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Can compulsory voting reduce information acquisition?

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ABSTRACT

An election with full turnout is supposed to achieve an outcome that perfectly reflects the majority's preference. This result requires voters to be perfectly informed about their preferences and to vote accordingly. I show that incentivizing participation with an abstention fine does not necessarily incentivize information acquisition. While a small abstention fine always increases information acquisition compared to Voluntary Voting, a high abstention fine that achieves full turnout increases information acquisition only if voting costs are high. If voting costs are low, the opposite is true: Less individuals acquire information under Compulsory Voting with full turnout than under Voluntary Voting.

1. Introduction

Maximizing turnout through Compulsory Voting is supposed to achieve a policy outcome that accurately reflects all citizens' preferences, thus maximizing the quality of the collective decision (Lijphart, 1997; Börgers, 2004; Chapman, 2019). If casting a ballot is costly, high turnout leads to high participation cost. Thus, the decision between making participation in an election voluntary or compulsory involves a trade-off between the quality and the social cost of the collective decision (cf., e.g., Börgers, 2004). If, however, voting costs are sufficiently small,[1] the trade-off between the quality and the cost of the collective decision seemingly disappears. Therefore, why not reduce voting costs and make participation compulsory?

High turnout can achieve a high quality of the collective decision only if the participating voters are sufficiently informed about the political alternatives to form a preference. Acquiring such information, however, is costly, such that some citizens might not have enough relevant knowledge to correctly assess which alternative they prefer.[2] It has been argued that the obligation to vote can increase political interest and involvement, thereby increasing information acquisition (Lijphart, 1997; de Leon and Rizzi, 2016). In that case, maximizing turnout indeed maximizes the quality of the collective decision. If, by contrast, Compulsory Voting does not incentivize information acquisition, uninformed individuals might feel compelled to vote without being able to tell the alternatives apart. They have two options: spoil their ballot or randomly select one of the alternatives. In some situations, however, it is impossible to spoil a ballot, e.g., when voting is public or when the ballot is cast electronically. And even when it is possible, spoiling a ballot is often not perceived as a legitimate voting choice (Ambrus et al., 2017). Thus, when they are compelled to participate, uninformed

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[1] Modern voting systems reduce participation cost substantially by allowing for early, postal, and in some countries even electronic voting (Gronke et al., 2007; Tsahkna, 2013; Hodler et al., 2015).

[2] Voters typically rely on mass media to acquire information about politicians, parties, and policy issues. Because of the spread of the Internet, the costs of accessing information have decreased substantially. But the opportunity costs of information acquisition increase as other sources of entertainment become available over the Internet (Gavazza et al., 2018). Moreover, not only the availability of information matters for becoming informed but also personal motivation and cognitive ability (Barabas et al., 2014).

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citizens might cast a valid, random vote. In that case, the share of random votes increases with the participation of uninformed voters until, eventually, "the election outcome itself will not be more likely to reflect the interests of the majority than, say, a fair coin toss" (Martinelli, 2006, p. 226). Therefore, Compulsory Voting might reduce the quality of the collective decision compared to Voluntary Voting, where uninformed individuals can abstain.

In this paper, I address the following question: If voters are initially uninformed about the available alternatives and if information acquisition is costly, should participation in an election be voluntary or compulsory? I specifically investigate how the introduction of an abstention fine affects an individual's incentives to acquire information which directly affect the quality of the collective decision. I also study the effect of the introduction of Compulsory Voting on expected social welfare.

I consider a private values model with two alternatives and costly voting. Initially, all individuals are uninformed about which alternative they prefer, but can acquire a costly, perfectly informative signal that reveals their preferred alternative. An individual is ex-ante equally likely to favor each alternative, i.e., preferences are neutral.[3] I explicitly model Compulsory Voting by introducing an (expected) abstention fine $f > 0$ that sanctions non-voters.[4] In the case of imperfect enforcement, the abstention fine relevant for the participation decision is not the fine f determined by legislation, but rather the expected fine $\tilde{f}$ based on the enforcement probability α, i.e., $\tilde{f} \equiv \alpha f$. Thus, the distinction between Voluntary and Compulsory Voting is continuous rather than binary, which is why my model allows for an abstention fine $f > 0$ which does not induce behavior significantly different from Voluntary Voting if $f \approx 0$.

An individual's rational decision to cast an informed vote is driven by the probability that her vote will be decisive for the outcome. Thus, my model applies to collective decisions in small electorates – as for example in clubs, boards of companies, or committees in universities, parties or parliaments.[5] I show that an uninformed voter's expected benefit of casting a pivotal vote is zero because – independent of which alternative she chooses – it is equally likely that the chosen alternative is her preferred alternative or her non-preferred alternative. Therefore, because voting is costly, an uninformed individual strictly prefers to abstain under Voluntary Voting. Under Compulsory Voting, however, depending on the level of the fine, an uninformed individual might be incentivized to participate in the election. In that case, because her expected benefit of casting a pivotal vote is zero, she is indifferent between the alternatives and I assume that she casts a valid vote by randomly selecting one of the alternatives.[6] If she is unbiased, she will flip a fair coin and select each alternative with equal probability. But an uninformed voter's random choice might be unconsciously affected by factors unrelated to the concrete policy positions. In that case, an uninformed voter is unconsciously biased in her random decision and selects one of the alternatives with an exogenously higher probability.[7] These factors are assumed to affect a voter's decision only as long as she is uninformed and makes a random choice: While the random choice of an uninformed voter is likely to be a fast choice based on impulses, the conscious choice of an informed voter is likely to be a careful decision based on deliberation.

My main result is that incentivizing participation with an abstention fine does not necessarily incentivize information acquisition. As long as the abstention fine is sufficiently small, uninformed individuals abstain from the election, such that Compulsory Voting increases information acquisition by increasing participation from informed voters only. If, however, the abstention fine is sufficiently high, uninformed individuals participate in the election by casting a random, valid vote, and full turnout is reached. In this case, Compulsory Voting increases information acquisition compared to Voluntary Voting only if the voting costs are high, and instead reduces information acquisition if the voting costs are sufficiently small. The intuition behind this result relies on the fact that under Compulsory Voting with full participation, the expected benefit of being pivotal is lower than under Voluntary Voting, and at the same time, the voting costs do not play a role in the voting decision anymore. If voting costs are high, only individuals with sufficiently low information costs participate under Voluntary Voting. Under Compulsory Voting with full participation, a marginal individual with low information costs who abstained under Voluntary Voting because her voting costs were too high, will acquire information and vote accordingly because the expected benefits of being pivotal exceed her information costs despite the reduction in the probability of being pivotal. If voting costs are sufficiently small, even individuals with high information costs participate under Voluntary Voting. Under Compulsory Voting with full participation, a marginal individual with high information costs who acquired information and voted accordingly under Voluntary Voting because her voting costs were small, will remain uninformed but participates by casting a random vote because her information costs exceed the expected benefits of being pivotal because of the reduced probability of being

[3] A previous version of the model considered an extension with non-neutral preferences, see Rohde (2022).

[4] In practice, Compulsory Voting is defined as "the legal obligation to attend the polls at election time" (Birch, 2009, p. 2). When the legal obligation to vote is combined with formal sanctions, these sanctions range from a fine of CHF 6 (about USD 7 in February 2024) in a Swiss Canton (Staatskanzlei Kanton Schaffhausen, 2019), to fines of AUD 20 (USD 13) for first-time abstentions in Australia (Western Australian Electoral Commission, 2024), and to fines of EUR 1000 (USD 1077) for repeated abstentions in Luxembourg (Luxembourg, 2003). Nevertheless, even for high abstention fines, enforcement might be weak. In Luxembourg, the payment of abstention fines has not been enforced since 1964 (Luxembourg Times, 2014). In Australia, non-voters can provide an excuse for not voting to avoid paying the abstention fine. As a result, only few non-voters are sanctioned (Birch, 2009).

[5] Rational voting models cannot explain significant turnout in large electorates. The probability of casting a pivotal vote approaches zero as the size of the electorate increases, and hence the probability of voting approaches zero as well. Nevertheless, the absolute number of voters remains non-zero in the limit (Taylor and Yildirim, 2010a). The same mechanism involving the probability of casting a pivotal vote drives the probability of acquiring information in my model, implying that a similar limit result concerning the number of informed voters in large elections is likely to hold.

[6] It is easy to see that the results of my analysis continue to hold even if some voters spoil their ballot, as long as some uninformed voters still cast a valid vote.

[7] First, in a decision between two candidates, unconscious biases can arise from stereotypical perceptions of their race, age, gender, religion, or other characteristics. Such unconscious biases regarding these characteristics are structurally widespread in society, such that it is plausible to assume that all uninformed voters are affected by the bias in the same direction. Second, advertisements for the candidates are omnipresent during an election campaign. One of the two candidates might have more noticeable election posters, such that an uninformed voter might be influenced by the recognition effect of that candidate's name on the ballot. It is plausible to assume that the eye-catching effect of the election posters affects all uninformed voters in the same manner, causing uninformed voters to be structurally biased towards the more noticeable candidate.

pivotal. As a result, with sufficiently low voting costs, there are some individuals who acquired information and voted accordingly under Voluntary Voting, but remain uninformed and cast a random vote under Compulsory Voting.

In the limit with zero voting costs, i.e., a degenerate cost distribution, it is impossible to incentivize information acquisition via an abstention fine. Instead, Compulsory Voting with any abstention fine always reduces the probability that an individual acquires information compared to Voluntary Voting. The intuition behind this result relies on the fact that, as the voting costs approach zero, only an individual's information costs play a role in her participation decision. Therefore, uninformed individuals participate in the election as soon as Compulsory Voting with any abstention fine $f > 0$ is introduced. In consequence, a marginal individual whose information costs were just low enough to acquire information under Voluntary Voting will remain uninformed but participate under Compulsory Voting with any abstention fine $f > 0$ because her information costs now exceed the reduced benefits of acquiring information.

Moreover, I show that the bias of uninformed voters has a detrimental effect on information acquisition under Compulsory Voting with full participation: The probability of acquiring information under Compulsory Voting with a high abstention fine that leads to full turnout is decreasing in the bias of uninformed voters. In particular, I find that if uninformed voters are fully biased and always select a particular alternative, not acquiring information is an equilibrium.

When Compulsory Voting only incentivizes participation but not information acquisition, the preference of the majority cannot necessarily be inferred from the outcome of the collective decision anymore. As a result, maximizing participation does not necessarily maximize the quality of the collective decision.

Finally, I find that Compulsory Voting reduces expected social welfare because of the negative externality of voting. This result is in line with the literature on costly voting in private values settings: Börgers (2004) shows that in the neutral preference setting, Voluntary Voting Pareto-dominates Compulsory Voting with full participation. Grüner and Tröger (2019) show that Voluntary Voting with the simple majority rule is optimal in the neutral preference setting and that Voluntary Voting dominates Compulsory Voting with full participation even when the participation cost is small. My result goes one step further by showing that *any* positive abstention fine $f > 0$ is sub-optimal. Therefore, my model cannot justify Compulsory Voting with any abstention fine $f > 0$ even if full participation is not reached.

2. Related literature

My model is related to three strands of literature: The literature on endogenous information acquisition of voters, the literature on compulsory voting with an explicit abstention fine, and the literature on costly voting with private values.

Endogenous information acquisition is mainly studied in *common* value models in which the election serves as an information aggregation mechanism.[8] If acquiring political information is costly, regardless of whether the information signal is perfectly informative (Mukhopadhaya, 2003; Persico, 2004) or whether it is imperfectly informative with increasing quality at increasing cost (Martinelli, 2006, 2007), information is a public good and the incentive to free-ride on other voters leads to under-investment in information. This is fundamentally different from the mechanisms involved in the private value setting. Moreover, these papers assume that voting is costless, such that all individuals vote. In that case, voting is implicitly considered as compulsory, but the participation decision cannot be studied. Only a few papers exist that explicitly model the incentives for participation under Compulsory Voting.

Tyson (2016) addresses a research question that is very similar to mine in a *common* value framework. He studies endogenous information acquisition when both voting and information acquisition are costly. He shows that an abstention tax leads to a higher level of informed voting but also creates an incentive for uninformed voters to participate in the election. In his setting, it is optimal for uninformed individuals who participate in the election to spoil their ballot, while in my setting, uninformed voters are indifferent between spoiling their ballot and casting a random vote. Because invalid votes do not affect the collective decision, Tyson finds that incentivizing voting increases the probability that the correct alternative is chosen collectively.[9] In contrast, I assume that uninformed voters cast a valid, random vote.[10] Because valid votes affect the collective decision and in particular the probability of being pivotal for other voters, uninformed valid votes can reduce the probability of acquiring information of other voters.

The intuition behind most of my results follows from Börgers (2004) and Taylor and Yildirim (2010a). Börgers (2004) compares Voluntary Voting to Compulsory Voting with full turnout, but does not explicitly consider how participation is enforced. Börgers' main result is that there is a *negative externality of voting*: By voting, an individual reduces the probability of being pivotal, and hence also the expected benefits of voting for all other voters. Börgers shows that because of the negative externality of voting, participation is inefficiently high under Voluntary Voting, such that Compulsory Voting further reduces expected social welfare. In his conclusion,

8 An exception is the experiment by Ou and Tyson (2023) in which they measure the willingness to participate in an election in a common value setting in which some participants (so-called "expert voters") are exogenously informed about the state of the world and about any partisan bias. Because expert voters are perfectly informed, the purpose of the election is not information aggregation, but defeating partisan bias. To study the participation decision, instead of asking participants to make a discrete vote choice, Ou and Tyson elicit each participant's willingness to vote, i.e., the maximum voting costs she is willing to incur. They show that revealing the number of expert voters in the electorate reduces the willingness to vote among expert voters, thereby reducing the probability of choosing the correct alternative collectively – which is the quality of democratic choice.

9 Note that, e.g., under electronic voting, it is impossible to spoil a ballot. Not allowing uninformed voters to spoil their ballot in Tyson's (2016) model would force uninformed voters who participate in the election to cast a valid, random vote. Because valid votes – in contrast to spoiled ballots – directly affect the election outcome, increased participation from uninformed voters under Compulsory Voting reduces the probability that the correct alternative is chosen collectively.

10 Jakee and Sun (2006) also suppose that uninformed voters vote randomly, but they take an expressive voting approach, which contrasts with my rational choice framework.

Börgers claims that his results remain unchanged if the voting costs are reinterpreted as the cost of information acquisition. He assumes that uninformed individuals abstain under Voluntary Voting and that Compulsory Voting forces all individuals to become informed, but points out that mandatory information acquisition would not be implementable. I explicitly address this point by analyzing the incentives to acquire costly information under Voluntary and Compulsory Voting. I show that Compulsory Voting might reduce information acquisition, but the negative externality of voting still leads to lower expected social welfare under Compulsory Voting than under Voluntary Voting.

Taylor and Yildirim (2010a) consider a private values model with non-neutral preferences, but they do not study information acquisition and consider only Voluntary Voting. While voters always exert a negative externality of voting on other voters, there can be a positive externality of voters on non-voters if the assumption of neutral preferences is relaxed. If one alternative is ex-ante preferred by the majority of voters (Krasa and Polborn, 2009; Taylor and Yildirim, 2010a) or if preferences are correlated (Goeree and Großer, 2007), the members of the majority have an incentive to free-ride on other voters, such that participation becomes inefficiently low. Taylor and Yildirim (2010a) formalize the *underdog effect*: Under non-neutral preferences, members of the expected minority are more likely to vote than voters of the expected majority. They show that, nevertheless, the ex-ante preferred alternative remains more likely to win the election. Goeree and Großer (2007) and Taylor and Yildirim (2010b) show that providing information on the electorate's preference increases participation among the minority group, and therefore reduces the probability that the alternative favored by the majority wins the election, as well as social welfare.

Krasa and Polborn (2009) study the effect of a subsidy for voters in a costly voting model with private values and non-neutral preferences, but they do not consider information acquisition. They show that, because voters of the majority exert a positive externality on non-voters with the same preference, turnout is inefficiently low under Voluntary Voting. Therefore, a subsidy for voting increases expected welfare and improves the quality of the collective decision. Similar to Krasa and Polborn (2009), I use an explicit abstention fine to model Compulsory Voting, but I focus on the case with neutral preferences and, more importantly, I introduce costly information acquisition.

To the best of my knowledge, there exists no work on endogenous information acquisition of voters in a purely private value setting. Thus, I contribute to the literature by analyzing the effects of an abstention fine on the participation and information acquisition decision of voters. I introduce a novel perspective on the effect of information acquisition under Compulsory Voting by assuming that uninformed voters cast a valid, potentially biased vote.

3. The model

There are $n \geq 3$ individuals $i \in \{1, 2, ..., n\}$ who have to make a collective policy decision x from the set of alternatives $X = \{A, B\}$. The outcome is determined by simple majority rule. In case of a tie, both alternatives are chosen with equal probability.

Let $r_i \in X$ denote the alternative favored by individual $i \in \{1, 2, ..., n\}$.[11] The preference r_i of individual i is assumed to be stochastically independent of the preference r_j of individual $j \neq i$. Ex-ante, an individual is equally likely to favor each alternative, i.e., preferences are *neutral*.

An individual is initially uninformed about her preferred alternative r_i and she does not automatically observe r_i at the interim stage. If she wants to learn which alternative she prefers, she can acquire a costly signal that reveals r_i. Let c_i denote the stochastic information costs of individual i. For each i, the information costs c_i are drawn independently from the CDF G which has the support $[\underline{c}, \overline{c}]$ where $0 \leq \underline{c} < \overline{c}$. Let g denote the PDF associated with G and assume that g is positive on all of the support. The information costs c_i of individual i are assumed to be stochastically independent of her preferred alternative r_i, and of the information costs c_j of individual $j \neq i$. Information acquisition is a binary decision, i.e., individual i can either acquire a perfectly informative signal or remain uninformed, such that she only knows that she, as well as all other individuals, favors alternative A with probability $\frac{1}{2}$.

Let k_i denote the stochastic voting costs, i.e., the cost of casting a ballot, of individual i.[12] The voting costs are not yet known to the individual when making her information acquisition decision. For each i, the voting costs k_i are drawn independently from the CDF H which is the same for all individuals and has the support $[\underline{k}, \overline{k}]$ with $0 \leq \underline{k} < \overline{k}$. Assume that $\underline{k} < \frac{1}{2}$ to rule out trivial equilibria where nobody votes. Let h denote the PDF associated with H and assume that h is positive on all of the support. The voting costs k_i of individual i are assumed to be stochastically independent of her preferred alternative r_i, her information costs c_i, and of the voting costs k_j of individual $j \neq i$.

To incentivize participation, abstention is sanctioned with a fine f. The fine f can be more generally considered an expected fine that also accounts for the probability of enforcement. I will call the case without a fine "Voluntary Voting", and the case where $f > 0$ "Compulsory Voting".

If individual i's preferred alternative r_i is chosen collectively, her utility is normalized to 1. If the other alternative is chosen, i's utility is normalized to zero. I assume that voters do not receive any intrinsic utility from the voting act itself.[13] Table 1 summarizes the ex-post payoff of an individual i, where $\mathbb{1}\{x = r_i\}$ is an indicator function that takes the value 1 if the collective outcome x is equal to i's preferred alternative r_i and zero otherwise.

[11] I rule out the possibility that an individual is indifferent between the two alternatives.

[12] I present a simplified version of the model in Appendix B. The simplified model assumes that all voters have the same voting costs, which are already known at the information stage, and considers uninformed voters to be unbiased.

[13] For the framework where voters receive intrinsic utility from fulfilling their civic duty of participating in the election see, e.g., Feddersen and Sandroni (2006).

Table 1
Ex-post payoff of individual i.

	Participate	Abstain
Informed	$\mathbb{1}\{x=r_i\}-(c_i+k_i)$	$\mathbb{1}\{x=r_i\}-c_i-f$
Uninformed	$\mathbb{1}\{x=r_i\}-k_i$	$\mathbb{1}\{x=r_i\}-f$

If an individual who knows her preferred alternative casts a ballot, voting against her preferred alternative is a weakly dominated strategy. Hence, informed voters vote sincerely for their preferred alternative. Uninformed individuals can participate in the election, although they do not know which alternative they favor, by casting a valid, random vote. I assume that no individual casts an invalid vote.

The timing of the game can be summarized as follows:

1. For each individual $i \in \{1,2,...,n\}$, nature draws the information costs $c_i \in [\underline{c},\overline{c}]$ and the voting costs $k_i \in [\underline{k},\overline{k}]$. Nature also draws i's preferred alternative r_i from the set of alternatives $X=\{A,B\}$ with $Pr(r_i=A)=\frac{1}{2}$.
2. Each individual privately observes her information costs c_i, but she neither observes her preference r_i nor her voting costs k_i.
3. *Information stage*: All individuals simultaneously decide whether to acquire information or not. The decision is private information. If individual i acquires information she privately observes her preference r_i.
4. Each individual privately observes her voting costs k_i.
5. *Voting stage*: All individuals simultaneously decide whether to vote or abstain.
6. The collective policy outcome $x \in X$ is realized by simple majority rule.
7. Payoffs are realized.

At the voting stage, we have three different political groups of individuals: Those who are informed and favor alternative A, those who are informed and favor B, and those who are uninformed (denoted by U). Let $\theta \in \Theta \equiv \{A,B,U\}$ denote the political group to which an individual belongs. Individuals in the same group face an identical decision problem so that, as is common in the literature, I can focus on type-symmetric strategies. A voting strategy must be a cutoff strategy with (potentially different) cutoff values $\hat{k}_\theta \in [\underline{k},\overline{k}]$ for each group $\theta \in \{A,B,U\}$. An individual i in group θ casts a ballot if and only if her voting costs k_i are sufficiently low, i.e., if $k_i \leq \hat{k}_\theta$, and abstains otherwise. The voting cost cutoff values pin down the voting probabilities

$$p_\theta = Pr(k_i \leq \hat{k}_\theta) = H(\hat{k}_\theta)$$

for each group $\theta \in \{A,B,U\}$. Note that $p_\theta \in [0,1]$ with $p_\theta=0$ if $\hat{k}_\theta \leq \underline{k}$ and $p_\theta=1$ if $\hat{k}_\theta \geq \overline{k}$.

At the information stage, all individuals face an identical decision problem. An information acquisition strategy must be a cutoff strategy with a common cutoff value $\hat{c} \in [\underline{c},\overline{c}]$ for all individuals such that an individual i acquires information if and only if her information costs c_i are sufficiently low, i.e., if $c_i \leq \hat{c}$, and remains uninformed otherwise. The information cost cutoff value pins down the probability of acquiring information

$$q \equiv Pr(c_i \leq \hat{c}) = G(\hat{c})$$

where $q \in [0,1]$ with $q=0$ if $\hat{c} \leq \underline{c}$ and $q=1$ if $\hat{c} \geq \overline{c}$.

