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Truly Costly Search and Word-Of-Mouth Communication

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Abstract

In markets with search frictions, consumers can acquire information about goods either through costly search or from friends via word-of-mouth (WOM) communication. How do sellers' market power react to a very large increase in the number of consumers' friends with whom they engage in WOM? The answer to the question depends on whether consumers are freely endowed with price information. If acquiring price quotes is costly, equilibrium prices are dispersed and the expected price is higher than the marginal cost of production. This implies that firms retain market power even if price information is disseminated among a very large number of consumers due to technological progress, such as social networking websites.

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1 Introduction

It is well documented that consumers can acquire information about goods not only on their own but also through their personal contacts, or friends, acquaintances.[1] Information dissemination through friends via word-of-mouth (WOM) communication is important due to recent technological growth, such as developments of social websites, online forums, and instant messengers. Consumers can exchange information with a greater number of friends as technology progresses further. Naturally, this puts forward a question, how do markets respond to an increase in the number of consumers' personal contacts?

Certainly, the answer to this question depends on many characteristics of a market under consideration. To illustrate the relationship between consumers' number of friends and prices, the existing literature considers the following canonical environment. In a duopoly market, symmetric firms produce homogeneous goods and compete in prices. Consumers do not know prices. They can acquire price quotes through costly search or may receive such information from friends, who have searched, for free. The existing literature shows that, if each consumer accesses a price quote of a random seller for free, prices converge to the sellers' marginal cost of production as the number of consumers' friends increases (Galeotti (2010)). Yet, we show in this paper that if consumers are not endowed with any free price quote, prices are dispersed across sellers and the expected price is greater than the production marginal cost even when the number consumers' friends increases without limits.

The intuition behind our main result is as follows. Sellers' market power and the level of price dispersion are determined by the share of consumers who only observe one price, also known as *locked-in* or *captive* consumers (e.g., Shilony (1977), Varian (1980), Armstrong and Vickers (2019)). Since captive consumers cannot "walk away" to a competing seller, sellers' market power increases with the share of captive consumers. Price dispersion exists as long as the share of captive consumers does not vanish or not all consumers become captive. An increase in the number of consumers' friends has two opposing impacts on the share of captive consumers. First is the direct competitive

[1] Due to Stigler (1961), most studies on consumer search focus on acquisition of information by consumers on their own. A pioneer study by Katz and Lazarsfeld (1955) show that consumers' personal contacts are a key reason why they buy goods. Later empirical studies back this point, e.g. Godes and Mayzlin (2004), Chen et al. (2011).

effect. Search intensity of consumers being fixed, the share of captive consumers falls with the number of friends because some of the captive consumers now have friends from whom they can receive another price quote. This puts competitive pressure on firms. There is also an indirect effect, which is anti-competitive. The share of consumers, who search and acquire price quotes through costly search, decreases with the number of friends. This is a standard free-riding effect. As information on prices is a public good for consumers, an increase in the number of personal contacts leads to a lower level of information acquisition. As a result less information diffuses throughout consumers. This leads to a higher share of captive consumers and, thus, raises sellers' market power.

In an extreme case when the number of consumers' personal contacts increases without bounds, the two effects discussed in the previous paragraph interact in a way so that sellers retain some market power. The reason is that, if searching for prices is costly, the share of consumers who search vanishes as the number of consumers' friends gets very large. Despite the fact that *almost* no consumer searches, very little amount of price information is still obtained. This information diffuses throughout a very large number of consumers so that there is a strictly positive share of captive consumers as well as that of price-comparing consumers. As a result, price dispersion remains. Since sellers never price below the production marginal cost, the presence of price dispersion means that the expected price is higher than the production marginal cost.

Our result differs from that of Galeotti (2010) because, in Galeotti (2010) each buyer knows a price quote of a firm without searching. This means that even if consumers do not search, they may still compare prices because they receive price information from friends. Therefore, when the number of consumers' friends gets very high so that buyers stop searching, all consumers compare prices because, in addition to a free price quote a consumer observes, each buyer almost certainly receives one more price quote from friends. If all consumers compare prices, firms find it optimal to price at the production marginal cost.

