// NO.23-051 | 11/2023

# DISCUSSION PAPER

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## Revenue Maximization with Partially Verifiable Information

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First Version: March 9, 2022
Last Update: November 8, 2023

**Abstract**

I consider a seller selling a good to bidders with two-dimensional private information: their valuation for a good and their characteristic. While valuations are non-verifiable, characteristics are partially verifiable and convey information about the distribution of a bidder's valuation. I derive the revenue-maximizing mechanism and show that it can be implemented by introducing a communication stage before an auction. I show that granting bidders a right to remain anonymous, i.e., to refuse participation in the communication stage, leaves the optimal mechanism unchanged and provides no benefits for the bidders.

**Keywords:** Mechanism Design, Auctions, Partially Verifiable Types, Communication

**JEL Classification:** D44, D82, D83

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*I thank Vitali Gretschko, Volker Nocke and Thomas Tröger for their valuable insights. Further, I thank seminar audiences at Mannheim and Toulouse for their feedback. This work was supported by the University of Mannheim's Graduate School of Economic and Social Sciences. Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 (Project B01) is gratefully acknowledged.

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# 1 Introduction

A seller wants to sell a good to a number of bidders. In his seminal paper, Myerson (1981) showed that the optimal auction employs reserve prices and bidding subsidies. They explicitly depend on the distributions of the valuations of the bidders, which are given exogenously. If the bidders anonymously participate in the auction, there is no directly observable information to condition on, and thus no way to derive distinct distributions of valuations. In this case, the optimal auction treats all bidders equally. Suppose there is some additional, private information that correlates with the bidders' valuations, and that it is possible to (partially) verify this information once it has been volunteered by bidders. Can the seller benefit from eliciting this additional information from the bidders, even if the information is not directly part of their utility function? Consider some examples of such situations:

**Procurement:** Consider an auction for the procurement of a good. Typically, the bidders submit their offers to produce the good, and the best offer wins. The bidding strategy of the bidders will generally depend on their cost for the production of the good, which is private information. Suppose the good can be produced with modern machines, at a lower (marginal) cost or with old machines at a higher cost. Can the seller incentivize the bidders to show her their machines? Can she use this information to receive a better offer in the auction?[1]

**Energy Auctions:** In the energy markets, there are frequent energy balancing auctions to balance out energy supply and demand. Suppose there is an auction in which the bidders offer to supply additional energy. The energy can be produced using gas, coal, solar power or wind. Can the seller incentivize the bidders to offer detailed information about their mode of production?

**Wealth in Auctions:** Consider an auction for a piece of art. Suppose richer bidders', through more disposable income, are, on average, willing to pay more money for the piece. Additionally, the neighborhood in which they live is a good indicator of wealth. Can the seller elicit the bidders' addresses? Can she use the address information in the auction to generate higher revenue?

All these examples have in common the existence of information which correlates with the valuation of the bidder and is thus relevant to the seller. However, note that this information is typically unobservable to the seller. In procurement,

[1]In practice, the buyer often issues a Request for Quote (RfQ) or a Request for Proposal (RfP) to the suppliers. These requests typically can provide such additional information. My paper shows how this information can be used in a procurement auction.

observing the machines that a supplier will use to produce the good is not possible. It is impossible to directly observe what kind of energy source is used to create electricity in energy auctions. It is also not possible to directly observe a bidder's wealth in an art auction. But if a bidder volunteers this information, it may be possible to (partially) verify it. In a procurement auction, the seller cannot verify the machines that are used in production ex ante, but a bidder may invite the seller to show her the machines used to produce the goods. Similarly, an energy provider can provide a detailed production overview, that is not publicly available, of how exactly the energy is produced. In the art auction, assume that it is not possible for the seller to observe a bidder's address ex ante. But when she is provided an address by a bidder, can confirm whether the given address is true or false. For example, the seller could ask the bidder to show her a valid ID document to verify the address. If the bidder lives in a particularly wealthy neighborhood, it is less likely that he is poor. If a bidder lives in a comparatively poor neighborhood, it is less likely that he is rich.

Intuitively, this additional information is useful for the seller and can be used to discriminate between the bidders in an optimal mechanism. My paper connects two strains of the mechanism design literature: First, the literature of selling a good to buyers in the presence of non-verifiable valuations, as considered in Myerson (1981). Second, the literature of mechanism design with partially verifiable types, as first considered by Green & Laffont (1986).

For the non-verifiable component of my model, I assume that the bidders' preferences are described by a quasi-linear utility function $u_i = \theta_i x_i + t_i$ where $\theta_i$ is the valuation of each bidder for the good, $x_i$ denotes the probability with which each bidder receives the good, and $t_i$ are the transfers each bidder receives or pays in the mechanism. I assume that the valuation $\theta_i$ is private, non-verifiable information.

For the partially verifiable of the model, I assume that every bidder has a characteristic $c_i \in C$. Characteristics have no direct impact on a bidder's utility function, but they are informative about the distribution of a bidder's valuation. There is no one-to-one relationship between a certain characteristic and any given valuation. Instead, there is some correlation between characteristics and valuations such that characteristics are informative about the valuations in a statistical sense. The characteristic $c_i$ is private, partially verifiable information and $C$ is a finite set containing all possible characteristics. To provide tractable results, I assume that, conditional on the characteristics, it is possible to order the distributions of the valuations according to the *hazard rate order*. Partial verifiability is in the sense of Green & Laffont (1986). For a bidder with characteristic $c$, there exists a partition of the set $C$ into two sets: first, a set containing the characteristics that cannot be verified to be different from $c$ and second, a set of those characteristics that are verifiably different from $c$.

After restricting the search for an optimal mechanism to direct mechanisms through an adjusted revelation principle, I show that incentive compatibility boils down to four conditions. The first two are the well-known monotonicity and integrability condition that follow from the application of Milgrom & Segal (2002). The third and fourth conditions relate to the bidders' characteristics. The third condition concerns the ex-interim allocation probability of a bidder that truthfully reports his characteristic. The allocation probability for this first bidder cannot be lower than that of another, second bidder, if the first bidder can present the evidence requested from the second bidder. This condition intuitively follows, as the characteristics of a bidder are not directly relevant for the utility. If this condition is violated, the former bidder can mimic the latter, which would be a profitable deviation. Therefore, bidders can only be treated unequally if one bidder is asked to present some particular evidence that the other bidder is not able to produce. The fourth condition simply states that a bidder can truthfully report his characteristic and valuation without being asked to present evidence that the bidder is unable to come up with.

Using the integrability condition, the expected revenue generated from the bidders corresponds to their virtual valuation, conditional on their characteristic. However, incentive compatibility now demands the grouping of bidders according to their characteristics. Therefore, the optimal mechanism has to find the optimal grouping structure. Given that the CDFs associated with the characteristics can be ordered according to the hazard rate order, the optimal mechanism groups bidders with characteristics close to one another in terms of the hazard rate order. The exact grouping, however, depends on the exact verifiability structure of the characteristics.

To argue how my mechanism can be integrated into existing auction formats, I show that a two-stage communication plus auction mechanism implements the revenue-maximizing social choice function. In a first stage, the bidders communicate with the seller about their characteristics. The seller then explicitly conditions the auction rules of the auction in the second stage on this communication. I show that the auction in the second stage maximizes the expected revenue, given the equilibrium of the communication stage. Therefore, it is not necessary for the seller to have commitment power regarding the auction rules as a result of the communication stage. In practice, the pre-auction communication can be implemented as easily as asking the bidders to fill out a questionnaire about their characteristics before the auction.

In the baseline model of the two stage implementation, it is not possible for bidders to refuse communication or engage in babbling with the seller. To alleviate concerns about this restriction, I discuss an extension in which I introduce a right to remain anonymous for the bidders. Every bidder can refuse to communicate in the pre-auction communication stage. I show that no bidder benefits from such a right to remain anonymous. I provide an intuitive unraveling result when bidders are granted a right to remain anonymous. Bidders with particularly desirable characteristics intentionally choose to communicate to separate themselves from bidders with less desirable characteristics. This incentive to engage in communication causes an unraveling effect such that in equilibrium, only those bidders with the least desirable characteristics are indifferent between actually remaining anonymous and communicating about their characteristics.

Partially verifiable information presents some technical challenges. As pointed out in Green & Laffont (1986), the revelation principle does not generally apply to environments with partially verifiable private information. They show that truthful implementation using the revelation principle is only without loss of generality if the structure of the partition of the set of characteristics $C$ satisfies a *nested range condition*.[2] If this condition is violated, there are social choice functions that are implementable in a direct mechanism but not truthfully implementable. Singh & Wittman (2001) argue that the nested range condition in Green & Laffont (1986) is too restrictive and excludes many interesting economic applications. In my model, the nested range condition is not necessary and will generally be violated.

To restore the revelation principle for my framework, I follow a more recent ap-

[2]They define the nested range condition as follows: Consider three distinct characteristics $c_1, c_2, c_3 \in C$. Let $\phi$ denote the partition of $C$, such that $c' \in \phi(c)$ denotes that a bidder of characteristic $c$ can report characteristic $c'$. Then the nested range condition is satisfied if: $c_2 \in \phi(c_1)$ and $c_3 \in \phi(c_2) \Rightarrow c_3 \in \phi(c_1)$

proach developed by Strausz (2016). In a methodological contribution, he argues that the failure of the revelation principle in frameworks with partially verifiable types is caused by the modelling approach of Green & Laffont (1986). Then, Strausz (2016) shows how to restore the revelation principle through what he refers to as the *extended environment*: Typically, the social choice function is defined as a mapping from the set of private information into the set of outcomes. In his new approach, he extends social choice functions to also map into the set of partially verifiable characteristics. This addition to the social choice function can be understood as a requirement for the bidders to present *evidence* within the mechanism. Evidence has also been considered by other authors: Kartik & Tercieux (2012) study implementation when bidders can generate evidence for their types at non-prohibitive costs. Ben-Porath & Lipman (2012) extend social choice functions to not just depend on the bidders' preferences, but allow them to submit evidence to support their claims. What sets these papers apart from mine is the general research question: while they consider the general question of implementability, I use their results to characterize the set of implementable social choice functions in my environment. Then, I determine the implementable social choice function that maximizes revenue.

There are other papers that focus on deriving revenue-maximizing mechanisms when information is partially verifiable. Ball & Kattwinkel (2019) derive revenue-maximizing mechanisms for a range of applications in a setting where the principal can use a probabilistic test with binary outcomes to verify the bidders' types. Tests with deterministic outcomes correspond to how partial verifiability is modelled in Green & Laffont (1986), as well as my paper. Generally, their framework allows for tests that are not restricted to deterministic outcomes. However, the authentication rate characterization in Ball & Kattwinkel (2019) reduces to the nested range condition if tests are deterministic. As my model generally violates the nested range condition, it cannot be nested in their approach. Further, in the auction application within their paper, they consider one dimensional, partially verifiable, private information: the bidders' valuations. My model considers two-dimensional private information instead: non-verifiable valuations and partially verifiable characteristics. This two-dimensional approach stems from a practical concern. Partially verifiable valuations in the auction environment demand some test that allows to verify that a bidder is willing to pay exactly some particular amount of money, say $100, for a good. However, it seems incredibly difficult to verify a bidders' exact valuation for a good. Verifying some informative characteristics fits a wide range of applications, as pointed out in the examples at the beginning of this paper.