In the following, I will solve the game using a backward induction logic.

The voting stage

First, let us derive the expected benefit of casting an informed vote. Individual i is pivotal only if her vote creates or breaks a tie. In both cases, she gains $\frac{1}{2}$ in expected utility. The probability of being pivotal depends on the other individuals' expected behavior and on whether individual i votes for A or for B. Let $\Pi_A(\mathbf{p},q)$ denote the probability that an individual who votes for A is pivotal and $\Pi_B(\mathbf{p},q)$ the probability that an individual who votes for B is pivotal if all other individuals participate with probabilities $\mathbf{p} \equiv (p_A,p_B,p_U)$ and if all others acquire information with probability q. Let $\mathcal{B}_A(\mathbf{p},q)$ denote the expected benefit from casting a pivotal vote for A, which is

$$\mathcal{B}_A(\mathbf{p},q) = \frac{1}{2}\Pi_A(\mathbf{p},q) \tag{1}$$

and let $\mathcal{B}_B(\mathbf{p},q)$ denote the expected benefit from casting a pivotal vote for B, which is

$$\mathcal{B}_B(\mathbf{p},q) = \frac{1}{2}\Pi_B(\mathbf{p},q). \tag{2}$$

Next, let us derive the expected benefit of casting an uninformed, valid vote, $\mathcal{B}_U(\mathbf{p},q)$. Because an uninformed voter casts a valid vote, she has to select either A or B without knowing which alternative she favors. If an uninformed voter i casts a pivotal vote for A, she knows that with probability $\frac{1}{2}$, she indeed favors A and her expected utility increases by $\frac{1}{2}$. But with probability $\frac{1}{2}$, she favors B and her expected utility decreases by $\frac{1}{2}$. The same logic applies if voter i casts a pivotal vote for B. It is evident immediately that her expected benefit of casting an uninformed, valid vote is always zero, i.e., $\mathcal{B}_U(\mathbf{p},q)=0$ for all $\mathbf{p},q$.[14] Because uninformed voters who participate in the election are indifferent between selecting alternative A or B, I need to make an assumption about behavior under indifference. Let $\lambda \in [0,1]$ denote the probability with which an uninformed voter selects alternative A and $1-\lambda$ the probability with which an uninformed voter selects alternative B.[15] The most straightforward way to select an alternative despite indifference would be for an uninformed individual to flip a fair coin, which results in selecting A and B with equal probability. In that case, uninformed voters are called *unbiased* and $\lambda = \frac{1}{2}$. One could, however, imagine that an uninformed voter in the voting booth does not actually flip a fair coin but instead makes a fast choice based on impulses. In that case, her random choice might be unconsciously affected by factors unrelated to the concrete policy positions. In that case, uninformed voters are called *biased* towards alternative A, and $\lambda > \frac{1}{2}$.

Next, let us calculate the probability $\Pi(\mathbf{p},q)$ that individual i is pivotal. Given the voting probabilities $\mathbf{p}$ and the information acquisition probability q, let $\phi_A(\mathbf{p},q)$ denote the ex-ante expected probability that an individual votes for alternative A, and $\phi_B(\mathbf{p},q)$ the ex-ante expected probability that an individual votes for alternative B, where

$$\phi_A(\mathbf{p},q) = \frac{1}{2}qp_A + \lambda(1-q)p_U \tag{3}$$

and

$$\phi_B(\mathbf{p},q) = \frac{1}{2}qp_B + (1-\lambda)(1-q)p_U. \tag{4}$$

The ex-ante expected probability that an individual abstains is $1-\phi_A-\phi_B$.

A voter is pivotal if her vote either creates a tie or breaks a tie. Hence, given the voting probabilities $\mathbf{p}$ and the information acquisition probability q, the probability that an A-vote is pivotal is given by (following Taylor and Yildirim, 2010a)

$$\begin{aligned}\Pi_A(\mathbf{p},q) = &\sum_{l=0}^{\lfloor\frac{n-1}{2}\rfloor}\binom{n-1}{l,l,n-1-2l}\phi_A^l\phi_B^l(1-\phi_A-\phi_B)^{n-1-2l}\\ &+\sum_{l=0}^{\lfloor\frac{n-2}{2}\rfloor}\binom{n-1}{l,l+1,n-2-2l}\phi_A^l\phi_B^{l+1}(1-\phi_A-\phi_B)^{n-2-2l}.\end{aligned} \tag{5}$$

Analogously, the probability that a B-vote is pivotal is given by

$$\begin{aligned}\Pi_B(\mathbf{p},q) = &\sum_{l=0}^{\lfloor\frac{n-1}{2}\rfloor}\binom{n-1}{l,l,n-1-2l}\phi_A^l\phi_B^l(1-\phi_A-\phi_B)^{n-1-2l}\\ &+\sum_{l=0}^{\lfloor\frac{n-2}{2}\rfloor}\binom{n-1}{l,l+1,n-2-2l}\phi_A^{l+1}\phi_B^{l}(1-\phi_A-\phi_B)^{n-2-2l}.\end{aligned} \tag{6}$$

From Taylor and Yildirim (2010a), we have $\Pi_A - \Pi_B = sign(\phi_B-\phi_A)$.

Given the probability that her vote will be pivotal, an individual casts a ballot if and only if her expected payoff from voting exceeds her expected payoff from abstaining. Thus, an individual with voting costs k_i in group $\theta \in \{A,B,U\}$ votes if and only if

$$k_i \le \mathcal{B}_\theta(\mathbf{p},q) + f \equiv \varphi_\theta(\mathbf{p},q) \tag{7}$$

and abstains otherwise.[16]

[14] Note that under non-neutral preferences, where alternative A is ex-ante more likely to be favored by the majority, uninformed voters derive a positive expected benefit of casting a valid vote for alternative A and it is a weakly dominant strategy for uninformed voters who participate in the election to vote for alternative A with probability 1. While in the neutral preferences setting I exogenously assume that uninformed voters cast a valid vote, this behavior arises endogenously in the non-neutral preferences setting, such that no uninformed voter ever spoils their ballot. Moreover, the bias towards alternative A arises endogenously as well. See Rohde (2022) for a previous version of the model allowing for non-neutral preferences.

[15] Note that the model could be easily extended to allow for uninformed voters spoiling their ballot with probability $\nu \in [0,1)$. In that case, the voting decision of uninformed voters can be considered as a two-step decision: First, an uninformed voter decides whether to cast a valid vote or spoil her ballot. If she casts a valid vote, she has to select A or B. If an uninformed voter is unbiased, she flips a fair coin. In that case, an uninformed, unbiased voter selects alternative A and B with equal probability $\frac{1-\nu}{2}$. If an uninformed voter is biased towards alternative A, she selects alternative A with probability $\frac{1-\nu}{2} < \lambda \le 1-\nu$ and alternative B with probability $1-\nu-\lambda$. Because an invalid vote does not affect the outcome of the election, ν does not affect the probability of being pivotal for other voters directly, but only via the reduction in the probability with which an uninformed voter casting a valid vote selects each alternative. Therefore, the negative effect of increased participation from uninformed voters under Compulsory Voting remains entirely driven by those uninformed voters who cast a valid vote. Hence, the results of my analysis continue to hold for all $\nu < 1$, i.e., as long as *some* uninformed voters cast a valid vote. If $\nu = 1$, i.e., all uninformed voters who participate in the election spoil their ballot, the participation of uninformed voters does not affect the election outcome compared to their abstention. It can be seen immediately from equations (3) to (6) that in this case, the probability of being pivotal for an informed voter remains entirely unaffected by uninformed voters. Hence, if no uninformed voter casts a valid vote, there is no negative effect of increased participation from uninformed voters on information acquisition under Compulsory Voting.

[16] Note that an individual in group $\theta \in \{A,B,U\}$ is indifferent between voting and abstaining if equation (7) holds with equality. Since the voting costs k_i are a continuous random variable, this is a probability zero event and can be ignored for the following analysis.

In any symmetric pure strategy Bayesian Nash equilibrium, the equilibrium voting cost cutoff value k_θ^* for the group of voters $\theta \in \{A, B, U\}$ must satisfy

$$\begin{aligned} &k_\theta^* = \varphi_\theta(\mathbf{p}^*, q) \text{ if } k_\theta^* \in (\underline{k}, \overline{k}) \\ \text{or } &k_\theta^* \geq \varphi_\theta(\mathbf{p}^*, q) \text{ if } k_\theta^* = \underline{k} \\ \text{or } &k_\theta^* \leq \varphi_\theta(\mathbf{p}^*, q) \text{ if } k_\theta^* = \overline{k}, \end{aligned} \tag{8}$$

where $\mathbf{p}^* = (p_A^*, p_B^*, p_U^*)$ are the equilibrium voting probabilities implied by the equilibrium voting cost cutoffs, and q is the information acquisition probability implied by some information cost cutoff $\hat{c}$.

Because the CDF H is strictly increasing on all of the support, finding equilibrium voting cost cutoff values $(k_A^*, k_B^*, k_U^*) \in [\underline{k}, \overline{k}]^3$ is equivalent to finding equilibrium voting probabilities $(p_A^*, p_B^*, p_U^*) = (H(k_A^*), H(k_B^*), H(k_U^*)) \in [0, 1]^3$. Hence, using that $H'(y) > 0$ for all $y \in (\underline{k}, \overline{k})$ and that $H(y) = 0$ for $y \leq \underline{k}$ and $H(y) = 1$ for $y \geq \overline{k}$, we can re-write the above conditions in one single condition: In any symmetric pure strategy Bayesian Nash equilibrium, the equilibrium probabilities of voting for all groups of voters $\theta \in \{A, B, U\}$ need to satisfy

$$p_\theta^* = H(\varphi_\theta(\mathbf{p}^*, q)) \tag{9}$$

for any information acquisition probability $q \in [0, 1]$.

The equilibrium behavior of uninformed voters at the voting stage can be derived directly from equation (7). Under Voluntary Voting, uninformed voters strictly prefer to abstain because voting is costly and the expected benefit of casting an uninformed vote is always zero. Under Compulsory Voting, uninformed voters participate if and only if their voting costs are smaller than the abstention fine, i.e., if $k_i < f$. Therefore, the equilibrium probability of voting for an uninformed individual is $p_U^* = H(f)$, which is positive if the abstention fine is sufficiently high, i.e., if $f \geq \underline{k}$.

The information stage

At the information stage, an individual acquires information if her expected payoff of doing so, given the respective probability of casting an informed vote after learning whether she favors A or B, exceeds the expected payoff of remaining uninformed, given the probability of casting an uninformed vote.

For any information cost cutoff $\hat{c}$ and a corresponding vector of equilibrium voting cost cutoffs (k_A^*, k_B^*, k_U^*), an individual i with information costs c_i acquires information about her preferred alternative, if and only if

$$\begin{aligned} &\frac{1}{2}\left[\int_{\underline{k}}^{k_A^*} \left(\frac{1}{2}\Pi_A(\mathbf{p}^*, q) - y\right) h(y)dy + \int_{k_A^*}^{\overline{k}} (-f)h(y)dy\right] \\ &+ \frac{1}{2}\left[\int_{\underline{k}}^{k_B^*} \left(\frac{1}{2}\Pi_B(\mathbf{p}^*, q) - y\right) h(y)dy + \int_{k_B^*}^{\overline{k}} (-f)h(y)dy\right] - c_i \\ \geq &\int_{\underline{k}}^{k_U^*} (-y)h(y)dy + \int_{k_U^*}^{\overline{k}} (-f)h(y)dy. \end{aligned} \tag{10}$$

The first line is the expected payoff if individual i acquires information and finds out that she prefers alternative A: If her voting costs are below k_A^*, she casts her vote for A and, if she is pivotal, gains $\frac{1}{2}$ in expected utility, but also pays the voting costs k_i. If her voting costs are above k_A^*, she abstains and pays the fine f. Analogously, the second line is the expected payoff if i acquires information and finds out that she prefers alternative B. The third line is the expected payoff if i remains uninformed.

Using integration by parts, the condition can be re-written such that an individual i with information costs c_i acquires information about her preferred alternative, if and only if

$$\begin{aligned} c_i \leq &\frac{1}{2}\left[\left(\frac{1}{2}\Pi_A - k_A^* + f\right)p_A^* + \int_{\underline{k}}^{k_A^*} H(y)dy\right] \\ &+ \frac{1}{2}\left[\left(\frac{1}{2}\Pi_B - k_B^* + f\right)p_B^* + \int_{\underline{k}}^{k_B^*} H(y)dy\right] \\ &- \left(f - k_U^*\right)p_U^* - \int_{\underline{k}}^{k_U^*} H(y)dy \end{aligned} \tag{11}$$

and remains uninformed otherwise. Let $\Phi(\mathbf{p}^*, q)$ denote the right-hand side of condition (11), which can be interpreted as the expected benefit of acquiring information. Note that if $k_A^*, k_B^*, k_U^* \in (\underline{k}, \overline{k})$, condition (11) can be further simplified to

$$c_i \leq \frac{1}{2}\int_{\underline{k}}^{k_A^*} H(y)dy + \frac{1}{2}\int_{\underline{k}}^{k_B^*} H(y)dy - \int_{\underline{k}}^{k_U^*} H(y)dy. \tag{12}$$

In any symmetric pure strategy Bayesian Nash equilibrium, the equilibrium information cost cutoff c^* must satisfy

$$\begin{aligned} &c^* = \Phi(\mathbf{p}^*, q) \text{ if } c^* \in (\underline{c}, \overline{c}) \\ &\text{or } c^* \geq \Phi(\mathbf{p}^*, q) \text{ if } c^* = \underline{c} \\ &\text{or } c^* \leq \Phi(\mathbf{p}^*, q) \text{ if } c^* = \overline{c}. \end{aligned} \tag{13}$$

Because the CDF G is strictly increasing on all of the support, finding the equilibrium information cost cutoff value $c^* \in [\underline{c}, \overline{c}]$ is equivalent to finding the equilibrium information acquisition probability $q^* = G(c^*) \in [0, 1]$. Hence, using that $G'(y) > 0$ for all $y \in (\underline{c}, \overline{c})$ and that $G(y) = 0$ for $y \leq \underline{c}$ and $G(y) = 1$ for $y \geq \overline{c}$, we can re-write the above conditions in one single condition: In any symmetric pure strategy Bayesian Nash equilibrium, the equilibrium probability q^* of acquiring information needs to satisfy

$$q^* = G(\Phi(\mathbf{p}^*, q^*)) \tag{14}$$

given the equilibrium voting probabilities $\mathbf{p}^*$.

Proposition 1 shows the existence of an equilibrium.

Proposition 1. *There exists a type-symmetric pure-strategy Bayesian Nash equilibrium in which the following conditions are satisfied simultaneously for the equilibrium probabilities of voting, $\boldsymbol{p}^* = (p_A^*, p_B^*, p_U^*)$, and the equilibrium probability of acquiring information, q^*:*

$$\begin{aligned} q^* &= G(\Phi(\boldsymbol{p}^*, q^*)) \\ p_A^* &= H(\varphi_A(\boldsymbol{p}^*, q^*)) \\ p_B^* &= H(\varphi_B(\boldsymbol{p}^*, q^*)) \\ p_U^* &= H(\varphi_U(\boldsymbol{p}^*, q^*)). \end{aligned}$$

Equilibrium properties

To understand the effect of an abstention fine on the probability of acquiring information later, it is important to first derive some basic properties of voting behavior in equilibrium.

Remark 1. The type-symmetric pure-strategy Bayesian Nash equilibrium has the following properties:

(i) If uninformed voters are unbiased ($\lambda = \frac{1}{2}$), then $0 < p_A^* = p_B^*$ in equilibrium.
(ii) If uninformed voters are biased ($\lambda > \frac{1}{2}$), then
 (a) if $f \leq \underline{k}$, $p_U^* = 0$ and $p_A^* = p_B^* > 0$.
 (b) if $f \in (\underline{k}, \overline{k})$, $0 < p_U^* < p_A^* \leq p_B^* \leq 1$, and if $p_A^* < 1$, then $p_A^* < p_B^*$.
 (c) if $f \geq \overline{k}$, $p_U^* = p_A^* = p_B^* = 1$.

Intuitively, if either uninformed voters are unbiased, or if they are biased but do not participate, there is no ex-ante expected majority for either of the two alternatives, such that A- and B-voters are equally likely to vote in equilibrium (as in Börgers, 2004). If biased, uninformed voters participate in the election, there is an ex-ante expected majority for alternative A, such that the probability of being pivotal is lower for A-voters than for B-voters. Therefore, informed individuals who favor B are more likely to participate than those who favor A – which is the underdog effect (cf., e.g., Taylor and Yildirim, 2010a).

Proposition 2. *The type-symmetric pure-strategy Bayesian Nash equilibrium is unique if*

$$\frac{1-\lambda}{\lambda} \geq 1 - \frac{1}{\lfloor \frac{n}{2} \rfloor}.$$

Note that Proposition 2 implies that, if uninformed voters are unbiased, i.e., if $\lambda = \frac{1}{2}$, the equilibrium is unique, which is the result from Börgers (2004). The uniqueness result can be extended to the case where uninformed voters are biased only if the bias λ is sufficiently close to $\frac{1}{2}$, such that voting probabilities remain sufficiently symmetric. To show this, I draw on the result from Taylor and Yildirim (2010a) for the case with non-neutral preferences, which says that there exists at most one equilibrium that satisfies $1 \geq \frac{\phi_B^*}{\phi_A^*} \geq 1 - \frac{1}{\lfloor \frac{n}{2} \rfloor}$. If uninformed voters are biased and participate, the expected probability that an individual votes for A, ϕ_A^*, is always higher than the ex-ante expected probability that an individual votes for B, ϕ_B^*. A-voters always impose a negative externality of voting on all other individuals. B-voters, however, might impose a positive externality of voting on other voters with the same preference, which can cause the existence of multiple equilibria (Taylor and Yildirim, 2010a). The positive externality arises only if the gap in voting probabilities is sufficiently large, which in the neutral preference setting is possible only if the bias of the uninformed voters is sufficiently large. If the bias of the uninformed voters is not too large, i.e., if λ is sufficiently close to $\frac{1}{2}$, the gap in voting probabilities is small and B-voters impose a negative externality of voting on all other individuals. In that case, equilibrium voting behavior is sufficiently symmetric for the equilibrium to remain unique. Thus, I find that the uniqueness result from the symmetric setting with unbiased uninformed voters is robust to small perturbations in the bias of uninformed voters.

Information acquisition

In the following, I will compare the probability of acquiring information under Voluntary and Compulsory Voting. In particular, I will analyze how the probability of acquiring information is affected on the one hand by the introduction of a marginal abstention fine $0 < f < \underline{k}$ which does not necessarily lead to full turnout, and on the other hand by the introduction of a high abstention fine $f \geq \overline{k}$ which leads to full turnout.

First, consider the introduction of a marginal abstention fine $0 < f < \underline{k}$. Recall from Remark 1 that $p_U^* = 0$ and $p_A^* = p_B^* > 0$ for all $\lambda \geq \frac{1}{2}$. Therefore, let $p_I^* \equiv p_A^* = p_B^*$ denote the equilibrium probability of voting for an informed individual.

Proposition 3. *If $0 \leq f < \underline{k}$, the probability of acquiring information weakly increases in the abstention fine f. It increases strictly if $q^* \in (0,1)$ and $p_I^* \in (0,1)$.*

This result is based on the fact that as long as $f < \underline{k}$, uninformed individuals strictly prefer to abstain. In that case, the incentives to acquire information are driven by the probability of voting for informed individuals only. If $p_I^* = 1$ at $f = 0$, i.e., all informed individuals vote under Voluntary Voting, the introduction of a marginal abstention fine does not affect the probability of voting for informed individuals, p_I^*, and hence it cannot affect the probability of acquiring information either. If, however, $p_I^* < 1$ at $f = 0$, the introduction of a marginal abstention fine incentivizes participation of informed individuals and therefore incentivizes information acquisition as well.

Next, for the comparison between Voluntary Voting and Compulsory Voting with a high abstention fine $f \geq \overline{k}$, let us start with some observations about the probability of casting a pivotal vote under Compulsory Voting with $f \geq \overline{k}$. Recall from Remark 1 that $f \geq \overline{k}$ leads to full participation, i.e., $p_A^* = p_B^* = p_U^* = 1$. Then, the voting probabilities are

$$\phi_A(\mathbf{1}, q^*) = \frac{1}{2}q^* + \lambda(1 - q^*)$$

and

$$\phi_B(\mathbf{1}, q^*) = \frac{1}{2}q^* + (1 - \lambda)(1 - q^*)$$

and the probabilities of being pivotal are given by

$$\Pi_A(\mathbf{1}, q^*) = \begin{cases} \binom{n-1}{\frac{n-1}{2}} \phi_A^{\frac{n-1}{2}} \phi_B^{\frac{n-1}{2}} & \text{if } n \text{ odd} \\ \binom{n-1}{\frac{n}{2}-1} \phi_A^{\frac{n}{2}-1} \phi_B^{\frac{n}{2}} & \text{if } n \text{ even} \end{cases} \tag{15}$$

and

$$\Pi_B(\mathbf{1}, q^*) = \begin{cases} \binom{n-1}{\frac{n-1}{2}} \phi_A^{\frac{n-1}{2}} \phi_B^{\frac{n-1}{2}} & \text{if } n \text{ odd} \\ \binom{n-1}{\frac{n}{2}-1} \phi_A^{\frac{n}{2}} \phi_B^{\frac{n}{2}-1} & \text{if } n \text{ even} \end{cases} \tag{16}$$

where, for ease of notation, $\phi_A = \phi_A(\mathbf{1}, q^*)$ and $\phi_B = \phi_B(\mathbf{1}, q^*)$. Moreover, the expected benefit of acquiring information is

$$\Phi(\mathbf{1}, q^*) = \frac{1}{4}\left(\Pi_A(\mathbf{1}, q^*) + \Pi_B(\mathbf{1}, q^*)\right). \tag{17}$$

Now, first note that if $q^* = 0$, $\phi_A(\mathbf{1}, 0) = \lambda$ and $\phi_B(\mathbf{1}, 0) = 1 - \lambda$. Thus, if $\frac{1}{2} \leq \lambda < 1$, we have $0 < \Pi_A(\mathbf{1}, 0) \leq \Pi_B(\mathbf{1}, 0)$, and hence $\Phi(\mathbf{1}, 0) > 0$. If, however, $\lambda = 1$, we have $\Pi_A(\mathbf{1}, 0) = \Pi_B(\mathbf{1}, 0) = 0$, and hence $\Phi(\mathbf{1}, 0) = 0$.

