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Tail Risk Managed Investment Strategies

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Introduction

Risk managed investment strategies have emerged as an important topic in practice and in academics. Especially during adverse market periods, like the global financial crisis or the recent corona crisis, the demand for tail risk mitigation tools increases. This results since investors are typically crash-averse and weight losses higher than gains of the same magnitude, i.e. investors are willing to pay high fees to avoid crashes (Bollerslev and Todorov, 2011, Chabi-Yo et al., 2018).[1] Unfortunately, due to an increase of correlations between different assets in extreme crash periods, diversification is an insufficient tool to achieve a good downside risk protection.[2] In other words, even well-diversified portfolios exhibit extremely high losses during crises and drawdowns of these portfolios are typically much higher than anticipated by investors. Furthermore, the occurrence of extreme crashes also leads to a portfolio return distribution with an extremely high left tail risk, indicated by a highly negative skewness and high kurtosis.[3] Since investors typically prefer higher levels of skewness and lower levels of kurtosis (Kraus and Litzenberger, 1976, Scott and Horvath, 1980), non-managed portfolios are unappealing for these investors, especially in crash periods. In other words, "investors are tail-risk averse"

[1] This observation is similar to the safety-first theory examined by Arzac and Bawa (1977). This theory "builds on the assumption that investors maximize the expected return while limiting the probability of suffering a particularly large loss below a predetermined admissible level" (Van Oordt and Zhou, 2016, p. 688).

[2] See, for example, Ang and Chen (2002), Ang and Bekaert (2002), Butler and Joaquin (2002), Chabi-Yo et al. (2018), Guidolin and Timmermann (2008), Karolyi and Stulz (1996), Patton (2004), Poon et al. (2004) and Longin and Solnik (2001) for studies on the high co-crash risk and increase of correlations of several assets during crash periods. Similarly, Jondeau and Rockinger (2003) find that an extremely negative skewness of different assets typically occurs simultaneously. The presence of asymmetric correlations, i.e. higher correlations in crash periods, leads to a wrong risk assessment of investors. That is, the investors' equity exposure is typically too high in bear markets and too low in calm markets (Ang and Chen, 2002).

[3] Generally, an asset's skewness is highly linked to an asset's crash risk. For example, Chen et al. (2001, p. 348) state: "when we speak of "forecasting crashes" [...], we are adopting a narrow and euphemistic definition of the word "crashes," associating it solely with the conditional skewness of the return distribution." Further, Chen et al. (2001) state that conditional skewness can be interpreted as a crash expectation measure. Thus, skewness risk and crash risk are highly linked with each other: a high negative skewness makes crashes more likely and severe, whereas severe crashes make return distributions more negatively skewed.

and have a high demand to hedge against this higher moment risk (Dreyer and Hubrich, 2019, p. 47).[4] Consequently, driven by the investors' demand to hedge against financial tail risks, tail risk mitigation tools are needed in order to make equity investments appealing for investors. Furthermore, the reduction of extremely negative returns does not only fit well to most investors' preferences, but is also a main driver of an investment strategy's long-term performance. For example, a drawdown of 50% has to be compensated by a return of 100%, whereas a drawdown of 20% requires a return of only 25%. Thus, avoiding extreme crash periods eventually results in a higher long-term performance.

The failure of diversification to hedge against financial tail risks and the importance of drawdown mitigation for investors has led to numerous portfolio optimization methods that are frequently examined in the literature and that are also applied by practitioners. However, most of these portfolio allocation methods, which aim to control a portfolio's risk, often fail to reduce drawdowns during crises. For example, the well-known mean-variance approach typically performs bad in practice, since this approach suffers under a high estimation risk (DeMiguel et al., 2009b, Jagannathan and Ma, 2003).[5] This holds particularly during crises where estimation risk is typically high. Similarly, during the recent corona crisis, the frequently used risk parity approach suffered losses that were much higher than anticipated by investors. Hence, also the risk parity approach, which is much more robust to estimation risk, fails as a crash risk mitigation tool.[6] The reason for the failure of these portfolio allocation methods is the above mentioned

[4]Most investors are willing to accept a lower return potential in order to reduce the probability of extremely negative returns, i.e. "investors are willing to give up some of the right tail to reduce the left tail" (Harvey et al., 2018, p. 15).

[5]See Tu and Zhou (2011), Garlappi et al. (2006), DeMiguel et al. (2009a), Kan and Zhou (2007) and Kirby and Ostdiek (2012) for further studies on mean-variance portfolios. The high estimation risk of the mean-variance approach mainly arises since the mean return is hard to estimate (Merton, 1980, Moreira and Muir, 2019). Thus, the estimation risk of this strategy can be significantly reduced by portfolio allocation approaches that are solely based on the assets' covariance matrix or volatilities (Fleming et al., 2001, 2003, Han, 2005, Kim et al., 2016, Moskowitz et al., 2012, Taylor, 2014, Zakamulin, 2015). Another disadvantage of the mean-variance optimization is that this approach is implicitly based on the assumption of normally distributed returns or investors with quadratic preferences. Since these assumptions are typically not fulfilled in practice, investment strategies based on higher moments or tail risk measures are more appealing (Basak and Shapiro, 2001, Cuoco et al., 2008, Ghysels et al., 2016, Jarrow and Zhao, 2006, Jondeau and Rockinger, 2006, 2012, Packham et al., 2017, Wang et al., 2012, Xiong and Idzorek, 2011).

[6]Risk parity is typically applied to several asset classes and equalizes the risk contribution of each asset in the portfolio. By doing this, risk parity aims to reduce losses during crises and is frequently applied by practitioners (Asness et al., 2012, Baltas, 2015, Maillard et al., 2010). See, for example, the "S&P Risk Parity Index - 12% Target Volatility (TR)" index, which "seeks to measure the performance of a multi-asset risk parity strategy that allocates risk equally among equity, fixed income, and commodities futures contracts, while targeting

observation that correlations increase in crises and most assets typically crash together. Due to this increase in correlations, drawdowns cannot be successfully reduced by changing the relative weights of the assets in the portfolio while the exposure to the portfolio is kept constant. Thus, these approaches fail to limit a portfolio's crash risk, especially when a crash protection is most needed. In contrast, a well performing investment strategy should dynamically reduce the exposure to the portfolio during a crash period and should then subsequently increase the portfolio exposure once the risk starts to decline. In other words, a good investment strategy should time the (short-term) risk of the portfolio, where the exposure to the portfolio should be inverse to the portfolio's risk. The benefits of these risk timing strategies have been frequently examined in the literature.[7] Generally, risk based market timing strategies are also frequently used by practitioners (Christoffersen and Diebold, 2006, Copeland and Copeland, 1999).[8] In particular, timing a portfolio's short-term risk also fits well to the observation that investors are risk-averse, i.e. investors dislike an extremely fluctuating and high volatility.[9] By managing a portfolio's risk on a frequent basis, portfolio returns become more normal and are more in line with the preferences of risk-averse investors. Interestingly, (short-term) risk timing is also beneficial for risk-averse investors with a long investment horizon (Moreira and Muir, 2019).[10]

This PhD thesis comprises three papers that examine several tail risk mitigation approaches.

a volatility level of 12%". Although this approach should reduce losses during crash periods, the "S&P Risk Parity Index - 12% Target Volatility (TR)" index suffers a drawdown of 29.14% during the recent corona crisis (see https://www.spglobal.com/spdji/en/indices/strategy/sp-risk-parity-index-12-target-volatility for further details and data on this index). Other risk parity funds lost even more than 40% during the corona crisis (https://www.markovprocesses.com/blog/risk-parity-funds-in-the-coronavirus-market-rout/).

[7]Volatility timing has been examined by Benson et al. (2014), Bollerslev et al. (2018), Dachraoui (2018), Dreyer and Hubrich (2019), Gormsen and Jensen (2017), Harvey et al. (2018), Marquering and Verbeek (2004) and Perchet et al. (2016) for long-only portfolios, whereas Baltas (2015), Barroso and Maio (2018), Barroso and Santa-Clara (2015), Cederburg et al. (2020), Daniel and Moskowitz (2016), Du Plessis and Hallerbach (2017) and Moreira and Muir (2017) examine volatility timing for long-short strategies.

[8]See, for example, the fund offered by Man AHL that targets a constant level of portfolio volatility (https://www.man.com/ahltargetrisk) and the research of Morningstar Inc who present a style switching strategy based on the market's volatility (https://www.morningstar.com/articles/925094/a-momentum-and-low-volatility-switching-strategy).

[9]Risk-averse investors are willing to pay high fees to hedge against high levels of volatility and high volatility changes (Adrian and Rosenberg, 2008, Ang et al., 2006b). This high demand for a moderate and stable volatility has led to the invention of new financial instruments, such as variance swaps (Bollerslev and Todorov, 2011, Footnote 11).

[10]The investors' evaluation period is typically much shorter than the investors' investment horizon (Benartzi and Thaler, 1995). Thus, even long-term investors are concerned about short-term risk and should therefore time short-term risk.

All three papers examine an approach called risk targeting, which is applied to several data sets in this thesis. The aim of risk targeting is to build a portfolio that includes risky assets and has a constant level of portfolio risk over time. This goal is achieved by constantly readjusting the exposure to the portfolio of risky assets based on a forecast of the portfolio's risk, where the portfolio's exposure is chosen inversely to the portfolio's risk. By timing the portfolio's risk, this strategy has the advantage that it fits well to the risk- and loss-averse nature of investors as summarized above. In particular, this simple approach reduces a portfolio's crash risk and is hardly influenced by the increase of correlations during crash periods. This is the case, since most crashes occur in periods of an increased risk (Ang and Bekaert, 2002, Liu et al., 2003, Muir, 2017). Thus, the portfolio exposure of risk targeting is reduced during a crash period and then subsequently increased when the risk starts to decline. In particular, by timing the portfolio's risk, this strategy enhances the portfolio's risk-return profile and produces high utility gains for investors. This arises since there exists a flat or even negative risk-return relation for most assets, i.e. a higher risk is not related to higher future returns. Hence, periods of an increased risk are followed by periods with an inferior risk-return profile (Dachraoui, 2018, Dopfel and Ramkumar, 2013). However, even when risk and future returns are positively related, volatility targeting can be advantageous (Moreira and Muir, 2017, 2019).[11] Due to these advantages of risk targeting, this strategy has recently emerged as an important and frequently examined topic in the financial literature, which has been applied to many investment strategies, such as long-only portfolios, multi-asset portfolios and long-short factor portfolios, like momentum and "Betting against Beta". However, so far, the literature focuses on risk targeting strategies that target a constant level of volatility, which has several drawbacks since volatility is a suboptimal risk measure. Furthermore, most financial studies examine risk targeting strategies based on simple and unconditional risk forecasting models. Since the performance of risk targeting

[11] For example, Moreira and Muir (2019, p. 509) find that an increase of volatility coincides with a higher future return, but that "this increase in expected returns is much more persistent than the increase in volatility. Investors can avoid the short-term increase in volatility by first reducing their exposure to equities when volatility initially increases and capture the increase in expected returns by coming back to the market as volatility comes down." Similarly, Cederburg et al. (2020) state: "Volatility management is likely to be successful if volatility is persistent and the risk-return relation is flat. [...] If lagged volatility is negatively related to average return for a given strategy, volatility management becomes even more attractive. A positive risk-return trade-off, in contrast, makes volatility management less effective." The risk-return relation for several portfolio strategies has been examined by Barroso and Maio (2019).

strongly depends on the accuracy of the portfolio risk forecast, the performance of risk targeting can further be improved by using more sophisticated risk models (Bollerslev et al., 2018). Moreover, risk targeting is typically used as a market timing tool to manage a whole portfolio's risk, where the underlying portfolio typically uses simple weighting schemes, such as equal- or value-weightings. Thus, this approach only regards the whole portfolio's risk, whereas the risk profile of the individual assets in the portfolio is not incorporated. In three different papers, this PhD thesis extends the risk targeting approach in several directions.

In the first paper, we apply risk targeting to long-only equity portfolios with daily rebalancing and show how the volatility targeting strategy can be extended to strategies that target a constant level of tail risk measured by Value at Risk (VaR) or Conditional Value at Risk (CVaR). This extension is appealing since volatility managing is, at least implicitly, based on the assumption of normally distributed returns or investors with quadratic preferences. Since return distributions are typically highly non-normal with a non-zero skewness and fat tails, volatility underestimates a portfolio's crash potential and tail risk measures are more suitable to assess a portfolio's risk. Furthermore, tail risk based investment strategies are also more suitable for investors who have preferences for higher moments or investors who are loss-averse. We show that the tail risk targeting strategies outperform the volatility targeting strategies by achieving a higher Sharpe Ratio and high utility gains for mean-variance investors, CRRA investors and loss-averse investors. This result is particularly driven since tail risk targeting is much more successful in reducing the drawdowns while the upside potential is also captured. In particular, when forecasting volatility and tail risk, we compare simple forecasting models, which are mostly used in the financial literature, to more advanced models that dynamically forecast risk. In line with Bollerslev et al. (2018), we find that more advanced forecasting models are more accurate and, by achieving high performance and utility gains, are more valuable for investors. Finally, we develop strategies that switch between volatility and CVaR targeting, where volatility targeting is used in uptrending periods and CVaR targeting is used in downtrending periods.[12]

[12]Generally, different portfolio risk management approaches can perform quite differently in different market environments, i.e. a strategy that works well in down periods does not necessarily work well in up periods. Consequently, in order to capture the advantages of both risk targeting approaches, combining both strategies is a natural extension. The combination of different weighting schemes or investment styles has been frequently examined in the literature (Barberis and Shleifer, 2003, Barroso and Maio, 2019, Daniel et al., 2017, DeMiguel et al., 2009b,

These switching strategies further reduce the portfolio's crash risk and successfully capture the portfolio's upside potential and eventually lead to an even higher risk-adjusted return. We further show that the returns of this switching strategy cannot be explained by volatility targeting alone. In contrast, volatility targeting becomes unprofitable once we control for the returns of the switching strategy.

In the second paper, we apply our volatility and tail risk targeting strategies to the long-short momentum portfolio that buys past winners and sells past losers. The momentum strategy is known to produce high returns that are accompanied with a high crash risk and a return distribution with a fat left tail, which makes the non-managed momentum strategy unappealing for investors. This especially holds since momentum crashes typically occur in periods when investors are highly averse to losses (Min and Kim, 2016). For that reason, momentum's high crash risk and possibilities to mitigate this high crash risk have recently emerged to a very important topic in the financial literature.[13] Applying our risk targeting approaches to the momentum portfolio extends the research of Barroso and Santa-Clara (2015), Daniel and Moskowitz (2016), Cederburg et al. (2020) and Moreira and Muir (2017) who apply volatility targeting based on simple volatility models to the momentum portfolio. In the first step, we show that applying more sophisticated and more accurate volatility models to the momentum portfolio leads to significant performance and utility gains. In particular, since the momentum portfolio is reallocated on a monthly basis, we explicitly show how monthly volatility can be forecasted based on models that are fitted to daily data. In the second step, we apply our volatility and CVaR switching approach to the momentum portfolio, where monthly CVaR estimates are again obtained by fitting the forecasting models to daily data. When estimating momentum's tail risk, we explicitly account for the extreme and highly time-varying higher moments of the momentum portfolio (Bali et al., 2008, Jondeau and Rockinger, 2003).[14] These volatil-

Garlappi et al., 2006, Kan and Zhou, 2007, Tu and Zhou, 2011, Wang, 2005).

[13]The risk of the momentum portfolio and approaches to manage momentum's risk have been examined by Barroso (2016), Barroso and Maio (2018), Barroso and Santa-Clara (2015), Boguth et al. (2011), Cederburg et al. (2020), Chabot et al. (2014), Chordia and Shivakumar (2002), Cooper et al. (2004), Daniel et al. (2017), Daniel and Moskowitz (2016), Du Plessis and Hallerbach (2017), Goyal and Jegadeesh (2017), Griffin et al. (2003), Grobys et al. (2018), Grobys and Kolari (2020), Grundy and Martin (2001), Jacobs et al. (2015), Kim et al. (2016), Min and Kim (2016), Moreira and Muir (2017), Ruenzi and Weigert (2018), Stivers and Sun (2010), Wang and Xu (2015) among others.

[14]We show that momentum's conditional skewness and kurtosis are highly time-varying and there exist periods

ity and CVaR switching strategies significantly outperform the volatility managed momentum strategy of Barroso and Santa-Clara (2015).[15] This result holds in terms of the risk-adjusted return as well as utility gains for mean-variance investors, CRRA investors and loss-averse investors. Our findings hold for several momentum crash indicators, subperiods and data sets. Furthermore, the returns of the switching approach cannot be explained by volatility targeting alone, whereas volatility targeting becomes unprofitable once we control for the returns of the switching strategy. In particular, the volatility and CVaR switching approach also works well to manage the risk of the Betting against Beta portfolio of Frazzini and Pedersen (2014), which also is an important topic in the financial literature.

The third paper again shows how to manage momentum's risk, but, in contrast to the second paper that focuses on momentum's portfolio risk, this paper focuses on the risk of the assets that are contained in the momentum portfolio. A similar approach has been examined by Du Plessis and Hallerbach (2017), Clare et al. (2014) and Goyal and Jegadeesh (2017) who weight assets of the momentum portfolio inversely to their volatility.[16] However, applying the inverse volatility weighting to the momentum portfolio has two important drawbacks. First, as stated above, using volatility is only suitable when asset returns are normally distributed or when investors have quadratic preferences. Both conditions are typically not fulfilled in practice. Second, and even more important, the momentum portfolio consists of long and short positions and the risk of a long or short position should be managed differently (Baltas, 2015, Bollerslev et al., 2020, Giot and Laurent, 2003). In other words, by applying the inverse volatility weighting to both the winners and losers portfolios, the risk-adjusted performance of both portfolios is

when these higher moments take extreme values or may even not exist. This higher moment risk is not captured by volatility targeting alone (Daniel et al., 2017, Gormsen and Jensen, 2017). The extreme levels of skewness and kurtosis occur since the winners' and losers' skewness moves in opposite directions, whereas the kurtosis comoves.

[15]The approach of Barroso and Santa-Clara (2015) is based on the assumption that momentum returns follow a random walk. We use the test of Lo and MacKinlay (1988) to show that the random walk hypothesis does not hold for the momentum portfolio. As a consequence, monthly risk cannot be obtained by simply scaling up daily risk by $\sqrt{21}$. However, since this square root of time rule (SRTR) is frequently applied in practice (Berkowitz and O'Brien, 2002, Danielsson and Zigrand, 2006), we also use the SRTR combined with our conditional risk forecasts and find surprisingly good results for this approach. A possible explanation for the good performance of the SRTR could be that this approach is quite robust against estimation risk, which should be high for the highly non-normally distributed momentum returns.

[16]This inverse volatility weighting is also frequently used for the time series momentum (TSMOM) strategy (Baltas, 2015, Kim et al., 2016, Moskowitz et al., 2012) and long-only portfolios (Asness et al., 2012, Kirby and Ostdiek, 2012).

increased. Thus, the benefits of buying the enhanced winners is offset by shorting the enhanced losers. In contrast, the aim of a weighting scheme applied to a long-short portfolio should be to enhance the performance of the long leg and to *worsen* the performance of the short leg. For these two reasons, we extend this approach by weighting the assets of the momentum portfolio inversely to their tail risk or systematic risk. Tail risk measures quantify an asset's own risk, whereas systematic risk measures quantify an asset's comovement with the (equally weighted) momentum portfolio. We then weight winners by their (systematic) left tail risk and losers by their (systematic) right tail risk.[17] Since the systematic tail risk weighting should be superior in crash periods, but disadvantageous in uptrending markets, we develop a strategy that switches between both approaches based on several momentum crash indicators. We find that this (systematic) tail risk switching approach outperforms the equally and volatility weighted momentum portfolio in terms of significantly higher Sharpe Ratios as well as economically high and statistically significant utility gains for mean-variance, CRRA and loss-averse investors. Furthermore, we show how this (systematic) tail risk weighting approach can be combined with the risk targeting strategy. This combined approach simultaneously manages individual asset risk and portfolio risk.[18] We find that managing portfolio risk is more important than managing individual asset risk, but that the strategy that simultaneously manages both kinds of risk provides the best risk-return profile and the highest utility gains.[19] These findings are quite robust against many modifications of this strategy. Finally, we show that the equally and volatility weightings become unprofitable when we control for the (systematic) tail risk weighting, whereas the opposite is not true. Furthermore, the different weighting schemes combined with risk targeting produce returns that cannot be explained by the weighting schemes

[17]In other words, in the winners portfolio, assets with a higher crash risk or assets with a higher co-crash risk with the momentum portfolio obtain lower weights. In contrast, in the losers portfolio, assets with a high upside potential or assets that rise when the momentum portfolio crashes obtain lower weights.

[18]Du Plessis and Hallerbach (2017) show that both approaches work well for the momentum strategy, but the authors do not combine both approaches. Moreira and Muir (2017, Sec. II.D) show that managing a portfolio's individual constituents' risk is different to risk targeting that manages the whole portfolio's risk. Simultaneously managing individual asset risk and portfolio risk has been examined by Moreira and Muir (2017, Sec. I.E), Harvey et al. (2018) and Zakamulin (2015). However, the authors only use volatility to manage both kinds of risk and do not apply this approach to the momentum portfolio.

[19]The observation that risk targeting is more important than risk weighting is again in line with the observation of increasing correlations in crash periods, i.e. simply changing the weights of a portfolio does not significantly reduce drawdowns in crash periods since all assets crash together.

alone, confirming the finding that risk targeting produces statistically significant performance gains.

In total, this PhD thesis contributes in several ways to the literature. First, we extend volatility targeting to tail risk targeting, i.e. we develop strategies that target a constant level of tail risk measured by Value at Risk (VaR) or Conditional Value at Risk (CVaR). Second, we show how a constant level of portfolio risk can be obtained based on advanced forecasting models, whereas the literature so far mainly focuses on simple forecasting models. This holds especially for risk targeting applied to the momentum strategy where portfolio weights are rebalanced monthly, but the forecasting models are fitted to daily data. Third, we show how the accuracy of risk targeting, which is an important driver of the success of risk targeting, can be backtested using sophisticated backtesting methods. Fourth, by switching between volatility and CVaR targeting, we further contribute to the literature that combines different portfolio risk management approaches. In particular, our switching approach is based on simple tools and could also be interesting for practitioners. Fifth, we calculate the economic value for loss-averse investors, whereas the economic value is so far only calculated for mean-variance and CRRA investors. However, the assumption of loss aversion is more realistic since the portfolio allocation of real investors is much better described by loss aversion, whereas the equity exposure in the mean-variance and CRRA framework is much higher than in practice (Ang et al., 2005, Benartzi and Thaler, 1995). Sixth, we develop a new weighting scheme that can be used to weight assets of a long-short portfolio. This is important, since "[s]imply inverting the long-only solution for the assets with a short position is completely incorrect" (Baltas, 2015). Our new weighting scheme aims to improve the performance of the long leg while the performance of the short leg is worsened. This is achieved by weighting assets in the long leg based on their (systematic) left tail risk, whereas assets in the short leg are weighted by their (systematic) right tail risk. Seventh, we develop a portfolio risk management approach that manages the (systematic) tail risk of the individual assets in the portfolio and simultaneously manages the whole portfolio's risk. This combined approach separates the asset allocation decision from the market timing decision and could also be an appealing approach for practitioners.

Chapter 1

Tail Risk Targeting: Target VaR and CVaR Strategies

1.1 Introduction

During extreme market crashes, due to an increase of correlations, diversification fails as a risk management tool. Especially when financial markets exhibit huge downturn periods, assets typically crash together, and thus lower the benefit of diversification just when it is most needed (Ang and Bekaert, 2002, Butler and Joaquin, 2002, Chabi-Yo et al., 2018, Guidolin and Timmermann, 2008, Karolyi and Stulz, 1996, Patton, 2004, Poon et al., 2004). Investors typically overestimate the benefits of diversification in bear markets and underestimate return potentials in bull markets. This leads to too high equity exposures in bear markets, whereas the equity allocation is too low in bull markets (Ang and Chen, 2002, Longin and Solnik, 2001).

For that reason, more tactical tools, such as volatility targeting, have become popular in the financial industry and academic literature.[1] The aim of volatility targeting is to build a portfolio, consisting of a risky and a riskless asset, that has a (predetermined) constant level of portfolio volatility over time. In order to achieve this constant level of portfolio volatility, the target volatility strategy allocates money between the risky and the riskless asset, based on a forecast of the risky asset's volatility: if the risky asset's volatility is expected to be high, the weight of the risky asset is decreased and vice versa (see Bollerslev et al. (2018) for example).

[1]For academic research on target volatility strategies see Barroso and Santa-Clara (2015), Bollerslev et al. (2018), Moreira and Muir (2017), Barroso and Maio (2018), Moreira and Muir (2019), Cederburg et al. (2020) among others. For research from practitioners see, e.g., Banerjee et al. (2016), Dreyer and Hubrich (2019), Hocquard et al. (2013), Benson et al. (2014) and Perchet et al. (2016).

By choosing a portfolio's exposure inversely to the portfolio's volatility, left tail risk can be significantly reduced (Dreyer and Hubrich, 2019, Harvey et al., 2018). Generally, the economic value of volatility timing, measured by significant performance and utility gains for investors who time volatility in multi-asset portfolios, has been extensively examined by Fleming et al. (2001), Fleming et al. (2003), Han (2005), Kirby and Ostdiek (2012) and Taylor (2014).[2] In a single asset scenario, using long-only equity portfolios, the economic value of volatility timing is examined by Dreyer and Hubrich (2019), Marquering and Verbeek (2004), Bollerslev et al. (2018) and Moreira and Muir (2017). The authors find vast utility gains of volatility timing and that volatility timing is superior to return timing. Dreyer and Hubrich (2019) find that the utility gains of volatility targeting mainly come from a reduction of left tail risk. Thus, volatility targeting works especially well for assets with non-normally distributed returns, like equities or dynamic trading strategies (Perchet et al., 2016). Barroso and Santa-Clara (2015), Barroso and Maio (2018), Cederburg et al. (2020) and Moreira and Muir (2017) examine volatility targeting applied to different long-short factor portfolios and find significant improvements in the risk-adjusted performance, especially for strategies with a high left tail risk, like momentum or betting against beta.[3] This also highlights a nice characteristic of volatility targeting. Volatility targeting can be used for any underlying investment strategy, i.e. volatility targeting can be separated from the fund manager's asset allocation choice, where the asset allocation is chosen first and this portfolio is then overlayed by a volatility targeting strategy as market timing tool (Harvey et al., 2018, Hocquard et al., 2013, Moreira and Muir, 2017, Zakamulin, 2015). Busse (1999) examines the impact of volatility timing for the institutional fund industry and concludes

[2]Fleming et al. (2001), Fleming et al. (2003), Han (2005), Kirby and Ostdiek (2012) and Taylor (2014) assess the economic value of volatility timing in a multivariate setting. This approach is slightly different to volatility targeting but demonstrates that investment decisions that rely on volatility (or more precisely covariances) solely work well in empirical applications.

[3]Barroso and Santa-Clara (2015) successfully use a target volatility strategy to manage the risk of the momentum portfolio and show that targeting a constant level of volatility extremely reduces the drawdowns of the momentum portfolio, the so called "momentum crashes". This high crash risk reduction translates into a superior risk-adjusted performance compared to the non-managed momentum portfolio (see also Daniel and Moskowitz (2016) and Rickenberg (2020a)). Moreira and Muir (2017) use a volatility timing strategy for different factor portfolios and show that the risk-adjusted performance of the volatility managed portfolios is superior to the performance of the non-managed portfolios. This finding is most pronounced for the momentum strategy. Barroso and Maio (2018) use volatility targeting for several factor strategies and find huge improvements of the volatility targeting strategies for all strategies except for the size factor. The best results are found for the momentum strategy and the "betting against beta" strategy of Frazzini and Pedersen (2014). This finding is confirmed by Cederburg et al. (2020).

that "funds that reduce systematic risk when conditional market volatility is high earn higher risk-adjusted returns" and that funds who time volatility the most are associated with higher Sharpe Ratios (Busse, 1999, p. 1010 and 1027). Generally, volatility is frequently used as a market timing tool by practitioners (Christoffersen and Diebold, 2006, Copeland and Copeland, 1999). This does not only hold for investors with short investment horizons. Moreira and Muir (2019) assess the economic value of volatility timing for long-term investors and the authors find that even long-term investors should time short-term volatility. This supports the finding of Benartzi and Thaler (1995) that even long-term investors have short evaluation periods and are concerned about short-term risk.

So far, studies on risk targeting focus on volatility as a risk measure: the weight of the risky asset is a function of the risky asset's volatility. However, since asset returns are typically skewed, fat-tailed and non-normally distributed, the choice of volatility as a measure of market risk is not appropriate (see Szegö (2002), Poon and Granger (2003), Kuester et al. (2006, p. 56) and Bali et al. (2009)). Xiong and Idzorek (2011), Guidolin and Timmermann (2008), Jondeau and Rockinger (2006), Jondeau and Rockinger (2012) and Ghysels et al. (2016) examine the impact of skewness and fat-tails on the asset allocation and show that incorporating higher moments, as done by adequately measuring downside risk, is beneficial compared to mean-variance optimization. Furthermore, most investors have preferences for higher levels of skewness and lower kurtosis (see Kraus and Litzenberger (1976), Scott and Horvath (1980), Guidolin and Timmermann (2008) among others). Bali et al. (2009) and Kelly and Jiang (2014) find that an increase of tail risk predicts higher kurtosis and lower (or more negative) skewness. Hence, investors who dislike a negative skewness and a high kurtosis should better manage a portfolio's downside risk. This also fits well to the observation that investors are more concerned about downside risks instead of return deviations (Bollerslev et al., 2015, Kelly and Jiang, 2014, Lee and Rao, 1988).[4] Similarly, most investors weight losses higher than gains, which implies that avoiding huge losses is crucial in order to increase a loss-averse investor's utility (Aït-Sahalia and Brandt, 2001, Ang et al., 2005, 2006a, Benartzi and Thaler, 1995). Avoiding crashes is

[4]See Arzac and Bawa (1977) who examine the portfolio allocation of safety-first investors who are concerned about financial tail events. See also Van Oordt and Zhou (2016) for an asset pricing study based on this theory.

particularly important since most investors are crash-averse and have a demand for portfolio insurance methods, especially in times of extremely negative returns (Bollerslev and Todorov, 2011, Chabi-Yo et al., 2018). Therefore, timing an asset's downside risk instead of volatility fits better to most investors' preferences. Furthermore, Benson et al. (2014) find that the superior performance of volatility targeting does result from mitigating drawdowns. Similarly, Dreyer and Hubrich (2019) and Harvey et al. (2018) find that volatility targeting reduces the likelihood of extremely negative returns, which is an important source of the outperformance of volatility targeting. Consequently, if drawdown protection is a main driver of the superior (risk-adjusted) performance of risk targeting, choosing the risky asset's weight based on a forecast of the risky asset's downside risk should be more successful in mitigating drawdowns and should eventually result in a superior risk-adjusted performance.

In this paper, we show how the idea of volatility targeting can be extended to tail risk targeting, where tail risk is measured by Value at Risk (VaR) or Conditional Value at Risk (CVaR).[5] These strategies aim to keep the VaR or CVaR of the portfolio constant over time by shifting money between the risky and the riskless asset, based on a forecast of the risky asset's tail risk. This approach translates into a strategy that increases the weight of the risky asset if the risky asset's tail risk is expected to be low and vice versa. Basak and Shapiro (2001), Alexander and Baptista (2004), Cuoco et al. (2008), Agarwal and Naik (2004) and Wang et al. (2012) demonstrate the benefits of downside risk timing instead of volatility timing in a portfolio allocation setting. To compare the economic of volatility and downside risk targeting, we follow the literature and assess the economic value of risk targeting for a mean-variance investor. Further, to incorporate preferences for higher moments, like skewness and kurtosis, we additionally assess the economic value for a CRRA investor (Dreyer and Hubrich, 2019, Jondeau and Rockinger, 2012). Finally, since most investors weight losses higher than gains, we assess the economic value of risk targeting for loss-averse investors. We find that tail risk targeting strategies deliver high utility gains compared to a static portfolio allocation. This is in line with Cuoco et al. (2008) who find that frequently reallocating portfolio weights based on estimates of downside

[5] These risk measures are frequently used by practitioners and are important for regulators and banks (Berkowitz et al., 2011, Berkowitz and O'Brien, 2002, Du and Escanciano, 2016). Further, VaR and CVaR are also used in studies on the cross-section of returns (Atilgan et al., 2020).

risk is superior to static portfolio allocations. In particular, an investor should manage portfolio risk based on a conditional risk model using a dynamic volatility model, like the GARCH(1,1) or EWMA model. Simple risk estimation models like the Historical Standard Deviation, as used in Moreira and Muir (2017), Barroso and Maio (2018) and Barroso and Santa-Clara (2015), or Historical Simulation typically fail to significantly increase an investor's utility and produce lower Sharpe Ratios than the conditional approaches. Moreover, we find that the economic value of CVaR timing is significantly higher than the economic value of volatility timing, especially when investors are highly risk- or loss-averse and in times of bear markets. This finding also holds during the recent corona crisis. Interestingly, even mean-variance investors should manage CVaR instead of volatility. For example, we find that a mean-variance investor is willing to pay a fee of about 0.8% per year to have access to a volatility targeting strategy. However, the same investor would even pay 4.253% per year to have access to the CVaR managed strategy. In contrast, a loss-averse investor is not willing to pay a positive fee for volatility targeting, but the same investor would pay up to 18% per year to have access to the CVaR targeting strategy.

Since estimating downside risk is more sophisticated than estimating volatility, we additionally show how the target VaR and target CVaR strategies can be approximated by a target volatility strategy. Further, we demonstrate how the accuracy of the target volatility, target VaR and target CVaR strategies can be backtested. In order to assess the accuracy of volatility targeting, we resort to the approaches of Diebold and Mariano (1995), White (2000), Hansen (2005), Romano and Wolf (2005), Hansen et al. (2003), Hansen et al. (2011), Hsu et al. (2010), Barras et al. (2010) and Bajgrowicz and Scaillet (2012) that test for equal or superior predictive ability. In order to assess the accuracy of VaR and CVaR targeting, we use the VaR backtest of Christoffersen (1998) and the CVaR backtests of McNeil and Frey (2000) and Embrechts et al. (2005). With these backtests in hand, we assess the accuracy of approximating a target VaR or target CVaR strategy by a target volatility strategy, i.e. we answer the question if controlling volatility is sufficient when downside risk is targeted. We find that for investors who are interested in targeting a constant VaR or CVaR over time, controlling volatility is not sufficient. Similarly, in order to target a constant level of volatility, an investor should manage volatility directly instead of downside risk. Generally, risk should be managed by a dynamic risk model, based on a dynamic volatility model, like the EWMA or GARCH(1,1) model. In contrast, using a static risk model, like the Historical Standard Deviation or Historical Simulation, fails to target the portfolio risk at a constant level, achieves a worse risk-adjusted performance and produces lower utilities for investor. In line with Bollerslev et al. (2018), we find a positive relation between forecasting accuracy, and hence a more constant portfolio risk, and risk-adjusted performance and utility gains (see also Perchet et al. (2016) and references therein).

Finally, we develop strategies that switch between volatility and CVaR targeting, based on an estimate of the expected market regime. If the market is expected to be in a down-market, CVaR targeting is used, whereas the portfolio's risk is managed by volatility if an up-market is expected. To determine up- and down-markets, we use technical trading rules (Bajgrowicz and Scaillet, 2012, Moskowitz et al., 2012) and the portfolio's expected volatility. We find that these switching strategies further increase the risk-adjusted return and the investors' utility. For example, a mean-variance investor is willing to pay 5.689% per year to switch to a strategy that dynamically switches between volatility and CVaR targeting. Further, a loss-averse investor is even willing to pay 21.135% per year to have access to this strategy. The benefits of this switching approach also hold during the recent corona crisis and in the long-run. Over the last 88 years, a 100\$ investment in the market would result in a portfolio value of $357{,}591$\$. By using the volatility targeting strategy, this amount can be raised to $4{,}420{,}160$\$. However, by switching between volatility and CVaR targeting, the wealth would even increase to $48{,}535{,}249$\$. Thus, switching between volatility and CVaR targeting is much more profitable than volatility targeting alone. Similarly, we find that volatility targeting produces a negative alpha, once we control for CVaR targeting or the switching strategy. In contrast, the returns of CVaR targeting and the switching strategy cannot be explained by volatility targeting.

This paper is structured as follows. In Section 1.2, we present the target volatility framework and review the literature on volatility targeting. Section 1.3 presents the target VaR and CVaR strategies and shows how VaR and CVaR are estimated. Furthermore, we show how the target VaR and CVaR strategies can be approximated by a target volatility strategy. Section 1.4

demonstrates how the accuracy of volatility, VaR and CVaR targeting can be tested. Section 1.5 shows the empirical results and Section 1.6 concludes the paper. The Appendix contains additional results and shows how the strategies have performed in the long-run and during the recent corona crisis.

1.2 Target Volatility Strategy

Throughout the paper, we consider a risky asset, e.g. an equity index, with price process $\{S_t\}_{t\in\{0,\ldots,T\}}$ over the period $[0,T], T \in \mathbb{N}$, and we define the return of the risky asset over the period $[t-1,t]$, representing one day, as

$$R_t := \frac{S_t}{S_{t-1}} - 1. \tag{1.2.1}$$

Further, we consider a riskless asset with returns $\{R_t^f\}_{t\in\{0,\ldots T\}}$. R_t^f describes the return of the riskless asset over the period $[t-1,t]$ and we assume that R_t^f is known at time $t-1$.[6] The day t return R_t^P of the portfolio that invests a weight w_t in the risky asset and $1-w_t$ in the riskless asset is then given by

$$R_t^P := w_t \cdot R_t + (1-w_t) \cdot R_t^f. \tag{1.2.2}$$

The aim of the target volatility strategy is to determine the weight w_t for each day t such that the portfolio volatility is constant over time and equals a predefined value. We denote the *portfolio volatility*, i.e. the (conditional) standard deviation of the portfolio return R_t^P conditioned on the information $\mathcal{F}_{t-1}$ that is available at time $t-1$, by $\sigma_t^P := \sqrt{\operatorname{var}(R_t^P \mid \mathcal{F}_{t-1})}$, where $\operatorname{var}\big(R_t^P \mid \mathcal{F}_{t-1}\big)$ denotes the (conditional) portfolio variance (see Hansen and Lunde (2005, p. 875)). In order to achieve a constant volatility level σ_{target} for σ_t^P over time, the weight of the risky asset has to be chosen as

$$w_t = \frac{\sigma_{\text{target}}}{\sigma_t}, \tag{1.2.3}$$

where σ_{target} is the desired volatility target and $\sigma_t := \sqrt{\operatorname{var}(R_t \mid \mathcal{F}_{t-1})}$ is the (conditional) volatility of the risky asset at day t (see Bollerslev et al. (2018, p. 2757) for example). By

[6] More formally, we assume that R_t^f is measurable with respect to $\mathcal{F}_{t-1}$, where $\mathcal{F}_{t-1}$ is the σ-algebra generated by the variables that are observed up to time $t-1$ (Hansen and Lunde, 2005, p. 875). Hence, $\mathcal{F}_{t-1}$ contains all relevant information available at time $t-1$.

construction, the day t weight w_t is known at day $t-1$ since σ_t is $\mathcal{F}_{t-1}$-measurable. The use of volatility targeting has several advantages, which are summarized in Appendix A.[7]

To implement a target volatility strategy, the volatility of the risky asset σ_t in Equation (1.2.3) is needed, which is unobservable in practice. Therefore, based on the available information at time $t-1$, the volatility for day t has to be forecasted.[8] We denote this (one-step ahead) forecast by $\hat{\sigma}_t$. Based on this volatility forecast, the weight w_t of the risky asset is given by

$$w_t = \frac{\sigma_{\text{target}}}{\hat{\sigma}_t}. \tag{1.2.4}$$

Consequently, the success of the target volatility strategy strongly depends on the quality of the volatility forecast $\hat{\sigma}_t$.[9] Benson et al. (2014) show that a target volatility strategy with perfect foresight, i.e. a strategy that knows the next period's volatility, outperforms the benchmark by more than 10% per year with a lower volatility and is successful in producing a constant volatility, indicated by an almost zero volatility of volatility (see also Bollerslev et al. (2018)). Moreover, Moreira and Muir (2017) and Bollerslev et al. (2018) show that using advanced volatility forecasting models in a volatility targeting strategy improves the risk-adjusted performance and heightens utility gains compared to simple and less accurate forecasting models. Taylor (2014) and Fleming et al. (2003) find a similar observation for a multivariate volatility timing strategy. In particular, Bollerslev et al. (2018) find a positive relation between forecasting accuracy and utility gains for investors who use volatility targeting. Furthermore, Dreyer and Hubrich (2019) find that a stabilization of portfolio volatility is linked to a lower crash risk of the portfolio. Thus, a more constant portfolio volatility is rewarded by higher risk-adjusted returns and utility gains as well as lower drawdowns (see also Perchet et al. (2016) and references therein).[10] Similarly, Marquering and Verbeek (2004) find that periods where volatility can be predicted well correspond to periods where volatility timing generates high utility gains.

[7]Volatility targeting is also quite frequently used by practitioners. See, for example, the fund offered by Man AHL (`https://www.man.com/ahltargetrisk`).

[8]See Bollerslev et al. (1992), Taylor (2005, Sec. 2), Poon and Granger (2003) and Hansen and Lunde (2005) for surveys on volatility forecasting.

[9]Obviously, the volatility of the target volatility strategy is only constant over time and equals σ_{target} if and only if the volatility forecast $\hat{\sigma}_t$ equals the true (ex-post) realized volatility σ_t on each day t.

[10]In a cross-sectional setting, Baltussen et al. (2018) find that assets with a high volatility of volatility (vol-of-vol) underperform assets with a more constant volatility. This especially holds during down-markets, where high vol-of-vol assets underperform low vol-of-vol assets by 0.83% per month. Further, high vol-of-vol assets also exhibit higher downside risk.

In total, target volatility strategies based on more accurate volatility forecasting models should obtain better risk-return profiles than strategies that use inaccurate volatility forecasts. In particular, simply using past volatility, as is it is frequently done in the literature, is not sufficient for volatility targeting. This is also confirmed by the finding of Dopfel and Ramkumar (2013, p. 31). The authors find that high volatility regimes concurrently occur with negative returns and significantly lower Sharpe Ratios compared to regimes with a normal volatility. However, the authors also show that this result reverses when returns of regimes with a high or normal volatility in the *previous* period are compared.[11] Similarly, Dachraoui (2018) finds a negative relation between σ_t and R_t but no relation between σ_{t-1} and R_t. This result highlights that accurately *forecasting* future volatility σ_t by $\hat{\sigma}_t$ is crucial when portfolio volatility should be managed, since simply measuring today's volatility is not sufficient to determine tomorrow's weight of the risky asset.[12] Due to the importance of accurate forecasting models that achieve a stable portfolio volatility, we present methods that test the accuracy of different target volatility strategies in Section 1.4.1.

For practical implementations, simple forecasting methods, like the Historical Standard Deviation (HSD) or the Exponential Weighted Moving Average (EWMA), which was proposed by the RiskMetrics™ group, can be used. Nevertheless, as argued above, more advanced – and potentially more accurate – methods, like the GARCH(1,1) model of Bollerslev (1986), could also be interesting. In this paper, we compare these three volatility models, where the HSD statically measures today's volatility, which is used as a forecast for tomorrow's volatility. Hence, this model does not consider the aforementioned issue of *forecasting* next day's volatility. In contrast, the EWMA and GARCH(1,1) models dynamically forecast next day's volatility, and thus should result in a more constant portfolio volatility and a higher risk-adjusted performance.[13]

[11]Interestingly, although returns of periods following a high volatility period are higher than returns following a low volatility period, Sharpe Ratios are slightly higher for periods following a low volatility period. Thus, the higher volatility is not compensated by an adequately higher return and these periods are unappealing for investors (Moreira and Muir, 2017).

[12]As mentioned in Appendix A, volatility targeting is still advantageous when there is no negative risk-return relation between σ_{t-1} and R_t. In other words, volatility targeting is still advantageous even when volatility is simply measured by past realized volatility up to day $t-1$. However, a strategy based on an accurate forecast of σ_t further enhances the risk-return profile of volatility targeting.

[13]In the EWMA and GARCH(1,1) model, past negative and positive returns have the same impact on future volatility. A well-known stylized fact, the so-called *leverage effect*, states that past negative returns influence future volatility more than past positive returns. We also used the GJR-GARCH model of Glosten et al. (1993) and the

All three models have several advantages and disadvantages. A possible extension could be to combine several forecasting models as suggested by Taylor (2014).

The day t volatility using the HSD is estimated by

$$\hat{\sigma}_t = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(R_{t-i} - \hat{\mu}_t)^2}, \tag{1.2.5}$$

where $\hat{\mu}_t = \frac{1}{m}\sum_{i=1}^{m} R_{t-i}$ is an estimate of the expected mean return. For the EWMA and the GARCH(1,1) models, it is assumed that the day t return of the risky asset can be described by

$$R_t = \sigma_t \cdot Z_t, \tag{1.2.6}$$

where Z_t is iid with mean zero, variance one and cumulative distribution function F_Z (see McNeil and Frey (2000, p. 275)). As usual, when working with daily returns, we assume that the expected mean return is zero. This is a quite weak assumption, since (absolute) daily returns are close to zero. Further, accurately estimating the expected daily return is not feasible (see Merton (1980), Fleming et al. (2001, p. 332), Fleming et al. (2003, p. 476), Kirby and Ostdiek (2012) among others). Christoffersen and Diebold (2006) show that the conditional mean is not forecastable, since returns R_t conditioned on $\mathcal{F}_{t-1}$ do not fluctuate over time. Further, Hansen and Lunde (2005) compare different mean specifications and find that all lead to an almost identical performance of the volatility models.

For the EWMA model, the volatility forecast $\hat{\sigma}_t$ is given by

$$\hat{\sigma}_t = \sqrt{(1-\lambda)\cdot R_{t-1}^2 + \lambda \cdot \hat{\sigma}_{t-1}^2}, \tag{1.2.7}$$

where λ is typically chosen as 0.94 when working with daily returns (Christoffersen, 2012, p. 70). The advantage of the EWMA model is that no parameters have to be estimated, what makes this model interesting for practical applications (Halbleib and Pohlmeier, 2012). However, frequently re-estimating the model parameters, as it is done for the GARCH(1,1) model, should also result in a more accurate volatility forecast. The volatility forecast in the GARCH

EGARCH model of Nelson (1991) that account for the leverage effect, but results were quite similar to the results of the EWMA and GARCH(1,1) model. This is in line with Taylor (2014) who comes to the same conclusion in a multivariate setting. See also Poon and Granger (2003) and Hansen and Lunde (2005) for a comparison of different volatility forecasting models.

model is given by

$$\hat{\sigma}_t = \sqrt{\hat{\omega} + \hat{\alpha}\, R_{t-1}^2 + \hat{\beta}\, \hat{\sigma}_{t-1}^2}, \tag{1.2.8}$$

where the parameters $\hat{\omega}, \hat{\alpha}, \hat{\beta}$ are estimated via Quasi Maximum Likelihood, i.e. we assume that the innovations Z_t in Equation (1.2.6) are iid standard normally distributed.[14]

Another field of current research, that could be of high interest in the context of target volatility strategies, is forecasting volatility based on the theory of *realized volatility* that measures volatility using high-frequency-data (see Andersen et al. (2001) for example). Due to its simplicity, the Heterogeneous Autoregressive model of Realized Volatility (HAR-RV), proposed by Corsi (2009), fits well to the target volatility framework (see Taylor (2014) who finds good results of the HAR model in a multivariate setting). Bollerslev et al. (2018) extend the HAR-RV model in several directions and use these modifications in a volatility targeting framework. The authors find good results of these models compared to models that rely on daily data. For example, an investor who uses a volatility targeting strategy would pay an annualized fee of 0.46% to switch from a simple strategy to a high-frequency-data based strategy. Similarly, Fleming et al. (2003) examine the economic value of high-frequency-data based estimates of daily volatility. They find that using high-frequency-data based volatility measures instead of daily data based measures can substantially increase the economic value of volatility timing in a multivariate mean-variance context (Fleming et al., 2003, p. 495-496).[15] This again demonstrates that the quality of a volatility targeting strategy strongly depends on the accuracy of the inherent volatility forecasting model.

For the implementation of the target volatility strategy, we follow Barroso and Santa-Clara (2015) who use an annualized volatility target σ_{target} of 12%. Typically, target volatility levels used in the literature range from 5% to 40% as annualized volatility target. Clearly, the higher

[14]The GARCH(p,q) model is defined for any lag order p and q. Bollerslev et al. (1992, p. 22) state that small lag orders are sufficient to model the volatility of equity returns in empirical applications (see also Kellner and Rösch (2016) in the context of VaR and CVaR forecasting). Since target volatility strategies are of high interest for practical implementations, we restrict ourselves to the lag orders $p = 1$ and $q = 1$.

[15]The authors use a mean-variance framework with a constant mean, which essentially translates into a volatility timing strategy, i.e. the weights of the assets are determined by estimates of conditional volatility and correlation solely. Although the authors use a multivariate setting – based on stocks, bonds, gold and cash – their findings that volatility timing adds economic value and is positively influenced by more accurate models is highly related to our approach based on stocks and cash.

the volatility target the higher the exposure to the risky asset. Therefore, risk-averse or loss-averse investors will prefer a lower target volatility level, whereas risk-seeking investors will choose a high volatility target.[16] Bollerslev et al. (2018) show how the volatility target can be derived as a function of the investor's risk aversion. An appealing alternative to choosing a fixed target volatility level was introduced by Wang et al. (2012) in a slightly different setting. The authors propose to switch between two target levels, where a high (low) target level is used when a low (high) risk regime is expected. Another alternative would be to use a time-varying volatility target that equals the long-term volatility (Dreyer and Hubrich, 2019).

Since the volatility of the risky asset is usually not constant over time, the weight of the risky asset has to be rebalanced every day, which can lead to high transaction costs. However, Dreyer and Hubrich (2019), Moreira and Muir (2017) and Bollerslev et al. (2018) find that volatility targeting is still beneficial, even when realistic transaction costs are considered. Harvey et al. (2018, Exhibit 8) also find that transaction costs hardly influence the Sharpe Ratio of volatility targeting. Similarly, Marquering and Verbeek (2004) find that transaction costs only marginally impact the utility gains of dynamic trading strategies if short sales and leverage in the risky asset are not allowed. By definition, risk targeting is a long-only strategy and by choosing a moderate target volatility level, the strategy is seldom leveraged. Thus, in order to better assess the accuracy of different risk models, we reallocate the weight of the risky asset on a daily basis as it is also done in Dreyer and Hubrich (2019). Nevertheless, several possibilities can be used to lower the turnover and, as a consequence, the transaction costs of risk targeting (see for example Kirby and Ostdiek (2012), Moreira and Muir (2017) and Bollerslev et al. (2018)).[17]

[16] By choosing a quite low volatility target, the target volatility strategy usually does not need leverage. Since some investors are leverage constrained, the weight w_t is often capped by a maximum allowed weight. For example, Strub (2013), Moreira and Muir (2017) and the S&P Dow Jones Risk Control Indices use a cap of 150% (Banerjee et al., 2016). Das and Uppal (2004) and Liu et al. (2003) find that investors should face potential jump risk by not leveraging the risky asset, i.e. they should choose an equity cap of 100% (see also Poon et al. (2004)). We focus in this paper on uncapped target risk strategies since we are also interested in the accuracy of different forecasting methods. Using an equity cap would distort this examination. However, we show additional results for leverage constrained investors in Appendix D.4. See also Moreira and Muir (2017) on how an equity cap of 100% and 150% affects the utility gains of a mean-variance investor compared to the unconstrained strategy. The authors find substantial utility gains of volatility timing, even after a tight equity cap is set.

[17] One possibility to reduce transaction costs is to reallocate the weight less frequently, e.g. monthly or quarterly. Moreira and Muir (2017, Sec II.F) compare several rebalancing intervals and find a superior performance of the volatility timing strategy when portfolio weights are adjusted monthly instead of quarterly or annually. Zakamulin (2015, Exhibit 2) and Perchet et al. (2016) also find better results for strategies with shorter rebalancing intervals. However, in an earlier version of their paper, Bollerslev et al. (2018) find a trade-off between forecasting accuracy

We follow the idea of Perchet et al. (2016) and show additional results of risk targeting for three different reallocation buffers in Section D.3.

1.3 Targeting a Constant Level of Tail Risk: Target VaR and CVaR Strategies

1.3.1 Managing Volatility versus Managing Tail Risk

As motivated in the previous section, due to the risk-averse nature of most investors, the demand for risk-managed investment strategies is very high. Risk management emerged as a major topic within the financial industry and is becoming more important for portfolio managers (Berkowitz and O'Brien, 2002, Christoffersen and Diebold, 2000). In Appendix A, we have summarized several justifications and advantages of volatility targeting as a tool to manage a portfolio's risk. Thus, dynamically scaling the exposure to a portfolio of risky asset is an appealing portfolio risk management tool. However, managing volatility does not necessarily mean managing risk (Poon and Granger, 2003, Szegö, 2002). In this section, we summarize several disadvantages of volatility as a risk measure and we argue why managing a portfolio's downside risk is more appropriate.

Return distributions are typically skewed and fat-tailed (see Farinelli et al. (2008) among others).[18] A negatively skewed return distribution coincides with a higher probability of extremely negative returns, whereas a positively skewed return distribution coincides with a higher probability of extremely positive returns. A fat-tailed distribution implies that extreme (positive or negative) returns are more likely than would be expected if returns were normally distributed (see Campbell and Hentschel (1992) among others). Hence, a negatively skewed and fat-tailed distribution makes extremely negative returns much more likely than anticipated by a normal

and transaction costs and conclude that it may be better not to trade every change in the optimal weight. The authors find better utility gains for the strategies that adjust the weight less frequently, especially when transaction costs are high and/or when models induce high day-to-day changes in the optimal weight. Kirby and Ostdiek (2012) also find good results of volatility timing strategies that react less sensitive to volatility changes. In order to reduce transaction costs, Taylor (2014) presents a method that decreases the changes in the optimal weight.

[18]Campbell and Hentschel (1992) explain the existence of negatively skewed and fat-tailed return distributions by the volatility feedback effect and the arrival of news. More precisely, news, both positive and negative, increase volatility and thus lower stock prices. Negative news additionally cause a stock price decline, whereas positive news dampen the volatility feedback induced stock price decline. Hence, the combination of these effects produces a negative skewness and a high excess kurtosis.

distribution. Interestingly, Gormsen and Jensen (2017) find that skewness becomes more negative when kurtosis increases, i.e. both moments comove in directions that increase left tail risk. Furthermore, the authors find that times of a negative skewness and/or high kurtosis are typically followed by low future returns. Managing volatility, which is at least implicitly based on the assumption of normally distributed returns, can thus lead to too low weights in times of huge positive returns and too high weights in times of extremely negative returns. For that reason, Harvey and Siddique (2000, p. 1293) suggest that instead of a mean-variance framework, a mean-variance-skewness framework should be used in an asset allocation analysis (see also Ghysels et al. (2016)). Guidolin and Timmermann (2008) also show that accounting for higher moments, like skewness and kurtosis, strongly affects the investor's asset allocation. Thus, higher moments should be incorporated in asset allocation decisions (see also Patton (2004), Ang et al. (2006a) and Jondeau and Rockinger (2012)). This is also confirmed by Xiong and Idzorek (2011) who highlight that accounting for skewness and kurtosis is crucial and superior to mean-variance optimization, especially in times of extremely negative returns. A simple way to incorporate higher moments in asset allocation decisions is to use downside risk measures as portfolio risk management tool. Downside risk measures typically increase when skewness becomes more negative and/or kurtosis increases (Bali et al., 2009). In particular, managing a portfolio's risk based on downside risk measures has the advantage that higher moments are considered, without relying on noisy estimates of skewness and kurtosis (Ghysels et al., 2016). Downside risk based portfolio allocation methods have been frequently used in the financial literature and have been compared to mean-variance optimized portfolios. For example, Farinelli et al. (2008) show that maximizing the Sharpe Ratio, i.e. maximizing the mean-variance trade-off, leads to a lower portfolio performance than maximizing the mean-downside risk trade-off. Jarrow and Zhao (2006) compare mean-variance optimized portfolios with mean-downside risk optimized portfolios and find huge differences in both portfolios when asset return distributions are non-normally distributed. Similarly, using hedge fund data, Agarwal and Naik (2004) compare a mean-downside risk framework with the mean-variance framework. The authors demonstrate that the mean-variance framework significantly underestimates the funds' downside risk and produces much higher losses during downturn periods. Generally, managing volatility is only suitable if asset returns are normally distributed or if investors have quadratic utility (see Agarwal and Naik (2004) and Bali et al. (2009) and references therein). This is confirmed by Jondeau and Rockinger (2006) who show that mean-variance portfolios and portfolio allocations that account for higher moments are nearly indistinguishable when returns are approximately normally distributed. However, both approaches produce significantly diverse allocations for non-normally distributed returns. Similarly, Packham et al. (2017) manage a portfolio's risk using the difference between Value at Risk (VaR) forecasts based on a normality assumption and distributions that account for fat tails and skewness. The authors find huge improvements of this approach compared to buy-and-hold and other tail risk-protection strategies. Thus, portfolio allocation methods that incorporate the assets' non-normalities are superior to volatility based allocation methods. Furthermore, Campbell and Hentschel (1992), Jondeau and Rockinger (2003), Harvey and Siddique (1999) and Bali et al. (2008) show that conditional skewness and kurtosis are highly time-varying. Thus, frequently reallocating the risky asset's weight based on an estimate of the current downside risk, and hence incorporating the time-variation in higher moments, seems crucial. Volatility based portfolios or simple static portfolio allocations do not incorporate the time-variation in higher moments. Jondeau and Rockinger (2012) show that incorporating the time-variation in skewness and kurtosis is crucial in portfolio selection problems and that higher moment timing outperforms volatility timing. Similarly, Cuoco et al. (2008) demonstrate that portfolio weights should be readjusted frequently based on estimates of the assets' tail risk.

Besides the existence of skewed and fat-tailed return distributions and the importance of incorporating this observation in asset allocation decisions, Scott and Horvath (1980) theoretically show that, under some assumptions, investors have preferences for higher (or positive) skewness and lower kurtosis (see also Guidolin and Timmermann (2008) and Bali et al. (2009)).[19]

[19] See also Kraus and Litzenberger (1976), Harvey and Siddique (2000) and Patton (2004) on the preference of positive skewness. Kraus and Litzenberger (1976) extend the traditional CAPM to a three moment CAPM that includes mean, variance and skewness. Harvey and Siddique (2000) extend this model to a conditional version. See also Section I.C in Harvey and Siddique (2000) on the geometry of the three moment efficient portfolios, where investors demand higher expected returns for holding negatively skewed assets. Guidolin and Timmermann (2008) examine the optimal asset allocation under four-moment preferences and regime switching and demonstrate that the asset allocation under four-moment preferences differs from the asset allocation of a mean-variance investor.

More generally, investors have a preference for odd moments, e.g. higher returns and positive skewness, but dislike even moments like variance and kurtosis. Bali et al. (2009) show that higher downside risk predicts lower future skewness. Similarly, Kelly and Jiang (2014) show that an increase of tail risk predicts higher kurtosis and lower skewness of future returns. Hence, investors with preferences as in Scott and Horvath (1980) should lower the exposure to the risky asset if tail risk – not necessarily volatility – increases. Generally, downside risk measures increase if the return distribution is leptokurtic or negatively skewed (Bali et al., 2009, Ghysels et al., 2016). Consequently, by managing downside risk instead of volatility, a higher kurtosis and/or a more negative skewness of the risky asset's return distribution induces a lower weight of the risky asset and fits well to these investors' preferences. In a utility based setting, Bali et al. (2009, p. 892) find that "investors dislike VaR". Furthermore, investors do not only have preferences for higher moments, but are also concerned about the occurrence of extreme crashes. Bollerslev and Todorov (2011, p. 2187) find that the compensation of tail risk – called "crash-o-phobia" by the authors – is extremely high and much higher than the compensation for volatility, i.e. investors fear tail risk much more than volatility (see also Bollerslev et al. (2015) and Chabi-Yo et al. (2018)). This is also confirmed by the earlier work of Lee and Rao (1988) who find that investors are more concerned about downside risk and that managing volatility is only sufficient when asset returns follow a symmetric distribution (see also Szegö (2002) and Strub (2013)).[20] Hence, investors are not concerned about return deviations from a mean but about extremely negative returns, which are described by higher moments and rare tail events (see Lempérière et al. (2017) and references therein). Furthermore, most investors are loss-averse, i.e. they weight losses higher than gains (Benartzi and Thaler, 1995). Loss-averse investors have a high demand for portfolio insurance methods that avoid huge losses and these investors seek for risk reduction methods, especially in times of high market downturns (Aït-Sahalia and Brandt, 2001, Ang et al., 2006a, Bollerslev and Todorov, 2011, Chabi-Yo et al., 2018). Consequently, for loss-averse investors, controlling downside risk instead of volatility,

See also Jondeau and Rockinger (2006), Jondeau and Rockinger (2012) and Lempérière et al. (2017) and references therein on preferences for higher moments and implications on asset allocation decisions.

[20]For most investors, "risk" is associated with low or even negative returns. Describing risk by volatility does not differentiate between positive or negative returns (see Lee and Rao (1988, p. 452) or Poon and Granger (2003, p. 480)).

i.e. controlling negative returns instead of return deviations, is crucial in order to increase their utility (see Aït-Sahalia and Brandt (2001, p. 1298), Ang et al. (2005), Ang et al. (2006a) and references therein). Aït-Sahalia and Brandt (2001, p. 1315) state that the theory of loss aversion is related to the literature on downside risk-based investment decisions. Similarly, Jarrow and Zhao (2006) motivate that loss-averse investors should manage downside risk instead of volatility when asset return distributions are non-normally distributed. Timing downside risk instead of volatility also fits well to safety-first investors who are concerned about avoiding financial disasters (see Arzac and Bawa (1977), Bali et al. (2009), Van Oordt and Zhou (2016) and references therein).

As summarized in Appendix A, due to the increase of correlations in bear markets, assets typically crash together. This reduces the benefits of diversification just when it is most needed, i.e. diversification is an inappropriate portfolio risk management tool (Ang and Chen, 2002, Butler and Joaquin, 2002, Karolyi and Stulz, 1996, Longin and Solnik, 2001, Poon et al., 2004). For example, Chabi-Yo et al. (2018) find a stronger asymptotic dependence in the left tail of stock returns than in the right tail, i.e. stocks tend to crash simultaneously. In particular, the left tail dependence increases in periods of market crashes (Chabi-Yo et al., 2018, Figure 2). Therefore, lowering the exposure to the risky asset in bear markets is crucial in order to manage the risk of the portfolio. However, by using Extreme Value Theory (EVT) and thus measuring tail risk, Longin and Solnik (2001) show that increases in correlations do not necessarily coincide with increases in volatility, but with the occurrence of huge negative returns. Thus, the portfolio's exposure in bear markets should be better determined by the portfolio's tail risk instead of the portfolio's volatility. This is line with Poon et al. (2004) and Kelly and Jiang (2014) who find that volatility standardized returns still exhibit significant tail dependency and tail risk. Additionally to the simultaneous increase of correlations, Jondeau and Rockinger (2003) show that skewness and kurtosis of different risky assets also comove, i.e. large (negative) returns in different risky assets tend to occur simultaneously. As a consequence, simply combining several risky assets or managing volatility does not necessarily reduce the occurrence of extremely negative returns. Similarly, Gormsen and Jensen (2017) find that skewness and kurtosis of the same asset typically co-move, i.e. kurtosis increases when skewness becomes more negative, which eventually increases the asset's crash risk. These periods often occur when market volatility is low, i.e. in low volatile periods, risk "hides in the tails". Ghysels et al. (2016) also find that skewness is typically hidden in the tails and that skewness in the tails has a high impact on portfolio allocations. Based on this finding, Gormsen and Jensen (2017) show that volatility targeting strategies still exhibit a high tail risk, i.e. managing volatility does not mean managing extremely negative returns. Thus, volatility targeting is an inappropriate crash risk mitigation tool. This especially holds for periods with an extremely high crash risk. For example, Liu et al. (2003) and Das and Uppal (2004) find that in times of huge price jumps, like the global financial crisis, negative skewness and kurtosis are higher than in normal times, which is not captured by volatility targeting. Similarly, Jarrow and Zhao (2006) show that the portfolio allocation of volatility and downside risk managed strategies can be vastly different when returns exhibit price jumps. These rare tail events are not predictable and cannot be completely avoided by managing risk dynamically (Bollerslev and Todorov, 2011).[21] However, rare events occur in the tail of the loss distribution and are often accompanied with changes in moments higher than volatility (Poon et al., 2004, p. 582). To better manage the potential event risk, an estimation method that reflects the current market condition, measured by a dynamic volatility model, combined with an estimation method that directly models the tail of the distribution, like EVT, should be used instead of volatility alone (Longin, 2000).

Another advantage of volatility targeting is that choosing the weight of the risky asset inversely to the asset's volatility is a simple tool to reduce left tail risk and drawdowns (Dreyer and Hubrich, 2019, Harvey et al., 2018). Benson et al. (2014, p. 96) state that, even in the absence of a negative risk-return relation, the mitigation of drawdowns is a main driver of the profitability of volatility targeting.[22] In order to achieve a high long-term performance, due to the asymmetric behavior of compounded returns, mitigating highly negative returns is more

[21] Systematic event risk, like unpredictable jumps, affect the allocation between the risky and the riskless asset (see Poon et al. (2004, p. 602) and Das and Uppal (2004)). In order to account for the potential of unpredictable price jumps, Liu et al. (2003) suggest that investors should avoid leveraged positions.

[22] Benson et al. (2014) compare arithmetic and geometric returns and find that the enhanced risk-return profile of volatility targeting comes from avoiding huge negative returns and not from a negative relation between risk and future returns. Dachraoui (2018) theoretically shows that a negative risk-return relation is not needed to provide an enhanced risk-return profile of risk targeting.

crucial than achieving highly positive returns.[23] This is also confirmed by the results of Barroso and Santa-Clara (2015) who find that the superior performance of the volatility managed momentum strategy is significantly driven by the huge drawdown reduction (see also Moreira and Muir (2017) and Barroso and Maio (2018)). Thus, drawdown reduction is a main driver of the superior performance of the target volatility strategy. Since asset returns are usually non-elliptically distributed, managing volatility underestimates the potential of extreme losses (Szegö, 2002, p. 1255). Consequently, managing downside risk instead of volatility should be more successful in mitigating drawdowns and should eventually result in an even better (risk-adjusted) performance compared to the target volatility and buy-and-hold strategies.[24]

As summarized above, the demand for tail risk hedging strategies is high, since these strategies fit well to the preferences of most investors and, by reducing left tail risk and drawdowns, these approaches deliver an enhanced risk-return profile. The main approaches to reduce the tail risk of a portfolio are derivative based and cash based strategies (see Strub (2013, p. 1), Asvanunt et al. (2015) and Happersberger et al. (2019)). Derivative based strategies achieve downside risk protection by buying or selling derivatives, e.g. options or futures, on the risky asset. Cash based strategies, as the here presented target risk strategies, dynamically allocate the wealth invested in the risky and riskless asset, based on the expected risk of the risky asset and are related to portfolio insurance strategies like CPPI (Happersberger et al., 2019). Asvanunt et al. (2015, Exhibit 4) find that strategies that dynamically readjust their portfolio allocation outperform strategies that use options as tail risk hedging instrument. The reason for this finding is that an option-based hedging approach works well during crises, but is too expensive in periods following a crisis (Asvanunt et al., 2015, Exhibit 9). Thus, cash based tail risk hedging strategies are appealing for practical implementations. However, so far, almost all studies on cash based tail risk hedging strategies focused on allocating money based on a forecast of

[23]For example, a return of -50% has to be compensated by a return of 100% to obtain a compounded return of zero. In contrast, returns of -25% and 50% produce a compounded return of 12.5%. Thus, avoiding extremely high negative returns is beneficial for long-term investors, since high losses have to be compensated by even higher returns. As stated above, this also fits well to the loss aversion of most investors.

[24]This is confirmed by Strub (2013, p. 6) who finds that "larger than normal tail risk is partly responsible for the outsized drawdowns experienced in market downturns [...], thus being able to accurately measure and control it is likely to yield significant improvements in risk adjusted performance" (see also Hocquard et al. (2013)).

volatility instead of a forecast of tail risk.[25] Similarly, there exists a huge literature on the economic value of volatility timing, whereas the economic value of *downside risk timing* is hardly examined (Basak and Shapiro, 2001). Cuoco et al. (2008) find that dynamically reallocating the amount invested in several assets based on downside risk is beneficial and superior to static approaches or approaches that do not account for downside risk. Therefore, we will later assess the economic value of downside risk timing and compare it to the economic value of volatility timing.

To account for the above mentioned drawbacks of the target volatility strategy, we next present the target Value at Risk (target VaR) and target Conditional Value at Risk (target CVaR) strategies. These strategies aim to achieve a constant VaR or CVaR of the portfolio over time. VaR is a widely used tool to measure market risk (Alexander and Baptista, 2004, Bali et al., 2008, Berkowitz et al., 2011, Berkowitz and O'Brien, 2002, Cuoco et al., 2008). However, CVaR has recently become more important from a regulatory and practical view (Du and Escanciano, 2016). By construction, both strategies, the target VaR and target CVaR strategy, automatically manage the downside risk of the risky asset, and thus correct for the drawbacks of the target volatility strategy.[26]

1.3.2 Target VaR Strategy

This section derives the optimal weight for the target VaR strategy and shows how the VaR of the risky asset can be estimated. We again consider an investor who invests w_t in a risky asset and $1-w_t$ in a riskless asset. The goal of the target VaR strategy is to determine w_t such that the

[25] Strub (2013) and Happersberger et al. (2019) use a cash based tail risk hedging strategy that relies on a similar weighting as in the target volatility strategy, but replaces the volatility in Equation (1.2.4) by an estimate of the risky asset's downside risk. Essentially, as we will see later, these strategies do not aim to target a constant level of portfolio risk over time, and hence do not belong to the class of risk targeting strategies. See also Basak and Shapiro (2001), Alexander and Baptista (2004), Cuoco et al. (2008) and Packham et al. (2017) for other tail risk based investment strategies.

[26] These tail risk targeting strategies are similar to the approach of Basak and Shapiro (2001, p. 376) and Cuoco et al. (2008) who incorporate downside risk measures in an asset allocation framework, but instead of targeting a constant level of tail risk, the authors require the downside risk to be below some prespecified limit (see also Wang et al. (2012) and Alexander and Baptista (2004)). Similar to our tail risk targeting strategies, this downside risk managed strategy also allocates wealth between a riskless asset and an (optimal) portfolio of risky assets (see Cuoco et al. (2008, Remark 3) for example). Basak and Shapiro (2001, p. 376) call this approach a softer form of portfolio insurance. We will motivate in Section 1.3.4 that the target VaR and target CVaR strategies can be derived as optimal trading strategies under downside risk limits.

portfolio achieves a constant Value at Risk over time. By definition, the VaR at a significance level α is the maximum loss, defined as the negative daily return, that is only exceeded with a probability of $100 \cdot \alpha\%$ (see Szegö (2002), Yamai and Yoshiba (2005) among others). In order to achieve a constant Value at Risk level $\mathrm{VaR}_\alpha^{\mathrm{target}}$, the investor specifies the desired (daily) Value at Risk level, i.e. the critical loss or loss threshold the investor is willing to accept, as well as the corresponding significance level α, i.e. the exceedance probability. For example, a target VaR level $\mathrm{VaR}_\alpha^{\mathrm{target}}$ of 1% with a corresponding significance level α of 5% translates into a strategy, where daily returns below -1% only occur with a probability of 5%.[27] In other words, daily returns should be higher than -1% with a probability of 95%.[28] Hence, the target VaR strategy has the advantage that it manages extreme losses instead of loss deviations. In particular, investors can choose a target VaR strategy that fits well to their acceptable loss limit, where the choice of α and $\mathrm{VaR}_\alpha^{\mathrm{target}}$ strongly depends on the investor's preferences and degree of risk aversion (Alexander and Baptista, 2004).

Since tail risk measures are typically defined based on loss variables, we define the daily portfolio loss at day t as

$$L_t^P := -R_t^P. \tag{1.3.1}$$

Similarly, the day t loss of the risky asset is defined as $L_t := -R_t$. Thus, the portfolio loss can be written as

$$L_t^P = w_t \cdot L_t - (1 - w_t) \cdot R_t^f. \tag{1.3.2}$$

[27]By defining the target VaR strategy in terms of a potential loss of wealth, this strategy can be easily interpreted by retail investors, whereas a given volatility target level should be less clear for most retail investors. VaR based investment strategies are already available for retail investors. See, for example, the strategies offered by Scalable Capital (`http://www.scalable.capital`).

[28]By choosing low values of $\mathrm{VaR}_\alpha^{\mathrm{target}}$ and α, this strategy can also be used by hedge fund managers as an alternative to absolute return strategies. These strategies typically have absolute return targets that are independent of the current market environment, whereas most mutual fund managers have relative return targets that are compared to a benchmark asset (see Fung and Hsieh (1997) and Agarwal and Naik (2004)). For example, assuming 250 trading days per year and by choosing $\mathrm{VaR}_\alpha^{\mathrm{target}} = 0.5\%$ and $\alpha = 0.4\%$, a daily return below -0.5% should only occur once a year (see Figure III in Appendix D for a performance chart of this strategy). Thus, the target VaR strategy aims to constantly produce returns with limited downside risk, regardless of the current market environment. This strategy should be advantageous to other hedge funds strategies since some hedge funds strategies exhibit huge losses during market downturns and bear significant tail risk (Agarwal and Naik, 2004). Similarly, Agarwal and Naik (2004) find low correlations between hedge funds and the market in up-markets, but high positive correlations during market downturn periods and that hedge funds often resemble a short put payoff profile. Investors who are interested in an absolute return strategy should therefore better use a conservative tail risk targeting strategy.

The day t VaR of the portfolio for a significance level α, denoted by $\mathrm{VaR}_\alpha^{P,t}$, is defined through the relation[29]

$$\mathrm{P}\big(L_t^P \leqslant \mathrm{VaR}_\alpha^{P,t} \mid \mathcal{F}_{t-1}\big) = 1 - \alpha. \tag{1.3.3}$$

Therefore, the portfolio VaR is given by the $(1-\alpha)$-quantile of the (conditional) portfolio loss distribution, denoted by $F^{-1}_{L_t^P|\mathcal{F}_{t-1}}(1-\alpha)$, i.e. $\mathrm{VaR}_\alpha^{P,t} = F^{-1}_{L_t^P|\mathcal{F}_{t-1}}(1-\alpha)$. In Appendix B, we show that the portfolio VaR is given by

$$\mathrm{VaR}_\alpha^{P,t} = w_t \cdot \mathrm{VaR}_\alpha^t - (1-w_t) \cdot R_t^f, \tag{1.3.4}$$

where $\mathrm{VaR}_\alpha^t := F^{-1}_{L_t|\mathcal{F}_{t-1}}(1-\alpha)$ denotes the day t VaR of the risky asset.[30] In order to achieve a constant portfolio VaR level $\mathrm{VaR}_\alpha^{\mathrm{target}}$ over time, i.e. $\mathrm{VaR}_\alpha^{P,t} = \mathrm{VaR}_\alpha^{\mathrm{target}}$ for all t, the weight of the risky asset has to be chosen as

$$w_t = \frac{\mathrm{VaR}_\alpha^{\mathrm{target}} + R_t^f}{\mathrm{VaR}_\alpha^t + R_t^f}. \tag{1.3.5}$$

By construction, since VaR_α^t and R_t^f are $\mathcal{F}_{t-1}$-measurable, the weight w_t is known at time $t-1$. Furthermore, the weight of the risky asset is increased if the downside risk of the risky asset, measured by its VaR, is expected to be low and vice versa. Thus, the tail risk of the portfolio is managed by allocating money between the risky and the riskless asset, based on the risky asset's VaR.[31] When a market crash becomes more likely, the amount invested in the risky asset is reduced. When market risk declines, the amount invested in the risky asset is subsequently increased.[32]

[29]Throughout the paper, we assume that the loss variables L_t^P and L_t are continuously distributed.

[30]The representation in Equation (1.3.4) can be directly seen by positive homogeneity and translation invariance of VaR (Szegö, 2002, p. 1259-1260).

[31]Many tail risk hedging strategies that aim to reduce a portfolio's tail risk only work well when markets exhibit huge drawdowns. In up-markets, these tail risk hedging strategies usually perform worse than a simple buy-and-hold strategy, which translates into a worse overall performance (Hocquard et al., 2013). The target VaR strategy has the advantage that this strategy increases the weight of the risky asset as downside shrinks, and hence captures the upside potential while downside risk is managed (Wang et al., 2012, p. 38). Dopfel and Ramkumar (2013) show that the periods following high risk periods are the most attractive ones (see also Muir (2017)). Hence, similar to portfolio insurance strategies, risk targeting delivers an option-like return profile (see also Fung and Hsieh (1997) who found a similar behavior for dynamic trading strategies used by hedge fund managers).

[32]The target VaR strategy manages a portfolio's crash risk by dynamically scaling the exposure to the risky asset. Another approach to manage a portfolio's crash risk would be to dynamically change the portfolio allocation. For example, Chabi-Yo et al. (2018) show in a cross-sectional setting that assets with a lower crash sensitivity outperform during times of market distress but underperform when markets are calm (see also Van Oordt and Zhou (2016)). Thus, during a crash period, the amount invested in crash-sensitive assets should be decreased and should then subsequently be increased when the crash risk declines. This strategy could additionally be combined with the target VaR strategy (see Rickenberg (2020c) who uses a similar approach for long-short strategies).

Similar to the volatility, the VaR of the risky asset is not observable, and hence a forecast of the risky asset's VaR is needed.[33] We use again several models to estimate the risky asset's VaR. As first approach, we use the unconditional Historical Simulation (HS) based on a rolling window of n days, i.e. we estimate VaR_α^t by the empirical $(1-\alpha)$-quantile of the past n daily losses (see Kuester et al. (2006, p. 56-57) or Halbleib and Pohlmeier (2012)). More formally, for a sample $l_{t-n}, ..., l_{t-1}$ of n realized losses, the day t VaR is given by

$$\widehat{\mathrm{VaR}}_\alpha^t = l_{(\lceil n(1-\alpha) \rceil), t-1}, \tag{1.3.6}$$

where $l_{(1),t-1} \leqslant ... \leqslant l_{(n),t-1}$ denotes the order statistics of the sample $l_{t-n}, ..., l_{t-1}$. This estimation method is frequently used by banks and practitioners (Berkowitz et al., 2011, Berkowitz and O'Brien, 2002). Furthermore, Atilgan et al. (2020) also use this VaR estimation method and find that stocks with a higher VaR underperform stocks with a lower VaR. Thus, in a cross-sectional setting, there exists a negative VaR-return relation.[34]

Historical Simulation relies on the assumption that the loss distribution can be estimated by the empirical distribution of past losses (McNeil and Frey, 2000, p. 273). Hence, Historical Simulation is based on the assumption that losses are iid. In practice, this assumption does not hold for losses of most risky assets, since asset returns (or losses respectively) are known to exhibit a time-varying volatility and volatility clustering (Pritsker, 2006, p. 563). Further, Pritsker (2006) shows that Historical Simulation does not respond to the 1987 crash. Thus, Historical Simulation is an inappropriate tool to manage extreme market crashes. Generally, most VaR estimation models that are frequently used in the financial industry, like Historical Simulation, work well in calm periods but fail to produce accurate risk forecast in times of high downside risk, just when reliable forecasts are most needed (Berkowitz et al., 2011, Halbleib and Pohlmeier, 2012). As a consequence, using such static estimation models as portfolio risk management tool can translate in high probabilities of extreme losses (Cuoco et al., 2008). Hence, using Historical Simulation in the context of a target VaR strategy can translate in a high exposure to the risky asset in times when financial markets are very risky, although a

[33] See Taylor (2005, Sec. 3) and Kuester et al. (2006) for a survey of VaR estimation models.

[34] Bali et al. (2009) cannot confirm this finding in a time-series setting. However, as mentioned earlier, a negative risk-return relation is not needed for the risk targeting strategy.

good risk-managed investment strategy should exhibit a low weight in the risky asset during times of high market risk. For that reason, a fast adapting estimation model that reflects the current market environment is crucial for a good performance of the target VaR strategy (see also Taylor (2014) and Bollerslev et al. (2018) who find a similar result for volatility managed portfolios). However, estimating VaR by Historical Simulation is easy, straightforward and is the current industry standard for estimating VaR (see Berkowitz et al. (2011) and references therein). Thus, this approach is particularly interesting for index providers and practitioners who are interested in simple target VaR strategies. Consequently, the target VaR strategy based on Historical Simulation deals as a benchmark strategy for more complex target VaR strategies.

Due to the disadvantages of the HS approach, we use three other VaR forecasting models that are based on a volatility forecast of the EWMA or GARCH(1,1) model. When estimating quantile risk measures, McNeil and Frey (2000, p. 273-274) propose to reflect the current volatility background, estimated by a dynamic volatility model, and to account for heavy tails in the conditional loss distribution (see also Longin (2000)). Christoffersen and Diebold (2000) find that volatility is highly forecastable for short horizons of less than 10 days. Thus, these volatility forecasts are highly valuable and should be incorporated when short-term risk is managed. In order to incorporate the risky asset's volatility in the VaR estimation, we assume that returns are given as in Equation (1.2.6). Under this assumption, the day t VaR of the risky asset is given by

$$\mathrm{VaR}_\alpha^t = \sigma_t \cdot F_{L^*}^{-1}(1-\alpha), \tag{1.3.7}$$

where $L_t^* := -Z_t$ is a random variable representing a standardized loss with expectation zero, variance one and $F_{L^*}^{-1}(1-\alpha)$ denotes the $(1-\alpha)$-quantile of L_t^*.[35] We also refer to dynamic risk models that account for the current volatility as conditional models and static models, like Historical Simulation or HSD, that are based on the assumption that returns are iid as unconditional models (Longin, 2000). The forecast $\widehat{\mathrm{VaR}}_\alpha^t$ for the day t VaR based on the information

[35]By assumption, the quantile $F_{L^*}^{-1}(1-\alpha)$ of the standardized loss $L_t^* := -Z_t = -R_t/\sigma_t$ does not depend on t (McNeil and Frey, 2000, p. 276). As an extension of this assumption, we follow Jondeau and Rockinger (2003) and Bali et al. (2008) and use a more sophisticated approach below that does not assume that Z_t is iid.

at time $t-1$ is then given by

$$\widehat{\mathrm{VaR}}_{\alpha}^{t} = \hat{\sigma}_t \cdot \hat{F}_{L^*,t}^{-1}(1-\alpha), \tag{1.3.8}$$

where $\hat{F}_{L^*,t}^{-1}(1-\alpha)$ denotes the estimator of $F_{L^*}^{-1}(1-\alpha)$, given the available information at day $t-1$. $\widehat{\mathrm{VaR}}_{\alpha}^{t}$ is then estimated in the following way using a two-stage approach as described by McNeil and Frey (2000, p. 277). In the first stage, we estimate the volatility $\hat{\sigma}_t$ using the EWMA or the GARCH(1,1) model given in Equation (1.2.7) or (1.2.8), respectively. The parameters of the GARCH(1,1) model are estimated using a Quasi Maximum Likelihood (QML) approach, i.e. assuming a standard normal distribution for the innovations Z_t. In the second stage, the standardized losses, i.e. $l_t^* = -R_t/\hat{\sigma}_t$, are calculated, which are used to calculate $\hat{F}_{L^*,t}^{-1}(1-\alpha)$.[36] In this context, VaR is often estimated by assuming a standard normal distribution for Z_t, i.e. $\hat{F}_{L^*,t}^{-1}(1-\alpha)$ is given by the $(1-\alpha)$-quantile of the standard normal distribution. However, even after standardizing returns or losses by a time-varying volatility, these observations still exhibit a non-zero skewness and fatter tails than the normal distribution (see Campbell and Hentschel (1992), Bollerslev et al. (1992), Glosten et al. (1993), Harvey and Siddique (1999), Ghysels et al. (2016), Jondeau and Rockinger (2003) and Bali et al. (2008)).[37] Similarly, Kelly and Jiang (2014) find that volatility standardized returns still exhibit significant tail risk. Therefore, we estimate $\hat{F}_{L^*,t}^{-1}(1-\alpha)$ based on a sample of n past standardized losses, denoted by $l_{t-n}^*, ..., l_{t-1}^*$, using three different methods that account for this stylized fact.

First, we use the Filtered Historical Simulation (FHS) approach (Barone-Adesi et al., 2008, 1999), i.e. we estimate $\hat{F}_{L^*,t}^{-1}(1-\alpha)$ by the empirical $(1-\alpha)$-quantile of the standardized losses $l_{t-n}^*, ..., l_{t-1}^*$ (Kuester et al., 2006, p. 57). The estimator for the day t VaR of the risky asset is then given by

$$\widehat{\mathrm{VaR}}_{\alpha}^{t} = \hat{\sigma}_t \cdot l_{(\lceil n(1-\alpha)\rceil),t-1}^*, \tag{1.3.9}$$

where $l_{(1),t-1}^* \leqslant ... \leqslant l_{(n),t-1}^*$ denotes the order statistics of the sample $l_{t-n}^*, ..., l_{t-1}^*$.[38] The

[36]The volatility σ_t, as described in Section 1.2, is calculated using daily returns and represents a forecast for the return volatility. However, from $\mathrm{var}(R_t|\mathcal{F}_{t-1}) = \mathrm{var}(-R_t|\mathcal{F}_{t-1})$ it follows that the volatility forecast $\hat{\sigma}_t$ can be directly used as a forecast for the volatility of the losses.

[37]This stylized fact holds for standardized equity returns but not for foreign exchange rates (Bollerslev et al., 1992, p. 38).

[38]Pritsker (2006) states that the choice of n is not straightforward. However, $n = 1000$ is frequently used in

FHS approach easily combines the conditional heteroscedasticity and the non-normality of asset returns in a simple estimation method without relying on any distributional assumption (Giannopoulos and Tunaru, 2005, p. 983).

As second estimation method, we use the Extreme Value Theory (EVT) approach of McNeil and Frey (2000).[39] The EVT approach is based on the assumption that the tail of the distribution of the standardized losses can be described by a Generalized Pareto Distribution (GPD).[40] The tail of this distribution is defined in terms of a threshold u.[41] Based on this threshold, the EVT approach assumes that standardized losses above the threshold u follow a GPD that is defined as

$$G_{\xi,\beta}(y) = \begin{cases} 1-(1+\xi y/\beta)^{-1/\xi}, & \text{if } \xi \neq 0 \\ 1-\exp(-y/\beta), & \text{if } \xi = 0, \end{cases} \tag{1.3.10}$$

with $\beta > 0$. The support of this distribution is given by $y \geqslant 0$ if $\xi \geqslant 0$ and $0 \leqslant y \leqslant -\beta/\xi$ if $\xi < 0$ (McNeil and Frey, 2000, p. 280). The parameter ξ is usually called the *shape* parameter and β is called the *scale* parameter (McNeil et al., 2015, p. 147). For the estimation of $\hat{F}^{-1}_{L^*,t}(1-\alpha)$, we again assume that the sample contains n standardized losses. The estimator $\hat{F}^{-1}_{L^*,t}(1-\alpha)$ is then given by

$$\hat{F}^{-1}_{L^*,t}(1-\alpha) = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{\alpha n}{N_u}\right)^{-\hat{\xi}} - 1\right), \tag{1.3.11}$$

where $\hat{\beta}$ and $\hat{\xi}$ are the Maximum Likelihood estimates and N_u denotes the number of standardized losses that exceed the threshold u (McNeil et al., 2015, p. 154 and 349). The VaR forecast

applications (see Kuester et al. (2006) or Christoffersen (2012)). See also Halbleib and Pohlmeier (2012) on how the window size impacts estimation results of VaR forecasts.

[39] See also McNeil et al. (2015, Section 5.2.6) for a survey on estimating quantile risk measures when a GARCH volatility model is used in the first stage and Kuester et al. (2006, Section 1.4) for a good summary on how to estimate VaR using EVT. EVT is also used by Allen et al. (2012), Poon et al. (2004), Longin and Solnik (2001), Kelly and Jiang (2014), Longin (2000) and Van Oordt and Zhou (2016) in other related financial topics.

[40] The EVT approach assumes that losses are iid. Hence, when working with short horizons, as done in this paper, standardizing losses by a time-varying volatility is crucial for the EVT approach. A well-known stylized fact states that daily losses are far away from iid, whereas the iid assumption fits quite well to standardized losses (Kuester et al., 2006, p. 62). When working with longer time horizons, the EVT approach can be directly applied to the non-standardized losses (McNeil and Frey, 2000).

[41] One drawback of EVT is the choice of the threshold u (Kellner and Rösch, 2016). If u is chosen too high, the estimation of the parameters is based on only few exceedance observations, making the estimation less precise. Choosing u too low contradicts the approximation in Equation (1.3.10), since this approximation only holds for the tails of the distribution (see Longin and Solnik (2001, Sec. II.A), Kuester et al. (2006, p. 62) and Yamai and Yoshiba (2005, p. 1008)). Longin and Solnik (2001, Appendix 1) show how u can be optimally chosen based on Monte Carlo Simulations. Packham et al. (2017, p. 740) find that their VaR-based tail risk protection strategy is robust against changes in u. As in Kellner and Rösch (2016), we choose the threshold u as the 90%-quantile.

is then given by Equations (1.3.8) and (1.3.11).

We will next use a further extension of the models presented above. The Historical Simulation approach assumes that returns are iid, which is not realistic in practice. In contrast, by assuming that only volatility standardized returns are iid, the EVT and FHS approaches are more realistic. However, several studies show that even this assumption is too restrictive, since even volatility standardized returns exhibit autoregressive patterns in the conditional skewness and kurtosis (Bali et al., 2008, Harvey and Siddique, 1999, Jondeau and Rockinger, 2003). Thus, we follow Jondeau and Rockinger (2003) and Bali et al. (2008) and use the EWMA and GARCH based approach combined with the skewed t distribution of Hansen (1994), where conditional skewness and kurtosis are modeled autoregressively. Similar to Equation (1.2.6), we assume that the daily return can be described by

$$R_t = \sigma_t \cdot Z_t,\ Z_t \sim stsk(\eta_t, \lambda_t), \tag{1.3.12}$$

where $Z_t \sim stsk(\eta_t, \lambda_t)$ means that Z_t is skewed t distributed with mean zero, variance one and time-varying parameters η_t and λ_t. The skewed t distribution of Hansen (1994) is characterized by the pdf

$$f_{stsk}\left(z \mid \eta_t, \lambda_t\right) = \begin{cases} bc\left(1 + \frac{1}{\eta_t - 2}\left(\frac{bz+a}{1-\lambda_t}\right)^2\right)^{-(\eta_t+1)/2} & \text{if } z < -\frac{a}{b} \\ bc\left(1 + \frac{1}{\eta_t - 2}\left(\frac{bz+a}{1+\lambda_t}\right)^2\right)^{-(\eta_t+1)/2} & \text{if } z \geqslant -\frac{a}{b} \end{cases}, \tag{1.3.13}$$

where

$$a := 4\lambda_t c \frac{\eta_t - 2}{\eta_t - 1}, \quad b^2 := 1 + 3\lambda_t^2 - a^2, \quad c := \frac{\Gamma\left(\frac{\eta_t+1}{2}\right)}{\sqrt{\pi(\eta_t - 2)}\Gamma\left(\frac{\eta_t}{2}\right)}.$$

The parameters of this distribution are restricted to $\eta_t > 2$ and $-1 < \lambda_t < 1$ (see Hansen (1994, p. 710) and Jondeau and Rockinger (2003, p. 1702)). Further, for $\lambda_t = 0$, this distribution is symmetric and equals the standardized t distribution. For $\lambda_t > 0$ ($\lambda_t < 0$), the distribution is positively (negatively) skewed (Hansen, 1994). Moreover, skewness exists for $\eta_t > 3$ and kurtosis exists for $\eta_t > 4$ (Jondeau and Rockinger, 2003). Jondeau and Rockinger (2003) show that, although η_t is often referred as the parameter that determines kurtosis and λ_t is referred as the skewness parameter, both parameters, η_t and λ_t, affect both moments, skewness and kurtosis. In particular, the relation between the parameters and higher moments is highly non-linear. The parameters of the skewed t distribution are then modeled autoregressively, where we first model unrestricted parameters by

$$\tilde{\eta}_t = a_1 + b_1 R_{t-1} + c_1 \tilde{\eta}_{t-1}, \tag{1.3.14}$$

$$\tilde{\lambda}_t = a_2 + b_2 R_{t-1} + c_2 \tilde{\lambda}_{t-1}. \tag{1.3.15}$$

To guarantee that the standardized skewed t distribution is well defined, the parameters have to be rescaled to fulfill the conditions $\eta_t > 2$ and $-1 < \lambda_t < 1$. We follow Jondeau and Rockinger (2003) and Bali et al. (2008) and use a logistic transformation to guarantee that these restrictions hold. The parameters η_t and λ_t are then given by

$$\eta_t = 2 + \exp\left(\tilde{\eta}_t\right) \tag{1.3.16}$$

$$\lambda_t = \frac{2}{1 + \exp\left(-\tilde{\lambda}_t\right)} - 1. \tag{1.3.17}$$

The α-quantile of the skewed t distribution is given by

$$F_{stsk}^{-1}\left(\alpha \mid \eta_t, \lambda_t\right) = \begin{cases} \frac{1}{b}\left((1-\lambda_t)\sqrt{\frac{\eta_t-2}{\eta_t}} F_t^{-1}\left(\frac{\alpha}{1-\lambda_t}|\eta_t\right) - a\right) & \text{if } \alpha < \frac{1-\lambda_t}{2} \\ \frac{1}{b}\left((1+\lambda_t)\sqrt{\frac{\eta_t-2}{\eta_t}} F_t^{-1}\left(\frac{\alpha+\lambda_t}{1+\lambda_t}|\eta_t\right) - a\right) & \text{if } \alpha \geqslant \frac{1-\lambda_t}{2}, \end{cases} \tag{1.3.18}$$

where $F_t^{-1}(z|\eta_t)$ is the inverse of the t distribution's cdf $F_t(z|\eta_t) = \int_{-\infty}^{z} f_t(u|\eta_t)du$ (Jondeau and Rockinger, 2003). The t distribution's pdf with η_t degrees of freedom is given by

$$f_t(z|\eta_t) = \frac{\Gamma\left(\frac{\eta_t+1}{2}\right)}{\Gamma\left(\frac{\eta_t}{2}\right)\sqrt{\pi\eta_t}} \left(1 + \frac{z^2}{\eta_t}\right)^{-(\eta_t+1)/2}, \tag{1.3.19}$$

where $\Gamma(\cdot)$ denotes the Gamma function. The VaR forecast for day t is then again given by Equation (1.3.8), where

$$\hat{F}_{L*,t}^{-1}(1-\alpha) = -F_{stsk}^{-1}\left(\alpha \mid \hat{\eta}_t, \hat{\lambda}_t\right), \tag{1.3.20}$$

and $\hat{\eta}_t$ and $\hat{\lambda}_t$ denote the Maximum Likelihood estimates of η_t and λ_t.[42]

Kuester et al. (2006) compare several VaR forecasting approaches and find that the GARCH-based EVT, FHS and skewed t distribution approaches always belong to the best conditional

[42] The minus one in front of the quantile and the significance level α results since we fit the skewed t distribution to standardized returns instead of standardized losses.

models, where the authors only use the skewed t distribution with constant parameters instead of time-varying parameters. The authors state that unconditional models, like Historical Simulation, fail to produce accurate VaR forecast and that only conditionally heteroskedastic models deliver acceptable VaR forecasts. Therefore, the authors prefer a conditional approach that accounts for the volatility dynamics. Furthermore, the authors find that the VaR violations of dynamic models are reasonably independent over time, which usually does not hold for Historical Simulation. This demonstrates the importance of a fast adapting model, which is crucial for the target risk strategies, since wrong risk timing translates into a high exposure to the risky asset when the market's downside risk is high and vice versa.[43] Moreover, McNeil and Frey (2000, p. 283) state that using a symmetric distribution, like the normal or the t-distribution, underestimates the loss potential (see also Szegö (2002)). Similarly, Kellner and Rösch (2016) find that only models that account for fat tails and/or skewness deliver accurate VaR forecasts. Since standardized losses typically follow an asymmetric distribution, using an approach like EVT, FHS or the skewed t distribution is a better choice for modeling the right tail of the standardized losses (Xiong and Idzorek, 2011). In particular, directly modeling the right tail of the loss distribution, instead of modeling the whole distribution, usually gives a better fit and is superior when tail risk is forecasted (Longin, 2000). McNeil and Frey (2000, p. 290-291) compare the GARCH-EVT approach with the unconditional EVT, the GARCH-normal and the GARCH-t models and find that, in most cases, the GARCH-EVT model is superior to the benchmark models and that the GARCH-EVT model is the only model that is not rejected in all 15 cases. Packham et al. (2017) use several alternative generalized distributions for Z_t and find that only the GPD works well for tail risk management. Halbleib and Pohlmeier (2012) also find convincing results of combining a dynamic volatility model, like the GARCH(1,1) model, with an estimate of $F_{L*}^{-1}(1-\alpha)$ that accounts for skewness, like the EVT approach or the skewed t-distribution. Similarly, Bali et al. (2008) find good results of combining a dynamic volatility model with the skewed t-distribution. As a consequence, combining a dynamic volatility model with the EVT, FHS or skewed t approach should result in a portfolio VaR that is closer to the

[43] We will come back to this point in Section 1.4.2, when we show how the accuracy of the target VaR strategies can be backtested.

desired target VaR level than the portfolio VaR of the simple Historical Simulation approach.[44] As stated above, since a more stable portfolio risk is typically rewarded by higher performance gains, we expect a better risk-return profile for the conditional approaches. We answer the question which model delivers the most accurate portfolio VaR later, when we assess the accuracy of several target VaR strategies using the backtesting method described in Section 1.4.2.

Since R_t^f is (typically) small compared to $\mathrm{VaR}_\alpha^{\mathrm{target}}$ and VaR_α^t, the weight w_t of the target VaR strategy can be approximated by[45]

$$w_t \approx \frac{\mathrm{VaR}_\alpha^{\mathrm{target}}}{\mathrm{VaR}_\alpha^t}. \tag{1.3.21}$$

The structure in Equation (1.3.21) is similar to the weight of the target volatility strategy, given in Equation (1.2.3), but the volatility of the risky asset is replaced by the VaR of the risky asset. The weighting in Equation (1.3.21) was also used by Happersberger et al. (2019). By using the approximation in Equation (1.3.21) and the decomposition of the VaR in Equation (1.3.7), the weight of the risky asset of the target VaR strategy can be approximated by

$$w_t \approx \frac{\mathrm{VaR}_\alpha^{\mathrm{target}}}{\mathrm{VaR}_\alpha^t} = \frac{F_{L*}^{-1}(1-\alpha) \cdot \dfrac{\mathrm{VaR}_\alpha^{\mathrm{target}}}{F_{L*}^{-1}(1-\alpha)}}{\sigma_t \cdot F_{L*}^{-1}(1-\alpha)} = \frac{\sigma_{\mathrm{target}}}{\sigma_t}, \tag{1.3.22}$$

with $\sigma_{\mathrm{target}} := \mathrm{VaR}_\alpha^{\mathrm{target}} / F_{L*}^{-1}(1-\alpha)$. Therefore, the weight of a target VaR strategy can be approximated by the weight of a target volatility strategy, where the target volatility level is determined by the target VaR level and the $(1-\alpha)$-quantile of the standardized losses. Hence, one could argue that every target VaR strategy, based on the decomposition in Equation (1.3.7), can be approximated by a target volatility strategy.[46] Further, a constant portfolio volatility can also be achieved by controlling VaR instead of volatility. Taylor (2005) shows that incorporating higher moments in volatility forecasts is beneficial. This results since the shape of the conditional return distribution is not fix over time as shown by Jondeau and Rockinger (2003)

[44] A possible extension to further improve the accuracy and robustness of VaR targeting in a simple manner could be to combine several VaR forecasting models as presented in Halbleib and Pohlmeier (2012). Allen et al. (2012) also use the average of different VaR forecasts made with different forecasting methods.

[45] For our sample, the mean of R_t^f and VaR_α^t is 0.007% and 4.0766%, respectively. Thus, for our $\mathrm{VaR}_\alpha^{\mathrm{target}}$ of 1.947% and these averages, the weight based on Equation (1.3.5) would be 0.4785, whereas the approximated weight would be 0.4776.

[46] This is a contrarian approach to Taylor (2005) who uses VaR forecasts based on the CAViaR model and Historical Simulation to obtain estimates of conditional volatility.

and Bali et al. (2008). However, for several reasons, the approximation in Equation (1.3.22) has to be regarded carefully. First, Equation (1.3.21) is just an approximation, which is only exact for $R_t^f = 0$. Second, to transform a target VaR strategy into the corresponding target volatility strategy, the distribution of the standardized losses, or at least the quantile $F_{L*}^{-1}(1-\alpha)$, has to be known. In practice, both are unknown and this transformation is not directly feasible. As a rough approximation, quantiles of the standard normal distribution can be used. In this case, the target VaR strategy can be approximated by the target volatility strategy, using a target volatility level of

$$\sigma_{\text{target}} = \frac{\text{VaR}_\alpha^{\text{target}}}{N_{1-\alpha}}, \tag{1.3.23}$$

where $N_{1-\alpha}$ denotes the $(1-\alpha)$-quantile of the standard normal distribution.[47]

1.3.3 Target CVaR Strategy

The target VaR strategy, presented in the previous section, has the advantage that the weight of the risky asset is a function of the expected downside risk instead of expected volatility. As stated above, focusing on downside risk management instead of volatility management has several advantages. VaR is the current industry standard when downside risk is measured and managed (Bali et al., 2008, Berkowitz et al., 2011, Berkowitz and O'Brien, 2002). However, the Conditional Value at Risk (CVaR) is becoming more important and is establishing as the more relevant risk measure for managing market risk and from a regulatory perspective (Du and Escanciano, 2016, Kellner and Rösch, 2016). The reason for this development is that the CVaR corrects for several drawbacks of VaR and the CVaR is often claimed as the better risk measure.[48] For example, the VaR has been criticized in the academic literature due to its lack of subadditivity and the disregarding of extreme losses, which are of main interest in risk management (see Artzner et al. (1999), Giannopoulos and Tunaru (2005, p. 980), McNeil and Frey (2000, p. 291-292) or Yamai and Yoshiba (2005, p. 998)). More precisely, VaR only contains

[47] In our notation, the volatility target σ_{target} is denoted as an annualized volatility, whereas the VaR target is chosen as a daily loss. Thus, a target VaR strategy with $\alpha = 5\%$ and $\text{VaR}_\alpha^{\text{target}} = 1\%$ can be approximated by a target volatility strategy with an (annualized) volatility target of $\sigma_{\text{target}} = (0.01/1.645) \cdot \sqrt{252} = 9.6\%$.

[48] See Yamai and Yoshiba (2005) for a good comparison of VaR and CVaR. Moreover, see Szegö (2002, p. 1261) for a list of drawbacks of VaR. See Du and Escanciano (2016) for a good motivation of why CVaR is becoming the more relevant risk measure for managing downside risk.

information on a certain quantile, whereas CVaR contains information on the whole right tail of the loss distribution (Du and Escanciano, 2016, p. 942). Berkowitz and O'Brien (2002) who examine the VaR models of six commercial banks demonstrate that the size of a VaR violation can be surprisingly large. This is confirmed by the study of Du and Escanciano (2016) who find that VaR responds less to extreme losses, such as those experienced during the recent financial crisis. Hence, VaR may underestimate risk in times of market stress, i.e. times of high asset price fluctuations (Yamai and Yoshiba, 2005, p. 998). Similarly, in a portfolio context, managing VaR means that only the exceedance probability is managed instead of the expected loss magnitude (see Basak and Shapiro (2001, p. 385) and Aït-Sahalia and Brandt (2001)). In contrast, by managing CVaR, both the exceedance probability and the size of extreme losses are managed. Basak and Shapiro (2001) and Alexander and Baptista (2004) find that managing CVaR is superior to managing VaR in an asset allocation context, especially if a risk-free asset is available.[49] Due the drawbacks of the VaR, we next extend the target VaR strategy to the target CVaR strategy, that aims to have a constant portfolio CVaR over time. As before, the weight of the risky asset is then given as a function of the expected CVaR of the risky asset, i.e. if the CVaR of the risky asset is expected to be low, the weight of the risky asset is increased and vice versa.

The CVaR is defined as the average loss in the worst $100 \cdot \alpha\%$ cases, i.e. the cases where the loss exceeds the VaR (see Acerbi and Tasche (2002, p. 1488) or Yamai and Yoshiba (2005, p. 999)). More formally, we define the portfolio CVaR, denoted by $\text{CVaR}_\alpha^{P,t}$, as

$$\text{CVaR}_\alpha^{P,t} = \mathbb{E}\big(L_t^P \mid L_t^P \geqslant \text{VaR}_\alpha^{P,t}, \mathcal{F}_{t-1}\big)\,. \tag{1.3.24}$$

In Appendix B.2, we show that the portfolio CVaR is given by

$$\text{CVaR}_\alpha^{P,t} = w_t \cdot \text{CVaR}_\alpha^t - (1 - w_t) \cdot R_t^f, \tag{1.3.25}$$

[49]Basak and Shapiro (2001) demonstrate that, in the context of asset allocation decisions, losses of VaR managed portfolios can exceed the desired VaR extremely (see also Alexander and Baptista (2004), Berkowitz et al. (2011), Cuoco et al. (2008) and references therein). However, Cuoco et al. (2008) show that this observation does no longer hold once risk is managed dynamically, i.e. taking the actual information into account and reevaluating the risk level dynamically, instead of managing risk by a static model (see also Berkowitz et al. (2011)). This highlights the importance of managing portfolio risk dynamically, as done by risk targeting.

where $\mathrm{CVaR}_\alpha^t := \mathbb{E}\big(L_t \,|\, L_t \geqslant \mathrm{VaR}_\alpha^t, \mathcal{F}_{t-1}\big)$ denotes the day t CVaR of the risky asset.[50] In order to achieve a constant portfolio CVaR of $\mathrm{CVaR}_\alpha^{\mathrm{target}}$ over time, i.e. $\mathrm{CVaR}_\alpha^{P,t} = \mathrm{CVaR}_\alpha^{\mathrm{target}}$ for all t, the weight of the risky asset has to be chosen as

$$w_t = \frac{\mathrm{CVaR}_\alpha^{\mathrm{target}} + R_t^f}{\mathrm{CVaR}_\alpha^t + R_t^f}. \tag{1.3.26}$$

Due to the definition of the CVaR, the target CVaR strategy manages expected losses, where the acceptable loss magnitude can be governed by the investor who chooses the values α and $\mathrm{CVaR}_\alpha^{\mathrm{target}}$. For example, a target CVaR level $\mathrm{CVaR}_\alpha^{\mathrm{target}}$ of 2% with a corresponding significance level α of 5% translates into a strategy with an average loss of 2% on the 5% worst days. In other words, the target CVaR strategy's average return on the worst 5 out of 100 days will be -2%.[51] The choices of α and $\mathrm{CVaR}_\alpha^{\mathrm{target}}$ again strongly depend on the individual investor's preferences and risk aversion (Alexander and Baptista, 2004).

As in the previous sections, since the CVaR of the risky asset is not observable, a forecast of CVaR_α^t is needed. We use the same estimation methods that we also use for the VaR of the risky asset. First, and especially interesting for practical implementations, we use Historical Simulation. For this method, we again assume that a data set of n realized losses $l_{t-1}, .., l_{t-n}$ with order statistics $l_{(1),t-1} \leqslant l_{(2),t-1} \leqslant ... \leqslant l_{(n),t-1}$ exists. Based on the ordered losses, we estimate CVaR_α^t by [52]

$$\widehat{\mathrm{CVaR}}_\alpha^t = \frac{1}{n - \lceil n(1-\alpha) \rceil + 1} \cdot \sum_{j=\lceil n(1-\alpha) \rceil}^{n} l_{(j),t-1}. \tag{1.3.27}$$

This estimator is motivated by Acerbi and Tasche (2002, Proposition 4.1),[53] who demonstrate that the estimator in Equation (1.3.27) is only unbiased for n converging to infinity. For small n, the estimator in Equation (1.3.27) is biased. Methods that account for this estimation bias are presented in Ko et al. (2009) among others. As before, the quality of the target CVaR

[50] The representation in Equation (1.3.25) again follows by positive homogeneity and translation invariance of CVaR.

[51] By choosing $\mathrm{CVaR}_\alpha^{\mathrm{target}}$ and α adequately, this strategy can also be an alternative to absolute return or other hedge fund strategies as examined in Fung and Hsieh (1997) and Agarwal and Naik (2004).

[52] See Ko et al. (2009, p. 719) or Giannopoulos and Tunaru (2005, p. 985-986).

[53] In this paper, we assume that losses are continuously distributed. This is confirmed by Giannopoulos and Tunaru (2005, p. 982) who state that only continuous probability distributions are used in practice. In this case, the CVaR, defined as Tail Conditional Expectation (TCE) in Acerbi and Tasche (2002, Definition 2.3), is equal to the Expected Shortfall (ES), defined in Acerbi and Tasche (2002, Corollary 5.3).

strategy strongly depends on the accuracy of the CVaR estimation.[54] However, since Historical Simulation deals as a benchmark model that is particularly interesting for practitioners, we keep the estimation as simple as possible.

For the return decomposition of Equation (1.2.6), the CVaR of the risky asset is given by

$$\mathrm{CVaR}_\alpha^t = \sigma_t \cdot \mathrm{CVaR}_\alpha^*, \tag{1.3.28}$$

where we define $\mathrm{CVaR}_\alpha^* := \mathbb{E}\big(L^* \mid L^* \geqslant F_{L^*}^{-1}(1-\alpha)\big)$ and L^* is again a continuously distributed random variable, representing a standardized loss, with expectation zero, variance one and $F_{L^*}^{-1}(1-\alpha)$ denotes the $(1-\alpha)$-quantile of L^* (see McNeil and Frey (2000, p. 276) for example). As for the VaR, we next present the estimation of the CVaR based on a two-stage approach, where the volatility is estimated in a first stage by one of the volatility models presented in Equation (1.2.7) or (1.2.8). In the second stage, we again consider a sample $l_{t-n}^*, \dots, l_{t-1}^*$ of n standardized losses with order statistics $l_{(1),t-1}^* \leqslant \dots \leqslant l_{(n),t-1}^*$. We denote the estimator for $\mathrm{CVaR}_\alpha^* = \mathbb{E}\big(L^* \mid L^* \geqslant F_{L^*}^{-1}(1-\alpha)\big)$ based on the available information at day $t-1$ by $\widehat{\mathrm{CVaR}}_\alpha^{t,*}$. Hence, the estimator for the CVaR of the risky asset on day t, denoted by $\widehat{\mathrm{CVaR}}_\alpha^t$, is given by

$$\widehat{\mathrm{CVaR}}_\alpha^t = \hat{\sigma}_t \cdot \widehat{\mathrm{CVaR}}_\alpha^{t,*}. \tag{1.3.29}$$

By using the FHS approach, the estimator $\widehat{\mathrm{CVaR}}_\alpha^{t,*}$ is given by Equation (1.3.27), where the j-th order statistic $l_{(j),t-1}$ of the loss variables is replaced by the j-th order statistic $l_{(j),t-1}^*$ of the standardized losses (see Giannopoulos and Tunaru (2005) for example). By using the EVT approach, the estimator $\widehat{\mathrm{CVaR}}_\alpha^{t,*}$ is given by

$$\widehat{\mathrm{CVaR}}_\alpha^{t,*} = \frac{\hat{F}_{L^*,t}^{-1}(1-\alpha)}{1-\hat{\xi}} + \frac{\hat{\beta}-\hat{\xi}u}{1-\hat{\xi}}, \tag{1.3.30}$$

where the estimator $\hat{F}_{L^*,t}^{-1}(1-\alpha)$ is given in Equation (1.3.11), u denotes the predetermined threshold and the parameters $\hat{\xi}$ and $\hat{\beta}$ are the QML estimators (see McNeil and Frey (2000, p. 293) or McNeil et al. (2015, p. 154)).

[54]This is in line with Yamai and Yoshiba (2005, p. 999) who find that the "effectiveness of expected shortfall, however, depends on the accuracy of estimation."

Finally, we also use the skewed t distribution with time-varying parameters to forecast next day's CVaR. Christoffersen (2012) and Rickenberg (2020a, Appendix A) show that, for a random variable $Z_t \sim stsk(\eta_t, \lambda_t)$, we have

$$\mathbb{E}\big(Z_t|Z_t < F_{stsk}^{-1}(\alpha|\,\eta_t,\ \lambda_t)\big) = \begin{cases} \frac{1}{\alpha}\frac{(1-\lambda_t)^2}{b}\left(f_{st}\left(z^{(-)}|\eta_t\right)\cdot\frac{\eta_t-2+\left(z^{(-)}\right)^2}{1-\eta_t} - \frac{a\cdot F_{st}\left(z^{(-)}|\eta_t\right)}{1-\lambda_t}\right) & \text{for } F_{stsk}^{-1}(\alpha|\,\eta_t,\ \lambda_t) < -\frac{a}{b} \\ \frac{1}{\alpha}\frac{(1+\lambda_t)^2}{b}\left(f_{st}\left(z^{(+)}|\eta_t\right)\cdot\frac{\eta_t-2+\left(z^{(+)}\right)^2}{1-\eta_t} + \frac{a\cdot\left(1-F_{st}\left(z^{(+)}|\eta_t\right)\right)}{1+\lambda_t}\right) & \text{for } F_{stsk}^{-1}(\alpha|\,\eta_t,\ \lambda_t) \geqslant -\frac{a}{b}, \end{cases} \tag{1.3.31}$$

where $z^{(-)}$ and $z^{(+)}$ are given by

$$z^{(-)} = \frac{b\cdot F_{stsk}^{-1}(\alpha|\,\eta_t,\ \lambda_t) + a}{1-\lambda_t}, \quad z^{(+)} = \frac{b\cdot F_{stsk}^{-1}(\alpha|\,\eta_t,\ \lambda_t) + a}{1+\lambda_t}.$$

Further, $f_{st}(z|\eta_t)$ and $F_{st}(z|\eta_t) = \int_{-\infty}^{z} f_{st}(u|\eta_t)du$ correspond to the pdf and cdf of the *standardized t distribution* with mean zero and variance one. The pdf of the standardized t distribution is given by (see Bollerslev (1987, p. 543) and Hansen (1994, p. 709))

$$f_{st}(z|\eta_t) = \frac{\Gamma\left(\frac{\eta_t+1}{2}\right)}{\Gamma\left(\frac{\eta_t}{2}\right)\sqrt{\pi(\eta_t-2)}}\left(1+\frac{z^2}{\eta_t-2}\right)^{-(\eta_t+1)/2}. \tag{1.3.32}$$

For the cdf of the t and standardized t distributions it holds $F_{st}(z|\eta_t) = F_t\left(\sqrt{\frac{\eta_t}{\eta_t-2}}z|\eta_t\right)$.[55] Bollerslev (1987) uses the standardized t distribution in the context of the GARCH(1,1) model and finds that the GARCH(1,1)-t model is superior to both, the GARCH(1,1)-normal and the unconditional t distribution. The forecast of day t's CVaR is then given by Equation (1.3.29). In this case, $\widehat{\text{CVaR}}_{\alpha}^{t,*}$ is given by

$$\widehat{\text{CVaR}}_{\alpha}^{t,*} = -\mathbb{E}\left(Z_t|Z_t < F_{stsk}^{-1}\left(\alpha|\,\hat{\eta}_t,\ \hat{\lambda}_t\right)\right), \tag{1.3.33}$$

where $\hat{\eta}_t$ and $\hat{\lambda}_t$ are again the Maximum Likelihood estimates of η_t and λ_t.[56]

McNeil and Frey (2000, p. 292) find that the quality of the CVaR estimation strongly depends on the approach that is used to model the tails of the loss distribution. Correctly modeling

[55] This relation is advantageous since the cdf of the t distribution is often available in most software packages, whereas the cdf of the standardized t distribution is not available (Jondeau and Rockinger, 2003).

[56] The minus one in front of the expectation in Equation (1.3.33) again results, since we fit the skewed t distribution to standardized returns instead of standardized losses. From Equations (1.3.20) and (1.3.33), it directly follows $-\mathbb{E}\left(Z_t|Z_t < F_{stsk}^{-1}\left(\alpha|\,\hat{\eta}_t,\ \hat{\lambda}_t\right)\right) = \mathbb{E}\left(L_t^* \mid L_t^* \geqslant F_{L_t^*}^{-1}(1-\alpha)\right)$, where $L_t^* = -Z_t$.

the tails, which is also crucial for the estimation of the VaR, becomes even more important when CVaR is estimated (Yamai and Yoshiba, 2005). Therefore, correctly modeling the tails of the loss distribution is a central issue in achieving a constant portfolio CVaR over time. As for the VaR, Kellner and Rösch (2016) find that only models that account for fat tails and/or skewness are able to produce accurate CVaR forecasts. As stated above, a more accurate risk forecast, and hence a more constant portfolio risk, is typically rewarded with a superior risk-return profile and high utility gains.

As in Equation (1.3.21), since R_t^f is (typically) small compared to the CVaR values, the weight of the risky asset can be approximated by

$$w_t \approx \frac{\mathrm{CVaR}_\alpha^{\mathrm{target}}}{\mathrm{CVaR}_\alpha^t}. \tag{1.3.34}$$

By using Equation (1.3.34), similar to Equation (1.3.22), we can approximate a target CVaR strategy by a target volatility strategy. The weight of the risky asset for this target volatility strategy is then given by

$$w_t \approx \frac{\mathrm{CVaR}_\alpha^{\mathrm{target}}}{\mathrm{CVaR}_\alpha^t} = \frac{\mathbb{E}\big(L^* \mid L^* \geqslant F_{L^*}^{-1}(1-\alpha)\big) \cdot \dfrac{\mathrm{CVaR}_\alpha^{\mathrm{target}}}{\mathbb{E}\big(L^* \mid L^* \geqslant F_{L^*}^{-1}(1-\alpha)\big)}}{\mathbb{E}\big(L^* \mid L^* \geqslant F_{L^*}^{-1}(1-\alpha)\big) \cdot \sigma_t} = \frac{\sigma_{\mathrm{target}}}{\sigma_t}, \tag{1.3.35}$$

with $\sigma_{\mathrm{target}} = \mathrm{CVaR}_\alpha^{\mathrm{target}} / \mathbb{E}\big(L^* \mid L^* \geqslant F_{L^*}^{-1}(1-\alpha)\big)$. However, since the distribution of the standardized losses, or at least $\mathbb{E}\big(L^* \mid L^* \geqslant F_{L^*}^{-1}(1-\alpha)\big)$, is not known in practice, the volatility target σ_{target} cannot be directly calculated. An approximation can be done by using a standard normal distribution for L^*. In this case, $\mathbb{E}\big(L^* \mid L^* \geqslant F_{L^*}^{-1}(1-\alpha)\big)$ is given by $\frac{\varphi(N_{1-\alpha})}{\alpha}$, where φ and $N_{1-\alpha}$ denote the density function and the $(1-\alpha)$-quantile of the standard normal distribution, respectively. The volatility target is then given by

$$\sigma_{\mathrm{target}} = \frac{\mathrm{CVaR}_\alpha^{\mathrm{target}}}{\varphi(N_{1-\alpha})/\alpha}. \tag{1.3.36}$$

For example, a target CVaR strategy with a significance level of $\alpha = 5\%$ and desired CVaR target $\mathrm{CVaR}_\alpha^{\mathrm{target}} = 2\%$ can be approximated by a target volatility strategy with an annualized volatility target of $\sigma_{\mathrm{target}} = \frac{0.02}{2.063} \cdot \sqrt{252} = 15.4\%$. Further, a constant volatility of 15.4% can be achieved by managing the risky asset's CVaR, and thus incorporating information on the risky asset's skewness and kurtosis. Moreover, by Equations (1.3.7) and (1.3.34), we obtain

$$w_t \approx \frac{\mathrm{CVaR}_\alpha^{\mathrm{target}}}{\mathrm{CVaR}_\alpha^t} = \frac{\mathrm{CVaR}_\alpha^{\mathrm{target}}}{\sigma_t \cdot F_{L^*}^{-1}(1-\alpha) \cdot \frac{\mathbb{E}\left(L^* \mid L^* \geqslant F_{L^*}^{-1}(1-\alpha)\right)}{F_{L^*}^{-1}(1-\alpha)}} = \frac{\mathrm{VaR}_\alpha^{\mathrm{target}}}{\mathrm{VaR}_\alpha^t}, \tag{1.3.37}$$

with $\mathrm{VaR}_\alpha^{\mathrm{target}} = \frac{F_{L^*}^{-1}(1-\alpha)}{\mathbb{E}\left(L^* | L^* \geqslant F_{L^*}^{-1}(1-\alpha)\right)} \cdot \mathrm{CVaR}_\alpha^{\mathrm{target}}$. Therefore, a target CVaR strategy can also be approximated by a target VaR strategy with an adjusted target VaR level.[57] However, the approximation of the target CVaR strategy by a target volatility strategy, given in Equation (1.3.35), is appealing since forecasting volatility is much easier than forecasting CVaR, but this argument is only partly valid for the approximation by a target VaR strategy, given in Equation (1.3.37).[58] Nevertheless, Equation (1.3.37) is helpful for comparing target VaR and target CVaR strategies. By assuming a standard normal distribution for L^*, we obtain the comparable target VaR level

$$\mathrm{VaR}_\alpha^{\mathrm{target}} = \frac{N_{1-\alpha}}{\varphi(N_{1-\alpha})/\alpha} \cdot \mathrm{CVaR}_\alpha^{\mathrm{target}}. \tag{1.3.38}$$

A target CVaR strategy with $\alpha = 5\%$ and $\mathrm{CVaR}_\alpha^{\mathrm{target}} = 2\%$ should then be compared to a target VaR strategy with $\alpha = 5\%$ and $\mathrm{VaR}_\alpha^{\mathrm{target}} = \frac{1.645}{2.063} \cdot 0.02 = 1.6\%$, which is again approximated by a target volatility strategy with $\sigma_{\mathrm{target}} = \frac{0.016}{1.645} \cdot \sqrt{252} = 15.4\%$.

This approach of comparing a target CVaR strategy with a target volatility or target VaR strategy is similar to the approach of Strub (2013). The author starts with a predefined volatility target and transforms this volatility target to a CVaR target by using a normality assumption. The weight of the risky asset is then obtained as the ratio of the transformed CVaR target and the forecast of the risky asset's CVaR. Strub (2013, p. 16) finds that the volatility and CVaR managed strategies offer a substantial drawdown protection, especially in the years when the underlying index suffers the most. By comparing a volatility managed strategy with a CVaR

[57] This approach is similar to Cuoco et al. (2008) who show how a VaR limit can be transformed in a CVaR limit and vice versa.

[58] Yamai and Yoshiba (2005, p. 1012) state that the estimation error for CVaR is larger than for VaR, especially when the return distribution exhibits fat tails. Similarly, Kellner and Rösch (2016) find that the model risk for CVaR is higher than for VaR, which is mainly driven by fat tails in the return distribution. Especially in times of financial market turmoils, like the recent global financial crisis or corona crisis, CVaR forecasts among different models are more volatile than VaR forecasts. Thus, the approximation of a target CVaR strategy by a target VaR strategy could be appealing for extremely fat-tailed asset return distributions.

managed strategy, Strub (2013, p. 17) finds that managing CVaR translates into a better risk-adjusted performance and lower drawdowns. Moreover, the author finds that, even after transaction costs, the risk managed strategies still deliver convincing performances, which makes the risk managed strategies appealing for practical applications and an interesting alternative to hedge fund strategies as examined in Fung and Hsieh (1997) and Agarwal and Naik (2004). Similarly, the CVaR managed strategy of Wang et al. (2012) reduces drawdowns without sacrificing returns, and hence captures the upside potential while downside risk is limited (Wang et al., 2012, p. 38).[59] This is in line with Basak and Shapiro (2001) who also find convincing results of managing expected losses, as done by managing CVaR, and the authors conclude that managing expected losses is superior to managing exceedance probabilities, as done by managing VaR (see also Aït-Sahalia and Brandt (2001, p. 1316)).

1.3.4 VaR and CVaR Targeting as Optimal Trading Strategies under Risk Limits

In this section, we motivate the VaR and CVaR targeting strategies from another perspective as optimal trading strategies when a trader faces an absolute risk limit. This is similar to the examination of Basak and Shapiro (2001), Wang et al. (2012), Cuoco et al. (2008) and Alexander and Baptista (2004).[60] We again consider a trader who invests w_t in the risky and $1 - w_t$ in the riskless asset and define the trader's portfolio value by $W_t := W_{t-1} \cdot (1 + R_t^P)$, $W_0 > 0$. Further, we define the absolute loss in t by $L_t^{abs} := W_{t-1} - W_t = -W_{t-1} \cdot R_t^P$. We now consider a portfolio optimization problem under an absolute risk limit $\overline{\mathrm{VaR}}_t$, given by

$$\max_{w_t} \mathbb{E}\big(R_t^P \mid \mathcal{F}_{t-1}\big) \quad \text{s.t.} \quad \mathrm{VaR}_\alpha^{t,abs} \leqslant \overline{\mathrm{VaR}}_t, \tag{1.3.39}$$

where $\mathrm{VaR}_\alpha^{t,abs} = W_{t-1} \cdot \mathrm{VaR}_\alpha^{P,t}$ denotes the VaR of the absolute loss L_t^{abs}. The risk limit $\mathrm{VaR}_\alpha^{t,abs} \leqslant \overline{\mathrm{VaR}}_t$ can then be rewritten as $\mathrm{VaR}_\alpha^{P,t} \leqslant \overline{\mathrm{VaR}}_t / W_{t-1}$. From Equation (1.3.4) it follows that the risk limit holds if

$$w_t \leqslant \frac{\overline{\mathrm{VaR}}_t / W_{t-1} + R_t^f}{\mathrm{VaR}_\alpha^t + R_t^f}. \tag{1.3.40}$$

[59] Wang et al. (2012) call their strategy a target CVaR strategy. However, the authors do not target a constant level of risk over time, but allow a maximum level of risk (see also Basak and Shapiro (2001) and Alexander and Baptista (2004)).

[60] I thank Peter Albrecht and Markus Huggenberger for this helpful comment.

Under the assumption $\mathbb{E}(R_t \mid \mathcal{F}_{t-1}) > R_t^f$, which is typically fulfilled in practice (Benartzi and Thaler, 1995), the expected portfolio return $\mathbb{E}\big(R_t^P \mid \mathcal{F}_{t-1}\big)$ is increasing in w_t. Hence, the trader chooses the highest possible equity exposure that still fulfills the risk limit $\mathrm{VaR}_\alpha^{t,abs} \leqslant \overline{\mathrm{VaR}}_t$. Thus, w_t is given by

$$w_t = \frac{\overline{\mathrm{VaR}}_t / W_{t-1} + R_t^f}{\mathrm{VaR}_\alpha^t + R_t^f}. \tag{1.3.41}$$

Consequently, by choosing a constant relative risk limit $\overline{\mathrm{VaR}}_t / W_{t-1} = \mathrm{VaR}_\alpha^{\mathrm{target}}$, the target VaR strategy follows as optimal dynamic trading strategy under a downside risk limit. By the same arguments as above, the weighting for the target CVaR strategy can be obtained if an investor faces a CVaR limit.

1.4 Assessing the Accuracy of Risk Targeting

In this section, we present methods that can be used to test the accuracy of the target risk strategies, i.e. we present tests that can be used to test if the different risk models are successful in targeting a constant level of portfolio risk over time. A constant portfolio risk is important for several reasons. First, a constant risk of the strategies should be achieved by definition of risk targeting. Second, an investor who chose a fund that targets a volatility level that fits to the investor's risk preferences would sell this fund if the fund achieves a significantly higher volatility than expected. Similarly, an investor who expects only a limited number of days where the portfolio return is smaller than $-\mathrm{VaR}_\alpha^{\mathrm{target}}$ would also divest if the fund exhibits too many extremely negative returns. Third, risk-averse investors are willing to pay for hedges against a changing portfolio volatility (Adrian and Rosenberg, 2008, Ang et al., 2006b, Bollerslev and Todorov, 2011). These investors are willing to pay higher fees for strategies with a more constant portfolio risk. Fourth, having a constant level of portfolio risk over time is frequently used by practitioners (Barroso and Santa-Clara, 2015, p. 112). Fifth, several studies show that a higher forecasting accuracy, and hence a more constant portfolio risk, coincides with higher (risk-adjusted) performance and utility gains (Bollerslev et al., 2018, Marquering and Verbeek, 2004, Moreira and Muir, 2017, Perchet et al., 2016, Taylor, 2014). Consequently, a fund that fails to target a constant risk over time typically achieves a suboptimal risk-return profile. For example, Bollerslev et al. (2018, p. 2732) write:

> "the investor achieves the maximum utility by successfully targeting a constant risk level, while the utility decreases with the volatility-of-volatility. Hence, risk models that help the investor achieve more accurate volatility forecasts are associated with higher levels of utility"

Bollerslev et al. (2018) find that an investor who uses volatility targeting is willing to pay a fee of 0.48% per year to switch from an inaccurate to a more accurate volatility model. The authors find that there exists a positive, non-linear relation between forecasting accuracy of volatility models and utility benefits. Further, they find that a model with perfect foresight, i.e. a model that produces a totally constant portfolio volatility over time, exhibits the highest utility benefit (see also Benson et al. (2014)). Moreover, Dreyer and Hubrich (2019) find that a portfolio volatility stabilization, i.e. a lower volatility of volatility, is linked to a higher tail risk reduction. Similarly, in a cross-sectional setting, Baltussen et al. (2018) find that assets with a high volatility of volatility (vol-of-vol) underperform assets with a more constant volatility. Further, higher vol-of-vol assets also exhibit higher downside risk and the vol-of-vol is linked to an asset's kurtosis. This especially holds during down-markets where high vol-of-vol assets underperform low vol-of-vol assets by 0.83% per month. Consequently, forecasting accuracy and a stable risk is an important driver of the investor's benefit from risk targeting and should therefore be tested. Besides backtesting the accuracy of volatility targeting, we additionally show how the accuracy of VaR and CVaR targeting can be tested. To assess the accuracy of several volatility models, Bollerslev et al. (2018) use the R^2 as well as the DM-test of Diebold and Mariano (1995), which tests for equal predictive ability. However, both methods have several disadvantages. Therefore, in order to assess the accuracy of volatility targeting, we use more powerful tools that will be presented in the next section.

1.4.1 Assessing the Accuracy of Volatility Targeting

Although several studies on volatility targeting have been made, only a few studies statistically assess if it is possible to achieve the desired volatility target over time. To assess the accuracy of volatility targeting for some set of models $\mathcal{M}$, we measure the portfolio variance of model k on day t by $RV_{k,t}^2 := w_{k,t}^2 \cdot RV_t^2$, where $w_{k,t}$ is the weight of strategy k, $k \in \mathcal{M}$, on day t and RV_t denotes the Realized Volatility on day t of the risky asset (see Andersen et al. (2001), Patton (2011) or Bollerslev et al. (2018) for a definition of RV_t).[61] Motivated by Hansen and Lunde (2005) and Patton (2011), we define the QLIKE loss function of model k on day t by

$$L_{k,t} := L\left(RV_{k,t}^2, \sigma_{\text{target,d}}^2\right) := \frac{RV_{k,t}^2}{\sigma_{\text{target,d}}^2} - \ln\left(\frac{RV_{k,t}^2}{\sigma_{\text{target,d}}^2}\right) - 1, \tag{1.4.1}$$

where the daily volatility target is given by $\sigma_{\text{target,d}} = \sigma_{\text{target}}/\sqrt{252}$ (see Christoffersen (2012, p. 85) and Taylor (2014, p. 475)). Patton (2011) shows that the QLIKE and the MSE loss functions are robust against noise in the volatility proxy. The MSE relies on the absolute forecast error, whereas the QLIKE relies on the relative forecast error. We choose the QLIKE instead of the MSE since the QLIKE penalizes models that underestimate risk, and hence produce a portfolio volatility that is too high. We follow Christoffersen (2012) and use a slightly different representation than the definition used by Hansen and Lunde (2005) and Patton (2011). This representation has the advantage that $L\left(RV_{k,t}^2, \sigma_{\text{target,d}}^2\right) = L\left(RV_t^2, \sigma_t^2\right)$ holds, which is the usual choice in the volatility evaluation literature. Moreover, our loss function is normalized in the sense that $L_{k,t} = 0$ holds if the portfolio volatility on day t equals the desired volatility target, whereas the representation of Hansen and Lunde (2005) and Patton (2011) is not normalized. In particular, by choosing $C(z) = \frac{1}{z}, \tilde{C}(z) = \log(z)$ and $B(z) = -\log(z)$, our representation still fulfills Proposition 1 of Patton (2011). Thus, our representation is a robust loss function in the sense of Patton (2011, Definition 1), and hence is robust against noise in the volatility proxy. Patton (2011) shows that using squared daily returns instead of Realized Volatility leads to quite similar conclusions. We also used squared daily returns instead of the Realized Volatility and found similar results for both methods.

In order to assess the accuracy of the volatility models, we define the relative loss between

[61] The Realized Volatility data are downloaded from the Oxford Man Realized Library (`https://realized.oxford-man.ox.ac.uk/`). We follow Hansen and Lunde (2005) and scale the Realized Volatility to a measure of the close-to-close volatility of day t. Bollerslev et al. (2018) simply add the squared overnight return to the Realized Volatility to obtain a measure for the whole day's variance. However, both methods deliver similar results.

model i and j as $X_{ij,t} = L_{i,t} - L_{j,t}$ and the average relative loss between these models as

$$\overline{X}_{ij} = \frac{1}{T}\sum_{t=1}^{T} X_{ij,t}. \tag{1.4.2}$$

The basic idea of testing for predictive accuracy is that a positive value of $\overline{X}_{ij}$ indicates that model j is more accurate than model i, i.e. model j is more successful in targeting a constant level of volatility.

To test for the accuracy of the different target volatility strategies, we first use the test for equal predictive ability (DM-test) of Diebold and Mariano (1995), which was also used by Patton (2011) and Bollerslev et al. (2018). Further, we use the Reality Check (RC-test) of White (2000) and Sullivan et al. (1999) and its extension, the test for Superior Predictive Ability (SPA-test) of Hansen (2005) and Hansen and Lunde (2005). In contrast to the DM-test, both tests, the RC- and SPA-test, test for superior predictive ability and can also be applied to more than two models simultaneously. Both tests test the null-hypothesis that a chosen benchmark model is more accurate than all the remaining models. Moreover, we use the stepwise extensions of the RC-test and SPA-test that are presented in Romano and Wolf (2005) and Hsu et al. (2010). We denote these stepwise extensions by Step-RC and Step-SPA. The stepwise approaches can be used to construct sets of models that are superior to a chosen benchmark model. Similarly, we also use the algorithm based on the False Discovery Rate (FDR) that is presented in Barras et al. (2010) and Bajgrowicz and Scaillet (2012). The authors show how the FDR can be used to identify models that are superior to a chosen benchmark model. The set of superior models, constructed by the stepwise approaches or the FDR approach, contains the models that produce a more constant portfolio volatility than the chosen benchmark model. Finally, we use the Model Confidence Set (MCS) of Hansen et al. (2011) and Hansen et al. (2003), where we mainly follow Hansen et al. (2003) who also applied this algorithm to assess the accuracy of volatility models. The MCS also identifies a set of superior models and has the advantage that no benchmark model is needed. A short summary of the tests can be found in Rickenberg (2020a, Appendix C).

For the DM-test, the Step-RC, the Step-SPA and the FDR approach, a certain benchmark model has to be chosen. These tests then compare the accuracy of all other models to this benchmark model. As benchmark model we choose the easiest one, which is the HSD. This model is frequently used in the volatility targeting literature and is similar to the model used by Barroso and Santa-Clara (2015), Barroso and Maio (2018), Cederburg et al. (2020) and Moreira and Muir (2017). When applying the RC- and SPA-test, we choose each model once as the benchmark and test if this benchmark is outperformed by at least one other model. Romano and Wolf (2005) and Bajgrowicz and Scaillet (2012) argue that applying the RC- and SPA-test to different benchmarks has several disadvantages that are corrected by the Step-RC, Step-SPA, the MCS and the FDR approaches.

1.4.2 Assessing the Accuracy of VaR Targeting

In Section 1.3, we have presented several VaR forecasting methods and we have shown how these VaR forecasting methods can be used to derive the weight w_t of the risky asset in a target VaR strategy. Moreover, we have shown how a target VaR strategy can be approximated by a target volatility strategy with an adjusted target volatility level given in Equation (1.3.22). Next, we want to assess the quality of the different forecasting methods and we want to compare the "true" target VaR strategies, based on a proper VaR forecast for the risky asset, with the approximated target VaR strategies, based on the risky asset's volatility solely. In other words, we want to assess if the different target VaR strategies succeed to produce a constant portfolio VaR over time and if controlling volatility is sufficient for this task. Similarly, Christoffersen and Diebold (2000) show how VaR backtesting methods can be used to backtest the accuracy of volatility models.

To assess the quality of the target VaR strategies, we define the hit variables

$$H_t^P = \begin{cases} 1, & \text{if } L_t^P > \mathrm{VaR}_\alpha^{\mathrm{target}} \\ 0, & \text{if } L_t^P \leqslant \mathrm{VaR}_\alpha^{\mathrm{target}}, \end{cases} \tag{1.4.3}$$

i.e. H_t^P is equal to one if the portfolio loss is higher than the VaR target $\mathrm{VaR}_\alpha^{\mathrm{target}}$, called a hit, and zero else. An accurate target VaR strategy should exhibit two abilities. First, the percentage of days when the portfolio loss is higher than the predefined VaR target, i.e. the proportion of hits in the hit-series $\{H_t^P\}_{t=1}^T$, should be equal to the desired significance level α. Second, the days when the portfolio loss is higher than the VaR target should occur randomly over time and should not be clustered (see Berkowitz and O'Brien (2002, p. 1101) and Berkowitz et al. (2011, p. 2217)). Assume that the hits of a target VaR strategy occur clustered on many subsequent days, i.e. the portfolio losses are higher than the predefined VaR target on every day in a certain period. As a consequence, investors would remove money from a fund using this strategy, since this strategy seems to fail in having a constant VaR over time.[62]

To test these two abilities, we resort to the VaR backtesting method of Christoffersen (1998), which is one of the most widely used VaR backtests in the academic literature (Du and Escanciano, 2016).[63] In Appendix C.1, we show that the variable H_t^P is equivalent to

$$H_t = \begin{cases} 1, & \text{if } L_t > \mathrm{VaR}_\alpha^t \\ 0, & \text{if } L_t \leqslant \mathrm{VaR}_\alpha^t, \end{cases} \tag{1.4.4}$$

i.e. H_t^P is equal to the hit variable based on the losses and VaRs of the risky asset solely, which is used in the backtest of Christoffersen (1998). Consequently, the backtesting approach of Christoffersen (1998) can be directly adopted for the variables H_t^P. Moreover, this result directly provides critical values which allow us to draw conclusions on the accuracy of the target VaR strategies.[64] The backtest of the target VaR strategy is then formed with the variables

$$\hat{H}_t^P = \begin{cases} 1, & \text{if } l_t^P > \mathrm{VaR}_\alpha^{\text{target}} \\ 0, & \text{if } l_t^P \leqslant \mathrm{VaR}_\alpha^{\text{target}}, \end{cases} \tag{1.4.5}$$

where l_t^P is the realized portfolio loss on day t. The first above mentioned ability, i.e. the correct hit proportion, is then tested with the unconditional coverage test. The second ability, i.e. the independence of the hits, is tested with the test of independence and both abilities are simultaneously tested by the conditional coverage test (Christoffersen, 1998).

1.4.3 Assessing the Accuracy of CVaR Targeting

In order to assess the accuracy of the target CVaR strategies, we again use backtesting methods that were developed for evaluating different CVaR forecasting methods. For backtesting CVaR,

[62] Besides this economic importance of independent hits, this ability should also hold by definition of VaR. See, for example, McNeil et al. (2015, Lemma 9.5) who show that the process of hit variables is a process of iid Bernoulli random variables with probability α (see also Christoffersen (1998), Berkowitz and O'Brien (2002) and Berkowitz et al. (2011)).

[63] See also Berkowitz and O'Brien (2002), Berkowitz et al. (2011, p. 2217) and Kuester et al. (2006, Sec. 2) for a short overview of this backtesting procedure.

[64] Christoffersen (1998) shows that, under the null hypothesis, the test statistic asymptotically follows a χ^2 distribution.

there does not exist a common backtesting procedure (Du and Escanciano, 2016). Furthermore, backtesting CVaR is more challenging than backtesting VaR. Therefore, we will use two different CVaR backtesting procedures that help us to draw more sound conclusions on the accuracy of the target CVaR strategies.[65] For this purpose, as first CVaR backtest, we use the CVaR backtesting procedure described in McNeil and Frey (2000, Section 4.3). This backtesting method compares the loss of the risky asset with the CVaR of the risky asset and is based on the result that, under the condition that the loss L_t exceeds VaR_α^t, the variables

$$X_t = \frac{L_t - \mathrm{CVaR}_\alpha^t}{\sigma_t} = L_t^* - \mathrm{CVaR}_\alpha^{t,*} \tag{1.4.6}$$

are iid with expectation zero. Based on this result, a backtesting procedure using a distribution free bootstrap is derived. However, in this paper, we are interested in the (normalized) difference between the portfolio loss and target CVaR level $\mathrm{CVaR}_\alpha^{\mathrm{target}}$, i.e. we are interested in the ratio

$$X_t^P = \frac{L_t^P - \mathrm{CVaR}_\alpha^{\mathrm{target}}}{\sqrt{\mathrm{var}(R_t^P \mid \mathcal{F}_{t-1})}}, \tag{1.4.7}$$

where we normalize these differences by the portfolio volatility. In Appendix C.2, we show that X_t^P equals X_t and thus, given $L_t^P - \mathrm{VaR}_\alpha^{t,P} > 0$, X_t^P should be iid with expectation zero as well. Hence, we can adopt the backtesting procedure of McNeil and Frey (2000, Section 4.3) for the variables X_t^P. The backtest is then implemented by using the realizations

$$x_t^P = \frac{l_t^P - \mathrm{CVaR}_\alpha^{\mathrm{target}}}{w_t \cdot \hat{\sigma}_t}, \tag{1.4.8}$$

where l_t^P denotes the realized day t portfolio loss based on the weight w_t. If the weight w_t of the risky asset is estimated correctly, the sample

$$\left\{ x_t^P : t = 1, ..., T, l_t^P > \widehat{\mathrm{VaR}}_\alpha^{t,P} \right\} \tag{1.4.9}$$

should behave like an iid sample with mean zero.[66]

[65] Both backtests used in this paper are unconditional backtests, which are less powerful than conditional backtests (Du and Escanciano, 2016). However, as opposed to the VaR backtesting literature, there does not exist a widely used conditional CVaR backtesting method.

[66] We standardize the strategies that rely on the Historical Simulation by the HSD volatility. Moreover, backtesting the target CVaR strategies for which a proper VaR forecast, and hence a portfolio VaR, exists is straightforward. For the strategies that are only based on a volatility forecast, the time series of the portfolio VaR is not available. Since we calculate the target volatility level by assuming a normal distribution for Z_t in these cases, we solve this

As second target CVaR backtesting procedure, we use the backtest derived in Embrechts et al. (2005). We again consider the days where the portfolio loss is higher than the portfolio VaR, i.e. we consider the days where $L_t^P > \mathrm{VaR}_\alpha^{t,P}$ holds. In these cases, stemming from the definition of the CVaR, the mean between the portfolio loss and the portfolio CVaR should be zero. Since the portfolio CVaR should be equal to $\mathrm{CVaR}_\alpha^{\mathrm{target}}$ over time, the measure

$$V_1 = \frac{\sum_{t=1}^{T} \left(L_t^P - \mathrm{CVaR}_\alpha^{\mathrm{target}}\right) \cdot \mathbb{1}_{\{L_t^P > \mathrm{VaR}_\alpha^{t,P}\}}}{\sum_{t=1}^{T} \mathbb{1}_{\{L_t^P > \mathrm{VaR}_\alpha^{t,P}\}}} \tag{1.4.10}$$

should exhibit a low absolute value (Embrechts et al., 2005, p. 72).[67] Nevertheless, Embrechts et al. (2005) argue that the measure V_1 has the drawback that it relies on an estimate of the portfolio VaR. In the definition of the measure V_1, the worst cases are defined as the days when the portfolio loss exceeds the estimated portfolio VaR. If the risky asset's VaR forecast, and hence by Equation (1.3.4) the portfolio VaR, is not credible, the validity of the measure V_1 is doubtful. To account for this observation, the authors propose a second measure V_2 that does not rely on a VaR forecast. The motivation of this measure stems from the interpretation that the CVaR is the expected loss in the $100 \cdot \alpha\%$ "worst" cases. Therefore, we denote by $D_t := L_t^P - \mathrm{CVaR}_\alpha^{\mathrm{target}}$ the difference between the portfolio loss and the CVaR target. Based on these differences, we define the worst cases as the $100 \cdot \alpha\%$ highest differences D_t, i.e. we define the worst cases as the cases when the target CVaR level is exceeded the most. This has the advantage that the worst cases do not depend on an estimate of the VaR, where we do not know if this estimate is credible. We denote the $(1-\alpha)$-quantile of $\{D_t\}_{t=1}^T$ by $D^{1-\alpha}$ and calculate V_2 by

$$V_2 = \frac{\sum_{t=1}^{T} D_t \cdot \mathbb{1}_{\{D_t > D^{1-\alpha}\}}}{\sum_{t=1}^{T} \mathbb{1}_{\{D_t > D^{1-\alpha}\}}}. \tag{1.4.11}$$

As before, the absolute value of V_2 should be low for a successful target CVaR strategy. As a third measure, denoted by V, Embrechts et al. (2005, p. 72) combine the measures V_1 and V_2

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problem in the following way. If a target CVaR strategy relies on a volatility forecast $\hat{\sigma}_t$ solely, we estimate the corresponding VaR by $\widehat{\mathrm{VaR}}_\alpha^t = \hat{\sigma}_t \cdot N_{1-\alpha}$, i.e. again assuming that Z_t follows a standard normal distribution. In this case, the portfolio VaR is given by $\widehat{\mathrm{VaR}}_\alpha^{t,P} = w_t \cdot \hat{\sigma}_t \cdot N_{1-\alpha} - (1-w_t) \cdot R_t^f$. An alternative would be to use the VaR target $\mathrm{VaR}_\alpha^{\mathrm{target}}$ as proxy for the portfolio VaR, i.e. $\widehat{\mathrm{VaR}}_\alpha^{t,P} = \mathrm{VaR}_\alpha^{\mathrm{target}}$.

[67] For the volatility based strategies, we again use $\widehat{\mathrm{VaR}}_\alpha^{t,P} = w_t \cdot \hat{\sigma}_t \cdot N_{1-\alpha} - (1-w_t) \cdot R_t^f$ as forecast for the portfolio VaR in this backtest.

and the authors define

$$V = \frac{|V_1| + |V_2|}{2}, \tag{1.4.12}$$

which should again be low for a good target CVaR strategy.

1.5 Empirical Results

1.5.1 Data

To evaluate the performance of the different target risk strategies and to backtest the ability of achieving a constant level of portfolio risk over time, we use data for the DAX Performance Index as risky asset. As proxy for the risk-free rate, we use the three month Euribor.[68] The data range from 01.01.2000 to 31.12.2018 and are obtained from Datastream. Although many studies on investment or fund strategies use monthly data, we use daily data, since daily data better capture the dynamics of the financial markets and are more closely to the manner how funds are managed (see Busse (1999, p. 1015) and Karolyi and Stulz (1996, p. 952)). Further, even long-term investors typically have short evaluation horizons (Benartzi and Thaler, 1995) and should also time short-term volatility (Moreira and Muir, 2019). Moreover, in order to better manage potential extreme events, focusing on daily return data is also beneficial, since extreme price changes can occur during short time intervals (Longin, 2000, p. 1104). Most studies on risk targeting – or more precisely volatility targeting – use data for the S&P 500, whereas risk targeting for German stocks is so far only rarely examined.[69] Some additional results for US data and small caps, proxied by the S&P 500 and the German small cap index SDAX, are given in Appendix D.

The chosen period from 2000 to 2018 is marked by changing periods of low and high risk and contains the collapse of the tech bubble, the global financial crisis and the European debt

[68] This is similar to Marquering and Verbeek (2004) who use the S&P 500 as risky asset and the three month US T-bill rate as risk-free asset to examine the economic value of volatility timing in the US market. For risk targeting, it is crucial to use a highly liquid asset as underlying risky asset since times of increasing volatility, which induce a portfolio reallocation, typically coincide with times of lower market liquidity (Ang et al., 2006b).

[69] Packham et al. (2017) examine data for German stocks as well, but in a slightly different setting. Barroso and Santa-Clara (2015) examine volatility targeting for a momentum portfolio consisting of German stocks. Ang et al. (2009) examine the (cross-sectional) low volatility anomaly for an international data set that also includes Germany.

crisis, but also times of continuously uptrending markets. This illustrates how risk targeting works in different market environments and whether the models are successful in adapting to changing market regimes. Dopfel and Ramkumar (2013) demonstrate the importance of portfolio risk management during in the global financial crisis, where volatility managing delivers higher returns with lower volatility. However, the authors also show that a good strategy should increase the equity exposure immediately after the crisis. Thus, a good strategy should adapt quite fast to the changes in market risk in our sample. In total, a well performing strategy should limit the downside while the upside potential is captured, as it is found for many hedge fund strategies (Fung and Hsieh, 1997). Additionally to our main results, Appendix D.6 shows how risk targeting works for a longer data set that covers about 88 years. Further, Appendix D.7 shows out-of-sample results for the recent corona crisis.

As in Kellner and Rösch (2016), we use an estimation window of $n = 1000$ days for Historical Simulation, FHS, EVT, the skewed t distribution and for estimating the GARCH(1,1) parameters. The HSD is estimated with an estimation window of $m = 30$ days. As benchmark portfolios for the risk targeting strategies, we use two buy-and-hold investment strategies. The first benchmark strategy is fully invested in the risky asset, i.e. $w_t = 1$ for all t. The second strategy initially invests $w_0 = 60\%$ of wealth in the risky asset and the remaining $1 - w_0 = 40\%$ in the risk-free asset, without rebalancing the weights over time. Benartzi and Thaler (1995) state that portfolios that contain approximately 50$\%$ stocks and 50% bonds are optimal for loss-averse investors. Similarly, Ang et al. (2005, Fig. 3) find that such portfolios are also held by moderately risk-averse investors. Further, 60/40 portfolios are frequently used by pension fund managers (Benartzi and Thaler, 1995, p. 87) for whom risk targeting can be an interesting alternative. In particular, a 60/40 portfolio should also be in line with the risk profile of an average investor (Asvanunt et al., 2015, Footnote 2).

In order to better manage extreme losses and to reduce drawdowns, we choose a low significance level of $\alpha = 0.5\%$ for the target VaR and CVaR strategies. Low significance levels are frequently used in practice and are also important from a regulatory perspective. For example, the Bank of International Settlements has set the significance level to 1% for measuring market risk and to only 0.1% for credit risk. Further, a significance level of $\alpha = 0.5\%$ is also set to calculate the Solvency Capital Requirement under Solvency II. Bali et al. (2008) also use a significance level of 0.5% in a VaR forecasting setting. In particular, Happersberger et al. (2019) find better result for downside risk managed strategies when a lower α is chosen. Further, Ghysels et al. (2016) find that skewness information is hidden in the distribution's tails and that this "tail skewness" is important to determine the optimal portfolio allocation. Thus, in order to better capture skewness risk, lower significance levels should be chosen. Additional results for significance levels of $\alpha = 1\%, 2.5\%$ and 5% are given in Appendix D.2. Finally, as in Barroso and Santa-Clara (2015) and Barroso and Maio (2018), we choose an annualized volatility target of $\sigma_{\text{target}} = 12\%$. By using Equations (1.3.23) and (1.3.38), we obtain VaR and CVaR target levels of $\text{VaR}_\alpha^{\text{target}} = 1.9471\%$ and $\text{CVaR}_\alpha^{\text{target}} = 2.1861\%$ for a significance level of $\alpha = 0.5\%$.[70]

1.5.2 Testing the Accuracy of Risk Targeting

We start the empirical part by assessing the accuracy of the different target risk strategies. By definition, the aim of risk targeting is to achieve a predefined level of portfolio risk constantly over time. In particular, we are interested in the question if more advanced models produce a more constant portfolio risk over time and what kind of risk – volatility, VaR or CVaR – an investor should manage if the investor targets a predefined level of volatility, VaR or CVaR. Further, we are interested in the question if managing volatility is sufficient or if incorporating higher moments, as done by managing VaR and CVaR, leads to a higher accuracy as found by Taylor (2005) in a different setting. Testing the accuracy of the different risk models is also important from an economical perspective, since previous studies have shown that a higher forecasting accuracy coincides with a higher economic value in terms of a higher risk-adjusted performance and high utility gains (Bollerslev et al., 2018, Fleming et al., 2003, Marquering and

[70] We have chosen the same α for both the target VaR and target CVaR strategies but different target risk levels. Another possibility would be to choose the same target level, but different significance levels as it is done in Alexander and Baptista (2004, p. 1262), i.e. $\text{VaR}_\alpha^{\text{target}} = \text{CVaR}_{\tilde{\alpha}}^{\text{target}}$ with $\alpha < \tilde{\alpha}$. Du and Escanciano (2016) suggest that the significance level for CVaR should be about twice the significance level of VaR, i.e. $2\alpha \approx \tilde{\alpha}$. For example, to guarantee that both strategies have the same target risk level when Equations (1.3.23) and (1.3.38) are used, a significance level of $\tilde{\alpha} = 5\%$ for the target CVaR strategy requires a significance level of about $\alpha = 1.96\%$ for the target VaR strategy.

Verbeek, 2004, Moreira and Muir, 2017, Taylor, 2014). Further, a more constant portfolio risk is also related to a lower tail risk (Dreyer and Hubrich, 2019). Consequently, a high forecasting accuracy, and hence a more constant portfolio risk, is beneficial for risk targeting investors. We first test the accuracy of the strategies when the investor's aim is to target a certain level of portfolio volatility over time. Whenever a benchmark model is needed, we choose the HSD model as benchmark, which we denote by model 0. Thus, we assess if more advanced models are more successful in targeting a constant level of volatility than the model used in Barroso and Santa-Clara (2015), Moreira and Muir (2017), Dreyer and Hubrich (2019) and Barroso and Maio (2018). This model is then tested against the remaining models $k = 1, ..., 16$. Bollerslev et al. (2018) find that more advanced models produce more accurate forecasts and higher utility gains than static forecasting models, like the HSD model.

Table I. Testing the Accuracy of Volatility Targeting

This table contains the results of the tests of predictive accuracy presented in Section 1.4.1. $\overline{L}_k^{norm} = \frac{\frac{1}{T}\sum_{t=1}^{T} L_{k,t}}{\frac{1}{T}\sum_{t=1}^{T} L_{0,t}}$ defines the average loss of model k, normalized by the loss of model 0 and is given in percent. DM-test stands for the test statistic of the Diebold and Mariano (1995) test. The null-hypothesis of equal predictive ability is rejected for $|\text{DM-test}| > 1.64$, where positive values indicate that model k is more accurate than the HSD model. Bold numbers of DM-test indicate that the model is significantly superior to the HSD model. $p^{RC,n}$ and p^{RC} stand for the naive p-value and p-value of the RC-test (Sullivan et al., 1999, White, 2000). $p^{SPA,n}$ and $p^{SPA,c}$ stand for the naive p-value and p-value of the SPA-test (Hansen, 2005, Hansen and Lunde, 2005). $p^{SPA,l}$ and $p^{SPA,u}$ give lower and upper bounds for the p-value of the SPA-test. Bold numbers of these tests indicate that the null-hypothesis that model k is the best model cannot be rejected at a test level of 10%. All p-values are given in percent.

Model	$\overline{L}_k^{norm}$	DM-test	$p^{RC,n}$	p^{RC}	$p^{SPA,n}$	$p^{SPA,l}$	$p^{SPA,c}$	$p^{SPA,u}$
Vola Hist	100.00	-	0.00	0.32	0.00	0.00	0.00	0.00
Vola EWMA	83.47	**6.28**	2.73	**63.60**	2.73	2.73	2.73	**12.30**
Vola GARCH	80.88	**4.32**	**100.00**	**100.00**	**100.00**	**100.00**	**100.00**	**100.00**
VaR Hist	278.33	-9.90	0.00	0.00	0.00	0.00	0.00	0.00
VaR EWMA FHS	98.44	0.31	0.00	0.66	0.00	0.00	0.00	0.00
VaR EWMA EVT	108.13	-1.36	0.00	0.00	0.00	0.00	0.00	0.00
VaR EWMA Stsk	111.72	-2.47	0.00	0.00	0.00	0.00	0.00	0.00
VaR GARCH FHS	89.25	**2.00**	0.00	**12.18**	0.00	0.00	0.00	0.00
VaR GARCH EVT	98.76	0.20	0.00	0.32	0.00	0.00	0.00	0.00
VaR GARCH Stsk	119.79	-2.88	0.00	0.00	0.00	0.00	0.00	0.00
CVaR Hist	273.18	-10.63	0.00	0.00	0.00	0.00	0.00	0.00
CVaR EWMA FHS	119.44	-2.83	0.00	0.00	0.00	0.00	0.00	0.00
CVaR EWMA EVT	124.49	-3.47	0.00	0.00	0.00	0.00	0.00	0.00
CVaR EWMA Stsk	142.35	-6.71	0.00	0.00	0.00	0.00	0.00	0.00
CVaR GARCH FHS	100.38	-0.06	0.00	0.10	0.00	0.00	0.00	0.00
CVaR GARCH EVT	106.77	-1.03	0.00	0.00	0.00	0.00	0.00	0.00
CVaR GARCH Stsk	149.67	-6.90	0.00	0.00	0.00	0.00	0.00	0.00

Tables I and II show the results for the tests presented in Section 1.4.1, where Table I shows the results for the DM-, RC- and SPA-test. The first column of Table I contains the average loss of all models, normalized by the average loss of the HSD model, i.e. $\overline{L}_k^{norm} = \frac{\frac{1}{T}\sum_{t=1}^{T} L_{k,t}}{\frac{1}{T}\sum_{t=1}^{T} L_{0,t}}$,

$k = 1, ..., 16$. A normalized loss smaller than 100% indicates that model k is (on average) more accurate than the HSD model, whereas values greater than 100% indicate that the HSD model is more successful in achieving a constant portfolio volatility. Table I clearly shows that the dynamic volatility models, i.e. the EWMA and GARCH based target volatility strategies, are the most accurate models, whereas managing VaR or CVaR typically leads to a less accurate portfolio volatility. In particular, when the aim is to target a constant level of volatility, the Historical Simulation managed strategies (VaR-HS and CVaR-HS) are the least accurate models. Further, the DM-test indicates that most of the CVaR models are significantly less accurate in targeting a constant level of volatility, indicated by values of less than -1.64. When using the RC-test, only three strategies – EWMA, GARCH and VaR-GARCH-FHS – cannot be rejected at a test level of 10%. The RC-test tests if a chosen benchmark model is at least as accurate as all the remaining models. If the null-hypothesis of a model cannot be rejected, i.e. the p-value is higher than the chosen test level of 10%, there is no indication that any other model is more successful in targeting a constant level of portfolio volatility over time than this model. The SPA-test, which extends the RC-test by using a studentized test statistic and a sample dependent null distribution, is typically more powerful in determining inferior models (Hansen, 2005, Hansen and Lunde, 2005). This is confirmed by our results, since more null-hypotheses are rejected. The SPA-test rejects all null-hypotheses of superior predictive ability, except for the null-hypothesis when the GARCH model is used as benchmark model. As a conclusion, Table I shows that the dynamic volatility models produce the most accurate portfolio volatility.

Table II shows the sets of superior models identified by the stepwise RC-test, the stepwise SPA-test, the MCS and the FDR approach. Whenever a benchmark model is needed, we choose the HSD model as benchmark strategy. The MCS has the advantage that no benchmark model has to be chosen. The MCS, which is an extension of the SPA-test, produces similar results to the SPA-test for all reasonable test levels. For both approaches, only the GARCH model is indicated as an accurate model. The Step-RC and Step-SPA produce larger sets than the MCS and these sets contain the EWMA, GARCH and VaR-GARCH-FHS models. This result is similar to the result of the RC-test. In particular, there are no differences between the sets

Table II. Sets of Accurate Volatility Targeting Models
This table contains the results of the stepwise RC-test of Romano and Wolf (2005), the stepwise SPA-test of Hsu et al. (2010), the MCS of Hansen et al. (2003) and Hansen et al. (2011) as well as the FDR method of Barras et al. (2010) and Bajgrowicz and Scaillet (2012). p^R and p^{SQ} stand for the p-values of the MCS and are given in percent. Bold values indicate that the model is contained in the MCS for a test level of 10%. Step-RC and Step-RCst show the step in which the model is added to the set of superior models using the stepwise multiple testing of Romano and Wolf (2005), where Step-RCst uses a studentized test statistic. Step-SPA and Step-SPAst show the step in which the model is added to the set of superior models using the stepwise multiple testing of Hsu et al. (2010), where Step-SPAst uses a studentized test statistic. A value of zero means that the model is not added to the set of superior models. The tests are performed for a test-level of 10%. The last column contains the step in which the model is added to the set of superior models targeting an FDR^+ of 10%. A value of zero indicates that the model is not contained in the superior set.

Model	p^R	p^{SQ}	Step-RC	Step-RCst	Step-SPA	Step-SPAst	$FDR^+ = 10\%$
Vola Hist	0.00	0.00	-	-	-	-	-
Vola EWMA	4.90	4.90	**1**	**1**	**1**	**1**	**1**
Vola GARCH	**100.00**	**100.00**	**1**	**1**	**1**	**1**	**2**
VaR Hist	0.00	0.00	0	0	0	0	0
VaR EWMA FHS	0.00	0.00	0	0	0	0	0
VaR EWMA EVT	0.00	0.00	0	0	0	0	0
VaR EWMA Stsk	0.00	0.00	0	0	0	0	0
VaR GARCH FHS	0.00	0.00	**1**	**1**	**1**	**1**	**3**
VaR GARCH EVT	0.00	0.00	0	0	0	0	0
VaR GARCH Stsk	0.00	0.00	0	0	0	0	0
CVaR Hist	0.00	0.00	0	0	0	0	0
CVaR EWMA FHS	0.00	0.00	0	0	0	0	0
CVaR EWMA EVT	0.00	0.00	0	0	0	0	0
CVaR EWMA Stsk	0.00	0.00	0	0	0	0	0
CVaR GARCH FHS	0.00	0.00	0	0	0	0	0
CVaR GARCH EVT	0.00	0.00	0	0	0	0	0
CVaR GARCH Stsk	0.00	0.00	0	0	0	0	0

of the Step-RC and Step-SPA test. Further, studentizing does not lead to different results. The FDR approach, which is known to typically produce sets of superior models that are at least as large as the sets of the Step-RC and Step-SPA approaches, chooses the same models as superior models. For this approach, the two volatility models (EWMA and GARCH) are chosen in the first two steps, which strengthens the earlier findings that the dynamic volatility models produce the most constant portfolio volatility.

To summarize the results of Tables I and II, we find convincing results of the EWMA, GARCH and VaR-GARCH-FHS model, where the GARCH model delivers the best results. Hence, an investor who wants to achieve a constant portfolio volatility over time should manage volatility directly by a dynamic risk model. Managing downside risk typically fails to target a constant level of volatility. Further, unconditional models, i.e. HSD or Historical Simulation, produce a portfolio volatility that significantly deviates from the desired volatility target. Since a higher forecasting accuracy typically coincides with a higher risk-adjusted performance and high utility gains, we expect better performance results for conditional models. Bollerslev et al. (2018) also find that static models, like the HSD model, are inaccurate and, due to their inaccuracy, produce lower utility gains for an investor who targets a constant level of volatility.

Table III reports results for the VaR backtest of Christoffersen (1998) that is summarized in Section 1.4.2, where we report p-values for the unconditional and conditional coverage test for significance levels of $\alpha = 0.5\%, 1\%, 2.5\%$ and 5%. These significance levels are also frequently used in the literature on VaR forecasting (see Bali et al. (2008) for example). The VaR backtesting results demonstrate that for all significance levels controlling volatility is not sufficient when an investor's aim is to target a constant portfolio VaR over time. In contrast, managing CVaR is feasible for an investor who targets a constant VaR over time. However, VaR based strategies are more successful in targeting a constant portfolio VaR over time than strategies that manage CVaR. Only two of the VaR based strategies that rely on a conditional volatility model can be rejected for a significance level of $\alpha = 0.5\%$ and a test level of 10%. Further, for higher significance levels of α, unconditional models based on the Historical Simulation also fail to target a constant VaR, whereas these models cannot be rejected for low significance levels. A possible explanation for this result is that low significance levels produce only a limited number of hits. Historical Simulation is known for producing adequate hit ratios, but these hits are usually clustered over time. Hence, when testing for unconditional coverage, Historical Simulation usually delivers convincing results. However, due to the failure of producing independent hits, Historical Simulation is often rejected once the independence or conditional coverage test is applied (Kuester et al., 2006). Since the independence test of Christoffersen (1998) only regards successive hits, low significance levels, and hence only very few hits over the whole sample, imply that the independence test fails to detect the lack of independence. This explains why Historical Simulation seems to perform well for low levels of α. Pritsker (2006) also finds that VaR backtests fail to identify inferior models when only few exceedances occurred over the sample.

Table IV shows the backtesting results for the two CVaR backtests that were summarized in

Table III. VaR Backtesting Results

The table reports the backtesting results of the Christoffersen (1998) VaR backtest for significance levels of $\alpha = 0.5\%, 1\%, 2.5\%$ and 5%. p_{uc} and p_{cc} denote the p-values for the unconditional coverage and conditional coverage test and are given in percent. Bold numbers mark the models that are not rejected at a test level of 10%.

	$\alpha = 0.5\%$		$\alpha = 1\%$		$\alpha = 2.5\%$		$\alpha = 5\%$	
Model	p_{uc}	p_{cc}	p_{uc}	p_{cc}	p_{uc}	p_{cc}	p_{uc}	p_{cc}
Vola Hist	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
Vola EWMA	0.01	0.03	0.00	0.00	0.00	0.01	0.02	0.04
Vola GARCH	1.46	2.90	2.09	6.87	0.10	0.40	5.98	**18.22**
VaR Hist	**82.09**	**25.17**	**68.80**	**27.71**	**55.22**	0.00	**51.14**	0.00
VaR EWMA FHS	**43.75**	**27.08**	**41.01**	**63.71**	**49.33**	**25.13**	**64.51**	**59.47**
VaR EWMA EVT	**27.94**	**10.06**	**22.13**	**30.36**	**67.98**	**71.83**	**15.16**	8.21
VaR EWMA Stsk	**66.34**	**21.64**	**79.56**	**80.76**	3.53	5.07	0.17	0.34
VaR GARCH FHS	**56.13**	**29.04**	**58.70**	**74.75**	**61.45**	**86.73**	**64.51**	**90.32**
VaR GARCH EVT	**19.21**	6.95	**28.47**	**37.41**	**96.32**	**99.90**	**39.42**	**72.83**
VaR GARCH Stsk	**19.21**	6.95	**12.52**	**18.43**	**74.79**	**85.83**	**26.64**	**50.57**
CVaR Hist	**19.21**	**39.97**	4.45	1.13	0.61	0.08	0.00	0.00
CVaR EWMA FHS	**27.94**	**10.06**	**12.52**	**18.43**	6.43	**12.41**	2.57	4.24
CVaR EWMA EVT	**12.57**	4.51	1.96	3.17	0.32	1.12	0.99	2.10
CVaR EWMA Stsk	**19.21**	6.95	**12.52**	**18.43**	**28.10**	**52.96**	**69.05**	**14.61**
CVaR GARCH FHS	**27.94**	**10.06**	**35.88**	**44.91**	**11.92**	**26.42**	8.03	**19.06**
CVaR GARCH EVT	4.57	1.55	3.00	4.80	1.46	5.08	5.06	**12.92**
CVaR GARCH Stsk	4.57	1.55	0.15	0.24	0.16	0.68	0.99	2.55

Section 1.4.3. We again choose the four significance levels $\alpha = 0.5\%, 1\%, 2.5\%$ and 5%. The p-value of the backtest of McNeil and Frey (2000) is denoted by p_{CVaR}. Nearly all target volatility and target VaR strategies fail to accurately target the portfolio CVaR and are rejected at a test level of 10%. In contrast, only two of the target CVaR strategies can be rejected for significance levels of $\alpha = 0.5\%$ and $\alpha = 5\%$. This indicates that, by controlling the CVaR of the risky asset, it is possible to achieve a constant portfolio CVaR over time. This finding is also supported by the results of the CVaR backtest of Embrechts et al. (2005), which exhibits the lowest values for the CVaR managed strategies. Further, the values V of the dynamically managed target CVaR strategies are systematically lower than the values of the remaining models, indicating that the CVaR of the risky asset should be managed dynamically. Interestingly, the CVaR-HS approach cannot be rejected by the backtest of McNeil and Frey (2000). However, this backtest is an unconditional backtest which only tests if the produced CVaR is correct on average (Du and Escanciano, 2016). As mentioned above, Historical Simulation is typically rejected once a conditional backtest is applied. The lower values of V of the conditionally managed strategies indicate a higher accuracy of conditional models.

The backtesting results presented in this section demonstrate two important issues. First,

Table IV. CVaR Backtesting Results
This table reports the backtesting results of the McNeil and Frey (2000) and Embrechts et al. (2005) CVaR backtests for significance levels of $\alpha = 0.5\%, 1\%, 2.5\%$ and 5%. V denotes the measure of the Embrechts et al. (2005) backtest, given in Equation (1.4.12). Bold numbers mark the lowest value of V. p_{CVaR} denotes the p-value of the backtest of McNeil and Frey (2000) and is given in percent. Bold numbers mark the models that are not rejected at a test level of 10%.

	$\alpha = 0.5\%$		$\alpha = 1\%$		$\alpha = 2.5\%$		$\alpha = 5\%$	
Model	V	p_{CVaR}	V	p_{CVaR}	V	p_{CVaR}	V	p_{CVaR}
Vola Hist	0.5825	1.11	0.4257	0.39	0.2840	0.04	0.2097	0.00
Vola EWMA	0.4934	1.85	0.3164	1.93	0.2068	0.42	0.1527	0.09
Vola GARCH	0.3337	**10.03**	0.2150	7.84	0.1163	9.48	0.0917	1.66
VaR Hist	0.5699	3.30	0.4179	3.69	0.3849	0.20	0.3169	0.00
VaR EWMA FHS	0.3621	2.30	0.2282	2.62	0.0843	**11.46**	0.0891	1.05
VaR EWMA EVT	0.3145	3.94	0.1842	3.94	0.0898	8.12	0.0957	1.16
VaR EWMA Stsk	0.3356	5.53	0.2182	3.53	0.1506	1.37	0.1472	0.06
VaR GARCH FHS	0.2825	**10.42**	0.1369	**13.93**	0.0681	**16.24**	0.0688	3.30
VaR GARCH EVT	0.2318	**14.41**	0.1094	**16.87**	0.0637	**14.92**	0.0738	2.50
VaR GARCH Stsk	0.1955	**26.53**	0.1219	**13.24**	0.0732	**12.96**	0.0867	1.60
CVaR Hist	0.3052	**12.97**	0.1376	**51.87**	0.0848	**77.77**	0.0526	**15.62**
CVaR EWMA FHS	0.1814	**28.01**	0.0772	**63.80**	0.0200	**93.16**	0.0085	**90.92**
CVaR EWMA EVT	0.2305	5.81	0.0857	**17.86**	0.0209	**35.76**	0.0186	**14.09**
CVaR EWMA Stsk	0.1913	**27.28**	0.0894	**34.94**	0.0390	**54.87**	0.0384	**47.23**
CVaR GARCH FHS	0.1602	**44.38**	0.0691	**57.07**	**0.0133**	**99.67**	**0.0070**	**94.12**
CVaR GARCH EVT	0.1936	**17.85**	0.0508	**41.94**	0.0211	**49.32**	0.0139	**32.10**
CVaR GARCH Stsk	**0.0634**	**70.90**	**0.0387**	**73.45**	0.0455	**14.36**	0.0315	4.93

if an investor is interested in targeting portfolio risk in terms of volatility, VaR or CVaR, the investor should directly manage volatility, VaR or CVaR, respectively. In particular, when the aim is to target a certain level of tail risk, it is not sufficient to manage volatility. Second, when portfolio risk is managed, the investor should use a fast-adapting dynamic risk model instead of unconditional models, like HSD or Historical Simulation, as it is done by Barroso and Santa-Clara (2015), Barroso and Maio (2018) and Moreira and Muir (2017). The next section examines the performance of risk targeting. Based on the backtesting results from this section, we expect a superior performance for the strategies that are based on a conditional risk model.

1.5.3 Performance of Risk Targeting

We next assess the performance of the different risk targeting strategies and the two benchmark portfolios. Results of this performance analysis over the whole sample are given in Table V. All target risk strategies, except for the Historical Simulation managed target VaR strategy, exhibit higher returns than the 60/40 portfolio with a risk, measured by volatility, drawdown, VaR or CVaR, that is lower – or comparable in the case of the volatility managed strategies – than the risk of the 60/40 portfolio. Further, the risk targeting strategies deliver higher returns with lower risk compared to the DAX. Therefore, dynamically managing portfolio risk can significantly reduce the portfolio's risk without simultaneously sacrificing returns (see also Fung and Hsieh (1997) who found a similar behavior for dynamic trading strategies used by hedge funds). This is also reflected by higher Sharpe Ratios for the dynamically managed target risk strategies compared to the Sharpe Ratios of the two benchmark portfolios. Moreover, within the (dynamically) managed target risk strategies, returns are quite similar. However, the downside risk managed strategies take significantly less risk than the volatility managed strategies. The highest Sharpe Ratio is found for the CVaR-EWMA-Stsk strategy, which is about 287.5% higher than the Sharpe Ratio of the DAX and 51.96% higher than the Sharpe Ratio of the HSD model. The Sharpe Ratio of the best volatility managed strategy is still 205% higher than the Sharpe Ratio of the DAX. The Sharpe Ratio of the best CVaR managed strategy is $0.155/0.122 - 1 = 27.05\%$ higher than the Sharpe Ratio of the best volatility managed strategy. In particular, Sharpe Ratios of the dynamically managed CVaR strategies are all higher than the Sharpe Ratios of the volatility managed strategies. This can also be seen by the modified Sharpe Ratio, which measures the risk-adjusted annualized excess return (see Jondeau and Rockinger (2012) for a definition of the modified Sharpe Ratio). The best results in terms of the Sharpe Ratios are found for the strategies that are based on the skewed t distribution of Jondeau and Rockinger (2003) and Bali et al. (2008). Generally, in order to increase the risk-adjusted performance, risk should be managed by a dynamic risk model and not by a static risk model, like HSD or Historical Simulation. The Sharpe Ratios of the statically managed strategies are significantly lower than the Sharpe Ratios of the dynamically managed strategies. This finding is in line with Bollerslev et al. (2018) since models that produce a more constant portfolio risk over time, as shown in Section 1.5.2, also yield an enhanced risk-adjusted performance.

Although the differences in the Sharpe Ratio seem small, results of Table V indicate significant performance gains of portfolio risk management, especially when downside risk is managed. This is the case, since our strategies are highly correlated, which results in very small standard errors for the relative Sharpe Ratios as highlighted in Kirby and Ostdiek (2012). For example, the average correlation between all risk targeting models is 97.34% and the maximum correlation between two strategies is 99.98%. This high correlation between the risk targeting strategies demonstrates that even small differences in the Sharpe Ratios indicate a striking improvement in the tail risk targeting strategies' performance. For example, Kirby and Ostdiek (2012) find Sharpe Ratios for their strategies in the range of 0.47 to 0.49, compared to the benchmark's Sharpe Ratio of 0.46, and they conclude that, due to the high correlation of the strategies, "[t]hese differences translate into significant performance gains". The performance gains of CVaR targeting compared to volatility targeting are even higher in magnitude than the gains found by Kirby and Ostdiek (2012), demonstrating the vast performance gains of CVaR targeting compared to volatility targeting. To test if any model produces a statistically higher Sharpe Ratio than the HSD managed model, we use the corrected version of the Sharpe Ratio test of Jobson and Korkie (1981), which is also used by DeMiguel et al. (2009b, p. 1928) and Cederburg et al. (2020). This test could also be applied to more than one strategy simultaneously as shown by Jobson and Korkie (1981, Sec. II.C). However, in Section 1.5.4, we use more sophisticated approaches to test for higher performance gains of all portfolios simultaneously. For that reason, we only test each strategy's Sharpe Ratio against the Sharpe Ratio of the HSD model. The test of Jobson and Korkie (1981) indicates that only the VaR-EWMA-Stsk model exhibits a Sharpe Ratio that is significantly higher than the Sharpe Ratio of the HSD model when using a test level of 10%.

We now turn to the drawdown protection ability of risk targeting. Several studies demonstrate that volatility targeting is an easy but successful drawdown and tail risk reduction method (see, for example, Benson et al. (2014), Barroso and Santa-Clara (2015), Dreyer and Hubrich (2019), Harvey et al. (2018) and Moreira and Muir (2017)). As expected, all risk targeting strategies and the 60/40 portfolio are successful in reducing the DAX's drawdown. The maximum drawdown (MDD) of the risk targeting strategies and the 60/40 portfolio is about half of the maximum drawdown of the DAX. Furthermore, the risk targeting strategies exhibit a slightly higher drawdown reduction than the 60/40 portfolio. This can be seen by ΔMDD,

Table V. Performance Results of Risk Targeting

This table shows the performance results of all target risk strategies and the two benchmark portfolios over the whole period. Return and Vola stand for the annualized return and volatility, respectively. SR stands for the annualized Sharpe Ratio, z_{JK} stands for the test statistic of the corrected version of the test of Jobson and Korkie (1981) and mSR is the modified Sharpe Ratio defined in Jondeau and Rockinger (2012). MDD and ΔMDD stand for the maximum drawdown and the reduction of the maximum drawdown compared to the maximum drawdown of the DAX. Calmar stands for the drawdown-adjusted return and is defined as in Farinelli et al. (2008) and Eling and Schuhmacher (2007). VaR and CVaR are the in-sample VaR and CVaR, which are estimated with Historical Simulation using all data. Min and Max stand for the minimum and maximum daily return, respectively. Return, Vola, MDD, ΔMDD, Min and Max are given in percent. Bold numbers of z_{JK} correspond to positive values that are significant at the 10% level, i.e. $z_{JK} \geqslant 1.6449$.

Model	Return	Vola	SR	z_{JK}	mSR	MDD	ΔMDD	Calmar	VaR	CVaR	Min	Max
Vola Hist	3.12	12.84	0.102	-	1.49	41.23	41.46	0.032	1.38	1.82	-5.64	5.14
Vola EWMA	3.34	12.50	0.122	0.96	1.97	40.52	42.46	0.038	1.34	1.76	-5.28	4.98
Vola GARCH	3.19	11.96	0.116	0.39	1.82	39.23	44.29	0.035	1.29	1.67	-5.29	4.03
VaR Hist	2.07	9.69	0.029	-0.68	-0.25	31.62	55.10	0.009	0.97	1.44	-4.99	5.29
VaR EWMA FHS	3.23	11.09	0.128	0.87	2.12	37.09	47.33	0.038	1.19	1.56	-4.33	4.24
VaR EWMA EVT	3.23	10.43	0.136	1.29	2.30	35.28	49.91	0.040	1.11	1.47	-4.24	4.01
VaR EWMA Stsk	3.43	10.83	0.150	**1.69**	2.63	35.07	50.20	0.046	1.16	1.53	-4.76	4.11
VaR GARCH FHS	3.11	11.33	0.115	0.34	1.81	38.32	45.59	0.034	1.22	1.59	-4.67	3.52
VaR GARCH EVT	3.13	10.63	0.125	0.62	2.04	36.55	48.10	0.036	1.15	1.49	-4.63	3.37
VaR GARCH Stsk	3.34	10.39	0.147	1.10	2.58	34.68	50.75	0.044	1.11	1.46	-4.84	3.32
CVaR Hist	2.43	9.26	0.069	-0.33	0.70	28.51	59.52	0.022	0.95	1.37	-5.12	4.17
CVaR EWMA FHS	3.22	10.16	0.139	1.13	2.38	34.35	51.22	0.041	1.07	1.43	-4.01	3.79
CVaR EWMA EVT	3.23	9.93	0.143	1.32	2.48	33.80	52.00	0.042	1.05	1.40	-3.90	3.74
CVaR EWMA Stsk	3.38	10.14	0.155	1.53	2.77	33.12	52.97	0.048	1.09	1.44	-4.67	3.70
CVaR GARCH FHS	3.23	10.68	0.134	0.79	2.25	37.00	47.47	0.039	1.14	1.49	-4.58	3.26
CVaR GARCH EVT	3.26	10.33	0.141	0.99	2.42	35.51	49.58	0.041	1.10	1.44	-4.41	3.19
CVaR GARCH Stsk	3.21	9.74	0.145	0.94	2.51	33.02	53.11	0.043	1.04	1.38	-4.75	2.98
DAX	2.73	23.46	0.040	-0.57	-	70.42	-	0.013	2.36	3.46	-8.49	11.40
60/40	2.37	11.94	0.048	-0.59	0.21	40.07	43.10	0.014	1.24	1.74	-4.32	5.10

which measures the percentage drawdown reduction compared to the drawdown of the DAX. Managing downside risk, especially managing CVaR, instead of volatility results in a higher drawdown reduction without simultaneously sacrificing returns. Interestingly, the Historical Simulation managed strategies exhibit the highest drawdown reduction. However, this superior drawdown protection is accompanied with significantly lower returns. This is confirmed by the Calmar Ratio, which measures the drawdown-adjusted return and takes the highest values for the dynamically managed target CVaR strategies, whereas the Calmar Ratios of the Historical Simulation managed strategies are significantly lower.[71]

[71] Since asset returns are usually non-normally distributed, performance measurement based on the Sharpe Ratio solely can lead to wrong conclusions (see Farinelli et al. (2008) or Eling and Schuhmacher (2007) for example). The Sharpe Ratio is only suitable for elliptical distributions, a class of distributions that contains the normal distribution (Eling and Schuhmacher, 2007, p. 2633). Therefore, besides the Sharpe Ratio, we additionally use the Calmar Ratio. This measure replaces the volatility in the Sharpe Ratio by the maximum drawdown. See Eling and Schuhmacher (2007) for definitions and a motivation of enhanced risk-adjusted performance measures. Further,

In line with the results of the maximum drawdown, the highest (least negative) minimum daily return is achieved by the CVaR managed strategies. The minimum return of the VaR managed strategies is also comparable to the minimum return of the CVaR managed strategies, but slightly lower. In contrast, the minimum return of the volatility managed strategies is significantly more negative than the minimum return of the downside risk managed strategies. Further, the minimum daily return of the Historical Simulation based strategies is significantly more negative than the minimum return of the dynamically managed strategies. This is somewhat surprising, since the Historical Simulation based strategies exhibit the lowest drawdowns, which indicates that these models are the most conservative. Further, the Historical Simulation based strategies also have the lowest average equity exposure, which is not shown here. However, the lower average equity exposure of the Historical Simulation managed strategies is consistent with Berkowitz and O'Brien (2002) who find that banks, who often use Historical Simulation as risk measurement tool, typically exhibit too conservative, i.e. too high, risk estimates that translate in lower equity weights of the strategies that are managed by Historical Simulation. Further, Berkowitz and O'Brien (2002) find that, although commercial banks' internal risk models produce more conservative risk estimates than a GARCH based VaR model, the banks' models deliver comparable – or even more – VaR violations than the GARCH based model. This explains the somewhat surprising result of a lower equity exposure and drawdown, but a more negative minimum return of the Historical Simulation managed strategies. That is, the Historical Simulation based strategies are more conservative on average, but fail to correctly manage downside risk just when downside risk protection is most needed. This result again highlights the need of a fast adapting risk model when portfolio risk is managed. As before, a higher forecasting accuracy, as examined in Section 1.5.2, coincides with a superior performance in terms of higher minimum returns. However, even the dynamically managed strategies exhibit minimum daily returns that are similar in magnitude to the minimum return of the 60/40 portfolio. This highlights the fact that unpredictable negative price jumps cannot

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Jobson and Korkie (1981, Sec. I) gives an overview on several widely used performance measures. We also used other performance measures that incorporate the non-normalities of the strategies' returns. However, results were quite similar to the Sharpe Ratio and Calmar Ratio and we do not show these measures here. This finding is in line with Eling and Schuhmacher (2007) who also find a similar ranking order across hedge funds when several risk-adjusted performance measures are used.

be completely avoided by risk targeting but, as supposed by Longin (2000), these jumps are best managed by models using EVT. By comparing the minimum return of the EVT, FHS and skewed t distribution based approaches that use the same volatility model, the minimum return of the EVT based models is always higher (less negative) than the minimum return of the other models. As expected, the results found for the minimum daily return reverse when the maximum daily return is compared. In this case, the volatility managed strategies exhibit higher maximum returns than the downside risk managed strategies. This indicates that downside risk timing seems superior in crash periods, but volatility timing is superior in bull markets. This motivates a strategy that switches between downside risk targeting in down-periods and volatility targeting in up-periods, which will be examined in Section 1.5.5.

The performance evaluation in Table V does not consider transaction costs. However, many studies demonstrate that volatility managing is still beneficial, even when realistic transaction costs are considered (see Moreira and Muir (2017), Kirby and Ostdiek (2012), Fleming et al. (2003), Fleming et al. (2001), Marquering and Verbeek (2004), Harvey et al. (2018) and Bollerslev et al. (2018)). In unreported results, we find that most downside risk managed strategies exhibit lower turnovers than the volatility managed strategies (see Kirby and Ostdiek (2012, p. 442) for a definition of the turnover). Hence, the superiority of downside risk managed strategies, especially CVaR managed strategies, compared to the volatility managed strategies would be even more striking if realistic transaction costs were considered. However, the skewed t distribution based strategy produces a higher turnover compared to the FHS and EVT based approaches. Hence, the outperformance of the skewed t distribution based strategy over the FHS and EVT based strategies will be lower after transaction costs. Nevertheless, in Appendix D.3, we show that risk targeting is also beneficial when a rebalancing buffer is used. Using a rebalancing buffer lowers the strategies' turnover, which translates into lower transaction costs. Thus, risk targeting strategies can also be used in practice and should be superior to non-managed strategies, even when transaction costs are considered.

In order to better assess how risk targeting works in different market environments and if risk targeting mitigates extremely negative returns, we next consider the days when the underlying

Table VI. Lowest and Highest DAX Returns
This table shows the returns for the days when the DAX exhibits the most extreme (positive or negative) returns. Panel A shows the five days with the lowest DAX returns as well as the corresponding returns of the target risk strategies and the 60/40 portfolio on these days. Panel B shows the five days with the highest DAX returns as well as the corresponding returns of the target risk strategies and the 60/40 portfolio on these days. All entries correspond to daily returns and are given in percent.

Model	Panel A: Low DAX Returns (%)					Panel B: High DAX Returns (%)				
Vola Hist	-4.125	-5.638	-2.751	-2.134	-1.119	1.425	1.952	1.774	2.180	3.204
Vola EWMA	-3.466	-5.279	-2.664	-1.858	-1.229	1.523	1.952	2.100	2.193	2.679
Vola GARCH	-3.114	-5.292	-2.622	-1.654	-1.292	1.690	1.912	2.444	2.264	2.301
VaR Hist	-3.211	-4.989	-4.108	-3.275	-2.690	2.548	2.680	3.456	4.467	5.291
VaR EWMA FHS	-3.157	-4.331	-2.192	-1.368	-0.903	1.126	1.976	1.553	1.625	1.985
VaR EWMA EVT	-3.038	-4.235	-2.001	-1.365	-0.901	1.125	1.742	1.547	1.622	1.976
VaR EWMA Stsk	-3.072	-4.764	-1.962	-1.610	-0.866	1.528	1.666	1.940	1.863	2.276
VaR GARCH FHS	-2.998	-4.668	-2.453	-1.399	-1.101	1.468	1.916	2.082	1.913	1.956
VaR GARCH EVT	-2.945	-4.633	-2.145	-1.330	-1.040	1.376	1.809	1.981	1.833	1.855
VaR GARCH Stsk	-2.799	-4.838	-2.016	-1.463	-1.076	1.554	1.625	2.170	1.740	1.919
CVaR Hist	-3.037	-5.123	-3.415	-2.757	-2.326	2.494	2.691	3.380	3.869	4.165
CVaR EWMA FHS	-2.987	-4.006	-1.771	-1.216	-0.802	1.005	1.731	1.385	1.449	1.770
CVaR EWMA EVT	-2.998	-3.895	-1.769	-1.214	-0.799	0.999	1.684	1.380	1.444	1.765
CVaR EWMA Stsk	-2.969	-4.666	-1.482	-1.558	-0.730	1.510	1.580	1.904	1.788	2.239
CVaR GARCH FHS	-2.964	-4.583	-1.986	-1.223	-0.963	1.270	1.836	1.835	1.697	1.718
CVaR GARCH EVT	-2.907	-4.407	-1.923	-1.197	-0.943	1.246	1.778	1.799	1.663	1.684
CVaR GARCH Stsk	-2.715	-4.749	-1.469	-1.412	-1.006	1.528	1.526	2.100	1.657	1.885
DAX	-8.492	-7.164	-7.073	-7.012	-6.838	7.632	7.845	10.344	11.277	11.402
60/40	-4.167	-4.098	-3.575	-3.243	-3.253	3.330	3.290	4.364	4.903	5.097

risky asset, i.e. the DAX, suffers the highest losses or obtains the highest gains. Table VI reports the five lowest and five highest daily DAX returns in conjunction with the corresponding returns of the target risk strategies and the 60/40 portfolio. On the days with the worst DAX returns, the target risk strategies and the 60/40 portfolio deliver significantly higher, i.e. less negative, daily returns. The highest reduction of the negative returns is achieved by the risk targeting strategies, especially for the strategies that manage the portfolio's CVaR. In line with our earlier results, the highest reduction is again achieved by the dynamically managed strategies. In contrast, the reduction of the HS managed tail risk targeting strategies is significantly lower. Thus, in order to mitigate extreme crashes, a fast adapting risk model should be used to manage portfolio risk. Similarly, the returns of the 60/40 portfolio are typically more negative than the returns of the risk targeting strategies. One exception is the day with the second lowest DAX return, where the target risk strategies' losses are higher than the loss of the 60/40 portfolio. This day indicates a day with an unpredictable negative price jump in the DAX, as examined in Liu et al. (2003) and Das and Uppal (2004). In line with Longin (2000), the best results for this day within the risk targeting strategies is achieved by the EVT based strategies, i.e. (unpredictable) negative price jumps are best managed by EVT. In total, Panel A of Table VI show that extremely negative returns can be significantly reduced by targeting a constant level of portfolio risk, especially when CVaR is managed.

The aforementioned results reverse when the highest daily DAX returns are regarded. In this case, the DAX delivers higher returns than all the remaining models.[72] The lowest returns are achieved by the risk targeting strategies, especially when CVaR is used as a risk measure. The low returns of the risk targeting strategies are in line with the observation that the highest daily returns typically occur during extreme crash periods, so called "bear market rallies". As a consequence, the exposure of the risk targeting strategies to the risky asset on these days is quite low and the risk targeting strategies fail to capture these extremely high returns. However, in order to achieve a high long-term performance, avoiding highly negative returns is more crucial than achieving highly positive returns.[73] Furthermore, the reduction of extremely negative returns also fits better to the preferences of investors who treat losses and gains asymmetrically and weight losses higher than gains. Aït-Sahalia and Brandt (2001) conjecture that loss aversion is highly related to downside risk managed portfolio strategies. This is confirmed by our results, since CVaR targeting delivers the most convincing mitigation of extreme losses, which fits well to the preferences of loss-averse investors.

Based on the results of Table VI, we find that volatility and downside risk targeting behave quite differently in different market environments. In uptrending markets, volatility targeting delivers higher returns, whereas downside risk targeting is more convincing in crash periods. This again motivates a strategy that switches between CVaR and volatility targeting as examined later. To strengthen this observation, we next assess risk targeting in two subsamples, one high risk and one low risk period. Table VII shows the performance of the strategies in the period from 15.07.2008 to 15.07.2011, i.e. during the height of the financial crisis and the time following the financial crisis. We split this period in two subsamples. Splitting the sample in two subsamples is appealing, since Dreyer and Hubrich (2019, Fig. 2) and Dopfel and Ramkumar

[72] The results of Table VI are also influenced by the chosen volatility target. We have chosen a quite low volatility target. To better capture the upside potential of high DAX returns and to lower the underperformance on days with high DAX returns, risk-seeking or less risk-averse investors should use a higher risk target.

[73] For example, a return of -5% has to be compensated by a return of 5.26%. Thus, high negative returns have to be compensated by even higher positive returns.

Table VII. Performance Results During and After the Financial Crisis
Panel A shows the performance of all strategies from 15.07.2008 to 15.07.2009, i.e. during the height of the financial crisis. Panel B shows the performance of all strategies from 16.07.2009 to 15.07.2011, i.e. the time following the financial crisis and before the European debt crisis. See Table V for a description of Return, Volatility, SR, MDD, Min, Max. - marks a negative Sharpe Ratio.

	Panel A: 15.07.2008 - 15.07.2009						Panel B: 16.07.2009 - 15.07.2011					
Model	Return	Volatility	SR	MDD	Min	Max	Return	Volatility	SR	MDD	Min	Max
Vola Hist	-3.51	13.16	-	16.93	-2.75	3.20	9.84	12.46	0.709	12.35	-3.03	3.16
Vola EWMA	-3.41	12.83	-	16.91	-2.66	2.93	10.49	12.21	0.776	11.91	-2.85	2.86
Vola GARCH	-4.07	13.56	-	17.71	-2.62	2.98	10.22	11.46	0.803	9.97	-2.35	2.45
VaR Hist	-8.53	17.20	-	22.38	-4.11	5.29	7.12	6.11	1.003	4.17	-1.10	1.76
VaR EWMA FHS	-1.22	10.29	-	12.94	-2.19	2.41	9.09	11.12	0.727	11.52	-2.78	2.60
VaR EWMA EVT	-1.53	9.63	-	12.36	-2.00	2.20	8.20	9.49	0.759	9.66	-2.29	2.21
VaR EWMA Stsk	-0.16	10.81	-	12.02	-1.96	2.42	8.48	10.10	0.740	10.68	-2.69	2.74
VaR GARCH FHS	-2.93	12.23	-	15.71	-2.45	2.76	9.46	10.98	0.770	9.95	-2.39	2.45
VaR GARCH EVT	-2.55	11.08	-	14.26	-2.15	2.42	8.70	9.59	0.803	8.48	-1.99	2.05
VaR GARCH Stsk	-0.08	11.49	-	12.52	-2.02	3.26	8.70	9.51	0.809	8.96	-2.28	2.47
CVaR Hist	-5.92	14.99	-	19.01	-3.42	4.17	6.99	5.98	1.003	4.09	-1.08	1.72
CVaR EWMA FHS	-0.96	8.56	-	10.79	-1.77	1.96	7.28	8.76	0.718	9.55	-2.28	2.02
CVaR EWMA EVT	-0.93	8.55	-	10.76	-1.77	1.96	7.46	8.62	0.751	8.95	-2.12	2.00
CVaR EWMA Stsk	0.67	9.72	-	10.73	-1.56	2.24	7.49	9.23	0.704	10.08	-2.60	2.70
CVaR GARCH FHS	-2.13	10.24	-	13.09	-1.99	2.25	8.09	9.12	0.778	8.52	-2.00	1.94
CVaR GARCH EVT	-1.95	10.01	-	12.74	-1.92	2.18	8.04	8.84	0.797	7.97	-1.87	1.88
CVaR GARCH Stsk	0.05	10.13	-	11.52	-1.75	2.10	8.00	8.70	0.805	8.45	-2.20	2.44
DAX	-18.36	41.48	-	44.53	-7.07	11.40	19.41	18.43	0.993	12.29	-3.33	5.30
60/40	-8.15	18.80	-	23.13	-3.58	5.10	9.89	9.48	0.936	6.86	-1.72	2.64

(2013) find that the profitability of risk targeting can be highly different for periods that include the global financial crisis or start after the global financial crisis. The first subperiod, given in Panel A, covers the financial crisis and ranges from 15.07.2008 to 15.07.2009. This period is marked by highly negative returns and high risk. During the financial crisis, the dynamically managed risk targeting strategies have significantly higher (less negative) returns than the two benchmark portfolios. Furthermore, the higher returns of the risk targeting strategies are accompanied by lower risk measured by volatility, drawdown and minimum return. For example, the DAX and the 60/40 portfolio exhibit an (annualized) return of -18.36% and -8.15% as well as a volatility of 41.48% and 18.80%, respectively. In contrast, return and volatility of the EWMA managed target volatility strategy are -3.41% and 12.83%, respectively. However, the CVaR-EWMA-Stsk strategy is even more convincing and achieves a positive return of 0.67% with a volatility of only 9.72%. Hence, CVaR targeting significantly outperforms volatility targeting and the two benchmark portfolios, as long as CVaR is managed by a dynamic risk model. This outperformance is accompanied by a high drawdown reduction. In contrast, the statically managed target VaR and target CVaR strategies (VaR-HS and CVaR-HS) exhibit significantly lower returns with higher risk than the dynamically managed strategies. Furthermore, there are significant differences between the EWMA and GARCH managed strategies. The EWMA managed strategies achieve higher returns with lower risk than the GARCH managed strategies. A possible explanation for the better results of the EWMA model could be the higher estimation risk for the GARCH model during highly volatile periods.