In environments without transfers, Ben-Porath et al. (2014) study the optimal mechanism for a principal who allocates objects to bidders, whose valuation is private information but can be verified at a cost. Li (2020) solves for the optimal mechanism in a setting where the principal can inspect a bidder's report at a cost and impose punishments on false reports. Erlanson & Kleiner (2020) study how a principal should optimally choose between implementing a new policy and maintaining the status quo when information relevant for the decision is privately held by bidders, but can be verified at a cost. However, as all of these papers preclude monetary transfers, they cannot be applied to a bidder-seller situation.

The remainder of the paper is organized as follows: Section 2 presents the model and derives the optimal mechanism. Section 3 discusses the two-stage implementation and the right to remain anonymous. Section 4 concludes.

# 2 Model

## 2.1 Description

Consider a seller (she) and $N \geq 1$ bidders (he) with unit demand. The seller owns $K \geq 1$ units of a homogeneous good. She does not gain utility from the consumption of the goods and is purely interested in revenue maximization. A bidder's utility function over a particular allocation and payment is equal to $\theta_i x_i + t_i$, where $\theta_i \in [\underline{\theta}, \overline{\theta}]$ represents the valuation of each bidder for the good, $x_i$ denotes the probability with which each bidder receives a good and $t_i$ denotes the transfer each bidder receives or pays in the mechanism. I assume that the valuation $\theta_i$ is private, non-verifiable information. As a novel feature of my model, every bidder also has a privately known, partially verifiable characteristic $c_i$ where $c_i \in C$ and $C$ is a finite set containing all possible characteristics.

A characteristic $c_i$ does not directly impact a bidder's utility. However, it is informative about the distribution of the bidder's valuation. In an application, characteristics are meant to capture real-life characteristics of the bidders that allow drawing statistical conclusions about the bidders' valuations. For example, the type of machines used to produce a product in the procurement example or the wealth of a bidder in the art auction example. Each characteristic $c_i \in C$ is associated with a CDF $F_{c_i} \in \mathcal{F}$ that governs the distribution of the valuations $\theta_i$. $\mathcal{F}$ denotes the set of all CDFs that are associated with a characteristic in $C$. When $c_i$ and $c_i'$ are distinct characteristics, their associated CDFs $F_{c_i}$ and $F_{c_i'}$ differ from each other on a set of valuations with strictly positive measure. I assume that all CDFs $F \in \mathcal{F}$ are continuously differentiable and admit strictly positive densities $f > 0$.[3] The virtual valuation $J(\theta_i)$ of a bidder is defined as $J(\theta_i) = \theta_i - \frac{1-F(\theta_i)}{f(\theta_i)}$ as in Myerson (1981). For simplicity, I assume that all distribution functions are associated with non-decreasing virtual valuations. Conditional on the characteristics $c_i$ and $c_j$ of two distinct bidders, the valuations $\theta_i$ and $\theta_j$ are distributed independently. There is a common initial prior $\Delta$ over the set of characteristics $C$ and hence also over the set $\mathcal{F}$. The prior $\Delta$ assigns a probability $\delta(c_i)$ to each characteristic $c_i \in C$.

For any distribution $F \in \mathcal{F}$, the hazard rate is defined as $\frac{f(\theta)}{1-F(\theta)}$. The hazard rate order $\succsim_{hr}$ relates distributions $F$ and $G$ (noted as $F \succsim_{hr} G$) if $\frac{f(\theta)}{1-F(\theta)} \leq \frac{g(\theta)}{1-G(\theta)}$. I make the following assumption:

**Assumption 1** *The hazard rate order $\succsim_{hr}$ establishes a linear order over $\mathcal{F}$. In particular, for any $F, G \in \mathcal{F}$ it holds that $F \succsim_{hr} G$ or $G \succsim_{hr} F$.*

Without loss of generality, I label the characteristics from $1, 2, ..., |C|$ s.t. $F_i \succsim_{hr} F_j$ iff $i \geq j$. To build some intuition for the hazard rate order, consider the likelihood ratio order $\succsim_{lr}$. For any two CDFs $F$ and $G$ let $F \succsim_{lr} G$ iff $\frac{f(\theta_i)}{g(\theta_i)}$ is increasing in $\theta_i$. As an example, any CDFs $F$ and $G$ with increasing density $f$ and decreasing density $g$ satisfy the likelihood ratio order. Note that it is a well established result that $F \succsim_{lr} G \Rightarrow F \succsim_{hr} G$. Therefore, the likelihood ratio order is sufficient for the hazard rate order.[4] The likelihood ratio order can be interpreted as follows in the context of this paper. Let $c_i$ and $c_i'$ be characteristics such that $F_{c_i} \succsim_{lr} F_{c_i'}$. Then, bidders are more likely to be of characteristic $c_i$ compared to $c_i'$, the higher the valuation that is considered. There are a variety of situations for which this assumption seems reasonable. Reconsider the examples from the introduction: in the procurement example, it seems intuitive that cheaper (marginal) production costs are more likely for a firm using more modern machines. In the energy auction, marginal costs for energy production using wind or solar power are likely

[3]Environments with differing type spaces for the bidders can be approximated through distributions with arbitrarily small densities on certain types in $[\underline{\theta}, \overline{\theta}]$

[4]For a detailed treatment of stochastic orders and further necessary and sufficient conditions for the hazard rate and the likelihood ratio order see Shaked & Shanthikumar (2007)

lower than using fossil fuels. In the art auction example, wealthier bidders are likely willing to pay more through having more disposable income.

The characteristics are private information. It is not possible for the seller to gather information about a bidder's characteristic ex ante. However, once a bidder reports a particular characteristic, he can submit evidence to support his claim and only then can this evidence be verified by the seller. To model this, I use a correspondence $\phi : C \twoheadrightarrow 2^C$. It is a primitive of the model that captures the degree to which characteristics are partially verifiable and whether evidence that is presented by a bidder can be rejected as objectively false or not. Every characteristic $c_i$ is assigned a set of characteristics $\phi(c_i) \subseteq C$. For every reported characteristic $\hat{c}_i$ such that $\hat{c}_i \in \phi(c_i)$, the bidder can produce evidence that cannot be rejected as objectively false. For every reported characteristic $\hat{c}_i \notin \phi(c_i)$ the bidder is unable to produce sufficient evidence to support his claim. I assume that the procedure that is used to judge whether evidence is objectively false is commonly known, hence $\phi$ is common knowledge. Further, there is no uncertainty in its outcome. I assume that neither the generation of evidence, nor the verification procedure, is associated with any costs for neither the seller nor the bidders. This assumption can be justified in situations where these costs are negligible compared to the value of the goods up for auction. For example, the costs of generating reports are negligible in a multi-million dollar procurement auction.

Note that the set $\phi(c_i)$ explicitly depends on the true characteristic $c_i$ of the bidder. Depending on his true characteristic, a bidder may find it more difficult to produce evidence to back up certain claims $\hat{c}_i$. To illustrate this point, recall an example from the introduction. Consider the procurement auction and bidder that produces using the most modern machines available on the market. Naturally, he will have a harder time coming up with evidence that he is producing using old machines than a bidder who is actually using old machines. In general, if $\phi(c_i) = C$ for all $c_i$, the characteristics are completely unverifiable. If $\phi(c_i) = \{c_i\}$, the characteristics are perfectly verifiable, and if $\phi(c_i) \subset C$, the characteristics are partially verifiable. To allow for some tractable results, I assume the following structure regarding the partial verifiability of the characteristics:

**Assumption 2** *Truthful disclosure is possible. That is, for all $c \in C$ it holds that $c \in \phi(c)$*

**Assumption 3** *For each characteristic $c \in C$ there is a lower bound $\underline{\phi}(c)$ and an upper bound $\overline{\phi}(c)$ such that $\phi(c) = \{c' \in C | \underline{\phi}(c) \leq c' \leq \overline{\phi}(c)\}$*

**Assumption 4** *The bounds are monotone. Let $c < c'$ be two characteristics, then it holds that $\underline{\phi}(c) \leq \underline{\phi}(c')$ and $\overline{\phi}(c) \leq \overline{\phi}(c')$*

Combining these assumptions highlights the idea that there is a meaningful order included in the labels, such that labels that are further away from each other are more distinct. The further away a particular characteristic is from the bidder's true characteristic, the harder it will be to generate credible evidence for that characteristic. However, if is possible to generate evidence for a characteristic further away from the true characteristic, it must also be possible to generate evidence for a characteristic closer to the truth. Thus, disclosing any characteristic is possible as long as they remain in the bounds set by $\phi(c)$.

**Remark:** Assumptions 2-4 do not generally guarantee that Green & Laffont's *nested range condition* is satisfied. They define the nested range condition as follows: For any three distinct elements $c_1, c_2, c_3 \in C$, if $c_2 \in \phi(c_1)$ and $c_3 \in \phi(c_2)$ then $c_3 \in \phi(c_1)$. Consider the following example $\phi(c_1) = \{c_1, c_2\}, \phi(c_2) = \{c_2, c_3\}, \phi(c_3) = \{c_3\}$. It is easy to verify that this example satisfies assumptions 2-4, but violates the nested range condition. If we replace $\phi(c_1)$ with $\phi'(c_1) = \{c_1, c_2, c_3\}$, it is easy to verify that the example satisfies assumptions 2-4 and the nested range condition. This highlights that my model allows for more general partial verifiability structures than those given by the nested range condition.

## 2.2 Mechanisms and the Revelation Principle for Partially Verifiable Types

The goods are allocated through a mechanism. An arbitrary mechanism is denoted by $g = (M, V, x, t)$. Its first component is a set of unverifiable cheap talk messages $M$. Second, there is a set $V \subseteq C$ of partially verifiable messages. Note that the mechanism does not necessarily have to allow all partially verifiable messages to be sent. The seller may benefit from excluding some messages from the mechanism, such that $V$ may generally be strictly smaller than $C$. The third component is an allocation rule $x$ that maps all possible combinations of messages into allocations of the goods. The fourth component is a transfer rule $t$ that maps all possible combinations of messages into transfers. A *direct mechanism* is a mechanism in which $M = \Theta$ and $V = C$.

