereby circumvent the risk of being sanctioned.
to apply for the referred vacancy ceases and the unemployed worker continues his job search without being at risk of receiving a sanction. For the unemployed worker the expected value of receiving a VR hence is

\[
R_{vr}(s, ps) = \int \max \left\{ A_{vr}(w), p_{doc} U(\max\{s - 1, 0\}, ps) + (1 - p_{doc}) A_{vr}(w) \right\} dF_{vr}(w),
\]

(2.1)

where \(A_{vr}(w)\) is the value of applying for a VR with attached wage \(w\).\(^7\)

It is important to note that if an unemployed worker applies for a VR, there is a positive probability that the employer rejects him so that he does not receive a job offer. In this case the unemployed worker remains unemployed and is not sanctioned. We denote the probability that a job offer is received upon applying for a VR by \(\lambda_{vr}\) (i.e. the probability of being rejected by the employer is \(1 - \lambda_{vr}\)). In case the unemployed worker fails to hand in a sick note, it is always optimal for him to apply for the referred vacancy and learn whether he is offered the job.\(^8\) If he indeed receives a job offer he may accept and start the job at the offered wage or reject, in which case he is at risk of receiving a sanction. This risk is realized with probability \(p_{sanc}\), where \(p_{sanc} < 1\) reflects the possibility that the responsible caseworker may use his substantial discretionary leeway in deciding if a sanction is imposed or not. If the unemployed worker receives a sanction, no benefits are payed out to him for the next \(K\) time periods. In terms of the state variables this means that \(s\) is increased by \(K\). Furthermore state variable \(ps\) is increased by 1, bringing the unemployed worker one step closer to a terminal sanction. Formally, the value of applying for a referred vacancy with attached wage offer \(w\) equals

\[
A_{vr}(w) = \lambda_{vr} \max \left\{ E(w, ps), p_{sanc} U(K, ps + 1) + (1 - p_{sanc}) U(\max\{s - 1, 0\}, ps) \right\} + (1 - \lambda_{vr}) U(\max\{s - 1, 0\}).
\]

(2.2)

\(^7\)For better readability we suppress that \(A_{vr}(w)\) also depends on \(s\) and \(ps\).

\(^8\)If substantial marginal costs of applying for a VR are introduced into the model, it may become optimal for the worker to refuse to apply and thereby risk a sanction before learning if he is offered to fill the vacancy. We assume that the marginal cost of applying for a vacancy is sufficiently small so that it is always favorable to apply and learn the employer’s decision first.
Terminal Sanctions

Whenever an unemployed worker receives a sanction it may happen that his accumulated number of sanctions exceeds the terminal sanction threshold, i.e. $ps \geq \bar{ps}$. When this happens, a terminal sanction is imposed, meaning the unemployed worker loses benefit eligibility and continues his job search without collecting benefits or receiving VRs. The value of being unemployed and terminally sanctioned thus is

$$\Phi = \beta \left( (1 - p_{sick}) p_{jo} \int \max \{ E(w, \bar{ps}), \Phi \} dF_{jo}(w) + (1 - p_{jo}(1 - p_{sick})) \Phi \right).$$

Value of Unemployment

The expected discounted lifetime utility of an unemployed worker in state $(s, ps)$ is given by the Bellman equation

$$U(s, ps) = \begin{cases} b \mathbf{1}_{s=0} + \beta (1 - p_{sick}) \left( p_{jo} R_{jo}(s, ps) + p_{vr} R_{vr}(s, ps) \right) \\ + (1 - p_{jo} - p_{vr}) U(\max\{s - 1, 0\}, ps) + \beta p_{sick} U(\max\{s - 1, 0\}, ps) \end{cases}$$

if $ps < \bar{ps}$. Implicit in $R_{jo}(s, ps)$ and $R_{vr}(s, ps)$ are the optimal decisions the unemployed worker makes about accepting job offers that he receives on the labor market or through VRs as well as his optimal decisions about strategically calling in sick after receiving a VR. If $ps \geq \bar{ps}$ the unemployed worker is terminally sanctioned and the value of unemployment equals $U(s, ps) = \Phi$.

Value of Employment

The expected discounted lifetime utility of an employed worker depends on the per period wage and an exogenous job destruction rate $\delta$. If a job is destroyed and the worker returns to unemployment it makes an important difference whether he gets a fresh start with his past sanctions $ps$ reset to 0 or whether $ps$ persists at the pre-employment level. Having $ps$ persist at the pre-employment level would imply that benefit eligibility once lost cannot be regained, thus overstating not only the likelihood of receiving a terminal sanction, but also the resulting utility loss. However if $ps$ is reset to 0 after any period of employment
(however short), the threat of receiving a terminal sanction is strongly understated relative to the real institutional setting. We stick as close to the real setup as possible by assuming that when a job is destroyed $ps$ is reset to 0, only if the worker has been employed for more than $\tau$ time periods. The value of employment thus becomes dependent on employment duration. We define $\tau$ as number of employment periods necessary to establish a new claim to unemployment benefits, i.e. after $\tau$ periods in employment $ps$ is set to 0. The value of being employed at wage $w$, given $ps$ and $\tau$ thus is

$$
\bar{E}(w, ps, \tau) = \begin{cases} 
  w + \beta(\delta U(0, ps) + (1 - \delta)E(w, ps, \tau - 1)) & \text{if } \tau > 0 , \\
  w + \beta(\delta U(0, 0) + (1 - \delta)E(w, 0, 0)) & \text{if } \tau = 0 .
\end{cases}
$$

The value of becoming employed at wage $w$ and given past sanctions $ps$ then equals

$$
E(w, ps) = \frac{
\bar{E}(w, 0, 0)}{ \text{if } ps = 0 }, \\
\bar{E}(w, ps, \tau) & \text{if } ps > 0 .
$$

**Reservation wages**

It is straightforward to show that the value of employment is strictly increasing in $w$. It hence follows that the unemployed worker adopts a reservation wage strategy when deciding whether to accept or reject job offers. The worker’s strategy is completely characterized by reservation wages for regular job offers $\bar{w}_{jo}(s, ps)$ and for job offers obtained through VRs $\bar{w}_{vr}(s, ps)$ for each combination of state variables $s \in \{0, \ldots, K\}$ and $ps \in \{0, \ldots, \bar{ps}\}$ and a reservation wage $\bar{w}_\Phi$ that characterizes decision-making of terminally sanctioned individuals.

### 2.5 Estimation

We estimate the model by maximum likelihood (ML), fitting the joint distribution of the observable data. Recall that we observe unbalanced panel data on employment status, the occurrence of VRs, reported sicknesses and imposed sanctions. Denote the vector of relevant observables for individual $i$ in time period $t$ by $Z_{it} = (e_{it+1}, e_{it}, vr_{it}, sick_{it}, sanc_{it}, s_{it}, ps_{it})$. 

61
where $e_{it}$ is an indicator for employment status (1 for employed 0 for unemployed). Data on the state variables $s_{it}$ and $ps_{it}$ are derived from the individual specific history of past $sanc_{it}$ realizations. In time periods when an individual accepts a job, $Z_{it}$ additionally includes the accepted wage, $w^{acc}_{it}$.  

Wage Offer Distributions

For the estimation we impose a parametric form on the wage offer distributions, i.e. we require that $F_{jo} = F_{\gamma_{jo}}$ and $F_{vr} = F_{\gamma_{vr}}$ are specified up to finite dimensional unknown parameters $\gamma_{jo}$ and $\gamma_{vr}$. In principle any parametric distribution can be used as long as it satisfies the Flinn and Heckman (1982) recoverability condition. In our estimations we specify $F_{jo}$ and $F_{vr}$ to be log-normal, hence the unknown parameters of the wage offer distributions are $\gamma_{jo} = (\mu_{jo}, \sigma_{jo})$ and $\gamma_{vr} = (\mu_{vr}, \sigma_{vr})$.

Measurement Error

We allow for measurement error in accepted wages to reduce the sensitivity of our estimates to the lowest sampled accepted wage. Allowing for measurement error furthermore prevents the likelihood function from falling to zero, whenever the wage of a sampled individual is smaller than the reservation wage implied by our model. As we use administrative data for our estimation the wages we observe are not prone to the usual reporting errors that are to be expected in survey data. However to obtain monthly wages, we scale up daily payments by a constant, which introduces measurement error. We assume the measurement error enters log-wages additively as is standard in the literature on empirical search models (cf. Wolpin 1987), i.e. $\ln(\tilde{w}^{acc}) = \ln(w^{acc}) + \epsilon$, where $\epsilon$ is normally distributed with mean zero and variance $\sigma^2_{\epsilon}$. The measurement error variance $\sigma^2_{\epsilon}$ is treated as unknown parameter, i.e. estimated along with the structural model parameters.

Likelihood function

For the ML estimation we fix the discount factor at $\beta = 0.997$. Note that we observe the exact unemployment benefits that an individual receives and thus we do not need to estimate

---

\footnote{To be more formal we could include an additional element $w^{acc}_{it} \cdot 1(e_{i,t+1} > e_{i,t})$ in $Z_{it}$.}
All remaining parameters are estimated. The complete vector of unknown parameters is

$$\theta = (\mu_{jo}, \sigma_{jo}, \mu_{vr}, \sigma_{vr}, p_{jo}, p_{vr}, \lambda_{vr}, p_{sick}, p_{doc}, p_{sanc}, \delta, \sigma_{\epsilon}).$$

Given our data for individuals $i = 1, ..., N$, where each individual is observed for a sequence of time periods $t = 1, ..., T$, the likelihood function equals

$$\mathcal{L} = \prod_{i=1}^{N} \prod_{t=1}^{T} h_{it}(Z_{it} | \theta)$$

For a derivation of the likelihood contributions $h_{it}(Z_{it} | \theta)$ see appendix B.1.4.

**Heterogeneity**

We introduce heterogeneity by allowing a subset of the structural parameters to vary across individuals. To account for observed heterogeneity we assume the relationship between observables $X_i$ and structural parameters can be captured by standard parametric functional forms. We specify two separate functional forms depending on the structural parameter’s range of admissible values. For $\mu_{jo}$ and $\mu_{vr}$, which take only positive values, we specify the dependence on $X_i$ by

$$\mu_{jo} = \exp(\zeta_1'X_i), \quad \mu_{vr} = \exp(\zeta_2'X_i).$$

For $p_{jo}$, $p_{vr}$, $\lambda_{vr}$, and $\delta$, which take values in $[0, 1]$, we specify dependence on $X_i$ by

$$p_{jo} = (1 + \exp(-\zeta_3'X_i))^{-1}, \quad p_{vr} = (1 + \exp(-\zeta_4'X_i))^{-1},$$
$$\lambda_{vr} = (1 + \exp(-\zeta_5'X_i))^{-1}, \quad \delta = (1 + \exp(-\zeta_6'X_i))^{-1}.$$ 

For the estimation we include in $X_i$ age, dummy variables indicating health restrictions and completion of apprenticeship training as well as a constant. For computational tractability we discretize age into 10 year bins, spanning the range from 28 to 58 years. For variables that vary over time we focus on measurements in the first sampled time period to ensure parameter stability within individual. As we observe the exact amount of benefits each sampled individual receives, we can furthermore account for heterogeneity in benefits. In

\(^{10}\) How we make use of the benefit data is described in more detail below in the section on observed heterogeneity.
particular we allow the benefit level \( b \) in our structural model to be individual specific and set it equal to the monthly benefits received in the first sample period. We discretize benefits into bins of width 250 spanning the range between 500 and 1500 Euros.

We account for unobserved heterogeneity by introducing a latent factor \( \nu \) that takes values in a discrete set \( \{v_1, \ldots, v_M\} \). The probability that \( \nu \) takes realization \( v_m \) is denoted by \( \pi_m \). We impose a normalization on \( \nu_M \) such that \( \mathbb{E}[\nu] = 0 \). The latent factor \( \nu \) is assumed to enter a subset of the structural parameters, namely in \( p_{\text{sick}} \), \( p_{\text{doc}} \) and \( p_{\text{sanc}} \), with different factor loadings, thereby introducing additional heterogeneity that is unrelated to \( X_i \). Formally we specify

\[
    p_{\text{sick}} = (1 + \exp(-X_i'\zeta_7 - \gamma_1\nu))^{-1}, \tag{2.4}
\]
\[
    p_{\text{doc}} = (1 + \exp(-X_i'\zeta_8 - \gamma_2\nu))^{-1}, \tag{2.5}
\]
\[
    p_{\text{sanc}} = (1 + \exp(-X_i'\zeta_9 - \gamma_3\nu))^{-1}. \tag{2.6}
\]

The advantage of using a one-factor specification relative to the unrestricted finite mixture model by Heckman and Singer (1984) is a reduction in the number of unknown parameters and a substantial reduction in computation time, as the one-factor specification requires computing a one-dimensional rather than a multidimensional integral. As the factor loadings \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) may take arbitrary values, the one-factor specification is not restricting the impact of unobserved heterogeneity to be similar across parameters. However it imposes a relation between variance and covariance of the structural parameters within the population.\(^\text{11}\)

The likelihood function for the model specification with observed and unobserved heterogeneity accounts for the dependence of structural model parameters on \( X_i \) and \( \nu \). To account for unobserved heterogeneity each individual’s likelihood contribution is averaged over unobserved types. The likelihood function then equals

\[
    \mathcal{L} = \prod_{i=1}^{N} \left( \sum_{m=1}^{M} \pi_m \prod_{t=1}^{T_i} h_{it}(Z_{it} | \theta(v_m, X_i)) \right), \tag{2.7}
\]

where the dependence of the structural parameters \( \theta \) on \( X_i \) and \( \nu \) is governed by the parameters \( \zeta_1, \zeta_2, \ldots, \zeta_9 \) and \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) respectively. We subsume these parameters into vectors \( \zeta \) and \( \nu \).

\(^\text{11}\)For example two parameters that both vary in the population conditional on \( X_i \) are necessarily (positively or negatively) correlated. See van den Berg (2001) for a discussion of the one-factor specification of unobserved heterogeneity in the context of the multi-spell mixed proportional hazard model.
\( \gamma \) and denote by \( \pi \) the vector containing the probabilities \( \pi_1, \ldots, \pi_M \). Maximum likelihood estimation for the specification with observed and unobserved heterogeneity is performed by maximizing \( \mathcal{L} \) over \( \zeta, \gamma, \pi, \sigma_{jo}, \sigma_{vr} \) and \( \sigma_e \).

**Identification**

In order to highlight how each of the structural model parameters is identified, we present equations that link the parameters of our model to empirical moments of the observed data. By showing that a set of empirical moments uniquely maps into model parameter values, we demonstrate that the structural model is identified in the formal econometric sense (see e.g. French and Taber 2011). Identification is a precondition for consistent estimation. Moreover we hope that pointing to links between the observed data and the model parameters provides intuition about which variation in the data identifies the structural parameters. We make use of the identification result by Flinn and Heckman (1982), which can be applied to our setting to obtain identification of reservation wages and wage offer distributions. According to their argument the lowest sampled wage accepted by an unemployed worker in state \((s, ps)\) after receiving / not receiving a VR identifies the reservation wages \( \overline{w}_{vr}(s, ps) \) and \( \overline{w}_{jo}(s, ps) \), respectively. Given identification of the reservation wages, the wage offer distributions \( F_{jo} \) and \( F_{vr} \) are identified from the respective distribution of accepted wages if \( F_{jo} \) and \( F_{vr} \) are recoverable.\(^{12}\) The remaining structural parameters are identified from transitions between unemployment and employment together with joint observations of VRs, sanctions and sickness absences. We denote by \( vr, sanc \) and \( sick \) variables that indicate arrivals of VRs, sanctions and sickness absences, respectively.\(^{13}\) Conditional on the state \((s, ps)\) our model

---

\(^{12}\)For estimation we will restrict \( F_{jo}, F_{vr} \) to be log-normal. The log-normal distribution is recoverable (see Flinn and Heckman 1982).