Second, note that if $q^* = 1$, $\phi_A(\mathbf{1}, 1) = \phi_B(\mathbf{1}, 1) \equiv \phi(\mathbf{1}, 1) = \frac{1}{2}$. Hence, $\Pi_A(\mathbf{1}, 1) = \Pi_B(\mathbf{1}, 1) \equiv \Pi(\mathbf{1}, 1)$ which is given by

$$\Pi(\mathbf{1}, 1) = \begin{cases} \binom{n-1}{\frac{n-1}{2}} \frac{1}{2}^{n-1} & \text{if } n \text{ odd} \\ \binom{n-1}{\frac{n}{2}-1} \frac{1}{2}^{n-1} & \text{if } n \text{ even} \end{cases} \tag{18}$$

where $\Pi(\mathbf{1}, 1) > 0$ such that $\Phi(\mathbf{1}, 1) = \frac{1}{2}\Pi(\mathbf{1}, 1) > 0$.

Let us continue with some observations about the expected benefit of casting an informed vote under Voluntary Voting. Recall that, under Voluntary Voting, $p_U^* = 0$ and $p_A^* = p_B^* \equiv p_I^* > 0$, and also $\phi_A(p_I^*, q^*) = \phi_B(p_I^*, q^*) \equiv \phi(p_I^*, q^*) = \frac{1}{2}q^* p_I^*$ and $\Pi_A(p_I^*, q^*) = \Pi_B(p_I^*, q^*) \equiv \Pi$. Hence,

$$\Phi(p_I^*, q^*) = \left(\frac{1}{2}\Pi(p_I^*, q^*) - k_I^*\right) p_I^* + \int_{\underline{k}}^{k_I^*} H(y)dy. \tag{19}$$

If we have full participation under Voluntary Voting, i.e., if $p_I^* = 1$ and $q^* = 1$, then $\phi = \frac{1}{2}$. Thus, the probability of being pivotal in that case is given by equation (18).

Armed with the characterization of the probabilities of being pivotal and the expected benefit of casting an informed vote, I can now proceed to comparing the probability of acquiring information under Compulsory Voting with full participation to Voluntary Voting.

Proposition 4. *Let $\frac{1}{2} \le \lambda < 1$. Let $\underline{c} < \frac{1}{4}(\Pi_A(\mathbf{1}, 0) + \Pi_B(\mathbf{1}, 0))$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1}, 1)$. Consider voting costs $k_i \in [\underline{k}, \overline{k}]$ where $\overline{k} = \underline{k} + \kappa$. There exists a unique threshold $\underline{\tilde{k}} \in (0, \frac{1}{2} - \underline{c})$ and $\kappa \in (0, \frac{1}{2}\Pi(\mathbf{1}, 1) - \underline{c})$ sufficiently small, such that for low voting costs $\underline{k} < \underline{\tilde{k}}$, the probability of acquiring information under Compulsory Voting with a high abstention fine $f \ge \overline{k}$ is strictly lower than under Voluntary Voting, while for high voting costs $\underline{k} > \underline{\tilde{k}}$, it is strictly higher than under Voluntary Voting.*

Note that the conditions $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1}, 0) + \Pi_B(\mathbf{1}, 0)\right)$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1}, 1)$ ensure that $q^* \in (0, 1)$ in any equilibrium under Compulsory Voting with a high abstention fine $f \ge \overline{k}$. Otherwise, if $\underline{c} > \frac{1}{4}\left(\Pi_A(\mathbf{1}, 0) + \Pi_B(\mathbf{1}, 0)\right)$, there exists an equilibrium in which $q^* = 0$, and if $\overline{c} < \frac{1}{2}\Pi(\mathbf{1}, 1)$, there exists an equilibrium in which $q^* = 1$ under Compulsory Voting with a high abstention fine $f \ge \overline{k}$. Moreover, $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1}, 0) + \Pi_B(\mathbf{1}, 0)\right)$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1}, 1)$ ensure that there exists at least one stable equilibrium.

The intuition behind Proposition 4 is the following: Under Voluntary Voting, uninformed individuals abstain, while Compulsory Voting with $f \ge \overline{k}$ leads to full participation even from uninformed voters. If voting costs are high, participation from informed individuals is low under Voluntary Voting. In that case, Compulsory Voting with $f \ge \overline{k}$ increases participation from both informed and uninformed individuals compared to Voluntary Voting. The expected benefit of acquiring information is increasing in the probability of casting an informed vote, but decreasing in the probability of casting an uninformed vote. Therefore, Compulsory Voting with $f \ge \overline{k}$ increases information acquisition if the increase in participation from informed voters is sufficiently large, i.e., if voting costs are sufficiently high. If, however, voting costs are low, participation from informed individuals is already high under Voluntary Voting. In that case, Compulsory Voting with $f \ge \overline{k}$ cannot increase participation from informed individuals much further, and the negative effect of increased participation from uninformed individuals predominates, such that information acquisition decreases compared to Voluntary Voting.

This effect becomes stronger as the bias of uninformed voters increases:

Proposition 5. *Let $\frac{1}{2} < \lambda < 1$. Let $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1}, 0) + \Pi_B(\mathbf{1}, 0)\right)$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1}, 1)$. Consider a stable equilibrium under Compulsory Voting with a high abstention fine $f \ge \overline{k}$. Then the probability of acquiring information is strictly decreasing in the bias λ of uninformed voters.*

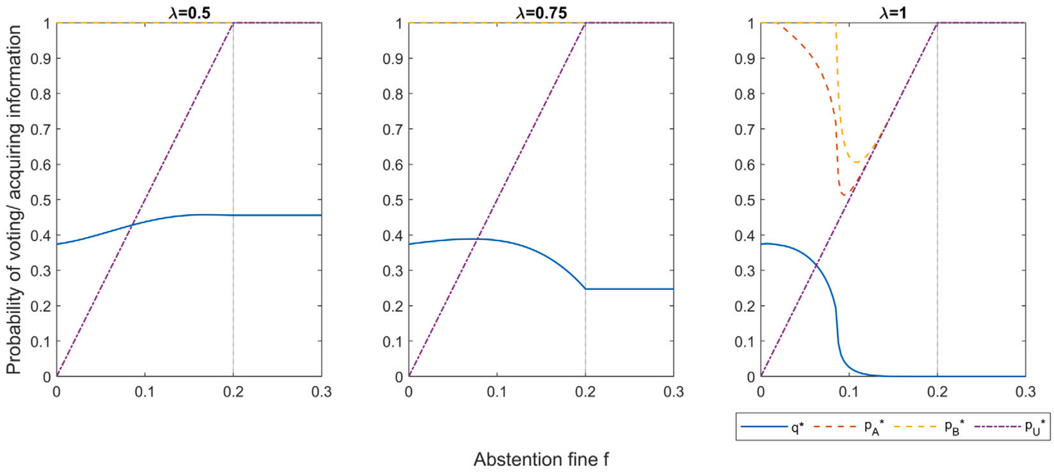
Intuitively, as uninformed voters become more likely to vote for alternative A, the expected majority for A becomes larger under Compulsory Voting with full participation. Therefore, the probability of casting a pivotal vote decreases, which reduces the incentives to acquire information. As a result, there exists an equilibrium in which no individual acquires information under Compulsory Voting with full participation if uninformed voters are fully biased, i.e., $\lambda = 1$:

Proposition 6. *Let $\lambda = 1$. Then not acquiring information is an equilibrium under Compulsory Voting with a high abstention fine $f \ge \overline{k}$.*

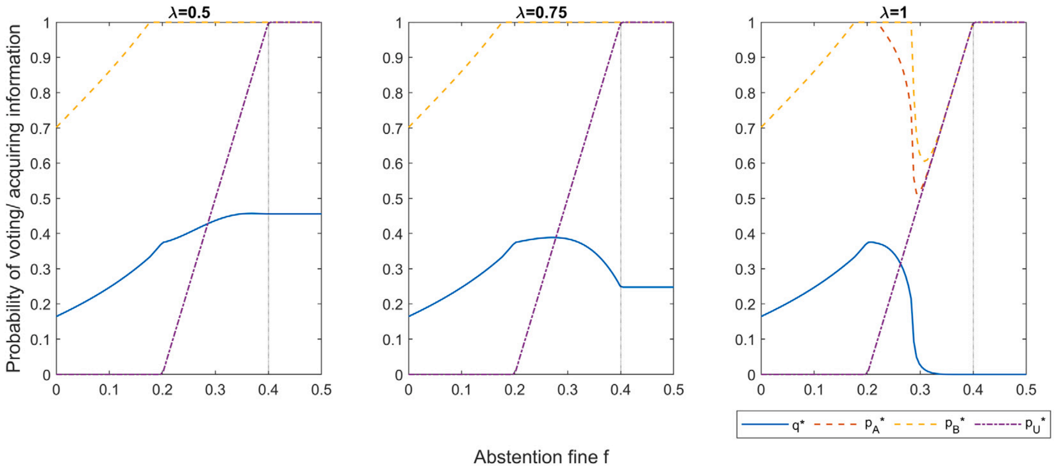
To illustrate the effect of Compulsory Voting on the equilibrium probability of acquiring information, I solve for the type-symmetric pure-strategy Bayesian Nash equilibrium numerically using a Gauss-Seidel algorithm. The equilibrium is unique in all of my numerical examples.

Fig. 1 displays the effect of introducing Compulsory Voting on the equilibrium probabilities of voting and on the equilibrium probability of acquiring information. I consider unbiased uninformed voters ($\lambda = 0.5$) and biased uninformed voters ($\lambda = 0.75$ and $\lambda = 1$). In panel (a), the voting costs are low, while in panel (b), the voting costs are high, in the sense that the distribution of voting costs in (b) first-order stochastically dominates the distribution in (a).

All figures illustrate the properties of the equilibrium voting probabilities as described in Remark 1. The figures in panel (b) are in line with the result from Proposition 3: A small abstention fine $0 < f < \underline{k}$ increases participation from informed individuals only, and therefore also increases information acquisition. The comparison between the figures for $\lambda = 0.75$ in panel (a) and (b) displays the result from Proposition 4: For low voting costs (panel (a)), Compulsory Voting with a high abstention fine $f \ge \overline{k}$ reduces the probability of acquiring information compared to Voluntary Voting, while for high voting costs (panel (b)), it increases the probability of acquiring information compared to Voluntary Voting.



(a) $k_i \sim U[0, 0.2]$



(b) $k_i \sim U[0.2, 0.4]$

Fig. 1. The effect of an abstention fine f on the equilibrium probabilities of voting, p_A^*, p_B^*, p_U^*, and on the equilibrium probability of acquiring information, q^*, in the neutral preference setting with differing strength $\lambda \geq \frac{1}{2}$ of the bias of uninformed voters towards alternative A. The information costs are uniformly distributed on the interval $[0, 0.3]$. In panel (a) the voting costs are uniformly distributed on the interval $[0, 0.2]$ and in panel (b) the voting costs are uniformly distributed on the interval $[0.2, 0.4]$. The vertical line indicates the upper bound of the voting costs, $\overline{k}$. There are $n = 9$ individuals in the electorate. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

As expected from Proposition 5, the probability of acquiring information under Compulsory Voting with a high abstention fine $f \geq \overline{k}$ is strictly decreasing in the bias of uninformed voters, λ. Moreover, the figures show that, as expected from Proposition 6, if uninformed individuals are fully biased ($\lambda = 1$), no individual acquires information under Compulsory Voting with a high abstention fine $f \geq \overline{k}$.

Welfare

The expected utility of an individual consists of the expected utility given that the individual acquires information, plus the expected utility given that the individual does not acquire information. In both cases, she can either cast a vote (informed or uninformed), or abstain. Hence, expected utility given the equilibrium probabilities of voting, $\mathbf{p}^* = (p_A^*, p_B^*, p_U^*)$, and the equilibrium probability of acquiring information, q^*, is given by

$$U(\mathbf{p}^*, q^*) = \frac{1}{2} Pr(A \text{ wins}) + \frac{1}{2} Pr(B \text{ wins})$$
$$+ \int_{\underline{c}}^{c^*} \left[\frac{1}{2} \left[\int_{\underline{k}}^{k_A^*} \left(\frac{1}{2} \Pi_A(\mathbf{p}^*, q^*) - k \right) h(k) \mathrm{d}k - \int_{k_A^*}^{\overline{k}} f h(k) \mathrm{d}k \right] \right. \tag{20}$$

$$+\frac{1}{2}\left[\int_{\underline{k}}^{k_B^*}\left(\frac{1}{2}\Pi_B(\mathbf{p}^*,q^*)-k\right)h(k)\mathrm{d}k-\int_{k_B^*}^{\overline{k}} f\,h(k)\mathrm{d}k\right]-c\Bigg]g(c)\mathrm{d}c$$
$$+\int_{c^*}^{\overline{c}}\left[-\int_{\underline{k}}^{k_U^*} kh(k)\mathrm{d}k-\int_{k_U^*}^{\overline{k}} f\,h(k)\mathrm{d}k\right]g(c)\mathrm{d}c$$

where the first line represents the individual's expected utility if she casts a vote but is not pivotal, or if she abstains. Intuitively, with probability $\frac{1}{2}$, she favors A, and hence she will get a payoff of 1 only if A wins and 0 otherwise. With probability $\frac{1}{2}$, she favors B, and hence she will get a payoff of 1 only if B wins and 0 otherwise. The second and third line represent her expected utility if she acquires information while the fourth line represents her expected utility if she remains uninformed. Moreover, we have (adapted from Taylor and Yildirim, 2010a)

$$\begin{aligned} Pr(A\text{ wins}) = {} & \frac{1}{2}\sum_{l=0}^{\lfloor\frac{n}{2}\rfloor}\binom{n}{l,l,n-2l}\phi_A^l\phi_B^l(1-\phi_A-\phi_B)^{n-2l} \\ & +\sum_{l=1}^{\lfloor\frac{n+1}{2}\rfloor}\sum_{l'=0}^{l-1}\binom{n}{l,l',n-l-l';}\phi_A^l\phi_B^{l'}(1-\phi_A-\phi_B)^{n-l-l'} \\ & +\sum_{l=\lfloor\frac{n+1}{2}\rfloor+1}^{n}\sum_{l'=0}^{n-l}\binom{n}{l,l',n-l-l';}\phi_A^l\phi_B^{l'}(1-\phi_A-\phi_B)^{n-l-l'} \end{aligned} \tag{21}$$

where, for ease of notation, $\phi_A \equiv \phi_A(\mathbf{p}^*,q^*)$ and $\phi_B \equiv \phi_B(\mathbf{p}^*,q^*)$, and $Pr(B\text{ wins}) = 1 - Pr(A\text{ wins})$.

Note that expected utility can be rewritten as

$$\begin{aligned} U(\mathbf{p}^*,q^*) = {} & \frac{1}{2}+q^*\frac{1}{2}\Bigg[\Bigg[\frac{1}{2}\Pi_A(\mathbf{p}^*,q^*)p_A^*-\int_{\underline{k}}^{k_A^*}kh(k)\mathrm{d}k-f(1-p_A^*)\Bigg] \\ & +\Bigg[\frac{1}{2}\Pi_B(\mathbf{p}^*,q^*)p_B^*-\int_{\underline{k}}^{k_B^*}kh(k)\mathrm{d}k-f(1-p_B^*)\Bigg]\Bigg] \\ & -(1-q^*)\Bigg[\int_{\underline{k}}^{k_U^*}kh(k)\mathrm{d}k+f(1-p_U^*)\Bigg] \\ & -\int_{\underline{c}}^{c^*}cg(c)\mathrm{d}c. \end{aligned} \tag{22}$$

Under the assumption that the expected revenue generated from the abstention fine is redistributed to the individuals, expected social welfare can be written as

$$\begin{aligned} W(\mathbf{p}^*,q^*) = n\Bigg(& \frac{1}{2}+q^*\frac{1}{2}\Bigg[\Bigg[\frac{1}{2}\Pi_A(\mathbf{p}^*,q^*)p_A^*-\int_{\underline{k}}^{k_A^*}kh(k)\mathrm{d}k\Bigg] \\ & +\Bigg[\frac{1}{2}\Pi_B(\mathbf{p}^*,q^*)p_B^*-\int_{\underline{k}}^{k_B^*}kh(k)\mathrm{d}k\Bigg]\Bigg] \\ & -(1-q^*)\Bigg[\int_{\underline{k}}^{k_U^*}kh(k)\mathrm{d}k\Bigg] \\ & -\int_{\underline{c}}^{c^*}cg(c)\mathrm{d}c.\Bigg). \end{aligned} \tag{23}$$

In the following, I will analyze how expected social welfare is affected on the one hand by the introduction of a marginal abstention fine $0 < f < \underline{k}$ which does not necessarily lead to full turnout, and on the other hand by the introduction of a high abstention fine $f \geq \overline{k}$ which leads to full turnout. I will also study the effect of the bias λ of uninformed voters on expected social welfare.

First, consider the case of a marginal abstention fine $0 < f < \underline{k}$. Recall from Remark 1 that then, $p_U^* = 0$ and $p_A^* = p_B^* > 0$ for all $\lambda \geq \frac{1}{2}$. Therefore, let $p_I^* \equiv p_A^* = p_B^*$ denote the equilibrium probability of voting for an informed individual. In that case, expected social welfare is given by

$$W(p_I^*, q^*) = n\left(\frac{1}{2} + q^*\Phi(p_I^*, q^*) - \int_{\underline{c}}^{c^*} cg(c)\mathrm{d}c\right) \tag{24}$$

where $\Phi(p_I^*, q^*)$ is the expected benefit of acquiring information as given by equation (19).

Proposition 7. *If $0 \leq f < \underline{k}$, expected social welfare weakly decreases in the abstention fine f. It decreases strictly if $q^* \in (0,1)$ and $p_I^* \in (0,1)$.*

The reduction in social welfare is driven by the negative externality of voting (see also Börgers, 2004): Because uninformed individuals abstain and A- and B-voters participate with equal probability, the increase in turnout from informed voters reduces the expected benefit of acquiring information. At the same time, both information and voting costs increase. Thus, again, smaller expected benefits and higher participation cost imply a reduction in expected social welfare.

Next, consider the case of a high abstention fine $f \geq \overline{k}$. Recall from Remark 1 that we have full turnout in that case, i.e., $p_U^* = p_A^* = p_B^* = 1$. Hence, $\phi_A = \frac{1}{2}q^* + \lambda(1-q^*)$ and $\phi_B = \frac{1}{2}q^* + (1-\lambda)(1-q^*)$. Hence, we have $\phi_A \geq \phi_B$ and therefore $\Pi_A \leq \Pi_B$, with strict inequality if $\lambda > \frac{1}{2}$. In that case, expected social welfare is given by

$$W(\mathbf{1}, q^*) = n\left(\frac{1}{2} + q^*\Phi(\mathbf{1}, q^*) - \int_{\underline{k}}^{\overline{k}} kh(k)\mathrm{d}k - \int_{\underline{c}}^{c^*} cg(c)\mathrm{d}c\right) \tag{25}$$

where $\Phi(\mathbf{1}, q^*) = \frac{1}{4}(\Pi_A(\mathbf{1}, q^*) + \Pi_B(\mathbf{1}, q^*))$.

Proposition 8. *Let $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1},0) + \Pi_B(\mathbf{1},0)\right)$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1},1)$. The introduction of a high abstention fine $f \geq \overline{k}$ strictly reduces expected social welfare compared to Voluntary Voting.*

Recall from Proposition 4 that Compulsory Voting with a high abstention fine $f \geq \overline{k}$ does not necessarily increase the probability of acquiring information compared to Voluntary Voting, but might instead reduce information acquisition if the voting costs are sufficiently small.

If uninformed voters are unbiased ($\lambda = \frac{1}{2}$), the increase in participation from both informed and uninformed voters under Compulsory Voting exerts a negative externality on all other voters. Therefore, and because of full turnout, the expected benefits of casting a pivotal, informed vote are smaller under Compulsory Voting with $f \geq \overline{k}$ than under Voluntary Voting, and the expected voting costs are higher. The expected information costs, however, decrease if the probability of acquiring information decreases compared to Voluntary Voting. Proposition 8 shows that the reduction in benefits always outweighs the reduction in information costs, such that expected social welfare decreases.

If uninformed voters are biased ($\lambda > \frac{1}{2}$), recall that B-voters are more likely to vote than A-voters. The increase in participation from B-voters under Compulsory Voting can exert a positive externality on other voters. At the same time, participation from – informed and uninformed – A-voters increases as well, thereby exerting a negative externality on others. Proposition 8 shows that the increase in voting costs and the negative externality from the increase in A-votes outweighs the positive externality from the increase in B-votes and the potential reduction in information costs. Therefore, again, expected social welfare is lower under Compulsory Voting with a high abstention fine $f \geq \overline{k}$ than under Voluntary Voting.

This effect becomes even stronger as the bias λ of uninformed voters increases:

Proposition 9. *Let $\frac{1}{2} < \lambda < 1$. Let $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1},0) + \Pi_B(\mathbf{1},0)\right)$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1},1)$. Consider a stable equilibrium under Compulsory Voting with a high abstention fine $f \geq \overline{k}$. Then expected social welfare is strictly decreasing in the bias λ of uninformed voters.*

Intuitively, an increase in the bias λ of uninformed voters increases the probability that any individual votes for alternative A under Compulsory Voting with a high abstention fine $f \geq \overline{k}$. Hence, both A- and B-voters are less likely to cast a pivotal vote, and the expected benefits of acquiring information decrease for all voters. We know from Proposition 5 that then, the probability of acquiring information under Compulsory Voting with a high abstention fine $f \geq \overline{k}$ decreases, thereby reducing expected information costs. Proposition 9 shows that the reduction in expected benefits outweighs the reduction in expected information costs.

4. Conclusion

The most important result of my analysis is that – in contrast to the prevailing view in the literature – Compulsory Voting with full participation does not necessarily achieve a collective outcome that accurately reflects the majority's preference. If individuals are initially uninformed about their preferred alternative and acquiring information is costly, this result would require that incentivizing participation with an abstention fine also incentivizes information acquisition.

I show that while a small abstention fine that does not achieve full turnout indeed always increases information acquisition, a high abstention fine that achieves full turnout does so only if the voting costs are sufficiently high. If the voting costs are low, the opposite is true: A high abstention fine that achieves full turnout reduces information acquisition compared to Voluntary Voting. In particular, in the limit with zero voting costs, it is impossible to incentivize information acquisition with an abstention fine. If uninformed voters are biased, the incentives to acquire information are reduced further.

