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DISCUSSION PAPER

// KARL-MARTIN EHRHART, ANSELM EICKE, LION HIRTH, FABIAN OCKER, MARION OTT, INGMAR SCHLECHT, AND RUNXI WANG

Analysis of a Capacity-Based Redispatch Mechanism

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Karl-Martin Ehrhart[a,b] Anselm Eicke[c] Lion Hirth[c] Fabian Ocker[d]
Marion Ott[e] Ingmar Schlecht[c,f] Runxi Wang[a]

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Abstract

This paper discusses a capacity-based redispatch mechanism in which awarded market participants are compensated for their availability for redispatch, rather than activation. The rationale is to develop a market design that prevents so-called "inc-dec gaming" when including flexible consumers with a market-based approach. We conduct a game-theoretical analysis of a capacity-based redispatch mechanism. Our analysis reveals that despite its intention, the capacity-based redispatch is prone to undesirable behavior of market participants. The reason is that the availability payment incentivizes participants to change their energy consumption (generation) behavior. However, this also applies to undesired participants who increase the redispatch requirement through participation. Under certain assumptions, the additional redispatch potential equals the additional redispatch demand it creates. Consequently, the mechanism does not resolve network constraints, while causing costs for the compensation payments. Furthermore, we study three alternative implementation options, none of which resolves the underlying problem. It follows from our analysis that a mechanism can only be promising if it is capable to distinguish between the potential participants to exclude the undesirable ones.

JEL Classification: D43, D44, L13, Q41, Q48

Keywords: Energy market, Congestion management, Capacity-based redispatch, Game theory, Auctions

[a]Karlsruhe Institute of Technology (KIT), ECON, Neuer Zirkel 3, 76131 Karlsruhe, Germany. E-mail: ehrhart@kit.edu (K.-M. Ehrhart), runxi.wang@kit.edu (R. Wang).

[b]Takon GmbH, Schwalbenweg 5, 95445 Bayreuth, Germany.

[c]Neon Neue Energieökonomik GmbH, Karl-Marx-Platz 12, 12043 Berlin, Germany. E-mail: eicke@neon.energy (A. Eicke), hirth@neon.energy (L. Hirth), schlecht@neon.energy (I. Schlecht).

[d]TenneT TSO GmbH, Bernecker Str. 70, 95448 Bayreuth, Germany. E-Mail: fabian.ocker@tennet.eu.

[e]ZEW Mannheim. ZEW – Leibniz Centre for European Economic Research, L 7, 1, 68161 Mannheim, Germany. E-Mail: marion.ott@zew.de.

[f]ZHAW School of Management and Law, Winterthur, Switzerland.

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1 Introduction

European power markets are organized in pricing zones. Within each zone, the power market is cleared as if the zone was free of network congestion. System operators, such as transmission system operators and distribution system operators, subsequently resolve the resulting network constraints through the redispatch of generators: Congested network elements are relieved by reducing the oversupply of electricity upstream of the constraint (downward redispatch) and reducing the scarcity downstream of the bottleneck (upward redispatch). In cost-based redispatch, which is predominantly applied in European systems, downward dispatched generators are compensated for their foregone profits and upward dispatched generators are paid their operational costs (EU, 2019). To do so, the system operator estimates the operating costs of the affected generators based on the plants' efficiency, fuel costs, and the costs of emission certificates.

In recent years, strain on transmission networks has increased, mostly due to the rising share of variable wind and solar energy production that are located far away from consumption centers. As a result, costs of managing the existing network have reached unprecedented levels in many European countries (ACER & CEER, 2021). This trend is expected to continue (Goop et al., 2017). Reasons are the phase out of thermal power production technologies close to consumption centers, such as coal-fired power plants, as well as the rare inclusion and thus utilization of flexible consumers and storage in the redispatch mechanism (Davi-Arderius & Schittekatte, 2023). Theoretically, flexible consumers and storage could similarly provide redispatch services, e.g., by reducing electricity consumption downstream of the congested network element. An increased participation of flexible consumers in the redispatch mechanism would increase competition and thereby most likely reduce costs. The exclusion of flexible demand and storage from providing redispatch services was no major limitation in times of inflexible load and mostly flexible thermal generation. In future, however, the capacity of flexible thermal generators will decline and demand will become much more flexible (IEA, 2023). Not including these assets in the redispatch mechanism thus forgoes a huge potential. However, the cost-based redispatch approach renders it nearly impossible for these market participants to provide redispatch services. This is because adequate compensation of costs and foregone profits is hardly possible for flexible consumers and storage technologies given that their willingness to pay for electricity is private knowledge of the demand entity and might vary strongly over time. Thus, an adequate compensation based on estimations by the system operator is impossible.

Hence, a key question for a future-proof electricity market design is how to harness the huge potential of flexible demand and storage for redispatch services. One option is market-based redispatch (Jin et al., 2020; Radecke et al., 2019), which is also promoted by European regulation as the default mechanism (EU, 2019). One design option is that system operators procure redispatch resources from market participants through auctions between spot market closure and the actual delivery of energy. This is compensated through an additional energy payment (energy-based redispatch). The benefit of this approach is that market participants indicate their costs for deviating from spot market schedules. This would open the door for all types of market participants. A fierce academic debate has revolved around this design option, which is also termed (local) markets for flexibility. Proponents highlight the potential to open redispatch for flexible consumers and storage. Critics argue that such a market-based approach incentivizes so-called “inc-dec gaming” (Dijk & Willems, 2011; Hirth & Schlecht, 2019; Holmberg & Lazarczyk, 2015). Anticipating that the system operator will operate a market for redispatch, market participants on both sides of the constraint face incentives to adjust their bids in spot markets to benefit from the subsequent redispatch market. Thereby, they aggravate congestion (Ehrhart et al., 2022; Grimm et al., 2022). Empirical studies on inc-dec gaming and market power in redispatch are by Graf et al. (2023) and Palovic et al. (2022).

An alternative design option for market-based redispatch is a compensation for redispatch services based on long-term contracts. In this concept, market participants receive no direct compensation for changing generation or consumption schedules, i.e., no payment for the provision of upward or downward redispatch energy. Instead, the compensation is independent from the actual activation, e.g., through a monthly or weekly payment based on the available capacity. This availability payment can be interpreted as a compensation for providing capacity for redispatch services, which is why we term this approach “capacity-based redispatch”. The intuition behind this idea is that such a capacity-based compensation allows flexible consumers to provide redispatch services, but does not provide incentives for undesirable and distorting strategic behavior such as inc-dec gaming (ENTSO-E, 2021; Neon & Consentec, 2019).

A capacity-based redispatch mechanism has, to the best of our knowledge, not yet been studied in literature. This is the gap that this paper fills. We analyze such a mechanism using a game-theoretical model and study whether it can tap the potential of flexible demand for redispatch services while preventing undesirable behavior by market participants.

