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Scaling, Unwinding and Greening QE in a Calibrated Portfolio Balance Model

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Scaling, unwinding and greening QE in a calibrated portfolio balance model*

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Abstract

We develop a portfolio balance model to analyze the impact of euro area quantitative easing (QE) on asset yields. Our model features two countries each populated by two agents representing their respective banking and mututal fund sectors. Agents, which differ in their preferences for assets, can trade currencies, bonds and equities. In simulations of the calibrated model we find that 10-year euro area bond returns decline by 31 basis points in response to €1 trillion in central bank bond purchases, which is in line with the empirical literature. QE leads to a substantial flattening of the yield curve and increasing the maturity of purchased bonds increases the average yield impact. When QE is unwound, yields increase quicker than the central bank balance sheet shrinks. This is because the yield impact scales non-linearly with increasing asset purchases. When assessing the potential impact of green QE, we find that it is slightly less effective in reducing bond yields than conventional QE. However, the spread between green and brown bond yields decreases with conventional QE while it increases with green QE.

JEL classification: C63 - G11 - E52

Keywords: euro area QE - portfolio balancing channel - yield curve - green QE

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1 Introduction

In the current low interest rate environment quantitative easing (QE) has become an essential tool in many central banks' monetary policy toolboxes. Buying bonds with newly created money can stimulate corporate borrowing by lowering long-term yields (Lo Duca et al., 2016) and prevent financial market panics (Vissing-Jorgensen, 2021). It has been extensively used to combat the economic fallout from the global financial crisis and the Covid-19 pandemic. In 2021 the aggregate volume of central bank asset purchases approaches 30% of global GDP. Despite its massive scale, the precise impact of QE on expected returns and the determinants of that impact remain elusive. While event studies around announcements of QE programs have dissipated most doubts[1] that central bank asset purchases substantially lower long-term yields through both the signaling and portfolio balance channel, they have limitations. Few data points, partially anticipated announcements and the assumption of immediate and rational asset pricing in response to announcements make precise impact assessments difficult (Bhattarai and Neely, 2016). A better understanding and quantification of the impact is necessary not least because central banks around the world consider unwinding QE as inflation increases in the wake of economic recovery from the Covid-19 crisis. Furthermore, new potential uses for QE are being discussed (see e.g. Krogstrup and Oman, 2019). Green QE, for example, may provide an entrance for central banks to participate in greening the economy, by creating funding incentives for firms to invest in "green" projects. Whether this is sensible or legal may be a point of contention, but its feasibility can be addressed regardless.

We develop a model with which we can assess the impact of QE on domestic and foreign asset yields through the portfolio balance channel. By simulating the model calibrated to the euro area, we can analyze the impact of asset purchases on the yield curve, understand how the impact of QE scales with increasing purchasing volumes and estimate the temporal distribution of yield increases as QE is passively unwound. Furthermore, we take a look at the likely effects of two measures discussed in the context of green monetary policy. That is, the extension of collateral eligibility criteria to encompass more "green" assets and the purchase of green bonds by the central bank.

Our main results are that €1 trillion in central bank bond purchases with an average maturity of 6.25 years reduce the average (weighted sum of all maturities) domestic bond yields by about 18.4 basis points, while the expected return on domestic equities declines by approximately 2.6 basis points. The Euro depreciates by approximately 0.06% and spillovers from euro area QE to foreign asset returns are negligible. We find that QE substantially flattens the domestic yield curve, with the 10, 5 and 1-year bond yields dropping on average by 31, 16 and 2.5 basis points, respectively. When compared with the empirically estimated impact on 10-year government bonds, our estimate is consistent with the 27-64 basis point range reported in different event studies (Andrade et al., 2016). Increasing the maturity of purchased assets increases the impact of QE. Our simulations furthermore suggest that the impact of QE increases nonlinearly with increasing purchasing volumes. Maturity and volume effects play a role in estimating return effects of unwinding QE. The overall bond holdings of the European Central Bank (ECB), which amount to approximately €4.2 trillion in July 2021, are estimated to be depressing average bond yields by 94 basis points. Passively unwinding these holdings, i.e. not reinvesting principal payments of maturing bonds, would more than halve the impact within 4 years. In our assessment of extending collateral eligibility criteria to green assets, we find, as empirical evidence confirms (see e.g. Macaire and Naef, 2021), that this would reduce the yield of those assets. However, the interaction of extended collateral eligibility criteria and QE is inconclusive. Banks' increasing preference for eligible assets can either reduce or increase the impact of QE.

[1] See e.g.Woodford (2012), Williamson (2017) and Greenlaw et al. (2018) for more skeptical takes.

This depends on banks' willingness to swap eligible assets for domestic currency. Furthermore, we find that there is a trade-off between green and conventional QE. Assuming that green and conventional bonds only differ in terms of corresponding preferences, our simulations suggest, on the one hand, that purchasing the preferred green bonds will reduce the overall impact of QE. On the other hand, green QE increases the spread between green and conventional yields (the greenium), while conventional QE reduces it. With other words, green QE is a suitable policy to incentivize investments in green projects, while it is less effective in providing stimulus to the general economy.

Our paper is related to theoretical work studying portfolio balance effects on asset returns. Portfolio rebalancing has been identified as one of two main channels through which QE can have an impact on yields. The signaling channel, where central bank purchases have an effect on expected future short-term interest rates, being the other main channel (see e.g. Bauer and Rudebusch, 2014). The portfolio balance channel can be traced back to the development of the Tobin-Markowitz mean-variance model of portfolio selection (Tobin, 1958). In this type of model risk-averse investors want to be compensated for the exposure to risk factors, which essentially establishes a link between the return of an asset and its supplied quantity. Changes in asset supply or demand thereby affect the risk premium even when expectations about future short-term interest rates remain constant. Since central bank asset purchases do not directly change the overall supply of assets, but lead to an exchange of bonds for currency, the effectiveness of QE hinges on the substitutability of bonds and currency. Portfolio balance models have been gaining popularity recently as they hold testable implications not only for the transmission of central bank asset purchases (Greenwood and Vayanos, 2014), but for exchange rate dynamics (Hau and Rey, 2008; Gabaix and Maggiori, 2015), the term structure of interest rates (Vayanos and Vila, 2021) and can account for puzzling empirical deviations from theory derived from the no-arbitrage condition (Greenwood et al., 2020; Gourinchas et al., 2021).

We develop a slightly different flavor of portfolio balance models. While at its core lies the same mean-variance portfolio optimization found in all portfolio models, we allow for more asset and agent heterogeneity. Both are important in the context of QE, since the substitutability of assets is not only determined by risk factors and their correlations, but also by investors, which differ in their business models. Empirical observations such as the home bias of banks, the preference of pension funds for long-term bonds or European households' distaste for equities can affect the substitutability of assets and needs to be considered. Our paper thus relates to a literature that acknowledges the role of investor heterogeneity in asset pricing (see e.g Koijen and Yogo, 2019), not least in the context of understanding return differences between more or less "green" assets (Pedersen et al., 2020; Zerbib, 2020; Pástor et al., 2020). Due to data limitations we can only calibrate agents representing banks and funds. Since both domestic and foreign agents trade in the assets purchased by the central bank, our model setup includes two regions: the euro area and the rest of the world. Several portfolio balance models that focus on exchange rate dynamics also include such a setup (see e.g. Hau and Rey, 2008; Gabaix and Maggiori, 2015).

Greenwood et al. (2020) and Gourinchas et al. (2021), building on the seminal preferred habitat model of Vayanos and Vila (2021), develop two-country models that are most closely related to ours. Both use their models to explain well known empirical deviations from the expectations hypothesis and the uncovered interest rate parity theory. There are several differences between their models and ours. First, they differ in the way agents are modeled. Greenwood et al. (2020) and Gourinchas et al. (2021) model a portfolio optimizing arbitrageur agent that integrates markets, while other investors invest in a single asset (their preferred habitat) and do not consider substituting between assets when their preferred option is unfavorably priced. Because of the salient importance of asset substitutability for the impact of QE all our agents optimize a portfolio comprising of all assets, but have different preferences for those assets. In this regard our approach is more related to the literature on asset pricing with ”tastes” (see e.g. Fama and French, 2007; Pástor et al., 2020; Pedersen et al., 2020; Zerbib, 2020). A second distinction pertains to the way assets are modeled. Assets in our model are portfolios representing currency, bonds and equities. Explicitly modeling currency is necessary since central bank asset purchases reduce the supply of bonds while increasing the supply of currency. The portfolios within our model differ in terms of maturity, default probability and payouts (interest/coupon/dividend payments), all of which determine their risk structure. For example, currency is low risk as it nominally cannot fluctuate in price (it has no maturity), cannot default and pays a known interest rate (the deposit rate). Bonds, in contrast to equities, have smaller than infinite maturity. This leads to principal repayments, which reduce the risk of holding bonds vs. equities as the bond price is anchored to its nominal value (limiting price fluctuations). In our modeling of assets, the risk emerges from the asset characteristics. While our bond portfolios have a constant average maturity, we are able to derive the term structure of interest rates in simulations. Contrasting with our approach, Greenwood et al. (2020) models a short-term bond (equivalent to our currency) that matures over night and a long-term bond modeled as a default-free perpetuity. Gourinchas et al. (2021) include the entire term structure of interest rates by modeling a continuum of bonds. A third distinction between our model and the models developed by Greenwood et al. (2020) and Gourinchas et al. (2021) are the risk factors, which determine asset prices. Mean reverting stochastic nominal interest rates and stochastic asset supply shocks introduce risk into the models of Greenwood et al. (2020) and Gourinchas et al. (2021), while we focus on default probability and inflation as risk factors. Thereby we assume that nominal interest rates stay constant and changes to the supply of assets is deterministically determined by central bank purchases. Average changes to asset yields within our setup thus reflect changes in risk premia and not changes in interest rate expectations. Stochastic inflation with a constant mean makes currency a risky asset.

Due to the additional model complexity introduced by including heterogeneous assets and optimizing agents as well as two currency areas, we cannot directly compute the equilibrium outcomes of QE, but simulate the model. Agents' adaptive expectations of risk thereby ensure that the consequences of stochastic default and inflation processes are fully accounted for.

Since we calibrate our model and use it to derive impact estimates of QE, our paper relates to a large empirical literature with varying methodologies. The most common empirical approach to estimating the impact of QE on yields is to look at high frequency data around announcement events. The literature has been reviewed by various authors (see e.g. Andrade et al., 2016; Bhattarai and Neely, 2016; Dell'Ariccia et al., 2018; Kuttner, 2018). Most closely related to our paper is the approach developed by Koijen et al. (2021), who use security-level holdings data for several investor sectors to estimate the impact of euro area QE from March 2015 to December 2017. Specifically, they estimate a demand system for euro area government bonds, which takes into account heterogeneous investor preferences. With instrumental variables they then relate portfolio rebalancing to yield changes. On average it is estimated that QE until the end of 2017 reduced yields by 65 basis points. Furthermore, they find that foreign investors reduced their exposure to euro area governement bonds most in response to QE. Unfortunately, Koijen et al. (2021) do not have security holdings data for different foreign investor sectors. It is thus unclear who is selling euro area bonds. Also, data on currency holdings are not tracked with their approach. Our model, however, suggests it is not necessarily the investors who sell the most bonds that are responsible for the impact of QE, but the investors who rebalance their portfolios from bonds to currency.

Our analysis of green QE builds on the assumption that investors' preferences towards green assets differs from preferences towards conventional assets. This assumption is based on a body of empirical evidence suggesting that investors draw non-financial utility from investing in assets that have a positive environmental impact (see e.g. Hong and Kacperczyk, 2009; Riedl and Smeets, 2017). Theoretical models show that such additional utility drives a wedge between the returns of conventional and green assets, which is sometimes referred to as the greenium (see e.g Pástor et al., 2020; Pedersen et al., 2020; Zerbib, 2020). While not entirely conclusive, empirical evidence increasingly seems to confirm this theoretical finding (see e.g. Zerbib, 2019).[2] Because a greenium improves funding conditions for green projects relative to brown projects, it can arguably accelerate the transition towards a carbon neutral economy. The role of monetary policy in mitigating climate change has hence become an important and controversial policy debate (see e.g. Tooze, 2019). The two main monetary policies proposed to support the transition towards carbon neutrality are changing the criteria for collateral eligibility and tilting QE towards green assets (see Krogstrup and Oman, 2019, for a review in this context). While Schoenmaker (2021) shows how to effectively tilt the ECB asset and collateral framework towards green assets, little is known about the impact this would have on bond yields and the greenium. We are aware of only two papers that attempt such an assessment: Macaire and Naef (2021) empirically estimate that the extension of collateral eligibility criteria by the People's Bank of China to include green bonds increased the greenium by 46 basis points.[3] Ferrari and Nispi Landi (2021) show within a calibrated DSGE model in which green and brown bonds are imperfect substitutes that a temporary increase in green asset holdings increases the greenium and mitigates detrimental emissions. However, the temporary nature of QE in their model limits its impact on the stock of pollution.

The remainder of the paper is organized as follows. In Section 2 we present our full model. Section 3 discusses how portfolio rebalancing affects asset returns in a simplified representative agent model. In Section 4 we calibrate the full model and present simulation results in Section 5. Section 6 concludes.

2 Model

The model comprises two countries, Domestic (D) and Foreign (F), each populated by two agents, which will be calibrated to represent the banking and mututal fund sector of their respective country.[4] An agent d can trade three types of assets, Cash (C), Bonds (B) and Equities (E), which are available for both countries, i.e. there is a total of six assets. Each period t represents a trading day, with 250 trading days in a year.

2.1 Risk Factors

Assets are exposed to two exogenous risk factors: default risk and inflation risk. Domestic and foreign inflation are modeled as correlated, normally distributed random variables:

$$\begin{aligned} \pi_t^D &\sim \mathcal{N}(\bar{\pi}^D, \sigma_\pi^D) \\ \pi_t^F &\sim \mathcal{N}(\bar{\pi}^F, \sigma_\pi^F), \end{aligned} \tag{2.1}$$

[2] Also, for recent issues of German government twin bonds, i.e. bonds that only differ in the way the respective raised money is spend, the market yield of the green bond is consistently below its conventional twin (Finanzagentur, 2021).

[3] Recently, the ECB has also moved to accept sustainablility-linked bonds as collateral (ECB, 2020).