To establish the revelation principle for my model, I follow the approach laid out by Strausz (2016). As the approach is relatively recent, I will briefly present the main definition and result. Using the language of Strausz (2016), I will refer to the environment as defined in Green & Laffont (1986) as the *initial environment*. The environment as defined in Strausz (2016) is referred to as the *extended environment*. Loosely speaking, the environments differ through the introduction of evidence.[5] Denote by $\mathcal{X}$ the set of all feasible physical allocations of the goods, and $\mathcal{T}$ describes the set of all feasible transfers to each of the bidders. The definition of a Bayesian incentive-compatible, direct mechanism is applied to the extended environment:

**Definition 1** *A Bayesian incentive compatible, direct mechanism in the extended environment is a tuple $\hat{g} = (x, t, \hat{c})$ with an allocation rule $x : \Theta^N \times C^N \to \mathcal{X}$, a transfer rule $t : \Theta^N \times C^N \to \mathcal{T}$ and an evidence rule $\hat{c} : \Theta \times C \to C$ such that*

$$\theta_i X(\theta_i, c_i) + T(\theta_i, c_i) \geq \theta_i X(\theta_i', c_i') + T(\theta_i', c_i') - P \cdot \mathbb{1}\{\hat{c}(\theta_i', c_i') \notin \phi(c_i)\} \quad (1)$$

*for all $(\theta_i', c_i') \in \Theta \times C$, where $X(\theta_i, c_i) = \mathbb{E}_{-i}[x_i(\theta_i, c_i, \theta_{-i}, c_{-i})]$ denotes the ex interim allocation probability and $T(\theta_i, c_i) = \mathbb{E}_{-i}[t_i(\theta_i, c_i, \theta_{-i}, c_{-i})]$ denotes the ex interim expected payment.*

The revelation principle can be re-established in this extended environment through standard arguments. Given that the revelation principle holds for this extended environment, it is vital to establish a connection between the extended environment and the initial environment.

**Proposition 1 (Strausz (2016))** *Consider the initial environment and its extension. If there exists some mechanism $g$ which implements the social choice function $f : \Theta^N \times C^N \to \mathcal{X} \times \mathcal{T}$ in the initial environment, then there exists a function $\hat{c} : \Theta \times C \to C$ such that the extended social choice function $\hat{f}(\cdot) = (f(\cdot), \{\hat{c}(\cdot)\}_{i=1}^N)$ is implementable in an Bayesian incentive compatible, direct mechanism in the extended environment.*

**Proof.** See appendix. ■

This proposition connects Green & Laffont's initial environment with Strausz' extended environment. It establishes that any social choice function that can

[5] A more detailed explanation, including all the technical definitions, can be found in section A.1 of the appendix.

be implemented by some mechanism in the initial environment can be truthfully implemented by a direct mechanism in the extended environment with a suitable evidence function. Therefore, I can focus the derivation of the optimal mechanism on direct, incentive-compatible mechanisms in the extended environment without loss of generality. Intuitively, the proposition is made possible through the addition of the evidence function $\hat{c}$. Using this function, it is possible to take the equilibrium disclosure behavior with respect to the partially verifiable characteristics in any mechanism in the initial environment and define it as the required evidence rule for the extended environment.

## 2.3 Incentive Compatibility and Expected Revenue

The previous section has established a revelation principle for this setup. To proceed, I first offer a full formal description of the maximization problem.

$$\max_{\{x(\theta,c),t(\theta,c),\hat{c}(\theta_i,c_i)\}} \mathbb{E}\left[\sum_{i=1}^{N} -t_i(\theta,c)\right] \tag{2}$$

$$\text{s.t. (IC) } \theta_i X(\theta_i,c_i) + T(\theta_i,c_i) \geq \theta_i X(\theta_i',c_i') + T(\theta_i',c_i') - P \cdot \mathbb{1}\{\hat{c}(\theta_i',c_i') \notin \phi(c_i)\} \tag{3}$$

$$\text{(IR) } \theta_i X(\theta_i,c_i) + T(\theta_i,c_i) \geq 0 \tag{4}$$

The seller wants to maximize her expected revenue from the allocation of the goods. However, she is restricted to Bayesian incentive-compatible, direct mechanisms that respect individual rationality in the extended environment without loss of generality. As a next step, I further characterize the incentive compatibility constraints.

**Proposition 2** *A direct mechanism $\hat{g} = (x,t,\hat{c})$ in the extended environment is Bayesian incentive compatible if and only if the following conditions hold:*

1. *Integrability*

$$\hat{U}(\theta_i,c_i) = \hat{U}(\underline{\theta},c_i) + \int_{\underline{\theta}}^{\theta_i} X(s,c_i)ds$$

2. *Monotonicity, that is $\theta_i > \theta_i'$ implies $X(\theta_i,c_i) \geq X(\theta_i',c_i)$*

3. *Optimality with respect to $c_i$, that is $X(\theta_i,c_i) \geq X(\theta_i,c_i')$ for all $(\theta_i,c_i')$ such that $\hat{c}(\theta_i,c_i') \in \phi(c_i)$.*

4. *Feasible evidence for truthful disclosure:* $\hat{c}(\theta_i, c_i) \in \phi(c_i)$ *for all* $\theta_i \in [\underline{\theta}, \overline{\theta}]$ *and* $c_i \in C$

**Proof.** See appendix. ■

Incentive compatibility boils down to four conditions—an integrability condition and a monotonicity condition akin to the literature's standard constraints. The third and fourth conditions are novel and relate to the characteristics. The third condition requires that the ex interim allocation probability of the good for a bidder $i$ with valuation and characteristic $(\theta_i, c_i)$ may not be lower than that of a bidder with valuation and characteristic $(\theta_i, c_i')$ if the required evidence $\hat{c}(\theta_i, c_i')$ can also be submitted by a bidder with characteristic $c_i$. The intuition for this constraint is that the utility function of the bidders is independent of their characteristics. The utility derived from the consumption of the good solely depends on their valuation for the good. Therefore, an incentive-compatible mechanism cannot assign the good more often to a bidder of a certain characteristic $c_i'$ compared to a bidder of characteristic $c_i$ if the latter bidder can provide the evidence demanded by the former bidder. The fourth condition states that it must be possible for a bidder to disclose his valuation and characteristic truthfully without being asked to submit evidence that the bidder cannot feasibly submit. It restricts the evidence rule for bidders that are telling the truth, such that they may not be asked for evidence which they cannot generate. By definition of the approach of Strausz (2016), a bidder who discloses evidence that is verifiably false faces a severe punishment.[6] Thus, a failure of this condition will result in non-truthful disclosure.

Using standard arguments that make use of the integrability condition, the expected transfer conditional on a specific characteristic $c_i$ is given by:

$$\mathbb{E}[T(\theta_i, c_i)|c_i] = \int_{\underline{\theta}}^{\overline{\theta}} X(\theta_i, c_i) J(\theta_i, c_i) f(\theta_i|c_i) d\theta_i \tag{5}$$

where $J(\theta_i, c_i) = \theta_i - \frac{1-F(\theta_i|c_i)}{f(\theta_i|c_i)}$. This expression for the expected revenue is very similar to the usual condition in the literature, with the difference being that it is conditional on a specific characteristic $c_i$. In particular, the virtual valuation is calculated using the conditional distribution and density functions. Recall that

[6] For my paper, a sufficiently severe punishment is to exclude such bidders from participating in the auction.

the probability with which a characteristic $c_i$ occurs is denoted by $\delta(c_i)$. Then I employ the law of iterated expectations to determine the unconditional expected transfers of a bidder.

$$\mathbb{E}[T(\theta_i, c_i)] = \sum_{c_i \in C} \left( \delta(c_i) \int_{\underline{\theta}}^{\overline{\theta}} X(\theta_i, c_i) J(\theta_i, c_i) f(\theta_i | c_i) d\theta_i \right) \tag{6}$$

The expected total revenue of the seller then equals

$$\mathbb{E}\left[\sum_{i=1}^{N} T(\theta_i, c_i)\right] = \int_{[\underline{\theta}, \overline{\theta}]^N} \left( \sum_{c \in C^N} \left( \sum_{i=1}^{N} x_i(\theta, c) J(\theta_i, c_i) \right) \delta(c_1) f(\theta_1 | c_1) \cdots \delta(c_N) f(\theta_N | c_N) \right) d\theta \tag{7}$$

Now, in principle, it is possible to engage in point wise maximization. However, condition 3 of proposition 2 has to be respected.[7] It is formulated in terms of the ex interim allocation probabilities. Therefore, this introduces some interdependence that has to be addressed first.

## 2.4 Deriving the Optimal Mechanism

In this section, I derive the revenue-maximizing mechanism. As argued above, point wise maximization is not quite possible yet due to the interdependence introduced by condition 3 of proposition 2. In the following lemma, I show how to extend this condition onto a point wise basis.

**Lemma 1** *Let $g^* = (x^*, t^*, \hat{c}^*)$ be a revenue-maximizing, incentive compatible mechanism. Fix a valuation $\theta_i$ and characteristics $c_i, c_i'$. Then $x_i^*(\theta_i, c_i, \theta_{-i}, c_{-i}) = 1$ and $\hat{c}^*(\theta_i, c_i) \in \phi(c_i') \Rightarrow x_i^*(\theta_i, c_i', \theta_{-i}, c_{-i}) = 1$ without loss of generality.*

**Proof.** See appendix. ■

Lemma 1 shows that the seller cannot gain any additional revenue by trying to separate bidders of different characteristics if bidders of both characteristics can submit the required evidence. This can be proven by examining the revenue of separating bidders of these two characteristics and comparing it to the revenue in which these characteristics are not separated. However, as the proof shows, the revenue of separating the bidders of the different characteristics cannot exceed that of treating the bidders equally. To gain some intuition for why this is true,

[7] Optimality with respect to $c_i$, that is $X(\theta_i, c_i) \geq X(\theta_i, c_i')$ for all $(\theta_i, c_i')$ such that $\hat{c}(\theta_i, c_i') \in \phi(c_i)$.

recall that the characteristics of a bidder are not part of his utility function. The only possibility to treat one bidder differently from the other is through the required evidence. However, if both bidders can produce the required evidence, a different treatment is simply not possible.

Using Lemma 1, a point wise approach to finding the optimal mechanism is possible. We can fix some profile of valuations and then design the optimal evidence rule for the mechanism. The optimal evidence rule then depends on the grouping of different characteristics that it achieves, and the optimal grouping will critically depend on assumption 1.[8] To see why, recall the definition of the virtual valuation $J_i(\theta_i, c_i)$ of a bidder $i$ conditional on his characteristic $c_i$:

$$J(\theta_i, c_i) = \theta_i - \underbrace{\frac{1 - F(\theta_i|c_i)}{f(\theta_i|c_i)}}_{\text{inverse hazard rate}} \tag{8}$$

Now consider two characteristics $c_i$ and $c_j$ such that $F(\theta_i|c_i) \succsim_{hr} F(\theta_j|c_j)$. Then comparing the virtual valuations we get:

$$F(\theta_i|c_i) \succsim_{hr} F(\theta_j|c_j) \Rightarrow \frac{f(\theta_i|c_i)}{1 - F(\theta_i|c_i)} \leq \frac{f(\theta_j|c_j)}{1 - F(\theta_j|c_j)} \Rightarrow J(\theta_i|c_i) \leq J(\theta_j|c_j) \tag{9}$$

Being able to order the CDFs $F(\theta_i|c_i)$ and $F(\theta_j|c_j)$ using the hazard ratio order, allows a uniform ordering over the virtual valuations associated with those CDFs. Recall that Myerson (1981) established that the virtual valuation of a bidder is the maximum revenue that the seller can extract from a bidder through the allocation of the good. Thus, in the benchmark of commonly known characteristics, the seller prefers the allocation of the good to bidders with characteristics that have lower ranks in the hazard rate order, as they have higher virtual valuations. In the setting of partially verifiable characteristics, however, the seller has to elicit the characteristics of the bidders first. It turns out that the intuition from the common knowledge case carries over to the partially verifiable case. The seller will use the evidence rule to get close to what she would want to do were the characteristics commonly known. This intuition is distilled into the following algorithm:

**The optimal mechanism:** Fix an arbitrary profile of valuations $\theta$. Then proceed as follows:

[8] The hazard rate order $\succsim_{hr}$ establishes a linear order over $\mathcal{F}$. In particular, for any $F, G \in \mathcal{F}$ it holds that $F \succsim_{hr} G$ or $G \succsim_{hr} F$.

