Essays on Price Discrimination in Search Markets

Inauguraldissertation zur Erlangung des akademischen Grades eines Doktors der Wirtschaftswissenschaften der Universität Mannheim

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Frühjahrs-/Sommersemester 2023

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Tag der mündlichen Prüfung: 04. Mai 2023

Acknowledgements

First and foremost, I would like to thank Volker Nocke for his insightful guidance and his encouragement throughout my years as his student. I have learned much of what I know from him. His support has granted me incredible opportunities that have enabled me to grow as a researcher and a person. Being his student has been a true pleasure. I would also like to thank Ernst-Ludwig von Thadden. His profound insights and the important questions he prompted me to consider have greatly improved the research contained in this thesis. I am deeply indebted to both and sincerely hope to make them proud.

Moreover, I would like to thank Dirk Bergemann, who advised me during my time at Yale University and beyond. I have learned a tremendous amount during our discussions. The months I spent contributing to the Digital Economy Project under his direction have been among the best of my life. I would also like to thank Nicolas Schutz for his support during my years in Mannheim and for his extensive guidance during the job market.

My research has also greatly benefitted from the conversations I have had with Hans-Peter Grüner, Marina Halac, Maximilian Schäfer, Fiona Scott Morton, Thomas Tröger, Jonas von Wangenheim, and Jidong Zhou. I am very grateful to all of them.

I thank my coauthors Marco Reuter and Marcel Preuss. Collaborating with Marco was a wonderful experience, during which I have learned very much and we became friends. Similarly, working with Marcel has significantly deepened my understanding of the topics I study in this thesis. His comments on my other work, for which I am very grateful, have been extremely valuable. Working with both of them has been great fun.

I would also like to thank my friends Federico and Tsung-Hsien. I will always cherish the memories we made together during our Ph.D. journeys.

Further, I am very grateful to Alison, Berenice, Elena, Henry, Jacopo, Jan, Julia, Laura, Laura, Oliver, Marco, Max, Niko, Nam, and Sascha.

Finally, I would like to thank my parents for their enduring love and support. This achievement would not have been possible without their continuous encouragement and inspiration.

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Chapter 1

General Introduction

Price discrimination refers to pricing policies according to which different consumers pay different prices for the same good. The implementation of such pricing schemes is becoming increasingly viable in the digital economy, given that the granular consumer data needed to effectively implement it is becoming available to firms. In fact, there is mounting empirical evidence of price discrimination in online markets.[1] As a result, antitrust authorities around the world have become worried about these business practices. In 2016, the secretariat of the OECD's competition committee has recognized that "there are particular reasons to worry that price discrimination in digital markets will be harmful."[2] Recently, the European Union (EU) has implemented additional compliance rules targeted at firms who engage in online price discrimination.[3]

This thesis studies price discrimination in search markets, i.e. markets in which consumers cannot costlessly access the offers of all firms in the market. Understanding price discrimination in the presence of search frictions is important, given that search costs in online markets are known to be substantial.[4] Moreover, this form of research also establishes whether reductions of search frictions, which may be induced by price transparency regulation or may result from new developments such as augmented reality, will be pro-competitive in markets where prices are personalized.

This thesis consists of three self-contained chapters. In Chapter 2, titled *Competitive Price Discrimination, Imperfect Information, and Consumer Search*, I set up a homogenous goods model in which all firms have access to information about consumers' valuations: When being visited by a consumer, any firm receives a signal about the consumer's valuation. The competitive effects of the price discrimination enabled by the availability of this information fundamentally depend on the level of search frictions. Consumers visit multiple firms in equilibrium if and only if search costs are at intermediate levels. When

[1] See, for example, Hannak et al. (2014), Larson et al. (2015), and Escobari et al. (2019).

[2] For details, please see OECD Secretariat (2016).

[3] This is outlined in Directive 2019/2161 of the European Parliament and the Council. For details, please see European Parliament (2019).

[4] See, for example, Koulayev (2014), De los Santos (2018), and Jolivet and Turon (2019).

search costs are low, all consumers just visit one firm. Consumer welfare is highest when search costs are negligible. However, reductions of search frictions can lead to higher prices by increasing the amount of consumers who visit multiple firms in equilibrium.

In Chapter 3, titled *Search, Data, and Market Power*, I study the relationship between data and market power in the presence of search frictions. I set up a duopoly model of a final goods market in which one firm receives a noisy signal about the valuation of any arriving consumer, while its rival receives no information. In contrast to the previous chapter, one firm thus has an informational advantage. The main message of this chapter is that the search choices of consumers strongly amplify the transmission of data advantages into competitive advantages through a selection effect: Because the firm with data price discriminates, consumers with low valuations prefer to visit the firm with data, while consumers with high valuations prefer to visit the firm without data. These search patterns push up the price the firm without data optimally sets, which is to the benefit of the firm with access to superior data. Perhaps surprisingly, reductions of search frictions exacerbate the dominant position of the firm with data. The establishment of a right to data portability can address the competitive imbalances caused by data advantages.

Chapter 4, titled *Search Disclosure*, is joint work with Marcel Preuss. We investigate the possibility that sellers can inform their rivals that a given consumer has obtained an offer from them. The provision of this information, which we call *search disclosure*, is enabled by the use of online tracking technologies such as cookies and fingerprinting. Formally, we integrate the possibility of search disclosure into the classic Wolinsky (1986) model. We show that firms can only have incentives to conduct search disclosure regarding consumers for whom they have not previously received disclosure from their rival. This form of information exchange is anti-competitive and induces higher industry profits. However, it will not necessarily occur in equilibrium. Firms only conduct search disclosure if prices cannot be revised or if search costs are low. The insights we obtain have implications for the optimal regulation of tracking in online markets.

Chapter 2

Competitive Price Discrimination, Imperfect Information, and Consumer Search

2.1 Introduction

The issue of online price discrimination has gained increased attention by legal authorities around the world in the last years, reflecting the growing body of empirical evidence for its prevalence.[1] In 2016, the secretariat of the OECD's competition committee recognized that "there are particular reasons to worry that price discrimination in digital markets will be harmful".[2] In the European Union (EU), new compliance rules targeted at firms engaging in online price discrimination have taken effect in 2022, complementing the general privacy regulation established in the GDPR and the DMA.[3]

This chapter studies price discrimination based on information about consumers' valuations in markets with search frictions. Understanding how the personalization of prices interacts with consumers' search choices is important, given that search frictions in online markets are known to be substantial.[4] Moreover, this analysis also establishes whether price transparency regulation, which has always been a core area of competition policy, can effectively mitigate potential negative effects of price discrimination.[5]

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[1]Hannak et al. (2014) show that e-commerce platforms differentiate prices by whether a consumer uses iOS or Android. Larson et al. (2015) demonstrate that prices for Princeton's SAT packages depend on the demographic characteristics of a consumer's ZIP code. Escobari et al. (2019) document that airline ticket prices are higher during business hours, when business travelers are more likely to buy.

[2]For details, please see OECD Secretariat (2016).

[3]This is outlined in Directive 2019/2161 of the European Parliament and the Council. For details, please see European Parliament (2019).

[4]See, for example, Koulayev (2014), De los Santos (2018), and Jolivet and Turon (2019).

[5]Price transparency is addressed in several directives of the EU, such as the 1998 Unit Prices Directive and the 2005 Unfair Commercial Practices Directive. For details, please see European Parliament (1998) and European Parliament (2005).

The main message of this chapter is that the competitive effects of price discrimination depend on the level of search frictions in a market. The equilibrium prices are lowest and consumer welfare is highest when search costs are low because low search costs endow consumers with a strong threat of searching. However, the effects of search cost reductions are non-monotonic. At intermediate levels of search costs, the presence of a sufficiently informative signal enables the existence of an equilibrium in which consumers visit multiple firms. In this equilibrium, reductions of search frictions lead to higher equilibrium prices. This is because consumers who arrive at a firm after visiting its rival(s) entail demand that is fully inelastic around the lowest equilibrium price, and reductions of search frictions increase the amount of consumers who visit multiple firms in equilibrium.

I consider a final goods market with search frictions. There is a unit mass of consumers who want to buy one unit of a homogeneous good, which is produced by a finite number of firms. Consumers have heterogeneous valuations for the good, which are private information to each consumer. They acquire consumption opportunities via sequential search and every consumer can costlessly visit one firm, but has to pay a search cost per additional firm that is visited.

The firms have information about the valuations of consumers. When a firm is visited by a consumer, the firm receives a noisy private signal about the valuation of this consumer. There are two possible signal realizations: low or high. The probability distribution of the signal is a step-function: Consumers with a valuation in the higher half of the interval of possible valuations are more likely to generate the high signal than the low signal. By contrast, consumers with a valuation in the lower half of the valuation spectrum are more likely to generate the low signal. The firms will use the available information to price discriminate: They will offer a relatively low price to all consumers who generate the low signal and will quote higher prices to consumers who generate the high signal.

The equilibrium price dispersion induced by the availability of said information endows consumers with incentives to search: Consumers who receive a comparatively high price at a firm may find it worthwhile to visit another firm in the hope of attaining a lower price there. Importantly, the strength of a consumer's incentives to search depend on her valuation. Consumers with very low valuations (i.e. with a valuation below the lowest equilibrium price) will never find it worthwhile to visit multiple firms. Moreover, consumers with very high valuations will also not find it worthwhile to search in pursuit of a lower price, because the probability of generating the favorable low signal is small for them. Thus, consumers with intermediate valuations have the highest incentives to search, which matches the empirical pattern documented by Byrne and Martin (2021).[6]

Different levels of search costs generate structurally different equilibria. In a nutshell, this is based on the fact that consumers can constrain the firms' prices with the threat of

[6]The authors show that there is an inverse U-shaped relationship between income and search intensity.

searching. This is because any firm has incentives to deter consumers who visit them from continuing to search. If search costs are sufficiently low, consumers are endowed with a strong threat of searching. The firms will, in response, optimally set comparatively low prices. Formally, the highest equilibrium price will be set in such a way that the consumers with the highest search incentives are exactly indifferent between continuing to search or not when receiving this price.

There is a non-monotonic relationship between the level of search costs and the number of firms consumers visit in equilibrium. When search costs are high, any firm sets the prices it would offer to consumers if it were a monopolist and consumers do not find it optimal to continue searching, even when receiving the highest equilibrium price. Perhaps surprisingly, all consumers also only visit one firm in equilibrium when search costs are small. Intuitively, the mere threat of searching is sufficient to push down prices (and their difference) so much that the actual act of visiting multiple firms is not optimal.

At intermediate levels of search costs, consumers will visit multiple firms in equilibrium. This holds by the following logic: When search costs are at intermediate levels, consumers cannot effectively constrain the prices of firms with the threat of searching, so firms set comparatively high prices. However, because search costs are not prohibitively high, consumers with intermediate valuations, who have the highest incentives to search, will find it optimal to continue searching if they receive an unfavorably high price.

Consumer welfare, which I measure by ex-ante consumer utility, is maximal when search costs are zero. However, the effects of changes in search costs are also non-monotonic. Reductions of search costs affect prices through two possible channels, namely by (i) changing the search incentives of consumers and by (ii) potentially expanding the set of consumers who search on the equilibrium path.[7] Expansions of the set of consumers who search on the equilibrium path will induce firms to raise the lowest equilibrium price. This is because any consumer who arrives at a firm after visiting its rival(s) would directly buy when offered the lowest equilibrium price.[8]

At low levels of search costs, only the first channel is active, because consumers do not visit multiple firms in equilibrium. Thus, an increase of search costs will lead to higher prices because the ability of consumers to restrict the firms' prices by threatening to search is reduced.

There is a threshold level of search costs at which a marginal increase of search costs will trigger a switch of the equilibrium that is played. When this happens, both equilibrium prices (or their average levels) will jump up discontinuously and consumer welfare is substantially reduced. This upward jump in prices is accompanied by a discontinuous increase in the amount of consumers that visit multiple firms on the equilibrium path.

[7] I say that a consumer searches on the equilibrium path if and only if she would continue searching when offered the highest equilibrium price.

[8] Otherwise, paying a search cost to visit an additional firm would not have been optimal.

At this point of discontinuity, both aforementioned channels are active. Consumers lose their ability to sustain low prices with the credible threat of searching, which induces an upward jump in the prices the firms set when observing the high signal realization. This, in turn, triggers search by a strictly positive measure of intermediate-valuation consumers. Because these consumers generate locally price inelastic demand around the lowest equilibrium price, the latter also jumps up discontinuously.

In any equilibrium in which consumers visit multiple firms, an increase of search costs leads to a reduction of the lowest equilibrium price. This result is driven by the second working channel: Less consumers arrive at any firm after visiting its rival(s), which reduces the upward pressure this consumer group exerts on said price.

In section 2.5, I consider generalized signal distributions. I show that the structure of any pure-strategy equilibrium mirrors its counterpart in the baseline model when restricting attention to binary signal distributions that are (i) continuously differentiable, (ii) strictly monotonic, and (iii) generate signal probabilities between zero and one for any valuation.

The rest of this chapter proceeds as follows: I lay out the related literature in section 2. In section 3, I set up the theoretical model, which is solved in section 4. I study the aforementioned extension in section 5 and conclude in section 6.

2.2 Related literature

My work is related to the developing strand of theoretical research which connects price discrimination to endogenous consumer search choices.[9] Fabra and Reguant (2020) study a simultaneous search setting where firms observe a consumer's desired quantity and price discriminate based on this information. Armstrong and Zhou (2016) and Preuss (2022) consider models where firms condition prices on a consumer's search history.[10] Mauring (2022) and Atayev (2022) study a setting with shoppers and non-shoppers as defined in Burdett and Judd (1983) and Stahl (1989). Mauring (2022) and Atayev (2022) assume that firms receive imperfect information about the affiliation of a particular consumer to the groups of shoppers and non-shoppers. Bergemann et al. (2021) consider a model where competing firms receive imperfect information about a consumer's search technology and/or the number of price offers a consumer obtains or has previously obtained.[11] In contrast to all these papers, I study a model in which firms receive information about

[9] My results are also related to the well-known Diamond paradox established in Diamond (1971). I show that the presence of a sufficiently informative signal about consumer valuations is sufficient to generate equilibria with on-path search when search costs are at intermediate levels.

[10] Armstrong and Vickers (2019) analyse a setting where firms face exogenously captive and non-captive consumers and receive information about this status.

[11] Garrett et al. (2019) consider a model of second-degree price discrimination in which consumers differ in their choice sets. Braghieri (2019) studies a model where firms condition prices on how a consumer reaches a firm — through an intermediary or via a sequential search process.

consumers' valuations.

Marshall (2020) is the only other paper which studies price discrimination based on information about valuations together with search. However, while Marshall (2020) sets up and empirically calibrates a structural model, he provides no analytical equilibrium characterization. Moreover, there are important differences in setup: In contrast to my work, Marshall (2020) assumes that sellers have perfect information about all components of a consumer's preferences except for search costs and considers a different search setup. In Marshall (2020), recall is impossible: When a consumer decides to search, she will never return to purchase at the firm she interacted with. In my framework, recall is free.

Moreover, my work is related to a handful of papers which study price discrimination based on noisy information about preferences in competitive settings.[12] Esteves (2014) analyses a Hotelling-style framework where firms receive noisy information about the horizontal preference parameters of consumers. Peiseler et al. (2022) analyse collusion within a model that is quite similar to the one analysed in Esteves (2014). Clavorà Braulin (2021) studies a horizontal differentiation setting where consumer preferences vary in two dimensions. In Clavorà Braulin (2021), firms have perfect information about the realizations of one dimension of consumer preferences, but not both.[13] Belleflamme et al. (2020) study a homogeneous goods model where two competing firms have differential access to an imperfect profiling technology. My work differs from all these papers in the sense that I study a model with search frictions, which the preceding papers do not.[14]

2.3 Framework

There is a unit mass of consumers, who each want to buy one unit of an indivisible and homogenous good. There are N active firms indexed $j \in \{1, 2, ..., N\}$ who produce this good at zero marginal cost. Consumers visit firms through sequential search. They can costlessly visit one firm, but visiting any firm after the first incurs search costs $c > 0$ per firm that is additionally visited. When a consumer visits a firm and decides to continue searching, the consumer can still purchase the good from the initially visited firm at the price that was previously offered to her without further cost.

Consumers are heterogeneous in their valuations for the good (v), which are private

[12] My work also has ties to the theoretical contributions which study price discrimination based on imperfect information in monopoly settings, such as Belleflamme and Vergote (2016), de Cornière and Montes (2017), and Valletti and Wu (2020).

[13] All the listed papers also assume that the market is fully covered, which I do not. Rhodes and Zhou (2022) show that this assumption is not without loss of generality when studying the welfare effects of price discrimination based on perfect information.

[14] Ali et al. (2023) study optimal information disclosure by consumers in a model of price discrimination. Guembel and Hege (2021) consider a model in which competing firms receive noisy signals about the most favored products of consumers and use these signals to offer products designed to match the individual consumers' tastes.

information to each consumer. The distribution of these valuations, namely the uniform distribution on $[0, 1]$, is common knowledge. When a consumer with valuation v buys the good at price p, the utility of the consumer is:

$$u(v, p) = v - p \tag{2.3.1}$$

The firms have access to information about consumers. When a firm j is visited by a consumer, this firm receives a private binary signal $\tilde{v}_j \in \left\{\tilde{v}^L, \tilde{v}^H\right\}$ about the valuation of this consumer. The probability distribution of the signal is

$$Pr(\tilde{v}^H|v) = \begin{cases} \alpha & \text{if } v \geq 0.5 \\ 1-\alpha & \text{if } v < 0.5 \end{cases} \tag{2.3.2}$$

I assume that $\alpha \in (0.5, 1)$. All firms know nothing about any consumer's search history. Every firm can offer a different price to any arriving consumer. As in Diamond (1971), firms thus cannot attract consumers through downward deviations from equilibrium prices.

The exact timing of the game is as follows: After discovering their valuation, any consumer randomly visits some firm first (at zero cost). When the consumer visits this firm, nature draws a signal according to the aforementioned distribution and privately reveals the outcome to the firm that is visited first.[15] Based on the signal it observes, this firm offers the consumer a price. After observing this price offer, the consumer decides whether or not she wants to visit an additional firm at cost $c > 0$.

If the consumer decides to visit an additional firm, nature draws another signal with the aforementioned distribution and privately reveals the signal to the firm which is visited second. Conditional on the consumer's valuation, this signal is drawn independently of the signal received by the firm which was visited first. Based on the signal it observes, the firm that is visited second offers the consumer a price.

The consumer can sequentially visit all N firms. Any firm that is visited will observe a signal with the aforementioned properties and will, based on this signal, offer the consumer a price. The consumer always chooses which firm to visit next randomly.[16] The game ends when all firms have been sampled or the consumer stops the search process. Then, the consumer decides from which firm to buy the product, or not to buy the good at all, in which case the consumer receives zero utility.

I study perfect Bayesian equilibria and restrict attention to symmetric equilibria. Any firm can be called to act in two information sets: It can be visited by a consumer and

[15] Formally, the equilibrium strategy of nature is thus to reveal the signal $\tilde{v}^L$ with probability $Pr(\tilde{v}^L|v)$ and the signal $\tilde{v}^H$ with probability $Pr(\tilde{v}^H|v)$ whenever prompted.

[16] This is without loss of generality because I restrict attention to symmetric equilibria and all firms are ex-ante identical.

observe the low signal or it may be visited by a consumer and observe the high signal. Thus, a pure strategy of any firm is a price tuple (p^L, p^H): The firm offers the price p^L (p^H) to any consumer who visits it and generates the low (high) signal. A consumer's search strategy must optimally define whether or not to continue search, conditional on the search history. Because I restrict attention to symmetric equilibria and the optimal search rule will thus be myopic, a consumer's search strategy can be described by a cutoff $\hat{p}(v)$: A consumer with valuation v will visit another firm if and only if the lowest price she has in hand is above $\hat{p}(v)$.

Both information sets of the firms are on the equilibrium path, because all consumers choose which firm to visit first randomly. The firms' beliefs are thus pinned down by Bayes' rule. Firms form beliefs over (i) the valuation of the consumer, (ii) the consumer's search queue, and (iii) the prices the consumer has received from the other firms (if any).

Given the timing outlined above, the off-path beliefs consumers form do not affect their decisions, implying that these beliefs do not need to be restricted throughout the analysis. At any point during the game, the only part of the history a consumer does not know is what signals were generated at previously visited firms. However, these do not affect her incentives to search, conditional on the best price in hand. This is because her utility of visiting the next firm only depends on the strategy of nature and the equilibrium strategy of firms, both of which are taken as given under sequential rationality.

Even under slight perturbations of the timing, the off-path beliefs of consumers do not play a central role. Suppose firstly that nature privately draws N signals for any firm at the beginning of the game, but only reveals any firm's signal to the firm when it is visited by the consumer. Then, assuming that the consumer's beliefs regarding the choices of nature are passive will ensure that all results from the equilibrium analysis I present continue to hold.[17] The only timing under which consumers would have to form beliefs about the choices of firms would be if firms, before being visited, privately set their price strategies. In such a framework, imposing that consumers' beliefs are passive with respect to the strategy of other firms would imply that all results of my equilibrium analysis extend.

In both alternative setups, the assumption of passive beliefs seems reasonable, given the well-known idea that you cannot signal what you don't know. Consider the first alternative timing: Nature is not a strategic player and firms do not observe the signals that nature draws for other firms. Hence, a given firm's choices cannot be informative about these signals. Now consider the second alternative timing. In this framework, firms do not observe their rival's strategies, so their pricing decisions cannot be informative about the former.

[17] Formally, the consumers' beliefs would have to be passive in the following sense: When receiving an off-equilibrium price offer, the consumer continues to believe that *nature* followed its equilibrium strategy, i.e. has drawn signals according to the aforementioned distribution for any firm.

Before moving forward with the analysis of the competitive equilibria, I consider the monopoly benchmark. To that end, I define $\Pi^M(p_j|\tilde{v}^k)$ as the expected profit a monopolist with access to the aforementioned signal stucture obtains when offering the price p_j to a consumer who generates the signal $\tilde{v}^k$, with global maximizers $\{p^{k,M}\}_{k\in\{L,H\}}$ given by:

$$p^{k,M} = arg\max_{p_j} \underbrace{\int_{p_j}^{1} Pr(\tilde{v}^k|v)dv}_{:=\Pi^M(p_j|\tilde{v}^k)} \tag{2.3.3}$$

The monopoly profit functions are strictly concave piecewise and the implied optimal monopoly prices are $p^{L,M} = 1/4\alpha$ and $p^{H,M} = 0.5$.[18]

Having defined the monopoly prices, we can also briefly consider the case with frictionless search, i.e. when $c = 0$. Then, all firms will offer the same uniform price in equilibrium. Any uniform price below $p^{L,M}$ can be supported as an equilibrium.

I define whether there is search on the equilibrium path in a given equilibrium using the set of consumer valuations for which the probability of visiting multiple firms in equilibrium is strictly positive. I say that there is search on the equilibrium path if and only if this set has positive measure. Finally, I impose the following tie-breaking rule:

Assumption 1 *Consumers continue searching if and only if it is strictly optimal for them to do so.*

2.4 Equilibrium analysis

2.4.1 Pure-strategy equilibria

In this section, I characterize the symmetric pure-strategy equilibria that exist in the aforementioned framework. To begin with, I define the optimization calculus of the firms and the consumers. Because the consumers' optimal search rule is myopic, a consumer with valuation v whose best price in hand is p_j will visit another firm if and only if:

$$\sum_{k\in\{L,H\}} Pr(\tilde{v}^k|v)\max\{v - p^k, 0\} - c - \max\{v - p_j, 0\} > 0 \tag{2.4.1}$$

Now consider the optimization problem of the firms, which optimally set their two prices p^L and p^H, given the consumers' search behaviour represented by the function $\hat{p}(v)$. In an equilibrium without on-path search, a firm maximizes the following profit function

[18]The monopoly low signal price $p^{L,M}$ is falling in α because a more precise signal implies that the average valuation of consumers that generate the low signal decreases.

through choice of p^k when observing the signal $\tilde{v}^k$, with $k \in \{L, H\}$:

$$\Pi^C(p_j; \tilde{v}^k) = p_j \left[\int_{p_j}^{1} \frac{1}{N} \mathbb{1}[p_j \leq \hat{p}(v)] Pr(\tilde{v}^k|v) dv \right] \tag{2.4.2}$$

In a symmetric equilibrium with search on the equilibrium path, this profit function becomes:

$$p_j \Bigg[\int_{p_j}^{1} \frac{1}{N} \big[\mathbb{1}[p_j \leq \hat{p}(v)] + \mathbb{1}[p_j > \hat{p}(v)] \mathbb{1}[p_j < p^H] \big(Pr(\tilde{v}^H|v) \big)^{N-1} \big] Pr(\tilde{v}^k|v) dv + \sum_{j=2}^{N} \int_{p_j}^{1}$$

$$\frac{\mathbb{1}[p^H > \hat{p}(v)]}{N} \big[\mathbb{1}[p_j \leq \hat{p}(v)] Pr(\tilde{v}^H|v)^{j-1} + \mathbb{1}[p_j > \hat{p}(v)] \mathbb{1}[p_j < p^H] Pr(\tilde{v}^H|v)^{N-1} \big] Pr(\tilde{v}^k|v) dv \Bigg] \tag{2.4.3}$$

Any symmetric equilibrium in pure strategies is characterized by the following proposition:

Proposition 1 (Equilibrium structure)
In any symmetric pure-strategy equilibrium, $p^L < p^H$ must hold. Moreover, the set of consumers with $v \geq p^H$ that (i) have a strictly positive probability of being offered p^H and (ii) search when the best price in hand is p^H must have measure zero.

In a nutshell, the first result holds because the signal is strictly informative about the consumers' valuations (since $\alpha > 0.5$ holds). Thus, firms would not be optimizing in a hypothetical equilibrium in which $p^H \leq p^L$. The second result must hold by the following logic: Consider a symmetric pure strategy equilibrium and suppose, for a contradiction, that there is a strictly positive measure of consumers with $v > p^H$ and $\hat{p}(v) < p^H$ who receive the price p^H with strictly positive probability (at any firm). Any such consumer who receives p^H at the initial firm will continue searching until obtaining a lower price or there are no more firms to sample. Thus, there would be a strictly positive measure of consumers with $v > p^H$ who receive p^H at all N firms. However, this would induce some firm to slightly undercut p^H, because this ensures that the sale is made to all these consumers, which represents a contradiction.

The insights of this proposition imply that, if there is search on the equilibrium path, only consumers with intermediate valuations will visit multiple firms.[19] No consumer with $v \leq p^L$ will find it optimal to continue searching after receiving either price. Moreover, any consumer with $v \geq p^H$ must not find it optimal to continue searching after receiving p^H, since this would endow the firms with incentives to undercut this price. Hence, only

[19] Naturally, no consumer would find it optimal to continue searching after receiving p^L, because search costs are strictly positive.

consumers with $v \in [p^L, p^H]$ can potentially search on the equilibrium path. This search behaviour matches the empirical pattern established in Byrne and Martin (2021), who document that there is an inverse U-shaped relationship between search intensity and income.

Based on this, we can further characterize any symmetric pure-strategy equilibrium with on-path search:

Lemma 1 (Equilibrium search patterns)
In a symmetric pure-strategy equilibrium with search on the equilibrium path, (i) $p^H = 0.5$, (ii) $\frac{c}{\alpha} + p^L < 0.5 \leq \frac{c}{1-\alpha} + p^L$, and (iii) $p^L = p^{L,S}$ must hold, where:

$$p^{L,S} = \frac{\alpha + 2(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)(0.5\alpha - c)}{4\alpha^2 + 2\alpha(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)} \tag{2.4.4}$$

The equilibrium high signal price p^H must be exactly equal to 0.5 if there is search on the equilibrium path. Suppose, for a contradiction, that $p^H < 0.5$ and there is search on the equilibrium path. Then, there are two possibilities: (i) A consumer with $v = p^H < 0.5$ strictly prefers to search when receiving p^H or (ii) a consumer with $v = p^H$ weakly prefers to not search when receiving p^H. In the first case, all consumers with $v \in [p^H, 0.5)$ will search when receiving p^H, which is a contradiction by the insights of proposition 1. In the second case, no consumer will find it optimal to search on the equilibrium path, since the incentives to search are maximal for consumers with $v \in [p^H, 0.5)$, a contradiction. By similar arguments, no equilibrium with $p^H > 0.5$ and on-path search can exist either.

The chain of inequalities listed in the second point must hold by the following logic: In an equilibrium with on-path search, high-valuation consumers (i.e. consumers with $v > 0.5$) must not find it optimal to continue search when their best price in hand is p^H. Thus, their cutoff price must be above p^H, which implies that search costs must be high enough to ensure that $0.5 \leq p^L + c/(1-\alpha)$ holds. By contrast, some consumers with intermediate valuations must find it optimal to search after receiving p^H. This is guaranteed if and only if $p^L + c/\alpha < 0.5$, because the cutoff price of consumers with $v \in (p^L + c/\alpha, 0.5)$ would then be equal to $p^L + c/\alpha$, i.e. below $p^H = 0.5$.

To establish that $p^L = p^{L,S}$ as defined in equation (2.4.4) must hold, we need to characterize the equilibrium search behaviour of consumers. There are three different valuation intervals to consider, namely (i) $v \in [0, p^L + c/\alpha]$, (ii) $v \in (p^L + c/\alpha, 0.5)$, and (iii) $v \in [0.5, 1]$. Consumers with $v \leq p^L + c/\alpha$ will not visit multiple firms, because their gains of search are strictly negative for any best price in hand. By contrast, consumers with $v \in (p^L + c/\alpha, 0.5)$ will find it optimal to continue searching when their best price in hand is p^H. Consumers with $v \in [0.5, 1]$ will find it optimal to refrain from searching in equilibrium, because $0.5 \leq p^L + c/(1-\alpha)$. For convenience, I now visualize the sequentially rational cutoff price $\hat{p}(v)$ function for different consumer groups in this equilibrium:

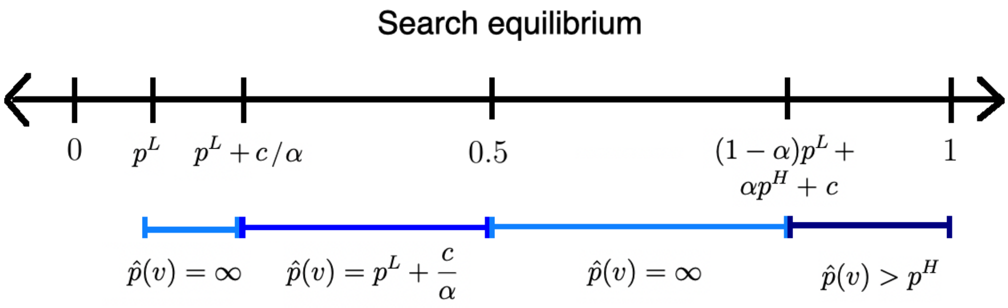

Figure 2.1: Price cutoffs in the search equilibrium

Having noted this, consider the optimization problem of a firm that receives the low signal. When offering a price $p_j \in [0, p^L + c/\alpha]$, no consumer will continue searching and all consumers who arrive at this firm after searching will buy. Consumers with $v \in (p^L + c/\alpha, 0.5)$ who did not start their search process at this firm arrive after searching if and only if they generate the high signal at all firms they have previously visited. Thus, the low signal profit function is continuously differentiable when $p_j \in [0, p^L + c/\alpha]$ and is given by:

$$\Pi^C(p_j; \tilde{v}^L) = p_j \underbrace{\int_{p_j}^{1} (1/N) Pr(\tilde{v}^L|v) dv}_{\text{First arriver demand}} + \underbrace{p_j \sum_{j=2}^{N} \left[\int_{c/\alpha + p^L}^{0.5} (1/N) Pr(\tilde{v}^L|v) Pr(\tilde{v}^H|v)^{j-1} dv \right]}_{\text{Demand from consumers who have searched}} \tag{2.4.5}$$

Setting up the first-order condition that p^L must solve and rearranging terms yields the expression given in equation (2.4.4). Note that $p^{L,S} > p^{L,M}$ must hold. This follows from the fact that consumers who arrive at a firm after searching generate inelastic demand around the equilibrium low signal price, which pushes up this price.

The characterization of possible equilibrium candidates without on-path search is more straightforward. One such candidate is the monopoly outcome, which can be sustained as a competitive equilibrium when search costs are high enough. Then, firms will optimally set the prices $p^{L,M}$ and $p^{H,M}$ and no consumer will find it worthwhile to visit multiple firms in equilibrium. I call this equilibrium candidate the monopoly price equilibrium.

There is another candidate for a symmetric equilibrium without search on the equilibrium path. In this equilibrium, $p^L = p^{L,M}$ must hold and the high signal price is equal to $p^L + c/\alpha$, i.e. is set to make consumers with the highest incentives to search exactly indifferent between continuing to search or not. I define this equilibrium candidate as the search deterrence equilibrium and now visualize the cutoff price function $\hat{p}(v)$ for this equilibrium:

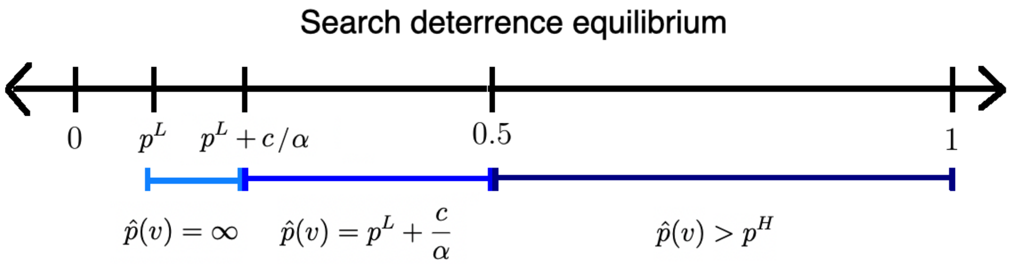

Figure 2.2: Price cutoffs in the search deterrence equilibrium

There are no other candidates for a symmetric pure-strategy equilibrium. The arguments pertaining to lemma 1 imply that there is a unique candidate for a symmetric equilibrium with on-path search. In an equilibrium without on-path search, $p^L = p^{L,M}$ must hold and p^H must either be equal to $p^L + c/\alpha$ or 0.5. The low signal price must be equal to $p^{L,M}$ because no consumer continues searching when offered a price in an open ball around the equilibrium p^L. It would never be optimal to set a high signal price above $p^{H,M} = 0.5$, because a downward deviation to $p^{H,M}$ would guarantee higher profits. Setting any price below 0.5 can only be optimal for a firm if marginal upward deviations would trigger search by a large amount of consumers, which requires that $p^H = p^L + c/\alpha$. These notions are formalized in the following proposition.

Proposition 2 (Pure-strategy equilibrium candidates)
There are three candidates for a symmetric pure-strategy equilibrium, namely:

- *The monopoly price equilibrium, in which* $(p^L, p^H) = (p^{L,M}, p^{H,M})$.
- *The search deterrence equilibrium, in which* $(p^L, p^H) = (p^{L,M}, p^{L,M} + c/\alpha)$.
- *The search equilibrium, in which* $(p^L, p^H) = (p^{L,S}, p^{H,M})$.

Thus, there potentially exist structurally different equilibria in this model. I now establish when the aforementioned equilibrium candidates exist:

Proposition 3 (Existence regions: pure-strategy equilibria)
The existence regions for the symmetric pure-strategy equilibrium candidates are as follows:

- *The monopoly price equilibrium exists if and only if* $c \geq \alpha(0.5 - p^{L,M})$.
- *The search equilibrium exists if and only if (i)* $c \in \big[(1-\alpha)(0.5-p^{L,S}), \alpha(0.5-p^{L,S})\big)$ *<u>and</u>* $\Pi^C(p^{L,S} + c/\alpha; \tilde{v}^H) \leq \Pi^M(0.5, \tilde{v}^H)$.
- *The search deterrence equilibrium exists if and only if (i)* $c \in \big(0, 0.5(0.5 - p^{L,M})\big)$ *<u>and</u>* $\Pi^C(p^{L,M} + 2c; \tilde{v}^H) \leq \Pi^M(p^{L,M} + c/\alpha; \tilde{v}^H)$.

Given that I have derived closed-form expressions for $p^{L,M}$, $p^{H,M}$, and $p^{L,S}$, one can analytically verify which equilibrium exists for any parameter combination.

Before visualizing the parameter regions for which these equilibrium candidates actually exist, it is useful to consider the ordering of the different equilibrium prices, holding the parameters fixed. The low signal price in the search equilibrium ($p^{L,S}$) must be strictly above the monopoly low signal price $p^{L,M}$. Moreover, the high signal price in the search deterrence equilibrium must be strictly below the monopoly high signal price ($p^{H,M} = 0.5$), because this equilibrium would not exist otherwise.

I now visualize the results of proposition 3 in the following figure, in which I plot different values of signal precision (α) on the x-axis and different search costs (c) on the y-axis. A given graph corresponds to a fixed level of firms ($N \in \{2, 3, 4\}$) The content can be interpreted as follows: Blue dots indicate that the unique equilibrium in pure strategies is the monopoly price equilibrium. Yellow dots indicate that the search equilibrium exists and is unique (within the set of symmetric pure-strategy equilibria). Light green dots indicate that both the search deterrence and the search equilibrium exist. Dark green dots indicate that the search deterrence equilibrium exists and is unique (again, within the set of symmetric pure-strategy equilibria).

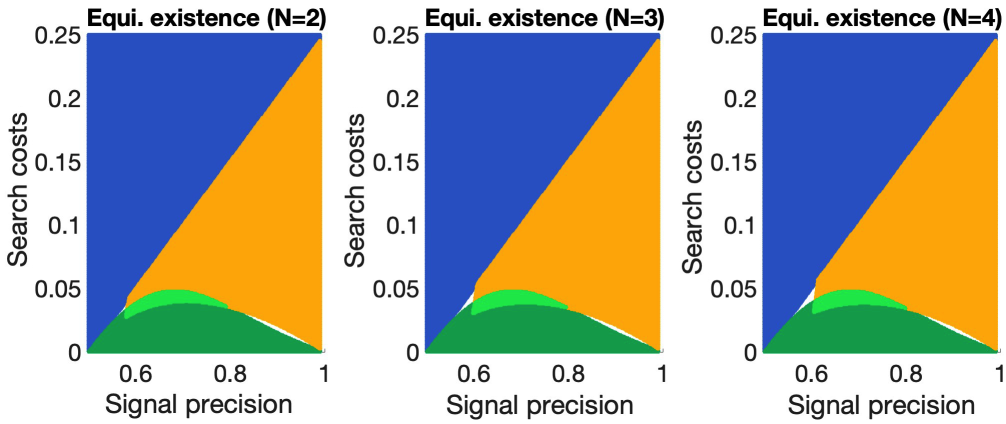

Figure 2.3: Existence regions: pure-strategy equilibria

The key message of this graph is that there is a non-monotonic relationship between the level of search frictions and how many firms consumers visit in equilibrium. When search costs are low or high, all consumers will only visit one firm in equilibrium. However, when search costs are at intermediate levels, there is search on the equilibrium path.

When search costs are sufficiently high, the monopoly outcome will emerge. At any possible equilibrium price, consumers would never find it optimal to search. Thus, any firm will just be visited by consumers who randomly arrive there first. Because consumers cannot effectively constrain prices with a credible threat of searching, all firms will optimally charge the monopoly prices.

The search equilibrium exists if and only if search costs are at an intermediate level. To see why this holds, recall the way in which consumers must search in an equilibrium with on-path search (see proposition 1): High-valuation consumers must not find it optimal to continue searching when receiving p^H, while consumers with intermediate valuations must find it optimal to continue searching if their best price in hand is p^H. Because the incentives to search are highest for consumers with $v \in (p^{L,S} + c/\alpha, 0.5)$, intermediate search costs are necessary and sufficient to sustain this search behaviour in equilibrium.

The search deterrence equilibrium exists if search costs are sufficiently low. This holds by the following logic: In general, firms have a desire to deter consumers who visit them from continuing to search, because consumers who visit a given firm's rival will buy at this firm with lower probability. Thus, consumers can constrain the prices of the firms with the threat of searching. When search costs are low, this threat of searching is very strong, which induces the firms to optimally set comparatively low prices. Because the firms' prices are small, consumers will not search on the equilibrium path. Intuitively, the mere threat of searching is sufficient to push down prices (and their difference) so much that the actual act of visiting multiple firms is not optimal.

Finally, there are small regions in which no symmetric pure-strategy equilibrium exists. For such parameters, mixed-strategy equilibria will be played, which I characterize in the next subsection.

2.4.2 Mixed-strategy equilibria

Now, I move on to characterize the set of symmetric mixed-strategy equilibria (MSE) that can exist in the baseline model. First, note the results of the following lemma:

Lemma 2 (Mixed-strategy equilibria: structure)
In any symmetric MSE:

- *The firms offer a deterministic price p^L when observing the low signal. This price is the lowest price that is offered in equilibrium.*
- *The probability that a firm offers a price above 0.5 after either signal is 0. Consumers with $v > 0.5$ will not visit multiple firms in equilibrium.*

The first result follows from the fact that the demand generated by consumers who arrive at a firm after searching will be fully inelastic around the lowest equilibrium price. This is because it can only be worthwhile for a consumer to search if she would directly buy when offered the lowest equilibrium price. Thus, the firms' profit functions will be strictly concave around the lowest equilibrium price, which implies that this price must be played with positive probability. Moreover, this lowest price must be offered after the low signal and no other price can be offered to consumers who generate this signal, because the low signal profit function has a unique maximum in any mixed-strategy equilibrium.

Secondly, the highest equilibrium price (call this p^{max}) has to be weakly below 0.5.[20] If $p^{max} > 0.5$, all consumers with $v \geq p^{max}$ have identical search incentives and there are just two possible outcomes: Either all consumers with $v \geq p^{max}$ search upon receiving p^{max}, or none of them search. If all of them search, there are either undercutting incentives from p^{max} (if p^{max} is played with positive probability) or p^{max} yields zero profits (if p^{max} is played with zero probability). If none of them search at this price p^{max}, they would also not search when offered the price $p_j = 0.5$, so firms would find it optimal to deviate downwards from p^{max} to 0.5. In any case, we obtain a contradiction.

We now work towards constructing an equilibrium in mixed strategies with elements $(p^L, \underline{p}^H, \bar{p}^H, F^H(p))$: When observing the low signal, firms set the price p^L. When observing the high signal, firms draw prices from the distribution $F^H(p)$. I define the convex hull of the support of this distribution as $[\underline{p}^H, \bar{p}^H]$.

First, one can show that $p^L + c/\alpha \leq \underline{p}^H$ must hold in such an equilibrium. Otherwise, there exists an interval of prices above $\underline{p}^H$ for which no consumer would continue searching. But then, the high signal profit function of a firm would be strictly increasing for prices around $\underline{p}^H$, a contradiction. This result implies that all consumers with $v \in (p^L + c/\alpha, 0.5)$ will continue searching when receiving any any price $p_j \in (p^L + c/\alpha, \bar{p}^H]$. By lemma 2, no other consumers can search on-path.

When considering an equilibrium in mixed strategies, it is hence important to distinguish whether the price $p^L + c/\alpha$, which deters search by all consumers, is played with positive probability or not. If it is played with zero probability, all consumers with $v \in (p^L + c/\alpha, 0.5)$ will continue searching after visiting some firm if and only if they generate the high signal at this firm. Thus, the measure of consumers who arrive at any firm after searching in such an MSE must be the same as in the search equilibrium. If this price is played with positive probability (i.e. $F^H(p^L + c/\alpha) > 0$), then the measure of consumers who search on the equilibrium path would be smaller, ceteris paribus. This is because consumers with $v \in (p^L + c/\alpha, 0.5)$ will continue searching after visiting some firm if and only if they generate the high signal and receive a price above $p^L + c/\alpha$.

This distinction also matters for the derivation of the equilibrium low signal price. Formally, the low signal profit function is given by the following for all prices in an open ball around the equilibrium p^L:

$$\underbrace{p_j \int_{p_j}^{1} \frac{1}{N} Pr(\tilde{v}^L|v) dv}_{\text{First arriver demand}} + \underbrace{p_j \sum_{j=2}^{N} \left[\int_{c/\alpha + p^L}^{0.5} \frac{1}{N} Pr(\tilde{v}^L|v) \left[Pr(\tilde{v}^H|v)(1 - F^H(p^L + c/\alpha)) \right]^{j-1} dv \right]}_{\text{Demand from consumers who have searched}} \tag{2.4.6}$$

[20]Formally, I define the highest price as the supremum of the support of the price distribution from which firms draw prices when they observe the high signal.

As a result, the equilibrium low signal price must solve:

$$\left.\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\right|_{p^L} + \sum_{j=2}^{N}\int_{c/\alpha+p^L}^{0.5}\frac{1}{N}Pr(\tilde{v}^L|v)\big[Pr(\tilde{v}^H|v)(1-F^H(p^L+c/\alpha))\big]^{j-1}dv = 0 \tag{2.4.7}$$

If $F^H(p^L + c/\alpha) = 0$, the low signal profit function is equal to the function given in equation (2.4.5) for all prices in an open ball around the equilibrium p^L. Then, the equilibrium p^L must be equal to $p^{L,S}$. If $F^H(p^L + c/\alpha) > 0$, the upward pricing pressure created by consumers who sample multiple firms is reduced, which means that $p^L \in [p^{L,M}, p^{L,S}]$ must hold in such a mixed-strategy equilibrium.

Now consider the optimization problem a firm faces when it is visited by a consumer who generates the high signal. Suppose a firm offers a price $p_j \in [0, p^L + c/\alpha]$. The resulting high signal profits would be:

$$\Pi^C(p_j;\tilde{v}^H) = p_j\int_{p_j}^{1}\frac{1}{N}Pr(\tilde{v}^H|v)dv+$$

$$p_j\sum_{j=2}^{N}\left[\int_{c/\alpha+p^L}^{0.5}\frac{1}{N}Pr(\tilde{v}^H|v)\big[Pr(\tilde{v}^H|v)(1-F^H(p^L+c/\alpha))\big]^{j-1}dv\right] \tag{2.4.8}$$

By contrast, offering any price $p_j \in (p^L + c/\alpha, \bar{p}^H]$ triggers search by all consumers with $v \in (p^L + c/\alpha, 0.5)$. Thus, the high signal profits from setting such a price are:

$$p_j\int_{0.5}^{1}\frac{1}{N}Pr(\tilde{v}^H|v)dv + p_j\sum_{j=1}^{N}\left[\int_{p_j}^{0.5}\frac{1}{N}Pr(\tilde{v}^H|v)\big[Pr(\tilde{v}^H|v)(1-F^H(p_j))\big]^{N-1}dv\right] \tag{2.4.9}$$

By the results of lemma 2 and because the price distribution $F^H(p)$ cannot have an atom at $\bar{p}^H$, the profits a firm makes when offering the price $\bar{p}^H$ to consumers who generate the high signal are equal to $(1/2N)\alpha\bar{p}^H$. Any price p_j in the the support of $F^H(p_j)$ needs to yield the same profits in order for the mixing indifference condition to be satisfied. For any $p_j > p^L + c/\alpha$, $F^H(p_j)$ must thus solve:

$$F^H(p_j;\bar{p}^H) = 1 - \left(\frac{\alpha}{2N(1-\alpha)^N}\frac{\bar{p}^H - p_j}{p_j(0.5-p_j)}\right)^{1/(N-1)} \tag{2.4.10}$$

I formalize the aforementioned discussion in the following proposition, recalling that I have defined $[\underline{p}^H, \bar{p}^H]$ as the convex hull of the support of $F^H(p_j)$:

Proposition 4 (Mixed-strategy equilibrium candidates)
Any symmetric MSE must have one of the following structures:

1. *The price $p^L + c/\alpha$ is offered with zero probability. The distribution $F^H(p_j)$ is atom-*

less and gapless, with $p^L + c/\alpha < \underline{p}^H$ *and* $\bar{p}^H < 0.5$.

2. *The price* $p^L + c/\alpha$ *is offered with positive probability, i.e.* $F^H(p^L + c/\alpha) > 0$. *There exists a* $\tilde{p}^H > p^L + c/\alpha$ *such that no price* $p_j \in (p^L + c/\alpha, \tilde{p}^H)$ *will be offered. The distribution* $F^H(p_j)$ *is gapless on* $[\tilde{p}^H, \bar{p}^H]$, *with* $\bar{p}^H \leq 0.5$.

I derive candidates for an equilibrium with either structure. I name the candidate that satisfies the first structure the *mixed search equilibrium*, because the structure of this equilibrium closely resembles the search equilibrium. In this equilibrium candidate, $p^L = p^{L,S}$ holds and $\underline{p}^H$ solves equation (2.4.10), noting that $F^H(\underline{p}^H; \bar{p}^H) = 0$ must hold. For any $p_j \in [\underline{p}^H, \bar{p}^H]$, $F^H(p_j; \bar{p}^H)$ needs to satisfy equation (2.4.10). The price $\bar{p}^H < 0.5$ must make consumers with $v > 0.5$ exactly indifferent between continuing to search and not.[21] Thus, it must satisfy the following:

$$\bar{p}^H = (1-\alpha)p^{L,S} + \alpha \int_{\underline{p}^H}^{\bar{p}^H} p dF^H(p) + c \tag{2.4.11}$$

A combination $(p^{L,S}, \underline{p}^H, \bar{p}^H, F^H(p))$ that satisfies these conditions constitutes an equilibrium if $p^{L,S} + c/\alpha < \underline{p}^H < \bar{p}^H < 0.5$ and $\Pi^C(p^{L,S} + c/\alpha; \tilde{v}^H) \leq \Pi^M(\bar{p}^H; \tilde{v}^H)$ jointly hold.

When deriving a candidate for an equilibrium that follows the second structure, I restrict attention to equilibria in which $\tilde{p}^H = \bar{p}^H = 0.5$ holds for simplicity. Such an equilibrium candidate is thus fully characterized by $F^H(p^L + c/\alpha)$, since this pins down the equilibrium p^L, which must solve equation (2.4.7). The probability $F^H(p^L + c/\alpha)$ must be set such that $\Pi^C(p^L + c/\alpha; \tilde{v}^H) = \Pi^C(0.5; \tilde{v}^H)$. I name the resulting equilibrium candidate the *partial search deterrence equilibrium* because this equilibrium in mixed strategies can be viewed as a hybrid of the search deterrence and the monopoly price equilibrium.

More precisely, the partial search deterrence equilibrium takes the following form: Consider a probability $F^H(p^L + c/\alpha)$ for which $\Pi^C(p^L + c/\alpha; \tilde{v}^H) = \Pi^C(0.5; \tilde{v}^H)$ holds, where p^L solves equation (2.4.7) and the two profits were defined in equations (2.4.8) and (2.4.9). Assume that firms offer the price p^L when observing the low signal and, when observing the high signal, offer the price $p^L + c/\alpha$ with probability $F^H(p^L + c/\alpha)$ and the price 0.5 with probability $1 - F^H(p^L + c/\alpha)$. This pricing strategy constitutes an equilibrium if $p^L + c/\alpha < 0.5$ and $0.5 < (1-\alpha)p^L + \alpha((p^L + c/\alpha)F^H(p^L + c/\alpha) + 0.5(1 - F^H(p^L + c/\alpha))) + c$ jointly hold.

I numerically calculate these equilibrium candidates and verify when they constitute equilibria. This analysis also suggests that, fixing the parameters, there is always a unique

[21] If consumers with $v > 0.5$ strictly prefer to search when receiving $\bar{p}^H$, setting this price yields zero profits, a contradiction. If consumers with $v < 0.5$ strictly prefer to not search when being offered this price, there is a profitable upward deviation.

candidate for an equilibrium in mixed strategies. The results are visualized in the following figure, which can be interpreted as follows: At orange dots, the mixed search equilibrium exists. At green dots, the partial search deterrence equilibrium exists.

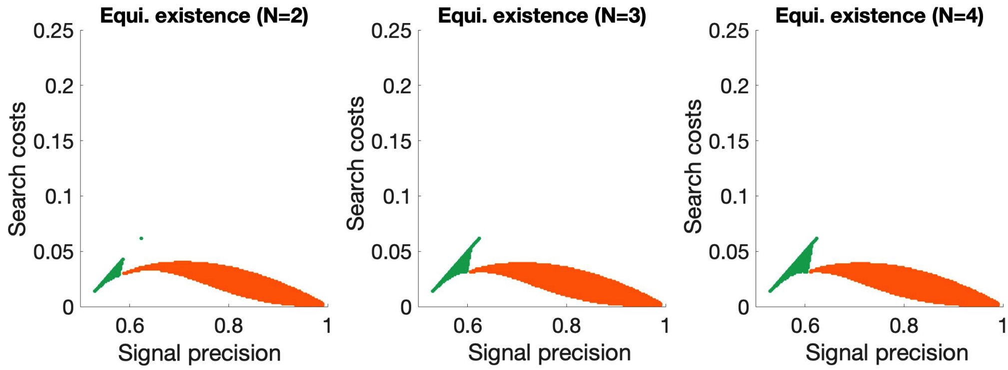

Figure 2.4: Existence regions: mixed-strategy equilibria

2.4.3 Equilibrium predictions and comparative statics

A corollary of the previous results is that the equilibria of the aforementioned model converge to the Diamond equilibrium when the signal becomes uninformative.

Corollary 1 (Uninformative signals)
As $\alpha \to 0.5$*, the only equilibrium that exists is the monopoly equilibrium and* $\lim_{\alpha \to 0.5} |p^{H,M} - p^{L,M}| = 0$.

The previous analysis has shown that there will be parameter regions for which multiple equilibria exist. To deal with this, I impose the following assumption:

Assumption 2 *When multiple equilibria exist, an equilibrium in which industry profits are maximal will be played.*

To clarify the implications of this assumption, note the results of the following corollary:

Corollary 2 (Ordering of profits)
When $p^{L,M} + c/\alpha \geq 0.5$*, the monopoly price equilibrium is unique. For any parameter combination at which* $p^{L,M} + c/\alpha < 0.5$*, firm profits would be highest in the search equilibrium. Firm profits in any mixed-strategy equilibrium would be higher than in the search deterrence equilibrium.*

This corollary establishes that equilibrium multiplicity can only be an issue when $p^{L,M} + c/\alpha < 0.5$. For these parameters, four potential equilibria can exist: the search equilibrium, the search deterrence equilibrium, the mixed search equilibrium, and the partial search deterrence equilibrium. The numerical existence results pertaining to the analysis of the mixed-strategy equilibria have suggested that the two mixed-strategy equilibria never jointly exist. Using the results of the above corollary, we can establish the following under assumption 2: When the search equilibrium exists, it will be played. If this equilibrium does not exist but an equilibrium in mixed strategies exists, the latter will be played. Otherwise, the search deterrence equilibrium will be played.

I now visualize which equilibrium will be played for different parameter combinations in the following graph, where I plot different values of signal precision (α) on the x-axis and different search costs (c) on the y-axis. Yellow dots indicate that the search equilibrium exists. Orange dots signify that the mixed search equilibrium will be played, while the partial search deterrence equilibrium is played at pink points. Green dots indicate that the search deterrence equilibrium is the only equilibrium that exists. Blue dots indicate that the monopoly equilibrium exists. I visualize this content when there are two ($N = 2$), three ($N = 3$) and four ($N = 4$) active firms, respectively:

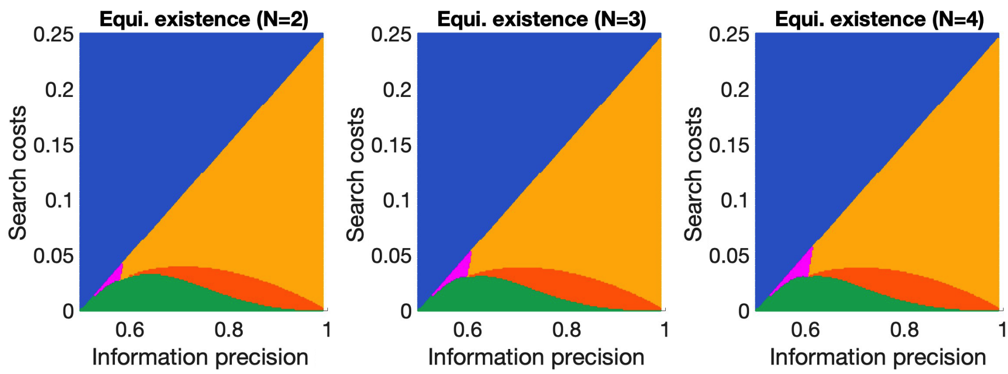

Figure 2.5: Existence regions: all equilibria

Recall that there are parameter regions for which no symmetric pure-strategy equilibrium exists, as shown in figure 2.3. In these regions, a mixed-strategy equilibrium will emerge. Consider the space of parameters in between the parameter regions for which the search and the search deterrence equilibrium exist, respectively. For these parameters (roughly, $c \in [0.01, 0.03]$ and $\alpha \in [0.75, 0.95]$), the search incentives of consumers with $v > 0.5$ are too weak to sustain the search deterrence equilibrium prices, but too strong to accommodate the existence of the search equilibrium. In the mixed search equilibrium, all prices that are offered after the high signal are below the high signal price in the search equilibrium. In a nutshell, prices in the mixed search equilibrium are thus adjusted downwards such that high-valuation consumers would not find it optimal to search on the equilibrium path, thus enabling the existence of this equilibrium.

Now consider the space of parameters in between the parameter regions for which the monopoly price and the search deterrence equilibrium exist, respectively. For these parameters (roughly, $c \in [0.03, 0.045]$ and $\alpha \in [0.55, 0.6]$), the search incentives of consumers are strong enough to avoid the monopoly outcome, but too weak to sustain the search deterrence equilibrium prices. When setting the latter, firms would always have incentives to deviate upwards from $p^{L,M} + c/\alpha$, because the cutoff price of high valuation consumers is too high. In the partial search deterrence equilibrium, this issue is avoided by the following logic: The presence of consumers who visit multiple firms on the equilibrium path grants firms substantially higher profits from setting the price $p^L + c/\alpha$ after the high signal, thus enabling the existence of this equilibrium.

Now I investigate the comparative statics of prices in the different equilibria. I begin by formally defining the comparative statics in the search equilibrium.

Corollary 3 (Comparative statics: search equilibrium)
In the search equilibrium, the high signal price is independent of c and N, and the low signal price is falling in c and rising in N.

Standard intuition regarding the effect of changes in search costs on prices suggests the following: An increase of search costs should, on average, reduce the number of firms a consumer has in her choice set, thus reducing competitive pressure and raising prices. The opposite holds true in the search equilibrium: The price $p^{L,S}$ falls in search costs. To see why, recall that any consumer that arrives at a firm after searching generates fully inelastic demand at prices $p_j \in (0, c/\alpha + p^{L,S})$. By contrast, the demand created by consumers that arrive at a firm first is price-sensitive. When search costs rise, less consumers search on the equilibrium path, so their weight in the firm's optimization problem falls. As a consequence, the optimal low signal price $p^{L,S}$ falls. Similar logic underlies the result that $p^{L,S}$ is increasing in the number of active firms. As N increases, consumers that arrive after searching receive higher weight in the firm's optimization problem, which creates additional upward pressure on the price $p^{L,S}$.[22]

Next, I study how the equilibrium prices respond to parameter changes when these changes may switch the equilibrium that is being played. When parameter changes move the market from one equilibrium into another, this can have a considerable effect on prices. I will focus on the effects of changes in search costs on the equilibrium outcomes. In visualizing these, I fix $N = 2$ and consider three levels of $\alpha \in \{0.6, 0.7, 0.8\}$. The corresponding effects are visualized in figure 2.6. Search costs are plotted on the x-axis. The blue line represents p^L, while the red line represents the average price set after the high signal.

[22]These changes do not affect the equilibrium high signal price, because no consumer who arrives after searching would buy at this price.

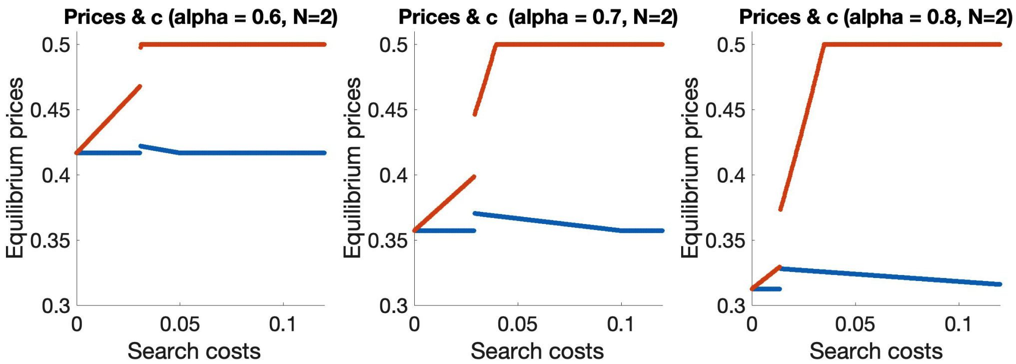

Figure 2.6: Comparative statics — search costs

Changes in search costs affect prices through two channels in this model. Firstly, increases of search costs reduce the search incentives of all consumers. Reductions of the consumers' search incentives lead to increases in the prices offered after the high signal in any mixed-strategy equilibrium and in the search deterrence equilibrium. Secondly, increases of search costs may reduce the measure of consumers that search on-path. This channel underlies the negative effect of increases in c on $p^{L,S}$, because consumers who arrive at a firm after searching exert upward pressure on the low signal price.

Moreover, when the market reverts from the search deterrence equilibrium to any equilibrium with on-path search, there is an upward jump of prices that is accompanied by a discontinuous increase in the measure of consumers that search on the equilibrium path. At this point of discontinuity, the following happens: Consumers lose the ability to sustain the search deterrence equilibrium prices with the threat of searching, which induces firms to set much higher prices when they observe the high signal. These higher prices trigger search by consumers with intermediate valuations, which induces an upward jump in p^L.

2.5 Generalized signal distributions

In this section, I study a framework that retains all the specifications of the baseline model, with the exception that firms now receive a binary signal with an arbitrary probability distribution $Pr(\tilde{v}^H|v)$. As before, I define p^L and p^H as the prices that firms offer to consumers who generate the low and the high signal in a pure-strategy equilibrium, respectively.

In a nutshell, I will show that the structure of potential pure-strategy equilibria in these generalized settings is the same as in the baseline model under some regularity conditions on $Pr(\tilde{v}^L|v)$.

First, I note that the fundamental structure of any symmetric pure-strategy equilibrium must be the same as in the baseline model:

Proposition 5 (Generalized signals: equilibrium structure)
Consider any price p^k that is offered in a symmetric pure-strategy equilibrium. The set of consumers with $v \geq p^k$ that (i) have a strictly positive probability of being offered p^k in equilibrium and (ii) search when receiving p^k must have measure zero.

Suppose, for a contradiction, that there is a strictly positive measure of consumers with $\nu \geq p^k$ who search upon receiving this price and receive this price with positive probability. Then, there will be a strictly positive measure of consumers who receive this price at all firms, and some firm will have undercutting motives that break the equilibrium.

Going forward, I will impose the following assumptions on $Pr(\tilde{v}^H|v)$:

Assumption 3 *The function $Pr(\tilde{v}^H|v)$ is continuously differentiable, strictly increasing, and satisfies $Pr(\tilde{v}^H|v) \in (0,1)$ for all $v \in [0,1]$.*

I begin the equilibrium analysis by characterizing the consumer's optimal search rule. A consumer with valuation v whose best price in hand is p_j will continue searching if and only if:

$$\gamma(v;p_j) = \sum_{k \in \{L,H\}} Pr(\tilde{v}^k|v) \max\{v - p^k, 0\} - c - \max\{v - p_j, 0\} > 0 \tag{2.5.1}$$

One can show that $p^L < p^H$ must hold in equilibrium, because the average valuation of consumers who generate the low signal is comparatively low. As a result, consumers will never continue searching if they receive a price close enough to p^L or if their valuation is sufficiently close to p^L. On the equilibrium path, consumers would thus only potentially continue searching after being offered the high signal price p^H. When the consumer's best price in hand is p^H, her gains of searching (i.e. the gain in utility that is attained by visiting another firm rather than stopping the search process) are:

$$\gamma(v;p^H) = Pr(\tilde{v}^L|v)\big[\max\{v - p^L, 0\} - \max\{v - p^H, 0\}\big] - c \tag{2.5.2}$$

A necessary condition for these to be positive is that $Pr(\tilde{v}^L|v)(v - p^L) - c > 0$. Thus, one can define the set $\hat{V}(p^L)$, which captures what consumers *can* search on-path:

$$\hat{V}(p^L) = \left\{ v \in [0,1] : Pr(\tilde{v}^L|v)(v - p^L) - c > 0 \right\} \tag{2.5.3}$$

One can use these insights when pinning down the potential equilibrium prices. Firstly, note that the demand implied by searchers remains fully inelastic around the equilibrium price p^L. Thus, the structure of profits around p^L is exactly the same as in the baseline model. Formally, the competitive profit functions will take the following form for prices

$p_j \in [0, \inf \hat{V}(p^L)]$:

$$\Pi^C(p_j;\tilde{v}^k) = p_j \int_{p_j}^{1} \frac{Pr(\tilde{v}^k|v)}{N} dv + p_j \underbrace{\sum_{j=2}^{N} \int_{v \in \hat{V}(p^L)\cap[p^L,p^H]} (1/N)\big[Pr(\tilde{v}^H|v)\big]^{j-1} Pr(\tilde{v}^k|v)\, dv}_{:=S^k(p^L,p^H)} \tag{2.5.4}$$

Because $p^L < \inf \hat{V}(p^L)$ must hold by the definition of $\hat{V}(p^L)$ and $\Pi^C(p_j;\tilde{v}^L)$ is differentiable around p^L, the equilibrium p^L in these generalized settings is pinned down by a first-order condition that is analogous to its counterpart in the baseline setting.

Similar notions hold true for the equilibrium high signal price. Recall that, in the baseline model, a consumer could only visit multiple firms in equilibrium if her valuation was in the set $(c/\alpha + p^L, 0.5)$, where $\inf \hat{V}(p^L) = c/\alpha + p^L$. The high signal price in the search deterrence equilibrium, namely $\inf \hat{V}(p^{L,M})$, satisfied $Pr(\tilde{v}^L| \inf \hat{V}(p^{L,M}))(\inf \hat{V}(p^{L,M}) - p^{L,M}) - c = 0$. In all other equilibria, the high signal price was a maximizer of $\Pi^M(p_j;\tilde{v}^H)$. The following proposition formalizes that this dichotomy is retained under the listed assumptions on $Pr(\tilde{v}^H|v)$:

Proposition 6 (Generalized signals: equilibrium characterization)
Suppose that assumption 3 holds. In a symmetric pure-strategy equilibrium, the low signal price p^L must satisfy:

$$\left.\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\right|_{p_j=p^L} + S^L(p^L,p^H) = 0 \tag{2.5.5}$$

The high signal price p^H must either be a local maximizer of $\Pi^M(p_j;\tilde{v}^H)$ <u>or</u> satisfy:

$$Pr(\tilde{v}^L|p^H)(p^H - p^L) - c = 0 \tag{2.5.6}$$

The equilibrium p^L must satisfy expression (2.5.5) by previous arguments. Now consider the equilibrium p^H. Given that competitive profits are equal to $\Pi^M(p^H;\tilde{v}^H)$ at p^H, a natural candidate for the equilibrium high signal price is a maximizer of the monopoly high signal profit function.

Now consider an equilibrium candidate in which p^H is not a local maximizer of $\Pi^M(p_j;\tilde{v}^H)$. Suppose that $Pr(\tilde{v}^L|p^H)(p^H - p^L) - c > 0$. By continuity of $Pr(\tilde{v}^L|v)$, there would exist an open interval of valuations above p^H for which the gains of searching when receiving p^H are strictly positive. This consumer group would create undercutting incentives that break the equilibrium. Suppose alternatively that $Pr(\tilde{v}^L|p^H)(p^H - p^L) - c < 0$. Then, there exists an $\epsilon > 0$ such that consumers with valuations $v \in [p^H - \epsilon, 1]$ would not continue searching if their best price in hand is in an open ball around p^H. As a result, the competitive high signal profit function is equal to $\Pi^M(p_j, \tilde{v}^H)$ in an open ball around p^H. But because p^H does not maximize $\Pi^M(p_j, \tilde{v}^H)$, there is a profitable deviation, a contradiction. Hence, a high signal price that does not satisfy the corresponding first-order condition must satisfy equation (2.5.6).

2.6 Conclusion

I have studied price discrimination based on imperfect information about consumer's valuations in a homogeneous goods model with search frictions. Whenever a consumer visits a firm, this firm receives a binary and informative signal about the consumer's valuation. I have highlighted that different search costs give rise to fundamentally different equilibria. My results refute the notion that a high volume of equilibrium search generally reflects high levels of competitive pressure or low search costs. It crucially matters which consumers choose to search in equilibrium. When the only consumers who search on-path have intermediate valuations, a feature matching the empirical phenomenon documented by Byrne and Martin (2021), equilibrium search is indicative of intermediate search costs and allows firms to sustain high prices.

My work sheds light on the effects of potential regulatory interventions in markets where firms can price discriminate. Pushing search costs down to negligible levels is a very effective way of regulating these markets. This maximizes consumer welfare and renders firms unable to price discriminate, which may be desirable in itself. However, the effects of search cost reductions on prices and consumer welfare are non-monotonic. When search cost reductions trigger increases in the amount of consumers who visit multiple firms in equilibrium, this tends to increase prices, because consumers who arrive at a firm after searching generate inelastic demand around the lowest equilibrium price.

Chapter 3

Search, Data, and Market Power

3.1 Introduction

This paper studies the relationship between data and market power. Data is becoming increasingly relevant in the digital age and is accumulating unevenly — some firms are building up significant advantages in terms of the scope and precision of the data they possess.[1] In order to ensure the proper functioning of digital markets, it is hence imperative to understand how such data advantages will translate into competitive advantages and foster market dominance. This question has gathered significant attention by policymakers (European Commission, 2020) and researchers (Kirpalani & Philippon, 2021; Bergemann & Bonatti, 2022; Eeckhout & Veldkamp, 2022) alike. I study said relationship in a theoretical model of price discrimination with search frictions, in which individual-level consumer data is used to personalize prices and consumers optimally choose which firms to visit.[2]

I show that consumers' search choices substantially amplify the transmission of data advantages into competitive advantages. Even arbitrarily small data advantages can make it optimal for nearly all consumers to only visit the firm with a data advantage, thus granting this firm market shares close to one. Such extreme forms of market dominance will reduce consumer welfare, for example by deterring entry or by reducing the incentives of firms to innovate. To guide policy, I study the optimal regulation in such contexts. Whereas reductions of search frictions will exacerbate the dominant position of a firm with superior data, the establishment of a right to data portability (as defined in the EU GDPR and the DMA) is an effective way of correcting the competitive imbalances caused by data advantages.[3]

[1] See, for example, Statista (2021) and Statista (2022).

