// NO.23-038 | 10/2023

DISCUSSION PAPER

// ALEXANDER HECZKO, THOMAS KITTSTEINER, AND MARION OTT

The Benefits of Auctioneer Competition: Merging Auctions and Adding Auctioneers

The Benefits of Auctioneer Competition: Merging Auctions and Adding Auctioneers☆

Alexander Heczko[a], Thomas Kittsteiner[b], Marion Ott[c]

[a] SRH Hochschule in Nordrhein-Westfalen, Platz der Deutschen Einheit 1, 59065 Hamm, Germany
[b] RWTH Aachen University, Templergraben 64, 52062 Aachen, Germany
[c] ZEW – Leibniz Centre for European Economic Research, L7, 1, 68161 Mannheim, Germany

September 20, 2023

Abstract

In a lab experiment, we analyze the benefits of increasing competition on auction platforms hosting multiple auctioneers of a homogeneous good. We find that increasing competition by merging separated individual auctions increases market efficiency and also buyers' payoffs, while there is no evidence of an increase in the auctioneers' expected revenues. Furthermore, competing auctioneers decrease reserve prices significantly when the number of competitors increases. Then, auctioneers' revenues decrease whereas buyers' payoffs and efficiency are enhanced. Different to previous findings for the monopolistic seller case, competing auctioneers do not increase reserve prices significantly when the number of buyers increases. For our theoretical model, we provide closed-form equilibrium reserve-price functions of competing auctioneers.

Keywords: Competing auctions, merging markets, parallel auctions, reserve price, experiment
JEL: D44, D47, D82

☆Financial support from the Deutsche Forschungsgemeinschaft (DFG) under grants KI 1915/1-1 and OT 487/2-1 is gratefully acknowledged.

Email addresses: alexander.heczko@srh.de (Alexander Heczko), thomas.kittsteiner@rwth-aachen.de (Thomas Kittsteiner), marion.ott@zew.de (Marion Ott)

1. Introduction

Many online market platforms, such as eBay, attract only few sellers and buyers for at least some of the items exchanged on the platform (see Anwar et al. 2006, Andersson et al. 2012, Bajari and Hortacsu 2003, Haruvy and Popkowski Leszczyc 2010). In such small markets, each market participant can have a significant impact on the price, market efficiency, participants' payoffs and therefore a platform's revenue.[1] As a consequence, understanding the impact of competition on both sides of a market is crucial to understand the effect of design changes and other measures aimed at enhancing competition. We focus on the impact of two such measures. Firstly, we consider the impact of merging two markets, thus effectively reducing each buyer's switching cost to zero. This fosters competition, even without increasing the total number of buyers and sellers. Secondly, we investigate the impact of increasing each the number of buyers and sellers.

We consider auction markets in which sellers (the auctioneers) have an incentive to distort reserve prices (away from the own valuation or cost) whereas buyers follow a simple optimal bidding strategy, which can involve switching between different auctions they observe. Applications are online auction platforms in which different sellers (of identical items) run parallel separate ascending (English) auctions and can each set an individual reserve price for their respective items.

In a laboratory experiment, we test our theoretical predictions on the impact of competition on participant's payoffs and market efficiency. The novel feature of our experiment is that we allow for competition both among buyers and among sellers in a market with simple rules that resemble those on common internet market platforms. More specifically, each seller offers one item in competing (parallel) ascending auctions to buyers who each have unit demand. First, sellers compete by setting their respective reserve prices (independently of each other); then, buyers, who can switch between auctions, compete for the offered items.

Our main experimental findings are as follows:

1. Tearing down barriers between separate auctions is beneficial to its participants – the total gains from trade are significantly higher in the merged markets than in the separate markets combined, sellers are not worse off, and buyers are significantly better off. Even though gains from trade can be lower in the merged markets than in the separate markets combined, when sellers choose the same positive mark-ups on their costs, we find that on average this matching effect is positive.
2. A seller reacts to an increase in the number of sellers by setting her reserve price more

[1] Large markets with a uniform-pricing rule typically do not provide significant incentives to distort the price because each participant is unlikely to determine the price.

aggressively (i.e., closer to costs). As a consequence, a seller's mean profit decreases and total gains from trade and a buyer's mean profit increase.

3. We do not find evidence that, keeping the number of sellers constant, sellers' reserve prices increase in the number of buyers. Whereas this observation is consistent with equilibrium behavior it differs from previous experimental findings for monopolistic sellers whose reserve prices are increasing in the number of bidders (Davis et al. 2011).

For our theoretical model, we provide closed-form equilibrium reserve-price functions for the family of Generalized Pareto Distributions with monotone non-decreasing hazard rate (which includes the uniform distribution).[2] The equilibrium strategies are linear and independent of the number of buyers.

To our knowledge, this study provides the first experiment to systematically analyze the effect of competition on reserve price setting in competing ascending auctions. Hoppe (2008a,b) experimentally compares the performance of parallel auctions with reserve price with the performances of a centralized double auction and overlapping auctions. Models in which auctioneers of a single item compete for buyers with private values and unit-demand who cannot switch between auctions have been theoretically analyzed by Burguet and Sakovics (1999), Virág (2010), and Pai (2010). Our study is related to a specific double auction market design (Williams 1991) and therefore is related to the experimental literature on double auctions of which there exists a huge variety. Previous experimental studies in that literature have mainly focused on the convergence of prices to those in a competitive equilibrium for various double auction clearinghouse designs and continuous double auction markets used in financial markets (see, e.g., Plott and Smith 2008).

2. Theoretical Considerations and Hypotheses

2.1. The Model

Consider a market with m sellers and n buyers. Each seller offers one of m indivisible homogeneous items and each buyer requires one item. A seller j's cost, $j \in \{1, 2, \ldots, m\}$, for the item is c_j and a buyer i's valuation, $i \in \{1, 2, \ldots, n\}$, for the item is v_i. Costs and valuations are independently drawn from Generalized Pareto Distributions with shape

[2] The only example of a closed-form equilibrium in a related setting with arbitrary numbers of sellers and buyers that we are aware of is that by Williams (1991) for the buyer's bid double auction with uniform distributions. Generalized Pareto Distributions are also used by Ausubel et al. (2014) and Marszalec et al. (2020) to derive closed-from equilibrium bidding strategies in multi-unit auctions.

parameter $\xi < 0$, support $[0, -100/\xi]$, cdf F_ξ, and pdf f_ξ:

$$F_\xi(x) = \left(1-(1+\xi x)^{-\frac{1}{\xi}}\right)/100 \qquad (1)$$
$$f_\xi(x) = (1+\xi x)^{-\frac{1+\xi}{\xi}}/100$$

For $\xi = -1$, this is the uniform distribution on $[0, 100]$. A seller's payoff is her profit $\pi_j = p - c_j$ if she trades at the price p and is $\pi_j = 0$ if she does not trade. A buyer's payoff is $v_i - p$ if he buys an item and pays p, and is zero otherwise. Payoff functions and distributions of costs and valuations are common knowledge. The realizations of costs and valuations are private information.

We focus on the sellers' decisions and examine the following *auctions market*. First, sellers simultaneously each set a reserve price in their single-item ascending (English) auction. This reserve price is the standing bid in the auction when the bidding begins. Buyers take turns in bidding. When it is a buyer's turn, he bids straightforwardly as follows.[3] If he is not the highest bidder in any auction, he randomly chooses one of the auctions with the lowest standing bid, and, if existent, restricts the random selection to those of these auctions that have not yet received any bids. In the chosen auction, the buyer bids the lowest feasible bid if this does not exceed his valuation and otherwise does not bid at all. The lowest feasible bid is the reserve price if his is the first bid in the auction and otherwise is one increment above the standing bid. If he bids, he becomes the new highest bidder in this auction and the new standing bid in this auction equals his bid. All auctions end when no new bids are submitted. The high bidder in an auction wins and pays a price equal to the standing bid.