\(^{13}\)Time subscripts are omitted for convenience.
implies the following relationships between data moments and structural parameters.

\[ P(e_{t+1} = 0|e_t = 1) = \delta \]
\[ P(vr_t = 1|e_t = 0) = p_{vr} \]
\[ P(sick_t = 0, vr_t = 0, sanct_t = 0, e_{t+1} = 0|e_t = 0) = (1 - p_{sick})(1 - p_{jo}(1 - F_{jo}(\overline{w}_{jo})) - p_{vr}) \]
\[ P(sick_t = 1, vr_t = 0, sanct_t = 0, e_{t+1} = 0|e_t = 0) = p_{sick}(1 - p_{vr}) \]
\[ P(sick_t = 1, vr_t = 1, sanct_t = 0, e_{t+1} = 0|e_t = 0) = p_{vr}(p_{sick} + (1 - p_{sick})F_{vr}(\overline{w}_{jo})p_{doc}) \]
\[ P(sick_t = 0, vr_t = 1, sanct_t = 1, e_{t+1} = 0|e_t = 0) = (1 - p_{sick})p_{vr}(1 - p_{doc})\lambda_{vr}F_{vr}(\overline{w}_{vr})p_{sanc} \]
\[ P(sick_t = 0, vr_t = 1, sanct_t = 0, e_{t+1} = 0|e_t = 0) = (1 - p_{sick})p_{vr}(\lambda_{vr}F_{vr}(\overline{w}_{vr})(1 - p_{doc}) \cdot (1 - p_{sanc}) + (1 - \lambda_{vr})(1 - p_{doc}F_{vr}(\overline{w}_{jo})) \]

It is straightforward to show that under this system of equations the left hand side empirical moments uniquely determine the right hand side structural model parameters. For estimation we use a slightly richer model specification that additionally accounts for parameter heterogeneity and measurement error in observed wages. We estimate the model by maximum likelihood, i.e. we fit the whole joint distribution of the observed variables. The above empirical moments coincide with a subset of the likelihood contributions that show up in the likelihood function.

### 2.6 Estimation Results

This section presents our parameter estimates. We provide estimates for a basic empirical specification that does not account for parameter heterogeneity and a full specification that does include both observed and unobserved parameter heterogeneity.\(^\text{14}\) Table 2.1 presents parameter estimates for the basic specification without heterogeneity, while Table 2.2 presents estimates for the full specification. For ease of interpretation of the estimation results with heterogeneity Tables B.2 and B.3 further provide the implied mean structural parameter

\(^\text{14}\)To be precise the basic specification accounts for heterogeneity in unemployment benefit levels, but does not include heterogeneity in any of the structural model parameters. The full specification accounts for both, parameter heterogeneity and heterogeneity in unemployment benefits.
values and the implied parameter point estimates for individuals of median age (38 years) and for each combination of $X_i$ and $\nu$.

A first thing that is notable from the parameter estimates in Table 2.1 that both mean and variance for the VR wage offer distribution are lower relative to the wage offer distribution of regular job offers, indicating that job offers obtained through VRs are on average less attractive and less dispersed than regular job offers. Another regularity across specifications is that the offer rate for regular job offers is generally lower than the VR arrival rate ($p_{jo} < p_{vr}$).

Note however that VR-recipients may still be rejected by the prospective employer (as is reflected by $\lambda_{vr} < 1$) and hence the rate at which VRs effectively yield job offers is lower than $p_{vr}$. The small estimate of the measurement error variance across both specifications suggests that measurement error in wages plays a limited role.

In the full specification the impact of age, health restrictions and apprenticeship training on each model parameter is significantly different from zero, indicating that it is relevant to account for observed heterogeneity. For all parameters that additionally include the unobserved factor $\nu$, the estimated impact of the latent factor is significantly different from zero and sizable. This implies that unobserved heterogeneity contributes significantly to the variation in $p_{doc}$ and $p_{sanc}$ across the sampled population. As observed and unobserved heterogeneity thus seem to play a significant role, we focus on the full empirical specification that includes parameter heterogeneity in the further analysis.

**Model Fit**

To assess how well our estimated model fits the observed data Table 2.3 contrasts data moments with the corresponding predicted values from the estimated model. Our model generally fits the data well, but somewhat overstates the rate at which workers are sanctioned, although both model and data predict a very low rate below 2%.

As further evidence on the model fit Figure 2.1 presents kernel estimates of the densities of observed and simulated accepted wages for regular job offers and VRs respectively. The graphs show that our model fits the distribution of accepted wages well.
Table 2.1: Parameter estimates, basic specification.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E(w_{jo}) )</td>
<td>2006</td>
<td>2.73</td>
</tr>
<tr>
<td>( E(w_{vr}) )</td>
<td>1948</td>
<td>3.45</td>
</tr>
<tr>
<td>( Sd(w_{jo}) )</td>
<td>161</td>
<td>0.63</td>
</tr>
<tr>
<td>( Sd(w_{vr}) )</td>
<td>555</td>
<td>2.94</td>
</tr>
<tr>
<td>( p_{jo} )</td>
<td>0.04</td>
<td>( 0.05 \times 10^{-3} )</td>
</tr>
<tr>
<td>( p_{vr} )</td>
<td>0.12</td>
<td>( 0.17 \times 10^{-3} )</td>
</tr>
<tr>
<td>( p_{sick} )</td>
<td>0.02</td>
<td>( 0.08 \times 10^{-3} )</td>
</tr>
<tr>
<td>( p_{doc} )</td>
<td>0.04</td>
<td>( 1.23 \times 10^{-3} )</td>
</tr>
<tr>
<td>( p_{sanc} )</td>
<td>0.31</td>
<td>( 0.35 \times 10^{-3} )</td>
</tr>
<tr>
<td>( \lambda_{vr} )</td>
<td>0.15</td>
<td>( 0.29 \times 10^{-3} )</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.05</td>
<td>( 0.29 \times 10^{-3} )</td>
</tr>
<tr>
<td>( \sigma_{\epsilon} )</td>
<td>0.30</td>
<td>( 0.04 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

*Notes:* Standard errors are computed using the outer product of the score.

Figure 2.1: Model fit: accepted wages

Panel A: Regular job offers

Panel B: VRs

*Notes:* Monthly accepted wages in Euro plotted separately for jobs taken up in a month in which a VR occurred (panel B)/ no VR occurred (panel A). All curves are smoothed using a normal kernel and a bandwidth of 250 (Euros).
Table 2.2: Parameter estimates, full specification

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>SE ·10^4</th>
<th>Parameter</th>
<th>Estimate</th>
<th>SE ·10^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(w_{jo})$:</td>
<td></td>
<td></td>
<td>$E(w_{vr})$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>7.384</td>
<td>0.011</td>
<td>Intercept</td>
<td>7.251</td>
<td>0.050</td>
</tr>
<tr>
<td>Age (divided by 10)</td>
<td>0.018</td>
<td>0.016</td>
<td>Age (divided by 10)</td>
<td>0.021</td>
<td>0.024</td>
</tr>
<tr>
<td>Apprenticeship</td>
<td>0.067</td>
<td>0.004</td>
<td>Apprenticeship</td>
<td>0.055</td>
<td>0.006</td>
</tr>
<tr>
<td>Health restrictions</td>
<td>-0.025</td>
<td>0.010</td>
<td>Health restrictions</td>
<td>-0.097</td>
<td>0.013</td>
</tr>
<tr>
<td>$Sd(w_{jo})$:</td>
<td>6.222</td>
<td>0.018</td>
<td>$Sd(w_{vr})$:</td>
<td>6.519</td>
<td>0.018</td>
</tr>
<tr>
<td>$\sigma_{e}$:</td>
<td>-1.545</td>
<td>0.167</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{jo}$:</td>
<td></td>
<td></td>
<td>$p_{vr}$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.574</td>
<td>0.042</td>
<td>Intercept</td>
<td>-1.371</td>
<td>0.062</td>
</tr>
<tr>
<td>Age (divided by 10)</td>
<td>0.018</td>
<td>0.007</td>
<td>Age (divided by 10)</td>
<td>-0.153</td>
<td>0.016</td>
</tr>
<tr>
<td>Apprenticeship</td>
<td>0.059</td>
<td>0.043</td>
<td>Apprenticeship</td>
<td>0.392</td>
<td>0.034</td>
</tr>
<tr>
<td>Health restrictions</td>
<td>-0.014</td>
<td>0.047</td>
<td>Health restrictions</td>
<td>-0.225</td>
<td>0.042</td>
</tr>
<tr>
<td>$p_{sick}$:</td>
<td></td>
<td></td>
<td>$\lambda_{vr}$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-4.831</td>
<td>0.147</td>
<td>Intercept</td>
<td>-1.643</td>
<td>0.056</td>
</tr>
<tr>
<td>Age (divided by 10)</td>
<td>0.180</td>
<td>0.035</td>
<td>Age (divided by 10)</td>
<td>-0.020</td>
<td>0.129</td>
</tr>
<tr>
<td>Apprenticeship</td>
<td>0.085</td>
<td>0.082</td>
<td>Apprenticeship</td>
<td>0.069</td>
<td>0.029</td>
</tr>
<tr>
<td>Health restrictions</td>
<td>0.331</td>
<td>0.079</td>
<td>Health restrictions</td>
<td>-0.056</td>
<td>0.111</td>
</tr>
<tr>
<td>$\delta$:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.632</td>
<td>0.032</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>-0.005</td>
<td>0.007</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apprenticeship</td>
<td>-0.036</td>
<td>0.032</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Health restrictions</td>
<td>0.029</td>
<td>0.034</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{sanc}$:</td>
<td></td>
<td></td>
<td>$p_{doc}$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-1.363</td>
<td>0.059</td>
<td>Intercept</td>
<td>-3.857</td>
<td>0.260</td>
</tr>
<tr>
<td>Age (divided by 10)</td>
<td>0.098</td>
<td>0.015</td>
<td>Age (divided by 10)</td>
<td>0.021</td>
<td>0.115</td>
</tr>
<tr>
<td>Apprenticeship</td>
<td>-0.131</td>
<td>0.052</td>
<td>Apprenticeship</td>
<td>-0.231</td>
<td>0.261</td>
</tr>
<tr>
<td>Health restrictions</td>
<td>0.181</td>
<td>0.071</td>
<td>Health restrictions</td>
<td>-0.194</td>
<td>0.472</td>
</tr>
<tr>
<td>$\gamma_{sanc}$:</td>
<td>1.028</td>
<td>0.036</td>
<td>$\gamma_{doc}$:</td>
<td>0.942</td>
<td>0.195</td>
</tr>
<tr>
<td>$\nu$:</td>
<td></td>
<td></td>
<td>$\pi$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_1$</td>
<td>3.554</td>
<td>0.057</td>
<td>$\pi_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_2$</td>
<td>-0.934</td>
<td>0.124</td>
<td>$\pi_2$</td>
<td>0.435</td>
<td>2.489</td>
</tr>
<tr>
<td>$v_3$</td>
<td>0.399</td>
<td>0.047</td>
<td>$\pi_3$</td>
<td>0.508</td>
<td>1.539</td>
</tr>
</tbody>
</table>

Notes: Standard errors are computed using the outer product of the score.
Table 2.3: Model fit

<table>
<thead>
<tr>
<th>Months starting in unemployment</th>
<th>Data</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>VR</td>
<td>13.15%</td>
<td>11.67%</td>
</tr>
<tr>
<td>Sickness absence (≥ 2 weeks)</td>
<td>2.00%</td>
<td>2.05%</td>
</tr>
<tr>
<td>Job take-up</td>
<td>4.95%</td>
<td>5.02%</td>
</tr>
<tr>
<td>Avg. accepted wage</td>
<td>2130</td>
<td>2195</td>
</tr>
</tbody>
</table>

| VR received                                          |            |            |
| Sickness absence (≥ 2 weeks)                         | 3.34%      | 3.57%      |
| Job take-up                                          | 14.57%     | 14.39%     |
| Sanction                                             | 0.33%      | 1.39%      |

Notes: Data moments are computed from a sample of 69,788 workers observed from 2000 to 2002. The sample restrictions described in 2.3 apply. Model moments are computed from simulations draws for 20,000 workers. We simulate histories of 108 time periods (months) for each worker.

Implied Reservation Wages

As a key implication the estimated model yields reservation wages for each agent type. These reservation wages are dependent on the number of remaining sanction periods $s$ and number of past sanctions $ps$ of an individual. In Figure 2.2 we present reservation wages for regular job offers and job offers obtained through VRs as function of $(s, ps)$ for different agent types. Recall that for the given institutional setting $s \in \{0, 1, 2, 3\}$ and $ps \in \{0, 1\}$ and thus $ps = 1$ if $s > 0$. Figure 2.2 shows that for individuals who have never been sanctioned the reservation wage for regular job offers is only slightly higher than the reservation wage for job offers obtained through VRs. In contrast for individuals who have previously been sanctioned there is a persisting positive gap between $w_{jo}$ and $w_{vr}$, because facing the risk of receiving a terminal sanction makes these individuals accept much lower wage offers for job offers obtained through VRs than for regular job offers.
Figure 2.2: Implied reservation wages, $h$: health restrictions, $a$: apprenticeship

Panel A: $h = 0$, $a = 0$

Panel B: $h = 0$, $a = 1$

Panel C: $h = 1$, $a = 0$

Panel D: $h = 1$, $a = 1$

Notes: Reservation wages by $(s, ps)$ plotted separately for jobs taken up in a month with/without a VR. Plotted are reservation wages for agents with median benefit level (1000 Euro) and of median age (38) and for the intermediate unobserved type $\nu$. Each of the plots corresponds to a different observable type in terms of health restrictions ($h$) and apprenticeship training ($a$).
2.7 Policy Simulations

In this section we use the estimated structural model to study how counterfactual policy changes impact job search outcomes and sick reporting. In particular we simulate two types of policy changes: First, we examine changes in sanction enforcement. Increasing sanction enforcement corresponds to instructing caseworkers use their discretionary leeway less and sanction unemployed workers who do not apply for VRs (or reject resulting job offers) more frequently. In our model changes in sanction enforcement are simulated by varying $p_{\text{sanc}}$. Second, we consider changes in the vacancy referral rate $p_{\text{vr}}$, i.e., we study what would change if vacancy referrals were sent out at higher rates. For each counterfactual policy change we examine effects on job finding rates, average unemployment duration and post-unemployment wages and the rate at which unemployed workers receive sanctions.

Varying Sanction Enforcement  Table 2.4 displays results on the impact of changing sanction enforcement on job search outcomes. We consider two extreme policy scenarios, one in which we abandon sanctions altogether ($p_{\text{sanc}} = 0$) and one in which we move to perfect sanction enforcement with zero discretion for caseworkers ($p_{\text{sanc}} = 1$). Furthermore we examine three intermediate scenarios in which sanction enforcement is doubled, tripled and quadrupled.\footnote{Recall that sanction enforcement at baseline on average is very low. Yet, for a fraction of the population doubling, tripling and quadrupling $p_{\text{sanc}}$ yields values greater than 1. For these individuals we fix counterfactual sanction enforcement at 1 (full enforcement).} The results presented in Table 2.4 show that increasing sanction enforcement leads to an increase in the overall job finding rate and correspondingly reduces average unemployment duration. Quantitatively, quadrupling the sanction enforcement rate $p_{\text{sanc}}$ leads to a 5% increase in the overall job finding rate, to an 11% increase in the job finding rate in months when a VR was received and to a reduction in average unemployment duration by 0.2 months (around 6 days). In comparison, moving to full sanction enforcement ($p_{\text{sanc}} = 1$) would reduce the average unemployment duration by 0.6 months (around 2 weeks and 4 days). Table 2.4 further shows that the average accepted wage falls slightly, by 1.3% (30 Euro), as sanction enforcement is quadrupled.

