Journal of Economic Interaction and Coordination
https://doi.org/10.1007/s11403-024-00434-y

REGULAR ARTICLE

Dynamic tax evasion and growth with heterogeneous agents

Francesco Menoncin[1] · **Andrea Modena**[2]

Received: 31 January 2024 / Accepted: 23 October 2024

Abstract
We develop a tractable model economy in which public capital improves aggregate productivity, and the taxpayers have heterogeneous evasion opportunities. By issuing bonds, compliant taxpayers supply the evaders with an instrument to hedge against auditing risks, thereby expanding their evasion capacity. The wealth share of tax evaders relates negatively to the economy's productivity but has a hump-shaped relationship with the growth rate of aggregate capital. The fiscal policy that maximizes welfare differs from the one that maximizes tax revenues because the latter does not account for the redistribution of wealth (and risk) between compliant and evasive taxpayers.

Keywords Dynamic tax evasion · General equilibrium · Growth · Heterogeneous agents

JEL Classification E20 · G11 · H26

We thank the editor, Giorgio Ricchiuti, and two anonymous referees for their constructive comments. Our gratitude also goes to Ernst-Ludwig von Thadden for the helpful discussions. Modena acknowledges support from the German Science Foundation (DFG) through CRC TR 224 (Project C03).

✉ Andrea Modena
andrea.modena@uni-mannheim.de

Francesco Menoncin
francesco.menoncin@unibs.it

[1] Department of Economics and Management, University of Brescia, Brescia, Italy

[2] Department of Economics, University of Mannheim, Mannheim, Germany

Published online: 09 December 2024

1 Introduction

The idea that tax evasion generates capital misallocation and hinders economic growth appears in the works of Fullerton and Karayannis (1994) and Roubini and Sala-i-Martin (1995). More recently, Ordonez (2014) and López (2017) have developed models in which tax evasion redistributes resources towards firms that reduce their size and productivity to remain undetected. Close to these studies, Di Nola et al. (2021) show that tax evaders can be less productive than tax-compliant firms precisely because of their fiscal advantage. Along the same lines, Menoncin et al. (2023) show that tax evasion exacerbates capital misallocation due to leverage constraints, generating a negative relation between aggregate productivity and the shadow economy.

An important aspect not accounted for in this literature is that several taxpayers have limited or no evasion opportunities due to institutional constraints (i.e., policies, laws, and regulations). Employees, for example, cannot avoid taxes because their compensation is taxed at the source. Conversely, self-employed workers are always able to under-report their earnings.

With this in mind, this paper develops a tractable macroeconomic model featuring a unit mass of households (taxpayers) and a tax-collecting public sector. Households are risk-averse and allocate their net worth between risky (but productive) capital and risk-free bonds to maximize the inter-temporal utility of consumption. The production technology involves both private and (tax-financed) public capital. The model's key feature is that only a fraction of taxpayers can conceal a portion of their income from the tax authority. Doing so reduces their fiscal burden but exposes them to the possibility of being audited and fined. In line with the incumbent literature (see references below), households do not experience direct dis-utility from tax evasion. However, the shape of the utility function does account for the households' risk aversion. Those who are more risk-averse experience a greater reduction in utility when evading compared to others.

As a first step, we derive households' optimal consumption, capital investment, and tax evasion policy in closed form. Then, we solve the model for its competitive Markovian equilibrium and show that macroeconomic aggregates can be expressed as functions of the (endogenous) net worth share of tax-compliant households. Households' tax evasion decisions depend *directly* on the intensity of the auditing process and the magnitude of the fine and *indirectly* on the relative net worth of tax-compliant households through the equilibrium risk-free rate.

In this context, we demonstrate that, by selling bonds, tax-compliant households provide tax evaders with a hedging instrument against auditing risks, thereby fostering their incentives to evade. Then, we show that the economy is more productive when tax-compliant households hold a higher net worth share. At the same time, however, this share exhibits a hump-shaped relationship with the growth rate of aggregate capital. Moreover, we find that in the presence of heterogeneous evasion opportunities, the tax rate that maximizes public revenues is higher than in an economy with only tax evaders.

In the paper's final section, we examine the effects of changes in the tax rate, auditing intensity, and fines on households' steady-state welfare. Our simulations indicate that the welfare-maximizing tax rate differs from the one maximizing tax revenue. The discrepancy arises because the latter overlooks the redistribution of net worth (and risk) among household types. Finally, we find that the net worth share of tax-compliant households under the welfare-maximizing tax rate remains relatively stable across various combinations of evasion auditing and fine parameters.

In our model, tax evasion decisions occur in a general equilibrium dynamic environment. This way, we generalise previous works developed either in a static (e.g., Dessy and Pallage, 2003) or in a dynamic but partial equilibrium framework (Lin and Yang 2001; Dzhumashev and Gahramanov 2011; Bernasconi et al. 2015; Levaggi and Menoncin 2013, 2016). Concerning the link between evasion and growth, we relate to the work of Chen (2003), which first studies tax evasion in a representative-agent macro model with public capital. Concerning the relationship between tax evasion and household heterogeneity, we are close to the works of Maffezzoli (2011) and Di Nola et al. (2021). Unlike these works, tax evasion in our model is impossible for some agents due to institutional constraints.

The paper proceeds as follows. Section 2 presents the model and characterizes its competitive (general) equilibrium. Section 3 solves the model numerically and presents the main results. Section 4 concludes. Proofs and derivations are collected in the appendix.