When Compulsory Voting only incentivizes participation but not information acquisition, the preference of the majority cannot necessarily be inferred from the outcome of the collective decision anymore.

Moreover, I show that because of the negative externality of voting, expected social welfare under Compulsory Voting is lower than under Voluntary Voting. This result holds even in the limit with zero voting costs. Under Compulsory Voting with full participation, expected social welfare is further reduced when the bias of uninformed voters increases.

Therefore – coming back to my initial thought experiment – I show that, compared to Voluntary Voting, nearly costless but mandatory elections have a detrimental effect both on the quality of the collective decision and on expected social welfare.

Future research could allow for a correlation between information costs and preferences. This might be the case if less educated individuals, for whom it is more costly to acquire policy-specific information, have systematically different policy preferences than more educated individuals. In that case, under Voluntary Voting where only informed individuals participate in the election, the outcome of the election is biased toward the preference of those with low information costs. Compulsory Voting might make participation between voters with different preferences more balanced and achieve a policy outcome that reflects the preferences of the entire electorate, not just of the subgroup of individuals with low information costs.

Declaration of competing interest

None.

Data availability

No data was used for the research described in the article.

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Appendix A. Proofs

Proof of Proposition 1.

I need to show that there exists a type-symmetric pure-strategy Bayesian Nash equilibrium, in which the following conditions are satisfied simultaneously for the equilibrium probabilities of voting, $\mathbf{p}^* = (p_A^*, p_B^*, p_U^*)$, and the equilibrium probability of acquiring information, q^*:

$$q^* = G(\Phi(\mathbf{p}^*, q^*))$$
$$p_A^* = H(\varphi_A(\mathbf{p}^*, q^*))$$
$$p_B^* = H(\varphi_B(\mathbf{p}^*, q^*))$$
$$p_U^* = H(\varphi_U(\mathbf{p}^*, q^*))$$

To show that such an equilibrium exists, define

$$\xi(\mathbf{p}, q) = \left(G(\Phi(\mathbf{p}, q), H(\varphi_A(\mathbf{p}, q)), H(\varphi_B(\mathbf{p}, q)), H(\varphi_U(\mathbf{p}, q)) \right). \tag{A.1}$$

From conditions (9) and (14), it is clear that the equilibrium probabilities of voting, $\mathbf{p}^* = (p_A^*, p_B^*, p_U^*)$, and the equilibrium probability of acquiring information, q^*, are a fixed point of ξ. Since ξ maps the compact and convex set $[0, 1]^4$ into itself, and since ξ is continuous, we have, by Brouwer's Fixed Point Theorem, such a fixed point of ξ exists. □

Proof of Remark 1.

(i) Let $\lambda = \frac{1}{2}$. Then $\phi_A(\mathbf{p}^*, q^*) = \frac{1}{2}\left(q^* p_A^* + (1-q^*)p_U^*\right)$ and $\phi_B(\mathbf{p}^*, q^*) = \frac{1}{2}\left(q^* p_B^* + (1-q^*)p_U^*\right)$. First I want to show that $p_A^* = p_B^*$ in equilibrium. Suppose for a contradiction that $p_A^* > p_B^*$. Then, $\phi_A > \phi_B$. Then, because $\Pi_A - \Pi_B = sign(\phi_B - \phi_A)$, we have $\Pi_A < \Pi_B$. But then, by the equilibrium definition of p_A^* and p_B^*, $p_A^* = H\left(\frac{1}{2}\Pi_A + f\right) \le H\left(\frac{1}{2}\Pi_B + f\right) = p_B^*$, which is a contradiction. Similarly, we get a contradiction if we assume $p_A^* > p_B^*$. Therefore, $p_A^* = p_B^*$ in equilibrium. Second, I need to show that $p_A^*, p_B^* > 0$. Suppose for a contradiction that $p_A^* = p_B^* \equiv p_I^* = 0$. Then $\phi_A = \phi_B = 0$ and $\Pi_A = \Pi_B = 1$. But then, by $\underline{k} < \frac{1}{2}$, $p_I^* = H(\frac{1}{2} + f) > H(\underline{k}) = 0$, which is a contradiction. Hence, $p_A^*, p_B^* > 0$.

(ii) Let $\lambda > \frac{1}{2}$.

(a) Let $f \le \underline{k}$. Then $p_U^* = H(f) = 0$. I want to show that $p_A^* = p_B^*$ in equilibrium. To do so, I need to consider two cases: either $q^* = 0$ or $q^* > 0$. First, consider the case where $q^* = 0$. Then $\phi_A = \phi_B = 0$ by $p_U^* = 0$, and $\Pi_A = \Pi_B = 1$. Hence, $p_A^* = H(\frac{1}{2} + f) = p_B^*$. Second, consider the case where $q^* > 0$. Suppose for a contradiction that $p_A^* < p_B^*$ in equilibrium. Then $\phi_A > \phi_B$ and $\Pi_A < \Pi_B$. But then, $p_A^* = H\left(\frac{1}{2}\Pi_A + f\right) \le H\left(\frac{1}{2}\Pi_B + f\right) = p_B^*$, which is a contradiction. Similarly, we get a contradiction if we assume $p_A^* > p_B^*$. Therefore, $p_A^* = p_B^*$ in equilibrium. It remains to be shown that $p_A^*, p_B^* > 0$. Suppose for a contradiction that $p_A^* = p_B^* \equiv p_I^* = 0$. Then $\phi_A = \phi_B = 0$ and $\Pi_A = \Pi_B = 1$. But then, by $\underline{k} < \frac{1}{2}$, $p_I^* = H(\frac{1}{2} + f) > H(\underline{k}) = 0$, which is a contradiction. Hence, $p_A^*, p_B^* > 0$.

(b) Let $f \in (\underline{k}, \overline{k})$. Then $p_U^*, p_A^*, p_B^* > 0$ and $p_U^* < 1$. First, $p_U^* < p_A^*$ follows directly from the equilibrium definition of p_U^* and p_A^* and $\Pi_A > 0$: $p_U^* = H(f) < H(\frac{1}{2}\Pi_A + f) = p_A^*$.

Second, I want to show that $p_A^* \le p_B^*$. Suppose for a contradiction that $p_A^* > p_B^*$. Then, for all $q^* \ge 0$, $\phi_A > \phi_B$ because $\lambda > \frac{1}{2}$ and $p_U^* > 0$. Thus, $\Pi_A < \Pi_B$. But then, $p_A^* = H(\frac{1}{2}\Pi_A + f) \le H(\frac{1}{2}\Pi_B + f) = p_B^*$, which is a contradiction. Hence, $p_A^* \le p_B^*$.

Third, I want to show that if $p_A^* < 1$, then $p_A^* < p_B^*$. Suppose for a contradiction that $p_A^* < 1$ but $p_A^* = p_B^*$. Then, for all $q^* \ge 0$, $\phi_A > \phi_B$ because $\lambda > \frac{1}{2}$ and $p_U^* > 0$. Thus, $\Pi_A < \Pi_B$. But then, $p_A^* = H(\frac{1}{2}\Pi_A + f) < H(\frac{1}{2}\Pi_B + f) = p_B^*$, which is a contradiction.

(c) Let $f \ge \overline{k}$. It follows directly that $p_U^* = H(f) = H(\overline{k}) = 1$ and $p_A^* = H(\frac{1}{2}\Pi_A + f) = H(\overline{k}) = 1$ and $p_B^* = H(\frac{1}{2}\Pi_B + f) = H(\overline{k}) = 1$. □

Proof of Proposition 2.

Let $\lambda \ge \frac{1}{2}$. I want to show that, if $\frac{1-\lambda}{\lambda} \ge 1 - \frac{1}{\lfloor \frac{n}{2} \rfloor}$, the symmetric pure-strategy Bayesian Nash equilibrium is unique.

Let $\phi_A^* \equiv \phi_A(\mathbf{p}^*, q^*) = \frac{1}{2}q^* p_A^* + \lambda(1-q^*)p_U^*$ and $\phi_B^* \equiv \phi_B(\mathbf{p}^*, q^*) = \frac{1}{2}q^* p_B^* + (1-\lambda)(1-q^*)p_U^*$. From Taylor and Yildirim (2010a) we have that there exists at most one equilibrium that satisfies $1 \ge \frac{\phi_B^*}{\phi_A^*} \ge 1 - \frac{1}{\lfloor \frac{n}{2} \rfloor}$.

Now, I want to show that $\frac{\phi_B^*}{\phi_A^*} \ge \frac{1-\lambda}{\lambda}$. Suppose for a contradiction that $\frac{\phi_B^*}{\phi_A^*} < \frac{1-\lambda}{\lambda}$. Plugging in $\phi_A^* = \phi_A(\mathbf{p}^*, q^*)$ and $\phi_B^* = \phi_B(\mathbf{p}^*, q^*)$ as above, this is

$$\frac{\frac{1}{2}q^* p_B^* + (1-q^*)(1-\lambda)p_U^*}{\frac{1}{2}q^* p_A^* + (1-q^*)\lambda p_U^*} < \frac{1-\lambda}{\lambda}$$

which, rearranging, is equivalent to

$$\frac{p_B^*}{p_A^*} < \frac{1-\lambda}{\lambda}.$$

This is a contradiction, because we have $p_B^* \ge p_A^*$ in equilibrium such that $\frac{p_B^*}{p_A^*} \ge 1$ while $\frac{1-\lambda}{\lambda} < 1$ because $\lambda > \frac{1}{2}$.

Therefore we must have that $\frac{\phi_B^*}{\phi_A^*} \ge \frac{1-\lambda}{\lambda}$ always in equilibrium. Then, if $\frac{1-\lambda}{\lambda} \ge 1 - \frac{1}{\lfloor \frac{n}{2} \rfloor}$, we have $\frac{\phi_B^*}{\phi_A^*} \ge 1 - \frac{1}{\lfloor \frac{n}{2} \rfloor}$ in any equilibrium, and because at most one equilibrium with this property can exist, we can conclude that the equilibrium is unique. □

Proof of Proposition 3.

Let $\lambda \ge \frac{1}{2}$. Suppose $0 \le f < \underline{k}$. I need to show that the probability of acquiring information weakly increases in the abstention fine f, and that it increases strictly if $0 < q^* < 1$ and $p_I^* < 1$.

To do so, consider the total differential $\frac{\mathrm{d}q^*}{\mathrm{d}f}$, which is given by (where I most of the time suppress the arguments $(\mathbf{p}^*, q^*)$ for ease of notation)

$$\begin{aligned}\frac{\mathrm{d}q^*}{\mathrm{d}f} &= \frac{\mathrm{d}}{\mathrm{d}f} G(\Phi(\mathbf{p}^*, q^*)) \\ &= g(\Phi(\mathbf{p}^*, q^*)) \left[\frac{\partial \Phi}{\partial p_A^*}\frac{\mathrm{d}p_A^*}{\mathrm{d}f} + \frac{\partial \Phi}{\partial p_B^*}\frac{\mathrm{d}p_B^*}{\mathrm{d}f} + \frac{\partial \Phi}{\partial p_U^*}\frac{\mathrm{d}p_U^*}{\mathrm{d}f} \right].\end{aligned}$$

The total differential $\frac{\mathrm{d}p_A^*}{\mathrm{d}f}$ is given by

$$\begin{aligned}\frac{\mathrm{d}p_A^*}{\mathrm{d}f} &= \frac{\mathrm{d}}{\mathrm{d}f}H(\varphi(\mathbf{p}^*,q^*))\\ &= h(\varphi(\mathbf{p}^*,q^*))\left[\frac{1}{2}\frac{\partial\Pi_A}{\partial\phi_A}\left(\frac{\partial\phi_A}{\partial p_A^*}\frac{\mathrm{d}p_A^*}{\mathrm{d}f}+\frac{\partial\phi_A}{\partial p_U^*}\frac{\mathrm{d}p_U^*}{\mathrm{d}f}+\frac{\partial\phi_A}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}f}\right)\right.\\ &\quad\left.+\frac{1}{2}\frac{\partial\Pi_A}{\partial\phi_B}\left(\frac{\partial\phi_B}{\partial p_B^*}\frac{\mathrm{d}p_B^*}{\mathrm{d}f}+\frac{\partial\phi_B}{\partial p_U^*}\frac{\mathrm{d}p_U^*}{\mathrm{d}f}+\frac{\partial\phi_B}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}f}\right)+1\right]\end{aligned}$$

and the total differential $\frac{\mathrm{d}p_B^*}{\mathrm{d}f}$ is given by

$$\begin{aligned}\frac{\mathrm{d}p_B^*}{\mathrm{d}f} &= \frac{\mathrm{d}}{\mathrm{d}f}H(\varphi(\mathbf{p}^*,q^*))\\ &= h(\varphi(\mathbf{p}^*,q^*))\left[\frac{1}{2}\frac{\partial\Pi_B}{\partial\phi_A}\left(\frac{\partial\phi_A}{\partial p_A^*}\frac{\mathrm{d}p_A^*}{\mathrm{d}f}+\frac{\partial\phi_A}{\partial p_U^*}\frac{\mathrm{d}p_U^*}{\mathrm{d}f}+\frac{\partial\phi_A}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}f}\right)\right.\\ &\quad\left.+\frac{1}{2}\frac{\partial\Pi_B}{\partial\phi_B}\left(\frac{\partial\phi_B}{\partial p_B^*}\frac{\mathrm{d}p_B^*}{\mathrm{d}f}+\frac{\partial\phi_B}{\partial p_U^*}\frac{\mathrm{d}p_U^*}{\mathrm{d}f}+\frac{\partial\phi_B}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}f}\right)+1\right].\end{aligned}$$

Note that from Proposition 1 as long as $0 \le f < \underline{k}$ we have $p_U^* = 0$. Thus, $\phi_A = \phi_B$ and $\Pi_A = \Pi_B$, which implies $\frac{\mathrm{d}p_A^*}{\mathrm{d}f} = \frac{\mathrm{d}p_B^*}{\mathrm{d}f}$. To simplify notation, let $p_I^* \equiv p_A^* = p_B^*$ and $\phi \equiv \phi_A = \phi_B = \frac{1}{2}q^* p_I^*$ and $\Pi \equiv \Pi_A = \Pi_B$.

Rearranging yields

$$\frac{\mathrm{d}p_I^*}{\mathrm{d}f} = \frac{h(\varphi(\mathbf{p}^*,q^*))\left[\frac{1}{2}\frac{\partial\Pi}{\partial\phi}\left(\frac{\partial\phi}{\partial p_U}\frac{\mathrm{d}p_U^*}{\mathrm{d}f}+\frac{\partial\phi}{\partial q}\frac{\mathrm{d}q^*}{\mathrm{d}f}\right)+1\right]}{1-h(\varphi(\mathbf{p}^*,q^*))\frac{1}{2}\frac{\partial\Pi}{\partial\phi}\frac{\partial\phi}{\partial p_I}}. \tag{A.2}$$

Moreover, $\frac{\partial\Phi}{\partial p_A^*} = \frac{1}{2}p_A^*\frac{1}{h(k_A^*)}$ and $\frac{\partial\Phi}{\partial p_B^*} = \frac{1}{2}p_B^*\frac{1}{h(k_B^*)}$. Thus, we can write that $\frac{\partial\Phi}{\partial p_I^*} = \frac{1}{2}p_I^*\frac{1}{h(k_I^*)}$.

Plugging in $\frac{\mathrm{d}p_A^*}{\mathrm{d}f} = \frac{\mathrm{d}p_B^*}{\mathrm{d}f} = \frac{\mathrm{d}p_I^*}{\mathrm{d}f}$, $\frac{\mathrm{d}q^*}{\mathrm{d}f}$ becomes

$$\frac{\mathrm{d}q^*}{\mathrm{d}f} = g(\Phi(\mathbf{p}^*,q^*))\left[2\frac{\partial\Phi}{\partial p_I^*}\frac{\mathrm{d}p_I^*}{\mathrm{d}f}+\frac{\partial\Phi}{\partial p_U^*}\frac{\mathrm{d}p_U^*}{\mathrm{d}f}\right]. \tag{A.3}$$

Plugging in $\frac{\mathrm{d}p_I^*}{\mathrm{d}f}$ from above and rearranging yields

$$\frac{\mathrm{d}q^*}{\mathrm{d}f} = \frac{g(\Phi(\mathbf{p}^*,q^*))\left[2\frac{\partial\Phi}{\partial p_I}h(\varphi(\mathbf{p}^*,q^*))\left(\frac{1}{2}\frac{\partial\Pi}{\partial\phi}\frac{\partial\phi}{\partial p_U^*}\frac{\mathrm{d}p_U^*}{\mathrm{d}f}+1\right)+\frac{\partial\Phi}{\mathrm{d}p_U}\frac{\mathrm{d}p_U^*}{df}\left(1-h(\varphi(\mathbf{p}^*,q^*))\frac{1}{2}\frac{\partial\Pi}{\partial\phi}\frac{\partial\phi}{\partial p_I^*}\right)\right]}{1-h(\varphi(\mathbf{p}^*,q^*))\frac{1}{2}\frac{\partial\Pi}{\partial\phi}\left[\frac{\partial\phi}{\partial p_I^*}+g(\Phi(\mathbf{p}^*,q^*))2\frac{\partial\Phi}{\partial p_I^*}\frac{\partial\phi}{\partial q^*}\right]}$$

In order to evaluate $\frac{\mathrm{d}q^*}{\mathrm{d}f}$ at $f = 0$, recall that $p_U^* = H(f)$ and $\underline{k} > 0$ such that $\frac{\mathrm{d}p_U^*}{\mathrm{d}f}\Big|_{0\le f<\underline{k}} = h(f)\Big|_{0\le f<\underline{k}} = h(0) = 0$. Thus,

$$\frac{\mathrm{d}q^*}{\mathrm{d}f}\Big|_{0\le f<\underline{k}} = \frac{g(\Phi(\mathbf{p}^*,q^*))h(\varphi(\mathbf{p}^*,q^*))2\frac{\partial\Phi}{\partial p_I^*}}{1-h(\varphi(\mathbf{p}^*,q^*))\frac{1}{2}\frac{\partial\Pi}{\partial\phi}\left[\frac{\partial\phi}{\partial p_I}+g(\Phi(\mathbf{p}^*,q^*))2\frac{\partial\Phi}{\partial p_I}\frac{\partial\phi}{\partial q}\right]}\Big|_{0\le f<\underline{k}}. \tag{A.4}$$

To sign $\frac{\mathrm{d}q^*}{\mathrm{d}f}\Big|_{0\le f<\underline{k}}$, I now need to derive the signs of its individual parts.

Lemma A.1. $\frac{\partial\Pi(\mathbf{p}^*,q^*)}{\partial\phi} < 0$ *for all* q^*.

Proof. If $\phi_A(\mathbf{p}^*,q^*) = \phi_B(\mathbf{p}^*,q^*) \equiv \phi(\mathbf{p}^*,q^*)$ the probability of being pivotal is given by

$$\begin{aligned}\Pi(\mathbf{p}^*,q^*) = &\sum_{l=0}^{\lfloor\frac{n-1}{2}\rfloor}\binom{n-1}{l,l,n-1-2l}\phi^{2l}(1-2\phi)^{n-1-2l}\\ &+\sum_{l=0}^{\lfloor\frac{n-2}{2}\rfloor}\binom{n-1}{l,l+1,n-2-2l}\phi^{2l+1}(1-2\phi)^{n-2-2l}.\end{aligned} \tag{A.5}$$

The derivative of Π with respect to ϕ is

$$\begin{aligned}\frac{\partial \Pi(\mathbf{p}^*, q^*)}{\partial \phi} = &\sum_{l=1}^{\lfloor \frac{n-1}{2} \rfloor} \binom{n-1}{l, l, n-1-2l} 2l\ \phi^{2l-1}(1-2\phi)^{n-1-2l} \\ &- 2\sum_{l=0}^{\lfloor \frac{n-2}{2} \rfloor} \binom{n-1}{l, l, n-1-2l} (n-1-2l)\phi^{2l}(1-2\phi)^{n-2-2l} \\ &+ \sum_{l=0}^{\lfloor \frac{n-2}{2} \rfloor} \binom{n-1}{l, l+1, n-1-2l} (2l+1)\phi^{2l}(1-2\phi)^{n-2-2l} \\ &- 2\sum_{l=0}^{\lfloor \frac{n-3}{2} \rfloor} \binom{n-1}{l, l+1, n-2-2l} (n-2-2l)\phi^{2l+1}(1-2\phi)^{n-3-2l}.\end{aligned}$$

Rearranging,

$$\begin{aligned}\frac{\partial \Pi(\mathbf{p}^*, q^*)}{\partial \phi} = &2\sum_{l=1}^{\lfloor \frac{n-1}{2} \rfloor} \frac{(n-1)!}{(l-1)!l!(n-1-2l)!}\phi^{2l-1}(1-2\phi)^{n-1-2l} \\ &- 2\sum_{l=0}^{\lfloor \frac{n-2}{2} \rfloor} \frac{(n-1)!}{l!l!(n-2-2l)!}\phi^{2l}(1-2\phi)^{n-2-2l} \\ &+ \sum_{l=0}^{\lfloor \frac{n-2}{2} \rfloor} \frac{(n-1)!}{l!(l+1)!(n-2-2l)!}(2l+1)\phi^{2l}(1-2\phi)^{n-2-2l} \\ &- 2\sum_{l=0}^{\lfloor \frac{n-2}{2} \rfloor} \frac{(n-1)!}{l!(l+1)!(n-3-2l)!}\phi^{2l+1}(1-2\phi)^{n-3-2l}\end{aligned}$$

where the first and fourth term cancel out. Therefore,

$$\frac{\partial \Pi(\mathbf{p}^*, q)}{\partial \phi} = \sum_{l=0}^{\lfloor \frac{n-2}{2} \rfloor} \frac{(n-1)!}{l!l!(n-2-2l)!}\phi^{2l}(1-2\phi)^{n-2-2l}\left[\frac{2l+1}{l+1} - 2\right]$$

which is negative because $\frac{2l+1}{l+1} - 2 = -\frac{1}{l+1} < 0$ for all $l \geq 0$. Hence, $\frac{\partial \Pi(\mathbf{p}^*, q^*)}{\partial \phi} < 0$. □

We also have that $\frac{\partial \phi(\mathbf{p}^*, q^*)}{\partial p_I^*} = \frac{1}{2}q^*$ which is strictly positive if $q^* > 0$, and $\frac{\partial \phi(\mathbf{p}^*, q^*)}{\partial q^*} = \frac{1}{2}p_I^*$ which is strictly positive because $p_I^* > 0$. Moreover, $\frac{\partial \Phi}{\partial p_I^*} = \frac{1}{2}p_I^* \frac{1}{h(k_I^*)}$ is again strictly positive because $p_I^* > 0$ and $h(y) > 0$ for all $y \in [\underline{k}, \overline{k}]$.