1. Set $\tilde{C} = C$, $c_i = 1$.
2. Group all bidders with characteristics $c_i' \in \tilde{C}$ such that $c_i \in \phi(c_i')$ into a group $G_{c_i}$.
   Set $\tilde{C} = C \setminus G_{c_i}$, $c_i = c_i + 1$
3. Repeat step 2 until $\tilde{C} = \emptyset$
4. Calculate the expected virtual valuation $J(\theta_i, G_{c_i})$ for each bidder in each group as

$$J(\theta_i, G_{c_i}) = \frac{1}{\sum_{c_i' \in G_{c_i}} \delta(c_i')} \sum_{c_i' \in G_{c_i}} \delta(c_i') J(\theta_i, c_i') \tag{10}$$

5. Assign the good to bidder $i$ if and only if $J(\theta_i, G_{c_i}) \geq \max_{M:N-1} J(\theta_j, G_{c_j})$ and $J(\theta_i, G_{c_i}) \geq 0$. Where $\max_{M:N-1} J(\theta_j, G_{c_j})$ denotes the $M$'th highest virtual valuation of the other $N-1$ bidders.

**Proposition 3** *Assigning the goods according to the algorithm constitutes the revenue-maximizing, incentive-compatible mechanism.*

**Proof.** See appendix. ■

The proposition claims two properties of the algorithm: Revenue maximization and incentive compatibility. First, I will provide a discussion why the algorithm maximizes revenue. Recall that the hazard rate order allows us to rank the virtual valuation of the bidders conditional on their characteristic such that $J(\theta_i, c_i = 1) \geq J(\theta_i, c_i = 2) \geq ... \geq J(\theta_i, c_i = |C|)$. Thus, if characteristics were observable, the seller would prioritize assigning the good to bidders with lower characteristics ceteris paribus. Fix a bidder $i$ and a profile or valuations and characteristics. Consider the most profitable way for the seller to assign the good to any other bidder $j \neq i$. Now define by $\bar{c}$ the characteristic such that the seller prefers assigning the good to bidder $i$ if $c_i \leq \bar{c}$ and prefers assigning the good to bidder $j$ if $\bar{c} \leq c_i$. For now, suppose that $\bar{c}$ is interior, i.e. that $1 < \bar{c} < |C|$.

If characteristics are not observable, but merely partially verifiable, the seller needs to distinguish the cases in which bidder $i$ has characteristic $c_i < \bar{c}$ from those in which he has characteristic $c_i'$ with $\bar{c} < c_i'$. Note that lemma 1 established that it is only optimal to assign the good to bidder $i$ with characteristic $c_i$ and not characteristic $c_i'$ if $\hat{c}(\theta_i, c_i) \notin \phi(c_i')$. Since the bounds of the evidence that can be generated are monotone by assumption 4, the only way to distinguish the two characteristics is by requiring evidence such that $\hat{c}(\theta_i, c_i) < \underline{\phi}(c_i')$. The most efficient way to achieve this is to set $\hat{c}(\theta_i, c_i) = \underline{\phi}(c_i)$ as it is done in the algorithm. Since this procedure generally creates groups of characteristics, the allocation of the good to a bidder then generates revenue equal to the expected virtual valuation of a bidder with valuation $\theta_i$ in that particular group. Then the optimal allocation rule assigns the goods to the bidders that have the highest expected virtual valuations depending on their group.

Second, I will discuss why the algorithm is incentive compatible. Recall that there are 4 conditions that characterize an incentive compatible mechanism.
Consider the two standard conditions for incentive compatibility: Integrability is satisfied, as it has been used in deriving the virtual valuation. Monotonicity is satisfied, as the virtual valuations are non-decreasing by assumption and a higher virtual valuation leads to a higher probability of being assigned a good.
Now consider the two conditions that relate to the truthful disclosure of the characteristics. Note that the algorithm assigns the bidders to groups, such as to maximize their expected virtual valuations. Recall the third condition of incentive compatibility: Optimality with respect to $c_i$, that is $X(\theta_i, c_i) \geq X(\theta_i, c_i')$ for all $(\theta_i, c_i')$ such that $\hat{c}(\theta_i, c_i') \in \phi(c_i)$. Given that bidders are assigned to groups such as to maximize their expected virtual valuation, and assignment is determined by virtual valuations, it is obvious that this condition is satisfied. A bidder that would deviate to reporting a different characteristic would lower his probability of receiving the good, and therefore deviation is not optimal. The fourth condition is: Feasible evidence for truthful disclosure: $\hat{c}(\theta_i, c_i) \in \phi(c_i)$ for all $\theta_i \in [\underline{\theta}, \overline{\theta}]$ and $c_i \in C$. Clearly, this condition is satisfied by construction.

Note that the proposition generates two interesting cases as corollaries:

**Corollary 1** *If characteristics are perfectly verifiable, each group contains exactly one characteristic and the optimal mechanism is the Myerson auction with heterogeneous priors.*

If characteristics are perfectly verifiable, it holds that $\phi(c_i) = \{c_i\}$ for all $c_i \in C$. No bidder can generate evidence for any other characteristic and participation in the auction is individually rational. Therefore, the bidders reveal their true characteristics and the optimal mechanism is equivalent to Myerson's optimal auction. At the other extreme are completely unverifiable characteristics:

**Corollary 2** *If characteristics are completely unverifiable, all characteristics are grouped into a single group and the optimal mechanism is a Myerson auction with symmetric priors.*

If characteristics are completely unverifiable, it holds that $\phi(c_i) = C$ for all $c_i \in C$. Since characteristics themselves are not part of the bidder's utility functions, it is impossible to try and discriminate between them in the mechanism. These corollaries link the assumption of symmetric / heterogeneous priors to partially verifiable characteristics and allow a practical interpretation: If there are observable differences between bidders or unobservable, but partially verifiable characteristics, it is possible to discriminate between bidders in an auction to increase revenue. If, on the other hand, there are no observable differences between bidders and any unobservable characteristics are completely unverifiable, it is not feasible to discriminate between bidders in an auction to increase revenue.

# 3 Two-Stage Implementation & A Right to Remain Anonymous

## 3.1 A Two-Stage Implementation

This section discusses the implementation of the optimal mechanism as a two-stage mechanism. It highlights that it is optimal to extend an existing auction format by including pre-auction communication. In a first stage, the bidders and the seller communicate about the bidders' characteristics. In a second stage, the seller sells the goods in an auction, with rules that explicitly depend on the communication of the first stage. The social choice function implemented by this alternative two-stage mechanism is equivalent to that implemented by the direct mechanism derived previously. In practice, such pre-auction communication could be implemented, for example, by asking the bidders to fill in a questionnaire that inquires about their characteristics before the auction. In procurement, such communication is common place: Procurement projects frequently issue a Request for Proposal (RfP) or a Request for Quote (RfQ). These requests describe the procurement project and solicit responses by prospective suppliers. Within those replies, suppliers describe their proposed solutions to the procurement problem in some detail, effectively communicating about their characteristics in the sense of the model.

Formally, the game consists of a *communication stage* in which the bidders communicate about their characteristics by disclosing some characteristic $\hat{c}_i \in C$. In the second stage - the *auction stage* - the seller makes the auction rules depend explicitly on the communication stage. Conditional on some equilibrium beliefs of the communication stage, it is optimal for the seller to use a Myerson auction that makes use of these beliefs are priors.[9] However, bidders anticipate this and take it into consideration when deciding on their optimal communication strategy. Note that truthful disclosure of characteristics does not necessarily constitute equilibrium behavior. Therefore, I introduce beliefs that describe the distribution of the valuations $\theta_i$ conditional on some reported characteristic $\hat{c}_i$. For any disclosure $\hat{c}_i$, denote the associated belief $\hat{F}(\theta_i|\hat{c}_i)$ with $\hat{F}_{c_i}$.

In a Perfect Bayesian equilibrium, these beliefs have to be derived given the bidders' strategies using Bayes rule whenever possible. As usual, off-equilibrium path beliefs in Perfect Bayesian equilibria may be arbitrary. However, later on, I will introduce a concept called *belief monotonicity.* The beliefs satisfy belief monotonicity, if the beliefs attached to particular characteristics can be ordered using the hazard rate order in the same order as the priors. I show that all beliefs on the equilibrium path naturally satisfy belief monotonicity, and extend this property to the off-equilibrium path beliefs by assumption. This restriction of the off-equilibrium path beliefs corresponds to the seller's ability to choose her preferred equilibrium in the mechanism design framework.

Next, I investigate the incentives that the auction stage creates for the communication stage. To derive these incentives, I will present several helpful results.

**Lemma 2** *Suppose the auction stage uses Myerson's mechanism. Let $F$ and $G$ be two CDFs such that $F \succsim_{hr} G$. Then any bidder weakly prefers to be assigned distribution $G$ over $F$.*

**Proof.** See appendix. ■

Lemma 2 highlights that the hazard rate order is useful for determining the bidder's optimal behavior. It implies weakly dominant strategies for the bidders

[9]In a Myerson auction with $K$ units of a homogeneous good, a bidder receives the good iff his virtual valuation is positive and among the $K$ highest virtual valuations of the $N$ bidders.

in the communication stage: disclose the characteristic with the largest hazard rate possible, i.e., the characteristic with the smallest possible label attached to it. In equilibrium, it may happen that bidders of different true characteristics $c_i, c_i'$ pool on the same characteristic $\hat{c}_i$. Then the distribution of the valuation $\theta_i$ conditional on the report $\hat{c}_i$ is a mixing distribution. To deal with that, I present a useful lemma for mixing distributions:

**Lemma 3** *Let $\alpha \in [0,1]$ and $F$ and $G$ be such that $F \succsim_{hr} G$. Then for $H = \alpha F + (1-\alpha)G$ we have that $F \succsim_{hr} H \succsim_{hr} G$*

**Proof.** See appendix. ■

The hazard rate order is preserved under mixing. Note that the result extends to a mixture of more than two distributions. If there are more than two distributions, the hazard rate order will rate the mixing distribution somewhere between the most and least favorable distribution included in the mixture. The exact order depends on the exact probability weights in the mixture.