The second subperiod, ranging from 16.07.2009 to 15.07.2011, covers the time following the financial crisis, but excludes the European financial debt crisis. This period is marked by a continuously uptrending market with high returns and low risk. Results for the second subperiod are given in Panel B. For this period, the DAX clearly outperforms the remaining strategies. This is in line with Dreyer and Hubrich (2019, Fig. 2) who also find that volatility targeting is outperformed by the non-managed portfolio, once the sample starts after the global financial crisis. Further, in this period, the different target risk strategies perform significantly diverse. The unconditional models, VaR-HS and CVaR-HS, perform very well in this calm market and, by taking less risk, exhibit high Sharpe Ratios. Moreover, the volatility targeting strategies produce higher returns than the downside risk targeting strategies. This again motivates a strategy that switches between volatility and CVaR targeting, where volatility targeting is only used in uptrending markets. However, the higher return of volatility targeting is also accompanied by higher risk. Consequently, the Sharpe Ratios of the dynamically managed target volatility, VaR and CVaR strategies are quite similar. In contrast, the Sharpe Ratios of the two benchmark portfolios are slightly higher than the Sharpe Ratios of the target risk strategies. However, the differences in the risk-adjusted performance are only small compared to the differences in Panel A. This demonstrates that risk targeting strategies are able to lower the downside risk in extreme crashes, but these strategies still (partly) capture the upside potential of the DAX. In particular, the downside risk managed strategies significantly outperform the remaining strategies in bear markets, but only slightly underperform the volatility targeting strategies in uptrending markets.

1.5.4 The Economic Value of Risk Targeting

In the previous section, we assessed the (risk-adjusted) performance and drawdown protection ability of risk targeting and found that CVaR targeting is superior to volatility targeting. However, our conclusions are so far based on *unconditional* risk-adjusted performance measures, like the Sharpe Ratio and Calmar Ratio. The performance evaluation based on unconditional risk measures has several disadvantages. First, these measures do not account for a time-varying volatility (Han, 2005, Marquering and Verbeek, 2004).[74] As a consequence, unconditional performance measures are not appropriate for strategies that time volatility (Boguth et al., 2011, Cederburg and O'Doherty, 2016). Second, the Sharpe Ratio does not account for higher moments. Hence, this measure is suboptimal for dynamic trading strategies that reduce a portfolio's tail risk, such as the strategies examined here (Dreyer and Hubrich, 2019, Sec. 5). In particular, unconditional performance measures do not incorporate skewness preferences of investors (Schneider et al., 2020). Third, when using an unconditional performance evaluation measure, performance evaluation can be biased since the rebalancing interval does not coincide with the evaluation period (Boguth et al., 2011, Footnote 6). Similarly, Benartzi and Thaler (1995) show that even long-term investors have quite short evaluation periods. Dreyer and Hubrich (2019) find that performance measurement is highly influenced by the performance evaluation frequency. Due to these disadvantages of the Sharpe Ratio, we next assess the economic value of volatility, VaR and CVaR timing, where we define the economic value as the annualized fee an investor is willing to pay to switch from a static portfolio allocation to a risk-managed portfolio. The economic value of volatility timing has been frequently examined in the literature.[75] These studies find huge improvements of volatility timing in terms of high utility gains for mean-variance investors. For example, Fleming et al. (2001), Fleming et al. (2003), Kirby and Ostdiek (2012), Han (2005) and Taylor (2014) examine utility gains in a multivariate framework using several asset classes, whereas Marquering and Verbeek (2004), Moreira and Muir (2017),

[74] Marquering and Verbeek (2004, p. 419-421) write: "It is important to realize that the Sharpe ratio does not appropriately take into account time-varying volatility. The risk of the dynamic strategies is typically overestimated by the sample standard deviation, particularly in the presence of volatility timing, because the ex post (unconditional) standard deviation is an inappropriate measure for the (conditional) risk an investor was facing at each point in time. This indicates a potentially severe disadvantage of the use of Sharpe ratios to evaluate dynamic strategies."

[75] See Fleming et al. (2001), Fleming et al. (2003), Kirby and Ostdiek (2012), Marquering and Verbeek (2004), Taylor (2014), Han (2005), Moreira and Muir (2017) and Bollerslev et al. (2018) for studies on the economic value of volatility timing for mean-variance investors. Calculating the economic value, defined as the fee an investor is willing to pay to switch from one strategy to another strategy, is similar to calculating the certainty equivalent return as done by Ang and Bekaert (2002), Dreyer and Hubrich (2019), Cederburg et al. (2020), Ghysels et al. (2016), Das and Uppal (2004), Guidolin and Timmermann (2008), DeMiguel et al. (2009b) and Moreira and Muir (2019).

Moreira and Muir (2019) and Bollerslev et al. (2018) work with only one risky asset. Further, Jondeau and Rockinger (2006) examine the economic value of portfolio strategies that incorporate higher moments and the authors find that the opportunity costs of ignoring higher moments can become very large when asset returns are non-normally distributed or investors are highly risk-averse. Similarly, Jondeau and Rockinger (2012) assess the economic value of dynamic timing strategies that also incorporate higher moments, like skewness and kurtosis, and find a higher economic value for these strategies compared to strategies that only time volatility. Ghysels et al. (2016) find that investors are willing to pay high fees to switch from a mean-variance optimization to a mean-variance-skewness optimization. These studies again demonstrate that portfolio allocations that incorporate information on higher moments are highly valuable for investors and increase the investors' utility.

In most studies, the economic value is defined as the maximum fee (in percent) a mean-variance investor is willing to pay to switch from one strategy to another strategy. However, as stated in Section 1.3.1, investors typically have preferences for moments higher than volatility and the mean-variance framework is not very realistic. For that reason, we further follow Jondeau and Rockinger (2012) and Dreyer and Hubrich (2019) and also calculate the economic value for an investor with constant relative risk aversion (CRRA). CRRA utility is frequently used in portfolio selection problems.[76] Guidolin and Timmermann (2008), Jondeau and Rockinger (2012) and Bali et al. (2009) show that, for reasonable levels of risk aversion, CRRA utility implies that investors prefer higher skewness and lower kurtosis, which is in line with the study of Scott and Horvath (1980).[77] In particular, "[t]he CRRA utility function implies that investors are tail-risk averse." (Dreyer and Hubrich, 2019, p. 47). For that reason,

[76] See, for example, Ang and Bekaert (2002), Liu et al. (2003), Das and Uppal (2004), Aït-Sahalia and Brandt (2001), Guidolin and Timmermann (2008), Ghysels et al. (2016) for studies that examine the portfolio allocation for CRRA investors.

[77] Several studies show that the CRRA utility framework is mainly driven by the preferences for mean, variance, skewness and kurtosis. For example, Guidolin and Timmermann (2008) compare the asset allocation under CRRA preferences with portfolio allocations under four moment preferences and find only minor differences between both approaches. Hence, portfolio selection under CRRA utility is mainly driven by preferences for the first four moments. A similar result also holds for investors with constant absolute risk aversion (CARA) as shown by Bali et al. (2009) and Jondeau and Rockinger (2006). Jondeau and Rockinger (2006) show that portfolio allocations under CARA utility is mainly driven by preferences for the first four moments and that portfolio allocations under CARA and CRRA utility produce similar results. For that reason, we do not calculate the economic value for CARA utility.

by calculating the economic value of risk targeting for an investor with CRRA utility, we explicitly take preferences for higher moments into account and consider the tail risk reduction of the strategies. Finally, in order to regard that investors weight losses higher than gains, we additionally calculate the economic value for loss-averse investors. The portfolio selection for loss-averse investors has been examined by Benartzi and Thaler (1995), Aït-Sahalia and Brandt (2001) and Ang et al. (2005). Aït-Sahalia and Brandt (2001) compare optimal portfolios for mean-variance investors, CRRA investors and loss-averse investors. The authors find that mean-variance and CRRA preferences produce only slightly different optimal portfolio selections (see also Guidolin and Timmermann (2008)), but loss aversion leads to a significantly different optimal portfolio. Similarly, Ang et al. (2005) examine the portfolio selection for CRRA investors and investors with disappointment aversion, who also treat gains and losses asymmetrically. The authors find more realistic asset allocations under disappointment aversion and that disappointment aversion can resemble portfolio allocations under CRRA preferences, whereas the opposite does not hold. Furthermore, several studies show that equity holdings of real investors are typically much lower than predicted for mean-variance or CRRA investors, whereas loss aversion successfully explains the low equity exposure of real investors (see Benartzi and Thaler (1995) and Ang et al. (2005)). Thus, the loss aversion framework seems to be more realistic than the mean-variance and CRRA frameworks. In total, we expect quite similar results for the mean-variance and CRRA investors, but quite different (and more realistic) results for the loss-averse investor.

As first method to calculate the economic value of risk targeting, we follow Fleming et al. (2001), Fleming et al. (2003), Han (2005) and Kirby and Ostdiek (2012) and assume that the investor's true utility function can be approximated by quadratic utility. For this investor, the realized day t utility is given by

$$U_{MV}(R_{t,a}) = W_{t-1}(1 + R_{t,a}) - \frac{1}{2}\gamma_{abs} W_{t-1}^2 (1 + R_{t,a})^2, \tag{1.5.1}$$

where γ_{abs} is the investor's absolute risk aversion, W_{t-1} denotes the investor's wealth on day $t-1$ and $R_{t,a}$ denotes the day t return of strategy a. We call an investor with preferences as in Equation (1.5.1) a mean-variance investor, since this approach is highly related to the mean-variance theory (Fleming et al., 2001, p. 334). By assuming that this investor has a constant relative risk aversion γ, Equation (1.5.1) can be rewritten as

$$U_{MV}(R_{t,a}) = W_{t-1}\left((1+R_{t,a}) - \frac{\gamma}{2(1+\gamma)}(1+R_{t,a})^2\right). \tag{1.5.2}$$

The economic value of a strategy a is then given by the percentage fee Δ_{MV} the investor with utility in Equation (1.5.1) is willing to pay to switch from the 60/40 portfolio to the strategy a. The fee Δ_{MV} is defined by equating the expected utilities

$$\mathbb{E}(U_{MV}(R_{t,a} - \Delta_{MV})) = \mathbb{E}(U_{MV}(R_{t,b}))\,, \tag{1.5.3}$$

where $R_{t,b}$ denotes the return of the 60/40 portfolio. The expected utility in Equation (1.5.3) is then estimated by the average realized utility. Hence, the fee Δ_{MV} is calculated by solving

$$\overline{U}_{MV}\left(R_{1,a} - \Delta_{MV}, ..., R_{T,a} - \Delta_{MV}\right) = \overline{U}_{MV}\left(R_{1,b}, ..., R_{T,b}\right), \tag{1.5.4}$$

where $\overline{U}_{MV}(R_1, ..., R_T) = \frac{1}{T}\sum_{t=1}^{T}(1+R_t) - \frac{\gamma}{2(1+\gamma)}(1+R_t)^2$. We calculate the fee Δ_{MV} for levels of risk aversion given by $\gamma = 2, 5, 10$ and 15, which are in line with previous studies using this approach (see Marquering and Verbeek (2004), Aït-Sahalia and Brandt (2001), Jondeau and Rockinger (2006) and Jondeau and Rockinger (2012) for example).

In the case of the CRRA investor, the realized day t utility is given by

$$U_{CRRA}(R_{t,a}) = \begin{cases} \frac{(1+R_{t,a})^{(1-\gamma)}}{1-\gamma}, & \text{if } \gamma > 1 \\ \ln(1+R_{t,a}), & \text{if } \gamma = 1. \end{cases} \tag{1.5.5}$$

Since we choose the same levels of γ as above, the investor's utility simplifies to the case $U_{CRRA}(R_{t,a}) = \frac{(1+R_{t,a})^{(1-\gamma)}}{1-\gamma}$. Following Jondeau and Rockinger (2012), the economic value for an investor with CRRA utility is defined by equating the expected utilities

$$\mathbb{E}(U_{CRRA}(R_{t,a} - \Delta_{CRRA})) = \mathbb{E}(U_{CRRA}(R_{t,b}))\,, \tag{1.5.6}$$

which is again estimated by the average realized utility. The percentage fee Δ_{CRRA} is then calculated by solving

$$\overline{U}_{CRRA}\left(R_{1,a} - \Delta_{CRRA}, ..., R_{T,a} - \Delta_{CRRA}\right) = \overline{U}_{CRRA}\left(R_{1,b}, ..., R_{T,b}\right), \tag{1.5.7}$$

where $\overline{U}_{CRRA}(R_1, ..., R_T) = \frac{1}{T}\sum_{t=1}^{T}\frac{(1+R_t)^{(1-\gamma)}}{1-\gamma}$.

Lastly, to account for the loss aversion of investors, we use a utility function that gives more weight on negative returns. Following Aït-Sahalia and Brandt (2001) and Benartzi and Thaler (1995), we define the investor's day t utility by

$$U_{LA}(R_{t,a}) = \begin{cases} (R_{t,a})^b, & \text{if } R_{t,a} \geqslant 0 \\ -l(-R_{t,a})^b, & \text{if } R_{t,a} < 0, \end{cases} \tag{1.5.8}$$

where $l > 1$ determines the investor's loss aversion and b measures the degree of risk seeking for negative returns and risk aversion for positive returns (see Aït-Sahalia and Brandt (2001, p. 1314) or Benartzi and Thaler (1995, p. 79)).[78] Typical values of l and b are in the range of $l = 2.25$ and $b = 0.88$, which are motivated empirically. Similar to Aït-Sahalia and Brandt (2001), we choose the four combinations of $l = 2.0, 3.0$ and $b = 0.8, 1$. A loss aversion of $l = 2$ implies that the disutility of a loss is twice as great as the utility of a positive return of the same magnitude (Benartzi and Thaler, 1995, p. 74).[79] The economic value for a loss-averse investor is then given by equating the expected utilities

$$\mathbb{E}(U_{LA}(R_{t,a} - \Delta_{LA})) = \mathbb{E}(U_{LA}(R_{t,b}))\,. \tag{1.5.9}$$

As above, we calculate Δ_{LA} by solving

$$\overline{U}_{LA}\left(R_{1,a} - \Delta_{LA}, ..., R_{T,a} - \Delta_{LA}\right) = \overline{U}_{LA}\left(R_{1,b}, ..., R_{T,b}\right), \tag{1.5.10}$$

where $\overline{U}_{LA}(R_1, ..., R_T) = \frac{1}{T}\sum_{t=1}^{T} R_t^b \cdot \mathbb{1}_{\{R_t \geqslant 0\}} - l(-R_t)^b \cdot \mathbb{1}_{\{R_t<0\}}$.

Table VIII shows the values Δ_i for the three utility functions, i.e. Δ_i is defined as the annualized percentage fee a mean-variance, CRRA or loss-averse investor is willing to pay to switch from the 60/40 strategy to one of the risk timing strategies. In addition, by choosing the same subperiods as in Table VII, we also examine the economic value of risk timing during and after the financial crisis. For these subperiods, we only report the results for a risk aversion of $\gamma = 5$

[78] We also used the risk-free rate instead of a zero return to define the cut-off point which determines a loss or a gain. Results for the economic value were nearly identical for both choices and the risk-free rate based results are not reported here.

[79] Another possibility to assess the economic value of an investor with unexpected utility would be to use preferences of an ambiguity-averse investor as in Aït-Sahalia and Brandt (2001) and Jondeau and Rockinger (2012) or preferences of a disappointment-averse investor as in Ang et al. (2005). See also Jondeau and Rockinger (2012, Footnote 17) for a list of studies that incorporate ambiguity aversion in asset allocation.

and $\gamma = 10$ in the case of the mean-variance and CRRA investor as well as $l = 2$ and $l = 3$ combined with $b = 0.8$ for the loss-averse investor.

Panel A of Table VIII shows the economic value for a mean-variance investor. The economic value over the whole sample is positive for almost all risk targeting strategies and levels of risk aversion. Further, we find that downside risk timing delivers a significantly higher economic value than volatility timing and that managing CVaR delivers the highest economic value. In other words, a mean-variance investor should manage CVaR instead of volatility. Interestingly, as in Marquering and Verbeek (2004), we find that the economic value of volatility timing is decreasing in the level of risk aversion γ, but this result reverses when the economic value of downside risk timing is assessed. In this case, the economic value is increasing in the risk aversion, i.e. for highly risk-averse mean-variance investors, CVaR timing becomes more important. For example, a mean-variance investor with a risk aversion of $\gamma = 15$ would pay an annualized fee of 0.760% to switch from the 60/40 portfolio to the GARCH managed strategy, but the same investor is not willing to pay a positive fee to switch to the HSD managed strategy. However, the same investor would even pay an annualized fee of 4.253% to switch from the 60/40 portfolio to the CVaR-GARCH-Stsk strategy. The differences between volatility and downside risk timing become even more striking in the crash period. During the financial crisis, an investor with a risk aversion of $\gamma = 5$ would pay an annualized fee of 9.176% to switch from the 60/40 portfolio to the EWMA managed strategy. However, the same investor is even willing to pay an annualized fee of 15.384% to switch to the CVaR-EWMA-Stsk strategy. Thus, in crash periods, mean-variance investors are willing to pay extremely high fees to switch to a risk targeting strategy, where the willingness to pay for CVaR managed strategies is significantly higher than the willingness to pay for volatility managed strategies. Hence, investors are willing to pay extremely high fees to mitigate crashes, which is best done by managing CVaR. This is in line with the results of Bollerslev and Todorov (2011) and Chabi-Yo et al. (2018) that investors are crash-averse. Furthermore, during the financial crisis, we find that the EWMA model is again superior to the GARCH model. This result is in line with the results of Table VII and could be explained by the fact that the EWMA model is not influenced by estimation risk. Further, dur-

Table VIII. Economic Value of Risk Targeting

This table shows the economic value, given as annualized percentage fee Δ_i an investor is willing to pay to switch from the 60/40 portfolio to a risk timing strategy for a given utility function U_i, $i \in \{\text{MV}, \text{CRRA}, \text{LA}\}$. Panel A shows results for a mean-variance investor. Panel B shows results for an investor with CRRA utility. Panel C shows results for a loss-averse investor. γ determines the investor's risk aversion. l determines the investor's loss aversion and b measures the investor's degree of risk seeking for negative returns and risk aversion for positive returns.

	Whole Sample				Crash		Recovery	
Panel A: Δ_{MV}	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 5$	$\gamma = 10$	$\gamma = 5$	$\gamma = 10$
Vola Hist	0.618	0.280	-0.282	-0.841	8.886	13.843	-1.339	-2.948
Vola EWMA	0.878	0.671	0.326	-0.018	9.176	14.387	-0.636	-2.099
Vola GARCH	0.802	0.792	0.776	0.760	8.021	12.644	-0.525	-1.554
VaR Hist	-0.049	0.682	1.912	3.158	0.731	2.172	-1.493	-0.186
VaR EWMA FHS	0.940	1.237	1.735	2.236	12.968	20.081	-1.396	-2.227
VaR EWMA EVT	1.009	1.521	2.381	3.248	12.906	20.407	-1.540	-1.542
VaR EWMA Stsk	1.168	1.552	2.194	2.841	13.930	20.780	-1.527	-1.828
VaR GARCH FHS	0.799	1.015	1.375	1.737	10.057	15.743	-0.995	-1.752
VaR GARCH EVT	0.898	1.346	2.098	2.856	11.070	17.580	-1.121	-1.171
VaR GARCH Stsk	1.126	1.653	2.537	3.430	13.672	20.057	-1.099	-1.114
CVaR Hist	0.347	1.206	2.653	4.121	5.073	8.469	-1.581	-0.236
CVaR EWMA FHS	1.031	1.629	2.634	3.648	14.001	22.157	-2.121	-1.798
CVaR EWMA EVT	1.066	1.733	2.856	3.992	14.038	22.200	-1.900	-1.513
CVaR EWMA Stsk	1.196	1.800	2.816	3.841	15.384	22.992	-2.093	-1.977
CVaR GARCH FHS	0.991	1.424	2.150	2.881	11.954	19.037	-1.503	-1.337
CVaR GARCH EVT	1.057	1.602	2.517	3.440	12.253	19.490	-1.453	-1.161
CVaR GARCH Stsk	1.068	1.794	3.016	4.253	14.491	21.799	-1.441	-1.088
Panel B: Δ_{CRRA}	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 5$	$\gamma = 10$	$\gamma = 5$	$\gamma = 10$
Vola Hist	0.731	0.387	-0.196	-0.792	7.883	12.686	-1.018	-2.645
Vola EWMA	0.947	0.735	0.374	0.003	8.122	13.166	-0.344	-1.820
Vola GARCH	0.805	0.792	0.763	0.728	7.079	11.543	-0.320	-1.355
VaR Hist	-0.292	0.438	1.668	2.917	0.430	1.810	-1.753	-0.452
VaR EWMA FHS	0.841	1.135	1.625	2.117	11.550	18.457	-1.232	-2.074
VaR EWMA EVT	0.839	1.348	2.202	3.065	11.418	18.707	-1.542	-1.551
VaR EWMA Stsk	1.041	1.422	2.057	2.697	12.566	19.230	-1.469	-1.775
VaR GARCH FHS	0.728	0.940	1.292	1.645	8.910	14.410	-0.845	-1.610
VaR GARCH EVT	0.749	1.194	1.939	2.692	9.768	16.084	-1.112	-1.167
VaR GARCH Stsk	0.952	1.476	2.355	3.247	12.401	18.629	-1.097	-1.115
CVaR Hist	0.062	0.918	2.365	3.837	4.384	7.688	-1.849	-0.510
CVaR EWMA FHS	0.833	1.427	2.426	3.438	12.390	20.322	-2.187	-1.873
CVaR EWMA EVT	0.844	1.509	2.627	3.762	12.426	20.363	-1.979	-1.599
CVaR EWMA Stsk	0.995	1.596	2.606	3.630	13.877	21.285	-2.118	-2.006
CVaR GARCH FHS	0.847	1.277	1.996	2.721	10.543	17.423	-1.538	-1.377
CVaR GARCH EVT	0.876	1.418	2.327	3.248	10.814	17.844	-1.513	-1.226
CVaR GARCH Stsk	0.828	1.550	2.768	4.005	13.040	20.150	-1.512	-1.162
	$b = 0.8$		$b = 1$		$b = 0.8$		$b = 0.8$	
Panel C: Δ_{LA}	$l = 2$	$l = 3$	$l = 2$	$l = 3$	$l = 2$	$l = 3$	$l = 2$	$l = 3$
Vola Hist	-4.053	-6.592	-4.703	-7.482	19.580	28.428	-9.136	-13.846
Vola EWMA	-3.186	-5.370	-3.601	-5.924	20.476	29.797	-8.288	-12.768
Vola GARCH	-1.780	-3.149	-1.858	-3.215	17.199	25.045	-6.375	-9.759
VaR Hist	8.009	12.568	10.125	15.791	6.462	10.148	9.090	15.170
VaR EWMA FHS	1.423	1.749	1.978	2.617	31.116	45.090	-5.690	-8.333
VaR EWMA EVT	3.564	5.064	4.635	6.710	32.736	47.798	-1.133	-1.114
VaR EWMA Stsk	2.641	3.537	3.426	4.733	30.469	43.812	-2.489	-3.272
VaR GARCH FHS	0.251	0.013	0.613	0.591	22.955	33.377	-5.268	-7.836
VaR GARCH EVT	2.574	3.590	3.473	4.971	26.938	39.426	-1.381	-1.666
VaR GARCH Stsk	3.851	5.479	5.013	7.230	28.531	40.709	-0.666	-0.561
CVaR Hist	9.139	14.167	11.688	18.014	13.800	20.473	9.395	15.689
CVaR EWMA FHS	4.589	6.648	5.887	8.645	36.478	53.296	0.674	2.023
CVaR EWMA EVT	5.332	7.800	6.836	10.110	36.525	53.359	1.188	2.758
CVaR EWMA Stsk	5.171	7.479	6.541	9.562	34.719	50.002	-0.055	0.858
CVaR GARCH FHS	2.565	3.526	3.439	4.864	30.050	43.969	-0.239	0.328
CVaR GARCH EVT	3.752	5.352	4.918	7.129	30.919	45.186	0.620	1.669
CVaR GARCH Stsk	6.149	9.106	7.867	11.715	32.692	47.158	1.759	3.466

ing the crash period, investors are willing to pay much lower fees to switch to the static VaR-HS and CVaR-HS models. Hence, static models fail to achieve a good downside risk protection just when it is most needed. This is again in line with our earlier findings that accurate risk models are superior to static risk models, especially in crash periods. As expected, these results reverse when we regard the period that starts immediately after the financial crisis. In this period, all risk targeting strategies exhibit a negative economic value. This is again in line with our earlier findings. However, the utility gains in the crash period are significantly higher in magnitude than the utility losses in the calm period. Panel B shows the economic value of risk timing for an investor with CRRA utility. In this case, we also incorporate preferences for higher moments and consider that investors dislike left tail risk. However, results are quite similar to the results of the mean-variance investor shown in Panel A. For CRRA investors, managing CVaR is superior to managing volatility, especially in times of bear markets and for highly risk-averse investors. However, results reverse during the uptrending period. As mentioned above, the observation that results for the mean-variance and CRRA investors are quite similar is in line with the studies on the optimal portfolio allocation of mean-variance and CRRA investors. These studies find quite similar portfolios for both investors, whereas the optimal portfolio of loss-averse investors is highly different and more in line with portfolios in practice. Thus, we expect different and more realistic results for the economic value of the loss-averse investor.

Panel C shows the economic value of risk targeting for a loss-averse investor. The economic value for a loss-averse investor is significantly different to the economic value of a mean-variance or CRRA investor. As stated above, this result is in line with Aït-Sahalia and Brandt (2001) who find similar results when mean-variance or CRRA preferences are used in portfolio selection problems, but vastly different allocations for loss-averse investors. Interestingly, over the whole period, the economic value of the target volatility strategies is negative, regardless of the level of loss aversion and volatility model. In other words, a loss-averse investor would pay a positive fee to switch away from a target volatility strategy to the 60/40 portfolio. In contrast, the economic value of downside risk targeting is always positive and typically very high. As before, we find that CVaR targeting produces the highest economic value, i.e.

a loss-averse investor should time CVaR or at least VaR instead volatility. Somewhat surprising, we find a higher economic value for the unconditional models (VaR-HS and CVaR-HS). However, this finding can be explained by the lower average equity exposure of these strategies. Hence, these strategies are more conservative and should be more appealing for loss-averse investors who have a preference for conservative strategies. The extremely high economic value for loss-averse investors who manage downside risk can partly be explained by the daily evaluation period used in the economic value calculation. Benartzi and Thaler (1995) show that loss aversion is more pronounced for shorter evaluation periods, i.e. the shorter the evaluation period for a loss-averse investor the less attractive are investments with higher risk. Similar horizon effects have been found by Aït-Sahalia and Brandt (2001) for loss aversion, but not for mean-variance and CRRA preferences. The authors conclude that loss aversion implies that short-term investors are extremely risk-averse, whereas long-term investors become more risk-neutral.[80] During the financial crisis, the economic value of risk targeting becomes extremely high, i.e. a loss-averse investor is willing to pay extremely high fees for downside risk protection during crash periods. This again confirms the result of Bollerslev and Todorov (2011) and Chabi-Yo et al. (2018) that investors are crash-averse. For example, a loss-averse investor with parameters $b = 0.8$ and $l = 3$ would pay an annualized fee of 29.797% to switch from the 60/40 portfolio to the EWMA managed target volatility strategy. However, the same investor would even pay a fee of 53.359% per year to switch to a CVaR managed strategy. Furthermore, during the crash period, we find a significantly higher economic value for dynamically managed strategies. This again indicates that more accurate risk models are more successful in managing extremely negative returns in crash periods. Interestingly, opposed to the results of the mean-variance and CRRA investor, we even find a positive economic value of almost all

[80]Dreyer and Hubrich (2019) also find that performance evaluation measures can be highly different when different evaluation periods are used. We also calculated the economic value for a loss-averse investor by first aggregating the daily returns to monthly returns. As expected, the economic value for a loss-averse investor calculated with monthly returns is smaller in magnitude than the economic value calculated with daily data. Nevertheless, the economic value of volatility targeting is still negative for all combinations and the economic value of downside risk targeting is still positive for all combinations, except for the VaR-GARCH-FHS model for $b = 0.8$. The highest economic value is again obtained by the strategies that manage CVaR. Furthermore, the economic value for a CRRA investor is nearly unchanged when monthly returns instead of daily returns are used. This result is also found by Aït-Sahalia and Brandt (2001) for the optimal portfolio choice under CRRA preferences and loss aversion.

CVaR targeting strategies in the uptrending market, whereas the economic value of volatility targeting is negative and high in magnitude. In other words, a loss-averse investor should time CVaR instead of volatility, regardless of whether the market is in a bull or a bear regime.

The fees given in Table VIII are extremely high compared to the fees found by Bollerslev et al. (2018). The authors argue that even their fees, in the range of 0.5%, are extremely beneficial for investors. This highlights the advantage of risk targeting, especially CVaR targeting, found for our data set. However, there are several differences between our study and the study of Bollerslev et al. (2018), which explain the differences in the magnitude of the fees. First, Bollerslev et al. (2018) calculate utility gains of several volatility targeting strategies, relying on different volatility models, against a benchmark volatility targeting strategy. In other words, the authors choose a certain target volatility strategy as benchmark model, whereas we choose the 60/40 portfolio as benchmark, which is similar to Marquering and Verbeek (2004). Second, the authors only compare the differences between several volatility forecasting models, whereas we also compare the differences between volatility and downside risk targeting. In line with the results of Bollerslev et al. (2018), differences within the volatility targeting strategies are only small, whereas the differences between volatility and CVaR targeting are significantly higher. For example, a mean-variance investor with a risk aversion of $\gamma = 5$ would pay a fee of $0.792\% - 0.280\% = 0.512\%$ per year to switch from the HSD managed strategy to the GARCH managed strategy. This result is comparable to the finding of Bollerslev et al. (2018) and again demonstrates the positive relation between forecasting accuracy – or equivalently a more constant portfolio volatility – and utility gains. However, the same investor would even pay $1.800\% - 0.280\% = 1.52\%$ per year to switch from the HSD managed strategy to the CVaR-EWMA-Stsk strategy. Third, Bollerslev et al. (2018) rebalance the weight of the volatility targeting strategy monthly, whereas we use daily rebalancing. Since the authors show that a higher accuracy typically coincides with higher utility gains, daily rebalancing should also produce a higher economic value. Since daily rebalancing also induces higher transaction costs, the extremely high fees of Table VIII should be lower in practice.[81] Nevertheless, in Section D.3, we show that risk targeting is still beneficial when portfolio weights are only rebalanced

[81] Tail risk targeting for monthly rebalancing is examined in Rickenberg (2020a).

when the DAX's risk changes dramatically. Fourth, in addition to mean-variance preferences, we also calculate the economic value for loss-averse investors, who are willing to pay extremely high fees for a crash risk reduction that is best done by CVaR targeting.

Finally, we test if the economic value found in Table VIII is also statistically significant. Bollerslev et al. (2018) use the DM-test to statistically compare the utility benefit of several volatility models used in a volatility targeting strategy. Taylor (2014, Sec. 2.2) presents a conditional test, that extends the DM-test, to assess if advanced forecasting models produce higher utility gains than simple forecasting models. Kirby and Ostdiek (2012) use a bootstrap based test to assess the significance of utility gains. A similar approach is also used by DeMiguel et al. (2009b) and Cederburg et al. (2020) to test for differences in the certainty equivalent return for mean-variance investors. We follow these approaches and apply the tests that were presented in Section 1.4.1 to test if a strategy produces a significantly higher utility. These tests are also frequently used to test for a superior (risk-adjusted) performance of technical trading rules or mutual funds (see Sullivan et al. (1999), Hsu et al. (2010), Barras et al. (2010), Bajgrowicz and Scaillet (2012) among others). Results of these tests are shown in Table IX, where we only show results for $\gamma = 10$ for the mean-variance and CRRA investor as well as $l = 2$ and $b = 0.8$ for the loss-averse investor. Whenever a benchmark model is needed, we choose the 60/40 portfolio as benchmark. Panel A shows results for the mean-variance investor. The DM-test indicates that almost all downside risk targeting strategies produce higher utilities than the 60/40 portfolio, whereas all target volatility strategies do not produce statistically higher utilities. The RC-test fails to reject any null hypothesis, which again demonstrates the weaknesses of the RC-test. In contrast, the SPA-test rejects the null hypotheses of all target volatility and target VaR strategies, whereas the null hypotheses of most target CVaR strategies cannot be rejected. Thus, the SPA-test indicates that the target CVaR strategies are significantly more valuable for mean-variance investors than volatility and VaR targeting. Results for the MCS are quite similar to the results of the DM-test. None of the target volatility strategies is contained in the MCS, which is also confirmed by the stepwise approaches, where we only show results for the studentized versions. The Step-SPA approach identifies almost all downside risk targeting

Table IX. Testing the Utility Gain of Risk Targeting

This table shows the results of the tests presented in Section 1.4.1 used to test the significance of the utility gains. Panel A shows results for a mean-variance investor with $\gamma = 10$. Panel B shows results for a CRRA investor with $\gamma = 10$. Panel C shows results for a loss-averse investor with $b = 0.8$ and $l = 2$. The description of the columns is given in Tables I and II.

Panel A: MV	DM-test	p^{RC}	p^{SPA}	p^{SQ}	Step-RCst	Step-SPAst	$FDR^+ = 10\%$
Vola Hist	-0.24	**19.69**	0.04	0.08	0	0	0
Vola EWMA	0.29	**26.15**	0.07	0.30	0	0	0
Vola GARCH	0.79	**30.32**	0.11	0.15	0	0	**15**
VaR Hist	**2.35**	**55.93**	1.75	**21.36**	**1**	**1**	**3**
VaR EWMA FHS	1.58	**47.12**	0.62	4.15	0	**2**	**13**
VaR EWMA EVT	**2.16**	**64.79**	0.97	**21.36**	**1**	**1**	**9**
VaR EWMA Stsk	**1.96**	**58.15**	0.77	**16.17**	**1**	**1**	**12**
VaR GARCH FHS	1.39	**39.00**	0.20	0.99	0	0	**14**
VaR GARCH EVT	**2.09**	**56.18**	0.55	7.74	**1**	**1**	**10**
VaR GARCH Stsk	**2.46**	**76.40**	3.02	**36.14**	**1**	**1**	**5**
CVaR Hist	**3.16**	**74.85**	**42.98**	**92.67**	**1**	**1**	**1**
CVaR EWMA FHS	**2.29**	**75.56**	1.18	**39.16**	**1**	**1**	**8**
CVaR EWMA EVT	**2.49**	**86.18**	**55.07**	**92.67**	**1**	**1**	**4**
CVaR EWMA Stsk	**2.40**	**85.74**	**47.75**	**92.67**	**1**	**1**	**7**
CVaR GARCH FHS	**2.09**	**58.51**	0.10	**10.22**	**1**	**1**	**11**
CVaR GARCH EVT	**2.41**	**75.18**	**21.02**	**36.14**	**1**	**1**	**6**
CVaR GARCH Stsk	**2.73**	**100.00**	**100.00**	**100.00**	**1**	**1**	**2**

Panel B: CRRA	DM-test	p^{RC}	p^{SPA}	p^{SQ}	Step-RCst	Step-SPAst	$FDR^+ = 10\%$
Vola Hist	-0.27	**20.49**	0.01	0.06	0	0	0
Vola EWMA	0.27	**26.74**	0.03	0.31	0	0	0
Vola GARCH	0.76	**31.12**	0.04	0.16	0	0	**15**
VaR Hist	**2.34**	**56.40**	1.87	**22.34**	**1**	**1**	**7**
VaR EWMA FHS	1.57	**47.61**	0.66	4.16	0	**2**	**13**
VaR EWMA EVT	**2.15**	**65.11**	0.70	**22.34**	**1**	**1**	**9**
VaR EWMA Stsk	**1.95**	**58.54**	0.95	**16.36**	**2**	**1**	**12**
VaR GARCH FHS	1.37	**39.83**	0.10	0.91	0	0	**14**
VaR GARCH EVT	**2.08**	**56.38**	0.26	7.42	**1**	**1**	**10**
VaR GARCH Stsk	**2.45**	**76.84**	3.40	**36.21**	**1**	**1**	**6**
CVaR Hist	**3.14**	**74.35**	**42.39**	**92.73**	**1**	**1**	**1**
CVaR EWMA FHS	**2.29**	**76.22**	1.20	**40.30**	**1**	**1**	**8**
CVaR EWMA EVT	**2.48**	**86.93**	**55.26**	**92.73**	**1**	**1**	**3**
CVaR EWMA Stsk	**2.39**	**85.67**	**47.91**	**92.73**	**1**	**1**	**4**
CVaR GARCH FHS	**2.08**	**58.86**	0.02	**10.47**	**1**	**1**	**11**
CVaR GARCH EVT	**2.41**	**76.08**	**21.72**	**36.21**	**1**	**1**	**5**
CVaR GARCH Stsk	**2.73**	**100.00**	**100.00**	**100.00**	**1**	**1**	**2**

Panel C: LA	DM-test	p^{RC}	p^{SPA}	p^{SQ}	Step-RCst	Step-SPAst	$FDR^+ = 10\%$
Vola Hist	-2.72	0.00	0.00	0.00	0	0	0
Vola EWMA	-2.25	0.00	0.00	0.00	0	0	0
Vola GARCH	-1.45	0.00	0.00	0.00	0	0	0
VaR Hist	**9.09**	**52.46**	0.00	0.18	**1**	**1**	**1**
VaR EWMA FHS	0.94	0.21	0.00	0.00	0	0	**13**
VaR EWMA EVT	**2.35**	2.54	0.00	0.00	**1**	**1**	**9**
VaR EWMA Stsk	**1.79**	0.81	0.00	0.00	**1**	**1**	**12**
VaR GARCH FHS	0.20	0.05	0.00	0.00	0	0	0
VaR GARCH EVT	**1.96**	0.64	0.00	0.00	**1**	**1**	**10**
VaR GARCH Stsk	**2.96**	2.69	0.00	0.00	**1**	**1**	**6**
CVaR Hist	**10.68**	**100.00**	**100.00**	**100.00**	**1**	**1**	**2**
CVaR EWMA FHS	**2.91**	**6.05**	0.00	0.00	**1**	**1**	**7**
CVaR EWMA EVT	**3.39**	**10.13**	0.05	0.18	**1**	**1**	**5**
CVaR EWMA Stsk	**3.38**	8.81	0.06	0.17	**1**	**1**	**3**
CVaR GARCH FHS	**1.90**	0.64	0.00	0.00	**1**	**1**	**11**
CVaR GARCH EVT	**2.73**	2.70	0.00	0.00	**1**	**1**	**8**
CVaR GARCH Stsk	**4.49**	**15.07**	0.03	0.18	**1**	**1**	**4**

strategies as superior, whereas the target volatility strategies are not identified as superior. As expected, the FDR approach produces the largest set of superior models and picks all models, except for the HSD and EWMA model. Panel B shows results for the CRRA investor. As in Table VIII, these results are again quite similar to the results of the mean-variance investor.

Panel C shows results for the loss-averse investor. The DM-test again indicates that almost all downside risk targeting strategies produce statistically higher utility gains than the 60/40 portfolio. In contrast, the volatility targeting strategies produce lower utilities, where the utilities of the HSD and EWMA based strategies are even statistically lower with a test statistic that is lower than -1.64. Results for the RC-test, SPA-test and the MCS approach are very different to the findings of the DM-test. These tests indicate that the Historical Simulation based target CVaR strategy clearly outperforms the remaining models. This result is also in line with the high economic value of this strategy shown in Table VIII. In contrast, the stepwise approaches and the FDR approach produce large sets of optimal models and pick (almost) all downside risk targeting strategies, whereas none of the target volatility strategies is chosen. The FDR approach further shows that the target CVaR strategies are typically picked in the first steps. The differences between the results of the MCS and the stepwise approaches can be explained by their construction. The stepwise approaches identify superior models (compared to the 60/40 portfolio) and then test the remaining models in the next steps. In contrast, the MCS eliminates bad performing models and then tests all the remaining models in the next steps. Hence, in the MCS approach, a good performing model remains in the test set until the last step. Thus, if one model clearly outperforms the remaining models, all other models are identified as inferior to this model. Due to the significantly higher economic value of the CVaR-HS model for a loss-averse investor, all other models are clearly eliminated in the first steps. In contrast, the stepwise approaches pick the HS based CVaR model and all other models that also produce a significantly higher utility than the 60/40 portfolio in the first steps. Thus, the stepwise approach picks all models that are superior to the 60/40 portfolio, whereas the MCS picks *the best* model(s) among all models. This also explains the different findings of the DM-test and SPA-test. The DM-test tests a models against the 60/40 portfolio, whereas the SPA-test tests a model against all remaining models. In total, results of Panel C demonstrate that loss-averse investors should time downside risk instead of volatility, where CVaR timing produces the best results. This again confirms the suggestion of Aït-Sahalia and Brandt (2001, p. 1315-1316) that loss aversion is highly related to CVaR based portfolio constructions, as examined by Basak and Shapiro (2001).

1.5.5 Switching Strategies

Results so far indicate that volatility targeting produces higher returns in uptrending markets, whereas CVaR targeting provides a better drawdown protection. However, in uptrending markets, the CVaR targeting approach is typically too conservative. For that reason, we next examine strategies that switch between volatility and CVaR targeting, based on indicators that indicate if the following day is an up- or down-day. Thus, we combine the superiority of the volatility targeting strategy in calm markets with the superiority of CVaR targeting in crash periods. Combining different portfolio strategies is frequently examined in the literature (DeMiguel et al., 2009b, Garlappi et al., 2006, Kan and Zhou, 2007, Tu and Zhou, 2011). Similarly, Wang et al. (2012) switch between different target risk levels, where a more conservative target is chosen if a crash regime is expected. This is similar to our approach of switching to a more conservative strategy when a down-market is expected. Furthermore, Taylor (2014) proposes to switch between several forecasting models using an estimate of the current market environment, which is again similar to switching between different target risk strategies. A combined strategy that manages portfolio risk by the portfolio's CVaR in times of bear markets, but switches to a volatility based strategy in bull markets, should be successful in capturing the upside potential while simultaneously drawdowns are mitigated. Another possibility would be to buy the risky asset, i.e. $w_t = 1$, in bull markets and use a CVaR based strategy in bear markets. However, controlling an asset's risk can even be advantageous in bull markets (Barroso and Santa-Clara, 2015, Table 6). Following Tu and Zhou (2011), we define the weight of day t as

$$w_t^{switch} = \delta_t \cdot w_t^{CVaR} + (1 - \delta_t) \cdot w_t^{vol}, \tag{1.5.11}$$

where $\delta_t \in \mathbb{R}$ is the weight placed on the target CVaR strategy, w_t^{CVaR} is the day t weight of the CVaR targeting strategy and w_t^{vol} is the day t weight of the volatility targeting strategy. Several possibilities to define the crash indicator δ_t are feasible. For example, a regime-switching process as in Ang and Bekaert (2002), Guidolin and Timmermann (2008) and Wang et al. (2012) could be used to determine bull and bear regimes. However, since risk targeting is also relevant for practical implementations, we will rely on simple models to determine δ_t. Our first two switching strategies use a crash indicator δ_t that equals one if a negative return on day t is expected and zero else, given information up to day $t-1$. Hence, these approaches use either volatility or CVaR targeting. In order to determine $\delta_t \in \{0,1\}$, we use methods from the literature on technical analysis (Bajgrowicz and Scaillet, 2012, Hsu et al., 2010, Moskowitz et al., 2012, Sullivan et al., 1999), where we use the two most prominent methods, i.e. Moving Averages (MA) and Time Series Momentum (TSMOM). Based on the MA approach, the indicator δ_t is given by

$$\delta_t = \begin{cases} 1, & \text{if } S_{t-1} \leqslant \mathrm{MA}_{t-1,n} \\ 0, & \text{if } S_{t-1} > \mathrm{MA}_{t-1,n}, \end{cases} \tag{1.5.12}$$

where $\mathrm{MA}_{t-1,n} = \frac{1}{n}\sum_{i=1}^{n} S_{t-i}$ denotes the Moving Average with a length of n days. Hence, if the risky asset is in an uptrend, given by $S_{t-1} > \mathrm{MA}_{t-1,n}$, the portfolio is managed by the portfolio's volatility. In contrast, if the risky asset is in a downtrend, given by $S_{t-1} \leqslant \mathrm{MA}_{t-1,n}$, the portfolio is managed by the more conservative CVaR targeting approach.

Based on the TSMOM approach of Moskowitz et al. (2012), the indicator δ_t is given by

$$\delta_t = \begin{cases} 1, & \text{if } S_{t-1} \leqslant S_{t-1-n} \\ 0, & \text{if } S_{t-1} > S_{t-1-n}. \end{cases} \tag{1.5.13}$$

Hence, the portfolio on day t is managed by volatility if the price of day $t-1$ is higher than the price of day $t-1-n$, i.e. we use volatility targeting when the risky asset is in an uptrend. In contrast, during a downtrend, i.e. $S_{t-1} \leqslant S_{t-1-n}$, the more conservative CVaR targeting approach is used. We use $n = 200$ days for the MA and TSMOM approach, which is the most common choice made by the literature and practitioners.[82]

[82]Moskowitz et al. (2012) find good results for the TSMOM strategy for periods between one and 36 months. This corresponds to a window size between 21 and 756 days. We also used other lengths and found good results for other choices of n. For example, choosing $n = 150$ produces even higher risk-adjusted returns compared to $n = 200$. However, since $n = 200$ is the most relevant length, we only show results for this choice.

Table X. Performance Results of Risk Targeting: Switching Strategies
This table shows performance results of the strategies that switch between the EWMA volatility targeting strategy and the CVaR targeting strategies for three different indicators δ_t. Panel A shows results for the indicator δ_t based on a 200 day Time Series Momentum (TSMOM) rule. Panel B shows results for the indicator δ_t based on a 200 day Moving Average (MA) rule. Panel C shows results for the indicator δ_t based on the volatility forecast $\hat{\sigma}_t$ of the EWMA model. The description of the columns is given in Table V.

Panel A: TSMOM Indicator	Return	Vola	SR	z_{JK}	MDD	Calmar	VaR	CVaR	Min	Max
Vola Hist	3.12	12.84	0.102	-	41.23	0.032	1.38	1.82	-5.64	5.14
Index	2.73	23.46	0.040	-0.57	70.42	0.013	2.36	3.46	-8.49	11.40
60/40	2.37	11.94	0.048	-0.59	40.07	0.014	1.24	1.74	-4.32	5.10
EWMA/CVaR Hist	4.36	12.33	0.206	**1.94**	31.65	0.080	1.29	1.75	-5.12	4.98
EWMA/CVaR EWMA FHS	3.92	11.71	0.180	**2.22**	37.03	0.057	1.24	1.65	-4.01	4.98
EWMA/CVaR EWMA EVT	3.96	11.64	0.184	**2.29**	36.62	0.059	1.23	1.64	-3.91	4.98
EWMA/CVaR EWMA Stsk	4.24	11.73	0.206	**3.00**	34.96	0.069	1.24	1.65	-4.67	4.98
EWMA/CVaR GARCH FHS	3.83	12.07	0.167	**2.02**	38.81	0.052	1.28	1.68	-4.58	4.98
EWMA/CVaR GARCH EVT	3.93	11.93	0.177	**2.26**	37.68	0.056	1.26	1.67	-4.41	4.98
EWMA/CVaR GARCH Stsk	4.19	11.76	0.202	**2.61**	35.38	0.067	1.24	1.65	-4.75	4.98

Panel B: MA Indicator	Return	Vola	SR	z_{JK}	MDD	Calmar	VaR	CVaR	Min	Max
Vola Hist	3.12	12.84	0.102	-	41.23	0.032	1.38	1.82	-5.64	5.14
Index	2.73	23.46	0.040	-0.57	70.42	0.013	2.36	3.46	-8.49	11.40
60/40	2.37	11.94	0.048	-0.59	40.07	0.014	1.24	1.74	-4.32	5.10
EWMA/CVaR Hist	4.28	12.31	0.199	**1.75**	31.08	0.079	1.29	1.75	-5.12	4.98
EWMA/CVaR EWMA FHS	3.87	11.66	0.176	**2.09**	36.38	0.056	1.23	1.64	-4.01	4.98
EWMA/CVaR EWMA EVT	3.92	11.58	0.181	**2.17**	35.98	0.058	1.22	1.63	-3.91	4.98
EWMA/CVaR EWMA Stsk	4.07	11.72	0.192	**2.62**	34.86	0.064	1.24	1.66	-4.67	4.98
EWMA/CVaR GARCH FHS	3.85	12.00	0.169	**2.02**	37.97	0.053	1.28	1.67	-4.58	4.98
EWMA/CVaR GARCH EVT	3.94	11.86	0.179	**2.23**	36.82	0.058	1.26	1.65	-4.41	4.98
EWMA/CVaR GARCH Stsk	4.08	11.71	0.193	**2.36**	34.48	0.066	1.24	1.65	-4.75	4.98

Panel C: Volatility Indicator	Return	Vola	SR	z_{JK}	MDD	Calmar	VaR	CVaR	Min	Max
Vola Hist	3.12	12.84	0.102	-	41.23	0.032	1.38	1.82	-5.64	5.14
Index	2.73	23.46	0.040	-0.57	70.42	0.013	2.36	3.46	-8.49	11.40
60/40	2.37	11.94	0.048	-0.59	40.07	0.014	1.24	1.74	-4.32	5.10
EWMA/CVaR Hist	2.42	12.26	0.051	-0.28	36.28	0.017	0.94	1.85	-7.60	10.87
EWMA/CVaR EWMA FHS	3.17	9.03	0.152	0.57	30.55	0.045	0.95	1.33	-3.55	4.17
EWMA/CVaR EWMA EVT	3.14	8.68	0.154	0.58	29.17	0.046	0.91	1.28	-3.40	4.14
EWMA/CVaR EWMA Stsk	3.80	9.13	0.217	1.30	25.64	0.077	0.96	1.33	-4.45	4.11
EWMA/CVaR GARCH FHS	3.25	10.23	0.141	0.52	36.28	0.040	1.08	1.47	-4.33	3.81
EWMA/CVaR GARCH EVT	3.25	9.58	0.151	0.65	32.89	0.044	1.02	1.38	-4.10	3.77
EWMA/CVaR GARCH Stsk	3.36	8.99	0.173	0.68	28.06	0.055	0.93	1.28	-4.56	8.40

The two indicators defined above are dummy variables, taking a value of $\delta_t = 1$ if a negative return is likely and zero else. As a consequence, the weight of day t is either given by the volatility targeting strategy *or* the CVaR targeting strategy. We next define a third indicator, where the weight of day t is given as a combination of the volatility and CVaR targeting strategies. This is similar to the approach of combining different forecasting methods (Allen et al., 2012, Halbleib and Pohlmeier, 2012, Taylor, 2014). If market risk increases, measured by expected volatility of day t, we place more weight on the CVaR targeting strategy, whereas CVaR targeting becomes less important when market risk decreases.[83] More formally, we define δ_t as

$$\delta_t = \frac{\hat{\sigma}_t}{\sigma_{\text{target}}}, \tag{1.5.14}$$

where σ_{target} is the chosen volatility target and $\hat{\sigma}_t$ is the volatility forecast of one of the volatility models. Generally, market timing strategies based on expected volatility are frequently used by practitioners (Christoffersen and Diebold, 2006, Copeland and Copeland, 1999). Defining δ_t with respect to the chosen volatility target is appealing since more risk-averse investors choose lower levels of σ_{target}, which implies higher values of δ_t. This fits well to our earlier finding that more risk-averse investors obtain higher utility gains from CVaR targeting compared to volatility targeting. Hence, by choosing δ_t as a function of σ_{target}, more risk-averse investors place higher weights on CVaR targeting, whereas risk-seeking investors place higher weights on volatility targeting. Thus, the portfolio of a risk-averse investor is more conservative than the portfolio of a risk-seeking investor.

The weight of the switching strategy using the indicator given in Equation (1.5.14) can be rewritten as

$$w_t^{switch} = w_t^{vol} + \left(\frac{w_t^{CVaR}}{w_t^{vol}} - 1 \right). \tag{1.5.15}$$

Hence, this switching strategy is similar to the volatility targeting strategy with weight w_t^{vol}, but the switching strategy places more (less) weight on the risky asset when the weight of the CVaR targeting strategy is higher (lower) than the weight of the volatility targeting strategy. This strategy is similar to the approach of Packham et al. (2017) who examine tail risk hedging strategies based on the difference of VaR forecasts under a normality assumption and forecasting methods that take non-normalities into account. By definition, the CVaR takes non-normalities into account and w_t^{CVaR} should be higher (lower) than w_t^{vol} when the market is in an bull (bear) regime with lower (higher) left tail risk. Thus, this switching strategy should be similar to the volatility targeting strategy, but should react more sensitive to changes in the market environment, where the weight is lowered in a down-market and increased in an up-market, governed by changes in higher moments, like skewness and kurtosis.

[83] We also used an indicator based on the two indicators defined above, given by $\delta_t = (\delta_t^{MA} + \delta_t^{TSMOM})/2$. Thus, this strategy uses either volatility targeting, CVaR targeting or an equally weighted combination of both strategies. However, results were quite similar to the previous approaches and are not reported.

Table XI. Economic Value of Risk Targeting: Switching Strategies
This table shows the economic value, given as annualized percentage fee Δ_i an investor is willing to pay to switch from the 60/40 portfolio to a strategy that switches between volatility and CVaR targeting, for a given utility function U_i, $i \in \{\text{MV}, \text{CRRA}, \text{LA}\}$. Panel A shows the economic value for a mean-variance investor. Panel B shows the economic value for an investor with CRRA utility. Panel C shows the economic value for a loss-averse investor. γ indicates the investor's risk aversion. l determines the investor's loss aversion and b measures the investor's degree of risk seeking for negative returns and risk aversion for positive returns.

	Whole Sample				Crash		Recovery	
Panel A: Δ_{MV}	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 5$	$\gamma = 10$	$\gamma = 5$	$\gamma = 10$
EWMA/CVaR Hist	0.013	-0.105	-0.302	-0.498	-16.012	-26.657	-4.204	-2.298
EWMA/CVaR EWMA FHS	1.097	2.026	3.593	5.183	19.340	29.305	-4.034	-3.127
EWMA/CVaR EWMA EVT	1.095	2.119	3.849	5.606	19.426	29.379	-3.844	-2.866
EWMA/CVaR EWMA Stsk	1.697	2.605	4.136	5.689	29.427	39.157	-4.048	-3.429
EWMA/CVaR GARCH FHS	1.056	1.632	2.601	3.578	15.483	24.424	-3.070	-2.366
EWMA/CVaR GARCH EVT	1.120	1.894	3.198	4.517	16.074	25.207	-3.162	-2.312
EWMA/CVaR GARCH Stsk	1.283	2.224	3.812	5.422	25.367	32.485	-3.284	-2.399
Panel B: Δ_{CRRA}	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	$\gamma = 5$	$\gamma = 10$	$\gamma = 5$	$\gamma = 10$
EWMA/CVaR Hist	0.054	-0.045	-0.194	-0.381	-13.517	-24.043	-4.581	-2.689
EWMA/CVaR EWMA FHS	0.790	1.714	3.277	4.870	17.388	27.079	-4.216	-3.322
EWMA/CVaR EWMA EVT	0.756	1.776	3.502	5.266	17.476	27.155	-4.040	-3.074
EWMA/CVaR EWMA Stsk	1.396	2.301	3.831	5.391	27.527	37.061	-4.173	-3.560
EWMA/CVaR GARCH FHS	0.865	1.438	2.401	3.376	13.723	22.415	-3.212	-2.517
EWMA/CVaR GARCH EVT	0.864	1.635	2.934	4.256	14.279	23.158	-3.332	-2.491
EWMA/CVaR GARCH Stsk	0.972	1.917	3.530	5.192	24.114	31.687	-3.461	-2.582
	$b = 0.8$		$b = 1$		$b = 0.8$		$b = 0.8$	
Panel C: Δ_{LA}	$l = 2$	$l = 3$	$l = 2$	$l = 3$	$l = 2$	$l = 3$	$l = 2$	$l = 3$
EWMA/CVaR Hist	9.677	14.863	11.314	17.176	-10.945	-15.003	14.550	24.395
EWMA/CVaR EWMA FHS	9.891	14.798	12.402	18.670	49.383	71.451	4.782	9.462
EWMA/CVaR EWMA EVT	11.059	16.621	13.999	21.135	49.497	71.493	5.333	10.248
EWMA/CVaR EWMA Stsk	10.397	15.350	13.112	19.398	52.726	72.999	3.289	7.092
EWMA/CVaR GARCH FHS	5.107	7.432	6.407	9.413	42.252	62.125	3.018	6.225
EWMA/CVaR GARCH EVT	7.255	10.771	9.161	13.674	43.604	64.214	4.131	8.045
EWMA/CVaR GARCH Stsk	10.306	15.553	13.425	20.117	46.414	64.530	5.446	10.048

Results for the three indicators are given in Table X, where we only show results for the strategies that switch between the EWMA model and the CVaR targeting strategies. For a better comparison to our previous results, we also show results for the HSD based target volatility strategy, the DAX and the 60/40 portfolio. Panel A shows results for the indicator δ_t based on the TSMOM strategy. Compared to the HSD model, switching between the EWMA based strategy and the target CVaR strategies successfully heightens the return while simultaneously the volatility is reduced. The switching strategies provide an enhanced risk-return profile, indicated by a higher Sharpe Ratio and Calmar Ratio, than the individual strategies given in Table V. For example, the strategy that switches between the EWMA model and the CVaR-EWMA-Stsk strategy increases the Sharpe Ratio of the HSD model by $0.206/0.102 - 1 = 101.96\%$. The high increase of the Sharpe Ratio can also be seen by the Sharpe Ratio test of Jobson and Korkie (1981). All switching strategies provide a statistically higher Sharpe Ratio than the HSD model, whereas only one model in Table V was able to provide a statistically higher Sharpe Ratio. Further, the switching strategies also provide a higher drawdown protection, indicated by the lower MDD and minimum return. Thus, the switching approach seems to capture the benefits of volatility and CVaR targeting and produces higher returns with lower risk. Panel B shows results for the indicator based on the 200 day Moving Average, which are quite similar to the TSMOM based results. As before, all switching strategies produce significantly higher Sharpe Ratios than the HSD model. Panel C shows results for the volatility based indicator δ_t. Interestingly, although some strategies based on this indicator produce the highest Sharpe Ratios in Table X, none of these strategies produces a significantly higher Sharpe Ratio with Jobson and Korkie (1981) test statistics lower than 1.64. However, this can be explained by a lower correlation of these switching strategies with the HSD strategy compared to the other indicators. Furthermore, the switching strategies based on the volatility based indicator δ_t exhibit the lowest drawdowns among the three indicators, which is in line with Equation (1.5.15) that this strategy is similar to the volatility targeting strategy, but more sensitive to up- and down-markets.

Table XI shows the economic value of the switching strategies that use the volatility based indicator δ_t for the mean-variance, CRRA and loss-averse investors. We only show results for the volatility based indicator δ_t, since Table X indicates that none of the switching strategies based on this indicator produces significant performance gains for the test of Jobson and Korkie (1981). Based on the results of Table XI, we will test in Table XII if these utility gains are statistically significant. Thus, our results in Table XII are conservative and results for the TSMOM and MA indicator are superior to the results shown here. The economic value is again calculated with respect to the 60/40 portfolio, i.e. the numbers in this table correspond to the annual percentage fee an investor is willing to pay to switch away from the 60/40 portfolio to one of the switching strategies. Consequently, these numbers can be directly compared to the results of Table VIII. Results of Panels A and B in Table XI are similar to the results of Table

VIII, but higher in magnitude. Thus, mean-variance and CRRA investors are willing to pay higher fees for the switching strategies over the whole sample and the crash period compared to the individual strategies. However, during the calm period, these investors prefer the 60/40 portfolio. Thus, a possible extension of our switching approach would be to switch between the CVaR managed strategy and a non-managed static portfolio. Panel C shows results for the loss-averse investor. Results are again similar to Table VIII, but higher in magnitude for the whole period and the period capturing the financial crisis. Interestingly, during the crisis period, the economic value of the strategy that switches to the Historical Simulation based target CVaR strategy is negative, although this strategy was quite convincing in Table VIII. However, this result is in line with our earlier findings that, especially on days with an extremely negative return, dynamically managed strategies are superior to a statically managed strategies. Since the switching strategy places higher weights on CVaR targeting when a crash is expected, the HS based switching strategy should perform worse during extreme crashes. During the calm period, the economic value of all switching strategies becomes positive and high in magnitude. This holds especially for the strategy that switches to the Historical Simulation based strategy. Thus, loss-averse investors are willing to pay high fees to have access to a strategy that switches between volatility and CVaR targeting, even when the market is trending upwards. In contrast, a loss-averse investor is not willing to pay a positive fee to use the EWMA based target volatility strategy as shown in Table VIII. In particular, Table XI again shows that conditional models produce higher utility gains than unconditional models. This is again in line with our earlier findings that there is a link between forecasting accuracy and utility gains for risk targeting investors.

To summarize Table XI, switching between volatility and CVaR targeting heightens utility gains for all three investors compared to the static 60/40 portfolio. Further, utility gains of the switching strategies are higher in magnitude compared to the economic value of the individual risk targeting strategies as shown in Table VIII. This confirms the earlier finding of Table X that the switching approach further enhances the risk-return profile. We will next test if these utility increases are also statistically significant. Table XII shows results for the tests that test

Table XII. Testing the Utility Gain of Risk Targeting: Switching Strategies

This table shows the results of the tests presented in Section 1.4.1 used to test for the significance of the utility gains. Panel A shows results for a mean-variance investor with $\gamma = 10$. Panel B shows results for a CRRA investor with $\gamma = 10$. Panel C shows results for a loss-averse investor with $b = 0.8$ and $l = 2$. The description of the columns is given in Tables I and II.

Panel A: Mean-Variance	DM-test	p^{RC}	p^{SPA}	p^{SQ}	Step-RCst	Step-SPAst	$FDR^{+} = 10\%$
Vola Hist	-0.23	3.66	0.30	0.80	0	0	0
Vola EWMA	0.28	5.91	0.57	1.64	0	0	0
Vola GARCH	0.77	7.51	1.21	2.81	0	0	0
EWMA/CVaR Hist	-0.14	8.43	**11.47**	**11.39**	0	0	0
EWMA/CVaR EWMA FHS	**2.17**	**74.39**	9.12	**48.84**	**1**	**1**	**5**
EWMA/CVaR EWMA EVT	**2.27**	**82.19**	**45.80**	**84.39**	**1**	**1**	**2**
EWMA/CVaR EWMA Stsk	**2.23**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**
EWMA/CVaR GARCH FHS	**1.82**	**41.73**	0.69	**14.90**	**1**	**1**	**6**
EWMA/CVaR GARCH EVT	**2.16**	**64.85**	**27.44**	**46.11**	**1**	**1**	**3**
EWMA/CVaR GARCH Stsk	**2.25**	**79.42**	**43.03**	**84.39**	**1**	**1**	**4**
Panel B: CRRA	DM-test	p^{RC}	p^{SPA}	p^{SQ}	Step-RCst	Step-SPAst	$FDR^{+} = 10\%$
Vola Hist	-0.26	3.97	0.24	0.76	0	0	0
Vola EWMA	0.26	5.99	0.60	1.45	0	0	0
Vola GARCH	0.75	7.57	1.13	2.57	0	0	0
EWMA/CVaR Hist	-0.11	9.01	**12.07**	**11.47**	0	0	0
EWMA/CVaR EWMA FHS	**2.17**	**74.66**	9.26	**48.31**	**1**	**1**	**5**
EWMA/CVaR EWMA EVT	**2.28**	**82.64**	**47.38**	**87.53**	**1**	**1**	**2**
EWMA/CVaR EWMA Stsk	**2.23**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**
EWMA/CVaR GARCH FHS	**1.82**	**42.04**	0.60	**14.00**	**1**	**1**	**6**
EWMA/CVaR GARCH EVT	**2.16**	**66.10**	**28.55**	**46.16**	**1**	**1**	**3**
EWMA/CVaR GARCH Stsk	**2.23**	**80.82**	**46.72**	**87.53**	**1**	**1**	**4**
Panel C: Loss Aversion	DM-test	p^{RC}	p^{SPA}	p^{SQ}	Step-RCst	Step-SPAst	$FDR^{+} = 10\%$
Vola Hist	-2.63	0.00	0.00	0.00	0	0	0
Vola EWMA	-2.16	0.00	0.00	0.00	0	0	0
Vola GARCH	-1.40	0.00	0.00	0.00	0	0	0
EWMA/CVaR Hist	**5.03**	**43.80**	**36.99**	**69.53**	**1**	**1**	**1**
EWMA/CVaR EWMA FHS	**4.33**	**43.64**	0.00	0.22	**1**	**1**	**2**
EWMA/CVaR EWMA EVT	**4.76**	**100.00**	**100.00**	**100.00**	**1**	**1**	**3**
EWMA/CVaR EWMA Stsk	**4.64**	**64.23**	**32.40**	**69.53**	**1**	**1**	**4**
EWMA/CVaR GARCH FHS	**2.71**	0.03	0.00	0.00	**1**	**1**	**7**
EWMA/CVaR GARCH EVT	**3.70**	**2.36**	0.00	0.02	**1**	**1**	**6**
EWMA/CVaR GARCH Stsk	**5.32**	**66.24**	**41.12**	**69.53**	**1**	**1**	**5**

for statistically significant utility gains of the switching strategies. For a better comparison, we also include the three volatility targeting strategies. Table XII shows that the switching strategies produce statistically significant utility increases, whereas the volatility targeting strategies do not. For all three investors, most switching strategies produce significantly higher utilities compared to the 60/40 portfolio, whereas the volatility targeting strategies do not exhibit statistically significant utility increases. In line with our earlier findings, the most convincing results are again found for the strategies that switch to a conditionally managed CVaR strategy. Consequently, results of Table XII show that switching between volatility and CVaR targeting produces statistically significant higher utilities for all three investors, whereas the test of Jobson and Korkie (1981) does not indicate that the switching strategies based on the volatility indicator δ_t exhibit statistically higher Sharpe Ratios. This again highlights that the Sharpe Ratio is a suboptimal performance measure for dynamic trading strategies (Han, 2005, Marquering and Verbeek, 2004).

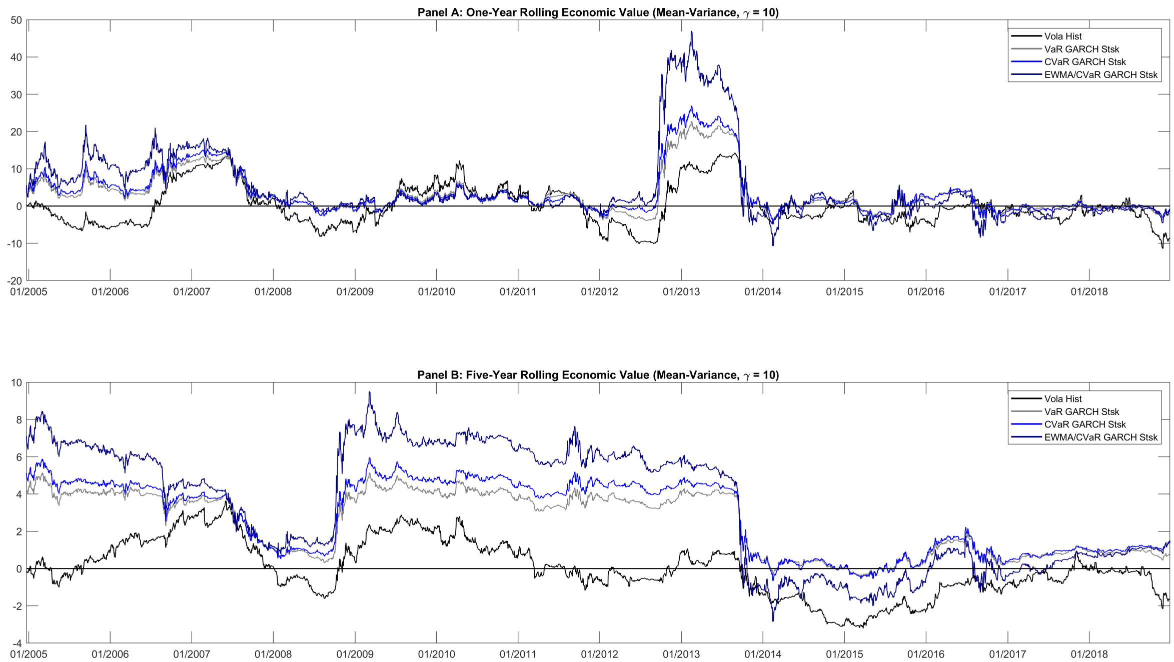
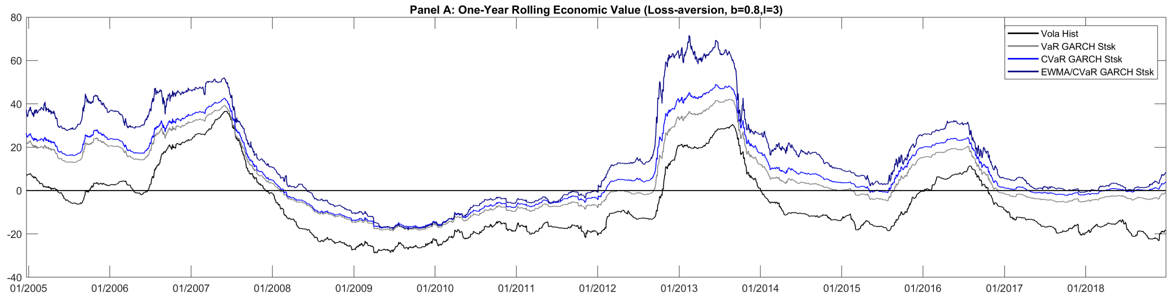


Figure I. One and Five Year Rolling Economic Value for a Mean-Variance Investor. This figure plots the one and five year rolling economic value measured by Δ_{MV} with respect to the 60/40 portfolio for a mean-variance investor with a risk aversion of $\gamma = 10$. Panel A shows the economic value for an investor with an investment horizon of one year, whereas Panel B shows the economic value for an investor with an investment horizon of five years. The dates on the x axes correspond to the end dates of the one or five year investment horizon.

We have so far focused our analysis on the economic value calculated over the whole sample. However, most investors typically have short evaluation periods (Benartzi and Thaler, 1995). Further, timing short-term risk is also beneficial for long-term investors (Moreira and Muir, 2019), i.e. even long-term investors should be concerned about short-term utility gains. For that reason, similar to Figure 2 of Marquering and Verbeek (2004), we next plot in Figure I the rolling one and five year economic value for a mean-variance investor who uses our risk targeting and switching strategies. Thus, this figure plots the rolling annualized fee a mean-variance investor with an investment horizon of one or five years is willing to pay to switch from the 60/40 portfolio to the risk targeting strategies. Panel A shows the rolling annualized percentage fee for an investor with a risk aversion of $\gamma = 10$ and an investment horizon of one year, whereas Panel B shows the rolling economic value for an investor with an investment horizon of five years. Figure I demonstrates that a mean-variance investor with an investment horizon of one year is almost always willing to pay a positive fee to have access to risk targeting. This holds especially for the strategies that target a constant level of downside risk and the switching strategy. In line with our earlier findings, the utility gains in the crises are substantially higher than the utility losses in the low risk periods. Hence, investors are willing to pay very high fees to avoid crash periods, whereas their utility loss of lower returns in uptrending markets is significantly lower. In particular, during crash periods, the economic value of downside risk timing and the switching strategy is significantly higher than the economic value of volatility timing, whereas the economic value of volatility targeting, downside risk timing and the switching strategy is comparable in calm periods. Interestingly, during the crises periods, we find that the switching strategy outperforms all other strategies. Thus, even during crises, switching between volatility and CVaR targeting outperforms downside risk targeting. Results in Panel B are similar to the results in Panel A, but the differences between volatility targeting, downside risk targeting and the switching strategy become even larger. Thus, for investors with longer investment horizons, downside risk targeting becomes far more important than volatility targeting. This again holds especially for the strategy that switches between volatility and CVaR targeting. In particular, we find that, on average, the switching strategy produces the highest economic value, followed by CVaR targeting. In contrast, volatility targeting exhibits the lowest (average) economic value. Results for the CRRA investor are again similar to the results of Figure I and are not shown here.

Figure II shows the rolling one and five year economic value for a loss-averse investor with parameters $b = 0.8$ and $l = 3$. Panel A shows results for an investor with an investment horizon of one year. The economic value of downside risk targeting for a loss-averse investor is always higher than the economic value of volatility targeting. As in Figure I, the economic value significantly increases for the periods that contain a crisis. Switching between volatility and CVaR targeting again outperforms all the remaining strategies, especially during crises. Panel B



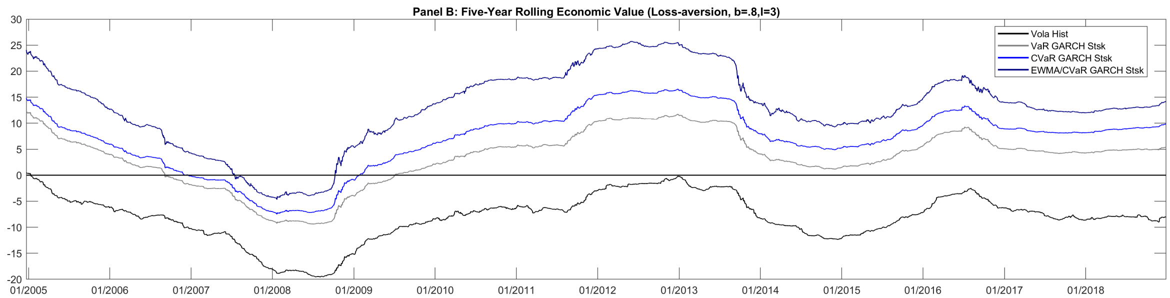


Figure II. One and Five Year Rolling Economic Value for a Loss-Averse Investor. This figure plots the one and five year rolling economic value measured by Δ_{LA} with respect to the 60/40 portfolio for a loss-averse investor with parameters $b = 0.8$ and $l = 3$. Panel A shows the economic value for an investor with an investment horizon of one year, whereas Panel B shows the economic value for an investor with an investment horizon of five years. The dates on the x axes correspond to the end dates of the one or five year investment horizon.

shows results for a loss-averse investor with an investment horizon of five years. Interestingly, the economic value of volatility targeting is always negative, i.e. a loss-averse investor with an investment horizon of five years is never willing to pay a positive fee to switch from the 60/40 portfolio to the HSD managed strategy. This is opposed to the finding of Moreira and Muir (2019) that even long-term investors should time short-term volatility.[84] In contrast, the economic value of downside risk targeting and the switching strategy is only negative for a short period. Thus, a loss-averse investor with an investment horizon of five years should almost always target downside risk or, even more advantageous, this investor should switch between volatility and CVaR targeting.

Finally, we compare the performance of the different risk targeting strategies by running time-series regressions of the returns from one strategy on the returns of the other strategies. Following Moreira and Muir (2017, Table I), we first run regressions of the risk-managed port-

[84]In contrast to our examination in Figure II, Moreira and Muir (2019) do not assess the economic value for loss-averse investors.

folios on the non-managed portfolio. Further, following Daniel and Moskowitz (2016, Table 8), we additionally regress the risk-managed strategies on the other risk-managed strategies. We follow the authors and rescale all strategies to the same level of volatility before running the regressions. Table XIII reports the annualized percentage alphas with the corresponding t-statistics.[85] We calculate the alphas for the switching strategies that use the same indicators as in Table X.

Table XIII. Spanning Tests: Portfolio Alphas
This table shows results of spanning test for the DAX and the risk targeting strategies. We run time-series regressions of each portfolio on the remaining strategies, where we use the DAX, the volatility targeting strategy, the CVaR targeting strategy and the strategy that switches between volatility and CVaR targeting. Panel A uses the TSMOM based indicator δ_t for the switching strategy, Panel B uses the MA based indicator and Panel C uses the volatility based indicator. We report annualized percentage alphas with corresponding t-statistics in parentheses. Bold numbers mark alphas that are significantly positive at the 10% level, whereas red numbers mark alphas that are significantly negative at the 10% level.

	DAX		Volatility		CVaR		Switching	
Panel A: TSMOM Indicator	α	$t-$stat	α	$t-$stat	α	$t-$stat	α	$t-$stat
DAX	-	-	-2.005	(-0.397)	-5.704	(-1.227)	-6.495	(-1.197)
Volatility	5.511	(0.990)	-	-	-3.421	(-1.446)	**-5.331**	(-2.699)
CVaR	**9.399**	(1.782)	**4.285**	(1.772)	-	-	-1.539	(-0.690)
Switching	**12.063**	(1.921)	**6.187**	(2.924)	2.253	(0.991)	-	-
Panel B: MA Indicator	α	$t-$stat	α	$t-$stat	α	$t-$stat	α	$t-$stat
DAX	-	-	-2.005	(-0.397)	-5.704	(-1.227)	-6.139	(-1.134)
Volatility	5.511	(0.990)	-	-	-3.421	(-1.446)	**-4.900**	(-2.407)
CVaR	**9.399**	(1.782)	**4.285**	(1.772)	-	-	-1.090	(-0.479)
Switching	**11.545**	(1.860)	**5.710**	(2.646)	1.796	(0.780)	-	-
Panel C: Volatility Indicator	α	$t-$stat	α	$t-$stat	α	$t-$stat	α	$t-$stat
DAX	-	-	-2.005	(-0.397)	-5.704	(-1.227)	-4.298	(-0.586)
Volatility	5.511	(0.990)	-	-	-3.421	(-1.446)	-3.629	(-0.713)
CVaR	**9.399**	(1.782)	**4.285**	(1.772)	-	-	-0.750	(-0.188)
Switching	13.941	(1.578)	8.033	(1.383)	3.464	(0.746)	-	-

Results in Table XIII show that the DAX always has a negative alpha when it is controlled for the performance of the risk targeting strategies. In contrast, all risk targeting strategies produce positive and economically large alphas when it is controlled for the performance of the DAX. In this case, the volatility targeting strategy produces an economically high alpha of 5.511%, which is in line with Moreira and Muir (2017, Table I) who find an alpha of 4.86% for the US market.

[85] Cederburg et al. (2020) state that the approach of Moreira and Muir (2017) has several disadvantages. Moreover, Boguth et al. (2011) and Cederburg and O'Doherty (2016) state that unconditional alphas are not appropriate to assess the performance of strategies that time volatility. Furthermore, Schneider et al. (2020) show that the unconditional alpha does not incorporate skewness preferences of investors. However, these regression results can be seen as a further robustness check of our economic value based findings. The economic value approach corrects for several disadvantage of the unconditional alpha and is a more powerful tool to compare the performance of dynamic trading strategies.

However, the alpha is not statistically significant with a t-statistic of 0.99.[86] In contrast, the performance of CVaR targeting and the switching strategies cannot be explained by the DAX with an alpha of 9.399% for the CVaR targeting strategy and alphas between 11.545% and 13.941% for the switching strategies. These alphas are statistically significant at the 10% level for the TSMOM and MA based switching strategies with t-statistics of 1.921 and 1.860. Interestingly, among the switching strategies, the switching strategy using the volatility based indicator δ_t produces the highest alpha but the lowest t-statistic. This can be explained by the lower correlation of this strategy with the DAX. When we control for the volatility targeting strategy, the CVaR targeting and switching strategies produce economically high and mostly significant alphas. For example, with an alpha of 4.285% and a t-statistic of 1.772, CVaR targeting cannot be explained by volatility targeting. In contrast, when we control for the performance of the CVaR targeting strategy, the alpha of the volatility targeting strategy becomes negative. Furthermore, when we control for the switching approach, the volatility targeting strategy's alpha is even significantly negative in most cases and economically high in magnitude with values between -3.629% and -5.331%. Thus, not switching away from volatility targeting in periods when a negative return is expected produces a significantly inferior performance. Similarly, the CVaR targeting strategy's alpha becomes negative, once we control for the switching approach.

In total, when we control for the switching strategy, all other strategies produce negative alphas. In contrast, the switching strategy's performance cannot be explained by volatility targeting, i.e. the switching approach produces economically high and statistically significant alphas in this case. The alphas of the switching strategies with respect to the CVaR targeting strategy are also positive, but not statistically significant. Thus, switching away from CVaR targeting in periods when the market is expected to be in an uptrend further improves the strategy's performance, but a huge part of the good performance of the switching strategy can be explained by CVaR targeting. Altogether, results in Table XIII confirm our earlier findings that risk targeting outperforms the non-managed portfolio. Furthermore, strategies that account for the DAX's tail risk outperform the volatility targeting strategy. However, the strategy that switches between volatility and CVaR targeting performs the best and produces economically high alphas with

[86]We show in Appendix D.6 that the alphas are highly significant when a longer data set is used.

respect to the remaining strategies.

1.6 Conclusion

This paper studies dynamic trading strategies that target a predefined level of risk measured by volatility, Value at Risk (VaR) or Conditional Value at Risk (CVaR). We derive weights for these trading strategies and present several methods to estimate volatility, VaR and CVaR. Based on a data set for the German stock market, we find that risk targeting offers an enhanced risk-return profile, a better drawdown protection and significant utility gains compared to a buy-and-hold equity investment and a static portfolio consisting of equities and bonds. Most convincing results are found for the strategies that target a constant level of portfolio CVaR over time. In particular, we find that mean-variance investors, CRRA investors and loss-averse investors should time downside risk, measured by CVaR, instead of volatility. This result especially holds for highly risk-averse or loss-averse investors and during crises. In particular, a loss-averse investor is not willing to pay a positive fee for volatility targeting, but the same investor would pay extremely high fees to have access to CVaR targeting. Generally, we find that risk should be managed by conditional risk models instead of unconditional models, as done by Barroso and Santa-Clara (2015), Barroso and Maio (2018) and Moreira and Muir (2017). This is in line with the result of Bollerslev et al. (2018) that a higher forecasting accuracy, and hence a more constant portfolio risk of risk targeting, typically coincides with higher performance benefits compared to static and less accurate forecasting models.

The risk-return profile and utility gains of risk targeting can further be improved by switching between volatility and CVaR targeting, where CVaR targeting is only used when a negative market return is expected. Based on three different crash indicators, we show that these switching strategies produce higher returns with lower risk compared to the volatility targeting strategies. Further, the mean-variance, CRRA and loss-averse investors are willing to pay high fees to have access to these switching strategies. When the volatility targeting and switching strategies are simultaneously compared to the utility of a static portfolio allocation, utility gains of the switching strategies are statistically significant, whereas the utility gains of volatility targeting are insignificant. In particular, the returns of the switching strategy cannot be explained by volatility targeting, whereas volatility targeting becomes unprofitable, once we control for CVaR targeting or the switching strategies. Furthermore, the superiority of both strategies, the CVaR managed and switching strategies, holds in the long run and during the recent corona crisis.

Appendix to Chapter 1

A Advantages of Volatility Targeting

This section summarizes several reasons why investors should target a constant level of volatility. See also Dreyer and Hubrich (2019) and references therein for further advantages of volatility targeting. Most of the advantages that will be summarized here also hold for the target VaR and target CVaR strategies presented in Section 1.3.

First, using the weight of Equation (1.2.3) implies that the weight of the risky asset is decreased in times of high volatility and increased in low volatile times. Since volatility is often associated with risk and investors are typically risk-averse (Scott and Horvath, 1980), targeting a constant level of volatility fits well to these investors' preferences. By choosing an adequate volatility target σ_{target}, investors can choose an investment strategy that fits well to their preferences and risk aversion (Bollerslev et al., 2018). Similarly, Zakamulin (2015) and Moreira and Muir (2017) show that mean-variance investors should, under some assumptions, optimally choose the weight of the risky asset as $w_t = (\sigma_{\text{target}}/\sigma_t)^2$ (see also Dopfel and Ramkumar (2013)).[87] Based on this result, Kirby and Ostdiek (2012) suggest that investors should additionally decrease the sensitivity of w_t to volatility changes, which leads to the weight given in Equation (1.2.3). Decreasing the sensitivity of w_t to volatility changes has the advantage that transaction costs are lowered and this strategy is more profitable in practice.[88]

[87] The mean-variance framework is only suitable for elliptical distributions. Since asset returns usually do not follow an elliptical distribution, this weighting is not optimal for realistic return distributions (Szegö, 2002, p.1254). We will revisit this issue in Section 1.3, where we present a similar weighting scheme based on risk measures that account for non-normalities in the asset return distribution.

[88] Since volatility can only be estimated with an estimation error, Kirby and Ostdiek (2012) suggest to scale the weight $w_t = (\sigma_{\text{target}}/\sigma_t)^2$ by a parameter η, called tuning parameter, that determines how aggressively the weight w_t reacts to changes in σ_t. This approach lowers the portfolio's turnover and, as a consequence, reduces transaction costs. By choosing $\eta = 0.5$, we obtain the weight of the target volatility strategy (see also Zakamulin (2015, p. 91)). Moreira and Muir (2017) compare the weight of the volatility and variance managed strategies. The authors find similar results, but less extreme weights and lower transaction costs for the volatility managed

Second, especially during bear markets, which are associated with increases in volatility and correlations, investors seek for risk reduction methods (Ang and Bekaert, 2002).[89] The last financial crises were all accompanied by higher than normal volatilities (Liu et al., 2003, Moreira and Muir, 2017). In particular, times of an extremely high volatility typically coincide with times of a downward moving market (Campbell and Hentschel, 1992, French et al., 1987). Similarly, Moreira and Muir (2017) find that the probability of a recession is higher in times of high a market volatility. This is confirmed by Muir (2017) who shows that asset prices decline and stock market volatility increases in financial crises and recessions, but these effects reverse subsequently. Thus, times of a significantly higher volatility coincide with declining asset prices and these times should be avoided by investors. In contrast, the times following a crisis, which are marked by a declining volatility, offer an appealing risk-return profile for investors (Dopfel and Ramkumar, 2013, Moreira and Muir, 2019). Further, since volatilities and correlations between different equity markets increase simultaneously during bear markets, drawdowns in crises cannot simply be managed by diversification (Ang and Bekaert, 2002, Ang and Chen, 2002, Butler and Joaquin, 2002, Karolyi and Stulz, 1996, Longin and Solnik, 2001, Patton, 2004). For example, Chabi-Yo et al. (2018) find that extremely negative returns of stocks are more related than extremely positive returns, i.e. stocks tend to crash simultaneously. In particular, the authors show that the correlation of extremely negative returns among different stocks increases in crash periods (Chabi-Yo et al., 2018, Figure 2). Similarly, Bollerslev et al. (2018) find co-movements, spillover effects and simultaneous spikes of volatilities between equities, bonds, commodities and currencies. Thus, risk characteristics between assets and asset classes are quite similar, especially in crash periods. Furthermore, Jondeau and Rockinger (2003) find that also higher moments, like (negative) skewness and kurtosis, increase simultaneously be-

strategy. Zakamulin (2015, Exhibit 2) also find better results for the volatility managed strategy compared to the variance managed strategy.

[89] Liu et al. (2003) find that most events with extremely negative returns are accompanied with high increases in volatility. Guidolin and Timmermann (2008) find a bear regime with low returns, negative alphas, high volatilities and highly correlated assets and a bull regime with higher returns, positive alphas, lower volatilities and less correlated returns (see also Wang et al. (2012, p. 27) and Hocquard et al. (2013)). Similarly, Ang and Bekaert (2002, p. 1139) find “a normal regime with low correlations, low volatilities, and a bear regime with higher correlations, higher volatilities, and lower conditional means.” The bear states occurred during financial crises and/or global recessions that were accompanied with high market volatilities, indicating that periods of market distress are associated with high volatilities and low returns (Muir, 2017).

tween markets during bear regimes. This indicates that the probability of an occurrence of large (negative) returns cannot be reduced by simply combining several risky assets. As stated above, these crash periods typically coincide with periods of an increased volatility. As a consequence, during high risk periods, the portfolio should be managed by simultaneously decreasing the exposure to a portfolio of risky assets and increasing the exposure to the riskless asset, as done by the target volatility strategy.[90] This also solves the problem identified by Ang and Chen (2002) and Longin and Solnik (2001) that investors incorrectly assess the benefits of diversification, and thus typically hold too much equities in bear markets, whereas they are underinvested in bull markets. However, by managing volatility, an investor is not protected against unpredictable tail events, marked by periods with extreme jumps in asset prices.[91]

Third, an often proclaimed justification of volatility targeting is the relation between volatility and future return. Although classical finance models, like the CAPM, indicate that higher risk should be compensated by higher expected returns (see Merton (1980) for example), many empirical studies find a negative relation between volatility and returns, i.e. a higher volatility coincides with lower or negative future returns (Glosten et al., 1993).[92] A possible explanation for the negative volatility-return relation is the *volatility feedback* effect, which is sometimes also called time-varying risk premium effect and is opposed to the well-known leverage effect

[90] This result is also found by Ang and Bekaert (2002) who examine the optimal portfolio allocation when assets' correlations increase in crash regimes. By incorporating a risk-free asset, they find that, in the normal regime, the risky asset should be leveraged by being short in the risk-free asset, whereas money should be shifted to the risk-free asset in the bear regime. Furthermore, the authors find significant drawbacks of ignoring information about the regime, once the possibility of shifting money to the risk-free asset is introduced (see also Patton (2004)).

[91] See Liu et al. (2003) for a study on how jump risk, in both equity prices and volatility, affects the dynamic asset allocation between a risky and a riskless asset. In order to face jump risk, investors should avoid leveraged positions in the risky asset. Hence, an equity cap of 100% or a low volatility target should be used (see also Das and Uppal (2004)). Alternatively, investors should manage extreme losses, measured by downside risk, instead of return deviations. Jarrow and Zhao (2006) show that managing volatility differs from managing downside risk when asset returns exhibit jump risk.

[92] A similar observation has also been found in cross-sectional analyses. See, for example, Frazzini and Pedersen (2014) who show that buying low beta assets and selling high beta assets produces high returns. Similarly, Ang et al. (2006b) and Ang et al. (2009) show that assets with a high past sensitivity to volatility changes, high idiosyncratic volatility or high total volatility have significantly lower returns than assets with a low past sensitivity to volatility changes, low idiosyncratic volatility or low total volatility, respectively. Moreira and Muir (2017, Sec. II.D) show that this effect is different to the time-series volatility effect examined here. Haugen and Heins (1975) find that the risk-return relation strongly depends on the sample period and whether the sample period is dominated by a bull or bear regime. Using a long data set, the authors find that "over the long run, stock portfolios with lesser variance in monthly returns have experienced greater average returns than their "riskier" counterparts" (Haugen and Heins, 1975, p. 782).

(see Glosten et al. (1993, p. 1786) for an explanation of the leverage effect).[93] Based on this observation, an increase in volatility induces an immediate stock price decline. In other words, if tomorrow's volatility σ_{t+1} is expected to be higher than today's volatility σ_t, then tomorrow's weight w_{t+1} should be lower than today's weight w_t.[94] However, results in the academic literature on the relation between volatility and future returns are very mixed and a relation between volatility and returns is hard to confirm. Lundblad (2007) shows that very long data sets are needed when the relation of volatility and future returns is examined. The author, using a data set ranging from 1836 to 2003, finds a positive relation between volatility and return. Bali and Peng (2006) using high-frequency-data based volatility measures find a significant and positive relation,[95] whereas Bollerslev et al. (2006), using similar volatility measures, find an insignificant or even negative relation. Bollerslev and Zhou (2006, p. 124-125) state that the risk-return relation in empirical investigations strongly depends on the volatility measure used in this investigation, which partly explains the inconsistent results in the academic literature (see also Glosten et al. (1993), Ghysels et al. (2005) and Bollerslev et al. (2013)). Furthermore, Adrian and Rosenberg (2008) show that the risk-return relation strongly depends on the examined time frequency of volatility. In line with the volatility feedback effect, the authors find a negative volatility-return relation for short-term volatility, but a positive relation for long-term volatility.

To summarize results in the academic literature, the relation between volatility and future

[93] Bollerslev et al. (2006, p. 354) describe the volatility feedback effect as: "If volatility is priced, an anticipated increase in volatility would raise the required rate of return, in turn necessitating an immediate stock-price decline to allow for higher returns. Therefore, the causality underlying the volatility feedback effect runs from volatility to prices, as opposed to the leverage effect that hinges on the reverse causal relationship" (see also Campbell and Hentschel (1992), Bekaert and Wu (2000) and Glosten et al. (1993) for an explanation of the leverage and volatility feedback effect). For additional studies on the relation between volatility and return, see also French et al. (1987), Bali and Peng (2006), Bollerslev et al. (2006), Bollerslev and Zhou (2006), Ghysels et al. (2005), Lundblad (2007), Bollerslev et al. (1992) among others. See Muir (2017) for an examination why risk premiums or expected returns vary over time, rise modestly in recessions and spike in financial crises. See Glosten et al. (1993) and Bekaert and Wu (2000) on how the leverage effect influences results on the volatility feedback effect. Bekaert and Wu (2000, p. 7) find that "the leverage effects reinforces the volatility feedback effect" and both effects interact. Both effects and their interaction are also nicely visualized in Bekaert and Wu (2000, Figure 1).

[94] The volatility feedback effect is reflected by the construction of the target volatility weighting given in Equation (1.2.3). More formally, from $\sigma_{t+1} > \sigma_t$ it follows $w_{t+1} = \sigma_{\text{target}}/\sigma_{t+1} < \sigma_{\text{target}}/\sigma_t = w_t$, i.e. an increase in volatility induces a decrease in the weight of the risky asset. Due to this relation, Harvey et al. (2018) find that, under the leverage effect, volatility targeting induces momentum, i.e negative returns induce higher future volatilities and lower future weights of the risky asset. The authors find that this observations explains a part of the increase of the Sharpe Ratio of the volatility targeting strategy.

[95] Similarly, Ghysels et al. (2005) using daily data to measure monthly volatility by advanced volatility measures find a positive and significant relation.

returns is hard to identify and results in the literature are too mixed to draw a distinct conclusion (see also Harvey and Siddique (1999) and references therein). Backus and Gregory (1993) theoretically confirm this observation. Similarly, Glosten et al. (1993) also argue that a positive and a negative relation would be consistent with theory. However, in order to achieve an enhanced risk-return profile by volatility targeting, a negative relation between volatility and future return is not needed. Moreira and Muir (2017) show that the relation between volatility and future *risk-adjusted* returns should be of main interest for volatility timing strategies instead of the risk-return relation (see also Dopfel and Ramkumar (2013)). Moreira and Muir (2017) theoretically show that an alpha of zero is obtained if movements of expected returns and volatility coincide. In particular, the authors find that the positive alpha of the volatility managed strategy is mainly driven by the negative relation between volatility and volatility-adjusted returns. In other words, volatility targeting produces positive alphas, since an increase in volatility is not compensated by an adequate increase in expected return. This is also empirically confirmed by the authors: although the authors cannot confirm a negative volatility-return relation, they find that volatility timing increases the Sharpe Ratio. The reason for the increasing Sharpe Ratio is that "changes in volatility are not offset by proportional changes in expected returns" (Moreira and Muir, 2017, p. 1611). In total, a high volatility in $t-1$ is related to a low Sharpe Ratio in t. Hence, high volatility periods exhibit an unattractive risk-return profile and should be avoided by investors. Similarly, Dachraoui (2018, Eq. (2)) shows that the Sharpe Ratio of the target volatility strategy is given by the Sharpe Ratio of the risky asset and the correlation between the volatility and the risk-adjusted return of the risky asset. In particular, if volatility and risk-adjusted returns of the risky asset are negatively correlated, the Sharpe Ratio of the target volatility strategy is higher than the Sharpe Ratio of the risky asset. A sufficient condition for this negative correlation is that volatility and return are negatively correlated or uncorrelated. This is confirmed by Barroso and Maio (2018) who find that volatility targeting works well since risk and future returns are nearly uncorrelated and risk is highly forecastable due to its persistent nature. Similarly, Harvey et al. (2018) find no clear pattern between the volatility of day $t-1$ and the return of day t. However, due to the persistence of volatility, they find that a high volatility in $t-1$ is related to a high volatility in t, which translates into a low Sharpe Ratio in t. Hence, an investor should be higher invested in the risky asset if the risky asset's volatility is low and vice versa, i.e. the investor should time volatility. Interestingly, even when volatility and return are weakly positively correlated, volatility targeting can still be advantageous. For example, Moreira and Muir (2019) find that an increase of volatility coincides with higher expected returns, but that the increase in expected return is much more persistent than the increase in volatility. Thus, investors should reduce the weight of the risky asset if short-term volatility increases, but the investors should then subsequently increase the exposure to the risky asset when volatility begins to decline.

The finding that investors should time (short-term) volatility does not fundamentally contradict the assumption that higher risk is compensated by higher expected returns. In the long run, assets with higher volatility typically earn higher risk premiums, but risk premiums typically fluctuate over time (Lempérière et al., 2017, Muir, 2017). Thus, long-term investors benefit from investing in riskier assets like equities, whereas increases of short-term volatility of the same asset are related to low or negative returns (Adrian and Rosenberg, 2008). Hence, in the long run, stocks typically have higher long-term returns than bonds and investors with a long investment horizon should participate in the stock market. Similarly, Blitz et al. (2019) state that "[t]he relation between risk and return only seems to be positive across entire asset classes, since stocks have higher returns than bonds, and corporate bond returns are higher than government bond returns, in the long run". However, most investors fail to capture the long-term potential of stocks, due to too short evaluation periods and the higher volatility of stocks (Benartzi and Thaler, 1995). Benartzi and Thaler (1995) show that the evaluation period of long-term investors is typically much shorter than their investment horizon. In other words, investors with an investment horizon of years act like investors with a horizon of several months. This hinders these investor to fully participate from the long-term performance potential of stocks, since investors with a short evaluation period are more sensitive to changes in market volatility (Moreira and Muir, 2019). For these long-term investors, timing short-term volatility can also be beneficial to capture the long-term potential of stocks and simultaneously manage short-term risk. Ang and Bekaert (2002) show that even for investors with longer horizons it is possible to act myopically, as done by risk targeting, instead of solving complex long-term portfolio problems. Consequently, risk targeting is an easy method to make the long-term potential of equity investments available for investors with short evaluation periods. In particular, even highly risk-averse investors can benefit from the huge long-term return potential of risky assets, where investors can choose the risk level they are willing to accept. Further, besides making stock market investments available for all investors, volatility targeting can even enhance the risk-adjusted performance for long-term investors by dynamically timing the risky asset's short-term risk (Moreira and Muir, 2019).

Fourth, many studies have shown that volatility timing can add substantial economic value in terms of higher risk-adjusted returns and high utility gains. Fleming et al. (2001), Fleming et al. (2003), Han (2005), Kirby and Ostdiek (2012) and Taylor (2014) examine the economic value of volatility timing strategies, i.e. strategies that rely on estimates of the covariance matrix solely, in a multivariate setting and the authors find that these strategies are superior to non-managed portfolios, even after transaction costs.[96] Although these studies use multivariate data sets, these studies demonstrate that volatility timing is typically related to higher risk-adjusted returns and high utility gains for mean-variance investors. Similarly, in a univariate setting, Marquering and Verbeek (2004) find substantial increases in the Sharpe Ratio and utility if volatility timing is added to return timing. Moreira and Muir (2019) confirm this finding for investors with a long investment-horizon. Moreira and Muir (2017) and Bollerslev et al. (2018), using a similar framework as in our paper, also demonstrate the vast utility gains and Sharpe Ratio increases of volatility targeting. Furthermore, Busse (1999) examines volatility timing used by mutual funds and finds higher Sharpe Ratios for funds using volatility timing. Additionally, Daniel and Moskowitz (2016), Cederburg et al. (2020), Barroso and Santa-Clara (2015), Moreira and Muir (2017) and Barroso and Maio (2018) demonstrate the vast potential of volatility timing overlayed on several portfolio strategies, especially in terms of drawdown reduction and an

[96]These studies examine the economic value of volatility timing for long-only portfolios. Volatility timing can also be applied to long-short strategies. For example, Moskowitz et al. (2012) and Kim et al. (2016) use volatility timing to manage the risk of the time series momentum (TSMOM) strategy. Similarly, Asness et al. (2013) and Goyal and Jegadeesh (2017) use volatility timing to weight the assets of the cross-sectional momentum strategy.

improvement of risk-adjusted returns. This especially holds for portfolios with a high left tail risk, such as momentum or betting against beta. Generally, volatility targeting works best for assets that deviate strongly from normality (Perchet et al., 2016, Exhibit 11). This is shown by Perchet et al. (2016) in a Monte Carlo Study and the authors find that the gains of volatility targeting are higher for assets that strongly deviate from normally distributed returns, exhibit high levels of volatility clustering or have fatter tails. Similarly, Harvey et al. (2018) find that volatility targeting works well for risky assets, like equities or portfolios that contain equities, but not for assets with lower risk, like bonds. Applying volatility targeting to equities usually produces "benchmark-comparable levels of return with lower risk" (Benson et al., 2014, p. 89).

Fifth, the target volatility strategy focuses on the risk, measured by volatility of the risky asset, and ignores the information about future returns (Bollerslev et al., 2018). Furthermore, a forecast of the whole return distribution is not needed, which is a tenuous task (Aït-Sahalia and Brandt, 2001). This is appealing, since future volatility can be estimated much more precisely than the whole distribution or future returns, which minimizes the estimation risk of this approach (Merton, 1980). Kirby and Ostdiek (2012) show that portfolio allocations that rely on estimates of returns and volatilities exhibit very high estimation risk, whereas estimation risk is only small for volatility based allocations.[97] For that reason, portfolio strategies that are based on a forecast of the asset's return typically perform bad in practice. For example, Moreira and Muir (2017) find higher utility gains for volatility timing strategies than for expected return timing strategies. Marquering and Verbeek (2004) examine both, return and volatility timing, and find that timing returns and volatility is superior to strategies that only time returns. Similarly, Moreira and Muir (2019) find that return timing strategies are highly influenced by estimation risk, whereas volatility timing strategies are quite robust against estimation risk. As a consequence, volatility timing outperforms return timing in practice.

Sixth, investors are typically crash-averse and dislike periods of highly negative returns (Bollerslev and Todorov, 2011, Chabi-Yo et al., 2018, Van Oordt and Zhou, 2016). Volatility timing has proven to be a good and easy drawdown protection method, which makes this

[97] Due to the high estimation risk, especially for mean returns, portfolio allocations under estimation risk are frequently examined in the financial literature (DeMiguel et al., 2009b, Garlappi et al., 2006, Kan and Zhou, 2007, Tu and Zhou, 2011).

approach appealing for investors who typically dislike huge drawdowns (Barroso and Maio, 2018, Barroso and Santa-Clara, 2015, Benson et al., 2014, Moreira and Muir, 2017, Perchet et al., 2016). Generally, volatility targeting is a simple but effective tail risk hedging instrument and significantly reduces left tail risk and fits well to investors with higher order preferences (Dreyer and Hubrich, 2019). Similarly, Harvey et al. (2018) find that volatility targeting successfully reduces the likelihood of extremely negative returns. Thus, volatility targeting also fits well to the loss-aversion of most investors (Aït-Sahalia and Brandt, 2001, Benartzi and Thaler, 1995). In particular, risk targeting is an easy way to manage portfolio risk dynamically. Cuoco et al. (2008) highlight the importance of managing portfolio risk dynamically, i.e. reevaluating portfolio weights frequently.

Seventh, risk-averse investors typically want to hedge against changes in volatility (see Ang et al. (2006b), Adrian and Rosenberg (2008) and references therein).[98] Adrian and Rosenberg (2008) find that investors are willing to pay for methods that protect them from changes in volatility. Moreover, in a cross-sectional setting, Baltussen et al. (2018) find that assets with a high volatility of volatility (vol-of-vol) underperform assets with a more constant volatility. Thus, assets with a more constant volatility produce higher returns than assets that exhibit higher volatility changes. Furthermore, assets with higher volatility changes also exhibit higher downside risk. Similarly, a highly fluctuating portfolio volatility is a driver of the portfolio's tail risk and a stabilization of the portfolio volatility is rewarded by a lower crash risk (Dreyer and Hubrich, 2019). Thus, investors strongly benefit from a stabilization of portfolio volatility. The investors' demand to hedge against these changes in volatility has led to the introduction of many new financial instruments, like variance swaps (Bollerslev and Todorov, 2011, Footnote 11). Volatility targeting is an easy way to hedge against this volatility risk without using any financial derivatives.

Eighth, liabilities of institutional investors, like insurance companies or pension funds, are often less volatile than investments in risky assets. Targeting a constant level of volatility can help to match the volatility of the investments with the volatility of the liabilities. For example,

[98] Generally, the variance risk premium, which measures the "compensation for the risk associated with temporal changes in the variation of the price level" is examined by Bollerslev and Todorov (2011) and Bollerslev et al. (2015).

Banerjee et al. (2016, p. 1) write that "a risk control strategy may provide a smoother path of asset returns and could more closely align the performance of the institution's assets to the characteristics of its liabilities."

Ninth, a skill of a portfolio manager can typically be separated into his ability to time the market and the ability to pick the right stocks (see Agarwal and Naik (2004) and references therein). Risk targeting can be used separately as market timing tool, which is independent of the asset selection process (Zakamulin, 2015). In practice, market timing and volatility timing are fundamentally related as documented by Christoffersen and Diebold (2006) and Copeland and Copeland (1999). The authors state that market timing strategies based on measures of volatility are frequently used by practitioners. Hence, portfolio managers can focus on picking the right assets without accounting for the current market environment, which is separately managed by a risk targeting strategy. Similarly, Cederburg et al. (2020), Daniel and Moskowitz (2016), Barroso and Santa-Clara (2015), Barroso and Maio (2018), Rickenberg (2020a) and Moreira and Muir (2017) use volatility targeting for several portfolio strategies, where the asset allocation is determined in the first step and these portfolios are then managed by volatility targeting. Moreira and Muir (2017, Sec. I.E), Rickenberg (2020a) and Zakamulin (2015) also examine volatility targeting applied to volatility weighted portfolios, where the portfolio allocation is chosen first based on the assets' volatility.

B Portfolio Risk

B.1 Portfolio Value at Risk

In this section, we derive the Value at Risk (VaR) for the portfolio loss given in Equation (1.3.2). We denote the conditional cumulative distribution function (cdf) of the risky asset's loss L_t, based on the information $\mathcal{F}_{t-1}$ available at time $t-1$, by $F_{L_t|\mathcal{F}_{t-1}}$. Moreover, we assume that $F_{L_t|\mathcal{F}_{t-1}}$ is continuous and strictly increasing and we denote the corresponding $(1-\alpha)$-quantile by $F^{-1}_{L_t|\mathcal{F}_{t-1}}(1-\alpha)$. For a positive weight w_t, the day t VaR of the portfolio loss, denoted by

$\mathrm{VaR}_\alpha^{P,t}$, is given by

$$\begin{aligned}
& \mathrm{P}\big(L_t^P \leqslant \mathrm{VaR}_\alpha^{P,t} \mid \mathcal{F}_{t-1}\big) = 1-\alpha \\
\Leftrightarrow\quad & \mathrm{P}\Big(w_t \cdot L_t - (1-w_t)\cdot R_t^f \leqslant \mathrm{VaR}_\alpha^{P,t} \mid \mathcal{F}_{t-1}\Big) = 1-\alpha \\
\Leftrightarrow\quad & \mathrm{P}\Big(L_t \leqslant (\mathrm{VaR}_\alpha^{P,t} + (1-w_t)\cdot R_t^f)/w_t \mid \mathcal{F}_{t-1}\Big) = 1-\alpha \\
\Leftrightarrow\quad & F_{L_t|\mathcal{F}_{t-1}}\Big((\mathrm{VaR}_\alpha^{P,t} + (1-w_t)\cdot R_t^f)/w_t\Big) = 1-\alpha \\
\Leftrightarrow\quad & (\mathrm{VaR}_\alpha^{P,t} + (1-w_t)\cdot R_t^f)/w_t = F^{-1}_{L_t|\mathcal{F}_{t-1}}(1-\alpha) \\
\Leftrightarrow\quad & \mathrm{VaR}_\alpha^{P,t} = w_t \cdot F^{-1}_{L_t|\mathcal{F}_{t-1}}(1-\alpha) - (1-w_t)\cdot R_t^f.
\end{aligned} \tag{B.1}$$

Since the VaR of the risky asset, denoted by VaR_α^t, is given by the $(1-\alpha)$-quantile of the risky asset's (conditional) loss distribution, i.e. $\mathrm{VaR}_\alpha^t = F^{-1}_{L_t|\mathcal{F}_{t-1}}(1-\alpha)$, the VaR of the portfolio is given by

$$\mathrm{VaR}_\alpha^{P,t} = w_t \cdot \mathrm{VaR}_\alpha^t - (1-w_t)\cdot R_t^f. \tag{B.2}$$

B.2 Portfolio Conditional Value at Risk

In this section, we derive the Conditional Value at Risk (CVaR) for the portfolio loss given in Equation (1.3.2). The CVaR of the portfolio loss L_t^P, denoted by $\mathrm{CVaR}_\alpha^{P,t}$, is given by

$$\mathrm{CVaR}_\alpha^{P,t} = \mathbb{E}\big(L_t^P \mid L_t^P \geqslant \mathrm{VaR}_\alpha^{P,t}, \mathcal{F}_{t-1}\big) \tag{B.3}$$

$$= \mathbb{E}\Big(w_t \cdot L_t - (1-w_t)\cdot R_t^f \mid w_t \cdot L_t - (1-w_t)\cdot R_t^f \geqslant \mathrm{VaR}_\alpha^{P,t}, \mathcal{F}_{t-1}\Big) \tag{B.4}$$

From Equation (B.2), i.e. $\mathrm{VaR}_\alpha^{P,t} = w_t \cdot \mathrm{VaR}_\alpha^t - (1-w_t)\cdot R_t^f$, and since the weight w_t and the riskless return R_t^f are $\mathcal{F}_{t-1}$-measurable, it follows

$$\begin{aligned}
\mathrm{CVaR}_\alpha^{P,t} &= \mathbb{E}\Big(w_t \cdot L_t - (1-w_t)\cdot R_t^f \mid L_t \geqslant \mathrm{VaR}_\alpha^t, \mathcal{F}_{t-1}\Big) \\
&= w_t \cdot \mathbb{E}\big(L_t \mid L_t \geqslant \mathrm{VaR}_\alpha^t, \mathcal{F}_{t-1}\big) - (1-w_t)\cdot R_t^f \\
&= w_t \cdot \mathrm{CVaR}_\alpha^t - (1-w_t)\cdot R_t^f,
\end{aligned} \tag{B.5}$$

where $\mathrm{CVaR}_\alpha^t := \mathbb{E}\big(L_t \mid L_t \geqslant \mathrm{VaR}_\alpha^t, \mathcal{F}_{t-1}\big)$ denotes the CVaR of the risky asset.

C Backtesting Target Risk Strategies

C.1 Backtesting Target VaR Strategies

By definition, the variable H_t^P is equal to one if $L_t^P - \mathrm{VaR}_\alpha^{\text{target}} > 0$ and zero else. From Equation (1.3.2) it follows that the portfolio loss is given by

$$L_t^P = w_t \cdot L_t - (1 - w_t) \cdot R_f^f. \tag{C.1}$$

Moreover, given the weight w_t, the portfolio VaR equals the predefined VaR level $\mathrm{VaR}_\alpha^{\text{target}}$, and hence from Equation (B.2) we obtain

$$\mathrm{VaR}_\alpha^{\text{target}} = \mathrm{VaR}_\alpha^{P,t} = w_t \cdot \mathrm{VaR}_\alpha^t - (1 - w_t) \cdot R_t^f. \tag{C.2}$$

Consequently, we have

$$L_t^P - \mathrm{VaR}_\alpha^{\text{target}} = w_t \cdot L_t - w_t \cdot \mathrm{VaR}_\alpha^t = w_t \cdot (L_t - \mathrm{VaR}_\alpha^t). \tag{C.3}$$

Since the weight w_t is strictly positive, it follows

$$L_t^P - \mathrm{VaR}_\alpha^{\text{target}} > 0 \Leftrightarrow L_t - \mathrm{VaR}_\alpha^t > 0. \tag{C.4}$$

Therefore, the variable H_t^P is equal to H_t.

C.2 Backtesting Target CVaR Strategies

Given the weight w_t, the target CVaR equals the portfolio CVaR, and hence from Equation (B.5) it follows

$$\mathrm{CVaR}_\alpha^{\text{target}} = \mathrm{CVaR}_\alpha^{t,P} = w_t \cdot \mathrm{CVaR}_\alpha^t - (1 - w_t) \cdot R_t^f. \tag{C.5}$$

Therefore, the difference between the portfolio loss and the target CVaR is given by

$$L_t^P - \mathrm{CVaR}_\alpha^{\text{target}} = w_t \cdot L_t - w_t \cdot \mathrm{CVaR}_\alpha^t = w_t \cdot (L_t - \mathrm{CVaR}_\alpha^t). \tag{C.6}$$

Moreover, from Equation (1.2.2) and since w_t and R_t^f are $\mathcal{F}_{t-1}$-measurable, we obtain

$$\sqrt{\mathrm{var}(R_t^P \mid \mathcal{F}_{t-1})} = \sqrt{\mathrm{var}(w_t \cdot R_t \mid \mathcal{F}_{t-1})} = w_t \cdot \sqrt{\mathrm{var}(R_t \mid \mathcal{F}_{t-1})} = w_t \cdot \sigma_t. \tag{C.7}$$

Consequently, from Equations (C.6), (C.7) and (1.3.29) it follows

$$\frac{L_t^P - \mathrm{CVaR}_\alpha^{\text{target}}}{\sqrt{\mathrm{var}(R_t^P \mid \mathcal{F}_{t-1})}} = \frac{w_t \cdot (L_t - \mathrm{CVaR}_\alpha^t)}{w_t \cdot \sigma_t} = \frac{L_t - \mathrm{CVaR}_\alpha^t}{\sigma_t} = L_t^* - \mathrm{CVaR}_\alpha^{t,*}. \tag{C.8}$$

D Additional Results

This section shows additional results for the risk targeting strategies. In Section D.1, we demonstrate that tail risk targeting can be used as an alternative to absolute return strategies. In Section D.2, we show results for other significance levels α. Section D.3 examines the profitability of risk targeting when a rebalancing buffer is used. In Section D.4, we examine risk targeting for leverage constrained investors. Section D.5 shows results for the US and small caps, proxied by the S&P 500 and the SDAX. In Section D.6, we examine the profitability of risk targeting in the long run using almost 100 years of data. Finally, in Section D.7, we conduct an out-of-sample study and demonstrate the benefits of risk targeting during the recent corona crisis. Throughout this section, we only show results for the volatility and CVaR targeting strategies as well as the strategies that switch between the EWMA model and the CVaR targeting strategies based on the TSMOM indicator. Further, we only show results of the strategies' return, volatility, Sharpe Ratio, maximum drawdown and the economic value Δ_{MV} of a mean-variance investor with a moderate risk aversion of $\gamma = 5$. The economic value Δ_{MV} measures the annualized percentage fee an investor is willing to pay to switch from the 60/40 portfolio to a risk targeting strategy. Additionally, we use the Jobson and Korkie (1981) and Diebold and Mariano (1995) tests to test for the significance of the Sharpe Ratio and utility increases.[99]

D.1 Tail Risk Targeting as Absolute Return Strategy

This section demonstrates that downside risk targeting can be used as an alternative to absolute return and other hedge fund strategies as examined in Fung and Hsieh (1997) and Agarwal and Naik (2004). In Figure III, we plot a target VaR strategy with a VaR target of $\mathrm{VaR}_{\alpha}^{\mathrm{target}} = 0.5\%$ and a significance level of $\alpha = 0.4\%$. Hence, a daily return lower than -0.5% should occur only once a year. Days with a return lower than -0.5% are marked with a red cross. As can be seen from the figure, the strategy is successful in mitigating extremely negative returns

[99] We use the HSD managed strategy as benchmark for both tests. By doing this, we can assess how different characteristics affect the profitability of risk targeting using volatility or CVaR as risk measure. Thus, the tests show if a risk targeting strategy outperforms the HSD based volatility targeting strategy. This is opposed to Tables IX and XII, where we used the 60/40 portfolio as benchmark for the Diebold and Mariano (1995) test. Our conclusions were similar to the case when the DM-test is calculated with respect to the 60/40 portfolio.

and produces positive or moderately negative returns with a very high probability. The target VaR strategy's long-term return is as high as the return of the 60/40 portfolio and only slightly lower than the return of the DAX. However, the target risk strategy also takes much less risk with significantly lower drawdowns. Hence, this strategy is appealing for highly risk-averse or loss-averse investors who want to benefit from the long-term potential of equity markets. We only show results for the VaR-EWMA-FHS strategy in Figure III since this strategy is easy to estimate and implement, and hence could be interesting for practitioners. Other risk targeting strategies produce similar or even superior performance charts.

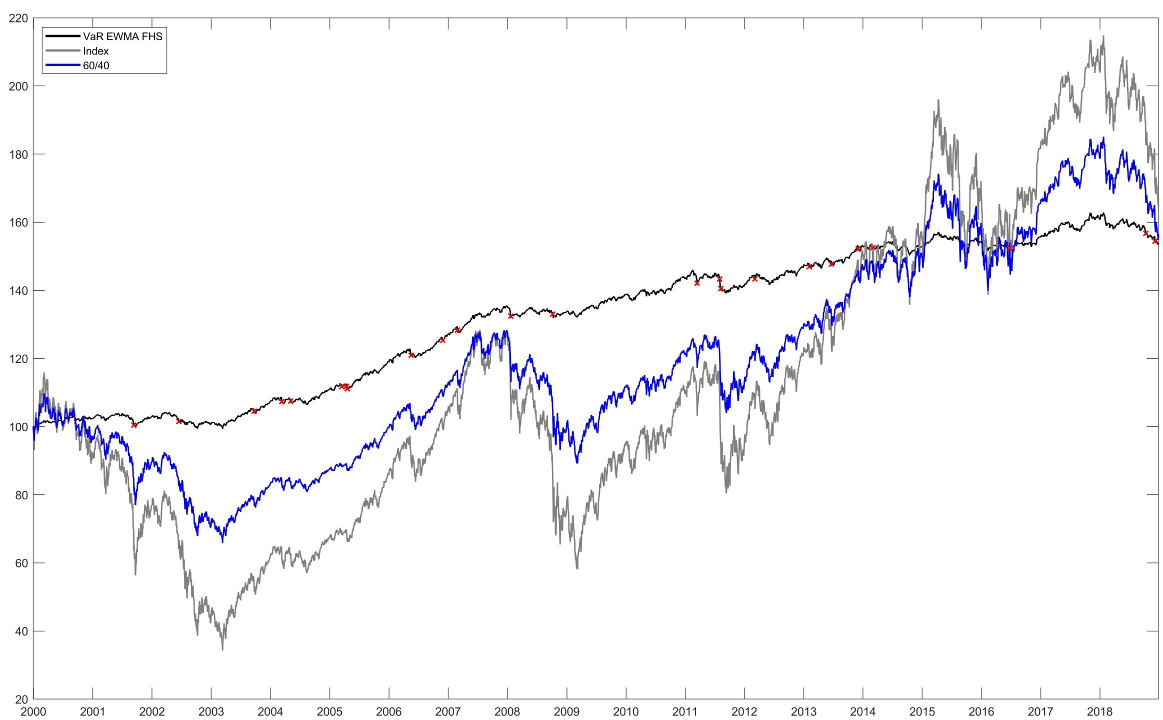


Figure III. Cumulative Return of VaR Targeting. This figure plots the cumulative return of the DAX, the 60/40 portfolio and a target VaR strategy. The target VaR strategy uses the EWMA volatility model combined with Filtered Historical Simulation (FHS), a VaR target of $\text{VaR}_\alpha^{\text{target}} = 0.5\%$ and a significance level of $\alpha = 0.4\%$. Days when the portfolio return is lower than -0.5% are marked with a red cross.

D.2 Tail Risk Targeting for Different Significance Levels

This section examines performance results of the tail risk targeting strategies for different significance levels α. In Table XIV, we show additional performance results of volatility and CVaR targeting for significance levels of $1\%, 2.5\%$ and 5%. The chosen significance levels are frequently used in the literature on VaR and CVaR forecasting (see Bali et al. (2008) for example). The target CVaR level is recalculated to match the chosen volatility target by using Equation (1.3.38). In line with the results of Tables V and VIII, we find that downside risk targeting is superior to volatility targeting in terms of higher Sharpe Ratios, lower drawdowns and a higher economic value. Further, also in line with the previous results, we find that managing risk by conditional models outperforms the strategies based on HSD or Historical Simulation. This result is most pronounced for high significance levels. Generally, downside risk targeting becomes less attractive if a higher significance level α is chosen. This result is in line with Ghysels et al. (2016) and Happersberger et al. (2019). We find that tail risk targeting resembles volatility targeting when higher significance levels are chosen. This finding is quite intuitive, since a lower α means that extreme losses in the far left tail are managed. In contrast, using a higher α means that also moderate losses are taken into account. In total, results of Table XIV highlight that investors should manage extreme losses by using CVaR combined with a low value of α. A similar results has also been found by Basak and Shapiro (2001). However, even for a significance level of $\alpha = 5\%$, CVaR targeting is typically superior to volatility targeting. Interestingly, the strategies based on the skewed t distribution are very stable in terms of the Sharpe Ratio for different levels of α. However, the drawdown and the economic value indicate that low levels of α are also superior for these models.

In Table XV, we show results for the strategies that switch between volatility and CVaR targeting for the same significance levels. Throughout the section, we use the EWMA volatility model and the TSMOM based indicator δ_t. In line with our main results, we find that the strategies that switch between volatility and CVaR targeting outperform the non-managed strategy or the strategies that use either volatility or CVaR targeting. However, in line with Table XIV, we find better results for the strategies that use lower significance levels α. This is again quite intuitive, since the switching approach typically uses CVaR targeting only in adverse market periods. These crash periods are best managed by tail risk measures that use low significance levels and manage *extreme* losses instead of moderate losses. Moreover, we again find that conditional models produce the most convincing results. All switching strategies based on con-

Table XIV. Performance Results for Different Significance Levels

This table shows additional performance results for the volatility and CVaR targeting strategies using the DAX and three different significance levels α. Panel A shows results for $\alpha = 1\%$, Panel B shows results for $\alpha = 2.5\%$ and Panel C shows results for $\alpha = 5\%$. Return and Volatility denote the annualized return and volatility in percent. SR denotes the annualized Sharpe Ratio. z_{JK} denotes the test statistic of the Jobson and Korkie (1981) test. MDD and ΔMDD denote the maximum drawdown and the reduction of the maximum drawdown in relation to the drawdown of the risky asset. $\Delta_{MV}^{\gamma=5}$ denotes the economic value for a mean-variance investor with risk aversion $\gamma = 5$. DM-test denotes the Diebold and Mariano (1995) test that tests if a strategy produces a significantly higher utility for a mean-variance investor with risk aversion $\gamma = 5$, where we use the HSD strategy as benchmark. Return, Volatility, MDD, ΔMDD and $\Delta_{MV}^{\gamma=5}$ are given in percent. Values of z_{JK} and DM-test that are higher than 1.645 are given in bold.

	Panel A: Significance Level of $\alpha = 1\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.115	12.843	0.102	-	41.228	41.458	0.280	-
Vola EWMA	3.336	12.499	0.122	0.965	40.522	42.459	0.671	**1.720**
Vola GARCH	3.191	11.965	0.116	0.389	39.233	44.290	0.792	1.123
CVaR Hist	2.316	9.550	0.055	-0.462	30.298	56.978	0.982	0.493
CVaR EWMA FHS	3.193	10.541	0.132	1.006	35.754	49.231	1.443	**1.820**
CVaR EWMA EVT	3.230	10.232	0.139	1.311	34.685	50.749	1.609	**1.891**
CVaR EWMA Stsk	3.408	10.529	0.152	1.612	34.268	51.340	1.659	**2.110**
CVaR GARCH FHS	3.174	10.875	0.126	0.616	37.396	46.898	1.279	1.509
CVaR GARCH EVT	3.191	10.516	0.132	0.789	36.157	48.659	1.452	1.624
CVaR GARCH Stsk	3.273	10.100	0.145	1.015	34.017	51.697	1.707	**1.754**
	Panel B: Significance Level of $\alpha = 2.5\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.115	12.843	0.102	-	41.228	41.458	0.280	-
Vola EWMA	3.336	12.499	0.122	0.965	40.522	42.459	0.671	**1.720**
Vola GARCH	3.191	11.965	0.116	0.389	39.233	44.290	0.792	1.123
CVaR Hist	2.061	10.223	0.027	-0.714	33.217	52.833	0.462	0.130
CVaR EWMA FHS	3.227	10.975	0.129	1.062	37.056	47.382	1.287	**1.927**
CVaR EWMA EVT	3.230	10.694	0.133	1.239	36.002	48.878	1.413	**1.955**
CVaR EWMA Stsk	3.438	11.075	0.147	**1.671**	35.830	49.123	1.449	**2.261**
CVaR GARCH FHS	3.137	11.082	0.120	0.488	37.919	46.156	1.151	1.437
CVaR GARCH EVT	3.114	10.830	0.121	0.519	37.086	47.338	1.240	1.490
CVaR GARCH Stsk	3.354	10.614	0.146	1.079	35.353	49.800	1.570	**1.840**
	Panel C: Significance Level of $\alpha = 5\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.115	12.843	0.102	-	41.228	41.458	0.280	-
Vola EWMA	3.336	12.499	0.122	0.965	40.522	42.459	0.671	**1.720**
Vola GARCH	3.191	11.965	0.116	0.389	39.233	44.290	0.792	1.123
CVaR Hist	2.027	10.958	0.022	-0.745	35.590	49.464	0.116	-0.119
CVaR EWMA FHS	3.217	11.194	0.126	1.023	37.421	46.864	1.179	**1.960**
CVaR EWMA EVT	3.231	11.084	0.129	1.136	37.018	47.436	1.242	**1.995**
CVaR EWMA Stsk	3.461	11.515	0.143	1.622	37.056	47.382	1.270	**2.368**
CVaR GARCH FHS	3.101	11.237	0.115	0.370	38.026	46.004	1.046	1.350
CVaR GARCH EVT	3.081	11.121	0.115	0.357	37.773	46.363	1.079	1.368
CVaR GARCH Stsk	3.414	11.032	0.145	1.082	36.386	48.332	1.445	**1.874**

ditional risk models produce statistically higher Sharpe Ratios for significance levels of 1% and 2.5%. In contrast, none of the volatility targeting or HS based strategies produces a significantly higher Sharpe Ratio. Similarly, all switching strategies that are based on a conditional CVaR model produce economically high utility gains for the mean-variance investor for all three significance levels, whereas the HS based switching strategy only slightly heightens the investor's utility. Furthermore, the utility increases of the conditional models are all statistically significant for all three significance levels.

Table XV. Performance Results for Different Significance Levels: Switching Strategies
This table shows additional performance results for the strategies that switch between volatility and CVaR targeting using the DAX and three different significance levels α. Panel A shows results for $\alpha = 1\%$, Panel B shows results for $\alpha = 2.5\%$ and Panel C shows results for $\alpha = 5\%$. The description of the columns is given in Table XIV.

	Panel A: Significance Level of $\alpha = 1\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.115	12.843	0.102	-	41.228	41.458	0.280	-
DAX	2.726	23.457	0.040	-0.569	70.424	-	-7.523	-2.548
EWMA/CVaR Hist	4.197	12.490	0.190	1.641	32.334	54.086	1.514	1.634
EWMA/CVaR EWMA FHS	3.805	11.842	0.168	**2.109**	37.938	46.130	1.452	**2.720**
EWMA/CVaR EWMA EVT	3.875	11.728	0.175	**2.233**	37.051	47.389	1.575	**2.787**
EWMA/CVaR EWMA Stsk	4.110	11.845	0.193	**2.932**	35.823	49.133	1.749	**3.160**
EWMA/CVaR GARCH FHS	3.741	12.158	0.158	**1.844**	39.070	44.521	1.236	**2.250**
EWMA/CVaR GARCH EVT	3.827	12.005	0.167	**2.082**	38.000	46.042	1.395	**2.469**
EWMA/CVaR GARCH Stsk	4.104	11.882	0.192	**2.535**	36.104	48.733	1.724	**2.724**
	Panel B: Significance Level of $\alpha = 2.5\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.115	12.843	0.102	-	41.228	41.458	0.280	-
DAX	2.726	23.457	0.040	-0.569	70.424	-	-7.523	-2.548
EWMA/CVaR Hist	3.865	12.888	0.159	0.985	34.401	51.151	0.986	0.903
EWMA/CVaR EWMA FHS	3.692	11.974	0.157	**1.994**	38.565	45.239	1.278	**2.711**
EWMA/CVaR EWMA EVT	3.746	11.870	0.163	**2.096**	37.696	46.473	1.381	**2.755**
EWMA/CVaR EWMA Stsk	3.930	12.022	0.176	**2.705**	37.008	47.450	1.487	**3.083**
EWMA/CVaR GARCH FHS	3.669	12.233	0.152	**1.691**	39.224	44.303	1.129	**2.133**
EWMA/CVaR GARCH EVT	3.692	12.131	0.155	**1.789**	38.460	45.387	1.201	**2.254**
EWMA/CVaR GARCH Stsk	3.979	12.065	0.179	**2.315**	37.075	47.355	1.513	**2.601**
	Panel C: Significance Level of $\alpha = 5\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.115	12.843	0.102	-	41.228	41.458	0.280	-
DAX	2.726	23.457	0.040	-0.569	70.424	-	-7.523	-2.548
EWMA/CVaR Hist	3.671	13.301	0.140	0.601	36.618	48.004	0.580	0.361
EWMA/CVaR EWMA FHS	3.607	12.034	0.149	**1.839**	38.528	45.292	1.166	**2.634**
EWMA/CVaR EWMA EVT	3.642	11.991	0.152	**1.929**	38.207	45.747	1.221	**2.692**
EWMA/CVaR EWMA Stsk	3.789	12.171	0.162	**2.355**	37.942	46.124	1.276	**2.921**
EWMA/CVaR GARCH FHS	3.589	12.284	0.145	1.497	39.044	44.559	1.026	**1.971**
EWMA/CVaR GARCH EVT	3.599	12.240	0.146	1.543	38.787	44.924	1.057	**2.040**
EWMA/CVaR GARCH Stsk	3.878	12.222	0.169	**2.035**	37.820	46.297	1.338	**2.433**

In total, this section confirms our earlier results for other significance levels α. In particular, switching between volatility and CVaR targeting produces the most convincing risk-return profile for all significance levels. This holds especially when risk is estimated with a conditional risk model. However, results for lower significance levels are advantageous compared to higher levels of α. Thus, investors should manage extreme losses instead of moderate losses or return deviations.

D.3 Tail Risk Targeting for Different Rebalancing Buffers

This section examines the performance of the risk targeting strategies when a rebalancing buffer η is used. Our main results rely on strategies where the weight of the risky asset is readjusted every day. In practice, readjusting the weight of the risky asset every day can lead to high transaction costs, which makes risk targeting less profitable. However, transaction costs can be significantly reduced by readjusting the weights less frequently or only in periods when risk changes dramatically. Bollerslev et al. (2018) find that reallocating portfolio weights less frequently reduces transaction costs without producing an inferior performance. Similarly, Perchet et al. (2016, Exhibit 5) find that daily, weekly and monthly rebalancing produce similar results. Further, Perchet et al. (2016, p. 36) suggest that a "reduction of turnover can be achieved with daily monitoring of volatility and rebalancing only when the volatility changes significantly". For that reason, we show in Table XVI results for the volatility and CVaR targeting strategies for rebalancing buffers η of $5\%, 10\%$ and 15%, i.e. we define a corridor of $10\%, 20\%$ and 30% around the current weight where no rebalancing is done. Using a rebalancing buffer means that portfolio weights are only readjusted when the optimal portfolio weight w_t, under a given risk forecast, lies outside this corridor around the current weight. Thus, portfolio weights are only readjusted if portfolio risk changes substantially. This has the advantage that portfolio risk is monitored every day, but transaction costs are significantly lowered by only reacting to extreme changes. We denote the day t weight under a rebalancing buffer η by w_t^η and the day t weight *before* rebalancing on day t by $\tilde{w}_t^\eta$. Thus, as in Kirby and Ostdiek (2012, Eq. (3)), $\tilde{w}_t^\eta$ is defined by

$$\tilde{w}_t^\eta := \frac{w_{t-1}^\eta(1+R_{t-1})}{w_{t-1}^\eta(1+R_{t-1}) + (1-w_{t-1}^\eta)(1+R_{t-1}^f)}. \tag{D.1}$$

The day t weight using a rebalancing buffer of η is then given by

$$w_t^\eta = \begin{cases} \tilde{w}_t^\eta, & \text{if} \quad (1-\eta)\tilde{w}_t^\eta \leqslant w_t \leqslant (1+\eta)\tilde{w}_t^\eta \\ w_t, & \text{else} \end{cases}, \tag{D.2}$$

where w_t is the optimal weight for the target volatility, target VaR or target CVaR strategy. The approach of Equation (D.2) is similar to the approach suggested by Perchet et al. (2016, p. 33).

Table XVI. Performance Results for Different Rebalancing Buffers
This table shows additional performance results for the volatility and CVaR targeting strategies using the DAX and rebalancing buffers as in Equation (D.2). Panel A uses a rebalancing buffer of $\eta = 5\%$, Panel B uses a rebalancing buffer of $\eta = 10\%$ and Panel C uses a rebalancing buffer of $\eta = 15\%$. The description of the columns is given in Table XIV.

	Panel A: Rebalancing Buffer of $\eta = 5\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.116	12.849	0.102	-	41.268	41.401	0.278	-
Vola EWMA	3.272	12.394	0.118	0.750	39.915	43.322	0.661	1.565
Vola GARCH	3.073	11.870	0.107	0.138	39.069	44.523	0.723	0.963
CVaR Hist	2.385	9.275	0.064	-0.377	28.870	59.005	1.154	0.607
CVaR EWMA FHS	3.196	10.072	0.138	1.087	33.834	51.957	1.642	**1.776**
CVaR EWMA EVT	3.179	9.844	0.139	1.180	33.289	52.731	1.717	**1.778**
CVaR EWMA Stsk	3.376	10.140	0.154	1.476	32.926	53.246	1.791	**1.973**
CVaR GARCH FHS	3.123	10.595	0.124	0.554	37.003	47.457	1.351	1.478
CVaR GARCH EVT	3.157	10.248	0.132	0.754	35.400	49.733	1.531	1.596
CVaR GARCH Stsk	3.218	9.729	0.145	0.941	32.881	53.309	1.801	**1.675**
	Panel B: Rebalancing Buffer of $\eta = 10\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.158	12.968	0.104	-	41.597	40.933	0.258	-
Vola EWMA	3.177	12.378	0.111	0.274	39.791	43.498	0.576	1.099
Vola GARCH	3.170	11.813	0.115	0.295	37.597	46.613	0.845	1.142
CVaR Hist	2.321	9.293	0.057	-0.467	28.628	59.349	1.085	0.559
CVaR EWMA FHS	3.089	10.036	0.128	0.689	33.869	51.907	1.552	1.614
CVaR EWMA EVT	3.065	9.810	0.129	0.732	33.409	52.560	1.619	1.606
CVaR EWMA Stsk	3.312	10.152	0.148	**1.169**	33.051	53.069	1.724	**1.849**
CVaR GARCH FHS	3.262	10.516	0.138	0.799	35.435	49.684	1.522	1.604
CVaR GARCH EVT	3.304	10.178	0.147	1.018	33.863	51.916	1.705	**1.705**
CVaR GARCH Stsk	3.178	9.750	0.141	0.756	33.303	52.711	1.754	1.582
	Panel C: Rebalancing Buffer of $\eta = 15\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.029	13.034	0.094	-	41.222	41.466	0.098	-
Vola EWMA	3.135	12.436	0.107	0.514	40.977	41.814	0.507	1.352
Vola GARCH	3.032	11.787	0.104	0.259	38.728	45.008	0.722	1.163
CVaR Hist	2.380	9.482	0.062	-0.319	29.599	57.971	1.071	0.665
CVaR EWMA FHS	3.141	10.064	0.133	1.117	34.126	51.542	1.591	**1.854**
CVaR EWMA EVT	3.176	9.857	0.139	1.331	33.570	52.332	1.710	**1.886**
CVaR EWMA Stsk	3.276	10.101	0.145	1.349	33.537	52.379	1.709	**1.913**
CVaR GARCH FHS	3.125	10.523	0.125	0.723	35.662	49.361	1.383	1.625
CVaR GARCH EVT	3.143	10.180	0.131	0.870	34.336	51.243	1.546	**1.677**
CVaR GARCH Stsk	2.964	9.722	0.120	0.527	33.145	52.935	1.554	1.509

Results for the target volatility and target CVaR strategies for the three rebalancing buffers are shown in Table XVI. These results are similar to our main results that rebalance the weight on a daily basis. The Sharpe Ratios for the strategies using a rebalancing buffer of $\eta = 5\%$ and $\eta = 10\%$ are similar to the Sharpe Ratios with daily rebalancing as shown in Table V. This finding is in line with Perchet et al. (2016, Exhibit 5) who also find nearly identical results for daily and weekly rebalancing. Similarly, Bollerslev et al. (2018) conclude that it may be better to not trade every change in the optimal weight. They find better utility gains for the strategies that adjust portfolio weights less frequently, especially when transaction costs are high and/or when models induce high day-to-day changes in the optimal weight. The authors combine the target volatility strategy with an approach that adjusts the weight only partially. Kirby and Ostdiek (2012) also find good results of models that are less sensitive to volatility changes. However, for a high rebalancing buffer of $\eta = 15\%$, we find that Sharpe Ratios (before transaction costs) are significantly lower. Nevertheless, risk targeting, especially CVaR targeting, still provides a convincing risk-return profile compared to the non-managed strategies.

In total, as in Table V, we again find the best results for the CVaR targeting strategies, regardless of the used rebalancing buffer. This holds especially when risk is managed by a conditional risk model. Thus, risk targeting, especially CVaR targeting, is still advantageous even when portfolio weights are readjusted less frequently. However, as expected, the strategies using a rebalancing buffer are slightly less profitable than the non-constrained strategies, at least before transaction costs.

In Table XVII, we show results for the switching strategies using the three rebalancing buffers.[100] In line with our previous results, switching between volatility and CVaR targeting significantly outperforms the non-managed and volatility managed strategies. Nearly all switching strategies exhibit significantly higher Sharpe Ratios and utilities. The best results are again found for the strategies that use conditional risk models. All switching strategies that use a conditional CVaR model produce statistically significant utility increases, even for high rebalancing buffers.

In total, this section shows that risk targeting is still advantageous to non-managed strategies when a rebalancing buffer is used. This is important since the rebalancing buffer can reduce transaction costs dramatically, which shows that risk targeting is an appealing tail risk hedging tool for practitioners. This holds particularly for the strategies that switch between volatility CVaR targeting, especially when conditional risk models are used.

[100]We use rebalancing buffers that only determine the weights of the volatility and CVaR targeting strategies, whereas the switching approach of Equation (1.5.11) is not affected by the rebalancing buffer. Thus, we reallocate the portfolio weight every time when the indicator δ_t changes, even when the portfolio weight changes are small. More formally, the weight of the switching strategy with a rebalancing buffer η is given by $w_t^{switch,\eta} = \delta_t \cdot w_t^{CVaR,\eta} + (1 - \delta_t) \cdot w_t^{vol,\eta}$, where $w_t^{vol,\eta}$ and $w_t^{CVaR,\eta}$ are given by Equation (D.2). An alternative would be to apply the rebalancing buffer directly to the weights of the switching strategy.

Table XVII. Performance Results for Different Rebalancing Buffers: Switching Strategies
This table shows additional performance results for the strategies that switch between volatility and CVaR targeting using the DAX and rebalancing buffers as in Equation (D.2). Panel A uses a rebalancing buffer of $\eta = 5\%$, Panel B uses a rebalancing buffer of $\eta = 10\%$ and Panel C uses a rebalancing buffer of $\eta = 15\%$. The description of the columns is given in Table XIV.

	Panel A: Rebalancing Buffer of $\eta = 5\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.116	12.849	0.102	-	41.268	41.401	0.278	-
DAX	2.726	23.457	0.040	-0.570	70.424	-	-7.523	-2.557
EWMA/CVaR Hist	4.273	12.263	0.200	**1.806**	31.468	55.316	1.703	**1.863**
EWMA/CVaR EWMA FHS	3.867	11.613	0.176	**2.112**	36.333	48.409	1.621	**2.617**
EWMA/CVaR EWMA EVT	3.898	11.538	0.180	**2.162**	35.903	49.019	1.687	**2.655**
EWMA/CVaR EWMA Stsk	4.188	11.652	0.203	**2.917**	34.565	50.919	1.918	**3.083**
EWMA/CVaR GARCH FHS	3.758	11.969	0.162	**1.851**	38.535	45.281	1.345	**2.308**
EWMA/CVaR GARCH EVT	3.859	11.829	0.173	**2.103**	37.309	47.022	1.511	**2.505**
EWMA/CVaR GARCH Stsk	4.152	11.680	0.199	**2.548**	35.066	50.207	1.868	**2.724**
	Panel B: Rebalancing Buffer of $\eta = 10\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.158	12.968	0.104	-	41.597	40.933	0.258	-
DAX	2.726	23.457	0.040	-0.587	70.424	-	-7.523	-2.564
EWMA/CVaR Hist	4.066	12.237	0.183	1.414	31.378	55.444	1.513	1.495
EWMA/CVaR EWMA FHS	3.740	11.589	0.166	**1.698**	36.290	48.469	1.509	**2.232**
EWMA/CVaR EWMA EVT	3.765	11.511	0.169	**1.745**	35.953	48.949	1.570	**2.259**
EWMA/CVaR EWMA Stsk	4.026	11.628	0.190	**2.398**	34.472	51.050	1.770	**2.665**
EWMA/CVaR GARCH FHS	3.778	11.905	0.165	**1.746**	36.920	47.574	1.395	**2.096**
EWMA/CVaR GARCH EVT	3.863	11.766	0.174	**1.951**	35.751	49.235	1.544	**2.235**
EWMA/CVaR GARCH Stsk	3.968	11.667	0.184	**2.010**	35.211	50.001	1.695	**2.274**
	Panel C: Rebalancing Buffer of $\eta = 15\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.029	13.034	0.094	-	41.222	41.466	0.098	-
DAX	2.726	23.457	0.040	-0.495	70.424	-	-7.523	-2.524
EWMA/CVaR Hist	4.144	12.373	0.188	**1.685**	31.792	54.856	1.521	**1.780**
EWMA/CVaR EWMA FHS	3.703	11.621	0.162	**1.836**	37.148	47.252	1.457	**2.429**
EWMA/CVaR EWMA EVT	3.752	11.556	0.168	**1.940**	36.627	47.991	1.536	**2.463**
EWMA/CVaR EWMA Stsk	3.952	11.666	0.183	**2.431**	35.993	48.890	1.680	**2.787**
EWMA/CVaR GARCH FHS	3.569	11.954	0.147	1.477	38.752	44.974	1.167	**2.157**
EWMA/CVaR GARCH EVT	3.666	11.819	0.157	**1.698**	37.517	46.728	1.327	**2.273**
EWMA/CVaR GARCH Stsk	3.843	11.690	0.173	**1.952**	36.231	48.554	1.562	**2.321**

D.4 Tail Risk Targeting for Different Leverage Constraints

This section examines the profitability of risk targeting for investors who are leverage constrained. Our main results are based on unconstrained portfolio weights. However, in practice, many investors have short-sale and leverage constraints (see Frazzini and Pedersen (2014) and references therein). The portfolio weights w_t are positive by construction, but risk targeting investors could be forced to use leverage in periods with low market risk. Following Perchet et al. (2016, Exhibit 5) and Moreira and Muir (2017, Table IV), we additionally assess the impact of leverage constraints on the profitability of risk targeting. The day t weight w_t^c under a leverage constraint is given by

$$w_t^c = \min(w_t, c), \tag{D.3}$$

where w_t is the optimal weight without leverage constraint. We choose values c of $1, 1.5$ and 2, i.e. no leverage, a maximum leverage of 50% and a maximum leverage of 100%. This is in line with the choices of Perchet et al. (2016) who use a cap of $c = 1$ and $c = 2$ as well as Moreira and Muir (2017, Table IV-V) who use $c = 1$ and $c = 1.5$. Perchet et al. (2016, Exhibit 5) find that a cap of $c = 2$ produces nearly identical results compared to the unconstrained strategy, whereas a cap of $c = 1$ reduces the return, volatility and drawdown and produces a slightly lower Sharpe Ratio. Similarly, Moreira and Muir (2017, Table V) find similar Sharpe Ratios for the non-constrained and leverage constrained strategies. Results for the leverage constrained target volatility and target CVaR strategies are shown in Table XVIII. In line with Perchet et al. (2016) and Moreira and Muir (2017), we find that risk targeting is still advantageous when a maximum equity exposure is used. Interestingly, we find that CVaR targeting is much less influenced by the leverage constraints. In particular, CVaR targeting produces identical results for equity caps of $c = 1.5$ and $c = 2$ that are in line with the results of the unconstrained strategies in Table V. Thus, CVaR targeting needs much lower levels of leverage than the volatility targeting strategies. As a consequence, the advantages of CVaR targeting are even more pronounced for leverage constrained investors. Furthermore, as expected, the drawdown reduction ability of all risk targeting strategies is not influenced by the maximum equity exposure.

In Table XIX, we show results for the switching strategies under the three leverage constraints. In line with Table XVIII, we find that the switching approach is less influenced by the maximum equity exposure than the HSD managed strategy. All switching strategies produce significantly higher Sharpe Ratios and utilities for all three equity caps and significantly outperform the non-managed and HSD managed strategies, i.e. all switching strategies produce statistically significant Sharpe Ratio and utility increases for all three equity caps. Furthermore, the drawdown reduction ability is again not influenced by leverage constraints.

In total, results in this section show that investors also benefit from risk targeting when they have (tight) leverage constraints. This holds especially for the strategies that target a constant

Table XVIII. Performance Results for Different Leverage Constraints

This table shows additional performance results for the volatility and CVaR targeting strategies using the DAX and leverage constraints as in Equation (D.3). Panel A uses a maximum equity exposure of $c = 100\%$, Panel B uses a maximum equity exposure of $c = 150\%$ and Panel C uses a maximum equity exposure of $c = 200\%$. The description of the columns is given in Table XIV.

	Panel A: Maximum Equity Exposure of $c = 100\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	2.954	12.409	0.093	-	41.228	41.458	0.343	-
Vola EWMA	3.102	12.198	0.106	0.761	40.522	42.459	0.592	1.340
Vola GARCH	3.136	11.908	0.112	0.634	39.233	44.290	0.766	1.079
CVaR Hist	2.431	9.260	0.069	-0.254	28.509	59.519	1.206	0.639
CVaR EWMA FHS	3.149	10.080	0.133	1.216	34.351	51.223	1.593	**1.806**
CVaR EWMA EVT	3.158	9.867	0.137	1.359	33.801	52.004	1.688	**1.832**
CVaR EWMA Stsk	3.307	10.083	0.149	1.597	33.121	52.970	1.747	**2.017**
CVaR GARCH FHS	3.237	10.670	0.134	1.160	36.998	47.465	1.430	**1.700**
CVaR GARCH EVT	3.265	10.324	0.141	1.382	35.505	49.583	1.605	**1.806**
CVaR GARCH Stsk	3.230	9.733	0.146	1.308	33.020	53.112	1.813	**1.783**
	Panel B: Maximum Equity Exposure of $c = 150\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	2.989	12.785	0.093	-	41.228	41.458	0.188	-
Vola EWMA	3.318	12.491	0.121	1.420	40.522	42.459	0.658	**2.099**
Vola GARCH	3.191	11.965	0.116	0.688	39.233	44.290	0.792	**1.366**
CVaR Hist	2.431	9.260	0.069	-0.238	28.509	59.519	1.206	0.711
CVaR EWMA FHS	3.219	10.159	0.139	1.447	34.351	51.223	1.629	**1.956**
CVaR EWMA EVT	3.231	9.932	0.143	**1.646**	33.801	52.004	1.733	**1.980**
CVaR EWMA Stsk	3.382	10.141	0.155	**1.823**	33.121	52.970	1.797	**2.146**
CVaR GARCH FHS	3.234	10.678	0.134	1.069	36.998	47.465	1.424	**1.757**
CVaR GARCH EVT	3.263	10.329	0.141	1.289	35.505	49.583	1.602	**1.852**
CVaR GARCH Stsk	3.214	9.738	0.145	1.190	33.020	53.112	1.794	**1.811**
	Panel C: Maximum Equity Exposure of $c = 200\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.085	12.830	0.100	-	41.228	41.458	0.258	-
Vola EWMA	3.336	12.499	0.122	1.091	40.522	42.459	0.671	**1.846**
Vola GARCH	3.191	11.965	0.116	0.458	39.233	44.290	0.792	1.185
CVaR Hist	2.431	9.260	0.069	-0.305	28.509	59.519	1.206	0.657
CVaR EWMA FHS	3.219	10.159	0.139	1.212	34.351	51.223	1.629	**1.849**
CVaR EWMA EVT	3.231	9.932	0.143	1.402	33.801	52.004	1.733	**1.878**
CVaR EWMA Stsk	3.385	10.141	0.155	1.603	33.121	52.970	1.800	**2.046**
CVaR GARCH FHS	3.234	10.678	0.134	0.855	36.998	47.465	1.424	1.636
CVaR GARCH EVT	3.263	10.329	0.141	1.063	35.505	49.583	1.602	**1.740**
CVaR GARCH Stsk	3.214	9.738	0.145	1.000	33.020	53.112	1.794	**1.716**

level of tail risk or the strategies that switch between volatility and CVaR targeting. In contrast, the strategy that is based on the HSD model is much more influenced by a maximum equity exposure. Thus, the outperformance of the CVaR targeting and switching strategies is even more pronounced for investors with leverage constraints.

Table XIX. Performance Results for Different Leverage Constraints: Switching Strategies
This table shows additional performance results for the strategies that switch between volatility and CVaR targeting using the DAX and leverage constraints as in Equation (D.3). Panel A uses a maximum equity exposure of $c = 100\%$, Panel B uses a maximum equity exposure of $c = 150\%$ and Panel C uses a maximum equity exposure of $c = 200\%$. The description of the columns is given in Table XIV.

	Panel A: Maximum Equity Exposure of $c = 100\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	2.954	12.409	0.093	-	41.228	41.458	0.343	-
DAX	2.726	23.457	0.040	-0.522	70.424	-	-7.523	-2.579
EWMA/CVaR Hist	4.121	12.030	0.191	**1.835**	31.650	55.058	1.669	**1.795**
EWMA/CVaR EWMA FHS	3.686	11.394	0.164	**2.063**	37.035	47.412	1.546	**2.544**
EWMA/CVaR EWMA EVT	3.723	11.318	0.169	**2.123**	36.619	48.002	1.618	**2.600**
EWMA/CVaR EWMA Stsk	3.999	11.409	0.191	**2.870**	34.956	50.363	1.846	**3.004**
EWMA/CVaR GARCH FHS	3.589	11.758	0.151	**1.870**	38.808	44.894	1.281	**2.172**
EWMA/CVaR GARCH EVT	3.687	11.613	0.161	**2.113**	37.680	46.495	1.446	**2.393**
EWMA/CVaR GARCH Stsk	3.952	11.443	0.186	**2.476**	35.379	49.763	1.785	**2.608**
	Panel B: Maximum Equity Exposure of $c = 150\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	2.989	12.785	0.093	-	41.228	41.458	0.188	-
DAX	2.726	23.457	0.040	-0.491	70.424	-	-7.523	-2.520
EWMA/CVaR Hist	4.345	12.323	0.204	**2.095**	31.650	55.058	1.742	**2.056**
EWMA/CVaR EWMA FHS	3.905	11.704	0.178	**2.466**	37.035	47.412	1.615	**2.919**
EWMA/CVaR EWMA EVT	3.946	11.629	0.183	**2.525**	36.619	48.002	1.691	**2.962**
EWMA/CVaR EWMA Stsk	4.222	11.718	0.205	**3.252**	34.956	50.363	1.919	**3.341**
EWMA/CVaR GARCH FHS	3.812	12.058	0.165	**2.296**	38.808	44.894	1.354	**2.570**
EWMA/CVaR GARCH EVT	3.910	11.917	0.176	**2.524**	37.680	46.495	1.518	**2.761**
EWMA/CVaR GARCH Stsk	4.176	11.750	0.200	**2.836**	35.379	49.763	1.858	**2.924**
	Panel C: Maximum Equity Exposure of $c = 200\%$							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	3.085	12.830	0.100	-	41.228	41.458	0.258	-
DAX	2.726	23.457	0.040	-0.551	70.424	-	-7.523	-2.542
EWMA/CVaR Hist	4.362	12.331	0.206	**1.981**	31.650	55.058	1.755	**1.978**
EWMA/CVaR EWMA FHS	3.923	11.713	0.180	**2.295**	37.035	47.412	1.629	**2.798**
EWMA/CVaR EWMA EVT	3.964	11.638	0.184	**2.361**	36.619	48.002	1.705	**2.845**
EWMA/CVaR EWMA Stsk	4.240	11.726	0.206	**3.076**	34.956	50.363	1.932	**3.224**
EWMA/CVaR GARCH FHS	3.830	12.066	0.167	**2.098**	38.808	44.894	1.367	**2.442**
EWMA/CVaR GARCH EVT	3.928	11.925	0.177	**2.335**	37.680	46.495	1.532	**2.639**
EWMA/CVaR GARCH Stsk	4.194	11.759	0.202	**2.678**	35.379	49.763	1.872	**2.819**

D.5 Tail Risk Targeting for US Data and Small Caps

Table XX shows additional performance results for US data and small caps, proxied by the S&P 500 and the German small cap index SDAX, respectively. The data are also obtained from Datastream. Following Marquering and Verbeek (2004), we use the three month treasury bill rate as risk-free rate for the US data. Panel A contains results for the S&P 500, which are mainly in line with the results of Tables V and VIII for the DAX. The dynamically managed target risk strategies exhibit higher returns than the 60/40 portfolio with comparable risk. Further, returns of the dynamically managed strategies are also comparable to the return of the

S&P 500, but with only about half of the volatility. The Historical Simulation based strategy performs again significantly worse than the dynamically managed strategies. This can also be seen by the Sharpe Ratios of the strategies. The highest Sharpe Ratios are obtained by the dynamically managed CVaR targeting strategies, followed by the volatility managed strategies. The lowest Sharpe Ratios are obtained by the HS managed strategy, the 60/40 portfolio and the S&P 500. However, the Sharpe Ratio test of Jobson and Korkie (1981) indicates that only the strategy based on the EWMA model combined with the skewed t distribution produces a statistically higher Sharpe Ratio. Similar results also hold for the maximum drawdown. The highest drawdown reduction, given by ΔMDD, is obtained by the dynamically managed CVaR strategies, whereas statically or volatility managed strategies are less successful in reducing the drawdown. Results for the economic value are also in line with the findings for the Sharpe Ratio. The economic value for the volatility managed strategies is low or even negative, whereas the dynamically managed target CVaR strategies produce high economic values.

In Panel B, we show results for the German small cap index SDAX. Interestingly, the dynamically managed CVaR strategies exhibit higher returns with lower risk than the 60/40 portfolio and the SDAX. The volatility managed strategies produce even higher levels of return but also exhibit significantly higher risk. Thus, as before, the highest Sharpe Ratios are obtained by the dynamically managed target CVaR strategies. The Jobson and Korkie (1981) test shows that four target CVaR strategies, but none of the target volatility strategies, exhibit statistically significant higher Sharpe Ratios than the HSD managed strategy. Similarly, the drawdown reduction is again the highest for the CVaR managed strategies, whereas the drawdown reduction of the volatility managed strategies is only small. Interestingly, the drawdown of the HSD managed strategy is even higher than the drawdown of the SDAX. The economic value of risk targeting is again the highest for the CVaR managed strategies. However, the increase in the investor's utility is not statistically significant. Interestingly, we find that the skewed t distribution does not work well for the SDAX, although this strategy works well for the DAX and the S&P 500. Hence, different estimation methods can perform quite differently when different assets are used. A possibility to obtain more robust results for different assets would be to combine

Table XX. Performance Results for the S&P 500 and SDAX

This table shows additional performance results of the volatility and CVaR targeting strategies for the S&P 500 and the SDAX for the period 01.01.2000 to 31.12.2018. Panel A shows results for the S&P 500, whereas Panel B shows results for the SDAX. The description of the columns is given in Table XIV.

	Panel A: Results for the S&P 500							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	4.760	13.091	0.236	-	36.587	36.766	-0.059	-
Vola EWMA	4.920	12.652	0.257	0.894	34.976	39.550	0.321	1.399
Vola GARCH	4.359	11.948	0.226	-0.210	33.524	42.059	0.129	0.325
CVaR Hist	3.083	9.103	0.159	-0.652	33.242	42.546	0.097	0.103
CVaR EWMA FHS	4.471	9.591	0.293	1.225	23.061	60.144	1.260	1.448
CVaR EWMA EVT	4.369	9.248	0.293	1.134	22.515	61.087	1.292	1.357
CVaR EWMA Stsk	5.123	9.934	0.347	**2.525**	25.619	55.722	1.755	**2.150**
CVaR GARCH FHS	4.527	10.050	0.285	0.924	26.064	54.953	1.132	1.336
CVaR GARCH EVT	4.404	9.736	0.282	0.863	24.509	57.640	1.138	1.289
CVaR GARCH Stsk	4.550	9.383	0.308	1.222	27.602	52.294	1.417	1.502
S&P 500	4.338	19.220	0.139	-0.751	57.859	-	-4.335	-1.585
60/40	3.395	10.139	0.173	-0.570	33.295	42.455	-	0.045
	Panel B: Results for the SDAX							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	8.475	13.524	0.486	-	69.219	-2.524	1.658	-
Vola EWMA	8.389	12.655	0.513	1.060	64.101	5.057	2.040	0.593
Vola GARCH	8.572	12.109	0.551	1.190	57.488	14.851	2.490	0.648
CVaR Hist	4.727	8.172	0.354	-1.296	38.189	43.436	0.451	-0.438
CVaR EWMA FHS	6.809	8.667	0.570	**2.278**	46.291	31.436	2.277	0.299
CVaR EWMA EVT	6.673	8.389	0.573	**2.270**	45.036	33.296	2.245	0.271
CVaR EWMA Stsk	5.822	9.139	0.434	-1.112	47.413	29.774	1.162	-0.250
CVaR GARCH FHS	7.671	9.204	0.628	**2.554**	43.768	35.173	2.905	0.588
CVaR GARCH EVT	7.487	8.901	0.630	**2.531**	42.341	37.286	2.843	0.536
CVaR GARCH Stsk	5.503	8.861	0.413	-1.118	42.852	36.529	0.958	-0.308
SDAX	6.489	15.792	0.293	-1.707	67.515	-	-1.523	-1.190
60/40	5.033	10.190	0.313	-1.535	47.612	29.479	-	-0.612

several forecasting methods (Allen et al., 2012, Halbleib and Pohlmeier, 2012, Taylor, 2014). In summary, the additional results for the S&P 500 and the SDAX confirm our earlier results for the DAX, i.e. portfolio risk is best managed by the portfolio's CVaR and portfolio risk should be estimated conditionally.

We next assess if our switching approach also works well for the S&P 500 and the SDAX. Results of the switching strategies are shown in Table XXI. The switching strategies are again successful in producing higher returns compared to the individual strategies with lower levels of volatility and drawdown. As a consequence, these strategies provide an enhanced risk-return profile with high Sharpe Ratio increases. The increase of the Sharpe Ratio is statistically significant for all dynamically managed strategies for the S&P 500. For the SDAX, all switching strategies exhibit significantly higher Sharpe Ratios. This finding is confirmed by the economic

Table XXI. Performance Results for the S&P 500 and SDAX: Switching Strategies
This table shows additional performance results for the strategies that switch between volatility and CVaR targeting using the S&P 500 and the SDAX. Panel A shows results for the S&P 500, whereas Panel B shows results for the SDAX. The description of the columns is given in Table XX.

	Panel A: Results for the S&P 500							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	4.760	13.091	0.236	-	36.587	36.766	-0.059	-
S&P 500	4.338	19.220	0.139	-0.751	57.859	-	-4.335	-1.694
EWMA/CVaR Hist	5.189	12.757	0.276	0.774	32.293	44.187	0.524	0.780
EWMA/CVaR EWMA FHS	5.424	11.787	0.318	**1.952**	26.331	54.492	1.230	**2.186**
EWMA/CVaR EWMA EVT	5.483	11.731	0.324	**1.980**	25.862	55.301	1.313	**2.189**
EWMA/CVaR EWMA Stsk	5.690	12.056	0.332	**2.674**	27.650	52.212	1.355	**2.627**
EWMA/CVaR GARCH FHS	5.506	12.123	0.316	**2.112**	27.504	52.465	1.145	**2.249**
EWMA/CVaR GARCH EVT	5.508	12.071	0.317	**2.114**	27.753	52.034	1.173	**2.280**
EWMA/CVaR GARCH Stsk	5.531	12.146	0.317	**2.150**	29.165	49.593	1.159	**2.155**
	Panel B: Results for the SDAX							
Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	8.475	13.524	0.486	-	69.219	-2.524	1.658	-
SDAX	6.489	15.792	0.293	-1.707	67.515	-	-1.523	-1.176
EWMA/CVaR Hist	10.282	11.870	0.703	**3.616**	46.276	31.458	4.224	1.538
EWMA/CVaR EWMA FHS	10.610	11.550	0.751	**5.068**	51.583	23.598	4.691	**1.996**
EWMA/CVaR EWMA EVT	10.700	11.501	0.762	**5.061**	50.807	24.746	4.800	**1.997**
EWMA/CVaR EWMA Stsk	9.910	11.734	0.680	**4.116**	51.845	23.210	3.939	1.606
EWMA/CVaR GARCH FHS	10.896	11.802	0.759	**5.322**	50.495	25.209	4.838	**2.104**
EWMA/CVaR GARCH EVT	11.004	11.736	0.772	**5.381**	49.798	26.241	4.972	**2.120**
EWMA/CVaR GARCH Stsk	10.340	11.772	0.714	**4.388**	49.516	26.660	4.327	**1.723**

value. All switching strategies produce economically high economic values and most utility increases are statistically significant. Hence, our simple switching approach does not only work well for the DAX, but also for the S&P 500 and the SDAX.

In total, results in this section show that risk targeting does not only work for the DAX, but also for US data and small caps. This holds especially for strategies that manage a portfolio's risk by dynamic risk models, where the best results are found for the strategies that manage a portfolio's tail risk. The portfolio's performance can further be improved by switching between volatility and CVaR targeting. In particular, mean-variance investors are willing to pay economically high and statistically significant fees to have access to these switching strategies.

D.6 Tail Risk Targeting in the Long Run

We have so far only examined a period of 18 years, which was marked by several crises. To assess if risk targeting is also beneficial in the long run, we use data for the US market from 1929 to 2018. Data for the US market and the risk-free rate are obtained from the website of Kenneth

French.[101] This data set is also used by Moreira and Muir (2017) and Dreyer and Hubrich (2019). Results for the long sample are shown in Table XXII, where we only show results for the switching strategies. As argued above, although CVaR targeting is superior to volatility targeting for our main data set, this does not necessarily hold in the long run, since CVaR targeting could be too conservative in uptrending periods. Furthermore, as before, we only show results for the switching strategies based on the EWMA model and the TSMOM crash indicator. Results for the GARCH(1,1) model and the other crash indicators were again quite similar, but slightly less profitable. In line with Moreira and Muir (2017) and our earlier findings, we find that volatility targeting significantly enhances the risk-return profile by producing a higher Sharpe Ratio with lower drawdowns. This finding is also in line with Moreira and Muir (2019) who show that long-term investors should time short-term volatility. However, in line with our main results, we find that the performance of risk targeting can further be enhanced by switching between volatility and CVaR targeting. All switching strategies increase the portfolio's Sharpe Ratio and these increases are all statistically significant. Similarly, mean-variance investors are willing to pay high fees to switch away from volatility targeting to the switching approach. This improved risk-return profile is also accompanied by significantly lower drawdowns of the switching strategies. Interestingly, although the skewed t distribution works well for the S&P 500 in the short sample, the same approach does not work well for the long US sample. As mentioned above, a further extension could be to use an average of several CVaR forecasting methods (Allen et al., 2012).

Similar to Moreira and Muir (2017, Figure 3), Figure IV shows the cumulative return of the US market, the 60/40 portfolio, the HSD managed strategy and the strategy that switches between volatility and CVaR targeting for a 100$ investment. For a better comparison, we rescale all strategies to the volatility of the US market. We only show results for the switching strategy that switches between the EWMA volatility model and the CVaR-EWMA-FHS model for the TSMOM indicator. This approach is easy to implement and does not need any estimated parameters. Thus, this approach could be appealing for practical implementations. Figure IV shows the clear outperformance of the risk targeting strategies. Both risk targeting strategies,

[101] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

Table XXII. Performance Results for the US Market in the Long Run: Switching Strategies
This table shows additional performance results for the strategies that switch between volatility and CVaR targeting using data of the US market for the period November 1929 to December 2018. The description of the columns is given in Table XIV.

Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	9.779	13.283	0.481	-	55.886	30.248	1.241	-
US Market	9.215	16.839	0.347	-2.218	80.121	-	-1.424	-2.425
60/40	8.620	13.597	0.387	-1.634	54.299	32.229	-	-1.465
EWMA/CVaR Hist	10.617	12.259	0.588	**3.876**	42.971	46.368	2.549	**2.871**
EWMA/CVaR EWMA FHS	10.580	11.801	0.607	**5.918**	39.002	51.321	2.740	**3.890**
EWMA/CVaR EWMA EVT	10.678	11.751	0.618	**6.183**	38.721	51.672	2.855	**4.051**
EWMA/CVaR EWMA Stsk	10.046	12.027	0.553	**3.785**	45.484	43.232	2.133	**2.560**
EWMA/CVaR GARCH FHS	10.239	12.084	0.566	**4.152**	44.918	43.938	2.285	**2.957**
EWMA/CVaR GARCH EVT	10.345	12.023	0.577	**4.578**	43.646	45.524	2.413	**3.215**
EWMA/CVaR GARCH Stsk	9.817	12.035	0.534	**2.463**	50.572	36.881	1.917	**1.759**

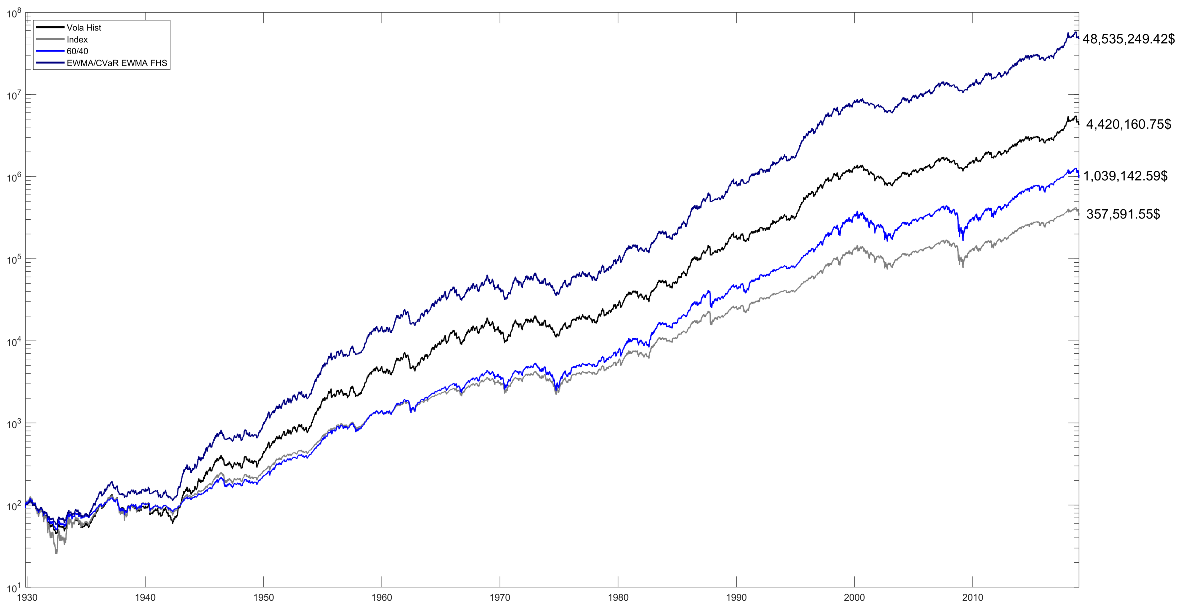


Figure IV. Cumulative Return of Risk Targeting. This figure plots the cumulative return of the US market, the 60/40 portfolio, the HSD target volatility strategy and a strategy that switches between volatility and CVaR targeting for the period 1929 to 2018. All strategies are rescaled to the same level of volatility.

the HSD managed strategy and the switching strategy, successfully capture the upside potential of the market while downside risk is limited. Nevertheless, the strategy that switches between volatility and CVaR targeting clearly outperforms the HSD managed strategy. A 100$ investment in the market portfolio would result in a terminal wealth of 357,591.55$. Invested in the target volatility strategy, the terminal wealth would increase to 4,420,160.75$. However, the strategy that switches between volatility and CVaR targeting produces a final wealth of even

48,535,249.42\$. This is in line with the results of Moreira and Muir (2019) that even long-term investors should time short-term risk. In particular, the outperformance of the switching strategy is quite steady over time, i.e. switching between volatility and CVaR targeting is advantageous in bull and bear regimes. Dreyer and Hubrich (2019) also find that risk targeting applied to the long US data set works well in several sub periods.

Table XXIII. Spanning Tests: Portfolio Alphas
This table shows results of spanning test of the risk targeting strategies for the US market. We run time-series regressions of each portfolio on the remaining strategies, where we use the US market, the HSD based volatility targeting strategy, the CVaR targeting strategy and the strategy that switches between volatility and CVaR targeting based on the TSMOM indicator δ_t. We report annualized percentage alphas with corresponding t-statistics in parentheses. Alphas that are significantly positive at the 10% level are given in bold. Alphas that are significantly negative at the 10% level are given in red.

	US Market		Volatility		CVaR		Switching	
TSMOM indicator	α	$t-$stat	α	$t-$stat	α	$t-$stat	α	$t-$stat
US Market	-	-	-2.204	(-0.697)	-3.808	(-1.196)	**-6.922**	(-2.001)
Volatility	**15.177**	(3.984)	-	-	-1.046	(-0.664)	**-7.122**	(-5.503)
CVaR	**18.132**	(4.611)	**3.571**	(2.198)	-	-	**-4.586**	(-2.899)
Switching	**27.088**	(6.100)	**9.737**	(6.822)	**7.639**	(4.668)	-	-

Finally, following Daniel and Moskowitz (2016) and Moreira and Muir (2017), we next repeat the examination of Table XIII and run time-series regressions for the non-managed and risk-managed portfolios. Results for these spanning tests are shown in Table XXIII. In line with our main results, we find that the market's alpha is always negative, once we control for the returns of the risk targeting strategies. When we control for the switching strategy, the alpha is even negative and statistically significant with a t-statistic of -2.001. In contrast, all risk targeting strategies have positive and statistically significant alphas with respect to the market.[102] The highest alpha of 27.088% with a t-statistic of 6.100 is obtained by the switching strategy. The alpha of the volatility targeting strategy becomes negative when we control for the returns of the CVaR targeting or switching strategy. In particular, when we control for the switching approach, the alpha is even significantly negative with a t-statistic of -5.503. Furthermore, CVaR targeting cannot be explained by volatility targeting, but exhibits a significantly negative alpha with respect to the switching approach. In contrast, the performance of the switching strategy

[102]The alpha for the volatility targeting strategy is higher than the alpha found by Moreira and Muir (2017). A possible explanation for this finding could be that we use daily rebalancing, whereas Moreira and Muir (2017) use monthly rebalancing. As shown in Section D.3, increasing the rebalancing interval reduces the profitability of volatility targeting.

cannot be explained by any other strategy and the switching strategy's alphas are economically high with t-statistics between 4.668 and 6.822. In total, all strategies exhibit significantly negative alphas when we control for the switching strategy, whereas the switching strategy has significantly positive alphas in all cases.

D.7 Out-Of-Sample Study: Corona Crisis

Finally, this section examines the performance of risk targeting during the recent corona crisis. This crisis is marked by one of the fastest drawdowns in history. Financial markets all over the world crashed over 30% within several weeks. For example, between 19.02.2020 and 18.03.2020, the DAX exhibits a maximum drawdown of 38.78%. This fast and severe crash was not only limited to equities, but nearly all asset classes crashed abruptly and simultaneously. This high co-crash behavior of nearly all asset classes made an adequate tail risk protection very changeling during this period. Most widely-used portfolio risk management approaches that aim to mitigate a portfolio's crash risk, such as the risk parity approach, failed as portfolio risk management tools. For example, the "S&P Risk Parity Index - 12% Target Volatility (TR)", a risk parity index that also targets a volatility of 12%, exhibits a maximum drawdown of 29.14% during the corona crisis.[103] Thus, the recent corona crisis challenges currently used tail risk hedging strategies and demonstrates the importance of a fast adapting crash risk mitigation tool. Consequently, assessing the performance of risk targeting during this challenging time gives further insights on the ability of risk targeting to mitigate extreme crashes. Moreover, assessing the performance of risk targeting during the recent corona crisis is particularly interesting, since this examination can be seen as an out-of-sample test of our earlier results. The first version of this paper was written in the years 2016 to 2018. Thus, applying the same methods as examined in the main part to data from the recent corona crisis demonstrates the profitability of risk targeting, without the potential of data mining. Since our main results are based on data that end in December 2018, we use data for the period that ranges from January

[103] Further details and data on this index can be obtained from `https://www.spglobal.com/spdji/en/indices/strategy/sp-risk-parity-index-12-target-volatility`. The aim of the index is described as: "The S&P Risk Parity Index – 12% Target Volatility seeks to measure the performance of a multi-asset risk parity strategy that allocates risk equally among equity, fixed income, and commodities futures contracts, while targeting a volatility level of 12%".

2019 to April 2020. This period is marked by an uptrending market in 2019 and the beginning of 2020. However, this calm and uptrending market ended abruptly by the severe crash in February and March 2020, initiated by the spread of the corona virus in Europe. Thus, this period is quite challenging for portfolio risk management approaches, since a good portfolio risk management tool has to adapt very fast to changing market environments.

Table XXIV. Performance Results for the DAX in the Corona Crisis
This table shows additional performance results for the DAX, the 60/40 portfolio, the volatility targeting strategies and the CVaR targeting strategies. The data range from 01.01.2019 to 30.04.2020. The description of the columns is given in Table XIV.

Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	-0.182	14.206	0.013	-	19.983	48.469	-0.598	-
Vola EWMA	2.066	13.422	0.181	**1.774**	18.673	51.847	2.078	**2.155**
Vola GARCH	3.507	12.908	0.301	**1.908**	18.036	53.492	3.799	**1.986**
CVaR Hist	-0.878	11.173	-0.046	-0.166	20.867	46.191	0.234	0.193
CVaR EWMA FHS	2.670	10.726	0.283	**2.075**	13.742	64.564	4.020	0.904
CVaR EWMA EVT	2.312	10.426	0.257	**2.010**	13.713	64.638	3.789	0.838
CVaR EWMA Stsk	2.911	9.673	0.339	**1.867**	13.107	66.202	4.711	0.769
CVaR GARCH FHS	3.578	10.560	0.374	**2.301**	14.088	63.671	5.014	1.086
CVaR GARCH EVT	3.450	10.200	0.375	**2.339**	13.684	64.712	5.041	1.008
CVaR GARCH Stsk	5.472	9.588	0.611	**1.991**	12.506	67.751	7.352	0.937
DAX	2.149	25.640	0.098	0.172	38.779	-	-7.093	-0.403
60/40	1.148	15.434	0.098	0.185	25.721	33.675	-	0.140

Table XXIV shows performance results for the DAX, the 60/40 portfolio, the volatility targeting strategies and the CVaR targeting strategies for the period starting in January 2019 and ending in April 2020. The DAX produces a slightly positive return with a high volatility and a drawdown of 38.779%. The bad risk-return profile during this period can also be seen by the low Sharpe Ratio of 0.098. The 60/40 portfolio produces a lower return with a lower volatility. This results in an equally high Sharpe Ratio of 0.098. Interestingly, the HSD managed strategy exhibits a negative return with a very low Sharpe Ratio.[104] In contrast, the EWMA and GARCH(1,1) managed strategies produce higher returns with lower levels of volatility. Both strategies exhibit significantly higher Sharpe Ratios than the HSD managed strategy. Similarly, the conditionally managed CVaR targeting strategies also produce statistically significant Sharpe Ratio increases. However, the HS managed CVaR strategy produces a negative return and a negative Sharpe Ratio. Thus, as before, more accurate risk models are rewarded with

[104] The Sharpe Ratio of this strategy is positive since the annualized return of -0.182% is less negative than the return of the risk-free asset in this period.

higher performance and utility gains. All risk targeting strategies are successful in reducing the portfolio's drawdown. For example, by using a CVaR targeting strategy, the DAX's drawdown of 38.779% can be reduced to only 12.506%. This high drawdown reduction of CVaR targeting is accompanied by a higher return. Furthermore, mean-variance investors are willing to pay high fees to switch to a conditionally managed target risk strategy. The highest utility gains are again found for the CVaR targeting strategies. In total, results for the corona crisis indicate that risk targeting can enhance a portfolio's return while simultaneously crash risk is significantly reduced. In line with our earlier results, conditionally managed strategies outperform strategies that manage portfolio risk based on unconditional risk models. In particular, the HSD based target volatility strategy fails to enhance the risk-return profile during the corona crisis and underperforms the non-managed portfolio.

Table XXV. Performance Results for the DAX in the Corona Crisis: Switching Strategies
This table shows additional performance results for the strategies that switch between volatility and CVaR targeting using the DAX. The switching strategies use the EWMA volatility model and the TSMOM based indicator δ_t. The data range from 01.01.2019 to 30.04.2020. The description of the columns is given in Table XIV.

Model	Return	Volatility	SR	z_{JK}	MDD	ΔMDD	$\Delta_{MV}^{\gamma=5}$	DM-test
Vola Hist	-0.182	14.206	0.013	-	19.983	48.469	-0.598	-
DAX	2.149	25.640	0.098	0.172	38.779	-	-7.093	-0.459
60/40	1.148	15.434	0.098	0.185	25.721	33.675	-	0.212
EWMA/CVaR Hist	-1.535	13.504	-0.087	-0.419	21.756	43.897	-1.564	-0.384
EWMA/CVaR EWMA FHS	1.553	12.290	0.156	0.915	16.376	57.771	2.155	1.102
EWMA/CVaR EWMA EVT	1.170	12.136	0.126	0.704	16.337	57.872	1.845	0.936
EWMA/CVaR EWMA Stsk	2.399	12.002	0.231	1.089	15.875	59.065	3.149	0.847
EWMA/CVaR GARCH FHS	2.328	12.326	0.219	1.242	16.319	57.918	2.916	1.151
EWMA/CVaR GARCH EVT	2.038	12.170	0.198	1.066	16.192	58.246	2.703	1.025
EWMA/CVaR GARCH Stsk	4.307	12.151	0.385	1.358	15.441	60.183	4.995	0.955

In Table XXV, we show results for the strategies that switch between the EWMA model and the CVaR targeting strategies based on the TSMOM indicator δ_t. All strategies that switch to a conditionally managed CVaR strategy clearly outperform the non-managed and HSD managed strategies. Furthermore, the switching strategies typically have higher Sharpe Ratios than the EWMA managed strategy. Thus, switching away from volatility targeting when a crash is expected produces a superior risk-return profile. Nevertheless, the switching strategies have slightly lower Sharpe Ratios than the CVaR managed strategies.[105] This finding is quite intu-

[105]Results for the GARCH based switching strategies or the strategies that use the volatility based indicator δ_t

itive, since CVaR targeting is expected to perform well in an extreme crash period as examined here. In contrast, as shown in the previous section, the switching approach is beneficial in the long run, since using CVaR targeting on every day would be too conservative. Thus, results of Tables XXIV and XXV are in line with our earlier findings.

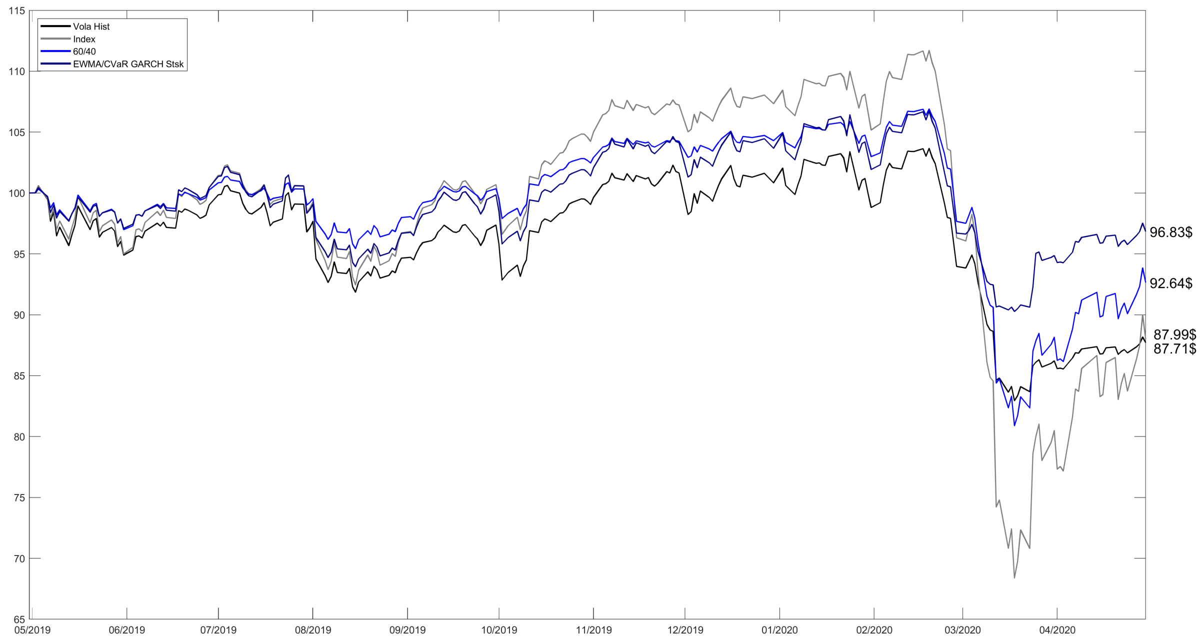


Figure V. Cumulative Return of Risk Targeting: Corona Crisis. This figure plots the cumulative return of the DAX, the 60/40 portfolio, the HSD based target volatility strategy and a strategy that switches between volatility and CVaR targeting for the one year period that ranges from May 2019 to April 2020.

In order to further demonstrate the profitability of the different strategies during the corona crisis, we show in Figure V the cumulative return of the DAX, the 60/40 portfolio, the HSD managed strategy and a switching strategy for the one year period from 01.05.2019 to 30.04.2020. This figure shows that the 60/40 portfolio and the risk targeting strategies successfully reduce the portfolio's drawdown, where the best drawdown reduction is found for the switching strategy. The bad performance of the HSD managed strategy during the corona crisis mainly occurs since this strategy fails to capture the upside potential of the DAX. In contrast, the switching strategy successfully captures the upside potential in the calm period and significantly reduces the drawdown in the crash period. This again shows that a conditional risk model that success-

are superior to the results shown here. However, in order to be in line with the results shown before, we only show results for the EWMA model and the TSMOM indicator.

fully adapts to changing market environments is needed when portfolio risk in managed.

In summary, results in this section show that our risk targeting strategies also perform well during the recent corona crisis and successfully reduce the DAX's crash risk. This holds especially for dynamically managed strategies. Unconditionally managed strategies can be used as an adequate tail risk mitigation tool, but these strategies fail to capture the portfolio's upside potential in calm markets. The switching strategy also performs well during this crisis but is, as expected, outperformed by the CVaR targeting strategies. This is in line with the motivation of the switching strategy that CVaR targeting is advantageous in crises but too conservative in the long run. However, switching between volatility and CVaR targeting outperforms the strategy that manages volatility on every day. Thus, the switching strategy is optimal for long-term investors who want to capture an asset's upside potential while simultaneously downside risk is limited in crash periods.

Chapter 2

Risk Managed Momentum

2.1 Introduction

Since the seminal paper of Jegadeesh and Titman (1993) many studies have documented the huge return potential of the momentum strategy that buys past winners and sells past losers.[1] Jegadeesh and Titman (1993) find that stocks which performed well in the past tend to outperform stocks with a low previous performance. The academic literature documents that momentum investing delivers abnormally high returns, Sharpe Ratios and alphas compared to the market and other factor portfolios.[2] This result is not only limited to US stocks, but also holds internationally and for almost every asset class.[3] However, the huge return potential of the momentum strategy is typically accompanied by high risk, especially in the left tail. For example, Barroso and Santa-Clara (2015) find that the momentum strategy exhibits a volatility of 27.53%, whereas the market's volatility of 18.96% is significantly lower. In particular, momentum's volatility is highly volatile and sometimes takes extreme values (Barroso and Santa-Clara, 2015, Fig. 2). This high and unstable volatility makes the momentum strategy unappealing for risk-averse investors who have a preference for a low and stable volatility. Further, besides the

[1] The definition of the (cross-sectional) momentum strategy examined in this paper is different to the time series momentum strategy of Moskowitz et al. (2012), which is more related to the field of trend-following (see also Goyal and Jegadeesh (2017) and Kim et al. (2016) for a comparison of both approaches).

[2] Several studies show that momentum returns cannot be explained by traditional models, like the CAPM or three factor model (Fama and French, 1996).

[3] See Jegadeesh and Titman (1993), Jegadeesh and Titman (2001), Hong et al. (2000) and Lesmond et al. (2004) for momentum in the US stock market, Rouwenhorst (1998) for momentum in Europe, Griffin et al. (2003) and Fama and French (2012) for international momentum, Moskowitz and Grinblatt (1999), Lewellen (2002), Chordia and Shivakumar (2002) and Grobys et al. (2018) for industry momentum, Lewellen (2002) for momentum of investment styles, Novy-Marx (2012) and Asness et al. (2013) for momentum in different markets and asset classes, Richards (1997), Chan et al. (2000) and Bhojraj and Swaminathan (2006) for country momentum and Carhart (1997) for momentum of funds.

high (average) volatility of momentum investing, several recent studies document a vast crash risk of this investment strategy.[4] Barroso and Santa-Clara (2015) state that momentum investing translates into a portfolio with an extremely negatively skewed and fat tailed return distribution, which induces periods with extremely high drawdowns. For example, Barroso and Santa-Clara (2015, Table 1) find a skewness of -2.47, a kurtosis of 18.24 and a minimum monthly return of -78.96% for the value-weighted momentum strategy. In contrast, the market portfolio exhibits a positive skewness of 0.17, a kurtosis of only 7.35 and a minimum monthly return of -29.04%. These values are even more extreme for the equally weighted momentum strategy examined in this paper. These characteristics of the momentum portfolio imply a fat left tail of the momentum portfolio's return distribution, which makes extremely negative returns, so called "momentum crashes", more likely and severe than predicted by the normal distribution. Similarly, Moreira and Muir (2017) and Barroso and Maio (2018) compare several portfolio strategies and find that the momentum portfolio exhibits the most pronounced risk profile in terms of volatility, skewness, kurtosis and drawdowns. These "momentum crashes", which are extensively examined in Grundy and Martin (2001), Daniel and Moskowitz (2016) and Daniel et al. (2017), are a big drawback of momentum investing. For example, Barroso and Santa-Clara (2015) document a two-month-return of the momentum portfolio of -91.59% in 1932. Hence, a sharp crash can reverse gains that were built over years in only two months. In addition, Barroso and Santa-Clara (2015) show that the recovery from a momentum crash can last up to 31 years. Furthermore, Daniel et al. (2017) find eight months with losses higher than 30% as well as six months with losses higher than 40%, but no month with gains higher than 30%. Besides the risk aversion of most investors, investors are typically loss-averse, i.e. they weight losses much higher than gains of the same magnitude (see Benartzi and Thaler (1995), Aït-Sahalia and Brandt (2001), Ang et al. (2006a) and Berkelaar et al. (2004)).[5] Interestingly, Min and Kim (2016) find that momentum crashes typically occur in times when investors have a high

[4]See Grundy and Martin (2001), Daniel et al. (2017), Barroso and Santa-Clara (2015), Daniel and Moskowitz (2016), Moreira and Muir (2017), Barroso and Maio (2018), Ruenzi and Weigert (2018) and Chabot et al. (2014) for studies on the huge crash risk of momentum investing.

[5]Similar to the loss aversion of investors, investors are typically crash-averse and are willing to pay high fees to avoid crashes (Bollerslev and Todorov, 2011, Chabi-Yo et al., 2018, Van Oordt and Zhou, 2016). This observation fits well to the safety first theory of Arzac and Bawa (1977).

marginal value of wealth and weight losses higher than gains. Consequently, due to momentum's high crash risk, the huge long-term potential of momentum investing is unavailable for most investors and managing momentum's risk is crucial in order to make momentum strategies available for investors who are risk- and/or loss-averse.

Several approaches to manage the risk of the momentum strategy have been presented in the literature. Grundy and Martin (2001), Martens and Van Oord (2014) and Barroso (2016) show that the momentum portfolio has a highly time-varying beta, which is negative after market declines. Grundy and Martin (2001) show that by hedging this time-varying risk, momentum crashes can be attenuated. However, Daniel and Moskowitz (2016), Martens and Van Oord (2014), Barroso (2016) show that this approach does not work well out-of-sample, since past betas hardly predict future betas as also found by Ang et al. (2006a). For that reason, Daniel and Moskowitz (2016) propose a volatility based approach to manage momentum's risk. This approach successfully dampens momentum crashes and significantly enhances the risk-return profile of the momentum portfolio. Similarly, Barroso and Santa-Clara (2015) scale the amount invested in the momentum portfolio by its Realized Volatility (RV), which has the advantage that the volatility of the momentum portfolio is constant over time and equals a predetermined value.[6] By targeting a constant level of volatility, the risk of the momentum strategy is managed by reducing the exposure to the momentum portfolio when momentum's volatility is high and vice versa.[7] Barroso and Maio (2019) find that there exists a negative risk-return relation for the momentum strategy, whereas most other portfolio strategies exhibit a positive relation. In other words, periods of an increased momentum volatility are related to low momentum returns. Hence, although a negative risk-return relation is not needed to increase the Sharpe Ratio of the

[6] This approach is also used by Moreira and Muir (2017) to manage the risk of several factor portfolios. The authors find that volatility managing works best for the momentum strategy. Similarly, Barroso and Maio (2018) and Cederburg et al. (2020) find that volatility targeting works well for strategies that strongly deviate from normally distributed returns, like momentum or the Betting against Beta strategy of Frazzini and Pedersen (2014). Grobys et al. (2018) and Du Plessis and Hallerbach (2017) show that volatility targeting also mitigates the crash risk of the industry momentum strategy of Moskowitz and Grinblatt (1999) and delivers a higher risk-adjusted performance compared to the non-managed strategy.

[7] Since the momentum portfolio is a zero-investment strategy, the weight invested in this portfolio can be scaled arbitrarily. The amount invested in the winners portfolio is financed by selling the losers portfolio. A non-managed momentum strategy is usually 1$ long in the winners portfolio and 1$ short in the losers portfolio. For the risk-managed momentum strategy, the amount invested long and short fluctuates over time and is increased if momentum's volatility is low and vice versa. See, for example, Figure 4 of Barroso and Santa-Clara (2015) on how the exposure of the volatility managed strategy fluctuates over time.

volatility managed portfolio (Cederburg et al., 2020, Dachraoui, 2018, Harvey et al., 2018, Moreira and Muir, 2017), this observation shows that managing momentum's volatility is an appealing approach. Generally, the benefits of volatility targeting have already been shown for equity portfolios by Marquering and Verbeek (2004), Bollerslev et al. (2018), Harvey et al. (2018), Moreira and Muir (2019) and Rickenberg (2020b).[8] Furthermore, Moreira and Muir (2019) examine volatility timing for long-term investors.

Although volatility targeting, as done by Barroso and Santa-Clara (2015) using the RV model, seems to be a promising approach to manage momentum crashes, this method has several drawbacks that are particularly relevant for the momentum strategy (see Rickenberg (2020b) for a general discussion of the drawbacks of volatility targeting). For example, the RV method does not properly forecast next month's volatility and relies on the assumption that momentum returns follow a random walk.[9] For that reason, we first present advanced volatility models that are able to accurately forecast risk and are more successful in targeting a constant level of risk. Rickenberg (2020b) shows that conditional approaches, like the GARCH(1,1) model of Bollerslev (1986), are more successful in targeting a constant level of risk compared to unconditional approaches, like the RV model that is used in Barroso and Santa-Clara (2015). Since a higher forecasting accuracy, or equivalently a more stable portfolio volatility, coincides with higher utility gains for risk targeting investors (Bollerslev et al., 2018), these models should also deliver an enhanced risk-return profile and higher utility gains. Furthermore, a portfolio volatility stabilization is also related to lower left tail risk and drawdowns

[8]Volatility targeting is used to manage the risk of a whole portfolio. Alternatively, volatility managing can also be used to weight a portfolio's constituents inversely to their volatility. The benefits of these volatility managed portfolios have been shown by Fleming et al. (2001), Fleming et al. (2003), Kirby and Ostdiek (2012), Han (2005) and Taylor (2014). These studies demonstrate that choosing an asset's weight inversely to its volatility produces significant utility gains, higher risk-adjusted returns and lower drawdowns. Goyal and Jegadeesh (2017) show that volatility managing could also be used to weight the assets within the winners and losers portfolio inversely to their volatility. The authors find that this approach is superior to traditional weighting schemes that are usually used for the momentum portfolio. The same weighting scheme was also used by Moskowitz et al. (2012) and Kim et al. (2016) for the time series momentum strategy. Zakamulin (2015), Moreira and Muir (2017, Sec. I.E) and Harvey et al. (2018) show that both approaches, managing volatility on an asset-level and portfolio-level, can easily be combined. The authors find that both approaches add value and that combining both approaches is appealing. Generally, Moreira and Muir (2017, Sec. II.D) show that the volatility targeting strategy, which is used in our paper, is different to the inverse volatility weighting. Du Plessis and Hallerbach (2017) and Rickenberg (2020c) apply both approaches to the industry momentum strategy.

[9]We use the variance ratio test of Lo and MacKinlay (1988) to show that the random walk hypothesis is clearly rejected for the momentum portfolio.

(Dreyer and Hubrich, 2019, Harvey et al., 2018). Based on the Realized Volatility theory, we use the HAR-RV model of Corsi (2009), which was also successfully used in Bollerslev et al. (2018). Additionally, we use the EWMA model, the GARCH(1,1) model of Bollerslev (1986) and the GJR-GARCH model of Glosten et al. (1993) as conditional volatility models. Daniel and Moskowitz (2016) successfully use the GJR-GARCH model to manage momentum's risk. The GARCH(1,1) model was also used in an earlier version of Barroso and Maio (2018) to manage the monthly risk of the momentum portfolio, but the authors find no improvements of this model. However, in order to forecast monthly volatility, Barroso and Maio (2018) fit the GARCH(1,1) model to monthly data. Fitting volatility models to monthly returns requires huge amounts of data to obtain accurate parameter estimates and ignores important information that is contained in daily returns. For that reason, we fit the conditional volatility models to daily data and show how monthly volatility can be estimated by these models. Furthermore, we use the result of Drost and Nijman (1993) and Meddahi and Renault (2004) on the temporal aggregation of the GARCH(1,1) model, i.e. we model monthly volatility based on the GARCH(1,1) model fitted to daily data. We find that conditional volatility models are more accurate in targeting a constant level of portfolio volatility and that this higher accuracy is rewarded by a higher risk-adjusted performance, lower drawdowns and high utility gains as also found by Bollerslev et al. (2018).

A second disadvantage of the volatility targeting approach of Barroso and Santa-Clara (2015) is that higher moments and the high non-normality of the momentum portfolio are ignored. The high negative skewness combined with the high kurtosis induces a high probability of extremely negative returns, which is not captured by managing volatility. Generally, managing a portfolio's volatility is only sufficient if the portfolio's returns are normally distributed. To assess the importance of higher moments when momentum's risk is managed, we examine the distributional properties, especially conditional skewness and kurtosis, of the momentum portfolio. The importance of incorporating (conditional) skewness and kurtosis has been shown in many fields of finance, e.g. asset pricing (Dittmar, 2002, Harvey and Siddique, 2000, Kraus and Litzenberger, 1976), portfolio selection (see Wang et al. (2012), Guidolin and Timmermann

(2008), Ghysels et al. (2016) among others), option pricing (Barone-Adesi et al., 2008) and risk management (Bali et al., 2008). We show that conditional skewness of the momentum portfolio is highly time-varying, mostly negative and sometimes takes extreme values or may even not exist. Further, conditional kurtosis is highly time-varying and there are periods when momentum's kurtosis takes extreme values or may not exist. We show that the extreme skewness of the momentum portfolio results since the winners' and losers' skewness moves in opposite directions, i.e. times of an extremely low skewness of the winners coincide with times of a high skewness of the losers and vice versa. Hence, buying winners and selling losers produces a highly time-varying skewness that can be extremely negative during momentum crashes. Further, we find that the kurtosis of winners and losers comove, i.e. periods of a high kurtosis of the winners coincide with periods of a high kurtosis of the losers and vice versa. Due to this relation of the winners' and losers' skewness and kurtosis, the momentum portfolio's return distribution is highly non-normal with an extremely fat left tail. This high crash risk of the momentum portfolio is not incorporated by the volatility targeting portfolio, which makes volatility targeting an insufficient tail risk mitigation tool for the momentum portfolio.[10] To account for the significant non-normality of momentum returns, we use the target Value at Risk (VaR) and target Conditional Value at Risk (CVaR) strategies of Rickenberg (2020b) that target a constant level of VaR and CVaR of the momentum portfolio.[11] When forecasting momentum's downside risk, we especially account for time-varying conditional skewness and kurtosis, as shown by Jondeau and Rockinger (2003) and Bali et al. (2008) based on the skewed t distribution of Hansen (1994). We estimate VaR and CVaR both unconditionally as in Bali et al. (2009) and conditionally by combining the time-varying skewed t distribution with the GARCH(1,1) and GJR-GARCH model. Similar to Wong and So (2003) and So and Wong (2012), we fit the model

[10] A similar observation also holds for the returns of hedge funds. Since hedge funds' returns are highly non-normal, the often used mean-variance approach is insufficient when analyzing hedge funds. As an alternative to the mean-variance approach, approaches based on downside risk measures that incorporate higher moments should be used (Agarwal and Naik, 2004).