2 Model

The continuous-time economy, with time $t \in [0, \infty)$, is populated by a continuum of households indexed as $i \in \mathbb{I}$, a representative firm, and the public sector. Households are born at time zero with initial net worth $n_{0,i}$ and either classify as tax compliant ($i = h$) or tax evaders ($i = e$). Their net worth is continuously (and frictionlessly) allocated between a bond $b_{t,i}$ and private capital $k_{t,i}$. The former asset yields the risk-free rate r_t, which will be determined in equilibrium. The latter can be rented to the representative firm at the competitive rate of A_t. However, its total (log) returns are uncertain and fluctuate with constant volatility σ^2. Therefore, holding capital yields

$$dk_{i,t} = k_{i,t}\left(A_t dt + \sigma dZ_t\right), \tag{1}$$

in which Z_t denotes a Brownian Motion defined on the filtered probability space $(\Omega, \mathbb{P}, \mathcal{H})$.

Without loss of generality, income from the bond is tax-free.[1] Conversely, the public sector levies a proportional tax $\tau \in (0, 1)$ on capital revenues, which is used to finance the supply of public capital G_t. As in Barro (1990), the firm takes G_t as given and uses aggregate private capital $K_t := \int_{\mathbb{I}} k_{t,i} di$ to produce output

[1] Taxing bonds would be immaterial because the risk-free interest rate would adjust in equilibrium.

$$Y_t = \alpha K_t^{\beta} G_t^{1-\beta}, \tag{2}$$

in which $\alpha > 0$ and $\beta \in [0, 1]$ are positive constants parametrizing TFP and the output elasticity to public capital. To characterize the rental rate of capital, we conjecture that

$$G_t = g_t K_t,$$

in which g_t expresses the (endogenous) supply of public capital per unit of private capital; we will verify that this condition holds in equilibrium.[2] This conjecture allows us to rewrite Eq. (2) as

$$Y_t = \alpha g_t^{1-\beta} K_t. \tag{3}$$

Equipped with this equation, the firm's zero-profits condition implies that $A_t = \alpha g_t^{1-\beta}$.

The difference between tax-compliant and tax-evading households is that the former (e.g., employees) are taxed at source and, thus, cannot conceal their income from the public agency. Accordingly, by imposing the budget constraint $n_{t,h} = b_{t,h} + k_{t,h}$, their net worth evolves with dynamics

$$dn_{t,h} = \underbrace{\left(n_{t,h} - k_{t,h}\right)}_{=b_{t,h}} r_t dt + dk_{t,h}(1-\tau) - c_{t,h} dt, \tag{4}$$

in which $c_{t,h}$ labels instantaneous consumption flows.

As in Levaggi and Menoncin (2016), households who self-report their income (e.g., self-employed) may conceal from the public agency a certain amount of capital $\tilde{k}_{t,e}$ to avoid tax payments. By doing so, they face the possibility of being audited and fined. Auditing events are modelled as independent Poisson processes with constant intensity $\lambda > 0$ and denoted as $\Pi_{t,e}$. Auditing fines $\eta(\tau) \in [0, 1]$ are a share of evaded income and take the following functional form:

$$\eta(\tau) := \eta_0 + \eta_1 \tau.$$

This choice includes the cases in which (i) fines are proportional to value of evaded risky assets ($\eta(\tau) = \eta_0$), as in Allingham and Sandmo (1972; (ii) fines are proportional to the evaded taxes ($\eta(\tau) = \eta_1 \tau$), as in Yitzhaki (1974).

In summary, tax evaders' net worth satisfies the budget constraint $n_{t,e} = b_{t,e} + k_{t,e} + \tilde{k}_{t,e}$ and evolves with dynamics

$$dn_{t,e} = \underbrace{(n_{t,e} - k_{t,e} - \tilde{k}_{t,e})}_{=b_{t,e}} r_t dt + dk_{t,e}(1-\tau) - c_{t,e} dt + d\tilde{k}_{t,e} - \eta(\tau)\tilde{k}_{t,e} d\Pi_{t,e}. \tag{5}$$

[2] In this context, G can be interpreted as a "pure" public good (i.e., non-rivalrous and non-excludable), such as broadband and mobility infrastructures, which benefits individual firms proportionally to the volume of their activity.

2.1 Households' problem

All households have log preferences and discount future utility at the constant rate ρ. Their optimization problem is

$$\max_{\{c_{t,i},k_{t,i},\tilde{k}_{t,i}\}} \mathbb{E}_0\left[\int_0^{\infty} e^{-\rho t}\ln c_{t,i}dt\right], \text{ with } i \in \{h, e\}, \tag{6}$$

subject to either Eq. (4) or Eq. (5).