As an immediate consequence of this bidding behavior, all trades occur at the same price p^* (possibly plus one increment). This price equals the $m + 1$-highest among the buyers' valuations and the sellers' reserve prices. Buyers with valuations above the price and sellers with reserve prices below the price trade. In particular, no buyer trades whose

[3]This straightforward bidding behavior is implemented by automated bidders in the experiment. It is a simple and plausible behavior, and buyers in our setting have incentives to adopt it because a buyer's payoff with this strategy is within an increment of his payoff in the associated sealed-bid Vickrey auction (Parkes 2006). A refined version of this behavior occurs as part of a perfect Bayesian equilibrium in a closely related setting with multiple parallel second-price proxy auctions (Peters and Severinov 2006). Empirical evidence from eBay and Tradera auctions supports that bidders switch between auctions of similar items, and that they tend to bid in the auction with the lower standing bid (Andersson et al. 2012, Anwar et al. 2006). Comparable behavior has been observed in an experiment by Ott (2009) in a related setting with heterogeneous items and unit demand, and in a setting with multi-unit demand and complementarities by Kagel et al. (2014). While many bids of human bidders in experiments are in line with straightforward bidding, deviations occur for example by following cues in selecting the bundle on which to bid, by bidding aggressively to increase others' payments (Kagel et al. 2014), or by jump bidding of impatient bidders in single-unit ascending auctions (Isaac et al. 2005).

valuation determines the price. In our theoretical analysis, we neglect price variations that result from the ascending auctions' increments (which converge to zero if the increment approaches zero).

Other markets that for sellers are strategically equivalent to our auction market, as they result in the same trades at prices equal to p^* and provide the same incentives to sellers, are the decentralized market with ascending second-price proxy auctions used by eBay (Peters and Severinov 2006) and the centralized seller's bid double auction (e.g., Williams 1991).[4]

2.2. Sellers' equilibrium reserve prices

A competing seller, just as a monopolist, faces a trade-off: increasing the reserve price increases the expected price paid conditional on the sale of the object and reduces the probability of a sale. A sale is less likely because either no buyer is willing to bid above the seller's reserve price or – differently to the monopolistic case – because some buyers still willing to bid above the seller's increased reserve price switch to sellers with lower reserve prices.

Profit-maximizing competing sellers simultaneously set reserve prices as follows.

Proposition 1. *Consider an auctions market with m sellers and n buyers. A seller's reserve price in a symmetric Bayesian equilibrium is given by*

$$r_j(c_j) = {}^{(100+mc_j)}/_{(m-\xi)} \text{ for all } j \in \{1, 2, \ldots, m\}.$$

A seller's expected profit in this equilibrium decreases in m.

Proposition 1 implies that a seller's mark-up $r_j(c_j) - c_j = {}^{(100+\xi c_j)}/_{(m-\xi)}$ is non-negative and decreases in the number of sellers. Note that a seller's mark-up is independent of the number of buyers. For a monopolistic seller, it is well known that the auction with this reserve price is an optimal mechanism (see Myerson 1981, Riley and Samuelson 1981). The proof of Proposition 1 is given in Appendix A.

[4] Peters and Severinov (2006) analyze the buyers' side and find a perfect Bayesian equilibrium in which all prices equal p^* when reserve prices, valuations, and bids lie on a discrete grid. For the sellers' side, they find that the optimal reserve price equals the costs on the grid if there are sufficiently many sellers and buyers. Thus, they do not consider our setting with continuous reserve prices and arbitrary low number of sellers. Williams (1991) analyzes the buyer's bid double auction. In this discrete-time double auction or clearinghouse, the price with n buyers and m sellers equals the m-highest (or $n + 1$-smallest) bid among all bids by buyers and sellers, sellers report their costs truthfully and buyers bid $b_i(v_i) = {}^{100nv_i}/_{(n+1)}$ in equilibrium when $c_j, v_i \sim U[0, 100]$ for all $j \in \{1, 2, \ldots, m\}$ and $i \in \{1, 2 \ldots, n\}$. In a seller's bid double auction, in contrast, the price equals the $m + 1$-highest bid, a buyer never determines the price if he trades, and buyers therefore bid $b_i(v_i) = v_i$.

2.3. Efficiency and Payoffs When Markets are Merged

Merging two markets changes efficiency and market participants' payoffs. To analyze merged markets, we assume uniform distributions, which we will use in the experiment, and compare two markets – each with one seller and one buyer – with a single market consisting of two sellers and two buyers. In particular, this merger of two identical markets does not change the buyer-seller ratio.

The effect of the merger on market efficiency is twofold. Even if the merger had no impact on reserve price setting, different matches of buyers and sellers can occur; we refer to this as the *matching effect* on gains from trade. In addition, increased seller competition results in smaller seller mark-ups in the merged markets, which for all realizations of costs and values weakly increases gains from trade in the merged markets over that in separate markets; we refer to this as the *competition effect* on gains from trade. Whereas the competition effect is positive, the same is not clear for the matching effect. This is because on the one hand, a merger creates additional matching possibilities that increase efficiency. On the other hand, there exist realizations of valuations and costs for which trading both items would be efficient and would happen in separated markets whereas the trade of only one item would occur in the merged markets, even if reserve prices were set to the same amount as in the separated markets (see Appendix B).[5] We show that overall this matching effect (and therefore the total effect) increases total gains from trade in expectation (see tables 5 and B.7).

Both buyers and sellers in expectation are better off after the merger (see Table B.7). While individual buyers might be worse off,[6] overall they benefit from increased seller competition and better matching opportunities (whereas their bidding strategy is unaffected by the presence of the additional buyer). Sellers suffer from a competition effect due to stronger seller competition and expect only 92% of their maximum joint profit, whereas they could achieve this maximum joint profit if they used their reserve-price functions of the separate markets.[7] However, this negative effect is overcompensated by a positive matching effect,

[5]Note that if sellers set reserve prices equal to their costs, this latter case could not arise and gains from trade would increase for all realizations of costs and values.

[6]As in the example above and in Appendix B, where one of the buyers trades before the merger but does not trade after the merger.

[7]The two sellers could achieve their maximum joint payoff 22.5 (or 11.3 per seller) if they both set their reserve price to $r_j(c_j) = {}^{(100+c_j)}/_2$. This corresponds to the optimal mechanism for a single seller of two items: Bulow and Roberts (1989) state that the payoff-maximizing mechanism for a seller of multiple units and increasing marginal costs is to "Sell k units to the buyers with the k highest marginal revenues [i.e., virtual values $v_i - {}^{1-F(v_i)}/_{f(v_i)}$], where the kth-highest marginal revenue is greater than the marginal cost of the kth unit and the $(k+1)$st marginal revenue is less than the corresponding marginal cost. Each of the k buyers pays the value on his demand curve associated with the greater of the kth marginal cost and the $(k+1)$st marginal revenue, in other words, the minimum value the buyer would have had to have to qualify to receive a unit."

so that a seller's profit is higher in the merged markets than in the separate markets.

2.4. Hypotheses

The first hypothesis concerns the merger of two identical markets (see Proposition 1 and Subsection 2.3).

Hypothesis 1. *If two markets, each with one seller and one buyer, are merged, sellers in the merged markets choose lower mark-ups than sellers in the separate markets. In the merged markets, total gains from trade are higher than the sum of the two separate markets, and both a seller's mean profit and a buyer's mean payoff are higher.*

The second hypothesis concerns the effect of increased seller competition (see Proposition 1; the effects on total gains and on buyers' payoffs in equilibrium follow directly from the effect on reserve prices).[8]

Hypothesis 2. *For a given number of buyers, the higher the number of sellers, the lower are the mark-ups and a seller's profit, and the higher are a buyer's payoff and the total gains from trade.*

The third hypothesis concerns the effect of increased buyer competition. According to our theoretical analysis, the mark-up is independent of the number of buyers. However, experimental evidence from the monopolistic seller case (Davis et al. 2011) suggests that sellers set higher reserve prices when more buyers compete. We base our prediction on this empirical evidence and hypothesize that this generalizes to our case of competing sellers.

Hypothesis 3. *If the number of sellers is fixed, competing sellers' mark-ups increase with the number of buyers.*

3. Experimental Design

In order to keep the design as simple as possible (while still resembling real-life markets) and to obtain clean theoretical predictions we make the following two experimental design choices. First, we computerize the bidding behavior of buyers. Computerized buyers bid straightforwardly as described in Subsection 2.1. This allows a comparison with previous research of the monopolistic case (Davis et al. 2011) and gives us control over sellers' beliefs about buyers' behavior. Thus, the automation of buyers reduces strategic uncertainty of the sellers, who can focus on their interaction with the other seller. Second, costs and valuations

[8] Unlike the setup in Hypothesis 1, the compared markets involve a different number of goods and therefore, if reserve prices are set lower when there are more sellers, efficiency gains and increases in buyer's payoffs are straightforward for the larger markets. We nevertheless analyze these for the sake of completeness.

