# Essays in Financial Markets and Market Design

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# Contents

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# Preface

This dissertation, titled "Essays in Financial Markets and Market Design," consists of three self-contained chapters. It contributes to our understanding of how informational and behavioral frictions shape financial market outcomes and how information can be used to improve market performance. In brief, the first chapter studies risk subscription practices and market design in the reinsurance industry. The second chapter addresses equilibrium existence in quote-driven mechanisms in securities markets, with connections to screening models in insurance markets. The third chapter explores how the endogenous allocation of limited investor attention affects market characteristics in order-driven mechanisms.

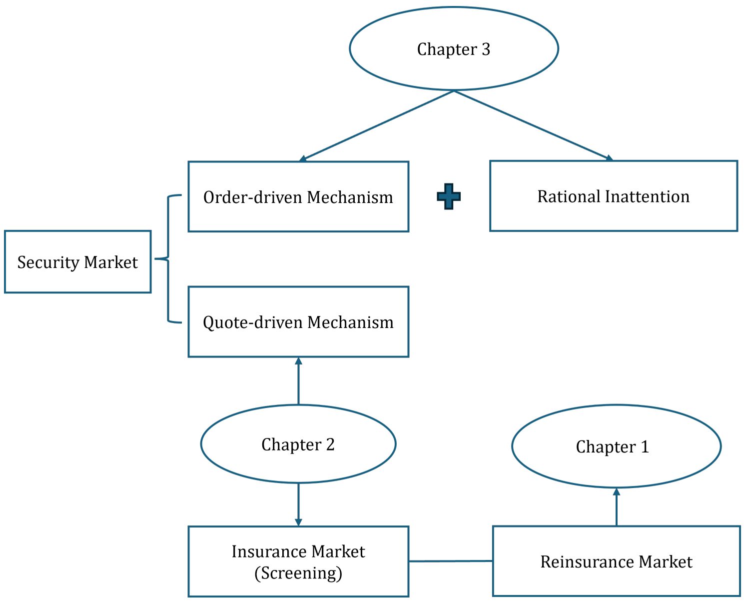


Figure 1: Overview of the Dissertation Structure

**Chapter 1** is titled "To Follow the Lead or Retrocede to Followers? An Auction Model of the Reinsurance Market." This chapter models competing reinsurance syndicates vying for underwriting risk in a common-value setting. In the market-wide follow-the-lead practice, the lead reinsurer makes an offer directly to clients based on their risk assessment, while followers, who typically provide capital and capacity, are locked into a single unit price determined at the tender stage. Whether this premium alignment feature benefits the client remains underexplored. Inspired by the design of spectrum auctions, we restructure the allocation process into multiple stages, with information revelation occurring between these stages, referred to as the retrocession case. In this scenario, the lead reinsurer makes an offer for the entire business and cedes partial risk to the followers. Similar risk allocation and capital-saving outcomes are achieved. We compare this situation with the follow-the-lead practice and find that, under the follow-the-lead scenario, lead reinsurers shade their offers to avoid the winner's curse, allowing followers to benefit. Conversely, in the case of retrocession, lead reinsurers make more aggressive offers to signal information. This design benefits the initial client insurer through an information transmission channel.

**Chapter 2** is a joint work with Ernst-Ludwig von Thadden. This chapter, titled "Revisiting Equilibrium in Quote-Driven Markets," examines the equilibrium problem in modeling quote-driven mechanisms in securities markets. The interaction between market makers and investors is formulated as a two-dimensional screening game in which competitive market makers offer price schedules to investors with informational and liquidity motives. The classic finding states that a separating equilibrium exists when informational asymmetry is not too severe, that is, when the liquidity motive dominates the informational motive. In contrast, we show that a separating outcome does not exist, even when the informational motive is arbitrarily small. The root cause lies in a cross-subsidization deviation: a market maker can profitably offer a partial-pooling price schedule that attracts a subset of investors to pool while leaving others separated. By strategically cross-subsidizing between subpools, in which the gains from a continuum of lower-cost types outweigh the losses from a continuum of higher-cost types, the deviating market maker secures strictly positive profits, thereby undermining equilibrium existence. We discuss potential remedies, including modifications to the game that help restore a full information transmission outcome.

**Chapter 3** is titled "Limited Attention, Information Choice, and Market Microstructure." This chapter examines how investors' endogenous information choices affect trading behavior and market outcomes, such as liquidity and volatility, in securities markets. We introduce attention allocation into an order-driven market using an entropy-based approach. A limited-attention investor allocates attention across macro-level and firm-specific news, subject to an information flow constraint. She then submits a market order alongside random orders from liquidity traders to market makers, who quote a price to clear the market. Our model (i) captures the comovement between investor attention and asset price volatility; (ii) provides a microfoundation for the crowding-out effect between macro-level and firm-level news in attention allocation; and (iii) explains the attenuation of price responsiveness to firm earnings news during periods of heightened aggregate uncertainty.

# Chapter 1

# To Follow the Lead or Retrocede to Followers? An Auction Model of the Reinsurance Market

## 1.1 Introduction

Rising threats, such as catastrophic climate change, and pandemics fuel a surge in demand for reinsurance.[1] In response, reinsurers often form syndicates to diversify risks.[2] A syndicate has a lead reinsurer and followers. The role of the lead is to determine the terms and conditions based on expertise and experience. The role of followers is to provide capital and the capacity to hold risk. Terms and conditions are typically uniform for all reinsurers.[3] This is referred to as the *follow-the-lead* practice, or premiums alignment. Does this practice truly benefit clients, who are insurers in this market? Are there other ways to organize the market to generate better terms for client insurers?

To address these questions, this study develops a theoretical model to analyze the follow-the-lead practice and propose a new design, hereafter referred to as the

[1]Deloitte reports that for US non-life insurance in 2022, it was the eighth consecutive year featuring at least 10 US catastrophes, causing over US$1 billion in losses. Property-catastrophe reinsurance costs for primary non-life carriers were driven up by 30.1% in 2023, which was double the prior year's hike of 14.8%. The US demand for catastrophe reinsurance alone is expected to grow as much as 15% by 2024, putting further pressure on prices.

[2]As of 31 December 2022, there were 77 syndicates, 8 special purpose arrangements, and 7 syndicates in a box (SIAB) at Lloyd's, a leading insurance and reinsurance marketplace located in London, United Kingdom. Source: https://www.lloyds.com/about-lloyds/our-market/lloyds-market

[3]See https://www.investopedia.com/terms/l/lead-reinsurer.asp. See also https://www.investopedia.com/terms/c/coreinsurance.asp

*retrocession case*.[4] We show that if the allocation process is restructured into multi-stages, in which the lead reinsurer makes an offer for the business first and cedes risks to followers, with information disclosed in between, this may not only favor the lead reinsurer but, more importantly, benefit the initial client insurer.

Our contributions to the literature are as follows. First, our modeling framework is novel in reinsurance economics. We introduce asymmetric information between reinsurers and model their competition as a first-price common-value auction, which is a natural choice given their information structure. This contrasts with existing studies that model price formation through Cournot-type interactions (e.g., Powers (2001); Boulatov and Dieckmann (2013)), Stackelberg-type interactions (e.g., Bäuerle and Glauner (2018); Chen et al. (2020)), or within a general equilibrium framework (e.g., Borch (1992); Bernis (2002); Chi and Tan (2013); Boonen et al. (2021)). Second, to the best of our knowledge, this is the first study to analyze this market practice and design a reinsurance market. Third, our model is based on auctions with resale, where the object being sold is divisible in our setting. Our results may also be applicable to other markets with divisible goods like energy markets (Anatolitis et al., 2022). Fourth, in addition to other financial markets featuring common values, such as IPO auctions where investors have to estimate the future cash flows of firms (Sherman, 2005), or securities markets (Yuan, 2022), we emphasize the common value feature in the insurance and reinsurance market, given that the risk can be estimated ex-ante and realized and fixed ex-post.

Below, we briefly outline the model, key results, and underlying intuitions. The model is depicted in Figure 1.1. The lead reinsurer makes an offer on behalf of the syndicate to the client. Other syndicates also make offers, and the client selects the most favorable one to underwrite the risk. Followers then subscribe to the pre-contractual shares at the same price as their lead reinsurer. In the follow-the-lead case, lead reinsurers shade their offers in the tender stage to protect themselves against the *winner's curse*,[5] earning a positive surplus. Since followers subscribe to

[4] Retrocession is a transaction in which a reinsurer transfers risks it has already insured to other reinsurers. After signing a treaty with the client insurer, the retrocedent (the original reinsurer) cedes part of the risks it has assumed to retrocessionaires. Source: https://blog.ccr-re.com/en/what-is-retrocession

[5] The "winner's curse" is a phenomenon often observed in auctions and competitive bidding

a portion of the business at the same unit price as the lead, they benefit from the leader's expertise in pricing and also earn positive payoffs in equilibrium.

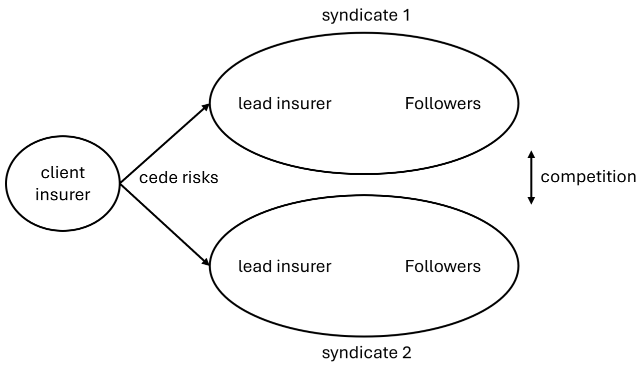


Figure 1.1: An illustration of the model

In contrast, in the retrocession design, the allocation process is divided into multiple stages, with offers disclosure occurring in between. The lead reinsurer makes an offer for the entire business and then sells part of the risk to followers to save capital. The client insurer, as the initial auctioneer, collects offers and reveals them to the follow market to promote transparency, allowing followers to have some information for evaluating the risk. In the follow market, followers with unit demand purchase the remaining risk in a first-price auction. Ultimately, the lead reinsurer retains a share while the followers hold the remaining risk. This approach achieves similar objectives in terms of risk allocation and capital savings as the follow-the-lead case.

The main result is that clients benefit more from the retrocession case than from the follow-the-lead case. In the retrocession case, a lead reinsurer balances the potential to exploit private information by retaining more shares and increasing capital savings by retaining less. Given the informational linkage across two stages, a lead reinsurer makes a more aggressive offer. The additional signaling component arises from the disclosure of bids, reducing uncertainty and limiting reinsurers' ability to leverage private information for profit. In the second stage, with offers from the first stage made public, followers' payoffs are reduced and drop to zero as there is

situations. It occurs when the winning bidder ends up overpaying for an asset due to overly optimistic estimates. This behavior is considered irrational in auctions. One behavioral concept to model this behavior is the "cursed equilibrium", proposed by Eyster and Rabin (2005).

no private information to exploit. Their surplus is extracted and transferred to the client through the lead reinsurer's aggressive bidding in the first stage.

**Related Literature.** Our study relates to the literature on economics of reinsurance. The pioneering work by Borch (1992) studies optimal risk-sharing among reinsurers to rationalize the syndication structure in the general equilibrium framework. Plantin (2006) provides a rationale for reinsurers arising endogenously from risk managers by focusing on their ability to mitigate the moral hazard problem faced by the cedant. Studies on the strategic interaction between insurers and reinsurers, typically in actuarial pricing. Zhu et al. (2023) study how competing reinsurers strategically set prices using the Stackelberg model. However, little attention has been given to follow-the-lead practices from an economic perspective. This study is the first to explore market-wide practices and design this market.

We are related to the literature on common-value auctions with a resale market. Bukhchandani and Huang (1989) model speculators bidding to resell investors to study the debatable question of which payment rule, uniform pricing or discrimination, is advantageous in treasury bill auctions. Haile (2003) studies the first-stage winner reselling a single-unit good to first-stage losers instead of a third party in our setting. In our model, the object is divisible and we allow the bidder to resell arbitrary shares. This is relevant not only to the insurance market, where risks are inherently divisible, but also to other markets, such as the energy market (e.g., Anatolitis et al. (2022)). In the auctions with resale literature, there are different motives for resale, such as cooperation on collusion through resale (Garratt et al., 2009), misperception of the resale market (Georganas, 2011), asymmetry leading to inefficiency (Hafalir and Krishna, 2008), and delaying the resale to achieve an expected gain (Khurana, 2024). In our model, the resale motive is the lead reinsurer's need to save capital. We endogenize the retention ratio by introducing capital constraints. The lead reinsurer balances ceding more to conserve capital with ceding less to retain greater uncertainty, which preserves the potential to exploit private information.

Our paper relates to auctions with signaling concerns. Perry et al. (2000) propose a two-round selling procedure in which only the two highest buyers are allowed to participate in the second round and bid above their first-round bid. This is equivalent to an English auction in an interdependent value setting. The revenue comparison between our model and an English auction was ambiguous. In independent private value settings, several studies have examine how different post-auction competitions, such as Cournot or Bertrand competition or disclosure policies, shape first-stage bidding behavior (e.g., Jehiel and Moldovanu (2000); Rhodes-Kropf and Katzman (2001); Varma (2003); Goeree (2003)). Calzolari and Pavan (2006) study optimal mechanisms under resale to third parties and inter-bidder resale. Dworczak (2020) characterizes the optimal mechanism with an aftermarket in a private value setting within the set of cutoff mechanisms. Bos and Pollrich (2022) analyze the optimal disclosure policy when bidders have concave or convex signaling concerns.

**Plan for the Paper.** The rest of the paper is organized as follows: Section 1.2 outlines the model and justifies the key assumptions in the follow-the-lead and retrocession cases. Section 1.3 analyzes the equilibrium strategies and compares the payoffs of the client insurer and reinsurers in the two settings. Section 1.4 is an extension section, which shows that the main results are robust against the disclosure of private information by the client insurer and the reserve price. Section 1.5 discusses the design. Section 1.6 concludes the paper. Proof and background information are provided in the appendices.

## 1.2 Model

**Competing Syndicates.** Consider a setting where $n \geq 2$ reinsurance syndicates compete to underwrite reinsurance risks denoted by a random variable $V$ for a client insurer in a first-price auction. Each lead reinsurer $i$ has a private signal $X_i$, where $i = 1, 2, ..., n$. A signal can be understood as their assessment of risk. $V$ and all signals $X_1, X_2, \dots, X_n$ are assumed to be affiliated, as defined in Definition 1.1.

**Definition 1.1.** *(Affiliation Condition) For all $\boldsymbol{x}_1, \boldsymbol{x}_2 \in [\underline{x}, \bar{x}]^n$, and $v_1, v_2 \in [\underline{v}, \bar{v}]^n$, the random variables $X_1, X_2, \dots, X_n$ and $V$ are said to be strictly affiliated if the* *following condition holds:*

$$f\left((v_1, \boldsymbol{x_1}) \vee (v_2, \boldsymbol{x}_2)\right) \cdot f\left((v_1, \boldsymbol{x_1}) \wedge (v_2, \boldsymbol{x_2})\right) > f\left(v_1, \boldsymbol{x_1}\right) \cdot f\left(v_2, \boldsymbol{x}_2\right),$$

*where " $\vee$" denotes component-wise maximum and " $\wedge$" denotes component-wise minimum.*

Let $f(v,x)$ denote the joint density function of $V$ and the vector of signals $X = (X_1, X_2, ..., X_n)$. It is assumed that $f$, strictly positive, with full support and twice continuously differentiable on $[\underline{v}, \overline{v}] \times [\underline{x}, \overline{x}]^n$, is symmetric in the last $n$ arguments. A bidding strategy is a measurable function denoted as $b(X_i) : [\underline{x}, \overline{x}] \to \mathbb{R}$, for $i = 1, 2, ..., n$.

Small followers are assumed to be uninformed.[6] The role of followers is to provide capital and capacity to hold the risks. They do not approach clients directly to compete with the lead because they are either less experienced or are smaller firms that cannot take on a large share of the risks.[7] Their roles can vary depending on the types of associated risk; here, we focus on one specific business. All reinsurers are assumed to be risk-neutral.[8] Each syndicate, consisting of a lead reinsurer and multiple followers, is assumed to be able to provide full coverage for the client insurer.

Affiliation is a strong form of positive correlation. Intuitively, it means if a subset of $X_i's$ are all large, then this implies an increased likelihood that the remaining $X_j's$ are also large. Suppose the insurance companies Amlin, Beazley and Catlin believe that a flood is more likely to occur in southern Germany, then it is likely that their competitor, Hiscox, would also tend to hold a similar belief. The lemmas implied bylied the affiliation condition used in the analysis of equilibrium are presented below.

**Lemma 1.1** (Milgrom and Weber, 1982, Theorem 3)**.** *If random variables $Z_1, \ldots, Z_k$*

[6]The assumption of an exogenous information structure will be discussed later in the discussion section. We maintain a minimal assumption in this section. In the extension section, we allow followers to have information, provided certain technical conditions hold. The main results remain qualitatively unaffected.

[7]Source: https://blog.ccr-re.com/en/what-is-a-follower

[8]This assumption makes the analysis tractable. Introducing risk aversion or non-linearity of utility in monetary transfer would render the differential equation in the leader's decision problem non-linear and hard to solve analytically.

*are affiliated, and $g_1, \ldots, g_k$ are all increasing functions, then $g_1(Z_1), \ldots, g_k(Z_k)$ are also affiliated.*

**Lemma 1.2** (Milgrom and Weber, 1982, Theorem 4). *If random variables $Z_1, \ldots, Z_k$ are affiliated, and $g_1, \ldots, g_k$ are all increasing functions, then $g_1(Z_1), \ldots, g_k(Z_k)$ are also affiliated.*

**Lemma 1.3** (Milgrom and Weber, 1982, Theorem 5). *Let $Z_1, \ldots, Z_k$ be affiliated, and let $H$ be any increasing function. Then the function $h$ defined by*

$$h(a_1, b_1; \ldots; a_k, b_k) = E\left[H(Z_1, \ldots, Z_k) \mid a_1 \leq Z_1 \leq b_1, \ldots, a_k \leq Z_k \leq b_k\right]$$

*is increasing in all of its arguments. In particular, for $l = 1, \ldots, k$, the functions $h_l(z_1, \ldots, z_l) = E\left[H(Z_1, \ldots, Z_k) \mid z_1, \ldots, z_l\right]$ are all increasing.*

Throughout the analysis, we keep the following technical assumption.

