Three Essays on the Macroeconomics of Banking

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Preface

This dissertation is composed of three chapters on macroeconomics and banking. The main objective of these chapters is to analyze how the banking sector affects the macroeconomy. In Chapter 1 I explore how the optimal capital requirement varies with the level of competition in the banking sector. In Chapter 2 I investigate how banking competition affects the business cycle and the transmission of uncertainty shocks. Finally, in Chapter 3 I analyze the business cycle implications of macroprudential regulation.

Chapter 1: Capital Requirements, Bank Competition and Stability

In the past 40 years, there has been a trend of increasing bank concentration and declining bank competition in the United States. In the first chapter, I develop a two-period general equilibrium model with imperfect competition in the banking sector to investigate how the level of competition affects the impact of capital requirements and the optimal capital requirement.

In this model, a reduction in banking competition can have two effects on bank stability. On the one hand, greater competition can lead to reduced bank margins, which in turn increases the risk of default. This effect is known as the margin effect. On the other hand, stronger competition results in lower loan rates that reduce borrowers’ risk incentives and ultimately decrease bank risk. This is known as the risk-shifting effect. These two effects can create a U-shaped relationship between competition and bank risk. When competition is low, the risk-shifting effect dominates, leading to a decrease in bank risk as competition increases. However, when competition is high, the margin effect dominates and bank risk increases with competition.

Capital requirements affect both the risk-shifting and the margin effect. They exacerbate the risk-shifting effect, as higher bank funding costs lead to higher borrower risk-taking. This effect is more pronounced when competition is higher since banks pass the higher funding costs to borrowers to a greater extent. Capital requirements also reduce the margin effect, as they provide equity buffers to banks. The reduction in the margin effect is more pronounced in highly competitive banking sectors, as banks have very small buffers due to small margins. According to my model, the impact on the margin effect is stronger, thus capital requirements reduce bank risk more when competition is higher.

I find that the optimal capital requirement can decrease with competition when risk-shifting is strong. As competition increases, banks pass more of their higher financing costs to borrowers,
and capital requirements become more effective in reducing bank risk when competition is high.

Chapter 2: Imperfect Banking Competition and the Propagation of Uncertainty Shocks

The recent conflict in Ukraine and the Covid-19 pandemic have resulted in a sharp increase in different measures of uncertainty. Additionally, many countries have experienced a rise in bank concentration. In Chapter 2, I investigate how the level of banking competition affects the propagation of uncertainty shocks. Using a panel dataset spanning 44 countries from 2000Q1 to 2020Q1, I find a negative correlation between the impact of uncertainty shocks on output growth and the level of banking competition. To identify exogenous variations in uncertainty, I use disaster shocks from Baker et al. (forthcoming).

To investigate how banking competition affects business cycle fluctuations and the transmission of uncertainty shocks, I develop a New Keynesian business cycle model with financial frictions and imperfect competition in the banking sector. In this model, entrepreneurs have limited net worth to buy physical capital so they need to borrow from bankers. Bankers compete à la Cournot to supply loans to entrepreneurs internalizing the demand for loans and the probability of borrower default.

Entrepreneurs face both idiosyncratic and aggregate shocks, which result in heterogeneous returns on their capital stock. If the return is insufficient to repay loans, entrepreneurs default. Uncertainty is defined as the cross-sectional dispersion of idiosyncratic shocks. As uncertainty increases, the probability of low returns rises and entrepreneurs are more likely to default. Due to financial frictions, banks respond to heightened uncertainty by reducing credit supply, constraining entrepreneurs’ ability to acquire capital and resulting in decreased investment and a contraction of output.

In this model, lower banking competition affects the transmission of uncertainty shocks to entrepreneurial returns through two channels. The first channel is the risk-shifting effect. This effect describes the impact of competition on borrower risk-taking. When banking competition is lower, the borrowing rate is higher, leading entrepreneurs to take on more risk and to increase their probability of default. I show that, when entrepreneurs take more risk, an increase in uncertainty leads to a larger rise in the default rate.

The second channel is the pass-through effect. This effect describes the impact of competition on the extent to which bankers pass shocks through to borrowers. With less competition, bankers have more market power, which reduces the extent to which they pass shocks through to borrowers. Uncertainty shocks increase non-performing loans and monitoring costs incurred by bankers. As competition in the banking sector increases, bankers respond by decreasing their loan supply more due to their lower market power.

When there is a lower level of banking competition due to a reduced number of competitors, the risk-shifting effect dominates. As a result, uncertainty shocks have a stronger impact on borrower
defaults, which exacerbates the reduction in credit supply, investment, and output.

Chapter 3: Firm Risk Shocks and the Banking Accelerator

The recent war in Ukraine and the Covid-19 pandemic have exposed many firms to an elevated risk of default. Without sufficient bank capitalization, a wave of corporate defaults may trigger asset sales and lead to a contraction in credit provision, which could exacerbate the recession. In the third chapter, which is joint work with Vivien Lewis, Stéphane Moyen, and Stefania Villa, we investigate the effects of firm risk shocks on the banking sector and how macroprudential policy can affect the propagation of risk shocks to the economy.

We first empirically demonstrate that firm risk shocks are transmitted to the financial sector as an increased risk of bank defaults. Next, we capture this transmission in a model that combines New Keynesian price-setting frictions with financial market imperfections. Our model features a loan contract between entrepreneurs and banks, both of which are subject to idiosyncratic default risk. A risk shock, defined as a second-moment shock to entrepreneur return, triggers a wave of corporate defaults that leads to losses on banks’ balance sheets. The rise in bank losses increases bank defaults, shrinks credit supply, and induces a demand-driven recession.

We study the implications of macroprudential policy for the business cycle. Specifically, we examine the effect of imposing a penalty on excessive bank leverage as a form of macroprudential policy. If banks fail to meet the capital requirement, they are required to pay a penalty to a bank resolution authority. Our findings indicate that both a higher penalty and a higher capital requirement can help to reduce the impact of risk shocks on bank defaults by increasing bank equity buffers.

In addition, we investigate the potential benefits of a countercyclical capital requirement. This policy is intended to counteract procyclicality in the financial system by lowering the capital requirement during a recession, which allows banks to sustain the economy by providing more loans to borrowers. We find that a countercyclical capital requirement results in less credit reduction after a risk shock, thus, sustaining investment and mitigating negative effects on output. However, this policy also leads to a stronger increase in bank defaults due to higher bank leverage.
Chapter 1

Capital Requirements, Bank Competition and Stability

1 Introduction

In the past 40 years, the United States has experienced a trend of increasing bank concentration and declining bank competition.\textsuperscript{1} The theoretical literature argues that bank competition can have two effects on bank stability: a margin effect and a risk-shifting effect. On the one hand, increased competition reduces the margin between the return on assets and financing costs, inducing banks to take more risk (margin effect).\textsuperscript{2} On the other hand, higher bank competition can lead to lower loan rates, lower asset risk, and ultimately, safer banks (risk-shifting effect).\textsuperscript{3}

The impact of bank competition on bank stability raises an important question: should regulators adjust the capital requirement accordingly? In this paper, I address this question by developing a two-period general equilibrium model that incorporates imperfect banking competition. My model builds on the work of Martinez-Miera and Repullo (2010), but with the addition of households that supply deposits and bank equity to banks, as well as imperfect competition in the deposits market. In this simple economy, households make consumption-savings decisions and can save using deposits and bank equity. Banks collect households’ savings and provide loans to firms that require resources to fund projects. The returns on these projects are stochastic, and the probabilities of default are pairwise imperfectly correlated, as modeled by Vasicek (2002). Furthermore, the return in case of success is increasing in risk. I assume that banks compete à la Cournot for loans and deposits. Deposits are insured and banks must satisfy a capital requirement that is binding since the cost of equity exceeds that of deposits.

According to Martinez-Miera and Repullo (2010), in the absence of bank equity, the margin effect and the risk-shifting effect lead to a U-shaped relationship between competition and bank

\textsuperscript{1}See Corbae and D’Erasmo (2021).
\textsuperscript{2}See for example Matutes and Vives (2000), Hellmann et al. (2000) and Repullo (2004).
\textsuperscript{3}See Boyd and de Nicoló (2005).
risk. Specifically, when competition is low, the risk-shifting effect dominates, leading to a decrease in bank risk as competition increases. Conversely, when competition is high, the margin effect dominates, resulting in an increase in bank risk with competition.

However, this U-shaped relationship may not hold when banks issue equity. Equity serves as a buffer against loan losses and reduces the margin effect. As a result, bank equity is more effective in reducing the risk of default when competition is high, as banks default in this case due to their small margins. Thus, if the capital requirement is high enough, bank risk can decrease with competition. This occurs when the level of bank equity is sufficient to make the margin effect less strong than the risk-shifting effect.

The relationship between competition and bank stability has implications for the optimal capital requirement. When the risk-shifting effect is strong, the results suggest that the optimal capital requirement decreases with competition. This is because higher capital requirements are more effective in reducing bank risk but also more costly when competition is high. However, when the risk-shifting is low, the relationship between competition and the probability of bank failure becomes U-shaped, resulting in a U-shaped relationship between the optimal capital requirement and competition.

The paper is organized as follows. Section 2 provides an overview of the related literature. Section 3 outlines the model and defines the equilibrium. Section 4 describes the model’s parameterization, and Section 5 presents the results of the model. Finally, Section 6 concludes and summarizes the paper’s main findings.

2 Related literature

This paper makes three main contributions to the literature. Firstly, it contributes to the strand of literature that examines the relationship between competition and stability. While previous studies, such as Boyd and de Nicoló (2005) and Martínez-Miera and Repullo (2010), focused on partial equilibrium models of Cournot competition in the deposit and/or loan markets, this paper considers a general equilibrium model where banks can issue equity and are subject to a binding capital requirement. The model shows that the capital requirement affects the competition-stability relationship, as bank equity is more effective in reducing bank risk when competition is high. This implies that bank risk can decrease with competition when the capital requirement is high enough, contrary to the results of Martínez-Miera and Repullo (2010).

Secondly, the paper contributes to the literature on the optimal capital requirement. Previous research in this area includes works by Van den Heuvel (2008), Mendicino et al. (2018), and Malherbe (2020), among others. Similar to these studies, my paper examines the role of the capital requirement in mitigating the moral hazard problem caused by deposit insurance while also limiting banks’ lending capacity. However, this paper focuses on an imperfectly competitive banking sector, where the effects of the capital requirement and its optimal level change with
competition. When competition is high, the pass-through of the costs of the capital requirement is higher, but the capital requirement is more effective in reducing bank risk.

Thirdly, this paper contributes to the existing literature that examines the impact of capital requirements in models of bank competition. Previous studies in this area include Corbae and D’Erasmo (2021) and Jamilov (2021). In this paper I focus on how bank competition affects the impact of capital requirements and their optimal level.

3 Model

In this section I develop a two-period model \((t = 1, 2)\) with four classes of agents: firms, banks, households and a deposit insurance agency. I consider a continuum of firms with projects that require a unit investment. These firms are penniless and must obtain the resources needed to run their projects through loans provided by banks. Banks raise resources from households by issuing deposits and bank shares. Deposits are insured by the deposit insurance agency, which charges lump sum taxes to households in order to balance its budget. In the first period, firms receive the resources they need to run their projects from banks, and in the second period, they repay their loans with interest.

3.1 Firms

I consider a continuum of mass \(m\) of penniless firms owned by households. Each firm has the option of generating a safe return \(u\), which varies across firms and does not require an initial investment. The value of this outside option is known only by the firm and the fraction of firms that have an outside option below \(u\) is denoted by \(G(u)\).

In the first period each firm is endowed with a project that requires a unit investment cost. Firms borrow from banks at the interest rate \(r\) to finance the investment cost. The return on a project \(R\) is stochastic and depends on the probability of failure \(p_i\), which is privately chosen by each firm. Specifically, \(R(p_i)\) is given by

\[
R(p_i) = \begin{cases} 
1 + \alpha(p_i), & \text{with probability } 1 - p_i, \\
1 - \lambda, & \text{with probability } p_i,
\end{cases}
\]

where \(\alpha(p_i)\) is the return on the project in case of success, and \(\lambda\) is the loss given failure.

To ensure an interior solution to the firm’s problem, the function \(\alpha(p_i)\) is assumed to be positive, increasing, and concave in \(p_i\), and satisfies \(\alpha(0) < \alpha'(0)\) and \(\alpha(0) - \alpha'(0) < r < \alpha(1)\). These assumptions imply a risk-return trade-off for the firm.

Firms operate under limited liability and choose the level of risk in their projects to maximize expected profits. If the project fails, its return is insufficient to repay the banks.\(^4\) In this case, as

\(^4\)This occurs under the assumption that \(r > -\lambda\).
a result of limited liability, firms default and their profits become zero, while the banks recover only a fraction \(1 - \lambda\) of the loan. The expected profits of a firm are given by the return in case of success, minus the interest payment \(\alpha(p_i) - r\), times the probability of success \(1 - p_i\). The expected profits of a firm is

\[
\nu(r) = \max_{p_i} \mathbb{E}[\Pi_{e,i}] = \max_{p_i} (1 - p_i)(\alpha(p_i) - r).
\]

The first order condition of the problem is\(^5\)

\[
(1 - p_i)\alpha'(p_i) - \alpha(p_i) + r = 0. \tag{1.1}
\]

Note that all firms behave the same way in equilibrium, since they differ only in their outside options. Therefore, the solution does not depend on the index \(i\), and I will drop it from here on.

By differentiating (1.1) it is possible to derive the relationship between choice of risk and the borrowing rate, which is given by

\[
p'(r) = \frac{1}{2\alpha'(p) - (1 - p)\alpha''(p)} > 0. \tag{1.2}
\]

This implies that firm risk increases as the loan rate rises. As in Martinez-Miera and Repullo (2010) I refer to this relationship as the risk-shifting effect.

It’s worth noting that not all firms will start a project, only those whose outside option is lower than the expected return on their project. Since each firm requires a unit loan and the size of the firms is \(m\), the loan demand \(L^d\) is given by \(mG(\nu(r))\).

Projects failures are pairwise imperfectly correlated as modelled by Vasicek (2002).\(^6\) It is possible to show that the fraction of insolvent firms \(x\) is distributed according to the distribution\(^7\)

\[
F(x) = \Phi \left( \frac{\sqrt{1 - \rho} \Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}} \right),
\]

where \(\rho\) is the coefficient of pairwise correlation and \(\Phi\) is the standard normal CDF. Note that \(F(x)\) is a function of the risk chosen by the firms. Specifically, if \(p = 0\), the distribution has a single mass point at \(x = 0\), and if \(p = 1\), the distribution has a single mass point at \(x = 1\). In fact, if \(p = 0\), none of the projects fails and \(x = 0\); similarly, if \(p = 1\), \(x = 1\). Furthermore, it can be shown that \(\mathbb{E}(x) = p\).

\(^5\)Note that the solution must be an interior solution. The corner \(p = 0\) is not a solution because \(\alpha(0) - \alpha'(0) < r\), while the corner \(p = 1\) is not a solution because \(r < \alpha(1)\).

\(^6\)In order to introduce correlation of failures across firms in a tractable way, it is assumed that for each firm, the realization of failure depends on the random variable \(y_i\). It is assumed that these variables are jointly normal distributed with pairwise correlation \(\rho\). More information on the distribution of failures is provided in Appendix A.

\(^7\)For the derivations see Appendix A.
3.2 Banks

The banking sector is composed by \( N \) homogeneous banks that finance their lending activities to firm through deposits and equity. These banks compete in a Cournot fashion for loans and deposits, with deposit insurance guaranteeing the latter. However, due to deposit insurance, the risk associated with deposits is not reflected in their cost, which leads to high leverage ratios for banks. To mitigate this risk and the associated cost of deposit insurance, a capital requirement of \( \phi \) is imposed by the deposit insurance agency.

Each bank issues deposits \( d_j \) and bank equity \( e_j^S \) to fund its lending activity. Deposits pay an interest rate \( i \) in the second period, while bank equity entitles the holder to a portion of the bank's dividend. The amount of equity issued by bank \( j \) determines its equity-to-loan ratio

\[
\theta_j = \frac{e_j^S}{l_j},
\]

where \( l_j \) is the amount of loans supplied by bank \( j \). The capital requirement ensures that \( \theta_j \geq \phi \).

Each bank's budget constraint requires that its loans are funded by its equity and deposits, so that

\[
l_j = e_j + d_j.
\]

Since banks compete à la Cournot, they internalize the effect of their decisions on the demand for loans and the supply of deposits and equity. Given the fraction of defaulting loans \( x \), bank \( j \) earns \((1 + r)(1 - x)l_j\) from performing loans, \((1 - \lambda)xl_j\) from nonperforming loans, and must pay out \((1 + i)(1 - \theta_j)l_j\) to depositors, where \( r \) and \( i \) are respectively the inverse loan demand and inverse deposit supply functions. The bank's dividend is then given by

\[
\Delta_j(l_j, l_{-j}, e_j, e_{-j}) = \max \left[(1 + r(L))(1 - x)l_j + (1 - \lambda)xl_j - (1 + i(l_j, l_{-j}, e_j, e_{-j}))(1 - \theta_j(l_j, e_j))l_j, 0\right],
\]

where \( l_{-j} \) and \( e_{-j} \) are respectively the supply of loans and equity issuance of all the other banks, \( L = \sum_{j=1}^{N} l_j \) and the max operator is a result of limited liability.\(^8\)

The occurrence of bank default depends on the realization of \( x \). For realizations of \( x \) that are larger than a cutoff level \( \hat{x}_j \), banks declare default. The cutoff is given by

\[
\hat{x}_j = \frac{1 + r - (1 + i)(1 - \theta_j)}{r + \lambda}.
\]

When a bank defaults, the deposit insurance agency monitors its assets and obtains their value minus a monitoring cost. Specifically, if bank \( j \) fails, the value obtained by the deposit insurance agency

\(^8\)Note that the return on loans is a function of the aggregate supply of loans and that the deposit rate depends on the aggregate supply of loans and equity issuance of each bank. Since banks compete à la Cournot for loans and deposits, they internalize the effect of their decisions on loan demand and deposit supply.
The deposit insurance agency is

\[(1 - \chi)((1 + r)(1 - x) + (1 - \lambda)x)l_j,\]

where \(\chi\) is the fraction of assets lost due to monitoring costs. Since the deposit insurance agency does not make a profit, a lump sum tax is necessary to cover the costs of deposit insurance. The tax is given by

\[T^D = \sum_{j=1}^{N} \mathbb{I}_{x_j < x}[(1 + i)(l_j - e_j) - (1 - \chi)((1 + r)(1 - x) + (1 - \lambda)x)l_j].\]

Each bank’s objective is to maximize the present value of its expected stream of dividend payments by choosing the optimal level of loans \(l_j\) and equity issuance \(e_j\), given the inverse supply functions of deposits and bank equity, the inverse demand of loans, and the supply of loans and equity issuance of the other banks. This can be expressed as follows

\[
\max_{l_j, e_j} \mathbb{E}\{\beta \Delta_j(l_j, l_{-j}, e_j, e_{-j}) - e_j\},
\]

subject to the capital requirement\(^9\)

\[\theta_j(l_j, e_j) \geq \phi.\]

The probability of bank default can be determined by calculating the probability that the default rate is higher than the cutoff \(\hat{x}_j\). This probability is given by the expression\(^{10}\)

\[q_j = \Phi\left(\frac{\Phi^{-1}(\rho) - \sqrt{1 - \rho} \Phi^{-1}(\hat{x}_j)}{\sqrt{\rho}}\right).\]

It is important to note that in a symmetric equilibrium if one bank fails, all banks fail since they invest in the same portfolio of loans.\(^{11}\)

### 3.3 Households

The economy is populated by a unit mass of risk-averse households. At \(t = 1\), households receive an endowment \(Y_1\), which can be consumed \((C_1)\) or saved in financial products such as deposits \((D)\) or equity in bank \(j\) \((e_j)\).\(^{12}\) At \(t = 2\) households receive a second endowment \(Y_2\), the return on their savings and the profits of the firms \(\Pi^e\). The resources obtained in the second period are used for consumption \((C_2)\) and to pay lump-sum taxes \((T^D)\). Households maximize the expected

\(^9\)The capital requirement is based on the amount of loans as there is only one risk class.
\(^{10}\)Appendix A provides detailed steps for calculating the probability of bank failure.
\(^{11}\)The return on each banks portfolio is stochastic due to the imperfect correlation of loan returns. Each bank invests in every loan and holds a portion of the aggregate loan portfolio. This implies that the fraction of loans that default is the same across banks’ portfolios and in a symmetric equilibrium every bank has the same probability of default.
\(^{12}\)Note that households can invest in every bank.
present value of their utility

$$\max_{C_1, C_2, D, \sum_{j=1}^N e_j} u(C_1) + \beta \mathbb{E}[u(C_2)],$$

subject to the constraints

$$C_1 + D + \sum_{j=1}^N e_j = Y_1,$$

$$C_2 = Y_2 + D(1 + i) + \sum_{j=1}^N \frac{e_j}{\bar{e}_j} \Delta_j + \Pi^e - T^D,$$

where $\bar{e}_j$ is the amount of equity issued by bank $j$ and $D = \sum_{j=1}^N d_j$.

The first-order conditions with respect to $D$ and $e_j$ determine the supply of deposits and bank equity of bank $j$, respectively. They are given by

$$u'(C_1) - \beta(1 + i)\mathbb{E}[u'(C_2)] = 0,$$

$$u'(C_1) - \frac{\mathbb{E}[u'(C_2)\Delta_j]}{\bar{e}_j} \leq 0.$$

### 3.4 Symmetric equilibrium

Given a function for the return of a project ($\alpha(p)$), a distribution for the firms’ outside option $G$ and a capital requirement $\phi$, a symmetric equilibrium is a set of quantities $\{l_j, e_j, d_j\}_{j=1}^N$, a set of prices $\{r, i\}$ and a probability of failure $p$ such that:

- At time $t = 1$, given the borrowing rate $r$, firms maximize profits by choosing the probability of failure their project $p$, which determines loan demand. At time $t = 2$, a share of projects fails. Entrepreneurs whose projects fail declare default.

- At time $t = 1$, given the deposit rate $i$ and banks’ equity issuance $\{\bar{e}_j\}_{j=1}^N$, households maximize the expected present value of their utility by choosing equity supply for every bank $\{e_j\}_{j=1}^N$ and deposit supply $D$. At time $t = 2$ households consume their endowment, savings and profits of firms.

- At time $t = 1$ banks maximize the present value of their expected stream of dividends by choosing their loan supply $l_j$, deposit demand $d_j$ and equity issuance $e_j$. At time $t = 2$ a portion of their loans defaults. If the return on their portfolio is less than the cost of deposits, banks default, otherwise, they pay dividends to households.

- At time $t = 1$ loan, deposit and equity markets clear.
At time \( t = 2 \), the budget constraint of the deposit insurance is balanced in every state.

## 4 Parametrization

As in Martinez-Miera and Repullo (2010), I assume a linear risk-shifting function

\[
p(r) = a + br,
\]

which implies that the return of a project in case of success, a function of the firm probability of default, has the following functional form

\[
\alpha(p) = \frac{1 - 2a + p}{2b}.
\]

I assume that \( G(u) \), the distribution of firms’ outside options, is the uniform distribution \([0, \bar{u}]\), the resulting inverse loan demand function is\(^{13}\)

\[
r(L) = \frac{1 - a - \sqrt{2bL\bar{u}}}{b}.
\]

Table 2.1 summarizes the parameters used, their meaning in the model and their values. The value of the loss given default parameter \( \lambda \) and the default correlation across projects \( \rho \) are obtained from the Basel Committee on Banking Supervision (2020, CRE31.4 and CRE32.5). Both the values are derived from the Basel Framework, the first one is the loss given default for senior claims on banks, corporates and sovereigns that are not secured by collateral, the second one is a value that lays within the lowest (0.12) and highest (0.24) possible correlation for bank, corporate and sovereign exposures in the risk weight function. As in Clerc et al. (2018) the fraction of assets lost in case of bank default \( \chi \) is set to 0.3. The utility function of the households is supposed to be exponential

\[
u(C) = \frac{1 - e^{-\gamma C}}{\gamma},
\]

with a parameter of constant relative risk aversion \( \gamma = 0.001 \) in line with Babcock et al. (1993). Finally, the endowments \( Y_1 \) and \( Y_2 \) are equal to 145 and 100, respectively. I set the discount factor \( \beta \) to 0.99 and \( \bar{u} \) to 0.1.

## 5 Results

In this model capital requirements have two welfare effects: they decrease bank default risk but also reduce their ability to provide loans. The strength of these effects varies with competition and borrowers’ risk-shifting behavior, thereby shaping the relationship between competition and the optimal capital requirement. In Section 5.1 I analyze the impact of banking competition on bank

\(^{13}\)It can be shown using the fact that \( L^d = mG(\nu(r)) \).
5. RESULTS

Table 1.1: Parameters of the model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Intercept risk-shifting function</td>
<td>0.05</td>
</tr>
<tr>
<td>$b$</td>
<td>Risk-shifting parameter</td>
<td>0.7</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>Maximum value outside option</td>
<td>0.1</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Loss given default</td>
<td>0.45</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Default correlation across projects</td>
<td>0.2</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Monitoring costs failed banks</td>
<td>0.3</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Absolute risk aversion household</td>
<td>0.001</td>
</tr>
<tr>
<td>$Y_1$</td>
<td>Endowment first date</td>
<td>145</td>
</tr>
<tr>
<td>$Y_2$</td>
<td>Endowment second date</td>
<td>100</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
</tbody>
</table>

default risk and how this relationship changes with the introduction of a capital requirement. Finally, in Section 5.2 I explore the relationship between competition and the optimal capital requirement.

5.1 Competition and capital requirements’ impact on bank risk

The parametrization described in the previous section implies that, due to deposit insurance, banks do not find it optimal to issue equity as the cost of deposits is lower than that of equity and does not reflect the risk of bank default. When the capital requirement is binding, the bank’s maximization problem can be simplified to

$$\max_{L_j} l_j h(L),$$

where

$$h(L) = (r + \lambda) \int_0^{\hat{x}} F(x) dx - \phi.$$

To ensure an interior solution to the problem, I assume that $h'(L) < 0$ and $h''(L) < 0$. Under these assumptions, there is a unique symmetric Cournot equilibrium that satisfies the first order condition of the bank problem

$$\frac{L}{n} h'(L) + h(L) = 0.$$

Similarly to Martinez-Miera and Repullo (2010), it is possible to derive the effect of bank competition on aggregate loan supply, borrowing rate and borrower risk-taking.

**Proposition 1.** When the capital requirement is binding, an increase in the number of banks results in an increase in the aggregate level of loans $L$, a decrease in the loan rate $r$ and a decrease in borrower risk $p$. 
Proof. Differentiating (1.8) and using the assumptions \( h'(L) < 0 \) and \( h''(L) < 0 \)

\[
\frac{dL}{dn} = -\frac{h(L)}{Lh''(L) + (n+1)h'(L)} > 0.
\]

Since \( L(r) = mG(\nu(r)) \), \( L'(r) = mG'(\nu(r))\nu'(r) < 0 \) implying that \( r'(L) < 0 \). Therefore

\[
\frac{dr}{dn} = r'(L) \frac{dL}{dN} < 0.
\]

Finally, because of (1.2)

\[
\frac{dp}{dn} = p'(r)r'(L) \frac{dL}{dN} < 0.
\]

When the capital requirement is binding, the relationship between competition and bank risk is given by

\[
\frac{dq}{dN} = q'(L) \frac{dL}{dN},
\]

where the second term is positive due to Proposition 1 and the first term is given by

\[
q'(L) = \phi(\cdot) \left( \frac{\partial \Phi^{-1}(p(r(L)))}{\partial p} p'(r(L)) r'(L) - \sqrt{1 - \rho \frac{\partial \Phi^{-1}(\hat{x}(L))}{\partial \hat{x}(L)} \hat{x}'(L)} \right).
\]  

(1.9)

Similar to Martinez-Miera and Repullo (2010), the sign of the relationship between competition and stability is determined by the sign of \( q'(L) \). This sign depends on the relative strength of two effects of competition on stability. The first effect, known as the risk-shifting effect, is negative and is determined by the first term inside the brackets of (1.9). This effect arises because when competition increases, loan rates decline, and borrowers take less risk. As a result, banks face lower borrower risk and are, therefore, safer.\(^{14}\) The second effect is the margin effect, which is determined by the second term inside the brackets of (1.9). This effect is positive and reflects the fact that when competition increases, bank margins decline, reducing the fraction of nonperforming loans required to make banks default.

In Figure 1.1, we can see how the relationship between competition, measured by the number of banks, and bank risk changes with the capital requirement. The light blue line represents the case without any capital requirements, while the orange and green lines represent the cases with a 5% and 10% capital requirement, respectively.

When there are no capital requirements, the relationship between competition and bank risk is U-shaped. At low levels of competition, the risk-shifting effect dominates and bank risk decreases as competition increases. Conversely, at high levels of competition, the margin effect dominates.

\(^{14}\)The magnitude of this effect is increasing in the risk-shifting parameter \( b \).
and bank risk increases as competition increases.

The introduction of capital requirements results in a flatter or even decreasing relationship between bank risk and competition. Specifically, when the capital requirement is set at 10%, a negative relationship between competition and bank risk emerges.

Figure 1.1: Competition and bank risk for different values of the capital requirement

![Graph showing the relationship between bank competition and the probability of bank failure for different values of the capital requirement \( \phi \).]

Notes. The graph shows the relationship between bank competition and the probability of bank failure for different values of the capital requirement \( \phi \).

The relationship between bank competition and bank risk is influenced by the introduction of a capital requirement, as it affects the risk-shifting and margin effects. Figure 1.2 illustrates the impact of competition and capital requirements on loan supply. As competition increases, banks’ loan supply also increases, as their market power decreases. However, the equilibrium level of loans decreases with the level of capital requirement. This is because, due to deposit insurance, equity is more expensive than deposits, resulting in higher funding costs for banks when capital requirements increase. Consequently, there is a reduction in loan supply. It is important to note that the reduction in loan supply is larger with higher levels of competition and is due to the need for banks to pass on the higher costs resulting from capital requirements to borrowers. This results in a stronger cut in loan supply when competition is high, as banks have smaller margins and need to pass on a greater proportion of the costs.

Figure 1.3 illustrates the relationship between borrower risk, competition, and the level of the capital requirement. As shown in Proposition 1, borrower risk decreases with competition in the banking sector because borrowers face lower borrowing costs. Conversely, the introduction of capital requirements leads to higher borrower risk. This happens because the higher borrowing rate leads to a lower supply of loans and a higher borrowing cost for firms. As a result, firms take more risk to offset the higher borrowing costs. Moreover, since the effect of higher capital
Figure 1.2: Competition and loan supply for different values of the capital requirement

Notes. The graph shows the relationship between bank competition and loan supply for different values of the capital requirement $\phi$.

requirements on the loan supply is stronger in more competitive markets, the increase in firm risk-taking due to higher capital requirements is stronger the higher is the level of competition.