[11] A similar approach of incorporating higher moments when momentum's risk is managed would be to combine the volatility targeting strategy with the approach of Taylor (2005). The author shows how estimates of volatility can be obtained from VaR estimates. As a consequence, this approach also incorporates higher moments when volatility is forecasted. However, combining volatility targeting with this approach is quite similar to the target VaR strategy used in our paper.

to daily data and use simulations to obtain forecasts for monthly downside risk.[12] Additionally, we forecast momentum's downside risk using Filtered Historical Simulation, which is also used by Barone-Adesi et al. (2008) and Engle (2011). Further, and particularly interesting for practical implementations, we also use the simple Historical Simulation approach, combined with the square root of time rule. This approach forecasts next month's risk by simply scaling up forecasts of daily downside risk by $\sqrt{21}$ (Danielsson and Zigrand, 2006, Wang et al., 2011).

Rickenberg (2020b) shows that targeting a constant level of downside risk, especially in terms of CVaR, is typically superior in crash periods, whereas volatility targeting exhibits higher returns in uptrending markets. Since the momentum portfolio trends upwards most of the time and momentum crashes are typically severe, short-lived and partly forecastable, we additionally develop strategies that switch between volatility and CVaR targeting. When there are no signs of a momentum crash, the portfolio is managed by volatility, whereas CVaR targeting is used when the probability of a momentum crash is high. Cooper et al. (2004), Daniel and Moskowitz (2016) and Wang and Xu (2015) show that momentum crashes typically occur when the past return of the market is negative and/or past market volatility is high. Further, Barroso and Santa-Clara (2015) and Barroso and Maio (2019) show that momentum's volatility is also successful in predicting momentum crashes. For that reason, we use several crash indicators based on the past market return, past market volatility and expected momentum volatility that indicate if momentum's risk is managed by volatility or CVaR. We find that the strategies that switch between volatility and CVaR targeting further enhance the risk-adjusted performance compared to the strategies that only manage volatility. In particular, by exhibit higher returns with lower risk, the RV managed strategy of Barroso and Santa-Clara (2015) is clearly outperformed by these switching strategies. Especially momentum's left tail risk, and as a consequence momentum's crash risk, is significantly reduced by the switching strategies. These results hold for several subperiods and data sets.

[12] Wong and So (2003) and So and Wong (2012) use the skewed generalized t distribution of Theodossiou (1998), which has one additional parameter compared to the skewed t distribution of Hansen (1994). Nevertheless, by choosing this parameter adequately, as done in Wong and So (2003) and So and Wong (2012), the distribution of Theodossiou (1998) simplifies to the distribution of Hansen (1994). Thus, this approach is similar to our approach. However, Wong and So (2003) and So and Wong (2012) do not account for the autoregressive pattern in conditional skewness and kurtosis as done by our approach.

Since unconditional performance measures, like the Sharpe Ratio, have several disadvantages and can only be applied when returns are normally distributed, we use more sophisticated methods to assess momentum's performance. In particular, the Sharpe Ratio is an inadequate performance measure for dynamic trading strategies, like the risk targeting strategies examined here (Han, 2005, Marquering and Verbeek, 2004).[13] For that reason, we assess the economic value of the switching strategies in a utility based setting, i.e. we calculate the annualized fee an investor with a given utility function is willing to pay to switch from the RV managed strategy to one of the strategies that switches between volatility and CVaR targeting. When calculating the economic value, we regard three different types of investors. First, we follow the approach of Fleming et al. (2001), Fleming et al. (2003), Han (2005) and Kirby and Ostdiek (2012) and calculate the economic value for an investor with quadratic utility. Second, since investors typically dislike a negative skewness and high kurtosis (Kraus and Litzenberger, 1976, Scott and Horvath, 1980), we additionally calculate the economic value for an investor with constant relative risk aversion (CRRA) as also done by Dreyer and Hubrich (2019), Jondeau and Rockinger (2012), Ghysels et al. (2016) and Guidolin and Timmermann (2008). Third, since investors typically weight losses higher than gains, we additionally calculate the economic value for loss-averse investors (Aït-Sahalia and Brandt, 2001, Benartzi and Thaler, 1995, Berkelaar et al., 2004). We find that the switching strategies deliver economically large and statistically significant utility gains for all three types of investors. This holds especially for investors that are highly risk- or loss-averse and during periods of a momentum crash.

This paper is structured as follows. Section 2.2 reviews the literature on momentum's high crash risk. Section 2.3 describes the volatility targeting approach of Barroso and Santa-Clara (2015) and extends this approach by using more advanced volatility models. Section 2.4 presents the VaR and CVaR targeting strategies and presents the models used to forecast monthly downside risk. Section 2.5 develops the strategies that switch between volatility and CVaR targeting, based on several momentum crash indicators. Section 2.6 shows the empirical

[13] This observation also holds for other unconditional performance measures, such as the unconditional alpha. See Boguth et al. (2011), Cederburg and O'Doherty (2016), Cederburg et al. (2020) and Schneider et al. (2020) for studies on the drawbacks of unconditional alphas. However, due to the importance of these alphas, we additionally use alphas as performance evaluation tools.

results and Section 2.7 concludes the paper.

2.2 Crash Risk of Momentum

This section summarizes the literature and the source of the high crash risk of the momentum strategy that buys assets with a high past return and sells assets with a low past return. Further details on the construction of the momentum portfolio are summarized in Appendix A. Moreover, Appendix A also reviews the literature on momentum and shows that the momentum portfolio produces a high return but is typically invested in riskier assets, i.e. assets with a small market capitalization, high beta and high volatility. Hence, the abnormally high long-term performance of the momentum strategy is accompanied with a high risk and infrequent periods of extremely high losses, so called "momentum crashes". For example, Barroso and Santa-Clara (2015, Table 1) show that the momentum portfolio exhibits an annualized volatility of 27.53%, whereas the market exhibits a volatility of only 18.96%. Moreover, the volatility of the momentum portfolio is highly unstable over time (Barroso and Santa-Clara, 2015, Fig. 2). This high and fluctuating volatility of the momentum strategy makes the huge return potential of momentum investing unavailable for highly risk-averse investors. Additionally, Min and Kim (2016) state that the momentum strategy is unappealing for risk-averse investors, since losses of the momentum portfolio typically occur in times when investors are highly risk-averse. Furthermore, Barroso and Santa-Clara (2015) find that momentum investing translates into a portfolio with an extremely negatively skewed and fat tailed return distribution.[14] This high left tail risk translates into a strategy with extremely high drawdowns. For example, Barroso and Santa-Clara (2015) find that the momentum portfolio exhibits a skewness of -2.47 and a kurtosis of 18.24, whereas the market is positively skewed and significantly less fat-tailed with a skewness of 0.17 and a kurtosis of 7.35. This implies a fat left tail of the momentum portfolio's

[14]The high negative skewness of the momentum portfolio results since winners are typically negatively skewed, whereas losers are typically positively skewed and the momentum portfolio is long the winners and short the losers (see Harvey and Siddique (2000) and Daniel and Moskowitz (2016, Table 1)). Similarly, Chen et al. (2001) find that large cap stocks, i.e. stocks that are typically in the winners portfolio, have a lower skewness than stocks with a low market capitalization. In particular, Jacobs et al. (2015) find that skewness is an important determinant of momentum returns and that a huge part of the high momentum returns can be explained as compensation for skewness risk. In other words, winners outperform losers since winners exhibit a lower skewness than losers.

return distribution, which makes extremely negative returns much more likely than anticipated by the normal distribution. Daniel and Moskowitz (2016) show that the strong positive average return and high Sharpe Ratio of momentum is punctuated with occasional crashes.[15] For example, Barroso and Santa-Clara (2015) find that the minimum monthly return of the momentum portfolio is -78.96% compared to -29.04% of the market portfolio. Similarly, Daniel and Moskowitz (2016, Table 2) list the returns of the 15 worst months of the momentum portfolio, which range from -24.04% to -74.36%.[16] Interestingly, the market return was positive in all of these months. Hence, while a momentum investors suffers an extremely high loss, an investor who simply invests in the market portfolio simultaneously earns a positive return. Further, Barroso and Santa-Clara (2015) document a two-month momentum return of -91.59% in 1932. Hence, a sharp crash of the momentum portfolio can reverse almost all gains that were built over years in only two months. Furthermore, Daniel et al. (2017) find eight months with losses higher than 30% as well as six months with losses higher than 40%, but no month with gains higher than 30%.[17] In particular, Barroso and Santa-Clara (2015) show that the recovery from a momentum crash can last up to 31 years. In total, these results show that momentum investing, although this strategy produces high average returns, is not applicable in practice unless the crash risk of this strategy is significantly reduced. Since most investors are loss-averse (Benartzi and Thaler, 1995), it is questionable if the high returns of momentum investing compensate for these extreme crashes. Min and Kim (2016) show that momentum crashes typically occur in times of bad markets, i.e. times when investors require a high risk premium and the marginal utility of wealth is higher. Thus, momentum crashes occur just when investors weight losses higher than gains. For that reason, Min and Kim (2016) argue that the momentum strategy bears an extreme downside risk for investors, unless the severity of momentum crashes is

[15] Daniel and Moskowitz (2016) and Daniel et al. (2017, Sec. 1) examine the drivers of these momentum crashes. We will come back to this point in Section 2.5.

[16] Daniel and Moskowitz (2016) find a slightly higher minimum monthly return compared to Barroso and Santa-Clara (2015), although both studies use the $t-12$ to $t-2$ ranking and value-weighted winners and losers portfolios. However, Daniel and Moskowitz (2016) use slightly different portfolio breakpoints as explained in Section 3 of the document `http://www.kentdaniel.net/data/momentum/mom_data.pdf`. Daniel and Moskowitz (2016) also find a lower skewness of -4.70 compared to the skewness of -2.47 found by Barroso and Santa-Clara (2015). Thus, the risk characteristics of the momentum portfolio strongly depend on the portfolio's construction method. Similarly, we use equally weighted winners and losers portfolios and find a higher risk for this construction method compared to the value-weighted approach of Barroso and Santa-Clara (2015).

[17] A similar but less extreme result also holds for stock market returns as shown by Chen et al. (2001).

reduced. Harvey and Siddique (2000) also show that the momentum strategy is highly related to (systematic) risk. Furthermore, Ruenzi and Weigert (2018) find that a substantial part of the abnormal momentum returns can be explained as compensation for the strategy's high crash risk. This results since the momentum strategy loads heavily on crash-sensitive assets. Similarly, Jacobs et al. (2015) state that the high momentum returns are a compensation for momentum's high skewness risk. Consequently, although the momentum portfolio offers an extremely high long-term performance, this performance cannot be captured by risk- and loss-averse investors. Therefore, managing the risk of momentum strategies seems crucial in order to make momentum strategies available for investors. For that reason, we next present several possibilities how the risk of the momentum strategy can be managed.

2.3 Volatility Managed Momentum Strategy

In the previous section, we have argued that the abnormally high return of the momentum strategy is accompanied with a high volatility, a high left tail risk and a high probability of extremely negative returns, so called momentum crashes. So far, several approaches to mitigate these momentum crashes have been proposed in the literature. Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) show that the mitigation of momentum crashes is a main driver of the performance of risk-managed momentum strategies. Grundy and Martin (2001), Cooper et al. (2004) and Daniel and Moskowitz (2016) argue that momentum crashes typically occur when the market suddenly rises after a longer period of negative market returns. The reason for this finding is that in times of a declining market, the momentum portfolio is long low beta stocks and short high beta stocks, which produces a highly negative beta of the momentum portfolio. Thus, a sharp increase of the market induces an extremely negative return of the momentum portfolio.[18] Grundy and Martin (2001) show that the beta of the momentum portfolio is highly time-varying and that hedging against this time-varying beta risk can significantly improve the performance of the momentum portfolio. However, their approach is

[18]Daniel and Moskowitz (2016) find that past losers typically outperform past winners when the market rebounds after a long period of a declining market. Since the momentum portfolio is short the past losers, the portfolio return becomes highly negative in these periods, which induces a momentum crash.

based on forward-looking betas and is not implementable in practice. Daniel and Moskowitz (2016) show that an approach based on ex-ante estimated betas does not significantly improve the performance of the hedged portfolio (see also Ang et al. (2006a) who find that past betas hardly predict future betas).[19] As a consequence, other tail risk hedging methods are needed in order to reduce momentum's crash risk.

Barroso and Santa-Clara (2015) show that the volatility of the momentum portfolio is highly time-varying and predictable over time (see also Barroso and Maio (2019, Table 1 and 2)). Furthermore, the authors decompose momentum's risk and find that the momentum specific risk is far more important than market risk. Therefore, Barroso and Santa-Clara (2015) propose to manage momentum's risk by overlaying the momentum strategy with a strategy that targets a constant level of volatility over time.[20] Volatility targeting is an easy but effective approach to reduce the crash risk of the momentum strategy. In contrast to the traditional momentum strategy, where each month 1$ is invested long in the winners and short in the losers portfolio, the dollar exposure of the volatility targeting strategy is time-varying, based on a forecast of momentum's volatility (Barroso and Santa-Clara, 2015, Fig. 2). Applying volatility targeting to the momentum portfolio is advantageous, since Barroso and Santa-Clara (2015) show that momentum returns are low when the expected volatility of the momentum strategy is high and vice versa.[21] In particular, Barroso and Maio (2019) show that there exists a negative relation between the momentum portfolio's volatility and return, whereas the volatility-return relation is positive for most other factors. Although a negative risk-return relation is not needed to increase the Sharpe Ratio by volatility targeting (Dachraoui, 2018, Harvey et al., 2018, Moreira

[19] An alternative to the approach of Grundy and Martin (2001) and Daniel and Moskowitz (2016) would be to use conditional betas as in Bali et al. (2017b).

[20] A nice characteristics of risk targeting is that this strategy can be overlayed on other portfolio strategies, i.e. the asset selection process can be separated from the portfolio risk management process (see Harvey et al. (2018), Zakamulin (2015), Rickenberg (2020b) and references therein). Another possibility of managing the risk of the momentum portfolio is to manage the risk of each constituent of the momentum portfolio. Goyal and Jegadeesh (2017) and Du Plessis and Hallerbach (2017) show that weighting each asset by its volatility increases the return, alpha and Sharpe Ratio of the momentum portfolio. This approach of weighting individual assets inversely to the assets' volatility is also used by Moskowitz et al. (2012), Fleming et al. (2001), Fleming et al. (2003), Kirby and Ostdiek (2012), Kim et al. (2016) and Asness et al. (2013). Moreira and Muir (2017, Sec. I.E) show that both approaches, volatility targeting and volatility weighting, are different. Rickenberg (2020c) simultaneously applies both strategies to the momentum portfolio.

[21] Similarly, Barroso (2016) finds that periods of a high beta typically coincide with periods of high volatility and low returns. However, since volatility is much more predictable than market beta, the risk-managed approach based on momentum's volatility is more appealing.

and Muir, 2017, 2019),[22] this observation shows that managing momentum's volatility is an appealing approach to increase momentum's return while simultaneously controlling momentum's risk.[23] In particular, Barroso and Santa-Clara (2015) find that a high volatility of the momentum strategy is a better momentum crash predictor than the bear market state variable examined by Daniel and Moskowitz (2016). In total, if the volatility of the momentum portfolio is expected to be high, the amount invested in the momentum portfolio should be lowered, whereas the invested amount should be increased if expected volatility is low.[24] This volatility managed momentum portfolio has a significantly improved risk-return profile with lower drawdowns compared to the non-managed momentum portfolio. A similar observation is also found by Du Plessis and Hallerbach (2017) and Grobys et al. (2018) who apply the volatility targeting approach to the industry momentum strategy of Moskowitz and Grinblatt (1999).

In this section, we shortly review the volatility targeting approach used in Daniel and Moskowitz (2016), Moreira and Muir (2017), Barroso and Santa-Clara (2015) and Barroso and Maio (2018). Furthermore, we extend this approach by using more advanced and possibly more accurate volatility models. Bollerslev et al. (2018) show that there exists a positive relation between forecasting accuracy, or equivalently a more constant portfolio risk, and utility gains for investors who target a constant level of portfolio volatility. Moreover, a more stable portfolio volatility is typically related to a lower crash risk (Dreyer and Hubrich, 2019). Consequently,

[22]Interestingly, volatility targeting can even be advantageous when there exists a positive relation between volatility and return. For example, Moreira and Muir (2019) find that an increase in volatility coincides with an increase in expected returns. However, the increase in expected returns is much more persistent than the increase in the volatility. Hence, investors should first decrease their exposure to the portfolio when the volatility initially starts to increase, but they should then subsequently increase their exposure when the volatility starts to decline. In other words, even when volatility and return are positively related, investors should time short-term volatility.

[23]For example, Cederburg et al. (2020) state: "Volatility management is likely to be successful if volatility is persistent and the risk-return relation is flat. In this scenario, a portfolio's conditional Sharpe ratio is negatively associated with its lagged volatility, and investors can capitalize on these dynamics in the conditional risk-return trade-off by taking more aggressive investment positions following low-volatility periods. If lagged volatility is negatively related to average return for a given strategy, volatility management becomes even more attractive. A positive risk-return trade-off, in contrast, makes volatility management less effective."

[24]By timing momentum's volatility and return, Daniel and Moskowitz (2016) use a slightly different approach to the approach of Barroso and Santa-Clara (2015). Daniel and Moskowitz (2016) compare this strategy to the volatility targeting approach and the authors find that the volatility and return timing strategy yields a slightly higher risk-adjusted performance than the volatility targeting strategy. However, timing volatility and return also exhibits more extreme and volatile weights as can be seen in Daniel and Moskowitz (2016, Fig. 5). Thus, the strategy that times return and volatility also produces higher transaction costs. Marquering and Verbeek (2004) and Moreira and Muir (2019) also find that volatility timing outperforms return timing, since return timing is much more influenced by estimation risk than volatility timing.

more advanced and more accurate volatility models should also exhibit higher risk-adjusted returns, significant utility gains for investors and less severe momentum crashes.

Throughout the paper, we consider the process $\{S_t\}_{t\in[0,T]}$ of prices over the period $[0, T], T \in \mathbb{N}$. The process $\{S_t\}_{t\in[0,T]}$ represents the prices of the portfolio that is long 1$ in the past winners and short 1$ in the past losers as described in Appendix A. The process $\{S_t\}_{t\in\{0,...,T\}}$ represents the process of monthly prices, where the interval $[t-1, t]$ represents one month. We define the (arithmetic) month t return of the momentum portfolio as

$$R_t := \frac{S_t}{S_{t-1}} - 1, \quad t = 1, ..., T. \tag{2.3.1}$$

Moreover, as in Barroso and Santa-Clara (2015), we assume that each month consists of $h = 21$ trading days and we consider the daily prices

$$S_{t,i} = S_{t+\frac{i}{h}}, \quad i = 1, ..., h, \tag{2.3.2}$$

where $S_{t,i}$ represents the closing price of day i in month t. Further, we define the daily (arithmetic) return as

$$R_{t,i} = \frac{S_{t,i}}{S_{t,i-1}} - 1, \quad i = 1, ..., h,\ t = 1, ..., T, \tag{2.3.3}$$

where we define $S_{t,0} = S_{t-1,h}$. Therefore, R_t represents the monthly return of month t, whereas $R_{t,i}$ represents the daily return of day i in month t.[25]

Barroso and Santa-Clara (2015), Moreira and Muir (2017) and Barroso and Maio (2018) propose to manage the crash risk of the momentum portfolio by targeting a constant level of portfolio volatility over time. Since buying past winners and selling past losers is a zero-investment strategy, the weight an investor is long in the winners and short in the losers portfolio can be scaled arbitrarily. Therefore, the authors propose to scale each month's weight invested in the winners and losers portfolio by the inverse of the momentum portfolio's past realized volatility. The monthly return R_t^* of the volatility managed momentum portfolio is then given by

$$R_t^* = \frac{\sigma_{\text{target}}}{\sigma_t} R_t, \tag{2.3.4}$$

[25] Our notation is similar to the notation that is frequently used in the literature on realized volatility, i.e. measuring daily volatility based on high-frequency data (see Patton (2011) for example).

where $\sigma_t = \sqrt{\text{var}(R_t \mid \mathcal{F}_{t-1})}$ is the (conditional) volatility of month t.[26] The volatility target σ_{target} can be chosen by the investor according to the investor's own risk aversion (Bollerslev et al., 2018). As in Barroso and Santa-Clara (2015), we target an annualized volatility of $\sigma_{\text{target}} = 12\%$. The aim of volatility targeting is to keep the portfolio volatility constant over time and equal to the chosen volatility target σ_{target}.[27] Generally, volatility targeting has several advantages compared to static portfolio allocations. For example, accurately targeting a constant level of portfolio risk is important, since a more constant portfolio volatility is typically rewarded with higher utility gains (Bollerslev et al., 2018). Similarly, a more stable portfolio volatility is linked to a lower tail risk (Dreyer and Hubrich, 2019). Further, investors are willing to pay high fees for insurance against a time-varying volatility (Adrian and Rosenberg, 2008, Ang et al., 2006b). Moreover, Adrian and Rosenberg (2008) show that bearing long-term volatility is compensated by a higher return, whereas an increase of short-term volatility typically coincides with negative returns.[28] From a practical view, Barroso and Santa-Clara (2015, p. 116) argue that "[r]unning a long-short strategy to have constant volatility is closer to what real investors (such as hedge funds) try to do than keeping a constant amount invested in the long and short legs of the strategy." In particular, several studies demonstrate that managing a portfolio's volatility increases the risk-adjusted performance and heightens utility gains (see Fleming et al. (2001), Fleming et al. (2003), Kirby and Ostdiek (2012), Moreira and Muir (2017), Bollerslev et al. (2018) among others). Furthermore, Marquering and Verbeek (2004) and Han (2005) find that, under realistic transaction costs, strategies that time volatility are superior to buy-and-

[26] $\mathcal{F}_{t-1}$ denotes the σ-algebra containing the available information up to the end of month $t-1$. In particular, $\mathcal{F}_{t-1}$ contains all daily returns known at the end of month $t-1$, i.e. $\mathcal{F}_{t-1} = \sigma\left(\{R_{j,i} : j = 1, ..., t-1; i = 1, ..., h\}\right)$, but could possibly contain more information, like information on other assets (see Meddahi and Renault (2004, p. 359) and Patton (2011)).

[27] Moreira and Muir (2017) use variance instead of volatility to scale the weight of the momentum portfolio. This strategy does not target a constant level of volatility over time, but is strongly related to this strategy. By managing the momentum portfolio's variance, the weight is adjusted more aggressively compared to the volatility targeting strategy. Hence, transaction costs are higher for the variance managed strategy (Kirby and Ostdiek, 2012). Kirby and Ostdiek (2012) and Moreira and Muir (2017) show that both approaches follow – under certain conditions – from solving for the optimal weight of a mean-variance investor (see also Aït-Sahalia and Brandt (2001)).

[28] Adrian and Rosenberg (2008) find that short-run and long-run volatility is priced quite differently. Thus, the relation of future returns with short-run and long-run volatility is different. Adrian and Rosenberg (2008, Table I) find a negative coefficient for short-run volatility but a positive coefficient for long-run volatility. Therefore, future returns are positively related to long-run volatility, but negatively to short-run volatility. This result is important in the context of risk targeting, since the high risk of momentum is rewarded by a huge return potential, but the performance can additionally be improved by timing short-run volatility. Moreira and Muir (2019) show that even long-term investors should manage short-term volatility.

hold portfolios and strategies that contemporaneously time return and volatility.[29] Generally, Rickenberg (2020b, Appendix A) discusses several reasons why investors should dynamically manage a portfolio's risk based on a forecast the portfolio's volatility.

To manage a portfolio's volatility as shown in Equation (2.3.4), a forecast of next month's volatility is needed. That is, based on the available information $\mathcal{F}_{t-1}$, the month t volatility σ_t has to be estimated. Barroso and Santa-Clara (2015) estimate σ_t by the past six months' Realized Volatility (RV), i.e. the square root of the sum of the previous six months' squared daily returns. Similarly, Moreira and Muir (2017) manage portfolio risk by using the Realized Variance, measured as the sum of the last month's squared daily returns. However, Ghysels et al. (2005) argue that one month of daily data is not sufficient to accurately measure monthly volatility (see also Bali et al. (2009) and Figlewski (1997)). In a similar setting, Chen et al. (2001) also use six months of daily data to estimate monthly volatility and skewness and argue that one month of daily data is not sufficient to provide accurate estimates. In particular, Ang et al. (2006b, p. 294) find that the negative volatility-return relation is more pronounced when volatility is measured with longer data sets of more than one month of daily data. Generally, Engle (2004) show that using too less data to estimate the unconditional volatility produces a very noisy estimate, whereas estimates based on too long estimation windows fail to adapt to changing market environments. Grobys et al. (2018) apply the RV estimator of Barroso and Santa-Clara (2015) to the industry momentum strategy of Moskowitz and Grinblatt (1999). The authors estimate Realized Volatility based on one, three and six months of data and find the best results for the estimator that uses only one month of data, as it is used in Moreira and Muir (2017). However, Grobys et al. (2018) find that the results are quite robust for all estimators. Thus, the optimal choice of the estimation window for the RV estimator is not clear. Nevertheless, we follow Barroso and Santa-Clara (2015), Daniel and Moskowitz (2016) and Barroso and Maio (2018) and use daily data of the past six months to estimate monthly

[29]Similarly, Moreira and Muir (2019, p. 509) state that "ignoring variation in volatility is very costly, and the benefits to timing volatility are significantly larger than the benefits to timing expected returns". Moreira and Muir (2017) also find higher performance gains of volatility timing compared to return timing.

volatility.[30] The month t volatility σ_t, used in Equation (2.3.4), is then estimated by

$$\sigma_t = \sqrt{\frac{1}{6}\sum_{j=1}^{6}\sum_{i=1}^{h} R_{t-j,i}^2} = \sqrt{\frac{1}{6}\sum_{i=1}^{6h} \tilde{R}_{(t-1)h-i+1}^2}, \tag{2.3.5}$$

where we define $\tilde{R}_s := R_{\lceil \frac{s}{h} \rceil, s-h(\lceil \frac{s}{h} \rceil -1)}$.[31] The estimator in Equation (2.3.5) assumes that the autocorrelation between daily returns is zero. A similar estimator that accounts for the autocorrelation in daily returns is presented in French et al. (1987) and Ghysels et al. (2005). However, as stated by Barroso and Santa-Clara (2015) and Moreira and Muir (2017), accounting for the autocorrelation does not significantly improve the performance of the risk-managed momentum strategy. For that reason, we rely on the estimator that simply estimates monthly volatility using the sum of squared daily returns. Estimating monthly volatility with daily returns has already been done by Merton (1980), French et al. (1987) and Ghysels et al. (2005). This approach of estimating monthly volatility by the sum of squared daily returns is similar to the theory of realized volatility that estimates daily volatility by the sum of squared intra-day returns (see Andersen et al. (2001), Patton (2011), Bollerslev et al. (2018) among others). Incorporating higher frequency data in measuring lower frequency volatility is found to be beneficial in many studies (see Andersen and Bollerslev (1997) and Andersen et al. (1999) for example).[32] In particular, higher frequency data based volatility estimates are also beneficial in portfolio allocation problems (see Fleming et al. (2003), Bollerslev et al. (2018) among others).

The estimator in Equation (2.3.5) assumes that volatility of month t can simply be forecasted by measuring past month's volatility. Although Barroso and Santa-Clara (2015) show that the volatility of the momentum portfolio is highly forecastable, the authors do not use a model to *forecast* the momentum portfolio's volatility. In contrast, Moreira and Muir (2017) use a simple AR(1) specification to forecast next month's variance and find similar results in terms of Sharpe Ratios compared to the model using past month's Realized Variance. However, the authors find that using a variance forecast instead of ex-post measured variance reduces transaction costs

[30] We also used other choices but found quite similar results.

[31] We follow Barroso and Santa-Clara (2015) and use raw daily returns instead of demeaned returns for the estimator in Equation (2.3.5). For the conditional volatility models presented later, we define $\tilde{R}_s$ as the demeaned return.

[32] See Adrian and Rosenberg (2008, Footnote 6) for a list of studies that use high-frequency data to estimate low-frequency volatility. The authors also use daily data in order to estimate monthly volatility.

and produces less extreme weights. Moreover, Moreira and Muir (2017) show (in an internet appendix) that the performance of the volatility managed portfolio can further be improved by using more sophisticated forecasting models. In particular, Bollerslev et al. (2018) and Rickenberg (2020b) show that volatility estimates of unconditional models are highly inaccurate, fail to target a constant level of risk over time and produce a worse risk-return profile as well as lower utility gains for risk targeting investors. Generally, Bollerslev et al. (2018) find a positive relation between forecasting accuracy and performance gains for volatility targeting investors. As a consequence, using a forecast of next month's volatility should produce a superior risk-adjusted performance compared to the realized estimator of Equation (2.3.5). Nevertheless, although risk-managed momentum strategies based on advanced volatility forecasting models should produce superior results, this approach is so far only rarely examined in the momentum literature. Similar to our approach, Daniel and Moskowitz (2016) examine risk-managed momentum strategies based on the GJR-GARCH model of Glosten et al. (1993). Furthermore, in an earlier version of their paper, Barroso and Maio (2018) use the GARCH(1,1) model of Bollerslev (1986) but find no improvements of this model. However, the authors fit the model to monthly data instead of daily data.[33] Fitting conditional volatility models to monthly data if higher frequency data are available has two important drawbacks (Ederington and Guan, 2010). First, calibrating the model to monthly returns dismisses all information that is contained in daily returns. Second, to accurately estimate the parameters of the GARCH(1,1) model, long data sets are required, which are not available when monthly observations are used. The parameters of GARCH models are typically estimated with at least 500 observations (Kuester et al., 2006). Hence, fitting the GARCH(1,1) model to monthly returns would require more than 40 years of data to obtain accurate estimates, whereas only two years of data are needed when the model is fitted to daily returns. Figlewski (1997) finds that the GARCH(1,1) model fitted to five years of monthly data delivers poor results due to bad parameter estimates, but the same model delivers good results when it is fitted to daily data.[34] As a consequence, when next month's

[33] Barroso and Maio (2018) use an expanding window approach starting with a window of only 160 observations. Using 160 observations in the Maximum Likelihood estimation of the GARCH(1,1) model will likely produce inaccurate parameter estimates (Figlewski, 1997).

[34] Similarly, by fitting volatility models to daily and monthly returns, Campbell and Hentschel (1992, Sec. 3.2) find that accurately estimating the model parameters with monthly returns sometimes fails, whereas no problems

volatility is forecasted, the volatility models should be fitted to daily data.

Generally, when estimating monthly volatility several approaches can be used. First, the model can be directly fitted to monthly return data in order to forecast monthly volatility. Second, daily data can be used to forecast the one-day ahead volatility which can then be scaled up to obtain a forecast of the monthly volatility. Third, the model can be fitted to daily returns which can then be used to model monthly volatility by iterating daily forecasts (see Figlewski (1997), Andersen et al. (1999), Meddahi and Renault (2004) and Kole et al. (2017)). The first and second approaches have the advantage that forecasts of monthly volatility are typically directly available, whereas in the third case, information on the temporal aggregation and the term structure of risk have to be known. However, as mentioned above, fitting models to monthly data requires huge data sets and discards all information available in daily data. Further, the second approach dismisses information on the term structure of risk. Consequently, the third approach, although more challenging, should be more efficient and accurate. Kole et al. (2017) compare the three approaches and find that iterated forecasts deliver the most accurate forecasts. Hence, incorporating information on short-term volatility is beneficial when longer-term volatility is forecasted.[35] The high misspecification of risk obtained by managing portfolio risk based on static volatility models or models that are fitted to monthly data could also result in a lower risk-adjusted performance, lower utility gains and higher left tail risk compared to more sophisticated and probably more accurate volatility models as found by Bollerslev et al. (2018). For that reason, we next present several alternatives to Equation (2.3.5) that properly forecast

occur when daily data are used (see also Figlewski (1997)). Adrian and Rosenberg (2008, p. 3001) also estimate monthly volatility and skewness using daily data “in order to improve the estimation precision”. Similarly, Andersen et al. (1999) find that switching to higher frequency data is more accurate when monthly volatility is estimated. Embrechts et al. (2005) also find that fitting models to higher frequency data is superior when longer-term risk is estimated. Further, fitting GARCH models to lower frequency returns, like monthly or quarterly returns, typically leads to parameter estimates that violate the stationary assumption of GARCH models (Embrechts et al., 2005). Generally, conditional volatility forecasts using daily or monthly data are conditioned on two different information sets, where the information set using daily data is much larger (see Wong and So (2003) and Andersen et al. (1999)).

[35]In contrast, information on longer-term volatility, like yearly volatility, can also be incorporated in order to model higher frequency volatility, like daily volatility (Engle and Rangel, 2008). This model could then also be used to forecast monthly volatility. Similarly, Adrian and Rosenberg (2008) develop a volatility model that simultaneously models short-run and long-run volatility. The authors fit this model to daily data in order to estimate monthly volatility. Adrian and Rosenberg (2008) show that short-run volatility captures skewness risk, whereas long-run volatility captures business cycle risk.

monthly volatility and are based on daily returns.[36]

A first and easy extension of the estimator in Equation (2.3.5) is the Heterogeneous Autoregressive (HAR) model of Corsi (2009). This model was also used by Bollerslev et al. (2018) to estimate the one-month ahead volatility based on daily, weekly and monthly volatility estimates. Taylor (2014) also uses this model in a multivariate volatility timing strategy and finds good results of this model. Although this model is easy to estimate, it typically delivers convincing results, what makes this approach interesting for practical implementations. The basic HAR model estimates next month's volatility by using several measures of past volatility, aggregated over different time horizons. The h-day ahead average Realized Volatility is then obtained by

$$RV_{t|t-1} = \beta_0 + \beta_D RV_{t-1}^D + \beta_W RV_{t-1}^W + \beta_M RV_{t-1}^M + \varepsilon_t, \tag{2.3.6}$$

where we define $RV_{t-1}^D = R_{t-1,h}^2$, $RV_{t-1}^W = \frac{1}{5}\left(R_{t-1,h}^2 + R_{t-1,h-1}^2 + ... + R_{t-1,h-4}^2\right)$ and $RV_{t-1}^M = \frac{1}{h}\left(R_{t-1,h}^2 + R_{t-1,h-1}^2 + ... + R_{t-1,1}^2\right)$.[37] We estimate the parameters in Equation (2.3.6) by OLS under the restriction that all parameters are non-negative to guarantee a positive estimate for $RV_{t|t-1}$. The estimator for the month t volatility based on the information up to month $t-1$ is then given by $\sigma_t = \sqrt{h \cdot RV_{t|t-1}}$.[38] A possible extension of this model could be the more complex MIDAS model used in Ghysels et al. (2005) and Ghysels et al. (2016). Further, Bollerslev et al. (2018) present several extensions of the basic HAR model, including the MIDAS, which could also be used to manage the risk of the momentum portfolio. A similar model, based on the range estimator, was also used by Hsieh (1993, p. 53) to forecast volatility. However, since one advantage of the HAR model is its simplicity, we restrict ourselves to the simplest form.

[36]Whereas estimating short-term volatility is straightforward and frequently examined in the academic literature, estimating longer-term volatility is more challenging and only rarely examined (see Christoffersen and Diebold (2000) and Ederington and Guan (2010)). However, long horizon volatility forecasts are important in many fields of finance, including risk management, option pricing and portfolio management (see Figlewski (1997), Engle (2004), Andersen et al. (1999), Ederington and Guan (2010) and Taylor (2005)). Figlewski (1997) gives a nice review on forecasting (long-term) volatility used for option pricing. Engle (2004) also nicely summarizes (long-term) volatility forecasting with applications to risk management and option pricing. See also Poon and Granger (2003) for a review of volatility forecasts, including a list of studies that also examine longer-term volatility forecasting based on higher frequency data.

[37]The HAR model is typically estimated with volatility measures RV_{t-1}^D, RV_{t-1}^W and RV_{t-1}^M based on intraday-data. In an earlier version of their paper, Bollerslev et al. (2018) state that the model can also be estimated based on daily data as done in our paper.

[38]Bollerslev et al. (2018) propose to directly forecast the volatility measure that is needed. Hence, instead of forecasting a measure of daily variance, which is then transformed to a measure of monthly volatility, one can also directly forecast monthly volatility or even the reciprocal of monthly volatility, i.e. $\frac{1}{\sigma_t}$, which is needed in Equation (2.3.4). We also estimated these quantities and found similar results.

Rickenberg (2020b) shows that risk managed portfolio strategies that use dynamic volatility models, like the EWMA model and the GARCH model of Bollerslev (1986), produce more convincing results in terms of higher Sharpe Ratios and higher utility gains compared to strategies based on static volatility models, like the RV model in Equation (2.3.5). Similarly, Christoffersen and Diebold (1997) demonstrate the importance of incorporating the conditional volatility in simulating h-day ahead returns. Generally, Hsieh (1993) finds that unconditional estimates are better suited for long-term applications, e.g. yearly horizons, whereas conditional estimates work well for short- and medium-term applications. Further, the author finds vast differences between capital requirements of unconditional and conditional risk forecasts, where conditional estimates seem better suited than unconditional estimates. We therefore show how dynamic volatility models can be used to manage the monthly risk of the momentum strategy. Interestingly, Barroso and Maio (2018) find that the GARCH managed strategy's Sharpe Ratio is significantly lower than the Sharpe Ratio of the RV managed strategy. However, as mentioned above, the authors fit the GARCH model to monthly data, which has the drawbacks that information in daily returns are not considered and parameter estimates are highly inaccurate. For that reason, we next show how conditional volatility models, i.e. the EWMA, GARCH and GJR-GARCH model, can be fitted to daily data and how estimates of monthly volatility are obtained for these models.[39]

For the dynamic volatility models, we assume that daily returns can be described by

$$R_{t,i} = \mu_{t,i} + \sigma_{t,i} \cdot Z_{t,i}, \tag{2.3.7}$$

where $Z_{t,i}$ is iid with mean zero, variance one and cumulative distribution function F_Z (see McNeil and Frey (2000, p. 275)). In Equation (2.3.7), the parameters $\mu_{t,i}$ and $\sigma_{t,i}$ represent the (conditional) mean and volatility of the daily return $R_{t,i}$, i.e. $\mu_{t,i} = \mathbb{E}(R_{t,i} \mid \mathcal{F}_{t,i-1})$ and $\sigma_{t,i} = \sqrt{\operatorname{var}(R_{t,i} \mid \mathcal{F}_{t,i-1})}$, where $\mathcal{F}_{t,i-1}$ contains all information up to time $t + \frac{i-1}{h}$. We follow

[39]As mentioned above, Figlewski (1997) shows that fitting the GARCH(1,1) model to monthly data fails to accurately forecast monthly volatility, whereas the GARCH(1,1) model fitted to daily data is superior to the RV model in forecasting monthly volatility. The problem of inaccurate parameter estimates does not hold for the EWMA model, since this model's parameters are typically set to a fixed value. Therefore, we additionally calibrate the EWMA model to monthly data to obtain a forecast of the monthly volatility. By doing this, we can assess the importance of incorporating the information in higher frequency data when the momentum strategy's risk is managed.

Bollerslev (1987) and assume a constant mean $\mu_{t,i} = \mu$ and define the demeaned daily return as $\overline{R}_{t,i} := R_{t,i} - \mu$. Fitting volatility models to the demeaned returns is frequently done in the academic literature (see Taylor (2005), Wong and So (2003), Ederington and Guan (2010), So and Wong (2012) among others).[40] Another possibility would be to assume an autoregressive process for the conditional mean.[41] However, Christoffersen and Diebold (2006) find that conditional returns are not forecastable, i.e. returns conditioned on the current information do not fluctuate over time (see also Merton (1980)). Similarly, Hsieh (1993) finds that the conditional mean is not predictable, whereas volatility is highly predictable. Han (2005) shows that monthly managed volatility timing strategies are superior to strategies that contemporaneously time returns and volatility. Furthermore, incorporating a model for the mean return should increase estimation risk and transaction costs (Daniel and Moskowitz, 2016, Moreira and Muir, 2019). Hence, when risk is managed on a monthly basis, modeling returns in a time-varying manner should result in a worse performance compared to assuming a constant mean, at least after transaction costs.[42] Further, Hansen and Lunde (2005) compare several mean specifications and find only minor differences between an autoregressive mean specification, a constant mean and a zero mean (see also Ederington and Guan (2010, Footnote 11)).

A simple and frequently used specification for the volatility in Equation (2.3.7) is the EWMA model. The EWMA model fitted to daily data is given by

$$\tilde{\sigma}_s^2 = (1-\lambda) \cdot \tilde{R}_{s-1}^2 + \lambda \cdot \tilde{\sigma}_{s-1}^2, \quad s = 2, ..., Th \tag{2.3.8}$$

where $\tilde{\sigma}_s := \sigma_{\lceil \frac{s}{h} \rceil, s-h(\lceil \frac{s}{h} \rceil - 1)}$ and $\tilde{R}_s := \overline{R}_{\lceil \frac{s}{h} \rceil, s-h(\lceil \frac{s}{h} \rceil - 1)} := R_{\lceil \frac{s}{h} \rceil, s-h(\lceil \frac{s}{h} \rceil - 1)} - \mu$. The parameter λ is typically chosen as $\lambda = 0.94$ when the model is fitted to daily data (see Wong and So (2003) or Christoffersen (2012, p. 70)). Then, the volatility forecast for the first day of

[40] Instead of fitting volatility models to the demeaned returns, a mean return of zero, i.e. $\mu_{t,i} = 0$, could also be assumed. Assuming a zero mean is frequently done in the literature when volatility models, fitted to daily data, are used to forecast the one-day ahead volatility. This assumption is sufficient for one-day ahead volatility forecasts. However, incorporating information on the mean return becomes crucial when longer-term risk is forecasted. This especially holds when longer-term downside risk is forecasted, as done in the next section.

[41] See, for example, Baillie and Bollerslev (1992) on more complex assumptions on the mean return to estimate the h-day ahead volatility. The authors fit an ARMA-GARCH(1,1) model to daily data in order to forecast the h-day ahead volatility.

[42] Han (2005) shows that this result does not necessarily hold when risk is managed on a daily basis. In this case, incorporating an autoregressive process for the mean return could result in a superior strategy, at least before transaction costs.

month t, $\sigma_{t,1}$, is given by $\tilde{\sigma}_{h(t-1)+1} = \sqrt{(1-\lambda)\tilde{R}^2_{h(t-1)} + \lambda \cdot \tilde{\sigma}^2_{h(t-1)}}$ or equivalently $\sigma_{t,1} = \sqrt{(1-\lambda)R^2_{t-1,h} + \lambda \cdot \sigma^2_{t-1,h}}$. The EWMA model gradually reduces the influence of past data when future volatility is estimated, whereas the Realized Volatility model in Equation (2.3.5) weights the past $6h$ daily returns equally and then radically ignores the older returns (Fan and Gu, 2003, p. 264). Thus, we expect that the EWMA model adapts faster to a changing market risk. Similarly, Grobys et al. (2018) suggest that giving more weight to more recent observations is advantageous when the monthly risk of the industry momentum strategy is managed. In particular, since this model does not need any parameter estimates, it is interesting for practical implementations. However, the parameter λ could also be obtained in a data-driven and time-varying manner as shown by Fan and Gu (2003).

In the EWMA model, returns are typically assumed to be iid normally distributed. Consequently, the volatility of the h-day aggregate return R_t is simply given by scaling up the one-day ahead volatility forecast by $\sqrt{h}$ (see Wong and So (2003) and Christoffersen (2012, p. 72)). Hence, the forecast for the month t volatility is given by

$$\sigma_t = \sqrt{h} \cdot \sigma_{t,1}. \tag{2.3.9}$$

Scaling up the daily volatility forecast by $\sqrt{h}$ to obtain a monthly volatility forecast is known as the square root of time rule (SRTR). The SRTR is a popular method in the financial industry and is recommended by banking supervisors (Danielsson and Zigrand, 2006). Generally, scaling up estimates of short-term volatility to obtain long-term volatility forecasts is a common approach, where the industry practice is the SRTR (Christoffersen and Diebold, 2000).[43] Although correct for the EWMA model, the SRTR is also often applied to other estimates of volatility or downside risk that are not based on an iid assumption. In particular, the SRTR is also implicitly used by Barroso and Santa-Clara (2015), Moreira and Muir (2017) and Barroso and Maio (2018) to obtain an estimate of monthly volatility based on an estimate of daily volatility. However, despite the widespread use of the SRTR, simply scaling up daily volatility by $\sqrt{h}$ is highly misleading and only holds under very strict conditions as shown by Diebold

[43] Instead of scaling one-day ahead estimates by $h^{1/2}$, other scaling rules can also be used. For example, by using extreme value theory, the one-day ahead volatility forecast can be scaled by $h^{1/\alpha}$, where α denotes the tail index. The exponent $1/\alpha$ is typically estimated by the Hill estimator fitted to daily data (Embrechts et al., 2005).

et al. (1998) and Danielsson and Zigrand (2006). Scaling rules imply that the distribution of daily and monthly returns have the same shape, which is typically not fulfilled for most return series (Kole et al., 2017, Neuberger, 2012). Further, short-run volatility is much less persistent than long-run volatility. Hence, simply scaling up daily volatility ignores "the risk that risk will change" (Engle, 2011, p. 442). Generally, the SRTR can be applied without reservation if asset returns follow a random walk. However, the random walk hypothesis is typically not true for financial return time-series, which can be shown by the variance ratio test of Lo and MacKinlay (1988).[44] The variance ratio test of Lo and MacKinlay (1988) is summarized in Appendix C and applied to the momentum returns in Section 2.6.2. As expected, the random walk hypothesis is clearly rejected for the momentum portfolio. As a consequence, since momentum returns do not follow a random walk, the SRTR and particularly the RV model of Barroso and Santa-Clara (2015), Moreira and Muir (2017) and Barroso and Maio (2018) are not appropriate to manage the risk of the momentum strategy. Furthermore, Saadi and Rahman (2008) find that the SRTR underestimates risk and that scaled volatility has a higher standard deviation. Similarly, Diebold et al. (1998) show that the SRTR typically magnifies volatility fluctuations. Hence, applying the SRTR in a volatility targeting strategy should lead to a too high (average) equity exposure, wrong risk timing and high transaction costs. Generally, Embrechts et al. (2005) find that the SRTR only works well when longer-term volatility, e.g. yearly volatility, is forecasted. Consequently, other approaches to manage monthly risk could be more accurate than the SRTR, which should result in a superior risk-return profile.

The advantage of the EWMA model is that no parameters have to be estimated, which makes this model interesting for practical applications (Halbleib and Pohlmeier, 2012). In particular, the EWMA model is a simple way to manage risk dynamically, which is advantageous for risk managed portfolios (Rickenberg, 2020b). For example, Grobys et al. (2018) find that volatility estimators based on more recent data are better in managing momentum's risk. For that reason, the authors suggest to use models that emphasize recent observations, like the EWMA model,

[44]Lo and MacKinlay (1988) statistically test the random walk hypothesis using weekly and monthly returns. If prices follow a random walk, then monthly variance should be four times greater than weekly variance. This relation is statistically rejected by Lo and MacKinlay (1988) for equity returns indicating that the random walk hypothesis does not hold for equity returns. Hsieh (1993) and Saadi and Rahman (2008) also test the random walk hypothesis and clearly reject this hypothesis.

when monthly risk is estimated. However, more complex models that reestimate the model parameters frequently are potentially more accurate. Further, as mentioned above, the random walk hypothesis is typically rejected for financial returns. Hence, simply scaling up forecasts of short-term volatility by $\sqrt{h}$, as done in Equation (2.3.9), is insufficient to obtain a forecast for the monthly volatility. For these reasons, we additionally use the GARCH(1,1) model proposed by Bollerslev (1986) and the GJR-GARCH model of Glosten et al. (1993) calibrated to daily data, where the monthly volatility forecasts are obtained by explicitly considering the time series properties contained in the financial return series. Andersen et al. (1999) and Figlewski (1997) show that the GARCH(1,1) model fitted to higher frequency data, e.g. daily returns, is able to accurately forecast lower frequency volatility, e.g. monthly volatility.

The GARCH(1,1) model fitted to daily returns is given by

$$\tilde{\sigma}_s^2 = \omega + \alpha\, \tilde{R}_{s-1}^2 + \beta\, \tilde{\sigma}_{s-1}^2. \tag{2.3.10}$$

The parameters ω, α, β are estimated by Quasi Maximum Likelihood (QML) under the assumption that the daily innovations $Z_{t,i}$ in Equation (2.3.7) are iid standard normally distributed. The GARCH(1,1) parameters are restricted to $\omega > 0$, $\alpha \geqslant 0$, $\beta \geqslant 0$ and $\alpha + \beta < 1$, where the last restriction guarantees covariance stationarity (Baillie and Bollerslev, 1992, Bollerslev, 1986). The volatility forecast for the first day of month t, based on the information $\mathcal{F}_{t-1}$ up to month $t-1$, is then given by $\sigma_{t,1} = \tilde{\sigma}_{h(t-1)+1}$. One possibility to obtain a forecast for the monthly volatility, which is also used in this paper, is to just scale up $\sigma_{t,1}$ by $\sqrt{h}$ as done in the EWMA model. However, the SRTR does not hold for the GARCH(1,1) model and has several disadvantages as summarized above. Therefore, longer-term volatility forecasts, based on the GARCH(1,1) or GJR-GARCH models fitted to daily data, are typically obtained by successive forward substitution (Ederington and Guan, 2010, Kole et al., 2017). By subsequently using Equation (2.3.10), the volatility forecast for the month t volatility can be calculated by

$$\sigma_t^2 = h \cdot \sigma^2 + \frac{1 - (\alpha + \beta)^h}{1 - \alpha - \beta} \cdot \left(\sigma_{t,1}^2 - \sigma^2\right), \tag{2.3.11}$$

where $\sigma^2 = \frac{\omega}{1-\alpha-\beta}$ is the unconditional variance of the GARCH(1,1) process (see Wong and So (2003, Eq. (4)) or Taylor (2005, Eq. (1))). The formula in Equation (2.3.11) directly follows from simply summing up the individual one-day ahead to h-day ahead daily variance forecasts (see Ghysels et al. (2005), Ederington and Guan (2010), Taylor (2005) or Christoffersen (2012, p. 72)).[45] This approach is similar to the $\sqrt{h}$ scaling but explicitly accounts for the time-series behavior in volatility.[46] In particular, the EWMA model ignores the long-run variance σ^2, which is important when longer-term volatility is forecasted.[47] If next day's volatility is estimated to be high, the EWMA model assumes that all days in this month are highly volatile days. In contrast, in the GARCH model, future days' volatility could revert to the long-run volatility (Christoffersen, 2012). Incorporating the unconditional variance σ^2 is of minor importance for one-day ahead forecasts, but regarding σ^2 becomes more important for longer-term volatility forecasts, since volatility is typically mean-reverting (Ederington and Guan, 2010, Engle, 2004). The unconditional volatility σ can be interpreted "as a trend around which the conditional volatility fluctuates" (Engle and Rangel, 2008, p. 1191). Furthermore, Engle and Rangel (2008) extend the GARCH model in a way that σ^2 is also modeled and is allowed to slowly fluctuate over time in order to capture slow moving macroeconomic developments. A similar model is developed by Adrian and Rosenberg (2008), where short-term volatility captures the market's skewness risk and long-term volatility captures business cycle risk.

Both models, the EWMA and the GARCH(1,1) model, are symmetric volatility models, i.e. they do not incorporate the asymmetric relation between returns and future volatility. Asym-

[45] Baillie and Bollerslev (1992) show that under the assumption $\alpha + \beta < 1$, i.e. covariance stationarity of the GARCH process, the forecast for the variance of day k in month t based on $\mathcal{F}_{t-1}$ is given by $\sigma^2_{t,k} = \sigma^2 + (\alpha + \beta)^{k-1}\left(\sigma^2_{t,1} - \sigma^2\right)$. The variance forecast for the month t variance is then obtained by summing up the daily variance forecasts within month t, i.e. $\sigma^2_t = \sum_{k=1}^{h} \sigma^2_{t,k}$, which is based on the assumption of zero autocorrelations. The summation of volatility over a certain period, i.e. $\sigma^2_t = \sum_{k=1}^{h} \sigma^2_{t,k}$, is sometimes called the integrated volatility, which plays an important role in many fields in the financial literature (see Andersen et al. (1999), Ederington and Guan (2010) and references therein). Using the geometric sum and the assumption $\alpha + \beta < 1$, Equation (2.3.11) follows directly from $\sigma^2_t = \sum_{k=1}^{h} \sigma^2 + (\alpha + \beta)^{k-1}\left(\sigma^2_{t,1} - \sigma^2\right)$. This result can also be extended to the ARMA(k, l)-GARCH(p, q) model (Baillie and Bollerslev, 1992, p. 99).

[46] In a similar setting, Neuberger (2012) finds that the time-series behavior should be regarded when monthly skewness is estimated based on daily data. Instead of scaling up daily skewness to obtain monthly skewness, the author finds that "[i]f high-frequency returns are to be used to improve the estimate of the skewness of low-frequency returns, it must be done in a way that reflects the serial dependencies that are manifest in the data."

[47] More generally, Baillie and Bollerslev (1992) compare forecasts of the GARCH(1,1) and the IGARCH(1,1) model, where $\sigma^2_{t,k}$ is given by $\omega(k-1) + \sigma^2_{t,1}$. The monthly variance is then given by $\sigma^2_t = \sum_{k=1}^{h} \sigma^2_{t,k} = \frac{h(h-1)}{2}\omega + h\sigma^2_{t,1}$ (Taylor, 2005, Eq. (2)). The EWMA model is a special case of the IGARCH(1,1) model assuming $\omega = 0$. In this case, conditioned on $\mathcal{F}_{t-1}$, it follows $\sigma^2_{t,k} = \sigma^2_{t,1}$ and thus $\sigma^2_t = h\sigma^2_{t,1}$. Hence, when using the EWMA model for estimating the h day volatility, only current information is important, whereas the long-run variance is not regarded.

metric volatility, also called leverage effect, corresponds to the observation that past negative returns typically increase volatility more than past positive returns of the same magnitude (see Nelson (1991), Campbell and Hentschel (1992) and Glosten et al. (1993) among others). Incorporating asymmetric volatility becomes crucial when longer-term volatility is forecasted, whereas the leverage effect is less important for one-day ahead volatility forecasts (Colacito and Engle, 2010, Ederington and Guan, 2010, Taylor, 2005). Under the leverage effect, even if one-day ahead returns are symmetrically distributed, longer-term returns can be asymmetric (see Adrian and Rosenberg (2008), Colacito and Engle (2010, Figure 4), Engle (2011, Sec. 3) and Neuberger (2012)). Thus, asymmetric volatility models should better capture the negative skewness of monthly momentum returns. To incorporate the effect that volatility responds asymmetrically to past returns, we additionally use the GJR-GARCH model of Glosten et al. (1993). Taylor (2005) finds good results for the GJR-GARCH to forecast monthly volatility (see also Lönnbark (2016) and references therein). Similarly, Kole et al. (2017) find good results for the GJR-GARCH model when 10-day downside risk is forecasted. Generally, Rosenberg and Engle (2002) fit several volatility models to daily returns in order to simulate h-day future price paths and find that the GJR-GARCH model delivers the most convincing results.[48] Harvey and Siddique (1999) and Lönnbark (2016) also use the GJR-GARCH model for modeling daily and monthly volatility. Furthermore, similar to our approach, Daniel and Moskowitz (2016) use the GJR-GARCH model fitted to daily data to manage the momentum portfolio's monthly risk.[49]

Following Daniel and Moskowitz (2016, Eq. (8)) and Lönnbark (2016, p. 951), we define the one-day ahead volatility of the GJR-GARCH model as

$$\tilde{\sigma}_s^2 = \omega + (\alpha + \gamma \delta_{s-1}) \cdot \tilde{R}_{s-1}^2 + \beta \, \tilde{\sigma}_{s-1}^2, \tag{2.3.12}$$

[48] Rosenberg and Engle (2002) and Barone-Adesi et al. (2008) show how the GJR-GARCH model, fitted to daily data, can be used to forecast h-day volatility using draws from historical standardized returns. This approach is called Filtered Historical Simulation (FHS) and has the advantage that non-normalities, like skewness and kurtosis in the daily returns, are taken into account when the h-day volatility is estimated. Incorporating non-normalities in volatility forecasts can increase the forecasting accuracy (Taylor, 2005). Barone-Adesi et al. (2008) show that the GJR-GARCH-FHS model outperforms other models that do not consider conditional volatility and/or conditional non-normalities. We will use the FHS approach in the next section when we estimate h-day downside risk measures.

[49] Daniel and Moskowitz (2016) use a slightly different approach to obtain a forecast of the monthly volatility based on the GJR-GARCH model's forecast of the daily volatility, which is similar to the HAR model shown above. This procedure is shortly summarized in Daniel and Moskowitz (2016, Appendix D).

where $\delta_{s-1} := \mathbb{1}_{\{\tilde{R}_{s-1}<0\}}$ captures the leverage effect. The parameters ω, α, γ and β are again obtained by QML. Using similar arguments as for the GARCH(1,1) model and assuming that $Z_{t,i}$ follows a symmetric distribution, it can be shown that the month t volatility of the GJR-GARCH model is given by

$$\sigma_t^2 = h \cdot \sigma^2 + \frac{1 - (\alpha + \beta + \frac{\gamma}{2})^h}{1 - (\alpha + \beta + \frac{\gamma}{2})} \cdot \left(\sigma_{t,1}^2 - \sigma^2\right), \tag{2.3.13}$$

where $\sigma^2 = \frac{\omega}{1-(\alpha+\beta+\frac{\gamma}{2})}$ is the unconditional variance of the GJR-GARCH model (Lönnbark, 2016, p. 951).[50] Ederington and Guan (2010) find that both models, the GARCH and GJR-GARCH model, perform nearly equally well when the 20-day volatility is forecasted.[51] Taylor (2005) also compares the GARCH and GJR-GARCH models in the context of longer-term volatility forecasts and finds better results for the GJR-GARCH model. Similarly, Colacito and Engle (2010) compare longer-term forecasts of the GARCH and GJR-GARCH model used in an asset allocation context and find that using the GJR-GARCH delivers sizable gains for investors. Investors are willing to pay high fees to have access to the GJR-GARCH model instead of the GARCH model if the investors reallocate the weights of a portfolio of risky assets every 20 days (see Figure 6 in Colacito and Engle (2010)). Hence, portfolio decisions based on long-horizon volatility forecasts should take asymmetric volatility into account.

Similar methods as presented above can also be used in a multivariate setting. Volatility timing of multi-asset portfolios has frequently been examined in the literature. For example, Fleming et al. (2001), Fleming et al. (2003) and Kirby and Ostdiek (2012) use simple volatility

[50] See also Colacito and Engle (2010, p. 21) and Ederington and Guan (2010) for a formula for the monthly volatility of the GJR-GARCH model fitted to daily data. Further, Taylor (2005, p. 714) also uses the GJR-GARCH fitted to daily data in order to forecast longer-term volatility and finds good results of this approach. However, the author uses a slightly different definition of the GJR-GARCH model compared to our definition. Ederington and Guan (2010) additional show how the month t volatility forecast can be obtained for the EGARCH model of Nelson (1991) fitted to daily data. The EGARCH model also incorporates the leverage effect, but instead of modeling volatility, the EGARCH model models the logarithm of the volatility. Ederington and Guan (2010) find that the EGARCH model also successfully forecasts monthly volatility when it is fitted to daily data. In contrast, Hsieh (1993) finds that the EGARCH model is good for one-day ahead forecasts but does not work well for h-day forecasts. Adrian and Rosenberg (2008) also use a modified EGARCH model fitted to daily data in order to model monthly volatility.

[51] Ederington and Guan (2010) evaluate the forecasting accuracy based on loss functions. However, the authors do not test if there are statistically significant differences. We will use more advanced backtesting methods, presented in Appendix D, to assess which of our volatility models is most successful in forecasting next month's volatility by testing which model is most successful in targeting a constant level of portfolio volatility over time. Further, we assess if incorporating higher moments, as it is also examined by Taylor (2005), is beneficial when a constant portfolio volatility is targeted.

estimates, which are similar to the RV estimator in Equation (2.3.5). Han (2005) and Taylor (2014) extend this examination to more advanced volatility models. Taylor (2014) finds good results for the HAR model and for conditional volatility models. In particular, Han (2005) examines multivariate volatility timing strategies for three different types of investors who rebalance their portfolio on a daily, weekly and monthly basis. All strategies rely on estimates of the covariance matrix using daily returns. Similar to our approaches, Han (2005, Sec. 3.2) shows how the weekly and monthly covariance matrix can be estimated by a model fitted to daily data, where the monthly covariance matrix is estimated by the sum of the daily forecasts within one month. The author finds that an investor who rebalances portfolio weights on a monthly basis is willing to pay an annualized fee of 5% to switch from a static portfolio to a volatility managed portfolio.

Several possible refinements of the models that are presented above could also be used for the volatility targeting strategy. For example, the spline-GARCH model of Engle and Rangel (2008) or the model of Adrian and Rosenberg (2008) that incorporate information on higher- and lower-frequency volatility could be used. Further, the extended versions of the GARCH and GJR-GARCH model as presented in Ederington and Guan (2010) could also be used. The authors modify the GARCH and GJR-GARCH model in a way that older observations become more important. Giving more weight to older information becomes more important if longer-term volatility is forecasted. For example, when forecasting volatility of day $t + 1$ based on information up to day t, due to the persistence of volatility, the observation of day t is more important than the observation of day $t - 1$. This is regarded by standard volatility models and is reasonable for one-day ahead forecasts. However, this result does not generally hold if the volatility of day $t + h$, based on information of day t, is forecasted (see also Andersen et al. (1999)). In this case, if h is large, the return of day $t - 1$ becomes *relatively* more important and should obtain a similar weight as the return of day t. However, the authors also find convincing results of standard volatility models fitted to daily data to forecast monthly volatility. Moreover, Grobys et al. (2018) find that managing the risk of the industry momentum strategy is best done when more recent daily return data are used to estimate monthly volatility. Therefore, we restrict ourselves to the basic GARCH and GJR-GARCH models presented above. Another possibility would be to use the method presented in Taylor (2005). The author shows how h-day ahead volatility forecasts can be made based on forecasts of certain quantiles.[52] This approach has the advantage that information on higher moments is incorporated in the volatility forecasts. Incorporating higher moments is beneficial since the shape of the conditional return distribution is typically highly non-normal and time-varying. The author finds good results of incorporating higher moments in longer-term volatility forecasts. However, we will use a similar approach in the next section, when we show how the portfolio volatility can be held constant by managing a portfolio's downside risk.

A second possibility to exploit the information of daily returns when monthly volatility in the GARCH(1,1) model is forecasted has been proposed by Drost and Nijman (1993) (see also Drost and Werker (1996), Diebold et al. (1998), Meddahi and Renault (2004) and Embrechts et al. (2005, Sec. 5)). The authors show that – under certain conditions – the GARCH(1,1) model is closed under temporal aggregation.[53] In other words, if daily returns follow a GARCH(1,1) process, then monthly returns also follow a GARCH(1,1) process where the monthly GARCH parameters can be calculated using the daily parameters (Wong and So, 2003). The month t volatility can then be estimated by

$$\sigma_t^2 = \omega_M + \alpha_M \overline{R}_{t-1}^2 + \beta_M \sigma_{t-1}^2, \tag{2.3.14}$$

where $\overline{R}_{t-1} := R_{t-1} - \mathbb{E}(R_{t-1} \mid \mathcal{F}_{t-2}) = R_{t-1} - h \cdot \mu$ defines the demeaned monthly return.

[52] A similar approach is also used by Ghysels et al. (2016) to obtain conditional skewness forecasts that are more robust to outliers.

[53] A model is "closed under temporal aggregation if the model keeps the same structure, with possibly different parameter values, for any data frequency" (Meddahi and Renault, 2004, p. 356). Drost and Nijman (1993) show that weak GARCH models are closed under temporal aggregation. Meddahi and Renault (2004) extend the examination of Drost and Nijman (1993) to a wider class of GARCH processes under milder moment restrictions and including the well-known leverage effect. Similarly, Drost and Werker (1996) examine the relation between the discrete GARCH(1,1) model and continuous time models, including models with continuous paths and jumps. In particular, they show how parameters of the GARCH(1,1) model can be obtained based on parameter estimates of continuous time models. Although the GARCH(1,1) model, without further assumptions, is not closed under temporal aggregation, we follow Diebold et al. (1998) and use the results of Drost and Nijman (1993). Meddahi and Renault (2004) show in a simulation study that using temporal aggregation for the GARCH(1,1) model is somewhat misleading. However, the authors choose $h = 400$, whereas previous studies show that using smaller values of h leads to small biases (see Meddahi and Renault (2004, Sec. 3.4) and a previous version of Drost and Nijman (1993)).

The monthly GARCH(1,1) parameters are given by

$$\omega_M = h\omega\frac{1-(\beta+\alpha)^h}{1-(\beta+\alpha)}, \quad \alpha_M = (\beta+\alpha)^h - \beta_M$$

and $|\beta_M| < 1$ is obtained as the solution of

$$0 = x(\beta,\alpha,\kappa,h)\beta_M^2 - \beta_M + x(\beta,\alpha,\kappa,h),$$

where

$$x(\beta,\alpha,\kappa,h) = \frac{a(\beta,\alpha,\kappa,h)(\beta+\alpha)^h - b(\beta,\alpha,h)}{a(\beta,\alpha,\kappa,h)\left(1+(\beta+\alpha)^{2h}\right) - 2b(\beta,\alpha,h)}$$

and

$$\begin{aligned} a(\beta,\alpha,\kappa,h) &= h(1-\beta)^2 + 2h(h-1)\frac{(1-\beta-\alpha)^2(1-\beta^2-2\beta\alpha)}{(\kappa-1)(1-(\beta+\alpha)^2)} \\ &\quad + 4\frac{\left(h-1-h(\beta+\alpha)+(\beta+\alpha)^h\right)\cdot(\alpha-\beta\alpha(\beta+\alpha))}{1-(\beta+\alpha)^2}, \\ b(\beta,\alpha,h) &= (\alpha-\beta\alpha(\beta+\alpha))\,\frac{1-(\beta+\alpha)^{2h}}{1-(\beta+\alpha)^2}, \quad \kappa = 3\frac{1-(\beta+\alpha)^2}{1-(\beta+\alpha)^2-2\alpha^2}. \end{aligned}$$

The kurtosis κ of the daily returns is calculated for conditional normality (Bollerslev, 1986). However, this assumption can also be relaxed for other distributions (see Embrechts et al. (2005) for the case of conditionally t distributed returns).

The model presented above highlights that the simple SRTR is highly misleading to obtain a monthly volatility forecast using daily data. This is confirmed by Diebold et al. (1998) in a simulation study. Scaling one-day volatility by the SRTR magnifies volatility fluctuations, whereas the true h-day volatilities are much less volatile, since temporal aggregation should dampen volatility fluctuations.[54] Applied to the target volatility strategy, the SRTR implies too many weight adjustments, which lead to needless and very high transaction costs. This also holds for the RV model of Barroso and Santa-Clara (2015) and Moreira and Muir (2017), which is also implicitly based on the SRTR. Further, although $\sqrt{h}$-scaling could be correct on average,

[54]Diebold et al. (1998) and Embrechts et al. (2005) argue that α_M and β_M tend to zero as $h \to \infty$, implying that long-run volatility fluctuations disappear in the Drost and Nijman (1993) model. In other words, long-run volatility is asymptotically constant, thus, long-term returns follow a random walk under this model (Embrechts et al., 2005). This is in line with the observation that returns are typically unconditionally normally distributed as the aggregation interval increases (Diebold et al., 1998). In contrast, by using the SRTR, volatility fluctuations increase as $h \to \infty$.

Diebold et al. (1998) find that the SRTR produces times where volatility is too high and times where volatility is too low. Consequently, used in the volatility targeting context, the SRTR produces times when the portfolio allocation is too high and times when the portfolio allocation is lower than desired.

The model given in Equation (2.3.14) explicitly models monthly volatility based on the monthly return $\overline{R}_t$, where the model parameters are estimated based on daily data. As mentioned above, this approach is appealing, since in this case information on higher frequency data are taken into account and more observations are available to obtain accurate parameter estimates. In contrast, fitting the GARCH model directly to monthly data is problematic, since this approach requires long data sets to obtain accurate parameter estimates. This problem does not exist for the EWMA model, since no parameters have to be estimated for this model. Hence, the EWMA model can directly be used for monthly data as frequently done by practitioners. The month t variance for the EWMA model fitted to monthly data is then given by

$$\sigma_t^2 = (1-\lambda)\cdot \overline{R}_{t-1}^2 + \lambda \cdot \sigma_{t-1}^2, \quad t = 1, ..., T, \tag{2.3.15}$$

where the parameter λ for the monthly EWMA model is usually set to $\lambda = 0.97$ (So and Wong, 2012). We use this approach to assess if this model performs equally well as the other models that are based on daily data.

Although incorporating daily data in estimating monthly volatility and relying on conditional models should increase the forecasting accuracy, the approaches presented above also have several drawbacks. For example, the model of Drost and Nijman (1993) relies on the assumption of a weak GARCH model. Temporal aggregation only holds under very strict assumptions that are unlikely fulfilled in practice. Further, this model assumes that the GARCH(1,1) model correctly describes the one-day return (Saadi and Rahman, 2008). Hence, if the GARCH model is wrong in describing daily volatility, monthly volatility is also badly modeled, where the misspecification increases in h. This drawback also holds for the iterated models given in Equations (2.3.11) and (2.3.13), but not for the SRTR.[55] Further, most models presented in this section do not provide any information on the distribution of the h-day return. As a

[55]Thus, the SRTR should be more robust to parameter uncertainty, which should be an issue for the highly non-normally distributed momentum portfolio.

consequence, these models cannot directly be used to estimate h-day downside risk measures. Embrechts et al. (2005) use further assumptions on the conditional return distribution to calculate h-day downside risk based on these models. We will come back to this point in the next section.

2.4 Tail Risk Managed Momentum Strategy

In Section 2.2, we have argued that the momentum strategy produces abnormally high returns, which are accompanied with infrequent times of extremely negative returns. In Section 2.3, we summarized an approach, volatility targeting, that was used by several studies to manage the crash risk of momentum. Further, we extended the volatility targeting approach of Barroso and Santa-Clara (2015) by using more advanced and probably more accurate volatility forecasting models. By using the volatility targeting approach, the risk of the momentum strategy can be significantly reduced without sacrificing returns. That is, the volatility managed strategy has much lower drawdowns, a higher (less negative) skewness, a lower kurtosis and a higher Sharpe Ratio compared to the non-managed strategy. For example, Barroso and Santa-Clara (2015) find a significantly higher skewness (-0.42 versus -2.47) and Sharpe Ratio (0.97 versus 0.53) as well as a much lower volatility (16.95% versus 27.53%), kurtosis (2.68 versus 18.24) and minimum monthly return (-28.40% versus -78.96%) of the volatility managed portfolio compared to the non-managed strategy. This result highlights that the risk of the momentum portfolio can be managed by dynamically reallocating the weight invested in the portfolio, based on momentum's own risk. However, all studies on managing the crash risk of the momentum strategy rely on volatility as a risk measure, and hence ignore the momentum portfolio's higher moments, like skewness and kurtosis, which can potentially be valuable. For example, Taylor (2005) shows that volatility forecasting models that incorporate higher moments are more accurate than standard volatility models.[56] Furthermore, a huge strand of the financial literature documents the importance of incorporating moments higher than volatility, like skewness

[56]Ghysels et al. (2016) use a similar approach based on quantile forecasts to estimate monthly and quarterly skewness based on daily data, which is then used in a portfolio context. Estimating quantiles is typically much more robust to outliers than estimating realized moments. Thus, in order to estimate volatility, skewness or kurtosis, extracting information of certain quantiles can be an appealing and more robust approach.

and kurtosis, in many fields of finance, like asset pricing (Dittmar, 2002, Harvey and Siddique, 2000, Kraus and Litzenberger, 1976), portfolio selection (Ghysels et al., 2016, Guidolin and Timmermann, 2008, Wang et al., 2012), option pricing (Barone-Adesi et al., 2008) and risk management (Bali et al., 2008). In particular, incorporating skewness in a portfolio allocation context can substantially increase utility gains for investors (Ghysels et al., 2016, Guidolin and Timmermann, 2008, Jondeau and Rockinger, 2006, 2012).[57]

As stated above, the momentum portfolio is known to be extremely negatively skewed and fat-tailed. This highly negative skewness arises since the higher returns of the winners are accompanied with a substantially lower skewness compared to the skewness of the losers (Harvey and Siddique, 2000, p. 1288). Similarly, Chen et al. (2001) find that a high past performance negatively predicts future skewness. Hence, buying past winners and selling past losers produces returns that have a high expected mean, but are extremely negatively skewed. That is, the huge return potential of the momentum strategy occurs jointly with a high probability of extremely negative returns. Thus, since momentum's negative skewness is a main driver of the probability of a momentum crash, information on higher moments should be accounted for when momentum crashes are managed. For example, Chen et al. (2001, p. 348) associate the word crash "solely with the conditional skewness of the return distribution". Hence, when the authors forecast crashes they mean forecasting conditional skewness and they find that conditional skewness can also be interpreted as a measure of the crash expectation (see also the references in Chen et al. (2001)). Furthermore, the kurtosis of the winners and losers portfolios differs widely and is typically quite high (see Table V of Harvey and Siddique (2000)). As a consequence, the momentum portfolio exhibits a high volatility, highly negative skewness and high kurtosis (Barroso and Maio, 2019, Table 3). The highly negative skewness and highly positive kurtosis imply that extremely negative returns are much more likely for the momentum strategy than anticipated by a normal distribution. Hence, this high probability of extremely negative returns, i.e. the momentum crashes, is not accounted for by managing volatility, since

[57] Earlier studies on this field focus on coskewness of an asset with the market (Harvey and Siddique, 2000, Kraus and Litzenberger, 1976), whereas more recent studies mainly focus on an asset's own skewness. Both definitions of skewness lead to improved portfolio allocations and utility gains compared to mean-variance optimized portfolios (Ghysels et al., 2016).

managing volatility is, at least implicitly, based on the assumption of normally distributed returns. This is confirmed by Daniel et al. (2017) who show that momentum's volatility, measured by Realized Volatility or by a GARCH(1,1) model, is not sufficient to identify turbulent market periods in which the momentum strategy suffers the highest losses. A possible explanation for this finding is that "left tail risk is related to left skewness of returns, and there are no a priori reasons to believe that changes in left skewness move in lock step with changes in the volatility of momentum strategy returns" (Daniel et al., 2017, p. 21). This has also been shown by Ghysels et al. (2016) who find that information on skewness is typically hidden in the tails and that skewness in the tails has a higher impact on portfolio allocations. Similarly, Gormsen and Jensen (2017, p. 21) find that the "[v]ariance is negatively correlated to the negative of skewness [and] kurtosis" and that "higher-moment risks are high at times when the market is perceived to be safe and calm as measured by variance. Said differently, risk doesn't go away – it hides in the tails." Neuberger (2012) also finds that a higher volatility coincides with a less negatively skewed return distribution. Similarly, Chen et al. (2001) find that a higher volatility does not necessarily predict a more negative skewness. Moreover, Brooks et al. (2005) find that times of a high volatility do not necessarily coincide with times of a high kurtosis.[58] In particular, Harvey and Siddique (1999), Harvey and Siddique (2000), Jondeau and Rockinger (2003), Brooks et al. (2005), Bali et al. (2008) and Ghysels et al. (2016) show that conditional skewness and kurtosis are highly time-varying, even after standardizing returns with a time-varying volatility. As a consequence, a higher volatility is not necessarily related to a lower skewness or higher kurtosis and volatility is not able to capture these higher moment risks. Thus, managing volatility can produce returns that are more extreme than desired, since high weights of the target volatility strategy can be chosen in times of a high crash risk. In contrast, Cuoco et al. (2008) show that dynamically readjusting portfolio weights based on the portfolio's downside risk is important to achieve a good risk-return profile of risk-managed portfolio strategies. Consequently, momentum's risk should be better managed based on an estimate of momentum's downside risk.[59] In particular, when estimating momentum's downside risk, the time-variation in higher

[58]A similar result also holds in the cross-section. Brooks et al. (2005) find that assets with a higher conditional volatility can have a lower conditional kurtosis than assets with a lower conditional volatility.

[59]See also Alexander and Baptista (2004) and Rickenberg (2020b) for a general discussion of the advantages of

moments should be taken into account (Bali et al., 2008). For example, Harvey and Siddique (1999) and Jondeau and Rockinger (2003) show that models that do not account for skewness and kurtosis fail to accurately fit the true return distribution, which has important consequences for asset allocation decisions (see also Harvey and Siddique (2000)). Generally, ignoring information on higher moments typically underestimates multi-period downside risk (Guidolin and Timmermann, 2006). This observation is even more pronounced for the momentum strategy, since momentum returns are extremely negatively skewed and fat-tailed. In total, managing the volatility of the momentum portfolio ignores the high non-normality of momentum's returns and is a suboptimal tail risk hedging approach. In contrast, by managing momentum's downside risk, the substantial skewness and kurtosis and the time-variation in these moments is captured and this approach should deliver superior results, especially during periods of a momentum crash.[60]

Although the RV managed momentum portfolio is less negatively skewed and fat-tailed than the non-managed strategy, this portfolio still exhibits a high left tail risk. Investors typically have preferences for higher (or positive) skewness and lower kurtosis (see Scott and Horvath (1980), Kraus and Litzenberger (1976), Guidolin and Timmermann (2008) among others). Hence, both strategies, the non-managed and RV managed momentum portfolios, are unappealing for investors with preferences for higher moments. In order to better fit to these investors' preferences, managing the momentum portfolio's risk based on other risk measures than volatility is more appealing. For example, downside risk measures, like Value at Risk (VaR) or Conditional Value at Risk (CVaR), increase when skewness decreases and/or kurtosis increases (Bali et al., 2009, Ghysels et al., 2016). Thus, a downside risk managed momentum strategy decreases the weight invested in the momentum portfolio when left tail risk increases, which is in line with these investors' preferences.[61] Besides preferences for moments higher than volatility, most in-

managing downside risk instead of managing volatility.

[60]Similar results have already been shown for hedge fund returns, which are also known to be highly non-normally distributed. Using volatility as risk measure for hedge funds translates into a high misspecification of risk and fails in asset allocation decisions involving hedge funds (Agarwal and Naik, 2004).

[61]Instead of using a downside risk based portfolio strategy, portfolio selection models that directly incorporate higher moments could also be used. For example, Ghysels et al. (2016) find that an investor who dislikes negative skewness is willing to pay about 6% per year to switch from a mean-variance optimized portfolio to a mean-variance-skewness optimized portfolio. Incorporating higher moments in portfolio selections by maximizing expected utility problems is frequently examined in the literature (see Aït-Sahalia and Brandt (2001) and Guidolin

vestors are typically averse to losses, i.e. they weight losses higher than gains of the same magnitude (Aït-Sahalia and Brandt, 2001, Ang et al., 2006a, Benartzi and Thaler, 1995, Berkelaar et al., 2004). Similarly, the results of Bollerslev et al. (2015) indicate that momentum investors are significantly crash-averse.[62] This arises since the momentum strategy is typically invested in assets with a high crash-sensitivity (Ruenzi and Weigert, 2018). Furthermore, Chabot et al. (2014) find that momentum investors are typically leveraged and are therefore extremely sensible to drawdowns. The RV managed strategy of Barroso and Santa-Clara (2015) still exhibits a minimum monthly return of -28.40% and high probability of extremely negative returns, which makes the RV managed strategy unappealing for loss- and crash-averse investors. Thus, instead of managing return deviations, the occurrence of large negative returns should be managed to better fit to the loss-averse investors' preferences. Rickenberg (2020b) shows that loss-averse investors are willing to pay extremely high fees to have access to downside risk managed strategies, whereas loss-averse investors prefer static portfolio allocations over volatility managed portfolios. For these reasons, we next show how these observations can be used to manage the monthly risk of the momentum portfolio, which should particularly be an appealing approach to mitigate momentum crashes.

Rickenberg (2020b) shows how a portfolio's Value at Risk (VaR) or Conditional Value at Risk (CVaR) can be targeted at a constant level over time. These strategies have the advantage that momentum's downside risk is timed, which has several advantages over volatility timing, as summarized above.[63] In particular, these risk measures are also important for practitioners in many fields. In practice, VaR has been the standard approach of measuring market risk by the financial industry and regulators for many years. However, CVaR has recently developed to

and Timmermann (2008) among others). However, these strategies do not target a constant level of portfolio risk over time and are harder to estimate than our simple downside risk based strategy. Further, skewness is hard to estimate directly and is extremely influenced by outliers (Ghysels et al., 2016, Kim and White, 2004, Neuberger, 2012). Therefore, portfolio decisions based on quantile risk measures are more robust.

[62] This result does not only hold for momentum investors. Generally, most investors are crash-averse and are willing to pay high fees to avoid crash periods (Bollerslev and Todorov, 2011, Chabi-Yo et al., 2018, Van Oordt and Zhou, 2016). This observation is similar to the theory of safety-first investors, who are concerned about avoiding financial tail events (Arzac and Bawa, 1977).

[63] VaR and CVaR cannot only be used as a market timing tool, but also as an asset allocation tool. For example, in a cross-section setting, Atilgan et al. (2020) find a negative relation between return and downside risk, i.e. assets with a higher VaR or CVaR obtain lower returns. Thus, VaR and CVaR can also be used to weight the assets in the momentum portfolio (Rickenberg, 2020c).

the more relevant risk measure (Du and Escanciano, 2016). In a portfolio context, controlling a portfolio's VaR means controlling loss frequencies, whereas controlling a portfolio's CVaR means controlling the severity of extremely negative returns, which is more appealing in order to manage momentum crashes (see Basak and Shapiro (2001) and Giannopoulos and Tunaru (2005) for example).[64]

Applied to the momentum strategy, a constant VaR or CVaR can be achieved by scaling the weight invested each month in the momentum portfolio by the inverse of the monthly VaR or CVaR denoted by VaR_α^t and CVaR_α^t, respectively. In this case, α is the significance level which is chosen by the investor in dependence of the investor's risk preference. The month t return of the VaR and CVaR managed momentum portfolio is then given by

$$R_t^* = \frac{\mathrm{VaR}_\alpha^{\mathrm{target}}}{\mathrm{VaR}_\alpha^t} \cdot R_t \quad \text{or} \quad R_t^* = \frac{\mathrm{CVaR}_\alpha^{\mathrm{target}}}{\mathrm{CVaR}_\alpha^t} \cdot R_t, \tag{2.4.1}$$

where $\mathrm{VaR}_\alpha^{\mathrm{target}}$ and $\mathrm{CVaR}_\alpha^{\mathrm{target}}$ denote the desired VaR and CVaR target, respectively. The VaR or CVaR target is simultaneously chosen with the significance level α. For example, the target VaR and CVaR strategies with a significance level of $\alpha = 1\%$ and target levels $\mathrm{VaR}_\alpha^{\mathrm{target}} = 3\%$ and $\mathrm{CVaR}_\alpha^{\mathrm{target}} = 5\%$ can be interpreted as follows. For the target VaR strategy, the monthly return of the portfolio should be higher than -3% with a probability of 99%. In other words, the monthly momentum return should be lower than -3% in only one out of 100 months. For the target CVaR strategy, the average return on the 1% worst months should be -5%. Hence, a loss-averse investor should choose low levels of α and/or $\mathrm{VaR}_\alpha^{\mathrm{target}}$ or $\mathrm{CVaR}_\alpha^{\mathrm{target}}$.[65] Further, an investor who wants to manage extreme losses should use the target CVaR strategy (Basak and Shapiro, 2001). An investor who is willing to accept extreme losses on only a limited number of months, where the loss magnitude is less important, could also use the target VaR strategy. Rickenberg (2020b) compares the target volatility, target VaR and target CVaR strategies and finds that the target CVaR strategy delivers the most convincing results, especially during crash

[64] See Giannopoulos and Tunaru (2005) and Yamai and Yoshiba (2005) and references therein for a discussion of the superiority of CVaR over VaR. See Basak and Shapiro (2001), Alexander and Baptista (2004) and Rickenberg (2020b) for a discussion why CVaR is superior to VaR when a portfolio's risk is managed.

[65] A low α also fits to the observation of Ghysels et al. (2016) that information on skewness is typically hidden in the tails and that skewness in the tails has a higher impact on portfolio allocations. Hence, a low significance level of α should be chosen to capture momentum's skewness risk. Rickenberg (2020b) also shows that low levels of α are superior for the target VaR and CVaR strategies.

periods. Thus, in order to mitigate momentum crashes, managing the portfolio's CVaR should be superior to managing VaR, since the VaR is typically too low in periods of extreme movements (see Basak and Shapiro (2001), Giannopoulos and Tunaru (2005) and references therein). As in Ghysels et al. (2016), the VaR and CVaR targeting approaches directly define portfolio weights in dependence on quantile risk measures. Hence, these strategies incorporate information on higher moments without explicitly estimating these quantities.[66] In contrast, standard portfolio choice approaches obtain portfolio weights through maximizing an expected utility problem (Aït-Sahalia and Brandt, 2001, Guidolin and Timmermann, 2008). Thus, our approach is more robust to estimation risk, which is an important determinant of the success of asset allocation approaches in practice.[67]

Downside risk measures, like VaR and CVaR, are typically defined for loss variables. Therefore, we define the month t loss of the momentum strategy by

$$L_t := -R_t. \tag{2.4.2}$$

Similarly, we define the daily loss of day i in month t by

$$L_{t,i} := -R_{t,i} \tag{2.4.3}$$

and further $\tilde{L}_s := L_{\lceil \frac{s}{h} \rceil, s-h(\lceil \frac{s}{h} \rceil - 1)}$. The month t VaR is then given by the loss distribution's $(1-\alpha)$-quantile, i.e.

$$\mathrm{VaR}_\alpha^t := F^{-1}_{L_t|\mathcal{F}_{t-1}}(1-\alpha), \tag{2.4.4}$$

where $F^{-1}_{L_t|\mathcal{F}_{t-1}}$ denotes the inverse of the conditional cdf of L_t and $\mathcal{F}_{t-1}$ denotes the information set up to month $t-1$ (Guidolin and Timmermann, 2006, p. 293). The month t CVaR is defined as

$$\mathrm{CVaR}_\alpha^t := \mathbb{E}\left(L_t \,|\, L_t \geqslant \mathrm{VaR}_\alpha^t, \mathcal{F}_{t-1}\right). \tag{2.4.5}$$

[66] Skewness is hard to estimate directly and skewness estimators are extremely influenced by outliers (Ghysels et al., 2016, Kim and White, 2004, Neuberger, 2012). One possibility to obtain more accurate estimates of skewness is to use option prices. Another possibility, as done in Ghysels et al. (2016) and Kim and White (2004), is to use forecasts of quantile risk measures which can be used to extract information on skewness. Using quantile risk measures to estimate skewness is far less sensitive to outliers than commonly used skewness estimation methods.

[67] See, for example, Kirby and Ostdiek (2012), Moreira and Muir (2019) and DeMiguel et al. (2009b) how estimation risk influences risk based portfolio allocations.

Most studies examine the estimation of downside risk measures for relatively short horizons, e.g. one-day ahead forecasts. However, for strategic asset allocation decisions, longer horizons should be considered (Guidolin and Timmermann, 2006).[68] Guidolin and Timmermann (2006) consider the term structure of VaR and CVaR under several econometric specifications. The authors show that different forecasting horizons and econometric specifications can lead to significantly different h-period risk forecasts, which translate in vastly different asset allocation decisions (see Figure 5 in Guidolin and Timmermann (2006)). Similarly, Embrechts et al. (2005) present several approaches how h-day VaR and CVaR can be estimated by models fitted to higher frequency data and the authors find that these approaches produce quite different risk forecasts. Thus, different forecasting models can produce quite different results and accurately modeling the return distribution seems crucial in order to obtain an accurate h-day downside risk forecast. In particular, Embrechts et al. (2005) find that fitting models to higher frequency data is superior when longer-term downside risk is estimated. Ghysels et al. (2016) also find that using daily data to estimate monthly quantiles is superior, since more data are available and the forecasting precision is increased. The authors confirm this conjecture in a Monte-Carlo simulation. Therefore, we present several econometric specifications fitted to daily data and show how these can be used to forecast monthly downside risk. As stated above, since momentum returns are known to be extremely negatively skewed and fat-tailed, we will explicitly take this observation into account. Further, as shown by Harvey and Siddique (1999), Jondeau and Rockinger (2003), Brooks et al. (2005) and Bali et al. (2008), higher moments are typically highly time-varying, which we will also consider by autoregressively modeling higher moments. In addition, we present simple and easy to implement models that could be of interest for practical implementations.[69] A further extensions could be to combine forecasts of several

[68]Measuring downside risk over multiple days is important from several perspectives. For example, 10-day VaR and CVaR are important from a regulatory perspective and are widely used in the financial industry (Berkowitz et al., 2011, Berkowitz and O'Brien, 2002, Saadi and Rahman, 2008). In practice, these 10-day downside risk measures are typically obtained using the SRTR, since the SRTR was explicitly advised by the Basel Committee on Banking Supervision in 1996 (Danielsson and Zigrand, 2006, Kole et al., 2017).

[69]We restrict ourselves to univariate forecasting models that forecast portfolio risk using past portfolio returns. As an alternative, portfolio risk could also be estimated based on past returns of the individual assets that belong to the momentum portfolio, which is then aggregated to a measure of portfolio risk. However, Kole et al. (2017) find that the degree of portfolio aggregation is not that important, but that using higher frequency data is more important in order to obtain accurate h-day forecasts of portfolio risk.

forecasting models (Allen et al., 2012).

We start by presenting two unconditional models that assume that the loss variables are iid. In this case, conditioning on the information set $\mathcal{F}_{t-1}$ is not needed and VaR and CVaR are calculated using the unconditional loss distribution. These models are typically easy to implement and are particularly interesting for practical implementations. The most prominent estimation model is the Historical Simulation (HS), which is commonly used by banks (Berkowitz et al., 2011, Berkowitz and O'Brien, 2002). The HS approach assumes that the monthly VaR and CVaR can be estimated by using the empirical cdf of the loss variables. Following the approach how HS is typically implemented in the financial industry, we use daily data to estimate monthly VaR and CVaR.[70] The estimates for VaR and CVaR of the momentum portfolio for the first day of month t, given information up to the end of month $t-1$, are then given by

$$\mathrm{VaR}_{\alpha}^{t,1} = L_{(\lceil n(1-\alpha)\rceil),t-1} \tag{2.4.6}$$

and

$$\mathrm{CVaR}_{\alpha}^{t,1} = \frac{1}{n - \lceil n(1-\alpha)\rceil + 1} \cdot \sum_{j=\lceil n(1-\alpha)\rceil}^{n} L_{(j),t-1}, \tag{2.4.7}$$

where $L_{(1),t-1} \leqslant ... \leqslant L_{(n),t-1}$ denotes the order statistics of the sample $\tilde{L}_{h(t-1)-n+1}, ..., \tilde{L}_{h(t-1)}$ of n daily losses. The month t VaR and CVaR are then simply given by $\mathrm{VaR}_{\alpha}^{t} = \sqrt{h} \cdot \mathrm{VaR}_{\alpha}^{t,1}$ and $\mathrm{CVaR}_{\alpha}^{t} = \sqrt{h} \cdot \mathrm{CVaR}_{\alpha}^{t,1}$.[71] This approach estimates monthly VaR and CVaR by first estimating the one-day ahead VaR or CVaR by the empirical estimator using daily losses, which is then scaled by the square root of time rule, i.e. multiplying the one-day ahead VaR and CVaR forecasts by $\sqrt{h}$ (Danielsson and Zigrand, 2006).[72] Using HS assumes that losses are iid and that

[70] As for volatility, estimating monthly downside risk with daily data is advantageous since information contained in daily data are taken into account and more data are available (Danielsson and Zigrand, 2006, McNeil et al., 2015). Estimating monthly VaR or CVaR with 500 observations, a number which is needed to obtain an accurate estimate, requires only about two years of daily data but more than 40 years of monthly data. Similarly, Ghysels et al. (2016) estimate monthly quantiles based on daily data in order to estimate monthly skewness. Kole et al. (2017) find that 10-day downside risk is best estimated based on daily data. This holds in a univariate and a multivariate setting.

[71] Another simple extension of the HS to estimate VaR and CVaR could be to use a regression approach, similar to Bali et al. (2009, Eq. 16). Further, a similar approach as the HAR model from Equation (2.3.6) could also be used to estimate VaR and CVaR in a simple way. Similarly, Ghysels et al. (2016) use the MIDAS model of Ghysels et al. (2005) to forecast the monthly VaR based on daily data.

[72] The one-day ahead VaR and CVaR estimates can also be scaled by $h^{1/\alpha}$, where $1/\alpha$ is obtained by the Hill

the distribution of monthly losses can be described by the distribution of daily losses. Taylor (2005) shows that the HS estimates of VaR can also be used to forecast monthly volatility. However, the author finds that this approach does not produce accurate volatility forecasts, but that volatility forecasts based on VaR estimates that incorporate information on skewness and kurtosis are highly accurate. The reason for that finding is that the conditional return distribution is not constant over time (Bali et al., 2008, Jondeau and Rockinger, 2003, Taylor, 2005). Thus, in order to accurately forecast monthly downside risk, more advanced methods are needed.

Unconditional return distributions of asset returns are typically highly non-normal and cannot be characterized by mean and variance alone. Using Historical Simulation does not directly take time-varying higher moments into account. In particular, many downside risk estimation methods used in the financial industry are based on the assumption of normally distributed returns. However, this approach is inconsistent with the empirical finding of non-normally distributed returns and translates into a high underestimation of extremely negative returns, like momentum crashes (Bali et al., 2008, p. 270). For example, if returns were normally distributed, a daily return of -5% should occur only once every 1000 years, whereas such a loss occurs approximately once every two years for stock returns (Brooks et al., 2005, p. 400). For the momentum portfolio, we find that a daily return lower than -5% occurs (on average) more than once per year. Therefore, as second unconditional model, we assume that daily returns are iid skewed t distributed with mean μ and variance σ^2, i.e. we assume that the daily return can be described by

$$R_{t,i} = \mu + \sigma \cdot Z, Z \sim stsk(\eta, \lambda), \tag{2.4.8}$$

where $Z \sim stsk(\eta, \lambda)$ means that Z follows a *standardized skewed* t *distribution* with parameters η and λ (Bali et al., 2009).[73] The standardized skewed t distribution has expectation zero

estimator relying on extreme value theory (Cotter, 2007, Embrechts et al., 2005). By doing this, the scaling rule takes information of the return distribution's tail into account. This scaling rule can be applied to raw or standardized returns as shown by Cotter (2007). The second case has the advantage that the current market environment is taken into account. We will later regard models that consider the current market environment, measured by the conditional volatility.

[73]This approach is similar to Theodossiou (1998) who fits the skewed generalized t distribution to the unconditional empirical distribution of stock returns. The skewed generalized t distribution is an extension of the standardized skewed t distribution used in our paper. Theodossiou (1998) finds that the skewed generalized t distribution, which explicitly models skewness and tail-behavior of the data, fits well to the observed data. However,

and variance one. This distribution was presented by Hansen (1994) and is characterized by the pdf

$$f_{stsk}\left(z \mid \eta, \lambda\right) = \begin{cases} bc\left(1 + \frac{1}{\eta-2}\left(\frac{bz+a}{1-\lambda}\right)^2\right)^{-(\eta+1)/2} & \text{if } z < -\frac{a}{b} \\ bc\left(1 + \frac{1}{\eta-2}\left(\frac{bz+a}{1+\lambda}\right)^2\right)^{-(\eta+1)/2} & \text{if } z \geqslant -\frac{a}{b}, \end{cases} \tag{2.4.9}$$

where

$$a := 4\lambda c\frac{\eta-2}{\eta-1}, \quad b^2 := 1 + 3\lambda^2 - a^2, \quad c := \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)}\Gamma\left(\frac{\eta}{2}\right)}.$$

The parameters of this distribution are restricted to $\eta > 2$ and $-1 < \lambda < 1$ (see Hansen (1994, p. 710) and Jondeau and Rockinger (2003, p. 1702)). Further, for $\lambda = 0$, this distribution is symmetric and equals the standardized t distribution presented below. For $\lambda > 0$, the distribution's mode is to the left of zero and the distribution is positively skewed. In contrast, if $\lambda < 0$, the distribution is negatively skewed (Hansen, 1994). Moreover, skewness exists for $\eta > 3$ and kurtosis exists for $\eta > 4$ (Jondeau and Rockinger, 2003). Jondeau and Rockinger (2003) show that although η is often referred as the parameter that determines kurtosis and λ determines skewness, both parameters, η and λ, affect both moments, skewness and kurtosis. In particular, the relation between the parameters and higher moments is highly non-linear. Eling (2014) analyzes the goodness-of-fit of several distributions and finds that a skewed t distribution fits the distribution of asset returns very well in comparison to 12 benchmark distributions. In particular, Eling (2014) finds that the skewed t distribution is performing well in describing hedge fund returns and provides good forecasts of VaR and CVaR for hedge fund returns. Further, the author finds that the normal and skew-normal distribution perform badly when fitted to asset returns. Hence, skewness *and* kurtosis should be considered when asset returns are described.[74]

In the case that daily (unconditional) returns follow a skewed t distribution, VaR for month t is given by

$$\text{VaR}_{\alpha}^{t} = -\left(h \cdot \mu + \sqrt{h} \cdot \sigma \cdot F_{stsk}^{-1}\left(\alpha \middle| \, \eta, \, \lambda\right)\right), \tag{2.4.10}$$

we follow Bali et al. (2009) and fit the standardized skewed t distribution to daily returns in order to calculate monthly VaR and CVaR.

[74] Eling (2014) uses a skewed t distribution that is different to the skewed t distribution of Hansen (1994). However, the results of Eling (2014) demonstrate the importance of modeling skewness and kurtosis when describing returns and estimating downside risk.

where $F_{stsk}^{-1}(\alpha|\,\eta,\,\lambda)$ denotes the α-quantile of the skewed t distribution with parameters η and λ (see also Danielsson and Zigrand (2006, Eq. 4) for the case of the normal distribution). Similarly, the CVaR of month t is given by

$$\text{CVaR}_{\alpha}^{t} = -\left(h\mu + \sqrt{h}\cdot\sigma\cdot\mathbb{E}\big(Z|Z < F_{stsk}^{-1}(\alpha|\,\eta,\,\lambda)\big)\right). \tag{2.4.11}$$

In Equations (2.4.10) and (2.4.11), we again use the SRTR to obtain the monthly VaR and CVaR based on an estimate of the daily VaR and CVaR.[75] Danielsson and Zigrand (2006) call this rule the "mean-corrected square root of time rule", since this rule corrects for a non-zero mean. The parameters μ and σ are simultaneously estimated with the parameters η and λ and are obtained by Maximum Likelihood estimation, where the parameters are re-estimated every month. This approach incorporates information on skewness and kurtosis in a parsimonious and easy to implement way. The shape of the distribution varies over time, since parameters are re-estimated every month. The Maximum Likelihood approach for the unconditional distribution is given in Bali et al. (2009) and similarly in Theodossiou (1998) for the skewed generalized t distribution.[76] Bali et al. (2008, Section 3.1) also use a similar approach for the skewed generalized t distribution of Theodossiou (1998).

The α-quantile $F_{stsk}^{-1}(\alpha|\,\eta,\,\lambda)$ of the standardized skewed t distribution, which is needed in Equation (2.4.10), is given by

$$F_{stsk}^{-1}(\alpha\mid\eta,\lambda) = \begin{cases} \frac{1}{b}\left((1-\lambda)\sqrt{\frac{\eta-2}{\eta}}F_t^{-1}(\frac{\alpha}{1-\lambda}|\eta) - a\right) & \text{if } \alpha < \frac{1-\lambda}{2} \\ \frac{1}{b}\left((1+\lambda)\sqrt{\frac{\eta-2}{\eta}}F_t^{-1}(\frac{\alpha+\lambda}{1+\lambda}|\eta) - a\right) & \text{if } \alpha \geqslant \frac{1-\lambda}{2}, \end{cases} \tag{2.4.12}$$

where $F_t^{-1}(z|\eta)$ is the inverse of the t distribution's cdf $F_t(z|\eta) = \int_{-\infty}^{z} f_t(u|\eta)du$ (Jondeau and Rockinger, 2003). The t distribution's pdf with η degrees of freedom is given by

$$f_t(z|\eta) = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\Gamma\left(\frac{\eta}{2}\right)\sqrt{\pi\eta}}\left(1+\frac{z^2}{\eta}\right)^{-(\eta+1)/2}, \tag{2.4.13}$$

[75] For the scaling of volatility, the iid assumption is sufficient. However, this result does not translate to the scaling of downside risk. When scaling the one-day VaR and CVaR by $\sqrt{h}$ to obtain the monthly estimates, it is additionally needed that returns are normally distributed with a mean of zero (Danielsson and Zigrand, 2006, Embrechts et al., 2005). Hence, scaling VaR and CVaR by $\sqrt{h}$ only holds under very restrictive assumptions, which usually do not hold in practice. However, the SRTR is frequently applied to obtain an estimate of the h-day VaR and CVaR, even when these conditions are not fulfilled. We follow this convention and also use the SRTR under the (unconditional) skewed t assumption.

[76] The log-likelihood in Bali et al. (2009, Footnote 5) contains a small error. The term $n\ln\Gamma(\nu-2)$ in Bali et al. (2009) has to be replaced by $\frac{n}{2}\ln(\nu-2)$ to provide the correct log-likelihood.

where $\Gamma(\cdot)$ denotes the Gamma function.[77] From Appendix B, it follows[78]

$$\mathbb{E}\big(Z|Z < F_{stsk}^{-1}(\alpha|\,\eta,\,\lambda)\big) = \begin{cases} \frac{1}{\alpha}\frac{(1-\lambda)^2}{b}\left(f_{st}\left(z^{(-)}|\eta\right)\cdot\frac{\eta-2+\left(z^{(-)}\right)^2}{1-\eta} - \frac{a\cdot F_{st}\left(z^{(-)}|\eta\right)}{1-\lambda}\right) & \text{for } F_{stsk}^{-1}(\alpha|\,\eta,\,\lambda) < -\frac{a}{b} \\ \frac{1}{\alpha}\frac{(1+\lambda)^2}{b}\left(f_{st}\left(z^{(+)}|\eta\right)\cdot\frac{\eta-2+\left(z^{(+)}\right)^2}{1-\eta} + \frac{a\cdot\left(1-F_{st}\left(z^{(+)}|\eta\right)\right)}{1+\lambda}\right) & \text{for } F_{stsk}^{-1}(\alpha|\,\eta,\,\lambda) \geqslant -\frac{a}{b}, \end{cases} \tag{2.4.14}$$

where we define

$$z^{(-)} = \frac{b\cdot F_{stsk}^{-1}(\alpha|\,\eta,\,\lambda)+a}{1-\lambda}, \quad z^{(+)} = \frac{b\cdot F_{stsk}^{-1}(\alpha|\,\eta,\,\lambda)+a}{1+\lambda}.$$

Further, $f_{st}(z|\eta)$ and $F_{st}(z|\eta) = \int_{-\infty}^{z} f_{st}(u|\eta)du$ correspond to the pdf and cdf of the *standardized t distribution* with mean zero and variance one. The pdf of the standardized t distribution is given by (see Bollerslev (1987, p. 543) and Hansen (1994, p. 709))

$$f_{st}(z|\eta) = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\Gamma\left(\frac{\eta}{2}\right)\sqrt{\pi(\eta-2)}}\left(1+\frac{z^2}{\eta-2}\right)^{-(\eta+1)/2}. \tag{2.4.15}$$

For the cdf of the t and standardized t distributions, it holds $F_{st}(z|\eta) = F_t\left(\sqrt{\frac{\eta}{\eta-2}}z|\eta\right)$.[79]

Rickenberg (2020b) shows that unconditional models fail to accurately target a certain level of risk and that dynamic models, which rely on a conditional volatility model, produce a more accurate portfolio risk, an enhanced risk-return profile and higher utility gains.[80] Further, Guidolin and Timmermann (2006) show that unconditional and conditional h-period ahead VaR and CVaR forecasts can vary substantially, which lead to quite different portfolio allocations. Generally, when a portfolio is reallocated every month, conditional approaches are needed, whereas for portfolios that are reallocated infrequently, like once a year, unconditional approaches are typically superior (Cotter, 2007). Similarly, Hansen (1994) argues that if the

[77]The t distribution is a symmetric distribution where the tail behavior is governed by the parameter η. The distribution as given in Equation (2.4.13) has a variance of $\frac{\eta}{\eta-2}$ for $\eta > 2$. The standardized version of this distribution with variance one is given in Equation (2.4.15). Bollerslev (1987) introduced the standardized skewed t distribution in the context of the GARCH model.

[78]See also Lönnbark (2016, p. 952 and 960) who also derives this result for $F_{stsk}^{-1}(\alpha|\,\eta,\,\lambda) < -a/b$. However, the second case, $F_{stsk}^{-1}(\alpha|\,\eta,\,\lambda) \geqslant -\frac{a}{b}$, is not shown by the author.

[79]This relation is advantageous since the cdf of the t distribution is often available in most software packages, whereas the cdf of the standardized t distribution is not available (Jondeau and Rockinger, 2003).

[80]Rickenberg (2020b) shows that conditional models are more accurate in forecasting risk, i.e. conditional models produce a more constant portfolio risk of strategies that target a constant level of risk. Bollerslev et al. (2018) show that there exists a positive relation between forecasting accuracy and utility gains of risk targeting investors. Consequently, the more accurate conditional risk models are also more valuable in terms of Sharpe Ratio and utility increases for investors who target a constant level of risk.

interest is to predict future values that depend on the conditional return distribution, e.g. next month's risk, it is important to correctly model the conditional return distribution. Therefore, besides the two above presented unconditional approaches – Historical Simulation and skewed t distribution fitted to daily returns combined with the SRTR – we next present conditional approaches that are based on a dynamic volatility model. These approaches explicitly model the conditional return distribution by conditionally modeling volatility, skewness and kurtosis, which should be more successful than simply managing volatility or managing risk unconditionally as presented above. Although the GARCH(1,1) model is able to generate fat tails of the unconditional return distribution, even when conditional normality is assumed as done in Section 2.3, this approach typically fails to correctly model the tail behavior of the return distribution (Bollerslev, 1987).[81] For example, Fan and Gu (2003) and Wong and So (2003, p. 1027) show that the distribution of the aggregate h-day return based on $\mathcal{F}_{t-1}$ deviates substantially from the normal distribution.[82] Consequently, momentum's conditional non-normality should be incorporated when monthly downside risk is estimated. In particular, Neuberger (2012) argues that skewness in monthly returns has two different sources. First, skewness coming from asymmetric volatility, i.e. the leverage effect, which produces skewness of monthly returns even when daily returns are symmetrically distributed (see also Colacito and Engle (2010) and Chen et al. (2001, Footnote 2)). Second, skewness of higher frequency returns, like skewness of daily returns, which also translates into skewness of monthly returns. Hence, although asymmetric volatility models, like the GJR-GARCH model, are able to model skewness of monthly returns, skewness in daily returns should also be incorporated. Thus, a conditional distribution that models skewness and kurtosis of standardized daily returns is needed.

Besides considering the *conditional* non-normalities of daily returns when monthly risk is

[81]Bollerslev (1987) uses the standardized t distribution in the context of the GARCH(1,1) model and finds that the GARCH(1,1)-t model is superior to the GARCH(1,1)-normal model and the unconditional standardized t distribution.

[82]Similarly, Baillie and Bollerslev (1992) derive formulas for the h-day ahead higher moments in the GARCH(1,1) model. The authors show that incorporating higher moments, and hence (conditional) non-normality, becomes more important when quantiles of the h-day ahead return are estimated for horizons of $h > 1$. That is, accounting for higher moments becomes more important when monthly VaR and CVaR are estimated compared to the case when one-day ahead VaR and CVaR are estimated. This result holds particularly in times of a high volatility, i.e. in times that typically coincide with momentum crashes. Additionally, Baillie and Bollerslev (1992) find that unconditional models perform much worse than conditional models in times of a high volatility.

estimated, the term-structure of risk is also crucial for h-day downside risk forecast. For example, Kole et al. (2017, p. 662) find that skewness and kurtosis of daily, weekly and monthly returns differ and that extreme and negative returns cluster over time, which is not captured by simple scaling rules. Similarly, Neuberger (2012) finds that skewness increases with the aggregation period, but does not nicely scale like variance. Ghysels et al. (2016) show that the term-structure of skewness can be highly different for different assets, and hence simple scaling rules do not apply for skewness. Moreover, Wong and So (2003, p. 1021) find that the kurtosis of the aggregate return increases with the holding period h, i.e. monthly returns are heavier tailed than daily returns. Generally, Harvey and Siddique (1999, p. 466) find that "daily and monthly returns [...] appear to have quite different properties". Scaling rules and unconditional approaches ignore that the return distribution changes over time (Cotter, 2007). For that reason, besides scaling a one-day ahead risk forecast into an h-day risk forecast by simple scaling rules like the SRTR, we additionally use simulation methods. Explicitly modeling the h-day return's distribution is crucial, since Danielsson and Zigrand (2006) show that the SRTR typically underestimates risk, which translates into too high weights invested in the momentum portfolio. In particular, the SRTR highly underestimates downside risk if the asset exhibits jumps, e.g. momentum crashes. Kole et al. (2017) find better results of iterated approaches to forecast multi-period downside risk compared to forecasts based on scaling rules. Similarly, McNeil and Frey (2000) and Lönnbark (2016) find that the SRTR performs badly in forecasting multi-period VaR and CVaR. The SRTR is only suitable when longer-term risk, like yearly risk, is forecasted. For example, Embrechts et al. (2005) find good results of the SRTR for longer forecasting horizons, like one year, which corresponds to $h = 252$ (see also Cotter (2007)).

One drawback of not using simple scaling rules, like the SRTR, is that the h-day VaR and CVaR forecasts are not directly available in closed form. Although the GARCH(1,1) model can easily provide a forecast of the h-day cumulative return, the distribution of this return is still unknown, even if the (conditional) distribution of the daily returns is known (see Christoffersen (2012, p. 73), Lönnbark (2016) and references therein). This holds particularly due to the dependence of higher order moments (Baillie and Bollerslev, 1992). Hence, when estimating the monthly VaR and CVaR based on the GARCH models fitted to daily data, monthly downside risk measures are not directly available. This makes the estimation of monthly VaR and CVaR much more challenging than the estimation of one-day ahead risk measures. To solve this problem, several approaches have been developed in the academic literature. A common and often used approach is the Filtered Historical Simulation (FHS), which is used by Rosenberg and Engle (2002), Pritsker (2006), Giannopoulos and Tunaru (2005), Barone-Adesi et al. (2008) and Engle (2011). Another possibility is to simulate h-day aggregate returns using a dynamic parametric model for the daily returns (Christoffersen, 2012, Engle, 2011, Lönnbark, 2016, So and Wong, 2012, Wong and So, 2003). Both approaches model monthly returns based on models fitted to daily data. Downside risk measures, like VaR and CVaR, can then simply be calculated based on simulated future monthly returns. In this paper, we use both approaches, FHS and the standardized skewed t distribution, combined with a dynamic volatility model, which is shortly summarized below.

Similar to Wong and So (2003), So and Wong (2012) and Lönnbark (2016), we use a dynamic volatility model combined with the skewed t-distribution of Hansen (1994), where we extend this approach by explicitly accounting for autoregressive patterns in the conditional skewness and kurtosis. By using Equation (2.4.8) – or the conditional version thereof used by Wong and So (2003) and So and Wong (2012) – skewness and kurtosis can be modeled in a time-varying way by frequently re-estimating the parameters η and λ (Hansen, 1994). However, several studies have demonstrated that autoregressively modeling conditional skewness and kurtosis is possible and typically produces superior results (Bali et al., 2008, Brooks et al., 2005, Harvey and Siddique, 1999, Jondeau and Rockinger, 2003). We therefore extend the model used in Wong and So (2003) and So and Wong (2012) by autoregressively modeling the parameters η and λ as shown in Jondeau and Rockinger (2003) and Bali et al. (2008).[83] Similar to Equation (2.4.8), we start by modeling daily returns based on the skewed t distribution. More

[83] Bali et al. (2008) also use this approach to forecast the one-day ahead VaR. Wong and So (2003) and So and Wong (2012) forecast the h-day VaR and CVaR, but the authors do not explicitly account for autoregressive conditional skewness and kurtosis. That is, instead of modeling the shape of the standardized skewed t distribution in a time-varying way by autoregressively modeling the parameters η and λ, the authors assume that these parameters are constant. As far as we know, estimating h-day VaR and CVaR under the skewed t distribution that accounts for autoregressive conditional skewness and kurtosis has not been examined so far.

formally, we assume that the return of day i in month t is given by

$$R_{t,i} = \mu + \sigma_{t,i} \cdot Z_{t,i},\ Z_{t,i} \sim stsk(\eta_{t,i}, \lambda_{t,i}), \tag{2.4.16}$$

where $\sigma_{t,i} = \tilde{\sigma}_{h(t-1)+i}$ is given by the GARCH(1,1) or GJR-GARCH process from Equation (2.3.10) or (2.3.12), respectively (Jondeau and Rockinger, 2003).[84] We follow Wong and So (2003) and So and Wong (2012) and again assume a constant mean in Equation (2.4.16).[85] The parameters $\eta_{t,i}$ and $\lambda_{t,i}$ are then modeled autoregressively by first modeling unrestricted parameters. The unrestricted parameters are given by

$$\tilde{\eta}_t = a_1 + b_1 \tilde{R}_{t-1} + c_1 \tilde{\eta}_{t-1}, \tag{2.4.17}$$

$$\tilde{\lambda}_t = a_2 + b_2 \tilde{R}_{t-1} + c_2 \tilde{\lambda}_{t-1}, \tag{2.4.18}$$

where $\tilde{R}_s := R_{\lceil \frac{s}{h} \rceil, s-h(\lceil \frac{s}{h} \rceil - 1)} - \mu$.[86] To guarantee that the standardized skewed t distribution is well defined, the parameters have to be rescaled to fulfill the conditions $\eta_{t,i} > 2$ and $-1 < \lambda_{t,i} < 1$. We follow Jondeau and Rockinger (2003) and Bali et al. (2008) and use a logistic transformation to guarantee that these restrictions hold. The parameters $\eta_{t,i}$ and $\lambda_{t,i}$ are then given by

$$\eta_{t,i} = 2 + \exp\left(\tilde{\eta}_{h(t-1)+i}\right) \tag{2.4.19}$$

$$\lambda_{t,i} = \frac{2}{1 + \exp\left(-\tilde{\lambda}_{h(t-1)+i}\right)} - 1. \tag{2.4.20}$$

The main difference between Equation (2.4.8) and Equation (2.4.16) is that in Equation (2.4.8) it is assumed that returns are iid, whereas this assumption is not needed in Equation (2.4.16). The iid assumption in Equation (2.4.16) is even more relaxed, since Equation (2.4.16)

[84] Similarly, Bali et al. (2008), Wong and So (2003) and So and Wong (2012) use the skewed generalized t distribution of Theodossiou (1998). As mentioned above, the skewed generalized t distribution is similar to the distribution of Hansen (1994) used in this paper, but the skewed generalized t distribution has an additional parameter κ that governs the peakedness of the distribution. In particular, the skewed t distribution is a special case of the skewed generalized t distribution by choosing $\kappa = 2$. Since Wong and So (2003) and So and Wong (2012) set $\kappa = 2$, the distribution used by the authors reduces to the distribution of Hansen (1994) used in our paper.

[85] When estimating one-day ahead downside risk, a zero mean is often assumed. For short horizons, incorporating a non-zero mean return is not that important when downside risk is estimated. However, this task becomes more important when longer-term downside risk is estimated (Danielsson and Zigrand, 2006, Embrechts et al., 2005, Engle, 2011).

[86] Bali et al. (2008) use the standardized return $\tilde{Z}_s$ instead of the demeaned return $\tilde{R}_s$ as done by Jondeau and Rockinger (2003). However, both approaches deliver similar results.

does not even assume that volatility standardized returns are iid, as it is frequently done when downside risk is estimated conditionally. That is, Equation (2.4.16) models conditional skewness and kurtosis in a time-varying manner, whereas the shape of the distribution given in Equation (2.4.8) only varies over time since η and λ are re-estimated every month.

The SRTR is also often combined with models that conditionally forecast risk. In this case, downside risk of month t can be obtained by scaling up the forecast of day one in month t, conditioned on $\mathcal{F}_{t-1}$, with the SRTR.[87] More formally, month t VaR and CVaR could simply be estimated by

$$\mathrm{VaR}_{\alpha}^{t} = -\sqrt{h}\sigma_{t,1} F_{stsk}^{-1}\left(\alpha \middle|\, \eta_{t,1},\ \lambda_{t,1}\right) - h\mu \tag{2.4.21}$$

and

$$\mathrm{CVaR}_{\alpha}^{t} = -\sqrt{h}\sigma_{t,1}\mathbb{E}\big(Z_{t,1}|Z_{t,1} < F_{stsk}^{-1}\left(\alpha \middle|\, \eta_{t,1},\ \lambda_{t,1}\right)\big) - h\mu, \tag{2.4.22}$$

where the parameters $\eta_{t,1}$ and $\lambda_{t,1}$ are the Maximum Likelihood estimates of Equations (2.4.19) and (2.4.20) using a sample of volatility standardized returns. By estimating the h-day VaR, Wong and So (2003) find that combining GARCH-type models with the SRTR produces bad VaR forecasts, whereas using the true h-day volatility forecast significantly improves the VaR forecast. Further, Cotter (2007) shows that combining a conditional VaR forecasting approach with the SRTR typically produces inaccurate results, where downside risk is highly misspecified for large values of h.[88] Consequently, we next present two more powerful simulation based approaches that are also frequently used in the literature.

As stated above, based on Equation (2.4.16), a closed form solution for the h-day return's cdf is typically unavailable in practice, since high-dimension integration would be needed. We therefore follow Wong and So (2003) and So and Wong (2012) and use the dynamic volatility models combined with the assumption of conditionally standardized skewed t distributed daily returns. Based on this assumption, we simulate monthly returns using the model fitted

[87] See Wong and So (2003, p. 1022) for the case of conditional normality. In practice, the assumption of conditional normality is typically combined with the EWMA model. That is, h-day downside risk is forecasted by first estimating the one-day ahead volatility and then using the SRTR. The estimates for VaR and CVaR are then obtained by assuming conditional normality (Fan and Gu, 2003).

[88] For that reason, the authors propose to scale conditional estimates by $h^{1/\alpha}$ instead of $h^{1/2}$, where α denotes the tail index.

to daily returns. That is, we rely on the assumption that the daily returns can be described by the GARCH(1,1) or GJR-GARCH process with standardized skewed t distributed innovations, given in Equation (2.4.16). Based on this model, an approximation of the distribution of monthly returns is obtained by simulating $N = 10{,}000$ future realizations in the following way, illustrated for the GARCH(1,1) model. The procedure for the GJR-GARCH model is similar and is not illustrated here. At the end of month $t-1$, we use the past 48 months of daily returns, which corresponds to 1056 daily returns, to estimate the parameters of the GARCH(1,1) model and the parameters of the standardized skewed t distribution as well as the unconditional mean μ. At the end of month $t-1$, the last daily return of month $t-1$, $R_{t-1,h}$, and the demeaned return, $\overline{R}_{t-1,h} = R_{t-1,h} - \mu$, are known. Based on the parameters of the GARCH(1,1) model, we can therefore estimate the daily variance for the first day of month t by $\sigma_{t,1}^2 = \omega + \alpha \cdot \overline{R}_{t-1,h}^2 + \beta \cdot \sigma_{t-1,h}^2$. For simulation step $i = 1$, we simulate a standardized skewed t distributed random variable $Z_{t,1}^{(i)} \sim stsk(\eta_{t,1}, \lambda_{t,1})$.[89] Based on $\sigma_{t,1}$ and $Z_{t,1}^{(i)}$ we can then calculate $R_{t,1}^{(i)} = \mu + \sigma_{t,1} \cdot Z_{t,1}^{(i)}$. Then, using $R_{t,1}^{(i)}$, μ and $\sigma_{t,1}$, we calculate $\sigma_{t,2}$ to obtain $R_{t,2}^{(i)} = \mu + \sigma_{t,2} \cdot Z_{t,2}^{(i)}$, where $Z_{t,2}^{(i)} \sim stsk(\eta_{t,2}, \lambda_{t,2})$. The parameters $\eta_{t,2}$ and $\lambda_{t,2}$ are obtained from Equations (2.4.17)–(2.4.20). By repeating this procedure for the remaining days in month t, we can simulate h daily returns $R_{t,1}^{(i)}, ..., R_{t,h}^{(i)}$ and we obtain the monthly return $R_t^{(i)}$ by

$$R_t^{(i)} = \prod_{j=1}^{h} (1 + R_{t,j}^{(i)}) - 1. \tag{2.4.23}$$

This procedure is then repeated for $i = 2, ..., N$ to obtain $N = 10{,}000$ simulated monthly losses $L_t^{(1)} = -R_t^{(1)}, ..., L_t^{(10{,}000)} = -R_t^{(10{,}000)}$. Based on this sample of monthly losses, we can simply calculate the monthly VaR and CVaR by Equations (2.4.6) and (2.4.7), respectively, where we replace the daily losses in these equations by the simulated monthly losses $L_t^{(1)}, ..., L_t^{(N)}$. Consequently, by starting from a model fitted to daily data, we obtain a time series of returns that reflects the term-structure of risk of the momentum returns by explicitly considering autore-

[89] The standardized skewed t distributed random variable can simply be simulated by first simulating a uniformly distributed random variable U on the interval $[0,1]$. In this case, the random variable $F_{stsk}^{-1}(U \mid \eta, \lambda)$ follows a standardized skewed t distribution with parameters η and λ (Jondeau and Rockinger, 2003). In Section 2.4 and Appendix C, Jondeau and Rockinger (2003) show how future price paths can be simulated based on this approach. Brooks et al. (2005, Sec. 1.5) also suggest that their approach of autoregressively modeling conditional variance and kurtosis can be used to model future price paths that can be used to calculate downside risk measures. However, the authors do not use these simulations to calculate longer-term downside risk measures.

gressive conditional skewness and kurtosis. These return series incorporate that risk can change over time, an important fact that is not regarded by simply scaling up daily risk by $\sqrt{h}$ (Engle, 2011, p. 442). Furthermore, the simulation method does not need any assumptions about the h-day aggregate return (So and Wong, 2012). In particular, if the number of replicates N tends to infinity, this estimator converges to the true VaR and CVaR (Wong and So, 2003). Wong and So (2003), So and Wong (2012) and Lönnbark (2016) find good results of this approach to estimate VaR and CVaR and show that most other models underestimate risk, especially for low levels of α. In particular, the authors find that the SRTR is much less precise in estimating monthly risk than the simulation method. Wong and So (2003), Alexander et al. (2013) and Lönnbark (2016) additionally propose alternative estimators that produce similar results to the simulation approach by matching the moments of the data with the moments of the skewed t distribution. The authors find that the differences between this approach and the simulation approach are only small and both approaches work very well compared to other specifications. Nevertheless, matching moments of the data and the skewed t distribution does not reflect autoregressive patterns in the conditional skewness and kurtosis as done by our approach. The results of Bali et al. (2008), Jondeau and Rockinger (2003) and Neuberger (2012) indicate that the serial dependencies in the data, especially in higher moments, should be regarded when monthly risk is estimated. For that reason, we only use the simulation approach presented above.

As second simulation based method, we estimate monthly VaR and CVaR based on Filtered Historical Simulation (FHS), which is also used by Engle (2004), Barone-Adesi et al. (2008) and Engle (2011) to simulate h-day returns based on a dynamic volatility model. The FHS approach is similar to the approach presented above, but instead of drawing innovations from a standardized skewed t distribution, the FHS draws innovations (with replacement) from volatility standardized historical returns.[90] Thus, one drawback of the FHS approach compared to the approach presented above is that higher moments are not modeled autoregressively. Neverthe-

[90] As in Pritsker (2006), we estimate the GARCH(1,1) and GJR-GARCH parameters by assuming that innovations are normally distributed. The volatilities of these models are then used to calculate standardized returns, which are used for the FHS approach. An alternative would be to estimate the GARCH(1,1) and GJR-GARCH parameters based on skewed t distributed returns and then use filtered returns from these volatility models for the FHS. Kuester et al. (2006) use FHS for standardized returns based on GARCH-normal and GARCH-skewed t standardized returns and find that both approaches produce different results.

less, despite its simplicity, the FHS approach typically performs quite well. As above, we again simulate $N = 10{,}000$ realizations to estimate VaR and CVaR.[91] The FHS approach is nicely summarized in Giannopoulos and Tunaru (2005, Sec. 4.2) and Pritsker (2006, Sec. 4) who use the GARCH-FHS approach to estimate the 10-day downside risk by first modeling 10-day return paths, which are then used to estimate the empirical VaR and CVaR of these 10-day return paths.[92] Hsieh (1993, Sec. V.B) also uses FHS to simulate future paths, in order to estimate h-day downside risk used for capital requirements of a trader and the author compares this to an unconditional simulation method. Moreover, Rosenberg and Engle (2002) and Barone-Adesi et al. (2008) use the FHS approach to calculate pricing kernels and option prices based on longer-term volatility forecasts. Rosenberg and Engle (2002) and Barone-Adesi et al. (2008) combine the FHS with the GJR-GARCH model of Glosten et al. (1993) and find good results of this method. Generally, Rosenberg and Engle (2002) fit several volatility models to daily data and find that the FHS approach combined with the GJR-GARCH model provides the most convincing results when options based on longer-term volatility forecasts are priced. Kole et al. (2017) also find good results of FHS combined with the GJR-GARCH model to forecast multi-period downside risk. In particular, Barone-Adesi et al. (2008) show that the FHS approach is more accurate than other competing models that do not model the volatility dynamically and/or do not consider non-normalities.

The GARCH-FHS approach easily combines a parametric model for the volatility dynamics with a non-parametric model for the standardized returns, whereas the GARCH model combined with the skewed t distribution is a fully parametric approach (Pritsker, 2006).[93] Although no distributional assumption is needed, the FHS approach considers the actual market environment by relying on a dynamic volatility model (Giannopoulos and Tunaru, 2005). Further, the FHS approach is an easy method that considers non-normalities in the return distribution (Barone-Adesi et al., 2008). Generally, the FHS also has the advantage that the CVaR is a co-

[91] Barone-Adesi et al. (2008) additionally simulate $N = 20{,}000$ future price paths, but the authors find only minor differences between $N = 20{,}000$ and $N = 10{,}000$ simulations.

[92] The CVaR could alternatively be estimated as an average of several VaRs over different levels of α (see Giannopoulos and Tunaru (2005) and references therein).

[93] By combining a parametric approach for the volatility and a non-parametric approach to model the conditional distribution, this approach belongs to the class of semi-parametric estimation methods. See Fan and Gu (2003) for a study on other semi-parametric approaches that can be used to estimate the h-day VaR.

herent risk measure under the FHS approach (Giannopoulos and Tunaru, 2005). In total, the FHS approach is an easy but effective alternative to the more complex simulation approach based on the skewed t distribution.

Besides the FHS and standardized skewed t distribution, other simulation approaches that are based on a dynamic volatility model are also frequently used in the academic literature. For example, Engle (2004), Engle (2011) and Christoffersen (2012) assume that innovations are normally distributed, i.e. $Z_{t,j}^{(i)} \sim N(0,1)$, $j = 1, ..., h$. Further, other distributional assumptions on $Z_{t,j}^{(i)}$ can be made, such as a standardized t distribution (Christoffersen, 2012). Similarly, Baillie and Bollerslev (1992, Sec. 7) discusses how quantiles can be obtained by using a GARCH(1,1) model fitted to daily data and using a Cornish-Fisher approach. This approach explicitly incorporates information on higher moments and is also used by Lönnbark (2016). Wang et al. (2012) use an extreme value theory (EVT) based simulation approach to forecast monthly CVaR based on daily data. Similarly, in order to forecast the h-day VaR and CVaR, McNeil and Frey (2000) fit a GARCH model combined with extreme value theory to daily data. The authors show that their simulation approach is more accurate in forecasting longer-term downside risk compared to the SRTR.[94] One advantage of the EVT approach is that extreme events can be modeled even if such events are not apparent in the estimation window.[95] In contrast, the FHS approach is not able to account for these extreme events, unless such events occurred in the estimation window (Cotter, 2007, Pritsker, 2006). Moreover, Guidolin and Timmermann (2006) show how simulation based approaches can be used in a multivariate setting. Further, the authors show how h-day ahead VaR and CVaR can be calculated by a simulation approach using a regime switching model (Guidolin and Timmermann, 2006, p. 294-295). Finally, the simulation approaches used here or in the literature could also be combined with the EWMA model, which could be interesting for practical implementations, especially when the EWMA model is combined with FHS (Rickenberg, 2020b). However, Wong and So (2003) find that the GARCH(1,1) model

[94] When estimating monthly downside risk based on daily data, two approaches exist for EVT. First, a GARCH based simulation approach similar to the approach presented above (Cotter, 2007, McNeil and Frey, 2000). Second, a scaling approach where one-day ahead estimates are scaled by $h^{1/\alpha}$ (Cotter, 2007).

[95] Rickenberg (2020b) uses risk targeting with daily rebalancing and shows that combining the GARCH(1,1) model with EVT delivers convincing results in a target VaR and target CVaR setting. Nevertheless, the FHS and the skewed t distribution also perform well in this context, especially in crash periods. In particular, all three approaches clearly outperform the Historical Simulation based strategies.

outperforms the EWMA model when the 10-day VaR is forecasted.

Although the simulation based approaches presented above should be superior to the unconditional models or the SRTR, these approaches also have several disadvantages. First, since several thousand paths have to be simulated each month, these approaches are quite time consuming. Second, iterative approaches, although typically more accurate, are also more prone to misspecifications, whereas approaches like the SRTR are quite robust against misspecifications (see Lönnbark (2016), Kole et al. (2017) and references therein). For the simulation approaches it is crucial that the model for the one-step ahead forecast is well specified, since "errors in the specification can be amplified by the forecasting horizon. Small errors in a single-period forecast can build up to a large error in a multi-step forecast" (Kole et al., 2017, p. 650). Hence, if the model for the daily return is badly suited, simulations of the monthly return, and hence estimates for monthly downside risk, will likely be inaccurate too. Due to the high non-normality of momentum returns, the more advanced models suffer under high estimation risk, whereas the simple SRTR should be more robust to errors in the specification.

Following Rickenberg (2020b), the weight of the target VaR and target CVaR strategies can also be approximated by

$$\mathrm{VaR}_{\alpha}^{\mathrm{target}} = \frac{\sigma_{\mathrm{target}} \cdot N_{1-\alpha}}{\sqrt{12}} \quad \text{and} \quad \mathrm{CVaR}_{\alpha}^{\mathrm{target}} = \frac{\sigma_{\mathrm{target}} \varphi(N_{1-\alpha})}{\sqrt{12}\alpha}, \tag{2.4.24}$$

where $N_{1-\alpha}$ and φ denote the $(1-\alpha)$-quantile and the pdf of the standard normal distribution. This result shows that the target VaR and CVaR strategies can also be approximated by a target volatility strategy, where the volatility target is given by Equation (2.4.24). We use these equations to derive our target VaR and target CVaR levels based on our chosen volatility of $\sigma_{\mathrm{target}} = 12\%$. By choosing a significance level of $\alpha = 0.5\%$, we obtain VaR and CVaR targets of $\mathrm{VaR}_{\alpha}^{\mathrm{target}} = 8.92\%$ and $\mathrm{CVaR}_{\alpha}^{\mathrm{target}} = 10.02\%$. Choosing a low α of 0.5% has several advantages. For example, Rickenberg (2020b) shows that low levels of α are beneficial in downside risk targeting strategies. Further, Wong and So (2003) find that incorporating higher moments becomes more important for lower levels of α, since the non-normality of returns is more pronounced as we move further into the tails (see also Bali et al. (2008)). This is confirmed by Ghysels et al. (2016) who find that conditional skewness is hidden in the tails of the distribution. Hence, differences between volatility and downside risk targeting should be more pronounced for small levels of α. Moreover, managing small exceedance probabilities should be more successful in managing extreme losses, i.e. momentum crashes, which is a main objective of this paper.

2.5 Switching between Volatility and Downside Risk Targeting

In Sections 2.3 and 2.4, we have shown how the risk of the momentum portfolio can be dynamically managed by targeting a constant level of volatility or downside risk. We next develop an approach that combines both risk targeting strategies. As mentioned above, the momentum portfolio exhibits a high volatility, which makes this investment strategy unappealing for risk-averse investors, unless the momentum portfolio's risk is targeted at a predefined level. Hence, managing each month's risk is crucial for the momentum strategy. Further, we have argued that a main driver of the superior performance of the risk-managed momentum strategy is the mitigation of momentum crashes, i.e. periods of large negative returns, as shown by Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016). However, the momentum portfolio trends upwards under a high volatility most of the time, whereas these crash periods are typically extreme but only short-lived and very infrequently.[96] Rickenberg (2020b) shows that downside risk targeting, especially CVaR targeting, performs well in turbulent times and is successful in mitigating drawdowns. In contrast, volatility targeting underestimates the risk in crash periods. However, in uptrending markets, downside risk targeting is typically too conservative and is outperformed by volatility targeting. Therefore, solely managing momentum's volatility should be successful in capturing the upside potential of momentum with limited and predefined risk, but should be suboptimal during momentum crashes. In contrast, solely managing momentum's downside risk should be successful in mitigating momentum crashes, but should fail to adequately capture the huge returns of the momentum strategy in calm periods. Hence, since momentum crashes

[96] Jondeau and Rockinger (2003, p. 1701) find that for equity returns "most of the tail-fatness of financial data is generated by large repeatedly occurring events of a given sign." Hence, the huge kurtosis of momentum could mainly be driven by rare but extreme periods of negative returns, such as the losses that occur during momentum crashes.

are extremely rare events, it seems obvious that, over the long-run, the downside risk managed momentum strategy does not outperform the volatility managed strategy. For that reason, in order to enhance the risk-return profile of momentum investing, we develop several strategies that exploit this consideration by switching between both risk targeting strategies based on an estimate of the market regime, where CVaR targeting is only used when a momentum crash is likely.

Switching between different investment strategies has frequently been examined in the academic literature and it has been shown that these switching strategies can increase the performance compared to the individual investment styles. For example, Barberis and Shleifer (2003) and Wang (2005) present strategies that switch between different investment styles based on the past performance of the individual style portfolios. When one style is expected to underperform, measured by the style's past performance, an investor should switch to another style that is expected to outperform.[97] Barroso and Maio (2019, Sec. 6) use a similar strategy based on an investment style's expected future performance, which is estimated using the style's past volatility. This strategy is long the factor, e.g. momentum, if the factor's return is expected to be positive and short else. Hence, this approach aims to invest in the momentum strategy, but switches to a contrarian strategy if a negative momentum return, indicated by a high volatility, is expected.[98] Similarly, Min and Kim (2016) propose to switch from momentum to a contrarian strategy when a bad market state, and hence a high probability of a momentum crash, is expected. Wang and Xu (2015, p. 84) also propose a strategy that switches from momentum

[97] This approach is based on the assumption that the past performance of an investment strategy is an indicator for the (short-term) future performance. Moskowitz et al. (2012) show that buying assets with a positive past performance and selling assets with a negative past performance produces high returns. Hence, an asset's past performance is a good indicator for the asset's short-term future performance as it is also exploited in the trend-following literature (see Sullivan et al. (1999) and Bajgrowicz and Scaillet (2012) among others). Wang (2005, p. 351) state that "style rotation is comparable in spirit to many studies on technical trading rules".

[98] Barroso and Maio (2019) show that there exists a negative relation between momentum's past volatility and future returns. Therefore, next month's return is typically negative when momentum's past volatility is high. Hence, if momentum's volatility significantly increases, the strategy switches to a contrarian strategy, whereas the strategy invests in the momentum portfolio when momentum's volatility is low. Generally, market timing strategies based on volatility signals are frequently applied in practice (Christoffersen and Diebold, 2006). For example, Copeland and Copeland (1999) examine style switching strategies based on signals of volatility. When market volatility increases, their strategy switches from a risky style to a more conservative style, which serves as a hedge for the riskier investment style. Furthermore, see the research of Morningstar Inc. (`https://www.morningstar.com/articles/925094/a-momentum-and-low-volatility-switching-strategy`), where a strategy is proposed that switches between a momentum and a low volatility strategy, based on the market's volatility.

to a contrarian strategy when the market volatility is high and past market return is negative, which the authors call a volatility down market. The authors write: "Given the large negative payoff in volatility down markets it is natural to reverse the momentum-trading in these volatility periods." Similarly, Asness et al. (2013) show that momentum and value investment styles are negatively correlated. In other words, value is a natural hedge for the momentum strategy. The authors show that combining value and momentum with fixed weights dampens momentum crashes. Daniel et al. (2017, Sec. 5.2) extend the approach of Asness et al. (2013) to a strategy that dynamically switches between both investment styles. The momentum portfolio suffers extreme losses – momentum crashes – in turbulent market regimes, whereas the value strategy typically performs well in these times. Hence, when the market is expected to be in a turbulent time, the combined strategy switches from momentum to the value strategy, where Daniel et al. (2017) use a regime switching approach to determine the probability of a crash period. If the regime switching model indicates a high probability of a momentum crash, the dynamic strategy switches to the value style to hedge against a momentum crash.

Besides switching between different investment styles, other switching strategies are also feasible. For example, Chabot et al. (2014, p. 16) propose to switch between the momentum portfolio and the risk-free rate based on an estimate of the momentum crash probability. Moreover, the momentum portfolio could also be hedged by other portfolios, like the market or the size portfolio, based on momentum's beta with these portfolios (Grundy and Martin, 2001, Martens and Van Oord, 2014).[99] Further, Wang et al. (2012) use a regime switching model in order to switch between different risk targets: if a crash regime is expected their strategy switches to a lower risk target, whereas a higher risk target is chosen when a calm regime is expected. Furthermore, Taylor (2014) finds that volatility models perform differently in different market environments, e.g. a model that performs well in a high volatility regime does not necessarily perform well in a low volatility regime. As a consequence, the author proposes to switch between different volatility models to manage the portfolio's risk based on the expected regime of the portfolio. Moreover, different portfolio allocation methods can also be combined.

[99]Hedging the momentum portfolio based on unconditional beta estimates does not work well in practice (Daniel and Moskowitz, 2016). Alternatively, more sophisticated estimation methods of conditional beta could be used (Bali et al., 2017b).

For example, Garlappi et al. (2006), Kan and Zhou (2007), DeMiguel et al. (2009b) and Tu and Zhou (2011) combine different weighting schemes, like the equally weighted, minimum-variance and mean-variance portfolio. Hence, instead of using the same weighting scheme over time, an investor could also dynamically switch between different weighting schemes. The rational behind this approach is that some weighting schemes may be better in crash periods, whereas other weighting schemes outperform in calm periods. In total, several studies have shown that switching between different styles, risk targets, forecasting models or weighting schemes can significantly improve the performance of a portfolio. Inspired by these studies, we develop a strategy that dynamically switches between volatility and CVaR targeting, where CVaR targeting is only used when a momentum crash becomes likely. More formally, following Tu and Zhou (2011), the month t weight of the switching strategy is given by

$$w_t^{switch} = \delta_t \cdot w_t^{CVaR} + (1 - \delta_t) \cdot w_t^{vol}, \tag{2.5.1}$$

where $\delta_t \in [0, 1]$, w_t^{CVaR} is the month t weight of the CVaR targeting strategy and w_t^{vol} is the month t weight of the volatility targeting strategy.[100]

Although this study does not consider transaction costs, we expect that the weighting scheme in Equation (2.5.1) should be appealing, even after realistic transaction costs. By using a convex combination of the weights of the volatility and CVaR targeting strategies, and since the weights of the volatility and CVaR targeting strategies should be quite similar, the switching strategy should exhibit similar transaction costs as the volatility managed momentum strategy. For example, the correlations between the weights of the switching strategies and the weights of the corresponding target volatility strategy are always higher than 0.95. For some strategies, the correlation is even higher than 0.99. Barroso and Santa-Clara (2015, p. 112) find that "the turnover of the risk-managed strategy is very close to the turnover of the raw momentum strategy, so the transaction costs of both strategies are very similar." Due to the high correlation of the weights of the switching strategies and volatility targeting strategies, this result should

[100] We only switch between the volatility and the CVaR targeting approaches, since Rickenberg (2020b) shows that within the three risk targeting strategies, volatility targeting works best in calm periods and CVaR targeting works best in crash periods. Other possibilities would be to switch between volatility and VaR targeting or VaR and CVaR targeting.

also hold for the switching strategies, especially for the strategy that switches to the unconditional CVaR strategies. Hence, the transaction costs of the non-managed, volatility managed and switching strategies should be quite similar. Hong et al. (2000) and Jegadeesh and Titman (2001) find that the non-managed, and thus also the risk-managed momentum strategies, are profitable even after transaction costs were considered, whereas Lesmond et al. (2004) find that the non-managed momentum strategy is not profitable after transaction costs. However, since the risk-managed momentum strategies, especially the switching strategies, produce significantly higher returns than the non-managed strategy, transaction costs are less of a concern for the risk-managed strategies (Barroso and Santa-Clara, 2015, p. 112).

To implement the strategy given in Equation (2.5.1), the weights δ_t have to be determined, where δ_t should be high when a momentum crash in month t is likely and low when there are no signs of a momentum crash. Several approaches to determine δ_t are possible. One possibility to indicate if a momentum crash in month t is likely or not is to use the past return, volatility or market beta of the momentum portfolio.[101] Christoffersen and Diebold (2006) find that conditional returns are not forecastable, whereas signs of returns and volatility are forecastable (see also Colacito and Engle (2010, p. 13)). Therefore, market timing strategies based on volatility changes or the past performance are frequently used by practitioners (Copeland and Copeland, 1999). This approach is particularly appealing for the momentum portfolio, since Barroso and Maio (2019) show that there exists a negative relation between momentum's risk, measured as realized volatility, and future momentum returns, whereas there exists a positive risk-return relation for most other factor portfolios. Hence, δ_t should increase if momentum's volatility increases, since a momentum crash becomes more likely for a higher volatility. Similarly, Barroso and Santa-Clara (2015) show that momentum's volatility is more successful in predicting momentum crashes than other momentum crash indicators based on the past market return and market volatility, as it is examined in Daniel and Moskowitz (2016). For that reason, as first possibility, we switch to the CVaR targeting strategy if momentum's volatility is (expected) to

[101] As mentioned above, the momentum portfolio's beta is highly time-varying and momentum crashes typically occur when the momentum portfolio has a negative beta. Hence, a bear market of the momentum portfolio could be defined as a month when the momentum portfolio has a negative beta. However, Barroso and Santa-Clara (2015) show that momentum's beta captures only a small part of the strategy's risk and that volatility is a superior approach to measure momentum's riskiness.

be high. In contrast, we use volatility targeting when momentum's volatility is expected to be low.

Several possibilities to define high and low volatility regimes are feasible. We define high and low volatility regimes with respect to the chosen volatility target: if expected volatility $\hat{\sigma}_t$ is higher (lower) than a chosen threshold, given as a function of the volatility target, month t is indicated as a high (low) volatility regime. This approach has the advantage that the riskiness of the switching strategy can be chosen by the investor. A risk-averse investor typically prefers CVaR managing over volatility managing as shown by Rickenberg (2020b). This investor also chooses a lower risk target, whereas a risk-seeking investor chooses a high risk target. Hence, by defining turbulent and calm periods with respect to the volatility target, the portfolio of a risk-seeking investor is mainly volatility managed, whereas the portfolio of a risk-averse investor is mainly CVaR managed.[102] Since we chose a quite low volatility target, we define δ_t as

$$\delta_t = \begin{cases} 1, & \text{if } \hat{\sigma}_t > 1.5\sigma_{\text{target}} \\ 0, & \text{if } \hat{\sigma}_t \leqslant 1.5\sigma_{\text{target}}, \end{cases} \tag{2.5.2}$$

where $\hat{\sigma}_t$ is a forecast of month t's volatility.[103]

Another possibility to define a crash regime, i.e. $\delta_t = 1$, would be to use momentum's past performance as done by the time series momentum strategy of Moskowitz et al. (2012) or other technical trading rules (Bajgrowicz and Scaillet, 2012, Sullivan et al., 1999). For example, if the past twelve months' performance of the momentum portfolio is negative, δ_t could be set to one and zero else.[104] This approach would be similar to the style switching strategies of Barberis and Shleifer (2003) and Wang (2005). The time series momentum strategy is related to the (cross-sectional) momentum strategy, but is a different phenomenon as shown by Moskowitz et al. (2012). Both approaches are based on the assumption that assets move in trends. Nevertheless, momentum uses the relative performances of several assets, whereas the time series momentum strategy relies on the past performance of a single asset or portfolio.[105] Both strategies, time

[102]As an alternative, low and high risk periods could also be defined as the relation between short-term and log-term volatility (Copeland and Copeland, 1999, Rickenberg, 2020c). A high risk regime could be defined when short-term volatility is higher than long-term volatility.

[103]We also used σ_{target} instead of $1.5\sigma_{\text{target}}$. However, both definitions deliver similar results.

[104]Moskowitz et al. (2012) and Goyal and Jegadeesh (2017, Table 1) find good results of the time series momentum strategy with a 12 months ranking and one month holding period.

[105]Both strategies use a different threshold to determine buy and sell signals. The (cross-sectional) momentum

series and cross-sectional momentum, are extensively compared by Moskowitz et al. (2012, Sec. 5), Kim et al. (2016) and Goyal and Jegadeesh (2017). Due to the different nature of time series and cross-sectional momentum, both approaches can easily be combined, where the cross-sectional momentum approach can be used to determine the portfolio composition and the time series momentum approach, applied to the cross-sectional momentum portfolio, can be used for managing the momentum portfolio's risk. Moskowitz et al. (2012) show that time series momentum is successful in determining downtrending and uptrending periods, where especially extreme events are well identified. However, this result that holds for equities, commodities and most style portfolios does not hold for the momentum portfolio. Chabot et al. (2014) argue that momentum crashes typically occur after periods of a strong momentum performance and that the probability of a momentum crash increases with past performance. Hence, time series momentum applied to the momentum portfolio will likely give false signals of momentum crashes. For that reason, we do not examine this approach here. As an alternative, following Chabot et al. (2014), a bear market could be defined if the momentum portfolio's drawdown exceeds a certain threshold.

Additionally to the past return or volatility of the momentum portfolio, the past return and past volatility of the market can also be used to identify momentum crashes.[106] Daniel and Moskowitz (2016) use the past market return and past market volatility as signal for a momentum crash. Daniel and Moskowitz (2016) and Cooper et al. (2004) show that momentum crashes typically occur when the past market return is negative and/or past market volatility is high (see also Daniel et al. (2017)).[107] Similarly, Wang and Xu (2015) show that past market volatility

strategy uses the cross-sectional mean return or a certain quantile of all past returns of all assets, whereas the time series momentum strategy uses a return of zero (Goyal and Jegadeesh, 2017).

[106] Instead of the past market volatility, the market's past return dispersion, measured as the cross-sectional volatility of individual stock returns could also be used. Stivers and Sun (2010) show that a high return dispersion of the market indicates future negative momentum returns. In particular, Wang and Xu (2015) show that the return dispersion of Stivers and Sun (2010) is highly correlated to the market volatility, although both methods are based on a different calculation methods, i.e. cross-sectional and time series returns, respectively. Further, Wang and Xu (2015) find that market volatility is a better predictor of future momentum returns than return dispersion. Du Plessis and Hallerbach (2017) also find that the volatility is superior to the return dispersion for the industry momentum strategy. Hence, a return dispersion based approach is not examined here.

[107] Grundy and Martin (2001), Martens and Van Oord (2014) and Daniel and Moskowitz (2016) show that the momentum strategy has a negative beta when the previous market return is negative. Momentum crashes then occur when the market rises after a long period of negative returns. Cooper et al. (2004) use the previous one-year, two-year and three-year return as measure for the previous market return. The authors show that the previous market return is a better predictor of future momentum returns than several frequently used macroeconomic variables.

can predict future returns of the momentum portfolio and the authors find a negative relation between market volatility and momentum returns.[108] Stivers and Sun (2010, Table 4) also show that momentum returns are positively related to the previous market return and negatively to market uncertainty. Daniel and Moskowitz (2016, Table 2) show that 14 out of the 15 worst months of the momentum portfolio occurred when the previous (two-year) market return was negative. Further, Daniel and Moskowitz (2016, Table 5) show that momentum returns decrease if market volatility increases. Similarly, Daniel et al. (2017, Table 2) show that 12 out of the 13 worst monthly momentum returns occurred when the market's past twelve months' return was negative and past market volatility was high. Further, Daniel et al. (2017, Table 3 and 14) find that momentum crashes occur and momentum returns are negatively skewed when the past market return is negative or market volatility is high. In particular, momentum returns are procyclical (see Stivers and Sun (2010) and references therein), i.e. if the market return is expected to be negative – which can be forecasted by the time series momentum strategy of Moskowitz et al. (2012) – the momentum return is also expected to be negative. Chabot et al. (2014) also find that momentum returns are cyclical and momentum crashes are partly forecastable. Similarly, Chordia and Shivakumar (2002) find that momentum returns are positive during economic expansions, times that typically coincide with periods of positive market returns and a low market volatility, and negative during recessions, times that typically coincide with negative market returns and a high market volatility (see also Wang and Xu (2015) and Martens and Van Oord (2014)). This result is confirmed and extended by Min and Kim (2016) who show that momentum yields negative returns in bad markets in which investors require high risk premiums and have a high marginal value of wealth. Summarized, large losses of the momentum return typically cluster in subsequent months and occur when markets rebound after long crash periods, which can be identified by a negative past market return or high market volatility.[109]

See also Griffin et al. (2003) who find that macroeconomic variables can hardly explain international momentum returns.

[108] A similar result has also been shown for market returns, i.e. a high market volatility predicts a low or even negative future market return (see Campbell and Hentschel (1992), Glosten et al. (1993) among others). This result does also hold in a cross-sectional setting, i.e. stocks with higher volatility underperform stocks with lower volatility (Ang et al., 2006b). In the literature, the relation between market volatility and momentum returns is hardly examined, whereas the relation between market volatility and market returns is widely examined (see the discussion in Rickenberg (2020b)).

[109] This observation was initially shown by Jegadeesh and Titman (1993), who find that the momentum "strategy

Based on the above presented relation between past market return or past market volatility and future momentum returns, we next develop two additional strategies that switch between volatility and CVaR targeting. As second crash indicator, we use the past 36 months' market return as indicator for a momentum crash.[110] If this return is negative, and hence a momentum crash becomes more likely, we manage the portfolio by targeting a constant level of CVaR. If the past market return is non-negative, we manage the momentum portfolio's volatility to better capture the upside potential of the momentum portfolio. More formally, the weight δ_t of Equation (2.5.1) is given by

$$\delta_t = \begin{cases} 1, & \text{if } R^{market}_{t-1:t-36} < 0 \\ 0, & \text{if } R^{market}_{t-1:t-36} \geqslant 0, \end{cases} \tag{2.5.3}$$

where $R^{market}_{t-1:t-36}$ is the market return in the months $t-36$ to $t-1$.

As third indicator to switch between volatility and CVaR targeting, we use the past market volatility. This approach is similar to Copeland and Copeland (1999) who also use market volatility to switch between different investment styles. Wang and Xu (2015) also suggest to switch away from the momentum strategy when market volatility is high. As is Daniel and Moskowitz (2016), we use the past six months' Realized Volatility RV^{market}_{t-1} as measure for the past market volatility. RV^{market}_{t-1} is defined as in Equation (2.3.5), where the momentum returns are replaced by market returns. Since a high past market volatility indicates that a momentum crash is more likely, we switch to CVaR targeting if RV^{market}_{t-1} is high, whereas we use volatility targeting if RV^{market}_{t-1} is low. As before, we define a high volatility regime when the market volatility is higher than $1.5\sigma_{\text{target}}$. The weight δ_t of Equation (2.5.1) is then given by

$$\delta_t = \begin{cases} 1, & \text{if } RV^{market}_{t-1} > 1.5\sigma_{\text{target}} \\ 0, & \text{if } RV^{market}_{t-1} \leqslant 1.5\sigma_{\text{target}}. \end{cases} \tag{2.5.4}$$

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tends to select high- (low-) beta stocks following a market increase (decrease) and hence tends to perform poorly during market reversals" and that "[i]n the 1930s there were four other months in which the [momentum] strategy lost over 40%. Each occurred when the market increased substantially."

[110] Cooper et al. (2004) use the past one-year, two-year and three-year market return to determine up and down markets. In contrast, Wang and Xu (2015) use the past six months' performance to indicate up and down periods. We also used other lengths but found best results for the three-year indicator. However, all lengths are successful in predicting momentum crashes. Another alternative to define the crash indicator δ_t would be to use information on other factors' recent performance or volatility, such as the size or value factors. Hedging the momentum portfolio based on the Fama-French factors typically produces more stable returns (Grundy and Martin, 2001, Martens and Van Oord, 2014).

Other possibilities to define high volatility regimes based on $\hat{\sigma}_t$ or RV_{t-1}^{market} as in Equation (2.5.2) or (2.5.4) would be to define δ_t in relation to volatility changes as in Copeland and Copeland (1999) and Ang et al. (2006b), i.e. instead of using the volatility target as reference point, the (long-term) average volatility of the past months could be used. For example, Copeland and Copeland (1999) define a high volatility regime if current volatility is x percent higher than the past three months' average volatility, where they find good results of choosing x by 20% or 30%. Similarly, Wang and Xu (2015, p. 84) define a high volatility regime if the volatility measured over the last 12 months is higher than the volatility measured over the last 36 months. This would be similar to Dreyer and Hubrich (2019) who examine volatility targeting when the volatility target equals the long-term volatility. Rickenberg (2020c) uses a similar approach for the industry momentum strategy. However, as stated above, defining the crash indicator in relation to the volatility target is appealing, since this approach fits well to the risk aversion of investors.

Wang and Xu (2015) find that momentum returns are most negative when the past market volatility is high *and* the past market return is negative (see also Daniel and Moskowitz (2016)). For that reason, we additionally use several crash indicators that combine the crash indicators above. Hence, a crash regime can also be defined when both crash indicators in Equations (2.5.3) and (2.5.4) are equal to one. Other possibilities of combining the three crash indicators are also feasible and will be defined and used in the empirical part.

Several extensions of our approach of switching between volatility and CVaR targeting are also possible. For example, instead of switching between two different risk targeting strategies, an investor could also use the non-managed momentum portfolio, where the investor switches to the downside risk managed strategy when a momentum crash becomes likely. However, Barroso and Santa-Clara (2015, Sec. 7) show that volatility targeting improves the risk-adjusted performance of the momentum portfolio even in periods when there is no momentum crash. Further, as argued above, the high and time-varying volatility of the momentum portfolio makes the non-managed momentum strategy unappealing for risk-averse investors, even in non-crash periods. Hence, for risk-averse investors, it is advantageous to switch from the downside risk managed strategy to the volatility managed strategy when there are no indications of a momentum crash. In this case, momentum's risk is managed in every month. Moreover, instead of choosing $\delta_t \in \{0, 1\}$, the weight δ_t could also be defined to vary between zero and one as in Tu and Zhou (2011). For example, similar to Daniel et al. (2017) and Wang et al. (2012), a regime-switching process could be used. The weight δ_t could then be chosen as the probability that there is a crash regime in the next month. Moreover, δ_t could be chosen by $\delta_t = \frac{\hat{\sigma}_t}{\sigma_{\text{target}}}$ or $\delta_t = \frac{RV_{t-1}^{market}}{\sigma_{\text{target}}}$.

Our approach of switching between volatility and CVaR targeting is different to the approaches of Barroso and Santa-Clara (2015) and Moreira and Muir (2017), who only use information on momentum's volatility to manage the momentum portfolio's risk. Daniel and Moskowitz (2016) show that the momentum portfolio's return and volatility are both highly forecastable, which is used by their dynamic strategy. Our approach also exploits the return forecastability of the momentum portfolio – but in a different way as in Daniel and Moskowitz (2016) – by switching to a more conservative risk targeting strategy when a negative momentum return is expected. Thus, we incorporate information on momentum's expected mean return, without relying on a point forecast of the mean. Portfolio allocation methods that incorporate a forecast of the mean return typically suffer under high estimation risk and perform bad in practice (Garlappi et al., 2006, Moreira and Muir, 2019, Tu and Zhou, 2011).

To assess if the outperformance of a strategy that switches to CVaR targeting is only influenced by switching away from the volatility managed momentum strategy during a momentum crash, we additionally define a strategy that switches between the (volatility managed) momentum and contrarian strategies. The month t weight of this strategy is given by

$$w_t^{switch} = \delta_t \cdot (-w_t^{vol}) + (1 - \delta_t) \cdot w_t^{vol}. \tag{2.5.5}$$

Hence, if the crash indicators δ_t accurately forecast momentum crashes, this strategy should outperform the strategies that switch between volatility and CVaR targeting. However, the strategy given in Equation (2.5.5) suffers extremely if δ_t gives false signals of a momentum crash. Thus, our CVaR switching approach is more robust against estimation risk.

2.6 Empirical Results

In this section, we examine the distributional properties of daily and monthly momentum returns. In particular, we examine the time-variation of conditional skewness and kurtosis, which is a main driver of momentum crashes. Further, we test for the random walk hypothesis, which underlies the SRTR and the RV model used by Barroso and Santa-Clara (2015) and Moreira and Muir (2017). We then test the accuracy of the different risk targeting strategies and answer the questions if more advanced models are more successful in targeting a constant level of volatility than the simple RV model and if incorporating higher moments is beneficial when a constant level of risk is targeted. Additionally, we assess the accuracy of the strategies that switch between volatility and CVaR targeting. We then assess the risk-adjusted performance of the different risk targeting strategies. In particular, we examine which strategy is best in mitigating drawdowns and if strategies that switch between volatility and CVaR targeting perform better than the RV managed strategy of Barroso and Santa-Clara (2015). Furthermore, we assess if a higher forecasting accuracy translates into a higher risk-adjusted performance as suggested by Bollerslev et al. (2018). Moreover, we calculate the economic value of risk targeting, i.e. the annual fee an investor is willing to pay to switch from the RV managed momentum strategy to a strategy that switches between volatility and CVaR targeting. We calculate the economic value for mean-variance investors, CRRA investors who dislike negative skewness and high kurtosis as well as loss-averse investors. Finally, we conduct spanning tests in the manner of Daniel and Moskowitz (2016) and Moreira and Muir (2017) to assess if the non-managed or risk-managed strategies can be explained by the returns of the Fama-French three factor model and the other strategies.

We obtain daily and monthly data for the market and the momentum portfolio from the website of Kenneth French.[111] Our data range from January 1927 to December 2018. Hence, compared to Barroso and Santa-Clara (2015), we have seven additional years of data to assess the value of risk targeting applied to the momentum strategy. We use the momentum portfolio where stocks in the winners and losers portfolios are equally weighted. Compared to

[111] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

the value-weighted momentum portfolio used by Barroso and Santa-Clara (2015), the equally weighted strategy weights smaller stocks relatively higher. This translates into a momentum strategy where momentum crashes are more pronounced compared to the value-weighted strategy. Hence, this weighting scheme better highlights which risk targeting strategy is best in mitigating momentum crashes. The equally weighted momentum portfolio is also frequently examined in the literature (Chordia and Shivakumar, 2002, Grundy and Martin, 2001, Hong et al., 2000, Jegadeesh and Titman, 1993, 2001, Lesmond et al., 2004). The appendix shows additional results for the momentum factor portfolio of Fama and French (2012), which is also used by Ruenzi and Weigert (2018). For this portfolio, the winners and losers portfolios are constructed using size-return double-sorted portfolios, and thus using stocks with small and big market capitalizations. Further, we also examine the momentum strategy for German stocks in the period January 1994 to April 2016. The equally weighted winners and losers portfolios for Germany are obtained from the Humboldt University.[112] Additionally, we show further results of risk targeting applied to other long-short strategies. For example, Barroso and Maio (2018), Cederburg et al. (2020) and Moreira and Muir (2017) examine risk targeting for several portfolio strategies and find that this approach works best for the momentum strategy and the Betting against Beta (BAB) strategy of Frazzini and Pedersen (2014). For that reason, we also apply our approach to the BAB portfolio in the US and Germany. Data for the BAB portfolios are obtained form the website of AQR.[113] Furthermore, and particularly interesting for practical implementations, we apply our risk targeting strategies to the industry momentum strategy (Grundy and Martin, 2001, Lewellen, 2002, Moskowitz and Grinblatt, 1999), as it has been done by Du Plessis and Hallerbach (2017), Grobys et al. (2018) and Rickenberg (2020c). Industry data are again obtained from the website of Kenneth French. Finally, since Barroso and Maio (2018), Cederburg et al. (2020) and Moreira and Muir (2017) find that the RV approach does not work well for size portfolios, we apply our switching approach to the small minus big (SMB) factor that buys small stocks and sells big stocks (Fama and French, 1993, 2012). Data for this strategy are again obtained from the website of Kenneth French.

[112] https://www.wiwi.hu-berlin.de/de/professuren/bwl/bb/daten/fama-french-factors-germany.

[113] https://www.aqr.com/Insights/Datasets.

2.6.1 Distributional Properties of Momentum Returns

This section starts the empirical part by examining the distributional properties of daily and monthly momentum returns. Similar to Barroso and Santa-Clara (2015, Table 1), we start by comparing the descriptive statistics of the returns of the market, winners, losers and momentum portfolios. Since the momentum strategy is a zero-investment strategy, which does not hold for the market, winners and losers portfolios, we subtract the risk-free rate from the winners, losers and market portfolios.[114] Table I shows the first four (standardized) moments, the minimum and maximum monthly return as well as the Jarque-Bera (JB) test that tests if the return series follow a normal distribution.[115] As in Barroso and Santa-Clara (2015), we find that the momentum strategy produces a higher return than the market portfolio which is accompanied with significantly higher risk, especially in the left tail. That is, compared to the market, the momentum portfolio has a significantly higher risk of large negative returns, indicated by the high negative skewness and high kurtosis. Compared to the value-weighted momentum strategy used by Barroso and Santa-Clara (2015, Table 1), the equally weighted strategy exhibits significantly higher left tail risk and a lower return. As a consequence, managing portfolio risk becomes more important for the equally weighted momentum strategy than for the value-weighted momentum strategy. Further, as expected, we find a lower mean return of the losers portfolio compared to the winners portfolio, which is accompanied by a significantly lower skewness of the winners compared to the skewness of the losers. Hence, buying winners and selling losers produces a portfolio that is extremely negatively skewed with a skewness of -4.26. Combined with the high kurtosis of 42.20 this indicates a high crash risk, which can be seen by the minimum monthly return of -89.70%. This minimum monthly return is significantly lower than the market's minimum monthly return of -29.13%. As expected, the JB-test strongly rejects the normality assumption for all return series, indicating that volatility managing is not sufficient for all four portfolios, especially for the momentum portfolio.

[114] Alternatively, following Daniel and Moskowitz (2016, Appendix A.1), the momentum portfolio could also be defined as the portfolio that buys the winners, sells the losers and is invested in the risk-free rate.

[115] For a better comparison with the results of Barroso and Santa-Clara (2015, Table 1), we also calculate the mean return as arithmetic mean. In the following tables, we calculate the mean return as geometric mean, which is more realistic.

Estimating moments higher than the second based on realized estimates is typically very sensitive to outliers (Ghysels et al., 2016, Kim and White, 2004, Neuberger, 2012). For that reason, we additionally use other measures of a distribution's asymmetry and fat-tailness that are based on quantiles of the distribution.[116] These measures are more robust to outliers than estimates based on realized moments. This approach of estimating higher moments based on quantiles is similar to the aforementioned method of Taylor (2005) who estimates volatility based on quantiles. Following Kim and White (2004) and Ghysels et al. (2016), we estimate the asymmetry of a distribution by

$$Sk_{\alpha}^{qu} = \frac{(q_{\alpha}(R_t) - q_{0.5}(R_t)) - (q_{0.5}(R_t) - q_{1-\alpha}(R_t))}{q_{\alpha}(R_t) - q_{1-\alpha}(R_t)}, \tag{2.6.1}$$

where $q_{\alpha}(R_t)$ denotes the α-quantile of R_t. The measure Sk_{α}^{qu} is normalized to values between -1 and 1, where a negative value indicates a left skewed distribution (Ghysels et al., 2016, p. 2151).[117] Following Kim and White (2004) and Ghysels et al. (2016), we choose $\alpha = 0.75$. Further, in order to emphasize extreme realizations, we follow Taylor (2005) and also choose $\alpha = 0.99$.[118] To quantify a distribution's fat-tailness, we also use a quantile based risk measure, which is defined by (Kim and White, 2004)

$$Ku^{qu} = \frac{(q_{7/8}(R_t) - q_{5/8}(R_t)) + (q_{3/8}(R_t) - q_{1/8}(R_t))}{q_{6/8}(R_t) - q_{2/8}(R_t)}. \tag{2.6.2}$$

Kim and White (2004, p. 60) show that the robust kurtosis equals 1.23 for the normal distribution. Hence, when returns are normally distributed, we expect $Sk_{\alpha}^{qu} = 0$ and $Ku^{qu} = 1.23$.

Results of the robust estimates of skewness and kurtosis in Table I show that the ranking of the portfolios can be quite different when different estimators are used. Interestingly, when the skewness estimator with $\alpha = 0.75$ is used, the momentum portfolio is the least negatively skewed portfolio. This shows that the high negative skewness of the momentum portfolio is mainly influenced by extreme return realizations in the left tail, i.e. the momentum crashes. Hence, the momentum strategy typically produces moderate returns, which are accompanied by

[116] I thank Peter Albrecht for this suggestion.

[117] Based on a Cornish-Fisher approximation, Ghysels et al. (2016, Sec. I.B) show how this normalized skewness measure can be transformed to a measure of the third moment.

[118] In order to emphasize skewness in the tails, Ghysels et al. (2016) also use $\alpha = 0.95$ and a weighted measure using quantiles for $\alpha \in [0.5, 1]$.

Table I. Descriptive Statistics
This table reports the descriptive statistics of the monthly returns of the market, winners, losers and momentum portfolios over the full sample. For a better comparison, we subtract the risk-free rate from the market, winners and losers portfolios. Mean and Vola denote the annualized (arithmetic) mean return and volatility, respectively. Skew and Kurt denote the realized skewness and (excess) kurtosis. Min and Max denote the minimum and maximum monthly return. JB denotes the value of the test statistic of the Jarque-Bera test. $\text{Sk}^{qu}_{0.75}$, $\text{Sk}^{qu}_{0.99}$ and Ku^{qu} denote quantile-based measures of skewness and kurtosis. The robust kurtosis of the normal distribution is given by $\text{Ku}^{qu} = 1.23$ (Kim and White, 2004, p. 60). Mean, Vola, Min and Max are given in percent.

Portfolio	Mean	Vola	Skew	Kurt	Min	Max	JB	$\text{Sk}^{qu}_{0.75}$	$\text{Sk}^{qu}_{0.99}$	Ku^{qu}
Market	7.77	18.52	0.19	10.81	-29.13	38.85	2814.99	-0.063	-0.141	1.290
Winners	17.90	25.35	0.12	8.84	-32.92	56.56	1569.47	-0.073	-0.152	1.337
Losers	7.93	38.28	2.85	25.56	-38.91	113.95	24894.65	-0.045	0.215	1.459
Momentum	9.97	26.32	-4.26	42.20	-89.70	22.24	74015.40	-0.012	-0.312	1.461

occasionally extremely negative returns that make this strategy unappealing for investors. As expected, the skewness estimate using $\alpha = 0.99$, and thus emphasizing extreme realizations, indicates that the momentum portfolio is the most negatively skewed portfolio. This estimator is more in line with the realized skewness estimator. Thus, the high negative skewness of -4.26 is mainly driven by extreme realizations. We also calculated the realized skewness by first eliminating extreme returns in the lower and upper $1\%, 2.5\%, 5\%$ and 10% quantile. In these cases, the realized skewness increases to $-1.197, -0.763, -0.334$ and -0.115, respectively. This again shows that momentum's left tail risk is mainly given by few extreme realizations. The robust kurtosis estimator is in line with the realized kurtosis estimator, but indicates a higher kurtosis of the winners compared to the market. Further, the difference between losers and the momentum portfolio is very small, whereas the realized estimator indicates a high difference. As expected, all estimates of the fat-tailness are higher than the 1.23 of the normal distribution and the market's kurtosis is the closest to this value. Furthermore, the robust kurtosis of the momentum portfolio is less extreme than the realized kurtosis. However, by eliminating the highest and lowest $1\%, 2.5\%, 5\%$ and 10% returns, the realized (excess) kurtosis of the momentum portfolio reduces to $6.823, 4.81, 3.151$ and 2.346, respectively. Thus, similar to the skewness, the high kurtosis of the momentum portfolio is mainly influenced by few extreme return realizations.[119] Hence, if these few extreme (negative) realizations could be mitigated, the

[119]This observation is also in line with Kim and White (2004, Table 3) who find that realized (negative) skewness and kurtosis decrease dramatically if only a few extreme observations are removed. For example, the authors find that negative skewness and excess kurtosis of the S&P 500 decline from -2.39 and 53.62 to only -0.04 and 3.44

momentum portfolio would be an appealing investment strategy for most investors with almost normal returns.

The high non-normality of daily and monthly momentum returns can also be seen in Figure I, which shows the empirical density of the momentum portfolio. Further, similar to Theodossiou (1998) and Daniel et al. (2017, Fig. 1), we also show the density of the normal and skewed t distribution fitted to the empirical data. When fitting the skewed t distribution to the momentum returns as shown in Equation (2.4.8), the parameter λ is negative for both the daily ($\lambda = -0.0924$) and monthly ($\lambda = -0.1108$) returns indicating a negatively skewed distribution. Further, the degree of freedom parameter η is very low for daily ($\eta = 2.3631$) and monthly ($\eta = 2.1753$) returns, which coincides with fatter tails of the return distribution (Jondeau and Rockinger, 2003, Theodossiou, 1998). In particular, the parameters λ and η are statistically significant for daily and monthly returns. The low values of $\eta < 3$ indicate that higher moments may not exist for the skewed t distribution. This is in line with Eling (2014) who also finds low values for the degree of freedom parameter η of a skewed t distribution, when fitting the distribution to stock returns.[120] Nevertheless, the non-existence of higher moments is mainly influenced by few outliers.[121] Obviously, the skewed t distribution gives a very good fit of the realized return observations, whereas the normal distribution clearly fails to provide an accurate fit of the observed data. Furthermore, the characteristics of daily data are much better captured than the characteristics of monthly data. This indicates that directly fitting the skewed t distribution to monthly returns is suboptimal. In contrast, the skewed t distribution should be fitted to daily data, which can then be used to measure monthly risk based on the methods presented before. Moreover, the monthly return distribution is more negatively skewed than the

if only six returns were removed. Jondeau and Rockinger (2003) also find that the kurtosis is mainly influenced by a few extreme realizations. In contrast, the quantile based measures of skewness and kurtosis are more robust to the exclusion of outliers.

[120] Eling (2014) presents several approaches to solve the problem of non-existing higher moments. As mentioned above, Eling (2014) uses a different version of the skewed t distribution. However, the approaches presented by Eling (2014) can directly be applied to our distribution. Since we are mainly interested in assessing the goodness-of-fit in this section, we do not apply these approaches.

[121] We also fitted the skewed t distribution to monthly returns when the highest and lowest $1\%, 2.5\%, 5\%$ and 10% returns were eliminated. In these cases, the estimates of η are given by $2.998, 4.865, 340.792$ and 341.892. Thus, the distribution becomes less fat-tailed with an existing skewness and kurtosis for the last three cases. The parameter λ remains negative in all cases, but becomes smaller in magnitude, indicating a less negatively skewed distribution when the most extreme realizations are eliminated.

daily data, demonstrating that daily and monthly returns have quite different properties (Harvey and Siddique, 1999). That is, simply scaling tail risk measures estimated with daily data by a constant to obtain estimates of monthly tail risk should fail to capture the differences in both return distributions. This is in line with Neuberger (2012) and Ghysels et al. (2016) who suggest that skewness cannot simply be scaled with a constant like volatility. Figure I also shows that the height of the empirical distribution is well fitted by the skewed t distribution, indicating that an additional parameter to model the height, as in the skewed generalized t distribution of Theodossiou (1998), is not needed.

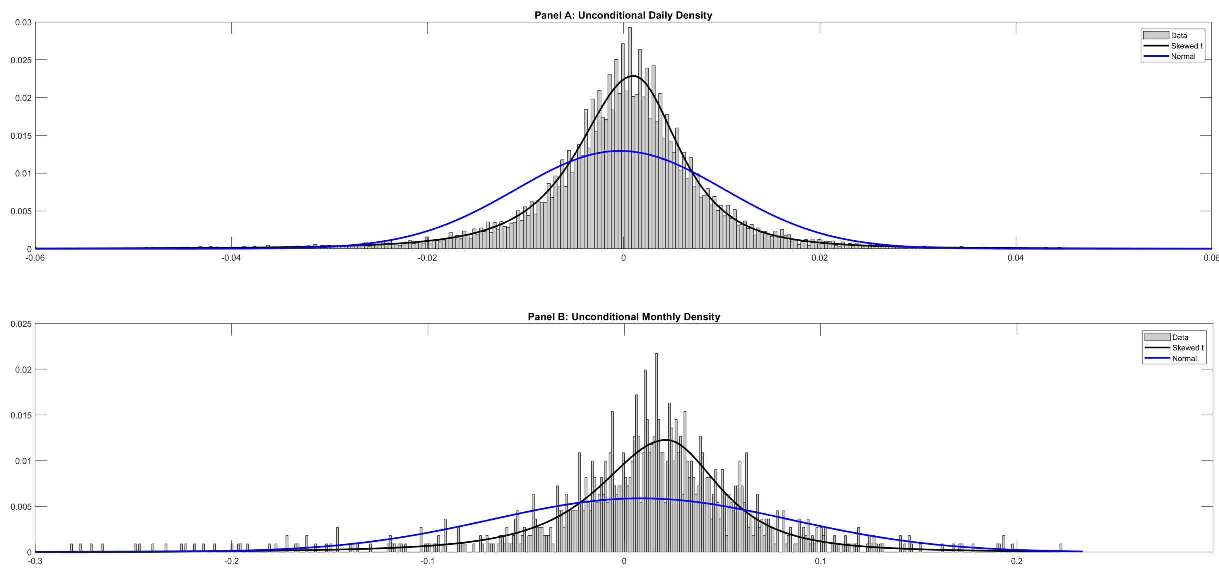

Figure I. Unconditional Return Distributions of Daily and Monthly Momentum Returns. This figure shows the empirical density of momentum returns together with the density of the normal and skewed t distribution. The density of the normal and skewed t distribution are plotted using maximum likelihood estimates. Panel A shows results for daily data, whereas Panel B shows results for monthly data.

Results so far indicate that incorporating skewness and kurtosis gives a significantly better fit of the observed momentum returns compared to the normal distribution. We also statistically tested this statement by using the Likelihood Ratio (LR) test, the Akaike information criterion (AIC), the Bayesian information criterion (BIC) and the Kolmogorov Smirnov (KS) test, which were also used by Theodossiou (1998), Bali et al. (2008) and Eling (2014). All tests clearly reject the normality of the momentum returns and demonstrate the importance of incorporating skewness and kurtosis when describing momentum returns. However, we are not mainly interested in reproducing the *whole* return distribution, but in capturing characteristics of extremely negative returns. In other words, when managing momentum crashes, we are mainly interested in adequately fitting the left tail of the return distribution. A simple tool to visualize the ability of a distribution to capture the characteristics of the tails of an observed sample of returns is the Quantile-Quantile-plot (QQ-plot), which is nicely explained in Christoffersen (2012, Sec. 6.3). The QQ-plot plots the quantiles of the realized daily and monthly returns against the quantiles of the normal or skewed t distribution. This tool is also used by Eling (2014) and Cotter (2007). The QQ-plots for the unconditional daily and monthly returns are given in Figure II.

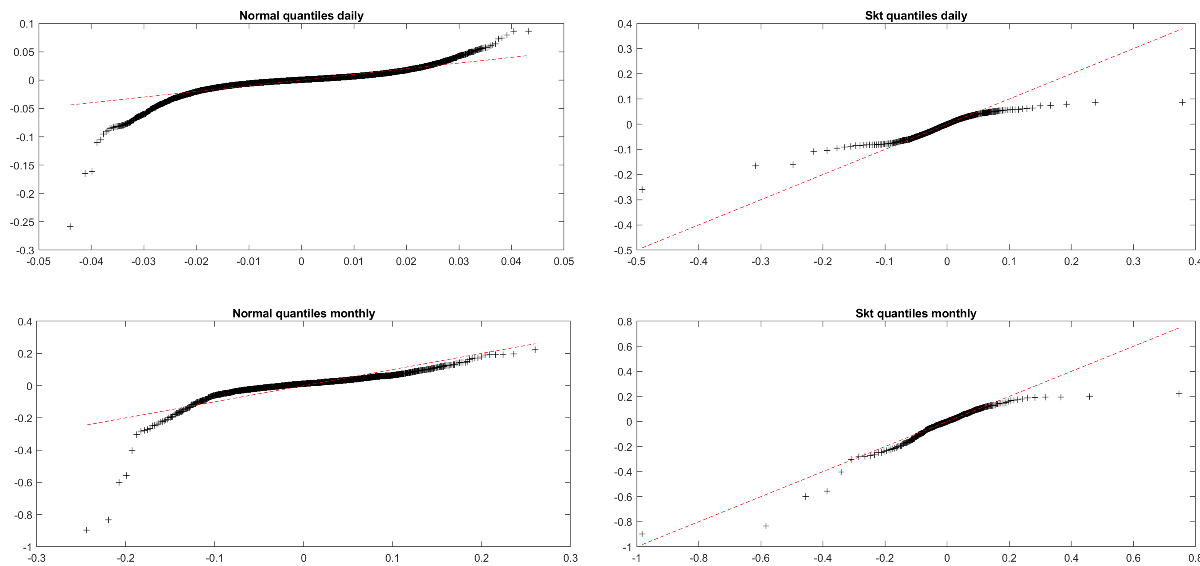

Figure II. Quantile-Quantile-Plot.
This figure shows the Quantile-Quantile-plot (QQ-plot) for daily and monthly momentum returns. The left panels plot the empirical quantiles against the quantiles of a normal distribution. The right panels plot the empirical quantiles against the quantiles of the skewed t distribution.

The results of Figure II highlight that both unconditional distributions, the normal and the skewed t distribution, well describe the middle part of the return distribution. This indicates that periods where returns are small in magnitude can simply be managed by volatility.[122] However, both distributions fail to adequately match the tails of the observed returns. For monthly returns, the normal distribution gives a quite good fit of the right tail but obviously fails to account for the high crash risk of the momentum returns, shown by the high deviation in the left tail. The

[122]This is also in line with our previous results that eliminating only a few extreme returns makes the momentum returns almost normally distributed.

skewed t distribution gives a slightly better fit of the left tail but fails to capture the characteristics in the right tail. As a conclusion, results in Figure II show that unconditional return distributions fail to adequately describe the tails of the momentum return distribution. For that reason, accounting for a time-varying volatility that captures the actual market environment seems crucial when portfolio risk is managed. Therefore, we next assess if conditional models that are based on a dynamic volatility model are more successful in capturing the extreme momentum returns. Thus, we next examine the conditional return distribution of the daily and monthly momentum returns based on the model given in Equation (2.4.16). If controlling volatility by a dynamic volatility model is sufficient to manage momentum crashes, then returns standardized by a time-varying volatility should be approximately normally distributed. Barroso and Maio (2018) show in an earlier version of their paper that the GARCH(1,1) model is able to target a certain level of volatility if returns are only modestly skewed and fat-tailed, but fails when returns are highly non-normal, such as the returns of momentum portfolio. Hence, a conditional volatility model is able to capture modest deviations from the normal distribution, but fails to capture extreme skewness and kurtosis. This finding is confirmed by Ghysels et al. (2016), Brooks et al. (2005) and Bali et al. (2008) who find that returns that are standardized by conditional volatility still exhibit significant non-normalities. Thus, conditional skewness and kurtosis should also be regarded when momentum's risk is managed.

To illustrate this finding, Table II shows the descriptive statistics of mean and volatility standardized momentum returns, where we use the conditional volatility of the GARCH(1,1) and GJR-GARCH model to standardize returns. Since we fit our estimation models to daily data, we additionally show results for daily returns. These standardized returns should be approximately normally distributed, if controlling momentum's volatility is sufficient. However, Table II demonstrates that standardized returns are still highly negatively skewed and fat-tailed. This result also holds for the robust estimates of skewness and kurtosis. This can also be seen by the JB-test that clearly rejects the normality assumption. Bali et al. (2008, Table 1) also show that returns that are standardized by a time-varying volatility are not normally distributed and that a conditional distribution that accounts for non-normalities is needed. Similarly, Brooks et al. (2005), using specification tests, finds that the GARCH(1,1) model is not able to capture all temporal dependencies in asset returns. Hence, using a distribution that incorporates the non-normality of standardized returns, as done in Equation (2.4.16), is needed to better manage momentum crashes. Furthermore, standardized monthly returns deviate much more from the normal distribution. Consequently, fitting the volatility models directly to monthly data fails to capture the high non-normality of monthly momentum returns.

Table II. Descriptive Statistics: Standardized Returns
This table reports the descriptive statistics of daily and monthly standardized momentum returns. Panel A reports results for daily returns, whereas Panel B uses monthly returns. The GARCH(1,1) and GJR-GARCH models are fitted to all available daily or monthly returns. The description of the columns is given in Table I.

Model	Mean	Vola	Skew	Kurt	Min	Max	JB	$Sk^{qu}_{0.75}$	$Sk^{qu}_{0.99}$	Ku^{qu}
	Panel A: Standardized Daily Momentum Returns									
GARCH	0.00	1.07	-0.68	10.08	-15.97	7.31	52592	-0.032	-0.096	1.359
GJR-GARCH	0.00	1.01	-0.72	9.42	-18.00	5.02	43807	-0.018	-0.104	1.302
	Panel B: Standardized Monthly Momentum Returns									
GARCH	0.06	1.06	-2.21	21.71	-12.15	3.36	17005	-0.053	-0.189	1.466
GJR-GARCH	0.07	1.09	-2.03	19.84	-12.21	3.66	13808	-0.064	-0.157	1.494

By applying the QQ-plot to the standardized returns, we next assess if the dynamic volatility models are able to capture the non-normalities of the momentum returns, especially in the tails. The QQ-plots of standardized daily and monthly returns are given in Figure III, where we plot the empirical quantiles against the quantiles of the normal and skewed t distribution with constant parameters η and λ. We only show results for the GARCH(1,1) standardized returns. Figure III shows that the dynamic volatility model is able to capture a significant part of the non-normalities in the momentum returns. This can be seen, since standardized returns are much better described by a normal distribution than the non-standardized returns in Figure II. This is in line with Theodossiou (1998, p. 1659) who finds that the conditional return distribution is expected to be less skewed and leptokurtic than the unconditional return distribution. However, the GARCH(1,1) model alone fails to capture the tail-behavior of the momentum returns, as already shown by Ghysels et al. (2016) for equity returns. For monthly returns, the right tail is well fitted by the model, whereas the left tail, i.e. the extremely negative returns, are not captured by the volatility model alone. Thus, volatility is an appealing method to manage momentum's risk in periods with positive or moderately negative returns, but is an inadequate risk management tool just when it is most needed, during periods of extremely low returns. In contrast, the skewed t distribution is very convincing in capturing the tail behavior of the standardized returns, both daily and monthly. As a conclusion, momentum crashes are not optimally managed by controlling volatility alone, as also suggested by Daniel et al. (2017). In contrast, managing conditional volatility as well as conditional skewness and kurtosis should work well in momentum crash periods.

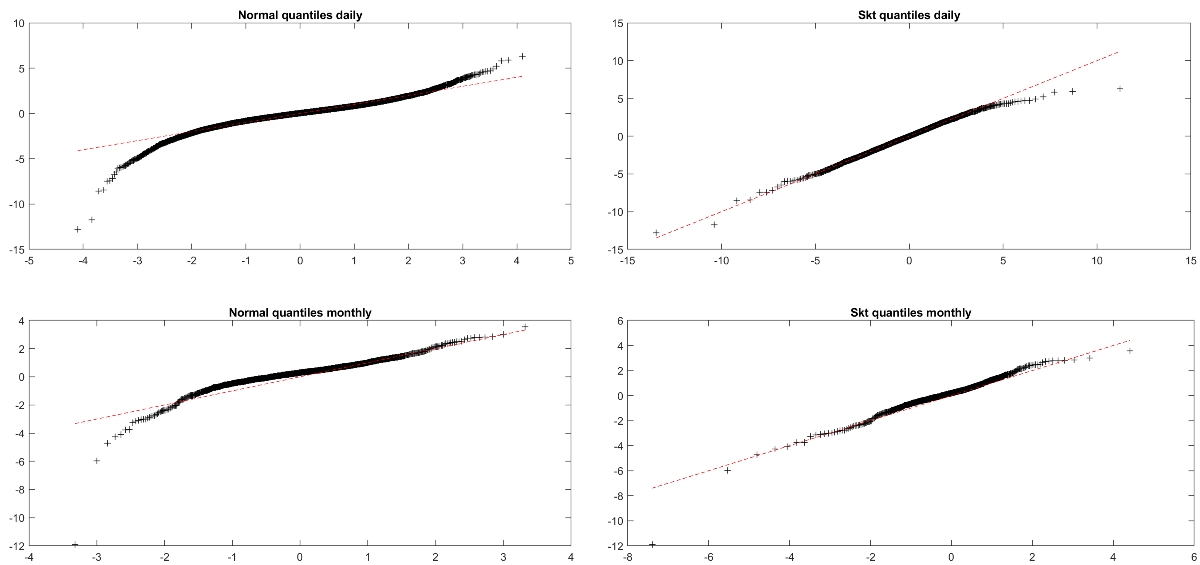

Figure III. Conditional Quantile-Quantile-Plot.
This figure shows the Quantile-Quantile-plot (QQ-plot) for mean-volatility standardized daily and monthly momentum returns. The left panels plot the empirical quantiles against the quantiles of a normal distribution. The right panels plot the empirical quantiles against the quantiles of the skewed t distribution.

Since risk targeting strategies readjust the exposure to the momentum strategy each month, information on the time-series behavior of higher moments is also highly important. Managing volatility assumes that conditional skewness and kurtosis are constant over time, which is usually not full-filled for asset returns (Bali et al., 2008, Brooks et al., 2005, Ghysels et al., 2016, Harvey and Siddique, 1999, Jondeau and Rockinger, 2003). For that reason, we follow Jondeau and Rockinger (2003, Fig. 6-7) and Bali et al. (2008, Fig. 1) and plot the time-series of the return, conditional volatility, conditional skewness and conditional kurtosis by fitting the model given in Equation (2.4.16) to daily and monthly returns, where we use the GJR-GARCH model as volatility model. Results are quite similar for the GARCH(1,1) model.[123] Jondeau and Rockinger (2003) show that the conditional skewness and conditional (excess) kurtosis are given by

$$\text{Skew} = \frac{m_3 - 3am_2 + 2a^3}{b^3} \quad \text{and} \quad \text{Kurt} = \frac{m_4 - 4am_3 + 6a^2m_2 - 3a^4}{b^4} - 3, \tag{2.6.3}$$

where

$$m_2 = 1 + 3\lambda^2 \tag{2.6.4}$$

$$m_3 = 16c\lambda(1+\lambda^2)\frac{(\eta-2)^2}{(\eta-1)(\eta-3)} \text{ for } \eta > 3 \tag{2.6.5}$$

$$m_4 = 3\frac{\eta-2}{\eta-4}\left(1 + 10\lambda^2 + 5\lambda^4\right) \text{ for } \eta > 4. \tag{2.6.6}$$

The time-series of the conditional skewness and (excess) kurtosis should be constantly zero if controlling momentum's risk by volatility is sufficient. If the conditional skewness and (excess) kurtosis are time-varying and non-zero, volatility managing alone is insufficient to manage momentum's risk. In this case, momentum's risk should be managed by additionally considering momentum's conditional non-normalities, as it is done by the downside risk targeting strategies.

Figure IV shows the time-series of the first four (standardized) moments for the model that is fitted to daily data. In contrast, Figure V shows results for the model fitted to monthly returns. We follow Jondeau and Rockinger (2003) and mark days when skewness or kurtosis does not exist with a cross.[124] Both figures demonstrate that momentum's volatility is highly time-varying and sometimes takes extreme values. Barroso and Santa-Clara (2015, Fig. 2) also find a highly time-varying and extreme monthly volatility for the value-weighted momentum strategy ranging between 3.04% and 127.87% on an annualized basis. Hence, controlling volatility of the portfolio is crucial since these extreme realizations of portfolio volatility make the momentum portfolio unattractive for risk-averse investors.[125] Further, Figures IV and V demonstrate

[123] A similar observation was also found by Bali et al. (2008, Footnote 11).

[124] See also Jondeau and Rockinger (2003, Table 3 and 4) and Bali et al. (2008) who fit a similar model to several equity and currency returns. Further, Bali et al. (2008, Table 3 and 4) additionally fit the skewed t distribution with constant parameters η and λ as well as several other volatility models to equity returns. The authors find that the model with time-varying skewness and kurtosis parameters gives a better fit than the model with constant parameters. See also Ghysels et al. (2016, Figure 1), Harvey and Siddique (1999, Figure 1) and Neuberger (2012, Figure 1) who show that skewness is highly time-varying.

[125] Many investors are willing to pay for hedges against a changing volatility (Adrian and Rosenberg, 2008, Ang et al., 2006b, Bollerslev and Todorov, 2011). Hence, these investors strongly benefit from targeting momentum's volatility at a constant level.

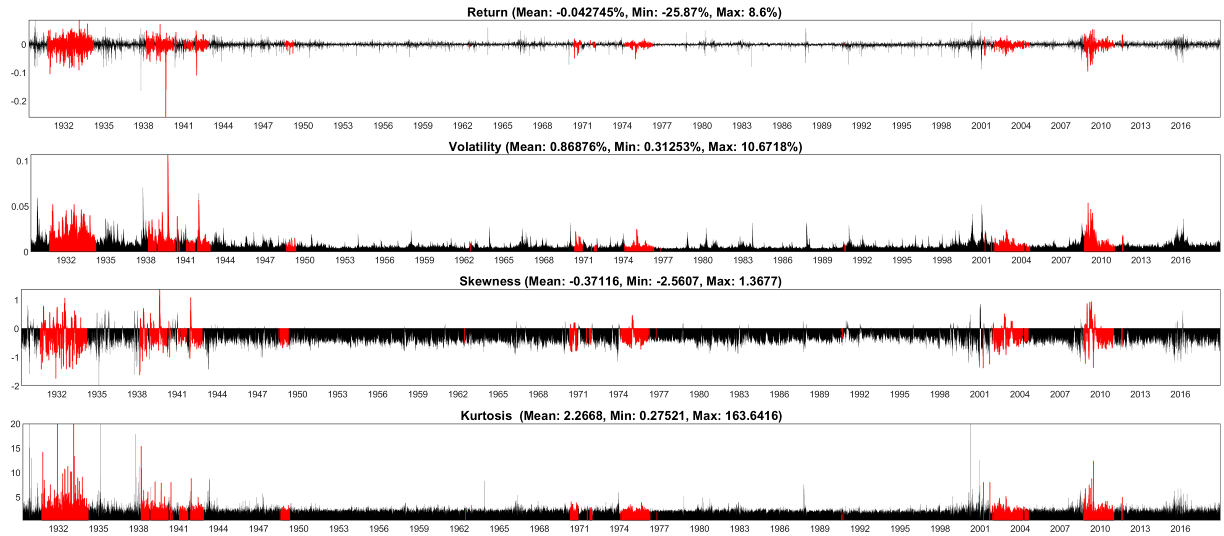

Figure IV. Daily Conditional Moments. This figure shows the daily return, conditional volatility, conditional skewness and conditional kurtosis. Conditional volatility, conditional skewness and conditional kurtosis are calculated for the model in Equation (2.4.16) using the GJR-GARCH model. Days when skewness or kurtosis do not exist are marked with an x. Days when the previous 36 months' market return is negative are shown in red.

that conditional skewness and kurtosis are also highly time-varying, highly non-normal and there are periods when momentum's skewness and kurtosis do not exist or take extreme values. A similar pattern was found by Brooks et al. (2005, Figure 1 and 4), Bali et al. (2008, Fig. 1) and Jondeau and Rockinger (2003, Figure 6) for stock returns. We additionally mark periods when the previous 36 months' market return is negative in red. As discussed in Section 2.5, these periods typically coincide with periods of extremely negative momentum returns and are used for our switching approach. Figures IV and V highlight that the volatility is typically very high in these periods, which is in line with the huge drawdown reduction of volatility targeting found by Barroso and Santa-Clara (2015). However, volatility alone is not sufficient in managing momentum crashes, as can be seen by the extreme outcomes of conditional skewness and conditional kurtosis. During times with a previous negative market return, skewness is typically extremely negative and kurtosis is typically very high. This high crash risk is not captured by the RV model of Barroso and Santa-Clara (2015). Interestingly, there even exist several periods with an extremely high kurtosis although volatility is low in these periods. These periods are also accompanied with a negative skewness. This highlights that volatility alone does not capture the tail risk characteristics of the momentum portfolio and can sometimes even give false signals. In other words, during periods of a momentum crash, the RV model of Barroso and Santa-Clara (2015) significantly underestimates momentum's risk and the RV managed momentum strategy's exposure is too high, i.e. simply controlling volatility will likely fail to mitigate momentum crashes as already found by Daniel et al. (2017). In contrast, frequently reallocating the weight invested in the momentum portfolio based on risk measures that incorporate conditional skewness and kurtosis is crucial, especially in periods with a high probability of a momentum crash (see also Cuoco et al. (2008)).

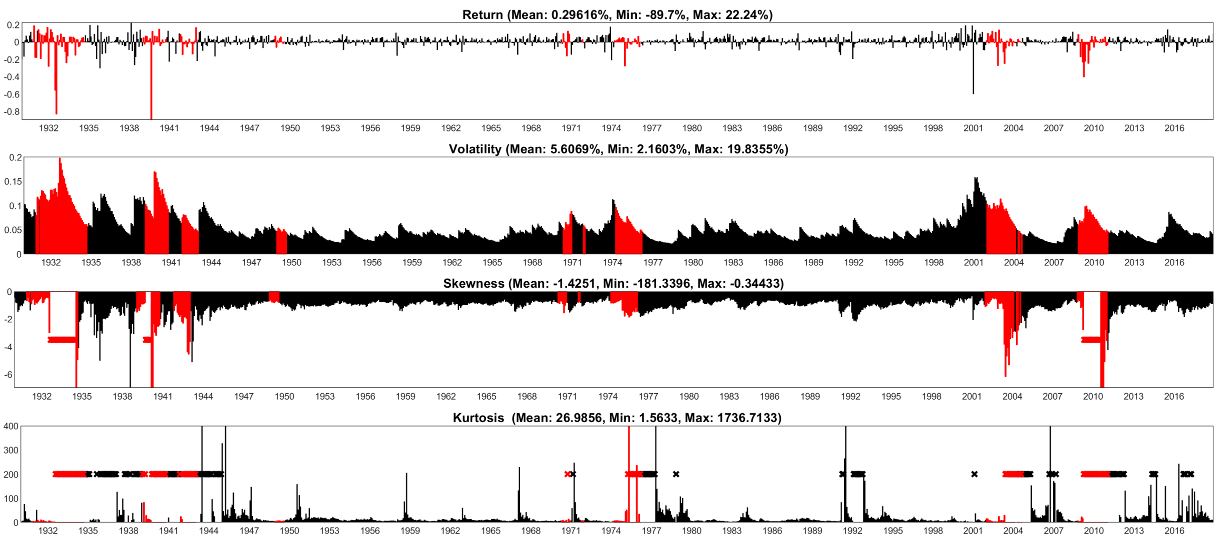

Figure V. Monthly Conditional Moments. This figure shows the monthly return, conditional volatility, conditional skewness and conditional kurtosis. Conditional volatility, conditional skewness and conditional kurtosis are calculated for the model in Equation (2.4.16) using the GJR-GARCH model. Months when skewness or kurtosis do not exist are marked with an x. Months when the previous 36 months' market return is negative are shown in red.