Table 1: Treatments, number of sessions, number of subjects, and number of rounds in the treatments

Treatment	Sellers	Buyers	# Sessions	# Subjects	# Rounds
1S1B	1	1	1	30	40
2S1B	2	1	1	30	40
1S2B	1	2	1	30	40
2S2B	2	2	1	30	40
2S5B	2	5	1	30	40

are uniformly (and independently) distributed. This distribution is easiest to understand for subjects.

We restrict attention to markets with very small numbers of participants, in particular we only consider the case of one or two sellers in any given market. This is to guarantee that our setup is sufficiently distinct from competitive markets in which participants' behavior hardly affects prices. It also reflects that we are interested in establishing the direction of effects of competition rather than their magnitude. To test our hypotheses we use four treatments in which one or two sellers each encounter one or two buyers. To allow for a better comparison of reserve price setting of two competing sellers in the presence of various degrees of buyer competition to the well analyzed case of a monopolistic seller, we in addition consider markets where two sellers encounter five buyers. We name our five treatments according to the numbers of sellers (S) and buyers (B) 1S1B, 1S2B, 2S1B, 2S2B, and 2S5B (see Table 1).

Subjects received written instructions that were then read out aloud. The automated bidding behavior of buyers was explained to subjects verbally and by means of a demonstration video. The video illustrated the setting using several examples for different orderings of reserve prices and bids. Subjects had to answer questions on the rules. Then, they participated in 40 identical rounds of auctions.

In each round, subjects took the role of a seller and set a starting price, i.e., reserve price, in their ascending auction. Buyers were automated and bid straightforwardly, as defined in Section 2.1, in random order. The auctions thus all ended at a price equal to the $m+1$-highest among buyers' valuations and sellers' reserve prices (with a potential deviation by plus one increment of one experimental currency unit (CU) in the case of two sellers and more than one bidder due to the random choice of the auction). Subjects knew their own distribution of the costs and its realization, the distribution of costs of a competing seller if present, and the distributions of the valuations of the buyer(s). Costs of sellers and valuations of buyers were independently drawn from uniform distributions $U\{1, 2, \ldots, 100\}$ in each round. Costs were opportunity costs, e.g., from an outside option for selling the already produced good at a price equal to these opportunity costs, and were therefore called values. A seller's earning in a period would equal his value in case of no trade and would equal the price in case of a sale.[9] In case of two sellers, sellers were rematched randomly at the beginning of each round. In each round, subjects learned their own costs, submitted their reserve price, outcomes were calculated and each subject was shown the own payoff and, in case of a sale, the sales price in their auction. With this information feedback, subjects in case of two sellers could neither observe the other seller's reserve price (because the price can be determined by a buyer or a seller) nor the other seller's costs.

At the end of the sessions, subjects filled out a questionnaire (comments, age, gender, degree program). They were paid according to five publicly randomly selected rounds plus a guaranteed show-up fee of four Euro. The conversion rate was 0.04 Euro/CU. The mean, minimum and maximum total earnings were 15.87 Euro, 8.80 Euro, and 21.00 Euro. Sessions lasted for one to one and a half hours and were conducted in the lab AiXperiment in Aachen using zTree (Fischbacher 2007). The 150 participants were recruited from a pool of students of various fields of study using ORSEE (Greiner 2015).

4. Results

In addition to non-parametric tests (Wilcoxon-Mann-Whitney tests[10]), we do regression analyses to infer how sellers' costs were used to determine reserve prices. The linear equilibrium strategy of the seller provides predictions for the coefficients. In the analyses, we replace observed reserve prices above 101, which all result in no sale, by 101 to reduce potential effects of outliers.[11] We apply a 5% significance level to all tests. Decisions of different subjects are treated as independent. In the two-seller treatments this is justified because sellers with random rematching, randomly drawn costs and values in each round, and the information feedback they receive after each round (see Section 3) cannot derive reliable information about the opponent's behavior.

The observed and predicted reserve-price mark-ups on costs and the profits are summarized in Table 2. Subjects on average chose lower mark-ups than predicted in all treatments.

[9] With this framing, every seller earns a positive payoff in each round (unless she sells below value), which has been conjectured to reduce the influence of cash balance effects (Kagel and Levin 2016) and aggressive bidding caused by anticipated regret (Turocy and Watson 2012).

[10] The Wilcoxon-Mann-Whitney test requires equal distributions of the parameter in the populations. If this assumption is violated, rejecting the null hypothesis does not allow concluding on a location shift (i.e., a difference in mean or median). For all Wilcoxon-Mann-Whitney tests on our hypotheses (see tables 6 and 4), we therefore also conducted one-tailed tests of stochastic equality with H_0: $P(X_1 > X_2) = P(X_1 < X_2)$ suggested by Schlag (2008) that do not require any assumption on distributions. The conclusions based on these tests are identical to those based on Wilcoxon-Mann-Whitney tests.

[11] The highest number accepted by the experimental software is 999. We replaced one 999 in 1S2B, and one 120 and one 999 in 2S1B.

Table 2: Observed mean mark-ups and profits. Equilibrium prediction for realized costs in parentheses.

Treatment	Mean mark-up $r_j - c_j$	Mean profit π_j
1S1B	11.7 (eq.: 24.6)	4.7 (eq.: 8.4)
2S1B	9.2 (eq.: 16.2)	3.2 (eq.: 5.6)
1S2B	12.2 (eq.: 24.3)	11.1 (eq.: 13.9)
2S2B	5.3 (eq.: 16.5)	6.0 (eq.: 10.4)
2S5B	9.7 (eq.: 16.4)	19.3 (eq.: 21.1)

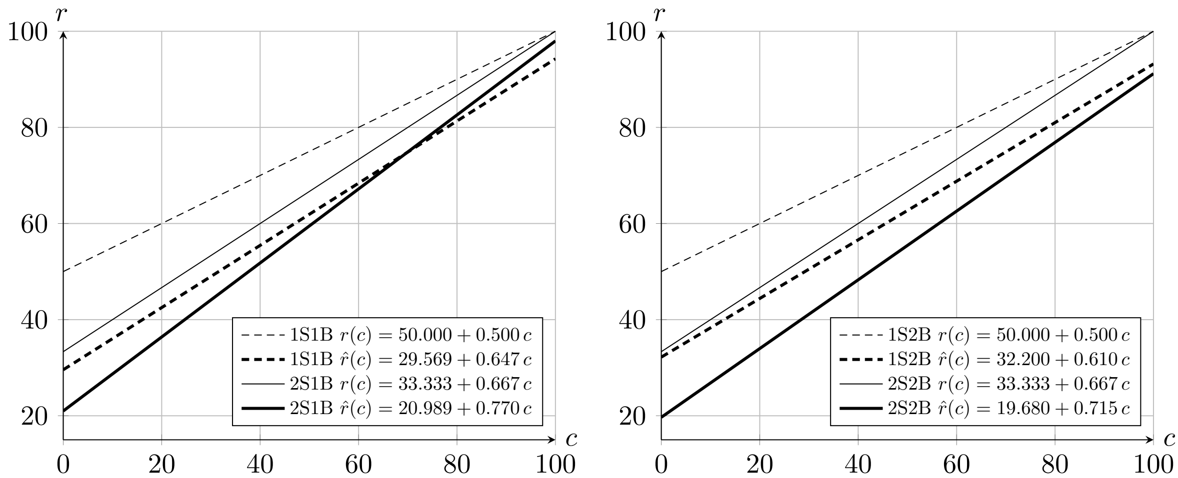

Figure 1: Equilibrium ($r(c)$) and estimated ($\hat{r}(c)$) reserve-price functions in 1S1B and in 2S1B (LHS) and in 1S2B and in 2S2B (RHS)

Estimated bidding functions are displayed in Figure 1. We estimate the model $r = \beta_0 + \beta_1 d_{S2} + \beta_2 c + \beta_3 c \times d_{S2}$, where the treatment dummy is $d_{S2} = 0$ in a treatment with one seller, in a basic linear regression as well as with random effects, both with cluster-robust standard errors (see Table C.8).

4.1. Changing Competition by Merging Markets

Who profits from a merger of markets? Theory predicts that total gains from trade increase and that both market sides benefit. To compare total gains from trade in markets with one agent on each market side with merged markets (that then have two agents per market side), we calculate the mean total gains from trade divided by the number of market participants in the 40 markets in which a subject participates. To compare the sellers' profits, we calculate the mean profit per subject over the 40 rounds, to compare the buyers' payoffs (or the total gains from trade) we calculate the mean payoff per buyer (or the mean total gains from trade) in the 40 markets in which a subject participates (see Table 3).