The beliefs associated with reporting a certain characteristic are of central importance for understanding the bidders' behavior. Therefore, I investigate the belief structure on and off the equilibrium path more closely. To achieve this, I introduce the notion of *belief monotonicity*:

**Definition 2 (Belief Monotonicity)** *Let $\hat{F}_{c_i}$ and $\hat{F}_{c_i'}$ be two beliefs that are associated with the disclosure of any characteristics $c_i$ and $c_i'$ such that $c_i < c_i'$. The beliefs satisfy belief monotonicity if $\hat{F}_{c_i'} \succsim_{hr} \hat{F}_{c_i}$*

Beliefs that satisfy belief monotonicity preserve the initial order of the characteristics and their associated distributions under the hazard ratio order. Note that belief monotonicity has to be satisfied for characteristics disclosed on the equilibrium path in any equilibrium.

**Lemma 4** *Any beliefs that are on the equilibrium path satisfy belief monotonicity.*

**Proof.** See appendix. ■

Intuitively, belief monotonicity on the equilibrium path is driven by the weakly dominant strategies of the bidders to disclose the lowest characteristic possible. Bidders with lower characteristics can report lower characteristics by the monotonicity assumption on the bounds of the partially verifiable messages. Given that belief monotonicity must hold on the equilibrium path, I extend the concept to the off-equilibrium path beliefs by assumption.

**Assumption 5** *Belief monotonicity holds off the equilibrium path.*

This assumption, together with lemma 4, establishes that the initial order of the characteristics under the hazard rate order, i.e. $F_{|C|} \succsim_{hr} F_{|C|-1} \succsim_{hr} ... \succsim_{hr} F_1$ must carry over to the beliefs in equilibrium, that is $\hat{F}_{|C|} \succsim_{hr} \hat{F}_{|C|-1} \succsim_{hr} ... \succsim_{hr} \hat{F}_1$. This order on the equilibrium beliefs is useful to determine the bidder's behavior when disclosing their characteristics. As lemma 2 established, such beliefs imply a (weakly) dominant disclosure strategy for each bidder: each bidder should disclose the lowest characteristic possible.

Given these preliminary results, it is straightforward to establish the equilibrium of the two stage game in the following proposition:

**Proposition 4** *The communication stage of the optimal mechanism induces equilibrium beliefs about characteristics as follows:*

$$\hat{F}_c = \sum_{c' \in D(c)} \delta(c') F_{c'}$$

*where* $D(c) := \{c' \in C | c = \underline{\phi}(c')\}$, $\hat{F}_c = F_{|C|}$ *if* $D(c) = \emptyset$ *and* $\delta(c)$ *denotes the mass allocated to* $c \in C$ *under the initial belief* $\Delta$*. In equilibrium, any bidder discloses the minimum possible characteristic that he can generate evidence for, i.e.* $\hat{c}(c) = \underline{\phi}(c)$

*The Myerson auction uses virtual valuations* $J(\theta_i, \hat{c}_i)$ *that are determined by the disclosed valuations* $\theta_i$ *and characteristic* $\hat{c}_i$ *as follows:*

$$J(\theta_i, \hat{c}_i) = \theta_i - \frac{1 - \hat{F}_{\hat{c}_i}(\theta_i)}{\hat{f}_{\hat{c}_i}(\theta_i)}$$

**Proof.** See appendix. ■

Note that the disclosure behavior in this two-stage mechanism is the same as the evidence rule of the revenue maximizing direct mechanism. Further, the virtual valuations that are used for allocating the good are defined analogously. Therefore, the two-stage mechanism implements the revenue maximizing social choice function.

As the seller uses the communication stage to increase her revenue, there may be the practical concern, that bidders will be refuse to communicate with the seller or attempt to engage in babbling. The next section examines this case more closely.

### 3.2 A Right to Remain Anonymous

This section extends the model by giving each bidder a right to remain anonymous in the communication stage. That is, each bidder is free to remain anonymous or to refuse to communicate. In terms of the model, this equals an addition of a characteristic $a$ to the set of all characteristics $C$ such that $a \in \phi(c_i)$ for every bidder of every characteristic $c_i \in C$. Moreover, this characteristic $a$ is available to all bidders regardless of their true characteristic, and when a bidder discloses this characteristic $a$, it is interpreted as remaining anonymous.

At first glance, it seems that such a right to remain anonymous should benefit the bidders. In particular, those with characteristics $c_i$ that are undesirable in terms of the hazard rate order. However, I will show that introducing the right to remain anonymous is inconsequential and provides no benefit to the bidders. While it seems appealing for bidders with particularly undesirable characteristics to choose to remain anonymous, it is not optimal for bidders with more desirable characteristics to pool with them in anonymity. Bidders with more desirable characteristics make the strategic choice to take part in communication to separate themselves from the others that find anonymity more desirable.

However, this leads to an unraveling effect. Whenever bidders of some characteristic find it optimal to leave anonymity, the beliefs over those that remain anonymous have to be updated. Given the updated beliefs, there are now other bidders among the remaining anonymous bidders that find it desirable to leave anonymity. This logic continues onward, such that in the end, only bidders of the least desirable characteristics will remain. If they cannot pool with other bidders on disclosing a specific characteristic, they will find themselves indifferent in between the disclosure of some characteristic, which is disclosed exclusively by bidders of those characteristics and remaining anonymous.

The presence of such an unraveling effect depends explicitly on the beliefs on and off the equilibrium path. A bidder finds it desirable to separate from anonymity if a characteristic with a more beneficial belief is available. However, consider a situation in which all bidders pool on anonymity. Then, the beliefs attached to any other characteristics are off-equilibrium path beliefs. So far, I restricted those beliefs to follow belief monotonicity. But for the discussion of anonymity, I require some further restrictions for off-equilibrium path beliefs. To see why, note that off-equilibrium path beliefs that attach the worst belief to all characteristics, i.e., $\hat{F}_c = F_{|C|}$ for all $c \in C$ also satisfy belief monotonicity. However, given these off-equilibrium path beliefs, pooling in anonymity is an equilibrium for all bidders.

To address this issue, I start with considering the worst on path equilibrium beliefs that can be attached to a characteristic:

**Lemma 5** *The worst on path equilibrium belief $\hat{F}_{c_i}$ that can be sustained for the disclosure of any characteristics $c_i$ such that $c_i = \underline{\phi}(c_i')$ for some $c_i' \in C$ is*

$$\hat{F}_{c_i} = \sum_{\{c_i' | c_i = \underline{\phi}(c_i')\}} \delta(c_i') F_{c_i'}$$

**Proof.** See appendix. ■

Now I extend this belief structure to the off-path beliefs by assumption.

**Assumption 6** *The off path beliefs $\hat{F}_{c_i}$ for any characteristics $c_i$ such that $c_i = \underline{\phi}(c_i')$ for some $c_i' \in C$ are not worse than*

$$\hat{F}_{c_i} = \sum_{\{c_i' | c_i = \underline{\phi}(c_i')\}} \delta(c_i') F_{c_i'}$$

*In the sense of the hazard ratio order.*

As the last step before establishing the proposition, I must consider the ex-ante belief attached to anonymity. Recall the initial beliefs $F_{|C|} \succsim_{hr} F_{|C|-1} \succsim_{hr} ... \succsim_{hr} F_1$ and the common initial prior $\Delta$ that was associated with the set of possible characteristics $C$. This common initial prior represents the initial belief about a bidder that is anonymous, such that I assign $F_a = \sum_{c_i \in C} \delta(c_i) F_{c_i}$.

Given these preliminaries, I present a proposition that formalizes the intuition of unraveling and establishes that a right to anonymity is not beneficial for the bidders.

**Proposition 5** *Suppose that each bidder has the right to remain anonymous. No bidder benefits from the right to remain anonymous.*

**Proof.** See appendix. ■

The presence of a right to remain anonymous is inconsequential. Even though anonymity may seem appealing at first glance, in equilibrium it is not. If true anonymity were to occur in the equilibrium, it would imply pooling of the bidders on the choice of remaining anonymous. However, such a pooling behavior is not optimal, as it requires bidders with more favorable characteristics to pool with those of less favorable characteristics. But then bidders with more favorable characteristics have an incentive to separate themselves from the rest, which causes unraveling.

# 4 Conclusion

In this paper, I considered a seller who wishes to sell multiple units of a homogeneous good to a group of bidders. Bidders have privately known, unverifiable valuations and privately known, partially verifiable characteristics. I use Strausz (2016)'s methodological contribution to recover the revelation principle for this framework. The structure of the partially verifiable characteristics that I consider is richer than the nested range condition of Green & Laffont (1986). I have shown that the revenue-maximizing mechanism is a Myerson auction that groups bidders according to their characteristics and the particular verifiability structure. It can be implemented in two stages: First, a communication stage about the bidders' characteristics, according to which beliefs about the distribution of the bidder's valuations are formed. Second, an auction stage in which these beliefs are used to play Myerson's optimal auction mechanism.

Further, the paper highlighted that introducing a right to remain anonymous for the bidders is inconsequential. If the bidders are allowed to refuse participation in the communication stage, the optimal mechanism is unchanged, and in particular, no bidder benefits from the right to remain anonymous. This is due to an unraveling effect: bidders with beneficial characteristics find it optimal to take part in the communication to avoid pooling with bidders that have less desirable characteristics.

# A Appendix

## A.1 Explanation of the initial and extended environment

The *initial environment* describes the environment as explained in Green & Laffont (1986). The *extended environment* describes the environment as explained in Strausz (2016).

**Outcomes:** In the *initial environment*, the set of outcomes is defined by the combination of physical outcomes and transfers: $\mathcal{X} \times \mathcal{T}$. In the *extended environment*, the set of outcomes is defined by $\mathcal{X} \times \mathcal{T} \times C^N$. In addition to defining the physical allocation of the good and the transfers, the outcome in the extended environment specifies a partially verifiable message for each bidder: evidence.
**Message Sets:** In the *initial environment*, the set of partially verifiable messages that can be sent in the mechanism depends on the true characteristic $c$ of a bidder and is equal to $\phi(c)$. In the *extended environment*, the set of partially verifiable messages that can be sent is independent of the true characteristic of a bidder and is given by some set $V \subseteq C$.[10]
**Utility Functions:** In the *initial environment*, the utility function of a bidder over the set of outcomes in the initial environment is given by

$$u_i = \theta_i x_i + t_i \tag{11}$$

In the *extended environment*, the utility function of a bidder over the set of outcomes in the extended environment is given by

$$\hat{u}_i = \begin{cases} \theta_i x_i + t_i & \text{if } \hat{c} \in \phi(c_i) \\ \theta_i x_i + t_i - P & \text{if } \hat{c} \notin \phi(c_i) \end{cases} \tag{12}$$

where $\hat{c}$ is some evidence in the form of a partially verifiable message and $P$ is a sufficiently large punishment that ensures that a bidder will not try to submit evidence that can be objectively rejected as false.[11] Clearly, there is effectively no difference between a bidder simply not being able to send a specific message, as in Green & Laffont (1986) and not wanting to send a message that is strictly

[10] For technical reasons, it is important to extend the set of messages that can be sent by a bidder in this way. By definition, in a Bayesian game, the set of possible actions of a bidder may not depend on his type.