Our paper contributes to the growing literature on consumer search and WOM, and the closest papers to ours are Atayev and Janssen (2019), Campbell et al. (2020), and Galeotti (2004)). Atayev and Janssen (2019), which unlike the current paper studies sequential search markets, shows that price dispersion persists when the number of consumers' personal contacts links increases. Campbell et al. (2020) examines the relationship between network structure of consumers' personal contacts and product quality. Galeotti (2004), which is an earlier version of Galeotti (2010), employs the same model as that in our current paper, but the author does not examine an impact of an (unbounded) increase in the number of buyers' personal contacts on market outcomes.

The rest of our paper is organized as follows. In the next section, we revisit Galeotti (2004) and examine the limiting case when the number of consumers' friends increases without bounds. In Section 3, we show that our main result holds even when forming a personal link is costly and, thus, consumers have to strategically decide whether to form links. The final section concludes.

2 Model and Analysis

In this section, we analyze a model of Galeotti (2004). There are two firms, or sellers, that produce homogeneous goods at a marginal cost normalized to zero. The firms compete in prices. Let $F_j(p)$ represent the probability that firm j charges a price that is not greater than p.

On the demand side, there is a countably infinite number of consumers, normalized to one. Each consumer demands a unit of the good, which she evaluates at $v > 0$. Prior to searching, a consumer does not know prices and, to make a purchase, she must learn at least one price quote. To do that, a buyer can search firms, which is costly. Alternatively, a buyer can obtain price information from her friends at no cost, given her friends acquired price quotes themselves. The cost of searching a firm is given by $c > 0$. Search is simultaneous, meaning that a consumer requests price quotes from $n \in \{0, 1, 2\}$ firms simultaneously (implying that when $n = 0$ she just waits), after which search is terminated. Let q_n be the probability that a consumer samples n price quotes: $\sum_{n=0}^{2} q_n = 1$. Each consumer is assumed to have k number of friends, or personal links, who may potentially inform the consumer about prices, where $1 \leq k < \infty$.[2]

Timing of the game is as follows. First, firms simultaneously set prices. Second, without knowing prices, consumers choose number of firms to search. Third, after all consumers finish their search, they share price information with their friends. Consumers

[2] As the paper is concerned with information diffusion among consumers, an uninteresting case of $k = 0$ is omitted, yet it can be easily incorporated.

who learn at least a single price—either through own search or from friend(s)—may make purchase. We employ Nash Equilibrium (NE) as a solution concept.

Galeotti (2004) shows that for sufficiently large k, there exists a stable NE where consumers randomize between searching one firm and not searching, i.e. $q_0, q_1 > 0$ and $q_0 + q_1 = 1$. Letting $q \equiv q_1$ for the rest of the paper so that $q_0 = 1 - q$ and $\eta \equiv \frac{\left(1-\frac{q}{2}\right)^{k+1} - (1-q)^{k+1}}{1+(1-q)^{k+1} - 2\left(1-\frac{q}{2}\right)^{k+1}}$, we restate the result in the following proposition.

Proposition 1 (Galeotti (2004) Theorem 5.1) *For any $v > 0$ and $k \geq 1$, there exists $[\underline{c}(k), \overline{c}] \subset (0, v)$ such that for any $c \in [\underline{c}(k), \overline{c}]$ there exists a stable NE given by $(F(p), q)$ where*

$$F(p) = 1 - \eta\left(\frac{v}{p} - 1\right), \text{ with support } \left[\frac{\eta}{1+\eta}v, v\right], \tag{1}$$

and q solves

$$c = (1-q)^k(v - E[p]) + \left(\left(1 - \frac{q}{2}\right)^k - (1-q)^k\right)\big(E[p] - E[\min\{p_1, p_2\}]\big). \tag{2}$$

We refer to the appendix of Galeotti (2004) for the proof. The intuition is as follows. First, the author assumes that buyers randomize between searching one firm and not searching. Given this search strategy, he determines sellers' pricing policies, which is given in (1). Note that, for any strictly positive shares of searching consumers and non-searching consumers, the price distribution is non-degenerate, meaning that prices are dispersed. Second, the author verifies whether, given such dispersed prices and buyers' number of friends, it is optimal for buyers randomize between searching one firm and not searching. He shows that this is indeed the case if the search cost is not too high and not too low. Moreover, the authors shows that there exists either one or two such equilibria, but only one of them is stable.