The underestimation of momentum's risk by the RV model does not only hold for the crash periods. Interestingly, in contrast to the findings of Ghysels et al. (2016, Figure 1), Harvey and Siddique (1999, Figure 1) and Neuberger (2012, Figure 1) who calculate monthly skewness of equities using daily data and find that skewness fluctuates between positive and negative values, we find that the skewness of the momentum portfolio is also highly time-varying but negative almost always. A negative skewness of momentum in almost every month indicates that the probability of extremely negative returns is substantially underestimated by managing momentum's volatility. A similar result was also found for hedge fund returns (Agarwal and Naik, 2004).

To better understand the source of the high fluctuation and extreme values in conditional skewness and kurtosis, we next assess the winners and losers portfolios' comovement of skewness and kurtosis. Following the approach of Jondeau and Rockinger (2003, Table 8), we calculate conditional skewness and kurtosis of the winners and losers portfolios for each month and classify these realizations into quartiles. We use the returns of the winners and losers portfolios in excess of the risk-free rate. As in Jondeau and Rockinger (2003), we classify months where skewness and kurtosis do not exist in the highest quartile.[126] We then calculate the frequency of joint realizations of skewness and kurtosis for the winners and losers portfolios. The frequency matrices are given in Table III. For example, the value of 15.14 for skewness indicates that in 15.14% of the realizations, the winners' skewness belongs to the lowest quartile, i.e. the winners' skewness is extremely low, whereas in the same month the losers' skewness belongs to the highest quartile, i.e. the losers' skewness is extremely high.

Table III. Comovements of Conditional Skewness and Kurtosis
This table shows frequency matrices for conditional skewness and conditional kurtosis of the winners and losers portfolios, where we subtract the risk-free rate from the winners and losers portfolios. The left matrix shows results for conditional skewness, whereas the right matrix shows results for conditional kurtosis. Winners(1) (Losers(1)) denotes the quartile when conditional skewness or conditional kurtosis takes a value in the lowest quartile of the winners (losers) portfolio. Winners(4) (Losers(4)) denotes the quartile when conditional skewness or conditional kurtosis takes a value in the highest quartile of the winners (losers) portfolio. Months when conditional skewness and conditional kurtosis do not exist belong to the highest quartile.

	Skewness				Kurtosis			
	Losers(1)	Losers(2)	Losers(3)	Losers(4)	Losers(1)	Losers(2)	Losers(3)	Losers(4)
Winners(1)	0.73	2.72	6.44	15.14	17.59	5.80	1.36	0.27
Winners(4)	1.81	6.80	9.34	6.98	5.89	11.51	5.98	1.54
Winners(3)	6.71	10.06	6.71	1.54	1.27	6.17	11.24	6.35
Winners(4)	15.78	5.35	2.54	1.36	0.27	1.45	6.44	16.86

The results of Table III for skewness resemble the result of Jondeau and Rockinger (2003, Table 8) but with a transposed frequency matrix. That is, we find that periods of a low skewness of the winners portfolio occur simultaneously with periods of a high skewness of the losers portfolio, whereas Jondeau and Rockinger (2003) find that periods of a low skewness typically

[126] We also classified a non-existing skewness in the lowest quartile, but found quite similar results.

occur simultaneously for different equity indices.[127] An investor who is long in the winners and short in the losers is concerned about a contemporaneous occurrence of a negative skewness of the winners and a positive skewness of the losers portfolio. Similarly, periods of a high skewness for the winners portfolio occur simultaneously with periods a low skewness for the losers portfolio. The extremely high elements (1,4) and (4,1) imply that large realizations of the opposite sign tend to occur simultaneously for the skewness. The off-diagonal elements of the skewness matrix sum up to 50.32%, which is significantly higher than the 25% that would hold if there were no correlation between the realizations of the winners' and losers' skewness. This indicates that large negative returns of the winners portfolio coincide with large positive returns of the losers portfolio. Hence, buying winners and selling losers results in a portfolio with a highly time-varying and extreme skewness, which is a main driver of the momentum crashes. In contrast, the diagonal elements sum up to only 15.6%. Thus, extreme realizations in the same quartile tend to occur only very rarely, i.e. an extremely negative skewness of the winners portfolio is only rarely compensated by shorting losers with an extremely negative skewness. This result illustrates the source of the high left tail risk of the momentum portfolio and the momentum crashes. Results for the comovement of the winners' and losers' kurtosis are also different to the findings of Jondeau and Rockinger (2003). We find that the conditional kurtosis of the winners and losers portfolios is highly related, whereas Jondeau and Rockinger (2003) find only low comovements of the kurtosis for equity indices. In particular, as indicated by the elements (1,1) and (4,4), periods of an extremely high or low kurtosis typically occur simultaneously. The sum of the diagonal elements of 57.2% is significantly higher than the 25%, which would hold if there were no relation between the kurtosis of the winners and losers. This shows that the kurtosis of winners and losers typically comoves and that buying winners and selling losers results in a highly time-varying and extreme kurtosis of the momentum portfolio. In total, the winners' and losers' skewness and kurtosis are highly related and comove in directions that are highly unfavorable for momentum investors who are long the winners and short the losers. This high comovement of the winners' and losers' higher moments is the source of momentum's

[127]The finding of Jondeau and Rockinger (2003) is in line with the observation that different assets typically crash together. In contrast, our findings indicate that a crash of the winners is typically related to an extremely positive return of the losers and vice versa.

high crash risk.[128]

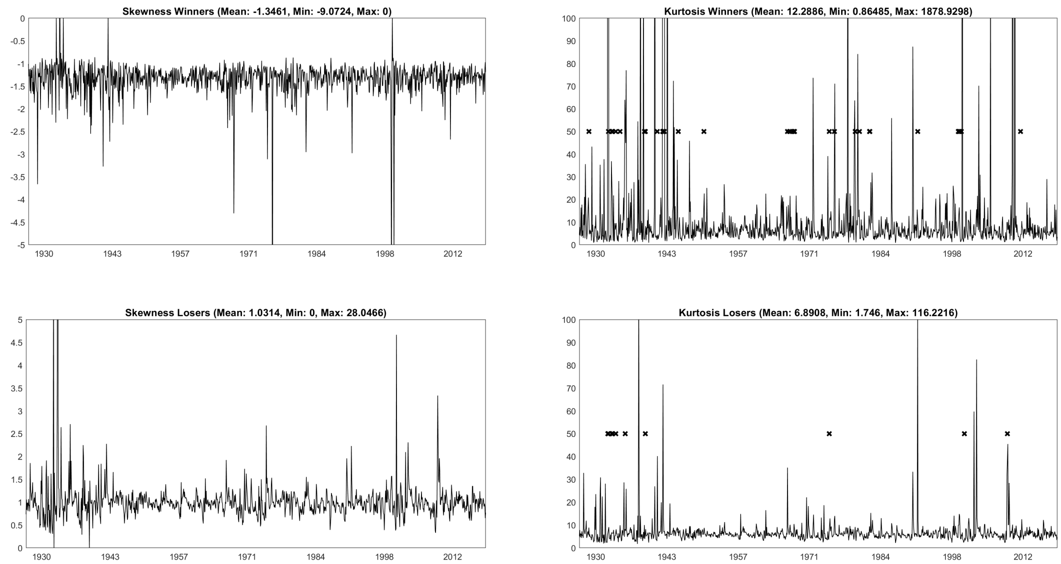

Figure VI. Monthly Conditional Skewness and Kurtosis of Winners and Losers. This figure shows the monthly conditional skewness and conditional kurtosis for the winners and losers portfolios. The returns of the winners and losers portfolios are in excess of the risk-free rate. The upper panels show the conditional skewness and conditional kurtosis for the winners portfolio. The lower panels show the conditional skewness and conditional kurtosis for the losers portfolio. Months where skewness or kurtosis do not exist are marked with an x.

The results of Table III are also visualized in Figure VI that shows the conditional skewness and kurtosis of the winners and losers portfolios. Both portfolios are again in excess of the risk-free rate. This figure shows that the conditional skewness of the winners is mostly negative, whereas the conditional skewness of the losers is mostly positive. Hence, buying winners and selling losers translates into a portfolio with a highly negative skewness. In line with Table III, months where the winners' skewness is extremely negative typically coincide with months where the losers' skewness is highly positive. Furthermore, as in Table III, the conditional kurtosis of winners and losers comoves, especially in periods with extreme realizations, i.e. periods of an extremely high or extremely low kurtosis of the winners and losers portfolios typically occur simultaneously. For both portfolios, conditional kurtosis is highly time-varying, sometimes takes extreme values and sometimes does not exist.

[128] Due this unfavorable comovement of momentum's higher moment risk, momentum's risk could also be managed by separately managing the winners and losers portfolios' risk. This approach is examined in Rickenberg (2020c).

2.6.2 Random Walk Hypothesis

In this section, we test if momentum returns follow a random walk. If the random walk hypothesis holds, monthly volatility can simply be obtained by using the SRTR, i.e. multiplying daily volatility by $\sqrt{h}$. Similarly, monthly VaR and CVaR can then also be calculated as in Equations (2.4.10) and (2.4.11). Further, the method used in Moreira and Muir (2017), Barroso and Santa-Clara (2015) and Barroso and Maio (2018) is also based on the assumption that the momentum returns follow a random walk. If the random walk hypothesis is rejected, more advanced models are needed to manage momentum's risk. We follow Lo and MacKinlay (1988) and use a variance ratio test to test the random walk hypothesis for the momentum portfolio. The variance ratio test is shortly summarized in Appendix C. Similar tests of the random walk hypothesis are also shown in Hsieh (1993) and Saadi and Rahman (2008).[129]

Table IV. Variance Ratio Test
This table show results for the Variance Ratio test of Lo and MacKinlay (1988). Panel A uses daily returns as base period, whereas Panel B uses weekly returns as base period. Bold entries mark values that are higher than 1.96.

Start	End	n_{obs}	$J_r + 1$	z_1	$\overline{M}_r + 1$	z_2	z_3
Panel A: Daily Returns							
19261103	20181210	24276	2.424	**35.076**	2.206	**36.841**	**15.943**
19261103	19701027	12138	2.269	**22.111**	1.930	**20.073**	**9.244**
19701028	20181210	12138	2.763	**30.702**	2.799	**38.841**	**18.240**
Panel B: Weekly Returns							
Start	End	n_{obs}	$J_r + 1$	z_1	$\overline{M}_r + 1$	z_2	z_3
19261108	20181214	4856	1.645	**18.342**	1.477	**17.783**	**6.590**
19261108	19701029	2428	1.693	**13.948**	1.428	**11.275**	**4.138**
19701105	20181214	2428	1.564	**11.355**	1.553	**14.577**	**6.371**

Results for the variance ratio test are given in Table IV. Panel A uses daily returns as base period against monthly returns, i.e. it is tested if scaling up daily volatility by $\sqrt{21}$ is sufficient to obtain monthly volatility. As in Lo and MacKinlay (1988), we additionally split the observed time period into two subperiods. Clearly, both variance ratios are significantly dif-

[129]Engle (2011, Sec. 4) also presents a similar test that can be applied to test if models that are fitted to daily data are able to capture the long-term skewness, e.g. skewness of monthly returns. This test compares the skewness of monthly returns and simulated returns as described in Section 2.4. Engle (2011) finds that asymmetric conditional volatility models, like the GJR-GARCH model, are able to capture a part of the long-term skewness. This is similar to the finding of Adrian and Rosenberg (2008) that the short-run volatility of an EGARCH model can capture skewness risk of monthly returns.

ferent from one, indicating that the random walk hypothesis does not hold for the momentum returns. The outcomes of z_1, z_2 and z_3 clearly reject the random walk hypothesis for all tests and (sub)periods when daily data are used as base period, as done in managing momentum's monthly risk. However, we find higher values compared to Lo and MacKinlay (1988). There are several possible explanations for this finding. First, Lo and MacKinlay (1988) show that the random walk hypothesis is clearly rejected for smaller firms, whereas the hypothesis cannot be rejected for all subperiods for larger firms. Thus, the evidence against the random walk hypothesis is much stronger for small-sized firms, and hence also for the momentum portfolio, since the momentum portfolio is typically invested in small-sized firms as summarized in Appendix A. For that reason, we should expect higher values for the momentum portfolio compared to the findings of Lo and MacKinlay (1988). Second, Lo and MacKinlay (1988) find that the values of the test statistic increase when the base observation period is lowered. Since we use daily data, opposed to weekly data as in Lo and MacKinlay (1988), we would expect significantly higher values. For a better comparison and since Lo and MacKinlay (1988) argue that using daily data results in biased estimates due to effects like non-trading, bid-ask spread and asynchronous prices, we repeat the procedure above by using weekly returns as base period. That is, we use the variance ratio test based on variances calculated with weekly and monthly returns. If returns follow a random walk, the variance calculated with monthly data should be $h = 4$ times higher than the variance calculated with weekly data. Results for this test are given in Panel B. As expected, the values for z_1, z_2 and z_3 are lower than for the test when daily data are used as base period. However, the random walk hypothesis is still rejected for all tests and subperiods. Further, similar to Lo and MacKinlay (1988), we find higher values of z_2 compared to z_3 and that the outcomes of the variance ratio test varies when different periods are tested. As a conclusion, the results of the variance ratio test give strong evidence against the random walk hypothesis. Consequently, managing momentum's risk based on the RV model of Barroso and Santa-Clara (2015) should deliver inferior results compared to more advanced models that reflect the term-structure of risk.

2.6.3 Accuracy of Risk Targeting Strategies

We next test if the different estimation models are able to target a certain level of volatility over time. In particular, we assess if more advanced models are more successful in targeting a constant volatility than the simple RV model. Further, we examine the importance of incorporating higher moments when the portfolio's volatility is targeted. Taylor (2005) finds that models that incorporate information on higher moments produce more accurate volatility forecasts than standard volatility models. Moreover, we are interested in the question if switching between volatility and CVaR targeting has any influence on the ability of targeting a constant level of volatility. Accurately targeting a constant level of portfolio risk over time is important for several reasons. For example, Bollerslev et al. (2018) show that a volatility targeting investor achieves higher utility gains from more accurate volatility models.[130] Further, risk-averse investors are willing to pay high fees for insurance against a changing volatility. Thus, risk-averse investors benefit from a more constant volatility (Adrian and Rosenberg, 2008, Ang et al., 2006b, Bollerslev and Todorov, 2011). This holds especially for practitioners who use long-short strategies. Barroso and Santa-Clara (2015, p. 116) write that "[r]unning a long-short strategy to have constant volatility is closer to what real investors (as hedge funds) try to do than keeping a constant amount invested in the long and short legs of the strategy." Furthermore, a stabilization of the portfolio volatility is typically related to a lower tail risk (Dreyer and Hubrich, 2019). Rickenberg (2020b, Sec. 4) summarizes further reasons why it is important to test for a constant risk of risk targeting strategies.

To test the accuracy of volatility targeting, we use several testing procedures that are frequently used in the academic literature, namely the DM-test of Diebold and Mariano (1995), the RC-test of White (2000) and Sullivan et al. (1999), the SPA-test of Hansen and Lunde (2005) and Hansen (2005), the stepwise RC-test of Romano and Wolf (2005), the stepwise SPA-test of Hsu et al. (2010), the FDR approach of Barras et al. (2010) and Bajgrowicz and Scaillet (2012) and the MCS of Hansen et al. (2003) and Hansen et al. (2011). The methods to assess the ac-

[130] Bollerslev et al. (2018) conclude that "the investor's utility is directly related to the volatility: the investor achieves the maximum utility by successfully targeting a constant risk level, while the utility decreases with the volatility-of-volatility. Hence, risk models that help the investor achieve more accurate volatility forecasts are associated with higher levels of utility."

curacy of volatility targeting are summarized in Appendix D. In particular, these tests cannot only be applied to assess the accuracy of volatility targeting. For example, Hansen (2005, Example 2) suggests that the SPA-test can be applied to backtest the accuracy of VaR forecasts. Kole et al. (2017) use the DM-test and the MCS to assess the accuracy of h-day VaR forecasts. Further, the methods can also be used to assess the performance of funds (Barras et al., 2010) and technical trading rules (Bajgrowicz and Scaillet, 2012, Hsu et al., 2010, Hsu and Kuan, 2005, Sullivan et al., 1999). Similarly, Goyal and Wahal (2015) use the test of Romano and Wolf (2005) to assess the performance of different momentum strategies. Moreover, Kirby and Ostdiek (2012), Taylor (2014), Bollerslev et al. (2018), DeMiguel et al. (2009b), Cederburg et al. (2020) and Rickenberg (2020b) use similar approaches to test if trading strategies produce significant utility gains. We will also use these tests to test for significant utility gains in Section 2.6.5.

Results of the tests are given in Table V, where we only show results for the volatility models, the CVaR models and the models that switch between volatility and CVaR targeting for only one crash indicator and the EWMA model. The tests show that more advanced volatility models, i.e. models based on Equation (2.3.7) and the HAR model, typically achieve a more constant volatility than the simple RV model. The models that use the past squared monthly return to forecast next month's volatility, i.e. the monthly EWMA model and the model of Drost and Nijman (1993), are the least accurate volatility models. This shows that incorporating higher frequency data to forecast monthly volatility increases the accuracy of the forecasting model. The CVaR models are less accurate in targeting a constant level of volatility. This is in line with Rickenberg (2020b) who also finds that an investor who wants to achieve a constant volatility over time should directly manage volatility. Interestingly, although managing CVaR is less successful in targeting a constant level of volatility than managing volatility, the switching strategies that manage momentum's CVaR in crash periods also do a good job and are typically more accurate than the RV model, as long as a conditional CVaR model is used.

As expected, results for the different backtesting procedures are quite robust, but some tests produce different results. For example, the DM-test favors the strategies that switch be-

Table V. Testing the Accuracy of Volatility Targeting

This table shows results of the tests of predictive accuracy presented in Appendix D. $\overline{L}_k^{norm} = \frac{\frac{1}{n}\sum_{t=1}^{n} L_{k,t}}{\frac{1}{n}\sum_{t=1}^{n} L_{0,t}}$ defines the average loss of model k normalized by the loss of model 0 and is given in percent. z_{DM} stands for the test statistic of the Diebold and Mariano (1995) test. The null-hypothesis of equal predictive ability at a test level of 10% is rejected for $|z_{DM}| > 1.64$, where positive values indicate that model k is more accurate than the RV model. Bold numbers of z_{DM} indicate that the model is significantly superior to the RV model. p^{RC} and p^{SPA} stand for the p-value of the RC-test of White (2000) and Sullivan et al. (1999) as well as the SPA-test of Hansen (2005) and Hansen and Lunde (2005). Bold numbers of these tests indicate that the null-hypothesis that model k is the best model cannot be rejected at a test level of 10%. p^R and p^{SQ} stand for the p-values of the MCS of Hansen et al. (2003) and Hansen et al. (2011). Step-RC, Step-RCst, Step-SPA, Step-SPAst denote the step in which a model is selected by the stepwise approaches of Romano and Wolf (2005) and Hsu et al. (2010). st indicates that the studentized version is used. FDR denotes the portfolio that targets an FDR^+ of 10%. All p-values are given in percent. switch stands for the switching strategy that switches between the EWMA model and the CVaR models.

Model	$\overline{L}_k^{norm}$	z_{DM}	p^{RC}	p^{SPA}	p^R	p^{SQ}	Step-RC	Step-RCst	Step-SPA	Step-SPAst	FDR
RV	100.00	-	**35.32**	0.42	4.42	5.43	0	0	0	0	-
HAR	94.34	1.08	**64.85**	**12.17**	**33.27**	**15.19**	0	0	0	0	**12**
EWMA-SRTR	84.67	**3.27**	**100.00**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**	**1**	**1**
GARCH-SRTR	95.12	0.88	**49.93**	**15.74**	**33.27**	**15.19**	0	0	0	0	**13**
GJR-SRTR	93.11	1.21	**57.08**	**26.04**	**33.46**	**22.52**	0	0	0	0	**9**
GARCH	90.65	**1.75**	**85.98**	**32.92**	**33.84**	**25.78**	0	0	0	**1**	**8**
GJR	92.92	1.06	**69.57**	**22.81**	**33.46**	**22.52**	0	0	0	0	**10**
Drost-Nijman	115.99	-1.74	3.92	0.02	0.49	1.26	0	0	0	0	0
EWMA Monthly	208.14	-5.66	0.00	0.00	0.00	0.00	0	0	0	0	0
CVaR HS	222.33	-6.71	0.00	0.00	0.00	0.00	0	0	0	0	0
CVaR-Skt-unc	227.23	-5.55	0.00	0.00	0.00	0.00	0	0	0	0	0
CVaR-GARCH-SRTR	123.38	-2.53	0.34	0.00	0.00	0.00	0	0	0	0	0
CVaR-GJR-SRTR	129.20	-2.81	1.33	0.00	0.05	0.09	0	0	0	0	0
CVaR-GARCH-Skt	133.58	-2.97	0.00	0.00	0.00	0.00	0	0	0	0	0
CVaR-GJR-Skt	155.42	-4.36	0.00	0.00	0.00	0.00	0	0	0	0	0
CVaR-GARCH-FHS	144.02	-3.21	0.00	0.00	0.00	0.00	0	0	0	0	0
CVaR-GJR-FHS	172.02	-4.51	0.00	0.00	0.00	0.00	0	0	0	0	0
CVaR HS-switch	93.84	0.96	**61.27**	5.29	**33.27**	**15.19**	0	0	0	0	**11**
CVaR Uncond-switch	101.10	-0.11	**33.06**	2.98	**20.66**	9.00	0	0	0	0	0
CVaR GARCH-switch	86.96	**2.50**	**95.18**	**38.48**	**35.03**	**33.24**	0	**1**	**1**	**1**	**4**
CVaR GJR-switch	87.60	**2.30**	**93.01**	**25.14**	**33.84**	**25.78**	0	**1**	**1**	**1**	**6**
CVaR GARCH Skt-switch	87.72	**2.51**	**92.96**	**12.61**	**33.84**	**25.78**	0	**1**	**1**	**1**	**5**
CVaR GJR Skt-switch	89.17	**2.17**	**88.01**	7.68	**33.46**	**22.52**	0	**1**	**1**	**1**	**7**
CVaR GARCH FHS-switch	85.98	**2.95**	**97.80**	**51.83**	**35.03**	**34.54**	0	**1**	**1**	**1**	**2**
CVaR GJR FHS-switch	87.34	**2.67**	**94.24**	**14.07**	**33.84**	**25.78**	0	**1**	**1**	**1**	**3**

tween volatility and CVaR targeting over the RV model. Furthermore, the EWMA-SRTR and GARCH(1,1) model is also successful in targeting a constant level of volatility and clearly outperforms the RV model. The remaining daily data based volatility models are also more accurate than the RV model, indicated by a positive value of the test statistic, but the higher accuracy is not statistically significant for these models. The RC-test also favors advanced volatility models and the switching strategies, but fails to reject inferior models. In contrast, the SPA-test and the MCS reject more inferior models, especially the strategies that switch to an unconditionally managed CVaR strategy and the RV approach. This result is in line with the findings of Hsu and Kuan (2005), Hansen (2005) and Hansen and Lunde (2005) that these approaches have more power in detecting inferior models than the RC-test. The stepwise approaches also favor the strategies that switch between volatility and CVaR targeting, as long as CVaR is managed by a conditional model. Further, the EWMA and GARCH(1,1) model are also picked by this approach. As expected, and motivated by the Monte Carlo Simulation of Bajgrowicz and Scaillet (2012), the FDR approach detects more superior models than the stepwise approach of Romano and Wolf (2005). The FDR approach chooses all switching strategies, except for the strategy that switches to the unconditionally managed skewed t distribution. Within the volatility models, all models that are based on daily returns are picked by the FDR approach. Interestingly, Table V shows that the SRTR rule's performance is generally very good, especially for the EWMA-SRTR model. A possible explanation for this finding could be that, due to the high non-normality of momentum returns, the advanced volatility models suffer under high estimation risk. An estimation error in the daily specification magnifies when monthly volatility is estimated based on an iterative approach. Hence, estimating monthly volatility with the SRTR is more robust against estimation risk compared to more advanced approaches. This conjecture also fits well to the good performance of the EWMA, which is much less influenced by estimation risk than the GARCH or GJR-GARCH model.

The FDR approach additionally provides estimates of the fraction of models that are equally, less or more accurate than the benchmark RV model. For that reason, we repeat the FDR approach for the RV model, the volatility models that use daily data and the models that switch between the EWMA model and the dynamic CVaR managed strategies. For these models, and by choosing the RV model as benchmark, the estimates of π_0, π_A^- and π_A^+ are given by $0, 0$ and 1, respectively. This shows that all of these models outperform the benchmark RV model of Barroso and Santa-Clara (2015) in providing a more accurate portfolio volatility over time. Thus, as expected, more advanced risk models are more successful in targeting a constant level of risk for the momentum portfolio. In contrast, the RV model of Barroso and Santa-Clara (2015), which is based on the random walk hypothesis, is significantly less accurate.

In total, Table V shows that more advanced volatility models are more successful in targeting a constant level of portfolio volatility. However, managing the momentum portfolio's CVaR over time, and thus incorporating higher moments, does not improve the accuracy. In contrast, managing CVaR only in times when a momentum crash is more likely additionally improves the accuracy of the volatility targeting strategy. In the next two sections, we will assess if this higher forecasting accuracy is also related to a higher performance and utility as it is found by Bollerslev et al. (2018).

2.6.4 Performance of Risk Targeting Strategies

We next assess the performance of the momentum portfolio and the strategies that target a constant level of volatility, VaR and CVaR. Performance results for all portfolios are given in Table VI, which shows that risk targeting significantly increases the return while simultaneously risk, especially in the left tail, is reduced compared to the non-managed momentum strategy. This can be seen by the Sharpe Ratio that is higher for all risk targeting strategies. Further, risk targeting significantly reduces the vast crash risk of the momentum portfolio. This can be seen by a significantly higher (less negative) skewness, lower kurtosis, lower drawdown and lower minimum monthly return. Thus, the risk-managed momentum strategies are much more appealing for investors who have preferences for higher moments and/or are loss-averse. For example, the maximum drawdown of the momentum strategy is 99.31%, i.e. a momentum investor could loose almost all the invested money in a certain period. However, the drawdown can significantly be reduced by risk targeting. For example, the RV managed strategy exhibits a maximum drawdown of only 63.98%, which can further be lowered to only 34.24% by targeting a constant level of CVaR.[131] Since a high drawdown reduction is a main driver of a strategy's long-term performance, all risk targeting strategies significantly increase momentum's (risk-adjusted) return. Interestingly, the simple RV model of Barroso and Santa-Clara (2015) and Moreira and Muir (2017) does a quite good job in increasing momentum's risk-adjusted performance. This is in line with Poon and Granger (2003) who review many studies on volatility modeling and find surprisingly good results for simple volatility models. Similarly, Figlewski (1997, p. 11) finds that simplified forecasting models can be very successful, especially when longer-term

[131] As mentioned earlier, mitigating drawdowns is not only important for loss-averse investors, but is also crucial to achieve a good long-term performance. For example, drawdowns of 99.31%, 63.98% and 34.24% have to be compensated by returns of 14,392.75$\%$, 177.62% and 52.07%, respectively.

volatility is forecasted. As a result, the Sharpe Ratio of the RV managed momentum portfolio is significantly higher than the Sharpe Ratio of the non-managed strategy indicated by a value of 10.08 of the Jobson and Korkie (1981) test statistic.[132] However, the risk-adjusted performance can additionally be increased by using more advanced volatility models. This observation is in line with the findings of Bollerslev et al. (2018) that a higher forecasting accuracy coincides with higher performance gains. The highest Sharpe Ratios are typically achieved by the models that are found to be more accurate in Table V. Three of the conditional volatility models produce significantly higher Sharpe Ratios than the RV managed strategy, indicated by a Jobson and Korkie (1981) test statistic higher than 1.64. Interestingly, the iterated GARCH and GJR-GARCH models, given in Equations (2.3.11) and (2.3.13), produce a lower risk-adjusted performance than the simple SRTR. Nevertheless, this result is again in line with the findings of the previous section, where we find that the simple SRTR is also more successful in targeting a constant level of volatility than the iterated models. This higher forecasting accuracy of the SRTR compared to the RV model and the iterated models is now related to an enhanced risk-return profile. As stated above, a possible explanation for the bad performance of the iterated models could be that the conditional volatility models, although they work well for one-day ahead forecasts, are not designed to forecast many steps ahead (Figlewski, 1997, p. 13). Hence, they deliver poor results when portfolio weights are re-adjusted monthly. A similar result was also found by Han (2005). Furthermore, the iterated models are much more influenced by estimation risk, which should be particularly high for portfolios with highly non-normal returns, such as the momentum portfolio.

As expected, the downside risk targeting strategies produce a significantly lower mean return than the volatility managed strategies, but, on the other hand, also take less risk in terms of a lower volatility, lower drawdown and a higher minimum return. This finding is in line with Rickenberg (2020b) who finds that downside risk targeting is successful in mitigating extremely negative returns, but is too conservative in uptrending markets. Consequently, since momentum

[132]The test of Jobson and Korkie (1981) tests if a strategy produces a significantly higher Sharpe Ratio than a certain benchmark model, where we use the corrected version of the test that is also used by DeMiguel et al. (2009b, Footnote 16) and Cederburg et al. (2020). We choose the RV managed strategy as benchmark strategy to test for differences in the Sharpe Ratio.

Table VI. Performance Results of Risk Targeting

This table shows the performance results of all risk targeting strategies and the momentum portfolio over the whole period. Return and Vola correspond to the annualized return and volatility, respectively. Skew and Kurt denote the realized skewness and kurtosis. SR stands for the annualized Sharpe Ratio. z_{JK} stands for the corrected test statistic of the Jobson and Korkie (1981) test. MDD stands for the maximum drawdown. VaR and CVaR are the in-sample VaR and CVaR, which are estimated by the unconditional skewed t distribution as given in Equations (2.4.10) and (2.4.11), respectively. Calmar denotes the ratio of the annualized return and the maximum drawdown. Min and Max stand for the minimum and maximum monthly return, respectively. Return, Vola, MDD, VaR, CVaR, Min and Max are given in percent. Bold numbers mark a Jobson and Korkie (1981) test that is higher than 1.64.

Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
Momentum	4.11	26.31	-4.38	43.59	0.156	-10.08	99.31	21.34	41.79	0.041	-89.70	22.24
RV	15.48	19.65	-1.48	12.38	0.788	-	63.98	14.81	22.29	0.242	-49.34	19.29
HAR	13.28	17.74	-1.31	10.21	0.748	-0.99	62.57	13.96	21.22	0.212	-42.35	18.29
EWMA-SRTR	18.15	19.61	-1.26	11.03	0.925	**4.25**	65.66	14.13	19.70	0.276	-50.35	21.78
GARCH-SRTR	16.98	19.09	-1.26	11.46	0.890	**2.50**	57.73	13.84	19.57	0.294	-50.49	18.76
GJR-SRTR	17.79	19.01	-1.25	11.94	0.936	**3.56**	58.67	13.41	18.75	0.303	-51.14	18.55
GARCH	13.87	17.92	-1.35	11.09	0.774	-0.24	56.56	13.82	20.56	0.245	-45.84	18.36
GJR	14.26	17.17	-1.36	12.22	0.830	1.22	56.09	12.84	18.77	0.254	-45.83	18.16
Drost-Nijman	12.36	17.14	-1.33	11.51	0.721	-1.44	51.89	13.70	21.91	0.238	-41.39	19.92
EWMA Monthly	7.52	12.62	-1.80	14.08	0.596	-3.32	51.70	10.38	18.80	0.145	-28.68	16.65
VaR HS	8.43	13.24	-1.58	12.08	0.636	-3.04	48.04	10.89	18.56	0.175	-29.11	15.20
VaR-Skt-unc	9.13	14.39	-1.64	12.81	0.634	-3.06	43.33	11.66	20.33	0.211	-32.40	14.82
VaR-GARCH-SRTR	13.47	15.64	-1.10	9.54	0.861	**1.84**	45.02	11.69	17.24	0.299	-37.72	16.75
VaR-GJR-SRTR	14.12	15.93	-1.12	9.82	0.887	**2.32**	46.39	11.67	17.09	0.304	-38.71	17.09
VaR-GARCH-Skt	11.91	15.40	-1.20	9.56	0.773	-0.07	45.91	12.06	18.62	0.259	-35.26	16.36
VaR-GJR-Skt	11.31	14.35	-1.41	11.90	0.788	0.31	49.56	10.91	16.55	0.228	-36.59	15.90
VaR-GARCH-FHS	11.19	15.39	-1.48	11.56	0.727	-1.14	52.94	12.16	18.77	0.211	-39.40	15.94
VaR-GJR-FHS	10.65	14.36	-1.77	15.04	0.742	-0.68	55.77	10.97	16.75	0.191	-40.34	15.95
CVaR HS	6.81	11.41	-1.84	14.46	0.597	-3.38	45.59	9.47	16.51	0.149	-26.17	14.25
CVaR-Skt-unc	7.62	11.46	-1.53	12.31	0.665	-2.05	34.24	9.27	16.19	0.223	-24.60	13.11
CVaR-GARCH-SRTR	11.87	14.13	-1.22	10.32	0.840	1.45	42.29	10.66	15.72	0.281	-35.48	14.38
CVaR-GJR-SRTR	12.60	14.55	-1.25	10.68	0.866	**1.94**	44.30	10.74	15.74	0.284	-36.76	14.39
CVaR-GARCH-Skt	10.75	13.61	-1.15	9.32	0.790	0.39	41.40	10.65	16.46	0.260	-30.55	15.28
CVaR-GJR-Skt	9.89	12.71	-1.39	12.03	0.778	0.18	45.59	9.78	15.05	0.217	-32.36	15.36
CVaR-GARCH-FHS	9.53	13.34	-1.56	12.29	0.714	-1.19	48.71	10.55	16.36	0.196	-34.86	15.65
CVaR-GJR-FHS	8.70	12.40	-1.94	17.10	0.701	-1.27	52.22	9.55	14.93	0.167	-36.24	15.36

crashes are very rare and short-lived events, whereas the momentum strategy trends upwards most of the time, downside risk targeting produces lower returns in the long-run. For that reason, the Sharpe Ratios of the downside risk targeting strategies are lower than the Sharpe Ratios of the strategies managed by a conditional volatility model. Generally, by construction, timing volatility instead of timing downside risk should lead to a higher Sharpe Ratio (Jondeau and Rockinger, 2012, p. 108). Therefore, other performance evaluation methods that take non-normalities into account should be considered. We will come back to this point in the next section.

Table VI also shows that the conditional risk models exhibit higher Sharpe Ratios than the unconditional models. This result is in line with the findings of Rickenberg (2020b) who also finds that conditional models outperform unconditional models when risk targeting is applied to long-only equity portfolios. Moreover, as stated above, this result is again in line with Bollerslev et al. (2018) who show that more accurate risk models also produce higher risk-adjusted returns. Nevertheless, even the conditionally managed VaR and CVaR targeting strategies produce Sharpe Ratios that are only slightly higher than the Sharpe Ratio of the RV managed momentum portfolio. As mentioned earlier, a possible explanation for this finding could be that these strategies are too conservative in the long-run, which can be solved by switching between volatility and CVaR targeting as presented in Section 2.5.

Table VII. Performance Results of Risk Targeting: 01.01.1938-01.01.1943
This table shows the performance results of the momentum portfolio, the volatility targeting strategies and the CVaR targeting strategies for the period 01.01.1938-01.01.1943. Return and Vola correspond to the annualized return and volatility, respectively. Skew and Kurt denote the realized skewness and kurtosis. MDD and ΔMDD stand for the maximum drawdown and the reduction of the maximum drawdown with respect to the momentum portfolio. VaR and CVaR are the in-sample VaR and CVaR, which are estimated by the unconditional skewed t distribution as given in Equations (2.4.10) and (2.4.11), respectively. Min and Max stand for the minimum and maximum monthly return, respectively. Return, Vola, MDD, ΔMDD,VaR, CVaR, Min and Max are given in percent.

Model	Return	Vola	Skew	Kurt	MDD	ΔMDD	VaR	CVaR	Min	Max
Momentum	-34.57	51.38	-3.760	22.430	90.39	-	46.30	95.75	-89.70	22.24
RV	-11.64	28.44	-3.705	22.031	49.34	45.42	20.49	42.28	-49.34	15.03
HAR	-7.76	23.81	-3.959	24.200	42.35	53.15	18.81	38.82	-42.35	10.70
EWMA-SRTR	-10.25	28.93	-3.813	22.471	50.35	44.30	22.13	45.80	-50.35	12.88
GARCH-SRTR	-7.71	28.21	-4.091	25.213	50.49	44.14	19.56	40.23	-50.49	12.07
GJR-SRTR	-7.50	28.31	-4.199	26.152	51.14	43.42	18.82	38.64	-51.14	11.93
GARCH	-7.45	25.29	-4.190	26.200	45.84	49.29	18.17	37.41	-45.84	11.38
GJR	-7.13	24.96	-4.360	27.572	45.83	49.30	17.03	35.03	-45.83	10.92
Drost-Nijman	-7.32	22.66	-4.212	26.699	41.39	54.21	17.95	37.07	-41.39	10.88
EWMA Monthly	-5.10	15.49	-4.406	27.904	28.73	68.21	11.97	24.75	-28.68	7.27
CVaR HS	-4.17	12.74	-4.521	28.524	23.73	73.74	9.46	19.57	-23.73	5.51
CVaR-Skt-unc	-3.63	11.96	-3.700	21.776	21.18	76.57	10.37	21.45	-20.74	5.67
CVaR-GARCH-SR	-5.46	19.04	-4.581	28.961	35.48	60.75	13.00	26.82	-35.48	7.09
CVaR-GJR-SR	-5.42	19.41	-4.792	30.829	36.76	59.34	12.68	26.13	-36.76	6.91
CVaR-GARCH-Skt	-4.27	16.89	-4.167	25.939	30.55	66.20	12.09	24.91	-30.55	7.67
CVaR-GJR-Skt	-4.79	17.14	-4.713	30.473	32.36	64.20	10.82	22.28	-32.36	7.42
CVaR-GARCH-FHS	-5.85	18.84	-4.475	28.091	34.86	61.43	13.73	28.35	-34.86	7.29
CVaR-GJR-FHS	-6.13	18.95	-4.903	31.812	36.24	59.91	12.42	25.64	-36.24	7.24

Results so far indicate that tail risk targeting is more successful in mitigated momentum crashes than the volatility managed strategies. To further assess the drawdown protection ability of tail risk targeting, we next compare the performance of the momentum portfolio, the volatility managed strategies and the CVaR managed strategies during a momentum crash. Table VII shows the performance of these strategies for the period from 01.01.1938 to 01.01.1943, i.e. a period of five years which is marked by a severe momentum crash. The momentum crash can easily be seen by the performance of the non-managed momentum strategy. The momentum portfolio exhibits an annualized return and volatility of -34.57% and 51.38%, respectively. Further, the maximum drawdown of 90.39% and the minimum monthly return of -85.70% shows that a momentum investor would have lost almost all the invested money during this period. In contrast, the volatility managed strategies successfully reduce the losses during this period and also take significantly less risk. Although the simple RV model is again doing a quite good job, the risk-return profile can additionally be improved by using advanced volatility models. This shows that the relation between forecasting accuracy, as shown in Table V, and performance gains particularly holds during adverse market conditions. Nevertheless, the risk-return profile during a momentum crash can further be improved by using a strategy that manages momentum's CVaR. The CVaR targeting strategies produce higher returns with significantly lower risk compared to the RV managed strategy. For example, by using CVaR targeting, the loss can be reduced to only -3.63% with a volatility of only 11.96%, whereas the RV model exhibits an annualized return and volatility of -11.64% and 28.44%. This result shows that CVaR targeting works well in crash periods but does not result in an overall enhanced risk-return profile, since the CVaR targeting strategy is too conservative in uptrending markets. This observation is also highlighted in Table VIII, which shows the performance of the momentum portfolio, the volatility managed portfolios and the CVaR managed portfolios on months when the crash indicator, described in Section 2.5, indicates a crash ($\delta_t = 1$) or a calm period ($\delta_t = 0$). We only present results for one crash indicator, but results for other crash indicators are quite similar. Results for the months when a momentum crash is likely are given in Panel A. The crash indicator is doing a quite good job in indicating crash months, highlighted by an annualized return of -50.83% of the momentum portfolio. Furthermore, the high negative return is accompanied by an extremely high volatility of 57.60%. The RV managed strategy again delivers a significantly less negative performance with lower risk, but the performance can still be improved by using more advanced volatility models. However, similar to Table VII, the loss can further be reduced by managing the portfolio's CVaR, where these strategies additionally take less risk. Panel B shows results for the months when a momentum crash is less likely ($\delta_t = 0$). As expected, with an annualized return of 14.01%, the non-managed momentum portfolio performs well in these months. Interestingly, this return can further be increased by managing the portfolio's risk. This is in line with Barroso and Santa-Clara (2015) who also find that risk targeting delivers an enhanced performance not only in crash periods but also in calm periods. The RV managed strategy increases the annualized return to 20.11% without taking more risk. Further, this strategy significantly reduces left tail risk of the non-managed momentum strategy. However, both the return and the reduction of risk, especially in the left tail, can further be improved by using a more advanced volatility model. In contrast, the CVaR targeting strategies produce significantly lower returns but also take less risk. In terms of the Sharpe Ratio, the CVaR managed strategies perform equally well to the RV managed strategy but slightly worse than the dynamically managed volatility strategies. That is, only the dynamically managed target volatility strategies produce a statistically higher Sharpe Ratio than the RV managed strategy in the months with $\delta_t = 0$. In total, Table VII confirms our earlier finding that CVaR targeting is superior in months when a momentum crash is likely, but volatility targeting is superior in uptrending periods, especially if an advanced volatility model is used. For that reason, we next assess the performance of the strategies that switch between volatility and CVaR targeting as explained in Section 2.5. We only report results for the strategies that switch between the EWMA model and the CVaR models. Results for the strategies that use the RV model, the GARCH model or the GJR-GARCH model as volatility model are quite similar.

Table IX shows the performance results of the strategies that switch between volatility and CVaR targeting, where we use the three crash indicators based on the past market return, past market volatility and expected momentum volatility as explained in Section 2.5. Additionally, we examine the strategy that switches between the RV managed momentum and contrarian portfolio. Table IX shows that all switching strategies, regardless of the used CVaR model and crash indicator, significantly improve the risk-return profile by producing higher returns with lower risk compared to the RV model of Barroso and Santa-Clara (2015). All strategies that switch between volatility and CVaR targeting produce significantly higher Sharpe Ratios than the RV managed strategy, indicated by very high values of the Jobson and Korkie (1981) test

Table VIII. Performance Results of Risk Targeting on Crash and Non-Crash Months

This table shows in Panel A the performance of the momentum portfolio, the volatility managed strategies and the CVaR managed strategies on months when the crash indicator indicates a crash ($\delta_t = 1$) and in Panel B the performance on months when the crash indicator does not indicate a crash ($\delta_t = 0$). The description of the columns is given in Table VI.

	Panel A: Months with $\delta_t = 1$											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
Momentum	-50.83	57.60	-2.74	12.97	-	-	99.94	70.62	132.60	-0.509	-89.70	18.97
RV	-16.50	26.29	-3.04	17.08	-	-	87.25	32.85	68.11	-0.189	-49.34	9.52
HAR	-14.29	25.62	-2.18	11.03	-	-	85.48	29.22	50.29	-0.167	-42.35	12.17
EWMA-SRTR	-13.66	25.61	-3.13	19.25	-	-	85.06	25.67	40.08	-0.161	-50.35	9.63
GARCH-SRTR	-11.33	26.27	-2.84	17.82	-	-	81.68	24.39	36.82	-0.139	-50.49	10.77
GJR-SRTR	-10.90	26.26	-2.96	18.84	-	-	80.58	23.94	36.01	-0.135	-51.14	10.86
GARCH	-12.56	25.27	-2.54	14.47	-	-	83.40	24.81	38.11	-0.151	-45.84	10.77
GJR	-11.26	24.23	-2.81	16.96	-	-	80.37	22.71	34.06	-0.140	-45.83	10.81
Drost-Nijman	-15.20	24.61	-2.54	12.52	-	-	85.76	32.51	66.67	-0.177	-41.39	11.03
EWMA Monthly	-13.59	21.39	-1.66	7.62	-	-	81.71	24.30	50.14	-0.166	-28.68	16.65
CVaR HS	-11.39	17.05	-2.24	9.32	-	-	74.58	23.15	47.73	-0.153	-23.73	6.11
CVaR-Skt-unc	-9.08	14.69	-2.37	10.93	-	-	64.99	17.52	36.19	-0.140	-21.20	7.13
CVaR-GARCH-SR	-8.42	19.30	-2.54	15.10	-	-	69.45	19.23	30.70	-0.121	-35.48	8.69
CVaR-GJR-SR	-9.47	19.93	-2.63	15.34	-	-	72.68	20.79	34.83	-0.130	-36.76	8.03
CVaR-GARCH-Skt	-8.57	17.82	-2.31	12.14	-	-	69.72	18.31	28.86	-0.123	-30.55	7.70
CVaR-GJR-Skt	-8.87	17.67	-2.75	15.52	-	-	70.03	17.29	26.53	-0.127	-32.36	7.62
CVaR-GARCH-FHS	-10.96	20.72	-2.30	11.58	-	-	78.18	21.36	32.88	-0.140	-34.86	9.10
CVaR-GJR-FHS	-10.65	20.36	-2.70	14.53	-	-	76.88	21.09	34.09	-0.139	-36.24	8.94
	Panel B: Months with $\delta_t = 0$											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
Momentum	14.01	18.83	-2.02	22.70	0.744	-5.71	59.95	14.59	24.23	0.234	-59.95	22.24
RV	20.11	18.43	-0.79	7.38	1.091	-	35.38	13.05	18.81	0.568	-35.38	19.29
HAR	17.17	16.33	-0.63	5.93	1.051	-0.71	32.44	11.87	17.20	0.529	-21.70	18.29
EWMA-SRTR	22.72	18.51	-0.54	5.19	1.227	**3.68**	33.75	12.53	16.86	0.673	-25.98	21.78
GARCH-SRTR	20.97	17.85	-0.48	4.91	1.175	**1.94**	30.65	12.23	16.80	0.684	-21.55	18.76
GJR-SRTR	21.84	17.75	-0.40	4.76	1.231	**3.04**	28.79	11.84	16.03	0.759	-21.68	18.55
GARCH	17.57	16.63	-0.63	5.59	1.056	-0.58	38.85	12.06	17.33	0.452	-21.39	18.36
GJR	17.81	15.93	-0.52	5.48	1.118	0.84	26.12	11.24	15.94	0.682	-21.17	18.16
Drost-Nijman	16.25	15.78	-0.52	6.65	1.030	-0.95	38.25	11.60	17.54	0.425	-22.58	19.92
EWMA Monthly	10.40	10.90	-1.07	11.13	0.954	-1.64	25.57	7.96	12.80	0.407	-25.57	12.41
CVaR HS	9.25	10.37	-1.26	13.12	0.892	-2.51	26.17	7.75	12.62	0.354	-26.17	14.25
CVaR-Skt-unc	9.84	10.88	-1.19	11.47	0.904	-2.43	24.60	8.03	13.26	0.400	-24.60	13.11
CVaR-GARCH-SR	14.61	13.23	-0.56	5.20	1.104	0.72	21.78	9.40	13.41	0.671	-16.86	14.38
CVaR-GJR-SR	15.61	13.59	-0.54	5.34	1.148	1.44	25.55	9.39	13.23	0.611	-19.72	14.39
CVaR-GARCH-Skt	13.35	12.87	-0.66	6.43	1.037	-0.49	25.20	9.38	14.05	0.530	-18.98	15.28
CVaR-GJR-Skt	12.41	11.84	-0.65	6.68	1.048	-0.16	24.87	8.53	12.68	0.499	-19.06	15.36
CVaR-GARCH-FHS	12.31	11.99	-0.66	5.88	1.026	-0.56	21.86	8.80	12.93	0.563	-17.49	15.65
CVaR-GJR-FHS	11.30	10.91	-0.69	6.64	1.036	-0.26	21.84	7.85	11.58	0.518	-17.57	15.36

statistic. In particular, by achieving a significantly higher (less negative) skewness and lower kurtosis than the RV managed strategy, the switching strategies are successful in mitigating momentum's crash risk. This reduction of left tail risk can also be seen by the maximum drawdown and minimum monthly return. Thus, switching away from the volatility managed momentum portfolio when a momentum crash is expected successfully reduces momentum's crash risk. Similarly, the RV managed strategy that switches between the momentum and the contrarian strategy outperforms the RV managed momentum strategy. This holds especially since this approach significantly reduces left tail risk. However, the strategy that switches to the contrarian portfolio produces a higher Sharpe Ratio that is not statistically significant. Hence, switching to CVaR targeting instead of switching to the contrarian portfolio is even more advantageous. The reason for this finding can be seen in Figures IV and V. Although the crash indicator is doing a good job in signaling a momentum crash, there are also many false signals where the momentum portfolio exhibits a positive return. Thus, the strategy that switches to the RV managed contrarian portfolio is much more influenced by false signals of δ_t. Moreover, results of Table IX indicate that this observation holds for all three crash indicators and not only for the market return based indicator used in Figures IV and V. Generally, we find that all three crash indicators produce quite similar results, which shows that our simple switching approach is quite robust to the definition of the crash indicator.

As mentioned in Section 2.5, the crash indicator can also be defined based on more than one momentum crash predictor. For that reason, Table X shows results for the RV managed strategy and the strategies that switch between volatility and CVaR targeting for three other crash indicators, that are combinations of the crash indicators used in Table IX. First, as examined by Daniel and Moskowitz (2016) and Wang and Xu (2015), we use a crash indicator δ_t that indicates a crash if previous market volatility is high *and* previous market returns is negative. Second, we use an indicator that indicates a crash if previous market volatility is high *or* previous market return is negative. As can be seen in Figures IV and V, the past market return successfully predicts most periods with extremely negative returns, extremely negative or non-existing skewness and high or non-existing kurtosis. However, there are also momentum crash periods that are not captured by the past market return alone. Therefore, defining a crash when market volatility is high or market return is negative increases the probability of capturing all momentum crashes. Third, we use a crash indicator that uses information from the market and the momentum portfolio. This indicator indicates a crash if past market return is negative *or* expected momentum volatility is high.[133] In comparison to Table IX, we find that combining several indicators delivers slightly better results. In particular, we find that combining informa-

[133]We also used other combination of the three indicators examined in Table IX. Since all combinations deliver very similar returns, we only report three combinations. In the following tables and figures we only report results for one crash indicator, but results are very robust when other crash indicators are chosen.

Table IX. Performance Results of the Switching Strategies: Single Indicators
This table shows the performance results for the RV managed momentum portfolio and the switching strategies. Panel A shows results for the crash indicator δ_t that equals one if past market volatility is high. Panel B shows results for the crash indicator δ_t that equals one if past market return is negative. Panel C shows results for the crash indicator δ_t that equals one if expected momentum volatility is high. The description of the columns is given in Table VI.

	Panel A: Market Volatility Crash Indicator											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
RV	15.48	19.65	-1.48	12.38	0.788	-	63.98	14.81	22.29	0.242	-49.34	19.29
CVaR HS	17.49	18.39	-0.71	6.16	0.951	**4.27**	58.40	13.06	18.53	0.299	-26.17	21.78
CVaR-Skt-unc	17.76	18.13	-0.63	5.93	0.979	**4.64**	49.17	12.58	17.79	0.361	-25.98	21.78
CVaR-GARCH-SRTR	18.83	18.52	-0.76	6.76	1.017	**5.71**	52.19	12.91	17.71	0.361	-35.48	21.78
CVaR-GJR-SRTR	18.74	18.59	-0.79	7.04	1.008	**5.57**	53.28	13.03	17.97	0.352	-36.76	21.78
CVaR-GARCH-Skt	18.56	18.45	-0.68	6.01	1.006	**5.36**	51.59	13.00	17.98	0.360	-30.55	21.78
CVaR-GJR-Skt	18.41	18.28	-0.70	6.46	1.007	**5.38**	55.38	12.80	17.82	0.332	-32.36	21.78
CVaR-GARCH-FHS	18.29	18.74	-0.80	6.63	0.976	**4.94**	57.85	13.39	18.55	0.316	-34.86	21.78
CVaR-GJR-FHS	18.24	18.48	-0.84	7.27	0.987	**4.93**	61.22	13.07	18.25	0.298	-36.24	21.78
RV-Mom/Contrarian	17.04	19.57	0.59	10.46	0.871	0.60	39.52	12.97	18.39	0.431	-22.81	49.34
	Panel B: Market Return Crash Indicator											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
RV	15.48	19.65	-1.48	12.38	0.788	-	63.98	14.81	22.29	0.242	-49.34	19.29
CVaR HS	18.37	18.21	-0.65	5.94	1.008	**5.56**	43.24	12.72	18.07	0.425	-25.98	21.78
CVaR-Skt-unc	18.68	17.97	-0.58	5.81	1.039	**5.97**	33.94	12.24	17.28	0.550	-25.98	21.78
CVaR-GARCH-SRTR	18.87	18.52	-0.78	6.93	1.019	**6.14**	39.14	12.92	17.83	0.482	-35.48	21.78
CVaR-GJR-SRTR	18.71	18.61	-0.82	7.18	1.005	**5.96**	40.93	13.06	18.11	0.457	-36.76	21.78
CVaR-GARCH-Skt	18.77	18.30	-0.69	6.26	1.025	**5.95**	41.33	12.79	17.81	0.454	-30.55	21.78
CVaR-GJR-Skt	18.70	18.27	-0.73	6.64	1.023	**6.06**	43.06	12.68	17.73	0.434	-32.36	21.78
CVaR-GARCH-FHS	18.48	18.68	-0.82	6.85	0.989	**5.52**	47.45	13.29	18.59	0.389	-34.86	21.78
CVaR-GJR-FHS	18.47	18.60	-0.85	7.33	0.993	**5.52**	50.96	13.10	18.41	0.362	-36.24	21.78
RV-Mom/Contrarian	17.88	19.51	0.19	10.97	0.916	1.08	54.99	13.05	18.72	0.325	-35.38	49.34
	Panel C: Momentum Volatility Crash Indicator											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
RV	15.48	19.65	-1.48	12.38	0.788	-	63.98	14.81	22.29	0.242	-49.34	19.29
CVaR HS	18.36	18.57	-0.70	6.03	0.989	**5.19**	52.42	13.15	18.37	0.350	-26.17	21.78
CVaR-Skt-unc	18.78	18.28	-0.62	5.84	1.027	**5.74**	40.97	12.62	17.55	0.458	-25.98	21.78
CVaR-GARCH-SRTR	19.24	18.52	-0.74	6.79	1.039	**6.18**	46.60	12.82	17.55	0.413	-35.48	21.78
CVaR-GJR-SRTR	19.16	18.59	-0.77	7.06	1.031	**6.02**	47.92	12.90	17.78	0.400	-36.76	21.78
CVaR-GARCH-Skt	19.03	18.52	-0.66	5.97	1.027	**5.73**	46.10	12.98	17.86	0.413	-30.55	21.78
CVaR-GJR-Skt	19.14	18.40	-0.69	6.37	1.040	**6.05**	50.34	12.74	17.56	0.380	-32.36	21.78
CVaR-GARCH-FHS	18.80	18.82	-0.77	6.55	0.999	**5.44**	52.75	13.33	18.34	0.356	-34.86	21.78
CVaR-GJR-FHS	18.96	18.66	-0.81	7.09	1.016	**5.57**	56.54	13.03	18.00	0.335	-36.24	21.78
RV-Mom/Contrarian	18.81	19.45	0.61	10.59	0.967	1.34	43.30	13.20	19.12	0.434	-22.81	49.34

tion on the market and momentum portfolio delivers the best risk-return profile. Thus, as before, all strategies that switch between volatility and CVaR targeting exhibit very high values of the Jobson and Korkie (1981) test statistic, whereas the strategy that switches to the contrarian portfolio does not exhibit a significantly higher Sharpe Ratio except for one crash indicator. In total, Tables IX and X show that switching between volatility and CVaR targeting can significantly enhance the risk-return of the momentum portfolio and our switching approach is surprisingly robust to the choice of the crash indicator.

To further demonstrate the differences between the different approaches, Figure VII shows

Table X. Performance Results of the Switching Strategies: Combined Indicators
This table shows performance results for the RV managed momentum portfolio and the switching strategies. Panel A shows results for the crash indicator δ_t that equals one if past market volatility is high and past market return is negative. Panel B shows results for the crash indicator δ_t that equals one if past market volatility is high or past market return is negative. Panel C shows results for the crash indicator δ_t that equals one if expected momentum volatility is high or past market return is negative. The description of the columns is given in Table VI.

	Panel A: Market Volatility and Market Return Crash Indicator											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
RV	15.48	19.65	-1.48	12.38	0.788	-	63.98	14.81	22.29	0.242	-49.34	19.29
CVaR HS	18.48	18.59	-0.65	5.62	0.994	**5.45**	50.91	13.19	18.33	0.363	-25.98	21.78
CVaR-Skt-unc	18.81	18.34	-0.58	5.50	1.026	**5.87**	38.92	12.71	17.57	0.483	-25.98	21.78
CVaR-GARCH-SRTR	18.91	18.77	-0.77	6.67	1.007	**5.88**	46.47	13.22	18.13	0.407	-35.48	21.78
CVaR-GJR-SRTR	18.76	18.86	-0.81	6.92	0.995	**5.72**	47.69	13.35	18.39	0.393	-36.76	21.78
CVaR-GARCH-Skt	18.88	18.62	-0.69	5.97	1.014	**5.77**	46.60	13.16	18.11	0.405	-30.55	21.78
CVaR-GJR-Skt	18.84	18.61	-0.72	6.31	1.012	**5.94**	50.61	13.09	18.04	0.372	-32.36	21.78
CVaR-GARCH-FHS	18.54	18.97	-0.81	6.56	0.978	**5.29**	53.94	13.64	18.89	0.344	-34.86	21.78
CVaR-GJR-FHS	18.59	18.92	-0.84	6.97	0.983	**5.39**	57.47	13.49	18.71	0.323	-36.24	21.78
RV-Mom/Contrarian	19.11	19.43	0.17	11.13	0.984	**1.76**	35.42	13.04	18.88	0.539	-35.38	49.34
	Panel B: Market Volatility or Market Return Crash Indicator											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
RV	15.48	19.65	-1.48	12.38	0.788	-	63.98	14.81	22.29	0.242	-49.34	19.29
CVaR HS	17.37	18.01	-0.72	6.54	0.965	**4.43**	50.35	12.58	18.27	0.345	-26.17	21.78
CVaR-Skt-unc	17.63	17.76	-0.63	6.29	0.993	**4.80**	39.98	12.10	17.52	0.441	-25.98	21.78
CVaR-GARCH-SRTR	18.79	18.26	-0.77	7.03	1.029	**5.98**	45.62	12.60	17.39	0.412	-35.48	21.78
CVaR-GJR-SRTR	18.70	18.34	-0.80	7.31	1.019	**5.81**	47.24	12.73	17.66	0.396	-36.76	21.78
CVaR-GARCH-Skt	18.45	18.13	-0.68	6.30	1.017	**5.55**	44.25	12.62	17.68	0.417	-30.55	21.78
CVaR-GJR-Skt	18.26	17.93	-0.71	6.82	1.018	**5.53**	48.12	12.39	17.51	0.380	-32.36	21.78
CVaR-GARCH-FHS	18.22	18.45	-0.81	6.93	0.988	**5.18**	51.83	13.03	18.24	0.352	-34.86	21.78
CVaR-GJR-FHS	18.12	18.15	-0.85	7.66	0.998	**5.10**	55.13	12.66	17.90	0.329	-36.24	21.78
RV-Mom/Contrarian	15.83	19.64	0.61	10.36	0.806	0.13	53.41	12.96	18.23	0.296	-22.81	49.34
	Panel C: Momentum Volatility or Market Return Crash Indicator											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
RV	15.48	19.65	-1.48	12.38	0.788	-	63.98	14.81	22.29	0.242	-49.34	19.29
CVaR HS	18.18	17.96	-0.67	6.41	1.012	**5.52**	42.53	12.37	17.69	0.427	-26.17	21.78
CVaR-Skt-unc	18.56	17.66	-0.57	6.18	1.051	**6.03**	33.94	11.80	16.79	0.547	-25.98	21.78
CVaR-GARCH-SRTR	19.17	18.17	-0.72	6.97	1.055	**6.58**	39.85	12.42	17.06	0.481	-35.48	21.78
CVaR-GJR-SRTR	19.05	18.26	-0.75	7.26	1.043	**6.34**	41.83	12.54	17.35	0.455	-36.76	21.78
CVaR-GARCH-Skt	18.95	18.04	-0.63	6.21	1.050	**6.24**	41.33	12.42	17.26	0.458	-30.55	21.78
CVaR-GJR-Skt	18.95	17.87	-0.65	6.70	1.060	**6.43**	42.18	12.13	16.94	0.449	-32.36	21.78
CVaR-GARCH-FHS	18.74	18.40	-0.76	6.83	1.018	**5.94**	45.92	12.87	17.88	0.408	-34.86	21.78
CVaR-GJR-FHS	18.78	18.19	-0.80	7.47	1.032	**5.91**	49.63	12.52	17.52	0.378	-36.24	21.78
RV-Mom/Contrarian	17.20	19.56	0.67	10.40	0.879	0.64	49.08	13.04	18.53	0.350	-22.81	49.34

the cumulative performance of the momentum portfolio, the RV managed portfolio and a portfolio that switches between volatility and CVaR targeting for a one dollar investment. As in Daniel and Moskowitz (2016, Fig. 6), we rescale all strategies to the same annualized volatility of 19%. Further, since the momentum strategies are zero-investment strategies, we follow Daniel and Moskowitz (2016, Appendix A.1) and add the risk-free rate to the portfolio return (see also Jacobs et al. (2015)). Investing one dollar in the momentum portfolio results in a portfolio value of $1{,}143.15$\$ over the whole period. Invested in the RV managed strategy, the investor would increase the portfolio value to $4{,}274{,}327.59$\$. However, if the investor would have invested one dollar in the switching strategy, the portfolio value would increase to even 198,152,426.83$ at the end of the period.[134] As can be seen in the performance chart, this outperformance is mainly driven by mitigating the drawdown periods while capturing the upside potential.

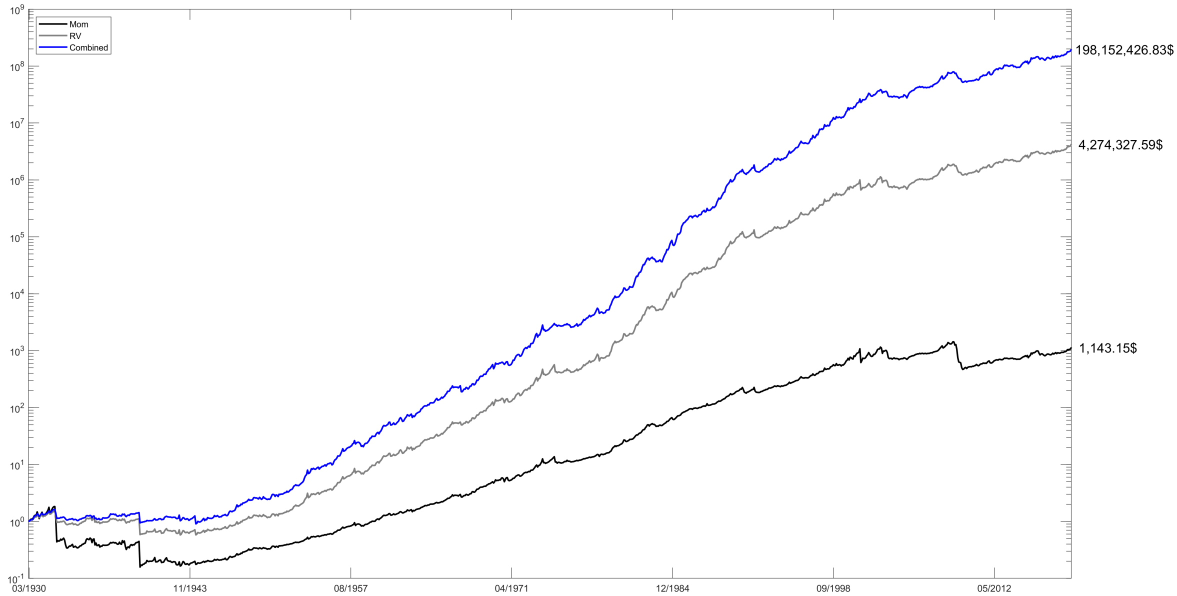

Figure VII. Performance of Risk Targeting. This figure plots the cumulative return of a one dollar investment in the momentum portfolio, the RV managed momentum portfolio and a portfolio that switches between volatility and CVaR targeting over the whole period, where we add the risk-free rate to these portfolios. All strategies are rescaled to the same volatility.

To further illustrate the drawdown reduction ability of the switching approach, Table XI shows the performance of the momentum portfolio, the RV managed strategy and the switching strategies during the crash period 01.01.1938-01.01.1943. Interestingly, compared to the CVaR targeting strategies examined in Table VII, the switching strategies exhibit an even higher (less negative) return and are significantly less negatively skewed. Hence, by better mitigating the crash risk and simultaneously capturing the upside potential, switching between volatility and CVaR targeting is superior to strategies that manage only volatility or CVaR, even during a crash period. Interestingly, the skewness and kurtosis of the RV managed strategy is similar to

[134] Jacobs et al. (2015, Fig. 1) also show that the long-term performance of the momentum portfolio can be increased by buying winners that are highly negatively skewed and selling losers that are less negatively or positively skewed. This strategy exhibits a significantly higher performance than the standard momentum strategy. However, this strategy also takes significantly more risk, especially left tail risk. In contrast, our strategy that switches between volatility and CVaR targeting achieves a higher performance while simultaneously left tail risk *is reduced*.

the skewness and kurtosis of the non-managed strategy. Hence, managing volatility does not provide an adequate downside risk protection during the crash period. In contrast, the strategies that switch between volatility and CVaR targeting exhibit a higher skewness, lower kurtosis, lower drawdown and higher minimum return, and thus provide a better drawdown protection in a crisis. The best result during the momentum crash is found for the strategy that switches to the contrarian portfolio. However, as seen before, this outperformance is only limited to the crash period and does not hold over long-run.

Table XI. Performance Results of the Switching Strategies: 01.01.1938-01.01.1943.
This table shows performance results of the momentum portfolio, the RV managed momentum portfolio and the switching strategies over the period 01.01.1938-01.01.1943. The description of the columns is given in Table VII.

Model	Return	Volatility	Skew	Kurt	MDD	ΔMDD	VaR	CVaR	Min	Max
Momentum	-34.57	51.38	-3.760	22.430	90.39	-	46.30	95.75	-89.70	22.24
RV	-11.64	28.44	-3.705	22.031	50.27	44.38	20.49	42.28	-49.34	15.03
CVaR HS	-1.97	18.29	-2.140	11.346	23.73	73.74	9.93	20.24	-23.73	12.88
CVaR-Skt-unc	-1.69	17.54	-1.843	10.013	21.18	76.57	10.61	21.65	-20.74	12.88
CVaR-GARCH-SRTR	-4.04	22.32	-3.135	17.420	35.48	60.75	13.03	26.63	-35.48	12.88
CVaR-GJR-SRTR	-4.27	22.64	-3.303	18.644	36.76	59.34	12.90	26.38	-36.76	12.88
CVaR-GARCH-Skt	-2.63	20.76	-2.644	14.156	30.55	66.20	12.44	25.37	-30.55	12.88
CVaR-GJR-Skt	-2.97	21.09	-2.914	15.987	32.36	64.20	11.25	22.92	-32.36	12.88
CVaR-GARCH-FHS	-3.91	22.21	-3.036	16.752	34.86	61.43	13.67	27.94	-34.86	12.88
CVaR-GJR-FHS	-4.20	22.44	-3.259	18.380	36.24	59.91	12.43	25.37	-36.24	12.88
RV-Mom/Contrarian	12.49	28.17	3.336	21.816	19.41	78.52	12.10	23.42	-19.41	49.34

The findings of Table XI can also be seen in Figure VIII that shows the cumulative performance of the momentum strategy, the RV managed strategy and a strategy that switches between volatility and CVaR targeting during the crash period examined in Table XI. A $1\$$ dollar investment in the momentum portfolio would result in a portfolio of only $0.12\$$ after five years. If the $1\$$ would have been invested in the RV managed strategy instead, the investor would have a portfolio value of at least $0.54\$$, i.e. the investor would have lost about half of the initially invested money. However, invested in the switching strategy, the portfolio value at the end of the crash period would even be $0.82\$$. That is, an investor who invests in the switching strategy would have 583.33% more money than the momentum investor and 51.85% more money than the investor who invests in the RV managed strategy. This result is striking since investors typically have a higher marginal utility of wealth during these crash periods, i.e. they weight losses significantly higher than gains (Min and Kim, 2016). More importantly, avoiding drawdowns is a main driver of a strategy's long-term performance. The momentum investors needs a return of 733.33% to make up for the loss during this crisis. In contrast, the RV managed strategy only needs a return of 85.18%, whereas a return of only 21.95% is sufficient for the switching strategy.

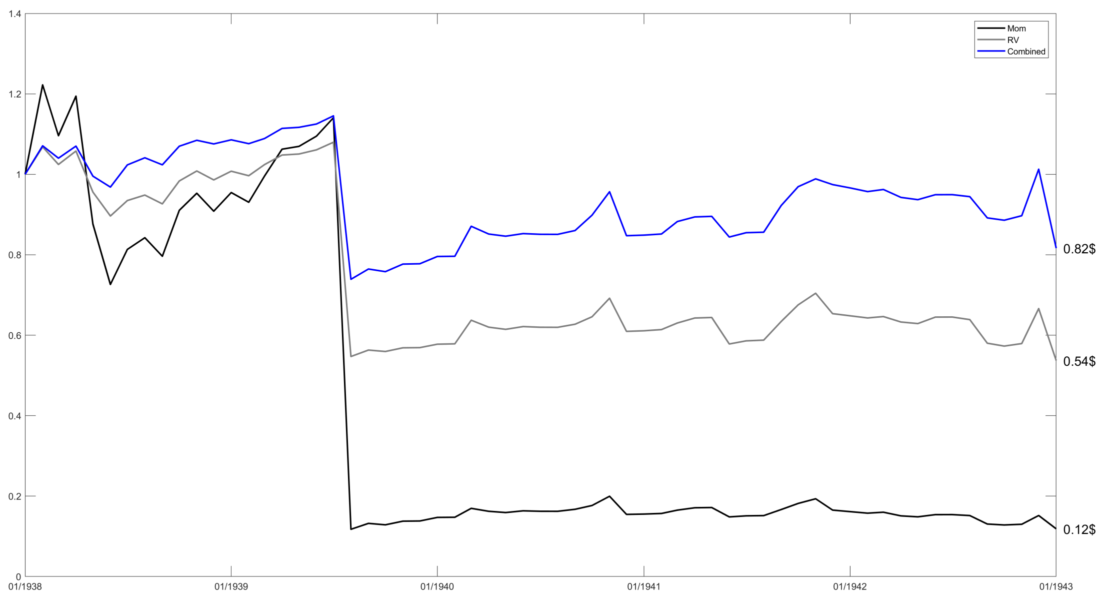

Figure VIII. Performance of Risk Targeting: 01.01.1938-01.01.1943. This figure plots the cumulative return of a one dollar investment in the momentum portfolio, the RV managed momentum portfolio and the portfolio that switches between volatility and CVaR targeting over the period 01.01.1938-01.01.1943.

The switching strategies' ability to mitigate momentum crashes while capturing the upside potential of the momentum strategy can also be seen in Table XII, which shows the five lowest and highest monthly momentum returns along with the corresponding returns of the RV managed and switching strategies in the same months. Since the low momentum returns are significantly higher in magnitude than the high momentum returns, mitigating the crashes is far more important than capturing the high positive returns. Furthermore, as stated above, for achieving a high long-term performance, mitigating negative returns has a higher impact than capturing the upside potential.[135] The RV managed momentum strategy of Barroso and Santa-Clara (2015) successfully reduces momentum crashes while the upside potential is also partly

[135]For example, a return of -50% has to be compensated by a return of 100% to obtain a compounded return of zero. In contrast, returns of -25% and 50% lead to a compounded return of 12.5%. Furthermore, mitigating extremely negative returns also fits well to the loss aversion of most investors.

captured. However, in line with the results of Table XI and Figure VIII, the switching strategies provide a significantly improved crash mitigation. The losses of the switching strategies are all lower compared to the losses of the non-managed and the RV managed momentum portfolio. In contrast, on the days with the highest momentum returns, the switching strategies achieve equally high returns as the RV managed strategy. Hence, the superiority of the switching strategies is mainly influenced by switching to a CVaR managed strategy in a crash period, whereas the upside potential is equally well captured. However, the superiority of the dynamic volatility model can also be seen by the third lowest momentum return, which is not captured by the crash indicator and is managed by volatility. Although both the RV managed strategy and the switching strategy manage the portfolio's volatility in this month, the loss of the switching strategy is about half of the loss of the RV managed strategy. Thus, by using a more accurate volatility model, momentum crashes can be significantly reduced compared to the non-managed and RV managed momentum portfolio. This again highlights the relation between forecasting accuracy and performance gains, especially in down markets. Table XII also demonstrates the good performance of the crash indicator δ_t, since four of the five worst crash months are identified by this indicator, whereas all of the months with an extremely high return are marked as a non-crash month. Thus, despite its simplicity, the crash indicator gives quite reliable indications if next month will be a crash or non-crash month, at least for the most extreme outcomes.

Table XII. Sorted Returns
This table shows the monthly returns of the momentum portfolio, the RV managed portfolio and the switching strategies on months when the momentum portfolio exhibits the five lowest and five highest returns. All entries are given in percent.

	Panel A: Low Returns					Panel B: High Returns				
Momentum	-89.700	-83.250	-59.950	-55.740	-40.360	19.140	19.300	19.340	19.770	22.240
RV	-49.337	-23.732	-35.382	-17.646	-11.875	14.808	8.090	7.695	9.262	6.824
CVaR HS	-23.735	-21.495	-20.222	-14.392	-11.910	12.786	8.066	7.995	8.480	10.211
CVaR-Skt-unc	-20.744	-12.993	-20.222	-9.115	-8.719	12.786	8.066	7.995	8.480	10.211
CVaR-GARCH-SRTR	-35.475	-10.924	-20.222	-12.363	-7.086	12.786	8.066	7.995	8.480	10.211
CVaR-GJR-SRTR	-36.755	-14.859	-20.222	-10.363	-6.433	12.786	8.066	7.995	8.480	10.211
CVaR-GARCH-Skt	-30.554	-14.184	-20.222	-12.870	-8.388	12.786	8.066	7.995	8.480	10.211
CVaR-GJR-Skt	-32.361	-17.846	-20.222	-13.352	-6.432	12.786	8.066	7.995	8.480	10.211
CVaR-GARCH-FHS	-34.864	-17.600	-20.222	-16.076	-9.334	12.786	8.066	7.995	8.480	10.211
CVaR-GJR-FHS	-36.238	-21.598	-20.222	-16.549	-7.166	12.786	8.066	7.995	8.480	10.211
RV-Mom/Contrarian	49.337	23.732	-35.382	17.646	11.875	14.808	8.090	7.695	9.262	6.824

Similar to Barroso and Santa-Clara (2015, Fig. 5), who show the empirical pdf of the momentum and risk-managed momentum strategy, we show in Figure IX the empirical cumulative density function (cdf) for returns smaller than -10%. This figure again highlights that the probability of suffering extreme losses is vastly reduced by the risk targeting strategies. In line with our previous results, the RV managed strategy significantly reduces the probability of obtaining extremely low returns. This does not only hold for the extremely negative returns, but also for moderate negative returns in the range of -10% to -20%. However, this probability can further be reduced by using the strategy that switches between volatility and CVaR targeting.

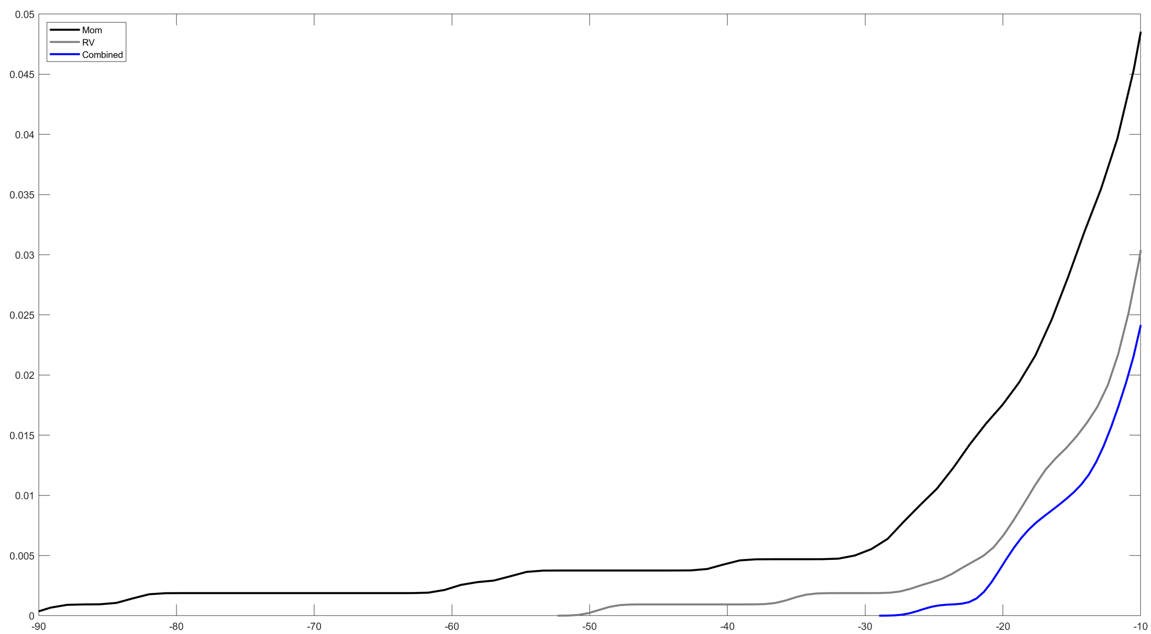

Figure IX. Empirical Cumulative Density Function. This figure plots the cumulative density function (cdf) of the momentum portfolio, RV managed portfolio and the portfolio that switches between volatility and CVaR targeting for returns smaller than -10%.

To further assess how the switching strategies perform in different market states, we follow Jegadeesh and Titman (2001), Hong et al. (2000, Table VIII) and Barroso and Santa-Clara (2015, Table 6) and split the whole sample into four subsamples. By doing this, we obtain different time periods, where the first and last subsamples are dominated by huge momentum crashes. In contrast, the second and third subsamples are characterized by a momentum portfolio that mainly trends upwards without any pronounced crash period. Thus, this subsample analysis points out how the momentum, RV managed and switching strategies behave in different market environments. The results of Table XIII demonstrate that risk targeting significantly improves the risk-adjusted performance of the momentum strategy in all subsamples. This confirms the earlier finding of Barroso and Santa-Clara (2015) that managing the momentum portfolio's risk is superior in crash and calm periods. However, the risk-adjusted performance can further be improved by using a strategy that switches between volatility and CVaR targeting. We find that for all subsamples, the switching strategies produce the highest risk-adjusted performance with statistically significant Sharpe Ratio increases, indicated by high values of the Jobson and Korkie (1981) test statistic. For example, in the first subsample, the momentum portfolio exhibits a negative return. By managing momentum's volatility, the return can be enhanced in order to achieve a positive return while simultaneously the risk is reduced. More precisely, the RV managed strategy achieves a Sharpe Ratio of 0.065, whereas the momentum's Sharpe Ratio is negative. However, by using a switching strategy, the performance can further be enhanced to achieve an even higher return while simultaneously the volatility is reduced. In total, this translates into significantly higher Sharpe Ratios between 0.196 and 0.255 with values of the Jobson and Korkie (1981) test statistic higher than 1.74. Further, by producing a higher skewness, lower kurtosis and higher minimum return, the switching strategies significantly reduce the crash risk compared to both the non-managed and RV managed strategies. The results for the other three subsamples are also in line with this finding. The switching strategies exhibit higher returns than the non-managed and RV managed strategies for all subsamples. Further, in three of four subsamples, the switching strategies also exhibit a significantly lower volatility, whereas left tail risk is massively reduced in all subsamples. Thus, in all subsamples, all switching strategies exhibit significantly higher Sharpe Ratios with Jobson and Korkie (1981) test statistics higher than 1.64. In total, results of Table XIII demonstrate that the superiority of the switching strategies is not only influenced by the momentum crashes but also holds in periods without severe crashes.

As a conclusion, results of this section show that strategies that switch between volatility and CVaR targeting are successful in providing an enhanced risk-return profile compared to the non-managed and RV managed momentum strategy. This holds since switching strategies provide a superior crash protection while simultaneously the upside potential is captured. Further,

Table XIII. Performance Results of Risk Targeting in Different Subsamples
This table shows the performance of the momentum portfolio, the RV managed strategy and the switching strategies in four subsamples. The description of the columns is given in Table VI.

	Panel A: 01.03.1930-30.04.1952											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
Momentum	-13.23	39.47	-4.00	28.83	-0.335	-4.82	99.31	33.71	68.65	-0.133	-89.70	22.24
RV	1.28	19.72	-3.26	25.67	0.065	-	63.98	16.46	27.37	0.020	-49.34	15.03
CVaR HS	3.46	16.92	-1.84	10.64	0.204	**1.74**	50.35	14.77	27.05	0.069	-25.98	12.88
CVaR-Skt-unc	4.03	15.89	-1.62	10.43	0.253	**2.01**	39.98	13.82	25.92	0.101	-25.98	12.88
CVaR-GARCH-SRTR	4.23	17.57	-2.31	15.54	0.241	**2.65**	45.62	13.85	23.39	0.093	-35.48	12.88
CVaR-GJR-SRTR	4.05	17.79	-2.39	16.27	0.228	**2.60**	47.24	14.05	24.01	0.086	-36.76	12.88
CVaR-GARCH-Skt	4.42	17.29	-1.97	12.24	0.255	**2.77**	44.25	14.06	23.82	0.100	-30.55	12.88
CVaR-GJR-Skt	4.09	17.58	-2.12	13.17	0.233	**2.59**	48.12	14.28	24.70	0.085	-32.36	12.88
CVaR-GARCH-FHS	3.99	18.19	-2.19	13.71	0.219	**2.65**	51.83	14.81	25.01	0.077	-34.86	12.88
CVaR-GJR-FHS	3.62	18.51	-2.33	14.61	0.196	**2.32**	55.13	15.02	25.89	0.066	-36.24	12.88
RV-Mom/Contrarian	6.45	19.61	2.43	24.35	0.329	0.79	32.35	11.08	16.90	0.199	-19.69	49.34

	Panel B: 01.05.1952-30.06.1974											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
Momentum	17.02	15.86	-1.10	7.38	1.074	-2.73	25.21	13.68	24.70	0.675	-20.93	17.80
RV	25.18	18.94	-0.73	5.56	1.330	-	23.72	15.17	24.27	1.062	-20.06	19.29
CVaR HS	28.23	19.04	-0.70	4.96	1.483	**2.10**	22.87	13.98	20.08	1.234	-20.72	18.66
CVaR-Skt-unc	28.16	19.14	-0.72	4.92	1.471	**2.00**	22.87	14.14	20.28	1.231	-20.72	18.66
CVaR-GARCH-SRTR	28.56	19.22	-0.71	4.84	1.486	**2.20**	22.87	14.16	20.11	1.249	-20.72	18.66
CVaR-GJR-SRTR	28.71	19.10	-0.71	4.90	1.503	**2.41**	22.87	14.12	20.14	1.255	-20.72	18.66
CVaR-GARCH-Skt	28.21	19.13	-0.72	4.95	1.475	**1.95**	22.87	14.09	20.21	1.233	-20.72	18.66
CVaR-GJR-Skt	28.22	18.99	-0.70	4.99	1.486	**2.03**	22.87	13.92	20.05	1.234	-20.72	18.66
CVaR-GARCH-FHS	28.27	19.04	-0.71	5.00	1.485	**2.05**	22.87	14.04	20.26	1.236	-20.72	18.66
CVaR-GJR-FHS	28.42	18.87	-0.69	5.05	1.506	**2.21**	22.87	13.68	19.73	1.243	-20.72	18.66
RV-Mom/Contrarian	25.08	18.95	-0.60	5.36	1.324	-0.04	29.39	14.62	22.56	0.853	-20.06	19.29

	Panel C: 01.07.1974-31.08.1996											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
Momentum	12.32	15.25	-1.75	11.65	0.808	-3.94	34.47	12.73	20.48	0.357	-27.73	12.74
RV	26.14	20.83	-0.80	6.25	1.255	-	33.35	14.39	20.42	0.784	-26.41	18.60
CVaR HS	28.43	19.27	-0.29	4.47	1.476	**2.76**	27.80	12.42	16.42	1.023	-18.54	21.78
CVaR-Skt-unc	28.33	19.59	-0.39	4.79	1.446	**2.48**	30.98	12.88	17.18	0.914	-21.20	21.78
CVaR-GARCH-SRTR	29.52	19.29	-0.22	4.02	1.531	**2.96**	24.98	12.36	15.89	1.182	-18.54	21.78
CVaR-GJR-SRTR	29.66	19.49	-0.21	4.00	1.522	**2.90**	28.47	12.51	16.14	1.042	-18.54	21.78
CVaR-GARCH-Skt	29.55	19.19	-0.22	4.18	1.539	**2.81**	25.37	12.20	15.86	1.165	-18.54	21.78
CVaR-GJR-Skt	29.14	19.16	-0.19	4.16	1.520	**2.70**	24.98	12.05	15.65	1.166	-18.54	21.78
CVaR-GARCH-FHS	29.10	19.19	-0.21	4.10	1.516	**2.94**	24.98	12.23	15.81	1.165	-18.54	21.78
CVaR-GJR-FHS	29.17	19.00	-0.17	4.17	1.535	**2.88**	24.98	11.83	15.29	1.168	-18.54	21.78
RV-Mom/Contrarian	24.69	20.95	-0.13	5.25	1.179	-0.36	29.50	13.41	18.15	0.837	-22.81	26.41

	Panel D: 01.09.1996-31.12.2018											
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	VaR	CVaR	Calmar	Min	Max
Momentum	3.01	26.88	-2.83	20.02	0.112	-5.36	82.24	22.62	40.64	0.037	-59.95	19.30
RV	11.23	18.46	-1.35	11.22	0.609	-	42.25	13.41	18.74	0.266	-35.38	17.38
CVaR HS	11.42	15.83	-0.67	7.84	0.721	1.36	37.57	10.85	15.52	0.304	-26.17	14.54
CVaR-Skt-unc	11.91	15.25	-0.49	7.48	0.781	**1.94**	28.43	9.96	14.17	0.419	-24.60	14.54
CVaR-GARCH-SRTR	14.74	16.11	-0.20	3.90	0.915	**3.34**	36.29	10.62	13.66	0.406	-13.74	14.54
CVaR-GJR-SRTR	14.33	16.13	-0.24	3.98	0.889	**3.02**	38.50	10.80	13.97	0.372	-13.96	14.54
CVaR-GARCH-Skt	13.52	16.05	-0.20	4.10	0.842	**2.71**	41.33	10.95	14.46	0.327	-15.46	14.54
CVaR-GJR-Skt	13.52	15.00	-0.02	4.00	0.901	**3.16**	33.04	9.89	12.90	0.409	-13.21	14.54
CVaR-GARCH-FHS	13.45	16.60	-0.35	4.17	0.810	**2.26**	45.59	11.63	15.31	0.295	-15.39	14.54
CVaR-GJR-FHS	13.28	15.27	-0.18	4.10	0.869	**2.65**	37.29	10.52	13.77	0.356	-14.22	14.54
RV-Mom/Contrarian	8.46	18.59	0.84	9.45	0.455	-0.41	53.41	12.69	17.04	0.158	-17.38	35.38

advanced volatility models also generate a sizeable value for an investor who manages the risk of the momentum portfolio. This result again demonstrates the relation between forecasting accuracy and performance gains, and thus supports the findings of Bollerslev et al. (2018). In line with Table V, the more accurate volatility and switching strategies also provide an enhanced risk-return profile compared to the less accurate RV model.

2.6.5 The Economic Value of Risk Targeting

The previous section demonstrates that strategies that switch between volatility and CVaR targeting provide an enhanced risk-return profile compared to non-managed and RV managed momentum portfolios. So far, our main conclusion were made by using the Sharpe Ratio as measure to assess the risk-adjusted performance. However, the high negative skewness and high kurtosis of the momentum strategy – of both the non-managed and RV managed strategy – make this strategy unappealing for investors who have preferences for higher moments. Investors typically prefer portfolios that are less left-skewed and less fat-tailed over portfolio with a high left tail risk (Harvey and Siddique, 2000, Kraus and Litzenberger, 1976, Scott and Horvath, 1980). Thus, higher order preferences should be incorporated when a strategy's performance is evaluated (Schneider et al., 2020). This observation is not regarded by the Sharpe Ratio, which assumes that returns are normally distributed (Amin and Kat, 2003).[136] Further, standard evaluation models, like the Sharpe Ratio or unconditional alpha, do not capture the investors' risk aversion (Chabot et al., 2014).[137] In particular, the Sharpe Ratio is an unconditional performance measure that does not account for the time-varying risk an investor is faced over time.[138] Further, most investors are loss-averse, i.e. they weight losses higher than gains of the same magnitude (Aït-Sahalia and Brandt, 2001, Benartzi and Thaler, 1995). In particular,

[136]For that reason, to better assess the performance of hedge funds, Amin and Kat (2003) develop a performance evaluation measure that does not need any distributional assumptions. The approach of Amin and Kat (2003) assumes that investors are only interested in the end-of-period wealth and not in the intermediate portfolio value. This assumption is typically unproblematic for hedge fund investors as argued by the authors, but is not valid for momentum investors.

[137]For example, Amin and Kat (2003, p. 253) write: "The fact that an investment offers a superior risk-return profile does not automatically mean that investors should buy into it as it may not fit their preferences".

[138]For example, Han (2005, p. 246) write: "However, the Sharpe Ratio does not take into account time-varying conditional volatility because the sample [standard deviation] overestimates the conditional risk an investor faces when she follows dynamic strategies. Consequently, the realized Sharpe ratio underestimates the performance of dynamic strategies." Similarly, Marquering and Verbeek (2004, p. 419-421) state: "It is important to realize that the Sharpe ratio does not appropriately take into account time-varying volatility. The risk of the dynamic strategies is typically overestimated by the sample standard deviation, particularly in the presence of volatility timing, because the ex post (unconditional) standard deviation is an inappropriate measure for the (conditional) risk an investor was facing at each point in time. This indicates a potentially severe disadvantage of the use of Sharpe ratios to evaluate dynamic strategies." Generally, unconditional performance measures are not suitable for strategies that time volatility (Boguth et al., 2011, Cederburg and O'Doherty, 2016).

loss aversion is much more successful in explaining the low fraction of retail investors' wealth invested in stocks, and thus is a more realistic choice than mean-variance preferences, as it is assumed for the Sharpe Ratio (Benartzi and Thaler, 1995, Berkelaar et al., 2004). Moreover, momentum investors are typically leveraged and are therefore sensible to drawdowns, which is also not reflected by the Sharpe Ratio (Chabot et al., 2014). Regarding the loss aversion is especially important for momentum investors, since momentum crashes typically occur in periods when investors have a high marginal utility of wealth as shown by Min and Kim (2016). Hence, a measure that explicitly accounts for the severity of losses, especially in crash periods, should be used. Further, by construction, timing volatility instead of timing downside risk should lead to a higher Sharpe Ratio (Jondeau and Rockinger, 2012, p. 108). Therefore, other performance measures that take non-normalities into account should be considered (Dreyer and Hubrich, 2019). To account for these facts, we next assess the economic value of the switching strategies for three different types of investors. The economic value is defined as the annualized percentage fee an investor is willing to pay to switch from a certain benchmark strategy to another strategy, where we use the RV managed momentum strategy of Barroso and Santa-Clara (2015) as the benchmark.

As first method to assess the economic value of risk targeting, we follow the approach of Fleming et al. (2001), Fleming et al. (2003), Han (2005) and Kirby and Ostdiek (2012) and assume that the investor's true utility function can be approximated by quadratic utility. Under this approximation, the investor's realized utility in month t is given by

$$U_{MV}(R_{t,a}) = W_{t-1}(1 + R_{t,a}) - \frac{1}{2}\gamma_{abs}W_{t-1}^2(1 + R_{t,a})^2, \tag{2.6.7}$$

where γ_{abs} is the investor's absolute risk aversion, W_{t-1} denotes the investor's wealth in month $t-1$ and $R_{t,a}$ denotes the month t return of strategy a. Since this investor only has preferences for the first two moments, we call an investor with this utility a mean-variance investor.[139] By assuming that the investor's relative risk aversion $\gamma = \gamma_{abs}W_{t-1}/(1-\gamma_{abs}W_{t-1})$ is constant over

[139]Fleming et al. (2001, p. 334) state that "[t]his measure is based on the close relation between mean-variance analysis and quadratic utility." See also Marquering and Verbeek (2004), Aït-Sahalia and Brandt (2001), DeMiguel et al. (2009b) and Bollerslev et al. (2018) who use a similar performance measure that is solely based on the mean, variance and risk aversion.

time, Equation (2.6.7) can be rewritten as

$$U_{MV}(R_{t,a}) = W_{t-1}\left((1+R_{t,a}) - \frac{\gamma}{2(1+\gamma)}(1+R_{t,a})^2\right), \tag{2.6.8}$$

where we choose values for γ of $2, 5, 10$ and 15 that are similar to the choices of previous studies. These choices of risk aversion are also in line with the finding of Rosenberg and Engle (2002) using options data. The economic value of strategy a is then given by the percentage fee Δ_{MV} the investor with utility in Equation (2.6.8) is willing to pay to switch from the RV managed strategy to the strategy a. The fee Δ_{MV} is calculated by equating the expected utilities

$$\mathbb{E}(U_{MV}(R_{t,a} - \Delta_{MV})) = \mathbb{E}(U_{MV}(R_{t,RV})), \tag{2.6.9}$$

where $R_{t,RV}$ denotes the return of the RV managed strategy.[140] We estimate the expected utility in Equation (2.6.9) by the average realized utility and calculate Δ_{MV} by solving

$$\overline{U}_{MV}\left(R_{1,a} - \Delta_{MV}, ..., R_{T,a} - \Delta_{MV}\right) = \overline{U}_{MV}\left(R_{1,RV}, ..., R_{T,RV}\right), \tag{2.6.10}$$

where $\overline{U}_{MV}(R_1, ..., R_T) = \sum_{t=1}^{T}(1+R_t) - \frac{\gamma}{2(1+\gamma)}(1+R_t)^2$.[141]

As stated above, investors typically have preferences for moments higher than the second moment. In particular, most investors dislike a negative skewness and high kurtosis. A common utility that reflects this finding is the constant relative risk aversion (CRRA), which is used by Aït-Sahalia and Brandt (2001), Ghysels et al. (2016), Bali et al. (2009), Dreyer and Hubrich (2019) and Guidolin and Timmermann (2008). The authors show that, for $\gamma > 1$, a CRRA investor dislikes a negative skewness and high kurtosis, which is in line with the common finding of Kraus and Litzenberger (1976) and Scott and Horvath (1980). Furthermore, Guidolin and Timmermann (2008) find that the portfolio selection under CRRA utility is mainly driven by

[140] An alternative to calculating the economic value by equating expected utilities would be to use the certainty equivalent value (CEV), i.e. the sure return that provides the same utility as achieved by the dynamic trading strategy (Ghysels et al., 2016, Footnote 27). The CEV was used by Ghysels et al. (2016), Aït-Sahalia and Brandt (2001), Guidolin and Timmermann (2008), Cederburg et al. (2020), Dreyer and Hubrich (2019), Moreira and Muir (2017) and Moreira and Muir (2019). However, Jondeau and Rockinger (2012) find similar results of calculating the CEV and the approach used here. Further, another possibility would be to evaluate the performance using the whole distribution as done by Amin and Kat (2003).

[141] Aït-Sahalia and Brandt (2001), Marquering and Verbeek (2004) and Bollerslev et al. (2018) use a quite similar approach for an investor who only has preferences for return and variance. In that case, the investor's expected utility is given by $\mathbb{E}(U_{MV}(R_t)) = \mathbb{E}(W_{t-1}(1+R_t)) - \frac{\gamma_{abs}}{2}\text{var}(W_{t-1}(1+R_t)) = W_{t-1}\left(1 + \mathbb{E}(R_t) - \frac{\gamma}{2}\text{var}(R_t)\right)$, where $\gamma = \gamma_{abs}W_{t-1}$ is the investor's relative risk aversion.

preferences for the first four moments. The CRRA utility is given by

$$U_{CRRA}(R_{t,a}) = \begin{cases} \frac{(1+R_{t,a})^{(1-\gamma)}}{1-\gamma}, & \text{if } \gamma > 1 \\ \ln(1+R_{t,a}), & \text{if } \gamma = 1, \end{cases} \tag{2.6.11}$$

where we use the same values for γ as above. Thus, the utility in Equation (2.6.11) simplifies to $U_{CRRA}(R_{t,a}) = \frac{(1+R_{t,a})^{(1-\gamma)}}{1-\gamma}$. Similar to the case above, we follow Jondeau and Rockinger (2012) and calculate the economic value for an investor with CRRA utility by equating the expected utilities

$$\mathbb{E}(U_{CRRA}(R_{t,a} - \Delta_{CRRA})) = \mathbb{E}(U_{CRRA}(R_{t,RV})), \tag{2.6.12}$$

which is again estimated by the average realized utility. The percentage fee Δ_{CRRA} is then calculated by solving

$$\overline{U}_{CRRA}(R_{1,a} - \Delta_{CRRA}, ..., R_{T,a} - \Delta_{CRRA}) = \overline{U}_{CRRA}(R_{1,RV}, ..., R_{T,RV}), \tag{2.6.13}$$

where $\overline{U}_{CRRA}(R_1, ..., R_T) = \sum_{t=1}^{T} \frac{(1+R_t)^{(1-\gamma)}}{1-\gamma}$.

To account for the loss aversion of investors, we use a third utility function that gives more weight on negative returns. We follow Aït-Sahalia and Brandt (2001) and Benartzi and Thaler (1995) and choose the following utility function

$$U_{LA}(R_{t,a}) = \begin{cases} (R_{t,a})^b, & \text{if } R_{t,a} \geqslant 0 \\ -l(-R_{t,a})^b, & \text{if } R_{t,a} < 0, \end{cases} \tag{2.6.14}$$

where we choose values of 0.8 and 1 for b as well as 2 and 3 for l (see Aït-Sahalia and Brandt (2001) who also use these parameters).[142] Benartzi and Thaler (1995) also use similar values of l and b. In particular, Berkelaar et al. (2004) estimate the investors' loss aversion based on stock market returns and confirm these levels of loss aversion. The economic value for a loss-averse investor with utility given in Equation (2.6.14) is again calculated by equating the expected utilities

$$\mathbb{E}(U_{LA}(R_{t,a} - \Delta_{LA})) = \mathbb{E}(U_{LA}(R_{t,RV})), \tag{2.6.15}$$

[142] The utility function in Equation (2.6.14) could also be modified in several directions. For example, as in Berkelaar et al. (2004, Eq. (8)), two different values for b could be chosen, depending on the sign of the return. Further, as in Berkelaar et al. (2004, Eq. (9)), a CRRA like utility function, where investors treat losses and gains differently, could be used. Moreover, instead of choosing a reference return of zero, a dynamic reference point that depends on the investor's current wealth could be used (Berkelaar et al., 2004). Alternatively, the risk-free rate could be used as reference point. However, since the momentum strategy is a zero-investment strategy, we use a reference point of zero.

which is again estimated by the average realized utility

$$\overline{U}_{LA}\left(R_{1,a}-\Delta_{LA},...,R_{T,a}-\Delta_{LA}\right)=\overline{U}_{LA}\left(R_{1,RV},...,R_{T,RV}\right), \tag{2.6.16}$$

where $\overline{U}_{LA}(R_1,...,R_T)=\sum_{t=1}^{T}(R_t)^b\cdot\mathbb{1}_{\{R_t\geqslant 0\}}-l(-R_t)^b\cdot\mathbb{1}_{\{R_t<0\}}$.[143] Berkelaar et al. (2004) state that the optimal portfolio of a loss-averse investor is similar to the portfolio choice of a CRRA investor with a VaR constraint as examined in Basak and Shapiro (2001). Thus, a loss-averse investor is typically more conservative than a CRRA investor. Generally, loss aversion seems to be more realistic, since the asset allocation of real investors are more in line with the asset allocation found for loss aversion. In contrast, the equity exposure found for the mean-variance or CRRA framework is typically higher than the equity exposure of real investors.

Table XIV shows the economic value of the different switching strategies in terms of an annualized fee an investor is willing to pay to switch away from the RV managed strategy of Barroso and Santa-Clara (2015) to one of the switching strategies. Besides the values of γ, b and l as stated above, we additionally calculate the economic value for time-varying parameters $\gamma_t^{switch}=15\cdot\delta_t+2\cdot(1-\delta_t)$, $b_t^{switch}=1\cdot\delta_t+0.8\cdot(1-\delta_t)$ and $l_t^{switch}=3\cdot\delta_t+2\cdot(1-\delta_t)$. The rational behind this is the observation of Min and Kim (2016) who find that momentum crashes, indicated by $\delta_t=1$, typically occur in periods when investors are more risk-averse and more concerned about losses. Adrian and Rosenberg (2008, Eq. (2)) also use a time-varying risk-aversion parameter. Further, besides showing the economic value of the switching strategies over the whole period, we additionally show the economic value for the crash period from 01.01.1938 to 01.01.1943.

Panel A of Table XIV shows the annualized percentage fee a mean-variance investor is willing to pay to switch away from the RV managed strategy to one of the switching strategies. Regardless of the strategy and risk aversion, a mean-variance investor is always willing to pay a positive fee to switch away from the RV managed strategy. Further, since the strategies that switch between volatility and CVaR targeting successfully reduce the volatility without sacrific-

[143] We calculate the average realized utility in Equation (2.6.16) by using the empirical return distribution. Another possibility would be to use distorted probabilities as explained in Benartzi and Thaler (1995) and Aït-Sahalia and Brandt (2001, Sec. 7). However, Benartzi and Thaler (1995) find that accounting for loss aversion, i.e. a value of $l>1$, is more important than using distorted probabilities. Berkelaar et al. (2004) also do not use distorted probabilities.

ing returns, the percentage fee Δ_{MV} increases with the level of risk aversion γ. In contrast, the willingness to pay to switch to the strategy that switches between the momentum and contrarian portfolio is only low. As expected, during the crash period, the willingness of a mean-variance investor to pay for the switching strategies significantly increases. This holds especially for the strategy that switches between the momentum and contrarian portfolio. This finding is in line with Table XI since switching to the contrarian strategy during a momentum crash produces high returns, whereas the other strategies exhibit negative returns. However, the outperformance of this strategy only holds in the crash period and not over the whole sample, since switching to the contrarian strategy suffers from several false signals of the crash indicator δ_t. Thus, results of the mean-variance investors are in line with the findings of the Sharpe Ratio and demonstrate the value of our switching strategy.

Panel B of Table XIV shows the economic value of the switching strategies for an investor with CRRA utility.[144] Over the whole period, the main findings are similar to the case of the mean-variance investor, but the willingness to pay for the switching strategies becomes higher for high levels of risk aversion. This results since the switching strategies vastly reduce left tail risk by exhibiting a higher skewness and lower kurtosis while simultaneously exhibiting a higher return than the RV managed strategy, as can be seen in Tables IX and X. As stated above, the CRRA framework considers that investors are left tail risk averse (Dreyer and Hubrich, 2019), which leads to economically high utility gains of our switching strategies. Ghysels et al. (2016) also find very high utility gains of a CRRA investor with a risk aversion of $\gamma = 5$. For example, the authors find that this investor would pay about 6% per year to switch from a mean-variance optimized portfolio to a mean-variance-skewness optimized portfolio. The highest fees in Table XIV are found for the risk aversion parameter that is high in crash periods and low in non-crash periods. Interestingly, the economic value during the crash period is slightly lower than the economic value over the whole sample. However, this finding is also in line with Table XI, since the reduction in the negative skewness and kurtosis of the switching strategies is only small during the crash period. In contrast, the economic value of the strategy that switches

[144]To better assess the influence of moments higher than volatility and since the strategies significantly differ in their achieved level of volatility, we rescale the strategies to an annualized volatility of σ_{target} before calculating the economic value for a CRRA investor.

Table XIV. Economic Value of Risk Targeting

This table shows the economic value of the switching strategies with respect to the RV managed strategy. Panel A shows the results for a mean variance investor with utility function (2.6.8). Panel B shows the results for a CRRA investor with utility function (2.6.11). Panel C shows the results for a loss-averse investor with utility function (2.6.14). The crash period ranges from 01.01.1938 to 01.01.1943. The entries in the table correspond to the annualized percentage fee an investor is willing to pay to switch from the RV managed strategy to a switching strategy.

Model	Whole Period					Crash Period				
Panel A: MV	$\gamma=2$	$\gamma=5$	$\gamma=10$	$\gamma=15$	γ_t^{switch}	$\gamma=2$	$\gamma=5$	$\gamma=10$	$\gamma=15$	γ_t^{switch}
CVaR HS	2.601	3.682	5.694	7.960	3.881	11.456	19.659	32.268	42.887	30.313
CVaR-Skt-unc	2.986	4.249	6.598	9.228	4.139	11.934	20.537	33.743	44.823	31.713
CVaR-GARCH-SRTR	3.430	4.387	6.170	8.187	3.916	9.080	14.459	22.814	30.068	21.039
CVaR-GJR-SRTR	3.313	4.216	5.900	7.809	3.840	8.888	14.049	22.072	29.055	20.313
CVaR-GARCH-Skt	3.254	4.291	6.221	8.400	3.963	10.601	17.155	27.291	35.975	25.261
CVaR-GJR-Skt	3.288	4.429	6.552	8.939	4.074	10.136	16.481	26.299	34.729	24.380
CVaR-GARCH-FHS	3.015	3.825	5.335	7.053	3.688	9.018	14.459	22.907	30.236	21.166
CVaR-GJR-FHS	3.093	4.037	5.796	7.789	3.838	8.678	13.990	22.243	29.412	20.587
RV-Mom/Contrarian	1.342	1.404	1.521	1.658	0.324	20.420	20.541	20.733	20.915	13.765
Panel B: CRRA	$\gamma=2$	$\gamma=5$	$\gamma=10$	$\gamma=15$	γ_t^{switch}	$\gamma=2$	$\gamma=5$	$\gamma=10$	$\gamma=15$	γ_t^{switch}
CVaR HS	2.350	2.657	4.750	13.270	15.818	2.197	2.503	4.516	9.952	15.389
CVaR-Skt-unc	2.734	3.042	5.300	14.113	17.198	2.273	2.734	5.300	12.098	18.945
CVaR-GARCH-SRTR	2.734	3.042	4.907	12.432	14.537	2.044	2.120	2.580	3.970	5.457
CVaR-GJR-SRTR	2.657	2.888	4.672	11.766	13.775	2.044	2.044	2.120	2.580	3.893
CVaR-GARCH-Skt	2.734	3.042	5.142	13.691	16.161	2.427	2.657	3.970	7.523	10.116
CVaR-GJR-Skt	2.811	3.119	5.142	13.270	15.904	2.197	2.350	3.042	5.142	7.362
CVaR-GARCH-FHS	2.350	2.657	4.594	12.265	14.113	1.968	2.120	2.811	4.672	6.247
CVaR-GJR-FHS	2.503	2.811	4.516	11.600	13.606	1.815	1.815	2.044	2.657	4.048
RV-Mom/Contrarian	0.979	1.510	4.516	14.963	15.818	8.569	10.198	17.285	33.508	34.784
Panel C: Loss Aversion	$b=0.8$		$b=1$		b_t^{switch}	$b=0.8$		$b=1$		b_t^{switch}
	$l=2$	$l=3$	$l=2$	$l=3$	l_t^{switch}	$l=2$	$l=3$	$l=2$	$l=3$	l_t^{switch}
CVaR HS	2.170	2.658	3.057	3.704	3.320	7.502	9.835	13.051	16.188	15.874
CVaR-Skt-unc	2.573	3.204	3.642	4.466	3.931	7.472	9.847	13.229	16.364	16.040
CVaR-GARCH-SRTR	2.554	2.792	3.496	3.874	3.454	6.271	7.841	9.971	12.138	11.941
CVaR-GJR-SRTR	2.468	2.692	3.377	3.736	3.344	6.359	7.957	9.965	12.153	11.954
CVaR-GARCH-Skt	2.449	2.750	3.398	3.851	3.436	6.886	8.716	11.409	13.828	13.585
CVaR-GJR-Skt	2.623	3.048	3.629	4.231	3.740	7.041	8.993	11.456	14.025	13.777
CVaR-GARCH-FHS	2.156	2.296	2.960	3.218	2.930	5.983	7.482	9.710	11.854	11.664
CVaR-GJR-FHS	2.412	2.712	3.279	3.724	3.334	6.214	7.919	9.945	12.276	12.078
RV-Mom/Contrarian	-0.305	-0.317	1.413	1.475	1.173	9.115	8.952	18.741	18.102	17.541

between the momentum and contrarian portfolio becomes significantly higher during the crash period. This finding is again in line with Table XI since this strategy achieves a high positive skewness, whereas all other portfolios are highly left skewed.

Panel C of Table XIV shows the results for a loss-averse investor. The economic value for a loss-averse investor is again positive for all strategies that switch between volatility and CVaR targeting, regardless of the values of b and l. In contrast, the strategy that switches between the momentum and the contrarian strategy only provides a positive but low economic value for $b = 1$ and b_t^{switch}. Interestingly, the economic value for a loss-averse investor is quite stable for different values of b and l and different switching strategies. Further, the economic value is significantly higher during the crash period and becomes extremely high for the strategy that switches between the momentum and contrarian portfolio. Interestingly, Table XIV shows that the economic value for the loss-averse investor is lower than the economic value for the mean-variance and CRRA investor. This can be explained by the fact that loss aversion is highly influenced by the evaluation period, whereas the CRRA and mean-variance approach are hardly influenced by the evaluation period (Aït-Sahalia and Brandt, 2001, Benartzi and Thaler, 1995). Rickenberg (2020b) finds a higher economic value for loss-averse investors when the portfolio is evaluated daily. Dreyer and Hubrich (2019) also find that the evaluation period has a high impact on the risk-adjusted performance of a portfolio. Generally, the evaluation period of investors is much smaller than the investors' investment horizon (Benartzi and Thaler, 1995). Thus, in practice, even long-term investors would evaluate their portfolio more frequently, which would lead to a higher economic value for the loss-averse investor than stated in Panel C of Table XIV.

To summarize Table XIV, all three types of investors are willing to pay high fees to switch away from the RV managed strategy of Barroso and Santa-Clara (2015) to a strategy that switches between volatility and CVaR targeting. The strategy that switches between the momentum and the contrarian portfolio works well in the crash period, but is less attractive over the whole sample. In total, results for the economic value further strengthen our earlier findings and demonstrate the good performance of our simple switching approach, which is an appealing and superior alternative to the volatility targeting approach.

Similar to Marquering and Verbeek (2004), we next show in Figure X the five and ten year rolling economic value of a strategy that switches between volatility and CVaR targeting for the mean-variance, CRRA and loss-averse investor. That is, Figure X shows how much an investor with a medium investment horizon is willing to pay to switch away from the RV managed strategy to one of the switching strategies. Using five and ten year rolling windows is appealing, since even long-term investors typically have relatively short evaluation periods (Benartzi and Thaler, 1995). Panel A shows the economic value for the three investors, where we choose values of $\gamma = 10, b = 1$ and $l = 2$ and a rolling window of five years.[145] This plot shows that the economic value of the switching strategy is almost always positive. Furthermore,

[145]Results for other levels of risk aversion and loss aversion were quite similar and are not shown here.

in periods when the RV managed strategy is superior to the switching strategy, the economic loss is only small in magnitude ranging around -2% per year. In contrast, in times when the RV managed strategy is inferior to the switching strategy, especially in times of a momentum crash, the economic value is significantly higher in magnitude ranging up to more than 10% per year. Panel B shows the economic value calculated over an investment horizon of ten years. The results are quite similar to the findings of Panel A, but the economic loss of the switching strategies in times when the RV managed strategy outperforms becomes even smaller. Thus, for investors with an investment horizon of ten years, the switching strategies are superior to the RV managed strategy in almost every market state. Moreira and Muir (2019) show that even investors with longer investment horizons should time short-term volatility. However, results of Figure X show that timing volatility *and* downside risk is even more appealing for investors with investment horizons of several years.

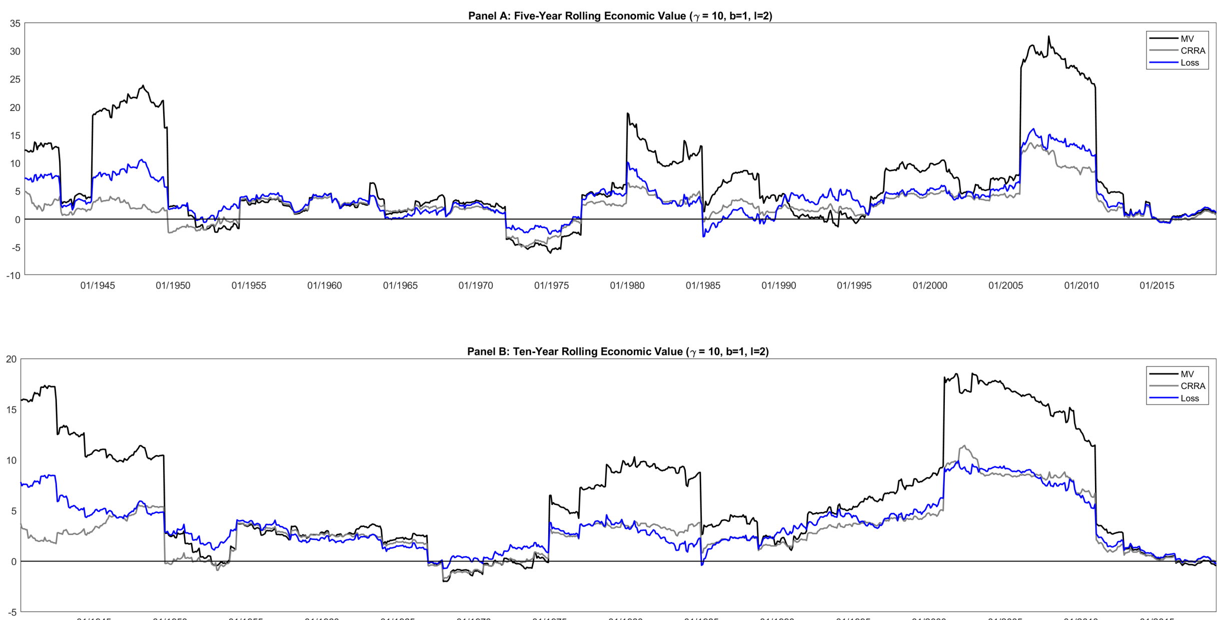

Figure X. Five and Ten Year Rolling Economic Value. This figure shows the five and ten year rolling economic value of a strategy that switches between volatility and CVaR targeting with respect to the RV managed strategy for a mean-variance investor, a CRRA investor and a loss-averse investor with parameters $\gamma = 10, b = 1$ and $l = 2$. Panel A plots the rolling utility for a window of five years. Panel B plots the rolling utility for a window of ten years.

Lastly, we test if the utility gains achieved by the strategies that switch away from the RV

Table XV. Tests for Statistical Significant Utility Gains
This table shows results for the tests of a statistically significant utility differential of the switching strategies compared to the RV managed strategy. The tests are based on the utilities for a risk aversion of $\gamma = 10$ as well as parameters of $b = 1$ and $l = 2$ for the loss-averse investor. The description of the columns is given in Table V.

Panel A: MV	z_{DM}	p^{RC}	p^{SPA}	p^{R}	p^{SQ}	Step-RC	Step-RCst	Step-SPA	Step-SPAst	FDR
RV	-	0.61	0.48	1.88	1.31	-	-	-	-	-
CVaR HS	**2.96**	**64.12**	**15.91**	**46.56**	**49.51**	**1**	**1**	**1**	**1**	**8**
CVaR-Skt-unc	**3.15**	**100.00**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**	**1**	**5**
CVaR-GARCH-SRTR	**3.18**	**79.17**	**30.38**	**71.46**	**75.63**	**1**	**1**	**1**	**1**	**2**
CVaR-GJR-SRTR	**3.10**	**67.05**	**19.10**	**49.11**	**53.98**	**1**	**1**	**1**	**1**	**3**
CVaR-GARCH-Skt	**3.04**	**79.76**	**40.17**	**71.46**	**75.63**	**1**	**1**	**1**	**1**	**6**
CVaR-GJR-Skt	**3.37**	**98.28**	**60.87**	**94.17**	**94.17**	**1**	**1**	**1**	**1**	**1**
CVaR-GARCH-FHS	**2.85**	**46.38**	4.17	**11.35**	**27.29**	**1**	**1**	**1**	**1**	**7**
CVaR-GJR-FHS	**3.18**	**63.86**	**10.19**	**32.88**	**48.53**	**1**	**1**	**1**	**1**	**4**
RV-Mom/Contrarian	**1.75**	2.51	0.94	7.83	9.13	0	0	0	0	**9**
Panel B: CRRA	z_{DM}	p^{RC}	p^{SPA}	p^{R}	p^{SQ}	Step-RC	Step-RCst	Step-SPA	Step-SPAst	FDR
RV	-	8.22	3.43	1.42	5.08	-	-	-	-	-
CVaR HS	**1.79**	**74.07**	**21.32**	**50.31**	**68.38**	**1**	**1**	**1**	**1**	**8**
CVaR-Skt-unc	**1.91**	**100.00**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**	**1**	**7**
CVaR-GARCH-SRTR	**2.30**	**86.98**	**40.45**	**91.76**	**91.64**	**1**	**1**	**1**	**1**	**2**
CVaR-GJR-SRTR	**2.29**	**77.75**	**20.00**	**50.31**	**70.56**	**2**	**1**	**2**	**1**	**1**
CVaR-GARCH-Skt	**2.08**	**95.00**	**80.94**	**98.78**	**98.32**	**1**	**1**	**1**	**1**	**6**
CVaR-GJR-Skt	**2.23**	**98.28**	**75.30**	**98.78**	**98.32**	**1**	**1**	**1**	**1**	**4**
CVaR-GARCH-FHS	**2.07**	**73.00**	**11.17**	**12.20**	**45.99**	0	**1**	0	**1**	**5**
CVaR-GJR-FHS	**2.23**	**72.73**	**19.68**	**33.77**	**58.02**	0	**1**	0	**1**	**3**
RV-Mom/Contrarian	1.58	**56.60**	**39.15**	**91.76**	**91.64**	0	0	0	0	**9**
Panel C: Loss Aversion	z_{DM}	p^{RC}	p^{SPA}	p^{R}	p^{SQ}	Step-RC	Step-RCst	Step-SPA	Step-SPAst	FDR
RV	-	4.75	0.04	0.11	0.64	-	-	-	-	-
CVaR HS	**4.38**	**61.62**	**11.76**	**39.62**	**54.70**	**1**	**1**	**1**	**1**	**7**
CVaR-Skt-unc	**4.76**	**100.00**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**	**1**	**4**
CVaR-GARCH-SRTR	**4.12**	**87.22**	**51.99**	**83.49**	**86.24**	**1**	**1**	**1**	**1**	**1**
CVaR-GJR-SRTR	**4.00**	**77.17**	**40.52**	**83.49**	**81.81**	**1**	**1**	**1**	**1**	**5**
CVaR-GARCH-Skt	**4.11**	**79.82**	**38.91**	**83.49**	**82.81**	**1**	**1**	**1**	**1**	**2**
CVaR-GJR-Skt	**4.86**	**99.33**	**73.35**	**96.26**	**96.26**	**1**	**1**	**1**	**1**	**3**
CVaR-GARCH-FHS	**3.68**	**48.52**	8.35	**24.92**	**31.45**	**1**	**1**	**1**	**1**	**8**
CVaR-GJR-FHS	**4.46**	**70.88**	**11.05**	**39.62**	**68.12**	**1**	**1**	**1**	**1**	**6**
RV-Mom/Contrarian	**1.75**	**18.67**	**11.99**	**24.92**	**31.45**	0	0	0	0	**9**

managed momentum strategy are also statistically significant. To assess the significance, we apply the tests summarized in Appendix D to the utility differential, as also done by Bollerslev et al. (2018) who use the DM-test to test for the significance of the utility increase. A similar approach to test for the statistical significance of utility increases is also used by Kirby and Ostdiek (2012), DeMiguel et al. (2009b, Footnote 18), Taylor (2014) and Cederburg et al. (2020). Whenever a benchmark model is needed, we use the RV managed strategy as benchmark. Results of the tests are given in Table XV, where Panel A shows results for the mean-variance investor with $\gamma = 10$. The DM-test indicates that all models that switch between volatility and CVaR targeting exhibit a statistically higher utility for a mean-variance investor. The utility increase of the strategy that switches between the momentum and the contrarian strategy is also significant at the 10%, but not at the 5% level. By using a test level of 10%, the RC-test only rejects the null-hypothesis of the RV managed strategy and the strategy that switches to the contrarian portfolio. The SPA-test, which typically identifies more inferior models than the RC-tests, rejects one additional model but comes to a similar conclusion. The MCS also only eliminates the RV managed strategy and the strategy that switches to the contrarian portfolio, whereas none of the CVaR switching strategies is eliminated. The stepwise approaches identify all strategies that switch between volatility and CVaR targeting as superior, whereas the strategy that switches to the contrarian is not picked. The FDR approach that targets an FDR^{+} of 10% identifies all switching models as superior to the RV managed strategy, where the contrarian switching strategy is picked in the last step. Thus, results in Panel A clearly show that switching away from the volatility targeting strategy when a momentum crash is likely significantly increases an investor's utility, where the most pronounced utility increases are obtained for the CVaR switching strategies.

Panel B shows results for the CRRA investor with $\gamma = 10$. Results are again similar to the results of Panel A. The switching strategies typically produce statistically significant utility gains in comparison to the RV managed strategy. The RC-test, SPA-test and the MCS give clear evidence against the RV managed strategy. All stepwise tests identify most strategies that switch between volatility and CVaR targeting as superior, whereas the strategy that switches to the contrarian portfolio is not contained in the set of superior models. Again, the FDR approach identifies all switching strategies as superior to the RV managed strategy, where the strategy that switches to the contrarian portfolio is picked in the last step. In total, all tests clearly show that an investor's utility can be significantly increased by using our switching approach instead of the RV managed strategy. Panel C shows results for a loss-averse investor with parameters $b = 1$ and $l = 2$. Results are again in line with Panels A and B. The strategies that switch between volatility and CVaR targeting show statistically significant utility gains. The strategy that switches between the RV managed momentum and contrarian portfolio also performs well, but the tests indicate a superior performance of the strategies that switch between volatility and CVaR targeting. Thus, the findings for the loss-averse investor further strengthen our earlier findings and show that a loss-averse investor benefits from switching between volatility and CVaR targeting.

2.6.6 Spanning Tests

Finally, in order to demonstrate how the non-managed and the different risk-managed momentum strategies are related, we next run time-series regressions of one strategy on the returns of the other strategies. Following Moreira and Muir (2017, Table 1), we regress the returns of the risk-managed momentum strategies on the non-managed momentum portfolio. Further, following Daniel and Moskowitz (2016, Table 8), we additionally regress each strategy on the returns of the other non-managed or risk-managed strategies as well as other factors, like the market or the Fama and French (1993) three factor model. Following the authors, we rescale all strategies to the same level of volatility. Results of these time-series regressions are shown in Table XVI, where we report the annualized alphas with the corresponding t-statistics.[146] In line with earlier studies, we find that the returns of the non-managed momentum portfolio cannot be explained by the CAPM or the three factor model. However, the alpha of the non-managed strategy becomes highly negative when we control for the risk-managed strategies. These negative alphas are even statistically significant with t-statistics of -2.404 and -2.523 for the volatility targeting and switching strategy, respectively. In contrast, confirming the findings of Moreira and Muir (2017) and Daniel and Moskowitz (2016), the volatility managed momentum strategy cannot be explained by the CAPM, the three factor model and the non-managed momentum portfolio. However, when we control for the switching strategy, the alpha of the volatility managed momentum portfolio becomes negative. This alpha is also statistically significant with a t-statistic of -2.761. In contrast, the returns of the switching strategy cannot be explained by the remaining strategies. All alphas of this strategy are economically high and statistically

[146]This time-series regression approach has several disadvantages as shown by Boguth et al. (2011), Cederburg and O'Doherty (2016), Cederburg et al. (2020) and Schneider et al. (2020). For example, the unconditional alphas do not account for volatility timing and higher order preferences of investors. These disadvantages are corrected by the economic value approach examined earlier. Thus, our economic value based findings are more meaningful, but results of Table XVI can be seen as a further robustness check.

significant with t-statistics between 5.003 and 11.562. In particular, with an alpha of 4.936%, the switching strategy cannot be explained by the RV managed strategy. Thus, Table XVI further strengthen our earlier findings that a strategy that switches between volatility and CVaR targeting outperforms the RV managed strategy and is highly valuable for investors.

Table XVI. Spanning Tests: Portfolio Alphas

This table shows portfolio alphas and t-statistics for the non-managed and risk-managed momentum portfolios. The alphas are the annualized and percentage intercepts of regressions of momentum returns on several other portfolios. t-statistics are given in parentheses. Returns are regressed on the CAPM and the Fama and French (1993) three factor model (FF3). The three factor model is further extended by returns of the non-managed and risk-managed momentum strategies. Mom stands for the non-managed momentum strategy, RV stands for the volatility managed momentum strategy, Switch stands for the strategy that switches between volatility and CVaR targeting, whereas Rem means that the remaining two strategies are included. All strategies are rescaled to the same level of volatility before running the regressions. An alpha with a corresponding t-statistic that is higher than 1.64 is given in bold. An alpha with a corresponding t-statistic that is smaller than -1.64 is given in red.

Model	CAPM	FF3	FF3 + Mom	FF3 + RV	FF3 + Switch	FF3 + Rem
Momentum	**9.827**	**12.350**	-	-2.931	-3.731	-1.306
	(6.240)	(7.644)	-	(-2.404)	(-2.523)	(-1.178)
RV	**18.538**	**20.569**	**8.318**	-	-2.224	-0.884
	(9.291)	(10.472)	(6.310)	-	(-2.761)	(-1.594)
Switching	**23.414**	**25.303**	**13.846**	**4.936**	-	**4.387**
	(10.517)	(11.562)	(8.225)	(5.003)	-	(5.081)

To summarize the empirical part, momentum returns do not follow a random walk and momentum returns are highly non-normal with an extremely high crash risk. Based on these observations we find that switching between volatility and CVaR targeting is a promising strategy to mitigate momentum crashes and to heighten the utility of most types of investors. These utility increases are statistically significant and are not limited to certain market environments. In other words, medium-term investors benefit from the switching approach in bull and bear markets. Thus, our switching strategies do not only produce significant higher Sharpe Ratios, as indicated by the Jobson and Korkie (1981) test, but also provide significant utility gains. Furthermore, our switching strategy cannot be explained by the RV managed momentum strategy of Barroso and Santa-Clara (2015), but the RV managed strategy becomes unprofitable, once we control for the switching strategy.

2.7 Conclusion

This paper studies the unconditional and conditional distribution of the momentum strategy and shows that momentum's conditional skewness and kurtosis are highly time-varying, take extreme values and there are periods when these moments may even not exist. We especially show that momentum's conditional skewness is negative almost all of the time and most extreme values or non-existing higher moments occur during momentum crash periods. We further show that the extreme outcomes of conditional skewness and kurtosis arise since the skewness of the winners and losers portfolios moves in opposite directions, whereas the kurtosis of both portfolios comoves. We further show that the random walk hypothesis does not hold for the momentum portfolio, which makes portfolio risk management based on the Realized Volatility (RV) method as done by Barroso and Santa-Clara (2015) and Moreira and Muir (2017) inappropriate.

Based on these observations, we show that momentum's portfolio risk should be better managed by advanced volatility models. In line with the findings of Bollerslev et al. (2018), we show that these advanced volatility models are more accurate than the RV model, which makes these models highly valuable for investors who target a constant level of momentum's volatility. We further show how a constant level of momentum's downside risk can be targeted. Based on several crash indicators, we then develop strategies that manage momentum's volatility in calm periods and momentum's downside risk when a momentum crash is likely. Compared to the RV managed momentum strategy, these strategies exhibit higher returns with less risk, which leads to statistically higher Sharpe Ratios of the switching strategies compared to the Sharpe Ratio of the RV managed strategy. In particular, by switching to downside risk targeting when a momentum crash is likely, these switching strategies significantly reduce momentum's crash risk without sacrificing returns. Hence, these strategies vastly reduce momentum's high left tail risk while the mass in the right tail is increased, which is highly valuable for investors.

To assess the economic value of the switching strategies, we calculate the annual percentage fee an investor is willing to pay to switch from the RV managed strategy to the strategies that switch between volatility and downside risk targeting. We calculate this fee for mean-variance investors, investors with preferences for higher skewness and lower kurtosis as well as for loss-averse investors. We find that the economic value of these switching strategies is high and statistically significant for all three types of investors. In particular, the economic value of switching to a downside risk targeting strategy becomes extremely high during a momentum crash. Finally, we show that our switching strategy cannot be explained by the returns of the RV managed momentum portfolio. In contrast, the RV managed momentum portfolio is unprofitable once we control for the returns of the switching strategy.

Appendix to Chapter 2

A Momentum Strategy: Construction Method and Literature Review

In this section, we shortly present the momentum strategy and review the literature on momentum investing. Several studies demonstrate that the momentum portfolio, i.e. the portfolio that buys past winners and sells past losers produces abnormally high returns, which challenge standard asset pricing models. For the momentum portfolio, assets are pilled into deciles based on their past J months' performance. The decile containing the assets with the lowest past return is called the losers portfolio and the decile containing the assets with the highest past return is called the winners portfolio.[147] The momentum portfolio – also called winner minus losers (WML) portfolio – is then built by being 1\$ long the winners and 1\$ short the losers portfolio (see Jegadeesh and Titman (1993) and Rouwenhorst (1998, p. 269) for example).[148] This portfolio is then held for K months and the process is repeated in the next month. As in Fama and French (1996), we choose $J = 12$, $K = 1$ and use a one month gap in the ranking period, i.e. assets in month t are ranked based on the performance between months $t - 12$ to $t - 2$.[149] Fama

[147] Other definitions of winners and losers are also possible. For example, in order "to place less emphasis on the tails of the performance distribution" Hong et al. (2000, p. 274) define winners as the 30% best performing stocks and losers as the 30% worst performing stocks. Further, instead of only buying and selling the extreme performers, other studies on momentum buy and sell all assets, where an asset is defined as winner (loser) if the stock's performance is higher (lower) than the cross-sectional average performance (Chan et al., 2000, Conrad and Kaul, 1998, Goyal and Jegadeesh, 2017, Lewellen, 2002, Moskowitz and Grinblatt, 1999).

[148] Momentum investing is based on the observation that stocks that performed well in the past will outperform stocks that performed poorly in the past. Therefore, momentum investing is related to trend-following rules and time series momentum, as examined in Moskowitz et al. (2012), Bajgrowicz and Scaillet (2012) and Sullivan et al. (1999). The similarities and differences between cross-sectional momentum and time series momentum are nicely presented in Moskowitz et al. (2012), Kim et al. (2016) and Goyal and Jegadeesh (2017).

[149] The same method is also used by Barroso and Santa-Clara (2015), Carhart (1997), Grundy and Martin (2001) and Daniel and Moskowitz (2016). Jegadeesh and Titman (1993) examine momentum portfolios where the ranking and holding period are equal, i.e. $J = K$, where their most frequently used ranking and holding periods are six months. The momentum portfolio then consists of K overlapping portfolios, where each portfolio obtains a weight of $1/K$ (see also Rouwenhorst (1998) and Moskowitz and Grinblatt (1999, Sec. III.A)). The different construction

and French (1996, Tables VI-VII) show that longer ranking periods do not work well for the momentum strategy. The assets within the winners and losers portfolio can be equally weighted (Chordia and Shivakumar, 2002, Grundy and Martin, 2001, Hong et al., 2000, Jegadeesh and Titman, 1993, 2001, Lesmond et al., 2004), value-weighted (Barroso and Santa-Clara, 2015, Novy-Marx, 2012, Richards, 1997), weighted based on past performance (Chan et al., 2000, Conrad and Kaul, 1998, Goyal and Jegadeesh, 2017, Lewellen, 2002, Moskowitz and Grinblatt, 1999) and volatility weighted (Asness et al., 2013, Goyal and Jegadeesh, 2017). Further, Fama and French (2012) construct a momentum factor using double-sorted portfolios based on size and past return. These momentum strategies have been extensively examined in the academic literature and it has been shown that momentum investing exhibits abnormally high returns for medium-term holding periods K between one month and twelve months. For longer holding periods of more than one year, the momentum effect reverses and the portfolio yields low or even negative returns (see Jegadeesh and Titman (1993, Table VII), Rouwenhorst (1998, Table VI), Jegadeesh and Titman (2001, Fig. 3), Conrad and Kaul (1998) and Richards (1997)). This result also holds when momentum is applied to mutual funds (Carhart, 1997).

The high performance of the momentum strategy has been first shown for the US market by Jegadeesh and Titman (1993). This finding has then been confirmed by several other studies (see Jegadeesh and Titman (2001) for example). Jegadeesh and Titman (1993) find that the momentum strategy earns a highly significant return and the authors show that the high performance holds for several subperiods and for all months except for January (see also Jegadeesh and Titman (2001, Table. 2)). Jegadeesh and Titman (2001) reexamine the momentum strategy for a larger time span and the authors confirm the earlier findings of Jegadeesh and Titman (1993).

methods of Jegadeesh and Titman (1993) and Fama and French (1996) are also compared by Min and Kim (2016). Fama and French (1996) and Min and Kim (2016) show that the performance of both approaches are quite similar. Novy-Marx (2012) also uses a one month holding period and finds that momentum returns are mainly driven by the medium-term past performance of months $t-12$ to $t-7$ and not by the recent past's performance of months $t-6$ to $t-2$ (see also Moskowitz and Grinblatt (1999)). Goyal and Wahal (2015) show that this result holds for US stocks, but is no international phenomenon. Further, the authors find that the results of Novy-Marx (2012) are driven by the negative impact of the $t-2$ return and the positive impact of the $t-12$ return. Hence, the strategy of Jegadeesh and Titman (1993, 2001) that uses the past six months' return as ranking criteria does not capture the valuable medium-term past returns. Similar to Novy-Marx (2012), Grobys et al. (2018) apply different ranking methods to the industry momentum strategy and the authors also use a month holding period. Interestingly, Moskowitz and Grinblatt (1999) show that only the momentum strategy using a 12 months ranking period cannot be explained by industry momentum, whereas momentum strategies based on other ranking periods are explained by industry momentum.

Chabot et al. (2014) show that momentum also works for data ranging from 1867 to 1907, i.e. data that start and end before the data used by Jegadeesh and Titman (1993). More recently, Barroso and Santa-Clara (2015, Table 1) using data that range from 1927 to 2011 find an annualized return of 14.46% and a Sharpe Ratio of 0.53 for the momentum portfolio, compared to an annualized return of 7.35% and a Sharpe Ratio of 0.39 achieved by the market. Rouwenhorst (1998) shows that the high performance of momentum also holds in Europe. Griffin et al. (2003) examine momentum internationally (see Griffin et al. (2003, Table I) for a summary of the used data). Fama and French (2012) find that equity momentum is found almost everywhere except for Japan. Similarly, Asness et al. (2013, Table I) find positive and significant alphas for US, UK and European equities, but a small and insignificant alpha for Japanese equities. Moskowitz and Grinblatt (1999), Lewellen (2002), Du Plessis and Hallerbach (2017), Grobys et al. (2018), Rickenberg (2020c) and Chordia and Shivakumar (2002) show that the momentum strategy also works for industries. Lewellen (2002) and Rickenberg (2020c) show that momentum can also be applied to investment styles. Richards (1997), Bhojraj and Swaminathan (2006), Rickenberg (2020c) and Chan et al. (2000) find momentum for country indices. Asness et al. (2013) find momentum effects in different markets and asset classes including international equities, equity indices, bonds, currencies and commodities. Novy-Marx (2012) finds momentum for international equities, commodities, currencies, industries and investment styles. Further, momentum can also be applied to mutual funds, i.e. funds that performed well in the recent past tend to perform well in subsequent periods (Carhart, 1997). Hence, the momentum effect is found for almost every market and asset class. In particular, the strong presence and interest in the momentum effect is not only limited to the academic literature, since fund managers "show a tendency to buy stocks that have increased in price over the previous quarter" (Jegadeesh and Titman, 1993, p. 66).

The vast performance of the momentum strategy cannot be explained by standard asset pricing models. For example, Fama and French (1996) show that momentum returns cannot be explained by the CAPM or the Fama-French three factor model, whereas many other market anomalies can be explained by the three factor model (see also Barroso (2016, Sec. 3)). Similarly, Jegadeesh and Titman (2001, Table IV) find positive and highly significant alphas of the momentum strategy with respect to the CAPM and the Fama-French three factor model. Furthermore, Barroso and Santa-Clara (2015) find a monthly alpha of 1.75% with respect to the Fama-French three factor model. This result also holds for several subperiods and market capitalizations (see Tables VI–VIII in Jegadeesh and Titman (2001)).[150] Hence, momentum returns cannot be explained by market or factor risk. Fama and French (1996, Sec. VI.D) present several explanations why the Fama-French three factor model fails to capture momentum returns. The observation that momentum returns cannot be explained by standard asset pricing models has led to the development of a four factor model that includes a factor for momentum (Carhart, 1997). Several other possible explanations for the momentum effect have been made in the literature, e.g. behavioral explanations, data snooping and compensation for macroeconomic risks (see Jegadeesh and Titman (1993), Rouwenhorst (1998), Jegadeesh and Titman (2001), Chordia and Shivakumar (2002), Griffin et al. (2003), Hong et al. (2000) and references therein).[151] Since momentum investing also works before and after the initial study of Jegadeesh and Titman (1993) and since momentum investing does not only work for US stocks in a certain time period but is apparent in almost every market, asset class and time-period, data snooping has been eliminated as a possible explanation (see Jegadeesh and Titman (2001) and Asness et al. (2013)). Moreover, in contrast to the findings of Chordia and Shivakumar (2002), Griffin et al. (2003) show that macroeconomic risks fail to explain the high momentum returns (see also Asness et al. (2013)). In contrast, Moskowitz and Grinblatt (1999) show that a substantial part

[150]The alpha of the momentum strategy is not an adequate measure to assess the performance of the momentum strategy, since this measure does not adequately account for the high crash risk of the momentum strategy. For example, Chabot et al. (2014, p. 1) write that "investors' aversion to such large losses may not be adequately captured by standard asset pricing models and the high historical alpha is due to inadequate risk adjustment, and is compensation for exposure to such crash risk." This finding is confirmed by Ruenzi and Weigert (2018) who find that the high returns of the momentum portfolio can be explained as a compensation for the high crash risk of the momentum portfolio. Similarly, Jacobs et al. (2015) find that the high returns of the momentum strategy are compensation for the high negative skewness of the momentum portfolio. Generally, unconditional alphas have several disadvantages as performance measure (Boguth et al., 2011, Cederburg and O'Doherty, 2016, Cederburg et al., 2020, Schneider et al., 2020). We will therefore use more sophisticated performance measures that consider momentum's crash and skewness risk.

[151]See Jegadeesh and Titman (2001, Sec. II) and Jacobs et al. (2015, Sec. 3) for a discussion of several explanations and studies of why the momentum effect exists for medium-term holding periods but reverses for longer holding periods. See Min and Kim (2016, Footnote 2) for a short summary and list of studies on the source of the profitability of momentum and Ruenzi and Weigert (2018) for a short summary and list of studies on the main drivers of momentum returns.

of momentum returns can be explained by industry momentum. Jegadeesh and Titman (2001) find that behavioral explanations are best in explaining the momentum effect, whereas Jacobs et al. (2015) and Ruenzi and Weigert (2018) suggest that momentum returns can be explained by momentum's skewness and crash risk. In total, there does not exist a common explanation for the high profitably of momentum and momentum's returns are still puzzling.

Jegadeesh and Titman (1993), Rouwenhorst (1998) and Jegadeesh and Titman (2001) find that the momentum portfolio typically contains firms with a smaller market capitalization, where the stocks in the losers portfolio are typically smaller than the assets in the winners portfolio. However, the authors show that the momentum effect is not limited to assets with a low market capitalization, but is also found for medium-sized and large-sized firms (see Jegadeesh and Titman (2001, Table 1) for example). Hong et al. (2000, Figure 2) illustrate the relation between firm size and momentum profits and find that momentum works for almost all size deciles. Nevertheless, Rouwenhorst (1998) shows that the momentum effect is more pronounced for small-sized firms (see also Fama and French (2012)). Further, the momentum portfolio typically has a negative or close to zero market beta on average and the assets in the winners and losers portfolios typically have high betas (see Jegadeesh and Titman (1993, Table II), Rouwenhorst (1998, Table 2) and Jegadeesh and Titman (2001, Table III)). Grundy and Martin (2001) show that the beta of the momentum strategy is highly time-varying (see also Barroso (2016) and Martens and Van Oord (2014)). In particular, the beta is positive following periods when the market increased and negative following periods with a negative market return.[152] However, Jegadeesh and Titman (1993) and Rouwenhorst (1998) find that momentum also works when assets are first sorted into high, medium and low beta stocks. Moreover, Rouwenhorst (1998) documents that the average volatility of the assets in the winners and losers portfolios is typically 30% to 40% higher than the average volatility of the assets in the middle

[152]Similarly, Barroso (2016) calculate the beta of the momentum portfolio as the weighted beta of the assets in the momentum portfolio and shows that this beta estimate is highly time-varying and depends on the past market return. See Barroso (2016, Figure 2) for the relation of momentum's beta on the past market return. The relation between the past market return and momentum's beta follows from the construction of the momentum portfolio. For example, in periods of a longer market decline, winners will be stocks with a highly negative beta, whereas loser stocks have a highly positive beta. By buying the winners and selling the losers, momentum's beta will be highly negative when the market's past return is negative. Similarly, when the market trends upwards, momentum's beta will be positive, since the winners have high betas, whereas the losers have low betas in these periods.

part, which are not included in the momentum portfolio. Hence, the momentum strategy typically picks assets that are riskier – measured by market capitalization, market beta or volatility – than the average asset. This results in a high risk of the momentum portfolio as summarized in Section 2.2. The aim of this paper is to manage the high risk of the momentum portfolio.

B Standardized Skewed t Distribution

Christoffersen (2012) shows that for $Z \sim stsk(\eta,\lambda)$ and $z < -\frac{a}{b}$

$$\mathbb{E}\big(Z\cdot\mathbb{1}_{\{Z<z\}}\big) = \frac{c}{b}(1-\lambda)^2\frac{\eta-2}{1-\eta}\left(1+\frac{1}{\eta-2}\left(z^{(-)}\right)^2\right)^{\frac{1-\eta}{2}} - \frac{a}{b}(1-\lambda)F_{st}\left(z^{(-)}|\eta\right) \quad \text{(B.1)}$$

holds, where $z^{(-)} = \frac{bz+a}{1-\lambda}$. This can also be conveniently rewritten as

$$\mathbb{E}\big(Z\cdot\mathbb{1}_{\{Z<z\}}\big) = \frac{(1-\lambda)^2}{b}\left(f_{st}\left(z^{(-)}|\eta\right)\cdot\frac{\eta-2+\left(z^{(-)}\right)^2}{1-\eta} - \frac{a\cdot F_{st}\left(z^{(-)}|\eta\right)}{1-\lambda}\right), \quad \text{(B.2)}$$

where $f_{st}\left(z|\eta\right)$ and $F_{st}\left(z|\eta\right)$ are the pdf and cdf of the standardized skewed t distribution, respectively.

We next derive a formula for $\mathbb{E}\big(Z\cdot\mathbb{1}_{\{Z<z\}}\big)$ for the case $z \geqslant -\frac{a}{b}$, which is not shown in Christoffersen (2012). We use that $\mathbb{E}(Z) = 0$, and hence

$$\mathbb{E}\big(Z\cdot\mathbb{1}_{\{Z<z\}}\big) = -\mathbb{E}\big(Z\cdot\mathbb{1}_{\{Z\geqslant z\}}\big) \quad \text{(B.3)}$$

holds. Using similar arguments as in Christoffersen (2012), we obtain

$$\mathbb{E}\big(Z\cdot\mathbb{1}_{\{Z\geqslant z\}}\big) = \int_z^{+\infty} u\cdot f_{stsk}(u|\eta,\lambda)du = bc\int_z^{+\infty} u\left(1+\frac{1}{\eta-2}\left(\frac{bu+a}{1+\lambda}\right)^2\right)^{-\frac{1+\eta}{2}} du.$$

By a change of variable, $x = \frac{bz+a}{1+\lambda}$, we get

$$\mathbb{E}\big(Z\cdot\mathbb{1}_{\{Z\geqslant z\}}\big) = \frac{c\cdot(1+\lambda)}{b}\int_{\frac{bz+a}{1+\lambda}}^{+\infty} x(1+\lambda)\left(1+\frac{x^2}{\eta-2}\right)^{-\frac{1+\eta}{2}} - a\left(1+\frac{x^2}{\eta-2}\right)^{-\frac{1+\eta}{2}} dx,$$

which yields

$$\mathbb{E}\big(Z\cdot\mathbb{1}_{\{Z\geqslant z\}}\big) = -\frac{c(1+\lambda)^2}{b}\cdot\frac{\eta-2}{1-\eta}\cdot\left(1+\frac{\left(z^{(+)}\right)^2}{\eta-2}\right)^{\frac{1-\eta}{2}} - \frac{a}{b}(1+\lambda)\cdot\left(1-F_{st}\left(z^{(+)}\right)\right),$$

where we define $z^{(+)} = \frac{bz+a}{1+\lambda}$. From $c \cdot \left(1 + \frac{\left(z^{(+)}\right)^2}{\eta-2}\right)^{\frac{1-\eta}{2}} = f_{st}\left(z^{(+)}\right) \cdot \left(1 + \frac{\left(z^{(+)}\right)^2}{\eta-2}\right)$ we obtain

$$\mathbb{E}\big(Z \cdot \mathbb{1}_{\{Z \geqslant z\}}\big) = -\frac{(1+\lambda)^2}{b} \cdot \left(f_{st}\left(z^{(+)}\right) \cdot \frac{\eta - 2 + \left(z^{(+)}\right)^2}{1-\eta} + \frac{a \cdot \left(1 - F_{st}\left(z^{(+)}\right)\right)}{1+\lambda} \right).$$

Therefore, we get

$$\mathbb{E}\big(Z \cdot \mathbb{1}_{\{Z < z\}}\big) = \frac{(1+\lambda)^2}{b} \cdot \left(f_{st}\left(z^{(+)}\right) \cdot \frac{\eta - 2 + \left(z^{(+)}\right)^2}{1-\eta} + \frac{a \cdot \left(1 - F_{st}\left(z^{(+)}\right)\right)}{1+\lambda} \right). \quad \text{(B.4)}$$

C Variance Ratio Test

This section shortly reviews the variance ratio test of Lo and MacKinlay (1988) that can be used to test the random walk hypothesis. For a time series $\{\tilde{R}_t\}_{t=1,\ldots,nh}$ and $\{R_t\}_{t=1,\ldots,n}$ of daily and monthly returns we define

$$\mu = \frac{1}{nh}\sum_{t=1}^{nh} \tilde{R}_t, \quad \sigma_d^2 = \frac{1}{nh}\sum_{t=1}^{nh}(\tilde{R}_t - \mu)^2, \quad \sigma_m^2 = \frac{1}{nh}\sum_{t=1}^{n}\left(R_t - h\mu\right)^2 \quad \text{(C.1a)}$$

and

$$J_r = \frac{\sigma_m^2}{\sigma_s^2} - 1. \quad \text{(C.1b)}$$

The variance ratio of monthly and daily volatility, $J_r + 1 = \frac{\sigma_m^2}{\sigma_s^2}$, should be equal to one, if the momentum returns follow a random walk. Moreover, Lo and MacKinlay (1988, Theorem 1) show that $z_1 := \frac{\sqrt{nh}J_r}{\sqrt{2(h-1)}}$ is asymptotically standard normally distributed, which can be used to statistically test the random walk hypothesis. Furthermore, Lo and MacKinlay (1988) argue that using non-overlapping data and unbiased variance estimators can improve the efficiency of the estimators, which translates into a more powerful test. Therefore, following the authors, we modify the variance estimators in Equation (C.1) and define

$$\overline{\sigma}_d^2 = \frac{1}{nh-1}\sum_{t=1}^{nh}(\tilde{R}_t - \mu)^2, \quad \overline{\sigma}_m^2 = \frac{1}{m}\sum_{t=h}^{nh}\left(R_{t:t-h} - h\mu\right)^2, \quad \text{(C.2)}$$

where $R_{t:t-h} := \sum_{k=0}^{h-1} \tilde{R}_{t-k}$ and $m := h(nh - h + 1)\left(1 - \frac{h}{nh}\right)$. The variance ratio test statistic is then given by

$$\overline{M}_r = \frac{\overline{\sigma}_m^2}{\overline{\sigma}_d^2} - 1, \quad \text{(C.3)}$$

which again should be equal to zero if the random walk hypothesis holds. Further, Lo and MacKinlay (1988, Theorem 2) show that $z_2 := \frac{\sqrt{nh}\cdot\overline{M}_r}{\sqrt{2(2h-1)(h-1)/3h}}$ is asymptotically standard normally distributed, which can be used to test the random walk hypothesis. To guarantee that the random walk hypothesis is not rejected due to heteroscedasticity, Lo and MacKinlay (1988) develop another test based on Equation (C.3). Under a slightly modified null hypothesis, Lo and MacKinlay (1988, Theorem 3) show that $z_3 := \frac{\sqrt{nh}\cdot\overline{M}_r}{\sqrt{\theta}}$ is asymptotically standard normally distributed, where[153]

$$\theta = \sum_{j=1}^{h-1} \left(\frac{2(h-j)}{h}\right)^2 \delta(j), \tag{C.4a}$$

$$\delta(j) = \frac{nh \sum_{t=j+1}^{nh} \left(\tilde{R}_t - \mu\right)^2 \left(\tilde{R}_{t-j} - \mu\right)^2}{\left(\sum_{t=1}^{nh} \left(\tilde{R}_t - \mu\right)^2\right)^2}. \tag{C.4b}$$

The three tests are applied to the momentum portfolio in Section 2.6.2.

D Comparing Predictive Accuracy

In this section, we describe several methods that can be used to compare the predictive ability of several forecasting methods. In the academic literature, there exist several possibilities how the accuracy of competing forecasting models can be tested. For example, it can be tested if two or more forecasts are equally accurate. Further, instead of testing for equal predictive ability, it can also be tested for superior predictive ability, i.e. it can be tested if a chosen benchmark is outperformed by any other alternative forecast. When comparing several forecasts, we can compare two or more competing forecasts simultaneously. Testing several null hypotheses typically increases the probability of rejecting a true null hypothesis. To account for this, a common method, which is usually used in simultaneously testing several individual hypotheses, is the Bonferroni correction. The Bonferroni correction simply divides the confidence level by the number of individual hypotheses that are tested. This guarantees that the confidence level of the simultaneous test does not exceed the desired confidence level (Bajgrowicz and Scaillet, 2012, p. 475). However, several studies found that this correction is far too conservative and

[153] Equation (19) in Lo and MacKinlay (1988) contains a small error, which is corrected in Equation (C.4b).

has less power than more advanced testing approaches. For that reason, we will present several more advanced testing approaches, which are frequently used in the literature. We will present methods that compare all alternative models to a chosen benchmark and also methods that are free of choosing a benchmark model. Finally, we present several algorithms that can be used to construct a set of superior forecasts.

For an accurate volatility targeting strategy, the portfolio volatility of month t should be constant and equal to $\sigma_{\text{target},m} := \sigma_{\text{target}}/\sqrt{12}$ for each month $t = 1, ..., n$, where n denotes the number of months used to test the accuracy. We denote the weight that is invested in the momentum portfolio of strategy k in month t by $w_{k,t}$, where we consider $l+1$ models $k = 0, 1, ..., l$. We measure the Realized Variance of strategy k in month t by

$$RV_{k,t}^2 = w_{k,t}^2 \cdot \sum_{i=1}^{h} R_{t,i}^2, \quad k = 0, 1, ..., l, \; t = 1, ..., n. \tag{D.1}$$

If a forecasting method is accurate, $RV_{k,t}^2$ should be constant and equal to $\sigma_{\text{target},m}^2$ in every month. Models that produce portfolio volatilities that are higher or lower than the volatility target and/or models that produce a highly volatile portfolio volatility are less favorable and indicate that this model fails to achieve the aim of targeting a constant level of volatility. Similar to Hansen and Lunde (2005) and Patton (2011), we define the sequences of losses by

$$L_{k,t} := L\left(RV_{k,t}^2, \sigma_{\text{target},m}^2\right) := \frac{RV_{k,t}^2}{\sigma_{\text{target},m}^2} - \ln\left(\frac{RV_{k,t}^2}{\sigma_{\text{target},m}^2}\right) - 1, \tag{D.2}$$

where we use the QLIKE loss function. The QLIKE loss function is also frequently used in the volatility forecasting literature. Hansen and Lunde (2005) and Patton (2011) also use other loss functions that can be used to test for the forecasting accuracy. However, Patton (2011) shows that only a certain class of loss functions, including the QLIKE loss function that is used in our paper, are robust to noise in the volatility proxy. The idea of the backtesting procedures summarized in this section is that the losses $L_{k,t}$ should be low for an accurate forecasting model k. We next present several approaches that can be used to test this hypothesis.

D.1 Diebold-Mariano Test (DM-test)

The DM-test presented in Diebold and Mariano (1995) tests if two competing models' forecasts are equally accurate. This test is frequently used when two models are compared. For example, this procedure is used by Patton (2011) to compare competing volatility models, by Kole et al. (2017) to assess the accuracy of h-day VaR forecasts and by Bollerslev et al. (2018) to test for significant utility increases of volatility targeting strategies. The test relies on the relative performance of two competing forecasts, which is defined by

$$X_{k,t} := L_{0,t} - L_{k,t}, \quad k = 1, ..., l, t = 1, ..., n. \tag{D.3}$$

The variable $X_{k,t}$ measures the performance – in terms of the chosen loss function – of model k against the benchmark model 0 in month t (Hansen, 2005, p. 367). A positive value of $X_{k,t}$ indicates that model k performed better – in terms of a smaller loss – than the benchmark model 0 in month t, i.e. model k's portfolio volatility was closer to the desired volatility target than the benchmark model's volatility. For a fixed $k = 1, ..., l$, the DM-test has the null hypothesis $H_0 : \mathbb{E}(X_{k,t}) = 0$ for all $t = 1, ..., n$. Hence, the DM-test can only be used to simultaneously compare two models. As the benchmark model 0 we use the Realized Volatility strategy used in Barroso and Santa-Clara (2015) and Moreira and Muir (2017) and we define $\mathcal{M}_0 = \{1, ..., l\}$. For a fixed $k \in \mathcal{M}_0$, we define the average relative performance of model k against the benchmark model on the whole sample as

$$\overline{X}_k := \frac{1}{n} \sum_{t=1}^{n} X_{k,t}. \tag{D.4}$$

The variables $X_{k,t}$ and $\overline{X}_k$ contain all the information that are needed to assess if any model is statistically equally – or less – accurate compared to the benchmark model. Consequently, all tests that are presented in the next sections use these variables as starting point.

Diebold and Mariano (1995) show that the average loss differential $\overline{X}_k$ is asymptotically normally distributed. In particular, the test statistic

$$T_n^{DM} = \frac{\overline{X}_k}{\hat{\omega}_k} \tag{D.5}$$

follows asymptotically a normal distribution, where $\hat{\omega}_k$ is an estimator for the asymptotic variance. Diebold and Mariano (1995) show that the asymptotic variance can be estimated by

$$\hat{\omega}_k = \sqrt{\frac{1}{n} \left(\hat{\gamma}_{0,k} + 2 \sum_{i=1}^{h-1} \hat{\gamma}_{i,k} \right)}, \tag{D.6}$$

where $\hat{\gamma}_{i,k}$ is an estimator for the i-th autocovariance of the loss differential, which can be estimated by the sample autocovariance

$$\hat{\gamma}_{i,k} = \frac{1}{n} \sum_{t=i+1}^{n} \left(X_{k,t} - \overline{X}_k\right) \left(X_{k,t-i} - \overline{X}_k\right). \tag{D.7}$$

Since the test statistic T_n^{DM} is asymptotically normally distributed, the null hypothesis of equal predictive ability cannot be rejected at a test level of 10% if $-1.64 < T_n^{DM} < 1.64$. If $|T_n^{DM}| > 1.64$, the null hypothesis is rejected at a 10% test level. In particular, a lower average loss of model 0 compared to model k translates into a negative test statistic T_n^{DM}. Thus, a negative T_n^{DM} indicates that the benchmark model is more successful in targeting a constant volatility of the momentum portfolio than model k. Similarly, a positive test statistic indicates that the competing model k produces a more accurate portfolio volatility than the benchmark model (Patton, 2011).

D.2 Reality Check (RC-test)

The test of equal predictive ability of Diebold and Mariano (1995) presented in the previous section simultaneously compares only two models. White (2000) extends this test to the Reality Check test (RC-test). The RC-test is also nicely summarized in Sullivan et al. (1999, Appendix B). The RC-test is based on a multiple null hypothesis, i.e. the benchmark model can be tested simultaneously against several alternative models. Further, the test of White (2000) does not test for equal predictive ability but for superior predictive ability. Thus, it is tested if the benchmark model is not outperformed by any alternative model. We follow the notation of Hansen and Lunde (2005) and define the vector $\mathbf{X}_t := (X_{1,t}, ..., X_{l,t})'$, which contains the relative performances of the l models against the benchmark model 0 in month t. The null hypothesis of the RC-test is then formulated as

$$H_0 : \boldsymbol{\lambda} \leqslant 0, \tag{D.8}$$

where $\boldsymbol{\lambda} := (\lambda_1, ..., \lambda_l)'$ is given by $\lambda_k := \mathbb{E}(X_{k,t})$. Thus, we have a multiple null hypothesis that is the intersection of l null hypotheses $H_0^k : \lambda_k \leqslant 0$ for $k = 1, ..., l$ (White, 2000). Consequently, it is tested if the chosen benchmark model is at least as good as all the remaining l models. In particular, it is tested that even the best alternative model is no better than the chosen benchmark. The alternative hypothesis is that the best of the l models is superior to the benchmark model (White, 2000, p. 1101). If at least one $\lambda_k > 0$ exists, there is strong support against the null hypothesis, since in this case model k is more accurate than the benchmark model (Hansen and Lunde, 2005, p. 879). In other words, the null hypothesis is rejected if there exists at least one model that is more accurate than the benchmark model. White (2000) argues that the test statistic of the RC-test is given by

$$T_n^{RC} := \max_{k=1,\dots,l} \sqrt{n}\, \overline{X}_k, \tag{D.9}$$

which is asymptotically normally distributed. To obtain p-values for this test, White (2000) proposes two possibilities, a Monte Carlo or a bootstrap implementation. We follow White (2000) and use the stationary bootstrap of Politis and Romano (1994) to obtain the p-values p^{RC} of the RC-test. The stationary bootstrap samples blocks of randomly varying length, where the average block length has to be chosen in advance. The bootstrap approach is also used and nicely described in Sullivan et al. (1999, Appendix C), Hansen and Lunde (2005), Hansen (2005) and Romano and Wolf (2005).[154]

By using the stationary bootstrap, we draw resamples of $\overline{\mathbf{X}} := \left(\overline{X}_1, ..., \overline{X}_l\right)'$, i.e. we obtain resamples of the vector that contains the average relative performances of the l models against the benchmark. Following Hansen (2005), we first simulate uniformly distributed variables $u_{b,t}$ and $v_{b,t}$ on $(0, 1]$ for $b = 1, ..., B$ and $t = 1, ..., n$. For a fixed b and $t = 1$, we define $\tau_{b,1} = \lceil n \cdot u_{b,1} \rceil$, where $\lceil \cdot \rceil$ denotes the ceiling function. For $t = 2, ..., n$, we define

$$\tau_{b,t} = \begin{cases} \lceil n \cdot u_{b,t} \rceil, & \text{if } v_{b,t} < q \\ \mathbb{1}_{\{\tau_{b,t-1} < n\}} \tau_{b,t-1} + 1, & \text{if } v_{b,t} \geqslant q. \end{cases} \tag{D.10}$$

Hence, for each $b = 1, ..., B$ we obtain sampled time indices $\{\tau_{b,1}, ..., \tau_{b,n}\}$. We follow Hansen and Lunde (2005) and set $B = 10{,}000$ and $q = 0.1$.[155] Based on $\tau_{b,t}$, we generate pseudo time-series of $\mathbf{X}_t$ by defining $\mathbf{X}^*_{b,t} := \left(X_{1,\tau_{b,t}}, ..., X_{l,\tau_{b,t}}\right)'$ for $b = 1, ..., B$ and $t = 1, ..., n$. These

[154] Romano and Wolf (2005, Appendix B) give a good overview over several bootstrap methods, including the stationary bootstrap used in our paper, that are frequently used in the literature on testing for predictive ability.

[155] Sullivan et al. (1999) and Hsu and Kuan (2005) also use $q = 0.1$, whereas White (2000) uses $q = 0.5$. However, the authors show that the p-values of the RC-test are not very sensitive to the choice of q (see also Hsu et al. (2010)). Furthermore, Equation (D.10) assumes that the time index 1 follows on the time index n, what is called "wrap-up" resampling (see Hsu and Kuan (2005, Footnote 2)).

resamples can be used to define

$$\overline{\mathbf{X}}_b^* := \frac{1}{n}\sum_{t=1}^{n}\mathbf{X}_{b,t}^* = \left(\overline{X}_{b,1}^*, ..., \overline{X}_{b,l}^*\right)', \tag{D.11}$$

where $\overline{X}_{b,k}^* = \frac{1}{n}\sum_{t=1}^{n} X_{k,\tau_{b,t}}$. This provides us with resamples of $\overline{\mathbf{X}} = (\overline{X}_1, ..., \overline{X}_l)'$, which are used to calculate p-values of the RC-test. Following Sullivan et al. (1999) and White (2000), we define

$$T_{b,n}^{RC*} := \max_{k=1,...,l} n^{1/2}\left(\overline{X}_{b,k}^* - \overline{X}_k\right), \quad b = 1, ..., B, \tag{D.12}$$

which can then be used to calculate p-values by

$$p^{RC} := \frac{\sum_{b=1}^{B} \mathbb{1}_{\{T_{b,n}^{RC*} > T_n^{RC}\}}}{B}. \tag{D.13}$$

Hence, p-values can simply be calculated by comparing the test statistic T_n^{RC} to the quantiles of the bootstrap observations $T_{b,n}^{RC*}$, which enables us to approximate the distribution of T_n^{RC} (Sullivan et al., 1999). Since a low test statistic gives no evidence against the null hypothesis, we set $p^{RC} = 1$ if $T_n^{RC} \leqslant 0$. White (2000, p. 1110) argue that the calculation of the p-value can be refined by using order statistics and interpolation. However, we follow White (2000), Hansen (2005) and Hansen and Lunde (2005) and use the method presented above to calculate p-values. As in Sullivan et al. (1999, p. 1659) and White (2000), we also calculate the naive p-values $p^{RC,naive}$. The naive p-value is calculated by applying the bootstrap procedure only to the *best* alternative model instead of *all* alternative models. However, conclusions based on $p^{RC,naive}$ were similar to conclusions based on p^{RC}, and hence results for $p^{RC,naive}$ are not reported here. A low value of p^{RC} indicates that the benchmark model is outperformed by at least one alternative model. Applied to the target volatility strategies, a low value of p^{RC} indicates that there is at least one risk estimation model that produces a portfolio volatility that is more closely to the desired volatility target on each month.

D.3 Superior Predictive Ability (SPA-test)

The RC-test of White (2000) has two main disadvantages, which are corrected by Hansen (2005) in the following way (see also Hansen and Lunde (2005) and Hsu and Kuan (2005)). First, the test-statistic of the RC-test is standardized by an estimator of the asymptotic standard deviation. Romano and Wolf (2005) also find that studentizing the RC test statistic of White (2000) increases the power of the RC-test.[156] Second, when using the stationary bootstrap, the bootstrap variables are re-centered to fulfill the null hypothesis. This is important to avoid that bad performing alternative models influence the power of the testing procedure. Hansen and Lunde (2005) call this procedure a test for superior predictive ability (SPA-test). Hansen (2005) and Hansen and Lunde (2005) show that the SPA-test has more power than the RC-test (see also Hsu and Kuan (2005) and Hsu et al. (2010)).

Since the SPA-test is an extension of the RC-test, the p-values of the SPA-test can be obtained by a similar approach to the one that is described in the previous section. However, as stated above, the SPA-test uses two extensions to the RC-test in order to improve its power. Hansen and Lunde (2005) and Hansen (2005) argue that the test statistic of the SPA-test is given by

$$T_n^{SPA} := \max_{k=1,...,l} \frac{\sqrt{n}\,\overline{X}_k}{\hat{\omega}_k}, \tag{D.14}$$

where $\hat{\omega}_k$ is an estimator for the asymptotic standard deviation of $\sqrt{n}\,\overline{X}_k$. Hansen and Lunde (2005) argue that the correction for $\hat{\omega}_k$ is crucial in order to not determine a wrong model, which makes this test more powerful compared to alternative tests like the RC-test. As for the RC-test, a high test statistic T_n^{SPA} implies that at least one model significantly outperforms the benchmark model, i.e. if T_n^{SPA} is high the null hypothesis $H_0 : \boldsymbol{\lambda} \leqslant 0$ is not plausible. To obtain an estimate of $\hat{\omega}_k$ and to calculate p-values for the test, we again use the stationary bootstrap. Based on the time indices $\{\tau_{b,1}, ..., \tau_{b,n}\}$, $b = 1, ..., B$, obtained by the stationary bootstrap, we

[156] Romano and Wolf (2005, Sec. 4.2) discusses three advantages of studentization and they conclude that "when the standard deviations of the basic test statistics [...] are different, the [test statistics] live on different scales. Comparing one basic test statistic to another is then like comparing apples to oranges. If one wants to compare apples to apples, one should use the studentized test statistics" (Romano and Wolf, 2005, p. 1255). See also Table VIII of Romano and Wolf (2005) that demonstrates that a studentized and non-studentized test statistic can produce completely different results.

estimate $\hat{\omega}_k$ by[157]

$$\hat{\omega}_k = \sqrt{\frac{n}{B}\sum_{b=1}^{B}\left(\overline{X}^*_{b,k} - \overline{X}_k\right)^2}, \quad k = 1, ..., l. \tag{D.15}$$

In order to calculate the p-values, Hansen and Lunde (2005) and Hansen (2005) argue that the bootstrap variables have to be re-centered to satisfy the null hypothesis. This is done by defining

$$\overline{Z}^{*,c}_{b,k} = \overline{X}^*_{b,k} - \overline{X}_k \cdot \mathbb{1}_{\{\overline{X}_k > -A_{k,n}\}}, \tag{D.16}$$

where $A_{k,n} := \frac{1}{4}n^{-1/4}\hat{\omega}_k$.[158] Since different choices of $A_{k,n}$ lead to different p-values, Hansen (2005) proposes to additionally calculate upper and lower bounds for the p-value based on

$$\begin{aligned} \overline{Z}^{*,l}_{b,k} &= \overline{X}^*_{b,k} - \max\left(\overline{X}_k, 0\right), \\ \overline{Z}^{*,u}_{b,k} &= \overline{X}^*_{b,k} - \overline{X}_k, \end{aligned} \tag{D.17}$$

where a wide range between these bounds indicates that there exist irrelevant alternative models. However, Hansen and Lunde (2005) and Hansen (2005) state that main conclusions should be made with the p-values calculated based on $\overline{Z}^{*,c}_{b,k}$, which is also supported by Hansen (2005, Corollary 2). The distribution of T_n^{SPA} can then be approximated by the empirical distribution of

$$T^{SPA*,i}_{b,n} := \max_{k=1,\ldots,l} \frac{\sqrt{n}\,\overline{Z}^{*,i}_{b,k}}{\hat{\omega}_k}, \quad b = 1, ..., B, i = l, c, u. \tag{D.18}$$

Similar to the RC-test, the p-values are then given by

$$p^{SPA,i} := \frac{\sum_{b=1}^{B} \mathbb{1}_{\{T^{SPA*,i}_{b,n} > T^{SPA}_n\}}}{B}, \quad i = l, c, u, \tag{D.19}$$

where we set $p^{SPA,i} = 1$ if $T_n^{SPA} \leqslant 0$, since a low test statistic gives no evidence against the null hypothesis. As in Hansen and Lunde (2005), we also calculate the naive p-values motivated by Sullivan et al. (1999) and White (2000) as described in the previous section. In

[157] As an alternative, Hansen (2005, p. 372) proposes to estimate $\hat{\omega}_k$ by $\hat{\omega}_k^2 = \hat{\gamma}_{0,k} + 2\sum_{i=1}^{n-1}\kappa(n,i)\hat{\gamma}_{i,k}$, where $\hat{\gamma}_{i,k} = \frac{1}{n}\sum_{j=1}^{n-i}\left(X_{k,j} - \overline{X}_k\right)\left(X_{k,j+i} - \overline{X}_k\right)$ and $\kappa(n,i) = \frac{n-1}{n}(1-q)^i + \frac{i}{n}(1-q)^{n-i}$ for $i = 0, 1, ..., n-1$. (see also Hsu et al. (2010) who also use this estimator). Hansen and Lunde (2005) and Hsu and Kuan (2005) use the estimator given in Equation (D.15).

[158] Hansen and Lunde (2005) argue that the choice of $A_{k,n}$ is not unique, which can lead to different p-values. However, we follow the authors and choose $A_{k,n} = \frac{1}{4}n^{-1/4}\hat{\omega}_k$, which is also used by Hsu and Kuan (2005). Hansen (2005), for example, chooses $A_{k,n} = \sqrt{\frac{2\hat{\omega}_k^2}{n}\log\log n}$, which is motivated by the law of the iterated logarithm (see also Hsu et al. (2010) who use this choice). However, both approaches deliver similar results.

particular, p-values of the RC-test can simply be obtained by setting $\hat{\omega}_k = 1$ for $k = 1, ..., l$ in Equations (D.14) and (D.18) and using the bootstrapped test statistic based on the variable $\overline{Z}^{*,u}_{b,k} = \overline{X}^{*}_{b,k} - \overline{X}_k$. As before, a low value of $p^{SPA,i}$ indicates that the benchmark model is outperformed by at least one model, and hence there exists at least one method that provides a more constant portfolio volatility.

D.4 Stepwise Reality Check (Step-RC)

Romano and Wolf (2005) extend the RC-test of White (2000) in two directions. First, similar to the SPA-test they use a studentized test statistic and find that doing this significantly improves the power of the test. Second, they subsequently apply the test to determine a set of superior models by simultaneously controlling the familywise error rate (FWE). The FWE is defined as the probability that at least one model is incorrectly assessed as superior. The advantage of this procedure compared to the RC-test and SPA-test is that the two latter tests only state if the chosen benchmark is outperformed by any other model. However, these tests do not provide any information on the remaining models, i.e. it is not known if there are any other models that outperform the benchmark. By subsequently applying the RC- or SPA-test and choosing each model once as the benchmark, we can construct a set of models that are not statistically outperformed by any other model. Unfortunately, this procedure totally ignores the FWE. A common solution for this problem is to adjust the confidence level of the test as done by the Bonferroni correction. However, Romano and Wolf (2005) show that this procedure has less power compared to their approach of stepwise multiple testing.[159] By applying the stepwise testing approach, we can identify superior strategies in more than one step and we can produce a potentially larger set of superior models than the set constructed by the RC-test combined with the Bonferroni correction. We follow Hsu et al. (2010) and call this approach of stepwise multiple testing using the RC-test statistic – both non-studentized and studentized – the *Step-RC* test.

[159]Romano and Wolf (2005, Appendix D) show that multiple testing is superior to joint testing as done in the RC- and SPA-test. In Section 2.3, Romano and Wolf (2005) present several methods for multiple testing that are frequently used.

For a set of models $\mathcal{M}$, the Step-RC test tests the null hypotheses

$$H_{0,\mathcal{M},k} : \mathbb{E}(X_{k,t}) < 0, k \in \mathcal{M}. \tag{D.20}$$

The Step-RC test then tests each individual hypothesis $H_{0,\mathcal{M},k}$, which is different to testing a joint hypothesis as done in the RC-test and SPA-test and highlighted in Romano and Wolf (2005, Appendix D). A rejected null hypothesis $H_{0,\mathcal{M},k}$ indicates that model k is superior to the benchmark model 0. The aim of the Step-RC test is to reject as many null hypotheses as possible by simultaneously controlling the FWE. In other words, we want to identify as many superior models as possible by simultaneously limiting the probability ε that at least one true null hypothesis is rejected.

Similar to the RC- and SPA-test, the non-studentized and studentized test statistics are given by

$$T_{n,k}^{Step-RC} = \overline{X}_k \quad \text{and} \quad T_{n,k}^{Step-RC,stud} = \frac{\overline{X}_k}{\hat{\omega}_k}. \tag{D.21}$$

Romano and Wolf (2005) find that applying the studentized test statistic is more powerful than applying the non-studentized test statistic (see also Hsu et al. (2010)). The Step-RC test then identifies superior models in several steps, which is nicely presented in Romano and Wolf (2005) and Hsu et al. (2010). For a given set of models $\mathcal{M} = \{1, ..., m\}$, $m = |\mathcal{M}|$, it is tested for which of the m models the null hypothesis can be rejected. In the first step, we set $\mathcal{M} = \mathcal{M}_0$, i.e. $m = l$ and we construct the joint confidence regions

$$[T_{n,1}^{Step-RC} - \hat{c}_{1-\varepsilon}, \infty) \times ... \times [T_{n,m}^{Step-RC} - \hat{c}_{1-\varepsilon}, \infty) \tag{D.22}$$

and

$$[T_{n,1}^{Step-RC,stud} - \hat{d}_{1-\varepsilon}, \infty) \times ... \times [T_{n,m}^{Step-RC,stud} - \hat{d}_{1-\varepsilon}, \infty). \tag{D.23}$$

The parameters $\hat{c}_{1-\varepsilon}$ and $\hat{d}_{1-\varepsilon}$ are estimated by using samples $\{\tau_{b,1}, ..., \tau_{b,n}\}, b = 1, ..., B$, of the stationary bootstrap, which are again used to construct resamples $\overline{\mathbf{X}}_b^* = \left(\overline{X}_{b,1}^*, ..., \overline{X}_{b,l}^*\right)'$ as defined in Equation (D.11). These resamples are used to calculate

$$max_{n,b} = \max_{j \in \mathcal{M}} \left(\overline{X}_{b,j}^* - \overline{X}_j\right) \text{ and } max_{n,b}^{stud} = \max_{j \in \mathcal{M}} \left(\frac{\overline{X}_{b,j}^* - \overline{X}_j}{\hat{\omega}_j}\right), \ b = 1, ..., B. \tag{D.24}$$

The parameters $\hat{c}_{1-\varepsilon}$ and $\hat{d}_{1-\varepsilon}$ are then calculated as the $(1-\varepsilon)$-quantile of $max_{n,b}$ and $max_{n,b}^{stud}$, respectively.[160] The null hypothesis of any model j in $\mathcal{M}$ is rejected if $0 \notin [T_{n,j}^{Step-RC} - \hat{c}_{1-\varepsilon}, \infty)$ for the non-studentized test holds or $0 \notin [T_{n,j}^{Step-RC,stud} - \hat{d}_{\varepsilon}, \infty)$ for the studentized version holds. Hence, a model is identified as superior to the chosen benchmark if $T_{n,j}^{Step-RC} > \hat{c}_{1-\varepsilon}$ or $T_{n,j}^{Step-RC,stud} > \hat{d}_{1-\varepsilon}$ holds. As long as there is at least one hull-hypothesis that can be rejected, the testing procedure is subsequently repeated and the rejected models are added to the set of superior models. In the first step, i.e. $\mathcal{M} = \mathcal{M}_0$, we denote the sets of superior models by $\mathcal{M}_{1-\varepsilon,0}^{Step-RC} := \{j \in \mathcal{M}_0 : T_{n,j}^{Step-RC} > \hat{c}_{1-\varepsilon}\}$ and $\mathcal{M}_{1-\varepsilon,0}^{Step-RC,stud} := \{j \in \mathcal{M}_0 : T_{n,j}^{Step-RC,stud} > \hat{d}_{1-\varepsilon}\}$ and we define the sets of the remaining models by $\mathcal{M}_1^{Step-RC} = \mathcal{M}_0 \backslash \mathcal{M}_{1-\varepsilon,0}^{Step-RC}$ and $\mathcal{M}_1^{Step-RC,stud} = \mathcal{M}_0 \backslash \mathcal{M}_{1-\varepsilon,0}^{Step-RC,stud}$, respectively. The above presented procedure is then repeated by choosing $\mathcal{M} = \mathcal{M}_1^{Step-RC}$ or $\mathcal{M} = \mathcal{M}_1^{Step-RC,stud}$. If there are any superior models in the second step, we add these models to the set of superior models from the first step and denote these sets by $\mathcal{M}_{1-\varepsilon,1}^{Step-RC}$ and $\mathcal{M}_{1-\varepsilon,1}^{Step-RC,stud}$, i.e. $\mathcal{M}_{1-\varepsilon,1}^{Step-RC} := \mathcal{M}_{1-\varepsilon,0}^{Step-RC} \cup \{j \in \mathcal{M}_1^{Step-RC} : T_{n,j}^{Step-RC} > \hat{c}_{1-\varepsilon}\}$ and $\mathcal{M}_{1-\varepsilon,1}^{Step-RC,stud} := \mathcal{M}_{1-\varepsilon,0}^{Step-RC,stud} \cup \{j \in \mathcal{M}_1^{Step-RC,stud} : T_{n,j}^{Step-RC,stud} > \hat{d}_{1-\varepsilon}\}$, where $\hat{c}_{1-\varepsilon}$ and $\hat{d}_{1-\varepsilon}$ are recalculated in every step. Generally, in the i-th step, as long as there is at least one rejected hypothesis, the test is repeated for the set of models $\mathcal{M}_i^{Step-RC} = \mathcal{M}_{i-1}^{Step-RC} \backslash \mathcal{M}_{1-\varepsilon,i-1}^{Step-RC}$ and $\mathcal{M}_i^{Step-RC,stud} = \mathcal{M}_{i-1}^{Step-RC,stud} \backslash \mathcal{M}_{1-\varepsilon,i-1}^{Step-RC,stud}$, where the set of superior models is given by $\mathcal{M}_{1-\varepsilon,i}^{Step-RC} := \mathcal{M}_{1-\varepsilon,i-1}^{Step-RC} \cup \{j \in \mathcal{M}_i^{Step-RC} : T_{n,j}^{Step-RC} > \hat{c}_{1-\varepsilon}\}$ and $\mathcal{M}_{1-\varepsilon,i}^{Step-RC,stud} := \mathcal{M}_{1-\varepsilon,i-1}^{Step-RC,stud} \cup \{j \in \mathcal{M}_i^{Step-RC,stud} : T_{n,j}^{Step-RC,stud} > \hat{d}_{1-\varepsilon}\}$. If in any step none of the null hypotheses $H_{0,\mathcal{M},k}, k \in \mathcal{M}$ can be rejected, the Step-RC test stops and the current set of superior models is defined as the output of this approach.

D.5 Stepwise Superior Predictive Ability (Step-SPA)

Hsu et al. (2010) extend the idea of Romano and Wolf (2005) by using a stepwise multiple testing approach based on the SPA-test of Hansen (2005) instead of the RC-test. The authors

[160] Romano and Wolf (2005) propose to use two distinct estimators $\hat{w}_j$ for the studentized test statistic $T_{n,j}^{Step-RC,stud}$ and in the bootstrap approach to obtain the samples $max_{n,b}^{stud}$. However, we follow Hansen (2005) and use the same estimator for both quantities. Hsu et al. (2010) also use the same estimator for both quantities, where the authors use the HAC estimator described in Section D.3 instead of the bootstrap based estimator used in our paper.

find that the Step-SPA has more power than the Step-RC, which is quite intuitive since the SPA-test is known to be more powerful than the RC-test. The superiority of the Step-SPA is theoretically shown by Hsu et al. (2010, Theorem 3) and also confirmed by a Monte-Carlo Simulation. Since Romano and Wolf (2005) also use a studentized version of the test statistic, the difference between the Step-RC and Step-SPA test lies in using a sample-dependent null distribution as explained in Section D.3. Hence, the Step-SPA uses the same algorithm as described in the previous section, where $\overline{X}^*_{b,j} - \overline{X}_j$ in Equation (D.24) is replaced by

$$\overline{Z}^{*,c}_{b,j} = \overline{X}^*_{b,j} - \overline{X}_j \cdot \mathbb{1}_{\{\overline{X}_j > -A_{j,n}\}}. \tag{D.25}$$

Although, we only use the studentized and re-centered version of the SPA-test in Section D.3, we follow Hsu et al. (2010) and also use the non-studentized but re-centered version in the Step-SPA test. This allows us to assess if the superiority of the Step-SPA approach comes from studentizing the test statistic or from using a data-dependent null-distribution.

D.6 False Discovery Rate (FDR)

Barras et al. (2010) and Bajgrowicz and Scaillet (2012) state that the Step-RC approach of Romano and Wolf (2005), which controls the FWE, is often too restrictive and sometimes fails to identify superior models, once a model is erroneously detected as superior. Therefore, they present an easy and straightforward approach based on the False Discovery Rate (FDR), which is defined as the proportion of models where the null hypothesis of equal predictive ability has been rejected, although this model is truly null. Barras et al. (2010) and Bajgrowicz and Scaillet (2012) extend the FDR to the FDR^+, which is defined as the proportion of false discoveries, i.e. models that have been erroneously chosen as superior. Following Barras et al. (2010) and Bajgrowicz and Scaillet (2012), a model k is called *significantly positive* if two conditions are fulfilled. First, the null hypothesis of equal predictability, $H_0 : \mathbb{E}(X_{k,t}) = 0$, is rejected. Second, the test-statistic $T_n^{FDR,k} = \overline{X}_k$ is positive, i.e. the benchmark's loss is (on average) higher than the loss of model k, and hence model k is more successful in targeting a constant level of volatility than the benchmark. Following Barras et al. (2010) and Bajgrowicz and Scaillet (2012), we denote by R^+ the number of models that are found to be significantly positive, and by F^+ the number of models that have been chosen as significantly positive, but are not truly superior to the benchmark. The FDR^+ is then given by $FDR^+ = F^+/R^+$. In particular, an FDR^+ of 10% means that among the models that have been identified as superior 10% have been erroneously chosen as superior. Barras et al. (2010) and Bajgrowicz and Scaillet (2012) show how the FDR^+ can be estimated, where this estimator is denoted by $\widehat{FDR}^+ = \hat{F}^+/\hat{R}^+$. An advantage of the FDR approach is that it is easily implemented once the (two-sided) p-values of the null hypothesis of equal predictability have been calculated. In contrast to the RC-test of White (2000), which uses one-sided p-values, the FDR method is based on testing for equal predictive ability instead of superior predictive ability. To calculate the two sided p-values we again use the stationary bootstrap of Politis and Romano (1994) and we define

$$T_{b,n}^{FDR*,k} = \overline{X}_{b,k}^* - \overline{X}_k, \tag{D.26}$$

where we again use resamples $\overline{X}_{b,k}^* = \frac{1}{n}\sum_{t=1}^n X_{k,\tau_{b,t}}$ for a given bootstrap sample $\{\tau_{b,1}, ..., \tau_{b,n}\}$, $b = 1, ..., B$. The two-sided p-values are then given by

$$p_i^{FDR} = 2 \cdot \min\left(\frac{1}{B}\sum_{b=1}^{B} \mathbb{1}_{\{T_{b,n}^{FDR*,i} > T_n^{FDR,i}\}}, \frac{1}{B}\sum_{b=1}^{B} \mathbb{1}_{\{T_{b,n}^{FDR*,i} < T_n^{FDR,i}\}}\right), i = 1, .., l, \tag{D.27}$$

where we use an equal-tailed test, as explained in the Internet Appendix of Barras et al. (2010). Based on these p-values, a model is called *significant* if its p-value is smaller than a chosen threshold. Barras et al. (2010) and Bajgrowicz and Scaillet (2012) then show that an estimate of FDR^+ is given by

$$\widehat{FDR}^+ = \frac{\hat{F}^+}{\hat{R}^+} = \frac{\frac{1}{2}\hat{\pi}_0 l\gamma}{n_+^{FDR}}, \tag{D.28}$$

where γ is the chosen threshold, n_+^{FDR} is the number of statistically positive models, i.e. $n_+^{FDR} = |\{k = 1, ..., l \mid p_k \leqslant \gamma, T_n^{FDR,k} > 0\}|$ and $\hat{\pi}_0$ is an estimate of π_0, the proportion of models that fulfill the null hypothesis of equal predictive ability. We follow Bajgrowicz and Scaillet (2012) and choose $\gamma = 0.4$. Barras et al. (2010) present a simple bootstrap approach to determine a data-driven value for γ, but they find that values of γ between 0.35 and 0.45 produce quite similar results compared to the value obtained by the bootstrap. Bajgrowicz and Scaillet (2012) show that π_0 can be estimated by

$$\hat{\pi}_0(\lambda) = \frac{n_0^{FDR}}{l(1-\lambda)}, \tag{D.29}$$

where $n_0^{FDR} = |\{k = 1, ..., l : p_k > \lambda\}|$. We follow Bajgrowicz and Scaillet (2012) and choose $\lambda = 0.6$. Barras et al. (2010) show how the value λ can alternatively be chosen by a simple bootstrap approach. However, they find that $\hat{\pi}_0$ is not very sensitive to the choice of λ and that values of λ between 0.5 and 0.6 typically deliver similar estimates of π_0 as the bootstrap approach. Further, Barras et al. (2010) and Bajgrowicz and Scaillet (2012, Appendix D) show how the proportion of significantly positive and negative models, π_A^+ and π_A^-, can be estimated. The estimators of π_A^+ and π_A^- are given by

$$\hat{\pi}_A^+ = \frac{\hat{T}^+(\gamma)}{l} \quad \text{and} \quad \hat{\pi}_A^- = \frac{\hat{T}^-(\gamma)}{l}, \tag{D.30}$$

where $\hat{T}^+(\gamma)$ and $\hat{T}^-(\gamma)$ are estimators for the number of models with a p-value smaller than the chosen threshold γ and a positive or negative relative performance, respectively. Bajgrowicz and Scaillet (2012) show that $\hat{T}^+(\gamma)$ and $\hat{T}^-(\gamma)$ are given by

$$\hat{T}^+(\gamma) = n_+^{FDR} - \frac{1}{2}\hat{\pi}_0 l\gamma \quad \text{and} \quad \hat{T}^-(\gamma) = n_-^{FDR} - \frac{1}{2}\hat{\pi}_0 l\gamma, \tag{D.31}$$

where $n_-^{FDR} = |\{k = 1, ..., l \mid p_k \leqslant \gamma, T_n^{FDR,k} < 0\}|$.

Bajgrowicz and Scaillet (2012) and Barras et al. (2010) show how the FDR^+ can be used to built a set of superior models with limited FDR^+. In other words, similar to the Step-RC and Step-SPA, we want to identify as many superior models as possible, where the amount of erroneously chosen models is limited by the predetermined FDR^+. Following Bajgrowicz and Scaillet (2012), we choose an FDR^+ target of 10%. The algorithm to identify the set of superior models starts by selecting the model with the smallest p-value among the models that have a positive $T_n^{FDR,k}$. In the next step, among the remaining models that have not been collected by the algorithm and that have a positive $T_n^{FDR,k}$, again the model with the lowest p-value is chosen. Then, the FDR^+ of this set of models is calculated. This process is then repeated as long as the portfolio FDR^+ is lower than the desired FDR^+ target of 10%. Increasing the number of models that are identified as significantly positive also bears the potential that more non-significant models are erroneously chosen. However, the algorithm collects the best performing models and guarantees that the FDR^+ of these models is limited by the desired FDR^+ target of 10%. In other words, this algorithm identifies superior models but guarantees that at least

90% of the identified models are truly superior and not chosen due to luck. Bajgrowicz and Scaillet (2012) show in a simulation study that the FDR approach has more power in detecting outperforming models than the RC-test and the Step-RC test.

D.7 The Model Confidence Set (MCS)

The RC- and SPA-test, presented in Sections D.2 and D.3, simultaneously compare a chosen benchmark to all remaining models. A drawback of these approaches is that one has to define a certain benchmark model. This also translates to the stepwise extensions presented in Sections D.4 and D.5 and also holds for the FDR approach presented above. In addition, the RC- and SPA-test only test if the benchmark model is not outperformed by the best alternative model. However, these tests cannot identify the best models of all available models. One possibility, as done in Section 2.6, is to choose each model once as the benchmark. This provides us with a set of models where the null hypothesis has not been rejected. Hence, this procedure constructs a set of good performing models. However, Hansen et al. (2011) argue that in this case, the test level of the individual SPA-tests has to be adjusted using the Bonferroni correction or similar approaches. Thus, this approach of subsequently applying the RC- and SPA-test induces a high loss of power. Instead of choosing each model once as the benchmark model and then testing this model against the remaining models, the Model Confidence Set (MCS) approach of Hansen et al. (2003) and Hansen et al. (2011) offers a more elegant way of comparing all available models. The MCS procedure constructs a set of models, called the Model Confidence Set, such that the best performing model will be contained in this set with a given level of confidence. Thus, the MCS approach is analogous to the confidence interval of a parameter. Hansen et al. (2011, p. 474-475) argue that the MCS approach is similar to the approach of subsequently choosing each model once as the benchmark and using the SPA-test, but more powerful. The MCS is then similar to the set of all benchmark models that have not been identified to be inferior to the remaining models by the SPA-test. An advantage of the MCS compared to subsequently applying the SPA-test is that no benchmark has to be chosen, and hence all models are tested simultaneously. Further, the MCS relies on testing equalities instead of inequalities, which also improves the power of the MCS compared to subsequently testing for superior predictive ability. Consequently, the MCS is a similar but more elegant and powerful way to compare all models simultaneously.

In order to apply the MCS procedure, we define $\mathcal{M}_0 = \{0, 1, ..., l\}$ as the set of all possible models. As stated above, in contrast to the approaches presented before, we now also include the benchmark model 0 into the models that are tested for superiority. The MCS is then denoted by $\mathcal{M}^*$ and contains the best model(s) for a given confidence level. Hansen et al. (2003) and Hansen et al. (2011) present an approach to estimate the MCS. In particular, the cardinality of the set $\mathcal{M}^*$ depends on the quality of the data, since a model is only eliminated if it performs significantly worse than the remaining models. The cardinality of the MCS will be higher, if some - or maybe all - models produce portfolio volatilities that are nearly equally accurate. In contrast, the cardinality will be low, if there are only a few models – or even only one model – that achieve the desired volatility target more accurately than the remaining models. Hence, instead of choosing only one model without reflecting the differences in the accuracy between all models, the MCS takes information on all models into account when the superior models are identified.

In the first stage, the MCS approach uses an equivalence test, similar to the test of Diebold and Mariano (1995). If this test is rejected, i.e. not all models in $\mathcal{M}_0$ produce equally accurate forecasts, a bad model is eliminated in the second stage by an elimination rule. This procedure is then subsequently repeated until the equivalence test cannot be rejected anymore. In this case, there is no evidence that there exists a bad performing model within the remaining models and none of the remaining models can be eliminated. The set of remaining model(s) is called the MCS. By eliminating bad performing models in an early stage, the MCS method is also robust against adding poor performing models to $\mathcal{M}_0$ (Hansen et al., 2003). This is one of the main drawbacks of the Reality Check of White (2000) and was previously fixed by the SPA-test of Hansen (2005). Further, the MCS procedure also provides p-values p_i^{MCS} for all $i \in \mathcal{M}^*$, where a low p-value makes it unlikely that model i is the best model. Thus, the MCS procedure eliminates bad performing models and gives us additional information on the remaining models.

In order to determine the MCS, we first define the relative performances for all models of

$\mathcal{M}_0$ by

$$X_{ij,t} = L_{i,t} - L_{j,t}, \text{ for all } i, j \in \mathcal{M}_0, t = 1, ..., n. \tag{D.32}$$

The aim of the MCS procedure is to determine the set of superior models $\mathcal{M}^*$, which is given by

$$\mathcal{M}^* = \{ i \in \mathcal{M}_0 : \mathbb{E}(X_{ij,t}) \leqslant 0 \text{ for all } j \in \mathcal{M}_0 \}. \tag{D.33}$$

For a given set of models $\mathcal{M} \subseteq \mathcal{M}_0$, the MCS procedure subsequently tests for equal predictive ability by testing the null hypothesis

$$H_{0,\mathcal{M}} : \mathbb{E}(X_{ij,t}) = 0, \text{ for all } i, j \in \mathcal{M}, \tag{D.34}$$

which is similar to the test of Diebold and Mariano (1995) presented in Section D.1. If this null hypothesis is rejected, the worst performing model is eliminated from $\mathcal{M}$. To calculate p-values for each model in $\mathcal{M}_0$ and to assess if the null hypothesis is rejected and which model is eliminated, we use the following algorithm (Hansen et al., 2003, p. 845).[161] First, we define by

$$\overline{X}_{ij} = \frac{1}{n} \sum_{t=1}^{n} X_{ij,t} \tag{D.35}$$

the average relative performance of model i against model j. For a given set $\mathcal{M}$, we next test for equal predictive ability, assess which model will be eliminated and calculate the p-value of the eliminated model. In the first step, we set $\mathcal{M} = \mathcal{M}_0$ and we denote the number of models in $\mathcal{M}$ by m, i.e. $m = |\mathcal{M}|$. To test if the null hypothesis holds and to calculate the p-value of the model that is removed in this step, we rely on two test statistics (Hansen et al., 2003, p. 845): The range statistic T_n^R and the semi-quadratic statistic T_n^{SQ}, which are given by

$$T_n^R = \max_{i,j \in \mathcal{M}} \frac{|\overline{X}_{ij}|}{\sqrt{\widehat{var}(\overline{X}_{ij})}} \quad \text{and} \quad T_n^{SQ} = \sum_{\substack{i,j \in \mathcal{M} \\ i<j}} \frac{(\overline{X}_{ij})^2}{\widehat{var}(\overline{X}_{ij})}, \tag{D.36}$$

[161] A detailed description of the bootstrap approach used in the MCS procedure is given in the online appendix of Hansen et al. (2011) and in Hansen et al. (2003). We mainly follow Hansen et al. (2003) who also use the MCS procedure to assess the accuracy of volatility models.

where T_n^R is the more conservative test statistic. The estimate $\widehat{var}(\overline{X}_{ij})$ is again obtained by using the stationary bootstrap of Politis and Romano (1994). For a given bootstrapped sample $\{\tau_{b,1}, ..., \tau_{b,n}\}, b = 1, ..., B$, we define

$$\overline{X}^*_{b,ij} = \frac{1}{n}\sum_{t=1}^{n} X^*_{b,ij,t}, \tag{D.37}$$

where $X^*_{b,ij,t} = X_{ij,\tau_{b,t}}$. These quantities are then used to calculate

$$\widehat{var}(\overline{X}_{ij}) = \frac{1}{B}\sum_{b=1}^{B}\left(\overline{X}^*_{b,ij} - \overline{X}_{ij}\right)^2 \quad \text{for all } i, j \in \mathcal{M}. \tag{D.38}$$

The test statistics for the bootstrapped samples are then given by

$$T^{R,*}_{b,n} = \max_{i,j\in\mathcal{M}} \frac{|\overline{X}^*_{b,ij} - \overline{X}_{ij}|}{\sqrt{\widehat{var}(\overline{X}_{ij})}} \quad \text{and} \quad T^{SQ,*}_{b,n} = \sum_{\substack{i,j\in\mathcal{M}\\ i<j}} \frac{\left(\overline{X}^*_{b,ij} - \overline{X}_{ij}\right)^2}{\widehat{var}(\overline{X}_{ij})}, \tag{D.39}$$

which are used to estimate the empirical distribution of T_n^R and T_n^{SQ}. The p-values for the hypothesis of testing the equal predictive ability of the models in $\mathcal{M}$ are then given by

$$P^R_{H_{0,\mathcal{M}}} = \frac{\sum_{b=1}^{B} \mathbb{1}_{\{T^{R,*}_{b,n} > T^R_n\}}}{B} \quad \text{and} \quad P^{SQ}_{H_{0,\mathcal{M}}} = \frac{\sum_{b=1}^{B} \mathbb{1}_{\{T^{SQ,*}_{b,n} > T^{SQ}_n\}}}{B}. \tag{D.40}$$

Calculating an individual p-value for each null hypothesis is common in testing multiple hypotheses (Romano and Wolf, 2005, Sec. 2.3).