Table 3: Mean observed total gains from trade per agent. Mean gains from trade per agent in equilibrium (eq.) and in the efficient outcome (eff.) for the realized costs in parentheses. Seller's profit and buyer's payoff observed and in equilibrium for the realized costs (eq.).

Treatment	Total gains (per agent)	Seller's profit	Buyer's payoff
1S1B	7.3 (eq.: 6.3, eff.: 8.3)	4.7 (eq.: 8.4)	9.8 (eq.: 4.2)
2S2B	9.2 (eq.: 9.3, eff.: 10.1)	6.0 (eq.: 10.4)	12.4 (eq.: 8.2)

Table 4: Wilcoxon-Mann-Whitney tests on Hypothesis 1 (one-tailed p-values, N=N_1=N_2).

Variable	H_0	H_1	N	U	p
Mark-up	$(r-c)_{\text{1S1B}} = (r-c)_{\text{2S2B}}$	$(r-c)_{\text{2S2B}} < (r-c)_{\text{1S1B}}$	30	242	0.001
π^{S}	$\pi^{\text{S}}_{\text{1S1B}} = \pi^{\text{S}}_{\text{2S2B}}$	$\pi^{\text{S}}_{\text{1S1B}} < \pi^{\text{S}}_{\text{2S2B}}$	30	386.5	0.176
π^{B}	$\pi^{\text{B}}_{\text{1S1B}} = \pi^{\text{B}}_{\text{2S2B}}$	$\pi^{\text{B}}_{\text{1S1B}} < \pi^{\text{B}}_{\text{2S2B}}$	30	240.5	0.001
Total gains	$((\pi^{\text{S}}+\pi^{\text{B}})/2)_{\text{1S1B}} =$ $((\pi^{\text{S}}_1+\pi^{\text{S}}_2+\pi^{\text{B}}_1+\pi^{\text{B}}_2)/4)_{\text{2S2B}}$	$((\pi^{\text{S}}+\pi^{\text{B}})/2)_{\text{1S1B}} <$ $((\pi^{\text{S}}_1+\pi^{\text{S}}_2+\pi^{\text{B}}_1+\pi^{\text{B}}_2)/4)_{\text{2S2B}}$	30	193.5	0.000

π^{S}: the seller's profit. π^{B}: the buyer's payoff.

The data partly support Hypothesis 1 (see Table 4). There is a significant competition effect because sellers in the merged markets choose their reserve prices on average closer to their costs than in the separate markets. Mean total gains from trade and the buyers' mean payoff are significantly and by 27% higher in the merged markets than in the separate markets. However, the sellers' mean profits do not significantly differ.

To further investigate the effect of merging markets, we use the estimated reserve-price functions for 1S1B and 2S2B to decompose the overall effect into the matching effect and the competition effect. The matching effect is the difference between payoffs in 1S1B and 2S2B when reserve prices were set as in 1S1B. The competition effect is the difference between payoffs in 2S2B when reserve prices were set as in 1S1B and when reserve prices are set as in 2S2B. Both effects have the predicted direction, i.e., the matching effect increases sellers' profits and buyers' payoffs, whereas the competition effect decreases the sellers' profits and increases the buyers' payoffs and overall gains from trade (see Table 5 for the size of the effects; see Table B.7 for estimated and equilibrium outcomes). The matching effect has a stronger positive influence on the gains from trade than the competition effect. Overall, the competition effect and therefore the total effect is smaller than in equilibrium.[12] Buyers mainly profit from the competition effect and get a smaller share of the matching

[12]Note, however, that the smaller increase in gains from trade when comparing estimated effects and equilibrium is due to the higher level of gains from trade in 1S1B with the estimated function of 1S1B than in equilibrium. Gains from trade in 1S1B with estimated reserve prices are 7.7, so the maximum increase is 2.3 because they are limited by gains from trade in the efficient outcome, which are 10.0 (see Table B.7). Thus, the increase of 3.0 in equilibrium is not possible when starting from the estimated outcome of 1S1B.

Table 5: Estimated matching and competition effects and effects in equilibrium (eq.).

Variable	Matching effect	Competition effect	Overall effect
Seller's profit	3.1 (eq.: 2.9)	−1.8 (eq.: −0.9)	1.3 (eq.: 2.0)
Buyer's payoff	0.5 (eq.: 0.8)	2.4 (eq.: 3.1)	3.0 (eq.: 4.0)
Total gains (per agent)	1.8 (eq.: 1.9)	0.3 (eq.: 1.1)	2.1 (eq.: 3.0)

Table 6: Wilcoxon-Mann-Whitney tests on Hypothesis 2 (one-tailed p-values, N=N_1=N_2).

Variable	H_0	H_1	N	U	p
Mark-up	$(r-c)_{1S1B} = (r-c)_{2S1B}$	$(r-c)_{2S1B} < (r-c)_{1S1B}$	30	264.5	0.003
Seller's profit	$\pi^S_{1S1B} = \pi^S_{2S1B}$	$\pi^S_{2S1B} < \pi^S_{1S1B}$	30	235.5	0.001
Buyer's payoff	$\pi^B_{1S1B} = \pi^B_{2S1B}$	$\pi^B_{1S1B} < \pi^B_{2S1B}$	30	124	0.000
Total gains	$(\pi^S + \pi^B)_{1S1B} =$ $(\pi^S_1 + \pi^S_2 + \pi^B)_{2S1B}$	$(\pi^S + \pi^B)_{1S1B} <$ $(\pi^S_1 + \pi^S_2 + \pi^B)_{2S1B}$	30	46.5	0.000
Mark-up	$(r-c)_{1S2B} = (r-c)_{2S2B}$	$(r-c)_{1S2B} > (r-c)_{2S2B}$	30	250	0.002
Seller's profit	$\pi_{1S2B} = \pi_{2S2B}$	$\pi_{1S2B} > \pi_{2S2B}$	30	163	0.000
Buyer's payoff	$\pi^B_{1S2B} = \pi^B_{2S2B}$	$\pi^B_{1S2B} < \pi^B_{2S2B}$	30	22	0.000
Total gains	$(\pi^S + \pi^B_1 + \pi^B_2)_{1S2B} =$ $(\pi^S_1 + \pi^S_2 + \pi^B_1 + \pi^B_2)_{2S2B}$	$(\pi^S + \pi^B_1 + \pi^B_2)_{1S2B} <$ $(\pi^S_1 + \pi^S_2 + \pi^B_1 + \pi^B_2)_{2S2B}$	30	23.5	0.000

effect, though the competition effect is smaller than in equilibrium. For sellers, the negative competition effect does not fully offset the positive matching effect, but is stronger than in equilibrium.

4.2. Increasing Seller Competition

Our data supports Hypothesis 2. Both with one and two buyers, the mark-ups and profits of two competing sellers are significantly lower than those of monopolistic sellers. Also, as predicted, a buyer's payoff and total gains from trade are higher when there are more sellers. The results of the non-parametrical tests are given in Table 6.

Comparing treatments with one buyer using the estimated bidding functions, we find, as predicted, that the reserve-price function with two sellers starts at a lower level and is steeper than with one seller ($r(0) = 29.6$ vs. $r(0) = 21.0$ and slopes of 0.65 vs. 0.77).[13] Figure 1 shows that the difference between one and two sellers' mean mark-ups is mainly driven by mark-ups on low costs, as the regression lines cross.

In the case of two buyers, as predicted, the reserve price function with two sellers has a smaller intercept and larger slope than with one seller (32.2 vs. 19.7 and slopes of 0.61

[13]The signs of β_1 and β_3 are negative and positive, as predicted. z-tests reject $\beta_1 = 0$ and $\beta_3 = 0$ (see Table C.8).

vs. 0.72),[14] and lies below that of a monopolistic seller.

Comparing observed reserve prices with equilibrium reserve prices, Figure 1 reveals that all four estimated bid-functions lie below the equilibrium bid-functions. The difference is significant in treatment 2S2B, where the intercept is lower than predicted and we find no evidence for different slopes. However, with monopolistic sellers, the intercepts are smaller but the functions are steeper than predicted. In treatment 2S1B, the function is also steeper than predicted but there is no evidence for a difference in intercepts.[15]

4.3. Increasing Buyer Competition

Our data does not support Hypothesis 3. There is no evidence that the mark-up of competing sellers increases with the number of buyers (see Table 2).[16] However, sellers profit from buyer competition and have a significantly higher payoff when the number of buyers increases.[17]

5. Conclusion

This paper explores how market performance measures such as efficiency and payoffs can be affected by a market designer with some control over the intensity of competition and buyers' switching costs.