[2] There is mounting empirical evidence for price discrimination in online markets — see Hannak et al. (2014), Larson et al. (2015), and Escobari et al. (2019). Regulatory bodies around the world are becoming concerned about this business practice — see OECD Secretariat (2016) and European Parliament (2019).

[3] For details, see article 20 of the European Union General Data Protection Regulation (GDPR) and article 6 of the EU Digital Markets Act (DMA).

I consider a duopoly model of a final goods market with search frictions. Every consumer can costlessly visit one firm, but has to pay a search cost to visit another firm after the first. Some consumers are *searchers*: They have equal valuation for the good of either firm and want to buy the good at the lowest possible price. The remaining consumers are *captive consumers*, who can only buy the good at the firm they are captive to. The valuation of any consumer is private information to the consumer.

The two firms have different degrees of information about consumers' valuations. One firm in the market, referred to as the *firm with data*, exogenously receives a private signal about the valuation of every consumer who visits it. This signal can take on two realizations: low or high. The high signal realization becomes more likely to occur when a consumer's valuation rises. Using this signal, the firm with data will price discriminate: It will offer a relatively low price (the low signal price) to all consumers who arrive and generate the low signal and a higher price (the high signal price) to all other arriving consumers. The other firm, referred to as the *firm without data*, receives no information about any consumer and will thus offer the same price to all arriving consumers.[4]

As a benchmark, I solve a variant of the above model in which every consumer can only visit one firm in Section 4.1. Then, the decision problem of any searcher boils down to choosing which firm to visit. Given that the firm with data price discriminates, searchers with low valuations prefer to visit the firm with data, while searchers with high valuations visit the firm without data. This is because consumers with low (high) valuations are likely to be identified as such and receive a comparatively low (high) price at the firm with data.

This search behaviour affects prices through a *selection effect*. Because searchers with low valuations visit the firm with data and vice versa, the average valuation of consumers who visit the firm without data is larger than the average valuation of consumers who visit the firm with data. Thus, these search patterns entail upward pressure on the uniform price of the firm without data and downward pressure on the prices of the firm with data.

A key message of this paper is that this selection effect amplifies the transmission of data advantages into competitive advantages. Simply put, this effect imposes a competitive externality on the firm without data: It pushes up the uniform price the firm without data would optimally set, which is to the benefit of the firm with data because it incentivizes searchers to visit and buy at this firm. In fact, a large majority of searchers will just visit the firm with data in equilibrium — only searchers with very high valuations will optimally visit the firm without data. Moreover, the market share of the firm with data converges to one as the share of searchers approaches one, regardless of the signal structure.

Why does the market only equilibrate when the firm without data is just visited by its captive consumers and searchers with very high valuations? Intuitively, the selection

[4] I focus on equilibria in which firms play pure strategies. In addition, I show that firms play pure strategies in any equilibrium in which prices are drawn from distributions with connected support.

effect becomes weak enough to enable equilibrium existence: When the mass of searchers who visit the firm without data is small, the distribution of consumer valuations is very similar at the two firms.[5] As a consequence, the optimal uniform price of the firm without data will be between the prices set by the firm with data. However, the selection effect is still active, which means that the optimal uniform price of the firm without data will lie just below the highest price of the firm with data (the high signal price), but significantly above the lowest price of the firm with data (the low signal price). These prices sustain the aforementioned search behaviour as an equilibrium: It is optimal for all searchers, except those with very high valuations, to visit the firm with data, because the potential benefit of receiving the low signal price at this firm is comparatively large.

In Section 4.2, I show that all previous insights go through when consumers can visit both firms, albeit under slightly stronger restrictions on the share of searchers. Formally, I solve the aforementioned model when the costs of visiting a second firm are arbitrary, while the analysis in Section 4.1 only considers the case in which these search costs are prohibitive.

To begin with, I show that no consumer will visit both firms in equilibrium if the share of searchers is not too low.[6] This result is based on two separate arguments: Firstly, any searcher who initially visits the *firm without data* in equilibrium would never continue searching, because the price this firm offers is non-stochastic.[7] Secondly, there exists no equilibrium in which some searchers continue searching after visiting the *firm with data* if there are enough searchers in the market. This is because searchers who arrive at the firm without data after visiting its rival exert upward pressure on the uniform price of this firm.[8] When the share of searchers is large enough, the price the firm without data would set in such a hypothetical equilibrium is thus so high that it is not worthwhile for any consumer to pay a search cost in pursuit of this price.

In equilibrium, all consumers thus only visit one firm and all results that were derived within the baseline model extend verbatim. The firm with data price discriminates and hence, the selection effect is active. As before, a large majority of searchers will thus only visit the firm with data. Moreover, the market share of the firm with data approaches one as the share of searchers goes to one, regardless of the signal structure.

Reductions of search frictions can only exacerbate the dominant position of the firm with data. When search costs are above a certain threshold, the possibility of searching

[5] This is because the distribution of valuations is the same for searchers and captive consumers.

[6] I define ρ as the share of searchers in the market. Assuming that $\rho \geq 0.2$ is sufficient for this result when the consumers' valuations are uniformly distributed and when restricting attention to linear signal distributions, independent of the exact level of search costs.

[7] Any searcher who finds it optimal to continue searching after visiting the firm without data would not initially visit this firm in equilibrium. She would be strictly better off by visiting the firm with data first and searching thereafter if and only if a high price is obtained, since this endows her with an option value.

[8] Note that the firm without data offers a uniform price and there are search costs. Thus, any searcher would only continue searching after visiting the firm with data if she would buy at the firm without data.

plays no role and changes in search costs do not affect the equilibrium outcomes. At sufficiently low search costs, reductions of search costs induce even more searchers to visit the firm with data. Intuitively, searchers constrain the prices of the firm with data with the threat of searching when search costs are sufficiently small. By strengthening this threat, reductions of search costs will induce the firm with data to lower its prices.[9] The reduced prices at the firm with data raise the incentives of searchers to visit this firm, thus granting it even higher market shares. Given that search costs online will likely decrease further in the future (think of augmented reality), access to superior data may thus become even more consequential.

In Section 5, I argue that the market dominance which arises from data advantages within my framework creates a need for regulatory interventions. In short, this is because the accompanying distortions can raise the average price level and will impede innovation and entry. By previous arguments, policies that decrease search frictions or merely reduce the informational advantage of a firm with superior data will not solve these issues.

Thus, I study the effects of two policies designed to curb data advantages on an individual level: the establishment of a right to anonymity and a right to data portability. A right to anonymity allows consumers to ensure that the firm with data receives no signal about them. Conversely, a right to data portability enables consumers to transfer the information the firm with data has about them to the firm without data. Whereas the former is inconsequential, the establishment of a right to data portability can be very effective. No consumer would exercise their right to anonymity, because this would be indicative of having a high valuation. By contrast, the incentives to exercise one's right to data portability are highest for low-valuation consumers. Through an unraveling effect, the establishment of a costless right to data portability can thus induce all searchers to visit the firm without data in equilibrium.

In Section 6, I present the results of various extensions of the baseline model. All results from the baseline model extend even if the firm with data receives a signal with an arbitrary finite number of realizations or a continuous signal, as long as the signal remains noisy. Moreover, the previous insights also apply when both firms receive signals about the valuations of visiting consumers, but the signal of one firm is less precise, or when consumers' preferences admit quality differentiation as in Mussa and Rosen (1978).

The rest of the paper proceeds as follows: I offer a detailed literature review in Section 2. In Section 3, I set up the theoretical framework, which is solved in Section 4. Sections 5 and 6 contain the analysis of the aforementioned policy proposals and extensions. I conclude and argue why my insights apply more generally, for example in insurance markets, in Section 7.

[9]There is no equivalent effect which influences the price of the firm without data. This is because searchers who visit this firm in equilibrium strictly prefer to refrain from searching when receiving its equilibrium price. Thus, searchers cannot effectively constrain the decisions of the firm without data using the threat of search.

3.2 Related literature

The findings I establish are novel because all previous work on the competitive effects of data advantages does not focus on the role of consumers' search choices. In preceding papers, there are either no search frictions (e.g. Eeckhout & Veldkamp, 2022; Rhodes & Zhou, 2022), search is random (Freedman and Sagredo, 2022), or there is no consumer heterogeneity that affects whether consumers visit an entity with better data or not (Kirpalani & Philippon, 2021; Bergemann & Bonatti, 2022). Thus, the selection effect that drives the relationship between data access and competitive advantages in my model is absent in previous work.

Several recent papers study the competitive effects of data advantages. In Belleflamme et al. (2020), a firm probabilistically either knows a consumer's valuation or knows nothing about the consumer. Bounie et al. (2021), Gu et al. (2019), Garcia (2022), and Delbono et al. (2022) study models where firms receive non-stochastic information about consumer preferences and some firms receive more informative data (e.g. a finer partition of the Hotelling line).[10] Rhodes and Zhou (2022) consider a setting in which some firms conduct first-degree price discrimination, whereas their rivals can only offer uniform prices.[11] He et al. (2023) consider a model of credit market competition in which some lenders have access to superior data about the creditworthiness of borrowers. Eeckhout and Veldkamp (2022) study a model in which better data reduces demand risk, thus inducing firms with data advantages to invest more into reducing marginal costs and attaining scale. In contrast to my work, there are no search frictions in all the aforementioned contributions.

The papers that are closest to mine within this area are Kirpalani and Philippon (2021), Freedman and Sagredo (2022), and Bergemann and Bonatti (2022), because these papers consider frameworks with search frictions.

Freedman and Sagredo (2022) examine a model of quality differentiation in which a unit mass of sellers offer quality-price menus to consumers. The firms observe signals about consumers' tastes for quality and different firms have access to signals with varying precision levels. Consumers are randomly matched with either one or two sellers. The key distinction to my work thus lies in the fact that consumers' choice sets are unrelated to their preferences in Freedman and Sagredo (2022) — in their model, consumers neither choose how many firms nor which kind of firms to visit. The heterogeneous search patterns that are central in my model are thus absent in Freedman and Sagredo (2022).

In Kirpalani and Philippon (2021), consumers choose whether to search for a good on a platform or an outside market. The platform has access to better data, which allows firms on the platform to generate a match with a higher probability. In contrast to my

[10] Clavorà Braulin (2021) considers a framework in which consumer preferences vary in two dimensions and firms may acquire different information about the components of a consumer's preferences.

[11] Guembel and Hege (2021) and Osório (2023) consider settings in which firms have different abilities to target their products to the individual preferences of consumers, but there is no price discrimination.

work, there is no consumer heterogeneity in Kirpalani and Philippon (2021) that affects the relative utility of search on the platform vs. searching on the outside market. In equilibrium, all consumers must hence be indifferent between searching on the platform or on the outside market. Thus, the aforementioned separating search behavior of consumers in my model is also absent in Kirpalani and Philippon (2021). In addition, the prices that consumers pay on the platform and on the outside market are the same in Kirpalani and Philippon (2021), i.e. no seller can conduct finer price discrimination in this model.

Bergemann and Bonatti (2022) study a model in which a platform has data about consumer preferences and uses this to match consumers and firms. Firms can sell through the platform in exchange for a fee, but do not acquire the platform's data. In contrast to my work, all firms have access to the same information in Bergemann and Bonatti (2022) and make symmetric offers in equilibrium. Moreover, while consumers can decide how many firms to visit outside of the platform, they cannot choose whether to access the platform or not.

My work also relates to the growing literature that studies price discrimination in search markets. Armstrong and Zhou (2016) and Preuss (2022) consider models where firms condition prices on a consumer's search history.[12] Fabra and Reguant (2020) study a simultaneous search setting where firms observe a consumer's desired quantity and price discriminate based on this information. Mauring (2022) and Atayev (2022) study a setting with shoppers and non-shoppers as defined in Burdett and Judd (1983) and Stahl (1989). Mauring (2022) and Atayev (2022) assume that firms receive imperfect information about the affiliation of a particular consumer to the groups of shoppers and non-shoppers. Marshall (2020) is the only other paper which considers a model of price discrimination based on information about valuations together with search, as this paper does. In all the listed contributions, consumers do not engage in directed search and no firm has a data advantage.[13]

Bergemann et al. (2021) study a homogenous goods model with search frictions in which competing firms receive information about the number of price offers a consumer obtains. In Bergemann et al. (2021), different firms may observe signals with varying levels of informativeness. In contrast to my work, all consumers have the same valuation in Bergemann et al. (2021) and consumers do not engage in directed search.

Ke et al. (2022) study the information design problem of an intermediary that connects sellers with consumers. In this model, every consumer just has a match at one seller. Ex ante, both the consumer and the sellers do not know with which seller the consumer has a match. By contrast, the intermediary perfectly knows said information and designs a

[12] Garrett et al. (2019) consider a model of second-degree price discrimination in which consumers differ in their choice sets, but firms do not have information about consumers. Braghieri (2019) studies a search model in which consumers decide whether or not to reveal their horizontal characteristic to firms.

[13] My work is also related to Esteves (2014) and Peiseler et al. (2022), who study price discrimination based on imperfect information about preferences in competitive settings without search frictions.

public information structure about this. Consumers engage in directed search by visiting firms according to the intermediary's recommendations. However, all firms are ex ante symmetric in Ke et al. (2022) and the intermediary's signals are public, so no firm has an informational advantage and all firms obtain the same expected outcomes.

3.3 Framework

There is a unit mass of consumers, who each want to buy at most one unit of an indivisible good that is produced by two firms at zero marginal cost. Consumers can costlessly visit one firm, but visiting a second firm after the first incurs search costs $c > 0$. There are two different groups of consumers, namely *captive consumers* and *searchers*. Captive consumers can only buy at the firm they are captive to and have zero valuation for the good of the other firm. By contrast, searchers have equal valuation for the good of either firm. The distribution of a consumer's valuation (v), which I denote by $G(v)$, is once continuously differentiable, has support $[0, 1]$, and is identical for searchers and captive consumers. Searchers make up a share $\rho \in (0, 1)$ of the total mass of consumers, while a share $0.5(1 - \rho)$ of consumers is captive to either firm. If a consumer with valuation v buys the good at price p, the utility of the consumer is:

$$u(v, p) = v - p \tag{3.3.1}$$

The two firms have differential access to information about consumers. One firm, which I call the *firm with data*, exogenously receives a binary private signal $\tilde{v} \in \left\{\tilde{v}^L, \tilde{v}^H\right\}$ about the valuation of any consumer who visits it. I define the probability distribution of this signal, which only depends on the consumer's valuation v, as $Pr(\tilde{v}^H|v)$, where $Pr(\tilde{v}^L|v) := 1 - Pr(\tilde{v}^H|v)$. As I will formalize later, I restrict attention to probability distributions that are monotonic in v. I define the signal $\tilde{v}^H$, which becomes more likely to occur when a consumer's valuation increases, as the high signal. The other firm, which I name the *firm without data*, receives no signal about the valuations of consumers.

Both firms can offer a different price to any consumer who visits. Thus, the game's timing is as follows: At the beginning, every consumer observes her valuation (and whether she is a searcher or captive to some firm) and optimally decides which firm to visit first. The firm that is visited first offers a price to the consumer. Based on her valuation and this price offer, the consumer then decides whether to visit the other firm at cost $c > 0$. If the consumer visits a second firm, this firm offers the consumer a price upon arrival. Crucially, both firms receive no information about any consumer's search history (i.e. they do not know whether an arriving consumer visits them first or second) and do not know whether a consumer is captive or a searcher. This setup implies that, as in Diamond (1971), firms cannot induce more consumers to visit them via downward deviations from equilibrium prices.

I study perfect Bayesian equilibria. Throughout the analysis, I mainly focus on equilibria in which firms play pure strategies. A pure strategy of the firm without data is a uniform price, which I call p^{nd}. A pure strategy of the firm with data is a price tuple (p^L, p^H). This firm offers the price p^L (p^H) to all consumers that visit it and generate the low (high) signal.[14] The strategy of a searcher must define which firm to visit first, based on her valuation. This decision is captured by a measurable function $s : [0,1] \to [0,1]$, where $s(v)$ is the probability that a searcher with valuation v visits the firm with data first. Moreover, the strategy of a searcher must also codify after which initial price offers they would continue searching, conditional on the firm that is visited first. Captive consumers always visit the firm they are captive to and do not search thereafter.

In the model, consumers know which firm has a data advantage. This assumption can be motivated along two dimensions. Firstly, knowledge of this fact can arise through learning. Over time, consumers can communicate with their peers and learn which firm sets stochastic prices and which firm sets a uniform price, allowing them to infer which firm uses data to personalize prices. Secondly, such awareness might result from regulation. The European Union, for example, has recently implemented regulation that mandates firms which engage in personalized pricing to inform any visiting consumer about this fact.[15] The benefits of measures that increase consumer awareness of personalized pricing have also been stressed by the OECD's competition committee.[16]

Before moving forward with the analysis, I consider the monopoly benchmark. I define $\Pi^{k,M}(p_j)$ as the profit a monopolist with access to the aforementioned information structure makes when offering the price p_j to consumers who generate the signal $\tilde{v}^k$, with global maximizers $\{p^{k,M}\}_{k\in\{L,H\}}$ given by:

$$p^{k,M} = arg \max_{p_j} \underbrace{p_j \int_{p_j}^{1} Pr(\tilde{v}^k|v)g(v)dv,}_{:=\Pi^{k,M}(p_j)} \quad k \in \{L, H\} \tag{3.3.2}$$

Similarly, I define $\Pi^{nd,M}(p_j)$ as the profit a monopolist without access to a signal would make when offering the price p_j, with a global maximizer $p^{nd,M}$ given by:

$$p^{nd,M} = arg \max_{p_j} \underbrace{p_j \int_{p_j}^{1} g(v)dv}_{:=\Pi^{nd,M}(p_j)} \tag{3.3.3}$$

In the analysis that follows, I impose the following assumptions on $Pr(\tilde{v}^H|v)$ and $G(v)$:

[14]The assumption that $\rho < 1$, i.e. that every firm has captive consumers, ensures that all information sets of both firms are on the equilibrium path, which rules out the existence of perfect Bayesian equilibria that are sustained by implausible off-path punishments.

[15]For details, please examine Directive 2019/2161 of the European Parliament.

[16]See article 5 in OECD Secretariat (2016).

Assumption 4 *The function $Pr(\tilde{v}^H|v)$ is strictly increasing, continuously differentiable, and satisfies $Pr(\tilde{v}^H|v) \in (0,1)$ for all $v \in [0,1]$. Moreover, $\Pi^{L,M}(p_j)$, $\Pi^{H,M}(p_j)$ and $\Pi^{nd,M}(p_j)$ are strictly concave in p_j.*

Under this assumption, $p^{L,M} < p^{nd,M} < p^{H,M}$ holds: When observing the low (high) signal, a monopolist will set a lower (higher) price than when he has no information about a consumer. This holds because the average valuation of consumers who generate the low (high) signal is relatively low (high).

I place no functional form restrictions on $Pr(\tilde{v}^H|v)$. Thus, my analysis also covers cases in which the signal $\tilde{v}$ is almost uninformative. Moreover, it is also possible that the firm with data receives a signal which induces it to set higher average prices than in the absence of information about consumers' valuations. To illustrate the connection between assumptions and primitives, I will consider examples in which $v \sim U[0,1]$ and the signal's probability distribution is linear during the analysis. When $v \sim U[0,1]$ and $Pr(\tilde{v}^H|v)$ is linear, assumption 4 is satisfied. A linear $Pr(\tilde{v}^H|v)$ with precision $\alpha \in (0,1)$ is given by:

$$Pr(\tilde{v}^H|v) = 0.5 + \alpha(v - 0.5) \tag{3.3.4}$$

In addition, I impose a tie-breaking rule on the behaviour of searchers.

Assumption 5 *Suppose that $\underline{p}$ is the lowest price offered by either firm. Any searcher with $v \geq \underline{p}$ who obtains equal expected utility by visiting either firm first visits both firms first with equal probability.*

In section 4.1, I solve the specified model under the restriction that $c \to \infty$. In section 4.2, I solve this model for arbitrary $c > 0$. I call the former framework the *baseline model* and the latter the *sequential search framework*.

3.4 Equilibrium analysis

3.4.1 Baseline model

Consider first the baseline model, in which it is prohibitively costly for searchers to visit a second firm ($c \to \infty$). In this framework, the only relevant choice that searchers have to make is which firm to visit. If firms play pure strategies, a searcher with valuation v prefers to visit the firm with data if and only if:

$$Pr(\tilde{v}^L|v)\max\{v - p^L, 0\} + Pr(\tilde{v}^H|v)\max\{v - p^H, 0\} \geq \max\{v - p^{nd}, 0\} \tag{3.4.1}$$

The strategy of searchers is represented by a function $s(v)$, where $s(v)$ is the probability that a searcher with valuation v visits the firm with data. Given the searchers' behaviour, the firm with data maximizes the following profit function through choice of the price p_j when observing the signal $\tilde{v}^k$, with $k \in \{L, H\}$:

$$\Pi^k(p_j; s(v)) = p_j \Bigg[\underbrace{\rho \int_{p_j}^1 s(v) Pr(\tilde{v}^k|v) g(v) dv}_{\text{searcher demand}} + \underbrace{0.5(1-\rho) \int_{p_j}^1 Pr(\tilde{v}^k|v) g(v) dv}_{\text{captive consumer demand}} \Bigg] \quad (3.4.2)$$

Analogously, the firm without data maximizes the following profit function:

$$\Pi^{nd}(p_j; s(v)) = p_j \Bigg[\underbrace{\rho \int_{p_j}^1 (1-s(v)) g(v) dv}_{\text{searcher demand}} + \underbrace{0.5(1-\rho) \int_{p_j}^1 g(v) dv}_{\text{captive consumer demand}} \Bigg] \quad (3.4.3)$$

I begin by characterizing equilibria in which firms play pure strategies. In such equilibria, the uniform price of the firm without data must lie between the prices of the firm with data. Moreover, the strategy of searchers is described by a cutoff rule:

Lemma 3 (Equilibrium search patterns)
Consider the baseline model. In an equilibrium in which firms play pure strategies:

- *The ordering $p^L < p^{nd} < p^H$ must hold.*
- *There exists a $\bar{v} > p^L$ such that all searchers with $v \in (p^L, \bar{v})$ visit the firm with data and all searchers with $v \in (\bar{v}, 1]$ visit the firm without data.*

Simply put, the first result holds because the optimal prices of the firms satisfy the ordering $p^L < p^{nd} < p^H$ if the valuations of consumers who visit either firm follow the same distribution. This holds, for example, if all searchers visit a given firm.

In equilibrium, $p^L < p^H$ must hold. To see this, note first that it is never optimal for the firm with data to set a price p^H that is strictly below p^L.[17] Thus, we can restrict attention to equilibrium candidates in which $p^L \leq p^H$. The only candidate for an equilibrium in which $p^L = p^H$ holds is an equilibrium in which all firms set the same uniform price. i.e. in which $p^L = p^H = p^{nd}$. But then, searchers with a valuation above the lowest equilibrium price visit both firms with equal probability (by the tie-breaking rule described in assumption 5), and the optimal prices of the firms would satisfy $p^{nd} < p^H$ or $p^L < p^H$, a contradiction.

Similar arguments establish that $p^{nd} \in (p^L, p^H)$ must hold in equilibrium. For example, suppose that $p^{nd} \leq p^L$. Then, all searchers with $v > p^{nd}$ visit the firm without data, implying that $p^{nd} \geq p^{nd,M}$ must hold. But then, the firm with data has a profitable downward deviation from p^L, since it only sells to captive consumers at p^L and $p^L \geq p^{nd,M} > p^{L,M}$.

[17] If $p^H < p^L$, there would either be a downward deviation from p^L to p^H when observing $\tilde{v}^L$ or vice versa.

When deciding which firm to visit, any searcher thus faces a tradeoff: By visiting the firm with data, she will attain the lowest price p^L with probability $Pr(\tilde{v}^L|v)$, but she may also obtain an unfavorable outcome if she generates the high signal and is thus offered p^H. Because the probability of receiving p^L is strictly falling in v, the optimal behaviour of searchers is characterized by a cutoff $\bar{v} > p^L$.

The equilibrium search behaviour established above will affect the optimal prices (and their ordering) through a *selection effect*: Searchers visit the firm without data if their valuation is comparatively high and vice versa. Thus, the average valuation of consumers who visit the firm without data is *higher* than the average valuation of consumers who visit the firm with data. This effect entails upward pressure on the uniform price of the firm without data and downward pressure on the prices of the firm with data.

An equilibrium in which firms play pure strategies is described by a vector $(p^L, p^H, p^{nd}, \bar{v})$. Before characterizing such equilibria, it is instructive to consider the best response functions of firms. Firms optimally set prices, given the search behaviour represented by $\bar{v}$. To fix ideas, suppose that all searchers with $v < \bar{v}$ visit the firm with data and that searchers with valuation $v > \bar{v}$ visit the firm without data, where $\bar{v} \in [0,1]$. Then, the firm with data maximizes the following objective through choice of p_j when observing the signal $\tilde{v}^k$, with $k \in \{L, H\}$:

$$\Pi^k(p_j;\bar{v}) = p_j\Bigg[\underbrace{\rho\mathbb{1}[p_j \leq \bar{v}]\int_{p_j}^{\bar{v}} Pr(\tilde{v}^k|v)g(v)dv}_{\text{searcher demand}} + \underbrace{0.5(1-\rho)\int_{p_j}^{1} Pr(\tilde{v}^k|v)g(v)dv}_{\text{captive consumer demand}}\Bigg] \tag{3.4.4}$$

The firm without data maximizes the following objective function:

$$\Pi^{nd}(p_j;\bar{v}) = p_j\Bigg[\underbrace{\rho\int_{\bar{v}}^{1} \mathbb{1}[p_j \leq v]g(v)dv}_{\text{searcher demand}} + \underbrace{0.5(1-\rho)\int_{0}^{1} \mathbb{1}[p_j \leq v]g(v)dv}_{\text{captive consumer demand}}\Bigg] \tag{3.4.5}$$

I define the optimal prices of the firm with data as $p^{L,*}(\bar{v}) = arg\max_{p_j\in[0,1]} \Pi^L(p_j;\bar{v})$ and $p^{H,*}(\bar{v}) = arg\max_{p_j\in[0,1]} \Pi^H(p_j;\bar{v})$. Similarly, I define $p^{nd,*}(\bar{v}) = arg\max_{p_j\in[0,1]} \Pi^{nd}(p_j;\bar{v})$.

In the following two graphs, I visualize these best response functions for a given parametric example in which $\rho = 0.5$, $v \sim U[0,1]$, and $Pr(\tilde{v}^H|v) = 0.5 + 0.7(v - 0.5)$. The functions $p^{L,*}(\bar{v})$, $p^{H,*}(\bar{v})$, and $p^{nd,*}(\bar{v})$ are plotted in blue, red, and yellow, respectively:

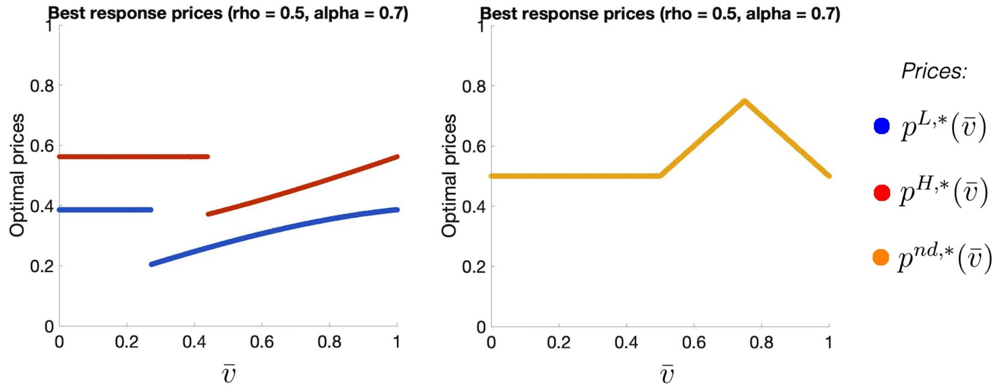

Figure 3.1: Best response functions

Consider first the optimal prices of the firm with data and recall that this firm is visited by searchers with valuation in $[0, \bar{v}]$. For low values of $\bar{v}$, this firm can only sell to searchers by setting very low prices, which yields low total profits. When $\bar{v}$ is low, it is hence optimal to forego these consumers entirely and to set prices that maximize the profits that accrue from captive consumers, namely $p^{L,M}$ and $p^{H,M}$, respectively. As $\bar{v}$ increases, it becomes optimal to set a price strictly below $\bar{v}$, thereby making the sale to some searchers. For such $\bar{v}$, the optimal prices of the firm with data are rising in $\bar{v}$, because the average valuation of consumers who visit the firm with data is rising in $\bar{v}$.

Now consider the optimal uniform price of the firm without data. Recall that this firm is visited by searchers with valuations in the interval $[\bar{v}, 1]$. The profits this firm attains from its searchers would be maximized by setting a price weakly above $\bar{v}$. By contrast, the profits this firm attains from its captive consumers are maximized by setting the price $p^{nd,M}$, which equals 0.5 in this example. When $\bar{v} \leq 0.5$, setting the price 0.5 also maximizes the profits that accrue from searchers. Thus, the optimal price $p^{nd,*}(\bar{v})$ is equal to 0.5 when $\bar{v} < 0.5$.

When $\bar{v} \in [0.5, 1]$, the optimal price of the firm without data depends on the mass of searchers who arrive at this firm and the corresponding strength of the selection effect. Given that these consumers entail upward pressure on the uniform price of this firm, this price will be comparatively low (high) when the mass of arriving searchers is small (large). When $\bar{v} \in [0.5, 0.5(1 + \rho)]$, the mass of searchers who arrive at the firm without data is *large*, which implies that $p^{nd,*}(\bar{v})$ will be equal to $\bar{v}$. For $\bar{v} \in (0.5(1 + \rho), 1]$, the mass of searchers who arrive at the firm without data becomes *small*, which means that the optimal price $p^{nd,*}(\bar{v})$ will be strictly below $\bar{v}$. Moreover, $p^{nd,*}(\bar{v})$ is now falling in $\bar{v}$, because the average valuation of consumers who visit the firm without data is *falling* in $\bar{v}$ in this interval.[18]

[18] In general, the average valuation of searchers who arrive at the firm without data is rising in $\bar{v}$, while their mass is falling in $\bar{v}$. Thus, increases in $\bar{v}$ entail opposing effects on the average valuation of all consumers who visit the firm without data. When $v \sim U[0, 1]$, the latter effect dominates for $\bar{v} \in [0.5(1 + \rho), 1]$.

For general valuation distributions, the following insight can be taken away: When the mass of searchers who arrive at the firm without data is large (relative to the mass of its captive consumers), this firm will find it optimal to set a price above $\bar{v}$. The firm without data will only find it optimal to set a price below $\bar{v}$ if the mass of searchers who arrive at this firm is small (i.e. $\bar{v}$ is large). For an arbitrary valuation distribution, the optimal price $p^{nd,*}(\bar{v})$ would thus only be below $\bar{v}$ if $\bar{v} \geq \bar{v}^{nd}$, which is defined as follows:

$$\rho\big[1 - G(\bar{v}^{nd})\big] + 0.5(1-\rho)\big[1 - G(\bar{v}^{nd}) - (\bar{v}^{nd})g(\bar{v}^{nd})\big] = 0 \tag{3.4.6}$$

When $v \sim U[0,1]$ as in the previous example, $\bar{v}^{nd} = 0.5(1+\rho)$, which is the point at which the function $p^{nd,*}(\bar{v})$ has its second kink. These considerations imply that a majority of searchers visit the firm with data in equilibrium:

Proposition 7 (Competitive advantages)
Consider the baseline model. In an equilibrium in which firms play pure strategies, the cutoff $\bar{v}$ must satisfy $\bar{v} \geq \bar{v}^{nd}$.

Intuitively, any hypothetical equilibrium in which $\bar{v} < \bar{v}^{nd}$ holds is ruled out by an incompatibility between optimal search behavior and optimal pricing by the firm without data. To see this, note firstly that optimality of the searchers' choices requires that $p^{nd} < \bar{v}$ must hold in equilibrium. This is because any searcher with valuation just above p^{nd} would strictly prefer to visit the *firm with data* (since $p^L < p^{nd}$ must be true in an equilibrium by lemma 3). Thus, the ordering $p^L < p^{nd} < \bar{v}$ must be satisfied in an equilibrium in which firms play pure strategies.

However, previous results have established that setting a price $p^{nd} \in (p^L, \bar{v})$ cannot be optimal for the firm without data when $\bar{v} < \bar{v}^{nd}$. For any $\bar{v}$ and any p^L, the profits of this firm will be equal to $\Pi^{nd}(p_j; \bar{v})$ when $p_j \in (p^L, \bar{v})$. If $\bar{v} < \bar{v}^{nd}$, the profits of this firm are thus strictly increasing in the price at any possible equilibrium $p^{nd} \in (p^L, \bar{v})$, because the upward pricing pressure created by the large mass of arriving searchers is too strong, a contradiction.

Having defined the key properties of any equilibrium in which firms play pure strategies, I now establish the existence of such an equilibrium.

Proposition 8 (Equilibrium existence)
In the baseline model, there always exists an equilibrium in which firms play pure strategies.

The proof of proposition 8 is by construction. I show that there always exists a $\bar{v}^* \in [\bar{v}^{nd}, 1]$ that induces optimal prices (given by $p^{L,*}(\bar{v}^*)$, $p^{H,*}(\bar{v}^*)$, and $p^{nd,*}(\bar{v}^*)$) which, in turn, make it optimal for searchers to visit the firm without data if and only if their valuation is above $\bar{v}^*$. I will find such a $\bar{v}^*$ using a fixed point approach.

Continuity of the firms' best response functions plays an important role in the proof of proposition 8. Without further assumptions, the functions $p^{L,*}(\bar{v})$ and $p^{nd,*}(\bar{v})$ will both be continuous on the interval $\bar{v} \in [\bar{v}^{nd}, 1]$. However, the function $p^{H,*}(\bar{v})$ is not necessarily continuous for these $\bar{v}$ if $\Pi^H(p^{nd,M}; \bar{v}^{nd}) \leq 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$ i.e. when the share of searchers (ρ) is too small. This entails the main technical challenge in proving this proposition. I relegate the formal arguments which show existence of an equilibrium in these constellations to the appendix and focus on the case in which $\Pi^H(p^{nd,M}; \bar{v}^{nd}) > 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$ holds in the following discussion.[19] Under this assumption, the optimal $p^{H,*}(\bar{v})$ will lie strictly below $\bar{v}$ for any $\bar{v} \in [\bar{v}^{nd}, 1]$.[20] Thus, the optimal price must satisfy a first-order condition, which guarantees continuity of the function $p^{H,*}(\bar{v})$ on $[\bar{v}^{nd}, 1]$.

To characterize the optimal search behavior of consumers, I define the following function:

$$\hat{v}(p^L, p^H, p^{nd}) := \sup\left\{v \in [0,1] : \underbrace{Pr(\tilde{v}^L|v)p^L + Pr(\tilde{v}^H|v)p^H}_{\text{exp. price at firm with data}} < p^{nd}\right\} \tag{3.4.7}$$

Conditional on (p^L, p^H, p^{nd}), all searchers will obtain a lower expected price at the firm with data if and only if their valuation is below $\hat{v}(p^L, p^H, p^{nd})$. Plugging in the best-response price functions into $\hat{v}(p^L, p^H, p^{nd})$ yields:

$$\hat{v}^B(\bar{v}) := \hat{v}\big(p^{L,*}(\bar{v}), p^{H,*}(\bar{v}), p^{nd,*}(\bar{v})\big) \tag{3.4.8}$$

A value $\bar{v}^* \geq \bar{v}^{nd}$ at which $\bar{v}^B(\bar{v}^*) = \bar{v}^*$, together with the implied optimal prices, constitutes an equilibrium. To see this, suppose that searchers visit the firm without data if $v > \bar{v}^*$ and the firm with data if $v < \bar{v}^*$, where $\hat{v}^B(\bar{v}^*) = \bar{v}^*$. Given this search behaviour, the firm without data optimally sets the price $p^{nd,*}(\bar{v}^*)$. The optimal prices of the firm with data are $p^{L,*}(\bar{v}^*)$ and $p^{H,*}(\bar{v}^*)$. Searchers optimally visit the firm where they receive the lower expected price (conditional on their valuation v). Thus, it is optimal for searchers to visit firms according to the cutoff rule implied by $\bar{v}^*$, because $\bar{v}^* = \hat{v}^B(\bar{v}^*)$, which implies that the combination $(p^{L,*}(\bar{v}^*), p^{H,*}(\bar{v}^*), p^{nd,*}(\bar{v}^*), \bar{v}^*)$ constitutes an equilibrium.

Thus, proving that an equilibrium in pure strategies exists amounts to establishing the existence of a solution to the equation $\hat{v}^B(\bar{v}) - \bar{v} = 0$ in the interval $[\bar{v}^{nd}, 1]$. The existence of an appropriate fixed point can be verified by applying the intermediate value theorem to this equation, together with the boundary conditions (i) $\hat{v}^B(\bar{v}^{nd}) > \bar{v}^{nd}$ and

[19] When $v \sim U[0,1]$, this property holds for any linear signal distribution if $\rho \geq 0.13$.

[20] Consider any $\bar{v} \geq \bar{v}^{nd}$. The high signal profits from any price $p_j \geq \bar{v}$ are bounded from above by $0.5(1-\rho)\Pi^{H,M}(p^{H,M})$. By setting a price $p_j < \bar{v}$ (e.g. $p_j = p^{nd,M}$) when observing $\tilde{v}^H$, the firm can attain higher profits, because $\Pi^H(p^{nd,M}; \bar{v}) \geq \Pi^H(p^{nd,M}; \bar{v}^{nd}) > 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$ holds for any $\bar{v} \geq \bar{v}^{nd}$.

(ii) $\hat{v}^B(1) \leq 1$. At $\bar{v} = \bar{v}^{nd}$, $p^{nd,*}(\bar{v}^{nd}) = \bar{v}^{nd}$ holds, while both optimal prices of the firm with data are strictly below $\bar{v}$. This establishes that $\hat{v}^B(\bar{v}^{nd}) = 1$. The second boundary condition, namely $\hat{v}^B(1) \leq 1$, holds because $\hat{v}^B(\bar{v})$ is the supremum of a set with elements that cannot be larger than 1. Moreover, $\hat{v}^B(\bar{v})$ is continuous on $\bar{v} \in [\bar{v}^{nd}, 1]$ because all price functions are continuous in $\bar{v}$. Thus, a solution to $\hat{v}^B(\bar{v}) - \bar{v} = 0$ exists in the interval $[\bar{v}^{nd}, 1]$.

To build further intuition, I present a numerical example. Suppose that $v \sim U[0,1]$, $\rho = 0.5$, and that $Pr(\tilde{v}^H|v) = 0.5 + 0.7(v - 0.5)$. For all possible equilibrium values of $\bar{v}$ on the x-axis[21], I have plotted the resulting $p^{L,*}(\bar{v})$ in blue, $p^{H,*}(\bar{v})$ in red, and $p^{nd,*}(\bar{v})$ in yellow, respectively, in the following graph. The function $\hat{v}^B(\bar{v})$ is plotted in green:

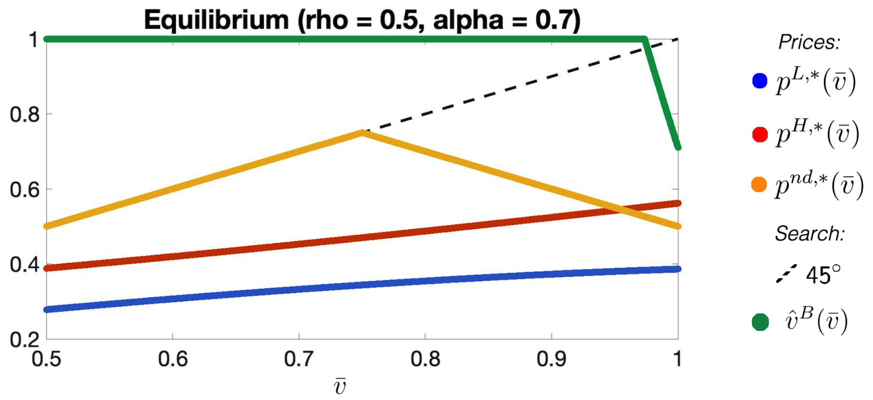

Figure 3.2: Visualization — equilibrium existence

The point $\bar{v}$ at which $\hat{v}^B(\bar{v})$ crosses the 45-degree line constitutes an equilibrium. When $\bar{v} \leq \bar{v}^{nd}$ (the term $\bar{v}^{nd}$ is equal to $0.5(1+\rho) = 0.75$ in this example), the selection effect is too strong to sustain an equilibrium. This manifests in the fact that the optimal uniform price of the firm without data lies above both prices the firm with data would set, so all searchers would prefer to visit the firm with data (i.e. $\hat{v}^B(\bar{v}) = 1$).

As $\bar{v}$ moves closer to 1, the selection effect becomes progressively weaker, i.e. the average valuations of consumers who visit either firm converge to each other.[22] This is accompanied by increases in the optimal prices of the firm with data and decreases in the optimal uniform price of the firm without data. These price changes will induce more searchers to visit the firm without data, which is represented by a falling $\hat{v}^B(\bar{v})$. When $\bar{v} \approx 1$, the optimal p^{nd} will lie just below the optimal p^H, while the optimal p^L lies substantially below these two prices. These prices, in turn, make the search behaviour represented by such high levels of $\bar{v}$ optimal. Only consumers with very high valuations, who are very likely to receive the high price at the firm with data, will optimally visit the firm without data.

[21]As argued previously, we can directly exclude equilibrium candidates in which $\bar{v} < 0.5$.

[22]To see this, consider the case where $\bar{v} = 1$. Then, the valuation distribution of consumers who visit either firm is exactly equal.

Note that there may potentially exist multiple equilibria in which firms play pure strategies. This multiplicity can arise from two sources: Firstly, $\hat{v}^B(\bar{v})$ can jump upwards when the function $p^{H,*}(\bar{v})$ is discontinuous on $[\bar{v}^{nd}, 1]$. Secondly, the search behaviour of searchers with $v < p^L$ is not pinned down in equilibrium, which means that $\Pi(p_j; \tilde{v}^L)$ may have a kink at the equilibrium p^L.

However, this multiplicity is largely inconsequential for the analysis of market concentration, because $\bar{v} \geq \bar{v}^{nd}$ holds true in any equilibrium in which firms play pure strategies. Moreover, the issue of multiplicity is easily solved by imposing two assumptions, namely that (i) $\Pi^H(p^{nd,M}; \bar{v}^{nd}) > 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$ holds (i.e. that there are enough searchers in the market), and (ii) that searchers with a valuation in an open interval below the lowest equilibrium price visit the firm that offers this price. This is formalized in proposition 12.

This completes the characterization of equilibria in which firms play pure strategies. Now, I consider equilibria in which at least one firm plays a mixed strategy. I restrict attention to equilibria in which firms draw prices from distributions with connected support.[23] As defined in Burdett and Judd (1983), a distribution $H(p)$ has connected (i.e. convex) support if $H(p_1) \neq H(p_2)$ holds for any distinct prices p_1, p_2 in the convex hull of its support. There exists no such equilibrium in which firms mix.

Proposition 9 (No mixing)
Consider the baseline model and restrict attention to equilibria in which firms draw prices from distributions with connected support. In any such equilibrium, firms play pure strategies.

This result is based on the following logic: I define the lowest price set by the firm without data and the firm with data as $\underline{p}^{nd}$ and $\underline{p}^{d}$, respectively. In an equilibrium in which firms mix, $\underline{p}^d = \underline{p}^{nd}$ must hold. Under our tie-breaking rule, there exists an interval of prices above this lowest price for which the profit functions of both firms are strictly concave. Thus, this lowest price $\underline{p}^{nd}$ must be offered with probability 1 by the firm without data. If the firm with data mixes, it only sells to its captive consumers for any price above $\underline{p}^{nd}$. This would imply that its profits are equal to $\Pi^{k,M}(p_j)$ for any price p_j it offers, which is a strictly concave function for either signal $\tilde{v}^k$, a contradiction to the mixing indifference condition.

Thus, we can restrict attention to equilibria in which firms play pure strategies, which I have characterized. In such equilibria, the firm with data has significant competitive advantages, as reiterated by the following corollary:

[23] This restriction applies to any information set separately. To clarify this restriction, note that an equilibrium in which the firm with data draws prices from the interval $[0.3, 0.4]$ when observing $\tilde{v}^L$ and draws prices from $[0.5, 0.6]$ when observing $\tilde{v}^H$ is admissible. However, an equilibrium in which this firm draws prices from a distribution with support $[0.3, 0.4] \cup [0.5, 0.6]$ when observing $\tilde{v}^L$ is inadmissible.

Corollary 4 (Market dominance)
Consider the baseline model. The equilibrium market share of the firm with data approaches 1 as $\rho \to 1$.

Recall that ρ is the share of searchers in the market. As $\rho \to 1$, the share of captive consumers approaches 0. In equilibrium, the measure of searchers who buy at the firm without data also approaches 0, because $\bar{v} \geq \bar{v}^{nd}$ and this lower bound converges to 1 as $\rho \to 1$. This is true even when there are multiple equilibria in which firms play pure strategies, because $\bar{v} \geq \bar{v}^{nd}$ holds in such any equilibrium. Thus, the equilibrium demand received by the firm without data approaches 0 as $\rho \to 1$, which implies that the market share of the firm with data approaches 1.

To build further intuition for this result, I now visualize the equilibrium prices and search cutoffs for different values of ρ. I assume that $v \sim U[0,1]$. A given graph corresponds to a fixed linear signal distribution, with $\alpha \in \{0.25, 0.6, 0.95\}$, while different levels of ρ are plotted on the x-axis of each graph. The color scheme of prices is as before, and the equilibrium levels of $\bar{v}$ are plotted in lilac.

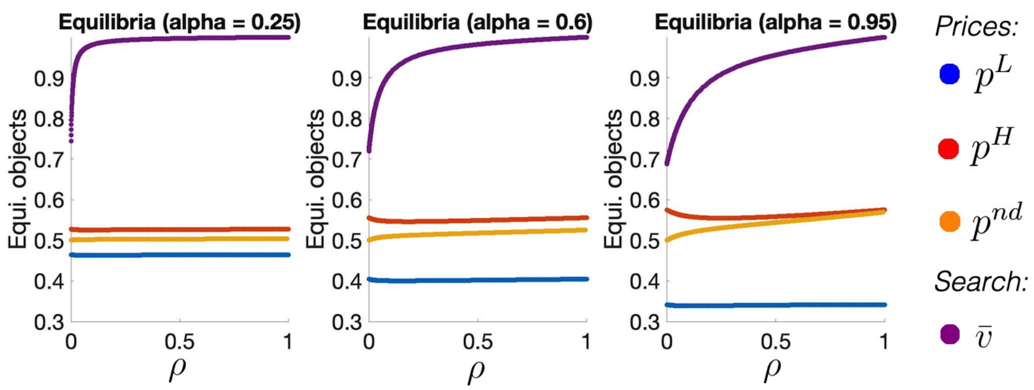

Figure 3.3: Baseline model — comparative statics (ρ)

When $\rho \to 1$, corollary 4 has established that $\bar{v} \to 1$. In conjunction, the uniform price of the firm without data approaches $Pr(\tilde{v}^L|1)p^{L,M} + Pr(\tilde{v}^H|1)p^{H,M}$, which is the expected price a searcher with valuation 1 would receive at a monopolist with access to data. To see why this must hold, note that the optimal low and high signal prices of the firm with data converge to $p^{L,M}$ and $p^{H,M}$ as $\bar{v}$ approaches 1, respectively. In order for the search behaviour represented by such a high level of $\bar{v}$ to be optimal, the uniform price of the firm without data has to be above the expected price at the firm with data (conditional on the valuation) for almost all searchers. This is guaranteed when the uniform price of this firm approaches $Pr(\tilde{v}^L|1)p^{L,M} + Pr(\tilde{v}^H|1)p^{H,M}$. Such a price is optimal for the firm without data because the slope of $p^{nd,*}(\bar{v})$ on $\bar{v} \in [\bar{v}^{nd}, 1]$ becomes very large as $\rho \to 1$.

I have established that arbitrarily small data advantages translate into substantial competitive advantages through directed consumer search. This result is underscored by considering what happens when no firm receives an informative signal. In this benchmark, both firms set the same uniform price in equilibrium and will thus receive exactly half of the market under the tie-breaking rule defined in assumption 5.

3.4.2 Sequential search framework

In this section, I show that all the results from the baseline model go through even if searchers can visit a second firm, albeit under slightly stronger restrictions on ρ. Formally, I no longer assume that c is prohibitively high, but consider an arbitrary $c > 0$. In terms of policy, the results I establish within this section also highlight that reductions of search frictions tend to further benefit the firm with a data advantage.

I begin the analysis by characterizing equilibria in which firms play pure strategies. As before, such an equilibrium needs to define the low signal and high signal price (p^L and p^H, respectively) of the firm with data, as well as the uniform price of the firm without data (p^{nd}). The strategy of searchers now specifies, for a given v, (i) which firm to visit first (captured by a function $s(v)$, as in the baseline model), (ii) after what price offers to continue searching after visiting the firm with data first, and (iii) after what price offers to continue searching after visiting the firm without data first.[24] Because searchers are forward-looking, they take into account under what conditions they would continue searching after sampling the first firm when deciding which firm to initially visit.

To express whether there is search on the equilibrium path, I define the probability with which a searcher with valuation v visits both firms in an equilibrium as $b(v)$. Consider the set $\{v \in [0,1] : b(v) > 0\}$. I say that there is search on the equilibrium path if and only if this set has strictly positive measure. When the share of searchers (ρ) is sufficiently large, there will be no search on the equilibrium path, independent of the exact value of search costs c. This is formalized by the following assumption and accompanying proposition:

Assumption 6 *Suppose that $p^{nd,s} + c > p^{H,M}$, where $p^{nd,s}$ solves the following:*

$$\left[\rho \int_{p^{nd,s}+c}^{1} Pr(\tilde{v}^H|v)g(v)dv + 0.5(1-\rho)\int_{p^{nd,s}}^{1} g(v)dv\right] = 0.5(1-\rho)p^{nd,s}g(p^{nd,s}) \quad (3.4.9)$$

Proposition 10 (No search beyond the first firm)
Suppose that assumption 6 holds. There exists no equilibrium in which firms play pure strategies and there is search on the equilibrium path.

[24]Off-path beliefs play no role in the analysis. All information sets of the firms are on the equilibrium path. Any searcher is only uncertain which node the game has reached when visiting the firm with data — then, she does not know which signal was generated. However, this does affect her incentives to continue searching, since these are fully pinned down by the initial price offer and the equilibrium price p^{nd}.

Assumption 6 requires that enough consumers engage in directed search, i.e. that ρ is high enough, as is underscored by the following remark:

Remark 1 *If $v \sim U[0,1]$ and $Pr(\tilde{v}^H|v)$ is linear, assumption 6 is satisfied if $\rho \geq 0.2$.*

The proof of proposition 10 consists of three steps: Firstly, $p^L < p^H$ must hold in any equilibrium in which firms play pure strategies and there is search on the equilibrium path. If $p^L = p^H$, any searcher would directly visit the firm which offers the lower uniform price and there would be no reason to search thereafter. If $p^H < p^L$, the firm with data would not be optimizing. Thus, the following arguments consider equilibria with $p^L < p^H$ and establish that (i) no searcher who initially visits the firm without data in equilibrium would continue searching and (ii) that, under assumption 6, there exists no equilibrium in which searchers would continue searching after initially visiting the firm with data.

Result (i) follows from a contrapositive argument and requires no assumptions — any searcher who finds it weakly optimal to continue searching after visiting the firm without data (and receiving p^{nd}) would never optimally visit this firm first. This holds by the following logic: By visiting the firm without data first and searching thereafter, the best price this consumer will have in hand after search is p^L with probability $Pr(\tilde{v}^L|v)$ and p^{nd} with probability $Pr(\tilde{v}^H|v)$, while the search cost $c > 0$ is surely paid. Alternatively, the consumer could visit the firm with data first and continue searching if and only if p^H is received. The latter approach would achieve strictly higher expected utility than visiting the firm without data first and searching thereafter, because it yields the same distribution of prices, but saves search costs. By contraposition, any consumer who visits the firm without data first in equilibrium would not search thereafter.

Now consider point (ii). Equilibria in which consumers search after visiting the firm with data cannot exist when ρ is high enough. Intuitively, this is based on the following logic: Searchers who arrive at the firm without data second put upward pressure on p^{nd}. This is because visiting this firm second (i.e. paying the search cost $c > 0$) is only optimal for consumers who would buy at p^{nd}. When the share of searchers (ρ) is high, the upward pressure these consumers exert on p^{nd} is strong. Then, p^{nd} would be very high in such a hypothetical equilibrium — so high, in fact, that no searcher would find it optimal to pay a search cost in pursuit of this price.

Now, I turn my attention to equilibria without on-path search. Under an assumption on ρ, all the results established for the baseline model go through verbatim for these equilibria:

Assumption 7 *Assume that $\Pi^H(p^{nd,M}; \bar{v}^{nd}) > 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$.*

Proposition 11 (Sequential search framework: equilibrium characterization) *In an equilibrium in which firms play pure strategies and there is no search on the equilibrium path:*

- *There exists a* $\bar{v} > p^L$ *such that all searchers with* $v \in (p^L, \bar{v})$ *visit the firm with data first and all searchers with* $v \in (\bar{v}, 1]$ *visit the firm without data first.*
- *The cutoff* $\bar{v}$ *must satisfy* $\bar{v} \geq \bar{v}^{nd}$*.*

Under assumption 7, such an equilibrium exists.

Remark 2 *If* $v \sim U[0,1]$ *and* $Pr(\tilde{v}^H|v)$ *is linear, assumption 7 is satisfied if* $\rho \geq 0.13$*.*

Consider an equilibrium in which firms play pure strategies and there is no search on the equilibrium path. As before, the equilibrium prices must satisfy the ordering $p^L < p^{nd} < p^H$. Thus, the strategy of searchers will be a cutoff rule, because the distribution of prices at the firm with data becomes strictly less favorable as a consumer's valuation rises. Moreover, there exists no such equilibrium in which $\bar{v} < \bar{v}^{nd}$ holds, because the firm without data would never optimally set p^{nd} below $\bar{v}$ in such a hypothetical equilibrium. However, optimal search by consumers implies that $p^{nd} < \bar{v}$ must hold in equilibrium, because searchers with valuation just above p^{nd} strictly prefer to visit the firm with data.

The proof that an equilibrium without on-path search exists for any $c > 0$ under assumption 7 is by construction. First, consider the equilibrium derived for the baseline model (in which c was prohibitively high). I define the components of this equilibrium as $(p^{L,1}, p^{H,1}, p^{nd,1}, \bar{v}^1)$, where $\bar{v}^1 = \hat{v}^B(\bar{v}^1)$, $p^{L,1} = p^{L,*}(\bar{v}^1)$, $p^{H,1} = p^{H,*}(\bar{v}^1)$, and $p^{nd,1} = p^{nd,*}(\bar{v}^1)$. The arguments pertaining to proposition 8 establish that such a combination exists.

If search costs are so high that $p^{H,1} \leq p^{nd,1} + c$, this combination of prices and $\bar{v}$ remains an equilibrium. Then, searchers would never find it optimal to search after visiting the first firm, which implies that it is optimal to visit firms according to the search rule implied by $\bar{v}^1$. Given this search behaviour, firms will find it optimal to set the prices $p^{L,1}, p^{H,1}$ and $p^{nd,1}$, respectively, establishing that this vector of prices and $\bar{v}^1$ constitutes an equilibrium.

Thus, it only remains to establish that an equilibrium of the desired form exists when $p^{H,1} > p^{nd,1} + c$. Consider an equilibrium candidate $(p^{L,2}, p^{H,2}, p^{nd,2}, \bar{v}^2)$, in which $p^{L,2} = p^{L,*}(\bar{v}^2)$, $p^{nd,2} = p^{nd,*}(\bar{v}^2)$, $p^{H,2} = p^{nd,2} + c$, and $\bar{v}^2$ is a solution to the following equation:

$$\bar{v}^2 - \underbrace{\hat{v}\big(p^{L,*}(\bar{v}^2), p^{nd,*}(\bar{v}^2) + c, p^{nd,*}(\bar{v}^2)\big)}_{:=\hat{v}^S(\bar{v}^2)} = 0 \tag{3.4.10}$$

There exists a $\bar{v}^2 \in [\bar{v}^{nd}, 1]$ that solves this equation. This holds because (i) $\hat{v}^S(\bar{v}^1) \geq \bar{v}^1$, (ii) $\hat{v}^S(1) \leq 1$, and (iii) $\hat{v}^S(\bar{v})$ is continuous on $[\bar{v}^1, 1]$ under assumption 7. The first result holds because $\hat{v}^S(\bar{v}^1) \geq \hat{v}^B(\bar{v}^1) = \bar{v}^1$. This reflects the following notion: When the firm with data sets a high signal price equal to $p^{nd,1} + c$ instead of the higher $p^{H,1}$, more searchers will prefer to visit the firm with data, i.e. $\hat{v}^S(\bar{v}^1) \geq \hat{v}^B(\bar{v}^1)$. The latter two results hold by the arguments made in the discussion of proposition 8.

To see why such a $\bar{v}^2$ constitutes an equilibrium, consider the implied search behaviour of searchers: As before, searchers will maximize their expected utility by initially visiting the firm that offers them (based on their valuation) the lower expected price. Because $p^{H,2} = p^{nd,2} + c$, it is weakly optimal to refrain from searching after visiting the firm with data. Moreover, one can show that searchers with $v > \bar{v}^2$ would not search after visiting the firm without data. Thus, it is optimal for searchers to visit the firm with data if and only if their valuation is below $\bar{v}^2$ and to refrain from searching thereafter.

It remains to show that the prices $(p^{L,2}, p^{H,2}, p^{nd,2})$ are optimal for firms if searchers visit firms according to the rule implied by $\bar{v}^2$. There will be no profitable deviations from $p^{L,2}$ and $p^{nd,2}$, because these prices are global maximizers of the respective profit functions when no consumer would ever leave to search, which are weakly above true profits for any price.

There will be no profitable deviations from $p^{H,2}$ under assumption 7 by the following logic: Because search costs are so low that $p^{H,1} > p^{nd,1} + c$, the ordering $p^{H,2} < p^{H,*}(\bar{v}^2)$ will hold. Intuitively, this represents the notion that searchers push down the high signal price of the firm with data below the unconstrained optimal price using the threat of searching. By strict concavity of the respective profit function, there are thus no profitable downward deviations from $p^{H,2}$. Moreover, assumption 7 guarantees that there will not be any profitable upward deviations (for which the firm with data would only sell to captive consumers). This is because equilibrium profits are bounded from *below* by $\Pi^H(p^{nd,M}, \bar{v}^{nd})$, while the profits from any deviation above $p^{H,2}$ are bounded from *above* by $0.5(1-\rho)\Pi^{H,M}(p^{H,M})$.

Equilibrium uniqueness (both within the baseline and the sequential search framework) requires a tie-breaking rule restricting the behaviour of searchers who have a valuation just below the lowest price that is offered by either firm, which I call $\underline{p}$:

Assumption 8 *There exists an $\epsilon > 0$ s.t., regardless of the exact value of the lowest price offered by either firm ($\underline{p}$), all searchers with $v \in [\underline{p} - \epsilon, \underline{p})$ initially visit a firm which offers the price $\underline{p}$ with weakly higher probability.*

Proposition 12 (Equilibrium uniqueness)
Under assumptions 6, 7, and 8, there exists a <u>*unique*</u> *equilibrium in which firms play pure strategies.*

The tie-breaking rule guarantees that the low signal profit function is differentiable around the lowest equilibrium price p^L, which must thus be equal to $p^{L,*}(\bar{v})$. This eliminates one potential source of equilibrium multiplicity. Moreover, assumption 7 ensures that all functions $p^{L,*}(\bar{v})$, $p^{H,*}(\bar{v})$, and $p^{nd,*}(\bar{v})$ are continuous on $[\bar{v}^{nd}, 1]$, which implies that $\hat{v}^S(\bar{v})$ and $\hat{v}^B(\bar{v})$ can never jump up on the interval $[\bar{v}^{nd}, 1]$, eliminating the other possible source of equilibrium multiplicity.

As before, one can rule out the existence of equilibria in which firms mix (within the set of equilibria in which firms draw prices from distributions with connected support):

Proposition 13 (Sequential search framework: no mixing)
In any equilibrium in which firms draw prices from distributions with connected support, all firms play pure strategies.

Summing up, the key results from the baseline model are retained. In equilibrium, a large majority of searchers only visit the firm with data. Moreover, the market share of the firm with data approaches 1, independent of the signal distribution, as $\rho \to 1$.

Corollary 5 (Sequential search framework: market dominance)
The equilibrium market share of the firm with data approaches 1 as $\rho \to 1$.

When ρ (the share of searchers) approaches 1, both assumptions 6 and 7 will hold, independent of the signal distribution. Thus, an equilibrium will exist. All consumers just visit one firm and $\bar{v} \geq \bar{v}^{nd}$ must hold, which implies the result because $\bar{v}^{nd} \to \rho$ as $\rho \to 1$.

It remains to study how changes in search costs (c) affect the equilibrium outcomes. Within the equilibrium established for the baseline model, search cost reductions play no role. When c becomes small, reductions of search costs exacerbate market dominance:

Corollary 6 (Comparative statics: search costs)
The equilibrium $\bar{v}$ *is unaffected by changes in* c *if* $c > p^{H,1} - p^{nd,1}$ *and is weakly decreasing in* c *if* $c \leq p^{H,1} - p^{nd,1}$. *If* $v \sim U[0,1]$ *and* $Pr(\tilde{v}^H|v)$ *is linear, the market share (sales based) of the firm with data is thus falling in* c *when* $c \leq p^{H,1} - p^{nd,1}$ *and independent of* c *otherwise.*

I visualize these effects in the following graph, in which I plot the equilibrium quantities for different levels of search costs (on the x-axis) and α-ρ combinations.

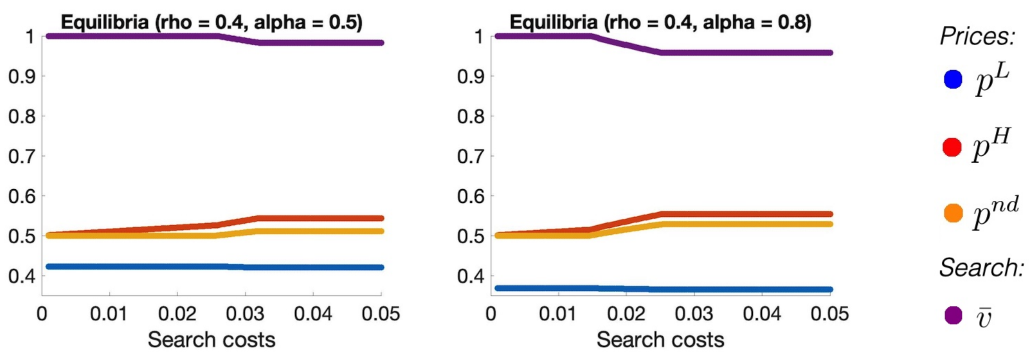

Figure 3.4: Comparative statics — search costs

When search costs are sufficiently high (i.e., $c \geq 0.03$), the equilibrium $(p^{L,1}, p^{H,1}, p^{nd,1}, \bar{v}^1)$ from the baseline model is played, in which the possibility of searching is not relevant. When c becomes sufficiently small, the equilibrium quantities are given by $(p^{L,2}, p^{H,2}, p^{nd,2}, \bar{v}^2)$. Then, search cost reductions lead to lower price levels, but exacerbate the problem of market dominance. Intuitively, searchers are now able to constrain the high signal price of the firm with data with the threat of searching, which implies that this price will approach p^{nd} as search costs fall. This increases the incentives of searchers to visit the firm with data. In particular, all searchers will prefer to only visit the firm with data (i.e. $\bar{v} = 1$) when c becomes sufficiently small, because p^L always remains substantially below p^{nd} and p^H.

Finally, I consider the effects of changes in information precision (α). I visualize the equilibrium quantities for different levels of α (on the x-axis) and c-ρ combinations.

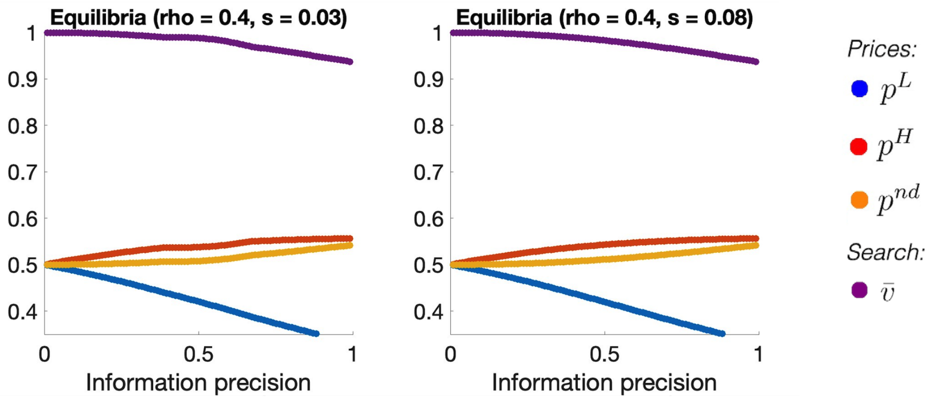

Figure 3.5: Comparative statics — information precision

As the signal of the firm with data becomes more informative (i.e. when this firm's data advantage becomes larger), the degree of market dominance enjoyed by this firm falls.[25] This holds by the following logic: When the precision of the signal (α) rises, searchers with high valuations are more likely to be recognized by the firm with data, in which case they receive an unfavorably high price. This reduces their incentives to visit the firm with data, which, in equilibrium, induces more searchers to visit the firm without data.

[25]There are two kinks in the evolution of the equilibrium objects when $c = 0.03$. This is because the difference between $p^{H,1}$ and $p^{nd,1}$ is maximal (and thus, can be above a given level of c) when α is at intermediate levels. Thus, the equilibrium of category 1 is played whenever information precision is relatively low or high, while the equilibrium of category 2 is played for intermediate levels of information precision.

3.5 Welfare and policy recommendations

3.5.1 Data and consumer welfare

The effects of data advantages imply a need for regulatory interventions for two reasons. Firstly, personalized pricing by the firm with data may lead to higher average prices, thereby reducing consumer welfare. More importantly, the market dominance resulting from data advantages (no matter how small these data advantages are) can reduce consumer welfare by discouraging entry, distorting competition, and by reducing the incentives to innovate.

The personalized pricing that the firm with data implements can lower consumer welfare as such. For example, suppose that the firm with data receives a binary signal that is effective at identifying high-valuation consumers, where $Pr(\tilde{v}^H|v) = 0.5$ if $v < 0.6$ and $Pr(\tilde{v}^H|v) = 1$ if $v \geq 0.6$. If all searchers visit the firm with data, a searcher's ex ante expected utility is 0.1025, while it equals 0.125 when no firm has data. When ρ is high and $\bar{v}$ is thus close to 1, consumer welfare in the competitive equilibrium with data will hence be lower than when no firm has data. The equilibrium dynamics induce consumers to flock to the firm with data, even though it effectively charges higher prices than its rival in the monopoly benchmark.

The market dominance enabled by data advantages can deter entry. This is best conceptualized by augmenting my model with an initial entry stage. There are two firms: the incumbent and the potential entrant, who has no data about consumers. Initially, the entrant has to decide whether or not to pay a fixed cost to enter the market, while the incumbent has to pay no such cost. After the entry decision, the product market competition game from the baseline model is played. If the incumbent has no data, both firms receive half of the market if the entrant enters. If the incumbent has a data advantage, the entrant is visited by a much lower mass of consumers, which makes entry less profitable. Thus, data advantages may discourage entry, which is to the detriment of consumers who have a strong preference for the entrant's product (e.g. the captive consumers in my model).

The presence of data can significantly distort competition. To see this, consider a duopoly with a high-quality and a low-quality firm. The valuations that searchers (and the corresponding captive consumers) have for the product of the high-quality firm, call these v, are uniformly distributed on $[0, 1]$. The valuation that any searcher has for the product of the low-quality firm is given by $v - \mu$. In accordance, the valuations that captive consumers have for the product of the low-quality firm are uniformly drawn from $[-\mu, 1 - \mu]$, where $\mu \geq 0$ captures the extent of the quality difference.

Suppose that no firm has data, but that $\mu > 0$. In a monopoly benchmark, the low-quality firm would set the price $0.5(1-\mu)$, while the high-quality firm sets the price 0.5. In the competitive equilibrium, searchers thus only visit the high-quality firm.[26] Endowing the low-quality firm with data changes this prediction. To see this, define $p^{L,\mu}$ and $p^{H,\mu}$ as the prices this firm would set in the monopoly benchmark when receiving the low and high signal, respectively. If $p^{L,\mu} + \mu < 0.5 < p^{H,\mu} + \mu$ holds, the equilibrium predictions from the baseline model are retained — a large majority of searchers only visit the low-quality firm, because it has data. This represents a significant distortion of competition.

Empirical evidence by Li et al. (2021) shows that shielding firms from competitive pressures reduces their incentives to innovate. The competitive distortions caused by data advantages have similar effects. To see this, reconsider the aforementioned example with a high-quality and a low-quality firm and consider the incentives of the low-quality firm to reduce μ, e.g. by conducting product innovation. When this firm has no data, reducing μ to 0 will increase the market share of this firm from $0.5(1-\rho)$ to 0.5, while the benefits of innovation are much smaller for this firm if it has a data advantage. This is to the detriment of consumers, who would benefit from innovation.

3.5.2 Policy implications

The preceding analysis has established the need for policy interventions when firms have unequal access to information about consumer preferences in markets with search frictions. However, the comparative statics results I have derived show that reduced market concentration cannot be attained by policy measures which reduce search frictions or which merely reduce the informational advantage of a firm with superior data.

Another way of depriving the firm with data of its advantage is to endow consumers with a right to anonymity. I study the effects of such a policy by integrating this possibility into the baseline framework — now, any searcher can pay a cost $e \geq 0$ before obtaining a price quote at the firm with data to ensure that this firm receives no signal about them, i.e. to become anonymous. Everything else is as in the baseline model. Any searcher thus has three possible choices: (i) visit the firm without data, (ii) visit the firm with data and choose to become anonymous, or (iii) visit the firm with data and refrain from becoming anonymous.

The analysis requires a tie-breaking rule. I assume that whenever two of the approaches listed above entail the offering of an identical uniform price, both these choices will be selected by searchers with equal probability.