The results presented in Figure 1.3 imply that capital requirements exacerbate the risk-shifting effect, as higher bank funding costs lead to higher borrower risk-taking. This effect is more pronounced in more competitive markets, where banks have smaller profits and need to pass on the higher funding costs to borrowers to a greater extent.$^{15}$

Figure 1.4 illustrates the impact of capital requirements and competition on the bank default cutoff. As competition increases, bank margins decrease, which implies that the share of nonperforming loans required to trigger a bank default decreases. In other words, bank margins act as a cushion against losses from nonperforming loans, and the smaller the cushion, the more fragile the bank becomes.

Similarly, bank equity increases the bank default cutoff by providing additional buffers. The effect of equity is especially strong in highly competitive banking sectors, where margins are already very small. In such cases, the rise in the default cutoff due to capital requirements is more pronounced.

The results depicted in Figure 1.4 suggest that bank equity acts as a buffer for banks, thereby reducing the margin effect. Moreover, the reduction in the margin effect is more pronounced in highly competitive banking sectors, where banks have very small buffers due to small bank margins. Therefore, capital requirements can effectively reduce the risk of default and promote

$^{15}$Note that the magnitude of this force is a function of $b$. Specifically, as the risk-shifting parameter increases, borrower risk rises more sharply in response to a rise in the capital requirement.
5. RESULTS

Figure 1.3: Competition and borrower risk for different values of the capital requirement

![Figure 1.3: Competition and borrower risk for different values of the capital requirement](image)

Notes. The graph shows the relationship between bank competition and borrower risk for different values of the capital requirement $\phi$.

Bank stability, especially in highly competitive banking sectors where the margin effect is stronger.

Figure 1.4: Competition and banks’ default cutoff for different values of the capital requirement

![Figure 1.4: Competition and banks’ default cutoff for different values of the capital requirement](image)

Notes. The graph shows the relationship between bank competition and the banks’ default cutoff for different values of the capital requirement $\phi$.

The parametrization introduced in Section 4 indicates that the effect of capital requirements on the risk-shifting effect is smaller than on the margin effect. Thus, capital requirements are more effective in reducing bank risk when competition is higher. This result is robust to the level
of the risk-shifting parameter as discussed in Appendix B.

5.2 The optimal capital requirement and competition

The optimal capital requirement is the one that maximizes welfare, which is defined as the expected lifetime utility of the household

\[ u(C_1) + \beta \mathbb{E}[u(C_2)]. \]

Capital requirements have two effects on welfare. On the one hand, they decrease bank risk, but on the other hand, they reduce banks’ ability to provide loans. As discussed in Section 5.1, the effectiveness of capital requirements in reducing bank risk is greater when competition is high. However, the costs of capital requirements also change with competition. When competition increases, the pass-through of the banks’ higher financing costs to the borrowers becomes larger, and banks respond to an increase in the capital requirement by cutting loan supply more. Therefore, the costs of capital requirements increase with competition.

Figure 1.5 illustrates the relationship between the optimal capital requirement and the level of competition for different values of the risk-shifting parameter, \( b \). The results suggest that the optimal capital requirement decreases with competition when the risk-shifting effect is strong, as capital requirements are more effective in reducing bank risk but also more costly when competition is high. However, when the risk-shifting parameter is low, the relationship between competition and the probability of bank failure becomes U-shaped, resulting in a U-shaped relationship between the optimal capital requirement and competition.

Figure 1.5 also shows how the optimal capital requirement varies with the risk-shifting parameter. The risk-shifting parameter impacts both the costs and benefits of increasing the capital requirement. Higher risk-shifting leads to an increase in borrowers’ risk-taking behavior, which in turn increases the costs of implementing capital requirements. However, it also increases bank risk, which in turn increases the benefits of implementing tighter capital requirements.

An increase in the capital requirement results in higher financing costs for banks. When competition is higher, the pass-through of these costs is also higher, leading banks to cut their loan supply by a larger amount. This larger cut in loan supply leads to a larger increase in the loan rate and in the risk faced by borrowers. Thus, the negative impact of the risk-shifting parameter on the optimal capital requirement is stronger when competition is high. On the other hand, the positive impact of the risk-shifting parameter on the optimal capital requirement is weaker when competition is high, because of the low risk-shifting effect. Therefore, when competition is high, the positive effect of the risk-shifting parameter is outweighed by the negative effect, causing the optimal capital requirement to decrease as the risk-shifting parameter increases.

When competition is low, the risk-shifting effect is stronger, but the pass-through of financing costs to borrowers is smaller. If the risk-shifting parameter \( b \) is low, the risk-shifting effect is negligible and it is optimal to set a high capital requirement since its cost is small. As \( b \) increases,
the optimal capital requirement decreases as its cost increases but the risk-shifting effect and the probability of bank default are small. Finally, when \( b \) is high and increases further, bank risk becomes high and the marginal benefit of increasing the capital requirement becomes higher than the marginal cost, leading to an increase in the optimal capital requirement.

Figure 1.5: Competition and optimal capital requirement for different risk-shifting

![Graph showing the relationship between bank competition and the optimal capital requirement for different values of the risk-shifting parameter \( b \).](image)

Notes. The graph shows the relationship between bank competition and the optimal capital requirement for different values of the risk-shifting parameter \( b \).

6 Conclusion

In this chapter, I examine the impact of capital requirements in a general equilibrium model where banks compete à la Cournot for deposits and loans. The introduction of capital requirements can alter the relationship between competition and stability, which depends on the relative strength of two forces: the risk-shifting effect and the margin effect. The risk-shifting effect is the positive effect of bank competition on bank stability. This effect arises because when competition increases, loan rates decrease and borrowers take on less risk. In contrast, the margin effect is the negative effect of competition on bank stability. This effect reflects the fact that when competition increases, bank margins decline, leading to a reduction in the fraction of nonperforming loans required to cause banks to default. The introduction of a capital requirement affects the risk-shifting and margin effects and influences the relationship between bank competition and bank risk. Capital requirements exacerbate the risk-shifting effect, as higher bank funding costs lead to higher borrower risk-taking, while they reduce the margin effect as they provide an additional buffer to banks. The effects of capital requirements on bank risk are stronger the higher is the level of banking competition.
Furthermore, I examine how the optimal capital requirement changes with the level of competition. The optimal capital requirement decreases with competition when risk-shifting is strong, as capital requirements are more effective in reducing bank risk, but they are also more costly when competition is high. As competition increases, banks pass on more of their higher financing costs to borrowers, resulting in a larger reduction in loan supply in response to an increase in capital requirements.
Appendix A  Probability of bank default

Let $y_i$ be a random variable that determines if the project of firm $i$ fails or not. I assume that the variables $y_i$ are jointly normally distributed with an expected value of $-\Phi^{-1}(p)$ and a unit variance. It is possible to decompose $y_i$ as follows

$$y_i = \sqrt{\rho} z + \sqrt{1-\rho} \epsilon_i - \Phi^{-1}(p),$$

where $\rho$ is the coefficient of pairwise correlation, $\Phi^{-1}$ is the inverse of the cumulative distribution function (CDF) of the standard normal distribution, $z$ is a common-risk factor and $\epsilon_i$ is a firm-specific risk factor that is independent of $z$. Both $\epsilon_i$ and $z$ are distributed according to a standard normal distribution. A project fails if $y_i < 0$ and the probability of this event is

$$Pr[\sqrt{\rho} z + \sqrt{1-\rho} \epsilon_i < \Phi^{-1}(p)] = \Phi(\Phi^{-1}(p)) = p,$$

where the first equality holds because the variables $\sqrt{\rho} z + \sqrt{1-\rho} \epsilon_i$ are distributed according to a joint standard normal distribution. By fixing the common-risk factor, the conditional probability of failure of a project can be expressed as

$$\gamma(z) = Pr[\sqrt{\rho} z + \sqrt{1-\rho} \epsilon_i < \Phi^{-1}(p)|z] = \Phi \left( \frac{\Phi^{-1}(p) - \sqrt{\rho} z}{\sqrt{1-\rho}} \right). \quad (1.A.1)$$

Let $S$ denote the percentage loss of a portfolio of projects and $S_i$ a variable that is 1 if the project $i$ fails and 0 otherwise. This implies that $Pr[S_i = 1|z] = \gamma(z)$. Since the variables $S_i$, conditional on $z$, are independently distributed with finite variances, the law of large numbers implies that $S$ conditional on $z$ converges to its expected value, $\gamma(z)$, as the number of projects goes to infinity.

The probability that $S$ is smaller than or equal to $x$ is

$$F(x) = Pr[S \leq x] = Pr[\gamma(z) \leq x] = Pr[z \geq \gamma^{-1}(x)] = \Phi(-\gamma^{-1}(x)). \quad (1.A.2)$$

Inverting the expression for $\gamma(z)$ given by (1.A.1), we can rewrite $F(x)$ as

$$F(x) = \Phi \left( \frac{\sqrt{1-\rho} \Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}} \right).$$

The probability of bank default $q_j$ is the probability that the loan default rate is larger than the cutoff $\hat{x}_j$ defined in (1.3). The probability of bank failure can be found using (1.A.2)

$$q_j = Pr[x > \hat{x}_j] = Pr[S > \hat{x}_j] = Pr[\gamma(z) > \hat{x}_j] = \Phi(\gamma^{-1}(\hat{x}_j)).$$
Finally, inverting (1.A.1), the probability of bank failure can be written as

\[ q_j = \Phi \left( \frac{\Phi^{-1}(p) - \sqrt{1 - \hat{p}} \Phi^{-1}(\hat{x}_j)}{\sqrt{\hat{p}}} \right). \]
Appendix B  Robustness

This section confirms the main findings presented in Section 5.1 for various levels of the risk-shifting parameter.

Figure 1.B.1 illustrates how the relationship between bank risk and competition changes with the capital requirement when the risk-shifting parameter is set to 0.4. Consistent with the baseline results, the capital requirement reduces the risk of bank failure more as the level of competition increases. However, note that in this case, the relationship between competition and bank risk does not decrease as in the baseline scenario when the capital requirement is set to 10%. This is due to the lower risk-shifting effect, which makes a less competitive banking sector safer compared to the baseline model.

Figure 1.B.1: Relationship between competition, bank risk and capital requirements for low $b$

![Graph showing the relationship between competition, bank risk, and capital requirements for different values of the capital requirement.](image)

Notes. The graph shows the relationship between bank competition and the probability of bank failure for different values of the capital requirement $\phi$. The risk-shifting parameter $b$ is set to 0.4.

Figure 1.B.2 illustrates how the relationship between bank risk and competition changes with the capital requirement when the risk-shifting parameter is set to 1. The results are similar to the results of the baseline model. In this case, the relationship between competition and bank risk also decreases when the capital requirement is set to 5%. This is due to the higher risk-shifting effect, which makes a less competitive banking sector riskier compared to the baseline model.
Figure 1.B.2: Relationship between competition, bank risk and capital requirements for high $b$

Notes. The graph shows the relationship between bank competition and the probability of bank failure for different values of the capital requirement $\phi$. The risk-shifting parameter $b$ is set to 1.
Chapter 2

Imperfect Banking Competition and the Propagation of Uncertainty Shocks

1 Introduction

The recent conflict in Ukraine and the Covid-19 pandemic have led to a sharp increase in many measures of uncertainty.1 When borrowers are subject to financial frictions, uncertainty shocks increase borrower defaults and lead to a contraction in the supply of loans and GDP.2 Credit markets play a crucial role in understanding the transmission of uncertainty shocks from borrowers to the economy. Structural changes in credit markets can affect how these shocks are transmitted. In this paper, I study how changes in banking competition, such as the recent fall in competition in the US banking sector, affect the propagation of uncertainty shocks.

The U.S. banking sector is highly concentrated. Since 2000, there has been a decrease in the number of commercial banks and an increase in bank asset concentration. In 2020, there were half as many commercial banks as there were in 2000 and the share of assets held by the three largest banks rose from 21% to 35%.3

In this paper, I provide empirical evidence on the correlation between the causal impact of uncertainty shocks on real output growth and the level of competition in the banking sector. I use disaster shocks such as natural disasters, terrorist attacks, political coups and revolutions that occurred in 44 countries between 2000Q1 and 2020Q1 as instruments for changes in first and second moments. My findings demonstrate that second moment shocks have a more severe impact on output growth when banking competition is lower.

To study the impact of banking competition on business cycle fluctuations and the effect of the recent decline in competition on the transmission of uncertainty shocks, I develop a New

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1Caldara et al. (2022), Ferrara et al. (2022) and Anayi et al. (2022) document an increase in uncertainty after the Russian invasion of Ukraine. Altig et al. (2020) and Baker et al. (2020) document an increase in uncertainty triggered by the Covid-19 pandemic.

2See for example Christiano et al. (2014), Caldara et al. (2016) and Alessandri and Muntaz (2019).

3See Figure 2.A.1 in Appendix A.
Keynesian business cycle model with financial frictions and imperfect competition in the banking sector. The main feature of this model is that bankers compete à la Cournot to provide loans to entrepreneurs. In this economy, there are $N$ bankers who invest their equity and deposits in loans to entrepreneurs. Entrepreneurs own and maintain physical capital but have insufficient net worth. They borrow from bankers to buy capital goods. Bankers choose optimally their loan supply internalizing loan demand and borrower default probability.

Entrepreneurs face both idiosyncratic and aggregate shocks. Idiosyncratic shocks result in heterogeneous returns on entrepreneurs capital stock. In some cases, the realized return may be insufficient to repay loans, leading to default. The cross-sectional dispersion of idiosyncratic shocks defines the level of uncertainty in the economy. As uncertainty increases, the probability of low returns and subsequent default rises. Financial frictions cause banks to respond to heightened uncertainty by reducing credit supply. This constrains entrepreneurs ability to acquire capital and results in decreased investment and a contraction of output.

The model is developed in two stages. In the first stage, I study the choices of entrepreneurs in a partial equilibrium framework. This allows me to present the first channel through which competition within the banking sector can influence the transmission of uncertainty shocks. Bankers in less competitive banking sectors use their higher market power to charge higher borrowing rates to borrowers. I show that as borrowing rates increase, so does risk-taking and the probability of default among entrepreneurs. Moreover, when entrepreneurs take on more risk, an increase in uncertainty leads to a larger rise in their default rate. This channel is called risk-shifting effect.

In the second stage, I incorporate the entrepreneurial sector in a calibrated general equilibrium model with an oligopolistic banking sector. This introduces a second channel through which bankers market power affects their response to shocks. Specifically, as bankers’ market power increases, bankers become less likely to pass shocks on to their borrowers. An uncertainty shock increases the number of non-performing loans and the monitoring costs incurred by bankers. In response to these increased costs, bankers decrease their loan supply. However, the size of this decrease is smaller for bankers with greater market power. I call this channel the pass-through effect.

The impact of banking competition on the transmission of uncertainty shocks is complex due to the presence of two opposing channels. To determine which channel dominates, I calibrate the general equilibrium model to match several US credit market statistics. Then, I study the implications of changes in competition resulting from variations in the number of competitors and the rise of a few dominant bankers.

When banking competition decreases due to a reduction in the number of bankers, the risk-shifting effect causes a stronger response in the default rate of entrepreneurs following uncertainty shocks. Bankers respond by reducing their loan supply more substantially and entrepreneurs face a larger contraction in their financial resources. This results in a larger credit crunch, causing investment to fall more and leading to a greater contraction in GDP. As a result, the risk-shifting
effect is stronger than the pass-through effect, and uncertainty shocks result in larger business cycle fluctuations when competition is lower. By calibrating the fall in competition to the increase in banking concentration in the US over the last 20 years, I find that a one-standard-deviation uncertainty shock implies a fall in GDP that is 0.1 percentage points larger.

In an extension of the model, I investigate the implications of heterogeneity among bankers. I assume that bankers have different marginal costs of providing loans, which leads to differences in market shares. Bankers with lower intermediation costs can more easily provide loans to entrepreneurs and thus gain larger market shares. This results in a more concentrated banking sector and higher borrowing rates for entrepreneurs. In response to uncertainty shocks, smaller bankers reduce their loan offerings and increase their markups more than larger bankers due to the pass-through effect. However, heterogeneity has a limited impact on business cycle fluctuations.

Related literature. The paper contributes to the literature on imperfect competition in the banking industry, financial frictions, uncertainty shocks and the role of banking competition in the transmission of shocks. My main contribution is connecting the literature on financial frictions, uncertainty shocks and the market structure of the banking industry. Specifically, building on the existing literature on uncertainty shocks and financial frictions, the paper examines how the impact of uncertainty shocks is affected by the market structure of the banking industry.

Imperfect competition in the banking industry. This paper builds on the extensive theoretical literature on imperfect competition in the banking industry. Boyd and de Nicoló (2005) identify the risk-shifting effect by developing a static model of imperfect competition in the banking industry. They find that less competitive banking sectors charge higher borrowing rates but have riskier portfolios because borrowers optimally respond to higher borrowing rates by taking on more risk. However, Martinez-Miera and Repullo (2010) and Hakenes and Schnabel (2011) respectively find that less competitive banking sectors have larger buffers against non-performing loans due to their larger profits and an incentive to reduce portfolio risk to protect their charter value. As a result, the relationship between banking competition and financial stability may be nonlinear.

The primary contribution of my work to this literature is the development of a DSGE model that incorporates the channel identified by Boyd and de Nicoló (2005). This channel is supported by the empirical evidence of Schaeck and Cihák (2014), Akins et al. (2016) and Berger et al. (2017). Furthermore, I introduce a novel channel, the pass-through effect.

Financial frictions. The existing literature on financial frictions has studied the implications of such frictions on the transmission of shocks, often through the assumption of costly state verification or agency problems. Notable examples of studies that have introduced financial frictions through costly state verification frameworks are Townsend (1979), Carlstrom and Fuerst (1997), Bernanke et al. (1999), Christiano et al. (2014) and Clerc et al. (2018). On the other
CHAPTER 2. IMPERFECT BANKING COMPETITION AND UNCERTAINTY

hand, Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) introduced financial frictions by adopting agency problems. Similarly to Kühl (2017), my paper combines the two approaches. In the model there is an agency problem because entrepreneurs can divert part of their assets after borrowing from banks. At the same time, banks have to pay a monitoring cost in order to observe the entrepreneur’s realized return.

My paper contributes to the existing literature on financial frictions by introducing imperfect banking competition in the banking sector. Differently from previous studies, I assume that loans are provided to entrepreneurs by an oligopolistic banking sector. This assumption implies that bankers charge a markup on the borrowing rate as observed by Corbae and D’Erasmo (2021). The introduction of imperfect banking competition in this economy creates an additional financial friction due to its impact on the borrowing rate.

**Uncertainty shocks.** The literature on uncertainty shocks suggests that uncertainty shocks play an important role in driving business cycle fluctuations, as demonstrated by numerous theoretical and empirical papers such as Bloom (2009), Christiano et al. (2014), Caldara et al. (2016), Basu and Bundick (2017), Alessandri and Mummert (2019) and Baker et al. (forthcoming). In the literature the consensus is that uncertainty shocks have important negative effects on output.

My contribution to this literature is twofold. First, following the work of Baker et al. (forthcoming), I provide empirical evidence that uncertainty shocks have stronger negative effects on output when banking competition is lower.

Second, building on the work of Christiano et al. (2014), I contribute to the theoretical literature by developing a DSGE model that incorporates both financial frictions and imperfect banking competition. I use the model to study the implications of imperfect competition in the banking sector for the transmission of uncertainty shocks. Consistent with the empirical evidence, the model shows that uncertainty shocks have more severe contractionary effects on output when the banking sector is less competitive. The driving force is the risk-shifting effect, which makes borrowers more vulnerable when the banking sector is more concentrated. Consequently, an uncertainty shock leads to a greater increase in non-performing loans and a stronger cut in lending when the banking sector is less competitive. This further exacerbates the negative impact of uncertainty shocks.

**Role of banking competition in the transmission of shocks.** This paper contributes to the macro-finance literature on the transmission of shocks through the banking sector, specifically focusing on the role of banking competition.

Prior studies, including Scharfstein and Sunderam (2016), Gödl-Hanisch (2022), and Cuciniello and Signoretti (2018), investigate the implications of imperfect competition in the banking sector for the transmission of monetary policy shocks. However, these studies have produced mixed conclusions. Specifically, while Scharfstein and Sunderam (2016) find that high concentration in the U.S. banking sector leads to lower transmission of monetary policy shocks, which is consistent with
their model of Cournot competition, the models of monopolistic competition in the banking sector developed by Gödl-Hanisch (2022) and Cuciniello and Signoretti (2018) suggest that monetary policy shocks have stronger effects when competition is lower. The latter finding is in line with the empirical evidence presented by Gödl-Hanisch (2022).

Other studies focused on other shocks. Jamilov and Monacelli (2021) develop a quantitative macroeconomic model with heterogeneous monopolistic financial intermediaries and study how banking competition affects the transmission of a capital quality shock. They find that credit market power decreases the impact of capital quality shocks. Villa (2020) builds a model where banks compete à la Cournot for loans and deposits, and argues that a sudden rise in the aggregate firms default probability has stronger negative effects when banking competition is lower.

My contribution to this literature is twofold. First, I introduce a new propagation channel in this set of models, the risk-shifting effect. Second, I study the propagation mechanism of a different shock, an uncertainty shock.

**Outline.** The paper is structured as follows. In Section 2 I provide empirical evidence on the effect of banking competition for the propagation of uncertainty shocks. Section 3 outlines the borrower side of the model and introduces the risk-shifting effect in a partial equilibrium framework. Section 4 presents the general equilibrium model. Section 5 displays the calibration and the results of the quantitative model. In this section I show quantitatively how the level of competition affects the transmission of uncertainty shocks. Finally, Section 6 concludes.

# 2 Empirical evidence

In this section I employ a panel dataset of 44 countries between 2000Q1 and 2020Q1 to empirically investigate the impact of banking competition on the transmission of uncertainty shocks. The section is structured as follows: Section 2.1 describes the data and Section 2.2 describes the regression model and the results. Appendix B.1 provides further information on the dataset and Appendix B.2 presents the robustness tests.

## 2.1 Data description

In my analysis I use data from 44 countries spanning the period 2000Q1-2020Q1. For each country I collect quarterly data on real GDP growth, first and second moments of national stock market returns, disaster shocks and yearly data on banking concentration. Real GDP growth is obtained from the International Financial Statistics of IMF or, if not available, from OECD. Data on first and second moments and disaster shocks are obtained from Baker et al. (forthcoming). Finally, banking concentration is obtained from the World Bank.

The first moment of stock market returns is the return of the broadest national index, while

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4The countries used in this analysis are listed in Table 2.B.1 in Appendix B.1.
the second moment is the logarithm of the quarterly standard deviations of daily stock returns. I use the second moment measure as a proxy for uncertainty.

The disaster shocks considered in this analysis include four types of events: natural disasters, terrorist attacks, coups and revolutions. For each category, a value of one is assigned if a disaster shock has occurred. To generate the final indexes, the events are weighted by the increase in media coverage during the 15-days period following the shock compared to the 15-days period preceding the event. Media coverage is defined by the number of articles published in English-language newspapers based in the United States that mention the affected country.

Banking competition is proxied by the 3-bank asset concentration ratio which is defined as the assets of the three largest commercial banks as a share of total commercial banking assets.\(^5\) This information is available only on annual basis. In this section the 3-bank asset concentration ratio is linearly interpolated to obtain a quarterly measure.\(^6\) A more concentrated banking sector indicates lower banking competition.

Descriptive statistics for the dataset can be found in Table 2.B.2 of Appendix B.1.

### 2.2 Banking competition and the impact of uncertainty shocks.

In this section I describe the regression model and I report the empirical results. In section 2.2.1 I describe the regression model and in 2.2.2 I present the results.

#### 2.2.1 Regression model

In order to estimate the effect of an increase in uncertainty on output growth, and to investigate how the level of banking competition affects the impact of uncertainty shocks, I estimate the following regression model

\[
y_{i,t+h} = \alpha_i + \tau_t + \beta^R \tilde{R}_{i,t} + \beta^V \tilde{V}_{i,t} + \beta^C \tilde{C}_{i,t} + \beta^{RC} \tilde{R}_{i,t} \tilde{C}_{i,t} + \beta^{VC} \tilde{V}_{i,t} \tilde{C}_{i,t} + \epsilon_{i,t}.\]

where \(y_{i,t+h}\) is the growth rate of real GDP from period \(t-1\) to period \(t+h\) for country \(i\), \(\alpha_i\) captures country fixed effects, \(\tau_t\) captures time fixed effects, \(\tilde{R}_{i,t}\) is the country demeaned measure of first moment of national stock market returns, \(\tilde{V}_{i,t}\) is the country demeaned measure of uncertainty and \(\tilde{C}_{i,t}\) is the country demeaned 3-bank asset concentration ratio.

This model extends the one proposed by Baker et al. (forthcoming) by adding banking concentration and interactions terms between banking concentration and and first and second moments. The interactions are included to isolate the effect of concentration on the impact of first and second moment shocks. Additionally, the model controls for non-linear effects of country characteristics by demeaning the variables at the country level.

\(^5\)As shown in Appendix B.2, the results are similar using the 5-bank asset concentration level

\(^6\)The results hold also keeping the level of concentration constant within a year. Appendix B.2 shows the results of this robustness test.
The coefficients $\beta^V$ and $\beta^{VC}$ measure the impact of an increase in uncertainty on real output growth. Specifically, $\beta^V$ captures the impact of an uncertainty shock on output growth when banking concentration is at the country mean, while $\beta^{VC}$ captures how the impact of uncertainty shocks varies with banking concentration. If $\beta^{VC}$ is negative, an increase in uncertainty has a more severe negative effect on output growth when concentration is higher.

Similarly to Baker et al. (forthcoming), I instrument first and second moment variables and their interaction with concentration using disaster shocks. This instrumental variable approach allows me to study the causal impact of second moment shocks on output growth and how it is correlated with the level of competition in the banking sector. Furthermore, because of the media weighting of the disaster shocks the regression gives higher weight to more important shocks.

As in Baker et al. (forthcoming) there is a potential issue with this identification strategy. The stock market level and volatility variables proxy for different channels through which disaster shocks have economic impact. The underlying exclusion restriction is that these effects impact economic activity only through shifts in the first and second moments of stock returns.

### 2.2.2 Results

Figure 2.1 shows the impact of a one-standard deviation uncertainty shock on real output growth at different levels of banking concentration. The blue line depicts the impulse response of output growth when concentration is at the country average and the blue dashed lines are the 90% confidence interval. The figure reveals a significant negative effect of an uncertainty shock on output growth.

The yellow line represents the impulse response of output growth when concentration is one standard deviation above the country average and the yellow dashed lines are the 90% confidence interval. In this case the fall in output growth is stronger and significant for a longer period.

Figure 2.2 plots the difference in the output growth responses between the average banking concentration specification (blue line in Figure 2.1) and the high banking concentration specification (yellow line in Figure 2.1). The graph shows that the decline in output growth is significantly more pronounced in countries with higher banking concentration.

The results shown in this section are robust to variations in the measure of concentration. In particular, the findings hold when keeping the level of concentration constant within a year, using the 5-bank asset concentration instead of the 3-bank asset concentration and controlling for endogeneity by replacing concentration with its lag. Additionally, the results hold even after removing the 2009 Global Recession from the sample. The last robustness test shows that the findings are not driven by the impact of the global recession that occurred in that year. The robustness tests are displayed in Appendix B.2.

In the following sections, I develop a general equilibrium model that replicates the empirical

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7 The instruments used are the disaster shocks and their interaction with with demeaned concentration.

8 A standard deviation corresponds to 10.21 percentage points.
Figure 2.1: Response of output growth to a one-standard deviation uncertainty shock.

Notes. The graph shows the responses of output growth to a one-standard deviation uncertainty shock for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation higher than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.
Figure 2.2: Effect of competition on output growth response to an uncertainty shock.

Notes. The graph shows the difference between the two specifications plotted in Figure 2.1. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.
findings presented in this section. This is accomplished through a two-step process. First, I introduce the entrepreneurial sector in a partial equilibrium framework. Next, I incorporate the banking sector and integrate the credit market into a standard DSGE model. The model is calibrated to reflect the US economy and used to examine how banking competition affects business cycle fluctuations and to assess the impact of the recent decline in banking competition on the transmission of uncertainty shocks.

3 Entrepreneurial sector and risk-shifting effect

In this section I introduce and analyze the entrepreneurial sector in a partial equilibrium setup and I characterize the risk-shifting effect, which is at the heart of the results shown in the paper.

The entrepreneurial sector is modeled similarly to Clerc et al. (2018). Specifically, there is a continuum of risk-neutral entrepreneurs, each of them is indexed by \( j \in (0, 1) \). Each entrepreneur lives for two consecutive periods. Every entrepreneur born at time \( t \) has financial resources given by inherited wealth from the previous generation of entrepreneurs \( n_t^{E,j} \) and loans \( b_t^j \) from the banking sector. Entrepreneurs use their financial resources to buy capital goods from capital good producers. The purchased capital goods are then rented out to final goods producers.

Entrepreneurs born at time \( t \) derive utility from donating a part of their terminal wealth in the form of dividends to the households \( c_{t+1}^{E,j} \), and the rest to the next generation of entrepreneurs as retained earnings, according to the utility function \((c_{t+1}^{E,j})^{\chi^E}(n_{t+1}^{E,j})^{1-\chi^E}\). At time \( t + 1 \), the maximization problem for the entrepreneur born at time \( t \) is

\[
\max_{c_{t+1}^{E,j}, n_{t+1}^{E,j}} (c_{t+1}^{E,j})^{\chi^E}(n_{t+1}^{E,j})^{1-\chi^E},
\]

subject to the budget constraint

\[
c_{t+1}^{E,j} + n_{t+1}^{E,j} \leq W_{t+1}^{E,j},
\]

where \( W_{t+1}^{E,j} \) is the terminal wealth of entrepreneur \( j \) born at time \( t \).

The first order conditions lead to the dividend payment rule \( c_{t+1}^{E,j} = \chi^E W_{t+1}^{E,j} \) and the earning retention rule \( n_{t+1}^{E,j} = (1 - \chi^E) W_{t+1}^{E,j} \).