Thus, taking all parts together, I have shown that $\frac{\mathrm{d}q^*}{\mathrm{d}f}\Big|_{0 \leq f < \underline{k}} \geq 0$, i.e., the probability of acquiring information is non-decreasing in the abstention fine f as long as $0 \leq f < \underline{k}$.

If $0 < q^* < 1$ and $p_I^* < 1$, we can be sure that $g(\Phi(\mathbf{p}^*, q^*)) > 0$ and $h(\varphi(\mathbf{p}^*, q^*)) > 0$, such that $\frac{\mathrm{d}q^*}{\mathrm{d}f}\Big|_{0 \leq f < \underline{k}} > 0$ with strict inequality, i.e., the probability of acquiring information is *strictly* increasing in the abstention fine f as long as $0 \leq f < \underline{k}$. □

Proof of Proposition 4.

To simplify notation, let q^{*V} denote the probability of acquiring information under Voluntary Voting, and let q^{*C} denote the probability of acquiring information under Compulsory Voting with full participation ($f \geq \overline{k}$). Let $\frac{1}{2} \leq \lambda < 1$ and $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1}, 0) + \Pi_B(\mathbf{1}, 0)\right)$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1}, 1)$. Then $\underline{c} < \Phi(\mathbf{1}, q(\underline{c}))$ and $\overline{c} > \Phi(\mathbf{1}, q(\overline{c}))$, which implies $q^{*C} \in (0, 1)$.

Recall that

$$\phi_A(\mathbf{1}, q^{*C}) = \frac{1}{2}q^{*C} + \lambda(1 - q^{*C})$$

and

$$\phi_B(\mathbf{1}, q^{*C}) = \frac{1}{2}q^{*C} + (1 - \lambda)(1 - q^{*C}).$$

Before continuing to the proof of the Proposition, I want to show that $\Phi(q(c))$ is weakly increasing in c, therefore implying that the equilibrium is not necessarily unique if $\lambda > \frac{1}{2}$.

Lemma A.2. $\Phi(\mathbf{1}, q(\hat{c}))$ *is weakly increasing in* $\hat{c}$ *on* $(\underline{c}, \overline{c})$.

Proof. The derivative of $\Phi(\mathbf{1}, q(\hat{c}))$ with respect to the information cost cutoff $\hat{c}$ is

$$\frac{\partial \Phi(\mathbf{1}, q(\hat{c}))}{\partial \hat{c}} = \frac{1}{4}\left[\left(\frac{\partial \Pi_A}{\partial \phi_A} + \frac{\partial \Pi_B}{\partial \phi_A}\right)\frac{\partial \phi_A}{\partial q}\frac{\partial q}{\partial \hat{c}} + \left(\frac{\partial \Pi_A}{\partial \phi_B} + \frac{\partial \Pi_B}{\partial \phi_B}\right)\frac{\partial \phi_B}{\partial q}\frac{\partial q}{\partial \hat{c}}\right] \tag{A.6}$$

where, for ease of notation, $\Pi_A = \Pi_A(\mathbf{1}, q(\hat{c}))$, $\Pi_B = \Pi_B(\mathbf{1}, q(\hat{c}))$, $\phi_A = \phi_A(\mathbf{1}, q(\hat{c}))$ and $\phi_B = \phi_B(\mathbf{1}, q(\hat{c}))$.

To sign this expression, we need to consider the individual parts. First, $\frac{\partial q}{\partial \hat{c}} = h(\hat{c}) > 0$ for all $c \in (\underline{c}, \overline{c})$. Second, $\frac{\partial \phi_A(\mathbf{1}, q(\hat{c}))}{\partial q} = \frac{1}{2} - \lambda \leq 0$ and $\frac{\partial \phi_B(\mathbf{1}, q(\hat{c}))}{\partial q} = \lambda - \frac{1}{2} \geq 0$. Third, if n is odd, $\Pi_A(\mathbf{1}, q(\hat{c})) = \Pi_B(\mathbf{1}, q(\hat{c})) \equiv \Pi(\mathbf{1}, q(\hat{c}))$. Then, using $\phi_A = 1 - \phi_B$,

$$\frac{\partial \Pi(\mathbf{1}, q(\hat{c}))}{\partial \phi_A} = \binom{n-1}{\frac{n-1}{2}}\frac{n-1}{2}\phi_A^{\frac{n-1}{2}-1}\phi_B^{\frac{n-1}{2}-1}[\phi_B - \phi_A] \tag{A.7}$$

and

$$\frac{\partial \Pi(\mathbf{1}, q(\hat{c}))}{\partial \phi_B} = \binom{n-1}{\frac{n-1}{2}}\frac{n-1}{2}\phi_A^{\frac{n-1}{2}-1}\phi_B^{\frac{n-1}{2}-1}[\phi_A - \phi_B]. \tag{A.8}$$

Thus, $\frac{\partial \Pi(\mathbf{1}, q(\hat{c}))}{\partial \phi_A} \leq 0$ and $\frac{\partial \Pi(\mathbf{1}, q(\hat{c}))}{\partial \phi_B} \geq 0$ by $\phi_A \geq \phi_B$. If n is even,

$$\frac{\partial \Pi_A(\mathbf{1}, q(\hat{c}))}{\partial \phi_A} + \frac{\partial \Pi_B(\mathbf{1}, q(\hat{c}))}{\partial \phi_A} = \binom{n-1}{\frac{n}{2}-1}\left(\frac{n}{2}-1\right)\phi_A^{\frac{n}{2}-2}\phi_B^{\frac{n}{2}-2}\left[\phi_B^2 - \phi_A^2\right] \tag{A.9}$$

which is weakly negative by $\phi_A \geq \phi_B$, and

$$\frac{\partial \Pi_A(\mathbf{1}, q(\hat{c}))}{\partial \phi_B} + \frac{\partial \Pi_B(\mathbf{1}, q(\hat{c}))}{\partial \phi_B} = \binom{n-1}{\frac{n}{2}-1}\left(\frac{n}{2}-1\right)\phi_A^{\frac{n}{2}-2}\phi_B^{\frac{n}{2}-2}\left[\phi_A^2 - \phi_B^2\right] \tag{A.10}$$

which is weakly positive by $\phi_A \geq \phi_B$. Therefore, putting all parts together, $\frac{\partial \Phi(\mathbf{1}, q(\hat{c}))}{\partial \hat{c}} \geq 0$. □

If $\lambda = \frac{1}{2}$, we have $\phi_A = \phi_B$ and therefore $\frac{\partial \Phi(\mathbf{1}, q(\hat{c}))}{\partial \hat{c}} = 0$. In that case, the function $\Phi(\mathbf{1}, q(\hat{c}))$ crosses the 45° line exactly once on the interval $(\underline{c}, \overline{c})$, implying that the equilibrium is unique.

If $\lambda > \frac{1}{2}$, we have $\phi_A > \phi_B$ for all $q^{*C} < 1$. Therefore $\frac{\partial \Phi(\mathbf{1}, q(\hat{c}))}{\partial \hat{c}} > 0$. In that case, the function $\Phi(\mathbf{1}, q(\hat{c}))$ crosses the 45° line at least once on the interval $(\underline{c}, \overline{c})$, but the equilibrium is not necessarily unique. Because $\frac{1}{2} \leq \lambda < 1$ and $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1}, 0) + \Pi_B(\mathbf{1}, 0)\right)$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1}, 1)$, we know that the function $\Phi(\mathbf{1}, q(\hat{c}))$ crosses the 45° line at least once from above, which yields a stable equilibrium. If the function $\Phi(\mathbf{1}, q(\hat{c}))$ additionally crosses the 45° line at least once from below, this equilibrium is unstable. But then, there exists another stable equilibrium, because the function $\Phi(\mathbf{1}, q(\hat{c}))$ must cross the 45° line again from above.

Now, consider $k_i \in [\underline{k}, \overline{k}]$ where $\overline{k} = \underline{k} + \kappa$. I need to show that there exists a unique $\underline{k}' \in (0, \frac{1}{2} - \underline{c})$ and $\kappa \in (0, \frac{1}{2}\Pi(\mathbf{1}, 1) - \underline{c})$ such that for all $\underline{k} < \underline{k}'$, we have $q^{*V} > q^{*C}$ while for all $\underline{k} > \underline{k}'$, we have $q^{*V} < q^{*C}$. Note that q^{*C} is not affected by the voting costs, because $p_U^* = p_A^* = p_B^* = 1$ in any case. Thus, we can focus on how the voting costs affect q^{*V}.

First, let $\underline{k} = \frac{1}{2} - \underline{c}$. Then, under Voluntary Voting,

$$\begin{aligned}
\Phi(p_I^{*V}, q^{*V}) &= \left(\frac{1}{2}\Pi(p_I^{*V}, q^{*V}) - k_I^{*V}\right)p_I^* + \int_{\underline{k}}^{k_I^{*V}} H(y)dy \\
&< \frac{1}{2}\Pi(p_I^{*V}, q^{*V}) - \underline{k} \\
&\leq \frac{1}{2} - \underline{k} \\
&= \underline{c}
\end{aligned}$$

where the second line follows from $p_I^{*V} \geq 0$ and $\int_{\underline{k}}^{k_I^{*V}} H(y)dy < k_I^{*V} - \underline{k}$, and the third line follows from $\Pi(p_I^{*V}, q^{*V}) \leq 1$. Hence, $q^{*V} = G(\Phi(p_I^{*V}, q^{*V}) = G(\underline{c}) = 0 < q^{*C}$.

Second, let $\underline{k} = 0$ and $\overline{k} = \kappa < \frac{1}{2}\Pi(\mathbf{1}, 1) - \underline{c}$. Recall that, if $p_I^{*V} = 1$ under Voluntary Voting, the probability of being pivotal is $\Pi(1, 1)$ as well, as defined by equation (18). Note that because $p_U^{*V} = 0$, ϕ is increasing in q^{*V}. Thus, because $\frac{\partial \Pi}{\partial \phi} < 0$, $\frac{1}{2}\Pi(1, 1) < \frac{1}{2}\Pi(1, q^{*V})$ for all $q^{*V} < 1$. Moreover, for these minimal voting costs we must have $p_I^{*V} = 1$ under Voluntary Voting because $\overline{k} < \frac{1}{2}\Pi(\mathbf{1}, 1)$. Then,

$$\begin{aligned}\Phi(1,q^{*V}) &= \frac{1}{2}\Pi(1,q^{*V}) - \overline{k} + \int_{\underline{k}}^{\overline{k}} H(y)dy \\ &> \frac{1}{2}\Pi(1,1) - \overline{k} \\ &> \underline{c}\end{aligned}$$

where the second line uses $\int_{\underline{k}}^{\overline{k}} H(y)dy > 0$. Hence, $q^{*V} = G(\Phi(1,q^{*V})) > G(\underline{c}) = 0$. From the assumption $\overline{c} > \frac{1}{2}\Pi(\mathbf{1},1)$ follows that $q^{*V} < 1$.

I want to show that, for these minimal voting costs, $q^{*V} > q^{*C}$. Suppose for a contradiction that $q^{*V} < q^{*C}$. Then $\phi_A(1,q^{*V}) = \frac{1}{2}q^{*V} < q^{*C} + \lambda(1-q^{*C}) = \phi_A(\mathbf{1},q^{*C})$ and $\phi_B(1,q^{*V}) = \frac{1}{2}q^{*V} < q^{*C} + (1-\lambda)(1-q^{*C}) = \phi_B(\mathbf{1},q^{*C})$. But then

$$\begin{aligned}\Phi(1,q^{*V}) &= \frac{1}{2}\Pi(1,q^{*V}) - \overline{k} + \int_{\underline{k}}^{\overline{k}} H(y)dy \\ &> \frac{1}{2}\Pi(1,q^{*V}) - \overline{k} \\ &> \frac{1}{2}\Pi(1,q^{*C}) - \overline{k} \\ &> \frac{1}{4}(\Pi_A(\mathbf{1},q^{*C}) + \Pi_B(\mathbf{1},q^{*C})) - \overline{k} \\ &= \Phi(\mathbf{1},q^{*C}) - \overline{k}\end{aligned}$$

where the second line follows from $\int_{\underline{k}}^{\overline{k}} H(y)dy > 0$, the third line follows from $\Pi(1,q)$ being strictly decreasing in q, and the fourth line follows from $\Pi_B(\mathbf{1},q^{*C}) > \Pi_A(\mathbf{1},q^{*C})$. Thus, for $\overline{k} = \kappa = 0$, we have $q^{*V} = G(\Phi(1,q^{*V})) > G(\Phi(\mathbf{1},q^{*C})) = q^{*C}$, which is a contradiction to $q^{*V} < q^{*C}$. By continuity of the expression above, the same holds true for $\kappa > 0$ sufficiently small. Hence, we must have that, there exists a $\kappa > 0$ sufficiently small, such that for voting costs $k_i \in [0,\kappa]$, $q^{*V} > q^{*C}$.

To summarize, we so far have that there exists a $\kappa \in (0, \frac{1}{2}\Pi(\mathbf{1},1) - \underline{c})$ sufficiently small such that for $k_i \in [0,\kappa]$, $q^{*V} > q^{*C}$, while for $k_i \in [\frac{1}{2} - \underline{c}, \frac{1}{2} - \underline{c} + \kappa]$, $q^{*V} < q^{*C}$. To complete the proof, it remains to be shown that q^{*V} is decreasing as the voting costs increase from $k_i \in [0,\kappa]$ to $k_i \in [\frac{1}{2} - \underline{c}, \frac{1}{2} - \underline{c} + \kappa]$.

Lemma A.3. *Under Voluntary Voting, consider an increase in the voting costs by $\delta > 0$, such that $k_i' = k_i + \delta$ for all voters i, and $\underline{k}' = \underline{k} + \delta < \frac{1}{2}$ and $\overline{k}' = \overline{k} + \delta$. Then the probability of acquiring information under Voluntary Voting is weakly decreasing as the voting costs increase.*

Proof. Note that k_i' has the CDF H' with support $[\underline{k} + \delta, \overline{k} + \delta]$ and $H'(y) = H(y - \delta)$.[17] Under the increased voting costs, let $p_I^{*\prime}$ denote the probability that an informed voter votes, with $p_I^{*\prime} = H'(k_I^{*\prime}) = H(\frac{1}{2}\Pi(\mathbf{p}^{*\prime}, q^{*\prime}) - \delta)$, where $p_I^{*\prime} > 0$ by $\underline{k}' < \frac{1}{2}$. Similarly, let $q^{*\prime}$ denote the probability of acquiring information under the increased voting costs.

First, consider the case where $k_I^* < \overline{k}$ under Voluntary Voting (recall that $k_I^* > \underline{k}$ by $\underline{k} < \frac{1}{2}$). I want to show that $k_I^{*\prime} \leq k_I^* + \delta$. Suppose for a contradiction that $k_I^{*\prime} > k_I^* + \delta$. Then $p_I^{*\prime} = H'(k_I^{*\prime}) > H'(k_I^* + \delta) = H(k_I^*) = p_I^*$. Then, by Π strictly decreasing in p_I, $\Pi(p_I^{*\prime}) < \Pi(p_I^*)$. Note that $k_I^* > \underline{k}$ and $k_I^{*\prime} \geq k_I^* + \delta$ imply $k_I^{*\prime} > \underline{k} + \delta$. But then, $k_I^* = \frac{1}{2}\Pi(p_I^*) > \frac{1}{2}\Pi(p_I^{*\prime}) > \frac{1}{2}\Pi(p_I^{*\prime}) - \delta \geq k_I^{*\prime}$, which is a contradiction to $k_I^{*\prime} > k_I^* + \delta$. Therefore, we can conclude that $k_I^{*\prime} \leq k_I^* + \delta$.

Note that $k_I^* < \overline{k}$ and $k_I^{*\prime} \leq k_I^* + \delta$ imply that $k_I^{*\prime} < \overline{k} + \delta$. Moreover, the assumption $\underline{k}' < \frac{1}{2}$ implies $k_I^{*\prime} > \underline{k} + \delta$. Therefore, $k_I^{*\prime} \in (\underline{k} + \delta, \overline{k} + \delta)$. Then, the expected benefit of acquiring information under the increased voting costs k_i' is

$$\begin{aligned}\Phi(k_I^{*\prime}) &= \int_{\underline{k}+\delta}^{k_I^{*\prime}} H'(y)\mathrm{d}y \\ &= \int_{\underline{k}+\delta}^{k_I^{*\prime}} H(y-\delta)\mathrm{d}y\end{aligned}$$

[17] Note that the CDF of the increased voting costs, H' first-order stochastically dominates H and, since k_i represents a loss, all individuals strictly prefer H over H'.

$$
\begin{aligned}
&\leq \int_{\underline{k}+\delta}^{k_I^*+\delta} H(y-\delta)\mathrm{d}y \\
&= \int_{\underline{k}}^{k_I^*} H(y)\mathrm{d}y \\
&= \Phi(k_I^*)
\end{aligned}
$$

where the third line follows from $k_I^{*\prime} \leq k_I^* + \delta$. From G increasing it follows that $G(\Phi(k_I^{*\prime})) \leq G\left(\Phi(k_I^*)\right)$, and hence $q^{*\prime} \leq q^*$.

Second, consider the case where $k_I^* = \overline{k}$ under Voluntary Voting. Then, we need to distinguish two further cases:
(i) $k_I^{*\prime} < \overline{k} + \delta$. Then, the expected benefit of acquiring information under the increased voting costs k_i' is

$$
\begin{aligned}
\Phi(k_I^{*\prime}) &= \int_{\underline{k}+\delta}^{k_I^{*\prime}} H'(y)\mathrm{d}y \\
&= \int_{\underline{k}+\delta}^{k_I^{*\prime}} H(y-\delta)\mathrm{d}y \\
&< \int_{\underline{k}+\delta}^{\overline{k}+\delta} H(y-\delta)\mathrm{d}y \\
&= \int_{\underline{k}}^{\overline{k}} H(y)\mathrm{d}y \\
&\leq \frac{1}{2}\Pi(\mathbf{p}^*, q^*) - \overline{k} + \int_{\underline{k}}^{\overline{k}} H(y)\mathrm{d}y \\
&= \Phi(k_I^*)
\end{aligned}
$$

Again, from G increasing it follows that $G(\Phi(k_I^{*\prime})) \leq G\left(\Phi(k_I^*)\right)$, and hence $q^{*\prime} \leq q^*$.

(ii) $k_I^{*\prime} = \overline{k} + \delta > k_I^*$. Then $p_I^{*\prime} = p_I^* = 1$. Suppose for a contradiction that $q^{*\prime} > q^*$. Then $\phi' = \frac{1}{2}q^{*\prime} > \frac{1}{2}q^* = \phi$, and hence $\Pi(\mathbf{p}^{*\prime}, q^{*\prime}) < \Pi(\mathbf{p}^*, q^*)$. But then, the expected benefit of acquiring information under the increased voting costs k_i' is

$$
\begin{aligned}
\Phi(k_I^{*\prime}) &= \frac{1}{2}\Pi(\mathbf{p}^{*\prime}, q^{*\prime}) - (\overline{k} + \delta) + \int_{\underline{k}+\delta}^{\overline{k}+\delta} H'(y)\mathrm{d}y \\
&< \frac{1}{2}\Pi(\mathbf{p}^{*\prime}, q^{*\prime}) - \overline{k} + \int_{\underline{k}+\delta}^{\overline{k}+\delta} H(y-\delta)\mathrm{d}y \\
&< \frac{1}{2}\Pi(\mathbf{p}^{*}, q^{*}) - \overline{k} + \int_{\underline{k}}^{\overline{k}} H(y)\mathrm{d}y \\
&= \Phi(k_I^*)
\end{aligned}
$$

where the third line follows from $\Pi(\mathbf{p}^{*\prime}, q^{*\prime}) < \Pi(\mathbf{p}^*, q^*)$. But this implies $G(\Phi(k_I^{*\prime})) \leq G(\Phi(k_I^*))$ which is a contradiction to $q^{*\prime} > q^*$. Therefore, we need to have $q^{*\prime} \leq q^*$ in this case as well.

To summarize, we have in all cases that $\Phi(k_I^{*\prime}) \leq \Phi(k_I^*)$ such that $q^{*\prime} \leq q^*$, i.e., the probability of acquiring information under Voluntary Voting weakly decreases as the voting costs increase from k_i to $k_i' = k_i + \delta$ for all voters i. □

From Lemma A.3 it follows that q^{*V} weakly decreases as the voting costs increase from k_i to $k_i' = k_i + \delta$. All in all, I have shown that there exists a $\kappa \in (0, \frac{1}{2}\Pi(\mathbf{1}, 1) - \underline{c})$ sufficiently small such that for $k_i \in [0, \kappa]$, $q^{*V} > q^{*C}$, while for $k_i \in [\frac{1}{2} - \underline{c}, \frac{1}{2} - \underline{c} + \kappa]$, $q^{*V} < q^{*C}$.

Moreover, q^{*V} is decreasing as the voting costs increase from $k_i \in [0,\kappa]$ to $k_i \in [\frac{1}{2} - \underline{c}, \frac{1}{2} - \underline{c} + \kappa]$. Therefore, we can conclude that there exists a unique threshold $\underline{\tilde{k}} \in (0, \frac{1}{2} - \underline{c})$, such that $q^{*V} = q^{*C}$ when $k_i \in [\underline{\tilde{k}}, \underline{\tilde{k}} + \kappa]$, and $q^{*V} > q^{*C}$ for all $\underline{k} < \underline{\tilde{k}}$ while $q^{*V} < q^{*C}$ for all $\underline{k} < \underline{\tilde{k}}$. □

Proof of Proposition 5.

Let $\frac{1}{2} < \lambda < 1$. Again, let q^{*C} denote the probability of acquiring information under Compulsory Voting with $f \geq \overline{k}$. Let $\overline{c} > \frac{1}{2}\Pi(\mathbf{1}, 1)$, which implies $q^{*C} < 1$, and let $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1},0) + \Pi_B(\mathbf{1},0)\right)$, which implies $q^{*C} > 0$. The voting probabilities are

$$\phi_A(\mathbf{1}, q^*) = \frac{1}{2}q^* + \lambda(1-q^*)$$

and

$$\phi_B(\mathbf{1}, q^*) = \frac{1}{2}q^* + (1-\lambda)(1-q^*).$$

Now, consider an increase in the bias of uninformed voters, such that $\lambda' > \lambda > \frac{1}{2}$. Let $q^{*C\prime}$ denote the probability of acquiring information under the increased bias λ'. I want to show that, in any stable equilibrium, $q^{*C\prime} < q^{*C}$. To do so, I will show that $\Phi(\mathbf{1}, q(\hat{c}))$ is decreasing in λ for all $\hat{c}$, such that $c^{*\prime} < c^*$.