Our analysis reveals significant shortcomings of the studied design of a capacity-based redispatch. Most problematic is that also an availability payment incentivizes some market participants to adjust their behavior at the spot market in a way that aggravates network congestion. The surprising result of our analysis is that the additional redispatch potential from this mechanism is countervailed by the additional redispatch need if participating units act rationally. In other words, the studied mechanism just mitigates the network constraints it creates. We study several alternative implementations and mitigation options, but none resolves the underlying problem.

The essence of the problem is that the capacity-based mechanism under consideration pays participants to do something they would not do without a payment, while incentivizing and treating desirable and undesirable participants alike. In the case of downward redispatch, for example, the desired consumers are those who are willing to increase their electricity consumption in return for an availability payment, which they would not do without a payment. On the other hand, there are the undesired consumers who are willing to reduce their consumption for a possible activation, which they would not do without availability payment. Our analysis illustrates this essential problem using a simple model. However, the problem applies in principle to all mechanisms where the two types of participants cannot be identified and distinguished. It follows that a mechanism can only be promising if it is able to solve this task, i.e., to differentiate between the potential participants along the lines of "the good ones go into the pot, the bad ones go into your crop."

The paper is organized as follows. First, we introduce the concept of capacity-based redispatch. Section 3 presents the model with continuous bids and discusses its shortcomings. Alternative implementations and measures are analyzed in Section 4. Section 5 concludes.

2 Concept of the capacity-based redispatch

In this section we present the basic idea and assumptions of the redispatch mechanism with a capacity-based compensation. We briefly present and discuss the four most relevant design choices, which include

- the compensation for the redispatch service,
- the design of the auction used to contract redispatch service providers,
- constraints on the availability of redispatch service providers, and
- the activation criteria.

Compensation. Market parties need a compensation for providing a redispatch service; changing their consumption or generation schedule, which resulted from spot market clearing, comes at a cost. Unlike energy-based redispatch, the capacity-based redispatch does not compensate each activation. Instead, contracted market participants receive a payment conditional on availability.

Auction design. We propose to use an auction to identify the cheapest possible redispatch service providers and to determine availability payments. The cost for being activated depends on the spot market price. For example, consumers that are requested to increase consumption must buy electricity at a spot market price.[1] Hence, each consumer's cost of activation increases with rising spot market prices. We therefore propose that each market

[1] Only price-sensitive consumers will participate in the capacity-based redispatch.

participant bids a function that describes the requested availability payment conditional on the spot market price. Consequently, the pool of redispatch providers that can be activated and that receive an availability payment depends on the spot market price. At high spot market prices, other units will be contracted than at low prices. This approach minimizes the inefficiency of activating entities whose willingness to pay diverges strongly from the market price. For each spot market price, the auctioneer sets a level of capacity it aims to contract and awards the respective number of bids. This auction volume should reflect the expected redispatch requirement (both measured in capacity units). We apply an auction that determines availability payments using the marginal pricing rule, i.e., all bidders receive the same payment at a given spot market price. In Section 4.1, we show that pay-as-bid pricing leads to the same expected outcome.

Constraints on availability. Requirements on the availability of awarded market participants involve a trade-off. From the system operator's perspective, market participants awarded in the auction for redispatch services would ideally always be available. However, this has two problematic consequences. First, it imposes high restrictions on market participants. Flexible industrial consumers, for example, may sometimes be able to shift production but not always. Imposing strict restrictions on availability strongly limits the pool of bidders. Second, forcing market parties to remain available for redispatch is inefficient. It would require them to remain in the congestion aggravating operation mode most of the time. For example, consumers in the surplus region that are awarded to be available for an increase in consumption must remain non-consuming. This means they aggravate network constraints when not activated. Therefore, we suggest that awarded market participants decide themselves when they are available for activation and when not. As a consequence, there is no guarantee that all awarded bids will be available for activation, suggesting that a capacity-based redispatch can only extend and not replace other redispatch mechanisms. To prevent free-riding of non-available units, only available market parties receive the capacity-based compensation. In Section 4.2, we discuss and anlayze two alternative forms of the availability rule.

Activation criteria. When activating bids, the system operator randomly selects among all awarded and available units at the given spot market price. There is no direct compensation for activation. However, it makes sense to limit the number of activation per participant to increase predictability of costs. Otherwise, participating consumers would need to include large security margins in their bids.

Figure 1 visualizes the analyzed process in a chronological order. Before real-time, system operators decide on the volume of redispatch services they would like to procure in a certain region. They conduct an auction at which market participants offer their capacity-based redispatch bids. The outcome of the auction determines the availability payment. At the day-ahead market, contracted market participants decide whether they are available for redispatch services, i.e., they do not consume electricity and receive the agreed compensation, or not. In real-time, system operators activate (some of) the available, contracted market participants.

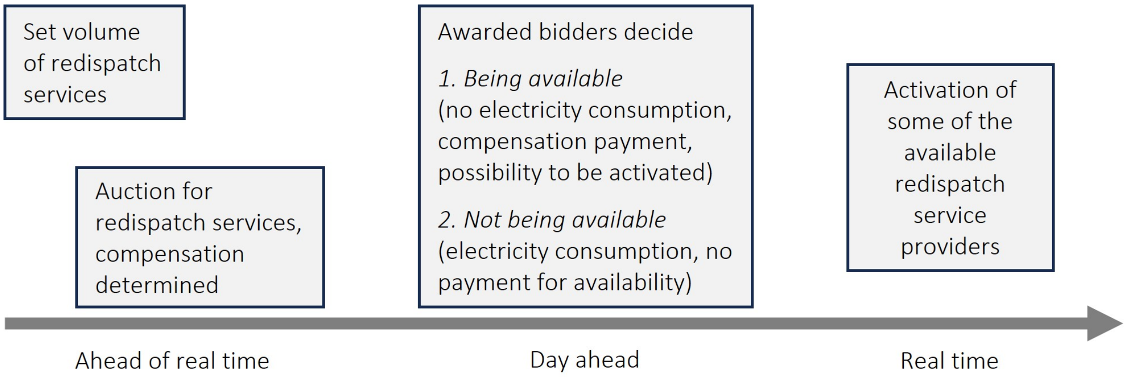

Figure 1: Sequence of the studied redispatch model

3 Model with continuous bids

In this section, we translate the concept presented in the previous section into a game-theoretical model. This allows us to study the properties of such a market design and the resulting incentives for market participants. With reference to the introduction and the Section 2, we first analyze consumers in downward redispatch, followed by the other three markets including generators and upward redispatch.

3.1 Electricity consumers in downward redispatch

There are N consumers participating in the capacity auction for downward redispatch. The consumers' willingness to pay for electricity are modeled as i.i.d. random variables V_i, $i \in \{1, 2, ..., N\}$, with the cumulative distribution function F with full support on the normalized interval $[0, 1]$. The consumers can thus increase consumption according to price incentives. The probability density function is denoted by f. Distributions are common knowledge. Each consumer i has private information of the own willingness to pay v_i as the realization of V_i. The spot market price S, like V_i, has full support on $[0, 1]$. Its cumulative distribution function is denoted by G and the probability density function is denoted by g. The realization s of S denotes the spot market price for a given redispatch event. For simplicity, we assume that each consumer provides exactly one unit of capacity.