**Assumption 1.1.** *$E[V|X_1, \ldots, X_n] := g(x_1, \ldots, x_n)$ is supermodular in signals; i.e., $\frac{\partial^2 g}{\partial x_i \partial x_j} \geq 0$.*

The assumption roughly means signals are information complements to value. This is standard in common value models with resale and ensures the monotonicity of bidding strategies. This assumption does not contradict strict affiliation; to illustrate, we construct a numerical example based on the normal distribution.

*A Numerical Example.* Let $V$, $X_1$, and $X_2$ be jointly normally distributed random variables with means $\mu_V$, $\mu_{X_1}$, and $\mu_{X_2}$ and a covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_V^2 & \sigma_{VX_1} & \sigma_{VX_2} \\ \sigma_{VX_1} & \sigma_{X_1}^2 & \sigma_{X_1X_2} \\ \sigma_{VX_2} & \sigma_{X_1X_2} & \sigma_{X_2}^2 \end{pmatrix}.$$

By the projection theorem,[9] the conditional expectation $E[V|X_1, X_2]$ is a linear function of $X_1$ and $X_2$: $E[V|X_1, X_2] = \beta_1 X_1 + \beta_2 X_2$. The vector $\beta = (\beta_1, \beta_2)^T$ is given by $\beta = \Sigma_{VX}\Sigma_{XX}^{-1}$, where $\Sigma_{VX} = (\sigma_{VX_1}, \sigma_{VX_2})^T$ and $\Sigma_{XX}$ is the covariance matrix of $X_1$ and $X_2$. The joint density function is $f(v, x_1, x_2) = \frac{1}{(2\pi)^{3/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}\mathbf{Z}^T\Sigma^{-1}\mathbf{Z}\right)$,

[9] See, e.g., DeGroot (2005)

where $\mathbf{Z} = (v, x_1, x_2)^T$. Taking the natural logarithm of $f$, we obtain $\ln f(v, x_1, x_2) = -\frac{3}{2}\log(2\pi) - \frac{1}{2}\log|\Sigma| - \frac{1}{2}\mathbf{Z}^T\Sigma^{-1}\mathbf{Z}$. Expanding the last term gives $\mathbf{Z}^T\Sigma^{-1}\mathbf{Z} = a_{11}v^2 + a_{22}x_1^2 + a_{33}x_2^2 + 2a_{12}vx_1 + 2a_{13}vx_2 + 2a_{23}x_1x_2$, where $a_{ij}$ are elements of $\Sigma^{-1}$, the inverse of the covariance matrix. Hence, the cross-partial derivative is $\frac{\partial^2}{\partial x_1 \partial x_2}(\ln f) = -a_{23}$, as the first two terms in $\ln f$ are constant.

We assume each variable has zero mean (i.e., $\mu_V = \mu_{X_1} = \mu_{X_2} = 0$) and specific values for the covariance matrix:

$$\Sigma = \begin{pmatrix} 1 & 0.8 & 0.6 \\ 0.8 & 1 & 0.5 \\ 0.6 & 0.5 & 1 \end{pmatrix}.$$

The conditional expectation is $E[V|X_1, X_2] = \frac{2}{3}X_1 + \frac{1}{3}X_2$, so the cross-partial derivative is zero and Assumption 1 is satisfied. The inverse of $\Sigma$ can be computed as

$$\Sigma^{-1} = \begin{pmatrix} 2.439 & -1.707 & -0.805 \\ -1.707 & 2.927 & -0.878 \\ -0.805 & -0.878 & 1.707 \end{pmatrix},$$

where the off-diagonal elements are negative. The cross-partial derivative is $\frac{\partial^2 \ln f}{\partial x_1 \partial x_2} = -(-0.878) = 0.878$, which shows that strict affiliation holds. Hence, the constructed example satisfies both assumptions.

**Capital-constrained Reinsurers.** Reinsurers are capital-constrained. The amount of capital needed to underwrite the entire risk is denoted by $I$.[10] The leader $i$'s retention ratio is denoted by $\alpha_i$. We assume that the lead reinsurer $i$ has an initial net capital $K < I$ for all $i$. We consider a symmetric case for simplicity. If this assumption is violated, one reinsurer would suffice to provide the coverage, which is rare in the reinsurance market. Lead reinsurers can tap followers who jointly have sufficient financial capacity to provide coverage. Such logic resembles that in syndicated loans.[11]

[10] For current capital requirements in the insurance industry in Europe, see Solvency II: https://www.eiopa.europa.eu/browse/regulation-and-policy/solvency-ii_en

[11] The details of loan syndication are provided in Appendices.

**Structure of the Game in the Follow-the-Lead Case.** The sequence of moves under the follow-the-lead practice is as follows.

(1) Pre-contractual Stage. The lead reinsurer $i$ decides on the retention ratio $\alpha_i^*$.

(2) Tender Stage. The lead reinsurer $i$ makes an offer $b(X_i)$ to compete for underwriting the risk in a first-price auction. The winner is the one with the best offer, and the payment is the winner's offer.[12]

(3) Subscription Stage. Suppose syndicate $i$ wins. Then, the leader $i$ subscribes to an $\alpha_i$ share of the risk, and its followers jointly subscribe to the remaining share, $1-\alpha_i$, at the same unit price determined during the tender stage.

(4) All payoffs are realized.

**Payoffs in the Follow-the-lead Case.** In the first-price auction, the price $\tilde{p}$ is the winner's offer: $\tilde{p} = b(X_i)$.[13] Assuming that the winning offer is made by $i = 1$. Given the price $\tilde{p}$ and retained share $\alpha_1$, the wining lead reinsurer 1's payoff is $\tilde{\pi}_L = \alpha_1(V - \tilde{p})$. Since the unit price of risk is mandated uniformly, the followers' payoff is $\tilde{\pi}_F = (1-\alpha_1)(V-\tilde{p})$. The reinsurance syndicate 1's payoff is $\tilde{\pi}_1 = V - \tilde{p}$. The expected payoff of lead reinsurer 1, conditional on winning, is $E\tilde{\pi}_L = \alpha_1 E[V - b(X_1)|X_1 = x, Y_1 < x]$, where $Y_1$ is highest order statistics of $(X_2, \ldots, X_n)$, i.e., the highest competing signal.[14] The client insurer, as the initial auctioneer, with payoff $\tilde{\pi}_C = \tilde{p}$ is better off with a higher $\tilde{p}$.

**Structure of the Game in the Retrocession Case.** In the retrocession case, the follow market is no longer locked into the same unit price as determined in the

[12] Ties (if any) are assumed to be broken at random.

[13] One can consider a negative $\tilde{p}$ to mean the premium cost paid by the client insurer to get coverage from reinsurers, so a higher negative $\tilde{p}$ means a lower premium cost in absolute value to get full coverage. We use a standard forward auction here to simplify comparison with the literature; the results do not change qualitatively in a reverse auction, but the analysis introduces additional complexity.

[14] The implicit assumption is that bids are monotonic in signals, which can be verified after explicitly deriving it. This is the logic in Milgrom and Weber (1982) and applies similarly in our retrocession game. Moreover, they establish the existence of equilibrium by explicitly identifying a candidate using the first-order differential equation derived from the incentive compatibility condition, and showing that the equilibrium payoff is positive so that individual rationality holds. We follow the same approach.

tender stage. Suppose there are $m_i$ followers in the winning syndicate $i$. As in the follow-the-lead case, the leader $i$ determines the retention ratio $\alpha_i$ prior to the contract. The remaining share is split into $k < m_i$ units and sold to followers in a first-price auction, with each follower assumed to have unit demand for simplicity.[15] The retrocession price for the shares ceded to its followers is denoted by $p_{re}$. Between the two stages, it is assumed that all bids in the tender stage are revealed publicly.[16] The sequence of moves is as follows. The difference is that the lead reinsurer secures the entire business and then cedes portions to the followers, with bidding information disclosed between the stages.

(1) Pre-contractual Stage. The lead reinsurer $i$ decides the retention ratio $\alpha_i^*$.

(2) Tender Stage. The lead reinsurer $i$ makes an offer $b(X_i)$ to compete for underwriting the risk in a first-price auction. The winner is the one with the best offer, and the payment is the winner's offer.

(3) All offers are disclosed after the tender stage.

(4) Subscription Stage. The winning leader $i$ sells the remaining $k$ units of risk to $m_i$ followers in a first-price auction.

(5) All payoffs are realized.

**Payoffs in the Retrocession Case.** In the retrocession case, followers hold $1-\alpha_i$ share of risk, their ex-post payoff is $\hat{\pi}_F = (1-\alpha_i)V - p_{re}$. The lead reinsurer $i$ keeps $\alpha_i$ share of the business, receives $p_{re}$ for reselling $1-\alpha_i$ share of the business, and pays $\hat{p}$ for the entire business initially, so the lead reinsurer's payoff is $\hat{\pi}_L = \alpha_i V + p_{re} - \hat{p}$. Assume that the winning bid is made by $i = 1$. Since the price

[15] Suppose two or more units are sold to two followers. In this case, both followers bidding zero is an equilibrium point, as they would still acquire the share at the lowest cost. However, if only one unit is sold to two followers, bidding zero is no longer an equilibrium since one would have an incentive to deviate. Thus, we require the number of bidders to exceed the number of units to prevent this situation.

[16] In government auctions, it is mandated that all bids are revealed afterward. Relaxing this assumption would significantly complicate the derivation of continuation payoffs. We do not address the optimal disclosure policy of bids in this paper. In the independent value setting within a class of cutoff mechanisms, Dworczak (2020) characterizes the optimal disclosure policy to the aftermarket. Full disclosure of bids is also an assumption made in the literature, such as Bukhchandani and Huang (1989) and Haile (2003).

$\hat{p}$ is the winner's offer $b(X_1)$, the leader's interim payoff conditional on winning is $E\hat{\pi}_L = E[\alpha_1 V + p_{re} - b(X_1)|X_1 = x, Y_1 < x]$. The followers' joint expected payoff is given by $E\hat{\pi}_F = E[(1-\alpha_1)V - p_{re}|X_1, \ldots, X_n]$. Similar to the follow-the-lead case, the client insurer, as the initial auctioneer, benefits from a high price and his payoff is $\hat{\pi}_C = \hat{p}$.

**Solution Concept.** We focus on separating equilibrium, in which bidders of different types submit distinct bids, and followers' beliefs are Bayesian updated wherever possible.

## 1.3 Equilibrium Analysis

In this section, we analyze the equilibria in the follow-the-lead and retrocession case. We then compare the price for the client insurer and the payoffs of reinsurers in the two cases.

### 1.3.1 Equilibrium Price in the Follow-the-lead Case

**Tender Stage.** The lead reinsurer 1 chooses $b(X_1)$ to maximize his expected payoff conditional on winning, $E\tilde{\pi}_L = \alpha_1 E[V - b(X_1)|X_1 = x, Y_1 < x]$. This is the standard first-price common value auction with affiliated signals (see, e.g., Krishna (2009)).

**Pre-contractual Stage.** The expected joint equilibrium payoff of reinsurers in the winning is nonnegative, otherwise bidding zero would be preferable. For the lead reinsurer, retaining more shares leads to a weakly higher payoff, so the leader 1 chooses to pool his capital to retain a share $\alpha_1^* = K/I < 1$. The equilibrium result is summarized in the following proposition.

**Proposition 1.1.** *In the follow-the-lead case, the equilibrium price $\tilde{p}$ is equal to $\tilde{b}(x)$. The winning lead retains $\alpha^* = K/I$ share of business, and $\tilde{b}(x)$ is the symmetric equilibrium bidding strategy of the lead with signal $x$, defined as:*

$$\tilde{b}(x) = q(x, x) - \int_{\underline{x}}^{x} L(s \mid x) dq(s, s),$$

*with*

$$q(x,y) = E[V|X_1 = x, Y_1 = y];$$
$$L(s \mid x) = \exp\left\{-\int_s^x \frac{f_{Y_1}(t \mid t)}{F_{Y_1}(t \mid t)}dt\right\}.$$

*Proof.* See Krishna (2009). □

The second term, $\int_{\underline{x}}^{x} L(s \mid x)dq(s,s)$, represents bid shading. Due to this term, the equilibrium bid is less than the expected value conditional on winning, expressed as $\tilde{b}(X_1) < E[V \mid X_1 = x, Y_1 < x]$.[17] A rational lead reinsurer shade his offer to protect himself from the winner's curse. In the follow-the-lead case, followers benefit from the lead reinsurer's expertise and also gain positive payoffs in equilibrium. In the next section, we study the retrocession case.

### 1.3.2 Equilibrium Price in the Retrocession Case

We analyze the game using backward induction. We first analyze the retrocession price in the subscription stage, then derive the equilibrium bidding strategy of the lead reinsurers in the tender stage. Finally, we determine the pre-contractual retention ratio.

**Retrocession Price in the Subscription Stage.** In the subscription stage, followers can learn information from bid revelation if the bidding strategy is separating. Given their information, the retrocession price of the risk in the secondary market is the $1-\alpha$ share of the expected value $V$, conditional on all publicly available information from the first-round bids, i.e., $p_{\text{re}} = (1-\alpha)E[V \mid X_1, Y_1, \ldots, Y_{n-1}]$, where $Y_1, \ldots, Y_{n-1}$ are the order statistics from the highest to the lowest of signals $(X_2, \ldots, X_n)$.

**Bidding Strategy in the Tender Stage.** Suppose that lead reinsurers $i = 2, \ldots, n$ adopt strategy $\hat{b}$ and lead reinsurer 1 receives information $X_1 = x$ and submits a bid equal to $b$. Then if the lead reinsurer 1 wins and followers believe that he is following $\hat{b}$, the retrocession price will be $(1-\alpha)E\left[V \mid X_1 = \hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}\right] := (1-\alpha)q\left(\hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}\right)$, where $\hat{b}^{-1}$ denotes the inverse of $\hat{b}$. Define $p(x', x, y) = E\left[q\left(\hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}\right) \mid X_1 = x, Y_{n-1} = y\right]$, which is the expected retrocession

[17] See p.100 in Krishna (2009) for details.

price conditional on lead 1's true signal $x$, highest competing signal $y$, and the followers' perception of lead 1's signal $x'$ when the lead 1 wins. By the assumption of strict affiliation, both $p$ and $q$ are increasing in each of their arguments. The payoff of lead reinsurer 1, denoted as $\pi(b|x)$, is the resale price of the $1-\alpha$ share, plus the retained value of the $\alpha$ share, minus the initial price paid, which can be written in the integral form:

$$\begin{aligned}\pi(b \mid x) &\equiv E\left[\left((1-\alpha)q\left(\hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}\right) + \alpha V - b\right) \cdot \mathbf{1}\left(b \geq \hat{b}\left(Y_1\right)\right) \mid X_1 = x\right] \\ &= E\left[E\left[\left((1-\alpha)q\left(\hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}\right) + \alpha V - b\right) \cdot \mathbf{1}\left(b \geq \hat{b}\left(Y_1\right)\right) \mid X_1, Y_1\right] \mid X_1 = x\right] \\ &= E\left[\left((1-\alpha)p\left(\hat{b}^{-1}(b), X_1, Y_1\right) + \alpha q(x,y) - b\right) \cdot \mathbf{1}\left(b \geq \hat{b}\left(Y_1\right)\right) \mid X_1 = x\right] \\ &= \int_{\underline{x}}^{\hat{b}^{-1}(b)} \left[(1-\alpha)p\left(\hat{b}^{-1}(b), x, y\right) + \alpha q(x,y) - b\right] \cdot f_{Y_1}(y \mid x)dy.\end{aligned}$$

The second equality applies the law of iterated expectation. The third follows from the definition of $q(x,y)$, and the last equation is in integral form, where $f_{Y_1}(y \mid x)$ is the conditional density of the order statistic $Y_1$. The equilibrium bidding strategy $\hat{b}(x)$ for solving the optimization problem is presented below, where $\beta = 1-\alpha$ is the retrocession share to followers.

$$\begin{aligned}\hat{b}(x) &= (1-\alpha)p(x,x,x) + \alpha q(x,x) - \int_{\underline{x}}^{x} L(s \mid x)dp(s,s,s) + (1-\alpha)\int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)}dL(s,x) \\ &= p(x,x,x) - \int_{\underline{x}}^{x} L(s \mid x)dp(s,s,s) + \beta\int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)}dL(s,x),\end{aligned}$$

where

$$\begin{aligned}L(s \mid x) &= \exp\left\{-\int_{s}^{x} \frac{f_{Y_1}(t \mid t)}{F_{Y_1}(t \mid t)}dt\right\}, \\ J(s) &= \int_{\underline{x}}^{s} p_1(s,s,y) \cdot f_{Y_1}(y \mid s)dy.\end{aligned}$$

**Retention Ratio in the Pre-contractual stage.** The optimal retention ratio is $\alpha^* = K/I$. This follows from the observation that the signaling component in $\hat{p} = \hat{b}(x)$ price increases proportionally with the shares $\beta$ ceded to the followers, or equivalently, decreases with the retention ratio $\alpha$. The payoffs for the client insurer and reinsurers are $\hat{p}$ and $V - \hat{p}$, respectively. Since the followers' payoffs are zero, the lead reinsurer's payoff is $V - \hat{p}$, which decreases as the ceding share $\beta$ increases. On the one hand, ceding more leads to more aggressive bidding, reduces the potential to leverage private information for profit, and results in lower profits. On the other hand, ceding more shares results in capital savings. Hence, given the capital constraint, the lead reinsurer at most retains a share $\alpha^* = K/I$, i.e., they pool their capital into the business.