Proposition 1 (Optimal policy) *When $i = h$ (without evasion), the optimal policy solving the problem in Eq. (6) is*

$$c_{t,h}/n_{t,h} = \rho, \tag{7}$$

$$k_{t,i}/n_{t,i} := \theta_{t,h} = \frac{A_t(1-\tau) - r_t}{\sigma^2(1-\tau)^2}; \tag{8}$$

When $i = e$ (with evasion), the optimal policy is

$$c_{t,e}/n_{t,e} = \rho, \tag{9}$$

$$k_{t,e}/n_{t,e} := \theta_{t,e} = \frac{A_t(1-\tau) - r_t}{\sigma^2(1-\tau)^2} - \frac{1}{\eta(\tau)(1-\tau)} + \frac{\lambda}{r_t\tau}, \tag{10}$$

$$\tilde{k}_{t,e}/n_{t,e} := \tilde{\theta}_{t,e} = \frac{1}{\eta(\tau)}\left(1 - \frac{\lambda\eta(\tau)}{r_t}\frac{1-\tau}{\tau}\right). \tag{11}$$

Proof See Appendix A.1. □

Consumption rates are constant and equal to the subjective discount rate ρ. Portfolio choices vary over time, depending on the household type. Tax-compliant households implement a mean-variance strategy. Conversely, as in Levaggi and Menoncin (2016), tax evaders trade off the risk of being audited and fined for higher expected (tax-free) returns on capital.[3]

To be economically meaningful, the optimal tax evasion rate in Eq. (11) shall lie within the interval [0, 1]. Accordingly, the tax rate and auditing parameters must satisfy the following restriction:

$$\frac{\lambda\eta(\tau)}{\tau}(1-\tau) \le r_t \le \frac{\lambda\eta(\tau)}{\tau}\frac{1-\tau}{1-\eta(\tau)}. \tag{12}$$

[3] Bond holdings do not appear as controls in Eq. (6); they are identified residually as $b_{t,h}/n_{t,h} = 1 - \theta_{t,h}$ and $b_{t,e}/n_{t,e} = 1 - \theta_{t,e} - \tilde{\theta}_{t,e}$ by using households' budget constraints.

Since the risk-free rate is determined in general equilibrium, we cannot verify Eq. (12) ex-ante. We will thus do it ex-post after solving the model numerically.

2.2 General equilibrium

In this section, we solve the model for its competitive equilibrium and characterize it through the dynamics of a suitable state variable.

Definition 1 (*Competitive equilibrium*) A competitive equilibrium is a map from histories of shocks $\{Z_t\}$ to macroeconomic aggregates such that (i) households solve the problem in Eq. (6); (ii) the bonds and capital markets clear; (iii) the supply of public capital in Eq. (2) equals total taxes plus auditing revenues.

The first clearing condition requires bonds to be in zero net supply:

$$\left(1-\theta_{t,h}\right)N_{t,h}+(1-\theta_{t,e}-\tilde{\theta}_{t,e})N_{t,e}=0, \tag{13}$$

in which $N_{t,h} := \int_{\square} n_{t,h} dh \; N_{t,e} := \int_{\square} n_{t,e} de$. The second is that total capital equals aggregate net worth:

$$K_t = N_{t,h} + N_{t,e}. \tag{14}$$

The government issues no debt and has no primary deficit. For simplicity, the government faces no costs to conduct audits. Therefore, the supply of public capital equals total taxes plus (average) auditing revenues:

$$G_t = \tau A_t\left(N_{t,h}\theta_{t,h} + N_{t,e}\theta_{t,e}\right) + \lambda\eta(\tau)\tilde{\theta}_{t,e}N_{e,t}. \tag{15}$$

Next, we guess and verify that the equilibrium quantities can be expressed as a function of a single state variable ϕ. In particular, we look for an equilibrium that is Markovian in ϕ and that is consistent with Definition 1. Within this structure, we adopt as a state variable the relative wealth share of tax-compliant households:

$$\phi := \frac{N_{t,h}}{N_{t,h}+N_{t,e}} \in [0,1]. \tag{16}$$

The conjecture will be verified because, as in most standard macroeconomic models, households' optimal strategies in Proposition 1 are linear in their net worth $n_{t,i}$. Accordingly, aggregation is trivial due to the Law of Large Numbers (Uhlig 1996), and the equilibrium is unaffected by *within* heterogeneity. As a result of this structure, the supply of public goods G_t is also linear in capital, which is consistent with the conjecture in Eq. (3).

The following proposition demonstrates that a competitive (and deterministic) equilibrium exists if there is a solution to a bi-variate system of algebraic equations for r and g. Moreover, it pins down the dynamics of the unique variable in Eq. (16). For clear notation, we henceforth drop all time subscripts t.

Proposition 2 (Equilibrium characterization) *For* $\phi \in [0, 1]$, *the following holds*:

1. *The state variable in Eq.* (16) *has the following law of motion*

$$\begin{aligned}\frac{d\ln\phi}{dt} &= r(\phi) + \theta_h(\phi)\big[\big(A - \sigma_K(\phi)\sigma\big)(1-\tau) - r(\phi)\big] \\ &\quad - \rho - \iota(\phi) + \sigma_K(\phi)^2,\end{aligned} \tag{17}$$

in which the aggregate capital volatility $\sigma_K(\phi)$ and investment rate $\iota(\phi)$ equal

$$\begin{aligned}\sigma_K(\phi) &= \sigma\phi\theta_h(\phi)(1-\tau) + \sigma(1-\phi)\big[\theta_e(\phi) + \theta_h(\phi)(1-\tau)\big], \\ \iota(\phi) &= A[(1-\tau)(\theta_h(\phi)\phi + \theta_e(\phi)(1-\phi)) + (1-\phi)\tilde{\theta}_e(\phi)] \\ &\quad - A(1-\phi)\tilde{\theta}_e(\phi)\lambda\eta(\tau) - \rho.\end{aligned}$$