In the capacity auction, each consumer i submits a bidding function $\ell_i(\cdot)$, which we refer to as continuous bids, for availability payments depending on the spot market price. Thus, for each spot market price s at the time of future redispatch events, the bidding function specifies the consumer's capacity bid $\ell_i(s)$. For each spot market price s, the auctioned capacity n_s is determined by the auctioneer and depends on the expected redispatch requirement as well as the activation probability $p(s)$. For each s, the n_s lowest capacity bids are awarded. In case of a uniform-price auction, the lowest rejected bid determines the auction price, i.e., the availability payment ℓ_A for all awarded consumers.

Since a consumer can optimize the bids in the bidding function $\ell_i(\cdot)$ to any spot market price s, and v_i and p are assumed not to change from the capacity auction to the redispatch events, an awarded consumer will always prefer to be available.

For a given downward redispatch event at the spot market price s, the n_s awarded consumers decide for themselves whether they are available for activation or not. Only the available consumers among the awarded consumers receive the availability payment ℓ_A as compensation. Among the available consumers, the system operator randomly selects consumers for activation. The activation probability is denoted $p(s)$. Consumers might estimate $p(s)$, e.g., based on historical data. If activated, consumer i must buy and consume electricity at the market price s, which has a value of v_i for the consumer.

Consider a particular awarded consumer in a redispatch event at the spot market price s. Consumers are symmetric, so for the ease of presentation, we omit the consumer index i. The revenue π of a consumer from an event in which the consumer is not available and does not consume electricity is normalized to zero, i.e., $\pi = 0$. If the awarded consumer is available, the consumer receives the availability payment ℓ_A and will be activated with the probability $p(s)$. Thus, the expected revenue is $\pi = p(s)(v - s) + \ell_A$.

If the consumer did not participate or did not win in the redispatch auction, the consumer would consume and buy energy at the spot market only if the willingness to pay exceeded the spot market price. Therefore, the revenue of such a consumer in the spot market, denoted by π_0, would be $v - s$ if $v > s$ and 0 if $v \leq s$.

In the capacity auction, the consumer submits the bidding function $\ell(\cdot)$, which maps a capacity bid to each spot market price s. For its derivation, we consider the indifference price function $\ell^*(\cdot)$. The indifference price $\ell^*(s)$ is the availability payment at which the consumer is indifferent between being awarded in the redispatch auction and not being awarded in the redispatch auction at price s. In the case of not being awarded, the consumer can only participate in the spot market. The indifference price $\ell^*(s)$ is derived by equating the revenues from both scenarios, $\pi = \pi_0$. For a consumer with $v \leq s$, we get $p(s)(v-s)+\ell^*(s) = 0 \iff \ell^*(s) = p(s)(s-v)$; for a consumer with $v > s$, we get $p(s)(v-s)+\ell^*(s) = v-s \iff \ell^*(s) = (1-p(s))(v-s)$. Under uniform pricing, it is optimal for the consumer to bid the indifference price at each s (e.g., Ausubel et al., 2014). Thus, the bidding function of a consumer with v is given by the bidder's indifference price function

$$\ell^*(s) = \begin{cases} (1-p(s))(v-s) \text{ if } s < v, \\ p(s)(s-v) \text{ if } s \geq v. \end{cases} \tag{1}$$

As shown in Figure 2, the indifference price function for a given v and constant $p(s)$ is V-shaped in s, with the slope $-(1-p(s))$ for $s < v$ and the slope $p(s)$ for $s \geq v$. Even if $p'(s) < 0$, the shape with a minimum of zero at $s = v$ persists because the activation probability is always positive.

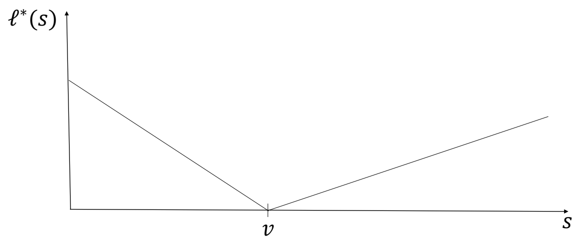

Figure 2: V-shaped indifference price function $\ell^*(s)$ of a consumer with v and $p(s) \equiv 1/3$

Since in the uniform-price auction consumers bid their indifference price functions (1), for a given s, the bids of all consumer types v form a V around $v = s$ if we assume a uniform distribution F (see Figure 3). A consumer with $v = s$, which is the vertex, bids zero. Consequently, the n_s consumers with v surrounding s on the left and right will win in the auction at the uniform auction price ℓ_A.

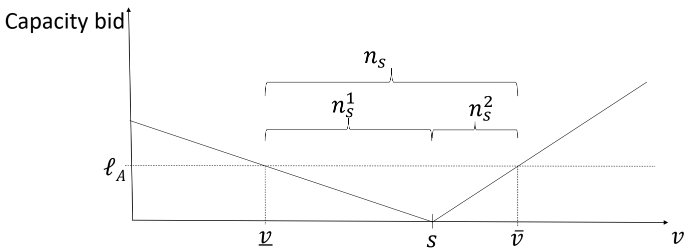

Figure 3: V-shaped curve of the bids at s of all consumer types $v \in [0, 1]$ (for $p = 1/3$ and $f(v) = 1$). n_s, n_s^1, and n_s^2 are the expected numbers of bids that fall into the respective interval.

The formalization of the indifference price function reveals a first relevant insight: the auction for redispatch services provides undesirable incentives for consumers with $v > s$ on the spot market. Consumers with $v \leq s$ (the valuations to the left of s in Figure 3) are desired in the redispatch mechanism. At a spot market price of s, they should not consume electricity and be available for upward redispatch. By contrast, consumers with $v > s$ are undesired in the redispatch mechanism (the valuations to the right of s in

Figure 3). Because their willingness to pay exceeds the spot market price, these consumers should already be consuming electricity and not participate in redispatch. Their availability for redispatch reduces the demand for electricity in the spot market, which increases the redispatch requirement. The total capacity of awarded consumers $n_s = n_s^1 + n_s^2$ is thus composed of the capacity n_s^1 of desired consumers with $v \leq s$ and the capacity n_s^2 of undesired consumers with $v > s$.

Next, we explore the properties of awarded bids above and below s and the properties of n_s^1 and n_s^2. Denote the expected redispatch requirement at price s by γ_s. As we consider a given spot market price s, denote the activation probability by $p = p(s)$. The n_s awarded bids are used for redispatch, and each bid is used with expected probability p. Thus, let $n_s p = \gamma_s$, where γ_s is a constant and n_s and p are inversely proportional. Consider a uniform distribution of consumers' willingness to pay, i.e., $f(v) = 1$ for all $v \in [0, 1]$. Then, in a uniform-price auction, for $v < s$, the bids as a function of v are given by $-p(v - s)$ and for $v > s$ by $(1 - p)(v - s)$, with the bidder with valuation s bidding zero (cp. the bidding function in (1)). Figure 3 displays this function for $p = 1/3$.

