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A Small Volume Reduction that Melts Down the Market: Auctions with Endogenous Rationing

A Small Volume Reduction that Melts Down the Market: Auctions with Endogenous Rationing*

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Abstract

Auctions with endogenous rationing have been introduced to stimulate competition. Such (procurement) auctions reduce the volume put out to tender when competition is low. This paper finds a strong negative effect of endogenous rationing on participation when bid-preparation is costly, counteracting the aim to stimulate competition. For multiple auctioneer's objectives mentioned in directives, we derive optimal mechanisms, which differ due to different evaluation of the tradeoff between participation and bid-preparation costs. Thus, the auctioneer needs to decide on an objective. However, reducing bid-preparation costs improves the optimal values of multiple objective functions.

Keywords: auction, participation, market design, optimal mechanism, renewable energy support
JEL: D82, Q48, D47, D44

1. Introduction

A specter is haunting the world of auctions, especially in the energy sector – the specter of endogenous rationing. In multi-unit auctions with endogenous rationing, bids do not only determine the prices but also the auction volume, i.e., how much is auctioned. This paper analyzes multi-unit procurement auctions with endogenous rationing in an environment of costly participation and single-unit supply. Additionally, we derive optimal mechanisms for different auctioneer's objectives that have been announced in markets in which auctions with endogenous rationing are used.

Procurement auctions find a growing number of applications throughout different fields in both industry and public procurement. Since the auctioneer buys goods

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or services, it is vital to ensure the offers' quality or seriousness. The auctioneer therefore often requests a certain kind of project preparation as a requirement for participation. To fulfill these requirements, bidders have to undertake costly measures, i.e., firms have to invest in their project even before they know whether they can actually realize it.[1] We call these costs bid-preparation costs. Only firms whose expected profit from the auction covers their bid-preparation costs will participate in the auction.

A worldwide large and growing field of application of multi-unit auctions with costly bid-preparation are auctions for renewable energy support, whose yearly total award prices reach a twelve-figure dollar amount.[2] In these auctions, bid-preparation costs play an essential role and are substantial. They mainly arise because bidders have to meet physical requirements by submitting a (partial) approval for building a plant on a specific site (Wigand et al., 2016; Mora et al., 2017). For onshore wind projects, the costs of the physical requirements are between two and ten percent of the invested amount (Wallasch et al., 2015; Quentin, 2015; Kitzing and Wendring, 2016). The field of auctions for renewable energy support became very large and is still growing because, for example, the State Aid Guidelines of the European Commission (2014) make auctions obligatory for all new support schemes for which member states wish to obtain state aid approval. To achieve the ambitious development goals for renewable energy, the auctioned volumes must increase.[3] However, this demand for new renewable energy sites requires a correspondingly high supply. Since May 2018, the auctions for onshore wind in Germany experienced either an under-subscription or a narrow over-subscription (BNetzA, 2019). Similar trends can be seen in other countries such as Brazil and France (Robert and Simon, 2019). Due to the lack of competition in these auctions, (almost) all submitted bids were successful and prices increased to the level of the reserve price. This development questions the use of auctions. They were introduced to provide support at market-

[1]Typically, the required measures are also necessary for realizing a project. Such measures are for example the development of a prototype in the industry sector or the collection of construction permits in the building sector.

[2]Worldwide, an estimated total capacity of 111 gigawatt of renewable energy was auctioned in 2017–2018 (IRENA, 2019). For estimating the monetary value, we set an average price of USD 50 per MWh, an average duration of support of 20 years, and an average of 2000 full load hours per year (mix of different renewable energy sources). This leads to a monetary value of USD 222 billion in 2017–2018.

[3]Transforming the world's energy systems toward affordable and clean energy is one of the sustainable development goals that all United Nations Member States adopted in 2015 (United Nations, 2015). One of the main measures to achieve this goal is to increase the share of renewable energy in the energy mix. For example, Germany plans to increase the share of renewable energy in the gross power consumption to 65% by 2030 (BReg, 2019a).

based payments, to allocate support efficiently, and to learn about the cost development in a competitive environment of renewable energy production. However, now the concern arises that in future auctions all participating projects' support will be equal to the reserve price, although the costs are lower and there is a belief that more projects with lower costs would be feasible.

Auctions with endogenous rationing have been suggested as a solution to the problems of low competition and high auction prices (IRENA, 2015; Müller, 2018; BReg, 2019b). Endogenous rationing means that the auction volume is adapted to the observed supply according to pre-specified rules. It is supposed to generate competition and, thus, keep prices low by preventing that all bidders win even if the number of bids is below the number of goods. Two concepts of endogenous rationing are in the focus: auctions with endogenous volume adaption and auctions with endogenous reserve price.

In auctions with endogenous volume, the final auction volume is reduced below the original auction volume if the number of bids is low. For example, the auction may stipulate that only a pre-announced share (e.g., 80 %) of the original auction volume will be awarded to bidders if the total bid volume does not exceed the original auction volume. In this case, the highest 20 % of bids will not win. This kind of rationing will be applied for example in Germany (BReg, 2019b), France (Ministre de l'Europe, 2018), and Ukraine (Legislation of Ukraine, 2019). Similar measures are applied in Brazil (IRENA, 2015), Greece (Papachristou et al., 2017), Kazakhstan (Abylkairova, 2018), Mexico (Jiménez, 2016), and Switzerland (Bundesamt für Energie, 2019).

In the second approach – the auction with endogenous reserve price – the submitted bids determine a reserve price below the original reserve price. Only bids below the endogenous reserve price are successful. Variations differ in the way the bids determine the endogenous reserve price, e.g., by the bids' mean or median. Endogenous reserve prices are applied in France (Ministre de l'Europe, 2019) and Peru (Comité, 2015).

Our analysis applies to markets in which preparing a bid is costly. A firm participates in the auction only if her expected profit from the auction exceeds the bid-preparation costs. Introducing endogenous rationing hits exactly this sensitive point of an auction with costly bid preparation. By ensuring that there will always be at least one losing bidder – the property that motivates its application – endogenous rationing removes any participation incentive for the weakest bidder, causing any bidder that expects to be the weakest participant to stay out. As a result, there may be no participation incentive for any firm.

Our theoretical analysis reveals that endogenous rationing has a strong negative effect on participation. Whereas many countries discuss or decided to introduce endogenous rationing, there is little experience with it because it has not yet been implemented or applied repeatedly or the data are not available. An example of an auction with endogenous volume adaption that has been conducted for several years is the auction for the support of energy efficiency projects and programs in Switzerland (Bundesamt für Energie, 2019). This auction indeed experienced a relative decrease in the bid volume after the introduction of the rationing rule.

Proponents of endogenous rationing posit that it ensures against undesirable extreme auction outcomes. In the context of renewable energy auctions the desire for such insurance may originate in the following two facts. First, the heterogenous community of political decision makers pursues multiple objectives such as maximizing social welfare and minimizing costs at the same time. Second, the social value of renewable energy plants is not clearly defined.[4] In Section 4, we define the four most prevalent objectives for procurement auctions and derive for each objective the optimal mechanism. The optimal mechanisms for the two common objectives to maximize the auctioneer's surplus or social welfare can each be implemented in two ways. One design is an auction with an optimal reserve price. In the other design, all participants are paid their bid-preparation costs for participating in an auction with a (lower) reserve price. Both designs incentivize the same participation in the auction. This participation differs from the participation in auctions with endogenous rationing, which therefore are not optimal with respect to these two main objectives.

The fact that it is optimal to (implicitly or explicitly) refund the bid-preparation

[4] Kreiss et al. (2019) point out that the State Aid Guidelines of the European Commission (2014), which are the superior directive for the support of renewable energies in the European Union, stipulate related but different goals. Their prioritization is however unclear. Among these goals are the minimization of the support payments and the minimization of the overall costs to achieve the renewables expansion goals. The ambiguity between these two goals is also found in the German law (BReg, 2017). The minimization of the support payments is stipulated, e.g., in the Netherlands and the United Kingdom, and also proposed for developing countries, in some cases as stand-alone objective without referring to the expansion goal. The minimization of the overall costs to achieve the expansion goal is stated, e.g., in California (US) and Mexico (Kreiss et al., 2019; IRENA, 2013). Minimizing the overall (social) costs is related to maximizing the social welfare, which is also a goal in national laws (e.g., BMU, 2016; BReg, 2017; Kazakh Government, 2009). Implementing this goal requires determining a monetary value of renewable energy, which, however, is missing in the laws and directives. The maximization of the consumer surplus or low prices for the customers are also postulated (e.g., BMU, 2016; IRENA, 2013; Hochberg and Poudineh, 2018; Kreiss et al., 2019). A further goal is to achieve the targeted expansion of renewable energy, which implies meeting the demand in the renewable energy auctions. This goal is stated, e.g., in Germany, Kazakhstan, Brazil, Mexico, and also proposed for developing countries (BReg, 2017; Kazakh Government, 2009; Hochberg and Poudineh, 2018; IRENA, 2013).

costs emphasizes their influence on the market outcome. Indeed, we find that lowering the bid-preparation costs increases competition and improves the auctioneer's surplus and social welfare. In the context of renewable energy auctions, current press reports and governmental statements suggest that lower bid-preparation costs can be achieved by simplifying preparation procedures and by taking measures to reduce the number of legal disputes that delay or prevent projects.

This paper is related to the literature on sales auctions with variable supply and to the literature on auctions with bid-preparation costs. Several works analyze auctions with variable supply whereas, to the best of our knowledge, we are the first to analyze auctions with endogenous rationing. Damianov and Becker (2010) analyze sales auctions with variable supply, in which the monopolistic seller chooses an optimal auction volume given the bids. This models a practice commonly applied in treasury auctions, in which the auction volume or the rule to determine the auction volume is not fixed before the auction (Nyborg et al., 2002). This practice originates in the wish to avoid low-price equilibria. Since bidders in treasury auctions have multi-unit demand and are allowed to submit non-increasing demand functions, they have an incentive to coordinate on low-price equilibria by strategically reducing their demand. Damianov (2005) and Damianov and Becker (2010) show that the seller can eliminate low-price equilibria by adjusting the supply to the demand.

There are crucial differences between treasury auctions and auctions for renewable energy support as well as between the ex-post supply adjustment and endogenous rationing as analyzed in this paper. Supply adjustment and endogenous rationing differ in that supply adjustment permits that all bids are successful whereas the motivation for endogenous rationing is to assure that there is at least one losing bid. In particular, in a price-discriminatory (pay-as-bid) auction with supply adjustment an optimizing monopsonist would accept all bids below his constant item value. In contrast to auctions for renewable energy support, treasury auctions are offered frequently, some even on a daily basis. Bidders repeatedly demand multiple units of the good and their bid-preparation costs are insignificant. In renewable energy auctions and other procurement auctions, projects are awarded that will be realized exactly once, most of the bidders participate with only one project,[5] and their bid-preparation costs are crucial. There is often only one or very few chances for a bidder to win.[6]

[5]The percentage of single-project bidders in renewable energy auctions is often above 90% (BNetzA, 2019). One reason is that usually for each project a legally independent project company is founded.

[6]Renewable energy auctions usually take place at most a few times a year and re-participation entails further costs due to, e.g., rescheduling of the project or renewal of permissions.