To illustrate the main idea, we present Figure 1. The horizontal axis represents the share of consumers who search one firm, i.e., q, and the vertical axis stands for the expected benefit as well as the cost of searching a firm. The solid curve represents the expected benefit of searching and the dashed lines stand for different levels of the search cost. For instance, if the search cost is given by c_1, the solid and the dashed lines intersect twice and each intersection represents an NE. Observe that an NE with $q \in (0, 1)$ exists for search costs smaller than $\bar{c}$. However, a stable NE exists only if the search cost is between $\underline{c}(k)$

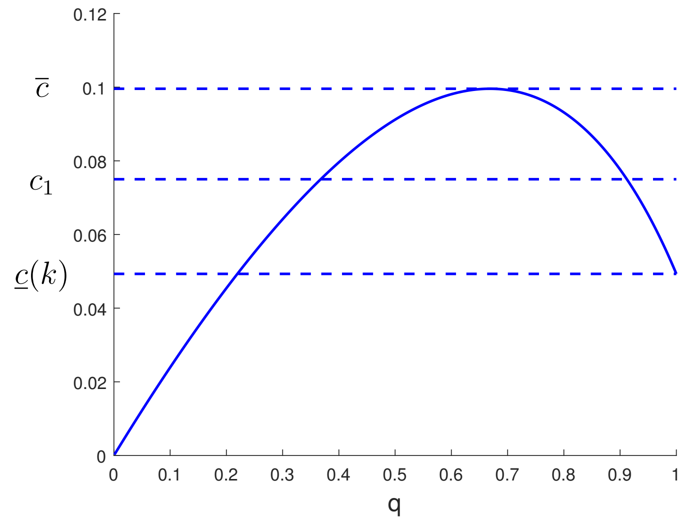

Figure 1: Illustration of equilibria existence $v = 1$, $k = 1$, and $c_1 = 0.075$

and $\overline{c}$ (like in many models of simultaneous consumer search such as Burdett and Judd (1983), Fershtman and Fishman (1992), Janssen and Moraga-Gonzalez (2004), Atayev (2019)). To see that, suppose that the actual search cost is given by c_1 so that there are two NEs. Then, only the NE which corresponds to the higher share of searching consumers, i.e., higher q, is locally stable. This is because if the actual share of searching consumers is higher (lower) than the equilibrium one, the expected benefit of searching is lower (higher) than the cost of doing so. Therefore, consumers have incentive to search less (more) and the actual share of searching consumers converges to the equilibrium one. Applying a similar argument, we can see that the NE that corresponds to the lower share of searching consumers is unstable.[3]

Regarding the question of interest, we state the main result in the next proposition.

Proposition 2 *In a stable NE with $q \in (0, 1)$, as $k \to \infty$ the lower bound of the search cost interval $\underline{c}(k)$ converges to zero, the share of searching consumers approaches zero, while price dispersion remains and the expected price is higher than the production marginal cost.*

The proof is in the appendix and the intuition is as follows. As consumers have more friends who can potentially share price information, buyers' incentive to acquire price

[3]There always exists a stable equilibrium without search (see, Galeotti (2004)), known as Diamond paradox due to Diamond (1971). In this equilibrium, there is no trade. However, we are not interested in this unrealistic equilibrium.

quotes through costly search falls. In the limit when $k \to \infty$ the share of searching consumers almost vanishes. To understand why price dispersion persists, we first note that the level of price dispersion is given by η. Observe that η is a ratio of captive consumers to that of price-comparing consumers. Thus, if η either approaches zero or goes to infinity, price dispersion disappears. In the appendix, we show that if $\eta \to 0$ or $\eta \to \infty$, the expected benefit of searching, which is given by the right-hand side of (2), converges to zero. However, this violates (2) and, hence, cannot be the case. Finally, as firms do not price below the production marginal cost, the presence of price dispersion means that the expected price is higher than the marginal cost of production.