[11] In equilibrium, it is sufficient to punishment a bidder that submits objectively false evidence by excluding him from the auction.

dominated as in Strausz (2016).
**Social Choice Functions:** In the *initial environment*, a social choice function is a mapping

$$f : \Theta^N \times C^N \to \mathcal{X} \times \mathcal{T} \tag{13}$$

In the *extended environment*, a social choice function is a mapping

$$\hat{f} : \Theta^N \times C^N \to \mathcal{X} \times \mathcal{T} \times C^N \tag{14}$$

The first two components of the social choice function are mappings from the private information of the bidders into allocations and transfers. The third component is a departure from the usual definition of a social choice function. It is a mapping from the private information of the bidders into the set of verifiable messages. This third component can be understood as an evidence rule. For each pair of valuations and characteristics $(\theta_i, c_i)$, it assign some partially verifiable evidence $c_i \in C$, that the bidder has to submit.

## A.2 Proof of proposition 1

The proof follows the structure given in Strausz (2016), with some slight adaptations to my framework.
Suppose some mechanism with allocation rule x, and transfer rule t implements $f$ in the initial environment. Consider some bidder $i$. Then for the tuple $(\theta_i, c_i)$, given the equilibrium strategies of the bidders $-i$, some strategy leading to the outcome $f(\theta, c)$ is optimal and in particular some verifiable message $\hat{c}(\theta_i, c_i) \in \phi(c_i)$ that bidder $i$ sends when outcome $f(\theta, c)$ is reached in equilibrium is optimal. Consider the direct mechanism $\hat{g} = (f, \hat{c})$ with $\hat{c}(\theta_i, c_i)$ being exactly the mapping that describes the part of the optimal strategy with regards to the verifiable messages in the equilibrium of the initial environment given mechanism $g$. Suppose all bidders $-i$ truthfully reveal their valuations and characteristics and follow the evidence rule $\hat{c}$. Fix some tuple $(\theta_i, c_i)$. Bayesian incentive compatibility holds for any $c_i'$ s.t. $\hat{c}(\theta_i, c_i') \notin \phi(c_i)$. Moreover, the optimality of the strategy leading to the implementation of $f(\theta, c)$ and sending the verifiable message $\hat{c}(\theta_i, c_i)$ implies that Bayesian incentive compatibility holds for any $(\theta_i', c_i')$ such that $\hat{c}(\theta_i', c_i') \in \phi(c)$. Therefore, we have incentive compatibility

## A.3 Proof of proposition 2

To make the proof more legible, I list conditions 1-4 of the proposition once again:

1. Integrability
$$\hat{U}(\theta_i, c_i) = \hat{U}(\underline{\theta}, c_i) + \int_{\underline{\theta}}^{\theta_i} X(s, c_i)ds$$
2. Monotonicity, that is $\theta_i > \theta_i'$ implies $X(\theta_i, c_i) \geq X(\theta_i', c_i)$
3. Optimality with respect to $c_i$, that is $X(\theta_i, c_i) \geq X(\theta_i, c_i')$ for all $(\theta_i, c_i')$ such that $\hat{c}(\theta_i, c_i') \in \phi(c_i)$.
4. Feasible evidence for truthful disclosure: $\hat{c}(\theta_i, c_i) \in \phi(c_i)$ for all $\theta_i \in [\underline{\theta}, \overline{\theta}]$ and $c_i \in C$

First, I show that 1, 2, 3 and 4 imply incentive compatibility. Consider some bidder with true valuation and characteristic $(\theta_i, c_i)$ and some possible deviations $\theta_i'$ with $\theta_i' < \theta_i$ and $c_i' \in C$. Note that condition 4 guarantees that incentive compatibility would trivially be satisfied if $\hat{c}(\theta_i', c_i') \notin \phi(c_i)$. Therefore, I can restrict to the case such that $\hat{c}(\theta_i', c_i') \in \phi(c_i)$. Now consider

$$
\begin{aligned}
&X(\theta_i, c_i)\theta_i + T(\theta_i, c_i) - (X(\theta_i', c_i')\theta_i + T(\theta_i', c_i')) && (15)\\
=&X(\theta_i, c_i)\theta_i + T(\theta_i, c_i) - (X(\theta_i', c_i')\theta_i + T(\theta_i', c_i')) + X(\theta_i', c_i')\theta_i' + T(\theta_i', c_i') - (X(\theta_i', c_i')\theta_i' + T(\theta_i', c_i')) && (16)\\
=&U(\theta_i, c_i) - U(\theta_i', c_i') - (\theta_i - \theta_i')X(\theta_i', c_i') && (17)\\
\overset{1}{=}&\int_{\underline{\theta}}^{\theta_i} X(s, c_i)ds - \int_{\underline{\theta}}^{\theta_i'} X(s, c_i')ds - (\theta_i - \theta_i')X(\theta_i', c_i') && (18)\\
\overset{3,4}{\geq}&\int_{\theta_i'}^{\theta_i} X(s, c_i)ds - (\theta_i - \theta_i')X(\theta_i', c_i) && (19)\\
\overset{2}{\geq}&0 && (20)
\end{aligned}
$$

Note that by condition 4 it holds that $\hat{c}(\theta_i', c_i) \in \phi(c_i)$ as the set of verifiable messages does not depend on $\theta_i'$ but only on $c_i$ and it must be possible for a bidder with the true value-characteristic pair $(\theta_i', c_i)$ to report their valuation truthfully without punishment. But then if both $\hat{c}(\theta_i', c_i') \in \phi(c_i)$ and $\hat{c}(\theta_i', c_i) \in \phi(c_i)$, condition 3 implies that $X(\theta_i', c_i) \geq X(\theta_i', c_i')$.

A similar argument can be made for $\theta_i' > \theta_i$. Consider some bidder with true valuation $\theta_i$ and true characteristic $c_i$ and some possible deviations $\theta_i'$ with $\theta_i' > \theta_i$ and $c_i' \in C$. Note that incentive compatibility would trivially be satisfied if $\hat{c}(\theta_i', c_i') \notin \phi(c_i)$. Therefore, I can restrict to the case such that $\hat{c}(\theta_i', c_i') \in \phi(c_i)$. Now consider

$$
\begin{aligned}
& X(\theta_i, c_i)\theta_i + T(\theta_i, c_i) - (X(\theta_i', c_i')\theta_i + T(\theta_i', c_i')) && (21)\\
=& X(\theta_i, c_i)\theta_i + T(\theta_i, c_i) - (X(\theta_i', c_i')\theta_i + T(\theta_i', c_i')) + X(\theta_i', c_i')\theta_i' + T(\theta_i', c_i') - (X(\theta_i', c_i')\theta_i' + T(\theta_i', c_i')) && (22)\\
=& U(\theta_i, c_i) - U(\theta_i', c_i') - (\theta_i - \theta_i')X(\theta_i', c_i') && (23)\\
\overset{1}{=}& -\left[\int_{\underline{\theta}}^{\theta_i'} X(s, c_i')ds - \int_{\underline{\theta}}^{\theta_i} X(s, c_i)ds - (\theta_i' - \theta_i)X(\theta_i', c_i')\right] && (24)\\
\overset{3,4}{\geq}& -\left[\int_{\theta_i}^{\theta_i'} X(s, c_i')ds - (\theta_i' - \theta_i)X(\theta_i, c_i')\right] && (25)\\
\overset{2}{\geq}& 0 && (26)
\end{aligned}
$$

Second, I show that IC implies 1, 2, 3, and 4. First, condition 4 obviously has to hold to allow each bidder to report his valuation and type truthfully without being subjected to punishment. If condition 4 failed, then either there would be some pair of misreports $(\theta_i', c_i')$ that allows the bidder to avoid the punishment, or if one considers the fringe case where there would be no possible report that allows the bidder to avoid punishment, the individual rationality constraints will end up being violated. Now, consider 2. Note that incentive compatibility implies that

$$
\begin{aligned}
X(\theta_i, c_i)\theta_i + T(\theta_i, c_i) &\geq X(\theta_i', c_i)\theta_i + T(\theta_i', c_i) && (27)\\
X(\theta_i', c_i)\theta_i' + T(\theta_i', c_i) &\geq X(\theta_i, c_i)\theta_i' + T(\theta_i, c_i) && (28)
\end{aligned}
$$

Rearrange

$$
(\theta_i - \theta_i')(X(\theta_i, c_i) - X(\theta_i', c_i)) \geq 0 \tag{29}
$$

Which yields monotonicity.

Next, the integrability condition, 1. follows from the fact that the utility itself is independent of $c_i$ and the application of Milgrom & Segal (2002). To see condition 3, consider the following: Suppose there is a $c_i'$ such that $\hat{c}(\theta_i, c_i') \in \phi(c_i)$ and a set of valuations $\tilde{\Theta}_i$ with positive measure such that $X(\theta_i, c_i) < X(\theta_i, c_i')$. Consider a value $\theta_i$ such that $\tilde{\Theta}_i \subseteq [\underline{\theta}, \theta_i]$. I have already shown that IC implies 1, such that the expected utility from truthful reporting equals

$$\int_{\underline{\theta}}^{\theta_i} X(s, c_i) ds < \int_{\tilde{\Theta}} X(s, c_i') ds + \int_{[\underline{\theta}, \theta_i] \setminus \tilde{\Theta}} X(s, c_i) ds \tag{30}$$

Which shows that truthful reporting of $c_i$ at values $\theta_i \in \tilde{\Theta}$ is not optimal and is a contradiction.

## A.4 Proof of lemma 1

Proof by contradiction. Suppose the statement does not hold. Then there exists $c_i, c_i'$ and some $\theta_i$ such that $x_i^*(\theta_i, c_i, \theta_{-i}, c_{-i}) = 1$ and $\hat{c}^*(\theta_i, c_i) \in \phi(c_i')$ but $x_i^*(\theta_i, c_i', \theta_{-i}, c_{-i}) = 0$. Consider some $\epsilon > 0$, small, and a ball with radius $\epsilon$ around $\theta_{-i}$. Note as all the virtual valuations are continuous, for small enough $\epsilon$ the virtual valuations of all bidders $-i$ can be approximated as constant, subject to a bounded error that vanishes as $\epsilon \to 0$. Since by assumption the mechanism is revenue-maximizing, the allocation $x_i^*(\theta_i, c_i, \theta_{-i}, c_{-i}) = 1$ is optimal at that particular profile and, for small enough $\epsilon$, in a neighborhood around the valuations of the other bidders in the profile. However, then in the whole neighborhood it holds that $x_i(\theta_i, c_i, \theta_{-i}, c_{-i}) = 1$ but $x_i(\theta_i, c_i', \theta_{-i}, c_{-i}) = 0$. However, this implies $X(\theta_i, c_i) > X(\theta_i, c_i')$ for that neighborhood. To not violate condition 3 of incentive compatibility, there must be another neighborhood with radius $\eta > 0$, small, around the valuations of the other bidders for some profile $(\theta_i, c_i, \theta_{-i}', c_{-i}')$ in which bidder $i$ is awarded the good for the pair $(\theta_i, c_i')$ but not for $(\theta_i, c_i)$.