Several works investigate standard auctions with bid-preparation costs and voluntary participation in single-unit settings where bidders learn their private information before deciding on participation. Stegemann (1996) shows payoff equivalence of the symmetric equilibria of the second-price auction and the pay-as-bid auction. He, as well as Tan and Yilankaya (2006) and Celik and Yilankaya (2009) also analyze asymmetric equilibria and asymmetric mechanisms whereas our analysis focuses on symmetric equilibria in symmetric mechanisms. Samuelson (1985) derives the socially optimal reserve price and the auctioneer's surplus-maximizing reserve price. Li and Zheng (2009) find empirical evidence for predictions of the model, for example, a lower entry probability of bidders when the number of potential bidders increases. Menezes and Monteiro (2000) derive an optimal (revenue-maximizing) sales mechanism. Their optimal mechanism, translated into a procurement setting, refunds the bid-preparation costs to all participating bidders and conducts a second-price auction with a reserve price equal to the highest costs of a participant, the cutoff type. We contribute to this literature by extending the analysis to multi-unit auctions and by providing a unifying view. Our results imply that the auction with surplus-maximizing reserve price by Samuelson (1985) implements the same optimal mechanism as the one by Menezes and Monteiro (2000) and that Samuelson's socially optimal reserve price is one way to implement the socially optimal symmetric mechanism.

2. Basic Model

We consider a multi-unit procurement auction for k units of a good, $k \geq 1$. The set of potential bidders consists of n risk-neutral firms each with single-unit supply, $n \geq 1$. We consider both $n \leq k$ and $n > k$ because endogenous rationing has been suggested in particular for auctions with low or even no competition. Firms are symmetric and have independent private costs for supplying the good. The firms' private costs $x_1, x_2, \ldots, x_n$ are independently drawn from the distribution F with density f and full support on $[\underline{x}, \overline{x}]$, $0 \leq \underline{x} < \bar{x}$. Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and $\mathbf{x}_{-i} = (x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$. Furthermore, let $F_{(k,n)}$ denote the distribution function of the k-th lowest of n independent signals and let $X_{(k,n)}$ denote the associated random variable. Thus, $F_{(k,n-1)} = \sum_{i=k}^{n-1} \binom{n-1}{i} F(x)^i (1 - F(x))^{n-1-i}$ if $n > k$, and we define $F_{(k,n-1)} = 0$ if $n \leq k$.

Our model of a multi-unit procurement auction follows the approach of Samuelson (1985) for a single-unit procurement auction. Firms simultaneously decide on their participation in the auction and on their bidding strategy. The bidding strategy is relevant only if the firm participates because only participating firms can bid and win in the auction. We consider only pure strategies. Let m denote the number of firms that participate in the auction. Clearly, $m \leq n$.

To participate in the auction, each firm has to incur bid-preparation costs $c > 0$, e.g., to meet qualification requirements set by the auctioneer. These are sunk costs for the firms when the auction starts and they are not paid to the auctioneer. A firm knows c and her private costs x_i when she decides about her participation. If firm i participates in the auction, she has an expected profit $\pi(x_i)$ from the auction. Her profit from the auction is $p - x_i$ if she wins a good at the payment p and is zero, otherwise. Firm i aims to maximize her expected payoff $\Pi(x_i)$ from the game, where $\Pi(x_i) = \pi(x_i) - c$ if she participates in the auction and $\Pi(x_i) = 0$ if she does not participate.

Our analysis, which comprises many multi-unit procurement auction formats, assumes the following properties:

(P1) The n firms simultaneously decide whether or not to participate in the auction and commit to a bid if they participate. Thus, when bidding, the firms know n but do not know m.[7]

(P2) Bids may not exceed a reserve price $r \in \mathbb{R}_+$, $r > \underline{x} + c$, set by the auctioneer.[8]

(P3) The k lowest bids win if $m \geq k$ (ties at the k-th lowest bid are broken randomly); all other firms obtain nothing. If $m < k$, all m bids win.

(P4) The reserve price r is the maximum payment from the auction, and for each firm there exists a bid such that the firm's payment with this bid is r if $m \leq k$.

(P5) Participating firms apply a symmetric bidding function $\beta(x_i, r)$. If $n > k$, $\beta(x_i, r)$ is strictly increasing in x_i. If $n \leq k$, $\beta(x_i, r)$ is weakly increasing in x_i. Let $\boldsymbol{\beta}(\mathbf{x}, r) = (\beta(x_1, r), \beta(x_2, r), \ldots, \beta(x_n, r))$. A symmetric equilibrium $\boldsymbol{\beta}(\mathbf{x}, r)$ exists.

An auction with properties P1–P5 is called *standard auction (STD auction)*. STD auctions comprise sealed-bid procurement auctions with different payment rules: the *pay-as-bid auction* in which all winning bidders receive their bids, and the *uniform-price auction* in which the lowest rejected bid determines the price that all winning bidders receive.[9] P3 and P5 imply that the goods are supplied by the firms with the

[7] This assumption is in line with the actual auctions for renewable energy support in Germany, where the number of admitted pre-registered firms n is known, but the number of participating firms m is unknown. Menezes and Monteiro (2000) prove revenue equivalence of first- and second-price auction with known (i.e., revealed participation) and unknown number of bidders in the single-unit sales auction with bid-preparation costs.

[8] If $r < \underline{x} + c$, no firm will participate in the auction.

[9] Dynamic auctions that each are equivalent to one of the sealed-bid auctions are the descending-clock (English) auction or the ascending-clock (Dutch) auction (Krishna, 2010).

lowest costs, i.e., the allocation is efficient conditional on the participating firms. However, according to P1 and P3, it is possible that not all k goods are supplied, even if the costs of k or more firms are below r. This is because the preparation costs c prevent all firms with costs x_i above a *cutoff cost level* $\hat{x}$ from participating in the auction.[10]

Denote by $\hat{i}$ a firm whose costs are at the cutoff costs $\hat{x}$. Firm $\hat{i}$ submits the highest bid, which wins only if a maximum of $k-1$ other firms bid. Therefore, she adjusts her bid to these cases and bids to ensure the payment r in case she wins. For example, in the pay-as-bid auction, $\hat{i}$'s optimal bid is $\beta(\hat{x}, r) = r$, whereas in the uniform-price auction $\beta(\hat{x}, r) = \hat{x}$ (or, weakly dominated, $\beta(\hat{x}, r) = r$). If $\hat{x} < \bar{x}$, firm $\hat{i}$ is indifferent between participating and not participating, and the cutoff costs $\hat{x}$ are uniquely determined by $\Pi(\hat{x}, r, c, n) = 0$, where

$$\begin{aligned}\Pi(\hat{x}, r, c, n) &= (r - \hat{x})\left(1 - F_{(k,n-1)}(\hat{x})\right) - c \\ &= (r - \hat{x})\textstyle\sum_{i=0}^{\min\{k,n\}-1}\binom{n-1}{i}F(\hat{x})^i\left(1 - F(\hat{x})\right)^{n-1-i} - c\,. \end{aligned} \tag{1}$$

The cutoff costs $\hat{x}$ determine a firm's ex-ante participation probability $F(\hat{x})$. The following lemma collects properties of $F(\hat{x})$. A proof is given in the appendix.

Lemma 1. *For a firm's ex-ante participation probability $F(\hat{x})$ the following hold:*

- $F(\hat{x}) \begin{cases} < 1, & \text{if } n > k \text{ or } r < \bar{x} + c\,, \\ = 1, & \text{otherwise.} \end{cases}$
- $\frac{dF(\hat{x})}{dr} > 0$ *and* $\frac{dF(\hat{x})}{dc} < 0$ *if* $n > k$ *or* $r < \bar{x} + c$.
  $F(\hat{x})$ *increases in* k *if* $n > k$. $F(\hat{x})$ *decreases in* n *if* $n \geq k$.

Under mild conditions we have $\hat{x} < \bar{x}$, that is, high-cost firms do not participate. If $n > k$, no reserve price can induce full participation because type $\bar{x}$'s profit from the auction is zero if all firms participate. If $n \leq k$, each bid wins but the high-cost firms forgo the auction if the reserve price does not cover their supply costs and bid-preparation costs.

The intuition behind the reaction of a firm's ex-ante participation probability $F(\hat{x})$ and the cutoff $\hat{x}$ to changes in r, k, c, and n in Lemma 1 is as follows. Increasing the reserve price or (if $n > k$) the number of goods increases the marginal bidder's expected profit from the auction via an increased profit in case of winning or an

[10] A cutoff level $\hat{x}$ that separates participating firms with costs $x_i \leq \hat{x}$ from non-participating firms with $x_i > \hat{x}$ exists because P5 implies $\pi(x_i) > \pi(x_j)$ for $x_i < x_j$ in equilibrium. If this did not hold, firm i could profitably deviate by bidding like the higher-cost firm j.

increased winning probability. This results in higher cutoff costs $\hat{x}$, which implies a higher expected number of participants $\mathrm{E}[m]$. Bid-preparation costs c have to be absorbed by the expected profits from the auction. Therefore, a higher c lowers the cutoff level. A growth of the pool of firms reduces the marginal bidder's expected profits from the auction if $n \geq k$. Thus, $\hat{x}$ decreases in n.[11]

The cutoff type $\hat{x}$ is the same in all STD auctions and she is the worst-off type among the participants. Payoff equivalence holds for all STD auctions.[12]

Lemma 2. *STD auctions are payoff equivalent.*

That is, in a symmetric equilibrium, a firm with costs x has the same expected profit in all STD auctions and the auctioneer has the same expected surplus in all STD auctions. The symmetric bidding equilibrium in a pay-as-bid auction is, for all $x \in [\underline{x}, \hat{x}]$,

$$\beta^{PaB}(x,r) = \frac{(1-G(\hat{x}))r + \int_x^{\hat{x}} y g(y) dy}{1-G(x)} = x + \frac{(1-G(\hat{x}))(r-\hat{x}) + \int_x^{\hat{x}} 1 - G(y) dy}{1-G(x)},$$

where $G(x) = F_{(k,n-1)}(x)$. A symmetric bidding equilibrium in the uniform-price auction is $\beta^{UP}(x,r) = x$ for all $x \in [\underline{x}, \hat{x}]$. Lemma 2 and the equilibria are proven in the appendix.[13]

If there is no competition, $n \leq k$, the equilibrium payment of STD auctions equals r, either because all bidders bid r in anticipation of the low number of bids or because the payment rule determines a uniform payment of r.

Instruments of endogenous rationing have been suggested as a means to avoid high payments in cases with $n \leq k$. However, the next section shows that the negative effect of endogenous rationing on participation in all cases of n and k is sufficient to erase any desired effect on payments.

3. Endogenous Rationing (ER)

In this section, we incorporate instruments of endogenous rationing (ER) into the STD auctions of Section 2. When we compare auctions with ER (ER auctions) and auctions without ER (STD auctions), we assume the same reserve price r, number of goods k, number of firms n, and payment rule, unless specified otherwise.

[11] Li and Zheng (2009) find empirical evidence of this "entry effect": a negative relationship between the number of potential bidders n and the participation probability.

[12] See Stegemann (1996)and Menezes and Monteiro (2000) for payoff equivalence between single-unit first- and second-price auctions with bid-preparation costs.

[13] In a descending clock auction, $\beta(x,r) = x$ for $x \in [\underline{x}, \hat{x}]$ is a firm's optimal exit price.

ER auctions have properties P1, P2, and P5. They violate P4 because they are designed to prevent an auction price r when $n \leq k$. They also violate P3: albeit the lowest bids win, less than m bids win if $m \leq k$ and less than k bids may win even though more bids have been submitted. Thus, ER auctions assign goods differently than STD auctions and give a different payment to the worst-off type. Therefore, payoff equivalence to the STD auctions fails (e.g., Krishna, 2010, Section 3.2.2).

ER auctions share some basic properties of STD auctions, e.g., a bidder's probability of winning increases if she reduces her bid and a winning bidder is paid at least her bid. They differ from STD auctions in that the auction volume is variable. As an important consequence, there will always be at least one losing bidder in the auction (if at least one firm participates and there is no volume floor), irrespective of the relationship between k and n. This is a crucial property if participation is costly.

Auctions with endogenous volume and with endogenous reserve price can have different properties. In auctions with endogenous reserve price, decreasing a bid can even reduce the auction volume, whereas with endogenous volume, the auction volume depends on the number but not on the size of the bids. However, some endogenous reserve price rules, like quantile rules, can be translated into endogenous volume rules.