At time \( t \), entrepreneurs maximize their expected future wealth by choosing how much capital \( K_t^j \) to buy at price \( q \) and how much to borrow \( b_{t+1}^j \) from the bankers

\[
\max_{K_t^j, b_{t+1}^j} E_t(W_{t+1}^{E,j}).
\]

The optimization problem is subject to the resource constraint

\[
q K_t^j - b_{t+1}^j = n_t^{E,j},
\] (2.1)
and to an incentive constraint. Similarly to Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Kühl (2017), there is a moral hazard problem: at time $t$ every entrepreneur can divert a fraction $\lambda$ of available funds. To ensure that entrepreneurs do not divert funds, the following incentive constraint must hold:

$$\lambda \frac{qK_{t+1}^j}{\Pi_{t+1}} \leq E_t(W_{t+1}^{E,j}).$$

Future wealth is defined as

$$W_{t+1}^{E,j} = \max \left[ \omega_{t+1}^j R_{t+1}^E qK_{t}^j - R_{t+1}^F b_{t+1}^j, 0 \right] \frac{\Pi_{t+1}}{\Pi_{t+1}}.$$ (2.3)

Future wealth is determined by the return from renting capital to final goods producers net of borrowing costs. The borrowing costs are determined by the borrowing rate $R_{t+1}^F$ times the amount borrowed. The return from renting capital is determined by the product of the amount of capital rented, its price $q$, the gross return per efficiency unit of capital $R_{t+1}^E$ and an idiosyncratic shock $\omega_{t+1}^j$. The return from lending capital and the borrowing costs are both discounted by the gross inflation rate $\Pi_{t+1} = P_{t+1}/P_t$. I assume that $R_{t+1}^E$ is a decreasing function in capital.

The idiosyncratic shock $\omega_{t+1}^j$ is a shock to the entrepreneur’s efficiency units of capital. This shock is assumed to be independently and identically distributed across entrepreneurs and to follow a log-normal distribution with mean one and standard deviation $\sigma_t = \sigma_\zeta t$. The cumulative distribution function and the probability density function of the idiosyncratic shock are denoted by $F(\cdot)$ and $f(\cdot)$, respectively. Uncertainty is defined as $\sigma_t$, while $\zeta_t$ represents an uncertainty shock that follows an AR(1) process

$$\ln \zeta_t = \rho \ln \zeta_{t-1} + \varepsilon_t,$$ (2.4)

where $0 < \rho < 1$ and $\sigma_{\varepsilon}^2$ is the standard deviation of the iid shock $\varepsilon_t$.

Entrepreneurs and bankers enter into a financial contract where the loan repayment depends on the realization of a random productivity shock $\omega_{t+1}^j$. If the shock is above a default cutoff $\bar{\omega}_{t+1}^j$, the entrepreneur has enough resources to pay the bankers $R_{t+1}^F b_{t+1}^j$, otherwise the entrepreneur defaults. The default cutoff is given by

$$\bar{\omega}_{t+1}^j = \frac{R_{t+1}^F b_{t+1}^j}{R_{t+1}^E qK_{t}^j}.$$ (2.5)

In contrast to Bernanke et al. (1999), the default cutoff $\bar{\omega}_{t+1}^j$ varies with the realization of $R_{t+1}^E$. 

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9Note that since bankers make positive profits, the financial contract cannot be derived using bankers’ zero profit condition as in Bernanke et al. (1999). As shown in this section, the incentive constraint pins down the capital demand.

10Note that in Section 4 the price of capital will be determined by supply and demand of capital and will not be constant.

11Note that entrepreneurs choose their probability of default by choosing $K_{t}^j$ and $b_{t+1}^j$. 
The probability of default of an entrepreneur is

\[ F_{t+1}^j = F(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} f(\omega_{t+1}^j) d\omega_{t+1}^j = \Phi \left( \frac{\log(\bar{\omega}_{t+1}^j) + 0.5\sigma_{t+1}^2}{\sigma_{t+1}} \right). \] (2.6)

In case of default, the entrepreneur obtains nothing and the bankers must pay a monitoring cost that is discussed more in detail in Section 4.

Appendix C.1.1 proves that the incentive constraint is binding in an active credit market. Therefore, the loan demand and the demand for capital are implicitly determined by the incentive participation constraint

\[ (1 - \Gamma(\bar{\omega}_{t+1}))R_{t+1}^E = \lambda, \] (2.7)

where \( 1 - \Gamma(\bar{\omega}_{t+1}) = 1 - \Gamma_{t+1}^j \) is the expected share of return that entrepreneurs retain after paying borrowing costs. Since all entrepreneurs face the same borrowing rate and expected return, the model can be aggregated by dropping the indices \( j \) from now on.

Propositions 1 and 2 demonstrate that an increase in the borrowing rate leads to a decrease in loan demand. This results in lower entrepreneurial leverage but higher default risk due to limited liability. Entrepreneurs reduce their demand for loans and leverage as borrowing rates rise. However, limited liability limits their potential losses in case of default, so they do not reduce their leverage enough to offset the higher borrowing rate. This results in a positive relationship between the borrowing rate and both the default rate of entrepreneurs and their default cutoff.

**Theorem 1.** Loan demand is a decreasing function of the loan rate.

**Theorem 2.** The default rate of the entrepreneurs and their default cutoff increase with the borrowing rate.

The proofs of Propositions 1 and 2 are provided in Appendix C.1.2.

Furthermore, Proposition 3 shows that an increase in uncertainty has a greater impact on entrepreneurial defaults when the default cutoff is higher. As entrepreneurs take more risk with a higher default cutoff, an increase in uncertainty has a larger impact on default rates.

**Theorem 3.** If \( R_{t+1}^E \leq \frac{\lambda}{1 - \Gamma(e^{-0.5\sigma})} \), an increase in uncertainty results in a larger rise in the default rate of entrepreneurs when the default cutoff is higher.

The proof of Proposition 3 is provided in Appendix C.1.2.\(^\text{12}\)

When the banking sector is less competitive, bankers tend to charge higher borrowing rates, leading to a higher default rate of entrepreneurs, as demonstrated in Proposition 2. Similarly to

\(^{12}\)The condition \( R_{t+1}^E \leq \frac{\lambda}{1 - \Gamma(e^{-0.5\sigma})} \) implies that \( F(\bar{\omega}_{t+1}) \leq \Phi(-1) \approx 0.1587 \). The condition of Proposition 3 is satisfied in the general equilibrium model.
Martinez-Miera and Repullo (2010), I call the effect of lower banking competition on borrower risk-taking, the risk-shifting effect.\(^\text{13}\)

4 General equilibrium

In this section, I will provide an overview of the remaining components of the model. The credit market is a crucial element for analyzing the transmission of uncertainty shocks. In Section 3, I characterized the credit demand of entrepreneurs and the risk-shifting effect. Here, I introduce the supply of credit by bankers and the pass-through effect.

Moreover, this section describes the remaining parts of the model that are standard. Intermediate goods producers utilize capital and labor to produce intermediate goods, which are then purchased by final goods producers who bundle them together to produce the final good. Finally, the central bank adjusts the policy rate according to a Taylor rule.

4.1 Bankers

There is a fixed number of bankers \(N\) competing in a Cournot fashion for loans. Each banker is indexed by \(i\) and lives across two consecutive periods. Bankers born at time \(t\) have equity in the form of inherited wealth from the previous generation of bankers \(n^{F,i}_t\) and borrow deposits \(d^i_t\) from households. They use these resources to provide loans to entrepreneurs.

At time \(t + 1\) bankers derive utility by donating part of their final wealth to households in the form of dividends \(c^{F,i}_{t+1}\) and by leaving the rest as retained earnings to the next generation of bankers according to the utility function \((c^{F,i}_{t+1})^{\chi^F} (n^{F,i}_{t+1})^{1-\chi^F}\). Therefore, the maximization problem of each banker at time \(t + 1\) is given by

\[
\max_{c^{F,i}_{t+1}, n^{F,i}_{t+1}} (c^{F,i}_{t+1})^{\chi^F} (n^{F,i}_{t+1})^{1-\chi^F},
\]

subject to the resource constraint

\[
c^{F,i}_{t+1} + n^{F,i}_{t+1} \leq W^{F,i}_{t+1},
\]

where \(W^{F,i}_{t+1}\) is the final wealth of the banker \(i\) born at time \(t\).

The first order conditions lead to the dividend payment rule

\[
c^{F,i}_{t+1} = \chi^F W^{F,i}_{t+1}, \quad (2.8)
\]

and the earning retention rule

\[
n^{F,i}_{t+1} = (1 - \chi^F) W^{F,i}_{t+1}. \quad (2.9)
\]

\(^{13}\)Similarly to Martinez-Miera and Repullo (2010), entrepreneurs in this model choose a higher default probability as their borrowing rate increases. In this case, the probability of default is determined through the choice of leverage.
The future wealth of each banker is
\[ W_{t+1}^{F,i} = \frac{\tilde{R}_{t+1}(b_t)b_t^i - R^D_t d_t^i - \gamma^i b_t^i}{\Pi_{t+1}}. \]

Bankers’ future wealth is determined by the return they earn from lending to entrepreneurs net of deposit and intermediation costs. The return from lending is calculated as the amount of loans multiplied by the return per unit of loans \( \tilde{R}_{t+1} \). The cost of deposits is given by the deposit rate \( R^D_t \) multiplied by the amount of deposits. In addition, each banker pays a per loan intermediation cost \( \gamma^i \), which can vary across bankers. All of these factors are discounted by the gross inflation rate \( \Pi_{t+1} \).

The return per unit of loans is
\[ \tilde{R}_{t+1} = (1 - F_{t+1})R^F_{t+1} + (1 - \xi) \int_0^\omega_{t+1} \omega_{t+1} f(\omega_{t+1})d\omega_{t+1} \frac{R^E_{t+1}q_t K_t}{b_t}. \] (2.10)

The first term of Equation 2.10 represents the return from performing loans, while the second term represents the return from non-performing loans. When a loan defaults bankers incur a monitoring cost \( \xi \) to observe the entrepreneur’s realized return on capital. This cost is a proportion of the realized gross payoff to the entrepreneurs. Note that all bankers receive the same return from non-performing loans because they have equal seniority.\(^{14}\)

At time \( t \), each banker chooses how much to lend to entrepreneurs and borrow from households, taking into account the decisions of other bankers. The objective is to maximize future wealth
\[ \max_{\{b_t^i,d_t^i\}} \frac{\tilde{R}_{t+1}(b_t)b_t^i - R^D_t d_t^i - \gamma^i b_t^i}{\Pi_{t+1}}, \]
subject to the balance sheet constraint
\[ n_{t+1}^{F,i} + d_t^i \geq b_t^i, \] (2.11)
and the loan demand (2.7) due to imperfect competition. The first order condition of the maximization problem, after substituting the balance sheet constraint, is
\[ \frac{\partial \tilde{R}_{t+1}}{\partial b_t} b_t^i + \tilde{R}_{t+1} - R^D_t - \gamma^i = 0. \]

It’s worth noting that, due to imperfect competition, the optimal choices of each banker depend on the impact of their decisions on the return they receive from lending to entrepreneurs. The impact of bankers’ decisions depends on the slope of the demand curve and on the level of competition in the banking sector.

\(^{14}\)Note also that bankers grant loans to every entrepreneur since they have the same level of risk
The level of competition not only affects bankers’ profits, but also the extent to which they pass shocks through to borrowers. In less competitive markets, bankers have higher profits, but pass shocks through to borrowers by a lesser extent, as profits absorb some of the impact. An uncertainty shock increases non-performing loans and monitoring costs for bankers. More competitive bankers decrease their loan supply more than less competitive bankers, due to lower market power. I define the pass-through effect the effect of banking competition on the extent to which bankers pass shocks through to borrowers.

If every banker has the same intermediation cost, the equilibrium is symmetric and the first order conditions of the bankers can be aggregated to

\[
\frac{\partial \tilde{R}_{t+1}}{\partial b_t} b_t + \tilde{R}_{t+1} - \tilde{R}_D - \gamma = 0.
\]

In this case, the level of competition increases with the number of bankers. As the number of bankers increases, the impact of the decisions of a single banker on the return bankers receive from lending to entrepreneurs decreases leading to a fall in the market power of bankers.

4.2 Rest of the model

The rest of the model follows a standard New Keynesian framework. Households maximize their utility choosing consumption, labor supply deposits supply. The production sector comprises final, intermediate, and capital goods producers. Final goods producers are perfectly competitive and use intermediate goods to produce consumption bundles using a constant-elasticity-of-substitution technology. Final goods are sold to households and to capital producers. Intermediate goods producers use capital and labor to produce intermediate goods with a Cobb-Douglas technology, setting prices subject to quadratic adjustment costs. This leads to a standard New Keynesian Phillips curve. Capital goods producers buy the final good, convert it into capital, and sell it to entrepreneurs. The model is closed by a central bank that sets the policy rate following a monetary policy rule.

4.2.1 Gross return on capital

The price of capital \(q_t\) is determined by the equilibrium of demand and supply of capital.

The gross return on capital is

\[
R_t^E = r_t^K + (1 - \delta) q_t \frac{\Pi_t}{q_{t-1}}.
\]

The gross return on capital is given by the sum of the real rental rate on capital \(r_t^K\) and the real capital gains net of depreciation \((1 - \delta) q_t\), divided by the real price per unit of capital in period \(t-1\). Finally, the return is expressed in nominal terms and multiplied by the inflation rate.
4.2.2 Households

Households are infinitely lived and maximize their expected lifetime utility,

\[ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \ln c_t - \frac{l_t^{1+\eta}}{1+\eta} \right), \tag{2.12} \]

where \( \beta \in (0, 1) \) is the discount factor, \( c_t \) is consumption, \( l_t \) is labor supply, \( \varphi > 0 \) is the relative weight on labor disutility and \( \eta \geq 0 \) is the inverse Frisch elasticity of labor supply. Households choose consumption, labor supply and deposit supply to maximize (3.33) subject to the budget constraint,

\[ c_t + d_t \leq w_t l_t + \frac{R_{it}^D d_{t-1}}{\Pi_t} + \Xi^K_t + \sum_{i=1}^{N} \chi^F W_t^{F,i} + \chi^E W_t^E + \Xi^P_t, \tag{2.13} \]

where \( w_t \) is the real wage, \( R_{it}^D \) is the gross interest rate on deposits paid in period \( t \), \( \Xi^K_t \) and \( \Xi^P_t \) are profits earned by capital goods producers and intermediate goods producers, respectively, and \( \sum_{i=1}^{N} \chi^F W_t^{F,i} \) and \( \chi^E W_t^E \) are the dividends received by households from bankers and entrepreneurs respectively. The first order conditions of the optimization problem lead to a labor supply equation,

\[ w_t = \varphi l_t^{\eta} / \Lambda_t, \] and an Euler equation,

\[ 1 = \mathbb{E}_t \{ \beta_{t,t+1} R_{it+1}^D / \Pi_{t+1} \}, \]

where \( \beta_{t,t+1} = \beta^s \Lambda_{t+s} / \Lambda_t \) is the household’s stochastic discount factor and \( \Lambda_t = 1/c_t \) is the Lagrange multiplier on the budget constraint.

4.2.3 Final goods producers

Final goods producers bundle the intermediate goods \( Y_{it} \), with \( i \in (0, 1) \), taking as given their price \( P_{it} \), and sell the output \( Y_t \) at the competitive price \( P_t \). Final goods producers choose the amount of inputs \( Y_{it} \) that maximizes profits \( P_t Y_t - \int_0^1 Y_{it} P_{it} d_i \), subject to the production function

\[ Y_t = \left( \int_0^1 Y_{it}^{(\varepsilon-1)/\varepsilon} d_i \right)^{\varepsilon/(\varepsilon-1)}, \]

where \( \varepsilon > 1 \) is the elasticity of substitution between intermediate goods. The resulting demand for intermediate good \( i \) is \( Y_{it}^d = (P_{it}/P_t)^{-\varepsilon} Y_t \). The price of final output, which is interpreted as the price index, is given by \( P_t = (\int_0^1 P_{it}^{1-\varepsilon} d_i)^{1/(1-\varepsilon)} \). In a symmetric equilibrium, the price of a variety and the price index coincide, \( P_t = P_{it} \).

4.2.4 Intermediate goods producers

Intermediate goods producers use capital and labor to produce intermediate goods according to a Cobb-Douglas production function. Because of the assumption of constant returns to scale the production function can be aggregated. Each producer produces a differentiated good using \( Y_{it} = A_t K_{it-1}^\alpha l_{it}^{1-\alpha} \), where \( \alpha \in (0, 1) \) is the capital share in production, \( A_t \) is aggregate technology, \( K_{it-1} \) is capital and \( l_{it} \) is labor. Intermediate goods producers choose the amount of inputs to maximize profits given by \( P_{it} Y_{it}/P_t - r_t^K K_{it-1} - w_t l_{it} \), where the real rental rate on capital \( r_t^K \) and the real wage \( w_t \) are taken as given, subject to the technological constraint and the demand
4. GENERAL EQUILIBRIUM

The optimization problem results in a labor demand and a capital demand that are
\[ w_t l_t = (1 - \alpha) s_t Y_t \] and \( r_t K_{t-1} = \alpha s_t Y_t \), respectively, where the Lagrange multiplier on the
demand constraint, \( s_t \), represents real marginal costs. By combining the two demands, it is
possible to obtain an expression for real marginal costs that is symmetric across producers,

\[
s_t = \frac{w_t^{1-\alpha}(r_t^K)^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} A_t. \tag{2.14}
\]

Firm \( i \) sets an optimal path for its product price \( P_{it} \) to maximize the present discounted value
of future profits, subject to the demand constraint and to price adjustment costs,

\[
\mathbb{E}_t \sum_{s=0}^{\infty} \beta_t \beta_{t+s+1} \left[ \frac{P_{it+s} Y_{it+s}}{P_{t+s}} - \frac{\kappa_p}{2} \left( \frac{P_{it+s}}{P_{it+s-1}} - 1 \right)^2 Y_{it+s} + s_{it+s} (Y_{it+s} - Y_{it+s}^d) \right]. \tag{2.15}
\]

Price adjustment costs are given by the second term in square brackets in (3.36); they depend
on firm revenues and on last period’s aggregate inflation rate. The parameter \( \kappa_p > 0 \) scales the
price adjustment costs. Under symmetry, all firms produce the same amount of output, and the
firm’s price \( P_{it} \) equals the aggregate price level \( P_t \), such that the price setting condition is

\[
\kappa_p \Pi_t (\Pi_t - 1) = \varepsilon s_t - (\varepsilon - 1) + \kappa_p \mathbb{E}_t \left\{ \beta_{t,t+1} \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \right\}. \tag{2.16}
\]

Under symmetry across intermediate goods producers, profits (in real terms) are
\( \Xi_t^P = Y_t - r_t^K K_{t-1} - w_t l_t - 0.5 \cdot \kappa_p (\Pi_t - 1)^2 Y_t \).

4.2.5 Capital goods production

Capital goods producers choose paths for investment \( I_t \) to maximize the expected present value of
future profits given by \( \mathbb{E}_t \sum_{s=0}^{\infty} \beta_t \beta_{t+s} [g_{t+s} I_{t+s} - (1 + g_{t+s}) I_{t+s}] \). The term \( g_t = 0.5 \cdot \kappa I_t I_{t-1} - 1)^2 \)
captures investment adjustment costs as in Christiano et al. (2014). Capital accumulation is
defined as

\[ I_t = K_t - (1 - \delta) K_{t-1}, \tag{2.17} \]

where \( \delta \in (0, 1) \) is the capital depreciation rate. The maximization problem leads to the optimality
condition for investment

\[
1 = q_t - \frac{\kappa I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa I \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \mathbb{E}_t \left\{ \beta_{t,t+1} \kappa I \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\}. \tag{2.18}
\]

In period \( t \) the profits of capital producers in real terms are \( \Xi_t^K = q_t I_t - (1 + g_t) I_t \).
4.2.6 Central bank

I assume the central bank sets the policy rate according to a standard Taylor rule. The monetary policy rule depends on its own lag, inflation and GDP growth. The respective feedback coefficients are \( \tau_R \), \( \tau_\Pi \) and \( \tau_y \) such that:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\tau_R} \left[ \left( \frac{\Pi_t}{\Pi} \right)^{\tau_\Pi} \left( \frac{GDP_t}{GDP_{t-1}} \right)^{\tau_y} \right]^{1-\tau_R},
\]

where \( GDP \) is defined as output net of default costs.

Since the deposit rate is risk-free, the policy rate and the deposit rate are identical, \( R_t = R^D_t \).

4.2.7 Market clearing

The production of consumption goods must be equal to the sum of goods demanded by households, goods used for investment, resources lost when adjusting prices and investment, as well as resources lost in the recovery of funds associated with defaults and due to intermediation costs,

\[
Y_t = c_t + (1 + g_t) I_t + \frac{\kappa_p}{2} (\Pi_t - 1)^2 Y_t + \mu^E C^E_t \frac{R^E_t q_{t-1} K_{t-1}}{\Pi_t} + \sum_{i=1}^{N} \gamma^i b^i_t.
\]

Labor demand must equal labor supply

\[
(1 - \alpha) s_t Y_t / l_t = \phi_t l^H_t / \Lambda_t.
\]

4.3 Symmetric equilibrium

A symmetric equilibrium is a set of allocations \{\( l_t, K_t, I_t, c_t, Y_t, n^E_t, b_t, n^F_t, d_t \}\}_{t=0}^{\infty}, prices \{q_t, w_t, r^K_t, \Pi_t, s_t\}_{t=0}^{\infty} and rates of return \{R^F_t, R^E_t, R^D_t, \bar{R}_t, R_t\}_{t=0}^{\infty} for which given shocks to entrepreneurial uncertainty \{\varsigma_t\}_{t=0}^{\infty}:

- Entrepreneurs maximize expected future wealth,
- Producers and bankers maximize profits,
- Households maximize utility,
- The central bank sets the policy rate according to the Taylor rule
- All markets clear.
5 Results

This section presents the calibration of the general equilibrium model and discusses how the transmission of an uncertainty shock in this economy changes with the level of banking competition. First, I show the implications of a change in competition due to a change in the number of competitors. Second, I examine the implications of heterogeneity among bankers.

5.1 Calibration

Table 2.1 presents the parameter values used for calibrating the model to the period 2010Q1-2019Q4. The discount factor $\beta$ is chosen to match the average yearly Federal Funds Effective rate of 0.6%, while the capital share in production $\alpha$ and the depreciation rate of capital are the same as in Christiano et al. (2014). The fraction of resources lost due to entrepreneur defaults $\xi$ matches the charge-off rate on business loans. The dividend payout ratios of entrepreneurs $\chi^E$ and bankers $\chi^F$ are selected to match the leverage of non-financial corporate business and the ratio of banker equity over assets, respectively. The proportion of assets that can be diverted by entrepreneurs $\lambda$ is chosen to match the ratio between non-financial corporate business loans and GDP. The intermediation cost $\gamma$ matches the average markup of bankers used by Jamilov and Monacelli (2021). The number of bankers $N$ is chosen such that the 3-bank asset concentration ratio is 33%, which is close to the data (35.15%). The autocorrelation of the uncertainty shock $\rho$, and the inverse Frish labor elasticity $\eta$ are obtained from Christiano et al. (2014). The parameter that determines the substitutability between intermediate goods $\epsilon$ is taken from Christensen and Dib (2008) to match a markup of 1.2. The price adjustment cost is taken from Smets and Wouters (2007) and the investment adjustment cost is from Carlstrom et al. (2014). The weight on labor disutility is chosen to normalize labor supply to 1. For the coefficients of the Taylor rule, the conventional Taylor rule parameters are used as in Gertler and Karadi (2011). The smoothing parameter is set to 0.8, the coefficient of the Taylor rule for inflation is 1.5 and the coefficient for GDP growth is 0.5/4. In the following a period corresponds to a quarter.

5.2 Implications of a reduction in the number of bankers

In this section, I examine how the transmission of uncertainty shocks is affected by changes in banking competition resulting from variations in the number of bankers. Figure 2.3 illustrates the responses of important variables in the model to a one-standard deviation uncertainty shock for different levels of competition.\footnote{Note that the size of the shock is such that bankers' do not default.}

Consider first the light blue solid line, which corresponds to the baseline level of competition. An uncertainty shock raises the default rate of entrepreneurs by increasing the share of entrepreneurs with productivity below the default cutoff. This increases credit risk, leading bankers to reduce loan supply and increase the loan rate. The higher loan rate further increases the default...
Table 2.1: Calibration of the baseline model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9985</td>
<td>Federal funds rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in production</td>
<td>0.4</td>
<td>Christiano et al. (2014)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate capital</td>
<td>0.025</td>
<td>Christiano et al. (2014)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Entrepreneur bankruptcy cost</td>
<td>0.3519</td>
<td>Charge-Off Rate Business Loans</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Steady-state uncertainty</td>
<td>0.2541</td>
<td>Delinquency Rate Business loans</td>
</tr>
<tr>
<td>$\chi^E$</td>
<td>Dividend payout entrepreneurs</td>
<td>0.0812</td>
<td>Non-financial Corporate Business Leverage</td>
</tr>
<tr>
<td>$\chi^F$</td>
<td>Dividend payout bankers</td>
<td>0.3466</td>
<td>Banker equity ratio = 12%</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Proportion divertible assets entrepreneurs</td>
<td>0.8110</td>
<td>Non-financial Corporate Business Loans/GDP</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Banker intermediation cost</td>
<td>0.0431</td>
<td>Markup bankers</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of bankers</td>
<td>9</td>
<td>3-Bank asset concentration</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Autocorrelation risk shock</td>
<td>0.97</td>
<td>Christiano et al. (2014)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inverse Frisch labor elasticity</td>
<td>1</td>
<td>Christiano et al. (2014)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Substitutability between goods</td>
<td>6</td>
<td>Christensen and Dib (2008)</td>
</tr>
<tr>
<td>$\kappa_p$</td>
<td>Price adjustment cost</td>
<td>20</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>$\kappa_I$</td>
<td>Investment adjustment cost</td>
<td>2.43</td>
<td>Carlstrom et al. (2014)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Weight on labor disutility</td>
<td>0.5718</td>
<td>Labor supply = $l = 1$</td>
</tr>
<tr>
<td>$\tau_R$</td>
<td>Coeff. TR for lag policy rate</td>
<td>0.8</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>$\tau_I$</td>
<td>Coeff. TR for inflation</td>
<td>1.5</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>$\tau_y$</td>
<td>Coeff. TR for GDP</td>
<td>0.5/4</td>
<td>Gertler and Karadi (2011)</td>
</tr>
</tbody>
</table>

Notes. The table describes the calibration of the baseline model.

rate of entrepreneurs. Due to the spike in defaults, bankers equity falls and their leverage increases. With reduced loan supply, entrepreneurs have less resources to buy capital and investment falls. Due to the fall in investment, also GDP decreases and inflation falls: we observe a demand-driven downturn as in Christiano et al. (2014). Finally, the central bank reacts to the falls in output and inflation by cutting the policy rate.

Consider now the red dashed and the blue dot-dashed lines which corresponds to models with 2 and 100 bankers, respectively. The former represents a highly concentrated banking sector, while the latter represents a nearly perfectly competitive one.

As discussed in Section 3, less competitive banking sectors experience higher default rates among entrepreneurs and greater borrower risk-taking. Therefore, after an uncertainty shock, the default rate for entrepreneurs increases more in economies with less competitive banking sectors. Despite the lower pass-through, bankers in less competitive sectors reduce loan supply more due to the stronger rise in credit risk. However, the larger losses incurred by less competitive banking sectors due to the stronger rise in entrepreneurial defaults lead to a smaller fall in equity and smaller increase in leverage because less competitive bankers have larger equity buffers. The stronger rise in entrepreneurial defaults and the stronger fall in loan supply leads to a stronger fall in investment and GDP when competition is lower. Finally, because of the stronger recession, consumption, inflation and the deposit rate also fall by more.

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16 This is consistent with the empirical evidence of Berger et al. (2017)
Figure 2.3: Impulse responses to an uncertainty shock varying the number of bankers.

Notes. The graph shows how several variables in the model respond to a one-standard deviation uncertainty shock for different levels of competition. The light blue solid lines represent the baseline model’s impulse responses, while the red dashed lines show the impulse responses of an economy with a highly concentrated banking sector. The blue dot-dashed lines display the impulse responses of an economy with a banking sector that is nearly perfectly competitive.

5.3 The recent fall in banking competition

In this section I quantify the business cycle implications of the recent decline in banking competition in the United States. The number of commercial banks in the US has decreased by approximately 50% between 2000 and 2020, mainly due to bank mergers. This consolidation trend may have contributed to the increase in concentration and reduction in banking competition identified by Corbae and D’Erasmo (2021). To understand the implications of this trend, I use the model to analyze how the responses to uncertainty shocks have changed over the past two decades.

Specifically, in Figure 2.4, I compare the baseline impulse responses (light blue dashed line) with the impulse responses of a variant of the model in which I set the number of bankers to match the share of assets held by the three largest banks in 2000 (blue dot-dashed lines). The figure shows that after the recent fall in banking competition, the effects of uncertainty shocks on the US economy are stronger. Specifically, a standard deviation uncertainty shock implies an increase in the default rate of entrepreneurs that is more than a percentage point higher at its peak and a

---

17Labonte and Scott (2021) provides an analysis of this trend.
decrease in GDP that is 0.1 percentage stronger.

Figure 2.4: Comparison impulse responses: Impact of recent fall in banking competition.

Notes. The graph shows how several variables in the model respond to a one-standard deviation uncertainty shock for different levels of competition. The light blue solid lines represent the impulse responses of the baseline model, while the blue dot-dashed lines display the impulse responses of an economy in which the number of banks is set to match the share of assets held by the three largest banks in the US in 2000.

5.4 Heterogeneity in the banking sector

In this section, I investigate the implication of heterogeneity among bankers. I assume that bankers have different marginal costs of providing loans. Specifically, I assume that every banker inherits an intermediation cost $\gamma^i$ when it is born and passes it on to the next generation of bankers in the following period. Bankers that face lower intermediation costs find it easier to provide loans to entrepreneurs and, consequently, obtain larger market shares. As a result of the more concentrated banking sector, entrepreneurs face higher borrowing rates.

To generate a distribution of bankers with a few large and many small bankers, as in Li (2019), I assume that the first bankers draw $\gamma^i$ from a reverse bounded Pareto distribution. Further details on the distribution can be found in Appendix D.

The reverse bounded Pareto distribution is characterized by three parameters: the lower and upper bounds of the distribution and the shape parameter. Consistent with Li (2019), I set the shape parameter to 0.1. The lower and upper bounds are calibrated such that the sum of the resources lost due to intermediation costs is equal to the baseline model, and the standard
deviation in markups is equal to $\sigma^\mu$. These assumptions imply that as $\sigma^\mu$ increases, concentration in the banking sector increases, while competition decreases. In fact, the higher $\sigma^\mu$, the larger the differences in productivity across banks.

The model is solved 1000 times each time drawing a distribution of $\gamma^j$ for the first generations of bankers. After solving the model, I average across replications the steady-state variables and the impulse responses.

Figure 2.5 shows the steady-state effects of banker heterogeneity on banker variables. In this figure $\sigma^\mu = 0.7$. The left panel plots the average share of assets (light blue dashed line) and the average markup (red solid line) of the nine bankers in the model. Bankers are sorted from the most productive to the least productive.