To shed light on the underlying mechanism we examine how reservation wages respond to the considered changes in sanction enforcement. Intuitively, in response to an increase in the risk of receiving a sanction in the future, unemployed workers are willing to accept
Table 2.4: Changing sanction enforcement, simulation results

<table>
<thead>
<tr>
<th>$p_{sanc}$</th>
<th>Unemployed</th>
<th>VR received</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Job take-up</td>
<td>Job take-up</td>
</tr>
<tr>
<td>$0$</td>
<td>4.97%</td>
<td>13.91%</td>
</tr>
<tr>
<td>$p_{sanc}$</td>
<td>5.02%</td>
<td>14.39%</td>
</tr>
<tr>
<td>$2p_{sanc}$</td>
<td>5.05%</td>
<td>15.03%</td>
</tr>
<tr>
<td>$3p_{sanc}$</td>
<td>5.15%</td>
<td>15.60%</td>
</tr>
<tr>
<td>$4p_{sanc}$</td>
<td>5.27%</td>
<td>16.09%</td>
</tr>
<tr>
<td>$1$</td>
<td>5.60%</td>
<td>18.33%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Avg. accepted wage</th>
<th>Avg. unemp. duration (months)</th>
<th>VR received</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2208</td>
<td>9.90</td>
<td>Job take-up</td>
</tr>
<tr>
<td>$p_{sanc}$</td>
<td>2195</td>
<td>9.88</td>
<td>13.91%</td>
</tr>
<tr>
<td>$2p_{sanc}$</td>
<td>2187</td>
<td>9.81</td>
<td>14.39%</td>
</tr>
<tr>
<td>$3p_{sanc}$</td>
<td>2175</td>
<td>9.77</td>
<td>15.03%</td>
</tr>
<tr>
<td>$4p_{sanc}$</td>
<td>2166</td>
<td>9.67</td>
<td>15.60%</td>
</tr>
<tr>
<td>$1$</td>
<td>2131</td>
<td>9.28</td>
<td>16.09%</td>
</tr>
</tbody>
</table>

By this mechanism, job finding rates increase and the distribution of accepted wages receives more mass at its lower end, leading to a reduction in average accepted wages in response to increases in sanction enforcement. Figure 2.3 displays the magnitude by which unemployed workers of the modal type in the considered population adjust their reservation wages when sanction enforcement is quadrupled. Figure 2.3 shows that quadrupling sanction enforcement leads to a modest reduction in reservation wages for regular wage offers ($\overline{w}_{jo}$) and a strong reduction in reservation wages for offers obtained through VRs ($\overline{w}_{vr}$). The drop in $\overline{w}_{vr}$ in response to quadrupling sanction enforcement is more pronounced for job searchers who have received a sanction in the past ($ps = 1$) and who thus would receive a terminal sanction if they were to be sanctioned again.

Figure 2.3: Increasing sanction enforcement, reservation wages

Panel A: Regular job offers

Panel B: VRs
Varying the Vacancy Referral Rate  Next, we consider changes in the VR rate $p_{vr}$. Table 2.5 presents outcomes for counterfactual experiments in which the VR rate is increased and decreased by 25% and 50%, respectively. Table 2.5 shows that in the considered experiments increasing the VR rate elevates the overall job finding rate, but decreases the job finding rate in months when a VR was received. Quantitatively, increasing the VR rate by a factor of 1.25 leads to a reduction in average unemployment duration by 0.32 months. The overall job finding rate rises by 6%, while the job finding rate for months when a VR was received falls by 3% as the VR rate is increased by a factor of 1.25.

**Table 2.5:** Changing the VR rate, simulation results

<table>
<thead>
<tr>
<th>$p_{vr}$</th>
<th>0.5$p_{vr}$</th>
<th>0.75$p_{vr}$</th>
<th>$p_{vr}$</th>
<th>1.25$p_{vr}$</th>
<th>1.5$p_{vr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unemployed</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VR</td>
<td>5.84%</td>
<td>8.75%</td>
<td>11.67%</td>
<td>14.62%</td>
<td>17.61%</td>
</tr>
<tr>
<td>Job take-up</td>
<td>4.39%</td>
<td>4.68%</td>
<td>5.02%</td>
<td>5.32%</td>
<td>5.62%</td>
</tr>
<tr>
<td>Avg. accepted wage</td>
<td>2168</td>
<td>2184</td>
<td>2195</td>
<td>2213</td>
<td>2235</td>
</tr>
<tr>
<td>Avg. unemp. duration (months)</td>
<td>10.59</td>
<td>10.21</td>
<td>9.88</td>
<td>9.56</td>
<td>9.16</td>
</tr>
<tr>
<td><strong>VR received</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job take-up</td>
<td>15.68%</td>
<td>14.83%</td>
<td>14.39%</td>
<td>14.06%</td>
<td>13.94%</td>
</tr>
<tr>
<td>Sanction</td>
<td>1.25%</td>
<td>1.35%</td>
<td>1.39%</td>
<td>1.45%</td>
<td>1.45%</td>
</tr>
</tbody>
</table>

It may at first seem surprising that VRs elevate overall job finding while reducing job finding in months when a VR is received. The reason is that VRs have two counteracting effects on job search behavior. On the one hand as the VR rate is increased the risk of receiving a sanction in the future increases. This decreases the option value of search and thus pushes towards lower reservation wages. On the other hand higher VR-rates increase the amount of job offers that unemployed workers can expect to sample in the future. This increases the option value of search and as a consequence pushes towards higher reservation wages. To examine which of these two opposing forces dominates, we examine the impact of increasing the VR rate on reservation wages.

Looking at the whole population we find that in the considered parameter range increasing the VR rate generally pushes towards higher reservation wages, both for regular job offers and for job offers obtained through VRs, and for all considered agent types. As the VR rate is increased unemployed workers thus become generally more selective about the range of job offers they are willing to accept, i.e., moral hazard increases when the VR rate is raised.
Figure 2.4: Sending more VRs, reservation wages

Panel A: Regular job offers

Panel B: VRs

Figure 2.4 displays reservation wages for the modal type in the population (in terms of observed heterogeneity $X$, and unobserved heterogeneity $ν$). The figure shows that reservation wages rise as the VR rate is increased, meaning that the force pushing towards a higher option value of search, because more job offers are sampled, dominates. This rise in reservation wages in response to a higher VR rate is more pronounced for for regular job offers than for VRs.

The fact that increasing the VR rate leads to higher reservation wages explains the declining job finding rates in months when a VR as $p_{vr}$ is increased: unemployed job searchers reject resulting job offers more often as they can expect to sample more job offers in the future, i.e., moral hazard increases. At the same time, despite higher moral hazard, the overall job finding rate increases, when $p_{vr}$ is increased. This is because of the mechanical effect that ceteris paribus more VRs (and more resulting job offers) lead to more transitions into employment. This mechanical effect overrides the decline in the rate at which job offers that are accepted.

**Eliminating VR Induced Sick Reporting** We next turn to examining the extent to which unemployed job searchers call in sick to circumvent VRs and by how much this affects job search outcomes. Our model allows to decompose the observed sickness absence rate into a baseline sick rate and a VR induced sick rate. Conditional on agent type (i.e. conditional on $X_i$ and $ν$) the overall probability to report sick for a particular individual in a given period
equals

\[
P( \text{sick report} \mid X_i, \nu) = p_{\text{sick}} + P( \text{VR induced sick report} \mid X_i, \nu) \\
= p_{\text{sick}} + (1 - p_{\text{sick}})p_{\text{vr}}F_{\text{vr}}(w_{j0})p_{\text{doc}}
\]

(2.8)

where all right hand side parameter values are implicitly conditioned on \(X_i\) and \(\nu\).

Looking at the overall unemployed population we find that VR induced sick reporting accounts for a substantial share of overall sick reporting. In particular

\[
\frac{P( \text{VR induced sick report})}{P( \text{sick report})} = \frac{\sum_{m=1}^{M} \sum_{x \in X} \pi_m P(X_i = x) P( \text{VR induced sick report} \mid X_i, \nu = \nu_m)}{\sum_{m=1}^{M} \sum_{x \in X} \pi_m P(X_i = x) P( \text{sick report} \mid X_i, \nu = \nu_m)} = 9.6\%,
\]

(2.9)

i.e., according to our estimated model 9.6% of all observed sick reports among unemployed individuals occur because individuals try to circumvent a VR. This number corresponds closely to the empirical finding of van den Berg et al. (2019b), who find that 9% of all sick reports are VR induced.

VR induced sick reporting hence accounts for a substantial share of overall sick reporting. In order to quantify to what extent VR induced sick reporting affects job search behavior we simulate a counterfactual scenario in which only individuals who are actually sick can obtain a sick note, i.e., in which VR induced sick reporting is completely shut down \(p_{\text{doc}} = 0\). While this counterfactual change does not immediately relate to a real world policy measure, this scenario can be interpreted as medical doctors becoming perfect in screening out individuals who ask for a sick note but in fact are not sick.

Table 2.6 displays sick reporting rates and job search outcomes for the counterfactual scenario in which VR induced sick reporting is shut down. In line with our above calculation based on (2.6) shutting down VR induced sick reporting reduces overall sick reporting by 9.6%. This overall effect is entirely driven by reduced sick reporting in months when a VR is received. As Table 2.6 shows sick reporting in these months by 9%, when VR induced sick
reporting is eliminated.

**Table 2.6: Eliminating VR induced sick reporting**

<table>
<thead>
<tr>
<th>$p_{doc}$</th>
<th>$p_{doc}$</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unemployed</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sickness absence ($\geq 2$ weeks)</td>
<td>2.05%</td>
<td>1.87%</td>
</tr>
<tr>
<td>Job take-up</td>
<td>5.02%</td>
<td>5.05%</td>
</tr>
<tr>
<td>Avg. accepted wage</td>
<td>2195</td>
<td>2188</td>
</tr>
<tr>
<td>Avg. unemp. duration (months)</td>
<td>9.88</td>
<td>9.82</td>
</tr>
<tr>
<td><strong>VR received</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sickness absence</td>
<td>3.57%</td>
<td>1.84%</td>
</tr>
<tr>
<td>Job take-up</td>
<td>14.39%</td>
<td>14.67%</td>
</tr>
<tr>
<td>Sanction</td>
<td>1.39%</td>
<td>1.56%</td>
</tr>
</tbody>
</table>

Despite the sizable effects on sick reporting we find that, shutting down VR induced sick reporting has only modest effects on job search outcomes, when looking at averages across the whole population. In periods when a VR is received we find a very modest increase in the job finding rate by 0.6% which translates into a slight reduction in average unemployment duration by 0.2 months (around 6 days).

Albeit average effects of eliminating VR induced sick reporting on job search outcomes are small, the magnitude of these effects strongly varies across individuals and is sizable for some subgroups in the considered population. To illustrate this we repeat our analysis, focusing only on the 20% subpopulation, with the highest probability of obtaining a sick note (i.e., the type with the highest $p_{doc}$).

Counterfactual outcomes for this subpopulation are displayed in Table 2.7. The presented results show that for this unobserved type VR induced sick reporting constitutes a large share of overall sick reporting. As a consequence for this type eliminating VR induced sick reporting does have a sizable impact on job search outcomes. In particular, shutting down VR induced sick reporting leads to a reduction by a factor of 2.1 in overall sick reporting and to a reduction by a factor of 9.1 in sick reporting in months when a VR was received. In terms of job search outcomes eliminating VR induced sick reporting leads to a 9.4% increase in the overall job finding rate, which corresponds to a 0.8 months reduction in average unemployment duration, and an 13.6% increase in the job finding rate in months when a VR was received. What is more, as job searchers lose the possibility of circumventing VRs by
reporting sick, the number of individuals who receive a sanction increases. Quantitatively the fraction of individuals who are sanctioned given they received a VR increases by a factor of 1.45.

Table 2.7: Eliminating VR induced sick reporting, $p_{doc}$ top 20%

<table>
<thead>
<tr>
<th>$\bar{p}_{doc}$</th>
<th>$p_{doc}$</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unemployed</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sickness absence (≥ 2 weeks)</td>
<td>3.81%</td>
<td>1.79%</td>
</tr>
<tr>
<td>Job take-up</td>
<td>5.13%</td>
<td>5.61%</td>
</tr>
<tr>
<td>Avg. unemp. duration (months)</td>
<td>9.98</td>
<td>9.21</td>
</tr>
<tr>
<td>Avg. accepted wage</td>
<td>2171</td>
<td>2118</td>
</tr>
<tr>
<td><strong>VR received</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sickness absence</td>
<td>18.60%</td>
<td>2.04%</td>
</tr>
<tr>
<td>Job take-up</td>
<td>16.08%</td>
<td>18.26%</td>
</tr>
<tr>
<td>Sanction</td>
<td>4.55%</td>
<td>6.58%</td>
</tr>
</tbody>
</table>

2.8 Conclusion

In this paper we study VRs and punitive sanctions, accounting for the possibility that workers may strategically report sick to avoid sanctions. We develop and estimate a structural job search model in which unemployed workers are forward looking and adjust their search behavior to receiving VRs or sanctions. Upon receiving a low wage VR, unemployed workers may rationally seek to get a sick note from their doctor to circumvent receiving a sanction.

We study a variety of counterfactual policy changes. We find that increasing sanction enforcement reduces moral hazard and considerably increases job finding rates. The underlying model mechanism is that unemployed workers decrease their reservation wages in response to facing an increased sanction risk. In contrast, increasing the VR rate increases moral hazard. Increasing the VR rate incentivizes unemployed workers to raise their reservation wages. Nevertheless, a higher VR rate leads to higher job finding rates, as the higher frequency at which VRs arrive mechanically leads to higher job take-up, even at increased reservation wages.

We find that VR induced sick reporting accounts for a substantial share of overall sick reporting. According to our estimated model, 9.6% of all observed sick reports are induced by VRs. Looking at averages across the whole worker population we find modest effects of
shutting down VR induced sick reporting on job search outcomes. However, there is substantial heterogeneity in the population. For the 20\% of workers with the highest propensity of VR induced sick reporting we find that eliminating VR induced sick reporting would reduce the mean unemployment duration by 0.8 months.

The structural framework used in this chapter abstracts from equilibrium effects of VRs and sanctions. It can be conjectured that in reality some of the policies we study may have considerable equilibrium effects. Especially increasing the VR rate may crowd out other workers who applied for the referred vacancies. Moreover firms may respond in their wage setting and vacancy posting behavior to policy changes related to VRs and sanctions. The next chapter develops a search and matching model that accounts for equilibrium effects of VRs to address this topic.
Chapter 3

The Equilibrium Effects of Vacancy Referrals

3.1 Introduction

The overarching motivation for using VRs as part of active labor market policy (ALMP) is bringing together unemployed workers and firms who otherwise would not have matched. The previous chapter provided an analysis of how VRs and sanctions impact individual level job search outcomes. An open question in evaluating VRs is to what extent their impact is reduced by equilibrium effects.\textsuperscript{1} In this chapter we study three channels by which we conjecture equilibrium effects to play a role. First, we ask to what extent assigning VRs to a group of unemployed workers decreases the chances to exit unemployment for other unemployed workers. Second, we consider how unemployed workers’ own job search effort may be affected by VRs. Third, we study to what degree firms vacancy posting behavior is affected by VRs.

The existing empirical evidence suggests that VRs have a positive impact on the job finding probabilities of VR recipients (see, e.g., Bollens and Cockx (2017), van den Berg et al. (2019b)).\textsuperscript{2} Further evidence is provided by Fougère et al. (2009), who estimate a partial

\textsuperscript{1}See Abbring and Heckman (2007) for a survey of the literature on general equilibrium evaluation of ALMPs.

\textsuperscript{2}Bollens and Cockx (2017) find that in Flanders VRs have strongly elevate job finding probabilities, both in the short and the long run. Using German data, find that VRs are associated with higher job finding probabilities. Using data from Sweden, Engström et al. (2012) in contrast, find that only a third of all VRs result in job applications, but that intensifying monitoring increases the fraction of VRs that result in job applications.
equilibrium job search model to investigate the extent to which VRs disincentivize job search effort. They find moderate disincentive effects, but show that the net employment effect of VRs, despite disincentive effects, is positive and large in magnitude.