An equilibrium in this extension consists of a price p^a that the firm with data offers to all consumers who become anonymous, in addition to the prices (p^L, p^H, p^{nd}) introduced previously. The establishment of a right to anonymity will be inconsequential:

[26] This is because any searcher will obtain the utility $\max\{v - 0.5, 0\}$ at the high-quality firm, which is strictly larger than the utility she would obtain at the low-quality firm, namely $\max\{v - 0.5(1+\mu), 0\}$.

Proposition 14 (Ineffective anonymity)
Consider the baseline model, augmented with the right to anonymity. For any $e \geq 0$, the set of consumers who exercise this right has measure zero.

The intuition behind this result mirrors the insights of Belleflamme and Vergote (2016), who derive a similar result in a monopoly setting. Only consumers with comparatively high valuations would ever want to exercise their right to anonymity. Low-valuation consumers, by contrast, benefit from the possibility that a firm profiles them. In equilibrium, firms will thus offer high prices to consumers who choose to become anonymous, which makes it detrimental for consumers to exercise this right.

By contrast, the establishment of a right to data portability (as expressed in the EU GDPR and the DMA) can be very effective. I show this by integrating a right to data portability into the baseline framework. Suppose that any searcher can, before obtaining a price quote, costlessly copy all the information the firm with data has about her and transfer this to the firm without data. A searcher now has three choices: As before, she can (i) visit the firm with data or (ii) obtain a price offer at the firm without data without porting her data. In addition, she can now (iii) obtain a price offer at the firm without data after porting her data. Formally, porting the data implies that the firm without data will, upon being visited, receive a signal about the consumer's valuation. The distribution of this signal is $Pr(\tilde{v}^H|v)$, just as for the firm with data.

A pure strategy of the firm with data remains a price tuple (p^L, p^H), while a pure strategy of the firm without data is now a vector $(p^{L,nd}, p^{H,nd}, p^{nd})$. This firm offers the price p^{nd} to all consumers who visit it but do not port their data and the prices $p^{L,nd}$ and $p^{H,nd}$ to all consumers who port their data and generate the low and high signal, respectively.

Endowing searchers with the ability to costlessly exercise their right to data portability can eliminate the advantage of the firm with data:

Proposition 15 (Data portability)
Consider the baseline model, augmented with a right to data portability. There exists an equilibrium in which <u>all</u> searchers visit the firm without data.

This equilibrium has the following form: All searchers visit the firm without data. The firm with data is only visited by its captive consumers and will thus optimally set the monopoly prices, namely $p^{L,M}$ and $p^{H,M}$. Searchers exercise their right to data portability if and only if their valuation is below a cutoff v^t. If their valuation is above v^t, they visit the firm without data but don't port their data. This cutoff v^t solves:

$$v^t = \sup \left\{ v \in [0,1] : Pr(\tilde{v}^H|v)p^{H,nd} + Pr(\tilde{v}^L|v)p^{L,nd} - p^{nd} \leq 0 \right\} \tag{3.5.1}$$

Because $v^t \leq 1$, the prices that the firm without data would offer to consumers who port their data are lower than their monopoly counterparts, i.e. $p^{L,nd} \leq p^{L,M}$ and $p^{H,nd} \leq p^{H,M}$. Since $p^{L,nd} \leq p^{L,M}$ and $p^{H,nd} \leq p^{H,M}$, visiting the firm without data and porting one's data yields higher expected utility than visiting the firm with data. Thus, it is optimal for all searchers to visit the firm without data.

Calculating the equilibrium values of v^t shows that v^t is generally below 1. This is crucial, because it implies that the equilibrium prices satisfy $p^{L,nd} < p^L$ and $p^{H,nd} < p^H$, making it *strictly* optimal for searchers to visit the firm without data. This insight establishes that a right to data portability can effectively counteract the competitive effects of data advantages even when exercising this right is costly or generates a less informative signal.

Naturally, there also exists an equilibrium in which no searchers exercise their right to data portability, the respective information sets of the firm without data are off the equilibrium path, and the firm's beliefs are such that it is optimal for searchers to not exercise this right. Then, the equilibrium outcomes will be the same as in the baseline model.

3.6 Extensions

In this section, I discuss the results of various extensions. Because the analysis of the sequential search framework indicates that restricting attention to prohibitively high search costs is without loss of generality when the share of searchers is high enough, I assume that searchers can only visit one firm in all the extensions I study. I provide a detailed documentation and proofs in sections B.3 and B.4.

In section B.3.1, I consider the equilibrium outcomes when the firm with data receives a finite signal with $K \geq 2$ possible realizations. The previous results extend under two assumptions: (i) First, I assume that the probability distribution of the signal is such that the implied hazard rates of the conditional valuation distributions are always ordered (and in the same way), regardless of the way in which searchers visit firms. Moreover, (ii) I assume that the signals invoke price discrimination in the sense that the effective price a consumer pays will be rising in her valuation. Under these assumptions, the equilibrium strategy of searchers remains a cutoff rule. Because the pricing calculus of the firm without data is the same as in the baseline model, $\bar{v} \geq \bar{v}^{nd}$ must hold in equilibrium. Moreover, an equilibrium in which firms play pure strategies exists.

In section B.3.2, I assume that the firm with data receives a continuous signal $\tilde{v} = v + \epsilon$ about the valuation of any arriving consumer (v), where the noise term ϵ is uniformly distributed on the interval $[-\bar{\epsilon}, \bar{\epsilon}]$. For simplicity, I assume that $\bar{\epsilon} \in (0, 1/8)$ and that $v \sim U[0, 1]$. All other specifications from the baseline model are retained. Because the signal is not perfect, any searcher can attain positive utility by visiting the firm with data.

However, the fact that the firm with data price discriminates implies that the strategy of searchers will once again be a cutoff rule, and the results of propositions 7 and 8 are retained. Thus, the insights from the main analysis are retained, so long as the firm with data receives an imperfect signal. Interestingly, calculating the equilibrium quantities shows that $\bar{v} \to \bar{v}^{nd}$ as $\bar{\epsilon} \to 0$.

In section B.3.3, I extend the baseline model by integrating quality differentiation into the analysis. I assume that consumers have preferences as in Mussa and Rosen (1978): When buying a good with quality q at the price p, a consumer's utility is $u(q,p) = \theta q - p$, where $\theta \sim U[0,1]$ and is private information to the consumer. The firm with data offers two different price-quality menus in equilibrium, depending on the observed signal. I restrict attention to equilibria in which the strategy of searchers is a cutoff rule (i.e. there exists a $\bar{\theta}$ such that all searchers with $\theta < \bar{\theta}$ visit a given firm and vice versa) or all searchers randomize, which I call simple equilibria. When the share of searchers is high enough ($\rho \geq 0.34$ is sufficient when considering linear signal distributions), all previous results are retained.

In section B.3.4, I consider situations in which both firms receive a signal about the valuations of visiting consumers, but the signal of one firm (the firm with better data) is more precise. For simplicity, I assume that $v \sim U[0,1]$ and once again restrict attention to simple equilibria. In any simple equilibrium, searchers with a valuation below the cutoff will visit the firm with better data and vice versa. This cutoff will be bounded from below. I analytically characterize equilibria in which firms play pure strategies and provide a condition that guarantees uniqueness (if such an equilibrium exists). Numerical analysis reveals that such an equilibrium always exists whenever the signal distribution is linear and that the market share of the firm with better data converges to 1 as $\rho \to 1$.

3.7 Conclusion

I have analyzed the relationship between data and market power in a duopoly model of directed search and personalized pricing. One of the firms in the market has a data advantage — in the baseline model, this firm receives a signal about the valuation of any consumer who visits it, while its rival receives no such information. Consumers can costlessly visit one firm but have to pay a search cost to visit a second firm after the first. There are two groups of consumers, namely *captive consumers* and *searchers*. Searchers have equal valuation for the good of both firms and, based on their valuation, optimally choose which firms to visit.

Directed consumer search strongly facilitates the transmission of data advantages into competitive advantages. In equilibrium, a large majority of searchers only visit the firm with data. The firm without data is just visited by searchers with very high valuations. As the share of searchers goes to 1, so does the market share of the firm with data.

While I have considered a framework in which data is only used to price discriminate, the insights apply more generally. Consider, for instance, an insurance market: Consumers with low risk benefit if a firm has information about their traits, because this would translate into more favorable contract terms. Thus, these consumers would all prefer to visit a firm with a data advantage, which improves the overall risk profile of consumers who visit this firm. The generally better contract terms this firm offers as a result will, in turn, attract even more consumers, mirroring the unraveling channels present in my model.

More generally speaking, the selection effect and the resulting competitive benefits manifest whenever low-valuation consumers systematically prefer a firm with better data. This feature can also emerge through privacy concerns. Empirical evidence by Lin (2022) establishes that consumers with higher wealth tend to value privacy more (i.e. have a larger disutility when firms attain access to their data). In markets where the wealth of consumers is positively correlated with their valuations, the familiar search patterns would thus also emerge in the absence of price discrimination, giving rise to the selection effect and its competitive implications.

Chapter 4

Search Disclosure

4.1 Introduction

We investigate the exchange of user data between firms in search markets.[1] Advances in tracking technologies have made it ever easier for online sellers to collect and share data about consumers. In fact, almost every website on the internet relies on such technologies to recognize the same user over time (Englehardt and Narayanan, 2016). By exchanging consumer identifiers with rivals, firms have a means to inform their rivals that a certain buyer has obtained an offer from them.[2] This form of information sharing may be collusive and harm consumers if it enables firms to coordinate prices. In addition, it provides firms with accurate information about a consumer's shopping history and thereby facilitates price discrimination, which regulatory bodies around the world are increasingly worried about.[3]

In this article, we analyze the effects of such information sharing, which we call search disclosure. Specifically, we ask when search disclosure occurs in equilibrium and whether search disclosure is anti-competitive and harms consumers. We show that search disclosure prevails in equilibrium only if search costs are sufficiently small or if firms cannot adjust a price offer made to a given consumer. Otherwise, firms do not share said information even though industry profits are higher if firms use search disclosure. Evidence of search history-based price discrimination is indeed limited despite its technical feasibility.[4] Our analysis, which shows that firms share the necessary information only under limited circumstances, thus provides an explanation for this phenomenon.

Formally, we consider the possibility of search disclosure within the sequential search

[1]This chapter is joint work with Marcel Preuss.

[2]These identifiers can be obtained via cookies, tracking pixels, digital fingerprinting and consumer sign-in.

[3]In 2016, the OECD's competition committee recognized that "there are particular reasons to worry that price discrimination in digital markets will be harmful" (OECD Secretariat, 2016). The EU has recently adopted new compliance rules for firms engaging in online price discrimination (European Parliament, 2019).

[4]See, for example, Mikians et al. (2012).

framework by Wolinsky (1986), in which consumers engage in costly sequential search to discover the prices of goods and how much they value them. Specifically, consumers randomly pick which firm to visit first and, based on the firm's price offer and their willingness-to-pay for that firm's good, decide whether to visit a second firm or not. In the model, firms can disclose a consumer's visit to their rival — this is possible when they have received search disclosure regarding the same consumer before as well as when they have not. Search disclosure thus endogenizes the sellers' beliefs about the search history of consumers and can give rise to rich forms of search history-dependent pricing, which we allow for. This includes the possible revision of prices for consumers who continue to search after getting an initial price quote.

In equilibrium, firms never disclose a consumer's visit to their rival after having received disclosure for the consumer. This is because any such consumer must have visited the rival first and continued to search. By conducting search disclosure in this situation, a firm would inform its rival about this fact. Since consumers only continue searching if they have a low willingness-to-pay for the initially inspected product, this induces the rival to revise its price downward. Such downward price revisions harm the disclosing seller, which is why there exists no equilibrium in which firms use search disclosure after having received it before.

Consequently, firms would only ever disclose a buyer's visit if they have not received search disclosure regarding the same buyer before – we call this information sharing strategy *partial disclosure*. If firms use partial disclosure in equilibrium, they do not know when a consumer continues to search after visiting them. However, a firm that encounters a buyer without having received disclosure about this buyer before will believe to be visited first. In addition, a firm that encounters a buyer after having received disclosure about this buyer knows that it is visited second. That is, partial disclosure enables sellers to price discriminate based on the consumer's search order, which leads to higher industry profits but lower consumer welfare.

However, even though industry profits are higher under partial disclosure, the latter does not arise in equilibrium unless search costs are small. If a firm uses partial disclosure, its rival will quote a higher price to a consumer when being visited second, which benefits the disclosing firm.[5] However, deviating by withholding search disclosure has a surprising benefit as well. Without receiving search disclosure, the rival will always believe that it is visited first and, thus, use search disclosure even if it is actually visited second. Consequently, by withholding search disclosure, a firm will be informed (by its rival) if the buyer continues to search. This allows the deviating seller to screen its buyers and to set a lower price for buyers who continue to search. Since buyers do not expect any price revisions, there are no Coasian dynamics as in Gul et al. (1986). The ability to screen

[5]If the rival knows that it is visited second, it understands that the consumer had a low valuation for the disclosing firm's product. This puts the rival in a favorable position, inducing it to set a high price.

buyers in said way will thus grant a firm strictly higher profits, creating strong incentives to withhold search disclosure.

If search costs are sufficiently large, partial disclosure is not an equilibrium outcome because being able to screen buyers is more valuable than inducing the rival to charge a higher price. If search costs are high, buyers only continue to search if the net utility from buying the first product they sampled is close to or less than zero. Thus, almost no consumer who continues to search would eventually buy the first product at the initially offered price, regardless of the other firm's price. The cost of deviating to not disclosing, which is that a firm's rival will charge a lower price when being visited second, is therefore negligible when search costs are high. By contrast, being able to screen buyers is very profitable in this case because setting a lower price for consumers who continue searching is the only way to earn any profits from them.

If an equilibrium with partial disclosure does not exist, we find that firms use no disclosure in equilibrium, which implies they cannot price discriminate. Firms do not use search disclosure because of the risk of triggering a downward price revision by their rival, which will happen if the consumer has sampled the rival first. In particular, this detrimental effect of deviating to disclosure dominates the beneficial effect of inducing the rival to set a higher price when it is visited second.

The interaction of search costs and search disclosure has major implications for welfare and consumer surplus because both are lower with partial disclosure than without any search disclosure. Thus, a reduction in search costs can lower both total welfare and consumer surplus if it induces sellers to conduct search disclosure in equilibrium. Future advances in technology will likely lower search costs (think of augmented reality) while also enhancing the feasibility of search disclosure. Our research shows that the combination of both can have adverse surplus effects by facilitating a collusive information exchange.

Interestingly, the equilibrium without disclosure is not the best possible outcome from a consumer surplus and welfare perspective. We numerically show that total and consumer surplus is highest if firms always disclose. The intuition for this result is that firms will revise prices downward for any buyer who searches. This encourages search, which additionally raises buyer welfare by improving the average match quality. Because this outcome is never reached if the decision to conduct search disclosure is left in the hands of the firms, regulation that mandates the full provision of search history information might improve outcomes.

Another important result is that the feasibility of price revisions weakly raises total and consumer surplus. This is because what discourages search disclosure is the possibility that the rival might revise its price downward. We show that if price revisions are impossible, firms always conduct search disclosure in equilibrium, leading to price discrimination that reduces consumer surplus and total welfare. In practice, making revised offers to consumers often requires re-targeting. Our analysis thus implies that privacy regulation which limits the ability of firms to conduct re-targeting can potentially lead to more data sharing, with adverse effects on consumer surplus and welfare.

The rest of the paper proceeds as follows: Section 2 reviews the related literature. Section 3 presents the model and Section 4 contains the equilibrium analysis and the comparative statics. In Section 5, we discuss the policy implications of our work. Section 6 concludes.

4.2 Related literature

We make two main contributions to the literature. First, we are the first who analyze the possibility that firms can inform their rivals about the visit of a given buyer. Second, the information that sellers may obtain in our framework gives rise to rich forms of price discrimination, some of which have not been studied before.

We thus contribute to the consumer search literature, in particular to the work on sequential consumer search for differentiated products, which builds on the workhorse model by Wolinsky (1986) and Anderson and Renault (1999). Since information sharing can inform rival sellers about an arriving buyer's search path, our analysis is related to Armstrong et al. (2009) and Zhou (2011), who study prominence and ordered search, respectively. We add to this literature by showing when an outcome comparable to ordered search can emerge endogenously as a result of sellers' information sharing choices.[6]

In addition, search disclosure in our model may enable sellers to revise prices for consumers who continue to search other sellers before making a purchase decision. The idea of discriminating against such consumers is reminiscent of Armstrong and Zhou (2016). The authors explore the phenomenon of search deterrence, i.e., when sellers commit to higher prices for returning consumers.[7] The key differences to that paper are that we 1) allow for discrimination not only against returning consumers but also against consumers who visit the rival first, 2) study discrimination that is based on endogenously provided information, and 3) do not allow firms to commit to future prices.

Search disclosure allows individual firms to price discriminate based on the inferred search history of the buyer. A handful of other recent papers study price discrimination in search markets. Fabra and Reguant (2020) study a simultaneous search model in which firms price discriminate based on perfect information about the quantity that consumers demand. Preuss (2022) studies price discrimination based on the search behavior of consumers, like this paper. Mauring (2022) considers firms which can discriminate against

[6]Ordered search or, similarly, search with prominence, are also studied by Armstrong (2017), Moraga-González and Petrikaitė (2013), and Haan and Moraga-González (2011).

[7]Pan and Zhao (2022) experimentally investigate the role of commitment power for search deterrence. Other related work is by Zhu (2012), who studies a sequential bargaining framework with repeat contacts in a market for over-the-counter financial securities.

consumers using information about whether a given consumer is a shopper or a non-shopper.[8] In Bergemann et al. (2021), competing firms receive noisy signals about the size of the consumers' choice sets and the consumers' search costs.[9] None of these papers consider the endogenous exchange of information about consumers' search histories.

This paper also contributes to the literature on information exchange between competitors. The question when rivals benefit from sharing their information with one another was first addressed by Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), and Gal-Or (1985), who studied the effects of agreements to exchange private information about demand conditions, as well as by Shapiro (1986) and Gal-Or (1986), who consider firms that can share information about private costs. Focusing on information about individual consumers, Chen et al. (2001) study settings where firms receive imperfect information about buyers' consideration sets that they can share. More recent work on the strategic sharing of consumer data includes Kim and Choi (2010), Zhao (2012), and Choe et al. (2023). These papers, however, neither consider the exchange of endogenously collected information like this article, nor do they study environments in which buyers sample offers sequentially.

The first work on the topic of sharing endogenously collected consumer information is by Taylor (2004), who studies a multi-period model in which sellers can sell their customer lists to one another.[10] Relatedly, Liu and Serfes (2006) study a two-period Hotelling model in which firms can share preference information they have acquired for all buyers that initially purchase at their firm.[11] In an online advertising context, Johnson et al. (2022) study the conditions under which online sellers agree to share unique identifiers of their websites' visitors with ad exchanges to facilitate re-targeting. In contrast to the above papers, we focus on the sharing of search-related information in a sequential search model.

4.3 Framework

In this section, we introduce a model of sequential search and information sharing, which is based on Wolinsky (1986). Two firms indexed $j \in \{A, B\}$ each produce a horizontally differentiated and indivisible good at constant marginal cost, which is normalized to zero. A representative consumer wants to buy at most one unit of the good.[12] The consumption

[8] While not directly addressing price discrimination, De Corniere (2016) studies a model in which consumers differ based on their search query, providing sellers information they use when setting prices. Similarly, consumers in Yang (2013) differ ex ante and thus search within different pools of firms, again giving firms information relevant to their pricing decision.

[9] Garrett et al. (2019) consider a model of second-degree price discrimination in which consumers differ in their choice sets, but firms do not have information about consumers.

[10] De Nijs (2017) considers a related model of a three-firm oligopoly.

[11] Extensions are studied by Choe et al. (2022) and Lin et al. (2021), among others.

[12] Our results continue to hold for a unit mass of consumers because we allow sellers to price discriminate. Laying out the model for a representative consumer is merely for conciseness.

utility this consumer attains when buying firm j's good is given by the match value u_j, which is uniformly and independently drawn from the unit interval. This distribution of match values, which we denote by F, is common knowledge.

The consumer does not know the realizations $\{u_j\}_{j=A,B}$ at the beginning of the game and has to discover these match values as well as prices via sequential search. When visiting any firm for the first time, she incurs a search cost $s > 0$ to inspect the firm's product, which means to discover the product's match value and price. The consumer has free recall, i.e., she can costlessly return to purchase at a firm that she has previously visited. Without loss of generality, the order in which the consumer visits firms is random.[13] We impose the tie-breaking rule that consumers stop searching when they are indifferent.

In addition to this relatively standard set-up, a firm can, upon being visited by the consumer, disclose to its rival that the consumer has inspected its product. We call the sharing of this information, which enables discriminatory pricing, search disclosure. We assume that search disclosure must be truthful so that firms cannot misreport a buyer's visit.[14] Firms do not observe the prices set by the rival firm, nor any match values. In the absence of search disclosure, firms do not know anything about the consumer's search path. For example, they do not know whether a consumer who inspects their product has visited the rival before. Consumers do not observe the firms' disclosure choices. The exact timing is explained below.

The game begins whenever the consumer starts searching. Without loss of generality, let firm A be the first seller the consumer samples. Upon sampling firm A, firm A sets its price p_A and the consumer observes p_A together with her match value u_A. At this stage, firm A also decides whether to disclose the buyer's visit to its rival or to withhold this information. This choice is captured by the variable $d_A \in \{D, ND\}$, where $d_A = D$ indicates that firm A has disclosed and $d_A = ND$ that it has not. The effect of disclosure is that, if the consumer continues to sample firm B, then firm B knows that the consumer visited firm A before.[15] Without observing firm A's disclosure decision, the consumer decides whether to buy (and receive net utility $u_A - p_A$), to continue searching, or to stop searching without a purchase.

If she continues and samples firm B, firm B quotes a price p_B, which the consumer then observes together with her match value u_B. At the same time, firm B decides whether to use search disclosure itself or not ($d_B \in \{D, ND\}$). Notably, firm A knows for sure that the consumer continued to sample firm B if and only if $d_B = D$.

Next, the consumer can, without incurring additional cost, again check firm A, and

[13]This holds because firms are ex-ante identical and we restrict attention to symmetric equilibria.

[14]This is a plausible assumption in our context since search disclosure is about sharing some unique identifiers, which a firm would have to guess correctly if it wanted to fake search disclosure.

[15]For example, firm A might install a cookie on the consumer's browser that is readable to firm B. Or, it collects identifiable information and shares these with firm B.

firm A can make a new price quote p'_A, which may differ from the original price p_A. Whether firm A wants to revise its price depends on whether or not it has learned new information about the consumer. If firm A is visited by the consumer again without having received disclosure, it has learned nothing. Thus, it will not change its price. By contrast, if firm A has received disclosure for the consumer, it will optimally revise its price. Afterwards, the consumer makes her decision immediately. Specifically, she chooses firm A if $u_a - p'_A > \max\{0, u_B - p_B\}$, firm B if $u_B - p_B > \max\{0, u_A - p'_A\}$ and makes no purchase otherwise.[16]

Given this description of the game, we can formalize the information sets in which a firm can be called to act using the following notation:

- $\mathcal{H}(j) = R$ if firm j has received disclosure and has not met the buyer before.
- $\mathcal{H}(j) = NR$ if firm j has not received disclosure and has not met the buyer before.
- $\mathcal{H}(j) = D \times p_j \times R$ if firm j has received disclosure and has met the buyer before, at which point firm j disclosed the visit to its rival (D) and offered the price p_j.
- $\mathcal{H}(j) = ND \times p_j \times R$ if firm j has received disclosure and has met the buyer before, at which point firm j did not disclose the visit to its rival (ND) and offered the price p_j.

A firm's strategy thus has to define (i) what prices to offer if in the information sets $\mathcal{H}(j) = NR$ and $\mathcal{H}(j) = R$, (ii) whether or not to disclose if in the information sets $\mathcal{H}(j) = NR$ and $\mathcal{H}(j) = R$, and (iii) what revision price to set in any information set $\mathcal{H}(j) = D \times p_j \times R$ and $\mathcal{H}(j) = ND \times p_j \times R$.

Characterizing all relevant information sets as above makes clear that we abstract from the passing of time. This reflects the notion that making inferences about the consumer's preferences based on the passage of time alone is challenging for sellers because consumers differ greatly in how long it takes them before they continue their search. As Ursu et al. (2021) document, consumers often take (quite long) breaks in the search process. As a result, merely observing that a buyer has not bought after some time has passed since she received the offer is not very informative about whether she has indeed continued to search. By contrast, receiving search disclosure resolves any uncertainty about whether the consumer has continued to search and is thus more informative than the passing of time could be. Consequently, neglecting the possibility of learning from the passage of

[16] That is, we implicitly rule out the possibility that the second visited firm, firm B in this example, can revise its price as well. The rationale for this assumption is that the consumer has perfect information about all match values at this point and thus makes a decision relatively quickly (she does not even need to leave B's website in order to see updated prices from A). In addition, the analysis would be equivalent if we assumed that firm A never discloses to firm B if the consumer returns. The intuition we build throughout the forthcoming analysis strongly suggests that firm A would indeed never disclose to firm B in this case.

time does not affect the predictions of our model qualitatively, but greatly simplifies the analysis.

Moreover, we note that returning to a previously visited firm before making a purchase decision weakly dominates not returning. This is because free recall implies that returning either has no value (if the consumer does not buy) or a positive value (if the consumer buys). Moreover, there are information sets in which the revised price may be lower than the original price, making returning a weakly dominant strategy even for consumers who would not buy at the original price. We therefore specify that the buyer will always return to any previously visited firm before making a purchase here and throughout the main analysis.[17] We also study a model in which some consumers incur a cost to return to a previously visited firm (and thus do not always return) in Appendix Section C.3.

As a solution concept, we use perfect Bayesian equilibrium (PBE). In a PBE, the buyer's and firms' beliefs must be consistent with Bayes' rule. In information sets that are off the equilibrium path, however, Bayes' rule does not apply. To discipline beliefs, we impose the following standard assumptions on off-equilibrium beliefs: Firstly, the buyer's beliefs are passive — whenever the buyer is offered an off-equilibrium price, she believes that firms have not otherwise deviated from their equilibrium strategies. Secondly, we assume that firms hold passive beliefs about their rivals' prices as well. That is, a firm that unexpectedly receives disclosure or has charged an off-equilibrium price continues to believe that the other firm has followed the equilibrium pricing strategy.

Thirdly, we need to make an assumption regarding the beliefs that a firm forms about the possible match values of the buyer in any off-path information set. We specify that these beliefs must be consistent as well. Consistency requires that (1) the firm believes that the buyer it faces has searched according to her equilibrium search strategy and (2) that the firm takes into account what it believes (or knows) about the prices the consumer has received along the search path.

4.4 Equilibrium analysis

There are three candidates for a symmetric pure-strategy PBE, namely (1) an equilibrium in which firms never disclose to their rivals, (2) an equilibrium in which firms disclose to their competitors if and only if they have not previously received disclosure, and (3) an equilibrium in which firms always disclose to their competitors.[18] We refer to

[17]When searching for products online, it takes just one click to return to a previously visited seller to check their offer again. Returning to a previously visited seller is thus different from sampling a new one, which requires finding the seller and inspecting the good. Compared to the search cost associated with the latter two actions, a click is essentially free. Moreover, in online markets free recall is facilitated by re-targeting, which provides consumers with the opportunity to easily return to a previously visited website.

[18]An equilibrium in which firms only disclose after having received disclosure, but not when receiving no previous disclosure would be outcome-equivalent to the no disclosure equilibrium.

these equilibrium candidates as (1) the *no disclosure equilibria*, (2) the *partial disclosure equilibria*, and (3) the *full disclosure equilibria*, respectively. We distinguish novel equilibrium objects in these different equilibrium candidates via superscripts, namely "n" (no disclosure), "d" (partial disclosure), and "f" (full disclosure). In the analysis that follows, we will characterize these three equilibrium candidates and determine when they exist.

Throughout the analysis, we restrict attention to equilibria with active search, which requires that search costs be not too large. Specifically, we restrict attention to search costs for which the buyer would participate in the market if there is no search disclosure, i.e., we assume that $s \leq 1/8$.

At the end of the section, we also briefly consider possible equilibria in which firms randomize over their disclosure choices. We argue that such equilibria will not exist unless search costs are small and that, even when they exist, the insights we establish when restricting attention to pure-strategy equilibria extend.

4.4.1 No disclosure equilibria

In a *no disclosure equilibrium*, only one information set is on the equilibrium path, namely $\mathcal{H}(j) = NR$, implying that firms never receive disclosure before meeting a buyer. Consequently, firms charge a uniform price p^* in any symmetric equilibrium. In particular, firms do not discriminate against consumers who continue to search since they cannot observe search behavior.

Consumers anticipate that firms do not price discriminate in equilibrium. Thus, their optimal search rule is given by a simple cutoff strategy: continue searching if and only if $u_j < w^n(p_j)$ (if j is the first seller sampled). Note that consumers do not stop searching without a purchase after sampling the first seller because our assumption of active search ($s \leq 1/8$) implies that $w^n(p_j) \geq p_j$ in equilibrium. To derive $w^n(p_j)$, suppose a consumer enjoys utility r if she stops searching. In this case, she is indifferent between receiving the incremental utility from sampling another seller $-j$ at cost $s > 0$ and consuming r if r satisfies

$$\mathbb{E}_{u_{-j}}[\max\{u_{-j} - p^e_{-j} - r,\ 0\}] = s, \tag{4.4.1}$$

where p^e_{-j} denotes the anticipated price at seller $-j$. Solving for r yields $r = w^* - p^e_{-j}$, where $w^* = 1 - \sqrt{2s}$, because match values are drawn from a uniform distribution on $[0, 1]$. To obtain the cutoff $w^n(p_j)$, note that the consumer buys from firm j if $u_j - p_j \geq r = w^* - p^*$, where we used that $p^e_{-j} = p^*$ in equilibrium. Thus, the critical match value satisfies

$$w^n(p_j) = w^* - p^* + p_j. \tag{4.4.2}$$

The ensuing pricing game is, of course, equivalent to the problem sellers face in the original Wolinsky (1986) model. The unique equilibrium price p^* thus solves

$$p^* = \frac{1-(p^*)^2}{1+w^*}. \tag{4.4.3}$$

To support this equilibrium, firms must not have an incentive to disclose when being visited by a buyer. The effect of deviating to disclosure depends on whether a buyer visits the deviating firm, say firm A, first or second. If the buyer visits firm A first and continues to search, then firm A's deviation to disclosure makes firm B reach the off-path information set $\mathcal{H}(B) = R$ (it receives disclosure for an unknown buyer). Consequently, disclosure informs firm B that it is visited second. Thus, B learns that the buyer's match at the rival firm, given by u_A, lies below $w^n(p_A)$, as she would not have searched otherwise. The profit function firm B maximizes in this case, which we denote by $\Pi^2(p_B)$, is therefore given by

$$\Pi^2(p_B) = p_B\left[\frac{1}{2}F(w^*)\left[1-F(w^n(p_B))\right] + \frac{1}{2}\int_{p_B}^{w^n(p_B)} F(p^*+u_B-p_B)du_B\right], \tag{4.4.4}$$

where we account for firm B's passive beliefs that the consumer received the price p^* at firm A, which implies that $w^n(p_A) = w^*$, i.e., the search cutoff consumers use in equilibrium.

By contrast, if the buyer visits firm A second, then firm B reaches the off-path information set $\mathcal{H}(B) = ND \times p^* \times R$ (it has received disclosure for a buyer to whom it offered the equilibrium price p^* before and whose visit it did not disclose to A). That is, disclosure by firm A informs firm B that the buyer has continued to search after sampling B, allowing firm B to infer that the buyer's match value u_B satisfies $u_B < w^n(p^*) = w^*$. Firm B's expected profit function in this information set, which we denote by $\Pi^3(p_B)$, is therefore given by

$$\Pi^3(p_B) = \frac{1}{2}p_B\int_{p_B}^{w^*} F(u_B-p_B+p^*)du_B. \tag{4.4.5}$$

Search disclosure by A thus endows firm B with valuable information to price discriminate against buyers with low match values for B. As it turns out, the ensuring price discrimination is detrimental to A's profits. We learn this from Lemma 4 which characterizes the prices p_2^n and p_3^n that maximize Π^2 and Π^3, respectively.

Lemma 4 *The optimal prices p_2^n and p_3^n following a rival's deviation to disclosure are*

$$p_2^n = \frac{1}{2}\left(1-\left(w^*-p^*\right)\right) + \frac{1}{4}\left((w^*) - \frac{(p^*)^2}{w^*}\right) \tag{4.4.6}$$

$$p_3^n = \frac{2}{3}(w^* + p^*) - \frac{1}{3}\sqrt{(w^*)^2 + 2w^*p^* + 4(p^*)^2} \tag{4.4.7}$$

These prices are uniquely determined and satisfy the ordering $p_3^n < p^* < p_2^n$.

The price p_2^n is strictly above p^* because receiving disclosure for a previously unknown buyer lets firm B know that this buyer has visited A first. Thus, if the buyer shows up at firm B, this indicates that she did not obtain a good match at firm A (formally, that $u_A < w^*$). This puts firm B in a competitively favorable position in which it can charge a higher price. The effect is reminiscent of environments in which consumers search firms in a certain and known order as in Zhou (2011).[19]

By contrast, the price p_3^n is below p^*. This is because A's search disclosure informs B that the buyer has continued to search, which B would otherwise not be able to observe. More precisely, B originally sets a uniform price p^* to maximize the joint profits from (i) buyers who arrive at B first and buy immediately, (ii) buyers who arrive at B first and return later, and (iii) buyers who sample A first. Upon receiving (unexpected) search disclosure about a buyer it met before, however, B knows that it faces a buyer from group (ii). Buyers in this group must have a low match value at firm B as they would not have continued to search otherwise. This induces firm B to revise its price downward. Notably, the ordering of the revised and the equilibrium price is different from the one found in Armstrong and Zhou (2016). This is because Armstrong and Zhou (2016) consider a setting in which firms can commit to future prices prices for buyers who continue to search.[20]

Thus, a deviation to search disclosure is beneficial for the deviating firm if the buyer visits this firm first and detrimental if the buyer visits the deviating firm second.[21] To evaluate this trade-off, consider the profit function of firm A if it deviates. Because the buyer neither anticipates nor observes any disclosure, she expects firm B to charge p^* so that her search rule after arriving at firm A is still characterized by the function $w^n(p_A)$. Thus, the profit function of firm A if it deviates to disclosure, which we denote by Π^1, is given by:

$$\Pi^1(p_A) = \underbrace{\frac{1}{2}p_A\left[\left[1 - F(w^n(p_A))\right] + \int_{p_A}^{w^n(p_A)} F(p_2^n + u_A - p_A)du_A\right]}_{\text{expected profits from a consumer who samples } A \text{ first}} +$$

[19]Nonetheless, there is a difference between the derivation of p_2^n in our analysis and the derivation of the price the firm visited second would charge in Zhou (2011). This is because there is no price discrimination in equilibrium here, implying that firms' and consumers' expectations, which determine p_2^n, differ.

[20]In their analysis of the monopoly case, the authors include an example without commitment. In this example, the return price is actually higher than the initial price as well. Their result obtains because of the strong asymmetry between the monopolist's offer and the outside option. Specifically, the distribution of the outside option is significantly more attractive than the distribution of the seller's net utility.

[21]Note also that firm B does not disclose "back" to firm A in the event that A was visited first because this would induce A to revise its price downward. We show this formally in the proof of Proposition 16.

$$\underbrace{\frac{1}{2}p_A\left[F(w^*)\big(1-F(w^*-p_3^n+p_A)\big)+\int_{p_A}^{w^*-p_3^n+p_A}F(p_3^n+u_A-p_A)du_A\right]}_{\text{expected profits from a consumer who samples } B \text{ first}} \tag{4.4.8}$$

To understand the expected profits from a consumer who starts at firm B, notice that $u_B < w^*$ must hold if the buyer shows up at firm A. Thus, such a buyer will surely consume at firm A if $u_A - p_A > w^* - p_3^n$, as reflected in the first term. If the buyer's net surplus at firm A is below $w^* - p_3^n$, she still consumes at firm A if $u_A - p_A$ is greater than zero and greater than $u_B - p_3^n$, which holds with probability $F(p_3^n + u_A - p_A)$. If this event holds for a buyer with $u_A < w^* - p_3^n + p_A$, the buyer will sample firm A after visiting firm B.[22] We find that the adverse effect of disclosure strictly dominates for any $s > 0$.

Proposition 16 *There always exists a unique no disclosure equilibrium in which sellers charge p^*. In this equilibrium, deviating by non-disclosure is strictly unprofitable.*

There are two reasons why an equilibrium with no disclosure can be sustained for any level of search costs. The first is that the rival's price reduction $(p^* - p_3^n)$ for buyers that sampled the rival first exceeds the rival's price increase $(p_2^n - p^*)$ for buyers who sampled the disclosing seller first. Figure 4.1 visualizes this fact, which holds by the following logic: When firm B receives search disclosure about a buyer it has not seen before, the only inference this firm can make when the buyer arrives is that $u_A < w^*$, which concerns the rival's product. By contrast, when B receives search disclosure about a buyer it has seen before, it learns that $u_B < w^*$, which concerns its own product. Because search disclosure is more informative about the buyer's demand for the own product in the latter case, the subsequent price reduction is greater in magnitude than the price increase in the former.

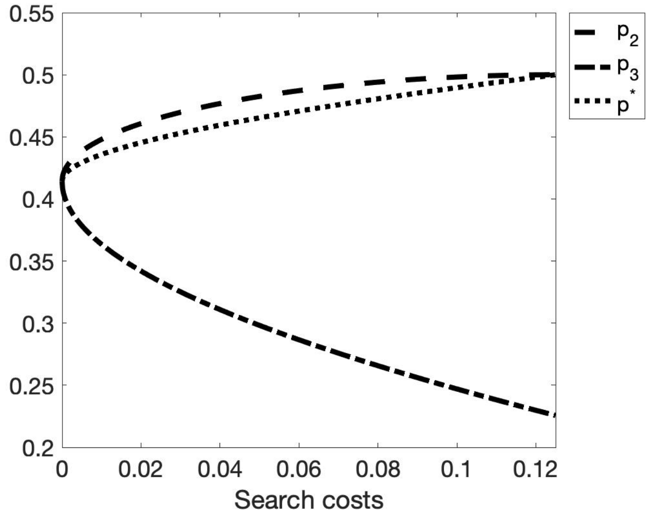

Figure 4.1: Equilibrium (Wolinsky) price and off-path prices

[22] This is because $u_B < p_3^n + u_A - p_A < p_3^n + (w^* - p_3^n + p_A) - p_A = w^*$, where the latter equals $w^n(p^*)$.

The second effect underlying Proposition 16 is that disclosure by firm A will reduce demand from a consumer who sampled B first more than it increases demand from a consumer who samples A first, even if the changes in p_2^n and p_3^n were equal in magnitude. This holds by the following logic: Suppose $p_2^n = p^* + \delta$ and $p_3^n = p^* - \delta$. If the buyer starts at firm B (the non-deviating firm), B's revised price of $p^* - \delta$ instead of p^* means that the buyer will be $1/2\delta$ less likely to choose firm A for sure. By contrast, if the buyer starts at firm A, a price of $p^* + \delta$ instead of p^* at firm B has no comparable effect. This is because the probability that such a buyer surely buys at firm A, namely $1 - F(w^* - p^* + p_A)$, is unaffected by search disclosure, given that buyers continue to expect the price p^* at firm B. Thus, buyers are more likely (by a difference of $1/2\delta$) to fall into the category in which their demand decreases.

4.4.2 Partial disclosure equilibria

We now consider equilibria in which firms disclose to their competitors if and only if they have not received disclosure beforehand. This disclosure strategy implies that firms are certain about whether they are visited first or second in the two information sets that are on path in such an equilibrium. If a firm faces an unknown buyer for whom no disclosure was received (i.e. $\mathcal{H}(j) = NR$), this firm knows that it is being visited first. If a firm faces an unknown buyer for whom disclosure was received (i.e. $\mathcal{H}(j) = R$), this firm knows that it is being visited second.

Consequently, the symmetric pure strategy equilibrium features two on-path prices p_1^* (set by the seller sampled first) and p_2^* (set by the seller sampled second). Consumers anticipate that sellers do not know whether they continue to search or not and, thus, do not expect prices to be revised in equilibrium. The optimal search rule thus still uses a simple cutoff value. Using previous notation, this cutoff value is given by

$$w^d(p_j) = w^* - p_2^* + p_j \tag{4.4.9}$$

so that a consumer buys from firm j (when j is visited first) and without sampling $-j$ if and only if $u_j \geq w^d(p_j)$.

This set-up is, of course, comparable to a model of ordered search. We can therefore invoke existing results from Armstrong et al. (2009) to characterize the partial disclosure equilibrium prices p_1^* and p_2^*. Importantly, Armstrong et al. (2009) show that $p_2^* > p_1^*$. Moreover, the cutoff for an active search market to exist is still $s \leq 1/8$.[23]

There are two information sets in which each seller $j \in \{A, B\}$ can deviate from the partial disclosure equilibrium strategy. Without loss of generality, suppose the buyer samples firm A first. That is, firm A will be in the information set $\mathcal{H}(A) = NR$ when the

[23]The reason is that as $s \to 1/8$, p_1^* and p_2^* converge to p^* so that the buyer expects the same prices (and thus the same surplus from search) with and without disclosure.

game begins as the buyer samples firm A. In this information set, firm A must not want to deviate to non-disclosure, or there is no partial disclosure equilibrium. In addition, if the consumer continues to sample firm B and firm A sticks to its equilibrium strategy, then B is in the information set $\mathcal{H}(B) = R$ as it will have received search disclosure from A. In this information set, the partial disclosure strategy dictates that firm B must not disclose back to firm A when the buyer arrives. Notably, if $\mathcal{H}(B) = R$, deviating to disclosure informs A that the buyer has continued to search. As argued before, this will lead seller A to revise its price downward, making such a deviation generally unprofitable.

The effects of a deviation to non-disclosure when no disclosure was received previously are more complex. Suppose that firm A does not disclose a buyer's visit when $\mathcal{H}(A) = NR$. If the buyer continues to search, then firm B faces a buyer for whom it has not received disclosure before (B's information set is thus $\mathcal{H}(B) = NR$ instead of $\mathcal{H}(B) = R$). Firm B will thus incorrectly believe that it is the first firm the buyer visits and offer p_1^* instead of p_2^*. In addition, firm A's deviation has a second effect on A's profits via the influence on B's subsequent disclosure decisions. Because firm B erroneously believes to be visited first, this firm will, following the equilibrium play, use search disclosure itself upon being visited by the buyer. This, in turn, will let firm A know if a consumer has continued to search. Since only buyers with match values below $w^d(p_A)$ continue to search, deviating to non-disclosure practically allows A to sequentially screen its buyers.

In sum, deviating to not disclosing when a consumer arrives at firm A first (firm A knows this in the partial disclosure equilibrium) has two opposing effects. On the one hand, it makes firm B charge a lower price, which lowers firm A's profits. On the other hand, it allows A to sequentially screen its buyers and to revise the price for buyers that continued to search. Notice that buyers do not expect any price revisions in a partial disclosure equilibrium and, thus, would never sample the other seller (firm B) only to get a lower price at firm A. Thus, even though the buyer's optimal search rule leads to negative selection, there are no Coasian dynamics as in Gul, Sonnenschein, and Wilson (1986). Revising the price for consumers who continue searching is therefore strictly profitable.

To evaluate which effect dominates, we first characterize how firms revise their initial prices when deviating to non-disclosure. To that end, we must derive the price a firm offers in each information set $\mathcal{H}(j) = ND \times p_1 \times R$, which depends on the firm's initial price p_1. If firm A charges an initial price p_1 when deviating to non-disclosure, the buyer continues to search firm B if and only if $u_A < w^d(p_1) = w^* - p_2^* + p_1$. Additionally, firm A knows that firm B sets the equilibrium price p_1^* after the deviation to non-disclosure (if the buyer continues to search). Thus, the revised price p_3 maximizes $\Pi^{3,d}(p_3|p_1)$:

$$\Pi^{3,d}(p_3|p_1) = p_3 \int_{p_3}^{w^d(p_1)} \frac{1}{2} F(u_A - p_3 + p_1^*) du_A. \tag{4.4.10}$$

The optimal price in $\mathcal{H}(j) = ND \times p_1 \times R$, which we denote by $p_3^d(p_1)$, is a function of p_1. Next, we analyze the profit function that A maximizes when choosing an initial price p_1 if it deviates to non-disclosure, knowing that it will be able to screen buyers based on whether they continue to search or not. This profit function, which we call $\Pi^{1,d}(p_1)$, is given by:

$$\Pi^{1,d}(p_1) = p_1 \frac{1}{2}\left[1 - F(w^d(p_1))\right] + p_3^d(p_1) \int_{p_3^d(p_1)}^{w^d(p_1)} \frac{1}{2} F(p_1^* + u_A - p_3^d(p_1)) du_A \tag{4.4.11}$$

Let p_1^d maximize $\Pi^{1,d}(p_1)$. By analyzing the first-order conditions that $p_3^d(p_1)$ and p_1^d must satisfy in the subgame following a deviation to non-disclosure, we obtain the following result.

Lemma 5 *Suppose firm j deviates at $\mathcal{H}(j) = NR$ to $d_j = ND$. Then, the optimal initial price p_1^d and the optimal revision price function $p_3^d(p_1)$ satisfy*

$$p_3^d(p_1) = (2/3)(w^d(p_1) + p_1^*) - (1/3)\sqrt{(w^d(p_1))^2 + 2w^d(p_1)p_1^* + 4(p_1^*)^2} \tag{4.4.12}$$

$$p_1^d = 1 - w^d(p_1^d) - \left(p_3(p_1^d)\right)^2 + p_3(p_1^d)\left(w^d(p_1^d) + p_1^*\right). \tag{4.4.13}$$

Both p_1^d and $p_3^d(p_1)$ are uniquely determined.

The uniqueness of p_1^d and $p_3^d(p_1)$ is important because it implies that the partial disclosure equilibrium exists if and only if the deviation profits given p_1^d and $p_3^d(p_1^d)$ do not exceed the equilibrium profits given p_1^* and p_2^*. We plot the four prices $(p_1^*, p_2^*, p_1^d, p_3^d(p_1^d))$ for different values of search costs in Figure 4.2.

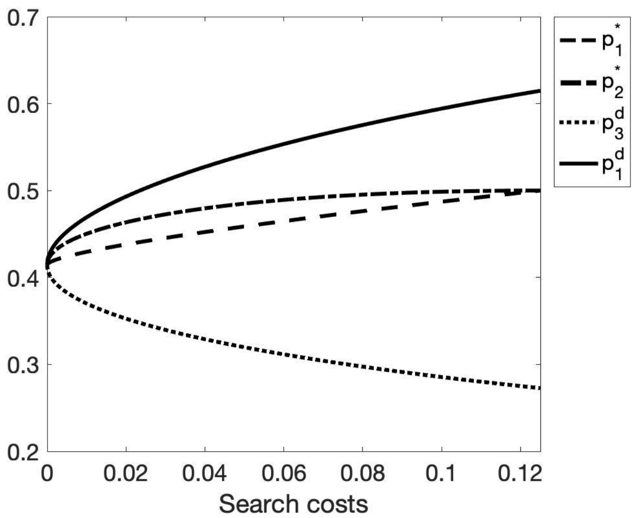

Figure 4.2: Equilibrium and deviation prices

With the system of equations for p_1^d and $p_3^d(p_1)$ as well as for p_1^* and p_2^*, we can characterize analytically when the partial disclosure equilibrium exists.

Proposition 17 *There exists a threshold $\bar{s} > 0$ such that the partial disclosure equilibrium exists if $s \leq \bar{s}$. In this equilibrium, prices are uniquely determined. There also exists a $\bar{s}' \geq \bar{s}$ such that a partial disclosure equilibrium does not exist if $s \geq \bar{s}'$.*

Our numerical analysis complements the proposition by showing that $\bar{s} = \bar{s}'$. That is, the partial disclosure equilibrium exists up to a unique cutoff value $\bar{s} \approx 0.01$.

To understand why the partial disclosure equilibrium cannot be sustained if search costs are high, consider the case in which search is very costly. In particular, suppose it is so costly that a buyer would search after sampling the first firm, say firm A, only if u_A is approximately equal to or smaller than p_A. Then, any buyer who continues to search is very unlikely to buy from firm A after sampling firm B, at least at A's original price. This mitigates the detrimental effect of deviating to non-disclosure because buyers who search further will most likely not buy from firm A regardless of whether B charges p_1^* or p_2^*. By contrast, learning that a buyer continued to search is very profitable for A if search costs are high. This is because a revised price of p_3^d can lead to significant demand from a buyer who continued to search, which is almost zero if A does not revise its price. Consequently, the beneficial effect of a deviation by non-disclosure dominates when search costs are high.

To understand why the partial disclosure equilibrium does exist if search costs are small, however, examine Figure 4.2 and note how the prices compare to each other. The gap between p_2^* and p_1^* increases the fastest in s when s is small. As argued before, the larger this difference, the greater is the cost of deviating from partial disclosure. Intuitively, the ratio of the $p_2^* - p_1^*$ gap and the $p_1^d - p_3^d(p_1^d)$ gap is largest when search costs are small, which means that the detrimental effect of a deviation by non-disclosure dominates when search costs are small.

4.4.3 Full disclosure equilibria

Consider the third possible equilibrium candidate, in which firms disclose to their competitors regardless of whether they have received disclosure before or not, i.e. $d_j = D$ if $\mathcal{H}(j) = NR$ or $\mathcal{H}(j) = R$. If both firms stick to this disclosure strategy, a firm can reach any of the following information sets: $\{NR\}$, $\{R\}$, and $\{D \times p_1 \times R\}$, where p_1 is the firm's arbitrary initial price. If $\mathcal{H}(j) = NR$, firm j believes it is visited first. We denote the equilibrium price firm j would set in this information set by p_1^f, where the superscript f refers to "full" disclosure. If $\mathcal{H}(j) = R$, firm j believes it is visited second and sets the price p_2^f in equilibrium. Lastly, firm j knows that it was visited first but that the buyer has also sampled firm $-j$ if $\mathcal{H}(j) = D \times p_1 \times R$, in which case firm j will offer a revised price given by $p_3^f(p_1)$.

To characterize the equilibrium prices, we must first solve for the consumers' optimal search rule. If firms follow the full disclosure strategy, prices will be revised on the equilibrium path. As a result, a consumer who has sampled one firm already anticipates that if she samples the other firm as well, then the price at the initially visited firm will change from p_1 to $p_3^f(p_1)$. The optimal search rule in this case is thus non-standard and must first be derived.

Lemma 6 *For any initial price p_1, there exists a $w^f(p_1)$ such that consumers will continue searching if and only if their initial match value is below $w^f(p_1)$. In equilibrium, $p_3^f(p_1) < w^f(p_1)$. If this cutoff is interior (strictly below 1), it solves:*

$$w^f(p_1) - p_1 = \underbrace{\int_0^{w^f(p_1)-p_3^f(p_1)+p_2^f} (w^f(p_1) - p_3^f(p_1))du + \int_{w^f(p_1)-p_3^f(p_1)+p_2^f}^1 (u - p_2^f)du - s}_{\text{Expected utility conditional on searching}} \tag{4.4.14}$$

Suppose that the consumer visits firm A first. The first integral on the right-hand side of (4.4.14) captures the expected value of being able to buy from firm A at the anticipated revised price $p_3^f(p_1)$ while the second integral captures the expected value of being able to buy from firm B at the anticipated price p_2^f. Given a value $w^f(p_1)$ as determined by (4.4.14), the consumer will, in equilibrium, continue to search if and only if her match value at the first firm is below $w^f(p_1)$.[24] For future reference, we define the equilibrium value of this cutoff as $w^f := w^f(p_1^f)$.

Taking note of the consumer's optimal search rule, we are ready to derive the equilibrium prices. If firm A receives disclosure for a buyer whom it quoted the price p_1 before, A knows that $u_A < w^f(p_1)$ because the buyer continued to search. In the information set $\mathcal{H}(A) = D \times p_1 \times R$, firm A maximizes the following profit function through choice of p_3:

$$\Pi^{3,f}(p_3|p_1) = \frac{1}{2}p_3 \int_{p_3}^{w^f(p_1)} F(u_A - p_3 + p_2^f)du_A. \tag{4.4.15}$$

We have already defined the solution to this as $p_3^f(p_1)$. Now consider the situation of a firm, say B, which receives disclosure for a previously unknown buyer ($\mathcal{H}(B) = R$). When this consumer shows up at firm B, the firm believes that the match value of the consumer at firm A satisfies $u_A < w^f$. Additionally, firm B expects that by following the equilibrium strategy of disclosing back to firm A, the price at which the consumer can buy from firm A is the revised equilibrium price p_3^f, where $p_3^f := p_3^f(p_1^f)$. Thus, firm B

[24]We show in the appendix that the search cutoff must be interior in an equilibrium.

sets p_2 to maximize:

$$\Pi^{2,f}(p_2) = \frac{1}{2}p_2 F(w^f)\big[1 - F(w^f - p_3^f + p_2)\big] + \frac{1}{2}p_2 \int_{p_2}^{w^f - p_3^f + p_2} F(p_3^f + u_B - p_2)du_B, \tag{4.4.16}$$

the solution to which we have already defined as p_2^f.[25]

Finally, consider a firm, say A, which is visited by a buyer about whom no disclosure was received yet so that A's information set is $\mathcal{H}(A) = NR$. In a full disclosure equilibrium, this information set is only reached if the buyer did not visit another firm before. Thus, firm A knows that it is the first firm this buyer visits. When setting its price p_1, firm A takes into account how p_1 affects the consumer's search decision (captured by the function $w^f(p_1)$) as well as the price $p_3^f(p_1)$, which A will revise its original price to if it subsequently receives disclosure from B. Formally, firm A's optimal price in this case (given by p_1^f) maximizes:

$$\Pi^{1,f}(p_1) = \frac{1}{2}\big[1 - F(w^f(p_1))\big]p_1 + \underbrace{\frac{1}{2}p_3^f(p_1) \int_{p_3^f(p_1)}^{w^f(p_1)} F(p_2^f + u_A - p_3^f(p_1))du_A}_{=\Pi^{3,f}(p_3|p_1)} \tag{4.4.17}$$

Pinning down the prices that maximize (4.4.15) – (4.4.17) allows us to describe necessary conditions that the prices in a full disclosure equilibrium must satisfy.

Lemma 7 *Consider a full disclosure equilibrium. Given w^f, the equilibrium prices (p_1^f, p_2^f, p_3^f) must jointly solve the following system of equations:*

$$p_3^f = \frac{2}{3}(w^f + p_2^f) - \frac{1}{3}\sqrt{\left(w^f + p_2^f\right)^2 + 3\left(p_2^f\right)^2} \tag{4.4.18}$$

$$p_2^f = \frac{1}{2}\left(1 - w^f + p_3^f\right) + \frac{1}{4}w^f - \frac{1}{4}w^f\left(p_3^f\right)^2 \tag{4.4.19}$$

$$p_1^f = (1 - w^f)\left(\frac{\partial w^f(p_1^f)}{\partial p_1}\right)^{-1} + p_3^f\left(p_2^f + w^f - p_3^f\right) \tag{4.4.20}$$

We show that, for any potential equilibrium search cutoff $w^f \in [0, 1]$, there exists a unique vector of prices that jointly solves these three equations. This allows us to establish that full disclosure cannot constitute an equilibrium.

Proposition 18 *A full disclosure equilibrium does not exist.*

[25] The function $\Pi^{2,f}(p_2)$ correctly depicts the profits of firm B in the information set $\mathcal{H}(B) = R$ if $w^f - p_3^f + p_2 \leq 1$. We verify that this condition must hold true for prices p_2 in an open ball around p_2^f.

Intuitively, a full disclosure equilibrium requires that $p_1^f \leq p_3^f$. To see this, consider the decision situation of a firm who faces a buyer it has already received disclosure about so that this firm knows that it is visited second. In a full disclosure equilibrium, this firm is supposed to disclose the buyer's visit to its competitor. However, doing so is strictly unprofitable if the price revision induced by this disclosure decision is downward, i.e., it is unprofitable if $p_3^f < p_1^f$. This is because a lower price at the rival firm entails a reduction in the demand of the disclosing firm. However, the negative selection effect implies that the optimal prices satisfy $p_3^f < p_1^f$.

Formally, we show that, for any for any $w^f < 1$, the unique joint solution to the equations (4.4.18) - (4.4.20) has the property $p_3^f < p_1^f$, which thus cannot support a full disclosure equilibrium. In addition, if $w^f = 1$, all prices would be exactly equal because a search cutoff of $w^f = 1$ means that buyers sample both sellers before making a purchase decision regardless of their match values, rendering any information obtained from search disclosure irrelevant. However, if all prices are the same, then consumers with sufficiently high match values at the first firm they visit must not find it profitable to continue to search since there are strictly positive search costs. In other words, consumers with initial match values below 1 would buy immediately, contradicting the specification that $w^f = 1$.

4.4.4 Comparative statics

We here seek to understand the relationship between the incidence of search disclosure and search costs. To deal with equilibrium multiplicity, we make the following assumption.

Assumption 9 *If multiple equilibria exist, we remove any equilibrium which is strictly Pareto dominated by another equilibrium, in terms of the payoffs of the firms.*

Assumption 9 seems justified in our context because tracking a consumer is a game that is usually repeated several thousand times per day. As a result, coordinating on their preferred equilibrium should be feasible for sellers. Moreover, our model describes an environment in which consumers do not arrive all at once but over time. While we do not model this explicitly, we imagine that if firms play the equilibrium strategy of their preferred equilibrium, consumers will quickly notice and adjust equilibrium beliefs accordingly. Since each consumers' individual mass is zero, ignoring the beliefs of a few initial consumers and simply playing the equilibrium strategy of their preferred equilibrium is essentially costless for firms.

The previous analysis has shown that we can restrict attention to the partial and the no disclosure equilibrium. Using Assumption 9 requires knowing which equilibrium firms prefer if both exist. To this end, recall that profits in the no disclosure equilibrium equal profits if search is random while profits in the partial disclosure equilibrium equal average profits if search is ordered. Thus, we know from Armstrong et al. (2009) that the partial disclosure equilibrium is preferred from the firms' perspective if $s \leq 0.021$. Because the partial disclosure equilibrium exists when search costs are sufficiently small by Proposition 17, while the no disclosure equilibrium exists for all search costs by Proposition 16 (and full disclosure never prevails), the next result follows immediately from the preceding discussion.

Proposition 19 *There is search disclosure in equilibrium if s is sufficiently small and no search disclosure if s is sufficiently high.*

Based on previous calculations that show that the partial disclosure equilibrium exists when search costs are below $\bar{s} \approx 0.01$, we can in fact conclude that there is a unique cutoff value $\bar{s} \approx 0.01$ which determines whether there is search disclosure in equilibrium or not.

Recall that the partial disclosure equilibrium achieves the same welfare and consumer surplus as the equilibrium of an ordered search model whereas the no disclosure equilibrium is outcome-equivalent to the standard random search equilibrium. We can thus leverage results from Armstrong et al. (2009) once more, who show that prominence (ordered search) lowers total surplus and consumer surplus compared to random search. Thus, the next result is an immediate corollary to Proposition 19.

Corollary 7 *A marginal reduction in search costs that triggers a shift to the partial disclosure equilibrium will lower consumer surplus and total welfare.*

4.4.5 Mixed-strategy equilibria

In this subsection, we consider equilibria in which firms mix over their disclosure choices. We argue that, even within such equilibria, search disclosure can only prevail when search costs are low enough.

Recall that there are two different information sets in which a firm has to decide whether to disclose or not: (i) when it faces an unknown buyer for whom no disclosure was received ($\mathcal{H}(j) = NR$) and (ii) when it faces an unknown buyer for whom disclosure was received ($\mathcal{H}(j) = R$). In the second information set, a firm has no incentives to disclose. It knows that it is visited second, which means that disclosure will merely induce the rival to revise its price. Even if firms mix over their disclosure decisions, the negative selection implied by consumers' search decisions will still be present. This implies that price revisions would be downward, which makes disclosing in $\mathcal{H}(j) = R$ suboptimal. Thus, there will be no equilibrium in which firms mix over their disclosure decision in the information set $\mathcal{H}(j) = R$.

Now consider the incentives of firms to mix over their disclosure decision in the information set $\mathcal{H}(j) = NR$. In this information set, a firm does not know whether the consumer has visited the firm first or second.[26] When the consumer visits the firm first, disclosure may have a positive or a negative effect on the disclosing firm's profits. As in the partial disclosure equilibrium, disclosure in this situation raises the rival's price, but denies the firm the opportunity to revise its price for consumers who continue to search. Which effect dominates depends on the level of search costs. As before, disclosure will have a negative effect on the disclosing firm's profits if the consumer visits this firm second, because it induces a downward price revision by the rival.

When search costs are high, we thus conjecture that it will be strictly suboptimal for any firm to disclose in $\mathcal{H}(j) = NR$. This follows from the logic established during the analysis of the partial disclosure equilibrium: Even when facing a consumer who visits the firm first, disclosure is disadvantageous when search costs are high. This is because the value of attaining a chance to revise one's price is high, while the benefits of reducing the price offered by the rival are minimal.

Thus, it is strictly optimal to withhold disclosure in $\mathcal{H}(j) = NR$ when search costs are high. This implies that there is no equilibrium in which firms mix over their disclosure decisions when search costs are sufficiently high. When search costs are low, there potentially exists an equilibrium in which firms mix over these choices when $\mathcal{H}(j) = NR$. Such an equilibrium would be a hybrid between the partial and the no disclosure equilibrium. Then, all our comparative statics results would be retained even if this equilibrium were to be played when search costs are low.

4.5 Policy implications

In this section, we study the equilibrium outcomes that emerge when modifying the framework we outlined and solved previously. We consider the equilibrium outcomes when firms cannot revise prices (Section 4.5.1) and when firms exogenously receive full search history information (Section 4.5.2). This analysis shows that a policymaker interested in maximizing consumer welfare should ensure that price revisions by firms are feasible and consumers can easily observe revised prices. In addition, the sharing of search history related information should not be left up to firms. This is because consumer welfare is maximal when firms exogenously receive full search history information (in the model with price revisions), an outcome which is impossible under voluntary search disclosure by the preceding analysis.

[26] Because the rival mixes over its disclosure decision, it could be that the consumer visited the rival first, but said rival did not disclose as part of its equilibrium strategy.

4.5.1 Banning price revisions

In this section, we solve the aforementioned model when firms cannot revise prices. Under this specification, there are just two relevant candidates for a symmetric pure-strategy PBE, namely (1) the no disclosure equilibrium and (2) the partial disclosure equilibrium. The full disclosure and the partial disclosure equilibrium are outcome-equivalent because a firm that receives disclosure for a known buyer has no more choices to make.

In a nutshell, the benefits of non-disclosure in the main analysis emerged from the possibility of price revisions, either by the rival firm or by the firm itself. When this possibility is eliminated, disclosing becomes strictly profitable regardless of search costs. Thus, when price revisions are impossible, the equilibrium result from the previous section basically flips.

Proposition 20 *If firms cannot revise prices, the no disclosure equilibrium does not exist, while the partial disclosure equilibrium always exists.*

To see why the no disclosure equilibrium never exists when price revisions are impossible, recall the trade-off firms face in the no disclosure equilibrium of the baseline model. If a firm discloses and that firm is the first firm the consumer visits, its rival will set a higher price (compared to the price the rival sets if the firm does not disclose). If a firm discloses and that firm is the second firm the consumer visits, the rival learns that it should revise its price downward. In the baseline model, the detrimental effect of potentially triggering a downward price revision dominates the benefit that accrues if a consumer visits the disclosing firm first. If price revisions are impossible, however, the second channel is shut down, and only the beneficial effect of disclosure remains. This implies that firms would always deviate from the no disclosure strategy.

By contrast, the partial disclosure equilibrium always exists when price revisions are impossible. The two equilibrium prices, namely p_1^* and p_2^*, are equal to the equilibrium prices from Armstrong et al. (2009). As before, a firm that deviates by non-disclosure when being visited first will make its rival wrongly believe to be visited first. The non-disclosure deviation thus reduces the rival's price from p_2^* to p_1^*. In the baseline model, such a deviation also offered the benefit of being notified by the rival if a consumer continued to search, which allowed the deviating firm to screen consumers. When price revisions are impossible, however, screening buyer types with different prices is not feasible. Thus, only the detrimental effect of the deviation to non-disclosure remains, rendering partial disclosure an equilibrium strategy for any level of search costs.

Consequently, if price revisions are impossible, there is search history-based price discrimination in equilibrium. Consumers will be charged different prices at either firm, depending on whether they visit this firm first or second. By previous arguments, consumer welfare in such an equilibrium is lower than in the no disclosure uniform price equilibrium.

The policy implications of this section are thus twofold: Firstly, the feasibility of price revisions weakly raises consumer welfare. Secondly, note that we modeled the effects of price revisions if firms have no commitment power. Thus, from a policy perspective, ensuring the feasibility of price revisions has to go hand in hand with provisions ensuring that firms cannot discourage search by committing to high prices for consumers who continue to search as in Armstrong and Zhou (2016).

4.5.2 Exogenous search disclosure

In this section, we suppose that a third party guarantees that each firm is informed about a buyer's entire search history, i.e., about all search decisions. This could be achieved, for example, by regulation that mandates search disclosure at all times. Alternatively, an online platform on which firms sell their products and consumers search, or more generally any large data intermediary, could make such search history information available to sellers.

As in Section 4.4.3, there are three prices a consumer can be offered on the equilibrium path, which we call $p_1^{f,*}, p_2^{f,*}$, and $p_3^{f,*}$. The prices $p_1^{f,*}$ and $p_2^{f,*}$ will be offered by a firm when a consumer visits this firm first and second, respectively. The price $p_3^{f,*}$ is offered by the first visited firm after the consumer samples the second firm as well. By Lemma 6, the consumers' optimal search behavior can be characterized by a cutoff rule $w^f(p_1)$ as defined in (4.4.14), where p_1 is an arbitrary initial price offered by the first visited seller.

It remains to derive the equilibrium objects for different levels of search costs. In equilibrium, there must be a search cutoff $w^{f,*}$ such that consumers find it optimal to continue searching if and only if the first seller's match value is below $w^{f,*}$, as well as a vector of prices $(p_1^{f,*}, p_2^{f,*}, p_3^{f,*})$ that jointly solve (4.4.18), (4.4.19), and (4.4.20), given $w^{f,*} = w^f(p_1^{f,*})$. Using numerical methods, we are able to compute the joint solution for any $s \in [0, 1/8]$. Moreover, we verify that this combination of prices, together with the search cutoff $w^f(p_1)$ as defined in equation (4.4.14) and the optimal revision price function $p_3^f(p_1)$ for an arbitrary initial price p_1, satisfy the sufficient conditions for an equilibrium. We visualize the on-path objects $(p_1^{f,*}, p_2^{f,*}, p_3^{f,*}, w_f^*)$ for any $s \in [0, 1/8]$ in Figure 4.3.

The figure shows that $p_2^{f,*} > p_1^{f,*} > p_3^{f,*}$ for all $s > 0$, which aligns with the intuition we have developed in the previous sections. Contrary to some of the previous results, however, the prices $p_2^{f,*}$ and $p_1^{f,*}$ do not approach the monopoly price of $1/2$ as search frictions grow large. This is because consumers now anticipate that continuing to search leads the initially visited firm to lower its price to $p_3^{f,*}$, which creates additional incentives to search and, thus, more competition. This notion is also reflected by the fact that the search cutoff w_f^* lies firmly above $p_1^{f,*}$ even if search costs are large. That is, while in the baseline model consumers would always buy immediately from the first visited firm

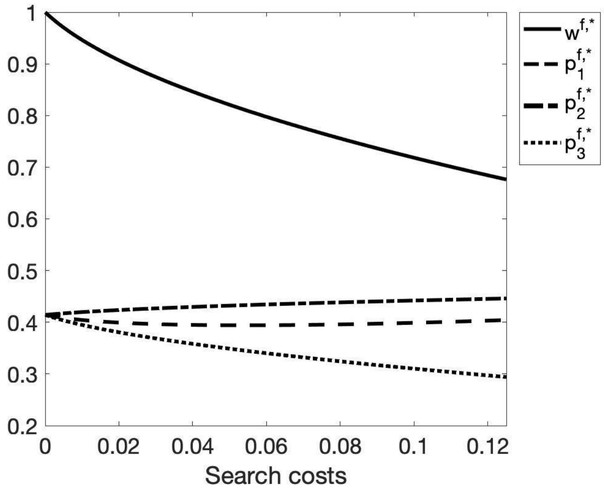

Figure 4.3: Equilibrium with exogenous information

(without continuing to search) at any price below their match value if $s \to 1/8$, they only buy immediately in this extension if the offered price is sufficiently smaller than their match value.

With all these results in hand, we now discuss how consumer surplus and total welfare are affected by the exogenous provision of search history information. Specifically, we are interested in how the outcome with exogenous search history information compares with the outcome of the game with endogenous search disclosure.

The exogenous provision of search history information increases consumer surplus. We visualize this in Figure 4.4, in which we compare the surplus in the no disclosure equilibrium to the consumer surplus under exogenous provision of search history information. In the no disclosure equilibrium, firms set the uniform price p^*, which means that consumer surplus in this equilibrium is equal to consumer surplus in the Wolinsky (1986) equilibrium.

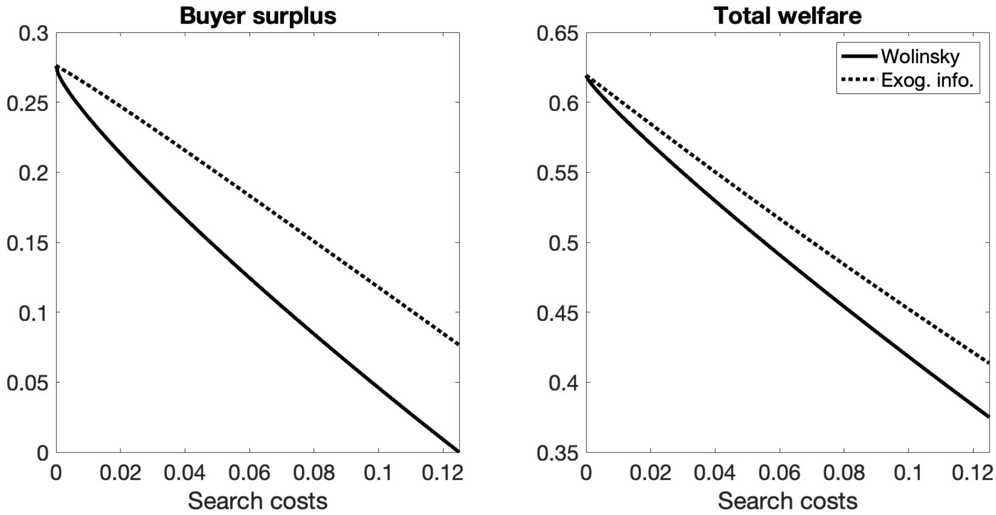

Figure 4.4: Welfare under exogenous information

Consumer surplus is higher under the exogenous provision of search history information than in the Wolinsky (1986) equilibrium. This is because the revision prices are comparatively low, which is favorable for consumers in general and also implies that even consumers with low match values make a purchase eventually. Moreover, low revision prices encourage search, which improves the average match quality of the purchased good. Both of these effects raise consumer surplus. In addition, these effects also imply that total welfare is higher when firms have exogenous access to full search history information.