[4] The model can easily be extended to comprise more agents per country.

with $\rho_\pi = \mathrm{Corr}(\pi_t^D, \pi_t^F) \in [0,1]$.[5] The default rate $\Omega_{a,t}$ for asset a at time t is derived from default events, which we model as a Poisson distribution with mean $\lambda_{a,t}$:

$$\begin{aligned} &\Omega_{a,t} \sim \omega_a \cdot \mathrm{Pois}(\lambda_{a,t}), \quad \text{with} \\ &\ln(\lambda_{a,t+1}) = \ln(\bar{\lambda}_a) + \phi_\Omega \left(\ln(\lambda_{a,t}) - \ln(\bar{\lambda}_a)\right) + \epsilon_{a,t}^\Omega \end{aligned} \tag{2.2}$$

where ω_a is an asset specific scaling factor taking into account the average size of a default event and the loss given default. The logarithm of the mean number of default events $\lambda_{a,t}$ follows an AR(1) process, where $\phi_\Omega \in [0,1)$ and $\epsilon_{a,t}^\Omega \sim \mathcal{N}(0, \sigma_a^\Omega)$. Default rates of any two assets can be correlated, i.e. $\rho_{a_1,a_2}^\Omega = \mathrm{Corr}(\epsilon_{a_1,t}^\Omega, \epsilon_{a_2,t}^\Omega) \in [0,1]$.[6]

2.2 Assets

Asset a in our model represents a portfolio of assets rather than an individual asset. The portfolio's daily real return $r_{d,a,t}$ is a function of last period's price $P_{d,a,t-1}$, the inflation rate π_t and the overnight change in the portfolio's nominal valuation $\Delta V_{d,a,t}$ in terms of agent d's home currency:

$$r_{d,a,t} = \frac{1 + \Delta V_{d,a,t}/P_{d,a,t-1}}{1 + \pi_t} - 1 \tag{2.3}$$

The change in valuations depends on four factors: (1) the repayment of maturing portfolio shares, (2) the change in the price of outstanding portfolio shares, (3) the interest, coupon or dividend payment and (4) the loss incurred due to defaulting assets within the portfolio:

$$\Delta V_{d,a,t} = \underbrace{mat_{a,t}(N_{d,a} - P_{d,a,t-1})}_{\text{repayment}} + \underbrace{out_{a,t}(P_{d,a,t} - P_{d,a,t-1})}_{\text{price change}} + \underbrace{all_{a,t}(N_{d,a} \cdot i_{a,t})}_{\text{interest}} - \underbrace{\Omega_{a,t} P_{d,a,t-1}}_{\text{defaults}}, \tag{2.4}$$

where $N_{d,a}$ is the nominal value of a portfolio share, $P_{d,a,t}$ is the current price and $i_{a,t}$ is the interest/coupon/dividend payed on a portfolio share. The nominal value and price of an asset in Eq. (2.4) depend on where agent d is domiciled and where asset a is issued:

$$N_{d,a} = \begin{cases} 1 & \text{if } d \text{ and } a \text{ in the same country} \\ 1 \cdot X_t^{F\to D} & \text{if } d \text{ in } D \text{ and } a \text{ in } F \\ 1 \cdot X_t^{D\to F} & \text{if } d \text{ in } F \text{ and } a \text{ in } D \end{cases} \tag{2.5}$$

$$P_{d,a,t} = \begin{cases} P_{a,t} & \text{if } d \text{ and } a \text{ in the same country} \\ P_{a,t} \cdot X_t^{F\to D} & \text{if } d \text{ in } D \text{ and } a \text{ in } F \\ P_{a,t} \cdot X_t^{D\to F} & \text{if } d \text{ in } F \text{ and } a \text{ in } D \end{cases} \tag{2.6}$$

The exchange rate $X^{D\to F} = 1/X^{F\to D}$ defines the amount of units of country F's currency can be purchased with one unit of country D's currency. An increase in $X^{D\to F}$ thus corresponds to an appreciation of country F's currency vis-à-vis country D's currency.

A constant repayment rate $(1 - m_a)$ on outstanding portfolio shares imposes a maturity structure on asset portfolios. The maturity parameter $m_a \in [0,1]$ determines how quickly an

[5] We assume constant mean inflation, in order to avoid an impact of expected changes to real interest rates on the structure of the yield curve. For the same reason we do not consider changes in the nominal level of interest rates as a risk factor.

[6] Note that we model default rates in this way because it provides a good match to publicly available data on global default events. Using logarithms in the AR(1) process prevents a negative number of default events.

asset position on an agent's balance sheet diminishes.[7] The portions of performing outstanding and maturity portfolio shares can thus be defined as follows:

$$\begin{aligned} out_{a,t} &\equiv m_a(1-\Omega_{a,t}) \\ mat_{a,t} &\equiv (1-m_a)(1-\Omega_{a,t}) \\ all_{a,t} &\equiv out_{a,t} + mat_{a,t} = (1-\Omega_{a,t}) \end{aligned}$$

With the inclusion of the maturity parameter, Eq. (2.4) can describe valuation changes of all three asset types within our model. A currency C is characterized by $m_C = 0$, $\Omega_{C,t} = 0$ and $P_{C,t} = N_C$. This leaves the interest rate paid for holding currency as the only factor determining its nominal valuation change between periods.[8] On the other extreme, equities E are characterized by $m_E = 1$, $\Omega_{E,t} \in \mathbb{R}^+$ and $P_{E,t} \in \mathbb{R}^+$, i.e. repayment does not play a role in its valuation. Note that the interest factor is determined by dividend payments, which unlike the currency rate can be subject to fluctuations. Similar to an equity portfolio, a bond portfolio B is exposed to defaults and price movements. However, a bond portfolio, if not entirely composed of perpetual bonds, is characterized by $m_B \in (0,1)$, which introduces the repayment factor into the valuation. The lower m_B, the bigger the impact of repayments on the valuation and the smaller the impact of price changes. This shift in valuation factors is crucial for the endogenous term structure of bond returns within our model. Since there is more uncertainty attached to price movements in comparison to repayments, increasing the average maturity of the bond portfolio will increase the riskiness of that portfolio. All else equal, agents will demand a higher risk premium for a portfolio with a longer average maturity.

2.3 Agents

Domestic and foreign banks and funds are budget constrained. The central bank and the underwriter are exogenous agents, which do not optimize, but act mechanically.

2.3.1 Banks and funds

Bank and fund agents are endowed with wealth $W_{d,0}$, which constrains their budget:

$$W_{d,t} - W_{d,t-1} = \sum_a Q_{d,a,t-1} \Delta V_{d,a,t} - O_{d,t}, \tag{2.7}$$

where $Q_{d,a,t}$ is the quantity of asset a held by agent d and $O_{d,t}$ are payouts to shareholders. In order to avoid dealing with growing balance sheets over time, agents payout profits from interest $(N_a \cdot i_{a,t})$ and repayments $(N_a - P_{a,t-1})$ to shareholders. For the sake of simplicity, these profits are payed out in the currency in which they have accrued, i.e. $O_{d,t} = O^D_{d,t} X^{D\to d}_t + O^F_{d,t} X^{F\to d}$, with $X^{D\to d}_t$ $(X^{F\to d}_t)$ being the exchange rate between the domestic (foreign) country and the country of domicile of agent d.[9] Defining $\mathcal{D}$ as the set of all domestic assets and $\mathcal{F}$ as the set of

[7] In Section 4 we show that when calibrating the maturity parameter to the data, it fits mutual funds bond holdings remarkably well. In Section 5.3 we show how to extract the term structure of bond returns from portfolio returns. Hatchondo and Martinez (2009) pioneered the one-parameter maturity model within a standard macro model. However, rather than assuming a constant repayment rate on bonds, they assume a perpetual bond with constantly declining coupon payments.

[8] We assume that institutional investors do receive or pay interest on currency holdings. In our calibration, we set the currency rate to the deposit facility rate of the central bank. Evidence suggests that many banks do pass on even negative deposit facility rate to their corporate depositors Altavilla et al. (2019).

[9] $X^{D\to D}_t = X^{F\to F}_t = 1$

all foreign assets, payouts in domestic and foreign currency are computed as follows:

$$O_{d,t}^{\{D,F\}} = \sum_{a \in \{\mathcal{D},\mathcal{F}\}} Q_{d,a,t-1} \left(mat_{a,t}(N_a - P_{a,t-1}) + all_{a,t}(N_a \cdot i_{a,t}) - \Omega_{a,t} P_{a,t-1} \right) \tag{2.8}$$

This leaves prices and exchange rate movements as the only valuation factors affecting the wealth of an agent. We restrict payouts to positive values. When payouts are negative, agents do not receive additional funds, but rather retain future positive payouts until losses have been compensated.

Bank and fund agents are risk averse and make portfolio decisions according to a mean-variance utility functions:

$$\begin{aligned} \mathbf{w}_{d,t}^{*} &= \arg\max_{\mathbf{w}} \mathbf{w}' \mathrm{E}_{d,t}[\mathbf{r}_{t+1}] - 0.5 \mathbf{w}' \left((\boldsymbol{\beta}_d \boldsymbol{\beta}_d') \odot \mathop{\mathrm{E}}_{d,t} [\boldsymbol{\Sigma}_{t+1}] \right) \mathbf{w} \\ \text{s.t.} &\qquad \mathbf{w} \geq 0 \quad \text{and} \quad \mathbf{w}'\mathbf{1} = 1, \end{aligned} \tag{2.9}$$

where $\mathbf{w}$ is a 6×1 vector of portfolio weights $w_{d,a,t}$ constrained by a no-short selling assumption ($\mathbf{w} \geq 0$) and the agent's budget ($\mathbf{w}'\mathbf{1} = 1$).[10] Furthermore, $\mathrm{E}_{d,t}[\cdot]$ is an expectations operator defining agent d's expectation at time t, $\mathbf{r}_t$ is a 6×1 vector of real asset returns $r_{d,a,t}$ and $\boldsymbol{\Sigma}_t$ is the 6×6 covariance matrix of real asset returns. Investor heterogeneity is introduced by business model factors $\beta_{d,a}$, which can differ for every agent d and asset a. Each factor can be interpreted as a business-model induced aversion towards a specific asset. With $\boldsymbol{\beta}_d$ being the 6×1 vector of business model factors, the factors enter the optimization problem through an element-wise multiplication of the 6×6 matrix $\boldsymbol{\beta}_d \boldsymbol{\beta}_d'$ with the expectation of the covariance matrix $\mathrm{E}_{d,t}[\boldsymbol{\Sigma}_{t+1}]$. The factors thereby play a role in determining the substitutability of assets and can account for differences in investment behavior between agents. For example, a home bias can be induced by higher business model factors on the respective foreign assets. Higher business model factors on equities can e.g. account for banks' preferences for bonds.

The demand for any asset a depends on its price $P_{d,a,t}$ in agent d's home currency, the desired portfolio composition $w_{d,a,t}^{*}$, agent wealth $W_{d,t}$ and the pre-trade quantity of that asset $Q_{d,a,t}'$ on the agent's balance sheet:

$$\Delta Q_{d,a,t} = \frac{w_{d,a,t}^{*} W_{d,t}}{P_{d,a,t}} - Q_{d,a,t}' \tag{2.10}$$

Pre-trade quantities differ from those held in the previous period $Q_{d,a,t-1}$ because parts of a portfolio a can mature or default. For domestic and foreign currency positions, pre-trade quantities furthermore take into account interest and principal payments as well as payouts:

$$\Delta Q_{d,a,t}' = \begin{cases} out_{a,t} Q_{d,a,t-1} & \text{if } a \text{ is a bond or equity} \\ Q_{d,C^D,t-1} + \sum_{a \in \mathcal{D}} Q_{d,a,t-1} N_a (mat_{a,t} + all_{a,t} i_{a,t}) - O_{d,t}^{D} & \text{if } a \text{ is domestic cash } C^D \\ Q_{d,C^F,t-1} + \sum_{a \in \mathcal{F}} Q_{d,a,t-1} N_a (mat_{a,t} + all_{a,t} i_{a,t}) - O_{d,t}^{F} & \text{if } a \text{ is foreign cash } C^F \end{cases} \tag{2.11}$$

2.3.2 Central bank and underwriter

We introduce two non-optimizing agents. A domestic central bank agent (CB) targets holding an exogenously specified quantity $Q_{\mathrm{CB},a}^{*}$ for each domestic asset portfolio $a \in \mathcal{D}$. Taking into

[10] We include the short selling constraint for the sake of simplicity, as it avoids dealing with leveraged balance sheets. A realistic inclusion of short selling would require dealing with short selling fees and margin accounts.

account its inventory of assets, the central bank's demand for an asset is given by:

$$\Delta Q_{\mathrm{CB},a,t} = Q^*_{\mathrm{CB},a} - out_{a,t} Q_{\mathrm{CB},a,t-1} \tag{2.12}$$

The central bank agent pays the market price $P_{a,t}$ in domestic currency.

The underwriter agent (U) endows the model financial market with specific quantities of each asset $Q_{a,0}$. By re-issuing defaulting or maturing assets it furthermore makes sure that the available quantities for each asset stay constant:

$$\Delta Q_{\mathrm{U},a,t} = -Q_{a,0}(mat_{a,t} + \Omega_{a,t}) \tag{2.13}$$

2.4 Expectations

We assume that agents know the underlying stochastic processes for inflation and asset defaults, i.e. $\mathrm{E}_{d,t}[\pi^D_t] = \bar{\pi}^D$, $\mathrm{E}_{d,t}[\pi^F_t] = \bar{\pi}^F$ and $\mathrm{E}_{d,t}[\Omega_{a,t}] = \omega_a \cdot \lambda_{a,t}$. They do not know, however, how these processes translate to asset prices and their fluctuations. Prices are assumed to be efficient, i.e. $\mathrm{E}_{d,t}[P_{a,t+1}] = P_{a,t}$ and agents form adaptive expectations about the covariance of returns. With $\hat{\mathrm{M}}_t[x, \phi] \equiv (1-\phi)\hat{\mathrm{M}}_{t-1}[x, \phi] + \phi x_t$ defining the exponentially weighted moving average of variable x and $\phi \in [0, 1]$, agents expect the covariance between return r_{d,a_1} and r_{d,a_2} to be

$$\hat{\mathrm{Cov}}_{d,t}(r_{d,a_1}, r_{d,a_2}) := \hat{\mathrm{M}}_t \left[\left(r_{d,a_1,t-1} - \hat{\mathrm{M}}_{t-1}[r_{d,a_1}, \phi] \right) \left(r_{d,a_2,t-1} - \hat{\mathrm{M}}_{t-1}[r_{d,a_2}, \phi] \right), \phi \right]. \tag{2.14}$$

Note that the returns have an agent-specific index d because domestic and foreign agents will have different expectations about the covariance of asset a. If agent and asset are not domiciled in the same country, exchange rate risk will affect return fluctuations.