Increasing the number of sellers has the effect of lowering sellers' mark-ups and payoffs (and, therefore, also increasing gains from trade). Thus, attracting additional auctioneers increases market efficiency for two reasons: a decrease in reserve prices and an increase in trading opportunities.

Market efficiency can also be improved without the need to attract additional auctioneers. This is possible if bidders are not aware of (or find it too costly to participate in) simultaneous auctions of substitutes and a designer can change that by effectively merging auctions. This could be achieved by advertising similar or identical objects that are for sale at the same time, or by having an automated bidding agent bid simultaneously in several actions. Obviously, our results cannot indicate whether attracting additional sellers or

[14] The signs of β_1 and β_3 are negative and positive, as predicted. z-tests reject $\beta_1 = 0$ and $\beta_3 = 0$ at the 5% level with one-tailed tests (see Table C.8); $p = 0.076/2$ for the hypothesis $\beta_3 = 0$.

[15] F-tests on null hypotheses on intercepts and slopes:
1S1B: $\beta_0 = 50$: $F(1, 59) = 84.65$, $p = 0.0000$; $\beta_2 = 1/2$: $F(1, 59) = 31.19$, $p = 0.0000$.
2S1B: $\beta_0 + \beta_1 = 100/3$: $F(1, 59) = 1.01$, $p = 0.3188$; $\beta_2 + \beta_3 = 2/3$: $F(1, 59) = 22.09$, $p = 0.0000$.
1S2B: $\beta_0 = 50$: $F(1, 59) = 54.06$, $p = 0.0000$; $\beta_2 = 1/2$: $F(1, 59) = 17.92$, $p = 0.0001$.
2S2B: $\beta_0 + \beta_1 = 100/3$: $F(1, 59) = 4.35$, $p = 0.0414$. $\beta_2 + \beta_3 = 2/3$: $F(1, 59) = 1.60$, $p = 0.2106$.

[16] Kruskal-Wallis test: H_0: $(r - c)_{2S1B} = (r - c)_{2S2B} = (r - c)_{2S5B}$, $p = 0.52$. In a regression analysis with treatment dummies to detect differences in intercepts and slopes between 2S1B and 2S2B as well as 2S1B and 2S5B, none of the treatment dummies differs significantly from zero at the 5% level in two-tailed t-tests.

[17] Jonckheere-Terpstra test H_0: $\pi_{\text{2S5B}} = \pi_{\text{2S2B}} = \pi_{\text{2S1B}}$, H_1: $\pi_{\text{2S5B}} \geq \pi_{\text{2S2B}} \geq \pi_{\text{2S1B}}$ with at least one strict inequality, $p = 0.01$.

merging auctions should be the preferred way to enhance efficiency because these measures are neither exclusive nor does their implementation come at similar costs.

In a theoretical analysis, we derive equilibrium predictions on the impact of changes in competition on the aforementioned market performance measures. Whereas our experimental findings are qualitatively in line with these equilibrium predictions, the magnitude of the observed mark-up in many cases is not. Different explanations for such deviations are possible (for example, sellers could be risk-averse or regret averse) and have been analyzed in the context of bidding behavior in first-price auctions (where buyers face a similar trade-off as the sellers in our model) or reserve-price setting (absent auctioneer competition). As our experiment was not designed to test the validity of different behavioral or standard models, we abstain from adding to these explanations, which we believe are all possible and potentially valid in our setup as well.

References

Andersson, T., Andersson, C., Andersson, F., 2012. An empirical investigation of efficiency and price uniformity in competing auctions. Economics Letters 116, 99–101.

Anwar, S., McMillan, R., Zheng, M., 2006. Bidding behavior in competing auctions: Evidence from eBay. European Economic Review 50, 307–322.

Ausubel, L.M., Cramton, P., Pycia, M., Rostek, M., Weretka, M., 2014. Demand reduction and inefficiency in multi-unit auctions. Review of Economic Studies 81, 1366–1400.

Bajari, P., Hortacsu, A., 2003. The winner's curse, reserve prices, and endogenous entry: Empirical insights from eBay auctions. RAND Journal of Economics 34, 329–355.

Bulow, J., Roberts, J., 1989. The simple economics of optimal auctions. The Journal of Political Economy 97, 1060–1090.

Burguet, R., Sakovics, J., 1999. Imperfect competition in auction designs. International Economic Review 40, 231–247.

Davis, A.M., Katok, E., Kwasnica, A.M., 2011. Do auctioneers pick optimal reserve prices? Management Science 57, 177–192.

Fischbacher, U., 2007. z-Tree: Zurich toolbox for ready-made economic experiments. Experimental Economics 10, 171–178. doi:10.1007/s10683-006-9159-4.

Greiner, B., 2015. Subject pool recruitment procedures: Organizing experiments with ORSEE. Journal of the Economic Science Association 1, 114–125. doi:10.1007/s40881-015-0004-4.

Haruvy, E., Popkowski Leszczyc, P.T., 2010. Search and choice in online consumer auctions. Marketing Science 29, 1152–1164.

Hoppe, T., 2008a. An experimental analysis of parallel multiple auctions. Technical Report. Otto-von-Guericke University Magdeburg.

Hoppe, T., 2008b. Just a small delay? Bidding behavior and efficiency in overlapping multiple auctions. Technical Report. Otto-von-Guericke University Magdeburg.

Isaac, R.M., Salmon, T.C., Zillante, A., 2005. An experimental test of alternative models of bidding in ascending auctions. International Jounrnal of Game Theory 33, 287–313. doi:10.1007/s00182-005-0203-y.

Kagel, J.H., Levin, D., 2016. Auctions: A survey of experimental research, in: Kagel, J.H., Roth, A.E. (Eds.), The Handbook of Experimental Economics, Volume 2. Princeton University Press. chapter 9, pp. 563–637.

Kagel, J.H., Lien, Y., Milgrom, P., 2014. Ascending prices and package bidding: Further experimental analysis. Games and Economic Behavior 85, 210–231. doi:10.1016/j.geb.2014.02.006.

Marszalec, D., Teytelboym, A., Laksa, S., 2020. EPIC fail: How below-bid pricing backfires in multiunit auctions. URL: https://t8el.com/wp-content/uploads/2020/10/EFail_2020-16.pdf.

Myerson, R.B., 1981. Optimal auction design. Mathematics of Operations Research 6, 58–73.

Ott, M., 2009. Second-Price Proxy Auctions in Bidder-Seller Networks. Thesis. Universität Karlsruhe.

Pai, M.M., 2010. Competition in mechanisms. SIGecom Exchanges 9, 1–5.

Parkes, D.C., 2006. Iterative combinatorial auctions, in: Cramton, P., Shoham, Y., Steinberg, R. (Eds.), Combinatorial Auctions. Cambridge, Mass: MIT Press, pp. 41–78.

Peters, M., Severinov, S., 2006. Internet auctions with many traders. Journal of Economic Theory 130, 220–245.

Plott, C.R., Smith, V.L. (Eds.), 2008. Handbook of Experimental Economics Results. volume 1. North Holland.

Riley, J.G., Samuelson, W.F., 1981. Optimal auctions. The American Economic Review 71, 381–392.

Schlag, K.H., 2008. A New Method for Constructing Exact Tests without Making any Assumptions. Working Paper. Department of Economics and Business, Universitat Pompeu Fabra, Barcelona.

Turocy, T.L., Watson, E., 2012. Reservation values and regret in laboratory first-price auctions: Context and bidding behavior. Southern Economic Journal 78, 1163–1180.

Virág, G., 2010. Competing auctions: Finite markets and convergence. Theoretical Economics 5, 241–274.

Williams, S.R., 1991. Existence and convergence of equilibria in the Buyer's Bid Double Auction. The Review of Economic Studies 58, 351–374.