Our goal in this paper is to investigate to what extent the documented positive effects of VRs on individual job finding probabilities translate into an increase in the economy wide employment rate. To this end we develop and estimate a dynamic economic model that accounts for various channels through which equilibrium effects may operate.

In reality unemployed workers send out multiple applications, but typically only search for one job. Similarly firms face a queue of multiple applications for a given job opening. When a worker receives a VR and applies, one application is added to the queue of applications the firm faces for this vacancy. This will typically reduce the chances to get hired for all other applicants. The net impact of one VR on the economywide employment rate may be zero, if the VR crowds out one other applicant who as a consequence remains unmatched. At the other extreme, a VR may lead to the filling of one vacancy that otherwise would have stayed empty, either because no other workers applied or because crowded out applicants find employment elsewhere. Our structural model reflects this reasoning and allows us to quantify crowding out effects. Furthermore, our model accounts for changes in firms’ vacancy posting behavior in response to VRs and for disincentive effects of VRs on job search effort, as in Fougère et al. (2009).

Specifically we develop a dynamic search and matching model in which unemployed workers and firms match at random according to an urn-ball matching technology (see, e.g., Rogerson et al. (2005) and Petrongolo and Pissarides (2001)). Workers send out multiple applications to (random) firms, and each firm hires one worker from its applicant pool, if it received at least one suitable application.3 In each time period each unemployed worker in our model chooses how many applications to send out to firms and on top may receive a VR from the PES. A VR increases the job finding probability of the unemployed worker who receives it, but reduces the chances to be hired for all other unemployed workers who applied for the vacancy. The magnitude of crowding out depends on the tightness of the labor market and on the number of applications that the VR recipient and other workers who applied for the vacancy sent out.

3Our model builds on the search and matching framework with multiple applications developed in Albrecht et al. (2004), Albrecht et al. (2006) and Gautier et al. (2016).
The model is estimated by generalized method of moments estimation, targeting data moments from German survey micro-data that contain information on unemployed workers job search behavior and post-unemployment outcomes as well as data on firm hiring decisions. The worker data that we use, among other things, are informative about the number of applications and VRs for a 2008 inflow sample into unemployment. The used firm data contain information on the number of hires and canceled searches in the last year, as well as the applicant pool for the last job opening at the surveyed firm, including how many candidates were referred through a VR and whether the hired candidate was one of them.

We use the estimated model to conduct counterfactual policy experiments in which the VR rate, i.e., the mass of workers receiving a VR in a given time period, is altered. Based on these policy experiments we study the impact of VRs on the economywide employment rate and analyze the magnitude of equilibrium effects. We use our model to study to what degree a simple randomized experiment, by ignoring equilibrium effects, would overestimate the positive impact of increasing the VR rate on job finding probabilities.

We find that equilibrium effects moderately reduce the effectiveness of VRs, but that VRs, despite equilibrium effects, have a modest positive impact on the economywide employment rate.

At the status quo policy the VR rate is at 58% (i.e., 58% of unemployed workers receive a VR in any given month). We find that increasing the VR rate from 58% to 100% reduces the unemployment rate from 9.8% to 9.6%.

For a single given worker, receiving a VR ceteris paribus elevates the the job finding probability by 3.1 percentage points. A reform that increases the VR rate from its status quo value to 80% increases the average job finding rate by 2.8 percentage points, a considerably smaller effect than the individual level effect. The equilibrium effects accountable for the discrepancy between the reform effect and the individual level effect are crowding out in the hiring process as well as changes in firms’ vacancy posting behavior. Accordingly, a simple randomized experiment would overstate the effect of the reform by a factor of 1.14.

The contribution of this chapter is twofold. First, we develop a structural model of the labor market that includes VRs and accounts for three main channels by which equilibrium effects may operate, crowding out effects in the hiring process, disincentivized job search effort and firms changing their vacancy posting behavior. Second, we estimate our model using worker and firm data and use the estimated model to quantify the discrepancy between
individual level effects of VRs, that have been highlighted in previous empirical studies, and
the aggregate effect of VRs on the economywide employment rate.

We thereby contribute to a small set of papers that have studied the equilibrium effects of
ALMPs. In particular, Crépon et al. (2013) and Gautier et al. (2018) have provided empirical
evidence on the equilibrium effects of ALMPs. Crépon et al. (2013) interpret their empirical
findings through the lens of a stylized search and matching model, in which equilibrium
effects arise exclusively because firms change their vacancy posting behavior in response to
ALMP changes. Gautier et al. (2018) estimate a structural model to analyze counterfactual
policy scenarios in which search costs are reduced.

Relative to these previous papers we provide the first study of the equilibrium effects of
VRs and of the underlying channels through which equilibrium effects operate.

The remainder of this paper proceeds as follows. Section 3.2 provides a brief description of
the institutional background. Section 3.3 develops the model, section 3.4 describes the data
and section 3.5 the model estimation. Section 3.6 provides results from policy simulations
and section 3.7 concludes.

3.2 Institutional Background

In most OECD countries recipients of unemployment insurance benefits are subject to a range
of job search monitoring and job search assistance practices (see Grubb (2001)). VRs are one
of the most important of these practices (see Immervoll and Knotz (2018a)). By making use
of VRs unemployment insurance agencies seek to support individual benefit recipients’ own
job search effort, but also to activate benefit recipients whose own search effort is deemed
insufficient.

In Germany UI benefit recipients may receive VRs starting immediately after they start
claiming benefits. Caseworkers at the public employment services (Bundesagentur für Arbeit)
select job vacancies from the agency’s database and matches them to UI benefit recipients.5
The selected benefit recipients are then notified by regular mail about the vacancy, are
typically given a brief description about the job opening and are asked to apply for the

4In these papers the considered ALMPs subsume job search assistance, training measures as well as VRs.
5The same vacancy may be referred to several UI benefit recipients and benefit recipients sometimes
receive several VRs at the same time.
referred position(s) (this description typically does not include the wage).\textsuperscript{6} Not applying for a referred vacancy or rejecting a resulting job offer may lead to sanction (a temporary benefit reduction), although enforcement of such sanctions is infrequent (see van den Berg et al. (2019a)).

There are two standard procedures by which job openings may enter as vacancies into the database of the PES. Firms may directly contact the PES and request that a specific vacancy is used for making VRs. Moreover, to find suitable vacancies caseworkers browse public job advertisements such as newspaper- and online-ads, i.e., in principal most publicly advertised job openings may end up being used by the PES for making VRs. There are two restrictions that caseworkers need to abide when making a VR. The job has to be within 2.5 hours commuting distance and has to pay at least the same amount that the benefit recipient receives in UI benefits. There are no other restrictions on the type of jobs that a UI benefit recipient may receive as VRs. In particular job openings referred as VRs do not need to match the pre-unemployment occupation of the UI benefit recipient who is assigned the vacancy.

\textbf{3.3 Model}

Our model is a dynamic search and matching model in which unemployed workers in each time period send out multiple applications, which are received by firms. Each time period each firm makes one job offer to one worker in their applicant pool. They decide which wage to offer to this worker and if the worker accepts a match is created and the worker starts producing at this firm. If the worker rejects the wage offer, either because he received a preferable offer from another firm or because staying unemployed is preferable to him, the firms position remains vacant for one time period. VRs are applications made by the unemployment agency on behalf of individual workers. There are several equilibrium effects and margins of crowding out that our model accounts for. First, A VR ceteris paribus increases the probability of being matched for the worker who receives the VR, but incurs a negative externality by decreasing the likelihood of being matched for all other unemployed workers. Second, VRs may crowd out individual job search effort, i.e., upon receiving a VR an

\textsuperscript{6}A VR typically contains information about the occupation, the working hours and the date of potential job start (see van den Berg et al. (2019b)).
unemployed worker may decide to write less job applications. Third, there is free entry and firm entry and wages may respond to the number of applications and VRs in the economy.

Formally, our model is set in discrete time and the time horizon is infinite. There is a unit mass of workers in the economy and a measure of vacancies, which is endogenously determined. All agents have linear preferences and discount the future at rate \( \beta \). Firms are homogeneous and workers are heterogeneous in their costs of writing applications, but are homogeneous in their productivity, i.e., whenever a firm and a worker are matched they produce a constant per period output \( y \).

**Hiring process** In each time period firms receive multiple applications from different workers. We assume that search is undirected, i.e. each firm is equally likely to receive a given application. Denote by \( p_{\alpha,\iota} \) the population share of workers writing \( \alpha \in \{0, 1, 2, \ldots, A\} \) applications and receiving \( \iota \in \{0, 1, 2, \ldots, J\} \) VRs. The queue length, i.e., the expected total number of applications for a given firm is\(^7\)

\[
\lambda = \frac{u}{v} \sum_{\iota=0}^{J} \sum_{\alpha=0}^{A} (\alpha + \iota) p_{\alpha,\iota} = \frac{u}{v} \sum_{\iota=0}^{J} p_{\iota} \sum_{\alpha=0}^{A} (\alpha + \iota) p_{\alpha|\iota}. \tag{3.1}
\]

As workers are homogeneous in their productivity, firms pick a candidate at random. **Albrecht et al. (2004)** show that in this case the probability that a given application is successful is\(^8\)

\[
\psi = \frac{1}{\lambda} \left(1 - e^{-\lambda}\right).
\]

For a worker who sends out \( \alpha \) applications and receives \( \iota \) VRs the the probability to receive \( k < \alpha + \iota \) job offers then is given by

\[
\mu(k|\alpha + \iota) = \binom{\alpha + \iota}{k} \psi^k (1 - \psi)^{\alpha + \iota - k}.
\]

\(^7\)The total number of applications is the sum of regular applications and VRs that a firm receives.

\(^8\)For a detailed exposition of urn-ball matching with multiple applications see **Albrecht et al. (2006)**.
**Workers**  When unemployed, workers receive unemployment benefits $b$ per period. Each period they choose how many applications to write and upon receiving job offers, whether to accept or reject. The probability of receiving $k$ job offers, $\mu(k|\alpha + \iota)$, depends on the sum of the number of job applications that an unemployed worker sends out, $\alpha$, and the number of VRs he receives, $\iota$.

Upon writing $\alpha$ job applications and receiving $\iota$ VRs the unemployed worker incurs application costs $C(a, \iota) = c \cdot (\alpha + \phi \iota)^2$. $c$, a parameter that scales the application costs, is heterogeneous in the worker population and is redrawn in each time period (i.e. for a given worker $c$ is i.i.d. across time). We assume $0 < \phi < 1$, i.e., applying for VRs is costly, but less costly than applying for jobs that are not referred by the public employment services. The underlying assumption is that applying for VRs is costly to some extent, as time costs and psychic costs of preparing for a job interview are incurred. We argue however that applying for VRs is less costly than applying for non-referred jobs, as the search process of looking through job ads, figuring out which jobs are suitable and deciding where to apply is undertaken by the public employment services.

The Bellman equation for an unemployed worker conditional on the number of applications and VRs, $\alpha$ and $\iota$, and the application cost parameter $c$ is

$$V_U(\alpha | c, \iota) = b - C(\alpha, \iota) + \beta \left( \sum_{k=1}^{\alpha} \mu(k|\alpha, \iota) \int_0^\infty \max\{V_U, V_E(w)\} dF^k(w) + \mu(0|\alpha, \iota)V_U \right)$$

(3.2)

where $F(w)$ is a wage offer distribution that workers take as given. For a given worker the optimal number of applications is

$$\alpha(\iota, c) = \arg\max_\alpha V(\alpha|c, \iota)$$

(3.3)

The unconditional value of unemployment is given by the Bellman Equation that takes into account that workers optimally choose the number of applications they send out and computed prior to the realization of the search cost parameter, i.e., taking expectations over
In terms of timing we assume that in a given time period workers decide how many applications to send out after learning the number of VRs they receive.

Employed workers receive a constant wage \( w \) each time period for the duration of their employment spell and are laid off at rate \( \delta \). The value of employment thus is given by

\[
V_E(w) = w + \beta((1 - \delta)V_E(w) + \delta V_U) \tag{3.5}
\]

It is straightforward to show that \( V_E \) is strictly increasing in \( w \). A workers decision rule in accepting and rejecting job offers thus can be summarized by a reservation wage \( \bar{w} \).

Using (3.5) together with (3.4) and rearranging, yields the reservation wage equation

\[
\bar{w} = b + \beta \max_\alpha \int_0^\infty \int_0^\infty \sum_{\alpha} \mu(k|\alpha + \iota) \int_0^\infty \frac{w - \bar{w}}{1 - \beta(1 - \delta)} dF_k(w) - C(\alpha, \iota) \tag{3.6}
\]

**Firms** Firms in each time period decide whether to post a vacancy and, if they decide to post a vacancy, which wage to post. The value a firm attributes to being matched with a worker and paying wage \( w \) is

\[
V_M(w) = y - w + \beta(1 - \delta)V_M(w) = \frac{y - w}{1 - \beta(1 - \delta)} \tag{3.7}
\]

where \( y \) is the productive value of a match. The value of posting a vacancy is given by

\[
V_F(w) = -k + \beta m_F(w)V_M(w),
\]

where \( k \) is the cost of posting a vacancy and \( m_F(w) \) is the endogenous probability for a given firm to be matched with a worker. Gautier et al. (2016) show that the matching probability for a firm is given by

\[
m_F(w) = \frac{u}{v} \sum_{k=1}^{A+J} kq_k F^{k-1}(w),
\]
where $q_k$ is the population share of unemployed workers receiving $k$ job offers,

$$q_k = \sum_{i=0}^{J} \sum_{\alpha=0}^{A} \mu(k|\alpha, i)p_{\alpha,i}$$

$$= \sum_{i=0}^{J} p_i \sum_{\alpha=0}^{A} \mu(k|\alpha, i)p_{\alpha[i,i]}.$$ (3.8)

We assume free market entry, i.e. it is optimal for new firms to enter the market and post vacancies until the value of posting a vacancy is driven down to zero, i.e., until $V_F(w) = 0$ for all $w$ or equivalently

$$0 = -k + \beta m_F(w) V_M(w)$$

$$= -k + \beta m_F(w) \frac{y - w}{1 - \beta(1 - \delta)},$$ (3.9)

where the second equality is obtained by inserting (3.7). From a firms perspective it is never optimal to post wages below the workers reservation wage $w$. Denote the smallest wage posted with positive probability by $w_{\text{min}}$. This wage equals the reservation wage, i.e., $w_{\text{min}} = \omega$. The firm posting the smallest wage will only attract unemployed workers who receive no other job offer, i.e., $m_F(\omega) = q_1$. Using $V_F(w) = V_F(\omega) = 0$ can be used together with $m_F(\omega) = q_1$ to characterize the wage offer distribution by

$$\sum_{k=1}^{A+J} kq_k F^{k-1}(w)(y - w) = q_1(y - \omega).$$ (3.10)

For given $y$, $q_k$ and $\omega$, (3.10) can be used to solve for the wage offer distribution $F$ point-wise at any $w$.

**Steady State** For the economy to be in steady state, flows into unemployment need to equal flows out of unemployment, i.e.,

$$0 = \delta e - m_W u,$$ (3.11)

so that the steady state unemployment is given by $u = \frac{\delta}{\delta + m_W}$.

**Equilibrium** A steady state equilibrium of the economy is given by a queue length $\lambda$, a policy function, $\alpha(\iota, c)$, a reservation wage $\omega$, a measure of vacancies $v$ and a wage offer
distribution $F$, such that (3.1), (3.3), (3.6), (3.9), (3.10) and (3.11) are satisfied.

**Distribution of Accepted Wages**  In the described setup a closed form relationship between the equilibrium wage offer distribution $F$ and the distribution of accepted wages can be derived. Making use of this relationship will allow us to compute moments from the accepted wage distribution analytically. This will prove useful for the structural estimation of the model as it allows us to target moments of the distribution of accepted wages rather than moments of the (unobserved) wage offer distribution. Denote the distribution of accepted wages by $G(w)$. Then in equilibrium it holds that

$$G(w) = \frac{\sum_{k=1}^{A+J} q_k F^k(w)}{1 - q_0}$$

(see Gautier et al. (2016)). We use this relationship in computing moments of the distribution of accepted wages implied by our model by using $F(w)$ to compute $G(w)$ at a set of approximation points and applying numerical integration techniques to obtain the desired moments.