3 Endogenous Link Formation

In reality, obtaining information via WOM is costly in a sense that one has to spend time to request information from friends. For instance, texting a message/writing an email to friends or posting a question on an online forum requires some time. At the same time, the current state of technology allows us to send the same message (or email) to a group of friends simultaneously and any post on an online forum can be seen by all users of the forum. We address this issue in this section. Particularly, we assume that each consumer can request price information from all her k links by incurring cost $l \geq 0$. Link formation is a binary decision: either a consumer forms k links or does not form any links. The link is directed, meaning that if consumer i forms a link with consumer j, information flows from j to i only. Let ω be the probability that a consumer invests in forming k number of personal links. The rest of the model is the same as that in the previous section.

Timing of the game is as follows. First, firms simultaneously set prices. Second, without knowing prices, consumers simultaneously decide whether to form links and how many firms to search. After search is complete, price information (if any) flows according to link formation. A consumer who observes at least one price can make a purchase. We use Nash equilibrium (NE) as a solution concept, and focus on symmetric equilibria.

To derive an equilibrium, we first suppose that all consumers establish k links. Conditional on that assumption, we find consumers' optimal search strategy and firms' optimal pricing policy. We know, however, that if all consumers establish k links, in equilibrium consumers randomize between not searching and searching one firm for $c \in [\underline{c}(k), \overline{c}]$ and firms price according to (1) (recall Proposition 1).

Next, given these search and pricing strategies, we determine conditions under which consumers indeed prefer establishing k links. It is easy to check that a searching consumer prefers to form k links if

$$\left(1-\left(1-\frac{q}{2}\right)^k\right)(E[p]-E[\min\{p_1,p_2\}])>l. \quad (3)$$

Similarly, a non-searching consumer finds it worthwhile to form links if

$$\left(1-(1-q)^k\right)(v-E[p])+\left(1+(1-q)^k-2\left(1-\frac{q}{2}\right)^k\right)(E[p]-E[\min\{p_1,p_2\}])>l. \quad (4)$$

In the appendix, we show that (4) is implied by (3). The intuition is that, for a given k, the probability that a searching consumer receives the second price quote via WOM is lower than the probability that a non-searching consumer receives any price information from friends. Therefore, if a searching consumer who observes one price finds it worthwhile to form links, a non-searching consumer who does not observe any price definitely prefers to form links. We show that, in equilibrium, searching consumers certainly form links if the cost of doing so is sufficiently small. We state this result in the following proposition.

Proposition 3 *For any $v>0$ and $k\geq 1$, there exists $[\underline{c}(k),\overline{c}]\subset(0,v)$ and $\overline{l}(k)>0$ such that for $c\in[\underline{c}(k),\overline{c}]$ and $l\leq\overline{l}(k)$, there exists a stable NE given by $(F(p),q,\omega)$, where $F(p)$ is given by (1), q is determined by (2), and $\omega=1$.*

Furthermore, as $k\to\infty$, $\overline{l}(k)$ remains strictly positive, the share of searching consumers converges to zero, and price dispersion persists with the expected price being higher than the production marginal cost.

The proof is in the appendix. Notice that the equilibrium in the proposition has similar characteristics as that in the previous section. The only difference is the condition on l. We can think of the model in Section 2 as a special case of the current model where $l=0$ (and k is fixed). The reason why the equilibrium exists for sufficiently small l is straightforward: if it was too costly to establish links, consumers would not form any links.

For the limiting case of $k\to\infty$, the NE inherits the properties of that in Proposition 2, namely price dispersion remains and the firms retain some market power. The intuition behind it is similar to that in the previous section, if consumers' incentive to form links does not decrease with k. Consumers' incentive to form links increases with k, as the probability of obtaining price information via WOM increases. As a result, an increase in k does not mitigate consumers' incentive to form links and, hence, in the limit as $k \to \infty$, prices are dispersed just like in the previous section.