3.1. Endogenous Auction Volume (EAV)

One option to endogenously ration the awards is an endogenous adaption of the auction volume to the number of bidders, which we refer to in short as *endogenous auction volume (EAV)*. Concretely, if m bids are submitted, the number of winning bids is determined according to a commonly known function

$$\kappa(m) \begin{cases} = k & \text{if } m > \bar{\mu}\,, \\ < \min\{k, m\} & \text{if } \underline{\mu} < m \leq \bar{\mu}\,, \\ = m & \text{if } m \leq \underline{\mu}\,. \end{cases}$$

The integer parameters $\underline{\mu}$ and $\bar{\mu}$ with $0 \leq \underline{\mu} < k \leq \bar{\mu}$ mark the limits of the range of the number of bids m in which the original auction volume $\min\{k, m\}$ is reduced. For the actual auction volume $\kappa(m)$ holds $\kappa(m) < \kappa(m+1)$ for all $m \in \{\underline{\mu}, \underline{\mu}+1, \ldots, \bar{\mu}-1\}$. Payments in the EAV auction are determined as in the related STD auction. For example, in an uniform-price auction, the $\kappa(m)$ winning bidders are paid the $(\kappa(m)+1)$-th lowest bid.

Examples of EAV auctions are planned auctions for innovation tenders in support of renewable energy sources in Germany. 80 % of the original volume will be awarded to bidders if $m \leq k$ (BReg, 2019b). Thus, $\kappa(m) = \lceil 0.8m \rceil$ if $m \leq \bar{\mu} = k$ and, implicitly, $\underline{\mu} = 4$, i.e., $\kappa(m) = m$ if $m \leq 4$.[14,15]

The endogenous rationing of the auction volume intends on keeping the payments low in case of low supply. However, the endogenous volume adaptation creates a strong adverse effect on participation.

Proposition 1. *In an EAV auction with volume k and limit $\underline{\mu}$, the cutoff costs $\hat{x}$ and the participants are the same as in a STD auction with the auction volume $\underline{\mu}$.*

Thus, even if an EAV auction puts a much larger volume out to tender than a STD auction, the expected number of bids may be the same in both auctions.[16]

The participants are determined by the cutoff costs $\hat{x}$, and these are the same in the EAV auction with volume k and in the STD auction with volume $\underline{\mu}$. In the EAV auction, the bidder with the cutoff costs $\hat{x}$ submits the highest bid and will win only if the total number of bidders is not above $\underline{\mu}$. Otherwise, the limited number of goods ($k < m$) or the rationing ($\kappa(m) < m$) prevents her from receiving a good. Because her bid wins only if she faces no competition, her payment is r (depending on the payment rule, either because she bids r or because the uniform price is equal to the reserve price). Therefore, she has the same expected profit as from a STD auction with the auction volume $\underline{\mu}$ (see (1)) and participates if and only if

$$(r - \hat{x}) \sum_{i=0}^{\min\{\underline{\mu},n\}-1} \binom{n-1}{i} F(\hat{x})^i \big(1 - F(\hat{x})\big)^{n-1-i} - c \geq 0. \tag{2}$$

This proves Proposition 1.

Comparing the EAV auction with an original (maximal) volume k and the STD auction with volume k with respect to the expected number of participants (i.e., bids), number of goods awarded, expected price(s), and bidders' profits (payoffs) yields the following results. The expected number of participating firms is lower in the EAV auction than in the STD auction because the cutoff costs are lower. The expected number of goods awarded is lower because fewer bids are expected

[14] According to the rule in Germany, no bid volume is cut in parts but the last winning bid is fully awarded. This results in an implicit volume floor of $\underline{\mu} = \max\{m : \lceil 0.8m \rceil = m\} = 4$. Unlike in our model, bid volumes may be heterogenous. Then, the lower bound on the number of winning bids can vary, but at least one bid will win.

[15] The abrupt switch from the volume k when $m = k + 1$ to the volume $0.8k$ when $m = k$ could be avoided by changing $\bar{\mu}$ to $1.25k$, i.e., $\kappa(m) = \lceil 0.8m \rceil$ if $m \leq 1.25k$.

[16] Referring to the 80 %-rule of Germany's auctions for innovation tenders, which implies $\underline{\mu} = 4$, this result suggests a small number of projects awarded in these auctions because in Germany's renewable energy auctions, which are conducted as STD auctions, the number of projects awarded is usually above twenty (BNetzA, 2019).

and it is possible that the number of bids falls in between $\underline{\mu}$ and k, such that the volume is reduced. With uniform pricing, in both EAV and STD auctions, bidders bid their costs. The expected cost of the price-determining bidder and, thus, the expected payment to each winning bidder is lower in the EAV auction than in the STD auction. Hence, the winning bidders' expected profits (payoffs) are lower in the EAV auction than in the STD auction. Since the payoff equivalence theorem applies to different EAV auction formats, these results also hold for the pay-as-bid auction and other auctions like the uniform-price auction in which the highest accepted bid determines the uniform price.[17]

Swiss Case

The Swiss auction for energy efficiency projects and programs implements a budget auction with EAV (Bundesamt für Energie, 2019). The auctioneer announces a budget for the auction. If more than 120 % of the announced budget is demanded by the bidders, the announced budget is awarded. If less than 120 % of the announced budget is demanded, then the awarded budget is reduced to 83.33 % of the demanded budget. The EAV rule has been introduced in 2013. Since 2013, the relative demand (ratio of demanded budget to announced budget) has continuously decreased (see Figure 1).[18]

3.2. Endogenous Reserve Price (ERP)

An *endogenous reserve price (ERP)* is derived from the submitted bids.

In the first step, the firms decide about their participation. Then, participating firms submit their bids, which may not exceed the default reserve price r. The bids of the $m \leq n$ participating firms are denoted by $\mathbf{b} = (b_1, b_2, \ldots, b_m)$. Next, the ERP $\varrho(\mathbf{b})$ is calculated on the basis of $\mathbf{b}$, where $\min_{i\in\{1,\ldots,m\}}\{b_i\} \leq \varrho(\mathbf{b}) \leq \max_{i\in\{1,\ldots,m\}}\{b_i\}$ and $1 \geq \frac{\partial\varrho(\mathbf{b})}{\partial b_i} \geq 0$ for all $i \in \{1, \ldots, m\}$. For the award, only bids $b < \varrho(\mathbf{b})$ are taken into consideration, or alternatively, $b \leq \varrho(\mathbf{b})$ and $b <$

[17] If $\underline{\mu} = 0$, no firm participates and payoff equivalence applies trivially. If $\underline{\mu} > 0$, $\hat{x}$ is the same in these auctions as in the (lowest-rejected-bid) uniform-price auction. Participants are in an auction with bidders' types distributed i.i.d. on $[\underline{x}, \hat{x}]$. In all auctions, the bids of the same lowest-ranked types win in a symmetric monotone equilibrium for all $\mathbf{x}$, and the expected payoff of the worst-off type $\hat{x}$ is the lowest payoff that incentivizes participation, c. Thus, payoff equivalence applies.

[18] Figure 1 shows the development over the years of the demanded budget and the awarded budget relative to the announced budget for projects and programs in total. There are separate announced budgets for projects and programs plus, until 2014, a third budget for the best projects or programs that are not awarded under their respective specific budget. In 2015, a second auction round and budget for projects has been introduced. The total announced budget increased from 9 million Swiss francs in 2010 to 50 million Swiss francs in 2018. The awarded budget can exceed the announced budget because the last project is fully awarded.

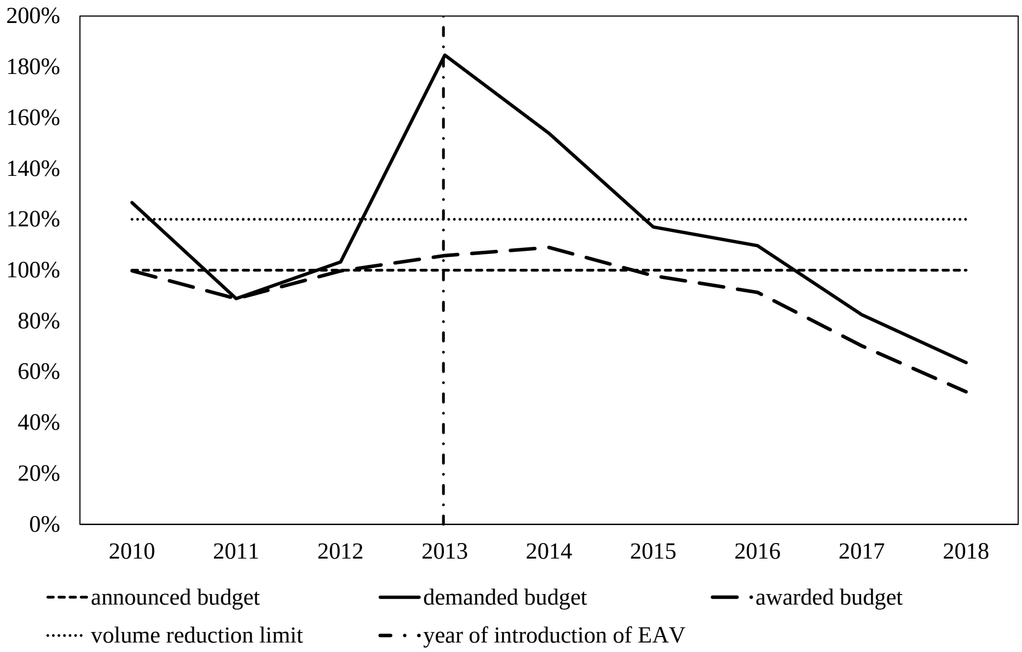

Figure 1: Development of the relative demanded and awarded budgets in the Swiss auction for energy efficiency projects and programs

$\max_{i\in\{1,\dots,m\}}\{b_i\}$.[19] We call these *accepted bids* and denote their number by m', $m' \leq m$. The payment rule is applied as if only the accepted bids were submitted and as if the ERP was the reserve price.[20]

We complement the ERP rule by a floor $\underline{\mu}$ and a ceiling $\bar{\mu}$, with $0 \leq \underline{\mu} < k \leq \bar{\mu}$. (The case of no floor or ceiling is included by $\underline{\mu} = 0$ or $\bar{\mu} = \infty$, respectively.) Floor and ceiling override rationing. If $m \geq \underline{\mu} > m'$, then the $\underline{\mu}$ lowest bids win (with random tie-breaking) and the reserve price equals the highest of the winning bids. If $m \leq \underline{\mu}$ or $m > \bar{\mu}$, all bids are accepted and the payment rule is applied with the reserve price $\max_{i\in\{1,\dots,m\}}\{b_i\}$ or r, respectively.

We focus on ERP rules with the property that for all $b_i \in (\underline{x}, \bar{x})$ there exist combinations of the other bidders' bids $\mathbf{b}_{-i}$ such that $\frac{\partial \varrho(\mathbf{b})}{\partial b_i} > 0$. Examples for ERP rules $\varrho(\mathbf{b})$ are the mean rules that determine the ERP based on the mean of the submitted bids, $\varrho(\mathbf{b}) = \alpha \frac{1}{m}\sum_{i=1}^{m} b_i$ with $\alpha \in (0,1]$, and the median rules that determine the ERP based on the median of the submitted bids, $\varrho(\mathbf{b}) = \alpha \cdot \text{median}(\mathbf{b})$ with $\alpha \in (0,1]$.[21]

[19] To enforce rationing if $n \leq k$ or $m \leq k$, the ERP winner-determination rule must prevent that all bids win if all bidders bid the same, e.g., r. This is met by both forms of the ERP rule.