Recall that for any $\hat{c}$, the expected benefit of casting an informed vote under Compulsory Voting with $f \geq \overline{k}$ is

$$\Phi(\mathbf{1}, q(\hat{c})) = \frac{1}{4}\left(\Pi_A(\mathbf{1}, q(\hat{c})) + \Pi_B(\mathbf{1}, q(\hat{c}))\right).$$

Then

$$\frac{\partial \Phi(\mathbf{1}, q(\hat{c}))}{\partial \lambda} = \frac{1}{4}\left[\left(\frac{\partial \Pi_A}{\partial \phi_A} + \frac{\partial \Pi_B}{\partial \phi_A}\right)\frac{\partial \phi_A}{\partial \lambda} + \left(\frac{\partial \Pi_A}{\partial \phi_B} + \frac{\partial \Pi_B}{\partial \phi_B}\right)\frac{\partial \phi_B}{\partial \lambda}\right].$$

From the proof of Lemma A.2 (equations (A.7) – (A.10)) and because $\phi_A > \phi_B$, we have $\frac{\partial \Pi_A(\mathbf{1},q(\hat{c}))}{\partial \phi_A} + \frac{\partial \Pi_B(\mathbf{1},q(\hat{c}))}{\partial \phi_A} < 0$, and that $\frac{\partial \Pi_A(\mathbf{1},q(\hat{c}))}{\partial \phi_B k} + \frac{\partial \Pi_B(\mathbf{1},q(\hat{c}))}{\partial \phi_B} > 0$. Moreover, $\frac{\partial \phi_A}{\partial \lambda} = 1 - q(\hat{c})$ and $\frac{\partial \phi_B}{\partial \lambda} = -(1 - q(\hat{c}))$. Thus, $\Phi(\mathbf{1}, q(\hat{c}))$ is strictly decreasing in λ for all $\hat{c} \in [\underline{c}, \overline{c})$. At $\overline{c}$, $\Phi(\mathbf{1}, 1)$ is constant in λ.

Therefore, an increase in λ corresponds to a downwards rotation of $\Phi(\mathbf{1}, q(\hat{c}))$ around $\overline{c}$. This means that for any $\lambda' > \lambda$, we have $\Phi'(\mathbf{1}, q(\hat{c})) < \Phi(\mathbf{1}, q(\hat{c}))$. Hence, we can conclude that, in any stable equilibrium, i.e., where the function $\Phi(\mathbf{1}, q(\hat{c}))$ crosses the 45° line from above, the new intersection for $\lambda' > \lambda$ is at $c' < c^*$, and hence $q^{*C\prime} < q^{*C}$. If the function does not cross the 45° line anymore because $\Phi'(\mathbf{1}, 0) < \underline{c}$, then $q^{*C\prime} = 0 < q^{*C}$. □

Proof of Proposition 6.

Let $\lambda = 1$. Under Compulsory Voting with $f \geq \overline{k}$, $\phi_A(\mathbf{1}, q^*) = 1 - \frac{1}{2}q^*$ and $\phi_B(\mathbf{1}, q^*) = \frac{1}{2}q^*$. Hence, $\phi_A(\mathbf{1}, 0) = 1$ and $\phi_B(\mathbf{1}, q^*) = 0$. Therefore, $\Pi_A(\mathbf{1}, 0) = \Pi_B(\mathbf{1}, 0) = 0$, which in turn implies $\Phi(\mathbf{1}, 0) = 0$. Therefore, for any $\underline{c} \geq 0$, we have $\Phi(\mathbf{1}, 0) \leq \underline{c}$. Therefore, $q^{*C} = 0$ is an equilibrium. □

Proof of Proposition 7.

Let $0 \leq f < \underline{k}$. I want to show that expected social welfare weakly decreases in the abstention fine f and decreases strictly if $0 < q^* < 1$ and $p_I^* < 1$. Recall that $0 \leq f < \underline{k}$ implies $p_U^* = 0$. Let $p_I^* \equiv p_A^* = p_B^*$ denote the equilibrium probability of voting for an informed individual. Note that then $\phi \equiv \phi_A = \phi_B = \frac{1}{2}q^* p_I^*$ and therefore $\Pi_A = \Pi_B \equiv \Pi$. Also recall from Proposition 3 that $\frac{\mathrm{d}q^*}{\mathrm{d}f}\Big|_{0 \leq f < \underline{k}} \geq 0$, with strict inequality if $0 < q^* < 1$ and $p_I^* < 1$. Before I proceed to the proof of the Proposition, I need to show that the probability of voting is increasing in the abstention fine f as well.

Lemma A.4. $\frac{\mathrm{d}p_I^*}{\mathrm{d}f}\Big|_{0 \leq f < \underline{k}} \geq 0$, *with strict inequality if* $0 < q^* < 1$ *and* $p_I^* < 1$.

Proof. From the proof of Proposition 3 (equation (A.3)), and $\frac{\mathrm{d}p_U^*}{\mathrm{d}f}|_{0 \leq f < \underline{k}} = 0$, we have

$$\frac{\mathrm{d}q^*}{\mathrm{d}f} = g(\Phi(\mathbf{p}^*, q^*))2\frac{\partial \Phi}{\partial p_I^*}\frac{\mathrm{d}p_I^*}{\mathrm{d}f}.$$

It is clear that $\frac{\mathrm{d}p_I^*}{\mathrm{d}f} < 0$ leads to a contradiction to $\frac{\mathrm{d}q^*}{\mathrm{d}f} > 0$. Hence, we can conclude that $\frac{\mathrm{d}p_I^*}{\mathrm{d}f}\Big|_{0 \leq f < \underline{k}} \geq 0$. If $0 < q^* < 1$ and $p_I^* < 1$, we have $\frac{\mathrm{d}q^*}{\mathrm{d}f} > 0$, and hence we must have $\frac{\mathrm{d}p_I^*}{\mathrm{d}f}\Big|_{0 \leq f < \underline{k}} > 0$ with strict inequality as well. □

Now, I need to show that expected social welfare is always weakly decreasing in the abstention fine f, and is strictly decreasing if $0 < q^* < 1$ and $p_I^* < 1$. To do so, consider the total differential $\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f}$, which is given by

$$\begin{aligned}\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f} = n\Bigg[& q^* p_I^* \frac{1}{2}\frac{\partial \Pi}{\partial \phi}\left[\frac{\partial \phi}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}f} + \frac{\partial \phi}{\partial p_I^*}\frac{\mathrm{d}p_I^*}{\mathrm{d}f}\right] \\ & + q^* \frac{1}{2}\Pi(\mathbf{p}^*,q^*)\frac{\mathrm{d}p_I^*}{\mathrm{d}f} + p_I^*\frac{1}{2}\Pi(\mathbf{p}^*,q^*)\frac{\mathrm{d}q^*}{\mathrm{d}f} \\ & - \frac{\mathrm{d}q^*}{\mathrm{d}f}\int_{\underline{k}}^{k_I^*} k h(k)\mathrm{d}k - q^* k_I^* h(k_I^*)\frac{\mathrm{d}k_I^*}{\mathrm{d}f} \\ & - c^* g(c^*)\frac{\mathrm{d}c^*}{\mathrm{d}f}\Bigg].\end{aligned}$$

First, consider the case where $0 < q^* < 1$ and $0 < p_I^* < 1$. Then, using that $\frac{\mathrm{d}p_I^*}{\mathrm{d}f} = h(k_I^*)\frac{\mathrm{d}k_I^*}{\mathrm{d}f}$ and $\frac{\mathrm{d}q^*}{\mathrm{d}f} = h(c^*)\frac{\mathrm{d}c^*}{\mathrm{d}f}$, the previous equation can be rewritten as

$$\begin{aligned}\left.\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f}\right|_{\substack{0<q^*<1,\\ 0<p_I^*<1}} &= n\Bigg[q^* p_I^* \frac{1}{2}\frac{\partial \Pi}{\partial \phi}\left[\frac{\partial \phi}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}f} + \frac{\partial \phi}{\partial p_I^*}\frac{\mathrm{d}p_I^*}{\mathrm{d}f}\right] \\ &\quad + q^*\frac{\mathrm{d}p_I^*}{\mathrm{d}f}\left[\frac{1}{2}\Pi(\mathbf{p}^*,q^*) - k_I^*\right] + p_I^*\frac{\mathrm{d}q^*}{\mathrm{d}f}\left[\frac{1}{2}\Pi(\mathbf{p}^*,q^*) - c^*\right] \\ &\quad - \frac{\mathrm{d}q^*}{\mathrm{d}f}\left[k_I^* p_I^* - \int_{\underline{k}}^{k_I^*} H(k)\mathrm{d}k\right]\Bigg] \\ &= n\Bigg[q^* p_I^* \frac{1}{2}\frac{\partial \Pi}{\partial \phi}\left[\frac{\partial \phi}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}f} + \frac{\partial \phi}{\partial p_I^*}\frac{\mathrm{d}p_I^*}{\mathrm{d}f}\right] \\ &\quad - f\left[q^*\frac{\mathrm{d}p_I^*}{\mathrm{d}f} + p_I^*\frac{\mathrm{d}q^*}{\mathrm{d}f}\right]\Bigg] \end{aligned} \tag{A.11}$$

where the second equality follows from the fact that $p_I^* < 1$ implies $k_I^* = \frac{1}{2}\Pi(\mathbf{p}^*,q^*) + f$ and $0 < q^* < 1$ implies $c^* = \Phi(\mathbf{p}^*,q^*) = \int_{\underline{k}}^{k_I^*} H(k)\mathrm{d}k$. Then, using that $\frac{\partial \Pi}{\partial \phi} < 0$ and $\frac{\mathrm{d}p_I^*}{\mathrm{d}f} > 0$ and $\frac{\mathrm{d}q^*}{\mathrm{d}f} > 0$ for all $0 < q^* < 1$ and $0 < p_I^* < 1$, we have directly that $\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f} < 0$.

To show that otherwise, $\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f} \le 0$, consider three separate cases. Also recall that by $\underline{k} < \frac{1}{2}$, we have $p_I^* > 0$.

(i) $q^* = 0$ and $0 < p_I^* < 1$. Then

$$\begin{aligned}\left.\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f}\right|_{\substack{q^*=0,\\ 0<p_I^*<1}} &= n\left[\frac{\mathrm{d}q^*}{\mathrm{d}f}\left[p_I^*\left(\frac{1}{2}\Pi(\mathbf{p}^*,q^*) - k_I^*\right) - \int_{\underline{k}}^{k_I^*} H(k)\mathrm{d}k\right]\right] \\ &= n\left[\frac{\mathrm{d}q^*}{\mathrm{d}f}\left[-p_I^* f - \int_{\underline{k}}^{k_I^*} H(k)\mathrm{d}k\right]\right]\end{aligned} \tag{A.12}$$

where the second equality follows from the fact that $p_I^* < 1$ implies $k_I^* = \frac{1}{2}\Pi(\mathbf{p}^*,q^*) + f$. Because $\frac{\mathrm{d}q^*}{\mathrm{d}f} \ge 0$, we can conclude that $\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f} \le 0$.

(ii) $q^* = 0$ and $p_I^* = 1$. Then $k_I^* = \overline{k} \le \frac{1}{2}\Pi(\mathbf{p}^*,q^*) + f$. If this holds with equality, $\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f}$ is the same as in equation (A.12). If instead, $\overline{k} < \frac{1}{2}\Pi(\mathbf{p}^*,q^*) + f$ with strict inequality, we have $g(\varphi(\mathbf{p}^*,q^*)) = g(\Pi(\mathbf{p}^*,q^*) + f) = 0$, and hence, from the proof of Proposition 3 (equation (A.4)), $\frac{\mathrm{d}q^*}{\mathrm{d}f} = 0$, such that $\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f} = 0$.

(iii) $0 < q^* < 1$ and $p_I^* = 1$. Then $c^* = \Phi(\mathbf{p}^*,q^*) = \int_{\underline{k}}^{k_I^*} H(k)\mathrm{d}k$, but again $k_I^* = \overline{k} \le \frac{1}{2}\Pi(\mathbf{p}^*,q^*) + f$. If this holds with equality, $\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f}$ is the same as in equation (A.11). If instead, $\overline{k} < \frac{1}{2}\Pi(\mathbf{p}^*,q^*) + f$ with strict inequality, we have by the same argument as in (ii) that $\frac{\mathrm{d}q^*}{\mathrm{d}f} = 0$ and, moreover, $\frac{\mathrm{d}p_I^*}{\mathrm{d}f} = 0$, such that $\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f} = 0$.

All in all, I have shown that $\frac{\mathrm{d}W(\mathbf{p}^*,q^*)}{\mathrm{d}f} \le 0$, with strict inequality if $0 < q^* < 1$ and $0 < p_I^* < 1$. □

Proof of Proposition 8.

Let $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1},0) + \Pi_B(\mathbf{1},0)\right)$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1},1)$. I want to show that Compulsory Voting with a high abstention fine $f \ge \overline{k}$ strictly reduces expected social welfare compared to Voluntary Voting. Let q^{*V} denote the equilibrium probability of acquiring information under Voluntary Voting, and c^{*V} the corresponding equilibrium information cost threshold. Analogously, let q^{*C} denote the equilibrium probability of acquiring information under Compulsory Voting with a high abstention fine $f \ge \overline{k}$, and c^{*C} the corresponding equilibrium information cost threshold. Recall that if $\frac{1}{2} \le \lambda < 1$, $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1},0) + \Pi_B(\mathbf{1},0)\right)$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1},1)$ imply $c^{*C} \in (\underline{c},\overline{c})$. This in turn implies $c^{*C} = \Phi(\mathbf{1},q^{*C}) = \frac{1}{4}\left(\Pi_A(\mathbf{1},q^{*C}) + \Pi_B(\mathbf{1},q^{*C})\right)$. If $\lambda = 1$, we have from Proposition 6 that $q^{*C} = 0$ is an equilibrium. Then $q^{*C} \le q^{*V}$. Moreover, $\overline{c} > \frac{1}{2}\Pi(\mathbf{1},1)$ implies $q^{*C} < 1$ for $\lambda = 1$ as well. Note that because $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1},0) + \Pi_B(\mathbf{1},0)\right) < \frac{1}{2} = \frac{1}{2}\Pi(p_I^{*V},0)$, we also have $c^{*V} > \underline{c}$.

Note that if $q^{*V} = q^{*C} \equiv q^*$, i.e., $c^{*V} = c^{*C} \equiv c^*$, we must have $\Phi(\mathbf{1},q^*) = \Phi(p_I^{*V},q^*)$, and hence $U(\mathbf{1},q^*) - U(\mathbf{p}^{*V},q^*) = -\int_{\underline{k}}^{\overline{k}} kh(k)\mathrm{d}k$ which is clearly negative. Therefore, consider now the remaining two cases.

(i) Suppose $q^{*V} < q^{*C}$. This implies $q^{*V} < 1$, i.e., $c^{*V} \in (\underline{c},\overline{c})$, and hence

$$c^{*V} = \Phi(p_I^{*V},q^{*V}) = \left(\frac{1}{2}\Pi(p_I^{*V},q^{*V}) - k_I^{*V}\right)p_I^{*V} + \int_{\underline{k}}^{k_I^{*V}} H(k)\mathrm{d}k.$$

Then,

$$\begin{aligned}
U(\mathbf{1},q^{*C}) - U(\mathbf{p}^{*V},q^{*V}) &= q^{*C}\Phi(\mathbf{1},q^{*C}) - \int_{\underline{k}}^{\overline{k}} kh(k)\mathrm{d}k - \int_{c^{*V}}^{c^{*C}} cg(c)\mathrm{d}c \\
&\quad - q^{*V}\Phi(p_I^{*V},q^{*V}) \\
&= q^{*C}\Phi(\mathbf{1},q^{*C}) - \int_{\underline{k}}^{\overline{k}} kh(k)\mathrm{d}k - c^{*C}q^{*C} + c^{*V}q^{*V} \\
&\quad + \int_{c^{*V}}^{c^{*C}} G(c)\mathrm{d}c - q^{*V}\Phi(p_I^{*V},q^{*V}) \\
&< -\overline{k} + \int_{\underline{k}}^{\overline{k}} H(k)\mathrm{d}k + c^{*C} - c^{*V} \\
&= -\overline{k} + \int_{\underline{k}}^{\overline{k}} H(k)\mathrm{d}k + c^{*C} \\
&\quad - \left(\frac{1}{2}\Pi(p_I^{*V},q^{*V}) - k_I^{*V}\right)p_I^{*V} - \int_{\underline{k}}^{k_I^{*V}} H(k)\mathrm{d}k \\
&< -\overline{k} + c^{*C} - \frac{1}{2}\Pi(p_I^{*V},q^{*V}) - k_I^{*V} + \int_{k_I^{*V}}^{\overline{k}} H(k)\mathrm{d}k \\
&< c^{*C} - \frac{1}{2}\Pi(p_I^{*V},q^{*V}) \\
&= \frac{1}{4}\left(\Pi_A(\mathbf{1},q^{*C}) + \Pi_B(\mathbf{1},q^{*C})\right) - \frac{1}{2}\Pi(p_I^{*V},q^{*V})
\end{aligned}$$

where I use integration by parts as well as the fact that $G(c) \le 1$ for all c and $H(k) \le 1$ for all k. To see that the expression in the last line is strictly negative, first note that $\phi(p_I^{*V},q^{*V}) = \frac{1}{2}q^{*V}p_I^{*V} \le \frac{1}{2}q^{*V} < \frac{1}{2}q^{*C} + (1-\lambda)(1-q^{*C}) = \phi_B(\mathbf{1},q^{*C}) \le \phi_A(\mathbf{1},q^{*C})$. Second, recall that (from Taylor and Yildirim, 2010a), $\frac{\partial \Pi_B}{\partial \phi_A} < 0$. Third, recall from Lemma A.1, that, in the case where $\Pi_A = \Pi_B = \Pi$ we have $\frac{\partial \Pi}{\partial \phi} < 0$

as well. Fourth, recall that $\Pi_A(\mathbf{1}, q^{*C}) \leq \Pi_B(\mathbf{1}, q^{*C})$. Taking all these observations together, we have $\frac{1}{4}\left(\Pi_A(\mathbf{1}, q^{*C}) + \Pi_B(\mathbf{1}, q^{*C})\right) \leq \frac{1}{2}\Pi_B(\mathbf{1}, q^{*C}) < \frac{1}{2}\Pi(p_I^{*V}, q^{*V})$. Therefore, $U(\mathbf{1}, q^{*C}) < U(\mathbf{p}^{*V}, q^{*V})$ if $q^{*V} < q^{*C}$.

(ii) Suppose $q^{*V} > q^{*C}$. Because $0 < q^{*V} \leq 1$ we have

$$c^{*V} \leq \Phi(p_I^{*V}, q^{*V}) = \left(\frac{1}{2}\Pi(p_I^{*V}, q^{*V}) - k_I^{*V}\right) p_I^{*V} + \int_{\underline{k}}^{k_I^{*V}} H(k)\mathrm{d}k,$$

which holds with equality if $c^{*V} \in (\underline{c}, \overline{c})$.

Then,

$$\begin{aligned}
U(\mathbf{1}, q^{*C}) - U(\mathbf{p}^{*V}, q^{*V}) &= q^{*C}\Phi(\mathbf{1}, q^{*C}) - \int_{\underline{k}}^{\overline{k}} kh(k)\mathrm{d}k + \int_{c^{*C}}^{c^{*V}} cg(c)\mathrm{d}c \\
&\quad - q^{*V}\Phi(p_I^{*V}, q^{*V}) \\
&= q^{*C}\Phi(\mathbf{1}, q^{*C}) - \int_{\underline{k}}^{\overline{k}} kh(k)\mathrm{d}k + c^{*V}q^{*V} - c^{*C}q^{*C} \\
&\quad - \int_{c^{*C}}^{c^{*V}} G(c)\mathrm{d}c - q^{*V}\Phi(p_I^{*V}, q^{*V}) \\
&\leq -\int_{\underline{k}}^{\overline{k}} kh(k)\mathrm{d}k - \int_{c^{*C}}^{c^{*V}} G(c)\mathrm{d}c
\end{aligned}$$

which is clearly negative. Thus, $U(\mathbf{1}, q^{*C}) < U(\mathbf{p}^{*V}, q^{*V})$ if $q^{*V} > q^{*C}$ as well.

All in all, from $U(\mathbf{1}, q^{*C}) < U(\mathbf{p}^{*V}, q^{*V})$ follows directly that expected social welfare is strictly lower under Compulsory Voting with a high abstention fine $f \geq \overline{k}$ than under Voluntary Voting. □

Proof of Proposition 9.

Let $\frac{1}{2} < \lambda < 1$ and $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1}, 0) + \Pi_B(\mathbf{1}, 0)\right)$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1}, 1)$. Consider $f \geq \overline{k}$. I want to show that, for any stable equilibrium, expected social welfare is strictly decreasing in the bias λ of uninformed voters. Recall from Proposition 4 that $\underline{c} < \frac{1}{4}\left(\Pi_A(\mathbf{1}, 0) + \Pi_B(\mathbf{1}, 0)\right)$ and $\overline{c} > \frac{1}{2}\Pi(\mathbf{1}, 1)$ imply $c^* \in (\underline{c}, \overline{c})$. This in turn implies $c^* = \Phi(\mathbf{1}, q^*)$. Then

$$\begin{aligned}
\frac{\mathrm{d}W(\mathbf{1}, q^*)}{\mathrm{d}\lambda} &= n\left(\frac{\mathrm{d}q^*}{\mathrm{d}\lambda}\Phi(\mathbf{1}, q^*) + q^*\frac{\partial\Phi(\mathbf{1}, q^*)}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}\lambda} - c^*g(c^*)\frac{\mathrm{d}c^*}{\mathrm{d}\lambda}\right) \\
&= n\left(q^*\frac{\partial\Phi(\mathbf{1}, q^*)}{\partial q^*}\frac{\mathrm{d}q^*}{\mathrm{d}\lambda}\right)
\end{aligned}$$

where the second equality follows from $c^* = \Phi(\mathbf{1}, q^*)$ and $\frac{\mathrm{d}q^*}{\mathrm{d}\lambda} = \frac{\mathrm{d}G(c^*)}{\mathrm{d}\lambda} = g(c^*)\frac{\mathrm{d}c^*}{\mathrm{d}\lambda}$. From Lemma A.2 we have $\frac{\partial\Phi(\mathbf{1}, q^*)}{\partial q^*} > 0$ for $\lambda > \frac{1}{2}$. Moreover, recall from Proposition 5 that q^* is strictly decreasing in λ in any stable equilibrium. Therefore, $\frac{\mathrm{d}W(\mathbf{1}, q^*)}{\mathrm{d}\lambda} < 0$. □

Appendix B. A simple benchmark model

The following simplified model is the most parsimonious version of the model and achieves the same results as the main model while allowing for a more intuitive understanding of the mechanisms driving the results, which is why it can be considered a benchmark model.