The left panel of Figure 2.5 shows that because of higher productivity, bankers with a low intermediation cost have a larger market share than their less productive counterparts. As a result, they can charge higher markups, obtain more profits and accumulate more equity. As a result of these differences, smaller bankers tend to have higher leverage, as depicted in the right panel of Figure 2.5.

Figure 2.6 displays the average impulse responses of the model with heterogeneous bankers for various levels of $\sigma^\mu$. The results indicate that the rise of a few dominant bankers does not substantially impact the responses of aggregate variables since they are similar to the responses of the baseline model. This suggests that the empirical findings are not driven by the rise of a few dominant banks.

Figure 2.7 shows how bankers’ responses to an uncertainty shock vary according to their asset size. The red dashed lines represent the impulse responses of the most productive banker, while the light blue solid lines display the impulse responses of the median banker. The blue dot-dashed lines correspond to the impulse responses of the least productive banker. In this figure, $\sigma^\mu = 0.7$.

The impulse responses reveal that smaller bankers are more severely affected by uncertainty shocks. They reduce their loan supply to a greater extent and experience a more substantial decline in equity compared to larger bankers. Due to their lower market power, smaller bankers have a higher pass-through of shocks to borrowers and need to transfer more of the shock onto them. As a result, their markup increases by more.

6 Conclusion

The literature on uncertainty argues that uncertainty shocks play a crucial role in driving business cycles. In light of the recent decline in banking competition, I study how lower competition in the banking sector affects the propagation of uncertainty shocks.

Empirically, I find a negative correlation between the impact of uncertainty shocks on real output growth and banking sector competition. I construct a calibrated New Keynesian dynamic

\[\text{Credit markup} = \frac{(1-p^E)(R^F - 1)}{R^{\mu^\mu} - 1 + \gamma} - 1\] as in Corbae and D’Erasmo (2021)
stochastic general equilibrium model that incorporates financial frictions and imperfect competition in the banking sector to capture this result.

Banking competition can change due to mergers that reduce the number of competitors or an increase in market share concentration among a few bankers. With fewer competitors, bankers charge higher borrowing rates to entrepreneurs due to reduced competition. This increases borrowers risk-taking due to limited liability - a channel known as the risk-shifting effect.

An uncertainty shock increases entrepreneurial defaults to a greater extent in less competitive banking sectors due to increased risk-taking by entrepreneurs. This leads to a stronger increase in credit risk and a stronger reduction in bankers loan supply. As a result, investment and output fall more after an uncertainty shock in economies with less competitive banking sectors.

I also explore the implications of heterogeneity in productivity among bankers. Heterogeneity results in the concentration of market share among a few productive bankers. My results show that larger bankers have more market power and charge higher markups. They are also less affected by uncertainty shocks due to their higher market power, which results in a lower pass-through of shocks to borrowers.
Figure 2.6: Impulse response functions to an uncertainty shock - Heterogeneous bankers

Notes. The graph illustrates the responses of several variables in the model to a one-standard-deviation uncertainty shock at different levels of $\sigma^\mu$. The light blue solid lines represent the impulse responses of the baseline model, while the red dashed lines display the impulse responses of an economy where $\sigma^\mu$ is 0.4. The blue dot-dashed lines correspond to the impulse responses of an economy where $\sigma^\mu$ is 0.7.

However, the introduction of heterogeneity does not substantially impact the responses of aggregate variables. The responses of an economy with heterogeneous bankers are similar to those of the baseline model. This suggests that the empirical findings are driven by a reduction in the number of competitors rather than by the rise of a few dominant banks.

It would be interesting to investigate the impact of monetary policy on business cycle stabilization and how this varies with the level of banking competition. Previous literature has suggested a tradeoff between output and inflation stabilization, as stabilizing output tends to generate inflation. However, in this framework where uncertainty shocks impact inflation and output in the same direction, the tradeoff between these two objectives may be reduced. Nonetheless, the magnitude of this tradeoff may be influenced by competition, as banking competition affects the size of financial frictions. Therefore, it is important to investigate how the level of competition in the banking sector impacts the tradeoff between output and inflation stabilization.
Figure 2.7: Impulse response functions - Bankers variables

Notes. The graph illustrates the responses of several banker variables in the model to a one-standard-deviation uncertainty shock when $\sigma^u$ is 0.7. The red dashed lines represent the impulse responses of the most productive banker, while the light blue solid lines display the impulse responses of the banker with median productivity. The blue dot-dashed lines correspond to the impulse responses of the least productive banker.
Appendix A  Evolution of banking competition

Figure 2.A.1 shows the evolution of the number of commercial banks and the 3-Bank asset concentration for the United States. The number of banks is retrieved from FRED, the 3-Bank asset concentration is obtained from World Bank. The number of banks has been decreasing since 2000, while the 3-Bank asset concentration has been increasing. This suggests that banking competition has been falling in recent years.

Figure 2.A.1: Number of banks and bankers concentration

Notes. Sample period: January 2000 to January 2020. The number of banks is measured as the number of commercial banks (FRED). Bank concentration is measured as the assets of three largest commercial banks as a share of total commercial banking assets (World Bank).
Appendix B  Additional information empirical evidence

B.1  Data description

In this section, I provide additional details about the dataset used in the empirical analysis.

The disaster shocks are obtained from Baker et al. (forthcoming) and are available for 59 countries from 1970Q1 to 2020Q1. However, information regarding banking concentration is only available from 2000 limiting the sample period to 2000Q1 to 2020Q1.

The reduction in the sample period means that some countries did not experience any shock during the period from 2000Q1 to 2020Q1. As a result, these countries are dropped from the analysis. Additionally, countries with GDP data available only at a yearly frequency are also dropped. Finally, observations with a concentration level equal to 100% are removed from the sample.

Table 2.B.1 lists the countries used in the analysis.

Table 2.B.2 presents the descriptive statistics of the dataset. It is worth noting that compared to Baker et al. (forthcoming), the number of observations is smaller and fewer disaster shocks are available due to sample availability.

The disaster shocks are defined as follows:

Natural Disasters: Extreme weather events such as, droughts, earthquakes, insect infestations, pandemics, floods, extreme temperatures, avalanches, landslides, storms, volcanoes, fires, and hurricanes.

Terrorist Attacks: Bombings and other non-state-sponsored attacks.

Coups: Military action which results in the seizure of executive authority taken by an opposition group from within the government.

Revolutions: A violent uprising or revolution seeking to replace the government or substantially change the governance of a given region.

To construct the disaster shock variables, for each category, country, and quarter, the shock variable is set to 1 if there was at least one disaster shock of that category in that quarter. The weights of these shocks are determined by the increase in media coverage 15 days after the event compared to 15 days before the event.

The increase in media coverage is defined as the percentage increase in the number of articles related to the event that were published in English-language newspapers based in the United States, comparing the 15-day period after the event to the 15-day period before the event.

B.2  Robustness tests

This section presents the results of the robustness tests. The first test is reported in Appendix B.2.1, where banking concentration is kept constant within each year instead of being interpolated. The second test is reported in Appendix B.2.2 where the 5-Bank asset concentration ratio is used
Table 2.B.1: Countries

<table>
<thead>
<tr>
<th>Asia &amp; Pacific</th>
<th>Europe &amp; North America</th>
<th>LatAm &amp; Caribbean</th>
<th>MENA</th>
<th>SSAF</th>
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<tr>
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<td></td>
<td>United States</td>
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</tbody>
</table>

Notes. List of the countries used in the empirical analysis.

as a proxy for the level of banking competition. Appendix B.2.3 shows that the main results hold when controlling for concentration endogeneity by replacing concentration with its lag. Finally Appendix B.2.4 shows that the results hold even after removing the 2009 Global Recession from the sample.

B.2.1 Constant concentration within each year

In this section, I present the results of the first robustness test, where banking concentration is kept constant within each year instead of interpolating the level of concentration.

Figure 2.B.1 displays the effect of an exogenous one-standard-deviation increase in uncertainty on real output growth for two different levels of banking concentration. The blue line indicates the impulse response of output growth when banking concentration is at the country average, while the blue dashed lines represent the 90% confidence intervals. The figure shows that an exogenous
increase in uncertainty has no significant negative impact on output growth.

The yellow line represents the impulse response of output growth when banking concentration is one standard deviation above the country average. The yellow dashed lines represent the 90% confidence interval. In this case, the fall in output growth is stronger and significant.

Figure 2.B.2 depicts the difference in output growth responses between the average concentration and the high concentration specifications. It shows that the decline in output growth is significantly stronger when banking concentration is higher.

### B.2.2 5-bank asset concentration

In this section, I present the results of the second robustness test, where the level of banking competition is measured by the 5-Bank asset concentration ratio, the share of assets held by the five largest banks.

Figure 2.B.3 shows the effect of an exogenous one-standard-deviation increase in uncertainty on output growth for different levels of banking concentration. The blue line indicates the impulse response of output growth when banking concentration is at the country average, while the blue dashed lines represent the 90% confidence intervals. The figure shows that an exogenous increase in uncertainty has no significant negative impact on output growth.

The yellow line represents the impulse response of output growth when concentration is one standard deviation above the country average. The yellow dashed lines are the 90% confidence interval. In this case, the fall in output growth is stronger.

Figure 2.B.4 depicts the difference in output growth responses between the average concentration and the high concentration specifications. It shows that the decline in output growth is significantly stronger when banking concentration is higher.
Figure 2.B.1: Impulse responses with constant concentration within a year

Notes. The graph shows the responses of output growth to a one-standard-deviation uncertainty shock for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation higher than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1. The level of banking concentration is constant within each year.

B.2.3 Lagged concentration

In this section, I present the results of the third robustness test, where the level of banking competition is measured by the 3-Bank asset concentration ratio lagged by one quarter. The measure of concentration is lagged by one quarter to control for the possible endogeneity of concentration.

Figure 2.B.5 shows the effect of an exogenous one-standard-deviation increase in uncertainty on output growth for different levels of banking concentration. The blue line indicates the impulse response of output growth when banking concentration is at the country average, while the blue dashed lines represent the 90% confidence intervals. The figure shows that an exogenous increase
in uncertainty has no significant negative impact on output growth.

The yellow line represents the impulse response of output growth when concentration is one standard deviation above the country average. The yellow dashed lines are the 90% confidence interval. In this case, the fall in output growth is stronger.

Figure 2.B.6 depicts the difference in output growth responses between the average concentration and the high concentration specifications. It shows that the decline in output growth is significantly stronger when banking concentration is higher.
Figure 2.B.3: Impulse responses with 5-Bank Asset Concentration

Notes. The graph shows the responses of output growth to a one-standard-deviation uncertainty shock for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation higher than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1. The level of banking competition is measured by the 5-Bank asset concentration ratio.

B.2.4 Impact of the 2009 Global Recession

In this section, I present the results of the fourth robustness test. In this test, I exclude the year 2009 from the sample. This test shows that the findings are not driven by the impact of the global recession that occurred in that year.

Figure 2.B.7 shows the effect of an exogenous one-standard-deviation increase in uncertainty on output growth for different levels of banking concentration. The blue line indicates the impulse response of output growth when banking concentration is at the country average, while the blue dashed lines represent the 90% confidence intervals. The figure shows that an exogenous increase
in uncertainty has no significant negative impact on output growth.

The yellow line represents the impulse response of output growth when concentration is one standard deviation above the country average. The yellow dashed lines are the 90% confidence interval. In this case, the fall in output growth is stronger.

Figure 2.B.8 depicts the difference in output growth responses between the average concentration and the high concentration specifications. It shows that the decline in output growth is significantly stronger when banking concentration is higher.
Figure 2.B.5: Impulse responses with lagged concentration

Notes. The graph shows the responses of output growth to a one-standard-deviation uncertainty shock for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation higher than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1. The level of banking competition is measured by the 3-Bank asset concentration ratio lagged by one quarter.
Figure 2.B.6: Effect of competition with lagged concentration

Notes. The graph shows the difference between the two specifications plotted in Figure 2.B.5. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.
Figure 2.B.7: Impulse responses without the Great Recession

Notes. The graph shows the responses of output growth to a one-standard-deviation uncertainty shock for different levels of banking concentration. In the response with high banking concentration, banking concentration is 1 standard deviation higher than the country average. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1. The year 2009 is removed from the sample.
Figure 2.B.8: Effect of competition without the Great Recession

Notes. The graph shows the difference between the two specifications plotted in Figure 2.B.7. 90% confidence intervals computed using delta-method. The sample period is 2000Q1-2020Q1.
Appendix C  
Entrepreneurial optimization problem

C.1 Loan demand and its properties

In this section, I derive and describe the properties of the loan demand function. Specifically, in Section C.1.1, I derive the loan demand function, and in Section C.1.2, I outline its properties.

C.1.1 Derivation of loan demand

In this section, I derive the loan demand function.

After substituting the resource constraint of the entrepreneurs (2.1), the Lagrangian of the maximization of the entrepreneurs is

$$\mathcal{L}(K^j_t, \Lambda_t) = E_t(W^E_{t+1}) + \Lambda \left( \frac{q K^j_t}{\Pi_{t+1}} - E_t(W^E_{t+1}) \right). \quad (2.C.1)$$

It is possible to rewrite the expected future wealth as:

$$E_t(W^E_{j,t+1}) = \int_{\omega_{t+1}}^{\infty} \omega_{t+1} R^E_{t+1} q K^j_t f(\omega_{t+1}) d\omega_{t+1} - (1 - F(\omega_{t+1}) R^F_{t+1} b_t^j) \frac{1}{\Pi_{t+1}}, \quad (2.C.2)$$

where $f(\cdot)$ and $F(\cdot)$ are the probability density function and cumulative distribution function, respectively, of the distribution of $\omega_{t+1}$.

Using the definition of the default cutoff (2.5), (2.C.2) can be simplified as

$$\int_{\omega_{t+1}}^{\infty} (\omega_{t+1} - \omega_{t+1}) R^E_{t+1} q K^j_t f(\omega_{t+1}) d\omega_{t+1}. \quad (2.C.3)$$

I can rewrite the term $\int_{\omega_{t+1}}^{\infty} (\omega_{t+1} - \omega_{t+1}) f(\omega_{t+1}) d\omega_{t+1}$ as

$$\int_{\omega_{t+1}}^{\infty} (\omega_{t+1} - \omega_{t+1}) f(\omega_{t+1}) d\omega_{t+1} = \int_{\omega_{t+1}}^{\infty} \omega_{t+1} f(\omega_{t+1}) d\omega_{t+1} - \omega_{t+1} \int_{\omega_{t+1}}^{\infty} f(\omega_{t+1}) d\omega_{t+1}$$

$$= 1 - \left( \int_{0}^{\omega_{t+1}} \omega_{t+1} f(\omega_{t+1}) d\omega_{t+1} + \omega_{t+1} \int_{\omega_{t+1}}^{\infty} f(\omega_{t+1}) d\omega_{t+1} \right)$$

$$= 1 - \left( G(\omega_{t+1}) + (1 - F(\omega_{t+1})) \omega_{t+1} \right) \quad \text{if} \quad G(\omega_{t+1}) \geq 0, \quad (2.C.4)$$
It is possible to express \( \Gamma(\overline{\omega}_{t+1}^j) \) as

\[
\Gamma(\overline{\omega}_{t+1}^j) = \int_0^{\overline{\omega}_{t+1}^j} \omega_{t+1}^j f(\omega_{t+1}^j) d\omega_{t+1}^j + \overline{\omega}_{t+1}^j \int_{\overline{\omega}_{t+1}^j}^{\infty} f(\omega_{t+1}^j) d\omega_{t+1}^j \\
= \omega_{t+1}^j F(\overline{\omega}_{t+1}^j) - \int_0^{\overline{\omega}_{t+1}^j} F(\omega_{t+1}^j) d\omega_{t+1}^j + \overline{\omega}_{t+1}^j (1 - F(\overline{\omega}_{t+1}^j)) \\
= \omega_{t+1}^j - \int_0^{\overline{\omega}_{t+1}^j} F(\omega_{t+1}^j) d\omega_{t+1}^j. \tag{2.C.5}
\]

Combining (2.C.3) and (2.C.4), expected future wealth can be written as

\[
E_t(W_{t+1}^{E,j}) = (1 - \Gamma(\overline{\omega}_{t+1}^j)) R_{t+1}^{E} q K_{t}^j \Pi_{t+1}. \tag{2.C.6}
\]

Substituting in the Lagrangian of the maximization problem of the entrepreneurs, we have

\[
\mathcal{L}(K_{t}^j, \Lambda_t) = (1 - \Gamma(\overline{\omega}_{t+1}^j)) R_{t+1}^{E} q K_{t}^j \Pi_{t+1} + \Lambda_t \left( \frac{q K_{t}^j}{\Pi_{t+1}} - \frac{(1 - \Gamma(\overline{\omega}_{t+1}^j)) R_{t+1}^{E} q K_{t}^j}{\Pi_{t+1}} \right).
\]

The first-order conditions of the Lagrangian are

\[
\frac{\partial \mathcal{L}}{\partial K_{t}^j} = \frac{\partial E_t(W_{t+1}^{E,j})}{\partial K_{t}^j} + \Lambda_t \left( \frac{q}{\Pi_{t+1}} - \frac{\partial E_t(W_{t+1}^{E,j})}{\partial K_{t}^j} \right) = 0, \tag{2.C.7}
\]

\[
\frac{\partial \mathcal{L}}{\partial \Lambda_t} = \frac{q K_{t}^j}{\Pi_{t+1}} - E_t(W_{t+1}^{E,j}) = 0. \tag{2.C.8}
\]

The first derivative of expected future wealth with respect to capital is \(^{19}\)

\[
\frac{\partial E_t(W_{t+1}^{E,j})}{\partial K_{t}^j} = \left( 1 - \Gamma(\overline{\omega}_{t+1}^j) \right) R_{t+1}^{E} q - \Gamma'(\overline{\omega}_{t+1}^j) R_{t+1}^{E} q K_{t}^j \Pi_{t+1} \right) \frac{1}{\Pi_{t+1}}. \tag{2.C.9}
\]

We first focus on the second term of (2.C.9). We can obtain the expression for \( \Gamma'(\overline{\omega}_{t+1}^j) \) by differentiating (2.C.4). This yields

\[
\Gamma'(\overline{\omega}_{t+1}^j) = G'(\overline{\omega}_{t+1}^j) - f(\overline{\omega}_{t+1}^j) \overline{\omega}_{t+1}^j + (1 - F(\overline{\omega}_{t+1}^j)), \tag{2.C.10}
\]

where \( G'(\overline{\omega}_{t+1}^j) \) is obtained by differentiating the definition of \( G(\overline{\omega}_{t+1}^j) \) included in (2.C.4),

\[
G'(\overline{\omega}_{t+1}^j) = \frac{\partial}{\partial \overline{\omega}_{t+1}^j} \int_0^{\overline{\omega}_{t+1}^j} \omega_{t+1}^j f(\omega_{t+1}^j) d\omega_{t+1}^j = \overline{\omega}_{t+1}^j f(\overline{\omega}_{t+1}^j). \tag{2.C.11}
\]

\(^{19}\)Although \( R_{t+1}^{E} \) is a function of capital, I assume entrepreneurs treat the return on capital as exogenous.
Combining (2.C.11) and (2.C.10), \( \Gamma'(\bar{w}_{t+1}^j) \) can be simplified to

\[
\Gamma'(\bar{w}_{t+1}^j) = 1 - F(\bar{w}_{t+1}^j) \geq 0. \tag{2.C.12}
\]

Using the definition of the default cutoff (2.5) and the resource constraint of the entrepreneur (2.1), we can derive an expression for the term \( \frac{\partial \bar{w}_{t+1}^j}{\partial K_t^j} \)

\[
\frac{\partial \bar{w}_{t+1}^j}{\partial K_t^j} = \frac{R_t^E q K_t^j - b_t^j}{R_t^E q K_t^j} = \frac{R_t^E n_t^E j}{R_t^E q K_t^j} \geq 0. \tag{2.C.13}
\]

Substituting (2.C.12) and (2.C.13) into (2.C.9) we obtain

\[
\frac{\partial E_t(W_{t+1}^{E,j})}{\partial K_t^j} = \left( (1 - \Gamma(\bar{w}_{t+1}^j)) R_t^E q - (1 - F^E(\bar{w}_{t+1}^j)) \frac{R_t^E n_t^E j}{K_t^j} \right) \frac{1}{\Pi_{t+1}}.
\]

Using the definition of \( \Gamma(\bar{w}_{t+1}^j) \) derived in (2.C.4) and the definition of the default cutoff (2.5)

\[
\frac{\partial E(W_{t+1}^{E,j})}{\partial K_t^j} = \left( R_t^E q - (1 - F(\bar{w}_{t+1}^j)) R_t^E q \bar{w}_{t+1}^j - R_t^E q G(\bar{w}_{t+1}^j) - (1 - F^E(\bar{w}_{t+1}^j)) \frac{R_t^E n_t^E j}{K_t^j} \right) \frac{1}{\Pi_{t+1}}
\]

\[
= \left( R_t^E q - (1 - F^E(\bar{w}_{t+1}^j)) R_t^E \frac{b_t^j + n_t^E j}{K_t^j} - R_t^E q G(\bar{w}_{t+1}^j) \right) \frac{1}{\Pi_{t+1}}
\]

\[
= \left( R_t^E (1 - G(\bar{w}_{t+1}^j)) - R_t^E (1 - F^E(\bar{w}_{t+1}^j)) \right) \frac{q}{\Pi_{t+1}}. \tag{2.C.14}
\]

Substituting in the first order condition of the Lagrangian with respect to capital (2.C.7)

\[
0 = R_t^E (1 - G(\bar{w}_{t+1}^j)) - R_t^E (1 - F^E(\bar{w}_{t+1}^j)) + \Lambda_t (\lambda - R_t^E (1 - G(\bar{w}_{t+1}^j)) + R_t^E (1 - F^E(\bar{w}_{t+1}^j))), \tag{2.C.15}
\]

implying that

\[
\Lambda_t = \frac{R_t^E (1 - G(\bar{w}_{t+1}^j)) - R_t^E (1 - F^E(\bar{w}_{t+1}^j))}{R_t^E (1 - G(\bar{w}_{t+1}^j)) - R_t^E (1 - F^E(\bar{w}_{t+1}^j)) - \lambda}
\]

\[
= 1 + \frac{\lambda}{R_t^E (1 - G(\bar{w}_{t+1}^j)) - R_t^E (1 - F^E(\bar{w}_{t+1}^j)) - \lambda}. \tag{2.C.16}
\]

The incentive constraint binds when the Lagrange multiplier \( \Lambda_t \) is positive. This occurs when the expected return earned by the entrepreneurs \( R_t^E (1 - G(\bar{w}_{t+1}^j)) \) exceeds the expected cost of borrowing \( R_t^F (1 - F^E(\bar{w}_{t+1}^j)) \). In this case, entrepreneurial equity is scarce, and entrepreneurs find it optimal to borrow from bankers, as indicated by a positive (2.C.14). This implies that, in an active credit market, the loan demand is implicitly defined by the incentive participation
constraint

\[(1 - \Gamma(\bar{\omega}_{t+1}))R^E_{t+1} = \lambda.\]

C.1.2 Properties of the Loan Demand

This section presents the properties of the loan demand function that was derived in Section C.1.1.

Proof of Proposition 1

Proposition 1 states that loan demand decreases as the borrowing rate increases. Let \( I \) be defined as

\[I_t \equiv (1 - \Gamma(\bar{\omega}_{t+1}))R^E_{t+1} - \lambda.\]

The derivative of the loan demand with respect to the borrowing rate is given by

\[
\frac{dB_t}{dR^F_t} = -\frac{\partial I_t}{\partial R^F_t} - \frac{\partial I_t}{\partial b_t}.
\] (2.C.17)

The numerator of (2.C.17) can be expressed as

\[
\frac{\partial I_t}{\partial R^F_t} = -R^E_{t+1}\Gamma'(\bar{\omega}_{t+1})\frac{\partial \bar{\omega}_{t+1}}{\partial R^F_t}.
\] (2.C.18)

From the definition of the default cutoff (2.5), the term \( \frac{\partial \bar{\omega}_{t+1}}{\partial R^F_t} \) is equal to

\[
\frac{\partial \bar{\omega}_{t+1}}{\partial R^F_t} = \frac{b_t}{R^E_{t+1}qK_t}.
\] (2.C.19)

Substituting the expressions for \( \Gamma'(\bar{\omega}_{t+1}) \) and \( \frac{\partial \bar{\omega}_{t+1}}{\partial R^F_t} \) derived in (2.C.12) and (2.C.19) respectively, into (2.C.18)

\[
\frac{\partial I_t}{\partial R^F_t} = -R^E_{t+1}(1 - F(\bar{\omega}_{t+1}))\frac{b_t}{R^E_{t+1}qK_t}.
\]

\[
= -(1 - F(\bar{\omega}_{t+1}))\frac{b_t}{qK_t} \leq 0.
\] (2.C.20)

The denominator of (2.C.17) can be expressed as

\[
\frac{\partial I_t}{\partial b_t} = -R^E_{t+1}\Gamma'(\bar{\omega}_{t+1})\frac{\partial \bar{\omega}_{t+1}}{\partial b_t} + (1 - \Gamma(\bar{\omega}_{t+1}))R^E_{t+1}.
\] (2.C.21)

The term \( \frac{\partial \bar{\omega}_{t+1}}{\partial b_t} \) is equal to

\[
\frac{\partial \bar{\omega}_{t+1}}{\partial b_t} = \frac{R^E_t}{q}\frac{R^E_{t+1}qK_t - (R^E_{t+1}K_t + R^E_t)b_t}{(R^E_{t+1}K_t)^2} = \frac{R^E_t}{q}\frac{R^E_{t+1}n_t - R^E_{t+1}K_t}{(R^E_{t+1}K_t)^2} \geq 0.
\] (2.C.22)
Substituting the expressions for $\Gamma'(\omega_{t+1})$ and $\frac{\partial \omega_{t+1}}{\partial b_t}$ derived in (2.C.12) and (2.C.22) respectively, into (2.C.21)

$$\frac{\partial T_t}{\partial b_t} = -R^E_{t+1}(1 - F(\omega_{t+1})) \left( \frac{R^E_t R^E_{t+1} n^E_t - R^E_t K_t}{(R^E_{t+1} K_t)^2} + (1 - \Gamma(\omega_{t+1})) R^E_{t+1} \right) \leq 0. \quad (2.C.23)$$

Substituting (2.C.20) and (2.C.23) in (2.C.17)

$$\frac{db_t}{dR^F_t} = \frac{(1 - F(\omega_{t+1})) \frac{b_t}{qK_t}}{R^E_{t+1}(1 - F(\omega_{t+1})) \frac{R^E_t R^E_{t+1} n^E_t - R^E_t K_t}{(R^E_{t+1} K_t)^2} - (1 - \Gamma(\omega_{t+1})) R^E_{t+1}} \leq 0. \quad (2.C.24)$$

Since $\frac{db_t}{dR^F_t} \leq 0$, loan demand is a decreasing function of the borrowing rate.

**Proof of Proposition 2**

Proposition 2 states that the default rate of entrepreneurs rises with the borrowing rate. This is because the default rate $F(\omega_{t+1})$, is an increasing function of $\omega_{t+1}$. This can be seen by taking the derivative of the default rate (3.4) with respect to the default threshold

$$F'(\omega_{t+1}) = f(\omega_{t+1}) = \frac{1}{\omega_{t+1} \sigma_t} \phi \left( \frac{\log(\omega_{t+1}) + 0.5 \sigma^2_t}{\sigma_t} \right) \geq 0, \quad (2.C.25)$$

where $\phi(\cdot)$ is the p.d.f. of the standard normal distribution.

In order to show that the entrepreneurial default rate increases with the borrowing rate, it is necessary to show that the default threshold $\omega_{t+1}$ increases with the borrowing rate. Using the loan demand function (2.7), the default threshold can be expressed as

$$\Gamma(\omega_{t+1}) = 1 - \frac{\lambda}{R^E_{t+1}}. \quad (2.C.26)$$

Inverting $\Gamma(\omega_{t+1})$

$$\omega_{t+1} = \Gamma^{-1} \left( 1 - \frac{\lambda}{R^E_{t+1}} \right). \quad (2.C.27)$$

The derivative of (2.C.25) can be expressed as

$$\frac{d\omega_{t+1}}{dR^F_t} = \frac{1}{1 - F \left( 1 - \frac{\lambda}{R^E_{t+1}} \right) (R^E_{t+1})^2} \frac{1}{q} \frac{R^E_t}{R^E_{t+1}} \frac{\partial b_t}{dR^F_t} \geq 0. \quad (2.C.28)$$

Since the default rate increases with the default threshold that is increasing in the loan rate, the default rate increases with the loan rate.

**Proof of Proposition 3**

Proposition 3 states that if $R^E_{t+1} \leq \frac{\lambda}{1 - \Gamma(e^{-\sigma} - 0.5 \sigma^2)}$, a rise in uncertainty leads to a stronger rise
in the default rate of the entrepreneurs when the default cutoff is higher.