2. *The risk-free rate* $r(\phi)$ *and public capital supply* $g(\phi)$ *satisfy the following system*:

$$\begin{cases} 0 = (1-\phi)\Big[\tau\alpha g^{1-\beta}\Big(\frac{\alpha_\tau g^{1-\beta}-r}{\sigma_\tau^2} - \frac{(1-\tau)^{-1}}{\eta(\tau)} + \frac{\lambda}{r\tau}\Big) + \lambda - \frac{\lambda^2\eta(\tau)}{r}\Big(\frac{1-\tau}{\tau}\Big)\Big] + \\ \quad +\tau\alpha g^{1-\beta}\Big(\frac{\alpha_\tau g^{1-\beta}-r}{\sigma_\tau^2}\Big)\phi - g \\ 0 = \Big(1 - \frac{\alpha_\tau g^{1-\beta}-r}{\sigma_\tau^2}\Big)\phi + \Big[1 - \frac{\lambda}{r\tau} - \frac{\alpha_\tau g^{1-\beta}-r}{\sigma_\tau^2} + \frac{1}{\eta(\tau)}\Big(\frac{\tau}{1-\tau}\Big)\Big](1-\phi) \end{cases} \tag{18}$$

where $\sigma_\tau := (1-\tau)\sigma$ and $\alpha_\tau := (1-\tau)\alpha$.

Proof See Appendix A.2. □

As is common in standard representative-agent macroeconomic models, equilibrium objects are independent of the number ("mass") of households within each type. In this regard, our model can be thought of as an economy featuring two representative agents with different ex-ante characteristics. As a result, the equilibrium always converges to the same steady state, regardless of how evaders' and non-evaders net worth is initially distributed.[4]

Another remark concerns the choice of modelling independent auditing processes across tax evaders. Under this assumption, the total amount of fines reducing tax evaders' net worth enters the market clearing condition as a deterministic rather than a "jump" process.

The third implication of Proposition 2 is that, even though the economy features aggregate uncertainty, the state variable has a deterministic law of motion.

Lemma 1 (Steady state) *The state variable* ϕ *has a steady state* Φ, *which satisfies*

[4] Doubling the mass of tax-compliant households relative to tax evaders, for example, would result in an equivalent equilibrium where each household holds half the individual net worth.

$$\frac{d\ln\phi}{dt} = 0 \iff r(\Phi) + \theta_h(\Phi)[(A(\Phi) - \sigma_K(\Phi)\sigma)(1-\tau) - r(\Phi)] \\ = \rho + \iota(\Phi) - \sigma_K^2(\Phi). \tag{19}$$

As we are not able to further characterize the steady-state equilibrium and its transition dynamics analytically, we now explore them numerically.

3 Numerical analysis and discussion

Following Bernasconi et al. (2020), our numerical exercise adopts the following parameters: $\alpha = 0.45$, $\beta = 0.9$, $\tau = 0.35$, $\lambda = 0.1$, $\rho = 0.02$, $\sigma = 0.3$, $\eta_0 = 0$, and $\eta_1 = 0.55$.[5] Considering these values, the parametric restrictions in Eq. (12) are satisfied if $r(0) > 0.092$ and $r(1) < 0.204$.

The blue solid lines in Fig. 1 show households' portfolios and the risk-free rate as functions of ϕ. The red stars mark the steady state Φ. What stands out is that tax-compliant households finance capital holdings by issuing bonds (Panels (a) and (b)). By doing so, they supply evaders with a hedging instrument against auditing risk. Accordingly, risk-free rates are lower than what they would be in a homogeneous-agent economy ($\Phi = 1$, Panel (c)). Additional net worth in the hands of tax-compliant households corresponds to a higher supply of hedging instruments, which allows for higher evasion rates (Panel (e)).

Figure 2 reports the macroeconomic aggregates. In line with the result of Lemma 1, the state variable drifts deterministically towards a steady-state level Φ (Panel (a)) (i.e., Eq. (17) is positive when ϕ is small, and vice versa). The supply of the public capital (per unit of private capital) g, which determines the economy's TFP A, is strictly increasing in ϕ because a lower share of capital in the hands of tax evaders is associated with a broader tax base and a lower tax evasion in the aggregate (Panels (b) and (c)). The investment rate of capital ι is a hump-shaped function of ϕ (Panel (c)). This is because when tax evaders are relatively few (ϕ is large), many resources are subtracted from private investments due to taxation. Conversely, when tax evaders are many (ϕ is small), a higher share of aggregate capital is concealed from taxes, thereby scaling down productivity (Panel (b)). The volatility of aggregate capital σ_K is overall decreasing in ϕ because, due to the presence of taxes, evaders' portfolios are more volatile than those of their tax-compliant peers.

3.1 The Laffer curve

As a next step of the analysis, we study the effect of households' heterogeneity on the economy's Laffer curve. The curve can be (implicitly) derived by substituting households' optimal policy in the clearing condition for public capital and dividing by K, obtaining

[5] Equivalently, we could set $\eta_0 = 0$ and $\eta_1 = 1.57$, implying that the evaders shall pay back all due taxes plus a 60% penalty upon auditing.