4 Conclusion

In the paper, we have revisited the role of costly information acquisition by consumers and its dissemination among consumers via WOM. Specifically, we showed that, when the number of consumers' personal links increases without limits, firms earn positive profits and price dispersion persists if searching for prices is costly. Also the result holds even when we allow for endogenous personal link formation.

We need to note that we considered a specific type of link formation: a consumer either forms k links or does not form any links. From a theoretical point, such modeling is restrictive as it implicitly assumes the following type of cost structure of link formation. The cost of forming one link is positive and that of forming additional links (up to k number) is free. However, the cost of forming a link in addition to k links is prohibitively high. A different way of modeling link formation is to assume linear or convex cost of link formation, which is common in the existing literature (e.g., Galeotti and Goyal (2010)). Yet this introduces difficulties as, generically, a symmetric equilibrium does not exist because searching consumers have less incentive to form links than non-searching consumers. As a result, undertaking comparative static analysis becomes difficult. This is certainly an area for future research.

A Proofs

A.1 Proofs of Proposition 1 and 2

For the proof of Proposition 1, we mainly summarize the proof provided in Galeotti (2004). This will be helpful for us for the proof of the Proposition 2.

First, we note some facts that we will use in the proof: $E[p] = v - \int_{\frac{\eta v}{1+\eta}}^{v} F(p)dp = v\eta \ln\left(1 + \frac{1}{\eta}\right)$ and $E[\min\{p_1, p_2\}] = v - 2\int_{\frac{\eta v}{1+\eta}}^{v} F(p)dp + \int_{\frac{\eta v}{1+\eta}}^{v} [F(p)]^2 dp$ so that

$$E[p] - E[\min\{p_1, p_2\}] = \eta v \left((1 + 2\eta) \ln\left(1 + \frac{1}{\eta}\right) - 2\right).$$

Also, it is easy to check that $\lim_{q \downarrow 0} \eta = \infty$ and $\lim_{q \uparrow 1} \eta = \frac{1}{2(2^k - 1)}$.

Second, we refer to Galeotti (2004) which shows that

$$\begin{aligned} \underline{c}(k) &= \lim_{q \uparrow 1} \left\{ (1-q)^k (v - E[p]) + \left(\left(1 - \frac{q}{2}\right)^k - (1-q)^k\right) \left(E[p] - E[\min\{p_1, p_2\}]\right) \right\} \\ &= \frac{v}{2^k(2^k - 1)} \left(\frac{2^{k-1} \ln\left(2^{k+1} - 1\right)}{2^k - 1} - 1 \right), \end{aligned}$$

where it is easy to check that $0 < \underline{c}(k) < v$. The author also shows that that the derivative of the RHS of (2) w.r.t. q evaluated at $q \uparrow 1$ is negative. From that facts that $0 < \underline{c}(k) < v$ and the RHS of (2) is decreasing in q in the neighborhood of $q \to 1$, there exists a stable NE for values of c that is slightly above $\underline{c}(k)$. This completes the proof Proposition 1.

Now, we analyze the limiting case for the proof of Proposition 2. It is clear that

$$\begin{aligned} \lim_{k \to \infty} \underline{c}(k) &= v \lim_{k \to \infty} \frac{\ln(1 + 2(2^k - 1))}{2(2^k - 1)^2} - \lim_{k \to \infty} \frac{v}{2^k(2^k - 1)} = v \lim_{x \to \infty} \frac{\ln(1 + 2x)}{2x^2} \\ &\overset{\text{l'Hopital}}{=} \lim_{x \to \infty} \frac{1}{2x(1 + 2x)} = 0, \end{aligned}$$

where $x = 2^k - 1$.