The n_s lowest of the submitted bids win and in a uniform-price auction the (n_s+1)-lowest bid determines the price ℓ_A. The valuations that underlie the awarded bids and the price-determining bid will therefore lie in an interval around s. With the uniform distribution, the expected boundaries $\underline{v}$ and $\bar{v}$ of this interval fulfill $-p(\underline{v} - s) = (1 - p)(\bar{v} - s)$ (with the auction volume n_s determining the level, i.e., the expected auction price).[2] Thus, the subintervals below and above s have the relation $(s - \underline{v})/(\bar{v} - s) = (1 - p)/p$. With a uniform distribution, the expected numbers of awarded bids below and above s have the same relation, $n_s^1/n_s^2 = (1 - p)/p$.

Using $n_s^1/n_s^2 = (1 - p)/p$, $n_s^1 + n_s^2 = n_s$, and $n_s p = \gamma_s$, the expected capacities n_s^1 and n_s^2 are

$$\begin{aligned} n_s^1 &= n_s(1 - p) = n_s - \gamma_s, \\ n_s^2 &= n_s p = \gamma_s. \end{aligned}$$

The number of desired consumers increases with n_s, while the number of undesired consumers is constant and is equal to the total redispatch requirement. The total redispatch requirement γ_s, which is to be resolved by the mechanism, can be divided into the net redispatch provision γ_s^0 and the additional redispatch requirement to the system caused by the undesired awarded consumers n_s^2, $\gamma_s = \gamma_s^0 + n_s^2$. Notably, $n_s^2 = n_s p = \gamma_s$. This indicates that the participation of the undesired consumers in the analyzed redispatch mechanism

[2] We restrict ourselves in what follows to inner solutions, that is, we exclude the cases that $\underline{v}$ is zero or $\bar{v}$ is one. From Figure 3 it is easy to see that if $\underline{v}$ or $\bar{v}$ hit the valuation boundaries, more consumers above or below s, respectively, will be awarded in order to have n_s awarded consumers.

generates just as much additional required redispatch capacity for the system as that which is resolved by the redispatch mechanism. As a consequence, the mechanism's net contribution to the redispatch requirement is zero, i.e., $\gamma_s^0 = 0$. This is due to the bidding incentives of the undesired consumers and the fact that the auctioneer lacks the means to distinguish undesired from desired consumers due to their private information about their valuations.

Note that this result is independent of p. The lower p, the higher n_s^1/n_s^2. Since n_s^2 remains constant, an increased ratio of desired consumers to undesired consumers implies more desired consumers n_s^1 in the larger group of n_s awarded consumers to fulfill the redispatch requirement. However, this only means that the redispatch requirement is fulfilled with desired consumers with a higher relative probability, but the capacity of undesired consumers remains constant and equal to the redispatch requirement.

The expected auction price can, by $\ell_A = -p(\underline{v} - s) = (1-p)(\bar{v} - s)$, for all $p \in (0,1)$ be written as $\ell_A = p(1-p)(\bar{v} - \underline{v})$. The expected width of the interval $\bar{v} - \underline{v}$ (the distance between the price-determining bidders' expected values in the cases that a bidder with $v < s$ or a bidder with $v > s$ determines the price) equals the expected distance between the lowest and highest of $n_s + 2$ neighboring draws of N draws from a uniform distribution, which is $(n_s+1)/(N+1)$. Therefore, the auction price $\ell_A = p(1-p)(\bar{v} - \underline{v}) = (1-p)(\gamma_s + p)/(N+1)$ increases as p decreases (for any redispatch requirement $\gamma_s > 1 - 2p$), resulting in higher redispatch payments to both the desired consumers and the undesired consumers, thereby contributing to an increase in the total redispatch costs. Conversely, a higher value of p leads to a higher ratio of undesired consumers to desired consumers, resulting in reduced total redispatch costs (see Appendix A.1). In the extreme scenario where $p = 1$, undesired consumers bid zero, only undesired consumers win, and the auction price is zero. Thus, in this case, the redispatch mechanism solves the self-generated redispatch requirement γ_0 at zero cost but, as for any p, does not contribute to resolving the redispatch requirement γ_s.

Replacing the uniform distribution (or any distribution with symmetric density around s) by a distribution that has a higher density of bids to the left of s than to the right of s, gives $n_s^1/n_s^2 > (1-p)/p$, $n_s^1 > n_s - \gamma_s$, and $n_s^2 < \gamma_s$ (as the indifference function in (1) is unaffected by F). Thus, the number of undesired consumers would be lower than γ_s and the problem mitigated. On the other hand, if the distribution has a lower density of bids to the left of s than to the right of s, we get $n_s^2 > \gamma_s$ and thus an even more extreme negative outcome with more than γ_s undesired consumers.[3]

The model has revealed that the capacity-based mechanism will attract desired and undesired consumers. For distributions with symmetric density around s, we find that the mechanism provides no net redispatch capacity while causing redispatch costs.

[3] An example of a distribution where the density to the left of any s is higher (lower) than to the right is the Beta distribution with parameters $\alpha = 1$ and $\beta > 2$ ($\alpha > 2$ and $\beta = 1$); another example is the standard power distribution with shape parameter smaller (larger) than one.

Our model uses a specific reference price (the spot market price) and, in part, a uniform distribution of values, in order to be able to be more comprehensive and to track the trade-offs. However, the fundamental issue is more general for capacity-based redispatch mechanisms. Consider consumers that signed a redispatch capacity contract and face a price of energy, which could be the spot market price or an individually contracted price. Among the consumers that were willing to sign the capacity contract to receive a specific payment ℓ for being available for redispatch (which could also be a penalty for not being available combined with a positive upfront payment for constant availability), there are available (undesired) consumers that wish to be activated, if their current willingness to pay is above their energy price, and available (desired) consumers that do not wish to be activated, if their current willingness to pay is below their energy price. Without knowing their willingness to pay, they cannot be distinguished. However, any consumer that is available only due to the availability payment ℓ harms the system: these consumers would consume energy, thereby relaxing the constraint, if the capacity mechanism did not hold them back. We find that it does not require extreme assumptions to make this effect so severe that it makes the capacity-based mechanism a costly mechanism that does not contribute anything to resolving constraints.

3.2 Other redispatch markets

In this section, we consider consumers in upward redispatch and generators in downward and upward redispatch.

Electricity consumers in upward redispatch The analysis and results of downward redispatch in Section 3.1 can also be applied to the same type of consumers for upward redispatch. Consumers submit a capacity bidding function for the compensation payments depending on the spot market price. For each spot market price, the auctioned capacity is determined by the auctioneer and depends on the expected upward redispatch requirement and the activation probability. To cover this, the lowest capacity bids are awarded. For a given upward redispatch event, the awarded consumers decide whether they are available for activation or not. Available means that the consumer's plant is running and consuming power and thus available for shutting down when being activated. Only the available consumers among the awarded consumers receive the payment as compensation. Among the available consumers, the system operator randomly selects consumers for activation.