Unlike the price, the level of the exchange rate does not have an impact on the return of an asset.[11] Lacking an inverse relationship between the exchange rate level and the return of an asset, expectations of an efficient foreign exchange market (i.e. $\mathrm{E}_{d,t}[X^{a\to d}_{t+1}] = X^{a\to d}_t$) can lead to an excessively volatile exchange rate. Following a large literature (see e.g. Rogoff, 1996, for an early review), we assume that the purchasing power parity (PPP) is an anchor for the long-run real exchange rate. Agents therefore expect:

$$\mathop{\mathrm{E}}_{d,t}[X^{D\to F}_{t+1}] = X^{D\to F}_t + \eta(\mathrm{PPP}^{D\to F} - X^{D\to F}_t) \tag{2.15}$$

$$\mathop{\mathrm{E}}_{d,t}[X^{F\to D}_{t+1}] = \frac{1}{\mathrm{E}_{d,t}[X^{D\to F}_{t+1}]}, \tag{2.16}$$

where η is the expected speed of reversion to the PPP, which, for the sake of simplicity, we assume is constant over time.

2.5 Solving the model

Since expectations about risks are adaptive, we simulate the model in order to allow for risk factors to be adequately represented in covariance expectations. We develop an algorithm to determine each price $P_{a,t}$ and the exchange rate $X^{F\to D}_t$ for each period t so that excess demand for assets ($\Delta Q_{a,t} = \sum_d \Delta Q_{d,a,t} + \Delta Q_{\mathrm{CB},a,t} + \Delta Q_{\mathrm{U},a,t}$) and net capital flows between the two countries in our model approach zero. To achieve this, the algorithm updates all prices iteratively

[11] Technically this can be seen by substituting in Eq. (2.4) the home currency nominal value $N_{d,a}$ and price $P_{d,a,t}$ with their local currency representations from Eqs. (2.6) and (2.5), setting $P_{a,t} = P_{a,t-1}$, $X^{a\to d}_t = X^{a\to d}_{t-1}$ and computing the return via Eq. (2.3). While the return remains a function of the price, the exchange rate cancels out when dividing by $P_{d,a,t-1} = P_{a,t-1} \cdot X^{a\to d}_{t-1}$.

until all markets clear to a satisfactory extent. Since the number of iterations the algorithm needs to find $P_{a,t}$ grows exponentially as excess demand approaches zero, we allow markets to fall short of completely clearing. Specifically, we assume that the market clearance condition is satisfied when excess demand or supply for every asset is below 1% of outstanding assets (i.e. $\Delta Q_{a,t}/Q_{a,0} < 0.01$).[12]

The algorithm, which is inspired by algorithms commonly used for calibrating neural network parameters[13] follows 7 steps, which are executed for each asset and the exchange rate in each iteration:

1. compute excess demand $\Delta Q_{a,t}$ for current prices.
2. $G_{a,t} \leftarrow \begin{cases} 1 & \text{if } \frac{\Delta Q_{a,t}}{Q_{a,0}} > 1 \\ -1 & \text{if } \frac{\Delta Q_{a,t}}{Q_{a,0}} < 1 \\ \frac{\Delta Q_{a,t}}{Q_{a,0}} & \text{else} \end{cases}$ (update relative demand variable)
3. $J_{a,t} \leftarrow \text{sgn}(G_{a,t}G_{a,t'})$ (update jump indicator)
4. $G_{a,t'} \leftarrow G_{a,t}$ (store current relative demand variable)
5. $\hat{J}_{a,t} \leftarrow \phi_J \hat{J}_{a,t} + (1 - \phi_J)J_{a,t}$ (update EWMA of jump indicator)
6. $\alpha_{a,t} \leftarrow \begin{cases} 0.1 & \text{if } \alpha_{a,t}\exp(\hat{J}_{a,t} - \theta) > 0.1 \\ \alpha_{a,t}\exp(\hat{J}_{a,t} - \theta) & \text{else} \end{cases}$ (update learning rate)
7. $\log(P_{a,t}) \leftarrow \log(P_{a,t}) + \alpha_{a,t}G_{a,t}$ (update price)

In essence, in each iteration the algorithm updates the logarithm of an asset's price according to the excess demand $G_{a,t}$ for that asset. Assuming the demand curve for any asset is downward sloping, excess demand for an asset increases its price, while excess supply decreases it.[14] The rate at which a price is updated is defined by the learning rate $\alpha_{a,t}$. Intuitively, when the learning rate is too large excess demand will switch signs between iterations. Therefore, we reduce $\alpha_{a,t}$ when excess demand jumps from positive to negative or from negative to positive (i.e. $J_{a,t} = -1$). When excess demand, on the other hand, stays positive or negative between iterations (i.e. $J_{a,t} = 1$), then the learning rate is increased. In order to avoid frequent fluctuations of the learning rate, instead of using the jump indicator $J_{a,t}$ to update the learning rate, an exponentially weighted moving average of the jump indicator $\hat{J}_{a,t}$ is compared to a threshold θ. Only if $\hat{J}_{a,t}$ exceeds the threshold, will the learning rate increase. Otherwise it will decrease. For the algorithm to work best, the learning rate should approach its optimal (most efficient) value from below. While a lower-than-optimal rate will lead to convergence, a higher-than-optimal rate will not. We find that $\theta = 0.9$ and $\phi_J = 0.8$ work best for our purpose. Furthermore, it improves the stability of the algorithm to clip the relative demand variable $G_{a,t}$ to range between -1 and $+1$, and limit the learning rate to not exceed 0.1.

[12] Overall excess demand is distributed across agents proportional to their demand. For example, a 1% overall excess demand for an asset, will cause every agent with a positive demand for that asset to only purchase 99% or their demanded quantity.

[13] Neural networks feature many interdependent parameters, which need to be updated simultaneously. Like the ADAM algorithm developed by Kingma and Ba (2014), which has become a standard algorithm for neural network calibration, our algorithm includes individual adaptive learning rates.

[14] More commonly, $G_{a,t}$ would be defined as the gradient of an objective function penalizing deviations from market clearing prices. However, especially the no short selling constraint in Eq. (2.9) leads to "flat" regions of the objective function and frequent optimization failures. Using excess demand instead of the gradient has proven to be much more reliable within our model.

For the exchange rate, the relative demand variable $G_{X,t}$ reflects net capital flows instead of excess demand. Step 2. of the algorithm therefore is replaced by

$$G_{X,t} \leftarrow \begin{cases} 1 & \text{if } \frac{\Delta Q_{X,t}}{\sum_d W_{d,0}} > 1 \\ -1 & \text{if } \frac{\Delta Q_{X,t}}{\sum_d W_{d,0}} < 1 \\ \frac{\Delta Q_{a,t}}{Q_{a,0}} & \text{else,} \end{cases}$$

with net capital flows $\Delta Q_{X,t}$ in terms of domestic currency being defined as domestic agents ($d \in \mathcal{D}$) demand for foreign assets ($a \in \mathcal{F}$) minus foreign agents ($d \in \mathcal{F}$) demand in domestic assets ($a \in \mathcal{D}$):

$$\Delta Q_{X,t} \equiv X_t^{F \to D} \sum_{d \in \mathcal{D}} \sum_{a \in \mathcal{F}} \Delta Q_{d,a,t} P_{a,t} - \sum_{d \in \mathcal{F}} \sum_{a \in \mathcal{D}} \Delta Q_{d,a,t} P_{a,t}$$

3 A representative agent model of portfolio rebalancing

The full model laid out in the previous section includes feedbacks between returns, prices, the exchange rate and balance sheets and therefore allows for a more comprehensive modeling of heterogeneous assets and agents. However, the basic workings of portfolio rebalancing, which drive the impact of central bank asset purchases on returns, can best be understood by looking at only the portfolio optimization of agents. For this purpose we simplify the model in this section and analytically look at central bank asset purchases in a model with a representative domestic agent who holds three domestic assets (cash and two other assets). When there is only one representative agent, each portfolio weight w_a is constant for asset $a \in \{C, A0, A1\}$. We derive from the first order condition of the mean variance portfolio optimization the determinants of asset returns.

$$r_a = \lambda + w_C \rho_{a,C} \sqrt{\xi_a \xi_C} + w_{A0} \rho_{a,A0} \sqrt{\xi_a \xi_{A0}} + w_{A1} \rho_{a,A1} \sqrt{\xi_a \xi_{A1}}.$$

where $\xi_a \equiv \sigma_a \cdot \beta_a$ is the multiplication of an asset's return standard deviation and the representative agent's business model factor β_a towards that asset, ρ_{a_1,a_2} is the return correlation between assets a_1 and a_2 and λ is the lagrange multiplier.

We assume that central bank asset purchases induce a shift in portfolio weights from $A0$ to cash, with qe defining the size of the shift in percentage points relative to total assets. Since the return on cash r_C cannot change with asset purchases (denoted by superscript q), we can derive the lagrange multipliers from the equation for expected currency return.

$$\begin{aligned} \lambda^q &= r_C - (w_C + qe)\xi_C + (w_{A0} - qe)\rho_{C,A0}\sqrt{\xi_C \xi_{A0}} + w_{A1}\rho_{C,A1}\sqrt{\xi_C \xi_{A1}} \\ &= \lambda - qe(\xi_C - \rho_{C,A0}\sqrt{\xi_C \xi_{A0}}) \end{aligned}$$

From this we can derive expressions for the change in expected returns for assets $A0$ and $A1$:

$$\begin{aligned} r_{A0}^q - r_{A0} &= qe(2\rho_{C,A0}\sqrt{\xi_C \xi_{A0}} - \xi_{A0} - \xi_C) \\ r_{A1}^q - r_{A1} &= qe(\rho_{C,A0}\sqrt{\xi_C \xi_{A0}} + \rho_{C,A1}\sqrt{\xi_C \xi_{A1}} - \rho_{A0,A1}\sqrt{\xi_{A0} \xi_{A1}} - \xi_C) \end{aligned}$$

Expressing, for the sake of clearer presentation, ξ_a as multiples of each other, i.e. $b \equiv \xi_{A0}/\xi_C$ and $e \equiv \xi_{A1}/\xi_{A0}$:

$$\Delta r_{A0} = r_{A0}^q - r_{A0} = qe \cdot \xi_C(2\rho_{C,A0}\sqrt{b} - b - 1) \tag{3.1}$$

$$\Delta r_{A1} = r_{A1}^q - r_{A1} = qe \cdot \xi_C(\rho_{C,A0}\sqrt{b} + \rho_{C,A1}\sqrt{b \cdot e} - \rho_{A0,A1} b\sqrt{e} - 1) \tag{3.2}$$

Several insights result from Equations (3.1) and (3.2):

- **Generally**, i.e. regardless of the asset characteristics purchased by the central bank, the equations show that the concentration of the purchased asset on agents' balance sheet matters. The greater the concentration (the larger the parameter qe), the bigger the impact on the purchased asset $A0$ and on the not-purchased asset $A1$. This suggests that buying assests such as governement bonds that are highly concentrated on the balance sheets of commercial banks is ceteris paribus likely to have a bigger impact than purchasing corporate bonds that are less closely held. Furthermore, the impact (at least in the representative agent case) increases linearly with the volume of purchases, while the impact in basis points is independent of the return level of cash and $A0$.

- The **return impact on the purchased asset** has an upper bound of zero. This is the case when cash and $A0$ are perfect substitutes, i.e. $\rho_{C,A0} = 1$ and $b = 1$. Otherwise the return of asset $A0$ always declines when purchased by the central bank (see Figure 3). Thereby the impact decreases with an increasing return correlation between cash and $A0$. Furthermore, for $b > 1$, which means that the purchased asset $A0$ is perceived to be of higher risk (i.e. $\sigma_{A0} \cdot \beta_{A0} > \sigma_C \cdot \beta_C$), the impact increases with increasing b. Intuitively, the purchase of higher risk assets reduces the exposure to the corresponding risk factors, which leads to lower risk premia. In a scenario where cash is perceived to be riskier than the purchased asset ($b < 1$) and $\rho_{C,A0} > 0$, the impact can increase with decreasing b. The rational for this counterintuitive effect is that agents need to be incentivized for increasing their risk exposure by selling to the central bank. Since the interest rate of cash is fixed, the return of $A0$ must fall for agents to accept the higher risk.

- The **return impact on other assets** can be either positive or negative. If cash and the assets $A0$ and $A1$ are all uncorrelated (i.e. $\rho_{C,A0} = \rho_{C,A1} = \rho_{A0,A1} = 0$), the return on $A1$ will decline when the asset $A0$ is purchased by the central bank. Thereby the impact increases with increasing asset purchases and increasing aversion ξ_C towards cash. The impact on $A1$, in this case, does not depend on its characteristic. Whether $A1$ is a safe short term bond or a risky stock, whether the business model of the representative agent favors or opposes the asset, its yield will invariably decline by $-qe \cdot \xi_C$, which generates the necessary incentive for the agent to hold the extra cash created by central bank purchases. If the return on cash is not correlated to the returns of $A0$ and $A1$, but $\rho_{A0,A1} > 0$, the impact on $A1$ increases with increasing $\rho_{A0,A1}$, b and e. With other words, the higher the risk and business model opposition towards $A1$, the higher its impact (i.e. $\Delta r_{A1} = -qe \cdot \xi_C(1 + \rho_{A0,A1} b\sqrt{e})$). This relation has two important implications for the effectiveness of asset purchases. First, it implies that central bank asset purchases will flatten the yield curve of the assets whose risk factors are correlated with those of the purchased asset. This results from the assumption that σ_{A1} and therefore e will typically increase with maturity. Second, the higher the risk and business model opposition of the purchased asset $A0$, the bigger the impact on any correlated assets. This implies that in order to maximize the impact of QE on a specific asset, it does not necessarily make sense to purchase that asset. For example, if the central bank wanted to accelerate the transition to a carbon neutral economy by reducing yields of green bonds through asset purchases, it should counter-intuitively purchase conventional rather than green bonds.[15] The higher the correlation between cash and $A0$ and the greater the correlation between cash and $A1$, the lower the impact of asset purchases on $A1$-returns. For the case that all returns are perfectly

[15] We thereby assume that conventional and green bonds are perfectly correlated as they are issued by the same entity, while green bonds are favored by most business models, i.e. have a lower business model factor.Note that purchasing conventional bonds will maximize the absolute impact on yields, but may not maximize the spread between conventional and green bonds.

correlated (i.e. $\rho_{C,A0} = \rho_{C,A1} = \rho_{A0,A1} = 1$), the impact depends on how the risk of $A0$ and $A1$ is perceived relative to the risk of holding cash (i.e. $\Delta r_{A1} = qe \cdot \xi_C (\sqrt{\frac{\xi_{A0}}{\xi_C}} - 1)(1 - \sqrt{\frac{\xi_{A1}}{\xi_C}})$). When central bank asset purchases decrease the overall risk exposure (i.e. $\xi_{A0} > \xi_C$) then the yield of correlated assets will decrease with $\xi_{A1} > \xi_C$ and increase with $\xi_{A1} < \xi_C$. Intuitively, the overall reduction in risk exposure increases the risk-bearing capacity of the representative agent which increases demand for risky assets, while reducing the demand for safe assets. If purchases, however, increase the perceived risk exposure (i.e. $\xi_{A0} < \xi_C$) risk premia will increase for high-risk assets ($\xi_{A1} > \xi_C$) and decrease for low-risk assets ($\xi_{A1} < \xi_C$).