Next, we need show that $q \to 0$ when $k \to \infty$ by contradiction. Suppose that $\lim_{k \to \infty} q > 0$. It implies that $\lim_{k \to \infty} \eta = 0$. Then, however, we have the RHS of (2) as

$$v \lim_{k \to \infty} \left[(1-q)^k \left(1 - \eta \ln\left(\tfrac{1}{\eta} + 1\right)\right) + \left(\left(1 - \tfrac{q}{2}\right)^k - (1-q)^k\right) \eta \left((1 + 2\eta) \ln\left(1 + \tfrac{1}{\eta}\right) - 2\right) \right] = 0,$$

since

$$\begin{aligned}
\lim_{k\to\infty} \eta \ln\left(\frac{1}{\eta}+1\right) &= \lim_{\eta\to 0} \eta \ln\left(\frac{1}{\eta}+1\right) \\
&= \lim_{z\to\infty} \frac{\ln(z+1)}{z} \\
&\overset{\text{l'Hopital}}{=} \lim_{z\to\infty} \frac{1}{z+1} \\
&= 0,
\end{aligned}$$

$$\begin{aligned}
\lim_{k\to\infty} \eta \left(1-\eta \ln\left(\frac{1}{\eta}+1\right)\right) &= \lim_{\eta\to 0} \eta \left(1-\eta \ln\left(\frac{1}{\eta}+1\right)\right) \\
&= \lim_{z\to\infty} \frac{1-\frac{\ln(z+1)}{z}}{z} = \lim_{z\to\infty} \frac{z-\ln(z+1)}{z^2} \\
&\overset{\text{l'Hopital}}{=} \lim_{z\to\infty} \frac{1-\frac{1}{z+1}}{2z} = \lim_{z\to\infty} \frac{z}{2z(z+1)} \\
&= 0,
\end{aligned}$$

and

$$\begin{aligned}
\lim_{k\to\infty} \eta \left((1+2\eta)\ln\left(1+\frac{1}{\eta}\right)-2\right) &= \lim_{\eta\to 0} \left[\eta \ln\left(1+\frac{1}{\eta}\right) - 2\eta\left(1-\eta\ln\left(\frac{1}{\eta}+1\right)\right)\right] \\
&= 0-2\times 0 = 0,
\end{aligned}$$

where $z = 1/\eta$. Thus, (2) is violated, a contradiction. Hence, must be $\lim_{k\to\infty} q = 0$.

Finally, we show that η remains strictly positive and finite in order to prove that the price dispersion remains when $k \to \infty$. We will prove it by contradiction that η cannot go to 0 or ∞ when $k \to \infty$. The former case have been proven in the previous paragraph, and to prove the latter we suppose that $\lim_{k\to\infty} \eta = \infty$. Note that this implies that $0 < \lim_{k\to\infty}(1-q)^k \leq 1$ and $0 < \lim_{k\to\infty}\left(1-\frac{q}{2}\right)^k \leq 1$. Then, we can evaluate the RHS of the indifference condition as

$$v \lim_{k\to\infty}\left[(1-q)^k\left(1-\eta\ln\left(\tfrac{1}{\eta}+1\right)\right)+\left(\left(1-\tfrac{q}{2}\right)^k-(1-q)^k\right)\eta\left((1+2\eta)\ln\left(1+\tfrac{1}{\eta}\right)-2\right)\right]=0,$$

since

$$\begin{aligned}\lim_{k\to\infty}\eta\ln\left(\frac{1}{\eta}+1\right) &= \lim_{\eta\to\infty}\eta\ln\left(\frac{1}{\eta}+1\right)\\ &= \lim_{z\to 0}\frac{\ln(z+1)}{z}\\ &\overset{\text{l'Hopital}}{=} \lim_{z\to 0}\frac{1}{z+1}\\ &= 1,\end{aligned}$$