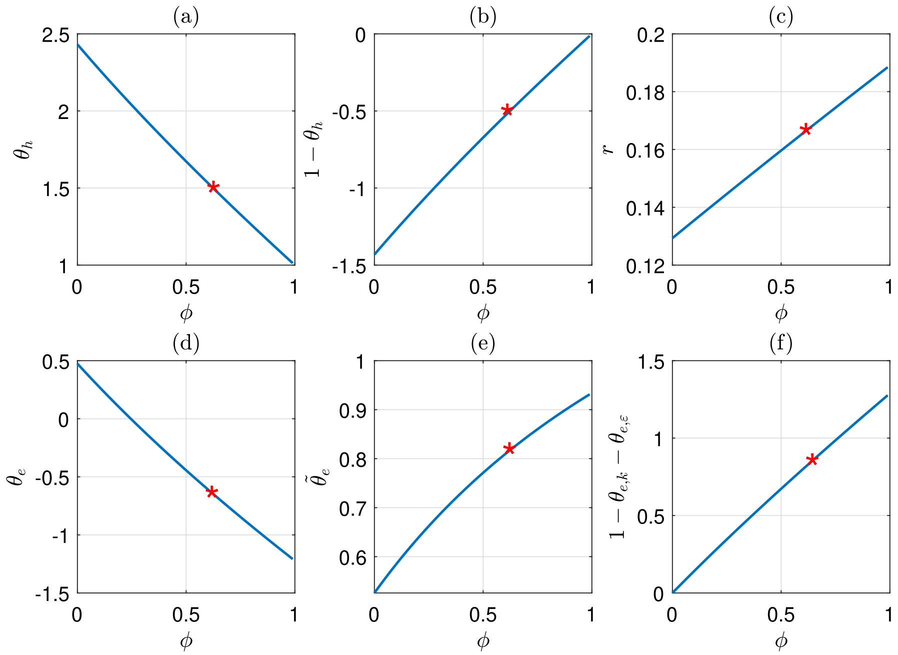

Fig. 1 Households' optimal capital allocation and the equilibrium risk-free rate as functions of ϕ (solid blue lines) and its steady state (red stars) (color figure online)

$$\alpha g(\Phi)^{(1-\beta)} = (\tau\alpha)^{-1}\frac{g(\Phi) - \lambda\eta\tilde{\theta}_e(\Phi)) + \Phi\lambda\eta\tilde{\theta}_e(\Phi)}{\theta_h(\Phi) - \theta_e(\Phi))\Phi + \theta_e(\Phi)},$$

which must be evaluated numerically.

The solid blue lines in Fig. 3 plot the Laffer curve in our baseline model (Panel (a)) and the corresponding steady-state level Φ (Panel (b)) as functions of τ. For comparison, the red dashed line depicts the same curve in a representative-agent ("benchmark") economy where all households can evade taxes. The curve slopes upward in both economies when τ is low because more taxes improve aggregate productivity by fostering the supply of public capital (Panel (a)). After reaching its maximum, the curve slopes downward because increasing levels of tax evasion end up eroding the size of the tax base. Including tax-compliant households in the economy amplifies these trends, thereby increasing the tax rate level that maximizes public revenues. When τ is small, the curve is steeper than in the benchmark economy because tax-compliant households make the tax base less sensitive to variations in the fiscal policy. When τ is larger, the curve is steeper because tax evaders take over an increasingly higher aggregate capital share (Panel (b)).

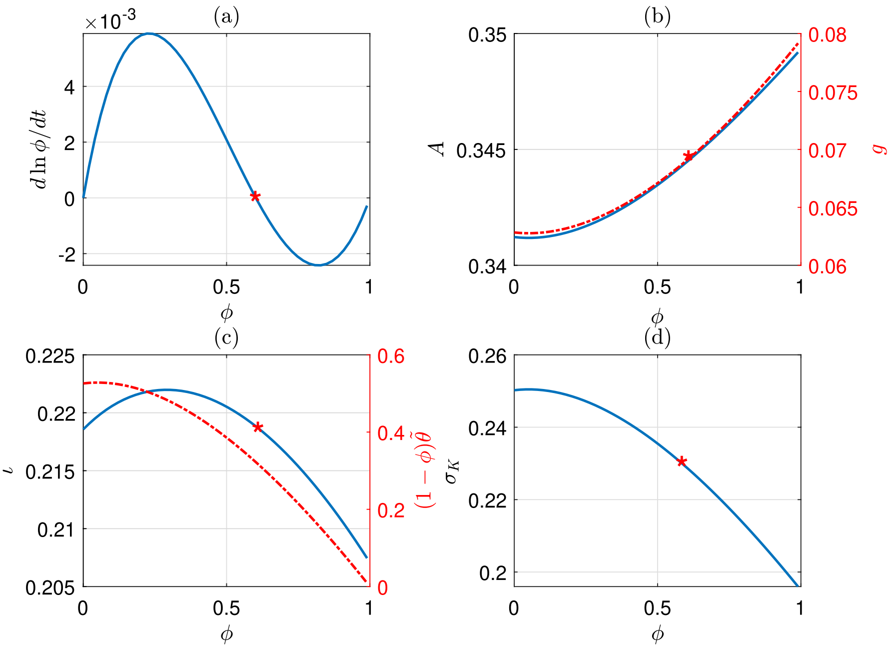

Fig. 2 Solid blue lines: State variable dynamics (Panel (**a**)), Total Factor Productivity (Panel (**b**)), capital investment rate (Panel (**c**)), and capital volatility (Panel (**d**))) as functions of ϕ and their steady states (red stars). Red dashed lines: Public-to-private capital ratio (Panel (**b**)) and aggregate tax evasion (Panel (**b**)) (color figure online)