There are $n \geq 3$ individuals $i \in \{1, 2, ..., n\}$ who have to make a collective policy decision x from the set of alternatives $X = \{A, B\}$. The outcome is determined by simple majority rule. In case of a tie, both alternatives are chosen with equal probability.

Let $r_i \in X$ denote the alternative which is strictly preferred by individual i.[18] The preference r_i of individual i is assumed to be stochastically independent of the preference r_j of individual $j \neq i$. Ex-ante, an individual is equally likely to favor each alternative, i.e., preferences are *neutral*. An individual is initially uninformed about her preferred alternative r_i and she does not automatically

[18] I rule out the possibility that an individual is indifferent between the two alternatives.

Table B.2
Ex-post payoff of individual i.

	Participate	Abstain
Informed	$\mathbb{1}\{x=r_i\}-(c_i+\varepsilon)$	$\mathbb{1}\{x=r_i\}-c_i-f$
Uninformed	$\mathbb{1}\{x=r_i\}-\varepsilon$	$\mathbb{1}\{x=r_i\}-f$

observe r_i at the interim stage. If she wants to learn which alternative she prefers, she can acquire a costly, perfectly informative signal that reveals r_i. Let c_i denote the stochastic information costs of individual i. For each i, the information costs c_i are drawn independently from the cumulative distribution function (CDF) G which is the same for all individuals and has the support $[\underline{c},\overline{c}]$ where $0 \le \underline{c} < \overline{c}$. Let g denote the probability density function (PDF) associated with G and assume that g is positive on all of the support. The information costs c_i of individual i are assumed to be stochastically independent of her preferred alternative r_i, and of the information costs c_j of individual $j \neq i$.

Let ε denote the voting costs, i.e., the cost of casting a ballot, which are deterministic and the same for all individuals. The voting costs are assumed to be known by each individual when they make their decision to acquire information. Therefore, the information acquisition decision and the voting decision can be treated as a bundle.

In order to incentivize participation, abstention is sanctioned with a fine f. I will call the case without a fine "Voluntary Voting", and the case where $f > 0$ "Compulsory Voting".

If individual i's preferred alternative r_i is chosen collectively, her utility is normalized to 1. If the other alternative is chosen, i's utility is normalized to zero. Table B.2 summarizes the ex-post payoff of an individual i, where $\mathbb{1}\{x=r_i\}$ is an indicator function that takes the value 1 if the collective outcome x is equal to i's preferred alternative r_i and zero otherwise.

For an informed individual, it is a weakly dominant strategy to vote sincerely for her preferred alternative. Therefore, *if* an individual is informed, she participates and votes for her favored alternative. For an uninformed individual, I assume that *if* she participates, she casts a valid vote and does not spoil her ballot.[19]

The timing of the game can be summarized as follows:

1. For each individual $i \in \{1,2,...,n\}$, nature draws the information costs $c_i \in [\underline{c},\overline{c}]$. Nature also draws i's preferred alternative r_i from the set of alternatives X with $Pr(r_i = A) = Pr(r_i = B) = \frac{1}{2}$.
2. Each individual privately observes her information costs c_i and the voting costs ε, but not her preference r_i.
3. *Information stage*: All individuals simultaneously decide whether to acquire information or not. The decision is private information. If individual i acquires information, she privately observes her preference r_i.
4. *Voting stage*: All individuals simultaneously decide whether to vote or abstain.
5. The collective policy outcome $x \in X$ is realized by simple majority rule.
6. Payoffs are realized.

First, let us derive the expected benefit of casting an informed vote. The expected payoff of casting an informed vote depends on the probability of being pivotal, which in turn depends on how many other voters participate. Individual i is pivotal only if her vote creates or breaks a tie. In both cases, she gains $\frac{1}{2}$ in expected utility. Let $\Pi(p)$ denote the probability that individual i is pivotal if all others participate with probability p and let $B(p)$ denote the expected benefit of casting an informed, pivotal vote, which is

$$B(p) = \frac{1}{2}\Pi(p) \tag{B.1}$$

where $B(p) \le \frac{1}{2}$ for all $p \in [0,1]$. From the perspective of individual i, it does not matter whether the other individuals who participate are informed or not: If they are informed, they vote for their preferred alternative and i knows that they favor each alternative with probability 1/2. If they are not informed, they also vote for each alternative with probability 1/2. Therefore, from Börgers (2004), the probability that individual i is pivotal is

$$\Pi(p) = \sum_{l=0}^{n-1} \binom{n-1}{l} p^l (1-p)^{n-1-l} \pi(l) \tag{B.2}$$

where

$$\pi(l) = \begin{cases} \binom{l}{\frac{l-1}{2}} \left(\frac{1}{2}\right)^l & \text{if } l \text{ is odd} \\ \binom{l}{\frac{l}{2}} \left(\frac{1}{2}\right)^l & \text{if } l \text{ is even} \end{cases} \tag{B.3}$$

[19] As explained for the main model in Section 3, the results of my analysis continue to hold if some uninformed voters spoil their ballot, as long as at least *some* uninformed voters cast a valid vote.

is the probability that i is pivotal, conditional on $l \leq n-1$ other voters participating. Börgers (2004) shows that $\Pi(p)$ is differentiable and strictly decreasing in p for all $p \in (0,1)$. Intuitively, the higher the probability with which each individual participates, the less likely it is that i will be pivotal.

If all other individuals participate in the election, the benefit of casting an informed, pivotal vote is

$$B(1) = \begin{cases} \binom{n-1}{\frac{n-1}{2}} \left(\frac{1}{2}\right)^n & \text{if } n \text{ is odd} \\ \binom{n-1}{\frac{n}{2}-1} \left(\frac{1}{2}\right)^n & \text{if } n \text{ is even.} \end{cases} \tag{B.4}$$

Next, let us derive the expected benefit of casting an uninformed vote. It is easy to see that, as in the main model in section 3, if individual i does not know which alternative she prefers but nevertheless casts a valid vote, her expected benefit of being pivotal is zero. Because an uninformed voter casts a valid vote, she has to select either A or B without knowing which alternative she favors. If an uninformed voter i casts a pivotal vote for A, she knows that with probability $\frac{1}{2}$, she indeed favors A and her expected utility increases by $\frac{1}{2}$. But with probability $\frac{1}{2}$, she favors B and her expected utility decreases by $\frac{1}{2}$. The same logic applies if voter i casts a pivotal vote for B. Because an uninformed voter who participates in the election is indifferent between selecting alternative A or B, I assume that she flips a fair coin, i.e., that she casts a valid vote by selecting one of the two alternatives with equal probability.

In the following, to characterize the equilibrium, I will consider two different cases: first, the case of a small abstention fine (or no abstention fine) $0 \leq f < \varepsilon$, where only informed individuals participate, and second, the case of a high abstention fine $f > \varepsilon$, which achieves full participation.[20]

B.1. Voluntary voting and compulsory voting with a small abstention fine

Let $0 \leq f < \varepsilon$. Because the expected benefit of casting an uninformed vote is zero and voting is costly, uninformed individuals abstain. Thus, an individual i with information costs c_i acquires information about her preferred alternative and votes accordingly if and only if

$$c_i \leq B(p) - \varepsilon + f \tag{B.5}$$

and remains uninformed and abstains otherwise.[21] Hence, individual i acquires information only if her information costs are sufficiently low, which yields the following equilibrium result:

Proposition B.1. *The unique symmetric Bayesian Nash equilibrium under Voluntary Voting with $f = 0$ or under Compulsory Voting with a small abstention fine $0 < f < \varepsilon$ has the following properties:*

(i) If $\overline{c} + \varepsilon - f \leq B(1)$, all individuals acquire information about their preferred alternative r_i and vote for this alternative.

(ii) If $\underline{c} + \varepsilon - f \geq \frac{1}{2}$, all individuals remain uninformed and abstain.

(iii) Otherwise, i.e., if $\underline{c} + \varepsilon - f < \frac{1}{2}$ and $\overline{c} + \varepsilon - f > B(1)$, there exists a unique equilibrium cutoff value $c^* \in (\underline{c} + \varepsilon - f, \overline{c} + \varepsilon - f)$ such that an individual i with information costs c_i acquires information about her preferred alternative r_i and casts her vote accordingly if and only if $c_i \leq c^* - \varepsilon + f$, and abstains otherwise.

Let p^* denote the equilibrium probability that individual i casts an informed vote, where

$$p^* \equiv Pr(c_i \leq c^* - \varepsilon + f) = G(c^* - \varepsilon + f) \tag{B.6}$$

and $1 - p^*$ is the equilibrium probability that an individual does not acquire information and abstains from the election. The effects of the voting costs ε and of the abstention fine f on the equilibrium probability of casting an informed vote are straightforward:

Remark B.1. Let $0 \leq f < \varepsilon$. Then the equilibrium probability of casting an informed vote is

(i) weakly decreasing in the voting costs ε and

(ii) weakly increasing in the abstention fine f.

In an interior equilibrium, both relationships are strict.

Intuitively, an increase in the voting costs ε makes it more likely that the participation cost of individual i, which are her information costs c_i plus the voting costs ε, exceeds the benefits of casting an informed, pivotal vote and therefore make it *less* likely for her to become informed. An increase in the abstention fine f makes it more expensive for i to remain uninformed and abstain and therefore makes it *more* likely for her to become informed and vote accordingly.

[20] In the case where $f = \varepsilon$, uninformed individuals are indifferent between abstaining and participating. If we suppose that the voting costs ε are a random variable which is drawn from a continuous probability function, this is a probability zero event and therefore does not need to be considered.

[21] Note that if condition (B.5) holds with equality, individual i is indifferent between getting informed or not. I assume here that in that case, i acquires information. Because this is a probability zero event, it does not matter for the further analysis.

From Remark B.1 follows directly that the probability of casting an informed vote is strictly higher under Compulsory Voting with a small abstention fine $0 < f < \varepsilon$ than under Voluntary Voting. Because casting an uninformed vote is strictly dominated by abstaining under Compulsory Voting with a small abstention fine $0 < f < \varepsilon$, incentivizing participation at the same time incentivizes information acquisition.

B.2. Compulsory voting with a high abstention fine

Let $f > \varepsilon$. Now, uninformed individuals participate in the election, and we have full turnout. The expected benefit of casting an informed, pivotal vote under full participation is given by $B(1)$. Recall that the expected benefit of casting an informed, pivotal vote $B(p)$ is strictly decreasing in the probability of voting p, such that $B(1)$ is its lowest value. Thus, under full participation, an individual i with information costs c_i acquires information about her preferred alternative and votes accordingly if and only if

$$c_i \leq B(1) \tag{B.7}$$

and remains uninformed but casts a random vote otherwise. Thus, we have the following equilibrium result:

Proposition B.2. *The unique symmetric Bayesian Nash equilibrium under Compulsory Voting with a high abstention fine $f > \varepsilon$ has the following properties:*
(i) If $\overline{c} \leq B(1)$, all individuals acquire information about their preferred alternative r_i and vote for this alternative.
(ii) If $\underline{c} \geq B(1)$, all individuals remain uninformed but cast a random vote.
(iii) Otherwise, if $\underline{c} < B(1) < \overline{c}$, all individuals participate, but an individual i with information costs c_i acquires information about her preferred alternative r_i and votes accordingly if and only if $c_i \leq B(1)$, and remains uninformed but casts a random vote otherwise.

Hence, although an individual always participates in the election, an individual does not necessarily acquire information. Let q^* denote the equilibrium probability that individual i acquires information, where

$$q^* \equiv Pr(c_i \leq B(1)) = G(B(1))$$

and $1 - q^*$ is the probability that an individual participates in the election without acquiring information.[22] Note that neither the voting costs ε nor the level of the abstention fine f affects the equilibrium probability of acquiring information anymore.

The following Proposition shows that – in contrast to a small abstention fine $0 < f < \varepsilon$ – a high abstention fine $f > \varepsilon$ does not necessarily incentivize information acquisition, which is the central result of this analysis.

Proposition B.3. *Let $\underline{c} < B(1) < \overline{c}$. There exists a unique voting costs threshold $\tilde{\varepsilon} \in (0, \frac{1}{2} - \underline{c})$ such that for all $\varepsilon < \tilde{\varepsilon}$, Compulsory Voting with a high abstention fine $f > \varepsilon$ strictly reduces the probability of acquiring information, while for all $\varepsilon > \tilde{\varepsilon}$ it strictly increases the probability of acquiring information compared to Voluntary Voting.*

The effect of a high abstention fine $f > \varepsilon$ on information acquisition depends on the voting costs: Compulsory Voting with a high abstention fine $f > \varepsilon$ increases the probability that an individual acquires information only if the voting costs are high, but reduces the probability that an individual acquires information if the voting costs are low. In the latter case there are some individuals who would have acquired information under Voluntary Voting, but rationally decide not to acquire information anymore under Compulsory Voting. Moreover, it follows from Proposition B.3 that, in the limit, as $\varepsilon \to 0$, it is never possible to incentivize information acquisition with an abstention fine. Instead, Compulsory Voting then always reduces the probability that an individual acquires information compared to Voluntary Voting.

Intuitively, the result from Proposition B.3 can be explained as follows. High voting costs $\varepsilon > \tilde{\varepsilon}$ allow only individuals with sufficiently low information costs to participate in the election under Voluntary Voting. As soon as the high abstention fine $f > \varepsilon$ is introduced, the voting costs do not play a role in the voting decision anymore, as they are fully compensated for by not having to pay the abstention fine. Now consider a marginal individual whose information costs are low but, in sum with the voting costs, are just too high for her to vote under Voluntary Voting. Then, under Compulsory Voting, the expected benefits of casting an informed vote – despite being reduced due to the negative externality of voting – exceed the information costs. Hence, the marginal individual acquires information and votes accordingly. As a result, with sufficiently high voting costs, there are some individuals who abstain under Voluntary Voting, but cast an informed vote under Compulsory Voting.

In contrast to that, low voting costs $\varepsilon < \tilde{\varepsilon}$ allow even individuals with high information costs to participate in the election under Voluntary Voting. Now consider a marginal individual whose information costs are high, but just sufficiently low so that this individual acquires information and votes under Voluntary Voting. Under Compulsory Voting, however, full participation reduces the expected benefit of casting an informed vote, and her information costs exceed the expected benefit of casting an informed vote. Hence, the

[22] Note that p^* and q^* both denote the equilibrium probability that an individual acquires information and votes accordingly. But the complementary events are different: Under a small abstention fine $0 \leq f < \varepsilon$, $1 - p^*$ is the equilibrium probability that an individual does not acquire information and abstains from the election. Under a high abstention fine $f > \varepsilon$ with full turnout, $1 - q^*$ is the equilibrium probability that an individual does not acquire information but participates in the election by casting a random vote.

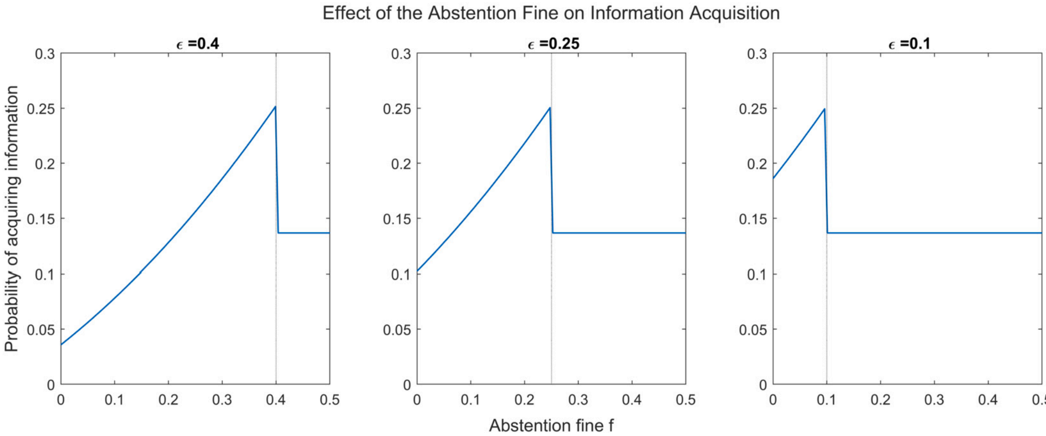

Fig. B.2. The effect of an abstention fine $0 \leq f \leq 0.5$ on the equilibrium probability of acquiring information for different voting costs $\varepsilon \in \{0.4, 0.25, 0.1\}$. The information costs are uniformly distributed on the interval $[0, 1]$. There are $n = 9$ individuals in the electorate.

marginal individual will not acquire information under Compulsory Voting anymore. As a result, with sufficiently low voting costs, there are some individuals who acquired information and voted accordingly under Voluntary Voting, but remain uninformed and cast a random vote under Compulsory Voting.

Fig. B.2 shows the effects of an abstention fine f on the equilibrium probability of acquiring information for different voting costs and illustrates the results from Remark B.1 and Proposition B.3: As long as the abstention fine f under Compulsory Voting is sufficiently small to not exceed the voting costs ε, Compulsory Voting does not achieve full participation but increases the equilibrium probability of acquiring information compared to Voluntary Voting. As soon as the abstention fine f exceeds the voting costs ε, such that full participation is reached, Compulsory Voting increases the equilibrium probability of acquiring information compared to Voluntary Voting only when the voting costs are sufficiently high. For small voting costs, Compulsory Voting with full participation reduces the equilibrium probability of acquiring information compared to Voluntary Voting.

B.3. Welfare

To analyze how the introduction of an abstention fine affects expected social welfare, we again need to distinguish the two cases of a small abstention fine $0 < f < \varepsilon$ and a high abstention fine $f > \varepsilon$.

Recall that if $f < \varepsilon$, only informed voters participate. Then, the expected utility of an individual consists of the expected utility given that the individual acquires information and votes accordingly, plus the expected utility given that the individual does not acquire information and abstains. Hence, expected utility is given by (adapted from Börgers, 2004)

$$U(p^*) = \int_{\underline{c}}^{c^*-\varepsilon+f} \left[\frac{1}{2} + B(p^*) - c - \varepsilon\right] g(c)\mathrm{d}c + \int_{c^*-\varepsilon+f}^{\overline{c}} \left[\frac{1}{2} - f\right] g(c)\mathrm{d}c$$

$$= \frac{1}{2} + \left[B(p^*) - \varepsilon\right] p^* - \int_{\underline{c}}^{c^*-\varepsilon+f} c g(c)\mathrm{d}c - f(1-p^*). \tag{B.8}$$

Because from the perspective of individual i, all other voters are equally likely to vote for A or B, her expected utility from the election result is $\frac{1}{2}$. In addition, if voter i participates in the election, she receives the expected benefit of casting an informed, pivotal vote $B(p^*)$ but pays both the voting costs ε and her information costs $\underline{c} < c_i < c^* - \varepsilon + f$. If voter i does not participate, she pays the abstention fine f. Note that, using integration by parts, equation (B.8) can be simplified to

$$U(p^*) = \frac{1}{2} - \left[B(p^*) - c^*\right] p^* + \int_{\underline{c}}^{c^*-\varepsilon+f} G(c)\mathrm{d}c - f.$$

Under the assumption that the expected revenue generated from the abstention fine is re-distributed to the individuals, expected social welfare can be written as

$$W(p^*) = n\left(\frac{1}{2} + \left[B(p^*) - \varepsilon\right] p^* - \int_{\underline{c}}^{c^*-\varepsilon+f} c g(c)\mathrm{d}c\right). \tag{B.9}$$

Proposition B.4. *Let $0 \leq f < \varepsilon$. Then expected social welfare is weakly decreasing in the abstention fine f. In an interior equilibrium, it is strictly decreasing.*

This result is in line with the result by Börgers (2004), and relies on the same intuition[23]: Because of the negative externality of voting, the expected benefits of acquiring information and voting are decreasing as participation increases in response to an increase in the abstention fine. At the same time, both information and voting costs increase. Thus, smaller expected benefits and higher overall participation cost imply a reduction in expected social welfare.

If $f > \varepsilon$, all individuals vote, but not all acquire information. Then, the expected utility of an individual consists of the expected utility given that the individual acquires information and votes accordingly, plus the expected utility given that the individual does not acquire information and casts a random vote. Hence, expected utility is given by (again adapted from Börgers, 2004)

$$U(q^*) = \int_{\underline{c}}^{B(1)} \left[\frac{1}{2} + B(1) - c - \varepsilon\right] g(c)\mathrm{d}c + \int_{B(1)}^{\overline{c}} \left[\frac{1}{2} - \varepsilon\right] g(c)\mathrm{d}c$$
$$= \frac{1}{2} - \varepsilon + B(1)q^* - \int_{\underline{c}}^{B(1)} cg(c)\mathrm{d}c. \tag{B.10}$$

Because from the perspective of individual i, all other voters are equally likely to vote for A or B, her expected utility from the election result is $\frac{1}{2}$. In addition, voter i always participates in the election and pays the voting costs ε, but she receives the expected benefit of casting an informed, pivotal vote $B(1)$ only if she acquires information. In that case, she pays her information costs $\underline{c} < c_i < B(1)$. Note that, using integration by parts, equation (B.10) can be simplified to

$$U(q^*) = \frac{1}{2} - \varepsilon + \int_{\underline{c}}^{B(1)} G(c)\mathrm{d}c.$$

Because under full participation, no individual pays the abstention fine, expected social welfare under Compulsory Voting with a high abstention fine $f > \varepsilon$ can be simply written as

$$W(q^*) = nU(q^*).$$

Proposition B.5. *Whenever Voluntary Voting does not achieve full turnout, the introduction of a high abstention fine $f > \varepsilon$ strictly reduces expected social welfare compared to Voluntary Voting.*

Because Compulsory Voting with $f > \varepsilon$ induces full turnout, the expected benefits of casting a pivotal, informed vote are smaller under Compulsory Voting with $f > \varepsilon$ than under Voluntary Voting, as long as there is less than full turnout under Voluntary Voting.[24] Moreover, the overall voting cost is higher.