and

$$\begin{aligned}\lim_{k\to\infty}\eta\left(1-\eta\ln\left(\frac{1}{\eta}+1\right)\right) &= \lim_{\eta\to\infty}\eta\left(1-\eta\ln\left(\frac{1}{\eta}+1\right)\right)\\ &= \lim_{z\to 0}\frac{1-\frac{\ln(z+1)}{z}}{z} = \lim_{z\to 0}\frac{z-\ln(z+1)}{z^2}\\ &\overset{\text{l'Hopital}}{=} \lim_{z\to 0}\frac{1-\frac{1}{z+1}}{2z} = \lim_{z\to 0}\frac{z}{2z(z+1)}\\ &= \frac{1}{2},\end{aligned}$$

and

$$\begin{aligned}\lim_{k\to\infty}\eta\left((1+2\eta)\ln\left(1+\frac{1}{\eta}\right)-2\right) &= \lim_{\eta\to\infty}\left[\eta\ln\left(1+\frac{1}{\eta}\right)-2\eta\left(1-\eta\ln\left(\frac{1}{\eta}+1\right)\right)\right]\\ &= 1-2\left(\frac{1}{2}\right)=0.\end{aligned}$$

Again the indifference condition is violated, which proves that it cannot be $\lim_{k\to\infty}\eta=\infty$. Hence, η must be strictly positive and finite when $k\to\infty$, which completes the proof.

A.2 Proof of Proposition 3

For the proof, we first assume that all consumers form k links and derive conditions under which consumers randomize over searching one firm and not searching. Later, we verify whether consumers indeed form k links, given their search strategies.

We know from Section 2 that if consumers have k links, they randomize over searching one firm and not searching in a stable equilibrium for $c\in[\underline{c}(k),\overline{c}]$ (see Proposition 2). Also firms' pricing policy is given by (1) and consumers' search strategy by (2) and $q_0+q_1=1$.

Next, we determine conditions under which consumers choose to form links. We note that a searching consumer strictly prefers to form a link if

$$v - E[p] + \left(1 - \left(1 - \frac{q}{2}\right)^k\right)(E[p] - E[\min\{p_1, p_2\}]) - c - l > v - E[p] - c.$$

The LHS of the equation represents a searching consumer's payoff that forming k links yields and the RHS that not forming any links yields. The inequality can be simplified to (3). It is easy to see that a non-searching consumer strictly prefers to establish links if (4) holds.

Now, we show that (4) is implied by (3), which means that we can ignore (4) from our further analysis. It is easy to see that this is the case if the LHS of (3) is less than the LHS of (4). This is the case if

$$\left(1 - (1-q)^k\right)(v - E[p]) > \left(\left(1 - \frac{q}{2}\right)^k - (1-q)^k\right)(E[p] - E[\min\{p_1, p_2\}]).$$

As $1 - (1-q)^k > \left(1 - \frac{q}{2}\right)^k - (1-q)^k$ for any $k \geq 1$ and $q \in (0,1)$, the inequality certainly holds if the following is true:

$$v - E[p] > E[p] - E[\min\{p_1, p_2\}].$$

We use the facts applied in the proof of Proposition 2 to rewrite the inequality as

$$1 - \eta \ln\left(1 + \frac{1}{\eta}\right) > \eta\left((1+2\eta)\ln\left(1 + \frac{1}{\eta}\right) - 2\right),$$

or

$$\ln\left(1 + \frac{1}{\eta}\right) < \frac{1+2\eta}{2\eta(1+\eta)}.$$

As both the LHS and the RHS of the inequality go to ∞ when $\eta \to 0$ and both sides converge to 0 when $\eta \to \infty$, the inequality holds if the derivative of the LHS is less negative than that of the RHS. The derivative of the LHS is $-\frac{1}{\eta(1+\eta)} = -\frac{2\eta^2+2\eta}{2\eta^2(1+\eta)^2}$ and that of the RHS is $-\frac{2\eta^2+2\eta+1}{2\eta^2(1+\eta)^2}$. The former is less negative than the latter for $\eta > 0$, which means that the inequality holds. This proves that the condition in (4) is implied by (3).

Then, the cutoff value of forming links, denoted by $\bar{l}(k)$, solves

$$\left(1-\left(1-\frac{q}{2}\right)^k\right)(E[p]-E[\min\{p_1,p_2\}])=l.$$

The LHS is positive for any $q \in (0,1)$ and independent of l. Since the LHS is strictly increasing in l, there exists a unique solution to the equation. This establishes the existence and uniqueness of $\bar{l}(k) > 0$.