**Proposition 1.2.** *In the retrocession case, the price of the entire risk, $\hat{p}$ is equal to $\hat{b}(x)$, where $\hat{b}(x)$ is the symmetric equilibrium bidding strategy of lead reinsurer with signal $x$, defined as:*

$$\begin{aligned}\hat{b}(x) &= (1-\alpha^*)p(x,x,x) + \alpha^* q(x,x) - \int_{\underline{x}}^{x} L(s \mid x) dp(s,s,s) + (1-\alpha^*)\int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s \mid x) \\ &= p(x,x,x) - \int_{\underline{x}}^{x} L(s \mid x) dp(s,s,s) + \beta^* \int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s \mid x),\end{aligned}$$

*where*

$$\begin{aligned}&\alpha^* = K/I, \\ &L(s \mid x) = \exp\left\{-\int_{s}^{x} \frac{f_{Y_1}(t \mid t)}{F_{Y_1}(t \mid t)} dt\right\}, \\ &J(s) = \int_{\underline{x}}^{s} p_1(s,s,y) \cdot f_{Y_1}(y \mid s) dy.\end{aligned}$$

*Proof.* See the Appendices. □

In this equilibrium, $J(s)$ measures the average of the responsiveness of the retrocession price to the perceived signal. The third term $S(\beta^*, x) := \beta^* \int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s|s)} dL(s \mid x)$ captures the informational linkage between the primary and secondary markets, where $\beta^*$ is the retrocesion shares to followers. The retrocession price factors into the lead reinsurer's decision problem initially and causes them to bid more aggressively. It is the disclosure of bids across markets that gives the lead reinsurer an incentive to signal.

Technically, his result can be viewed as a weighted average of two extreme settings. In the no-resale setting ($\alpha = 1$ or $\beta = 0$), the bidding strategy in Proposition

1.2 corresponds to $\tilde{b}(x) = q(x,x) - \int_{\underline{x}}^{x} L(s \mid x)\, dq(s,s)$, as in the follow-the-lead case. In this setting, since the followers' payoff is zero, the lead reinsurer's payoff is the syndicate's payoff $E\tilde{\pi}_1$ without resale.

In the full resale setting ($\alpha = 0$ or $\beta = 1$), the strategy in Proposition 1.2 becomes $\hat{b}^{Full}(x) = p(x,x,x) - \int_{\underline{x}}^{x} L(s \mid x)\, dp(s,s,s) + \int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s|s)}\, dL(s|x)$. In this setting, the lead reinsurer's payoff is the syndicates's payoff $E\hat{\pi}_1^{Full}$ since the followers's payoff is zero. Taken together, when partial risk $\alpha$ is held by lead reinsurer and the rest $\beta$ is ceded, the equilibrium bidding strategy $\hat{b}(x)$ is a weighted average of the two settings, i.e., $\hat{b}(x) = \alpha\tilde{b}(x) + \beta\hat{b}^{Full}(x)$ as reflected in Proposition 1.2. Also, the payoff of the lead reinsurer is a weighted average of their payoffs in the two settings, i.e., $\hat{\pi}_L = \alpha\tilde{\pi}_1 + \beta\hat{\pi}_1^{Full}$, where $\alpha + \beta = 1$.

**Corollary 1.1.** *Under Assumption 1.1, the equilibrium bidding function $\hat{b}(x)$ is monotone increasing in signal $x$.*

*Proof.* See the Appendices. □

The separating bidding strategy is monotonic, enabling reinsurers in the follow market to invert lead reinsurers' bids from the tender stage to glean information.

### 1.3.3 Comparison of Payoffs in the Follow-the-lead Case and Retrocession Case

Given the equilibrium results in the follow-the-lead case (Proposition 1.1) and retrocession case (Proposition 1.2), we can compare the payoffs of client and reinsurers in the two cases.

**Proposition 1.3.** *The client insurer is better off in the retrocession case than in the follow-the-lead case, i.e., $\tilde{\pi}_C > \hat{\pi}_C$. Reinsurers are better off in the follow-the-lead case, i.e., $\hat{\pi}_1 > \tilde{\pi}_1$. Specifically, the lead reinsurer is better off in the retrocession case, i.e., $\tilde{\pi}_L > \hat{\pi}_L$, while the followers are better off in the follow-the-lead case, i.e., $\hat{\pi}_F > \tilde{\pi}_F$.*

*Proof.* See the Appendices. □

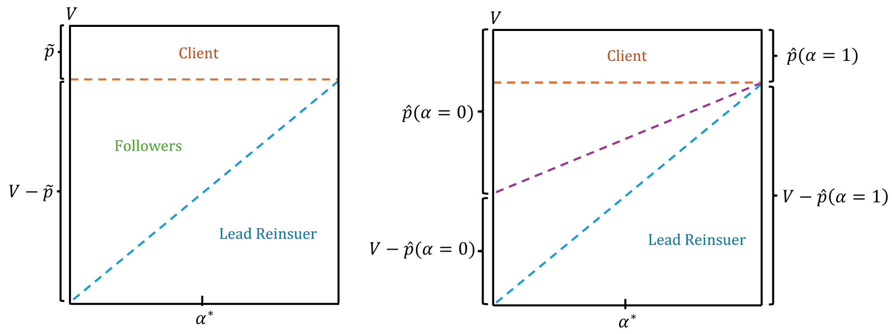


Figure 1.2: Payoffs Comparison in the Two Organizational Structures

The main results of the paper can be summarized in Figure 1.2. $\alpha^*$ is the equilibrium retention ratio. The left panel show the follow-the-lead case. The upper rectangle represents the client's payoff varying with $\alpha$, the middle triangle corresponds to the followers' payoff, and the lower triangle denotes the lead reinsurer's payoff.

The right panel shows the retrocession case. The purple dotted line corresponds to the price function $\hat{p}$ derived in Proposition 1.2, which decreases with the pre-contractual retention ratio $\alpha$. At the right boundary ($\alpha = 1$), the price converges to $\tilde{p}$, as in the follow-the-lead case. At the left boundary ($\alpha = 0$), it is full resale. The followers' payoffs are divided by this pricing function into two small triangles, which are then allocated to the client and the lead reinsurer, respectively. The upper trapezoid above $\hat{p}$ represents the client's payoff, while the lower trapezoid below $\hat{p}$ corresponds to the lead reinsurer's payoff. In the retrocession case, the followers' payoffs are zero.

Intuitively, for followers, their rent is zero in the retrocession case, whereas in the follow-the-lead case, they benefit from the lead reinsurer's expertise, shielding themselves from the winner's curse. Their rent comes from sharing the lead reinsurer's private knowledge of the risk. For the lead reinsurer, although they bid aggressively, this is only for the portion ceded to the followers, while their retention share remains unaffected. The partial surplus transferred from the followers compensates for their aggressive bidding on the ceded share, leaving them better off. For the client insurer, the additional signaling component in the first-stage offers improves their position. The extra surplus for the client insurer is extracted from the reinsurance syndicates, achieved through reselling risk and signaling to the follower market.

## 1.4 Several Extensions

This section provides a robustness check when some assumptions of the model are altered.

### 1.4.1 Announcement of Client Insurer's Information

In the main section, we assume that the client insurer does not have information because it is the reinsurers who assess, understand, and price the risk. We then relax this assumption and introduce a disclosure stage at the beginning of the games, allowing for pre-trading communication between the client insurer and reinsurers. We assume that the client has private information $X_0$ that satisfies the affiliation condition defined in Definition 1.1 and supermodularity condition defined in Assumption 1.1, i,e., $E\left[V|X_1,\ldots,X_n\right] := g\left(x_1,\ldots,x_n;x_0\right)$ is supermodular in signals; i.e., $\frac{\partial^2 g}{\partial x_i \partial x_j} \geq 0$. We show that the disclosure of the client insurer's private information does not qualitatively change the main results from the previous section.

#### Equilibrium Price in the Follow-the-Lead Case

To study the client insurer's disclosure policy, we first derive the equilibrium price when $X_0$ is revealed. Assume w.l.o.g. that syndicate 1 wins the business. In a similar way, the joint interim expected payoff conditional on winning, is given by $E\left[V-b(X_1)\mid X_1=x, Y_1<X_1;X_0\right]$, where $Y_1$ is the highest signal among the competitors, i.e., $Y_1 = max(X_2,\ldots,X_n)$. When the client insurer reveals publicly $X_0$, the equilibrium price of the reinsurance contract is summarized in the following proposition.

**Proposition 1.4.** *Under the follow-the-lead case, when $X_0$ is revealed, the equilibrium price of the entire contract $\tilde{p}$ is equal to $\tilde{b}_I(X_1;X_0)$. $\alpha^* = K/I$, and $\tilde{b}_I(X_1;X_0)$*

*is the symmetric equilibrium bidding strategy of leader 1 with signal $x$, defined as:*

$$\tilde{b}_I(X_1; X_0) = q(x, x; x_0) - \int_{\underline{x}}^{x} L(s \mid x; x_0) dq(s, s; x_0),$$

*with*

$$q(x, y; x_0) = E[V|X_1 = x, Y_1 = y; X_0 = x_0];$$
$$L(s \mid x; x_0) = \exp\left\{ -\int_s^x \frac{f_{Y_1}(t \mid t; x_0)}{F_{Y_1}(t \mid t; x_0)} dt \right\}.$$

*Proof.* See the Appendices. □

To show that publicly revealing $X_0$ before trading benefits the client, we need the revenue ranking result or the linkage principle as shown in lemma 1.4. Let $W(z, x)$ be the expected price paid by bidder 1 if he is the winning bidder when he receives a signal $x$ but bids as if his signal were $z$, i.e., $W(z, x) = b(z)$. Let $W_2(z, x)$ denote the partial derivative of the function $W(z, x)$ with respect to the second argument. Then the following result holds.

**Lemma 1.4.** *Let A and B be two auctions in which the highest bidder wins. Suppose that each auction has a symmetric and increasing equilibrium such that (1) for all $x$, $W_2^A(x, x) \geq W_2^B(x, x)$; (2) $W^A(\underline{x}, \underline{x}) = E[V \mid X_1 = \underline{x}, Y_1 = \underline{x}] = W^B(\underline{x}, \underline{x})$. Then the expected revenue in A is at least as large as that in B.*

*Proof.* See Proposition 7.1 in Krishna (2009). □

In our setting, auctions A and B correspond to the scenarios with and without revealing $X_0$, respectively. Based on lemma 1.4, we obtain the following result.

**Proposition 1.5.** *Under the follow-the-lead case, publicly revealing $X_0$ benefits the client.*

*Proof.* See the Appendices. □

Based on the above results, we know that the client insurer would disclose $X_0$ to reinsurers in the disclosure stage. One might wonder would the client insurer censor information, say, she discloses information if it is above some threshold and withhold it otherwise. A simple unraveling argument under passive beliefs of reinsurers rules out such a possibility.[18]

### Equilibrium Price in the Retrocession Case

Given that the client insurer has private information $X_0$, we consider the impact of revealing $X_0$ on the equilibrium price of the reinsurance business. Let $\hat{b}(x; x_0)$ be the symmetric bidding strategy in the tender stage conditional on $X_0 = x_0$. Then if the lead reinsurer 1 wins and followers believe that he is following $\hat{b}(X_1; X_0)$, the retrocession price will be $(1-\alpha)q\left(\hat{b}^{-1}(b; x_0), Y_1, \ldots, Y_{n-1}; X_0\right) = E\left[(1-\alpha)V \mid X_1 = \hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}; X_0\right]$, where $\hat{b}^{-1}(\cdot; x_0)$ denotes the inverse of $\hat{b}(\cdot; x_0)$.

Define $p\left(x', x, y; x_0\right) = E\left[q\left(x', Y_1, \ldots, Y_{n-1}; X_0\right) \mid X_1 = x, Y_1 = y\right]$, which is the expected retrocession price conditional on $X_1$ and $Y_1$ when the lead 1 wins and followers believe that bidder 1's private signal is equal to $x'$. The payoff of lead reinsurer 1, denoted as $\pi(b \mid x; x_0)$, by a similar way, can be written in the following integral form:

$$\begin{aligned}\pi(b \mid x; x_0) &\equiv E\left[\left((1-\alpha)q\left(\hat{b}^{-1}(b), Y_1; X_0\right) + \alpha V - b\right) \cdot \mathbf{1}\left(b \geq \hat{b}\left(Y_1\right)\right) \mid X_1 = x; X_0 = x_0\right] \\ &= \int_{\underline{x}}^{\hat{b}^{-1}(b)} \left[(1-\alpha)p\left(\hat{b}^{-1}(b), x, y; x_0\right) + \alpha q(x, y; x_0) - b\right] \cdot f_{Y_1}(y \mid x; x_0)dy.\end{aligned}$$

In the Appendices, we derive the first-order condition using the Leibniz integral rule, solve the differential equation, and demonstrate that the bidding strategy leads to a maximum payoff, not a minimum. From this, we obtain the following equilibrium result.

**Proposition 1.6.** *In the retrocession case, when $X_0$ is revealed, the winning leader keeps a share $\alpha^* = K/I$ of risks and cedes the rest $\beta^* = 1 - \alpha^*$ to followers. The price of the entire business $\hat{p}$ is equal to $\hat{b}(x_1; x_0)$, where $\hat{b}(x; x_0)$ is the symmetric*

[18] Suppose $X_0$ is uniformly distributed on $[0, 1]$ and such a cutoff $x^* \in [0, 1]$ exists. Upon not receiving information, reinsurers believe that $x$ is uniformly in $[0, x^*]$ with an average $x^*/2$. Thus, a client insurer with value $x \in (x^*/2, 1]$ would disclose to avoid being perceived as a low type. After $n$ rounds of reasoning, a client insurer with value $x \in (x^*/2^n, 1]$ would disclose. Hence, full disclosure is optimal when there are no other frictions, such as disclosure costs (Dye (1985)), uncertainty of information endowment(Wagenhofer (1990)), etc.

*equilibrium bidding strategy of lead reinsurer with signal $x$, defined as:*

$$\begin{aligned}\hat{b}(x;x_0) &= (1-\alpha^*)p(x,x,x;x_0) + \alpha^* q(x,x;x_0) - \int_{\underline{x}}^{x} L(s \mid x;x_0)\, dp(s,s,s;x_0) \\ &\qquad + \beta^* \int_{\underline{x}}^{x} \frac{J(s;x_0)}{f_{Y_1}(s \mid s;x_0)}\, dL(s,x;x_0) \\ &= p(x,x,x;x_0) - \int_{\underline{x}}^{x} L(s \mid x;x_0)\, dp(s,s,s;x_0) + \beta^* \int_{\underline{x}}^{x} \frac{J(s;x_0)}{f_{Y_1}(s \mid s;x_0)}\, dL(s,x;x_0)\end{aligned}$$

*where*

$$\begin{aligned}L(s \mid x;x_0) &= \exp\left\{-\int_s^x \frac{f_{Y_1}(t \mid t;x_0)}{F_{Y_1}(t \mid t;x_0)} dt\right\}, \\ J(s;x_0) &= \int_{\underline{x}}^{s} p_1(s,s,y;x_0) \cdot f_{Y_1}(y \mid s;x_0) dy.\end{aligned}$$

*Proof.* See the Appendices. □

Based on the linkage principle, we have the following revenue ranking result.

**Proposition 1.7.** *In the retrocession case, publicly revealing $X_0$ benefits the clients.*

*Proof.* See the Appendices. □

Based on the above result, we know that under the retrocession case, the client insurer reveals $X_0$ publicly to reinsurers, and the price of the reinsurance business is $\hat{p}$ as defined in Proposition 1.6. The clients are better off since the equilibrium price in Proposition 1.6 is higher than $\tilde{p}$ in Proposition 1.4 by the signaling component.

### 1.4.2 Followers' Information

We relax the assumption that followers are uninformed. Our main results remain unaffected if the followers have some information, provided that their information is garbled by that of the lead, as defined in Definition 1.2.

**Definition 1.2.** *(Garbling Condition) A random variable $Z_{n+k}$ is a garbling of $(Z_1, Z_2, \ldots, Z_n)$ if the joint density of $V, Z_1, Z_2, \ldots, Z_n, Z_{n+k}$ can be written as $g(V, Z_1, \ldots, Z_n) \cdot h(Z_{n+k} \mid Z_1, \ldots, Z_n)$, where $g$ and $h$ are joint density and conditional density of respective variables.*

It is a strong sufficient statistic condition.[19] Should a following reinsurer denoted as $n+k$ ground her estimate $Z_{n+k}$ on the information accessible to lead reinsurers, the garbling condition is satisfied. In an extreme case where followers are uninformed and leaders are informed, this condition holds as well, because empty information sets are subsets of any non-empty set. One implication from the garbling condition is lemma 1.5.

**Lemma 1.5.** *If a random variable $Z_{n+k}$ is a garbling of random variables $Z_1, Z_2, \ldots,$ $Z_n$, then for $k = 1, 2, \ldots, m$, it holds that*

$$E\left[V \mid Z_1, Z_2, \ldots, Z_n, Z_{n+k}\right] = E\left[V \mid Z_1, Z_2, \ldots, Z_n\right].$$

*Proof.* See the Appendices. □

lemma 1.5 says that the evaluation of reinsurance risk $V$ at a more precise information set $(Z_1, Z_2, \ldots, Z_n)$ remains unchanged when an additional piece of coarse information $Z_{n+k}$ is introduced. To see that the retrocession price is not affected, note that the signals of followers are garbled by those of lead reinsurers. For an individual follower, her evaluation of the risk conditional on her private signal $X_{n+k}$ and the lead reinsurers' signals, is equivalent to evaluating it based on the lead reinsurers' signals by lemma 1.5, i.e., $E\left[V \mid X_1, Y_1, \ldots, Y_{n-1}, X_{n+k}\right] = E\left[V \mid X_1, Y_1, \ldots, Y_{n-1}\right]$, where $k = 1, 2, \ldots, m$. Given that their value remains unaffected, the analysis in the main section remains unchanged.