The effect of an increase of the default cutoff on the default rate is given by the derivative of (3.4) with respect to \( \bar{\omega}_{t+1} \)

\[
F'(\bar{\omega}_{t+1}) = \frac{1}{\bar{\omega}_{t+1} \sigma_{t+1}} \phi \left( \frac{\log(\bar{\omega}_{t+1}) + 0.5 \sigma_{t+1}^2}{\sigma_{t+1}} \right). \tag{2.C.27}
\]

In order to show that an increase in uncertainty has a stronger effect on the default rate when the default cutoff is higher, \( \frac{dF'_{t+1}}{d\sigma_{t+1}} \) must to be positive

\[
\frac{dF'_{t+1}}{d\sigma_{t+1}} = -\frac{d\bar{\omega}_{t+1}}{d\sigma_{t+1}} \sigma_{t+1} + \bar{\omega}_{t+1} \phi' \left( \frac{\log(\bar{\omega}_{t+1}) + 0.5 \sigma_{t+1}^2}{\sigma_{t+1}} \right) + \phi' \left( \frac{\log(\bar{\omega}_{t+1}) + 0.5 \sigma_{t+1}^2}{\sigma_{t+1}} \right) \frac{1}{\bar{\omega}_{t+1} \sigma_{t+1}} \frac{d\omega_{t+1}}{d\sigma_{t+1}} \sigma_{t+1} + 0.5 \sigma_{t+1}^2 - \log(\bar{\omega}_{t+1}). \tag{2.C.28}
\]

The term \( \phi'(x) \) can be written as

\[
\phi'(x) = -\frac{1}{\sqrt{2\pi}} xe^{-0.5x^2}. \tag{2.C.29}
\]

The term \( \phi(x) \) can be written as

\[
\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2}. \tag{2.C.30}
\]

Substituting (2.C.29) and (2.C.30) into (2.C.28)

\[
\frac{dF'_{t+1}}{d\sigma_{t+1}} = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2} \frac{1}{\bar{\omega} \sigma^2} \left( -\frac{d\bar{\omega}_{t+1}}{d\sigma_{t+1}} \sigma_{t+1} + \bar{\omega}_{t+1} \phi' \left( \frac{\log(\bar{\omega}_{t+1}) + 0.5 \sigma_{t+1}^2}{\sigma_{t+1}} \right) + \phi' \left( \frac{\log(\bar{\omega}_{t+1}) + 0.5 \sigma_{t+1}^2}{\sigma_{t+1}} \right) \frac{1}{\bar{\omega}_{t+1} \sigma_{t+1}} \frac{d\omega_{t+1}}{d\sigma_{t+1}} \sigma_{t+1} + 0.5 \sigma_{t+1}^2 - \log(\bar{\omega}_{t+1}) \right). \tag{2.C.31}
\]

Equation 2.C.31 can be written as

\[
\frac{dF'_{t+1}}{d\sigma_{t+1}} = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2} \frac{1}{\bar{\omega} \sigma^2} \left( -\frac{d\bar{\omega}_{t+1}}{d\sigma_{t+1}} \sigma_{t+1} + \frac{\log(\bar{\omega}_{t+1}) + 0.5 \sigma_{t+1}^2}{\bar{\omega}_{t+1}} - \frac{\log(\bar{\omega}_{t+1}) + 0.5 \sigma_{t+1}^2}{\bar{\omega}_{t+1}} \frac{1}{d\bar{\omega}_{t+1}} \frac{d\sigma_{t+1}}{d\sigma_{t+1}} \sigma_{t+1} \right). \tag{2.C.32}
\]

Equation 2.C.32 is positive when \( R^E_{t+1} \leq \frac{\lambda}{1-\Gamma(e^{-0.5x^2})} \). In order to see that this assumption...
is sufficient, note that

\[- \frac{d\overline{\sigma}_{t+1}}{d\sigma_{t+1}} \sigma_{t+1} + \frac{1}{\overline{\sigma}_{t+1}} \frac{d\overline{\sigma}_{t+1}}{d\sigma_{t+1}}, \]  

(2.C.33)

is positive if \( \frac{d\overline{\sigma}_{t+1}}{d\sigma_{t+1}} \geq 0 \). The effect of uncertainty on the default cutoff is

\[ \frac{d\overline{\sigma}_{t+1}}{d\sigma_{t+1}} = \frac{R^E_t}{q} \frac{db_t}{d\sigma_{t+1}} \frac{R^E_{t+1} q K_t - R^E b - R^E_{t+1} K_t b_t}{(R^E_{t+1} K_t)^2} = \frac{R^E_t}{q} \frac{db_t}{d\sigma_{t+1}} \frac{R^E_{t+1} n^E - R^E_{t+1} K_t b_t}{(R^E_{t+1} K_t)^2}. \]

The effect of uncertainty on the default cutoff is positive if \( \frac{db_t}{d\sigma_{t+1}} \geq 0 \). The effect of uncertainty on loan demand is

\[ \frac{db_t}{d\sigma_{t+1}} = - \frac{\partial \overline{\sigma}_{t+1}}{\partial \sigma_{t+1}}. \]  

(2.C.34)

The numerator of (2.C.34) can be expressed as

\[ \frac{\partial L_t}{\partial \sigma_{t+1}} = - \frac{\partial \Gamma(\overline{\sigma}_{t+1})}{\partial \sigma_{t+1}} R^E_{t+1}. \]  

(2.C.35)

Substituting the definition of \( \Gamma(\overline{\sigma}_{t+1}) \), the term \( \frac{\partial \Gamma(\overline{\sigma}_{t+1})}{\partial \sigma_{t+1}} \) in (2.C.35) can be expressed as

\[ \frac{\partial \Gamma(\overline{\sigma}_{t+1})}{\partial \sigma_{t+1}} = - \frac{\partial F(\overline{\sigma}_{t+1})}{\partial \sigma_{t+1}} \overline{\sigma}_{t+1} + \frac{\partial G(\overline{\sigma}_{t+1})}{\partial \sigma_{t+1}} \\
= - F'(\overline{\sigma}_{t+1}) \overline{\sigma}_{t+1}^2 + G'(\overline{\sigma}_{t+1}) \overline{\sigma}_{t+1} + 5 \overline{\sigma}_{t+1}^2 - \frac{5 \overline{\sigma}_{t+1}}{\sigma_{t+1}} \\
= - F'(\overline{\sigma}_{t+1}) \overline{\sigma}_{t+1}^2 \sigma_{t+1} \leq 0. \]

Substituting the last expression into (2.C.35)

\[ \frac{\partial L_t}{\partial \sigma_{t+1}} = F'(\overline{\sigma}_{t+1}) \overline{\sigma}_{t+1}^2 \sigma_{t+1} + R^E_{t+1}. \]  

(2.C.36)

Substituting (2.C.36) and (2.C.23) into (2.C.34)

\[ \frac{db_t}{d\sigma_{t+1}} = - \frac{F'(\overline{\sigma}_{t+1}) \overline{\sigma}_{t+1}^2 \sigma_{t+1} + R^E_{t+1}}{R^E_{t+1}(1 - F(\overline{\sigma}_{t+1})) \frac{R^E_{t+1} n^E - R^E_{t+1} K_t}{(R^E_{t+1} K_t)^2} - \frac{\lambda}{1 - \Gamma(\overline{\sigma}_{t+1}) R^E_{t+1}} \geq 0. \]

Therefore, \( \left( - \frac{d\overline{\sigma}_{t+1}}{d\sigma_{t+1}} \overline{\sigma}_{t+1} + \frac{1}{\overline{\sigma}_{t+1}} \frac{d\overline{\sigma}_{t+1}}{d\sigma_{t+1}} \right) \geq 0. \)

The assumption \( R^E_{t+1} \leq \frac{\lambda}{1 - \Gamma(\overline{\sigma}_{t+1} - 0.5 \overline{\sigma}_{t+1})} \) implies that

\[ R^E_{t+1} \leq \frac{\lambda}{1 - \Gamma(\overline{\sigma}_{t+1} - 0.5 \overline{\sigma}_{t+1})}. \]
\[
1 - \frac{\lambda}{R_{t+1}^E} \leq \Gamma \left( e^{-\sigma_{t+1} - 0.5\sigma_{t+1}^2} \right)
\]

\[
\Gamma^{-1} \left( 1 - \frac{\lambda}{R_{t+1}^E} \right) \leq e^{-\sigma_{t+1} - 0.5\sigma_{t+1}^2}
\]

\[
\log(\omega_{t+1}) \leq -\sigma_{t+1} - 0.5\sigma_{t+1}^2
\]

\[
\frac{\log(\omega_{t+1}) + 0.5\sigma_{t+1}^2}{\sigma_{t+1}} \leq -1.
\] (2.C.37)

Because of (2.C.37) and since
\[
- \frac{d\omega_{t+1}\sigma_{t+1}}{\sigma_{t+1}^2} + \frac{1}{\omega_{t+1}} \frac{d\omega_{t+1}}{d\sigma_{t+1}} \geq 0
\]

\[
- \frac{d\omega_{t+1}\sigma_{t+1}}{\omega_{t+1}} - \log(\omega_{t+1}) + 0.5\sigma_{t+1}^2 \frac{1}{\omega_{t+1}} \frac{d\omega_{t+1}}{d\sigma_{t+1}} \geq 0.
\] (2.C.38)

Moreover, because of (2.C.37)

\[
-1 + \frac{\log(\omega_{t+1}) + 0.5\sigma_{t+1}^2}{\sigma_{t}} \log(\omega_{t+1}) - 0.5\sigma_{t+1}^2 \sigma_{t} \geq 0.
\] (2.C.39)

Finally, because of (2.C.38) and (2.C.39), (2.C.28) is positive and a rise in uncertainty leads to a stronger rise in the default rate of the entrepreneurs when the default cutoff is higher.
Appendix D  Reverse bounded Pareto distribution

Suppose that $\gamma$ follows a Pareto distribution with scale parameter $a > 0$ and support $\gamma \in [\gamma_s, \infty)$. Its p.d.f. and its c.d.f. are

$$f_\gamma(\gamma) = \frac{a \gamma_s^a}{\gamma^{a+1}},$$
$$F_\gamma(\gamma) = 1 - \left(\frac{\gamma_s}{\gamma}\right)^a.$$

A bounded Pareto distribution is a distribution obtained from restricting the domain of the Pareto distribution. Let $S$ and $H$ be the lower bound and the upper bounds of the bounded Pareto distribution. The resulting p.d.f. and c.d.f. are

$$f_{\gamma B}(\gamma) = \frac{f_\gamma(\gamma)}{F_\gamma(H) - F_\gamma(S)} = \frac{a \gamma_s^a}{\gamma^{a+1} - \left[1 - \left(\frac{\gamma_s}{S}\right)^a\right]} = \frac{aS^a \gamma^{-a-1}}{1 - \left(\frac{S}{H}\right)^a},$$
$$F_{\gamma B}(\gamma) = \frac{F_\gamma(\gamma) - F_\gamma(S)}{F_\gamma(H) - F_\gamma(S)} = \frac{1 - \left(\frac{\gamma_s}{\gamma}\right)^a - \left[1 - \left(\frac{\gamma_s}{S}\right)^a\right]}{1 - \left(\frac{\gamma_s}{H}\right)^a - \left[1 - \left(\frac{\gamma_s}{S}\right)^a\right]} = \frac{1 - S^a \gamma^{-a}}{1 - \left(\frac{S}{H}\right)^a}.$$

This distribution is characterized by a positive skewness and a long right tail. A market share distribution that features many small bankers and a few large bankers can be obtained by flipping the distribution around the y-axis and shifting it to the right by $S + H$. This leads to a reverse bounded Pareto distribution whose domain is $(S, H)$. The p.d.f. and the c.d.f. of this distribution are

$$f_{\gamma BR}(\gamma) \equiv f_{\gamma B}(-\gamma + H + S) = \frac{aS^a (-\gamma + H + S)^{-a-1}}{1 - \left(\frac{S}{H}\right)^a},$$
$$F_{\gamma BR}(\gamma) = \int_S^\gamma \frac{aS^a (-\gamma + H + S)^{-a-1}}{1 - \left(\frac{S}{H}\right)^a} d\gamma = \frac{S^a (-\gamma + H + S)^{-a} - S^a H^{-a}}{1 - \left(\frac{S}{H}\right)^a}.$$
Chapter 3

Firm Risk and the Banking Accelerator

Joint with Vivien Lewis, Stéphane Moyen and Stefania Villa

1 Introduction

As a result of the Covid-19 pandemic and the lockdown measures that followed, many firms were faced with a heightened risk of default, with elevated exit rates in some sectors and especially among small firms (Crane et al., 2020). In late 2020, various indicators pointed to an increase in expected corporate defaults, as shown by Greenwood et al. (2020). Macroprudential regulators identified corporate insolvencies as a major threat to financial stability. With inadequate bank capitalization, a wave of corporate defaults may trigger asset sales and a reduction in credit provision, which in turn exacerbates the recession (Gourinchas et al., 2021).

This paper analyzes empirically and theoretically the transmission of risk shocks from firms to the economy. First, we empirically show that firm risk shocks are transmitted to the financial sector in the form of greater bank default risk. Second, we capture this transmission in a model that combines New Keynesian price setting frictions with financial market imperfections. In particular, we show how a fragile and highly leveraged banking sector exacerbates the adverse effects of firm risk shocks. Furthermore, we investigate how macroprudential policies can help to stabilize the macroeconomy in the face of such shocks.

Credit demand and financial intermediation are modelled as follows. Similarly to Bernanke et al. (1999), henceforth BGG, entrepreneurs have insufficient net worth to buy capital and therefore borrow from banks. Entrepreneurs are subject to idiosyncratic default risk, which gives rise to a costly state verification problem. When an entrepreneur declares default, banks incur monitoring costs in order to observe the entrepreneur’s realized return on capital. As in Zhang (2009), Benes and Kumhof (2015) and Clerc et al. (2018), we depart from BGG by stipulating a default threshold that is contingent on aggregate shocks. In BGG, debt contracts do not have this contingency,
such that the entrepreneur’s net worth varies together with aggregate risk. Since the financial intermediary is then perfectly insulated from such risk, its balance sheet plays no role. Here, in contrast, banks suffer balance sheet losses if entrepreneurial defaults are higher than expected.

Banks have limited liability. When a bank fails, it is monitored by a bank resolution authority, an action which destroys part of the bank’s remaining assets. Bank defaults do not, however, affect the return on deposits. Full deposit insurance - financed through lump sum taxes on households - removes any incentive for depositors to monitor the banks’ activities. Thus, the deposit rate is equal to the policy rate. At the same time, bank equity is limited to the accumulated wealth of bankers, who are the only agents allowed to invest in banks. This results in a high equity return per unit invested. As a consequence of expensive equity and cheap deposit funding, banks have an incentive to maximize leverage. Due to limited liability, banks do not internalize the cost of increased banking sector fragility. Macroprudential policy imposes a penalty on excessive bank leverage, thereby limiting the amount of resources lost due to bank failures. If banks are not sufficiently capitalized, they have to pay a penalty to the bank resolution authority.

Our results show how a fragile and highly leveraged banking sector exacerbates the adverse effects of firm risk shocks. Furthermore, we find that a countercyclical capital buffer is effective at stabilizing the business cycle in response to such shocks. However, such a policy increases bank default risk.

The remainder of the paper is structured as follows. Our empirical evidence is presented in Section 2. Section 3 outlines the model. Section 4 analyzes the transmission of firm risk shocks to the economy and the financial sector, with a focus on the role of bank leverage. We investigate the effectiveness of macroprudential policy, which takes the form of a penalty on excessive bank leverage. Additionally, we study the implications of introducing a countercyclical capital buffer. Finally, Section 5 concludes.

2 Empirical evidence

Risk shocks have been identified as an important driving force of business cycle fluctuations (Christiano et al., 2014). What do we mean by ‘risk shocks’? A micro-based view of macroeconomics takes into consideration that individual producers with different levels of productivity coexist. At a given point in time, we might think of a productivity distribution across firms, whose standard deviation provides a measure of risk.\(^2\) The idea behind this notion of firm risk is that a greater standard deviation implies greater uncertainty regarding firms’ output. In that sense, heightened firm risk necessarily implies greater macroeconomic volatility and a contractionary effect on output. Indeed, Chugh (2016) shows that average productivity and the cross-sectional standard deviation of idiosyncratic productivity are inversely correlated. In line with this, Kehrig (2011)

\(^2\)This notion of risk differs from e.g. Bloom (2009) and Basu and Bundick (2017), where ‘uncertainty shocks’ are defined as a time-varying variance of aggregate productivity.
shows that the dispersion of productivity across US plants rises in recessions, Bloom et al. (2018) shows that the establishment-level dispersion in TFP shocks and in output growth is strongly countercyclical.

In our empirical analysis, we measure firm risk as the firm cross-sectional implied volatility of Dew-Becker and Giglio (2020), consistent with the theoretical concept outlined above. Bank risk is proxied by the spot funding spread (SFS) from Jondeau et al. (2020). The spot funding spread is given by the three-month IBOR-OIS spread, where IBOR stands for Interbank Offered Rate and OIS for Overnight Interest Swap. A detailed description of the data can be found in Appendix A.1.

We estimate a four-variable vector autoregression (VAR) at the monthly frequency with lag length 2. The variable vector contains the logarithm of real GDP, the logarithm of the price index (measured by the GDP deflator), firm risk and bank risk. Data on real GDP are available only at the quarterly frequency, and thus we impute the missing values by means of the Chow and Lin (1971) method. We perform a decomposition of the VAR residual matrix to identify the underlying structural shocks by imposing a lower triangular structure (Cholesky decomposition). The third shock is then labelled a ‘firm risk shock’; the identifying assumption is that output and prices are predetermined and do not respond contemporaneously to the shock, while bank risk does. For a more detailed description of the model and its assumptions, please refer to Appendix A.2.

Figure 3.1 shows that an increase in firm risk leads to a significant fall in output and inflation in the US over the sample from January 2005 to June 2020. This is consistent with the view that adverse firm risk shocks induce a demand-driven recession. Moreover, bank risk rises significantly, which indicates that firm risk carries over to the banking sector in the form of a higher implicit bank default probability.

In a set of additional estimation exercises reported in Appendix A.3, we show that our main results are robust to a number of alternative specifications. First, the US is not a special case, since we observe similar responses to firm risk shocks also in the Euro area. Second, the specific measures of firm and bank risk do not matter for the overall pattern.

1. Firm risk is alternatively measured as the excess bond premium for corporates computed by Gilchrist and Zakrajek (2012a).³ As a third proxy for firm risk, we use ‘spread per unit of leverage’ (SPL) calculated at the Bundesbank. This time series is obtained by combining end-of-the-month CDS-spread data in basis points from Markit and quarterly end-of-the-period debt and equity data from Bloomberg for all the firms in the EURO Stoxx 50 and Dow Jones 30. After computing the SPL for every firm as the ratio of CDS-spread to debt divided by equity, the aggregate SPL is given by the median across firms. The main message of Figure 3.1 is unchanged for all these specifications.

³The corresponding reference for the Euro area is Gilchrist and Mojon (2018).
Figure 3.1: IRFs to a one standard deviation firm risk shock


2. As an alternative measure of bank risk, we use the excess bond premium for financials (Gilchrist and Zakrajek, 2012b). Measuring bank risk using the forward (rather than spot) credit spread provided by Jondeau et al. (2020) also makes little difference.

Third, replacing GDP with industrial production leaves our results intact. Finally, when we include the policy interest rate in the VAR, we find that it drops significantly in response to the shock. This indicates that the central bank responds in an accommodating way to the demand-driven contraction.

A similar transmission pattern of firm-level risk shocks is found in the VAR study on German data in Bachmann and Bayer (2013), in Gilchrist et al. (2014), and in the DSGE model estimated on US data by Christiano et al. (2014). The aforementioned papers concentrate on the macroeconomic impact of firm risk shocks; the effects on the banking sector more specifically have, to our knowledge, not been studied yet.

In the following section, our aim is to develop a business cycle model that can replicate the patterns uncovered here.

3 Model

We now sketch our model that features a costly state verification problem both for entrepreneurs and for banks. Banks monitor failed entrepreneurs and a bank resolution authority monitors failed
banks. Given the non-state-contingent nature of the loan contract, entrepreneurial defaults affect bank balance sheets. We first discuss the non-financial sector; second, we explain the workings of the financial sector. Third, we present the monetary and macroprudential policy rules. Finally, the rest of the model contains the household sector, goods production and market clearing.

3.1 Non-financial sector

This section discusses in detail the loan contract between entrepreneurs and banks. Townsend (1979) analyzes a costly state verification problem where the entrepreneur’s return cannot be observed by the lender without incurring a monitoring cost. He shows that the optimal contract in the presence of idiosyncratic risk is a standard debt contract in which the repayment does not depend on the entrepreneur’s project outcome. This argument is used in the financial accelerator model of Bernanke et al. (1999), where the debt contract between the borrower and the lender specifies a fixed repayment rate. In the case of default, the lender engages in costly monitoring and seizes the entrepreneur’s remaining capital.

As in Bernanke et al. (1999), the risk to the entrepreneur has an aggregate as well as an idiosyncratic component. The latter depends on the aggregate return to capital, which is observable. Carlstrom et al. (2014) ask ‘why should the loan contract call for costly monitoring when the event that leads to a poor return is observable by all parties?’ Indeed, Carlstrom et al. (2016) show that the privately optimal contract includes indexation to the aggregate return to capital, which they call $R^k$-indexation. They argue that this type of contract comes close to financial contracts observed in practice. Furthermore, Carlstrom et al. (2014) estimate a high degree of indexation in a medium-scale business cycle model. Consistent with these findings, we stipulate a financial contract whereby the entrepreneur’s default threshold depends on the aggregate return to capital.

**Entrepreneurs.** There is a continuum of risk-neutral entrepreneurs indicated by the superscript ‘E’. They combine net worth and bank loans to purchase capital from the capital production sector and rent it to intermediate goods producers. Entrepreneurs face a probability $1 - \chi^E$ of staying in business in the next period, where $\chi^E \in (0, 1)$. Let $W^E_t$ be entrepreneurial wealth accumulated from operating firms. Entrepreneurs have zero labor income. Incumbent entrepreneurs’ net worth is the wealth held by entrepreneurs at $t$ who are still in business in $t + 1$, that is $(1 - \chi^E)W^E_{t+1}$. Entrepreneurs who fail return to their household, bringing with them their residual wealth, i.e. $\chi^E W^E_{t+1}$. Of this, a fraction $\iota$ is provided to new entrepreneurs as startup financing. Thus, total entrepreneurial net worth is given by

$$n^E_{t+1} = (1 - \chi^E + \iota \chi^E) W^E_{t+1}, \quad (3.1)$$

and entrepreneurial profits retained by the households are $\Xi^{EH}_{t+1} = (1 - \iota) \chi^E W^E_{t+1}$.

Aggregate entrepreneurial wealth in period $t + 1$, measured in terms of final consumption
goods, is given by the value of the capital stock bought in the previous period, \( q_t K_t \), multiplied by the ex-post nominal return on capital \( R_{t+1} \), multiplied by the fraction of returns left to the entrepreneur, \( 1 - \Gamma_{t+1}^E \), discounted by the gross rate of inflation, \( \Pi_{t+1} = P_{t+1}/P_t \), that is,

\[
W_{t+1}^E = (1 - \Gamma_{t+1}^E) \frac{R_{t+1}^E q_t K_t}{\Pi_{t+1}}. \tag{3.2}
\]

The discussion of the contracting problem between entrepreneurs and banks below contains a derivation of \( \Gamma_{t+1}^E \).

Entrepreneur \( j \) purchases capital \( K_t^j \) at the real price \( q_t \) per unit. The amount \( q_t K_t^j \) spent on capital goods exceeds her net worth \( n_t^E j \). She borrows the remainder,

\[
b_t^j = q_t K_t^j - n_t^E j, \tag{3.3}
\]

from the full range of banks, which in turn obtain funds from depositors and equity holders (‘bankers’). Capital is chosen at \( t \) and used for production at \( t + 1 \). It has an ex-post gross return \( \omega_{t+1}^E R_{t+1}^E \), where \( R_{t+1}^E \) is the aggregate return on capital (as stated above) and \( \omega_{t+1}^E \) is an idiosyncratic disturbance. The idiosyncratic productivity disturbance is iid log-normally distributed with mean \( \mathbb{E}\{\omega_{t+1}^E\} = 1 \) and a time-varying standard deviation \( \sigma_t^E = \sigma^E \varsigma_t^E \), where \( \varsigma_t^E \) is a firm risk shock. Moreover, \( \varsigma_t^E \) is supposed to follow an AR(1) process such that \( \ln \varsigma_t^E = \rho E \ln \varsigma_{t-1}^E + \varepsilon_t^E \).

The probability of default for an individual entrepreneur is given by the respective cumulative distribution function evaluated at the threshold \( \omega_{t+1}^E \), to be specified below,

\[
F_{t+1}^E = F^E(\omega_{t+1}^E) = \int_0^\omega_{t+1}^E f^E(\omega_{t+1}^E)d\omega_{t+1}^E, \tag{3.4}
\]

where \( f^E(\cdot) \) is the respective probability density function.

The ex-post gross return to entrepreneurs, in terms of consumption, of holding a unit of capital from \( t \) to \( t + 1 \) is given by the rental rate on capital, \( r_{t+1}^K \), plus the capital gain net of depreciation, \( (1 - \delta) q_{t+1} \), divided by the real price of capital, in period \( t \). In nominal terms, this is:

\[
R_{t+1}^E = \frac{r_{t+1}^K + (1 - \delta) q_{t+1} \Pi_{t+1}}{q_t}. \tag{3.5}
\]

The financial contract, which we turn to next, determines how the project return is divided between the entrepreneur and the bank.

**Financial contract.** After the financial contract is signed, the entrepreneurs’ idiosyncratic productivity shock realizes. Those entrepreneurs whose productivity is below the threshold,

\[
\omega_{t+1}^E = \frac{Z_t b_t}{R_{t+1}^E q_t K_t} = \frac{x_t^E}{R_{t+1}^E}, \tag{3.6}
\]
3. MODEL

\[ x^E_t \equiv Z_t b_t / (q_t K_t) \] is the entrepreneur’s loan-to-value ratio, the contractual debt repayment divided by the value of capital purchased and \( Z_t \) is the contractual repayment rate. Here, the cutoff \( \omega^E_{t+1} \) is contingent on the realization of the aggregate state \( R^E_{t+1} \), such that aggregate shocks produce fluctuations in firm default rates, which in turn impinge on bank balance sheets.

The details of the financial contract are as follows. In the default case, the entrepreneur has to turn the whole return \( \omega^E_{t+1} R^E_{t+1} q_t K_t \) over to the bank. Of this, a fraction \( \mu^E \) is lost as a monitoring cost that the bank needs to incur to verify the entrepreneur’s project return. In the non-default case, the bank receives only the contractual payment \( \omega^E_{t+1} R^E_{t+1} q_t K_t \). The remainder, \( \omega^E_{t+1} - \omega^E_{t+1} \), goes to the residual claimant, the entrepreneur. Consequently, if the entrepreneur does not default, the payment to the bank is independent of the realization of the idiosyncratic shock but contingent on the aggregate return \( R^E_{t+1} \).

Following the notation in Bernanke et al. (1999), we define the share of the project return \( R^E_{t+1} q_t K_t \) accruing to the bank, gross of monitoring costs, as

\[
\Gamma^E_{t+1} = \Gamma^E(\omega^E_{t+1}) \equiv \int_0^{\omega^E_{t+1}} \omega^E_{t+1} f^E(\omega^E_{t+1}) d\omega^E_{t+1} + (1 - F^E_{t+1}) \omega^E_{t+1},
\] (3.7)
such that remainder, \( 1 - \Gamma^E_{t+1} \), represents the share of the return which is left for the entrepreneur. The share of the project return subject to firm defaults is defined as

\[
G^E_{t+1} = G^E(\omega^E_{t+1}) \equiv \int_0^{\omega^E_{t+1}} \omega^E_{t+1} f^E(\omega^E_{t+1}) d\omega^E_{t+1}.
\] (3.8)

Being risk-neutral, the entrepreneur cares only about the expected return on his investment given by

\[
\mathbb{E}_t \left\{ \left[ 1 - \Gamma^E \left( \frac{x^E_t}{R^E_{t+1}} \right) \right] R^E_{t+1} q_t K_t \right\},
\] (3.9)

where the expectation is taken with respect to the random variable \( R^E_{t+1} \).

The bank’s ex-post gross return on loans, in nominal terms, is given by

\[
R^F_{t+1} = (\Gamma^E_{t+1} - \mu^E G^E_{t+1}) \frac{R^E_{t+1} q_t K_t}{b_t}.
\] (3.10)

In order for the bank to agree to the contract, the return that the bank earns from lending funds to the entrepreneur must be at least as high as the return the bank would obtain from lending to a (fictitious) riskless firm,

\[
\mathbb{E}_t \left\{ (1 - F^E_{t+1}) \omega^E_{t+1} + (1 - \mu^E) \int_0^{\omega^E_{t+1}} \omega^E_{t+1} f^E(\omega^E_{t+1}) d\omega^E_{t+1} R^E_{t+1} q_t K_t \right\} \geq \mathbb{E}_t \{ R^F_{t+1} b_t \}.
\] (3.11)
Aggregating capital holdings and net worth over entrepreneurs, we define $K_t = \sum_j K_{t}^{j} d j$ and $n_t^{E} = \int_j n_t^{Ej} d j$, and using the borrowing requirement (3.3), we can replace $b_t$ with $(q_t \int_j K_{t}^{j} d j - n_t^{E})$ in the bank’s participation constraint (3.11).

Using the above results we are able to derive the financial contract. The entrepreneur’s objective is given by

$$\max_{x_t^{Ej}, K_{t+1}^{j}} \mathbb{E}_t \left\{ \left[ 1 - \Gamma_t^{E} \left( \frac{x_t^{Ej}}{R_t^{E}} \right) \right] R_{t+1}^{E} q_t^{j} K_{t}^{j} \right\},$$

subject to the bank’s participation constraint written as follows,

$$\mathbb{E}_t \left[ \Gamma_t^{E} \left( \frac{x_t^{Ej}}{R_t^{E}} \right) - \mu_t^{E} G_t^{E} \left( \frac{x_t^{Ej}}{R_t^{E}} \right) \right] R_{t+1}^{E} q_t^{j} K_{t}^{j} = \mathbb{E}_t \left\{ R_{t+1}^{F} (q_t \int_j K_{t}^{j} d j - n_t^{E}) \right\}. \tag{3.13}$$

The optimality conditions of the contracting problem are

$$\mathbb{E}_t \{ -\Gamma_{t+1}^{Ej} + \xi_t^{j} \left( \Gamma_{t+1}^{Ej} - \mu_t^{E} G_{t+1}^{Ej} \right) \} = 0, \tag{3.14}$$

$$\mathbb{E}_t \{ (1 - \Gamma_{t+1}^{E}) R_{t+1}^{E} + \xi_t^{j} \left[ (\Gamma_{t+1}^{E} - \mu_t^{E} G_{t+1}^{E}) \right] R_{t+1}^{E} - R_{t+1}^{F} \} = 0, \tag{3.15}$$

where $\xi_t^{j}$ is the Lagrange multiplier on the bank’s participation constraint (3.13).

### 3.2 Financial sector

The financial sector consists of a range of banks with idiosyncratic productivity. Banks receive equity funding from bankers and deposit funding from households. Their assets are the loans which they provide to the entrepreneurs. Deposits are fully insured; depositors therefore have no incentive to monitor a bank’s activities and receive the risk-free return that coincides with the policy rate. Since bankers are the only agents in the economy allowed to hold bank equity, the size of total equity funding is restricted to the bankers’ accumulated wealth. This restriction keeps the equity return - per unit of equity held - high. Bankers have limited liability and can walk away if a bank defaults. As deposit funding is cheap and equity funding is expensive, banks therefore have an incentive to maximize leverage and will hold only the minimum amount of capital as required by the macroprudential authority. Those financial institutions that are unable to pay depositors using their returns on corporate loans fail; they are monitored by a tax-funded bank resolution authority.