Finally, for the limiting case of $k \to \infty$, we provide the proofs as follows. We, first, assume that in equilibrium all consumers form links and show that q converges to 0 and price dispersion remains with $E[p]$ being higher than zero as $k \to \infty$. Then, we demonstrate that consumers indeed prefer forming links. Assume that consumers always form links. Then, from the proof of Proposition 2, we know that $\lim_{k\to\infty} q = 0$ and $0 < \lim\limits_{k\to\infty} \eta < \infty$. The latter means that the price dispersion remains, which, along with the fact that firms do not price below production marginal cost, implies that $\lim\limits_{k\to\infty} E[p] > 0$.

Now, we show that, given consumers' search behavior and firms' pricing policies as above, consumers indeed prefer forming links as $k \to \infty$. Notice that this is the case if $\lim\limits_{k\to\infty} \bar{l}(k) > 0$, or if the LHS of (3) is positive and does not converge to zero as $k \to \infty$. As $\lim\limits_{k\to\infty} [1-(1-q/2)^k] \geq 0$ and $\lim_{k\to\infty}(E[p]-E[\min\{p_1,p_2\}] > 0$ (or price dispersion remains), the LHS of (3) is positive. It does not converge to zero if $\lim\limits_{k\to\infty} (1-q/2)^k < 1$. We prove the last inequality by contradiction. Suppose $\lim\limits_{k\to\infty} (1-q/2)^k = 1$. Then, it must be that $\lim\limits_{k\to\infty} (1-q)^k = 1$ as otherwise it means that

$$\lim_{k\to\infty} \eta = \lim_{k\to\infty} \frac{\left(1-\frac{q}{2}\right)^{k+1}-(1-q)^{k+1}}{1+(1-q)^{k+1}-2\left(1-\frac{q}{2}\right)^{k+1}} = \frac{1-\lim\limits_{k\to\infty}(1-q)^{k+1}}{-1+\lim\limits_{k\to\infty}(1-q)^{k+1}} = -1,$$

which cannot be the case. However, that $\lim\limits_{k\to\infty} (1-q/2)^k = 1$ and $\lim\limits_{k\to\infty} (1-q)^k = 1$ also mean that

$$
\begin{aligned}
\lim_{k\to\infty}\eta &= \lim_{k\to\infty}\frac{\left(1-\frac{q}{2}\right)^{k+1}-(1-q)^{k+1}}{1+(1-q)^{k+1}-2\left(1-\frac{q}{2}\right)^{k+1}} \\
&\overset{\text{l'Hopital}}{=} \lim_{k\to\infty}\frac{\left(1-\frac{q}{2}\right)^{k+1}\left[\ln\left(1-\frac{q}{2}\right)-\frac{kq'}{2(1-\frac{q}{2})}\right]-(1-q)^{k+1}\left[\ln(1-q)-\frac{kq'}{(1-q)}\right]}{(1-q)^{k+1}\left[\ln(1-q)-\frac{kq'}{(1-q)}\right]-2\left(1-\frac{q}{2}\right)^{k+1}\left[\ln\left(1-\frac{q}{2}\right)-\frac{kq'}{2(1-\frac{q}{2})}\right]} \\
&= \lim_{k\to\infty}\frac{-\frac{kq'}{2}+kq'}{-kq'+kq'}=\infty,
\end{aligned}
$$

a contradiction. Thus, it must be that $\lim\limits_{k\to\infty}(1-q/2)^k < 1$. Then, the facts that $\lim_{k\to\infty}(E[p]-E[\min\{p_1,p_2\}] > 0$ and $\lim\limits_{k\to\infty}(1-q/2)^k < 1$ prove that the LHS of (3) is strictly positive as $k\to\infty$, or that $\lim\limits_{k\to\infty}\bar{l}(k) > 0$.

The proof of the proposition is complete.

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