Finally, recall that the no disclosure equilibrium, in which consumer welfare is the same as in the Wolinsky (1986) equilibrium, exists for all search costs under endogenous search disclosure (if price revisions are feasible). However, if search costs are sufficiently small or price revisions are impossible, the partial disclosure equilibrium exists and will be selected because it is preferred by firms. Notably, previous arguments have established that buyer surplus in the partial disclosure equilibrium is even lower than in the Wolinsky (1986) equilibrium. Thus, exogenous information provision makes buyers weakly better off compared to the outcome under endogenous search disclosure, regardless of our equilibrium selection criterion.

4.6 Conclusion

We have studied the incentives of firms to exchange information about consumers in a sequential search framework. When being visited by the buyer, a firm can notify its rival — we refer to this as *search disclosure*. Search disclosure benefits the disclosing firm if it is visited first by the buyer, but is detrimental if the buyer visits the disclosing firm second and firms are able to adjust the price they offer to a given consumer.

The possibility of price revisions is thus of central importance for the incidence of search disclosure. If revising prices is not feasible, firms will always disclose to their competitors in equilibrium. This prediction is reversed if revising prices is feasible. Then, we show that an equilibrium without disclosure is the unique pure-strategy equilibrium for a large range of search costs. Firms will only conduct search disclosure in equilibrium when search costs are sufficiently small, even though industry profits can be raised by sharing said information. Thus, the possibility of price revisions prevents price discrimination and thus weakly raises consumer welfare, even though prices are never revised in equilibrium.

An important implication of our work is that policymakers should codify an explicit right for price revisions in the markets we study and make the possibility of them common knowledge. The importance of price revisions highlights a critical benefit of firms being able to re-target visitors because re-targeting is a means to inform consumers about revised prices. Moreover, firms must not be able to commit to future prices as in Armstrong and Zhou (2016). Arguably, ensuring and announcing such a right may be easier than prohibiting communication between firms.

Another obstacle to firms being able to offer revised prices might arise if some consumers can only see the revised price at a cost. We therefore study a scenario in which a positive (and possibly large) fraction of consumers face recall costs in Appendix Section C.3. We find that the no disclosure equilibrium continues to exist for a wide range of search costs, except if search costs are small. The reason it does not exist for small search costs is that the downside of disclosing, which is the potential downward price revision by a rival, weighs less when some consumers do not see revised offers due to costly recall. By the same token, we argue that the partial disclosure equilibrium continues to exist for small search costs (with a widening range as more consumers face costly recall).[27]

Finally, we note that third parties in the search models we study could ensure access to detailed search history information for the participating firms, offering a substitute for voluntary search disclosure by firms. We show that buyers would benefit from the exogenous availability of this information if prices are revisable, and are indifferent otherwise.

[27]We limit ourselves to offering a verbal discussion and conjectures regarding the partial and full disclosure equilibrium because the analysis is not very tractable.

Appendix A

Chapter 2: Proofs

A.1 Proofs — Section 2.4.

Throughout the appendix, I use the terminology $Pr^H(v) := Pr(\tilde{v}^H|v)$ and $Pr^L(v) := Pr(\tilde{v}^L|v)$ for ease of exposition.

Proof of Proposition 1:

Part 1: Consider any equilibrium price p^k (which is offered after a signal $\tilde{v}^k$). The set $\{v \geq p^k : \hat{p}(v) > p^k, Pr(\tilde{v}^k|v) > 0\}$ must have zero measure.

Consider a symmetric pure strategy equilibrium (p^L, p^H). Suppose, for a contradiction, that the set $\{v \geq p^k : \hat{p}(v) > p^k, Pr(p^k|v) > 0\}$ has strictly positive measure for some price $p^k \in \{p^L, p^H\}$. Thus, there is a strictly positive measure of consumers with $v \geq p^k$ who receive the price p^k from all firms (since receiving p^k always triggers search because search is myopic), which is given by:

$$\int_{\{v \geq p^k : \hat{p}(v) < p^K, Pr(p^k|v) > 0\}} \left[Pr(p^k|v)\right]^N dv > 0 \tag{A.1.1}$$

When setting the price p^k, some firm will only make the sale to these consumers with probability below 1, no matter of how ties are broken. When marginally undercutting this price, this firm will sell to all these consumers, representing an upward jump in this component of demand. All other components of demand can only weakly increase after a decrease in price: Any such decrease in price will (i) reduce the search incentives of consumers and (ii) allow more consumers to buy. Thus, marginally undercutting p^k will imply a discontinuous upward jump of total demand, entailing a profitable deviation.

Part 2: In equilibrium, $p^L < p^H$ must hold.

Suppose that $p^L = p^H := p^*$ holds in equilibrium. No consumer will search on the equilibrium path, i.e. the equilibrium profits are given by $\Pi^M(p^*; \tilde{v}^L)$ and $\Pi^M(p^*; \tilde{v}^H)$, respectively. Because $c > 0$. no consumer will leave to search for prices in an open ball above p^*. Thus, $p^* \geq p^H$ must hold. But then, there is a profitable downward deviation after the low signal, a contradiction.

Suppose that $p^H < p^L$ holds in equilibrium. If there is no search on the equilibrium path, it is evident that it is either profitable to deviate from p^L to p^H when observing $\tilde{v}^L$ or vice versa. Suppose there is search on the equilibrium path. Then, no consumer would find it optimal to search in an open ball around p^H. Searchers who arrive at a firm after searching and generate $\tilde{v}^H$ put upward pressure on this price. Hence, $p^H \geq p^{H,M}$ must hold. By the result of part 1, any firm will make profits equal to $\Pi^M(p^L; \tilde{v}^L)$ when observing $\tilde{v}^L$. No consumer would leave to search when this firm deviates to $p^{L,M} < p^H$, which means that this deviation would yield low signal profits equal to $\Pi^M(p^{L,M}; \tilde{v}^L)$. Hence, the deviation is strictly profitable, a contradiction.

■

Proof of Lemma 1:

Part 1: In any symmetric pure-strategy equilibrium, $p^H = p^L + c/\alpha$ or $p^H = 0.5$ must hold.

Recall that $\alpha < 1$. We have to consider two different subcases, namely possible equilibria in which $p^L + c/\alpha < 0.5$ and possible equilibria in which $p^L + c/\alpha \geq 0.5$. Before moving forward, note that $c/\alpha + p^L < c/(1-\alpha) + p^L$ holds because $\alpha > 0.5$.

Subcase 1: Suppose that $c/\alpha + p^L \leq 0.5$. There are two possible equilibrium high signal prices, namely $p^H = p^L + c/\alpha$ and $p^H = 0.5$. No other p^H can be supported in equilibrium.

Suppose $p^H < c/\alpha + p^L$. No consumer will search at p^H and the cutoff price of consumers with $v < 0.5$ is $\hat{p}(v) = \alpha p^L + (1-\alpha)p^H + c > p^H$. Thus, profits are equal to monopoly profits in the price interval $p_j \in [p^H, \alpha p^L + (1-\alpha)p^H + c]$. An upward deviation from p^H is profitable.

Suppose $p^H \in (c/\alpha + p^L, 0.5)$. Then, consumers with $v \in (p^H, 0.5)$ search at p^H, a contradiction.

Suppose $p^H > 0.5$. All consumers with $v \geq p^H$ have identical incentives to search. Thus, there are just two possibilities. If consumers with $v > p^H$ find it optimal to search after p^H, there is a contradiction to proposition 1. Thus, consumers with $v > p^H$ must find it optimal to refrain from search when being offered p^H. Then, there is a profitable downward deviation to $p^{H,M}$, because consumers with $v \in [0.5, p^H]$ have lower incentives to search than consumers with $v > p^H$.

Subcase 2: Suppose that $c/\alpha + p^L > 0.5$. In equilibrium, $p^H = 0.5$ must hold.

Suppose $p^H < 0.5$. The cutoff price of all consumers will be above $p^L + c/\alpha > p^H$. Thus, you would have a profitable marginal upward deviation after the high signal.

Suppose alternatively that $p^H \in (0.5, 1]$. This cannot be an equilibrium. Either, consumers with $v > p^H$ search on-path (a contradiction to proposition 1) or there is a profitable downward deviation to $p^{H,M}$, because consumers with $v \in [0.5, p^H]$ have lower incentives to search than consumers with $v > p^H$.

Part 2: If there is search on the equilibrium path, $p^H = 0.5$ must hold.

If $p^H = p^L + c/\alpha$, there will be no on-path search. To see this, consider an equilibrium with an arbitrary p^L and $p^H = p^L + c/\alpha$. Note that $p^H = p^L + c/\alpha < 0.5$ must hold by previous results. Consumers with $v < p^H$ will strictly prefer to not search at $p^H = c/\alpha + p^L < 0.5$. Consumers with $v \geq p^H$ must not search on the equilibrium path by previous logic.

Part 3: In an equilibrium with on-path search, the ordering $p^L + c/\alpha < 0.5 \leq p^L + c/(1-\alpha)$ must hold.

If $p^H = 0.5 \leq c/\alpha + p^L$, no consumer will search on path. If $0.5 > c/(1-\alpha) + p^L$, consumers with $v > 0.5$ would prefer to continue search when offered p^H, which breaks the equilibrium.

Part 4: Calculating the sequentially rational search strategy of consumers:

We consider different intervals of consumer valuations.

(i) $v \leq p^L$: These consumers would never search, i.e. $\hat{p}(v) = \infty$ holds for them.

(ii): $v \in (p^L, 0.5)$: Search is strictly optimal for prices $p_j \in [p^L, p^H]$ if and only if:

$$\alpha(v-p^L)+(1-\alpha)\max\{v-p_j, 0\}-c > \max\{v-p_j, 0\} \iff \alpha(v-p^L)-c > \alpha\max\{v-p_j, 0\}$$

Since the RHS is positive, a necessary condition for search to occur at these prices is $\alpha[v - p^L] - c > 0$. If this is true, the indifference price is pinned down by:

$$\alpha(v - p^L) - c = \alpha(v - p_j) \iff \hat{p}(v) = p^L + c/\alpha < p^H \tag{A.1.2}$$

Our assumption implies that there exist $v \in [p^L + c/\alpha, 0.5]$ for which this necessary condition is fulfilled and the cutoff price is hence $\hat{p}(v) = p^L + c/\alpha$. For all consumers with $v \leq p^L + c/\alpha$, search is never optimal (not even for $p_j \geq p^H$) since their gains of search are $\alpha(v - p^L) - c$.

(iii) $v > p^H = 0.5$: Such a consumer will find it strictly optimal to search for prices $p_j \in [p^L, p^H]$ if and only if:

$$(1-\alpha)(v - p^L) + \alpha(v - p_j) - c > (v - p_j)$$

Supposing that the price cutoff is below p^H, it will be: $\hat{p}(v) = c/(1-\alpha) + p^L$. Our assumption was that $0.5 = p^H < \frac{c}{1-\alpha} + p^L$, which means that this cannot be the correct search cutoff, i.e. these consumers will never search at $p_j \leq p^H$.

Part 5: Closed-form solution for p^L.

Consider an equilibrium (p^L, p^H) with search, in which $p^L + c/\alpha < 0.5 \leq c/(1-\alpha) + p^L$ must hold. For prices $p_j \in (0, c/\alpha + p^L]$, the low signal objective function is:

$$\Pi^C(p_j; \tilde{v}^L) = p_j \int_{p_j}^{1} (1/N) Pr(\tilde{v}^L|v) dv + p_j \underbrace{\sum_{j=2}^{N} \left[\int_{c/\alpha + p^L}^{0.5} (1/N) Pr(\tilde{v}^L|v) Pr(\tilde{v}^H|v)^{j-1} dv \right]}_{S^L(\alpha, c; p^L)} \tag{A.1.3}$$

To evaluate this, note that:

$$S^L(\alpha, c; p^L) = \left(\frac{(1-\alpha)\big(1 - (1-\alpha)^{N-1}\big)}{1 - (1-\alpha)} \right) (1/N) \Big[(0.5 - p^L)\alpha - c \Big] \tag{A.1.4}$$

An equilibrium p^L must thus satisfy the following:

$$\left. \frac{\partial \Pi^C(p_j; \tilde{v}^L)}{\partial p_j} \right|_{p^L} = 0 \iff$$

$$0.5(1/N) - 2\alpha(1/N)p^L + S(\alpha, c; p^L) = 0 \iff p^L = \frac{1}{4\alpha} + \frac{S(\alpha, c; p^L)}{(2/N)\alpha} \tag{A.1.5}$$

Note strict concavity of the objective function for $p_j \in [0, p^L + c/\alpha]$. Solving for p^L yields:

$$p^L = \frac{\alpha + 2(1-\alpha)\big(1 - (1-\alpha)^{N-1}\big)(0.5\alpha - c)}{4\alpha^2 + 2\alpha(1-\alpha)\big(1 - (1-\alpha)^{N-1}\big)} \tag{A.1.6}$$

Note that $\frac{\partial p^L}{\partial c} > -1$, because $\alpha > 1-\alpha$ and $\left(1-(1-\alpha)^{N-1}\right) < 1$.

■

Proof of Proposition 2:

Previous arguments have established that there just exists a single candidate for an equilibrium with search on the equilibrium path. In this equilibrium, $p^H = 0.5$ and $p^L = p^{L,S}$ must hold. The latter is uniquely determined because $\Pi^M(p_j;\tilde{v}^L)$ is strictly concave for any $p_j < 0.5$. I refer to this equilibrium as the search equilibrium.

Now consider possible equilibria without on-path search. In any such equilibrium, $p^L = p^{L,M}$ must hold, because competitive low signal profits are equal to $\Pi^M(p_j;\tilde{v}^L)$ for prices in an open ball around the equilibrium p^L. By previous arguments, p^H must either be equal to $p^L + c/\alpha$ or 0.5 in a symmetric pure-strategy equilibrium. I refer to the equilibrium candidate $(p^L, p^H) = (p^{L,M}, p^{L,M} + c/\alpha)$ as the search deterrence equilibrium. I label the equilibrium candidate $(p^L, p^H) = (p^{L,M}, p^{H,M})$ the monopoly price equilibrium.

■

Proof of Proposition 3:

Part 1: Establishing existence conditions for the monopoly price equilibrium:

The monopoly price equilibrium exists if and only if $0.5 \leq p^{L,M} + c/\alpha$. Under this condition, the cutoff prices of all consumers are above $p^{H,M} = 0.5$ when the firms' prices are $(p^{L,M}, p^{H,M})$. Then, there are no deviations from the equilibrium prices, as competitive profits are below monopoly profits everywhere (because there is no search on the equilibrum path). If said condition is violated, there exists a positive measure of agents with $v \in (p^L + c/\alpha, 0.5)$ that search when receiving $p^{H,M} = 0.5$ when the firms' prices are $(p^{L,M}, p^{H,M})$, which makes it suboptimal for firms to set $p^{L,M}$ after $\tilde{v}^L$, implying that the monopoly price equilibrium does not exist.

Part 2: Establishing existence conditions for the search deterrence equilibrium:

In this equilibrium, there is no search on-path, which implies that the competitive profit functions are bounded from above by $\Pi^M(p_j;\tilde{v}^k)$. Since $p^{L,M}$ maximizes $\Pi^M(p_j;\tilde{v}^L)$, there are no deviations from the equilibrium $p^L = p^{L,M}$.

Consider deviations from $p^H < 0.5$. There will not be any profitable deviations to $p_j < p^H$, since profits are equal to $\Pi^M(p_j;\tilde{v}^H)$ when $p_j \in [0, p^H]$ and $\Pi^M(p_j;\tilde{v}^H)$ is strictly increasing in this interval. Now consider a deviation to $p_j = 0.5$. To ensure that this is not profitable, consumers with $v \geq 0.5$ will move on to search when offered the out-of-equilibrium price $p_j = 0.5$. This requires:

$$(1-\alpha)(v-p^L) + \alpha(v-p^H) - c > (v-0.5) \iff 0.5 > (1-\alpha)p^L + \alpha p^H + c$$

Thus, $c < 0.5(0.5 - p^{L,M})$ must hold. Otherwise, a deviation to $p_j = 0.5$ would yield $\Pi^M(0.5, \tilde{v}^H)$ and would be profitable since $\Pi^M(p_j, \tilde{v}^H)$ is strictly increasing in $[0, 0.5]$.

The cutoff price of consumers with $v \in [0.5, 1]$ is $\hat{p}(v) = (1-\alpha)p^L + \alpha p^H + c \in (p^H, 0.5)$. Thus, no deviations to prices $p_j \in ((1-\alpha)p^L + \alpha p^H + c, 1)$ will be profitable, since profits will be zero as all consumers move on to search and never return.

Now consider price deviations in the interval $p_j \in (p^H, (1-\alpha)p^L + \alpha p^H + c)$.[1] In this interval of prices, all consumers with $v \leq 0.5$ that can search (i.e. $v \in (p^L + c/\alpha, 0.5)$) will search and will not return. All consumers with $v \geq 0.5$ do not search at these prices and buy directly because their cutoff price is below 0.5 by assumption. Thus, profits $\Pi^C(p_j; \tilde{v}^H)$ in the price interval $p_j \in (p^H, (1-\alpha)p^L + \alpha p^H + c)$ are increasing and satisfy:

$$\Pi^C(p_j; \tilde{v}^H) = p_j \int_{0.5}^{1} (1/N) Pr(\tilde{v}^H | v) dv \tag{A.1.7}$$

Thus, the most profitable deviation is to $p_j = (1-\alpha)p^L + \alpha p^H + c < 0.5$, at which:

$$\Pi^C((1-\alpha)p^L + \alpha p^H + c; \tilde{v}^H) = \big((1-\alpha)p^L + \alpha p^H + c\big)(0.5/N)\alpha \tag{A.1.8}$$

Note that $(1-\alpha)p^L + \alpha p^H + c = (1-\alpha)p^L + \alpha\big(p^L + c/\alpha\big) + c = p^L + 2c$. Equilibrium profits are:

$$\Pi^C(c/\alpha + p^L; \tilde{v}^H) = \underbrace{\big(c/\alpha + p^L\big)\big[(1/N)(1-\alpha)\big(0.5 - c/\alpha - p^L\big) + (0.5/N)\alpha\big]}_{=\Pi^M(c/\alpha + p^L; \tilde{v}^H)} \tag{A.1.9}$$

Thus, a necessary condition for equilibrium existence (which is also sufficient given that said cutoff price is below 0.5) is $\Pi^M(c/\alpha + p^L; \tilde{v}^H) \geq \Pi^C((1-\alpha)p^L + \alpha p^H + c; \tilde{v}^H)$, i.e.:

$$\big(c/\alpha + p^L\big)\big[(1-\alpha)\big(0.5 - c/\alpha - p^L\big) + 0.5\alpha\big] \geq 0.5\alpha\big((1-\alpha)p^L + \alpha p^H + c\big) \tag{A.1.10}$$

Part 3: Existence conditions for the search equilibrium

Part 3a: The search equilibrium exists if and only if $p^{L,S} + c/\alpha < 0.5 \leq p^{L,S} + c/(1-\alpha)$ and $\Pi^C(p^{L,S} + c/\alpha; \tilde{v}^H) \geq \Pi^M(0.5; \tilde{v}^H)$

If the ordering or the no-deviation condition fail, the equilibrium cannot exist by previous arguments.

[1] This interval is non-degenerate since $p^H = \alpha p^L + (1-\alpha)p^H + c$ by construction and $p^L < p^H$.

Suppose both conditions are true. If firms set the prices $(p^{L,S}, p^{H,M})$, the sequentially rational search strategy of consumers is:

$$\hat{p}(v) = \begin{cases} \infty, & \text{if } v < p^{L,S} + c/\alpha \\ p^{L,S} + c/\alpha, & \text{if } v \in [p^{L,S} + c/\alpha, 0.5) \\ \infty, & \text{if } v \in [0.5, \alpha p^H + (1-\alpha)p^L + c) \\ \alpha p^H + (1-\alpha)p^L + c, & \text{if } v \in [\alpha p^H + (1-\alpha)p^L + c, 1] \end{cases} \tag{A.1.11}$$

I will show two results in the following: Given this search strategy, firms will have no profitable deviations from p^L. The most profitable deviation from p^H is to $p^L + c/\alpha$, which is not profitable by assumption. Thus, we have an equilibrium.

<u>Part 3b:</u> There are no profitable deviations from p^L

Consider first prices $p_j \in [0, c/\alpha + p^L]$. Note that the objective function is strictly concave in this price range. Thus, there will be no deviations in this interval.

Secondly, consider $p_j \in (c/\alpha + p^L, 0.5]$. For all these prices, $\Pi^C(p_j; \tilde{v}^k) < \Pi^M(p_j; \tilde{v}^k)$ holds for both $\tilde{v}^k$ because:

$$p_j\left[\int_{0.5}^{1} \frac{1}{N} Pr(\tilde{v}^k|v)dv + \int_{p_j}^{0.5} \frac{1}{N} Pr(\tilde{v}^H|v)^{N-1} Pr(\tilde{v}^k|v)dv + \sum_{f=2}^{N} \int_{p_j}^{0.5} \frac{1}{N} Pr(\tilde{v}^k|v) Pr(\tilde{v}^H|v)^{N-1}dv\right]$$

$$<$$

$$p_j \int_{p_j}^{0.5} (1/N) Pr(\tilde{v}^k|v)dv + p_j \int_{0.5}^{1} (1/N) Pr(\tilde{v}^k|v)dv \iff N\big(1-\alpha\big)^{N-1} < 1 \tag{A.1.12}$$

This inequality holds for all relevant N since $1 - \alpha < 0.5$.

For $p_j \geq 0.5$, $\Pi^C(p_j; \tilde{v}^k) \leq \Pi^M(p_j; \tilde{v}^k)$ holds since the sale will not be made to any searchers.

Thus, $\Pi^C(p_j; \tilde{v}^k) \leq \Pi^M(p_j; \tilde{v}^k)$ holds for any $p_j \in (c/\alpha + p^L, 1]$ and any $\tilde{v}^k$. By contrast, note that $\Pi^C(p_j; \tilde{v}^L) > \Pi^M(p_j; \tilde{v}^L)$ holds for all prices $p_j \leq c/\alpha + p^L$. Moreover, note that $p^{L,S} > p^{L,M}$ because:

$$p^{L,S} = \frac{1}{4\alpha} + \frac{M(\alpha, p^{L,S})}{(2/N)\alpha} > \frac{1}{4\alpha} = p^{L,M} \tag{A.1.13}$$

Thus, $p^{L,M} < p^{L,S} < p^{L,S} + c/\alpha$ holds. This and the fact that $p^{L,S}$ maximizes $\Pi^C(p_j; \tilde{v}^L)$ on $p_j \in (0, c/\alpha + p^{L,S}]$ yields $\Pi^C(p^{L,S}; \tilde{v}^L) \geq \Pi^C(p^{L,M}; \tilde{v}^L) > \Pi^M(p^{L,M}; \tilde{v}^L)$. Since $p^{L,M}$ maximizes $\Pi^M(p_j; \tilde{v}^L)$ over the entire domain, we have the following for all prices $p_j \in$

$(c/\alpha + p^{L,S}, 1)$:

$$\Pi^C(p^{L,S}; \tilde{v}^L) > \Pi^M(p^{L,M}; \tilde{v}^L) \geq \Pi^M(p_j; \tilde{v}^L) \geq \Pi^C(p_j; \tilde{v}^L) \tag{A.1.14}$$

This shows there are no profitable deviations from $p^{L,S}$.

Part 3c: The most profitable deviation from the equilibrium p^H is to $p^L + c/\alpha$.

As argued before, $\Pi^C(p_j|\tilde{v}^H) \leq \Pi^M(p_j|\tilde{v}^H)$ holds in the price interval $p_j \in (c/\alpha + p^L, 1)$. Since $p^H = p^{H,M}$ maximizes $\Pi^M(p_j; \tilde{v}^H)$, there are no profitable deviations in this region.

Thus, consider a deviation in $p_j \in (0, c/\alpha + p^L]$, where $\Pi^C(p_j; \tilde{v}^H)$ is given by:

$$\Pi^C(p_j; \tilde{v}^H) = \Pi^M(p_j; \tilde{v}^H) + p_j S^H(c, \alpha; p^L) \tag{A.1.15}$$

Recall that $\Pi^M(p_j; \tilde{v}^H)$ is strictly rising for all prices $p_j \leq 0.5$. Thus, $p_j = c/\alpha + p^{L,S}$ is the most profitable deviation. Equilibrium profits (which have to be higher) are $\Pi^M(0.5; \tilde{v}^H) = (1/4N)\alpha$.

■

Proof of Lemma 2:

Part 1: The probability that a firm plays a price strictly above 0.5 is 0:

Define $[p^{min}, p^{max}]$ as the convex hull of the support of prices firms draw after either signal. If p^{max} is weakly below 0.5, we are done.

Suppose instead that $p^{max} > 0.5$. Label the two signals as $\tilde{v}^{max}$ and $\tilde{v}^{min}$. There must exist a signal $\tilde{v}^{max}$ such that the supremum of the support of the price distribution offered after this signal lies above 0.5. Call the associated equilibrium price distributions F^{max} and F^{min}. The infima (suprema) of the support of the two distributions are $\underline{p}^{max}$ ($\bar{p}^{max}$) and $\underline{p}^{min}$ ($\bar{p}^{min}$) respectively, where $\bar{p}^{max} > 0.5$. I explicitly allow these distributions to have gaps and atoms. Upon receiving $\bar{p}^{max}$, all consumers with $v \geq \bar{p}^{max}$ have the same gains of search, namely:

$$\bar{p}^{max} - Pr(\tilde{v}^{min}|v) \int_{\underline{p}^{min}}^{\bar{p}^{min}} p dF^{min}(p) - Pr(\tilde{v}^{max}|v) \int_{\underline{p}^{max}}^{\bar{p}^{max}} p dF^{max}(p) - c \tag{A.1.16}$$

To see this, note that both $Pr(\tilde{v}^{min}|v)$ and $Pr(\tilde{v}^{max}|v)$ will be constant, given that these consumers all have $v \geq \bar{p}^{max} > 0.5$.

Suppose that consumers with $v \geq \bar{p}^{max}$ all strictly prefer to search when offered $\bar{p}^{max}$. If $\bar{p}^{max}$ is played with zero probability after both signals, they never return when being offered $\bar{p}^{max}$, thus implying that profits from setting this price are zero, a contradiction. If $\bar{p}^{max}$ is played with positive probability after some signal, there are undercutting motives from this price, breaking the equilibrium.

Thus, consumers with $v \geq \bar{p}^{max}$ must weakly prefer to not search for any $p_j \leq \bar{p}^{max}$. Consumers with $v \in [0.5, \bar{p}^{max})$ have lower search incentives and would also not search for prices $p_j \leq \bar{p}^{max}$, since their gains of search at $\bar{p}^{max}$ are:

$$Pr(\tilde{v}^{min}|v)\int_{\underline{p}^{min}}^{\min\{v,\bar{p}^{min}\}}(v-p)dF^{min}(p)+Pr(\tilde{v}^{max}|v)\int_{\underline{p}^{max}}^{\min\{v,\bar{p}^{max}\}}(v-p)dF^{max}(p)-c<0$$

As a result, no consumers with $v \in [0.5, 1]$ would arrive at any firm after searching and no such consumers would leave a firm to search at the prices $p_j \in [0.5, \bar{p}^{max}]$. Thus, the profits a firm would make when setting any price $p_j \in [0.5, \bar{p}^{max}]$ would be equal to $\Pi^M(p_j|\tilde{v}^{\max})$, which are strictly decreasing on $p_j \in [0.5, \bar{p}^{max}]$. This holds true, no matter whether $\tilde{v}^{max} = \tilde{v}^L$ or $\tilde{v}^{max} = \tilde{v}^H$.

If there are two or more different prices $p_j \in [0.5, \bar{p}^{max}]$ in the support of F^{max}, it would violate the mixing indifference condition. Thus, suppose that it is just $\bar{p}^{max}$ that is in the support of F^{max}, which means that this price would have to be played with positive probability. But then, there there is a profitable deviation towards 0.5, a contradiction.

By analogous arguments, $\bar{p}^{min} \leq 0.5$ must also hold. This implies the desired result.

Part 2: The lowest equilibrium price p^{min} must be played after the low signal and must be played with probability 1.

I label a signal for which the infimum of the support of prices is equal to p^{min} as $\tilde{v}^{min}$, with associated price distribution F^{min}. Suppose p^{min} is played with probability below 1 after this signal. For prices in $[p^{min}, p^{min} + c)$, no consumer will search. To see this, note that the gains of search at any initial price p_j are bounded from above by $\big[\max\{v - p^{min}\} - \max\{v - p_j, 0\}\big] - s$.

Similarly, no consumer with $v \in [p^{min}, p^{min} + c)$ would ever search on the equilibrium path. Thus, all consumers that arrive after search must have a valuation $v \geq p^{min} + c$ and must have received prices $p_j \geq p^{min} + c$ previously. When setting a price in the interval $[p^{min}, p^{min} + c]$, the sale will thus be made to all consumers who arrive after searching and no consumer leaves to search.

Define the mass of consumers who arrive at a firm after searching and generate the signal $\tilde{v}^{min}$ in equilibrium as $S^{min}(F^{min}, F^{max})$. Thus, the profit function for prices in $p_j \in [0, p^{min} + c]$ is $\Pi^C(p_j; \tilde{v}^{min}) = \Pi^M(p_j; \tilde{v}^{min}) + p_j S^{min}(F^{min}, F^{max})$.

This implies that p^{min} must be played after $\tilde{v}^L$. Suppose, for a contradiction, that $\tilde{v}^{min} = \tilde{v}^H$. We know, from part 1, that $p^{min} < 0.5$, and that prices in any open ball around this lowest price must be played with positive probability (else, our support would have been incorrectly defined). However, profits in a small enough open ball will be strictly increasing, which implies a contradiction.

Thus, p^{min} is played after the low signal. The low signal profits for prices around this are equal to $\Pi^C(p_j;\tilde{v}^L) = \Pi^M(p_j;\tilde{v}^L) + p_j S^L(F^L, F^H)$. By previous logic, prices in an open ball around p^{min} need to be played with positive probability after the low signal.

Since the associated profits are strictly concave, the price p^{min} needs to be played with positive probability after the low signal. It must hence satisfy a first-order condition $\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\Big|_{p^L} + S^L(F^L, F^H)$. Note that $\Pi^M(p_j;\tilde{v}^L) + p_j S^L(F^L, F^H)$ is strictly concave on $[0, 0.5]$. Thus, we have $\Pi^M(p^L;\tilde{v}^L) + p^L S^L(F^L, F^H) > \Pi^M(p_j;\tilde{v}^L) + p_j S^L(F^L, F^H)$ for any $p_j \in [0, 0.5] \setminus p^L$. This means that no other price below 0.5 can be in the support of F^L. The probability that a price above 0.5 is played must be zero. Hence, p^L must be played with probability 1 after the corresponding signal.

Part 3: Consumers with $v > 0.5$ cannot search on the equilibrium path:

We know that $\bar{p}^H = p^{max} \leq 0.5$. Suppose, for a contradiction, that consumers with $v \geq 0.5$ search on the equilibrium path. Then, they must search at $\bar{p}^H$. Since they have a higher probability of generating $\tilde{v}^L$, consumers with $v \in [\bar{p}^H, 0.5)$ would thus also search at $\bar{p}^H$. Thus, all consumers with $v > \bar{p}^H$ would search at $\bar{p}^H$. If $\bar{p}^H$ is played with zero probability, all consumers that leave to search never return. Thus, profits are zero, a contradiction. If $\bar{p}^H$ is played with positive probability, there will be undercutting motives, a contradiction.

■

Proof of Proposition 4:

Define the convex hull of the support of the high signal price distribution as $[\underline{p}^H, \bar{p}^H]$.

Part 1: In a MSE, $p^L + c/\alpha \leq \underline{p}^H$ must hold.

Previous arguments imply that $\bar{p}^H \leq 0.5$ must hold. Suppose, for a contradiction, that $\underline{p}^H < p^L + c/\alpha$. For prices in an open ball above $\underline{p}^H$, no consumer will leave to search. All consumers who arrive after searching buy. Recall that high signal monopoly profits are strictly rising on $p_j < 0.5$. Thus, high signal profits will be strictly increasing in the open ball around $\underline{p}^H$, which is a contradiction because prices in this open ball must be offered with positive probability.

Note that any price strictly above $p^L + c/\alpha$ will trigger search by all consumers with $v \in (p^L + c/\alpha, 0.5)$. The price $p^L c/\alpha$ will not trigger search (consumers are indifferent).

Part 2: Suppose $F^H(p^L + c/\alpha) = 0$. Then, $p^L + c/\alpha < \underline{p}^H$ must hold.

In general, $p^L + c/\alpha \leq \underline{p}^H < 0.5$ must hold in any MSE.

To understand the result, further note that the competitive profit function in any MSE will jump downwards at $p^L + c/\alpha$. To see this, note that the high signal profit function is the following for any $p_j \leq p^L + c/\alpha$:

$$\Pi^C(p_j; \tilde{v}^H) = p_j \Bigg[\int_{p_j}^{0.5} \frac{1}{N} Pr(\tilde{v}^H|v)dv + \int_{0.5}^{1} \frac{1}{N} Pr(\tilde{v}^H|v)dv \Bigg] +$$

$$p_j \sum_{j=2}^{N} \Bigg[\int_{c/\alpha+p^L}^{0.5} \frac{1}{N} Pr(\tilde{v}^H|v) \big[Pr(\tilde{v}^H|v)(1 - F^H(p^L + c/\alpha))\big]^{j-1} dv \Bigg] \qquad \text{(A.1.17)}$$

By contrast, high signal profits are the following for any $p_j \in (p^L + c/\alpha, \bar{p}^H]$:

$$\Pi^C(p_j; \tilde{v}^H) = p_j \Bigg[\int_{p_j}^{0.5} \frac{1}{N} Pr(\tilde{v}^H|v) \big[Pr(\tilde{v}^H|v)(1 - F^H(p_j))\big]^{N-1} dv + \int_{0.5}^{1} \frac{1}{N} Pr(\tilde{v}^H|v)dv \Bigg] +$$

$$p_j \sum_{j=2}^{N} \Bigg[\int_{p_j}^{0.5} \frac{1}{N} Pr(\tilde{v}^H|v) \big[Pr(\tilde{v}^H|v)(1 - F^H(p_j))\big]^{N-1} dv \Bigg] \qquad \text{(A.1.18)}$$

To see why this holds true, note that all consumers with $v \in (p^L + c/\alpha, 0.5)$ will search when offered any high signal price except $p^L + c/\alpha$. All signal probabilities are interior. Thus, we have $\lim_{p_j \uparrow p^L + c/\alpha} \Pi^C(p_j; \tilde{v}^H) > \lim_{p_j \downarrow p^L + c/\alpha} \Pi^C(p_j; \tilde{v}^H)$

Now suppose, for a contradiction, that $\underline{p}^H = p^L + c/\alpha$, but said price is played with zero probability. Then, for any open ball above $\underline{p}^H$, the probability that a price in this open ball is played must be strictly positive. But when setting the open ball small enough, a deviation to $p^L + c/\alpha$ would yield profits that are strictly higher than those for any price in the open ball, a contradiction.

Part 3: Suppose $F^H(p^L + c/\alpha) = 0$. Then, $\bar{p}^H < 0.5$ must hold.

Suppose $\bar{p}^H = 0.5$. In order to have an MSE, we need to have $\underline{p}^H < \bar{p}^H$. The price $\underline{p}^H$ must satisfy the following mixing indifference condition:

$$\underline{p}^H \big[(0.5 - \underline{p}^H)[1 - \alpha]^N + 0.5(1/N)\alpha\big] = 0.5(1/N)\alpha\bar{p}^H \qquad \text{(A.1.19)}$$

But if $\bar{p}^H = 0.5$, the only price $\underline{p}^H$ that can satisfy this requirement is $\underline{p}^H = 0.5$, a contradiction.

Part 4: Suppose $F^H(p^L + c/\alpha) = 0$. Then, the distribution $F^H(p_j)$ must be atomless and gapless on $[\underline{p}^H, \bar{p}^H]$.

It was established that $p^L + c/\alpha < \underline{p}^H$. This means that any price p_j in the support of F^H triggers search by all consumers with $v \in [p_j, 0.5]$.

Suppose, for a contradiction, that some price $p^* \in [\underline{p}^H, \bar{p}^H]$ is played with positive probability. Because $\bar{p}^H < 0.5$ must hold, there is a positive measure of consumers with $v \in [p^*, 0.5]$ who receive this price at all firms. This creates undercutting motives, a contradiction.

Suppose, for a contradiction, that there is a gap between p_1 and p_2 for $p_1, p_2 \in [\underline{p}^H, \bar{p}^H]$. Formally, this means that $p_1 < p_2$ and $F^H(p_1) = F^H(p_2)$. At both prices, the probability of beating your rival is hence $(1 - F^H(p_2))$ when generating the high signal. Thus, the profits of setting any price $p_j \in [p_1, p_2]$ are:

$$\Pi^H(p_j; \tilde{v}^H) = p_j \Bigg[\sum_{j=1}^{N} \int_{p_j}^{0.5} (1/N)[Pr(\tilde{v}^H|v)]^N (1 - F(p_2))^{N-1} dv + \int_{0.5}^{1} (1/N) Pr(\tilde{v}^H|v) dv \Bigg] \tag{A.1.20}$$

The derivatives of this w.r.t the price are:

$$\frac{\partial \Pi^H(p_j; \tilde{v}^H)}{\partial p_j} = \Bigg[\int_{p_j}^{0.5} [Pr(\tilde{v}^H|v)]^N (1 - F(p_2))^{N-1} dv + \int_{0.5}^{1} (1/N) Pr^H(v) dv \Bigg] - p_j [Pr(\tilde{v}^H|p_j)]^N (1 - F(p_2))^{N-1} \tag{A.1.21}$$

$$\frac{\partial^2 \Pi^H(p_j; \tilde{v}^H)}{\partial p_j^2} = -2[Pr(\tilde{v}^H|p_j)]^N (1 - F(p_2))^{N-1} - p_j \frac{\partial Pr^H(p_j)}{\partial p_j} N[(1 - F(p_2)) Pr(\tilde{v}^H|p_j)]^{N-1} \tag{A.1.22}$$

Thus, profits are strictly concave and differentiable on this interval. However, we know that a mixing indifference condition must hold at p_1 and p_2. If profits are weakly decreasing at p_1, they will be strictly lower at p_2, a contradiction. If they are strictly increasing, there would be a profitable deviation into the interval, a contradiction. Hence, there cannot be a gap in the distribution of prices.

Part 5: Suppose $F^H(p^L + c/\alpha) > 0$. There must exist a $\tilde{p}^H > p^L + c/\alpha$ s.t. no price in $(p^L + c/\alpha, \tilde{p}^H)$ will be offered. The distribution F^H must be gapless on $[\tilde{p}^H, \bar{p}^H]$.

This holds by the same logic as established in part 2. It must hold that $\underline{p}^H = p^L + c/\alpha < \bar{p}^H \leq 0.5$. At $p_j = p^L + c/\alpha$, the profit function jumps downward. Hence, prices just above this price must be played with zero probability. Else, there would be a profitable deviation to $p^L + c/\alpha$.

The result w.r.t to the gaplessness holds by logic analogous to the one presented in part 4, because all prices in $[\tilde{p}^H, \bar{p}^H]$ trigger search by all consumers with $v \in (p^L + c/\alpha, 0.5]$.

■

Existence conditions and further details: Mixed-strategy equilibria

Part 1: The mixed search equilibrium.

Firms set the price $p^{L,S}$ when observing $\tilde{v}^L$ and draw prices from $F^H(p_j)$ when observing the high signal, with $p^L + c/\alpha < \underline{p}^H < \bar{p}^H < 0.5$.

Previous arguments have established that all consumers with $v \in [p^L + c/\alpha, 0.5]$ will find it optimal to search for any price that is offered after the high signal. Consumers with $v < p^L + c/\alpha$ will never find it optimal to search. Consumers with $v > 0.5$ are indifferent between searching and not searching at $\bar{p}^H$, so will optimally also not search on the equilibrium path.

The price $p^{L,S}$ maximizes low signal profits on $[0, 0.5]$. It yields profits above $\Pi^M(p^{L,M}$ $;\tilde{v}^L)$. For prices $p_j \geq 0.5$, profits are below monopoly profits. Thus, there are no profitable deviations from $p^{L,S}$. The most profitable deviation from the high signal prices is to $p_j = p^{L,S} + c/\alpha$. To see this, consider the following regions of possible price deviations: (i) $[0, p^L + c/\alpha]$, (ii) $[p^L + c/\alpha, \underline{p}^H]$, (iii) $[\bar{p}^H, 1]$. I go through them now:

(i) $p_j \in [0, p^L + c/\alpha]$:

No consumer will search for these prices and the sale is made to all consumers that arrive after search. Thus, the most profitable deviation in this interval would be to $p_j = p^L + c/\alpha < 0.5$. The profits $\Pi^C(p^L + c/\alpha; \tilde{v}^H)$ at this deviation are:

$$0.5(1/N)(p^L + c/\alpha) - (1/N)(1-\alpha)(p^L + c/\alpha)^2 +$$

$$(p^L + c/\alpha)\big(0.5 - (p^L + c/\alpha)\big)(1/N)\left(\frac{(1-\alpha)^2\big(1-(1-\alpha)^{N-1}\big)}{1-(1-\alpha)}\right) \tag{A.1.23}$$

In order for the equilibrium to exist, these need to be below the equilibrium high signal profits, namely $0.5(1/N)\alpha\bar{p}^H$.

(ii) $p_j \in [p^L + c/\alpha, \underline{p}^H]$:

For these prices, all consumers with $v \in (p^L + c/\alpha, 0.5]$ that arrive at firm j will search. Thus, all such consumers will buy at firm j if and only if they generate the high signal at all N firms. Thus profits for these prices $p_j \in [p^L + c/\alpha, \underline{p}^H]$ are:

$$p_j \int_{0.5}^{1} (1/N)\alpha dv + p_j \left[\sum_{j=1}^{N} \int_{p_j}^{0.5} (1/N)(1-\alpha)^N dv \right] = 0.5(1/N)\alpha p_j + [0.5p_j - (p_j)^2](1-\alpha)^N \tag{A.1.24}$$

The derivative of this w.r.t. p_j will be strictly positive because $1 > N(1-\alpha)^{N-1}$. Thus, profits in this interval are strictly lower than profits at $\underline{p}^H$, which are the equilibrium high signal profits. Thus, there will be no profitable deviations in this price interval.

(iii) $p_j \in (\bar{p}^H, 1]$:

All consumers search and never return for these prices, since $\bar{p}^H$ was the cutoff price for consumers with $v > 0.5$. Thus, deviation profits are zero.

Part 2: The partial search deterrence equilibrium.

If this equilibrium, $F^H(p^L + c/\alpha)$ is set to satisfy $\Pi^C(p^L + c/\alpha; \tilde{v}^H) = \Pi^M(0.5; \tilde{v}^H)$. The price p^L solves (2.5.4). We assume that $p^L + c/\alpha < 0.5 \leq (1-\alpha)p^L + \alpha F^H(p^L + c/\alpha)(p^L + c/\alpha) + \alpha(1 - F^H(p^L + c/\alpha))(0.5) + c$ hold.

Given these prices and the associated conditions, all consumers with $v \in (p^L + c/\alpha, 0.5)$ will search when receiving a price $p_j > p^L + c/\alpha$. All consumers with $v > 0.5$ will not search on the equilibrium path. At $\bar{p}^H = 0.5$, they weakly prefer to refrain from searching, because:

$$(1-\alpha)(v-p^L)+\alpha F^H(p^L+c/\alpha)(v-p^L-c/\alpha)+\alpha(1-F^H(p^L+c/\alpha))(v-0.5)-c \leq (v-0.5) \iff$$

$$0.5 \leq (1-\alpha)p^L + \alpha F^H(p^L + c/\alpha)(p^L + c/\alpha) + \alpha(1 - F^H(p^L + c/\alpha))0.5 \tag{A.1.25}$$

The price p^L maximizes low signal profits on $[0, 0.5]$. It yields profits above $\Pi^M(p^{L,M}; \tilde{v}^L)$. For prices $p_j \geq 0.5$, profits are weakly below monopoly profits. Thus, there are no profitable deviations.

Now consider the high signal prices, which all yield $\Pi^H(0.5; \tilde{v}^H)$. For prices $p_j > 0.5$, profits are below monopoly profits, so there can't be a deviation into this region. By construction, $\Pi^H(0.5; \tilde{v}^H) = \Pi^H(p^L + c/\alpha; \tilde{v}^H)$. Profits are rising in p_j for $p_j < p^L + c/\alpha$, so there cannot be any deviations into this region. Now consider $p_j \in [p^L + c/\alpha, 0.5)$. For these prices, profits are below monopoly profits, because:

$$\Pi^H(p_j;\tilde{v}^H) = p_j\left[\int_{p_j}^{0.5}[Pr(\tilde{v}^H|v)]\big[Pr(\tilde{v}^H|v)(1-F(\underline{p}^H))\big]^{N-1}dv + \int_{0.5}^{1}(1/N)Pr(\tilde{v}^H|v)dv\right] <$$

$$p_j\left[\int_{p_j}^{0.5}(1/N)[Pr(\tilde{v}^H|v)]dv + \int_{0.5}^{1}(1/N)Pr(\tilde{v}^H|v)dv\right] \tag{A.1.26}$$

$$\iff$$

$$N(1-\alpha)^{N-1}[1-F(\underline{p}^H)]^{N-1} < 1$$

This holds by the result that $N(1-\alpha)^{N-1} < 1$. Thus, there cannot be any profitable deviations into this region either. These deviations would yield profits below $\Pi^M(p_j;\tilde{v}^H)$, which are below the equilibrium high signal profits.

Also note, for future reference, that p^L must solve:

$$\left.\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\right|_{p^L} + \sum_{j=2}^{N}\int_{c/\alpha+p^L}^{0.5}\frac{1}{N}\alpha\big[\underbrace{(1-\alpha)(1-F^H(p^L+c/\alpha))}_{:=y}\big]^{j-1}dv = 0 \tag{A.1.27}$$

$$\iff$$

$$(0.5)(1/N) - 2\alpha(1/N)p^L + \big[(0.5-p^L)\alpha - c\big](1/N)\left(\frac{[y][1-y^{N-1}]}{1-y}\right) = 0 \tag{A.1.28}$$

Moreover, the mixing indifference condition that $p^L + c/\alpha$ must satisfy is:

$$(p^L + c/\alpha)\left[\int_{0.5}^{1}\frac{1}{N}Pr(\tilde{v}^H|v)dv\right] +$$

$$(p^L + c/\alpha)\sum_{j=1}^{N}\left[\int_{c/\alpha+p^L}^{0.5}\frac{1}{N}Pr(\tilde{v}^H|v)\big[Pr(\tilde{v}^H|v)(1-F^H(p^L+c/\alpha))\big]^{j-1}dv\right] = 0.25(1/N)\alpha \tag{A.1.29}$$

$$\iff$$

$$(p^L + c/\alpha)\left[0.5(1/N)\alpha + (0.5 - p^L - c/\alpha)(1/N)(1-\alpha)\left(\frac{1-y^N}{1-y}\right)\right] = 0.25(1/N)\alpha \tag{A.1.30}$$

Proof of Corollary 1:

Consider any $s > 0$ and suppose that $\alpha \to 0.5$. Then, $p^{L,M} \to 0.5 = p^{H,M}$, which means that $\lim_{\alpha \to 0.5}[p^{L,M} + c/\alpha] = 0.5 + 2s > 0.5$. Thus, the monopoly equilibrium exists.

This implies that the search deterrence equilibrium cannot exist, because this requires that $p^{L,M} + c/\alpha < 0.5$. In the search equilibrium, we have $p^{L,S} > p^{L,M}$ (for any fixed α and s), which means that $\lim_{\alpha \to 0.5}[p^{L,S} + c/\alpha] \geq 0.5 + 2c > 0.5$ must also hold true. This rules out the existence of the search equilibrium. Similarly, recall that any MSE must satisfy the following properties: The highest price $\bar{p}^H$ must be weakly below 0.5 and the equilibrium low signal price (which is between $p^{L,M}$ and $p^{L,S}$) must satisfy $p^L + c/\alpha \leq \underline{p}^H$. When $p^{L,M} + c/\alpha > 0.5$, these two conditions cannot be jointly satisfied, implying that there exists no equilibrium in mixed strategies.

■

Proof of Corollary 2:

Part 1: If $p^{L,M} + c/\alpha \geq 0.5$, the monopoly equilibrium is the unique equilibrium that exists.

This equilibrium exists by previous logic. The search deterrence equilibrium cannot exist. In the search equilibrium or any equilibrium in which firms mix, the equilibrium price p^L will be above $p^{L,M}$. Thus, we have $p^L + c/\alpha > 0.5$. However, the highest equilibrium must be weakly below 0.5, which yields a contradiction by previous arguments.

Part 2: Profits in any mixed-strategy equilibrium would be below the profits in the search equilibrium and above profits in the search deterrence equilibrium.

Consider the mixed search equilibrium, in which $p^L = p^{L,S}$ and $p^L + c/\alpha < \underline{p}^H < \bar{p}^H < 0.5$ must hold. The measure of consumers who arrive at any firm after searching is the same as in the search equilibrium. Thus, equilibrium low signal profits are the same as in the search equilibrium, and thus above the low signal profits in the search deterrence equilibrium.

Moreover, the equilibrium high signal profits are equal to $\Pi^M(\bar{p}^H; \tilde{v}^H)$, which are below $\Pi^M(p^{H,M}; \tilde{v}^H)$, i.e. high signal profits in the search equilibrium. Now consider the equilibrium high signal profits in the search deterrence equilibrium, which are $\Pi^M(p^{L,M} + c/\alpha; \tilde{v}^H)$. Because $p^{L,M} + c/\alpha < p^{L,S} + c/\alpha < \bar{p}^H$ and because monopoly high signal profits are rising for any $p_j < 0.5$, high signal profits in the mixed search equilibrium are above those in the search deterrence equilibrium. This establishes the desired result for the mixed search equilibrium.

Now consider the partial search deterrence equilibrium, in which $\underline{p}^H = p^L + c/\alpha$ and this price is played with positive probability after the high signal. This means that the measure of consumers who arrive at any firm after searching is below the corresponding measure in the search equilibrium (ceteris paribus). As a result, the low signal price in this equilibrium satisfies $p^L \in [p^{L,M}, p^{L,S}]$. Because this price is available to set in the search equilibrium and low signal profits are higher for any relevant price, we know that low signal profits in the search equilibrium must be above those in the partial search deterrence equilibrium. However, low signal profits in the partial search deterrence equilibrium are still strictly above the monopoly low signal profits, i.e. they are higher than low signal profits in the search deterrence equilibrium.

High signal profits in the partial search deterrence equilibrium are weakly below those in the search equilibrium, because $\bar{p}^H \leq 0.5$. However, high signal profits are still above those in the search deterrence equilibrium. This is because $\bar{p}^H > p^L + c/\alpha \geq p^{L,M} + c/\alpha$. This implies the desired result.

■

Proof of Corollary 3:

The high signal price in the search equilibrium is 0.5, so independent of c and N. To see how $p^{L,S}$ is affected by N and c, note that $p^{L,S}$ has to satisfy:

$$\left.\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\right|_{p_j=p^{L,S}} + \underbrace{\left(\frac{\alpha(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)}{1-(1-\alpha)}\right)(1/N)\big[(0.5-p^{L,S})-c/\alpha\big]}_{=S^L(c,N,\alpha;p^{L,S})} = 0 \tag{A.1.31}$$

The fact that $\frac{\partial S^L(.)}{\partial c} < 0$ implies that a rise of c will lead to a fall in $p^{L,S}$ (because $p^{L,S} < 0.5$ must always hold and $\Pi^M(p_j;\tilde{v}^L)$ is strictly concave in this interval).

To consider the effect of a rise in N, recall that $p^{L,S}$ must also solve:

$$T(p^{L,S},\alpha) = p^{L,S} - \frac{1}{4\alpha} - \frac{S^L(c,N,\alpha;p^{L,S})}{(2/N)\alpha} = 0 \tag{A.1.32}$$

One can show that $\frac{\partial T}{\partial p^{L,S}} > 0$. Because $1-\alpha < 1 \implies log(1-\alpha) < 0$, we have that:

$$\frac{\partial T}{\partial N} = -\frac{(1-\alpha)}{2\alpha}\left(-\big(log(1-\alpha)\big)(1-\alpha)^{N-1}\right)\left[(0.5-p^{L,S})-c/\alpha\right] < 0 \tag{A.1.33}$$

Application of the implicit function theorem then implies that $p^{L,S}$ is rising in N. ■

A.2 Proofs — Section 2.5.

Proof of Proposition 5:

The proof of this result follows arguments that are analogous to those made in the proof of proposition 1.

■

Proof of Proposition 6:

Part 1: In equilibrium, $p^L < p^H$ must hold.

Because $Pr(\tilde{v}^H|v)$ is strictly increasing in v, $p^{L,M} < p^{H,M}$ must hold.

Suppose that $p^L = p^H = p^*$. There is no search on the equilibrium path. In an open ball of prices around p^*, no consumer will move on to search. Thus, profits in an open ball around p^* are equal to monopoly profits. As a result, $p^H = p^L$ cannot hold. There would be a profitable deviation from one of these prices, because at least one of them could not be a local maximizer of the corresponding monopoly profit function.

By analogous arguments, there cannot exist an equilibrium in which $p^H < p^L$.

Part 2: In an equilibrium with on-path search, the set of consumer valuations that search on the equilibrium path is $\hat{V}(p^L) \cap [p^L, p^H]$ (ignoring measure zero sets). Any consumer who arrives after searching will buy when offered a price in an open ball around p^L.

To see the first part, note that $\hat{V}(p^L)$ must be non-empty in an equilibrium with search. Continuity of the function $Pr(\tilde{v}^H|v)$ implies that $Pr(\tilde{v}^L|v)(v - p^L) - c = 0$ must hold at $v = \inf \hat{V}(p^L)$, which implies that $\inf \hat{V}(p^L) > p^L$. Any consumer with $v \in \hat{V}(p^L)$ and $v \in [p^L, p^H]$ will have strictly positive gains from search when offered p^H. Proposition 1 implies that consumers with $v > p^H$ cannot search on path. Consumers with $v \notin \hat{V}(p^L)$ cannot search on-path.

To see the second part, consider first an equilibrium with search, in which $\underline{v} := \inf \hat{V}(p^L) < p^H$ must hold. Any consumer with $v \in \hat{V}(p^L)$ has a price cutoff $\hat{p}(v) = p^L + \frac{s}{Pr(\tilde{v}^L|v)}$. Since $Pr(\tilde{v}^L|\underline{v})(\underline{v} - p^L) - c = 0$ holds, $\hat{p}(\underline{v}) = \underline{v}$ must hold. This, together with the fact that $\hat{p}'(v) > 0$, implies that $\hat{p}(v) > \underline{v}$ holds for any $v \in \hat{V}(p^L)$. Thus, no consumers with $v \in \hat{V}(p^L)$ search when receiving $p_j \leq \underline{v}$. No consumer with $v \notin \hat{V}(p^L)$ can search after $\underline{v}$, since $\underline{v} < p^H$ holds in an equilibrium with search. Moreover, any consumer who arrives after search has $v > \underline{v}$ and must have received $p^H > \underline{v}$, implying that any such consumer would buy when offered $p_j \leq \underline{v}$, which lies strictly above p^L. If we consider an equilibrium without on-path search, no such consumer exists.

Part 3: Arguing why p^L must satisfy the first-order condition.

Given that the signal probability functions are continuous, the monopoly low signal profit function is continuously differentiable. Since competitive low signal profits are given by the function in equation (2.5.4) in the price interval $p_j \in [0, \underline{v}]$, we know that this FOC must hold.

Part 4: Proving that $Pr(\tilde{v}^L|p^H)(p^H - p^L) - c = 0$ must hold when p^H is not a local maximizer of $\Pi^M(p_j; \tilde{v}^H)$:

(i) $Pr(\tilde{v}^L|p^H)(p^H - p^L) - c > 0$ cannot be true in a symmetric pure-strategy equilibrium.

Suppose, for a contradiction, that $Pr(\tilde{v}^L|p^H)(p^H - p^L) - c > 0$. By continuity of the gains of search, there exists an interval $(p^H, p^H + \delta)$ such that a consumer with $v \in (p^H, p^H + \delta)$ would search when receiving p^H. Together with the assumption that $Pr(\tilde{v}^H|v) \in (0, 1)$, this yields a contradiction to proposition 1.

(ii) If $Pr(\tilde{v}^L|p^H)(p^H - p^L) - c < 0$, the equilibrium p^H must satisfy $\left.\frac{\partial \Pi^M(p_j;\tilde{v}^H)}{\partial p_j}\right|_{p^H} = 0$.

Any consumer with $v \in [p^H, 1]$ searches when offered a price $p_j \geq p^H$ if and only if:

$$Pr(\tilde{v}^L|v)(v - p^L) + Pr(\tilde{v}^H|v)(v - p^H) - c > \max\{v - p_j, 0\} \tag{A.2.1}$$

In order for a consumer with $v \geq p^H$ to search at a price $p_j \geq p^H$, the LHS of this expression needs to be positive. Define the following set which contains all v that satisfy this condition:

$$\hat{V}^H(p) = \left\{v \in [0, 1] : v - p^H + Pr(\tilde{v}^L|v)(p^H - p^L) - c > 0\right\} \tag{A.2.2}$$

The cutoff price (if it is weakly above p^H) of agents with a valuation in the above set is $\hat{p}^H(v) = Pr(\tilde{v}^L|v)\big(p^L - p^H\big) + p^H + c$.

By continuity of $Pr(\tilde{v}^L|v)$, it holds that $\lim_{v\to p^H} Pr(\tilde{v}^L|v) = Pr(\tilde{v}^L|p^H)$. This implies that $\lim_{v\to p^H}\left[v - p^H + Pr(\tilde{v}^L|v)(p^H - p^L) - c\right] < 0$, since $Pr(\tilde{v}^L|p^H)(p^H - p^L) < c$ holds.

Continuity implies that you can find an open interval $[p^H - \delta, p^H + \delta]$ such that any $v \in [p^H - \delta, p^H + \delta]$ will satisfy $v \notin \hat{V}^H(p)$ and $v \notin \hat{V}^L(p)$ and thus $\hat{p}(v) = \infty$.

Find the first valuation $v' \leq 1$ above p^H that solves $v' - p^H + Pr(\tilde{v}^L|v')(p^H - p^L) - c = 0$. If this does not exist, $\hat{p}(v) = \infty$ holds for all $v > p^H$. If such a v' exists, continuity arguments imply that $\hat{p}(v) = \infty$ holds for all $v \in [p^H, v')$. Now consider valuations $v \geq v'$. Note that $\hat{p}(v') = v' > p^H$. All consumers with $v > v'$ will have a cutoff price above $\hat{p}(v')$, since $\hat{p}'(v) \geq 0$.

Summing up, consumers with $v \in [p^H, v')$ have $\hat{p}(v) = \infty$ and consumers with $v \in [v', 1]$ have $\hat{p}(v) \geq v' > p^H$. Thus, all consumers with $v > p^H$ will not search for prices $p_j \in [p^H, v']$ and hence not for prices below this either. This means that competitive profits are equal to monopoly profits for any price in the interval $[p^H, v']$.

Consider prices just below p^H. Consumers with $v \in [p^H - \delta, p^H]$ have $v \notin \hat{V}(p^L)$, i.e. they won't search for $p_j \leq p^H$. We have shown that no consumer with $v \geq p^H$ will search at $p_j \leq p^H$. In the price interval $[p^H - \delta, p^H]$, profits will thus also be monopoly profits.

Thus, competitive profits equal monopoly profits in an open ball around p^H. Differentiability of this function implies that the high signal price must satisfy said FOC.

$\blacksquare$

Appendix B

Chapter 3: Proofs and further material

B.1 Proofs — Section 3.4.

Proof of Lemma 3:

Part 1: $p^{L,M} < p^{nd,M} < p^{H,M}$ must hold.

By definition, $p^{nd,M}$ solves $\int_{p^{nd,M}}^{1} \frac{g(v)}{g(p^{nd,M})} dv - p^{nd,M} = 0$. At $p_j = p^{nd,M}$, we have:

$$\int_{p^{nd,M}}^{1} Pr^L(v)g(v)dv - p^{nd,M}\big(Pr^L(p^{nd,M})g(p^{nd,M})\big) < 0 \iff$$

$$\int_{p^{nd,M}}^{1} \underbrace{\frac{Pr^L(v)}{Pr^L(p^{nd,M})}}_{<1} \frac{g(v)}{g(p^{nd,M})} dv < p^{nd,M} \tag{B.1.1}$$

The inequality $Pr^L(v) < Pr^L(p^{nd,M})$ holds for all $v > p^{nd,M}$ by the assumption that $Pr^H(v)$ is strictly increasing in v. By strict concavity of the monopoly profit functions, $p^{L,M} < p^{nd,M}$ must hold. Analogous arguments imply that $p^{H,M} > p^{nd,M}$ must hold.

Part 2: In an equilibrium in which firms play pure strategies, $p^H < p^L$ cannot hold.

Consider an arbitrary equilibrium, in which the searcher's strategy is given by $s(v)$. I first define $\phi(v) := \rho s(v)g(v) + 0.5(1-\rho)g(v)$, which is measurable. Moreover, note that the conditional cumulative density functions for either signal $k \in \{L, H\}$ are:

$$F^k(x) = Pr(v \leq x|\tilde{v}^k) = \frac{1}{Pr(\tilde{v}^k)} \int_0^1 Pr(v \leq x \wedge \tilde{v}^k|v)\phi(v)dv = \frac{1}{Pr(\tilde{v}^k)} \int_0^x Pr(\tilde{v}^k|v)\phi(v)dv \tag{B.1.2}$$

We can define a probability density function $f^k(x) = (1/Pr(\tilde{v}^k))Pr(\tilde{v}^k|x)\phi(x)$ corresponding to $F^k(x)$. The hazard rates for these distributions are $h^k(x) = \frac{f^k(x)}{1-F^k(x)}$. Thus, $h^H(x) < h^L(x) \;\; \forall x \in (0, 1)$, i.e. $F^H(x)$ strictly hazard ratio dominates $F^L(x)$, since:

$$h^H(x) < h^L(x) \iff \int_x^1 \underbrace{(Pr(\tilde{v}^L|v)/Pr(\tilde{v}^l|x))}_{<1} \phi(v)dv < \int_x^1 \underbrace{(Pr(\tilde{v}^H|v)/Pr(\tilde{v}^H|x))}_{>1} \phi(v)dv \tag{B.1.3}$$

Suppose, for a contradiction, that $p^H < p^L$. Since p^L and p^H are available to set after any signal, there must be no profitable deviation from p^L to p^H after $\tilde{v}^L$ and no profitable deviation from p^H to p^L after $\tilde{v}^H$. But given the hazard ratio ordering established above, it will either be profitable to deviate from p^L to p^H when observing $\tilde{v}^L$ or vice versa, a contradiction.

Part 3: In an equilibrium in which firms play pure strategies, $p^L = p^H$ cannot hold.

Suppose $p^{nd} < p^L = p^H$. Then, all searchers with $v \geq p^{nd}$ visit the firm without data. In equilibrium, the firm with data thus only sells to its captive consumers. Thus, $p^L = p^{L,M}$ and $p^H = p^{H,M}$ must hold, which contradicts $p^L < p^H$.

Suppose $p^L = p^H < p^{nd}$. Then, all searchers with $v > p^H$ visit the firm with data. Thus, $p^H \geq p^{H,M}$ must hold (else, there is an upward deviation because for prices above p^H, high signal profits are $0.5(1 + \rho)\Pi^{H,M}(p_j)))$. Also, $p^{nd} = p^{nd,M}$ must hold since the firm without data only sells to captive consumers in equilibrium. Thus, $p^{nd} < p^{H,M} \leq p^H$, a contradiction.

Thus, suppose that $p^L = p^H = p^{nd}$. Then, all searchers with a valuation above the equilibrium price (call this $p^* := p^L = p^H = p^{nd}$) visit either firm with probability 0.5. This implies that $p^* \geq p^{H,M}$ must hold, since high signal profits for the prices $p_j \geq p^*$ are $0.5\Pi^{H,M}(p_j)$. For prices below the equilibrium price, this is not generally true, as the search strategy of consumers with $v \leq p^*$ is not pinned down and can be given by any measurable $s(v)$.

It must be optimal for the firm with data to set p^* after $\tilde{v}^L$ and for the firm without data to set p^* (even though, under random search, there would be profitable downward deviations to $p^{nd,M} < p^*$ for both). I show that these two conditions cannot jointly hold, a contradiction. Define $\varphi(v) := \rho s(v) + 0.5(1-\rho)$, with $1-\varphi(v) = \rho(1-s(v)) + 0.5(1-\rho)$.

Firstly, the following no-deviation condition needs to hold for the firm without data:

$$p^{nd,M}\int_{p^{nd,M}}^{p^*}(1-\varphi(v))g(v)dv \leq (p^*-p^{nd,M})\int_{p^*}^{1}(0.5)g(v)dv \tag{B.1.4}$$

Moreover, another no-deviation condition w.r.t. the optimal low signal action must hold:

$$p^{nd,M}\left[\int_{p^{nd,M}}^{p^*}\varphi(v)Pr^L(v)g(v)dv+\int_{p^*}^{1}Pr^L(v)(0.5)g(v)dv\right] \leq p^*\int_{p^*}^{1}Pr^L(v)(0.5)g(v)dv \tag{B.1.5}$$

Because $Pr^L(v)$ is strictly falling in v, the validity of condition (B.1.5) requires that the following condition must hold:

$$p^{nd,M}\int_{p^{nd,M}}^{p^*}\varphi(v)g(v)dv < (p^*-p^{nd,M})\int_{p^*}^{1}g(v)(0.5)dv \tag{B.1.6}$$

Adding up the two conditions yields $p^{nd,M}\int_{p^{nd,M}}^{1}g(v)dv < p^*\int_{p^*}^{1}g(v)dv$. This is a contradiction to the fact that $p^{nd,M}$ is a unique maximizer of the monopoly profit function of the firm without data, which holds under assumption 4.

Part 4: In an equilibrium in which firms play pure strategies, $p^L < p^{nd} < p^H$ must hold.

By previous arguments, $p^L < p^H$ must hold. Suppose, for a contradiction, that $p^{nd} \leq p^L$. Then, all searchers with $v > p^{nd}$ visit the firm without data. For all prices $p_j \geq p^{nd}$, the profit function of the firm without data is thus $0.5(1+\rho)\Pi^{nd,M}(p_j)$, which means that $p^{nd} \geq p^{nd,M}$ must hold (else, there is a profitable upward deviation). By implication, $p^{nd,M} \leq p^{nd} < p^L$ holds, which implies that $p^L > p^{L,M}$. Because the firm with data only sells to captive consumers in equilibrium, there is a profitable downward deviation from p^L, a contradiction.

By analogous arguments, $p^H \leq p^{nd}$ cannot hold. Then, $p^H = p^{H,M}$, which implies $p^{nd} > p^{nd,M}$. Thus, the firm without data would deviate, a contradiction.

Part 5: Existence of the cutoff $\bar{v}$.

In equilibrium, we must have $p^L < p^{nd} < p^H$. All consumers with $v \in (p^L, p^{nd}]$ visit the firm with data, since their utility at the firm without data is 0. Consider consumers with $v \in (p^{nd}, p^H]$, for whom the preference for the firm with data is $P^D(v) = \left[Pr^L(v)(v-p^L)\right] - (v-p^{nd})$. This is strictly falling in v. Now consider consumers with $v \in [p^H, 1]$. For them, the preference for the firm with data is $P^D(v) = \big[Pr^L(v)(v-p^L)+$

$Pr^H(v)(v-p^H)\big] - (v-p^{nd})$, which is continuous at p^H. For consumers with $v \in [p^H, 1]$, the preference for the firm with data is strictly falling in v.

Thus, there must be a unique $\bar{v}$, because all searchers with $v \in (p^L, p^{nd}]$ strictly prefer the firm with data and the preference for this firm is strictly decreasing in v thereafter.

■

Proof of Proposition 7:

Consider first the monopoly profit function of the firm without data, which is $\Pi^{nd,M}(p_j) = p_j[1-G(p_j)]$. The second derivative, which is strictly negative for all $p_j \in [0,1]$ by assumption, is: $\frac{\partial^2 \Pi^{nd,M}(p_j)}{\partial p_j^2} = -2g(p_j) - p_j g'(p_j)$.

In equilibrium, $p^{nd} < \bar{v}$ must hold. Suppose, for a contradiction, that $p^{nd} \geq \bar{v}$ holds. Because $p^L < p^{nd}$ must hold by lemma 3, consumers with $v \in (p^L, p^{nd}]$ strictly prefer to visit the firm with data. Since the expected utilities at the two firms are continuous in v, consumers with v just above p^{nd} will also strictly prefer to visit the firm with data. However, $\bar{v} \leq p^{nd}$ holds by assumption, which means that these consumers visit the firm without data, a contradiction.

Setting a price $p^{nd} < \bar{v}$ will only be optimal for the firm without data if $\bar{v} \geq \bar{v}^{nd}$. Suppose, for a contradiction, that we have an equilibrium in which $\bar{v} < \bar{v}^{nd}$, where:

$$\rho\big[1-G(\bar{v}^{nd})\big] + 0.5(1-\rho)\big[1-G(\bar{v}^{nd}) - (\bar{v}^{nd})g(\bar{v}^{nd})\big] = 0 \tag{B.1.7}$$

Recall that $\Pi^{nd,M}(p_j)$ is strictly concave, which implies that the function $\rho\big[1-G(\bar{v})\big] + 0.5(1-\rho)\big[1-G(\bar{v}) - (\bar{v})g(\bar{v})\big]$ is strictly falling in $\bar{v}$.