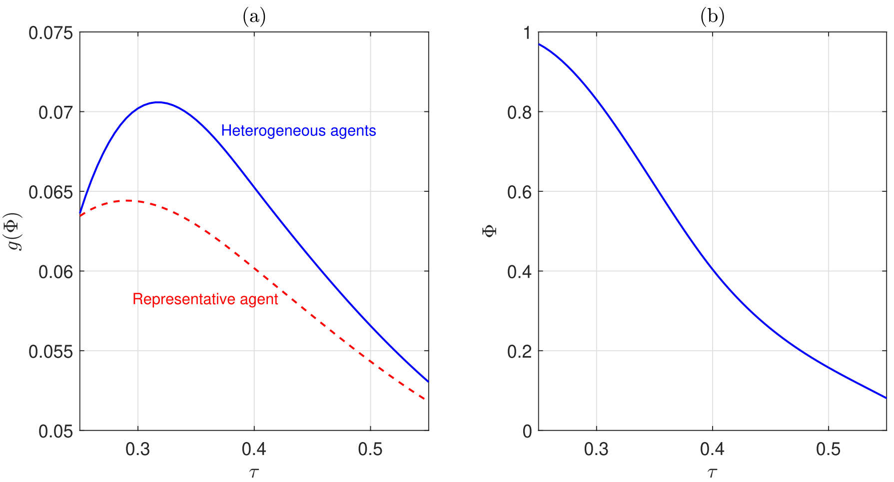

Fig. 3 Laffer curve (Panel (**a**)) and the relative wealth share of tax-compliant households in the steady state (Panel (**b**))

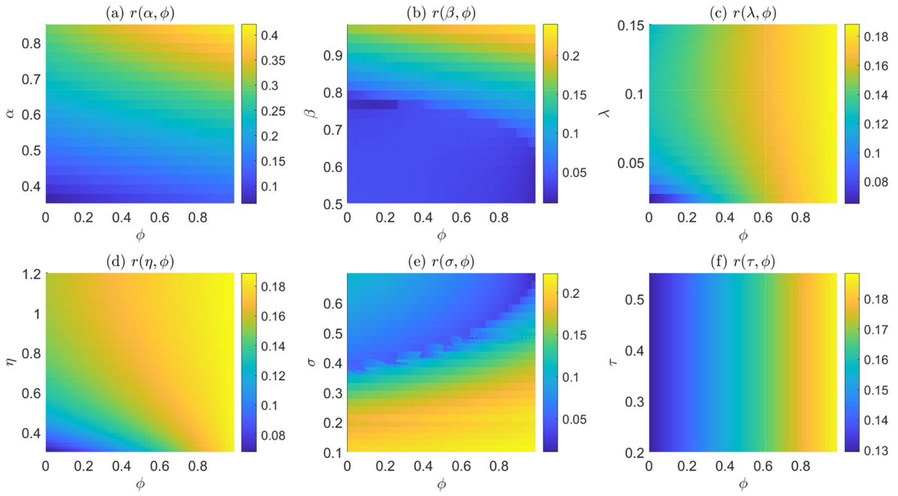

Fig. 4 Comparative statics analysis: equilibrium interest rate for all levels of the state variable different values of aggregate TFP (Panel (**a**)); output-to-public good elasticity (Panel (**b**)); auditing intensity (Panel (**c**)); auditing fine (Panel (**d**)); uncertainty (Panel (**e**)); and the tax rate (Panel (**f**))

3.2 Comparative statics

The result that heterogeneous tax evasion opportunities increase the propensity to evade taxes arises because when agents who can (cannot) evade taxes hold a larger share of aggregate net worth, the risk-free rate is lower (higher) in equilibrium. To verify this claim, we derive the optimal tax evasion in Eq. (11) with respect to the level of the state variable ϕ, obtaining

$$\frac{\partial \tilde{\theta}_e(\phi)}{\partial \phi} = \frac{\partial \tilde{\theta}_e(\phi, r(\phi))}{\partial r} \times \frac{\partial r(\phi)}{\partial \phi} = \frac{\lambda}{r(\phi)^2} \frac{1-\tau}{\tau} \frac{\partial r(\phi)}{\partial \phi}. \tag{20}$$

According to this equation, provided that $\tau \in (0,1)$ and $\lambda > 0$, tax evasion increases with ϕ as long as interest rates increase with ϕ. The strength of the relationship depends on the level of ϕ.

Unfortunately, we cannot verify analytically that $\partial r/\partial \phi > 0$ for all ϕ because that would require solving the system in Eq. (18) in closed form. To tackle this issue, the panels of Fig. 4 display the equilibrium level of $r(\phi)$ for $\phi \in [0,1]$ under different values of the following parameters: (a) aggregate TFP α; (b) output-to-public good elasticity β; (c) auditing intensity λ; (d) auditing fine η_0; (e) aggregate uncertainty; and (f) tax rate. According to these simulations, the risk-free interest rate increases with the share of the net worth of tax-compliant households, ϕ, across most specifications. The only exception occurs when, ceteris paribus, the aggregate volatility parameter σ is extremely large and, accordingly, $r(\phi)$ is extremely low (see Panel (e)); a case which violates the parametric restrictions in Eq. (12).