### 3.4 Data

We estimate our model using a combination of worker and firm data. For worker information we use the IZA Evaluation Dataset, a survey covering a large sample of workers who became unemployed between June 2007 and May 2008 in Germany.\(^9\) More specifically the survey covers a random sample of 17,396 newly unemployed workers aged 16 to 54 who are first surveyed two months after becoming unemployed and are tracked and interviewed repeatedly over time. Importantly for our context the survey asks unemployed workers about the number of applications they send out, the number of vacancy referrals they receive and contains information about unemployment duration and post-unemployment wages for unemployed workers who find a job.

For information on firm hiring we use the IAB Job Vacancy Survey, a representative survey among German firms about their recruiting process.\(^{10}\) The survey is conducted as

---

\(^9\)See Caliendo et al. (2011) and Arni et al. (2014) for a detailed description of the IZA Evaluation Dataset.

\(^{10}\)See Kubis (2018) for a detailed description of the IAB Job Vacancy Survey.
repeated cross-section and covers around firms from 2000 to 2014. The survey asks detailed questions about firms’ recruitment, the number of successful and unsuccessful candidate searches and the used search channels, including a question about whether the firms receive VRs. Furthermore, the survey asks questions about a firm’s last filled vacancy and last unsuccessful search, including questions about the applicant pool and the hired candidate.

**Sample** In selecting our sample we make use of a brief time window in which our two data sources overlap. The IZA Evaluation Dataset covers workers who became unemployed in 2007 and 2008 and follows them for up to three years. The IAB Job Vacancy Survey covers a repeated cross-section of firms for each year from 2000 to 2014. However, a question on how often firms were unsuccessful in filling a vacancy, a key variable that we make use of in our estimation, was only added starting in 2008. For the structural estimation of our model we thus restrict our attention to 2008, the only year for which we have all relevant information from both datasets we use.

For the IZA Evaluation Dataset we start from a sample of 6,520 workers who entered unemployment in 2008. We restrict this sample by dropping workers below age 18, and by dropping workers who report numbers of monthly applications or numbers of monthly VRs above the respective 98th percentile. We thus arrive at a subsample of 6,114 workers.

Our initial sample from the IAB Job Vacancy Survey covers 13,652 firms who were surveyed in 2008. We drop firms who report numbers of applicants or hires above the respective 98th percentile. The final sample we use from the IAB Job Vacancy Survey covers 13,428 firms.

**Descriptives** In order to provide an overview of the German labor market situation in 2008 we provide a range of summary statistics on the search behavior of unemployed workers, in Table 3.1, and on the hiring behavior of firms, in Table 3.2.

Table 3.1 provides descriptive statistics on worker behavior and worker characteristics from the 2008 wave of the IZA evaluation dataset. The summary statistics show that the average accepted monthly wage of employed workers in our sample by far exceeds the average monthly unemployment benefits claimed by unemployed workers (by a factor larger than 1.7), suggesting that the employment premium is large. Unemployed workers in our sample on average send out 12.7 applications and received 2 VRs in the last month before taking the survey. Interestingly workers who received at least one VR on average report 2.6 applications
more than workers who did not receive a VR, suggesting that VRs crowding out job search effort, is not a dominant force.\textsuperscript{11} Finally, in our inflow sample of unemployed workers is on average 37 years of age, 46% are female and 77% have been unemployed previously in their lives.

**Table 3.1: Summary statistics, IZA Evaluation Dataset**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment benefits</td>
<td>676.02</td>
<td>482.86</td>
</tr>
<tr>
<td>Accepted wage</td>
<td>1181.38</td>
<td>727.72</td>
</tr>
<tr>
<td># applications</td>
<td>12.74</td>
<td>17.76</td>
</tr>
<tr>
<td># applications (if # VRs= 0)</td>
<td>10.00</td>
<td>25.84</td>
</tr>
<tr>
<td># applications (if # VRs&gt; 0)</td>
<td>12.60</td>
<td>24.22</td>
</tr>
<tr>
<td># VRs</td>
<td>2.00</td>
<td>4.03</td>
</tr>
<tr>
<td>Age</td>
<td>37.71</td>
<td>14.53</td>
</tr>
<tr>
<td>Female</td>
<td>0.46</td>
<td></td>
</tr>
</tbody>
</table>

*Notes: Summary statistics from the 2008 wave of the IZA Evaluation Dataset. Statistics on Unemployed before, Age and Female are averages across the whole sample. Statistics on Accepted wage are conditional on employment and statistics on Unemployment benefits, # Applications and # VRs are conditional on unemployment. Data on accepted wages correspond to monthly wages. Data on # Applications and # VRs correspond to applications sent out and VRs received in the last month prior to the survey interview.*

Table 3.2 provides summary statistics on firms’ hiring decisions from the 2008 wave of the IAB Job vacancy survey. The displayed descriptive statistics show that a considerable number of candidate searches are unsuccessful and need to be canceled without filling a vacancy, relative to the number of searches that end in a hire. It is implied by these statistics that around 31% of all candidate searches are unsuccessful. On average firms report that for the last vacancy filled before the survey interview 15.5 applications were received 4.9 of which were referred by the PES as VRs. Thus on average 31.8% of all received applications were received through VRs.

\textsuperscript{11}The correlation between the number of applications and VRs, at Cov(\(\alpha, \iota\)) = 0.11, points in the same direction.
Table 3.2: Summary statistics, IAB Job Vacancy Survey

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td># unsuccessful searches</td>
<td>1.72</td>
<td>2.32</td>
</tr>
<tr>
<td># hires</td>
<td>3.86</td>
<td>10.43</td>
</tr>
<tr>
<td># applications received (last vacancy)</td>
<td>15.52</td>
<td>74.16</td>
</tr>
<tr>
<td># VRs received (last vacancy)</td>
<td>4.93</td>
<td>35.92</td>
</tr>
</tbody>
</table>

Notes: Summary statistics from the 2008 wave of the IAB Job Vacancy Survey. The no. of hires and unsuccessful searches relate to the last 12 months before the survey interview. Data on applications received and VRs received relate to the last vacancy filled at the surveyed firms. The number of applications received includes both VR and non-VR applications.

Figure 3.1 provides illustrations of the distributions of the number of monthly applications sent out and VRs received by workers, in panel A and B, and the number of applications and VRs received by firms, in panel C and D.

In panel A there is considerable bunching in workers who report round numbers for their numbers of applications, especially for numbers greater than ten. In our sample 6% of workers report not having sent out any applications. The median and the 25th and 75th percentile, respectively, are at 3, 10, and 20 applications. 4% report having sent out more than 40 applications.

In Panel B the empirical distribution of VRs received by workers does not exhibit bunching at round numbers. 41% report not having received a VR in the last month prior to the interview. 17% report having received one, 13% report two and 8% report having received 3 VRs. The 95th percentile is at 9 VRs.

The histograms in panel C and D both exhibit some degree of bunching at round numbers. Bunching is particularly pronounced for non-VR applications at round numbers greater than ten.

Perhaps surprisingly 19% of all firms report receiving only one application for their last job opening. The median, the 75th and the 90th percentile of the empirical distribution, respectively, are at 4, 11 and 26 applications, i.e. the empirical distribution is strongly skewed. 3.5% of firms report having received more than 40 applications for their last job opening.

Panel D shows that a large share of firms (31%) report receiving no VR-applications.
Extremely few firms (less than one percent) report having received more than 15 VR-applications for their last job opening. 42% report having received 1-5, 15% report 6-10 and 6% report having received 10-15 VR-applications.

**Figure 3.1:** No. of VRs and applications, worker- and firm-side

Panel A: # applications sent out per month (worker side)

Panel B: # VRs received per month (worker side)

Panel C: # applications received for a given vacancy (firm side)

Panel D: # applicants via VRs for a given vacancy (firm side)

*Notes:* Panel A and B are computed using the IZA Evaluation Dataset. Panel C and D are computed using the IAB Job Vacancy Survey, based on questions about the last vacancy that opened at the surveyed firms.
3.5 Estimation

To obtain estimated values for the structural model parameters we proceed in two steps. First a subset of the parameters are set externally without making use of our structural model. These parameters are fixed in accordance with variable values, that are directly observable in our data. The remaining parameters are estimated using the generalized method of moments (GMM), i.e., we search across the parameter space to find model parameters such that a set of moments that we compute using our structural model matches their empirical counterpart from the worker and firm data as close as possible.

3.5.1 Pre-set and Directly Estimated Parameters

The model parameters that we set externally and the values we fix them at are summarized in Table 3.3. In particular we fix the monthly discount factor at $\beta = 0.997$. We set the monthly unemployment benefits equal to $b = 849$, which corresponds to the average monthly unemployment insurance benefits observed for our sample in the IZA Evaluation Dataset. The maximum number of monthly applications are fixed at $A = 45$, which corresponds to the 95th percentile of the empirical distribution of monthly applications in the in the IZA Evaluation Dataset. We furthermore set $J = 1$, i.e., we impose that workers receive at most one VR in any given months. A policy scenario in this setting thus is fully characterized by the fraction of unemployed workers who receive a VR in a given month.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly discount factor, $\beta$:</td>
<td>0.997</td>
</tr>
<tr>
<td>Monthly unemployment benefits, $b$:</td>
<td>676</td>
</tr>
<tr>
<td>Max. number of monthly applications, $A$:</td>
<td>40</td>
</tr>
<tr>
<td>Max. number of monthly VRs, $J$:</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.3: Externally set parameters
3.5.2 Generalized Method of Moments Estimation

The remaining model parameters that we estimate by GMM estimation are the vacancy creation cost parameter, \(k\), the production value \(y\), the VR application cost scale parameter \(\phi\), the job destruction rate \(\delta\) and the parameter governing the distribution of application costs, \(\mu\). We denote the vector of structural parameters to be estimated by GMM estimation by \(\theta = (k, y, \phi, \delta, \mu)\). For a given parameter value, \(\theta\), we solve the structural model using dynamic programming.\(^{12}\) Having solved the model we compute a vector of model moments, \(m(\theta)\). GMM-estimation minimizes the distance between the vector of model moments, \(m(\theta)\), and its data counterpart, \(\widehat{m}\), i.e.,

\[
\hat{\theta} = \text{argmin} \ (m(\theta) - \widehat{m})' \widehat{W} (m(\theta) - \widehat{m}),
\]

(3.12)

where \(\hat{\theta}\) is our estimator of the structural model parameters and \(\widehat{W}\) is the diagonal matrix of the inversed variances of the empirical moments, \(\widehat{m}\).

In the estimation we target 14 empirical moments that can be approximated or computed directly as function of structural model parameters. Among the targeted moments are the share of individuals sending out \(\alpha = 0\) applications (one moment), the shares for which \(\alpha\) lies within either of six disjoint intervals (six moments) and the mean number of applications sent out by workers who received a VR (\(\iota = 1\)) and did not receive a VR (\(\iota = 0\)), respectively. Furthermore we target average number of applications received by firms, the employer match rate, the unemployment rate and the mean and standard deviation of the expected wage distribution.

To find the minimizer that solves (3.12) we invoke a Powell algorithm to find local minimal, and search over a grid of starting values to find the global minimizer.

3.5.3 Parameter Estimates and Model Fit

The GMM parameter estimates are presented in Table 3.4. Our estimate for the VR application cost scale parameter, \(\hat{\phi} = 0.27\), implies that the application costs for a job that is referred through a VR are 27% of the application costs for a non-referred job. We estimate that the per period output of a successful match is \(\hat{y} = 1888\). The implied reservation wage

\(^{12}\)To make the model solution tractable we approximate the wage offer distribution by a piecewise linear cumulative distribution function and use numerical techniques to approximate integrals.
at the estimated parameter values is $\bar{w}(\hat{\theta}) = 848$, i.e., upon a successful match firms realize $\hat{y} - \bar{w}(\hat{\theta}) = 1040$ in profits per time period. The estimated job separation rate, $\hat{\delta} = 0.06$ implies that matches on average last around 16.6 months. The costs of creating a vacancy, which in equilibrium equal the value of posting a vacancy, are estimated at $\hat{k} = 1256$. The mean of the application distribution is estimated at $\hat{\mu} = 6.74$.

For an assessment of the model fit in Table 3.5 and Figure 3.2 we contrast the targeted empirical moments with their model counterparts at the estimated parameter values.

The estimated model provides a good fit of most of the moments presented in Table 3.5, although the model is a bit sparse on the standard deviation of wages and the employer matching rate. By construction the model predicts that unemployed workers who receive a VR write fewer applications ($\mathbb{E}[\alpha|\iota = 0] > \mathbb{E}[\alpha|\iota = 1]$), while in the data this relationship is reversed. As a consequence, while the model fit is good for the mean number of applications conditional on not having received a VR ($\mathbb{E}[\alpha|\iota = 0]$), our model underpredicts the mean number of applications conditional on not having received a VR ($\mathbb{E}[\alpha|\iota = 1]$).

The model fit for the distribution of the number of applications, $\alpha$, is presented in Figure 3.2. The model under predicts the number of unemployed workers who send out zero applications, but reflects the general empirical pattern, according to which, first, the large majority of individuals send out or less applications, and second, mass of individuals is declining in the number of applications for $\alpha > 0$. 

Table 3.4: GMM parameter estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.27</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.06</td>
</tr>
<tr>
<td>$k$</td>
<td>1256</td>
</tr>
<tr>
<td>$y$</td>
<td>1888</td>
</tr>
<tr>
<td>$\mu$</td>
<td>6.74</td>
</tr>
</tbody>
</table>
Table 3.5: Model fit

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Queue length</td>
<td>10.59</td>
<td>11.05</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>7.8%</td>
<td>9.7%</td>
</tr>
<tr>
<td>$\mathbb{E}[\alpha</td>
<td>\iota = 0]$</td>
<td>10.0</td>
</tr>
<tr>
<td>$\mathbb{E}[\alpha</td>
<td>\iota = 1]$</td>
<td>12.6</td>
</tr>
<tr>
<td>Avg. accepted wage</td>
<td>1181</td>
<td>1619</td>
</tr>
<tr>
<td>S.d., accepted wages</td>
<td>727</td>
<td>93</td>
</tr>
<tr>
<td>Employer matching rate</td>
<td>0.69</td>
<td>0.58</td>
</tr>
</tbody>
</table>

*Notes:* Displayed are moments implied by our model at the estimated parameter values and their empirical counterparts computed from the IZA Evaluation Dataset and the IAB Job Vacancy Survey Data.

Figure 3.2: Model fit, no. of applications

![Model fit, no. of applications](image)

*Notes:* The displayed plot shows the frequencies of workers with number of application, $\alpha$, as implied by our model at the estimated parameter values and their empirical counterparts computed from data on unemployed workers’ monthly applications from the IZA Evaluation Dataset.
3.6 Policy Simulations

To study the impact of VRs on labor market outcomes and to quantify the equilibrium effects of VRs we examine counterfactual policy scenarios. A policy in our model is given by the VR rate, i.e., by the probability that a given worker receives a VR in a given time period, \( p_{vr} = P(t = 1). \)\(^{13}\)

3.6.1 The Impact of VRs on Aggregate Labor Market Outcomes

We solve our model for a range of values for \( p_{vr} \) that span the unit interval, keeping all other model parameters fixed at their estimated values. Figure 3.3 displays the impact of varying the VR-rate on various labor market outcomes.

Panel A shows that increasing the VR rate has a positive impact on the mean total number of applications (including VR applications) that workers send out. An extreme policy change from a scenario where nobody receives VRs \( (p_{vr} = 0) \), to a scenario in which everybody receives a VR with certainty in each period \( (p_{vr} = 1) \), would increase the mean number of applications (including VR applications) by 0.6. Additionally Panel A displays the mean number of applications, which were not brokered through a VR. This number is declining in the VR-rate, suggesting that in response to receiving a VR unemployed workers reduce the number of non-VR applications they send out. This behavior partially off-sets the positive impact of VRs on the mean number of applications, which is why the effect of a policy change from \( p_{vr} = 0 \) to \( p_{vr} = 1 \) on the total number of applications is smaller than one.