In equilibrium, the objective function of the firm with data for prices $p_j \in (p^L, \bar{v})$ is:

$$\Pi^{nd}(p_j;\bar{v}) = p_j\left[\rho\int_{\bar{v}}^{1} g(v)d\nu + \frac{1-\rho}{2}\int_{p_j}^{1} g(v)d\nu\right] \tag{B.1.8}$$

We consider any $p^{nd} \in (p^L, \bar{v})$ and any $\bar{v} < \bar{v}^{nd}$. The derivative of $\Pi^{nd}(p_j)$ at p^{nd} satisfies:

$$\left.\frac{\partial \Pi^{nd}(p_j)}{\partial p_j}\right|_{p^{nd}} = \rho\big[1-G(\bar{v})\big] + 0.5(1-\rho)\big[1-G(p^{nd}) - p^{nd}g(p^{nd})\big] >$$

$$\rho\big[1-G(\bar{v}^{nd})\big] + 0.5(1-\rho)\big[1-G(\bar{v}^{nd}) - (\bar{v}^{nd})g(\bar{v}^{nd})\big] = 0 \tag{B.1.9}$$

This is a contradiction. There would exist a profitable upward deviation.

■

Proof of Proposition 8:

Part 1: Preliminaries - definition and properties of $\bar{v}^{HC}$.

I define a cutoff $\bar{v}^{HC}$ that solves: $\max_{p_j \leq \bar{v}^{HC}} \Pi^H(p_j;\bar{v}^{HC}) = 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$. This cutoff captures whether the optimal price high signal price is below or above $\bar{v}$. If $\bar{v} \leq \bar{v}^{nd}$, then $p^{H,*}(\bar{v}) = p^{H,M}$. If $\bar{v} > \bar{v}^{nd}$, $p^{H,*}(\bar{v}) < \bar{v}$ and solves an appropriate FOC.

Note that $\max_{p_j \leq \bar{v}} \Pi^H(p_j;\bar{v})$ is strictly rising in $\bar{v}$. Also, we can establish that (i) $\bar{v}^{HC} < p^{H,M}$ and (ii) that, if $\bar{v} > \bar{v}^{HC}$, $arg\max_{p_j \leq \bar{v}} \Pi^H(p_j;\bar{v}) < \bar{v}$. To establish this, recall that:

$$\Pi^H(p_j;\bar{v}) = p_j\big[\rho\mathbb{1}[p_j \leq \bar{v}]\int_{p_j}^{\bar{v}} Pr(\tilde{v}^H|v)g(v)dv + 0.5(1-\rho)\int_{p_j}^{1} Pr(\tilde{v}^H|v)g(v)dv\big] \tag{B.1.10}$$

(i) $\bar{v}^{HC} < p^{H,M}$.

Suppose $\bar{v}^{HC} = p^{H,M}$. Then, the left derivative of $\Pi^H(p_j;\bar{v}^{HC})$ at $p_j = \bar{v}^{HC} = p^{H,M}$ would be strictly negative. Thus, profits could be strictly increased by a downward movement from $p_j = \bar{v}^{HC}$, a contradiction to the equality $\max_{p_j \leq \bar{v}^{HC}} \Pi^H(p_j;\bar{v}^{HC}) = 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$.

Suppose $\bar{v}^{HC} > p^{H,M}$. Then, setting $p_j = p^{H,M} < \bar{v}^{HC}$ is available within $[0,\bar{v}^{HC}]$. This would yield strictly higher profits than $0.5(1-\rho)\Pi^{H,M}(p^{H,M})$, since the sale will also be made to searchers, a contradiction to the equality $\max_{p_j \leq \bar{v}^{HC}} \Pi^H(p_j;\bar{v}^{HC}) = 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$.

(ii) If $\bar{v} \geq \bar{v}^{HC}$, i.e. $\max_{p_j \leq \bar{v}} \Pi^H(p_j;\bar{v}) \geq 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$, the locally optimal price $p_j \leq \bar{v}$ must be strictly below $\bar{v}$ and thus solve an appropriate FOC.

Suppose $\bar{v} \geq \bar{v}^{HC}$, but the optimal price is exactly equal to $\bar{v}$. Because $\bar{v} \geq \bar{v}^{HC} > p^{H,M}$, the left derivative of $\Pi^H(p_j;\bar{v})$ at $p_j = \bar{v}$ would be strictly negative, a contradiction.

Part 2: The functions $p^{L,*}(\bar{v})$ and $p^{nd,*}(\bar{v})$ are continuous on $[\bar{v}^{nd}, 1]$, while the function $p^{H,*}(\bar{v})$ is continuous on $[\bar{v}^{HC}, 1]$ and equal to $p^{H,M}$ for $\bar{v} < \bar{v}^{HC}$.

For any $\bar{v} \geq \bar{v}^{nd}$, we have $p^{L,M} < \bar{v}$, because $p^{nd,M} < \bar{v}^{nd}$. This means that $p^{L,*}(\bar{v}) < \bar{v}$ and must solve an appropriate first-order condition. Thus, it is continuous in $\bar{v}$.

Now consider $p^{nd,*}(\bar{v})$ and any $\bar{v} \geq \bar{v}^{nd}$. For prices $p_j > \bar{v}$, the derivative of profits is strictly negative, which implies that $p^{nd,*}(\bar{v}) \leq \bar{v}$ must hold. At $p_j = \bar{v}$, the left derivative of profits is given by:

$$\rho\big[1 - G(\bar{v})\big] + 0.5(1-\rho)\big[1 - G(\bar{v}) - (\bar{v})g(\bar{v})\big] \tag{B.1.11}$$

By strict concavity of $\Pi^{nd,M}(p_j)$, this term is falling in $\bar{v}$. It is hence strictly negative for any $\bar{v} > \bar{v}^{nd}$ and just zero for $\bar{v} = \bar{v}^{nd}$ (by the concavity assumption, we have $p^{nd,*}(\bar{v}^{nd}) = \bar{v}^{nd}$). For any $\bar{v} \geq \bar{v}^{nd}$, the optimal price must thus solve:

$$\rho\big[1 - G(\bar{v})\big] + 0.5(1-\rho)\big[1 - G(p^{nd,*}) - (p^{nd,*})g(p^{nd,*})\big] = 0 \tag{B.1.12}$$

The solution function will be continuous in $\bar{v}$. Moreover, it is falling in $\bar{v}$ because the LHS is falling in $\bar{v}$ and $p^{nd,*}$.

The fact that $p^{H,*}(\bar{v})$ is continuous on $\bar{v} \in [\bar{v}^{HC}, 1]$ follows from previous arguments. It must lie strictly below $\bar{v}$ and satisfy a first-order condition, making $p^{H,*}(\bar{v})$ continuous.

Part 3: If the firms' prices are given by $p^{L,*}(\bar{v})$, $p^{nd,*}(\bar{v})$, and $p^{H,*}(\bar{v})$, searchers optimally visit the firm with data iff $v < \hat{v}^G(\bar{v})$, where:

$$\hat{v}^G(\bar{v}) = \sup\left\{v \in [0,1] : \sum_{k\in\{L,H\}} Pr(\tilde{v}^k|v)\max\{v - p^{k,*}(\bar{v}), 0\} - (v - p^{nd,*}(\bar{v})) > 0\right\} \tag{B.1.13}$$

To see this, note that the preference for the firm with data, namely $P^D(v) = \sum_{k\in\{L,H\}} Pr(\tilde{v}^k|v)$ $\max\{v - p^{k,*}(\bar{v}), 0\} - (v - p^{nd,*}(\bar{v}))$ is strictly falling in v. This holds because $Pr(\tilde{v}^H|v)$ is strictly increasing in v and $p^{L,*}(\bar{v}) < p^{H,*}(\bar{v})$ holds for any $\bar{v}$.

To see that $p^{L,*}(\bar{v}) < p^{H,*}(\bar{v})$, recall that it was previously established that $p^{L,*}(\bar{v}) \leq p^{H,*}(\bar{v})$. Suppose, for a contradiction, that $p^{L,*}(\bar{v}) = p^{H,*}(\bar{v})$. If $p^{L,*}(\bar{v}) = p^{H,*}(\bar{v}) > \bar{v}$, the prices must equal their monopoly counterparts, a contradiction.

If $p^{L,*}(\bar{v}) = p^{H,*}(\bar{v}) < \bar{v}$, the prices $p^{k,*}(\bar{v})$ must satisfy corresponding first-order conditions. But since $Pr^H(v)$ is strictly increasing in v, $p^{L,*}(\bar{v}) < p^{H,*}(\bar{v})$ will hold, a contradiction.

Thus, suppose that $p^{L,*}(\bar{v}) = p^{H,*}(\bar{v}) = \bar{v}$. If $\bar{v} \geq \bar{v}^{HC}$, the optimal high signal price must lie strictly below $\bar{v}$ (see the arguments in part 1), a contradiction. If $\bar{v} < \bar{v}^{HC} < p^{H,M}$, the optimal high signal price is $p^{H,M}$, which lies above $\bar{v}$, a contradiction.

Part 4: An equilibrium in which firms play pure strategies exists.

I proove this result by showing that there exists a $\bar{v} \in [\bar{v}^{nd}, 1]$ such that $\hat{v}^G(\bar{v}) = \bar{v}$. Suppose such a fixed point exists and consumers search according to the implied cutoff rule. The prices $p^{L,*}(\bar{v})$, $p^{H,*}(\bar{v})$, and $p^{nd,*}(\bar{v})$ are optimal by construction. The postulated search behaviour will be optimal, given these prices. Thus, we have an equilibrium.

For any $\bar{v} \geq \bar{v}^{nd}$, both price functions $p^{L,*}(\bar{v})$ and $p^{nd,*}(\bar{v})$ will be continuous in $\bar{v}$. To see this, note that $p^{L,M} < p^{nd,M} < \bar{v}^{nd}$. This implies that both optimal prices will be below $\bar{v}$ and solve appropriate first-order conditions.

We will establish the existence of a fixed point of $v^G(\bar{v})$. To begin, note that the following two boundary conditions will be satisfied: (i) $v^G(\bar{v}^{nd}) > \bar{v}^{nd}$ and (ii) $v^G(1) \leq 1$.

The first condition holds because, at $\bar{v} = \bar{v}^{nd}$, the optimal price of the firm without data will be equal to $\bar{v}^{nd}$. Because $p^{L,*}(\bar{v}^{nd}) < \bar{v}^{nd}$, $p^{L,*}(\bar{v}^{nd}) < p^{nd,*}(\bar{v}^{nd})$ would hold. As a result, a consumer with $v = \bar{v}^{nd}$ would strictly prefer to visit the firm with data, and thus $v^G(\bar{v}^{nd}) > \bar{v}^{nd}$. The second condition, namely $v^G(1) \leq 1$, holds because all elements of $\left\{v \in [0,1] : \sum_{k\in\{L,H\}} Pr(\tilde{v}^k|v) \max\{v - p^{k,*}(\bar{v}), 0\} - (v - p^{nd,*}(\bar{v})) > 0\right\}$ are below 1.

Suppose $\bar{v}^{HC} < \bar{v}^{nd}$. Then, all functions $p^{L,*}(\bar{v}^{nd}), p^{nd,*}(\bar{v}^{nd})$, and $p^{H,*}(\bar{v}^{nd})$ are continuous on $[\bar{v}^{nd}, 1]$, which means that $v^G(\bar{v})$ is continuous on this interval. With our two boundary conditions, the intermediate value theorem guarantees the existence of a fixed point.

Now consider a situation in which $\bar{v}^{HC} \geq \bar{v}^{nd}$. Suppose, for a contradiction, that there exists no fixed point of $\hat{v}^G(\bar{v})$ on $[\bar{v}^{nd}, 1]$. This implies that $\hat{v}^G(\bar{v}) > \bar{v}$ must hold for any $v \in [\bar{v}^{nd}, \bar{v}^{HC}]$. At $\bar{v}^{HC}$, the optimal high signal price of the firm with data jumps down, which implies that $\lim_{\bar{v}\downarrow\bar{v}^{HC}} \hat{v}^G(\bar{v}) > \hat{v}^G(\bar{v}^{HC})$. As a result, $\hat{v}^G(\bar{v}) > \bar{v}$ holds for $\bar{v}$ in an open ball above $\bar{v}^{HC}$. Since all functions $p^{L,*}(\bar{v}^{nd}), p^{nd,*}(\bar{v}^{nd})$, and $p^{H,*}(\bar{v}^{nd})$ are continuous on $[\bar{v}^{HC}, 1]$, so is $\hat{v}^G(\bar{v})$. Because $\lim_{\bar{v}\downarrow\bar{v}^{HC}} \hat{v}^G(\bar{v}) > \bar{v}^{HC}$, the intermediate value theorem guarantees the existence of a fixed point.

■

Proof of Proposition 9:

A proof of a more general statement may be found in the proof of proposition 13. ■

Proof of Corollary 4:

I work with the equilibrium $\bar{v}$ for a given signal distribution as a function of ρ and call this $\bar{v}^*(\rho)$. An equilibrium with $p^L < p^H$ always exists. First, note that $\lim_{\rho\to1} \bar{v}^*(\rho) = 1$ holds by the squeeze theorem because, for any $\rho \in (0,1)$, we have that $\bar{v}^{nd}(\rho) \leq \bar{v}^*(\rho) \leq 1$ and $\lim_{\rho\to1} \bar{v}^{nd}(\rho) = 1$. To see that $\lim_{\rho\to1} \bar{v}^{nd}(\rho) = 1$, recall that, for any $\rho \in (0,1)$, $\bar{v}^{nd}(\rho)$ solves:

$$\underbrace{\rho\big[1 - G(\bar{v}^{nd}(\rho))\big]}_{LHS(\rho)} = \underbrace{-0.5(1-\rho)\big[1 - G(\bar{v}^{nd}(\rho)) - (\bar{v}^{nd}(\rho))g(\bar{v}^{nd}(\rho))\big]}_{RHS(\rho)} \tag{B.1.14}$$

This equality continues to hold as $\rho \to 1$. As $\rho \to 1$, the RHS goes to 0, no matter the limit of $\bar{v}^{nd}(\rho)$. This is because the distribution of valuations, namely $G(v)$, is continuous and has finite density. Thus, the LHS must also go to 0 as $\rho \to 1$. This implies that $\lim_{\rho\to1} \bar{v}^{nd}(\rho) = 1$, since $G(v)$ is continuous, $G(v) = 1$ only if $v \geq 1$, and $\bar{v}^{nd}(\rho) \leq 1$ for any $\rho \in (0,1)$.

The total demand that the firm without data receives in equilibrium is $D^{nd^*}(\rho) = \rho[1 - G(\bar{v}^*(\rho))] + 0.5(1-\rho)\int_{p^{nd}(\rho)}^{1} g(v)dv$. I have defined $p^{nd}(\rho)$ as the equilibrium price of the firm without data. Now consider the limit of $D^{nd^*}(\rho)$ as $\rho \to 1$, noting that all components of demand are continuous in ρ and that $\int_{p^{nd}}^{1} g(v)dv \in [0,1]$. Thus, we have $\lim_{\rho\to1} D^{nd^*}(\rho) = (1)(0) + (0)\int_{\lim_{\rho\to1} p^{nd}(\rho)}^{1} dv = 0$. Since the demand of the firm without data approaches 0 when $\rho \to 1$, the market share of the firm with data approaches 1 by any definition of the market share (sales or profit).

■

Statement and proof of lemma 8:

Lemma 8 *Consider the sequential search framework. In any equilibrium in which firms play pure strategies, the ordering $p^L < p^{nd} < p^H$ must hold and:*

- *There exists an $\epsilon > 0$ such that any searcher who visits the firm without data first in equilibrium will not search when offered a price $p_j \in [0, p^{nd} + \epsilon]$ at this firm.*
- *There exists a $\bar{v} > p^L$ such that all searchers with $v \in (p^L, \bar{v})$ visit the firm with data first and all searchers with $v \in (\bar{v}, 1]$ visit the firm without data first.*
- *The ordering $\bar{v} \geq \bar{v}^{nd}$ holds.*

Proof of Lemma 8:

Part 1: In equilibrium, $p^L < p^H$ must hold.

Suppose, for a contradiction, that $p^H < p^L$ in equilibrium. Previous arguments have established that $p^{nd} \in (p^H, p^L)$ would have to hold in such an equilibrium.

Suppose $p^L \leq p^{nd} + c$, i.e. no searcher will leave the firm with data to search at the equilibrium prices. Thus, for $p_j \in [p^H, p^L]$, all consumers who arrive at the firm with data buy there iff the price is below their v. Then, the structure of equilibrium profits equal the one defined in the proof of lemma 3 and there is either a deviation from p^L to p^H or vice versa.

Suppose instead that $p^L > p^{nd} + c$. Then, the firm with data only sells to its captive consumers at p^L and thus $p^L = p^{L,M}$. All searchers who arrive at the firm without data buy in an open ball around p^{nd} (first arrivers must have $v \geq p^{nd}$ and would never search thereafter by an option value logic, second arrivers would entail inelastic demand around p^{nd}). Hence, $p^{nd} \geq p^{nd,M}$ holds, and $p^{nd} \geq p^{nd,M} > p^L$, a contradiction.

Suppose, for a contradiction, that there exists an equilibrium in which $p^L = p^H$. The only possible equilibrium candidate is $p^L = p^H = p^{nd}$ (otherwise, all searchers with valuation above the lowest equilibrium price visit the same firm, which yields a contradiction).

Thus, consider an equilibrium in which $p^L = p^H = p^{nd} := p^*$. By our tie-breaking rule and because no consumer will leave to search for prices $p_j \leq p^* + c$, the equilibrium price must satisfy $p^* \leq p^{H,M}$ (else, there is a profitable upward deviation when observing $\tilde{v}^H$). Thus, the arguments made in the proof of lemma (3) imply that either the firm without data or the firm with data (when observing $\tilde{v}^L$) will have a profitable downward deviation to $p^{nd,M}$.

Part 2: In equilibrium, $p^{nd} \in (p^L, p^H)$ must hold.

This follows from the arguments made in the proof of lemma 3. If $p^{nd} \notin (p^L, p^H)$, all searchers with a valuation below the lowest equilibrium price visit the same firm, which implies that the postulated ordering of prices would not be optimal.

Part 3: Any searcher who optimally visits the firm without data first must find it strictly optimal to not search when receiving p^{nd}.

To see this, define $U^{nd,s}(v)$ and $U^{nd,ns}(v)$ as the expected utilities of visiting the firm without data first and searching or not searching, respectively. Define $U^{d,s}(v)$ as the expected utility of visiting the firm with data and searching if and only if p^H is received there.

Consider a consumer that optimally visits the firm without data first, who must have $\nu > p^{nd}$. Suppose, for a contradiction, that $Pr^L(v)(p^{nd} - p^L) - s \geq 0 \iff U^{nd,s}(v) \geq U^{nd,ns}(v)$ holds for such a consumer. Crucially, $U^{d,s}(v) > U^{nd,s}(v)$ will hold generally, because:

$$\underbrace{Pr^L(v)(v - p^L) + Pr^H(v)(v - p^{nd} - s)}_{U^{d,s}(v)} > \underbrace{Pr^L(v)(v - p^L) + Pr^H(v)(v - p^{nd}) - s}_{U^{nd,s}(v)} \tag{B.1.15}$$

The utility of visiting the firm without data first is $U^{nd,s}(v)$, while the utility of visiting the firm with data first is at least $U^{d,s}(v)$. It would thus be strictly optimal for this consumer to visit the firm with data first, a contradiction. Hence, $Pr^L(v)(p^{nd} - p^L) - s < 0$ must hold for any consumer that visits the firm without data first in equilibrium, which implies that there exists an $\epsilon > 0$ such that these consumers would also not search for prices $p_j \leq p^{nd} + \epsilon$.

Part 4: Uniqueness of cutoff $\bar{v}$ for equilibria with $p^{nd} + s \geq p^H$

We consider an equilibrium candidate and define a $\tilde{v}^I$ that solves $Pr^L(\tilde{v}^I)(p^{nd}-p^L)-s=0$. All consumers with $v \in (p^L, \tilde{v}^I)$ will surely visit the firm with data, because search would be optimal for them after visiting the firm without data (if $v > p^{nd}$). Similarly, consumers with $v \leq p^{nd}$ will visit the firm with data.

Thus, consider consumers with $v \in (\max\{|p^{nd}, \tilde{v}^I\}, 1)$ and recall that $p^H \leq p^{nd} + s$ holds by assumption. If $v < p^H$, their preference for the firm with data is $P^D(v) = Pr^L(v)(v-p^L) + Pr^H(v)(0) - (v-p^{nd})$. If $v \geq p^H$, their preference for the firm with data is $P^D(v) = Pr^L(v)(v-p^L) + Pr^H(v)(v-p^H) - (v-p^{nd})$. The preference for the firm with data is continuous at p^H and strictly falling in v for $v \in (\max\{|p^{nd}, \tilde{v}^I\}, 1)$. This implies the result.

Part 5: Uniqueness of cutoff $\bar{v}$ in equilibria with $p^{nd} + c < p^H$

Searchers leave the firm with data to search when receiving p^H if and only if $v > p^{nd} + c$. As before, $\tilde{v}^I$ solves $Pr^L(\tilde{v}^I)(p^{nd} - p^L) - s = 0$. Calculating the relative preferences for the firm with data for two separate cases, namely (i) $p^{nd} + c < \tilde{v}^I$ and (ii) $\tilde{v}^I \leq p^{nd} + c$ yields the desired result based on steps that mirror those taken in the previous part.

Part 6: Establishing that $\bar{v} \geq \bar{v}^{nd}$ holds true.

First, note that $p^{nd} < \bar{v}$ must hold. A searcher with v just above p^{nd} will not visit the firm without data first. If such a consumer would search thereafter, she would not visit the firm without data first (by the arguments of part 3). If she would not search therafter, her utility at the firm without data is $v - p^{nd}$, which converges to 0 as $v \to p^{nd}$. By contrast, their utility at the firm with data is at least $Pr^L(v)(v - p^L)$, which remains strictly positive for any such v. Since expected utilities are continuous in v, searchers with a valuation in an open ball above p^{nd} visit the firm with data, which implies the result.

Hence, $p^{nd} < \bar{v}$ must hold in an equilibrium (by the choices of searchers). However, if $\bar{v} < \bar{v}^{nd}$, such a price is not optimal for the firm without data. I establish this for two different kinds of equilibria, with (i) $p^H \leq p^{nd} + c$ and with (ii) $p^H > p^{nd} + c$.

(i) Case 1: Suppose we have an equilibrium in which $p^H \leq p^{nd} + c$.

To constitute an equilibrium, p^{nd} must lie strictly below $\bar{v}$. No arriving searcher will leave the firm without data for prices in an open ball around p^{nd}. A zero measure of searchers arrives at the firm without data after visiting its rival, because $p^H \leq p^{nd} + c$ (if $p^H = p^{nd} + c$, there would else be undercutting by the firm with data). Thus, for prices in an open ball around $p^{nd} < \bar{v}$, the profits of the firm without data are:

$$\Pi^{nd}(p_j;\bar{v}) = p_j\left[\rho\int_{\bar{v}}^{1} g(v)dv + 0.5(1-\rho)\int_{p_j}^{1} g(v)dv\right] \tag{B.1.16}$$

But for $\bar{v} < \bar{v}^{nd}$ and any $p^{nd} < \bar{v}$, the derivative at p^{nd} is strictly positive by the arguments made in the proof of proposition 7, a contradiction.

(ii) Case 2: $p^H > p^{nd} + c$.

If $\bar{v} \leq p^{nd} + c$, previous arguments directly imply the result, because the set of searchers who visit the firm with data first and search thereafter has measure zero (any such consumer must have $v > p^{nd} + c$ and $v < \bar{v}$, which cannot hold jointly).

If $\bar{v} > p^{nd} + c$, searchers with $v \in (p^{nd} + c, \bar{v})$ visit the firm with data first and then search iff they generate $\tilde{v}^H$. Since $p^H > p^{nd} + c$ holds by assumption, these consumers buy in an open ball around p^{nd}. In an open ball around p^{nd}, the profits at the firm without data are:

$$\Pi^{nd}(p_j;\bar{v}) = p_j\rho\int_{p^{nd}+c}^{\bar{v}} Pr^H(v)g(v)dv + p_j\rho\int_{\bar{v}}^{1} g(v)dv + p_j 0.5(1-\rho)\int_{p_j}^{1} g(v)dv \tag{B.1.17}$$

For any $\bar{v} < \bar{v}^{nd}$, the derivative of the second component is strictly positive for any $p_j < \bar{v}$. The derivative of the first component is positive. If $\bar{v} < \bar{v}^{nd}$, there would always be an upward deviation from any possible equilibrium p^{nd}. Hence, $\bar{v} \geq \bar{v}^{nd}$ must hold.

■

Proof of Proposition 10:

Any searcher who visits two firms with positive probability must either (i) visit the firm with data first with positive probability and search thereafter with positive probability or (ii) visit the firm without data first and search thereafter with positive probability.

Part 1: The set of consumers who visit the firm without data first (with positive probability) and search with positive probability thereafter must have measure zero.

This follows from the arguments made in the proof of lemma 8, part 3. Any searcher who visits the firm without data first in equilibrium would find it strictly optimal to not search thereafter.

Part 2: Under assumption 2, there exists no equilibrium in which a strictly positive measure of searchers visit the firm with data first and search thereafter with positive probability.

In such an equilibrium, $p^H > p^{nd} + c$ must hold. If $p^H < p^{nd} + c$, any searcher would find it strictly optimal to not search after any price the firm with data would offer to her in equilibrium, which implies the result. Suppose $p^H = p^{nd} + c$ holds in equilibrium and suppose, for a contradiction, that the set of searchers who visit the firm without data first and search thereafter has strictly positive measure. By lemma 8, it must hold that $p^L < p^{nd}$. Thus, p^L does not induce search. Any searcher who searches after visiting the firm with data must do so when receiving p^H. In a hypothetical equilibrium like this, the firm with data would prefer to undercut p^H, since this deters search by all searchers who visit both firms and hence do not buy at p^H (since $p^{nd} < p^H$), a contradiction.

Thus, suppose $p^{nd} + c < p^H$ holds in equilibrium and that the set of searchers who visit the firm without data first and search thereafter has strictly positive measure.

In such an equilibrium, the ordering $\bar{v} > p^{nd} + c$ must hold. To see this, suppose that $\bar{v} \leq p^{nd} + c$. By lemma 8, searchers who visit the firm with data first must have $v \in [0, \bar{v}]$. Moreover, searching after visiting the firm with data is only optimal if $v \geq p^{nd} + c$. Thus, the set of searchers who visit the firm with data first & search thereafter with positive probability is a subset of $[0, \bar{v}] \cap [p^{nd} + c, 1]$, which has zero measure because $\bar{v} \leq p^{nd} + c$, a contradiction.

Now let's consider the optimal pricing of the firm with data. We have proven that $p^{nd} + c < \bar{v}$ must hold. All searchers with $v \in [p^{nd} + c, \bar{v}]$ will visit the firm with data first and search when being offered p^H, which occurs with probability $Pr^H(v)$. Thus, the firm without data makes the sale to all these searchers at the price p^{nd} with probability $Pr^H(v)$. Since $p^{nd} + c < \bar{v}$, the firm without data will also make the sale to all searchers who initially visit it. For p_j in an open ball around p^{nd}, the profit function of the firm without data is hence:

$$p_j \left[\rho \int_{p^{nd}+s}^{\bar{v}} Pr^H(v) g(v) dv + \rho \int_{\bar{v}}^{1} g(v) dv + 0.5(1-\rho) \int_{p_j}^{1} g(v) dv \right] \tag{B.1.18}$$

Thus, an equilibrium p^{nd} must equal $p^{nd,3}(\bar{v})$, which solves:

$$\left[\rho \int_{p^{nd,3}+c}^{\bar{v}} Pr^H(v) g(v) dv + \rho \int_{\bar{v}}^{1} g(v) dv + 0.5(1-\rho) \int_{p^{nd,3}}^{1} g(v) dv \right] = 0.5(1-\rho) p^{nd,3} g(p^{nd,3}) \tag{B.1.19}$$

Finally, I make the following argument: Because $p^{nd,3}(1) + s > p^{HM}$ (which holds by assumption 3), such an equilibrium cannot exist.

In this equilibrium, $p^H = p^{H,M} > p^{nd} + c$ must be satisfied, where $p^{nd} = p^{nd,3}(\bar{v})$ must hold for the equilibrium level of $\bar{v}$, whatever this may be. Note that the function $p^{nd,3}(\bar{v})$ is falling in $\bar{v}$ (by concavity of the monopoly profit function of the firm without data). Thus, we have $p^{nd,3}(1) + c \leq p^{nd,3}(\bar{v}) + c$ for any possible $\bar{v}$. Moreover, note that $p^H = p^{H,M}$ must hold because the firm with data will only sell to captive consumers for prices in an open ball around the equilibrium p^H.

We have assumed that $p^{nd,3}(1) + c > p^{HM}$, noting that $p^{nd,3}(1) = p^{nd,s}$ a´s defined in assumption 6. Since $p^H = p^{H,M} < p^{nd,3}(\bar{v}) + c = p^{nd} + c$, this equilibrium cannot exist, because there exists no $\bar{v}$ at which the necessary conditions its existence are satisfied.

■

Proof of Proposition 11:

Part 1: The first two bullet points hold by lemma 5.

Part 2: When $\Pi^H(p^{nd,M}; \bar{v}^{nd}) > 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$ (assumption 7), the optimal $p^{H,*}(\bar{v})$ lies strictly below $\bar{v}$ for any $\bar{v} \geq \bar{v}^{nd}$ and will be strictly increasing in $\bar{v}$.

This follows from the definition of $\bar{v}^{HC}$ in the proof of proposition 8 and the accompanying discussion.

Part 3: Consider an equilibrium candidate in which $p^L < p^{nd} < p^H$, $p^H \leq p^{nd} + c$, $p^H < \bar{v}$, and $\hat{v}(p^L, p^H, p^{nd}) = \bar{v}$. It is optimal for searchers to visit the firm with data if and only if $v > \bar{v}$ and never search thereafter.

<u>Part 3a:</u> In such an equilibrium candidate, the cutoff $\tilde{v}^I(p^L, p^H, p^{nd})$ will lie strictly below $\hat{v}^I(p^L, p^H, p^{nd})$, where these cutoffs are defined as follows:

$$Pr^L(\tilde{v}^I(.))\big(p^{nd} - p^L\big) - s = 0 \quad ; \quad Pr^L(\hat{v}^I(.))p^L + Pr^H(\hat{v}^I(.))p^H - p^{nd} = 0 \qquad \text{(B.1.20)}$$

Any consumer with $v < \tilde{v}^I(.)$ would find it optimal to search after visiting the firm without data, provided that $v \geq p^{nd}$. Thus, all these consumers will prefer to visit the firm with data.

Note first that $Pr^L(v)p^L + Pr^H(v)p^H = p^{nd} \iff Pr^L(v)\big(p^{nd} - p^L\big) + Pr^H(v)\big(p^{nd} - p^H\big) = 0$. Now note that $p^{nd} + c \geq p^H$ by assumption, i.e. $p^{nd} - p^H \geq -s$. Thus:

$$0 = Pr^L(\tilde{v}^I)\big(p^{nd} - p^L\big) - s < Pr^L(\tilde{v}^I)\big(p^{nd} - p^L\big) + Pr^H(\tilde{v}^I)(p^{nd} - p^H) \qquad \text{(B.1.21)}$$

Since $\frac{\partial}{\partial v}\big[Pr^L(v)\big(p^{nd} - p^L\big) + Pr^H(v)\big(p^{nd} - p^H\big)\big] < 0$, we have $\tilde{v}^I(p^L, p^H, p^{nd}) < \hat{v}^I(p^L, p^H, p^{nd})$.

Part 3b: The postulated search behaviour is optimal.

Because $\hat{v}(p^L, p^H, p^{nd}) = \bar{v}$, $\bar{v}$ is either equal to $\hat{v}^I(p^L, p^H, p^{nd})$ or 1 (the latter being true if $\hat{v}^I(p^L, p^H, p^{nd}) \geq 1$). Define $p = (p^L, p^H, p^{nd})$. It was established that $\hat{v}^I(p) > \tilde{v}^I(p)$.

Suppose $\tilde{v}^I(p) \geq 1$ in equilibrium, which then implies that $\hat{v}^I(p) > 1$, and thus $\hat{v}(p) = 1 = \bar{v}$. For all consumers with $v < 1 \leq \tilde{v}^I(p)$, it is strictly optimal to visit the firm with data in equilibrium, i.e. to visit according to the rule represented by $\bar{v} = 1$. No searcher will search after visiting the firm with data since $p^H \leq p^{nd} + s$. No searcher who visits the firm without data first finds it optimal to search afterwards (since no such consumer exists).

Suppose $\tilde{v}^I(p) < 1$. Because $\tilde{v}^I(p) < \hat{v}^I(p)$ will also hold, $\hat{v}(p)$ is either 1 when $\hat{v}^I(p) \geq 1$ or $\hat{v}(p) = \hat{v}^I(p)$. In either case, $\tilde{v}^I(p) < \bar{v}$. Thus, any consumer with $v \geq \bar{v}$ finds it strictly optimal to not search after visiting the firm without data first. Because $p^H \leq p^{nd} + c$ and $\hat{v}(p) = \bar{v}$, she will visit the firm without data first and not search thereafter.

Any searcher with $v < \tilde{v}^I(p)$ visits the firm with data first and does not search thereafter. Any searchers with $v \in [\tilde{v}^I(p), \bar{v}]$ would not search after visiting either firm. Because $\bar{v} = \hat{v}(p)$, they hence optimally visit the firm with data.

Part 4: If $p^{H,1} \leq p^{nd,1} + c$, the vector $(p^{L,1}, p^{nd,1}, p^{H,1}, \bar{v}^1)$ is an equilibrium

Search: It is optimal for searchers to visit the firm with data if and only if $v > \bar{v}^1$ and never search thereafter.

The ordering $p^{L,1} < p^{nd,1} < p^{H,1}$ holds by construction, since $\hat{v}^B(\bar{v}^1) = \bar{v}^1$. The latter holds because assumption 7 guarantees that a solution to $\hat{v}^G(\bar{v}) = \bar{v}$ on $\bar{v} \in [\bar{v}^{nd}, 1]$ also solves $\hat{v}^B(\bar{v}) = \bar{v}$. By assumption 7, we also have $p^{H,1} < \bar{v}^1$. By specification, $p^{H,1} \leq p^{nd,1} + c$. Thus, the insights of part 3 apply and the result follows.

Pricing: There are no profitable deviations from the equilibrium prices, given that searchers split according to $\bar{v}^1$ and do not search thereafter (for equilibrium prices).

Consider first the firm without data. True competitive profits are bounded from above $\Pi^{nd}(p_j; \bar{v}^1)$. This is because no consumers arrive after search. For prices $p_j \in [0, p^{nd} + \epsilon]$, true profits equal this function. For prices sufficiently high, searchers leave this firm to search, implying that true profits are below $\Pi^{nd}(p_j; \bar{v}^1)$. By construction, $p^{nd,1}$ maximizes $\Pi^{nd}(p_j; \bar{v}^1)$, and so there will be no profitable deviations.

Analogous arguments show that the firm with data has no profitable deviations, because competitive profits are bounded from above by $\Pi^k(p_j; \bar{v}^1)$, conditional on the signal

$\tilde{v}^k$.

Part 5: If $p^{H,1} > p^{nd,1} + c$, there exists a $\bar{v}^2 \in [\bar{v}^{nd}, 1]$ s.t. $\hat{v}^S(\bar{v}^2) = \bar{v}^2$.

Recall that $p^{L,1} = p^{L,*}(\bar{v}^1)$, $p^{nd,1} = p^{nd,*}(\bar{v}^1)$, and $p^{H,1} = p^{H,*}(\bar{v}^1)$. In an equilibrium of category 2, $p^L = p^{L,*}(\bar{v})$, $p^{nd} = p^{nd,*}(\bar{v})$, $p^H = p^{nd,*}(\bar{v}) + c$. We are looking for a $\bar{v}$ that solves:

$$\bar{v} = \hat{v}^S(\bar{v}) := \sup \big\{ v \in [0,1] : \underbrace{Pr^L(v)p^{L,*}(\bar{v}) + Pr^H(v)(p^{nd,*}(\bar{v}) + c) - p^{nd,*}(\bar{v})}_{D^S(v;\bar{v}):=Pr^L(v)(p^{L,*}(\bar{v})-p^{nd,*}(\bar{v}))+Pr^H(v)s} < 0 \big\} \tag{B.1.22}$$

For any level of $\bar{v} \geq \bar{v}^{nd}$, we have $p^{L,*}(\bar{v}) < \bar{v}$ and $p^{nd,*}(\bar{v}) \leq \bar{v}$. As a result, these price functions will be continuous in $\bar{v}$. Since $p^{L,*}(\bar{v}) \leq p^{L,M} < p^{nd,M}$ and $p^{nd,*}(\bar{v}) \geq p^{nd,M}$, we have $p^{L,*}(\bar{v}) < p^{nd,*}(\bar{v})$ for any $\bar{v} \geq \bar{v}^{nd}$. Since $p^{H,1} > p^{nd,1} + s$, we have:

$$\underbrace{\hat{v}(p^{L,*}(\bar{v}^1), p^{nd,*}(\bar{v}^1) + c, p^{nd,*}(\bar{v}^1))}_{=\hat{v}^S(\bar{v}^1)} \geq \underbrace{\hat{v}(p^{L,*}(\bar{v}^1), p^{H,*}(\bar{v}^1), p^{nd,*}(\bar{v}^1))}_{=\hat{v}^B(\bar{v}^1)} \tag{B.1.23}$$

In words, this inequality means the following: When the firm with data sets the high signal price $p^{nd,*}(\bar{v}^1) + c$ instead of the higher $p^{H,*}(\bar{v}^1)$, more searchers arrive at the firm with data first (i.e. the LHS is greater), since the prices there are more attractive.

To see that there exists a desired fixed point, note that (i) $\hat{v}^S(1) \leq 1$ and (ii) $\hat{v}^S(\bar{v}^1) \geq \bar{v}^1$. The first point holds by construction. The second point holds because $\hat{v}^S(\bar{v}^1) \geq \hat{v}^B(\bar{v}^1)$ and $\hat{v}^B(\bar{v}^1) = \bar{v}^1$. Moreover, the function $\hat{v}^S(\bar{v})$ can be shown to be continuous on $[\bar{v}^{nd}, 1]$, because $p^{L,*}(\bar{v})$ and $p^{nd,*}(\bar{v})$ are continuous. Thus, the intermediate value theorem implies the result.

Part 6: Suppose $p^{H,1} > p^{nd,1} + c$. At $\bar{v}^2$, the following two conditions are satisfied: (i) $p^{H,2} < \bar{v}^2$ and (ii) $p^{H,2} < arg\max_{p_j} \Pi^H(p_j; \bar{v}^2) := p^{H,*}(\bar{v}^2)$.

<u>Part 6a:</u> If $p^{H,*}(\bar{v}^1) > p^{nd,*}(\bar{v}^1) + s$ holds, previous results imply that $\bar{v}^2 \geq \bar{v}^1$.

Suppose $\bar{v}^1 = 1$. Then, $\bar{v}^2 = \bar{v}^1$ must be true, by previous arguments. Suppose instead that $\bar{v}^1 < 1$. Then $Pr^L(v)p^{L,*}(\bar{v}^1) + Pr^H(v)p^{L,*}(\bar{v}^1) = p^{nd,*}(\bar{v}^1)$ must hold. Because $p^{H,*}(\bar{v}^1) > p^{nd,*}(\bar{v}^1) + s$, we have:

$$\hat{v}(p^{L,*}(\bar{v}^1), p^{nd,*}(\bar{v}^1) + c, p^{nd,*}(\bar{v}^1)) - \bar{v}^1 > \hat{v}(p^{L,*}(\bar{v}^1), p^{H,*}(\bar{v}^1), p^{nd,*}(\bar{v}^1)) - \bar{v}^1 = 0 \tag{B.1.24}$$

Note that $\hat{v}(p^{L,*}(\bar{v}), p^{nd,*}(\bar{v})+c, p^{nd,*}(\bar{v}))$ is weakly decreasing in $\bar{v}$ because $p^{L,*}(\bar{v})$ is rising in $\bar{v}$ and $p^{nd,*}(\bar{v})$ is falling in $\bar{v}$. Thus, the function $\hat{v}^S(p^{L,*}(\bar{v}), p^{nd,*}(\bar{v}) + c, p^{nd,*}(\bar{v})) - \bar{v}$ is strictly decreasing in $\bar{v}$ and is strictly positive at $\bar{v}^1$. Hence, $\bar{v}^2 \geq \bar{v}^1$ must hold.

Part 6b: Since $\bar{v}^2 \geq \bar{v}^1$, $p^{H,2} < arg\max_{p_j} \Pi^H(p_j; \bar{v}^2) = p^{H,*}(\bar{v}^2)$ and $p^{H,2} < \bar{v}^2$ holds.

Note that $\bar{v}^1 \in [\bar{v}^{nd}, 1]$. Because $\bar{v}^2 \geq \bar{v}^1 \geq \bar{v}^{nd}$, $arg\max_{p_j} \Pi^H(p_j; \bar{v}^2) = p^{H,*}(\bar{v}^2)$ will be strictly below $\bar{v}^2$ and solve a FOC. Because $\bar{v}^2 \geq \bar{v}^1$, we know that the prices satisfy: (i) $p^{k,*}(\bar{v}^2) \geq p^{k,*}(\bar{v}^1)$ and (ii) $p^{nd,*}(\bar{v}^2) \leq p^{nd,*}(\bar{v}^1)$. Thus:

$$p^{H,2} = p^{nd,*}(\bar{v}^2) + s \leq p^{nd,*}(\bar{v}^1) + c < p^{H,*}(\bar{v}^1) \leq p^{H,*}(\bar{v}^2) < \bar{v}^2 \tag{B.1.25}$$

Part 7: If $p^{H,1} > p^{nd,1} + c$, the vector $(p^{L,2}, p^{nd,2}, p^{H,2}, \bar{v}^2)$ is an equilibrium

Part 7a: At $\bar{v}^2$, $\Pi^H(p^{2,H}; \bar{v}^2) \geq 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$ holds by assumption 7.

Note that $p^{H,2} = p^{nd,*}(\bar{v}^2) + c > p^{nd,M}$, since $p^{nd,*}(\bar{v}^2) > p^{nd,M}$. High signal profits for $p_j < \bar{v}^2$, which includes $p^{H,2}$ since $p^{H,2} < p^{H,*}(\bar{v}^2) < \bar{v}^2$ are:

$$\Pi^H(p_j; \bar{v}^2) = p_j \rho \int_{p_j}^{\bar{v}^2} Pr^H(v) g(v) dv + p_j 0.5(1-\rho) \int_{p^H}^{1} Pr^H(v) g(v) dv \tag{B.1.26}$$

We know that this function is strictly concave on $p_j \in [0, \bar{v}^2]$ and that $p^{nd,M} < p^{H,2} < p^{H,*}(\bar{v}^2) < \bar{v}^2$. Thus, profits from setting $p^{nd,M}$, namely $\Pi^H(p^{nd,M}; \bar{v}^2)$, will be below equilibrium profits, namely $\Pi^H(p^{H,2}; \bar{v}^2)$. Moreover, we have $\bar{v}^2 > \bar{v}^{nd}$, which also implies that $\Pi^H(p^{nd,M}; \bar{v}^2) \geq \Pi^H(p^{nd,M}; \bar{v}^{nd})$. By assumption, the final component is above $\Pi^{H,M}(p^{H,M})$.

Part 7b: The search behavior represented by the cutoff $\bar{v}^2$ will be optimal by the arguments in part 3.

This is because $p^{L,2} < p^{nd,2} < p^{H,2}$ and $p^{H,2} = p^{nd,2} + c$ hold by construction, $p^{H,2} < \bar{v}^2$ by part 6, and $\hat{v}(p^{L,2}, p^{nd,2}, p^{H,2}) = \bar{v}^2$ holds by definition.

Part 7c: The prices $(p^{L,2}, p^{nd,2}, p^{H,2})$ are optimal for firms.

Since no consumer leaves to search on the equilibrium path, $\Pi^{nd}(p_j; \bar{v})$ and $\Pi^L(p_j; \bar{v})$ are upper bounds for the true respective objective functions. Since the former are both globally maximized by our prices for the given $\bar{v}^2 > \bar{v}^{nd}$, we know there can be no profitable deviations from them p^{nd} or p^L.

Now consider the optimal pricing calculus of the firm with data when observing $\tilde{v}^H$. Part 6 established that $p^{H,2} < arg\max_{p_j} \Pi^H(p_j;\bar{v}^2)$ and $p^{H,2} < \bar{v}^2$. Because $\Pi^H(p_j;\bar{v}^2)$ is strictly concave on $p_j \in [0,\bar{v}^2]$, there cannot be any profitable downward deviations $p_j < p^{H,2}$, because these would just yield profits of $\Pi^H(p_j;\bar{v}^2)$. Any upward deviation would, at best, yield profits equal to $0.5(1-\rho)\Pi^{H,M}(p^{H,M})$. This deviation is not profitable by the result in part 7a. No other possible deviations remain. Thus, it is optimal for firms to set said prices.

■

Proof of Proposition 12:

Part 1: In equilibrium, the low signal price of the firm with data must be given by $p^{L,*}(\bar{v})$ and the price of the firm without data must be given by $p^{nd,*}(\bar{v})$.

Under assumptions 6 and 7, the search strategy of searchers must be a cutoff rule. Moreover, $\bar{v} \geq \bar{v}^{nd}$ must hold, which implies that $p^{nd,*}(\bar{v}) < \bar{v}$ must hold and must solve $\frac{\partial \Pi^{nd}(p^{nd,*}(\bar{v});\bar{v})}{\partial p_j} = 0$. Similarly, the equilibrium price p^{nd} must be between p^L and $\bar{v}$, for which the profits of the firm without data are given by $\Pi^{nd}(p_j;\bar{v})$. Thus, the optimal price of the firm without data must solve the same first-order condition, to which there is a unique solution because $\Pi^{nd,M}(p_j)$ is strictly concave. Thus, p^{nd} must be equal to $p^{nd,*}(\bar{v})$.

For any $\bar{v} \geq \bar{v}^{nd}$ that we consider, the optimal low signal price must be below $\bar{v}$, because $\bar{v}^{nd} > p^{nd,M} > p^{L,M}$. Moreover, note that $p^L < p^{nd} < p^H$ must hold in an equilibrium in which firms play pure strategies. Thus, searchers with $v \in [p^L - \epsilon, \bar{v}]$ visit the firm with data, so this firm's profits in an open ball around p^L must be given by $\Pi^L(p_j;\bar{v})$. Thus, the optimal price p^L must satisfy a corresponding first-order condition, to which there will be a unique solution, namely $p^{L,*}(\bar{v})$.

Part 2: For any $\bar{v} \geq \bar{v}^{nd}$, both $\hat{v}^B(\bar{v})$ and $\hat{v}^S(\bar{v})$ are weakly falling in $\bar{v}$.

Our assumptions guarantee that, for any $\bar{v} \geq \bar{v}^{nd}$, the functions $p^{L,*}(\bar{v})$, $p^{H,*}(\bar{v})$, and $p^{nd,*}(\bar{v})$ are continuous in $\bar{v}$. Moreover, $p^{L,*}(\bar{v})$ and $p^{H,*}(\bar{v})$ are rising in $\bar{v}$, while $p^{nd,*}(\bar{v})$ is falling in $\bar{v}$. Now consider $\hat{v}^B(\bar{v})$, which is given by:

$$\hat{v}^B(\bar{v}) = \sup\left\{v \in [0,1] : Pr(\tilde{v}^L|v)p^{L,*}(\bar{v}) + Pr(\tilde{v}^H|v)p^{H,*}(\bar{v}) - p^{nd,*}(\bar{v}) < 0\right\} \quad \text{(B.1.27)}$$

Consider two $\bar{v}', \bar{v}''$, with $\bar{v}' < \bar{v}''$. The function in $Pr(\tilde{v}^L|v)p^{L,*}(\bar{v}) + Pr(\tilde{v}^H|v)p^{H,*}(\bar{v}) - p^{nd,*}(\bar{v})$ is rising in $\bar{v}$. Thus, any valuation $v \in \left\{v \in [0,1] : Pr(\tilde{v}^L|v)p^{L,*}(\bar{v}'') + Pr(\tilde{v}^H|v)p^{H,*}(\bar{v}'') - p^{nd,*}(\bar{v}'') < 0\right\}$ will also be in $\left\{v \in [0,1] : Pr(\tilde{v}^L|v)p^{L,*}(\bar{v}') + Pr(\tilde{v}^H|v)p^{H,*}(\bar{v}') - p^{nd,*}(\bar{v}') <\right.$

$0\big\}$. This implies that $\hat{v}^B(\bar{v}'') \leq \hat{v}^B(\bar{v}')$ since $\bar{v}' < \bar{v}''$.

Now consider $\hat{v}^S(\bar{v})$. Because the function $Pr(\tilde{v}^L|v)\big[p^{L,*}(\bar{v}) - p^{nd,*}(\bar{v})\big] + Pr(\tilde{v}^H|v)s$ is increasing in v and increasing in $\bar{v}$, analogous arguments imply that $\hat{v}^S(\bar{v})$ is falling in $\bar{v}$.

Part 3: When $p^{H,1} \leq p^{nd,1} + c$, $\bar{v} = \bar{v}^1$ must hold in equilibrium and the equilibrium prices are uniquely determined.

Consider any $\bar{v} < \bar{v}^1$. At $\bar{v}^1$, high signal profits are equal to $\Pi^H(p_j; \bar{v}^1)$ for $[p^{L,*}(\bar{v}^1), p^{nd,*}(\bar{v}^1)+c]$. Recall that $p^{nd,*}(\bar{v})$ is falling in $\bar{v}$ while $p^{H,*}(\bar{v})$ is rising in $\bar{v}$ as long as $\bar{v} \in [\bar{v}^{nd}, 1]$, which must hold in equilibrium. For any $\bar{v} \in [\bar{v}^{nd}, \bar{v}^1]$, we thus have: $p^{H,*}(\bar{v}) < p^{nd,*}(\bar{v}) + c$.

Thus, $p^{H,*}(\bar{v}) \in [p^{L,*}(\bar{v}), p^{nd,*}(\bar{v}) + c]$ holds for any $\bar{v} \in [\bar{v}^{nd}, \bar{v}^1]$. This means that $p^{H,*}(\bar{v})$ is the unique optimal price to set, because it strictly maximizes $\Pi^H(p_j; \bar{v})$. Since $\hat{v}^B(\bar{v})$ is weakly decreasing (by part 2), $\hat{v}^B(\bar{v}) > \bar{v}$ for any $\bar{v}$ under consideration, so we cannot have an equilibrium at these values $\bar{v} < \bar{v}^1$.

Consider any $\bar{v} > \bar{v}^1$. When $p^{nd,*}(\bar{v}) + c < p^{H,*}(\bar{v})$, the high signal price must be equal to $p^{nd,*}(\bar{v}) + c$ to constitute an equilibrium. If the price is above $p^{nd,*}(\bar{v}) + c$, it must optimally be $p^{H,M}$, since the sale would only be made to captive consumers. This cannot constitute an equilibrium, because $p^{H,M} \geq p^{H,*}(\bar{v}) > p^{nd,*}(\bar{v}) + c$, i.e. there would be search on the equilibrium path, a contradiction. A price below $p^{nd,*}(\bar{v}) + c < p^{H,*}(\bar{v})$ cannot be optimal, since profits would be equal to $\Pi^H(p_j; \bar{v})$, which are strictly increasing for any $p_j < p^{H,*}(\bar{v})$.

Find that $\bar{v}'$ for which $p^{nd,*}(\bar{v}') + c = p^{H,*}(\bar{v}')$. All $\bar{v} \in [\bar{v}^1, \bar{v}')$ could not have been equilibria, because the optimal high signal price is $p^{H,*}(\bar{v})$ and $\hat{v}^B(\bar{v}) < \bar{v}$. Now consider any $\bar{v} \geq \bar{v}'$. Because $\bar{v}' \geq \bar{v}^1$, we have $\hat{v}^S(\bar{v}') = \hat{v}^B(\bar{v}') \leq \bar{v}'$. We know that $\hat{v}^S(\bar{v})$ is weakly falling in $\bar{v}$, which implies that we cannot have a fixed point of $\hat{v}^S(\bar{v})$, and thus no equilibrium, in the interval $(\bar{v}', 1]$. It only remains to consider $\bar{v}'$. If this equals $\bar{v}^1$, it is an equilibrium. If $\bar{v}' > \bar{v}^1$, $\hat{v}^S(\bar{v}') = \hat{v}^B(\bar{v}') < \bar{v}'$, and we cannot have an equilibrium.

Thus, $\bar{v} = \bar{v}^1$ holds in equilibrium. The low signal price and the uniform price of the firm with data must be $p^{L,*}(\bar{v})$ and $p^{nd,*}(\bar{v})$ by previous logic. The high signal price must be $p^{H,*}(\bar{v})$. Any other price yields strictly lower profits. All these prices are uniquely determined.

Part 4: When $p^{H,1} > p^{nd,1} + c$, $\bar{v} = \bar{v}^2$ must hold in equilibrium. The equilibrium prices are uniquely determined.

Note that $p^{nd,*}(\bar{v}) + c$ is falling in $\bar{v}$ and $p^{H,*}(\bar{v})$ is rising for $\bar{v} \in [\bar{v}^{nd}, 1]$. Thus, find the $\bar{v}'$ such that: $p^{H,*}(\bar{v}') = p^{nd,*}(\bar{v}') + c$. Because $p^{H,1} > p^{nd,1} + c$, we know $\bar{v}^1 > \bar{v}'$. For any $\bar{v} < \bar{v}'$, $p^{H,*}(\bar{v}) < p^{nd,*}(\bar{v}) + c$, i.e. the optimal price is $p^{H,*}(\bar{v})$. Thus, no value $\bar{v} < \bar{v}'$ can be an equilibrium, because $\hat{v}^B(\bar{v}) > \bar{v}$ holds there.

Consider any $\bar{v} \in [\bar{v}', 1]$. The high signal price must be equal to $p^{nd,*}(\bar{v}) + c$ in equilibrium, i.e. $\hat{v}^S(\bar{v}) = \bar{v}$ would have to hold to constitute an equilibrium. The function $\hat{v}^S(\bar{v})$ is weakly falling, so there is at most one candidate for an equilibrium.

By previous arguments, the prices must be $p^{L,*}(\bar{v}^2)$, $p^{nd,*}(\bar{v}^2)$, and $p^{H,*}(\bar{v}^2)$.

■

Proof of Proposition 13:

Define $[\underline{p}^j, \bar{p}^j]$ as the convex hull of the support of the price distribution offered by either firm $j \in nd, d$. The search strategy of searchers is $s(v)$ and searchers continue searching after visiting firm j if and only if the price is above $\hat{p}^j(v)$. I show that there exists no equilibrium in which firms mix by considering all possible cases: (i) $\underline{p}^{nd} < \underline{p}^d$, (ii) $\underline{p}^{nd} > \underline{p}^d$, and (iii) $\underline{p}^{nd} = \underline{p}^d$.

Part 1: There exists no equilibrium in which firms mix and $\underline{p}^{nd} < \underline{p}^d$.

Suppose $\underline{p}^{nd} < \underline{p}^d$. The price $\underline{p}^{nd}$ is played with probability 1. Suppose, for a contradiction, that it is part of a mixed strategy. By the restriction of connected support, there exists multiple prices below $\underline{p}^d$ that are played by the firm without data.

All searchers with $v < \underline{p}^d$ will surely visit the firm without data first. For any $p_j < \underline{p}^d$, no searcher who arrives at the firm without data first will search. Consumers who arrive at the firm without data second has $v \geq \underline{p}^d$ and must have received a price strictly above $\underline{p}^d$. Thus, they entail inelastic demand when the firm without data offers a price $p_j \in [\underline{p}^{nd}, \underline{p}^d]$. Thus, the firm with data makes the following profits when setting any price $p_j \in [\underline{p}^{nd}, \underline{p}^d]$:

$$\Pi^{nd}(p_j) = p_j \Big[\rho \int_{p_j}^{\underline{p}^d} g(v)dv + \rho \int_{\underline{p}^d}^{1} \big[s(v)Pr(p^d > \hat{p}^s(v)) + (1 - s(v)) \big] g(v)dv + \\ 0.5(1-\rho) \int_{p_j}^{1} g(v)dv \Big] \tag{B.1.28}$$

This function is strictly concave (since $\Pi^{nd}(p_j)$ is strictly concave), which implies that the firm without data cannot make the same profits for the different prices in $[\underline{p}^{nd}, \underline{p}^d]$ it offers, a contradiction to mixing indifference. Thus, $\underline{p}^{nd}$ is played with probability 1.

Now consider the prices of the firm with data. Because $\underline{p}^{nd}$ is played with probability 1 and $\underline{p}^{nd} < \underline{p}^d$, all searchers with $v \geq \underline{p}^d$ never arrive at the firm with data. Thus, the firm with data just makes the sale to its captive consumers for any price $p_j > \underline{p}^d$ and makes scaled monopoly profits. This implies that the firm with data would not mix, since its monopoly profit functions are strictly concave. Thus, firms do not mix in an equilibrium of category (i).

Part 2: There exists no equilibrium in which firms mix and $\underline{p}^d < \underline{p}^{nd}$.

Suppose $\underline{p}^d < \underline{p}^{nd}$. As before, $\underline{p}^d$ must be played with probability 1 by the firm with data in the corresponding information set (no matter whether this price is played after seeing $\tilde{v}^L$ or $\tilde{v}^H$). This is because all searchers with valuation below $\underline{p}^{nd}$ visit the firm with data. All searchers with valuation above $\underline{p}^{nd}$ will generate inelastic demand for the firm with data around $\underline{p}^d$, because no such consumer would search when receiving a price below $\underline{p}^{nd}$.

Now consider the optimal pricing of the firm without data. No consumer who visits the firm without data first in equilibrium would search after receiving $\underline{p}^{nd}$. Suppose, for a contradiction, that a searcher with valuation v visits the firm without data first and finds it weakly optimal to search when receiving the price $\underline{p}^{nd}$, which means that she will search for any price she can receive at this firm. Because $\underline{p}^d < \underline{p}^{nd}$, it would be better for any such searcher to initially visit the firm with data. This is because this endows the consumer with an option value of saving search costs, which she can do when receiving $\underline{p}^d$, the best equilibrium price.

Thus, any searcher who visits the firm without data first must find it strictly optimal to not search at $\underline{p}^{nd}$. Because search preferences are continuous in the initial price, searchers will also not search if offered a price just above $\underline{p}^{nd}$ (by the dominated convergence theorem).

There exist $\epsilon > 0$ and $\delta > 0$ such that: (i) Searchers with $v \in [\underline{p}^d, \underline{p}^{nd} + \epsilon]$ visit the firm with data first. Setting ϵ small enough also implies that these consumers would never search thereafter, and (ii) searchers who visit the firm without data first don't search if offered $p_j \in [\underline{p}^{nd}, \underline{p}^{nd} + \delta]$. Moreover, searchers who arrive at the firm without data second buy if offered $p_j \in [\underline{p}^{nd}, \underline{p}^{nd} + c]$ (else, it would not be optimal to pay the search cost to visit this firm).

This establishes that $\underline{p}^{nd}$ will be played with probability 1. To see this, set $\tau = \min\{\epsilon, \delta, c\}$. For all $p_j \in [\underline{p}^{nd}, \underline{p}^{nd} + \tau]$, the profits of the firm without data are:

$$\Pi^{nd}(p_j) = p_j \left[\rho \int_{p^{nd}+\epsilon}^{1} \big[s(v) Pr(\hat{p}^s(v) > p^d) + (1 - s(v))(1) \big] g(v) dv + 0.5(1-\rho) \int_{p_j}^{1} g(v) dv \right] \tag{B.1.29}$$

The demand implied by searchers is fully inelastic for these prices. This means that the profits of the firm without data are strictly concave for all $p_j \in [\underline{p}^{nd}, \underline{p}^{nd} + \tau]$. If this firm mixes, it must offer multiple prices in this interval by the restriction of connected support. This violates the mixing indifference condition, a contradiction.

We have established that $\underline{p}^d$ and $\underline{p}^{nd}$ both have to be played with probability 1. Thus, the only possibility of mixing is that the firm with data mixes after one of the two signals. Define that the firm with data mixes after $\tilde{v}^m$ and that the convex hull of the support of the associated price distribution is $[\underline{p}^m, \bar{p}^m]$. No consumer who visits the firm without data first will search thereafter (by an option value logic, as above). Moreover, searchers who visit the firm with data first will search if $v > \underline{p}^{nd} + c$ and the price they receive is above this cutoff.

It cannot hold that $\underline{p}^{nd} + c < \bar{p}^m$. Then, all searchers will surely not consume at the firm with data for $p_j \in [\underline{p}^{nd} + c, \bar{p}^m]$, which means profits only accrue from captive consumers. Since these are strictly concave, there is a contradiction (by the restriction of connected support).

Finally, suppose that $\bar{p}^m \leq \underline{p}^{nd} + c$, i.e. that none of the prices played after $\tilde{v}^m$ trigger search. Then, we can show that $\underline{p}^d$, which is strictly lower than $\bar{p}^m$ by the assumption that the firm with data is mixing, must be played after the low signal. If $\underline{p}^d$ were played after $\tilde{v}^H$, there would be a contradiction by hazard ratio ordering arguments (since no price triggers search).

Since $\underline{p}^d < \bar{p}^m$ and $\underline{p}^d$ is played after $\tilde{v}^L$, the strategy of searchers ($s(v)$) will be a cutoff rule, because the price distribution at the firm with data becomes strictly less favorable as a consumer's valuation increases. Searchers will visit the firm with data only if their valuation is below $\bar{v}$. Because $\bar{p}^m \leq \bar{v}$ must hold (else the firm only sells to captive consumers for a subinterval of prices), profits from any price $p_j \in [\underline{p}^m, \bar{p}^m]$ are $\Pi^H(p_j; \bar{v})$ as defined in equation (3.4.4). But this is strictly concave, so the firm with data would also never mix.

Part 3: There exists no equilibrium in which $\underline{p}^d = \underline{p}^{nd}$

Suppose $\underline{p}^d = \underline{p}^{nd}$. For prices in an open ball above the lowest price, no consumer that arrives at any firm will leave to search (any consumers who arrive after searching directly buy). Consider such an interval of prices, and call it $[\underline{p}^d, \underline{p}^d + \epsilon]$, s.t. $\underline{p}^d + \epsilon < \bar{p}^d$.

Even if some individual prices in this interval are played with positive probability, the preferences that consumers with $v \in [\underline{p}^d, \underline{p}^d + \epsilon]$ have over which firm to visit will be continuous in v. This can be shown by applying the dominated convergence theorem.

Suppose that there exists a $v' \in [\underline{p}^d, \underline{p}^d + \epsilon]$ such that a searcher with valuation v' strictly prefers to visit the firm with data. Then, consumers with valuation in an open ball with radius δ around v' will also strictly prefer to visit the firm with data first. As a result, setting any price $p_j \in [v', \underline{p}^d + \epsilon]$ will yield the following profits for the firm with data:

$$p_j \Big[\rho \int_{p_j}^{v'+\delta} Pr^k(v) g(v) dv + \rho \int_{v'+\delta}^{1} \big[s(v) + (1 - s(v)) Pr(\hat{p}^{nd}(v) < p^{nd}) \big] Pr^k(v) g(v) dv +$$

$$0.5(1-\rho)\int_{p_j}^{1} Pr^k(v)g(v)dv\Big] \tag{B.1.30}$$

This profit function is strictly concave in this domain, a contradiction to mixing indifference.

Similar arguments rule out that some searchers with $v \in [\underline{p}^d, \underline{p}^d + \epsilon]$ strictly prefer to visit the firm without data. Thus, they must all be indifferent and randomize by our tie-breaking rule. But then, the firm without data would make the following profits for any price $p_j \in [\underline{p}^d, \underline{p}^d + \epsilon]$:

$$p_j\Big[\rho\int_{p_j}^{\underline{p}^d+\epsilon}(0.5)g(v)dv + \rho\int_{\underline{p}^d+\epsilon}^{1}\big[(1-s(v)) + s(v)Pr(\hat{p}^d(v) < p^d)\big]g(v)dv +$$

$$0.5(1-\rho)\int_{p_j}^{1} g(v)dv\Big] \tag{B.1.31}$$

But this profit function is strictly concave once more. Thus, the firm without data would have to set this lowest price with probability 1. If the firm with data sets this price with probability 1 as well, we have no MSE. Alternatively, it sets it with probability below 1. Then, all searchers with $v > \underline{p}^{nd}$ visit the firm without data and don't search. Thus, the firm with data would not mix, because it sells only to captive consumers for any of its prices.

■

Proof of Corollary 5:

As $\rho \to 1$, assumptions 6 and 7 both hold. First, note that $p^{nd,s}(\rho)$ as defined in equation (3.4.9) satisfies $p^{nd,s}(\rho) \to \max\{p^{nd,M}, 1-c\}$ as $\rho \to 1$. Thus, $\lim_{\rho\to 1}[p^{nd,s}(\rho) + c] \geq 1 > p^{H,M}$, i.e. assumption 6 is satisfied. Now consider assumption 7, namely $\Pi^H(p^{nd,M}, \bar{v}^{nd}) > 0.5(1-\rho)\Pi^{H,M}(p^{H,M})$. As $\rho \to 1$, the LHS goes to something strictly positive, while the RHS goes to 0. Thus, the assumption is satisfied as well.

As $\rho \to 1$, there is no search on the equilibrium path and $\bar{v} \geq \bar{v}^{nd}$ holds in equilibrium by the previous results. Thus, $\bar{v} \to 1$ as $\rho \to 1$, which implies the result.

■

Proof of Corollary 6:

Part 1: The equilibrium $\bar{v}$ weakly decreases in c.

First, consider $c > p^{H,1} - p^{nd,1}$. Then, the equilibrium $(p^{L,1}, p^{H,1}, p^{nd,1}, \bar{v}^1)$ is played, in which $\bar{v}$ is unaffected by c. Second, consider $c \leq p^{H,1} - p^{nd,1}$. Then, the equilibrium

$(p^{L,2}, p^{H,2}, p^{nd,2}, \bar{v}^2)$ is played, in which $\hat{v}(p^{L,*}(\bar{v}^2), p^{nd,*}(\bar{v}^2) + s, p^{nd,*}(\bar{v}^2)) - \bar{v}^2 = 0$.

Consider two values of c for which $c \leq p^{H,1} - p^{nd,1}$ and call them c' and c'', with $c'' > c'$. Define the resulting equilibrium levels of $\bar{v}$ as $\bar{v}^2(c'') := \bar{v}^{2,''}$ and $\bar{v}^2(c') := \bar{v}^{2,'}$. I show that $\bar{v}^2(c'') \leq \bar{v}^2(c')$. If $\bar{v}^{2,'} = 1$, the result is immediate.

Thus, suppose that $\bar{v}^{2,'} < 1$. Then, $\bar{v}^{2,'}$ must set the expected prices exactly equal. For $c'' > c'$, we thus have: $Pr^L(\bar{v}^{2,'})(p^{L,*}(\bar{v}^{2,'}) - p^{nd,*}(\bar{v}^{2,'})) + Pr^H(\bar{v}^{2,'})c'' > 0$. This implies that $\hat{v}^S(\bar{v}^{2,'}) - \bar{v}^{2,'} < 0$ at c''. Because said expression is falling in $\bar{v}$, it must hold that $\bar{v}^{2,''} < \bar{v}^{2,'}$.

Part 2: Market shares

When $c > p^{H,1} - p^{nd,1}$, changes of c do not affect the equilibrium outcomes.

Suppose $v \sim U[0,1]$ and that $Pr^L(v)$ is linear. Then, $g(p)p = p$ is rising in p. Moreover, the function $g(p)pPr^L(p)$ is rising in p for any $p < 0.5$ (note that $p^{L,M} < 0.5$ in our example).

As c rises, $\bar{v}$ falls. This reduces the high signal demand of the firm with data and increases the high signal price because $p^{nd,*}(\bar{v})$ is falling in $\bar{v}$. The high signal price further rises because c rises. Thus, the high signal demand of the firm with data falls. Moreover, the low signal demand of the firm with data falls for any relevant p. This reduction of demand will, by strict concavity of the low signal profit function, lead to a decrease in p^L. Note that $p^L \leq p^{L,M} < 0.5$ must hold. Because $g(p)pPr^L(p)$ is rising in p when $p \in [0, 0.5]$, and this equals demand by the FOC that p^L must satisfy, the equilibrium low signal demand falls.

Now consider the firm without data. Because $\bar{v}$ falls, the demand of the firm without data rises for any relevant price. This will trigger an increase of the price, which leads to higher equilibrium demand for the firm without data because $g(p)p$ is rising in p.

Thus, total demand of the firm with data is falling in c and the demand of the firm without data is rising. Thus, the sales based market share of the firm with data falls in c.

$\blacksquare$

B.2 Proofs — Section 3.5.

Proof of Proposition 14:

If $e > 0$, it is optimal to exercise this right only if $v \geq p^a + e$. Suppose the right to anonymity is exercised by a positive measure of consumers. Thus, the corresponding information set for the firm with data is on-path and this firm must believe that a consumer who has anonymized is a searcher and has $v \geq p^a + e$. Thus, there is a profitable upward deviation from p^a.

Now consider $e = 0$ and suppose that a strictly positive measure of consumers exercises the right to anonymity.

Suppose $p^{nd} < p^a$. Then, any searcher with $v > p^{nd}$ would not visit the firm with data and utilize their right to anonymity. If a consumer exercises this right, she must have $v < p^{nd}$. But then, setting the price p^a would be suboptimal, a contradiction.

Suppose $p^a \leq p^{nd}$. Then, all searchers weakly prefer to visit the firm with data. Suppose $p^L \neq p^H$. Then, $p^L < p^a < p^H$ must hold. But then, consumers with $v \in (p^L, p^a]$ will not exercise the right to anonymity. Thus, there is a profitable upward deviation from p^a, as the firm with data knows that any searcher who anonymizes (and has $v > p^L$) has a valuation strictly above p^a.

Thus, $p^L = p^H$ must hold. If p^a is not equal to p^L, either no consumer will anonymize (a contradiction to the premise) or all searchers anonymize (then there is a contradiction to the postulated ordering of prices). The final case is hence $p^L = p^H = p^a$. By assumption, consumers then randomize between anonymizing and not anonymizing. Then, we obtain several contradictions. For instance, $p^L = p^H$ would not be optimal.

$\blacksquare$

Proof of Proposition 15:

There are five equilibrium prices: the prices of the firm with data (p^L, p^H), the uniform price of the firm without data (p^{nd}) and the signal prices at this firm $(p^{L,nd}, p^{H,nd})$. I will construct an equilibrium in which (i) searchers with $v \in [0, v^t)$ visit the firm without data and port their data, and (ii) searchers with $v \in (v^t, 1]$ visit the firm without data but do not port their data.