### 1.4.3 Reserve Price

This section discusses the introduction of a reserve price or participation fee. These two are equivalent to excluding some bidders with lower values from participation. We exclude this in the main section for two reasons. First, from a technical perspective, it could lead to the nonexistence of monotone equilibria when signals are affiliated (See Landsberger (2007) for counterexamples). Second, in cases where

[19]Milgrom and Weber (1982) use this to show that when a less-informed bidder competes with better-informed bidders in a common value auction, she receives zero payoffs in an equilibrium.

equilibrium exists with a reserve price $r \in [\underline{v}, \bar{v}]$, the main result remains qualitatively unaffected. In the follow-the-lead case, given a reserve price $r$, any lead reinsurer with a signal $x$ below $r$ would not participate, as their payoff would be negative if they win. For lead reinsurers participating in the business, the boundary condition for the lowest type changes to $\hat{b}(r) = q(r, r)$. The rest of the derivation remains unchanged. Hence, the symmetric equilibrium bidding strategy for the lead reinsurer with signal $x$ is given by:

$$\tilde{b}(r, x) = q(x, x) - \int_r^x L(s \mid x) dq(s, s).$$

In the retrocession case, the same set of lead reinsurers with value $X < r$ are excluded. The boundary changes to $\hat{b}(r) = p(r, r, r)$, and the symmetric bidding strategy is given by:

$$\hat{b}(r, x) = p(x, x, x) - \int_r^x L(s \mid x) dp(s, s, s) + \beta^* \int_r^x \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s \mid x),$$

and the comparison of prices remains unaffected.

## 1.5 Discussion of the Model

Below, we discuss several assumptions of the model and the motivation of the design.

**Collusive Bidders.** Our model assumes no collusive behavior in either scenario, as reinsurers typically interact across various business lines and prioritize long-term reputations, discouraging deceptive or illegal actions. We acknowledge that preventing bidder collusion is a key aspect of auction design. In the retrocession scenario, the lead reinsurer profits more than in the follow-the-lead case. This increased profit incentive encourages the lead reinsurer's participation but may also heighten collusion risks, such as forming tacit alliances to demand higher premiums and share surplus after winning. Exploring a collusion-proof design is an intriguing avenue for future research.

**Exogenous Information Structure.** The information structure in our model is exogenously given. We assume that the leader has information while followers are uninformed, modeling followers' reliance on the leader's expertise to price risk. The leader's role often depends on specialized expertise; for example, Tokio Marine excels in assessing earthquake risks, while Taiping Re specializes in typhoons. Their roles as leaders and followers may interchange depending on the business context. Our model focuses on one specific business line, and the information assumption is a simplification of reality. A potential direction for future research could be to allow reinsurers to acquire information and study their information acquisition behavior under various organizational structures.

**Motivation of the Design.** The optimal selling mechanism for an object when bidders' values are correlated is well-established in theory. Crémer and McLean (1985) (hereafter CM) show that when bidders' values are slightly correlated, the entire rent can be extracted. McAfee et al. (1989) extends this result to cases where agents' types are continuously distributed. The details of this mechanism are provided in the Appendices.

Though the full surplus extraction result is insightful, it has been criticized as unrealistic and has not been observed in practice. Several theoretical explanations include bidders' risk aversion and limited liability (See Robert (1991)), information acquisition about competitors' types (See Bikhchandani (2010)), and non-robustness to bidders' beliefs (See Pham and Yamashita (2024)). Börgers (2015) argues that "one should view the Cremer-McLean result as a paradox rather than guidance for constructing practical mechanisms."

Our design is motivated by Milgrom's spectrum auction design for the Federal Communications Commission, specifically the Simultaneous Multi-Round Auction (SMRA) (Milgrom, 2000). In SMRA, the original single-round bidding process is restructured into multiple rounds, with some bidding information revealed between stages to encourage more aggressive bidding in subsequent rounds. Although our model is not as sophisticated as the SMRA, which involves repeated bidding and multiunit allocation, it borrows some of its key features. For example, we restructure the allocation process from static, one-time allocation to a two-round dynamic allocation, accounting for the information disparity between lead reinsurers and followers. Additionally, bidding information from the first round is disclosed before the opening of the second round, establishing informational linkages across markets to promote aggressive bidding. Moreover, this phased disclosure enhances transparency by allowing followers to better understand the risk environment, leading to more informed decision-making and improved price discovery.

Another merit of the retrocession design is that the lead reinsurer in the syndicate is more willing to participate, as their equilibrium payoffs are higher under this structure. Although the lead reinsurer bids more aggressively, the signaling component pertains only to the cession shares allocated to followers, which is compensated by followers in secondary markets. Our design may shed light on the organization of this market and other markets with common-value features for policymakers.

## 1.6 Concluding Remarks

This paper presents a model for studying the organization of the reinsurance market by comparing the current follow-the-lead practice with a proposed retrocession design that offers greater benefits to clients. In follow-the-lead practice, a capital-constrained lead reinsurer sets the terms of the offer, and followers subscribe to the remaining shares at a uniform price. The lead reinsurer benefits from private information about the underlying risk, while the followers earn rent by leveraging the lead's expertise in risk assessment.

In the proposed retrocession design, the allocation process is divided into two stages with information revelations occurring between them. In the first stage, the lead reinsurer balances leveraging their potential to exploit private information by retaining more shares and increasing capital savings by retaining fewer shares. In the second stage, with all information public, followers' payoffs drop to zero as there is no private information to exploit. Their surplus is extracted and transferred to the client through aggressive bidding by the lead reinsurer.

This retrocession design is not widely observed in practice, possibly for reasons similar to the initial introduction of the SMRA in spectrum auctions. There may be a lack of understanding regarding its mechanism, as restructuring a simple allocation process into a dynamic one introduces additional complexity and requires more effort to organize the subsequent interactions.

## 1.7 Appendices to Chapter 1

### 1.7.1 Proof of Proposition 1.2

*Proof.* Given the lead reinsurer's payoff, the first-order condition, derived using the Leibniz rule, is expressed as follows:

$$\begin{aligned}\hat{b}'(x) &= \left[(1-\alpha)p(x,x,x) + \alpha q(x,x) - \hat{b}(x)\right] \cdot \frac{f_{Y_1}(x \mid x)}{F_{Y_1}(x \mid x)} + (1-\alpha)\int_{\underline{x}}^{x} p_1(x,x,y) \cdot \frac{f_{Y_1}(y \mid x)}{F_{Y_1}(x \mid x)}\, dy \\ &= [p(x,x,x) - \hat{b}(x)] \cdot \frac{f_{Y_1}(x \mid x)}{F_{Y_1}(x \mid x)} + (1-\alpha)\int_{\underline{x}}^{x} p_1(x,x,y) \cdot \frac{f_{Y_1}(y \mid x)}{F_{Y_1}(x \mid x)}\, dy\end{aligned}$$

With the boundary condition of the lowest type $\hat{b}(\underline{x}) = p(\underline{x},\underline{x},\underline{x})$, the solution to the above differential equation is

$$\begin{aligned}\hat{b}(x) &= (1-\alpha)p(x,x,x) + \alpha q(x,x) - \int_{\underline{x}}^{x} L(s \mid x) dp(s,s,s) + (1-\alpha)\int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s,x) \\ &= p(x,x,x) - \int_{\underline{x}}^{x} L(s \mid x) dp(s,s,s) + (1-\alpha)\int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s,x),\end{aligned}$$

where

$$\begin{aligned}L(s \mid x) &= \exp\left\{-\int_{s}^{x} \frac{f_{Y_1}(t \mid t)}{F_{Y_1}(t \mid t)} dt\right\}, \\ J(s) &= \int_{\underline{x}}^{s} p_1(s,s,y) \cdot f_{Y_1}(y \mid s) dy.\end{aligned}$$

To demonstrate that the lead reinsurer indeed achieves the maximum profit, not the minimum, consider that if $x' < x$, her payoff when bidding $\hat{b}(x')$ is then $\pi\left(\hat{b}(x') \mid x\right)$. Note that $\frac{\partial \pi\left(\hat{b}(x') \mid x'\right)}{\partial \hat{b}} = 0$, and the following inequality holds.

$$\frac{\partial \pi\left(\hat{b}\left(x'\right) \mid x'\right)}{\partial b} = \frac{\left[p\left(x', x', x'\right) - \hat{b}(x)\right] \cdot f_{Y_1}\left(x' \mid x'\right) + \int_{\underline{x}}^{x'} (1-\alpha) p_1\left(x', x', y\right) \cdot f_{Y_1}\left(y \mid x'\right) dy}{\hat{b}'\left(x'\right)}$$

$$- F_{Y_1}\left(x' \mid x'\right)$$

$$= \left[\hat{b}'\left(x'\right)\right]^{-1} F_{Y_1}\left(x' \mid x'\right) \left\{ \begin{array}{c} \left(p\left(x', x', x'\right) - \hat{b}\left(x'\right)\right) \frac{f_{Y_1}(x' \mid x')}{F_{Y_1}(x' \mid x')} \\ + \int_{\underline{x}}^{x} (1-\alpha) p_1\left(x', x', y\right) \cdot \frac{f_{Y_1}(y \mid x')}{F_{Y_1}(x' \mid x')} dy - \hat{b}'\left(x'\right) \end{array} \right\}$$

$$\leq \left[\hat{b}'\left(x'\right)\right]^{-1} F_{Y_1}\left(x' \mid x'\right) \left\{ \begin{array}{c} \left(p\left(x', x, x'\right) - \hat{b}\left(x'\right)\right) \frac{f_{Y_1}(x' \mid x)}{F_{Y_1}(x' \mid x)} \\ + \int_{\underline{x}}^{x} (1-\alpha) p_1\left(x', x, y\right) \cdot \frac{f_{Y_1}(y \mid x)}{F_{Y_1}(x' \mid x)} dy - \hat{b}'\left(x'\right) \end{array} \right\}$$

$$= \left[\frac{F_{Y_1}\left(x' \mid x'\right)}{F_{Y_1}\left(x' \mid x\right)}\right] \left[\frac{\partial \pi\left(b\left(x'\right) \mid x\right)}{\partial \hat{b}}\right].$$

The above inequality holds because $F_{Y_1}(\cdot \mid x)$ dominates $F_{Y_1}(\cdot \mid x')$ in terms of reverse hazard rate for all $x' < x$ (see p. 287 in the Appendices of Krishna, 2009). Moreover, $p$ and its derivative $p_1$ are nondecreasing in their arguments due to affiliation and Assumption 1, and $\frac{F_{Y_1}(\cdot \mid x')}{F_{Y_1}(x' \mid x')}$ first-order dominates $\frac{F_{Y_1}(\cdot \mid x)}{F_{Y_1}(x' \mid x)}$, also by affiliation. Note that $\frac{\partial \pi(\hat{b}(x'))}{\partial \hat{b}} \geq 0$ implies that when the true signal of a lead reinsurer is $x$ and he bids $b(x')$ where $x' \leq x$, he would increase the bid to maximize his payoff. Symmetrically, it holds that $\frac{\partial \pi(\hat{b}(x'))}{\partial \hat{b}} \leq 0$ if $x' \geq x$. The lead would lower his bid $\hat{b}(x')$ to increase his payoff if $x' \leq x$. We must also have that for all $x$, $\alpha p(x,x,x) + (1-\alpha) q(x,x) - \hat{b}(x) = p(x,x,x) - \hat{b}(x) > 0$ holds for all $x$ in $[\underline{x}, \bar{x}]$, otherwise a bid of zero would be better. Note the following results hold.

$$\begin{aligned} p(x,x,x) - \hat{b}(x) &= \int_{\underline{x}}^{x} L(s \mid x)\, dp(s,s,s) - (1-\alpha) \int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)}\, dL(s,x) \\ &= \int_{\underline{x}}^{x} L(s \mid x) \left[p_1(s,s,s) + p_2(s,s,s) + p_3(s,s,s)\right] ds \\ &\quad - (1-\alpha) \int_{\underline{x}}^{x} L(s,x) \frac{J(s)}{F_{Y_1}(s \mid s)}\, ds \\ &= \int_{\underline{x}}^{x} L(s \mid x) \left[p_1(s,s,s) + p_2(s,s,s) + p_3(s,s,s) - (1-\alpha) \frac{J(s)}{F_{Y_1}(s \mid s)}\right] ds \\ &\geq \int_{\underline{x}}^{x} L(s \mid x) \left[p_2(s,s,s) + p_3(s,s,s)\right] ds > 0. \end{aligned}$$

The first equality uses total differentiation of $dp$ and the fact that $\frac{dL(s,x)}{L(s,x)} = \frac{f_{Y_1}(s \mid s)}{F_{Y_1}(s \mid s)} ds$.

The inequality uses $(1-\alpha)\frac{J(s)}{F_{Y_1}(s|s)} := \beta\frac{J(s)}{F_{Y_1}(s|s)} = \beta\int_{\underline{x}}^{s} p_1(s,s,y)\cdot\frac{f_{Y_1}(y|s)}{F_{Y_1}(s|s)}dy \leq p_1(s,s,s)$ for $y \leq s$ and $0 \leq \beta \leq 1$. The last inequality holds since $p_2 > 0$ and $p_3 > 0$ by affiliation. Hence, bidding $b(x)$ is indeed an equilibrium when the lead's signal is $x$. This completes the proof. □

### 1.7.2 Proof of Corollary 1.1

*Proof.* In the proof of Proposition 1.2, we establish that $p(x,x,x) - b(x) > 0$. Substituting this result into the expression for $\hat{b}'(x)$ and noting that $p_1(x,x,y) \geq 0$ by Assumption 1.1, we conclude that the derivative $\hat{b}'(x)$ is positive. This completes the proof. □

### 1.7.3 Proof of Propostion 1.3

*Proof.* First, for the client, in the follow-the-lead case, the client's payoff is $\hat{\pi}_C = \hat{p}$, while in the retrocession case, the client's payoff is $\tilde{\pi}_C = \tilde{p}$. Since $\tilde{p} - \hat{p} = S(\beta^*, x) > 0$, the client is better off in the retrocession case. This is illustrated in Figure 1.2, where the distance between the purple dotted line and the upper bound of the box is greater than that between the orange dotted line and the upper bound of the box.

Second, for the followers, in the follow-the-lead case, the followers' payoff is $\tilde{\pi}_F = (1-\alpha^*)(V - \tilde{p}) > 0$, whereas in the retrocession case, their payoff is zero, i.e., $\hat{\pi}_F = 0$. Therefore, followers are better off in the follow-the-lead case. This is illustrated in Figure 1.2, where the middle triangle is divided by the purple line into two parts in the right-hand panel, which are transferred to the client and the lead reinsurers.

Third, for the lead reinsurer, in the follow-the-lead case, the leader's payoff is $\tilde{\pi}_L = \alpha^*\tilde{\pi}_1 > 0$. In the retrocession case, the lead's payoff is $\hat{\pi}_L = \alpha^*\tilde{\pi}_1 + \beta^*\hat{\pi}_1^{\text{Full}} > \tilde{\pi}_L$, where $\alpha^* + \beta^* = 1$. The fact that $\hat{\pi}_1^{\text{Full}} > 0$ can be seen in the proof of Proposition 2 that the equilibrium payoff is positive if $\alpha = 0$ or $\beta = 1$. This is illustrated in Figure 1.2, where the lower trapezoid in the right-hand panel covers the lower triangle in the left-hand panel.

Fourth, for the reinsurers, in the common-value setting, their joint payoff, $V - p$, and the client's payoff, $p$, sum to a constant $V$ in both cases. If the client is better off in the retrocession case than in the follow-the-lead case, the reinsurers are correspondingly better off in the follow-the-lead case. This can be seen in Figure 1.2, where the lower rectangle in the left panel covers the trapezoid below the purple dotted line in the right panel. $\square$

### 1.7.4 Proof of Proposition 1.4

*Proof.* Taking the FOC of equilibrium payoff to get the equilibrium bidding strategy candidate

$$\tilde{b}(x) = q(x, x; x_0) - \int_{\underline{x}}^{x} L(s \mid x; x_0)\, dq(s, s; x_0).$$

We need to show that the bidding strategy candidate $\tilde{b}(x; x_0)$ leads to a maximum in payoff. If the leader bids $\tilde{b}(x'; x_0)$ instead when his signal is $x$, the payoff is $\pi(\tilde{b}(x'; x_0) \mid x)$. Taking the derivative:

$$\begin{aligned}
\frac{\partial \pi(\tilde{b}(x'; x_0) \mid x)}{\partial x'} &= (q(x, x'; x_0) - \tilde{b}(x'; x_0)) f_{Y_1}(x' \mid x; x_0) - \tilde{b}'(x; x_0) F_{Y_1}(x' \mid x; x_0) \\
&= F_{Y_1}(x' \mid x; x_0) \left[ (q(x, x'; x_0) - \tilde{b}(x'; x_0)) \frac{f_{Y_1}(x' \mid x; x_0)}{F_{Y_1}(x' \mid x; x_0)} - \tilde{b}'(x; x_0) \right].
\end{aligned}$$

If $x' < x$, then since $q(x, x'; x_0) > q(x', x'; x_0)$ and $\frac{f_{Y_1}(x'|x;x_0)}{F_{Y_1}(x'|x;x_0)} > \frac{f_{Y_1}(x'|x';x_0)}{F_{Y_1}(x'|x';x_0)}$, it holds that

$$\begin{aligned}
\frac{\partial \pi(\tilde{b}(x'; x_0) \mid x)}{\partial x'} &> F_{Y_1}(x' \mid x; x_0) \left[ (q(x', x'; x_0) - \tilde{b}(x'; x_0)) \frac{f_{Y_1}(x' \mid x'; x_0)}{F_{Y_1}(x' \mid x'; x_0)} - \tilde{b}'(x; x_0) \right] \\
&= \frac{\partial \pi(\tilde{b}(x'; x_0) \mid x')}{\partial x'} = 0.
\end{aligned}$$

In words, the payoff increases with a higher bid when $x' < x$. Similarly, we have that $\frac{\partial \pi(\tilde{b}(x';x_0)|x)}{\partial x'} < 0$ when $x' > x$. The payoff increases with a lower bid when $x' < x$. Hence, choosing $\tilde{b}(x; x_0)$ when the signal is $x$ indeed leads to a maximum payoff.