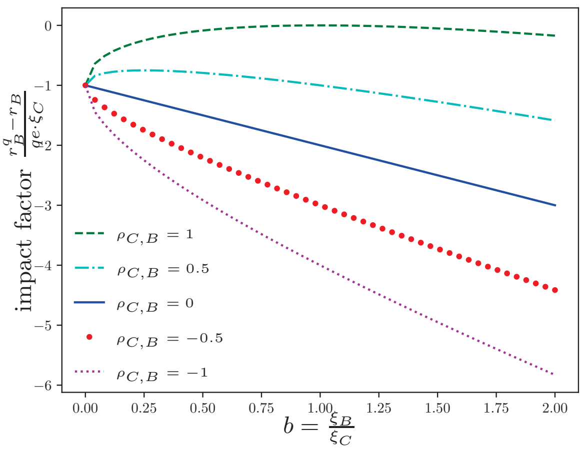


Figure 1: Impact factor of central bank bond purchases in dependence on the relative bond risk sensitivity b for different correlations between currency and bond returns.

4 Calibration

The two regions in our model are calibrated to data from the euro area (domestic country) and the rest-of-the-world (foreign country), which comprises 20 countries.[16] For the sake of simplicity and lacking better data, we take a mostly static calibration approach. This entails setting all observable variables with representations in our model to their respective values as measured at the end of 2014. Only the stochastic processes for inflation rates and default probabilities take into account time series data. We choose 2014 as the reference year since the ECB started its expanded asset purchasing program (APP) in March 2015.

Table 1 shows agents' initial balance sheet positions. Note that the underlying data is compiled and estimated from various sources and no claim is made to completeness. There are several differences between our data and the aggregate data in other studies looking at pre-APP asset holdings. While Eser et al. (2019) and Koijen et al. (2021) report more granular asset holdings

[16] US, UK, Australia, Brazil, Canada, Switzerland, China, Colombia, Hungary, Indonesia, India, Israel, Japan, Korea, Mexico, Norway, New Zealand, Russia, Turkey and South Africa. Of these countries, the US accounts for about 50% of the joint stock market capitalization and outstanding bonds volume.

Table 1: Balance sheet positions of funds and banks residing in the euro area (domestic) and in the rest-of-the-world (foreign) in € trillion. *Sources:* ECB Investment fund balance sheet statistics, CBD2, Bankscope, IMF Coordinated Portfolio Investment Survey, Morning Star Direct, Fred database and FSB Global Shadow Banking Monitoring Report.

	dom. funds	dom. banks	for. funds	for. banks	market size
dom. bonds	1.961	3.204	0.263	0.568	5.995
for. bonds	1.651	0.656	7.045	10.009	19.361
sum bonds	3.612	3.860	7.308	10.576	25.356
dom. equities	0.930	0.571	0.556	0.078	2.134
for. equities	1.821	0.075	8.461	1.692	12.049
sum equities	2.750	0.646	9.018	1.770	14.183
dom. currency	0.411	0.080	0.063	0.136	0.690
for. currency	0.218	0.016	1.202	2.400	3.836
sum currency	0.629	0.096	1.265	2.536	4.526
total assets	6.992	4.602	17.592	14.883	44.067

for euro area investors, both only report aggregate foreign investor holdings of euro area assets.[17] Both papers also do not include a breakdown of foreign assets. Since agent and asset heterogeneity is crucial in our context, we choose to work with our less precise, but broader portfolio data. Also missing in Eser et al. (2019) and Koijen et al. (2021) are investors' currency holdings. We need currency holdings to calibrate business model factors, which ultimately determine the substitutability between bonds and currency. However, our currency data, in particular for banks, is problematic. While all agents will hold some cash for purely transactional purposes, banks will also hold currency from institutional investors to whom they will pass on any accrued interest, even if negative (see Altavilla et al., 2019). Lacking a better understanding which part of banks' currency holdings are a result of portfolio optimization, we set their currency positions to their reserve holdings with the central bank. With respect to banks' bond and stock holdings, we only consider those that are classified as "trading assets" or "available for sale". We assume that balance sheet positions classified as "held to maturity" are not subject to banks' portfolio optimization. Considering that the ECB under the APP predominately purchased sovereign bonds, it is furthermore unfortunate that our data does not differentiate between sovereign and corporate debt holdings. Eser et al. (2019) and Koijen et al. (2021), on the other hand, are able to make this distinction, but do not exclude asset holdings classified as "held to maturity".

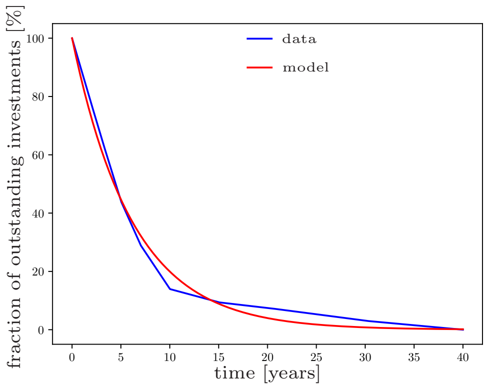
Since, in the benchmark calibration, we include only one bond portfolio per region, all agents hold bond portfolios with the same maturity structure. While this assumption is empirically clearly flawed, we do not think it has a large impact on our results. Eser et al. (2019) report that the average maturity for different investor groups they classify as being price-sensitive ranges from 5 to 7.3 years. In Table 2 we estimate the average maturity of bonds to be 6.25 years. Our one-parameter maturity model fits the data remarkably well, as can be seen when comparing the outstanding share of the bond portfolio over time (see the figure on the right of Table 2).

Table 3 contains all remaining parameters and initial values that we estimate/calibrate from data. Mean inflation expectations are sourced from the ECB's inflation forecast for the domestic country and the OECD's inflation forecast for the countries that make up the foreign region. Forecasts are made at the end of 2014 for the year 2015. Inflation volatility is estimated from

[17] Eser et al. (2019) assume that the foreign non-official sector is price-sensitive, while the foreign official sector is not. Koijen et al. (2021) do not make any assumptions about the price-sensitivity of investors.

Table 2: Portfolio maturity structure. The maturity structure of the bond portfolios are calibrated to match the average bond portfolio holdings of open-end investment funds resident in the euro area in December 2014. The data in the left panel is sampled from over 15,000 funds in the Morning Star database. We assume that the maturity dates are distributed equally within a maturity bin. The figure in the right panel compares the maturity data to the model portfolio with maturity parameter $m = 0.99936$, which corresponds to an average maturity of 6.25 years.

remaining maturity	% of total assets
0-1 year	11.5
1-3 years	22.7
3-5 years	21.7
5-7 years	15.0
7-10 years	15.1
10-15 years	4.6
15-20 years	1.9
20-30 years	4.4
30-40 years	3.1



20 years (1997-2017) of HICP inflation rate data for the respective regions. The default process is estimated with data from 20 years (1997-2017) of global default events published in the S&P 2017 Annual Global Corporate Default Study and Rating Transitions. Each region is assigned their share of global default events according to their relative total market capitalization. Model parameters are computed so that they minimize the quadratic distance in terms of first order autocorrelation and variance between the stochastic process and historic default events. Broadly in line with data from the S&P study, we assume that on average 2% of issuers default each year. Scaling factors for bonds are four times lower than for equities. We thereby assume that 50% of defaulting corporate debt can be recovered and that sovereign debt, which makes up half of the bond portfolio, does not default. Currency rates reflect the deposit facility of the respective central banks. Nominal yields for bonds reflect yields to maturity for domestic and foreign market S&P corporate and government bond indices. Equity yields are computed from the equity risk premia (as estimated by Absolute Strategy Research) plus the yield of the respective 10-year government bond. Rates and yields reflect end-of-2014 data and in the case of the foreign country are market cap weighted averages. We take the foreign exchange rate reversion speed from Rogoff (1996), while the exponentially moving average (EWMA) memory parameter used to update agents' covariance estimates is chosen to ensure the stability of the model.

Latent business model factors $\boldsymbol{\beta}_d$ are computed so that endogenous portfolio holdings of agents match the data given the processes for risk factors as well as end-of-2014 values for asset returns and the maturity structure of bonds. For the sake of simplicity we set all prices and the exchange rate to their nominal value and PPP, respectively (i.e. $P_{a,0} = X_0^{F \to D} = 1$). Then we use the algorithm from Section 2 to change the business model factors until each portfolio position deviates from its target value in Table 1 by no more than 1%. The relative demand variable $G_{d,a}$ for the business model factors tracks the difference between the initial portfolio positions and the position targeted by portfolio optimization, i.e. $\Delta Q_{d,a,0} = w_{d,a,0} S_{d,0} - Q_{d,a,0}$.

Updates of the business model factors build on the interpretation of a factor as an aversion towards the corresponding asset. A high relative demand for an asset thus leads to an increase in the business model factor. Steps 2 and 7 of the algorithm thus need to be replaced by:

2. $G_{d,a} \leftarrow \begin{cases} 1 & \text{if } \frac{w_{d,a,0}S_{d,0}-Q_{d,a,0}}{Q_{d,a,0}} > 1 \\ -1 & \text{if } \frac{\Delta Q_{d,a,0}}{Q_{d,a,0}} < 1 \\ \frac{\Delta Q_{d,a,0}}{Q_{d,a,0}} & \text{else} \end{cases}$

7. $\log(\beta_{d,a}) \leftarrow \log(\beta_{d,a}) + \alpha_{a,d,t}G_{d,a}$

The difficulty of calibrating business model factors lies in the circular relation between factors and model dynamics: business model factors affect the variance-covariance structure of assets, which affects portfolio decisions, which calls for a recalibration of business model factors. Therefore, after computing the business model factors we simulate the model for 100 periods and then recompute the factors. This procedure is repeated 1000 times. While most business model factors converge and remain reasonably stable, others fluctuate strongly. Tables 4 and 5 show the mean business model factors and their fluctuations within the last 100 repetitions of the calibration procedure. Note that while the specific value of a business model factor does not contain much information, the relative values of factors for an agent can be interpreted. As expected, bank agents, in contrast to fund agents, have higher business model induced aversions towards their respective home country equities than to bonds. Given equal risk and return characteristics, all agents except for domestic funds would prefer their respective home country's assets over the respective foreign assets. Thereby foreign agents, which represent mostly US investors, have a higher home bias than domestic banks, i.e. the relative difference between home and foreign business model factors is greater for foreign agents. Although business model factors for currencies are most volatile, all agents have a strong preference for the foreign currency (mostly dollars) relative to the domestic currency (euro). This may reflect the special role of the dollar as the leading international reserve currency. Compared to the domestic currency, agents mostly neglect the risk (inflation and exchange rate risk) associated with holding the foreign currency.

Table 3: Parameters of stochastic inflation and default rate processes, initial nominal interest rates and expectations parameters.

Category	Symbol	Description	Value
inflation process in Eq. (2.1)	$\bar{\pi}^D$	mean dom. inflation	1%
	$\bar{\pi}^F$	mean for. inflation	2.1%
	σ_π^D	dom. inflation volatility	1%
	σ_π^F	for. inflation volatility	0.7%
	$\mathrm{Corr}(\pi_t^D, \pi_t^F)$	inflation correlation	40%
default process in Eq. (2.2)	$\bar{\lambda}_{a\in\mathcal{D}}$	mean dom. default events	15/250
	$\bar{\lambda}_{a\in\mathcal{F}}$	mean for. default events	75/250
	$\phi_{\mathcal{D}}^{\Omega}$	AR parameter for dom. assets	0.0011
	$\phi_{\mathcal{F}}^{\Omega}$	AR parameter for for. assets	0.0014
	$\sigma_{\mathcal{D}}^{\Omega}$	dom. default shock volatility	0.00176
	$\sigma_{\mathcal{F}}^{\Omega}$	for. default shock volatility	0.01132
	ω_B	bonds scaling factor	0.005/250
	ω_E	equities scaling factor	0.02/250
	$\mathrm{Corr}(\epsilon_{B,t}^{\Omega}, \epsilon_{E,t}^{\Omega})$	bond-stock default rate corr.	0
	$\mathrm{Corr}(\epsilon_{\mathcal{D},t}^{\Omega}, \epsilon_{\mathcal{F},t}^{\Omega})$	dom.-for. default rate corr.	0
nominal rates	$r_C^{\mathcal{D}}$	dom. currency rate	-0.2%
	$r_C^{\mathcal{F}}$	for. currency rate	0.85%
	$r_B^{\mathcal{D}}$	dom. bond yield	0.93%
	$r_B^{\mathcal{F}}$	for. bond yield	1.92%
	$r_E^{\mathcal{D}}$	dom. equity yield	6.3%
	$r_E^{\mathcal{F}}$	for. equity yield	7.1%
expectation parameters in Eqs. (2.14) and (2.15)	η	fx reversion speed to PPP	15% p.a.
	ϕ	EWMA memory parameter	0.001

Table 4: Mean business model factors from 100 calibrations.

	dom. funds	dom. banks	for. funds	for. banks
dom. bonds	262.228	112.307	3423.45	1717.16
dom. equities	247.504	273.308	941.778	5119.94
for. bonds	178.323	305.705	110.256	69.945
for. equities	78.599	1070.64	43.785	180.816
dom. currency	7890.04	15536.9	5238.97	4801.78
for. currency	0.002	0.002	0.003	0.002

Table 5: Standard deviation (in percent) of business model factors from 100 calibrations.

	dom. funds	dom. banks	for. funds	for. banks
dom. bonds	4.5	2.8	19.5	7.8
dom. equities	7.5	6.3	11.4	13.8
for. bonds	7.4	6.1	6.8	4.5
for. equities	5.2	8.3	4.8	5.3
dom. currency	27.8	48.3	51.2	27.4
for. currency	22.9	36.9	106.7	39.8

5 Results

To analyze the impact of QE we simulate changing setups of the model with and without asset purchases by the central bank. Since agents' adaptive expectations of the variance-covariance structure of asset returns take time to adjust to changes in the experiment design, we sample variables at times $t \in \{2000, 2020, 2040, ..., 2980\}$ and then take the average value over those periods. In the benchmark simulations we consider 20 random seeds ($s \in \mathcal{S}$) and 100 different business model factor calibrations ($c \in \mathcal{C}$). In order to decrease the substantial simulation time, all other simulations are conducted with 10 random seeds ($s \in \mathcal{S}^*$) and 10 representative business model factor calibrations ($c \in \mathcal{C}^*$).