Panel B shows that the queue length is strictly increasing in the VR rate, i.e., the higher the VR rate, the higher is the competition for a given vacancy that any applicant faces.

Panel C and D display the average worker match rate in the economy and the aggregate unemployment rate for different values of \( p_{vr} \). Note that the worker match rate and the aggregate unemployment rate are linked to each other by a nonlinear relationship (given by (3.11)). Panel C and D show that increasing the VR rate lowers the aggregate unemployment rate (and increasing the worker matching rate), although the effect on the aggregate unemployment rate is quantitatively modest. Increasing the VR rate from \( p_{vr} = 0 \) to \( p_{vr} = 1 \) leads to a drop in the unemployment rate from 9.8% to 9.6%.

\(^{13}\)Recall that we imposed that a given worker can receive at most one VR in a given time period \( (J = 1) \).
Figure 3.3: Counterfactual policy scenarios

Panel A: Mean no. of applications

Panel B: Queue length

Panel C: Worker match rate

Panel D: Unemployment rate

Notes: Panel A and B are computed using the IZA Evaluation Dataset. Panel C and D are computed using the IAB Job Vacancy Survey, based on questions about the last vacancy that opened at the surveyed firms.
3.6.2 Measuring the Equilibrium Effects of VRs

To quantify the equilibrium effects of VRs we decompose the aggregate effect of increasing the VR rate on the worker matching rate, $m_W$. In particular we decompose the mean worker matching rate, $m_W$, into the matching rate for those workers who received a VR, $m_W|_{v_r=1}$, and for those who received no VR, $m_W|_{v_r=0}$,

$$m_W = p_{v_r}m_W|_{v_r=1} + (1 - p_{v_r})m_W|_{v_r=0}. \quad (3.13)$$

Taking derivatives with respect to $p_{v_r}$ yields the analogous decomposition of the impact of changing the VR rate, $p_{v_r}$, on the worker matching rate

$$\frac{\partial m_W}{\partial p_{v_r}} = m_W|_{v_r=1} - m_W|_{v_r=0} + p_{v_r} \frac{\partial m_W|_{v_r=1}}{\partial p_{v_r}} + (1 - p_{v_r}) \frac{\partial m_W|_{v_r=0}}{\partial p_{v_r}}. \quad (3.14)$$

In particular the impact of changing the VR rate on the worker matching rate is decomposed into three parts. First, the direct effect of assigning a VR to a worker on the matching rate of this worker, net of equilibrium effects. In (3.14) this direct effect is labeled $A$. Second, the
equilibrium effect on workers who receive a VR, labeled $B$ and third, the equilibrium effect on workers who do not receive a VR, labeled $C$.\footnote{In the language of the treatment evaluation literature (see, e.g., Imbens and Angrist (1994) and Heckman and Vytlacil (2007)) $A$ is the effect on compliers, net of equilibrium effects, $B$ is the equilibrium effect on the treated and $C$ is the equilibrium effect on the untreated.}

Figure 3.4 provides an illustration of the decomposition for our estimated model. The figure shows the decomposition for a change from the status quo policy, $p_{vr} = 0.575$, to $p_{vr}' = 0.8$, i.e., a 39\% increase in the VR rate. We find that the overall effect of this increase in the VR rate on the worker matching rate is $\Delta m_w = 0.0276$. This effect can be decomposed into the direct effect of increasing the VR rate, which equals $A = 0.0314$, the equilibrium effect on unemployed workers who do receive a VR, $B = -0.0039$, and the equilibrium effect on those who do not receive a VR, $C = -0.0038$.

The above decomposition can be used to illustrate the extent to which the impact of VRs on the worker matching rate would be overestimated in a simple experimental design, in which VRs are assigned randomly to workers. The effect measured by comparing the matching rates of workers who were randomly assigned a VR to workers who did not receive a VR would correspond to $A = m_{W|vr=1} - m_{W|vr=0}$.\footnote{More precisely, the measured effect would correspond to $A$ for small scale experiments. For larger scale experiments that substantially change $p_{vr}$ the measured effect would equal $m_{W|vr=1}(p'_{vr}) - m_{W|vr=0}(p'_{vr})$, where $p'_{vr}$ denotes the VR rate after the policy intervention.} The average treatment effect measured in such an experiment thus misses equilibrium effects on the overall matching rate, given by $p_{vr}B + (1 - p_{vr})C$. Our estimated model thus predicts that a randomized experiment would overstate the effect of a reform that increases the VR rate from 57.5\% to 80\% on the worker matching rate by 0.38 percentage points. This equals an overstatement of the true, $\Delta m_w = 0.0276$, effect by 14.2\%.

\section{Conclusion}

In this paper we analyze the equilibrium effects of VRs. We find that equilibrium effects reduce the impact of VRs on job finding rates substantially. This means that randomized experiments, that measure the individual level effect of VRs on the reemployment rate, substantially overstate the effect of VRs on the economywide employment rate.

Our results show that assigning VRs to workers imposes a negative externality on other workers, as it lowers their hiring probabilities. We show that, at the same time, firms react
by posting more vacancies to an increased VR rate, as the matching rate for a given vacancy increases. For unemployed workers we find that the effect of receiving a VR is 14.2% lower than it were in the absence of equilibrium effects.

Despite equilibrium effects that reduce the positive effects of VRs, we find that overall VRs have a positive effect on the economywide employment rate, even though this effect is modest. We find that moving from sending out no VRs to sending one VR to every unemployed worker every month would increase the employment rate by 0.17 percentage points. Our computations suggest that such a policy change would improve the budget constraint of the PES, given by expected revenues from UI contributions net of expected UI payments, by 5.2%. Increasing the VR rate thus could to some degree be self financing.

A first obvious extension of our setup would be to allow for multiple VRs in any given month. Another interesting extension would be to see how the analysis extends to settings where VRs can be targeted to particular worker groups, e.g. to workers with especially high search costs. A further extension in this direction would be to consider settings with heterogeneous workers and firms, where VRs can be targeted to workers and firms to create particularly productive matches.
Bibliography


Appendix A

Appendix to Chapter 1

A.1 Maintenance Payments, Details and Functional Forms

In this Appendix I present details on how maintenance payments are computed and the exact functional forms for computing child support and alimony payments. From 1980 to 2013 the policy parameters have been adjusted from year to year by the Danish state administration to account for inflation. Throughout the paper I use the year 2004 values of the Danish maintenance policy parameters and deflate wages (and other money amounts) taking 2004 as base year.¹

**Child support, functional form** Child support $cs$ depends on the number of children an ex-couple has and the non-custodial parents labor income. Suppose ex-spouse $s$ is the custodial parent of $n_s$ children. If the non-custodial ex-spouse $\tilde{s}$ earns annual labor income $I_{\tilde{s}}$ then the child support that $\tilde{s}$ needs to pay to $s$ is given by

$$cs(n_s, I_{\tilde{s}}, B) = nB \cdot \left(1 + \sum_{k=0}^{K} a_k 1\{b_k(n) \leq I_{\tilde{s}} < b_{k+1}(n)\}\right) \quad (A.1)$$

Where the year 2004 values of the parameters that enter into (A.1) are $B = 9420$ (DKK), $K = 5$ (i.e., child support varies across 6 income brackets across) as well as the values of $a_k$ and $b_k(n)$, which are given in Tables A.1 and A.2.

**Table A.1:** Child support parameters ¹

<table>
<thead>
<tr>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.25</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

*Notes: Source: Danish State Administration (Statsforvaltning).*

¹Information on policy parameters for past years was provided by the Danish State Administration (Statsforvaltning)
Table A.2: Child support parameters 2

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>b_0(n)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b_1(n)</td>
<td>320</td>
<td>340</td>
<td>370</td>
</tr>
<tr>
<td>b_2(n)</td>
<td>340</td>
<td>370</td>
<td>410</td>
</tr>
<tr>
<td>b_3(n)</td>
<td>370</td>
<td>410</td>
<td>460</td>
</tr>
<tr>
<td>b_4(n)</td>
<td>550</td>
<td>650</td>
<td>750</td>
</tr>
<tr>
<td>b_5(n)</td>
<td>1000</td>
<td>1250</td>
<td>1400</td>
</tr>
<tr>
<td>b_6(n)</td>
<td>+∞</td>
<td>+∞</td>
<td>+∞</td>
</tr>
</tbody>
</table>

Notes: Source: Danish State Administration (*Statsforvaltning*).

**Alimony, functional form**  Alimony payments depend on both spouses labor incomes. Denote by \( l \) the lower earner and by \( h \) the higher earner in terms of annual labor income net of child support payments and by \( \tilde{I}_l, \tilde{I}_h \) the respective annual labor incomes net of child support. Then the alimony payments that \( l \) is entitled to receive from \( h \) are given by

\[
alim(\tilde{I}_H, \tilde{I}_L) = \begin{cases} 
\tau \cdot (\tilde{I}_H - \tilde{I}_L) & \text{if } \tilde{I}_L \geq C_1 \text{ and } \tilde{I}_H - C_2 \geq \tau \cdot (\tilde{I}_H - \tilde{I}_L) \text{ and } C_3 - \tilde{I}_L \geq \tau \cdot (\tilde{I}_H - \tilde{I}_L) \\
\tau \cdot (\tilde{I}_H - C_1) & \text{if } \tilde{I}_L < C_1 \text{ and } \tilde{I}_H - C_2 \geq \tau \cdot (\tilde{I}_H - \tilde{I}_L) \text{ and } C_3 - \tilde{I}_L \geq \tau \cdot (\tilde{I}_H - \tilde{I}_L) \\
\max\{\tilde{I}_H - C_2, 0\} & \text{if } \tilde{I}_H - C_2 < \tau \cdot (\tilde{I}_H - \tilde{I}_L) \\
\max\{C_3 - \tilde{I}_L, 0\} & \text{if } C_3 - \tilde{I}_L < \tau \cdot (\tilde{I}_H - \tilde{I}_L)
\end{cases}
\]

(A.2)

By this functional form it is ensured that, 1. if the receiver’s labor income is below \( C_1 \), alimony payments are capped by \( \tau \cdot (I_s - C_1) \), 2. the maintenance payer’s labor earnings net of maintenance payments are at least \( C_2 \), 3. the maintenance receiver’s labor earnings plus maintenance payments are capped by \( C_3 \). The 2004 values for the parameters that enter into (A.2) are given by \( \tau = 0.2 \), \( C_1 = 90000 \), \( C_2 = 204000 \) and \( C_3 = 230000 \).
A.2 Computational Details

This appendix provides details on the numerical solution and the structural estimation of the model.

Model solution The model is solved by backwards recursion, i.e., for each time period $t$ the model agents’ problem is solved at a grid of points in the state space, taking the continuation values in $t + 1$ as given. I first solve the model for divorced couples (i.e., I solve for the values of divorce $V_{ft}^{div}, V_{mt}^{div}$) and then solve the decision problem of married couples, using the values of divorce as input.

Approximations For the model solution I solve the model for a discrete grid of points in the state space and use numerical approximation techniques to compute continuation values and best response functions of divorcees at points off the discrete grid. In particular I use linear interpolation to interpolate between points on the asset grid $A_t, A_{ft}, A_{mt}$ and the relative bargaining weight in married couples $\mu_{ft}$, and Gauss-Hermite quadrature (see Judd (1998)) to approximate integrals taken over the distribution of the wage shocks, $\epsilon_{st} \sim \mathcal{N}(0, \sigma_{\epsilon})$. For the approximation of the random walk according to which the “love shocks” $\xi_{ft}, \xi_{mt}$ evolve I use Rouwenhorst’s method for discretizing highly persistent processes (see Kopecky and Suen (2010) and Fella et al. (2017)).

Computation I implement the model solution in Python. As the state space is large (129,600 points for divorced couples and 945,000 points for married couples) the model solution is computationally demanding. I parallelize iterations over points the state space across 40 cores on a high performance cluster and use a just in time compiler to achieve further speed improvements. Using this setup one model solution takes between 20 and 25 minutes.

Estimation For the minimization of the MSM criterion function I use basin-hopping, a global optimization routine. The basin-hopping algorithm uses the Nelder-Mead algorithm for finding local minima and upon successful completion of the Nelder-Mead pertubes the coordinates of the obtained local minimum (stochastically) and reiterates the local minimization procedure several times. Upon completion of several local minimization steps the algorithm selects the smallest of the obtained local minima.
A.3 Timing of Events

Figure A.1: Timing of events for married couples

A.4 Child Custody

As child support payments are mandated to be made from the non-custodial to the custodial parent, it is important to understand which parent typically takes custody and how custody decisions correlate with other variables. To examine which variables are associated with custody decisions I estimate a multinomial probit model. The estimated conditional probabilities from the probit model are used as input in the structural estimation (see section 1.5). I use data on the primary residence of a couple’s children after divorce to define the dependent child custody variable. Information on the primary residence captures the relevant notion of child custody, as having *physical custody* of a child is what matters for the entitlement to receiving child support payments from the other parent.\(^2\)

I define the dependent variable by

\[
cust_i = \begin{cases} 
0 & \text{mother takes custody} \\
1 & \text{father takes custody.} 
\end{cases}
\]

\(^2\)If parents share *legal custody* but not *physical custody* child support payments are not affected. If parents share *physical custody* and the children spend equal time with each parent, no claim to child support payments is established. In Denmark shared physical custody is not uncommon, but occurs in less than 9% of all divorce cases (see Bjarnason and Arnarsson (2011)).
In 9% of all divorce cases in my sample couples with multiple children split custody, i.e., some children stay with each parent. I categorize these cases as follows. If parents split custody such that the majority of children stay with one parent I classify this parent as custodial parent. If parents split custody equally I randomly classify one parent as custodial parent with probability 0.5.

As right hand side variables $X_i$ I consider marriage duration ($t$) and number of children $n_t$ at the time of divorce. The estimated empirical model is summarized by

$$cust_i = 1 \{ \pi X_i > \nu_i \}$$
$$\nu_i \sim \mathcal{N}(0,1)$$

The coefficient estimates $\hat{\pi}$ are presented in Table A.4. The estimates show that a higher number of children is associated with a lower propensity of the father to take custody. Longer marriage duration in contrast is associated with a higher propensity of the father to take custody. Note that in more extensive empirical specifications, where age of the youngest child is added as right hand side variable the coefficient estimate of marriage duration becomes insignificant (see Table A.6). As the age of the youngest child and marriage duration are highly correlated at 0.68. It seems plausible that marriage duration mostly picks up the association between $cust_i$ and the age of the youngest child.