Analogous to Section 3.1, the indifference price $\ell^*(s)$ is derived by equating the revenues from the case of being awarded and the case of not being awarded and thus only participating in the spot market. For a consumer with $v < s$, we get $(1 - p(s))(v - s) + \ell^*(s) = 0 \Longleftrightarrow \ell^*(s) = (1 - p(s))(s - v)$; for a consumer with $v \geq s$, we get $(1 - p(s))(v - s) + \ell^*(s) =$

$v - s \Longleftrightarrow \ell^*(s) = p(s)(v - s)$. Under uniform pricing, it is optimal for the consumer to bid the indifference price at each s. Thus, the bidding function of a consumer with v is given by the bidder's indifference price function

$$\ell^*(s) = \begin{cases} p(s)(v - s) \text{ if } s \leq v, \\ (1 - p(s))(s - v) \text{ if } s > v, \end{cases}$$

which has its minimum of zero at $s = v$ and for constant $p(s)$ is V-shaped. With a uniform distribution F, this in turn leads to a V-shaped aggregate bidding function that precisely corresponds to the one shown in Figure 3 mirrored across $v = s$. Now the undesired bidders are those with $v < s$, of which n_s^2 are available. Consequently, the results are exactly the same as for downward redispatch: $n_s^2 = \gamma_s$, i.e., for any given auction price, the upward redispatch mechanism creates as much additional redispatch requirement as it can resolve. Therefore, its net contribution to the redispatch requirement is zero, $\gamma_s^0 = 0$.

Electricity generators Consider a set of price-taking electricity generators whose production costs (willingness to accept) are modeled by independent random variables C, analogous to the consumers' willingness to pay V. Apart from this difference, all other model settings remain the same as described in Section 3.1. Thus, a generator earns a profit of $s - c$ for producing electricity at costs $C = c$ and selling it on the spot market at price s. Generators also submit a capacity bidding function for the availability payments depending on the spot market price. As before, because of the bidding function $\ell(\cdot)$, awarded generators are always available.

Also, the results for downward and upward redispatch are the same as for the consumers. In both cases, there is a V-shaped aggregate bidding function that causes the redispatch mechanism to create as much additional redispatch demand as it can resolve.

For a given downward redispatch event, the awarded generators decide whether they are available for activation or not. Available means that the generator's plant is running and selling power and thus available for shutting down when being activated. Only the available generators among the awarded generators receive the availability payment as compensation. Among the available generators, the system operator randomly selects generators for activation. Analogous to 3.1, for downward redispatch, the indifference price $\ell^*(s)$ is derived by equating the revenues from the case of being awarded and the case of not being awarded and thus only participating in the spot market. For a generator with $c \leq s$, we get $(1 - p(s))(s - c) + \ell^*(s) = s - c \Longleftrightarrow \ell^*(s) = p(s)(s - c)$; for a generator with $c > s$, we get $(1 - p(s))(s - c) + \ell^*(s) = 0 \Longleftrightarrow \ell^*(s) = (1 - p(s))(s - c)$. Under uniform pricing, it is optimal for the generator to bid the indifference price at each s. Thus, the bidding function of a generator with c is given by the bidder's indifference price function

$$\ell^*(s) = \begin{cases} (1-p(s))(c-s) \text{ if } s < c, \\ p(s)(s-c) \text{ if } s \geq c, \end{cases}$$

which has its minimum of zero at $s = c$ and for constant $p(s)$ is V-shaped. With a uniform distribution F, this in turn leads to a V-shaped aggregate bidding function that precisely corresponds to the one shown in Figure 3, where v is replaced by c. Now the undesired bidders are those with $c > s$, of which n_s^2 are available. Consequently, the results are exactly the same as for the electricity generators in downward redispatch: $n_s^2 = \gamma_s$, i.e., for any given auction price, the downward redispatch mechanism creates as much additional redispatch requirement as it can resolve. Therefore, its net contribution to the redispatch requirement is zero, $\gamma_s^0 = 0$.

Similarly, for a given upward redispatch event, available means that the generator's plant is not running and not selling power and thus available for running when being activated. The bidding function of a generator with c is

$$\ell^*(s) = \begin{cases} p(s)(c-s) \text{ if } s \leq c, \\ (1-p(s))(s-c) \text{ if } s > c, \end{cases}$$

which has its minimum of zero at $s = c$ and for constant $p(s)$ is V-shaped. With a uniform distribution F, this in turn leads to a V-shaped aggregate bidding function. Now the undesired bidders are those with $c < s$. Consequently, the results are exactly the same as for the electricity consumers in upward redispatch: $n_s^2 = \gamma_s$ and $\gamma_s^0 = 0$.

3.3 Aggregation of consumers and generators

If we aggregate consumers and generators for both downward and upward events, the results still hold, i.e., the redispatch mechanism creates as much additional redispatch demand as it can resolve. Although different proportions of consumers and generators may be activated for redispatch events, this does not change the fact that both the activated consumers and generators do not make a net contribution to the redispatch requirement. This is because the zero net contribution result applies to each of the four submarkets for any redispatch requirement γ_s and any auction price ℓ_A.

It is a possible design to use the capacity-based redispatch for flexible consumers as part of a hybrid redispatch system, in which flexible generation is remunerated based on costs. In this case, the system operator has an incentive to activate the capacity from the market-based mechanism before remunerating other energy-adjustment measures, because the payment is based on availability but not on activation.

3.4 Limitations

Our analysis based on the abstract model under idealized conditions shows the core problem of the system: some market participants have an incentive to alter their consumption or generation behavior at the spot market to be eligible for the availability premium. What aggravates this problem is the fact that these undesired bidders have a high likelihood of being awarded in the auction because even a small availability payment suffices to change the behavior of market participants. If market participants do not react to the monetary incentives provided by the mechanism, then the undesirable behavior identified in the model is expected to occur in a less extreme form. However, this should also apply to the desired consumers.[4] Therefore, the underlying problem remains with consumers that understand that reducing consumption in order to receive the availability payment is profitable.

The model assumes a time-invariant willingness to pay of consumers. In reality, the value of additional electricity consumption at a given moment depends on many factors, such as upstream or downstream processes, the availability of workers, heat and material storage. Therefore the willingness to pay for electricity is likely to strongly vary over time. This leads to further inefficiencies in the capacity-based redispatch mechanism that are not even reflected in the model. A uniform payment will be too attractive in some moments (resulting in undesired availability) and too low in others (providing no incentive to offer flexibility to the redispatch system). In addition, market participants with a time-variant willingness to pay face the risk of being dis-proportionally often activated in hours in which the spot market price strongly deviates from their willingness to pay. Consequently, these consumers are expected to ask for a premium to compensate this risk. Hence, a capacity-based redispatch is tailored to flexible consumers that have a rather stable willingness to pay over time.[5]

In our model, we only consider consumers with additional consumption and not consumers with shiftable loads. The latter group is less suited for redispatch because their activation can only shift redispatch demand within a multi-hour redispatch period. This may reduce network constraints in some hours at the cost of increasing them at others. Modeling consumers with shiftable loads is more complicated due to the inter-temporal constraints. In addition, these market participants have to include the expected optimum time (lowest market price) for consumption in their calculation. The expected difference

[4]For example, Fowlie et al. (2021) investigate empirically whether and measure how strongly electricity consumers react to monetary incentives to adjust consumption.