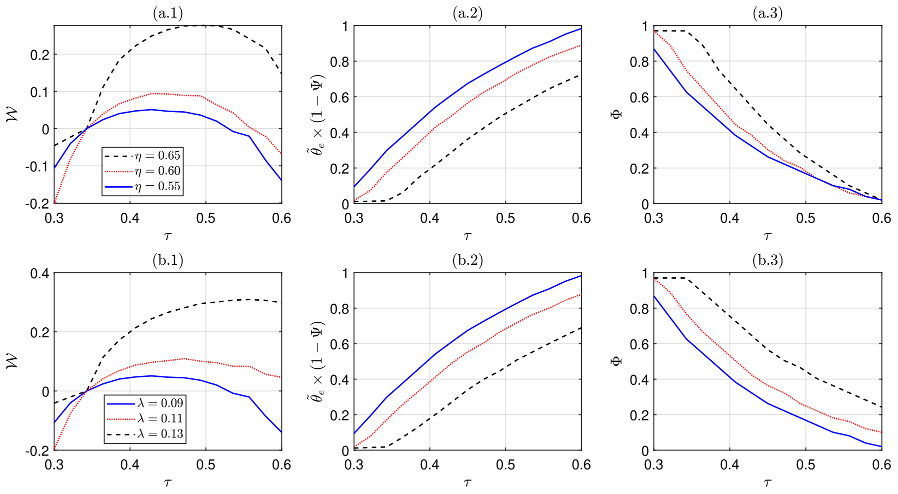

Fig. 5 Aggregate welfare function (% variations from the baseline parametrization; Panels (**a.1**) and (**b.1**)), tax evasion (Panels (**a.2**) and (**b.2**)), and net worth share of tax-compliant households (Panels (**a.3**) and (**b.3**)) for different fiscal parameters

Table 1 Equilibrium net worth share at the welfare-maximizing tax rate for different fiscal parameters in the steady state

Panel (a)			
η	0.55	0.60	0.65
Φ^*	0.2829	0.3233	0.3031
Panel (b)			
λ	0.09	0.11	0.13
Φ^*	–	0.3231	0.3234

3.3 Fiscal policy and welfare

This section evaluates the effect of different fiscal policies (i.e., variations in the τ, λ, and η parameters) on social welfare in the steady state.

In the aggregation process, the government takes into account that the agents' welfare depends on their heterogeneous tax evasion opportunities. In particular, the social welfare function is a weighted sum of the value functions of both taxpayers (W_h, whose weight is $0 \leq \Gamma \leq 1$) and evaders (W_e) in the steady state[6]:

$$\mathcal{W} := \Gamma \times W_e(\Phi) + (1-\Gamma) \times W_h(\Phi), \tag{21}$$

In our numerical simulations, the Government sets the tax rate τ to maximize Eq. (31). We assume that $\Gamma = 0.5$, i.e. the Government does not discriminate among households.

[6] Details on the derivation of the value functions can be found in Appendix A.3.

Figure 5 displays the simulated social welfare function, the associated level of tax evasion, and the steady-state level of ϕ as functions of the tax rate τ for different values of the fine rate η (Panels (a.1-3)) and the auditing intensity λ (Panels (b.1-3)). The social welfare function (Panels (a.1) and (b.1)) is expressed in percentage variations from the baseline parametrization ($\tau = 0.35$, $\eta = \eta_0 = 0.55$, $\lambda = 0.1$). Table 1 displays the "optimal" wealth share of tax-compliant households, dented as Φ^*, at the welfare-maximizing tax rate for different levels of η (Panel (a)) and λ (Panel (b)).

As a first result, our exercise highlights the welfare-maximizing level of tax evasion is not necessarily the same as that which maximizes tax revenues (see Fig. 3). This outcome occurs because the latter policy does not account for the fact that changes in tax rates redistribute not only net worth but also the auditing and financial risk between households of different types. The second result is that, as intuition suggests, higher auditing fines and intensities are associated with higher welfare-maximizing tax rates. This is because increasing η and λ makes tax evasion less attractive (Panels (a.2) and (b.2)), thereby reducing the endogenous differences between tax-compliant and tax-evading individuals (Panels (a.3) and (b.3)). Finally, we observe that for a given set of fiscal parameters (η, λ, τ), the "optimal" level net worth share Φ remains roughly constant (see Table 1).

4 Conclusions

We develop a tractable model of a production economy where taxpayers have heterogeneous evasion opportunities. We solve the model for its competitive equilibrium and show that, by issuing bonds, tax-compliant households supply evaders with hedging instruments against auditing risks, thereby increasing their evasion capacity.

In this framework, we find that aggregate productivity decreases in tax evaders' relative wealth share, but investments are hump-shaped. When tax evaders are few (many), capital grows slowly due to high taxation (low TFP). Next, we show that heterogeneous evasion opportunities increase the tax rate, which maximizes public revenues relative to the representative-agent economy featuring only tax evaders. Finally, we show that the tax rate that maximizes social welfare differs from the one that maximizes tax revenue because the latter does not consider that taxes redistribute net worth and risk between households with different tax evasion opportunities.