[20] That means, if $m' \leq k$, in the uniform-price auction, the price is equal to $\varrho(\mathbf{b})$.

[21] A different auction with median-price rule is the Medicare auction analyzed by Cramton et al. (2015) and widely criticized by auction experts (see http://www.cramton.umd.edu/papers2010-2014/further-comments-of-concerned-auction-experts-on-medicare-bidding.pdf, accessed 09/16/2019). One of the reasons why the Medicare auction was predicted to perform badly is

With an ERP, it is not an equilibrium that all bidders bid r.[22] Therefore, we focus on symmetric and strictly increasing bidding functions (P4) and assume the existence of an equilibrium in symmetric pure strategies.

Proposition 2. *In an auction with ERP $\varrho(\mathbf{b})$, the cutoff costs $\hat{x}$ and the participants are the same as in a STD auction with the auction volume $k = \underline{\mu}$.*

A firm $\hat{i}$ with the cutoff costs $\hat{x}$ receives a good if and only if the number of bidders m is $\underline{\mu}$ or less. If $m > \underline{\mu}$, her bid will not win because the other bidders submit lower bids, and thus, firm $\hat{i}$'s bid $b_{\hat{i}} = \max_{i\in\{1,\dots,m\}}\{b_i\}$ is not below $\varrho(\mathbf{b})$. If $m \leq \underline{\mu}$, firm $\hat{i}$'s bid wins and, furthermore, determines $\varrho(\mathbf{b})$ and, therefore, her payment. Thus, firm $\hat{i}$ participates and bids $b_{\hat{i}} = r$ iff (2) holds. This proves Proposition 2.

The fact that firm $\hat{i}$'s decision problem is the same in an EAV auction as in an ERP auction reveals a parallelism between the two instruments. If $\underline{\mu} > 0$, then $\hat{x} > \underline{x}$, and all firms with $\hat{x} \geq x_i \geq \underline{x}$ participate in the auction. If the ERP is a quantile of the bids, then there is an EAV rule under which the same firms win and their payments are the same. For example, the median rule for the ERP corresponds to an EAV of 50 %.[23] However, there are also differences, where the ERP rules do not correspond to any EAV rule. Consider, for example, the ERP that is equal to the mean. With this rule, an additional bid can increase or decrease the number of winning bidders, which is impossible under an EAV rule. Furthermore, in the uniform-price auction, bidders have an incentive to bid above their costs because an increase in their bid may increase the ERP and may therefore increase their payment. In contrast, bidders cannot influence their payment in the uniform-price EAV auction, in which it is optimal for the bidders to bid their costs. Payoff equivalence does not hold for different auctions (e.g., pay-as-bid vs. uniform pricing) with the same ERP rule because different payment rules may determine different sets of winners due to the differences in the bidding strategies.

This paper identifies low participation as a detrimental effect of endogenous rationing. Auctions with ER may also be plagued by strategic manipulations. ERP

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that it is not ex post individually rational in that winning bidders' payments can be below their bids: the k lowest bids win and the median of the winning bids is the payment to each winner. All auctions that we analyze are ex post individually rational. Further differences to our setting are that in the Medicare auction bids are non-binding, bids have to be above a bid floor (assumed to be below $\underline{x}$ and necessitated by the incentives evoked by the design), and bid-preparation costs are absent.

[22] Only if the bidders knew that $m \leq \underline{\mu}$, all bidders bidding r would be an equilibrium.

[23] Concretely, the median rule with $\alpha = 1$ that chooses the higher of the two middle values in case of an even number of bids as the ERP (i.e., $\rho(\mathbf{b}) = b_{(\lceil 0.5m \rceil)}$ where $b_{(j)}$ denotes the j-th lowest bid) corresponds to the 50 %-EAV rule that rejects the bid at the 50 % boundary in case of an odd number of bids (i.e., $\kappa(m) = \lfloor 0.5m \rfloor$).

auctions are susceptible to actions that artificially increase supply in order to profit from a higher probability of award and higher prices. A firm may participate with a serious bid and with an additional bid (either because multiple bids are feasible or under a different identity) that equals the reserve price to increase the probability of an award for the serious bid. The bid-preparation costs reduce the attractiveness or availability of such strategic supply expansion. Auctions with ER have further unfavorable properties. For example, with any form of ER, i.e., also with EAV, the average payment may increase when the number of bids increases.

4. Conflicting Objectives for Optimal Auction Design

The main reason for the suggestion of endogenous rationing (ER) is the supposed insurance against undesirable extreme outcomes, e.g., prices at the level of the reserve price (i.e., the highest possible price) in case of lack of competition (i.e., $n \leq k$). In the context of renewable energy auctions this is also because the auctioneer (i.e., the heterogenous community of political decision makers) often pursues multiple objectives that are not compatible (see Footnote 4). In this section, we emphasize the necessity to prioritize or weight the objectives. We will illustrate this by comparing different objectives typically brought forward in procurement auctions. We discuss four ex-ante objectives, which are prevalent in procurement auctions, particularly those organized by the government or other public institutions:

- O1 Maximize the auctioneer's expected surplus Π_0
- O2 Maximize the expected social welfare W (minimize the expected social costs)
- O3 Maximize the expected number of goods awarded K^a, given k
- O4 Minimize the auctioneer's expected payments for goods awarded

O1 and O2 require that prior to the auction the auctioneer assigns a value to acquiring goods. For example, in an auction for renewable energy support, the government's value for a good is the social value of the energy produced by the renewable energy plants. We assume that every acquired unit of the good has the same value v and that $c + \underline{x} < v$, so that the production of the goods increases social welfare.

An optimal auction with O1 maximizes the auctioneer's expected surplus Π_0. His expected surplus with the uniform-price rule (and any payoff-equivalent rule) is

$$\begin{aligned}\Pi_0 =&(v-r)\textstyle\sum_{i=1}^{\min\{k,n\}}\binom{n}{i} iF(\hat{x})^i\big(1-F(\hat{x})\big)^{n-i} \\ &+ k\left(v - \mathrm{E}[X_{(k+1,n)} \,|\, X_{(k+1,n)} \leq \hat{x}]\right) F_{(k+1,n)}(\hat{x})\,.\end{aligned} \tag{3}$$

An optimal auction with objective O2 maximizes the expected social welfare, which is equivalent to minimizing the expected social costs. In contrast to the auctioneer's surplus, the social welfare takes the firms' costs instead of the award prices into account. The components of the social welfare are the bidders' participation costs, the auction winners' costs to produce the good, and the auctioneer's value for every good awarded. The expected social welfare W is given by

$$\begin{aligned} W = &- ncF(\hat{x}) - \textstyle\sum_{i=1}^{\min\{k,n\}} E[X_{(i,n)}|X_{(i,n)} \leq \hat{x}]\, F_{(i,n)}(\hat{x}) \\ &+ v\Big(\min\{k,n\} - \textstyle\sum_{i=0}^{\min\{k,n\}-1} (\min\{k,n\} - i)\binom{n}{i} F(\hat{x})^i (1 - F(\hat{x}))^{n-i}\Big)\,. \end{aligned} \tag{4}$$

According to objective O3, the auctioneer aims to acquire as many as possible of the demanded goods k. The expected number of goods awarded K^a reveals how well an auction achieves this aim, where $K^a \leq k$. K^a is given by

$$K^a = \begin{cases} \sum_{i=1}^{k-1} \binom{n}{i} i F(\hat{x})^i \big(1 - F(\hat{x})\big)^{n-i} + kF_{(k,n)}(\hat{x}) & \text{if } n > k\,, \\ \sum_{i=1}^{n} \binom{n}{i} i F(\hat{x})^i \big(1 - F(\hat{x})\big)^{n-i} & \text{if } n \leq k\,. \end{cases} \tag{5}$$

The auctioneer's objective O4 is to minimize his expected payments, K^a multiplied by the average price. We include this objective because it has been brought forward (see Footnote 4). However, it is an implausible objective (see also Proposition 3 (O4)).

Proposition 3 presents optimal auctions for objectives O1 to O4. The uniform-price auction in the proposition could be replaced by any payoff equivalent auction. Proposition 3 is proven in the appendix.

Proposition 3. *An optimal symmetric mechanism for objective* Oj*,* $j \in \{1, 2, 3, 4\}$*, can be implemented by*

(Mr) a uniform-price auction with an optimal reserve price r_r^{Oj} *or*

(Mc) a uniform-price auction with a refund of c for all participating firms and an optimal reserve price r_c^{Oj}*.*

The optimal cutoff type $\hat{x}^{Oj}$ *is unique and is the same in Mr and Mc. The optimal cutoff type* $\hat{x}^{Oj}$ *and reserve prices* r_r^{Oj} *and* r_c^{Oj} *are the following.*

O1 *Assume* $x + \frac{F(x)}{f(x)}$ *is increasing.*

If $n > k$ *or* $v < \bar{x} + \frac{1}{f(\bar{x})} + c$	*Otherwise*
$\left(v - \hat{x}^{O1} - \frac{F(\hat{x}^{O1})}{f(\hat{x}^{O1})}\right)\left(1 - F_{(k,n-1)}(\hat{x}^{O1})\right) - c = 0$	$\hat{x}^{O1} = \bar{x}$
$r_r^{O1} = v - \frac{F(\hat{x}^{O1})}{f(\hat{x}^{O1})}$	$r_r^{O1} = \bar{x} + c$
$r_c^{O1} = \hat{x}^{O1}$	$r_c^{O1} = \hat{x}^{O1}$

O2	*If* $n > k$ *or* $v < \bar{x} + c$	*Otherwise*
	$(v - \hat{x}^{O2})\left(1 - F_{(k,n-1)}(\hat{x}^{O2})\right) - c = 0$	$\hat{x}^{O2} = \bar{x}$
	$r_r^{O2} = v$	$r_r^{O2} \geq \bar{x} + c$
	$r_c^{O2} = \hat{x}^{O2}$	$r_c^{O2} \geq \hat{x}^{O2}$

O3	*If* $n > k$	*Otherwise*
	$\hat{x}^{O3} = \bar{x}$	$\hat{x}^{O3} = \bar{x}$
	r_r^{O3} *does not exist*	$r_r^{O3} \geq \bar{x} + c$
	$r_c^{O3} = \hat{x}^{O3}$	$r_c^{O3} \geq \hat{x}^{O3}$

O4	
	$\hat{x}^{O3} = \underline{x}$
	$r_r^{O3} \leq \underline{x} + c$
	$r_c^{O3} \leq \underline{x} + c$

Proposition 3 enables us to compare the mechanisms that achieve objectives O1 to O4. In case of no competition ($n \leq k$) and a sufficiently high value v, objectives O1 to O3 are achieved by either mechanism Mr or Mc by choosing the lowest reserve price that incentivizes all firms to participate. In particular, these objectives are not achieved by excluding bidders, like endogenous rationing does. Only objective O4 is achieved by Mr or Mc with a reserve price that prevents any participation.[24]

In a competitive market ($n > k$) or if $v < \bar{x}+c$, the optimal cutoff type to achieve a specific objective is the same whether one implements Mr or Mc. However, the four objectives ask for different cutoffs. The optimal reserve prices r_r and r_c, if they exist, have the same order as the cutoff types.

Corollary 1 summarizes these relationships. It shows that achieving all objectives with the same auction design is impossible. In particular in a competitive market or if the value v is sufficiently low, each objective requires a specific auction design that is incompatible with the optimal design for other objectives.

Corollary 1. *Assume* $x + \frac{F(x)}{f(x)}$ *is increasing. If* $n \leq k$ *and* $v \geq \bar{x} + \frac{1}{f(\bar{x})} + c$ *then*

- $\hat{x}^{O3} = \hat{x}^{O2} = \hat{x}^{O1} = \bar{x} > \hat{x}^{O4} = \underline{x}$,
- $r_r = \bar{x} + c$ *and* $r_c = \bar{x}$ *achieve O1, O2, and O3 but not O4.*

[24] In order that r_r^{O4} and r_c^{O4} exist, we deviate from assumption (P2) and allow $r \leq \underline{x} + c$.