[5]Consumers with a time-variant willingness to pay need to take this variability into account when determining their bid for a price s, and even when awarded in the auction, they might not always prefer to be available for redispatch at s. For the system operator, this means that the available capacity might fluctuate. For consumers with a time-variant willingness to pay, this means greater uncertainty in calculating their optimal capacity bid, which makes participation in capacity-based redispatch less attractive. This applies to desired and undesired consumers.

between the price at the time of redispatch activation and the optimal price is included in their calculation as their opportunity cost. This higher level of uncertainty reduces the competitiveness of these consumers, making the capacity-based mechanism less attractive to them. This applies equally to both desired and undesired consumers.

4 Analysis of alternative implementations and measures

In this section, we discuss three alternative implementations of the capacity-based redispatch and model them when analytically tractable solutions exist. The final subsection analyzes three potential mitigation measures.

4.1 Pay-as-bid pricing

Applying the pay-as-bid rule instead of uniform pricing cannot be expected to mitigate the problem of undesired consumers. The bidding functions of the desired and undesired consumers are shifted upwards by bid mark-ups, which does not change the expected outcome. Under the conditions of the model, revenue equivalence applies (e.g., Krishna, 2010), meaning that the expected outcome is the same under uniform pricing and pay-as-bid.

4.2 Bids for spot market price intervals

In the model with continuous bids in Section 3, consumers condition their bids on the spot market price in a bidding function. We now consider the case where consumers instead condition their bids on spot market price intervals. Thus, there is one redispatch auction per interval. In the extreme case, the interval spans all possible spot market prices. A bid can only be activated for redispatch if the spot market price is in the interval for which that bid was awarded in the auction. In case of a redispatch event, the auctioneer randomly selects among the awarded and available consumers of the respective interval.

For this approach, we distinguish two forms of the availability rule: the free consumption decision and the free availability decision. Under the free consumption decision, the consumer can decide whether or not to consume energy at a given spot market price. If the consumer chooses not to consume, the consumer must be available for redispatch. Conversely, the free availability decision allows consumers to freely decide on their availability, with the option of not running the unit but also not being available for redispatch. In the model with bids conditioned on the spot market price (c.f. Section 3), the consumer has an incentive to be available at any spot market price if the availability payment is above the bidder's indifference price – which is always the case, since the consumer will not bid less. Thus, it does not matter whether the consumer has the right to decide on its availability for redispatch or not. So the choice of the availability rule does not matter in the model with continuous bids. In contrast, in the model with bids for spot market price intervals, the availability rule makes a difference.

4.2.1 Free consumption decision

The results presented below are derived from the analysis in Appendix A.2. Suppose the spot market prices are uniformly distributed on $[0, 1]$, i.e., $g(s) = 1$ for all $s \in [0, 1]$. Now, consider an interval of spot market prices $[\underline{s}, \overline{s}] \subseteq [0, 1]$. First, we study consumers with $v < \underline{s}$. If they win in the auction at an auction price above their indifference price, they will be available for redispatch if the spot market price falls in the interval. However, they will require a higher payment than a consumer with $v = \underline{s}$, which comes with a reduced probability of being awarded in the auction. The reason for this is that their indifference price ℓ^* is higher due to the larger difference between their willingness to pay and the spot market prices $s \in [\underline{s}, \overline{s}]$.

For a consumer with $v \in [\underline{s}, \overline{s}]$ exists an indifference price ℓ^* at which the consumer is indifferent between being awarded and not being awarded. If the consumer wins, this consumer will be available for redispatch if and only if $p(v-s)+\ell_A \geq v-s \iff v-\frac{\ell_A}{1-p} \leq s$. One can show that a bidder's indifference price ℓ^* decreases in v, so that consumers with higher v will win the auction. However, the higher v, the higher the spot market prices for which the consumer is available for redispatch. That is, the auction will tend to select consumers that are not available when needed. Moreover, available consumers may be of the undesired type, which generates additional redispatch requirement (see Section 3).

A consumer with $v > \overline{s}$ will be available if and only if $v - \frac{\ell_A}{1-p} \leq s$. Thus, for a given auction price ℓ_A, this consumer might rarely or never be available. However, the consumer will submit a low bid in the auction because the consumer always has the option to not be available and consume energy at a positive surplus. In a uniform-price auction, the consumer might even bid zero and then depending on the realized auction price ℓ_A decide about availability. Thus, these consumers exaggerate the selection of awarded consumers that then are not available for redispatch.[6] On the one hand, this makes redispatch hard to plan because the auctioneer does not know which share of the awarded consumers will actually be available for redispatch. On the other hand, whenever these consumers are available for redispatch, they are of the undesired type.

In summary, the introduction of bids for spot market intervals does not prevent the existence of undesired awarded consumers that generate additional redispatch requirements. Moreover, it introduces other undesirable properties compared to the model of continuous bids. The auction systematically tends to select consumers that are not available when needed (i.e., with v in or above the interval). While it is preferable if undesired consumers

[6]In a pay-as-bid auction, the bid will be low, not zero, but the adverse selection effect exists, too.

are not available and therefore do not receive the availability payment, their presence requires a larger auction volume, makes volume planning more demanding, and increases total payments. In addition, the amount of awarded consumers that are available for upward redispatch is not constant throughout the interval: more awarded consumers will be available at spot market prices at the upper boundary of the interval than at prices at the lower boundary. This is because each awarded consumer with $v > \underline{s}$ has a cutoff spot market price in $[\underline{s}, \bar{s}]$ above which the consumer is available, but not below; and awarded consumers with $v < \underline{s}$ are available at every $s \in [\underline{s}, \bar{s}]$. Finally, in uniform-price auctions, there are incentives to submit zero bids, which may lower the auction price but also result in lower availability for redispatch because the awarded zero-bid consumers are those with $v > \bar{s}$ who will never be available if the auction price ℓ_A is small.

4.2.2 Free availability decision

If a consumer is free to decide about its availability, then a consumer with $v > s$ will be available if and only if $p(v - s) + \ell_A \geq v - s \iff v - \frac{\ell_A}{1-p} \leq s$, and a consumer with $v \leq s$ will be available if and only if $p(v - s) + \ell_A \geq 0 \iff v + \frac{\ell_A}{p} \geq s$. A consumer with a given valuation will thus be available only if the spot market price is sufficiently close to the willingness to pay, $s - \frac{\ell_A}{p} \leq v \leq s + \frac{\ell_A}{1-p}$.