In the next step, we eliminate the worst performing model $e_{\mathcal{M}}$ in $\mathcal{M}$, which is given by

$$e_{\mathcal{M}} = \arg\max_{i\in\mathcal{M}} \frac{\overline{X}_{i\cdot}}{\sqrt{\widehat{var}(\overline{X}_{i\cdot})}}, \tag{D.41}$$

where $\overline{X}_{i\cdot} = \frac{1}{m-1}\sum_{j\in\mathcal{M}} \overline{X}_{ij}$. The estimate $\widehat{var}(\overline{X}_{i\cdot})$ is given by

$$\widehat{var}(\overline{X}_{i\cdot}) = \frac{1}{B}\sum_{b=1}^{B}\left(\overline{X}^*_{b,i\cdot} - \overline{X}_{i\cdot}\right)^2, \quad \text{for } i \in \mathcal{M}, \tag{D.42}$$

where $\overline{X}^*_{b,i\cdot} = \frac{1}{m-1}\sum_{j\in\mathcal{M}} \overline{X}^*_{b,ij}$. We then define $\mathcal{M}_{k+1} = \mathcal{M}_k \backslash \{e_{\mathcal{M}_k}\}$ and repeat the procedure for $\mathcal{M}_{k+1}$. Hence, in the first step, i.e. $\mathcal{M} = \mathcal{M}_k = \mathcal{M}_0$, we obtain $\mathcal{M}_1 = \mathcal{M}_0 \backslash \{e_{\mathcal{M}_0}\}$. The p-values for model $e_{\mathcal{M}_0}$, i.e. the model that is removed in the first step, are given by $p^R_{e_{\mathcal{M}_0}} = P^R_{H_{0,\mathcal{M}_0}}$ and $p^{SQ}_{e_{\mathcal{M}_0}} = P^{SQ}_{H_{0,\mathcal{M}_0}}$, respectively. The above presented procedure is then repeated for $\mathcal{M} = \mathcal{M}_k$, $k = 1, ..., l$, until no more model can be removed. The p-values for the model that is eliminated in step k are given by $p^R_{e_{\mathcal{M}_k}} = \max_{j \leqslant k} P^R_{H_{0,\mathcal{M}_j}}$ and $p^{SQ}_{e_{\mathcal{M}_k}} = \max_{j \leqslant k} P^{SQ}_{H_{0,\mathcal{M}_j}}$, respectively. In the last step, i.e. for the model that has not been removed in any step, we set the p-values equal to 1. More formally, we have $P^R_{H_{0,\mathcal{M}_{l+1}}} = 1$ and $P^{SQ}_{H_{0,\mathcal{M}_{l+1}}} = 1$, which is a reasonable choice, since the last surviving model is at least as good as itself (Hansen et al., 2011, p. 462). The estimate for the model confidence set for a confidence level ε is then given by

$$\widehat{\mathcal{M}}^{R,*}_{1-\varepsilon} = \left\{ j \in \mathcal{M}_0 : p^R_j > \varepsilon \right\} \quad \text{and} \quad \widehat{\mathcal{M}}^{SQ,*}_{1-\varepsilon} = \left\{ j \in \mathcal{M}_0 : p^{SQ}_j > \varepsilon \right\}, \tag{D.43}$$

which are estimates of the set of superior models $\mathcal{M}^*$. The model confidence sets $\widehat{\mathcal{M}}^{R,*}_{1-\varepsilon}$ and $\widehat{\mathcal{M}}^{SQ,*}_{1-\varepsilon}$ thus contain the best forecasting model with a probability of $1 - \varepsilon$. Thus, these sets separate the models in $\mathcal{M}_0$ into superior and inferior models. Since the range statistic is more conservative, we will typically have $\widehat{\mathcal{M}}^{SQ,*}_{1-\varepsilon} \subseteq \widehat{\mathcal{M}}^{R,*}_{1-\varepsilon}$. Hansen et al. (2011) show asymptotically that the probability that the estimated sets in Equation (D.43) contain the true set of superior models $\mathcal{M}^*$ is at least $1 - \varepsilon$.

Similar to the stepwise multiple tests of Romano and Wolf (2005) and Hsu et al. (2010) as well as the FDR approach of Bajgrowicz and Scaillet (2012) and Barras et al. (2010), the MCS approach identifies a set of superior models. However, there are some main differences between these approaches. First, the approaches of the previous sections identify models that are superior to a chosen benchmark, whereas the MCS approach does not require a certain benchmark. Second, the approaches of the previous sections identify the superior models out of $\mathcal{M}_0 = \{1, ..., l\}$, whereas the MCS approach eliminates bad performing models from $\mathcal{M}_0 \cup \{0\}$ until all bad performing models are eliminated. The remaining set of models then contains the good performing models. Romano and Wolf (2005, Footnote 12) call their approach of identifying superior models instead of eliminating bad performing models a step-down method, whereas the opposite is a step-up method. In the first case, the set of superior models increases in every step, whereas in the latter case, the set of models is trimmed until only superior models are left.

E Additional Performance Results

In this section, we present additional performance results for six different data sets. First, we use the momentum factor of Fama and French (2012), which is also used by Ruenzi and Weigert (2018). By doing this, we show that our main results also hold for different construction methods of the momentum portfolio. Our main results in Section 2.6 are based on the momentum portfolio where assets within the winners and losers portfolios are equally weighted. In contrast, the momentum factor uses a size-balanced construction method based on size-return double sorted portfolios. This construction method makes momentum crashes less severe. Second, we use the equally weighted momentum strategy for German equities. Ruenzi and Weigert (2018) find that the crash risk of momentum seems to be the highest in the US. Therefore, the benefit from switching to CVaR targeting in crash periods should also be more pronounced for US data. By calculating the strategies for the German market, we can assess if risk targeting strategies, and in particular the strategies that switch between volatility and CVaR targeting, are also beneficial for momentum strategies outside the US. Third, we show that our strategy also works for the US Betting against Beta (BAB) factor of Frazzini and Pedersen (2014). Fourth, we additionally show results for the BAB factor in Germany. The rational behind examining the BAB portfolio is that Moreira and Muir (2017), Cederburg et al. (2020) and Barroso and Maio (2018) examine risk targeting using the RV model for several portfolio strategies and find the best results for the momentum and BAB portfolios. Fifth, we also use risk targeting for the industry momentum strategy (Chordia and Shivakumar, 2002, Grundy and Martin, 2001, Moskowitz and Grinblatt, 1999, Novy-Marx, 2012). Grobys et al. (2018) and Du Plessis and Hallerbach (2017) show that the RV approach of Barroso and Santa-Clara (2015) applied to the industry momentum strategy provides an enhanced risk-return profile compared to the non-managed industry momentum strategy. Industry momentum has the advantage that this strategy can be easily built based on ETFs, and thus is a more practicable alternative to the stock based momentum strategy. Sixth, Barroso and Maio (2018), Cederburg et al. (2020) and Moreira and Muir (2017) find that the RV managed strategy underperforms the non-managed strategy for the small minus big (SMB) factor that buys small sized firms and sells big sized firms, whereas the authors find vast performance gains for most other portfolios. A possible explanation for this finding could be the high positive skewness of this factor, which is not captured by managing volatility. We therefore apply our risk targeting strategies that take non-normalities into account to the SMB factor to assess if our strategy also works for positively skewed strategies where the RV strategy fails to increase the risk-adjusted performance.

E.1 Additional Performance Results for Momentum: US Momentum Factor and Germany

Table XVII shows the performance results for the non-managed and RV managed momentum strategy as well as the switching strategies for the US momentum factor and the German momentum portfolio. For a better clarity, we only show results for one crash indicator for the US momentum factor and two crash indicators for the German momentum portfolio. Results for the other crash indicators are similar. Panel A shows results for the US momentum factor for the whole period and the crash period that ranges from 01.01.1938 to 01.01.1943.[162] As before, risk targeting significantly increases the risk-adjusted performance by achieving a higher return while simultaneously the risk is reduced, especially in the left tail. In line with our main results, all strategies that switch between volatility and CVaR targeting offer the most convincing risk-return profile by achieving the highest returns with less risk. As a consequence, the switching strategies significantly increase the Sharpe Ratio compared to the non-managed and RV managed strategy, indicated by high and statistically significant values of the Jobson and Korkie (1981) test statistic for all switching strategies. This higher Sharpe Ratio is accompanied by a significant left tail risk reduction. The importance of risk targeting is again highlighted during the crash period. The RV managed momentum strategy successfully reduces the loss and the volatility of the non-managed strategy during the crash period. However, the performance can be significantly improved by our switching approach. Interestingly, all switching strategies

[162] Interestingly, during the crash period, the return of the US momentum factor is significantly less negative than the performance of the equally weighted momentum strategy in Table VII. Thus, as expected, momentum crashes are much more pronounced for the equally weighted momentum portfolio. This can also be seen by the higher left tail risk of the equally weighted portfolio, which exhibits a skewness of -4.38. In contrast, the skewness of the momentum factor is only -3.061. Similarly, Barroso and Santa-Clara (2015) find a lower crash risk for the momentum strategy using value-weighted winners and losers portfolios. Momentum's crash risk can further be reduced by weighting assets within the winners and losers portfolios inversely to their risk (Rickenberg, 2020c).

exhibit positive returns during the crash period, whereas the non-managed and RV managed momentum strategies exhibit a negative return. Further, the switching strategies provide a significant downside risk protection, whereas the RV managed momentum strategy fails to reduce the high left tail risk of the non-managed momentum strategy. That is, the switching strategies significantly reduce momentum's negative skewness, kurtosis and minimum return, whereas the RV managed strategy's left tail risk is comparable to the non-managed strategy's left tail risk. Thus, the RV managed strategy fails as tail risk hedging instrument just when it is most needed. This is in line with Rickenberg (2020b) who also finds that unconditional models do not provide an adequate tail risk reduction in crash periods. Interestingly, one of the strategies that switches between volatility and CVaR targeting even produces positively skewed returns. As before, the strategy that switches to the contrarian strategy delivers the best results during the crash period, but fails to produce an outperformance in the long-run.

Panel B shows results for the momentum strategy in Germany. In contrast to the US momentum strategy, the German momentum portfolio exhibits a higher return but also produces higher risk. However, this finding is also driven by the different sample period, which is much shorter for the German sample. Due to our quite low volatility target of 12%, all risk targeting strategies produce lower returns but also take significantly less risk. In total, all strategies exhibit higher Sharpe Ratios, where again the highest Sharpe Ratios are found for the strategies that switch between volatility and CVaR targeting. Nevertheless, the increase in the Sharpe Ratio is not statistically significant. This is also in line with the finding of Ruenzi and Weigert (2018) that the crash risk of the momentum strategy is higher in the US. A lower crash risk in Germany makes downside risk management less important and decreases the benefit from our switching strategies for this data set. Interestingly, the RV managed strategy does not reduce momentum's left tail risk, whereas the switching strategies exhibit a substantial reduction of left tail risk. Hence, for the German momentum portfolio, managing volatility does not provide an adequate tail risk protection. The benefit of the strategies that switch between volatility and CVaR targeting is again highlighted during the crash period ranging from 01.12.2000 to 01.10.2001. As in Panel A, the non-managed and RV managed momentum strategy produce negative returns,

Table XVII. Performance Results of Risk Targeting: US Factor and Germany

This table shows performance results for the non-managed momentum portfolio, the RV managed momentum portfolio and the switching strategies for the US momentum factor and the equally weighted momentum portfolio for German equities. Panel A shows results for the US momentum factor. Panel B shows results for Germany, where the sample period is 01.01.1994 to 30.04.2016 and the market return based crash indicator is used. Panel C shows results for Germany and the one month TSMOM crash indicator. The crash period for Germany is from 01.12.2000 to 01.10.2001. The description of the columns is given in Table VI.

	Whole Period							Crash Period				
Panel A: US Mom Factor	Return	Vola	Skew	Kurt	SR	z_{JK}	Min	Return	Vola	Skew	Kurt	Min
Momentum	6.16	16.42	-3.061	30.665	0.375	-6.757	-52.26	-7.22	22.15	-2.441	12.724	-30.35
RV	14.95	17.70	-0.457	5.799	0.845	-	-28.22	-5.86	18.39	-2.390	13.995	-28.22
CVaR HS	15.64	16.82	-0.210	4.363	0.930	**2.379**	-19.34	3.44	11.76	-0.237	6.553	-11.48
CVaR-Skt-unc	15.65	16.51	-0.112	4.260	0.948	**2.630**	-16.57	4.45	10.20	0.796	6.090	-6.84
CVaR-GARCH-SRTR	16.42	17.31	-0.229	4.380	0.948	**2.889**	-19.83	1.90	14.28	-1.527	11.245	-19.83
CVaR-GJR-SRTR	16.39	17.35	-0.209	4.491	0.945	**2.634**	-20.45	1.64	14.26	-1.341	9.521	-18.71
CVaR-GARCH-Skt	16.11	17.12	-0.188	4.210	0.941	**2.819**	-17.92	1.37	13.99	-1.247	8.936	-17.42
CVaR-GJR-Skt	16.18	16.84	-0.165	4.109	0.961	**3.210**	-16.11	2.66	12.98	-0.733	7.325	-14.68
CVaR-GARCH-FHS	16.05	17.15	-0.203	4.247	0.936	**2.633**	-17.75	1.24	13.64	-1.128	9.077	-17.30
CVaR-GJR-FHS	16.11	16.84	-0.177	4.183	0.957	**3.057**	-16.15	2.41	12.75	-0.769	7.822	-14.89
RV-Mom/Contrarian	14.39	17.74	0.237	5.080	0.811	-0.261	-18.04	18.50	17.63	2.607	13.669	-6.26
Panel B: Germany	Return	Vola	Skew	Kurt	SR	z_{JK}	Min	Return	Vola	Skew	Kurt	Min
Momentum	27.20	28.71	-1.189	10.539	0.948	-3.526	-50.01	-9.38	84.83	-1.062	3.004	-50.01
RV	16.37	14.05	-1.163	8.307	1.166	-	-23.92	-5.51	32.52	-1.779	4.924	-23.92
CVaR HS	16.30	13.63	-0.464	7.083	1.196	0.396	-19.68	4.66	32.25	-0.931	3.396	-19.68
CVaR-Skt-unc	16.09	13.77	-0.510	7.091	1.168	0.050	-19.68	5.95	32.45	-0.937	3.367	-19.68
CVaR-GARCH-SRTR	17.75	14.09	-0.470	6.399	1.260	1.386	-19.68	3.85	32.20	-0.901	3.406	-19.68
CVaR-GJR-SRTR	17.76	14.08	-0.467	6.393	1.261	1.418	-19.68	3.54	32.17	-0.896	3.408	-19.68
CVaR-GARCH-Skt	17.59	14.42	-0.500	6.245	1.220	0.765	-19.68	7.12	32.86	-0.891	3.299	-19.68
CVaR-GJR-Skt	17.68	14.46	-0.494	6.197	1.222	0.794	-19.68	7.39	32.91	-0.895	3.290	-19.68
CVaR-GARCH-FHS	17.93	14.58	-0.480	6.082	1.229	0.924	-19.68	9.30	33.59	-0.823	3.182	-19.68
CVaR-GJR-FHS	18.02	14.65	-0.469	6.018	1.230	0.917	-19.68	9.88	33.66	-0.831	3.172	-19.68
RV-Mom/Contrarian	8.81	14.55	-0.706	6.773	0.605	-2.206	-23.92	-32.24	30.90	-1.082	4.185	-23.92
Panel C: Germany TSMOM	Return	Vola	Skew	Kurt	SR	z_{JK}	Min	Return	Vola	Skew	Kurt	Min
Momentum	27.20	28.71	-1.189	10.539	0.948	-3.526	-50.01	-9.38	84.83	-1.062	3.004	-50.01
RV	16.37	14.05	-1.163	8.307	1.166	-	-23.92	-5.51	32.52	-1.779	4.924	-23.92
CVaR HS	17.30	13.52	-0.517	6.651	1.280	1.401	-19.68	4.04	31.50	-1.122	3.424	-19.68
CVaR-Skt-unc	17.25	13.54	-0.537	6.741	1.274	1.317	-19.68	5.07	31.76	-1.097	3.392	-19.68
CVaR-GARCH-SRTR	17.20	13.69	-0.562	6.357	1.257	1.232	-19.68	-0.23	30.26	-1.228	3.690	-19.68
CVaR-GJR-SRTR	17.49	13.84	-0.511	6.243	1.263	1.359	-19.68	4.38	31.54	-1.122	3.427	-19.68
CVaR-GARCH-Skt	18.19	14.08	-0.553	6.108	1.292	**1.681**	-19.68	2.34	30.88	-1.195	3.557	-19.68
CVaR-GJR-Skt	18.15	14.17	-0.497	6.142	1.281	1.518	-19.68	6.87	32.51	-0.987	3.282	-19.68
CVaR-GARCH-FHS	18.52	14.28	-0.541	5.895	1.297	**1.870**	-19.68	4.02	31.40	-1.141	3.454	-19.68
CVaR-GJR-FHS	18.46	14.42	-0.454	6.033	1.280	1.543	-19.68	8.96	33.40	-0.853	3.201	-19.68
RV-Mom/Contrarian	5.15	14.71	-0.796	6.488	0.350	-2.996	-23.92	-23.39	31.89	-1.551	4.033	-23.92

whereas all strategies that switch between volatility and CVaR targeting exhibit positive returns. Further, the RV managed strategy does not reduce left tail risk during the crash and produces an even higher negative skewness and higher kurtosis than the non-managed portfolio. In contrast, the strategies that switch between volatility and CVaR targeting slightly reduce the negative skewness of the non-managed momentum portfolio and exhibit a lower left tail risk than the RV managed strategy. Interestingly, the strategy that switches to the contrarian portfolio performs the worst during the crash period and exhibits an extremely negative return. Hence, switching to the contrarian strategy during this momentum crash, although this approach is appealing in theory, fails due to false signals of the crash indicator for momentum in Germany. Thus, more research on the drivers of momentum crashes outside the US is needed. This can also be seen by the low Sharpe Ratio of this approach over the long-run. We therefore use a second crash indicator for the German momentum portfolio. Panel C contains results for an indicator based on the momentum portfolio's own performance using the TSMOM strategy of Moskowitz et al. (2012) with a lookback period of one month. This approach produces slightly higher Sharpe Ratios, where two strategies produce statistically higher Sharpe Ratios for a test level of 10%. Although results for this indicator are slightly better than results of Panel B, both indicators produce quite similar results and show that momentum crashes in Germany are different to momentum crashes in the US. Consequently, the crash indicator that works well for our US sample does not work equally well for the German sample.[163] Nevertheless, our CVaR switching strategy is quite robust to these false signals of δ_t and still produces an enhanced risk-return profile compared to the non-managed and RV managed momentum strategy.

Table XVIII shows the economic value of the switching strategies for the US momentum factor. Panel A shows results for the mean-variance investor over the whole period and the crash period. As before, the economic value of the strategies that switch between volatility and CVaR targeting is positive regardless of the level of risk aversion and CVaR strategy. In contrast, the strategy that switches between the RV managed momentum and contrarian portfolio does not increase the investor's utility. As expected, the economic value of the switching strategies significantly increases during the crash period. Hence, during a momentum crash, the investor significantly benefits from switching away from the volatility managed momentum portfolio. This especially holds for the strategy that switches to the contrarian portfolio. However, the willingness to pay for the strategies that switch to the CVaR managed portfolio are also very high and are not only limited to the crash period. Panel B shows results for the investor with CRRA utility. Results are again in line with the findings for the mean-variance investor, i.e. all CVaR switching strategies exhibit positive fees that increase with the level of risk aversion. Furthermore, the economic value significantly increases during the crash period and takes ex-

[163] Grobys et al. (2018) also find that the crash indicator that works well for the stock momentum strategy does not work equally well for industry momentum. Hence, momentum crashes strongly depend on the used data set.

Table XVIII. Economic Value of Risk Targeting: US Momentum Factor
This table shows the economic value of the switching strategies with respect to the RV managed strategy for the US momentum factor. Panel A shows results for a mean-variance investor with utility function (2.6.8). Panel B shows results for a CRRA investor with utility function (2.6.11). Panel C shows results for a loss-averse investor with utility function (2.6.14).

Model	Whole Period					Crash Period				
Panel A: MV	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}
CVaR HS	0.746	1.252	2.191	3.259	1.139	10.681	13.833	18.861	23.447	11.615
CVaR-Skt-unc	0.805	1.478	2.725	4.137	1.320	11.925	15.654	21.589	26.956	13.007
CVaR-GARCH-SRTR	1.346	1.572	1.991	2.474	0.975	8.753	10.835	14.174	17.273	9.155
CVaR-GJR-SRTR	1.315	1.521	1.905	2.348	0.950	8.471	10.560	13.909	17.017	8.946
CVaR-GARCH-Skt	1.108	1.441	2.061	2.772	1.037	8.221	10.420	13.945	17.209	8.933
CVaR-GJR-Skt	1.218	1.711	2.627	3.671	1.220	9.708	12.362	16.605	20.505	10.733
CVaR-GARCH-FHS	1.054	1.373	1.966	2.647	1.012	8.130	10.480	14.242	17.717	9.694
CVaR-GJR-FHS	1.159	1.650	2.561	3.600	1.206	9.476	12.216	16.597	20.617	10.972
RV-Mom/Contrarian	-0.539	-0.561	-0.603	-0.650	-0.131	25.511	25.999	26.788	27.544	12.055
Panel B: CRRA	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}
CVaR HS	0.904	0.979	1.282	2.197	2.965	7.202	7.844	10.526	16.679	18.769
CVaR-Skt-unc	1.055	1.131	1.586	2.657	3.660	8.974	9.789	13.354	21.341	24.696
CVaR-GARCH-SRTR	1.055	1.131	1.510	2.350	2.657	5.378	5.614	6.644	8.812	9.707
CVaR-GJR-SRTR	1.055	1.131	1.434	2.273	2.580	5.142	5.457	6.883	10.116	10.938
CVaR-GARCH-Skt	0.979	1.055	1.434	2.427	2.888	4.907	5.221	6.803	10.608	11.600
CVaR-GJR-Skt	1.207	1.282	1.739	2.734	3.428	6.168	6.644	8.812	13.944	15.389
CVaR-GARCH-FHS	0.979	1.055	1.434	2.350	2.734	4.829	5.142	6.803	10.608	11.682
CVaR-GJR-FHS	1.131	1.282	1.663	2.657	3.350	6.009	6.406	8.488	13.438	14.792
RV-Mom/Contrarian	-0.374	-0.150	0.451	1.739	1.739	16.765	17.894	22.691	32.923	34.194
Panel C: Loss Aversion	$b = 0.8$		$b = 1$		b_t^{switch}	$b = 0.8$		$b = 1$		b_t^{switch}
	$l = 2$	$l = 3$	$l = 2$	$l = 3$	l_t^{switch}	$l = 2$	$l = 3$	$l = 2$	$l = 3$	l_t^{switch}
CVaR HS	0.963	1.335	1.216	1.660	1.441	9.723	11.427	13.292	15.281	14.676
CVaR-Skt-unc	1.201	1.755	1.561	2.222	1.883	10.273	12.161	14.849	16.958	16.323
CVaR-GARCH-SRTR	1.062	1.125	1.369	1.461	1.284	7.960	9.178	10.471	11.875	11.401
CVaR-GJR-SRTR	1.013	1.058	1.321	1.397	1.235	7.686	8.795	9.912	11.205	10.759
CVaR-GARCH-Skt	0.918	1.048	1.208	1.383	1.224	7.759	9.092	10.156	11.707	11.239
CVaR-GJR-Skt	1.116	1.344	1.446	1.748	1.510	8.805	10.268	11.753	13.428	12.900
CVaR-GARCH-FHS	0.882	1.015	1.162	1.336	1.187	7.690	9.103	10.239	11.892	11.421
CVaR-GJR-FHS	1.124	1.386	1.448	1.784	1.538	8.791	10.357	11.802	13.556	13.033
RV-Mom/Contrarian	-1.353	-1.404	-0.568	-0.592	-0.462	19.504	19.413	24.613	24.364	22.980

treme values for a highly risk-averse investor during this crash. In contrast, the strategy that switches to the contrarian portfolio only provides a positive economic value for a highly risk-averse CRRA investor or during the crash period. Panel C shows results for the loss-averse investor, which are again in line with our earlier findings. The economic value is again positive for all strategies that switch between volatility and CVaR targeting, regardless of the level of loss aversion. In contrast, for the strategy that switches to the contrarian portfolio, the economic value is negative for all choices of b and l. During the crash period, the economic value of the switching strategies is again very high and is the highest for the strategy that switches to the contrarian portfolio.

Table XIX shows the economic value of the switching strategies for the German momentum

Table XIX. Economic Value of Risk Targeting: Germany

This table shows the economic value of the switching strategies with respect to the RV managed strategy for the German momentum strategy using the TSMOM indicator. Panel A shows results for a mean-variance investor with utility function (2.6.8). Panel B shows results for a CRRA investor with utility function (2.6.11). Panel C shows results for a loss-averse investor with utility function (2.6.14).

Model	Whole Period					Crash Period				
Panel A: MV	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}
CVaR HS	0.863	1.105	1.558	2.083	2.487	10.076	11.052	12.667	14.241	7.564
CVaR-Skt-unc	0.819	1.050	1.484	1.986	2.467	11.081	11.819	13.043	14.242	8.859
CVaR-GARCH-SRTR	0.754	0.920	1.230	1.590	2.384	5.957	7.993	11.338	14.528	0.619
CVaR-GJR-SRTR	0.979	1.073	1.250	1.456	2.356	10.429	11.368	12.921	14.438	7.680
CVaR-GARCH-Skt	1.559	1.544	1.514	1.480	2.363	8.482	10.009	12.526	14.952	4.607
CVaR-GJR-Skt	1.506	1.447	1.336	1.206	2.306	12.705	12.718	12.739	12.760	11.462
CVaR-GARCH-FHS	1.815	1.703	1.492	1.245	2.315	10.090	11.158	12.924	14.643	7.224
CVaR-GJR-FHS	1.740	1.562	1.227	0.832	2.230	14.576	13.682	12.185	10.659	14.487
RV-Mom/Contrarian	-9.851	-10.135	-10.671	-76.333	-2.522	-18.694	-18.231	-17.463	-16.708	-31.838
Panel B: CRRA	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}
CVaR HS	1.207	1.358	2.197	4.204	2.580	3.505	3.660	4.126	4.907	2.965
CVaR-Skt-unc	1.131	1.282	2.044	4.048	2.427	3.893	4.048	4.516	5.300	3.428
CVaR-GARCH-SRTR	0.979	1.131	1.891	4.048	2.120	1.815	1.891	2.273	2.965	1.055
CVaR-GJR-SRTR	1.055	1.207	2.044	4.282	2.044	3.660	3.738	4.204	4.985	3.119
CVaR-GARCH-Skt	1.358	1.510	2.350	4.594	1.968	2.888	2.965	3.350	4.048	2.197
CVaR-GJR-Skt	1.207	1.434	2.273	4.594	1.815	4.594	4.672	5.221	6.168	4.126
CVaR-GARCH-FHS	1.358	1.586	2.427	4.750	1.815	3.505	3.660	4.048	4.829	2.965
CVaR-GJR-FHS	1.207	1.434	2.350	4.750	1.586	5.300	5.457	6.009	7.122	4.985
RV-Mom/Contrarian	-8.500	-8.431	-7.945	-6.616	-8.223	-7.597	-7.527	-7.387	-7.037	-8.292
Panel C: Loss Aversion	$b = 0.8$		$b = 1$		b_t^{switch}	$b = 0.8$		$b = 1$		b_t^{switch}
	$l = 2$	$l = 3$	$l = 2$	$l = 3$	l_t^{switch}	$l = 2$	$l = 3$	$l = 2$	$l = 3$	l_t^{switch}
CVaR HS	1.239	1.753	1.686	2.282	1.927	5.973	5.453	8.412	7.884	5.701
CVaR-Skt-unc	1.219	1.738	1.656	2.256	1.907	6.831	6.229	9.380	8.752	6.588
CVaR-GARCH-SRTR	0.884	1.205	1.267	1.644	1.444	3.421	3.592	5.248	5.587	2.806
CVaR-GJR-SRTR	1.017	1.294	1.442	1.760	1.532	6.187	5.625	8.786	8.256	5.824
CVaR-GARCH-Skt	1.407	1.572	1.867	2.049	1.751	5.048	4.837	7.230	7.106	4.569
CVaR-GJR-Skt	1.363	1.537	1.846	2.031	1.737	7.648	6.684	10.705	9.675	7.498
CVaR-GARCH-FHS	1.439	1.465	1.919	1.940	1.668	6.116	5.654	8.538	8.099	5.774
CVaR-GJR-FHS	1.436	1.503	1.938	1.990	1.706	8.985	7.725	12.418	10.998	9.006
RV-Mom/Contrarian	-11.040	-11.565	-10.547	-11.076	-8.457	-19.688	-20.850	-20.653	-21.705	-24.414

portfolio using the TSMOM crash indicator. Results for the market return crash indicator are similar but slightly lower in magnitude. Panel A shows results for the mean-variance investor. The economic value is again positive for all strategies that switch between the volatility and CVaR managed strategies. The strategy that switches to the contrarian portfolio provides a negative economic value that is very high in magnitude, confirming the observation that further research on momentum crashes outside the US is needed.[164] During the crash period, the economic value is positive and high in magnitude for all strategies that switch between volatility and CVaR targeting. In contrast, the economic value of the strategy that switches to the con-

[164] A possibility could be to use indicators based on other factor portfolios as examined in Grundy and Martin (2001) and Martens and Van Oord (2014). Alternatively the regime switching approach of Daniel et al. (2017) could also be used.

trarian portfolio is again negative and high in magnitude. Panel B shows the economic value for the CRRA investor which is again positive for all strategies that switch between volatility and CVaR targeting for all levels of risk aversion. In contrast, the strategy that switches to the contrarian portfolio has again a negative economic value for all levels of risk aversion. The benefit of switching to the CVaR managed strategy is again highlighted during the crash period, where the CVaR switching strategies produce high economic values. Panel C shows results for the loss-averse investor. These are in line with the results for the mean-variance and CRRA investor. The economic value for the strategies that switch between volatility and CVaR targeting are again positive for all levels of loss aversion. This holds for both periods, but the economic value is again higher for the crash period. In contrast, the strategy that switches to the contrarian portfolio has again a negative economic value for all levels of loss aversion and both periods. In total, for all three investors, switching between volatility and CVaR targeting heightens the investors' utility regardless of the level of risk or loss aversion. Further, this result holds over the whole sample and during the crash period. In contrast, switching to the contrarian portfolio is no adequate tool for managing momentum crashes in Germany, since the momentum crash indicators give several false signals. Thus, the characteristics of momentum crashes strongly depend on the used data set.

E.2 Additional Performance Results for Betting against Beta

In this section, we show results for the Betting against Beta (BAB) factor of Frazzini and Pedersen (2014), where we use data for the US and German BAB portfolios. Moreira and Muir (2017), Cederburg et al. (2020) and Barroso and Maio (2018) show that the risk targeting approach of Barroso and Santa-Clara (2015) can successfully be applied to the BAB portfolio. The authors compare the RV approach for several portfolios and find that volatility targeting works best for the momentum and BAB portfolios. This results since the returns of these portfolios are highly non-normal with a high left tail risk. Thus, our switching approach should also work well for this strategy. For that reason, we first start by assessing the distributional properties of the US BAB portfolio's monthly returns. Figure XI shows conditional moments of the US BAB portfolio. As for the momentum portfolio, conditional volatility, skewness and kurtosis of the BAB portfolio are highly time-varying. Further, skewness of the BAB portfolio is negative almost always, but in contrast to the momentum portfolio, there is no month when the skewness does not exist. In contrast, there are several months when the conditional kurtosis does not exist or exhibits extreme realizations. Figure XI also shows that times of a high kurtosis do not necessarily coincide with times of a high volatility. Similarly, times of a high volatility do not necessarily coincide with times when conditional kurtosis is high. Hence, managing volatility does not capture all the (tail) risk of the BAB portfolio and our switching strategy that manages tail risk in crash periods should be more successful in mitigating the BAB portfolio's left tail risk.

Since the BAB portfolio of Frazzini and Pedersen (2014) is rescaled to a market beta of zero, a crash indicator based on past market return or market volatility should not provide any information on the crash risk of the BAB portfolio.[165] For that reason, we use a crash indicator based on the BAB portfolio's past return using the TSMOM strategy of Moskowitz et al. (2012). This indicator indicates a crash, i.e. $\delta_t = 1$, if the BAB portfolio's past twelve months' performance is negative.[166] Figure XI shows that this crash indicator works quite well for the BAB portfolio. This figure marks months when the past twelve months' return of the BAB portfolio is negative in red. This indicator captures most of the months when the BAB portfolio has a negative return and an increased risk. However, there are also periods with extremely negative returns and periods of high left tail risk that are not captured by this indicator. Thus, more research on the drivers of BAB crashes is needed, which would eventually lead to an improved performance of our switching strategy.

Table XX shows performance results for the US BAB portfolio, the RV managed strategy used by Moreira and Muir (2017), Cederburg et al. (2020) and Barroso and Maio (2018) as well as the switching strategies. We show performance results over the whole period and a

[165]Another possibility would be to use the construction method of the BAB portfolio that is used by Bali et al. (2017a) who examine a portfolio where stocks are first ranked by their beta. The BAB portfolio is then built by selling the 10% stocks with the highest beta and buying the 10% stocks with the lowest beta, without rescaling this portfolio to a beta of zero. For this strategy, the indicator based on the past market return should deliver good results since this BAB portfolio has a negative beta per construction.

[166]Moskowitz et al. (2012) and Goyal and Jegadeesh (2017) find good results of TSMOM for look back periods between one and 36 months. We also used other lengths between one and 36 months and found similar results to the 12 months look back period. However, the 12 months look back period is the most common choice in the trend following literature.

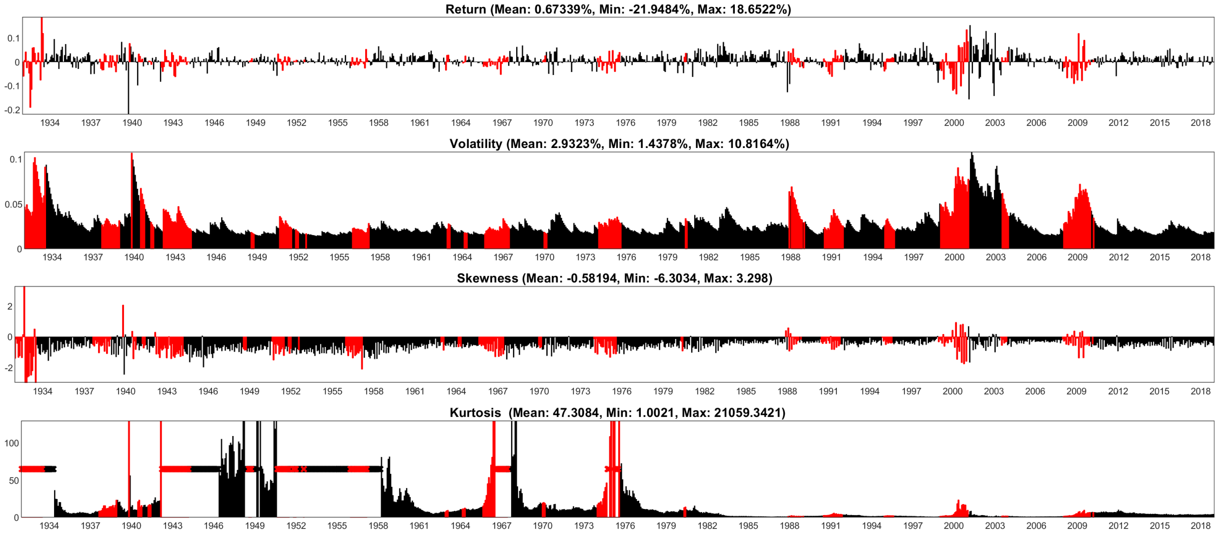

Figure XI. Monthly Conditional Moments: BAB. This figure shows the monthly return, conditional volatility, conditional skewness and conditional kurtosis of the BAB portfolio. Conditional volatility, conditional skewness and conditional kurtosis are calculated for the model in Equation (2.4.16) using the GJR-GARCH model. Months when skewness or kurtosis do not exist are marked with an x. Months when the previous 12 months' return of the BAB portfolio is negative are shown in red.

BAB crash period that ranges from 01.06.1941 to 01.06.1943. As for the momentum portfolio, risk targeting delivers higher Sharpe Ratios while simultaneously left tail risk is reduced. The RV managed strategy raises the Sharpe Ratio of the non-managed BAB portfolio from 0.808 to 0.954, which is statistically significant with a Jobson and Korkie (1981) test statistic of 1.958. However, the switching strategies produce even higher Sharpe Ratios that are all higher than one and significantly higher than the Sharpe Ratio of the RV managed strategy indicated by the test of Jobson and Korkie (1981). Further, the switching strategies are again successful in reducing left tail risk. The higher Sharpe Ratio of the switching strategies is also accompanied by a reduction of left tail risk. The strategy that switches to the RV managed short BAB portfolio has a slightly higher Sharpe Ratio than the RV managed BAB portfolio, but the increase is not statistically significant.[167] Thus, this strategy is again outperformed by the CVaR switching strategies. During the crash period, all risk targeting strategies (except for the strategy that switches to the short BAB portfolio) produce higher losses compared to the non-managed strategy. However, this result is mainly influenced by our volatility target of 12% and the low volatility of the BAB

[167] The short BAB portfolio is long high beta stocks and short low beta stocks.

portfolio. Thus, the risk targeting strategies are typically leveraged, which results in higher losses during the crash period. A loss-averse investor should therefore choose a lower risk target for the BAB portfolio. Nevertheless, results of the crash period highlight the failure of the RV approach to manage extreme down periods. The RV managed strategy produces an annualized return of only -21.33% with an annualized volatility of 26.74%. Thus, the annualized loss of this strategy is almost three times higher than the loss of the non-managed strategy. The loss of the RV managed strategy can be more than halved with a much lower volatility by using one of the switching strategies. This again shows that the RV managed strategy fails as a tail risk hedging instrument just when it is most needed. In contrast, our CVaR switching approach limits the strategy's tail risk during crash periods and successfully captures the upside potential in calm markets.

Table XX. Performance Results of Risk Targeting: US Betting against Beta
This table shows the performance results for the non-managed US BAB portfolio, the RV managed BAB portfolio and the strategies that switch between volatility and CVaR targeting. The sample period is 01.03.1934 to 31.12.2018 and the crash period is 01.06.1941 to 01.06.1943. The description of the columns is given in Table VI.

	Whole Period							Crash Period				
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	Min	Return	Vola	Skew	Kurt	Min
US BAB	8.57	10.61	-0.759	9.855	0.808	-1.958	-21.95	-8.34	11.30	-0.542	2.861	-8.31
RV	15.62	16.36	-0.657	5.789	0.954	-	-23.59	-21.33	26.74	-1.069	4.183	-23.59
CVaR HS	16.99	15.54	-0.365	6.380	1.093	**3.753**	-28.41	-9.40	11.46	-1.588	5.470	-11.22
CVaR-Skt-unc	17.00	15.51	-0.382	6.454	1.097	**3.901**	-28.41	-10.13	12.23	-1.393	4.567	-11.22
CVaR-GARCH-SRTR	17.12	15.66	-0.403	6.328	1.094	**4.093**	-28.41	-11.20	14.58	-1.105	3.647	-11.22
CVaR-GJR-SRTR	16.66	15.84	-0.454	6.320	1.051	**2.873**	-28.41	-11.66	14.52	-0.931	3.349	-11.22
CVaR-GARCH-Skt	17.08	15.72	-0.415	6.269	1.087	**3.828**	-28.41	-9.93	13.55	-1.083	3.696	-11.22
CVaR-GJR-Skt	16.82	15.81	-0.404	6.232	1.064	**3.255**	-28.41	-11.08	13.17	-1.157	3.828	-11.22
CVaR-GARCH-FHS	17.16	15.73	-0.407	6.228	1.091	**3.891**	-28.41	-9.43	12.86	-1.287	4.268	-11.22
CVaR-GJR-FHS	16.92	15.84	-0.400	6.195	1.068	**3.313**	-28.41	-10.39	12.50	-1.329	4.395	-11.22
RV-BAB/Short BAB	16.05	16.34	-0.390	5.516	0.983	0.247	-23.59	-5.83	27.37	-0.801	4.807	-23.59

Figure XII shows the performance of the non-managed BAB portfolio, the RV managed BAB portfolio and one strategy that switches between volatility and CVaR targeting. We again rescale all strategies to the same level of volatility. Further, since the BAB portfolio is a zero-investment strategy, we add the risk-free rate to the portfolio return. As for the momentum portfolio, the lowest portfolio value of $2{,}183{,}853.59\$$ is achieved by the non-managed BAB portfolio. The RV managed strategy increases this value to $21{,}708{,}629.35\$$. However, the strategy that switches between volatility and CVaR targeting produces a portfolio value of even $152{,}266{,}771.65\$$. Interestingly, during the first half, the RV managed strategy underperforms the non-managed strategy. Hence, the outperformance of the RV managed strategy compared to the non-managed strategy is only driven by the second half. Thus, the profitability of the RV managed portfolio strongly depends on the examined time period, as it is also shown by Dreyer and Hubrich (2019). In contrast, the switching strategy performs similar to the non-managed strategy in the first half and then clearly outperforms the other strategies.

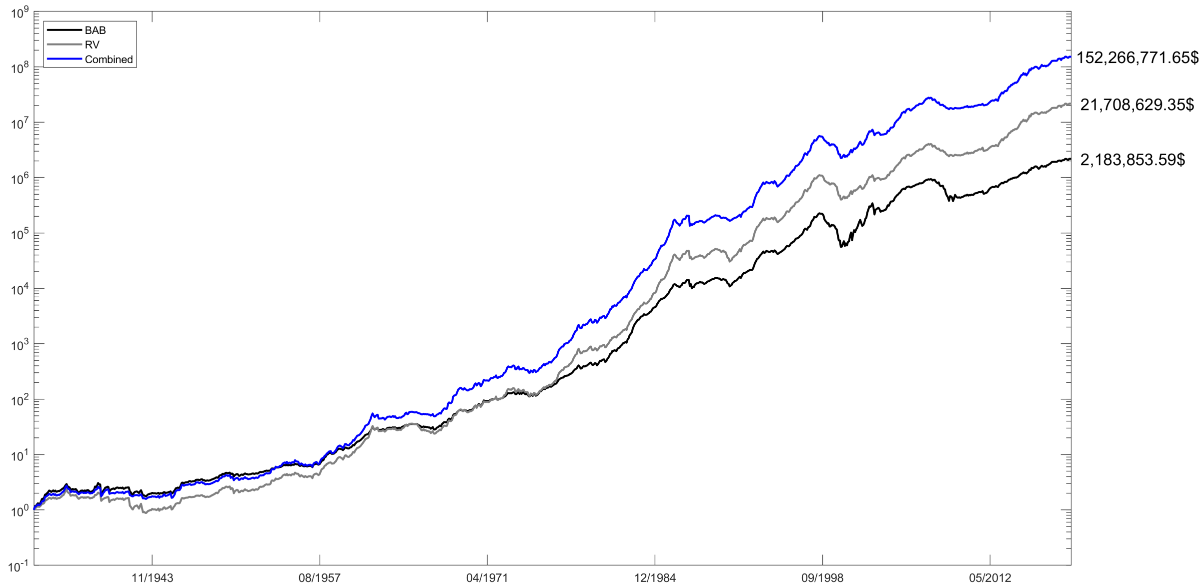

Figure XII. Performance of Risk Targeting: Betting against Beta. This figure plots the cumulative return of a one dollar investment in the BAB portfolio, the RV managed BAB portfolio and a portfolio that switches between volatility and CVaR targeting over the whole period, where we add the risk-free rate to these portfolios. All strategies are rescaled to the same volatility.

Table XXI shows the economic value of the switching strategies, measured as the annualized percentage fee an investor is willing to pay to switch away from the RV managed strategy. Compared to the momentum strategy, the economic value is lower for the BAB portfolio but still positive for all strategies that switch between volatility and CVaR targeting and for all types of investors. In contrast, the economic value of the strategy that switches to the short BAB portfolio is low in magnitude and even negative for a loss-averse investor with $b = 0.8$. During the crash period, the economic value for the mean-variance and loss-averse investor becomes extremely high for all switching strategies. This is in line with the results of Table XX and highlights the high misspecification of the RV managed strategy during periods of high losses. In other words, as stated above, the RV approach is an inappropriate tail risk hedging instrument, which gives a wrong risk assessment in crises. In contrast, our switching approach performs well during crises and over the long-run.

Table XXI. Economic Value of Risk Targeting: US Betting against Beta
This table shows the economic value of the switching strategies with respect to the RV managed strategy for the US BAB portfolio. Panel A shows the results for a mean-variance investor with utility function (2.6.8). Panel B shows the results for a CRRA investor with utility function (2.6.11). Panel C shows the results for a loss-averse investor with utility function (2.6.14). The crash period ranges from 01.06.1941 to 01.06.1943.

Model	Whole Period					Crash Period				
Panel A: MV	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}
CVaR HS	1.329	1.767	2.584	3.520	1.578	17.527	26.166	38.576	48.404	34.936
CVaR-Skt-unc	1.347	1.804	2.656	3.632	1.599	16.514	24.816	36.750	46.222	33.783
CVaR-GARCH-SRTR	1.426	1.806	2.512	3.324	1.539	14.845	22.132	32.634	41.032	30.994
CVaR-GJR-SRTR	0.988	1.268	1.790	2.393	1.362	14.273	21.549	32.034	40.417	30.681
CVaR-GARCH-Skt	1.382	1.728	2.372	3.113	1.499	16.592	24.401	35.643	44.600	32.876
CVaR-GJR-Skt	1.137	1.436	1.993	2.634	1.408	15.187	23.060	34.389	43.405	32.292
CVaR-GARCH-FHS	1.446	1.784	2.415	3.141	1.504	17.323	25.446	37.129	46.418	33.843
CVaR-GJR-FHS	1.218	1.501	2.030	2.640	1.408	16.148	24.329	36.093	45.436	33.367
RV-BAB/Short BAB	0.376	0.392	0.423	0.458	0.087	18.996	18.477	17.708	17.037	8.053
Panel B: CRRA	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$	γ_t^{switch}
CVaR HS	1.434	1.510	1.586	1.207	2.503	-0.673	-0.747	-1.193	-1.933	1.207
CVaR-Skt-unc	1.510	1.510	1.586	1.207	2.503	-0.673	-0.747	-0.971	-1.342	0.075
CVaR-GARCH-SRTR	1.434	1.510	1.510	1.207	2.273	0.150	0.150	0.150	0.300	-1.193
CVaR-GJR-SRTR	0.979	1.055	1.055	0.753	1.510	-0.300	-0.300	-0.150	0.225	-1.268
CVaR-GARCH-Skt	1.358	1.434	1.434	1.207	2.120	0.602	0.602	0.602	0.753	0.451
CVaR-GJR-Skt	1.131	1.207	1.282	1.055	1.815	-0.896	-0.896	-0.896	-0.896	-0.896
CVaR-GARCH-FHS	1.434	1.434	1.510	1.282	2.197	0.526	0.526	0.376	0.225	0.828
CVaR-GJR-FHS	1.207	1.207	1.282	1.131	1.815	-0.747	-0.822	-0.971	-1.193	-0.225
RV-BAB/Short BAB	0.300	0.376	0.526	0.904	0.979	8.327	8.408	8.488	8.569	9.544
Panel C: Loss Aversion	$b = 0.8$		$b = 1$		b_t^{switch}	$b = 0.8$		$b = 1$		b_t^{switch}
	$l = 2$	$l = 3$	$l = 2$	$l = 3$	l_t^{switch}	$l = 2$	$l = 3$	$l = 2$	$l = 3$	l_t^{switch}
CVaR HS	1.475	1.950	2.061	2.644	1.939	16.565	20.921	24.087	29.098	24.382
CVaR-Skt-unc	1.549	2.049	2.121	2.733	2.006	15.848	19.836	22.592	27.524	22.746
CVaR-GARCH-SRTR	1.467	1.847	2.011	2.488	1.819	14.014	17.348	19.465	23.596	19.075
CVaR-GJR-SRTR	1.027	1.326	1.447	1.820	1.317	13.271	16.551	18.681	22.715	18.295
CVaR-GARCH-Skt	1.379	1.718	1.897	2.323	1.695	15.275	18.934	21.592	26.052	21.270
CVaR-GJR-Skt	1.149	1.459	1.612	2.001	1.452	14.337	18.086	20.562	25.060	20.518
CVaR-GARCH-FHS	1.419	1.747	1.945	2.358	1.721	16.159	20.128	22.873	27.676	22.736
CVaR-GJR-FHS	1.178	1.464	1.656	2.016	1.463	15.317	19.360	21.982	26.771	22.062
RV-BAB/Short BAB	-0.293	-0.306	0.402	0.423	0.319	18.120	18.203	19.377	19.380	17.701

Table XXII summarizes results for the German BAB portfolio, where we only show the economic value for $\gamma = 10, b = 1$ and $l = 2$. The sample period for the German BAB portfolio starts in December 1992. For this data set, the non-managed and RV managed strategies produce similar Sharpe Ratios, whereas the Sharpe Ratios of the strategies that switch between volatility and CVaR targeting are higher, but not statistically significant. The strategy that switches to the short BAB strategy produces the lowest returns. Hence, further research on the drivers of BAB crashes in Germany is needed. Nevertheless, the strategies that switch between volatility and CVaR targeting are again very successful in reducing left tail risk. Interestingly, all strategies that switch between the volatility and CVaR managed strategies exhibit a positive skewness, whereas the skewness of the non-managed and RV managed strategies is negative. Thus, as before, our switching approach is successful in increasing the BAB portfolio's risk-adjusted performance while simultaneously left tail risk is reduced. The economic value for the German BAB portfolio is quite low, but positive for all strategies that switch between volatility and CVaR targeting. In contrast, the economic value of the strategy that switches to the short BAB portfolio is negative.

Table XXII. Performance Results of Risk Targeting: Betting against Beta for Germany
This table shows performance results for the non-managed BAB portfolio, the RV managed BAB portfolio and the switching strategies for the German BAB portfolio. The sample period starts in December 1992. The description of the columns is given in Table VI. The economic value is calculated for $\gamma = 10, b = 1$ and $l = 2$.

	Performance Results									Economic Value		
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Δ_{MV}	Δ_{CRRA}	Δ_{LA}
BAB Germany	11.73	17.55	-0.257	4.487	0.668	-0.345	36.46	0.322	-21.48	-	-	-
RV	8.05	11.88	-0.108	3.946	0.678	-	31.59	0.255	-11.50	-	-	-
CVaR HS	8.13	10.94	0.194	4.659	0.743	1.028	21.41	0.380	-11.94	1.357	0.979	1.181
CVaR-Skt-unc	8.04	10.94	0.178	4.619	0.734	1.002	22.04	0.365	-11.94	1.266	0.828	0.989
CVaR-GARCH-SRTR	8.32	10.95	0.185	4.580	0.760	1.376	21.53	0.386	-11.94	1.522	1.131	1.280
CVaR-GJR-SRTR	8.43	11.02	0.199	4.565	0.765	1.440	21.57	0.391	-11.94	1.532	1.207	1.326
CVaR-GARCH-Skt	8.40	11.37	0.113	4.289	0.738	1.142	22.98	0.365	-11.94	1.028	0.904	0.893
CVaR-GJR-Skt	8.42	11.54	0.087	4.244	0.730	0.996	23.71	0.355	-11.94	0.818	0.753	0.764
CVaR-GARCH-FHS	8.47	11.36	0.150	4.285	0.746	1.178	23.86	0.355	-11.94	1.115	0.979	1.029
CVaR-GJR-FHS	8.44	11.52	0.104	4.237	0.733	0.997	23.76	0.355	-11.94	0.865	0.828	0.812
RV-BAB/Short BAB	6.66	11.95	0.080	3.781	0.558	-0.541	34.24	0.195	-11.50	-1.400	-1.119	-1.373

In total, results in this section show that our simple switching strategy also works well for the BAB portfolio, both in Germany and in the US. The outperformance of the switching strategy compared to the RV managed strategy results since the switching strategy limits the crash risk during BAB crashes and captures the upside potential in calm periods. In contrast, the RV managed strategy fails to limit the downside risk during crash periods. As for the momentum portfolio, we find that the switching strategy is more beneficial when US data are used.

E.3 Additional Performance Results for Industry Momentum

In this section, we apply our risk targeting approaches to the industry momentum strategy (Chordia and Shivakumar, 2002, Grundy and Martin, 2001, Moskowitz and Grinblatt, 1999, Novy-Marx, 2012). The individual stock momentum strategy has the disadvantage that this strategy is typically invested in small sized and illiquid stocks, and hence produces high transaction costs (Lesmond et al., 2004). In contrast, industries have corresponding and highly liquid ETFs with low transaction costs (Han, 2005, Sec. 5.7). Further, the individual stock momentum strategy has to buy and sell large numbers of stocks every month, whereas the industry momentum strategy only contains a small number of industries. Consequently, industry momentum can be replicated with much lower transaction costs and is more relevant for practical implementations. Grobys et al. (2018) and Du Plessis and Hallerbach (2017) show that the risk of the industry momentum strategy can be successfully managed by the RV approach of Barroso and Santa-Clara (2015). For that reason, we will apply the RV estimator and our switching approach to the industry momentum portfolio.

Grobys et al. (2018) find that the momentum crash indicators used for the stock based momentum strategy do not work well for industry momentum. We therefore use a crash indicator based on the past performance of the industry momentum strategy, where we use the twelve months TSMOM indicator. Thus, we switch to the CVaR targeting strategy when momentum's past twelve months return is negative. Goyal and Jegadeesh (2017, Table 1) and Moskowitz et al. (2012) find good results for TSMOM with lookback periods between one and 36 months. For that reason, we also used other lookback periods between one and 36 months and found that these lookback periods also worked well.[168] However, results for other ranking periods are not shown here.

Results for the non-managed and RV managed industry momentum strategies as well as the switching strategies are shown in Table XXIII. The industry momentum strategy uses 30 equally weighted US industries, ranks industries based on their past twelve months' performance and assigns the 30% best and worst performing industries as winners and losers. Interestingly, the RV managed strategy produces a higher volatility than the non-managed strategy. This is in line with Grobys et al. (2018, Table 4) who also find a higher volatility of the RV managed strategy compared to the non-managed strategy. This result is driven by the lower volatility of the industry momentum portfolio combined with the volatility target of 12%. Nevertheless, despite the higher volatility of the risk-managed industry momentum strategy, risk targeting significantly

[168] Similarly, Moskowitz and Grinblatt (1999), Novy-Marx (2012) and Du Plessis and Hallerbach (2017) show that industry momentum also works well for other ranking periods between one and twelve months.

reduces left tail risk and exhibits a positive skewness, whereas the non-managed strategy has a skewness of -0.904. This finding is again in line with Grobys et al. (2018, Table 4) who also find that the RV managed industry momentum portfolio exhibits a positive skewness. Furthermore, the higher volatility of risk targeting is also accompanied by a significantly higher return.[169] Consequently, the Sharpe Ratio of the RV managed momentum strategy is significantly higher than the Sharpe Ratio of the non-managed industry momentum strategy, indicated by a Jobson and Korkie (1981) test statistic of 5.178. Moreover, the Sharpe Ratios of the switching strategies are all higher than the Sharpe Ratios of the non-managed and RV managed strategies. However, the increases of the switching strategies' Sharpe Ratios are not statistically significant. A possible explanation for this finding is that the industry momentum portfolio is less negatively skewed than the stock momentum strategy. As a consequence, momentum crashes of industry momentum are less severe than momentum crashes of stock momentum (Grobys et al., 2018, Table 7). Due to the lower crash risk, switching to the CVaR targeting strategy is less important for the industry momentum portfolio than for the stock momentum portfolio. Nevertheless, the switching strategies produce lower drawdowns and exhibit an improved risk-return profile compared to the RV managed strategy.

Table XXIII. Performance Results of Risk Targeting: Industry Momentum
This table shows performance results for the non-managed industry momentum portfolio, the RV managed industry momentum portfolio as well as the switching strategies applied to the industry momentum portfolio. The sample period starts in November 1930. The description of the columns is given in Table VI. The economic value is calculated for $\gamma = 10, b = 1$ and $l = 2$.

	Performance Results									Economic Value		
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Δ_{MV}	Δ_{CRRA}	Δ_{LA}
Industry Momentum	9.17	12.11	-0.904	11.364	0.758	-5.178	48.48	0.189	-25.96	-	-	-
RV	19.50	17.51	0.172	5.574	1.114	-	45.46	0.429	-24.92	-	-	-
CVaR HS	19.09	16.85	0.221	4.568	1.134	0.713	41.28	0.463	-17.92	0.899	0.376	0.193
CVaR-Skt-unc	19.14	16.86	0.230	4.595	1.135	0.762	41.55	0.461	-18.11	0.912	0.376	0.282
CVaR-GARCH-SRTR	19.71	17.20	0.144	4.278	1.146	1.168	40.81	0.483	-18.44	0.763	0.451	0.092
CVaR-GJR-SRTR	19.77	17.28	0.164	4.288	1.144	1.127	41.49	0.476	-18.21	0.665	0.451	0.118
CVaR-GARCH-Skt	19.62	17.18	0.169	4.477	1.142	1.076	42.69	0.460	-20.30	0.736	0.451	0.162
CVaR-GJR-Skt	19.63	17.35	0.190	4.674	1.131	0.669	43.83	0.448	-21.40	0.412	0.300	0.077
CVaR-GARCH-FHS	19.55	17.28	0.117	4.733	1.131	0.669	45.80	0.427	-24.27	0.478	0.225	0.047
CVaR-GJR-FHS	19.58	17.47	0.124	4.990	1.121	0.265	47.29	0.414	-25.82	0.147	0.075	-0.036
RV-Mom/Contrarian	15.18	17.82	0.237	5.354	0.852	-2.742	50.19	0.302	-24.92	-4.246	-2.594	-3.932

Similar to the findings of the Sharpe Ratio, the economic value of the CVaR switching

[169] The performance results in Grobys et al. (2018) are different to our results in Table XXIII. The reasons for these differences are that Grobys et al. (2018) use another industry data set and another cut-off point to determine winners and losers. Generally, the profitability of industry momentum strongly depends on the data set, cut-off point and ranking period (see Grundy and Martin (2001) and Rickenberg (2020c)).

strategies is positive but quite low in magnitude, where the highest fees are found for the mean-variance investors. The lower economic value for the CRRA and loss-averse investor are in line with the finding that both approaches, the RV managed strategy and the switching strategies, produce similar levels of skewness, kurtosis and maximum drawdown. In total, switching between volatility and CVaR targeting slightly enhances the performance of the industry momentum strategy, but switching to CVaR targeting in (expected) crash periods is less important for industry momentum than for stock momentum. A more accurate crash indicator for industry momentum could possibly further enhance the switching strategies' performance. As shown by Grobys et al. (2018) and Du Plessis and Hallerbach (2017), industry momentum crashes are far more difficult to predict than crashes of the stock momentum strategy. This can also be seen by the bad performance of the strategy that switches to the contrarian portfolio. Thus, further research beyond the analysis of Grobys et al. (2018) and Du Plessis and Hallerbach (2017) on industry momentum crashes is needed, which would eventually lead to an improved performance of our switching approach.[170]

E.4 Additional Performance Results for the Size Factor

Barroso and Maio (2018), Cederburg et al. (2020) and Moreira and Muir (2017) show that the RV approach works well for several factor portfolios. However, the authors show that volatility targeting does not work for the SMB (small minus big) factor. The SMB factor is long assets with a small market capitalization and short assets with a large market capitalization (Fama and French, 1993, 2012). The authors show that the RV approach applied to the SMB factor produces a worse risk-adjusted performance and a negative alpha. A reason for this finding could be the high positive skewness of the SMB factor portfolio. Generally, Cederburg et al. (2020) find that the benefits of risk targeting strongly depend on the used data set and that risk targeting performs well for portfolios that strongly deviate from normality, such as momentum or the BAB portfolio. Thus, a possible explanation for the bad performance of the RV managed SMB portfolio could be that the volatility does not distinguish between positive and negative

[170]One possibility would be to apply the regime switching approach of Daniel et al. (2017) to the industry momentum portfolio.

returns. A positive skewness indicates that positive returns are more likely than anticipated by a normal distribution. Volatility quantifies the positive returns as risk and therefore has a too low exposure to the SMB portfolio. In contrast, the CVaR targeting approach only quantifies negative returns as risk and should be superior in capturing the upside potential of the SMB portfolio. Further, although the SMB portfolio is positively skewed on average, there can still be periods with a high crash risk and high negative skewness. By switching to CVaR targeting in these periods, the SMB portfolio's performance could potentially be enhanced. For these reasons, we next apply our risk targeting approaches to the the SMB factor.

Table XXIV. Performance Results of Risk Targeting: Size Factor
This table shows performance results for the non-managed SMB factor, the RV managed SMB factor and the switching strategies for the SMB factor using the 12 months TSMOM crash indicator. The sample period starts in November 1930. The description of the columns is given in Table VI. The economic value is calculated for $\gamma = 10, b = 1$ and $l = 2$.

	Performance Results									Economic Value		
Model	Return	Vola	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Δ_{MV}	Δ_{CRRA}	Δ_{LA}
SMB Factor	2.41	11.09	1.996	22.708	0.218	1.601	55.04	0.044	-16.87	-	-	-
RV	2.35	17.39	0.454	5.319	0.135	-	82.40	0.029	-21.54	-	-	-
CVaR HS	3.64	16.20	0.364	5.385	0.225	**2.864**	75.58	0.048	-22.97	3.156	0.828	2.436
CVaR-Skt-unc	3.69	16.00	0.381	5.490	0.231	**3.037**	73.05	0.051	-22.97	3.500	0.904	2.599
CVaR-GARCH-SRTR	2.85	16.48	0.226	5.348	0.173	1.224	80.83	0.035	-22.97	1.942	0.150	1.373
CVaR-GJR-SRTR	2.86	16.59	0.236	5.266	0.172	1.236	79.83	0.036	-22.97	1.777	0.150	1.285
CVaR-GARCH-Skt	3.28	16.40	0.319	5.231	0.200	**2.195**	75.06	0.044	-22.97	2.477	0.526	1.751
CVaR-GJR-Skt	3.25	16.29	0.318	5.234	0.199	**2.074**	73.18	0.044	-22.97	2.624	0.526	1.834
CVaR-GARCH-FHS	3.39	16.20	0.305	5.245	0.209	**2.448**	73.47	0.046	-22.97	2.896	0.602	1.975
CVaR-GJR-FHS	3.34	16.22	0.326	5.247	0.206	**2.250**	72.73	0.046	-22.97	2.825	0.602	1.983
RV-SMB/Short SMB	5.02	17.32	0.058	5.446	0.290	1.123	75.77	0.066	-27.48	2.741	1.358	2.659

Results for the SMB factor are shown in Table XXIV, where we use the 12 months TSMOM crash indicator. In line with the findings of Moreira and Muir (2017), Cederburg et al. (2020) and Barroso and Maio (2018), we find that increasing the performance of the SMB factor by risk targeting is challenging. The RV managed strategy clearly underperforms the non-managed strategy with a Jobson and Korkie (1981) test statistic of 1.601. The RV managed strategy exhibits a lower return with a significantly higher volatility and drawdown. Thus, as expected, due to the positive skewness of the SMB factor, the RV managed portfolio does not capture the upside potential of the SMB factor. Even more important, during crash periods, the RV managed portfolio increases the SMB factor's crash risk, which leads to a maximum drawdown of 82.40%, compared to a drawdown of 55.04% of the non-managed strategy. In contrast, our switching strategy regards the SMB portfolio's non-normalities and significantly outperforms the RV managed strategy. In terms of the Sharpe Ratio and Calmar Ratio, the RV managed strategy is clearly outperformed by the switching strategies. This can also be seen by the high economic values, i.e. investors are willing to pay economically high fees to switch from the RV managed strategy to the switching approach. Furthermore, the switching strategies produce even higher returns than the non-managed SMB factor, but these strategies do not outperform the non-managed strategy in terms of the risk-adjusted return, since the switching strategies also take higher risk. Thus, as stated by Cederburg et al. (2020), the benefits of risk targeting strongly depend on the used data set and increasing the SMB portfolio's performance by risk targeting is quite challenging. In total, based on our results, our conjecture is that risk targeting, and especially our CVaR switching approach, works the best for portfolios with high left tail risk, such as the stock momentum strategy and the BAB portfolio. This is in line with the results of Harvey et al. (2018) who find that volatility targeting works well for "risk assets (e.g. equity and credit)". In contrast, for portfolios with a lower crash risk, risk targeting becomes less important.

To summarize results in Appendix E, risk targeting, especially strategies that switch between volatility and CVaR targeting, do not only work well for the equally weighted momentum strategy in the US, but also for other construction methods of the momentum portfolio, momentum in Germany, the BAB portfolio of Frazzini and Pedersen (2014) in the US and Germany as well as the industry momentum portfolio. In particular, switching between volatility and CVaR targeting successfully heightens returns and investors' utility while left tail risk is simultaneously reduced. Nevertheless, increasing the performance of the SMB factor by risk targeting is still challenging. However, switching between volatility and CVaR targeting clearly outperforms the RV approach of Barroso and Maio (2018) and Moreira and Muir (2017), even for the SMB factor.

Chapter 3

Tail Risk Weighted Momentum Strategies

3.1 Introduction

Jegadeesh and Titman (1993) find that stocks which performed well in the past tend to outperform stocks with a low previous months' performance. Following the seminal paper of Jegadeesh and Titman (1993), momentum investing has been extensively examined in the financial literature. For example, Jegadeesh and Titman (1993), Jegadeesh and Titman (2001), Hong et al. (2000), Novy-Marx (2012), Boguth et al. (2011), Fama and French (2016) and Lesmond et al. (2004) examine stock momentum in the US market, Rouwenhorst (1998) and Nijman et al. (2004) examine stock momentum in Europe, Griffin et al. (2003), Fama and French (2012) and Goyal and Wahal (2015) examine stock momentum internationally, Clare et al. (2014) examine commodity momentum and Asness et al. (2013) and Clare et al. (2016) examine momentum for international stocks, country indices, currencies, bonds and commodities. Moreover, several studies show that momentum investing also works well for portfolios of stocks instead of individual stocks. See, for example, Moskowitz and Grinblatt (1999), Lewellen (2002), Chordia and Shivakumar (2002), Grundy and Martin (2001), Stivers and Sun (2010), Swinkels (2002), Nijman et al. (2004) and Grobys et al. (2018) for momentum of industry portfolios, Asness et al. (2013), Richards (1997), Novy-Marx (2012), Goyal and Wahal (2015), Nijman et al. (2004) and Bhojraj and Swaminathan (2006) for momentum of country indices, Novy-Marx (2012), Stivers and Sun (2010) and Lewellen (2002) for momentum of investment styles, Carhart (1997) for momentum of mutual funds and Bali et al. (2012) for momentum of hedge funds. Portfolio based momentum strategies are appealing from a practical perspective, since individual stock momentum strategies are typically highly invested in small sized and illiquid firms. Therefore, momentum strategies using individual equities are not profitable once realistic transaction costs are considered (Korajczyk and Sadka, 2004, Lesmond et al., 2004). In contrast, portfolios that replicate industries and countries typically have corresponding and highly liquid ETFs with low costs (Han, 2005, O'Neal, 2000, Richards, 1997). Further, individual stock momentum strategies have to buy large numbers of stocks, whereas only few trades have to be done by momentum strategies using portfolios. Thus, in practice, momentum strategies based on portfolios are far less sensitive to transaction costs.[1] Moskowitz and Grinblatt (1999, Table 2) and George and Hwang (2004, Table 1) find that, even before transaction costs were considered, industry momentum is as profitable as individual momentum. Hence, from a practical view, momentum strategies based on portfolios are more appealing than momentum strategies using individual stocks.

One drawback of the momentum strategy is the occurrence of "momentum crashes", i.e. periods of extremely large negative returns (Daniel et al., 2017, Daniel and Moskowitz, 2016). For example, Barroso and Santa-Clara (2015) find a skewness of -2.47 and a kurtosis of 18.24 for the value-weighted individual stock momentum strategy. This high left tail risk implies a high likelihood of extremely negative returns, e.g. Barroso and Santa-Clara (2015) find a minimum monthly return of -78.96%, a minimum two months' return of -91.59% and that the recovery from a momentum crash can last up to 31 years. Similarly, using an equally weighted momentum strategy, Rickenberg (2020a) finds a skewness of -4.38, a kurtosis of 43.59, a minimum monthly return of -89.70% and a maximum drawdown of 99.31%. A drawdown of this size requires a return $14{,}392.75\%$ to compensate for such a huge loss. Momentum crashes are typically less severe for the industry momentum strategy, but even this strategy exhibits a high crash risk. For example, Grobys et al. (2018, Table 4) find a minimum monthly return of -62.75%, a skewness of -1.50 and a kurtosis of 22.91 for the industry momentum portfolio. Furthermore, Min and Kim (2016) find that momentum crashes typically occur in periods when

[1] See, for example, Lesmond et al. (2004, p. 375) who conclude that "returns associated with relative strength investing strategies (buying past winners and selling past losers) do not exceed trading costs", but the authors also state that this result does not necessarily hold for the industry momentum strategy. O'Neal (2000) find that industry momentum is profitable even after considering high transaction costs.

investors are highly risk-averse and the authors conclude that "momentum strategies expose investors to greater downside risk". This high left tail risk makes momentum investing unappealing for investors who dislike negatively skewed and fat-tailed return distributions (Guidolin and Timmermann, 2008, Kraus and Litzenberger, 1976, Scott and Horvath, 1980). Further, most investors are loss-averse, i.e. they weight losses higher than gains of the same magnitude (Aït-Sahalia and Brandt, 2001, Benartzi and Thaler, 1995) and investors are averse to crashes (Bollerslev and Todorov, 2011, Chabi-Yo et al., 2018, Van Oordt and Zhou, 2016). This holds especially for momentum investors since they typically hold leveraged positions (Chabot et al., 2014). Thus, the momentum strategy is unappealing for most investors unless momentum's risk is managed and momentum crashes are attenuated. In order to manage momentum's crash risk, several approaches have been used in the literature. Grundy and Martin (2001) and Martens and Van Oord (2014) use momentum's beta to hedge momentum's risk. Further, several studies suggest to switch to other assets like the value strategy, contrarian strategy or the risk-free rate if a momentum crash is likely (Barroso and Maio, 2019, Chabot et al., 2014, Daniel et al., 2017). Moreover, Barroso and Santa-Clara (2015), Moreira and Muir (2017), Barroso and Maio (2018), Cederburg et al. (2020), Du Plessis and Hallerbach (2017), Grobys et al. (2018), Grobys and Kolari (2020) and Daniel and Moskowitz (2016) use a volatility targeting approach to manage the risk of the momentum portfolio. Similarly, Rickenberg (2020a) also uses a volatility targeting approach but switches to a tail risk managed strategy when the probability of a momentum crash increases. Thus, managing the crash risk of the momentum strategy has recently emerged to an important topic in the financial literature. However, most approaches examined in the literature so far manage momentum's risk on a portfolio level, ignoring information on the assets that are contained in the momentum portfolio. That is, momentum strategies examined in the literature are mainly based on simple weighting schemes of the assets in the winners and losers portfolios, where assets within the winners and losers portfolio are typically equally weighted (Chordia and Shivakumar, 2002, Grundy and Martin, 2001, Hong et al., 2000, Jegadeesh and Titman, 1993, 2001, Lesmond et al., 2004, Moskowitz and Grinblatt, 1999, Rachev et al., 2007, Rickenberg, 2020a, Swinkels, 2002), value-weighted (Barroso and Santa-Clara, 2015, Novy-Marx, 2012, Richards, 1997) or weighted based on past performance (Bhojraj and Swaminathan, 2006, Chan et al., 2000, Conrad and Kaul, 1998, Lewellen, 2002, Moskowitz and Grinblatt, 1999).

More recently, Clare et al. (2014), Du Plessis and Hallerbach (2017) and Goyal and Jegadeesh (2017) show that weighting assets within the winners and losers portfolios inversely to their risk, measured by their past Realized Volatility, increases the (risk-adjusted) performance and lowers left tail risk compared to classical weighting schemes. This simple inverse risk weighting was also used by Moskowitz et al. (2012), Baltas (2015), Clare et al. (2014), Clare et al. (2016), Goyal and Jegadeesh (2017), Dudler et al. (2015), Du Plessis and Hallerbach (2017) and Kim et al. (2016) for the time series momentum (TSMOM) strategy and by Asness et al. (2013) to weight different momentum portfolios in a global momentum portfolio. Generally, the economic value of the inverse volatility weighting has been extensively examined in the literature (Fleming et al., 2001, 2003, Han, 2005, Kirby and Ostdiek, 2012, Taylor, 2014). Frazzini and Pedersen (2014), Asness et al. (2014) and Asness et al. (2020) also use a weighting scheme where assets with higher risk obtain lower weights than assets with lower risk, but the authors measure risk by beta and correlation instead of volatility. Inverse risk weightings are motivated by the "low risk anomaly", i.e. the finding that assets with lower risk outperform assets with higher risk (Ang et al., 2006b, 2009, Asness et al., 2020, Atilgan et al., 2020, Bali et al., 2017a, Baltussen et al., 2018, Cederburg and O'Doherty, 2016, Frazzini and Pedersen, 2014, Haugen and Heins, 1975, Schmielewski and Stoyanov, 2017, Schneider et al., 2020). Weighting assets in the momentum portfolio inversely to their volatility is appealing since the momentum strategy is typically invested in assets with a higher than average volatility (Rouwenhorst, 1998). Ignoring information on an asset's risk can lead to a portfolio where the portfolio's risk is mainly determined by a few assets. In particular, inverse risk weightings are similar to the well-known mean-variance and minimum variance approaches.[2] However, the mean-variance approach suffers under high estimation risk of mean returns (Merton, 1980)

[2]Minimum variance portfolios have recently become popular in the financial industry (Clarke et al., 2006). The minimum variance portfolio is a special case of the mean-variance portfolio and is the unique portfolio on the mean-variance efficient frontier that is independent of the mean return (Merton, 1972). A portfolio strategy that is again similar to the minimum variance portfolio is risk parity. Inverse volatility portfolios then follow from the risk parity approach by ignoring information on the assets' correlations or assuming that all correlations are equal (Baltas, 2015, Maillard et al., 2010).

and produces an inferior out-of-sample performance (DeMiguel et al., 2009a,b, Garlappi et al., 2006, Jagannathan and Ma, 2003, Kan and Zhou, 2007, Tu and Zhou, 2011).[3] In contrast to the mean-variance optimization, inverse risk weighting strategies have the advantage that no mean estimate is needed.[4] In particular, combining momentum with inverse risk weightings can also be seen as an alternative portfolio allocation approach that is far less sensitive to estimation risk, since no direct mean estimate is needed, but not all the information on the assets' return is ignored.[5]

Although estimating variances is far less prone to estimation risk than estimating mean returns, weighting assets by their estimated risk is also sensitive to misspecifications, especially when the number of assets increases (Kan and Zhou, 2007, Table 1). Thus, the inverse risk weightings are also sensitive to estimation errors of variances and can produce highly volatile portfolio weights (Kirby and Ostdiek, 2012). For that reason, we extend the inverse volatility weighting to a rank based weighting scheme that is more robust to estimation risk and also weights highly volatile assets lower than low volatile assets.[6] Forecasting an asset's cross-

[3] Alternatives to the classical mean-variance approach are mean-variance portfolios using more robust estimates of the mean return and the covariance matrix (DeMiguel et al., 2009a,b, Garlappi et al., 2006, Jagannathan and Ma, 2003, Kan and Zhou, 2007, Tu and Zhou, 2011). These studies mainly use Bayesian and shrinkage estimators. Furthermore, portfolio strategies that are combinations of different weighting schemes are also frequently used. For example, Kan and Zhou (2007, Sec. III) switch between the mean-variance and minimum variance portfolio, where more weight is placed on the minimum variance approach when estimation risk of the mean return increases (see also Tu and Zhou (2011), DeMiguel et al. (2009b) and Garlappi et al. (2006) for similar switching strategies).

[4] Other volatility based portfolio allocation methods that are independent of the mean return are "low volatility strategies" that rank assets by their risk and only buy assets with the lowest risk (Blitz, 2016, Blitz and Van Vliet, 2007, Blitz et al., 2019, Blitz and Vidojevic, 2017, Chow et al., 2014). These simple low volatility portfolios are frequently used by practitioners. The good performance of volatility based investment strategies has led to numerous financial products replicating these strategies. See, for example, the ETFs offered by iShares (`https://www.blackrock.com/us/individual/investment-ideas/what-is-factor-investing/minimum-volatility`) and the funds offered by Robeco (`https://www.robeco.com/en/themes/low-volatility-investing/`).

[5] This strategy selects the assets that are contained in the portfolio based on an estimate of the *relative mean* and then weights these assets solely based their risk characteristics. Forecasting relative returns and risk is far less sensitive to estimation risk than forecasting absolute returns (Christoffersen and Diebold, 2006). A long-only strategy that only buys the winners portfolio, where the assets in the winners portfolio are inverse risk weighted can also be interesting for practitioners with short-sale constraints. This strategy is appealing since shorting the losers portfolio induces high transaction costs (Clare et al., 2016, Korajczyk and Sadka, 2004). We also examine the long-only risk weighted winners portfolio in Appendix B.13 and compare these strategies to the mean-variance and minimum variance approach. We find that our two stage approach outperforms the mean-variance approach in terms of statistically higher Sharpe Ratios and high utility gains for investors. Combining information on an asset's return and risk in a long-only asset allocation setting has also been examined by Blitz and van Vliet (2018) and Clare et al. (2016).

[6] Although our rank based weighting does not weight assets inversely to their risk, we also subsume this weighting scheme under the inverse risk weighting. Hence, when we write "inverse risk weightings" or "weighting assets inversely to their risk" we mean either weighting assets according to the "true" inverse risk weighting or according

sectional risk rank is much easier than forecasting an asset's risk (Langlois, 2020). Similar rank based weighting schemes were also used by Asness et al. (2013), Asness et al. (2014), Asness et al. (2020), Frazzini and Pedersen (2014), Liu et al. (2018) and Schneider et al. (2020).

Applying the inverse volatility weighting to the momentum portfolio slightly improves the portfolio's risk-return profile. However, using volatility as a risk measure has several disadvantages and is (at least implicitly) based on the assumption that returns are normally distributed. In a portfolio setting, managing volatility means managing return deviations, whereas most investors are concerned about (extremely) negative returns, which should be the main objective of portfolio risk management (Agarwal and Naik, 2004, Alexander and Baptista, 2004, Basak and Shapiro, 2001, Lee and Rao, 1988). In particular, by managing an asset's volatility, higher moments like skewness and kurtosis are not taken into account. The importance of incorporating skewness and kurtosis have been shown in many fields of finance, e.g. asset pricing (Bali et al., 2009, Dittmar, 2002, Harvey and Siddique, 1999, 2000, Jondeau et al., 2019, Kraus and Litzenberger, 1976, Langlois, 2020, Schneider et al., 2020), portfolio selection (Ghysels et al., 2016, Guidolin and Timmermann, 2008, Wang et al., 2012), option pricing (Barone-Adesi et al., 2008) and risk management (Bali et al., 2008). Moreover, since momentum investing is a long-short strategy that buys winners and sells losers, an asset's risk should be managed differently based on the information of whether the asset is in the long or short leg. Thus, for an asset in the winners portfolio risk should be defined as downside risk, whereas for an asset in the losers portfolio risk should be defined as upside potential (Bollerslev et al., 2020, 2015, Giot and Laurent, 2003). This is not regarded by the inverse volatility weighting. The aim of risk weighting applied to the momentum strategy should be to *enhance* the performance of the winners portfolio and to *worsen* the performance of the losers portfolio. Since inverse volatility weighting typically enhances the performance of a portfolio, the performance gain of buying the enhanced winners portfolio is (partly) offset by shorting the enhanced losers portfolio.[7] We

to the rank based weighting.

[7] For example, Ang et al. (2006b, p. 292) suggest that "one way to improve the returns to a momentum strategy is to short past losers with high idiosyncratic volatility." The inverse volatility weighting approach of Clare et al. (2014), Du Plessis and Hallerbach (2017) and Goyal and Jegadeesh (2017) obviously does the opposite. In contrast, Asness et al. (2014), Asness et al. (2020) and Frazzini and Pedersen (2014) use a rank based weighting that regards if an asset is a long or short position. For example, Frazzini and Pedersen (2014, p. 9) examining a portfolio that buys low beta assets and sells high beta assets write: "In each portfolio, securities are weighted by the ranked

therefore extend the inverse volatility weighting to inverse risk weightings that take these disadvantages of the volatility weighting into account. Thus, we weight assets inversely to their risk based on risk measures that take non-normalities into account and that regard if an asset is a long or short position.[8] To measure an asset's tail risk we rely on several frequently used univariate risk measures, namely skewness (Amaya et al., 2015, Chen et al., 2001, Jondeau et al., 2019, Langlois, 2020), kurtosis (Amaya et al., 2015), Lower Partial Moments (Arzac and Bawa, 1977, Bawa and Lindenberg, 1977, Lee and Rao, 1988), Value at Risk (Allen et al., 2012, Atilgan et al., 2020, Bali et al., 2009, 2008) and Conditional Value at Risk (Alexander and Baptista, 2004, Allen et al., 2012, Bali et al., 2009, Basak and Shapiro, 2001). Further, to account for an asset's downside risk *and* upside potential we also examine "reward-to-risk" timing strategies that are similar to the strategies examined by Kirby and Ostdiek (2012) and Zakamulin (2017). These strategies weight assets by their difference of up and down semivariance (Bollerslev et al., 2019, Patton and Sheppard, 2015), down-to-up volatility (Chen et al., 2001), down-to-up skewness and R-Ratio (Rachev et al., 2007). These reward-to-risk timing strategies have the advantage that information on the assets' return potential is also regarded without relying on a noisy estimate of the assets' mean return.

Additionally to weighting the assets in the momentum portfolio based on estimates of their *own* risk using univariate risk measures, we further use weighting schemes based on systematic (tail) risk. Our systematic (tail) risk measures quantify an asset's comovement with the equally weighted momentum portfolio, where we additionally condition on bad states of the momentum portfolio and differentiate between long and short positions.[9] Thus, our systematic tail risk measures quantify how much an asset in the winners (losers) portfolio decreases (in-

betas (i.e., lower-beta securities have larger weights in the low-beta portfolio and higher-beta securities have larger weights in the high-beta portfolio)." Hence, lower risk assets are weighted higher in the long portfolio, whereas higher risk assets are weighted higher in the short portfolio.

[8] A long-only strategy that only buys the winners and applies a low risk weighting based on risk measures that account for non-normalities can also be an appealing alternative to mean-downside risk approaches as examined by Agarwal and Naik (2004), Cuoco et al. (2008), Basak and Shapiro (2001), Bawa and Lindenberg (1977), Lee and Rao (1988) and Price et al. (1982). This strategy is examined in Appendix B.13.

[9] Systematic (tail) risk is typically measured with respect to the market. However, since market crashes and momentum crashes typically do not occur simultaneously (Daniel and Moskowitz, 2016, Table 2), measuring systematic tail risk with respect to the momentum portfolio is more reasonable. Acharya et al. (2016), Engle et al. (2015) and Harvey and Siddique (2000) also measure systematic risk with respect to other benchmarks than the market portfolio.

creases) when the momentum portfolio exhibits a negative return. Based on these measures, assets in the winners (losers) portfolio that perform well (bad) during a momentum crash are weighted higher than assets that strongly decrease (increase) when the momentum portfolio suffers extremely negative returns. As systematic risk measures we use an asset's beta (Cederburg and O'Doherty, 2016, Frazzini and Pedersen, 2014, Lettau et al., 2014), correlation (Asness et al., 2020), coskewness (Harvey and Siddique, 2000, Kraus and Litzenberger, 1976, Langlois, 2020), cokurtosis (Dittmar, 2002), downside beta (Ang et al., 2006a, Lettau et al., 2014), downside correlation (Ang and Chen, 2002, Hong et al., 2007), LPM-beta (Bali et al., 2014, Bawa and Lindenberg, 1977, Lee and Rao, 1988), HTCR-beta (Bali et al., 2014), Tail-Sens (Agarwal et al., 2017, Chabi-Yo et al., 2018, Poon et al., 2004, Weigert, 2015), Tail-Risk (Agarwal et al., 2017), Tail-beta (Van Oordt and Zhou, 2017, 2016) and Marginal Expected Shortfall (Acharya et al., 2012, 2016, Brownlees and Engle, 2016, Engle et al., 2015).

Since the systematic tail risk measures condition on bad states of the momentum portfolio, this weighting scheme is expected to perform well in periods of a momentum crash. However, in calm periods, measuring systematic tail risk should not provide much information on an asset's risk. Even more important, a higher comovement of an asset with the momentum portfolio is desired in calm periods and downweighting these assets is disadvantageous. In contrast, the univariate risk measures do not condition on the return of the momentum portfolio and are independent of the state of the momentum portfolio. Therefore, we additionally examine an approach that switches between the weighting schemes based on univariate and systematic (tail) risk measures, where the systematic (tail) risk based weighting is only used when a momentum crash is likely. The benefits of combining several portfolio strategies have been shown by Kan and Zhou (2007), Garlappi et al. (2006), DeMiguel et al. (2009b), Tu and Zhou (2011), Rickenberg (2020b) and Rickenberg (2020a). In order to estimate the expected momentum state, we rely on the equally weighted momentum portfolio's own volatility, which is a negative predictor for momentum's future return (Barroso and Maio, 2019, Barroso and Santa-Clara, 2015, Grobys et al., 2018). Other alternatives to indicate momentum crashes are the simple time series momentum strategy of Moskowitz et al. (2012), past market return or past market volatility.[10]

Applying low risk weightings to the momentum strategy should provide an enhanced risk-return profile by incorporating information on each asset's risk in the momentum portfolio. However, since correlations between different assets typically increase simultaneously in bad market states, i.e. assets typically crash together, crash risk cannot completely be mitigated by different weighting schemes (Ang and Chen, 2002, Chabi-Yo et al., 2018, Poon et al., 2004). Therefore, the risk weighted momentum strategies still exhibit a high crash risk and other tail risk protection methods are needed. Barroso and Santa-Clara (2015), Cederburg et al. (2020), Barroso and Maio (2018), Moreira and Muir (2017) and Rickenberg (2020a) show that the momentum portfolio's risk can simply be managed by targeting a constant level of portfolio volatility. This approach has also been used to target the risk of the industry momentum portfolio (Du Plessis and Hallerbach, 2017, Grobys, 2018, Grobys and Kolari, 2020, Grobys et al., 2018). Barroso and Santa-Clara (2015) and Barroso and Maio (2019) show that a higher volatility of the momentum portfolio is correlated with negative future momentum returns.[11] Hence, in times of a high momentum volatility an investor should decrease the amount invested in the long and short leg. Thus, by simply reducing the exposure to the momentum portfolio when momentum's risk increases, momentum crashes can be attenuated. In particular, this approach is solely based on information of the whole portfolio and is independent of the portfolio allocation process. Thus, the low risk weightings and the volatility targeting approach can easily be combined as shown by Baltas (2015), Zakamulin (2015), Harvey et al. (2018) and Moreira and Muir (2017, I.E). In a first stage, the portfolio's composition is determined based on the low risk weighting. In a second stage, the portfolio's overall risk is managed based on the portfolio's

[10]The drivers of momentum crashes for the individual stock momentum strategy have been frequently examined in the literature (Cooper et al., 2004, Daniel et al., 2017, Daniel and Moskowitz, 2016, Grundy and Martin, 2001, Min and Kim, 2016, Wang and Xu, 2015). Momentum crashes typically occur when the past market return is negative and/or past market volatility is high. Grobys et al. (2018) find that these indicators do not well predict momentum crashes of the industry momentum strategy. In contrast, Du Plessis and Hallerbach (2017) find that a higher market volatility predicts negative industry momentum returns. We find that indicators using the equally weighted momentum portfolio's past return or volatility and indicators using the market's past return or volatility all produce good results. However, we find that the indicator based on the industry momentum's volatility produces the most convincing risk-return profile. Grobys et al. (2018) also find that the industry momentum portfolio's volatility is a good predictor of industry momentum crashes. Robustness results for alternative definitions of the crash indicator are shown in the appendix.

[11]Interestingly, volatility targeting can also be advantageous when volatility and return are unrelated or even positively related (Dachraoui, 2018, Moreira and Muir, 2017).

volatility. This approach simply separates the asset allocation process from the portfolio risk management process (Agarwal and Naik, 2004). Du Plessis and Hallerbach (2017) show that both approaches, volatility weighting and volatility targeting, enhance the industry momentum portfolio's performance. However, the authors do not use both strategies simultaneously. Combining both approaches is appealing since both capture different aspects of risk (Moreira and Muir, 2017, Sec. II.D).

The paper is structured as follows. Section 3.2 reviews the literature on industry momentum, momentum crashes and presents the equal and volatility weighting schemes used in the literature so far. Section 3.3 extends the inverse volatility weighting to inverse tail risk weightings using univariate tail risk measures. Section 3.4 develops inverse risk weightings based on systematic (tail) risk measures that measure the comovement of an asset with the (equally weighted) momentum portfolio. Section 3.5 presents portfolio strategies that are combinations of the univariate and systematic risk weightings. Section 3.6 shows how these weighting schemes can be combined with the risk targeting approach. Section 3.7 shows the empirical results and Section 3.8 concludes the paper. Additional empirical results and robustness checks are shown in the appendix.

3.2 Momentum Strategy: Equal and Volatility Weighting, Industry Momentum and Momentum Crashes

This section shortly summarizes the literature on momentum investing. We first summarize in Section 3.2.1 the equally weighted momentum strategy, which received a lot of attention since the seminal paper of Jegadeesh and Titman (1993). In Section 3.2.2, we summarize the literature on the industry momentum strategy of Moskowitz and Grinblatt (1999), which is mainly used in this paper and we compare this strategy to the individual stock based momentum strategy of Jegadeesh and Titman (1993). Section 3.2.3 summarizes the literature on momentum crashes and the high left tail risk of the momentum strategy. Finally, in Section 3.2.4 we present the novel approach introduced by Clare et al. (2014), Du Plessis and Hallerbach (2017) and Goyal and Jegadeesh (2017) that weights assets in the winners and losers portfolio inversely to their volatility.

3.2.1 Equally Weighted Momentum Strategy

This section presents the equally weighted momentum strategy of Jegadeesh and Titman (1993) and shortly reviews the literature on this strategy. Momentum investing is based on the assumption that stocks which performed well in the past will outperform stocks which performed poorly in the past.[12] As in Jegadeesh and Titman (1993), Rouwenhorst (1998, p. 269) and Jegadeesh and Titman (2001), we rank assets by their previous J months' performance. The top $100 \cdot p\%$, $p \in [0, 0.5]$, of these ranked assets are assigned as the winners portfolio and the bottom $100 \cdot p\%$ are assigned as the losers portfolio. The assets within both portfolios are then equally weighted.[13] A zero-investment strategy, called the winners minus losers (WML) portfolio, is then built by buying past winners which is financed by selling past losers.[14] This portfolio is then held for K months, where we choose a holding period K of one month. A holding period of one month is frequently used in the momentum literature (Barroso and Santa-Clara, 2015, Daniel and Moskowitz, 2016, Fama and French, 1996, 2012, Novy-Marx, 2012).[15] The mo-

[12]The observation that assets move in trends can be explained by several behavioral characteristics of investors. See Clare et al. (2016, Sec. 2) for a short summary of some these explanations.

[13]Equally weighted winners and losers portfolios are frequently used in the momentum literature (Chordia and Shivakumar, 2002, Fama and French, 1996, Grobys et al., 2018, Grundy and Martin, 2001, Hong et al., 2000, Jegadeesh and Titman, 1993, 2001, Korajczyk and Sadka, 2004, Lesmond et al., 2004, Moskowitz and Grinblatt, 1999, Rachev et al., 2007, Rickenberg, 2020a, Swinkels, 2002). Alternatives to the equally weighted winners and losers portfolios, which are also used in the literature, are value-weighted portfolios (Barroso and Santa-Clara, 2015, Novy-Marx, 2012, Richards, 1997), past return based weighted portfolios (Asness et al., 2013, Bhojraj and Swaminathan, 2006, Chan et al., 2000, Conrad and Kaul, 1998, Jegadeesh and Titman, 2002, Lewellen, 2002, Moskowitz and Grinblatt, 1999, Pan et al., 2004), weights based on market value and past return (Chan et al., 2000, Eq. (5)), liquidity weighted portfolios (Korajczyk and Sadka, 2004) or double sorted portfolios based on size and return (Fama and French, 1996, 2012).

[14]The (cross-sectional) momentum strategy is by construction similar to the time series momentum strategy (TSMOM) examined by Moskowitz et al. (2012), Du Plessis and Hallerbach (2017), Goyal and Jegadeesh (2017), Kim et al. (2016), Baltas (2015), Clare et al. (2014), Clare et al. (2016) and Dudler et al. (2015). Both strategies, cross-sectional momentum and TSMOM, are extensively compared by Du Plessis and Hallerbach (2017), Moskowitz et al. (2012, Sec. 5), Goyal and Jegadeesh (2017) and Kim et al. (2016). Conrad and Kaul (1998) find that a huge part of the profitability of momentum investing comes from cross-sectional differences in expected returns. However, Jegadeesh and Titman (2002) find that this conclusion is due to a bad research design and that momentum profits are mainly driven by time series momentum of the individual stocks. Moskowitz et al. (2012, Table 5.B) also find that (cross-sectional) momentum is driven by TSMOM and not by cross-sectional variation in mean returns. Moskowitz and Grinblatt (1999) and Pan et al. (2004) confirm this finding for momentum strategies based on industry portfolios.

[15]Jegadeesh and Titman (1993), Jegadeesh and Titman (2001) and Rouwenhorst (1998) use a ranking and holding period of six months. In this case, the momentum portfolio invests each month $1/6$ of the current wealth in six different momentum portfolios. The aggregate momentum portfolio's return is then given as the average return of six portfolios each starting one month apart (see Rouwenhorst (1998, p. 269) for example).

mentum effect typically holds for holding periods between one and twelve months, but reverses for holding periods longer than twelve months (Conrad and Kaul, 1998, Jegadeesh and Titman, 1993, 2001, Richards, 1997, Rouwenhorst, 1998).[16] Moskowitz et al. (2012) also find good results of the one month holding period for the TSMOM strategy and that the TSMOM effect is weaker for longer holding periods.

More formally, for a basket of $m \in \mathbb{N}$ assets, the winners and losers portfolios each contain $n = \lceil p \cdot m \rceil$ assets. We denote the month t return of asset $i \in \{1, ..., n\}$ in the winners and losers portfolios by $R_{i,t}^W$ and $R_{i,t}^L$, respectively. The returns R_t^W and R_t^L of the winners and losers portfolios in month t using equal weights are then given by

$$R_t^W = \sum_{i=1}^{n} w_{i,t}^W \cdot R_{i,t}^W = \frac{1}{n} \sum_{i=1}^{n} R_{i,t}^W \quad \text{and} \quad R_t^L = \sum_{i=1}^{n} w_{i,t}^L \cdot R_{i,t}^L = \frac{1}{n} \sum_{i=1}^{n} R_{i,t}^L, \tag{3.2.1}$$

where the weights in month t of asset i in the winners and losers portfolios are given by $w_{i,t}^W = \frac{1}{n}$ and $w_{i,t}^L = \frac{1}{n}$ (Goyal and Jegadeesh, 2017, Eq. (1)). The return R_t of the winners minus losers (WML) portfolio in month t is then given by

$$R_t^{WML} = R_t^W - R_t^L = \frac{1}{n} \sum_{i=1}^{n} R_{i,t}^W - R_{i,t}^L. \tag{3.2.2}$$

Equation (3.2.1) shows that the portfolio weights of the momentum portfolio are solely determined by the choice of p and the assets' past performance.[17]

The profitability of the momentum strategy as presented above has been first shown for individual stocks in the US (Boguth et al., 2011, Hong et al., 2000, Jegadeesh and Titman, 1993, 2001, Lesmond et al., 2004, Novy-Marx, 2012). However, the momentum effect also

[16]Similar to the momentum strategy based on the assets' past return, momentum could also be defined relative to the assets' 52 week high. George and Hwang (2004), Grobys (2018) and Gupta et al. (2010) examine a strategy that buys assets that are close to their 52 week high and sells assets that are far from their 52 week high. George and Hwang (2004, Table 1) find that this strategy is as profitable as the momentum strategy defined above. Interestingly, the long-term reversal, which is found for the return based momentum strategy, does not hold for the 52 week high strategy.

[17]The weight of an asset in the WML portfolio is $\frac{1}{n}$ if the asset belongs to the winners portfolio, $-\frac{1}{n}$ if the asset belongs to the losers portfolio and zero if the asset belongs to the middle part (see also Bhojraj and Swaminathan (2006, Footnote 9)). Other possibilities of determining the weights of the assets based on an asset's past return are examined by Chan et al. (2000), Conrad and Kaul (1998), Moskowitz and Grinblatt (1999), Bhojraj and Swaminathan (2006), Lewellen (2002), Jegadeesh and Titman (2002, p. 145), Moskowitz et al. (2012, p. 241) and Goyal and Jegadeesh (2017, Eq. (11)). In this case, an asset with a higher past performance is weighted higher in the winners portfolio. In contrast, an asset with a lower past performance has a higher weight (in absolute values) in the losers portfolio. Similarly, Asness et al. (2013) use a rank based weighting scheme by ranking assets based on their past performance. In contrast to our approach, where the momentum portfolio only consists of the extreme (positive or negative) performers, these approaches assign each asset as either a winner or loser.

holds for European stocks (Nijman et al., 2004, Rouwenhorst, 1998), for International stocks (Asness et al., 2013, Fama and French, 2012, Goyal and Wahal, 2015, Griffin et al., 2003), for currencies, bonds and commodities (Asness et al., 2013, Clare et al., 2014, 2016), for industry portfolios (Chordia and Shivakumar, 2002, Du Plessis and Hallerbach, 2017, Grobys et al., 2018, Grundy and Martin, 2001, Lewellen, 2002, Moskowitz and Grinblatt, 1999, Nijman et al., 2004, Novy-Marx, 2012, Swinkels, 2002), for country indices (Asness et al., 2013, Bhojraj and Swaminathan, 2006, Goyal and Wahal, 2015, Nijman et al., 2004, Novy-Marx, 2012, Richards, 1997, Stivers and Sun, 2010), for investment styles (Lewellen, 2002, Novy-Marx, 2012, Stivers and Sun, 2010), for mutual funds (Carhart, 1997) and for hedge funds (Bali et al., 2012).[18] Thus, momentum has frequently been examined in the literature and has been shown internationally and for almost every asset class. Further, momentum also works well for several choices of p, where the individual stock momentum strategy typically chooses p between 10% and 30%. For example, for the stock momentum strategy, Jegadeesh and Titman (1993), Jegadeesh and Titman (2001) and Rouwenhorst (1998) use $p = 10\%$, Kim et al. (2016), Goyal and Wahal (2015) and Novy-Marx (2012) use $p = 20\%$, whereas Hong et al. (2000) use $p = 30\%$ in order to "place less emphasis on the tails of the performance distribution." Moskowitz and Grinblatt (1999) also use a cut-off point of $p = 30\%$ for the individual stock momentum strategy. For the industry momentum strategy, Moskowitz and Grinblatt (1999) use 20 industries and choose $n = 3$. In contrast, Grobys et al. (2018) use 49 industries and a value of $p = 1/6$, whereas Du Plessis and Hallerbach (2017) use $p = 25\%$ for the 49 industries. For a momentum strategy using several international asset classes, Kim et al. (2016) use $p = 20\%$, whereas Asness et al. (2013) and Goyal and Jegadeesh (2017) assign each asset as either a winner or loser. Asness et al. (2013) define winners and losers based on an asset's past performance rank, whereas Goyal and Jegadeesh (2017) define winners (losers) as assets that perform better (worse) than the cross-sectional mean.

Although momentum works well for several data sets and cut-off points, the chosen data set and cut-off point p can substantially influence the profitability of momentum investing. For

[18] See Jegadeesh and Titman (2001), Jegadeesh and Titman (2002), Ruenzi and Weigert (2018) and Hong et al. (2000) who summarize several explanations for the profitability of momentum strategies. See Moskowitz et al. (2012, Footnote 1) for a list of further studies on (cross-sectional) momentum.

example, Hong et al. (2000) find that their momentum strategy performs quite differently compared to the strategy of Jegadeesh and Titman (1993). Korajczyk and Sadka (2004) compare the momentum strategy for different ranking and holding periods, data sets as well as weighting schemes and also find that the different strategies can perform quite differently. Similarly, Lesmond et al. (2004, Table 1) compare the impact of different data sets and cut-off points p on the profitability of the momentum strategy and also find that momentum returns can be quite different for different data sets and cut-off points p. In the main part of this paper, we use a data set of 30 equally weighed US industries and a cut-off point of $p = 30\%$. Since different data sets and cut-off points can lead to very different results, we show additional results in Appendix B for other data sets and other choices of p.

Equally weighted portfolios are not only used in the momentum literature, but are also relevant as an easy portfolio allocation method that is frequently used by practitioners. DeMiguel et al. (2009b) compare several portfolio allocation methods and find surprisingly good results of the equally weighted strategy, especially for large data sets. The advantage of equally weighted portfolios is that no parameters have to be estimated. However, other studies show that the performance of the equally weighted portfolio can further be enhanced by using other weighting schemes based on the assets' risk (see Kirby and Ostdiek (2012) for example). We will come back to this point in later sections. Nevertheless, DeMiguel et al. (2009b) propose to use the equally weighted strategy as benchmark for other strategies. In particular, DeMiguel et al. (2009b) show that the equally weighted portfolio performs well for assets with low idiosyncratic risk, such as the assets used in our study.

3.2.2 Industry Momentum

As mentioned in the previous section, the momentum strategy does not only work for individual stocks, but can also be applied to portfolios of stocks, e.g. industry portfolios, country indices, investment styles and mutual funds.[19] This paper examines these portfolio based momentum strategies and focuses in the main part on the industry momentum strategy, which

[19] Momentum has also been shown for non-stock assets, like currencies and commodities (see Asness et al. (2013), Clare et al. (2014) and references therein).

was introduced by Moskowitz and Grinblatt (1999).[20] The authors show that buying past winning industries and selling past losing industries produces very high returns, which cannot be explained by the Carhart (1997) four factor model.[21] Interestingly, Moskowitz and Grinblatt (1999) find that industry momentum can explain individual stock momentum, whereas the opposite does not hold. Furthermore, Novy-Marx (2012) finds that industry momentum is highly correlated with conventional stock momentum.[22] Grundy and Martin (2001) and Chordia and Shivakumar (2002) cannot confirm the finding of Moskowitz and Grinblatt (1999) and they conclude that stock momentum and industry momentum are different phenomena.[23] Nijman et al. (2004) confirm this finding for European momentum. However, Moskowitz and Grinblatt (1999) find that industry momentum is as profitable as individual stock momentum. Thus, industry momentum is an appealing alternative to individual stock momentum, especially for practitioners. Moskowitz and Grinblatt (1999, p. 1286) write that "if one were to trade on momentum, industry-based strategies appear to be more profitable and more implementable." George and Hwang (2004, Table 1) also find that the profitability of individual stock and in-

[20] Industry momentum has also been examined by Novy-Marx (2012), Grobys et al. (2018), Grobys (2018), Grobys and Kolari (2020), George and Hwang (2004), Gupta et al. (2010), Grundy and Martin (2001), Chordia and Shivakumar (2002), Lewellen (2002), Du Plessis and Hallerbach (2017), Swinkels (2002) and Stivers and Sun (2010, p. 993). The importance of industry portfolios is not only limited to the momentum literature. Industry portfolios also play an important role in many fields of finance, e.g. portfolio allocation (Behr et al., 2012, DeMiguel et al., 2009a,b, Garlappi et al., 2006, Harvey et al., 2018, Kirby and Ostdiek, 2012, Kritzman et al., 2010, Zakamulin, 2015, 2017), risk measurement (Van Oordt and Zhou, 2017) and asset pricing (Baltussen et al., 2018, Dittmar, 2002, Harvey and Siddique, 2000).

[21] Earlier studies found that individual stock momentum cannot be explained by the Fama and French (1993) three factor model (see Fama and French (1996) for example). For that reason, asset pricing models have been extended to a four factor model by including a factor for momentum (Carhart, 1997). Fama and French (2016) show that also a five factor model that incorporates a profitability and investment factor has problems in explaining momentum returns. We confirm this finding in Appendix B.11 for industry momentum.

[22] A possible explanation for this finding is that the individual stock momentum strategy picks winners and losers assets that are mainly in the same industries (Grundy and Martin, 2001, Sec. 4.3). A similar behavior is also found for low risk strategies that buy assets with a low past volatility. These strategies are often criticized by picking assets that are mainly concentrated in a few industries (Walkshäusl, 2014). However, Asness et al. (2014) show that low risk strategies also perform well when it is controlled for the industry exposure or when this strategy is directly built on industries.

[23] A possible explanation for the different results found by Moskowitz and Grinblatt (1999), Grundy and Martin (2001) and Chordia and Shivakumar (2002) could be the choice of different cut-off points. For example, Moskowitz and Grinblatt (1999) use $n = 3$ for the industry momentum strategy and $p = 30\%$ for the stock momentum strategy. Chordia and Shivakumar (2002) use $n = 2$ for the industry momentum strategy and $p = 10\%$ for the stock momentum strategy, whereas Grundy and Martin (2001) use $n = 3$ for industry momentum and $p = 10\%$ for stock momentum. Further, the studies use different ranking and holding periods. Thus, different cut-off points as well as different ranking and holding periods can lead to quite different conclusions. Grobys and Kolari (2020, p. 100) also find different results of industry momentum compared to Moskowitz and Grinblatt (1999) and suggest that a possible explanation is the use of different values of m and n.

dustry momentum is quite similar. From a practical perspective, portfolio based momentum strategies have several advantages compared to individual stock momentum strategies. First, Lesmond et al. (2004) find that momentum profits are not robust to transaction costs due to a very high number of transactions and since the strategy is mainly invested in small and highly illiquid assets. Grundy and Martin (2001) also find that momentum profits become statistically insignificant for quite low levels of transaction costs. Similarly, Korajczyk and Sadka (2004) find that buying (equally weighted) winners becomes unprofitable once price impact based transaction costs are considered. This results since the individual stock momentum strategy is typically invested in highly illiquid and small sized firms. Trading these assets on a monthly basis produces high transaction costs. Further, loser stocks are typically highly volatile and shorting costs for volatile assets are typically very high (see Blitz et al. (2019) and references therein). This especially holds for equally weighted momentum strategies (Korajczyk and Sadka, 2004). Korajczyk and Sadka (2004) compare equally weighted, value weighted and illiquidity weighted momentum strategies and find that the equally weighed strategy performs the best before transaction costs and the worst after transaction costs. In contrast, weighting assets inversely to their illiquidity produces higher returns after transaction costs. Thus, the high returns of the individual stock momentum strategy are not achievable in practice. In contrast, non-stock based momentum strategies can typically be built based on highly liquid futures and ETFs and are much less influenced by high transaction costs of highly illiquid individual stocks (Asness et al., 2013, Footnote 12). For example, most industry and country indices typically have corresponding and highly liquid ETFs with low costs (Han, 2005, O'Neal, 2000, Richards, 1997).[24] Second, the individual stock momentum strategy reallocates large numbers of assets each month, which induces a high turnover, and hence high transaction costs. In contrast, the portfolio based momentum strategy invests in a small number of assets, and hence produces low transaction costs. For example, Chordia and Shivakumar (2002) use an industry momentum strategy that buys and sells only the two best and worst performing industries. Similarly,

[24] Other possibilities to lower transaction costs of the individual stock momentum strategy are also possible. First, small and illiquid stocks can be excluded as explained in Bali and Cakici (2008, p. 43). Second, assets can be weighted inversely to their illiquidity (Korajczyk and Sadka, 2004). Third, stocks can be weighted inversely to their (idiosyncratic) volatility, since stocks with high (idiosyncratic) volatility are typically small and illiquid (Bali and Cakici, 2008). We will come back to the third point in Section 3.2.4.

Moskowitz and Grinblatt (1999) and Grundy and Martin (2001) use an industry momentum strategy based on only three industries in the winners and losers portfolio. Stivers and Sun (2010) define the 12 best and worst performing industries as winners and losers, respectively. Thus, industry momentum is far more profitable than individual stock momentum, once realistic transaction costs are considered.[25] Third, we will use different weighting schemes based on an asset's risk to weight the assets in the momentum portfolio. Small sized and illiquid assets that are only traded several times per day are prone to market microstructure issues and typically have large bid-ask spreads. Since we will estimate an asset's monthly risk based on daily data, it is likely that risk estimates are influenced by this issue and would lead to suboptimal and noisy portfolio weights of the individual stock momentum strategy. Bad data quality is less of a concern for assets that are portfolios themselves, like industry or country indices. Thus, pooling several assets into a portfolio reduces the impact of estimation risk (Jiang et al., 2020, p. 370). Further, a larger number of assets n used by the stock momentum strategy also increases the number of required estimates, and thus heightens estimation risk (Kan and Zhou, 2007). As a benchmark strategy to the risk weighted momentum strategies, we follow Moskowitz and Grinblatt (1999) and weight industries in the winners and losers portfolio equally as shown in the previous section.[26] The equally weighted momentum strategy has the advantage that no risk estimates are needed.

Moskowitz and Grinblatt (1999) show that industry momentum holds for several ranking and holding periods between one and twelve months. In particular, the authors find good results for strategies that use a one month holding period as also used in Stivers and Sun (2010),

[25] O'Neal (2000) shows that the industry momentum strategy is also profitable under realistic transaction costs by using industry mutual funds. Due to the rise of ETFs and online brokers, transaction costs are nowadays significantly lower than assumed by O'Neal (2000), what makes this strategy nowadays even more profitable. For example, iShares charges an annualized fee of about 0.4% per year for industry ETFs. This corresponds to a monthly fee of about 0.033%. In particular, ETFs can be bought without front-end load. Moreover, by using online brokers, like Interactive Brokers, broker fees can be reduced to zero (`https://www.interactivebrokers.com/en/index.php?f=42773`). Similarly, Charles Schwab, one of the largest brokerage firms in the US, recently removed all commission fees and offers commission free brokerage (`https://www.schwab.com/pricing-page`). In contrast, O'Neal (2000) uses an average annualized fund fee of 1.89%, a front-end load of 3% and broker fees of 7.5$.

[26] Applying equal weights to the assets in the winners and losers portfolios of the industry momentum strategy is also done by Gupta et al. (2010), Grobys (2018), Grobys et al. (2018), Novy-Marx (2012), Grundy and Martin (2001), Stivers and Sun (2010) and Chordia and Shivakumar (2002). In contrast, Lewellen (2002) uses a weighting based on the industries' past return.

Grobys et al. (2018), Novy-Marx (2012) and in our paper.[27] Furthermore, Moskowitz and Grinblatt (1999) find good results when industries are ranked based on the past twelve months' performance. Interestingly, the authors find that industry momentum also works well when industries are ranked by their last month's performance. This finding for industry momentum is different to the individual stock momentum strategy, since individual stocks exhibit a short-term reversal effect. The high performance of industry momentum for different ranking periods has also been confirmed by several studies on industry momentum. For example, Novy-Marx (2012, Sec. 5.2) also finds significant momentum in industries for a one month holding period and several ranking periods. In particular, the author finds good results by ranking industries based on the performance between months $t-12$ and $t-7$. Further, Novy-Marx (2012, p. 443) confirms the finding of Moskowitz and Grinblatt (1999) that "industries do exhibit momentum at very short (one month) horizons", i.e. using a one month ranking period. Similarly, Grobys et al. (2018) also use a one month holding period and find good results for the six and twelve months ranking period as well as the $t-12$ to $t-7$ ranking of Novy-Marx (2012). Du Plessis and Hallerbach (2017) use 49 industry portfolios combined with the one month and 12 months ranking period and find good results for both ranking periods. Stivers and Sun (2010) find good results of the six months ranking period. Furthermore, Grundy and Martin (2001, Table 4) compare different ranking periods and find that all rankings deliver good results as long as equally weighted industries are used.[28] Grundy and Martin (2001) find that particularly the ranking periods that include the last month, i.e. the $t-6$ to $t-1$ and $t-12$ to $t-1$ rankings, deliver good results. Nevertheless, the $t-12$ to $t-2$ ranking, which was introduced by Fama and French (1996) for the stock momentum strategy, also performs well. Thus, industry momen-

[27] In contrast, Chordia and Shivakumar (2002) and Grundy and Martin (2001) use a six months holding period as used by Jegadeesh and Titman (1993) for the stock momentum strategy. Swinkels (2002) also compares several ranking and holding periods for international industry momentum strategies.

[28] Moskowitz and Grinblatt (1999) and Grobys et al. (2018) use value-weighted industry portfolios but equally weighted winners and losers portfolios. In contrast, Grundy and Martin (2001) compare industry momentum strategies using equal- and value-weighted industries. These weightings refer to the industry portfolios used for the momentum strategy and not to the weighting of the winners and losers portfolios. Grundy and Martin (2001) weight industries in the winners and losers portfolios equally, regardless of whether equal- or value-weighted industries are used. The authors find that these momentum strategies produce quite different results and that the momentum strategy using equally weighted industries outperforms the value-weighted based strategy. This observation is confirmed by Lewellen (2002, Table 2) for industries and other data sets. Hence, the profitability of portfolio based momentum strategies is quite sensitive to the chosen data sets. Thus, as a robustness check, we will use several other data sets, including value-weighted industries, in the appendix.

tum works well for several ranking periods, but the profitability of industry momentum can be quite different for different ranking periods and data sets. The observation that different ranking and holding periods can produce very different results was also found for the TSMOM strategy (Dudler et al., 2015, Goyal and Jegadeesh, 2017, Moskowitz et al., 2012). We will use the $t-12$ to $t-1$ ranking in the main part, but we show additional results for several other ranking periods in the appendix. An alternative to the past return based ranking would be to apply the 52 week high approach of George and Hwang (2004) to industry portfolios as also done by Grobys (2018) and Gupta et al. (2010, Sec. 4.5). Ranking industries by the difference to their 52 week high produces a strategy that is different to the usual industry momentum strategy (Grobys, 2018, Table 2). However, this strategy is not examined here.

3.2.3 Momentum Crashes

The industry momentum strategy presented in the previous section produces high returns by buying past winning industries and selling past losing industries. However, the high return of industry momentum is also accompanied by high risk. O'Neal (2000) show that industry momentum strategies can outperform the market even after transaction costs, but the author concludes that "[t]hese strategies entailed greater total and systematic risk, however, than the index." In particular, momentum investing translates into a strategy with a high left tail risk, which induces a high likelihood of extremely negative returns, so called "momentum crashes". These momentum crashes have been frequently examined for the individual stock momentum strategy (Barroso and Santa-Clara, 2015, Cooper et al., 2004, Daniel et al., 2017, Daniel and Moskowitz, 2016, Grundy and Martin, 2001, Min and Kim, 2016, Wang and Xu, 2015). Momentum crashes are particularly undesirable for investors, since these crashes typically occur when the market exhibits a longer period of negative returns (Cooper et al., 2004, Daniel and Moskowitz, 2016, Grundy and Martin, 2001). In these periods, investors are highly averse to decreasing wealth. Chordia and Shivakumar (2002, p. 993) find that momentum returns of the stock momentum strategy are positive (negative) when the marginal utility of wealth is lower (higher). Similarly, Min and Kim (2016) find that momentum crashes of the stock momentum strategy typically occur in periods when investors have a high marginal utility of wealth, what makes momentum investing unappealing for investors. In particular, Chordia and Shivakumar (2002, p. 993) find that “momentum payoffs are negative during recessions and positive during expansions”. Further, Chordia and Shivakumar (2002, Sec. II.H) examine the relation between industry momentum and the business cycle and find that also industry momentum is related to the macroeconomy. Momentum crashes of the industry momentum strategy have been examined by Du Plessis and Hallerbach (2017), Grobys et al. (2018), Grobys (2018) and Grobys and Kolari (2020). Interestingly, Grobys et al. (2018) find that momentum crashes of the industry momentum portfolio are significantly less severe than momentum crashes of the stock momentum strategy. Hence, by using portfolios instead of individual stocks for the momentum strategy, left tail risk is significantly reduced. For example, using a value-weighted stock momentum strategy, Barroso and Santa-Clara (2015) find a volatility of 27.53%, a skewness of -2.47, a kurtosis of 18.24, a minimum monthly return of -78.96 and a maximum drawdown of 96.69%. Further, the authors find a return of -91.59% in only two months and that the recovery from a momentum crash can last up to 31 years. Daniel and Moskowitz (2016) find ten months with returns lower than -30% for the value-weighted stock momentum strategy, while the market has a positive return in these months. Using an equally weighted stock momentum strategy, Rickenberg (2020a) finds an even higher left tail risk compared to the value-weighted strategy. Rickenberg (2020a) shows that the stock momentum strategy exhibits a volatility of 26.31%, a skewness of -4.38, a kurtosis of 43.59, a minimum monthly return of -89.70% and a maximum drawdown of 99.31%. In contrast, the industry momentum strategy using the six months ranking period has a volatility of $5.98 \cdot \sqrt{12} = 20.71\%$, a skewness of -1.50, a kurtosis of 22.91 and a minimum monthly return of 62.75% (Grobys et al., 2018, Table 4). Furthermore, Grobys et al. (2018, Table 7) show that crashes of the industry momentum strategy are less severe than crashes of the stock momentum strategy. For example, the stock momentum strategy has more than 15 months with returns lower than -20%, whereas the industry momentum strategy has only five of these months. The average return on the 15 worst months is -41.56% for the stock momentum strategy and -25.43% for the industry momentum strategy. Further, stock and industry momentum crashes do not necessarily coincide, i.e. the worst months of the stock momentum strategy do not necessarily coincide with high losses of the industry momentum strategy. During some stock momentum crashes, industry momentum earns even positive returns. Summarized, industry momentum strategies do not only offer similar returns and lower transaction costs than the stock momentum strategy, they are also far less risky.

Although the industry momentum strategy is less risky than the stock momentum strategy, this strategy is still unattractive for most investors. Grobys et al. (2018, Figure 1) show that the volatility of the industry momentum strategy is highly volatile and fluctuates between 1.74% and 69.77% with an average of $6.51 \cdot \sqrt{12} = 22.55\%$. This high and volatile volatility of the industry momentum strategy makes momentum investing unappealing for risk-averse investors. Risk-averse investors are averse to a high volatility of volatility and are willing to pay high fees for insurance against volatility fluctuations (Adrian and Rosenberg, 2008, Ang et al., 2006b, Baltussen et al., 2018, Bollerslev and Todorov, 2011). Thus, the high return of momentum investing will unlikely be an adequate compensation for the high volatility risk of this strategy. Further, investors typically have preferences for moments higher than volatility and prefer a higher skewness and lower kurtosis (Dittmar, 2002, Harvey and Siddique, 2000, Kraus and Litzenberger, 1976, Scott and Horvath, 1980). Thus, investors are averse to strategies that produce a fat left tail as done by the industry momentum strategy. The high left tail risk of momentum also increases the likelihood of extreme losses, which is unappealing for loss-averse investors who weight losses higher than gains of the same magnitude (Aït-Sahalia and Brandt, 2001, Benartzi and Thaler, 1995). Similarly, investors are averse to crashes and are willing to pay high fees to hedge against extreme crashes (Arzac and Bawa, 1977, Bollerslev and Todorov, 2011, Chabi-Yo et al., 2018, Van Oordt and Zhou, 2016, Weigert, 2015). This is particularly important for "momentum investors who often employ leverage and are therefore sensitive to even small drawdowns" (Chabot et al., 2014). Further, the momentum strategy is highly related to left jump tail risk (Bollerslev et al., 2015, Fig. 9). Concluding, the high return of the industry momentum strategy is unavailable in practice for most investors, since investors are unlikely willing to accept the high (left tail) risk of the momentum strategy. Thus, in order to make momentum investing more appealing for investors, methods that reduce this high risk are needed.

3.2.4 Volatility Weighted Momentum Strategy

In the previous section, we summarized that the high return of the momentum strategy is accompanied with high risk, which makes this strategy unavailable for investors unless the strategy's risk is managed. The equally weighted momentum strategy presented in Section 3.2.1 has the disadvantage that risks of the assets in the winners and losers portfolios are not regarded. The assets in the momentum portfolio are typically risky in terms of a low market capitalization, a high market beta and a high volatility (Jegadeesh and Titman, 1993, 2001, Rouwenhorst, 1998). For example, for the individual stock momentum strategy, Rouwenhorst (1998) finds that the volatility of the assets in the winners and losers portfolios is on average 30% to 40% higher than the volatility of the assets that are not included in the momentum portfolio (see also Kirby and Ostdiek (2012, Figure 1.C)). This finding is quite intuitive, since a high volatility makes extreme (negative or positive) returns more likely (Jang and Kang, 2019, Table 1). Thus, assets with a higher volatility tend to appear more frequently in the extreme tails of the performance distribution. Further, Harvey and Siddique (2000, Table 1) show that different industries have quite different levels of volatility. For example, annualized volatilities range from 14.15% to 34.36% for different industries. Thus, by weighting each industry with the same cash amount, the industry momentum portfolio's volatility can potentially be dominated by only a few assets. In contrast, industries with lower volatility hardly contribute to the portfolio's risk. One way to reduce the high volatility of the momentum portfolio and the high risk contribution of only a few industries is to overweight low volatile industries and underweight high volatile industries. Interestingly, several studies show that low volatile assets outperform assets with a higher volatility, which is called the "low volatility anomaly".[29] Ang et al. (2006b, Table XI)

[29] The low volatility anomaly has been frequently examined in the literature (Ang et al., 2006b, 2009, Asness et al., 2014, Atilgan et al., 2020, Bali et al., 2017a, Bali and Cakici, 2008, Bali et al., 2011, 2014, 2017b, Blitz et al., 2019, Blitz and Vidojevic, 2017, Boyer et al., 2009, Chen and Petkova, 2012, Fama and French, 2016, Guo and Savickas, 2010, Liu et al., 2018, Schneider et al., 2020, Stambaugh et al., 2015). This anomaly contradicts traditional finance models, like the CAPM, which assume that higher risk is also compensated with higher returns. Haugen and Heins (1975) find that the risk-return relation strongly depends on the sample period and is different when the sample period is dominated by a bull or bear regime. Haugen and Heins (1975, p. 782) conclude: "The results of our empirical effort do not support the conventional hypothesis that risk – systematic or otherwise – generates a special reward. Indeed, our results indicate that, over the long run, stock portfolios with lesser variance

find that the low volatility anomaly holds in several subsamples and is apparent in bull and bear markets, recessions and expansions as well as in volatile and calm markets. This negative relation between volatility and (risk-adjusted) returns also holds for industry portfolios (Harvey et al., 2018, Kirby and Ostdiek, 2012, Zakamulin, 2017). Similarly, Jordan and Riley (2015) show that the low volatility anomaly holds for mutual funds, i.e. funds with lower volatility outperform funds with higher volatility in terms of raw and risk-adjusted returns as well as portfolio alphas. However, even in the absence of a negative volatility-return relation, volatility managed portfolios can be superior to equally weighted portfolios. For example, Kirby and Ostdiek (2012, Figure 1.D) find that volatility sorted portfolios have approximately the same mean returns, but quite different levels of volatility. In other words, risk-adjusted returns are much lower for highly volatile assets, since the higher risk is not rewarded by an adequately higher return.[30] Thus, volatility and Sharpe Ratio of different assets are negatively correlated (Zakamulin, 2017). Based on this observation, Kirby and Ostdiek (2012) show that a strategy that weights assets inversely to their volatility significantly increases the Sharpe Ratio and utility of mean-variance investors compared to an equally weighted strategy (see also Kritzman et al. (2010), Zakamulin (2015) and Zakamulin (2017)).

Based on the low volatility anomaly, underweighting highly volatile assets should produce more stable momentum returns and should reduce the high risk of the momentum portfolio in terms of a lower volatility and lower drawdowns, without simultaneously producing lower returns. One easy method to take the low volatility anomaly into account is to weight assets inversely to their volatility. The inverse volatility weighting is nicely summarized in Asness et al. (2012, Appendix A), Kirby and Ostdiek (2012) and Zakamulin (2015).[31] Applied to the momentum strategy, month t weights of asset i in the winners and losers portfolio, used in

in monthly returns have experienced greater average returns than their "riskier" counterparts." Similar to the low volatility anomaly, Baltussen et al. (2018) show that assets with higher uncertainty, measured by the volatility of volatility, also underperform assets with lower uncertainty.

[30]This observation does not only hold in a cross-sectional setting. For example, Moreira and Muir (2017) and Moreira and Muir (2019) show that a higher volatility of an asset is not rewarded by an adequately higher return of the same asset. Moreira and Muir (2017, Sec. II.D) compare this effect to the cross-sectional low risk anomaly and find that both effects are different. The negative relation between an asset's volatility and future return, also called volatility feedback effect, has been frequently examined in the literature (see Bekaert and Wu (2000), French et al. (1987) and Glosten et al. (1993) for example).

[31]Kirby and Ostdiek (2012) and Zakamulin (2015) show that the inverse volatility weighting follows (under certain conditions) from the optimal portfolio allocation of a mean-variance investor.

Equation (3.2.1), are given by

$$w_{i,t}^{W} = \frac{1/\sigma_{i,t}^{W}}{\sum_{j=1}^{n} 1/\sigma_{j,t}^{W}} \quad \text{and} \quad w_{i,t}^{L} = \frac{1/\sigma_{i,t}^{L}}{\sum_{j=1}^{n} 1/\sigma_{j,t}^{L}}, \tag{3.2.3}$$

where $\sigma_{i,t}^{W}$ and $\sigma_{i,t}^{L}$ denote the month t volatility of asset i in the winners and losers portfolios, respectively. We follow the common approach and estimate monthly volatility based on daily returns (Barroso and Santa-Clara, 2015, Fama and French, 2016, French et al., 1987, Jang and Kang, 2019, Merton, 1980, Moreira and Muir, 2017, 2019), where we use the past six months of daily data and assume that each month has $h = 21$ days. Robustness results for estimation windows between one and six months are shown in Appendix B.1. Ghysels et al. (2005) and Bali et al. (2009) also use one to six months of daily data to measure monthly volatility. The estimation of all risk measures used in this paper is summarized in Appendix C.

Besides the simple Realized Volatility (RV) estimator of Equation (C.1), several other possibilities to estimate an asset's monthly volatility could be used. For example, monthly volatility could be estimated based on monthly returns. However, Bali and Cakici (2008) find quite different results for volatility sorted portfolios that are built on risk estimates using daily or monthly returns. In particular, the authors find a negative risk-return relation when risk is estimated with daily data, but no relation when risk is estimated with monthly data. Generally, estimating monthly volatility with monthly data typically produces inaccurate estimates (Merton, 1980). Thus, monthly volatility should be estimated based on daily data. An alternative daily data based estimation method of monthly volatility, which is similar to the method used in Equation (C.1), would be to account for the daily mean and first-order autocorrelation as shown by French et al. (1987), Chen and Petkova (2012, Eq. (11)) and Jondeau et al. (2019, Eq.(2)). However, this approach typically produces similar results to the simple RV estimator (Barroso and Santa-Clara, 2015, Moreira and Muir, 2017). Similarly, a RV based forecasting model, such as the HAR model, could be used (Bollerslev et al., 2018, Patton and Sheppard, 2015). Alternatively, future volatility could also be estimated based on an industry's lagged characteristics, such as an industry's lagged volatility, skewness or momentum (see Ang et al. (2009, Sec. 6.4.2) or Chen and Petkova (2012)). Another alternative would be to follow Chen and Petkova (2012, Eq. (6)), Guo and Savickas (2010) and Jondeau et al. (2019) and estimate monthly volatility of a portfolio by first estimating each constituent's volatility. The (average) volatility of the portfolio is then given as the equal- or value-weighted average of the individual volatilities. Chen and Petkova (2012, Table 1) find that the average volatility is a negative predictor for future returns and a positive predictor for future volatility, and hence average volatility predicts periods that offer an unappealing risk-return profile. Thus, downweighting industries with a high average volatility could be an appealing approach. Another alternative could be to weight industries by their cross-sectional dispersion, calculated as the cross-sectional variance of all assets within an industry (Du Plessis and Hallerbach, 2017, Grobys, 2018, Stivers and Sun, 2010). Further, instead of using simple non-parametric estimation methods, more sophisticated volatility models that properly forecast an asset's volatility could be used. However, Fu (2009, p. 30) examines the risk-return relation for several volatility models and finds a positive relation between return and volatility of the same month, but a negative risk-return relation between volatility of month $t-1$ and return of month t. Thus, measuring volatility as Realized Volatility is appealing for the inverse risk weighting. Furthermore, instead of weighting industries based on their Realized Volatility, industries could also be weighted based on their volatility innovation (Adrian and Rosenberg, 2008, Ang et al., 2006b, Chang et al., 2013). Moreover, instead of using a (total) volatility based weighting, an alternative weighting based on the idiosyncratic volatility could be used (Ang et al., 2006b, 2009, Bali and Cakici, 2008, Fu, 2009, Liu et al., 2018, Stambaugh et al., 2015). However, Bali and Cakici (2008) find no improvements of portfolios that are weighted inversely to the assets' idiosyncratic volatility. Similarly, Asness et al. (2020) find that the low risk effect is mainly driven by leverage constraints, which gives evidence against a low idiosyncratic risk effect.[32] Finally, instead of weighting assets by their volatility, assets could also be weighted by their volatility of volatility (vol-of-vol). Baltussen et al. (2018) find that stocks with a high vol-of-vol underperform assets with a low vol-of-vol. However, the au-

[32] We also used weightings based on idiosyncratic volatility and find that the weightings based on (total) volatility outperform the weightings based on idiosyncratic volatility. We estimated idiosyncratic volatility as past idiosyncratic volatility (Ang et al., 2006b, 2009, Liu et al., 2018, Stambaugh et al., 2015) as well as expected idiosyncratic volatility (Bali and Cakici, 2008, Fu, 2009). Similarly, we find that an approach based on realized skewness (Amaya et al., 2015, Chang et al., 2013) outperforms a weighting based on idiosyncratic skewness (Boyer et al., 2009). These observations can also be explained by the findings of Bali and Cakici (2008) and Langlois (2020) that idiosyncratic risk is less important when portfolios like industry portfolios are used, which exhibit lower idiosyncratic risk than individual assets.

thors find that the vol-of-vol measure captures similar risk characteristics as the kurtosis. For reasons that will be discussed in Sections 3.3 and 3.4, weightings based on symmetric risk measures have several disadvantages. We also used several alternative estimation methods and definitions of volatility, but found quite similar results to the RV based weighting. Further, since risk-managed industry momentum strategies are also relevant for practitioners, we only show results for the simplest volatility estimation method.

The inverse volatility weighting scheme of Equation (3.2.3) has been frequently used in the financial literature and it has been shown that this simple weighting scheme produces very convincing results. We summarize in Appendix A further advantages of portfolio strategies that overweight low volatile assets. Generally, the economic value of volatility managing for long-only portfolios has been frequently shown in the literature (Fleming et al., 2001, 2003, Han, 2005, Kirby and Ostdiek, 2012, Taylor, 2014). Moreover, the inverse volatility weighting scheme of Equation (3.2.3) has also been applied to long-short strategies. In particular, the combination of an asset's momentum and risk is appealing, since both measures capture different characteristics. Thus, even after forming portfolios based on the assets' past performance, there still is a negative volatility-return relation (see Ang et al. (2006b) and Guo and Savickas (2010, p. 1643)). For example, Novy-Marx (2012) concludes: "Higher realized volatility is also associated, even after controlling for past performance, with lower expected returns." Similarly, Kirby and Ostdiek (2012, p. 462) apply volatility timing to different momentum portfolios and find that volatility timing significantly enhances the risk-return profile, even when transaction costs are considered. Thus, combing information on an asset's momentum and risk is appealing. The benefits of combining information on an asset's past performance and volatility has also been shown by Blitz and van Vliet (2018).[33] Similarly, Moskowitz et al. (2012), Goyal and Jegadeesh (2017), Du Plessis and Hallerbach (2017), Baltas (2015), Clare et al. (2014), Clare et al. (2016) and Kim et al. (2016) use the inverse volatility weighting to scale the assets in a multivariate TSMOM strategy. Kim et al. (2016) find that the good performance of

[33] Blitz and van Vliet (2018) examine a strategy that only buys assets with low volatility, high past return and high net payout yield. The authors find that combining information on these three characteristics produces high returns, even after controlling for momentum. This strategy is also investable via mutual funds (`https://www.robeco.com/en/strategies/equity/conservative-equity.html`).

TSMOM is mainly driven by the inverse volatility weighting, i.e. combining information on an asset's past return and risk is appealing. Asness et al. (2013) also apply the inverse volatility weighting to momentum and value portfolios, since different portfolios can have quite different levels of portfolio volatility. Weighting these portfolios inversely to their volatility reduces the risk that the portfolio's volatility is dominated by the volatility of a few portfolios. Further, the inverse volatility weighting of Equation (3.2.3) has also been used to weight the assets in the winners and losers portfolios of the (cross-sectional) momentum strategy. Applying the inverse volatility weighting to the (cross-sectional) momentum portfolio has been done by Clare et al. (2014) for commodity momentum, by Goyal and Jegadeesh (2017) for momentum using several international asset classes and by Du Plessis and Hallerbach (2017) for the industry momentum strategy. Clare et al. (2014) show that using the inverse volatility weighting instead of the equal-weighting has different impacts on the returns of the momentum portfolio. First, returns and Sharpe Ratios are slightly higher for the inverse volatility weighted portfolios. Second, volatility and drawdowns are reduced. Third, the returns of the inverse volatility weighted portfolios are less negatively skewed. However, the authors conclude: "Overall the results of adding the risk parity overlay to momentum investing have limited impact on the results but do lead to some overall improvement, especially with regard to maximum drawdowns" (Clare et al., 2014, p. 7). Similarly, Goyal and Jegadeesh (2017, Table 9 and 10) show that the volatility weighted momentum portfolio exhibits higher returns and portfolio alphas than the equally weighted momentum portfolio. Goyal and Jegadeesh (2017, Table 11) find that volatility weighting increases the Sharpe Ratio from 0.75 to 0.82 for the 12 months ranking period and that the increase in momentum's Sharpe Ratio is robust for other ranking periods. Moreover, Goyal and Jegadeesh (2017, Table 12.A) show that volatility weighting also increases momentum's return when volatility weighting is used for country momentum.[34] Du Plessis and Hallerbach (2017, Exhibit 6 and Exhibit 8.B) show that volatility weighting applied to industry momentum produces higher returns and reduces left tail risk. Thus, combining the volatility weighting with momentum can produce an enhanced risk-return profile, where especially downside risk is reduced.

[34]We also show results for country momentum in Appendix B.7.

The weighting scheme in Equation (3.2.3) has the advantage that no estimate of the assets' mean returns or correlations are needed.[35] By not incorporating an estimate of the assets' mean return, estimation risk is significantly reduced. Nevertheless, estimation risk is still an issue for the inverse volatility weighting since portfolio weights are determined by an estimate of the assets' risk. Garlappi et al. (2006, Footnote 4) find that the estimation error is more costly for mean returns and is less important for (co-)variances. Similarly, Moreira and Muir (2019) find that strategies that time mean and volatility suffer more under parameter uncertainty than strategies that only time volatility. However, estimating volatility can also be quite noisy and can lead to suboptimal portfolio weights (Kan and Zhou, 2007). Weigert (2015, Sec. 3.1) and Ang et al. (2006a) find that risk measurement is less precise for highly volatile assets due to higher measurement errors for these assets. Since assets in the winners and losers portfolios are typically highly volatile (Rouwenhorst, 1998), we expect a high estimation risk for the weighting scheme in Equation (3.2.3). This is especially the case, since this paper focuses on simple non-parametric estimation methods. For that reason, we extend the approach of Equation (3.2.3) and use a second weighting scheme that is less sensitive to estimation risk. Similar to the weighting scheme used by Asness et al. (2014), Asness et al. (2020), Frazzini and Pedersen (2014), Schneider et al. (2020) and Liu et al. (2018, Sec. 4) we use a rank based weighting approach.[36] The rank based weighting scheme has also been used to exploit the low volatility anomaly (Liu et al., 2018, Sec. 4.2). For this weighting scheme, we first measure month t risk of asset i in the winners and losers portfolio by $\sigma_{i,t}^{W}$ and $\sigma_{i,t}^{L}$, respectively. We then sort assets in the winners and losers portfolios in descending order and denote the rank of asset i in month t in the winners and losers portfolio by $\mathrm{rank}_{i,t}^{W}$ and $\mathrm{rank}_{i,t}^{L}$. That is, $\mathrm{rank}_{i,t}^{W}$ ($\mathrm{rank}_{i,t}^{L}$) is equal to n

[35] Other weighting schemes, that are similar to the inverse volatility weighting, are the frequently used mean-variance, minimum variance and risk parity portfolios. These portfolio strategies are summarized in Appendix A. The frequently used mean-variance optimization suffers under high estimation risk of mean returns, which leads to a bad out-of-sample performance (Kirby and Ostdiek, 2012). We show in Appendix B.13 that a strategy that focuses on the winners portfolio and weights the winners inversely to their volatility clearly outperforms the mean-variance optimization. The risk-weighted winners portfolio uses an estimate of the assets' relative mean and volatility. We therefore call this portfolio a *relative mean-risk* portfolio.

[36] The rank based weighting is nicely summarized in Asness et al. (2014, Appendix A). Asness et al. (2013, Eq. 1) also use a rank based weighting scheme for the momentum portfolio, where an asset's rank is determined by the assets' past performance. Hence, past performance determines both the weight of an asset and if an asset belongs to the winners or losers portfolio. We use past performance to identify if an asset belongs to the winners or losers portfolio, but in contrast to Asness et al. (2013), we use a rank weighting based on an asset's risk instead of return.

if asset i exhibits the lowest risk in the winners (losers) portfolio. Similarly, $\text{rank}_{i,t}^W$ ($\text{rank}_{i,t}^L$) is equal to one if asset i exhibits the highest risk in the winners (losers) portfolio in month t. The weights of the momentum portfolio used in Equation (3.2.1) are then given by

$$w_{i,t}^W = \frac{\text{rank}_{i,t}^W}{\sum_{j=1}^n \text{rank}_{j,t}^W} \quad \text{and} \quad w_{i,t}^L = \frac{\text{rank}_{i,t}^L}{\sum_{j=1}^n \text{rank}_{j,t}^L}. \tag{3.2.4}$$

Using the rank based weighting is advantageous compared to the inverse risk weighting since "using ranks [...] as portfolio weights helps to mitigate the influence of outliers" (Asness et al., 2013, p. 938). Thus, the rank based weighting should also produce lower transaction costs and should be more suitable for practical implementations.[37] Asness et al. (2020, Sec. 6) compare several weighting methods and find good results of the rank based weighting.

Langlois (2020, Sec. 2.5) theoretically shows that if one is interested in the assets' ordering, forecasting cross-sectional ranks is advantageous compared to forecasting an asset's risk. Langlois (2020, Sec. 5) confirms this in a Monte-Carlo Simulation. An alternative to the rank weighing used in our paper would be to directly forecast an asset's rank as shown in Langlois (2020, Eq. (4)). However, due to the high autocorrelation of volatility and other risk measures, the ranking of past sample estimates should be a quite good indicator for the assets' future risk ranking. Furthermore, similar to the inverse volatility weighting, we also considered an inverse rank weighting, i.e. the month t weight of an asset would be given by $w_{i,t} = \frac{1/(n+1-\text{rank}_{i,t})}{\sum_{j=1}^n 1/(n+1-\text{rank}_{j,t})}$. We only report results for the weighting in Equation (3.2.4) since the inverse rank weighting would produce more extreme weights than the weighting scheme in Equation (3.2.4).[38]

Several other alternative weighting schemes to the weightings in Equations (3.2.3) and (3.2.4) are also possible. For example, a Fama-French type construction method by using double sorted portfolios based on past return and volatility could be used (see Asness et al. (2020, Sec. 6.3) or Chen and Petkova (2012)). However, Asness et al. (2020, Sec. 6.4) find that the rank

[37] Another alternative to lower the impact of estimation risk and to lower transaction costs of a weighting scheme would be to partially readjust portfolio weights as done by Bollerslev et al. (2018) and Bollerslev et al. (2020, Eq. (19)). Interestingly, Bollerslev et al. (2020, Table 11) find that this partially adjustment approach produces an enhanced risk-return profile, even before transaction costs. A possible explanation for this result is that partially adjusting portfolio weights reduces the impact of estimation risk as it is also done by the rank weighting.

[38] For example, consider a portfolio that consists of ten assets. Using the inverse rank weighting would produce weights between 3.41% and 34.14% with a standard deviation of the weights of 9.47%. Thus, although this portfolio consists of ten assets, about one third of the money would be concentrated in only one asset and 51.21% would be invested in only two assets. In contrast, our risk weighting in Equation (3.2.4) produces weights between 1.82% and 18.18% with a standard deviation of the weights of only 5.5%.

weighting produces higher risk-adjusted returns than the Fama-French type weighting. Furthermore, different weighting schemes for the winners and losers portfolios could be used. One drawback of the inverse volatility weighting is that overweighting lower volatile assets in the momentum portfolio should increase the performance of both legs of the momentum portfolio. Thus, performance gains of buying the enhanced winners portfolio are (partly) offset by shorting the enhanced losers portfolio. Ang et al. (2006b, p. 292) state that "one way to improve the returns to a momentum strategy is to short past losers with high idiosyncratic volatility." We also used a strategy where highly volatile losers obtained higher weights. This strategy produces higher returns than the inverse volatility managed portfolio, but this higher return is also accompanied with higher risk. In total, this strategy produces a lower Sharpe Ratio, significantly increases left tail risk and is therefore not further examined here.

The weighting schemes that were presented in this section should lower momentum's drawdowns and are therefore an appealing method to manage momentum's risk. This approach is different to other portfolio risk management tools that were examined in the literature on momentum crashes. For example, a frequently applied method to reduce momentum's left tail risk is the risk targeting approach that dynamically scales the exposure to the momentum strategy based on the portfolio's risk (Barroso and Santa-Clara, 2015, Cederburg et al., 2020, Daniel and Moskowitz, 2016, Grobys et al., 2018, Moreira and Muir, 2017, Rickenberg, 2020a). This approach is highly different to our approach since risk targeting only considers the whole portfolio's risk. In contrast, our approach considers the risks of the individual assets in the winners and losers portfolios. Moreira and Muir (2017, Sec. II.D) also find that both approaches, cross-sectional and time series volatility managing, are different. In particular, both approaches can also be used simultaneously as shown in Section 3.6. Thus, this combined approach manages the volatilities of the individual assets *and* the whole portfolio.

3.3 Tail Risk Weighted Momentum

In Section 3.2, we summarized the literature on (industry) momentum and the inverse volatility weighting approach used by Clare et al. (2014), Du Plessis and Hallerbach (2017) and Goyal and Jegadeesh (2017) to weight assets in the winners and losers portfolios. The authors show that the volatility weighted momentum portfolio exhibits an enhanced risk-return profile compared to the equally weighted momentum strategy. However, the authors find that the risk-return profile is only slightly improved by applying the volatility based weighting scheme to the momentum portfolio. In this section, we will argue that the volatility based weighting scheme has two important disadvantages and we show how these disadvantages can be corrected to further enhance momentum's risk-return profile.

The first disadvantage of the inverse volatility weighting scheme generally holds for all portfolio allocation methods and is not unique for long-short strategies like momentum. Several studies show that using volatility as a portfolio risk management tool is disadvantageous, since using volatility as a risk measure is only suitable when asset returns are normally distributed or investors have quadratic utility (Agarwal and Naik, 2004, Alexander and Baptista, 2004). Both assumptions are typically not fulfilled in practice, which makes volatility a suboptimal portfolio risk management tool.[39] In particular, volatility does not incorporate higher moments like skewness and kurtosis. Asset returns are typically skewed and leptokurtic with time-varying higher moments (Bali et al., 2008, Harvey and Siddique, 1999, Jondeau and Rockinger, 2003). This holds especially for the returns of the momentum strategy as summarized in Section 3.2.3 and shown by Rickenberg (2020a). Thus, the momentum portfolio usually exhibits a high left tail and crash risk. So far, although it is well-known that momentum returns are highly non-normal, momentum strategies that incorporate this high non-normality have not reached much attention (Rachev et al., 2007). The high crash risk of momentum investing makes an adequate portfolio risk management crucial in order to make momentum investing available for investors. Unfortunately, volatility underestimates the probability of extremely low returns for highly left skewed portfolios like the momentum strategy.[40] The high negative skewness of the momentum portfolio arises since winners exhibit a lower skewness than losers and the winners' and losers'

[39] Similarly, the frequently used mean-variance approach is only suitable for quadratic utility or normally distributed returns, an assumption that typically does not hold in practice (see Bali et al. (2009), Agarwal and Naik (2004) and references therein).

[40] In financial data, extreme losses occur much more frequently than expected for normally distributed returns. These extreme losses are not captured by volatility, whereas downside risk measures are more successful in capturing extremely negative returns (Bali et al., 2009, Footnote 3).

skewness is highly related and moves in opposite directions (Rickenberg, 2020a). Thus, as frequently shown in the literature and summarized in Appendix A, winners and losers have quite different risk characteristics, which is a main driver of the high crash risk of the momentum portfolio. Consequently, the crash risk of the momentum portfolio could be reduced by increasing the skewness of the winners while the losers' skewness is reduced. The difference in the winners' and losers' skewness and other non-normal characteristics are not regarded by the inverse volatility weighting, and hence the inverse volatility weighting is a suboptimal weighting scheme to reduce momentum crashes. As a consequence, higher moments should be incorporated when portfolio weights are readjusted each month (Cuoco et al., 2008, Jondeau and Rockinger, 2012). Furthermore, most investors' preferences cannot be described by quadratic utility, since investors typically have preferences for higher skewness and lower kurtosis (Kraus and Litzenberger, 1976, Scott and Horvath, 1980). For investors with higher order preferences, it becomes more important to identify and avoid periods of market downturns (Guidolin and Timmermann, 2008). Thus, in order to better fit to these investors' preferences, risk should be measured by incorporating information on higher moments. Further, investors are typically loss-averse, i.e. they weight negative returns higher than positive returns of the same magnitude (Aït-Sahalia and Brandt, 2001, Benartzi and Thaler, 1995). Similarly, most investors are crash-averse, i.e. they are willing to pay high fees to avoid crashes (Bollerslev and Todorov, 2011, Chabi-Yo et al., 2018, Van Oordt and Zhou, 2016, Weigert, 2015). This holds especially for momentum investors, since these investors frequently use leverage (Chabot et al., 2014). Moreover, Bollerslev et al. (2015, p. 131) show that investors' fear, measured by left jump tail risk, is significantly priced for momentum investors. Generally, investors perceive risk as downside risk, whereas upside risk is seen as upside potential (Lee and Rao, 1988). In contrast, volatility quantifies both downside risk and upside potential as risk, and hence fails to capture the risk of suffering high losses. This holds especially for extreme losses such as momentum crashes. Bollerslev et al. (2015, Fig. 4) also find that a measure that captures left jump tail risk is highly different to volatility, i.e. measuring (extreme) losses is different to measuring return deviations. Thus, weighting assets based on volatility does not optimally fit to most investors' preferences, whereas weightings that incorporate non-normalities and the assets' loss potential are more realistic. Furthermore, weighting schemes based on risk measures that incorporate higher moments also fit well to preferences of safety-first investors, who are concerned with avoiding rare disasters (see Bali et al. (2009), Arzac and Bawa (1977), Van Oordt and Zhou (2016) and references therein).

The second disadvantage arises since momentum is a long-short strategy and volatility is a symmetric risk measure that does not distinguish between long or short positions. Giot and Laurent (2003) find that the (tail) risk of an asset can be quite different, depending on whether the asset is a long or a short position. Similarly, Atilgan et al. (2020, Sec. 5.6) find an asymmetry between assets' left and right tail risk. Bollerslev et al. (2015, Fig. 2) show that left jump tail risk and right jump tail risk are highly different, where left jump tail risk is significantly higher in magnitude than right jump tail risk. Bollerslev et al. (2020, Sec. 3) also show that systematic downside risk of long and short positions is not identical. Hence, volatility is not the adequate measure to quantify risk of the assets in the momentum portfolio, since the momentum portfolio contains long and short positions. As mentioned above, weighting low volatile assets higher than highly volatile assets typically improves the risk-adjusted performance and lowers drawdowns of the volatility managed portfolio compared to the equally weighted portfolio. However, since the momentum strategy is short the losers portfolio, an enhanced performance of the losers is not desired. Thus, the superior performance of the volatility weighted winners portfolio is (partly) offset by a superior performance of the volatility managed losers portfolio. In particular, simply applying a weighting scheme that works well for long-only portfolios to a long-short portfolio is insufficient. Similarly, Baltas (2015) state: "Simply inverting the long-only solution for the assets with a short position is completely incorrect". In contrast, a good weighting scheme should produce an *inferior* performance with *higher* drawdowns of the losers. Ang et al. (2006b, p. 292) also state that "one way to improve the returns to a momentum strategy is to short past losers with high idiosyncratic volatility". In a similar setting, Frazzini and Pedersen (2014), Asness et al. (2014), Asness et al. (2020) and Liu et al. (2018) examine long-short strategies where assets wither higher risk obtain lower (higher) weights in the long (short) portfolio than assets with lower risk. This weighting approach is more realistic and more in line with the low risk anomaly. Therefore, the performance of the inverse volatility approach of Clare et al. (2014), Du Plessis and Hallerbach (2017) and Goyal and Jegadeesh (2017) can further be improved by using a weighting scheme that reflects the information on whether an asset is a long or short position.[41] Instead of overweighting highly volatile losers, as suggested by Ang et al. (2006b), we use asymmetric risk measures that distinguish between long and short positions. In order to distinguish between long and short positions, we measure risk as left (right) tail risk for an asset in the winners (losers) portfolio.

Besides the advantages that a tail risk weighting approach incorporates non-normalities and information on whether an asset is a long or short position, this weighting approach also considers that winners and losers exhibit quite different tail risk characteristics. For example, Chen et al. (2001), Harvey and Siddique (2000) and Langlois (2020) find that winners exhibit a lower skewness than losers. In particular, the winners' and losers' skewness is highly time-varying and moves in opposite directions (Rickenberg, 2020a). Further, Atilgan et al. (2020, Table 1.B) and Bali et al. (2014, Table 2) find a negative relation between momentum and tail risk, i.e. losers exhibit a higher tail risk than winners. Bollerslev et al. (2015) find that jump tail risk is priced differently for winners and losers and investors' fear is especially priced for the losers portfolio. Similarly, Jang and Kang (2019) find that the crash risk of the assets in the momentum portfolio can be quite different, where losers exhibit a higher probability of extreme crashes. Thus, simply equal or volatility weighting momentum's assets would lead to a portfolio where the portfolio's crash risk is influenced by a few assets. Furthermore, weighting the assets by their tail risk is also advantageous, since the low risk anomaly, that has been shown for volatility, also holds for tail risk. For example, Jang and Kang (2019, Figure 2) show that stocks with a higher crash probability underperform stocks with a lower probability of extremely negative returns. The authors find that their crash probability measure quantifies left tail risk and

[41] This also holds for the volatility weighted TSMOM strategy examined by Clare et al. (2014), Clare et al. (2016), Du Plessis and Hallerbach (2017), Goyal and Jegadeesh (2017), Kim et al. (2016) and Moskowitz et al. (2012). The authors use volatility weighting for both long and short positions, since the assets in the portfolio have quite different levels of risk. By the same argument as above, the performance of the TSMOM strategy could potentially be improved by scaling the assets based on their tail risk, since this weighting approach differs between long and short positions. Baltas (2015) develops a volatility based weighting approach that works for the TSMOM strategy and is suitable for portfolios that contain long and short positions.

is related but different to volatility. Thus, assets with higher left tail risk exhibit lower returns as also shown by Atilgan et al. (2020), Bi and Zhu (2020) and Bali et al. (2014, Table 1), but this effect is different to the low volatility effect of Ang et al. (2006b). In particular, Bi and Zhu (2020, Table 5) find that the negative tail risk and return relation holds in periods of high and low market volatility, i.e. the negative tail risk-return relation holds in low and high risk regimes.

Based on the observations summarized above, risks of the assets in the winners and losers portfolios should be managed differently, where especially the assets' tail risk should be regarded. More precisely, in the winners portfolio, assets with a higher left tail risk should obtain lower weights whereas the opposite holds for the losers portfolio. This important observation is not captured by the equal and inverse volatility weighting. Furthermore, Jang and Kang (2019, Table 4) show that the crash probability is not captured by momentum. A similar observation also holds for momentum and skewness or other tail risk measures. As a consequence, combining information on an asset's momentum and tail risk is appealing, since both measures capture different characteristics. The combined approach that first sorts assets by their momentum and then weights winners and losers by their tail risk should lower momentum's crash risk compared to the equally and volatility weighted momentum strategy. Furthermore, the tail risk managed momentum portfolio should additionally produce higher returns than the other two weighting schemes.

To address the before mentioned drawbacks of the volatility weighted momentum portfolio, we next present several risk measures that incorporate non-normalities of the assets' return distribution and consider whether the asset is a long or short position. By adopting the inverse risk weighting scheme of Equation (3.2.3), month t weight of asset i in the winners and losers portfolio is given by

$$w_{i,t}^{W} = \frac{1/\mathcal{R}_{i,t}^{W}}{\sum_{j=1}^{n} 1/\mathcal{R}_{j,t}^{W}} \quad \text{and} \quad w_{i,t}^{L} = \frac{1/\mathcal{R}_{i,t}^{L}}{\sum_{j=1}^{n} 1/\mathcal{R}_{j,t}^{L}}, \tag{3.3.1}$$

where $\mathcal{R}_{i,t}^{W}$ and $\mathcal{R}_{i,t}^{L}$ denote the month t risk of asset i in the winners and losers portfolio, respectively. Using the inverse risk weighting based on risk measures that incorporate non-normalities means that momentum's tail risk is timed and this approach is a natural extension of the volatility timing strategies frequently examined in the literature. Jang and Kang (2019, Sec. 4.4) find that institutional investors who time the crash risk of their holdings earn higher returns, which makes tail risk timing an appealing strategy for the momentum portfolio. Furthermore, since assets with higher tail risk are typically less liquid (Atilgan et al., 2020, Bali et al., 2014), the inverse tail risk weighting should also reduce transaction costs. Generally, the benefits of tail risk timing have also been shown by Agarwal and Naik (2004), Alexander and Baptista (2004), Basak and Shapiro (2001), Cuoco et al. (2008), Jondeau and Rockinger (2012), Jondeau and Rockinger (2006), Rickenberg (2020b) and Rickenberg (2020a).

Since some of our risk measures presented below can become negative or zero, we rescale risk measures that are not strictly positive by definition by a logistic transformation. Hence, if a risk measure $\mathcal{R}_{i,t}$ can potentially be zero or negative, we use the weighting scheme $\frac{1/\exp(\mathcal{R}_{i,t})}{\sum_{j=1}^{n} 1/\exp(\mathcal{R}_{j,t})}$ instead of $\frac{1/\mathcal{R}_{i,t}}{\sum_{j=1}^{n} 1/\mathcal{R}_{j,t}}$.[42] For the rank based weighting in Equation (3.2.4), we rank assets in the winners and losers portfolios by their estimated risk $\mathcal{R}_{i,t}^{W}$ and $\mathcal{R}_{i,t}^{L}$.[43] For both weightings, we define the risk measure in a way that $\mathcal{R}_{i,t}^{W}$ measures an asset's *downside risk*, whereas $\mathcal{R}_{i,t}^{L}$ measures an asset's *upside risk*. Thus, in the winners portfolio, assets with a higher left tail risk obtain lower weights, whereas in the losers portfolio, assets with a higher right tail risk obtain lower weights. More formally, we define by

$$\tilde{R}_{i,t}^{L} := -R_{i,t}^{L} \tag{3.3.2}$$

the momentum investor's return coming from shorting asset i in the losers portfolio. Risk for an asset in the winners portfolio is then defined as the left tail risk of $R_{i,t}^{L}$, whereas risk for a loser asset is defined as the left tail risk of $\tilde{R}_{i,t}^{L}$.

One easy and straightforward method to incorporate non-normalities into the volatility weightings in Equations (3.2.3) and (3.2.4) would be to use volatility forecasts that capture

[42] Using $\exp(\mathcal{R}_{i,t})$ instead of $\mathcal{R}_{i,t}$ is similar to imposing a short-sale constraint in an optimization problem, which is found beneficial in portfolio allocation problems. A short-sale constraint has a shrinkage like effect, and thus is particularly important for estimates with high sampling errors (DeMiguel et al., 2009b, Jagannathan and Ma, 2003). Since we estimate monthly risk using non-parametric estimation methods that are likely to suffer under estimation risk, a short-sale constraint is appealing for our approach. Further, by imposing a short-sale constraint in the weighting scheme, buy and sell signals are solely made using the assets' momentum.

[43] Another alternative to the tail risk based inverse risk and rank weightings would be to use a Fama-French type weighting that sorts assets based on size and tail risk (Atilgan et al., 2020, Sec. 5.2).

non-normalities in the return distribution. Taylor (2005) presents several approaches how information on higher moments can be incorporated in volatility forecasts. These approaches exploit information on certain quantiles to forecast volatility. However, since we use other quantile based risk measures, this approach is not presented here. We also used the approach of Taylor (2005) and found quite similar results to the methods present later in this section.

Another easy method to incorporate non-normalities of the return distribution is to use weighting schemes that are directly based on higher moments like skewness and kurtosis. As for volatility, kurtosis has the disadvantage that risk is measured symmetrically, and thus the kurtosis does not distinguish between long and short positions. However, due to the importance of kurtosis in many fields of finance, we also use kurtosis based weighting schemes. In particular, portfolio allocations based on skewness and kurtosis are frequently examined in the literature (Ghysels et al., 2016, Guidolin and Timmermann, 2008, Jondeau and Rockinger, 2006, 2012, Jondeau et al., 2019, Langlois, 2020).[44] A high negative (positive) skewness and a high kurtosis make extremely negative (positive) returns more likely. Moreover, Jiang et al. (2020, Table 4) find that assets with a higher skewness also exhibit slightly higher returns. Hence, since investors have preferences for higher levels of skewness, assets with a lower (or negative) skewness offer an unattractive risk-return profile for investors. Thus, an asset with a high negative (positive) skewness and high kurtosis should be weighted lower in the winners (losers) portfolio. Following Ang and Chen (2002, Eq. 28), month t skewness for an asset i in the winners and losers portfolio is then defined by

$$\mathrm{Skew}_{i,t}^{W} = \frac{\mathbb{E}\Big(\big(R_{i,t}^{W} - \mathbb{E}\big(R_{i,t}^{W}\big)\big)^{3}\Big)}{\mathrm{var}(R_{i,t}^{W})^{3/2}} \quad \text{and} \quad \mathrm{Skew}_{i,t}^{L} = \frac{-\mathbb{E}\Big(\big(R_{i,t}^{L} - \mathbb{E}\big(R_{i,t}^{L}\big)\big)^{3}\Big)}{\mathrm{var}(R_{i,t}^{L})^{3/2}}. \tag{3.3.3}$$

To define a skewness based risk measure that downweights stocks wither higher risk, we multi-

[44] Skewness and kurtosis are also important for asset pricing (Bali et al., 2009, Dittmar, 2002, Harvey and Siddique, 2000, Kraus and Litzenberger, 1976), modeling return distributions (Hansen, 1994, Harvey and Siddique, 1999, Jondeau and Rockinger, 2003) and risk management (Bali et al., 2008). Further, skewness preferences can partly explain the (idiosyncratic) volatility puzzle mentioned in the previous section (see Boyer et al. (2009, p. 198), Schneider et al. (2020) and Amaya et al. (2015)).

ply skewness by -1 and define the risk measure as[45]

$$\mathcal{R}_{i,t}^{W} = \exp\left(-\text{Skew}_{i,t}^{W}\right) \quad \text{and} \quad \mathcal{R}_{i,t}^{L} = \exp\left(-\text{Skew}_{i,t}^{L}\right). \tag{3.3.4}$$

Since the skewness of $R_{i,t}^{L}$ equals the skewness of $\tilde{R}_{i,t}^{L}$ multiplied by -1, higher risk for asset i in the winners portfolio is measured by a lower (or negative) skewness, whereas higher risk for asset i in the losers portfolio is measured by a higher (or positive) skewness. Thus, in the winners (losers) portfolio, more negatively (positively) skewed assets are weighted lower. As for volatility, we estimate monthly skewness by the realized counterpart using daily data.[46] We follow Jang and Kang (2019) and estimate realized skewness using the past six months of daily data. The estimation of realized skewness is given in Equation (C.3) in Appendix C. However, other estimation lengths are also frequently used in the literature. For example, Kelly and Jiang (2014, p. 2861) estimate monthly skewness and kurtosis based on daily data of one month, Langlois (2020) estimate realized skewness using 12 months of daily data, Bali et al. (2012) estimate monthly skewness using the last 36 months of monthly returns, whereas Amaya et al. (2015) use several estimation windows between five days and 60 months of daily data. Using different estimation lengths to estimate realized skewness can lead to quite different conclusions (Amaya et al., 2015, Table 15). As a robustness check, we also use other lengths to measure skewness and kurtosis. These robustness results are shown in Table XVI in Appendix B.1.

Although Amaya et al. (2015, Table 1) and Langlois (2020) find that realized volatility captures some of the skewness and other risks, i.e. a higher realized volatility is related to lower skewness, higher kurtosis and higher systematic risk, measured by a lower coskewness and higher beta, Amaya et al. (2015, Table 4) find that realized skewness captures other information than volatility, beta and coskewness. Similarly, Boyer et al. (2009) and Stambaugh et al. (2015) also find a relation between idiosyncratic volatility and skewness, but they also state that both

[45]Chen et al. (2001, p. 353) also multiply skewness by -1 because "[b]y putting a minus sign in front of the third moment, we are adopting the convention that an increase in [the skewness measure] corresponds to a stock being more "crash prone", i.e. having a more left-skewed distribution."

[46]Calculating monthly skewness based on daily returns is similar to the concept of realized skewness used by Amaya et al. (2015) who estimate weekly moments based on high-frequency data. See also Neuberger (2012) on the estimation of skewness using high-frequency data. Amaya et al. (2015) find that these estimates capture a different kind of information than measures estimated with daily data. For example, realized skewness measured with high-frequency data captures price jumps. This does not (necessarily) hold for the measures calculated with daily data.

risks are different. Furthermore, Harvey and Siddique (2000, p. 1290) cannot find statistical differences between volatilities in the winners and losers portfolios, but between the skewness of both portfolios. This indicates that the volatility weighting presented in Section 3.2.4 is not suitable to manage the higher moment risks of the winners and losers. Managing the momentum portfolio's skewness risk is important for several reasons. First, Harvey and Siddique (2000, Table I) show that different industries have quite different levels of skewness. Some industries are negatively skewed, whereas others are positively skewed. Similarly, Ang and Chen (2002, Table 5) find that industries are typically negatively skewed and that skewness between different industries can vary extremely. Boyer et al. (2009) find that the industry affiliation is an important determinant of skewness. Atilgan et al. (2019, Table 1.B) confirm this finding for country indices and the authors find that some countries are negatively skewed, whereas others are positively skewed. Amaya et al. (2015) also find that realized skewness and kurtosis are highly different for different assets and that these higher moments are highly time-varying. The time-variation of skewness is also examined by Jondeau and Rockinger (2012), Jondeau and Rockinger (2003), Harvey and Siddique (1999), Rickenberg (2020a) and Bali et al. (2008), and thus portfolio weights should be readjusted frequently, as done by the inverse skewness weighting. Consequently, simply equal or volatility weighting industries implies that the portfolio's skewness risk is dominated by the skewness of a few industries. Second, momentum is related to skewness, i.e. winners are more negatively skewed than losers (Amaya et al., 2015, Chen et al., 2001, Harvey and Siddique, 2000, Langlois, 2020). This produces a high negative skewness of the momentum portfolio, which increases left tail risk and the probability of momentum crashes. By weighting assets in the momentum portfolio inversely to their skewness, winners should be more positively skewed and losers should be more negatively skewed. By buying the skewness weighted winners and shorting the skewness weighted losers, the high crash risk of the momentum portfolio can be reduced. Thus, combining information on the assets' momentum and skewness is an appealing approach to produce high returns with limited crash risk. Amaya et al. (2015, Sec. 5.2.3) also combine a skewness and a return based strategy and find that the combined strategy outperforms the individual strategies.[47] Third, investors typically

[47] Jacobs et al. (2015) also incorporate information on an asset's skewness into the momentum portfolio building

have preferences for higher skewness (Harvey and Siddique, 2000, Kraus and Litzenberger, 1976, Scott and Horvath, 1980), i.e. most investors “are willing to give up some of the right tail to reduce the left tail” (Harvey et al., 2018, Footnote 3). By downweighting negatively (positively) skewed assets in the winners (losers) portfolio, the momentum portfolio’s risk-return profile is more in line with investors’ preferences. Fourth, Chen et al. (2001) state that skewness is an easy method to measure an asset’s crash probability.[48] Since investors are typically crash-averse and are willing to pay high fees to hedge against extreme crashes (Bollerslev and Todorov, 2011, Chabi-Yo et al., 2018), investors highly benefit from the crash risk mitigation of the skewness weighted momentum portfolio. Similarly, a higher negative skewness indicates that (extremely) high negative returns are more likely than (extremely) high positive returns. Since investors weight losses higher than gains (Aït-Sahalia and Brandt, 2001, Benartzi and Thaler, 1995), skewness weighted portfolios better fit to investors being loss-averse. For the reasons summarized above, Jondeau et al. (2019, p. 29) conclude that “investor decisions are likely to be highly sensitive to the level of skewness”.

We restrict the estimation of skewness to the easy realized sample estimator as shown in Appendix C. Using the simple realized estimator makes this approach appealing for practical implementations. However, several extensions of this approach are also feasible. For example, Amaya et al. (2015) present several alternative estimation methods based on high-frequency and daily data to estimate realized skewness. Moreover, instead of estimating an industry’s skewness based on past returns of the industry, skewness could also be estimated by first estimating the skewness of each asset in that industry. The industry’s skewness can then be calculated as the (equally or value-weighted) average of the skewness of each asset (see Boyer et al. (2009, p. 184), Langlois (2020) and Jondeau et al. (2019)).[49] Further, instead of using past realized skewness, a forecast of next month’s expected skewness could be used. However, Boyer et al. (2009) find that past skewness is an important determinant of future skewness, and

process. However, their approach increases left tail risk, whereas our aim is to reduce momentum’s left tail risk.

[48]Jang and Kang (2019) also find that skewness is similar to a risk measure that captures the probability of extreme losses.

[49]Jondeau et al. (2019, Fig. 2) show that both measures of skewness, realized skewness and average skewness, capture quite different characteristics of risk. Atilgan et al. (2019, Table 1) also find that average skewness and skewness of countries can be quite different.

hence past skewness is a good indicator of future skewness. This especially holds when we are only interested in an asset's skewness rank as done by the rank weighting. Nevertheless, other variables like past momentum are also successful in forecasting skewness and could also be included when next month's skewness is estimated (Boyer et al., 2009, Chen et al., 2001, Langlois, 2020). Alternatively, approaches that model skewness autoregressively as in Harvey and Siddique (1999), Jondeau and Rockinger (2012), Jondeau and Rockinger (2003) and Bali et al. (2008) could also be used to forecast next month's skewness. Furthermore, since skewness estimates are very sensitive to few extreme realizations, quantile based estimates of skewness that are more robust to outliers could be used (Amaya et al., 2015, Ghysels et al., 2016, Jiang et al., 2020, Kim and White, 2004, Langlois, 2020).[50] However, by using our rank based weighting, portfolio weights are less sensitive to noisy estimates of skewness. We also used skewness estimates based on quantiles, but found better performance results for the weightings based on realized skewness. A possible explanation for this finding could be that realized moments emphasize observations in the far tails of the return distribution, and hence a realized skewness based asset allocation is more successful in managing momentum crashes. Ghysels et al. (2016) also find that emphasizing realizations in the tails of the distribution are superior to measures that focus on less extreme realizations in portfolio allocations. Similarly, an alternative measure of an asset's crash risk that emphasizes observations in the tails is the crash probability of Jang and Kang (2019, Eq. (1)). Jang and Kang (2019) show that this measure also captures left tail risk and is similar to skewness. To weight assets in the losers portfolio, the jackpot probability that measures the probability of extremely high returns could be used (see Jang and Kang (2019) and referenced therein). Jang and Kang (2019) find that institutional investors who time the crash probability of their holdings produce higher returns, which demonstrates the importance of tail risk timing. However, since this paper focuses on measures that can be estimated by simple non-parametric approaches and since skewness is highly related to the crash probability,

[50] See also Jondeau et al. (2019, Footnote 5) for several possibilities how the influence of outliers can be reduced when skewness is estimated. Another alternative that is less influenced by outliers would be to directly forecast the cross-sectional skewness rank as shown by Langlois (2020). The author finds that an asset's skewness rank is easier to predict than an asset's skewness and that an asset's past skewness rank is a good predictor for an asset's future rank. Thus, our simple non-parametric sample estimator should also be a good predictor for an asset's future skewness rank.

we leave this examination for future research. Moreover, information on skewness could also be extracted from option prices (Chang et al., 2013, Neuberger, 2012, Schneider et al., 2020). Furthermore, industries could also be weighted based on their skewness innovations (Chang et al., 2013) or their (expected) idiosyncratic skewness (Bali et al., 2011, Boyer et al., 2009, Jacobs et al., 2015, Langlois, 2020). However, as for the case of volatility and idiosyncratic volatility, we found better results for the realized measures of total skewness instead of idiosyncratic skewness. A possible explanation could again be that we use assets that are portfolios themselves instead of individual assets.[51] Finally, the maximum (or minimum) return, a measure that also captures idiosyncratic risk, could be used as a skewness measure (Bali et al., 2011, Boyer et al., 2009, Jacobs et al., 2015). Bali et al. (2011) find that the maximum return is also related to realized skewness, but both measures capture different characteristics.[52]

Due to the importance of kurtosis for many financial fields, we also use a kurtosis based weighting. The kurtosis is defined by

$$\mathrm{Kurt}_{i,t}^{W} = \frac{\mathbb{E}\Big(\big(R_{i,t}^{W} - \mathbb{E}(R_{i,t}^{W})\big)^4\Big)}{\mathrm{var}(R_{i,t}^{W})^2} \quad \text{and} \quad \mathrm{Kurt}_{i,t}^{L} = \frac{\mathbb{E}\Big(\big(R_{i,t}^{L} - \mathbb{E}(R_{i,t}^{L})\big)^4\Big)}{\mathrm{var}(R_{i,t}^{L})^2}. \tag{3.3.5}$$

The month t risk for asset i in the winners and losers portfolio is then given by $\mathcal{R}_{i,t}^{W} = \mathrm{Kurt}_{i,t}^{W}$ and $\mathcal{R}_{i,t}^{L} = \mathrm{Kurt}_{i,t}^{L}$. We again estimate kurtosis as realized kurtosis (Amaya et al., 2015) using the past six months of daily data as shown in Equation (C.5) in Appendix C. An alternative to this estimation method is a again a quantile based estimation method (Kim and White, 2004). However, as for skewness, we find better results for the realized estimator, which emphasizes observations in the far tails. The quantile based estimation of kurtosis is similar to the quantile

[51] Using idiosyncratic skewness measures does not work well for assets that are portfolios of stocks. For example, Langlois (2020, Footnote 11) state that "[e]stimating idiosyncratic skewness using portfolios is not possible because portfolio aggregation diversifies idiosyncratic risk." Similarly, Goyal and Jegadeesh (2017, Footnote 1) write: "Empirical regularities documented for indexes and asset classes need not carry over to individual stocks and vice versa because indexes diversify away firm-specific returns". Generally, total skewness and idiosyncratic skewness are typically quite similar (see Boyer et al. (2009, p. 187) and Langlois (2020)).

[52] Bali et al. (2011, Table 5) find that a high MAX is a proxy for high (idiosyncratic) volatility, high beta and low momentum. Hence, weighting assets by their MAX seems appealing. Following Asness et al. (2020) we also used a MAX and volatility standardized MAX (SMAX) weighting, but found that weighing assets in this way does not work as well as our remaining models. This finding is quite intuitive, since MAX is a measure of a certain stock's idiosyncratic risk based on lottery demands of investors. Thus, as expected, the MAX theory cannot directly be translated to industry portfolios. Asness et al. (2020, Footnote 8) also find that MAX does not work well for industries, whereas systematic risk measures work well for industries as shown by Asness et al. (2014). Therefore, it is better to measure an industry's risk as total or systematic risk instead of idiosyncratic risk. Weighting assets by their systematic risk is examined in the next section.

based estimation of volatility used by Taylor (2005).

Kurtosis measures the "extremes of the return distribution" (Amaya et al., 2015, p. 139), and hence the kurtosis based weighting should dampen momentum crashes. However, as for volatility, kurtosis is a symmetric risk measure. Thus, the kurtosis based weighting also reduces the likelihood of extremely positive returns, which is a disadvantage of symmetric risk measures. For example, Amaya et al. (2015) show that realized skewness and kurtosis, measured with high-frequency data, capture jumps of the assets. Realized skewness differs between positive and negative jumps, whereas realized kurtosis measures negative and positive jumps as risk.[53] Thus, skewness captures only the risk a momentum investor is concerned of, whereas kurtosis also captures an asset's upside potential as risk and does not distinguish between long and short positions.

The before presented skewness and kurtosis measures are hard to estimate directly, since realized estimates of higher moments are highly influenced by outliers and few extreme realizations (Ghysels et al., 2016, Kim and White, 2004). For that reason, we next present several other risk measures that incorporate information on skewness and kurtosis without explicitly relying on estimates of these quantities. In particular, the interaction of both moments is also important, e.g. investors are more concerned about a high kurtosis combined with a negative skewness, since this combination makes extremely negative returns more likely. In contrast, a high kurtosis combined with a positive skewness is less problematic. Similarly, a given level of negative skewness is more concerning for high levels of kurtosis than for low levels. A frequently used class of risk measures that incorporates information on higher moments and also distinguishes between long and short positions is the class of *Lower Partial Moments (LPM)*. Lower Partial Moments have been frequently used to expand the well-known mean-variance approach to a mean-LPM optimization (Bawa and Lindenberg, 1977, Lee and Rao, 1988, Price et al., 1982).[54] Further, Atilgan et al. (2019) and Bali et al. (2014) use LPM based measures in

[53]This statement has to be regarded carefully, since we measure monthly realized moments based on daily data, whereas Amaya et al. (2015) use high-frequency data. Amaya et al. (2015, p. 141-142) state that "[s]kewness estimates from moving windows of daily or weekly data are likely to have different averages than skewness measures constructed from intraday data" and "skewness (and kurtosis) measures computed from high-frequency data are likely to contain different information from those computed from daily data or from options". However, this result shows that skewness captures left tail risk, whereas kurtosis captures the risk in both tails.

[54]Bawa and Lindenberg (1977, Theorem 3) examine the optimal portfolio selection in a mean-LPM model and

a cross-sectional analysis. Bali et al. (2014, Table 1) and Atilgan et al. (2019, Table 3) show that stocks with higher tail risk, measured by LPM, underperform assets with lower tail risk. This result does not only hold for individual stocks but also for style portfolios and country indices of developed markets, where the negative LPM-return relation is especially pronounced and statistically significant for style portfolios (Atilgan et al., 2019, Tables 5 and 6). Thus, the low risk anomaly does not only hold for volatility but also for tail risk measured by LPM. This low LPM anomaly makes an inverse LPM weighting scheme appealing in order to enhance momentum's risk-return profile. In particular, Bali et al. (2014, Table 2) find a negative relation between an asset's momentum and LPM, which shows that assets in the momentum portfolio have quite different levels of tail risk. Thus, by using a simple equal or volatility weighting scheme, momentum's tail risk may be dominated by a few assets.

The LPM of order k for asset i in the winners and losers portfolio is defined by

$$\mathrm{LPM}^W_{i,t,k} = \mathbb{E}\big(\max(q - R^W_{i,t}, 0)^k\big) \quad \text{and} \quad \mathrm{LPM}^L_{i,t,k} = \mathbb{E}\big(\max(q + R^L_{i,t}, 0)^k\big)\,, \tag{3.3.6}$$

where q is a chosen threshold (see Lee and Rao (1988, Eq. (1)), Price et al. (1982, Eq. (1)) and Bali et al. (2014, Eq. (1))).[55] To better capture changes in the risk of the assets, we use a quite short estimation window of six months of daily data and a threshold of $q = 0$ in the main part. Furthermore, as in Atilgan et al. (2019) and Bali et al. (2014), we also used quantile based definitions of the threshold q and other estimation lengths. Another alternative would be to use the risk-free rate as cut-off point q (Bawa and Lindenberg, 1977). Bali et al. (2014, Table 4) find that LPM negatively predicts future returns for several cut-off points, where the negative relation is stronger for cut-off points in the center instead of the tail of the distribution. Results for other cut-off points were quite similar and are not shown here, but robustness results for other estimation lengths are shown in Appendix B.1. As choices for the order k, we follow the literature and choose orders of $k = 0, 1$ and 2. The LPM of order $k = 0$ is also called shortfall probability, the LPM of order $k = 1$ is called shortfall expectation and the LPM of order $k = 2$

show that this model reduces, under certain conditions like normally distributed returns, to the mean-variance portfolio selection rule. However, for non-normal and skewed distributions, both approaches can be quite different (Price et al., 1982).

[55]The LPM for asset i in the losers portfolio can be rewritten as $\mathbb{E}\big(\max(R^L_{i,t} - (-q), 0)^k\big)$. Thus, risk for the losers portfolio is measured by only regarding returns that are higher than $-q$, which corresponds to the Upper Partial Moment (UPM) with a threshold of $-q$.

is also known as shortfall variance. The shortfall variance is essentially identical to the well-known semivariance (Ang et al., 2006a, Bollerslev et al., 2019, Patton and Sheppard, 2015).[56] Bali et al. (2009, Eq. (18)) also use a risk measure that measures the variance of extremely negative returns, which is similar to the LPM of order 2. Using a semivariance based weighting is a natural extension of the volatility based weighting presented in the previous section. The semivariance measures the volatility of losses, and thus contains information on (extreme) losses without regarding high positive returns as risk. In contrast, symmetric risk measures like volatility "eliminate any information that may be contained in the sign of these returns" (Patton and Sheppard, 2015, p. 683). In particular, by measuring the volatility of losses, a higher tail risk also translates into a higher semivariance, and hence this approach is appealing to manage momentum crashes. Further, by defining the volatility only for losses, the semivariance also distinguishes between long and short positions. For an asset in the winners portfolio, risk is defined as the volatility of negative returns, whereas for an asset in the losers portfolio, risk is defined as the volatility of positive returns. Patton and Sheppard (2015) show that the volatility of positive and negative returns contains quite different information, thus the winners' and losers' risk should be managed differently. In particular, volatility of positive and negative returns contain other information than the normal RV measure. Hence, we expect different results of the volatility and semivariance based weightings and the semivariance weighting should be advantageous for long-short portfolios like momentum. As an alternative, positive and negative semivariance can also be used to forecast an industry's volatility in an HAR model (Patton and Sheppard, 2015, Eq. (16)). The winners and losers portfolio could then be weighted based on this volatility forecast. This approach is not examined here but, following Bollerslev et al. (2019) and Patton and Sheppard (2015), we also use two additional risk measures that are based on the semivariance and will be presented later.

Besides LPMs we also use two other quantile based risk measures, Value at Risk (VaR) and Conditional Value at Risk (CVaR), that are similar to LPMs but capture different aspects of risk as shown by Atilgan et al. (2020, Sec. 5.5). VaR and CVaR are frequently used in the

[56] Price et al. (1982, Footnote 1) state that "semivariance and lower partial moments are interchangeable terms. However, [...] the literature does distinguish between the two in terms of their theoretical origins". See also Price et al. (1982, Footnote 3) for a note on the different theoretical origins.

financial literature but these risk measures are also important from a practical view. VaR has been the most prominent risk measure in recent years and is important for portfolio managers (Agarwal and Naik, 2004, Alexander and Baptista, 2004, Cuoco et al., 2008, Schmielewski and Stoyanov, 2017) as well as for banks and regulators (Bali et al., 2009, 2008). However, CVaR now emerges as the more relevant risk measure for portfolio managers (Basak and Shapiro, 2001, Rickenberg, 2020b) as well as for banks and regulators (Du and Escanciano, 2016). The reason for this finding is that CVaR corrects for several disadvantages of VaR. For example, CVaR is a coherent risk measure, whereas VaR is only coherent for certain assumptions (Acerbi and Tasche, 2002). Further, the VaR, which is defined as a certain quantile, does not provide information on extreme losses (Basak and Shapiro, 2001, Yamai and Yoshiba, 2005). In contrast, the CVaR measures risk by incorporating information on the losses higher than the VaR, and thus captures information on the whole tail and is a good tool to quantify the severity of extreme losses.

Assuming that returns are continuously distributed, month t VaR for an asset i in the winners and losers portfolio is defined through the relation

$$\mathrm{P}\Big(-R_{i,t}^{W} > \mathrm{VaR}_{i,t}^{W,\alpha}\Big) = \alpha \quad \text{and} \quad \mathrm{P}\Big(R_{i,t}^{L} > \mathrm{VaR}_{i,t}^{L,\alpha}\Big) = \alpha, \tag{3.3.7}$$

where α is the chosen significance level. Thus, the VaR is defined as the maximum loss that is exceeded with a probability of $100\alpha\%$, where typically low levels of α are chosen. For asset i in the winners portfolio, we define a loss as $L_{i,t}^{W} := -R_{i,t}^{W}$, whereas a loss for the losers portfolio is defined as the return of the asset, i.e. $L_{i,t}^{L} := R_{i,t}^{L}$. Thus, the VaR for asset i in the winners and losers portfolio is simply given as the $(1-\alpha)-$quantile of the distribution of $-R_{i,t}^{W}$ and $R_{i,t}^{L}$, respectively. Hence, the VaR can also be written as $\mathrm{VaR}_{i,t}^{W,\alpha} = F_{-R_{i,t}^{W}}^{-1}(1-\alpha)$ and $\mathrm{VaR}_{i,t}^{L,\alpha} = F_{R_{i,t}^{L}}^{-1}(1-\alpha)$, where $F_{-R_{i,t}^{W}}^{-1}$ and $F_{R_{i,t}^{L}}^{-1}$ denote the inverse of the cdf of $-R_{i,t}^{W}$ and $R_{i,t}^{L}$, respectively. Consequently, risk for the winners portfolio is defined as left tail risk, whereas risk for the losers portfolio is defined as right tail risk. Giot and Laurent (2003) find that the VaR of long and short positions can differ extremely. For example, Giot and Laurent (2003, Fig.1-6) show that stocks typically have fat tails that are not symmetric. Atilgan et al. (2020, Sec. 5.6) also find that left and right tail risk are quite different. Hence, regarding the right or left tail is different and long and short positions should be managed differently. This important point is not regarded by weighting assets inversely to their volatility. Defining risk asymmetrically for winner and loser assets is more realistic, since investors are concerned about left tail risk for long position and right tail risk for short positions (Giot and Laurent, 2003). Thus, the VaR based weighting is more in line with the preferences of momentum investors.

As mentioned above, since VaR only captures the probability of extreme returns but not their severity, VaR has several disadvantages in a portfolio context (Alexander and Baptista, 2004, Basak and Shapiro, 2001, Rickenberg, 2020b). We therefore use the CVaR as alternative quantile based risk measure that corrects the drawbacks of VaR. The CVaR is defined as the average loss for months when the loss is higher than VaR. More formally, following Acharya et al. (2016, Eq. (1)), Bali et al. (2009, p. 901) and Rachev et al. (2007, p. 2329), month t CVaR for asset i in the winners and losers is defined by

$$\mathrm{CVaR}_{i,t}^{W,\alpha} = -\mathbb{E}\Big(R_{i,t}^{W} \mid -R_{i,t}^{W} > \mathrm{VaR}_{i,t}^{W,\alpha}\Big) \quad \text{and} \quad \mathrm{CVaR}_{i,t}^{L,\alpha} = \mathbb{E}\Big(R_{i,t}^{L} \mid R_{i,t}^{L} > \mathrm{VaR}_{i,t}^{L,\alpha}\Big)\,. \tag{3.3.8}$$

Therefore, for an asset in the winners portfolio, risk is defined as the average return of the $100 \cdot \alpha\%$ lowest returns multiplied by minus one. In contrast, risk for an asset in the losers portfolio is defined as the average return of the $100 \cdot \alpha\%$ highest returns. The CVaR for an asset in the losers portfolio, estimated non-parametrically, is similar to the MAX measure of Bali et al. (2017a, 2011).

We estimate VaR and CVaR non-parametrically as shown in Equations (C.7) and (C.8) using an α of 5% and the last 12 months of daily data. Bali et al. (2009) also estimate VaR non-parametrically using the last one to six months of daily data. We also used other estimation lengths and significance levels and found quite similar results to our main results. Furthermore, we also used several other alternative estimation methods of VaR and CVaR that are frequently used in the literature, but found no significantly better results of these models. For example, a simple alternative method that properly forecasts VaR and CVaR is presented in Bali et al. (2009, Eq. (14)). This approach is similar to the RV based volatility forecasting model used by Bollerslev et al. (2018). However, Bali et al. (2009) find similar results of this method to the simple approach of using month $t-1$ VaR as measure for next month's VaR. A reason for this finding is that Bali et al. (2009, Table 1.C) and Atilgan et al. (2020, Table 7) show that the non-parametric VaR is persistent, i.e. VaR measured in month $t-1$ is a good predictor for VaR in month t. Another simple non-parametric alternative would be to define risk as the as the difference between VaR of month $t-1$ and $t-2$, where higher values of this DeltaVaR measure indicate that an extreme loss occurred recently (Atilgan et al., 2020, Sec. 4.1). Further alternatives to the non-parametric estimator would be to use more sophisticated unconditional approaches or approaches that rely on a dynamic volatility model (see Rickenberg (2020b) and Rickenberg (2020a) and references therein). We also used downside risk estimates based on Extreme Value Theory (Allen et al., 2012, Schmielewski and Stoyanov, 2017) and the skewed t distribution of Hansen (1994), both unconditionally (Bali et al., 2009, Sec.II.A.3) and conditionally with time-varying parameters (Bali et al., 2008, Jondeau and Rockinger, 2003), but did not find significantly better results compared to the simple non-parametric approach. Allen et al. (2012) also use VaR and CVaR estimates based on a non-parametric approach, an EVT approach and an approach using a skewed distribution and the authors find quite similar results for all three methods. Further extensions would be to use VaR and CVaR forecasts that are estimated as averages of different VaR and CVaR forecasts using different estimation methods (Allen et al., 2012). Moreover, as in Allen et al. (2012) and Kelly and Jiang (2014), downside risk of industry i could also be estimated by using cross-sectional returns of all assets contained in industry i within month $t-1$.

The advantages of VaR and CVaR based trading strategies have been shown in several studies. For example, Rickenberg (2020b) and Rickenberg (2020a) uses VaR and CVaR targeting strategies that time a portfolio's tail risk and the author finds that these strategies outperform the volatility targeting strategies of Cederburg et al. (2020), Moreira and Muir (2017) and Barroso and Santa-Clara (2015). Alexander and Baptista (2004), Agarwal and Naik (2004), Basak and Shapiro (2001) and Cuoco et al. (2008) use mean-VaR and mean-CVaR optimization and show that these approaches are superior to mean-variance optimization. Similarly, Schmielewski and Stoyanov (2017) find good results for portfolios that invest in assets with a low VaR. An explanation of the advantages of the VaR and CVaR based approaches is that Atilgan et al. (2020)

find “a significantly negative cross-sectional relation between left-tail risk and future returns on individual stocks trading in the US and international countries” (see also Bi and Zhu (2020)). Atilgan et al. (2020) further show that the low VaR and low CVaR anomaly is different to the low volatility and low beta anomaly. In particular, Atilgan et al. (2020, Sec. 5.12) show that the low tail risk anomaly also holds for mutual funds, i.e. funds with higher left tail risk underperform funds with lower left tail risk. Moreover, Atilgan et al. (2019) find a negative risk-return relation for VaR and CVaR for international stocks, style portfolios and country indices of developed countries. This low tail risk anomaly is important for our inverse tail risk timing strategies and should lead to a superior risk-return profile of the tail risk managed momentum strategy. In particular, Atilgan et al. (2020, Table 2) show that the low tail risk anomaly cannot be explained by asset pricing models that include momentum. Further, Atilgan et al. (2020, Table 5) find that even after controlling for momentum in bivariate portfolio sorts, there is still a strong negative relation between tail risk and future return. Similarly, Bi and Zhu (2020, Table 6) find that the negative tail risk-return relation cannot be explained by momentum. Thus, an asset's momentum and tail risk quantify different characteristics and even after sorting assets based on their momentum, weighting assets inversely to their VaR and CVaR should produce higher (risk-adjusted) returns with lower crash risk.

The weighting schemes based on quantile risk measures as presented above have the advantage that information on extreme losses is taken into account, which should lower the high crash risk of momentum investing. Further, this weighting scheme also fits well to most investors' preferences, since risk measures like VaR and CVaR increase if negative skewness or kurtosis increases (Bali et al., 2009). Bali et al. (2009) find that investors who care about higher moments dislike higher values of VaR. In particular, Bali et al. (2009) find that VaR predicts negative future skewness and is a better predictor of future skewness than past skewness. Hence, industries that are undesired by investors obtain lower weights by the VaR and CVaR weighting. Moreover, due to the construction of quantile risk measures, non-normalities are regarded without explicitly relying on estimates of higher moments. Thus, portfolio weights obtained by these risk measures are more robust than portfolio weights that rely directly on higher moments

(Ghysels et al., 2016).[57] In particular, quantile risk measures quantify risk differently for assets in the winners and losers portfolio. For an asset in the winners portfolio, risk is defined as left tail risk, i.e. downside risk, whereas for an asset in the losers portfolio, risk is defined as right tail risk, i.e. upside potential. However, these approaches have the disadvantage that only a small part of the return distribution's information is used. For example, consider two assets in the winners portfolio that have the same level of downside risk, but different levels of upside potential. By weighting these assets inversely to their quantile risk, both assets obtain the same weight in the winners portfolio. However, the asset with higher upside potential should obtain a higher weight. Similarly, in the losers portfolio, assets with higher downside risk should obtain higher weights than assets with lower downside risk. To take this information into account, we next use weightings that exploit information on both tails of the return distribution. This approach is similar to the reward-to-risk timing strategy examined by Kirby and Ostdiek (2012) and Zakamulin (2017). Kirby and Ostdiek (2012, Eq. (12)) use a weighting scheme that is based on the ratio of the expected mean return and volatility. This approach is similar to a weighting scheme that weights assets by their (expected) Sharpe Ratio.[58] However, instead of using an estimate of the assets' mean return, we measure upside potential for an asset in the winners (losers) portfolio by right (left) tail risk. Kirby and Ostdiek (2012) also find that reward-to-risk timing strategies that rely on an indirect mean estimate are more robust than strategies that directly forecast an asset's mean return.[59]

As first measure that takes information on both tails into account, we use a measure examined by Bollerslev et al. (2019) and Patton and Sheppard (2015) that is based on the semivariance. Following Bollerslev et al. (2019) and Patton and Sheppard (2015, Eq. (6)), we define the up and down semivariance of asset i in the winners portfolio as

$$\mathrm{var}_{i,t}^{W,+} = \mathbb{E}\Big(\max\left(R_{i,t}^{W}, 0\right)^2\Big) \quad \text{and} \quad \mathrm{var}_{i,t}^{W,-} = \mathbb{E}\Big(\min\left(R_{i,t}^{W}, 0\right)^2\Big). \qquad (3.3.9)$$

[57] Ghysels et al. (2016) argue that a classical portfolio optimization that relies on mean, variance and skewness can produce quite noisy and time-varying portfolio weights. For that reason, Ghysels et al. (2016) use a quantile based estimation method to estimate skewness, which is more robust to outliers and produces more stable portfolio weights.

[58] Since portfolio weights for this approach can become negative, Kirby and Ostdiek (2012, Eq. (13) and (14)) use a similar weighting scheme where portfolio weights are defined to be non-negative.

[59] See also Appendix A where we summarize several arguments why portfolio allocations based on estimates of the absolute mean return are suboptimal.

Similarly, for asset i in the losers portfolio, up and down semivariance are given by

$$\mathrm{var}_{i,t}^{L,+} = \mathbb{E}\Big(\min\left(R_{i,t}^{L},0\right)^2\Big) \quad \text{and} \quad \mathrm{var}_{i,t}^{L,-} = \mathbb{E}\Big(\max\left(R_{i,t}^{L},0\right)^2\Big). \tag{3.3.10}$$

In particular, we have $\mathrm{var}_{i,t}^{L,+} = \mathbb{E}\Big(\max\left(-R_{i,t}^{L},0\right)^2\Big)$ and $\mathrm{var}_{i,t}^{L,-} = \mathbb{E}\Big(\min\left(-R_{i,t}^{L},0\right)^2\Big)$. Hence, the down and up semivariance decomposes the variance, used in the previous section, into "bad" and "good" variance, where bad variance for the winners (losers) is defined as the variance of negative (positive) returns. In order to define a risk measure that captures information on downside risk and upside potential, we follow Bollerslev et al. (2019) and Patton and Sheppard (2015, Eq. (8)) and use the spread of down and up semivariance, which the authors call a measure of signed jumps (SJ).[60] Calculating the spread between downside and upside risk measures is frequently done in the literature (Ang et al., 2006a, Van Oordt and Zhou, 2016). The SJ measure for the assets in the winners and losers portfolios is then given by

$$\mathrm{SJ}_{i,t}^{W} = \mathrm{var}_{i,t}^{W,-} - \mathrm{var}_{i,t}^{W,+} \quad \text{and} \quad \mathrm{SJ}_{i,t}^{L} = \mathrm{var}_{i,t}^{L,-} - \mathrm{var}_{i,t}^{L,+}. \tag{3.3.11}$$

Hence, the SJ measure indicates higher risk if down semivariance increases and/or up semivariance decreases. A positive SJ measure for an asset in the winners (losers) portfolio indicates that the asset's risk is dominated by downward (upward) jumps.[61] Thus, assets with a higher SJ risk should be weighted lower.[62] An alternative to define the spread between "bad volatility" and "good volatility" for winners and losers could be to define the spread only for periods when bad volatility is higher than good volatility, i.e. only for periods when risk is dominated by negative jumps (Patton and Sheppard, 2015, Eq. (19)). Further, the SJ measure could also

[60]When calculating this measure based on high-frequency data, the spread of down and up semivariance is a measure for signed jumps (SJ) that captures the risk of downside jumps. We multiply the SJ measure of Bollerslev et al. (2019) and Patton and Sheppard (2015) by -1 in order to guarantee that a higher value of this measure also corresponds to higher downside risk. Similarly, Bollerslev et al. (2015, p. 118) also develop a risk measure that is defined as the difference between left tail jumps and right tail jumps, and thus is defined in the way as our risk measure. Bollerslev et al. (2015, p. 118) argue that this jump tail measure quantifies investors' fear.

[61]The ability of the SJ to capture jump risk is mainly fulfilled when SJ is estimated with high-frequency data. We estimate the SJ with daily data where price jumps are less likely. However, the distinction between positive and negative jumps helps to demonstrate the ability of SJ to capture left and right tail risk. Bollerslev and Todorov (2011) and Bollerslev et al. (2015) also examine left jump tail risk and find that left jump tail risk captures investors' fear. However, the left jump tail risk measure examined by Bollerslev and Todorov (2011) and Bollerslev et al. (2015) is based on options data and not market data as the measure examined here.

[62]Patton and Sheppard (2015, p. 696) suggest: "Assessing the usefulness of realized semivariance and signed jump variation in concrete financial applications, such as portfolio management [...] represents an interesting area for future research."

be used to forecast an asset's volatility in an HAR model, since periods with negative jumps increase future volatility, whereas periods with positive jumps decrease future volatility (Patton and Sheppard, 2015, Eq. (18)). However, these approaches are not examined here.

The SJ measure can be interpreted as a measure of an asset's skewness (Bollerslev et al., 2019, Footnote 1), but has the advantage that this measure is more robust to outliers.[63] A negative value of SJ indicates that the distribution is negatively skewed. Since the SJ measure can be negative or zero, we define the risk measure used in Equation (3.3.1) by $\mathcal{R}_{i,t}^W = \exp\left(\mathrm{SJ}_{i,t}^W\right)$ and $\mathcal{R}_{i,t}^L = \exp\left(\mathrm{SJ}_{i,t}^L\right)$. Further, to account for different levels of variance of the different industries, we follow Bollerslev et al. (2019) and define by

$$\mathrm{RSJ}_{i,t}^W = \frac{\mathrm{SJ}_{i,t}^W}{\mathbb{E}\big((R_{i,t}^W)^2\big)} \quad \text{and} \quad \mathrm{RSJ}_{i,t}^L = \frac{\mathrm{SJ}_{i,t}^L}{\mathbb{E}\big((R_{i,t}^L)^2\big)} \tag{3.3.12}$$

a normalized signed jump measure. By normalizing the SJ measure, the relative signed jump (RSJ) measures lies between -1 and 1. Bollerslev et al. (2019, Table 1.B) find that the RSJ measure is highly correlated with the realized skewness measure. However, the authors find that both measures capture different aspects of risk. Further, RSJ risk captures other information than momentum, which makes the combination of an asset's momentum and RSJ appealing. The risk measures used to weight the industries within the momentum portfolio are again defined by $\mathcal{R}_{i,t}^W = \exp\left(\mathrm{RSJ}_{i,t}^W\right)$ and $\mathcal{R}_{i,t}^L = \exp\left(\mathrm{RSJ}_{i,t}^L\right)$.