This completes the proof. $\square$

### 1.7.5 Proof of Proposition 1.5

*Proof.* When $X_0$ is revealed, the expected payment of a winning leader when he receives a signal x but bids as if his signal were z (i.e., for all $X_0 = x_0$, he bids $\tilde{b}(z; x_0)$) is

$$W^A(z,x) = E\left[\tilde{b}_I(z;x_0) \mid X_1 = x\right],$$

so $W_2^A(z,x) \geq 0$, because $X_0$ and $X_1$ are affiliated.

When $X_0$ is not revealed, similarly we have that

$$W^B(z,x) = \tilde{b}(z),$$

so $W_2^B(z,x) = 0$.

Hence, $W_2^A(z,x) \geq W_2^B(z,x)$. Publicly revealing $X_0$ raises $\tilde{p}$. $\square$

### 1.7.6 Proof of Proposition 1.6

*Proof.* The proof is similar to that in Proposition 1.2. The FOC is derived using the Lebnitz's integral rule, and $\hat{b}(x;x_0)$ satisfies a first-order differential equation:

$$\begin{aligned}
\widehat{b}'(x) &= \left[(1-\alpha)p(x,x,x;x_0) + \alpha q(x,x;x_0) - \hat{b}(x;x_0)\right] \cdot \frac{f_{Y_1}(x \mid x;x_0)}{F_{Y_1}(x \mid x;x_0)} \\
&\qquad + (1-\alpha)\int_{\underline{x}}^{x} p_1(x,x,y;x_0) \cdot \frac{f_{Y_1}(y \mid x;x_0)}{F_{Y_1}(x \mid x;x_0)}\, dy \\
&= \left[p(x,x,x;x_0) - \hat{b}(x;x_0)\right] \cdot \frac{f_{Y_1}(x \mid x;x_0)}{F_{Y_1}(x \mid x;x_0)} \\
&\qquad + (1-\alpha)\int_{\underline{x}}^{x} p_1(x,x,y;x_0) \cdot \frac{f_{Y_1}(y \mid x;x_0)}{F_{Y_1}(x \mid x;x_0)}\, dy.
\end{aligned}$$

With the boundary condition of the lowest type $\hat{b}(\underline{x}; x_0) = p(\underline{x}, \underline{x}, \underline{x}; x_0)$, the solution to the above differential equation is

$$\begin{aligned}
\hat{b}(x; x_0) = (1-\alpha)p(x,x,x;x_0) + \alpha q(x,x;x_0) - \int_{\underline{x}}^{x} L(s \mid x; x_0) dp(s,s,s;x_0) \\
+ (1-\alpha) \int_{\underline{x}}^{x} \frac{J(s;x_0)}{f_{Y_1}(s \mid s; x_0)} dL(s,x;x_0) \\
= p(x,x,x;x_0) - \int_{\underline{x}}^{x} L(s \mid x; x_0) dp(s,s,s;x_0) + (1-\alpha) \int_{\underline{x}}^{x} \frac{J(s;x_0)}{f_{Y_1}(s \mid s; x_0)} dL(s,x;x_0),
\end{aligned}$$

where

$$\begin{aligned}
L(s \mid x; x_0) &= \exp\left\{ -\int_{s}^{x} \frac{f_{Y_1}(t \mid t; x_0)}{F_{Y_1}(t \mid t; x_0)} dt \right\}, \\
J(s; x_0) &= \int_{\underline{x}}^{s} p_1(s,s,y;x_0) \cdot f_{Y_1}(y \mid s; x_0) dy.
\end{aligned}$$

Given the equilibrium bidding strategy candidate, to show that the lead reinsurer indeed achieves the maximum profit, not the minimum, consider that if $x' < x$, their payoff when bidding $\hat{b}(x'; x_0)$ is then $\pi\left(\hat{b}(x'; x_0) \mid x\right)$. Note that $\frac{\partial \pi\left(\hat{b}(x';x_0) \mid x'\right)}{\partial \hat{b}} = 0$, and the following inequality holds.

$$\begin{aligned}
&\frac{\partial \pi\left(\hat{b}(x'; x_0) \mid x'\right)}{\partial \hat{b}} \\
&= \frac{\left[p(x',x',x';x_0) - \hat{b}(x)\right] \cdot f_{Y_1}(x' \mid x'; x_0) + \int_{\underline{x}}^{x'} (1-\alpha) p_1(x',x',y;x_0) \cdot f_{Y_1}(y \mid x'; x_0) dy}{\hat{b}'(x'; x_0)} \\
&- F_{Y_1}(x' \mid x'; x_0) \\
&= \left[\hat{b}'(x'; x_0)\right]^{-1} F_{Y_1}(x' \mid x'; x_0) \left\{ \begin{array}{c} \left(p(x',x',x';x_0) - \hat{b}(x';x_0)\right) \frac{f_{Y_1}(x' \mid x'; x_0)}{F_{Y_1}(x' \mid x'; x_0)} \\ + \int_{\underline{x}}^{x'} (1-\alpha) p_1(x',x',y;x_0) \cdot \frac{f_{Y_1}(y \mid x'; x_0)}{F_{Y_1}(x' \mid x'; x_0)} dy - \hat{b}'(x'; x_0) \end{array} \right\} \\
&\leq \left[\hat{b}'(x'; x_0)\right]^{-1} F_{Y_1}(x' \mid x'; x_0) \left\{ \begin{array}{c} \left(p(x',x,x';x_0) - \hat{b}(x';x_0)\right) \frac{f_{Y_1}(x' \mid x; x_0)}{F_{Y_1}(x' \mid x; x_0)} \\ + \int_{\underline{x}}^{x} (1-\alpha) p_1(x',x,y;x_0) \cdot \frac{f_{Y_1}(y \mid x; x_0)}{F_{Y_1}(x' \mid x; x_0)} dy - \hat{b}'(x'; x_0) \end{array} \right\} \\
&= \left[\frac{F_{Y_1}(x' \mid x'; x_0)}{F_{Y_1}(x' \mid x; x_0)}\right] \left[\frac{\partial \pi\left(\hat{b}(x'; x_0) \mid x\right)}{\partial \hat{b}}\right].
\end{aligned}$$

The above inequality holds because $F_{Y_1}(\cdot \mid x; x_0)$ dominates $F_{Y_1}(\cdot \mid x'; x_0)$ in terms of reverse hazard rate for all $x' < x$ (see p.287 in the Appendices of Krishna, 2009).

Moreover, $p$ and its derivative $p_1$ are nondecreasing in their arguments due to affiliation and Assumption 1.1, and $\frac{F_{Y_1}(\cdot|x';x_0)}{F_{Y_1}(x'|x';x_0)}$ first-order dominates $\frac{F_{Y_1}(\cdot|x;x_0)}{F_{Y_1}(x'|x;x_0)}$, also by affiliation. Note that $\frac{\partial\pi(\hat{b}(x'))}{\partial\hat{b}} \geq 0$ implies that when the true signal of a lead reinsurer is $x$ and he bids $\hat{b}(x';x_0)$ where $x' \leq x$, he would increase the bid to maximize his payoff. Symmetrically, it holds that $\frac{\partial\pi(\hat{b}(x';x_0))}{\partial\hat{b}} \leq 0$ if $x' \geq x$. The lead would lower his bid $\hat{b}(x')$ to increase his payoff if $x' \leq x$. We must also have that for all $x$, $\alpha p(x,x,x;x_0) + (1-\alpha)q(x,x;x_0) - \hat{b}(x;x_0) = p(x,x,x;x_0) - \hat{b}(x;x_0) > 0$ holds for all $x$ in $[\underline{x},\bar{x}]$, otherwise a bid of zero would be better.

$$
\begin{aligned}
p(x,x,x;x_0) - \hat{b}(x;x_0) &= \int_{\underline{x}}^{x} L(s \mid x;x_0)\, dp(s,s,s;x_0) - (1-\alpha)\int_{\underline{x}}^{x} \frac{J(s;x_0)}{f_{Y_1}(s \mid s;x_0)}\, dL(s,x;x_0) \\
&= \int_{\underline{x}}^{x} L(s \mid x;x_0)\,[p_1(s,s,s;x_0) + p_2(s,s,s;x_0) + p_3(s,s,s;x_0)]\, ds \\
&- (1-\alpha)\int_{\underline{x}}^{x} L(s,x;x_0)\frac{J(s;x_0)}{F_{Y_1}(s \mid s;x_0)}\, ds \\
&= \int_{\underline{x}}^{x} L(s \mid x;x_0)\left[p_1(s,s,s;x_0) + p_2(s,s,s;x_0) + p_3(s,s,s;x_0) - (1-\alpha)\frac{J(s;x_0)}{F_{Y_1}(s \mid s;x_0)}\right] ds \\
&\geq \int_{\underline{x}}^{x} L(s \mid x;x_0)\,[p_2(s,s,s;x_0) + p_3(s,s,s;x_0)]\, ds \geq 0.
\end{aligned}
$$

The first equality uses total differentiation of $dp$ and the fact that $\frac{dL(s,x;x_0)}{L(s,x;x_0)} = \frac{f_{Y_1}(s|s;x_0)}{F_{Y_1}(s|s;x_0)}ds$. The inequality uses $\frac{J(s;x_0)}{F_{Y_1}(s|s;x_0)} = \int_{\underline{x}}^{s} p_1(s,s,y;x_0)\cdot\frac{f_{Y_1}(y|s;x_0)}{F_{Y_1}(s|s;x_0)}dy \leq p_1(s,s,s;x_0)$ for $y \leq s$. The last inequality holds since $p_2 \geq 0$ and $p_3 \geq 0$ by affiliation. Hence, bidding $\hat{b}(x;x_0)$ is indeed an equilibrium when the lead's signal is $x$. This completes the proof. □

### 1.7.7 Proof of Corollary 1.2

*Proof.* The proof of corollary 1.2 is similar to that in the proof of corollary 1.1. In the proof of proposition 6, we establish that $p(x,x,x;x_0) - \hat{b}(x;x_0) > 0$. Substituting this result into the derivative of $\tilde{b}(x)$ and noting that $p_1(x,x,y;x_0) \geq 0$ by Assumption 1.1, we then conclude the derivative $\hat{b}'(x;x_0)$ is positive. This completes the proof. □

### 1.7.8 Proof of Proposition 1.7

*Proof.* When $X_0$ is revealed, the expected payment of a winning leader when he receives a signal x but bids as if his signal were z (i.e., for all $X_0 = x_0$, he bids $\hat{b}(z; x_0)$) is

$$W^A(z, x) = E\left[\hat{b}(z; x_0) \mid X_1 = x\right],$$

so $W_2^A(z, x) \geq 0$, because $X_0$ and $X_1$ are affiliated.

When $X_0$ is not revealed, similarly we have that

$$W^B(z, x) = \hat{b}(z),$$

so $W_2^B(z, x) = 0$. Hence, $W_2^A(z, x) \geq W_2^B(z, x)$.

Thus, publicly revealing $X_0$ raises $\hat{p}$ by lemma 1.4. □

### 1.7.9 Proof of Lemma 1.5

*Proof.*

$$\begin{aligned}
&E\left[V \mid Z_1, Z_2, \ldots, Z_n, Z_{n+k}\right] = \int_{\mathbb{R}} v \cdot f_{V|Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(v \mid z_1, z_2, \ldots, z_n, z_{n+k}\right) dv \\
&= \int_{\mathbb{R}} v \cdot \frac{f_{V,Z_1,z_2,\ldots,Z_n,Z_{n+k}}\left(v, z_1, z_2, \ldots, z_n, z_{n+k}\right)}{f_{Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(z_1, z_2, \ldots, z_n, z_{n+k}\right)} dv \\
&= \int_{\mathbb{R}} v \cdot \frac{f_{V,Z_1,Z_2,\ldots,Z_n}\left(v, z_1, z_2, \ldots, z_n\right) \cdot f_{Z_{n+k}|Z_1,Z_2\ldots,Z_n}\left(z_{n+k} \mid z_1, z_2, \ldots, z_n\right)}{f_{Z_1,Z_2\ldots,Z_n,Z_{n+k}}\left(z_1, z_2, \ldots, z_n, z_{n+k}\right)} dv \\
&= \int_{\mathbb{R}} v \cdot \frac{f_{V,Z_1,Z_2,\ldots,Z_n}\left(v, z_1, z_2, \ldots, z_n\right) \cdot \frac{f_{Z_1,Z_2,\ldots,Z_n,Z_{n+k}}(z_1,z_2,\ldots,z_n,z_{n+k})}{f_{Z_1,Z_2,\ldots,Z_n}(z_1,z_2,\ldots,z_n)}}{f_{Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(z_1, z_2, \ldots, z_n, z_{n+k}\right)} dv \\
&= \int_{\mathbb{R}} v \cdot \frac{f_{V,Z_1,Z_2,\ldots,Z_n}\left(v, z_1, z_2, \ldots, z_n\right)}{f_{Z_1,Z_2,\ldots,Z_n}\left(z_1, z_2, \ldots, z_n\right)} dv \\
&= E\left[V \mid Z_1, Z_2, \ldots, Z_n\right].
\end{aligned}$$

The first and last equalities correspond to the definitions of conditional expectation. The second equality uses

$$f_{V|Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(v \mid z_1, z_2, \ldots, z_n, z_{n+k}\right) = \frac{f_{V,Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(v, z_1, z_2, \ldots, z_n, z_{n+k}\right)}{f_{Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(z_1, z_2, \ldots, z_n, z_{n+k}\right)}.$$

The numerator in the third equality follows the definition of the garbling condition. The fourth equality uses

$$f_{Z_{n+k}|Z_1,Z_2,\ldots,Z_n}\left(z_{n+k} \mid z_1, z_2, \ldots, z_n\right) = \frac{f_{Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(z_1, z_2, \ldots, z_n, z_{n+k}\right)}{f_{Z_1,Z_2,\ldots,Z_n}\left(z_1, z_2, \ldots, z_n\right)}.$$

This completes the proof. □

### 1.7.10 Comparison of Syndicated Loan and Reinsurance Syndicate

This section contrasts syndicated loans with reinsurance syndicates, as these two are similar in organization, and the former is extensively studied in banking.

Below we briefly describe the basic background of syndicated loans and reinsurance syndicates. A syndicated loan is provided by a group of lenders to a single borrower, often for large-scale loans exceeding a single lender's capacity. It is formed by banks or financial institutions, with a lead bank arranging and managing the loan. The loan amount is divided among participants, limiting each lender's risk to their portion.

In a co-reinsurance syndicate, multiple reinsurers pool resources to underwrite large or complex risks. A lead reinsurer is responsible for determining the terms and conditions, which are binding for the follow market. Reinsurers each assume a portion of the risk according to their participation percentages. Large risks are spread to reduce the financial impact on any single insurer.

Both organizations share similarities, requiring multiple banks or reinsurers to jointly raise capital and share risks. They adopt a leader-follower structure to reduce transaction costs in business. Though the organizational structures have some commonalities between a syndicated loan and a co-insurance syndicate, there are crucial differences.

Firstly, the industry applications are different. Syndicated loans are used in banking and finance, while reinsurance syndicates serve the broader insurance and reinsurance industry.

Secondly, the underlying risk types involved in these two structures differ. A syndicated loan is subject to the credit risk of the borrower, while the default risk has been historically rare in insurance; one example we know is American International Group (AIG)'s near-failure in 2008 due to its exposure to credit default swaps (CDS) and the housing market collapse.[20] In contrast, the main risk related to pricing in

[20] AIG, a global company with about 1 trillion US Dollars in assets before the crisis, lost 99.2 billion in 2008. On September 16 of that year, the Federal Reserve Bank of New York stepped in with an 85 billion loan to keep the failing company from going under. See https://insight.kellogg.northwestern.edu/article/what-went-wrong-at-aig

reinsurance is the underwriting risk covered by the contracts, such as potential damages caused by a hurricane in Florida or an earthquake in Japan.

Thirdly, the pricing practices induced by covering different risks in these two industries differ. The pricing in syndicated loans is typically standardized, often based on benchmark interest rates such as London Inter-bank Offered Rate (LIBOR) or Secured Overnight Financing Rate (SOFR), plus an applicable margin.[21] Conversely, there is no such benchmark in reinsurance treaties. The price of a reinsurance policy varies case by case, depending on the specific business and the individual underwriter's assessment of the underwriting risk.

Lastly, the regulatory focuses differ between the insurance and banking sectors. In insurance, regulations aim to prevent unfair pricing and exclusionary practices, driven by the sector's reliance on competitive risk assessment and pricing. In contrast, banking regulations prioritize financial stability, risk management, and the prevention of systemic failures, viewing syndicated loans as a means to spread risk and increase lending capacity for large projects. While competition concerns are monitored, the overarching goal in banking is to ensure that collaborative lending practices do not compromise the resilience of the financial system.

[21] Source: https://www.srsacquiom.com/our-insights/syndicated-loan-market/

### 1.7.11 Reinsurance Business and Follow-the-lead Practice

This section offers background information on the reinsurance business and the follow-the-lead practice.

**Reinsurance Business and Syndicates.** Reinsurance can be classified into external reinsurance and internal reinsurance. The follow-the-lead practice discussed in this paper is used in external reinsurance, wherein cedants rely on professional reinsurers for risk cession. Internal reinsurance, or captive reinsurance, is when a parent company forms its own reinsurance entity to manage risks from its insurance operations internally, acting as a form of self-insurance instead of outsourcing to external reinsurance firms. External reinsurance typically constitutes the primary business line. As Hsiao and Shiu (2019) shows, in the UK life insurance industry, 80.24% of the insurers used at least one type of reinsurance. The participation rate for external reinsurance usage is 76.33%. This means non-affiliated professional external reinsurers play an important role in diversifying the risks.