Appendix A. Proof of Proposition 1

The following proof mirrors arguments used by Williams (1991) in his analysis of the buyer's bid double auction, because our sellers face the inverse of the strategic problem of William's buyers. In addition, it extends the analysis from the uniform distribution (Williams 1991) to the family of Generalized Pareto Distributions with monotone non-decreasing hazard rate.[18]

[18] This family consists of the Generalized Pareto Distributions with shape parameter $\xi < 0$ in (1) and with $\xi = 0$, which has the cdf $F(x) = 1 - e^{-x}$ for all $x \geq 0$. All arguments in the proof apply also to the latter

Proof of Proposition 1. Consider an auction market with n buyers, indexed by i, and $m-1$ sellers, indexed by j. Buyers' private values v_i and sellers' private costs c_j are independently distributed according to a Generalized Pareto Distribution F_ξ as given in (1). For convenience, we rescale the problem to the standard supports $[0, -1/\xi]$ instead of $[0, -100/\xi]$ and denote the cdf and pdf by $\tilde{F}_\xi$ and $\tilde{f}_\xi$. Note that, for all $\xi < 0$, the hazard rate $\tilde{f}_\xi(x)/(1-\tilde{F}_\xi(x)) = (1+\xi x)^{-1}$ with the derivative $-\xi(1+\xi x)^{-2}$ is monotone increasing in x.

We will show that the optimal reserve price of (w.l.o.g.) seller 1 is $r_1(c_1) = (1+mc_1)/(m-\xi)$ if $r_j(c_j) = (1+mc_j)/(m-\xi)$ for all $j \in \{2, 3, \ldots, m\}$ and if all buyers $i \in \{1, 2, \ldots, n\}$ follow the strategies ascribed to them in the main text. In what follows, assume the $m-1$ sellers $j > 1$ and the n buyers follow these strategies.

Recall that a buyer quits the market if the price in all auctions has reached his valuation. Thus, buyer i's strategy can be described by his maximum bid, which equals his valuation v_i. The probability that a buyer with v_i will quit at a price below a reserve price r_1 is $F_B(r_1) := \text{Prob}\{v_i \leq r_1\} = \tilde{F}_\xi(r_1)$ with the pdf $f_B(r_1) = \tilde{f}_\xi(r_1)$. The probability that a seller $j > 1$ with c_j sets a reserve price below r_1 is $F_S(r_1) := \text{Prob}\{r_*(c_j) \leq r_1\} = \text{Prob}\{c_j \leq r_*^{-1}(r_1)\} = \tilde{F}_\xi(r_*^{-1}(r_1))$ with the pdf $f_S(r_1) = \tilde{f}_\xi(r_*^{-1}(r_1)) \cdot \mathrm{d}\, r_*^{-1}(r_1)/\mathrm{d}\, r_1$.

Denote by $F_{(k:n+m-1)}$ and $f_{(k:n+m-1)}$ the cdf and the corresponding pdf of the k-th order statistics for $k \in \{1, 2, \ldots, n+m-1\}$ (where $k = 1$ denotes the distribution of the maximum) calculated from the n buyers' valuations and the reserve prices of the $m-1$ sellers $j > 1$.

If $m > 1$ and seller 1 sets a reserve price $r_1 \geq 1/(m-\xi)$, then

$$F_{(k:n+m-1)}(r_1) = \sum_{\substack{0 \leq i+j \leq k-1 \\ 0 \leq i \leq n \\ 0 \leq j \leq m-1}} \binom{n}{i}\binom{m-1}{j} \tilde{F}_\xi(r_1)^{n-i}(1-\tilde{F}_\xi(r_1))^i (\tilde{F}_\xi(r_*^{-1}(r_1)))^{m-1-j}(1-\tilde{F}_\xi(r_*^{-1}(r_1)))^j.$$

If $m > 1$ and $r_1 < 1/(m-\xi)$ or if $m = 1$, then seller 1 trades if at least one buyer bids more than r_1 (because the reserve prices of the other sellers, if there are any, are above r_1). We get for $k \in \{m, m+1, \ldots, n+m-1\}$

$$F_{(k:n+m-1)}(r_1) = \sum_{i=0}^{k-m} \binom{n}{i} \tilde{F}_\xi(r_1)^{n-i}(1-\tilde{F}_\xi(r_1))^i.$$

distribution but we exclude it for ease of exposition.

The expected payoff of seller 1 with costs c_1 setting a reserve price r_1 is given by[19]

$$\mathbb{E}\left[\pi_1(r_1,c_1)\right] = \big(F_{(m+1:n+m-1)}(r_1) - F_{(m:n+m-1)}(r_1)\big)(r_1-c_1) + \int_{r_1}^{-1/\xi} (x-c_1) f_{(m+1:n+m-1)}(x)\,\mathrm{d}x.$$

This function is differentiable for all r_1 if $m=1$. If $m>1$, it is continuous but not necessarily differentiable at $r_1 = 1/(m-\xi)$. For $m=1$ and for $m>1$ and $r_1 \neq 1/(m-\xi)$, the derivative is

$$\tfrac{\partial \mathbb{E}[\pi_1(r_1,v_1)]}{\partial r_1} = F_{(m+1:n+m-1)}(r_1) - F_{(m:n+m-1)}(r_1) - f_{(m:n+m-1)}(r_1)(r_1-c_1).$$

For $m>1$ and $0<r_1<1/(m-\xi)$, the expected utility is strictly increasing in r_1 for all c_1,

$$\begin{aligned}
\tfrac{\partial \mathbb{E}[\pi_1(r_1,v_1)]}{\partial r_1} &= F_{(m+1:n+m-1)}(r_1) - F_{(m:n+m-1)}(r_1) - f_{(m:n+m-1)}(r_1)(r_1-c_1)\\
&= n\tilde{F}_\xi(r_1)^{n-1}\Big(1-\tilde{F}_\xi(r_1) - \tilde{f}_\xi(r_1)(r_1-c_1)\Big)\\
&\geq n\tilde{F}_\xi(r_1)^{n-1}\Big(1-\tilde{F}_\xi(r_1) - \tilde{f}_\xi(r_1)r_1\Big)\\
&= n\tilde{F}_\xi(r_1)^{n-1}\Big((1+\xi r_1)^{-\frac{1}{\xi}} - (1+\xi r_1)^{-\frac{1+\xi}{\xi}} r_1\Big)\\
&= n\tilde{F}_\xi(r_1)^{n-1}(1+\xi r_1)^{-\frac{1}{\xi}}\big(1-(1+\xi r_1)^{-1}r_1\big) > 0.
\end{aligned}$$

The last inequality holds because $(1+\xi r_1)^{-\frac{1}{\xi}} \in (0,1]$ for $r_1>0$ by $\tilde{F}_\xi$ being a cdf, and because $1-(1+\xi r_1)^{-1}r_1 > 0$ for all $r_1 < 1/(m-\xi)$ (with $1+\xi r_1>0$ for all $r_1 < -1/\xi$). This implies that the best response of seller 1 to the given strategies of the other sellers and buyers is to set a reserve price $r_1 \geq 1/(m-\xi)$ for all c_1. Thus, for $m=1$ and for $m>1$ and $r_1 > 1/(m-\xi)$, the first-order condition is given by

$$r_1 - c_1 = \tfrac{F_{(m+1:n+m-1)}(r_1) - F_{(m:n+m-1)}(r_1)}{f_{(m:n+m-1)}(r_1)}. \tag{A.1}$$

The difference $\Delta F(r_1) := F_{(m+1:n+m-1)}(r_1) - F_{(m:n+m-1)}(r_1)$ simplifies to

$$\Delta F(r_1) = \sum_{\substack{i+j=m\\1\leq i\leq n\\0\leq j\leq m-1}} \tbinom{n}{i}\tbinom{m-1}{j}\tilde{F}_\xi(r_1)^{n-i}(1-\tilde{F}_\xi(r_1))^i \tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-1-j}\Big(1-\tilde{F}_\xi(r_*^{-1}(r_1))\Big)^j.$$

Assuming $n \geq m$ (for the case $n<m$, the steps are the same with $n-m$ and $n-m-1$ as

[19]Note that $F_{(k:n+m-1)}(r_1)$ is the probability that the k-th highest of the other $n+m-1$ participants has a valuation or reserve price below r_1. Thus, $F_{(m+1:n+m-1)}(r_1) - F_{(m:n+m-1)}(r_1)$ is the probability that exactly m of the others have a valuation or reserve price above r_1.

the lower bounds on the index j) and using $i = m - j$, this simplifies to

$$\Delta F(r_1) = \sum_{j=0}^{m-1} \binom{n}{m-j}\binom{m-1}{j} \tilde{F}_\xi(r_1)^{n-m+j}(1-\tilde{F}_\xi(r_1))^{m-j}\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-1-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^j . \tag{A.2}$$