**Banks.** Bank $i$ has productivity $\omega_{t+1}^{F_i}$. The random variable $\omega_{t+1}^{F_i}$ is log-normally distributed with mean one and standard deviation $\sigma^{F}$. The operating profits of bank $i$ in period $t + 1$ are given by the revenues from its lending activity minus the costs paid for deposits

$$OP_{t+1}^{F_i}(\omega_{t+1}^{F_i}) \equiv \omega_{t+1}^{F_i} R_{t+1}^{F} b_{t}^{i} - R_{t}^{D} d_{t}^{i},$$
where $R_{t+1}^F$ is the interest rate obtained from the lending activity, $R_{t}^D$ is the deposit rate and $d_i^t$ is the amount of deposits issued by the bank. Banks are subject to limited liability, i.e. bank’s operating profits cannot fall below zero. The bank fails if it is not able to pay depositor using its returns on corporate loans. Similar to the entrepreneurial sector, there exists a threshold productivity level below which bank $i$ fails,

$$\omega_{t+1}^F R_{t+1}^F b_i^t < R_{t}^D d_i^t.$$  (3.16)

The default threshold is the value of $\omega_{t+1}^F$ for which (3.16) holds with equality,

$$\omega_{t+1}^F = \frac{R_{t}^D d_i^t}{R_{t+1}^F b_i^t}.$$  (3.17)

Similarly to Benes and Kumhof (2015), a macroprudential regulator imposes capital requirements on banks. At the beginning of period $t + 1$, the authority imposes a penalty $\gamma^B b_i^t$ on the non-defaulted banks, with $\gamma^B \in (0, 1)$, if the bank operating profit is less than a fraction $\phi_t \in (0, 1)$ of the return on loans. Note that there is no limited liability with respect to the penalty. The penalty rule implies a threshold such that, if $\omega_{t+1}^F$ is below this threshold, the bank has to pay the penalty,

$$\omega_{t+1}^F R_{t+1}^F b_i^t - R_{t}^D d_i^t < \phi_t \omega_{t+1}^F R_{t+1}^F b_i^t.$$  (3.18)

The penalty threshold is the value of $\omega_{t+1}^F$ for which (3.18) holds with equality, i.e.

$$\omega_{t+1}^{\phi} = \frac{R_{t}^D d_i^t}{(1 - \phi_t) R_{t+1}^F b_i^t} = \frac{\omega_{t+1}^F}{1 - \phi}.$$  (3.19)

Note that $\phi_t$ represents the capital requirement because the left-hand side of (3.18) is equal to pre-penalty bank equity at the beginning of period $t + 1$, and, on the right-hand side, $\omega_{t+1}^F R_{t+1}^F b_i^t$ is the value of assets at the beginning of period $t + 1$. The definition of the capital requirement in (3.18) differs from Clerc et al. (2018) in that it is expected rather than current net worth that determines the adequacy of the capital buffer. This specification avoids the indeterminacy problem highlighted in Lewis and Roth (2018).\footnote{Rubio and Yao (2020) specify a similar constraint in the form of a loan-to-value ratios for households taking out mortgage loans. There, impatient households can borrow from patient households but have to satisfy an LTV ratio limiting the amount they can borrow to a certain fraction of the future expected value of their housing.}

Bank $i$’s profit in period $t + 1$ is

$$\Xi_{t+1}^{F_i} = \begin{cases} 
\omega_{t+1}^F R_{t+1}^F b_i^t - R_{t}^D d_i^t & \omega_{t+1}^{\phi} \leq \omega_{t+1}^F \\
\omega_{t+1}^F R_{t+1}^F b_i^t - R_{t}^D d_i^t - \gamma^B b_i^t & \omega_{t+1}^{\phi} \leq \omega_{t+1}^F < \omega_{t+1}^{\phi} \\
0 & \omega_{t+1}^F < \omega_{t+1}^{\phi} \end{cases}$$  (3.20)
In the following, we introduce notation that is analogous to the entrepreneurial sector. Let \( F_t^F \) denote the probability of bank default, such that

\[
F_{t+1}^F = F^F(\omega_{t+1}^F) \equiv \int_0^{\omega_{t+1}^F} f(\omega_{t+1})d\omega_{t+1}.
\]

(3.21)

The share of the return on loans subject to bank defaults is defined as

\[
G_{t+1}^F = G^F(\omega_{t+1}^F) \equiv \int_0^{\omega_{t+1}^F} \omega_{t+1} f(\omega_{t+1})d\omega_{t+1}.
\]

(3.22)

When the bank resolution authority monitors a failed bank, a fraction of this share, represented by \( \mu^F \), is lost.

Finally, the share of operating profits (gross of bank default costs) that will accrue to the depositors is

\[
\Gamma^F(\omega_{t+1}^F) = \int_0^{\omega_{t+1}^F} \omega_{t+1} f(\omega_{t+1})d\omega_{t+1} + \omega_{t+1} \int_{\omega_{t+1}^F}^{\infty} f(\omega_{t+1})d\omega_{t+1}.
\]

(3.23)

Figure (3.2) shows the bank productivity distribution and the two thresholds. If productivity is above the penalty threshold, \( \omega_i^F > \omega_i^\phi \), bank \( i \) fulfills the capital requirement and does not fail. If productivity is below the penalty threshold, \( \omega_i^F \leq \omega_i^\phi \), the bank does not fulfill the capital requirement and it has to pay a penalty equal to a proportion \( \gamma^F \) of the loans contracted in the previous period. If productivity is below the default threshold, \( \omega_i^F < \omega_i^F \), the bank fails and it is monitored by the bank resolution authority. Table 3.1 shows the division of the bank’s loan return in the three cases.

| \( \omega_t^F \) | Depositor Banker Bank resolution authority |
|----------------|-------------------------------|----------------------------------|
| \( \omega_t^\phi \) | \( R_t^{D_{t-1}} b_{t-1} \)  | \( \omega_t^F R_t^F b_{t-1} - R_t^{D_{t-1}} d_{t-1} > 0 \) | \( 0 \) |
| \( \omega_t^\phi \leq \omega_t^F < \omega_t^F \) | \( R_t^{D_{t-1}} b_{t-1} \)  | \( \omega_t^F R_t^F b_{t-1} - R_t^{D_{t-1}} d_{t-1} < 0 \) | \( \gamma^B b_{t-1} \) |
| \( \omega_t^F < \omega_t^\phi \) | \( R_t^{D_{t-1}} b_{t-1} \)  | \( \omega_t^F R_t^F b_{t-1} - R_t^{D_{t-1}} d_{t-1} = 0 \) | \( \omega_t^F R_t^F b_{t-1} - R_t^{D_{t-1}} d_{t-1} - \mu^F \omega_t^F R_t^F b_{t-1} \) |

Bank profits (3.20) can be aggregated across banks to yield

\[
\int_{\omega_t^F}^{\omega_{t+1}^F} (\omega_t^F R_t^F b_t^i - R_t^{D_t} d_t^i - \gamma^B b_t^i)dF^F(\omega_{t+1}^F) + \int_{\omega_t^F}^{\omega_{t+1}^F} (\omega_t^F R_t^F b_t^i - R_t^{D_t} d_t^i)dF^F(\omega_{t+1}^F).
\]

(3.24)

The first term in (3.24) are profits of banks with intermediate productivity that are required to pay the penalty. The second term in (3.24) are the profits of high-productivity banks. Profits of defaulting banks are zero. Using the default threshold (3.17) to replace \( R_t^D d_t^i \) with \( \omega_{t+1}^F R_{t+1}^F b_t^i \). 

Table 3.1: Division of bank loan return
and using the definition of $\Gamma^F(F_{i,t}^{F})$ in (3.23), we can rewrite aggregate bank profits as

$$\Xi_t^F = (1 - \Gamma_t^F(R_t^F b_t) - \gamma^B b_t[F_t^F(F_{t+1}^F) - F_t^F(F_{t+1}^F)].$$

The first term in (3.25) is the bank’s expected revenue after the bank has made interest payments to the depositors, but before the (possible) payment of the penalty. The second term represents the expected penalty payment.

Banks choose the volume of loans $b_t^i$ that maximizes profits (3.25), where we note the dependence of the threshold productivity levels, $\varpi_{t+1}^F$ and $\varpi_{t+1}^\phi$, on loans $b_t^i$. All banks behave the same in equilibrium, such that we drop the index $i$ from here on. Using simplified notation in (3.25), bank profits are given by

$$\Xi_t^F = (1 - \Gamma_t^F(R_t^F b_t) - \gamma^B b_t[F_t^F(F_{t+1}^F) - F_t^F(F_{t+1}^F)],$$

and the bank’s first order condition becomes

$$\mathbb{E}_t\left\{ R_t^F \left[ (1 - \Gamma_t^F) - \Gamma_t^F \frac{R_t^D n_t^B}{R_t^F b_t} \right] - \gamma^B \left[ F_{t+1}^F - F_{t+1}^F + \left( \frac{F_t^\phi}{1 - \phi_t} - F_t^F \right) \frac{R_t^D n_t^B}{R_t^D b_t} \right] \right\} = 0,$$

where $F_t^\phi = F^F(\varpi_t^\phi)$ and $F_t^F = F^F(\varpi_t^F)$.

**Bankers.** Following Gertler and Karadi (2011), households have a unit mass and consist of two types of people. A proportion $F$ of household members are bankers and the remaining $1 - F$ are
workers. Similar to the labor search-and-matching literature where only a fraction of household members are employed, consumption is nevertheless equalized across members through perfect intra-household risk sharing. Every period, some individuals switch between the two occupations. In particular, a person who is currently a banker has a constant probability $1 - \chi^B$ of remaining a banker in the next period, which is independent of the time already spent in the banking sector.\footnote{See, for instance, Merz (1995).} Every period, $(1 - \chi^B)F$ bankers thus quit banking and become workers. The same number of workers randomly become bankers, such that the proportions of bankers and workers within the household remain fixed. Bankers who quit transfer their wealth to their respective household. The household uses a fraction $\iota$ of this transfer to provide its new bankers with startup funds, as is described below.

A banker’s only investment opportunity is to provide equity to banks. We suppose that a banker holds a diversified portfolio of bank equity, by investing his net worth in all banks. Let $n^B_t$ denote the aggregate net worth of bankers. Bankers obtain an ex-post aggregate nominal return of $R^B_{t+1}$ on their investment, which determines their wealth in the next period,

$$W^B_{t+1} = \frac{R^B_{t+1}n^B_t}{\Pi_{t+1}}. \tag{3.28}$$

The return on equity is the ratio of bank profits to banker net worth,

$$R^B_{t+1} = \frac{\Xi^F_{t+1}}{n^B_t}. \tag{3.29}$$

Aggregate net worth of existing bankers is the wealth held by bankers at $t$ who are still around one period later, $(1 - \chi^B)W^B_{t+1}$. A banker who leaves the banking business turns his residual equity over to the household. Newly entering bankers receive startup funds from their respective households, which are a fraction $\iota$ of the value of exiting bankers’ wealth, i.e. $\iota\chi^B W^B_{t+1}$. Therefore, aggregate banker net worth is given by the sum of existing and new bankers’ net worth,

$$n^B_{t+1} = (1 - \chi^B + \iota\chi^B) W^B_{t+1}, \tag{3.30}$$

and bank profits retained by the households are $\Xi^FH_{t+1} = (1 - \iota)\chi^B W^B_{t+1}$.

### 3.3 Monetary and macroprudential policies

We now specify two types of macroeconomic policies: monetary policy and macroprudential policy. There are two dimensions in which these policies work: at the steady state and out of steady state. At the steady state, the policy maker chooses a target value for inflation, $\Pi$, and a bank capital

\footnote{The average lifetime of a banker is thus $1/(1 - \chi^B)$, where $\chi^B \in (0, 1)$. Bankers have a finite horizon such that they do not accumulate enough wealth to fund all investments without the need for external borrowing.}
requirement, \( \phi \). Out of steady state, inflation and the capital requirement are set according to feedback rules. We consider a monetary policy rule by which the central bank may adjust the policy rate in response to its own lag and inflation. The respective feedback coefficients are \( \tau_R \) and \( \tau_\Pi \), such that:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\tau_R} \left[ \left( \frac{\Pi_t}{\Pi} \right)^{\tau_\Pi} \right]^{1-\tau_R}.
\] (3.31)

Thanks to full deposit insurance financed through lump-sum taxation, the policy rate and the deposit rate are identical, \( R_t = R_t^D \). Macroprudential policy is given by a rule for the capital requirement,

\[
\frac{\phi_t}{\phi} = \left( \frac{\phi_{t-1}}{\phi} \right)^{\zeta_\phi} \left[ \left( \frac{b_t}{b} \right)^{\zeta_b} \right]^{1-\zeta_\phi}.
\] (3.32)

The objective of monetary policy is to stabilize inflation so as to minimize the price adjustment costs that firms face. The objective of macroprudential policy is to stabilize the bank default rate so as to minimize the bank resolution costs incurred by taxpayers in the case of bank failures.

### 3.4 Rest of the model

The remainder of the model is a standard New Keynesian setup. Households choose their optimal consumption and labor supply within the period, and their optimal capital deposits across periods. Within the production sector, we distinguish between final goods producers, intermediate goods producers, and capital goods producers. Final goods producers are perfectly competitive. They create consumption bundles by combining intermediate goods using a constant-elasticity-of-substitution technology and sell them to the household sector and to capital producers. Intermediate goods producers use capital and labor to produce, with a Cobb-Douglas technology, the goods used as inputs by the final goods producers. They set prices subject to quadratic adjustment costs, which introduces a New Keynesian Phillips curve in our model. Finally, capital goods producers buy the final good and convert it to capital, which they sell to the entrepreneurs.

**Households.** Households are infinitely lived and have expected lifetime utility,

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \ln c_t - \varphi \frac{l_t^{1+\eta}}{1+\eta} \right),
\] (3.33)

where \( \beta \in (0, 1) \) is the subjective discount factor, \( c_t \) is consumption, \( l_t \) is labor supply, \( \varphi > 0 \) is the relative weight on labor disutility and \( \eta \geq 0 \) is the inverse Frisch elasticity of labor supply. The household chooses paths for \( c_t, l_t \) and capital deposits \( d_t \) to maximize (3.33) subject to a sequence of budget constraints,

\[
c_t + d_t + t_t \leq w_t l_t + \frac{R_t^D d_{t-1}}{\Pi_t} + \Xi_t^K + \Xi_t^P + \Xi_t^{EH} + \Xi_t^{FH},
\] (3.34)
where $t_t$ are lump sum taxes (in terms of the final consumption good), $w_t$ is the real wage, $R_t^D$ is the gross interest rate on deposits paid in period $t$, $\Xi_t^K$ and $\Xi_t^P$ are capital goods producers’ and intermediate goods producers’ profits, respectively, which are redistributed to households in a lump sum fashion. Additionally, $\Xi_t^{EH}$ and $\Xi_t^{EH}$ represent the profits from exiting bankers and entrepreneurs, respectively, after deducting the startup funding granted to entrants.

The household’s first order optimality conditions can be simplified to a labor supply equation, $w_t = \varphi_t^d / \Lambda_t$, and a consumption Euler equation, $1 = \mathbb{E}_t \{ \beta_{t,t+1} R_{t+1}^D / \Pi_{t+1} \}$, where $\beta_{t,t+s} = \beta^s \Lambda_{t+s} / \Lambda_t$ is the household’s stochastic discount factor between $t$ and $t+s$ and the Lagrange multiplier on the budget constraint (3.34), $\Lambda_t = 1/c_t$, captures the shadow value of household wealth in real terms.

**Final goods producers.** A final goods firm bundles the differentiated industry goods $Y_{it}$, with $i \in (0,1)$, taking as given their price $P_{it}$, and sells the output $Y_t$ at the competitive price $P_t$. The optimization problem of the final goods firm is to choose the amount of inputs $\beta_t$ to maximize per-period profits given by $P_t Y_t - \int_0^1 Y_{it} d_{it}^\varepsilon$, subject to the production function $Y_t = \left( \int_0^1 Y_{it}^{(\varepsilon-1)/\varepsilon} d_{it}^\varepsilon \right)^{\varepsilon/(\varepsilon-1)}$, where $\varepsilon > 1$ is the elasticity of substitution between intermediate goods. The resulting demand for intermediate good $i$ is $Y_{it}^d = (P_{it}/P_t)^{-\varepsilon} Y_t$. The price of final output, which we interpret as the price index, is given by $P_t = \left( \int_0^1 P_{it}^{1-\varepsilon} d_{it} \right)^{1/(1-\varepsilon)}$. In a symmetric equilibrium, the price of a variety and the price index coincide, $P_t = P_{it}$.

**Intermediate goods producers.** Firms use capital and labor to produce intermediate goods according to a Cobb-Douglas production function. The assumption of constant returns to scale allows us to write the production function as an aggregate relationship. Each individual firm produces a differentiated good using $Y_{it} = AK_{it-1}^{\alpha} l_{it-1}^{1-\alpha}$, where $\alpha \in (0,1)$ is the capital share in production, $A$ is aggregate technology, $K_{it-1}$ is capital and $l_{it}$ is labor. Intermediate goods firms choose factor inputs to maximize per-period profits given by $P_{it} Y_{it} / P_t - r_t^K K_{it-1} - w_t l_{it}$, where the real rental rate on capital $r_t^K$ and the real wage $w_t$ are taken as given, subject to the technological constraint and the demand constraint. The resulting demands for capital and labor are $w_t l_{it} = (1-\alpha) s_{it} Y_{it}$ and $r_t^K K_{it-1} = \alpha s_{it} Y_{it}$, respectively, where the Lagrange multiplier on the demand constraint, $s_{it}$, represents real marginal costs. By combining the two factor demands, we obtain an expression showing that real marginal costs are symmetric across producers,

$$s_t = \frac{w_t^{1-\alpha} (r_t^K)^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha} A} 1.$$  \hspace{1cm} (3.35)

Firm $i$ sets an optimal path for its product price $P_{it}$ to maximize the present discounted value of future profits, subject to the demand constraint and to price adjustment costs,

$$\mathbb{E}_t \sum_{s=0}^\infty \beta_{t,t+s} \left[ \frac{P_{it+s} Y_{it+s}^d}{P_{it+s}} - \frac{\kappa_p}{2} \left( \frac{P_{it+s}}{P_{it+s-1}} - 1 \right)^2 Y_{it+s} + s_{it+s} (Y_{it+s} - Y_{it+s}^d) \right].$$  \hspace{1cm} (3.36)
Price adjustment costs are given by the second term in square brackets in (3.36); they depend on firm revenues and on last period’s aggregate inflation rate. The parameter $\kappa_p > 0$ scales the price adjustment costs. Under symmetry, all firms produce the same amount of output, and the firm’s price $P_t$ equals the aggregate price level $P_t$, such that the price setting condition is

$$\kappa_p \Pi_t (\Pi_t - 1) = \varepsilon s_t - (\varepsilon - 1) + \kappa_p \mathbb{E}_t \left\{ \beta_{t,t+1} \Pi_{t+1} \left( \frac{Y_{t+1}}{Y_t} - 1 \right) \right\} .$$  (3.37)

In (3.37), perfectly flexible prices are given by $\kappa_p \to 0$. Under symmetry across intermediate goods producers, profits (in real terms) are thus $\Xi^P_t = Y_t - r^K_t K_{t-1} - w_t l_t - 0.5 \cdot \kappa_p (\Pi_t - 1)^2 Y_t$.

**Capital goods production.** The representative capital-producing firm chooses a path for investment $I_t$ to maximize profits given by $\mathbb{E}_t \sum_{s=0}^{\infty} \beta_{t,s} [q_{t+s} I_{t+s} - (1 + g_{t+s}) I_{t+s}]$. The term $g_t = 0.5 \cdot \kappa_I (I_t / I_{t-1} - 1)^2$ captures investment adjustment costs as in Christiano et al. (2005). Capital accumulation is defined as:

$$I_t = K_t - (1 - \delta) K_{t-1},$$  (3.38)

where $\delta \in (0,1)$ is the capital depreciation rate. The optimality condition for investment is given by:

$$1 = q_t - \frac{\kappa_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa_I \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \mathbb{E}_t \left\{ \beta_{t,t+1} \kappa_I \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\} .$$  (3.39)

Capital producers’ period-$t$ profits, in real terms, are $\Xi^K_t = q_t I_t - (1 + g_t) I_t$.

**Bank resolution authority.** The losses of a failed bank are taken over by a bank resolution authority, which is funded through lump sum taxes on households, $t_t$ and the penalty payments of banks. As shown in Table 3.1, the outlays of the bank resolution authority are given by

$$\int_0^{\Xi^F_i} \left[ \mu^F \omega^F_i R^F_i b^i_{t-1} - (\omega^F_i R^F_i b^i_{t-1} - R^D_i d^i_{t-1}) \right] f^F(\omega^F_i) d\omega^F_i - \gamma^B b_{t-1} [F^\phi_t - F^F_t].$$

As shown in Appendix B.1, this implies that the lump sum taxes paid by the household are

$$t_t = \left[ \Xi^F_t - \left( \Gamma^F_t - \mu^F G^F_t \right) \right] \frac{R^F_t b_{t-1}}{\Pi_t} - \gamma^B b_{t-1} [F^\phi_t - F^F_t] .$$  (3.40)

**Market clearing.** Consumption goods produced must equal goods demanded by households, goods used for investment, resources lost when adjusting prices and investment, as well as resources
lost in the recovery of funds associated with entrepreneur and bank defaults,
\[
Y_t = c_t + (1 + g_t)I_t + \frac{\kappa_p}{2} (\Pi_t - 1)^2 Y_t + \mu^F G_t^F R_t^F b_{t-1} + \mu^E G_t^E R_t^E q_{t-1} K_{t-1}. \tag{3.41}
\]
Firms’ labor demand must equal labor supply, \((1 - \alpha)s_t Y_t / l_t = \varphi_t l_t^\eta / \Lambda_t\).

**Aggregate uncertainty.** As noted above, the random variable \(\omega_{t+1}^E\) has a log-normal distribution with mean one and standard deviation \(\sigma_t^E = \sigma^E \varsigma_t^E\), which introduces time variability of firm risk via the autoregressive processes,
\[
\ln \varsigma_t^E = \rho^E \ln \varsigma_{t-1}^E + \varepsilon_t^E, \tag{3.42}
\]
with \(\rho^E \in (0, 1)\). Let the parameter \(\sigma_\varsigma\) denote the standard deviations of the iid normal shock \(\varepsilon_t^E\).

We are now ready to provide a formal definition of equilibrium in our economy.

**Definition 1.** An equilibrium is a set of allocations \(\{l_t, K_t, I_t, c_t, Y_t, n_t^E, b_t, n_t^B, d_t, x_t^E\}_{t=0}^\infty\), prices \(\{w_t, r_t^K, q_t, \Pi_t, s_t\}_{t=0}^\infty\) and rates of return \(\{R_t^F, R_t^E, R_t^B\}_{t=0}^\infty\) for which, given the monetary and macroprudential policies \(\{R_t, \phi_t\}_{t=0}^\infty\) and shocks to firm risk \(\{\varsigma_t^E\}_{t=0}^\infty\) entrepreneurs maximize the expected return on their investment, firms and banks maximize profits, households maximize utility and all markets clear.

A summary of the model equations is provided in Appendix B.2.

### 3.5 Calibration

In the model, a time period is interpreted as one quarter. We normalize risk shocks in steady state by setting \(\varsigma_t^E = 1\). We also normalize labor, \(l = 1\), and set the weight on labor disutility, \(\varphi\), to meet this target. The calibration of our model parameters is summarized in Table 3.2. We set \(\Pi = 1\) to obtain a steady state with zero inflation. The subjective discount factor \(\beta\) is set to 0.99, implying a quarterly risk-free (gross) nominal interest rate of \(R = 1/0.99\). The inverse of the Frisch elasticity of labor supply is set to \(\eta = 1\), as in Christiano et al. (2014). This value lies in between the micro estimates of the Frisch elasticity, which are typically below 1, and the calibrated values used in macro studies, which tend to be above 1. As is standard in the literature (see Bernanke et al. (1999) and Carlstrom et al. (2016), among many others), the capital share in production is set to \(\alpha = 0.35\), while the depreciation rate is \(\delta = 0.025\), such that 10% of the capital stock has to be replaced each year. The substitution elasticity between goods varieties is \(\varepsilon = 6\), implying a gross steady state markup of \(\varepsilon / (\varepsilon - 1) = 1.2\) (Christensen and Dib, 2008). The Rotemberg price adjustment cost parameter is \(\kappa_p = 20\), which corresponds to a price duration of around 3 quarters in the Calvo model of staggered price adjustment; that value is in line with the duration implied by the posterior estimate of the Calvo parameter in Smets and Wouters (2007).\(^7\)

\(^7\) For the algebraic relationship between the Rotemberg and the Calvo parameter see Cantore et al. (2014).
The investment adjustment cost parameter is set to $\kappa_I = 2.43$, the estimate of Carlstrom et al. (2014) for the indexation-to-$R^k$ model. The financial parameters and interest rates are displayed in last part of Table 3.2.

We set the coefficients of the monetary policy rule for lagged policy rate and inflation to 0.8 and 1.5, respectively. The coefficient for the inflation rate is the value suggested by Taylor (1999) and Gertler and Karadi (2011).

We first discuss the financial parameters, before turning to the various interest rates and corresponding spreads in steady state. Following Bernanke et al. (1999), we calibrate the ratio of capital to net worth, $qK/n^E$, equal to 2; and (ii) a quarterly entrepreneur default rate of $F^E = 0.0075$, which corresponds to an annual default rate of 3%. We choose the fraction of realized payoffs lost in bankruptcy, $\mu^E$, to match the spread between the return on capital and the deposit rate, $R^E/R^D$ in the data. The spread is obtained taking the average of the spread of Gilchrist and Zakrajek (2012a) between January 2005 and June 2020 and is equal to 238 basis points per year. As far as the banking sector is concerned, we calibrate a steady state capital requirement for banks, i.e. the ratio of equity to loans, of 8%, that is $\phi = 0.08$ in line with the

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
<th>Target/Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.99$</td>
<td>Household discount factor</td>
<td>Bernanke et al. (1999)</td>
</tr>
<tr>
<td>$\eta = 1$</td>
<td>Inverse Frisch labor elasticity</td>
<td>Christiano et al. (2014)</td>
</tr>
<tr>
<td>$\alpha = 0.35$</td>
<td>Capital share in production</td>
<td>Bernanke et al. (1999)</td>
</tr>
<tr>
<td>$\delta = 0.025$</td>
<td>Capital depreciation rate</td>
<td>Bernanke et al. (1999)</td>
</tr>
<tr>
<td>$\varepsilon = 6$</td>
<td>Substitutability between goods</td>
<td>Christensen and Dib (2008)</td>
</tr>
<tr>
<td>$\kappa_p = 20$</td>
<td>Price adjustment cost</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>$\kappa_I = 2.43$</td>
<td>Investment adjustment cost</td>
<td>Carlstrom et al. (2014)</td>
</tr>
<tr>
<td>$\varphi = 0.662$</td>
<td>Weight on labor disutility</td>
<td>Labor supply = $l = 1$</td>
</tr>
<tr>
<td>$\chi^E = 0.019$</td>
<td>Entrepreneur exit rate</td>
<td>Entrepren. leverage = $qK/n^E = 2$</td>
</tr>
<tr>
<td>$\sigma^E = 0.271$</td>
<td>Entrepreneur risk volatility</td>
<td>Entrepreneurial default rate = $4F^E = 3%$ p.a.</td>
</tr>
<tr>
<td>$\mu^E = 0.08$</td>
<td>Entrepreneur monitoring cost</td>
<td>Entrepreneur return spread = $R^E/R^D - 1 = 238bp$</td>
</tr>
<tr>
<td>$\phi = 0.08$</td>
<td>Bank capital requirement</td>
<td>Basel Accords</td>
</tr>
<tr>
<td>$\mu^F = 0.3$</td>
<td>Bank monitoring cost</td>
<td>Clerc et al. (2018)</td>
</tr>
<tr>
<td>$\sigma^F = 0.055$</td>
<td>Bank risk volatility</td>
<td>Bank default rate = $4F^F = 0.9%$ p.a.</td>
</tr>
<tr>
<td>$\chi^B = 0.025$</td>
<td>Banker exit rate</td>
<td>Bank leverage = $b/n^B = 7$</td>
</tr>
<tr>
<td>$\gamma^B = 0.005$</td>
<td>Bank penalty</td>
<td>Bank equity return spread = $R^B/R^D - 1 = 600$ p.a.</td>
</tr>
<tr>
<td>$\iota = 0.002$</td>
<td>Transfer to new entr./bankers</td>
<td>Gertler and Kiyotaki (2010)</td>
</tr>
<tr>
<td>$\tau_R = 0.8$</td>
<td>Coeff. TR for lag policy rate</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>$\tau_I = 1.5$</td>
<td>Coeff. TR for inflation</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>$\zeta_b = 0$</td>
<td>Coeff. MP for capital requirement</td>
<td></td>
</tr>
</tbody>
</table>

Notes: See main text on the calibration of the US bank default rate and on the strategy to determine the shock parameters.
CHAPTER 3. FIRM RISK SHOCKS AND THE BANKING ACCELERATOR

Basel Accords. Bank monitoring costs are calibrated to $\mu^F = 0.3$ as in Clerc et al. (2018).\(^8\) Bank risk is equal to $\sigma^F = 0.055$ to target the bank default rate in the data. The banker turnover rate $\chi^B$ is calibrated at 0.025, to match a ratio of bank assets to bank net worth of 7. The value of $\chi^B$ is in the ballpark of the numbers found in the literature, e.g. Gertler and Kiyotaki (2010) and Angeloni and Faia (2013). As in Gertler and Karadi (2011), the proportional transfer to the entering entrepreneurs and bankers is set to $\iota = 0.002$. The bank capital requirement penalty is calibrated to 0.005 to target an equity return premium of 400 basis points.\(^9\)

In the following, we report the implied financial parameters and provide an interpretation for our results. In the corporate sector, we obtain a productivity cutoff of roughly one half, $\omega^E = 0.499$. In the banking sector, we find a default cutoff of $\omega^F = 0.855$ and a penalty cutoff of $\omega^\phi = 0.929$.

In our model, bank resolution costs are substantially higher than firm monitoring costs ($\mu^F > \mu^E$). This may reflect the greater opaqueness of bank balance sheets, which makes monitoring more difficult (Morgan, 2002). Moreover, the role of banks in financial intermediation suggests that the costs and externalities associated with bank failures are particularly high. E.g. Kupiec and Ramirez (2013) find that bank failures cause non-bank commercial failures and have long-lasting negative effects on economic growth. The implied probability of bank default $F^F$ lies below the value reported in de Walque et al. (2010) using the Z-score method to compute the probability that banks’ own funds are not sufficient to absorb losses, which yields 0.4% p.a., and the ratio of bank failures to the number of commercial banks, which is 0.9% p.a. for the period 1984-2015 according to the Federal Deposit Insurance Corporation.\(^10\) On the other hand, if we count bank closings rather than failures, we find a rate of 2.7% p.a. in US data.\(^11\) Our value therefore lies within this range of estimates. The implied bank leverage is calibrated at 7, an intermediate value between Gertler and Karadi (2011) and Gerali et al. (2010).