5.1 Benchmark

In our benchmark simulations the central bank either demands domestic bonds worth €1 trillion in nominal value or does not purchase assets at all. Tables 6, 7 and 8 show summary statistics of the impact on returns, prices and return volatilities.

As expected, we find that central bank purchases of domestic bonds lead to a decline in their yields. On average domestic bond yields decline by about 11 basis points, with 50 percent of all impacts ranging from approximately minus 9 to minus 13 basis points. Empirical studies estimate the impact range of a €1 trillion purchase (approximately 10% of euro area GDP in 2014) on a 10 year domestic government bond to lie between minus 27 and minus 64 basis points (see Andrade et al., 2016). However, these estimates cannot be directly compared with our benchmark result for several reasons. (1) We focus entirely on the effect of portfolio rebalancing, while empirical studies may also be picking up different impact channels such as the signaling channel or a liquidity channel. Within all impact channels central bank asset purchases lead to a reduction in yields. (2) We observe the impact on the entire bond portfolio, while empirical studies often report the impact on 10 year government bonds. Since the average maturity of our bond portfolio is substantially lower than 10 years, the empirical yield impact is expected to be more pronounced.[18] (3) Part of the subdued effect is due to an increase in return volatility. (4) While the ECB almost exclusively purchased government bonds within the APP, our bond portfolio represents a 50-50 mix of sovereign and corporate bonds. The effect of this assumption on the yield is unclear. A higher degree of market segregation would increase the impact of QE, while an arguably better substitutability between sovereign bonds and cash would reduce the yield impact of QE.

The yield impact on domestic equities and foreign assets are more than one order of magnitude lower than the impact on domestic bonds. Domestic equity yields decline on average by only

[18]In Section 5.3 we estimate the return impact on a 10-year bond to be on average 31 basis points.

0.17 basis points in response to a €1 trillion purchase of domestic bonds. This suggests that the substantial impact of QE on stock prices, which some studies find (see e.g. Chen et al., 2016; Fratzscher et al., 2016), is unlikely to be a result of portfolio rebalancing. Our result is more in line with Weale and Wieladek (2016), who, using a VAR approach, find no significant reaction of real share prices to the January 2015 QE announcement by the ECB. Note that the impact on domestic equities strongly depends on the correlation between bond and equity returns, with a higher correlation leading to a bigger impact (see Eq. 3.2).[19]

Interestingly, the impact of domestic bond purchases on foreign bond yields is slightly larger than zero. This seems counterintuitive, but can be explained by an increasing return volatility in response to QE. Table 8 shows that the volatility of expected returns increases for all domestic and foreign assets as well as the exchange rate. Agents will seek higher compensation for more volatile investments, which subdues the yield reducing impact of QE on all assets. Since default risk and inflation risk in the model are unaffected by central bank asset purchases, the increase in return volatility is nonfundamental. Its underlying source is the central bank's price-insensitive demand for domestic bonds.[20] Table 9 regresses changes in expected return on changes in volatility. The constant provides an estimate of by how much yields would decline if return volatility were to be unaffected by central bank purchases. Note that all portfolio returns decline in response to QE when controlling for changes in return volatility between simulations with and without QE. The average impact on bond returns increases to -18.4 basis points.

Central bank purchases of domestic bonds do impact exchange rates, as multiple studies have shown (see e.g. Neely, 2015; Bhattarai et al., 2021). In our benchmark simulations the Euro depreciates on average by 0.06% (see Table 7). The magnitude of the impact is in line with the empirical findings in Weale and Wieladek (2016). Since all agents expect the exchange rate to gradually revert back to its purchasing power parity, domestic agents reduce their expecations of annualized foreign currency returns by 0.88 basis points (see Table 6), while foreign agents increase their expectation of the domestic currency return by the same amount. This means that for the domestic agent the expected yields of all assets, bar domestic currency, decline, while foreign agents expect an increase in domestic currency and equity returns.

[19]Perfectly correlated default risk of bonds and equities causes equity yields to decline more than bond yields. This is the case because of a higher sensitivity of equity prices to changes in default probabilities due to their longer maturity and higher loss given default.

[20]Assuming downward sloping demand functions, a price change will lead all price sensitive agents to adjust their demand in a way that reduces excess demand or supply. For example, if an asset is in excess demand, an increase in its price will cause demand to decline and supply to increase. The inclusion of a price-insensitive agent, however, leads to a slower closing of the supply-demand differential. Larger price movements are thus necessary to clear markets. While a link between QE and nonfundamental volatility has not been established in the empirical literature, the above argument can help explain an increased volatility of stocks with a higher ETF ownership (see Ben-David et al., 2018). Since ETFs are easy and cheap to trade, they are likely to attract less sophisticated investors with less price sensitive demand functions.

Table 6: Impact of asset purchases on returns in basis points: Each data point R represents the average return of a simulation run defined by its calibration $c \in \mathcal{C}$, random seed $s \in \mathcal{S}$ and experiment $e \in \{0, 1\}$, which defines whether the central bank purchases assets ($e = 1$) or not ($e = 0$). Summary statistics are computed from the difference in sampled return of 2000 simulation pairs, i.e. $R_{c,s,e=1} - R_{c,s,e=0}$. The volume of asset purchases is €1 trillion.

	dom. bonds	dom. equities	for. bonds	for. equities	for. currency
mean	-10.97	-0.17	0.02	-0.3	-0.88
std	2.78	5.34	0.61	2.25	0.87
min	-19.9	-21.33	-2.81	-11.71	-5.45
25%	-12.95	-3.37	-0.3	-1.48	-1.21
50%	-11.36	-0.38	-0	-0.38	-0.71
75%	-9.27	2.93	0.32	0.76	-0.36
max	-0.89	21.51	3.16	12.3	2

Table 7: Impact of asset purchases on prices in percent: Each data point P represents the average price of a simulation run defined by its calibration $c \in \mathcal{C}$, random seed $s \in \mathcal{S}$ and experiment $e \in \{0, 1\}$, which defines whether the central bank purchases assets ($e = 1$) or not ($e = 0$). Summary statistics are computed from the percentage difference in sampled return of 2000 simulation pairs, i.e. $\frac{P_{c,s,e=1} - P_{c,s,e=0}}{P_{c,s,e=0}}$. The volume of asset purchases is €1 trillion.

	dom. bonds	dom. equities	for. bonds	for. equities	exchange rate
mean	0.64	0.02	-0	0.03	0.06
std	0.16	0.63	0.03	0.25	0.06
min	0.05	-2.52	-0.17	-1.4	-0.13
25%	0.54	-0.35	-0.02	-0.08	0.02
50%	0.66	0.04	0	0.04	0.05
75%	0.75	0.4	0.02	0.17	0.08
max	1.15	2.6	0.15	1.36	0.37

Table 8: Impact of asset purchases on volatility in percent: Each data point V represents the average expected return volatility of a simulation run defined by its calibration $c \in \mathcal{C}$, random seed $s \in \mathcal{S}$ and experiment $e \in \{0, 1\}$, which defines whether the central bank purchases assets ($e = 1$) or not ($e = 0$). Summary statistics are computed from the percentage difference in sampled volatilities of 2000 simulation pairs, i.e. $\frac{V_{c,s,e=1} - V_{c,s,e=0}}{V_{c,s,e=0}}$. The volume of asset purchases is €1 trillion.

	dom. bonds	dom. equities	for. bonds	for. equities	exchange rate
mean	3.67	0.22	0.02	0.02	1.34
std	1.19	0.42	0.12	0.23	9.37
min	1.53	-1.48	-0.74	-1.11	-26.44
25%	2.77	-0.03	-0.04	-0.1	-3.95
50%	3.46	0.2	0.02	0.01	-0.03
75%	4.4	0.45	0.09	0.13	5.51
max	8	1.86	0.47	1.52	82.73

Table 9: Impact of asset purchases on returns in basis points when controlling for changes in return volatility: Each data point in the regression represents the average return R or volatility V of a simulation run defined by its calibration $c \in \mathcal{C}$, random seed $s \in \mathcal{S}$ and and experiment $e \in \{0, 1\}$, which defines whether the central bank purchases assets ($e = 1$) or not ($e = 0$). We use the following regression equation: $\Delta R_{c,s} = \text{const} + \beta \Delta V_{c,s} + \epsilon_{c,s}$, with $\Delta R_{c,t} = R_{c,s,e=1} - R_{c,s,e=0}$ and $\Delta V_{c,s} = \frac{V_{c,s,e=1} - V_{c,s,e=0}}{V_{c,s,e=0}}$. Standard errors in parenthesis are robust.

	const	(SE)	β	(SE)	R^2
dom. bonds	-18.434***	(0.106)	2.034***	(0.031)	0.76
dom. equities	-2.618***	(0.066)	11.18***	(0.156)	0.76
for. bonds	-0.034***	(0.01)	3.124***	(0.1)	0.4
for. equities	-0.468***	(0.015)	9.123***	(0.085)	0.91
for. currency	-0.836***	(0.019)	-0.033***	(0.002)	0.13

In order to understand how the substitutability of domestic bonds for domestic currency affects yields, we plug in the balance sheet size, calibrated business model factors, variances and return correlations into Eq. 3.1 of the simplified model and compute the amount of domestic bonds agents are willing to substitute for domestic currency when the yield of the bonds is reduced by 1 basis point per year.[21] Table 10 shows the summary statistics of those amounts. Note that dividing 1000 by the sum of hypothetical bond sales reveals the yield reduction of the domestic bond implied by the simplified model. It ranges approximately from 17 to 25 basis points. A comparison with the full model's impact range from Table 6 (approximately -1 to -20 basis points) indicates that variation in random seeds and calibrations directly explains only a small part of the variation in QE impact. In fact most of the variation in impact is due to differences in return volatility between simulations with and without QE. These are a result of agents' adaptive expectations of return variance and covariances, which perpetuate small differences in return dynamics.

The most important insight from Table 10 is that domestic agents, when compared to foreign agents, are willing to substitute substantially larger amounts of domestic bonds for domestic currency. Therefore the business models of domestic agents are the more important drivers of QE effectiveness. This result is in stark contrast with the observation in Koijen et al. (2021) that predominantly foreign investors were reducing their exposure to domestic bonds in the wake of the APP. There are at least two explanations for this discrepancy: First, we may be omitting relevant foreign investors in our calibration. While the domestic bond holdings of our domestic agents approximately match those reported in Koijen et al. (2021), the domestic bond holdings of our foreign agents make up less than 20% of foreign investor holdings reported in Koijen et al. (2021). This means that foreign investors that are not banks or mutual funds hold a substantial portion of domestic bonds and are eager to exchange those bonds for domestic currency. If, as in our model, foreign investors expect the euro to appreciate as the exchange rate reverts to the PPP, it is plausible that they would want to increase their currency holdings in the wake of QE. A second explanation for the discrepancy between the data and our simulation results lies with the time period for which Koijen et al. (2021) report the rebalancing flows. Between

[21] Specifically, the amounts in Table 10 are computed by:

$$-\frac{\text{1bps p.a.} \cdot \text{TA}_{d,t}}{2\rho_{d,C,B}\sqrt{\beta_{d,B}\beta_{d,C}\sigma_{d,B,t}\sigma_{d,C,t}} - \beta_{d,C}\sigma_{d,C,t} - \beta_{d,B}\sigma_{d,B,t}},$$

where TA stand for total assets.

Table 10: Hypothetical domestic bond sales by agents to the central bank implied by simplified model in Eq. (3.1). Values, in billions of euro, indicate the volume of domestic bonds agents are willing to substitute for domestic currency when the domestic bond yield declines by 1 bps. Return correlations, return volatility and wealth are averages of a simulation run defined by its calibration $c \in \mathcal{C}$, random seed $s \in \mathcal{S}$ and experiment $e = 1$, in which the central bank purchases €1 trillion of domestic bonds. Summary statistics are computed from 2000 data points.

	dom. funds	dom. banks	for. funds	for. banks	overall demand
mean	17.1	21.41	3.63	5.94	48.08
std	0.7	2.36	0.84	0.46	3.08
min	15.22	14.87	2.44	4.88	39.87
25%	16.61	20.01	3.07	5.62	45.99
50%	17.04	21.63	3.43	5.92	47.88
75%	17.56	23.21	3.95	6.21	49.9
max	19.89	26.68	7.15	7.74	59.72

Table 11: Portfolio rebalancing after asset purchases with adapting covariance structure: Each data point represents the average asset position of a simulation run defined by its calibration $c \in \mathcal{C}$, random seed $s \in \mathcal{S}$ and experiment $e \in \{0, 1\}$, which defines whether the central bank purchases assets ($e = 1$) or not ($e = 0$). Mean and standard deviations (in parenthesis) are computed from the difference in average asset positions of 2000 simulation pairs, i.e. $Q_{c,s,e=1} - Q_{c,s,e=0}$.

	dom. funds		dom. banks		for. funds		for. banks	
dom. bonds	-342.6	(16.7)	-492.5	(22.4)	-55.7	(13.1)	-108.8	(12.2)
dom. equities	-2.2	(3.3)	2.0	(3.7)	0.2	(5.5)	0.1	(1.6)
for. bonds	-3.0	(26.0)	4.1	(10.9)	-0.5	(26.1)	-0.6	(26.8)
for. equities	4.9	(7.4)	1.2	(1.2)	-6.3	(6.4)	0.1	(3.6)
dom. currency	220.5	(140.1)	126.0	(119.9)	323.4	(179.4)	339.8	(98.2)
for. currency	126.4	(134.1)	364.9	(124.7)	-261.6	(179.8)	-229.7	(100.1)

2015Q1 and 2017Q4 interest rates in the US increased substantially giving foreign investors good reason to reduce their euro-exposure while increasing their dollar-exposure.[22] Changing economic conditions rather than QE itself, could thus be the reason for foreign investors' portfolio rebalancing.

Table 11 shows the average amounts and standard deviations of agents' portfolio rebalancing in response to QE. While the distribution of domestic bonds sold by different agents qualitatively matches those of the simplified model in Table 10, substantial rebalancing of currency positions also accompany QE. In particular, domestic agents diversify into the foreign currency in order to get rid of some of the domestic currency they obtained by selling bonds to the central bank. Foreign agents, on the other hand, further want to increase their domestic currency holdings by selling their own currency. This makes sense for them as they expect the depreciated domestic currency to bounce back eventually, allowing them to profit from their currency holdings.