<u>Setting up the equilibrium candidate:</u>

The prices $p^{L,nd}$ and $p^{H,nd}$ must, given v^t, solve $p^{k,nd}(v^t) = arg\max_{p_j} \left[p_j \int_{p_j}^{v^t} \rho Pr^k(v)g(v)dv\right]$. Thus, these optimal prices will be strictly below v^t. The price p^{nd} must maximize:

$$\Pi^{nd}(p_j; v^t) = p_j \left[\rho \int_{v^t}^{1} \mathbb{1}[p_j \leq v]g(v)dv + 0.5(1-\rho) \int_{0}^{1} \mathbb{1}[p_j \leq v]g(v)dv\right] \qquad \text{(B.2.1)}$$

In order for the search behavior we posited to be optimal, we need to have $p^{nd} < v^t$. This, in turn, implies that $v^t \geq \bar{v}^{nd}$ must hold. For any $v^t \geq \bar{v}^{nd}$, the price p^{nd} will equal $p^{nd,*}(v^t)$. Because all prices must be below v^t in equilibrium, v^t must solve:

$$v^t = \underbrace{\sup\left\{v \in [0,1] : Pr^H(v)p^{H,nd}(v^t) + Pr^L(v)p^{L,nd}(v^t) - p^{nd}(v^t) < 0\right\}}_{:=\hat{v}^T(v^t)} \qquad \text{(B.2.2)}$$

Previous arguments show that the function in this supremum is rising in v, which means we have a well-defined supremum. The function $\hat{v}^T(v^t)$ is continuous because all price functions are continuous in v^t. At $v^t = \bar{v}^{nd}$, we have $\hat{v}^T(\bar{v}^{nd}) = 1 > \bar{v}^{nd}$. At $v^t = 1$, we have $\hat{v}^T(1) \leq 1$ by definition. Thus, the intermediate value theorem guarantees that $v^t = \hat{v}^T(v^t)$ holds at some $v^t \geq \bar{v}^{nd}$.

Based on this, I construct the following candidate for an equilibrium: The firm with data sets the prices $(p^{L,M}, p^{H,M})$. The firm without data sets the prices $(p^{L,nd}(v^t), p^{H,nd}(v^t), p^{nd}(v^t))$, where $v^t = \hat{v}^T(v^t)$. Searchers with $v \in [0, v^t)$ visit the firm without data and port their data, and (ii) searchers with $v \in (v^t, 1]$ visit the firm without data but do not port their data.

Equilibrium verification:

The search behaviour in the posited equilibrium is optimal, given the prices: For all searchers with $v < v^t$, porting the data is strictly better than remaining anonymous at the firm without data. This is because the expected price when porting the data lies below p^{nd} for a consumer with $v = p^{H,nd}$ (because $p^{H,nd} < v^t$ must hold). For any consumer with $v > p^{H,nd}$, the preference for porting is strictly falling in v and switches sign at v^t. For searchers with $v > v^t$, it is better to remain anonymous at the firm without data than to port the data.

For any searcher, it is better to port the data to the firm without data than to visit the firm with data. To see this, recall that the firm with data sets $p^L = p^{L,M}$ and $p^H = p^{H,M}$. Since $v^t \leq 1$, we know that $p^{L,nd} \leq p^L = p^{L,M}$ and $p^{H,nd} \leq p^H = p^{H,M}$. Thus, any searcher prefers porting the data over visiting the firm with data.

Thus, all searchers with $v < v^t$ port the data. All searchers with $v > v^t$ prefer to visit the firm without data anonymously over porting the data, which they in turn prefer to visiting the firm with data. Hence, the postulated search behavior is optimal.

The postulated prices of the firms are optimal, given the equilibrium search behaviour. This holds by construction. Thus, said equilibrium candidate constitutes an equilibrium.

■

B.3 Extensions

B.3.1 Non-binary finite signals

Suppose that the firm with data receives a signal that can take $K \geq 2$ possible realizations, where the probability that a consumer with valuation v generates a signal $\tilde{v}^k$ is $Pr(\tilde{v}^k|v)$ and $\sum_{k=1}^{K} Pr(\tilde{v}^k|v) = 1$. Conditional on the search strategy of searchers, namely

$s(v)$, one can define the cumulative distribution function of a consumer's valuation, conditional on the consumer having generated the signal $\tilde{v}^k$, as $F^k(v; s(v))$:

$$F^k(x; s(v)) = \frac{1}{Pr(\tilde{v}^k)} \int_0^x Pr(\tilde{v}^k|v)[\rho s(v) + 0.5(1-\rho)]g(v)dv \tag{B.3.1}$$

The hazard ratios are $h^k(v; s(v)) = \frac{f^k(v;s(v))}{1-F^k(v;s(v))}$, where $f^k(v; s(v))$ are the corresponding densities. I define $\Pi^{k,M}(p_j)$ as the profit a monopolist with access to said information structure makes when offering the price p_j to consumers who generate $\tilde{v}^k$. The monopoly profit function of a firm without data is $\Pi^{nd,M}(p_j)$, as defined in equation (3). The optimal monopoly prices are $p^{nd,M} := arg \max_{p_j} \Pi^{nd,M}(p_j)$ and $p^{k,M} := arg \max_{p_j} \Pi^{k,M}(p_j)$.

Everything else is as in the baseline model. An equilibrium consists of the search strategy of consumers, the uniform price set by the firm without data (p^{nd}), and the prices set by the firm with data after any signal, namely $\{p^k\}_{k\in\{1,\ldots,K\}}$. I impose the tie-breaking rule laid out in assumption 2 and the following new assumptions:

Assumption 10 *Assume that $Pr(\tilde{v}^k|v)$ are all once continuously differentiable, satisfy $Pr(\tilde{v}^k|v) \in (0,1)$ $\forall v$ and that:*

- *For any measurable $s(v)$, the signals are hazard ratio ordered, i.e. $h^1(v; s(v)) > h^2(v; s(v)) > \ldots > h^K(v; s(v))$ holds for all $v \in [0,1]$.*
- *The monopoly profit functions $\Pi^M(p_j|\tilde{v}^1)$,...,$\Pi^M(p_j|\tilde{v}^K)$, and $\Pi^{nd,M}(p_j)$ are all strictly concave in the price and $p^{nd,M} \in (p^{1,M}, p^{K,M})$.*

Moreover, for any vector of prices $(p^1, \ldots, p^K)$ s.t. $p^1 \leq p^2 \leq \ldots \leq p^K$ and $p^1 \neq p^K$:

$$\frac{\partial}{\partial v}\left[\sum_{k=1}^{K} Pr(\tilde{v}^k|v) \max\{v - p^k, 0\}\right] < 1 \tag{B.3.2}$$

I label this framework the *extended data framework*. These assumptions imply that, in any competitive equilibrium, there will be price discrimination in the sense that consumers with higher valuations pay greater effective prices. Under these conditions, the equilibrium strategy of searchers will remain a cutoff rule when firms play pure strategies, which means that all results derived within the baseline model go through verbatim:

Proposition 21 (Non-binary finite signals: equilibrium)
Consider the extended data framework. In any equilibrium in which firms play pure strategies:

- *The ordering $p^{nd} \in (p^1, p^K)$ will hold.*
- *There exists a $\bar{v} > p^1$ such that all searchers with $v \in (p^1, \bar{v})$ visit the firm with data and all searchers with $v \in (\bar{v}, 1]$ visit the firm without data.*
- *The ordering $\bar{v} \geq \bar{v}^{nd}$ must hold.*

There exists an equilibrium in which firms play pure strategies.

B.3.2 Continuous signals

The previous insights all go through even when the firm with data receives continuous signals about consumer valuations, so long as the information is not perfect. I show this in a model which retains all the specifications from the baseline model, with two exceptions: First, the firm with data receives a continuous signal $\tilde{v} = v + \epsilon$ about the valuation of any arriving consumer (v), where the noise term ϵ is uniformly distributed on the interval $[-\bar{\epsilon}, \bar{\epsilon}]$. I assume that $\bar{\epsilon} \in (0, 1/8)$. Second, I now assume that $v \sim U[0, 1]$. I name this framework the *continuous signals framework.*

An equilibrium in which firms play pure strategies thus consists of (i) a uniform price (p^{nd}) of the firm without data, (ii) a function $p^d(\tilde{v})$ that defines what price the firm with data would offer after observing the signal $\tilde{v}$, and (iii) the search strategy of searchers, namely $s(v)$. In equilibrium, searchers separate exactly as before:

Lemma 9 (Continuous signals: search)
Consider the continuous signals framework. In equilibrium:

- *Any consumer with* $v > 0$ *obtains strictly positive expected utility at the firm with data.*
- *There exists a* $\bar{v}$ *such that all searchers with* $v \in (0, \bar{v})$ *visit the firm with data and all searchers with* $v \in (\bar{v}, 1]$ *visit the firm without data.*

Because $\bar{\epsilon} > 0$, the firm with data cannot perfectly price discriminate and any consumer with positive valuation can gain some utility (in expectation) by visiting this firm. Thus, all searchers with $v < p^{nd}$ will optimally visit the firm with data. However, the price distribution at the firm with data becomes strictly less favorable as a consumer's valuation increases. This implies that, as before, searchers with low valuations visit the firm with data and vice versa.

As before, this separating behavior will give rise to a selection effect. In equilibrium, a majority of searchers will thus visit the firm with data:

Proposition 22 (Continuous signals: equilibrium)
Consider the continuous signals framework. In equilibrium, $\bar{v} \geq 0.5(1 + \rho)$ *must hold and an equilibrium exists.*

Recall that, when $v \sim U[0, 1]$, the cutoff $\bar{v}^{nd}$ as defined in equation (10) equals $0.5(1+\rho)$. As before, any equilibrium candidate in which $\bar{v} < 0.5(1 + \rho)$ holds is ruled out by an incompatibility of optimal search and optimal pricing by the firm without data. Optimal behavior by searchers implies that $p^{nd} < \bar{v}$ must hold in equilibrium, since any consumer with positive valuation can attain strictly positive expected utility by visiting the firm with data. A searcher who is indifferent between both firms (i.e. a searcher with valuation $\bar{v}$) must hence receive strictly positive utility at the firm without data. However, the firm without data would optimally set a uniform price p^{nd} weakly above $\bar{v}$ when $\bar{v} < 0.5(1+\rho)$, which rules out any such equilibria.

The optimization calculus of the firm without data is the same as in the baseline model. Thus, the uniform price it will set is given by $p^{nd,*}(\bar{v})$. The assumption that $\bar{\epsilon} < 1/8$ implies that the firm with data will price according to the following function for any $\bar{v} \geq 0.5(1+\rho)$:

$$p^{d,*}(\tilde{v}) = \begin{cases} 0.5(\tilde{v} + \bar{\epsilon}) & \tilde{v} \in [-\bar{\epsilon}, 3\bar{\epsilon}] \\ (\tilde{v} - \bar{\epsilon}) & \tilde{v} \in [3\bar{\epsilon}, 1+\bar{\epsilon}] \end{cases} \tag{B.3.3}$$

The prices set by the firms respond continuously to changes in $\bar{v}$, which is sufficient to ensure equilibrium existence. It remains to study the properties of these equilibria in some more detail. To that end, I fix $\rho = 0.4$ and visualize the equilibrium values of $\bar{v}$ and p^{nd} for different levels of $\bar{\epsilon}$ (which are plotted on the x-axis) in the following graph:

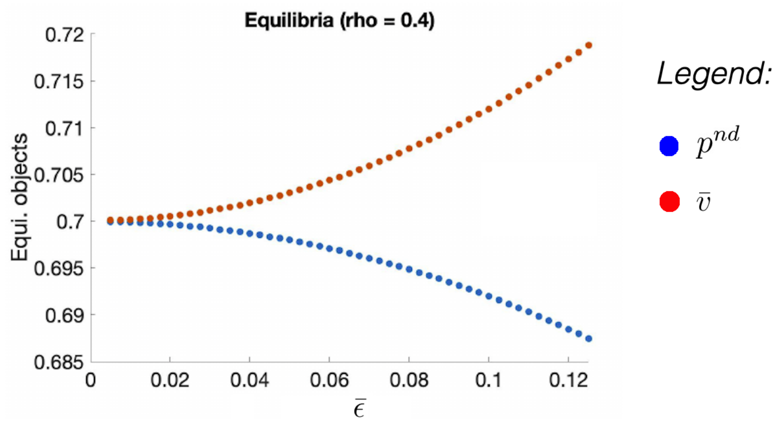

Figure B.1: Equilibria under continuous signals

As $\bar{\epsilon} \to 0$, the cutoff $\bar{v}$ and the price p^{nd} both converge to $0.5(1+\rho)$. To see why, suppose that $\bar{\epsilon} \approx 0$ and that searchers visit firms according to a cutoff rule with $\bar{v} \approx 0.5(1+\rho)$. The firm without data will optimally set the price $p^{nd,*}(\bar{v})$, which is approximately equal to $0.5(1+\rho)$ for this search rule. Because the firm with data is able to almost perfectly price discriminate, every consumer will attain close to zero expected utility by visiting this firm. While consumers with $v < p^{nd} \approx 0.5(1+\rho)$ would still prefer to visit the firm with data, almost all consumers with valuation above p^{nd} would prefer to visit the firm without data, because the utility they attain there is linearly rising in v. Thus, the search behavior represented by the cutoff $\bar{v} \approx 0.5(1+\rho)$ is optimal, because $p^{nd} \approx 0.5(1+\rho)$.

B.3.3 Quality differentiation

In this section, I integrate quality differentiation into the analysis by combining the previous search setup with the model of Mussa & Rosen (1978). The consumer's type is now denoted by $\theta \sim U[0,1]$. The firms can offer different quality levels $q \in [0,1]$. When paying the price p for a good with quality q, a consumer's net utility is:

$$u(q,p;\theta) = \theta q - p \tag{B.3.4}$$

There are two active firms, namely *the firm with data* and *the firm without data.* For any consumer who arrives, the firm with data receives a signal $\tilde{\theta} \in \{\tilde{\theta}^L, \tilde{\theta}^H\}$ about the consumer's type. The probability distribution of this signal is denoted by $Pr(\tilde{\theta} = \tilde{\theta}^H|\theta) := Pr^H(\theta)$, where $Pr^L(\theta) := 1 - Pr^H(\theta)$. The firm without data receives no information about any consumer. The provision of any quality level is costless. As in the baseline analysis, there are searchers and captive consumers, with shares $\rho \in (0,1)$ and $(1-\rho)$. Any consumer can only visit one firm. I label this model the *quality differentiation framework.*

By the revelation principle, it is without loss to restrict the strategy space of the firms to direct mechanisms. Thus, an equilibrium in this game consists of the following objects: (i) the search strategy of searchers, (ii) a quality-price menu $(q^{nd}(\theta), t^{nd}(\theta))$ offered by the firm without data, and two quality price menus $(q^L(\theta), t^L(\theta))$ and $(q^H(\theta), t^H(\theta))$ that the firm with data offers to consumers who generate the low signal and the high signal, respectively. I restrict the strategy space of the firms to menus in which the mapping from messages into qualities is a measurable function. I further restrict attention to *simple equilibria*:

Definition 1 *An equilibrium in the quality differentiation framework is simple if and only if (i) all searchers visit either firm with probability* 0.5 *<u>or</u> (ii) there exists a cutoff* $\bar{\theta}$ *such that all searchers with* $\theta < \bar{\theta}$ *visit a given firm and all searchers with* $\theta > \bar{\theta}$ *visit the other firm.*

Moreover, I impose some tie-breaking rules. I define the infimum of types that receive quality at the firm with data and at the firm without data as $\underline{\theta}^d$ and $\underline{\theta}^{nd}$, respectively.

Assumption 11 *If firms offer identical quality-price menus, searchers visit either firm with equal probability. For any other combination of menus, searchers with* $\theta \leq \min\{\underline{\theta}^d, \underline{\theta}^{nd}\}$ *visit the firm* j *with the lower* $\underline{\theta}^j$*. If* $\underline{\theta}^{nd} = \underline{\theta}^d$*, searchers with* $\theta \leq \min\{\underline{\theta}^d, \underline{\theta}^{nd}\}$ *visit the same firm.*

The main technical challenge in the following analysis stems from the fact that the density of types that arrive at either firm is not continuous — it has a jump discontinuity at $\bar{\theta}$. However, because this stark difference in the consumers' search choices can only occur at precisely one level of θ in a simple equilibrium, the types of consumers that arrive at the firms are still absolutely continuous random variables and admit a well-defined density.

To express these densities, I define the probability that a consumer arrives at the firm without data in equilibrium as $Pr(I^{nd})$ and the probability that a consumer arrives at the firm with data and generates the signal $\tilde{\theta}^k$ as $Pr(I^k)$. Because each firm has captive consumers, these probabilities are all strictly positive, i.e. all information sets of both firms are on the equilibrium path. To characterize the search behavior of consumers in a simple equilibrium, I define $g^H \in \{0, 0.5, 1\}$ and $g^L \in \{0, 0.5, 1\}$ as the probabilities with which a searcher with type $\theta > \bar{\theta}$ and $\theta < \bar{\theta}$ visits the firm with data, respectively. If all searchers visit either firm with equal probability, this is captured by setting $g^L = g^H = 0.5$ and choosing any $\bar{\theta}$. Defining $g := (g^L, g^H)$, the type of a consumer who visits the firm without data has the following probability density:

$$f^{nd}(\theta; \bar{\theta}, g) = \begin{cases} (1/Pr(I^{nd}))(\rho(1-g^L) + 0.5(1-\rho)) & \theta < \bar{\theta} \\ (1/Pr(I^{nd}))(\rho(1-g^H) + 0.5(1-\rho)) & \theta > \bar{\theta} \end{cases} \tag{B.3.5}$$

The type of a consumer who visits the firm with data and generates the signal $\tilde{\theta}^k$ has an analogously defined probability density, which I call $f^k(\theta; \bar{\theta}, g)$.

Given these densities, we can construct the virtual valuation functions. The virtual valuation function of consumers who visit the firm without data, which I call $J^{nd}(\theta; \bar{\theta}, g)$, is $J^{nd}(\theta; \bar{\theta}, g) = \theta - (1 - F^{nd}(\theta; \bar{\theta}, g))/f^{nd}(\theta; \bar{\theta}, g)$. The virtual valuation function of consumers who visit the firm with data and generate the signal $\tilde{\theta}^k$ is $J^k(\theta; \bar{\theta}, g) = \theta - (1 - F^k(\theta; \bar{\theta}, g))/f^k(\theta; \bar{\theta}, g)$. Note that $F^{nd}(\theta; \bar{\theta}, g)$ and $F^k(\theta; \bar{\theta}, g)$ are the cumulative density functions that accompany $f^{nd}(\theta; \bar{\theta}, g)$ and $f^k(\theta; \bar{\theta}, g)$, respectively. Moving forward, I impose the following assumptions:

Assumption 12 *The signal distribution $Pr^H(\theta)$ is continuous, strictly increasing, and maps into $(0,1)$ for any $\theta \in [0,1]$.*

Under these assumptions, the insights of Milgrom & Segal (2002) apply and the expected utility a consumer with type θ attains in an implementable mechanism can be expressed using the integrability condition. Thus, the expected revenue the firm without data obtains in an implementable mechanism is:

$$R^{nd}(q^{nd}(\theta); \bar{\theta}, g) = -U^{nd}(0) + \int_0^1 q^{nd}(\theta) J^{nd}(\theta; \bar{\theta}, g) f^{nd}(\theta; \bar{\theta}, g) d\theta \tag{B.3.6}$$

The expected revenue the firm with data obtains from a consumer that generates $\tilde{\theta}^k$ has an analogous form. I have defined $U^{nd}(0)$ as the utility the lowest type would attain in a mechanism set by the firm without data. The set of consumer types for which the virtual valuations are positive are partially characterized by the cutoffs $\hat{\theta}^{nd}, \hat{\theta}^{L}$, and $\hat{\theta}^{H}$, which are defined as follows:

$$\hat{\theta}^k = \inf\left\{\theta : J^k(\theta; \bar{\theta}, g) > 0\right\} \quad ; \quad \hat{\theta}^{nd} = \inf\left\{\theta : J^{nd}(\theta; \bar{\theta}, g) > 0\right\} \tag{B.3.7}$$

Note that the virtual valuation functions can jump down at $\bar{\theta}$, which means that the virtual valuations are not necessarily positive for all θ above said cutoffs. In addition, while the functions $J^{nd}(\theta; \bar{\theta}, g)$ and $J^{H}(\theta; \bar{\theta}, g)$ are both piecewise strictly increasing in θ by construction, the low signal virtual valuation function $J^{L}(\theta; \bar{\theta}, g)$ may be non-monotonic. To deal with the former problem in the equilibrium analysis, I set up the following assumption:

Assumption 13 *Fix $g_L = 0$ and $g_H = 1$. For any $\bar{\theta} \leq 0.5$, $J^L(\theta; \bar{\theta}, g) < 0 \ \forall \theta \leq \bar{\theta}$.*

Remark 3 *For any linear $Pr^H(v)$, assumption 13 is satisfied if $\rho \geq 0.34$.*

This assumption rules out equilibria in which searchers separate in a different way than previously, i.e. equilibria in which searchers with low θ visit the firm without data. Such equilibria could only be sustained if the firm without data implements an ironing mechanism, in which it starts providing quality at lower types than the firm with data (formally, $\hat{\theta}^{nd} < \hat{\theta}^{L}$). This assumption rules out such equilibria, given that any such equilibrium must feature $\hat{\theta}^{L} < \bar{\theta}$ and $\bar{\theta} \leq 0.5$, which is made impossible under said assumption. Moreover, there generally exists no equilibrium in which all searchers randomize between the firms. These notions are formalized in the following lemma:

Lemma 10 (Quality differentiation: search)
Consider the quality differentiation framework:

- *When all searchers visit firms randomly, the cutoffs satisfy $\hat{\theta}^{L} < \hat{\theta}^{nd} < \hat{\theta}^{H}$.*
- *Suppose assumption 13 holds as well. In a simple equilibrium, there exists a $\bar{\theta}$ such that searchers visit the firm with data if $\theta < \bar{\theta}$ and the firm without data if $\theta > \bar{\theta}$.*

The first result follows from the fact that consumers who generate the low signal have a distribution of types θ with less mass at high types and vice versa. This shifts up the distribution of virtual valuations. Intuitively, a firm would be more willing to offer positive quality to a consumer with a given θ when observing the low signal (rather than the high signal or no signal), because the mass of consumers with higher types for whom this decision would incur a revenue loss becomes smaller. The second result holds because the firm without data cannot attract low type consumers with an ironing mechanism in equilibrium.

The main result from the baseline analysis is retained in any simple equilibrium. This is formalized in the following proposition, together with the accompanying assumptions:

Assumption 14 *Fix $g^L = 1$ and $g^H = 0$.*

- *The function $J^L(\theta; \bar{\theta}, g)$ is piecewise strictly increasing in θ for $\theta \in [0, \bar{\theta})$ and $\theta \in (\bar{\theta}, 1]$.*
- *For any $\bar{\theta} \geq 0.5(1+\rho)$, $\lim_{\theta \downarrow \bar{\theta}} J^k(\theta; \bar{\theta}, g) > 0$ holds for both $k \in \{L, H\}$.*

Remark 4 *Assumption 14 is satisfied for any linear signal distribution.*

This assumption guarantees that, in equilibrium, all functions $J^x(\theta; \bar{\theta}, g)$ (with $x \in \{nd, L, H\}$) will be strictly negative if $\theta < \hat{\theta}^x$ and strictly positive if $\theta > \hat{\theta}^x$. Thus, the optimal $q^x(\theta)$ assigns quality 1 to all types above $\hat{\theta}^x$ and quality 0 to all types below this threshold. Such an equilibrium exists and retains the key property from the baseline analysis:

Proposition 23 (Quality differentiation: equilibrium)
Consider the quality differentiation framework. Under assumptions 13 and 14, $\bar{\theta} \geq 0.5(1+\rho)$ must hold in any simple equilibrium, and a simple equilibrium always exists.

Any value $\bar{\theta} < 0.5(1+\rho)$ cannot constitute an equilibrium, since $\bar{\theta} \leq \hat{\theta}^{nd}$ would hold for any such $\bar{\theta} < 0.5(1+\rho)$. This is not consistent with optimal consumer search behavior — because $\hat{\theta}^L < \hat{\theta}^{nd}$, searchers with θ just above $\hat{\theta}^{nd}$ would strictly prefer to visit the firm with data, but visit the firm without data in the supposed equilibrium. Thus, $\bar{\theta} \geq 0.5(1+\rho)$ must hold in a simple equilibrium, which exactly replicates the result from the baseline analysis. Such a simple equilibrium always exists under the stated assumptions. Thus, the main equilibrium result from the baseline model is retained in this extension, albeit under slightly stronger restrictions on ρ, as expressed in remark 3.

B.3.4 Endowing both firms with data

In this section, I study a framework in which both firms receive binary signals about the valuation of any consumer who visits. I specify that there is one firm that has access to better data than the other. I define the two firms as the *firm with better data* and the *firm with worse data*. The consumers' valuations are drawn uniformly from the unit interval. Everything else is as in the baseline model and all consumers can only visit one firm. I label the resulting framework the *dispersed data framework*.

The probability that a consumer with valuation v generates the high signal at the firm with better data is $Pr^{HB}(v)$ and $Pr^{HW}(v)$ at the firm with worse data. In the dispersed data framework, an equilibrium consists of a quadruple of prices $(p^{LB}, p^{HB}, p^{LW}, p^{HW})$ and the search strategy of searchers. The prices that the firm with better data offers to consumers that generate the low and high signal, respectively, are (p^{LB}, p^{HB}). The prices the firm with worse data offers in the respective information sets are (p^{LW}, p^{HW}).

To illustrate differences in signal precision, consider a monopolist with access to a signal with distribution $Pr^{HB}(v)$. I define the prices this firm would set after the low and high signal as $p^{LB,M}$ and $p^{HB,M}$, respectively. Analogously, the prices set by a monopolist who receives a signal with distribution $Pr^{HW}(v)$ in the respective information sets are defined as $p^{LW,M}$ and $p^{HW,M}$. In the analysis, I assume that the signal distributions are well behaved and that the firm with better data receives a more precise signal in the following sense:

Assumption 15 *Both functions $Pr^{HB}(v)$ and $Pr^{HW}(v)$ are strictly increasing in v, continuous, and map into $(0,1)$ for any v. The signal probability functions are such that:*

- *For any $v < 0.5$, $Pr^{HB}(v) < Pr^{HW}(v)$. For any $v > 0.5$, $Pr^{HB}(v) > Pr^{HW}(v)$.*
- *The prices that firms would set when all consumers randomize between firms satisfy the ordering $p^{LB,M} < p^{LW,M} < p^{HW,M} < p^{HB,M}$.*

In words, the signal which the firm with better data receives implies a higher chance of correctly recognizing whether a consumer's valuation is in the upper or lower half of the valuation interval. Moreover, the signal of the firm with better data is more informative in the sense that, when consumers randomly arrive at firms, this firm sets a lower price to consumers who generate the low signal and vice versa.

Moreover, I impose a tie-breaking rule concerning searchers with valuation below the lowest equilibrium price, which I call $p^{min} := \min\{p^{LW}, p^{HW}, p^{LB}, p^{HB}\}$.

Assumption 16 *Searchers with $v \leq p^{min}$ visit a firm that offers p^{min} with higher probability.*

The richness of the pricing possibilities enables the potential existence of equilibria with intractable search behaviour. To facilitate the analysis, I restrict attention to the following "simple" category of equilibria.

Definition 2 *An equilibrium is simple if and only if (i) all searchers visit each firm with probability 0.5 <u>or</u> (ii) there exists some $\bar{v}$ s.t. all searchers with valuation above $\bar{v}$ visit a particular firm and all searchers with valuation below $\bar{v}$ visit the other firm.*

The definition of a simple equilibrium does not impose restrictions on *which firm* consumers on either side of a cutoff $\bar{v}$ visit in an equilibrium where their strategy is a cutoff rule. Instead, I show that any simple equilibrium retains the structure of previous equilibria.[1] Moreover, the equilibrium $\bar{v}$ is bounded from below, as before.

Lemma 11 (Dispersed data: equilibrium characterization)
Consider the dispersed data framework. In a simple equilibrium:

[1]If both firms receive a signal with the same probability distribution, there is a unique simple equilibrium in which both firms follow the same pricing strategy and all searchers randomize between both firms.

- *The ordering* $p^{LB} \leq p^{LW}$ *holds. There exists a* $\bar{v}$ *such that searchers visit the firm with worse data if* $v > \bar{v}$ *and the firm with better data if* $v < \bar{v}$.
- $\bar{v} \geq \bar{v}^{LW}$ *must hold, where* $\bar{v}^{kW} \in (0,1)$ *solves the following for either* $k \in \{L, H\}$*:*

$$\rho \int_{\bar{v}^{kW}}^{1} Pr^{kW}(v)dv + 0.5(1-\rho)\left[\int_{\bar{v}^{kW}}^{1} Pr^{kW}(v)dv - \bar{v}^{kW} Pr^{kW}(\bar{v}^{kW})\right] = 0 \tag{B.3.8}$$

Low-valuation consumers prefer to visit the firm with better data, because they are more likely to generate the low signal, i.e. to be correctly identified, at this firm. As a result, consumers with low (high) valuations can expect a more favorable price distribution at the firm with better data (firm with worse data). The interpretation of $\bar{v}^{LW}$ is the same as in the baseline model:[2] For any $\bar{v} < \bar{v}^{LW}$, the firm with worse data would optimally set a low signal price (p^{LW}) that is weakly above $\bar{v}$. However, searchers with valuation just above p^{LW} would never visit the firm with worse data in a simple equilibrium, because the firm with better data will always offer the lowest equilibrium price. This means that $\bar{v} \leq p^{LW}$ cannot hold in a simple equilibrium, which rules out equilibria in which $\bar{v} < \bar{v}^{LW}$.

It remains to establish the existence of a simple equilibrium. For a given $\bar{v}$, the prices of the firm with better data need to maximize the following objective functions for the corresponding signal $\tilde{v}^k \in \{\tilde{v}^L, \tilde{v}^H\}$ in any such equilibrium:

$$\Pi^{kB}(p_j;\bar{v}) := p_j\left[\underbrace{\rho\int_0^{\bar{v}} Pr^{kB}(v)\mathbb{1}[p_j \leq v]dv}_{\text{searcher demand}} + \underbrace{0.5(1-\rho)\int_0^1 Pr^{kB}(v)\mathbb{1}[p_j \leq v]dv}_{\text{captive consumer demand}}\right] \tag{B.3.9}$$

The firm with worse data maximizes the following objective, given the signal $\tilde{v}^k \in \{\tilde{v}^L, \tilde{v}^H\}$:

$$\Pi^{kW}(p_j;\bar{v}) := p_j\left[\underbrace{\rho\int_{\bar{v}}^{1} Pr^{kW}(v)\mathbb{1}[p_j \leq v]dv}_{\text{searcher demand}} + \underbrace{0.5(1-\rho)\int_0^1 Pr^{kW}(v)\mathbb{1}[p_j \leq v]dv}_{\text{captive consumer demand}}\right] \tag{B.3.10}$$

I define the optimal prices of the firm with better data for a given $\bar{v}$ as $p^{LB,*}(\bar{v}) := arg\max_{p_j \in [0,1]} \Pi^{LB}(p_j;\bar{v})$ and $p^{HB,*}(\bar{v}) := arg\max_{p_j \in [0,1]} \Pi^{HB}(p_j;\bar{v})$. Analogously, I define the optimal prices of the firm with worse data as $p^{LW,*}(\bar{v}) := arg\max_{p_j \in [0,1]} \Pi^{LW}(p_j;\bar{v})$ and $p^{HW,*}(\bar{v}) := arg\max_{p_j \in [0,1]} \Pi^{HW}(p_j;\bar{v})$. In equilibrium, the search behavior of searchers will, as before, be determined by the expected prices they can anticipate at the two firms.

[2] In fact, this cutoff becomes $\bar{v}^{LW} = 0.5 + 0.5\rho$ when $Pr^{LW}(v) = Pr^{HW}(v)$, i.e. when the firm with worse data receives no informative signal, as in the baseline model.

For firm $j \in \{W, B\}$ and a fixed $\bar{v}$, these expected prices (conditional on v) are given by:

$$EP^j(v;\bar{v}) = Pr^{Lj}(v)p^{Lj,*}(\bar{v}) + Pr^{Hj}(v)p^{Hj,*}(\bar{v}) \tag{B.3.11}$$

For which searchers the expected price is lower at the firm with better data (given the equilibrium level of $\bar{v}$) is tracked by the following object:

$$\hat{v}^X(\bar{v}) = \sup\left\{v \in [0,1] : EP^B(v;\bar{v}) - EP^W(v;\bar{v}) < 0\right\} \tag{B.3.12}$$

Now, I define an assumption that ensures the existence of a viable candidate for a simple equilibrium. Afterwards, I establish when this candidate constitutes an equilibrium.

Assumption 17 *Define* $\bar{v}^D = \max\{\bar{v}^{LW}, p^{HB,M}\}$. *Suppose that* $EP^B(\bar{v}^D;\bar{v}^D) < EP^W(\bar{v}^D;\bar{v}^D)$.

Remark 4 *Assumption 17 is satisfied for any linear signal distribution.*

Proposition 24 (Dispersed data: equilibrium existence)
Consider the dispersed data framework. Under assumption 17, the following equation has a solution $\bar{v}^* \in [\bar{v}^{LW}, 1]$*:*

$$\bar{v}^* = \hat{v}^X(\bar{v}^*) \tag{B.3.13}$$

The combination $(p^{LB}(\bar{v}^*), p^{HB}(\bar{v}^*), p^{LW}(\bar{v}^*), p^{HW}(\bar{v}^*), \bar{v}^*)$ *is an equilibrium if, given* $(p^{LB}(\bar{v}^*), p^{HB}(\bar{v}^*), p^{LW}(\bar{v}^*), p^{HW}(\bar{v}^*))$*, it is weakly optimal for searchers to visit the firm with better data if and only if* $v \leq \bar{v}^*$*.*

In a hypothetical equilibrium of the above form, all prices of the firms are optimal by construction. Imposing optimality of the postulated search behavior is required, because the fact that $\bar{v} = \hat{v}^X(\bar{v})$ holds does not rule out the possibility that some consumers with $v < \bar{v}$ optimally visit the firm with worse data. This is because the search preferences of searchers have kinks at the equilibrium prices, which means that it may not necessarily be optimal for them to visit the firm where they receive the lower expected price.

However, numerical analysis shows that it is indeed optimal for searchers to visit the firm where they receive the lower expected price in equilibrium candidates of the above form, establishing that these combinations constitute equilibria. I study linear signal distributions as defined in equation (4), where the precision of the signal the firm with better data receives is α_b and the precision of the signal that its rival receives is α_w, with $\alpha_w < \alpha_b$. I consider $\rho \in \{0.05, 0.35, 0.65, 0.95\}$, $\alpha_w \in [0, 0.49]$ and $\alpha_b \in [0.5, 0.99]$ (with 25 grid points each). For different combinations of ρ, α_w, and α_b, I calculate the solution to equation (B.3.13). Given the implied prices, I then check whether it is optimal for all searchers with $v < \bar{v}^*$ to visit the firm with better data and vice versa for searchers with

$v > \bar{v}^*$. I show that this requirement is met, i.e. that said combination of prices and $\bar{v}$ constitutes an equilibrium, for any of the parameter combinations listed above.

Every graph corresponds to a fixed level of ρ. Different levels of α_b are plotted on the x-axis and different levels of α_w on the y-axis. For a given parameter combination, a green dot indicates that the price - search combination from equation (B.3.13) constitutes a perfect Bayesian equilibrium.

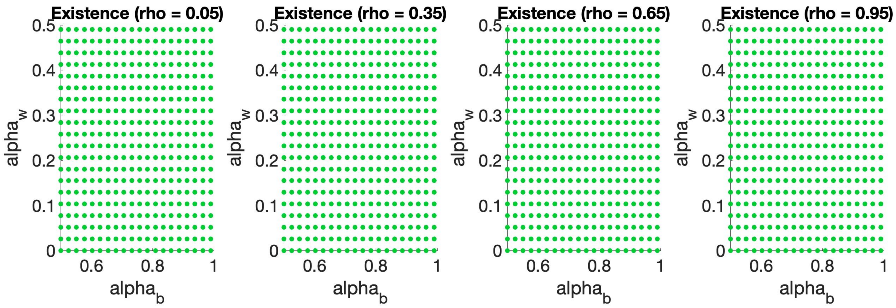

Figure B.2: Visualization — existence of a simple equilibrium

Finally, I visualize the comparative statics results of increases in α_w for different parameter combinations in the following figures:

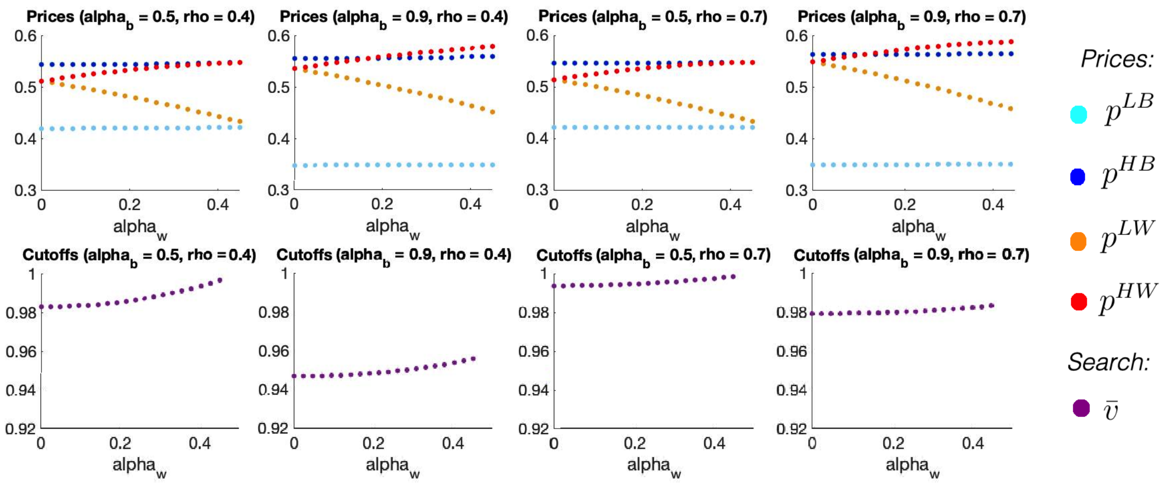

Figure B.3: Equilibrium objects in the dispersed data framework

Summing up, the key prediction from the baseline model also holds true in the dispersed data framework when restricting attention to simple equilibria. Any simple equilibrium retains the property that $\bar{v}$ is bounded from below. The numerical simulations indicate that a simple equilibrium always exists and that $\bar{v} \to 1$ as $\rho \to 1$.

B.4 Proofs — extensions

Proof of Proposition 21:

Part 1: In an equilibrium in which firms play pure strategies, $p^{nd} \in (p^1, p^K)$ holds.

The monopoly prices satisfy $p^{1,M} < p^{2,M} < ... < p^{K,M}$. In general, our assumption on the ordering of the hazard ratios implies that $p^1 \leq p^2 \leq ... \leq p^K$ holds in any equilibrium in which firms play pure strategies.

There exists no equilibrium in which the firm with data sets a uniform price, i.e. $p^1 = p^2 = ... = p^K$. The only possible such equilibrum is $p^1 = p^2 = ... = p^K = p^{nd} := p^*$. But then, the tie-breaking rule defined in assumption 2 applies, so all searchers with $v \geq p^*$ randomize between firms. Thus, $p^* \geq p^{K,M}$ holds. But because $p^{nd,M} < p^*$ and $p^{1,M} < p^*$, there is either a downward deviation from p^* to $p^{nd,M}$ for the firm without data or a deviation from p^* to $p^{1,M}$ for the firm with data when it observes $\tilde{v}^1$, a contradiction. Thus, $p^1 < p^K$ must hold in equilibrium.

Suppose that $p^{nd} \leq p^1$. There will exist a price above p^1 that any consumer will receive with strictly positive probability. Thus, any consumer with $v \geq p^{nd}$ will visit the firm without data, which implies that $p^{nd} \geq p^{nd,M}$ must hold. This implies that $p^1 \geq p^{nd} > p^{1,M}$. The firm with data only sells to its captive consumers at p^1, which means there is a profitable downward deviation for this firm, because monopoly profits are strictly maximized at $p^{1,M}$, a contradiction.

Suppose that $p^K \leq p^{nd}$. Because there will exist a price below p^K (we have ruled out uniform price equilibria), all searchers with a valuation above p^K will visit the firm with data. Thus, $p^K \geq p^{K,M} > p^{nd,M}$ must hold, which implies that there will be a downward deviation from p^{nd}, since the firm without data only sells to captive consumers at this price.

This establishes the desired ordering of prices: $p^{nd} \in (p^1, p^K)$ must hold.

Part 2: The strategy of searchers will be a cutoff rule.

In equilibrium, $p^1 < p^{nd}$ must hold. All consumers with $v \leq p^{nd}$ will strictly prefer to visit the firm with data. For all consumers with $v > p^{nd}$, the preference for the firm without data is as follows:

$$P^{nd}(v) = (v - p^{nd}) - \left[\sum_{k=1}^{K} Pr(\tilde{v}^k | v) \max\{v - p^k, 0\} \right] \implies \frac{\partial P^{nd}(v)}{\partial v} > 0 \qquad \text{(B.4.1)}$$

By assumption, the derivative of this object will be above 0, i.e. the preference for the firm without data will be strictly rising in v. This establishes the existence of a unique cutoff.

Part 3: In equilibrium, $\bar{v} \geq \bar{v}^{nd}$ must hold.

This holds by previous logic. Because $p^1 < p^{nd}$ and $Pr(\tilde{v}^1|v) > 0$ for any v, $p^{nd} < \bar{v}$ must hold in equilibrium, since a consumer with valuation $v = p^{nd}$ would find it strictly optimal to visit the firm with data. The firm without data must find it optimal to set $p^{nd} < \bar{v}$, which will only be true if $\bar{v} \geq \bar{v}^{nd}$. This is because, for $p_j \in (p^1, \bar{v})$, the profits of the firm without data are given by $\Pi^{nd}(p_j; \bar{v})$.

Part 4 An equilibrium in which firms play pure strategies exists.

For any possible cutoff search strategy, one can show that $p^1 \leq p^2 \leq ... \leq p^K$ and $p^1 \neq p^K$ will hold. As a result, a consumer's preference for the firm without data is strictly rising as $v \geq p^{nd}$ and we can describe the search behaviour of consumers using a cutoff that solves:

$$\hat{v}(p^1, ..., p^K, p^{nd}) = \sup \left\{ v \in [0,1] : \sum_{k=1}^{K} Pr(\tilde{v}^k|v) \max\{v - p^k, 0\} - (v - p^{nd}) > 0 \right\} \tag{B.4.2}$$

Similary, we can define:

$$v^F(\bar{v}) = \sup \left\{ v \in [0,1] : \sum_{k=1}^{K} Pr(\tilde{v}^k|v) \max\{v - p^{k,*}(\bar{v}), 0\} - (v - p^{nd,*}(\bar{v})) > 0 \right\} \tag{B.4.3}$$

Note that $p^{k,*}(\bar{v})$ are the optimal prices set by the firm with data, if searchers visit firms according to said cutoff rule. The optimal price of the firm without data is given by $p^{nd,*}(\bar{v})$.

We work towards showing the existence of a fixed point of $v^F(\bar{v})$. To begin, note that the following two boundary conditions will be satisfied: (i) $v^F(\bar{v}^{nd}) > \bar{v}^{nd}$ and (ii) $v^F(1) \leq 1$.

The first condition holds because, at $\bar{v} = \bar{v}^{nd}$, the optimal price of the firm without data will be equal to $\bar{v}^{nd}$, which lies above $p^{nd,M}$. Thus, we have $p^{1,M} < p^{nd,M} < \bar{v}^{nd}$, so the lowest signal price of the firm without data would optimally lie below $\bar{v}^{nd}$. As a result, a consumer with $v = \bar{v}^{nd}$ would strictly prefer to visit the firm with data, and thus $v^F(\bar{v}^{nd}) > \bar{v}^{nd}$. The second condition holds by construction.

There may be multiple points of discontinuity in this function, because the optimal price functions may jump, namely at the cutoffs $\bar{v}^{k,C}$, which are defined as follows:

$$\max_{p_j \leq \bar{v}^{k,C}} \Pi^k(p_j; \bar{v}^{k,C}) = 0.5(1-\rho)\Pi^{k,M}(p^{k,M}) \tag{B.4.4}$$

As before, we can argue that $\bar{v}^{k,C} < p^{k,M}$. It cannot be exactly equal, because then the left derivative at $\bar{v}^{k,C} = p^{k,M}$ would be strictly negative, implying a contradiction. By analogous arguments, $p^{k,M}$ cannot be below $\bar{v}^{k,C}$.

For all $\bar{v} \leq \bar{v}^{k,C}$, the optimal price will be equal to the monopoly price (since $\max_{p_j \leq \bar{v}} \Pi^k(p_j; \bar{v})$ is strictly falling in $\bar{v}$). For any $\bar{v} \in (\bar{v}^{k,C}, 1)$, the optimal price will be strictly below $p^{k,M}$, while it becomes exactly equal to the monopoly price when $\bar{v} = 1$.

Since $\bar{v}^{k,C} < 1$, we know that the optimal price $p^{k,*}(\bar{v})$ jumps from $p^{k,M}$ to something below this. This downward jump in prices raises the incentives of searchers to visit the firm with data, i.e. will induce an *upward* jump in $\hat{v}^F(\bar{v})$.

Thus, there will be up to K potential points of discontinuity on the relevant interval $[\bar{v}^{nd}, 1]$. At any such point of discontinuity, $\hat{v}^F(\bar{v})$ jumps upwards.

Suppose, for a contradiction, that there exists no fixed point of $\hat{v}^F(\bar{v})$ on $[\bar{v}^{nd}, 1]$. This implies that $\bar{v}^F(\bar{v}) > \bar{v}$ for any interval on which said function is continuous (this proof can be done by induction). Thus, you can find the largest point of discontinuity, which will still be strictly below 1. At that point, you will have $\bar{v}^F(\bar{v}) > \bar{v}$. The function is continuous up to 1, where said inequality flips. Thus, a fixed point must exist, a contradiction.

And thus, an equilibrium exists. The prices are optimal by the construction of $\hat{v}^F(\bar{v})$. The search choices are optimal by definition.

■

Proof of Lemma 9:

For a given valuation v, the random variable $\tilde{v}$ is uniformly distributed with mean v and support $[v-\bar{\epsilon}, v+\bar{\epsilon}]$. Hence, the conditional density is $f_{\tilde{v}|v} = 1/2\bar{\epsilon}$ for $\tilde{v} \in [v-\bar{\epsilon}, v+\bar{\epsilon}]$ and 0 otherwise.

Part 1: The price the firm with data will set is weakly rising in $\tilde{v}$.

I consider the cumulative density function of v, conditional on $\tilde{v}$. The density of arriving consumers valuations is $g(v)$, which is strictly positive and bounded throughout. The probability that the consumer's v is below x, conditional on this consumer generating $\tilde{v}^r$, is:

$$Pr(v \leq x|\tilde{v}^r) = \int_0^x f_{v|\tilde{v}}(v|\tilde{v}^r)dv = \int_0^x \frac{\mathbb{1}[v \in [\tilde{v}^r - \bar{\epsilon}, \tilde{v}^r + \bar{\epsilon}]](1/2\bar{\epsilon})g(v)}{f_{\tilde{v}}(\tilde{v}^r)}dv \tag{B.4.5}$$

Call the corresponding cdf $B(x|\tilde{v})$. The corresponding density is: $b(x|\tilde{v}) = (1/f_{\tilde{v}}(\tilde{v}))\mathbb{1}[x \in [\tilde{v} - \bar{\epsilon}, \tilde{v} + \bar{\epsilon}]](1/2\bar{\epsilon})g(x)$. We can define the hazard ratio:

$$h(x|\tilde{v}) = \frac{\mathbb{1}[x \in [\tilde{v} - \bar{\epsilon}, \tilde{v} + \bar{\epsilon}]](1/2\bar{\epsilon})g(x)}{\int_x^1 \mathbb{1}[v \in [\tilde{v} - \bar{\epsilon}, \tilde{v} + \bar{\epsilon}]](1/2\bar{\epsilon})g(v)dv} \tag{B.4.6}$$

Consider two signal realizations $\tilde{v}^1, \tilde{v}^2$ with $\tilde{v}^1 < \tilde{v}^2$. Suppose that there is an overlap between the supports. If there is no overlap, i.e. $\tilde{v}^1 + \bar{\epsilon} \leq \tilde{v}^2 - \bar{\epsilon}$, then $p(\tilde{v}^1) < p(\tilde{v}^2)$ must hold, because each price must be in the support of valuations corresponding to a signal. The interval of valuations where the two signals overlap is $[\tilde{v}^2 - \bar{\epsilon}, \tilde{v}^1 + \bar{\epsilon}]$.

Suppose, for a contradiction, that $p^d(\tilde{v}^2) := p^2 < p^d(\tilde{v}^1) := p^1$. The price p^2 must satisfy $p^2 \geq \tilde{v}^2 - \bar{\epsilon}$ - else, there is a profitable upward deviation. Similarly, $p^1 < \tilde{v}^1 + \bar{\epsilon}$ must hold. Thus, both prices must lie in $[\tilde{v}^2 - \bar{\epsilon}, \tilde{v}^1 + \bar{\epsilon}]$. For any $x \in [\tilde{v}^2 - \bar{\epsilon}, \tilde{v}^1 + \bar{\epsilon}]$, the hazard ratios satisfy $h(x|\tilde{v}^1) > h(x|\tilde{v}^2)$, because $\tilde{v}^2 > \tilde{v}^1$. But then, $p^d(\tilde{v})$ must be weakly rising in the signal $\tilde{v}$ - else, there is a profitable deviation from one of these prices.

Part 2: Consider any $v > 0$. There exists a $\tilde{v}' > \tilde{v}^{lb}(v) := v - \bar{\epsilon}$ such that $p^d(\tilde{v}') < v$.

Suppose there exists no such $\tilde{v}' > \tilde{v}^{lb}(v)$. By implication, $p^d(\tilde{v}') \geq v$ must hold for any $\tilde{v}' > \tilde{v}^{lb}(v)$. Consider the profits the firm with data would make after any $\tilde{v}'$ for some $p_j \in [\max\{\tilde{v}' - \bar{\epsilon}, 0\}, \tilde{v}' + \bar{\epsilon}]$, which are:

$$\Pi(p_j; \tilde{v}') = p_j \frac{1}{f_{\tilde{v}}(\tilde{v}')} \int_{p_j}^{\tilde{v}' + \bar{\epsilon}} (1/2\bar{\epsilon})g(v)dv \tag{B.4.7}$$

We can take the following limit of the postulated equilibrium profits:

$$\lim_{\tilde{v}' \to \tilde{v}^{lb}(v)} \Pi(p^d(\tilde{v}'); \tilde{v}') = \big(\lim_{\tilde{v}' \to \tilde{v}^{lb}(v)} p^d(\tilde{v}')\big) \frac{1}{f_{\tilde{v}}(\tilde{v}')} \int_{\lim_{\tilde{v}' \to \tilde{v}^{lb}(v)} p^d(\tilde{v}')}^{\tilde{v}^{lb}(v) + \bar{\epsilon}} (1/2\bar{\epsilon})g(v)dv = 0 \tag{B.4.8}$$

The latter condition holds because $\tilde{v}^{lb}(v) + \bar{\epsilon} = v$ and $\lim_{\tilde{v}' \to \tilde{v}^{lb}(v)} p^d(\tilde{v}') \geq v$ by assumption. Thus, profits converge to zero in this case as $\tilde{v} \to \tilde{v}^{lb}(v)$.

Alternatively, the firm with data could set the price $0.5\big(\tilde{v}' + \bar{\epsilon}\big)$, at which it would be guaranteed to make positive profits. For any $\tilde{v}$, the profits from this approach are:

$$\Pi(0.5\big(\tilde{v}' + \bar{\epsilon}\big); \tilde{v}') = 0.5\big(\tilde{v}' + \bar{\epsilon}\big) \left[\frac{1}{f_{\tilde{v}}(\tilde{v}')} \int_{\max\{\tilde{v}' - \bar{\epsilon}, 0\}}^{\tilde{v}' + \bar{\epsilon}} (1/2\bar{\epsilon})g(v)\mathbb{1}[v > 0.5(\tilde{v}' + \bar{\epsilon})]dv\right] \tag{B.4.9}$$

This remains strictly positive even in the limit at $\tilde{v}' \to \tilde{v}^{lb}(v)$ because $v > 0$. Thus, we would have a profitable deviation for some signal close enough to $\tilde{v}^{lb}(v)$, and thus a contradiction.

Part 3: For any $v > 0$, the expected utility of visiting the firm with data is strictly positive.

On the interval $\tilde{v} \in [\tilde{v}^{lb}(v), \tilde{v}']$, we will have $p^d(\tilde{v}) < v$ (by monotonicity of $p^d(\tilde{v})$). Thus, the expected utility of the consumer, namely $\int_{v-\bar{\epsilon}}^{v+\bar{\epsilon}} \max\{v - p^s(v), 0\}(1/2\bar{\epsilon})dv$, is positive.

Part 4: The expected utility of visiting the firm with data is Lipschitz continuous.

The utility of visiting the firm with data is $U^s(v) = \int_{v-\bar{\epsilon}}^{v+\bar{\epsilon}} \max\{v - p^d(\tilde{v}), 0\}(1/2\bar{\epsilon})d\tilde{v}$. There exists $K = \left(\frac{2}{2\bar{\epsilon}} + 1\right) \in \mathbb{R}^+$ such that $|U^d(v^1) - U^d(v^2)| \leq K|v^1 - v^2|$ for any v^1, v^2.

Part 5: By the previous results, the search rule of consumers is a cutoff rule.

Consider consumers with $v \leq p^{nd}$ receive 0 utility at the firm with data, but strictly positive utility at the firm with data (by previous arguments), so they all prefer the firm without data. If $v > p^{nd}$, the expected utility of visiting the firm without data is $U^{nd}(v) = v - p^{nd}$. It was shown that $U^s(v)$ is Lipschitz continuous, hence differentiable almost everywhere. The derivative of $U^s(v)$ is strictly below 1, since the price distribution at the firm with data changes in v. Thus, the preference for the firm with data, namely $U^s(v) - U^{nd}(v)$, is strictly falling in v. This establishes the result.

■

Proof of Proposition 22:

Part 1: If searchers search according to a cutoff rule, the optimal price function at the firm with data is:

$$p^d(\tilde{v}) = \begin{cases} 0.5(\tilde{v} + \bar{\epsilon}) & \tilde{v} \in [-\bar{\epsilon}, 3\bar{\epsilon}] \\ (\tilde{v} - \bar{\epsilon}) & \tilde{v} \in [3\bar{\epsilon}, 1 + \bar{\epsilon}] \end{cases} \tag{B.4.10}$$

<u>Part 1a:</u> Monopoly pricing

The maximization problem of a monopolist with data upon observing $\tilde{v}^1 \in [\bar{\epsilon}, 1 - \bar{\epsilon}]$ is to maximize the following through choice of $p_j \in [\tilde{v}^1 - \bar{\epsilon}, \tilde{v}^1 + \bar{\epsilon}]$:

$$\Pi(p_j; \tilde{v}^1) = p_j \int_{p_j}^{\tilde{v}^1 + \bar{\epsilon}} (1/2\bar{\epsilon})dv = p_j \left[\tilde{v}^1 + \bar{\epsilon} - p_j\right] \tag{B.4.11}$$

The first-order condition of this expression is equal to 0 at $p(\tilde{v}^1) = 0.5(\tilde{v}^1 + \bar{\epsilon})$. Whether this constitutes an interior price depends on $\bar{\epsilon}$. The lower bound is $\tilde{v}^1 - \bar{\epsilon}$. At $\tilde{v}^1 = 3\bar{\epsilon}$, we have that $p(3\bar{\epsilon}) = 2\bar{\epsilon}$, which is exactly equal to the lower bound. Thus, we can compute the following pricing schedule on $[\bar{\epsilon}, 1-\bar{\epsilon}]$ because $3\bar{\epsilon} < 1-\bar{\epsilon}$ by the fact that $\bar{\epsilon} < 0.25$:

$$p^*(\tilde{v}^1) = \begin{cases} 0.5(\tilde{v}^1 + \bar{\epsilon}) & \tilde{v}^1 \in [\bar{\epsilon}, 3\bar{\epsilon}] \\ (\tilde{v}^1 - \bar{\epsilon}) & \tilde{v}^1 \in [3\bar{\epsilon}, 1-\bar{\epsilon}] \end{cases} \tag{B.4.12}$$

Analogous arguments establish the optimal prices for $\tilde{v}^1 < \bar{\epsilon}$ and $\tilde{v}^1 > 1-\bar{\epsilon}$. Summing up, the optimal prices of a monopolist are given by the schedule listed at the beginning.

<u>Part 1b:</u> In the competitive equilibrium, the optimal price function remains unchanged.

In equilibrium, searchers will visit the firm with data if $v < \bar{v}$ and vice versa. Because they can all obtain strictly positive utility at the firm with data, $p^{nd} < \bar{v}$ must hold. Thus, searchers will push up p^{nd}, and thus $\bar{v} > p^{nd} \geq 0.5$ must hold in an equilibrium.

Consider a signal $\tilde{v}^1 < 3\bar{\epsilon}$, where $\tilde{v}^1 + \bar{\epsilon} < 4\bar{\epsilon} < 0.5 < \bar{v}$. Thus, all searchers with $v \in [\tilde{v}^1 - \bar{\epsilon}, \tilde{v}^1 + \bar{\epsilon}]$ arrive at the firm with data. The optimal price thus remains $0.5(\tilde{v}^1 + \bar{\epsilon})$.

Thus, consider a signal $\tilde{v}^1 > 3\bar{\epsilon}$, for which the upper bound of valuations may lie above $\bar{v}$. If $\tilde{v}^1 + \bar{\epsilon} \leq \bar{v}$, as before, then nothing changes - the optimal price is $\tilde{v}^1 - \bar{\epsilon}$.

Suppose instead that $\bar{v} < \tilde{v}^1 + \bar{\epsilon}$. Then, there are two further possibilities:

(i) $\bar{v} \leq \tilde{v}^1 - \bar{\epsilon}$: Then, no searchers would be arriving at this firm and generate $\tilde{v}^1$. This means that the optimal price is $\tilde{v}^1 - \bar{\epsilon}$ (the sale is only made to captive consumers).

(ii) $\bar{v} \in (\tilde{v}^1 - \bar{\epsilon}, \tilde{v}^1 + \bar{\epsilon})$: Some arriving searchers will generate $\tilde{v}^1$.

Suppose we have a price at which the sale is only made to captive consumers, which implies that $p_j \geq \bar{v} > \tilde{v}^1 - \bar{\epsilon}$. But because $\tilde{v}^1 > 3\bar{\epsilon}$, there is a downward deviation (since this raises the profits from captive consumers). Thus, consider a price $p_j < \bar{v}$, at which:

$$\Pi(p_j; \tilde{v}^1) = p_j \left[\rho \int_{p_j}^{\bar{v}} (1/2\bar{\epsilon}) dv + 0.5(1-\rho) \int_{p_j}^{\min\{\tilde{v}^1 + \bar{\epsilon}, 1\}} (1/2\bar{\epsilon}) dv \right] \tag{B.4.13}$$

Thus, the derivative at any p_j will be weakly below the monopoly case, which means that the optimal price must also be directly at the lower bound here.

Part 2: In equilibrium, $\bar{v} \geq 0.5(1+\rho)$ must hold.

For $\bar{v} < 0.5(1+\rho)$, we have $p^{nd,*}(\bar{v}) \geq \bar{v}$. However, this is not consistent with optimal search behaviour. Any searcher with $v > 0$ receives strictly positive utility at the firm with data. Thus, $p^{nd} < \bar{v}$ must hold in an equilibrium. But such a price would never be optimally set if $\bar{v} < 0.5(1+\rho)$.

Part 3: Establishing equilibrium existence.

For any $\bar{v} \geq 0.5(1+\rho)$, the firm with data will price according $p^*(\tilde{v})$. The strategy $s(v)$ is a cutoff rule. The price p^{nd} depends on $\bar{v}$. I work with the following object:

$$\hat{v}^C(\bar{v}) = \sup\left\{v \in [0,1] : \int_{v-\bar{\epsilon}}^{v+\bar{\epsilon}} \max\{v - p^d(\tilde{v}), 0\}(1/2\bar{\epsilon})d\tilde{v} - \max\{v - p^{nd}(\bar{v}), 0\} \geq 0\right\} \tag{B.4.14}$$

Since the function in this supremum is strictly falling in v, the supremum separates the groups of searchers who visit the firm with data from those who visit the firm without data.

As before, we work with boundary conditions and the intermediate value theorem. At $\bar{v} = 0.5(1+\rho)$, it holds that $p^{nd} = 0.5(1+\rho)$, which means that $\hat{v}^C(\bar{v}) > \bar{v}$ will hold at $\bar{v} = 0.5(1+\rho)$. At $\bar{v} = 1$, $\hat{v}^C(1) \leq 1$ holds by construction. Finally, the utility of visiting the firm with data will not be affected by changes in $\bar{v}$, but only p^{nd} responds to changes in $\bar{v}$. This establishes continuity of $\hat{v}^C(\bar{v})$, which confirms the existence of an equilibrium.

■

Proof of Lemma 10:

Part 1: Density of valuations at the firms

Note that the type $\theta \sim U[0,1]$ of any consumer is a random variable with density $f(\theta) = 1$. Moreover, the event "ds" denotes that a consumer is a searcher, while the event "lc" denotes the probability that a consumer is captive.

Consider any interval I. We can write the probability that $\theta^{nd} \in I$ as follows:

$$Pr(\theta^{nd} \in I) = Pr(\theta \in I | I^{nd}) = \frac{Pr(\theta \in I \wedge I^{nd})}{Pr(I^{nd})} = \frac{1}{Pr(I^{nd})}\int_0^1 Pr(\theta \in I \wedge I^{nd}|\theta)d\theta \tag{B.4.15}$$

Note that:

$$Pr(\theta \in I \wedge I^{nd}|\theta \wedge ds)Pr(ds|\theta) = \rho\mathbb{1}\big[\theta \in I\big]\big[\mathbb{1}[\theta > \bar{\theta}](1-g^H) + \mathbb{1}[\theta = \bar{\theta}](1-\bar{g}) + \mathbb{1}[\theta < \bar{\theta}](1-g^L)\big]$$

Similarly, we have that $Pr(\theta \in I \wedge I^{nd}|\theta \wedge lc)Pr(l|\theta) = 0.5(1-\rho)\mathbb{1}\big[\theta \in I\big]$. Thus:

$$Pr(\theta^{nd} \in I) = \int_I \Bigg(\underbrace{\frac{\rho\big[\mathbb{1}[\theta > \bar{\theta}](1-g^H) + \mathbb{1}[\theta = \bar{\theta}](1-\bar{g}) + \mathbb{1}[\theta < \bar{\theta}](1-g^L)\big] + 0.5(1-\rho)}{Pr(I^{nd})}}_{:=f^{nd}(\theta)}\Bigg)d\theta$$

Now consider the firm with data. The valuations of consumers that visit the firm with data and generate the signal $\tilde{v}^k$ is a random variable - call this θ^k. This is also an absolutely continuous random variable - the following holds for any interval I:

$$Pr(\theta^k \in I) = Pr(\theta \in I | I^k) = \frac{Pr(\theta \in I \wedge I^k)}{Pr(I^k)} = \frac{1}{Pr(I^k)} \int_0^1 Pr(\theta \in I \wedge I^k | \theta) d\theta \tag{B.4.16}$$

Note that:

$$Pr(\theta \in I \wedge I^k | \theta \wedge ds) Pr(ds|\theta) = \rho \mathbb{1}\big[\theta \in I\big] Pr^k(\theta) \big[\mathbb{1}[\theta > \bar{\theta}] g^H + \mathbb{1}[\theta = \bar{\theta}]\bar{g} + \mathbb{1}[\theta < \bar{\theta}] g^L\big] \tag{B.4.17}$$

Note further that $Pr(\theta \in I \wedge I^k|\theta \wedge lc)Pr(lc|\theta) = \mathbb{1}[\theta \in I]Pr^k(\theta)0.5(1-\rho)$. Thus, $Pr(\theta^k \in I)$ is:

$$\int_I \underbrace{\frac{1}{Pr(I^k)} \Big(\rho Pr^k(\theta)\big[\mathbb{1}[\theta > \bar{\theta}] g^H + \mathbb{1}[\theta = \bar{\theta}]\bar{g} + \mathbb{1}[\theta < \bar{\theta}] g^L\big] + Pr^k(\theta)0.5(1-\rho)\Big)}_{:=f^k(\theta)} d\theta \tag{B.4.18}$$

All densities are bounded from above and measurable, i.e. integrable, and thus well-defined.

Part 2: Calculating the virtual valuation functions

I omit the conditioning on $(\bar{\theta}, g)$ in the following arguments for ease of exposition.

Consider first the firm without data. For any $\theta < \bar{\theta}$, we can note that:

$$\begin{aligned} 1-F^{nd}(\theta) &= \frac{1}{Pr(I^{nd})}\int_\theta^{\bar{\theta}} \big(\rho(1-g^L)+0.5(1-\rho)\big)dx + \frac{1}{Pr(I^{nd})}\int_{\bar{\theta}}^1 \big(\rho(1-g^H)+0.5(1-\rho)\big)dx \\ \implies J^{nd}(\theta) &= \theta - \frac{\int_\theta^{\bar{\theta}} \big(\rho(1-g^L)+0.5(1-\rho)\big)dx + \int_{\bar{\theta}}^1 \big(\rho(1-g^H)+0.5(1-\rho)\big)dx}{\rho(1-g^L)+0.5(1-\rho)} \end{aligned} \tag{B.4.19}$$

For $\theta > \bar{\theta}$, the virtual valuation at the firm without data can be calculated as follows:

$$J^{nd}(\theta) = \theta - \frac{\int_\theta^1 \big(\rho(1-g^H)+0.5(1-\rho)\big)dx}{\rho(1-g^H)+0.5(1-\rho)} \tag{B.4.20}$$

One can show that $J^{nd}(\theta)$ is always piecewise strictly increasing.

Now let's calculate the virtual valuations at the firm with data. The virtual valuation takes the following form when $\theta < \bar{\theta}$:

$$J^k(\theta) = \theta - \frac{\int_\theta^{\bar{\theta}} \big(\rho g^L + 0.5(1-\rho)\big) Pr^k(x)dx + \int_{\bar{\theta}}^1 \big(\rho g^H + 0.5(1-\rho)\big) Pr^k(x)dx}{\big(\rho g^L + 0.5(1-\rho)\big) Pr^k(\theta)} \tag{B.4.21}$$

The virtual valuation takes the following form when $\theta > \bar{\theta}$:

$$J^k(\theta) = \theta - \frac{1 - F^k(\theta)}{f^k(\theta)} = \theta - \frac{\int_\theta^1 \big(\rho g^H + 0.5(1-\rho)\big) Pr^k(x)dx}{\big(\rho g^H + 0.5(1-\rho)\big) Pr^k(\theta)} \tag{B.4.22}$$

One can show that $J^H(\theta)$ is always piecewise strictly increasing.

Part 3: Setting up the expected revenues of the firms

Any mechanism is incentive compatible if and only of it satisfies both the integrability and the monotonicity condition. This follows because one can apply the envelope theorem from Milgrom & Segal (2002). Using standard arguments, the expected revenue becomes:

$$-\mathbb{E}[t(\theta)] = -U(0) + \int_0^1 q(\theta)\left(\theta - \frac{1 - F^{nd}(\theta)}{f^{nd}(\theta)}\right) f^{nd}(\theta)d\theta \tag{B.4.23}$$

Similar arguments prove that the firm with data, when observing a given signal, would also face an expected revenue function that is equal to:

$$-\mathbb{E}[t^k(\theta)] = -U^k(0) + \int_0^1 q^k(\theta)\left(\theta - \frac{1 - F^k(\theta)}{f^k(\theta)}\right) f^k(\theta)d\theta \tag{B.4.24}$$

Part 4: Ordering of cutoffs when consumers visit randomly:

The virtual valuations at the firm with data and without data are:

$$J^k(\theta) = \theta - \int_\theta^1 \big(Pr^k(x)/Pr^k(\theta)\big)dx \quad : \quad J^{nd}(\theta) = \theta - \int_\theta^1 (1)dx \tag{B.4.25}$$

Because $Pr^H(\theta)$ is strictly increasing, $Pr^L(x)/Pr^L(\theta) < 1 < Pr^H(x)/Pr^H(\theta)$ holds $\forall x > \theta$: This implies that, for any $\theta \in (0,1)$, the ordering $J^H(\theta) < J^{nd}(\theta) < J^L(\theta)$ holds.

To see this, note that the functions $J^{nd}(\theta)$ and $J^H(\theta)$ are monotonically increasing and continuous. Because $J^H(1) > 0$, we know that there must exist a $\hat{\theta}^H < 1$ such that $J^H(\hat{\theta}^H) = 0$ and that the virtual valuation is strictly positive for all $\theta \geq \hat{\theta}^H$. At $\theta > \hat{\theta}^H$, both other virtual valuations will be strictly positive, which implies that the associated cutoffs must both lie strictly below $\hat{\theta}^H$. That $\hat{\theta}^L < \hat{\theta}^{nd}$ must hold follows by analogous arguments.

Part 5: In any simple equilibrium, there must exist a $\bar{\theta}$ such that all consumers with $\theta < \bar{\theta}$ visit the firm with data and vice versa.

Part 5a: There exists no simple equilibrium in which all searchers randomize between firms.

In that setting, $\hat{\theta}^L < \hat{\theta}^{nd}$. The firm without data will offer quality 1 to all consumers with $\theta > \hat{\theta}^{nd}$ by monotonicity of $J^{nd}(\theta)$. Then, the firm with data must offer 0 quality to all consumers with $\theta \leq \hat{\theta}^{nd}$ - else, these consumers would not randomize. But then, we obtain a contradiction, as the utility of consumers with $\theta \geq \hat{\theta}^{nd}$ at the firm without data is $\theta - \hat{\theta}^{nd}$, but strictly below $(\theta - \hat{\theta}^{nd})$ at the firm with data since $\hat{\theta}^{nd} < \hat{\theta}^H$. Thus, they would not randomize, a contradiction.

Part 5b: Initial steps:

Suppose we are in a simple equilibrium where there exists no $\bar{\theta}$ such that all consumers with type above it visit the firm without data and vice versa. Thus, there must exist a $\bar{\theta}$ such that searchers with $\theta < \bar{\theta}$ visit the firm without data and all consumers with $\theta > \bar{\theta}$ visit the firm with data, i.e. $g^L = 0, g^H = 1$. In that case, we know that the virtual valuations of the firm with data jump up at $\bar{\theta}$, because:

$$\lim_{\theta \uparrow \bar{\theta}} J^k(\theta) < \lim_{\theta \downarrow \bar{\theta}} J^k(\theta) \iff 0.5(1-\rho) < \rho + 0.5(1-\rho) \tag{B.4.26}$$

Part 5c: $\hat{\theta}^L < \hat{\theta}^{nd}$ must hold in such a simple equilibrium with $g^L = 0$, $g^H = 1$.