Consider the profile $(\theta_i, c_i, \theta_{-i}, c_{-i})$. Denote by $J$ the maximum revenue that can be achieved by incentive compatible assignment of the good to another bidder, or possibly through keeping the good. Denote by $J'$ the revenue associated with the best, alternative incentive compatible alternate assignment given the profile $(\theta_i, c_i, \theta_{-i}', c_{-i}')$. Then the assignment rule $x_i(\theta_i, c_i, \theta_{-i}, c_{-i}) = 1$, $x_i(\theta_i, c_i', \theta_{-i}, c_{-i}) = 0$, $x_i(\theta_i, c_i, \theta_{-i}', c_{-i}') = 0$ and $x_i(\theta_i, c_i', \theta_{-i}', c_{-i}') = 1$ yields more revenue than the assignment of the good to bidder $i$ at both $(\theta_i, c_i)$ and $(\theta_i, c_i')$ given $(\theta_{-i}', c_{-i}')$ and assigning to the best alternative yielding $J$ at $(\theta_{-i}, c_{-i})$ if

$$Pr(B_\epsilon(\theta_{-i}), c_{-i})[f(\theta_i|c_i)\delta(c_i)(J(\theta_i, c_i) + J') + f(\theta_i|c_i')\delta(c_i')(J + J(\theta_i, c_i'))] \quad (31)$$
$$\geq Pr(B_\eta(\theta'_{-i}), c'_{-i})[f(\theta_i|c_i)\delta(c_i)(J + J(\theta_i, c_i)) + f(\theta_i|c_i')\delta(c_i')(J + J(\theta_i, c_i'))] \quad (32)$$

Note that for sufficiently small $\epsilon$ and $\eta$ the approximation errors will be small enough to be negligible. Further, through appropriate choice of $\epsilon$ and $\eta$ it is possible to set $Pr(B_\epsilon(\theta_{-i}), c_{-i}) = Pr(B_\eta(\theta'_{-i}), c'_{-i})$. There are some more subtle details to note. First, for the bidders $-i$ the changed allocation using the alternatives $J$ and $J'$ is incentive compatible by assumption. Second, the changed allocation is incentive compatible for bidder $i$ as the interim allocation probability $X(\theta_i, c_i)$ and $X(\theta_i, c_i')$ remains unchanged through the appropriate choice of $\epsilon$ and $\eta$. Thus, the inequality implies

$$J' - J \geq 0 \quad (33)$$

Now consider an alternate assignment rule, which assigns the good to bidder $i$ given the profile $(\theta_i, c_i, \theta_{-i}, c_{-i})$ and $(\theta_i, c_i', \theta_{-i}, c_{-i})$, but never assigns the good to bidder $i$ under alternative profile $(\theta_i, c_i, \theta'_{-i}, c'_{-i})$ and $(\theta_i, c_i', \theta'_{-i}, c'_{-i})$. Then the following inequality must hold

$$f(\theta_i|c_i)\delta(c_i)(J(\theta_i, c_i) + J') + f(\theta_i|c_i')\delta(c_i')(J + J(\theta_i, c_i')) \quad (34)$$
$$\geq f(\theta_i|c_i)\delta(c_i)(J(\theta_i, c_i) + J') + f(\theta_i|c_i')\delta(c_i')(J(\theta_i, c_i') + J') \quad (35)$$
$$\Rightarrow J' - J \leq 0 \quad (36)$$

Note that the only way both of these inequalities can be true at the same time, is if they hold with equality. However, this implies that the revenue of an incentive compatible mechanism that sets $x_i(\theta_i, c_i, \theta_{-i}, c_{-i}) = 1$ while setting $x_i(\theta_i, c_i', \theta_{-i}, c_{-i}) = 0$ is the same as that of a mechanism that sets $x_i(\theta_i, c_i, \theta_{-i}, c_{-i}) = 1 = x_i(\theta_i, c_i', \theta_{-i}, c_{-i}) = 1$. Therefore, I can restrict mechanisms to follow the assertion in the lemma without loss of generality.

## A.5 Proof of proposition 3

Fix some bidder $i$ with valuation $\theta_i$ and characteristic $c_i$. For now, suppose that characteristics are observable. Note that for any valuation $\theta_i \in \Theta$ the virtual valuations of the bidder can be ranked according to the hazard rate order, i.e. it holds that $J(\theta_i, c_i = 1) \geq J(\theta_i, c_i = 2) \geq ... \geq J(\theta_i, c_i = |C|)$. Note that this virtual valuation is exactly the revenue that the seller can extract through the allocation of the good to the bidder. Now consider the choice of the seller: assign the good to bidder $i$ or assign the good to some other bidder $j$ with some valuation and characteristic. It is clear that the larger the characteristic $c_i$, the smaller the virtual valuation of bidder $i$, and thus the seller may favor allocation of the good to bidder $j$. There will be some characteristic $\bar{c}$ such that if $c_i \leq \bar{c}$, the seller wants to allocate the good to bidder $i$, and if $c_i \geq \bar{c}$ the seller wants to allocate the good to bidder $j$. The larger the amount of revenue that the seller can receive through the allocation of the good to bidder $j$, the smaller the value $\bar{c}$.

Now consider what changes if characteristics are unobservable, but partially verifiable. The seller has to find an evidence rule $\hat{c}(\cdot)$ such that she can distinguish the characteristics $c_i \leq \bar{c}$ from the characteristics $\bar{c} \leq c_i$ if possible. Given assumption 4, namely that the upper and lower bounds of the characteristics for which a bidder can produce evidence are monotone in the true characteristics, it is clear that the optimal evidence rule asks every bidder to produce evidence for the lowest possible characteristic that they feasibly can. To see why, consider the two possible situations that may arise: let $c_i$ be the largest characteristic such that $c_i \leq \bar{c}$ and let $c_i'$ be the smallest characteristic such that $\bar{c} \leq c_i'$. If the evidence rule $\hat{c}(\cdot)$ can distinguish $c_i$ from $c_i'$, that is if $\underline{\phi}(c_i) < \underline{\phi}(c_i')$, then the seller can implement the same allocation as she would if characteristics were observable. If the evidence rule $\hat{c}(\cdot)$ is unable to distinguish $c_i$ from $c_i'$, i.e. if $\underline{\phi}(c_i) = \underline{\phi}(c_i')$, the monotone bounds assumption implies that $\phi(c_i) \subset \phi(c_i')$. However, then lemma 1 implies that if the seller assign the good to the bidder with characteristic $c_i$ she must also assign it to the bidder with characteristic $c_i'$ in a revenue maximizing mechanism.

Having established the optimal evidence rule $\hat{c}(\cdot)$, it is straightforward to calculate the expected virtual valuation of bidders that have been grouped together into a group $G_{c_i}$ by the evidence rule through

$$J(\theta_i, G_{c_i}) = \frac{1}{\sum_{c_i' \in G_{c_i}} \delta(c_i')} \sum_{c_i' \in G_{c_i}} \delta(c_i') J(\theta_i, c_i') \tag{37}$$

Then a point wise maximization implies that is optimal to assign the goods to the buyers with the largest expected virtual valuations as defined above.

## A.6 Proof of lemma 2

Recall that the utility of every bidder at their lowest type in Myerson's optimal auction is set to 0. Then the integrability condition of the incentive compatibility constraints in Myerson's optimal auction reads as:

$$U(\theta_i) = \int_{\underline{\theta}}^{\theta_i} X(\hat{c}_i, s) ds \tag{38}$$

Where $X(\hat{c}, s)$ denotes the interim allocation probability of bidder $i$ who reports characteristic $\hat{c}$ and valuation $s$. Recall that a bidder receives the good if and only if his virtual valuation $J(\theta_i, F)$, defined by

$$J(\theta_i, F) = \theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)} \tag{39}$$

is larger than 0 and any of the other bidders' virtual valuations. Let $F$ and $G$ be such that $F \succsim_{hr} G$. It is easily verified that $J(\theta_i, F) \leq J(\theta_i, G)$ for every $\theta_i$. This implies that $X(\theta_i, F) \leq X(\theta_i, G)$ for every $\theta_i$. Therefore, if the bidder is given the choice between choosing $F$ or $G$, it is (weakly) dominant to choose $G$ over $F$ for every valuation $\theta_i$.

## A.7 Proof of lemma 3

Note that a common alternative characterization of the hazard ratio order is $F \succsim_{hr} G$ iff $\frac{1-G(\theta)}{1-F(\theta)}$ is decreasing in $\theta$. Let $H = \alpha F + (1-\alpha)G$. First, show that $F \succsim_{hr} H$, i.e. that $\frac{1-H(\theta)}{1-F(\theta)}$ is decreasing in $\theta$.

$$\frac{\partial}{\partial \theta}\left(\frac{1 - H(\theta)}{1 - F(\theta)}\right) = \frac{-h(\theta)(1 - F(\theta)) + f(\theta)(1 - H(\theta))}{(1 - F(\theta))^2} \tag{40}$$

$$= \frac{-(\alpha f(\theta) + (1 - \alpha)g(\theta))(1 - F(\theta)) + f(\theta)(1 - (\alpha F(\theta) + (1 - \alpha)G(\theta))}{(1 - F(\theta))^2} \tag{41}$$

This is negative if the numerator is negative, i.e., if

$$-(\alpha f(\theta)+(1-\alpha)g(\theta))(1-F(\theta))+f(\theta)(1-(\alpha F(\theta)+(1-\alpha)G(\theta))\leq 0 \tag{42}$$

$$\iff \alpha(-f(\theta))(1-F(\theta))+f(\theta)(1-F(\theta))+(1-\alpha)(-g(\theta)(1-F(\theta))+f(\theta)(1-G(\theta))\leq 0 \tag{43}$$

$$\iff (1-\alpha)(-g(\theta)(1-F(\theta))+f(\theta)(1-G(\theta))\leq 0 \tag{44}$$

Note that the last inequality holds since $F \succsim_{hr} G$.

Second, show that $H \succsim_{hr} G$. Consider

$$\frac{\partial}{\partial\theta}\left(\frac{1-G(\theta)}{1-H(\theta)}\right)=\frac{-g(\theta)(1-H(\theta))+h(\theta)(1-G(\theta))}{(1-H(\theta))^2} \tag{45}$$

Again, this is negative if the numerator is negative, that is if

$$-g(\theta)(1-H(\theta))+h(\theta)(1-G(\theta))\leq 0 \tag{46}$$

$$\iff -g(\theta)(1-(\alpha F(\theta)+(1-\alpha)G(\theta)))+(\alpha f(\theta)+(1-\alpha)g(\theta))(1-G(\theta))\leq 0 \tag{47}$$

$$\iff \alpha(-g(\theta)(1-F(\theta))+f(\theta)(1-G(\theta)))\leq 0 \tag{48}$$

Where the inequality holds since $F \succsim_{hr} G$.