[22] Fama (1984) was the first to empirically show that the expected return on a domestic-long and foreign-short carry trade increases with the domestic minus foreign interest rate differential.

Table 12: Impact of different QE volumes on returns in basis points. We control for changes in return volatilities between simulations with and without QE. Each data point in the regression represents the average return R or volatility V of a simulation run defined by its calibration $c \in \mathcal{C}*$, random seed $s \in \mathcal{S}*$ and qe volume $qe \in \{0, 500, 1000, 1500, 2000, 2500, 3000, 3500, 4000\}$. We use the following regression equation: $\Delta R = \beta_1 \frac{qe \text{ [in bn. EUR]}}{500\text{bn. EUR}} + \beta_2 \left(\frac{qe \text{ [in bn. EUR]}}{500\text{bn EUR}}\right)^2 + \beta_3 \Delta V + \epsilon$, with $\Delta R = R_{c,s,qe=x} - R_{c,s,qe=0}$ and $\Delta V = \frac{V_{c,s,qe=x} - V_{c,s,qe=0}}{V_{c,s,qe=0}}$. Standard errors in parenthesis are robust.

	β_1	(SE)	β_2	(SE)	β_3	(SE)	R^2
dom. bond	-8.173***	(0.188)	-0.368***	(0.012)	1.96***	(0.109)	0.99
dom. equity	-1.345***	(0.099)	-0.077***	(0.016)	11.323***	(0.324)	0.85
for. bond	-0.094***	(0.02)	-0.0	(0.004)	3.654***	(0.115)	0.82
for. equity	-0.339***	(0.023)	0.001	(0.004)	9.464***	(0.118)	0.97
exp. for. currency	-0.319***	(0.037)	-0.009	(0.006)	-0.091***	(0.005)	0.85

5.2 Scaling QE

While the representative agent model in Section 3 implies a linear relationship between the volume of asset purchases and the impact on yields, Table 10 already indicates that heterogeneity can break that linearity. With increasing asset purchases, agents for which domestic bonds and domestic currency are inferior substitutes will need to be increasingly incentivized to sell their domestic bonds to the central bank. In order to see how assets react to increasing QE-volumes, we simulate our model for 8 different QE volumes ranging from €500 billion to €4 trillion. The results are summarized in Table 12, for which we regress the change in yield relative to the no-QE case on the volume. We include a quadratic term to detect a non-linear effect and control for changes in return volatility between simulations with and without QE. For domestic bond and equity portfolios central bank asset purchases have a significant non-linear effect. This is not the case for foreign asset portfolios and changes to expected foreign currency returns. According to the results in Table 12 we estimate the ECB's Asset Purchase Program (APP), under which bonds worth approximately €3 trillion (until June 2021) have been bought, to have reduced the yields of the euro area bond portfolio by approximately 62 basis points. The pandemic emergency purchase programme (PEPP), which by July 2021 has added about €1.2 trillion in bond purchases to the ECB's balance sheet, further reduced yields on average by 32 basis points.

5.3 Portfolio and Asset Maturity

The representative agent model in Section 3 suggests that increasing the maturity of purchased assets increases the overall impact of QE. We modify the benchmark setup of our model in order to analyze the impact of changing the maturity of purchased assets. Furthermore, we use the new setup to estimate the impact of asset purchases on the yield of an asset of a particular maturity rather than on the yield of a portfolio with an average maturity.

To obtain a new setup that remains comparable to the benchmark, we split the domestic and foreign bond portfolios into two equally sized portfolios, respectively. The split portfolios are assigned different average maturities, while the combined average maturity is kept constant at 6.25 trading years, i.e. $6.25 = 0.5(T_m^{\text{long}} + T_m^{\text{short}})$. Specifically, we choose $T_m^{\text{short}} \in \{1, 1.5, 2, 2.5, 3, 3.5, 4\}$ trading years and correspondingly $T_m^{\text{long}} \in \{11.5, 11, 10.5, 10, 9.5, 9, 8.5\}$

trading years.[23] All other parameters in the model stay the same. We thereby make the simplifying assumption that business model factors do not change with changing average portfolio maturity.[24] The different maturities of the portfolios will be reflected in the covariance estimates of agents, which adapt endogenously during simulations.

In simulations the central bank can purchase either the portfolio with the long or short average maturity. In Table 13 we report the impact asset purchases of varying average maturity have on the portfolios in the model. Note that for the bond portfolios we report the impact on the combined expected return of the short and long portfolios, i.e. $\frac{1}{2}(r^{\text{long}} + r^{\text{short}})$. Furthermore, we control for changes in return volatility. The benchmark result computed from Table 12 is included for comparison. Table 13 shows that the impact of QE tends to increase with increasing maturity of purchased assets. This effect is most clear for the domestic bond portfolio, where an increase in the average maturity of purchased assets by 1 year reduces the average domestic bond yield by approximately 2 basis points. Increasing the average maturity of purchased bonds also increases the impact on the domestic equity portfolio as well as the foreign portfolios. However, because the correlation between domestic bond returns and the other returns is less pronounced, the relationship between purchased maturity and impact is noisier.

We can use the setup of the split bond portfolios to approximate the impact QE has on the yield curve. Thereby we compare simulations without QE with simulations in which the central bank purchases €500 billion of the domestic high and low maturity portfolios. The simulations produce return data for the 7 bond portfolio pairs (with average maturity of T_m^{short} and T_m^{long}). Note that while the average maturity of the joint bond portfolios and the average maturity of the central bank asset purchases stays constant in all simulations, the maturity distribution of assets within the portfolios (i.e. the share of the joint portfolio that matures at a given date) is not the same. Each setup can therefore have unintended consequences on simulation outcomes and add noise to our measurements of domestic bond returns. To some extend we can control for these consequences by regressing bond returns on changes in domestic and foreign equity portfolio returns as well as exchange rate volatility. These variables would not be affected if the setups with different portfolio pairs would be perfectly comparable. Figure 2(a) shows the mean portfolio yields for different maturities with and without QE. The graph shows that the impact of QE increases with increasing maturity, i.e. the yield curve flattens with central bank asset purchases. However, the illustrated portfolio yield curves $r(m)$ cannot be equated to spot yield curves $r(t)$, which would allow us to estimate the impact of QE on a specific maturity such as a 10 year bond. Nevertheless there is a relation between $r(m)$ and $r(t)$. Specifically, the portfolio yield curve $r(m)$ can be expressed as the infinite sum of spot returns $r(t)$ weighted by the share of assets that matures at time t, i.e.

$$r(m) = \sum_{t=1}^{\infty} (1-m) m^{t-1} r(t). \tag{5.1}$$

We can approximate $r(t)$ by replacing it with the parametric yield curve model proposed by Svensson (1994):

$$r^{\text{Sv}}(t) = \beta_0 + \beta_1 \frac{1-\exp(\frac{-t}{\tau_1})}{\frac{t}{\tau_1}} + \beta_2 \left(\frac{1-\exp(\frac{-t}{\tau_1})}{\frac{t}{\tau_1}} - \exp(\frac{-t}{\tau_1}) \right) + \beta_3 \left(\frac{1-\exp(\frac{-t}{\tau_2})}{\frac{t}{\tau_2}} - \exp(\frac{-t}{\tau_2}) \right) \tag{5.2}$$

[23]Because portfolios with a similar risk-return profile can lead to problems in the portfolio optimization of agents, we refrain from including portfolios pairs with smaller differences in average maturity than 4.5 years.

[24]With detailed data on the maturity structure of investors' asset holdings, we could recalibrate business model factors to account for maturity preferences. However, this data is not available to us.

Table 13: Impact of asset purchases of varying average maturity on returns in basis points. We control for changes in return volatility by regressing the return impact on changes in return volatility. Each data point in the regression represents the average return R or volatility V of a simulation run defined by its calibration $c \in \mathcal{C}*$, random seed $s \in \mathcal{S}*$, the average maturity of the purchased portfolio $T_m \in \{1, 1.5, 2, 2.5, 3, 3.5, 4, 8.5, 9, 9.5, 10, 10.5, 11, 11.5\}$ [in years] and experiment $e \in \{0, 1\}$, which defines which domestic bonds the central bank purchases. We use the following regression equation: $\Delta R_{c,s,T_m} = \text{const} + \beta \Delta V_{c,s,T_m} + \epsilon_{c,s}$, with $\Delta R_{c,s,T_m} = R_{c,s,T_m,e=1} - R_{c,s,T_m,e=0}$ and $\Delta V_{c,s,T_m} = \frac{V_{c,s,T_m,e=1} - V_{c,s,T_m,e=0}}{V_{c,s,T_m,e=0}}$. The reported impact is the constant of the regression equation. For bond portfolios we report the average reduction in yield of both the long and short maturity portfolio (i.e. $\frac{1}{2}(\Delta R^{\text{long}}_{c,s,T_m} + R^{\text{short}}_{c,s,T_m})$). Standard errors in parenthesis are robust. Standard errors for the joint bond portfolios are bootstrap estimates.

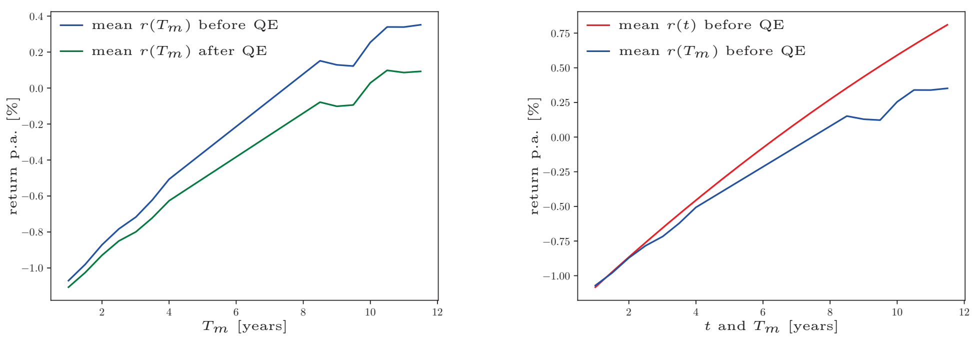
maturity [y]	dom. bond	(SE)	dom. equity	(SE)	for. bond	(SE)	for. equity	(SE)
1	-4.27	(0.32)	2.08	(1.25)	0.18	(0.15)	0.33	(0.44)
1.5	-4.77	(0.37)	-3.17	(1.88)	0.1	(0.15)	-0.29	(0.59)
2	-5.86	(0.33)	-5.93	(1.27)	0.75	(0.16)	-0.22	(0.47)
2.5	-7.14	(0.36)	-6.29	(1.09)	0.1	(0.22)	-1.82	(0.28)
3	-7.94	(0.42)	-3.34	(0.93)	-0.1	(0.26)	-0.75	(0.21)
3.5	-10.03	(0.53)	-4.06	(0.91)	-0.27	(0.15)	-1.34	(0.23)
4	-12.27	(0.43)	-2.91	(0.75)	-0.13	(0.09)	-0.7	(0.2)
6.25(bench.)	**-17.82**		**-3.00**		**-0.19**		**-0.68**	
8.5	-22.14	(0.62)	-8.24	(0.66)	-0.2	(0.09)	-1.58	(0.17)
9	-22.67	(0.54)	-6.7	(0.81)	-0.6	(0.1)	-1.82	(0.21)
9.5	-21.96	(0.56)	-7.96	(1.17)	-0.45	(0.24)	-1.85	(0.25)
10	-23.11	(0.61)	-10.21	(0.94)	-0.45	(0.19)	-2.49	(0.24)
10.5	-24.01	(0.73)	-8.49	(1.19)	-0.12	(0.15)	-2.12	(0.37)
11	-25.88	(1.09)	-12.26	(1.19)	-0.14	(0.14)	-2.34	(0.53)
11.5	-25.13	(0.66)	-7.79	(1.42)	-0.46	(0.1)	-1.96	(0.46)

The Svensson model is flexible enough to fit a wide range of term structures and is therefore used by many central banks, including the ECB, to estimate the yield curve from bond data. When inserting Eq. (5.2) into Eq. (5.1), the infinite sum converges and we can numerically solve for the parameters of the Svensson model that minimize the quadratic distance to the simulation returns $r(m)$:

$$\{\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\tau}_1, \hat{\tau}_2\} = \underset{\{\beta_0,\beta_1,\beta_2,\beta_3,\tau_1,\tau_2\}}{\arg\min} \sum_m \left(r(m) - \sum_{t=1}^{\infty} (1-m) m^{t-1} r^{\mathrm{Sv}}(t) \right)^2 . \tag{5.3}$$

Figure 2(b) compares the estimated spot yield curve (red line) to the portfolio yield curve without central bank asset purchases (blue line). Plausibly, the difference between spot returns and simulated portfolio returns is smaller at the short end of the yield curves. When t and T_m approach zero and the portfolio collapses into assets that mature overnight the returns of the spot and portfolio rates must be the same. Note that the concave form of the portfolio returns $r(m)$ can be explained with Eq. (2.4). While the contribution of the inherently uncertain price effect on portfolio return is linearly increasing with m, it is not a linear function in average maturity $T_m = \frac{1}{1-m}$. With $m'(T_m) > 0$ and $m''(T_m) < 0$, increases in the maturity parameter m become smaller with increasing average maturity. This gives rise to the concavity of term premia observed in normal times, i.e. when the yield curve is not inverted.

In addition to the yield curve estimated from simulated median portfolio returns, Figure 5.3 includes estimates for the 2.5 and 97.5 percentiles (dashed green lines) of portfolio returns as well as the ECB's end-of-2014 estimates for the euro area government bond term structure.[25] Since the bond portfolios in the model group sovereign and corporate debt together, the steeper term structure estimated from the simulated data is a plausible result.



(a) portfolio yield curves

(b) yield curve transformation

Figure 2: (a) Portfolio yield curves before and after central bank asset purchases. (b) Portfolio and spot yield curves before central bank asset purchases.

The impact of QE on spot yields is derived by estimating the spot yield curves from simulations with and without central bank asset purchases. Figure 5.3 shows the average impact and the 2.5-97.5 percentile range of the estimated impact of €1 trillion domestic bank purchases of an average maturity of 6.25 years. Note that estimating the parameters of the Svensson yield curve model allows us to extend our analysis beyond the average portfolio maturities simulated.

[25] The nominal yield curves for each day are estimated with the Svensson model and are published on the ECB's website (ECB, 2021). We subtract a 1% inflation expectation to obtain real values.