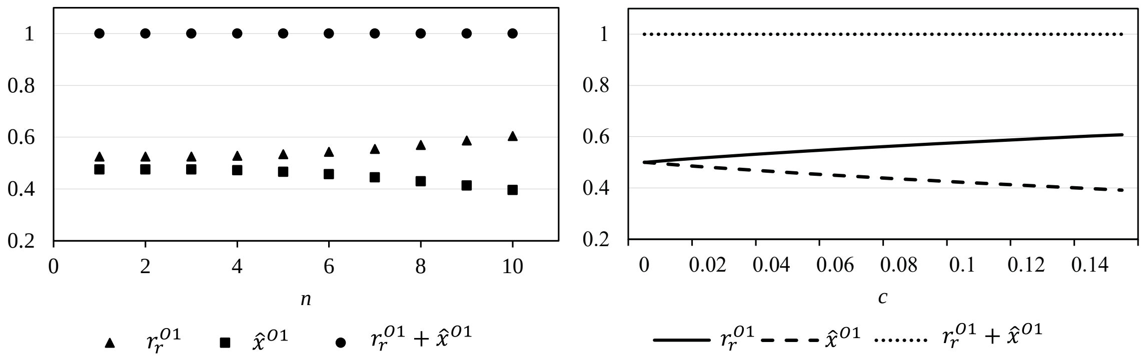

Figure 2: Reserve price r_r^{O1}, cutoff costs $\hat{x}^{O1}$, and their sum $\hat{x}^{O1} + r_r^{O1} = v$ when $x_i \overset{i.i.d}{\sim} U[0,1]$, $v = 1$, and $k = 3$. LHS: $c = 0.05$ and $n \in \{1, 2, \ldots, 10\}$. RHS: $n = 6$ and $c \in [0, 0.15]$.

If $n > k$ or $v < \bar{x} + c$ then

- $\hat{x}^{O3} > \hat{x}^{O2} > \hat{x}^{O1} > \hat{x}^{O4}$,
- $r_r^{O2} > r_r^{O1} > r_r^{O4}$, *and* $r_c^{O3} > r_c^{O2} > r_c^{O1} > r_c^{O4}$.

In case of low competition, the optimal mechanism aims at maximizing (for O1 to O3) or minimizing (for O4) participation. The case of strong competition or sufficiently low value v involves trade-offs between participation and bid-preparation costs. In the following paragraphs, we take a closer look at this case.

In Mc, the fixed payment of c induces participation by all firms with $x \leq \hat{x}$ and the reserve price $r_c = \hat{x}$ guarantees that firms with $x > \hat{x}$ do not participate.[25]

In an optimal mechanism Mr, the relationship between the reserve price and the cutoff type is more complex. For the reserve price r_r^{O1} and cutoff $\hat{x}^{O1}$ that maximize the auctioneer's surplus it holds that $v > r_r^{O1} > \hat{x}^{O1} > \underline{x}$. The optimal reserve price depends on the optimal cutoff, which varies with the number of firms. If n increases, $\hat{x}^{O1}$ decreases, and r_r^{O1} increases or decreases depending on whether the reverse hazard rate $F(x)/f(x)$ decreases or increases. In any case, by the assumption that $x + F(x)/f(x)$ increases, $\hat{x}^{O1}$ and r_r^{O1} diverge when n increases. The same applies to the effect of the costs c on reserve price and cutoff type. Figure 2 illustrates these properties of $\hat{x}^{O1}$ and r_r^{O1}, using a uniform distribution F on $[0, 1]$. With this distribution, the optimal reserve price and cutoff value add up to the auctioneer's value

$$r_r^{O1} + \hat{x}^{O1} = v - \frac{F(\hat{x}^{O1})}{f(\hat{x}^{O1})} + \hat{x}^{O1} = v.$$

[25]For $k = 1$, Menezes and Monteiro (2000) derive the optimal auction of type Mc for objective O1.

To attract socially optimal participation, the reserve price is set equal to a good's value.[26] Intuitively, at $r_r^{O2} = v$, the auctioneer's expected marginal value, $v(1 - F_{(k,n-1)}(\hat{x}^{O2}))$, equals the total costs of the marginal bidder, $c+\hat{x}^{O2}(1-F_{(k,n-1)}(\hat{x}^{O2}))$.

Although the expected number of goods awarded K^a is important both for the auctioneer's expected surplus and the expected social welfare, it is obvious that the optimal reserve price for maximizing K^a is the highest possible reserve price that does not exclude bidders from the auction. In a competitive market this can be achieved only by a mechanism Mc.

On the contrary, the reserve price that minimizes the auctioneer's payments is as low as possible in order to exclude bidders, to prevent any award, and to pay nothing. Clearly, this outcome is typically not pursued by an auctioneer. Thus, O4 is not a reasonable primary objective. Objective O4 may instead be subsumed under objective O1 because the auctioneer's surplus involves the sum of his payments.

Concluding, the four objectives conflict and cannot all be achieved with one reserve price. The optimal surplus is achieved with less participation than is socially optimal: the optimal reserve price r_r^{OS} is below the value of a good, which, however, is the optimal reserve price for O2. In contrast, O3 calls for a higher reserve price that avoids excluding bidders, which can only be achieved with a mechanism Mc. Objective O4 requires a reserve price below the firms' costs. These considerations show that, while it is possible to design the auction optimally for specific goals, it is impossible to create a panacea in form of a design that is optimal for multiple goals.

5. Policy Implications

The ER auctions do not achieve any of the three reasonable objectives O1 to O3. Assuming an auction with ER is the optimal choice, what may an auctioneer's objective look like? In the STD auctions, a firm will be successful if her costs are sufficiently low to participate and are among the k lowest costs. In an auction with ER, a firm's success in addition depends on how many other firms' costs are sufficiently low to participate. The auctioneer buys few goods if few costs are below the level for participation and competition is weak. He buys many goods and expresses a higher willingness to pay if competition is fierce. With an ER auction, the auctioneer adjusts to the competition level: in good times he is willing to pay more than in bad times. In an auction conducted by a government, like the renewable energy auctions, it is more likely that the auctioneer's aim is to govern the market

[26]For $k = 1$, Samuelson (1985) identifies the optimal reserve price $r_r^{O2} = v$. For the same setting, Stegemann (1996) shows that asymmetric equilibria of the second-price auction can improve efficiency.

than to adjust his willingness to pay to the market situation. Moreover, is it really wanted that there are less than k winners if all bidders bid $\underline{x}$ in an auction with EAV or if one bidder bids $\underline{x}$ and all others bid $\underline{x} + \varepsilon$ in an auction with ERP?

The optimal mechanisms point at the relevance of the bid-preparation costs. Indeed, reducing these costs supports the auctioneer's objectives O1 to O3.

Proposition 4. *The auctioneer's surplus (O1), social welfare (O2), and the number of goods awarded (O3) in the respective optimal mechanism increase if the bid-preparation costs decrease.*

Proposition 4 is proven in the appendix. Given an optimal mechanism Mr, reducing the bid-preparation costs increases competition because lower bid-preparation costs increase each firm's participation probability and the number of participants. Given an optimal mechanism Mc, lower bid-preparation costs allow reducing refund payments to participants and total bid-preparation costs without changing participation. Therefore or after adjusting the respective optimal mechanism to the lower c, the auctioneer's surplus, social welfare, and the number of goods awarded increase.

In our framework, a straightforward policy implication of Proposition 4 is to reduce the bid-preparation costs. Even if the objectives are in conflict, this measure contributes to three different objectives. Thus, any factors that unnecessarily increase bid-preparation costs should be eliminated. Such factors include administrative or formal obstacles that increase bid-preparation costs but cannot be influenced by bidders. For example, bid-preparation costs for onshore wind projects in Germany lie between two and ten percent of the invested amount although the auctions' participation requirements are the same for all bidders (Wallasch et al., 2015). The costs of meeting the requirements differ due to different measures bidders need to take and different obstacles they face. One of the reasons for the stagnating development is the rigorous refusal of onshore wind projects by part of the citizens (Quentin, 2019). Legal disputes on projects can increase bid-preparation costs dramatically. The German federal government advocates a compensation of citizens in order to reduce this rejection (ZEITonline, 2020), which would in turn reduce bid-preparation costs. Of course, as in this example, any measure that reduces the bid-preparation costs has to take the costs for its implementation into account. Furthermore, a measure that reduces bid-preparation costs by mitigating participation requirements would have to consider potential detrimental effects on project quality. Hence, it is not advised to abrogate the participation requirements that cause the bid-preparation costs, but to find a reasonable balance between entry barriers that ensure the seriousness of bids and attractive conditions for participating in the auction.

6. Conclusion

Endogenous rationing has been suggested as a means to increase competition in case of low participation in auctions. However, according to the analysis in this paper its primary effect in auctions with costly participation is a strong reduction of participation. As a consequence, endogenous rationing is not a component of optimal auctions.

The optimal auctions to maximize the auctioneer's surplus, social welfare, or the number of goods awarded differ if the number of potential bidders is high. Thus, the auctioneer then needs to prioritize objectives. If the number of potential bidders is lower than the number of goods and the auctioneer values the goods sufficiently high, the optimal auction is the same for the three objectives. The auctioneer then needs to maximize participation, which he can do by setting a sufficiently high reserve price and/or by refunding the bid-preparation costs. A policy measure that contributes to all three objectives with any number of potential bidders – supposing it absorbs its implementation costs and has no detrimental effect on project quality – is to reduce bid-preparation costs.

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Appendix A. Proofs

The proofs of Lemma 1 and Proposition 3 will make use of the following lemma.

Lemma 3.

$$\sum_{i=0}^{k} \tbinom{n}{i} F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x)) = -n\tbinom{n-1}{k} F(x)^k (1-F(x))^{n-k-1} \tag{A.1}$$

$$\sum_{i=0}^{k} \tbinom{n}{i}(i-k) F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x)) = \sum_{i=1}^{k} \tbinom{n}{i} i F(x)^{i-1}(1-F(x))^{n-i} \tag{A.2}$$

Proof of Lemma 3: We will use the identities

$$\tbinom{n}{i} i = n\tbinom{n-1}{i-1} \qquad \text{and} \qquad \tbinom{n}{i}(n-i) = n\tbinom{n-1}{i} \tag{A.3}$$

and the binomial theorem

$$\textstyle\sum_{i=0}^{n} \tbinom{n}{i} F(x)^i (1-F(x))^{n-i} = (F(x)+1-F(x))^n = 1\,. \tag{A.4}$$

Proof of (A.1) via simplifying a telescoping sum.

$$
\begin{aligned}
&\textstyle\sum_{i=0}^{k}\binom{n}{i}F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x)) \\
&\quad = \textstyle\sum_{i=0}^{k}\binom{n}{i}\left(iF(x)^{i-1}(1-F(x))^{n-i}-(n-i)F(x)^{i}(1-F(x))^{n-i-1}\right) \\
&\quad \underset{(A.3)}{=} n\left(\textstyle\sum_{i=1}^{k}\binom{n-1}{i-1}F(x)^{i-1}(1-F(x))^{n-i}-\sum_{i=0}^{k}\binom{n-1}{i}F(x)^{i}(1-F(x))^{n-i-1}\right) \\
&\quad = n\left(\textstyle\sum_{i=0}^{k-1}\binom{n-1}{i}F(x)^{i}(1-F(x))^{n-i-1}-\sum_{i=0}^{k}\binom{n-1}{i}F(x)^{i}(1-F(x))^{n-i-1}\right) \\
&\quad = -n\binom{n-1}{k}F(x)^{k}(1-F(x))^{n-k-1}
\end{aligned}
$$