The pdf $f_{(m:n+m-1)}(r_1)$ captures the case that one of n buyers quits at r_1 and exactly $m-1$ of the remaining $n-1$ valuations and $m-1$ reserve prices are above r_1 or that one of $m-1$ sellers sets his reserve price $r_*(c_j)$ at r_1 and exactly $m-1$ of the remaining n valuations and $m-2$ reserve prices are above r_1. The pdf $f_{(m:n+m-1)}(r_1)$ is therefore given by

$$n f_B(r_1) \sum_{\substack{i+j=m-1\\0\le i\le n-1\\0\le j\le m-1}} \binom{n-1}{i}\binom{m-1}{j}\tilde{F}_\xi(r_1)^{n-i-1}(1-\tilde{F}_\xi(r_1))^i\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-1-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^j$$

$$+ (m-1) f_S(r_1) \sum_{\substack{i+j=m-1\\1\le i\le n\\0\le j\le m-2}} \binom{n}{i}\binom{m-2}{j}\tilde{F}_\xi(r_1)^{n-i}(1-\tilde{F}_\xi(r_1))^i\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-2-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^j ,$$

which simplifies to

$$n\tilde{f}_\xi(r_1)\sum_{j=0}^{m-1}\binom{n-1}{m-1-j}\binom{m-1}{j}\tilde{F}_\xi(r_1)^{n-m+j}(1-\tilde{F}_\xi(r_1))^{m-1-j}\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-1-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^j$$

$$+ (m-1)\tilde{f}_\xi(r_*^{-1}(r_1))\frac{\mathrm{d}\, r_*^{-1}(r_1)}{\mathrm{d}\, r_1}\cdot$$

$$\cdot\sum_{j=0}^{m-2}\binom{n}{m-1-j}\binom{m-2}{j}\tilde{F}_\xi(r_1)^{n-m+j+1}(1-\tilde{F}_\xi(r_1))^{m-1-j}\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-2-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^j .$$

In a first step, replace in the first sum and the last sum, respectively,

$$\binom{n-1}{m-1-j} = \frac{m-j}{n}\binom{n}{m-j} \qquad \text{and} \qquad \binom{m-2}{j} = \frac{j+1}{m-1}\binom{m-1}{j+1},$$

in a second step, replace the index j in the last sum with $j+1$, and in a third step, factor out to express $f_{(m:n+m-1)}(r_1)$ by

$$
\begin{aligned}
& n\tilde{f}_\xi(r_1)\sum_{j=0}^{m-1}\tfrac{m-j}{n}\tbinom{n}{m-j}\tbinom{m-1}{j}\tilde{F}_\xi(r_1)^{n-m+j}(1-\tilde{F}_\xi(r_1))^{m-1-j}\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-1-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^j \\
&\quad + (m-1)\tilde{f}_\xi(r_*^{-1}(r_1))\tfrac{\mathrm{d}\,r_*^{-1}(r_1)}{\mathrm{d}\,r_1}\cdot \\
&\quad \cdot\sum_{j=0}^{m-2}\tbinom{n}{m-1-j}\tfrac{j+1}{m-1}\tbinom{m-1}{j+1}\tilde{F}_\xi(r_1)^{n-m+j+1}(1-\tilde{F}_\xi(r_1))^{m-1-j}\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-2-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^j \\
&= \tilde{f}_\xi(r_1)\sum_{j=0}^{m-1}(m-j)\tbinom{n}{m-j}\tbinom{m-1}{j}\tilde{F}_\xi(r_1)^{n-m+j}(1-\tilde{F}_\xi(r_1))^{m-1-j}\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-1-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^j \\
&\quad + \tilde{f}_\xi(r_*^{-1}(r_1))\tfrac{\mathrm{d}\,r_*^{-1}(r_1)}{\mathrm{d}\,r_1}\cdot \\
&\quad \cdot\sum_{j=1}^{m-1}j\tbinom{n}{m-j}\tbinom{m-1}{j}\tilde{F}_\xi(r_1)^{n-m+j}(1-\tilde{F}_\xi(r_1))^{m-j}\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-1-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^{j-1} \\
&= \tfrac{\tilde{f}_\xi(r_1)}{1-\tilde{F}_\xi(r_1)}\sum_{j=0}^{m-1}(m-j)\tbinom{n}{m-j}\tbinom{m-1}{j}\tilde{F}_\xi(r_1)^{n-m+j}(1-\tilde{F}_\xi(r_1))^{m-j}\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-1-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^j \\
&\quad + \tfrac{\tilde{f}_\xi(r_*^{-1}(r_1))}{(1-\tilde{F}_\xi(r_*^{-1}(r_1))}\tfrac{\mathrm{d}\,r_*^{-1}(r_1)}{\mathrm{d}\,r_1}\cdot \\
&\quad \cdot\sum_{j=1}^{m-1}j\tbinom{n}{m-j}\tbinom{m-1}{j}\tilde{F}_\xi(r_1)^{n-m+j}(1-\tilde{F}_\xi(r_1))^{m-j}\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-1-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^j.
\end{aligned}
$$

Using $r_*^{-1}(r_1) = {}^{((m-\xi)r_1-1)}/_m$, we get

$$
\tfrac{\tilde{f}_\xi(r_1)}{1-\tilde{F}_\xi(r_1)} = \tfrac{(1+\xi r_1)^{-\frac{1+\xi}{\xi}}}{(1+\xi r_1)^{-\frac{1}{\xi}}} = (1+\xi r_1)^{-1} = \frac{\left(1+\xi\frac{(m-\xi)r_1-1}{m}\right)^{-\frac{1+\xi}{\xi}}}{\left(1+\xi\frac{(m-\xi)r_1-1}{m}\right)^{-\frac{1}{\xi}}}\tfrac{m-\xi}{m} = \tfrac{\tilde{f}_\xi(r_*^{-1}(r_1))}{(1-\tilde{F}_\xi(r_*^{-1}(r_1))}\tfrac{\mathrm{d}\,r_*^{-1}(r_1)}{\mathrm{d}\,r_1}
$$

Thus, we can combine the two sums and express $f_{(m:n+m-1)}(r_1)$, using (A.2), by

$$
\begin{aligned}
& \tfrac{m\tilde{f}_\xi(r_1)}{1-\tilde{F}_\xi(r_1)}\sum_{j=0}^{m-1}\tbinom{n}{m-j}\tbinom{m-1}{j}\tilde{F}_\xi(r_1)^{n-m+j}(1-\tilde{F}_\xi(r_1))^{m-j}\tilde{F}_\xi\left(r_*^{-1}(r_1)\right)^{m-1-j}\left(1-\tilde{F}_\xi(r_*^{-1}(r_1))\right)^j \\
&= \tfrac{m\tilde{f}_\xi(r_1)}{1-\tilde{F}_\xi(r_1)}\left(F_{(m+1:n+m-1)}(r_1) - F_{(m:n+m-1)}(r_1)\right). \qquad \text{(A.3)}
\end{aligned}
$$

Using (A.3), the first-order condition (A.1) simplifies to

$$
r_1 - c_1 = \tfrac{F_{(m+1:n+m-1)}(r_1) - F_{(m:n+m-1)}(r_1)}{f_{(m:n+m-1)}(r_1)} = \tfrac{1-\tilde{F}_\xi(r_1)}{m\tilde{f}_\xi(r_1)} = \tfrac{1+\xi r_1}{m}. \qquad \text{(A.4)}
$$

Seller 1's optimal strategy is $r_1(c_1) = r_*(c_1) = {}^{(1+mc_1)}/_{(m-\xi)}$ because this also satisfies the second-order condition. For $m = 1$ and for $m > 1$ and $r_1 > {}^1\!/_{(m-\xi)}$, the second derivative is

$$\frac{\partial^2 \mathbb{E}[\pi_1(r_1, v_1)]}{\partial r_1^2} = f_{(m+1:n+m-1)}(r_1) - 2 f_{(m:n+m-1)}(r_1) - \frac{\partial f_{(m:n+m-1)}(r_1)}{\partial r_1}(r_1 - c_1).$$

Using that, if $r_j(c_j) = r_*(c_j)$, by (A.3) and (A.4),

$$\frac{\partial f_{(m:n+m-1)}(r_1)}{\partial r_1} = \left(\frac{m\tilde{f}_\xi(r_1)}{1-\tilde{F}_\xi(r_1)}\right)' \Delta F(r_1) + \frac{m\tilde{f}_\xi(r_1)}{1-\tilde{F}_\xi(r_1)} \left(f_{(m+1:n+m-1)}(r_1) - f_{(m:n+m-1)}(r_1)\right),$$

this second derivative is negative when evaluated at $r_1(c_1) = r_*(c_1)$ for all c_1:

$$\begin{aligned} & f_{(m+1:n+m-1)}(r_*(c_1)) - 2 f_{(m:n+m-1)}(r_*(c_1)) - \left.\frac{\partial f_{(m:n+m-1)}(r_1)}{\partial r_1}\right|_{r_1 = r_*(c_1)} \frac{1-\tilde{F}_\xi(r_*(c_1))}{m\tilde{f}_\xi(r_*(c_1))} \\ &= -f_{(m:n+m-1)}(r_*(c_1)) - \left(\frac{\tilde{f}_\xi(r_1)}{1-\tilde{F}_\xi(r_1)}\right)' \Delta F(r_1) \frac{1-\tilde{F}_\xi(r_1)}{\tilde{f}_\xi(r_1)} < 0 \end{aligned}$$

The final inequality holds because all factors are positive.