Following Chen et al. (2001, p. 354), we use another risk measure that is based on the relation between "good" and "bad" volatility. Chen et al. (2001) state that this measure, similar to the skewness, quantifies the asymmetry in the return distribution but does not incorporate an estimate of the third moment. This is advantageous since estimates of the third moment can be quite noisy (Ghysels et al., 2016, Kim and White, 2004). The measure of Chen et al. (2001), named down-to-up volatility (DuVol), is defined as the ratio of the down and up variance. In

[63] Jiang et al. (2020, Eq. (1)) also define a similar measure that captures the asymmetry of the return distribution, which is an alternative to the realized skewness but has the advantage that no estimate of the third moment is needed. This measure of Jiang et al. (2020) is defined as the difference between the probability of positive and negative returns. Thus, similar to the SJ measure, this skewness measure can be written as the difference of two LPMs of order $k = 0$ and measures the difference between information in the left and right tail. In our case, risk for an asset in the winners (losers) portfolio can be defined as the probability of negative (positive) returns minus the probability of positive (negative) returns. Jiang et al. (2020, Eq. (4)) additionally define a second entropy based asymmetry measure. Jiang et al. (2020) find that these measures capture other aspects of risk than the realized skewness measure.

contrast to the SJ and RSJ measure, the DuVol is a conditional risk measure.[64] Further, in contrast to the SJ measure and the measure used by Chen et al. (2001), we additionally extend the DuVol measure by only regarding observations in the far tails. Bali et al. (2009, Eq. (18)) also use a conditional measure of an asset's down variance by conditioning on returns in the far left tail. The DuVol measure for an asset i in the winners and losers portfolios is given by

$$\mathrm{DuVol}_{i,t}^{W} = \frac{\mathrm{var}\left(R_{i,t}^{W} | R_{i,t}^{W} < q_d\right)}{\mathrm{var}\left(R_{i,t}^{W} | R_{i,t}^{W} > q_u\right)} \quad \text{and} \quad \mathrm{DuVol}_{i,t}^{L} = \frac{\mathrm{var}\left(-R_{i,t}^{L} | - R_{i,t}^{L} < q_d\right)}{\mathrm{var}\left(-R_{i,t}^{L} | - R_{i,t}^{L} > q_u\right)}, \tag{3.3.13}$$

where q_d and q_u define the cut-off points that define down and up days. Thus, for an asset in the winners portfolio, higher values of DuVol correspond to a higher crash risk in relation to upside potential. In contrast, for an asset in the losers portfolio, higher values of DuVol correspond to a higher upside potential in relation to downside risk. In particular, higher values of DuVol indicate a left (right) skewed distribution for an asset in the winners (losers) portfolio, i.e. higher values of DuVol indicate that these assets contribute negatively to the momentum portfolio and should be weighted lower. To provide a sufficient amount of data, we use the past 12 months of daily returns to estimate DuVol. Further, we choose q_d as the 30% and q_u as the 70% quantile. In contrast, Chen et al. (2001) and the SJ measure presented above separate all days in down and up days based on the sign of the return, i.e. $q_d = q_u = 0$. We also used other estimation windows and cut-off points and found quite similar results.

We further extend the DuVol measure of Chen et al. (2001) presented above to a measure that captures the ratio of "good" and "bad" skewness. Thus, as the DuVol measure, this measure also captures the asymmetries of the return distribution but uses the third moment instead of the second. This measure, named down-to-up skewness (DuSkew), is defined as

$$\mathrm{DuSkew}_{i,t}^{W} = -\frac{\mathbb{E}\Big(\Big(R_{i,t}^{W} - \mathbb{E}\Big(R_{i,t}^{W} \mid R_{i,t}^{W} < q_d\Big)\Big)^3 \mid R_{i,t}^{W} < q_d\Big)}{\mathbb{E}\Big(\Big(R_{i,t}^{W} - \mathbb{E}\Big(R_{i,t}^{W} \mid R_{i,t}^{W} > q_u\Big)\Big)^3 \mid R_{i,t}^{W} > q_u\Big)} \tag{3.3.14}$$

[64]Consider an estimation sample of 250 observations for an asset and assume that this asset exhibits negative returns on only ten days. The down semivariance used for the SJ measure would then be estimated based on all 250 observation, where 240 observations would be set to zero. Thus, this measure is sensitive to the percentage of negative returns and the down semivariance increases with the percentage of negative returns. In contrast, assuming a cut-off point of zero, the DuVol measure would use a measure of down variance that would only be calculated based on the ten negative returns. Thus, the down variance is only determined by the magnitude of the negative returns and not by the percentage of negative returns. Hence, although both measure capture the asymmetry of the return distribution, they should capture other aspects of risk.

$$\mathrm{DuSkew}_{i,t}^{L} = -\frac{\mathbb{E}\Big(\Big(-R_{i,t}^{L} - \mathbb{E}\Big(-R_{i,t}^{L} \mid -R_{i,t}^{L} < q_d\Big)\Big)^3 \mid -R_{i,t}^{L} < q_d\Big)}{\mathbb{E}\Big(\Big(-R_{i,t}^{L} - \mathbb{E}\Big(-R_{i,t}^{L} \mid -R_{i,t}^{L} > q_u\Big)\Big)^3 \mid -R_{i,t}^{L} > q_u\Big)}, \tag{3.3.15}$$

where the multiplication with minus one guarantees that a higher DuSkew coincides with higher risk. The DuSkew of the losers portfolio can again be rewritten as

$$\mathrm{DuSkew}_{i,t}^{L} = -\frac{\mathbb{E}\Big(\Big(R_{i,t}^{L} - \mathbb{E}\Big(R_{i,t}^{L} \mid R_{i,t}^{L} > -q_d\Big)\Big)^3 \mid R_{i,t}^{L} > -q_d\Big)}{\mathbb{E}\Big(\Big(R_{i,t}^{L} - \mathbb{E}\Big(R_{i,t}^{L} \mid R_{i,t}^{L} < -q_u\Big)\Big)^3 \mid R_{i,t}^{L} < -q_u\Big)}. \tag{3.3.16}$$

Thus, the DuSkew measure quantifies a higher risk for an asset in the winners (losers) portfolio when the skewness in the left (right) tail is higher in magnitude than the skewness in the right (left) tail. Although the DuSkew should be positive for most asset return distributions, it can potentially be zero or negative. Thus, we also use the risk measures $\mathcal{R}_{i,t}^{W} = \exp\left(\mathrm{DuSkew}_{i,t}^{W}\right)$ and $\mathcal{R}_{i,t}^{L} = \exp\left(\mathrm{DuSkew}_{i,t}^{L}\right)$ for the DuSkew based weighting. An alternative to the weighting based on DuSkew would be to weight assets directly based on the skewness of the relevant tail. Schneider et al. (2020, Footnote 7) state that up and down skewness captures other information than total skewness.[65] Schneider et al. (2020, Figure 4 and 5) find that assets with a lower (more negative) down skewness underperform assets with higher (less negative) down skewness, where the authors estimate skewness based on options data. This result also holds when it is controlled for momentum. Thus, an inverse down skewness weighting could also be appealing for momentum investors.

Lastly, we use a reward-to-risk measure that is based on the CVaR. However, instead of weighting assets in the winners (losers) portfolio solely based on left (right) tail risk, we also incorporate information on the assets' other tail. As before, an asset with a higher right (left) tail risk should obtain a higher weight in the winners (losers) portfolio. A CVaR based measure that takes information on both tails into account is used by Rachev et al. (2007). We call this measure the Rachev-Ratio (R-Ratio). In order to guarantee that a higher value corresponds to higher risk, we define an asset's risk as the inverse of the measure used by Rachev et al. (2007,

[65] Consider an asset with a total skewness of zero. Assuming a cut-off point of zero, this zero skewness arises since up and down skewness are equally high in magnitude. However, loss-averse would prefer assets with a higher (less negative) down skewness, since this asset has a lower probability of extremely negative returns. This observation is not captured by total skewness.

Eq. (4)),[66] i.e.

$$\mathrm{RR}_{i,t}^{W} = \frac{\mathbb{E}\Big(-R_{i,t}^{W} \mid -R_{i,t}^{W} > \mathrm{VaR}_{i,t}^{W,\alpha}\Big)}{\mathbb{E}\Big(R_{i,t}^{W} \mid R_{i,t}^{W} > \mathrm{VaR}_{i,t}^{W,1-\alpha}\Big)} \quad \text{and} \quad \mathrm{RR}_{i,t}^{L} = \frac{\mathbb{E}\Big(R_{i,t}^{L} \mid R_{i,t}^{L} > \mathrm{VaR}_{i,t}^{L,\alpha}\Big)}{\mathbb{E}\Big(-R_{i,t}^{L} \mid -R_{i,t}^{L} > \mathrm{VaR}_{i,t}^{L,1-\alpha}\Big)}. \quad (3.3.17)$$

Hence, for an asset in the winners (losers) portfolio risk is defined as the average return in the far left (right) tail, whereas reward is defined as the average return in the far right (left) tail. The weighting based on the R-Ratio downweights assets with higher risk and/or lower reward potential for a momentum investor. The R-Ratio focuses on extreme returns, both positive and negative, whereas the middle part of the return distribution is not regarded.

Rachev et al. (2007) find good results of a momentum strategy that ranks assets based on their past R-Ratio instead of their past raw return. This approach is similar to Dudler et al. (2015) who rank assets by their volatility-standardized returns, which is similar to rankings based on the Sharpe Ratio. The authors find that these strategies exhibit lower tail risk and higher risk-adjusted returns than the strategy that ranks assets based on their past cumulative return. Nevertheless, Rachev et al. (2007) find that ranking assets based on performance measures using an estimate of the past mean return, like the Sharpe Ratio, perform significantly worse than the strategies that rank assets by their R-Ratio. Thus, using quantile based measures to quantify return potential is a promising reward measure and less noisy than estimates of the mean return. This result again demonstrates the importance of incorporating non-normalities in portfolio decisions and that portfolio decisions based on estimates of the absolute mean return are suboptimal in practice.

3.4 Systematic Tail Risk Weighted Momentum

The previous section shows how the constituents of the winners and losers portfolios can be weighted inversely to their tail risk. Weighting assets by their tail risk instead of volatility has the advantage that non-normalities are taken into account and the tail risk weighting also distinguishes between long and short positions. The inverse risk weightings used so far are based on the observation that higher risk is typically not compensated by an adequately higher

[66] Rachev et al. (2007) use the R-Ratio as a measure of an asset's risk-adjusted performance. Thus, the authors define the measure as the CVaR of the right tail (reward) divided by the CVaR of the left tail (risk).

return. This low risk anomaly has been shown for volatility (Ang et al., 2006b, 2009, Asness et al., 2012, Bali and Cakici, 2008, Blitz and Van Vliet, 2007, Fama and French, 2016) and tail risk (Atilgan et al., 2019, 2020, Bali et al., 2014, Bi and Zhu, 2020, Schmielewski and Stoyanov, 2017). Thus, by downweighting assets with higher downside (upside) risk in the winners (losers) portfolio, momentum crashes should be mitigated without producing lower returns. Nevertheless, the low risk anomaly does not only hold for univariate risk measures but also for systematic risk measures as frequently shown in the literature and summarized below. As in Agarwal et al. (2017), we refer to measures that quantify an asset's own risk as univariate risk measures and to measures that quantify the comovement between an asset and the momentum portfolio as systematic (tail) risk measures.[67] Similarly, Atilgan et al. (2019) and Atilgan et al. (2020) refer to measures like VaR and CVaR as downside risk or tail risk measures and to measures like downside beta as systematic downside risk or systematic tail risk measures. Agarwal et al. (2017) find that systematic tail risk captures other information than univariate risk measures, like skewness, kurtosis, VaR and CVaR (see also Atilgan et al. (2019, Table 2.B)).[68] Similarly, Acharya et al. (2016, Table 4 and Fig. 2) find that higher systematic risk indicates low returns in a crisis, whereas this observation does not necessarily hold for univariate risk measures. Adrian and Brunnermeier (2016, Sec. III.E) show that a VaR based systematic risk measure captures other characteristics than the univariate VaR. Bali et al. (2012) also show that univariate and systematic risk measures capture very different aspects of risk.

[67]Systematic risk measures are also important to quantify risks of financial institutions (Acharya et al., 2012, 2016, Adrian and Brunnermeier, 2016, Allen et al., 2012, Engle et al., 2015). See Brownlees and Engle (2016), Engle et al. (2015, Footnote 1), Adrian and Brunnermeier (2016, Sec. I) and Acharya et al. (2016, Footnote 4) for an overview on other studies on systematic risk that focus on the financial sector. Allen et al. (2012) show that systematic risk of the financial sector can forecast economic downturns, i.e. a sector's systematic risk contains valuable information for other assets. However, the authors find that this observation is only limited to the financial sector.

[68]In the literature, systematic (tail) risk measures are typically defined to measure the comovement with the market portfolio. However, since market and momentum crashes typically do not happen simultaneously (Daniel and Moskowitz, 2016, Table 2), we measure an asset's comovement with the (equally weighted) momentum portfolio. Harvey and Siddique (2000, p. 1278), Atilgan et al. (2018, p. 50) and Atilgan et al. (2019, Footnote 6) also calculate systematic risk with alternative benchmarks. Similarly, Acharya et al. (2016, p. 27) estimate banks' systematic risk as comovement risk with the the financial sector or the market portfolio. Engle et al. (2015) also estimate systematic (tail) risk with respect to different benchmarks. In particular, Engle et al. (2015, Figure 4 and 5) show that systematic risk can be different when measured with respect to different benchmarks. When measuring asset i's comovement with the momentum portfolio, we do not eliminate asset i from the momentum portfolio. This is opposed to Weigert (2015, Footnote 10) and Chabi-Yo et al. (2018) who remove asset i from the market portfolio when the comovement of asset i and the market is measured.

The authors show that systematic risk helps to explain hedge fund returns, whereas univariate tail risk measures, like skewness and kurtosis, as well as idiosyncratic risk measures are not successful in explaining hedge fund returns. Furthermore, Asness et al. (2020, Footnote 8) find that managing industries' idiosyncratic risk, measured by the MAX return (Bali et al., 2017a, 2011), does not work well, whereas managing the industries' beta works well. Concluding, it is better to measure an industry's risk as systematic risk (or univariate total risk) instead of idiosyncratic risk. Moreover, systematic risk captures other characteristics than the univariate risk measures presented in the previous section.

The low risk anomaly for systematic risk measures means that assets that co-vary more with the benchmark portfolio underperform assets with a lower sensitivity to the benchmark portfolio. For example, Frazzini and Pedersen (2014) show that a strategy that buys low beta assets, sells high beta assets and weights assets inversely to their beta produces high returns. This strategy is also called the "Betting against Beta (BAB)" strategy and its performance is frequently examined in the literature (Barroso and Maio, 2018, Bollerslev et al., 2020, Cederburg and O'Doherty, 2016, Cederburg et al., 2020, Fama and French, 2016, Liu et al., 2018, Moreira and Muir, 2017, Schneider et al., 2020).[69] Asness et al. (2014) find that the BAB strategy also works for industries, i.e. a strategy that buys low risk industries, sells high risk industries and weights industries by their beta produces high returns. Bali et al. (2017a, p. 2369-2370) state that the BAB anomaly "is one of the most persistent and widely studied anomalies in empirical research of security returns".[70] Bali et al. (2017a) and Liu et al. (2018, Table 2) confirm the beta

[69]Frazzini and Pedersen (2014) suggest that the beta anomaly results due to leverage constraints of investors (see also Asness et al. (2012)). Bali et al. (2017a) suggest that the beta anomaly is generated by lottery demands of investors. Schneider et al. (2020) find that the low beta anomaly can be explained by (co)skewness risk. Fama and French (2016) find that, similar to the low volatility anomaly, the betting against beta strategy overweights conservative and profitable firms that exhibit high returns. Fama and French (2016, p. 85) state that "the returns on low β stocks behave like those of profitable firms that invest conservatively, whereas the returns on high β stocks track those of less profitable firms that invest a lot." This finding is confirmed by Asness et al. (2020, Table 8). Liu et al. (2018) find that the beta anomaly arises due to the positive correlation of beta and (idiosyncratic) volatility combined with a negative risk-return relation for overpriced stocks as shown by Stambaugh et al. (2015). Thus, the low beta anomaly is related to the low volatility anomaly of Ang et al. (2006b) and Ang et al. (2009). Liu et al. (2018, p. 8) find that "there is little evidence of a beta anomaly once one controls for [idiosyncratic volatility]". However, Asness et al. (2020) cannot confirm the finding of Liu et al. (2018). See also Blitz et al. (2019) and Liu et al. (2018) and references therein for several other possible explanations of the beta anomaly.

[70]Cederburg and O'Doherty (2016) show that the low beta anomaly only holds when portfolio performance is evaluated unconditionally using unconditional performance measures such as the unconditional CAPM alpha. However, the authors cannot confirm a statistically significant outperformance of low beta stocks when portfolio performance is measured conditionally using the conditional approach of Boguth et al. (2011). This results since

anomaly by sorting stocks on past beta and shorting the 10% stocks with the highest beta and buying the 10% stocks with the lowest beta. This strategy is different to the BAB strategy of Frazzini and Pedersen (2014), since the BAB strategy has an additional levered net-long exposure (Liu et al., 2018, Eq. (9)). Jagannathan and Ma (2003) state that shorting high beta stocks and buying low beta stocks is similar to an unconstrained minimum variance strategy, a strategy that typically produces high (risk-adjusted) returns. Moreover, Asvanunt et al. (2015), Blitz and Van Vliet (2007), Blitz and Vidojevic (2017) and Blitz et al. (2019) find that simple low beta portfolios that only buy assets with the lowest beta perform well. Furthermore, Asness et al. (2020) show that the low risk anomaly also holds for correlation instead of beta, i.e. assets with a higher correlation with a benchmark portfolio underperform assets with a lower correlation. In particular, Bali et al. (2017b) and Bali et al. (2017a) find that the momentum effect and the low systematic risk effect are two distinct characteristics. Hence, even after forming the momentum portfolio, the assets' systematic tail risk contains additional information. Thus, weighting assets in the momentum portfolio by their systematic risk is appealing and should reduce the portfolio's risk without producing lower returns. Interestingly, Asness et al. (2020) find that exploiting the low systematic risk anomaly is more profitable in terms of higher risk-adjusted returns than exploiting the (idiosyncratic) risk anomaly like the low volatility anomaly.

As in the previous section, when measuring systematic risk, we define losses for the assets in the winners and losers portfolios asymmetrically, i.e. a loss for an asset in the winners portfolio is defined as a negative return, whereas a loss for an asset in the losers portfolio is defined as a positive return. Measuring systematic risk for the relevant tail, i.e. left systematic (tail) risk for the winners and right systematic (tail) risk for the losers, is important since systematic risks in both tails can be highly different. For example, Longin and Solnik (2001) reject the multivariate normality for the left tail but not for the right tail. In particular, they find higher systematic left tail risk than systematic right tail risk. Similarly, Ang and Chen (2002) and Hong et al. (2007) find that correlations in the left tail are higher than in the right tail. This holds especially for

unconditional performance measures are not suitable for strategies that time volatility. This shows that dynamic trading strategies, as examined in our paper, cannot be compared solely based on unconditional performance measures. For that reason, we will also assess the performance of our strategies using conditional approaches that also incorporate that strategies time volatility.

portfolios that are sorted based on their momentum (Hong et al., 2007, Table 3). Chabi-Yo et al. (2018) find that systematic tail risk for both tails can be highly different and systematic left tail risk is more pronounced than systematic right tail risk. Ang et al. (2006a, p. 1209) find that systematic "downside risk and upside risk are priced asymmetrically" and that "aversion to downside risk is priced more strongly, and more robustly, in the cross section than investors' attraction to upside potential" (Ang et al., 2006a, p. 1211). Bollerslev et al. (2020, Eq. (11)) also show that systematic risk should be defined differently for long and short positions. Similarly, Baltas (2015) suggests that correlation measures of long and short positions should have the opposite sign. The difference between left and right systematic tail risk is not regarded by the volatility weighting that is based on a normality assumption (Hong et al., 2007, p. 1563).

Weighting assets by their systematic risk means that assets in the winners (losers) portfolio that exhibit low (high) returns during momentum crashes are weighted lower. Thus, this approach should be especially successful in managing momentum crashes. However, weighting assets by their comovement risk should be inferior to other weighting schemes in periods when the momentum portfolio exhibits positive returns, since assets in the winners (losers) portfolio that produce high (low) returns in up-periods are weighted lower. Thus, comovement risk should be measured asymmetrically depending on the state of the momentum portfolio. For that reason, when measuring the comovement of an asset with the momentum portfolio, we further condition on the return of the momentum portfolio. Since we are mainly interested in mitigated (extremely) low returns of the momentum portfolio, we condition on (extremely) bad states of the momentum portfolio.[71] These measures, which we call systematic tail risk measures, quantify the crash-sensitivity of an asset with the momentum portfolio. Thus, in our case, these measures quantify how much industry i in the winners (losers) portfolio falls (rises) when the momentum portfolio suffers a momentum crash. Chabi-Yo et al. (2018) and Van Oordt and Zhou (2016) show that assets with a higher crash-sensitivity, i.e. assets that exhibit high negative returns when the market crashes, produce very low returns during market crashes. Van Oordt

[71] We will present in Section 3.5 another approach that manages momentum's risk by incorporating information on the momentum portfolio's expected state. This approach uses systematic (tail) risk measures only in periods when a momentum crash is likely. When a positive momentum return is likely, momentum's risk is managed by univariate risk measures.

and Zhou (2016) additionally show that assets with a higher crash-sensitivity do not outperform the less crash-sensitive assets over the whole sample. Similarly, Agarwal et al. (2017) find that funds who time crash risk, i.e. funds who reduce their position in crash-sensitive assets in crises, outperform funds who are bad systematic tail risk timers.[72] Thus, in order to mitigate momentum crashes, assets that strongly co-crash with the momentum portfolio should obtain lower weights, especially in times of a momentum crash.

Based on the systematic risk measures that will be defined below, we again use the inverse risk weighting in Equation (3.2.3) and the rank weighting in Equation (3.2.4). Weighting industries in the momentum portfolio by their systematic (tail) risk is important for several reasons. First, by weighting assets inversely to their systematic (tail) risk, the severity of momentum crashes should be significantly lowered. Inverse systematic risk weighting gives assets that provide a hedge against momentum crashes higher weights than assets that contribute negatively to the momentum return when the momentum portfolio crashes. Assets with a lower sensitivity to market declines typically outperform in bad market states, which should lower momentum crashes (Chabi-Yo et al., 2018, Van Oordt and Zhou, 2016). Second, weighting assets inversely to their systematic risk also fits well to most investors' preferences, since risk-averse investors prefer to invest in assets that perform well while the market is in a downturn period (Guidolin and Timmermann, 2008, p. 910). This is confirmed by Chabi-Yo et al. (2018), Bollerslev and Todorov (2011) and Van Oordt and Zhou (2016) who find that investors are willing to pay high fees for assets that hedge against extremely negative returns, i.e. investors are crash-averse. Third, different industries can have quite different levels of systematic risk. Hence, simply equal or volatility weighting the industries means that the momentum portfolio's systematic risk can be dominated by a few industries. For example, Asness et al. (2014) and Harvey and Siddique (2000, Table 1) find that different industries have quite different levels of beta and coskewness.[73] Similarly, Van Oordt and Zhou (2017) find that industries have different levels

[72] This is similar to the finding of Jang and Kang (2019) that assets with a high crash probability produce lower returns and that institutional investors who time the crash probability of their holdings produce higher returns. However, the crash probability measure of Jang and Kang (2019) is a univariate risk measure, whereas Agarwal et al. (2017) use a co-crash risk measure.

[73] As mentioned before, systematic risk in the literature is typically measured as comovement with the market and not with the momentum portfolio. However, if different industries have different levels of systematic risk with the market portfolio, it is likely that systematic risk of the industries with the momentum portfolio is also different.

of systematic tail risk measured by the tail beta, which will be defined below. Fourth, timing systematic risk typically produces high returns (Agarwal et al., 2017, Asness et al., 2020, 2014, Bali et al., 2017a, Frazzini and Pedersen, 2014).

As first measure to quantify an asset's systematic risk, we follow Frazzini and Pedersen (2014) and Asness et al. (2014) and use an asset's beta, which is the most frequently used measure to quantify systematic risk in the financial literature. In order to quantify the co-movement between an industry and the momentum portfolio, we measure an industry's beta with respect to the equally weighted momentum portfolio. The month t beta for asset i in the winners and losers portfolio is then defined by

$$\beta_{i,t}^{W} = \frac{\text{cov}(R_{i,t}^{W}, R_t^{M})}{\text{var}(R_t^{M})} \quad \text{and} \quad \beta_{i,t}^{L} = \frac{\text{cov}(-R_{i,t}^{L}, R_t^{M})}{\text{var}(R_t^{M})}, \tag{3.4.1}$$

where R_t^M denotes the month t return of the equally weighted momentum portfolio. Thus, a high beta for an asset in the winners (losers) portfolio means that the asset moves in the same (opposite) direction as the momentum portfolio. For example, if the momentum portfolio suffers an extremely high loss, a high beta means that an asset in the winners (losers) portfolio exhibits a highly negative (positive) return. Thus, by weighting high beta stocks lower, the winners (losers) portfolio should perform well (bad) during a momentum crash. This finding is in line with Levi and Welch (2019, Table 2) who find that beta is good measure to quantify an asset's hedging ability in crash periods.[74] This finding is most pronounced in extreme crash periods (Levi and Welch, 2019, Fig. 2). Thus, by weighting the assets of the momentum portfolio inversely to their beta, momentum crashes should be attenuated. Moreover, Bali et al. (2017a, Table 12) and Bali et al. (2017b) find that high beta assets typically have lower skewness, higher (idiosyncratic) volatility and higher systematic risk measured by other systematic risk measures. Thus, managing the momentum portfolio's beta also means managing the portfolio's (systematic) tail risk. Thus, the beta weighting is an appealing approach to manage momentum crashes and fits well to most investors' preferences.

[74] See also Figure 1 in Levi and Welch (2019). Further, Levi and Welch (2019) state that the beta "can predict subsequent stock returns quite well also in bear, extreme bear, and crashing markets. In such conditions, low-beta stocks outperformed high-beta stocks, just as predicted. This is good news for the usefulness of market beta as a measure of stocks' hedging abilities against market crashes. Low-beta stocks make portfolios "safer" in bear markets".

As in Ang et al. (2006a), Bali et al. (2017b) and Frazzini and Pedersen (2014), we estimate beta non-parametrically using the last 12 months of daily returns. The beta estimator is given in Equation (C.14). Several alternatives to the simple non-parametric beta estimator were also used in the literature. For example, different betas could be estimated based on several lagged momentum returns. Industry i's beta could then be defined as the sum of the different beta estimates as shown in Boguth et al. (2011, Footnote 23) and Cederburg and O'Doherty (2016, Eq. (5) and (6)). Liu et al. (2018, p. 3-4) also use this approach and several other estimation methods of an asset's beta (see Table A.1 in Liu et al. (2018)). Similarly, beta could also be measured with respect to several factor portfolios, e.g. the Carhart (1997) four factor model. The beta used for weighting the assets could then be estimated as the average of the four betas. This approach was used by Kirby and Ostdiek (2012, Eq. (17)) and the authors find good results of the averaged four factor beta. Further, instead of measuring beta with respect to the return of the momentum portfolio, beta could also be calculated with respect to volatility innovations as in Adrian and Rosenberg (2008), Ang et al. (2006b, Eq. (3)), Chang et al. (2013) and Chen and Petkova (2012). Assets with a higher sensitivity to volatility innovations produce lower returns than assets with a lower sensitivity to volatility innovations. This result also holds for industry portfolios (Chen and Petkova, 2012, Table 6). We also used a weighting based on this approach but found no significant better results compared to the simple beta in Equation (3.4.1). Further, beta could also be calculated with respect to innovations of higher moments, like skewness or kurtosis (Chang et al., 2013). Chang et al. (2013, Fig. 5) show that a strategy that buys (sells) assets with a low (high) sensitivity to skewness innovations produces high returns. The sensitivity to skewness innovations can be interpreted as a measure of jump tail risk and this effect is not captured by momentum or the sensitivity to volatility innovations. Another alternative to our approach would be to estimate beta conditionally as presented in Bali et al. (2017b, Eq. (3)) and Engle et al. (2015).[75] Further, Bali et al. (2017b, Sec. 2) present several alternative estimates of an asset's beta. Another possibility to estimate an asset's conditional beta would be to

[75] Interestingly, Bali et al. (2017b) find that the beta anomaly does not hold, once beta is estimated conditionally. In contrast, they find that the anomaly holds when beta is estimated unconditionally, where this finding is not statistically significant. However, when forming long-short portfolios, they do not downweight stocks with high betas as done by Frazzini and Pedersen (2014). This could be a possible explanation for the finding of Bali et al. (2017b).

estimate future beta based on lagged variables, like lagged volatility, lagged beta or past momentum (Boguth et al., 2011, Cederburg and O'Doherty, 2016). This approach could also be used to directly forecast an asset's beta rank as shown by Langlois (2020). However, Cederburg and O'Doherty (2016, p. 748) find that an asset's past beta rank is a good predictor for the asset's future beta rank, i.e. combining our rank weighting with the non-parametric estimation of beta should produce reliable portfolio weights without using complex estimation methods. To obtain more robust portfolio weights of the inverse risk weighting, shrinked betas as in Frazzini and Pedersen (2014), Asness et al. (2014, p. 27) and Schneider et al. (2020) could also be used. Finally, instead of estimating industry i's beta using past industry returns, beta could also be estimated as the average beta of all assets that are contained in industry i (see Boguth et al. (2011, Sec. 4.1) and Cederburg and O'Doherty (2016, p. 750)).

Generally, when estimating systematic risk, we use quite short estimation windows that are, however, not too short. Similarly, Ang et al. (2006a) and Chabi-Yo et al. (2018, p. 1074) also use 12 months of daily data to estimate systematic risk for two reasons. First, too short estimation windows, and thus too less data provide less accurate estimates of systematic risk. Second, since systematic risk is typically highly time-varying, short estimation windows should be used to capture the time-variation in systematic risk. Thus, 12 months of daily data is a good balance between estimation error and capturing time-varying systematic risk. Langlois (2020, p. 405) also use a 12 months estimation period to estimate systematic risk "because it provides a reasonable trade-off between having enough returns while allowing for variations over time." The time-variation of systematic risk is not taken into account by simple weighting schemes, such as the equal or volatility weighting schemes. As robustness check, we show in Appendix B.1 that our weighting schemes are also robust for different estimation windows between six and 60 months of daily data. Bali et al. (2017a, Footnote 6) also find that the beta anomaly is robust to alternative beta estimators.

Bali et al. (2017a, Table 3) find that the low beta anomaly still holds when assets are sorted first on momentum and then on beta. Similarly, Bali et al. (2017b) find that the momentum and beta effect are two different characteristics. Thus, weighting stocks within the winners and losers portfolio inversely to their beta should capture characteristics that are not captured by sorting stocks based on past performance and should further enhance the performance of the momentum portfolio. Beta weighted portfolios have also been used in the literature but, as far as we know, this weighting scheme has not been applied to momentum. The rank based beta weighting was successfully used by Frazzini and Pedersen (2014), Asness et al. (2014) and Asness et al. (2020) for long-short portfolios. The inverse beta weighting was used by Chow et al. (2014), but the authors do not rescale the beta to positive values. As mentioned above, this means that a low risk asset with a negative beta, i.e. an asset that actually should obtain a high weight in the portfolio, will be a short position. For that reason, since beta can be zero or negative, we again use risk measures defined by $\mathcal{R}_{i,t}^W = \exp\left(\beta_{i,t}^W\right)$ and $\mathcal{R}_{i,t}^L = \exp\left(\beta_{i,t}^L\right)$ to guarantee non-negative portfolio weights for the inverse risk weighting. In particular, the inverse beta weighting is similar to the volatility based risk parity approach. Maillard et al. (2010, Eq. (5)) show that the weights of the risk parity approach are given by an inverse beta weighting scheme. The difference between our weighting scheme and risk parity is that we measure beta with respect to the equally weighted momentum portfolio. In contrast, risk parity measures beta with respect to the portfolio that again depends on the weight of asset i. Thus, the risk parity weighting does not have a closed form solution and is much harder to estimate than our simple inverse beta weighting.

Similar to the low beta anomaly of Frazzini and Pedersen (2014) and Asness et al. (2014), Asness et al. (2020) show that also a "low correlation anomaly" holds. The month t correlation of asset i in the winners and losers portfolio with the momentum portfolio is given by

$$\rho_{i,t}^W = \operatorname{corr}(R_{i,t}^W, R_t^M) \quad \text{and} \quad \rho_{i,t}^L = \operatorname{corr}(-R_{i,t}^L, R_t^M), \tag{3.4.2}$$

where we again define correlations for an asset in the winners and losers portfolios asymmetrically. This is in line with Baltas (2015) who state: "The correlation structure of a long-only universe is very different to the correlation structure on a long-short universe." The author also states that the correlation of a short position has to switch the sign as done in Equation (3.4.2). The correlation weighting is appealing since Ang and Chen (2002) find that correlations are higher in down-periods, especially in extreme down-markets. That is, most stocks typically crash together and equally or volatility weighted portfolios do not provide an adequate crash protection. In contrast, weighting assets inversely to their correlations means that assets that provide a hedge against a momentum crash obtain higher weights. We again estimate correlations by the simple sample estimator given in Equation (C.15) using the past 12 months of daily data. Alternatively, following Jondeau et al. (2019, Footnote 11) and Chen and Petkova (2012, Eq. (7)) industry i's correlation could also be estimated by first estimating each constituent's correlation with the momentum portfolio and then calculating the average correlation of the individual correlations. Maillard et al. (2010, Eq (4)) use a similar inverse correlation weighting, where correlation of asset i is measured as the average correlation to all the remaining assets. However, we focus on the simple correlation of industry i with the momentum portfolio estimated as in Equation (C.15).

The correlation measure of Equation (3.4.2) is also linked to the beta measure of Equation (3.4.1). Using an asset's correlation, the beta can be written as

$$\beta_{i,t}^{W} = \rho_{i,t}^{W} \cdot \frac{\sigma_{i,t}^{W}}{\sigma_{t}^{M}} \quad \text{and} \quad \beta_{i,t}^{L} = \rho_{i,t}^{L} \cdot \frac{\sigma_{i,t}^{L}}{\sigma_{t}^{M}}, \tag{3.4.3}$$

where σ_t^M denotes the month t volatility of the equally weighted momentum portfolio (Asness et al., 2020, Frazzini and Pedersen, 2014). Hence, weightings based on beta and correlation should produce quite similar results, but beta also captures the observation that different industries have quite different levels of volatility. Thus, we expect that both approaches perform differently as also shown by Asness et al. (2020).

Besides beta and correlation other systematic risk measures that are also frequently used in the financial literature are the coskewness (Harvey and Siddique, 1999, 2000, Kraus and Litzenberger, 1976, Langlois, 2020) and cokurtosis (Dittmar, 2002). The month t coskewness for an asset in the winners and losers portfolio is defined by (see Harvey and Siddique (2000, Eq. (11)), Guidolin and Timmermann (2008, p.914), Ang et al. (2006a, p. (6)), Ang and Chen (2002, Eq. (29)) and Kraus and Litzenberger (1976))

$$\beta_{i,t}^{W,Skew} = \frac{\mathbb{E}\Big(\big(R_{i,t}^{W} - \mathbb{E}(R_{i,t}^{W})\big)\left(R_{t}^{M} - \mathbb{E}(R_{t}^{M})\right)^{2}\Big)}{\sqrt{var(R_{i,t}^{W})var(R_{t}^{M})}} \tag{3.4.4}$$

$$\beta_{i,t}^{L,Skew} = \frac{\mathbb{E}\Big(\big(-R_{i,t}^{L} - \mathbb{E}(-R_{i,t}^{L})\big)\left(R_{t}^{M} - \mathbb{E}(R_{t}^{M})\right)^{2}\Big)}{\sqrt{var(R_{i,t}^{L})var(R_{t}^{M})}}. \tag{3.4.5}$$

As for the skewness, in order to guarantee that higher values of $\mathcal{R}_{i,t}^{W}$ and $\mathcal{R}_{i,t}^{L}$ correspond to higher risk for a momentum investor, our coskewness based risk measure is given by $\mathcal{R}_{i,t}^{W} := \exp\left(-\beta_{i,t}^{W,Skew}\right)$ and $\mathcal{R}_{i,t}^{W} := \exp\left(-\beta_{i,t}^{L,Skew}\right)$.[76] For an asset in the winners (losers) portfolio, the coskewness measures the covariance of industry i's positive (negative) return with the squared momentum return. Hence, a high risk measure $\mathcal{R}_{i,t}^{W}$ or $\mathcal{R}_{i,t}^{L}$ indicates that industry i contributes negatively to the momentum portfolio's return in times of a high momentum volatility (Ang et al., 2006a, p. 1212). Barroso and Santa-Clara (2015), Barroso and Maio (2019) and Grobys et al. (2018) show that a high volatility of the momentum portfolio is typically related to a negative momentum return and is a good indicator for a momentum crash. Thus, by giving assets with a higher coskewness risk $\mathcal{R}_{i,t}^{W}$ or $\mathcal{R}_{i,t}^{L}$ lower weights, momentum crashes should be attenuated. Further, Harvey and Siddique (2000, Table I) show that winners have lower coskewness than losers, and thus, the momentum portfolio exhibits a negative coskewness which is undesired by investors. Langlois (2020) confirm that past momentum negatively predicts future coskewness, which leads to a high negative coskewness of the momentum portfolio. Schneider et al. (2020) also find that momentum is highly related to coskewness risk. Furthermore, Harvey and Siddique (2000, Table I) find that industries have quite different levels of coskewness. Similarly, Ang and Chen (2002, Table 5) find that industries are negatively coskewed and coskewness varies across industries. Thus, by simply equal or volatility weighting industries of the momentum portfolio, the momentum portfolio's high negative coskewness could be mainly influenced by only a few constituents. Hence, the high negative coskewness of the momentum portfolio, which is a driver of momentum crashes and is undesired by investors (Kraus and Litzenberger, 1976), can be reduced by weighting assets inversely to their coskewness risk.

We again estimate the coskewness with past daily returns of industry i and the equally weighted momentum portfolio using the simple sample estimator that is frequently applied in the literature and shown in Equation (C.16). An alternative to this approach would be to estimate

[76] We again define the risk measure used in our weighting scheme as the negative coskewness, such that assets with a higher coskewness also obtain higher weights. This is important since assets with a higher coskewness "are appealing because they offer defensive returns during bad times; these stocks provide downside risk protection" (Langlois, 2020, p. 399). Thus, weighting assets by their negative coskewness should successfully dampen momentum crashes.

the coskewness based on residuals from a regression on a constant and the market return (Harvey and Siddique, 2000, Langlois, 2020). Moreover, coskewness could also be estimated conditionally using lagged risk measures, such as an asset's lagged coskewness, beta or volatility as well as other information, such as an asset's momentum (Harvey and Siddique, 2000). Further, since we are mainly interested in an asset's cross-sectional rank, it would also be sufficient to forecast an asset's coskewness rank (Langlois, 2020, Eq. (4)). Langlois (2020) show that forecasting an asset's coskewness rank is more precise than directly forecasting an asset's coskewness. The author confirms this in a Monte-Carlo Simulation. However, since non-parametric estimators of systematic risk are typically highly persistent, past coskewness should be a good predictor for the future coskewness rank.

Besides coskewness, cokurtosis is also frequently used in the financial literature (see Harvey and Siddique (2000, Eq. (11)), Guidolin and Timmermann (2008, p.914), Ang et al. (2006a, Eq. (10)) and Dittmar (2002)). The cokurtosis for an asset in the winners and losers portfolio is defined by

$$\beta_{i,t}^{W,Kurt} = \frac{\mathbb{E}\Big(\big(R_{i,t}^W - \mathbb{E}(R_{i,t}^W)\big)\cdot\big(R_t^M - \mathbb{E}(R_t^M)\big)^3\Big)}{\sqrt{var(R_{i,t}^W)}var(R_t^M)^{3/2}} \tag{3.4.6}$$

$$\beta_{i,t}^{L,Kurt} = \frac{\mathbb{E}\Big(\big(-R_{i,t}^L - \mathbb{E}(-R_{i,t}^L)\big)\cdot\big(R_t^M - \mathbb{E}(R_t^M)\big)^3\Big)}{\sqrt{var(R_{i,t}^L)}var(R_t^M)^{3/2}}. \tag{3.4.7}$$

Since the cokurtosis can potentially be negative, we again define the risk measures used for the inverse weighting approach by $\mathcal{R}_{i,t}^W := \exp\left(\beta_{i,t}^{W,Kurt}\right)$ and $\mathcal{R}_{i,t}^L := \exp\left(\beta_{i,t}^{L,Kurt}\right)$. The cokurtosis based risk measure of an asset in the winners (losers) portfolio measures the comovement of the positive (negative) return with the third moment of the market. Hence, a high cokurtosis risk means that industry i's return contributes negatively to the momentum portfolio when the momentum portfolio is negatively skewed. As mentioned above, a negative skewness is a good crash probability measure (Chen et al., 2001). Thus, weighting assets inversely to their cokurtosis risk should reduce the severity of momentum crashes.

The systematic risk measures presented above measure the comovement of an asset with the momentum portfolio. However, during periods of an uptrending momentum portfolio, a higher systematic risk of an asset in the winners portfolio means that this asset rises with the momentum portfolio. Similarly, a higher systematic risk as defined above means that an asset in the losers portfolio decreases when the momentum portfolio increases. Thus, a higher systematic risk of assets in the winners and losers portfolios is appealing on days with a positive momentum return. This important point is not regarded by our weighting approaches, since assets that contribute positively to the momentum investor's return are weighted lower in uptrending periods. One way to deal with this state dependence is to define systematic risk by conditioning on bad states of the momentum portfolio. Conditioning on the state of the benchmark asset when systematic risk is measured is frequently done in the literature.[77] Measuring systematic risk by conditioning on bad states of the momentum portfolio also fits well to the finding that investors are loss- and crash-averse. Ang et al. (2006a), Lettau et al. (2014) and Farago and Tédongap (2018) find that besides beta other downside risk factors are priced when investors trade losses and gains asymmetrically. Similarly, Chabi-Yo et al. (2018), Van Oordt and Zhou (2016) and Weigert (2015) demonstrate the importance of systematic *tail* risk for crash-averse investors. Hence, weighting assets in the momentum portfolio based on systematic downside risk measures should be an appealing approach for loss-averse investors. In particular, this approach should be successful in reducing extreme losses, and thus should produce an enhanced risk-return profile by mitigating momentum crashes.

One frequently used systematic risk measure that conditions on the state of the benchmark portfolio is the downside beta (Ang et al., 2006a, Atilgan et al., 2018, 2019, Bawa and Lindenberg, 1977, Farago and Tédongap, 2018, Lettau et al., 2014). The downside beta is defined as the beta of asset i with the momentum portfolio, conditioned on bad states of the momentum portfolio. Conditioning on bad states is important since assets typically have higher sensitivities to unfavorable periods than to favorable periods (Atilgan et al., 2018, 2019, Bollerslev et al., 2020). More formally, the month t downside beta of asset i in the winners and losers portfolio is given by

$$\beta_{i,t}^{W,-} = \frac{\text{cov}(R_{i,t}^W, R_t^M | R_t^M < 0)}{\text{var}(R_t^M | R_t^M < 0)} \quad \text{and} \quad \beta_{i,t}^{L,-} = \frac{\text{cov}(-R_{i,t}^L, R_t^M | R_t^M < 0)}{\text{var}(R_t^M | R_t^M < 0)}, \tag{3.4.8}$$

where we use a cut-off point of zero to determine bad states of the momentum portfolio (Ang

[77] We also use other weighting schemes that take the state of the momentum portfolio into account. These approaches will be presented in the next section.

et al., 2006a, Eq. (12)). Ang et al. (2006a) and Atilgan et al. (2018) additionally use cut-off points that are equal to the mean return of the market or the risk-free rate instead of a zero return. However, the authors find similar results for different cut-off points (see also Atilgan et al. (2019, Footnote 7)). Alternatively, down-states could be defined as periods when momentum's return is more than one standard deviation lower than its mean return (Lettau et al., 2014, p. 203). Further, to emphasize the comovement in the tails, quantile based cut-off points can also be used (see Atilgan et al. (2018), Atilgan et al. (2019, Eq. (2)), Bali et al. (2014) and Chabi-Yo et al. (2018, Sec. III.D.1)). We will later present systematic tail risk measures that measure the comovement behavior for extremely negative outcomes. For that reason, we will not further examine the quantile based downside beta. We also used quantile based downside betas and found similar results to the remaining quantile based systematic risk measures and the usual downside beta. As in Ang et al. (2006a) and Atilgan et al. (2018), we estimate downside beta using the past 12 months of daily data. As mentioned above, too short estimation windows produce inaccurate estimates. Too long estimation windows produce noisy estimates since systematic risk is time-varying (Ang et al., 2006a, p. 1202). We show in Appendix B.1 that our results also hold for different estimation windows.

Another alternative to our weighting approach in Equation (3.4.8) could be to use a weighting scheme that uses upside beta for an industry in the losers portfolio, i.e. the sensitivity of positive industry returns conditioned on a positive momentum return. We also used this weighting scheme, but as expected, we found better results of our weighting scheme that conditions on negative momentum returns for the winners and losers portfolios. This is in line with Bollerslev et al. (2020) who find that the beta measures that condition on bad states of the benchmark portfolio contain more information than the measures that condition on positive benchmark returns. Alternatively, downside beta could also be defined as in Ang and Chen (2002, p. 461), Bollerslev et al. (2020), Longin and Solnik (2001) and Hong et al. (2007, p. 1554) by conditioning on bad states of industry i *and* the momentum portfolio. Risk for the winners is then defined by conditioning on negative returns of the momentum portfolio and industry i, whereas risk for the losers is defined by conditioning on positive returns of industry i and negative returns of the momentum portfolio.[78] Further, similar to the downside beta, downside volatility that conditions on bad states of the momentum portfolio could be used (Ang et al., 2006a, p. 1228). However, this measure would again not distinguish between long and short positions. In contrast, Ang and Chen (2002, p. 461) and Hong et al. (2007, p. 1554) define downside volatility by conditioning on bad states of asset i and the momentum portfolio. Applied to our approach, this measure would define downside volatility for an asset in the winners portfolio as volatility on days when industry i and the momentum portfolio co-crash. In contrast, for an asset in the losers portfolio, downside volatility would be defined as volatility on days when the momentum portfolio crashes and industry i rises. Since we use other systematic risk measures that are based on simultaneous crashes of an industry with the momentum portfolio, we do not report results for the downside volatility. Furthermore, to capture information on the whole distribution or at least on both tails, risk could also be measured as the spread between downside beta and the unconditional beta or upside beta (Ang et al., 2006a, Lettau et al., 2014). The beta spread measures upside potential relative to downside risk, and hence is similar to the reward-to-risk timing strategies presented in the previous section. Ang et al. (2006a, Table 4) show that both measures, upside and downside beta, capture different aspects of risk and are priced quite differently. We also used weightings based on the beta spread but we did not find significantly superior results compared to the downside beta weighting. Since the next section examines strategies that use systematic risk measures as a tool to manage momentum crashes, whereas in up-periods momentum's risk is managed by univariate risk measures, we concentrate on systematic risk measures that only capture the relevant tail's risk, since these measure are more valuable in momentum crash periods.

Farago and Tédongap (2018) develop three alternative systematic downside risk measures that are similar to the downside beta and that could also be used to weight assets of the winners and losers portfolios. One main difference between the measures of Farago and Tédongap

[78]These "semibetas" are also examined by Bollerslev et al. (2020) who decompose the usual beta into four semibetas. In the notation of Bollerslev et al. (2020, Eq. (2)), winners would be weighted by $\hat{\beta}^{\mathcal{N}}$ and losers by $\hat{\beta}^{\mathcal{M}-}$. Bollerslev et al. (2020) find that these semibetas, which condition on the returns of asset i and the benchmark portfolio, capture quite different information than downside beta and upside beta. Moreover, the authors find that the semibetas that condition on bad states of the benchmark portfolio contain more information than the betas that condition on positive benchmark returns.

(2018) and the downside beta of Ang et al. (2006a), Bawa and Lindenberg (1977) and Lettau et al. (2014) is the definition of down-days. Farago and Tédongap (2018) define down-days on both return and volatility of the benchmark portfolio, whereas the downside beta used here defines down-days solely based on a low return of the benchmark portfolio. The definition used by Farago and Tédongap (2018) is motivated by the theory of disappointment aversion. Farago and Tédongap (2018, Fig. 2) compare the definition of down-days based on return (Panel A) and down-days based on return and volatility (Panel B) and find that both definitions of down-days produce quite similar results. This can be explained since a decreasing return typically coincides with an increasing volatility (see Bekaert and Wu (2000), French et al. (1987), Glosten et al. (1993) and references therein). For that reason, we will not use the risk measures of Farago and Tédongap (2018), but we will later present an approach that manages the momentum portfolio's risk by incorporating information on the momentum portfolio's volatility.

Ang et al. (2006a) state that investors weight losses higher than gains and that investors are downside risk averse. Thus, investors are concerned about downside risks, such as the risk measured by downside beta. These downside risks are not captured by other risk measures that rely on the whole return distribution. For example, Ang et al. (2006a, p. 1224) find that a high downside beta is related but different to a negative coskewness, high volatility or high cokurtosis. Ang et al. (2006a, p. 1199) state that "skewness and other centered moments may not effectively capture aversion to risk across upside and downside movements in all situations. This is because they are based on unconditional approximations to a nonsmooth function. In contrast, the downside beta [...] conditions directly on a downside event that the market is less than [a certain threshold]." Lettau et al. (2014) find that a downside beta based CAPM model is more successful in explaining the returns of several asset classes compared to the usual CAPM. This finding holds for currencies, size and value sorted style portfolios, beta sorted portfolios, the BAB portfolio and industry portfolios. However, both models have difficulties in explaining momentum returns (Lettau et al., 2014, Sec. 6.5). Thus, momentum and (downside) beta seem to capture different characteristics. Nevertheless, Ang et al. (2006a, p. 1221) find that past winners typically exhibit higher downside risk, which translates into a momentum portfolio with high systematic downside risk. Thus, the equally weighted momentum portfolio can be highly exposed to systematic downside risk. Moreover, Ang et al. (2006a) find that portfolios of stocks, like the 25 size and value sorted portfolios (Fama and French, 1993), are exposed to downside market risk and can have quite different levels of downside risk. An equally weighted portfolio's downside risk could therefore be dominated by the downside risk of a few assets. In contrast, our rank and inverse risk weighting should produce a momentum portfolio that is better diversified with lower systematic downside risk and less severe momentum crashes. Furthermore, the downside beta weighted portfolio is also appealing since Bali et al. (2014, Table 3.B) find that downside beta negatively predicts future returns. Similarly, Atilgan et al. (2018, Exhibit 2) and Atilgan et al. (2019, Table 3) find that stocks with higher downside beta underperform stocks with lower downside beta. This result also holds even after stocks are sorted by their past return, i.e. the underperformance of high downside beta stocks also holds for the momentum portfolio (Atilgan et al., 2018, Exhibit 4). Further, Atilgan et al. (2019, Table 5-6) find a high and statistically significant negative relation between downside beta and return for portfolios sorted on size and value as well as country indices, i.e. the negative downside beta-return relation also holds for portfolios of stocks. Thus, the downside beta weighted momentum portfolio based on industries, style portfolios and country indices should obtain an enhanced risk-return profile.

One disadvantage of using simple non-parametric estimates of downside beta is that risk is not properly forecasted. Ang et al. (2006a, p. 1225) find that past downside beta alone does not accurately predict future downside beta, and hence the sample estimator provides a noisy estimate of industry i's future downside beta. However, Ang et al. (2006a) find that assets with low (high) past downside betas typically have low (high) future downside betas.[79] Bollerslev et al. (2020, Fig. 3.B) confirm this finding for the semibetas that condition on returns of asset i and the benchmark portfolio.[80] We therefore expect better results of the rank weighting in Equation

[79] Interestingly, Levi and Welch (2019, Sec. 3) find that the usual beta is a better predictor of future downside beta than past downside beta. Thus, downside beta could also be predicted based on past betas and other observations like an industry's momentum.

[80] The main results of Bollerslev et al. (2020) rely on semibetas estimated with high frequency data. However, the authors show in Section 4 that results for monthly semibetas estimated with daily data are very similar to the high frequency data based estimates.

(3.2.4) compared to the inverse risk weighting in Equation (3.2.3), since the inverse risk weighting is based on a point estimate of future downside beta, whereas the rank weighting is only based on the assets' risk order. An alternative would be to use more sophisticated estimation models for downside beta. However, due to the relevance of risk-managed industry momentum strategies for practitioners, we restrict ourselves on simple non-parametric estimation methods and leave the examination of dynamically managed strategies for future research.

Similar to the downside beta, correlation can also be estimated by conditioning on different states of the momentum portfolio. Longin and Solnik (2001) find that correlations behave differently in different market environments. Similarly, Ang and Chen (2002) and Hong et al. (2007) find that correlations are higher in down-markets, especially in extreme down-markets, i.e. most stocks typically crash together. This result also holds for momentum portfolios (Hong et al., 2007, Table 3). Since we are more concerned about high correlations with the momentum portfolio in momentum crash periods, we next measure the correlation of industry i and the momentum portfolio, conditioned on negative momentum returns (Ang et al., 2006a, p. 1228). The downside correlation is defined by

$$\rho_{i,t}^{W,-} = \text{corr}(R_{i,t}^{W}, R_t^M | R_t^M < 0) \quad \text{and} \quad \rho_{i,t}^{L,-} = \text{corr}(-R_{i,t}^{L}, R_t^M | R_t^M < 0), \tag{3.4.9}$$

where we again define correlations differently for an asset in the winners and losers portfolio. Weighting assets by their downside correlation means that assets with a good crash protection obtain higher weights. In particular, Ang and Chen (2002) find that different assets, like industry and style portfolios, have quite different levels of downside correlation. Further, correlations for these assets behave differently in up- and down-markets. Lettau et al. (2014, Table 4) confirm this fining for currencies and show that correlations are higher in down-markets. Interestingly, Ang and Chen (2002, p. 472) find that past performance is also related to different asymmetric correlations. Thus, assets in the momentum portfolio typically have high asymmetries in their correlation structure and weighting stocks with respect to their correlation helps to increase diversification and drawdown protection benefits, especially in extreme down-markets. In contrast, simply equal or volatility weighting industries lowers the diversification benefits, especially in down-markets like momentum crashes.

Ang and Chen (2002) and Longin and Solnik (2001) find that downside correlations capture different risk characteristics than the unconditional correlations, beta, skewness and coskewness. Hence, weightings based on downside correlations should produce different results to other weightings that do not measure systematic downside risk. Ang and Chen (2002, Sec. 2) demonstrate the importance of incorporating asymmetric correlations in the portfolio allocation of a CRRA investor who maximizes expected utility. Generally, Ang and Chen (2002, p. 485) suggest that asymmetric correlations should be incorporated in portfolio allocations and risk management since this information is highly valuable for investors who are very averse to losses and downside risk. This result is also confirmed by Hong et al. (2007) for investors who treat losses and gains asymmetrically. Hong et al. (2007, Table 8) find that disappointment averse investors are willing to pay high fees for asset allocations that incorporate asymmetric correlations. Hong et al. (2007, p. 1575) conclude: "incorporating assets' asymmetric characteristics can add substantial economic value in portfolio decisions".

Instead of using the downside correlation of Equation (3.4.9), several other possibilities to measure correlations in down-periods could be used. For example, downside correlations could be defined by conditioning on momentum's volatility instead of momentum's return. Moreover, downside correlation could be calculated as the spread between correlations in high and low volatile periods (Moreira and Muir, 2019, Eq. (20)). Further, exceedance correlations that condition on simultaneous crashes of industry i and the momentum portfolio could be used (see Ang and Chen (2002, Eq. (2)) and Hong et al. (2007, p. 1550)). We also used this definition and found similar results to the downside correlation that only conditions on bad states of the momentum portfolio. Another alternative would be to weight industries based on the H-statistics, which summarizes information on exceedance correlations for different cut-off points (see Ang and Chen (2002, Eq. (15) and (17)) and (Hong et al., 2007, Eq. (7))). A disadvantage of exceedance correlations compared to downside correlations is that risk estimates become quite noisy by conditioning on simultaneous crashes of industry i and the momentum portfolio, especially when the non-parametric sample estimator is used. For that reason, Longin and Solnik (2001) estimate exceedance correlations based on extreme value theory (EVT). Ang and Chen

(2002) and Hong et al. (2007) use non-parametrically estimated exceedance correlations, but the authors do not use a rolling window approach as done in our paper.

Similar to the downside beta, another frequently used systematic tail risk measure is the LPM-beta (Bali et al., 2014, Bawa and Lindenberg, 1977, Lee and Rao, 1988, Price et al., 1982). The LPM-beta for an asset in the winners and losers portfolio is defined by

$$\beta_{i,t}^{W,LPM} = \frac{\mathbb{E}\big((R_{i,t}^{W} - q) \cdot (R_t^M - q) \mid R_t^M < q\big)}{\mathbb{E}\big((R_t^M - q)^2 \mid R_t^M < q\big)} \quad \text{and} \quad \beta_{i,t}^{L,LPM} = \frac{\mathbb{E}\big((-R_{i,t}^{L} - q) \cdot (R_t^M - q) \mid R_t^M < q\big)}{\mathbb{E}\big((R_t^M - q)^2 \mid R_t^M < q\big)}, \tag{3.4.10}$$

where q is a threshold that defines bad states of the momentum portfolio. We choose $q = 0$ and use the past 12 months of daily data to estimate LPM-beta. In Appendix B.1, we show further results for other estimation windows. We also estimated LPM-beta by choosing q as the $5\%, 10\%, 20\%$ and 30% quantile, but found similar results to $q = 0$.[81] Choosing a quantile based cut-off point of q for the LPM-beta is similar to the quantile based downside beta used by Chabi-Yo et al. (2018, Sec.III.D.1). Thus, by construction, downside beta and LPM-beta are quite similar as also shown by Bali et al. (2014, Table 2). The authors find that the LPM-beta is highly correlated with downside beta, and hence both measures capture quite similar characteristics. Furthermore, the LPM-beta is also related to the usual beta, but measures other aspects of risk. Lee and Rao (1988) and Bawa and Lindenberg (1977) show that, under certain conditions, the LPM-beta simplifies to the traditional beta. However, these conditions are typically not fulfilled in practice. Especially when returns are non-normally distributed and exhibit a non-zero skewness, LPM-beta and the normal beta are different (Price et al., 1982, Table II). Bali et al. (2014, Table 2) find a positive correlation between beta and LPM-beta but this correlation is significantly lower than the correlation between LPM-beta and downside beta.[82] Thus, we expect that the beta and the LPM-beta capture different characteristics of risk and lead to different weightings, whereas LPM-beta and downside beta should produce quite similar results. In

[81] By choosing $q = 0$, the LPM-beta becomes similar to the downside beta where the difference between both measures is that downside beta uses demeaned returns, whereas LPM-beta uses raw returns. For quantile based cut-off points, the difference between both measures is that downside beta is standardized by the mean of extremely negative returns, whereas LPM-beta is standardized by the cut-off point. We find that results for both measures deliver similar results for different cut-off points and we focus on the simplest measures using $q = 0$.

[82] Bali et al. (2014, Table 2) also find a weak and negative correlation between LPM-beta and coskewness as well as a positive relation between LPM-beta and an asset's idiosyncratic volatility or univariate tail risk measured by the asset's LPM. Thus, many weighting schemes presented in this paper should produce similar results since the correlations found by Bali et al. (2014) have the expected sign. However, correlations are typically far away from one or minus one.

particular, weighting momentum's constituents by their LPM-beta is important, since an asset's past return is negatively correlated with an asset's LPM-beta (Bali et al., 2014, Table 2). This negative correlation translates to quite different levels of LPM-beta for the assets in the momentum portfolio. Further, Bali et al. (2014) find a slightly negative relation between LPM-beta and future returns, which should produce an enhanced risk-return profile of LPM-beta weighted portfolios.

The term in the numerator of the LPM-beta in Equation (3.4.10) is also known as the co-LPM that measures the co-semivariance of industry i and the momentum portfolio. The LPM-beta measures systematic risk as the ratio of the co-LPM and the LPM of the momentum portfolio. Bawa and Lindenberg (1977) and Lee and Rao (1988) note that this definition is similar to the usual beta, but industry i only contributes to systematic tail risk when the return of the momentum portfolio is lower than the chosen threshold q. In this case, systematic risk is increased if an industry's return in the winners (losers) portfolio is lower (higher) than the (negative) threshold, whereas systematic risk is lowered when the industry's return is higher (lower) than the (negative) threshold (Lee and Rao, 1988, p. 450). Thus, the LPM-beta measures the co-movement of asset i and the momentum portfolio based on the condition that the momentum portfolio suffers a loss. The LPM-beta in Equation (3.4.10) could also be defined for other orders. However, we follow Bali et al. (2014) and use the LPM-beta only for order $k = 2$ (Bawa and Lindenberg, 1977, Eq. (5)). The LPM-beta for other orders is given in Lee and Rao (1988, Theorem 4). The LPM-beta of orders $k = 3$ and $k = 4$ would be similar to the coskewness and cokurtosis if these measures additionally condition on bad states of the momentum portfolio. We also used weightings on downside coskewness and downside cokurtosis, i.e. coskewness and cokurtosis by additionally conditioning on low momentum returns, but again found quite similar results to other systematic risk measures. Interestingly, the downside coskewness produces slightly better results than the coskewness as shown in Table XVIII in Appendix B.1.

The systematic tail risk measures presented so far condition on bad states of the momentum portfolio. This is appealing since one aim of our weightings is to reduce momentum crashes. However, systematic risk measures are directional., i.e. these measures quantify quite different aspects of risk when the conditioning is switched. Adrian and Brunnermeier (2016, p. 1713) state that "conditioning radically changes the interpretation of the systemic risk measure". Bali et al. (2014, Eq. (13)) modify the LPM-beta by conditioning on bad states of asset i instead of bad states of the benchmark portfolio. The authors call this measure the Hybrid Tail Covariance Risk-beta (HTCR-beta), which is defined by

$$\beta_{i,t}^{W,HTCR} = \frac{\mathbb{E}\Big((R_{i,t}^W - q) \cdot (R_t^M - q) \mid R_{i,t}^W < q\Big)}{\mathbb{E}\Big((R_t^M - q)^2 \mid R_{i,t}^W < q\Big)} \quad \text{and} \quad \beta_{i,t}^{L,HTCR} = \frac{\mathbb{E}\Big((-R_{i,t}^L - q) \cdot (R_t^M - q) \mid -R_{i,t}^L < q\Big)}{\mathbb{E}\Big((R_t^M - q)^2 \mid -R_{i,t}^L < q\Big)}. \tag{3.4.11}$$

Thus, the HTCR-beta is similar to the quantile based univariate risk measures presented in Section 3.3, but measures risk as comovement risk of industry i and the momentum portfolio instead of univariate risk of industry i. Higher values of HTCR-beta for an asset in the winners (losers) portfolio indicate that the momentum portfolio exhibits lower returns in states with negative (positive) returns of industry i. Thus, industries with a high HTCR-beta should be weighted lower. This is important for the assets in the momentum portfolio, since momentum and HTCR-beta are highly related. Bali et al. (2014, p. 222) state: "Stocks with high H-TCR (low H-TCR) are generally past winners (losers)". Furthermore, the HTCR-beta, although conditioned on low industry returns instead of low returns of the benchmark, also captures systematic tail risk. Bali et al. (2014) find that a high HTCR risk coincides with higher beta, downside beta, LPM-beta and lower coskewness.[83] The HTCR-beta is again estimated using the past 12 months of daily data and a cut-off point of $q = 0$. Robustness results for different estimation windows are shown in Appendix B.1. As before, we also estimated the HTCR-beta for quantile based cut-off points q and found quite similar results.

The systematic risk measures presented so far measure the comovement of an asset with the momentum portfolio without conditioning on the state of the momentum portfolio or by additionally conditioning on a negative momentum return. Nevertheless, we are mainly interested in managing momentum crashes, i.e. periods of *extremely* negative momentum returns. Avoiding crashes is important since investors are crash-averse and have a demand for assets that provide some kind of crash protection (Bollerslev and Todorov, 2011, Chabi-Yo et al.,

[83]For their main results, Bali et al. (2014) use a different measure of an asset's hybrid tail covariance risk, which is the non-standardized version of the HTCR-beta. However, Bali et al. (2014, Table 6) show that both measures produce quite similar results, and hence the findings of Bali et al. (2014) should also hold for the HTCR-beta. We also used the non-standardized HTCR measure and also found similar results to the HTCR-beta.

2018, Van Oordt and Zhou, 2016). Hence, instead of conditioning on below zero returns of the momentum portfolio, conditioning on *extremely* negative returns is more appealing in order to manage momentum crashes. Poon et al. (2004, p. 583) find that risk measures like the correlation underestimate extremely risky events, such as momentum crashes. Thus, these measures are bad in quantifying the dependence in the far tails. Similarly, Chabi-Yo et al. (2018) state that the beta measures the comovement with the market but does not regard tail events, such as momentum crashes. Thus, we next present several systematic tail risk measures that condition on *extremely* low momentum returns.

One systematic tail risk measure that quantifies the sensitivity of asset i to extreme down-periods of the benchmark portfolio was presented by Van Oordt and Zhou (2016) and Van Oordt and Zhou (2017) and is called *Tail-beta* by the authors. This measure is motivated by the safety-first theory of Arzac and Bawa (1977).[84] The Tail-beta measures to what extinct an industry looses with the momentum portfolio, i.e. a Tail-beta of 0.5 (2) means that industry i looses 5% (20%) when the momentum portfolio exhibits a loss of 10% (Van Oordt and Zhou, 2016, p. 687). As before, we define losses asymmetrically for an asset in the winners and losers portfolio. Following Van Oordt and Zhou (2016, Eq (1)) and Van Oordt and Zhou (2017, Eq. (1)), the Tail-beta is given by

$$R_{i,t}^{W} = \beta_{i,t}^{W,Tail} \cdot R_t^M + \varepsilon_{i,t}^{W}, \quad \text{for} \quad R_t^M < -\mathrm{VaR}_t^{M,\alpha} \tag{3.4.12}$$

$$-R_{i,t}^{L} = \beta_{i,t}^{L,Tail} \cdot R_t^M + \varepsilon_{i,t}^{L}, \quad \text{for} \quad R_t^M < -\mathrm{VaR}_t^{M,\alpha}, \tag{3.4.13}$$

where $\varepsilon_{i,t}^{W}$ and $\varepsilon_{i,t}^{L}$ is an idiosyncratic error term of asset i in the winners and losers portfolio, respectively. $\mathrm{VaR}_t^{M,\alpha}$ denotes the month t VaR of the momentum portfolio for a significance level of α. Van Oordt and Zhou (2016) and Van Oordt and Zhou (2017) state that the Tail-beta can be estimated by a regression using only observations when the momentum portfolio suffers the most extreme losses. Alternatively, the authors develop an estimation method based on EVT that produces more reliable estimates. We apply the EVT based estimation of Tail-beta which is summarized in Appendix C. This estimation procedure provides an easy to implement

[84]The safety-first theory "builds on the assumption that investors maximize the expected return while limiting the probability of suffering a particularly large loss below a predetermined admissible level" (Van Oordt and Zhou, 2016, p. 688).

and non-parametric estimate of Tail-beta. Van Oordt and Zhou (2016, Table 2) find that non-parametric estimates of beta and Tail-beta are quite persistent. Similarly, Kelly and Jiang (2014) who use a similar risk measure find that systematic tail risk is quite stable over time with an AR(1) coefficient higher than 0.97.[85] Hence, past systematic tail risk is a good predictor for future systematic tail risk and Kelly and Jiang (2014, p. 2850) conclude that "the severity of extreme returns is highly predictable." Thus, simple non-parametric estimates of systematic tail risk also contain information on future systematic tail risk and can be used for our weighting schemes. We again use the last 12 months of daily data and a cut-off point of $\alpha = 10\%$ to estimate Tail-beta. Van Oordt and Zhou (2016, Footnote 8) use 60 months of daily data in their estimation, but find similar results for 12 months of daily data. Further, the authors find similar results for different cut-off points. We show additional results for other estimation lengths and cut-off points in Appendix B.1.

The Tail-beta is again similar in nature to the usual beta. Equation (C.24) in the appendix demonstrates that the Tail-beta can be decomposed into a measure for the comovement of both assets and the ratio of the risks of both assets. Thus, this decomposition is similar to the decomposition of the usual beta given in Equation (3.4.3). The difference to the usual beta is that risk is measured by tail risk instead of risk using the whole distribution and the dependency is measured as tail dependency using the Hill estimator instead of the linear correlation coefficient. Hence, weighting assets based on their Tail-beta should be superior in mitigating momentum crashes compared to the weighting schemes based on the usual beta.

Van Oordt and Zhou (2016) find in a cross-sectional setting that assets with high systematic tail risk suffer losses that are 2-3 times higher than the benchmark's losses when the benchmark portfolio is in a crash period (see Table 3 in Van Oordt and Zhou (2016) for example). Thus, high Tail-beta assets exhibit significantly higher losses in adverse market periods and should be weighted lower in order to mitigate momentum crashes. The authors write that "estimated tail betas help predict losses in future stock market crashes" (Van Oordt and Zhou, 2016, p. 696). In contrast, in calm periods, assets with higher Tail-beta do not significantly underperform. In

[85] Kelly and Jiang (2014) and Karagiannis and Tolikas (2019) also use an EVT based systematic tail risk measure, called *Tail risk*, that is similar to the Tail-beta. This measures is also used by Allen et al. (2012) and Chabi-Yo et al. (2018).

total, Van Oordt and Zhou (2016, p. 693) find that "shorting the high-tail-beta portfolio while taking a long position in the low-tail-beta portfolio would have provided significant protection against systematic tail risk, without bearing a cost in the long run. In fact, such a strategy would have led to a positive, albeit insignificant, average return." Further, Van Oordt and Zhou (2017, Table 2) find that Tail-beta highly varies among industries. Thus, some industries loose only half of the benchmark during a crash, whereas others loose twice as much as the benchmark. Similarly, Kelly and Jiang (2014, Table 8) find that systematic tail risk of different industries is correlated but differs among industries. Hence, in order to mitigate momentum crashes and to avoid that the momentum portfolio's crash risk is dominated by a few industries, a Tail-beta weighting is appealing. An alternative to the Tail-beta presented in Equation (3.4.12) would be to use the Tail-beta spread, defined as the difference of the Tail-beta and the usual beta (Van Oordt and Zhou, 2016, p. 690). However, results for the Tail-beta spread were quite similar and are not presented here. Moreover, as already stated above, the next section develops a strategy that uses the systematic tail risk measures as a tool to manage momentum crashes, where systematic tail risk is only used in periods when a momentum crash is likely. As a consequence, focusing on measures that quantify systematic risk in the tails is more beneficial for this approach.

Another frequently used measure that quantifies the co-crash risk of two assets is the *Tail-Sens* (TS) or *Lower Tail Dependency* (LTD). The TS is used by Chabi-Yo et al. (2018), Poon et al. (2004), Agarwal et al. (2017) and Weigert (2015) and is defined by

$$\mathrm{TS}_{i,t}^{W} = \lim_{\alpha \to 0} \mathrm{P}\Big(R_{i,t}^{W} < -\mathrm{VaR}_{i,t}^{W,\alpha} \mid R_{t}^{M} < -\mathrm{VaR}_{t}^{M,\alpha}\Big) \tag{3.4.14}$$

$$\mathrm{TS}_{i,t}^{L} = \lim_{\alpha \to 0} \mathrm{P}\Big(-R_{i,t}^{L} < -\mathrm{VaR}_{i,t}^{L,\alpha} \mid R_{t}^{M} < -\mathrm{VaR}_{t}^{M,\alpha}\Big)\,. \tag{3.4.15}$$

The Tail-Sens in our case is defined as the probability that industry i suffers an extreme loss at the same time the momentum portfolio suffers an extreme loss, where we define extreme losses as losses higher than the VaR. Weigert (2015, p. 136) find that "a quintile portfolio consisting of stocks with the strongest LTD underperforms a quintile portfolio consisting of stocks with the weakest LTD by more than 9% on a monthly basis during periods of heavy market downturns worldwide." Hence, low LTD assets offer a hedge against crashes and should be weighted higher in crash periods (Chabi-Yo et al., 2018, p. 1074).

Chabi-Yo et al. (2018, Sec. III.D.3) find that different industries exhibit different levels of LTD. Similarly, Weigert (2015, Table 1) find that also different countries exhibit quite different levels of LTD. Thus, different portfolios can have quite different levels of crash-sensitivity and an equally or volatility weighted portfolio's crash risk can be dominated by a few assets with high crash-sensitivity. Chabi-Yo et al. (2018, Table 1-2) find that the LTD is correlated but different to beta, downside beta, Tail-beta, coskewness and cokurtosis. Thus, LTD captures other characteristics of risk and should produce different results to the approaches presented above. We follow Agarwal et al. (2017) and estimate LTD non-parametrically as given in Equation (C.27). When estimating LTD, short estimation windows should be used. For example, Chabi-Yo et al. (2018, Sec.III.D.2) find better results of 12 months of daily data instead of 24 or 36 months.[86] This results since the authors show that LTD is highly time-varying. Thus, simply using equal-weights or too long estimation windows does not regard the time-varying crash-sensitivity of the assets in the momentum portfolio. For our main results, we use an estimation window of 12 month and a cut-off point q that equals the 30% quantile. Alternatively, the cut-off point q could also be estimated using a bootstrap approach (Poon et al., 2004). Additional results for other cut-off points and other estimation windows are again shown in Appendix B.1.

Measuring and using extreme tail dependency is an important determinant in portfolio risk management and other fields of finance as shown by Poon et al. (2004, Sec. 5-6). However, Agarwal et al. (2017, p. 615) argue that the TS only measures how likely it is that industry i suffers the most extreme losses at the same time the momentum portfolio suffers the most extreme losses. A disadvantage of this measure is that the severity of these extreme losses is not taken into account. To take the severity of extreme loss into account, Agarwal et al. (2017,

[86]Chabi-Yo et al. (2018) estimate the LTD using a copula approach. In contrast, Poon et al. (2004) and Agarwal et al. (2017) use simple to implement non-parametric approaches to estimate LTD. We used both estimation approaches, the non-parametric approach of Agarwal et al. (2017) and the copula-based approach of Chabi-Yo et al. (2018) and found that the simple non-parametric approach performs surprisingly well and is not outperformed by the copula approach. Poon et al. (2004) present several alternative approaches to estimate extreme tail dependency. For example, the authors present in Section 2 a non-parametric approach to estimate extreme tail dependency using the Hill estimator. Moreover, in Section 3, the authors show how extreme tail dependency can be estimated using parametric approaches. Furthermore, Chabi-Yo et al. (2018) state that the copula approach can also be implemented on volatility adjusted returns. However, Chabi-Yo et al. (2018, Footnote 13) conclude that both approaches deliver similar results.

p. 611) scale the lower tail dependency by the ratio of the CVaR of asset i and the benchmark portfolio. This approach is an extension of the usual beta in Equation (3.4.3). The TS based measure of Agarwal et al. (2017, Eq. (2)), which the authors call *Tail-Risk (TR)*, is defined by

$$\mathrm{TR}_{i,t}^{W} = \mathrm{TS}_{i,t}^{W} \cdot \frac{\mathrm{CVaR}_{i,t}^{W,\alpha}}{\mathrm{CVaR}_{t}^{M,\alpha}} \quad \text{and} \quad \mathrm{TR}_{i,t}^{L} = \mathrm{TS}_{i,t}^{L} \cdot \frac{\mathrm{CVaR}_{i,t}^{L,\alpha}}{\mathrm{CVaR}_{t}^{M,\alpha}}, \tag{3.4.16}$$

where $\mathrm{CVaR}_t^{M,\alpha} := \mathbb{E}\Big(-R_t^M \mid -R_t^M > \mathrm{VaR}_t^{M,\alpha}\Big)$ denotes the month t CVaR of the momentum portfolio. Agarwal et al. (2017) also estimate the Tail-Risk measure by replacing the CVaR with VaR. This measure is then similar to the Tail-beta estimated as in Equation (C.24), which is also given by a measure of the tail dependency and the ratio of the VaR of industry i and the momentum portfolio. However, since CVaR is typically superior to VaR, we only use the CVaR based Tail-Risk measure. Agarwal et al. (2017, p. 627) find that the Tail-Risk measure is related to other systematic risk measures like the downside beta. However, the authors find that their systematic tail risk measure captures other information than univariate risk measures, like skewness, kurtosis, VaR and CVaR. We follow Agarwal et al. (2017) and estimate Tail-Risk non-parametrically using $\alpha = 30\%$ and the last 12 months of daily data as shown in Equation (C.28). Agarwal et al. (2017) show that their estimation approach is robust to other estimation lengths, other values of α and a definition based on VaR instead of CVaR.[87] Robustness results for other estimation windows and cut-off points are again shown in Appendix B.1.

Agarwal et al. (2017, p. 630) find that also hedge funds time systematic tail risk by investing less in assets with a high systematic tail risk exposure before the financial crisis. Further, Agarwal et al. (2017, p. 632-633) find that hedge funds that time systematic tail risk outperform hedge funds that are bad systematic tail risk timers. Thus, timing systematic tail risk, especially in crises, is related to a higher performance. Similarly, Agarwal et al. (2017, Figure 1) find that the Tail-Risk measure is successful in predicting down-periods and that aggregate Tail-Risk is negatively correlated with the market return, i.e. higher aggregated Tail-Risk predicts lower market returns. Thus, systematic tail risk timing plays an important role in mitigating crashes.

Finally, we use another CVaR based systematic tail risk measure which is frequently exam-

[87] Agarwal et al. (2017) use $\alpha = 5\%$ and 24 monthly returns for their main results. Using 24 monthly returns corresponds to about one month of daily data. The authors show that their approach is robust to estimation lengths of 36 and 48 months as well as using the 10% and 20% quantile as cut-off point (see Agarwal et al. (2017, Sec. 4.2)).

ined in the literature. Brownlees and Engle (2016), Acharya et al. (2012), Engle et al. (2015), Allen et al. (2012) and Acharya et al. (2016) use a CVaR based systematic tail risk measure that measures the comovement of an asset with the benchmark portfolio and is a natural extension of the univariate CVaR defined in Equation (3.3.8). The CVaR presented in the previous section measures the average loss of industry i conditioned on the information that industry i suffers an extreme loss. Calculating the average loss of industry i conditioned on extreme losses of the momentum portfolio leads to the Marginal Expected Shortfall (MES). Acharya et al. (2016) find that MES is different to the CVaR that is solely based on asset i's distribution and to other systematic risk measures like beta. The MES is given by

$$\mathrm{MES}_{i,t}^{W,\alpha} = -\mathbb{E}\Big(R_{i,t}^{W} \mid -R_t^M > \mathrm{VaR}_t^{M,\alpha}\Big) \text{ and } \mathrm{MES}_{i,t}^{L,\alpha} = \mathbb{E}\Big(R_{i,t}^{L} \mid -R_t^M > \mathrm{VaR}_t^{M,\alpha}\Big). \tag{3.4.17}$$

By definition, the MES measures industry i's sensitivity to declines of the momentum portfolio, and hence the MES measures how exposed an industry is to tail events of the benchmark index (Acharya et al., 2016, p. 4). Weighting industries based on their MES should be particularly appealing during momentum crash periods. For example, Acharya et al. (2016, p. 22) find that "a higher MES is associated with a more negative return during the crisis".

Acharya et al. (2016, Table 5) show that using more recent data is beneficial when MES is estimated. Thus, we follow Acharya et al. (2016) and estimate MES non-parametrically using the last 12 months of daily data and $\alpha = 30\%$. Alternatively, instead of using a quantile based cut-off point, a fixed percentage loss, e.g. a return of -40% as in Engle et al. (2015), could be used. Further, as in Adrian and Brunnermeier (2016) who use a similar VaR based systematic risk measure, risk could also be defined as the difference between the MES for low levels of α and the MES using a higher value of $\alpha = 50\%$. Robustness results for other cut-off points and estimation lengths are again shown in Appendix B.1. As in Brownlees and Engle (2016, Table 8) we find that other cut-off points and other estimation procedures of MES produce quite similar results.

Following Acharya et al. (2016, Eq.(2) and (3)), the MES in Equation (3.4.17) can also be derived as the contribution of industry i to the portfolio CVaR. From Equation (3.2.2) we see that the momentum return is given by

$$R_t^M = \sum_{i=1}^{n} w_{i,t}^W \cdot R_{i,t}^W - w_{i,t}^L \cdot R_{i,t}^L. \tag{3.4.18}$$

Based on R_t^M and the momentum portfolio's CVaR, the MES of asset i in the winners and losers portfolio is given by

$$\text{MES}_{i,t}^{W,\alpha} = \frac{\partial \text{CVaR}_t^{M,\alpha}}{\partial w_{i,t}^W} \quad \text{and} \quad \text{MES}_{i,t}^{L,\alpha} = \frac{\partial \text{CVaR}_t^{M,\alpha}}{\partial w_{i,t}^L}. \tag{3.4.19}$$

Thus, the MES is defined as the sensitivity of portfolio risk to the weight of asset i. A high MES indicates that portfolio risk significantly increases if the weight of asset i is increased. Thus, in order to reduce momentum's risk, assets with a high MES should be weighted lower.

A possible refinement of our approach would be to estimate MES based on more advanced (conditional) models. Acharya et al. (2016, Table 5) show that using more advanced models is beneficial when MES is estimated. For example, Acharya et al. (2012) and Brownlees and Engle (2016) present an alternative estimation method by first fitting a multivariate GARCH model to the daily returns of asset i and the benchmark. This model is then used to simulate future price paths of both assets. MES is then calculated by applying the sample estimator to the simulated returns (see Appendix A of Brownlees and Engle (2016) for a more detailed description). This approach is more complex and time consuming but has the advantage that systematic tail risk is forecasted. Brownlees and Engle (2016) also show how MES can be calculated under a normality assumption and based on copulas (see Equation (4) and Appendix B of Brownlees and Engle (2016) for more details on the normal and copula approach, respectively). However, using the MES estimator based on the bivariate normal distribution would produce the same results as the inverse beta weighting presented above. We therefore do not examine this method here. The more complex methods based on the copula and multivariate GARCH models are appealing to manage momentum crashes. However, due to the importance of risk-managed momentum portfolios for practitioners, we rely on the simple non-parametric estimator used by Acharya et al. (2016) and leave the examination of more advanced estimation methods for future research.

A measure that is similar to the MES and that could also be used to weight the industries in the momentum portfolio is the CoVaR that also measures extreme tail dependency (see Acharya et al. (2012, p. 62) and Adrian and Brunnermeier (2016) for example). The CoVaR measures the VaR of the benchmark portfolio by conditioning on the state of industry i. Thus, using CoVaR would condition on bad outcomes of industry i instead of bad outcomes of the momentum portfolio. However, Adrian and Brunnermeier (2016, Sec.II.D) show that the CoVaR can also be defined when the conditioning is reversed, i.e. CoVaR could also be defined as the VaR of industry i conditioned on low momentum returns. Nevertheless, we do not use the CoVaR measure, since CVaR based risk measures are typically superior to VaR based risk measures in portfolio allocations (Basak and Shapiro, 2001, Rickenberg, 2020b). Additionally, Acharya et al. (2012, Eq. (12)) derive a simple closed form expression for CoVaR under a normality assumption which shows that this measure only depends on the correlation between the asset and the benchmark portfolio, whereas MES also considers asset i's volatility. Since different industries typically have different levels of volatility, these different levels of volatility should be reflected when industries in the momentum portfolio are weighted by their systematic risk. Due to the advantages of MES compared to CoVaR we only examine results for MES.[88] Furthermore, accurately estimating CoVaR non-parametrically would be quite challenging. Adrian and Brunnermeier (2016) show how CoVaR can be estimated using quantile regressions, multivariate dynamic volatility models, copulas, distributional assumptions or Bayesian methods. We again leave the examination of these advanced estimation methods for future research.

Although all of the measures defined in this and the previous section quantify an asset's (systematic tail) risk and are sometimes highly correlated, many of them capture different aspects of risk. For example, Bali et al. (2017a) find that the beta anomaly still holds when assets are first sorted on their skewness, downside beta, tail risk and (idiosyncratic) volatility. Bali et al. (2017b) find that assets with a higher beta typically have a lower coskewness, but that both measures capture different aspects of risk (Bali et al., 2017b, Table 4). Langlois (2020) finds that past momentum and (idiosyncratic) volatility negatively predict an asset's coskewness, but that volatility and coskewness are highly different. In particular, the author finds that skewness and coskewness capture different aspects of risk. Ang et al. (2006a, Table 3) find

[88]Adrian and Brunnermeier (2016, p. 1711) state that CoVaR can also be extended to a *Co Expected Shortfall* measure that is based on the CVaR. This measure would then again be quite similar to the MES presented above.

that downside beta and coskewness are similar but capture different characteristics. The authors conclude that “[i]n summary, downside beta risk and coskewness risk are different” (Ang et al., 2006a, p. 1213). This result is confirmed by Bollerslev et al. (2020, p. 24) who find that coskewness and cokurtosis “do contain additional information not accounted for by semibetas”. Similarly, Atilgan et al. (2018) find that downside beta is different to beta, upside beta, (idiosyncratic) volatility, momentum, skewness and coskewness. Chabi-Yo et al. (2018, p. 1081) conclude that “the risk associated with LTD is related but clearly different from risks associated with regular market beta, downside beta, coskewness, cokurtosis, and tail risk.” Van Oordt and Zhou (2016, Table 5) show that Tail-beta captures other information than volatility, skewness, kurtosis, downside beta, coskewness, cokurtosis and past return. Bali et al. (2014) find that idiosyncratic volatility, LPM, beta, coskewness, HTCR-beta and LPM-beta capture different aspects of risk. Atilgan et al. (2020) find a positive relation between tail risk and idiosyncratic volatility, beta and downside beta as well as a negative relation between tail risk and momentum and coskewness. However, the authors find that the low tail risk anomaly is different to the low volatility and low beta anomaly and they conclude that “left-tail risk has distinct, significant information orthogonal to market beta, downside beta, idiosyncratic volatility, lottery demand, co-skewness [...] and past return characteristics and it is a strong and robust [negative] predictor of future returns” (Atilgan et al., 2020, p. 734). Hence, all these measures, although somewhat related, capture different characteristics of risk and should also lead to different weightings. In particular, all these measures should contain additional information even when assets are first sorted on momentum.

3.5 Switching Strategies

The previous two sections show how the assets in the winners and losers portfolios can be weighted based on their (systematic) tail risk. These weighting schemes should produce higher returns while simultaneously momentum crashes are reduced. Section 3.3 develops weighting schemes based on univariate risk measures that quantify an asset’s own tail risk. In contrast, Section 3.4 uses weighting schemes based on systematic (tail) risk measures that measure an asset's comovement and co-crash risk with the (equally weighted) momentum portfolio. As mentioned above, both classes of risk measures typically capture different aspects of risk. For example, Agarwal et al. (2017, p. 627) find that systematic tail risk captures other information than univariate risk measures, like skewness, kurtosis, VaR and CVaR. Similarly, Van Oordt and Zhou (2016, Table 5) find that systematic tail risk captures other information than volatility, skewness and kurtosis. Bali et al. (2012) show that systematic risk can explain hedge fund returns, whereas tail risk measures, like skewness and kurtosis, fail to explain hedge fund returns. Adrian and Brunnermeier (2016) find that a VaR based systematic risk measure captures other information than the univariate VaR.

Timing the assets' univariate or systematic crash risk is important to reduce momentum crashes and to enhance the risk-return profile of the momentum portfolio. For example, Jang and Kang (2019) find that financial institutions that time the (univariate) crash probability of their holdings obtain higher returns. Similarly, Agarwal et al. (2017) find that funds that time their constituents' systematic crash risk during crises outperform funds that are bad crash risk timers. However, the benefits of systematic tail risk timing are mainly limited to down-periods. More importantly, a higher comovement risk of the assets in the momentum portfolio is desired during uptrending periods, and hence weighting assets inversely to their comovement risk is disadvantageous in these periods. Thus, the performance of the systematic tail risk weighting strongly depends on the state of the momentum portfolio and incorporating this state dependence can potentially increase momentum's performance. For example, Acharya et al. (2016, Table 4 and Fig. 2) find that systematic tail risk is a negative return predictor during crises, which does not necessarily hold for univariate risk measures and for non-crises periods. Agarwal et al. (2017, p. 620) state that "the impact of [systematic tail risk] on future returns is strongly positive in periods of positive market returns and negative when the market returns are negative". Weigert (2015) and Chabi-Yo et al. (2018) find that assets with lower tail dependency outperform assets with higher tail dependency in crash periods, i.e. assets with lower co-crash risk offer a good protection against extremely negative returns and should be weighted higher during momentum crashes. In contrast, assets with a higher tail dependency outperform less crash-sensitive assets in uptrending markets.[89] Similarly, Levi and Welch (2019, Fig. 1) and Asvanunt et al. (2015, Exhibit 1) show that low beta assets provide a good hedge against crashes. This observation holds especially in extreme crash periods (Levi and Welch, 2019, Fig. 2). However, Levi and Welch (2019) state that "in periods in which the [benchmark portfolio] appreciates, assets with high (all-days or down-days) beta should offer higher average rates of return". Hence, assets with high systematic risk perform well (bad) in bull (bear) periods, whereas assets with low systematic risk perform well (bad) in bear (bull) periods (see Asness et al. (2020, Fig. 1) and Levi and Welch (2019, Table 2)). Thus, although downweighting stocks with higher systematic tail risk should mitigate momentum crashes, this approach should perform worse in uptrending periods. Thus, using the systematic risk weighting in every month is not optimal over the long run. Generally, many studies show that different portfolio weighting methods can perform quite differently in different market environments. For example, Blitz and Van Vliet (2007, Exhibit 1) and Chow et al. (2014) find that low risk portfolios perform well in down-markets but bad in up-markets. Behr et al. (2012) confirm this finding for the minimum variance portfolio.

Based on the findings summarized above, the weighting schemes that use systematic risk measures should provide a good downside risk protection when the momentum portfolio suffers extreme losses. However, in uptrending markets, assets with a low sensitivity to the momentum portfolio should perform worse. In contrast, the weighting schemes based on univariate risk measures should be less successful in crash periods than the systematic risk weightings, but should successfully capture the upside potential in calm periods. This should especially hold for risk measures that capture information on both tails of the return distribution, like the skewness or reward-to-risk measures. For that reason, we next develop strategies that switch between both approaches, where the systematic risk weighting is only used when a momentum crash is likely. In periods when a positive momentum return is expected, we use a weighting scheme based on a univariate risk measure. Switching between different portfolio weightings or investment styles has been frequently examined in the literature (Chow et al., 1999, Copeland

[89] Van Oordt and Zhou (2016) also find that assets with a higher systematic tail risk significantly underperform in crash periods, but the authors do not find an outperformance in uptrending periods. However, their results are only significant in the crash period.

and Copeland, 1999, Daniel et al., 2017, DeMiguel et al., 2009b, Garlappi et al., 2006, Kan and Zhou, 2007, Rickenberg, 2020a,b, Tu and Zhou, 2011, Wang et al., 2012).[90] Similarly, Chow et al. (1999) argues that regarding risk characteristics of turbulent and calm periods separately is advantageous compared to strategies that use either risk characteristics of the calm or turbulent period. Using only the risk characteristics of turbulent times would be too conservative over the long run. In contrast, investors who only consider risk characteristics of calm periods and disregard information on extreme crash risk "may not survive to generate long-term performance if the portfolios cannot withstand exceptional periods of market turbulence" (Chow et al., 1999, p. 67).[91] Rickenberg (2020a) also uses a risk-managed momentum strategy that switches between two weighting schemes, where the conservative weighting scheme is only used when a momentum crash is likely. However, Rickenberg (2020a) only manages the momentum portfolio's risk using equally weighted winners and losers portfolios.

To take the momentum state dependence of the univariate and systematic (tail) risk weightings into account, we next develop a combined strategy that switches between both weighting schemes, where we rely on simple estimates of the expected momentum state to decide which weighting scheme is used.[92] More formally, following DeMiguel et al. (2009b, Eq. (9)) and Tu and Zhou (2011, Eq. (1)), the month t weight of asset i in the winners and losers portfolio is

[90] See, for example, DeMiguel et al. (2009b, Sec. 1.6) for a short summary on strategies that combine different weighting schemes. For example, Kan and Zhou (2007, Sec. III) and Garlappi et al. (2006) combine the minimum variance and the mean-variance portfolio, where the relative weights invested in both portfolios depend on the estimation error of the mean return. Similarly, DeMiguel et al. (2009b, Sec. 1.6.2) combine the equally weighted and minimum variance portfolio, whereas Tu and Zhou (2011) combine the equally weighted portfolio with four other portfolio strategies including the mean-variance portfolio.

[91] Chow et al. (1999) present a mean-variance approach based on two different covariance matrices, one estimated with "normal" observations and one estimated with outliers that capture risk characteristics in turbulent times. Chow et al. (1999, p. 70) summarize the disadvantage of the usual mean-variance approach or the approach of using either the normal or stressed covariance matrix as follows: "If we optimize based on the full-sample covariance matrix, the portfolio will be significantly suboptimal in a period of financial stress and, indeed, may not survive such a period without unpropitious adjustments. If we optimize based on the outlier covariance matrix, the portfolio's expected return for the full horizon will be lower than desired. What to do? As with many choices, the best solution is to compromise."

[92] An alternative would be to use different weighting schemes for both portfolios. For example, based on the finding of Blitz and Van Vliet (2007), Chow et al. (2014) and Behr et al. (2012) that volatility managed portfolios perform well in down-markets and bad in up-markets, winners (losers) could be volatility (equally) weighted in down-markets and equally (volatility) weighted in up-markets. However, we restrict ourselves on weighting schemes that use the same risk measure for both legs.

given by

$$w_{i,t}^{j,switch} = \delta_t \cdot w_{i,t}^{j,sys} + (1 - \delta_t) \cdot w_{i,t}^{j,uni}, \quad j \in \{W, L\}, \tag{3.5.1}$$

where $w_{i,t}^{j,uni}$ and $w_{i,t}^{j,sys}$ denote the month t weight using a univariate or systematic risk measure, respectively. The indicator $\delta_t \in \{0, 1\}$ indicates if a momentum crash in month t is likely or not, where δ_t is equal to one when the momentum crash probability is high and zero else. This approach is similar to the weighting presented in Chow et al. (1999, Eq. (B-1)) that weights risk characteristics of calm and crash periods by the probability that a calm or crash regime occurs.[93]

As mentioned above, to better capture the upside potential in up-markets ($\delta_t = 0$), we focus on univariate risk measures that capture information on both tails of the return distribution. In contrast, the quantile based risk measures, like LPM, VaR or CVaR, only capture information on downside (upside) risk for the winners (losers). Hence, these weighting schemes should perform well in down-periods, but should not contain much information in uptrending periods. For that reason, we focus on strategies using skewness or RSJ to determine $w_{i,t}^{j,uni}$. For example, in an uptrending market, winners with higher skewness, i.e. higher upside potential, are weighted higher. Similarly, using the RSJ measure, winner (loser) industries with a higher weight are typically industries where positive (negative) jumps are more likely than negative (positive) jumps. Jondeau et al. (2019, Figure 5) find that a skewness based trading strategy performs well in uptrending periods but still exhibits a high crash risk in crises. However, the high crash risk of a skewness based momentum portfolio is not a concern for our switching strategy. Even more important, Jiang et al. (2020, Sec. V.B) find that skewness is a positive return predictor in periods of low market volatility. Thus, the skewness weighting should work well in periods when market volatility is low, since assets with a higher skewness, which also obtain higher weights, exhibit higher returns in these periods. This is appealing for investors, since most investors would even accept lower returns in order to obtain higher levels of skewness. Since periods of

[93] Another alternative to our switching strategy in Equation (3.5.1) that switches between univariate and systematic tail risk would be to follow the approach of Chow et al. (1999) and define each risk measure for calm and crash periods. Thus, when estimating an industry's univariate and systematic tail risk all observations could be separated into normal observations and multivariate outliers (see Chow et al. (1999, Fig. 2-4) for an illustration of this separation process). Each risk measure could then be estimated based on these two samples and the portfolio risk could then be defined as a weighted average of these measures (Chow et al., 1999, Eq. (B-1)). Hence, instead of switching between univariate and systematic tail risk, one could also switch between normal and stressed risk, e.g. skewness in calm periods and skewness in crash periods.

a low market volatility typically coincide with periods of a low momentum crash probability (Daniel and Moskowitz, 2016, Du Plessis and Hallerbach, 2017, Wang and Xu, 2015), our switching strategy would switch to the skewness weighting, just when this approach offers an appealing risk-return profile. In contrast, in periods of a high market volatility, Jiang et al. (2020, Sec. V.B) find that skewness is a negative return predictor, i.e. assets with a higher skewness exhibit lower returns. Since periods with higher market volatility typically coincide with an increased momentum crash probability, our switching approach would use a systematic risk based weighting in these times. Thus, the switching approach would use the skewness weighting only when this weighting scheme produces an enhanced risk-return profile, but switches to a systematic tail risk weighting in times when the skewness weighting becomes unappealing. Since the measure of Jiang et al. (2020) is also quite similar to the RSJ measure, switching between an RSJ and systematic tail risk weighting, based on a momentum crash indicator, should also be very appealing. In total, our switching approach should produce higher returns than the equally weighted portfolio in calm periods ($\delta_t = 0$) while simultaneously momentum crashes are reduced in high risk periods ($\delta_t = 1$).

To determine whether a momentum crash – or a negative momentum return – is likely in the next month, several approaches have been presented in the financial literature. Generally, the drivers of momentum crashes for the individual stock momentum strategy have been examined by Grundy and Martin (2001), Daniel and Moskowitz (2016), Min and Kim (2016), Wang and Xu (2015) and Cooper et al. (2004). The authors show that momentum crashes typically occur when the market rebounds after a period of negative market returns or high market volatility. Hence, momentum crashes of the individual stock momentum strategy can be (partly) predicted by the past return and/or volatility of the market. However, Grobys et al. (2018, Table 7) find that industry momentum and stock momentum crashes are quite different and that these crash indicators do not work as well for industry momentum as for individual stock momentum. In contrast, Du Plessis and Hallerbach (2017) find that a high market volatility negatively predicts industry momentum returns. This finding is similar to the earlier finding of Wang and Xu (2015) and Daniel and Moskowitz (2016) that negative returns of the individual stock momentum strategy typically occur after periods of a high market volatility.

Instead of predicting momentum crashes based on past market volatility, the momentum crash indicator can also be defined based on the volatility of the (equally weighted) momentum strategy. Barroso and Santa-Clara (2015) and Barroso and Maio (2019) show that a high volatility of the individual stock based momentum portfolio predicts negative returns of the momentum portfolio. Du Plessis and Hallerbach (2017), Grobys et al. (2018) and Grobys and Kolari (2020) find a similar result for industry momentum and Grobys (2018) confirm this finding for the 52 week high momentum strategy. Based on this observation, we define a crash indicator that indicates a momentum crash in the next month if momentum's volatility is expected to be high, where we define "high" as a volatility that is higher than the chosen volatility target σ_{target} that will be used in the next section. Defining the crash indicator based on the volatility target σ_{target} has the advantage that an investor's portfolio allocation is directly influenced by the investor's risk-aversion, which should be incorporated in the portfolio allocation process (Chow et al., 1999).[94] For our main results we choose $\sigma_{\text{target}} = 8\%$, but we show results for other choices in Appendix B.8. We additionally show results for a time-varying threshold σ_{target} that is determined by the long run volatility. The crash indicator is then given by

$$\delta_t = \begin{cases} 1 & \text{if } RV_{t-1}^{WML} > \sigma_{\text{target}} \\ 0 & \text{if } RV_{t-1}^{WML} \leqslant \sigma_{\text{target}}, \end{cases} \tag{3.5.2}$$

where $RV_{t-1}^{WML} = \sqrt{\frac{1}{T}\sum_{k=1}^{T}\sum_{j=1}^{21}(r_{t-k,j}^{WML,eq})^2}$ denotes the Realized Volatility of the equally weighted momentum portfolio, $r_{t,j}^{WML,eq}$ denotes the daily return of the equally weighted momentum portfolio on day j in month t and T denotes the number of months used to estimate RV_{t-1}^{WML}. We use $T = 1$ for our main results but find that results are quite robust for different choices of T, which are shown in Appendix B.8. The appendix additionally shows results when past market volatility instead of past momentum volatility is used.

As an alternative to the volatility based crash indicator in Equation (3.5.2), momentum crashes could also be predicted based on the equally weighted momentum strategy's past per-

[94] A risk-averse investor would choose a lower volatility target (see Bollerslev et al. (2018, p. 2757) who derive the volatility target as a function of an investor's risk aversion), which would lead to a portfolio that is mainly managed by systematic tail risk. In contrast, a less risk-averse investor chooses a higher volatility target which leads to a more aggressive and mainly univariate tail risk managed portfolio. Thus, the portfolio of a more risk-averse investor will be more conservatively than the portfolio of a less risk-averse investor.

formance using the TSMOM strategy of Moskowitz et al. (2012). Moskowitz et al. (2012, p. 241) find that "TSMOM and [cross-sectional momentum] are related, but are not the same". Thus, applying TSMOM to the momentum portfolio should contain additional information on the expected state of the momentum portfolio. Moskowitz et al. (2012) find that an asset's own past performance is a positive predictor of its future return. Further, Moskowitz et al. (2012, Figure 4) and Asvanunt et al. (2015, Exhibit 3) show that TSMOM works best in extreme markets and is successful in determining crash periods. Harvey et al. (2018) also argue that trend-following rules, such as the TSMOM strategy, well forecast crashes. Thus, "time series momentum may be a hedge for extreme events" (Moskowitz et al., 2012, p. 230) and should be an adequate tool to predict momentum crash periods. However, instead of switching to a contrarian strategy, as suggested by TSMOM, TSMOM signals could be used to switch between univariate and systematic risk based weightings. Moskowitz et al. (2012, Table 2) find good results for the 12 months lookback and one month holding TSMOM strategy, but the authors show that other lookback periods also work well. Similarly, Goyal and Jegadeesh (2017, Table 1) find good results of TSMOM for a one month holding period and lookback periods between one and 60 months. The impact of the lookback period on the profitability of TSMOM has also been examined by Dudler et al. (2015) and Du Plessis and Hallerbach (2017). In particular, Moskowitz and Grinblatt (1999) and Novy-Marx (2012) find that an industry's past month's return contains valuable information on the industry's future return. Moreover, Goyal and Jegadeesh (2017, Table 1) find that the short-term reversal effect does not hold for the TSMOM strategy but for the individual stock based cross-sectional momentum strategy. Thus, short look back periods should work well for the TSMOM strategy applied to the industry momentum portfolio. We used several TSMOM strategies based on lookback periods between one month and 36 months to determine δ_t and found quite similar results for these lookback periods. Additional robustness results for the TSMOM based crash indicators are again shown in the appendix.

In total, to rule out that the switching approach only works for one definition of the crash indicator δ_t, we also used several other definitions of the crash indicator. These robustness results are shown in Appendix B.8. In Table LIII, we show results for the crash indicator in Equation

(3.5.2) when Realized Volatility is estimated based on the past six months of daily data as well as two other levels of σ_{target}. In Table LIV, we define the crash indicator of Equation (3.5.2) based on the past volatility of the market for three levels of σ_{target}.[95] This indicator is motivated by the finding of Daniel and Moskowitz (2016), Du Plessis and Hallerbach (2017) and Wang and Xu (2015) that a high market volatility is good indicator for a momentum crash. In Table LV, we define the momentum crash indicator when momentum's short-term volatility is higher than momentum's long-term volatility, where we use three different combinations of short- and long-term volatility. Finally, in Table LVI, we define the crash indicator based on three different TSMOM strategies applied to the equally weighted momentum strategy. Furthermore, combining information on several crash indicators also produces very good results. For example, combining information on momentum's volatility and the market's volatility or combining information on momentum's performance with information on volatility is also an appealing approach to predict industry momentum crashes. This is in line with Rickenberg (2020a) who also finds that combining several crash indicators produces good results. However, results for these combinations are not shown since these indicators produce very similar results to the remaining crash indicators.

3.6 Managing Individual and Portfolio Risk

The previous sections show how the momentum portfolio's risk can be managed by weighting the individual assets of the momentum portfolio inversely to their risk. However, several studies show that correlations between different assets significantly increase during crash periods, i.e. assets typically crash together (Ang and Chen, 2002, Chabi-Yo et al., 2018, Hong et al., 2007, Longin and Solnik, 2001, Poon et al., 2004, Weigert, 2015). Hence, diversification fails as a risk management tool in times when it is most needed. Momentum crashes can be attenuated by weighting assets inversely to their risk, but these crash periods can still be severe. Alternatively,

[95] Switching strategies based on the market's volatility are also frequently used by practitioners (Copeland and Copeland, 1999). See, for example, the research of Morningstar Inc. (`https://www.morningstar.com/articles/925094/a-momentum-and-low-volatility-switching-strategy`). The author proposes a market volatility based strategy that switches between a momentum and a low volatility strategy. In periods of a low market volatility, this switching strategy invests in a momentum portfolio and switches to a low volatility portfolio when market volatility increases.

the momentum portfolio's risk can also be managed by dynamically adjusting the exposure to the momentum strategy.[96] An easy method to manage a portfolio's overall risk is the volatility targeting strategy. The aim of volatility targeting is to frequently adjust the exposure to a strategy in order to achieve a constant level of portfolio volatility σ_{target} over time. Risk targeting is an appealing method to reduce left tail risk and drawdowns (Barroso and Santa-Clara, 2015, Harvey et al., 2018, Rickenberg, 2020a,b).[97] In particular, risk targeting has the advantage that portfolio risk management can be separated from the portfolio allocation process.[98] Hence, the risk targeting approach can easily be combined with the momentum strategy as frequently done in the literature (Barroso and Maio, 2018, Barroso and Santa-Clara, 2015, Daniel and Moskowitz, 2016, Du Plessis and Hallerbach, 2017, Grobys, 2018, Grobys and Kolari, 2020, Grobys et al., 2018, Moreira and Muir, 2017, Rickenberg, 2020a). Barroso and Santa-Clara (2015) and Barroso and Maio (2019) show that a higher volatility of the individual stock based momentum portfolio predicts a higher future volatility and lower future return of the momentum portfolio.[99] Thus, periods with a high volatility of the momentum portfolio are unappealing for investors since these periods coincide with significantly lower Sharpe Ratios and should be

[96]The momentum strategy is typically 1$ long the winners and 1$ short the losers. Since momentum is a zero investment strategy, the exposure to the strategy can be scaled arbitrarily. In order to manage momentum crashes, the dollar exposure should be reduced in periods of high risk and increased in periods of low risk. Another alternative to this approach is to hedge momentum's risk with other factor portfolios, like the market or size factor (Grundy and Martin, 2001, Martens and Van Oord, 2014) or the value factor that is negatively correlated with momentum (Asness et al., 2013).

[97]See Rickenberg (2020b, Appendix A) for a list of further advantages of risk targeting applied to equity portfolios. In particular, volatility targeting is highly related to the optimal portfolio choice of a mean-variance investor. For example, Zakamulin (2015, p. 91) state that "under some conditions, the volatility targeting strategy might present the optimal implementation of risk control over time".

[98]Agarwal and Naik (2004) show that the skill of a portfolio manager can be separated into his market timing and stock picking ability. Targeting the risk weighted momentum strategy's volatility means that the overall risk is managed by scaling the exposure to the portfolio, whereas the asset allocation is chosen based on the assets' individual past performance and risk.

[99]A similar observation also holds for long-only equity portfolios. In contrast to the risk-return relation of the momentum portfolio, the risk-return relation for equities has been frequently examined (see Bekaert and Wu (2000), French et al. (1987), Glosten et al. (1993) among others). See also Rickenberg (2020b) for further references on the risk-return relation for equities. A possible explanation for the negative relation between volatility and future return is the volatility feedback effect or time-varying risk premium effect. Bekaert and Wu (2000, p. 1-2) state: "If volatility is priced, an anticipated increase in volatility raises the required return on equity, leading to an immediate stock price decline." This effect is different to the well-known leverage effect that describes a negative relation between return and future volatility. Bekaert and Wu (2000) state that both effects interact and that the leverage effect strengthens the volatility feedback effect. See also Bekaert and Wu (2000, Fig. 1) for an illustration of both effects.

avoided by momentum investors.[100] Based on this observation, Cederburg et al. (2020), Daniel and Moskowitz (2016), Barroso and Maio (2018), Barroso and Santa-Clara (2015), Rickenberg (2020a) and Moreira and Muir (2017) show that volatility targeting works well for the individual stock momentum strategy and significantly increases momentum's (risk-adjusted) return. Du Plessis and Hallerbach (2017), Grobys et al. (2018, Table 2) and Grobys and Kolari (2020, Table 5) show that this result also holds for industry momentum, which makes volatility targeting an appealing approach to manage the industry momentum portfolio's portfolio risk. For example, Grobys and Kolari (2020) find that volatility targeting increases the return of the industry momentum strategy by 87% while simultaneously left tail risk is reduced. Similar results also hold when volatility targeting is applied to the 52 week high industry momentum strategy as done by Cederburg et al. (2020) and Grobys (2018), where again especially left tail risk is reduced by volatility targeting (Grobys, 2018, Table 1). However, these studies target the risk of equally or value-weighted momentum portfolios, and hence only manage momentum's portfolio risk without regarding individual asset risk.

Du Plessis and Hallerbach (2017) apply volatility targeting and volatility weighting to time series and cross-sectional industry momentum strategies. The authors find that both approaches, volatility targeting and inverse volatility weighting, add value by producing a higher Sharpe Ratio with lower downside risk. However, the authors do not apply both approaches simultaneously. Similarly, Kim et al. (2016) highlight the importance of volatility weighting for the time series momentum strategy and volatility targeting for the cross-sectional momentum strategy. Nonetheless, the authors do not apply both approaches to the momentum strategy. Generally,

[100]Volatility targeting does not only add value when risk and return are negatively correlated, as it is found for momentum. More generally, volatility targeting can even be advantageous when risk and return are uncorrelated. For example, when risk and return are unrelated, but volatility is persistent, periods with high volatilities are unappealing for investors since these periods offer an unattractive risk-return profile (Cederburg et al., 2020, Du Plessis and Hallerbach, 2017, Moreira and Muir, 2017, 2019). This result has been theoretically shown by Du Plessis and Hallerbach (2017) and also holds empirically as shown by Du Plessis and Hallerbach (2017), Moreira and Muir (2017) and Moreira and Muir (2019). Further, volatility targeting can even be advantageous when risk and return are positively related. For example, Moreira and Muir (2019, p. 509) find for their data set that an increase of volatility coincides with an increase of expected returns, but that "this increase in expected returns is much more persistent than the increase in volatility. Investors can avoid the short-term increase in volatility by first reducing their exposure to equities when volatility initially increases and capture the increase in expected returns by coming back to the market as volatility comes down." Nevertheless, due to the negative risk-return relation for momentum "volatility management becomes even more attractive" (Cederburg et al., 2020). The good results of volatility targeting applied to momentum can bee seen in Table 1, Table 2 and Table 4 of Cederburg et al. (2020).

simultaneously managing a portfolio's individual *and* portfolio risk has already been examined in the literature, but has not been applied to the momentum strategy. For example, Harvey et al. (2018) state that volatility managing can be used in a portfolio context on the asset level as done by Fleming et al. (2001), Fleming et al. (2003), Han (2005) and Kirby and Ostdiek (2012), the portfolio level as done by Barroso and Santa-Clara (2015), Daniel and Moskowitz (2016) and Moreira and Muir (2017) or both approaches can be used simultaneously as done by Baltas (2015), Moreira and Muir (2017, Sec. I.E) and Zakamulin (2015). Harvey et al. (2018, Exhibit 16) show that all three approaches outperform the non-managed portfolio and that the strategy that manages risk on an asset and portfolio level performs best. Similarly, Zakamulin (2015, Exhibit 3) shows that strategies that simultaneously manage a portfolio's individual asset risk *and* portfolio risk produce higher risk-adjusted returns than non-managed strategies or strategies that only manage a portfolio's individual asset risk. In particular, Zakamulin (2015, p. 86) state that "in a world in which the mean return and covariance matrix vary with time, it is insufficient to revise the composition of the optimal risky portfolio. In addition, as time passes, one must also revise the capital allocation". Moreira and Muir (2017, Sec. I.E) apply volatility targeting to mean-variance efficient factor portfolios by first calculating portfolio weights that maximize the portfolio's Sharpe Ratio. The risk of the mean-variance efficient portfolio is then managed by the portfolio's volatility. Moreira and Muir (2017, Table II) find that this strategy outperforms the strategy that only uses mean-variance efficient portfolio weights. Further, Moreira and Muir (2017, p. 1621) find that "[t]his finding is robust to including the momentum factor as well". Moreover, Cederburg et al. (2020), Barroso and Maio (2018) and Moreira and Muir (2017) find that volatility targeting also works well for the low beta and low volatility portfolio. This approach also regards an asset's individual risk and the whole portfolio's risk. Moreover, an approach that is very similar to the approach examined here is also presented in Baltas (2015). The author applies the inverse volatility and risk parity weighting scheme to the TSMOM strategy and then uses the volatility targeting approach to manage the whole portfolio's risk. The combination of risk targeting and risk weighting is appealing since both approaches capture different aspects. For example, Moreira and Muir (2017, Sec. II.D) compare both strategies and find that volatility targeting is "very different from strategies that explore a weak risk-return trade-off in the cross-section of stocks, which are often attributed to leverage constraints" and that "one can volatility time the cross-sectional anomaly." In particular, "time-series volatility managed portfolios are distinct from the low-beta anomaly documented in the cross-section." Thus, combining portfolio risk management in the time-series and the cross-section, as done by applying volatility targeting to our risk weightings, seems appealing.

The risk weightings presented in the previous sections do not change the amount invested long in the winners portfolio and short in the losers portfolio. From $\sum_{i=1}^{n} w_{i,t}^{W} = \sum_{i=1}^{n} w_{i,t}^{L} = 1$ it follows that the strategies examined so far invest each month 1$ long in the winners portfolio and 1$ short in the losers portfolio. Thus, the risk weighted momentum portfolio is still a zero-investment strategy. Therefore, the exposure to the momentum strategy can be scaled arbitrarily using the volatility targeting strategy of Barroso and Santa-Clara (2015, Eq. (6)). The month t return of this strategy is then given by

$$R_t^{WML*} = \frac{\sigma_{target}}{\hat{\sigma}_t} \cdot R_t^{WML} = \gamma_t \cdot R_t^{WML}, \tag{3.6.1}$$

where σ_{target} is the chosen volatility target, R_t^{WML} is the month t return of the momentum portfolio using one of our weighting schemes, $\hat{\sigma}_t$ is an estimate of the strategy's month t volatility and $\gamma_t = \sigma_{target}/\hat{\sigma}_t$ is the month t exposure to this strategy. Hence, by combining risk weighting with risk targeting, the month t portfolio weight of asset i in the winners and losers portfolio is given by $w_{i,t}^{W*} = \gamma_t \cdot w_{i,t}^{W}$ and $w_{i,t}^{L*} = \gamma_t \cdot w_{i,t}^{L}$, respectively. Thus, an asset's weight in the winners and losers portfolio is given by the relation of the asset's risk to the risks of the remaining assets in the portfolio ($w_{i,t}^{W}$ or $w_{i,t}^{L}$) as well as the exposure to the whole portfolio (γ_t). This approach is similar to the approach used by Baltas (2015, Eq. (4)) for the TSMOM strategy. In particular, from $\sum_{i=1}^{n} w_{i,t}^{W} = \sum_{i=1}^{n} w_{i,t}^{L} = 1$ we obtain $\sum_{i=1}^{n} w_{i,t}^{W*} = \sum_{i=1}^{n} w_{i,t}^{L*} = \gamma_t$. Thus, as shown in Barroso and Santa-Clara (2015, Fig. 4), the amount invested in the long and short leg of the momentum portfolio varies over time. As mentioned above, momentum's risk and return are negatively correlated and momentum's volatility is a good momentum crash indicator. Hence, by targeting a constant level of volatility, the exposure to the momentum strategy is typically increased when a positive momentum return is expected and decreased when a negative momentum return or a momentum crash is likely. Du Plessis and Hallerbach (2017) and Grobys et al. (2018) also apply the approach of Equation (3.6.1) to the industry momentum strategy and find that this approach produces a higher risk-adjusted return and reduces the portfolio's crash risk compared to the strategy with a fixed exposure (Grobys et al., 2018, Table 4). In particular, the authors find that the volatility managed industry momentum strategy has a significantly higher skewness, which can sometimes be even positive. Thus, the volatility targeted momentum strategy is much more appealing for investors who dislike lower levels of skewness and are crash-averse.

In order to forecast next month's volatility $\hat{\sigma}_t$, we first construct "pseudo" daily returns based on the weights that have materialized in the past. For a given weighting scheme, we define the "pseudo" return for asset i in the winners and losers portfolio on day j of month t as

$$\tilde{r}^W_{i,t,j} = w^W_{i,t} \cdot r^W_{i,t,j} \quad \text{and} \quad \tilde{r}^L_{i,t,j} = w^L_{i,t} \cdot r^L_{i,t,j}, \quad j = 1, ..., 21, \tag{3.6.2}$$

where $r^W_{i,t,j}$ and $r^L_{i,t,j}$ denote the return of asset i in the winners and losers portfolio on day j in month t. Hence, "pseudo" daily returns are calculated by assuming fixed portfolio weights within one month. The momentum portfolio's "pseudo" return on day j of month t is then given by

$$\tilde{r}_{t,j} = \sum_{i=1}^{n} \tilde{r}^W_{i,t,j} - \tilde{r}^L_{i,t,j} = \sum_{i=1}^{n} w^W_{i,t} \cdot r^W_{i,t,j} - w^L_{i,t} \cdot r^L_{i,t,j}. \tag{3.6.3}$$

To forecast month t's volatility, we use the Realized Volatility (RV) estimator of month $t-1$ that is also used by Barroso and Santa-Clara (2015), Moreira and Muir (2017) and Grobys et al. (2018).[101] The volatility forecast used in Equation (3.6.1) is then given by

$$\hat{\sigma}_t = \sqrt{\frac{1}{T} \sum_{k=1}^{T} \sum_{j=1}^{21} \tilde{r}^2_{t-k,j}}, \tag{3.6.4}$$

where we follow Barroso and Santa-Clara (2015) and choose $T = 6$ months for our main results. However, our results are also robust to choices between one and twelve months.

[101] Rickenberg (2020a) shows that targeting momentum's volatility based on advanced volatility models is superior to the simple RV model. However, since this paper focuses on simple non-parametric risk estimates, we rely on the simple RV estimator. We show in Appendix B.9 robustness results for the EWMA volatility model. Rickenberg (2020a,b) shows that this model performs well in a volatility targeting context.

Several alternatives to this approach are also feasible. For example, following the approach of Christoffersen (2012, p. 22) "pseudo" returns could also be defined as the returns "that would have materialized if today's portfolio allocation had been used through time". The "pseudo" returns for asset i in the winners and losers portfolio on day j of month t would then be given by

$$\tilde{r}^{W}_{i,t-k,j} = w^{W}_{i,t} \cdot r^{W}_{i,t-k,j} \quad \text{for} \quad k = 1, ..., T \text{ and } j = 1, ..., 21, \tag{3.6.5}$$

$$\tilde{r}^{L}_{i,t-k,j} = w^{L}_{i,t} \cdot r^{L}_{i,t-k,j} \quad \text{for} \quad k = 1, ..., T \text{ and } j = 1, ..., 21. \tag{3.6.6}$$

Further, following Zakamulin (2015, p. 94), month t's volatility could also be estimated in two steps. First, month t's covariance matrix of all asset could be estimated. Second, based on this covariance matrix, portfolio volatility of month t could be estimated using the covariance matrix from the first step and the month t vector of all weights as shown in Zakamulin (2015, Eq. (28)). Furthermore, instead of using pseudo returns, the exposure to each strategy could also be calculated based on the equally weighted momentum portfolio's volatility (see also Moreira and Muir (2017, Sec. II.E), Cederburg et al. (2020, Footnote 7) and references therein on scaling a strategy's exposure by the volatility of another strategy). Another alternative to the volatility targeting strategy based on Realized Volatility would be to scale the exposure by the cross-sectional dispersion, which is also a measure for market uncertainty and is negatively correlated with the momentum portfolio's return (Grobys, 2018, Stivers and Sun, 2010, Wang and Xu, 2015). However, Du Plessis and Hallerbach (2017) find that a dispersion based approach applied to the industry momentum strategy is less effective than the volatility based approach.

As a robustness check, we show in Section B.9 several alternatives to the volatility targeting approach presented here. In Table LVII, we show results for other levels of σ_{target} and for the EWMA volatility model. Further, since Rickenberg (2020a,b) shows that portfolio risk can also be managed by targeting a constant level of tail risk, measured by CVaR, we show in Tables LVIII and LIX results for the strategy that manages momentum's CVaR as well as results for the strategy that switches between volatility and CVaR targeting. For the switching strategy, we use the same crash indicator that is also used to switch between the univariate and systematic risk weightings. Thus, an asset's weight is given by the asset's univariate tail risk and the portfolio's volatility in times when a momentum crash is unlikely, whereas the asset's weight is given by the asset's systematic risk and the portfolio's tail risk when a momentum crash is likely. We estimate CVaR based on the simple Historical Simulation and the more advanced EWMA-FHS model combined with the SRTR rule (see Rickenberg (2020a) for more details on these methods).

3.7 Empirical Results

3.7.1 Data

We now examine the performance of the different weighting schemes applied to the industry momentum strategy. For our main results, we use daily and monthly returns of 30 equally weighted US industries obtained from the website of Kenneth French. We use equally weighted industries, since Grundy and Martin (2001, Table 4) and Moskowitz and Grinblatt (1999, Footnote 12) find better results for the momentum strategy using equally weighted industries instead of value-weighted industries. In Appendix B.4, we show additional results for value-weighted industries and confirm the finding of Grundy and Martin (2001) that industry momentum based on equally weighted industries is more profitable. To determine winners and losers, we rank industries based on their past performance between months $t-12$ and $t-1$ and we define winners and losers as the best and worst $p = 30\%$ performers. Appendix B shows additional results for other ranking periods, other US industry data sets, other cut-off points as well as Global and European industry portfolios. The reason for this robustness check is that several studies show that the performance of the industry momentum strategy can be quite different for different data sets, ranking periods and cut-off points. We further show in Appendix B additional results for other portfolio based momentum strategies using investment styles and country indices. Lewellen (2002) and Novy-Marx (2012) apply the momentum strategy to investment style portfolios. We show results for 25 and 100 double sorted portfolios based on size and value using US, European and International stocks (Fama and French, 1993, 2012). Further, we use several style portfolios based on profitability and investment (Fama and French, 2016). Generally, style portfolios are frequently used in portfolio allocation studies (DeMiguel et al., 2009a,b, Kan and

Zhou, 2007, Kirby and Ostdiek, 2012, Zakamulin, 2015).[102] Additionally, we show results for our momentum strategies applied to 25 and 61 country indices. Novy-Marx (2012), Asness et al. (2013), Richards (1997), Bhojraj and Swaminathan (2006) and Chan et al. (2000) show that momentum strategies also work for different country equity indices. Country indices are also frequently used in studies on portfolio allocation and asset pricing (Asness et al., 2020, Atilgan et al., 2019, DeMiguel et al., 2009b, Garlappi et al., 2006, Kirby and Ostdiek, 2012).[103]

The next sections show our main results for the 30 equally weighted US Industry portfolios using the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. Our data set ranges from November 1930 to December 2018. Section 3.7.2 starts by examining results of the inverse risk and rank weighting schemes using a single risk measure. Results for the strategies that switch between a univariate and systematic risk measure are shown in Section 3.7.3. Finally, in Section 3.7.4 we show results for the strategies that simultaneously manage the momentum portfolio's constituents' individual risk and the momentum portfolio's portfolio risk. Additional performance results are shown in Appendix B.

3.7.2 Tail Risk Weighted Momentum Strategies: Single Measures

In this section, we show results for the momentum strategies that are weighted by a single risk measure as shown in Section 3.2.4, Section 3.3 and Section 3.4. We compare these risk weighted momentum strategies to the strategy where industries in the winners and losers portfolios are equally weighted. Table I shows results for the strategies using the inverse risk weighting of Equation (3.3.1). This table shows that the inverse volatility weighting reduces the volatility of the momentum portfolio, but also produces a lower return. In contrast, the (systematic) tail risk weighted portfolios produce higher returns without producing higher levels of volatility. This can also be seen by the higher Sharpe Ratio of most (systematic) tail risk weighted strategies. To assess if the increase in the Sharpe Ratio is statistically significant, we use the testing

[102] For example, Kan and Zhou (2007, p. 646) state: "Because Fama and French's (1993) 25 portfolios, formed based on size- and book-to-market ratio, are the standard test assets in recent empirical asset pricing studies, we assume that the investor invests in these 25 portfolios."

[103] We obtain US industries and style portfolios from Kenneth French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). International and European industry data as well as country indices are obtained from Datastream.

procedure of Jobson and Korkie (1981).[104] The inverse volatility weighted momentum strategy produces a slightly higher Sharpe Ratio than the equally weighted strategy which is, however, not statistically significant. This result is in line with Clare et al. (2014) who also find only minor improvements of the volatility weighted momentum strategy. In contrast, the tail risk weighted strategies using univariate risk measures, except for the kurtosis and DuSkew based strategies, all produce higher Sharpe Ratios, where five strategies produce statistically higher Sharpe Ratios. As mentioned in earlier sections, the finding that the kurtosis based strategy does not enhance the risk-return profile is not surprising, since kurtosis measures risk symmetrically and is a bad measure to weight assets of a portfolio consisting of long and short positions. Best results among the univariate risk measures are found for the skewness and the RSJ measure that measure a distribution's asymmetry and incorporate information on both tails.

The systematic tail risk weighted strategies, except for the coskewness based strategy, also produce higher Sharpe Ratios. We show in Appendix B.1 that one possibility to improve the performance of the coskewness based weighting is to condition on negative returns of the momentum portfolio when coskewness risk is measured. We call this measure the *Downside Coskewness*, which is similar to the LPM-beta of order $k = 3$. The remaining systematic risk weighted strategies produce higher Sharpe Ratios, where most increases are also statistically significant. The increase of the MES managed strategy is only significant at a test level of 10%, i.e. $z_{JK} > 1.64$. In particular, Sharpe Ratios of the systematic risk weighted strategies are slightly higher than the Sharpe Ratios of the univariate risk weighted strategies. Thus, measuring an asset's comovement with the momentum portfolio and downweighting assets that strongly co-crash with the momentum portfolio is an appealing portfolio allocation approach. This finding is also interesting for practitioners since most portfolio allocation methods used in practice focus on univariate risk measures, where especially volatility is frequently used. Thus, practitioners should also pay attention to systematic risk based weightings as examined by Asness et al. (2014), Asness et al. (2020) and Frazzini and Pedersen (2014). Asness et al. (2020)

[104] The Jobson and Korkie (1981) approach is used to test for a significantly higher Sharpe Ratio and has been frequently applied in the literature (Blitz and Van Vliet, 2007, DeMiguel et al., 2009b, Jondeau et al., 2019, Zakamulin, 2015, 2017). Since the original test of Jobson and Korkie (1981) contains a small error, we use a modified version that corrects for this error (see Cederburg et al. (2020, Footnote 8), DeMiguel et al. (2009b, p. 1928) and Jondeau et al. (2019, p. 41) for a short summary of this testing procedure).

Table I. Performance Results: Inverse Risk Weighting

This table shows performance results of the equally and inverse risk weighted industry momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. Return and Volatility correspond to the annualized return and volatility, respectively. Skew and Kurt denote the realized skewness and kurtosis. SR stands for the annualized Sharpe Ratio. z_{JK} denotes the test statistic of the Jobson and Korkie (1981) test. MDD stands for the maximum drawdown. Calmar denotes the ratio of the annualized return and the maximum drawdown. Min and Max stand for the minimum and maximum monthly return, respectively. Return, Volatility, MDD, Min and Max are given in percent. Bold numbers of z_{JK} mark strategies that produce a statistically higher Sharpe Ratio than the equally weighted strategy for a test level of 5%, i.e. $z_{JK} > 1.96$.

Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	9.17	12.11	-0.904	11.364	0.758	-	48.48	0.189	-25.96	18.61
RV	8.91	11.33	-1.065	11.484	0.787	1.22	47.60	0.187	-23.75	15.94
LPM0	9.32	12.20	-0.845	11.026	0.764	**1.96**	48.30	0.193	-24.78	19.34
LPM1	9.18	12.10	-0.902	11.340	0.758	0.99	48.45	0.189	-25.88	18.61
LPM2	9.17	12.11	-0.904	11.363	0.758	0.21	48.48	0.189	-25.95	18.61
Skew	10.17	12.33	-0.369	9.743	0.825	**2.64**	37.84	0.269	-27.37	19.12
Kurt	8.45	11.53	-1.031	10.645	0.733	-0.47	50.47	0.167	-24.45	13.25
DuVol	9.62	12.29	-0.463	9.992	0.783	0.90	43.92	0.219	-25.32	18.72
DuSkew	9.31	12.91	-0.308	9.539	0.721	-0.77	44.66	0.208	-24.80	19.63
SJ	9.19	12.10	-0.895	11.328	0.759	1.39	48.32	0.190	-25.81	18.62
RSJ	10.69	12.32	-0.626	9.551	0.868	**4.84**	41.56	0.257	-25.72	18.42
VaR HS	9.26	11.42	-0.917	11.309	0.811	**2.21**	48.61	0.190	-24.13	17.43
CVaR HS	9.16	11.33	-0.941	11.146	0.808	**2.04**	48.24	0.190	-23.14	16.75
Rachev HS	9.40	12.11	-0.639	10.128	0.776	1.30	46.91	0.200	-24.70	18.52
Corr	9.36	11.35	-0.977	11.504	0.825	**4.86**	46.97	0.199	-25.09	17.20
Down Corr	9.27	11.56	-1.029	12.501	0.801	**3.39**	49.87	0.186	-27.38	17.84
Beta	9.55	10.13	-0.950	10.780	0.943	**4.29**	41.29	0.231	-20.32	15.53
Down Beta	9.55	10.60	-0.964	12.627	0.901	**2.82**	47.16	0.203	-26.10	15.21
CoSkew	9.29	12.33	-0.645	10.710	0.753	-0.20	50.53	0.184	-25.95	19.56
CoKurt	9.68	10.92	-1.298	14.356	0.887	**2.16**	51.56	0.188	-28.59	13.17
LPM Beta	10.00	10.16	-0.606	9.423	0.984	**4.47**	42.01	0.238	-18.21	16.14
HTCR Beta	9.32	10.29	-0.938	10.772	0.906	**3.57**	41.44	0.225	-20.46	14.59
Tail Beta	9.57	10.54	-0.715	10.119	0.907	**3.44**	41.97	0.228	-19.19	16.56
Tail Sens	9.23	11.77	-0.936	11.480	0.784	**3.85**	47.99	0.192	-25.54	17.98
Tail Risk	9.52	10.56	-0.815	10.356	0.901	**3.84**	43.90	0.217	-19.38	16.46
MES	9.18	12.08	-0.899	11.322	0.760	1.95	48.37	0.190	-25.82	18.61

also find that systematic risk based trading strategies produce higher returns than strategies that are based on (idiosyncratic) risk.

The inverse risk weighting used in Table I relies on point estimates of the industries' risk. Since we focus on non-parametric estimates, the inverse risk weighting is prone to estimation risk, and hence portfolio weights could potentially be influenced by estimation errors, which lead to highly variable portfolio weights and needless transaction costs. Therefore, we next examine results of the rank weighting that is less sensitive to estimation errors and should produce more robust portfolio weights. Results of the rank weighted industry momentum strategies are shown in Table II. As for the inverse risk weighting, the volatility weighted strategy produces a higher Sharpe Ratio than the equally weighted strategy, but this increase is not statistically significant. In contrast, most of the (systematic) tail risk weighted strategies exhibit an enhanced risk-return profile. Among the univariate risk measures, we again find the best results for the skewness and RSJ measures. Weightings based on these risk measures particularly reduce left tail risk while simultaneously a higher return is obtained. Further, weightings based on systematic risk measures produce the most convincing risk-return profile. All systematic risk based weightings, except for the coskewness based weighting, produce higher Sharpe Ratios. The increase in the Sharpe Ratio is statistically significant for all systematic risk measures, except for the downside correlation based weighting. Weightings based on systematic risk measures are again successful in reducing left tail risk while simultaneously increasing return potential. Interestingly, we find that the highest Sharpe Ratio is obtained for the MES weighting, which is the model that did not produce a statistically higher Sharpe Ratio when used for the inverse risk weighting. Thus, although the inverse risk and rank weightings produce an enhanced risk-return profile compared to the equally weighted strategy, both weighting schemes can sometimes produce quite different results.

In total, results of Table I and Table II demonstrate the importance of incorporating the assets' (systematic) tail risk and that symmetric risk measures, like volatility and kurtosis, do not work well for long-short strategies. In particular, weightings based on skewness, RSJ or systematic risk measures successfully reduce momentum's high negative skewness while simultaneously the strategies' return is increased. This finding is striking since most "investors are willing to give up some of the right tail to reduce the left tail" (Harvey et al., 2018, p. 15). Our weighting schemes do not only reduce the mass in the left tail, they also *increase* the mass in the right tail, i.e. these strategies are highly valuable for most investors. In contrast, the volatility weighted strategy increases left tail risk while simultaneously a lower return is obtained. This finding again highlights the disadvantage of using volatility as risk measure and that using volatility as a portfolio risk management tool does not necessarily manage a portfolio's loss potential, especially for long-short portfolios.

Since the rank weighting is more successful in reducing left tail risk and since these strategies are also less prone to estimation risk, the rank weighted strategies are more convincing and

Table II. Performance Results: Rank Weighting

This table shows performance results of the equally and rank weighted industry momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table I.

Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	9.17	12.11	-0.904	11.364	0.758	-	48.48	0.189	-25.96	18.61
RV	8.44	10.87	-1.192	11.828	0.776	0.48	45.89	0.184	-21.59	13.57
LPM0	10.58	13.62	-0.563	10.720	0.777	0.41	53.79	0.197	-30.24	22.73
LPM1	9.57	11.21	-0.750	10.769	0.853	**2.13**	47.34	0.202	-22.92	19.97
LPM2	8.97	10.97	-0.982	10.902	0.818	1.43	47.22	0.190	-22.04	15.90
Skew	10.80	12.65	-0.263	9.869	0.854	**2.31**	40.19	0.269	-29.32	20.11
Kurt	7.90	11.16	-1.051	10.707	0.708	-1.17	53.06	0.149	-22.93	13.49
DuVol	10.09	12.32	-0.386	9.187	0.819	1.47	37.96	0.266	-26.61	18.74
DuSkew	9.63	12.38	-0.481	9.811	0.778	0.49	42.69	0.226	-26.41	18.00
SJ	11.70	13.03	-0.436	9.894	0.898	**3.23**	37.95	0.308	-26.75	21.00
RSJ	12.02	12.83	-0.604	9.920	0.937	**4.08**	37.09	0.324	-27.89	18.04
VaR HS	9.18	11.04	-0.771	10.911	0.832	1.73	46.87	0.196	-21.91	17.97
CVaR HS	8.92	11.02	-1.138	12.414	0.810	1.25	49.26	0.181	-22.48	15.27
Rachev HS	9.88	12.41	-0.247	8.577	0.796	0.89	42.56	0.232	-25.25	18.18
Corr	9.33	9.88	-0.905	11.482	0.944	**3.53**	40.20	0.232	-20.22	16.07
Down Corr	9.15	10.94	-1.953	24.802	0.836	1.58	56.15	0.163	-36.45	16.00
Beta	9.46	9.74	-0.851	10.789	0.971	**4.11**	40.28	0.235	-19.80	14.10
Down Beta	9.38	10.40	-1.322	16.785	0.902	**2.91**	48.02	0.195	-29.48	15.94
CoSkew	8.91	12.35	-0.785	14.159	0.721	-0.84	56.03	0.159	-32.18	22.98
CoKurt	9.58	10.60	-1.685	18.225	0.904	**2.95**	50.84	0.188	-30.88	12.43
LPM Beta	9.84	9.92	-0.690	10.407	0.991	**4.52**	42.12	0.233	-19.75	15.81
HTCR Beta	9.31	9.80	-0.790	9.681	0.949	**3.90**	38.97	0.239	-17.45	14.78
Tail Beta	9.65	10.65	-0.630	10.203	0.906	**3.19**	44.28	0.218	-19.40	19.12
Tail Sens	9.62	10.10	-0.704	9.093	0.952	**4.02**	37.38	0.257	-19.81	14.00
Tail Risk	9.47	10.39	-0.626	10.394	0.912	**3.33**	43.79	0.216	-20.12	16.89
MES	10.06	9.95	-0.457	9.048	1.011	**4.83**	38.95	0.258	-16.58	17.20

seem more relevant for practitioners. For that reason, we further concentrate on this weighting scheme and only present results for the rank weighted strategies. Results for the inverse risk weighted strategies were quite similar, albeit slightly worse. As a robustness check of the results presented in this section, we further show in Appendix B.1 additional results for the rank weighted strategy using other estimation windows and cut-off points to determine extreme returns when (systematic) tail risk is estimated. Results in the appendix show that the risk weighted momentum strategies are also beneficial when estimation lengths between one month and 60 months of daily data are used. Further, cut-off points of $10\%, 20\%$ and 30% can be used when systematic risk is estimated. We find that our simple weighting approach is hardly influenced by the choice of the estimation window and cut-off point. In particular, our approach works well for all reasonable choices, but works best for short- and medium term estimation windows as well as cut-off points that emphasize observations in the far tail. Nevertheless, in the next sections we will only show results for the estimation windows and cut-off points used in this section.

3.7.3 Switching Between Univariate and Systematic Risk Measures

Results in Tables I and II highlight that the (systematic) tail risk weighted portfolios produce an enhanced risk-return profile compared to the equally and volatility weighted momentum strategies, where especially the systematic risk weighted strategies perform well by reducing momentum's risk while simultaneously a higher return is achieved. As mentioned in Section 3.5, the systematic risk weighted strategies should perform especially well in down-periods, but should not adequately capture momentum's upside potential in up-periods. In contrast, the weightings based on univariate risk measures should be less successful in mitigating extreme losses in extreme down-periods, but should be more successful in capturing the upside potential. For that reason, we next examine the performance of the switching strategies presented in Section 3.5 that weight industries based on a univariate risk measure when an up-period is expected and weight industries based on systematic tail risk measures in expected down-periods. To better capture the return potential in up-periods, risk measures that quantify risk based on the whole distribution or at least on both tails should be used. For that reason, we only show results for the switching strategies that use the skewness or RSJ measure as univariate risk measure. Furthermore, these risk measures work particularly well as shown in Tables I and II.

Table III shows results for the strategies that switch between the skewness or RSJ measure and the systematic risk measures, where we use the equally weighted momentum portfolio's volatility to determine if a down- or up-period is expected.[105] Panel A shows results for the strategies that switch between skewness and the systematic tail risk measures, whereas Panel B shows results for the strategies that use the RSJ measure as univariate risk measure. Compared to the results of Table II, switching between univariate and systematic risk measures typically produces higher returns and higher Sharpe Ratios than the strategies that rely on either the univariate *or* systematic risk based weightings. Furthermore, by switching between the two risk measures, left tail risk can further be reduced. Thus, using univariate risk measures in up-

[105] We show additional results for several other crash indicators in Appendix B.8. We find that the switching strategy also works well for several other definitions of δ_t. This finding is in line with Rickenberg (2020a) who also finds that switching strategies based on several momentum crash indicators produce quite similar results.

Table III. Performance Results: Switching Strategies
This table shows performance results of the strategies that switch between a univariate and systematic risk based weighting scheme using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. Panel A shows results for the strategies that use skewness as univariate risk measure, whereas Panel B shows results for the strategies that use the RSJ measure as univariate risk measure. The description of the columns is given in Table I.

	Panel A: Switching Based on Skewness Measure									
Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	9.17	12.11	-0.904	11.364	0.758	-	48.48	0.189	-25.96	18.61
RV	8.44	10.87	-1.192	11.828	0.776	0.48	45.89	0.184	-21.59	13.57
Skew/Corr	10.85	11.19	-0.493	8.685	0.969	**4.16**	40.28	0.269	-20.22	16.07
Skew/Down Corr	10.34	11.94	-1.427	18.755	0.866	**2.26**	56.61	0.183	-36.45	16.00
Skew/Beta	10.92	11.09	-0.442	8.175	0.985	**4.57**	39.56	0.276	-19.80	14.80
Skew/Down Beta	10.75	11.49	-0.885	12.690	0.935	**3.75**	47.82	0.225	-29.48	15.94
Skew/CoSkew	10.00	12.51	-0.803	13.624	0.799	0.94	55.96	0.179	-32.18	22.98
Skew/CoKurt	10.48	11.69	-1.155	13.701	0.897	**2.96**	51.39	0.204	-30.88	14.80
Skew/LPM Beta	11.11	11.12	-0.379	8.214	0.999	**4.89**	40.63	0.273	-19.75	15.81
Skew/HTCR Beta	10.74	11.05	-0.441	7.648	0.972	**4.50**	39.05	0.275	-17.45	14.80
Skew/Tail Beta	10.80	11.62	-0.405	8.469	0.929	**3.69**	42.61	0.253	-19.40	19.12
Skew/Tail Sens	10.91	11.16	-0.459	7.463	0.978	**4.63**	37.37	0.292	-19.81	14.80
Skew/Tail Risk	10.75	11.42	-0.392	8.507	0.942	**3.93**	41.16	0.261	-20.12	16.89
Skew/MES	11.12	11.08	-0.226	7.400	1.003	**4.91**	37.69	0.295	-16.58	17.20
	Panel B: Switching Based on RSJ Measure									
Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	9.17	12.11	-0.904	11.364	0.758	-	48.48	0.189	-25.96	18.61
RV	8.44	10.87	-1.192	11.828	0.776	0.48	45.89	0.184	-21.59	13.57
RSJ/Corr	11.91	11.12	-0.495	8.650	1.072	**6.10**	38.26	0.311	-20.22	16.07
RSJ/Down Corr	11.40	11.87	-1.453	19.070	0.960	**4.18**	55.09	0.207	-36.45	16.00
RSJ/Beta	11.99	11.01	-0.441	8.114	1.089	**6.58**	38.61	0.310	-19.80	14.57
RSJ/Down Beta	11.82	11.42	-0.898	12.811	1.034	**5.78**	47.84	0.247	-29.48	15.94
RSJ/CoSkew	11.05	12.45	-0.823	13.770	0.888	**2.98**	54.31	0.204	-32.18	22.98
RSJ/CoKurt	11.55	11.62	-1.174	13.874	0.994	**4.96**	50.27	0.230	-30.88	14.57
RSJ/LPM Beta	12.18	11.04	-0.377	8.152	1.103	**6.92**	39.26	0.310	-19.75	15.81
RSJ/HTCR Beta	11.81	10.98	-0.439	7.568	1.076	**6.59**	38.88	0.304	-17.45	14.78
RSJ/Tail Beta	11.87	11.55	-0.411	8.446	1.027	**5.74**	42.31	0.280	-19.40	19.12
RSJ/Tail Sens	11.98	11.09	-0.459	7.390	1.080	**6.72**	36.97	0.324	-19.81	14.57
RSJ/Tail Risk	11.82	11.35	-0.395	8.471	1.042	**6.00**	40.77	0.290	-20.12	16.89
RSJ/MES	12.19	11.01	-0.220	7.297	1.107	**6.92**	35.63	0.342	-16.58	17.20

periods and systematic risk measures in down-periods successfully captures the upside potential while simultaneously left tail risk is reduced. In line with Table II, we find slightly better results for the switching strategies using the RSJ measure compared to the skewness based strategies. All switching strategies that use the RSJ measure exhibit a significantly higher Sharpe Ratio and the values of the Jobson and Korkie (1981) test statistic are significantly higher than the values of the strategies that do not switch between univariate and systematic risk based weightings. For example, the values of the Jobson and Korkie (1981) test statistic for the strategies using the RSJ or MES measure in every month are 4.08 and 4.83, respectively. In contrast, the strategy that switches between these two measures produces a higher Sharpe Ratio with a Jobson and

Korkie (1981) value of 6.92. Interestingly, even the strategy that switches to the coskewness based strategy produces a significantly higher Sharpe Ratio with a Jobson and Korkie (1981) value of 2.98. In contrast, using the coskewness based weighting in every month produces a lower Sharpe Ratio with a Jobson and Korkie (1981) value of -0.84.

To visualize the differences between different weighting schemes, we show in Figure I the cumulative return of three momentum strategies using equal weights, volatility based weights and weights based on our switching approach. We follow Daniel and Moskowitz (2016), Jacobs et al. (2015) and Bollerslev et al. (2019, p. 15) and show the long-term performance for the portfolios that initially invest $1$$ the risk-free rate combined with the zero-investment momentum portfolio. For a better comparison, we rescale all strategies to the same level of volatility. Figure I shows that the equally and volatility weighted strategies perform nearly identical. The volatility weighted strategy produces are slightly higher terminal wealth, which is mainly driven by the middle part of the sample. The equally and volatility weighted strategies produce a terminal wealth of $1{,}690{,}351$$ and $2{,}085{,}431$$, respectively. In contrast, the strategy that is based on the switching approach clearly outperforms the other strategies. This outperformance is achieved by mitigating crash periods and simultaneously capturing the upside potential. The switching approach produces a terminal wealth of $323{,}124{,}537$$, which is about 191 times the terminal wealth of the volatility managed momentum portfolio. Thus, a long-term investor significantly benefits from timing short-term tail risk. Moreira and Muir (2019) find that a similar observation holds for long-only investors who time portfolio volatility. We will come back to the benefits of portfolio volatility timing in the next section.

To assess how our switching strategies perform in good and bad momentum regimes, we examine in Table IV the strategies' performance in the months with the most extreme positive and negative momentum returns. Table IV shows the five months where the equally weighted momentum portfolio exhibits the lowest returns as well as the five months with the highest returns. For each of these months, we additionally show returns of the volatility weighted strategy and the switching strategies. Panel A shows results for the skewness based strategies and Panel B shows results for the RSJ based strategies. Results in Table IV demonstrate that volatility

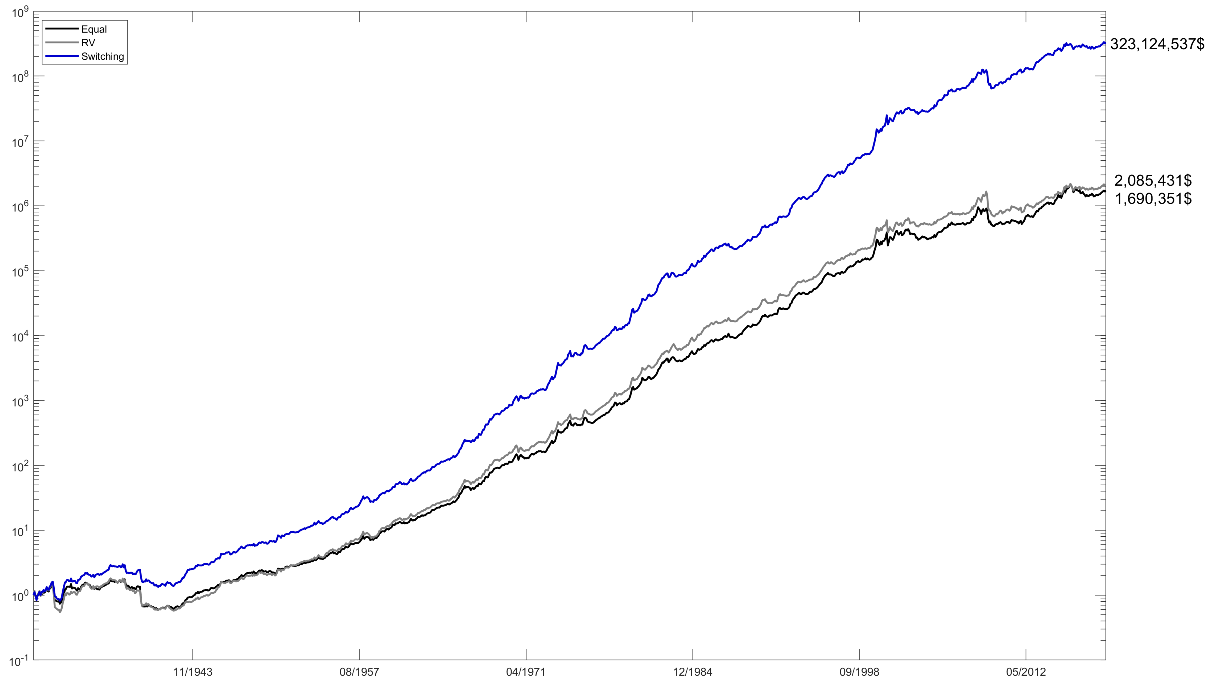

Figure I. Cumulative Return. This figure plots the cumulative return of the equally weighted, volatility weighted or switching based weighted momentum strategy combined with a one dollar investment in the risk-free rate. As in Daniel and Moskowitz (2016), we rescale all strategies to an annualized volatility of 19%.

weighting reduces the extremely negative returns, but produces higher losses in the fourth and fifth worst months. Further, the volatility weighted strategy produces significantly lower returns when the equally weighted strategy produces very high returns. In contrast, the switching strategies significantly reduce the losses on the worst months, but also successfully capture the upside potential in the months when the momentum portfolio performs well. Interestingly, we find that both the skewness and RSJ based strategies perform almost equally well in extreme periods. In the months with an extremely negative return both strategies perform the same, i.e. the five worst months are all captured by our momentum crash indicator. This again shows that momentum's own volatility is a good predictor of the momentum crash probability as also shown by Barroso and Santa-Clara (2015), Barroso and Maio (2019) and Rickenberg (2020a) for the individual stock momentum strategy and by Du Plessis and Hallerbach (2017), Grobys et al. (2018) and Grobys and Kolari (2020) for the industry momentum strategy. During the months with an extremely high return, all switching strategies also produce quite similar returns. Thus, the outperformance of the RSJ based strategies over the skewness based strategies is mainly influenced by the months with a mediocre performance.

Table IV. Sorted Returns
This table shows monthly returns of the equally weighted momentum portfolio, the volatility managed portfolio and the switching strategies on months when the momentum portfolio exhibits the five lowest and five highest returns. All entries are given in percent.

Panel A: Skewness	Low Returns					High Returns				
Equal	-25.96	-24.01	-20.34	-15.59	-12.76	11.25	11.66	12.35	14.57	18.61
RV	-18.02	-20.43	-17.23	-20.26	-21.59	8.43	7.80	8.39	12.00	13.57
Skew/Corr	-20.22	-15.83	-10.50	-19.16	-14.70	12.35	14.80	7.20	11.08	12.26
Skew/Down Corr	-36.45	-16.47	-10.70	-16.22	-16.98	12.35	14.80	7.59	11.81	16.00
Skew/Beta	-19.80	-16.68	-11.16	-16.58	-14.71	12.35	14.80	6.81	10.68	14.10
Skew/Down Beta	-29.48	-16.98	-13.05	-15.82	-15.04	12.35	14.80	7.05	9.84	15.94
Skew/CoSkew	-32.18	-22.13	-13.79	-10.02	-13.94	12.35	14.80	12.26	15.83	22.98
Skew/CoKurt	-30.88	-16.80	-11.66	-16.55	-18.75	12.35	14.80	8.17	12.43	11.17
Skew/LPM Beta	-19.75	-16.85	-11.03	-16.18	-14.50	12.35	14.80	6.87	10.28	15.81
Skew/HTCR Beta	-17.45	-16.55	-15.77	-15.85	-12.28	12.35	14.80	6.89	9.47	14.78
Skew/Tail Beta	-15.58	-19.08	-14.30	-19.40	-14.57	12.35	14.80	8.02	10.10	15.94
Skew/Tail Sens	-15.23	-15.57	-13.69	-19.81	-12.51	12.35	14.80	7.27	9.49	14.00
Skew/Tail Risk	-15.70	-18.29	-11.44	-20.12	-15.97	12.35	14.80	8.02	10.25	16.89
Skew/MES	-14.18	-16.42	-11.00	-16.58	-15.01	12.35	14.80	6.87	10.29	17.20
Panel B: RSJ	**Low Returns**					**High Returns**				
Equal	-25.96	-24.01	-20.34	-15.59	-12.76	11.25	11.66	12.35	14.57	18.61
RV	-18.02	-20.43	-17.23	-20.26	-21.59	8.43	7.80	8.39	12.00	13.57
RSJ/Corr	-20.22	-15.83	-10.50	-19.16	-14.70	11.80	14.57	7.20	11.08	12.26
RSJ/Down Corr	-36.45	-16.47	-10.70	-16.22	-16.98	11.80	14.57	7.59	11.81	16.00
RSJ/Beta	-19.80	-16.68	-11.16	-16.58	-14.71	11.80	14.57	6.81	10.68	14.10
RSJ/Down Beta	-29.48	-16.98	-13.05	-15.82	-15.04	11.80	14.57	7.05	9.84	15.94
RSJ/CoSkew	-32.18	-22.13	-13.79	-10.02	-13.94	11.80	14.57	12.26	15.83	22.98
RSJ/CoKurt	-30.88	-16.80	-11.66	-16.55	-18.75	11.80	14.57	8.17	12.43	11.17
RSJ/LPM Beta	-19.75	-16.85	-11.03	-16.18	-14.50	11.80	14.57	6.87	10.28	15.81
RSJ/HTCR Beta	-17.45	-16.55	-15.77	-15.85	-12.28	11.80	14.57	6.89	9.47	14.78
RSJ/Tail Beta	-15.58	-19.08	-14.30	-19.40	-14.57	11.80	14.57	8.02	10.10	15.94
RSJ/Tail Sens	-15.23	-15.57	-13.69	-19.81	-12.51	11.80	14.57	7.27	9.49	14.00
RSJ/Tail Risk	-15.70	-18.29	-11.44	-20.12	-15.97	11.80	14.57	8.02	10.25	16.89
RSJ/MES	-14.18	-16.42	-11.00	-16.58	-15.01	11.80	14.57	6.87	10.29	17.20

To further assess how the different weighting schemes perform in different market environments, Table V shows the performance in the two 15 years periods with the best and worst performance of the equally weighted momentum strategy. The 15 years period with the best performance ranges from November 1967 to October 1982, whereas the 15 years period with the worst performance ranges from June 1932 to May 1947. In the best 15 years, the momentum investor would have earned an annualized return of 15.54%, whereas the annualized return in the worst 15 years would only be 1.14% per year.[106] In line with Table II, the performance in these periods cannot be significantly improved by volatility weighting. Interestingly, although

[106]This result again demonstrates that industry momentum is less risky than stock momentum. Even in the worst case, an industry momentum investor achieves a positive return after 15 years, whereas Barroso and Santa-Clara (2015) show that a recovery from a momentum crash can last up to 31 years for the individual stock momentum strategy.

volatility weighting typically performs well in down-periods, the volatility weighted strategy even underperforms the equally weighted momentum portfolio in the worst 15 years. A possible explanation for this finding could be that the losers portfolio performs particularly bad in the down-period. Thus, volatility weighting applied to the losers portfolio should dampen the bad performance of the losers, which is disadvantageous for a momentum investor. We come back to this point in Table VII. In contrast, the switching strategies perform well in both regimes and clearly outperform the equally weighted strategy in up- and down-markets. The Jobson and Korkie (1981) values are typically higher than 1.96 in both periods. In particular, the good performance of the switching strategies is driven by mitigating drawdowns and simultaneously enhancing the performance in up-periods. Interestingly, we find that the beta and correlation based weightings are superior to the weightings based on downside correlation and downside beta when the 15 worst years are regarded. This is somewhat surprising since the downside risk measures condition on bad momentum states and should work well in down-periods. However, this result is in line with Levi and Welch (2019) who find that the usual beta is a better hedging instrument during crash periods than the downside beta.

Similar to Asness et al. (2013, Table VII), Barroso and Santa-Clara (2015, Table 6), Jegadeesh and Titman (1993, Table VI) and Jegadeesh and Titman (2001), we next examine the performance of the momentum strategies in different sub-samples.[107] We only show results for the RSJ based weightings, but results for the skewness based strategies are quite similar. Examining the profitability in different sub-samples is important since the profitability of long-short strategies can be quite different in different sample periods (see Grobys and Kolari (2020, p. 111) and references therein). Table VI shows results for the different momentum strategies where we split the whole sample into three equally spaced sub-samples. Splitting the whole sample into three sub-samples is appealing since the first and last sub-sample contain a momentum crash, whereas the second sub-sample is characterized by a calm and mainly uptrending period. This further demonstrates how the different weighting schemes perform in different market environments. Moreover, splitting the whole sample into three smaller samples is more

[107] Ang et al. (2006b, Table XI) also examine the low volatility anomaly in different sub-samples and find that the low volatility puzzle holds in bull and bear markets, recessions and expansions as well as volatile and calm markets.

Table V. Best and Worst 15 Years

This table shows performance results of the equally weighted momentum portfolio, the volatility managed portfolio and the switching strategies in the two 15 years periods where the equally weighted momentum strategy exhibits the best and worst performance. The description of the columns is given in Table I.

	Best 15 Years: 01.11.1967 - 31.10.1982					Worst 15 Years: 01.06.1932 - 31.05.1947				
Panel A: Skew	Return	Volatility	Skew	SR	z_{JK}	Return	Volatility	Skew	SR	z_{JK}
Equal	15.54	11.32	-0.015	1.373	-	1.14	14.29	-2.074	0.080	-
RV	14.07	9.84	-0.228	1.429	0.71	1.03	13.69	-1.782	0.076	-0.04
Skew/Corr	18.26	11.73	0.106	1.557	1.92	5.27	13.24	-0.795	0.398	**2.47**
Skew/Down Corr	18.31	11.81	0.059	1.551	1.92	2.39	15.83	-3.101	0.151	0.63
Skew/Beta	18.54	11.72	0.146	1.582	**2.22**	4.65	13.00	-0.966	0.358	**2.35**
Skew/Down Beta	18.80	11.83	0.131	1.588	**2.33**	3.27	14.40	-2.224	0.227	1.44
Skew/CoSkew	18.50	12.22	0.064	1.514	1.60	0.23	15.81	-2.125	0.015	-0.71
Skew/CoKurt	18.41	11.73	0.083	1.570	**2.09**	3.95	15.04	-2.491	0.263	1.67
Skew/LPM Beta	18.74	11.81	0.154	1.587	**2.35**	4.93	12.94	-0.937	0.381	**2.61**
Skew/HTCR Beta	18.53	11.87	0.150	1.561	**2.05**	4.01	12.83	-1.141	0.312	**2.37**
Skew/Tail Beta	18.42	12.15	0.038	1.516	1.64	4.37	13.66	-0.368	0.320	**1.96**
Skew/Tail Sens	17.83	11.92	0.082	1.496	1.34	4.59	12.39	-0.872	0.370	**2.76**
Skew/Tail Risk	18.29	11.90	0.093	1.537	1.85	4.24	13.01	-0.465	0.326	**2.02**
Skew/MES	18.78	11.83	0.145	1.588	**2.41**	4.69	12.35	-0.604	0.379	**2.38**
	Best 15 Years: 01.11.1967 - 31.10.1982					Worst 15 Years: 01.06.1932 - 31.05.1947				
Panel B: RSJ	Return	Volatility	Skew	SR	z_{JK}	Return	Volatility	Skew	SR	z_{JK}
Equal	15.54	11.32	-0.015	1.373	-	1.14	14.29	-2.074	0.080	-
RV	14.07	9.84	-0.228	1.429	0.71	1.03	13.69	-1.782	0.076	-0.04
RSJ/Corr	19.15	11.49	0.066	1.667	**3.12**	5.11	13.07	-0.843	0.391	**2.47**
RSJ/Down Corr	19.20	11.57	0.016	1.660	**3.16**	2.23	15.69	-3.196	0.142	0.56
RSJ/Beta	19.44	11.48	0.109	1.693	**3.46**	4.49	12.82	-1.025	0.350	**2.35**
RSJ/Down Beta	19.69	11.59	0.094	1.699	**3.59**	3.12	14.25	-2.312	0.219	1.39
RSJ/CoSkew	19.40	11.99	0.022	1.618	**2.87**	0.08	15.67	-2.197	0.005	-0.83
RSJ/CoKurt	19.30	11.49	0.042	1.680	**3.32**	3.79	14.89	-2.577	0.255	1.63
RSJ/LPM Beta	19.64	11.57	0.118	1.697	**3.63**	4.77	12.76	-0.994	0.374	**2.62**
RSJ/HTCR Beta	19.42	11.63	0.114	1.670	**3.30**	3.85	12.65	-1.210	0.304	**2.38**
RSJ/Tail Beta	19.31	11.92	-0.006	1.621	**2.94**	4.21	13.49	-0.397	0.312	1.93
RSJ/Tail Sens	18.72	11.68	0.040	1.602	**2.57**	4.43	12.21	-0.935	0.363	**2.80**
RSJ/Tail Risk	19.18	11.67	0.052	1.644	**3.15**	4.09	12.84	-0.504	0.318	**2.00**
RSJ/MES	19.67	11.59	0.109	1.698	**3.72**	4.53	12.17	-0.655	0.372	**2.38**

realistic since investors typically have short evaluation periods (Benartzi and Thaler, 1995). Table VI shows that the volatility weighted momentum portfolio does not significantly outperform the equally weighted momentum portfolio in any sub-period. In the third sub-sample, the volatility weighted portfolio does even underperform the equally weighted momentum strategy. Further, the volatility weighted strategy does not reduce momentum's left tail risk. In the second and third sub-sample, weighting assets by their volatility does even increase momentum's left tail risk and maximum drawdown. In contrast, the switching strategies generate higher Sharpe Ratios in all three sub-samples. This higher Sharpe Ratio is typically obtained by achieving higher returns with similar levels of volatility. The increase in the Sharpe Ratio is also statistically significant for most strategies and sub-samples. Further, the switching strategies typically reduce the portfolio's left tail risk by producing a higher skewness and lower drawdowns. In total, Table VI shows that our switching approach does not only outperform the other weighting schemes over the long run, but is also very robust in different sub-samples marked by either a crash period or a calm and uptrending period.

We have so far shown that the momentum strategies using the weighting schemes based on (systematic) tail risk measures outperform the equally and volatility weighted momentum strategies. We next assess if the outperformance of the switching strategies is driven by the long or short leg of the momentum portfolio. Several studies have shown that the profitability of the (equally weighted) momentum strategy is either driven by the long or short leg of the portfolio, where this finding strongly depends on the examined data set. For example, Hong et al. (2000) and Lesmond et al. (2004) find that the high returns of the individual stock based momentum strategy are mainly generated by shorting the losers portfolio. This holds especially when the least liquid loser stocks are shorted (Moskowitz and Grinblatt, 1999). Generally, the profits of equity based long-short anomalies are mainly driven by the short side (see Jang and Kang (2019) and references therein). In contrast, the profitability of portfolio based momentum strategies is mainly driven by the long side. For example, Moskowitz and Grinblatt (1999) find that the profitability of industry momentum is mainly driven by the winners portfolio. This observation is confirmed by Chan et al. (2000, Table 1) and Bhojraj and Swaminathan (2006) for the country momentum strategy. For that reason, we next examine the performance of the winners and losers portfolios separately. Obviously, the aim of a weighting scheme should be to improve the performance of the winners portfolio and to worsen the performance of the losers portfolio. Results for the winners and losers portfolios, both equally and risk weighted, are given in Table VII. Volatility weighting reduces the volatility of both portfolios without sacrificing returns. This reduction of volatility produces a higher Sharpe Ratio for the winners and losers portfolios. In particular, for the losers portfolio, volatility weighting reduces the portfolio's volatility *and* improves the portfolio's return. This leads to a high increase of the losers portfolio's Sharpe Ratio which is highly significant, given by a Jobson and Korkie (1981) value of 2.88. In contrast, the increase of the winners' Sharpe Ratio is not significant. In total, the benefits of buying the

Table VI. Performance Results in Different Sub-Samples

This table shows performance results of the equally weighted momentum portfolio, the volatility managed strategy and the switching strategies in three sub-samples. The description of the columns is given in Table I.

	Panel A: 01.11.1930 – 28.02.1960									
Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	5.24	12.36	-1.732	15.531	0.424	-	48.48	0.108	-25.96	14.57
RV	5.09	11.91	-1.526	12.316	0.428	0.06	45.89	0.111	-21.59	12.00
RSJ/Corr	7.58	11.49	-0.683	10.270	0.660	**2.78**	38.26	0.198	-20.22	16.07
RSJ/Down Corr	6.24	13.04	-2.938	30.741	0.479	0.69	55.09	0.113	-36.45	13.49
RSJ/Beta	7.58	11.32	-0.830	9.746	0.670	**3.13**	38.61	0.196	-19.80	12.88
RSJ/Down Beta	6.82	12.13	-1.983	19.982	0.562	1.91	47.84	0.143	-29.48	14.86
RSJ/CoSkew	5.12	13.11	-2.005	20.346	0.390	-0.51	54.31	0.094	-32.18	15.83
RSJ/CoKurt	6.84	12.67	-2.215	20.978	0.540	1.57	50.27	0.136	-30.88	12.43
RSJ/LPM Beta	7.72	11.24	-0.786	9.885	0.686	**3.38**	39.26	0.197	-19.75	14.79
RSJ/HTCR Beta	7.02	11.12	-0.985	8.709	0.631	**2.95**	38.88	0.180	-17.45	12.27
RSJ/Tail Beta	7.26	11.79	-0.461	8.867	0.615	**2.35**	42.31	0.172	-15.58	19.12
RSJ/Tail Sens	7.41	11.02	-0.733	7.020	0.672	**3.42**	36.97	0.200	-15.23	11.27
RSJ/Tail Risk	7.18	11.46	-0.490	8.331	0.627	**2.53**	40.77	0.176	-15.97	16.43
RSJ/MES	7.59	10.90	-0.545	7.404	0.696	**3.30**	35.63	0.213	-15.01	13.77
	Panel B: 01.03.1960 – 30.06.1989									
Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	12.78	9.97	-0.058	4.613	1.282	-	13.20	0.968	-9.73	11.25
RV	12.31	8.78	-0.200	4.610	1.403	1.71	13.38	0.920	-9.02	9.12
RSJ/Corr	15.31	10.22	0.101	4.798	1.498	**2.93**	13.88	1.103	-10.62	11.80
RSJ/Down Corr	15.37	10.27	0.064	4.841	1.497	**3.00**	14.14	1.087	-10.62	11.80
RSJ/Beta	15.39	10.21	0.134	4.749	1.508	**3.07**	13.90	1.107	-10.62	11.80
RSJ/Down Beta	15.50	10.29	0.126	4.763	1.506	**3.07**	14.25	1.087	-10.62	11.80
RSJ/CoSkew	15.51	10.57	0.064	4.694	1.467	**2.66**	15.80	0.982	-10.62	11.80
RSJ/CoKurt	15.27	10.24	0.089	4.754	1.492	**2.86**	14.41	1.060	-10.62	11.80
RSJ/LPM Beta	15.53	10.27	0.143	4.740	1.512	**3.17**	13.65	1.137	-10.62	11.80
RSJ/HTCR Beta	15.29	10.30	0.144	4.767	1.485	**2.80**	14.52	1.054	-10.62	11.80
RSJ/Tail Beta	15.50	10.46	0.051	4.972	1.483	**2.91**	12.97	1.195	-10.62	11.80
RSJ/Tail Sens	15.15	10.35	0.077	4.789	1.464	**2.58**	13.76	1.101	-10.62	11.80
RSJ/Tail Risk	15.49	10.36	0.080	4.761	1.496	**3.13**	13.81	1.122	-10.62	11.80
RSJ/MES	15.57	10.27	0.139	4.732	1.516	**3.27**	13.04	1.194	-10.62	11.80
	Panel C: 01.07.1989 – 31.12.2018									
Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	9.63	13.65	-0.547	9.007	0.705	-	33.86	0.284	-24.01	18.61
RV	8.02	11.59	-1.103	11.709	0.692	-0.11	37.97	0.211	-20.43	13.57
RSJ/Corr	12.99	11.53	-0.687	9.124	1.127	**4.32**	33.82	0.384	-19.16	14.57
RSJ/Down Corr	12.78	12.05	-0.387	7.684	1.060	**3.93**	31.43	0.407	-16.48	16.00
RSJ/Beta	13.13	11.40	-0.443	8.342	1.151	**4.57**	30.19	0.435	-16.68	14.57
RSJ/Down Beta	13.31	11.68	-0.318	8.079	1.140	**4.62**	30.01	0.443	-16.98	15.94
RSJ/CoSkew	12.80	13.36	-0.070	9.594	0.958	**2.99**	28.25	0.453	-22.13	22.98
RSJ/CoKurt	12.70	11.74	-0.600	7.648	1.081	**4.11**	32.52	0.391	-16.80	14.57
RSJ/LPM Beta	13.42	11.51	-0.341	8.417	1.167	**4.74**	29.80	0.451	-16.85	15.81
RSJ/HTCR Beta	13.28	11.39	-0.348	8.042	1.166	**4.78**	29.21	0.455	-16.55	14.78
RSJ/Tail Beta	12.99	12.26	-0.619	9.634	1.060	**4.18**	32.87	0.395	-19.40	15.94
RSJ/Tail Sens	13.53	11.78	-0.600	8.974	1.148	**4.68**	33.53	0.403	-19.81	14.57
RSJ/Tail Risk	12.93	12.07	-0.588	10.332	1.072	**4.18**	34.87	0.371	-20.12	16.89
RSJ/MES	13.56	11.73	-0.197	8.473	1.156	**4.68**	30.67	0.442	-16.58	17.20

enhanced winners is offset by shorting the enhanced losers. Thus, although volatility weighting produces an enhanced risk-return profile, this advantage does not translate into a better performance of long-short strategies. This finding is opposed to the finding of Clare et al. (2014)

for commodity momentum, where inverse volatility weighting worsens the performance of the losers. In contrast to the volatility weighting, the skewness based switching strategies significantly improve the performance of the winners portfolio, whereas the performance of the losers is only slightly improved. More strikingly, the RSJ based switching strategies significantly increase the winners' Sharpe Ratio, while the losers' Sharpe Ratio is decreased. Thus, an investor who is long the winners and short the losers strongly benefits from weighting the winners' and losers' constituents by their risk estimated with the RSJ based switching strategy. In total, results of Table VII show that the bad performance of the volatility managed strategy and the good performance of the switching strategies found in Table III is driven by the long and the short leg of the momentum strategy. Interestingly, although Table VII indicates that volatility weighting only slightly improves the performance of the winners, whereas the performance of the losers is significantly improved, we find in Table III that volatility weighting still slightly improves momentum's Sharpe Ratio. This shows that the performance of a long-short strategy does not only depend on the *average* performance of the long and short leg, but also on the timing of the realized returns of both portfolios.[108] Nevertheless, Table VII highlights that the weights of long-short portfolios should not be determined based on symmetric risk measures like volatility. Instead, asymmetric tail risk measures that differentiate between long and short positions should be used as suggested by Giot and Laurent (2003).

So far, conclusions on the long and short leg of the momentum portfolio were only made by regarding the winners' and losers' Sharpe Ratios. Besides mean and variance other risk characteristics are also important for long-short investors. Several studies show that winners and losers typically have quite different risk characteristics. For example, Harvey and Siddique (2000) find that winners typically have a lower skewness than losers. Further, Bollerslev

[108]Consider, for example, three portfolios A, B and C with returns in the first year of $R_1^A = -50\%, R_1^B = 25\%$ and $R_1^C = -20\%$, respectively. In the second year, portfolios A, B and C generate returns of $R_2^A = 100\%, R_2^B = -20\%$ and $R_2^C = 25\%$, respectively. Thus, all three portfolios have a compounded return of zero. However, buying one of the portfolios and selling another portfolio does not automatically produce a return of zero. For example, the strategy that buys portfolio A and sells portfolio B earns a compounded return of -45%, whereas the strategy that buys portfolio A and sells portfolio C earns a compounded return of 22.5%. Even more interesting, the investor who buys portfolio B and sells portfolio A would even loose more than 100%. Thus, the performance of a long-short strategy can vary extremely and strongly depends on whether positive and negative returns of the long and short leg occur simultaneously or oppositely. Simply comparing average returns of the long and short side can lead to false conclusions.

Table VII. Winners and Losers Portfolios
This table shows the performance of the equally and risk weighted winners and losers portfolios. The description of the columns is given in Table I.

	Winners Portfolio					Losers Portfolio				
Panel A: Skew	Return	Volatility	Skew	SR	z_{JK}	Return	Volatility	Skew	SR	z_{JK}
Equal	18.79	23.57	1.102	0.797	-	7.44	26.26	1.709	0.283	-
RV	18.44	22.67	1.203	0.813	1.03	8.12	24.99	1.809	0.325	**2.88**
Skew/Corr	20.22	23.59	1.234	0.857	**3.46**	7.43	25.31	1.491	0.294	0.70
Skew/Down Corr	20.24	23.53	1.229	0.860	**3.80**	7.73	25.82	1.754	0.299	1.10
Skew/Beta	20.10	23.59	1.135	0.852	**3.21**	7.27	25.35	1.483	0.287	0.25
Skew/Down Beta	20.27	23.50	1.163	0.863	**3.86**	7.46	25.59	1.662	0.292	0.60
Skew/CoSkew	19.91	23.71	1.282	0.839	**2.58**	7.60	26.42	2.003	0.288	0.31
Skew/CoKurt	20.18	23.47	1.067	0.860	**3.72**	7.58	25.75	1.679	0.294	0.79
Skew/LPM Beta	20.30	23.59	1.149	0.860	**3.72**	7.28	25.29	1.516	0.288	0.34
Skew/HTCR Beta	20.03	23.54	1.074	0.851	**3.16**	7.34	25.53	1.585	0.288	0.32
Skew/Tail Beta	20.13	23.25	1.126	0.866	**4.11**	7.32	25.10	1.480	0.292	0.57
Skew/Tail Sens	20.08	23.34	1.095	0.860	**3.77**	7.19	25.35	1.509	0.284	0.04
Skew/Tail Risk	20.03	23.28	1.115	0.860	**3.79**	7.33	25.04	1.462	0.293	0.63
Skew/MES	20.32	23.62	1.102	0.860	**3.69**	7.31	25.23	1.446	0.290	0.43
	Winners Portfolio					Losers Portfolio				
Panel B: RSJ	Return	Volatility	Skew	SR	z_{JK}	Return	Volatility	Skew	SR	z_{JK}
Equal	18.79	23.57	1.102	0.797	-	7.44	26.26	1.709	0.283	-
RV	18.44	22.67	1.203	0.813	1.03	8.12	24.99	1.809	0.325	**2.88**
RSJ/Corr	20.64	23.56	1.228	0.876	**4.56**	6.79	25.29	1.489	0.268	-0.98
RSJ/Down Corr	20.67	23.50	1.223	0.880	**4.93**	7.08	25.80	1.754	0.275	-0.60
RSJ/Beta	20.53	23.55	1.128	0.872	**4.32**	6.63	25.33	1.481	0.262	-1.45
RSJ/Down Beta	20.70	23.46	1.157	0.882	**4.97**	6.82	25.57	1.661	0.267	-1.18
RSJ/CoSkew	20.33	23.68	1.276	0.859	**3.74**	6.96	26.41	2.004	0.264	-1.47
RSJ/CoKurt	20.61	23.44	1.061	0.879	**4.84**	6.94	25.73	1.678	0.270	-0.97
RSJ/LPM Beta	20.73	23.56	1.143	0.880	**4.83**	6.64	25.27	1.513	0.263	-1.38
RSJ/HTCR Beta	20.46	23.51	1.067	0.870	**4.28**	6.70	25.51	1.583	0.263	-1.43
RSJ/Tail Beta	20.56	23.22	1.119	0.885	**5.25**	6.68	25.08	1.477	0.266	-1.09
RSJ/Tail Sens	20.50	23.31	1.089	0.880	**4.91**	6.55	25.33	1.507	0.259	-1.63
RSJ/Tail Risk	20.46	23.25	1.108	0.880	**4.92**	6.69	25.02	1.459	0.267	-1.00
RSJ/MES	20.75	23.59	1.096	0.879	**4.80**	6.67	25.21	1.443	0.264	-1.23

et al. (2015, p. 131) find that fear, measured as jump tail risk, is priced quite differently for the winners and losers portfolio. For that reason, we next examine how the different weighting schemes influence the skewness risk of the winners and losers, which is a main driver of momentum crashes. Results examined so far indicate that the switching strategies successfully reduce the momentum portfolio's left tail risk, whereas the volatility weighting does not reduce the portfolio's left tail risk. By examining the distributional properties of winners and losers separately, we can identify the source of this finding. First of all, we confirm the earlier findings of Harvey and Siddique (2000) and also find a lower skewness of the winners. Thus, buying winners and shorting losers produces a high negative skewness of the momentum portfolio, which is a main driver of momentum's high crash risk. Table VII shows that volatility weighting hardly affects the winners' and losers' skewness. This is quite intuitive, since volatility does not account for non-normalities in the return distribution. Thus, managing the winners' and losers' individual volatilities does not reduce the high left tail risk of the momentum strategy. This makes volatility weighting an inappropriate tool to manage momentum crashes. In contrast, the switching strategies hardly affect the winners' skewness but significantly reduce the losers' skewness. This reduction of the losers' skewness is appealing for a momentum investor who is short the losers portfolio. This shows that the left tail risk reduction of the switching approach is mainly driven by making the losers portfolio less positively skewed. This is in line with the construction of our weighting scheme, since losers with a lower skewness, i.e. higher crash risk, obtain higher weights.

We have so far shown that our switching approach based on (systematic) tail risk measures produces significantly higher returns and reduces left tail risk of the momentum strategy. Conclusions so far are mainly based on the strategies' Sharpe Ratio and skewness. By incorporating the investors' preferences for mean and variance as well as higher moments like skewness and kurtosis, we next assess how valuable the (systematic) tail risk switching approach is for investors. Schneider et al. (2020) show that incorporating investors' skewness preferences is important when a strategy's (risk-adjusted) performance is assessed. Further, we will especially regard the finding that investors are averse to losses and weight losses higher than gains. Assessing the economic value of different trading strategies, i.e. calculating the annualized percentage fee investors are willing to pay to switch from a benchmark strategy to a dynamic trading strategy, has frequently been done in the financial literature. For example, Fleming et al. (2001), Fleming et al. (2003), Han (2005), Kirby and Ostdiek (2012), Marquering and Verbeek (2004) and Bollerslev et al. (2018) examine the value of volatility timing strategies for mean-variance investors. Ang and Chen (2002, p. 449) and Hong et al. (2007, Eq. (48)) estimate the utility loss for a CRRA and disappointment averse investor who ignores information on asymmetric correlations. Jondeau and Rockinger (2012) and Ghysels et al. (2016) show the importance of incorporating skewness in the portfolio optimization for CRRA investors. Rickenberg (2020b) and Rickenberg (2020a) examines the economic value of tail risk targeting strategies for mean-variance, CRRA and loss-averse investors. We follow Rickenberg (2020b) and Rickenberg

(2020a) and estimate the economic value of our risk weighted momentum strategies for these three investors. The economic value is defined as the annualized percentage fee an investor is willing to pay to switch from the equally weighted momentum strategy to one of the risk weighted strategies. We use a risk aversion of $\gamma = 5$ for the mean-variance and CRRA investor as well as a loss aversion of $l = 2$ for the loss-averse investor. A loss aversion of two means that an investor's disutility of a loss is twice as great as the utility of a positive return (Aït-Sahalia and Brandt, 2001, Benartzi and Thaler, 1995). These levels of risk aversion and loss aversion are in line with the literature. We also used other levels of risk aversion and loss aversion and found quite similar results, which are not shown here.[109]

Assessing the performance based on an investor's utility has the advantage that the performance is evaluated conditionally. Cederburg and O'Doherty (2016) and Boguth et al. (2011) show that assessing a strategy's performance based on an unconditional measure, such as the strategy's CAPM alpha, can lead to wrong conclusions.[110] This holds especially for dynamic strategies that time volatility, such as the strategies examined in our paper. Boguth et al. (2011, p. 367) state that "volatility timing should be taken into account whenever evaluating investment performance". For example, Cederburg and O'Doherty (2016) find that performance evaluation based on the betting against beta's unconditional alpha is highly misleading and overstates the strategy's performance. Similarly, Boguth et al. (2011) find that the performance of momentum strategies should not be assessed based on unconditional risk measures. In contrast to the Sharpe Ratio, the economic value approach considers the risk an investor was faced each month and regards that risk is timed by our strategies.[111] Further, the economic value approach has the

[109] Instead of choosing fixed values of γ and l, these values can also be derived empirically. For example, Rosenberg and Engle (2002) derive the level of γ using options data. Kritzman et al. (2010, Appendix A) show how the risk aversion γ of a mean-variance investor can be calculated based on stock and bond return data. Benartzi and Thaler (1995) and Berkelaar et al. (2004) also derive the level of loss aversion using market data.

[110] We show in Section B.11 additional results based on the strategies' alpha. Since performance evaluation based on the alpha has several disadvantages as shown by Boguth et al. (2011), Cederburg and O'Doherty (2016), Cederburg et al. (2020) and Schneider et al. (2020), we focus in the main part on the economic value that corrects for the drawbacks of the portfolio alpha.

[111] For example, Han (2005, p. 246) state that "the Sharpe Ratio does not take into account time-varying conditional volatility because the sample [standard deviation] overestimates the conditional risk an investor faces when she follows dynamic strategies. Consequently, the realized Sharpe ratio underestimates the performance of dynamic strategies." Similarly, Marquering and Verbeek (2004, p. 419-421) write that "[i]t is important to realize that the Sharpe ratio does not appropriately take into account time-varying volatility. The risk of the dynamic strategies is typically overestimated by the sample standard deviation, particularly in the presence of volatility timing, because the ex post (unconditional) standard deviation is an inappropriate measure for the (conditional) risk an

Table VIII. Economic Value
This table shows the annualized percentage fee an investor is willing to pay to switch from the equally weighted momentum strategy to a risk weighted strategy. The fee is calculated for a mean-variance investor with risk aversion $\gamma = 5$, a CRRA investor with $\gamma = 5$ and a loss-averse investor with a loss aversion of $l = 2$. The fee is calculated over the whole period, the first half and the second half. Panel A shows results for the switching strategies based on skewness. Panel B shows results for the switching strategies using the RSJ measure. All strategies are rescaled to the same level of volatility.

	Whole Period			First Half			Second Half		
	Panel A: Switching Based on Skewness Measure								
Model	$\Delta_{MV}^{\gamma=5}$	$\Delta_{CRRA}^{\gamma=5}$	$\Delta_{LA}^{l=2}$	$\Delta_{MV}^{\gamma=5}$	$\Delta_{CRRA}^{\gamma=5}$	$\Delta_{LA}^{l=2}$	$\Delta_{MV}^{\gamma=5}$	$\Delta_{CRRA}^{\gamma=5}$	$\Delta_{LA}^{l=2}$
RV	0.168	0.075	0.186	-0.043	0.000	-0.125	0.407	0.225	0.528
Skew/Corr	2.341	2.503	2.275	2.524	2.888	2.304	2.127	2.120	2.230
Skew/Down Corr	1.217	0.753	1.399	0.801	0.150	1.060	1.706	1.815	1.775
Skew/Beta	2.514	2.734	2.446	2.638	2.965	2.398	2.362	2.427	2.485
Skew/Down Beta	1.972	1.891	2.007	1.682	1.586	1.622	2.285	2.427	2.422
Skew/CoSkew	0.473	0.451	0.562	0.121	0.000	0.245	0.849	0.979	0.900
Skew/CoKurt	1.553	1.358	1.643	1.353	1.131	1.557	1.782	1.815	1.728
Skew/LPM Beta	2.671	2.888	2.567	2.794	3.119	2.454	2.521	2.657	2.675
Skew/HTCR Beta	2.370	2.580	2.196	2.320	2.657	1.885	2.406	2.503	2.516
Skew/Tail Beta	1.896	2.120	1.869	1.900	2.350	1.626	1.877	1.891	2.117
Skew/Tail Sens	2.430	2.657	2.188	2.591	2.965	2.070	2.251	2.273	2.302
Skew/Tail Risk	2.032	2.273	1.991	2.180	2.580	1.848	1.862	1.891	2.133
Skew/MES	2.715	2.965	2.594	2.918	3.350	2.455	2.495	2.657	2.730
	Panel B: Switching Based on RSJ Measure								
Model	$\Delta_{MV}^{\gamma=5}$	$\Delta_{CRRA}^{\gamma=5}$	$\Delta_{LA}^{l=2}$	$\Delta_{MV}^{\gamma=5}$	$\Delta_{CRRA}^{\gamma=5}$	$\Delta_{LA}^{l=2}$	$\Delta_{MV}^{\gamma=5}$	$\Delta_{CRRA}^{\gamma=5}$	$\Delta_{LA}^{l=2}$
RV	0.168	0.075	0.186	-0.043	0.000	-0.125	0.407	0.225	0.528
RSJ/Corr	3.491	3.660	3.360	3.073	3.428	2.869	3.919	3.970	3.867
RSJ/Down Corr	2.270	1.815	2.403	1.271	0.602	1.557	3.431	3.505	3.357
RSJ/Beta	3.679	3.893	3.548	3.195	3.505	2.972	4.178	4.282	4.146
RSJ/Down Beta	3.083	3.042	3.063	2.193	2.044	2.152	4.067	4.204	4.066
RSJ/CoSkew	1.461	1.434	1.502	0.576	0.451	0.726	2.418	2.580	2.341
RSJ/CoKurt	2.636	2.427	2.673	1.844	1.586	2.074	3.538	3.583	3.331
RSJ/LPM Beta	3.837	4.048	3.667	3.355	3.738	3.027	4.331	4.438	4.335
RSJ/HTCR Beta	3.535	3.738	3.296	2.876	3.196	2.454	4.224	4.360	4.183
RSJ/Tail Beta	2.993	3.196	2.909	2.421	2.811	2.165	3.588	3.583	3.687
RSJ/Tail Sens	3.586	3.815	3.273	3.156	3.583	2.645	4.021	4.048	3.913
RSJ/Tail Risk	3.151	3.350	3.048	2.720	3.119	2.402	3.591	3.583	3.710
RSJ/MES	3.886	4.204	3.699	3.495	3.970	3.043	4.280	4.438	4.368

advantage that the portfolio evaluation period coincides with the portfolio reallocation period, which gives a more realistic assessment of a strategy's performance (Boguth et al., 2011). Evaluating the strategies' performance monthly is also more realistic since investors typically have short evaluation periods (Benartzi and Thaler, 1995).

Table VIII shows the economic value for the three investors, where we calculate the economic value over the whole period as well as over the first and second half. As expected, the economic value of the volatility managed strategy is only small for all three investors. This is

investor was facing at each point in time. This indicates a potentially severe disadvantage of the use of Sharpe ratios to evaluate dynamic strategies."

in line with our earlier findings that volatility weighting does not produce an enhanced risk-return profile for long-short strategies like momentum. In contrast, the economic value of the weighting schemes based on the (systematic) tail risk switching approach is typically high in magnitude. Interestingly, the fees are quite similar for all three types of investors, where the highest fees of the switching approach are typically found for the CRRA investor. This comes from the reduction of left tail risk as shown in Table III. We further find that the economic value is quite similar for the different samples. This is again in line with Table VI that the switching approach does work well in different sub-periods.

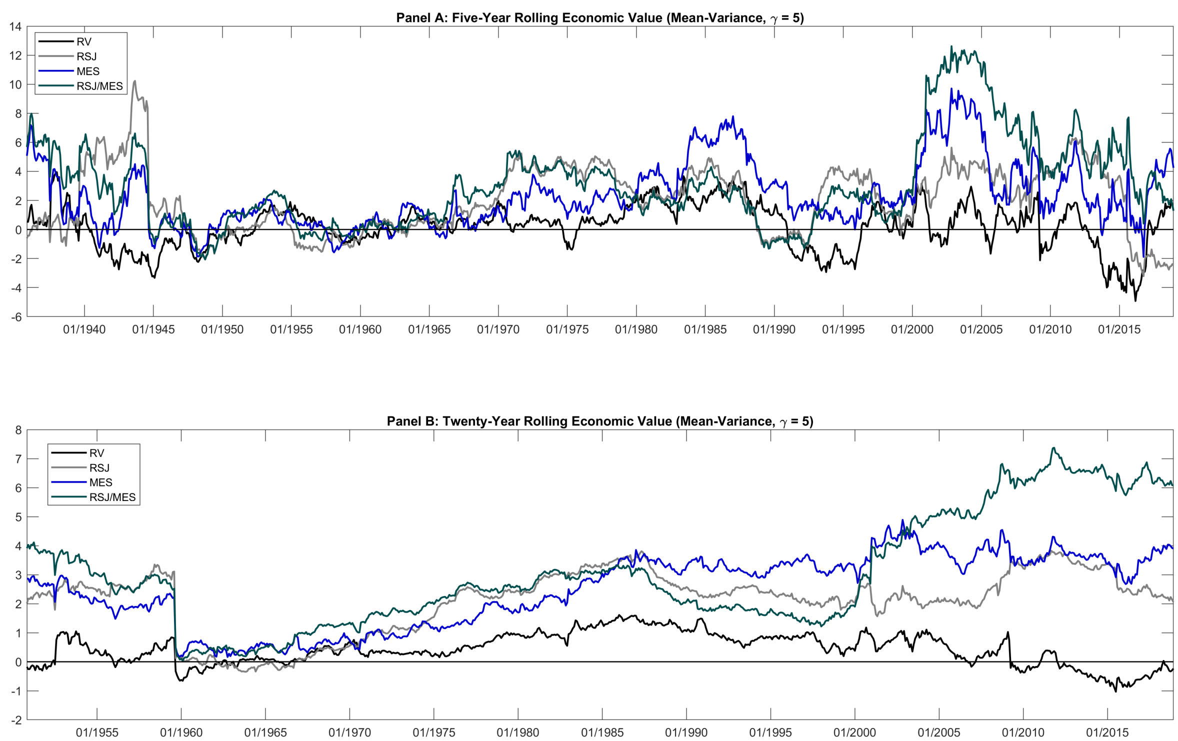

Figure II. Rolling Economic Value. This figure plots the rolling economic value for a mean-variance investor with risk aversion $\gamma = 5$ who invests in four different risk weighted momentum portfolios using volatility, RSJ and MES as risk measures or the approach that switches between RSJ and MES. Panel A shows the rolling economic value for an investor who invests for five years in these portfolios. Panel B shows the economic value for an investor who invests for 20 years in these portfolios.

Since investors typically have quite short evaluation periods (Benartzi and Thaler, 1995), we next show in Figure II the rolling economic value for a mean-variance investor who invests for five or 20 years in the risk weighted momentum strategies. The economic value is again defined as the annualized percentage fee an investor is willing to pay to switch from the equally weighted momentum portfolio to the risk weighted portfolios. The dates on the x-axis in Figure II mark the date when the investment period ends. Results for the CRRA and loss-averse investor are quite similar and are not shown here. Figure II compares the economic value of the volatility managed momentum strategy to the strategies using the RSJ and MES based weightings. To further assess if switching between these two risk measures is advantageous to using these risk measures directly, we also show the rolling economic value of the strategy that switches between the RSJ and MES weightings. Panel A of Figure II shows the rolling economic value for a mean-variance investor who invest for five years in these portfolios. Among the four strategies, the economic value of the volatility weighted strategy is the lowest and sometimes becomes even negative. In contrast, the (systematic) tail risk weighted strategies constantly produce a higher and mostly positive economic value. Interestingly, the switching approach is typically quite successful in capturing the advantages of the univariate and systematic risk weighted strategies. Panel B shows the rolling economic value for an investor who invest for 20 years in the different risk weighted strategies. The differences between the different weighting approaches become now more evident. The economic value of the volatility managed strategy is again quite low and becomes again negative in some periods. In contrast, the economic value of the (systematic) tail risk weighted strategies is always higher than the economic value of the volatility managed strategy and is positive most of the times. The switching approach is again successful in capturing the advantages of the univariate and systematic risk measures. However, there are still periods when the switching approach underperforms the RSJ and MES weighted portfolios. Thus, further research on the drivers of industry momentum crashes is needed, which could improve the switching strategy's performance.

Table VIII shows that the economic value of our switching strategies is quite high and Figure II shows that the economic value also holds for shorter investment periods. We next test if the utility increases found in Table VIII are also statistically significant. Testing for the significance of utility increases is frequently done in the financial literature. For example, Cederburg et al. (2020, Footnote 19), DeMiguel et al. (2009b), Kirby and Ostdiek (2012), Bollerslev et al. (2018) and Jondeau et al. (2019, p. 41) statistically test for differences in the utility of a mean-

Table IX. Test for Significant Utility Increases

This table shows results of the tests that test for statistically significant utility increases of the risk weighted strategies compared to the equally weighted momentum portfolio. DM-test stands for the test statistic of the Diebold and Mariano (1995) test. p^{RC} and p^{SPA} stand for the p-values of the RC- and SPA-test of Sullivan et al. (1999), White (2000), Hansen (2005) and Hansen and Lunde (2005). p^{SQ} stands for the p-value of the MCS (Hansen et al., 2003, 2011). Step-SPA and Step-SPAst stand for the set of optimal models of the non-studentized and studentized stepwise SPA approach (Hsu et al., 2010, Romano and Wolf, 2005). $FDR^+ = 5\%$ stands for the portfolio that targets an FDR^+ of 5% (Bajgrowicz and Scaillet, 2012, Barras et al., 2010). Bold numbers mark models that are superior to the equally weighted strategy or models that cannot be rejected by the RC- or SPA-test.

Panel A: MV	DM-test	p^{RC}	p^{SPA}	p^{SQ}	Step-SPA	Step-SPAst	$FDR^+ = 5\%$
Equal	-	0.00	0.00	0.01	-	-	-
RV	0.36	0.00	0.00	0.11	0	0	0
RSJ/Corr	**5.50**	**64.63**	**14.10**	**28.96**	**1**	**1**	**6**
RSJ/Down Corr	**3.20**	4.89	2.20	1.92	**1**	**1**	**11**
RSJ/Beta	**6.00**	**86.28**	**19.38**	**39.32**	**1**	**1**	**3**
RSJ/Down Beta	**4.90**	**24.96**	4.27	**17.31**	**1**	**1**	**8**
RSJ/CoSkew	**2.64**	0.27	0.10	0.83	**1**	**1**	**12**
RSJ/CoKurt	**3.94**	7.49	1.57	2.11	**1**	**1**	**10**
RSJ/LPM Beta	**6.11**	**97.29**	**42.43**	**79.62**	**1**	**1**	**2**
RSJ/HTCR Beta	**5.74**	**68.93**	**16.71**	**33.27**	**1**	**1**	**5**
RSJ/Tail Beta	**6.06**	**19.65**	0.23	2.30	**1**	**1**	**9**
RSJ/Tail Sens	**5.93**	**71.28**	**20.72**	**39.32**	**1**	**1**	**4**
RSJ/Tail Risk	**5.94**	**31.45**	0.34	6.39	**1**	**1**	**7**
RSJ/MES	**6.32**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**
Panel B: CRRA	DM-test	p^{RC}	p^{SPA}	p^{SQ}	Step-SPA	Step-SPAst	$FDR^+ = 5\%$
Equal	-	0.00	0.00	0.14	-	-	-
RV	0.16	0.03	0.00	0.68	0	0	0
RSJ/Corr	**5.15**	**68.16**	**13.69**	**26.24**	**1**	**1**	**6**
RSJ/Down Corr	1.53	9.07	7.54	5.15	**1**	**2**	**12**
RSJ/Beta	**5.72**	**84.54**	**15.39**	**31.57**	**1**	**1**	**3**
RSJ/Down Beta	**4.05**	**18.69**	8.42	**18.78**	**1**	**1**	**9**
RSJ/CoSkew	**2.02**	1.26	0.50	2.47	**2**	**1**	**11**
RSJ/CoKurt	**2.98**	8.53	4.60	5.15	**1**	**1**	**10**
RSJ/LPM Beta	**5.87**	**96.07**	**33.75**	**65.03**	**1**	**1**	**2**
RSJ/HTCR Beta	**5.50**	**70.21**	**12.49**	**30.31**	**1**	**1**	**5**
RSJ/Tail Beta	**5.69**	**38.27**	0.25	5.15	**1**	**1**	**8**
RSJ/Tail Sens	**5.41**	**69.35**	**14.47**	**31.57**	**1**	**1**	**4**
RSJ/Tail Risk	**5.45**	**45.62**	0.66	9.09	**1**	**1**	**7**
RSJ/MES	**5.77**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**
Panel C: Loss Aversion	DM-test	p^{RC}	p^{SPA}	p^{SQ}	Step-SPA	Step-SPAst	$FDR^+ = 5\%$
Equal	-	0.00	0.00	0.00	-	-	-
RV	0.41	0.00	0.00	0.01	0	0	0
RSJ/Corr	**5.20**	**68.41**	**18.91**	**37.37**	**1**	**1**	**4**
RSJ/Down Corr	**3.87**	6.91	1.30	1.59	**1**	**1**	**11**
RSJ/Beta	**5.47**	**90.52**	**38.50**	**71.50**	**1**	**1**	**3**
RSJ/Down Beta	**4.71**	**43.35**	5.12	**19.66**	**1**	**1**	**8**
RSJ/CoSkew	**2.86**	0.34	0.13	0.26	**1**	**1**	**12**
RSJ/CoKurt	**4.18**	**17.03**	1.81	2.84	**1**	**1**	**10**
RSJ/LPM Beta	**5.55**	**97.62**	**58.05**	**96.62**	**1**	**1**	**2**
RSJ/HTCR Beta	**5.02**	**61.69**	**11.77**	**24.85**	**1**	**1**	**5**
RSJ/Tail Beta	**5.32**	**26.30**	1.58	4.56	**1**	**1**	**9**
RSJ/Tail Sens	**5.43**	**56.90**	**14.36**	**24.85**	**1**	**1**	**6**
RSJ/Tail Risk	**5.33**	**30.30**	1.45	8.44	**1**	**1**	**7**
RSJ/MES	**5.76**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**

variance investor using volatility based investment strategies compared to the equally weighted portfolio. Rickenberg (2020b) and Rickenberg (2020a) tests for differences in the utility of mean-variance, CRRA and loss-averse investors who use volatility and tail risk managed portfolio strategies. We follow the approach of Rickenberg (2020b) and Rickenberg (2020a) and use several testing procedures to test for the significance of the differences in the utility of the three investors. Results of these tests are shown in Table IX, where we only show results for the RSJ based switching strategies. Results for the skewness based switching strategies were again quite similar. Panel A shows results for the mean-variance investor. The DM-test of Diebold and Mariano (1995), which is also used by Bollerslev et al. (2018), shows that all switching strategies produce a significantly higher utility for the mean-variance investor. Interestingly, although the volatility managed portfolio should fit well to the preferences of a mean-variance investor, the increase in the investor's utility is not significant. As mentioned earlier, a possible explanation for this finding is that volatility weighting is not appropriate for long-short strategies. The RC- and SPA-test of White (2000), Sullivan et al. (1999), Hansen (2005) and Hansen and Lunde (2005) also clearly reject the equally weighted and volatility weighted portfolio. This is confirmed by the MCS (Hansen et al., 2003, 2011) and step-SPA approach (Hsu et al., 2010, Romano and Wolf, 2005).[112] The FDR approach of Bajgrowicz and Scaillet (2012) and Barras et al. (2010) using an FDR target of 5% is also in line with the remaining tests and picks all switching strategies, whereas the volatility weighted strategy is not picked.[113] Panels B and C show results for the CRRA and loss-averse investor, respectively. Results for these investors are similar to the results of the mean-variance investor and show that the switching strategies clearly outperform the volatility and equally weighted strategy.