In addition to reinsurance, there are various Alternative Risk Transfer (ART) mechanisms like Insurance-Linked Securities (ILS), which offer additional options for managing risk. These financial instruments allow insurers to transfer risk to investors in the financial markets, similar to how banks distribute loan risks through securitization. For example, by issuing $1 million in catastrophe bonds, an insurer can raise the same amount from investors. If a catastrophe occurs, the principal is used to cover losses, and investors receive only coupon payments. Otherwise, the insurer repays the principal plus coupons. Despite their presence in diversifying risks, these instruments fall outside the scope of this paper.

Client insurers depend on professional reinsurers for both risk management and pricing, as reinsurers typically have a deeper understanding of underlying risks than their clients, drawing on historical data, experience, and new technology. For instance, newly emerging risks such as cybercrime require specialized knowledge and tools. Lloyd's syndicates use the Axio360 platform to develop solutions for cyber-physical damage coverage, leveraging it as a decision-making engine for comprehensive cyber risk management. This includes cybersecurity assessments, cyber risk quantification (CRQ), risk transfer, and cyber insurance analysis.[22] Individual clients may lack access to such advanced technology when evaluating the potential risks.

In external reinsurance, reinsurers often collaborate to spread risk more effectively. Notable examples include insurance and reinsurance syndicates like Hiscox ESG 3033, which, brokered by Aon, provided coverage for a new wind farm in Spain and a solar farm risk in the USA in 2023.[23] Furthermore, Beazley launched Syndicate 4321 in 2022, offering exclusive capacity for clients with high ESG ratings.[24]

**Follow-the-lead Practice.** Follow-the-lead practice locks the aftermarket on a single set of terms and conditions determined in the tender phase. Regarding the details, the Commission outlines some market elements contained in the subscription procedure:

(a) Alignment on the contractual terms offered by the lead (re)insurer.

(b) Revealing the price offered by the lead (re)insurer to the follow market.

(c) Potentially, guaranteeing to the lead that the price and conditions, and the share of the risk, that were agreed with it at the end of the first round, will not be changed to its detriment if participants in the follow market were to offer a lower price;

(d) Alignment on the premium.

Up to today, follow-the-lead remains a prevalent market practice, as evidenced by recent articles. One notes, "Follower reinsurers accept to participate in a reinsurance treaty in which the final terms and conditions have already been agreed, but they don't necessarily influence the terms and conditions involved."[25] Another states, "Despite not being in the lead, they (followers) enjoy the same level of compensation as the lead reinsurer."[26]

[22] See https://www.reinsurancene.ws/lloyds-of-london-investment-in-axio-to-support-company-growth-and-benefit-
[23] See https://www.hiscoxgroup.com/news/press-releases/2023/02-08-23
[24] See https://www.beazley.com/en-us/news-and-events/esg-syndicate-4321/
[25] See an article written by CCR Re in 2021 https://blog.ccr-re.com/en/what-is-a-follower
[26] See an article by insurance professionals in 2022: https://www.investopedia.com/terms/l/lead-reinsurer.asp

### 1.7.12 Optimal Selling Mechanism

Below, we discuss the implementation of the optimal selling mechanism, sufficient conditions for full surplus extraction, and its distinction from a second-price auction, based on McAfee et al. (1989).

**Implementation**

1. The insurer (seller) *randomly* selects two syndicates labeled $i$ and $j$ among $n$ competing syndicates.

2. The insurer then asks leader $j$ to report his signal, the realization is denoted as $x_j$, but offers the business to syndicate $i$ at a price $z(x_j)$, where $z(\cdot)$ is a price function that depends only on $j$'s report.

**Incentive Compatibility and Participation Constraint**

The first observation is that the mechanism above is weakly incentive-compatible (IC). This is because a reinsurer's payoff does not depend on its own actions: syndicate $i$'s payoff depends on syndicate $j$'s report, while the payoffs for all other participants are zero.

Second, we need to show the existence of a price function $z(\cdot)$ such that the participation constraints are satisfied, i.e., the buyer $i$'s expected payoff is nonnegative. Specifically, if his payoff is zero, the mechanism is optimal from the seller's perspective.

Denote the conditional distribution of each lead reinsurer $i$'s signal $x_i$ as $F(x_i|v)$, with density $f(x_i|v)$ continuous and strictly positive on $[0,1] \times [0,1]$.[27] Let the distribution of $v$ be $G(v)$. The payoff of leader $i$ is given by:

$$\pi_i = \int_0^1 \left[ v - \int_0^1 z(x) f(x \mid v) dx \right] \frac{f(x_i \mid v)}{\int_0^1 f(x_i \mid u)\, dG(u)} dG(v).$$

Thus, characterizing the optimal mechanism reduces to finding $z(\cdot)$ such that $\pi_i = 0$.

**Surplus Extraction as a Minimum Norm Problem** The full surplus extraction problem can be transformed into a minimum norm problem. Consider a Hilbert

[27] We consider $[0,1]$ w.l.o.g. since it is isomorphic to interval $[\underline{v}, \overline{v}]$.

space of square-integrable functions $L^2([0,1],G)$ and define the norm as $\|x\| = (x,x)^{\frac{1}{2}}$, where $(x,y) = \int_0^1 x(v)y(v)dG(v)$. Define a set $Y$ as:

$$Y = \left\{ y \in L^2([0,1],G) \mid \exists z \in L^2([0,1],G), y(v) = \int_0^1 z(x)f(x \mid v)dx \right\}.$$

Then we have the following results.

**Lemma 1.6** (McAfee et al., 1989, Lemma)**.** *$\exists z(\cdot) \in L^2([0,1],G)$ satisfying $\pi_i = 0$ if and only if $\min_{y\in Y} \|y - v\|$ has a solution.*

**Theorem 1.1** (McAfee et al., 1989, Main Theorem)**.** *$\forall \varepsilon > 0, \exists z \in L^2([0,1],G)$ s. t. $\pi_i \in [0,\varepsilon]$.*

The intuition behind leaving small rents $\varepsilon$ is that the mechanism is only weakly IC, meaning the buyer is not worse off if they lie. To ensure strict incentives, a cost shall be imposed on the seller to leave strict rents for the buyer to report truthfully.

**Sufficient Conditions for Full Surplus Extraction**

Balakrishnan (2012) states that every closed convex set in a Hilbert space has a unique element of minimum norm. Two examples provided by McAfee et al. (1989) satisfy the condition that the set $Y$ defined above is closed.

1. The conditional density satisfies a separability condition, i.e., $f(s \mid v) = \sum_{i=1}^{n} a_i(s)b_i(v)$.

2. The value $v$ has a finite support, i.e., $G = \{v_1, v_2, \dots, v_m\}$.

**Distinction from Second-Price Auction**

The mechanism discussed here resembles but is essentially different from a second-price auction (SPA) in key aspects:

1. A SPA is not optimal as it is dominated by a English auction in a common value setting.

2. In this mechanism, the two buyers are selected arbitrarily. In contrast, in a SPA, the highest and second-highest bidders are chosen by the buyers themselves based on their signals or bids. The buyers' knowledge of this prevents full surplus extraction.

# Chapter 2

# Revisiting Equilibrium in Quote-driven Markets

Joint with Elu von Thadden

## 2.1 Introduction

The quote-driven system is a basic trading system in securities and other financial markets.[1] Examples include the Stock Exchange Automated Quotation (SEAQ) system on the London Stock Exchange, the eSpeed government bond trading system, and the foreign exchange market. In these markets, market makers (dealers) post prices before order submission. The seminal modeling approach for quote-driven markets includes Glosten (1989), who studies traders with both informational and liquidity motives. Madhavan (1992) extends this to a multi-period setting and compares the quote-driven mechanism with the order-driven model in Kyle (1989), focusing on price efficiency and the cost of information.

Our paper challenges the conventional wisdom on equilibrium existence in quote-driven markets. We find that even when the sufficient conditions for a separating equilibrium—such as those in Glosten (1989) and Madhavan (1992)—are satisfied, a separating equilibrium still fail to exist in a two-dimensional competitive screening

[1]Another basic trading system is the order-driven market, where investors submit buy and sell orders before observing the price. Orders are either executed immediately (continuous auction) or accumulated for a period and executed simultaneously (periodic or call auction). An example of an order-driven system is the Stock Exchange Electronic Trading Service (SETS), which operates via a limit order book on the London Stock Exchange. The New York Stock Exchange and NASDAQ are considered hybrids of the two systems.

framework. This failure arises from a profitable partial-pooling cross-subsidization deviation, whereby a deviating market maker offers a price-quantity schedule that attracts a subset of investors. He cream-skims a continuum of low-cost investor types to subsidize a continuum of high-cost types, resulting in strictly positive expected profits.

In the model, a risk-averse informed trader has two motives for trading: an informational motive to exploit private information, and a liquidity motive to rebalance her position. The trader's type is modeled as a convex combination of these two motives. Competitive market makers move first by quoting a price schedule and are assumed to break even in expectation. After observing the quoted prices, the investor announces the quantity she wishes to buy or sell. The classic result states that the market shuts down when information asymmetry is too severe—specifically, when the informational motive dominates the liquidity motive. Conversely, when liquidity motives dominate, the market opens and a separating equilibrium exists.

Our argument draws a parallel with the equilibrium existence problem in Rothschild and Stiglitz (1978), where insurers offer contracts to high- and low-risk individuals. In a separating equilibrium, high-risk agents receive full insurance while low-risk agents receive partial insurance. Supporting this equilibrium requires a sufficiently large share of high-risk individuals to deter pooling and prevent cross-subsidization deviations.

The difficulty in establishing our equilibrium nonexistence result lies in constructing a partial-pooling, cross-subsidization deviation in a setting with continuously distributed and unbounded types. Unlike the binary types in the insurance literature, the investor's type is defined as a convex combination of trading motives, leading to an unbounded type space. We overcome this challenge by leveraging statistical properties of truncated Gaussian distributions, as most models in market microstructure—including the market-making model considered here—are based on a linear-Gaussian framework. Based on this, we derive sufficient conditions for the existence of a cross-subsidization deviation and provide a numerical example confirming that these conditions are nonempty. Furthermore, we show that such a deviation exists even when the informational motive is arbitrarily small compared to the liquidity motive.

Moreover, we discuss a signaling variant of the market-making model, in which the informed investor submits an order before observing the price quotation. In that setting, we prove the existence and uniqueness of a separating equilibrium under full information transmission.

**Related Literature** This paper relates to the market-making models in the market microstructure literature. Glosten (1989) is among the first to model convoluted trading motives. He contrasts a competitive screening setting with a monopolistic market maker framework and shows that monopoly power can mitigate the trading costs caused by adverse selection, thereby rationalizing the role of the specialist system on the NYSE. Madhavan (1992) extends Glosten's model to a multi-period setting and compares the quote-driven market with the order-driven market in Kyle (1989). He finds that the quote-driven system offers greater price efficiency than the continuous auction. Moreover, with free entry into market making, equilibria in the two mechanisms tend to converge.

The insight that markets may shut down when the informational motive dominates the liquidity motive also appears in other studies. Keim and Madhavan (1996) model block trading in the upstairs market and show that equilibrium exists only if the information motive is not too large. Naik et al. (1999) examine whether full disclosure in dealership markets improves the welfare of risk-averse investors, and find that greater transparency may actually worsen risk-sharing outcomes. Their equilibrium existence condition similarly requires that information asymmetry not be too severe.

More recent studies have focused on automated market-making in the context of digital platforms and decentralized finance. Lehar and Parlour (2025) compare AMMs with traditional market-making and show that while AMMs offer continuous liquidity without inventory risk and operate without requiring trust, they may lead to less efficient price discovery. Aoyagi and Ito (2024) analyze the coexistence of limit order books and automated market makers, showing how traders self-select across platforms and how this affects market outcomes.

Another strand of related literature is the equilibrium problem in insurance markets with adverse selection. Riley (2001) reviews the evolution of screening models beyond the RS framework, noting persistent challenges in equilibrium existence when multiple types or contract externalities are involved. Fixing the Nash equilibrium concept, Farinha Luz (2017) characterizes a unique mixed-strategy equilibrium in the Rothschild-Stiglitz setting with two types of insurees. He notes that for more than two types, the existence and characterization of equilibrium remains open.

Besides searching for mixed-strategy equilibrium, early contributions attempt to address nonexistence by modifying the equilibrium concept. Wilson (1977) proposes the anticipatory equilibrium (E2 equilibrium) and shows that when the Rothschild-Stiglitz (RS) equilibrium fails to exist, the E2 equilibrium yields a pooling outcome. This framework is extended by Miyazaki (1977) and Spence (1978), who introduce menus of contracts. The resulting Miyazaki-Wilson-Spence (MWS) equilibrium predicts a separating allocation with cross-subsidization and contracts that are jointly zero-profit and second-best efficient. Riley (1979) introduces the reactive equilibrium, which takes into account that each firm anticipates competitors' responses before offering a new contract.

A more recent study by Azevedo and Gottlieb (2019) shows that Riley equilibrium may fail to exist when insurees possess multi-dimensional private information. Azevedo and Gottlieb (2017) define their equilibrium concept as a refinement of the weak equilibrium and show that, in insurance markets with adverse selection, such an equilibrium always exists. Attar et al. (2014) study nonexclusive competition, in contrast to the exclusive contracting assumption in Rothschild and Stiglitz (1978), and demonstrate that the ability of agents to contract with multiple firms undermines standard screening mechanisms, potentially leading to equilibrium nonexistence or inefficiency. Attar et al. (2022) analyze a setting in which privately informed consumers may hold multiple contracts across insurers. They find that effective public intervention should target firms' pricing behavior rather than restricting consumer choice. The proposed regulatory framework achieves efficiency by penalizing cross-subsidization between contracts.

Mimra and Wambach (2014) reviews recent developments in addressing equilibrium issues in RS, including those that reconsider the microstructure of the insurance market, and notes that most contributions still focus on the case with only two types. While conceptually similar to the competitive screening model in insurance markets, we argue that the technical challenges in the quote-driven markets in this paper are more severe. First, the investor's action space—allowing both long and short positions—is unbounded in $\mathbb{R}$, whereas the insuree's choice of coverage typically lies in a bounded interval, such as $[0, 1]$. Second, types are modeled as convex combinations of interacting trading motives, yielding an unbounded type space with infinitely many distinct types. It remains to be explored whether the remedies developed in the insurance literature can be extended to address the existence problem identified in the market-making model.

**Plan for the Paper.** Section 2.2 presents the model and derives a separating equilibrium if it exists. Section 2.3 analyses how cross-subsidization deviations destroy equilibrium existence. Section 2.4 discusses equilibrium existence in the signaling version of market making. Section 2.5 concludes.

## 2.2 Model and Equilibrium Analysis

### 2.2.1 Players and Information

Consider a single asset with value $S$. Each risk-averse investor privately observes a noisy signal $s = S + \epsilon$ before trading, where $\epsilon$ is the noise term. An investor holds a risk-free asset, cash, denoted as $C$, and a position $W$ in a risky asset. The investor has two trading motives: to exploit her private signal $s$ and to adjust her position $W$. Risk-neutral market makers quote price schedules to investors. The unit trading price of the asset quoted by market makers is denoted as $p$. The state of nature, represented by $\omega = (W, s)$, is known only to the investor. Both the position $W$ and the asset value $S$ may take positive or negative values, allowing for short-selling. The quantity of the asset that the investor wishes to trade is represented by $x$, where $x > 0$ indicates that the investor purchases the asset from market makers, and $x < 0$ indicates that selling the asset to market makers. After a trade, the transfer $t$ from the investor to the market maker is $t = -px$.

The investor's final wealth is $C + (W + x)S + t$ after a trade takes place, so her expected utility is given by

$$u_A(x, t, \omega) = E_S[U(C + (W + x)S + t)|\omega]. \tag{2.1}$$

Additionally, the investor is risk-averse and her utility function is specified as the Constant Absolute Risk Aversion (CARA) form, i.e.,

$$U(c) = -\exp(-ac), \quad a > 0.$$

Market makers are assumed to be risk-neutral, and aim to maximize their expected profits. The payoff for a market maker, conditioned on the signal $s$, is described by the function:

$$u_P(x, t, \omega) = E_S[-t - xS|\omega]. \tag{2.2}$$

For tractability, it is assumed that $\varepsilon$, $W$, and $S$ are normally and independently distributed, i.e.,

$$\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2), \quad W \sim \mathcal{N}(0, \sigma_W^2), \quad S \sim \mathcal{N}(m_S, \sigma_S^2).$$

### 2.2.2 Structure of the Game and Solution Concept

The strategic interaction between market makers and investors is modeled as a competitive screening game. The timing of the game is as follows:

0. Nature draws the investor's type $\omega = (s, W)$.
1. Market makers offer price schedules $p(x)$ in a competitive market.
2. Each investor submits a quantity $x$ that she wishes to trade.
3. All values are realized and a transfer $t$ is made from the investor to market makers.

The natural solution concept in this setting is perfect Bayesian equilibrium, where players' beliefs are updated according to Bayes' rule wherever possible, and strategies are optimal given those beliefs. We focus on the full information transmission case, characterized by a separating equilibrium. We first analyze the investor's decision problem, then that of the market makers, and finally determine the equilibrium price–quantity schedule.

### 2.2.3 Equilibrium Analysis

Since the investor observes a noisy signal $s$ about the true asset value $S$, she needs to evaluate $S$ before making a trading decision. The following lemma describes the updating rule for Gaussian variables. It states that the conditional expectation is a convex combination of the unconditional expectation and the observation. The observation only affects the expectation, not the variance.