<table>
<thead>
<tr>
<th>Table A.3: Child custody, probit model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child custody:</td>
</tr>
<tr>
<td>Number of children ($n$)</td>
</tr>
<tr>
<td>Marriage duration ($t$)</td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
Table A.4: Child custody, probit model - prediction and marginal effects

<table>
<thead>
<tr>
<th></th>
<th>At avg. $X_i$</th>
<th>Sample avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>P( father takes custody )</td>
<td>0.0770$^{***}$</td>
<td>0.0795$^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0015)</td>
<td>(0.0015)</td>
</tr>
<tr>
<td>Partial effect, number of children ($n$)</td>
<td>-0.0167$^{***}$</td>
<td>-0.0168$^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0023)</td>
<td>(0.0023)</td>
</tr>
<tr>
<td>Partial effect, marriage duration ($t$)</td>
<td>0.0041$^{***}$</td>
<td>0.0042$^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>Observations</td>
<td>32313</td>
<td>32313</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.5: Child custody, multinomial probit

<table>
<thead>
<tr>
<th>Child custody:</th>
<th>cust$_i = 1$</th>
<th>cust$_i = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of children ($n$)</td>
<td>-0.237$^{***}$</td>
<td>0.702$^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0240)</td>
<td>(0.0200)</td>
</tr>
<tr>
<td>Marriage duration ($t$)</td>
<td>0.041$^{***}$</td>
<td>0.032$^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0030)</td>
<td>(0.0028)</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.861$^{***}$</td>
<td>-3.024$^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0540)</td>
<td>(0.0509)</td>
</tr>
<tr>
<td>Observations</td>
<td>32313</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
### Table A.6: Child custody, probit (extensive specification)

<table>
<thead>
<tr>
<th>Child: cust&lt;sub&gt;i&lt;/sub&gt;</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of children (n)</strong></td>
<td>-0.0425** (0.0181)</td>
<td></td>
</tr>
<tr>
<td><strong>Marriage duration (t)</strong></td>
<td>0.0025 (0.0031)</td>
<td></td>
</tr>
<tr>
<td><strong>Earnings f (in DKK 10,000s)</strong></td>
<td>0.0013* (0.0007)</td>
<td></td>
</tr>
<tr>
<td><strong>Earnings m (in DKK 10,000s)</strong></td>
<td>-0.0005 (0.0000)</td>
<td></td>
</tr>
<tr>
<td><strong>Age youngest child</strong></td>
<td>0.0426*** (0.0034)</td>
<td></td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>-1.735*** (0.0490)</td>
<td></td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>32313</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parantheses
* p < 0.1, ** p < 0.05, *** p < 0.01

### Table A.7: Child custody, multinomial probit

| Child custody: cust<sub>i</sub> = 1 cust<sub>i</sub> = 2 |
|-------------------------|------|------|
| **Number of children (n)** | -0.237*** (0.0240) | 0.702*** (0.0200) |
| **Marriage duration (t)** | 0.041*** (0.0030) | 0.032*** (0.0028) |
| **Constant** | -1.861*** (0.0540) | -3.024*** (0.0509) |
| **Observations** | 32313 |      |

Standard errors in parantheses
* p < 0.1, ** p < 0.05, *** p < 0.01

119
### Table A.8: Child custody, multinomial probit (extensive specification)

<table>
<thead>
<tr>
<th>Child custody:</th>
<th>$cust_i = 1$</th>
<th>$cust_i = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of children ($n$)</td>
<td>-0.125***</td>
<td>0.834***</td>
</tr>
<tr>
<td></td>
<td>(0.0271)</td>
<td>(0.0235)</td>
</tr>
<tr>
<td>Marriage duration ($t$)</td>
<td>0.001</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(0.00440)</td>
<td>(0.0040)</td>
</tr>
<tr>
<td>Earnings $f$ (in DKK 10,000s)</td>
<td>0.003***</td>
<td>0.006***</td>
</tr>
<tr>
<td></td>
<td>(0.0010)</td>
<td>(0.0009)</td>
</tr>
<tr>
<td>Earnings $m$ (in DKK 10,000s)</td>
<td>-0.001**</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>Age youngest child</td>
<td>0.064***</td>
<td>0.0585***</td>
</tr>
<tr>
<td></td>
<td>(0.0048)</td>
<td>(0.0044)</td>
</tr>
<tr>
<td>Constant</td>
<td>-2.273***</td>
<td>-3.567***</td>
</tr>
<tr>
<td></td>
<td>(0.0540)</td>
<td>(0.0509)</td>
</tr>
<tr>
<td>Observations</td>
<td>32313</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parantheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
Table A.9: Child custody, multinomial probit (extensive specification)

<table>
<thead>
<tr>
<th></th>
<th>cust&lt;sub&gt;i&lt;/sub&gt; = 1</th>
<th>cust&lt;sub&gt;i&lt;/sub&gt; = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of children (n)</td>
<td>-0.125***</td>
<td>0.834***</td>
</tr>
<tr>
<td></td>
<td>(0.0271)</td>
<td>(0.0235)</td>
</tr>
<tr>
<td>Marriage duration (t)</td>
<td>0.001</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(0.00440)</td>
<td>(0.0040)</td>
</tr>
<tr>
<td>Earnings f (in DKK 10,000s)</td>
<td>0.003***</td>
<td>0.006***</td>
</tr>
<tr>
<td></td>
<td>(0.0010)</td>
<td>(0.0009)</td>
</tr>
<tr>
<td>Earnings m (in DKK 10,000s)</td>
<td>-0.001**</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>Age youngest child</td>
<td>0.064***</td>
<td>0.0585***</td>
</tr>
<tr>
<td></td>
<td>(0.0048)</td>
<td>(0.0044)</td>
</tr>
<tr>
<td>Constant</td>
<td>-2.273***</td>
<td>-3.567***</td>
</tr>
<tr>
<td></td>
<td>(0.0540)</td>
<td>(0.0509)</td>
</tr>
</tbody>
</table>

Observations 32313

Standard errors in parantheses

* p < 0.1, ** p < 0.05, *** p < 0.01
### A.5 Model Fit

**Table A.10:** Model fit, work hours and housework hours

<table>
<thead>
<tr>
<th>Moment</th>
<th>Children</th>
<th>Model</th>
<th>Data</th>
<th>Std. dev. (data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours worked female (married)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>31.7</td>
<td>30.4</td>
<td>12.4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>30.7</td>
<td>30.3</td>
<td>11.1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>29.8</td>
<td>30.4</td>
<td>11.4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>28.4</td>
<td>28.3</td>
<td>13.1</td>
<td></td>
</tr>
<tr>
<td>Hours worked female (divorced)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>30.8</td>
<td>28.0</td>
<td>14.5</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>29.5</td>
<td>28.9</td>
<td>13.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>28.0</td>
<td>29.0</td>
<td>13.5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>27.0</td>
<td>25.5</td>
<td>15.2</td>
<td></td>
</tr>
<tr>
<td>Hours worked male (married)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>33.3</td>
<td>31.9</td>
<td>12.1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>33.2</td>
<td>33.2</td>
<td>10.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>32.8</td>
<td>33.7</td>
<td>10.6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>32.0</td>
<td>33.1</td>
<td>12.1</td>
<td></td>
</tr>
<tr>
<td>Hours worked male (divorced)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>30.1</td>
<td>28.5</td>
<td>14.6</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>31.4</td>
<td>31.2</td>
<td>12.9</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>31.8</td>
<td>31.9</td>
<td>12.3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>32.8</td>
<td>31.5</td>
<td>13.2</td>
<td></td>
</tr>
<tr>
<td>Housework hours female (married)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>15.8</td>
<td>13.6</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>17.0</td>
<td>16.5</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>≥ 2</td>
<td>18.7</td>
<td>19.3</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>Housework hours female (divorced)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>17.5</td>
<td>9.6</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>18.9</td>
<td>19.0</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>≥ 2</td>
<td>20.9</td>
<td>21.9</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>Housework hours male (married)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>9.6</td>
<td>10.5</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10.2</td>
<td>10.5</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>≥ 2</td>
<td>11.4</td>
<td>9.9</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>Housework hours male (divorced)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>13.8</td>
<td>8.0</td>
<td>6.9</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>12.9</td>
<td>11.1</td>
<td>6.9</td>
<td></td>
</tr>
<tr>
<td>≥ 2</td>
<td>12.1</td>
<td>13.5</td>
<td>6.9</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** Moments from model simulations for 20,000 couples at the MSM-estimated parameter values and targeted data moments. Data moments are computed from Danish administrative data (on 279,197 couples), with the exception of mean housework hours, which are obtained from the Danish Time Use Survey (which includes 2,105 households).
A.6 Figures

Figure A.2: Women’s weekly work around divorce, by number of children

Couples with 0 children

Couples with 1 child

Couples with 2 children

Couples with ≥3 children

Notes: Each figure contains coefficient estimates of 1.1 for women, separately by number of children. Included are all women in my sample, that are observed for at least 3 periods prior and 6 periods after getting divorced.
Figure A.3: Men’s weekly work around divorce, by number of children

Couples with 0 children

Couples with 1 child

Couples with 2 children

Couples with ≥3 children

Notes: Each figure contains coefficient estimates of 1.1 for men, separately by number of children. Included are all men in my sample, that are observed for at least 3 periods prior and 6 periods after getting divorced.
Appendix B

Appendix to Chapter 2

B.1 Derivations

B.1.1 Value of Employment

For the value of employment, $\tilde{E}(w, ps, \tau)$, we have

$$\tilde{E}(w, 0, 0) = w + \beta \left( (1 - \delta)E(w, 0, 0) + \delta U(0, 0) \right)$$  \hspace{1cm} (B.1)

$$= w + \beta \delta U(0, 0) \frac{1}{1 - \beta (1 - \delta)},$$  \hspace{1cm} (B.2)

for $ps = 0$ and $\tau = 0$ and

$$\tilde{E}(w, ps, \tau) = w \sum_{l=0}^{\tau - 1} \beta^l (1 - \delta)^l + \beta \delta U(0, ps) \sum_{l=0}^{\tau - 1} \beta^l (1 - \delta)^l + \beta^\tau (1 - \delta)^\tau \tilde{E}(w, 0, 0)$$

$$= w \frac{1 - \beta^\tau (1 - \delta)^\tau}{1 - \beta (1 - \delta)} + \beta \delta U(0, ps) \frac{1 - \beta^\tau (1 - \delta)^\tau}{1 - \beta (1 - \delta)} + \beta^\tau (1 - \delta)^\tau \tilde{E}(w, 0, 0)$$

$$= \frac{w}{1 - \beta (1 - \delta)} + \beta \delta U(0, ps) \frac{1 - \beta^\tau (1 - \delta)^\tau}{1 - \beta (1 - \delta)} + \beta^\tau (1 - \delta)^\tau \frac{\beta \delta U(0, 0)}{1 - \beta (1 - \delta)}$$  \hspace{1cm} (B.3)

for $ps > 0$. Reservation wages equalize the value of accepting and rejecting job offers. For regular job offers we thus have

$$E\left(\overline{w}_{jo}(s, ps)\right) = U(\max\{s - 1, 0\}, ps),$$  \hspace{1cm} (B.4)

for each $(s, ps)$ such that $ps < \overline{ps}$. Using (B.12) together with (B.2) yields

$$U(0, 0) = \frac{\overline{w}_{jo}(0, 0)}{1 - \beta}$$  \hspace{1cm} (B.5)
(B.3), (B.12) and (B.5) together imply

$$U(0, ps) = \frac{(1 - \beta)w_{jo}(0, ps) + \beta^{r+1}\delta(1 - \delta)^r w_{jo}(0, 0)}{(1 - \beta)(1 - \beta + \beta^{r+1}\delta(1 - \delta)^r)}. \quad (B.6)$$

Inserting (B.5) and (B.6) back into (B.2) and (B.3) respectively, yields

$$\hat{E}(w, 0, 0) = \frac{w}{1 - \beta(1 - \delta)} + \frac{\beta \delta w_{jo}(0, 0)}{(1 - \beta)(1 - \beta(1 - \delta))}. \quad (B.7)$$

### B.1.2 Terminally Sanctioned Unemployed

Terminally sanctioned unemployed workers search for a job while receiving reduced UI benefits, $b_{low}$ and do not receive VRs. The value of being terminally sanctioned hence is given by

$$\Phi = b_{low} + \beta p_{jo} \int \max\{E(w, ps), \Phi\} dF_{jo}(w) + (1 - p_{jo})\Phi. \quad (B.8)$$

Rearranging and inserting (B.3) into (B.8) yields

$$(1 - \beta)\Phi = b_{low} + \beta p_{jo} \int_{w_{\Phi}}^{+\infty} \frac{w - w_{\Phi}}{1 - \beta(1 - \delta)} dF_{jo}(w). \quad (B.9)$$

Using that $\Phi = U(0, ps) = E(w, ps)$ together with (B.3) yields

$$(1 - \beta + \beta^r(1 - \delta)^r)\Phi = w_{\Phi} + \beta^{r+1}(1 - \delta)^r \delta U(0, 0) \quad (B.10)$$

Inserting $\Phi$ from equation (B.10) into (B.9) yields the reservation wage equation for terminally sanctioned unemployed workers

$$\frac{(1 - \beta)w_{\Phi} + \beta^{r+1}(1 - \delta)^r \delta w_{jo}(0, 0)}{1 - \beta + \beta^r(1 - \delta)^r} = b_{low} + \beta p_{jo} \int_{w_{\Phi}}^{+\infty} \frac{w - w_{\Phi}}{1 - \beta(1 - \delta)} dF_{jo}(w). \quad (B.11)$$

Note that the reservation wage for regular job offers of unemployed workers with no past sanctions, $w_{jo}(0, 0)$, enters this equation. The reservation wage of terminally sanctioned unemployed workers, $w_{\Phi}$, thus cannot be solved for in isolation, but we need to solve equation (B.11) jointly with the rest of the model.
B.1.3 Derivation of the System of Reservation Wage Equations

Reservation wages equalize the value of accepting a job offer with the value of continuing to search for a job. For each combination of \(s, ps\), we thus have

\[
E(w_{jo}(s, ps)) = U(\max\{s - 1, 0\}, ps),
\]

(B.12)

and moreover for the reservation wages after receipt of a VR

\[
E(\overline{w}_{vr}(s, ps)) = \begin{cases} (1 - psanc)U(\max\{s - 1, 0\}, ps) + psancU(\overline{s}, ps + 1), & \text{if } ps < \overline{ps} - 1 \\ (1 - psanc)U(\max\{s - 1, 0\}) + psanc\Phi, & \text{if } ps = \overline{ps} - 1 \end{cases}
\]

(B.13)

\[
= \begin{cases} (1 - psanc)E(\overline{w}_{jo}(s, ps)) + psancE(\overline{w}_{jo}(\overline{s}, ps + 1)), & \text{if } ps < \overline{ps} - 1 \\ (1 - psanc)E(\overline{w}_{jo}(s, ps)) + psancE(\overline{w}_{\Phi}), & \text{if } ps = \overline{ps} - 1. \end{cases}
\]

(B.14)

For the value of unemployment, \(U\), for \(s > 0\), rearranging (2.3) yields

\[
U(s, ps) = \beta(1 - psick) + \int_{\overline{w}_{jo}(s, ps)}^{+\infty} \frac{w - \overline{w}_{jo}(s, ps)}{1 - \beta(1 - \delta)} dF_{jo}(w) + p_{vr}R_{vr}(s, ps)
\]

\[
+ (1 - p_{vr})E(\overline{w}_{jo}(s, ps), ps) + \beta psickE(\overline{w}_{jo}(s, ps), ps).
\]

By inserting (B.12) into (2.1) and rearranging it follows that

\[
R_{vr}(s, ps) = \int A_{vr}(w) dF_{vr}(w) + p_{doc} \int \max\{E(\overline{w}_{jo}(s, ps), ps) - A_{vr}(w), 0\} dF_{vr}(w).
\]

(B.15)

Consider the first expression on the right hand side sum of (B.15). By inserting (2.2) and rearranging we get

\[
\int A_{vr}(w) dF_{vr}(w) = \lambda_{vr} \int \max\{E(w, ps), E(\overline{w}_{vr}(s, ps), ps)\} dF_{vr}(w)
\]

\[
+ (1 - \lambda_{vr}) E(\overline{w}_{jo}(s, ps), ps)
\]

\[
= \lambda_{vr} \int_{\overline{w}_{vr}(s, ps)}^{+\infty} \frac{w - \overline{w}_{vr}(s, ps)}{1 - \beta(1 - \delta)} dF_{vr}(w) + \lambda_{vr}E(\overline{w}_{vr}(s, ps), ps)
\]

\[
+ (1 - \lambda_{vr}) E(\overline{w}_{jo}(s, ps), ps)
\]