To summarize this section, we have shown that equally and volatility weighted momentum portfolios are suboptimal. In contrast, weighting assets based on their (systematic) tail risk does not only reduce the momentum portfolio's left tail risk, but also produces higher returns. This enhanced risk-return profile can be seen by higher Sharpe Ratios and that investors are willing to pay high fees to switch to a (systematic) tail risk weighted strategy. Both the increase in the strategies' Sharpe Ratio and the utility increases are statistically significant and robust among

[112] Goyal and Wahal (2015) also use the stepwise approach of Romano and Wolf (2005) to test the differences of momentum's profitability when different ranking periods are used.

[113] We also calculated the FDR approach for an FDR target of $FDR^+ = 10\%$. This approach additionally picks the volatility weighted strategy as superior to the equally weighted strategy for the loss-averse investor. Results for the mean-variance and CRRA investor were the same for both FDR targets.

sub-samples as well as bull and bear markets.

3.7.4 Volatility Targeting

In the previous section, we examined strategies that manage momentum's individual asset risk without regarding the overall portfolio risk. In this section, we assess the performance of momentum portfolios that are overlayed by a strategy that targets a constant level of portfolio volatility. Volatility targeting is an easy but appealing method to manage a portfolio's risk by reducing the portfolio's left tail risk and drawdowns (Barroso and Maio, 2018, Barroso and Santa-Clara, 2015, Du Plessis and Hallerbach, 2017, Grobys and Kolari, 2020, Grobys et al., 2018, Rickenberg, 2020a,b). One advantage of volatility targeting is that this approach can easily be combined with other portfolio strategies that manage a portfolio's individual asset risk.[114] For example, Moreira and Muir (2017), Harvey et al. (2018) and Zakamulin (2015) target the volatility of volatility weighted portfolios and find that managing individual and portfolio risk typically outperforms non-managed strategies as well as strategies that manage either individual or portfolio risk. Harvey et al. (2018, p. 27) compare the following three portfolio risk management approaches: "unscaled at both the asset and portfolio level", "[v]olatility scaling at the asset level only" and "[v]olatility scaling at both the asset and portfolio level". Harvey et al. (2018, Exhibit 16) find that the last approach that accounts for risk at both levels, the asset and portfolio level, performs best, whereas the approach that does not account for any kind of risk performs the worst. To assess the benefits of individual and portfolio risk management applied to the momentum portfolio, we now examine four different approaches. First, we use the equally weighted momentum strategy that does not account for any kind of risk. Second, we use the volatility weighted momentum strategy presented in Section 3.2.4 that accounts for individual asset risk without accounting for portfolio risk. Third, we use the equally weighted momentum strategy overlayed by the volatility targeting strategy. This approach manages portfolio risk but not individual asset risk. Fourth, we combine the volatility and (systematic) tail

[114]Portfolio risk management can be separated in two parts: asset allocation and market timing (Agarwal and Naik, 2004). Our approach chooses the assets based on their performance and risk, whereas market timing is done by scaling the exposure to that portfolio using volatility targeting. See Rickenberg (2020b, Appendix A) for a list of further advantages of volatility targeting.

risk weighted strategies with the volatility targeting approach as presented in Section 3.6. These approaches account for individual asset and portfolio risk. In particular, this last approach is similar to the strategy of Baltas (2015). The author combines the TSMOM strategy with the risk parity weighting and the volatility targeting strategy.

Table X. Performance Results: Volatility Targeting
This table shows performance results of the equally weighted and volatility weighted momentum strategies without volatility targeting as well as the equally weighted and risk weighted momentum strategies that are overlayed by the target volatility (TV) strategy. The description of the columns is given in Table I.

	Panel A: Switching Based on Skewness Measure									
Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	9.13	11.98	-0.934	11.729	0.763	-	48.48	0.188	-25.96	18.61
RV	8.37	10.70	-1.241	12.339	0.782	0.49	45.89	0.182	-21.59	13.57
Equal (TV)	13.35	11.79	0.184	5.462	1.133	**6.27**	32.31	0.413	-16.61	15.95
RV (TV)	15.28	12.64	0.275	5.483	1.209	**5.95**	36.25	0.422	-17.46	17.87
Skew/Corr (TV)	14.49	11.36	0.315	5.045	1.276	**6.89**	31.39	0.462	-14.62	14.61
Skew/Down Corr (TV)	14.11	11.56	0.048	6.776	1.221	**6.39**	40.09	0.352	-22.54	14.61
Skew/Beta (TV)	14.49	11.49	0.282	5.237	1.261	**6.78**	32.42	0.447	-16.21	14.61
Skew/Down Beta (TV)	14.37	11.50	0.163	6.019	1.250	**6.69**	34.90	0.412	-20.16	14.61
Skew/CoSkew (TV)	13.58	11.51	0.361	5.623	1.179	**5.73**	33.48	0.406	-15.83	17.77
Skew/CoKurt (TV)	14.13	11.51	0.069	6.404	1.227	**6.48**	37.61	0.376	-21.56	14.61
Skew/LPM Beta (TV)	14.65	11.38	0.354	4.971	1.287	**7.02**	31.34	0.468	-13.27	14.61
Skew/HTCR Beta (TV)	14.32	11.36	0.364	4.815	1.260	**6.64**	29.70	0.482	-11.21	14.61
Skew/Tail Beta (TV)	14.38	11.59	0.328	4.929	1.242	**6.74**	33.02	0.436	-12.69	14.61
Skew/Tail Sens (TV)	14.53	11.30	0.416	4.745	1.285	**6.92**	27.58	0.527	-11.16	14.61
Skew/Tail Risk (TV)	14.45	11.54	0.385	4.940	1.252	**6.80**	32.60	0.443	-11.79	14.61
Skew/MES (TV)	14.70	11.41	0.433	4.856	1.288	**6.95**	29.68	0.495	-11.17	14.61
	Panel B: Switching Based on RSJ Measure									
Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	9.13	11.98	-0.934	11.729	0.763	-	48.48	0.188	-25.96	18.61
RV	8.37	10.70	-1.241	12.339	0.782	0.49	45.89	0.182	-21.59	13.57
Equal (TV)	13.35	11.79	0.184	5.462	1.133	**6.27**	32.31	0.413	-16.61	15.95
RV (TV)	15.28	12.64	0.275	5.483	1.209	**5.95**	36.25	0.422	-17.46	17.87
RSJ/Corr (TV)	15.56	11.29	0.293	4.832	1.378	**8.12**	28.48	0.546	-14.62	13.49
RSJ/Down Corr (TV)	15.12	11.50	0.009	6.635	1.315	**7.60**	37.91	0.399	-22.54	13.49
RSJ/Beta (TV)	15.57	11.41	0.255	5.028	1.364	**8.06**	30.61	0.509	-16.21	13.49
RSJ/Down Beta (TV)	15.41	11.43	0.131	5.850	1.348	**7.93**	34.67	0.444	-20.16	13.49
RSJ/CoSkew (TV)	14.52	11.46	0.305	5.412	1.267	**6.85**	31.80	0.457	-15.83	17.39
RSJ/CoKurt (TV)	15.14	11.46	0.031	6.248	1.322	**7.68**	35.81	0.423	-21.56	13.49
RSJ/LPM Beta (TV)	15.72	11.31	0.331	4.745	1.390	**8.27**	29.12	0.540	-13.27	13.49
RSJ/HTCR Beta (TV)	15.40	11.29	0.342	4.589	1.364	**7.90**	28.16	0.547	-11.76	13.49
RSJ/Tail Beta (TV)	15.43	11.52	0.288	4.735	1.339	**7.99**	32.09	0.481	-12.69	13.49
RSJ/Tail Sens (TV)	15.56	11.24	0.384	4.512	1.385	**8.11**	25.89	0.601	-11.73	13.49
RSJ/Tail Risk (TV)	15.50	11.47	0.345	4.722	1.351	**8.05**	31.25	0.496	-11.80	13.49
RSJ/MES (TV)	15.77	11.33	0.408	4.604	1.392	**8.21**	26.55	0.594	-11.74	13.82

Table X shows performance results of the strategies that use volatility targeting as well as the equally and volatility weighted momentum strategies without volatility targeting.[115] Table

[115] Results for the equally and volatility weighted strategies without volatility targeting are slightly different to the results shown in Table II, since the volatility targeting approach needs six months to estimate portfolio volatility. Thus, the sample examined in this section is six months shorter than the sample examined in the previous section.

X confirms the finding of earlier studies that volatility targeting enhances the risk-return profile of a given portfolio. Volatility targeting significantly increases the Sharpe Ratio of the equally weighted momentum strategy with an extremely high Jobson and Korkie (1981) value of 6.27. Thus, accounting for portfolio risk produces an enhanced risk-return profile compared to the strategy that does not account for any kind of risk. Further, volatility targeting significantly reduces left tail risk as can be seen by the higher skewness, lower kurtosis and lower drawdown. Interestingly, all strategies that use the target volatility overlay exhibit a positive skewness. This result is in line with Grobys et al. (2018) and Grobys and Kolari (2020). Volatility targeting more than doubles the equally weighted momentum portfolio's Calmar Ratio that quantifies the drawdown-adjusted return. In particular, accounting for portfolio risk is more beneficial than accounting for individual asset risk. The equally weighted strategy that targets a constant level of portfolio volatility significantly outperforms the strategy that manages the individual assets' volatility. Further, the strategy that manages individual asset *and* portfolio volatility produces a slightly higher Sharpe Ratio and skewness than the equally weighted strategy overlayed by the volatility targeting approach. Thus, simultaneously managing individual asset and portfolio risk outperforms the strategies that manage either individual or portfolio risk as found by Harvey et al. (2018) and Zakamulin (2015). However, the performance can further be enhanced by simultaneously managing the individual assets' (systematic) tail risk and portfolio volatility. This especially holds for the strategies using the RSJ based weighting approach. These strategies further increase the Sharpe Ratio and skewness and further decrease the kurtosis and maximum drawdown. The Sharpe Ratio of this approach is nearly twice the Sharpe of the equally or volatility weighted momentum portfolio. The Jobson and Korkie (1981) values for the RSJ based strategies lie between 6.85 and 8.27 and indicate a highly significant increase in momentum's Sharpe Ratio.

To further illustrate the differences between the four approaches, Figure III shows the cumulative return of five different strategies. The first two strategies are the equally and volatility weighted strategies without volatility targeting. Additionally, we show the cumulative return of three strategies using volatility targeting combined with either equally, volatility or (system-

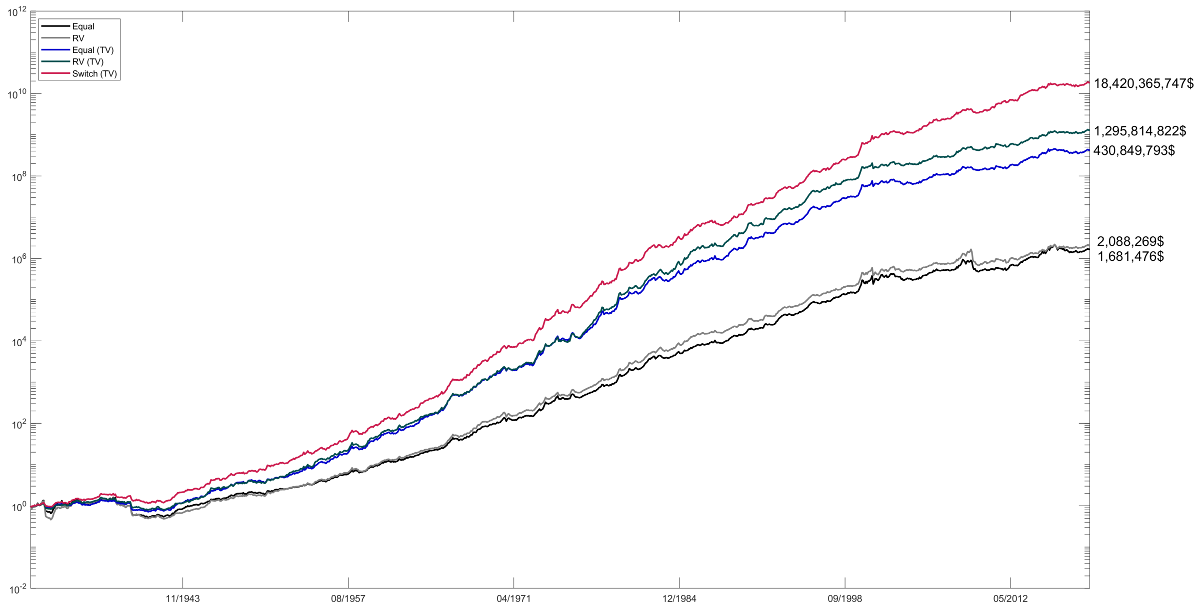

Figure III. Cumulative Return: Volatility Targeting. This figure plots the cumulative return of five momentum strategies combined with a one dollar investment in the risk-free rate. The five momentum strategies are the equally weighted and volatility weighted momentum strategy without volatility targeting as well as the equally, volatility and (systematic) tail risk weighted strategies combined with the target volatility (TV) approach. As in Daniel and Moskowitz (2016), we rescale all strategies to an annualized volatility of 19%.

atic) tail risk weighted industries. Since the different strategies have quite different levels of volatility, we rescale the strategies to the same level of volatility. The figure shows that the equally and volatility weighted strategies without volatility targeting are clearly outperformed by the strategies using volatility targeting. This outperformance is driven by mitigating crash periods and capturing the upside potential. Among the strategies with volatility targeting, the equally weighted strategy is outperformed by the risk weighted strategies, where the strategy based on the (systematic) tail risk weighting clearly outperforms the remaining strategies. Over the whole period, an investor who would have invested $1\$$ in the the risk-free rate combined with the equally weighted momentum strategy, would possess a terminal wealth of $1{,}681{,}476\$$. If the investor would have weighted industries by their volatility, this amount would increase to $2{,}088{,}269\$$. If the investor would have applied the volatility targeting approach to the equally weighted momentum portfolio, the investor's terminal wealth would increase to $430{,}849{,}793\$$. If the investor would additionally weight industries of the momentum portfolio by their volatility or (systematic) tail risk, the terminal wealth would even increase to $1{,}295{,}814{,}822\$$ or even

18,420,365,747\$. Thus, the terminal wealth of an investor who accounts for individual asset risk *and* portfolio risk is about 10,955 times higher than the terminal wealth of an investor who ignores all kinds of risk.

Table XI. Sorted Returns: Volatility Targeting

This table shows monthly returns of the equally weighted and volatility weighted momentum strategies without volatility targeting as well as the equally weighted and risk weighted momentum strategies that are overlayed by the target volatility (TV) strategy for the months when the momentum portfolio exhibits the five lowest and five highest returns. All entries are given in percent.

Model	Low Returns					High Returns				
Equal	-25.96	-24.01	-20.34	-15.59	-12.76	11.25	11.66	12.35	14.57	18.61
RV	-18.02	-20.43	-17.23	-20.26	-21.59	8.43	7.80	8.39	12.00	13.57
Equal (TV)	-16.27	-16.61	-7.62	-4.64	-4.57	15.95	9.91	8.40	4.08	15.14
RV (TV)	-13.65	-17.46	-7.28	-5.95	-8.46	16.53	7.68	8.38	4.19	12.54
RSJ/Corr (TV)	-14.62	-9.48	-4.04	-5.96	-5.33	13.49	11.44	8.56	4.25	10.44
RSJ/Down Corr (TV)	-22.54	-9.77	-4.08	-4.96	-6.24	13.49	11.16	7.86	4.24	12.20
RSJ/Beta (TV)	-16.21	-10.37	-4.58	-6.10	-5.58	13.49	12.16	8.02	4.20	11.82
RSJ/Down Beta (TV)	-20.16	-10.05	-4.84	-5.73	-5.44	13.49	11.02	7.52	3.57	12.22
RSJ/CoSkew (TV)	-15.83	-13.28	-4.79	-2.84	-4.73	13.49	11.20	7.20	3.12	17.39
RSJ/CoKurt (TV)	-21.56	-10.01	-4.82	-5.30	-7.18	13.49	11.29	8.86	4.50	9.38
RSJ/LPM Beta (TV)	-13.27	-10.08	-4.31	-5.92	-5.39	13.49	11.27	7.95	3.65	12.90
RSJ/HTCR Beta (TV)	-11.21	-9.96	-6.16	-5.73	-4.56	13.49	11.46	7.87	2.92	11.84
RSJ/Tail Beta (TV)	-12.69	-12.09	-5.78	-6.12	-5.62	13.49	12.50	9.00	3.55	12.59
RSJ/Tail Sens (TV)	-8.60	-9.74	-5.15	-5.76	-4.79	13.49	12.25	7.92	2.93	11.72
RSJ/Tail Risk (TV)	-11.79	-11.80	-4.65	-6.62	-6.10	13.49	13.03	8.62	3.87	13.46
RSJ/MES (TV)	-9.19	-10.19	-4.22	-6.12	-5.52	13.49	12.07	7.89	3.45	13.82

To assess how the volatility targeting approach performs in extremely good and bad market environments, Table XI shows the performance of the different strategies for the five months with the highest and lowest return of the equally weighted momentum strategy. From now on, we only show results for the RSJ based switching strategies. Results for the skewness based switching strategies were similar but slightly worse. Nevertheless, the strategies that use skewness as univariate risk measure also clearly outperform the equally and volatility weighted strategies. Table XI shows that volatility targeting successfully reduces the extremely low returns of the momentum portfolio. This finding is in line with Barroso and Santa-Clara (2015), Daniel and Moskowitz (2016) and Rickenberg (2020a) who show that volatility targeting is an appealing drawdown protection method for momentum investors. The highest reduction of extremely low returns is typically found for the switching based strategies combined with volatility targeting. Thus, simultaneously managing the individual assets' (systematic) tail risk and the momentum portfolio's volatility is a good approach to reduce momentum's loss potential.[116] Furthermore, periods with an extremely high momentum return are quite good captured by volatility targeting. Thus, the good performance of the target volatility approach is driven by reducing left tail risk without sacrificing much of the return potential. This makes volatility targeting appealing for investors, since most investors are willing to give up some of the right tail in order to reduce left tail risk (Harvey et al., 2018).

Table XII. Best and Worst 15 Years: Volatility Targeting
This table shows performance results of the equally and volatility weighted momentum portfolio without volatility targeting as well as the strategies that use the target volatility (TV) overlay in the two 15 years periods where the equally weighted momentum strategy exhibits the best and worst performance. The description of the columns is given in Table I.

	Best Months: 01.11.1967 – 31.10.1982					Worst Months: 01.06.1932 – 01.05.1947				
Model	Return	Volatility	Skew	SR	z_{JK}	Return	Volatility	Skew	SR	z_{JK}
Equal	15.54	11.32	-0.015	1.373	-	1.14	14.29	-2.074	0.080	-
RV	14.07	9.84	-0.228	1.429	0.71	1.03	13.69	-1.782	0.076	-0.04
Equal (TV)	25.12	15.50	0.265	1.621	**2.75**	3.59	9.20	-1.320	0.391	**2.77**
RV (TV)	28.00	16.12	0.272	1.738	**2.60**	4.23	9.99	-0.597	0.424	**2.25**
RSJ/Corr (TV)	27.78	14.65	0.275	1.896	**3.82**	5.88	9.32	-0.557	0.631	**3.48**
RSJ/Down Corr (TV)	27.55	14.63	0.252	1.884	**3.79**	4.84	10.33	-2.312	0.469	**2.62**
RSJ/Beta (TV)	28.01	14.64	0.288	1.913	**3.98**	5.39	9.66	-0.825	0.558	**3.18**
RSJ/Down Beta (TV)	28.16	14.63	0.278	1.925	**4.15**	5.17	9.98	-1.722	0.518	**2.98**
RSJ/CoSkew (TV)	27.62	14.70	0.298	1.878	**3.90**	3.09	8.99	-1.057	0.343	1.73
RSJ/CoKurt (TV)	27.66	14.55	0.268	1.901	**3.84**	5.19	10.28	-2.066	0.505	**2.93**
RSJ/LPM Beta (TV)	28.13	14.65	0.289	1.920	**4.13**	5.85	9.17	-0.371	0.638	**3.53**
RSJ/HTCR Beta (TV)	27.84	14.68	0.294	1.897	**3.96**	5.05	9.16	-0.192	0.552	**2.97**
RSJ/Tail Beta (TV)	27.75	14.83	0.235	1.872	**3.98**	5.26	9.46	-0.293	0.556	**3.10**
RSJ/Tail Sens (TV)	27.29	14.76	0.271	1.849	**3.59**	5.52	8.60	0.164	0.642	**3.29**
RSJ/Tail Risk (TV)	27.74	14.74	0.262	1.882	**3.91**	5.26	9.25	-0.123	0.568	**3.12**
RSJ/MES (TV)	28.19	14.67	0.282	1.921	**4.20**	5.59	8.81	0.106	0.634	**3.22**

Similar to Table V, we next show in Table XII results for the momentum strategies that are overlayed by volatility targeting in the two 15 years periods where the equally weighted momentum portfolio exhibits the highest and lowest average return. This table shows that volatility targeting is advantageous regardless of whether the momentum portfolio is in a bull or bear regime. During the bull period, all volatility targeting strategies produce a higher Sharpe Ratio and this increase is also statistically significant. The highest Sharpe Ratios are obtained by the strategies using the (systematic) tail risk weighting. Further, all volatility targeting strategies

[116] A possible extension of this approach would be to manage the portfolio's tail risk instead of volatility. Doing this should further reduce extremely high losses. Rickenberg (2020b) and Rickenberg (2020a) presents portfolio risk targeting strategies that manage the portfolio's volatility in up-markets and the portfolio's tail risk in down-markets. Rickenberg (2020a) applies this approach to the equally weighted momentum portfolio and finds that this approach outperforms volatility targeting by further reducing left tail risk without sacrificing return potential. However, in order to not combine too many different strategies, we only show results for the volatility targeting approach in the main part. Results for the tail risk targeting strategy are shown in Appendix B.10.

exhibit a positive skewness, whereas the skewness of the strategies without volatility targeting is negative. During the bear period, the strategies without volatility targeting have positive Sharpe Ratios, which are, however, very low. This low Sharpe Ratio can be massively increased by the volatility targeting approach. All portfolios that are overlayed with volatility targeting exhibit a statistically higher Sharpe Ratio than the equally weighted momentum portfolio without volatility targeting. The highest Sharpe Ratios are again achieved by the strategies using the (systematic) tail risk weighting. Further, left tail risk is again significantly reduced by volatility targeting. In particular, strategies that use the (systematic) tail risk weighting combined with volatility targeting provide the best left tail risk reduction. Interestingly, two of these strategies even obtain a positive skewness, whereas the skewness of the equally weighted momentum strategy is -2.074. The strategy that applies volatility targeting to the equally weighted momentum portfolio still has a skewness of -1.320. Thus, although volatility targeting is an appealing method to reduce the probability of momentum crashes, left tail risk can further be reduced by additionally weighting assets of the momentum portfolio by their (systematic) tail risk.

As in Table VI, we next examine in Table XIII the performance of the volatility targeting strategies in different sub-samples. Since the sample for the volatility targeting strategies is six months shorter, sub-samples are also slightly different to the sub-samples in Table VI. Table XIII shows that volatility targeting increases the Sharpe Ratio in all three sub-samples and that these increases are statistically significant for every weighting scheme and sub-sample. This is in line with Barroso and Santa-Clara (2015) and Rickenberg (2020a) who also find that volatility targeting is beneficial in different sub-samples. Further, in all sub-samples, Sharpe Ratios are the highest for the strategies that combine volatility targeting with the (systematic) tail risk weightings. As in Table X, volatility targeting significantly reduces left tail risk in all three sub-samples by producing a higher skewness, lower kurtosis and lower drawdowns. In two of the three sub-samples, volatility targeting even produces a positive skewness, whereas the skewness of the strategies without volatility targeting is negative. The crash risk reduction is again the highest for the models using the (systematic) tail risk weightings combined with volatility targeting. In particular, the drawdown reduction is most pronounced in the last sub-sample, where

Table XIII. Performance Results in Different Sub-Samples: Volatility Targeting
This table shows performance results of the equally weighted and volatility weighted momentum strategies without volatility targeting as well as the equally weighted and risk weighted momentum strategies that are overlayed by the target volatility (TV) strategy in three sub-samples. The description of the columns is given in Table I.

	Panel A: 01.05.1931 – 30.06.1960									
Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	5.22	11.93	-1.948	17.402	0.438	-	48.48	0.108	-25.96	14.57
RV	5.01	11.41	-1.737	13.998	0.440	0.03	45.89	0.109	-21.59	12.00
Equal (TV)	8.50	9.61	-0.557	6.974	0.885	**4.24**	32.31	0.263	-16.27	9.88
RV (TV)	9.51	10.45	-0.125	5.070	0.910	**3.63**	36.25	0.262	-13.65	11.13
RSJ/Corr (TV)	9.82	9.59	-0.195	6.143	1.023	**4.54**	28.48	0.345	-14.62	11.51
RSJ/Down Corr (TV)	9.30	10.14	-1.232	14.409	0.917	**3.98**	37.91	0.245	-22.54	11.51
RSJ/Beta (TV)	9.72	9.81	-0.368	6.971	0.991	**4.47**	30.61	0.318	-16.21	11.51
RSJ/Down Beta (TV)	9.45	9.96	-0.865	11.012	0.948	**4.20**	34.67	0.272	-20.16	11.51
RSJ/CoSkew (TV)	8.23	9.43	-0.382	7.457	0.873	**3.37**	31.80	0.259	-15.83	11.51
RSJ/CoKurt (TV)	9.37	10.09	-1.102	12.879	0.929	**4.12**	35.81	0.262	-21.56	11.51
RSJ/LPM Beta (TV)	9.93	9.53	-0.110	5.436	1.042	**4.68**	29.12	0.341	-13.27	11.51
RSJ/HTCR Beta (TV)	9.45	9.54	-0.020	4.705	0.990	**4.23**	28.16	0.336	-11.21	11.51
RSJ/Tail Beta (TV)	9.59	9.72	-0.098	5.025	0.986	**4.36**	32.09	0.299	-12.69	11.51
RSJ/Tail Sens (TV)	9.77	9.24	0.154	4.223	1.057	**4.53**	25.89	0.377	-8.60	11.51
RSJ/Tail Risk (TV)	9.61	9.60	0.005	4.772	1.000	**4.41**	31.25	0.307	-11.79	11.51
RSJ/MES (TV)	9.88	9.34	0.130	4.329	1.059	**4.55**	26.55	0.372	-9.19	11.51

	Panel B: 01.07.1960 – 31.08.1989									
Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	12.53	9.99	-0.043	4.599	1.254	-	13.20	0.949	-9.73	11.25
RV	12.04	8.81	-0.183	4.574	1.368	1.63	13.38	0.900	-9.02	9.12
Equal (TV)	20.86	14.16	0.235	3.773	1.473	**3.60**	19.14	1.090	-10.90	15.95
RV (TV)	24.03	14.90	0.166	4.327	1.613	**3.43**	19.08	1.259	-14.02	17.87
RSJ/Corr (TV)	22.35	13.52	0.264	3.900	1.653	**4.00**	17.27	1.294	-11.83	13.49
RSJ/Down Corr (TV)	22.22	13.50	0.251	3.912	1.646	**3.98**	17.56	1.265	-11.83	13.49
RSJ/Beta (TV)	22.39	13.52	0.274	3.886	1.656	**4.03**	17.32	1.292	-11.83	13.49
RSJ/Down Beta (TV)	22.39	13.51	0.268	3.896	1.657	**4.09**	17.63	1.270	-11.84	13.49
RSJ/CoSkew (TV)	22.34	13.56	0.279	3.715	1.647	**4.08**	18.95	1.179	-10.36	13.49
RSJ/CoKurt (TV)	22.12	13.46	0.265	3.898	1.643	**3.88**	17.77	1.245	-11.81	13.49
RSJ/LPM Beta (TV)	22.48	13.52	0.274	3.870	1.663	**4.15**	17.10	1.315	-11.77	13.49
RSJ/HTCR Beta (TV)	22.21	13.54	0.281	3.874	1.640	**3.93**	17.96	1.237	-11.76	13.49
RSJ/Tail Beta (TV)	22.42	13.61	0.235	3.893	1.647	**4.16**	16.48	1.361	-11.76	13.49
RSJ/Tail Sens (TV)	22.14	13.59	0.259	3.867	1.630	**3.87**	17.24	1.284	-11.73	13.49
RSJ/Tail Risk (TV)	22.47	13.59	0.244	3.860	1.653	**4.21**	17.32	1.298	-11.80	13.49
RSJ/MES (TV)	22.54	13.53	0.272	3.860	1.666	**4.23**	16.52	1.365	-11.74	13.49

	Panel C: 01.09.1989 – 31.12.2018									
Model	Return	Volatility	Skew	Kurt	SR	z_{JK}	MDD	Calmar	Min	Max
Equal	9.78	13.68	-0.554	8.989	0.715	-	33.86	0.289	-24.01	18.61
RV	8.15	11.61	-1.111	11.700	0.702	-0.10	37.97	0.215	-20.43	13.57
Equal (TV)	11.07	10.91	0.111	6.591	1.015	**3.56**	17.14	0.646	-16.61	15.14
RV (TV)	12.81	11.88	0.339	6.768	1.079	**2.95**	17.46	0.734	-17.46	16.67
RSJ/Corr (TV)	14.85	10.15	0.305	4.169	1.463	**5.79**	11.60	1.280	-9.48	11.44
RSJ/Down Corr (TV)	14.21	10.32	0.249	4.395	1.377	**5.28**	16.29	0.873	-9.77	12.20
RSJ/Beta (TV)	14.95	10.35	0.328	4.475	1.444	**5.60**	12.22	1.223	-10.37	12.16
RSJ/Down Beta (TV)	14.75	10.25	0.314	4.302	1.439	**5.62**	13.33	1.107	-10.05	12.22
RSJ/CoSkew (TV)	13.43	10.72	0.369	6.301	1.253	**4.42**	18.59	0.722	-13.28	17.39
RSJ/CoKurt (TV)	14.30	10.29	0.181	4.132	1.390	**5.37**	13.34	1.072	-10.01	11.29
RSJ/LPM Beta (TV)	15.09	10.27	0.358	4.498	1.470	**5.78**	11.78	1.281	-10.07	12.90
RSJ/HTCR Beta (TV)	14.91	10.17	0.308	4.315	1.465	**5.75**	13.30	1.121	-9.96	11.84
RSJ/Tail Beta (TV)	14.64	10.65	0.287	4.909	1.374	**5.52**	13.96	1.049	-12.09	12.59
RSJ/Tail Sens (TV)	15.09	10.21	0.349	4.256	1.479	**5.96**	10.82	1.395	-9.74	12.25
RSJ/Tail Risk (TV)	14.78	10.61	0.387	5.103	1.392	**5.56**	12.60	1.173	-11.80	13.46
RSJ/MES (TV)	15.24	10.49	0.447	4.780	1.453	**5.71**	12.15	1.254	-10.19	13.82

the drawdown of the switching strategies with volatility targeting is only about one third of the drawdown of the equally weighted momentum strategy without volatility targeting. This high drawdown reduction is also accompanied with higher returns. This can be seen by the Calmar Ratio of the switching strategies, which is about four times the Calmar Ratio of the equally weighted momentum strategy. In total, Table XI, Table XII and Table XIII show that simultaneously accounting for individual asset and portfolio risk is beneficial in different sub-samples and market environments. In particular, strategies that simultaneously manage the individual assets' (systematic) tail risk and the momentum portfolio's portfolio risk outperform strategies that ignore any kind of risk, strategies that account only for individual asset risk or strategies that only manage portfolio risk.

Table XIV. Economic Value: Volatility Targeting
This table shows the annualized percentage fee an investor is willing to pay to switch from the equally weighted momentum strategy to the volatility weighted strategy or a strategy that uses the target volatility (TV) approach. The fee is calculated for a mean-variance investor with risk aversion $\gamma = 5$, a CRRA investor with $\gamma = 5$ and a loss-averse investor with a loss aversion of $l = 2$. The fee is calculated over the whole period, the first half and the second half. All strategies are rescaled to the same level of volatility.

	Whole Period			First Half			Second Half		
Model	$\Delta_{MV}^{\gamma=5}$	$\Delta_{CRRA}^{\gamma=5}$	$\Delta_{LA}^{l=2}$	$\Delta_{MV}^{\gamma=5}$	$\Delta_{CRRA}^{\gamma=5}$	$\Delta_{LA}^{l=2}$	$\Delta_{MV}^{\gamma=5}$	$\Delta_{CRRA}^{\gamma=5}$	$\Delta_{LA}^{l=2}$
RV	0.173	0.075	0.180	-0.031	0.000	-0.131	0.430	0.225	0.553
Equal (TV)	4.067	4.516	3.469	4.912	5.457	4.018	3.227	3.583	2.926
RV (TV)	4.904	5.378	4.275	5.101	5.614	4.055	4.757	5.142	4.494
RSJ/Corr (TV)	6.792	7.282	6.203	6.708	7.282	5.945	6.849	7.202	6.459
RSJ/Down Corr (TV)	6.097	6.485	5.622	5.859	6.247	5.290	6.328	6.724	5.968
RSJ/Beta (TV)	6.639	7.122	6.108	6.464	7.043	5.742	6.796	7.202	6.482
RSJ/Down Beta (TV)	6.462	6.883	5.966	6.140	6.644	5.462	6.780	7.202	6.494
RSJ/CoSkew (TV)	5.556	6.009	5.030	5.681	6.247	5.011	5.411	5.772	5.043
RSJ/CoKurt (TV)	6.172	6.565	5.667	5.978	6.406	5.439	6.351	6.724	5.898
RSJ/LPM Beta (TV)	6.927	7.442	6.355	6.823	7.442	5.996	7.007	7.442	6.719
RSJ/HTCR Beta (TV)	6.636	7.122	6.048	6.404	7.043	5.573	6.853	7.282	6.537
RSJ/Tail Beta (TV)	6.355	6.803	5.787	6.204	6.803	5.387	6.489	6.883	6.195
RSJ/Tail Sens (TV)	6.868	7.362	6.218	6.780	7.442	5.910	6.936	7.362	6.526
RSJ/Tail Risk (TV)	6.493	6.963	5.915	6.448	7.043	5.599	6.516	6.883	6.234
RSJ/MES (TV)	6.950	7.442	6.396	6.943	7.603	6.097	6.939	7.362	6.696

We next assess how valuable the combination of managing individual asset and portfolio risk is for investors. To determine the economic value of this combined strategy, we again calculate the annualized percentage fee a mean-variance, CRRA or loss-averse investor is willing to pay to switch from the equally weighted momentum strategy to one of our risk-managed strategies. Since the strategies with and without volatility targeting have quite different levels of volatility, we first rescale all strategies to the same level of volatility. The economic value for the three investors is given in Table XIV, where we again calculate the economic value over the whole period, the first half and the second half. For comparison, we again show the economic value of the volatility weighted strategy, which is almost zero and even negative for the first half. In contrast, volatility targeting significantly increases the economic value, where the results are quite robust for the three samples. This is again in line with previous studies that the benefits of volatility targeting are not much influenced by the sample period (Barroso and Santa-Clara, 2015, Rickenberg, 2020a). Further, the economic value among the three investors is again quite similar, where the highest fees are found for the CRRA investor. This is line with the high left tail risk reduction of volatility targeting, which makes this approach highly valuable for CRRA investors who dislike negative skewness and high kurtosis (Guidolin and Timmermann, 2008). Moreover, in line with our previous findings, we again find the highest fees for the strategies that combine the (systematic) tail risk weighting with volatility targeting. Among the strategies that use volatility targeting, an investor is typically willing to pay an almost 50% higher fee for the strategies that use the (systematic) tail risk weighting approach compared to the equally or volatility weightings. In particular, fees are quite similar among the (systematic) tail risk weighted strategies.

As in Figure II, we next show in Figure IV the rolling economic value for a mean-variance investor who invests for five or 20 years in four different risk-managed momentum portfolios. The four risk-managed strategies are the volatility weighted portfolio without volatility targeting as well as the equally weighted, volatility weighted and (systematic) tail risk weighted strategies with volatility targeting. Panel A shows the economic value for the five years investment horizon, whereas Panel B shows results for the 20 years investment horizon. Figure IV shows that the rolling economic value of the strategies that manage the whole portfolio's risk is always higher than the economic value of the strategy that only manages the individual assets' volatilities. This especially holds for Panel B where the investor has a twenty years investment horizon. The highest economic value is typically achieved by the strategy that combines the (systematic) tail risk weighting with the volatility targeting approach. Panel B also shows that the economic value of the volatility weighted momentum strategy can become negative, whereas the eco-

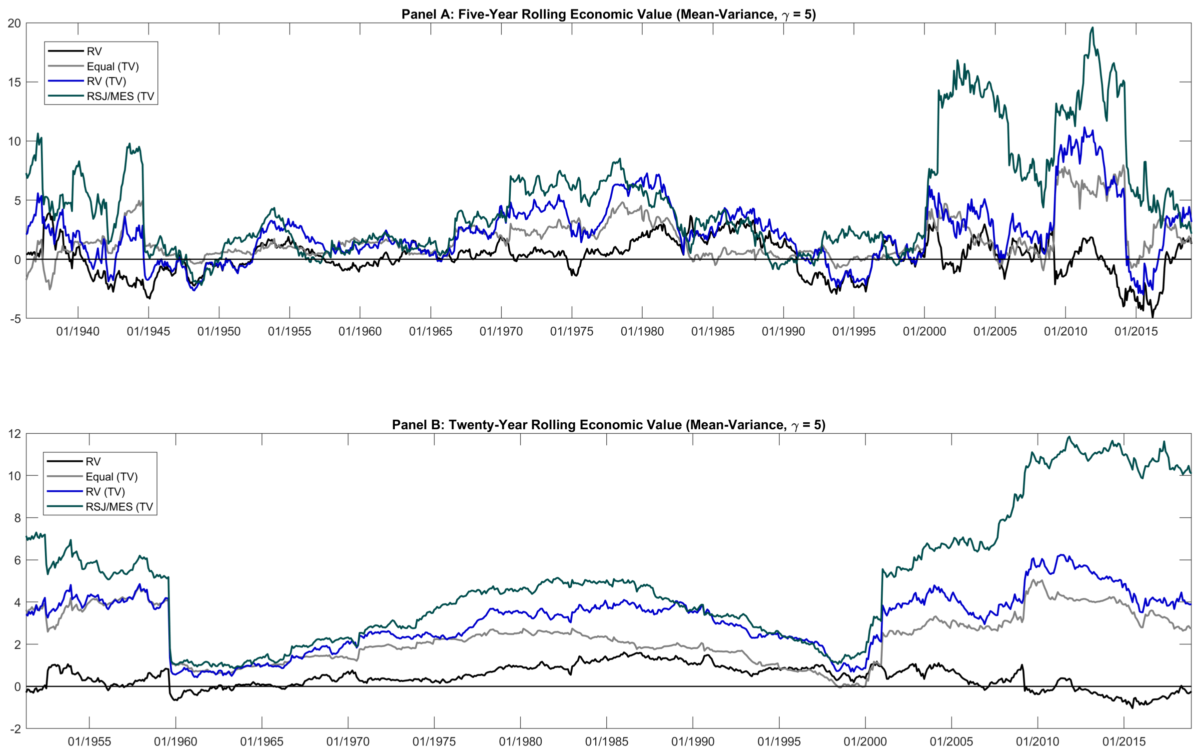

Figure IV. Rolling Economic Value: Volatility Targeting. This figure plots the rolling economic value for a mean-variance investor with risk aversion $\gamma = 5$ who invests in the volatility weighted momentum portfolio without volatility targeting as well as the equally weighted, volatility weighted and (systematic) tail risk weighted momentum portfolio with target volatility (TV) overlay. Panel A shows the rolling economic value for an investor who invests for five years in these portfolios. Panel B shows the economic value for an investor who invests for 20 years in these portfolios.

nomic value of the strategies that use volatility targeting is always positive. Thus, a long-term investor should time monthly portfolio volatility as also found by Moreira and Muir (2019).

Finally, to test if the economic value found in Table XIV is also statistically significant, we repeat the testing approach of Table IX for the volatility targeting strategies, where we rescale the strategies to the same level of volatility. Results for these tests are shown in Table XV. The DM-test, the stepwise SPA-test and the FDR approach indicate that all strategies that use volatility targeting produce statistically significant utility increases. However, the RC-test, the SPA-test and the MCS indicate that only the strategies that weight assets by their (systematic) tail risk combined with volatility targeting generate significant utility increases. This highlights that different testing approaches can lead to different results. The DM-test, the stepwise-SPA test and the FDR approach each test all strategies against the equally weighted momentum portfolio. Since the equally and volatility weighted portfolios combined with volatility targeting

Table XV. Test for Significant Utility Increases: Volatility Targeting
This table shows results of the tests that test for statistically significant utility increases of the risk managed strategies compared to the equally weighted momentum portfolio. The description of the columns is given in Table IX.

Panel A: MV	DM-test	p^{RC}	p^{SPA}	p^{SQ}	Step-SPA	Step-SPAst	$FDR^+ = 5\%$
Equal	-	0.00	0.00	0.00	-	-	-
RV	0.37	0.00	0.00	0.00	0	0	0
Equal (TV)	**5.82**	0.11	0.00	0.08	**1**	**1**	**14**
RV (TV)	**5.14**	1.68	0.39	1.37	**1**	**1**	**13**
RSJ/Corr (TV)	**7.94**	**90.94**	**35.00**	**79.57**	**1**	**1**	**4**
RSJ/Down Corr (TV)	**7.90**	**34.17**	3.17	3.44	**1**	**1**	**11**
RSJ/Beta (TV)	**8.05**	**75.50**	2.91	**19.69**	**1**	**1**	**5**
RSJ/Down Beta (TV)	**8.08**	**59.01**	4.53	**14.54**	**1**	**1**	**8**
RSJ/CoSkew (TV)	**6.65**	**13.07**	0.30	0.68	**1**	**1**	**12**
RSJ/CoKurt (TV)	**7.60**	**38.50**	2.78	3.89	**1**	**1**	**10**
RSJ/LPM Beta (TV)	**8.26**	**98.39**	**58.06**	**90.48**	**1**	**1**	**2**
RSJ/HTCR Beta (TV)	**7.61**	**72.27**	**10.23**	**36.99**	**1**	**1**	**6**
RSJ/Tail Beta (TV)	**8.06**	**50.42**	1.73	5.16	**1**	**1**	**9**
RSJ/Tail Sens (TV)	**7.91**	**89.90**	**45.33**	**90.48**	**1**	**1**	**3**
RSJ/Tail Risk (TV)	**7.94**	**63.00**	4.26	9.84	**1**	**1**	**7**
RSJ/MES (TV)	**8.28**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**
Panel B: CRRA	**DM-test**	**p^{RC}**	**p^{SPA}**	**p^{SQ}**	**Step-SPA**	**Step-SPAst**	**$FDR^+ = 5\%$**
Equal	-	0.00	0.00	0.00	-	-	-
RV	0.16	0.00	0.00	0.04	0	0	0
Equal (TV)	**5.62**	0.25	0.00	0.23	**1**	**1**	**14**
RV (TV)	**5.16**	3.30	0.56	1.99	**1**	**1**	**13**
RSJ/Corr (TV)	**7.50**	**89.83**	**32.35**	**72.81**	**1**	**1**	**4**
RSJ/Down Corr (TV)	**7.70**	**32.30**	4.81	4.01	**1**	**1**	**11**
RSJ/Beta (TV)	**7.65**	**74.19**	2.98	**17.83**	**1**	**1**	**6**
RSJ/Down Beta (TV)	**7.84**	**55.09**	6.64	**13.09**	**1**	**1**	**8**
RSJ/CoSkew (TV)	**6.50**	**16.20**	0.20	0.88	**1**	**1**	**12**
RSJ/CoKurt (TV)	**7.43**	**35.99**	3.97	4.24	**1**	**1**	**10**
RSJ/LPM Beta (TV)	**7.78**	**97.86**	**50.26**	**88.76**	**1**	**1**	**2**
RSJ/HTCR Beta (TV)	**7.21**	**72.65**	9.37	**34.04**	**1**	**1**	**5**
RSJ/Tail Beta (TV)	**7.58**	**51.86**	1.76	5.29	**1**	**1**	**9**
RSJ/Tail Sens (TV)	**7.41**	**90.16**	**44.84**	**88.76**	**1**	**1**	**3**
RSJ/Tail Risk (TV)	**7.47**	**64.49**	3.76	9.45	**1**	**1**	**7**
RSJ/MES (TV)	**7.66**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**
Panel C: Loss Aversion	**DM-test**	**p^{RC}**	**p^{SPA}**	**p^{SQ}**	**Step-SPA**	**Step-SPAst**	**$FDR^+ = 5\%$**
Equal	-	0.00	0.00	0.00	-	-	-
RV	0.39	0.00	0.00	0.00	0	0	0
Equal (TV)	**5.12**	0.00	0.00	0.00	**1**	**1**	**14**
RV (TV)	**4.73**	0.38	0.18	0.19	**1**	**1**	**13**
RSJ/Corr (TV)	**7.09**	**87.27**	**30.88**	**55.58**	**1**	**1**	**3**
RSJ/Down Corr (TV)	**7.15**	**37.93**	2.94	4.10	**1**	**1**	**11**
RSJ/Beta (TV)	**7.17**	**80.37**	**11.02**	**39.75**	**1**	**1**	**5**
RSJ/Down Beta (TV)	**7.12**	**66.79**	5.66	**28.52**	**1**	**1**	**7**
RSJ/CoSkew (TV)	**6.09**	9.08	0.55	0.78	**1**	**1**	**12**
RSJ/CoKurt (TV)	**6.70**	**40.49**	2.74	5.22	**1**	**1**	**10**
RSJ/LPM Beta (TV)	**7.31**	**98.07**	**55.11**	**92.38**	**1**	**1**	**2**
RSJ/HTCR Beta (TV)	**6.76**	**67.01**	**11.48**	**26.72**	**1**	**1**	**6**
RSJ/Tail Beta (TV)	**7.07**	**49.62**	3.99	8.75	**1**	**1**	**9**
RSJ/Tail Sens (TV)	**7.05**	**80.37**	**27.86**	**55.58**	**1**	**1**	**4**
RSJ/Tail Risk (TV)	**7.03**	**55.98**	3.95	**14.14**	**1**	**1**	**8**
RSJ/MES (TV)	**7.43**	**100.00**	**100.00**	**100.00**	**1**	**1**	**1**

produce a significantly higher utility than the equally weighted strategy without volatility targeting, these three tests mark the two models with volatility targeting as superior. In contrast, the RC-test, the SPA-test and the MCS test each model against *all* remaining models. Since the (systematic) tail risk weighted strategies outperform the equally and volatility weighted strategies, the two strategies are not identified as superior for these tests. In total, results of Table XV again demonstrate the advantages of weighting momentum's constituents by their (systematic) tail risk, even when the whole portfolio's risk in managed. Among the (systematic) tail risk weighted strategies, the strategy using MES as systematic tail risk measure produces the most convincing results. In line with our earlier findings, results are again quite similar for the three investors.

To summarize this section, we find that managing a portfolio's risk is typically more important than managing the portfolio's individual asset risk. This is in line with the observation that correlations increase in bear markets, i.e. assets typically co-crash (Ang and Chen, 2002, Chabi-Yo et al., 2018, Poon et al., 2004). Hence, downturn periods are best managed by reducing the exposure to the portfolio and not by diversification. However, we find that simultaneously managing individual asset *and* portfolio risk significantly outperforms strategies that ignore all kinds of risk, only manage individual asset risk or only account for portfolio risk but ignore individual asset risk. Additionally, we find that the (systematic) tail risk weightings are advantageous to equal or volatility weightings, even when the different weighting schemes are combined with the volatility targeting strategy.

3.7.5 Robustness Checks

We have so far shown that our (systematic) tail risk weighted momentum strategies significantly outperform the equally and volatility weighted strategies, especially when these weightings are combined with the target volatility approach. However, we have so far only examined one momentum strategy using 30 equally weighted US industry portfolios, the $t-12$ to $t-1$ months ranking period and a cut-off point of $p = 30\%$. Several studies show that the profitability of momentum investing can be quite different when different data sets are used, assets are ranked based on other ranking periods or winners and losers are defined based on other cut-off points. Similarly, risk based portfolio allocation methods are also highly influenced by different data set characteristics (Kirby and Ostdiek, 2012). Moreover, besides industry momentum, other portfolio based momentum strategies are also frequently examined. For example, Lewellen (2002), Stivers and Sun (2010) and Novy-Marx (2012) examine momentum of investment styles, whereas Chan et al. (2000), Asness et al. (2013), Bhojraj and Swaminathan (2006), Novy-Marx (2012), Nijman et al. (2004) and Richards (1997) examine momentum of country indices. Further, momentum also works for assets outside the US (Asness et al., 2013, Fama and French, 2012, Griffin et al., 2003, Nijman et al., 2004, Rouwenhorst, 1998, Swinkels, 2002). Besides using only one certain momentum strategy and data set, we have also shown results for one certain estimation method for each risk measure. Risk measures estimated with other estimation methods can sometimes produce quite different estimates of an asset's risk. Thus, different estimation methods can also produce quite different performance results for risk weighted portfolios. To rule out the possibility that our (systematic) tail risk based approach does only work for a certain momentum strategy and estimation method, we also examined several robustness results. These robustness results are shown in Appendix B and are shortly summarized here. A more detailed description of the robustness results can be found in Appendix B.

In Section B.1, we show that our (systematic) tail risk weightings also work well when risk is estimated based on alternative estimation windows and cut-off points, whereas the volatility weighted strategy does not significantly outperform the equally weighted strategy for different volatility estimates. Section B.2 shows that industry momentum also works well when four other other ranking periods are used. In particular, we find that the (systematic) tail risk weighted momentum strategies clearly outperform the equally weighted and volatility weighted strategies for all ranking periods. Section B.3 shows that industry momentum also works well for alternative cut-off points to determine winners and losers. We find that the (systematic) tail risk weighted momentum strategies significantly outperform the non-managed momentum strategy for all cut-off points. In Section B.4, we apply our approaches to the industry momentum strategy using different industry classifications. Instead of using 30 equally weighted US industries, we also use smaller and bigger sets of US industry portfolios between 5 and 49 US industries. Further, we show that our results are also robust to using value-weighted industries instead of equally weighted industries. In Section B.5, we show that industry momentum also performs well for industries outside the US by applying the strategies to International and European industries. We again find that the risk-managed momentum strategies significantly outperform the equally weighted momentum strategy. Section B.6 shows results for the momentum strategies applied to several style portfolios in the US, Internationally and in Europe. Confirming the finding of Stivers and Sun (2010), Lewellen (2002) and Novy-Marx (2012), we find that momentum also works well for investment styles. In line with our earlier findings, we show that accounting for the individual styles' risk and the momentum portfolio's risk significantly outperforms the equally weighted style momentum strategy. This holds especially when individual asset risk is managed by our (systematic) tail risk switching approach. Section B.7 applies momentum to two data sets consisting of country indices. Confirming the earlier results of Chan et al. (2000), Asness et al. (2013), Bhojraj and Swaminathan (2006), Novy-Marx (2012) and Richards (1997), we find that past winning countries outperform past losing countries. However, the performance of the country momentum strategy can again significantly be increased by accounting for the countries' individual risk. In Section B.8, we show that the switching approach presented in Section 3.5 is also robust to several other definitions of the crash indicator δ_t based on momentum's past volatility and past return as well as past market volatility. In Section B.9, we show additional results for the volatility targeting approach presented in Section 3.6 for other definitions of σ_{target} and other volatility models. Section B.10 shows that our results are robust to using the tail risk targeting strategy of Rickenberg (2020a,b) instead of the volatility targeting strategy of Barroso and Santa-Clara (2015) to manage portfolio risk. In Section B.11, we assess the profitability of the non-managed and risk-managed momentum strategies based on the strategies' alpha using the CAPM, the Fama and French (1993) three factor model and the Carhart (1997) four factor model. Moreover, we conduct spanning tests in the manner of Daniel and Moskowitz (2016, Sec. 4.4) and Moreira and Muir (2017) to control for the performance of the strategies that use other weighting schemes. Results in this section show that the profitability of the (systematic) tail risk weighted strategy cannot be explained by the factor models or the equally and volatility weighted momentum strategies. This finding is confirmed in Section B.12, where we assess the significance of our findings when other benchmarks that also use the target volatility overlay are chosen for the Diebold and Mariano (1995) and Jobson and Korkie (1981) tests. Results in this section confirm that the (systematic) tail risk weighting is superior to the remaining weighting schemes, even when the different weighting schemes are combined with volatility targeting. Finally, Appendix B.13 shows that the (systematic) tail risk weighting is also superior to the equal and volatility weightings when only the winners are regarded. In particular, this strategy clearly outperforms other portfolio methods, like the equally weighted, minimum variance and mean-variance portfolios, that are based on all available assets.

In total, our robustness results show that our simple (systematic) risk weighting approach also works well for several other portfolio based momentum strategies based on other data sets, other ranking periods and other cut-off points. Further, other estimation windows can also be used to estimate the assets' (systematic) tail risk. In particular, our weighting approach works best when it is combined with a risk targeting strategy that manages portfolio risk, measured by volatility or tail risk. The robustness of our results is striking, since DeMiguel et al. (2009b) find that portfolio allocation methods perform quite differently when other data sets are used. Further, Kirby and Ostdiek (2012) and Zakamulin (2015) find that risk based portfolio allocation methods based on industries typically do not outperform equally weighted portfolios. Similarly, Clare et al. (2016) find that risk based asset allocation methods do not outperform the equally weighted portfolio when only assets of the same asset class, in our paper equities, are used. Thus, in view of these earlier findings, the significant outperformance of our approach and the robustness of our results are striking. This is especially the case since our (systematic) tail risk approach is easy to understand and implement, and thus can be an appealing alternative for practitioners.

3.8 Conclusion

This paper studies different weighting schemes applied to industry, style and country momentum. Momentum strategies examined in the literature so far typically use simple weighting schemes that do not incorporate different risk levels of the assets that are contained in the momentum portfolio. Momentum strategies based on these simple weightings, like the equally weighted momentum strategy, exhibit a suboptimal risk-return profile, a fat left tail and a high crash risk. More recently, in order to improve momentum's risk-return profile, volatility weighted momentum strategies have been examined in the financial literature. However, we find that using volatility to determine portfolio weights is suboptimal for long-short strategies like momentum. The reason for this finding is that volatility weighting improves the performance of the long and short portfolio, i.e. the benefits of buying the enhanced winners portfolio is offset by shorting the enhanced losers portfolio. For that reason, we develop several weighting schemes based on univariate and systematic tail risk measures. These weighting schemes incorporate the assets' non-normalities and distinguish between long and short positions. In particular, we develop a strategy that manages momentum's univariate risk in low risk periods and switches to a systematic tail risk weighting when a momentum crash becomes likely. We find that this approach significantly outperforms the equally and volatility weighted momentum strategies. The good performance of this strategy results since the winners' performance is improved, whereas the losers performance is worsened. In total, the (systematic) tail risk weighted momentum strategies exhibit higher returns while simultaneously left tail risk is reduced. Furthermore, these weighting schemes are highly valuable for mean-variance, CRRA and loss-averse investors who are willing to pay high and statistically significant fees to have access to these weightings. In particular, the outperformance of the (systematic) tail risk weighted momentum strategy holds in different sub-samples as well as in bull and bear markets.

Since the weighting schemes examined in this paper only change the allocation among the assets in the winners and losers portfolios, the amount invested long in the winners and short in the losers is not changed and can be scaled arbitrarily. Thus, these risk weighted momentum portfolios can be overlayed by a target risk strategy that targets a constant level of portfolio risk. By doing this, this combined strategy simultaneously manages individual asset risk and portfolio risk. We find that accounting for both types of risk outperforms strategies that do not manage any kind of risk, only manage individual asset risk or only manage portfolio risk. In particular, the best risk-return profile is obtained by the strategies that manage individual (systematic) tail risk combined with the target risk approach.

Future research could employ our risk management approach to other long-short strategies. Further, similar to the BAB and BAC strategy of Frazzini and Pedersen (2014) and Asness et al. (2020), long-short strategies that bet against skewness, RSJ or other systematic tail risk measures, combined with the rank weighting, could also be examined in future research. Finally, our (systematic) tail risk weighting combined with the risk targeting approach could also be a promising method to reduce the high crash risk of the individual stock momentum strategy that is frequently examined in the financial literature.

Appendix to Chapter 3

A Advantages of Volatility Weighted Portfolios

This section summarizes several advantages of portfolio strategies that overweight low volatile assets. We further summarize studies that have used these strategies and how different portfolio allocation methods are connected. We also motivate why combining a volatility based portfolio allocation with momentum is appealing and why this approach can be an alternative to other portfolio optimization methods. As stated in Section 3.2.4, volatility based portfolio allocation methods are motivated by the low volatility anomaly that has been examined quite frequently and that states that low volatile assets obtain higher (risk-adjusted) returns than highly volatile assets.

Several explanations for the low volatility anomaly have been examined in the literature so far. For example, a possible explanation for the improved risk-return profile of low risk assets is leverage aversion of investors as examined by Asness et al. (2012), Frazzini and Pedersen (2014) and Asness et al. (2020) in a similar setting.[117] Another explanation is that low volatility investing heavily loads on other firm characteristics that are related to higher returns. Jordan and Riley (2015) use volatility as a new factor and extend the Carhart (1997) four factor model to a five factor model that incorporates a low volatility factor. Jordan and Riley (2015) find that the low volatility factor, applied to mutual funds, captures similar characteristics as the profitability and investment factors (Fama and French, 2016), i.e. "low volatility funds hold stocks in companies that are more profitable and invest more conservatively than companies whose stock is held by high volatility funds". Similarly, Fama and French (2016, p. 92) state that

[117] Asness et al. (2012, p. 50) state: "Because some investors choose to overweight riskier assets in order to avoid leverage, the price of riskier assets is elevated or, equivalently, the expected return on riskier assets is reduced. In contrast, the safer assets are underweighted by these investors and thus trade at low prices (i.e., offer high expected returns)."

"returns of low volatility stocks behave like those of firms that are profitable but conservative in terms of investment, whereas the returns of high volatility stocks behave like those of firms that are relatively unprofitable but nevertheless invest aggressively". This result is confirmed by Asness et al. (2020, Table 7). Similarly, low volatility portfolios typically load on value stocks (see Blitz (2016) and references therein). Further, Walkshäusl (2014, Exhibit 2) finds that low risk strategies heavily load on stocks of conservative industries, such as consumer staples, health care and utilities. Nevertheless, these loadings cannot completely explain the low risk anomaly and the relation between risk and return is still negative or flat even when it is controlled for profitability, investment, value or industry classification (Asness et al., 2014, Blitz, 2016, Blitz and Vidojevic, 2017). For example, Blitz (2016, p. 99) concludes "that the low-volatility effect is a distinct phenomenon that cannot be explained by the value effect". Other studies explain the finding of a low volatility anomaly by a bad research design of these studies, loadings on other risks or investors' preferences.[118] For example, Bali and Cakici (2008) examine the impact of several characteristics, like different data frequencies and weighting schemes, on the cross-sectional relation of return and volatility. The authors find that the relation between volatility and return can be highly different for these modifications. Fu (2009) state that using more advanced volatility forecasting models can solve the low volatility puzzle. Chen and Petkova (2012) show that the low volatility anomaly can be explained by exposure to changes of average market volatility. Amaya et al. (2015), Boyer et al. (2009) and Schneider et al. (2020) state that the low volatility anomaly can be explained by investors' (co)skewness preferences. Similarly, Bali et al. (2011) and Bali et al. (2017a) state that low risk anomalies can be explained by investors' lottery demand. See also Asness et al. (2012, Footnote 3), Asness et al. (2020), Blitz et al. (2019) and Chen and Petkova (2012, Sec. 6) for a review of further explanations of the low volatility anomaly. In total, many explanations have been made for the low volatility anomaly. However, the outperformance of low risk assets is still puzzling and exploiting this effect is

[118] This is also the case for other risk based anomalies. For example, Cederburg et al. (2020) state that the (time-series) volatility anomaly of Moreira and Muir (2017) can be explained by a bad performance evaluation measure that relies on unconditional portfolio alphas. Similarly, the low beta anomaly of Frazzini and Pedersen (2014) can also be solved by changing the research design. For example, Cederburg and O'Doherty (2016) and Schneider et al. (2020) solve the low beta anomaly by advanced performance evaluation methods, Bali et al. (2017a) explain the low beta anomaly by investors' lottery demand and Bali et al. (2017b) find that the low beta anomaly disappears once beta is estimated conditionally.

advantageous for investors.

The low volatility anomaly can easily be exploited by so called “low volatility portfolios” that only buy the least volatile assets from a given universe. These strategies typically produce convincing risk-adjusted returns and are frequently used by practitioners (Blitz and Van Vliet, 2007, Chow et al., 2014).[119] Further, Blitz and Van Vliet (2007) find that assets with lower volatility also have lower drawdowns. Similarly, Jang and Kang (2019, Table 1) find that an increase of volatility also increases the probability of extremely positive and negative returns, where the impact for extremely negative returns is higher. In total, buying the least volatile assets should produce an enhanced risk-return profile where especially drawdowns are attenuated. Nevertheless, the aim of this paper is to apply different weighting schemes to the winners and losers portfolio. Thus, the idea of low volatility portfolios cannot be applied directly to the momentum portfolio, but these findings demonstrate the advantage of overweighting low volatile assets.[120]

Besides the inverse volatility and rank weighting schemes presented in Section 3.2.4, other weighting schemes that exploit the low volatility anomaly are also frequently used in the academic literature and financial industry. The most frequently used portfolio allocation method that incorporates the assets' volatilities is the well-known mean-variance optimization. The mean-variance approach determines portfolio weights based on the assets' mean returns and covariance matrix. Although this optimization method is frequently applied, most studies state that mean-variance optimization does not perform well in out-of-sample applications. The reason for this finding is that mean returns are hard to estimate (Merton, 1980), which leads to

[119] Low volatility portfolios have the advantage that no portfolio optimization is needed. Chow et al. (2014) find a similar risk-adjusted performance for mean-variance optimized portfolios and low volatility portfolios. Similarly, Walkshäusl (2014, p. 55) “conclude that minimum-volatility, low-volatility and low-beta strategies seem to perform rather equally around the world”. Moreover, Chow et al. (2014) find that the mean-variance optimization typically has a higher turnover and thus produces higher transaction costs than the strategy that only buys the least volatile assets. In contrast, low risk portfolios can be implemented in practice with quite low transaction costs (Blitz et al., 2019). Zakamulin (2017) find that the profitability of the minimum variance portfolio is mainly driven by the low volatility effect. Thus, simple low volatility portfolios are an appealing alternative to more complex portfolio strategies and again highlight the advantage of overweighting low volatile assets.

[120] One way to adapt the idea of low volatility portfolios in the context of momentum portfolios would be to only buy assets with a high past return and a low volatility (Blitz and van Vliet, 2018). Similarly, the losers portfolio could contain assets with a low past return and high volatility as suggested by Ang et al. (2006b). This would be similar to the approach of Jacobs et al. (2015) who buy and sell assets based on their momentum and (skewness) risk.

suboptimal portfolio weights and high transaction costs.[121]

Due to the finding that mean-variance portfolios do not perform well in out-of-sample examinations, several alternatives that do not incorporate an estimate of mean returns have been developed in the financial literature. One frequently used portfolio optimization method that is directly linked to the mean-variance optimization and does not rely on an estimate of the assets' mean return is the minimum variance optimization. The minimum variance portfolio is the unique portfolio on the mean-variance efficient frontier that is independent of the mean-return (Merton, 1972). Thus, the minimum variance portfolio follows from the mean-variance portfolio by ignoring the mean return. This portfolio optimization typically outperforms the mean-variance approach since no mean return estimate is needed.

A third volatility based weighting scheme that is also examined in the literature and is frequently applied by practitioners is the risk parity approach (Asness et al., 2012, Asvanunt et al., 2015, Baltas, 2015, Maillard et al., 2010).[122] Risk parity is similar in nature to the frequently used minimum variance portfolio strategy and is a compromise of the equally weighted portfolio and the minimum variance optimization. Maillard et al. (2010, p. 60) state that the equally weighted portfolio has a limited diversification if risks of the assets are highly different and that minimum variance portfolios suffer under a high portfolio concentration and are typically invested in a limited number of assets. In contrast, risk parity aims to equalize the portfolio risk contribution of each asset. Hence, risk parity is a middle ground between the equally weighted portfolio and the minimum variance portfolio. As a consequence, risk parity portfolios are typically better diversified than equally weighted or minimum variance portfolios (Maillard et al., 2010). In particular, the inverse volatility weighting in Equation (3.2.3) is a special case of the risk parity approach.[123] Thus, the inverse risk weighting is expected to be better diversified than

[121] Merton (1980) state that the estimation accuracy of volatilities can be increased by using finer data frequencies. However, this result does not hold for mean returns, which makes the estimation of mean returns more challenging. Kritzman et al. (2010) state that the estimation accuracy of mean returns can be increased by expanding the estimation window and using long data samples.

[122] See Jagannathan and Ma (2003), DeMiguel et al. (2009a), DeMiguel et al. (2009b), Kan and Zhou (2007), Tu and Zhou (2011), Garlappi et al. (2006), Kritzman et al. (2010), Zakamulin (2015), Fleming et al. (2003), Fleming et al. (2001), Han (2005), Chow et al. (1999) and Behr et al. (2012) for studies on risk parity, minimum variance and mean-variance portfolios in the financial literature.

[123] Maillard et al. (2010) show that the inverse volatility weighting is a special case of the risk parity portfolio under the assumption that the correlations of the assets are equal (but not necessarily zero). Similarly, Kirby and Ostdiek (2012) and Zakamulin (2015, p. 90) show that an inverse variance weighting follows from the minimum

the minimum variance portfolio and this approach is also less sensitive to estimation risk and transaction costs. Due to the high importance of risk-managed industry momentum for practitioners, we concentrate in this paper on the two easiest portfolio allocation methods, i.e. equally weighted and inverse volatility weighted portfolios. Nevertheless, applying more complex portfolio optimization methods that also incorporate information on the assets' correlations could also be appealing. For example, Baltas (2015) applies the risk parity approach to long-short trend-following strategies and finds that incorporating the assets' correlations is advantageous when the portfolio's risk is managed. Similarly, the mean-variance and minimum variance portfolios could also be applied to the winners and losers portfolios. However, as stated in Sections 3.3 and 3.4, these approaches are based on volatility and have the disadvantage that it is not distinguished between long and short positions.[124] Therefore, another appealing alternative would be to apply downside risk based portfolio optimization methods to the winners and losers (see Alexander and Baptista (2004), Basak and Shapiro (2001) and Agarwal and Naik (2004) for further details on these approaches). These approaches would consider correlations as proposed by Baltas (2015) and would distinguish between long and short positions since risk could be defined as left (right) tail risk for an asset in the winners (losers) portfolio. We leave the examination of applying downside risk based portfolio optimization methods to the winners and losers portfolios for future research.

The portfolio optimization methods presented above and in Section 3.2.4 are frequently used in the literature and are compared to the equally weighted portfolio that is also used in this paper. For example, DeMiguel et al. (2009b) compare several portfolio allocation methods and find surprisingly good results of the equally weighted strategy and that the equally weighted portfolio is not statistically outperformed by the mean-variance portfolio and other risk based strategies. The authors claim that huge amounts of data are needed to provide accurate estimates of the assets' monthly risk. However, one drawback of the authors' examination is the use of monthly returns to estimate monthly risk. As stated above, the estimation precision can signifi-

variance strategy when all correlations are zero. The inverse volatility weighting of Equation (3.2.3) then follows if the sensitivity to volatility changes is reduced by a tuning parameter. By using a tuning parameter, the inverse volatility weighting typically produces lower transaction costs than the inverse variance weighting and minimum variance portfolio.

[124] Baltas (2015) extends the risk parity approach in a way that it can be applied to long-short portfolios.

cantly be increased by using daily data. For example, Jagannathan and Ma (2003, Table V) find good performance results of models that estimate monthly covariances by daily data and that this approach works equally well as several other more complex estimation methods. Fleming et al. (2001), Fleming et al. (2003), Han (2005) and Taylor (2014) also find good results of volatility based trading strategies that estimate the covariance matrix with higher frequency data. Thus, the bad performance of the volatility weighted strategies found by DeMiguel et al. (2009b) is mainly driven by a bad research design as extensively shown by Kirby and Ostdiek (2012) and Kritzman et al. (2010). For example, Clarke et al. (2006) find that minimum variance portfolios deliver equally high returns with significantly lower risk compared to the market portfolio. Zakamulin (2015) also finds good results of minimum variance portfolios with short sale constraints. Similarly, Maillard et al. (2010) find that the equally weighted portfolio is outperformed by the minimum variance and risk parity portfolio. In particular, Kirby and Ostdiek (2012) and Zakamulin (2015, Exhibit 3) find that simple inverse volatility weighting strategies that do not rely on portfolio optimization outperform the equally weighted portfolio and more complex strategies like the mean-variance approach. Similarly, Walkshäusl (2014, Exhibit 1) finds that low volatility, low beta and minimum variance portfolios outperform the market portfolio by producing higher returns with lower levels of volatility than the market. In total, volatility managed portfolios typically produce higher Sharpe Ratios compared to other static portfolio allocations and investors are willing to pay high fees to have access to a volatility managed portfolio (Fleming et al., 2001, 2003, Kirby and Ostdiek, 2012, Moreira and Muir, 2017, Taylor, 2014). Interestingly, this result does not only hold for short-term investors, but even long-term investors should time (short-term) volatility (Moreira and Muir, 2019, Table 4). The benefits of volatility managed portfolios also hold when these strategies are applied to portfolios, such as industry and style portfolios. For example, Kirby and Ostdiek (2012, Sec. V.B) examine the inverse volatility weighting applied to 10 US industry portfolios and find that the volatility managed portfolio outperforms the equally weighted portfolio without producing higher transaction costs. Applying the inverse volatility weighting to industries is appealing, since Harvey et al. (2018, Exhibit 18) show that volatility and return of the 10 US industries are negatively correlated, i.e. industries with a higher volatility produce lower returns and should be weighted lower (see also Zakamulin (2017)). Nevertheless, Zakamulin (2015, 2017) applies the inverse volatility weighting to industry and style portfolios and finds that this approach works well for style portfolios, but has difficulties with industry portfolios. Kirby and Ostdiek (2012) also find a weaker benefit of applying the inverse volatility weighting to industries compared to other assets and that the "data set that poses the biggest challenge to the timing strategies contains 10 industry portfolios". This results since different industries have quite similar mean returns. Thus, weighting industries inversely to their volatility outperforms the strategy that weights industries equally in terms of higher risk-adjusted returns, but industry portfolios are "notorious for being very difficult to use in portfolio optimization" (Zakamulin, 2015, p. 96).

The volatility based weighting schemes are typically applied to long-only portfolios. However, the inverse volatility weighting has not only been used as a simple asset allocation tool for long-only portfolios, but has also been used in many other areas. For example, Harvey and Siddique (2000, Table IV) use volatility weighting in an asset pricing context. Bali and Cakici (2008, Sec. V) examine the low volatility puzzle by additionally weighting assets inversely to their volatility. The authors find that different weighting schemes, including equal weighting and inverse risk weighting, can lead to quite different portfolio returns. Asness et al. (2014, p. 28) use the inverse volatility weighting to weight the assets in the betting against beta (BAB) portfolio.[125] Moskowitz et al. (2012), Kim et al. (2016), Du Plessis and Hallerbach (2017), Dudler et al. (2015), Baltas (2015), Clare et al. (2016) and Goyal and Jegadeesh (2017) apply the inverse volatility weighting to the time series momentum (TSMOM) strategy, which is a similar approach to applying the inverse volatility weighting to the (cross-sectional) momentum strategy.[126] Clare et al. (2016), Kim et al. (2016) and Goyal and Jegadeesh (2017) show

[125]The betting against beta strategy is long low beta assets and short high beta assets (Asness et al., 2014, Bali et al., 2017a, Cederburg and O'Doherty, 2016, Fama and French, 2016, Frazzini and Pedersen, 2014, Lettau et al., 2014, Schneider et al., 2020). This strategy is further examined in Section 3.4.

[126]The TSMOM strategy is similar to the cross-sectional momentum strategy that is examined in our paper and also relies on the assumption that a high past performance predicts a good future performance. However, TSMOM relies on an asset's own past performance to determine buy and sell signals, whereas the cross-sectional momentum strategy defines buy and sell signals relative to the performance of other assets. Thus, TSMOM is more related to the field of trend-following (Bajgrowicz and Scaillet, 2012, Sullivan et al., 1999). Both strategies, TSMOM and cross-sectional momentum, are extensively compared by Moskowitz et al. (2012), Goyal and Jegadeesh (2017) and Kim et al. (2016). Jegadeesh and Titman (2002) show that TSMOM and not cross-sectional differences in expected returns, as suggested by Conrad and Kaul (1998), is a main driver of the profitability of the cross-sectional

that the good performance of the time series momentum strategy of Moskowitz et al. (2012) is also driven by the use of the inverse volatility weighting.[127] Thus, these studies show that combining information on an asset's past performance and volatility works well in a portfolio context and outperforms a strategy that only incorporates information on the assets' past performance.[128] The benefits of combining information on an asset's past performance and volatility has also been shown by Blitz and van Vliet (2018). Similarly, Ang et al. (2006b) combine the low volatility effect with the momentum effect by first sorting assets into quintiles based on their past return and then sorting assets within each quintile by their (idiosyncratic) volatility. The results of Ang et al. (2006b) indicate that both effects, the momentum effect and the low volatility effect, capture different characteristics, and thus both effects can be combined.[129] In a similar way, Novy-Marx (2012, Table 16) shows that the low volatility anomaly also holds after controlling for momentum and the author concludes that "[h]igher realized volatility is also associated, even after controlling for past performance, with lower expected returns." Guo and Savickas (2010, p. 1643) also find that both effects are different and that the low volatility effect does not explain momentum returns. Thus, even after choosing assets based on their momentum, volatility contains additional information and highly volatile assets underperform assets with a lower volatility. This is also shown by Kirby and Ostdiek (2012, p. 462) who

momentum strategy. Goyal and Jegadeesh (2017) show that differences between TSMOM and momentum mainly occur since momentum is a zero-investment strategy, whereas TSMOM typically has a positive average market exposure.

[127]Clare et al. (2016, Table 1) compare four different strategies applied to several asset classes. The four strategies are the equally weighted strategy applied to all assets, TSMOM using equal weights, inverse volatility weighting applied to all assets and the combination of TSMOM with the inverse volatility weighting. The authors find that the equally weighted portfolio is clearly outperformed by the remaining strategies. Further, the two TSMOM approaches outperform the inverse volatility weighting applied to all assets. The strategy with the highest Sharpe Ratio and lowest drawdown is the strategy that combines the inverse volatility weighting with the TSMOM approach.

[128]Dudler et al. (2015) and Clare et al. (2016, Sec. 3.6) also use volatility weighting applied to the TSMOM strategy, but additionally examine buy and sell signals based on volatility adjusted past returns instead of raw returns. The authors find superior results and lower transaction costs for the strategy that uses volatility adjusted returns. Similarly, Rachev et al. (2007) examine momentum strategies by ranking assets based on their risk-adjusted performance.

[129]The approach of Ang et al. (2006b) is different to our approach since Ang et al. (2006b) use past return and volatility to determine in which quintile an asset belongs, where the assets within a quintile are value-weighted. We use past return to determine in which portfolio an asset belongs and use volatility to determine the asset's weight. Another approach would be to use an asset's risk for both, ranking and weighting the asset. For example, Frazzini and Pedersen (2014), Asness et al. (2020) and Asness et al. (2014) use past risk, measured by an assets' beta, correlation or volatility to determine both, the belonging to the long or short portfolio and the weighting of each asset within these portfolios.

apply volatility timing to 10 momentum portfolios and find that volatility timing significantly enhances the risk-return profile, even when transaction costs are considered. Although the approach of Kirby and Ostdiek (2012) is different to our approach, this result again shows that combing information on an asset's momentum and risk is appealing.[130] Furthermore, the inverse volatility weighting applied to an individual stock based momentum strategy also has two additional advantages. First, highly volatile stocks are typically small and illiquid, and thus suffer under high transaction costs (Bali and Cakici, 2008). The inverse risk weighting lowers the weight invested in these stocks, and thus lowers transaction costs of the momentum strategy that is typically highly invested in these stocks. This holds especially for the losers portfolio since shorting highly volatile losers produces high transaction costs (see Blitz et al. (2019) and references therein). Second, Bali and Cakici (2008, Table 6) find a negative risk-return relation for small stocks but no relation for large sized stocks. Thus, the negative risk-return relation is more pronounced for the momentum strategy, since this strategy is highly invested in small sized firms. This finding is confirmed by Barroso and Maio (2019) who test the risk-return relation for several factor portfolios and show that the negative risk-return relation is most pronounced for the momentum portfolio. However, these two reasons are not that important for the industry momentum strategy, but again show the benefits of combining information on past return and volatility.

For the reasons that were discussed above, the inverse volatility weighting is an appealing approach to manage momentum crashes. In particular, this approach is different to the risk targeting approach that is frequently used in the literature to manage momentum crashes (Barroso and Maio, 2018, Barroso and Santa-Clara, 2015, Cederburg et al., 2020, Daniel and Moskowitz, 2016, Grobys, 2018, Grobys and Kolari, 2020, Grobys et al., 2018, Moreira and Muir, 2017, Rickenberg, 2020a). Risk targeting manages momentum's risk on a portfolio level, whereas the inverse volatility weighting manages risk on an individual asset level. Thus, risk targeting totally ignores information on each asset in the momentum portfolio. Several studies show

[130] Kirby and Ostdiek (2012) divide their dataset in 10 momentum portfolios and weight these portfolios inversely to their volatility. This approach is similar to Asness et al. (2013) who also weight different momentum portfolios by their volatility to construct a global momentum portfolio. In contrast, we use information on an asset's momentum to form a winners and a losers portfolio. Assets within these two portfolios are then weighted by their volatility.

that risks of the assets in the winners and losers portfolio can be quite different. For example, winners and losers portfolios have quite different levels of volatility, skewness and kurtosis (Harvey and Siddique, 2000). In particular, Rickenberg (2020a) shows that the skewness of winners and losers moves in opposite directions. Further, Bollerslev et al. (2015, p. 131) find that investors' fear is priced differently for winners and losers. Generally, past performance is an important determinant of an asset's risk, which makes risk characteristics of winners and losers very different (Amaya et al., 2015, Chen et al., 2001, Harvey and Siddique, 2000, Langlois, 2020). In particular, momentum crashes mainly occur when the losers portfolio sharply rises which is not upset by an adequate rise of the winners portfolio (Daniel et al., 2017, Daniel and Moskowitz, 2016).[131] Hence, instead of managing the whole momentum portfolio's risk, the winners and losers portfolios' risk should be managed separately, as done by the inverse volatility weighting. Moreira and Muir (2017, Sec. II.D) show that accounting for risk in the cross-section, as done by inverse risk weighting, and accounting for risk on a portfolio level, as done by risk targeting, are two different empirical phenomena. This observation is also confirmed by Du Plessis and Hallerbach (2017) for the industry momentum strategy. The authors show that both approaches work well in order to manage the industry momentum portfolio's risk, but both approaches deliver different results. Additionally, both approaches can easily be combined as done by Baltas (2015), Moreira and Muir (2017), Zakamulin (2015) and Harvey et al. (2018). The authors show that managing both kinds of risk, i.e. individual asset risk and portfolio risk, is superior to strategies that manage either individual asset risk or portfolio risk. This is also confirmed by the results of Cederburg et al. (2020), Moreira and Muir (2017), Barroso and Maio (2018) and Rickenberg (2020a) who show that targeting the risk of the betting against beta (BAB) anomaly, i.e. the portfolio that buys low risk assets, sells high risk assets and weights assets inversely to their risk, substantially improves the risk-adjusted performance of the BAB portfolio.[132] Combing both approaches means managing the portfolio's risk by first

[131]Daniel and Moskowitz (2016) show that these periods can be predicted based on past market return and volatility. However, Wang and Xu (2015) find that market volatility predicts returns of the winners and losers portfolio asymmetrically. This again shows that winners and losers behave quite differently.

[132]By weighting assets with a higher beta lower than assets with a lower beta, the betting against beta portfolio of Frazzini and Pedersen (2014) is similar to the inverse volatility portfolio. However, Asness et al. (2020) compare the beta anomaly to the volatility anomaly and find that both capture different aspects of risk and that exploiting the low beta anomaly produces higher risk-adjusted returns.

managing the portfolio's constituents' individual risk and then managing the whole portfolio's risk. This approach is further examined in Section 3.6.

The approach of combining the momentum strategy with the inverse risk weighting is important for two strands in the financial literature. First, by weighting the assets of the momentum portfolio inversely to their risk, the risk-weighted portfolio should exhibit an enhanced risk-return profile, where especially momentum crashes should be attenuated. Hence, this approach fits well to the literature on momentum crash management (Barroso and Santa-Clara, 2015, Daniel and Moskowitz, 2016, Grundy and Martin, 2001, Moreira and Muir, 2017, Rickenberg, 2020a). Second, choosing the assets in the long and short portfolio based on their past *relative* performance and then applying the inverse risk weighting to these assets is an appealing and simple asset allocation tool based on the assets' *relative mean* and risk. Portfolio allocation methods examined in the literature mainly focus on models that incorporate an estimate of the assets' *absolute mean* and risk or approaches that only focus on the assets' risk. The approaches that are solely based on the assets' risk totally ignore information on the assets' performance. In contrast, the approaches based on an estimate of the assets' absolute mean, like the mean-variance approach, incorporate more information, but these approaches perform bad in out-of-sample studies and usually take extreme weights in the individual assets (Behr et al., 2012, DeMiguel et al., 2009a,b, Garlappi et al., 2006, Jagannathan and Ma, 2003, Kan and Zhou, 2007, Kirby and Ostdiek, 2012, Kritzman et al., 2010). The bad performance of these approaches results due to the high estimation risk of the mean return (Merton, 1980). For example, Garlappi et al. (2006) find that the mean-variance portfolio is outperformed by the minimum variance approach since the mean is hard to estimate.[133] DeMiguel et al. (2009b) find that ignoring the mean return in a portfolio optimization setting leads to less extreme weights. Kirby and Ostdiek (2012) also find that portfolio methods that incorporate an estimate of the absolute

[133]The high estimation risk of the mean-variance approach has led to several studies examining alternative estimation methods for the mean return and covariance matrix. Frequently used approaches to reduce estimation risk are shrinkage estimators and other Bayesian approaches (Garlappi et al., 2006, Kan and Zhou, 2007, Kirby and Ostdiek, 2012). See the appendix of Clarke et al. (2006) for a short summary of both approaches. Shrinkage estimators are also frequently applied to the covariance matrix of the minimum variance approach (DeMiguel et al., 2009a, Kan and Zhou, 2007). Further, imposing a short-sale constraint, which has a similar effect as using shrinkage estimators, can also reduce the extreme weights of the unconstrained mean-variance approach (Jagannathan and Ma, 2003).

mean produce high transaction costs. This result also holds for portfolio methods based on industry portfolios (Kirby and Ostdiek, 2012, Table 1). However, although the minimum variance approach produces less extreme weights than the mean-variance approach, Kirby and Ostdiek (2012) show that the minimum variance portfolio can still exhibit extreme weights. The authors find that the high misspecification of the mean-variance and minimum variance approach can be further reduced by the inverse volatility weighting, since this approach is more robust against estimation risk and eventually outperforms more complex portfolio optimization methods.[134] Similarly, Moreira and Muir (2019) find that parameter uncertainty is less important for volatility managed portfolios but has a higher impact on mean-volatility managed portfolios. Moreira and Muir (2019, p. 509) conclude that "ignoring variation in volatility is very costly, and the benefits to timing volatility are significantly larger than the benefits to timing expected returns".[135]

Based on the arguments summarized above, portfolio allocations based on estimates of the absolute mean return are suboptimal in practice. In contrast, the approaches that only focus on the assets' risk, mainly measured by variance, typically perform well out-of-sample (Fleming et al., 2001, 2003, Han, 2005, Kirby and Ostdiek, 2012), but discard information on the assets' performance that can potentially be valuable. Our approach, which could be called a *relative mean-risk approach*, is a middle ground between a mean-variance based portfolio allocation and a portfolio allocation that is solely based on the assets' volatility.[136] Christoffersen and Diebold

[134] Kirby and Ostdiek (2012, p. 456-457) compare the volatility timing strategy with a strategy that relies on an estimate of volatility and expected mean and find that volatility timing is superior for inaccurate estimates of the expected mean. Nevertheless, both strategies perform similar (at least before transaction costs) when the expected mean is estimated with a more complex model or more data (see also Kritzman et al. (2010)). This again highlights that mean-variance optimization strongly depends on the mean estimation method and that incorporating a mean estimate does not significantly enhance the portfolio's performance.

[135] Generally, estimation uncertainty is more of a concern for expected return timing strategies than for volatility timing strategies. Moreira and Muir (2019, Sec. 5.2) show that investors' utility gains of expected return timing strategies are highly influenced by estimation uncertainty, whereas the investors' utility gains of volatility timing strategies are less sensitive to estimation uncertainty. In particular, investors that use expected return timing strategies can have high utility losses when noisy mean estimates are used. In contrast, volatility timing increases the investors' utility even when volatility estimates are noisy. Moreira and Muir (2019, p. 524) conclude that "the benefits of timing expected returns are very sensitive to parameter uncertainty. [...] However, this result is not true with volatility. [...] Hence, the utility gains from volatility timing are far more robust to parameter uncertainty".

[136] Behr et al. (2012) also show how information on the relative performance of industries can be incorporated in portfolio allocation decisions. The authors start from a given portfolio optimization, like the mean-variance allocation, and modify portfolio weights based on the past relative performance of several industry portfolios. For a given portfolio allocation, they modify only the weights for the industries in the winners and losers portfolios, whereas the weights of the industries in the middle part are unchanged. The authors find that combining information on the

(2006) find that conditional (absolute) returns are not forecastable, whereas signs of returns and volatility are forecastable. This makes a relative mean-risk approach appealing for practical implementations, since this approach uses more information than the simple inverse volatility portfolio while simultaneously estimation risk is limited. In particular, this simple approach does not need an estimate of a large covariance matrix, which also reduces estimation risk.[137] Moreover, estimation risk can further be reduced by focusing on the risk-weighted winners portfolio. Moskowitz and Grinblatt (1999), Chan et al. (2000) and Bhojraj and Swaminathan (2006) find that the profitability of portfolio based momentum strategies is mainly driven by the returns of the winners portfolio.[138] This is also advantageous from a practical view, since shorting losers also produces higher transaction costs than buying winners (Korajczyk and Sadka, 2004, Lesmond et al., 2004). For that reasons, Korajczyk and Sadka (2004) and Clare et al. (2016) examine only the winners portfolio instead of the winners minus losers portfolio. Thus, for practical implementations, a strategy that only buys the winner industries and weights assets in the winners portfolio inversely to their risk could be an appealing alternative to more complex portfolio allocations. In particular, this strategy is only based on market data and does not need shorting of assets or any accounting data, which is important from a practical view (Blitz and van Vliet, 2018). Moreover, this approach is similar to the reward-to-risk timing strategies examined by Kirby and Ostdiek (2012). Kirby and Ostdiek (2012, p. 438) state that such simple portfolio allocation methods are advantageous since they "retain the most appealing features of the [equally weighted] strategy (no optimization, no covariance matrix inversion, and no short sales) while exploiting sample information about the reward and risk characteristics of the assets

industries' risk and momentum produces an enhanced portfolio allocation.

[137]For the mean-variance approach, an asset's weight is a function of the asset's mean return and the covariance matrix of all assets. For our approach, an asset's weight is only based on an estimate of whether the asset performs better or worse than a certain percentage, e.g. 90%, of all other assets. Further, only a limited number of risk forecasts for the assets in the winners and losers portfolios are needed. For example, the mean-variance approach applied to the 49 industry data set requires $49 \cdot (49 + 1)/2 = 1225$ parameter estimates for the covariance matrix and 49 estimates for the mean returns. In contrast, our approach with $p = 10\%$ requires only the estimates of the relative performance and ten risk estimates (five estimates for the winners and five for the losers portfolio). A long-only strategy that only buys the winners reduces the required risk estimates to only five. Kan and Zhou (2007) find that estimation risk of the covariance matrix can also lead to suboptimal portfolio weights, especially for large data sets.

[138]This contradicts the finding of the individual stock momentum strategy, where the profitability is mainly driven by shorting (illiquid and small sized) loser stocks (Hong et al., 2000, Lesmond et al., 2004). Generally, the profits of individual stock based long-short anomalies are mainly driven by the short side (see Jang and Kang (2019) and references therein).

under consideration." The strategy that buys the risk-managed winners portfolio is examined in Appendix B.13 and compared to the mean-variance and minimum variance portfolios as well as the strategy that weights all assets equally.

B Robustness Results

This section shows additional performance results for different momentum strategies using alternative estimation windows and/or cut-off points to quantify (systematic) risk, alternative ranking periods, alternative cut-off points to determine winners and losers as well as alternative data sets of US industries. Additionally, we show results for European and International industry momentum, results for several US, European and International style momentum strategies and results for country momentum. We further show results for other definitions of the momentum crash indicator δ_t, other risk targeting strategies including the tail risk targeting approach of Rickenberg (2020a,b) and results for long-only portfolios. Throughout this section, we only show results for the strategies' return, volatility, Sharpe Ratio and Jobson and Korkie (1981) test as well as the economic value for mean-variance, CRRA and loss-averse investors with the corresponding DM-test of Diebold and Mariano (1995). The Jobson and Korkie (1981) test and the DM-test are calculated with respect to the equally weighted momentum strategy. In Section B.11, we show additional results for portfolio alphas and in Section B.12 we show results for the Diebold and Mariano (1995) and Jobson and Korkie (1981) tests calculated with respect to other benchmark strategies that also use volatility targeting.

B.1 Alternative Estimators

In this section, we show results for the risk weighted momentum strategies using risk measures estimated with alternative estimation windows and/or cut-off points to define down days. Several estimation windows are frequently used in the financial literature to estimate (systematic) tail risk, which can sometimes lead to quite different results. Univariate risk measures, like Realized Volatility, are typically estimated using the last one to six months of daily data. For example, Ang et al. (2006b) and Ang et al. (2009) estimate volatility using the last month of daily data for their main results. Furthermore, the authors also check their results for other estimation lengths and find that the low volatility effect also holds for longer estimation periods and a one month gap (Ang et al., 2006b, Sec. II.F). A one month estimation window to estimate monthly volatility is also used by French et al. (1987), Moreira and Muir (2017) and Farago and Tédongap (2018). Grobys et al. (2018) compare results for realized volatility estimates using the last one to six months of daily data in a portfolio risk management setting and find that results are slightly different. The authors find that volatility managed portfolios perform the best when short estimation windows are used. Similarly, Ang et al. (2006b) and Ang et al. (2009, Table 5) find that the low risk effect is more pronounced when (idiosyncratic) volatility is estimated with short estimation windows, where the effect is most pronounced for one and three months. However, Ghysels et al. (2005) find that the risk-return relation can be quite different when different estimation windows are used and the authors suggest that "there is an optimal window size to estimate the risk-return trade-off" and that "[o]ne month's worth of daily data simply is not enough to reliably estimate the conditional variance" (Ghysels et al., 2005, p. 522). For that reason, longer estimation windows, like the six months period used in our paper, are frequently applied in the literature. For example, Barroso and Santa-Clara (2015) also use the past six months of daily data to estimate realized volatility. Fama and French (2016) find that the low volatility anomaly holds when volatility is estimated using the last 60 days of returns, i.e. they use about three months of data. Moreover, the low risk effect is also apparent when risk is estimated with longer samples such as 12 months. Furthermore, Blitz and Van Vliet (2007) and Blitz (2016) show that low volatility portfolios also perform well when volatility is estimated using data of the past three years. Using longer estimation windows to estimate monthly risk has the advantage that turnover and transaction costs of low risk portfolios are reduced (Blitz et al., 2019). In total, results in the literature suggest that it is not a priori clear which estimation window should be used to quantify volatility. Similarly, other univariate risk measures, like realized skewness, VaR, CVaR and LPM, are also frequently estimated based on alternative estimation windows. For example, Bali et al. (2009) estimate skewness, VaR, CVaR and LPM using the last one to six months of daily data. Bali et al. (2014) estimate LPM using the past six and twelve months of daily data. Amaya et al. (2015, Sec. 5.5) estimate realized skewness using the past week's, the past month's, the past six months', the past 12 months' and the past 60 months' daily returns and conclude that "results for historical skewness critically depend on the estimation window used". Chang et al. (2013, Sec. 4.7) also find that conclusions based on skewness and kurtosis estimates using different estimation windows can be highly different.

To rule out that results presented in Section 3.7 only hold for certain estimation windows of univariate risk measures, we next examine the performance of risk weighted momentum strategies using alternative estimation windows. In order to better assess how different estimation windows influence results of our weighting schemes, we only show results for the strategies that do not switch between univariate and systematic tail risk. However, the switching approach is also robust against using different estimation windows. Table XVI shows results for the volatility, skewness, LPM of order 1 and RSJ based momentum strategies estimated using the last one to six months of daily data. We focus on short- and medium-term estimation windows, since univariate risk is highly time-varying (Fu, 2009). Other estimation windows up to 60 months also work well but are not shown here. Further, similar results also hold for VaR, CVaR, LPM of other orders, down-to-up volatility, down-to-up skewness and R-Ratio. Additionally, weightings based on VaR, CVaR, LPM, down-to-up volatility, down-to-up skewness and R-Ratio also work well when other cut-off points are used to define down days or extreme realizations. For example, as in Bali et al. (2014) we also estimated LPM, down-to-up volatility and down-to-up skewness where down and up days are defined as returns in certain lower and upper quantiles. Atilgan et al. (2020) also show that the low tail risk anomaly holds when several cut-off points are used to define extreme losses.

Table XVI. Robustness Results: Univariate Risk Measures

This table shows performance results for the equally and risk weighted industry momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. Risk measures are estimated with daily data of the past one to six months. SR denotes the Sharpe Ratio and z_{JK} denotes the Jobson and Korkie (1981) test statistic.

Weighting	Equal		RV		LPM ($k = 1$)		Skew		RSJ	
Lengths	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}
1 Month	0.758	-	0.764	0.215	0.934	**3.796**	0.844	**2.105**	0.937	**4.076**
2 Months	0.758	-	0.723	-0.709	0.873	**2.519**	0.826	1.843	0.871	**2.602**
3 Months	0.758	-	0.746	-0.198	0.871	**2.486**	0.841	**2.219**	0.865	**2.514**
4 Months	0.758	-	0.761	0.144	0.865	**2.385**	0.883	**3.222**	0.887	**3.127**
5 Months	0.758	-	0.773	0.405	0.865	**2.395**	0.852	**2.326**	0.865	**2.434**
6 Months	0.758	-	0.776	0.478	0.853	**2.128**	0.854	**2.310**	0.832	1.653

Table XVI highlights that managing the winners' and losers' volatility does not significantly enhance momentum's risk-return profile, regardless of the chosen estimation length. The strategies that use volatility estimated with daily data of the last two or three months do even have a lower Sharpe Ratio than the equally weighted momentum strategy. In contrast, for the remaining risk measures, i.e. LPM, skewness and RSJ, the Sharpe Ratio is always higher than the Sharpe Ratio of the equally weighted strategy and this increase is statistically significant in most cases. Thus, results in Table XVI are mainly in line with our previous results and show that volatility weighting does not significantly enhance momentum's risk-return profile. In contrast, weightings based on tail risk measures produce statistically higher Sharpe Ratios for all risk measures and estimation windows except for two cases. In these two cases, the increases are only significant at the 10% level, i.e. $z_{JK} > 1.64$.

Similar to univariate risk measures, systematic risk measures used in the financial literature are also estimated using alternative estimation lengths. Since systematic risk measures quantify an asset's comovement with a benchmark portfolio, these risk measures are typically estimated based on longer estimation windows. This holds especially for the systematic tail risk measures that condition on a bad state of the benchmark portfolio. Frequently chosen window lengths to obtain systematic (tail) risk measures range from one to 60 months. For example, Ang et al. (2006a) estimate beta, downside beta, correlation, downside correlation, coskewness and cokurtosis using daily data of the past 12 months. Ang et al. (2006a, Sec. 2.4) show that their results also hold when they use the last two years of weekly data and other cut-off points to define down days for downside beta. Bali et al. (2014) use the past six and twelve months of daily data to estimate beta, coskewness, downside beta, LPM-beta and HTCR-beta. Similarly, Chabi-Yo et al. (2018) use the past 12 months of daily data to estimate beta, downside beta, coskewness, cokurtosis and lower tail dependency. In contrast, Van Oordt and Zhou (2016) estimate beta, downside beta, coskewness, cokurtosis and tail dependency using the past 60 months of daily data. Thus, several estimation windows to estimate systematic risk are used in the financial literature. Ang et al. (2006a, p. 1202) argue that using medium-term estimation windows, like the past 12 months, has two advantages over too short or too long estimation windows. First, the estimation of systematic (tail) risk needs a sufficient amount of data to provide reliable estimates. Second, since systematic tail risk is highly time-varying, too long estimation windows may cause the estimates to be noisy (see also Chabi-Yo et al. (2018)). Similarly, Langlois (2020, p. 405) use "a period of 12 months to measure risk measures such as coskewness because it provides a reasonable trade-off between having enough returns while allowing for variations over time." To assess the influence of the chosen estimation window on the results of the risk weighted momentum strategies, we next show results for systematic risk weighted strategies using estimation windows between six and 60 months.

Table XVII. Robustness Results: Correlation, Downside Correlation, Beta and Downside Beta
This table shows performance results for the equally and risk weighted industry momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. Risk measures are estimated with daily data of the past six to 60 months. SR denotes the Sharpe Ratio and z_{JK} denotes the Jobson and Korkie (1981) test statistic.

Model	Equal		Correlation		Downside Correlation		Beta		Downside Beta	
Lengths	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}
6 Months	0.758	-	0.897	**2.794**	0.863	**2.393**	0.924	**3.217**	0.947	**3.790**
9 Months	0.758	-	0.942	**3.720**	0.848	1.928	0.961	**3.909**	0.943	**3.808**
12 Months	0.758	-	0.944	**3.532**	0.836	1.582	0.971	**4.107**	0.902	**2.914**
24 Months	0.758	-	0.913	**3.092**	0.898	**2.693**	0.959	**3.956**	0.949	**3.665**
36 Months	0.758	-	0.890	**2.625**	0.917	**3.087**	0.918	**3.179**	0.956	**3.793**
48 Months	0.758	-	0.855	1.951	0.897	**2.691**	0.901	**2.928**	0.925	**3.218**
60 Months	0.758	-	0.849	1.864	0.888	**2.569**	0.896	**2.848**	0.919	**3.175**

Table XVII shows results for correlation, downside correlation, beta and downside beta using the past six to 60 months of daily data. Results of our systematic risk weighted strategies are again quite robust for different estimation lengths. The Sharpe Ratio is again higher in all cases and the increases in the Sharpe Ratios are significant in most cases. Nevertheless, the Sharpe Ratio is higher when systematic risk is estimated based on short- or medium-term estimation windows. Using the past four or five years of daily data also increases the Sharpe Ratio compared to the equally weighted strategy but the increase is only significant in three of four cases. This is in line with the finding of Ang et al. (2006a) that systematic tail risk is time-varying and should be estimated with short estimation windows. Table XVII also shows that weightings based on beta and downside beta are superior to the weightings based on correlation and downside correlation. As shown in Equation (3.4.3), the beta also considers the volatilities of industry i and the momentum portfolio. A similar decomposition also holds for downside beta (Ang et al., 2006a, Eq. (15)). Thus, additionally regarding the differences in the industries' univariate risk seems advantageous when industries are weighted by their systematic risk. Hong et al. (2007) also find that beta asymmetry is more pronounced than correlation asymmetry for portfolios that are sorted by their momentum. Thus, weighting assets by their downside beta, and hence also incorporating information on downside volatility, seems advantageous when industries are weighted by their risk.

Table XVIII shows further results for coskewness, cokurtosis, LPM-beta and HTCR-beta using the last six to 60 months of daily data. In particular, since Section 3.7 shows that the coskewness based weighting does not work well, we examine an alternative definition of coskewness by conditioning on negative momentum returns. We call this measure the *downside coskewness* and find that weighting industries by their downside coskewness performs very well compared to the usual definition of coskewness. Weighting industries based on their downside coskewness produces an enhanced risk-return profile, whereas the coskewness based weighted portfolio underperforms the equally weighted portfolio. However, although the Sharpe Ratio of the downside coskewness weighted momentum strategy is higher than the Sharpe Ratio of the equally weighted strategy in all cases, the increase is only statistically significant for the 24 months estimation window. In contrast, results for cokurtosis, LPM-beta and HTCR-beta are again quite robust for different estimation windows. The Sharpe Ratio is again higher than the Sharpe Ratio of the equally weighted strategy in all cases and the increases are statistically significant in all cases, except for the cokurtosis based weighting using the last 60 months of daily data. In total, Table XVIII again shows that our weighting approach works well for other estimation windows, where again short- and medium-term estimation windows outperform longer estimation windows.

Finally, besides choosing alternative estimation windows, alternative cut-off points to define extreme returns are also frequently applied in the financial literature on systematic tail risk. For example, Van Oordt and Zhou (2016) estimate Tail-beta using the past 60 months of daily data in order to provide enough data to obtain reliable estimates of systematic tail risk and the authors combine this estimation window with alternative cut-off points to determine extreme returns.

Table XVIII. Robustness Results: Coskewness, Cokurtosis, LPM-beta and HTCR-beta
This table shows performance results for the equally and risk weighted industry momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. Risk measures are estimated with daily data of the past six to 60 months. SR denotes the Sharpe Ratio and z_{JK} denotes the Jobson and Korkie (1981) test statistic.

Model	Equal		Down Coskewness		Cokurtosis		LPM-beta		HTCR-beta	
Lengths	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}
6 Months	0.758	-	0.758	0.058	0.903	**3.006**	1.008	**4.897**	0.934	**3.606**
9 Months	0.758	-	0.840	1.905	0.905	**3.107**	0.998	**4.743**	0.923	**3.409**
12 Months	0.758	-	0.792	0.734	0.904	**2.952**	0.991	**4.522**	0.949	**3.902**
24 Months	0.758	-	0.894	**2.731**	0.898	**2.803**	0.965	**4.049**	0.964	**4.208**
36 Months	0.758	-	0.851	1.840	0.892	**2.680**	0.933	**3.525**	0.954	**4.015**
48 Months	0.758	-	0.826	1.389	0.866	**2.219**	0.909	**3.022**	0.921	**3.408**
60 Months	0.758	-	0.814	1.167	0.845	1.797	0.910	**3.058**	0.926	**3.503**

Agarwal et al. (2017) estimate Tail-Sens and Tail-Risk using the past 24, 36 and 48 months of monthly data combined with cut-off points of $5\%, 10\%$ and 20%.[139] Chabi-Yo et al. (2018, Section III.D.1) extend the downside beta of Ang et al. (2006a) by conditioning on extremely low market returns in the lower $1\%, 2\%, 5\%$ and 10% quantile. Bali et al. (2014) estimate LPM-beta and HTCR-beta by conditioning on the lowest $5\%, 10\%$ and 20% observations and the authors combine the different cut-off points with different estimation windows. Acharya et al. (2016) use the past 12 months of daily returns and a cut-off of 5% to estimate MES. Kelly and Jiang (2014) estimate systematic tail risk based on the Hill estimator using a threshold of 5%. In particular, the choice of the cut-off point is important when systematic tail risk is estimated. For example, Farago and Tédongap (2018, p. 84) find that systematic tail risk highly depends on the chosen cut-off point and the authors find that cut-off points to define down days should be far in the left tail.[140]

To assess the influence of different estimation windows and cut-off points on the profitability of the systematic tail risk weighted strategies, Table XIX shows results for the strategies based on Tail-beta, Tail-Sens, Tail-Risk and MES using the past twelve to 60 months of daily data combined with cut-off points that equal the $10\%, 20\%$ and 30% quantile. As before, all

[139] In contrast to Agarwal et al. (2017), who use monthly returns to estimate systematic tail risk, we use daily returns. Estimating Tail-Sens and Tail-Risk based on their approach produces quite noisy estimates. For example, Agarwal et al. (2017, p. 615) argue that their systematic tail risk measures are calculated based on only two observations. Thus, their Tail-Sens measure is either zero, 0.5 or 1.

[140] See also Kelly and Jiang (2014, Footnote 11) on the choice of the threshold. Too low values of the threshold lead to noisy estimates, whereas too high values do not capture returns in the tails but also in the center of the distribution.

Table XIX. Robustness Results: Tail-beta, Tail-Sens, Tail-Risk and MES
This table shows performance results for the equally and risk weighted industry momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. Risk measures are estimated with daily data of the past twelve to 60 months with cut-off points between 10% and 30%. SR denotes the Sharpe Ratio and z_{JK} denotes the Jobson and Korkie (1981) test statistic.

Model	Equal		Tail-beta		Tail-Sens		Tail-Risk		MES	
Lengths/ Cut-Off	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}	SR	z_{JK}
12 Months/ 10qu	0.758	-	0.906	**3.190**	0.839	1.831	0.898	**3.032**	0.964	**4.090**
12 Months/ 20qu	0.758	-	0.886	**2.697**	0.873	**2.573**	0.903	**3.147**	0.990	**4.488**
12 Months/ 30qu	0.758	-	0.869	**2.331**	0.952	**4.020**	0.912	**3.331**	1.011	**4.828**
24 Months/ 10qu	0.758	-	0.881	**2.645**	0.912	**3.381**	0.908	**3.280**	0.952	**3.883**
24 Months/ 20qu	0.758	-	0.862	**2.261**	0.872	**2.551**	0.898	**3.040**	0.946	**3.807**
24 Months/ 30qu	0.758	-	0.838	1.741	0.858	**2.181**	0.859	**2.239**	0.956	**3.926**
36 Months/ 10qu	0.758	-	0.863	**2.207**	0.900	**3.013**	0.892	**2.924**	0.929	**3.502**
36 Months/ 20qu	0.758	-	0.848	1.953	0.848	**1.991**	0.877	**2.592**	0.932	**3.507**
36 Months/ 30qu	0.758	-	0.843	1.760	0.859	**2.208**	0.823	1.500	0.943	**3.686**
48 Months/ 10qu	0.758	-	0.848	**1.968**	0.873	**2.409**	0.865	**2.350**	0.922	**3.277**
48 Months/ 20qu	0.758	-	0.823	1.433	0.853	**2.059**	0.846	**1.988**	0.915	**3.203**
48 Months/ 30qu	0.758	-	0.808	1.067	0.816	1.292	0.810	1.202	0.911	**3.135**
60 Months/ 10qu	0.758	-	0.862	**2.276**	0.859	**2.172**	0.881	**2.706**	0.890	**2.712**
60 Months/ 20qu	0.758	-	0.814	1.242	0.817	1.323	0.850	**2.063**	0.906	**3.074**
60 Months/ 30qu	0.758	-	0.782	0.580	0.789	0.748	0.815	1.318	0.902	**2.960**

strategies produce higher Sharpe Ratios than the equally weighted momentum strategy. Further, the increases in the Sharpe Ratios are statistically significant in most cases. Nevertheless, results of Table XIX show that emphasizing return observations in the far tail produces an enhanced risk-return profile compared to models that also incorporate non-extreme observations. The worst results are found for the models that use a cut-off point of 30%, especially when this choice is combined with an estimation window of 60 months. In contrast, choosing a 10% cut-off point typically produces the highest Sharpe Ratios. The advantage of low cut-off points in a portfolio allocation setting has also been shown by Rickenberg (2020b). Similarly, Farago and Tédongap (2018, p. 84) also find that cut-off points to define down days should be far in the left tail. Moreover, as for the other risk measures, we find that short- and medium-term estimation windows typically produce higher Sharpe Ratios. Acharya et al. (2016, Table 5) also find that using more recent data is beneficial when MES is estimated, and thus shorter estimation windows should be used.

Concluding, results in this section demonstrate that the results shown in the main part also hold when risk is estimated using different estimation windows and cut-off points. In particular, weighting momentum's constituents based on their volatility does not significantly improve the performance of the momentum portfolio. In contrast, weighting momentum's constituents based on tail risk measures and systematic risk measures produces a statistically higher risk-adjusted performance.

B.2 Alternative Ranking Periods

In this section, we show additional performance results for alternative ranking periods. For our main results, we rank assets based on their performance between month $t-12$ and $t-1$. However, several studies show that momentum strategies also work well for several other ranking periods, but momentum's performance can be quite different when other ranking periods are used. Momentum strategies examined in the literature typically use several variations of the past one to twelve months' performance. For example, Jegadeesh and Titman (1993), Jegadeesh and Titman (2001), Boguth et al. (2011) and Rouwenhorst (1998) rank individual stocks based on their performance in the past six months. Jegadeesh and Titman (1993) and Rouwenhorst (1998) show additional results for rankings based on the past three, nine and twelve months. In contrast, Fama and French (1996), Fama and French (2012), Fama and French (2016), Rickenberg (2020a) and Barroso and Santa-Clara (2015) rank individual stocks based on the performance between month $t-12$ and $t-2$. Novy-Marx (2012) examines momentum for ranking periods of months $t-12$ to $t-7$ and $t-6$ to $t-2$ and shows that the profitability of the individual stock momentum strategy is mainly driven by the performance of months $t-12$ to $t-7$.[141] Similarly, several ranking periods are frequently used for the industry momentum strategy. For example, Moskowitz and Grinblatt (1999) rank industries based on the past six months' performance but the authors show that industry momentum also performs well when industries are ranked based on their past month's performance. This finding is interesting, since individual stocks exhibit a short-term reversal effect, i.e. the momentum effect reverses when individual stocks are ranked based on their past month's performance (see Goyal and Jegadeesh (2017, Table 1.B) and Moskowitz and Grinblatt (1999, Table VI)).[142] Novy-Marx (2012, p. 443) con-

[141] Goyal and Wahal (2015) confirm this finding for US stocks but not for stocks outside the US. Further, Goyal and Wahal (2015) show that the results of Novy-Marx (2012) are mainly driven by the negative impact of the month $t-2$ return and the positive impact of the month $t-12$ return.

[142] Goyal and Wahal (2015, Table 2) show that the return reversal of individual stocks also holds for the $t-2$ return. A possible explanation for the observation that the short-term reversal holds for individual stocks but not

firm the finding of Moskowitz and Grinblatt (1999) that "industries do exhibit momentum at very short (one month) horizons". Pan et al. (2004, Table 2) also find high autocorrelations of industries for short lags, which is a main driver of industry momentum. Bali et al. (2012) find a similar pattern for hedge funds, i.e. hedge funds exhibit momentum when they are ranked based on their past month's performance. Moreover, Novy-Marx (2012) finds that industry momentum also performs well when industries are ranked based on their $t-12$ to $t-7$ or $t-6$ to $t-1$ performance. Grundy and Martin (2001) examine industry momentum for the $t-7$ to $t-2$, $t-6$ to $t-1$, $t-12$ to $t-2$ and $t-12$ to $t-1$ ranking periods and find that the profitability of industry momentum can be quite different for these ranking periods. Chordia and Shivakumar (2002) and Stivers and Sun (2010) follow the approach of Moskowitz and Grinblatt (1999) and examine industry momentum for the past six months ranking period. Du Plessis and Hallerbach (2017) examine industry momentum using the past twelve months and past month ranking period. Grobys et al. (2018) and Grobys and Kolari (2020) also use different ranking periods and find that different rankings can produce quite different results of the industry momentum strategy. In particular, Grobys and Kolari (2020, Table 3) show that the correlation of industry momentum strategies based on different ranking periods is very low. Further, the crash risk of industry momentum using different ranking periods is also quite different and momentum crashes do not happen simultaneously (Grobys and Kolari, 2020, Table 7). The varying performance of different ranking periods can also nicely be seen in Grobys and Kolari (2020, Fig. II). Swinkels (2002) also find that different ranking periods can lead to quite different performance results of international momentum strategies. Thus, there is no common ranking period used for industry momentum and the profitability of industry momentum is highly influenced by the chosen ranking period.

To assess how different ranking periods affect the profitability of the non-managed and risk-managed momentum strategies, we next show results for four additional ranking periods. Table

for industries is that "the one-month return reversal for individual stocks is generated by microstructure effects (such as bid-ask bounce and liquidity effects), which are alleviated by forming industry portfolios" (Moskowitz and Grinblatt, 1999, p. 1274). Similarly, the January effect, i.e. the observation that momentum performs bad in January, also holds for the individual stock momentum strategy, but not for industry momentum (George and Hwang, 2004, Table 2). A possible explanation for this finding is that the January effect can be explained by tax loss selling which "is associated with capital losses of individual stocks, not the loss of the industry" (George and Hwang, 2004, Footnote 5).

Table XX. Robustness Results: $t-6$ **to** $t-1$ **Ranking**
This table shows performance results of the equally and risk weighted momentum strategies using 30 equally weighted US industries, the $t-6$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. Return and Volatility correspond to the annualized return and volatility, respectively. SR stands for the annualized Sharpe Ratio and z_{JK} denotes the corresponding value of the Jobson and Korkie (1981) test. $\Delta_{MV}^{\gamma=5}$, $\Delta_{CRRA}^{\gamma=5}$ and $\Delta_{LA}^{l=2}$ denote the economic value for a mean-variance, CRRA and loss-averse investor, respectively. DM-test stands for the corresponding DM-test of Diebold and Mariano (1995). Panel A shows results for the strategies without volatility targeting, whereas Panel B shows results for the strategies that use the target volatility (TV) overlay. The Jobson and Korkie (1981) test and DM-test are calculated with respect to the equally weighted momentum strategy. Values for these tests that are higher than 1.96 are given in bold, whereas values smaller than -1.96 are given in red.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	8.56	11.46	0.747	-	-	-	-	-	-	-
RV	7.26	10.39	0.699	-1.02	-0.547	-1.13	-0.673	-1.18	-0.659	-1.34
RSJ/Corr	9.76	10.61	0.919	**3.36**	1.813	**2.87**	1.815	**2.64**	1.417	**2.15**
RSJ/Down Corr	10.01	10.78	0.929	**3.54**	1.922	**3.65**	1.968	**3.35**	1.541	**2.87**
RSJ/Beta	9.72	10.36	0.938	**3.70**	2.010	**3.58**	2.044	**3.42**	1.515	**2.60**
RSJ/Down Beta	10.08	10.41	0.969	**4.26**	2.335	**4.55**	2.427	**4.40**	1.811	**3.35**
RSJ/CoSkew	10.56	11.56	0.914	**3.57**	1.769	**3.20**	1.968	**3.11**	1.485	**2.60**
RSJ/CoKurt	9.84	10.74	0.916	**3.39**	1.782	**3.17**	1.815	**3.04**	1.387	**2.28**
RSJ/LPM Beta	10.21	10.40	0.981	**4.43**	2.470	**4.51**	2.503	**4.49**	2.013	**3.39**
RSJ/HTCR Beta	9.91	10.57	0.937	**3.77**	2.004	**3.79**	2.044	**3.60**	1.566	**2.77**
RSJ/Tail Beta	9.89	10.87	0.909	**3.37**	1.712	**3.00**	1.739	**2.81**	1.307	**2.24**
RSJ/Tail Sens	9.91	10.84	0.914	**3.44**	1.765	**3.25**	1.739	**2.88**	1.423	**2.52**
RSJ/Tail Risk	9.93	10.81	0.919	**3.52**	1.810	**3.36**	1.815	**3.21**	1.461	**2.57**
RSJ/MES	10.08	10.57	0.954	**4.08**	2.182	**3.91**	2.197	**3.85**	1.776	**3.02**

	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	8.57	11.39	0.753	-	-	-	-	-	-	-
RV	7.28	10.28	0.708	-1.08	-0.509	-1.06	-0.598	-1.13	-0.638	-1.30
Equal (TV)	12.72	11.04	1.152	**6.39**	4.200	**5.74**	4.516	**5.79**	3.523	**4.57**
RV (TV)	13.41	11.76	1.140	**5.17**	4.076	**4.81**	4.360	**4.92**	3.297	**3.71**
RSJ/Corr (TV)	13.35	10.70	1.248	**6.41**	5.221	**6.20**	5.536	**6.31**	4.462	**4.79**
RSJ/Down Corr (TV)	13.41	10.66	1.258	**6.53**	5.330	**6.12**	5.693	**6.18**	4.581	**4.79**
RSJ/Beta (TV)	13.32	10.74	1.240	**6.26**	5.136	**6.53**	5.457	**6.61**	4.378	**5.11**
RSJ/Down Beta (TV)	13.52	10.65	1.269	**6.59**	5.451	**6.43**	5.772	**6.43**	4.678	**5.06**
RSJ/CoSkew (TV)	13.49	10.71	1.259	**6.50**	5.333	**6.36**	5.693	**6.27**	4.678	**5.00**
RSJ/CoKurt (TV)	13.27	10.64	1.247	**6.45**	5.214	**5.85**	5.536	**5.99**	4.418	**4.49**
RSJ/LPM Beta (TV)	13.64	10.71	1.274	**6.66**	5.504	**6.50**	5.851	**6.57**	4.766	**5.09**
RSJ/HTCR Beta (TV)	13.41	10.74	1.249	**6.41**	5.236	**6.71**	5.536	**6.71**	4.458	**5.27**
RSJ/Tail Beta (TV)	13.37	10.84	1.233	**6.40**	5.067	**5.61**	5.378	**5.72**	4.273	**4.37**
RSJ/Tail Sens (TV)	13.39	10.73	1.248	**6.51**	5.223	**6.35**	5.536	**6.46**	4.463	**4.92**
RSJ/Tail Risk (TV)	13.40	10.88	1.232	**6.38**	5.049	**5.95**	5.378	**6.05**	4.314	**4.67**
RSJ/MES (TV)	13.58	10.83	1.254	**6.53**	5.284	**6.51**	5.614	**6.64**	4.574	**5.16**

XX shows results for the $t-6$ to $t-1$ ranking period, Table XXI shows results for the $t-12$ to $t-7$ ranking period, Table XXII shows results for the $t-12$ to $t-2$ ranking period and Table XXIII shows results for the $t-6$ to $t-2$ ranking period. All tables show results for the strategies where only individual asset risk is managed as well as results for the strategies that additionally use the target volatility (TV) overlay. For the $t-6$ to $t-1$ ranking, the $t-6$

to $t-2$ ranking and the $t-12$ to $t-2$ ranking, we find that the volatility managed strategy underperforms the equally weighted strategy. In contrast, the (systematic) tail risk weighted strategies clearly outperform the equally and volatility weighted strategies. These increases in the Sharpe Ratio and utility are highly significant. For the $t-12$ to $t-7$ ranking, the volatility weighted strategy exhibits a higher Sharpe Ratio than the equally weighted strategy and a positive economic value. However, the increases in the Sharpe Ratio and utility are not statistically significant. In contrast, the (systematic) tail risk weighted strategies produce statistically significant Sharpe Ratio and utility increases. For all four ranking periods, we find that volatility targeting highly improves the risk-return profile for all weighting schemes by significantly increasing the strategies' Sharpe Ratio and economic value. Thus, accounting for portfolio risk is beneficial regardless of the chosen ranking period and weighting scheme. The highest Sharpe Ratios and economic values are again found for the (systematic) tail risk weighted strategies that are overlayed by volatility targeting.

In line with earlier studies, results in this section show that the profitability of momentum can be quite different when different ranking periods are used. In particular, we find that industry momentum works best when industries are ranked based on the most recent performance. This confirms the earlier finding of Novy-Marx (2012) and Moskowitz and Grinblatt (1999) who also find that industry momentum is mainly driven by the recent past's performance. This is opposed to the individual stock based momentum strategy where the profitability is mainly driven by the $t-12$ to $t-7$ performance (Novy-Marx, 2012).

To conclude this section, we find that the tail risk weighting and volatility targeting approach work well for all four additional ranking periods. In contrast, the profitability of the volatility weighted strategy strongly depends on the chosen ranking period and is less robust than the tail risk weighted strategies. These results confirm our findings of Section 3.7 that momentum's risk is best managed by simultaneously managing individual asset risk and portfolio risk, where individual asset risk is best managed based on (systematic) tail risk measures.

Table XXI. Robustness Results: $t-12$ to $t-7$ Ranking
This table shows performance results of the equally and risk weighted momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-7$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	5.75	11.60	0.496	-	-	-	-	-	-	-
RV	5.62	10.51	0.534	0.93	0.371	0.70	0.451	0.89	0.449	0.83
RSJ/Corr	7.82	11.40	0.686	**4.17**	2.080	**3.31**	2.197	**3.45**	1.936	**2.91**
RSJ/Down Corr	8.13	11.34	0.717	**4.92**	2.420	**4.01**	2.580	**4.05**	2.116	**3.37**
RSJ/Beta	8.15	11.29	0.722	**4.90**	2.472	**4.09**	2.657	**4.08**	2.333	**3.77**
RSJ/Down Beta	8.09	11.17	0.724	**5.00**	2.501	**4.13**	2.657	**4.16**	2.208	**3.57**
RSJ/CoSkew	8.16	11.58	0.705	**4.69**	2.295	**3.71**	2.503	**3.66**	1.887	**3.10**
RSJ/CoKurt	7.84	11.40	0.687	**4.24**	2.098	**3.29**	2.273	**3.40**	1.867	**2.74**
RSJ/LPM Beta	8.26	11.27	0.732	**5.20**	2.591	**4.22**	2.734	**4.20**	2.391	**3.87**
RSJ/HTCR Beta	8.13	11.25	0.723	**4.97**	2.478	**3.79**	2.657	**3.77**	2.352	**3.43**
RSJ/Tail Beta	8.16	11.73	0.695	**4.67**	2.196	**3.44**	2.350	**3.66**	2.136	**3.29**
RSJ/Tail Sens	8.17	11.49	0.711	**4.94**	2.362	**4.29**	2.503	**4.41**	2.168	**3.75**
RSJ/Tail Risk	8.20	11.50	0.713	**5.26**	2.385	**3.61**	2.503	**3.73**	2.182	**3.21**
RSJ/MES	8.36	11.43	0.731	**5.26**	2.581	**4.32**	2.657	**4.43**	2.425	**4.01**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	5.67	11.58	0.490	-	-	-	-	-	-	-
RV	5.56	10.47	0.531	0.84	0.396	0.75	0.526	0.94	0.478	0.89
Equal (TV)	7.43	11.41	0.651	**2.66**	1.751	**3.46**	2.044	**3.64**	0.750	1.15
RV (TV)	8.45	12.42	0.681	**2.62**	2.114	**2.63**	2.427	**2.90**	1.220	1.37
RSJ/Corr (TV)	9.39	11.49	0.817	**4.41**	3.588	**4.62**	3.893	**4.82**	2.563	**2.99**
RSJ/Down Corr (TV)	9.56	11.43	0.837	**4.60**	3.798	**5.25**	4.126	**5.35**	2.730	**3.37**
RSJ/Beta (TV)	9.69	11.55	0.838	**4.73**	3.822	**5.06**	4.126	**5.15**	2.830	**3.40**
RSJ/Down Beta (TV)	9.64	11.43	0.843	**4.67**	3.871	**5.04**	4.126	**5.08**	2.848	**3.34**
RSJ/CoSkew (TV)	9.39	11.31	0.830	**4.43**	3.719	**4.57**	4.048	**4.69**	2.625	**2.93**
RSJ/CoKurt (TV)	9.38	11.44	0.820	**4.45**	3.612	**4.77**	3.893	**4.96**	2.568	**3.06**
RSJ/LPM Beta (TV)	9.81	11.55	0.849	**4.80**	3.939	**5.25**	4.204	**5.33**	2.965	**3.57**
RSJ/HTCR Beta (TV)	9.55	11.48	0.832	**4.57**	3.746	**4.78**	4.048	**4.85**	2.780	**3.19**
RSJ/Tail Beta (TV)	9.81	11.59	0.847	**4.90**	3.918	**4.98**	4.204	**5.04**	2.886	**3.38**
RSJ/Tail Sens (TV)	9.71	11.46	0.848	**4.82**	3.923	**5.35**	4.204	**5.47**	2.841	**3.37**
RSJ/Tail Risk (TV)	9.77	11.63	0.840	**4.86**	3.844	**4.83**	4.126	**4.97**	2.818	**3.18**
RSJ/MES (TV)	9.88	11.57	0.854	**4.89**	3.998	**5.17**	4.282	**5.28**	3.011	**3.45**

B.3 Alternative Cut-Off Points

We next examine the impact of the chosen cut-off point to determine winners and losers on the profitability of the non-managed and risk-managed momentum strategies. Several cut-off points are frequently used in the momentum literature. For example, for the individual stock momentum strategy, Jegadeesh and Titman (1993), Jegadeesh and Titman (2001) and Rouwenhorst (1998) define winners and losers as the 10% best and worst performing stocks. In contrast, in order to "place less emphasis on the tails of the performance distribution", Hong et al. (2000, p. 274) use the 30% quantile as cut-off point to determine winners and losers. The authors also show that different cut-off points can produce quite different results. Generally, different

Table XXII. Robustness Results: $t-12$ to $t-2$ Ranking

This table shows performance results of the equally and risk weighted momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-2$ ranking period and a cut-off point of $p=30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	8.14	11.83	0.688	-	-	-	-	-	-	-
RV	7.34	10.70	0.686	0.00	-0.066	-0.13	-0.075	-0.20	-0.042	-0.08
RSJ/Corr	10.41	10.87	0.957	**5.47**	2.938	**5.05**	3.042	**4.87**	2.700	**4.54**
RSJ/Down Corr	10.05	11.05	0.910	**4.62**	2.417	**4.33**	2.503	**4.09**	2.104	**3.71**
RSJ/Beta	10.37	10.87	0.954	**5.41**	2.904	**5.07**	3.042	**4.75**	2.690	**4.57**
RSJ/Down Beta	10.29	10.93	0.941	**5.26**	2.759	**5.24**	2.888	**4.94**	2.456	**4.49**
RSJ/CoSkew	10.22	11.53	0.887	**4.17**	2.162	**4.85**	2.427	**5.26**	1.816	**3.80**
RSJ/CoKurt	10.41	11.09	0.939	**5.30**	2.733	**5.21**	2.888	**4.91**	2.479	**4.73**
RSJ/LPM Beta	10.55	10.79	0.978	**5.93**	3.158	**6.10**	3.350	**5.97**	2.845	**5.13**
RSJ/HTCR Beta	10.41	10.80	0.964	**5.75**	3.011	**5.74**	3.119	**5.57**	2.629	**4.77**
RSJ/Tail Beta	10.58	11.30	0.936	**5.29**	2.707	**5.43**	2.888	**5.14**	2.485	**4.68**
RSJ/Tail Sens	10.57	10.97	0.964	**5.57**	3.008	**5.00**	3.196	**4.64**	2.607	**4.54**
RSJ/Tail Risk	10.37	11.08	0.935	**5.43**	2.697	**5.54**	2.888	**5.32**	2.456	**4.85**
RSJ/MES	10.72	10.90	0.984	**6.06**	3.227	**5.58**	3.428	**5.36**	2.945	**4.80**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	8.08	11.73	0.688	-	-	-	-	-	-	-
RV	7.26	10.54	0.689	-0.07	-0.034	-0.07	-0.075	-0.16	-0.035	-0.07
Equal (TV)	11.70	11.67	1.002	**5.36**	3.404	**5.17**	3.815	**5.27**	2.826	**4.29**
RV (TV)	13.13	12.63	1.040	**4.70**	3.818	**3.81**	4.282	**4.02**	3.260	**3.33**
RSJ/Corr (TV)	13.58	11.23	1.209	**6.99**	5.674	**7.10**	6.089	**7.10**	4.988	**6.02**
RSJ/Down Corr (TV)	13.23	11.26	1.175	**6.57**	5.299	**6.80**	5.693	**6.94**	4.615	**5.77**
RSJ/Beta (TV)	13.44	11.37	1.182	**6.69**	5.371	**7.13**	5.772	**7.18**	4.726	**6.07**
RSJ/Down Beta (TV)	13.41	11.28	1.189	**6.68**	5.446	**7.10**	5.851	**7.21**	4.765	**5.99**
RSJ/CoSkew (TV)	13.04	11.22	1.162	**6.19**	5.149	**6.16**	5.614	**6.23**	4.506	**5.50**
RSJ/CoKurt (TV)	13.35	11.26	1.186	**6.74**	5.412	**7.00**	5.851	**6.89**	4.704	**6.01**
RSJ/LPM Beta (TV)	13.55	11.24	1.205	**6.84**	5.625	**7.51**	6.089	**7.51**	4.917	**6.30**
RSJ/HTCR Beta (TV)	13.49	11.23	1.201	**6.77**	5.582	**6.87**	6.009	**6.89**	4.842	**5.81**
RSJ/Tail Beta (TV)	13.44	11.44	1.175	**6.74**	5.292	**6.71**	5.693	**6.64**	4.592	**5.60**
RSJ/Tail Sens (TV)	13.62	11.21	1.215	**6.98**	5.735	**7.01**	6.168	**6.91**	4.939	**6.01**
RSJ/Tail Risk (TV)	13.44	11.44	1.175	**6.76**	5.299	**7.05**	5.772	**7.04**	4.641	**5.98**
RSJ/MES (TV)	13.67	11.32	1.208	**6.93**	5.653	**7.72**	6.089	**7.68**	4.963	**6.46**

cut-off points can lead to quite different portfolios for long-short strategies. For example, Bali and Cakici (2008) find quite different results for risk-sorted long-short portfolios when different cut-off points are used. To assess the influence of the chosen cut-off point on the profitability of the non-managed and risk-managed momentum strategies, we next choose three additional cut-off points between 10% and 40%. This range fits well to the cut-off points that are frequently used in the literature on industry momentum. For example, Grundy and Martin (2001) and Moskowitz and Grinblatt (1999) use 20 US industries and define winners and losers as the three best and worst performing industries, i.e. they choose $p=15\%$. In contrast, Chordia and Shivakumar (2002) also use 20 industries but define winners and losers as the two best and

Table XXIII. Robustness Results: $t-6$ to $t-2$ Ranking
This table shows performance results of the equally and risk weighted momentum strategies using 30 equally weighted US industries, the $t-6$ to $t-2$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.38	11.32	0.564	-	-	-	-	-	-	-
RV	5.13	10.16	0.505	-1.20	-0.684	-1.21	-0.673	-0.94	-0.905	-1.91
RSJ/Corr	8.62	10.48	0.822	**4.98**	2.730	**4.39**	2.888	**3.98**	2.308	**3.99**
RSJ/Down Corr	8.23	10.77	0.764	**4.16**	2.113	**3.63**	2.197	**3.33**	1.760	**3.06**
RSJ/Beta	8.71	10.33	0.843	**5.36**	2.951	**5.07**	3.119	**4.43**	2.554	**4.78**
RSJ/Down Beta	8.87	10.29	0.862	**5.73**	3.153	**5.58**	3.428	**4.93**	2.658	**4.89**
RSJ/CoSkew	8.59	11.33	0.759	**4.20**	2.068	**4.23**	2.350	**4.10**	1.733	**3.55**
RSJ/CoKurt	8.31	10.79	0.770	**4.24**	2.182	**3.83**	2.273	**3.50**	1.849	**3.42**
RSJ/LPM Beta	9.07	10.30	0.881	**6.07**	3.351	**5.66**	3.583	**5.21**	2.906	**5.03**
RSJ/HTCR Beta	8.60	10.35	0.831	**5.41**	2.819	**4.89**	3.042	**4.44**	2.441	**4.48**
RSJ/Tail Beta	8.24	10.93	0.754	**3.94**	2.009	**3.68**	2.120	**3.18**	1.759	**3.42**
RSJ/Tail Sens	8.52	10.73	0.794	**4.80**	2.434	**4.38**	2.580	**3.98**	2.142	**4.07**
RSJ/Tail Risk	8.22	10.67	0.770	**4.24**	2.175	**4.26**	2.350	**3.87**	1.807	**3.65**
RSJ/MES	9.01	10.52	0.856	**5.63**	3.089	**5.17**	3.273	**4.91**	2.772	**4.52**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.39	11.24	0.568	-	-	-	-	-	-	-
RV	5.16	10.06	0.513	-1.25	-0.637	-1.16	-0.673	-0.90	-0.868	-1.88
Equal (TV)	10.25	11.10	0.924	**6.06**	3.749	**4.56**	4.204	**4.49**	3.039	**3.38**
RV (TV)	10.33	11.81	0.875	**4.13**	3.247	**3.29**	3.660	**3.27**	2.333	**2.24**
RSJ/Corr (TV)	12.30	10.87	1.132	**7.46**	5.966	**6.91**	6.485	**6.52**	5.143	**5.70**
RSJ/Down Corr (TV)	11.96	10.90	1.097	**7.19**	5.590	**6.86**	6.009	**6.57**	4.817	**5.52**
RSJ/Beta (TV)	12.33	10.92	1.129	**7.47**	5.933	**6.98**	6.406	**6.58**	5.136	**5.76**
RSJ/Down Beta (TV)	12.39	10.81	1.146	**7.56**	6.111	**7.02**	6.644	**6.53**	5.316	**5.73**
RSJ/CoSkew (TV)	11.91	10.67	1.117	**7.23**	5.799	**6.54**	6.326	**6.22**	4.982	**5.19**
RSJ/CoKurt (TV)	12.00	10.90	1.101	**7.31**	5.638	**6.79**	6.089	**6.56**	4.863	**5.51**
RSJ/LPM Beta (TV)	12.54	10.88	1.153	**7.72**	6.186	**6.95**	6.644	**6.60**	5.392	**5.65**
RSJ/HTCR (TV)	12.24	10.90	1.124	**7.46**	5.876	**6.86**	6.326	**6.46**	5.125	**5.74**
RSJ/Tail Beta (TV)	11.96	11.06	1.082	**7.20**	5.433	**6.55**	5.930	**6.28**	4.675	**5.32**
RSJ/Tail Sens (TV)	12.11	10.93	1.108	**7.39**	5.706	**6.88**	6.168	**6.56**	4.951	**5.66**
RSJ/Tail Risk (TV)	11.85	11.04	1.074	**6.98**	5.344	**6.25**	5.851	**6.07**	4.556	**5.03**
RSJ/MES (TV)	12.50	10.96	1.140	**7.72**	6.053	**6.78**	6.565	**6.56**	5.286	**5.47**

worst performing industries, i.e. $p = 10\%$. Swinkels (2002) uses 40 industries and $p = 10\%$ as well as $p = 20\%$. Novy-Marx (2012) uses 49 industries and chooses a cut-off point of $p = 1/3$, whereas Grobys et al. (2018) and Du Plessis and Hallerbach (2017) use $p = 1/6$ and $p = 25\%$ for the 49 industries. Similarly, Stivers and Sun (2010) use 48 industries and a cut-off point of $p = 25\%$, whereas Grobys and Kolari (2020) use $p = 20\%$ for the 48 US industries.

Table XXIV shows results for the momentum strategy that uses a cut-off point of $p = 10\%$. Due to the higher volatility of this strategy, we use a volatility target of $\sigma_{\text{target}} = 12\%$ for this data set. Table XXV defines winners and losers as the $p = 20\%$ best and worst performers. Finally, Table XXVI uses a cut-off point of $p = 40\%$. Our main results of Section 3.7 are based

Table XXIV. Robustness Results: $p = 10\%$

This table shows performance results of the equally and risk weighted momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period, a cut-off point of $p = 10\%$ and a volatility target of 12%. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	13.15	21.59	0.609	-	-	-	-	-	-	-
RV	13.74	20.02	0.686	**2.22**	1.317	1.88	2.811	1.27	1.206	**2.14**
RSJ/Corr	14.88	20.49	0.726	**3.22**	2.117	**2.52**	3.970	1.71	1.686	**2.55**
RSJ/Down Corr	15.08	20.51	0.735	**3.24**	2.275	**2.32**	4.516	1.75	1.915	**2.75**
RSJ/Beta	14.86	20.34	0.731	**3.30**	2.197	**2.35**	3.970	1.64	1.767	**2.47**
RSJ/Down Beta	15.14	20.33	0.745	**3.63**	2.456	**3.24**	4.516	**2.01**	2.084	**3.69**
RSJ/CoSkew	14.40	21.27	0.677	1.76	1.194	1.11	3.815	1.34	0.887	1.18
RSJ/CoKurt	14.17	21.25	0.667	1.72	1.116	1.54	-0.896	-0.33	1.136	1.76
RSJ/LPM Beta	15.13	20.39	0.742	**3.49**	2.400	**3.00**	4.516	**1.98**	2.049	**3.39**
RSJ/HTCR Beta	15.46	20.40	0.758	**4.03**	2.718	**3.02**	4.750	**1.97**	2.379	**3.43**
RSJ/Tail Beta	14.99	20.57	0.729	**3.21**	2.180	**2.96**	3.815	1.68	1.884	**3.33**
RSJ/Tail Sens	14.93	20.81	0.717	**2.97**	1.983	**2.34**	3.273	1.32	1.727	**2.96**
RSJ/Tail Risk	14.71	20.62	0.713	**2.80**	1.873	**2.61**	3.893	1.76	1.549	**2.86**
RSJ/MES	14.91	20.52	0.726	**3.13**	2.120	**2.36**	4.048	1.70	1.741	**2.49**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	13.10	21.55	0.608	-	-	-	-	-	-	-
RV	13.72	19.98	0.687	1.95	1.344	1.91	2.888	1.29	1.250	**2.22**
Equal (TV)	15.48	16.58	0.934	**4.92**	5.921	**3.22**	10.362	**2.94**	4.581	**3.54**
RV (TV)	17.57	17.24	1.019	**5.45**	7.622	**4.03**	12.348	**3.39**	6.480	**4.65**
RSJ/Corr (TV)	16.99	16.81	1.011	**5.48**	7.469	**4.00**	11.849	**3.25**	6.095	**4.81**
RSJ/Down Corr (TV)	16.93	16.62	1.018	**5.47**	7.613	**3.87**	12.015	**3.20**	6.299	**4.52**
RSJ/Beta (TV)	17.04	16.75	1.017	**5.53**	7.592	**4.00**	11.932	**3.25**	6.223	**4.85**
RSJ/Down Beta (TV)	17.10	16.59	1.031	**5.67**	7.859	**4.26**	12.265	**3.41**	6.509	**5.02**
RSJ/CoSkew (TV)	16.06	16.43	0.978	**4.65**	6.782	**3.16**	11.517	**2.82**	5.623	**3.65**
RSJ/CoKurt (TV)	16.35	16.97	0.964	**5.03**	6.567	**3.89**	9.625	**3.66**	5.400	**4.38**
RSJ/LPM Beta (TV)	17.15	16.74	1.025	**5.60**	7.743	**4.15**	12.181	**3.38**	6.436	**5.06**
RSJ/HTCR Beta (TV)	17.32	16.68	1.038	**5.83**	8.012	**4.19**	12.432	**3.39**	6.711	**5.07**
RSJ/Tail Beta (TV)	17.05	16.83	1.013	**5.47**	7.519	**4.08**	11.849	**3.32**	6.205	**5.06**
RSJ/Tail Sens (TV)	16.90	16.74	1.009	**5.43**	7.446	**3.75**	11.517	**3.08**	6.091	**4.40**
RSJ/Tail Risk (TV)	16.87	16.82	1.002	**5.29**	7.306	**3.86**	11.600	**3.16**	5.954	**4.75**
RSJ/MES (TV)	17.07	16.84	1.013	**5.45**	7.521	**3.80**	11.849	**3.15**	6.172	**4.62**

on a cut-off point of 30%, i.e. winners and losers consist of the 9 best and worst performing industries. Choosing p equal to $10\%, 20\%$ and 40% means that winners and losers are defined as the $3, 6$ and 12 best and worst performing industries, respectively. For the strategy using $p = 10\%$, we find that all risk weighted strategies produce higher Sharpe Ratios and higher utilities except for one case. The highest Sharpe Ratios and economic values are again found for the (systematic) tail risk weightings. The increase in the Sharpe Ratio and utility is statistically significant for most risk weighted strategies, including the volatility weighted strategy. For the strategies using $p = 20\%$ and $p = 40\%$, we find that the volatility managed strategy does not outperform the equally weighted strategy. In contrast, the (systematic) tail risk weighted

Table XXV. Robustness Results: $p = 20\%$

This table shows performance results of the equally and risk weighted momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 20\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	11.02	15.37	0.717	-	-	-	-	-	-	-
RV	10.16	14.11	0.720	0.12	-0.019	-0.03	-0.374	-0.32	0.171	0.37
RSJ/Corr	12.10	13.82	0.876	**3.80**	2.154	**3.75**	2.197	**3.03**	1.919	**3.53**
RSJ/Down Corr	12.62	14.12	0.894	**4.43**	2.415	**3.85**	2.657	**4.29**	2.218	**3.34**
RSJ/Beta	12.15	13.68	0.888	**4.11**	2.326	**4.24**	2.427	**3.21**	2.102	**3.90**
RSJ/Down Beta	12.73	14.01	0.908	**4.65**	2.613	**4.61**	2.888	**4.55**	2.397	**4.17**
RSJ/CoSkew	12.17	14.85	0.820	**2.56**	1.381	**2.17**	2.044	**2.12**	1.136	1.86
RSJ/CoKurt	12.12	14.26	0.850	**3.33**	1.828	**2.60**	1.358	1.41	1.681	**2.65**
RSJ/LPM Beta	12.66	13.84	0.915	**4.65**	2.697	**4.80**	2.965	**3.41**	2.494	**4.56**
RSJ/HTCR Beta	12.24	13.76	0.889	**4.13**	2.340	**4.16**	2.503	**3.46**	2.072	**3.67**
RSJ/Tail Beta	12.35	14.21	0.869	**3.50**	2.067	**4.09**	2.273	**2.53**	1.969	**3.89**
RSJ/Tail Sens	12.08	13.96	0.865	**3.52**	2.004	**2.89**	2.273	**2.25**	1.740	**2.67**
RSJ/Tail Risk	12.01	14.15	0.848	**3.01**	1.770	**3.08**	2.120	**2.12**	1.644	**3.24**
RSJ/MES	12.35	13.77	0.897	**4.13**	2.443	**4.88**	2.811	**3.01**	2.194	**4.72**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	11.06	15.28	0.724	-	-	-	-	-	-	-
RV	10.20	13.97	0.730	0.04	0.029	0.05	-0.374	-0.30	0.210	0.46
Equal (TV)	12.30	11.67	1.054	**5.48**	4.483	**4.98**	5.693	**4.17**	3.633	**4.43**
RV (TV)	13.83	12.51	1.105	**5.22**	5.219	**4.68**	6.406	**4.17**	4.502	**4.57**
RSJ/Corr (TV)	13.35	11.24	1.187	**6.59**	6.398	**6.17**	7.523	**5.25**	5.303	**5.63**
RSJ/Down Corr (TV)	13.32	11.16	1.193	**6.65**	6.486	**6.37**	7.603	**5.50**	5.395	**5.68**
RSJ/Beta (TV)	13.37	11.27	1.186	**6.57**	6.385	**6.63**	7.523	**5.48**	5.290	**6.05**
RSJ/Down Beta (TV)	13.55	11.18	1.213	**6.85**	6.765	**7.09**	7.924	**5.98**	5.663	**6.41**
RSJ/CoSkew (TV)	12.49	11.06	1.129	**5.41**	5.549	**4.44**	6.963	**3.95**	4.667	**3.98**
RSJ/CoKurt (TV)	13.13	11.35	1.157	**6.32**	5.965	**6.14**	6.803	**5.51**	4.901	**5.32**
RSJ/LPM Beta (TV)	13.69	11.17	1.225	**7.05**	6.944	**6.89**	8.166	**5.59**	5.817	**6.37**
RSJ/HTCR Beta (TV)	13.31	11.16	1.193	**6.54**	6.477	**6.48**	7.683	**5.46**	5.343	**5.73**
RSJ/Tail Beta (TV)	13.44	11.37	1.183	**6.49**	6.327	**5.94**	7.523	**4.89**	5.278	**5.45**
RSJ/Tail Sens (TV)	13.23	11.19	1.182	**6.32**	6.325	**5.72**	7.603	**4.81**	5.218	**5.37**
RSJ/Tail Risk (TV)	13.32	11.40	1.168	**6.25**	6.119	**5.88**	7.362	**4.84**	5.070	**5.61**
RSJ/MES (TV)	13.49	11.22	1.203	**6.66**	6.621	**6.76**	7.924	**5.39**	5.514	**6.35**

strategies produce very high and statistically significant Sharpe Ratio and utility increases. In particular, we find that the (systematic) tail risk weightings perform the best for higher cut-off points. This is quite intuitive, since the winners and losers portfolios consist of only three industries for the $p = 10\%$ cut-off. Thus, differences between the weighting schemes are only small. In contrast, when higher values of p are chosen, winners and losers contain more assets and differences between different weighting schemes are higher. However, the (systematic) tail risk weighting approach outperforms the other weighting schemes, even for low cut-off points. Finally, the volatility targeting approach works well for all three cut-off points and significantly increases the Sharpe Ratio and the investors' utility. The highest Sharpe Ratios are again obtained for the strategies that combine volatility targeting with the (systematic) tail risk weightings.

Table XXVI. Robustness Results: $p = 40\%$
This table shows performance results of the equally and risk weighted momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 40\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	7.71	10.06	0.766	-	-	-	-	-	-	-
RV	6.81	9.06	0.751	-0.26	-0.163	-0.35	-0.225	-0.46	-0.038	-0.09
RSJ/Corr	10.10	9.46	1.068	**5.81**	2.831	**5.80**	2.888	**5.58**	2.644	**5.37**
RSJ/Down Corr	9.83	10.09	0.974	**4.11**	1.959	**3.69**	1.891	**3.02**	2.026	**3.99**
RSJ/Beta	10.05	9.23	1.089	**5.98**	3.025	**5.80**	3.119	**5.64**	2.752	**5.21**
RSJ/Down Beta	10.15	9.51	1.067	**5.80**	2.827	**5.53**	2.888	**5.40**	2.615	**4.93**
RSJ/CoSkew	10.10	10.28	0.982	**4.77**	2.032	**3.88**	2.120	**3.84**	1.989	**3.62**
RSJ/CoKurt	9.82	9.69	1.013	**4.93**	2.322	**4.78**	2.350	**4.55**	2.179	**4.55**
RSJ/LPM Beta	10.31	9.38	1.100	**6.19**	3.128	**6.09**	3.273	**5.91**	2.884	**5.49**
RSJ/HTCR Beta	10.11	9.30	1.087	**6.17**	3.011	**5.18**	3.119	**5.06**	2.707	**4.62**
RSJ/Tail Beta	9.98	9.94	1.004	**4.85**	2.231	**4.76**	2.273	**4.62**	2.154	**4.55**
RSJ/Tail Sens	10.22	9.44	1.082	**6.12**	2.967	**5.60**	3.119	**5.33**	2.679	**5.19**
RSJ/Tail Risk	10.00	9.69	1.032	**5.39**	2.493	**5.54**	2.580	**5.36**	2.332	**5.07**
RSJ/MES	10.43	9.30	1.121	**6.61**	3.332	**5.97**	3.505	**5.80**	3.025	**5.44**

	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	7.60	9.98	0.761	-	-	-	-	-	-	-
RV	6.70	8.93	0.750	-0.30	-0.130	-0.28	-0.225	-0.41	-0.019	-0.04
Equal (TV)	13.11	11.77	1.114	**5.96**	3.278	**5.68**	3.505	**5.73**	2.784	**4.92**
RV (TV)	14.85	12.45	1.192	**5.72**	3.990	**5.11**	4.204	**5.22**	3.449	**4.55**
RSJ/Corr (TV)	15.18	11.26	1.348	**7.43**	5.435	**7.09**	5.693	**7.01**	4.973	**6.16**
RSJ/Down Corr (TV)	14.84	11.43	1.298	**7.05**	4.969	**6.89**	5.142	**6.92**	4.535	**6.05**
RSJ/Beta (TV)	15.00	11.33	1.324	**7.12**	5.206	**6.93**	5.457	**6.91**	4.743	**6.04**
RSJ/Down Beta (TV)	15.03	11.31	1.329	**7.21**	5.257	**6.95**	5.536	**6.93**	4.762	**6.04**
RSJ/CoSkew (TV)	14.87	11.38	1.307	**7.09**	5.056	**6.37**	5.300	**6.37**	4.614	**5.67**
RSJ/CoKurt (TV)	14.75	11.29	1.307	**7.04**	5.055	**6.57**	5.300	**6.54**	4.579	**5.77**
RSJ/LPM Beta (TV)	15.23	11.30	1.348	**7.37**	5.429	**7.11**	5.693	**7.04**	4.937	**6.13**
RSJ/HTCR Beta (TV)	15.09	11.28	1.338	**7.25**	5.338	**6.77**	5.614	**6.75**	4.841	**5.89**
RSJ/Tail Beta (TV)	15.00	11.49	1.305	**7.27**	5.035	**7.11**	5.300	**7.09**	4.544	**6.12**
RSJ/Tail Sens (TV)	15.15	11.23	1.349	**7.38**	5.440	**6.95**	5.693	**6.88**	4.934	**6.05**
RSJ/Tail Risk (TV)	14.95	11.42	1.309	**7.25**	5.073	**7.00**	5.300	**6.96**	4.565	**6.11**
RSJ/MES (TV)	15.31	11.28	1.357	**7.46**	5.515	**7.14**	5.772	**7.07**	5.014	**6.18**

Results in this section show that the profitability of momentum strongly depends on the chosen cut-off point. We find that lower cut-off points typically produce higher returns but also exhibit a higher volatility. The best risk-return profile is found for the strategies that use higher cut-off points. Similarly, the benefits of our risk weightings are also higher for strategies that use higher cut-off points. In contrast, volatility targeting adds the highest value when it is combined with lower cut-off points. This finding is also quite intuitive, since risk targeting is more advantageous when it is overlayed on portfolios with higher risk (Harvey et al., 2018).

As stated above, momentum strategies based on lower cut-off points are typically riskier, which makes volatility targeting more important for these strategies. However, both approaches, risk weighting and risk targeting, work well for all cut-off points and clearly outperform the non-managed and volatility weighted portfolio.

B.4 Alternative US Industry Data Sets

This section examines the profitability of the non-managed and risk-managed momentum strategies when different US industry portfolios are used. For our main results, we use 30 equally weighted US industry portfolios. However, several other US industry portfolios are frequently used in the literature on industry momentum and portfolio optimization. For example, instead of using equally weighted industry portfolios, value-weighted industries could be used. Grundy and Martin (2001), Lewellen (2002, Table 2) and Moskowitz and Grinblatt (1999, Footnote 12) find that industry momentum produces quite different results when equally or value-weighted industries are used, where the momentum strategy based on equally weighted industries produces a superior risk-return profile. Lewellen (2002, Table 2) shows that a similar result also holds for momentum based on style portfolios, i.e. using equally or value-weighted style portfolios produces different results of momentum investing and momentum strategies based on equally weighted style portfolios are more profitable. Value-weighted industries were also used by Grobys et al. (2018), Grobys (2018) Grobys and Kolari (2020), Gupta et al. (2010), George and Hwang (2004), Swinkels (2002) and Moskowitz and Grinblatt (1999). Further, instead of using different weighting schemes to weight assets within one industry, different industry classification are also frequently used in the literature. For example, Lewellen (2002) use 15 industries, Moskowitz and Grinblatt (1999), Grundy and Martin (2001), George and Hwang (2004), Pan et al. (2004) and Chordia and Shivakumar (2002) use 20 industries, Swinkels (2002) uses 40 industries, Stivers and Sun (2010), Grobys (2018) and Grobys and Kolari (2020) use 48 industries, whereas Novy-Marx (2012), Grobys et al. (2018) and Du Plessis and Hallerbach (2017) use 49 industries when examining the profitability of industry momentum. Interestingly, Grobys and Kolari (2020, p. 100) find quite different results compared to Moskowitz and Grinblatt (1999) and state that this finding can be explained by using different industry data sets. In a similar setting, Behr et al. (2012) use 10, 17, 30 and 48 industries and find better results when 48 industry portfolios are used. Thus, the size of the chosen data set can influence the profitability of portfolio strategies. Generally, industry portfolios are important for many financial fields, like portfolio optimization and asset pricing, where also different industry portfolio data sets are used. For example, Lettau et al. (2014) use 5 industry portfolios, DeMiguel et al. (2009a) use 10 and 48 industries, Harvey and Siddique (2000) use 27 and 32 industries, Zakamulin (2015) uses 30 industry portfolios, Chen and Petkova (2012) and Chang et al. (2013) use 49 industries, whereas Kirby and Ostdiek (2012), DeMiguel et al. (2009b) and Harvey et al. (2018) examine portfolio strategies using 10 industry portfolios. In particular, some of these studies find quite different results when different data sets are used.

To assess how different industry classifications and value-weighted industries influence the profitability of the non-managed and risk-managed momentum strategies, we next examine performance results for different industry data sets. To assess the influence of the weighting scheme used to weight assets within the industry portfolios, we use 30 value-weighted US industry portfolios. To assess how the size of the industry data set influences the profitability of the non-managed and risk-managed strategies, we use six equally weighted data sets consisting of $5, 10, 12, 17, 38$ and 49 equally weighted US industries. These data sets are again obtained from Kenneth French's website. Similar to the choice of the cut-off point, different US industry classifications also affect the number of industries in the winners and losers portfolios. Using 30 equally weighted industries, as done in the main part of this paper, means that winners and losers consist of 9 industries each. In contrast, using $5, 10, 12, 17, 38$ and 49 industry portfolios means that winners and losers consist of $2, 3, 4, 6, 12$ and 15 industries each. Results for the value-weighted data set are shown in Table XXVII, whereas results for the six additional equally weighted data sets are shown in Table XXVIII, Table XXIX, Table XXX, Table XXXI, Table XXXII and Table XXXIII. For the 10 industry portfolios we use a volatility target of 12%, since this momentum strategy produces a quite high level of volatility.

In line with the finding of Grundy and Martin (2001), Table XXVII shows that industry momentum using value-weighted industries underperforms the strategy using equally weighted

Table XXVII. Robustness Results: 30 Value-Weighted US Industries
This table shows performance results of the equally and risk weighted momentum strategies using 30 value-weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.61	13.02	0.508	-	-	-	-	-	-	-
RV	5.91	12.08	0.489	-0.44	-0.286	-0.55	-0.150	-0.21	-0.341	-0.77
RSJ/Corr	7.97	11.97	0.666	**3.67**	1.871	**2.56**	2.120	**2.07**	1.595	**2.43**
RSJ/Down Corr	8.17	12.11	0.675	**3.82**	1.994	**2.75**	2.197	**2.70**	1.619	**2.33**
RSJ/Beta	7.79	11.70	0.666	**3.48**	1.860	**2.66**	2.120	**2.12**	1.515	**2.43**
RSJ/Down Beta	8.04	11.94	0.673	**3.63**	1.966	**2.38**	2.273	**2.08**	1.652	**2.28**
RSJ/CoSkew	7.95	13.01	0.611	**2.57**	1.259	**2.39**	1.358	**2.31**	1.197	**2.27**
RSJ/CoKurt	8.25	12.33	0.669	**3.83**	1.935	**2.79**	2.044	**2.56**	1.741	**2.57**
RSJ/LPM Beta	7.88	11.82	0.667	**3.54**	1.879	**2.65**	2.120	**2.26**	1.561	**2.40**
RSJ/HTCR Beta	8.03	11.88	0.676	**3.75**	1.994	**2.82**	2.197	**2.46**	1.732	**2.66**
RSJ/Tail Beta	7.18	12.31	0.584	**1.83**	0.880	**2.21**	1.055	1.65	0.640	1.86
RSJ/Tail Sens	8.06	12.02	0.670	**4.00**	1.929	**2.98**	2.273	**2.56**	1.597	**2.58**
RSJ/Tail Risk	7.66	12.11	0.633	**3.03**	1.472	**2.99**	1.739	**2.36**	1.085	**2.50**
RSJ/MES	7.80	11.87	0.657	**3.34**	1.764	**2.40**	2.044	**1.96**	1.415	**2.13**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.50	12.98	0.501	-	-	-	-	-	-	-
RV	5.77	12.02	0.480	-0.64	-0.314	-0.61	-0.225	-0.25	-0.364	-0.82
Equal (TV)	9.24	10.57	0.874	**5.71**	4.467	**5.36**	5.378	**3.89**	3.317	**4.68**
RV (TV)	9.22	11.15	0.827	**4.21**	3.900	**3.59**	4.750	**2.91**	2.926	**3.12**
RSJ/Corr (TV)	9.81	10.44	0.939	**5.79**	5.276	**4.75**	6.247	**3.75**	4.193	**4.37**
RSJ/Down Corr (TV)	9.90	10.38	0.954	**5.89**	5.456	**4.86**	6.406	**3.83**	4.344	**4.49**
RSJ/Beta (TV)	9.79	10.46	0.935	**5.65**	5.225	**4.64**	6.168	**3.69**	4.139	**4.24**
RSJ/Down Beta (TV)	9.83	10.40	0.945	**5.71**	5.349	**4.43**	6.326	**3.59**	4.297	**4.16**
RSJ/CoSkew (TV)	9.68	10.33	0.937	**5.63**	5.243	**4.90**	6.168	**3.83**	4.324	**4.53**
RSJ/CoKurt (TV)	10.01	10.50	0.953	**6.03**	5.451	**4.90**	6.406	**3.86**	4.380	**4.54**
RSJ/LPM Beta (TV)	9.79	10.46	0.937	**5.66**	5.242	**4.54**	6.168	**3.64**	4.168	**4.17**
RSJ/HTCR Beta (TV)	9.84	10.44	0.943	**5.77**	5.317	**4.64**	6.247	**3.76**	4.264	**4.24**
RSJ/Tail Beta (TV)	9.34	10.48	0.891	**5.11**	4.675	**4.78**	5.614	**3.70**	3.635	**4.22**
RSJ/Tail Sens (TV)	9.68	10.38	0.933	**5.68**	5.195	**4.95**	6.089	**3.88**	4.114	**4.30**
RSJ/Tail Risk (TV)	9.69	10.51	0.922	**5.60**	5.064	**4.84**	6.009	**3.75**	3.920	**4.41**
RSJ/MES (TV)	9.76	10.51	0.928	**5.56**	5.134	**4.50**	6.089	**3.59**	4.037	**4.06**

industries as examined in Section 3.7. Industry momentum based on value-weighted industries produces a lower return with a similar level of volatility as the momentum strategy using equally weighted industries.[143] In total, the Sharpe Ratio of the strategy using value-weighted industries is lower than the Sharpe Ratio of the strategy using equally weighted industries. In line with our earlier results, the performance of this strategy cannot be enhanced by weighting the value-weighted industries by their volatility. In contrast, the (systematic) tail risk weightings significantly enhance the performance of the industry momentum strategy. This holds especially for the strategies that combine the (systematic) tail risk weighting with the volatility

[143]The use of equally or value-weighted industries only determines how assets within one industry are weighted, but not how different industries are weighted in the winners and losers portfolios.

Table XXVIII. Robustness Results: 5 Equally Weighted US Industries
This table shows performance results of the equally and risk weighted momentum strategies using 5 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	7.15	11.58	0.618	-	-	-	-	-	-	-
RV	6.62	11.50	0.575	-1.50	-0.461	-1.11	-0.524	-1.14	-0.294	-0.72
RSJ/Corr	8.14	11.51	0.707	**2.93**	0.964	**2.20**	0.979	**2.05**	0.907	**1.99**
RSJ/Down Corr	8.38	11.63	0.720	**3.30**	1.108	**3.34**	1.207	**3.51**	1.137	**3.36**
RSJ/Beta	8.25	11.35	0.727	**3.36**	1.166	**3.15**	1.282	**3.09**	1.110	**2.62**
RSJ/Down Beta	8.49	11.41	0.744	**3.93**	1.358	**5.05**	1.510	**4.77**	1.308	**4.22**
RSJ/CoSkew	8.53	11.77	0.725	**3.57**	1.160	**3.60**	1.282	**3.73**	1.013	**3.04**
RSJ/CoKurt	8.27	11.47	0.721	**3.31**	1.114	**3.00**	1.131	**3.01**	1.026	**2.58**
RSJ/LPM Beta	8.39	11.38	0.737	**3.68**	1.284	**4.08**	1.434	**3.97**	1.246	**3.47**
RSJ/HTCR Beta	8.31	11.33	0.734	**3.56**	1.241	**3.67**	1.358	**3.54**	1.175	**3.10**
RSJ/Tail Beta	8.76	11.49	0.762	**4.73**	1.556	**5.00**	1.663	**4.99**	1.550	**4.70**
RSJ/Tail Sens	8.39	11.45	0.733	**3.71**	1.246	**3.13**	1.282	**3.10**	1.101	**2.61**
RSJ/Tail Risk	8.72	11.43	0.764	**4.70**	1.571	**4.33**	1.663	**4.25**	1.559	**3.92**
RSJ/MES	8.42	11.34	0.742	**3.85**	1.333	**3.63**	1.434	**3.45**	1.281	**3.09**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	7.11	11.56	0.615	-	-	-	-	-	-	-
RV	6.55	11.48	0.571	-1.58	-0.482	-1.15	-0.598	-1.17	-0.312	-0.75
Equal (TV)	10.73	12.53	0.856	**3.96**	2.643	**3.16**	2.580	**3.20**	1.636	**2.24**
RV (TV)	10.93	12.79	0.855	**3.63**	2.628	**3.04**	2.657	**3.07**	1.708	**2.17**
RSJ/Corr (TV)	12.20	12.69	0.962	**4.95**	3.777	**3.98**	3.738	**3.96**	2.867	**3.18**
RSJ/Down Corr (TV)	12.22	12.73	0.960	**4.93**	3.759	**4.20**	3.738	**4.17**	2.865	**3.37**
RSJ/Beta (TV)	12.15	12.70	0.957	**4.86**	3.730	**3.96**	3.738	**4.03**	2.830	**3.30**
RSJ/Down Beta (TV)	12.30	12.70	0.969	**5.02**	3.851	**4.28**	3.815	**4.35**	2.953	**3.60**
RSJ/CoSkew (TV)	12.22	12.75	0.959	**4.90**	3.745	**4.18**	3.738	**4.15**	2.843	**3.41**
RSJ/CoKurt (TV)	12.15	12.72	0.955	**4.85**	3.706	**4.03**	3.660	**4.04**	2.792	**3.24**
RSJ/LPM Beta (TV)	12.25	12.71	0.964	**4.94**	3.797	**4.12**	3.815	**4.18**	2.909	**3.45**
RSJ/HTCR Beta (TV)	12.16	12.70	0.958	**4.86**	3.740	**4.06**	3.738	**4.09**	2.841	**3.35**
RSJ/Tail Beta (TV)	12.51	12.70	0.985	**5.28**	4.029	**4.36**	4.048	**4.35**	3.160	**3.60**
RSJ/Tail Sens (TV)	12.20	12.73	0.958	**4.89**	3.742	**3.94**	3.738	**3.90**	2.799	**3.09**
RSJ/Tail Risk (TV)	12.49	12.70	0.983	**5.25**	4.012	**4.29**	3.970	**4.28**	3.127	**3.53**
RSJ/MES (TV)	12.28	12.70	0.967	**4.99**	3.829	**4.04**	3.815	**4.09**	2.939	**3.36**

targeting approach. Results for the remaining data sets using several equally weighted industry portfolios are mainly in line with our previous findings. For the data set consisting of $5, 10$ or 12 equally weighted industries, we find that the volatility weighted momentum portfolio underperforms the equally weighted momentum portfolio. This underperformance is even statistically significant for the 10 equally weighted US industries. In contrast, the (systematic) tail risk weighted strategies exhibit significantly higher Sharpe Ratios and utilities compared to the equally and volatility weighted strategies for all data sets. For the remaining data sets using $17, 38$ or 49 equally weighted industries, we find that all risk based weighting schemes enhance the risk-return profile of the industry momentum strategy, where again the highest per-

Table XXIX. Robustness Results: 10 Equally Weighted US Industries

This table shows performance results of the equally and risk weighted momentum strategies using 10 equally weighted US industries, the $t-12$ to $t-1$ ranking period, a cut-off point of $p = 30\%$ and a volatility target σ_{target} of 12%. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.07	15.04	0.603	-	-	-	-	-	-	-
RV	7.47	14.97	0.499	**-3.40**	-1.421	**-5.75**	-2.740	**-1.99**	-1.069	**-4.77**
RSJ/Corr	11.45	14.28	0.802	**5.85**	2.692	**6.29**	3.815	**4.02**	2.660	**7.22**
RSJ/Down Corr	11.33	14.34	0.790	**5.24**	2.535	**5.79**	3.583	**4.80**	2.474	**5.70**
RSJ/Beta	11.34	14.46	0.784	**5.24**	2.486	**6.18**	1.968	1.70	2.719	**7.38**
RSJ/Down Beta	11.07	14.53	0.762	**4.35**	2.195	**3.87**	0.677	0.28	2.401	**5.54**
RSJ/CoSkew	10.91	15.49	0.704	**2.95**	1.448	**2.19**	0.526	0.32	1.598	**2.70**
RSJ/CoKurt	11.40	14.36	0.794	**5.61**	2.585	**5.46**	3.660	**3.97**	2.513	**5.56**
RSJ/LPM Beta	11.45	14.49	0.790	**5.27**	2.566	**6.28**	2.120	1.71	2.776	**8.04**
RSJ/HTCR Beta	11.35	14.44	0.786	**5.22**	2.506	**5.60**	2.044	1.55	2.640	**6.63**
RSJ/Tail Beta	10.68	14.79	0.722	**3.52**	1.632	**2.92**	1.434	1.30	1.753	**3.46**
RSJ/Tail Sens	11.25	14.41	0.781	**5.23**	2.400	**5.16**	3.505	**3.77**	2.310	**5.50**
RSJ/Tail Risk	10.91	14.69	0.743	**4.17**	1.926	**3.47**	1.586	1.39	2.051	**4.56**
RSJ/MES	11.64	14.30	0.814	**5.93**	2.875	**8.14**	3.583	**5.11**	2.931	**9.49**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.04	14.95	0.605	-	-	-	-	-	-	-
RV	7.42	14.85	0.500	**-3.37**	-1.431	**-5.68**	-2.740	**-1.97**	-1.083	**-4.81**
Equal (TV)	12.05	13.18	0.914	**5.41**	4.174	**4.93**	5.457	**3.55**	3.188	**4.72**
RV (TV)	10.83	13.76	0.787	**2.73**	2.424	**2.56**	3.738	**2.31**	1.677	**2.20**
RSJ/Corr (TV)	14.02	13.13	1.067	**6.92**	6.308	**6.10**	8.085	**4.20**	5.439	**6.29**
RSJ/Down Corr (TV)	13.88	13.12	1.058	**6.68**	6.178	**6.14**	8.004	**4.22**	5.270	**6.22**
RSJ/Beta (TV)	13.92	13.34	1.043	**6.58**	5.988	**6.65**	6.963	**5.13**	5.343	**6.86**
RSJ/Down Beta (TV)	13.71	13.35	1.027	**6.24**	5.769	**6.04**	6.009	**4.30**	5.109	**6.28**
RSJ/CoSkew (TV)	13.34	13.29	1.004	**5.97**	5.428	**5.01**	6.724	**4.32**	4.647	**4.65**
RSJ/CoKurt (TV)	13.89	13.08	1.062	**6.78**	6.227	**6.00**	8.085	**4.05**	5.307	**6.13**
RSJ/LPM Beta (TV)	14.11	13.25	1.065	**6.82**	6.279	**6.60**	7.523	**4.95**	5.546	**6.94**
RSJ/HTCR Beta (TV)	13.98	13.34	1.048	**6.62**	6.053	**6.35**	7.043	**4.93**	5.349	**6.69**
RSJ/Tail Beta (TV)	13.47	13.27	1.015	**6.10**	5.585	**5.18**	7.043	**4.05**	4.818	**5.14**
RSJ/Tail Sens (TV)	13.91	13.17	1.057	**6.76**	6.155	**6.08**	7.924	**4.17**	5.246	**6.25**
RSJ/Tail Risk (TV)	13.62	13.38	1.018	**6.27**	5.635	**5.59**	6.644	**4.50**	4.928	**5.79**
RSJ/MES (TV)	14.22	13.17	1.080	**7.01**	6.483	**6.71**	8.085	**4.62**	5.642	**7.22**

formance and utility gains are found for the (systematic) tail risk weighted strategies. Although the volatility weighted strategy's Sharpe Ratio is higher than the Sharpe Ratio of the equally weighted strategy for all three data sets, the increase in the Sharpe Ratio is only significant for the 49 industries. The good performance of the volatility managed industry momentum strategy using 49 industries is in line with the finding of Du Plessis and Hallerbach (2017). Similar to the choice of the cut-off point, we find that the risk weighted momentum strategies perform the best when more industries are contained in the winners and losers portfolios. The best improvements of the risk-return profile are found for the 49 industry data set. This is again quite intuitive, since different weighting schemes have a higher impact when more assets are

Table XXX. Robustness Results: 12 Equally Weighted US Industries
This table shows performance results of the equally and risk weighted momentum strategies using 12 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta^{\gamma=5}_{MV}$	DM-test	$\Delta^{\gamma=5}_{CRRA}$	DM-test	$\Delta^{l=2}_{LA}$	DM-test
Equal	8.72	12.72	0.686	-	-	-	-	-	-	-
RV	7.79	12.10	0.644	-1.21	-0.528	-1.58	-0.524	-1.40	-0.523	-1.51
RSJ/Corr	11.31	12.10	0.935	**6.32**	2.918	**5.81**	2.811	**3.93**	2.710	**5.91**
RSJ/Down Corr	11.45	12.35	0.927	**6.04**	2.832	**5.34**	2.734	**3.44**	2.700	**5.79**
RSJ/Beta	11.30	12.13	0.931	**6.03**	2.874	**6.12**	2.657	**3.23**	2.791	**6.45**
RSJ/Down Beta	11.48	12.30	0.933	**5.80**	2.902	**5.60**	2.811	**3.73**	2.778	**6.31**
RSJ/CoSkew	10.74	13.09	0.821	**3.51**	1.605	**3.12**	1.510	**2.45**	1.421	**2.98**
RSJ/CoKurt	11.41	12.14	0.940	**6.39**	2.966	**4.77**	3.350	**4.03**	2.566	**4.64**
RSJ/LPM Beta	11.50	12.16	0.945	**6.38**	3.035	**6.41**	3.196	**4.87**	2.901	**6.67**
RSJ/HTCR Beta	11.52	11.85	0.971	**7.03**	3.340	**6.84**	3.428	**5.14**	3.040	**6.87**
RSJ/Tail Beta	11.09	12.53	0.885	**5.08**	2.326	**4.33**	2.657	**3.66**	2.206	**4.41**
RSJ/Tail Sens	11.28	12.10	0.932	**5.96**	2.870	**5.28**	3.273	**4.50**	2.566	**5.27**
RSJ/Tail Risk	11.13	12.37	0.900	**5.48**	2.494	**5.10**	2.811	**4.20**	2.325	**5.32**
RSJ/MES	11.80	11.99	0.983	**7.16**	3.467	**6.27**	4.048	**4.68**	3.219	**6.35**

	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta^{\gamma=5}_{MV}$	DM-test	$\Delta^{\gamma=5}_{CRRA}$	DM-test	$\Delta^{l=2}_{LA}$	DM-test
Equal	8.67	12.62	0.687	-	-	-	-	-	-	-
RV	7.71	11.97	0.644	-1.33	-0.529	-1.59	-0.524	-1.43	-0.535	-1.54
Equal (TV)	13.37	12.86	1.040	**6.11**	4.097	**5.49**	4.907	**4.58**	3.276	**5.42**
RV (TV)	12.57	13.41	0.937	**3.61**	2.908	**3.16**	3.815	**2.87**	2.131	**2.77**
RSJ/Corr (TV)	15.76	13.01	1.212	**7.41**	6.108	**6.29**	6.724	**5.79**	5.443	**6.47**
RSJ/Down Corr (TV)	15.84	12.98	1.220	**7.54**	6.200	**6.49**	6.883	**5.95**	5.495	**6.71**
RSJ/Beta (TV)	15.74	13.08	1.204	**7.25**	6.014	**6.22**	6.644	**5.75**	5.405	**6.58**
RSJ/Down Beta (TV)	15.87	13.02	1.219	**7.38**	6.187	**6.42**	6.883	**5.86**	5.464	**6.75**
RSJ/CoSkew (TV)	15.22	13.03	1.168	**6.80**	5.598	**5.41**	6.326	**5.14**	4.797	**5.42**
RSJ/CoKurt (TV)	15.79	12.81	1.232	**7.60**	6.341	**5.94**	7.202	**5.15**	5.460	**6.02**
RSJ/LPM Beta (TV)	15.96	12.99	1.228	**7.57**	6.298	**6.47**	7.043	**5.78**	5.608	**6.87**
RSJ/HTCR Beta (TV)	15.98	12.87	1.242	**7.62**	6.454	**6.46**	7.202	**5.70**	5.714	**6.77**
RSJ/Tail Beta (TV)	15.61	12.99	1.202	**7.37**	5.986	**5.99**	6.803	**5.31**	5.232	**6.21**
RSJ/Tail Sens (TV)	15.71	12.91	1.217	**7.36**	6.161	**6.28**	7.043	**5.42**	5.379	**6.45**
RSJ/Tail Risk (TV)	15.60	12.98	1.201	**7.32**	5.979	**5.80**	6.803	**5.18**	5.224	**6.03**
RSJ/MES (TV)	16.14	12.84	1.257	**7.83**	6.623	**6.31**	7.523	**5.25**	5.836	**6.73**

contained in the portfolio. Furthermore, for all six data sets of equally weighted industries, we find that volatility targeting again significantly enhances the risk-return profile, regardless of the weighting scheme. The good performance of the volatility targeting approach for the 49 industry portfolios has already been shown by Du Plessis and Hallerbach (2017) and Grobys et al. (2018). Results in this section show that volatility targeting also works well for other industry classifications. In line with our previous results, we find that the highest Sharpe Ratios and economic values are again found for the strategies that combine volatility targeting with the (systematic) tail risk weightings.

In total, results in this section confirm the finding of Hong et al. (2000) and Grundy and

Table XXXI. Robustness Results: 17 Equally Weighted US Industries

This table shows performance results of the equally and risk weighted momentum strategies using 17 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta^{\gamma=5}_{MV}$	DM-test	$\Delta^{\gamma=5}_{CRRA}$	DM-test	$\Delta^{l=2}_{LA}$	DM-test
Equal	7.35	10.78	0.682	-	-	-	-	-	-	-
RV	7.16	10.37	0.690	0.21	0.069	0.21	-0.075	-0.20	0.078	0.24
RSJ/Corr	9.34	10.58	0.883	**4.40**	2.013	**4.05**	2.197	**2.89**	1.923	**5.00**
RSJ/Down Corr	9.13	10.81	0.845	**3.72**	1.635	**3.97**	1.739	**4.04**	1.632	**4.21**
RSJ/Beta	9.03	10.95	0.825	**3.49**	1.458	**3.46**	1.207	**2.28**	1.611	**4.19**
RSJ/Down Beta	9.37	10.66	0.879	**4.45**	1.978	**4.77**	2.120	**4.81**	1.962	**4.94**
RSJ/CoSkew	9.16	10.98	0.834	**3.71**	1.530	**3.98**	1.815	**3.82**	1.410	**3.82**
RSJ/CoKurt	8.99	11.21	0.802	**2.81**	1.240	**3.14**	0.753	1.10	1.469	**4.43**
RSJ/LPM Beta	9.21	10.71	0.860	**4.24**	1.787	**3.94**	1.891	**3.71**	1.815	**4.44**
RSJ/HTCR Beta	8.99	10.81	0.832	**3.60**	1.512	**2.95**	1.434	**2.48**	1.614	**3.53**
RSJ/Tail Beta	9.06	10.86	0.834	**3.72**	1.534	**3.75**	1.586	**3.82**	1.542	**3.67**
RSJ/Tail Sens	8.97	11.19	0.802	**2.86**	1.223	**3.50**	1.131	**3.25**	1.411	**4.30**
RSJ/Tail Risk	9.04	10.92	0.828	**3.64**	1.475	**3.93**	1.510	**3.91**	1.560	**4.29**
RSJ/MES	9.28	10.68	0.869	**4.37**	1.879	**4.33**	2.044	**3.69**	1.863	**4.96**

	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta^{\gamma=5}_{MV}$	DM-test	$\Delta^{\gamma=5}_{CRRA}$	DM-test	$\Delta^{l=2}_{LA}$	DM-test
Equal	7.27	10.76	0.675	-	-	-	-	-	-	-
RV	7.07	10.35	0.683	0.15	0.064	0.19	-0.075	-0.22	0.068	0.21
Equal (TV)	12.75	12.66	1.007	**6.37**	3.350	**6.98**	3.660	**6.24**	3.042	**6.29**
RV (TV)	13.79	13.13	1.051	**5.57**	3.782	**5.48**	4.204	**4.93**	3.420	**5.72**
RSJ/Corr (TV)	14.37	12.18	1.180	**7.23**	5.054	**6.54**	5.536	**5.36**	4.678	**6.92**
RSJ/Down Corr (TV)	14.13	12.36	1.144	**6.93**	4.702	**7.05**	5.142	**6.16**	4.422	**7.39**
RSJ/Beta (TV)	14.08	12.50	1.127	**6.87**	4.538	**7.57**	4.907	**6.81**	4.306	**7.62**
RSJ/Down Beta (TV)	14.29	12.30	1.162	**7.06**	4.881	**7.17**	5.378	**6.14**	4.591	**7.24**
RSJ/CoSkew (TV)	14.12	12.20	1.157	**6.98**	4.834	**7.03**	5.300	**5.89**	4.489	**7.18**
RSJ/CoKurt (TV)	13.97	12.56	1.113	**6.68**	4.407	**7.38**	4.672	**7.02**	4.239	**7.35**
RSJ/LPM Beta (TV)	14.23	12.35	1.152	**7.06**	4.785	**7.18**	5.221	**6.07**	4.506	**7.48**
RSJ/HTCR Beta (TV)	14.09	12.38	1.138	**6.89**	4.645	**7.36**	5.064	**6.35**	4.388	**7.48**
RSJ/Tail Beta (TV)	14.02	12.37	1.133	**6.83**	4.598	**6.74**	5.064	**5.91**	4.317	**6.67**
RSJ/Tail Sens (TV)	13.92	12.52	1.112	**6.68**	4.388	**6.66**	4.829	**6.00**	4.185	**6.96**
RSJ/Tail Risk (TV)	13.97	12.41	1.125	**6.82**	4.520	**6.78**	4.985	**5.96**	4.281	**6.99**
RSJ/MES (TV)	14.24	12.33	1.155	**7.00**	4.808	**7.07**	5.300	**5.85**	4.500	**7.27**

Martin (2001) that the use of different data sets can lead to quite different performance results of momentum strategies. However, for all data sets, we find that the (systematic) tail risk weighted momentum portfolios outperform the non-managed and volatility managed strategies. Furthermore, volatility targeting enhances the risk-return profile regardless of the used US industry data set or weighting scheme. Thus, results in this section confirm our earlier findings for seven additional data sets.

Table XXXII. Robustness Results: 38 Equally Weighted US Industries
This table shows performance results of the equally and risk weighted momentum strategies using 38 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	7.84	11.77	0.666	-	-	-	-	-	-	-
RV	7.93	11.45	0.692	0.56	0.287	0.58	0.000	0.02	0.781	1.54
RSJ/Corr	9.53	11.02	0.865	**3.92**	2.167	**4.71**	1.968	**2.84**	2.208	**5.23**
RSJ/Down Corr	9.50	11.35	0.838	**3.38**	1.876	**3.44**	1.663	1.94	1.958	**3.96**
RSJ/Beta	9.87	11.10	0.889	**4.33**	2.441	**4.39**	2.044	**2.25**	2.667	**5.08**
RSJ/Down Beta	9.78	11.24	0.870	**3.71**	2.226	**3.45**	1.968	**2.05**	2.471	**3.82**
RSJ/CoSkew	9.24	12.31	0.750	1.66	0.934	1.77	1.055	1.73	1.085	**2.06**
RSJ/CoKurt	9.52	11.25	0.847	**3.44**	1.977	**3.50**	1.739	**1.96**	2.052	**3.98**
RSJ/LPM Beta	9.98	11.07	0.901	**4.50**	2.567	**4.46**	2.503	**3.39**	2.757	**4.63**
RSJ/HTCR Beta	9.75	11.05	0.882	**4.29**	2.358	**4.66**	2.197	**2.94**	2.500	**4.95**
RSJ/Tail Beta	9.78	11.57	0.845	**3.79**	1.962	**3.78**	1.815	**2.79**	2.318	**4.27**
RSJ/Tail Sens	9.63	11.33	0.850	**3.73**	2.005	**4.53**	2.120	**4.47**	2.132	**4.50**
RSJ/Tail Risk	9.89	11.59	0.853	**3.93**	2.045	**4.46**	2.044	**3.81**	2.337	**4.59**
RSJ/MES	10.15	11.10	0.915	**4.84**	2.711	**5.14**	2.734	**4.34**	2.885	**5.23**

	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	7.78	11.68	0.665	-	-	-	-	-	-	-
RV	7.88	11.31	0.697	0.63	0.336	0.66	0.075	0.07	0.818	1.57
Equal (TV)	10.68	11.30	0.945	**5.12**	3.018	**4.33**	3.350	**4.11**	2.762	**3.84**
RV (TV)	13.77	12.65	1.089	**6.05**	4.599	**6.09**	4.985	**5.96**	4.326	**5.38**
RSJ/Corr (TV)	11.99	10.93	1.097	**6.04**	4.686	**7.09**	4.985	**6.80**	4.434	**6.16**
RSJ/Down Corr (TV)	11.78	10.94	1.077	**5.80**	4.467	**6.48**	4.829	**6.14**	4.223	**5.69**
RSJ/Beta (TV)	12.32	11.33	1.088	**6.07**	4.597	**7.21**	4.750	**6.60**	4.559	**6.39**
RSJ/Down Beta (TV)	11.91	11.06	1.077	**5.76**	4.465	**6.46**	4.750	**6.22**	4.351	**5.63**
RSJ/CoSkew (TV)	11.06	10.82	1.023	**5.05**	3.864	**4.66**	4.282	**4.54**	3.632	**4.29**
RSJ/CoKurt (TV)	11.77	10.96	1.074	**5.75**	4.436	**6.82**	4.672	**6.58**	4.247	**5.97**
RSJ/LPM Beta (TV)	12.39	11.19	1.107	**6.18**	4.793	**6.75**	5.142	**6.48**	4.692	**5.93**
RSJ/HTCR Beta (TV)	11.95	11.00	1.086	**5.97**	4.565	**7.02**	4.829	**6.77**	4.399	**5.97**
RSJ/Tail Beta (TV)	12.38	11.40	1.085	**6.22**	4.560	**6.62**	4.907	**6.34**	4.520	**5.84**
RSJ/Tail Sens (TV)	12.01	10.99	1.093	**6.10**	4.641	**6.73**	5.064	**6.32**	4.451	**6.04**
RSJ/Tail Risk (TV)	12.48	11.48	1.087	**6.24**	4.576	**6.84**	4.907	**6.56**	4.516	**6.01**
RSJ/MES (TV)	12.56	11.20	1.121	**6.43**	4.944	**7.07**	5.300	**6.77**	4.844	**6.25**

B.5 International and European Industry Momentum

Results examined in the previous sections are so far based on US industry momentum. This is in line with the literature, since most studies on momentum typically focus on US data. However, momentum does not only work in the US, but is also an international phenomenon (Asness et al., 2013, Fama and French, 2012, Griffin et al., 2003, Rouwenhorst, 1998). Similarly, industry momentum is also profitable for non-US industries (see Gupta et al. (2010), Nijman et al. (2004), Swinkels (2002) and references therein). Swinkels (2002, p. 133) "conclude that industry momentum is a global phenomenon". Gupta et al. (2010, Table 5) find that also the 52 week high industry momentum strategy is profitable in most countries. Although it has been

Table XXXIII. Robustness Results: 49 Equally Weighted US Industries
This table shows performance results of the equally and risk weighted momentum strategies using 49 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p=30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	8.49	12.62	0.673	-	-	-	-	-	-	-
RV	8.75	11.30	0.775	**2.31**	1.125	1.79	1.739	1.49	1.152	1.92
RSJ/Corr	11.24	11.61	0.968	**5.80**	3.430	**4.03**	3.350	**2.01**	3.446	**4.48**
RSJ/Down Corr	10.89	12.15	0.896	**5.06**	2.611	**3.84**	1.968	**2.01**	2.816	**4.12**
RSJ/Beta	11.27	11.07	1.018	**6.67**	3.987	**6.16**	4.750	**4.20**	3.734	**5.69**
RSJ/Down Beta	10.83	11.37	0.952	**5.67**	3.232	**3.83**	3.350	**2.10**	3.149	**4.20**
RSJ/CoSkew	10.38	13.03	0.796	**2.84**	1.461	**2.80**	1.282	1.68	1.685	**3.00**
RSJ/CoKurt	10.77	12.41	0.868	**4.37**	2.303	**3.87**	1.434	1.62	2.660	**4.32**
RSJ/LPM Beta	11.41	10.97	1.040	**7.09**	4.254	**5.46**	5.064	**3.98**	3.949	**5.01**
RSJ/HTCR Beta	11.40	11.06	1.031	**7.07**	4.144	**5.98**	4.750	**4.69**	3.889	**5.38**
RSJ/Tail Beta	10.76	11.53	0.933	**5.62**	2.990	**4.73**	3.738	**3.33**	2.739	**4.29**
RSJ/Tail Sens	10.90	11.54	0.944	**5.87**	3.132	**4.21**	3.738	**3.09**	2.854	**4.05**
RSJ/Tail Risk	10.70	11.59	0.923	**5.35**	2.868	**3.97**	3.738	**2.85**	2.585	**3.68**
RSJ/MES	11.37	11.08	1.026	**6.87**	4.083	**5.68**	4.907	**4.27**	3.772	**5.09**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	8.58	12.48	0.688	-	-	-	-	-	-	-
RV	8.79	11.14	0.789	**2.08**	1.096	1.75	1.739	1.48	1.121	1.85
Equal (TV)	12.37	11.45	1.080	**6.69**	4.483	**5.62**	5.614	**4.00**	3.649	**4.99**
RV (TV)	15.70	12.99	1.209	**6.97**	5.979	**5.32**	7.122	**4.22**	5.196	**5.24**
RSJ/Corr (TV)	14.18	11.19	1.268	**7.97**	6.688	**7.59**	7.523	**5.04**	6.078	**7.10**
RSJ/Down Corr (TV)	13.76	11.00	1.251	**8.03**	6.489	**7.77**	7.442	**5.35**	5.776	**6.99**
RSJ/Beta (TV)	14.41	11.08	1.300	**8.32**	7.061	**8.36**	8.246	**5.65**	6.286	**7.56**
RSJ/Down Beta (TV)	13.73	11.04	1.244	**7.60**	6.415	**7.71**	7.362	**5.24**	5.734	**7.05**
RSJ/CoSkew (TV)	12.99	10.81	1.202	**7.18**	5.912	**6.69**	7.122	**4.97**	5.106	**5.98**
RSJ/CoKurt (TV)	13.74	11.16	1.232	**7.84**	6.268	**7.45**	7.122	**5.03**	5.654	**6.91**
RSJ/LPM Beta (TV)	14.37	10.91	1.317	**8.37**	7.258	**7.87**	8.488	**5.50**	6.417	**7.08**
RSJ/HTCR Beta (TV)	14.28	10.87	1.314	**8.32**	7.229	**8.59**	8.408	**5.87**	6.435	**7.45**
RSJ/Tail Beta (TV)	13.91	11.19	1.243	**8.06**	6.392	**7.42**	7.523	**5.09**	5.538	**6.95**
RSJ/Tail Sens (TV)	13.83	11.00	1.257	**7.87**	6.552	**7.19**	7.763	**5.01**	5.684	**6.57**
RSJ/Tail Risk (TV)	13.74	11.30	1.216	**7.67**	6.071	**6.60**	7.282	**4.64**	5.199	**6.22**
RSJ/MES (TV)	14.49	10.99	1.319	**8.38**	7.281	**7.74**	8.569	**5.46**	6.441	**6.95**

shown that industry momentum is profitable internationally, risk-managed industry momentum outside the US has not been examined so far. To assess the profitability of the non-managed and risk-managed industry momentum strategies outside the US, we next use International and European industry portfolios, which are obtained from Datastream. As in Swinkels (2002), we use the Datastream industry classification that provides a long sample of daily and monthly industry returns. The sample period ranges from 01.01.1978 to 31.12.2018. As for the US industries, we show results for different industry classifications. Tables XXXIV to XXXVII show results for the momentum strategy using International industry portfolios based on $10, 19, 40$ and 113 industries. Similarly, Tables XXXVIII to XLI show results for industry momentum using 10, 19, 40 and 113 European industry portfolios. For the momentum strategy based on 113 industries, we use a cut-off point of $p = 20\%$. Thus, the winners and losers portfolios consist of 3, 6, 12 and 22 industries for the 10, 19, 40 and 113 industry portfolios. As in our main results, we rank industries based on their $t-12$ to $t-1$ performance. Swinkels (2002) also find that ranking periods that include the last month's performance outperform the ranking periods that skip the last month. An alternative to the ranking method used here would be to rank European and International industries based on the past performance of the same US industries. Swinkels (2002, Table 6 and 9) finds good results for International industry momentum portfolios that are ranked based on the performance of the corresponding US industries.