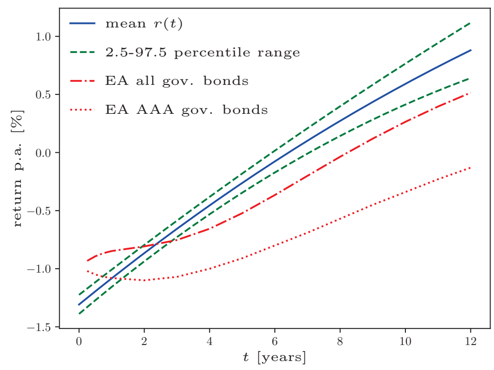


Figure 3: Domestic bond yield curves from the model and euro area government bond yield curves on 29.12.2014 from ECB (2021).

However, the estimates become less reliable as we look at higher maturities. For a 10 year bond, the yield reduction due to QE ranges from approximately 19-42 basis points with an average of around 31 basis points. This estimate lies within the range of 27-64 basis point range estimated for a 10 year government bond from event studies (Andrade et al., 2016). For a 5 year bond the estimated yield reduction ranges from 9-23 basis points, while the impact on a 1 year bond is estimated to range from positive 2 to negative 7 basis points. Our estimates thus imply a substantial flattening of the yield curve due to QE.

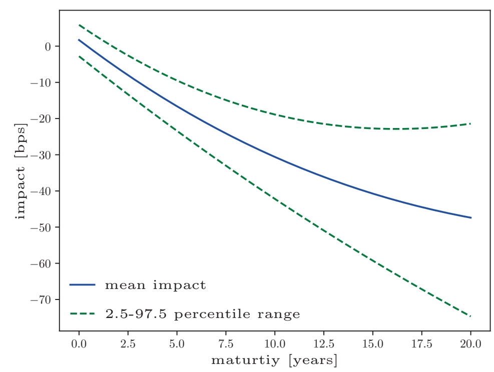


Figure 4: Impact of €1 trillion in bond purchases with an average maturity of 6.25 years on the domestic yield curve.

5.4 Unwinding QE

In the previous two sections we have shown that both the volume and average maturity of asset purchases have an effect on bond yields. When passively unwinding QE programs both the volume and average maturity of central bank bond holdings change over time. Note that our modeling of maturity as a constant repayment rate implies that a change in portfolio size does not alter the remaining average maturity. Since this assumption is unrealistic when considering the impact of unwinding QE, we compute the outstanding portfolio volume and remaining average maturity from the raw data used to calibrate the maturity parameter m in Table 2. Table 14 shows that in this case the average maturity of the unwinding portfolio first slightly declines and then increases substantially. In order to assess the impact of unwinding QE on average bond yields, we combine the results from the previous two sections. We start with bond holdings worth €4.2 trillion, which is approximately the combined volume of bonds the ECB purchased through the APP and PEPP until July 2021. Note that we do not perform additional simulations in this section and therefore need to make assumptions about the linear and quadratic terms reported in Table 12 for different average bond portfolio maturities. We assume that the factors change proportionately to the change in average yield impact reported in Table 13. For example, when considering an average maturity of 1 year, $\beta_1 = -8.173\frac{-4.27}{-17.82}$ and $\beta_2 = -0.368\frac{-4.27}{-17.82}$, where -17.82 is the average impact on the bond portfolio yield in the benchmark case.[26] Furthermore, we interpolate and where necessary linearly extrapolate average return impacts reported in Table 13 to compute the effect for the missing bond portfolio maturities. Table 14 shows the estimated increases of average domestic bond yields over 20 years of passively unwinding the €4.2 trillion in central bank purchases. Note that due to the non-linear impact of QE-size the expected increases in bond yields are largest in the first years of unwinding. After four years, approximately 55% of the yield reduction due to QE is reversed, while approximately 45% of the portfolio has matured. The fact that the portfolio's average remaining maturity does not strongly decline in the process of passively unwinding QE slightly subdues yield increases.

[26]When using the same scaling method for both parameters, the sum of squared differences between the computed impact (assuming purchases worth €1tr) and the simulated impact from Table 13 is minimized.

Table 14: Impact on domestic bond yields when unwinding €4.2 trillion of purchased bonds within year Δt after purchases stop. The yield impact is computed as the difference between the domestic bond yields between two consecutive years, i.e. $Y(\Delta t) = \text{yield}(\Delta t) - \text{yield}(\Delta t - 1)$. Yields take into account both the volume and maturity effect of purchases. Specifically, the yield is computed from scaling the parameters β_1 and β_2 from Table 12 by the average impact $\Delta R(M)$ on the domestic bond portfolio reported for varying average portfolio maturities in Table 13, i.e. $\beta_1(\Delta t) = \frac{-8.173}{-17.82}\Delta R(M(\Delta t))$ and $\beta_2(\Delta t) = \frac{-0.368}{-17.82}\Delta R(M(\Delta t))$ We linearly interpolate and extrapolate to get the average impact for maturities not simulated for Table 13. The volume $V(\Delta t)$ and average maturity $M(\Delta t)$ of bonds remaining the the central bank's portfolio in a given year are derived from the data in Table 2. With y_0 denoting the bond yield prior to QE, we estimate the domestic bond yield at time Δt to be $\text{yield}(\Delta t) = y_0 + \beta_1(\Delta t)\frac{V(\Delta t)}{€0.5\text{ tr.}} + \beta_2(\Delta t)\left(\frac{V(\Delta t)}{€0.5\text{ tr.}}\right)^2$.

delta time [y]	avg. out. maturity [y]	volume matured [€ bn.]	yield impact [bps]
1	6.45	482.15	15.85
2	6.32	475.58	13.86
3	6.32	475.56	12.07
4	6.47	456.45	10.22
5	6.95	456.39	8.71
6	7.27	315.91	6.25
7	8.02	315.52	5.36
8	8.6	212.08	3.92
9	9.74	211.8	4.81
10	12.08	211.8	1.76
11	11.9	38.78	1.3
12	11.76	38.3	1.22
13	11.67	38.3	1.13
14	11.66	38.3	1.04
15	11.74	38.3	0.96
16	11.22	16.09	1.09
17	10.7	16.03	0.68
18	10.17	16.03	0.82
19	9.64	16.02	0.72
20	9.1	16.03	0.27

5.5 Green QE

Two measures discussed in the context of green monetary policy have been the extension of collateral eligibility criteria to encompass more "green" assets and the increased direct purchases of green assets within central banks' asset purchasing programs. We can analyze the impact of both measures on asset returns by slightly deviating from the benchmark setup. Specifically, we assume that new collateral eligibility criteria would change the business model factor the domestic bank agent attaches to domestic bonds. Business model factors can also be varied to allow for a simple and general distinction between green and conventional (brown) assets. A reduction in the business model factor agents attach to a portion of a bond portfolio can be interpreted as an increase in non-financial utility investors seem to draw from green investments (Hong and Kacperczyk, 2009; Riedl and Smeets, 2017). This additional utility drives a wedge between otherwise identical green and brown asset returns, which is referred to as the greenium.

5.5.1 Collateral elibibility

In order to estimate the effect of changing collateral eligibility criteria on bond returns we simulate our model with varying values of the business model factor that the domestic bank agent attaches to domestic bonds. Specifically, we multiply the benchmark business model factor $\beta_{\text{dom.bank,dom.bond}}$ with $\nu \in \{1, 0.9, 0.8, 0.7, 0.6, 0.5\}$ so that the new factor $\beta'_{\text{dom.bank,dom.bond}} = \nu * \beta_{\text{dom.bank,dom.bond}}$. A lower business model factor imposes a more favorable view of the domestic bond portfolio on the domestic bank agent, which is justified as a larger portion of that portfolio can now be used in credit operations with the central bank.[27] Table 15 shows that a decrease of domestic banks' business model factor associated with domestic bonds, leads to a reduction in their yields. Intuitively, the domestic bank agent's lower aversion towards domestic bonds increases its demand for them. Higher demand leads to an increasing price, which lowers the expected yield. As shown in Table 15 the domestic bank agent increases its holdings of domestic bonds, while selling domestic and foreign currency as well as foreign bonds. Note that the domestic bank agent predominantly sells low risk assets to obtain domestic bonds, as these are better substitutes for each other. When $\nu \leq 0.7$ the domestic bank agent sells all of its domestic and foreign currency holdings. As the business model factor is further reduced the domestic bank agent increasingly sells other assets in order to buy domestic bonds. This leads to increasing yields for the foreign bond portfolio, while the yield impact on other assets becomes less negative. The general decline in yields when ν is close to 1 is explained by an increase in the overall risk tolerance of the domestic bank agent. The aversion towards domestic bond risk factors declines as the expanded collateral eligibility criteria generates additional utility for the domestic bank agent. When ν declines further the increase in supply of assets other than domestic bonds counteracts the effect of the domestic bank agent's increased risk tolerance.

While data confirms that extending collateral eligibility will lower the corresponding assets' yield (see e.g. Corradin and Rodriguez-Moreno, 2016; Macaire and Naef, 2021), it is unclear how such a policy would interact with central bank asset purchases. In the representative agent model in Section 3 a reduction in the business model factor of the assets purchased by the central bank will reduce the impact of QE for $b > 1$.[28] However, in the full model with heterogeneous agents the change in impact due to changes in the business model factor is more complex. Table 16 shows that a decreasing business model factor first leads to a reduction in impact on domestic

[27] Note that collateral eligibility of government bonds is already reflected in the bank agent's calibrated business model factor. Since the bond portfolio comprises both government and corporate bonds, a reduction in business model factor can be interpreted as reflecting an expansion of the eligibility criteria.

[28] When $b < 1$ and $\rho_{C,\text{A0}} > 0$ the impact could also increase. However, even for $\nu = 0.5$ we find that $b(\nu) = \frac{\nu\beta_{\text{dom.bank,dom.bond}}\sigma_{\text{dom.bank,dom.bond}}}{\beta_{\text{dom.bank,dom.curr}}\sigma_{\text{dom.bank,dom.curr}}}$ is greater than 1.

Table 15: Effect of different business model factors that the domestic bank attaches to domestic bonds on returns and domestic bank portfolio weights. Return impact is given in basis points and portfolio weight impact in percentage points. Each data point represents the average return R or portfolio weight W of a simulation run defined by its calibration $c \in \mathcal{C}*$, random seed $s \in \mathcal{S}*$ and multiplier $\nu \in \{1, 0.9, 0.8, 0.7, 0.6, 0.5\}$. When computing the return impact we control for changes in return volatility, i.e. $\Delta R_{c,s,\nu} = \text{const} + \beta \Delta V_{c,s,\nu} + \epsilon_{c,s,\nu}$ with $\Delta R_{c,s,\nu} = R_{c,s,\nu} - R_{c,s,\nu=1}$ and $\Delta V_{c,s,\nu} = \frac{V_{c,s,\nu} - V_{c,s,\nu=1}}{V_{c,s,\nu}}$. The constant of the regression equation is the reported mean return impact. The impact on asset weights is computed as $W_{c,s,\nu} - W_{c,s,\nu=1}$. The standard deviation of return and asset weight impact is stated in parenthesis.

	0.5	0.6	0.7	0.8	0.9
dom. bonds return	-11.34	-9.2	-6.71	-5.55	-4.35
	(6.81)	(6.83)	(5.85)	(5.82)	(3.38)
dom. equities return	0.41	-0.05	-1.1	-1.82	-1.52
	(4.15)	(4.16)	(3.98)	(3.87)	(2.59)
for. bonds return	0.87	0.63	0.32	0.08	-0.11
	(0.67)	(0.69)	(0.58)	(0.67)	(0.41)
for. equities return	-0.25	-0.3	-0.36	-0.34	-0.37
	(0.77)	(0.83)	(0.71)	(0.72)	(0.51)
dom. bond weight	7.43	6.1	4.81	3.56	2.13
	(1.28)	(1.26)	(1.24)	(1.19)	(0.8)
dom. equity weight	-0.44	-0.32	-0.2	-0.1	-0.03
	(0.1)	(0.1)	(0.1)	(0.1)	(0.08)
for. bond weight	-4.81	-3.61	-2.44	-1.33	-0.39
	(0.56)	(0.56)	(0.57)	(0.56)	(0.38)
for. equity weight	-0.03	-0.02	-0.01	-0.0	0.01
	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)
dom. currency weight	-0.89	-0.89	-0.89	-0.88	-0.71
	(1.07)	(1.07)	(1.07)	(1.05)	(0.81)
for. currency weight	-1.27	-1.27	-1.27	-1.26	-1.01
	(1.61)	(1.61)	(1.61)	(1.58)	(1.12)

Table 16: Impact of QE for different business model factors that the domestic bank attaches to domestic bonds on returns. Return impact is given in basis points. Each data point represents the average return R of a simulation run defined by its calibration $c \in \mathcal{C}*$, random seed $s \in \mathcal{S}*$, the multiplier $\nu \in \{1, 0.9, 0.8, 0.7, 0.6, 0.5\}$ and experiment $e \in \{0, 1\}$, which defines whether the central bank purchases assets ($e = 1$) or not ($e = 0$). When computing the return impact we control for changes in return volatility between simulations of different values for ν and between simulations with and without QE, i.e. $\Delta R_{c,s,\nu} = \text{const} + \beta_1 \Delta V_{c,s,\nu} + \beta_2 \Delta V_{c,s,\nu,e=0} + \beta_1 \Delta V_{c,s,\nu,e=1} + \epsilon_{c,s,\nu}$ with $\Delta R_{c,s,\nu} = R_{c,s,\nu,e=1} - R_{c,s,\nu,e=0}$, $\Delta V_{c,s,\nu} = \frac{V_{c,s,\nu,e=1} - V_{c,s,\nu=1,e=0}}{V_{c,s,\nu,e=0}}$ and $\Delta V_{c,s,\nu,e=x} = \frac{V_{c,s,\nu,e=x} - V_{c,s,\nu=1,e=x}}{V_{c,s,\nu,e=x}}$. The constant of the regression equation is the reported mean return impact. The standard deviation of returns is stated in parenthesis.