The risk-free rate corresponds to the deposit rate $R^D$ and to the policy rate $R$ and it is equal to 4.04% per year in steady state. The realized return on bank loans is $R^F$ and is equal to 5.08% per year in steady state. This return contains a discount which is related to the monitoring cost that the bank must incur when an entrepreneur declares default. The next higher rate of return is the return on capital, $R^E$ that is equal to 6.44% per year. The return on capital is higher than the realized loan return $R^F$, because it needs to compensate the entrepreneur for running the risk of default while it is not reduced by the monitoring cost. Finally, the return on equity earned by bankers $R^B$ is equal to 10.1% in steady state. This value exceeds the realized loan return, because it contains a compensation to bankers (or equity holders) for the risk of bank default.

A description of the steady state computation is provided in Appendix B.3.

---

\(^8\) Differently from the monitoring cost related to the entrepreneurial sector, bank monitoring costs $\mu^F$ do not affect the computation of the steady state financial variables. They appear only in the aggregate resource constraint.

\(^9\) The series of the spread is computed as the difference between the return on average equity for all U.S. banks and the 10-Year treasury constant maturity rate.

\(^10\) The annual number of bank failures in the US, starting in 1936, can be downloaded from www fdic gov.

3.6 Macroprudential policies at the steady state

Figure 3.3 displays the steady state effects of the bank penalty. The range of \( \gamma^b \) is from 0.005, its baseline value, to 0.1, a value around which the probability of paying the penalty is constantly at zero. As shown by the definition of bank profits (3.26), an increase in the bank penalty increases the costs banks face for being leveraged. Due to the higher costs, banks cut their supply of loans and thereby decrease their leverage. The fall in the loan supply also leads to a reduction in the amount of capital entrepreneurs buy from capital producers. Hence, firm leverage decreases as well. The reduction in bank and firm leverage implies a fall in the probabilities of default and in the probability of paying the penalty.

Figure 3.3: Steady state effects of different values of the bank penalty, \( \gamma^b \)

![Graphs showing the effects of different \( \gamma^b \) values on firm and bank failure rates, and bank and firm leverage.]

Figure 3.4 shows the steady state effects of the capital requirement. The range of \( \phi \) is dictated by the models region of stability and it is equal to \([0.08; 0.12]\).\(^{12}\) For a given level of bank leverage, a higher capital requirement increases the penalty threshold, as shown by the definition of the threshold (3.19). This increases the banks’ probability of paying the penalty. Hence, banks react by cutting their loan supply and decrease their leverage. As shown by the definition of the bank default threshold (3.17), the fall in leverage decreases the bank default threshold, thus decreasing the bank default rate. While equation (3.19) implies that there is a direct positive effect of the capital requirement on the penalty threshold, the negative general equilibrium effect due to the

\(^{12}\) Values of \( \phi \) higher than 0.12 give rise to unstable equilibria.
fall in bank leverage is larger. As a result, the probability of banks paying the penalty actually decreases with a higher capital requirement. A higher capital requirement implies that banks decrease their leverage by cutting their loan supply. The reduced amount of loans is translated into minor lending to firms, whose leverage decreases so does their failure rate.

4  Firm risk shocks and the banking sector

This section discusses the effects of bank fragility and capitalization on the transmission of firm risk shocks. First, we describe the transmission of the shock in the baseline model. We compare the predicted impulse responses to the ones generated by a variant of the model with a high level of bank penalty. Second, we investigate the effects of a higher bank capital requirement on the dynamics. Third, we study how risk shocks propagate with a countercyclical capital requirement.

4.1  The effect of bank penalty

Consider first the impulse responses of the baseline model, the one with low bank penalty, which are the red dashed lines in Figure 3.5. An exogenous increase in the standard deviation of project returns implies that investment projects become riskier and firms are thus more likely to default. The annual default rate of firms rises by more than 4 percentage points. As a result, the external finance premium (or firm risk spread) rises and entrepreneurs reduce their investment demand.
As investment falls, so do both output and inflation: we observe a demand-driven downturn. The central bank reacts by cutting the policy interest rate. Entrepreneurs’ reduced demand for capital implies that they borrow less. Due to higher firm defaults banks face higher losses, bank equity falls and the annual default rate of banks rises. The fall in equity is larger than the fall in loans and bank leverage increases.

To sum up, the shock generates responses similar to the ones reported in Christiano et al. (2014) and Nuño and Thomas (2017).

Let us now turn to the dynamic responses given by the dotted blue lines in Figure 3.5. The responses are originated by a model where banks face a higher penalty, meaning that it is more costly for them to be highly leveraged.

The response of the firm default rate and the external finance premium are similar to the previously discussed version of model; however, the impulse of the other variables differ. The effect of the shock on the macroeconomic variables, i.e. investment, output, inflation and the policy rate is reduced by the bank penalty. To understand the mechanisms behind this banking accelerator effect, we need to take a look at the banking sector variables.

The increase in firm risk has adverse consequences for banks, whose default probability increases on impact. However, when banks face a higher penalty, the rise in the bank default probability is mitigated due to the banks being better capitalized. The higher default rate among firms leads to a decline in bank profits, which, in turn, reduces the net worth of bankers. However, the higher penalty imposed on banks results in them being less vulnerable and experiencing a smaller reduction in net worth.
The fall in loans is similar to the baseline model. This is the result of two opposing forces. First, the higher bank penalty leads to a higher cost of being leveraged, causing banks to respond more strongly by reducing their loan supply. Second, with a higher bank penalty, banks need to cut their loan supply by less to maintain the same leverage because the decline in net worth due to the shock is lower.

Finally, the lower bank default rate attenuates the propagation of the firm risk shock to output and inflation. Hence, the central bank reduces the policy rate by less. The interpretation is that a higher bank penalty reduces bank fragility reducing the propagation of risk shocks.

4.2 The effect of capital requirements

We next consider the effect of a higher capital requirement on the propagation of firm risk shocks.

Consider the impulse responses of the model with a higher capital requirement, which are the dotted blue lines in Figure 3.6. Similarly to the responses of the model with a higher bank penalty, the effect of the shock on investment, output, inflation and the policy rate is reduced compared to the baseline model. A firm risk shock increases bank losses from nonperforming loans leading to a rise in bank default but because of the lower bank leverage, bank defaults rise less in the model with a higher capital requirement compared to the baseline model.
4. FIRM RISK SHOCKS AND THE BANKING SECTOR

Figure 3.7: IRFs with countercyclical capital requirement

4.3 Countercyclical capital requirement (CCyB)

We next consider a countercyclical capital requirement. We gauge the effectiveness of the policy framework in dampening business cycle fluctuations.

We allow for macroprudential policy to set a countercyclical bank capital requirement in response to changes in borrowing, such that $\zeta_b > 0$. The Basel III policy recommendation of a countercyclical capital buffer prescribes a rise in the capital requirement in response to a rise in the credit-to-GDP gap above a certain threshold, see Basel Committee on Banking Supervision (2010a,b).\(^{13}\)

Figure 3.7 shows that a countercyclical capital buffer is effective in dampening the negative effects of a firm risk shock on the macroeconomic variables. The drop in investment, output and inflation is reduced when the CCyB requirement is activated. This is because the reduction in the capital requirement allows banks to lend more than if the CCyB coefficient were zero. As a result, the drop in loans is reduced. The policy rate falls by less in response to the shock, which indicates that the burden on monetary policy to smooth the business cycle is alleviated by macroprudential policy.

However, Figure 3.7 reveals a drawback of the CCyB: an increase in bank fragility. The rise in bank defaults due to a risk shock is stronger when a CCyB is in place because banks increase their leverage to a larger extent to provide loans to entrepreneurs.

\(^{13}\)Tente et al. (2015, p.14) discuss how the CCyB rate is computed for Germany. The credit-to-GDP gap, at present, is computed using the HP filter Hodrick and Prescott (1997).
5 Conclusion

The Covid-19 pandemic has focused attention on heightened default risk facing firms. How do such shocks propagate through the economy and how is their transmission affected by the health of the banking sector? A vector autoregression analysis with US data shows that bank risk rises significantly in response to exogenous increases in firm risk, suggesting that banks are not fully insulated from the negative effects of corporate defaults. We analyze firm risk shocks in a business cycle model featuring leveraged firms and banks. Macroprudential regulation imposes a capital ratio on banks and charges a penalty on deviating from the required capital buffer. This implies banks have some ‘skin in the game’, which affects their lending decisions. We show that the penalty and the capital requirement are effective in decreasing bank leverage inducing a positive effect on bank stability. However, this has a negative effect on loan supply increasing the spread on the loan rate and the entrepreneurial spread. A penalty on banks that fall short of the regulatory capital requirement helps also to dampen the effects of firm risk shocks on the macroeconomy. We also consider the implications of a countercyclical capital requirement. We find that this policy is effective in reducing output and investment fluctuations. However, a countercyclical capital requirement leads to a stronger response of bank defaults to risk shocks.
## Appendix A  Empirical evidence

This section provides a detailed description of the data used in the VAR exercise, describes the VAR model and reports additional results.

### A.1 Data

Table 3.A.1 gives a detailed overview of the data series used in our analysis.

<table>
<thead>
<tr>
<th>Series</th>
<th>Source</th>
<th>Identifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real GDP</td>
<td>FRED</td>
<td>GDPC1</td>
</tr>
<tr>
<td>Industrial production</td>
<td>FRED</td>
<td>INDPRO</td>
</tr>
<tr>
<td>Consumer Price Index</td>
<td>FRED</td>
<td>CPIAUCSL</td>
</tr>
<tr>
<td>Firm cross-sectional implied volatility</td>
<td>Dew-Becker and Giglio (2020)</td>
<td>idio_iv</td>
</tr>
<tr>
<td>Spread per unit of leverage (SPL) (%)</td>
<td>Bundesbank</td>
<td>Internal data</td>
</tr>
<tr>
<td>Non-financial excess bond premium (%)</td>
<td>Gilchrist and Zakrajek (2012a)</td>
<td>gzspr_nf</td>
</tr>
<tr>
<td>Financial excess bond premium (%)</td>
<td>Gilchrist and Zakrajek (2012b)</td>
<td>gzspr_f</td>
</tr>
<tr>
<td>Spot funding spread (%)</td>
<td>Jondeau et al. (2020)</td>
<td>SFS_US</td>
</tr>
<tr>
<td>Forward funding spread (%)</td>
<td>Jondeau et al. (2020)</td>
<td>FFS_US</td>
</tr>
<tr>
<td>Federal funds rate (%)</td>
<td>FRED</td>
<td>FEDFUNDS</td>
</tr>
<tr>
<td>Euro Area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real GDP</td>
<td>Eurostat (retrieved from FRED)</td>
<td>CLVMEURSCAB1GQEAE19</td>
</tr>
<tr>
<td>Industrial production</td>
<td>ECB SDW</td>
<td>STS.M.1.B.Y.PROD.NS0010.4.000</td>
</tr>
<tr>
<td>HICP</td>
<td>ECB SDW</td>
<td>ICP.M.U2.Y.000000.3.INX</td>
</tr>
<tr>
<td>Spread per unit of leverage (SPL) (%)</td>
<td>Bundesbank</td>
<td>Internal data</td>
</tr>
<tr>
<td>Non-financial corporate excess bond premium (%)</td>
<td>Gilchrist and Mojon (2018)</td>
<td>spr_nfc_bund_ea</td>
</tr>
<tr>
<td>Financial corporate excess bond premium (%)</td>
<td>Gilchrist and Mojon (2018)</td>
<td>spr_bk_bund_ea</td>
</tr>
<tr>
<td>Spot funding spread (%)</td>
<td>Jondeau et al. (2020)</td>
<td>SFS_EA</td>
</tr>
<tr>
<td>Forward funding spread (%)</td>
<td>Jondeau et al. (2020)</td>
<td>FFS_EA</td>
</tr>
<tr>
<td>EONIA (%)</td>
<td>Bundesbank</td>
<td>BBK01.SU0304</td>
</tr>
</tbody>
</table>

Some of the series were transformed before estimation. Data on real GDP is not available at a monthly frequency. For this reason we imputed the missing values using the Chow and Lin (1971) method as it is done in Gambacorta et al. (2014) and Boeckx et al. (2017). We use seasonally and working day adjusted data for HICP and industrial production in the Euro Area. The data for CPI and industrial production in the US and real GDP for Euro Area and US are seasonally adjusted. The data for the firm cross-sectional implied volatility of Dew-Becker and Giglio (2020) is obtained from [http://www.dew-becker.org/](http://www.dew-becker.org/). The authors obtain the firm cross-sectional implied volatility following the steps below. They construct the linear projection of the return on stock \( i \), \( R_{i,t} \), on the market return \( R_{mkt,t} \) – where both returns are defined as the excess return over the risk-free rate – as

\[
R_{i,t} = \beta_i R_{mkt,t} + \varepsilon_{i,t}. \tag{3.A.1}
\]
The excess return on stock $i$ can be decomposed as the sum of the market return and a firm-specific component,

$$R_{i,t} = R_{mkt,t} + \epsilon_{i,t}. \quad (3.A.2)$$

Now, subtracting (3.A.2) from (3.A.1) and rearranging, we obtain

$$\epsilon_{i,t} = (\beta_i - 1)R_{mkt,t} + \bar{\epsilon}_{i,t}. \quad (3.A.3)$$

Applying the variance function to both sides of (3.A.2), the variance of the return on stock $i$ is

$$\text{Var}(R_{i,t}) = \text{Var}(R_{mkt,t}) + \text{Var}(\epsilon_{i,t}) + 2\text{Cov}(R_{mkt,t}, \epsilon_{i,t}). \quad (3.A.4)$$

Now, taking the covariance of both sides of (3.A.3) and the market return $R_{mkt,t}$, and exploiting the fact that $R_{mkt,t}$ and $\bar{\epsilon}_{i,t}$ are uncorrelated by virtue of the CAPM equation (3.A.1), we obtain

$$\text{Var}(R_{i,t}) = \text{Var}(R_{mkt,t}) + \text{Var}(\epsilon_{i,t}) + 2(\beta_i - 1)\text{Var}(R_{mkt,t}).$$

The weighted conditional variance for $R_{i,t}$ is

$$\sum_i \omega_i \text{Var}(R_{i,t}) = \text{Var}(R_{mkt,t}) + \sum_i \omega_i \text{Var}(\epsilon_{i,t}), \quad (3.A.5)$$

since $\sum_i \omega_i \beta_i = 1$.\(^{14}\) It is possible to rearrange (3.A.5) in order to obtain the firm cross-sectional implied volatility

$$\sum_i \omega_i \text{Var}(\epsilon_{i,t}) = \sum_i \omega_i \text{Var}(R_{i,t}) - \text{Var}(R_{mkt,t}),$$

where $\text{Var}(R_{mkt,t})$ is the S&P 500 option-implied volatility, $\text{Var}(R_{i,t})$ is the cross-sectional option-implied volatility and $\omega_{i,t}$ are market capitalization weights. This approximation is accurate if $\beta_i \approx 1$. The option-implied volatilities are obtained using the BlackScholes formula for European options ignoring dividends using options price data from the Berkeley Options Database (BODB) for 1/1980 to 6/1995, and Optionmetrics for 1/1996 to 12/2020.

The data on spread per unit of leverage (SPL) are obtained combining end-of-the-month CDS-spread data in basis points from Markit and quarterly end-of-the-period debt and equity data from Bloomberg for all the non-financial firms in the EURO Stoxx 50 and Dow Jones 30. After computing the SPL for every firm as the ratio of CDS-spread to debt divided by equity, the aggregate SPL is given by the median across firms. The corporate bond spread of Gilchrist and Mojon (2018) is obtained from [https://publications.banque-france.fr/en/economic-and-financial-publications-working-papers/credit-risk-euro-area](https://publications.banque-france.fr/en/economic-and-financial-publications-working-papers/credit-risk-euro-area).

\(^{14}\)Note that $\sum_i \omega_i \beta_i$ is the beta of the portfolio. Since the portfolio used is the market portfolio, the beta of the portfolio return with the market return is 1.
Figure 3.A.1: Variables as they enter the US VAR

Table 3.A.2: Summary statistics baseline period - US

<table>
<thead>
<tr>
<th></th>
<th>SPL</th>
<th>Corp. Spread</th>
<th>Bank Spread</th>
<th>SFS</th>
<th>FFS</th>
<th>CS Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>0.5075</td>
<td>2.110</td>
<td>1.855</td>
<td>0.1650</td>
<td>0.2069</td>
<td>0.1888</td>
</tr>
<tr>
<td>Mean</td>
<td>0.7312</td>
<td>2.376</td>
<td>2.4053</td>
<td>0.2923</td>
<td>0.2791</td>
<td>0.2059</td>
</tr>
<tr>
<td>Std</td>
<td>0.4581</td>
<td>1.092</td>
<td>1.902</td>
<td>0.3293</td>
<td>0.2132</td>
<td>0.0548</td>
</tr>
</tbody>
</table>


The non-financial and financial corporate excess bond premia of Gilchrist and Zakrajek (2012a) and Gilchrist and Zakrajek (2012b) are obtained from http://people.bu.edu/sgilchri/Data/data.htm. The spot and forward credit spreads of Jondeau et al. (2020) are obtained from https://sites.google.com/site/sahuceconomics/Bank-Funding-Cost-Indicators. The spot funding spread is given by the three-months IBOR (Interbank Offered Rate)-OIS (Overnight Interest Swap) spread.
### Table 3.A.3: Summary statistics - US

<table>
<thead>
<tr>
<th></th>
<th>SPL Corp. Spread</th>
<th>Bank Spread</th>
<th>Fed funds</th>
<th>SFS FFS</th>
<th>CS Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>0.4795</td>
<td>1.770</td>
<td>1.5661</td>
<td>3.250</td>
<td>0.1650</td>
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<tr>
<td>Mean</td>
<td>0.6953</td>
<td>2.054</td>
<td>1.7553</td>
<td>3.544</td>
<td>0.2923</td>
</tr>
<tr>
<td>St.d</td>
<td>0.4428</td>
<td>0.9469</td>
<td>1.206</td>
<td>2.79</td>
<td>0.329</td>
</tr>
</tbody>
</table>


### Table 3.A.4: Correlations baseline period - US

<table>
<thead>
<tr>
<th></th>
<th>SPL Corp. Spread</th>
<th>Bank Spread</th>
<th>SFS FFS</th>
<th>CS Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPL Corp. Spread</td>
<td>0.78****</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank Spread</td>
<td>0.78****</td>
<td>0.95****</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SFS</td>
<td>0.52****</td>
<td>0.79****</td>
<td>0.76****</td>
<td></td>
</tr>
<tr>
<td>FFS</td>
<td>0.63****</td>
<td>0.82****</td>
<td>0.84****</td>
<td>0.90****</td>
</tr>
<tr>
<td>CS Volatility</td>
<td>0.51****</td>
<td>0.85****</td>
<td>0.90****</td>
<td>0.80****</td>
</tr>
</tbody>
</table>

Notes: $p < 0.0001$ ****, $p < 0.001$ ***, $p < 0.01$ **, $p < 0.05$ *. Sample period: Jan 2005 - Jun 2020.

### Table 3.A.5: Correlations - US

<table>
<thead>
<tr>
<th></th>
<th>SPL Corp. Spread</th>
<th>Bank Spread</th>
<th>SFS FFS</th>
<th>CS Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPL Corp. spread</td>
<td>0.76****</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank spread</td>
<td>0.79****</td>
<td>0.83****</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SFS</td>
<td>0.52****</td>
<td>0.79****</td>
<td>0.76****</td>
<td></td>
</tr>
<tr>
<td>FFS</td>
<td>0.63****</td>
<td>0.82****</td>
<td>0.84****</td>
<td>0.90****</td>
</tr>
<tr>
<td>CS Volatility</td>
<td>0.49****</td>
<td>0.49****</td>
<td>0.41****</td>
<td>0.80****</td>
</tr>
</tbody>
</table>

Notes: $p < 0.0001$ ****, $p < 0.001$ ***, $p < 0.01$ **, $p < 0.05$ *. Sample period: Jan 1985 - Jun 2020.

### Table 3.A.6: Summary statistics baseline period - Euro Area

<table>
<thead>
<tr>
<th></th>
<th>SPL Corp. Spread</th>
<th>Bank Spread</th>
<th>SFS FFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>0.7980</td>
<td>1.340</td>
<td>1.455</td>
</tr>
<tr>
<td>Mean</td>
<td>0.8981</td>
<td>1.405</td>
<td>1.545</td>
</tr>
<tr>
<td>Std</td>
<td>0.5114</td>
<td>0.6268</td>
<td>0.8242</td>
</tr>
</tbody>
</table>


### Table 3.A.7: Summary statistics - Euro Area

<table>
<thead>
<tr>
<th></th>
<th>SPL Corp. Spr.</th>
<th>Bank Spr.</th>
<th>EONIA</th>
<th>SFS FFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>0.7210</td>
<td>1.295</td>
<td>1.210</td>
<td>0.815</td>
</tr>
<tr>
<td>Mean</td>
<td>0.8543</td>
<td>1.355</td>
<td>1.306</td>
<td>1.469</td>
</tr>
<tr>
<td>St.d.</td>
<td>0.4972</td>
<td>0.5790</td>
<td>0.7956</td>
<td>1.702</td>
</tr>
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</table>

Figure 3.A.2: Variables as they enter the Euro Area VAR

Table 3.A.8: Correlations baseline period - Euro Area

<table>
<thead>
<tr>
<th></th>
<th>SPL</th>
<th>Corp. Spread</th>
<th>Bank Spread</th>
<th>SFS</th>
<th>FFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPL</td>
<td>0.84****</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corp. Spread</td>
<td></td>
<td>0.90****</td>
<td>0.87****</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank Spread</td>
<td></td>
<td></td>
<td>0.68****</td>
<td>0.43****</td>
<td></td>
</tr>
<tr>
<td>SFS</td>
<td></td>
<td></td>
<td></td>
<td>0.53****</td>
<td>0.94****</td>
</tr>
<tr>
<td>FFS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: $p < 0.0001$ ****, $p < 0.001$ ***, $p < 0.01$ **, $p < 0.05$ *. Sample period: Jan 2005 - Jun 2020.
### Table 3.A.9: Correlations - Euro Area

<table>
<thead>
<tr>
<th></th>
<th>SPL</th>
<th>Corp. Spr.</th>
<th>Bank Spr.</th>
<th>SFS</th>
<th>FFS</th>
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<td>SPL</td>
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<td></td>
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<td>0.78****</td>
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<tr>
<td>Bank Spread</td>
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<td>0.68****</td>
<td>0.43****</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SFS</td>
<td>0.66****</td>
<td>0.71****</td>
<td>0.53****</td>
<td>0.94****</td>
<td></td>
</tr>
<tr>
<td>FFS</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

*Notes: $p < 0.0001$ ****; $p < 0.001$ ***, $p < 0.01$ **, $p < 0.05$ *. Sample period: Jan 1999 - Oct 2020.

## A.2 VAR model

We estimate a vector autoregression model, which in reduced-form is given by

$$ y_t = c + \sum_{j=1}^{p} B_j y_{t-j} + u_t, \quad \text{with } t = 1, \ldots, T, \quad (3.A.6) $$

where $y_t$ is an $N \times 1$ vector of endogenous variables, $u_t \sim N(0, \Sigma)$ is an $N \times 1$ vector of reduced form residuals, $c$ is an $N \times 1$ intercept vector and $B_j$ are $N \times N$ matrices containing the VAR slope coefficients. In the specifications we opted for a lag length of $p = 2$ that is the lag length suggested by the Hannan-Quinn criterion for the baseline model. We choose the Hannan-Quinn criterion because it provides a good compromise between model parsimony and in-sample fit.\(^{15}\)

We propose the following selection of endogenous variables:

$$ y_t = [y_t, p_t, s_{t}^{nf}, s_{t}^{f}]^\prime, $$

where $y_t$ denotes either output measured as GDP (baseline) or industrial production, $p_t$ denotes the price index, $s_{t}^{nf}$ denotes a proxy for corporate risk that can be either firm cross-sectional implied volatility (baseline), spread per unit of leverage or non-financial corporate excess bond premium, $s_{t}^{f}$ denotes a proxy for bank risk that can be either the spot funding spread (baseline), the forward funding spread or the financial corporate excess bond premium. Output and the price index are in logarithms.

To conduct a structural analysis, we identify the following model,

$$ A_0 y_t = a + \sum_{j=1}^{p} A_j y_{t-j} + e_t, \quad \text{with } t = 1, \ldots, T, \quad (3.A.7) $$

where $A_0$ is an $N \times N$ matrix such that $A_j = A_0 B_j$, $a = A_0 c$ and $e_t = A_0 u_t$ with $e_t \sim N(0, I_N)$, $I_N$ is the $N \times N$ identity matrix and $E(u_t u_t') = (A_0 A_0)^{-1} = \Sigma$ is the covariance matrix of the

\(^{15}\)Throughout the analysis and across different specifications, we keep the specification comparable by using the same lag length.
VAR residuals. Since the estimated model (3.6) does not allow us to identify the structural form (3.7) without additional assumptions, we impose identifying restrictions on the impulse response functions (IRFs) to shocks. The literature has developed several methods to determine $A_0$ based on economic considerations. We identify firm risk shocks using a zero-restrictions approach. In particular, we assume that output is not contemporaneously affected by movements in the price index, by firm risk shocks and by changes in financial corporate risk. Furthermore, we assume that the price level is not contemporaneously affected by firm risk shocks and movements in bank risk. Finally, we assume that firm risk does not contemporaneously react to changes in bank risk. The assumptions on $A_0$ can be summarized as

$$
\begin{pmatrix}
  u^f_t \\
  u^p_t \\
  u^{snf}_t \\
  u^{sf}_t
\end{pmatrix} =
\begin{bmatrix}
  a_{11} & 0 & 0 & 0 \\
  a_{21} & a_{22} & 0 & 0 \\
  a_{31} & a_{32} & a_{33} & 0 \\
  a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\begin{pmatrix}
  e^f_t \\
  e^p_t \\
  e^{snf}_t \\
  e^{sf}_t
\end{pmatrix}
\equiv A_0^{-1}
$$

In other words, the contemporaneous impact matrix $A_0^{-1}$ is restricted to be lower triangular.

### A.3 Results

**Baseline.** We estimate impulse response functions to a one-standard-deviation firm risk shock for the US and the Euro Area; they are plotted in Figures (3.3)-(3.10). Figure (3.3) shows the estimated impulse responses of the baseline specification. The solid blue lines represent the mean responses and the dashed red lines represent the 95% bootstrapped confidence intervals obtained with 10000 replications. The responses of GDP and of the price index are multiplied by 100 in order to obtain percentage changes. We assume that a standard deviation shock hits the economy in period 0, due to the identification scheme, GDP and the price index are allowed to respond only with a one-period lag.

Figure (3.3) shows that a positive standard deviation corporate risk shock in the US induces a fall in GDP and in the price level, while it increases corporate and bank risk. The effect on firm risk is significant for approximately ten months, while the effect on bank risk is significant for five months. Also the effects on GDP and the price index are significant: in both cases for several periods after the shock hits.

**Robustness.** Figures (3.4-3.10) display the estimated impulse responses using different specifications from the baseline. Figure (3.4) illustrates the results obtained by replacing firm cross-sectional implied volatility with the excess bond premium of Gilchrist and Zakrajek (2012a), while Figure (3.5) shows the corresponding results for the Euro Area. Moreover, Figures (3.6) and (3.7) present the results of the specifications that include GDP, price index, SPL and SFS.

Figures (3.A.8) and (3.A.9) display the results obtained by using specifications that differ from the baseline solely because of the measure of bank risk. Specifically, Figures (3.A.8) and (3.A.9) plot the results obtained by replacing SFS with FFS and financial corporate excess bond premium, respectively.

Figure (3.A.10) presents the results obtained by including the federal funds rate in the baseline specification. The federal funds rate is included at the third position of the vector of endogenous variables, between the price index and firm risk. The results of this specification are similar to the baseline: a positive standard deviation firm risk shock significantly increases bank risk and decreases both output and the price index. Additionally, the results show that a firm risk shock decreases significantly the federal funds rate.

Overall, the robustness exercises yield results that are similar to the baseline specification. A standard deviation increase in firm risk significantly increases bank risk and significantly decreases GDP and the price index.
Figure 3.A.4: IRFs with non-financial corporate excess bond premium - US


Figure 3.A.5: IRFs with non-financial corporate excess bond premium - Euro Area

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Figure 3.A.6: IRFs with SPL - US


Figure 3.A.7: IRFs with SPL - Euro Area

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Figure 3.A.8: IRFs with FFS - US


Figure 3.A.9: IRFs with financial excess bond premium - US

Appendix B  Model

This section provides further details on the model.

B.1 Bank resolution authority

Separating the different terms of 3.40, we obtain

\[ R_t^F b_{t-1}^i (\mu^F - 1) \int_0^{\bar{\omega}_t^F} \omega_t^F f^F(\omega_t^F) d\omega + R_{t-1}^D d_{t-1}^i \int_0^{\bar{\omega}_t^F} f^F(\omega_t^F) d\omega - \gamma^B b_{t-1} [F_t^\phi - F_t^F]. \]

Recall that we define

\[ F^F = F(\bar{\omega}^F) = \int_0^{\bar{\omega}^F} f^F(\omega^F) d\omega, \]

\[ G^F = G(\bar{\omega}^F) = \int_0^{\bar{\omega}^F} \omega f^F(\omega^F) d\omega, \]

\[ \Gamma^F = \Gamma(\bar{\omega}^F) = (1 - F^F(\bar{\omega}^F)) \bar{\omega}^F + G^F(\bar{\omega}^F). \]

Using the above notation, we can write

\[ R_t^F b_{t-1}^i (\mu^F - 1) G_t^F + R_{t-1}^D d_{t-1}^i F_t^F - \gamma^B b_{t-1} [F_t^\phi - F_t^F]. \]

Then, replacing \( R_{t-1}^D d_{t-1}^i \) with \( \bar{\omega}_t^Fi R_t^F b_{t-1}^i \) using (3.17), we obtain

\[ R_t^F b_{t-1}^i (\mu^F - 1) G_t^F + \bar{\omega}_t^Fi R_t^F b_{t-1}^i F_t^F - \gamma^B b_{t-1} [F_t^\phi - F_t^F]. \]

This can be written as

\[ R_t^F b_{t-1}^i [ (\mu^F - 1) G_t^F + \bar{\omega}_t^Fi F_t^F ] - \gamma^B b_{t-1} [F_t^\phi - F_t^F]. \]

Finally, using the definition of \( \Gamma^F \), we can replace \(-G_t^F + \bar{\omega}_t^Fi F_t^F\) with \(-\Gamma_t^F + \bar{\omega}_t^Fi\) to obtain,

\[ R_t^F b_{t-1}^i [ \mu^F G_t^F - \Gamma_t^F + \bar{\omega}_t^Fi ] - \gamma^B b_{t-1} [F_t^\phi - F_t^F]. \]

The balance sheet of the bank resolution authority, in terms of final consumption, is therefore

\[ t_t = [\bar{\omega}_t^F - (\Gamma_t^F - \mu^F G_t^F)] \frac{R_t^F b_{t-1}^i}{\Pi_t} - \frac{\gamma^B b_{t-1} [F_t^\phi - F_t^F]}{\Pi_t}. \]

The bank resolution authority makes no profits; thus, bank resolution costs (deposit insurance outlays) just cover its income.
B.2 Model summary

Endogenous variables (42)

Households (3): \(c_t, \beta_{t-1,t}, w_t\). Consumption goods production (4): \(l_t, r^K_t, s_t, \Pi_t\). Capital goods production (3): \(q_t, g_t, I_t\). Entrepreneurs (11): \(R^E_t, W^E_t, n^E_t, K_t, \varpi^E_t, F^E_t, F^{E^t}_t, G^E_t, G^{E^t}_t, \Gamma^E_t, \Gamma^{E^t}_t\). Financial contract (2): \(x^E_t, \xi_t\). Banks (12): \(b_t, d_t, R^F_t, \omega_t, \omega\phi_t, F^F_t, F^{F^t}_t, G^F_t, G^{F^t}_t, \Gamma^F_t, \Gamma^{F^t}_t, F\phi_t, F^{\phi^t}_t\). Bankers (3): \(W^B_t, n^B_t, R^B_t\). Market clearing (1): \(Y_t\). Policy (2): \(R^D_t, \phi_t\). Shocks (1): \(\varsigma^E_t\).