Assume, for a contradiction, that $\hat{\theta}^L \geq \hat{\theta}^{nd}$ in the supposed equilibrium with $g^L = 0$, $g^H = 1$.

First, note that $\hat{\theta}^L \leq \hat{\theta}^H$ holds once more. Suppose, for a contradiction, that $\hat{\theta}^H < \hat{\theta}^L$ holds in such an equilbrium. When $\hat{\theta}^H \neq \bar{\theta}$, we know that continuity of the virtual valuation function implies that $\hat{\theta}^L < \hat{\theta}^H$ must hold, a contradiction.

Thus, suppose that $\hat{\theta}^H = \bar{\theta}$. Suppose that $\hat{\theta}^H < \hat{\theta}^L$ holds. This is impossible, as any $\theta > \hat{\theta}^H = \bar{\theta}$ must satisfy $J^H(\theta) \geq 0$ by monotonicity of $J^H(\theta)$ - else, we would have a contradiction. But this implies that $J^L(\theta) > 0$ holds for any $\theta > \bar{\theta}$, which implies that $\hat{\theta}^L \leq \bar{\theta} = \hat{\theta}^H$ must hold in this case, a contradiction.

Thus, we know that $\hat{\theta}^L \leq \hat{\theta}^H$ must hold in any equilibrium of this type. Moreover, recall that we have assumed (for a contradiction) that $\hat{\theta}^{nd} \leq \hat{\theta}^L$ holds.

Note that $\hat{\theta}^{nd} \neq \bar{\theta}$ must hold, since the virtual valuation at the firm with data jumps down in this subcase. There are thus two subcases: (i) $\hat{\theta}^{nd} > \bar{\theta}$ and (ii) $\hat{\theta}^{nd} < \bar{\theta}$.

(i) Subcase 1: $\bar{\theta} < \hat{\theta}^{nd}$

Suppose that $\hat{\theta}^{nd} < \hat{\theta}^{L}$ and that $\bar{\theta} < \hat{\theta}^{nd}$ holds. Because $\bar{\theta} < \hat{\theta}^{nd}$, the firm without data assigns quality $q^{nd}(\theta) = 1$ to all $\theta > \hat{\theta}^{nd}$ and 0 to all other types - because the virtual valuation at the firm without data is always monotonic. Then, all consumers with $\theta \in (\hat{\theta}^{nd}, \hat{\theta}^{L})$ would get strictly positive utility only at the firm without data and would prefer this firm but visit the firm with data in equilibrium, a contradiction.

Suppose that $\hat{\theta}^{nd} = \hat{\theta}^{L}$ and that $\bar{\theta} < \hat{\theta}^{nd}$ holds. Suppose further that $\hat{\theta}^{L} = \hat{\theta}^{H}$. But then, we have $\bar{\theta} < \hat{\theta}^{nd} = \hat{\theta}^{L} = \hat{\theta}^{H}$, which cannot be true. Hence, $\hat{\theta}^{L} < \hat{\theta}^{H}$ must hold true.

Thus, we have the ordering $\bar{\theta} < \hat{\theta}^{nd} = \hat{\theta}^{L} < \hat{\theta}^{H}$. Consumers with $\theta \in (\hat{\theta}^{nd}, \hat{\theta}^{H})$ get the utility $\theta - \hat{\theta}^{nd}$ at the firm without data and at most the utility $Pr^{L}(\theta)(\theta - \hat{\theta}^{L})$ at the firm with data. Because $\hat{\theta}^{nd} = \hat{\theta}^{L}$, they all strictly prefer the firm without data, but visit the firm with data in equilibrium (since $\hat{\theta}^{nd} > \bar{\theta}$), a contradiction.

(ii) Subcase 2: $\hat{\theta}^{nd} < \bar{\theta}$ and there exist **no** $\theta > \bar{\theta}$ for which $J^{nd}(\theta) < 0$.

For all types $\theta > \bar{\theta}$, the virtual value $J^{nd}(\theta)$ must then be strictly positive (the converse creates a contradiction by monotonicity of $J^{nd}(\theta)$). In that case, the optimal mechanism of the firm without data sets $q^{nd}(\theta) = 1$ for all $\theta > \hat{\theta}^{nd}$ - once again, because the virtual valuation function at the firm without data is always monotonic.

Suppose $\hat{\theta}^{nd} < \hat{\theta}^{L}$. Then, consumers with $\theta > \bar{\theta}$ get the utility $\theta - \hat{\theta}^{nd} > 0$ at the firm without data. The utility they get at the firm with data is weakly smaller than $\theta - \hat{\theta}^{L} < \theta - \hat{\theta}^{nd}$. Thus, all consumers with $\theta > \bar{\theta}$ strictly prefer the firm without data, a contradiction.

Suppose $\hat{\theta}^{nd} = \hat{\theta}^{L}$. Then, we must have $\hat{\theta}^{nd} = \hat{\theta}^{L} < \bar{\theta}$ and hence $\hat{\theta}^{L} < \hat{\theta}^{H}$ must hold. Once again, consumers with $\theta > \bar{\theta}$ get the utility $\theta - \hat{\theta}^{nd} > 0$ at the firm without data. The utility they get at the firm with data is strictly smaller than $\theta - \hat{\theta}^{L}$, because $\hat{\theta}^{L} < \hat{\theta}^{H}$. Thus, all consumers with $\theta > \bar{\theta}$ prefer the firm without data, a contradiction.

(iii) Subcase 3: $\hat{\theta}^{nd} < \bar{\theta}$ and there exist **some** $\theta > \bar{\theta}$ for which $J^{nd}(\theta) < 0$.

In order for such $\theta > \bar{\theta}$ for which $J^{nd}(\theta) < 0$ to exist, the right limit of the virtual valuation at the firm without data must be weakly negative - by monotonicity of $J^{nd}(\theta)$. For any $\bar{\theta} > 0.5$, this right limit will be strictly positive, because it is $\lim_{\theta \downarrow \bar{\theta}} J^{nd}(\theta) = 2\bar{\theta} - 1 > 0$.

Thus, $\bar{\theta} \leq 0.5$ must hold in such an equilibrium. We are still assuming, for a contradiction, that $\hat{\theta}^{L} \geq \hat{\theta}^{nd}$, and that $g^{H} = 1$ (high valuation searchers visit firm with data).

Assume, within this subcase, that $\bar{\theta} < \hat{\theta}^{L}$, which also implies that $\hat{\theta}^{L} < \hat{\theta}^{H}$. Then, the provided quality schedule of the firm without data must satisfy $q^{nd}(\theta) = 0$ for any $\theta \in [0, \hat{\theta}^L)$. If this (expected) quality is strictly positive for any such type, then the cutoff $\bar{\theta}$ would not be below $\hat{\theta}^L$ - this follows from the search behaviour of consumers, since a consumer with $\theta = \hat{\theta}^L$ would attain strictly positive utility only at the firm without data.

Thus, the lowest type that gets quality in equilibrium is $\hat{\theta}^L$. By our equilibrium refinement, $\bar{\theta} < \hat{\theta}^L$ thus cannot hold - all consumers with $\theta < \hat{\theta}^L$ visit the same firm, a contradiction.

Suppose instead that $\bar{\theta} = \hat{\theta}^L$, which implies the ordering $\hat{\theta}^{nd} < \bar{\theta} = \hat{\theta}^L$. Once again, any type $\theta \in [0, \hat{\theta}^L) = [0, \bar{\theta})$ must receive $q^{nd}(\theta) = 0$ - else, the cutoff must lie strictly above $\hat{\theta}^L = \bar{\theta}$ by monotonicity of $q^{nd}(\theta)$ in an equilibrium and the optimal search behavior of consumers.

Thus, all types $\theta \geq \bar{\theta} = \hat{\theta}^L$ for which $J^{nd}(\theta) < 0$ must also get zero quality - if they get positive quality, there would be a profitable deviation by monotonicity of $J^{nd}(\theta)$.

Thus, $\underline{\theta}^{nd} > \bar{\theta}$ would hold. Then, either $\underline{\theta}^d \leq \bar{\theta}$ or $\underline{\theta}^d > \bar{\theta}$ must hold. If $\underline{\theta}^d \leq \bar{\theta}$, we have $\underline{\theta}^d \leq \bar{\theta} < \underline{\theta}^{nd}$, and thus all consumers with $\theta < \bar{\theta}$ visit the firm with data by our first tie-breaking rule, a contradiction. If $\underline{\theta}^d > \bar{\theta}$, our tie-breaking rules imply a contradiction as well.

Thus, we must have $\hat{\theta}^L < \bar{\theta}$ in this sort of problematic equilibrium. It was previously also established that the equilibrium under consideration must satisfy $\bar{\theta} \leq 0.5$. For any such $\bar{\theta}$, our assumption implies that $\hat{\theta}^L \geq \bar{\theta}$, which means that no such equilibrium can exist.

Thus, we are done. We have shown that $\hat{\theta}^L < \hat{\theta}^{nd}$ must hold.

<u>Part 5d:</u> There exists no simple equilibrium in which $g^L = 0$ and $g^H = 1$ (all types above $\bar{\theta}$ go to the firm with data).

We have established that $\hat{\theta}^L < \hat{\theta}^{nd}$ holds in a simple equilibrium in which there exists no $\bar{\theta}$ such that consumers with type above it go to the firm without data and vice versa. In this sort of equilibrium, it must also hold that $\hat{\theta}^{nd} \neq \bar{\theta}$. If all consumers visit the firm with data in equilibrium, we can write $\bar{\theta} = 1$ and all consumers with type below this visit the firm with data, a contradiction.

Suppose that $\bar{\theta} < \hat{\theta}^{nd}$, which means that $\underline{\theta}^{nd} = \hat{\theta}^{nd}$. Recall that all consumers with $\theta < \bar{\theta}$ supposedly visit the firm without data. Then, $\underline{\theta}^d$ must be weakly above $\hat{\theta}^{nd}$ - else, all consumers with $\theta < \bar{\theta}$ must visit the firm with data by our refinement. But this implies that all consumers with $\theta < \hat{\theta}^{nd}$ either all visit the firm without data (if $\underline{\theta}^{nd} < \underline{\theta}^d$) or $\underline{\theta}^d = \underline{\theta}^{nd}$ - in either case, they all visit the same firm by our tie-breaking rule, a contradiction.

Finally, suppose that $\hat{\theta}^{nd} < \bar{\theta}$ and hence $\hat{\theta}^L < \bar{\theta}$. No type $\theta \in (0, \hat{\theta}^{nd})$ can receive positive quality at the firm with data. Else, all types $\theta < \hat{\theta}^{nd}$ would visit the firm with data by our refinement but visit the firm without data in the supposed equilibrium, a contradiction. Thus, we must have $\hat{\theta}^{nd} \leq \underline{\theta}^d$.

Moreover, the fact that $\hat{\theta}^L < \bar{\theta}$ implies that $\bar{\theta} \geq 0.5$ must hold. Previous results imply that the virtual valuation of the firm without data thus stays weakly positive for all $\theta > \hat{\theta}^{nd}$. Thus, the firm without data will offer quality 1 to all $\theta > \hat{\theta}^{nd}$. Because $\hat{\theta}^{nd} = \underline{\theta}^{nd} \leq \underline{\theta}^d$all consumers weakly prefer the firm without data.

Suppose (within the subcase $\hat{\theta}^{nd} < \bar{\theta}$) that $\hat{\theta}^{nd} < \underline{\theta}^d$. Then, all consumers visit the firm without data and we could express the search behavior by $\bar{\theta} = 0$, where all consumers with type above $\bar{\theta}$ visit the firm without data, a contradiction.

Thus, suppose (within the subcase $\hat{\theta}^{nd} < \bar{\theta}$) that $\hat{\theta}^{nd} = \underline{\theta}^d$. Unless both firms offer exactly the same menu, all consumers with $\theta > \hat{\theta}^{nd}$ will strictly prefer the firm without data. If the preference is strict, all consumers will visit the firm without data, since $\bar{\theta} > 0$ holds in the supposed equilibrium and all consumers with $\theta > \hat{\theta}^{nd}$ strictly prefer the firm without data. But then, we could have expressed the strategy with a cutoff $\bar{\theta} = 0$ s.t. all consumers with $\theta < \bar{\theta}$ visit the firm with data and vice versa, a contradiction.

If the menus are the same, all searchers randomize, which cannot be a simple equilibrium.

■

Proof of Proposition 23:

Part 1: In a simple equilibrium, $\hat{\theta}^L < \hat{\theta}^{nd}$ holds

Suppose we are in a simple equilibrium, in which searchers with $\theta < \bar{\theta}$ visit the firm with data. Suppose, for the contradiction we seek, that $\hat{\theta}^{nd} \leq \hat{\theta}^L$. In the equilibrium we study, the virtual valuation functions at the firm with data jump down at $\bar{\theta}$, i.e. $\lim_{\theta \uparrow \bar{\theta}} J^k(\theta) > \lim_{\theta \downarrow \bar{\theta}} J^k(\theta)$.

Note first that $\hat{\theta}^L < \hat{\theta}^H$ must hold. This is because neither cutoff $\hat{\theta}^k$ can be exactly at $\bar{\theta}$ - then, $\lim_{\theta \downarrow \bar{\theta}} J^k(\theta) \geq 0$ would have to hold and hence the virtual corresponding valuation would also be strictly positive for values just below $\bar{\theta}$. Thus, $\hat{\theta}^L$ and $\hat{\theta}^H$ must be at points where the virtual valuation is continuous, i.e. they must set the corresponding virtual valuations to zero. A $\hat{\theta}^H$, we would thus have $J^H(\hat{\theta}^H) = 0$ and hence $J^L(\hat{\theta}^H) > 0$. Because $J^L(\hat{\theta}^H) > 0$ would hold there, thus would also hold for values just below $\hat{\theta}^H$. Hence, we know that $\hat{\theta}^L < \hat{\theta}^H$ must hold in an equilibrium where consumers seperate in this way.

The virtual valuation $J^{nd}(\theta)$ jumps up at $\bar{\theta}$, i.e. the utility any consumer attains at the firm with data is $\max\{\theta - \hat{\theta}^{nd}, 0\}$. Thus, all searchers with $\theta > \hat{\theta}^{nd}$ prefer the firm without data, since $\hat{\theta}^{nd} \leq \hat{\theta}^L < \hat{\theta}^H$. Thus, $\bar{\theta} \leq \hat{\theta}^{nd}$ must hold, Because $\hat{\theta}^{nd} \leq \hat{\theta}^L$ by assumption, we obtain a contradiction, since $J^{nd}(\theta; \bar{\theta}) < J^L(\theta; \bar{\theta})$ for all $\theta \geq \bar{\theta}$ and thus,

$\hat{\theta}^L < \hat{\theta}^{nd}$ would hold.

Part 2: There exists no simple equilibrium in which $\bar{\theta} \in [0.5, 0.5(1+\rho)]$.

Consider first $\bar{\theta} \in [0.5, 0.5(1+\rho)]$. We know that $\hat{\theta}^{nd} \geq \bar{\theta}$ holds for these values of $\bar{\theta}$, since $\lim_{\theta \uparrow \bar{\theta}} J^{nd}(\theta; \bar{\theta}) < 0$ for $\bar{\theta} < 0.5(1+\rho)$.

The virtual valuation $J^L(\theta)$ will be strictly positive for any $\theta > \hat{\theta}^L$. If $\hat{\theta}^L > \bar{\theta}$, this is true by monotonicity. The case $\hat{\theta}^L = \bar{\theta}$ cannot be true, because, in a simple equilibrium, the virtual valuations at the firm with data jump down.

Thus, consider the third case where $\hat{\theta}^L < \bar{\theta}$. While the virtual valuation will be strictly positive for any $\theta \in (\hat{\theta}^L, \bar{\theta})$, it may drop into the negative at $\bar{\theta}$. To discuss this, consider the right limit of $J^L(\theta)$ at $\bar{\theta}$. Recall that $J^L(\theta) = \theta - \int_\theta^1 \big(Pr^L(x)/Pr^L(\theta)\big)dx$ for any $\theta > \bar{\theta}$: We know that $Pr^L(x)/Pr^L(\theta) < 1$ holds for all $x > \theta$ at any θ. Thus, we have:

$$\lim_{\theta \downarrow \bar{\theta}} J^L(\theta) = \bar{\theta} - \int_{\bar{\theta}}^1 \big(Pr^L(x)/Pr^L(\bar{\theta})\big)dx > \bar{\theta} - \int_{\bar{\theta}}^1 \big(1\big)dx = \bar{\theta} - (1-\bar{\theta}) = 2\bar{\theta} - 1 \tag{B.4.27}$$

Because $\bar{\theta} \geq 0.5$, it follows that $\lim_{\theta \downarrow \bar{\theta}} J^L(\theta) > 0$. Thus, the low signal optimal mechanism is uniquely pinned down - all types above $\hat{\theta}^L$ will be assigned the quality level $q^L(\theta) = 1$.

Recall that $\hat{\theta}^L < \hat{\theta}^{nd}$ must hold in such an equilibrium - but this contradicts the statement $\hat{\theta}^{nd} \geq \bar{\theta}$. All consumers with type below $\hat{\theta}^{nd}$ surely prefer the firm with data. Moreover, consumers with $\theta = \hat{\theta}^{nd}$ attain utility 0 at the firm with data, but strictly positive utility at the firm without data - so they visit the firm with data, and so will consumers with a type just above $\hat{\theta}^{nd}$. In equilibrium, they visit the firm without data, a contradiction.

Part 3: There exists no simple equilibrium in which $\bar{\theta} \in [0, 0.5)$.

First, note that $\hat{\theta}^{nd} = 0.5$ holds for any such $\bar{\theta} < 0.5$. The virtual valuation at the firm without data jumps up at $\bar{\theta}$. Thus, we consider the right limit of $J^{nd}(\theta)$ at $\bar{\theta}$. Recall that this virtual valuation equals $J^{nd}(\theta) = \theta - \int_\theta^1 (1)dx$ for any $\theta > \bar{\theta}$. Thus, we have $\lim_{\theta \downarrow \bar{\theta}} J^{nd}(\theta) = \bar{\theta} - (1-\bar{\theta}) = 2\bar{\theta} - 1 < 0$. Moreover, we have that:

$$\lim_{\theta \downarrow \bar{\theta}} J^{nd}(\theta) = \bar{\theta} - \int_{\bar{\theta}}^1 (1)dx > \bar{\theta} - \int_{\bar{\theta}}^1 \left(\frac{\rho + 0.5(1-\rho)}{0.5(1-\rho)}\right)dx = \lim_{\theta \uparrow \bar{\theta}} J^{nd}(\theta) \tag{B.4.28}$$

This proves that $\hat{\theta}^{nd} > \bar{\theta}$ holds for such $\bar{\theta} < 0.5$. Moreover, $\hat{\theta}^{nd} = 0.5$ will hold exactly.

For any $\theta > \bar{\theta}$, the virtual valuation $J^L(\theta)$ is strictly greater than $J^{nd}(\theta)$. Thus, we have some $\tilde{\theta}^L > \bar{\theta}$ with: (i) $\tilde{\theta}^L < \hat{\theta}^{nd}$ and (ii) $J^L(\theta) > 0$ for any $\theta > \tilde{\theta}^L$.

Suppose we have a simple equilibrium in which $\bar{\theta} < 0.5$. In equilibrium, the firm with data must optimally assign strictly positive quality to a strictly positive measure of consumers with $\theta \in (\tilde{\theta}^L, \hat{\theta}^{nd})$.

Suppose, for a contradiction, that all consumers with $\theta \in (\tilde{\theta}^L, \hat{\theta}^{nd})$ receive the quality level 0 according to the mechanism $q^L(\theta)$. By monotonicity, all consumers with $\theta \leq \tilde{\theta}^L$ must also receive the quality level 0. But this is a contradiction - all consumers with $\theta > \tilde{\theta}^L$ have strictly positive virtual valuations. Thus, the firm is guaranteed to make more higher revenue by setting $q^L(\theta) = 1$ to all $\theta \in (\tilde{\theta}^L, 1)$. This also satisfies monotonicity, so it is feasible, and we cannot have an equilibrium.

Thus, there must exist a type $\theta \in (\tilde{\theta}^L, \hat{\theta}^{nd})$ that receives a strictly positive quality level $q^L(\theta)$ at the firm with data. By monotonicity, all types above this must also receive a strictly positive $q^L(\theta)$. Thus, the utility of a consumer with $\theta = \hat{\theta}^{nd} > \bar{\theta}$ at the firm with data will be strictly positive. The utility this consumer would receive at the firm without data would be zero - this is a contradiction, as consumers with θ just above $\bar{\theta}$ would similarly prefer the firm with data, but visit the firm without data in equilibrium.

Part 4: Under said assumptions, there exists a simple equilibrium with $\bar{\theta} \geq [0.5(1+\rho), 1]$.

Properties of the virtual valuations under the assumptions:

The virtual valuation of the firm without data always satisfies monotonicity. Moreover, this virtual valuation $J^{nd}(\theta)$ jumps up around $\bar{\theta}$ in the equilibrium we study.

When $\theta < \bar{\theta}$, we have:

$$\frac{\partial J^k}{\partial \theta} = 2+$$
$$\left(\frac{\int_\theta^{\bar{\theta}} \big(\rho g^L + 0.5(1-\rho)\big) Pr^k(x)dx + \int_{\bar{\theta}}^1 \big(\rho g^H + 0.5(1-\rho)\big) Pr^k(x)dx}{\big(\rho g^L + 0.5(1-\rho)\big) Pr^k(\theta)}\right) \left[\big(Pr^k(\theta)\big)^{-1} \frac{\partial Pr^k(\theta)}{\partial \theta}\right] \tag{B.4.29}$$

For $\theta > \bar{\theta}$, this derivative is:

$$\frac{\partial J^k}{\partial \theta} = 2 - \int_\theta^1 \big(Pr^k(x)/Pr^k(\theta)\big)dx \left[-\big(Pr^k(\theta)\big)^{-1} \frac{\partial Pr^k(\theta)}{\partial \theta}\right] \tag{B.4.30}$$

Thus, the high signal virtual valuation is generally piecewise increasing, while the low signal virtual valuation is rising under our assumptions for $\bar{\theta} \in [0.5(1+\rho), 1]$ and $(g^L = 1, g^H = 0)$.

It was further assumed that both virtual valuations do not jump into the negative region at $\bar{\theta}$ for $\bar{\theta} \geq 0.5(1+\rho)$. Formally, it was stated that $\lim_{\theta \downarrow \bar{\theta}} J^k(\theta; \bar{\theta}) > 0$. Recall that

$\lim_{\theta\uparrow\bar{\theta}} J^k(\theta;\bar{\theta}) > \lim_{\theta\downarrow\bar{\theta}} J^k(\theta;\bar{\theta})$ holds, which implies that $\hat{\theta}^k < \bar{\theta}$ must hold.

Summing it all up, these assumptions imply that the virtual valuation functions will be strictly positive for all types above the respective cutoffs $\hat{\theta}^{nd}$ and $\hat{\theta}^k$, which implies that the optimal mechanism just assigns quality 1 to all consumers with a positive virtual valuation.

General notions:

I describe the consumer's search behavior by:

$$\theta^*(\bar{\theta}) = \sup\left\{\theta \in [0,1] : Pr^L(\theta)\hat{\theta}^L(\bar{\theta}) + Pr^H(\theta)\hat{\theta}^H(\bar{\theta}) < \hat{\theta}^{nd}(\bar{\theta})\right\} \tag{B.4.31}$$

We have defined $\hat{\theta}^k(\bar{\theta}) = \inf\left\{\theta : J^k(\theta,\bar{\theta}) > 0\right\}$ and $\hat{\theta}^{nd}(\bar{\theta}) = \inf\left\{\theta : J^{nd}(\theta,\bar{\theta}) > 0\right\}$. Suppose we have found an $\bar{\theta} \geq 0.5(1+\rho)$.

To establish that this is an equilibrium, we make use of some ancillary results. By assumption, we have that $\hat{\theta}^H < \bar{\theta}$ holds for any $\bar{\theta} \geq 0.5(1+\rho)$. Previous arguments imply that $\hat{\theta}^{nd} \leq \bar{\theta}$ for any such $\bar{\theta} \geq 0.5(1+\rho)$. $\hat{\theta}^{nd}$ has to solve the following:

$$J^{nd}(\hat{\theta},\bar{\theta}) = \hat{\theta} - \frac{\int_{\hat{\theta}}^{\bar{\theta}} 0.5(1-\rho)dx + \int_{\bar{\theta}}^{1} 0.5(1+\rho)dx}{0.5(1-\rho)} = 0 \tag{B.4.32}$$

Thus, $\hat{\theta}^{nd}$ is falling in $\bar{\theta}$. Now consider $\hat{\theta}^k$, which has to solve the following since $\hat{\theta}^k < \bar{\theta}$:

$$\hat{\theta}^k - \int_{\hat{\theta}^k}^{\bar{\theta}} \left(Pr^k(x)/Pr^k(\hat{\theta}^k)\right)dx - \int_{\bar{\theta}}^{1} \left((1-\rho)/(1+\rho)\right)\left(Pr^k(x)/Pr^k(\hat{\theta}^k)\right)dx = 0 \tag{B.4.33}$$

Thus, we have $\frac{\partial\hat{\theta}^k}{\partial\bar{\theta}} = -\frac{\partial J/\partial\bar{\theta}}{\partial J/\partial\hat{\theta}^k} > 0$. Summing up, our assumptions imply that $\frac{\partial\hat{\theta}^k}{\partial\bar{\theta}} > 0$ and that $\frac{\partial\hat{\theta}^{nd}}{\partial\bar{\theta}} < 0$. Because $\hat{\theta}^L(1) < \hat{\theta}^{nd}(1)$, the following holds for any $\bar{\theta} \in [0.5(1+\rho), 1]$:

$$\hat{\theta}^L(\bar{\theta}) \leq \hat{\theta}^L(1) < \hat{\theta}^{nd}(1) \leq \hat{\theta}^{nd}(\bar{\theta}) \tag{B.4.34}$$

Thus, we must have $\hat{\theta}^L < \hat{\theta}^{nd}$ in the equilibrium candidate we have found.

Optimal menus:

Consider first the firm without data, for which $J^{nd}(\theta)$ is monotonic and jumps upward at $\bar{\theta}$. The virtual valuation $J^{nd}(\theta)$ is thus strictly positive if and only if $\theta > \hat{\theta}^{nd}$. Thus, their optimal menu is to offer $q^{nd}(\theta) = 1$ to all $\theta > \hat{\theta}^{nd}$ and quality 0 to all other types.

Now consider the firm with data. The virtual valuations are piecewise monotonic by assumption and don't jump into the negative region at $\bar{\theta}$, again by assumption. Thus, the virtual valuation $J^k(\theta)$ will be strictly positive iff $\theta > \hat{\theta}^k$, which means their optimal mechanism will also assign $q^k(\theta) = 1$ to all $\theta > \hat{\theta}^k$ and quality 0 to all other types.

Search:

Because $\hat{\theta}^L < \hat{\theta}^{nd}$, all searchers with $\theta < \hat{\theta}^{nd}$ visit the firm with data. The preference for the firm with data of consumers with $\theta \in [\hat{\theta}^{nd}, \hat{\theta}^H]$ is $P^d(\theta) = Pr^L(\theta)(\theta - \hat{\theta}^L) - (\theta - \hat{\theta}^{nd})$.

This is falling in θ. Because $\hat{\theta}^H$ lies strictly below $\theta^*(\bar{\theta}) = \bar{\theta}$ by assumption and the fact that the LHS in this sup-expression is strictly rising in θ, we have:

$$Pr^L(\hat{\theta}^H)\hat{\theta}^L + Pr^H(\hat{\theta}^H)\hat{\theta}^H < \hat{\theta}^{nd} \iff Pr^L(\theta)\big(\hat{\theta}^H - \hat{\theta}^L\big) - \big(\hat{\theta}^H - \hat{\theta}^{nd}\big) > 0 \tag{B.4.35}$$

Thus, the consumer with $\theta = \hat{\theta}^H$ prefers the firm with data, and so will all $\theta \in [\hat{\theta}^{nd}, \hat{\theta}^H]$.

Now consider $\theta \in [\hat{\theta}^H, \bar{\theta})$. We know that $q^H(\theta) = 1$ for any $\theta \in (\hat{\theta}^H, \bar{\theta})$ and $q^L(\theta) = 1$ for any $\theta \in (\hat{\theta}^L, \bar{\theta})$. Thus, the utility that this consumer attains at the firm with data is $U^d(\theta) = Pr^L(\theta)(\theta - \hat{\theta}^L) + Pr^H(\theta)(\theta - \hat{\theta}^H)$. Similarly, $q^{nd}(\theta) = 1$ for any $\theta \in (\hat{\theta}^{nd}, \bar{\theta})$, i.e. their utility at the firm without data is $\theta - \hat{\theta}^{nd}$. Thus, any such searchers prefer the firm with data because $\hat{\theta}^{nd} > Pr^L(\theta)\hat{\theta}^L + Pr^H(\theta)\hat{\theta}^H$. Consumers with $\theta > \bar{\theta}$ prefer to visit the firm without data by analogous arguments.

Thus: When $\bar{\theta} = \theta^*(\hat{\theta})$ and $\bar{\theta} > 0.5(1+\rho)$, we have an equilibrium.

Existence of a solution to $\bar{\theta} = \theta^*(\hat{\theta})$ on $\bar{\theta} > 0.5(1+\rho)$

It remains to show that such a value exists. To see this, recall first that $\hat{\theta}^k$ are both strictly rising in $\bar{\theta}$, while $\hat{\theta}^{nd}$ is strictly falling in $\bar{\theta}$. We work with the object $\bar{\theta}'$. At $\bar{\theta} = \bar{\theta}'$, we have:

$$Pr^L(1)\hat{\theta}^L(\bar{\theta}') + Pr^H(1)\hat{\theta}^H(\bar{\theta}') - \hat{\theta}^{nd}(\bar{\theta}') = 0 \tag{B.4.36}$$

Note that this function is strictly rising in θ, which implies that $\theta^*(\bar{\theta}) = 1$ for any such $\bar{\theta} \in [0.5(1+\rho), \bar{\theta}']$. Further note that $\bar{\theta}'$ must be strictly above $\bar{\theta} = 0.5(1+\rho)$, since:

$$\hat{\theta}^L(0.5(1+\rho)) < \hat{\theta}^H(0.5(1+\rho)) < 0.5(1+\rho) = \hat{\theta}^{nd}(0.5(1+\rho)) \tag{B.4.37}$$

For any $\bar{\theta} \in [\bar{\theta}', 1]$, we have $Pr^L(1)\hat{\theta}^L(\bar{\theta}') + Pr^H(1)\hat{\theta}^H(\bar{\theta}') - \hat{\theta}^{nd}(\bar{\theta}') \geq 0$. This implies that $\theta^*(\bar{\theta})$ must solve:

$$Pr^L(\theta^*)\hat{\theta}^L(\bar{\theta}') + Pr^H(\theta^*)\hat{\theta}^H(\bar{\theta}') - \hat{\theta}^{nd}(\bar{\theta}') = 0 \tag{B.4.38}$$

Thus: (i) At $\bar{\theta} = \bar{\theta}'$, we have $\theta^*(\bar{\theta}) \geq \bar{\theta}$. (ii) At $\bar{\theta} = 1$, $\theta^*(\bar{\theta}) \leq \bar{\theta}$. By assumption and our result, all cutoffs are strictly below $\bar{\theta}$ for $\bar{\theta} \geq 0.5(1+\rho)$, Thus, their solutions are continuous in $\bar{\theta}$, and so is $\theta^*(\bar{\theta})$. Application of the intermediate value theorem to the equation $\theta^*(\bar{\theta}) - \bar{\theta} = 0$ just laid out, together with the border conditions, guarantee existence of such a $\bar{\theta}$.

■

Proof of Lemma 11:

Because there is no search after visiting the first firm by assumption, previous arguments show that any equilibrium must satisfy the ordering $p^{LW} \leq p^{HW}$ and $p^{LB} \leq p^{HB}$.

Moreover, there exists no simple equilibrium in which all consumers randomize under assumption 4. If all consumers randomize, $p^{LB} = p^{LB,M} < p^{LW,M} = p^{LW}$ would be optimally set - but then, searchers with $v \in (p^{LB}, p^{LW})$ would not randomize.

Part 1: In a simple equilibrium, $p^{LB} \leq p^{LW}$ must hold.

Suppose, for a contradiction, that $p^{LW} < p^{LB}$. We know that all consumers with $v < p^{LB}$ will surely visit the firm with worse data.

Suppose that $p^{HW} \leq p^{LB}$ holds as well. Given that $p^{LW} \leq p^{HW}$ and $p^{LB} \leq p^{HB}$ must hold as well, $p^{LW} \leq p^{HW} \leq p^{LB} \leq p^{HB}$ holds. Then, all searchers will visit the firm with worse data (since $p^{LW} < p^{LB}$), which would then imply a contradiction, since $p^{LB} < p^{LW}$ holds true when the valuations of consumers are uniformly distributed by assumption 15.

Thus, the only possible equilibria with $p^{LW} < p^{LB}$ must satisfy $p^{LB} < p^{HW}$. Then, there are two possibilities now (i) $p^{LW} < p^{LB} < p^{HW} \leq p^{HB}$, and (ii) $p^{LW} < p^{LB} \leq p^{HB} < p^{HW}$. Neither of these can constitute an equilibrium, as I will show now.

(i) Ruling out $p^{LW} < p^{LB} < p^{HW} \leq p^{HB}$.

Note first that all consumers with $v \leq p^{LB}$ will surely visit the firm with worse data. The expected utilities of consumers with $v \in [p^{HB}, 1]$ are:

$$U^B(v) = v - \left(Pr^{HB}(v)p^{HB} + Pr^{LB}(v)p^{LB}\right) \tag{B.4.39}$$

$$U^W(v) = v - \left(Pr^{HW}(v)p^{HW} + Pr^{LW}(v)p^{LW}\right) \tag{B.4.40}$$

For a consumer with $v > 0.5$, the expected price at the firm with better data is higher:

$$Pr^{HB}(v)p^{HB} + Pr^{LB}(v)p^{LB} > Pr^{HW}(v)p^{HW} + Pr^{LW}(v)p^{LW} \tag{B.4.41}$$

But this yields a contradiction. Consumers with $v > \max\{p^{HB}, 0.5\}$ will prefer visiting the firm with worse data, because the expected price there is lower for them. In any equilibrium, $p^{HB} < 1$ must hold. Thus, both consumers with valuations $v \in (\max\{p^{HB}, 0.5\}, 1]$ and low-valuation consumers prefer the firm with worse data. Either the definition of the simple equilibrium fails or all consumers prefer the firm with worse data - in which case our results on the resulting optimal prices under uniform valuations imply that $p^{LB} < p^{LW}$, a contradiction.

(ii) Ruling out $p^{LW} < p^{LB} \leq p^{HB} < p^{HW}$.

Once again, all searchers with valuations $v \leq p^{LB}$ surely visit the firm with worse data. Thus, the cutoff $\bar{v}$ in a simple equilibrium must be such that all consumers with $v > \bar{v}$ visit the firm with better data.

Suppose, for a contradiction, that $\bar{v} < p^{HB}$. Then, no searchers will buy at the firm with worse data (since searchers only visit the firm with worse data if $v \leq \bar{v}$), i.e. $p^{HW} = p^{HW,M}$. The price of the firm with better data is strictly above $\bar{v}$, i.e. has to satisfy $p^{HB} = p^{HB,M}$. However, we know that $p^{HW,M} < p^{HB,M}$, which implies that $p^{HW} < p^{HB}$, a contradiction.

Suppose, instead, that $\bar{v} \in [p^{HB}, p^{HW})$. Then, searchers put upward pressure on p^{HB} and vice versa. Thus, the two prices will optimally satisfy $p^{HW} < p^{HB}$, a contradiction.

Thus, we have ruled out any candidate for an equilibrium in which $p^{LW} < p^{LB}$.

Part 2: In a simple equilibrium (where $p^{LB} \leq p^{LW}$ by part 1), the cutoff $\bar{v}$ must be such that all consumers with $v > \bar{v}$ visit the firm with worse data.

In such an equilibrium, all consumers with $v \leq p^{LW}$ surely visit the firm with better data. If all consumers weakly prefer the firm with better data, then their strategy can be described by cutoff rule $\bar{v} = 1$, where they visit the firm with better data if and only if $v \leq \bar{v}$.

Suppose there exists some consumer who strictly prefers the firm with worse data. Then, any cutoff $\bar{v}$ must be interior, i.e. $\bar{v} \in (0, 1]$. Because all consumers with $v \leq p^{LW}$ surely visit the firm with better data, there exists no $\bar{v}$ such that all consumers with $v < \bar{v}$ visit the firm with worse data and vice versa.

Part 3: If $\bar{v} < \bar{v}^{LW}$, $p^{LW}(\bar{v}) \geq \bar{v}$ will be optimally set by the firm with worse data.

Consider the function $H(\bar{v}) := \rho \int_{\bar{v}}^{1} Pr^{LW}(v)dv + 0.5(1-\rho)\left[\int_{\bar{v}}^{1} Pr^{LW}(v)dv - \bar{v}Pr^{LW}(\bar{v})\right]$. Our assumption on concavity of the low signal monopoly profit function implies that this function is strictly decreasing in $\bar{v}$. An appropriate $\bar{v}^{LW}$ that sets the above equation to zero exists by the intermediate value theorem.

Now consider a $\bar{v} < \bar{v}^{LW}$, at which $H(\bar{v}) > 0$. Consider the optimal pricing problem of the firm with worse data when observing the low signal. For prices $p_j < \bar{v}$, the profits are:

$$\Pi^{LW}(p_j) = \rho p_j \int_{\bar{v}}^{1} Pr^{LW}(v)dv + 0.5(1-\rho)p_j \int_{p_j}^{1} Pr^{LW}(v)dv \tag{B.4.42}$$

The derivative $\frac{\partial \Pi^{LW}(p_j)}{\partial p_j}$ is strictly decreasing in p_j by previous arguments. At $p_j = \bar{v}$, we know that the (left) derivative is strictly positive because $\bar{v} < \bar{v}^{LW}$. Thus, this derivative must be strictly positive for all $p_j < \bar{v}$. Thus, $p^{LW}(\bar{v}) \geq \bar{v}$ must hold.

Part 4: In a simple equilibrium, $\bar{v}^{LW} \leq \bar{v}$ must hold.

Suppose, for a contradiction, that $\bar{v} < \bar{v}^{LW}$. Then, we have established that $p^{LW}(\bar{v}) \geq \bar{v}$ will hold. Note that $p^{HW}(\bar{v}) \geq p^{LW}(\bar{v})$ will generally hold. Also recall that $p^{LB} \leq p^{LW}$ must hold in a simple equilibrium.

If $p^{LB} < p^{LW}$, a consumer with $v = p^{LW}$ will strictly prefer to visit the firm with better data - and by continuity arguments, so will consumers with valuation v just above p^{LW}. This represents a contradiction to the properties of $\bar{v}$. This is because $\bar{v} \leq p^{LW}$, but consumers with a valuation in an open ball above p^{LW}, i.e. with $v > \bar{v}$, would strictly prefer to visit the firm with better data but visit the firm with worse data in equilibrium.

Suppose that $p^{LB} = p^{LW}$. There exists no simple equilibrium in which a firm sets a uniform price. Thus, $p^{LB} < p^{HB}$ and $p^{LW} < p^{HW}$ must hold in such an equilibrium.

Suppose that $p^{LB} = p^{LW} < 0.5$. Then, all consumers with valuation in an open ball above p^{LW} will strictly prefer to visit the firm with better data, because they receive the low signal price there with a higher probability, a contradiction because $\bar{v} \leq p^{LW}$.

Suppose alternatively that $p^{LB} = p^{LW} \geq 0.5$. Then, consumers with $v < 0.5$ will visit the firm with better data (since they offer lowest the equilibrium price with higher probability to them) and consumers with v just above 0.5 will visit the firm with worse data, since they receive the low signal price with higher probability at the firm with worse data. Thus, this cutoff must then be exactly equal to $\bar{v} = 0.5$ in a simple equilibrium. Searchers visit the firm with better data if and only if $v < \bar{v} = 0.5$. But then, the optimal price of the firm with better data is strictly below 0.5, a contradiction.

■

Proof of Proposition 24:

Part 1: For any $\bar{v} \geq \max\{p^{HB}, \bar{v}^{LW}\}$, the function $\hat{v}^X(\bar{v})$ will be continuous.

The prices of the firm with better data are both strictly below $\bar{v}$ because $p^{HB} \leq \bar{v}$, which implies that $p^{L,*}(\bar{v})$ and $p^{H,*}(\bar{v})$ must solve the FOCs and be continuous.

We can also generally prove continuity of the prices of the firm with worse data. The low signal price has to be weakly below $\bar{v}$ and always solve a FOC that is continuous for any $\bar{v} \geq \bar{v}^{LW}$ - this establishes this part of the result. The high signal price is also continuous. It solves a FOC for all $\bar{v} \geq \bar{v}^{HW}$. For any $\bar{v} \in [p^{HW,M}, \bar{v}^{HW})$, the optimal price is $\bar{v}$. For any $\bar{v} < p^{HW,M}$, the optimal price is $p^{HW,M}$.

Part 2: Suppose $p^{HB} \leq \bar{v}^{LW}$. A solution $\bar{v} \in [\bar{v}^{LW}, 1]$ to $\hat{v}^X(\bar{v}) = \bar{v}$ exists.

Then, our assumption tells us that the expected price functions at $\bar{v} = \bar{v}^{LW}$ are such that $\hat{v}^X(\bar{v}^{LW}) > \bar{v}^{LW}$. For the prices at $\bar{v} = \bar{v}^{LW}$, we know that $EP^B(\bar{v}^{LW}; \bar{v}^{LW}) < EP^W(\bar{v}^{LW}; \bar{v}^{LW})$ by assumption, which implies that consumers with v in an open ball around $v = \bar{v}^{LW}$ would have a strictly lower expected price at the firm with better data, which establishes that the supremum of the corresponding set must lie above $\bar{v}^{LW}$. At $\bar{v} = 1$, we know that $\hat{v}^X(1) \leq 1$. Because $p^{HB} \leq \bar{v}^{LW}$, we are guaranteed continuity of this function in the interval $[\bar{v}^{LW}, 1]$ and hence existence of a solution.

Part 3: Suppose $\bar{v}^{LW} < p^{HB,M}$. A solution $\bar{v} \in [p^{HB,M}, 1]$ to $\hat{v}^X(\bar{v}) = \bar{v}$ exists.

Our assumption guarantees that $\hat{v}^X(p^{HB,M}) > p^{HB,M}$ by previous arguments. Thus, the supremum of the corresponding set must lie above $p^{HB,M}$. Continuity of the function proves existence of an appropriate solution, together with the fact that$\hat{v}^X(1) > 1$.

Part 4: Consider the $\bar{v} \in [\bar{v}^{LW}, 1]$ with $\hat{v}^X(\bar{v}) = \bar{v}$. By construction, the prices at this $\bar{v}$ are optimal. By assumption, the search behavior is optimal, i.e. we have an equilibrium.

■

B.5 Omitted results

B.5.1 Baseline model — no data advantages

Suppose no firm receives an informative signal in the baseline model, i.e. $Pr^H(v) = 0.5\ \forall v$. In the monopoly benchmark, all firms would set the price $p^{nd,M}$.

(i) Ruling out equilibria in which $p^L = p^H \neq p^{nd}$.

If $p^L < p^{nd}$, all searchers with $v > p^L$ visit the firm with data. Thus, $p^{nd} = p^{nd,M}$, since the firm without data is only visited by captive consumers. But then, there exists a profitable upward deviation from p^L. If $p^{nd} < p^L$, all searchers with $v > p^{nd}$ visit the firm without data. Thus, $p^L = p^{nd,M}$, and there is a profitable upward deviation from p^{nd}.

(ii) Ruling out equilibria with $p^L < p^H$ (analogous arguments rule out equilibria with $p^H < p^L$, since the signal is not informative):

Previous arguments establish that $p^L < p^{nd} < p^H$ must hold. As a result, there exists an $\epsilon > 0$ such that any searcher with $v \in (p^L, p^{nd} + \epsilon]$ will visit the firm with data. Thus, searchers put upward pressure on p^{nd}, and hence $p^{nd} \geq p^{nd,M}$ must hold. One can show that the average price at the two firms must be equal. Then, all searchers with $v \in (p^L, p^H)$ strictly prefer the firm with data, while those with $v \geq p^H$ are indifferent. Searchers put upward pressure on p^{nd}, which implies that $p^{nd} \geq p^{nd,M}$ and hence $p^H > p^{nd,M}$. But this is a contradiction, as searchers put downward pressure on this price, i.e. $p^H \leq p^{nd,M}$.

B.5.2 Dispersed data framework — no data advantages

First, one can show that $p^{LB} \leq p^{LW}$ must hold and that the reverse, namely $p^{LB} \geq p^{LW}$, must also hold true. This is based on arguments analogous to those made in the proof of lemma 11, part 1. Thus, $p^{LW} = p^{LB}$. Based on this, one can show that $p^{HB} = p^{HW}$ must hold.

Suppose that $p^{HB} < p^{HW}$, noting that both low signal prices must be the same. Then, all consumers with $v \geq p^{HB}$ will strictly prefer to visit the firm with better data. In order to constitute a simple equilibrium, the cutoff must be set in such a way that all searchers with $v > \bar{v}$ (where $\bar{v} \leq p^{HB}$) visit the firm with better data.

But by this logic, the firm with worse data will only make the sale to captive consumers in an open ball around p^{HW}, which implies that $p^{HW} = p^{HW,M}$. Moreover, because all searchers with $v \geq p^{HB}$ visit the firm with better data, we have $p^{HB} \geq p^{HB,M}$, as there would be an upward deviation otherwise. Hence, we have $p^{HB} \geq p^{HB,M} = p^{HW}$, a contradiction. Similar logic rules out the other case - hence, all prices have to be equal and all searchers randomize.

Appendix C

Chapter 4: Proofs and further material

C.1 Proofs — Section 4.4.

Proof of Lemma 4:

This proof consists of four parts. In the first part, we derive p_2^n and in the second part, we show that $p_2^n > p^*$. In part three, we derive p_3^n and part four verifies that $p_3^n < p^*$.

Part 1: Deriving p_2^n.

In $\mathcal{H}(B) = R$, the perceived profit function of firm B is:

$$\Pi_2(p_B) = p_B \underbrace{\left\{ \frac{1}{2} F(w^*)\big[1 - F(w^n(p_B))\big] + \frac{1}{2} \int_{p_B}^{w^n(p_B)} F(u_B + p^* - p_B) du_B \right\}}_{:= D^2(p_B)},$$

where $w^n(p_B) = w^* - p^* + p_B$ as defined in the main text. The derivative of $D^2(p_B)$ with respect to p_B reads:

$$\frac{\partial D^2(p_B)}{\partial p_B} = -\frac{1}{2} F(w^*) f(w^n(p_B)) + \frac{1}{2} \left[F(w^*) - F(p^*) - \int_{p_B}^{w^n(p_B)} f(u_B + p^* - p_B) du_B \right] = -\frac{1}{2} w^*$$

Similarly, demand simplifies to:

$$D^2(p_B) = \frac{1}{2} w^* \big[1 - \big(w^* - p^* + p_B\big)\big] + \frac{1}{2} \left[\frac{1}{2} (w^*)^2 - \frac{1}{2} (p^*)^2 \right]$$

The price p_2^n must solve the first-order condition $p_B \frac{\partial D^2(p_B)}{\partial p_B} + D^2(p_B) = 0$. Plugging in the above results yields:

$$p_2^n = \frac{1}{2}\left(1 - (w^* - p^*)\right) + \frac{1}{4}\left(w^* - \frac{(p^*)^2}{w^*}\right) \tag{C.1.1}$$

Part 2: Showing that $p_2^n - p^* > 0$.

Using (C.1.1), $p_2^n - p^* > 0$ if and only if

$$2 > w^* + 2p^* + \frac{p^{*2}}{w^*} \Leftrightarrow \sqrt{2w^*} > w^* + p^* \tag{C.1.2}$$

Substituting the equilibrium expression for p^* given by

$$p^* = -\frac{1}{2}(1 + w^*)\left(1 - \sqrt{1 + \frac{4}{(1 + w^*)^2}}\right) \tag{C.1.3}$$

which can be obtained from solving equation (4.4.3) explicitly, allows us to rewrite (C.1.2) as

$$\sqrt{2w^*} > -\frac{1}{2}(1 - w^*) + \frac{1}{2}(1 + w^*)\sqrt{1 + \frac{4}{(1 + w^*)^2}} \tag{C.1.4}$$

$$\Leftrightarrow \sqrt{2w^*} + \frac{1}{2}(1 - w^*) > \sqrt{\frac{1}{4}(1 + w^*)^2 + 1} \tag{C.1.5}$$

$$\Leftrightarrow \frac{1}{4}(1 - w^*)^2 + (1 - w^*)\sqrt{2w^*} + 2w^* > \frac{1}{4}(1 + w^*)^2 + 1 \tag{C.1.6}$$

$$\Leftrightarrow (1 - w^*)\sqrt{2w^*} > 1 - w^* \tag{C.1.7}$$

The last inequality holds if $1 > w^* > 1/2$. Since $w^* = 1 - \sqrt{2s}$ (which follows from the standard Wolinsky analysis), $w^* < 1$ holds for any positive search costs. Moreover, $w^* > 1/2$ for all $s < 1/8$, which is exactly the threshold above no equilibrium with active search exists ($w^* \geq p^*$ if and only if $s \leq 1/8$). This proves that $p_2^n - p^* > 0$ if $w^* \geq p^*$.

Part 3: Derivation of p_3^n.

If the consumer visited firm B before, disclosure by firm A leads to $\mathcal{H}(B) = ND \times p^* \times R$. Accordingly, B's profits are given by

$$\Pi^3(p_B) = p_B \int_{p_B}^{w^*} \frac{1}{2} F(u_B - p_B + p^*) du_B \tag{C.1.8}$$

as already derived in the main text (see equation (4.4.5)). Whenever p_B is such that $w^* - p_B + p^* < 1$, this profit function is:

$$\Pi^3(p_B) = p_B \int_{p_B}^{w^*} \frac{1}{2}(u_B - p_B + p^*) du_B \tag{C.1.9}$$

We conjecture that the optimal p_B falls in this interval (which we verify later). Then, p_B must solve the following first-order condition:

$$\int_{p_B}^{w^*} \frac{1}{2}(u_B - p_B + p^*)du_B + p_B\Big[-\frac{1}{2}F(p^*) + \int_{p_B}^{w^*}\frac{1}{2}(-1)du_B\Big] = 0$$

$$\iff$$

$$1.5(p_B)^2 + p_B\big[-w^* - 2p^* - w^*\big] + \big[\frac{1}{2}(w^*)^2 + w^*p^*\big] = 0$$

$$\iff$$

$$3(p_B)^2 - p_B\big[(4)(w^* + p^*)\big] + \big[(w^*)(w^* + 2p^*)\big]$$

Denote the solution to this first order condition by p_3^n. Then,

$$p_3^n = (2/3)(w^* + p^*) - (1/3)\sqrt{(w^*)^2 + 2w^*p^* + 4(p^*)^2} \tag{C.1.10}$$

where we have ignored the positive root because calculations show that the negative one is the only appropriate one. Note that this price is the global maximizer of $\Pi^3(p_B)$ as defined in (C.1.9).

Moreover, one can show that this price will always be in the region that we have restricted our attention to, namely $p_B > w^* + p^* - 1$. For prices p_B at which $p_B \leq w^* + p^* - 1$, true profits as defined in (C.1.8) are below the profit function defined in (C.1.9). Because p_3^n is the global maximizer of the latter and $p_3^n > w^* + p^* - 1$, the optimal price must equal p_3^n.

Part 4: Verifying the ordering $p_3^n < p^*$.

Using (C.1.10), $p^* - p_3^n > 0$ holds if and only if

$$\frac{1}{3}\sqrt{w^{*2} + 2w^*p^* + 4p^{*2}} > \frac{2}{3}w^* - \frac{1}{3}p^* \Leftrightarrow \sqrt{w^{*2} + 2w^*p^* + 4p^{*2}} > 2w^* - p^* \tag{C.1.11}$$

$$\Leftrightarrow w^{*2} + 2w^*p^* + 4p^{*2} > 4w^{*2} - 4w^*p^* + p^{*2} \tag{C.1.12}$$

$$\Leftrightarrow p^{*2} + 2w^*p^* > w^{*2} \tag{C.1.13}$$

The equilibrium condition pinning down p^* (from Wolinsky, 1986) tells us that

$$p^* = \frac{1 - p^{*2}}{1 + w^*} \Leftrightarrow w^* = \frac{1 - p^* - p^{*2}}{p^*}.$$

Substituting the expression for w^* into inequality (C.1.13) above yields

$$2 - 2p^* - p^{*2} > \left(\frac{1 - p^* - p^{*2}}{p^*}\right)^2 \Leftrightarrow 2p^* - 2p^{*4} + 3p^{*2} - 4p^{*3} > 1. \tag{C.1.14}$$

We know that $p^* \in (\sqrt{2} - 1, 1/2]$ in any equilibrium with active search. This follows from the necessary condition that $w^* \geq p^*$ and $w^* = 1 - \sqrt{2s}$. It can be verified that $2p^* - 2p^{*4} + 3p^{*2} -$

$4p^{*3} = 1$ for $p^* = \sqrt{2} - 1$. Thus, inequality (C.1.14) holds if

$$\partial_{p^*}\left(2p^* - 2p^{*4} + 3p^{*2} - 4p^{*3}\right) > 0 \text{ for all } p^* \in [\sqrt{2} - 1, 1/2] \tag{C.1.15}$$

$$\Leftrightarrow (2 - 8p^{*3}) + (6p^* - 12p^{*2}) > 0 \text{ for all } p^* \in [\sqrt{2} - 1, 1/2] \tag{C.1.16}$$

$$\Leftrightarrow (1 - 4p^{*3}) + 3p^*(1 - 2p^*) > 0 \text{ for all } p^* \in [\sqrt{2} - 1, 1/2] \tag{C.1.17}$$

Since $p^* \leq 1/2$, it is easy to verify that $1 - 4p^{*3} > 0$ and $3p^*(1 - 2p^*) \geq 0$, implying that the above inequality always holds. This completes the proof. ■

Proof of Proposition 16:

The proof has two parts. We first show that the profit function depicted in (4.4.8) is the correct one. This follows from the discussion in the main text subject to one additional observation: Firm B, after reaching $\mathcal{H}(B) = R$ due to firm A's deviation, must not use search disclosure itself. Search disclosure by firm B would inform firm A that the buyer continued to search. Then, firm A would reach the information set $\mathcal{H}(A) = D \times p_1 \times R$, in which it can revise its price for return consumers, which expression (4.4.8) does not allow for.

After establishing that $\mathcal{H}(A) = D \times p_1 \times R$ will not be reached, we show that disclosure in $\mathcal{H}(A) = NR$, through its effect on (4.4.8) via p_2 and p_3, is not profitable.

Part 1: $\mathcal{H}(A) = D \times p_1 \times R$ will not be reached (even) if $d_A = D$, because B would not find it profitable to disclose in $\mathcal{H}(B) = R$.

Recall that firm B believes that firm A offered the consumer the price p^*. Thus, it believes that $d_B = D$ (disclosing back) leads to $\mathcal{H}(A) = D \times p^* \times R$. In this information set, B anticipates that A expects B's price to equal p_2^n, since this is part of B's equilibrium strategy. Consequently, firm B believes that firm A would revise its price p_A to maximize the following profit function:

$$\Pi^{3,dd}(p_A) = p_A \int_{p_A}^{w^*} \frac{1}{2} F(u_A - p_A + p_2^n) du_A, \tag{C.1.18}$$

where we have accounted for the fact that, in the information set $\mathcal{H}(A) = D \times p^* \times R$, firm A must believe that the consumers initial match value satisfied $u_j \in [0, w^*]$. The optimal price in this information set, which we denote by p_3^{dd} to account for the "double" deviation, is given by:

$$p_3^{dd} = (2/3)(w^* + p_2) - (1/3)\sqrt{(w^*)^2 + 2w^*p_2 + 4(p_2)^2}, \tag{C.1.19}$$

where we have again ignored the positive root because calculations show that the negative root is the appropriate one. If $p_3^{dd} < p^*$, firm B would expect firm A to revise its price downward and firm B has no incentive to disclose "back" if $\mathcal{H}(B) = R$. At $w^* = 1$ or, equivalently, at $s = 0$, $p_3^{dd} = p^*$. To show that $p_3^{dd} < p^*$, it is thus sufficient to show that the derivatives with respect to w^* have opposite signs. We begin with p^*. By taking the derivative of p^* given in (C.1.3), we obtain

$$\frac{\partial p^*}{\partial w^*} = \frac{1 - \sqrt{1 + \frac{4}{(1+w^*)^2}}}{2\sqrt{1 + \frac{4}{(1+w^*)^2}}}, \tag{C.1.20}$$

which makes it easy to verify that $\partial p^*/\partial w^* \in (-1/2, 0)$. The derivative of p_3^{dd} is

$$\begin{aligned}\frac{\partial p_3^{dd}}{\partial w^*} =& \frac{2}{3} - \frac{w^* + p_2}{3\sqrt{(w^* + p_2)^2 + 3p_2^2}} + \left(\frac{2}{3} - \frac{w^* + 4p_2}{3\sqrt{(w^* + p_2)^2 + 3p_2^2}}\right)\frac{\partial p_2}{\partial w^*} &\text{(C.1.21)}\\ =& \left(\frac{2}{3} - \frac{w^* + p_2}{3\sqrt{(w^* + p_2)^2 + 3p_2^2}}\right)\left(1 + \frac{\partial p_2}{\partial w^*}\right) - \frac{p_2}{\sqrt{(w^* + p_2)^2 + 3p_2^2}}\frac{\partial p_2}{\partial w^*}. &\text{(C.1.22)}\end{aligned}$$

It can be verified that the first term above is greater than 1/3. Thus, $\partial p_3^{dd}/\partial w^* > 0$ follows if $\partial p_2/\partial w^* \in (-1, 0)$. Taking the derivative of p_2 (see Lemma 4 for the expression) w.r.t p^* yields:

$$\frac{\partial p_2}{\partial w^*} = \frac{1}{2}\left(1 - \frac{p^*}{w^*}\right)\frac{\partial p^*}{\partial w^*} - \frac{1}{4}\left(1 - \left(\frac{p^*}{w^*}\right)^2\right) = \frac{1}{2}\left(1 - \frac{p^*}{w^*}\right)\left(\frac{\partial p^*}{\partial w^*} - \frac{1}{2}\left(1 + \frac{p^*}{w^*}\right)\right). \tag{C.1.23}$$

Since $\partial p^*/\partial w^* < 0$ as shown above, $\partial p_2^n/\partial w^* < 0$ holds because $p^* < w^*$. To bound $\partial p_2^n/\partial w^*$ from below, observe that $\partial p^*/\partial w^* > -1/2$ implies that

$$\frac{\partial p_2^n}{\partial w^*} > \frac{1}{2}\left(1 - \frac{p^*}{w^*}\right)\left(-\frac{1}{2} - \frac{1}{2}\left(1 + \frac{p^*}{w^*}\right)\right) = -\frac{1}{2}\left(1 - \left(\frac{p^*}{w^*}\right)^2\right) > -\frac{1}{2}, \tag{C.1.24}$$

which is sufficient to prove that $\partial p_2^n/\partial w^* \in (-1, 0)$. Thus, $\partial p_3^{dd}/\partial w^* > 0$ whereas $\partial p^*/\partial w^* < 0$, which proves $p_3^{dd} < p^*$. The firm that receives disclosure will thus not disclose back, and the deviating firm has no chance to revise its price later. Expression (4.4.8) thus correctly represents the deviating firm's profits.

Part 2: A deviation to disclosure is strictly unprofitable at $\mathcal{H}(A) = NR$.

Knowing that (4.4.8) correctly represents the profits of the deviating firm, we next focus on analyzing the average effect of changing the rival's price from p^* to either p_3^n or p_2^n.

Let $D(p_A, p^*)$ represent firm A's demand if it does not disclose and charges a price p_A. Also, let $D^d(p_A, p^*, p_2^n, p_3^n)$ denote firm A's demand after disclosure (deviation), where p_2^n is the price firm B sets if $\mathcal{H}(B) = R$ and p_3^n the revised price if $\mathcal{H}(B) = ND \times p^* \times R$. Because firm B will not disclose back to firm A, disclosure by firm A never leads to $H(A) = D \times p_A \times R$ so that firm A will never revise its price. The demand $D^d(p_A, p^*, p_2^n, p_3^n)$ is expressed in equation (4.4.8) and is given by:

$$D^d(p_A, p^*, p_2^n, p_3^n) = \frac{1}{2}\left[\left[1 - w^n(p_A)\right] + \int_{p_A}^{w^n(p_A)} (p_2^n + u_A - p_A) du_A\right] +$$

$$\frac{1}{2}\left[(w^*)\big(1 - (w^* - p_3^n + p_A)\big) + \int_{p_A}^{w^* - p_3^n + p_A} (p_3^n + u_A - p_A) du_A\right] \tag{C.1.25}$$

One can show that the optimal revision price will lie in the range of prices for which $w^* - p_3^n + p_A < 1$ and $w^* - p^* + p_2^n < 1$, which implies that it is without loss to consider the demand function given in (C.1.25). Total profits are equal to $\max_{p_A} p_A D^d(p_A, p^*, p_2^n, p_3^n)$ if A deviates, while they are equal to $\max_{p_A} p_A D(p_A, p^*)$ in the no disclosure equilibrium. Thus, a deviation is strictly unprofitable if

$$D(p_A, p^*) < D^d(p_A, p^*, p_2^n, p_3^n) \text{ for all } p_A. \tag{C.1.26}$$

Let $p_2^n = p^* + \delta$ and $p_3^n = p^* - \delta - \varepsilon$. By Lemma 4, $\delta > 0$ holds but the sign of ε is unknown. Then, $D^d(p_A, p^*, p_2^n, p_3^n) = D^d(p_A, p^*, p^* + \delta, p^* - \delta - \varepsilon)$. It is easy to verify that $\partial_\varepsilon D^d < 0$. Thus, (C.1.26) holds if (i) $\varepsilon \geq 0$ and (ii) $\partial_\delta D^d < 0$ because $D(p_A, p^*) = D^d(p_A, p^*, p^*, p^*)$.

To see that (i) is true, notice that:

$$\varepsilon = (p^* - p_3^n) - \delta = (p^* - p_3^n) - (p_2^n - p^*) \tag{C.1.27}$$

$$= \frac{10}{12}p^* - \frac{5}{12}w^* - \frac{6}{12} + \frac{3}{12}\frac{(p^*)^2}{w^*} + \frac{4}{12}\sqrt{(w^*)^2 + 2w^*p^* + 4(p^*)^2} \tag{C.1.28}$$

Substituting $w^* = (1 - p^* - p^*)/p^*$, which follows from equation (4.4.3), into the expression above yields that $\varepsilon \geq 0$ if and only if

$$\frac{1}{12}\left(-1 - \frac{5}{p^*} + 15p^* - \frac{3(p^*)^3}{-1 + p^* + (p^*)^2} + 4\sqrt{1 + \frac{1}{(p^*)^2} - \frac{2}{p^*} + 3(p^*)^2}\right) \geq 0 \tag{C.1.29}$$

$$\Leftrightarrow \left(4\sqrt{1 + \frac{1}{(p^*)^2} - \frac{2}{p^*} + 3(p^*)^2}\right)^2 \geq \left(1 + \frac{5}{p^*} - 15p^* + \frac{3(p^*)^3}{-1 + p^* + (p^*)^2}\right)^2 \tag{C.1.30}$$

$$\Leftrightarrow 16\left(1 + \frac{1}{(p^*)^2} - \frac{2}{p^*} + 3(p^*)^2\right) \geq \frac{(5 - 4p^* - 21(p^*)^2 + 14(p^*)^3 + 12(p^*)^4)^2}{(p^*)^2(-1 + p^* + (p^*)^2)^2} \tag{C.1.31}$$

Additional steps show that inequality (C.1.31) holds if and only if

$$16 - 64p^* + 64(p^*)^2 + 32(p^*)^3 - 16(p^*)^4 - 96(p^*)^5 - 32(p^*)^6 + 96(p^*)^7 + 48(p^*)^8 \geq$$
$$25 - 40p^* - 194(p^*)^2 + 308(p^*)^3 + 449(p^*)^4 - 684(p^*)^5 - 308(p^*)^6 + 336(p^*)^7 + 144(p^*)^8$$
$$\Leftrightarrow -3(3 + 8p^* - 86(p^*)^2 + 92(p^*)^3 + 155(p^*)^4 - 196(p^*)^5 - 92(p^*)^6 + 80(p^*)^7 + 32(p^*)^8) \geq 0$$

One can easily verify that the left-hand side equals 0 if $p^* = \sqrt{2} - 1$ and 3/8 if $p^* = 1/2$. Thus, showing that the left-hand side of this expression is concave over $[\sqrt{2} - 1, 1/2]$ is sufficient to show that inequality (C.1.31) is true (by concavity, the boundary conditions imply that the derivative of the left-hand side is positive on this interval). The second derivative of the left-hand side is given by

$$-12\big(- 43 + 138p^* + 465(p^*)^2 - 980(p^*)^3 - 690(p^*)^4 + 840(p^*)^5 + 448(p^*)^6\big) \quad \text{(C.1.32)}$$

To show that (C.1.32) is negative, we show separately that (1) $465(p^*)^2 - 980(p^*)^3 + 448(p^*)^6$ and (2) $-43 + 138p^* - 690(p^*)^4 + 840(p^*)^5$ are both non-negative for all $p^* \in [\sqrt{2} - 1, 1/2]$. Consider term (1) first, which we show satisfies:

$$465(p^*)^2 - 980(p^*)^3 + 448(p^*)^6 \geq 0 \Leftrightarrow 465 \geq 980p^* - 448(p^*)^4 \quad \text{(C.1.33)}$$

It can be verified that inequality (C.1.33) holds at $p^* = 1/2$. Moreover, $980p^* - 448(p^*)^4$ increases in p^* for all $p^* \leq (35/64)^{1/3}$ (note $(35/64)^{1/3} > 1/2$), implying that (C.1.33) holds for all $p^* \in [\sqrt{2} - 1, 1/2]$. Consider term (2) next, which we show satisfies:

$$-43 + 138p^* - 690(p^*)^4 + 840(p^*)^5 \geq 0 \quad \text{(C.1.34)}$$

Again, it can be verified that inequality (C.1.34) holds (strictly) at $p^* = \sqrt{2} - 1$. Thus, it is sufficient to show that the left-hand side of (C.1.34) increases in p^*. By taking the derivative, we see that this conditions holds if and only if

$$6(23 - 460(p^*)^3 + 700(p^*)^4) \geq 0 \Leftrightarrow 23 \geq 460(p^*)^3 - 700(p^*)^4 \quad \text{(C.1.35)}$$

One can check that the function $460(p^*)^3 - 700(p^*)^4$ obtains its maximum at $p^* = 69/140$. Since $23 > 460(69/140)^3 - 700(69/140)^4$, we know that (C.1.35) holds for all $p^* \in [\sqrt{2} - 1, 1/2]$. This completes the proof of subpart (i).

We next prove (ii): $\partial_\delta D^d < 0$. Evaluating $D^d(p_A, p^*, p^*+\delta, p^*-\delta-\varepsilon)$, we obtain

$$\frac{1}{2}\Bigg(\underbrace{w^*(1-w^*+p^*-\delta-\varepsilon-p_A)+\int_{p_A}^{w^*-p^*+\delta+\varepsilon+p_A}(p^*-\delta-\varepsilon+u_A-p_A)\mathrm{d}u_A}_{\text{Modified searcher profits}}\Bigg)$$
$$+\frac{1}{2}\Bigg(\underbrace{(1-w^*+p^*-p_A)+\int_{p_A}^{w^*-p^*+p_A}(p^*+\delta+u_A-p_A)\mathrm{d}u_A}_{\text{Modified first arriver profits}}\Bigg).$$

The derivative of this demand function with respect to δ satisfies

$$\frac{1}{2}\Bigg((-1)w^*+(1)w^*+\int_{p_A}^{w^*-p^*+\delta+\varepsilon+p_A}(-1)\mathrm{d}u_A\Bigg)+\frac{1}{2}\int_{p_A}^{w^*-p^*+p_A}(1)\mathrm{d}u_A=-\frac{\delta+\varepsilon}{2}<0,$$

which shows that demand falls in δ for any $\delta>0$.

Part 3: Establishing uniqueness of the candidate equilibrium.