	0.5	0.6	0.7	0.8	0.9	1.0
dom. bonds return	-28.75	-28.79	-25.45	-15.4	-14.06	-18.27
	(11.37)	(10.39)	(16.07)	(20.95)	(9.71)	(4.83)
dom. equities return	-7.14	-6.41	-3.87	-2.02	-1.93	-2.17
	(3.44)	(4.84)	(5.62)	(5.35)	(4.61)	(2.86)
for. bonds return	-1.13	-1.08	-0.72	-0.33	-0.09	-0.1
	(0.42)	(0.56)	(0.52)	(0.69)	(0.57)	(0.4)
for. equities return	-1.32	-1.17	-0.86	-0.6	-0.38	-0.47
	(0.65)	(0.79)	(0.77)	(0.85)	(0.77)	(0.68)

bonds but then reverses as the business model factor is reduced further. Although domestic bond holdings increase and become better substitutes for domestic currency as ν decreases, the domestic bank agent seizes to be the prime seller of bonds to the central bank. This is shown in Table 17, which reports the percentage point changes in portfolio weights for all agents and assets between simulations with and without QE. Note that with decreasing ν the domestic bank agent becomes less and less inclined to increase its domestic currency holdings. On average, when $\nu = 0.5$ all proceeds form domestic bond sales by the domestic bank agent are used to purchase other assets, in particular foreign bonds. Substituting domestic for foreign bonds apparently is preferred to substituting domestic bonds for currency, even if this implies a stronger fall in domestic bond yields. Domestic funds and foreign banks, on the other hand, who own a much smaller share of outstanding domestic bonds, increase their bond sales to the central bank. Since domestic bonds and currency are relatively inferior substitutes for the domestic fund and foreign bank agents, the impact on domestic bond yields increases. In general and contrary to the mechanics of the representative agent model, selling domestic bonds does not imply that agents want to substitute bonds for currency. In fact, domestic agents use the proceeds to buy foreign assets, while foreign agents tend to exchange foreign currency for domestic currency. The impact of QE can best be explained by observing which agent is substituting which volume of domestic bonds for domestic currency. For example, while the domestic bank agent steadily reduces its domestic bond sales with decreasing ν, this does not imply that it also decreases the degree to which bonds and currency are substituted. For $\nu = \{0.9, 0.8\}$ the volume of domestic bonds that are substituted for domestic currency (3.32% and 2.9% of total assets for $\nu = 0.9$ and $\nu = 0.8$, respectively) is higher than the substitution volume for $\nu = 1$ (2.84% of total assets). This higher substitution volume leads to the lower impact of QE on yields for $\nu = 0.9$ and $\nu = 0.8$ when compared to the benchmark $\nu = 1$.

Table 17: Impact of QE for different business model factors that the domestic bank attaches to domestic bonds on portfolio weights. The impact on portfolio weights is given in percentage points. Each data point represents the average weight W of an asset in a simulation run defined by its calibration $c \in \mathcal{C}*$, random seed $s \in \mathcal{S}*$, the multiplier $\nu \in \{1, 0.9, 0.8, 0.7, 0.6, 0.5\}$ and experiment $e \in \{0, 1\}$, which defines whether the central bank purchases assets ($e = 1$) or not ($e = 0$). Mean and standard deviations (in parenthesis) are computed from differences of 100 simulation pairs, i.e. $W_{c,s,\nu,e=1} - W_{c,s,\nu,e=0}$.

	0.5	0.6	0.7	0.8	0.9	1.0
dom. bank - dom. currency	0.0	0.35	1.68	2.9	3.32	2.84
	(0.01)	(0.52)	(1.35)	(2.11)	(2.68)	(2.68)
dom. bank - dom. bonds	-3.75	-4.04	-5.42	-7.74	-9.72	-10.73
	(0.29)	(0.44)	(1.09)	(1.29)	(0.98)	(0.44)
dom. bank - dom. equity	0.29	0.26	0.15	0.05	-0.01	-0.03
	(0.08)	(0.08)	(0.1)	(0.1)	(0.08)	(0.09)
dom. bank - for. bonds	3.4	3.24	2.43	1.37	0.44	0.03
	(0.24)	(0.39)	(0.61)	(0.64)	(0.46)	(0.2)
dom. bank - for. equities	0.05	0.06	0.05	0.04	0.03	0.03
	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
dom. bank - for. currency	-0.0	0.14	1.11	3.37	5.95	7.86
	(0.01)	(0.38)	(1.41)	(2.58)	(3.1)	(2.8)
dom. fund - dom. currency	3.73	3.48	3.36	3.5	3.66	2.82
	(3.17)	(3.06)	(2.8)	(2.61)	(2.11)	(1.99)
dom. fund - dom. bonds	-8.02	-7.88	-7.13	-6.01	-5.08	-4.84
	(0.51)	(0.59)	(0.8)	(0.79)	(0.55)	(0.24)
dom. fund - dom. equities	-0.15	-0.15	-0.15	-0.12	-0.08	-0.07
	(0.09)	(0.09)	(0.09)	(0.09)	(0.07)	(0.07)
dom. fund - for. bonds	-0.26	-0.25	-0.24	-0.14	-0.09	0.1
	(0.29)	(0.3)	(0.34)	(0.35)	(0.33)	(0.39)
dom. fund - for. equities	0.03	0.04	0.05	0.05	0.03	0.07
	(0.1)	(0.11)	(0.12)	(0.13)	(0.11)	(0.15)
dom. fund - for. currency	4.66	4.76	4.11	2.72	1.57	1.92
	(3.13)	(3.09)	(2.95)	(2.73)	(2.12)	(1.97)
for. bank - dom. currency	2.94	2.91	2.59	2.21	2.05	2.35
	(0.86)	(0.89)	(0.83)	(0.65)	(0.56)	(0.67)
for. bank - dom. bonds	-1.09	-1.09	-1.05	-0.94	-0.81	-0.67
	(0.09)	(0.09)	(0.13)	(0.14)	(0.13)	(0.11)
for. bank - dom. equities	-0.0	-0.01	-0.01	-0.01	-0.01	0.0
	(0.01)	(0.01)	(0.02)	(0.02)	(0.01)	(0.01)
for. bank - for. bonds	-0.65	-0.59	-0.41	-0.24	-0.07	-0.07
	(0.17)	(0.23)	(0.24)	(0.24)	(0.22)	(0.17)
for. bank - for. equities	0.0	0.0	0.01	0.01	0.0	-0.01
	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)
for. bank - for. currency	-1.2	-1.23	-1.13	-1.03	-1.16	-1.61
	(0.85)	(0.88)	(0.79)	(0.65)	(0.57)	(0.7)
for. fund - dom. currency	1.82	1.86	1.81	1.75	1.7	1.89
	(1.02)	(0.99)	(0.99)	(1.0)	(0.91)	(0.95)
for. fund - dom. bonds	-0.49	-0.48	-0.49	-0.45	-0.39	-0.31
	(0.13)	(0.13)	(0.14)	(0.13)	(0.1)	(0.08)
for. fund - dom. equities	-0.02	-0.02	-0.02	-0.02	-0.01	0.0
	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.03)
for. fund - for. bonds	-0.37	-0.35	-0.24	-0.13	-0.04	-0.05
	(0.11)	(0.13)	(0.14)	(0.16)	(0.12)	(0.14)
for. fund - for. equities	-0.07	-0.08	-0.07	-0.06	-0.05	-0.06
	(0.06)	(0.08)	(0.07)	(0.08)	(0.07)	(0.07)
for. fund - for. currency	-0.88	-0.93	-0.99	-1.1	-1.21	-1.48
	(1.0)	(0.97)	(1.01)	(1.03)	(0.94)	(0.94)

5.5.2 Purchasing green bonds

Here we define green QE as the purchase of green bonds by the central bank. To create a distinction between green and brown bonds, we split the domestic and foreign bond portfolios into a green and brown bond portfolio, respectively. The only difference between the respective green and brown bond portfolios are the business model factors agents attach to them. The respective business model factors attached to green bonds are multiples $\nu \in \{0.95, 0.9, 0.85, 0.8\}$ of the factors attached to their brown siblings.[29]

When purchasing assets, the central bank chooses between purchasing €1 trillion (nominal value) of brown bonds or green bonds, or to be color blind in its purchases (i.e. €500bn. green and brown bonds, respectively). Table 18 shows the impact of the different purchasing strategies on the yield of the domestic green bond portfolio. Since brown and green bonds share all fundamental characteristics and only differ in terms of agents' preferences, it is not surprising that the impact of QE on the expected return of the green bond portfolio is rather similar for the different purchasing strategies. The difference between the impacts for a given ν is less than one basis point. Nevertheless, the result suggests that a slightly higher yield reduction for green bonds can be achieved when the central bank purchases brown rather than green bonds. This is true for all ν. This result is predicted by the representative agent model in Section 3 and is due to the higher aversion (larger business model factor) towards brown bonds. The purchase of brown bonds relative to purchases of green bonds frees up more risk bearing capacity on the balance sheets of agents, which leads to larger declines in risk premia.[30]

While the highest impact on overall asset returns can be achieved by purchasing assets with the highest risk and business model aversion, Table 19 shows that this strategy does not maximize firms' incentives to invest in green projects. On the contrary, the greenium observed without central bank asset purchases decreases when only brown assets or a mix of green and brown assets are purchased. The same mechanism that leads to a flattening of the yield curve in response to QE also reduces the spread between green and brown asset returns. The difference is that while long maturity bonds are riskier than short maturity bonds in terms of return volatility, brown bonds are viewed less favorably in terms of the agents' business strategies. If the central bank, however, exclusively purchases green assets, Table 19 suggests that the greenium can be increased. This result can under certain conditions also be derived from the representative agent model in Section 3: Assuming that A0 is the green asset and A1 is the brown asset, $\rho_{A0,A1} = 1$, $\rho_{C,A1} = \rho_{C,A0} = 0$ and $\xi_{A1} = \xi_{A0}/\nu$ we can write $\Delta r_{A0} = -qe \cdot \xi_C(1 + \frac{\xi_{A0}}{\xi_C}) < \Delta r_{A1} = -qe \cdot \xi_C(1 + \frac{\xi_{A0}/\sqrt{\nu}}{\xi_C})$ for $\nu < 1$. Note that lower correlations (i.e., $\rho_{A0,A1} < 1$) and higher correlations between assets and currency (i.e. $\rho_{C,A1} = \rho_{C,A0} > 0$) reduces or even reverses the impact of green QE on the greenium.

[29]Keeping risk factors, coupons and the maturity of brown and green bonds identical while varying the business model factors attached to them, is the simplest way to differentiate between green and brown bonds. The emission German government twin bonds provides evidence that even such a minimal distinction between green and conventional bonds produces a greenium.

[30]The increase in QE impact with decreasing ν can partly be explained by increasing return volatility and partly by a change in the willingness to substitute domestic bonds for domestic currency.

Table 18: Impact of central bank asset purchases on the return of domestic green bonds for different business model factors agents attach to green bonds. Return impact is given in basis points. Central bank purchases can either target green, brown or both (color blind QE) domestic bonds. We control for changes in return volatility by regressing the return impact on changes in return volatility. Each data point in the regression represents the average return R or volatility V of a simulation run defined by its calibration $c \in \mathcal{C}*$, random seed $s \in \mathcal{S}*$, the multiplier $\nu \in \{0.95, 0.9, 0.85, 0.8\}$ and experiment $e \in \{0, \text{green}, \text{brown}, \text{both}\}$, which defines which domestic bonds the central bank purchases. We use the following regression equation: $\Delta R_{c,s,\nu} = \text{const} + \beta \Delta V_{c,s,\nu} + \epsilon_{c,s,\nu}$, with $\Delta R_{c,s,\nu} = R_{c,s,\nu,e=x} - R_{c,s,\nu,e=0}$ and $\Delta V_{c,s,\nu} = \frac{V_{c,s,\nu,e=x} - V_{c,s,\nu,e=0}}{V_{c,s,\nu,e=0}}$. The reported impact is the constant of the regression equation. Standard errors in parenthesis are robust.

ν	color blind QE	(SE)	brown QE	(SE)	green QE	(SE)
0.95	-10.937	(0.764)	-10.771	(0.799)	-9.876	(0.79)
0.9	-12.304	(0.73)	-11.582	(0.798)	-12.146	(0.797)
0.85	-12.679	(0.692)	-13.414	(0.686)	-12.369	(0.787)
0.8	-12.924	(0.596)	-13.566	(0.615)	-12.834	(0.696)

Table 19: Greenium (spread between green and brown bond returns) for different business model factors agents attach to green bonds. The greenium is given in basis points. Central bank purchases can either target green, brown or both (color blind QE) domestic bonds. We control for changes in return volatility by regressing the greenium on changes in return volatility. Each data point in the regression represents the average return R or volatility V of a simulation run defined by its calibration $c \in \mathcal{C}*$, random seed $s \in \mathcal{S}*$, the multiplier $\nu \in \{0.95, 0.9, 0.85, 0.8\}$ and experiment $e \in \{0, \text{green}, \text{brown}, \text{both}\}$, which defines which domestic bonds the central bank purchases. We use the following regression equation: $\Delta R_{c,s,\nu} = \text{const} + \beta \Delta V_{c,s,\nu} + \epsilon_{c,s,\nu}$, with $\Delta R_{c,s,\nu} = R^{\text{green}}_{c,s,\nu,e=x} - R^{\text{brown}}_{c,s,e=x}$ and $\Delta V_{c,s,\nu} = \frac{V^{\text{green}}_{c,s,\nu,e=x} - V^{\text{brown}}_{c,s,e=x}}{V^{\text{brown}}_{c,s,e=x}}$. The reported greenium is the constant of the regression equation. Standard errors in parenthesis are robust.

ν	no QE	(SE)	color blind QE	(SE)	brown QE	(SE)	green QE	(SE)
0.95	-1.911	(0.043)	-1.856	(0.096)	-1.791	(0.114)	-1.945	(0.084)
0.9	-3.955	(0.13)	-3.667	(0.351)	-3.634	(0.345)	-3.859	(0.385)
0.85	-6.348	(0.232)	-4.64	(0.593)	-4.096	(0.579)	-6.966	(0.788)
0.8	-9.487	(0.357)	-6.362	(1.102)	-5.776	(0.98)	-11.013	(1.168)

6 Conclusion

We develop a portfolio balance model to assess the impact of QE on domestic and foreign asset yields. Because asset substitutability is crucial for determining the impact of QE on yields, our model features heterogeneous agents and assets. While assets become better substitutes when they have common exposure to risk factors, the business models of investors introduce a non-financial dimension to portfolio decisions. For example, banks' home bias or investors' preferences for green assets can be taken into account through agent-specific business model factors. We calibrate our model to represent banks and mutual funds in the euro area and the rest of the world, respectively. With simulations we find that the yield curve is substantially flattened by QE, that the yield impact scales non-linearly with the size of asset purchases and that unwinding the current ECB balance sheet would reverse the yield impact faster than ECB asset holdings. When analyzing the potential return impact of a green QE, our results are mixed. It is unclear whether extending collateral eligibility criteria to include green bonds would enhance or dampen the impact of QE. Furthermore, we find that conventional bond purchases are slightly more effective in reducing bond yields, including green bond yields. However, the greenium, i.e. the spread between green and brown bond yields, declines with the portion of brown assets purchased by the central bank.

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