Appendix

Proof of Proposition 1

Let $d\phi = \phi\left(\mu^{\phi} dt + \sigma^{\phi}\right) dZ$ be the dynamics of an arbitrary state variable. Then, the value function of tax evaders V solves the HJBE

$$
0 = \max_{c,k,\tilde{k}} \left\{ \log c + \frac{\partial V}{\partial n}\mu_n + \frac{1}{2}\frac{\partial^2 V}{\partial n^2}(\sigma_n)^2 + \frac{\partial^2 V}{\partial \phi \partial n} V\phi\sigma^{\phi}\sigma_n + \lambda V(n(1-\eta\tilde{k}/n), \phi) \right\}
$$
$$
+ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \phi}\phi\mu^{\phi} + \frac{1}{2}\frac{\partial^2 V}{\partial \phi^2}(\phi\sigma^{\phi})^2 - (\rho+\lambda)V, \tag{22}
$$

in which μ_n and σ_n are the drift and diffusion of Eq. (5). By considering the guess function $V(n, \phi) = v(\phi) + \rho^{-1} \ln n$, the FOCs are

$$
c: \quad c = \rho n, \tag{23}
$$

$$
\tilde{k}: \quad A - r = [k/n(1-\tau) + \tilde{k}/n]\sigma^2 + \eta\lambda(1-\eta\tilde{k}/n)^{-1}, \tag{24}
$$

$$
k: \quad [A(1-\tau) - r]/\sigma^2(1-\tau) = k/n(1-\tau) + \tilde{k}/n. \tag{25}
$$

Eqs. (23)-(25) can be rearranged to obtain those in Proposition 1. Optimal bond holdings satisfy $b = n - k - \tilde{k}$. By substituting these objects in Eq. (22) and rearranging, one obtains an ODE for the value of the unknown function v

$$
\rho v = \Theta/\rho + \frac{\partial v}{\partial t} + \frac{\partial v}{\partial \phi}\phi\mu^{\phi} + \frac{1}{2}\frac{\partial^2 v}{\partial \phi^2}(\phi\sigma^{\phi})^2, \tag{26}
$$

in which $\Theta = \frac{\log \rho - 1}{\rho} + r + \theta_e(A(1-\tau) - r) + \tilde{\theta}_e(A - r) - 0.5\sigma^2[\theta_e(1-\tau) + \tilde{\theta}_e]^2 + \lambda \log(1 - \eta\tilde{\theta}_e)$. The problem of tax-compliant households can be solved by following the same steps while setting $\tilde{k} = \lambda = 0$.

Proof of Proposition 2

Considering the optimal strategies in Proposition 2, households' aggregate net worths are

$$
\begin{aligned}
dN_e = {} & [r + \theta_e(A(1-\tau) - r) + \tilde{\theta}_e(A - r) - \rho - \tilde{\theta}_e\eta\lambda]dt \\
& + [\theta_e(1-\tau) + \tilde{\theta}_e]\sigma dZ,
\end{aligned} \tag{27}
$$

$$
dN_h = \left[r + \theta_h(A(1-\tau) - r) - \rho\right]dt + \theta_h(1-\tau)\sigma dZ. \tag{28}
$$

By using that $N_h + N_e = K$ and applying Itô's lemma to the definition of ϕ, one gets

$$
d\phi/\phi = dN_h/N_h - dK/K + dK^2/K^2 - dN_h dK/(N_h K), \tag{29}
$$

$$
dK/K = \phi dN_h/N_h + (1-\phi)dN_e/N_e. \tag{30}
$$

By substituting Eqs. (27)-(28) in Eqs. (29)-(30) and using that $\phi\theta_h + (1-\phi)(\tilde{\theta}_e + \theta_e) = 1$, the diffusion term of the state variable vanishes and the results of Proposition 2 follow suit.

We match the results of Proposition 1 with Eqs. (13)-(15) to obtain Eq. (18) as it appears in the main text.

Welfare in the steady state

In the steady state $\phi = \Phi$, macroeconomic aggregates and thus prices ($r(\Psi)$ and $g(\Psi)$) are constant. Accordingly, the PDE in Eq. (26) reduces to the following ODE:

$$\rho v - \frac{\partial v}{\partial t} = \Theta(\Psi)/\rho.$$

By imposing the transversality condition $\lim_{t\to\infty} v_t e^{-\rho t} < \infty$, this equation has the following unique solution:

$$v(\Psi) = \frac{\Theta(\Psi, r(\Psi), g(\Psi))}{\rho^2}. \tag{31}$$

Ignoring all constant terms, the welfare of the "representative" tax-evader as a function of ϕ is

$$\begin{aligned} W_e(\Phi) = \; & \frac{\ln(1-\Phi)}{\rho} + \frac{r(\Phi) + \theta_e(\Phi)(A(\Phi)(1-\tau) - r(\Phi)) + \tilde{\theta}_e(\phi)(A(\Phi) - r(\Phi))}{\rho^2} \\ & + \frac{\lambda \log(1 - \eta\tilde{\theta}_e(\Phi)) - 0.5\sigma^2[\theta_e(\Phi)(1-\tau) + \tilde{\theta}_e(\Phi)]^2}{\rho^2}. \end{aligned}$$

Similarly, the welfare of the representative tax-compliant household is

$$W_h(\Phi) = \frac{\ln\Phi}{\rho} + \frac{r(\phi) + \theta_h(\phi)(A(\Phi)(1-\tau) - r(\Phi)) - 0.5\sigma^2\theta_h^2(\Phi)(1-\tau)^2}{\rho^2}.$$

Equipped with these equations, we define the economy's aggregate ("social") welfare as it appears in the main text.

Funding Open Access funding enabled and organized by Projekt DEAL.

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