Now consider the second term on the right hand side sum of (B.15). From (B.12) and (2.2)
it follows that $A_{vr}(w) \geq E(\bar{w}_{jo}(s, ps), ps)$ if and only if $w \geq \bar{w}_{jo}(s, ps)$. The second term in equation (B.15) thus yields

$$p_{doc} \int \max\{E(\bar{w}_{jo}(s, ps), ps) - A_{vr}(w), 0\} dF_{vr}(w)$$

$$= p_{doc} \int_0 \max\{E(\bar{w}_{jo}(s, ps), ps) - A_{vr}(w), 0\} dF_{vr}(w)$$

$$= p_{doc} \lambda_{vr} \left( F_{vr}(\bar{w}_{jo}(s, ps)) \left( E(\bar{w}_{jo}(s, ps), ps) - E(\bar{w}_{vr}(s, ps), ps) \right) \right)$$

$$- \left( \int_{\bar{w}_{vr}(s, ps)}^{\bar{w}_{jo}(s, ps)} \frac{w - \bar{w}_{vr}(s, ps)}{1 - \beta(1 - \delta)} dF_{vr}(w) \right)$$

$$= p_{doc} \lambda_{vr} \left( F_{vr}(\bar{w}_{jo}(s, ps)) \frac{\bar{w}_{jo}(s, ps) - \bar{w}_{vr}(s, ps)}{1 - \beta(1 - \delta)} \right) - \frac{\bar{w}_{jo}(s, ps)}{\bar{w}_{vr}(s, ps)} \left( \int_{\bar{w}_{vr}(s, ps)}^{\bar{w}_{jo}(s, ps)} \frac{w - \bar{w}_{vr}(s, ps)}{1 - \beta(1 - \delta)} dF_{vr}(w) \right)$$

After inserting (B.12), (2.2) and rearranging we have (for $s > 0$)

$$E(\bar{w}_{jo}(s + 1, ps), ps) =$$

$$\beta(1 - p_{sick}) \left[ p_{jo} \int_{\bar{w}_{jo}(s, ps)}^{+\infty} \frac{w - \bar{w}_{jo}(s, ps)}{1 - \beta(1 - \delta)} dF_{jo}(w) + p_{vr} \lambda_{vr} \int_{\bar{w}_{vr}(s, ps)}^{+\infty} \frac{w - \bar{w}_{vr}(s, ps)}{1 - \beta(1 - \delta)} dF_{vr}(w) \right]$$

$$+ p_{doc} \left( F_{vr}(\bar{w}_{jo}(s, ps)) \frac{\bar{w}_{jo}(s, ps) - \bar{w}_{vr}(s, ps)}{1 - \beta(1 - \delta)} \right) - \left( \int_{\bar{w}_{vr}(s, ps)}^{\bar{w}_{jo}(s, ps)} \frac{w - \bar{w}_{vr}(s, ps)}{1 - \beta(1 - \delta)} dF_{vr}(w) \right)$$

$$- \lambda_{vr} p_{vr} \frac{\bar{w}_{jo}(s, ps) - \bar{w}_{vr}(s, ps)}{1 - \beta(1 - \delta)} \right] + \beta E(\bar{w}_{jo}(s, ps), ps). \quad (B.16)$$

Note from (2.3) that it holds that

$$U(0, ps) = U(1, ps) + b.$$ 

Together with (B.12) and (B.3) we thus have

$$\bar{w}_{jo}(0, ps) = \bar{w}_{jo}(1, ps) + (1 - \beta(1 - \delta))b. \quad (B.17)$$

We use equation (B.16) (for $s = 1, \ldots, K$ and $ps = 1, \ldots, \overline{ps} - 1$) equation (B.17), equation (B.14) (for $s = 0, \ldots, K$ and $ps = 1, \ldots, \overline{ps} - 1$) and equation (B.10) to solve for the reservation wages $\bar{w}_{jo}(s, ps), \bar{w}_{vr}(s, ps)$ for (for $s = 0, \ldots, K$ and $ps = 1, \ldots, \overline{ps} - 1$) and $\bar{w}_\Phi$. Taken together we thus have a system of $2 \times K \times (\overline{ps} - 1) + 1$ reservation wage equations that we solve for the same number of reservation wages.
B.1.4 Likelihood Contributions

In the following the individual subscript \( i \) is omitted for notational convenience. For transitions from unemployment to unemployment the likelihood contribution \( h_{t}^{uu} = h_t(e_t = 0, \, vr_t, \, sick_t, \, sanc_t, e_{t-1} = 0| \theta) \) is given by

\[
h_{t}^{uu} = \begin{cases} 
(1 - p_{sick})(1 - p_{jo})\lambda_{jo}(1 - F_{\gamma_{jo}}(w_{jo,s})) - p_{vr} & \text{if } (vr_t = 0, \, sick_t = 0, \, s_t = s) \\
p_{sick}(1 - p_{vr}) & \text{if } (vr_t = 0, \, sick_t = 1) \\
p_{vr}(p_{sick} + (1 - p_{sick})F_{\gamma_{vr}}(w_{jo,s})p_{doc}) & \text{if } (vr_t = 1, \, sick_t = 0, \, s_t = s) \\
p_{vr}(1 - p_{sick})F_{\gamma_{vr}}(w_{vr,s})(1 - p_{doc})\lambda_{vr}(1 - p_{sanc}) + (1 - p_{doc}F_{\gamma_{vr}}(w_{jo,s}))(1 - \lambda_{vr}) & \text{if } (vr_t = 1, \, sick_t = 0, \, sanc_t = 0, \, s_t = s) \\
p_{vr}(1 - p_{sick})F_{\gamma_{vr}}(w_{vr,s})(1 - p_{doc})\lambda_{vr}p_{sanc} & \text{if } (vr_t = 1, \, sick_t = 0, \, sanc_t = 1, \, s_t = s). 
\end{cases}
\]

For transitions from unemployment to employment \( h_{t}^{ue} = h_t(e_t = 1, \, vr_t, \, e_{t-1} = 0| \theta) \) we have

\[
h_{t}^{ue} = \begin{cases} 
(1 - p_{sick})p_{jo}\lambda_{jo} \int f_{\gamma_{jo}}(w) 1\{w \geq w_{jo,s}\} \frac{1}{\sigma_e} \phi\left(\frac{w - \tilde{w}_{acc}}{\sigma_e}\right)dw & \text{if } (vr_t = 0, \, \tilde{w}_{acc}, \, s_t = s) \\
(1 - p_{sick})p_{vr}\lambda_{vr} \int f_{\gamma_{vr}}(w) 1\{w \geq w_{vr,s}\} (1 - p_{doc}) 1\{w \leq w_{jo,s}\} \frac{1}{\sigma_e} \phi\left(\frac{w - \tilde{w}_{acc}}{\sigma_e}\right)dw & \text{if } (vr_t = 1, \, \tilde{w}_{acc}, \, s_t = s) 
\end{cases}
\]

and finally for transitions from employment to unemployment \( h_{t}^{cu} = h_t(e_t = 0, \, e_{t-1} = 1| \theta) \) and transitions from employment to employment \( h_{t}^{ce} = h_t(e_t = 1, \, e_{t-1} = 1| \theta) \) we have

\[
h_{t}^{cu} = 1 - h_{t}^{ce} = \delta.
\]
B.2 Figures

Figure B.1: Empirical distribution of accepted wages

Notes: Monthly wages in Euro. Plotted separately for jobs found in months in which a VR was received and months in which no VR was received. Based on a sample of 69,788 workers observed from 2000 to 2002. The sample restrictions described in 2.3 apply.

Figure B.2: Empirical distribution of UI benefits

Notes: Monthly UI benefits in Euro. Plotted separately for jobs found in months in which a VR was received and months in which no VR was received. Based on a sample of 69,788 workers observed from 2000 to 2002. The sample restrictions described in 2.3 apply.
Figure B.3: Job take-up after sanctions

Notes: Month $t = 0$ refers to the first month after a sanction is imposed. Displayed are job take-up rates $t$ months after a sanction is imposed and for non-sanctioned unemployed workers, together with 95% confidence intervals.
### B.3 Tables

**Table B.1:** Summary statistics, unemployment and job search outcomes

<table>
<thead>
<tr>
<th>Sample</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full sample</strong></td>
<td></td>
</tr>
<tr>
<td>Months starting in unemployment</td>
<td>59.65%</td>
</tr>
<tr>
<td>Months starting in employment</td>
<td>40.35%</td>
</tr>
<tr>
<td><strong>Months starting in unemployment</strong></td>
<td></td>
</tr>
<tr>
<td>VR</td>
<td>13.15%</td>
</tr>
<tr>
<td>Sickness absence (≥ 2 weeks)</td>
<td>2.00%</td>
</tr>
<tr>
<td>Job take-up</td>
<td>4.95%</td>
</tr>
<tr>
<td>Avg. accepted wage</td>
<td>2130</td>
</tr>
<tr>
<td><strong>VR received</strong></td>
<td></td>
</tr>
<tr>
<td>Sickness absence (≥ 2 weeks)</td>
<td>3.34%</td>
</tr>
<tr>
<td>Job take-up</td>
<td>14.57%</td>
</tr>
<tr>
<td>Sanction</td>
<td>0.33%</td>
</tr>
</tbody>
</table>

*Notes:* Summary statistics computed from a sample of 69,788 workers observed from 2000 to 2002. The sample restrictions described in 2.3 apply. A time unit is a month.

**Table B.2:** Average structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(w_{jo})$</td>
<td>1781</td>
</tr>
<tr>
<td>$E(w_{vr})$</td>
<td>1514</td>
</tr>
<tr>
<td>$p_{jo}$</td>
<td>0.078</td>
</tr>
<tr>
<td>$p_{vr}$</td>
<td>0.129</td>
</tr>
<tr>
<td>$p_{sick}$</td>
<td>0.022</td>
</tr>
<tr>
<td>$\lambda_{vr}$</td>
<td>0.151</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.066</td>
</tr>
<tr>
<td>$p_{doc}$</td>
<td>0.037</td>
</tr>
<tr>
<td>$p_{sanc}$</td>
<td>0.304</td>
</tr>
</tbody>
</table>

*Notes:* The table displays estimates of the structural model parameters averaged over the empirical distribution of observables, $X_i$, and the estimated distribution of the unobserved factor, $\nu$. 

132
Table B.3: Implied structural parameters

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>apprenticeship</th>
<th>health restricted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$E(w_{jo})$</td>
<td>1726</td>
<td>1684</td>
</tr>
<tr>
<td>$E(w_{vr})$</td>
<td>1526</td>
<td>1385</td>
</tr>
<tr>
<td>$p_{jo}$</td>
<td>0.075</td>
<td>0.074</td>
</tr>
<tr>
<td>$p_{vr}$</td>
<td>0.124</td>
<td>0.102</td>
</tr>
<tr>
<td>$p_{sick}$</td>
<td>0.016</td>
<td>0.022</td>
</tr>
<tr>
<td>$\lambda_{vr}$</td>
<td>0.152</td>
<td>0.145</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.066</td>
<td>0.068</td>
</tr>
<tr>
<td>$\nu = 3.554$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{doc}$</td>
<td>0.394</td>
<td>0.349</td>
</tr>
<tr>
<td>$p_{sanc}$</td>
<td>0.935</td>
<td>0.945</td>
</tr>
<tr>
<td>$\nu = -0.934$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{doc}$</td>
<td>0.009</td>
<td>0.008</td>
</tr>
<tr>
<td>$p_{sanc}$</td>
<td>0.125</td>
<td>0.146</td>
</tr>
<tr>
<td>$\nu = 0.399$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{doc}$</td>
<td>0.032</td>
<td>0.027</td>
</tr>
<tr>
<td>$p_{sanc}$</td>
<td>0.359</td>
<td>0.402</td>
</tr>
</tbody>
</table>

Notes: The table displays estimates of the structural model parameters for all combinations of observables, $X_i$, and the unobserved factor, $\nu$, fixing individual age at its median value (age 38).
Appendix C

Appendix to Chapter 3

Table C.1: Summary statistics firm hiring

<table>
<thead>
<tr>
<th>Sample</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Firms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vacancies</td>
<td>1.079</td>
<td>8.38</td>
</tr>
<tr>
<td>Vacancies reported at the BA</td>
<td>0.334</td>
<td>3.76</td>
</tr>
<tr>
<td>Firms with vacancies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Using VRs</td>
<td>22.6%</td>
<td></td>
</tr>
<tr>
<td>Not using VRs</td>
<td>77.4%</td>
<td></td>
</tr>
<tr>
<td>Firms using VRs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hiring VR candidate</td>
<td>26.8%</td>
<td></td>
</tr>
<tr>
<td>Hiring other candidate</td>
<td>65.2%</td>
<td></td>
</tr>
<tr>
<td>Hiring no candidate</td>
<td>8%</td>
<td></td>
</tr>
<tr>
<td>Firms not using VRs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hiring candidate</td>
<td>89.7%</td>
<td></td>
</tr>
<tr>
<td>Hiring no candidate</td>
<td>10.3%</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Summary statistics from the IAB Job Vacancy Survey. Computations based on a subsample of 13,428 firms surveyed in 2008. The sample restrictions described 3.4 apply.
Table C.2: Summary statistics, applicant pool

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applicants total</td>
<td>17.64</td>
<td>48.93</td>
</tr>
<tr>
<td>Applicants via VR</td>
<td>2.38</td>
<td>9.19</td>
</tr>
</tbody>
</table>

Notes: Summary statistics from the IAB Job Vacancy Survey. Computations based on a subsample of 13,428 firms surveyed in 2008. The sample restrictions described 3.4 apply.

Table C.3: Summary statistics, hired candidates

<table>
<thead>
<tr>
<th>Sample</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Firms not using BA hiring services</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>34.8</td>
<td>10.284</td>
</tr>
<tr>
<td>Previously unemployed</td>
<td>39.2%</td>
<td></td>
</tr>
<tr>
<td>Previously employed</td>
<td>41.6%</td>
<td></td>
</tr>
<tr>
<td>Previously education</td>
<td>13.2%</td>
<td></td>
</tr>
<tr>
<td>Previously outside labor force</td>
<td>3.3%</td>
<td></td>
</tr>
<tr>
<td><strong>Firms using BA hiring services</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>37.3</td>
<td>11.45</td>
</tr>
<tr>
<td>Previously unemployed</td>
<td>83.2%</td>
<td></td>
</tr>
<tr>
<td>Previously employed</td>
<td>10.9%</td>
<td></td>
</tr>
<tr>
<td>Previously education</td>
<td>3.8%</td>
<td></td>
</tr>
<tr>
<td>Previously outside labor force</td>
<td>0%</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Summary statistics from the IAB Job Vacancy Survey. Computations based on a subsample of 13,428 firms surveyed in 2008. The sample restrictions described 3.4 apply.
<table>
<thead>
<tr>
<th>Table C.4: Computation of targeted moments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data Moment</strong></td>
</tr>
<tr>
<td>Employer matching rate</td>
</tr>
<tr>
<td>Queue length</td>
</tr>
<tr>
<td>Unemployment rate</td>
</tr>
<tr>
<td>Share with $\alpha$ applications</td>
</tr>
<tr>
<td>Share with $\iota$ vacancy referrals</td>
</tr>
<tr>
<td>Correlation, # applications/ # VRs</td>
</tr>
<tr>
<td>Avg. accepted wage</td>
</tr>
<tr>
<td>S.d., accepted wages</td>
</tr>
</tbody>
</table>
Eidesstattliche Erklärung

Hiermit erkläre ich, die vorliegende Dissertation selbstständig angefertigt und mich keiner anderen als der in ihr angegebenen Hilfsmittel bedient zu haben. Insbesondere sind sämtliche Zitate aus anderen Quellen als solche gekennzeichnet und mit Quellenangaben versehen.

Mannheim, 10.06.2019

Hanno Foerster
<table>
<thead>
<tr>
<th>Year Range</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sep. 2016 - Dec. 2016</td>
<td>Visiting graduate student, University College London.</td>
</tr>
<tr>
<td>2011 - 2013</td>
<td>M.Sc. in Economics, University of Bonn.</td>
</tr>
<tr>
<td>Aug. 2011 - May 2012</td>
<td>Visiting graduate student, University of California Berkeley.</td>
</tr>
<tr>
<td>2008 - 2011</td>
<td>B.Sc. in Economics, University of Bonn.</td>
</tr>
<tr>
<td>2008</td>
<td>Abitur, Gymnasium Broich, Mülheim an der Ruhr.</td>
</tr>
</tbody>
</table>