Proof of (A.2) via induction. For $k=1$, $n(1-F(x))^{n-1}=n(1-F(x))^{n-1}$, which proves the base case. For the induction step assume (A.2) holds for k. Now look at $k+1$.

$$
\begin{aligned}
&\textstyle\sum_{i=0}^{k+1}\binom{n}{i}F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x))(i-k-1) \\
&\quad = \textstyle\sum_{i=0}^{k}\binom{n}{i}F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x))(i-k) \\
&\qquad - \textstyle\sum_{i=0}^{k}\binom{n}{i}F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x)) \\
&\quad \underset{IH}{=} \textstyle\sum_{i=0}^{k}\binom{n}{i}iF(x)^{i-1}(1-F(x))^{n-i}-\sum_{i=0}^{k}\binom{n}{i}iF(x)^{i-1}(1-F(x))^{n-i-1} \\
&\qquad + \textstyle\sum_{i=0}^{k}\binom{n}{i}nF(x)^{i}(1-F(x))^{n-i-1} \\
&\quad = -\textstyle\sum_{i=0}^{k}\binom{n}{i}iF(x)^{i}(1-F(x))^{n-i-1}+\sum_{i=0}^{k}\binom{n}{i}nF(x)^{i}(1-F(x))^{n-i-1} \\
&\quad = \textstyle\sum_{i=0}^{k}\binom{n}{i}(n-i)F(x)^{i}(1-F(x))^{n-i-1} \\
&\quad = \textstyle\sum_{i=1}^{k+1}\binom{n}{i-1}(n-i+1)F(x)^{i-1}(1-F(x))^{n-i} \\
&\quad = \textstyle\sum_{i=1}^{k+1}\binom{n}{i}iF(x)^{i-1}(1-F(x))^{n-i}
\end{aligned}
$$

■

Proof of Lemma 1: Note that $F(\hat{x})<1$ iff $\hat{x}<\bar{x}$ and that $F(\hat{x})$ increases iff $\hat{x}$ increases.

First we show that $\hat{x}<\bar{x}$ if $n>k$ or $r<\bar{x}+c$. If a firm with costs $\bar{x}$ participates, the other $n-1$ firms also participate. If a firm with costs $\bar{x}$ is wins a good, it receives a payment of r by (P4). A firm with $\bar{x}$ wins a good with probability 0 if $n>k$ and with probability 1 if $n\leq k$. A firm participates iff its expected profit from participating is non-negative. Thus, if $n>k$, a firm with $\bar{x}$ does not participate because $(r-\bar{x})\cdot 0-c<0$. If $n\leq k$, a firm with $\bar{x}$ does not participate iff $(r-\bar{x})\cdot 1-c<0$.

If, to the contrary, $n\leq k$ and $r\geq\bar{x}+c$, then the expected payoff of the worst-off type $\bar{x}$ is positive, $r-\bar{x}-c\geq 0$, and all firms participate, $\hat{x}=\bar{x}$ and $F(\hat{x})=1$.

To prove the remaining properties, consider the expected profit of the firm $\hat{i}$ who receives a good only if no more than $k-1$ other firms participate if $n > k$ or who receives a good for sure if she participates if $n \geq k$. Her expected profit is (see (1))

$$\begin{aligned}\Pi(\hat{x}, r, c, n) &= (r-\hat{x})(1-F_{(k,n-1)}(\hat{x})) - c \qquad \text{(A.5)}\\ &= \begin{cases}(r-\hat{x})\sum_{i=0}^{k-1}\binom{n-1}{i}F(\hat{x})^i\big(1-F(\hat{x})\big)^{n-i-1} - c & \text{if } n > k\\ r-\hat{x}-c & \text{if } n \leq k.\end{cases}\end{aligned}$$

Since firm $\hat{i}$ participates only if $\Pi(\hat{x}, r, c, n) \geq 0$ and since $c > 0$, it follows that $\hat{x} < r$.

If not all firms participate ($\hat{x} < \bar{x}$), firm $\hat{i}$ is indifferent between participating and not participating in the auction. Since firm $\hat{i}$ is indifferent if and only if her expected profit from participating is zero, the cutoff costs $\hat{x}$ are determined by

$$\Pi(\hat{x}, r, c, n) = 0\,. \qquad \text{(A.6)}$$

There exists a unique $\hat{x}$ that fulfills (A.6), since the derivative of (A.5) with respect to $\hat{x}$ is negative for all $\hat{x} \leq r$:[27]

$$\begin{aligned}\frac{\partial\Pi(\hat{x}, r, c, n)}{\partial\hat{x}} &= \begin{cases}-\sum_{i=0}^{k-1}\binom{n-1}{i}F(\hat{x})^i\big(1-F(\hat{x})\big)^{n-i-1} & \\ -(r-\hat{x})k\binom{n-1}{k}f(\hat{x})F(\hat{x})^{k-1}\big(1-F(\hat{x})\big)^{n-k-1} & \text{if } n > k\\ -1 & \text{if } n \leq k\end{cases}\\ &< 0\,. \qquad \text{(A.7)}\end{aligned}$$

To determine how $\hat{x}$ depends on r and c, we apply the implicit function theorem. With (A.6) and (A.7) we get

$$\begin{aligned}\frac{d\hat{x}}{dr} &= -\frac{\frac{\partial\Pi(\hat{x},r,c,n)}{\partial r}}{\frac{\partial\Pi(\hat{x},r,c,n)}{\partial\hat{x}}} = -\frac{\sum_{i=0}^{\min\{k,n\}-1}\binom{n-1}{i}F(\hat{x})^i\big(1-F(\hat{x})\big)^{n-i-1}}{\frac{\partial\Pi(\hat{x},r,c,n)}{\partial\hat{x}}} > 0\,,\\ \frac{d\hat{x}}{dc} &= -\frac{\frac{\partial\Pi(\hat{x},r,c,n)}{\partial c}}{\frac{\partial\Pi(\hat{x},r,c,n)}{\partial\hat{x}}} = \frac{1}{\frac{\partial\Pi(\hat{x},r,c,n)}{\partial\hat{x}}} < 0\,.\end{aligned}$$

The cutoff costs $\hat{x}$ increase in k if $n > k$ because the probability in (A.5) increases in k.

Further, $\hat{x}$ decreases in n if $n \geq k$. For given $\hat{x}$, the probability of getting a

[27] The second term of the derivative stems from simplifying a telescoping sum (see Lemma 3(A.1)).

good in (A.5) decreases in n because $F_{(k,n-1)}(\hat{x}) < F_{(k,n)}(\hat{x})$. Take the $\hat{x}$ that fulfills (A.6) for n firms. All firms with $x < \hat{x}$ would have a positive expected payoff from behaving like the firm with $\hat{x}$. If there were $n+1$ firms and the firm with costs $\hat{x}$ was the cutoff type, its probability of getting a good would be lower and the payoff in (A.5) would be negative. Thus, the cutoff costs with $n+1$ firms must be smaller than with n firms. ■

Proof of Lemma 2: Firms participate in the auction iff $x_i \leq \hat{x}$. If $n > k$, type $\hat{x}$ wins iff $m \leq k$ and, by (P4), type $\hat{x}$ can then bid to receive the maximum payment r. Type $\hat{x}$'s expected profit from the auction is, with $G(x) = F_{(k,n-1)}(x)$, $\pi(\hat{x}, r, c, n) = (r - \hat{x})(1 - G(\hat{x})) = c$ (see (1) and the proof of Lemma 1). If $n \leq k$, $\pi(\hat{x}, r, c, n) = r - \hat{x}$.

If $n > k$, we assume symmetric, strictly monotone equilibrium bidding functions (P5). Denote by $\pi(x, z)$ the expected profit of a firm of type $x \leq \hat{x}$ who bids as if her type was z. Let $p(z)$ denote the payment to a firm who bids like type z. To maximize

$$\pi(x, z) = p(z) - (1 - G(z))x \qquad \text{for all } x, z \in [\underline{x}, \hat{x}]\,,$$

we derive the first-order condition

$$\frac{\mathrm{d}}{\mathrm{d}z}\pi(x, z) = \frac{d}{dz}p(z) - g(z)x = 0 \qquad \text{for all } x, z \in [\underline{x}, \hat{x}]\,.$$

In equilibrium, $z = x$, and, thus

$$\begin{aligned} & \frac{\mathrm{d}}{\mathrm{d}y}p(y) = g(y)y && \text{for all } y \in [\underline{x}, \hat{x}] \\ \Longrightarrow\ & p(x) = \text{const} + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y \\ & \qquad = (1 - G(\hat{x}))\,r + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y && \text{for all } x \in [\underline{x}, \hat{x}]\,. \end{aligned}$$

Therefore, the expected profit

$$\pi(x, r, c, n) = (1 - G(\hat{x}))\,r + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y - (1 - G(x))\,x \qquad \text{for all } x \in [\underline{x}, \hat{x}]\,,$$

is the same for all auctions that assign goods to the same types (P5) and in which a firm can bid to receive r if $m \leq k$ (P4).

If $n \leq k$, $\pi(x, r, c, n) = r - x$ for all $x \in [\underline{x}, \hat{x}]$ in all auctions with property (P4). ■

Proof of equilibrium bidding functions: If $n > k$, $\pi(x,r,c,n) = (1-G(\hat{x}))\,r + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y - (1-G(x))\,x$ for all $x \in [\underline{x}, \hat{x}]$ by payoff equivalence (Lemma 2).

In a uniform-price auction, if all bidders bid according to $\beta(x,r) = x$, a firm with type $x \leq \hat{x}$ wins and has costs x if she is among the k lowest types, which has probability $1 - G(x)$. She receives the payment r if no more than k firms participate, which has probability $1 - G(\hat{x})$. Her payment is equal to her opponents' k-th lowest bid if more than k firms participate but she is among the k lowest types. Her expected payment from these cases is $\int_x^{\hat{x}} g(y)y\,\mathrm{d}y$. Thus, her expected profit is $(1-G(\hat{x}))\,r + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y - (1-G(x))\,x$.

In a pay-as-bid auction, if all bidders bid according to $\beta^{PaB}(x,r)$, a firm with type $x \leq \hat{x}$ wins, has costs x, and receives the payment $\beta^{PaB}(x,r)$ iff she is among the k lowest types, which has probability $1-G(x)$. Therefore, her expected payment is

$$\begin{aligned}(1-G(x))\beta^{PaB}(x,r) &= (1-G(x))\,x + (1-G(\hat{x}))(r-\hat{x}) + \int_x^{\hat{x}} (1-G(y))\,\mathrm{d}y \\ &= (1-G(\hat{x}))\,r + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y\,,\end{aligned}$$

where the last step uses partial integration, $\int_x^{\hat{x}} (1-G(y))\,\mathrm{d}y = [(1-G(y))y]_x^{\hat{x}} + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y$.

If $n \leq k$, $\pi(x,r,c,n) = r - x$ for all $x \in [\underline{x}, \hat{x}]$ by payoff equivalence. Bidders receive the payment r by bidding x (or r) in a uniform-price auction and by bidding r in a pay-as-bid auction. ■

Proof of Proposition 3: We will prove parts O1 to O4 of Proposition 3 consecutively.