Table XXXIV. Robustness Results: 10 Equally Weighted International Industries
This table shows performance results of the equally and risk weighted momentum strategies using 10 equally weighted International industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.79	12.57	0.540	-	-	-	-	-	-	-
RV	5.38	11.34	0.475	-1.37	-0.830	**-2.17**	-0.896	**-2.26**	-1.010	**-2.38**
RSJ/Corr	7.69	11.80	0.652	**2.26**	1.299	**2.64**	1.207	**2.43**	0.983	**1.84**
RSJ/Down Corr	8.05	11.90	0.676	**2.61**	1.594	**3.59**	1.586	**3.44**	1.407	**2.91**
RSJ/Beta	7.75	11.70	0.663	**2.50**	1.422	**2.76**	1.358	**2.59**	1.127	**1.99**
RSJ/Down Beta	7.91	11.83	0.668	**2.56**	1.494	**3.24**	1.434	**3.05**	1.262	**2.47**
RSJ/CoSkew	7.13	12.52	0.570	0.61	0.358	0.83	0.376	0.84	0.118	0.27
RSJ/CoKurt	7.71	11.86	0.650	**2.17**	1.281	**3.14**	1.282	**3.02**	0.998	**2.20**
RSJ/LPM Beta	8.00	11.67	0.686	**2.91**	1.696	**3.70**	1.663	**3.48**	1.441	**2.84**
RSJ/HTCR Beta	7.96	11.61	0.686	**3.00**	1.692	**3.18**	1.663	**3.01**	1.439	**2.51**
RSJ/Tail Beta	7.41	11.71	0.633	1.93	1.071	**2.16**	1.055	**2.10**	0.833	1.58
RSJ/Tail Sens	7.70	12.12	0.636	**1.98**	1.121	1.89	1.055	1.81	0.842	1.41
RSJ/Tail Risk	7.69	11.74	0.655	**2.39**	1.330	**2.85**	1.282	**2.73**	1.076	**2.13**
RSJ/MES	7.60	11.70	0.649	**2.25**	1.265	**2.60**	1.207	**2.44**	0.976	1.82

	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.79	12.59	0.539	-	-	-	-	-	-	-
RV	5.41	11.34	0.477	-1.45	-0.794	**-2.03**	-0.822	**-2.12**	-0.979	**-2.27**
Equal (TV)	7.70	10.19	0.756	**3.16**	2.481	**4.77**	2.580	**5.07**	1.997	**2.85**
RV (TV)	6.80	10.34	0.658	1.39	1.317	**1.96**	1.434	**2.14**	0.755	0.88
RSJ/Corr (TV)	8.65	10.09	0.857	**3.81**	3.704	**5.73**	3.738	**5.80**	3.085	**3.99**
RSJ/Down Corr (TV)	8.81	10.04	0.877	**3.94**	3.940	**5.09**	3.970	**5.17**	3.459	**3.83**
RSJ/Beta (TV)	8.65	10.09	0.858	**3.80**	3.712	**5.96**	3.815	**6.06**	3.140	**4.22**
RSJ/Down Beta (TV)	8.72	10.14	0.861	**3.80**	3.744	**5.09**	3.815	**5.17**	3.283	**3.79**
RSJ/CoSkew (TV)	7.63	10.05	0.759	**2.55**	2.519	**3.57**	2.580	**3.70**	2.017	**2.58**
RSJ/CoKurt (TV)	8.54	10.13	0.843	**3.58**	3.536	**5.18**	3.583	**5.25**	2.995	**3.60**
RSJ/LPM Beta (TV)	8.87	10.06	0.882	**4.07**	4.004	**5.87**	4.048	**5.97**	3.485	**4.24**
RSJ/HTCR Beta (TV)	8.77	10.01	0.876	**4.04**	3.930	**5.77**	3.970	**5.89**	3.381	**4.09**
RSJ/Tail Beta (TV)	8.21	10.04	0.818	**3.32**	3.227	**4.53**	3.350	**4.66**	2.705	**3.23**
RSJ/Tail Sens (TV)	8.58	10.19	0.842	**3.64**	3.525	**4.83**	3.583	**4.92**	2.944	**3.39**
RSJ/Tail Risk (TV)	8.47	10.07	0.841	**3.63**	3.510	**5.10**	3.583	**5.19**	2.972	**3.56**
RSJ/MES (TV)	8.51	10.07	0.845	**3.65**	3.560	**5.25**	3.660	**5.37**	3.002	**3.74**

Table XXXV. Robustness Results: 19 Equally Weighted International Industries
This table shows performance results of the equally and risk weighted momentum strategies using 19 equally weighted International industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.25	10.57	0.591	-	-	-	-	-	-	-
RV	5.52	9.37	0.589	-0.01	-0.069	-0.15	-0.075	-0.13	-0.100	-0.21
RSJ/Corr	7.85	9.96	0.788	**3.26**	1.946	**5.05**	1.968	**5.23**	1.727	**5.14**
RSJ/Down Corr	7.85	9.95	0.789	**3.38**	1.954	**5.65**	1.968	**5.75**	1.793	**5.65**
RSJ/Beta	7.73	9.76	0.792	**3.19**	1.981	**4.89**	2.044	**5.06**	1.736	**4.78**
RSJ/Down Beta	7.86	9.80	0.802	**3.43**	2.088	**4.96**	2.120	**5.07**	1.911	**4.94**
RSJ/CoSkew	7.88	10.62	0.742	**2.72**	1.505	**3.54**	1.510	**3.47**	1.434	**3.25**
RSJ/CoKurt	7.90	9.94	0.795	**3.40**	2.020	**5.40**	2.044	**5.48**	1.813	**5.23**
RSJ/LPM Beta	7.81	9.78	0.798	**3.33**	2.048	**5.76**	2.044	**5.92**	1.853	**5.87**
RSJ/HTCR Beta	7.79	9.73	0.801	**3.39**	2.072	**6.01**	2.120	**6.21**	1.825	**6.49**
RSJ/Tail Beta	7.95	9.99	0.795	**3.49**	2.024	**6.01**	2.044	**6.26**	1.832	**6.75**
RSJ/Tail Sens	7.89	10.14	0.779	**3.21**	1.859	**4.52**	1.891	**4.62**	1.714	**4.32**
RSJ/Tail Risk	7.93	10.05	0.789	**3.51**	1.964	**6.41**	1.968	**6.65**	1.852	**6.77**
RSJ/MES	7.78	9.91	0.786	**3.14**	1.924	**5.47**	1.968	**5.68**	1.722	**5.58**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.33	10.53	0.601	-	-	-	-	-	-	-
RV	5.60	9.34	0.599	-0.11	-0.061	-0.13	-0.075	-0.11	-0.097	-0.20
Equal (TV)	8.27	10.29	0.804	**3.02**	2.011	**4.17**	2.120	**4.22**	1.772	**2.57**
RV (TV)	8.02	10.26	0.782	**2.14**	1.794	**2.57**	1.891	**2.61**	1.443	1.62
RSJ/Corr (TV)	9.75	10.10	0.965	**4.18**	3.614	**6.18**	3.738	**6.29**	3.334	**4.91**
RSJ/Down Corr (TV)	9.66	10.00	0.967	**4.23**	3.626	**6.96**	3.738	**7.07**	3.384	**5.42**
RSJ/Beta (TV)	9.55	10.08	0.948	**3.94**	3.438	**6.08**	3.505	**6.18**	3.166	**4.94**
RSJ/Down Beta (TV)	9.70	10.02	0.967	**4.21**	3.636	**6.11**	3.738	**6.19**	3.419	**5.08**
RSJ/CoSkew (TV)	9.60	10.01	0.959	**4.30**	3.555	**6.82**	3.660	**6.88**	3.313	**5.12**
RSJ/CoKurt (TV)	9.72	10.05	0.967	**4.24**	3.635	**7.00**	3.738	**7.08**	3.354	**5.32**
RSJ/LPM Beta (TV)	9.69	10.06	0.963	**4.12**	3.595	**6.56**	3.660	**6.65**	3.367	**5.22**
RSJ/HTCR Beta (TV)	9.65	10.03	0.962	**4.12**	3.583	**6.88**	3.660	**6.97**	3.325	**5.43**
RSJ/Tail Beta (TV)	9.71	10.02	0.969	**4.25**	3.653	**7.37**	3.738	**7.50**	3.406	**5.89**
RSJ/Tail Sens (TV)	9.74	10.12	0.963	**4.19**	3.594	**6.41**	3.660	**6.51**	3.351	**4.93**
RSJ/Tail Risk (TV)	9.82	10.09	0.973	**4.38**	3.696	**7.53**	3.815	**7.59**	3.506	**5.82**
RSJ/MES (TV)	9.66	10.13	0.953	**4.03**	3.494	**6.12**	3.583	**6.20**	3.256	**4.95**

Results in this section show that industry momentum does not only work for US industries, but also for industries outside the US. For all eight data sets examined in this section, the industry momentum strategy produces high Sharpe Ratios. Thus, similar to the case of the individual stock momentum strategy, the industry momentum effect is also apparent internationally. Since industry momentum outside the US is only rarely examined, this is an interesting contribution to the momentum literature. Furthermore, since also the low risk anomaly holds internationally (Ang et al., 2009), we expect good results of the risk-managed industry momentum strategies outside the US. However, we find that for six of the eight data sets, the volatility weighted momentum strategy underperforms the equally weighted momentum strategy. Only for the Eu-

Table XXXVI. Robustness Results: 40 Equally Weighted International Industries
This table shows performance results of the equally and risk weighted momentum strategies using 40 equally weighted International industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.70	11.47	0.584	-	-	-	-	-	-	-
RV	4.75	9.96	0.477	**-2.17**	-1.227	**-2.17**	-1.268	**-2.19**	-1.266	**-2.22**
RSJ/Corr	8.67	10.22	0.848	**3.98**	2.819	**5.08**	2.888	**5.11**	2.623	**5.22**
RSJ/Down Corr	8.63	10.53	0.820	**3.69**	2.518	**4.89**	2.580	**4.89**	2.335	**4.99**
RSJ/Beta	8.21	10.05	0.816	**3.37**	2.468	**3.93**	2.503	**3.96**	2.226	**3.89**
RSJ/Down Beta	8.30	10.30	0.806	**3.30**	2.361	**5.72**	2.427	**5.81**	2.114	**5.64**
RSJ/CoSkew	8.46	11.41	0.741	**2.72**	1.694	**3.03**	1.739	**2.98**	1.486	**2.65**
RSJ/CoKurt	8.55	10.35	0.827	**3.70**	2.586	**5.50**	2.580	**5.47**	2.376	**5.51**
RSJ/LPM Beta	8.36	10.18	0.821	**3.41**	2.520	**4.81**	2.580	**4.89**	2.316	**4.79**
RSJ/HTCR Beta	8.44	10.12	0.834	**3.60**	2.665	**5.09**	2.734	**5.17**	2.433	**4.99**
RSJ/Tail Beta	8.43	10.43	0.809	**3.79**	2.397	**4.67**	2.427	**4.73**	2.194	**4.28**
RSJ/Tail Sens	8.76	10.44	0.838	**4.09**	2.716	**4.78**	2.734	**4.82**	2.548	**5.08**
RSJ/Tail Risk	8.06	10.42	0.774	**3.13**	2.012	**4.01**	2.044	**3.99**	1.837	**4.21**
RSJ/MES	8.44	10.18	0.830	**3.58**	2.616	**4.26**	2.657	**4.30**	2.376	**4.21**

	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.79	11.48	0.592	-	-	-	-	-	-	-
RV	4.81	9.96	0.483	**-2.40**	-1.246	**-2.19**	-1.268	**-2.21**	-1.276	**-2.21**
Equal (TV)	8.21	10.37	0.792	**2.96**	2.124	**3.81**	2.273	**3.85**	1.806	**2.33**
RV (TV)	6.98	10.76	0.649	0.65	0.582	0.72	0.677	0.81	0.322	0.33
RSJ/Corr (TV)	10.34	9.95	1.040	**4.78**	4.818	**5.03**	4.985	**5.07**	4.408	**4.15**
RSJ/Down Corr (TV)	10.12	9.99	1.013	**4.56**	4.530	**4.98**	4.672	**5.01**	4.140	**4.00**
RSJ/Beta (TV)	10.01	10.01	1.000	**4.33**	4.388	**4.85**	4.516	**4.91**	3.985	**4.11**
RSJ/Down Beta (TV)	9.89	10.03	0.986	**4.22**	4.234	**5.25**	4.360	**5.30**	3.836	**4.14**
RSJ/CoSkew (TV)	9.70	10.13	0.958	**4.09**	3.924	**5.27**	4.048	**5.29**	3.520	**3.86**
RSJ/CoKurt (TV)	10.20	9.93	1.027	**4.65**	4.678	**5.37**	4.829	**5.39**	4.246	**4.25**
RSJ/LPM Beta (TV)	10.03	10.02	1.001	**4.32**	4.399	**4.99**	4.516	**5.05**	4.033	**4.06**
RSJ/HTCR Beta (TV)	10.18	9.96	1.023	**4.49**	4.632	**5.48**	4.750	**5.52**	4.258	**4.38**
RSJ/Tail Beta (TV)	10.08	10.08	1.000	**4.61**	4.387	**5.66**	4.516	**5.71**	4.013	**4.51**
RSJ/Tail Sens (TV)	10.45	9.97	1.048	**4.93**	4.905	**5.50**	5.064	**5.54**	4.525	**4.52**
RSJ/Tail Risk (TV)	9.91	10.13	0.979	**4.34**	4.153	**5.91**	4.282	**5.96**	3.786	**4.94**
RSJ/MES (TV)	10.17	10.00	1.017	**4.49**	4.567	**5.01**	4.672	**5.06**	4.144	**4.16**

ropean momentum strategy using 19 and 113 industry portfolios, volatility weighting produces a higher Sharpe Ratio than the equally weighted portfolio. Nevertheless, the increase in the Sharpe Ratio is quite low and not statistically significant. For the international momentum strategy using 40 industry portfolios, the volatility weighted strategy's Sharpe Ratio is even significantly lower than the equally weighted strategy's Sharpe Ratio. Similarly, the economic value of the volatility weighted momentum strategy is very low or even negative in all eight cases. This again highlights the disadvantage of using volatility weighting for long-short strategies. In contrast, the (systematic) tail risk weighted strategies exhibit higher Sharpe Ratios and high economic values, which are statistically significant for all models and all eight data sets.

Table XXXVII. Robustness Results: 113 Equally Weighted International Industries

This table shows performance results of the equally and risk weighted momentum strategies using 113 equally weighted International industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p=20\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.19	14.68	0.626	-	-	-	-	-	-	-
RV	7.97	12.95	0.615	-0.16	-0.236	-0.40	-0.225	-0.35	-0.173	-0.29
RSJ/Corr	10.23	12.63	0.810	**2.80**	2.421	**4.15**	2.427	**4.31**	2.279	**3.51**
RSJ/Down Corr	10.33	13.13	0.786	**2.63**	2.114	**3.23**	2.120	**3.25**	2.033	**2.88**
RSJ/Beta	10.02	12.16	0.824	**2.87**	2.591	**4.77**	2.580	**4.92**	2.405	**3.90**
RSJ/Down Beta	10.03	12.54	0.800	**2.71**	2.275	**3.64**	2.350	**3.81**	2.069	**3.15**
RSJ/CoSkew	10.96	14.42	0.760	**2.50**	1.811	**2.73**	1.815	**2.68**	1.736	**2.44**
RSJ/CoKurt	9.97	12.85	0.776	**2.41**	1.964	**3.26**	1.968	**3.31**	1.769	**2.72**
RSJ/LPM Beta	10.23	12.36	0.828	**2.94**	2.649	**4.34**	2.657	**4.47**	2.468	**3.58**
RSJ/HTCR Beta	10.15	12.36	0.822	**2.86**	2.568	**4.21**	2.580	**4.28**	2.384	**3.47**
RSJ/Tail Beta	10.28	12.64	0.813	**3.11**	2.462	**4.14**	2.503	**4.15**	2.300	**3.53**
RSJ/Tail Sens	10.42	12.94	0.806	**2.93**	2.371	**7.60**	2.350	**7.77**	2.281	**5.13**
RSJ/Tail Risk	9.92	12.63	0.785	**2.75**	2.077	**3.95**	2.120	**3.96**	1.944	**3.33**
RSJ/MES	10.21	12.47	0.818	**2.82**	2.528	**4.97**	2.503	**5.07**	2.394	**4.13**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.21	14.68	0.628	-	-	-	-	-	-	-
RV	8.05	12.94	0.622	-0.25	-0.164	-0.27	-0.150	-0.22	-0.119	-0.20
Equal (TV)	8.09	10.21	0.793	**2.44**	2.061	**2.32**	2.273	**2.49**	1.528	1.55
RV (TV)	8.19	10.49	0.781	1.73	1.902	1.68	2.120	1.81	1.448	1.11
RSJ/Corr (TV)	9.90	10.14	0.977	**3.75**	4.628	**4.24**	4.829	**4.38**	4.006	**3.17**
RSJ/Down Corr (TV)	9.71	10.15	0.956	**3.63**	4.342	**3.90**	4.594	**4.07**	3.778	**2.96**
RSJ/Beta (TV)	10.00	10.15	0.985	**3.71**	4.752	**4.40**	4.985	**4.53**	4.131	**3.32**
RSJ/Down Beta (TV)	9.64	10.09	0.955	**3.49**	4.329	**4.24**	4.594	**4.46**	3.690	**3.17**
RSJ/CoSkew (TV)	9.22	9.97	0.925	**3.46**	3.900	**3.64**	4.126	**3.75**	3.533	**2.96**
RSJ/CoKurt (TV)	9.57	10.16	0.943	**3.44**	4.151	**3.96**	4.360	**4.13**	3.488	**2.83**
RSJ/LPM Beta (TV)	10.03	10.18	0.985	**3.71**	4.741	**4.35**	4.985	**4.48**	4.113	**3.27**
RSJ/HTCR Beta (TV)	9.89	10.04	0.985	**3.75**	4.746	**4.97**	4.985	**5.09**	4.080	**3.58**
RSJ/Tail Beta (TV)	9.95	10.21	0.974	**3.86**	4.590	**3.91**	4.829	**3.98**	4.023	**2.99**
RSJ/Tail Sens (TV)	10.06	10.14	0.992	**4.08**	4.849	**5.15**	5.064	**5.22**	4.227	**3.67**
RSJ/Tail Risk (TV)	9.79	10.27	0.954	**3.71**	4.317	**4.21**	4.516	**4.30**	3.748	**3.15**
RSJ/MES (TV)	10.05	10.19	0.987	**3.78**	4.771	**4.55**	4.985	**4.64**	4.141	**3.43**

Thus, although the low volatility effect holds internationally, applying the volatility weighting to international momentum strategies does not work well. In contrast, the (systematic) tail risk weighting is appealing for International and European industry momentum.[144] The volatility targeting strategy again significantly enhances the risk-return profile for all weighting schemes and data sets. This is in line with the finding that volatility targeting also works well for non-

[144] Ang et al. (2009) show that the low volatility puzzle holds internationally. The authors find that the low volatility effect holds for Japan, Europe, Asia, all countries and all countries ex US, but the authors find that this effect is most pronounced in the US. The international low volatility anomaly is confirmed by Blitz and Van Vliet (2007), Blitz et al. (2019), Guo and Savickas (2010, Table 7) and Walkshäusl (2014). However, as stated above, this does not imply that weighting highly volatile assets lower is beneficial for a long-short strategy, since the inverse volatility weighting also enhances the losers' performance. Nonetheless, our results indicate that a low (systematic) tail risk effect seems apparent internationally as also shown by Atilgan et al. (2018), Atilgan et al. (2019), Atilgan et al. (2020, Section 6), Bi and Zhu (2020), Asness et al. (2020) and Frazzini and Pedersen (2014).

Table XXXVIII. Robustness Results: 10 Equally Weighted European Industries

This table shows performance results of the equally and risk weighted momentum strategies using 10 equally weighted European industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta^{\gamma=5}_{MV}$	DM-test	$\Delta^{\gamma=5}_{CRRA}$	DM-test	$\Delta^{l=2}_{LA}$	DM-test
Equal	6.86	14.31	0.479	-	-	-	-	-	-	-
RV	6.11	12.83	0.476	-0.03	-0.130	-0.20	-0.075	-0.13	-0.266	-0.41
RSJ/Corr	7.30	12.93	0.565	1.75	1.078	1.51	1.131	1.49	0.827	1.24
RSJ/Down Corr	7.33	12.81	0.572	1.82	1.166	1.39	1.131	1.32	0.903	1.13
RSJ/Beta	7.41	12.76	0.581	**2.03**	1.279	1.78	1.358	1.88	1.029	1.50
RSJ/Down Beta	6.98	12.79	0.545	1.30	0.805	0.99	0.828	0.96	0.520	0.66
RSJ/CoSkew	6.83	14.01	0.488	0.16	0.097	0.15	0.000	0.01	-0.158	-0.22
RSJ/CoKurt	7.16	13.17	0.543	1.25	0.798	1.38	0.828	1.36	0.732	1.49
RSJ/LPM Beta	7.54	12.70	0.594	**2.32**	1.462	1.85	1.510	1.87	1.166	1.51
RSJ/HTCR Beta	7.71	12.74	0.605	**2.54**	1.608	**2.44**	1.663	**2.47**	1.341	**2.07**
RSJ/Tail Beta	7.58	12.95	0.585	**2.15**	1.349	1.81	1.434	1.91	1.016	1.49
RSJ/Tail Sens	7.19	12.94	0.556	1.61	0.951	1.17	0.904	1.15	0.674	0.87
RSJ/Tail Risk	7.61	12.91	0.590	**2.20**	1.409	1.81	1.434	1.88	1.123	1.51
RSJ/MES	7.68	12.82	0.599	**2.36**	1.528	1.87	1.586	1.87	1.293	1.65
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta^{\gamma=5}_{MV}$	DM-test	$\Delta^{\gamma=5}_{CRRA}$	DM-test	$\Delta^{l=2}_{LA}$	DM-test
Equal	7.16	14.16	0.505	-	-	-	-	-	-	-
RV	6.31	12.76	0.495	-0.34	-0.218	-0.33	-0.225	-0.30	-0.305	-0.46
Equal (TV)	6.99	9.56	0.731	**2.84**	2.809	**2.35**	2.965	**2.42**	1.925	1.34
RV (TV)	6.89	9.91	0.696	**1.96**	2.341	**2.24**	2.580	**2.36**	1.626	1.32
RSJ/Corr (TV)	7.80	9.63	0.811	**3.30**	3.901	**2.95**	4.126	**3.08**	3.024	**2.08**
RSJ/Down Corr (TV)	7.63	9.61	0.794	**3.03**	3.677	**2.64**	3.893	**2.79**	2.909	1.93
RSJ/Beta (TV)	7.85	9.65	0.813	**3.29**	3.936	**2.79**	4.126	**2.92**	3.115	**2.04**
RSJ/Down Beta (TV)	7.41	9.72	0.762	**2.67**	3.242	**2.33**	3.428	**2.50**	2.525	1.71
RSJ/CoSkew (TV)	6.88	9.32	0.738	**2.36**	2.898	**2.49**	3.042	**2.58**	1.894	1.43
RSJ/CoKurt (TV)	7.81	9.84	0.794	**3.01**	3.671	**3.03**	3.970	**3.25**	3.088	**2.28**
RSJ/LPM Beta (TV)	7.88	9.58	0.823	**3.41**	4.072	**3.04**	4.282	**3.17**	3.219	**2.19**
RSJ/HTCR Beta (TV)	7.96	9.62	0.828	**3.48**	4.136	**3.27**	4.360	**3.40**	3.294	**2.31**
RSJ/Tail Beta (TV)	7.73	9.64	0.802	**3.25**	3.782	**2.99**	3.970	**3.12**	2.875	**2.01**
RSJ/Tail Sens (TV)	7.53	9.54	0.789	**3.11**	3.602	**2.68**	3.815	**2.79**	2.703	1.85
RSJ/Tail Risk (TV)	7.96	9.65	0.825	**3.46**	4.101	**3.31**	4.282	**3.43**	3.258	**2.36**
RSJ/MES (TV)	8.03	9.63	0.834	**3.56**	4.222	**3.26**	4.438	**3.37**	3.382	**2.38**

US data (Barroso and Santa-Clara, 2015, Rickenberg, 2020a,b). The highest Sharpe Ratios and economic values are again found for the strategies that combine volatility targeting with the (systematic) tail risk weightings.

B.6 Style Momentum

We have so far only examined momentum strategies using several US, European and International industry portfolios. Lewellen (2002) shows that the momentum strategy can also be applied to style portfolios, like the 25 double sorted portfolios based on size and value (Fama and French, 1993). Lewellen (2002) finds that industry and style momentum are two different

Table XXXIX. Robustness Results: 19 Equally Weighted European Industries
This table shows performance results of the equally and risk weighted momentum strategies using 19 equally weighted European industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.84	11.07	0.618	-	-	-	-	-	-	-
RV	6.16	9.97	0.618	0.02	-0.047	-0.06	0.000	-0.04	-0.157	-0.21
RSJ/Corr	8.75	10.21	0.857	**3.40**	2.453	**4.71**	2.580	**4.81**	2.545	**4.26**
RSJ/Down Corr	8.70	10.08	0.863	**3.45**	2.521	**4.13**	2.657	**4.27**	2.458	**3.98**
RSJ/Beta	8.93	9.97	0.895	**3.76**	2.856	**4.13**	2.965	**4.21**	2.865	**3.74**
RSJ/Down Beta	8.94	9.87	0.905	**3.86**	2.955	**4.68**	3.119	**4.80**	2.924	**4.32**
RSJ/CoSkew	8.41	10.56	0.796	**2.62**	1.838	**2.95**	1.891	**2.84**	1.817	**2.45**
RSJ/CoKurt	8.60	10.15	0.847	**3.22**	2.349	**4.89**	2.427	**5.24**	2.302	**4.69**
RSJ/LPM Beta	9.17	9.92	0.924	**4.11**	3.158	**4.31**	3.273	**4.33**	3.144	**3.83**
RSJ/HTCR Beta	8.79	10.01	0.879	**3.61**	2.680	**4.31**	2.811	**4.35**	2.647	**3.72**
RSJ/Tail Beta	8.90	10.43	0.854	**3.42**	2.428	**3.77**	2.580	**3.94**	2.539	**3.44**
RSJ/Tail Sens	8.89	10.66	0.835	**3.15**	2.235	**2.94**	2.350	**2.91**	2.372	**2.74**
RSJ/Tail Risk	8.64	10.22	0.845	**3.31**	2.332	**3.32**	2.427	**3.46**	2.376	**3.05**
RSJ/MES	9.09	10.17	0.895	**3.79**	2.850	**3.62**	2.965	**3.63**	2.908	**3.19**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	7.17	11.05	0.648	-	-	-	-	-	-	-
RV	6.32	9.99	0.633	-0.30	-0.198	-0.27	-0.150	-0.25	-0.304	-0.41
Equal (TV)	9.01	9.92	0.909	**3.51**	2.673	**3.94**	2.811	**4.09**	2.213	**2.58**
RV (TV)	8.63	10.33	0.835	**1.91**	1.918	1.82	2.044	**1.98**	1.518	1.42
RSJ/Corr (TV)	10.25	9.83	1.043	**3.94**	4.076	**5.60**	4.204	**5.96**	3.887	**4.70**
RSJ/Down Corr (TV)	10.19	9.81	1.039	**3.89**	4.032	**5.54**	4.204	**5.91**	3.778	**5.01**
RSJ/Beta (TV)	10.29	9.81	1.049	**3.91**	4.137	**4.96**	4.282	**5.23**	3.946	**4.29**
RSJ/Down Beta (TV)	10.37	9.73	1.066	**4.08**	4.318	**5.45**	4.516	**5.75**	4.111	**4.87**
RSJ/CoSkew (TV)	10.05	9.76	1.030	**3.82**	3.946	**4.78**	4.126	**5.00**	3.742	**4.33**
RSJ/CoKurt (TV)	10.20	9.80	1.041	**3.88**	4.057	**5.55**	4.204	**5.95**	3.831	**4.81**
RSJ/LPM Beta (TV)	10.56	9.74	1.084	**4.23**	4.508	**5.08**	4.672	**5.29**	4.296	**4.39**
RSJ/HTCR Beta (TV)	10.30	9.77	1.055	**3.99**	4.201	**4.85**	4.360	**5.09**	3.973	**4.11**
RSJ/Tail Beta (TV)	10.28	9.98	1.030	**3.87**	3.944	**4.66**	4.126	**5.00**	3.820	**4.07**
RSJ/Tail Sens (TV)	10.40	9.88	1.053	**4.10**	4.184	**4.04**	4.360	**4.17**	3.963	**3.42**
RSJ/Tail Risk (TV)	10.13	9.91	1.022	**3.79**	3.858	**4.37**	4.048	**4.67**	3.701	**3.84**
RSJ/MES (TV)	10.42	9.85	1.058	**4.04**	4.235	**4.43**	4.360	**4.61**	4.035	**3.74**

phenomena and that both approaches deliver high returns. Style momentum has also been examined by Novy-Marx (2012) and Stivers and Sun (2010). Besides the usage of style portfolios in the momentum literature, style portfolios also play and important role in the asset pricing and portfolio allocation literature.[145] Kan and Zhou (2007, p. 646) state that the "25 portfolios, formed based on size- and book-to-market ratio, are the standard test assets in recent empirical asset pricing studies". Similarly, Lettau et al. (2014, p. 209) "employ the Fama and French

[145] See Atilgan et al. (2019), Blitz and Vidojevic (2017), Chang et al. (2013), Chen and Petkova (2012) Jiang et al. (2020), Guo and Savickas (2010), Hong et al. (2007), Lettau et al. (2014), Langlois (2020), Fama and French (2016), Asness et al. (2013), Kritzman et al. (2010), Zakamulin (2015), DeMiguel et al. (2009a), Bali et al. (2017b), Kirby and Ostdiek (2012), Bali et al. (2017b), Harvey and Siddique (2000), DeMiguel et al. (2009b), Kan and Zhou (2007) and Farago and Tédongap (2018) for several financial studies that use style portfolios.

Table XL. Robustness Results: 40 Equally Weighted European Industries
This table shows performance results of the equally and risk weighted momentum strategies using 40 equally weighted European industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	8.13	11.62	0.699	-	-	-	-	-	-	-
RV	6.60	9.99	0.661	-0.55	-0.472	-0.82	-0.449	-0.81	-0.302	-0.55
RSJ/Corr	10.94	10.31	1.062	**4.69**	3.913	**5.66**	3.970	**5.72**	3.892	**5.08**
RSJ/Down Corr	10.96	10.41	1.052	**4.51**	3.806	**5.86**	3.815	**5.63**	3.864	**4.93**
RSJ/Beta	10.56	10.15	1.040	**4.22**	3.668	**6.32**	3.738	**6.41**	3.660	**5.76**
RSJ/Down Beta	10.71	10.34	1.035	**4.15**	3.622	**6.07**	3.660	**5.86**	3.694	**5.13**
RSJ/CoSkew	10.28	11.17	0.920	**3.08**	2.375	**4.51**	2.427	**4.27**	2.202	**3.41**
RSJ/CoKurt	10.70	10.51	1.019	**4.01**	3.444	**5.16**	3.428	**5.19**	3.488	**4.59**
RSJ/LPM Beta	10.59	10.24	1.034	**4.15**	3.604	**6.29**	3.660	**6.19**	3.608	**5.48**
RSJ/HTCR Beta	10.40	10.30	1.010	**3.87**	3.341	**5.26**	3.350	**5.28**	3.330	**4.70**
RSJ/Tail Beta	10.33	10.59	0.975	**3.83**	2.968	**6.97**	3.042	**6.68**	2.877	**5.78**
RSJ/Tail Sens	11.05	10.65	1.038	**4.61**	3.650	**7.38**	3.660	**7.05**	3.548	**5.53**
RSJ/Tail Risk	10.34	10.53	0.982	**3.83**	3.045	**5.95**	3.119	**5.77**	3.011	**5.36**
RSJ/MES	10.58	10.34	1.023	**4.09**	3.484	**5.71**	3.505	**5.70**	3.415	**5.29**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	8.52	11.61	0.734	-	-	-	-	-	-	-
RV	6.90	10.00	0.690	-0.79	-0.532	-0.96	-0.524	-0.96	-0.370	-0.72
Equal (TV)	10.29	10.19	1.009	**3.61**	2.944	**3.13**	3.119	**3.13**	2.638	**2.33**
RV (TV)	9.69	10.54	0.920	1.80	1.968	1.65	2.197	1.78	1.847	1.40
RSJ/Corr (TV)	12.66	10.06	1.258	**4.92**	5.658	**5.18**	5.772	**5.16**	5.503	**4.31**
RSJ/Down Corr (TV)	12.39	10.07	1.230	**4.67**	5.353	**5.71**	5.457	**5.51**	5.300	**4.48**
RSJ/Beta (TV)	12.49	10.12	1.234	**4.65**	5.396	**5.44**	5.536	**5.42**	5.275	**4.51**
RSJ/Down Beta (TV)	12.26	10.19	1.204	**4.40**	5.065	**5.73**	5.142	**5.54**	5.031	**4.52**
RSJ/CoSkew (TV)	11.36	9.97	1.139	**3.91**	4.355	**4.13**	4.516	**4.02**	4.080	**3.25**
RSJ/CoKurt (TV)	12.41	10.11	1.227	**4.56**	5.323	**5.25**	5.457	**5.19**	5.256	**4.22**
RSJ/LPM Beta (TV)	12.40	10.13	1.224	**4.56**	5.292	**5.30**	5.378	**5.22**	5.183	**4.39**
RSJ/HTCR Beta (TV)	12.26	10.12	1.212	**4.46**	5.156	**4.87**	5.300	**4.83**	5.017	**4.05**
RSJ/Tail Beta (TV)	11.89	10.18	1.169	**4.23**	4.681	**4.62**	4.829	**4.59**	4.530	**3.92**
RSJ/Tail Sens (TV)	12.51	10.07	1.242	**4.88**	5.485	**5.20**	5.614	**5.13**	5.235	**4.19**
RSJ/Tail Risk (TV)	12.14	10.24	1.185	**4.37**	4.863	**4.63**	4.985	**4.60**	4.742	**4.02**
RSJ/MES (TV)	12.46	10.12	1.232	**4.66**	5.370	**5.02**	5.536	**5.01**	5.181	**4.26**

book-to-market and size-sorted portfolios because they are among the most commonly tested equity cross sections". Jagannathan and Ma (2003, Table IX) also use the 25 portfolios based on size and value in a portfolio setting and find that incorporating an estimate of the mean return does not work well for these portfolios. In contrast, Novy-Marx (2012), Stivers and Sun (2010) and Lewellen (2002) find that information on the styles' relative performance is a valuable asset allocation tool. This again shows that portfolio allocations based on relative mean estimates are advantageous to portfolio allocations based on absolute mean estimates.

Due to the importance of style portfolios in the financial literature and since Lewellen (2002) shows that style and industry momentum are quite different, we next examine as a further ro-

Table XLI. Robustness Results: 113 Equally Weighted European Industries
This table shows performance results of the equally and risk weighted momentum strategies using 113 equally weighted European industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p=20\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	10.31	14.56	0.708	-	-	-	-	-	-	-
RV	9.49	13.15	0.722	0.25	0.118	0.21	0.075	0.11	0.479	0.80
RSJ/Corr	12.25	13.04	0.939	**2.83**	3.034	**2.85**	3.119	**2.80**	3.194	**2.91**
RSJ/Down Corr	12.32	13.40	0.919	**2.68**	2.773	**3.92**	2.888	**3.82**	2.874	**3.50**
RSJ/Beta	12.12	12.80	0.947	**2.87**	3.126	**3.38**	3.196	**3.35**	3.242	**3.23**
RSJ/Down Beta	12.08	13.10	0.922	**2.64**	2.802	**4.16**	2.888	**4.06**	2.857	**3.58**
RSJ/CoSkew	11.15	14.67	0.760	0.74	0.677	0.80	0.828	0.96	0.552	0.59
RSJ/CoKurt	11.67	13.47	0.866	**2.03**	2.067	**2.48**	2.120	**2.44**	2.150	**2.31**
RSJ/LPM Beta	12.40	12.83	0.966	**3.08**	3.393	**4.17**	3.428	**4.13**	3.567	**3.91**
RSJ/HTCR Beta	12.56	13.00	0.966	**3.15**	3.395	**3.75**	3.428	**3.62**	3.425	**3.53**
RSJ/Tail Beta	12.17	13.53	0.899	**2.62**	2.505	**2.69**	2.580	**2.73**	2.700	**2.87**
RSJ/Tail Sens	12.20	13.76	0.887	**2.39**	2.347	1.82	2.350	1.79	2.430	1.92
RSJ/Tail Risk	12.29	13.22	0.930	**2.92**	2.908	**3.70**	2.965	**3.74**	3.104	**3.72**
RSJ/MES	12.70	12.90	0.984	**3.33**	3.639	**3.57**	3.660	**3.50**	3.714	**3.39**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	10.66	14.56	0.732	-	-	-	-	-	-	-
RV	9.81	13.15	0.746	0.14	0.126	0.22	0.075	0.11	0.492	0.81
Equal (TV)	10.27	9.75	1.053	**4.41**	4.211	**4.82**	4.438	**4.78**	3.576	**3.62**
RV (TV)	11.22	10.32	1.088	**3.46**	4.682	**3.20**	4.907	**3.19**	4.423	**2.88**
RSJ/Corr (TV)	11.58	10.04	1.154	**3.81**	5.597	**4.14**	5.772	**4.10**	5.763	**4.00**
RSJ/Down Corr (TV)	11.40	10.06	1.133	**3.70**	5.316	**4.18**	5.457	**4.10**	5.459	**3.94**
RSJ/Beta (TV)	11.70	10.12	1.155	**3.80**	5.622	**4.40**	5.772	**4.31**	5.755	**4.11**
RSJ/Down Beta (TV)	11.25	10.05	1.119	**3.52**	5.120	**3.94**	5.300	**3.87**	5.223	**3.74**
RSJ/CoSkew (TV)	9.73	9.87	0.986	**2.48**	3.284	**2.30**	3.505	**2.37**	3.138	**2.09**
RSJ/CoKurt (TV)	11.16	10.05	1.110	**3.49**	4.993	**4.14**	5.142	**4.10**	5.085	**3.84**
RSJ/LPM Beta (TV)	11.71	10.08	1.161	**3.85**	5.698	**4.59**	5.851	**4.50**	5.942	**4.34**
RSJ/HTCR Beta (TV)	11.85	9.96	1.189	**4.14**	6.090	**5.12**	6.247	**5.01**	6.083	**4.70**
RSJ/Tail Beta (TV)	10.98	9.99	1.099	**3.39**	4.851	**3.84**	4.907	**3.72**	4.893	**3.67**
RSJ/Tail Sens (TV)	11.31	10.02	1.129	**3.76**	5.262	**3.75**	5.457	**3.82**	5.254	**3.66**
RSJ/Tail Risk (TV)	11.53	10.27	1.123	**3.61**	5.173	**3.85**	5.300	**3.75**	5.383	**3.73**
RSJ/MES (TV)	11.97	10.04	1.192	**4.13**	6.135	**4.70**	6.247	**4.59**	6.238	**4.43**

bustness check the profitability of (risk-managed) style momentum.[146] Weighting different style portfolios by their risk is important, since different style portfolios can have quite different risk characteristics. For example, Harvey and Siddique (2000, Table I) find that the 25 size and value double sorted portfolios have quite different levels of skewness and coskewness. Similarly, Jiang et al. (2020, Table 1.B) find that style portfolios exhibit different levels of return distribution asymmetries. Ang and Chen (2002, p. 472) find that different size portfolios have different asymmetric correlations. Thus, similar to the industry momentum strategy, simply equal or volatility weighting the style portfolios in the winners and losers portfolios should be

[146] Style portfolios are again obtained from Kenneth French's website.

Table XLII. Robustness Results: 25 Double Sorted US Portfolios Based on Size and Value
This table shows performance results of the equally and risk weighted momentum strategies using 25 double sorted US portfolios based on size and value, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta^{\gamma=5}_{MV}$	DM-test	$\Delta^{\gamma=5}_{CRRA}$	DM-test	$\Delta^{l=2}_{LA}$	DM-test
Equal	4.59	12.43	0.369	-	-	-	-	-	-	-
RV	4.73	12.47	0.379	0.30	0.121	0.43	-0.150	-0.37	0.290	1.10
RSJ/Corr	6.10	12.52	0.487	**2.70**	1.381	**2.26**	1.968	**2.48**	1.408	**2.44**
RSJ/Down Corr	6.19	12.55	0.493	**2.94**	1.451	**2.29**	2.350	**2.25**	1.374	**2.31**
RSJ/Beta	6.28	12.07	0.520	**3.50**	1.741	**2.94**	2.503	**2.72**	1.664	**2.88**
RSJ/Down Beta	6.23	12.41	0.502	**3.09**	1.547	**2.64**	2.427	**2.47**	1.579	**2.73**
RSJ/CoSkew	5.07	13.07	0.388	0.55	0.257	0.47	0.451	0.71	0.208	0.36
RSJ/CoKurt	6.17	12.52	0.492	**2.90**	1.429	**2.21**	2.503	**2.02**	1.319	**2.36**
RSJ/LPM Beta	6.49	12.38	0.524	**3.44**	1.803	**3.13**	2.734	**2.73**	1.841	**2.88**
RSJ/HTCR Beta	6.16	12.12	0.508	**3.49**	1.615	**2.96**	2.120	**2.90**	1.534	**2.77**
RSJ/Tail Beta	6.31	13.07	0.483	**2.83**	1.374	**3.01**	1.586	**3.30**	1.578	**3.37**
RSJ/Tail Sens	5.97	12.76	0.468	**2.73**	1.192	**2.08**	0.979	1.20	1.259	**2.54**
RSJ/Tail Risk	6.27	13.03	0.481	**2.87**	1.348	**3.01**	1.586	**3.43**	1.495	**4.04**
RSJ/MES	6.51	12.46	0.523	**3.44**	1.795	**3.28**	2.503	**3.11**	1.859	**3.16**

	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta^{\gamma=5}_{MV}$	DM-test	$\Delta^{\gamma=5}_{CRRA}$	DM-test	$\Delta^{l=2}_{LA}$	DM-test
Equal	4.72	12.30	0.384	-	-	-	-	-	-	-
RV	4.79	12.38	0.387	0.11	0.043	0.14	-0.225	-0.49	0.226	0.74
Equal (TV)	8.13	12.93	0.629	**4.09**	2.880	**2.79**	3.970	**2.49**	1.753	1.78
RV (TV)	8.43	14.50	0.581	**3.41**	2.441	**2.76**	2.427	**2.35**	1.798	**2.15**
RSJ/Corr (TV)	9.39	12.89	0.729	**5.07**	4.048	**3.40**	5.064	**3.13**	3.014	**2.65**
RSJ/Down Corr (TV)	9.40	12.72	0.739	**5.12**	4.152	**3.27**	5.457	**2.86**	2.999	**2.48**
RSJ/Beta (TV)	9.58	12.83	0.746	**5.30**	4.246	**3.44**	5.536	**2.96**	3.147	**2.70**
RSJ/Down Beta (TV)	9.49	12.83	0.739	**5.20**	4.163	**3.33**	5.378	**2.92**	3.064	**2.57**
RSJ/CoSkew (TV)	8.93	12.89	0.692	**4.56**	3.616	**3.12**	4.750	**2.80**	2.614	**2.36**
RSJ/CoKurt (TV)	9.41	12.60	0.747	**5.16**	4.240	**3.40**	5.614	**2.83**	3.031	**2.61**
RSJ/LPM Beta (TV)	9.64	12.86	0.749	**5.36**	4.283	**3.54**	5.536	**3.05**	3.182	**2.77**
RSJ/HTCR Beta (TV)	9.41	12.87	0.731	**5.19**	4.076	**3.42**	5.142	**3.06**	3.015	**2.66**
RSJ/Tail Beta (TV)	9.58	13.28	0.722	**5.34**	3.991	**3.55**	4.829	**3.40**	3.068	**2.85**
RSJ/Tail Sens (TV)	9.36	13.01	0.719	**5.09**	3.951	**3.32**	4.829	**3.17**	2.979	**2.63**
RSJ/Tail Risk (TV)	9.61	13.33	0.721	**5.36**	3.986	**3.64**	4.750	**3.49**	3.083	**2.96**
RSJ/MES (TV)	9.71	12.94	0.750	**5.43**	4.300	**3.64**	5.457	**3.19**	3.237	**2.87**

suboptimal and should translate into a portfolio where risk may be dominated by a few styles' risk. Further, Atilgan et al. (2019, Table 5) find a strong and statistically significant negative relation between downside risk, measured by downside beta, LPM, VaR and CVaR, and future return for style portfolios. This negative relation makes an inverse risk weighting highly attractive for style momentum. We therefore apply our different weighting schemes to several style momentum strategies. Table XLII shows results for the equally weighted and risk-managed style momentum strategies using the 25 size and value double sorted portfolios of Fama and French (1993), which are the most frequently used style portfolios. In Tables XLIII to XLV we show additional results for style portfolios based on profitability and investment (Fama and

Table XLIII. Robustness Results: 25 Double Sorted US Portfolios Based on Size and Profitability
This table shows performance results of the equally and risk weighted momentum strategies using 25 double sorted US portfolios based on size and profitability, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

Panel A: Without Volatility Targeting										
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	4.14	9.70	0.427	-	-	-	-	-	-	-
RV	4.15	9.62	0.431	0.11	0.034	0.13	0.000	0.02	0.025	0.10
RSJ/Corr	5.56	9.43	0.589	**3.42**	1.499	**2.76**	1.510	**2.76**	1.241	**2.23**
RSJ/Down Corr	5.58	9.51	0.587	**3.43**	1.483	**2.84**	1.510	**2.86**	1.238	**2.29**
RSJ/Beta	5.44	9.20	0.592	**3.41**	1.519	**2.68**	1.586	**2.72**	1.214	**2.09**
RSJ/Down Beta	5.48	9.20	0.595	**3.46**	1.548	**2.62**	1.586	**2.69**	1.211	**1.98**
RSJ/CoSkew	5.46	10.24	0.534	**2.66**	1.017	**2.35**	0.979	**2.07**	1.013	**2.36**
RSJ/CoKurt	5.49	9.53	0.576	**3.19**	1.379	**2.46**	1.434	**2.49**	1.127	**1.97**
RSJ/LPM Beta	5.45	9.22	0.591	**3.42**	1.510	**2.63**	1.586	**2.70**	1.212	**2.08**
RSJ/HTCR Beta	5.45	9.21	0.592	**3.42**	1.519	**2.69**	1.586	**2.74**	1.202	**2.08**
RSJ/Tail Beta	5.57	9.84	0.565	**3.44**	1.295	**2.65**	1.282	**2.66**	1.168	**2.52**
RSJ/Tail Sens	5.73	9.52	0.602	**3.98**	1.620	**3.17**	1.663	**3.13**	1.349	**2.55**
RSJ/Tail Risk	5.48	9.67	0.566	**3.42**	1.294	**2.73**	1.358	**2.88**	1.094	**2.33**
RSJ/MES	5.47	9.31	0.587	**3.43**	1.481	**2.63**	1.510	**2.66**	1.224	**2.18**

Panel B: With Volatility Targeting										
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	4.03	9.72	0.415	-	-	-	-	-	-	-
RV	4.06	9.65	0.420	0.16	0.051	0.19	0.000	0.08	0.049	0.19
Equal (TV)	6.67	12.90	0.517	1.62	1.068	**2.49**	1.207	**2.79**	0.585	0.96
RV (TV)	6.90	13.00	0.531	1.75	1.205	**2.09**	1.358	**2.40**	0.802	1.19
RSJ/Corr (TV)	8.77	12.86	0.682	**3.36**	2.591	**3.29**	2.734	**3.39**	2.048	**2.27**
RSJ/Down Corr (TV)	8.81	12.84	0.686	**3.39**	2.629	**3.37**	2.811	**3.49**	2.076	**2.31**
RSJ/Beta (TV)	8.82	12.95	0.681	**3.41**	2.587	**3.22**	2.734	**3.35**	2.093	**2.30**
RSJ/Down Beta (TV)	8.91	12.90	0.691	**3.48**	2.673	**3.31**	2.811	**3.46**	2.152	**2.33**
RSJ/CoSkew (TV)	8.75	12.99	0.674	**3.36**	2.518	**3.44**	2.657	**3.57**	2.027	**2.38**
RSJ/CoKurt (TV)	8.74	12.81	0.683	**3.31**	2.594	**3.17**	2.734	**3.29**	2.025	**2.19**
RSJ/LPM Beta (TV)	8.87	12.96	0.684	**3.45**	2.613	**3.26**	2.734	**3.41**	2.123	**2.34**
RSJ/HTCR Beta (TV)	8.83	12.95	0.682	**3.41**	2.590	**3.21**	2.734	**3.34**	2.085	**2.28**
RSJ/Tail Beta (TV)	8.82	13.21	0.668	**3.39**	2.471	**3.47**	2.657	**3.77**	2.070	**2.64**
RSJ/Tail Sens (TV)	8.96	12.88	0.695	**3.56**	2.711	**3.50**	2.888	**3.59**	2.163	**2.41**
RSJ/Tail Risk (TV)	8.84	13.13	0.674	**3.43**	2.520	**3.52**	2.657	**3.79**	2.080	**2.61**
RSJ/MES (TV)	8.87	13.02	0.681	**3.47**	2.591	**3.24**	2.734	**3.39**	2.115	**2.37**

French, 2016).[147] Table XLIII shows results for 25 double sorted portfolios based on size and profitability, Table XLIV uses size and investment sorted portfolios, whereas Table XLV shows results for style portfolios sorted on profitability and investment. Further, we show additional results for style momentum based on non-US data. Table XLVI shows results for style momentum using International size and value sorted style portfolios, Table XLVII shows results for style momentum using International ex US size and value sorted style portfolios and Table XLVIII shows results for momentum using 25 size and value sorted European style portfolios (Fama and French, 2012). Further, in order to assess how style momentum works when larger data sets are

[147]Fama and French (2016) show that a five factor model that includes an investment and profitability factor can explain many anomalies, like the low volatility and low beta anomaly, but fails to explain momentum returns.

Table XLIV. Robustness Results: 25 Double Sorted US Portfolios Based on Size and Investment
This table shows performance results of the equally and risk weighted momentum strategies using 25 double sorted US portfolios based on size and investment, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	4.29	9.76	0.439	-	-	-	-	-	-	-
RV	3.54	10.22	0.346	**-2.75**	-0.850	-1.78	-0.971	-1.69	-0.611	-1.45
RSJ/Corr	6.01	9.63	0.624	**3.63**	1.722	**3.60**	1.815	**3.29**	1.492	**3.22**
RSJ/Down Corr	5.89	9.69	0.608	**3.40**	1.572	**3.59**	1.663	**3.40**	1.365	**3.19**
RSJ/Beta	5.60	9.71	0.577	**2.76**	1.286	**2.65**	1.282	**2.56**	1.189	**2.53**
RSJ/Down Beta	5.57	9.96	0.559	**2.51**	1.137	**2.25**	1.055	1.87	1.139	**2.48**
RSJ/CoSkew	5.84	10.43	0.559	**2.68**	1.158	**2.34**	1.055	1.79	1.284	**3.04**
RSJ/CoKurt	5.97	9.70	0.616	**3.49**	1.646	**3.62**	1.739	**3.35**	1.436	**3.25**
RSJ/LPM Beta	5.60	9.88	0.566	**2.63**	1.199	**2.47**	1.131	**2.16**	1.186	**2.60**
RSJ/HTCR Beta	5.66	9.72	0.582	**2.87**	1.341	**2.79**	1.282	**2.67**	1.252	**2.72**
RSJ/Tail Beta	5.60	10.19	0.549	**2.52**	1.053	**2.20**	0.904	1.72	1.105	**2.65**
RSJ/Tail Sens	6.04	9.70	0.623	**3.72**	1.714	**3.63**	1.815	**3.34**	1.512	**3.31**
RSJ/Tail Risk	5.67	10.10	0.562	**2.80**	1.162	**2.53**	1.131	**2.29**	1.162	**2.82**
RSJ/MES	5.65	9.86	0.573	**2.77**	1.260	**2.64**	1.207	**2.36**	1.233	**2.72**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	4.18	9.78	0.427	-	-	-	-	-	-	-
RV	3.45	10.25	0.336	**-2.56**	-0.825	-1.71	-0.971	-1.64	-0.578	-1.36
Equal (TV)	8.13	13.46	0.604	**2.83**	1.776	**2.38**	2.044	**2.29**	1.323	1.61
RV (TV)	7.01	14.05	0.499	1.29	0.832	1.19	1.055	1.34	0.585	0.78
RSJ/Corr (TV)	10.21	13.19	0.774	**4.09**	3.342	**3.60**	3.583	**3.40**	2.875	**2.92**
RSJ/Down Corr (TV)	10.10	13.22	0.764	**4.04**	3.245	**3.59**	3.505	**3.42**	2.789	**2.93**
RSJ/Beta (TV)	9.94	13.41	0.741	**3.94**	3.043	**3.61**	3.273	**3.54**	2.656	**2.97**
RSJ/Down Beta (TV)	9.93	13.51	0.735	**3.97**	2.992	**3.60**	3.196	**3.58**	2.635	**3.00**
RSJ/CoSkew (TV)	10.32	13.38	0.771	**4.37**	3.318	**3.74**	3.583	**3.63**	2.917	**3.12**
RSJ/CoKurt (TV)	10.14	13.18	0.769	**4.04**	3.294	**3.60**	3.583	**3.42**	2.833	**2.92**
RSJ/LPM Beta (TV)	9.96	13.48	0.739	**3.98**	3.026	**3.61**	3.273	**3.56**	2.659	**2.99**
RSJ/HTCR Beta (TV)	10.01	13.38	0.748	**4.00**	3.106	**3.65**	3.350	**3.58**	2.714	**3.00**
RSJ/Tail Beta (TV)	9.92	13.52	0.734	**4.04**	2.985	**3.76**	3.196	**3.73**	2.622	**3.16**
RSJ/Tail Sens (TV)	10.19	13.19	0.773	**4.09**	3.325	**3.64**	3.583	**3.45**	2.866	**2.95**
RSJ/Tail Risk (TV)	9.99	13.52	0.739	**4.09**	3.027	**3.72**	3.273	**3.68**	2.655	**3.12**
RSJ/MES (TV)	10.02	13.47	0.743	**4.04**	3.067	**3.66**	3.273	**3.59**	2.691	**3.02**

used, we show in Table XLIX results for 100 double sorted size and value portfolios in the US. Style momentum for 100 double sorted portfolios on size and value is also examined by Stivers and Sun (2010). For the 100 size and value sorted portfolios we use a cut-off point of $p = 20\%$. Results for the 30% cut-off point were quite similar, but slightly less profitable. Furthermore, instead of examining style momentum based on double sorted portfolios, style portfolios sorted on only one characteristic, like size or value, can also be used (Lewellen, 2002). In particular, several style portfolios sorted on one characteristic can be pooled together (Bali et al., 2017b, DeMiguel et al., 2009b, Farago and Tédongap, 2018, Kirby and Ostdiek, 2012). Following this idea, Table L shows results for 20 portfolios consisting of 10 equally weighted US portfolios

Table XLV. Robustness Results: 25 Double Sorted US Portfolios Based on Profitability and Investment

This table shows performance results of the equally and risk weighted momentum strategies using 25 double sorted US portfolios based on profitability and investment, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	5.15	7.65	0.674	-	-	-	-	-	-	-
RV	4.34	7.18	0.604	-1.87	-0.523	**-2.03**	-0.524	**-1.96**	-0.645	**-2.62**
RSJ/Corr	6.62	7.43	0.891	**4.15**	1.578	**2.60**	1.663	**2.52**	1.451	**2.40**
RSJ/Down Corr	6.56	7.52	0.872	**3.98**	1.447	**2.60**	1.510	**2.53**	1.341	**2.40**
RSJ/Beta	6.55	7.45	0.879	**3.96**	1.490	**2.46**	1.510	**2.37**	1.367	**2.26**
RSJ/Down Beta	6.45	7.59	0.850	**3.61**	1.288	**2.42**	1.282	**2.36**	1.215	**2.27**
RSJ/CoSkew	6.50	8.49	0.766	**2.17**	0.690	1.68	0.677	1.71	1.035	**2.23**
RSJ/CoKurt	6.54	7.50	0.872	**3.94**	1.445	**2.53**	1.510	**2.46**	1.342	**2.35**
RSJ/LPM Beta	6.55	7.50	0.873	**3.97**	1.452	**2.43**	1.510	**2.35**	1.345	**2.26**
RSJ/HTCR Beta	6.55	7.44	0.880	**3.99**	1.499	**2.45**	1.586	**2.37**	1.384	**2.27**
RSJ/Tail Beta	6.67	7.73	0.862	**4.12**	1.376	**2.40**	1.434	**2.31**	1.345	**2.36**
RSJ/Tail Sens	6.61	7.50	0.881	**4.18**	1.509	**2.48**	1.586	**2.40**	1.397	**2.29**
RSJ/Tail Risk	6.56	7.70	0.852	**3.96**	1.301	**2.38**	1.358	**2.30**	1.257	**2.32**
RSJ/MES	6.70	7.57	0.885	**4.31**	1.537	**2.36**	1.586	**2.27**	1.493	**2.25**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	5.07	7.67	0.661	-	-	-	-	-	-	-
RV	4.27	7.19	0.594	-1.89	-0.502	-1.94	-0.449	-1.86	-0.615	**-2.48**
Equal (TV)	12.29	13.28	0.925	**4.10**	1.994	**4.01**	2.044	**4.00**	1.476	**2.25**
RV (TV)	10.93	13.25	0.825	**2.33**	1.286	**2.63**	1.358	**2.72**	0.841	1.37
RSJ/Corr (TV)	13.94	12.65	1.102	**4.98**	3.228	**4.60**	3.350	**4.51**	2.779	**3.25**
RSJ/Down Corr (TV)	13.88	12.68	1.094	**4.96**	3.171	**4.58**	3.273	**4.50**	2.715	**3.22**
RSJ/Beta (TV)	13.91	12.68	1.097	**4.97**	3.193	**4.52**	3.273	**4.43**	2.744	**3.21**
RSJ/Down Beta (TV)	13.84	12.73	1.088	**4.95**	3.126	**4.55**	3.196	**4.46**	2.679	**3.23**
RSJ/CoSkew (TV)	14.00	13.18	1.062	**5.18**	2.948	**4.32**	3.042	**4.24**	2.638	**3.13**
RSJ/CoKurt (TV)	13.85	12.66	1.094	**4.93**	3.166	**4.59**	3.273	**4.51**	2.717	**3.24**
RSJ/LPM Beta (TV)	13.94	12.70	1.097	**5.01**	3.192	**4.48**	3.273	**4.39**	2.745	**3.19**
RSJ/HTCR Beta (TV)	13.91	12.68	1.097	**4.96**	3.187	**4.47**	3.273	**4.38**	2.745	**3.19**
RSJ/Tail Beta (TV)	14.17	12.79	1.108	**5.28**	3.269	**4.47**	3.350	**4.37**	2.836	**3.22**
RSJ/Tail Sens (TV)	13.97	12.69	1.102	**5.06**	3.222	**4.49**	3.350	**4.40**	2.771	**3.19**
RSJ/Tail Risk (TV)	14.05	12.78	1.100	**5.21**	3.209	**4.52**	3.273	**4.43**	2.772	**3.24**
RSJ/MES (TV)	14.13	12.75	1.108	**5.20**	3.269	**4.29**	3.350	**4.20**	2.849	**3.10**

sorted on size and 10 equally weighted US portfolios sorted on value. We also examined style momentum using only 10 size or 10 value portfolios, but found quite similar results, which are not shown here.

When examining style momentum, we again use the $t-12$ to $t-1$ ranking period, as we have done for the industry momentum strategy. Other ranking periods can also be used for style momentum. Lewellen (2002), Stivers and Sun (2010) and Novy-Marx (2012) show that style momentum works well for several ranking periods and that, similar to industry momentum, style momentum is also driven by the recent past's performance. Thus, the reversal effect that is known for the individual stock momentum strategy does not hold for the style momentum

Table XLVI. Robustness Results: 25 Double Sorted International Portfolios Based on Size and Value

This table shows performance results of the equally and risk weighted momentum strategies using 25 double sorted International portfolios based on size and value, the $t-12$ to $t-1$ ranking period and a cut-off point of $p=30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	5.29	9.14	0.579	-	-	-	-	-	-	-
RV	5.59	8.97	0.623	1.01	0.378	**2.53**	0.376	**2.67**	0.127	0.76
RSJ/Corr	7.85	8.87	0.886	**5.50**	2.667	**5.95**	2.657	**5.70**	2.577	**5.52**
RSJ/Down Corr	7.69	8.86	0.868	**5.28**	2.511	**6.86**	2.503	**6.74**	2.404	**6.71**
RSJ/Beta	7.69	8.77	0.877	**5.13**	2.590	**6.59**	2.580	**6.50**	2.486	**6.00**
RSJ/Down Beta	7.67	8.80	0.872	**5.04**	2.549	**6.63**	2.503	**6.58**	2.463	**6.20**
RSJ/CoSkew	7.51	9.33	0.805	**4.29**	1.976	**4.92**	1.968	**4.81**	1.968	**4.13**
RSJ/CoKurt	7.69	8.87	0.867	**5.23**	2.500	**6.89**	2.503	**6.78**	2.407	**6.90**
RSJ/LPM Beta	7.70	8.78	0.877	**5.13**	2.589	**6.30**	2.580	**6.21**	2.490	**5.63**
RSJ/HTCR Beta	7.75	8.86	0.874	**5.14**	2.565	**6.31**	2.580	**6.20**	2.499	**5.75**
RSJ/Tail Beta	7.40	9.02	0.820	**4.44**	2.098	**5.35**	2.120	**5.32**	1.974	**4.34**
RSJ/Tail Sens	7.76	9.07	0.856	**5.20**	2.409	**6.46**	2.427	**6.23**	2.429	**6.25**
RSJ/Tail Risk	7.59	8.93	0.850	**4.92**	2.353	**6.51**	2.350	**6.44**	2.264	**6.42**
RSJ/MES	7.69	8.87	0.867	**5.09**	2.502	**6.40**	2.503	**6.29**	2.416	**5.92**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	5.34	9.23	0.579	-	-	-	-	-	-	-
RV	5.66	9.05	0.626	1.07	0.412	**2.21**	0.376	**2.25**	0.174	0.78
Equal (TV)	10.88	12.18	0.893	**2.69**	2.807	**3.39**	2.888	**3.36**	1.701	1.47
RV (TV)	11.81	12.38	0.954	**3.13**	3.336	**4.52**	3.350	**4.43**	2.243	**2.10**
RSJ/Corr (TV)	13.89	11.50	1.207	**4.70**	5.507	**5.54**	5.614	**5.44**	4.477	**3.70**
RSJ/Down Corr (TV)	13.79	11.51	1.198	**4.64**	5.428	**5.64**	5.457	**5.54**	4.389	**3.79**
RSJ/Beta (TV)	13.84	11.59	1.194	**4.66**	5.393	**5.71**	5.457	**5.60**	4.378	**3.80**
RSJ/Down Beta (TV)	13.88	11.59	1.197	**4.67**	5.421	**5.60**	5.457	**5.50**	4.417	**3.74**
RSJ/CoSkew (TV)	13.97	11.55	1.210	**4.73**	5.528	**5.27**	5.614	**5.17**	4.550	**3.62**
RSJ/CoKurt (TV)	13.76	11.53	1.193	**4.65**	5.384	**5.70**	5.457	**5.60**	4.352	**3.84**
RSJ/LPM Beta (TV)	13.86	11.58	1.197	**4.66**	5.415	**5.57**	5.457	**5.46**	4.403	**3.70**
RSJ/HTCR Beta (TV)	13.91	11.62	1.197	**4.70**	5.420	**5.57**	5.457	**5.46**	4.416	**3.71**
RSJ/Tail Beta (TV)	13.64	11.66	1.170	**4.53**	5.188	**5.21**	5.221	**5.12**	4.141	**3.48**
RSJ/Tail Sens (TV)	13.87	11.59	1.197	**4.67**	5.414	**5.54**	5.457	**5.43**	4.419	**3.69**
RSJ/Tail Risk (TV)	13.79	11.64	1.185	**4.63**	5.309	**5.48**	5.378	**5.38**	4.281	**3.70**
RSJ/MES (TV)	13.87	11.63	1.193	**4.67**	5.382	**5.56**	5.457	**5.46**	4.367	**3.71**

strategy. We also used several other ranking periods, but found that the (risk-managed) style momentum strategies are quite robust to other ranking periods.

Results in this section confirm the finding of Lewellen (2002), Novy-Marx (2012) and Stivers and Sun (2010) that momentum also works well when it is applied to investment style portfolios. In contrast to the industry momentum strategy, we find that style momentum produces lower returns, but also exhibits lower levels of volatility. This is in line with Lewellen (2002, Table 2) who also finds that style momentum is less profitable than industry momentum.[148] For the 25 size and value double sorted US style portfolios, we find in Table XLII that,

[148]The results of Lewellen (2002) are not directly comparable to our results, since Lewellen (2002) uses a re-

Table XLVII. Robustness Results: 25 Double Sorted International ex US Portfolios Based on Size and Value

This table shows performance results of the equally and risk weighted momentum strategies using 25 double sorted International ex US portfolios based on size and value, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.22	7.26	0.857	-	-	-	-	-	-	-
RV	6.51	7.24	0.900	0.77	0.292	0.90	0.300	0.91	0.266	0.69
RSJ/Corr	7.77	7.36	1.055	**3.04**	1.361	**3.55**	1.358	**3.53**	1.251	**3.38**
RSJ/Down Corr	7.66	7.35	1.043	**2.86**	1.277	**3.37**	1.282	**3.34**	1.141	**3.10**
RSJ/Beta	7.76	7.31	1.062	**3.08**	1.409	**3.57**	1.434	**3.54**	1.273	**3.24**
RSJ/Down Beta	7.70	7.29	1.057	**2.99**	1.370	**3.45**	1.358	**3.43**	1.237	**3.11**
RSJ/CoSkew	7.66	7.56	1.013	**2.44**	1.069	**2.50**	1.055	**2.48**	0.978	**2.08**
RSJ/CoKurt	7.84	7.38	1.063	**3.18**	1.411	**3.63**	1.434	**3.61**	1.306	**3.38**
RSJ/LPM Beta	7.73	7.32	1.056	**3.00**	1.365	**3.46**	1.358	**3.44**	1.246	**3.14**
RSJ/HTCR Beta	7.75	7.37	1.052	**2.99**	1.337	**3.53**	1.358	**3.51**	1.241	**3.33**
RSJ/Tail Beta	7.67	7.46	1.028	**2.62**	1.172	**3.03**	1.131	**3.00**	1.077	**2.42**
RSJ/Tail Sens	7.81	7.52	1.039	**2.90**	1.247	**3.39**	1.282	**3.38**	1.203	**3.02**
RSJ/Tail Risk	7.71	7.44	1.035	**2.75**	1.226	**3.27**	1.207	**3.24**	1.134	**2.62**
RSJ/MES	7.75	7.35	1.055	**3.01**	1.356	**3.41**	1.358	**3.39**	1.244	**3.03**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.36	7.31	0.870	-	-	-	-	-	-	-
RV	6.69	7.29	0.918	0.85	0.332	0.96	0.300	0.97	0.321	0.79
Equal (TV)	13.12	11.50	1.140	**2.70**	1.854	**2.31**	1.891	**2.33**	1.490	1.66
RV (TV)	14.16	11.90	1.190	**2.78**	2.182	**4.35**	2.197	**4.41**	1.898	**2.55**
RSJ/Corr (TV)	14.33	11.02	1.301	**3.56**	2.924	**3.48**	2.965	**3.50**	2.463	**2.87**
RSJ/Down Corr (TV)	14.22	11.01	1.292	**3.48**	2.868	**3.41**	2.888	**3.42**	2.400	**2.79**
RSJ/Beta (TV)	14.38	11.03	1.303	**3.60**	2.943	**3.52**	2.965	**3.53**	2.481	**2.86**
RSJ/Down Beta (TV)	14.33	11.00	1.303	**3.56**	2.939	**3.47**	2.965	**3.48**	2.482	**2.82**
RSJ/CoSkew (TV)	14.33	10.95	1.308	**3.49**	2.972	**3.51**	2.965	**3.52**	2.588	**2.88**
RSJ/CoKurt (TV)	14.44	11.02	1.310	**3.65**	2.989	**3.56**	2.965	**3.57**	2.529	**2.87**
RSJ/LPM Beta (TV)	14.36	11.04	1.301	**3.57**	2.929	**3.50**	2.965	**3.51**	2.477	**2.83**
RSJ/HTCR Beta (TV)	14.40	11.07	1.301	**3.58**	2.924	**3.52**	2.965	**3.53**	2.485	**2.86**
RSJ/Tail Beta (TV)	14.36	11.06	1.298	**3.54**	2.908	**3.60**	2.888	**3.62**	2.483	**2.83**
RSJ/Tail Sens (TV)	14.39	11.11	1.296	**3.54**	2.889	**3.50**	2.888	**3.52**	2.471	**2.79**
RSJ/Tail Risk (TV)	14.38	11.12	1.293	**3.54**	2.874	**3.51**	2.888	**3.53**	2.449	**2.75**
RSJ/MES (TV)	14.39	11.06	1.301	**3.58**	2.929	**3.49**	2.965	**3.50**	2.486	**2.79**

similar to the results of the industry momentum strategy, all risk weightings produce higher Sharpe Ratios and a positive economic value. However, the increase is not statistically significant for the volatility weighted strategy. In contrast, all (systematic) tail risk weighted strategies, except for the coskewness based strategy, exhibit statistically higher Sharpe Ratios and utilities. Volatility targeting again significantly improves the risk-return profile regardless of the weighting scheme, where again the best results are found for the strategies that use the (systematic) tail risk weighting combined with volatility targeting. Results for the other US style portfolios,

turn based weighting scheme, i.e. every style portfolio is contained in the momentum portfolio and each style is weighted by the relative past performance.

Table XLVIII. Robustness Results: 25 Double Sorted European Portfolios Based on Size and Value
This table shows performance results of the equally and risk weighted momentum strategies using 25 double sorted European portfolios based on size and value, the $t-12$ to $t-1$ ranking period and a cut-off point of $p=30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	5.17	7.89	0.655	-	-	-	-	-	-	-
RV	5.48	7.26	0.755	1.91	0.739	**1.98**	0.753	**2.00**	0.582	1.48
RSJ/Corr	6.59	7.29	0.904	**3.36**	1.867	**3.61**	1.815	**3.57**	1.637	**3.33**
RSJ/Down Corr	6.79	7.26	0.936	**3.82**	2.110	**5.51**	2.120	**5.52**	1.921	**5.09**
RSJ/Beta	6.60	7.24	0.912	**3.43**	1.927	**3.73**	1.891	**3.71**	1.713	**3.51**
RSJ/Down Beta	6.82	7.28	0.937	**3.69**	2.122	**4.93**	2.120	**4.98**	1.999	**4.68**
RSJ/CoSkew	7.04	7.69	0.916	**3.80**	1.965	**4.48**	1.968	**4.45**	1.881	**3.90**
RSJ/CoKurt	6.58	7.38	0.891	**3.24**	1.771	**4.62**	1.739	**4.54**	1.620	**4.91**
RSJ/LPM Beta	6.69	7.19	0.931	**3.58**	2.074	**4.08**	2.044	**4.09**	1.874	**3.85**
RSJ/HTCR Beta	6.66	7.28	0.914	**3.48**	1.948	**4.02**	1.891	**4.01**	1.763	**3.81**
RSJ/Tail Beta	6.67	7.53	0.886	**3.30**	1.737	**2.58**	1.739	**2.58**	1.612	**2.36**
RSJ/Tail Sens	6.49	7.52	0.863	**2.88**	1.559	**2.19**	1.510	**2.17**	1.447	**2.08**
RSJ/Tail Risk	6.61	7.47	0.885	**3.27**	1.725	**2.58**	1.739	**2.58**	1.535	**2.27**
RSJ/MES	6.62	7.28	0.910	**3.36**	1.911	**3.80**	1.891	**3.81**	1.729	**3.49**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	5.15	7.97	0.646	-	-	-	-	-	-	-
RV	5.51	7.33	0.752	**2.00**	0.794	**2.13**	0.828	**2.15**	0.628	1.58
Equal (TV)	8.45	10.42	0.811	1.59	1.298	1.46	1.282	1.45	0.783	0.98
RV (TV)	9.90	10.83	0.914	**2.28**	2.070	**2.21**	2.120	**2.25**	1.664	1.90
RSJ/Corr (TV)	9.95	9.50	1.048	**3.15**	3.063	**3.52**	3.042	**3.48**	2.439	**3.79**
RSJ/Down Corr (TV)	10.19	9.51	1.071	**3.31**	3.236	**4.16**	3.273	**4.14**	2.681	**5.05**
RSJ/Beta (TV)	10.07	9.55	1.054	**3.27**	3.115	**3.65**	3.119	**3.61**	2.523	**3.90**
RSJ/Down Beta (TV)	10.29	9.57	1.075	**3.36**	3.270	**4.17**	3.273	**4.16**	2.788	**4.64**
RSJ/CoSkew (TV)	10.53	9.61	1.096	**3.52**	3.427	**4.19**	3.428	**4.16**	2.997	**4.81**
RSJ/CoKurt (TV)	9.90	9.55	1.036	**3.08**	2.979	**3.58**	2.965	**3.54**	2.411	**4.50**
RSJ/LPM Beta (TV)	10.14	9.49	1.069	**3.30**	3.223	**3.80**	3.196	**3.76**	2.658	**4.03**
RSJ/HTCR Beta (TV)	10.12	9.56	1.059	**3.26**	3.145	**3.79**	3.119	**3.76**	2.581	**4.06**
RSJ/Tail Beta (TV)	10.24	9.84	1.041	**3.19**	3.010	**3.11**	3.042	**3.10**	2.573	**2.93**
RSJ/Tail Sens (TV)	9.91	9.62	1.029	**3.03**	2.926	**3.04**	2.965	**3.02**	2.387	**3.06**
RSJ/Tail Risk (TV)	10.14	9.76	1.039	**3.19**	2.998	**3.26**	3.042	**3.25**	2.465	**3.10**
RSJ/MES (TV)	10.13	9.56	1.060	**3.25**	3.153	**3.78**	3.119	**3.75**	2.598	**3.90**

shown in Tables XLIII to XLV, are again quite similar. Volatility weighting does not significantly improve the risk-return profile, whereas the (systematic) tail risk weightings produce significant Sharpe Ratio and utility increases. The volatility weighted style momentum strategy does even significantly underperform the equally weighted strategy for the style momentum strategy based on size and investment or profitability and investment sorted portfolios. Table XLVI shows results for the 25 size and value sorted portfolios using International stocks instead of just US stocks. Similar to the US style strategy, all risk weighted strategies exhibit higher Sharpe Ratios than the equally weighted strategy as well as high and positive economic values. However, these increases are only statistically significant for the (systematic) tail risk weighted

Table XLIX. Robustness Results: 100 Double Sorted US Portfolios Based on Size and Value
This table shows performance results of the equally and risk weighted momentum strategies using 100 double sorted US portfolios based on size and value, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 20\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta^{\gamma=5}_{MV}$	DM-test	$\Delta^{\gamma=5}_{CRRA}$	DM-test	$\Delta^{l=2}_{LA}$	DM-test
Equal	4.43	13.09	0.339	-	-	-	-	-	-	-
RV	4.79	12.43	0.385	1.61	0.551	1.27	0.225	0.29	0.550	1.26
RSJ/Corr	5.30	12.35	0.429	**2.67**	1.084	1.76	1.358	1.74	0.748	1.40
RSJ/Down Corr	5.08	12.94	0.393	1.58	0.650	0.90	1.055	1.23	0.513	0.95
RSJ/Beta	5.41	11.67	0.463	**3.38**	1.477	**2.38**	1.663	**2.35**	1.009	1.65
RSJ/Down Beta	5.34	12.29	0.435	**2.63**	1.143	1.86	1.510	**2.43**	0.925	1.82
RSJ/CoSkew	5.25	13.94	0.376	1.23	0.515	1.33	0.451	0.96	0.605	1.50
RSJ/CoKurt	4.86	12.42	0.391	1.55	0.625	1.04	0.677	0.80	0.188	0.30
RSJ/LPM Beta	5.71	12.47	0.458	**2.96**	1.430	**2.12**	1.968	**2.97**	1.431	**3.09**
RSJ/HTCR Beta	5.16	11.93	0.433	**2.42**	1.100	1.51	1.358	1.77	0.745	1.13
RSJ/Tail Beta	5.80	12.80	0.453	**3.40**	1.404	**3.21**	1.586	**3.47**	1.283	**3.31**
RSJ/Tail Sens	5.43	12.87	0.422	**2.30**	1.011	1.51	1.434	1.76	0.870	1.71
RSJ/Tail Risk	5.85	12.97	0.451	**3.43**	1.382	**2.37**	1.739	**2.87**	1.338	**3.02**
RSJ/MES	5.86	12.69	0.462	**3.04**	1.483	**2.13**	2.120	**2.98**	1.579	**3.40**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta^{\gamma=5}_{MV}$	DM-test	$\Delta^{\gamma=5}_{CRRA}$	DM-test	$\Delta^{l=2}_{LA}$	DM-test
Equal	4.48	13.04	0.343	-	-	-	-	-	-	-
RV	4.78	12.41	0.385	1.37	0.501	1.11	0.150	0.22	0.500	1.09
Equal (TV)	7.26	12.01	0.605	**4.39**	3.214	**3.02**	3.583	**2.67**	1.782	1.84
RV (TV)	8.06	13.14	0.613	**4.10**	3.374	**3.43**	3.738	**3.19**	2.241	**2.35**
RSJ/Corr (TV)	7.82	11.86	0.660	**4.53**	3.897	**3.05**	4.360	**2.74**	2.520	**2.13**
RSJ/Down Corr (TV)	7.67	11.90	0.644	**4.35**	3.702	**2.82**	4.204	**2.56**	2.327	1.94
RSJ/Beta (TV)	8.06	11.65	0.692	**4.79**	4.283	**3.28**	4.907	**2.99**	2.818	**2.30**
RSJ/Down Beta (TV)	7.89	11.69	0.675	**4.63**	4.075	**3.14**	4.672	**2.90**	2.632	**2.17**
RSJ/CoSkew (TV)	7.74	11.94	0.648	**4.53**	3.748	**3.04**	4.282	**2.86**	2.361	**2.02**
RSJ/CoKurt (TV)	7.53	11.85	0.636	**4.10**	3.593	**2.80**	3.970	**2.49**	2.248	1.90
RSJ/LPM Beta (TV)	8.18	11.74	0.696	**4.96**	4.343	**3.36**	4.985	**3.07**	2.924	**2.42**
RSJ/HTCR Beta (TV)	7.80	11.66	0.669	**4.50**	3.999	**3.08**	4.594	**2.83**	2.557	**2.11**
RSJ/Tail Beta (TV)	8.09	11.99	0.675	**4.84**	4.095	**3.33**	4.672	**3.01**	2.690	**2.37**
RSJ/Tail Sens (TV)	7.78	11.91	0.653	**4.49**	3.814	**2.92**	4.360	**2.67**	2.400	**1.98**
RSJ/Tail Risk (TV)	8.15	12.06	0.676	**5.00**	4.107	**3.32**	4.594	**2.98**	2.718	**2.38**
RSJ/MES (TV)	8.27	11.79	0.701	**5.09**	4.407	**3.42**	5.064	**3.12**	2.975	**2.47**

portfolios. Table XLVII shows results for the International style portfolios where the US are excluded. These results are quite similar to the results of the International data set where US data are included. Results for the European size and value style portfolios, shown in Table XLVIII, are again quite similar and demonstrate that all risk weighted strategies increase momentum's Sharpe Ratio and heighten the investors' utility. For this data set, volatility weighting also works well but is again outperformed by the (systematic) tail risk weighted strategies. Volatility targeting further increases the strategies' Sharpe Ratio and utility gains, especially for the strategies using the (systematic) tail risk weighting. In Table XLIX, we show results for 100 size and value double sorted portfolios and find that all risk weighted strategies exhibit high

Table L. Robustness Results: 20 US Portfolios Based on Size and Value
This table shows performance results of the equally and risk weighted momentum strategies using 20 US portfolios consisting of 10 single sorted portfolios based on size and 10 single sorted portfolios based on value, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	5.96	13.73	0.434	-	-	-	-	-	-	-
RV	5.79	12.73	0.455	1.01	0.193	0.50	1.131	1.06	0.078	0.39
RSJ/Corr	6.80	11.55	0.589	**4.62**	1.905	**3.25**	3.350	1.59	0.780	1.18
RSJ/Down Corr	6.90	11.69	0.590	**4.96**	1.928	**3.38**	3.428	1.63	0.848	1.38
RSJ/Beta	6.82	11.60	0.587	**4.69**	1.884	**3.20**	3.350	1.62	0.844	1.34
RSJ/Down Beta	6.94	11.86	0.585	**4.94**	1.859	**3.21**	3.428	1.63	0.984	1.86
RSJ/CoSkew	7.70	14.63	0.526	**3.37**	1.176	1.92	3.350	1.38	1.238	**3.44**
RSJ/CoKurt	6.92	11.73	0.590	**4.83**	1.927	**3.25**	3.428	1.60	0.862	1.42
RSJ/LPM Beta	6.85	11.72	0.584	**4.81**	1.847	**3.22**	3.350	1.62	0.887	1.53
RSJ/HTCR Beta	6.83	11.81	0.578	**4.58**	1.768	**2.88**	3.350	1.58	0.845	1.34
RSJ/Tail Beta	7.18	12.60	0.570	**5.14**	1.688	**2.95**	3.273	1.67	1.081	**2.36**
RSJ/Tail Sens	6.97	11.96	0.583	**5.03**	1.846	**3.65**	3.119	1.70	0.912	1.61
RSJ/Tail Risk	7.13	12.35	0.578	**5.20**	1.783	**3.46**	3.273	1.72	1.070	**2.25**
RSJ/MES	6.91	12.02	0.575	**4.54**	1.730	**2.74**	3.428	1.56	0.953	1.77
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	6.09	13.65	0.446	-	-	-	-	-	-	-
RV	5.89	12.65	0.466	0.64	0.172	0.44	1.131	1.04	0.054	0.24
Equal (TV)	9.80	14.38	0.682	**3.25**	3.114	**2.73**	3.660	**2.13**	0.382	0.41
RV (TV)	9.94	14.03	0.708	**3.46**	3.427	**2.90**	4.516	1.89	0.515	0.53
RSJ/Corr (TV)	11.15	13.89	0.802	**4.24**	4.629	**4.22**	5.772	**2.31**	1.838	1.70
RSJ/Down Corr (TV)	11.20	13.89	0.806	**4.27**	4.678	**4.26**	5.772	**2.38**	1.885	1.78
RSJ/Beta (TV)	11.19	13.92	0.804	**4.28**	4.647	**4.25**	5.772	**2.35**	1.878	1.76
RSJ/Down Beta (TV)	11.22	13.90	0.807	**4.31**	4.694	**4.16**	5.772	**2.40**	1.940	1.86
RSJ/CoSkew (TV)	11.48	14.23	0.806	**4.41**	4.689	**4.10**	5.851	**2.47**	2.041	**2.12**
RSJ/CoKurt (TV)	11.23	13.88	0.809	**4.30**	4.714	**4.18**	5.851	**2.35**	1.906	1.81
RSJ/LPM Beta (TV)	11.18	13.91	0.804	**4.26**	4.649	**4.22**	5.772	**2.36**	1.888	1.79
RSJ/HTCR Beta (TV)	11.23	13.96	0.804	**4.33**	4.654	**4.27**	5.772	**2.35**	1.907	1.78
RSJ/Tail Beta (TV)	11.36	14.02	0.811	**4.44**	4.744	**4.48**	5.772	**2.54**	2.000	**1.98**
RSJ/Tail Sens (TV)	11.29	13.94	0.810	**4.39**	4.728	**4.28**	5.851	**2.36**	1.960	1.87
RSJ/Tail Risk (TV)	11.43	14.02	0.816	**4.49**	4.802	**4.42**	6.009	**2.40**	2.065	**2.04**
RSJ/MES (TV)	11.22	13.97	0.803	**4.32**	4.639	**4.24**	5.772	**2.36**	1.905	1.83

Sharpe Ratio and utility increases, where again the highest increases are found for the (systematic) tail risk weighted strategies. The increase in the Sharpe Ratio and the investors' utility is again not statistically significant for the volatility weighted strategy. Volatility targeting again further enhances the risk-return profile for all weighting schemes and works best when volatility targeting is combined with the (systematic) tail risk weightings. Finally, Table L shows results for the data set consisting of 10 portfolios sorted on size and 10 portfolios sorted on value. Results are again quite similar to our earlier findings, i.e. volatility weighting does not significantly enhance momentum's risk-return profile, whereas the (systematic) tail risk weightings produce high and mostly statistically significant Sharpe Ratio and utility increases. As before, the risk-return profile can further be enhanced by overlaying these strategies with the volatility targeting approach.

B.7 Country Momentum

As next robustness check, we apply non-managed and risk-managed momentum strategies to several country indices. Momentum of country equity indices has been shown by Chan et al. (2000), Clare et al. (2016, Table 5), Richards (1997), Asness et al. (2013), Nijman et al. (2004), Novy-Marx (2012), Goyal and Jegadeesh (2017, Table 11.A) and Bhojraj and Swaminathan (2006). Richards (1997, Table 1.B) and Bhojraj and Swaminathan (2006) show that country momentum holds for holding periods of up to twelve months. In particular, country indices are not only used in the momentum literature, but also in portfolio allocation and asset pricing studies (Asness et al., 2020, Atilgan et al., 2019, DeMiguel et al., 2009b, Garlappi et al., 2006, Kirby and Ostdiek, 2012). Generally, country indices are used in many fields and are also important for practitioners since "country indices represent the largest and the most frequently traded securities of any stock market" (Bhojraj and Swaminathan, 2006, p. 433). We use two different data sets consisting of 25 and 61 country indices obtained from Datastream.[149] We use country indices denominated in US dollar. An alternative would be to use indices in local currency, since Bhojraj and Swaminathan (2006, p. 433) find that "the profitability of [country momentum] strategies can be significantly improved by forming momentum portfolios based on past equity-indices returns measured in local currencies rather than in U.S. dollars". As for our main results, we use the $t-12$ to $t-1$ ranking and a cut-off point of $p = 30\%$. Nevertheless, country momentum also works for other ranking periods and cut-off points. For example, Bhojraj and Swaminathan (2006) use a six months ranking period combined with $p = 20\%$, Richards (1997) find country momentum for ranking periods between three and twelve months combined with $p = 20\%$, Asness et al. (2013) also use the past 12 months ranking and $p = 1/3$, Novy-Marx (2012) use the $t-12$ to $t-7$ and $t-6$ to $t-2$ rankings combined

[149] In the literature on country momentum, several data sets have been used. For example, Richards (1997) use 16 developed countries, whereas Bhojraj and Swaminathan (2006, Table 1) use 38 countries and the authors additionally confirm the earlier results of Richards (1997) using the same 16 countries. Chan et al. (2000) and Novy-Marx (2012) show that country momentum also works well when it is applied to 23 countries, whereas Asness et al. (2013) find country momentum for 18 country indices.

with $p = 1/3$, whereas Chan et al. (2000) use several ranking periods between one week and 26 weeks and assign each country as either a winner or a loser based on the countries' past return. We also used several other ranking periods and cut-off points, but again found that results of our weighting schemes are quite robust. Weighting countries in the momentum portfolio by their risk is important since different countries have quite different risk characteristics. For example, Baltas (2015, Fig. 2) finds that different country indices have quite different levels of volatility. Atilgan et al. (2019, p. 14) find that countries' skewness varies between -1.13 and 1.08 and kurtosis lies between 3.33 and 9.03. Further, Atilgan et al. (2019, Table 6) find a strong negative systematic risk and return relation for countries from developed and emerging markets as well as a negative univariate tail risk and return relation for developed markets. This negative risk-return relation makes the inverse risk weighting appealing for country momentum.

Results for the 25 country indices are shown in Table LI, whereas Table LII shows results for the 61 country indices. Since country momentum has a slightly higher volatility than industry and style momentum, we use higher volatility target levels for these two data sets. For both data sets, all risk weighted strategies outperform the equally weighted strategy, where only the (systematic) tail risk weighted strategies produce significantly higher Sharpe Ratios and utilities. Nevertheless, the performance and utility gains of our risk weightings are lower for country momentum than for style and industry momentum. This result is in line with Atilgan et al. (2019) who find a stronger negative (systematic) tail risk-return relation for style portfolios than for country indices. Moreover, the impact of volatility targeting is mixed for the country momentum strategies. For the data set consisting of 25 country indices, we again find the highest Sharpe Ratios and utility gains for the volatility targeting strategy combined with the (systematic) tail risk weightings. However, the impact of volatility targeting is only small for this data set. For the data set consisting of 61 country indices, we even find a slightly negative impact of volatility targeting. For this data set, we find the best risk-return profile for the (systematic) tail risk weighted portfolios without target volatility overlay. Thus, the profitability of volatility targeting strongly depends on the used data set as also shown by Cederburg et al. (2020).

Table LI. Robustness Results: 25 Equity Country Indices
This table shows performance results of the equally and risk weighted momentum strategies using 25 equity country indices, the $t-12$ to $t-1$ ranking period, a cut-off point of $p = 30\%$ and a volatility target of 12%. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	5.32	13.02	0.409	-	-	-	-	-	-	-
RV	5.49	11.85	0.463	0.87	0.617	0.99	0.602	0.90	0.606	1.05
RSJ/Corr	7.60	13.49	0.563	**2.42**	1.936	**2.42**	1.968	**2.34**	1.922	**2.42**
RSJ/Down Corr	7.12	13.53	0.526	1.91	1.484	**2.47**	1.510	**2.35**	1.399	**2.30**
RSJ/Beta	7.13	13.36	0.533	**1.98**	1.562	**2.28**	1.586	**2.21**	1.588	**2.20**
RSJ/Down Beta	7.04	13.30	0.530	1.89	1.514	**2.52**	1.510	**2.42**	1.406	**2.20**
RSJ/CoSkew	7.72	13.88	0.557	**2.52**	1.876	**3.06**	1.891	**2.98**	1.811	**2.92**
RSJ/CoKurt	7.39	13.39	0.552	**2.33**	1.796	**2.93**	1.815	**2.80**	1.773	**2.69**
RSJ/LPM Beta	6.96	13.22	0.526	1.80	1.471	**2.33**	1.434	**2.20**	1.404	**2.03**
RSJ/HTCR Beta	7.29	13.42	0.543	**2.10**	1.686	**2.32**	1.663	**2.25**	1.661	**2.19**
RSJ/Tail Beta	7.29	13.61	0.536	**2.06**	1.602	**2.43**	1.586	**2.37**	1.572	**2.42**
RSJ/Tail Sens	7.57	13.66	0.554	**2.39**	1.829	**2.69**	1.815	**2.63**	1.974	**2.83**
RSJ/Tail Risk	6.78	13.30	0.509	1.61	1.265	**2.04**	1.282	1.95	1.295	**1.96**
RSJ/MES	7.14	13.36	0.534	**1.97**	1.573	**2.19**	1.586	**2.11**	1.565	**2.15**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	5.11	13.02	0.393	-	-	-	-	-	-	-
RV	5.40	11.76	0.459	0.95	0.755	1.12	0.677	1.02	0.774	1.19
Equal (TV)	6.43	13.84	0.464	1.47	0.933	**1.96**	0.904	1.93	0.599	0.83
RV (TV)	6.96	13.32	0.522	1.64	1.623	**2.36**	1.663	**2.36**	1.138	1.41
RSJ/Corr (TV)	8.45	13.83	0.611	**2.88**	2.751	**2.98**	2.734	**2.93**	2.398	**2.30**
RSJ/Down Corr (TV)	8.08	13.85	0.583	**2.53**	2.406	**3.09**	2.427	**3.01**	2.050	**2.27**
RSJ/Beta (TV)	8.00	13.88	0.576	**2.42**	2.317	**2.75**	2.350	**2.71**	2.011	**2.06**
RSJ/Down Beta (TV)	7.91	13.76	0.575	**2.34**	2.297	**2.85**	2.273	**2.77**	1.942	**2.04**
RSJ/CoSkew (TV)	8.33	13.75	0.606	**2.83**	2.681	**3.00**	2.657	**2.90**	2.376	**2.32**
RSJ/CoKurt (TV)	8.17	13.65	0.598	**2.67**	2.583	**3.29**	2.580	**3.21**	2.239	**2.33**
RSJ/LPM Beta (TV)	7.87	13.79	0.571	**2.27**	2.245	**2.74**	2.197	**2.66**	1.912	1.95
RSJ/HTCR Beta (TV)	8.22	13.95	0.589	**2.56**	2.480	**2.72**	2.503	**2.68**	2.146	**2.07**
RSJ/Tail Beta (TV)	7.95	13.92	0.571	**2.37**	2.261	**2.53**	2.273	**2.49**	1.968	**2.04**
RSJ/Tail Sens (TV)	8.58	13.88	0.618	**3.02**	2.831	**3.19**	2.888	**3.15**	2.587	**2.52**
RSJ/Tail Risk (TV)	7.78	13.81	0.563	**2.27**	2.155	**2.63**	2.120	**2.58**	1.876	**1.99**
RSJ/MES (TV)	8.11	13.81	0.587	**2.54**	2.447	**2.93**	2.427	**2.88**	2.116	**2.21**

B.8 Alternative Crash Indicators

Our results in the main part and the robustness results shown so far highlight that our approach that switches between univariate and systematic tail risk significantly outperforms the equally and volatility weighted momentum portfolio. Our results are so far based on the crash indicator δ_t that indicates a momentum crash when the momentum portfolio's volatility, measured by past month's Realized Volatility, is higher than the chosen volatility target σ_{target}. Thus, the profitability of the switching approach is also influenced by the chosen volatility model and the chosen volatility target level. To rule out that the switching approach is only profitable for this certain definition of the crash indicator δ_t, we next show several robustness results for other

Table LII. Robustness Results: 61 Equity Country Indices
This table shows performance results of the equally and risk weighted momentum strategies using 61 equity country indices, the $t-12$ to $t-1$ ranking period, a cut-off point of $p = 30\%$ and a volatility target of 15%. The description of the columns is given in Table XX.

	Panel A: Without Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	7.27	14.91	0.488	-	-	-	-	-	-	-
RV	7.47	13.22	0.565	1.03	0.985	1.43	1.055	1.54	1.298	**2.13**
RSJ/Corr	9.07	15.19	0.597	1.50	1.531	**2.83**	1.663	**3.00**	1.614	**2.58**
RSJ/Down Corr	9.36	15.33	0.611	1.71	1.726	**2.12**	1.891	**2.16**	1.810	**1.99**
RSJ/Beta	9.69	15.00	0.646	**2.15**	2.202	**3.16**	2.350	**3.24**	2.272	**2.88**
RSJ/Down Beta	9.73	15.21	0.640	**2.05**	2.127	**2.46**	2.273	**2.48**	2.157	**2.34**
RSJ/CoSkew	8.99	15.80	0.569	1.20	1.179	1.42	1.282	1.46	1.303	1.52
RSJ/CoKurt	9.21	15.48	0.595	1.52	1.513	**2.81**	1.663	**3.00**	1.685	**2.73**
RSJ/LPM Beta	9.35	14.99	0.623	1.81	1.889	**2.62**	2.044	**2.69**	1.957	**2.36**
RSJ/HTCR Beta	9.45	15.00	0.630	1.91	1.985	**3.01**	2.120	**3.12**	2.004	**2.64**
RSJ/Tail Beta	9.99	15.35	0.651	**2.30**	2.290	**2.40**	2.427	**2.40**	2.429	**2.52**
RSJ/Tail Sens	9.11	15.27	0.597	1.53	1.528	**2.52**	1.663	**2.63**	1.676	**2.49**
RSJ/Tail Risk	10.12	15.13	0.669	**2.58**	2.528	**2.76**	2.657	**2.74**	2.705	**2.80**
RSJ/MES	9.77	14.99	0.652	**2.22**	2.285	**3.04**	2.427	**3.06**	2.370	**2.93**
	Panel B: With Volatility Targeting									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	7.18	14.94	0.481	-	-	-	-	-	-	-
RV	7.43	13.18	0.564	0.98	1.066	1.52	1.131	1.63	1.431	**2.29**
Equal (TV)	8.24	16.07	0.513	0.67	0.507	0.78	0.602	0.87	0.289	0.43
RV (TV)	9.28	15.40	0.602	1.33	1.717	1.34	1.891	1.42	1.519	1.12
RSJ/Corr (TV)	9.64	15.96	0.604	1.41	1.773	**3.05**	1.891	**3.04**	1.819	**2.48**
RSJ/Down Corr (TV)	9.80	15.94	0.615	1.52	1.920	**2.15**	2.044	**2.18**	2.021	**1.96**
RSJ/Beta (TV)	10.20	15.92	0.641	1.78	2.279	**2.85**	2.427	**2.85**	2.319	**2.40**
RSJ/Down Beta (TV)	10.23	16.00	0.640	1.76	2.267	**2.51**	2.427	**2.55**	2.331	**2.30**
RSJ/CoSkew (TV)	9.21	15.77	0.584	1.22	1.484	1.83	1.586	1.86	1.563	1.72
RSJ/CoKurt (TV)	9.63	15.99	0.602	1.40	1.743	**2.93**	1.891	**3.00**	1.809	**2.54**
RSJ/LPM Beta (TV)	10.01	15.93	0.629	1.62	2.110	**2.67**	2.273	**2.70**	2.192	**2.31**
RSJ/HTCR Beta (TV)	10.14	15.93	0.637	1.72	2.224	**3.07**	2.350	**3.08**	2.243	**2.54**
RSJ/Tail Beta (TV)	10.53	16.11	0.653	**1.97**	2.464	**2.45**	2.580	**2.46**	2.609	**2.39**
RSJ/Tail Sens (TV)	9.62	15.91	0.605	1.41	1.774	**2.68**	1.891	**2.71**	1.871	**2.33**
RSJ/Tail Risk (TV)	10.68	16.00	0.668	**2.15**	2.657	**2.48**	2.811	**2.49**	2.835	**2.40**
RSJ/MES (TV)	10.32	15.88	0.650	1.87	2.408	**2.87**	2.580	**2.85**	2.487	**2.54**

definitions of δ_t.

In Table LIII, we show results for three additional crash indicators that are similar to the crash indicator used so far. In Panel A, we show results for the crash indicator when RV_t^{WML} is estimated with the past six months of daily data, whereas Panels B and C use RV_t^{WML} estimated with the past month's daily data combined with $\sigma_{\text{target}} = 5\%$ and $\sigma_{\text{target}} = 12\%$, respectively. Results in this table show that our switching approach is also advantageous to the equally and volatility weighted momentum portfolio for other volatility models and volatility targets. Thus, our results shown so far are not limited to a certain definition of δ_t.

We next show in Table LIV results for the crash indicator δ_t based on past volatility of the

Table LIII. Robustness Results: Alternative Momentum Volatility Based Crash Indicators
This table shows performance results of the equally and risk weighted momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period, a cut-off point of $p = 30\%$ and several alternative definitions of the crash indicator δ_t based on momentum's past Realized Volatility. Panel A shows results for the crash indicator using the past six months of daily data to estimate portfolio volatility combined with $\sigma_{\text{target}} = 8\%$, whereas Panels B and C use the past month's daily data to estimate portfolio volatility combined with $\sigma_{\text{target}} = 5\%$ and $\sigma_{\text{target}} = 12\%$, respectively. The description of the columns is given in Table XX.

	Panel A: Past Six Months' Realized Volatility and $\sigma_{\text{target}} = 8\%$									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	11.36	11.09	1.024	**5.11**	2.956	**4.91**	3.119	**4.70**	2.904	**4.82**
RSJ/Down Corr	10.94	11.83	0.925	**3.35**	1.872	**2.63**	1.434	1.19	2.108	**3.35**
RSJ/Beta	11.46	10.95	1.047	**5.62**	3.208	**5.65**	3.428	**5.52**	3.129	**5.30**
RSJ/Down Beta	11.23	11.36	0.988	**4.67**	2.562	**4.25**	2.503	**3.48**	2.634	**4.28**
RSJ/CoSkew	10.56	12.39	0.852	**2.14**	1.065	1.71	0.979	1.33	1.241	**2.05**
RSJ/CoKurt	11.15	11.60	0.961	**4.19**	2.269	**3.62**	2.120	**2.72**	2.406	**4.05**
RSJ/LPM Beta	11.55	10.99	1.051	**5.76**	3.257	**5.61**	3.505	**5.53**	3.150	**5.19**
RSJ/HTCR Beta	11.26	10.96	1.027	**5.50**	2.982	**5.30**	3.196	**5.12**	2.796	**4.74**
RSJ/Tail Beta	11.20	11.50	0.974	**4.66**	2.399	**4.86**	2.580	**4.83**	2.313	**4.25**
RSJ/Tail Sens	11.37	11.03	1.031	**5.57**	3.024	**5.34**	3.273	**4.94**	2.771	**4.86**
RSJ/Tail Risk	11.04	11.26	0.980	**4.72**	2.460	**4.78**	2.657	**4.62**	2.362	**4.31**
RSJ/MES	11.52	10.92	1.054	**5.75**	3.291	**5.93**	3.583	**5.52**	3.138	**5.42**
	Panel B: Past Month's Realized Volatility and $\sigma_{\text{target}} = 5\%$									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	10.05	10.23	0.982	**4.22**	2.475	**4.46**	2.503	**4.11**	2.351	**4.31**
RSJ/Down Corr	9.75	11.23	0.869	**2.20**	1.234	1.74	0.526	0.38	1.447	**2.56**
RSJ/Beta	10.18	10.10	1.009	**4.81**	2.772	**5.47**	2.888	**5.25**	2.692	**5.24**
RSJ/Down Beta	10.18	10.68	0.953	**3.91**	2.159	**3.90**	1.891	**2.54**	2.204	**4.20**
RSJ/CoSkew	9.80	12.37	0.792	0.76	0.391	0.76	0.300	0.51	0.533	1.07
RSJ/CoKurt	10.04	10.87	0.924	**3.37**	1.836	**2.88**	1.434	1.65	1.909	**3.35**
RSJ/LPM Beta	10.52	10.22	1.029	**5.26**	3.002	**6.02**	3.119	**5.84**	2.913	**5.78**
RSJ/HTCR Beta	10.00	10.13	0.987	**4.65**	2.527	**6.05**	2.657	**5.74**	2.320	**5.29**
RSJ/Tail Beta	10.22	10.92	0.936	**3.80**	1.962	**4.26**	2.120	**4.21**	1.929	**4.00**
RSJ/Tail Sens	10.28	10.36	0.992	**4.80**	2.590	**5.37**	2.734	**4.85**	2.343	**4.96**
RSJ/Tail Risk	10.09	10.68	0.945	**3.96**	2.054	**4.05**	2.197	**3.88**	2.000	**3.78**
RSJ/MES	10.55	10.21	1.033	**5.26**	3.040	**6.20**	3.273	**5.61**	2.930	**5.75**
	Panel C: Past Month's Realized Volatility and $\sigma_{\text{target}} = 12\%$									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	12.20	11.58	1.054	**5.57**	3.289	**4.63**	3.583	**4.12**	2.992	**4.60**
RSJ/Down Corr	12.25	11.70	1.047	**5.73**	3.211	**4.76**	3.505	**4.32**	2.871	**4.61**
RSJ/Beta	12.34	11.49	1.074	**6.05**	3.515	**5.15**	3.815	**4.58**	3.210	**5.00**
RSJ/Down Beta	12.37	11.60	1.066	**6.12**	3.426	**5.14**	3.738	**4.61**	3.095	**4.93**
RSJ/CoSkew	12.01	12.13	0.990	**4.89**	2.577	**4.05**	2.888	**3.98**	2.373	**3.74**
RSJ/CoKurt	12.09	11.68	1.035	**5.50**	3.082	**4.45**	3.350	**4.03**	2.771	**4.32**
RSJ/LPM Beta	12.45	11.52	1.081	**6.27**	3.592	**5.25**	3.893	**4.69**	3.266	**5.11**
RSJ/HTCR Beta	12.28	11.49	1.069	**6.23**	3.455	**4.81**	3.738	**4.34**	3.110	**4.56**
RSJ/Tail Beta	12.29	11.88	1.034	**5.61**	3.072	**5.28**	3.350	**4.75**	2.875	**4.99**
RSJ/Tail Sens	12.29	11.57	1.062	**6.14**	3.385	**4.83**	3.660	**4.27**	3.027	**4.72**
RSJ/Tail Risk	12.15	11.77	1.032	**5.55**	3.051	**5.22**	3.273	**4.57**	2.838	**5.13**
RSJ/MES	12.35	11.54	1.070	**6.15**	3.469	**5.12**	3.815	**4.57**	3.154	**5.00**

market instead of past volatility of the momentum portfolio. Du Plessis and Hallerbach (2017) find a negative relation between the volatility of the market and the returns of the industry momentum portfolio. This negative relation has also been shown for the returns of the individual stock based momentum strategy (Daniel and Moskowitz, 2016, Wang and Xu, 2015). Thus, we show results for a crash indicator that indicates a crash ($\delta_t = 1$) if past market volatility is high, where we define high with respect to three different volatility levels between 5% and 12%. Results in Table LIV show that our (systematic) tail risk switching strategy also works well when momentum crashes are estimated by a high market volatility. This result holds for all three different levels of σ_{target}.

In Table LV, we show results for alternative definitions of δ_t based on momentum's volatility, where we define "low" and "high" volatility periods based on the relation of short- and long-term volatility (Copeland and Copeland, 1999). In other words, the threshold $\sigma_{target,t}$ is not fixed but varies over time based on past long-term volatility (Dreyer and Hubrich, 2019). This has the advantage that crash and non-crash periods are not defined based on an arbitrarily chosen constant. However, as stated in Section 3.5, defining δ_t with respect to a constant σ_{target} has the advantage that an investor's portfolio fits well to the investor's risk-aversion. For that reason, we use a constant σ_{target} for our main results, but results in Table LV again demonstrate that the profitability of our switching strategy is not driven by data mining. In Panel A, we define a crash regime when volatility estimated with the past month's daily returns is higher than the long-term volatility estimated with the past twelve months' daily returns. Panels B and C define short-term volatility by the Realized Volatility of the past three and six months, whereas long-term volatility is defined by the past 24 and 48 months' volatility, respectively. In line with our previous results, we find that the (systematic) tail risk switching strategies clearly outperform the equally and volatility weighted strategies.

Finally, in Table LVI we use crash indicators δ_t that are based on the equally weighted momentum portfolio's past performance using the TSMOM strategy of Moskowitz et al. (2012), where we use the one month, six months and twelve months TSMOM strategies in Panels A, B and C, respectively. These lengths are frequently used in the literature on TSMOM and are

Table LIV. Robustness Results: Alternative Market Volatility Based Crash Indicators
This table shows performance results of the equally and risk weighted momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period, a cut-off point of $p = 30\%$ and several alternative definitions of the crash indicator δ_t based on volatility estimated with daily market returns of the past month. Panels A, B and C show results for the crash indicator using $\sigma_{\text{target}} = 5\%$, $\sigma_{\text{target}} = 8\%$ and $\sigma_{\text{target}} = 12\%$, respectively. The description of the columns is given in Table XX.

Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Panel A: Past Month's Market Volatility and $\sigma_{\text{target}} = 5\%$										
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	9.44	9.91	0.953	**3.70**	2.136	**3.70**	2.120	**3.35**	2.171	**3.80**
RSJ/Down Corr	9.22	10.96	0.841	1.69	0.921	1.23	0.075	0.06	1.303	**2.15**
RSJ/Beta	9.57	9.77	0.980	**4.28**	2.447	**4.79**	2.503	**4.52**	2.539	**4.84**
RSJ/Down Beta	9.49	10.42	0.911	**3.09**	1.681	**2.87**	1.358	1.67	1.896	**3.36**
RSJ/CoSkew	8.96	12.33	0.726	-0.72	-0.343	-0.71	-0.374	-0.64	-0.182	-0.39
RSJ/CoKurt	9.65	10.62	0.909	**3.06**	1.670	**2.39**	1.207	1.24	1.884	**2.99**
RSJ/LPM Beta	9.94	9.94	1.000	**4.70**	2.674	**5.07**	2.734	**4.92**	2.747	**5.10**
RSJ/HTCR Beta	9.44	9.84	0.960	**4.12**	2.215	**5.36**	2.273	**5.06**	2.175	**5.03**
RSJ/Tail Beta	9.74	10.66	0.913	**3.35**	1.702	**3.75**	1.815	3.69	1.797	3.63
RSJ/Tail Sens	9.73	10.11	0.962	**4.23**	2.245	**4.23**	2.350	**3.88**	2.116	**4.13**
RSJ/Tail Risk	9.56	10.40	0.920	**3.49**	1.766	**3.49**	1.891	**3.29**	1.824	**3.47**
RSJ/MES	10.16	9.97	1.019	**4.97**	2.882	**6.17**	3.042	**5.52**	2.905	**5.89**
Panel B: Past Month's Market Volatility and $\sigma_{\text{target}} = 8\%$										
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	9.75	10.21	0.956	**3.79**	2.172	**3.96**	2.197	**3.61**	2.120	**4.00**
RSJ/Down Corr	9.54	11.18	0.853	1.95	1.059	1.58	0.300	0.25	1.333	**2.46**
RSJ/Beta	9.86	10.07	0.979	**4.32**	2.432	**4.87**	2.503	**4.61**	2.442	**4.71**
RSJ/Down Beta	9.79	10.67	0.918	**3.31**	1.766	**3.45**	1.510	**2.17**	1.892	**3.71**
RSJ/CoSkew	9.29	12.34	0.753	-0.12	-0.045	-0.09	-0.075	-0.17	0.095	0.20
RSJ/CoKurt	9.97	10.86	0.918	**3.29**	1.779	**2.81**	1.434	1.60	1.900	**3.28**
RSJ/LPM Beta	10.20	10.21	0.999	**4.77**	2.666	**5.03**	2.811	**4.89**	2.661	**4.85**
RSJ/HTCR Beta	9.80	10.13	0.967	**4.35**	2.304	**5.42**	2.427	**5.00**	2.212	**5.06**
RSJ/Tail Beta	10.07	10.88	0.925	**3.60**	1.838	**3.91**	1.968	**3.77**	1.873	**3.76**
RSJ/Tail Sens	10.03	10.31	0.973	**4.48**	2.364	**4.61**	2.503	**4.10**	2.142	**4.59**
RSJ/Tail Risk	9.84	10.64	0.925	**3.63**	1.836	**3.84**	1.968	**3.55**	1.840	**3.78**
RSJ/MES	10.35	10.22	1.013	**4.95**	2.814	**5.81**	3.042	**5.14**	2.768	**5.47**
Panel C: Past Month's Market Volatility and $\sigma_{\text{target}} = 12\%$										
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	10.79	11.03	0.978	**4.26**	2.434	**3.98**	2.503	**3.54**	2.344	**4.22**
RSJ/Down Corr	10.58	11.86	0.893	**2.79**	1.517	**2.36**	0.979	0.89	1.689	**3.30**
RSJ/Beta	10.89	10.93	0.997	**4.76**	2.648	**4.34**	2.734	**3.91**	2.571	**4.56**
RSJ/Down Beta	10.99	11.40	0.964	**4.33**	2.294	**4.07**	2.120	**3.18**	2.317	**4.44**
RSJ/CoSkew	10.21	12.65	0.807	1.13	0.571	1.16	0.526	0.83	0.715	1.58
RSJ/CoKurt	10.68	11.55	0.925	**3.47**	1.864	**3.08**	1.586	**2.09**	1.920	**3.65**
RSJ/LPM Beta	11.24	11.01	1.021	**5.32**	2.922	**4.70**	3.042	**4.28**	2.841	**4.92**
RSJ/HTCR Beta	10.73	10.92	0.983	**4.73**	2.487	**4.15**	2.580	**3.70**	2.328	**4.28**
RSJ/Tail Beta	10.98	11.58	0.948	**4.15**	2.108	**4.25**	2.273	**3.78**	2.103	**4.41**
RSJ/Tail Sens	10.84	11.04	0.982	**4.74**	2.477	**4.48**	2.580	**3.82**	2.226	**4.82**
RSJ/Tail Risk	10.83	11.37	0.953	**4.25**	2.162	**4.12**	2.273	**3.65**	2.124	**4.27**
RSJ/MES	11.19	10.99	1.018	**5.19**	2.884	**4.60**	3.119	**4.03**	2.789	**4.86**

successful in forecasting crashes. In this case, the crash indicator δ_t equals one if the equally weighted momentum portfolio's return in the past months is negative. Results in Table LVI

Table LV. Robustness Results: Alternative Momentum Volatility Based Crash Indicators

This table shows performance results of the equally and risk weighted momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period, a cut-off point of $p = 30\%$ and several alternative definitions of the crash indicator δ_t. Panel A shows results for the crash indicator that equals one if past month's volatility is higher than past twelve months' volatility. Panel B shows results for the crash indicator that equals one if past three months' volatility is higher than past 24 months' volatility. Panel C shows results for the crash indicator that equals one if past six months' volatility is higher than past 48 months' volatility. The description of the columns is given in Table XX.

	Panel A: One Month's Realized Volatility Higher Than Twelve Months' Realized Volatility									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	11.45	11.36	1.008	**4.64**	2.775	**3.94**	3.042	**3.47**	2.690	**4.09**
RSJ/Down Corr	11.60	11.52	1.006	**4.92**	2.752	**4.08**	3.042	**3.64**	2.618	**4.16**
RSJ/Beta	11.37	11.24	1.012	**4.77**	2.811	**4.12**	3.119	**3.65**	2.709	**4.18**
RSJ/Down Beta	11.54	11.36	1.016	**5.02**	2.858	**4.18**	3.196	**3.74**	2.714	**4.16**
RSJ/CoSkew	11.49	12.36	0.930	**3.63**	1.912	**3.06**	2.273	**2.99**	1.832	**2.97**
RSJ/CoKurt	11.31	11.44	0.989	**4.50**	2.562	**3.69**	2.811	**3.31**	2.435	**3.75**
RSJ/LPM Beta	11.72	11.29	1.037	**5.33**	3.100	**4.56**	3.428	**4.04**	2.993	**4.66**
RSJ/HTCR Beta	11.38	11.24	1.013	**4.96**	2.824	**3.93**	3.119	**3.53**	2.656	**3.85**
RSJ/Tail Beta	11.49	11.77	0.976	**4.49**	2.415	**4.80**	2.657	**4.09**	2.315	**4.92**
RSJ/Tail Sens	11.72	11.54	1.016	**5.19**	2.861	**3.94**	3.119	**3.50**	2.680	**3.99**
RSJ/Tail Risk	11.50	11.67	0.985	**4.68**	2.517	**4.51**	2.734	**3.86**	2.436	**4.64**
RSJ/MES	11.82	11.40	1.036	**5.37**	3.086	**4.61**	3.428	**4.04**	2.985	**4.74**

	Panel B: Three Months' Realized Volatility Higher Than 24 Months' Realized Volatility									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	11.13	10.86	1.025	**5.12**	2.959	**3.97**	3.196	**3.50**	2.773	**4.02**
RSJ/Down Corr	11.26	11.17	1.009	**4.99**	2.776	**4.02**	3.042	**3.68**	2.572	**3.95**
RSJ/Beta	11.26	10.75	1.048	**5.49**	3.212	**4.56**	3.505	**4.04**	3.043	**4.49**
RSJ/Down Beta	11.33	10.97	1.033	**5.33**	3.044	**4.17**	3.350	**3.79**	2.854	**4.09**
RSJ/CoSkew	11.35	12.05	0.941	**3.82**	2.031	**3.06**	2.350	**2.98**	1.957	**3.12**
RSJ/CoKurt	11.23	11.16	1.006	**4.97**	2.755	**4.06**	2.965	**3.63**	2.582	**4.13**
RSJ/LPM Beta	11.44	10.88	1.051	**5.63**	3.254	**4.65**	3.583	**4.13**	3.086	**4.62**
RSJ/HTCR Beta	11.28	10.89	1.035	**5.30**	3.074	**4.24**	3.350	**3.80**	2.909	**4.17**
RSJ/Tail Beta	11.35	11.40	0.995	**4.93**	2.631	**4.55**	2.888	**3.99**	2.547	**4.54**
RSJ/Tail Sens	11.33	11.18	1.014	**5.13**	2.839	**3.83**	3.119	**3.39**	2.632	**3.89**
RSJ/Tail Risk	11.07	11.26	0.983	**4.67**	2.488	**4.11**	2.734	**3.58**	2.384	**4.18**
RSJ/MES	11.38	10.98	1.036	**5.40**	3.082	**4.40**	3.428	**3.91**	2.925	**4.40**

	Panel C: Six Months' Realized Volatility Higher Than 48 Months' Realized Volatility									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	11.25	11.08	1.015	**5.07**	2.855	**5.03**	3.042	**4.53**	2.799	**5.20**
RSJ/Down Corr	10.84	11.85	0.915	**3.26**	1.760	**2.50**	1.282	1.13	2.002	**3.21**
RSJ/Beta	11.48	10.94	1.050	**5.86**	3.242	**6.03**	3.428	**5.46**	3.169	**5.92**
RSJ/Down Beta	11.10	11.40	0.973	**4.50**	2.397	**3.92**	2.350	**3.23**	2.470	**3.96**
RSJ/CoSkew	10.72	12.54	0.855	**2.23**	1.098	1.81	1.055	1.47	1.313	**2.20**
RSJ/CoKurt	10.91	11.60	0.941	**3.89**	2.040	**3.59**	1.891	**2.57**	2.154	**4.14**
RSJ/LPM Beta	11.51	11.06	1.041	**5.79**	3.148	**5.65**	3.350	**5.21**	3.073	**5.44**
RSJ/HTCR Beta	11.34	10.91	1.039	**5.87**	3.114	**5.46**	3.350	**4.91**	2.927	**5.12**
RSJ/Tail Beta	11.31	11.54	0.981	**4.81**	2.469	**5.78**	2.657	**5.16**	2.447	**5.68**
RSJ/Tail Sens	11.24	11.15	1.008	**5.38**	2.773	**4.79**	2.965	**4.25**	2.582	**4.82**
RSJ/Tail Risk	11.10	11.40	0.974	**4.68**	2.392	**5.40**	2.580	**4.67**	2.335	**5.52**
RSJ/MES	11.61	11.02	1.054	**6.04**	3.282	**5.66**	3.583	**4.97**	3.142	**5.52**

Table LVI. Robustness Results: Alternative TSMOM Based Crash Indicators
This table shows performance results of the equally and risk weighted momentum strategies using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period, a cut-off point of $p = 30\%$ and several alternative definitions of the crash indicator δ_t based on the TSMOM strategy applied to the momentum portfolio. Panels A, B and C show results for the crash indicator using the one, six and twelve months TSMOM strategy, respectively. The description of the columns is given in Table XX.

	Panel A: TSMOM Using Past One Month									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	11.38	12.21	0.932	**3.95**	1.949	**3.70**	1.968	**3.35**	2.272	**4.25**
RSJ/Down Corr	10.86	12.81	0.848	**2.10**	1.042	**2.22**	0.602	0.88	1.558	**3.53**
RSJ/Beta	11.47	12.08	0.949	**4.29**	2.139	**3.98**	2.120	**3.60**	2.488	**4.58**
RSJ/Down Beta	11.16	12.39	0.900	**3.31**	1.604	**3.42**	1.434	**2.89**	2.010	**4.12**
RSJ/CoSkew	10.54	13.00	0.811	1.31	0.630	1.22	0.526	0.83	0.893	1.79
RSJ/CoKurt	11.30	12.63	0.895	**3.25**	1.550	**3.56**	1.358	**2.93**	1.986	**4.34**
RSJ/LPM Beta	11.49	12.11	0.949	**4.30**	2.133	**4.06**	2.197	**3.77**	2.476	**4.57**
RSJ/HTCR Beta	11.57	12.02	0.963	**4.57**	2.288	**4.25**	2.350	**3.77**	2.586	**4.83**
RSJ/Tail Beta	11.21	12.33	0.909	**3.55**	1.695	**3.62**	1.739	**3.08**	1.952	**4.41**
RSJ/Tail Sens	11.52	12.24	0.941	**4.19**	2.044	**3.60**	2.120	**3.11**	2.304	**4.23**
RSJ/Tail Risk	11.02	12.30	0.896	**3.25**	1.541	**3.09**	1.586	**2.68**	1.831	**3.79**
RSJ/MES	11.54	12.07	0.956	**4.35**	2.211	**4.11**	2.273	**3.57**	2.498	**4.67**
	Panel B: TSMOM Using Past Six Months									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	11.46	12.19	0.940	**3.78**	2.036	**3.04**	2.120	**2.61**	2.206	**3.47**
RSJ/Down Corr	11.53	12.34	0.934	**3.76**	1.973	**3.69**	2.044	**3.14**	2.149	**4.16**
RSJ/Beta	11.48	12.15	0.945	**3.87**	2.092	**3.29**	2.197	**2.83**	2.325	**3.73**
RSJ/Down Beta	11.63	12.21	0.952	**4.10**	2.169	**3.73**	2.273	**3.18**	2.344	**4.11**
RSJ/CoSkew	11.01	12.71	0.866	**2.37**	1.221	**2.58**	1.358	**2.33**	1.228	**2.75**
RSJ/CoKurt	11.68	12.34	0.947	**4.03**	2.108	**3.40**	2.197	**2.95**	2.305	**3.83**
RSJ/LPM Beta	11.60	12.17	0.954	**4.08**	2.183	**3.51**	2.273	**3.04**	2.423	**3.92**
RSJ/HTCR Beta	11.59	12.16	0.953	**4.09**	2.179	**3.42**	2.273	**2.91**	2.372	**3.85**
RSJ/Tail Beta	11.39	12.29	0.927	**3.67**	1.890	**3.65**	1.968	**3.08**	2.120	**4.08**
RSJ/Tail Sens	11.83	12.24	0.967	**4.39**	2.334	**3.68**	2.427	**3.13**	2.495	**4.11**
RSJ/Tail Risk	11.30	12.26	0.922	**3.54**	1.829	**3.35**	1.891	**2.86**	2.042	**3.80**
RSJ/MES	11.59	12.18	0.952	**4.03**	2.163	**3.50**	2.197	**3.01**	2.399	**3.93**
	Panel C: TSMOM Using Past Twelve Months									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.17	12.11	0.758	-	-	-	-	-	-	-
RV	8.44	10.87	0.776	0.48	0.168	0.36	0.075	0.16	0.186	0.41
RSJ/Corr	12.02	12.39	0.970	**5.11**	2.371	**5.21**	2.427	**4.67**	2.509	**5.42**
RSJ/Down Corr	11.36	13.04	0.871	**2.76**	1.307	**2.82**	0.979	1.35	1.703	**3.94**
RSJ/Beta	11.94	12.38	0.964	**4.96**	2.306	**5.16**	2.350	**4.58**	2.485	**5.30**
RSJ/Down Beta	11.74	12.67	0.927	**4.21**	1.903	**4.57**	1.815	**4.12**	2.191	**4.84**
RSJ/CoSkew	11.20	13.18	0.850	**2.38**	1.062	**2.30**	0.904	1.71	1.242	**2.88**
RSJ/CoKurt	11.66	12.87	0.906	**3.72**	1.679	**3.99**	1.510	**3.37**	2.010	**4.67**
RSJ/LPM Beta	12.00	12.41	0.967	**5.05**	2.339	**5.22**	2.427	**4.71**	2.500	**5.28**
RSJ/HTCR Beta	12.00	12.34	0.973	**5.16**	2.398	**5.24**	2.503	**4.52**	2.525	**5.45**
RSJ/Tail Beta	11.89	12.45	0.955	**4.82**	2.206	**4.92**	2.273	**4.44**	2.328	**5.05**
RSJ/Tail Sens	12.47	12.32	1.012	**5.96**	2.836	**5.73**	2.965	**4.88**	2.930	**5.98**
RSJ/Tail Risk	12.04	12.39	0.971	**5.16**	2.383	**5.42**	2.503	**4.79**	2.521	**5.45**
RSJ/MES	12.08	12.32	0.980	**5.13**	2.485	**5.31**	2.580	**4.52**	2.610	**5.41**

again show that our switching strategy is superior to the equally and volatility weighted momentum portfolio, where we find the best results for the twelve months TSMOM strategy.

In total, results in this section show that our (systematic) tail risk switching strategies are not a result of data mining and are also advantageous to the equally and volatility weighted momentum portfolio for several crash indicators based on momentum's past return and volatility or past market volatility. Rickenberg (2020a) also shows that different definitions of the momentum crash indicator produce quite similar results when momentum's portfolio risk is managed by a strategy that switches between two risk targeting approaches. In particular, we find that crash indicators based on volatility of either the market or the momentum portfolio produce more convincing results. This finding is in line with Barroso and Santa-Clara (2015) who also find that volatility is more successful in forecasting crashes than other crash indicators based on past returns.

B.9 Alternative Volatility Targeting Strategies

In this section, we show that our results are also robust to other specifications of the volatility targeting approach. Our main results are based on a volatility target of $\sigma_{\text{target}} = 8\%$ combined with the Realized Volatility (RV) model to estimate monthly portfolio risk. In this section, we examine results for other volatility target levels and other volatility models to estimate monthly portfolio volatility. Rickenberg (2020b) and Rickenberg (2020a) shows that more advanced dynamic volatility models, like the EWMA or GARCH model, are more successful in targeting a constant level of portfolio volatility and typically produce a better risk-return profile than the RV estimator. To assess how different volatility targets and volatility models affect the risk-return profile of our risk-managed momentum strategies, Table LVII shows several robustness results. Panel A shows results for a volatility target of $\sigma_{\text{target}} = 5\%$, whereas Panel B shows results for a volatility target of $\sigma_{\text{target}} = 12\%$. In both cases, portfolio volatility is estimated by the RV model.[150] In contrast, Panel C shows results when portfolio volatility is managed by the EWMA model instead of the RV model. We follow Rickenberg (2020a) and fit the EWMA

[150]The different volatility targets do not only affect the volatility targeting approach, but also the assets' weights, since other volatility targets lead to other momentum crash indicators as shown in Section 3.5. We also examined strategies where the different volatility targets were only used to determine the exposure to the strategy, where the momentum crash indicator is still estimated with a volatility target of $\sigma_{\text{target}} = 8\%$. Results of these approaches were quite similar to the results presented here. In particular, this again shows that our switching approach is quite robust against other definitions of the crash indicator δ_t as also shown in Section B.8.

model to daily data and use the square root of time rule (SRTR) to obtain an estimate of the portfolio's monthly volatility. Rickenberg (2020a) shows that this model, although it is very easy to implement, performs quite well.

Table LVII highlights that the volatility targeted momentum strategies are robust to other choices of σ_{target} and the volatility model used to estimate portfolio risk. In all three cases, volatility targeting significantly increases the Sharpe Ratio and the investors' utility. In line with our previous findings, the highest Sharpe Ratios and utility gains are found for the strategies that combine the target volatility approach with the (systematic) tail risk weightings. As expected, the strategy using a lower volatility target is significantly less risky but also exhibits a lower return than the strategy using a higher volatility target. In total, both strategies produce similar Sharpe Ratios and economic values. Similarly, using the EWMA model instead of the RV model hardly affects the risk-return profile of the risk-managed momentum strategy and results in Panel C are quite similar to the results in Tables X and XIV. Concluding, the volatility targeting strategy used in our main part is also robust to other volatility targets and volatility models.

B.10 Tail Risk Targeting Strategies

Results are so far based on strategies that combine a given weighting scheme with the volatility targeting approach to manage portfolio risk. However, Rickenberg (2020a,b) shows that portfolio risk can also be managed by targeting a constant level of tail risk, measured by VaR or CVaR, where especially the CVaR targeting strategy produces a convincing risk-return profile. Moreover, Rickenberg (2020a,b) finds that switching between volatility and CVaR targeting can further enhance the risk-return profile. We chose the volatility targeting approach for our main results for several reasons. First, volatility targeting is so far the main approach to manage portfolio risk. Second, when managing a portfolio's monthly risk, the Realized Volatility model is easy to understand and implement. Rickenberg (2020a) argues that managing a portfolio's monthly risk based on daily data is not straightforward and can become quite complicated. Since risk-managed industry momentum is also highly relevant for practitioners, we focus on the easy RV model. Third, Rickenberg (2020a,b) finds that CVaR targeting works well when

Table LVII. Robustness Results: Different Volatility Targets and EWMA Model

This table shows performance results of volatility targeting strategies using volatility target levels of $\sigma_{\text{target}} = 5\%$ and $\sigma_{\text{target}} = 12\%$ in Panel A and B as well as the EWMA volatility model in Panel C. The industry momentum strategy uses 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The description of the columns is given in Table XX.

	Panel A: Volatility Targeting With RV Model and $\sigma_{\text{target}} = 5\%$									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.13	11.98	0.763	-	-	-	-	-	-	-
RV	8.37	10.70	0.782	0.34	0.173	0.36	0.075	0.16	0.180	0.39
Equal (TV)	8.34	7.37	1.131	**6.23**	4.067	**5.82**	4.516	**5.62**	3.469	**5.12**
RV (TV)	9.52	7.90	1.205	**5.96**	4.904	**5.14**	5.378	**5.16**	4.275	**4.73**
RSJ/Corr (TV)	9.34	7.04	1.328	**7.41**	6.354	**7.46**	6.803	**7.17**	5.552	**6.86**
RSJ/Down Corr (TV)	9.03	7.20	1.255	**6.71**	5.499	**6.91**	5.851	**6.92**	4.944	**6.50**
RSJ/Beta (TV)	9.51	7.21	1.320	**7.38**	6.254	**7.85**	6.644	**7.56**	5.582	**7.35**
RSJ/Down Beta (TV)	9.34	7.17	1.303	**7.22**	6.060	**8.06**	6.485	**7.90**	5.457	**7.47**
RSJ/CoSkew (TV)	8.49	7.15	1.187	**5.79**	4.705	**5.76**	5.221	**5.80**	4.281	**5.34**
RSJ/CoKurt (TV)	9.17	7.10	1.292	**7.28**	5.938	**7.71**	6.247	**7.61**	5.237	7.04
RSJ/LPM Beta (TV)	9.67	7.15	1.352	**7.69**	6.629	**8.87**	7.122	**8.36**	5.982	**8.35**
RSJ/HTCR Beta (TV)	9.26	7.04	1.315	**7.20**	6.200	**8.02**	6.644	**7.55**	5.442	**7.34**
RSJ/Tail Beta (TV)	9.45	7.39	1.278	**7.19**	5.762	**6.96**	6.247	**6.76**	5.107	**6.44**
RSJ/Tail Sens (TV)	9.35	7.06	1.324	**7.35**	6.308	**7.46**	6.803	**7.13**	5.590	**6.70**
RSJ/Tail Risk (TV)	9.44	7.38	1.279	**7.12**	5.779	**6.79**	6.247	**6.63**	5.098	**6.19**
RSJ/MES (TV)	9.72	7.22	1.347	**7.52**	6.563	**8.63**	7.043	**8.00**	5.958	**8.17**

	Panel B: Volatility Targeting With RV Model and $\sigma_{\text{target}} = 12\%$									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.13	11.98	0.763	-	-	-	-	-	-	-
RV	8.37	10.70	0.782	0.34	0.173	0.36	0.075	0.16	0.180	0.39
Equal (TV)	20.02	17.68	1.132	**6.23**	4.067	**5.82**	4.516	**5.62**	3.469	**5.12**
RV (TV)	23.00	18.96	1.213	**5.96**	4.904	**5.14**	5.378	**5.16**	4.275	**4.73**
RSJ/Corr (TV)	23.10	16.87	1.369	**7.59**	6.552	**6.66**	7.043	**6.22**	5.916	**6.08**
RSJ/Down Corr (TV)	23.15	16.89	1.371	**7.72**	6.565	**6.78**	7.122	**6.35**	5.905	**6.17**
RSJ/Beta (TV)	23.16	16.93	1.369	**7.66**	6.541	**6.91**	7.043	**6.43**	5.927	**6.27**
RSJ/Down Beta (TV)	23.23	16.91	1.374	**7.76**	6.594	**6.94**	7.122	**6.47**	5.956	**6.27**
RSJ/CoSkew (TV)	22.71	16.90	1.344	**7.34**	6.283	**6.50**	6.803	**6.18**	5.709	**5.93**
RSJ/CoKurt (TV)	22.87	16.88	1.355	**7.53**	6.402	**6.60**	6.883	**6.20**	5.748	**5.99**
RSJ/LPM Beta (TV)	23.29	16.89	1.379	**7.75**	6.649	**6.92**	7.202	**6.45**	6.029	**6.27**
RSJ/HTCR Beta (TV)	23.12	16.91	1.367	**7.62**	6.530	**6.69**	7.043	**6.27**	5.926	**6.11**
RSJ/Tail Beta (TV)	23.15	16.98	1.363	**7.76**	6.488	**7.05**	7.043	**6.61**	5.860	**6.31**
RSJ/Tail Sens (TV)	23.17	16.83	1.377	**7.69**	6.631	**6.78**	7.122	**6.34**	5.994	**6.18**
RSJ/Tail Risk (TV)	23.02	16.95	1.358	**7.67**	6.434	**7.00**	6.963	**6.53**	5.816	**6.34**
RSJ/MES (TV)	23.20	16.91	1.373	**7.68**	6.583	**6.88**	7.122	**6.41**	5.975	**6.27**

	Panel C: Volatility Targeting With EWMA Model and $\sigma_{\text{target}} = 8\%$									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.13	11.98	0.763	-	-	-	-	-	-	-
RV	8.37	10.70	0.782	0.34	0.173	0.36	0.075	0.16	0.180	0.39
Equal (TV)	13.53	11.91	1.135	**5.89**	4.096	**5.70**	4.516	**5.49**	3.237	**4.45**
RV (TV)	15.51	12.75	1.216	**5.88**	4.981	**5.38**	5.378	**5.41**	4.075	**4.52**
RSJ/Corr (TV)	16.84	12.89	1.306	**6.66**	5.966	**6.61**	6.326	**6.51**	5.559	**6.42**
RSJ/Down Corr (TV)	15.92	13.06	1.219	**5.90**	5.018	**7.01**	5.221	**7.07**	4.919	**7.28**
RSJ/Beta (TV)	17.65	13.44	1.313	**6.64**	6.036	**6.81**	6.406	**6.62**	5.826	**7.10**
RSJ/Down Beta (TV)	16.51	13.23	1.248	**5.97**	5.329	**7.32**	5.693	**7.60**	5.241	**7.23**
RSJ/CoSkew (TV)	12.93	12.55	1.030	**3.53**	2.935	**3.73**	3.350	**3.86**	2.909	**3.42**
RSJ/CoKurt (TV)	16.45	12.87	1.278	**6.64**	5.670	**6.54**	5.851	**6.55**	5.374	**6.54**
RSJ/LPM Beta (TV)	17.69	13.31	1.329	**6.76**	6.210	**7.25**	6.644	**7.24**	6.017	**7.24**
RSJ/HTCR Beta (TV)	16.37	12.59	1.300	**6.58**	5.902	**7.79**	6.326	**7.66**	5.478	**7.36**
RSJ/Tail Beta (TV)	17.49	13.88	1.260	**6.28**	5.444	**6.07**	5.851	**6.03**	5.321	**6.13**
RSJ/Tail Sens (TV)	16.70	12.83	1.302	**6.65**	5.920	**6.56**	6.326	**6.46**	5.551	**6.05**
RSJ/Tail Risk (TV)	17.61	13.85	1.272	**6.35**	5.575	**6.01**	6.009	**6.02**	5.345	**6.07**
RSJ/MES (TV)	18.18	13.44	1.353	**6.95**	6.465	**7.98**	6.963	**7.76**	6.266	**7.98**

risk is estimated with conditional parametric models. However, CVaR targeting performs quite poorly when CVaR is estimated unconditionally. Since this paper focuses on non-parametric approaches to estimate risk, using the RV model is appealing since this model quickly adapts to new market environments. In contrast, estimating CVaR based on the non-parametric Historical Simulation (HS) approach does not work well in a portfolio risk management setting, since the HS-CVaR typically adjusts quite slowly to changing market environments. Fourth, Rickenberg (2020a) finds that the benefits of tail risk targeting are lower for industry momentum. Fifth, since our different weighting schemes use many different risk measures and also switch between univariate and systematic tail risk weightings, we do not want to further complicate the risk-managed portfolio strategies and rely on the simplest model to estimate portfolio risk. Sixth, our risk weightings make momentum returns more normal. Tail risk targeting is typically more important for assets that strongly deviate from the normal distribution. For almost normally distributed returns, the differences between volatility and tail risk targeting are only small. As stated above, the left tail risk of industry momentum is much lower than for the stock momentum strategy used by Rickenberg (2020a). This holds especially for the risk weighted portfolios. Nevertheless, as a robustness check, this section examines the CVaR targeting approach of Rickenberg (2020a,b) applied to the industry momentum strategies.

In Table LVIII, we show results for the tail risk targeting strategies that use CVaR estimated with Historical Simulation. The HS estimator for CVaR is also used by Atilgan et al. (2019), Atilgan et al. (2020) and Bi and Zhu (2020). Panel A shows results for the portfolios that use CVaR targeting in every month, whereas Panel B shows the portfolios that switch between volatility and CVaR targeting based on the crash indicator of Section 3.5. Rickenberg (2020a,b) shows that using CVaR targeting in every month is typically too conservative and that switching between volatility and CVaR targeting can further enhance the portfolio's risk-return profile. Since we use the same crash indicator as for our strategies that switch between the univariate and systematic risk weightings, an industry's weight is given by the industry's univariate risk and the portfolio's volatility when no crash is expected ($\delta_t = 0$). In contrast, when a momentum crash in month t is expected ($\delta_t = 1$), the industry's weight is given by the industry's systematic tail risk and the portfolio's CVaR. Table LVIII shows that our main results are also robust when the tail risk targeting overlay is used, where we again find the best results when this approach is combined with the (systematic) tail risk weightings. In particular, we find slightly better results for the strategy that switches between volatility and CVaR targeting. This finding is in line with Rickenberg (2020a,b) who also finds that switching between volatility and CVaR targeting is superior to using CVaR in every month.

Table LVIII. Robustness Results: Tail Risk Targeting Using Historical Simulation
This table shows performance results of risk targeting strategies using momentum's tail risk measured by CVaR estimated with Historical Simulation. The momentum strategy uses 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. Panel A shows results for the strategy that uses CVaR targeting in every month. Panel B shows results for the strategy that switches between volatility and CVaR targeting. The description of the columns is given in Table XX.

	Panel A: CVaR Targeting Using Historical Simulation									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.16	11.92	0.769	-	-	-	-	-	-	-
RV	8.42	10.63	0.792	0.58	0.219	0.49	0.150	0.25	0.203	0.45
Equal (TV)	13.74	12.52	1.097	**5.03**	3.586	**4.97**	4.048	**4.85**	3.346	**4.47**
RV (TV)	15.31	13.38	1.144	**4.72**	4.093	**4.09**	4.516	**4.11**	3.816	**3.87**
RSJ/Corr (TV)	16.07	12.10	1.329	**7.16**	6.123	**7.00**	6.565	**6.63**	5.886	**6.52**
RSJ/Down Corr (TV)	15.63	12.35	1.265	**6.56**	5.427	**7.06**	5.772	**6.91**	5.316	**6.67**
RSJ/Beta (TV)	16.10	12.22	1.317	**7.09**	5.995	**7.21**	6.406	**6.86**	5.800	**6.68**
RSJ/Down Beta (TV)	15.89	12.29	1.293	**6.84**	5.735	**7.28**	6.089	**7.14**	5.606	**6.68**
RSJ/CoSkew (TV)	14.99	12.25	1.224	**5.97**	4.971	**5.91**	5.457	**5.80**	4.770	**5.60**
RSJ/CoKurt (TV)	15.66	12.30	1.273	**6.69**	5.516	**6.82**	5.851	**6.68**	5.375	**6.26**
RSJ/LPM Beta (TV)	16.27	12.18	1.335	**7.26**	6.192	**7.49**	6.644	**7.10**	5.991	**6.89**
RSJ/HTCR Beta (TV)	15.96	12.15	1.314	**6.97**	5.958	**6.92**	6.406	**6.59**	5.742	**6.39**
RSJ/Tail Beta (TV)	15.90	12.30	1.293	**6.94**	5.724	**7.03**	6.168	**6.63**	5.477	**6.49**
RSJ/Tail Sens (TV)	16.04	12.05	1.331	**7.13**	6.147	**7.03**	6.644	**6.61**	5.848	**6.53**
RSJ/Tail Risk (TV)	15.94	12.24	1.302	**7.02**	5.828	**6.99**	6.326	**6.62**	5.575	**6.50**
RSJ/MES (TV)	16.34	12.17	1.342	**7.29**	6.271	**7.51**	6.803	**6.97**	6.053	**7.02**

	Panel B: Volatility and CVaR Targeting Using Historical Simulation									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.16	11.92	0.769	-	-	-	-	-	-	-
RV	8.42	10.63	0.792	0.58	0.219	0.49	0.150	0.25	0.203	0.45
Equal (TV)	13.27	11.96	1.110	**5.75**	3.724	**5.06**	4.126	**5.00**	3.141	**4.39**
RV (TV)	15.12	12.78	1.182	**5.49**	4.517	**4.60**	4.985	**4.63**	3.928	**4.25**
RSJ/Corr (TV)	15.48	11.38	1.360	**7.79**	6.492	**7.66**	6.963	**7.20**	5.932	**6.97**
RSJ/Down Corr (TV)	14.96	11.68	1.281	**7.01**	5.614	**7.45**	5.851	**7.16**	5.220	**6.98**
RSJ/Beta (TV)	15.46	11.49	1.345	**7.70**	6.321	**7.81**	6.724	**7.40**	5.816	**7.09**
RSJ/Down Beta (TV)	15.23	11.59	1.314	**7.39**	5.982	**7.69**	6.326	**7.49**	5.554	**6.95**
RSJ/CoSkew (TV)	14.39	11.62	1.238	**6.44**	5.138	**6.33**	5.536	**6.22**	4.624	**5.88**
RSJ/CoKurt (TV)	15.07	11.60	1.299	**7.25**	5.814	**7.30**	6.089	**7.07**	5.372	**6.58**
RSJ/LPM Beta (TV)	15.59	11.45	1.362	**7.88**	6.511	**8.06**	6.963	**7.61**	5.987	**7.26**
RSJ/HTCR Beta (TV)	15.31	11.42	1.341	**7.58**	6.278	**7.44**	6.724	**7.05**	5.732	**6.71**
RSJ/Tail Beta (TV)	15.29	11.61	1.317	**7.56**	6.009	**7.62**	6.485	**7.13**	5.460	**6.87**
RSJ/Tail Sens (TV)	15.46	11.34	1.363	**7.79**	6.525	**7.73**	7.043	**7.20**	5.890	**6.97**
RSJ/Tail Risk (TV)	15.32	11.55	1.326	**7.63**	6.110	**7.66**	6.565	**7.20**	5.554	**6.93**
RSJ/MES (TV)	15.65	11.45	1.367	**7.88**	6.567	**8.14**	7.043	**7.49**	6.035	**7.47**

In Table LIX, we repeat the examination of Table LVIII, but we estimate CVaR based on the EWMA-FHS approach combined with the SRTR (see Rickenberg (2020a) for further details).

Results are quite similar to the results when CVaR is estimated with Historical Simulation and are also in line with the results when portfolio risk is estimated with the RV model. Further, in line with Rickenberg (2020a,b), we find that using the EWMA-FHS approach produces slightly higher risk-adjusted returns and higher utility gains than the HS approach. Thus, managing a portfolio's risk based on a dynamic risk model is typically advantageous to managing a portfolio's risk with an unconditional risk model.

Table LIX. Robustness Results: Tail Risk Targeting Using Filtered Historical Simulation
This table shows performance results of risk targeting strategies using momentum's tail risk measured by CVaR estimated with Filtered Historical Simulation. The momentum strategy uses 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. Panel A shows results for the strategy that uses CVaR targeting in every month. Panel B shows results for the strategy that switches between volatility and CVaR targeting. The description of the columns is given in Table XX.

	Panel A: CVaR Targeting Using Filtered Historical Simulation									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.16	11.92	0.769	-	-	-	-	-	-	-
RV	8.42	10.63	0.792	0.58	0.219	0.49	0.150	0.25	0.203	0.45
Equal (TV)	13.59	12.11	1.122	**5.37**	3.855	**5.30**	4.282	**5.08**	3.145	**4.31**
RV (TV)	15.84	13.11	1.208	**5.41**	4.794	**4.76**	5.221	**4.76**	4.080	**4.10**
RSJ/Corr (TV)	16.29	11.99	1.359	**7.25**	6.462	**6.87**	6.883	**6.58**	5.920	**6.13**
RSJ/Down Corr (TV)	15.60	12.39	1.259	**6.17**	5.373	**6.27**	5.300	**5.14**	5.087	**6.02**
RSJ/Beta (TV)	16.36	12.06	1.356	**7.27**	6.429	**6.89**	6.883	**6.59**	5.910	**6.16**
RSJ/Down Beta (TV)	15.93	12.20	1.306	**6.73**	5.883	**6.86**	6.089	**6.55**	5.493	**6.15**
RSJ/CoSkew (TV)	15.36	11.93	1.287	**6.54**	5.672	**6.51**	6.089	**6.28**	5.044	**5.78**
RSJ/CoKurt (TV)	15.87	12.24	1.297	**6.67**	5.783	**6.36**	5.930	**5.97**	5.400	**5.78**
RSJ/LPM Beta (TV)	16.38	12.05	1.360	**7.31**	6.466	**7.07**	6.883	**6.81**	5.945	**6.28**
RSJ/HTCR Beta (TV)	16.19	11.97	1.352	**7.17**	6.388	**6.79**	6.883	**6.50**	5.819	**6.01**
RSJ/Tail Beta (TV)	16.22	12.12	1.338	**7.22**	6.225	**7.08**	6.724	**6.71**	5.647	**6.23**
RSJ/Tail Sens (TV)	16.37	11.87	1.379	**7.43**	6.682	**7.29**	7.202	**6.85**	6.034	**6.40**
RSJ/Tail Risk (TV)	16.27	12.11	1.344	**7.28**	6.294	**7.00**	6.803	**6.65**	5.706	**6.19**
RSJ/MES (TV)	16.48	12.00	1.373	**7.42**	6.606	**7.22**	7.122	**6.79**	6.038	**6.45**
	Panel B: Volatility and CVaR Targeting Using Filtered Historical Simulation									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Equal	9.16	11.92	0.769	-	-	-	-	-	-	-
RV	8.42	10.63	0.792	0.58	0.219	0.49	0.150	0.25	0.203	0.45
Equal (TV)	13.44	11.77	1.142	**6.18**	4.081	**5.80**	4.516	**5.59**	3.457	**4.97**
RV (TV)	15.36	12.63	1.216	**5.89**	4.881	**5.00**	5.300	**5.02**	4.199	**4.46**
RSJ/Corr (TV)	15.99	11.63	1.375	**7.97**	6.652	**7.55**	7.043	**7.20**	6.067	**6.68**
RSJ/Down Corr (TV)	15.24	12.04	1.266	**6.72**	5.467	**6.89**	5.300	**5.26**	5.169	**6.67**
RSJ/Beta (TV)	16.08	11.73	1.372	**7.99**	6.607	**7.51**	7.043	**7.17**	6.045	**6.65**
RSJ/Down Beta (TV)	15.59	11.84	1.316	**7.34**	6.006	**7.50**	6.168	**7.07**	5.598	**6.67**
RSJ/CoSkew (TV)	14.78	11.55	1.279	**6.94**	5.595	**6.91**	6.009	**6.69**	4.999	**6.20**
RSJ/CoKurt (TV)	15.51	11.87	1.307	**7.29**	5.903	**6.99**	6.009	**6.45**	5.495	**6.31**
RSJ/LPM Beta (TV)	16.09	11.69	1.376	**8.04**	6.661	**7.76**	7.043	**7.47**	6.100	**6.83**
RSJ/HTCR Beta (TV)	15.86	11.61	1.366	**7.85**	6.547	**7.34**	6.963	**7.02**	5.933	**6.45**
RSJ/Tail Beta (TV)	15.87	11.79	1.346	**7.90**	6.322	**7.68**	6.803	**7.25**	5.739	**6.69**
RSJ/Tail Sens (TV)	16.02	11.50	1.392	**8.12**	6.841	**7.90**	7.362	**7.39**	6.152	**6.90**
RSJ/Tail Risk (TV)	15.96	11.79	1.353	**7.97**	6.404	**7.60**	6.883	**7.20**	5.794	**6.66**
RSJ/MES (TV)	16.20	11.65	1.390	**8.16**	6.813	**7.99**	7.282	**7.46**	6.207	**7.08**

In total, results in this section show that our (systematic) tail risk weighting approach can also be combined with tail risk targeting strategies. As before, the best risk-return profile is found for the strategies that combine the (systematic) tail risk weighting with the tail risk targeting approach. However, using CVaR targeting or the strategy that switches between volatility and CVaR targeting does not further enhance the risk-adjusted performance compared to the volatility targeting approach based on the RV model. As mentioned above, there are several possible explanations for this finding. First, Rickenberg (2020a,b) shows that tail risk targeting works best when risk is managed by conditional and advanced forecasting models. However, this section only shows results for simple models that are easy to implement. Second, Rickenberg (2020a,b) finds that switching between volatility and CVaR targeting works best when volatility is also estimated conditionally. Results in this section are based on strategies that switch between the simple RV model and simple CVaR models. Third, Rickenberg (2020a,b) shows that tail risk targeting works best for portfolios that strongly deviate from normality and have a high left tail risk. As stated above, the industry momentum's left tail risk is significantly lower than the left tail risk of the individual stock based momentum strategy. This holds especially for the (systematic) tail risk weighted strategies, since this weighting approach significantly reduces left tail risk and makes returns "more normal". Thus, differences between tail risk targeting and volatility targeting are expected to be low for industry momentum, especially for the (systematic) tail risk weighted strategies. Similarly, Rickenberg (2020a) also finds that the benefits of targeting the industry momentum portfolio's tail risk are lower than the benefits found for the individual stock based momentum strategy. Nevertheless, results in this section show that our approach is robust to strategies that manage the portfolio's tail risk. Future research could apply the (systematic) tail risk weightings, combined with the tail risk targeting overlay, to the individual stock based momentum portfolio.

B.11 Portfolio Alpha and Spanning Tests

Our performance evaluations were so far based on the strategies' Sharpe Ratio and economic value. Following Daniel and Moskowitz (2016, Sec. 4.4), we next "conduct spanning tests with respect to the other momentum strategies and other factors". A similar approach has also been used by Moreira and Muir (2017) to assess the profitability of volatility managed portfolios. The performance evaluation based on the strategies' alpha has several disadvantages (Boguth et al., 2011, Cederburg and O'Doherty, 2016, Cederburg et al., 2020, Schneider et al., 2020). For example, Boguth et al. (2011) and Cederburg and O'Doherty (2016) state that the unconditional alpha does not account for volatility timing that is an important component of dynamic trading strategies as examined in this paper. Further, Schneider et al. (2020) show that the usual alpha does not account for the (co)skewness preferences of investors. For these reasons, the economic value approach used in the main part is more realistic and more powerful than the simple portfolio alpha. However, due to the importance of portfolio alphas in the financial literate, we show in Table LX annualized percentage alphas and the corresponding t-statistics for several spanning tests. We calculate the alpha with respect to the CAPM, the Fama and French (1993) three factor model and the Carhart (1997) four factor model. In order to consider the observation that low risk strategies are typically highly exposed to the investment and profitability factor, we also regressed the returns on the Fama and French (2016) five factor model. Since results were quite similar and the returns of the five factor model are only available from 1963 onwards, these results are omitted. Further, to control for the performance of the different weighting schemes, we follow Daniel and Moskowitz (2016, Table 8) and regress the strategies' returns on the four factor model expanded by the returns of the remaining momentum strategies. For example, we regress the (systematic) tail risk weighted strategy's returns on the four factor model expanded by the returns of the equally weighted strategy's returns. Thus, we control for market, size, value, stock momentum and industry momentum. By doing this, we extract the value of the (systematic) tail risk weighting by controlling for the performance of well established factors that include the stock momentum factor and the performance of the equally weighted industry momentum strategy. The same procedure is repeated for the remaining weighting schemes. Before running the regressions, we follow Daniel and Moskowitz (2016) and Moreira and Muir (2017) and rescale all strategies to the same level of volatility.

In Panel A, we show results for the strategies without volatility targeting. Thus, Panel A contains the risk-adjusted performance of the different weighting schemes and shows if the performance of one strategy is captured by the performance of a strategy with another weighting scheme or other factors. We find that classical factor models cannot explain the returns of indus-

Table LX. Robustness Results: Spanning Tests

This table shows portfolio alphas and t-statistics for the non-managed and risk-managed industry momentum portfolios using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p = 30\%$. The portfolio alphas are the annualized and percentage intercepts of a regression of the industry momentum returns using different weighting schemes on several other portfolios. t-statistics are given in parentheses. Panel A shows results for the strategies without volatility targeting, whereas Panels B and C use the strategies that additionally use volatility targeting. Returns are regressed on the CAPM, the Fama and French (1993) three factor model (FF3) or the Carhart (1997) four factor model (FF4). The four factor model is further extended by including returns of the non-managed and risk-managed industry momentum strategies. Equal stands for the industry momentum strategy using equal-weights, Vol stands for the industry momentum strategy using volatility weights, whereas Tail is the strategy that uses the (systematic) tail risk weighting. Rem means that the remaining two weighting schemes are included in the regression. (TV) means that a strategy additionally uses the target volatility overlay. Following Daniel and Moskowitz (2016) and Moreira and Muir (2017), we rescale all strategies to the same annualized volatility of 19% before running the regressions. Alphas with a corresponding t-statistic that is higher than 1.96 are given in bold. Alphas with a corresponding t-statistic that is smaller than -1.96 are given in red.

	Panel A: Without Volatility Targeting						
Model	CAPM	FF3	FF4	FF4 + Equal	FF4 + Vol	FF4 + Tail	FF4 + Rem
Equal	**17.679**	**18.868**	**7.387**	-	1.248	**-2.737**	**-1.607**
	(8.475)	(9.004)	(5.064)	-	(1.462)	(-3.003)	(-2.149)
RV	**17.977**	**18.997**	**6.781**	0.998	-	**-2.284**	-0.883
	(8.812)	(9.316)	(4.913)	(1.280)	-	(-2.516)	(-1.073)
Tail Risk	**24.365**	**25.067**	**13.574**	**6.523**	**6.882**	-	**6.103**
	(10.614)	(11.014)	(8.721)	(6.882)	(7.108)	-	(6.892)
	Panel B: With Volatility Targeting						
Model	CAPM	FF3	FF4	FF4 + Equal	FF4 + Vol	FF4 + Tail	FF4 + Rem
Equal (TV)	**24.134**	**24.879**	**14.233**	**6.878**	**8.014**	**3.088**	**3.807**
	(9.702)	(9.983)	(7.256)	(5.936)	(5.169)	(2.070)	(2.721)
RV (TV)	**25.525**	**26.097**	**15.071**	**8.872**	**8.051**	**4.926**	**6.141**
	(9.823)	(10.110)	(7.699)	(6.424)	(5.570)	(3.222)	(4.257)
Tail Risk (TV)	**29.376**	**29.926**	**19.605**	**12.545**	**13.140**	**5.552**	**12.243**
	(11.550)	(11.780)	(9.871)	(8.821)	(8.345)	(4.455)	(8.599)
	Panel C: With Volatility Targeting and Controlling for Volatility Targeting						
Model	FF4 + Equal(TV)	FF4 + Vol(TV)	FF4 + Tail(TV)	FF4 + Rem(TV)	FF4 + Rem + Equal(TV)	FF4 + Rem + Vol(TV)	FF4 + Rem + Tail(TV)
Equal (TV)	-	0.758	**-1.862**	**-1.994**	-	**-1.871**	**-1.393**
	-	(1.027)	(-2.318)	(-3.166)	-	(-2.845)	(-2.015)
RV (TV)	**2.480**	-	0.275	**1.689**	1.423	-	1.220
	(3.313)	-	(0.288)	(2.259)	(1.925)	-	(1.451)
Tail Risk (TV)	**5.845**	**6.081**	-	**5.453**	**5.627**	**5.791**	-
	(6.859)	(6.027)	-	(6.519)	(6.827)	(6.182)	-

try momentum, regardless of the used weighting scheme. Annualized CAPM alphas range from 17.679% of the equally weighted momentum strategy to 24.365% of the (systematic) tail risk weighted strategy with corresponding t-statistics of 8.475 and 10.614. In particular, we find that industry momentum cannot be explained by stock momentum as can be seen by the high and statistically significant four factor alphas. This finding is in line with the results of Moskowitz and Grinblatt (1999) who also find that industry momentum is not subsumed by stock momentum. Interestingly, the alphas of the equally and volatility weighted momentum strategies are not significantly positive once it is controlled for the returns of another momentum strategy. In particular, when we control for the returns of the (systematic) tail risk weighted strategy, the portfolio alphas become even significantly negative with t-statistics of -2.737 and -2.284 for the equally and volatility weighted strategy, respectively. In contrast, the (systematic) tail risk weighted strategy's performance is not captured by the equally and volatility weighted strategies. Even when we control for both strategies simultaneously, the alpha of 6.103% is still economically high with a t-statistics of 6.892. In Panels B and C, we show results for the strategies that additionally use volatility targeting. Panel B shows results when it is controlled for the same portfolios as in Panel A. Thus, in this case, we do not control for the performance of the volatility targeting strategy, but for the industry momentum strategies using the different weighting schemes. In line with our previous findings, all strategies exhibit statistically significant and highly positive alphas, regardless of the weighting scheme. The highest alphas are obtained for the strategies that use the (systematic) tail risk weighting. This finding is in line with our previous findings that managing portfolio risk is more important than managing individual asset risk and that the best risk-return profile is found for the strategies that combine volatility targeting with the (systematic) tail risk weighting. Furthermore, results in Panel B are also in line with the findings of Daniel and Moskowitz (2016) and Moreira and Muir (2017) for the individual stock based momentum strategy. For example, Moreira and Muir (2017, Table 1) find an alpha of 10.52% for the volatility targeted momentum strategy with respect to the four factor model and Daniel and Moskowitz (2016, Table 8) find an alpha of 14.27% for the volatility targeted momentum strategy with respect to the market and the non-managed momentum strategy. We find alphas of 14.233% and 6.878% when we control for the stock momentum strategy or the stock and industry momentum strategies, respectively. In Panel C, we show results when we additionally control for the strategies that use volatility targeting. The strategy that uses equal-weights does not produce a significantly positive alpha in any case. Thus, the performance of this strategy can be explained by the performance of the remaining strategies. Similarly, the alpha of the volatility weighted strategy with volatility targeting is mostly insignificant, once we control for the (systematic) tail risk weighted strategy. In contrast, the strategy that uses the (systematic) tail risk weighting combined with volatility targeting produces economically high and statistically significant alphas in all cases with t-statistics higher than 6.027. Thus, combining the (systematic) tail risk weighting with volatility targeting produces returns that cannot be explained by volatility targeting alone. In total, results of Table LX support our earlier findings and highlight that the (systematic) tail risk weighting is superior to the remaining weighting schemes. In particular, the combination of the (systematic) tail risk weighted strategies with the volatility targeting approach produces statistically significant and economically large alphas.

B.12 Portfolio Performance: Additional Benchmarks

Our main results show that managing portfolio risk is more important than managing individual asset risk and that the combination of the (systematic) tail risk weighting with the volatility targeting approach produces the best risk-return profile. The previous section shows that the (systematic) risk weighting, combined with volatility targeting, is superior to the remaining strategies, even when we control for the performance of volatility targeting combined with the equal and volatility weighting. This has been shown by regressions that control for the impact of volatility targeting and other weighting schemes. Another method to assess the benefits of combining the (systematic) tail risk weighting with the target volatility approach is to calculate the Jobson and Korkie (1981) and Diebold and Mariano (1995) tests with respect to other benchmark portfolios. Our main results calculate these tests with respect to the equally weighted portfolio without volatility targeting, i.e. we test if the (systematic) tail risk weighted strategies, with or without volatility targeting, produce significantly higher Sharpe Ratios and utilities than the strategy that ignores all kinds of risk. In this section, we repeat this examination for other benchmark strategies that also use volatility targeting, i.e. we assess if the (systematic) tail risk weighted strategies with volatility targeting produce significantly higher Sharpe Ratios and utilities than other weighting schemes that also use volatility targeting.

Table LXI shows additional performance results for the strategies that use the target volatility overlay, where the Jobson and Korkie (1981) and Diebold and Mariano (1995) tests are calculated with respect to two other benchmarks. In Panel A, we calculate the tests with respect

Table LXI. Robustness Results: Performance Results With Alternative Benchmarks
This table shows performance results of risk targeting strategies for the momentum strategy using 30 equally weighted US industries, the $t-12$ to $t-1$ ranking period and a cut-off point of $p=30\%$. The description of the columns is given in Table XX, but we use other benchmarks to calculate the Jobson and Korkie (1981) and Diebold and Mariano (1995) tests. Panel A shows results for the tests that use the equally weighted portfolio with volatility targeting as benchmark. Panel B shows results for the tests that use the volatility weighted portfolio with volatility targeting as benchmark.

	Panel A: Equally Weighted Portfolio With Target Volatility Overlay As Benchmark									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Eq	9.13	11.98	0.763	**-6.23**	-3.921	**-5.82**	-4.336	**-5.62**	-3.455	**-5.12**
RV	8.37	10.70	0.782	**-4.83**	-3.754	**-5.04**	-4.264	**-4.87**	-3.271	**-4.18**
Eq (TV)	13.35	11.79	1.133	-	-	-	-	-	-	-
RV (TV)	15.28	12.64	1.209	1.58	0.807	1.70	0.828	1.73	0.834	1.73
RSJ/Corr (TV)	15.56	11.29	1.378	**4.89**	2.627	**4.37**	2.657	**4.33**	2.674	**4.33**
RSJ/Down Corr (TV)	15.12	11.50	1.315	3.73	1.958	3.18	1.891	**2.80**	2.082	**3.43**
RSJ/Beta (TV)	15.57	11.41	1.364	**4.65**	2.480	**4.10**	2.503	**4.03**	2.573	**4.08**
RSJ/Down Beta (TV)	15.41	11.43	1.348	**4.39**	2.309	**3.63**	2.273	**3.44**	2.412	**3.70**
RSJ/CoSkew (TV)	14.52	11.46	1.267	**2.92**	1.435	**2.86**	1.434	**2.84**	1.524	**2.95**
RSJ/CoKurt (TV)	15.14	11.46	1.322	**3.85**	2.029	**3.19**	1.968	**2.92**	2.129	**3.34**
RSJ/LPM Beta (TV)	15.72	11.31	1.390	**5.17**	2.758	**4.44**	2.811	**4.42**	2.804	**4.35**
RSJ/HTCR Beta (TV)	15.40	11.29	1.364	**4.64**	2.477	**4.10**	2.503	**4.10**	2.500	**3.96**
RSJ/Tail Beta (TV)	15.43	11.52	1.339	**4.35**	2.206	**4.15**	2.273	**4.13**	2.249	**3.99**
RSJ/Tail Sens (TV)	15.56	11.24	1.385	**5.07**	2.701	**4.66**	2.734	**4.65**	2.675	**4.52**
RSJ/Tail Risk (TV)	15.50	11.47	1.351	**4.57**	2.339	**4.37**	2.427	**4.35**	2.384	**4.14**
RSJ/MES (TV)	15.77	11.33	1.392	**5.16**	2.780	**4.72**	2.888	**4.67**	2.843	**4.61**

	Panel B: Volatility Weighted Portfolio With Target Volatility Overlay As Benchmark									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Eq	9.13	11.98	0.763	**-5.96**	-4.693	**-5.14**	-5.125	**-5.16**	-4.289	**-4.73**
RV	8.37	10.70	0.782	**-6.82**	-4.528	**-5.73**	-5.054	**-5.47**	-4.109	**-5.19**
Eq (TV)	13.35	11.79	1.133	-1.58	-0.801	-1.70	-0.822	-1.73	-0.836	-1.73
RV (TV)	15.28	12.64	1.209	-	-	-	-	-	-	-
RSJ/Corr (TV)	15.56	11.29	1.378	**2.66**	1.807	**2.75**	1.815	**2.70**	1.821	**2.82**
RSJ/Down Corr (TV)	15.12	11.50	1.315	1.68	1.142	1.66	1.055	1.39	1.228	1.85
RSJ/Beta (TV)	15.57	11.41	1.364	**2.49**	1.661	**2.48**	1.663	**2.41**	1.715	**2.59**
RSJ/Down Beta (TV)	15.41	11.43	1.348	**2.23**	1.491	**2.17**	1.434	**2.00**	1.553	**2.32**
RSJ/CoSkew (TV)	14.52	11.46	1.267	0.91	0.624	1.00	0.602	0.98	0.676	1.05
RSJ/CoKurt (TV)	15.14	11.46	1.322	1.77	1.213	1.79	1.131	1.56	1.276	1.95
RSJ/LPM Beta (TV)	15.72	11.31	1.390	**2.88**	1.936	**2.81**	1.968	**2.77**	1.947	**2.87**
RSJ/HTCR Beta (TV)	15.40	11.29	1.364	**2.43**	1.657	**2.51**	1.663	**2.50**	1.643	**2.49**
RSJ/Tail Beta (TV)	15.43	11.52	1.339	**2.21**	1.389	**2.41**	1.434	**2.36**	1.394	**2.44**
RSJ/Tail Sens (TV)	15.56	11.24	1.385	**2.80**	1.880	**2.88**	1.891	**2.87**	1.821	**2.87**
RSJ/Tail Risk (TV)	15.50	11.47	1.351	**2.40**	1.520	**2.53**	1.586	**2.50**	1.532	**2.56**
RSJ/MES (TV)	15.77	11.33	1.392	**2.90**	1.958	**2.87**	2.044	**2.85**	1.985	**2.94**

to the equally weighted portfolio with target volatility overlay, i.e. we control for the performance of the volatility targeting approach but ignore the risks of the individual assets. In Panel B, we use the volatility weighted strategy with target volatility overlay as benchmark, i.e. we control for volatility managing at the individual asset and portfolio level. Results in Table LXI confirm the findings of the previous section. The (systematic) tail risk weightings outperform the other two weighting schemes, even when we control for the impact of the target volatility overlay. Thus, although volatility targeting is more important than risk weighting, results in

Table LXI demonstrate that our (systematic) tail risk weighting approach still produces economically high and statistically significant performance gains, even when these weighting schemes are combined with the target volatility strategy. Thus, investors who time market risk by the target volatility strategy should additionally readjust their asset allocation based on the assets' (systematic) tail risk.

B.13 Long-Only Strategies

In this section, we follow Korajczyk and Sadka (2004) and Clare et al. (2016, Table 5) and focus on the long-only strategy that invests in the (risk-managed) winners portfolio. Focusing on the winners portfolio is advantageous from a practical view since many investors have short-sale constraints and shorting the losers portfolio typically generates high transaction costs (Korajczyk and Sadka, 2004, Lesmond et al., 2004). Thus, long-only portfolios are more relevant for practitioners and most anomalies are investable via "smart beta" ETFs (Blitz et al., 2019). Further, the profitability of industry momentum strategies is mainly driven by the winners portfolio (Behr et al., 2012, Moskowitz and Grinblatt, 1999, O'Neal, 2000).[151] We compare our (systematic) tail risk weighted strategy with the frequently used mean-variance and minimum variance approaches as well as with the equally weighted strategy using all industries (DeMiguel et al., 2009a,b, Jagannathan and Ma, 2003, Kirby and Ostdiek, 2012, Zakamulin, 2015). As in Kirby and Ostdiek (2012), we estimate the mean-variance portfolio in a way that the equally weighted portfolio's mean is targeted. Kirby and Ostdiek (2012) show that this strategy produces more reliable and less extreme portfolio weights than the method used by DeMiguel et al. (2009b). Results for these strategies are shown in Table LXII where we use the 49 equally weighted US industry portfolios. The Jobson and Korkie (1981) test statistic and the economic value are calculated with respect to the mean-variance portfolio.

Table LXII shows that using the equally weighted 49 industries clearly underperforms all other weighting schemes that incorporate the industries' risk. This is in line with Harvey et al.

[151] This results also holds for the country momentum strategy, i.e. the profitability of country momentum is mainly driven by the winners portfolio (Bhojraj and Swaminathan, 2006, Chan et al., 2000). Interestingly, this observation is opposed to the individual stock based momentum strategy, where the profitability is mainly driven by shorting (small and illiquid) loser stocks (Hong et al., 2000, Lesmond et al., 2004).

Table LXII. Performance of Long-Only Portfolio Strategies Using 49 Industries

This table shows performance results of the equally weighted, mean-variance and minimum variance strategies applied to 49 equally weighted US industries as well as the equally weighted and risk weighted winners portfolios using the $t-12$ to $t-1$ ranking period and $p = 30\%$. The description of the columns is given in Table XX.

	Performance Results: Long-Only Portfolios									
Model	Return	Volatility	SR	z_{JK}	$\Delta_{MV}^{\gamma=5}$	DM-test	$\Delta_{CRRA}^{\gamma=5}$	DM-test	$\Delta_{LA}^{l=2}$	DM-test
Mean-Variance (All)	14.39	20.67	0.696	-	-	-	-	-	-	-
Minimum-Variance (All)	13.73	19.54	0.703	0.49	0.057	0.11	0.000	-0.07	0.099	0.23
Equal (All)	13.48	24.49	0.550	**-6.35**	-2.383	**-3.69**	-2.374	**-4.36**	-2.754	**-5.25**
Equal (Winners)	18.87	24.04	0.785	**2.08**	1.822	**2.08**	1.586	**2.06**	0.997	1.29
RV (Winners)	18.78	22.67	0.828	**3.53**	2.528	**3.26**	2.273	**3.20**	1.703	**2.46**
RSJ/Corr (Winners)	20.51	23.43	0.875	**4.51**	3.426	**3.41**	3.042	**3.61**	2.391	**3.06**
RSJ/Down Corr (Winners)	20.58	23.54	0.874	**4.42**	3.400	**3.42**	3.042	**3.55**	2.473	**3.02**
RSJ/Beta (Winners)	20.39	23.56	0.865	**4.34**	3.253	**3.21**	2.888	**3.36**	2.257	**2.79**
RSJ/Down Beta (Winners)	20.35	23.54	0.865	**4.30**	3.234	**3.18**	2.888	**3.31**	2.316	**2.80**
RSJ/CoSkew (Winners)	20.37	24.58	0.829	**2.96**	2.603	**2.92**	2.580	**3.05**	2.097	**2.27**
RSJ/CoKurt (Winners)	20.45	23.47	0.871	**4.32**	3.355	**3.30**	2.965	**3.53**	2.309	**2.95**
RSJ/LPM Beta (Winners)	20.50	23.65	0.867	**4.39**	3.276	**3.16**	2.965	**3.32**	2.363	**2.82**
RSJ/HTCR Beta (Winners)	20.57	23.63	0.870	**4.40**	3.345	**3.17**	2.888	**3.37**	2.353	**2.86**
RSJ/Tail Beta (Winners)	20.12	23.07	0.872	**4.39**	3.346	**3.44**	3.042	**3.53**	2.365	**2.89**
RSJ/Tail Sens (Winners)	20.41	23.26	0.878	**4.43**	3.467	**3.13**	3.042	**3.28**	2.397	**2.81**
RSJ/Tail Risk (Winners)	20.09	23.09	0.870	**4.30**	3.307	**3.29**	2.965	**3.37**	2.343	**2.74**
RSJ/MES (Winners)	20.44	23.51	0.869	**4.37**	3.321	**2.99**	2.965	**3.14**	2.333	**2.65**

(2018, Exhibit 18) who find a negative risk-return relation for industries. We further find only minor differences between the mean-variance and minimum variance approach. A possible explanation for the small difference between both approaches could be the use of industry portfolios instead of individual stocks, which reduces estimation risk of the mean return. Further, differences among industries' mean returns are typically small, which reduces the influence of incorporating mean returns in the portfolio optimization process. Moskowitz and Grinblatt (1999, p. 1251) also find "little cross-sectional variation in our industry sample means" (see also Pan et al. (2004, Table 1)). Interestingly, buying only the winner industries instead of all 49 industries significantly enhances the risk-adjusted and raw return. Even the equally weighted winners portfolio clearly outperforms the mean-variance and minimum variance portfolios. Thus, as argued in Section 3.2.4, incorporating an estimate of the assets' *relative* mean is advantageous compared to approaches that use an absolute mean estimate or totally ignore any information on the assets' performance. Furthermore, the equally weighted winners portfolio is clearly outperformed by the risk weighted winners portfolios, where the best risk-return profile is obtained by the (systematic) tail risk weighted winners. However, the volatility weighted winners portfolio only slightly underperforms the (systematic) tail risk weighted winners portfolios. In line with Table VII, this again highlights that volatility weighting is an appealing approach to manage a long-only portfolio's risk as frequently shown in the literature (Fleming et al., 2001, 2003, Han, 2005, Kirby and Ostdiek, 2012). However, the benefits of volatility weighting do not translate to long-short strategies, since volatility is a symmetric risk measure and also enhances the performance of the short leg. Furthermore, the good performance of the risk weighted winners portfolios for the industry data set is striking, since earlier studies find that risk based portfolio allocations applied to industries are difficult to use (Kirby and Ostdiek, 2012, Zakamulin, 2015). For example, Zakamulin (2015, p. 96) use an inverse volatility weighting scheme combined with a volatility targeting approach applied to industry portfolios and conclude that "this dataset is notorious for being very difficult to use in portfolio optimization [...] and moreover, to date, no asset pricing model can explain the cross-section of returns on industry-sorted portfolios." Chang et al. (2013, Sec. 6) also find difficulties in using industry portfolios in an asset pricing context. Similarly, Kritzman et al. (2010, p. 35) state that industry portfolios are "notorious for the exceptional performance of the [equally weighted] portfolio".

Figure V further visualizes the differences between the approaches that focus on an absolute, relative or no mean estimate, i.e. Figure V shows the cumulative return of the equally weighted, mean-variance and minimum variance portfolios using all 49 industries as well as a (systematic) tail risk weighted winners portfolio. All strategies are rescaled to the same level of volatility. The equally weighted strategy using all 49 industries, which does not incorporate any information on the assets' risk or return, is clearly outperformed by the mean-variance and minimum variance approach. However, all strategies are clearly outperformed by the risk weighted winners portfolio. This outperformance is quite steady over time and is not driven by a single period. Thus, buying risk-managed winners is an appealing long-only portfolio strategy in bull and bear markets and is a promising alternative to more complex portfolio optimization methods.

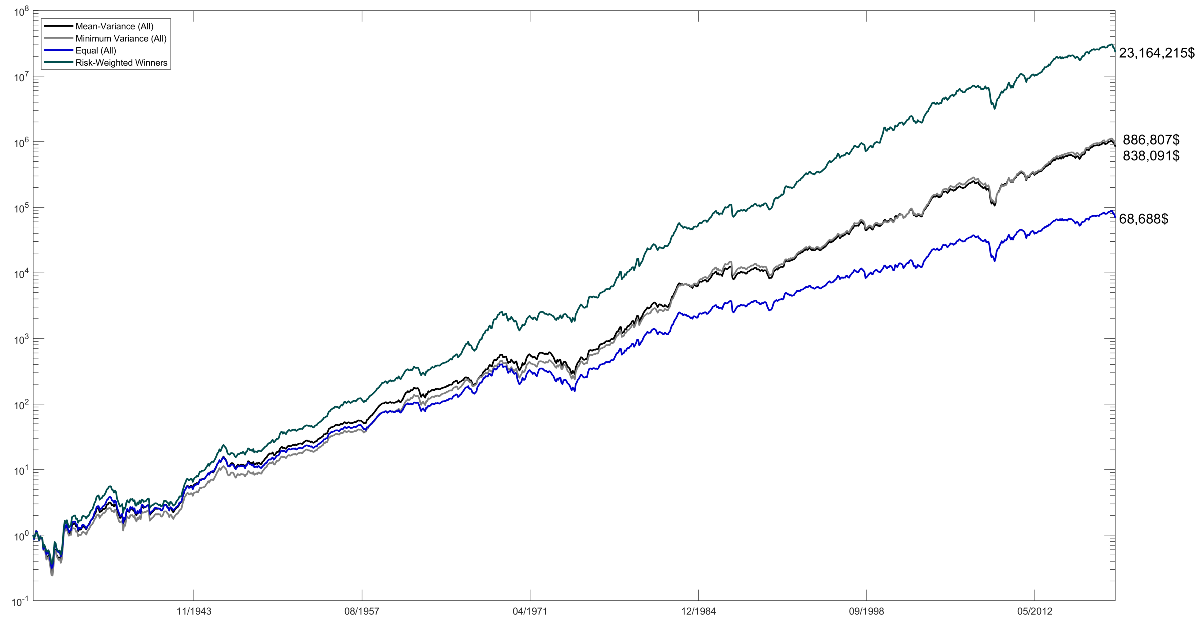

Figure V. Performance of Long-Only Portfolios. This figure plots the cumulative return of a one dollar investment in four long-only investment strategies using 49 equally weighted US industries. The four investment strategies are the equally weighted portfolio, the mean-variance portfolio and the minimum variance portfolio that invest in all 49 industries as well as the risk weighted winners portfolio. All strategies are rescaled to the same level of volatility.

C Estimation of Risk Measures

In this section, we present the estimation of the risk measures used in the empirical part. We estimate all risk measures using daily data. Estimating monthly risk with daily data is important to increase the estimation accuracy as shown by several studies. For example, DeMiguel et al. (2009b, Sec. 4) show that huge amounts of data should be used in order to provide good results of the minimum variance portfolio. Thus, using daily data to estimate monthly risk is important for risk-based portfolio allocations (Merton, 1980). Other approaches to estimate monthly risk would be to estimate risk based on monthly data (Agarwal et al., 2017, DeMiguel et al., 2009b), shrinkage estimators or other Bayesian approaches (DeMiguel et al., 2009a, Jagannathan and Ma, 2003), the use of high-frequency data (Amaya et al., 2015, Bollerslev et al., 2018, 2019, 2020, Fleming et al., 2003, Patton and Sheppard, 2015), estimation based on lagged risk measures and other information like momentum (Boguth et al., 2011, Cederburg and O'Doherty, 2016, Chen et al., 2001, Langlois, 2020) or the use of more complex conditional models (Bali et al., 2017b, Brownlees and Engle, 2016, Engle et al., 2015, Fu, 2009, Rickenberg, 2020a).[152] We concentrate in this paper on simple daily data based estimators of monthly risk, since these estimates are easy to implement and are therefore interesting for practitioners. Using daily data to estimate monthly risk is frequently done in the literature on asset pricing (Ang et al., 2006a, Boguth et al., 2011, Cederburg and O'Doherty, 2016, Langlois, 2020, Van Oordt and Zhou, 2016) and portfolio selection (Asness et al., 2014, DeMiguel et al., 2009a, Jagannathan and Ma, 2003, Kirby and Ostdiek, 2012). Further, since our strategies rely on assets that are portfolios themselves, sampling error of our simple non-parametric estimators is less important for this data set (Jagannathan and Ma, 2003, p. 1654). This does especially hold for the rank weighting, which is the main focus of this paper, since an asset's cross-sectional risk rank can be estimated more precisely than an asset's risk (Langlois, 2020). In particular, Jagannathan and Ma (2003) find that sample estimators of the monthly covariance matrix based on daily data perform equally well in portfolio allocations as more advanced estimation procedures.

In Section C.1, we show how univariate risk measures are estimated, whereas Section C.2 shows the estimation of systematic risk measures. For the estimation of month t univariate risk, we consider samples $\{r_{i,t-1-\frac{d-1}{h}}\}_{d=1}^{Th}$ of realized returns for asset i that are available at the end of month $t-1$. T denotes the number of months used to estimate the risk measures, $h = 21$ denotes the number of days per month and $r_{i,t-1-\frac{d-1}{h}}$ denotes the realized daily return of asset i on day $t-1-\frac{d-1}{h}$, where $r_{i,t-1}$ denotes the last daily return of month $t-1$. Thus, for $1 \leqslant d \leqslant 21$, $r_{i,t-1-\frac{d-1}{h}}$ denotes the daily return of asset i on day $h-d+1$ of month $t-1$. For an asset in the losers portfolio, $r_{i,t-1-\frac{d-1}{h}}$ is defined as the negative return of asset i on day $t-1-\frac{d-1}{h}$. For the estimation of systematic risk measures, we also consider realized daily returns $\{r_{mom,t-1-\frac{d-1}{h}}\}_{d=1}^{Th}$ of the equally weighted momentum portfolio. We estimate univariate and systematic risk based on past returns of an industry and the momentum portfolio. Alternatively, risk of an industry could also be estimated by first estimating the risks of all

[152] As in Frazzini and Pedersen (2014) and Asness et al. (2020) we also used simple shrinkage estimators for the different beta estimates. However, these estimators do not alter the weights of the ranking based approach and results of the inverse risk weighting were similar, regardless of whether the shrinked or non-shrinked betas were used. Alternatively, Jagannathan and Ma (2003, Appendix B) show how beta can be estimated by incorporating microstructure effects. However, the authors find no sizeable benefit of using this approach compared to the sample estimator using daily data.

assets within this industry. The risk of the industry is then estimated as the average risk of the constituents' risk (Boyer et al., 2009, Cederburg and O'Doherty, 2016, Chen and Petkova, 2012, Jondeau et al., 2019, Langlois, 2020).

C.1 Univariate Risk Measures

This section shortly lists the non-parametric estimation of univariate risk measures using the past T months of daily data.[153]

Volatility: Following Barroso and Santa-Clara (2015), Moreira and Muir (2017) and Grobys et al. (2018), we estimate month t volatility using the simple Realized Volatility estimator that is given by

$$\hat{\sigma}_{i,t,T} = \sqrt{\frac{1}{T}\sum_{d=1}^{Th} r^2_{i,t-1-\frac{d-1}{h}}}, \tag{C.1}$$

where T denotes the number of months used to estimate volatility of month t.

Skewness: Following Amaya et al. (2015, Eq. (3)), Chen et al. (2001, Eq. (1)), Bollerslev et al. (2019, Eq. (7)) and Jiang et al. (2020, Eq. (B-5)), the skewness of month t using one month of daily data is estimated by

$$\widehat{Skew}_{i,t,1} = \frac{\sqrt{h}\sum_{d=1}^{h} r^3_{i,t-1-\frac{d-1}{h}}}{\hat{\sigma}^3_{i,t,1}}, \tag{C.2}$$

where $\hat{\sigma}_{i,t,1}$ is the simple Realized Volatility of Equation (C.1). Further, following Amaya et al. (2015, Eq. (6)) and Bollerslev et al. (2019, Eq. (10)), the skewness estimate based on T months of daily data is given by

$$\widehat{Skew}_{i,t,T} = \frac{1}{T}\sum_{j=0}^{T-1} \widehat{Skew}_{i,t-j,1}. \tag{C.3}$$

The estimate for month t risk is then given by $\widehat{\mathcal{R}}_{i,t} = -1 \cdot \widehat{Skew}_{i,t,T}$.

Kurtosis: Following Amaya et al. (2015, Eq. (4)) and Bollerslev et al. (2019, Eq. (8)), the kurtosis of month t using one month of daily data is estimated by

$$\widehat{Kurt}_{i,t,1} = \frac{h\sum_{d=1}^{h} r^4_{i,t-1-\frac{d-1}{h}}}{\hat{\sigma}^4_{i,t,1}}, \tag{C.4}$$

[153] Instead of estimating an industry's univariate risk by using the industry's past T months of daily data, an industry's risk could also be estimated by using all daily data of all assets in this industry within month $t-1$ (Allen et al., 2012, Karagiannis and Tolikas, 2019, Kelly and Jiang, 2014).

where $\hat{\sigma}_{i,t,1}$ is the simple Realized Volatility of Equation (C.1). Further, following Amaya et al. (2015, Eq. (7)) and Bollerslev et al. (2019, Eq. (10)), the kurtosis estimate based on T months of daily data is given by

$$\widehat{Kurt}_{i,t,T} = \frac{1}{T} \sum_{j=0}^{T-1} \widehat{Kurt}_{i,t-j,1}. \tag{C.5}$$

LPM: Following Bali et al. (2014) and Price et al. (1982), we estimate month t LPM as

$$\widehat{\text{LPM}}_{i,t,k,T} = \frac{1}{Th} \sum_{d=1}^{Th} (q - r_{i,t-1-\frac{d-1}{h}})^k \cdot \mathbb{1}_{\{r_{i,t-1-\frac{d-1}{h}} < q\}}, \tag{C.6}$$

where k and q denote the order and the threshold, respectively.

VaR: As in Allen et al. (2012), Atilgan et al. (2020), Bali et al. (2009), Rachev et al. (2007) and Van Oordt and Zhou (2016), we estimate VaR for month t as the empirical quantile of the past T months' realized losses. We define by $\{l_{i,t-1-\frac{d-1}{h}}\}_{d=1}^{Th} := \{-r_{i,t-1-\frac{d-1}{h}}\}_{t=1}^{Th}$ the loss of asset i on day $t-1-\frac{d-1}{h}$ and denote by $\{l_{i,t-1,(d)}\}_{d=1}^{Th}$ the order statistics of $\{l_{i,t-1-\frac{d-1}{h}}\}_{d=1}^{Th}$, i.e. $l_{i,t-1,(1)} \leqslant ... \leqslant l_{i,t-1,(Th)}$. VaR for month t is then given by

$$\widehat{\text{VaR}}_{i,t,T}^{\alpha} = \sqrt{h} \cdot l_{i,t-1,(\lceil Th(1-\alpha) \rceil)}, \tag{C.7}$$

where α denotes the chosen significance level.[154]

CVaR: As in Allen et al. (2012, Eq. (9)), Bali et al. (2009), Rachev et al. (2007) and Agarwal et al. (2017), we estimate CVaR for month t as

$$\widehat{\text{CVaR}}_{i,t,T}^{\alpha} = \sqrt{h} \cdot \frac{1}{Th - \lceil Th(1-\alpha) \rceil + 1} \cdot \sum_{d=\lceil Th(1-\alpha) \rceil}^{Th} l_{i,t-1,(d)}, \tag{C.8}$$

where α denotes the chosen significance level.

SJ: Following Bollerslev et al. (2019, Eq. (5)), we estimate SJ by

$$\widehat{SJ}_{i,t,T} = \sum_{d=1}^{Th} r^2_{i,t-1-\frac{d-1}{h}} \cdot \mathbb{1}_{\{r_{i,t-1-\frac{d-1}{h}} < 0\}} - \sum_{d=1}^{Th} r^2_{i,t-1-\frac{d-1}{h}} \cdot \mathbb{1}_{\{r_{i,t-1-\frac{d-1}{h}} > 0\}}. \tag{C.9}$$

[154]The multiplication with $\sqrt{h}$ guarantees that VaR is defined as a measure of asset i's monthly tail risk. However, scaling VaR by a constant does not change our weightings.

RSJ: Following Bollerslev et al. (2019, Eq. (6)), we estimate RSJ by

$$\widehat{RSJ}_{i,t,T} = \frac{\widehat{SJ}_{i,t,T}}{RV_{i,t,T}}, \tag{C.10}$$

where $RV_{i,t,T} = \sum_{d=1}^{Th} r^2_{i,t-1-\frac{d-1}{h}}$ denotes the realized variance over T months.

DuVol: Following Chen et al. (2001, Eq. (2)), we estimate the down-to-up volatility (DuVol) of month t as

$$\widehat{DuVol}_{i,t,T} = \frac{\frac{1}{n_d-1}\sum_{d=1}^{Th} r^2_{i,t-1-\frac{d-1}{h}} \cdot \mathbb{1}_{\{r_{i,t-1-\frac{d-1}{h}} < q_d\}}}{\frac{1}{n_u-1}\sum_{d=1}^{Th} r^2_{i,t-1-\frac{d-1}{h}} \cdot \mathbb{1}_{\{r_{i,t-1-\frac{d-1}{h}} > q_u\}}}, \tag{C.11}$$

where q_d and q_u denote the thresholds that mark up- and down-days and $n_d := |\{d = 1, ..., Th : r_{i,t-1-\frac{d-1}{h}} < q_d\}|$ and $n_u := |\{d = 1, ..., Th; r_{i,t-1-\frac{d-1}{h}} > q_u\}|$ denote the number of down- and up-days. For our main results, we choose q_d and q_u as the 30% lowest and highest returns.

DuSkew: Similar to the DuVol, we estimate the down-to-up skewness (DuSkew) as

$$\widehat{DuSkew}_{i,t,T} = \frac{\frac{1}{n_d-1}\sum_{d=1}^{Th} r^3_{i,t-1-\frac{d-1}{h}} \cdot \mathbb{1}_{\{r_{i,t-1-\frac{d-1}{h}} < q_d\}}}{\frac{1}{n_u-1}\sum_{d=1}^{Th} r^3_{i,t-1-\frac{d-1}{h}} \cdot \mathbb{1}_{\{r_{i,t-1-\frac{d-1}{h}} > q_u\}}}, \tag{C.12}$$

where we choose q_d and q_u as the 30% lowest and highest returns. The estimate for month t risk is then given by $\hat{\mathcal{R}}_{i,t} = -1 \cdot \widehat{DuSkew}_{i,t,T}$.

R-Ratio: Similar to Rachev et al. (2007), we estimate the R-Ratio as

$$\widehat{\mathrm{RR}}^{\alpha}_{i,t,T} = \frac{\widehat{\mathrm{CVaR}}^{\alpha}_{i,t,T}}{\widehat{\mathrm{CVaR}}^{\alpha,ret}_{i,t,T}}, \tag{C.13}$$

where $\widehat{\mathrm{CVaR}}^{\alpha,ret}_{i,t,T}$ is estimated based on Equation (C.8) using realized ordered returns $r_{i,t-1,(d)}$ instead of realized ordered losses $l_{i,t-1,(d)}$.

C.2 Systematic Risk Measures

This section shortly lists the non-parametric estimation of systematic risk measures using the past T months of daily data of an industry and the equally weighted momentum portfolio.

Beta: As in Ang et al. (2006a, Eq. (B-7)), Atilgan et al. (2020, p. 728) and Bali et al. (2014, p. 243), we estimate the beta as

$$\hat{\beta}_{i,t,T} = \frac{\frac{1}{Th}\sum_{d=1}^{Th}\left(r_{i,t-1-\frac{d-1}{h}} - \mu_{i,t,T}\right) \cdot \left(r_{mom,t-1-\frac{d-1}{h}} - \mu_{mom,t,T}\right)}{\frac{1}{Th}\sum_{d=1}^{Th}\left(r_{mom,t-1-\frac{d-1}{h}} - \mu_{mom,t,T}\right)^2}, \tag{C.14}$$

where $\mu_{i,t,T} = \frac{1}{Th}\sum_{d=1}^{Th} r_{i,t-1-\frac{d-1}{h}}$ and $\mu_{mom,t,T} = \frac{1}{Th}\sum_{d=1}^{Th} r_{mom,t-1-\frac{d-1}{h}}$.

Correlation: Similar to the beta, we estimate the correlation by

$$\widehat{Corr}_{i,t,T} = \frac{\frac{1}{Th}\sum_{d=1}^{Th}\big(r_{i,t-1-\frac{d-1}{h}} - \mu_{i,t,T}\big)\cdot\big(r_{mom,t-1-\frac{d-1}{h}} - \mu_{mom,t,T}\big)}{\sqrt{\frac{1}{Th}\sum_{d=1}^{Th}\big(r_{mom,t-1-\frac{d-1}{h}} - \mu_{mom,t,T}\big)^2}\cdot\sqrt{\frac{1}{Th}\sum_{d=1}^{Th}\big(r_{i,t-1-\frac{d-1}{h}} - \mu_{i,t,T}\big)^2}}. \tag{C.15}$$

Coskewness: As in Ang et al. (2006a, Eq. (B-9)), Bi and Zhu (2020, Eq. (A-2)), Bali et al. (2014, Eq. (17)), Bollerslev et al. (2019, Eq. (A-2)), Bollerslev et al. (2020, Eq. (17)), Jiang et al. (2020, Eq. (B-9)) and Langlois (2020), we estimate the coskewness by

$$\widehat{CoSkew}_{i,t,T} = \frac{\frac{1}{Th}\sum_{d=1}^{Th}\big(r_{i,t-1-\frac{d-1}{h}} - \mu_{i,t,T}\big)\cdot\big(r_{mom,t-1-\frac{d-1}{h}} - \mu_{mom,t,T}\big)^2}{\sqrt{\frac{1}{Th}\sum_{d=1}^{Th}\big(r_{i,t-1-\frac{d-1}{h}} - \mu_{i,t,T}\big)^2}\cdot\Big(\frac{1}{Th}\sum_{d=1}^{Th}\big(r_{mom,t-1-\frac{d-1}{h}} - \mu_{mom,t,T}\big)^2\Big)}. \tag{C.16}$$

Cokurtosis: As in Ang et al. (2006a, Eq. (B-9)), Bollerslev et al. (2020, Eq. (18)), Jiang et al. (2020, Eq. (B-10)) and Bollerslev et al. (2019, Eq. (A-3)), we estimate the cokurtosis by

$$\widehat{CoKurt}_{i,t,T} = \frac{\frac{1}{Th}\sum_{d=1}^{Th}\big(r_{i,t-1-\frac{d-1}{h}} - \mu_{i,t,T}\big)\cdot\big(r_{mom,t-1-\frac{d-1}{h}} - \mu_{mom,t,T}\big)^3}{\sqrt{\frac{1}{Th}\sum_{d=1}^{Th}\big(r_{i,t-1-\frac{d-1}{h}} - \mu_{i,t,T}\big)^2}\cdot\Big(\frac{1}{Th}\sum_{d=1}^{Th}\big(r_{mom,t-1-\frac{d-1}{h}} - \mu_{mom,t,T}\big)^2\Big)^{3/2}}. \tag{C.17}$$

Downside Beta: As in Ang et al. (2006a, Eq. (B-8)), Atilgan et al. (2020, p. 728) and Bali et al. (2014, Eq. (15)), we estimate the downside beta as

$$\widehat{\beta}^{-}_{i,t,T} = \frac{\frac{1}{n_q}\sum_{d=1}^{Th}\big(r_{i,t-1-\frac{d-1}{h}} - \mu^{-}_{i,t,T}\big)\cdot\big(r_{mom,t-1-\frac{d-1}{h}} - \mu^{-}_{mom,t,T}\big)\cdot\mathbb{1}_{\{r_{mom,t-1-\frac{d-1}{h}}<q\}}}{\frac{1}{n_q}\sum_{d=1}^{Th}\big(r_{mom,t-1-\frac{d-1}{h}} - \mu^{-}_{mom,i,t}\big)^2\cdot\mathbb{1}_{\{r_{mom,t-1-\frac{d-1}{h}}<q\}}}, \tag{C.18}$$

where we define $n_q = \sum_{d=1}^{Th}\mathbb{1}_{\{r_{mom,t-1-\frac{d-1}{h}}<q\}}$, $\mu^{-}_{i,t,T} = \frac{1}{n_q}\sum_{d=1}^{Th} r_{i,t-1-\frac{d-1}{h}}\cdot\mathbb{1}_{\{r_{mom,t-1-\frac{d-1}{h}}<q\}}$ and $\mu^{-}_{mom,t,T} = \frac{1}{n_q}\sum_{d=1}^{Th} r_{mom,t-1-\frac{d-1}{h}}\cdot\mathbb{1}_{\{r_{mom,t-1-\frac{d-1}{h}}<q\}}$.

Downside Correlation: Similar to Hong et al. (2007, p. 1550), we estimate the downside correlation by

$$\widehat{Corr}^{-}_{i,t,T} = \frac{\frac{1}{n_q}\sum_{d=1}^{Th}\big(r_{i,t-1-\frac{d-1}{h}} - \mu^{-}_{i,t,T}\big)\cdot\big(r_{mom,t-1-\frac{d-1}{h}} - \mu^{-}_{mom,t,T}\big)\cdot\mathbb{1}_{\{r_{mom,t-1-\frac{d-1}{h}}<q\}}}{\sigma^{-}_{i,t,T}\cdot\sigma^{-}_{mom,t,T}}, \tag{C.19}$$

where

$$\sigma^{-}_{i,t,T} = \sqrt{\frac{1}{n_q}\sum_{d=1}^{Th}(r_{i,t-1-\frac{d-1}{h}} - \mu^{-}_{i,t,T})^2 \cdot \mathbb{1}_{\{r_{mom,t-1-\frac{d-1}{h}}<q\}}} \tag{C.20}$$

and

$$\sigma^{-}_{mom,t,T} = \sqrt{\frac{1}{n_q}\sum_{d=1}^{Th}(r_{mom,t-1-\frac{d-1}{h}} - \mu^{-}_{mom,t,T})^2 \cdot \mathbb{1}_{\{r_{mom,t-1-\frac{d-1}{h}}<q\}}} \tag{C.21}$$

denote the downside volatility of asset i and the momentum portfolio.

LPM-beta: As in Bali et al. (2014) and Price et al. (1982, Eq. (5)), we estimate the LPM-beta as

$$\widehat{\beta}^{LPM}_{i,t,T} = \frac{\frac{1}{n_q}\sum_{d=1}^{Th}(r_{i,t-1-\frac{d-1}{h}} - q)\cdot(r_{mom,t-1-\frac{d-1}{h}} - q)\cdot \mathbb{1}_{\{r_{mom,t-1-\frac{d-1}{h}}<q\}}}{\frac{1}{n_q}\sum_{d=1}^{Th}(r_{mom,t-1-\frac{d-1}{h}} - q)^2 \cdot \mathbb{1}_{\{r_{mom,t-1-\frac{d-1}{h}}<q\}}}, \tag{C.22}$$

where q denotes the chosen threshold.

HTCR-beta: As in Bali et al. (2014), we estimate the HTCR-beta as

$$\widehat{\beta}^{HTCR}_{i,t,T} = \frac{\frac{1}{n_{q,i}}\sum_{d=1}^{Th}(r_{i,t-1-\frac{d-1}{h}} - q)\cdot(r_{mom,t-1-\frac{d-1}{h}} - q)\cdot \mathbb{1}_{\{r_{i,t-1-\frac{d-1}{h}}<q\}}}{\frac{1}{n_{q,i}}\sum_{d=1}^{Th}(r_{mom,t-1-\frac{d-1}{h}} - q)^2 \cdot \mathbb{1}_{\{r_{i,t-1-\frac{d-1}{h}}<q\}}}, \tag{C.23}$$

where $n_{q,i} = \sum_{d=1}^{Th}\mathbb{1}_{\{r_{i,t-1-\frac{d-1}{h}}<q\}}$.

Tail-beta: Following the estimation procedure of Van Oordt and Zhou (2016) and Van Oordt and Zhou (2017, Sec. 1.2), we use the EVT based non-parametric estimate of Tail-beta. The authors show that, under certain conditions, Tail-beta can be estimated by

$$\hat{\beta}^{Tail}_{i,t,T} = \widehat{\tau}_{i,t,T}(\alpha)^{1/\hat{\xi}_{mom,t,T}} \cdot \frac{\widehat{\text{VaR}}^{\alpha}_{i,t,T}}{\widehat{\text{VaR}}^{\alpha}_{mom,t,T}}, \tag{C.24}$$

where $\widehat{\text{VaR}}^{\alpha}_{mom,t,T}$ is the VaR of the momentum portfolio and $1/\hat{\xi}_{mom,t,T}$ is estimated by the Hill estimator using losses of the momentum portfolio $\{l_{mom,t-1-\frac{d-1}{h}}\}_{d=1}^{Th} := \{-r_{mom,t-1-\frac{d-1}{h}}\}_{t=1}^{Th}$ with corresponding order statistics $\{l_{mom,t-1,(d)}\}_{d=1}^{Th}$. The Hill estimator is given by

$$\frac{1}{\hat{\xi}_{mom,t,T}} = \frac{1}{Th - \lceil Th(1-\alpha)\rceil + 1} \cdot \sum_{d=\lceil Th(1-\alpha)\rceil}^{Th} \ln\left(\frac{l_{mom,t-1,(d)}}{l_{mom,t-1,(\lceil Th(1-\alpha)\rceil)}}\right). \tag{C.25}$$

The Hill estimator is also used by Karagiannis and Tolikas (2019), Kelly and Jiang (2014) and Poon et al. (2004, Eq. (6)). Further, Van Oordt and Zhou (2016) and Van Oordt and Zhou (2017) show that $\widehat{\tau(\alpha)}$ can be estimated by

$$\hat{\tau}_{i,t,T}(\alpha) = \frac{1}{Th - \lceil Th(1-\alpha) \rceil + 1} \sum_{d=1}^{Th} \mathbb{1}_{\left\{l_{i,t-1-\frac{d-1}{h}} > \widehat{\text{VaR}}^{\alpha}_{i,t,T} \text{ and } l_{mom,t-1-\frac{d-1}{h}} > \widehat{\text{VaR}}^{\alpha}_{mom,t,T}\right\}}, \tag{C.26}$$

i.e. $\hat{\tau}_{i,t,T}(\alpha)$ measures the occurrence of joint crashes of asset i and the momentum portfolio. The structure of Tail-beta in Equation (C.24) is similar to the usual beta in Equation (3.4.3), but focuses on the tails instead of the whole distribution.

An alternative to the approach presented above would be to calculate $\hat{\beta}^{Tail}_{i,t,T}$ based on a conditional regression for the observations when the momentum portfolio suffers the most extreme losses (Van Oordt and Zhou, 2017, Footnote 4). However, Van Oordt and Zhou (2016) and Van Oordt and Zhou (2017) show that the simple non-parametric estimator gives more reliable estimates compared to the conditional regression approach.

Tail-Sens: As in Agarwal et al. (2017, p. 615), we estimate the Tail-Sens non-parametrically. The estimate for Tail-Sens is then given by

$$\widehat{TS}_{i,t,T} = \frac{\sum_{d=1}^{Th} \mathbb{1}_{\{-r_{i,t-1-\frac{d-1}{h}} > \text{VaR}^{\alpha}_{i,t,T}, -r_{mom,t-1-\frac{d-1}{h}} > \text{VaR}^{\alpha}_{m,t,T}\}}}{\sum_{d=1}^{Th} \mathbb{1}_{\{-r_{mom,t-1-\frac{d-1}{h}} > \text{VaR}^{\alpha}_{m,t,T}\}}}, \tag{C.27}$$

where $\widehat{\text{VaR}}^{\alpha}_{i,t,T}$ and $\widehat{\text{VaR}}^{\alpha}_{mom,t,T}$ are the VaR of asset i and the momentum portfolio, estimated based on Equation (C.7) for losses of asset i and the momentum portfolio, respectively. Alternatives to the estimator in Equation (C.27) are given in Chabi-Yo et al. (2018), Poon et al. (2004) and Weigert (2015).

Tail-Risk: As in Agarwal et al. (2017, p. 615), we estimate Tail-Risk non-parametrically. The estimate for Tail-Risk is given by

$$\widehat{TR}_{i,t,T} = \widehat{TS}_{i,t,T} \cdot \frac{\widehat{\text{CVaR}}^{\alpha}_{i,t,T}}{\widehat{\text{CVaR}}^{\alpha}_{mom,t,T}}, \tag{C.28}$$

where $\widehat{\text{CVaR}}^{\alpha}_{i,t,T}$ and $\widehat{\text{CVaR}}^{\alpha}_{mom,t,T}$ are the CVaR of asset i and the momentum portfolio, estimated based on Equation (C.8) for losses of asset i and the momentum portfolio, respectively.

This measure has a similar structure as the normal beta, which can be decomposed as shown in Equation (3.4.3), but the Tail-Risk measure replaces the correlation by a measure of co-crash risk. Moreover, volatility of asset i and the momentum portfolio are replaced by their CVaR. Hence, the Tail-Risk measure is a natural extension of the usual beta but focuses on tail events instead of the whole distribution. The Tail-Risk is again similar in nature to the Tail-beta of Equation (C.24).

MES: As in Acharya et al. (2016, Eq. (16)), we estimate the Marginal Expected Shortfall (MES) by

$$\widehat{\mathrm{MES}}^{\alpha}_{i,t,T} = \frac{1}{Th - \lceil Th(1-\alpha) \rceil + 1} \cdot \sum_{d=1}^{Th} -r_{i,t-1-\frac{d-1}{h}} \cdot \mathbb{1}_{\{-r_{mom,t-1-\frac{d-1}{h}} > \widehat{\mathrm{VaR}}^{\alpha}_{mom,t,T}\}}. \tag{C.29}$$

Brownlees and Engle (2016, Eq. (3)) also use the equation above applied to simulated returns of a multivariate GARCH model to calculate MES. Other estimation methods of MES can be found in Brownlees and Engle (2016) and Engle et al. (2015).

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