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Preface to the Special Issue on "Games, Dynamics and Optimization"

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From March 13 to March 15, 2018, a 3-day interdisciplinary Workshop on mathematical game theory, continuous optimization and dynamical system was held at the University of Vienna. This conference was the first edition of a a series of worhshops which is now already in its third edition, and it became a central element of the scientific work plan of the COST Action CA16228 "European Network for Game Theory". The workshop was a jointly organized event together with the Vienna Center for Operations Research (VCOR) and the Department of Mathematics of the University of Vienna. The motivation for the creation of this workshop was that recently there are two interesting evolutions happening in disciplines which used to be quite close together but somehow have seen a diverging evolution: dynamical systems derived from games are a well-established research field with many important applications in mathematical biology, physics, engineering and economics. Recently, dynamical systems have also witnessed important applications in optimization and machine learning. Here, the arsenal of mathematical tools for studying continuous time systems have been successfully applied in shedding new light on the asymptotic properties of optimization algorithms, and to help practitioners and researchers in designing new algorithms. It is an interesting, and very notable, coincidence that both fields have one founding father for their evolution: the Lotka–Volterra system. In evolutionary game theory, Lotka–Volterra systems have been studied since the 1970s and led to the creation of the celebrated replicator dynamics. In optimization, the large class of Barrier-dynamics contains as special case Lotka–Volterra systems, and the connection between Riemannian gradient flows, evolutionary game theory, and optimization is now well-documented. With all these in mind, the aim of the "Games, Dynamics and Optimization" (GDO) workshop series is to explore the analogies between dynamical systems in games and dynamical

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systems in optimization, to boost their evolution, popularize their use, and deepen and extend their theoretical foundations. With this purpose in mind, a special issue in the journal *Applied Mathematics and Optimization* has been announced, where representative papers on the theme of GDO are collected. This special issue consists of ten papers, all of which touching key topics in mathematical game theory, dynamical systems and nonlinear analysis. In what follows, we group the contributions according to these three fields.

Papers on Operator Splitting

- The work by Adly and Bourdin introduces a new decomposition formula for the resolvent operators of the sum of two monotone operators, a computational routine usually involved in the numerical treatment of monotone inclusions and PDEs via operator splitting algorithms.
- Dynamical systems inspired by operator splitting techniques is a very active field of research within optimization theory. The continuous time analysis often sheds new light on the asymptotic behavior of numerical algorithms, via a careful numerical discretization. In particular, continuous time analysis admits a direct analysis via Lyapunov techniques, and their structure gives us insights into the design of new algorithms. The most successful story in this area is the recent beautiful connection between Nesterov's accelerated algorithm and second-order dynamical systems. Along these lines, the paper by Attouch and Cabot introduces a fairly broad family of iterative forward-backward schemes, which can be related to second-order dynamical systems by a careful explicit numerical discretization. The choice of the step parameters in the discretized dynamics is a key issue to guarantee a good performance of the resulting scheme. This paper derives explicit modeling recommendations on step-sizes, proximal parameters and inertial parameters, to ensure stability of the discrete-time dynamical system. An application to zero-sum games sheds light on the connection between operator splitting and game theory.
- The paper by Csetnek, Malitsky and Tam proposes a new algorithm for finding a zero of the sum of two maximally monotone operators, where one is assumed to be single-valued and Lipschitz continuous, which arises from a non-standard discretization of a continuous dynamical system associated with the celebrated Douglas–Rachford algorithm. This is a truly novel results in this field, and interesting connections between discrete and continuous time dynamical systems become apparent.

Papers on Partial Differential Equations

- Differential games describe a competitive optimal control problem between players who jointly control an ordinary differential equation in order to steer the system towards certain states in which each player is guaranteed a certain payoff. In the setting of non-anticipative strategies determining the value of such a game when players' action spaces are constrained is a delicate issue. An even more challenging setting is studied in the paper by Bettiol, Quincampoix and Vinter, in which

players controls are even affected by coupled constraints. Such Generalized Nash games are classical in economics and operations research, and play an increasingly important role in PDE constrained optimization as well. Under a viability condition, the authors prove the existence of the game value in such a complex strategic environment.
- Continuing the line of challenging optimal control problems, the paper by Breiten, Kunisch and Pfeiffer, investigates stabilization of the 2-dimensional Navier–Stokes equation by classical numerical optimal control tools, namely the value function approximation. Key to this results is an interesting discovery related to the local continuity properties of the value function around the steady state. This allows the authors to perform a delicate linearization, which they use to produce an approximate feedback control.
- The limit behavior of singular perturbations problems of biLaplacian type is investigated in the paper by Dipierro, Karakhanyan and Valdinoci. This study provides new solutions to free boundary problems, and contains an interesting search game in continuous time and space.
- Gradient flows are the paradigm for dissipative evolution equations, which are also of key importance in games and optimization. One standard numerical discretization scheme for gradient flows is the classical Euler scheme. To preserve the dissipation dynamics along a time discretization, the paper by Jüngel, Stefanelli and Trussardi investigate two modifications of the discretized Euler scheme, and demonstrates important improvements with respect to the latter.
- The paper by Rodrigues and Santos aims to extend classical results on variational and quasi-variational inequalities over convex sets defined through gradient constraints, by replacing the gradient with a fractional gradient. The continuous dependence of the solution on the data is proved and generalized Lagrange multipliers rules are derived.

Papers on Optimization

- Finite sum models are the canonical model in machine learning and empirical risk minimization. In the paper by Dinh, Goberna and Volle, a finite sum model is investigated in an abstract setting, and new optimality conditions are derived. These abstract results are then used to derive interesting new results for the best-approximation problem.
- In large scale applications, the use of randomized algorithms is a standard tool to improve the iteration complexity. The paper by Nedic and Necoara deals with the minimization of a nonsmooth convex function with respect to a feasible set, which is represented as large intersection of functional constraints. In this challenging setting, a parallel and sequential stochastic subgradient algorithm based on minibatch estimators is studied.

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