**Lemma 2.1.** *If $X \sim \mathcal{N}(\mu_X, \sigma_X^2), Y \sim \mathcal{N}(0, \sigma_Y^2)$ are jointly normal and independent, $Z = X + Y$, and $z$ is an observation of $Z$, then*

$$X|z \sim \mathcal{N}\left(\frac{\sigma_Y^2 \mu_X + \sigma_X^2 \mu_z}{\sigma_Y^2 + \sigma_X^2}, \frac{\sigma_Y^2 \sigma_X^2}{\sigma_X^2 + \sigma_Y^2}\right).$$

Using the above updating rule, the investor's estimate of asset value is normally distributed as $S|s \sim N(m, \sigma^2)$ with

$$m = \frac{\sigma_\epsilon^2 m_S + \sigma_S^2 s}{\sigma_\epsilon^2 + \sigma_S^2}, \sigma^2 = \frac{\sigma_\epsilon^2 \sigma_S^2}{\sigma_\epsilon^2 + \sigma_S^2}.$$

Therefore, the investor's expected utility can be rewritten as $u_A(x, t, \omega) =$

$$
\begin{aligned}
&= -\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} exp\left(-a\left[C - px + (W + x)\xi\right]\right) exp\left(-\frac{(\xi - m)^2}{2\sigma^2}\right) d\xi \\
&= -\frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-a[C - px]\right) \int_{-\infty}^{+\infty} exp\left(-a(W + x)\xi + \frac{(\xi - m)^2}{2\sigma^2}\right) d\xi \\
&= -exp\left(-a\left(C - px + m(W + x) - \frac{1}{2}a(W + x)^2\sigma^2\right)\right) \\
&= -exp\left(-a\left(C - \frac{1}{2}a\sigma^2 W^2 + Wm\right)\right) exp\left(-a(m - p - a\sigma^2 W)x - \frac{1}{2}\sigma^2 x^2\right).
\end{aligned}
$$

The first term is a known constant to the investor. Hence, her utility is ordinally equivalent to

$$v_A(x, p, \theta) = (\theta - p)x - \frac{1}{2}a\sigma^2 x^2. \tag{2.3}$$

where $\theta$ is given by

$$\begin{aligned} \theta &= m - a\sigma^2 W \\ &= \frac{1}{\sigma_\epsilon^2 + \sigma_S^2}\left(\sigma_\epsilon^2 m_S + \sigma_S^2 s - a\sigma^2 x^2\right). \end{aligned} \tag{2.4}$$

The two-dimensional uncertainty of the position $W$ and signal $s$ can be condensed into a single variable denoted as $\theta$. This implies that the two trading motives, the informational motive to take advantage of signal $s$ and the liquidity motive to adjust her position $W$, are interrelated. The more exposure $W$ the agent already has to the asset, the less inclined she is to employ her private information in trading. The investor considers both trading motives not independently, but rather their projection onto one space, $\theta$. Hence, if information is transmitted in trading, it will be a statistic of $\theta$.

The market maker maximizes his expected profits $v_P(x, p, \theta) = E[(p - S)x|\theta]$ and needs to evaluate $E[S|\theta]$. By the law of iterated expectations,

$$\begin{aligned} E[S|\theta] &= E[E[S|s]|\theta] \\ &= E[m|\theta]. \end{aligned}$$

By lemma 2.1, $m|\theta$ is normally distributed with mean

$$\begin{aligned} E[m\,|\theta] &= (1-\beta)E[m] + \beta\theta \\ &= (1-\beta)m_S + \beta\theta, \end{aligned}$$

where

$$\beta = \frac{var(m)}{var(m) + a^2\sigma^4\sigma_W^2}$$
$$= \frac{\sigma_S^2 + \sigma_\varepsilon^2}{\sigma_S^2 + \sigma_\varepsilon^2 + a^2\sigma_W^2\sigma_\varepsilon^4}.$$

Hence, the expected payoff for the market maker is

$$v_P(x, p, \theta) = E\left[(p - S)x|\theta\right]$$
$$= E\left[(p - m)x|\theta\right]$$
$$= \left(p - \beta\theta - (1 - \beta)m_S\right)x.$$

Note that $\beta\theta + (1 - \beta)m_S$ represents the expected cost passed from the investor to the market maker, where $\beta$ is the weight placed on private information and $(1 - \beta)$ is the weight on the prior mean of the asset value.

### 2.2.4 Separating Equilibrium Candidate

We focus on full information transmission, where the investor's action is uniquely determined by her type in a one-to-one manner. In a separating equilibrium, the investor's decision perfectly reveals her type. Competition among market makers implies zero expected profits, resulting in $p(\theta) = \beta\theta + (1 - \beta)m_S$ for all $\theta \in \mathbb{R}$.

An investor of type $\theta$ chooses $\hat{\theta}$ to maximize her payoff given by:

$$V(\theta, \hat{\theta}, x) = \left(\theta - \beta\hat{\theta} - (1 - \beta)m_S\right) X(\hat{\theta}) - \frac{1}{2}dX(\hat{\theta})^2, \tag{2.5}$$

where $d = a\sigma^2$. In the absence of asymmetric information, the first-best allocation serves as a benchmark and is given by

$$X^{FB}(\theta) = \frac{1 - \beta}{d}(\theta - m_S). \tag{2.6}$$

When the agent's private information can be represented by a continuous real-valued random variable, the incentive-compatible separating strategy is typically characterized by some differential equation, if the strategy is known to be differentiable. However, since the equilibrium is unknown, differentiability can not be taken for granted. In this regard, the price schedule derived in Glosten (1989) necessitates that each trading schedule be differentiable, which was missing in the original paper, but can be resolved by using the theorems presented in Mailath and Von Thadden (2013).

Note that type $\theta = m_S$ must get her first-best quantity $x = 0$ in any incentive-compatibility separating price schedule.[2] If an investor's convoluted trading motive perfectly matches the prior mean asset price, she will lack any incentive to trade. Separation implies no other type gets $x = 0$. Hence, $\partial_{\widehat{\theta}} V(\theta, \theta, X(\theta)) = -\beta X(\theta) \neq 0$ for all $x \neq m_s$. Since $\partial^2_{\theta\widehat{\theta}} V \equiv 0$, Theorem 4.3 in Mailath and Von Thadden (2013) therefore implies that any incentive-compatible schedule $X$ must be differentiable on the open sets $(m_S, \infty)$ and $(-\infty, m_S)$. By Theorem 5 in Mailath and Von Thadden (2013), the schedule $X$ is continuous at $\theta = m_S$. Calculating the derivatives of $X$ on $(m_S, \infty)$ and $\theta \neq m_S$ from (DE) in Mailath and Von Thadden (2013) showing that the left-hand and right-hand derivatives exist and identical. Hence, $X$ is differentiable everywhere in $\mathcal{R}$. One therefore can take the first-order condition of Equation (2.5):

$$\left(\theta - \beta\widehat{\theta} - (1-\beta)m_S\right) X'(\widehat{\theta}) - \beta X(\widehat{\theta}) - dX'(\widehat{\theta})X(\widehat{\theta}) = 0.$$

Rearranging terms gives

$$(1-\beta)(\theta - m_S)X'(\theta) - \beta X(\theta) - dX'(\theta)X(\theta) = 0.$$

To get an explicit solution, multiplying with "integrating factor" $-|X(\theta)|^{-1/\beta}$:

$$\left[d\,|X|^{(\beta-1)/\beta} - sign(X)(1-\beta)(\theta - m_S)\,|X|^{-1/\beta}\right] X' + \beta\,|X|^{(\beta-1)/\beta} = 0.$$

Note that left-hand side has the form

$$\frac{d}{d\theta}\Phi(\theta, X(\theta)) = \Phi_\theta + \Phi_x X',$$

[2]Since $V(m_s, m_s, X(m_s)) \geq 0$ (type $\theta = m_s$ has the option of choosing $x = 0$), and $V(m_s, m_s, x) = -\frac{dx^2}{2}$, which is strictly negative if $x \neq m_s$, we have $X(m_s) = 0$.

for some $\Phi : \mathbb{R}^2 \to \mathbb{R}$. Integrating $\Phi_\theta$ and $\Phi_x$ (for $\beta \neq 1/2$):

$$\begin{aligned}
\Phi(\theta, x) &= \beta \left|x\right|^{(\beta-1)/\beta} \theta + h(x), \\
\Phi(\theta, x) &= \beta \left|x\right|^{(\beta-1)/\beta} \theta + dsign(x) \frac{\beta}{2\beta - 1} \left|x\right|^{(2\beta-1)/\beta} \\
&\quad -\beta m_S \left|x\right|^{(\beta-1)/\beta} + f(\theta)
\end{aligned}$$

with $h$ and $f$ arbitrary. Comparing the above two equations yields

$$\begin{aligned}
&f(\theta) = 0, \\
&h(x) = dsign(x) \frac{\beta}{2\beta - 1} \left|x\right|^{(2\beta-1)/\beta} - \beta m_S \left|x\right|^{(\beta-1)/\beta}.
\end{aligned}$$

Hence,

$$\Phi(\theta, x) = \beta \left|x\right|^{(\beta-1)/\beta} (\theta - m_S) + dsign(x) \frac{\beta}{2\beta - 1} \left|x\right|^{(2\beta-1)/\beta}.$$

The general solution of ODE (for $\beta \neq 1/2$) is given by

$$dX(\theta) - (1 - 2\beta)(\theta - m_S) = c \left|X(\theta)\right|^{(1-\beta)/\beta} \tag{2.7}$$

with $c \in \mathbb{R}$ constant. Since $X(\cdot)$ satisfying Equation (2.6) is strictly monotone on all of $\mathcal{R}$ only if $c = 0$, it holds that:

$$X(\theta) = \frac{1 - 2\beta}{d} (\theta - m_S). \tag{2.8}$$

If $\beta \geq \frac{1}{2}$, Equation (2.8) is not a solution, because the investor's problem has a minimum at $\hat{\theta} = \theta$. If $\beta < \frac{1}{2}$, the investor problem has a maximum at $\hat{\theta} = \theta$. The above results are summarized in the following proposition.

**Proposition 2.1.** *The competitive screening game has no equilibrium with maximum information transmission (fully separating with respect to $\theta$) if $\beta \geq \frac{1}{2}$, i.e. if*

$$\sigma_S^2 + \sigma_\varepsilon^2 \geq a^2 \sigma_W^2 \sigma_\varepsilon^4. \tag{2.9}$$

*If an equilibrium with maximum information transmission exists, it is given by*

$$X(\theta) = \frac{1-2\beta}{d}(\theta - m_S), \tag{2.10}$$
$$P(\theta) = \beta\theta + (1-\beta)m_S. \tag{2.11}$$

Compared with the first-best $X^{FB}$ in Equation (2.6), the equilibrium in Equation (2.11) involves too little trade due to private information. The bid-ask spread is also micro-founded in this model. To see this, combining Equations (2.11) and (2.10) and eliminating $\theta$ yields the following price-quantity schedule

$$P(x) = m_S + d\frac{\beta}{1-2\beta}x.$$

The bid price is $B(x) = P(x) - m_S = d\frac{\beta}{1-2\beta}x$ for $x > 0$, and the ask price is $A(|x|) = m_S - P(x) = d\frac{\beta}{1-2\beta}|x|$ for $x < 0$. Hence, the bid-ask spread is given by $D(x) = \frac{1}{2}(B(x) - A(x)) = d\frac{\beta}{1-2\beta}x$, which centers around mid-price $m_S$.

This model provides an information-based explanation for trading distortion, where unfavorable information structures can prevent trade. Specifically, when the noise about asset value is too large (large $\sigma_S^2$), or the private information about the position is too precise (small $\sigma_W^2$), no trade occurs. In the former case, excessive uncertainty about the asset value deters the trader from engaging in trading. In the latter case, if the trader knows her position too precisely and is still willing to trade, it is more likely that she possesses favorable information rather than merely seeking to adjust her position. Anticipating this, market makers become less willing to trade.

Note that Glosten formulates the results in a different way. In Proposition 1 of Glosten (1989), it is stated that "if $\alpha > 0.5$ (which is equivalent to $\beta < 0.5$), then there is a unique equilibrium pricing schedule." In the next section, we show that even if the claimed sufficient condition for equilibrium existence in Glosten (1989) holds, a separating equilibrium may still fail to exist due to cross-subsidization between contracts offered by a deviating market maker.

## 2.3 Non-Existence of Equilibrium

In this section, we first show that the claimed sufficient condition for equilibrium existence is necessary to rule out pooling deviation. We then demonstrate that even if this condition holds, a deviating market maker can offer a price schedule that involves cross-subsidization across different types of investors. Moreover, we show that such a deviation exists regardless of how small the information weight is. This profitable cross-subsidization deviation causes the separating market outcome derived in the last section to unravel. Numerical examples are provided to illustrate our findings.

### 2.3.1 Necessary of $\beta < \frac{1}{2}$ to Rule Out Pooling Deviation

The condition $\beta < \frac{1}{2}$ is crucial to exclude fully pooling deviation. The quadratic payoff for agents in the separating outcome is given by $V_A = \frac{1-2\beta}{d}(\theta - m_s)^2$, which results from substituting the equilibrium results (2.10) and (2.11) into the investor's payoff function (2.3). The quadratic payoff is depicted in the following Figure 2.1 for $\beta < \frac{1}{2}$.

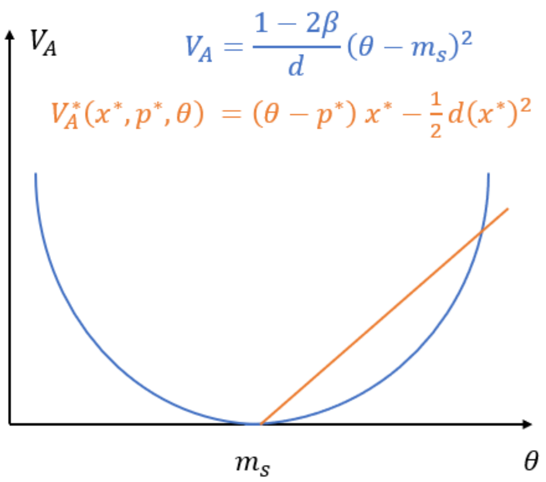


Figure 2.1: Equilibrium and Deviating Payoffs of Investors When $\beta < \frac{1}{2}$

Pooling deviation means that investors who wish to purchase the assets ($\theta > m_s$) or those who wish to sell the assets ($\theta < m_s$) pooling at the same contract $(p^*, x^*)$.

The payoff for investors opting for contract $(p^*, x^*)$ is expressed as

$$V_A^*(x^*, p^*, \theta) = (\theta - p^*)\, x^* - \frac{1}{2}d(x^*)^2 \tag{2.12}$$

The payoff function is linear in $\theta$. However, the linear deviating payoff line cannot consistently remain above the parabola for all $\theta$, as depicted in Figure 2.1. An exception arises when $\beta > \frac{1}{2}$, resulting in negative payoffs for agents (See Figure 2.2). In this case, the separating outcome is dominated by a simple pooling deviation, where agents choose not to trade, guaranteeing a payoff of zero. Although the claimed sufficient condition $\beta < \frac{1}{2}$ effectively rules out pooling, it does not exclude partial-pooling deviations involving cross-subsidies between contracts, as will be further illustrated in the subsequent section.

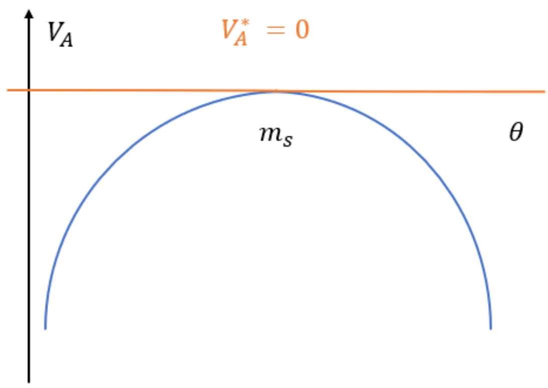


Figure 2.2: Equilibrium and Deviating Payoff of Investors when $\beta > \frac{1}{2}$

### 2.3.2 Construction of Partial-pooling Cross-subsidization Deviation

The idea of constructing a partial-pooling deviation is depicted in Figure 2.3. Suppose a type $\tilde{\theta}$ is attracted by some contract $(p^*, x^*)$, her payoff must be strictly higher than the separating outcome. The payoff of investors accepting the deviating contract is linear in type $\theta$ according to Equation (2.12). Given the convex nature of the original equilibrium payoff curve (represented by the blue hyperbola) and the linearity of the deviating payoff line (illustrated by the orange line), it is evident that types falling within the interval $(\underline{\theta}, \overline{\theta})$ achieve a higher payoff in the new contract compared to the original separating outcome. The slope of the straight deviating payoff line corresponds to the quantity $x^*$, while the price $p^*$ is associated with the intercept of the line. Given the payoff function, one can express the deviating contract $(p^*, x^*)$ in terms of $\underline{\theta}$ and $\overline{\theta}$.