On the equilibrium path, the uniform price p^* will be set, which is uniquely determined. All off-path prices, namely p_2^n, p_3^n and the revision price functions, are also uniquely determined by Lemma 4, implying the desired result. ■

Proof of Lemma 5:

We derive $p_3^d(p_1)$ first. As shown in the main text, the profit function of the deviating firm A in the information set $\mathcal{H}(A)=ND\times p_1\times R$ is given by (4.4.10):

$$\Pi^{3,d}(p_3|p_1)=p_3\int_{p_3}^{w^d(p_1)}\frac{1}{2}F(u_A-p_3+p_1^*)du_A=p_3\int_{p_3}^{w^d(p_1)}\frac{1}{2}(u_A-p_3+p_1^*)du_A$$

Consider a generic initial price p_1. We restrict attention to prices p_3 which satisfy $w^d(p_1)-p_3+p_1^*<1$ (and we have verified that the optimal revision price will satisfy this). This allows us to rewrite the above profit function with the second equality. The optimal price $p_3^d(p_1)$ needs to solve the following first-order condition:

$$\int_{p_3^d}^{w^d(p_1)}\frac{1}{2}(u_A-p_3^d+p_1^*)du_j-p_3^d\frac{1}{2}\big(p_1^*\big)+p_3^d\int_{p_3^d}^{w^d(p_1)}\frac{1}{2}(-1)du_A=0$$

$$\Longleftrightarrow$$

$$\frac{1}{2}(w^d(p_1)-p_3^d+p_1^*)^2-\frac{1}{2}(p_1^*)^2-p_3^dp_1^*-p_3^d\big(w^d(p_1)-p_3^d\big)=0 \qquad \text{(C.1.36)}$$

The unique solution to this equation that satisfies $p_3^d(p_1) \in [0,1]$ is given by

$$p_3^d(p_1) = (2/3)(w^d(p_1) + p_1^*) - (1/3)\sqrt{(w^d(p_1))^2 + 2w^d(p_1)p_1^* + 4(p_1^*)^2}. \tag{C.1.37}$$

We derive p_1^d next. Firm A's profit function if it does not disclose when $\mathcal{H}(A) = NR$ is shown in (4.4.11), which we repeat here for convenience.

$$\Pi^{1,d}(p_1) = p_1\frac{1}{2}\big[1 - F(w^d(p_1))\big] + p_3^d(p_1)\int_{p_3^d(p_j)}^{w^d(p_1)} \frac{1}{2}F(p_1^* + u_j - p_3^d(p_1))du_A$$

This expression is valid only if $p_3^d(p_1) < w^d(p_1)$, which must hold in a PBE (else, no profits are made when setting $p_3^d(p_1)$). The derivative of $\Pi^{1,d}(p_1)$ with respect to p_1 if $w^d(p_1) < 1$ is:

$$\frac{\partial \Pi^{1,d}(p_1)}{\partial p_1} = p_1\frac{1}{2}[-1] + \frac{1}{2}\big[1 - (w^d(p_1))\big] + p_3^d(p_1)\Bigg[\frac{1}{2}(p_1^* + w^d(p_1) - p_3^d(p_1))$$

$$-\frac{1}{2}(p_1^*)\frac{\partial p_3^d(p_1)}{\partial p_1} - \int_{p_3^d(p_1)}^{w^d(p_1)} \frac{1}{2}\frac{\partial p_3^d(p_1)}{\partial p_1}du_A\Bigg] + \frac{\partial p_3^d(p_1)}{\partial p_1}\Bigg[\int_{p_3^d(p_1)}^{w^d(p_1)} \frac{1}{2}(p_1^* + u_A - p_3^d(p_1))du_A\Bigg],$$

Note that the left derivative of profits is strictly negative if $w^d(p_1) = 1$ (since $p_1 > p_3^d(p_1)$ holds in that case), which implies that $w^d(p_1^d) < 1$ must hold at the optimal p_1^d. Thus, we obtain the following first-order condition for the optimal deviation price p_1^d :

$$\big[1 - p_1^d - w^d(p_1^d)\big] + p_3^d(p_1^d)(p_1^* + w^d(p_1^d) - p_3^d(p_1^d)) + \frac{\partial \Pi^{3,d}(p_3^d|p_1)}{\partial p_3}\frac{\partial p_3^d(p_1^d)}{\partial p_1} = 0 \tag{C.1.38}$$

By the Envelope theorem, $\partial \Pi^{3,d}(p_3^d|p_1)/\partial p_3 = 0$ so that (C.1.38) simplifies to

$$p_1^d = 1 - w^d(p_1^d) - \big(p_3^d(p_1^d)\big)^2 + p_3^d(p_1^d)\big(w^d(p_1^d) + p_1^*\big), \tag{C.1.39}$$

which equals the expression provided in the lemma. The solution to (C.1.39) is unique if the right-hand side of (C.1.39) decreases in p_1^d everywhere. Its derivative shows that this is true if and only if

$$1 > \frac{\partial p_3^d(p_1^d)}{\partial p_1}\left(w^d(p_1^d) + p_1^*\right) + p_3^d(p_1^d)\left(1 - 2\frac{\partial p_3^d(p_1^d)}{\partial p_1}\right). \tag{C.1.40}$$

We first show that this inequality holds if $\partial p_3^d/\partial p_1 \in (\frac{1}{3}, \frac{1}{2})$. Second, we verify that $\partial p_3^d/\partial p_1 \in (\frac{1}{3}, \frac{1}{2})$.
If $\partial p_3^d/\partial p_1 \in (\frac{1}{3}, \frac{1}{2})$, the right-hand side of (C.1.40) satisfies

$$\frac{\partial p_3^d(p_1^d)}{\partial p_1}\left(w^d(p_1^d) + p_1^*\right) + p_3^d(p_1^d)\left(1 - 2\frac{\partial p_3^d(p_1^d)}{\partial p_1}\right) < \frac{1}{2}\left(w^d(p_1^d) + p_1^*\right) + \frac{1}{3}p_3^d(p_1^d). \tag{C.1.41}$$

Additionally, we know that $w^d(p_1^d) < 1$, $p_1^* < 1/2$ as well as $p_3^d(p_1) < 1/2$. The latter follows from Remark 5 (presented at the end of this proof) upon substituting $w^d(p_1^d)$ with w and p_1^* with p. Thus, $\frac{1}{2}\left(w^d(p_1^d) + p_1^*\right) + \frac{1}{3}p_3^d(p_1^d) < \frac{3}{4} + \frac{1}{6} < 1$ if $\partial p_3^d/\partial p_1 \in (\frac{1}{3}, \frac{1}{2})$, which proves the first claim.

To verify that $\partial p_3^d/\partial p_1 \in (\frac{1}{3}, \frac{1}{2})$, we first calculate the derivative of p_3^d and find that

$$\frac{\partial p_3^d(p_1^d)}{\partial p_1} = \frac{1}{3}\left(2 - \frac{w^d(p_1^d) + p_1^*}{\sqrt{\left(w^d(p_1^d) + p_1^*\right)^2 + 3\left(p_1^*\right)^2}}\right) > \frac{1}{3}\left(2 - \frac{w^d(p_1^d) + p_1^*}{\sqrt{\left(w^d(p_1^d) + p_1^*\right)^2}}\right) = \frac{1}{3}. \tag{C.1.42}$$

Additionally, $\partial p_3^d/\partial p_1 < 1/2$ if and only if

$$\frac{w^d(p_1^d) + p_1^*}{\sqrt{\left(w^d(p_1^d) + p_1^*\right)^2 + 3\left(p_1^*\right)^2}} > \frac{1}{2} \iff \left(w^d(p_1^d) + p_1^*\right)^2 > \frac{1}{4}\left(w^d(p_1^d) + p_1^*\right)^2 + \frac{3}{4}\left(p_1^*\right)^2 \tag{C.1.43}$$

$$\iff \left(w^d(p_1^d) + p_1^*\right)^2 > \left(p_1^*\right)^2 \tag{C.1.44}$$

where the last inequality holds always. This proves the second claim that $\partial p_3^d/\partial p_1 < 1/2 \in (\frac{1}{3}, \frac{1}{2})$ and, thus, guarantees uniqueness of the optimal deviation price p_1^d. ∎

Remark 5 *Let*

$$p_3(w,p) = \frac{2}{3}(w+p) - \frac{1}{3}\sqrt{(w+p)^2 + 3p^2}, \tag{C.1.45}$$

$w \leq 1$ *and* $p \leq 1/2$*. Then* $p_3(w,p) \leq 1 - \dfrac{\sqrt{3}}{3} < 1/2.$

Proof of Remark 5:

Taking the partial derivatives, it is easy to see that p_3 increases in w and in p for all $w \leq 1$ and all $p \leq 1/2$:

$$\frac{p_3(w,p)}{\partial w} = \frac{2}{3} - \frac{p+w}{3\sqrt{(w+p)^2 + 3p^2}} > \frac{2}{3} - \frac{p+w}{3\sqrt{(w+p)^2}} = \frac{1}{3} > 0. \tag{C.1.46}$$

In addition,

$$\frac{p_3(w,p)}{\partial p} = \frac{2}{3} - \frac{4p+w}{3\sqrt{3p^2 + (w+p)^2}} > 0 \tag{C.1.47}$$

$$\Leftrightarrow 2\sqrt{3p^2 + (w+p)^2} > 4p + w \quad \Leftrightarrow \quad 3w^2 > 0, \tag{C.1.48}$$

which holds always if $w > 0$. Thus, p_3 is bounded from above by $p_3(1, 1/2) = 2/3(3/2) - 1/3\sqrt{3}$, which proves the claim. ■

Proof of Proposition 17:

The Proposition has two parts. First, we show that there exists a threshold $\bar{s}$ such that the partial disclosure equilibrium exists for all search costs below this threshold. Thereafter, we show that there exists another threshold $\bar{s}'$ such that the partial disclosure equilibrium does not exist for search costs above this threshold.

Part 1: There is a value $\bar{s}$ such that partial disclosure is an equilibrium if $s \leq \bar{s}$.

In a partial disclosure equilibrium, $d_j = D$ if $\mathcal{H}(j) = NR$ and $d_j = ND$ if $\mathcal{H}(j) = R$. To prove that the partial disclosure equilibrium exists, we show that deviating from these disclosure strategies is not profitable if s is sufficiently small.

Claim 1: Deviating to non-disclosure in the information set $\mathcal{H}(j) = NR$ is not profitable if $s < \hat{s}$ ($\hat{s} > 0$).

If firm j encounters a buyer while $\mathcal{H}(j) = NR$, then the buyer must have started her search at j, given that firm j's rival plays its equilibrium strategy. To evaluate the effect of deviating to $d_j = ND$ when $\mathcal{H}(j) = NR$, we can thus restrict attention to buyers who search in this order. In equilibrium, total profits from such buyers are given by

$$\Pi^{EQ} = \frac{1}{2}p_1^*\left[1 - F(w^d(p_1^*))\right] + \frac{1}{2}p_1^* \int_{p_1^*}^{w^d(p_1^*)} F(p_2^* + u_j - p_1^*)\mathrm{d}u_j, \tag{C.1.49}$$

where $w^d(p_1^*) = w^* - p_2^* + p_1^*$ is the equilibrium search cutoff. As argued in the main text, the equilibrium prices p_1^* and p_2^* can be obtained from Armstrong et al. (2009), who provide the following implicit equations for these prices when there are two firms:

$$p_1^* = \frac{1}{2}\big(1 - (w^* - p_2^*)\big) + \frac{1}{4}(w^*)^2 - \frac{1}{4}(p_2^*)^2 \tag{C.1.50}$$

$$p_2^* = \frac{1}{2}\big(1 - (w^* - p_2^*)\big) + \frac{1}{4}(w^* - p_2^* + p_1^*) - \frac{1}{4}\frac{(p_1^*)^2}{w^* - p_2^* + p_1^*}. \tag{C.1.51}$$

Armstrong et al. (2009) show that these prices are unique and satisfy $p_2^* \geq p_1^*$. In comparison, profits after deviating to non-disclosure are given by:

$$\Pi^{DEV}(p_1) = \frac{1}{2}p_1\left[1 - F(w^d(p_1))\right] + \frac{1}{2}p_3^d(p_1) \int_{p_3^d(p_1)}^{w^d(p_1)} F(p_1^* + u_j - p_3^d(p_1))\mathrm{d}u_j, \tag{C.1.52}$$

Lemma 5 provides implicit equations for the optimal prices $p_3^d(p_1)$ and p_1^d and guarantees that p_1^d and $p_3^d(p_1^d)$ are uniquely determined.

One can verify that $p_1^d = p_3^d(p_1) = p_1^* = p_2^* = \sqrt{2} - 1$ if $w^* = 1$ or, equivalently, $s = 0$. That is, $\Pi^{DEV} = \Pi^{EQ}$ if $s = 0$, where we define Π^{DEV} as the deviation profits for the optimal deviation price, i.e., $\Pi^{DEV} := \Pi^{DEV}(p_1^d)$. To show that a threshold $\hat{s} > 0$ exists such that $\Pi^{DEV} \leq \Pi^{EQ}$ for all $s \leq \hat{s}$, it is thus sufficient to show that

$$\left.\frac{\partial \Pi^{EQ}}{\partial s}\right|_{s=0} > \left.\frac{\partial \Pi^{DEV}}{\partial s}\right|_{s=0} \iff \left.\frac{\partial \Pi^{EQ}}{\partial w^*}\right|_{w^*=1} < \left.\frac{\partial \Pi^{DEV}}{\partial w^*}\right|_{w^*=1}. \tag{C.1.53}$$

To prove the validity of the second inequality in (C.1.53), take the derivative of Π^{EQ}:

$$\frac{\partial \Pi^{EQ}}{\partial w^*} = \left(\frac{p_1^*}{2} - \frac{p_1^* w^*}{2} + \frac{p_1^*(w^* - p_2^*)}{2}\right)\frac{\partial p_2^*}{\partial w^*} + \left(-\frac{p_1^*}{2} + \frac{p_1^* w^*}{2}\right), \tag{C.1.54}$$

where we have already used that $\frac{\partial \Pi^{EQ}}{\partial p_1^*}\frac{\partial p_1^*}{\partial w^*} = 0$ due to the Envelope Theorem. Since $p_1^* = p_2^* = \sqrt{2} - 1$ at $w^* = 1$, the derivative further simplifies to

$$\left.\frac{\partial \Pi^{EQ}}{\partial w^*}\right|_{w^*=1} = \frac{\sqrt{2}-1}{2}\left(2 - \sqrt{2}\right)\frac{\partial p_2^*}{\partial w^*} \tag{C.1.55}$$

The derivative of Π^{DEV} is given by

$$\begin{aligned}\frac{\partial \Pi^{DEV}}{\partial w^*} =& \left(\frac{p_1^d}{2} - \frac{p_3(w^* + p_1^* - p_2^* + p_1^d - p_3^d(p_1^d))}{2}\right)\frac{\partial p_2^*}{\partial w^*} \\ &+ \left(\frac{p_3^d(p_1^d)(w^* - p_2^* + p_1^d - p_3^d(p_1^d))}{2}\right)\frac{\partial p_1^*}{\partial w^*} + \left(-\frac{p_1^d}{2} + \frac{p_3^d(p_1^d)(w^* + p_1^* - p_2^* + p_1^d - p_3^d(p_1^d))}{2}\right)\end{aligned} \tag{C.1.56}$$

where we have again used the Envelope Theorem, which implies $\frac{\partial \Pi^{DEV}}{\partial p_1} = 0$ at p_1^d. At $w^* = 1$, the solution is again $p_1^d = p_1^* = p_2^* = p_3^d(p_1^d) = \sqrt{2} - 1$. Thus, the derivative further simplifies to

$$\left.\frac{\partial \Pi^{DEV}}{\partial w^*}\right|_{w^*=1} = \frac{\sqrt{2}-1}{2}\left(2 - \sqrt{2}\right)\frac{\partial p_1^*}{\partial w^*}. \tag{C.1.57}$$

Consequently, (C.1.53) holds if and only if $\partial p_2^*/\partial w^* < \partial p_1^*/\partial w^*$. We prove this inequality using the equations determining p_1^* and p_2^* given by (C.1.50) and (C.1.51) and the multi-variate version of the Implicit Function Theorem. Rewriting (C.1.50) and (C.1.51) implies the following system of implicit equations:

$$0 = -p_1^* + \frac{1}{2}(1 - w^* + p_2^*) + \frac{1}{4}\left(w^*\right)^2 - \frac{1}{4}\left(p_2\right)^* \tag{C.1.58}$$

$$0 = \frac{1}{2} + \frac{1}{4}(p_1^* - w^*) - \frac{3}{4}p_2^* + \frac{1}{4}\frac{\left(p_1^*\right)^2}{p_2^* - p_1^* - w^*} \tag{C.1.59}$$

By differentiating everything with respect to w^* and collecting terms, we get

$$(-1)\frac{\partial p_1^*}{\partial w^*} + \frac{1}{2}(1-p_2^*)\frac{\partial p_2^*}{\partial w^*} = \frac{1}{2}(1-w^*) \text{ as well as} \tag{C.1.60}$$

$$\begin{aligned}\frac{1}{4}\left(1+\frac{2p_1^*(p_2^*-p_1^*-w^*)+(p_1^*)^2}{(p_2^*-p_1^*-w^*)^2}\right)\frac{\partial p_1^*}{\partial w^*} - \left(\frac{3}{4}+\frac{1}{4}\frac{(p_1^*)^2}{(p_2^*-p_1^*-w^*)^2}\right)\frac{\partial p_2^*}{\partial w^*} \\ = \frac{1}{4}\left(1-\frac{(p_1^*)^2}{(p_2^*-p_1^*-w^*)^2}\right).\end{aligned} \tag{C.1.61}$$

At $(p_1^*, p_2^*, w^*) = (\sqrt{2}-1, \sqrt{2}-1, 1)$, this reduces to

$$\begin{pmatrix} -1 & \frac{1}{2}(2-\sqrt{2}) \\ \frac{1}{2}(3-2\sqrt{2}) & -\frac{1}{2}(3-\sqrt{2}) \end{pmatrix}\begin{pmatrix} \frac{\partial p_1^*}{\partial w^*} \\ \frac{\partial p_2^*}{\partial w^*} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{1}{2}(1-\sqrt{2}) \end{pmatrix} \tag{C.1.62}$$

It can also be verified that the derivatives exist by checking that the determinant of the first matrix above is non-zero. Solving the system of linear equations yields

$$\begin{pmatrix} \frac{\partial p_1^*}{\partial w^*} \\ \frac{\partial p_2^*}{\partial w^*} \end{pmatrix} = \begin{pmatrix} \frac{1}{17}(4\sqrt{2}-7) \\ \frac{1}{17}(\sqrt{2}-6) \end{pmatrix} \approx \begin{pmatrix} -0.08 \\ -0.27 \end{pmatrix}, \tag{C.1.63}$$

which shows that $\partial p_2^*/\partial w^* < \partial p_1^*/\partial w^*$ at $w^* = 1$, completing the proof of (C.1.53) and Claim 1.

Claim 2: deviating to disclosure when in the information set $\{R\}$ is not profitable.

Consider a firm $-j$ that is at the information set $\mathcal{H}(-j) = R$. Since $\mathcal{H}(-j) = R$ is on-path, firm $-j$ believes that firm j, which must have been visited before, set the price p_1^* and that the buyer continued to search if $u_j < w^d(p_1^*) = w^* - p_2^* + p_1^*$. If firm $-j$ follows the equilibrium strategy and does not disclose, firm j's price remains at p_1^*. But if firm $-j$ deviates to $d_{-j} = D$, then $\mathcal{H}(j) = D \times p_1^* \times R$, which leads firm j to revise its price to maximize the following profit function:

$$p_j \int_{p_j}^{w^d(p_1^*)} \frac{1}{2}F(u_j - p_j + p_2^*)du_j \tag{C.1.64}$$

Let p_3^{dr} denote the price that maximizes (C.1.64), where the superscripts reflect that this stage is preceded by j both using and receiving search disclosure itself (i.e., $\mathcal{H}(j) = D \times p_1^* \times R$). Clearly, $d_{-j} = D$ is not a profitable deviation for firm $-j$ in the information set $H(-j) = R$ if $p_3^{dr} \leq p_1^*$.

We seek to show that there is a value $\tilde{s} > 0$ such that $p_3^{dr} \leq p_1^*$ if $s \leq \tilde{s}$.[1] By taking the derivative of (C.1.64) with respect to p_j, we obtain the first-order condition that p_3^{dr} needs to satisfy. Similarly to $p_3^d(p_1)$ that we derived before, solving for p_3^d yields:

$$\begin{aligned} p_3^{dr} =& \frac{2}{3}(w^d(p_1^*) + p_2^*) - \frac{1}{3}\sqrt{(w^d(p_1^*))^2 + 2w^d(p_1^*)p_2^* + 4(p_2^*)^2} && \text{(C.1.65)} \\ =& \frac{2}{3}(w^* + p_1^*) - \frac{1}{3}\sqrt{(w^* + p_1^*)^2 + 3\,(p_2^*)^2} && \text{(C.1.66)} \end{aligned}$$

To show that $p_3^{dr} < p_1^*$ when s is sufficiently small, observe that $p_3^{dr} = p_1^* = p_2^* = \sqrt{2}-1$ at $s = 0$ or equivalently, at $w^* = 1$. Thus, the claim is true if

$$\left.\frac{\partial p_1^*}{\partial w^*}\right|_{w^*=1} < \left.\frac{\partial p_3^{dr}}{\partial w^*}\right|_{w^*=1}. \tag{C.1.67}$$

Since $\partial p_1^*/\partial w^*$ at $w^* = 1$ is already known from before, it only remains to calculate $\partial p_3^{dr}/\partial w^*$. Taking the derivative of (C.1.66) with respect to w^* while taking into account that p_1^* and p_2^* are functions of w^* as well yields:

$$\frac{\partial p_3^{dr}}{\partial w^*} = \left(\frac{2}{3} - \frac{1}{3}\frac{w^* + p_1^*}{\sqrt{(w^* + p_1^*)^2 + 3\,(p_2^*)^2}}\right)\left(1 + \frac{\partial p_1^*}{\partial w^*}\right) - \frac{p_2^*}{\sqrt{(w^* + p_1^*)^2 + 3\,(p_2^*)^2}}\frac{\partial p_2^*}{\partial w^*}. \tag{C.1.68}$$

To evaluate this expression at $w^* = 1$, recall that $p_1^* = p_2^* = \sqrt{2} - 1$ and $\partial p_1^*/\partial w^* = -1/17(4\sqrt{2} - 7)$ and $\partial p_2^*/\partial w^* = -1/17(\sqrt{2} - 6)$ (see (C.1.63) above) at $w^* = 1$. Thus

$$\frac{\partial p_3^{dr}}{\partial w^*} = 1/119(22 + 19\sqrt{2}) > \partial p_1^*/\partial w^*,$$

which completes the proof that there is a value $\tilde{s} > 0$ such that $p_3^{dr} \leq p_1^*$ if $s \leq \tilde{s}$. Consequently, deviating to $d_{-j} = D$ is not profitable when $\mathcal{H}(-j) = R$.

In sum, let $\bar{s} = \min(\tilde{s}, \hat{s})$. Then, $s \leq \bar{s}$ implies that deviating from the equilibrium disclosure strategy in either information set is not profitable. This establishes the existence of a partial disclosure equilibrium for all $s \leq \bar{s}$ and completes the proof of the first part.

Part 2: There is a value $\bar{s}' > 0$ such that partial disclosure is not an equilibrium if $s > \bar{s}'$.

In the partial disclosure equilibrium, we know that $p_1^* \leq p^* \leq p_2^*$, with $p_1^* \to p^*$ and $p_2^* \to p^*$ as $s \to 1/8$. The latter follows from the fact that the prices p_1^* and p_2^* are continuous in s and $p_1^* = p_2^* = p^*$ when $s = 1/8$. Recall also that the search cutoff $w^d(p_1)$

[1] Calculations show that $p_3^{dr} < p_1^*$ for all $s > 0$ but restricting attention to small s suffices here.

satisfies: $w^d(p_1) = w^* - p_2^* + p_1$. Because $p_2^* \to p^*$ as $s \to 1/8$, it holds that $w^d(p_1) - p_1 \to 0$ as $s \to 1/8$.

Consider the equilibrium profits of firm j when it sets the price p_1^* and discloses in $\mathcal{H}(j) = NR$:

$$\Pi^{1,*}(p_1^*) = p_1^* \frac{1}{2}\left[1 - F(w^d(p_1^*))\right] + p_1^* \int_{p_1^*}^{w^d(p_1^*)} \frac{1}{2} F(p_2^* + u_j - p_1^*) du_j$$

By continuity of the expression above and because $w^d(p_1) \to p_1$ as $s \to 1/8$, it follows that:

$$\lim_{s \to 1/8} \Pi^{1,*}(p_1^*) = \frac{1}{2} p_1^* [1 - p_1^*] \tag{C.1.69}$$

Suppose that firm j deviates to non-disclosure when $\mathcal{H}(j) = NR$, in which case it gets a chance to screen its buyers and receives profits as defined in equation (4.4.11). To establish that deviating to non-disclosure is profitable if $s \to 1/8$, it is sufficient to show that the deviation is profitable if firm j charges p_1^*. For $p_1 = p_1^*$, the profits a firm makes as $s \to 1/8$ (for which $w^d(p_1^*) \to p_1^*$) converge to

$$\lim_{s \to 1/8} \Pi^{1,d}(p_1^*) = p_1^* \frac{1}{2}\left[1 - p_1^*\right] + p_3^d(p_1^*) \int_{p_3^d(p_1^*)}^{p_1^*} \frac{1}{2} F(p_1^* + u_j - p_3^d(p_1^*)) du_j, \tag{C.1.70}$$

where we have simplified notation by neglecting the limit of p_1^* as $s \to 1/8$. It is easy to see that (C.1.70) is strictly greater than (C.1.69) as $s \to 1/8$ if $p_1^* > p_3^d(p_1^*)$ in the limit since this implies that the second term in (C.1.70) is strictly positive. To calculate the limit of $p_1^* - p_3^d(p_1^*)$, note that:

$$\lim_{s \to 1/8} \left(p_3^d(p_1^*)\right) = \frac{4}{3} \lim_{s \to 1/8} (p_1^*) - \frac{1}{3} \sqrt{7 \left(\lim_{s \to 1/8} (p_1^*)\right)^2} = \frac{4 - \sqrt{7}}{3} \lim_{s \to 1/8} (p_1^*)$$

where we again used that $w^d(p_1^*) \to p_1^*$ as $s \to 1/8$. It thus follows that

$$\lim_{s \to 1/8} \left(p_3^d(p_1^*) - p_1^*\right) = \frac{1 - \sqrt{7}}{3} \lim_{s \to 1/8} (p_1^*) < 0,$$

which shows that deviating to non-disclosure is strictly profitable as $s \lim 1/8$. This completes the proof. ■

Proof of Lemma 6:

Consider the search decision of a consumer who has visited firm j first. Note that consumers have passive beliefs. When receiving the (potentially off-equilibrium) price p_1, a consumer with match value u_j will thus search if and only if:

$$\int_0^{u_j-p_3^f(p_1)+p_2^f} \max\{u_j-p_3^f(p_1),0\}f(u_{-j})du_{-j}+\int_{u_j-p_3^f(p_1)+p_2^f}^1 \max\{u_{-j}-p_2^f,0\}f(u_{-j})du_{-j}-s >$$

$$\max\{u_j - p_1, 0\}, \tag{C.1.71}$$

where the left-hand side depicts the value of continuing to search and the right-hand side the value of not doing so. The optimal search strategy is a cutoff rule with cutoff $w^f(p_1)$. This holds by the following logic: Because we restrict attention to search costs which admit on-path search, a consumer always strictly prefers to continue search (and does not buy immediately) if $u_j \leq p_1$. Moreover, if $u_j > p_1$, a consumer's gains from search (the difference between the left-hand side and the right-hand side of equation (C.1.71) is strictly falling in u_j. To see this, note that the derivative of the gains of search for $u_j \geq p_1$ is bounded from above by

$$F\left(u_j - p_3^f(p_1) + p_2^f\right) - 1. \tag{C.1.72}$$

Thus, there must be a unique cutoff $w^f(p_1)$ so that consumers continue searching if and only if their initial match value is below $w^f(p_1)$.[2]

To show that $w^f(p_1)$ is given by the expression presented in the lemma, observe that the maximum functions on both sides of (C.1.71) vanish if $u_j = w^f(p_1)$. This follows from the fact that (i) $w^f(p_1) > p_1$ and (ii) $w^f(p_1) > p_3^f(p_1)$ must hold.

While (i) holds by previous arguments, (ii) is due to the following logic: In a PBE, $w^f(p_1) > p_3^f(p_1)$ must hold for any initial price p_1. To see this, suppose toward a contradiction that $w^f(p_1) \leq p_3^f(p_1)$. Since firms must have consistent beliefs in any PBE, firm j in the information set $\mathcal{H}(j) = D \times p_1 \times R$ will believe that all returning consumers have $u_j \leq w^f(p_1)$. When setting a price $p_3^f(p_1)$ above $w^f(p_1)$, the firm's profits in this information set would be zero. By contrast, it is easy to see that j could earn strictly positive profits in the information set $\mathcal{H}(j) = D \times p_1 \times R$ by charging a price of $p_3^f(p_1) < w^f(p_1)$, contradicting that $w^f(p_1) \leq p_3^f(p_1)$ holds in equilibrium.

Point (ii) also implies that, for any $u_{-j} > w^f(p_1) - p_3^f(p_1) + p_2^f$, we have that $\max\{u_{-j} - p_2^f, 0\} = u_{-j} - p_2^f$. Thus, regardless of whether $p_1 < p_3^f(p_1)$ or not, the cutoff $w^f(p_1)$, if it lies strictly below 1, sets the following equation equal to 0:

$$T(w^f, p_1) := (w^f - p_3^f(p_1) + p_2^f)(w^f - p_3^f(p_1)) + \int_{w^f-p_3^f(p_1)+p_2^f}^1 (u_{-j} - p_2^f)du_{-j} - s - (w^f - p_1) \tag{C.1.73}$$

[2]To ensure that a cutoff is always unique, we impose the tie-breaking rule that consumers do not continue to search if they are indifferent.

which completes the proof of the lemma.

For future reference, we use $T(w^f, p_1)$ to find the derivative of $w^f(p_1)$ w.r.t. p_1:

$$\frac{\partial T}{\partial p_1} = 1 - (w^f(p_1) - p_3^f(p_1) + p_2^f)\frac{\partial p_3^f(p_1)}{\partial p_1} \quad ; \quad \frac{\partial T}{\partial w^f} = (w^f(p_1) - p_3^f(p_1) + p_2^f) - 1 \tag{C.1.74}$$

This establishes that, so long as $w^f(p_1)$ is interior:

$$\frac{\partial w^f(p_1)}{\partial p_1} = \frac{1 - (w^f(p_1) - p_3^f(p_1) + p_2^f)\frac{\partial p_3^f(p_1)}{\partial p_1}}{1 - (w^f(p_1) - p_3^f(p_1) + p_2^f)}. \tag{C.1.75}$$

■

Proof of Lemma 7:

There are two cases, namely $p_1^f > p_3^f$ and $p_1^f \leq p_3^f$. The first case cannot be an equilibrium because no firm j would disclose in the information set $\mathcal{H}(j) = R$ if $p_1^f > p_3^f$. This is because $d_j = D$ when $\mathcal{H}(j) = R$ reduces the competitor's price (since $p_1^f > p_3^f$) and intensifies competition. It is therefore sufficient to study the case that $p_1^f \leq p_3^f$.

Part 1: Derivation of p_3^f and p_2^f.

To derive the optimal price $p_3^f(p_1)$ firm j offers in the information set $\mathcal{H}(j) = D \times p_1 \times R$, we compute the derivative of $\Pi^{3,f}(p_3|p_1)$ as defined in equation (4.4.15). One can show that $w^f(p_1^f) - p_3^f + p_2^f < 1$ must hold. Thus, the equilibrium price $p_3^f(p_1) < w^f(p_1)$ must satisfy the following first-order condition for p_1 in an open ball around p_1^f:

$$\int_{p_3}^{w^f(p_1)} \frac{1}{2}\left(u_j - p_3 + p_2^f\right) du_j - \frac{1}{2}p_3\left(w^f(p_1) - p_3 + p_2^f\right) = 0 \tag{C.1.76}$$

Thus, the price $p_3^f(p_1)$ is given by:

$$p_3^f(p_1) = \frac{2}{3}(w^f(p_1) + p_2^f) - \frac{1}{3}\sqrt{\left(w^f(p_1) + p_2^f\right)^2 + 3\left(p_2^f\right)^2} \tag{C.1.77}$$

Consider p_2^f next. We showed in the main text that the profit function of firm B in the information set $\mathcal{H}(B) = R$ for prices in an open ball around the equilibrium p_2^f is

$$\Pi^{2,f}(p_2) = p_2\frac{1}{2}F(w^f)\left[1 - F(w^f - p_3^f + p_2)\right] + p_2\int_{p_2}^{w^f - p_3^f + p_2} \frac{1}{2}F(p_3^f + u_B - p_2)du_B \tag{C.1.78}$$

The corresponding first-order condition is:

$$w^f\big[1-(w^f-p_3^f+p_2)\big]-w^f p_2+\int_{p^2}^{w^f-p_3^f+p_2}(p_3^f+u_B-p_2)du_B=0 \tag{C.1.79}$$

Thus, for a fixed w^f, the equilibrium price p_2^f must solve:

$$p_2^f=(1/2)\big[1-w^f+p_3^f\big]+\frac{1}{4}(w^f)-\frac{1}{4}\frac{(p_3^f)^2}{(w^f)} \tag{C.1.80}$$

Part 2: For any w^f, the equilibrium prices p_2^f and $p_3^f := p_3^f(p_1^f)$ are uniquely determined.

Using (C.1.77), the on-path revision price p_3^f for a fixed search cutoff w^f (where $w^f = w^f(p_1^f)$ must hold in equilibrium) is given by

$$p_3^f=\frac{2}{3}\left(w^f+p_2^f\right)-\frac{1}{3}\sqrt{\left(w^f+p_2^f\right)^2+3\left(p_2^f\right)^2} \tag{C.1.81}$$

For a fixed search cutoff w^f, the derivative of p_3^f with respect to p_2^f is:

$$\frac{\partial p_3^f}{\partial p_2^f}=(2/3)-(1/3)(1/2)\frac{2\big(w^f+p_2^f\big)+6p_2^f}{\sqrt{\big(w^f+p_2^f\big)^2+3\big(p_2^f\big)^2}}<2/3 \tag{C.1.82}$$

Moreover, for a fixed w^f, we have:

$$\frac{\partial p_2^f}{\partial p_3^f}=(1/2)-(1/2)\underbrace{(p_3^f/w^f)}_{<1}\in(0,1/2) \tag{C.1.83}$$

We can define the solution price p_2^f using a fixed-point expression:

$$T^2(p_2^f):=p_2^f-p_2\big(p_3(p_2^f)\big)=0 \implies \frac{\partial T^2}{\partial p_2^f}=1-\frac{\partial p_2}{\partial p_3}\frac{\partial p_3}{\partial p_2} \tag{C.1.84}$$

Note that $\frac{\partial p_3}{\partial p_2}<2/3$ and $\frac{\partial p_2}{\partial p_3}\in\big(0,1/2\big)$, which implies that $\frac{\partial p_2}{\partial p_3}\frac{\partial p_3}{\partial p_2}<(1/3)$. Thus, T^2 is strictly rising in p_2. Consequently, there is a unique solution for p_2^f, and by extension, for p_3^f, for any given w^f.

Part 3: Initial equilibrium characterization: $w^f<1$ must hold.

Suppose toward a contradiction that $w^f=1$ for some level of search costs $s>0$ and recall that $p_1^f\le p_3^f$ must hold in a full disclosure equilibrium. If $w^f=1$, it is easy to verify that the unique solutions for p_2^f and p_3^f are given by $p_2^f=p_3^f=0.4142$. In addition,

$p_2^f = p_3^f$ combined with $p_1^f \leq p_3^f$ and $s > 0$ implies that consumers with a match value close to 1 at the first firm would not continue to search, contradicting that $w^f = 1$. To see this, note that the gains of search are continuous in the initial match value and that a consumer with initial match value of 1 would strictly prefer to not continue searching.

Part 4: Derivation of p_1^f.

p_1^f must maximize $\Pi^{1,f}(p_1)$ as defined in equation (4.4.17). Using the Envelope theorem, which implies that $\partial\Pi^{3,f}(p_3|p_1)/\partial p_3 = 0$, the first-order condition that p_1^f must solve can be written as:

$$\frac{1}{2}(1 - w^f(p_1)) - \frac{1}{2}\frac{\partial w^f(p_1)}{\partial p_1}p_1 +$$

$$\frac{1}{2}p_3(p_2(w^f), w^f(p_1))\frac{\partial w^f(p_1)}{\partial p_1}\big(p_2(w^f) + w^f(p_1) - p_3(p_2(w^f), w^f(p_1))\big) = 0 \tag{C.1.85}$$

In an equilibrium, $w^f(p_1^f) = w^f$ by definition, which implies that p_1^f must solve:

$$p_1^f = (1 - w^f)\left(\frac{\partial w^f(p_1^f)}{\partial p_1}\right)^{-1} + p_3^f\big(p_2^f + w^f - p_3^f\big) \tag{C.1.86}$$

■

Proof of Proposition 18:

We seek to show that $p_3^f < p_1^f$ in any full disclosure equilibrium, implying a profitable deviation when $\mathcal{H}(j) = R$. We prove this in three parts.

Part 1: A solution for $\frac{\partial w^f(p_1)}{\partial p^1}$ at $p_1 = p_1^f$.

By optimal consumer search, it was established that the derivative of $w^f(p_1)$ w.r.t p_1 depended on $\frac{\partial p_3^f(p_1^f)}{\partial p_1}$. To pin down $\frac{\partial w^f(p_1^f)}{\partial p_1}$, recall that the deviation $p_3^f(p_1)$ solves, for p_1 around p_1^f, the following function:

$$p_3^f(p_1) = \frac{2}{3}\left(w^f(p^1) + p_2^f\right) - \frac{1}{3}\sqrt{\left(w^f(p^1) + p_2^f\right)^2 + 3\left(p_2^f\right)^2} \tag{C.1.87}$$

At $p_1 = p_1^f$, we have $w^f(p_1^f) = w^f$, which implies that:

$$\frac{\partial p_3^f(p_1^f)}{\partial p_1} = \frac{\partial w^f(p_1)}{\partial p_1}\left[\frac{2}{3} - \frac{1}{3}\frac{1}{2}\frac{2\big(w^f + p_2^f\big)}{\sqrt{\big(w^f + p_2^f\big)^2 + 3\big(p_2^f\big)^2}}\right] \tag{C.1.88}$$

Plugging this back into the expression for $\frac{\partial w^f(p_1)}{\partial p_1}$ given in equation (C.1.75), which was derived from consumers' optimal search behaviour, implies that, at $p_1 = p_1^f$, expression (C.1.88) becomes:

$$\frac{\partial w^f(p_1^f)}{\partial p_1}\left[1 - (w^f - p_3^f + p_2^f) + (w^f - p_3^f + p_2^f)\left(\frac{2}{3} - \frac{1}{3}\frac{\big(w^f + p_2^f\big)}{\sqrt{\big(w^f + p_2^f\big)^2 + 3\big(p_2^f\big)^2}}\right)\right] = 1 \tag{C.1.89}$$

The term in brackets is strictly positive, because $(w^f - p_3^f + p_2^f) < 1$ must hold in equilibrium. Thus, the derivative of $w_1^f(p_1)$ w.r.t. p_1 (evaluated at the equilibrium value p_1^f) is independent of the exact value of p_1^f. Moreover, it is strictly positive.

Part 2: Uniqueness of p_1^f.

The results from Part 1 imply that $\frac{\partial w^f(p_1^f)}{\partial p_1}$ will be strictly positive and independent of the exact value of p_1^f. This establishes that, for any w^f, p_1^f is uniquely pinned down and given by:

$$p_1^f = (1 - w^f)\left(\frac{\partial w^f(p_1^f)}{\partial p_1}\right)^{-1} + p_3^f\big(p_2^f + w^f - p_3^f\big) \tag{C.1.90}$$

Part 3: There exists no full disclosure equilibrium.

Previous arguments have established that $w^f < 1$ must hold in equilibrium. However, examining the unique joint solutions (p_1^f, p_2^f, p_3^f) establishes that, for any $w^f < 1$, the ordering $p_3^f < p_1^f$ will hold. This, however, is a contradiction. Under this ordering of equilibrium prices, firms would prefer to deviate and not disclose after receiving disclosure.

■

Proof of Proposition 19:

We know from Armstrong et al. (2009) that industry profits are higher under ordered search than under random search if $s \leq 0.021$. Now, we show that industry profits under random search are the same as industry profits in the partial disclosure equilibrium, which implies the result, together with the discussion surrounding the statement of this proposition.

Under ordered search, the firm which is visited first sets the price p_1^*, while the firm that is visited second sets the price p_2^*. Thus, total industry profits are:

$$p_1^*\left[\big(1-F(w^d(p_1^*))\big)+\int_{p_1^*}^{w^d(p_1^*)}F(p_2^*+u_j-p_1^*)du_j\right]+$$

$$p_2^*\left[F(w^d(p_1^*))\big(1-F(w^*)\big)\right]+\int_{p_2^*}^{w^*}F(u_{-j}-p_2^*+p_1^*)du_{-j} \qquad \text{(C.1.91)}$$

This is just twice the profits that any firm attains in the partial disclosure equilibrium, which implies the result.

■

Proof of Corollary 7:

This result follows from the insights of Armstrong et al. (2009). Consumer surplus in the partial disclosure equilibrium is equal to the consumer surplus in the ordered search equilibrium defined in Armstrong et al. (2009). To see this, note that the ex ante expected utility of the buyer under ordered search and in the partial disclosure equilibrium is:

$$\int_0^{w^d(p_1^*)}\int_0^1\big(\max\{u_j-p_1^*,u_{-j}-p_2^*,0\}-s\big)du_{-j}du_j+\int_{w^d(p_1^*)}^1(u_j-p_1^*)du_j-s \qquad \text{(C.1.92)}$$

Because industry profits are also the same (see the proof of the previous proposition), this implies that total surplus is also the same in the partial disclosure equilibrium and under ordered search. Armstrong et al. (2009) show that total welfare and consumer surplus under ordered search are below their counterparts in the Wolinsky (1986) equilibrium, which implies the result.

■

C.2 Proofs — Section 4.5.

Proof of Proposition 20:

The result directly follows from the discussion in the two paragraphs after the statement of the Proposition. ■

Consumer surplus calculations: We define consumer surplus as the ex-ante expected utility of the buyer that we rely on throughout the paper, in particular in Section 4.5.2. To calculate consumer surplus, define $u^s(u_j)$ and $u^{ns}(u_j)$ as the expected utilities of searching and not searching, respectively, for a buyer that draws an initial match value u_j.

Consider the following general formulation where we define p_1^f as the price the buyer would receive at the initial firm she visits and p_2^f and p_3^f as the other prices she could receive second (or when returning to the initially visited firm) on the search path. Note that this formulation nests all our equilibria. Recall further that there was always a unique search cutoff w^f such that buyers search (in equilibrium) if and only if their initial match value is below this cutoff. Noting this, the ex-ante utility (BS) of the buyer is:

$$BS = \int_0^{w^f} u^s(u_j)du_j + \int_{w^f}^1 u^{ns}(u_j)du_j, \tag{C.2.1}$$

where $u^{ns}(u_j) = \max\{u_j - p_1^f, 0\} - s$ and $u^s(u_j)$ is given by

$$u^s(u_j) = \int_0^{\min\{\max\{u_j-p_3^f,0\}+p_2^f,1\}} \max\{u_j - p_3^f, 0\}du_{-j} + \int_{\min\{\max\{u_j-p_3^f,0\}+p_2^f,1\}}^1 (u_{-j} - p_2^f)du_{-j} - 2s.$$

We use (C.2.1) to calculate buyer surplus for different search costs in Figure 4.4.

C.3 Extension: Search with costly recall

A key comparative static result of our analysis is that sellers do not use search disclosure in equilibrium if search costs are too large. This prediction hinges on the result that the no disclosure equilibrium exists when the partial disclosure equilibrium does not (the former always exists in the base model). The reason why no disclosure is an equilibrium in the baseline model regardless of search costs was that deviating to disclosure can induce the rival to revise its price downward, the negative effect of which outweighed any benefits from disclosing. The dominance of this negative effect on the deviating firm's profits is directly related to the free recall assumption. This is because free recall guarantees that every consumer learns about the revised price of the firm they visited first before they make a purchase decision, which is to the detriment of the disclosing firm.

In this extension, we therefore study the case in which a positive mass of consumers face strictly positive recall costs. While positive recall costs mitigate the negative effects of deviating to search disclosure, we document that our results are robust in the sense that no disclosure remains an equilibrium for a large share of parameter combinations, and in particular when search costs are not too low. This is exactly what was to be expected, given that the costs of triggering a price revision by one's rival are smallest for low search costs. This is because price revisions are small in magnitude when search costs are small, given that the decision to search is not very informative about a consumer's match value. We also argue why partial disclosure does not emerge as an equilibrium if search costs are too large.

To model costly recall, we extend the framework of Janssen and Parakhonyak (2014), who consider costly recall for all consumers. In our analysis, we assume that a share $1-\rho<1$ of all consumers face recall costs. Precisely, $1-\rho$ consumers must incur the cost $b>0$ if they want to return to the seller they visited first after having continued to search. The remaining $\rho>0$ share of consumers have free recall as in the baseline model. Everything else is identical to the baseline model as well.

We require one further amendment to the consistency requirement on off-path beliefs. We specify that a firm in an information set in which it can revise its price, following some deviation, believes that consumers who have returned must have found it optimal to return.

We focus on the no disclosure equilibrium in this framework with costly recall. In this equilibrium, there is just one equilibrium price, which we call p^c, given that firms know nothing about the consumers' search histories in equilibrium. That is, consumers expect to receive the equilibrium price p^c at any firm and expect no revisions of prices. Thus, consumers with free recall will search if their match value is below $w^0(p_j;p^c)=\left(1-\sqrt{2s}\right)-p^c+p_j$, as in the baseline model. The optimal cutoff for consumers with recall costs, however, is different. Consumers with costly recall search if their match value is below

$$w^b(p_j;p^c)=\left(1+b-\sqrt{2s+2b}\right)-p^c+p_j. \tag{C.3.1}$$

As before, we restrict attention to equilibria with active search. As in Janssen and Parakhonyak (2014), we also restrict attention to search costs under which there also is positive return demand by consumers with recall costs in equilibrium, i.e. $w^b(p^c;p^c)>p^c+b$[3].

In equilibrium, the firms obtain profits from both consumers with free recall and consumers with costly recall. The demand from the former group, which we denote by $D^0(p_j)$ here, has the same structure as defined in equation (4.4.8). Equilibrium demand from consumers with recall costs, which we here denote by $D^b(p_j)$, is novel and given by:

$$D^b(p_j)=\underbrace{\int_{w^b(p_j;p^c)}^{1}(1/2)du_j+\int_{p_j+b}^{w^b(p_j;p^c)}(1/2)F(u-p_j-b+p^c)du_j}_{\text{first arriver demand}}+$$

[3]The equivalent condition on b and s when there are no consumers with free recall can be found in proposition 6 in Janssen and Parakhonyak (2014).

$$\underbrace{w^b(p^c;p^c)\int_{w^b(p_j;p^c)-b}^{1}(1/2)du_j+\int_{p_j}^{w^b(p_j;p^c)-b}(1/2)F(u_j-p_j+b+p^c)du_j}_{\text{searcher demand}} \tag{C.3.2}$$

To understand this expression, consider first the demand from consumers who arrive at firm j first. Because $w^b(p_j;p^c) > p_j$ since we restrict attention equilibria with active search, any consumer who arrives at firm j first and obtains a match value above $w^b(p_j;p^c)$ will directly buy. Any consumer with $u_j < w^b(p_j;p^c)$ will search and ultimately buy at firm j if and only if it is worthwhile to return at all (i.e. $u_j > p_j + b$) *and* it is better to return than to purchase at the other firm (i.e. $u_j - p_j - b > u_{-j} - p^c$).

Next, consider consumers who visit firm $-j$ first. These consumers are expected to sample firm j only if $u_{-j} < w^b(p^c;p^c)$, given firm j believes that they have received the price p^c at firm $-j$. If their match at firm j is above $w^b(p_j;p^c) - b$, any such consumer will surely buy at firm j. If their match value u_j is below this cutoff, they will buy at firm j if this match value exceeds p_j and it is better to purchase at firm j than to return to firm $-j$, i.e., $u_j - p_j > u_{-j} - p^c - b$.

We define the equilibrium profit function of the firm as $\Pi(p_j;p^c)$, which is given by:

$$\Pi(p_j;p^c) = p_j\left[\rho D^0(p_j) + (1-\rho)D^b(p_j)\right] \tag{C.3.3}$$

The equilibrium price p^c in the no disclosure equilibrium must thus solve:

$$p^c = arg\max_{p_j}\Pi(p_j;p^c) \tag{C.3.4}$$

The no disclosure equilibrium exists if it is not worthwhile for a firm to deviate from the equilibrium by disclosing when an unknown buyer arrives at the firm. As in the baseline framework, such a deviation by firm j will have different effects, depending on whether the buyer has visited firm $-j$ before or not. If a buyer who arrives at firm j first (without firm j knowing) and firm j discloses, then firm $-j$ will offer a price p_2^c to the consumer. As before, this price p_2^c is above the equilibrium price p^c.

By contrast, if a buyer arrives at firm j after having visited firm $-j$ before, then search disclosure leads the rival firm $-j$ to revise its price. We define the price that a firm would choose in this information set as p_3^c. Recall that the revised price was always below the equilibrium price in the baseline framework. Due to presence of consumers with costly recall, this is no longer true in general. This is because consumers with recall costs who return to the firm they initially visited generate inelastic demand around the original price.[4]

[4]Note that we still assume passive beliefs here. This implies that a firm who receives disclosure still believes that its rival set the equilibrium price p^c, and that consumers searched according to the equilibrium search rules.

The profit function of a firm that receives disclosure for a known buyer is non-differentiable at the price p^c+b. This holds because, given the firm's beliefs, it anticipates that all consumers with recall costs who return to the firm surely buy when offered a return price below p^c+b, which is not true when the return price is above p^c+b.

For any price $p_j \leq p^c+b$, a firm j who receives disclosure for a known buyer believes it will make obtain the following profits:

$$\Pi^R(p_j;p^c) = p_j\Bigg[\underbrace{(1-\rho)\int_{p^c+b}^{w^b(p^c;p^c)}(u_j-b)du_j}_{\text{Consumers with recall costs}} + \underbrace{\rho\int_{p_j}^{w^0(p^c;p^c)}(u_j-p_j+p^c)du_j}_{\text{Consumers with free recall}}\Bigg] \quad \text{(C.3.5)}$$

To understand this expression, note the following. From firm j's points of view, all consumers with recall costs who return to firm j must (i) have a match value above p^c+b and (ii) must have a difference in match values, namely $u_j - u_{-j}$, that is greater than b. Result (ii) holds because firm j believes that the consumer received the price p^c at its rival. In that situation, it would only be worthwhile for a consumer with recall costs to return to firm j instead of buying at the rival if $u_j - b - p^c > u_{-j} - p^c$, i.e. if and only if $u_j - u_{-j} > b$.

These results imply that the demand from returning consumers with costly recall is fully inelastic for $p_j \leq p^c+b$. When offering a price $p_j \leq p^c+b$, the match value of any any such consumer who returns will exceed the price, by result (i). Moreover, firm j would also expect any such consumer to buy at firm j rather than at firm $-j$. This is because, by result (ii), the consumption utility at firm j (namely $u_j - p_j$) remains above the consumption utility at firm $-j$ (namely $u_j - p^c$) for any $p_j \leq p^c+b$.

At any price $p_j > p^c+b$, by contrast, the demand generated by returning consumers with recall costs is elastic. Then, the objective function $\Pi^R(p_j;p^c)$ is:

$$\Pi^R(p_j;p^c) = p_j\Bigg[\underbrace{(1-\rho)\int_{p_j}^{w^b(p^c;p^c)}(u_j-p_j+p^c)du_j}_{\text{Consumers with recall costs}} + \underbrace{\rho\int_{p_j}^{w^0(p^c;p^c)}(u_j-p_j+p^c)du_j}_{\text{Consumers with free recall}}\Bigg] \quad \text{(C.3.6)}$$

The two different consumer groups hence affect the optimal return price in different ways: Consumers with recall costs push up the optimal revision price p_3^c, while consumers with free recall exert downward pressure on this price. As a result, the relationship between p_3^c and s is non-monotonic. Using numerical methods, we compute p_3^c (and all other relevant equilibrium objects) for different parameter combinations. The results are visualized by the following graphs, which plot the relationship between p_3^c and s for different levels of ρ and b:

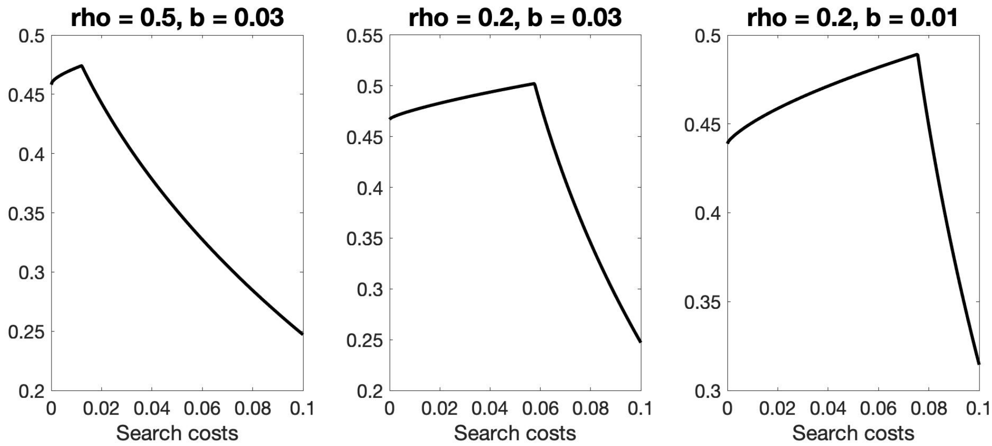

Figure C.1: Costly recall — return prices

For low levels of s, the price p_3^c equals $p^c + b$ and thus lies strictly above p^c. To see why this is optimal, recall that consumers with recall costs generate inelastic demand for prices $p_j \leq p^c + b$. These consumers thus push the optimal revision price of the firm up towards $p^c + b$, but no further than that, because the associated profits have a kink at this price. When search costs are low, the measure of consumers with costly recall who arrive at a firm is relatively high, which implies that the optimal revision price equals $p^c + b$. The optimal revision price p_3^c will fall below p^c only for sufficiently high search costs, at which the weight of returning consumers with free recall, which push the price below p^c, becomes high enough.

For low values of s, there is hence no detrimental effect of disclosure because disclosure always leads to upward changes in the rival's price. Thus, the no disclosure equilibrium does not exist for low search costs in this setup. Only when p_3^c falls sufficiently far below p^c and the detrimental effect of disclosure is sufficiently strong, no disclosure becomes an equilibrium. This is visualized by the following figure. In each graph, we compare the equilibrium profits $\Pi(p^c; p^c)$ (dotted line) to the profits that are attainable via a deviation to disclosure (solid line) for different combinations of ρ and b. If the equilibrium profits are above the deviation profits, no disclosure is an equilibrium.[5]

[5]The validity of this claim hinges on one additional observation. No firm must find it optimal to deviate by disclosing after having received disclosure. We numerically verified that this is the case if search costs are low enough such that $\Pi(p^c; p^c)$ is greater than the depicted deviation profits.

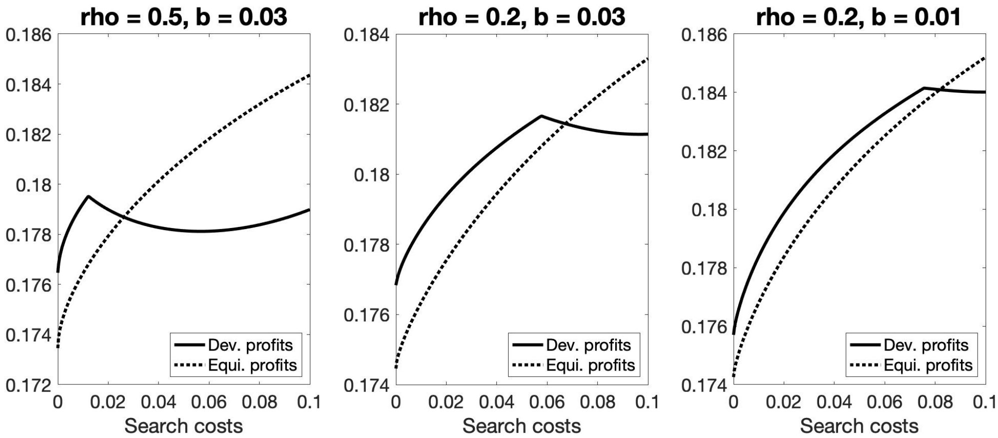

Figure C.2: Costly recall — deviation incentives

There are three main takeaways: First, the no disclosure equilibrium continues to exist for a wide range of search costs even when 50% of consumers have substantial return costs ($b = 0.03$).

Second, the presence of return costs does strongly counteract the existence of this equilibrium if search costs are too small, which has implications for the optimal regulation of the markets we describe. For instance, enabling price retargeting of consumer through tracking (i.e. raising ρ) may be quite beneficial, because it counteracts the incidence of search history based price discrimination.

Third, the analysis has reaffirmed that the incentives to conduct search disclosure are reduced when search costs increase. Intuitively, this notion would also carry over when analyzing the partial disclosure equilibrium in the extension with costly recall. We conjecture that the partial disclosure equilibrium will only exist when search costs are sufficiently small. When they are high, consumers who leave a firm to search will never return (this effect is only reinforced by recall costs). Thus, the benefits of search disclosure (i.e. increasing the price the rival would set upon being visited second) will be negligible even under costly recall. By contrast, the benefits of withholding disclosure (via receiving the ability to revise one's price) are still large even if search costs are high, because the resulting selection will be particularly pronounced. By similar arguments, the full disclosure equilibrium may exist if recall is costly, but only if search costs are very small.

C.3.1 Mathematical analysis: costly recall

Part 1: Equilibrium characterization

Janssen and Parakhonyak (2014) show that the reservation utility of a consumer with recall costs in a uniform price equilibrium (with equilibrium price p^c) is:

$$w^b(p_j) = \left(1 + b - \sqrt{2b + 2s}\right) - p^c + p_j \tag{C.3.7}$$

In equilibrium, consumers with free and costly recall will search if and only if their initial valuation is below $w^{*,0}$ and $w^{*,b}$, which are respectively defined as follows:

$$w^{*,0} = w^0(p^c) := 1 - \sqrt{2s} \quad ; \quad w^{*,b} = w^b(p^c) := 1 + b - \sqrt{2b + 2s} \tag{C.3.8}$$

Consider consumers for whom recall is free and who arrive at firm j:

- First arrivers with $u_j > w^0(p_j)$ will buy at firm j.
- First arrivers with $u_j \in [p_j, w^0(p_j)]$ buy at firm j iff $u_{-j} < u_j - p_j + p^c$.
- Second arrivers with $u_j > w^0(p_j)$ buy at firm j if and only if $u_{-j} < w^{*,0}$.
- Second arrivers with $u_j \in [p_j, w^0(p_j)]$ buy at firm j if and only if $u_{-j} < u_j - p_j + p^c$.

Now consider consumers for whom recall is costly.

- First arrivers with $u_j > w^b(p_j)$ will buy at firm j.
- First arrivers with $u_j \in [p_j, w^b(p_j)]$ buy at firm j iff $u_{-j} < u_j - p_j - b + p^c$ and $u_j - p_j - b > 0$.
- Second arrivers with $u_j > w^b(p_j)$ buy at firm j if and only if $u_{-j} < w^{b,*}$.
- Second arrivers with $u_j \in [p_j, w^b(p_j)]$ buy at firm j iff $u_{-j} < w^{b,*}$, and $u_{-j} < u_j - p_j + b + p^c$.

Firstly, consider the components of demand from consumers with strictly positive return costs $b > 0$. Consider first arrivers with $u_j > w^b(p_j)$. Demand from these consumers is:

$$D^1(p_j) = \int_{w^b(p_j)}^{1} (1/2) du_j \tag{C.3.9}$$

Consider second arrivers with $u_j \in [p_j, w^b(p_j) - b]$. The condition that $u_{-j} < u_j - p_j + b + p^c$ implies that they have searched (i.e. $u_{-j} < w^{b,*}$), because:

$$u_{-j} < w^b(p_j) - b - p_j + b + p^c = w^{b,*}$$

Now consider second arrivers with $u_j \in [w^b(p_j) - b, 1]$. For these consumers, search guarantees consumption. Search requires that $u_{-j} < w^{b,*}$. Consumption occurs at j if $u_j - p_j > u_{-j} - p^c - b \iff u_j - p_j + b > u_{-j} - p^c$. If these consumers searched, we have:

$$u_{-j} - p^c < w^{b,*} - p^c = w^b(p_j) - p_j \leq u_j - p_j + b$$

Thus, demand implied by second arrivers is:

$$D^S(p_j) = w^{b,*}\int_{w^b(p_j)-b}^{1}(1/2)du_j + \int_{p_j}^{w^b(p_j)-b}(1/2)(u_j - p_j + b + p^c)du_j \tag{C.3.10}$$

Thirdly, consider the demand that comes from agents with return costs that visit firm j first, search, and then return. This demand is:

$$D^R(p_j) = \int_{p_j+b}^{w^b(p_j)}(1/2)(u_j - p_j - b + p^c)du_j \tag{C.3.11}$$

One can show that demand from consumers with return cost b is thus:

$$D(p_j;b) = (1/2)\big(2p^c - 2p_j + 1 - (p^c)^2 - p_jb - p^c\sqrt{2b+2s} + p_j\sqrt{2b+2s}\big) \tag{C.3.12}$$

Thus, the equilibrium price will need to solve:

$$p_j\frac{\partial D(p_j)}{\partial p_j} + D(p_j) = 0 \iff$$

$$\big(2p^c - 2p_j + 1 - (p^c)^2 - p_j(1-\rho)b\big) - p^c\big(\rho\sqrt{2s} + (1-\rho)\sqrt{2b+2s}\big) + p_j\big(\rho\sqrt{2s} + (1-\rho)\sqrt{2b+2s}\big)$$

$$+p_j\Bigg(-2 - (1-\rho)b + \big(\rho\sqrt{2s} + (1-\rho)\sqrt{2b+2s}\big)\Bigg) = 0 \tag{C.3.13}$$

The equilibrium price is thus:

$$p^* = \frac{-d^* - \sqrt{(d^*)^2 - 4(a^*)(c^*)}}{2a^*} \tag{C.3.14}$$

We have defined $a^* = -1$, $c^* = 1$, and:

$$d^* = -2 - 2(1-\rho)b + \rho\sqrt{2s} + (1-\rho)\sqrt{2b+2s}$$

Naturally, this is only the correct equilibrium price as long as $p^c + b < w^{b,*} := w^b(p^c)$.

Part 2: Optimal second arriver pricing:

Suppose a firm j receives search disclosure for a previously unknown buyer, which implies that this consumer must have arrived second and must have $u_{-j} < w^{*,b}$ or $u_{-j} < w^{*,0}$, respectively. For second arrivers with return costs b, demand is:

$$D^S(p_j;b) = w^{b,*}\int_{w^b(p_j)-b}^{1}(1/2)du_j + \int_{p_j}^{w^b(p_j)-b}(1/2)(u_j - p_j + b + p^c)du_j \tag{C.3.15}$$

For second arrivers with free recall (share ρ), demand is:

$$D^S(p_j;0) = w^{0,*}\int_{w^0(p_j)}^{1}(1/2)du_j + \int_{p_j}^{w^0(p_j)}(1/2)(u_j - p_j + p^c)du_j \tag{C.3.16}$$

Thus, a firm who receives disclosure for a previously unknown buyer will maximize the following through choice of p_2:

$$p_2\big[\rho D^S(p_j;0) + (1-\rho)D^S(p_j;b)\big] \tag{C.3.17}$$

Part 3: Optimal revision of prices

Now consider the optimal price that a firm j would set when receiving disclosure for a consumer it has seen before. Any such consumer with return costs must have $u_j < w^{*,b}$ and must find it optimal to return (expecting to receive p^c at the initial firm they visit). This requires that (i) $u_j > p^c + b$ and (ii) $u_j - p^c - b > u_{-j} - p^c \iff u_{-j} < u_j - b$.

Consider prices $p_j \in [p^c, p^c + b]$. For any such price, a consumer who returns can buy (by condition (i)) and will buy, given that they still prefer to buy at firm j (by condition (ii)). Thus, demand (from consumers with return costs) for prices $p_j \in [p^c, p^c + b]$ is:

$$D^R(p_j, b) = \int_{p^c+b}^{w^{*,b}}\int_0^{u_j-b}(1/2)du_{-j}du_j \tag{C.3.18}$$

Moreover, demand (from consumers with return costs) for prices $p_j \in [p^c + b, 1]$ is:

$$D^R(p_j, b) = \int_{p_j}^{w^{*,b}}\int_0^{u_j-p_j+p^*}(1/2)du_{-j}du_j = \int_{p_j}^{w^{*,b}}(1/2)(u_j - p_j + p^c)du_j \tag{C.3.19}$$

Both these probabilities will be strictly interior.

Recalling that a share ρ of agents have no recall costs, total demand from returning consumers for prices $p_j \leq p^c + b$ is:

$$D^R(p_j) = \rho\int_{p_j}^{w^{*,0}}(1/2)(u_j - p_j + p^*)du_j + (1-\rho)\int_{p^*+b}^{w^{*,b}}(1/2)(u_j - b)du_j \tag{C.3.20}$$

Return demand can be similarly computed for $p_j > p^* + b$, noting that the component derived from consumers without return costs stays the same.

Part 4: The effects of a deviation by disclosure

Now suppose that firm j, who initially receives no disclosure, deviates by disclosing.

Consider any second arriver with return costs and $u_j \in [w^b(p_j) - b, 1]$. By previous arguments, any such consumer will arrive at firm j and not return to firm $-j$, because she does not anticipate a revision of the original price at firm $-j$. Thus, conducting search disclosure will not harm the disclosing firm when facing such a consumer.

Now consider any second arriver with return costs and $u_j \in [p_j, w^b(p_j) - b]$ and $u_{-j} \in [0, u_j - p_j + b + p^c]$. In the perception of any such consumer, it is not optimal to return to firm $-j$. Thus, conducting search disclosure will not harm the disclosing firm when facing such a consumer.

Any consumer with $u_j \in [p_j, w^b(p_j) - b]$ and $u_{-j} \in [u_j - p_j + b + p^c, 1]$ would in fact return to firm $-j$ (anticipating to receive the price p^c there). Any such consumer would thus buy at firm j iff and only if $u_j - p_j > u_{-j} - p_3$. When $p_3 < p^c$, no such consumer would ever buy at firm j. However, if $p_3 \geq p^c$, there is a chance that any such consumer buys at firm j. This occurs if $u_j - p_j > u_{-j} - p_3$. Thus, any such consumer will buy at firm j if $u_{-j} \in [u_j - p_j + p^c + b, u_j - p_j + p_3]$.

Thus, demand from second arrivers with return costs is:

$$D^S(p_j) = (1/2)w^{b,*}[1 - (w^b(p_j) - b)] + \int_{p_j}^{w^b(p_j)-b} (1/2)(u_j - p_j + b + p^c)du_j +$$

$$\int_{p_j}^{w^b(p_j)-b} \int_{u_j-p_j+p^c+b}^{u_j-p_j+p_3} (1/2)du_{-j}du_j \tag{C.3.21}$$

Now consider first arrivers with return costs. Consumers with $u_j > w^b(p_j)$ will never search, so demand from them is still given by:

$$D^1(p_j) = \int_{w^b(p_j)}^{1} (1/2)du_j \tag{C.3.22}$$

This is because $w^b(p_j) > p_j$. Thirdly, consider consumers with return costs $b > 0$ who arrive at firm j first, search, and return. Given the timing of search disclosure, they have received the price $p^{2,d}$. Because the rival firm will not engage in search disclosure, the demand implied by these consumers is:

$$D^R(p_j) = \int_{p_j+b}^{w^b(p_j)} (1/2)(u_j - p_j - b + p^2)du_j \tag{C.3.23}$$

Deviation demands from consumers without return costs have been derived previously — this is given in equation (4.4.11). We can use all these notions to compute the total demand (and thus the optimal price) set by a firm after deviating by non-disclosure. This profit is:

$$\Pi^{1,b}(p_j) = p_j(1-\rho)\frac{1}{2}\Bigg[\int_{w^b(p_j)}^{1}(1)du_j + \int_{p_j+b}^{w^b(p_j)}(u_j - p_j - b + p^2)du_j +$$

$$w^{b,*}[1-(w^b(p_j)-b)] + \int_{p_j}^{w^b(p_j)-b}(u_j - p_j + b + p^c)du_j + \int_{p_j}^{w^b(p_j)-b}\int_{u_j-p_j+p^c+b}^{u_j-p_j+p_3}(1)du_{-j}du_j\Bigg] +$$

$$p_j\frac{\rho}{2}\Bigg[\big[1-w^0(p_j)\big] + \int_{p_j}^{w^0(p_j)}(p_2 + u_j - p_j)du_j + w^{*,0}\big(1-(w^{*,0}-p_3+p_j)\big) +$$

$$\int_{p_j}^{w^{*,0}-p_3+p_j}(p_3 + u_j - p_j)du_j\Bigg] \tag{C.3.24}$$

Part 5: Deviations to disclosure after receiving disclosure.

Suppose a firm j receives disclosure for a buyer that it has not seen before. By the passive beliefs assumption, it believes that its rival $-j$ offered the price p^* and that consumers who visited the rival first continued searching if and only if their match there was below $w^{,0}$ and $w^{*,b}$, respectively. By not disclosing, the rival's price would remain unchanged. By disclosing, firm j gives its rival the chance to revise its price. From firm j's point of view, the rival will then maximize the following profit function:

$$\Pi^{dd}(p_{-j}) = \rho\int_{0}^{w^{*,0}}(u_{-j} - p_{-j} + p_2)\mathbb{1}[p_{-j} \leq u_{-j}]du_{-j} +$$

$$(1-\rho)\int_{p^*+b}^{w^{*,b}}\int_{0}^{u_{-j}-p^*-b+p_2}\mathbb{1}[u_{-j}-p_{-j}\geq 0]\mathbb{1}[u_{-j}-p_{-j}\geq u_j - p_2]du_j du_{-j} \tag{C.3.25}$$

The rival's $(-j)$ beliefs are passive and the rival knows that it initially disclosed. Receiving disclosure lets the rival know that it was visited first. The rival thus knows that firm j was in the information set $\mathcal{H}(j) = R$ after receiving disclosure by $-j$. By the assumption of passive beliefs, the rival firm $-j$ must believe that firm j offered the price p_2.

Upon offering a price $p_{-j} \leq p^c + b$, profits are thus:

$$\rho \int_0^{w^{*,0}} (u_{-j} - p_{-j} + p_2)\mathbb{1}[p_{-j} \leq u_{-j}]du_{-j} + (1-\rho) \int_{p^*+b}^{w^{*,b}} (u_{-j} - p^* - b + p_2)du_{-j} \tag{C.3.26}$$

This is because:

$$u_{-j} - p^c - b + p_2 \leq u_{-j} - p_{-j} + p_2$$

By contrast, if $p_{-j} > p^c + b$, profits are:

$$\rho \int_0^{w^{*,0}} (u_{-j} - p_{-j} + p_2)\mathbb{1}[p_{-j} \leq u_{-j}]du_{-j} + (1-\rho) \int_{p_{-j}}^{w^{*,b}} (u_{-j} - p_{-j} + p_2)du_{-j} \tag{C.3.27}$$

Appendix D

Statement/Erklärung

Bei der eingereichten Dissertation mit dem Titel "Essays on Price Discrimination in Search Markets" handelt es sich um mein eigenständig erstelltes Werk, das den Regeln guter wissenschaftlicher Praxis entspricht. Ich habe nur die angegebenen Quellen und Hilfsmittel benutzt und mich keiner unzulässigen Hilfe Dritter bedient. Insbesondere habe ich wörtliche und nicht wörtliche Zitate aus anderen Werken als solche kenntlich gemacht.

Carl-Christian Groh

Mannheim, den 28. Februar 2023.

Appendix E

Curriculum Vitae

Carl-Christian Groh

2017-2023: Universität Mannheim, Ph.D. studies in Economics.

2014-2016: Ludwig-Maximilians Universität München, M.Sc. in Economics.

2011-2014: Ludwig-Maximilians Universität München, B.Sc. in Volkswirtschaftslehre.

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