In the auction, the consumer merely bids for the option to receive the availability payment but has no binding duties and is not restricted in future consumption decisions. Therefore, in a uniform-price auction, the consumer can bid zero, receive this option, and depending on the auction price ℓ_A decide on future availability. However, if all consumers do this, the auction price will be $\ell_A = 0$ and no consumer will be available for redispatch.[7] Similarly, there is an equilibrium in a pay-as-bid auction in which all consumers bid zero and are not available. The significant unavailability of awarded bids thus adds to the issues identified in Section 4.2.1. Hence, in this setup, the auctioneer needs to enforce some form of availability of awarded bids to prevent such equilibria.

4.3 Potential mitigation measures

We analyze three potential mitigation measures to prevent the core problem of undesired bids (see Section 3). Note that as long as the undesired consumers still have an incentive to participate or the auctioneer cannot distinguish them from the desired ones, the redispatch mechanism is inefficient because it creates as much congestion as it resolves. Nevertheless, mitigation measures might reduce this inefficiency. In the following paragraphs, we

[7] The interval $s - \frac{\ell_A}{p} \leq v \leq s + \frac{\ell_A}{1-p}$ shrinks to $s = v$, in which case the consumer is indifferent about being available or not. This case occurs with probability zero. In all other cases, the consumer will not be available.

briefly discuss three ideas of mitigation measures and their potential effects. Due to the interdependence of the bids, however, this cannot be solved analytically, so we refrain from making concrete quantitative statements. A detailed analysis may be a promising objective for future studies.

4.3.1 Utilizing information from the submitted bidding function

In the model in Section 3, we identified a V-shaped bidding function, with bids monotonically increasing in the range of spot market prices for which the consumer's type v is a desired type, $v < s$, and bids monotonically decreasing in the range of spot market prices for which the bidder's type v is an undesired type, $v > s$ (see Equation (1)). As a first mitigation measure, the auctioneer might therefore use the information revealed by the submitted bidding function. However, this would strongly affect bidding behavior. If the consumers knew or anticipated the attempt of the auctioneer, they would adjust their calculations and bids to counteract the auctioneer's attempt to extract the full information rent.

4.3.2 Utilizing the different signs of the slope in the bidding functions

A second mitigation measure might be to distinguish the desired and undesired consumers by leveraging the different signs of the slope in the bidding functions. This approach states that if a consumer is available at a certain price s_0 with an availability payment ℓ_0, then the consumer must also be available (once or with a low activation probability) at a lower price $\tilde{s}$, where $\tilde{s} < s_0$, receiving the same availability payment ℓ_0. For the undesired consumers ($v > s_0$), the indifference price at $\tilde{s}$ is higher than at s_0, so they may suffer a loss at $\tilde{s}$ with ℓ_0. Meanwhile, for desired consumers with $v \leq \tilde{s} \leq s_0$, the indifference price at $\tilde{s}$ is lower than at s_0, potentially leading to additional profits. However, for desired consumers with $\tilde{s} < v \leq s_0$, the comparison of the indifference prices depends on specific parameters. Therefore, this measure is expected to reduce the incentives for strategic bidding among undesired consumers, while providing additional profit or loss for desired consumers, contingent on their willingness to pay within the price range. When calculating indifference prices, the consumers will consider potential additional profit or loss at a lower price, which creates interdependence between bids for different spot market prices. Consequently, numerically quantifying the explicit impact of this approach is challenging.

4.3.3 Requiring a monotonically increasing bidding function

Alternatively, and as third mitigation measure, the auctioneer may impose requirements on the bidding function. Anticipating the V-shape of the bidding function, the auctioneer might require that all consumers submit monotonically increasing bidding functions, aiming at getting rid of the undesired part of the bidding function. However, this would not solve the identified issues. Simply making the bidding function start at the vertex of the V is not optimal and the consumers face non-trivial trade-offs between enforcing the monotonic shape and the otherwise optimal bid. Finding an explicit bidding function even for the uniform-price auction is then hard because bids for different spot market prices are no longer independent. Moreover, such a monotonically increasing bidding function would start at spot market prices below the willingness to pay, thereby enforcing low bids by undesired consumers. Thereby, these bids would tend to be selected, again resulting in an adverse selection issue.

5 Summary and discussion

This paper discusses a capacity-based redispatch mechanism in which previously awarded market participants are compensated based on their availability for redispatch, but not for activation. The idea behind this approach is to develop a market design that prevents inc-dec gaming incentives, which occur when the compensation payment is based on redispatch activation. To study the behavior of rational market participants under a capacity-based redispatch mechanism, we propose a practical implementation, which we analyze using a game-theoretical model.

Our analysis reveals that the studied capacity-based redispatch is prone to undesirable behavior by market participants. The main reason is that some market participants are incentivized to alter their behavior at the spot market to be eligible for the availability payment. Such behavior does not require any foresight on network constraints, is risk-free and increases the need for redispatch. As the system operator has no information on the market participants' individual willingness to pay or buy electricity, it is unable to pick only desirable bids in the selection process. The main result of our analysis is that the additional redispatch potential the approach activates equals the additional redispatch need it creates. Thus, the studied redispatch mechanism does not contribute to resolve network constraints while causing additional costs for the availability payments.

This finding can be considered robust within the framework of our model. First, the result is independent of the probability of being activated when awarded in the redispatch mechanism. Moreover, although changes in the assumed uniform distribution of willingness to pay and its constancy over time as well as the constancy of the activation probability may affect this result, its main effect remains intact.

We discuss and model three alternative implementation options to study whether they resolve the problem of undesirable behavior by market participants. We show that pay-as-bid pricing leads to the same awarded bids in the auction. The possibility to submit bids for intervals of spot market prices instead of continuous bids also does not resolve the issue. It even features two additional undesirable properties: it systematically selects those bids of consumers which are most likely not available when needed, and it results in an uneven distribution of available consumers within the interval. Similarly, changing restrictions on the availability of awarded bids has little effect. Consequently, none of these alternative implementations solves the underlying problem. We also analyze three mitigation measures and their potential effects, but none of them demonstrates a clear advantage in mitigating the problem.

There is another point that supports our findings: Depending on the spot market price at the time of a redispatch event, a participant may be desired or undesired at a different spot market price. Hence, participants cannot be classified *a priori* as desired or undesired. Thus, under the assumption of flexible and profit-maximizing decision makers, market participants can be expected to be sometimes desired and sometimes undesired, which supports our assumption of symmetry in the calculations of the two types of participants.

The core of the identified problem is that the capacity-based mechanism pays market participants to do something that they would not do without payment, and targets and treats desired and undesired participants equally. Only if desired and undesired market participants can be treated differently, e.g., by setting different incentives, is it possible to design a capacity-based mechanism in such a way that it achieves the desired outcome. This includes incentivizing, identifying, and activating only those participants that alleviate network congestion and do not aggravate it. This is where future research should start when searching for such mechanisms. Section 4.3 presents first approaches in this direction, which take up the idea of differentiating between desired and undesired participants through special rules and incentives. However, these approaches also show that this is not an easy task and that these mechanisms are likely to be difficult or limited to analyze theoretically. This is where experimental analysis comes in to test the robustness and strengthen the internal and external validity of promising mechanisms that are expected to have the right properties and deliver the desired results.