Next, we show that a seller's expected equilibrium profit decreases in the number of sellers m if the number of buyers n is held constant. Let $r_{j*}(c_j, m) = {}^{(1+mc_j)}\!/_{(m-\xi)}$ be the equilibrium strategy of seller j when there are m sellers and assume all sellers and bidders choose their equilibrium strategies. Note that $r_{j*}(c_j, m)$ decreases in m for all $\xi < 0$ and all c_j in the support of the distribution. W.l.o.g., consider seller 1.

First, if seller 1 unilaterally deviates from his equilibrium strategy to $r_{1*}(c_1, m+1)$, his expected profit strictly decreases because $r_{j*}(c_j, m)$ is a unique best response.

Second, seller 1's expected profit decreases further if first, the remaining sellers also play $r_{j*}(c_j, m+1)$. To see this note that the price in our market always equals the $m+1$-highest value among all reserve prices and valuations, and that a seller trades if and only if his reserve price is weakly below the price. Fix seller 1's reserve price at $r_{1*}(c_1, m+1)$. If the remaining sellers $j \in \{2, 3, \ldots, m\}$ decrease their reserve price from $r_{j*}(c_j, m)$ to $r_{j*}(c_j, m+1)$, the $m+1$-highest among all reserve prices and valuations and therefore the price must (weakly) decrease. If the price decreases and seller 1's reserve price is fixed, either his profit from his trade decreases or he loses his trade (and if he does not trade at the higher price, he does not trade at the lower price). A trade is always profitable because the reserve price exceeds the costs. Thus, seller 1 is weakly worse off if the price decreases weakly.

Third, seller 1's expected profit decreases further if a new seller $m+1$ enters the market and plays according to $r_{m+1*}(c_{m+1}, m+1)$. Fix seller j's reserve price at $r_{j*}(c_j, m+1)$ for all $j \in \{1, 2, \ldots, m\}$. Assume seller $m+1$ enters the market with the reserve price $r_{m+1*}(c_{m+1}, m+1)$. The $m+2$-highest among all reserve prices and valuations is weakly lower than the $m+1$-highest among all reserve prices and valuations without that of seller $m+1$. Thus, the price decreases when seller $m+1$ enters. By the arguments of the previous paragraph seller 1's profit decreases weakly. ■

Appendix B. Matching and Competition Effects when Markets are Merged

When two one-seller-one-buyer markets are merged, total gains from trade per agent will be higher in the merged markets than in the single market (see Section 2.3 and Table B.7) due to a *matching effect* and a *competition effect*.

To see this matching effect, assume reserve prices were given by $(100+c_j)/2$ in the two one-seller-one-buyer markets (in which they are optimal for sellers) as well as in the two-sellers-two-buyers market after a merger. For most realized valuations and costs, the merged markets are efficient if the separate markets are efficient because additional matching possibilities increase efficiency. However, note that if both buyers' valuations exceed both sellers' costs and the lower costs and the higher valuation are very different while the higher costs and the lower valuation are close to each other, it is possible that the separate markets generate an efficient outcome while the merged markets do not.

Consider the following example with sellers $j \in \{1, 2\}$, buyers $i \in \{1, 2\}$, costs $c_1 = 40$ and $c_2 = 70$, and valuations $v_1 = 100$ and $v_2 = 75$. In the efficient outcome, two trades occur, with gains from trade of 65. With reserve prices $r_1 = 70$ and $r_2 = 85$, in the separate markets if seller 2 is in a market with buyer 1 and seller 1 is in a market with buyer 2, both trades take place whereas in the merged markets only seller 1 and buyer 1 trade, with total gains from trade of only 60.[20] On average, however, if sellers stick to $(100+c_j)/2$, a merger of markets improves gains from trade.

The competition effect from using the strictly lower reserve prices $r_j(c_j) = (100+2c_j)/3$ instead of $r_j(c_j) = (100+c_j)/2$ in the merged market increases gains from trade in the merged markets for all value and cost realizations. With both reserve-price functions, reserve prices always exceed costs. Thus, the lower reserve prices due to seller competition in the merged markets promote efficient trades. To quantify this effect, compare the total gains from trade per agent of 8.1 in the merged markets when sellers set reserve prices $r_j(c_j) = (100+c_j)/2$ as in the separate markets with the total gains from trade of 9.3 in equilibrium with $r_j(c_j) = (100+2c_j)/3$, an increase by 1.1 (see Table B.7 for the equilibrium properties and Table 5 for the size of the effect). This competition effect arises although the number of buyers also doubles when the markets are merged.

[20] Note that the same effect arises if the sellers adjust reserve prices: The same trades would take place if the sellers chose their different equilibrium reserve prices of $r_1 = 60$ and $r_2 = 80$ in the merged markets and of $r_1 = 70$ and $r_2 = 85$ in the separate markets.

Table B.7: Efficient outcomes, equilibrium outcomes, and estimated outcomes for 1S1B and 2S2B for $c, v \sim U[0, 100]$.

	1S1B	2S2B
Efficient outcome:		
A seller's profit	0.0	5.0
A buyer's payoff	16.7	15.0
Total gains (per agent)	8.3	10.0
Probability of a trade per seller	0.50	0.50
Equilibrium outcome:		
A seller's profit	8.3	10.4
A buyer's payoff	4.2	8.1
Total gains (per agent)	6.3	9.3
Probability of a (efficient) trade per seller	0.25	0.37
Outcome with estimated bidding functions:	$\hat{r}_{1S1B}$	$\hat{r}_{2S2B}$
A seller's profit	6.4	7.7
A buyer's payoff	9.0	12.0
Total gains (per agent)	7.7	9.8

Appendix C. Regression Results

Table C.8: Estimated reserve-price functions $\hat{r}(c)$ to compare one- and two-seller treatments

	Linear regression 1S1B vs. 2S1B r	 1S2B vs. 2S2B r	Random-effects GLS regression 1S1B vs. 2S1B r	 1S2B vs. 2S2B r
d_{2S}	$-8.580^{**}(-3.01)$ $[-14.293, -2.868]$	-12.520^{***} (-3.58) $[-19.516, -5.524]$	-8.015^{**} (-2.68)	-11.42^{***} (-3.23)
c	0.647^{***} (24.60) $[.594, .670]$	0.610^{***} (23.44) $[.558, .662]$	0.662^{***} (30.99)	0.626^{***} (26.35)
$d_{2S} \times c$	0.123^{***} (3.59) $[.0542, .191]$	0.105^{*} (2.27) $[.0123, .197]$	0.112^{***} (3.60)	0.0832 (1.77)
constant	29.569^{***} (13.32) $[25.126, 34.013]$	32.200^{***} (13.30) $[27.355, 37.044]$	28.826^{***} (12.27)	31.400^{***} (12.41)
N	2400	2400	2400	2400
$F(3, 59)$	642.89^{***}	330.93^{***}		
Wald $\chi^2(3)$			2150.01^{***}	1204.21^{***}
R^2	.770	.627	.770	.627

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$ (two-tailed); SE adjusted for 60 clusters
t statistics for the linear regression and z statistics in the random-effects regression in parentheses
95% confidence intervals

Download ZEW Discussion Papers:

https://www.zew.de/en/publications/zew-discussion-papers

or see:

https://www.ssrn.com/link/ZEW-Ctr-Euro-Econ-Research.html
https://ideas.repec.org/s/zbw/zewdip.html

//
IMPRINT

ZEW – Leibniz-Zentrum für Europäische Wirtschaftsforschung GmbH Mannheim

ZEW – Leibniz Centre for European Economic Research

L 7,1 · 68161 Mannheim · Germany
Phone +49 621 1235-01
info@zew.de · zew.de