## A.8 Proof of lemma 4

By contradiction. Let $c_i$ and $c_i'$ be the two smallest characteristics that are disclosed on the equilibrium path with $c_i < c_i'$ such that there are two beliefs $\hat{F}_{c_i}$ and $\hat{F}_{c_i'}$ and a valuation $\theta_i$, where $\frac{\hat{f}_{c_i}(\theta_i)}{1-\hat{F}_{c_i}(\theta_i)} < \frac{\hat{f}_{c_i'}(\theta_i)}{1-\hat{F}_{c_i'}(\theta_i)}$. Since the beliefs are on the equilibrium path, they have to be formed according to Bayes' rule. Fix this value of $\theta_i$ and denote the set of all characteristics that disclose their characteristic as $c_i$ by $D(c_i)$ and the set of all characteristics that disclose their characteristic as $c_i'$ as $D(c_i')$. As both of the beliefs are on the equilibrium path, neither $D(c_i)$ nor $D(c_i')$ are empty. Note that by the same argument as used in the proof of lemma 2, any bidder with valuation $\theta_i$ prefers to disclose characteristic $c_i'$ over characteristic $c_i$ if possible. However, as the beliefs are equilibrium beliefs, it must be impossible for any bidder to do so, that is for all characteristics $a \in D(c_i)$, it holds that $c_i' \notin M(a)$.

Now consider any bidder of characteristic $c''$ with $c_i'' > c_i' > c_i$. Suppose that $c_i'' \in D(c_i)$. By assumption 3 we know that $c_i' \in M(c_i'')$. However, then bidders of characteristic $c_i''$ should disclose characteristic $c_i'$ by lemma 2. Therefore, for any bidder of characteristic $c_i'' > c_i' > c_i$ we know that $c_i'' \notin D(c_i)$. Now consider any bidder of characteristic $c_i''$ with $c_i'' < c_i < c_i'$. Since $c_i$ and $c_i'$ are the two smallest characteristics that violate belief monotonicity, we know that $\hat{F}_{c_i} \succsim_{hr} \hat{F}_{c_i''}$. Since truthful disclosure of the characteristic is possible by assumption, i.e., $c_i'' \in M(c_i'')$, any bidder of such a characteristic is better off disclosing their characteristic truthfully rather than disclosing characteristic $c_i$. Therefore, for any bidder of characteristic $c_i''$ with $c_i'' < c_i < c_i'$ it holds that $c_i'' \notin D(c_i)$. Thus, the only bidder that will possibly disclose characteristic $c_i$ is the bidder that actually has characteristic $c_i$ and since it is disclosed on the equilibrium path we have that $D(c_i) = \{c_i\}$ and therefore $\hat{F}_{c_i} = F_{c_i}$. Note that however, for $\frac{\hat{f}_{c_i}(\theta_i)}{1-\hat{F}_{c_i}(\theta_i)} < \frac{\hat{f}_{c_i'}(\theta_i)}{1-\hat{F}_{c_i'}(\theta_i)}$ to hold true on the equilibrium path, there must exist some characteristics $c_i'' < c_i$ such that $c_i'' \in D(c_i')$. However, if $c_i' \in M(c_i'')$ by assumption 3 and 4 it holds that $c_i' \in \phi(c_i)$, which is a contradiction to $c_i' \notin \phi(c_i)$.

## A.9 Proof of lemma 5

Denote by $D(c_i)$ the set of all characteristics that disclose $c_i$ in equilibrium. By belief monotonicity and lemma 2, it holds that for any characteristic $c_i' \in C$ we have that $c_i > \underline{\phi}(c_i') \Rightarrow c_i' \notin D(c_i)$. Further, for any characteristic $c'$ such that $c_i < \underline{\phi}(c_i')$ it holds that $c_i' \notin D(c_i)$ as it would be impossible to disclose this characteristic. Therefore, the only characteristics $c_i'$ that can disclose $c_i$ in equilibrium are such that $c_i = \underline{\phi}(c_i')$. Note that they will also do so, that is $c_i = \underline{\phi}(c_i') \Rightarrow c_i' \in D(c_i)$. This holds as belief monotonicity and lemma 2 again imply that no bidder that can disclose $c_i$ would like to disclose any characteristic larger than $c_i$. Therefore, it holds that $D(c) = \{c_i' | c_i = \underline{\phi}(c_i')\}$ and the equilibrium belief must be equal to the one given in the lemma and, in particular, cannot be worse than that.

## A.10 Proof of proposition 4

Note that lemma 4 established that belief monotonicity holds in the equilibrium of the optimal mechanism. Further, assumption 5 establishes belief monotonicity for the off-equilibrium path beliefs. Further, lemma 2 has established that given a choice between two beliefs $F$ and $G$ with $G \succsim_{hr} F$, it is (weakly) optimal for a bidder to choose belief $F$. Given belief monotonicity on and off the equilibrium path, this is equivalent to choosing the lower characteristic. Hence, disclosing larger characteristics cannot be a profitable deviation. Finally, note that it is easy to verify that all the beliefs chosen on the equilibrium path, i.e., those where $D(c) \neq \emptyset$ satisfy belief monotonicity through the application of lemma3. This follows, as bidders always disclose the lowest possible characteristic, such that for characteristics $c_i < c_i'$ the set of bidders that disclose $c_i$, i.e., $D(c_i)$ features lower characteristics than the set $D(c_i')$.

## A.11 Proof of proposition 5

Suppose that a right to remain anonymous is introduced to the two-stage mechanism, that is, suppose there is a characteristic $a$ such that $a \in \phi(c_i)$ for all $c_i \in C$. The goal is to show that the equilibrium belief associated with anonymity, i.e., $\hat{F}_a$, in the optimal mechanism with the right to remain anonymous cannot be better than the worst equilibrium belief on the equilibrium path in the mechanism without the right to remain anonymous. I achieve this in two steps.

First, I show that the equilibrium belief of any characteristic that is disclosed on the equilibrium path must not be worse than $\hat{F}_a$. Let $c_i \neq a$ be any characteristic that is disclosed on the equilibrium path of the optimal mechanism, including the right to remain anonymous, and let $\hat{F}_{c_i}$ be the associated belief. Note that the belief associated with $a$ in any optimal mechanism must fit somewhere in the hazard rate order. If $a$ is disclosed on the equilibrium path, this holds by lemma 4, and if it is not disclosed on the equilibrium path, it holds by assumption. Now suppose that $\hat{F}_{c_i} \succsim_{hr} \hat{F}_a$. By lemma 2, we know that disclosing $a$ is preferred by any bidder over $c_i$. However, then $c_i$ will not be disclosed on the equilibrium path, a contradiction. This implies that the belief attached to anonymity has to be (weakly) worse than that of any characteristic $c_i \neq a$ if such a characteristic is disclosed on the equilibrium path

Second, I show that all characteristics $c_i$ such that there exists a characteristic $c_i'$ with $c_i = \underline{\phi}(c_i')$ are disclosed on the equilibrium path. Note that doing this establishes the claim. Those characteristics are the only ones disclosed on the equilibrium path in the optimal mechanism without the right to remain anonymous. They cannot have worse equilibrium beliefs than anonymity, as shown above. Start, by considering the set of those characteristics $\{c_i \in C | \exists c_i' \in C \text{ s.t. } c_i = \underline{\phi}(c_i')\}$. Note that the structure of the message sets by assumptions 3 and 4 implies that this set is equal to $\{1, 2, ..., \bar{c}\}$ for some $\bar{c} \in C$. I iterate through this set starting from the lowest characteristic and show that there is no equilibrium such that the characteristic is not disclosed on the equilibrium path. Begin with $c = 1$. Assume $c = 1$ is not disclosed on the equilibrium path. Now I consider two cases: First, suppose that no characteristic other than anonymity is disclosed on the equilibrium path. Then the equilibrium belief of anonymity is $\hat{F}_a = \sum_{c \in C} \delta(c) F_c$. Note that by assumption, the off-path belief $\hat{F}_1$ for $c = 1$ is not worse than $\sum_{\{c'|1=\underline{\phi}(c')\}} \delta(c') F(c')$. However, this implies that $\hat{F}_a \succsim \hat{F}_1$, as the characteristics in $\{c'|1 = \underline{\phi}(c')\}$ are a subset of more beneficial characteristics than those in $C$ itself.[12] Therefore, there is a profitable deviation and no equilibrium.

Second, suppose that some other characteristic $c_i \neq 1$ is disclosed on the equilibrium path. By belief monotonicity, the equilibrium belief attached to this characteristic has a worse position in the hazard rate order than the off-path belief attached to $c_i = 1$. As argued above, the equilibrium belief attached to anonymity must not be better than that of any characteristic disclosed on the equilibrium path. Therefore, any bidder of a characteristic $c_i$ such that $1 \in \phi(c_i)$ will deviate to the disclosure of $c_i = 1$. Together, both points cover all the cases, such that there is no equilibrium in which $c_i = 1$ is not disclosed on the equilibrium path.

Having established this, the next characteristic to iterate through is $c_i = 2$. However, given that $c_i = 1$ must be disclosed on the equilibrium path, it is possible to remove those characteristics $c_i \in C$ with $1 \in \phi(c_i)$ from the consideration and follow the same arguments made for $c_i = 1$. It is possible to follow this line of argumentation all the way up to $\bar{c}$. Therefore, in any equilibrium including the right to remain anonymous all characteristics $\{c_i \in C | \exists c_i' \in C \text{ s.t. } c_i = \underline{\phi}(c_i')\}$ are disclosed on the equilibrium path. Note that these are the only characteristics

[12] Technically the sets could be equal if characteristics are completely unverifiable, however, corollary 1 has shown that in this case no information is transmitted in equilibrium. In this sense, all bidders are already anonymous. A right to remain anonymous would not yield any benefit in any case

disclosed on the mechanism's equilibrium path without a right to remain anonymous and that the equilibrium beliefs have remained unchanged. However, the equilibrium belief of remaining anonymous is not better than any of the beliefs on the equilibrium path. Thus, no bidder has benefited from the right to remain anonymous.

## A.12 Proof that non-decreasing virtual valuations are preserved under mixing

Let $F$ and $G$ be such that the virtual valuations associated with them are non-decreasing. Let $\alpha \in [0,1]$ and $H = \alpha F + (1-\alpha)G$. Then show that

$$J_H(\theta) = \theta - \frac{1-H(\theta)}{h(\theta)} \tag{49}$$

Is non-decreasing in $\theta$. Note that McAfee & McMillan (1987) established that the virtual valuation is non-decreasing if and only if $1/(1-H(\theta))$ is convex. Therefore, consider

$$\frac{\partial^2}{\partial\theta^2}\left(\frac{1}{1-H(\theta)}\right) = 2h(\theta)^2(1-H(\theta))^{-3} + h'(\theta)(1-H(\theta))^{-2} \tag{50}$$

This is positive if

$$2h(\theta)^2 + h'(\theta)(1-H(\theta)) \geq 0 \tag{51}$$

$$\iff \alpha^2(2f(\theta)^2 + f'(\theta)(1-F(\theta))) + (1-\alpha)^2(2g(\theta) + g'(\theta)(1-G(\theta))) + 4\alpha(1-\alpha)g(\theta)f(\theta) \geq 0 \tag{52}$$

This inequality holds since $F$, and $G$ have non-decreasing virtual valuations by assumption and $g, f \geq 0$.

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