O1 Auctioneer's surplus. We use standard mechanism design arguments (e.g., Myerson, 1981; Krishna, 2010) to derive optimal mechanisms when firms have to bear participation costs in order to bid. Let $q^p(x_i)$, $q^p : [\underline{x}, \bar{x}] \to [0,1]$, define a firm's participation probability as a function of her costs x_i, let $q_i^g(\mathbf{x})$, $q_i^g : [\underline{x}, \bar{x}]^n \to [0,1]$ denote firm i's probability of getting an item when the costs $\mathbf{x}$ are announced conditional on i's participation, and let $p_i(\mathbf{x})$, $p_i : [\underline{x}, \bar{x}]^n \to \mathbb{R}$ denote the payment to firm i when the costs $\mathbf{x}$ are announced. The firms send messages $\mathbf{x}$ to the mechanism. The mechanism designer chooses a mechanism $(q^p, (q_1^g, q_2^g, \ldots, q_n^g), (p_1, p_2, \ldots, p_n))$ to maximize his expected surplus, taking the firms' (interim) individual rationality

(IR) and incentive compatibility (IC) constraints into account. His problem is

$$\max_{(q^p,(q_1^g,q_2^g,\ldots,q_n^g),(p_1,p_2,\ldots,p_n))} \int \sum_{i=1}^{n} q^p(x_i) q_i^g(\mathbf{x}) v - p_i(\mathbf{x}) \, \mathrm{d}H(\mathbf{x})$$

$$\text{s.t. (IR), (IC)}$$

$$0 \le q^p(x_i) \le 1, 0 \le q_i^g(\mathbf{x}) \le 1 \quad \forall i = 1,2,\ldots,n, \sum_{i=1}^{n} q_i^g(\mathbf{x}) \le k$$

where $H(\mathbf{x})$ denotes the joint distribution of the individual cost distributions, $H(\mathbf{x}) = \Pi_{i=1}^{n} F(x_i)$ and $H_{-i}(\mathbf{x}_{-i}) = \Pi_{j \neq i} F(x_j)$.

A firm i's (interim) expected payoff from reporting x_i when supply costs are x_i is

$$\Pi_i(x_i) = \int p_i(\mathbf{x}) - q^p(x_i) \left(q_i^g(\mathbf{x}) x_i + c \right) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \,. \tag{A.8}$$

The IR constraint and the IC constraint, which ensure truthful reporting of x_i, are

$$\Pi_i(x_i) \ge 0 \quad \forall x_i \in [\underline{x}, \bar{x}] \tag{IR}$$

$$\Pi_i(x_i) \ge \int p_i(x_i', \mathbf{x}_{-i}) - q^p(x_i') \left(q_i^g(x_i', \mathbf{x}_{-i}) x_i + c \right) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \; \forall x_i, x_i' \in [\underline{x}, \bar{x}] \,. \tag{IC}$$

Furthermore, define firm i's (interim) expected probability of getting a good conditional on i's participation

$$Q_i(x_i) := \int q_i^g(\mathbf{x}) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \,.$$

Condition (IC) implies, for all x_i and x_i'

$$\begin{aligned}
\Pi_i(x_i) &= \int p_i(x_i, \mathbf{x}_{-i}) - q^p(x_i) \left(q_i^g(x_i, \mathbf{x}_{-i}) x_i + c \right) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \\
&\ge \int p_i(x_i', \mathbf{x}_{-i}) - q^p(x_i') \left(q_i^g(x_i', \mathbf{x}_{-i}) x_i + c \right) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \\
&= \int p_i(x_i', \mathbf{x}_{-i}) - q^p(x_i') \left(q_i^g(x_i', \mathbf{x}_{-i}) x_i' + c \right) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \\
&\qquad + \int q^p(x_i') \, q_i^g(x_i', \mathbf{x}_{-i})(x_i' - x_i) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \\
&= \Pi_i(x_i') + q^p(x_i') Q_i(x_i')(x_i' - x_i) \,.
\end{aligned}$$

Thus, for $x_i > x_i'$,

$$\frac{\Pi_i(x_i) - \Pi_i(x_i')}{x_i - x_i'} \ge -q^p(x_i') Q_i(x_i')$$

and for $x_i < x_i'$,

$$\frac{\Pi_i(x_i) - \Pi_i(x_i')}{x_i - x_i'} \leq -q^p(x_i')Q_i(x_i')\,.$$

Therefore, (IC) implies that

$$\frac{\mathrm{d}\Pi_i(x_i)}{\mathrm{d}x_i} = -q^p(x_i)Q_i(x_i)$$

and, by integration,

$$\Pi_i(x_i) = \mathrm{const}_i + \int_{x_i}^{\bar{x}} q^p(z)Q_i(z)\,\mathrm{d}z. \tag{A.9}$$

Using (A.8) and (A.9) we can rewrite the auctioneer's expected surplus with incentive compatible payoffs of firms as

$$\begin{aligned}
\Pi_0 &= \int \sum_{i=1}^{n} q^p(x_i)q_i^g(\mathbf{x})v - p_i(\mathbf{x})\,\mathrm{d}H(\mathbf{x}) \\
&= \int \left[\sum_{i=1}^{n} q^p(x_i)q_i^g(\mathbf{x})v - p_i(\mathbf{x}) + \int p_i(\mathbf{x}) - q^p(x_i)\left(q_i^g(\mathbf{x})x_i + c\right)\mathrm{d}H_{-i}(\mathbf{x}_{-i}) \right] \mathrm{d}H(\mathbf{x}) \\
&\quad - \int \sum_{i=1}^{n} \Pi_i(x_i)\,\mathrm{d}H(\mathbf{x}) \\
&= \int \sum_{i=1}^{n} q^p(x_i)\left[q_i^g(\mathbf{x})(v - x_i) - c\right]\mathrm{d}H(\mathbf{x}) - \sum_{i=1}^{n} \mathrm{const}_i \\
&\quad - \int \sum_{i=1}^{n} \int_{x_i}^{\bar{x}} q^p(z)Q_i(z)\,\mathrm{d}z\,\mathrm{d}H(\mathbf{x}) \\
&= \int \sum_{i=1}^{n} q^p(x_i)\left[q_i^g(\mathbf{x})(v - x_i) - c\right]\mathrm{d}H(\mathbf{x}) - \sum_{i=1}^{n} \mathrm{const}_i \\
&\quad - \sum_{i=1}^{n} \int \frac{F(x_i)}{f(x_i)} q^p(x_i)Q_i(x_i)\,\mathrm{d}H(\mathbf{x}) \\
&= \int \sum_{i=1}^{n} q^p(x_i)\left[q_i^g(\mathbf{x})\left(v - x_i - \frac{F(x_i)}{f(x_i)}\right) - c\right]\mathrm{d}H(\mathbf{x}) - \sum_{i=1}^{n} \mathrm{const}_i\,,
\end{aligned}$$

where in the next-to-last step we interchanged the order of integration in the hindmost integral.[28] To maximize his surplus, the auctioneer will choose const_i as low as

[28] $\int \int_{x_i}^{\bar{x}} q^p(z)Q_i(z)\,\mathrm{d}z\,\mathrm{d}H(\mathbf{x}) = \int \int_{\underline{x}}^{\bar{x}} \int_{x_i}^{\bar{x}} q^p(z)Q_i(z)\,\mathrm{d}z\,\mathrm{d}F(x_i)\,\mathrm{d}H_{-i}(\mathbf{x}_{-i}) =$
$\int \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^{z} q^p(z)Q_i(z)\mathrm{d}F(x_i)\,\mathrm{d}z\,\mathrm{d}H_{-i}(\mathbf{x}_{-i}) = \int \int_{\underline{x}}^{\bar{x}} q^p(z)Q_i(z)F(z)\,\mathrm{d}z\,\mathrm{d}H_{-i}(\mathbf{x}_{-i}) =$
$= \int \int_{\underline{x}}^{\bar{x}} \frac{q^p(z)Q_i(z)F(z)}{f(z)}\,\mathrm{d}F(z)\,\mathrm{d}H_{-i}(\mathbf{x}_{-i}) = \int \frac{q^p(x_i)Q_i(x_i)F(x_i)}{f(x_i)}\,\mathrm{d}H(\mathbf{x})$.

possible which is zero because (IC) (i.e., (A.9)) and (IR) bound const_i to zero from below.

Thus, the auctioneer's problem can be written as

$$\max_{(q^p,(q_1^g,q_2^g,\ldots,q_n^g),(p_1,p_2,\ldots,p_n))} \int \sum_{i=1}^{n} q^p(x_i)\left[q_i^g(\mathbf{x})\left(v - x_i - \frac{F(x_i)}{f(x_i)}\right) - c\right] \mathrm{d}H(\mathbf{x})$$
$$\text{s.t.} \quad 0 \le q^p(x_i) \le 1, \quad 0 \le q_i^g(\mathbf{x}) \le 1 \quad \forall i = 1,2,\ldots,n, \quad \sum_{i=1}^{n} q_i^g(\mathbf{x}) \le k$$

We assume that $x_i + \frac{F(x_i)}{f(x_i)}$ is increasing in x_i. Thus, either there exists a unique $\tilde{x} < \bar{x}$ such that $v - \tilde{x} - \frac{F(\tilde{x})}{f(\tilde{x})} = 0$ and $v - x_i - \frac{F(x_i)}{f(x_i)} \ge 0$ for all $x_i \le \tilde{x}$, or we have that $v - x_i - \frac{F(x_i)}{f(x_i)} \ge 0$ for all $x_i \in [\underline{x}, \bar{x}]$, in which case $\tilde{x} = \bar{x}$. Conditional on participation, the auctioneer will choose $q_i^g(\mathbf{x}) = 1$ for the at most $\min\{n,k\}$ firms with the lowest $x_i \le \tilde{x}$, and $q_i^g(\mathbf{x}) = 0$ for the remaining firms. Thus, for firms that participate and have $x_i \le \tilde{x}$, we get

$$\begin{aligned} Q_i(x_i) = \int q_i^g(\mathbf{x})\mathrm{d}H_{-i}(\mathbf{x}_{-i}) &= \text{Prob}\{x_i \text{ is among the } \min\{n,k\} \text{ lowest costs}\} \\ &= \sum_{j=0}^{\min\{k,n\}-1} \tbinom{n-1}{j} F(x_i)^j (1 - F(x_i))^{n-j-1}. \end{aligned}$$

The auctioneer maximizes

$$\max_{(q^p,(p_1,p_2,\ldots,p_n))} \sum_{i=1}^{n} \int q^p(x_i)\left[Q_i(x_i)\left(v - x_i - \frac{F(x_i)}{f(x_i)}\right) - c\right] \mathrm{d}F(x_i)$$
$$\text{s.t.} \qquad 0 \le q^p(x_i) \le 1$$

by choosing $q^p(x_i) = 1$ if $Q_i(x_i)\left(v - x_i - \frac{F(x_i)}{f(x_i)}\right) \ge c$ and $q^p(x_i) = 0$ if the inverse holds. Because $x_i + \frac{F(x_i)}{f(x_i)}$ and $Q_i(x_i)$ are increasing in x_i for all $x_i < \bar{x}$, there exists a unique $\hat{x} \le \tilde{x}$ such that $q^p(x_i) = 1$ if $x_i \le \hat{x}$ and $q^p(x_i) = 0$ if $x_i > \hat{x}$. For $n > k$, $\hat{x}$ is determined by $Q_i(\hat{x})\left(v - \hat{x} - \frac{F(\hat{x})}{f(\hat{x})}\right) = c$. For $n \le k$, $Q_i(x_i) = 1$ for all $x_i \in [\underline{x}, \bar{x}]$ (by the binomial theorem (A.4)). Then, either $\hat{x}$ is the solution of $v - \hat{x} - \frac{F(\hat{x})}{f(\hat{x})} = c$, in which case $\hat{x} \le \bar{x}$, or we have that $v - \bar{x} - \frac{1}{f(\bar{x})} > c$, in which case $\hat{x} = \bar{x}$. Summarizing, and, for convenience, setting $Q_i(x_i) = 0$ if $q^p(x_i) = 0$, we have

$$q^p(x_i) = \begin{cases} 0 & \text{if } x_i > \hat{x} \\ 1 & \text{if } x_i \leq \hat{x} \end{cases} \tag{A.10}$$

$$Q_i(x_i) = \begin{cases} 0 & \text{if } x_i > \hat{x} \\ \sum_{j=0}^{\min\{k,n\}-1} \binom{n-1}{j} F(x_i)^j (1 - F(x_i))^{n-j-1} & \text{if } x_i \leq \hat{x}\,. \end{cases} \tag{A.11}$$