Model equations (42)

Household FOC labor (real wage, \(w_t\))

\[ w_t = \varphi l_t^\eta c_t \]

Household’s stochastic discount factor (\(\beta_{t-1,t}\))

\[ \beta_{t-1,t} = \beta \frac{c_{t-1}}{c_t} \]

Consumption Euler equation (household consumption, \(c_t\))

\[ 1 = R^D_t \mathbb{E}_t \left\{ \frac{\beta_{t+1}}{\Pi_{t+1}} \right\} \]

Production (labor supply, \(l_t\))

\[ Y_t = A_t K^{\alpha}_t l^{1-\alpha}_t \]

Labor demand combined with rental rate on capital (rental rate on capital, \(r^K_t\))

\[ w_l l_t = \frac{1 - \alpha}{\alpha} r^K_t K_{t-1} \]

Marginal costs (real marginal costs, \(s_t\))

\[ s_t = \frac{w_t^{1-\alpha}(r^K_t)^\alpha}{\alpha \alpha (1 - \alpha)^{1-\alpha}} \frac{1}{A_t} \]

New Keynesian Phillips Curve (inflation, \(\Pi_t\))

\[ \kappa_p \Pi_t (\Pi_t - 1) = \Pi_t \xi - (\xi - 1) + \kappa_p \mathbb{E}_t \left\{ \beta_{t+1} \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \right\} \]

Investment demand (real price of capital, \(q_t\))

\[ 0 = q_t - (1 + g_t) - \kappa_I \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \mathbb{E}_t \left\{ \beta_{t+1} \kappa_I \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\} \]
Investment adjustment cost \((g_t)\)
\[
g_t = \frac{\kappa I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2
\]

Law of motion for capital (investment, \(I_t\))
\[
K_t = I_t + (1 - \delta) K_{t-1}
\]

Borrowing (capital, \(K_t\))
\[
b_t = q_t K_t - n_t^E
\]

Entrepreneur net worth \((n_t^E)\)
\[
n_t^E = (1 - \chi^E + \iota \chi^E) \mathcal{W}_t^E
\]

Entrepreneur wealth \((\mathcal{W}_t^E)\)
\[
\mathcal{W}_t^E = (1 - \Gamma_t^E) \frac{R_t^E q_{t-1} K_{t-1}}{\Pi_t}
\]

Entrepreneur default condition (entrepreneur productivity cutoff, \(\varpi_t^E\))
\[
\varpi_t^E = \frac{x_t^{i-1}}{R_t^E}
\]

Gross return on capital holdings (return on capital holdings, \(R_t^E\))
\[
R_t^E = \frac{r_t^K + (1 - \delta) q_t}{q_{t-1}} R_t^E
\]

Entrepreneur productivity \((F_t^E, F_t^E', G_t^E, G_t^E', \Gamma_t^E, \Gamma_t^E')\)
\[
F_t^E = \Phi \left( \frac{\ln(\varpi_t^E) + \frac{1}{2} \left( \sigma_t^E \right)^2}{\sigma_t^E} \right)
\]
\[
F_t^E' = \frac{1}{\varpi_t^E \sigma_t^E} \Phi' \left( \frac{\ln(\varpi_t^E) + \frac{1}{2} \left( \sigma_t^E \right)^2}{\sigma_t^E} \right)
\]
\[
G_t^E = \Phi \left( \frac{\ln(\varpi_t^E) - \frac{1}{2} \left( \sigma_t^E \right)^2}{\sigma_t^E} \right)
\]
\[
G_t^E' = \frac{1}{\varpi_t^E \sigma_t^E} \Phi' \left( \frac{\ln(\varpi_t^E) - \frac{1}{2} \left( \sigma_t^E \right)^2}{\sigma_t^E} \right)
\]
\[
\Gamma_t^E = \varpi_t^E (1 - F_t^E) + G_t^E
\]
\[
\Gamma_t^E' = -\varpi_t^E F_t^E' + (1 - F_t^E) + G_t^E'
\]
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Financial contract 1 (entrepreneurial leverage, $x_t^E$)

$$
\mathbb{E}_t \left\{ -\Gamma_{t+1}^E + \xi_t (\Gamma_{t+1}^E - \mu_t^E G_{t+1}^E) \right\} = 0
$$

Financial contract 2 (Lagrangian multiplier on bank's participation constraint, $\xi_t$)

$$
\mathbb{E}_t \left\{ (1 - \Gamma_{t+1}^E) R_{t+1}^E + \xi_t [\Gamma_{t+1}^E - \mu_t^E G_{t+1}^E] R_{t+1}^E - R_{t+1}^F \right\} = 0
$$

Bank balance sheet (bank deposits, $d_t$)

$$
b_t = n_{t}^B + d_t
$$

Bank first order condition (bank loans, $b_t$)

$$
\mathbb{E}_t \left\{ R_{t+1}^F \left[ (1 - \Gamma_{t+1}^F) - \Gamma_{t+1}^F \frac{R_{t+1}^D n_{t}^B}{R_{t+1}^F b_t} \right] \right\} = \gamma^B \mathbb{E}_t \left\{ F_{t+1}^{\phi} - F_{t+1}^F + \left( \frac{F_{t+1}^{\phi'}}{1 - \phi_{t+1}} - F_{t+1}^F \right) \frac{R_{t+1}^D n_{t}^B}{R_{t+1}^F b_t} \right\}
$$

Bank default condition (bank productivity cutoff, $\bar{\omega}_t^F$)

$$
\bar{\omega}_t^F = \frac{R_{t-1}^D d_{t-1}}{R_{t-1}^F b_{t-1}}
$$

Penalty cutoff ($\bar{\omega}_t^\phi$)

$$
\bar{\omega}_t^\phi = \frac{R_{t-1}^D d_{t-1}}{(1 - \phi_{t-1}) R_{t-1}^F b_{t-1}}
$$

Bank return on loans, gross of monitoring costs (bank return on loans, $R_t^F$)

$$
R_t^F = (\Gamma_t^F - \mu_t^E G_t^E) \frac{R_t^F q_{t-1} K_{t-1}}{b_{t-1}}
$$

Bank productivity ($F_t^F$, $F_t^{\phi}$, $G_t^F$, $\Gamma_t^F$, $\Gamma_t^{\phi}$, $F_t^F$, $F_t^{\phi}$)

$$
F_t^F = \Phi \left( \ln \frac{\bar{\omega}_t^F + 1}{2} \left( \frac{\sigma_t^F}{\sigma_t^F} \right)^2 \right)
$$

$$
F_t^{\phi} = \frac{1}{\bar{\omega}_t^F \sigma_t^F} \Phi^\prime \left( \ln \frac{\bar{\omega}_t^F + 1}{2} \left( \frac{\sigma_t^F}{\sigma_t^F} \right)^2 \right)
$$

$$
G_t^F = \Phi \left( \ln \frac{\bar{\omega}_t^F - 1}{2} \left( \frac{\sigma_t^F}{\sigma_t^F} \right)^2 \right)
$$

$$
\Gamma_t^F = (1 - F_t^F) \bar{\omega}_t^F + G_t^F
$$
\[ \Gamma_t^F = -F_t^F \omega_t^F + (1 - F_t^F) + G_t^F \]
\[ F_t^\phi = \Phi \left( \ln \omega_t^\phi + \frac{1}{2} \left( \sigma_t^F \right)^2 \right) \]
\[ F_t^{\phi'} = \frac{1}{\omega_t^\phi \sigma_t^F} \Phi' \left( \ln \omega_t^\phi + \frac{1}{2} \left( \sigma_t^F \right)^2 \right) \]

Banker net worth \((n_t^B)\)
\[ n_t^B = (1 - \chi^B + \iota \chi^B) \mathcal{W}_t^B \]

Banker wealth \((\mathcal{W}_t^B)\)
\[ \mathcal{W}_t^B = \frac{R_t^B n_{t-1}^B}{\Pi_t} \]

Ex post gross rate of return on equity (banker return on equity, \(R_t^B\))
\[ R_t^B = b_{t-1} \left( 1 - \Gamma_t^F \right) \frac{R_t^F - \gamma^B (F_t^\phi - F_t^F)}{n_{t-1}^B} \]

Goods market clearing (output, \(Y_t\))
\[ Y_t = c_t + (1 + g_t) I_t + \frac{k_t}{2} (\Pi_t - 1)^2 Y_t + \mu^E G_t^E R_t^E q_{t-1} K_{t-1} - \frac{\mu^F G_t^F R_t^F b_{t-1}}{\Pi_t} \]

Monetary policy rule (policy rate, \(R_t^D\))
\[ \frac{R_t^D}{R_t^D} = \left( \frac{R_{t-1}^D}{R_t^D} \right)^\tau_R \left[ \left( \frac{\Pi_t}{\Pi} \right)^\tau_M \right]^{1-\tau_R} \]

Macropirudential policy rule (capital ratio, \(\phi_t\))
\[ \frac{\phi_t}{\phi} = \left( \frac{\phi_{t-1}}{\phi} \right)^\zeta_\phi \left[ \left( \frac{b_t}{b} \right)^\zeta_b \right]^{1-\zeta_\phi} \]

Entrepreneur risk shock \((\zeta_t^E)\)
\[ \ln \zeta_t^E = \rho_E \ln \zeta_{t-1}^E + \zeta_t^E \]
Table 3.B.10: Baseline model - summary

1. \( w_t = \varphi l_t c_t \)
2. \( \beta_{t-1,t} = \beta c_{t-1}/c_t \)
3. \( 1 = R_{tt}^E \mathbb{E}_t \{ \beta_{t,t+1}/\Pi_{t+1} \} \)
4. \( Y_t = A_t K_t^{\alpha} l_t^{1-\alpha} \)
5. \( \alpha w_t l_t = (1 - \alpha) r_{t}^E K_t \)
6. \( \alpha^2 (1 - \alpha)^{1-\alpha} s_t = u_t^{1-\alpha} (r_t^E)^{\alpha} / A_t \)
7. \( \kappa_t \Pi_t (\Pi_t - 1) = s_t - (\varepsilon - 1) + \kappa_t \mathbb{E}_t \{ \beta_{t,t+1} \Pi_{t+1} (\Pi_{t+1} - 1) Y_{t+1} / Y_t \} \)
8. \( \kappa_t (I_t / I_{t-1} - 1) I_{t-1} / I_t = \gamma_t - (1 + \gamma_t) + \mathbb{E}_t \{ \beta_{t,t+1} \kappa_t (I_{t+1} / I_t - 1) (I_{t+1} / I_t) \} \)
9. \( \gamma_t = 0.5 \kappa_t (I_t / I_{t-1} - 1)^2 \)
10. \( K_t = I_t + (1 - \delta) K_{t-1} \)
11. \( b_t = q_t K_t - n_t^E \)
12. \( n_t^E = (1 - \chi^E + \lambda \chi^E) \mathcal{W}_t^E \)
13. \( \mathcal{W}_t^E = (1 - \Gamma_t^E) R_t^E q_{t-1} K_t / \Pi_t \)
14. \( \mathcal{V}_t^E = x_t^E / R_t^E \)
15. \( R_t^E = [r_t^K + (1 - \delta) q_t] \Pi_t / q_{t-1} \)
16. \( F_t^E = \Phi ([\ln \mathcal{V}_t^E + 0.5 (\sigma_t^E)^2] / \sigma_t^E) \)
17. \( F_t^{E'} = \Phi' ([\ln \mathcal{V}_t^E + 0.5 (\sigma_t^E)^2] / \sigma_t^E) / (\mathcal{V}_t^E \sigma_t^E) \)
18. \( G_t^E = \Phi ([\ln \mathcal{V}_t^E - 0.5 (\sigma_t^E)^2] / \sigma_t^E) \)
19. \( G_t^{E'} = \Phi' ([\ln \mathcal{V}_t^E - 0.5 (\sigma_t^E)^2] / \sigma_t^E) / (\mathcal{V}_t^E \sigma_t^E) \)
20. \( \Gamma_t^E = G_t^E + \mathcal{V}_t^E (1 - F_t^E) \)
21. \( \Gamma_t^{E'} = G_t^{E'} + (1 - F_t^E) - \mathcal{V}_t^E F_t^{E'} \)
22. \( \mathbb{E}_t [\xi_t (\Gamma_t^{E'} - \mu_t G_t^{E'} + 1)] = \mathbb{E}_t \{ \Gamma_t^{E'} + 1 \} \)
23. \( \mathbb{E}_t \{ (1 - \Gamma_t^{E'}) R_t^E + \xi_t [(\Gamma_t^{E'} - \mu_t G_t^{E'}) R_t^E - R_t^{E'}] \} = 0 \)
24. \( d_t = b_t - n_t^B \)
25. \( \mathbb{E}_t \{ R_t^E [1 - (1 - \Gamma_t^{E'}) - \Gamma_t^{E'} R_t^E n_t^B / (R_t^E b_t)] \} = \gamma_t \mathbb{E}_t \{ F_t^{E'} + [F_t^{E'} - R_t^{E'} + F_t^{E'} - R_t^{E'} + 1 - \phi_t] - F_t^{E'} R_t^E n_t^B / (R_t^E b_t) \} \)
26. \( \mathcal{V}_t^E = R_t^E d_{t-1} / (R_t^E b_{t-1}) \)
27. \( \mathcal{V}_t^E = R_t^E d_{t-1} / (1 - \phi_t) R_t^{E'} b_{t-1} \)
28. \( R_t^E = (\Gamma_t^E - \mu_t G_t^E) R_t^E q_{t-1} K_t / b_{t-1} \)
29. \( F_t^E = \Phi ([\ln \mathcal{V}_t^E + 0.5 (\sigma_t^E)^2] / \sigma_t^E) \)
30. \( F_t^{E'} = \Phi' ([\ln \mathcal{V}_t^E + 0.5 (\sigma_t^E)^2] / \sigma_t^E) / (\mathcal{V}_t^E \sigma_t^E) \)
31. \( G_t^E = \Phi ([\ln \mathcal{V}_t^E - 0.5 (\sigma_t^E)^2] / \sigma_t^E) \)
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(32) $\Gamma_t^F = G_t^F + (1 - F_t^F)\pi_t^F$

(33) $\Gamma_t^{F'} = G_t^{F'} + (1 - F_t^{F'}) - F_t^{F'}\pi_t^F$

(34) $F_t^\phi = \Phi([\ln \omega_t^\phi + 0.5(\sigma_t^F)^2]/\sigma_t^F)$

(35) $F_t^{\phi'} = \Phi'([\ln \omega_t^\phi + 0.5(\sigma_t^F)^2]/\sigma_t^F)/([\omega_t^\phi]^\phi)$

(36) $n_t^B = (1 - \chi^B + c\chi^B)\psi_t^B$

(37) $\psi_t^B = [R_t^B n_t^{B-1}]/\Pi_t$

(38) $R_t^B = b_{t-1}[(1 - \Gamma_t^F)R_t^F - \gamma^B(F_t^\phi - F_t^{F'})]/n_t^{B-1}$

(39) $Y_t = c_t + (1 + g_t)I_t + 0.5\kappa_p(\Phi_t - 1)^2Y_t + \mu^EG_t^F R_t^F q_{t-1}K_{t-1}/\Pi_t + \mu^G G_t^F R_t^F b_{t-1}/\Pi_t$

(40) $\ln(R_t^F/R^D) = \tau_R \ln(R_t^D/R^D) + (1 - \tau_R)\ln(\Pi_t/\Pi)$

(41) $\ln(\phi_t/\phi) = \zeta_t \ln(b_t/b)$

(42) $\ln c_t^E = \rho E \ln c_{t-1}^E + \varepsilon_t^E$

Additional equations

$\pi_t^F = Z_t b_t/(q_tK_tR_t^F)$

$\Xi_t^F = (1 - \Gamma_t^F)R_t^F q_{t-1}K_{t-1}$

$\Xi_t^F = (1 - \Gamma_t^F)R_t^F b_{t-1} - \gamma^B b_{t-1}(F_t^\phi - F_t^{F'})$

$Y_t^{\text{net}} = c_t + (1 + g_t)I_t + 0.5\kappa_p(\Phi_t - 1)^2Y_t$

The system consists of 42 endogenous variables, $I_t$, $\beta_{t-1,t}$, $\Pi_t$, $Y_t$, $r_t^F$, $w_t$, $s_t$, $I_t$, $g_t$, $K_t$, $b_t$, $n_t^F$, $W_t^F$, $x_t^F$, $R_t^F$, $F_t^F$, $F_t^{F'}$, $G_t^F$, $G_t^{F'}$, $\Gamma_t^F$, $\Gamma_t^{F'}$, $\epsilon_t$, $\zeta_t$, $\xi_t$, $\omega_t^\phi$, $q_t$, $d_t$, $\pi_t^F$, $\Xi_t^F$, $\Xi_t^F$, $R_t^F$, $F_t^F$, $F_t^{F'}$, $G_t^F$, $G_t^{F'}$, $\Gamma_t^F$, $\Gamma_t^{F'}$, $\epsilon_t$, $\zeta_t$, and one exogenous processes, $\varepsilon_t^F$. We might define the additional variables $Z_t$, $\Xi_t^F$, $\Xi_t^F$, and $Y_t^{\text{net}}$. The functions $\Phi(\cdot)$ and $\Phi'(\cdot)$ denote, respectively, the cumulative distribution function and the probability density function of the standard normal distribution.

B.3 Steady state

Parameter group 1. Given a value for steady state inflation $\Pi$, technology $A$, we can solve for the following steady state variables recursively.

Consumption Euler equation (deposit rate, $R^D$)

$$R^D = \frac{\Pi}{\beta}$$

Investment demand (price of capital, $q$)

$$q = 1$$

NKPC (real marginal costs, $s$)

$$s = \frac{\varepsilon - 1}{\varepsilon} + \frac{\kappa_p}{\varepsilon} (1 - \beta) (\Pi - 1) \Pi$$
**Parameter group 2.** Given initial values for $\bar{\omega}^E$, $n^B$, $R^E$, $n^E$, $F^E$ and $l$ we solve recursively for the following variables.

Gross return on capital holdings (real rental rate on capital, $r^K$)

$$r^K = \left[ \frac{R^E}{\Pi} - (1 - \delta) \right] q$$

Capital demand / rental rate on capital (output-capital ratio, $Y/K$)

$$\frac{Y}{K} = \frac{1}{\alpha} \frac{r^K}{s}$$

Production (capital stock, $K$)

$$K = \left( \frac{1}{A \alpha s} l^{\alpha-1} \right)^{\frac{1}{\alpha-1}}$$

Law of motion for capital (investment, $I$)

$$I = \delta K$$

Definition of output-to-capital ratio (output, $Y$)

$$Y = \frac{1}{\alpha} \frac{r^K K}{s}$$

Labor demand (real wage, $w$)

$$w = (1 - \alpha) \frac{Y}{l}$$

Auxiliary variables - entrepreneur ($F^{E'}$, $G^E$, $G^{E'}$, $\Gamma^E$, $\Gamma^{E'}$).

$$F^{E'} = \frac{1}{\omega^E \sigma^E} \Phi' \left( \ln \bar{\omega}^E + \frac{1}{2} \left( \frac{\sigma^E}{\sigma^E} \right)^2 \right)$$

$$G^E = \Phi \left( \ln \bar{\omega}^E - \frac{1}{2} \left( \frac{\sigma^E}{\sigma^E} \right)^2 \right)$$

$$G^{E'} = \frac{1}{\omega^E \sigma^E} \Phi' \left( \ln \bar{\omega}^E - \frac{1}{2} \left( \frac{\sigma^E}{\sigma^E} \right)^2 \right)$$

$$\Gamma^E = \bar{\omega}^E (1 - F^E) + G^E$$

$$\Gamma^{E'} = -\bar{\omega}^E F^{E'} + (1 - F^E) + G^{E'}$$
Entrepreneur wealth accumulation (entrepreneurial wealth, $W^E$)

\[ W^E = (1 - \Gamma^E) \frac{R^E qK}{\Pi} \]

Entrepreneur default condition (entrepreneurial leverage, $x^E$)

\[ x^E = \omega^E R^E \]

Borrowing by entrepreneurs (entrepreneurial loans, $b$)

\[ b = qK - n^E \]

Banks’ return on loans (realized bank loan rate, $R^F$)

\[ R^F = (\Gamma^E - \mu^E G^E) \frac{R^E qK}{b} \]

Bank balance sheet (bank deposits, $d$)

\[ d = b - n^B \]

Penalty cutoff ($\omega^\phi$)

\[ \omega^\phi = \frac{R^D d}{(1 - \phi) R^F b} \]

Bank default cutoff ($\omega^F$)

\[ \omega^F = \frac{R^D d}{R^F b} \]

Probability of bank default ($F^F$)

\[ F^F = \Phi \left( \frac{\ln \omega^F + \frac{1}{2} (\sigma^F)^2}{\sigma^F} \right) \]

Auxiliary variables - bank ($F^{F'}$, $G^F$, $\Gamma^F$, $\Gamma^{F'}$, $F^\phi$, $F^{\phi'}$)

\[ F^{F'} = \frac{1}{\omega^F \sigma^F} \Phi' \left( \frac{\ln \omega^F + \frac{1}{2} (\sigma^F)^2}{\sigma^F} \right) \]

\[ G^F = \Phi \left( \frac{\ln \omega^F - \frac{1}{2} (\sigma^F)^2}{\sigma^F} \right) \]

\[ \Gamma^F = (1 - F^F) \omega^F + G^F \]

\[ \Gamma^{F'} = -F^{F'} \omega^F + (1 - F^F) + G^{F'} \]
\[ F^\phi = \Phi \left( \ln \omega^\phi + \frac{1}{2} \left( \sigma^F \right)^2 \right) \]

\[ F^{\phi'} = \frac{1}{\omega^\phi \sigma^F} \Phi' \left( \ln \omega^\phi + \frac{1}{2} \left( \sigma^F \right)^2 \right) \]

Bankers’ ex-post gross return on equity (realized return on equity, \( R^B \))

\[ R^B = b \left( 1 - \Gamma^F \right) R^F - \gamma^B (F^\phi - F^F) \]

Banker wealth accumulation (banker wealth, \( W^B \))

\[ W^B = \frac{R^B n^B}{\Pi} \]

Goods market clearing (consumption, \( c \))

\[ c = Y - I - \mu^E G^E R^E q K - \mu^F G^F R^F b \]

Financial contract FOC 1 (Lagrange multiplier on bank’s participation constraint, \( \xi \))

\[ \xi = \frac{\Gamma^E \mu^E G^E}{\Gamma^E - \mu^E G^E} \]

**Parameter group 3.** Finally, we solve numerically for \( \omega^E, n^B, R^E, n^E, F^E \) and \( l \).

Financial contract FOC 2

\[ 0 = (1 - \Gamma^E) R^E + \xi \left[ (\Gamma^E - \mu^E G^E) R^E - R^F \right] \]

Entrepreneur default rate

\[ 0 = -F^E + \Phi \left( \ln \omega^E + \frac{1}{2} \left( \sigma^E \right)^2 \right) \]

Aggregate entrepreneurial net worth

\[ n^E = (1 - \chi^E + \iota \chi^E) W^E \]

Aggregate banker net worth

\[ n_B = (1 - \chi^B + \iota \chi^B) W^B \]
Bank FOC

\[ R^F \left[ (1 - \Gamma^F) - \Gamma^F \frac{R^D n^B}{R^F b} \right] = \gamma^B \left\{ F^\phi - F^F + \left( \frac{F^{\phi'}}{1 - \phi} - F^{F'} \right) \frac{R^D n^B}{R^F b} \right\} \]

Labor supply

\[ w = \varphi l^n c \]
Table 3.B.11: Steady state computation

<table>
<thead>
<tr>
<th>Variables group 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ( R^D = \Pi/\beta )</td>
</tr>
<tr>
<td>(2) ( q = 1 )</td>
</tr>
<tr>
<td>(3) ( s = (\varepsilon - 1)/\varepsilon + \kappa_p(1 - \beta)(\Pi - 1)\Pi/\varepsilon )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4) ( r^K = [R^E/\Pi - (1 - \delta)]q )</td>
</tr>
<tr>
<td>(5) ( K = [\alpha^{-1}r^K/(\alpha s A)]^{1/(\alpha - 1)} )</td>
</tr>
<tr>
<td>(6) ( I = \delta K )</td>
</tr>
<tr>
<td>(7) ( Y = r^K K/(\alpha s) )</td>
</tr>
<tr>
<td>(8) ( w = (1 - \alpha)sY/l )</td>
</tr>
<tr>
<td>(9) ( b = qK - n^E )</td>
</tr>
<tr>
<td>(10) ( d = b - n^B )</td>
</tr>
<tr>
<td>(11) ( FEt = \Phi(\ln \omega^E + 0.5(\sigma^E)^2)/\sigma^E) )</td>
</tr>
<tr>
<td>(12) ( GE = \Phi(\ln \omega^E - 0.5(\sigma^E)^2)/\sigma^E) )</td>
</tr>
<tr>
<td>(13) ( G_E^t = \Phi(\ln \omega^E - 0.5(\sigma^E)^2)/\sigma^E) )</td>
</tr>
<tr>
<td>(14) ( E = GE + \omega^E(1 - F^E) )</td>
</tr>
<tr>
<td>(15) ( E = GE + (1 - F^E) - \omega^E F^E )</td>
</tr>
<tr>
<td>(16) ( \omega^E = (1 - \Omega^E)R^E qK/\Pi )</td>
</tr>
<tr>
<td>(17) ( \omega^E = \omega^E R^E )</td>
</tr>
<tr>
<td>(18) ( R^E = (\Omega^E - \mu^E G^E)R^E qK/b )</td>
</tr>
<tr>
<td>(19) ( \omega^E = R^D d/(R^F b) )</td>
</tr>
<tr>
<td>(20) ( \omega^E = R^D d/(1 - \phi R^F b) )</td>
</tr>
<tr>
<td>(21) ( F^E = \Phi(\ln \omega^E + 0.5(\sigma^E)^2)/\sigma^F )</td>
</tr>
<tr>
<td>(22) ( F^E = \Phi(\ln \omega^E + 0.5(\sigma^E)^2)/\sigma^F )</td>
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<tr>
<td>(23) ( F^E = \Phi(\ln \omega^E - 0.5(\sigma^E)^2)/\sigma^F )</td>
</tr>
<tr>
<td>(24) ( F^E = \Phi(\ln \omega^E - 0.5(\sigma^E)^2)/\sigma^E )</td>
</tr>
<tr>
<td>(25) ( G^E = \Phi(\ln \omega^E - 0.5(\sigma^E)^2)/\sigma^E )</td>
</tr>
<tr>
<td>(26) ( E^E = G^E + \omega^E(1 - F^E) )</td>
</tr>
<tr>
<td>(27) ( E^E = G^E + (1 - F^E) - \omega^E F^E )</td>
</tr>
<tr>
<td>(28) ( \xi = \Gamma^F/[(\Gamma^E - \mu^E G^E)] )</td>
</tr>
<tr>
<td>(29) ( R^B = b(1 - \Omega^E)R^E - \gamma^B(F^E - F^D)/n^B )</td>
</tr>
<tr>
<td>(30) ( W^B = R^B n^B/\Pi )</td>
</tr>
<tr>
<td>(31) ( c = Y - I - \mu^E G^E R^E qK/\Pi - \mu^F G^F R^E b/\Pi )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(32) ( 0 = (1 - \Gamma^E)R^E + \xi[(\Gamma^E - \mu^E G^E)R^E - R^F] )</td>
</tr>
<tr>
<td>(33) ( 0 = \gamma^B {F^E - F^E + [F^E/(1 - \phi) - F^F]R^D n^B/(R^F b)} - R^F[(1 - \Gamma^E) - \Gamma^F R^D n^B/(R^F b)] )</td>
</tr>
<tr>
<td>(34) ( n^E = (1 - \chi^E + t\chi^E) W^E )</td>
</tr>
<tr>
<td>(35) ( n^B = (1 - \chi^B + t\chi^B) W^B )</td>
</tr>
<tr>
<td>(36) ( 0 = F^E - \Phi(\ln(\omega^E) + 0.5(\sigma^E)^2)/\sigma^E) )</td>
</tr>
<tr>
<td>(37) ( 0 = \phi - w/(c(\Pi)) )</td>
</tr>
</tbody>
</table>

Notes. First, given the calibrated parameters, we compute the variables of group 1 recursively: \( R^D, q \) and \( s \). Second, given initial values for \( \omega^E, n^B, n^E, F^E, R^E \) and \( l \) we can compute the variables (4) to (31): \( r^K, K, I, Y, w, b, d, F^E, G^E, G^E, \Gamma^E, \Gamma^E, \omega^E, x^E, R^F, \omega^E, \omega^E, F^E, F^E, F^E, F^E, F^E, G^E, \Gamma^F, \Gamma^F, \xi, R^B, W^B \) and \( c \) using equations (4) to (31). Finally, we solve the six equations (32)-(37) numerically for \( \omega^E, n^B, n^E, F^E, R^E \) and \( l \).
Bibliography


BIBLIOGRAPHY


Eidesstattliche Erklärung

Hiermit erkläre ich, dass ich die Arbeit selbstständig angefertigt und die benutzten Hilfsmittel vollständig und deutlich angegeben habe.

Mannheim, am 11. Mai 2023

__________________________
Tommaso Gasparini
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