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A Appendix

A.1 Model with continuous bids

Let K_s^1 denote the redispatch costs of the desired consumers, K_s^2 the redispatch costs of the undesired consumers and $K_s^{12} = K_s^1 + K_s^2$ the total redispatch costs at the spot market price s. K_s^1, K_s^2 and K_s^{12} can be written as functions of $p(s)$ or n_s using $n_s^1 = n_s(1 - p(s))$, $n_s^2 = n_s p(s)$, and $l_A = p(s)(1 - p(s))(n_s + 1)/(N + 1)$.

$$K_s^1 = n_s^1 \cdot \ell_A = (1 - p(s))^2 \frac{\gamma_s(\gamma_s/p + 1)}{N + 1}$$
$$K_s^2 = p(s)(1 - p(s)) \frac{\gamma_s(\gamma_s/p + 1)}{N + 1}$$
$$K_s^{12} = (1 - p(s)) \frac{\gamma_s(\gamma_s/p + 1)}{N + 1}$$

K_s^1, K_s^2, and K_s^{12} are each monotonically decreasing in $p(s)$ (K_s^2 due to $\gamma_s > 1 - 2p$) and monotonically increasing in n_s. Regarding K_s^1, the scenario is clear: both n_s^1 and ℓ_A monotonically increase in n_s. As for K_s^2, n_s^2 remains constant, while ℓ_A increases in n_s. Consequently, the total redispatch costs increase in n_s and decrease in $p(s)$.

A.2 Bids for spot market price intervals

For a consumer with $v < \underline{s}$:

$$\pi_0 = 0$$

$$\pi = \int_{\underline{s}}^{\bar{s}} \left(p(v-s) + \ell\right) g(s) ds$$

$$\Rightarrow \ell^* = \frac{\int_{\underline{s}}^{\bar{s}} p(s-v) g(s) ds}{G(\bar{s}) - G(\underline{s})} = p\left(\frac{\bar{s}+\underline{s}}{2} - v\right)$$

For a consumer with $v \in [\underline{s}, \bar{s}]$:

$$\pi_0 = \int_{\underline{s}}^{v} (v-s) g(s) ds$$

$$\pi = \int_{\underline{s}}^{\max\{v - \frac{\ell}{1-p}, \underline{s}\}} (v-s) g(s) ds + \int_{\max\{v - \frac{\ell}{1-p}, \underline{s}\}}^{\bar{s}} \left(p(v-s) + \ell\right) g(s) ds$$

$$\Rightarrow \ell = \frac{(1-p) \int_{\max\{v - \frac{\ell}{1-p}, \underline{s}\}}^{\bar{s}} (v-s) g(s) ds - \int_{v}^{\bar{s}} (v-s) g(s) ds}{G(\bar{s}) - G(\max\{v - \frac{\ell}{1-p}, \underline{s}\})} \tag{A.1}$$

Letting $g(s) = 1$ as assumed in Section 4.2 and solving the differential equation (A.1), we have

$$\ell^* = \begin{cases} \frac{v^2 - 2((1-p)\underline{s} + p\bar{s})v + (1-p)\underline{s}^2 + p\bar{s}^2}{2(\bar{s} - \underline{s})}, & \text{if } v < v_0 \\ (\sqrt{1-p} - (1-p))(\bar{s} - v), & \text{if } v \geq v_0 \end{cases}$$

where $v_0 = \bar{s} - \bar{s}\sqrt{1-p} + \underline{s}\sqrt{1-p} \in [\underline{s}, \bar{s}]$.

For consumers with $v > \bar{s}$:

$$\pi_0 = \int_{\underline{s}}^{\bar{s}} (v-s) g(s) ds$$

$$\pi = \int_{\underline{s}}^{\min\{v - \frac{\ell}{1-p}, \bar{s}\}} (v-s) g(s) ds + \int_{\min\{v - \frac{\ell}{1-p}, \bar{s}\}}^{\bar{s}} \left(p(v-s) + \ell\right) g(s) ds$$

$$\Rightarrow \ell^* = 0$$

In sum, we have

$$\ell^* = \begin{cases} \frac{1}{2}p(\bar{s}+\underline{s}) - pv, & \text{if } v \in [0,\underline{s}) \\ \frac{v^2-2((1-p)\underline{s}+p\bar{s})v+(1-p)\underline{s}^2+p\bar{s}^2}{2(\bar{s}-\underline{s})}, & \text{if } v \in [\underline{s}, v_0) \\ (\sqrt{1-p}-(1-p))(\bar{s}-v), & \text{if } v \in [v_0, \bar{s}] \\ 0, & \text{if } v \in (\bar{s}, 1] \end{cases}$$

$$\frac{\partial \ell^*}{\partial v} = \begin{cases} -p \leq 0, & \text{if } v \in [0,\underline{s}) \\ \frac{v-(1-p)\underline{s}-p\bar{s}}{\bar{s}-\underline{s}} \leq 0, & \text{if } v \in [\underline{s}, v_0) \\ 1-p-\sqrt{1-p} \leq 0, & \text{if } v \in [v_0, \bar{s}] \\ 0, & \text{if } v \in (\bar{s}, 1] \end{cases}$$

$$\frac{\partial \ell^*}{\partial p} = \begin{cases} (\bar{s}-v)(1-\frac{1}{2\sqrt{1-p}}), & \text{if } p \in [0, 1-(\frac{\bar{s}-v}{\bar{s}-\underline{s}})^2) \\ \frac{1}{2}(\bar{s}+\underline{s}) - v, & \text{if } p \in [1-(\frac{\bar{s}-v}{\bar{s}-\underline{s}})^2, 1] \end{cases}$$

Besides, we have

$$\ell^*(v=\underline{s}) = \frac{1}{2}p(\bar{s}-\underline{s}),$$
$$\ell^*(v=v_0) = (1-p)(1-\sqrt{1-p})(\bar{s}-\underline{s}),$$
$$\ell^*(v=\bar{v}) = 0.$$

$$\frac{\partial \ell^*(v=\underline{s})}{\partial v} = -p,$$
$$\frac{\partial \ell^*(v=v_0)}{\partial v} = 1-p-\sqrt{1-p}.$$
$$\frac{\partial \ell^*(p=1-(\frac{\bar{s}-v}{\bar{s}-\underline{s}})^2)}{\partial p} = \frac{1}{2}(\bar{s}+\underline{s}) - v.$$

Thus, ℓ^* is continuous on $[0,1]$, partially differentiable in v on $\in [0,\bar{s}]$ and partially differentiable in p. ℓ^* is monotonically decreasing in v. ℓ^* is in most cases monotonically increasing in p with $\ell^*(p=0)=0$ and is monotonically decreasing in p if $p > \frac{3}{4}$ and $v > \frac{1}{2}(\bar{s}+\underline{s})$.

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ZEW – Leibniz-Zentrum für Europäische Wirtschaftsforschung GmbH Mannheim
ZEW – Leibniz Centre for European Economic Research

L 7,1 · 68161 Mannheim · Germany
Phone +49 621 1235-01
info@zew.de · zew.de