Recall that Compulsory Voting with a high abstention fine $f > \varepsilon$ does not necessarily increase the probability of acquiring information compared to Voluntary Voting, but might instead reduce information acquisition if the voting costs are sufficiently small (Proposition B.3). On the one hand, if Compulsory Voting with $f > \varepsilon$ increases the probability of acquiring information compared to Voluntary Voting, it also increases the overall information cost. Then the same logic applies as for the intuition of Proposition B.4: Smaller expected benefits and higher overall participation cost imply a reduction in expected social welfare. If on the other hand, Compulsory Voting with $f > \varepsilon$ reduces the probability of acquiring information compared to Voluntary Voting, it also reduces overall information cost. Proposition B.5 shows that the reduction in benefits always outweighs the reduction in overall information cost, such that welfare overall decreases.

Appendix C. Proofs for the benchmark model

Proof of Proposition B.1.

(i) and (ii) follow directly from everything that has been stated before.

(iii) Let $\underline{c} + \varepsilon - f < \frac{1}{2}$ and $\overline{c} + \varepsilon - f > B(1)$. A *voting strategy* takes the form $\sigma : [\underline{c} + \varepsilon - f, \overline{c} + \varepsilon - f] \to \{0, 1\}$ where $\sigma_i = 0$ means that individual i abstains and $\sigma_i = 1$ means that i casts an informed vote for her favored alternative r_i. An equilibrium voting strategy must be a cutoff strategy (as in Börgers, 2004) and there must be a common cutoff value $\hat{c}$ such that, for all individuals $i \in \{1, ..., n\}$,

[23] Börgers does not consider an explicit abstention fine, but considers Compulsory Voting to be an exogenous increase in the voting costs threshold to $\overline{c}$, such that all individuals vote. Then, however, if $\overline{c} > c^*$, some individuals vote although their cost exceeds the expected benefits of doing so. This yields an additional negative effect in Börgers' model, which does not occur in my case, where the abstention fine endogenously increases the equilibrium voting costs threshold.

[24] Note that if Voluntary Voting already achieves full turnout, the introduction of the abstention fine affects neither turnout nor information acquisition, and expected social welfare remains constant.

$\sigma_i = 1$ if $c_i \leq \hat{c} - \varepsilon + f$, and $\sigma_i = 0$ otherwise. For any cutoff value $\hat{c}$, the probability that individual i votes as implied by the cutoff voting strategy, is

$$p(\hat{c}) \equiv Pr(c_i \leq \hat{c} - \varepsilon + f) = G(\hat{c} - \varepsilon + f).$$

Note that $p(\hat{c}) \in [0,1]$ where $p(\hat{c}) = 1$ if $\hat{c} \geq \overline{c} + \varepsilon - f$ and $p(\hat{c}) = 0$ if $\hat{c} \leq \underline{c} + \varepsilon - f$. Through the equilibrium voting probability $p(\hat{c})$, the equilibrium benefit of voting $B(p(\hat{c}))$ is fixed and defines the cost threshold for which each individual participates. A value $\hat{c}$ is a threshold for an equilibrium cutoff voting strategy if and only if $B(p(\hat{c})) = \hat{c}$ for $\hat{c} \in (\underline{c} + \varepsilon - f, \overline{c} + \varepsilon - f)$ or $B(p(\hat{c})) \leq \hat{c}$ if $\hat{c} = \underline{c} + \varepsilon - f$ or $B(p(\hat{c})) \geq \hat{c}$ if $\hat{c} = \overline{c} + \varepsilon - f$. For existence and uniqueness of such an equilibrium threshold, we need to show that $B(p(\hat{c}))$ is differentiable and strictly decreasing in $\hat{c}$ on the interval $(\underline{c} + \varepsilon - f, \overline{c} + \varepsilon - f)$ (Börgers, 2004). Then, the function $B(p(\hat{c}))$ intersects with the 45° line exactly once on the interval $(\underline{c} + \varepsilon - f, \overline{c} + \varepsilon - f)$, so that we have exactly one point where $B(p(\hat{c})) = \hat{c}$. We already know that $B(p)$ is strictly decreasing in p and that $p = G(\hat{c} - \varepsilon + f)$. We can conclude from the fact that p is strictly increasing in $\hat{c}$ because $\frac{\partial G(\hat{c}-\varepsilon+f)}{\partial \hat{c}} = g(\hat{c} - \varepsilon + f) > 0$ for $\hat{c}$ in $(\underline{c} + \varepsilon - f, \overline{c} + \varepsilon - f)$ that $B(p(\hat{c}))$ is indeed strictly decreasing in $\hat{c}$ on this interval. □

Proof of Remark B.1.

Let $0 \leq f < \varepsilon$.

(i) Consider an increase in the voting costs, such that $\varepsilon' > \varepsilon$. Let $p^{*\prime}$ denote the equilibrium probability of voting under the increased voting costs ε', and $c^{*\prime}$ the corresponding equilibrium information cost cutoff value. I want to show that $p^{*\prime} \leq p^*$, with strict inequality if $p^* \in (0,1)$. If $p^* = 1$, it is obvious that $p^{*\prime} \leq p^*$. Therefore, consider now the remaining two cases $p^* = 0$ and $p^* \in (0,1)$.

First, consider $p^* = 0$. Then $c^* = \underline{c} + \varepsilon - f \geq \frac{1}{2}$. Then, for any $\varepsilon' > \varepsilon$ we have $\underline{c} + \varepsilon' - f > \frac{1}{2}$, such that $p^{*\prime} = 0$ as well.

Second, consider $p^* \in (0,1)$, i.e., $c^* \in (\underline{c} + \varepsilon - f, \overline{c} + \varepsilon - f)$. Recall that then $c^* = B(p^*)$. Now I want to show that $c^{*\prime} > c^*$. Suppose for a contradiction that $c^{*\prime} \leq c^*$. By the equilibrium definition of c^* and $B(\cdot)$ strictly decreasing, this is equivalent to $p^{*\prime} \geq p^*$, which, by the equilibrium definition of p is equivalent to $G(c^{*\prime} - \varepsilon' + f) \geq G(c^* - \varepsilon + f)$. By $G(\cdot)$ strictly increasing, this is equivalent to $c^{*\prime} - c^* \geq \varepsilon' - \varepsilon$. The left-hand side of this inequality is strictly negative, whereas the right-hand side is strictly positive, which yields a contradiction. Hence, we must have that $c^{*\prime} > c^*$. By the equilibrium definition of c^* and $B(\cdot)$ strictly decreasing, this is equivalent to $p^{*\prime} < p^*$.

(ii) Next, I need to show that the equilibrium probability of casting an informed vote, p^*, is weakly increasing in the abstention fine f, with strict inequality of $p^* \in (0,1)$. If $p^* = 0$, it is obvious that $p^{*\prime} \geq p^*$ for any $f' > f$. Therefore, consider now the remaining two cases $p^* = 1$ and $p^* \in (0,1)$.

First, consider $p^* = 1$. Then $c^* = \overline{c} + \varepsilon - f \leq B(1)$. Then, for any $f' > f$ we have $\underline{c} + \varepsilon - f' < B(1)$, such that $p^{*\prime} = 1$ as well.

Second, consider $p^* \in (0,1)$, i.e., $c^* \in (\underline{c} + \varepsilon - f, \overline{c} + \varepsilon - f)$. Recall that then $c^* = B(p^*)$. In equilibrium,

$$\frac{\mathrm{d}p^*}{\mathrm{d}f} = g(c^* - \varepsilon + f)\left[\frac{\mathrm{d}c^*}{\mathrm{d}f} + 1\right]$$

where, again from the implicit definition of c^*,

$$\frac{\mathrm{d}c^*}{\mathrm{d}f} = \frac{\partial B(p)}{\partial p}\left[\frac{\partial G(c^* - \varepsilon + f)}{\partial c^*}\frac{\mathrm{d}c^*}{\mathrm{d}f} + \frac{\partial G(c^* - \varepsilon + f)}{\partial f}\right].$$

Rearranging,

$$\frac{\mathrm{d}c^*}{\mathrm{d}f} = \frac{\frac{\partial B(p)}{\partial p}\frac{\partial G(c^*-\varepsilon+f)}{\partial f}}{1 - \frac{\partial B(p)}{\partial p}\frac{\partial G(c^*-\varepsilon+f)}{\partial c^*}} = \frac{\frac{\partial B(p)}{\partial p}g(c^* - \varepsilon + f)}{1 - \frac{\partial B(p)}{\partial p}g(c^* - \varepsilon + f)}$$

such that from $\frac{\partial B(p)}{\partial p} < 0$ and $g(c) > 0$ for $c \in (\underline{c}, \overline{c})$, we have $\frac{\mathrm{d}c^*}{\mathrm{d}f} \in (-1, 0)$. Therefore, $\frac{\mathrm{d}p^*}{\mathrm{d}f} > 0$ for all $c^* \in (\underline{c} + \varepsilon - f, \overline{c} + \varepsilon - f)$. □

Proof of Proposition B.2.

(i) and (ii) follow directly from everything that has been stated before.

(iii) Let $\underline{c} < B(1) < \overline{c}$. A *voting strategy* takes the form $\sigma : [\underline{c}, \overline{c}] \to \{0,1\}$ where $\sigma_i = 1$ means that individual i acquires information and hence votes for her favored alternative, and $\sigma_i = 0$ means that individual i remains uninformed and randomly votes for each alternative with equal probability.

The equilibrium voting strategy is a cutoff strategy with a common cutoff value $\hat{c}$ such that $\sigma_i = 1$ if $c_i \leq \hat{c}$ and $\sigma_i = 0$ otherwise. In particular, a value $\hat{c}$ is a threshold for an equilibrium cutoff voting strategy if and only if $B(1) = \hat{c}$ for $\hat{c} \in (\underline{c}, \overline{c})$ or $B(1) \leq \hat{c}$ if $\hat{c} = \underline{c}$ or $B(1) \geq \hat{c}$ if $\hat{c} = \overline{c}$. If $\underline{c} < B(1) < \overline{c}$, the function $B(1)$ crosses the 45° line exactly once on the interval $(\underline{c}, \overline{c})$ because $B(1)$ is simply a constant function. Therefore, we can conclude immediately that the equilibrium cutoff value exists and is unique. □

Proof of Proposition B.3.

Consider a high abstention fine $f > \varepsilon$ and let $\underline{c} < B(1) < \overline{c}$. Let p^{*V} denote the equilibrium probability of acquiring information under Voluntary Voting and q^* the equilibrium probability of acquiring information under Compulsory Voting. I need to show that there exists a unique $\tilde{\varepsilon} \in (0, \frac{1}{2} - \underline{c})$ such that for $\varepsilon < \tilde{\varepsilon}$, $q^* < p^{*V}$ while for $\varepsilon > \tilde{\varepsilon}$, $q^* > p^{*V}$.

First, consider $\varepsilon = 0$.[25] Then, by $\underline{c} < B(1) < \frac{1}{2}$ and $\overline{c} > B(1)$, $c^{*V} \in (\underline{c} + \varepsilon, \overline{c} + \varepsilon)$, and hence $c^{*V} = B(p^{*V})$. Note that because B is strictly decreasing in p^{*V} and $p^{*V} < 1$, we have $B(p^{*V}) > B(1)$. Because G is strictly increasing, it follows directly that $p^{*V} = G(B(p^{*V})) > G(B(1)) = q^*$.

Second, consider $\varepsilon = \frac{1}{2} - \underline{c}$. Then, $p^{*V} = 0$. Because $\underline{c} < B(1)$, it follows that $q^* = G(B(1)) > 0 = p^{*V}$.

Moreover, recall from Remark B.1 that p^{*V} is strictly decreasing in ε, while q^* is unaffected by ε. Therefore, because $p^{*V} > q^*$ at $\varepsilon = 0$ and $p^{*V} < q^*$ at $\varepsilon = \frac{1}{2} - \underline{c}$, we can conclude that there exists a unique $\tilde{\varepsilon} \in (0, \frac{1}{2} - \underline{c})$ where $p^{*V} = q^*$, and hence for all $\varepsilon < \tilde{\varepsilon}$, $q^* < p^{*V}$ while for all $\varepsilon > \tilde{\varepsilon}$, $q^* > p^{*V}$. □

Proof of Proposition B.4.

Let $0 \le f < \varepsilon$. Let $p^* = G(c^* - \varepsilon + f)$ denote the probability of acquiring information (and voting) for $f < \varepsilon$. I need to show that expected social welfare is weakly decreasing in the abstention fine f, and strictly increasing if $p^* \in (0,1)$. Note that if $p^* = 1$, an increase in the abstention fine f does not affect the probability of acquiring information (and voting), and therefore expected social welfare remains unaffected as well. Therefore, consider now the remaining two cases $p^* = 0$ and $p^* \in (0,1)$.

First, consider $p^* \in (0,1)$. Then, we have from Remark B.1 that $\frac{\mathrm{d}p^*}{\mathrm{d}f} > 0$. Moreover, recall that then $c^* = B(p^*)$.

$$\begin{aligned}
\frac{\mathrm{d}W(p^*)}{\mathrm{d}f} &= n\left[\left[B(p^*) - (c^* - \varepsilon + f) - \varepsilon\right] g(c^* - \varepsilon + f)\frac{\mathrm{d}(c^* - \varepsilon + f)}{\mathrm{d}f}\right. \\
&\quad \left. + \int_{\underline{c}}^{c^*-\varepsilon+f} \frac{\partial B(p^*)}{\partial p^*}\frac{\mathrm{d}p^*}{\mathrm{d}f} g(c)\mathrm{d}c\right] \\
&= n\left[-f g(c^* - \varepsilon + f)\frac{\mathrm{d}(c^* - \varepsilon + f)}{\mathrm{d}f} + \frac{\partial B(p^*)}{\partial p^*}\frac{\mathrm{d}p^*}{\mathrm{d}f}p^*\right] \\
&= n\left[\left(\frac{\partial B(p^*)}{\partial p^*}p^* - f\right)\frac{\mathrm{d}p^*}{\mathrm{d}f}\right]
\end{aligned}$$

where the second equality follows from $c^* = B(p^*)$ and $p^* = G(c^* - \varepsilon + f)$, and the third equality follows from the fact that $\frac{\mathrm{d}p^*}{\mathrm{d}f} = g(c^* - \varepsilon + f)\frac{\mathrm{d}(c^* - \varepsilon + f)}{\mathrm{d}f}$. Because $\frac{\partial B(p^*)}{\partial p^*} < 0$ and $\frac{\mathrm{d}p^*}{\mathrm{d}f} > 0$ for all $p^* \in (0,1)$, it follows that $\frac{\mathrm{d}W(p^*)}{\mathrm{d}f} < 0$.

Second, consider $p^* = 0$. Then $c^* = \underline{c} + \varepsilon - f \ge \frac{1}{2}$ and $\left.\frac{\mathrm{d}p^*}{\mathrm{d}f}\right|_{p^*=0} \ge 0$. Then

$$\left.\frac{\mathrm{d}W(p^*)}{\mathrm{d}f}\right|_{p^*=0} = n\left[\left(\frac{1}{2} - \underline{c} - \varepsilon\right)\left.\frac{\mathrm{d}p^*}{\mathrm{d}f}\right|_{p^*=0}\right]$$

where $\underline{c} + \varepsilon - f \ge \frac{1}{2}$ implies $\frac{1}{2} - \underline{c} - \varepsilon \le 0$. Hence, $\left.\frac{\mathrm{d}W(p^*)}{\mathrm{d}f}\right|_{p^*=0} \le 0$. □

Proof of Proposition B.5.

Let $p^{*V} = G(c^{*V} - \varepsilon)$ denote the probability of acquiring information (and voting) under Voluntary Voting. I want to show that, as long as $p^{*V} < 1$, expected social welfare compared is strictly lower under Compulsory Voting with a high abstention fine $f > \varepsilon$ than under Voluntary Voting. Let $q^* = G(B(1))$ denote the probability of acquiring information under Compulsory Voting with $f > \varepsilon$. I need to distinguish the three cases, where $q^* \in (0,1)$, $q^* = 0$ and $q^* = 1$.

(i) Consider $\underline{c} < B(1) < \overline{c}$ such that $q^* \in (0,1)$. Note that $B(1) < \overline{c}$ implies $p^{*V} < 1$. Therefore, either $p^{*V} = 0$ and $c^{*V} \ge B(p^{*V})$, or $p^{*V} \in (0,1)$ and $c^{*V} = B(p^{*V})$. Then,

$$\begin{aligned}
U(q^*) - U(p^{*V}) = {}& B(1)q^* - B(p^{*V})p^{*V} - \varepsilon(1 - p^{*V}) \\
& - \int_{\underline{c}}^{B(1)} c g(c)\mathrm{d}c + \int_{\underline{c}}^{c^{*V}-\varepsilon} c g(c)\mathrm{d}c
\end{aligned}$$

[25] Note that for $\varepsilon = 0$, uninformed voters are indifferent between casting a random vote and abstaining. Here, I assume that all uninformed voters abstain.

Recall from Proposition B.3 that there exists a unique voting costs threshold $\tilde{\varepsilon} \in (0, \frac{1}{2} - \underline{c})$ such that for all $\varepsilon < \tilde{\varepsilon}$, we have $q^* < p^{*V}$, while for all $\varepsilon > \tilde{\varepsilon}$, we have $q^* > p^{*V}$. Therefore, we need to distinguish these two cases.

First, consider $q^* > p^{*V}$, which is equivalent to $c^{*V} - \varepsilon < B(1)$. Therefore,

$$
\begin{aligned}
U(q^*) - U(p^{*V}) &= B(1)q^* - B(p^{*V})p^{*V} - \varepsilon(1 - p^{*V}) - \int_{c^{*V}-\varepsilon}^{B(1)} c g(c) \mathrm{d}c \\
&= B(1)q^* - B(p^{*V})p^{*V} - \varepsilon(1 - p^{*V}) \\
&\quad - \left[B(1)q^* - (c^{*V} - \varepsilon)p^{*V}\right] + \int_{c^{*V}-\varepsilon}^{B(1)} G(c) \mathrm{d}c \\
&= \left[c^{*V} - B(p^{*V})\right] p^{*V} - \varepsilon + \int_{c^{*V}-\varepsilon}^{B(1)} G(c) \mathrm{d}c \\
&< B(1) - c^{*V}
\end{aligned}
$$

where the second-to-last line follows from the fact that either $p^{*V} = 0$ or $c^{*V} = B(p^{*V})$ and from $G(c) \leq 1$ for all c. Because $B(1) < B(p^{*V}) \leq c^{*V}$ it follows that $U(q^*) - U(p^{*V}) < 0$.

Second, consider $q^* < p^{*V}$, which is equivalent to $B(1) < c^{*V} - \varepsilon$. Therefore,

$$
\begin{aligned}
U(q^*) - U(p^{*V}) &= B(1)q^* - B(p^{*V})p^{*V} - \varepsilon(1 - p^{*V}) + \int_{B(1)}^{c^{*V}-\varepsilon} c g(c) \mathrm{d}c \\
&= B(1)q^* - B(p^{*V})p^{*V} - \varepsilon(1 - p^{*V}) \\
&\quad + \left[(c^{*V} - \varepsilon)p^{*V} - B(1)q^*\right] - \int_{B(1)}^{c^{*V}-\varepsilon} G(c) \mathrm{d}c \\
&= \left[c^{*V} - B(p^{*V})\right] p^{*V} - \varepsilon - \int_{B(1)}^{c^{*V}-\varepsilon} G(c) \mathrm{d}c.
\end{aligned}
$$

Because either $p^{*V} = 0$ or $c^{*V} = B(p^{*V})$, it follows that $U(q^*) - U(p^{*V}) < 0$.

(ii) Consider $\underline{c} > B(1)$ such that $q^* = 0$. Hence, $q^* \leq p^{*V}$. Note that $\underline{c} > B(1)$ implies $B(1) < \overline{c}$, and hence $p^{*V} < 1$. Therefore, again, either $p^{*V} = 0$ and $c^{*V} \geq B(p^{*V})$, or $p^{*V} \in (0, 1)$ and $c^{*V} = B(p^{*V})$. Thus,

$$
\begin{aligned}
U(0) - U(p^{*V}) &= -B(p^{*V})p^{*V} - \varepsilon(1 - p^{*V}) + \int_{\underline{c}}^{c^{*V}-\varepsilon} c g(c) \mathrm{d}c \\
&= -B(p^{*V})p^{*V} - \varepsilon(1 - p^{*V}) + (c^{*V} - \varepsilon)p^{*V} - \int_{\underline{c}}^{c^{*V}-\varepsilon} G(c) \mathrm{d}c \\
&= \left[c^{*V} - B(p^{*V})\right] p^{*V} - \varepsilon - \int_{\underline{c}}^{c^{*V}-\varepsilon} G(c) \mathrm{d}c.
\end{aligned}
$$

As before, because either $p^{*V} = 0$ or $c^{*V} = B(p^{*V})$, it follows that $U(q^*) - U(p^{*V}) < 0$.

(iii) Consider $\overline{c} < B(1)$ such that $q^* = 1$. Hence, $q^* \geq p^{*V}$. If $p^{*V} = 1$ then obviously $U(q^*) = U(p^{*V})$. Thus, consider $p^{*V} < q^*$. Then,

$$
\begin{aligned}
U(1) - U(p^{*V}) &= B(1) - B(p^{*V})p^{*V} - \varepsilon(1 - p^{*V}) - \int_{c^{*V}-\varepsilon}^{\overline{c}} c g(c) \mathrm{d}c \\
&= B(1) - B(p^{*V})p^{*V} - \varepsilon(1 - p^{*V})
\end{aligned}
$$

$$- \left[\overline{c} - (c^{*V} - \varepsilon)p^{*V}\right] + \int_{c^{*V}-\varepsilon}^{\overline{c}} G(c)\mathrm{d}c$$

$$= B(1) - \left[c^{*V} - B(p^{*V})\right] p^{*V} - \varepsilon - \overline{c} + \int_{c^{*V}-\varepsilon}^{\overline{c}} G(c)\mathrm{d}c$$

$$< B(1) - c^{*V}$$

where the second-to-last line follows, as before, from the fact that either $p^{*V} = 0$ or $c^{*V} = B(p^{*V})$ and from $G(c) \leq 1$ for all c. Then, again, because $B(1) < B(p^{*V}) \leq c^{*V}$ it follows that $U(q^*) - U(p^{*V}) < 0$.

All in all, we have $U(q^*) < U(p^{*V})$ for any individual in all cases, as long as $p^{*V} < 1$. Therefore, we can conclude that $W(q^*) < W(p^{*V})$, i.e., expected social welfare is strictly lower under Compulsory Voting with a high abstention fine $f > \varepsilon$ compared to Voluntary Voting. □

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