It remains to determine payment functions $p_1, p_2, \ldots, p_n$ such that the payoff (A.8) satisfies incentive compatibility (A.9):

$$\Pi_i(x_i) = \int p_i(\mathbf{x}) - q^p(x_i)\,(q_i^g(\mathbf{x})x_i + c)\,\mathrm{d}H_{-i}(\mathbf{x}_{-i}) = \int_{x_i}^{\bar{x}} q^p(z)Q_i(z)\,\mathrm{d}z\,.$$

Plugging in (A.10) and (A.11) gives

$$\Pi_i(x_i) = \begin{cases} \int p_i(\mathbf{x})\mathrm{d}H_{-i}(\mathbf{x}_{-i}) = 0 & \text{if } x_i > \hat{x} \\ \int p_i(\mathbf{x})\mathrm{d}H_{-i}(\mathbf{x}_{-i}) - Q_i(x_i)x_i - c = \int_{x_i}^{\hat{x}} Q_i(z)\,\mathrm{d}z & \text{if } x_i \leq \hat{x} \end{cases}$$

and we get

$$\int p_i(\mathbf{x})\mathrm{d}H_{-i}(\mathbf{x}_{-i}) = \begin{cases} 0 & \text{if } x_i > \hat{x} \\ \int_{x_i}^{\hat{x}} Q_i(z)\,\mathrm{d}z + Q_i(x_i)x_i + c & \text{if } x_i \leq \hat{x}\,. \end{cases} \tag{A.12}$$

In the case of $n > k$, let $x_{(k,n-1)}$ denote the k-th lowest of i's opponents' realized costs $\mathbf{x}_{-i}$ if $k < n$ and let $F_{(k,n-1)}$ denote the distribution of the random variable $X_{(k,n-1)}$. Note that $F_{(k,n-1)}(z) = 1 - Q_i(z)$. In the case of $n \leq k$, for ease of notation define $y_{(k,n-1)} > \hat{x}$ and $F_{(k,n-1)}(z) = 1 - Q_i(z) = 0$ for all $z \in [\underline{x}, \bar{x}]$.

Two payment functions that fulfill (A.12) are, if $\hat{x} < \bar{x}$,

$$p_i(\mathbf{x}) = \begin{cases} 0 & \text{if } x_i > \min\{\hat{x}, y_{(k,n-1)}\} \\ v - \frac{F(\hat{x})}{f(\hat{x})} & \text{if } x_i \leq \hat{x} < y_{(k,n-1)} \\ y_{(k,n-1)} & \text{if } x_i \leq y_{(k,n-1)} \leq \hat{x} \end{cases} \tag{A.13}$$

or

$$p_i(\mathbf{x}) = \begin{cases} 0 & \text{if } x_i > \hat{x} \\ c & \text{if } y_{(k,n-1)} < x_i \leq \hat{x} \\ \min\{\hat{x}, y_{(k,n-1)}\} + c & \text{if } x_i \leq \min\{\hat{x}, y_{(k,n-1)}\}\,. \end{cases} \tag{A.14}$$

Obviously, both (A.13) and (A.14) fulfill (A.12) for $x_i > \hat{x}$. For $x_i \leq \hat{x}$ and (A.13), we get

$$\begin{aligned}\int p_i(\mathbf{x})\mathrm{d}H_{-i}(\mathbf{x}_{-i}) &= (1 - F_{(k,n-1)}(\hat{x}))\left(v - \frac{F(\hat{x})}{f(\hat{x})}\right) + \int_{x_i}^{\hat{x}} z\mathrm{d}F_{(k,n-1)}(z) \\ &= (1 - F_{(k,n-1)}(\hat{x}))\left(v - \frac{F(\hat{x})}{f(\hat{x})}\right) + \left[zF_{(k,n-1)}(z)\right]_{x_i}^{\hat{x}} \\ &\quad - \int_{x_i}^{\hat{x}} F_{(k,n-1)}(z)\mathrm{d}z \\ &= Q_i(\hat{x})\left(v - \frac{F(\hat{x})}{f(\hat{x})}\right) - Q_i(\hat{x})\hat{x} + Q_i(x_i)x_i + \int_{x_i}^{\hat{x}} Q_i(z)\mathrm{d}z \\ &= c + x_iQ_i(x_i) + \int_{x_i}^{\hat{x}} Q_i(z)\mathrm{d}z\,.\end{aligned}$$

For $x_i \leq \hat{x}$ and (A.14), we get

$$\begin{aligned}\int p_i(\mathbf{x})\mathrm{d}H_{-i}(\mathbf{x}_{-i}) &= c + \int_{x_i}^{\hat{x}} z\mathrm{d}F_{(k,n-1)}(z) + (1 - F_{(k,n-1)}(\hat{x}))\,\hat{x} \\ &= c + \left[zF_{(k,n-1)}(z)\right]_{x_i}^{\hat{x}} - \int_{x_i}^{\hat{x}} F_{(k,n-1)}(z)\mathrm{d}z + \hat{x} - F_{(k,n-1)}(\hat{x})\,\hat{x} \\ &= c - x_i(1 - Q_i(x_i)) - \int_{x_i}^{\hat{x}} 1 - Q_i(z)\mathrm{d}z + \hat{x} \\ &= c + x_iQ_i(x_i) + \int_{x_i}^{\hat{x}} Q_i(z)\mathrm{d}z\,.\end{aligned}$$

According to Stegemann (1996, Lemma 1), every direct mechanism in which each firm announces its type is associated with an outcome-equivalent semi-direct mechanism in which only participating firms announce their type.

In the semi-direct mechanism associated with (A.13), firms participate in the auction iff $x_i \leq \hat{x}$, participating firms reveal their true costs x_i, and the assignment and payments are determined by a uniform-price rule with the reserve price $v - \frac{F(\hat{x})}{f(\hat{x})}$. In the mechanism associated with (A.14), firms participate in the auction iff $x_i \leq \hat{x}$, participating firms reveal their true costs x_i and receive a payment of c, and assignment and further payments are determined by a uniform-price rule with the reserve price $\hat{x}$. Note that, in both cases, the optimal reserve price depends on $\hat{x}$, and, therefore, on n. In both cases, by payoff equivalence, the pricing rule could be replaced by any pricing rule with a monotonic symmetric equilibrium.[29]

[29] Stegemann (1996, Theorem 4) proves payoff-equivalence between symmetric equilibria of first- and second-price single-unit auctions with bid-preparation costs.

If $\hat{x} = \bar{x}$, a payment function that fulfills (A.12) is $p_i(\mathbf{x}) = c + \bar{x}$. Then, all firms participate and are paid (the reserve price) $c + \bar{x}$.

O2 Social welfare. We will first determine the socially optimal cutoff value $\hat{x}$, which by the assumption of symmetry is the same for all firms. Second, we will determine the reserve price that induces socially optimal participation. Third, we will describe a second mechanism that generates the same expected welfare.

First, assume $v < \bar{x} + c$ or $n > k$. The first-order condition of the problem to maximize W is $\partial W / \partial \hat{x} = 0$, where

$$\begin{aligned}\frac{\partial W}{\partial \hat{x}} =& - ncf(\hat{x}) - \sum_{i=1}^{\min\{k,n\}} \hat{x} f_{(i,n)}(\hat{x}) \\ & - vf(\hat{x}) \sum_{i=0}^{\min\{k,n\}-1} \tbinom{n}{i} (\min\{k,n\} - i) F(\hat{x})^{i-1} (1 - F(\hat{x}))^{n-i-1} (i - nF(\hat{x}))\,,\end{aligned}$$

which with $f_{(i,n)}(\hat{x}) = nf(\hat{x}) \binom{n-1}{i-1} F(\hat{x})^{i-1} (1 - F(\hat{x}))^{n-i}$ and Lemma 3(A.2) and (A.3) leads to

$$(v - \hat{x}) \sum_{i=1}^{\min\{k,n\}} \tbinom{n-1}{i-1} F(\hat{x})^{i-1} (1 - F(\hat{x}))^{n-i} - c = 0\,. \tag{A.15}$$

For $v = r$, Condition (A.15) equals Condition (1) to determine the cutoff costs $\hat{x}$. Thus, $r_r^{O2} = v$ attracts the participation that generates the social optimum.

Second, assume $v \geq \bar{x} + c$ and $n \leq k$. Then, participation by all firms is socially optimal and the auctioneer attracts participation by all firms with a reserve price $r_r^{O2} \geq \bar{x} + c$, e.g., $r_r^{O2} = v$.

One socially optimal mechanism is therefore a uniform-price auction with the reserve price $r_r^{O2} = v$. A second mechanism that also achieves socially optimal participation of all types $x \leq \hat{x}$ is a uniform-price auction with $r_c^{O2} = \hat{x}$ and an additional payment of c to all participants. In this mechanism, type $\hat{x}$ has an expected payoff from the auction of zero and is therefore the highest type that will participate.

O3 Number of goods awarded. The maximum number of goods awarded is k if $n > k$ or n if $n \leq k$. If $n \leq k$, by Lemma 1, $r \geq \bar{x} + c$ achieves full participation and the maximum number of goods awarded. If $n > k$, $\hat{x} < \bar{x}$. Thus, under symmetric participation a reserve price as high as possible maximizes the expected number of bidders, and, therefore, the expected number of goods awarded. However, there is no reserve price that achieves full participation.

An optimal mechanism is therefore a uniform-price auction with the reserve price $r_c^{O3} = \bar{x}$ and an additional payment of c to all participants. Then, all participants participate and the expected number of goods awarded is maximized. If $n \leq k$, a second optimal mechanism is a uniform-price auction with the reserve price $r_r^{O3} = \bar{x} + c$.

O4 Auctioneer's costs. The auctioneer minimizes his costs, that is, his payments, by the reserve price $r_r^{O4} \leq \underline{x} + c$. Then, no firm participates and the auctioneer pays nothing. Clearly, the auctioneer has no incentive to refund participation costs. ■

Proof of Proposition 4: Denote the optimal mechanism of the type Mc or Mr when the bid-preparation costs are c by M$c(c)$ and M$r(c)$, respectively. Let M$c(c)$ be characterized by the optimal cutoff $\hat{x}$ and the reserve price $r_c = \hat{x}$ given c and let M$r(c)$ be characterized by the optimal reserve price r_r given c. If the costs decrease to c', M$c(c)$ and M$r(c)$ are still available to the auctioneer.

We compare the values of the auctioneer's objective functions when the costs are c' and he chooses M$c(c)$ with the situation when the costs are c and he chooses M$c(c)$. In the former case his objective function O1 takes a higher value because the same firms participate but the amount c' he has to pay to each participant is lower. Similarly, in the former case, the objective function O2 takes a higher value because the participants and the auction assignment and payments (other than the reimbursed bid-preparation costs) are the same but the total bid-preparation costs are lower.

Next, consider the value of the auctioneer's objective function O3 when the costs are c and he chooses M$r(c)$. The auction is a STD auction with a reserve price and by Lemma 1, $\frac{dF(\hat{x})}{dc} < 0$ if $n > k$ or $r < \bar{x} + c$. Otherwise, $F(\hat{x})$ is constant in c. Thus, if c decreases to c', the number of participants increases weakly. When the number of participants increases then the number of goods awarded also increases.

When costs are c', the optimal mechanisms M$c(c')$ and M$r(c')$ by definition outperform M$c(c)$ and M$r(c)$. Therefore, the auctioneer with either of the objectives O1 to O3 is better off with an optimal mechanism when the bid-preparation costs are c' than with the optimal mechanism when the bid-preparation costs are c. ■

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