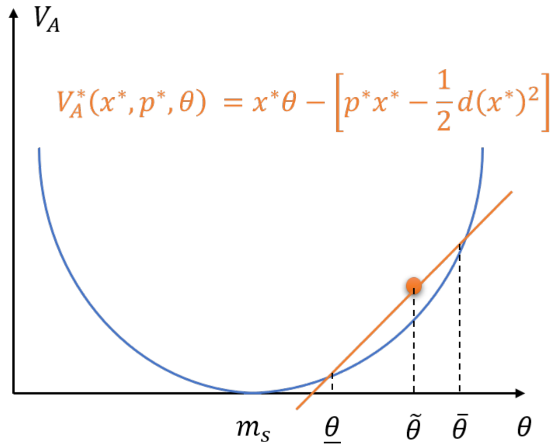


Figure 2.3: Illustration of partial-pooling deviating price schedule

**Lemma 2.2.** *Suppose a deviating market maker offers the following partial pooling price quantity schedule* $(p^*, x^*)$ :

$$
\begin{aligned}
p^* &= \frac{\underline{\theta}\overline{\theta} - m_s^2}{(\overline{\theta} - m_s) + (\underline{\theta} - m_s)} - \frac{1 - 2\beta}{4}[(\overline{\theta} - m_s) + (\underline{\theta} - m_s)] \\
x^* &= \frac{1 - 2\beta}{2d}(\overline{\theta} + \underline{\theta} - 2m_s),
\end{aligned}
$$

*then investors within* $(\underline{\theta}, \overline{\theta})$ *pool at the above contract, and those outside the interval stay at the original outcome.*

*Proof.* See the Appendices. □

The price schedule in Lemma 2.2 exhibits partial-pooling of investors within the interval $(\underline{\theta}, \overline{\theta})$ (which is a strict subset of $\mathcal{R}$), it remains to show that such type of deviation can generate strictly positive profits for the deviating market maker, i.e.,

$$
\int_{\underline{\theta}}^{\overline{\theta}} (p^* - (\beta\theta + (1 - \beta)m_s))x^* f(\theta)d\theta > 0. \tag{2.13}
$$

This profitability condition is equivalent to

$$\int_{\underline{\theta}}^{\overline{\theta}} (p^* - (1-\beta)m_s)x^* f(\theta)d\theta > \beta x^* \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta.$$

Since investors within $(\underline{\theta}, \overline{\theta})$ purchase $x^* > 0$ units of assets at $p^*$, the above condition reads

$$(p^* - (1-\beta)m_s) \int_{\underline{\theta}}^{\overline{\theta}} f(\theta)d\theta > \beta \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta.$$

Summarizing the terms involving $\theta$ on the right-hand side gives

$$p^* - (1-\beta)m_s > \frac{\beta \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} f(\theta)d\theta}.$$

By the definition of the conditional expectation, we have that

$$E[\theta|\underline{\theta} \leq \theta \leq \overline{\theta}] = \frac{\int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} f(\theta)d\theta}.$$

Therefore, the profitability condition (2.13) can be transformed to

$$p^* > \underbrace{(1-\beta)m_s + \beta E[\theta|\underline{\theta} \leq \theta \leq \overline{\theta}]}_{\text{average cost of investors}}. \tag{2.14}$$

Equation (2.13) is interpreted as follows: in order for the deviating market maker to generate strictly positive profits from traders willing to accept the contract, the uniform price $p^*$ charged must exceed the average cost passed on to him.

Before analyzing the above condition, we first illustrate how the partial-pooling price schedule presented in Lemma 2.2 can yield positive profits. Under such a price schedule, the deviating market maker cannot earn profits from all contract-accepting types $(\underline{\theta}, \overline{\theta})$. Instead, achieving overall positive profit requires the strategic use of cross-subsidization across contracts. This insight is formalized in the following lemma.

**Lemma 2.3.** *Cross-subsidies among contracts. For the deviating price schedule to* *be profitable, there must exist a well-defined cutoff type* $\theta_{cutoff} \in (\underline{\theta}, \overline{\theta})$ *such that the deviating market maker exactly breaks even, i.e.,* $p^* = \beta\theta_{cutoff} + (1-\beta)m_s$.

*Proof.* See the Appendices. □

Lemma 2.3 indicates that the deviating market maker reaps benefits from investors with comparatively lower costs $(\underline{\theta}, \theta_{\text{cutoff}})$, while incurring losses from those with relatively higher costs $(\theta_{\text{cutoff}}, \overline{\theta})$. If the accrued gains surpass the incurred losses, the deviating market maker can achieve a net positive profit.

To handle the profitability condition, we apply Lemma 2.4 to simplify the last term on the right-hand side of Equation (2.14). This approach enables a tractable analysis of the profitability condition, allowing us to leverage statistical results of the truncated normal distribution to establish a non-existence result later. Otherwise, numerical simulations of Equation (2.13) might be necessary, posing challenges due to the presence of unknown variables in the upper and lower limits of the integral.

**Lemma 2.4.** *Doubly Truncated Normal. Suppose that* $X$ *follows a standard normal distribution. Let* $Y = \mu + \sigma X$. *Then, conditional on* $Y \in A = [a_1, a_2]$, *where* $-\infty \leq a_1 \leq a_2 \leq \infty$, *the expected value of the doubly truncated normal distribution can be expressed as*

$$E[Y|A] = \mu - \sigma\frac{\phi(\alpha_2) - \phi(\alpha_1)}{\Phi(\alpha_2) - \Phi(\alpha_1)},$$

*where* $\alpha_k = \frac{a_k - \mu}{\sigma}$ *for* $k = 1, 2$.

*Proof.* See the Appendices. □

The first and second moments of the truncated normal distribution can be found on page 156 of Samuel (1994). The proof of Lemma 2.4 is provided in the Appendix for the sake of completeness. Relying on Lemma 2.4, and considering the fact that $\theta$ is a sum of normally distributed variables, making it also normal (See the details in the proof of Lemma 2.3), we can transform the profitability condition (2.13) into a simpler form, as demonstrated in the following proposition.

**Proposition 2.2.** *Existence of profitable deviating price schedule. The partial-pooling deviating price schedule $(p^*, x^*)$ defined in Lemma 2.2 is profitable if the following condition holds:*

$$\frac{\frac{\bar{X}+\underline{X}}{2} - \frac{\phi(\underline{X})-\phi(\bar{X})}{\Phi(\bar{X})-\Phi(\underline{X})}}{\frac{\bar{X}+\underline{X}}{2} - \frac{2}{\frac{1}{\bar{X}}+\frac{1}{\underline{X}}}} > \frac{1}{2\beta}, \tag{2.15}$$

*where*

$$\begin{aligned}
\bar{X} &= \frac{\overline{\theta} - m_s}{\sigma_\theta}, \\
\underline{X} &= \frac{\underline{\theta} - m_s}{\sigma_\theta}, \\
\phi(\xi) &= \frac{1}{\sqrt{2\pi}} exp\left(-\frac{\xi^2}{2}\right), \\
\Phi(\xi) &= \int_{-\infty}^{\xi} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\xi^2}{2}\right) d\xi.
\end{aligned}$$

*Proof.* See the Appendices. □

Note that $\underline{X}$ and $\bar{X}$ in condition (2.14) represent the normalized lower bound and upper bound, respectively, of the interval $(\bar{\theta}, \underline{\theta})$ attracted by the deviating contract. Here, $\phi$ and $\Phi$ denote the density and distribution functions of the standard normal, respectively. The condition involves two variables $\underline{X}$ and $\bar{X}$ (or equivalently $\underline{\theta}$ and $\overline{\theta}$) due to the deviating contract consisting of two elements: the price $p^*$ and the quantity $x^*$. Recall that the quantity $x^*$ corresponds to the slope of the line connecting $\underline{\theta}$ and $\overline{\theta}$ in Figure 2.2, while the price $p^*$ is related to the intercept.

A pertinent question is whether the set of contracts satisfying condition (2.15) is empty. We provide the following example to show that it is not, even when $\beta < \frac{1}{2}$. In other words, despite the claimed sufficient condition for equilibrium existence, it is still feasible to construct a profitable partial-pooling cross-subsidization deviation.

***Example.*** Suppose $\beta = \frac{1}{3} < \frac{1}{2}$, then a fully separating equilibrium should exist by the main proposition in Glosten (1989). Let $\underline{X} = 2$ and $\bar{X} = 3$. Consider a deviating market maker offering the price schedule $\left(p^* = m_s + \frac{47}{60}\sigma_\theta, x^* = \frac{5}{6d}\sigma_\theta\right)$. By construction, only the investors within the interval $(m_s + 2\sigma_\theta, m_s + 3\sigma_\theta)$ purchase $x^*$ units of assets from the deviating market maker at the price $p^*$. The deviating market marker earns strictly positive profits because

$$\frac{\frac{2+3}{2} - \frac{\phi(2)-\phi(3)}{\Phi(3)-\Phi(2)}}{\frac{2+3}{2} - \frac{2}{\left(\frac{1}{2}+\frac{1}{3}\right)}} > \frac{1}{2 \cdot \frac{1}{3}}$$

holds as $\phi(2) = 0.053991$, $\phi(3) = 0.004432$, and $\Phi(3) - \Phi(2) = 0.021400$, so $LHS = 1.842 > 1.5 = RHS$.

An intuitive illustration of the non-emptiness of the profitability condition (2.15) and the motivation of Proposition 2.3 can be observed through the accompanying Figure 2.4, which illustrates the graph of the bi-variate function

$$f(\underline{X}, \overline{X}) = \frac{\frac{\bar{X}+\underline{X}}{2} - \frac{\phi(\underline{X})-\phi(\bar{X})}{\Phi(\bar{X})-\Phi(\underline{X})}}{\frac{\bar{X}+\underline{X}}{2} - \frac{2}{\frac{1}{\bar{X}}+\frac{1}{\underline{X}}}}$$

(i.e., the $LHS$ of condition 2.15). Let us define the critical value $\beta^* = \frac{1}{2f}$. As the functional value $f$ takes on values of 5, 10, 15, ..., the proposed price schedule remains profitable for values of $\beta$ greater than $\beta^* = \frac{1}{10}, \frac{1}{20}, \frac{1}{30}, \ldots$. (the critical value $\beta^*$ seems to gradually approach zero.) A close examination of Figure 2.4 seems to suggest that $f$ increases as $\underline{X}$ approaches $\overline{X}$, and both values augment concurrently. Putting it differently, the partial-pooling deviating price schedule $(p^*, x^*)$ defined in Lemma 2.2 remains consistently profitable, regardless of how small the value of $\beta$ may be. We apply a Taylor series expansion to prove this observation and to establish the subsequent result in Proposition 2.3.

**Proposition 2.3.** *There is no fully separating outcome in this game irrespective of $\beta$.*

*Proof.* See the Appendices. □

The result is established by showing the consistent profitability of the cross-subsidization deviating price schedule $(p^*, x^*)$ from Lemma 2.2 for arbitrarily small values of $\beta$. The non-existence result in Proposition 2.3 arises from the deviating

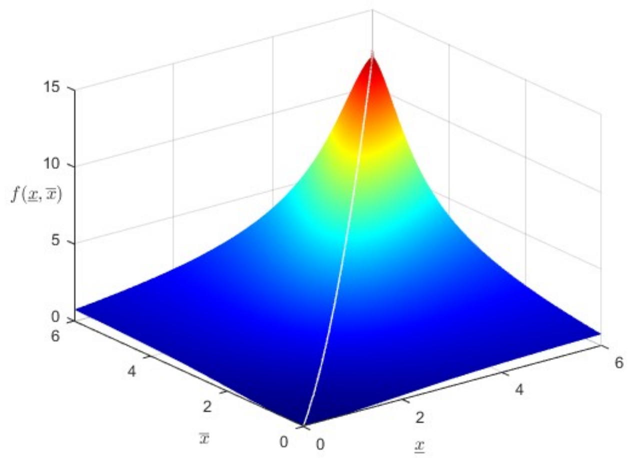


Figure 2.4: 3D plot of $f(\underline{x}, \overline{x})$

market maker's ability to offer a partial-pooling contract within the linear-quadratic setting, enabling them to attract a strict subset of investors and generate strictly positive profits through cross-subsidization across contracts. More precisely, the deviating market maker earns profits from investors with relatively low trading costs while incurring losses from those with relatively high costs. The profits from the former exceed the losses from the latter, resulting in a strictly positive net gain.

### 2.3.3 Equilibrium Nonexistence Issue in Madhavan (1992)

Another classical paper Madhavan (1992) has the same equilibrium nonexistence problem described in the last section. Madhavan (1992) contrasts the quote-driven mechanism in Glosten (1989) with the order-driven mechanism in Kyle (1989) in a multi-period setting. The multi-period quote-driven mechanism is a repetition of the single-period model in Glosten (1989), with the difference that market makers can adjust their price schedule over time. In Proposition 1 of his paper, it is claimed that if $\gamma < \frac{\rho^2}{\psi}$, an equilibrium exists. The parameter $\gamma$ measures the initial degree of information asymmetry, which, in the notation of our paper, corresponds to:

$$\gamma = \frac{\left(\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_S^2}\right)\frac{1}{\sigma_\epsilon^2}}{\frac{1}{\sigma_S^2}},$$

where $\sigma_\epsilon$ is the standard deviation of the noise in the signal, and $\sigma_S$ is the standard deviation of the true asset value. The parameter $\rho$ corresponds to $a$, the risk-aversion coefficient in the CARA utility function. The parameter $\psi$ is the precision of the initial position, i.e., $\frac{1}{\sigma_W^2}$. Given these definitions, the sufficient condition can be rearranged as:

$$\sigma_S^2 + \sigma_\epsilon^2 < a^2 \sigma_W^2 \sigma_\epsilon^4.$$

This is equivalent to $\beta < \frac{1}{2}$ when compared with Equation (2.9) in our notation. Since his $N$-period game is solved using backward induction, in the last trading period, the cross-subsidization deviation problem arises for the same reason.

## 2.4 Signaling Variant of the Market-Making Model

This section shows that a separating equilibrium exists and is unique in the signaling version of the market-making model described above.

Suppose the sequence of moves is altered as follows:

1. The informed investor submits a market order of size $x$.

2. Market makers observe the order size $x$ and then post a price $p(x)$.

We focus on settings with full information transmission and adopt separating equilibrium as the solution concept. Note that cross-subsidization deviations by market makers do not arise in this setting, since market makers move after the investor in the signaling game.

**Claim:** *The price-quantity schedule described in Proposition 2.1 constitutes the unique separating equilibrium when the liquidity motive dominates the informational motive (i.e., $\beta < \frac{1}{2}$).*

The argument proceeds as follows. Separation, combined with incentive compatibility, implies differentiability (see Section 6.2 of Mailath and Von Thadden (2013)). Therefore, any separating equilibrium must satisfy the first-order necessary condition. As shown in the previous section, the resulting first-order ODE has a unique solution given an initial boundary condition, and this solution serves as a candidate separating equilibrium. Moreover, the second-order sufficient condition holds if and only if $\beta < \frac{1}{2}$, ensuring that the candidate is indeed a separating equilibrium.

Modeling the interaction between traders and market makers as a signaling game has two merits. First, it preserves the insight that equilibrium with full information transmission fails to exist when the informational motive dominates the liquidity motive. Second, it guarantees the existence of a separating equilibrium when information asymmetry is not severe—that is, when there are sufficient non-informational motives to trade. The existence of such a separating equilibrium enables the model to generate testable predictions and opens the door to empirical investigation.

A variant of this signaling game is presented in the theoretical section of Keim and Madhavan (1996), where an initiator (investor) moves first to submit an order, followed by block traders (upstairs market makers), who facilitate the trade by locating potential counterparties to take the opposite side of the block transaction. Their sufficient condition $A < \frac{1}{2}$ (see Equation A.10 on page 35) corresponds to $\beta < \frac{1}{2}$ in Glosten (1989). In the empirical section, the authors test the predicted positive correlation between pre-trade price movements and trade size in block trading based on data on block transactions in the upstairs market. This pattern is interpreted as evidence of information leakage.

## 2.5 Concluding Remarks

This paper investigates the problem of equilibrium existence in quote-driven markets. The strategic interaction between market makers and risk-averse investors is modeled as a two-dimensional competitive screening game, in which market makers offer price schedules to investors with both liquidity and informational motives. Our main finding is that even when information asymmetry is not too severe (i.e., when liquidity motives dominate informational ones), a separating equilibrium still fails to exist. This market breakdown is driven by a profitable cross-subsidization deviation that involves partial pooling of investor types. The deviating market maker cream-skims a continuum of low-cost investors to subsidize a continuum of high-cost investors, resulting in strictly positive net gains.

This finding draws a parallel with the insurance literature and challenges the conventional wisdom that, under mild informational asymmetries, markets naturally open and separating equilibria are sustainable in quote-driven mechanisms. Notably, the mechanism persists even when the informational motive is arbitrarily small. In the signaling version of the market-making model, a separating equilibrium exists and is unique. Exploring alternative modeling structures or equilibrium notions to address the nonexistence result remains a promising avenue for future research.

## 2.6 Appendices to Chapter 2

### 2.6.1 Proof of Lemma 2.2

*Proof.* Given the equilibrium payoff equation $V^A = \frac{1-2\beta}{2d}(\theta - m_S)^2$ and two points $\left(\underline{\theta}, \frac{1-2\beta}{2d}(\underline{\theta} - m_S)^2\right)$ and $\left(\overline{\theta}, \frac{1-2\beta}{2d}\left(\overline{\theta} - m_S\right)^2\right)$ on the parabola, the slope of the deviating payoff line connecting these two points is

$$\begin{aligned} \text{slope} &= \frac{\frac{1-2\beta}{2d}\left[\left(\overline{\theta} - m_s\right)^2 - \left(\underline{\theta} - m_s\right)^2\right]}{\overline{\theta} - \underline{\theta}} \\ &= \frac{1-2\beta}{2d}\frac{(\overline{\theta} - m_s - \underline{\theta} + m_s)(\overline{\theta} - m_s + \underline{\theta} - m_s)}{\overline{\theta} - \underline{\theta}} \\ &= \frac{1-2\beta}{2d}\left(\overline{\theta} + \underline{\theta} - 2m_s\right) := x^*. \end{aligned}$$

Substituting the coordinate $\left(\underline{\theta}, \frac{1-2\beta}{2d}(\underline{\theta} - m_S)^2\right)$ into the deviating payoff line $V(x^*, p^*, \theta) = (\theta - p^*)x^* - \frac{1}{2}d(x^*)^2$ gives

$$\left(\underline{\theta} - p^*\right)x^* - \frac{1}{2}d(x^*)^2 = \frac{1-2\beta}{2d}\left(\underline{\theta} - m_s\right)^2.$$

So, the price is

$$\begin{aligned} p^* &= \underline{\theta} - \frac{1}{2}dx^* - \frac{1-2\beta}{2d}\frac{\left(\underline{\theta} - m_s\right)^2}{x^*} \\ &= \underline{\theta} - \frac{d}{2}\frac{1-2\beta}{2d}\left(\overline{\theta} + \underline{\theta} - 2m_s\right) - \frac{1-2\beta}{2d}\frac{\left(\underline{\theta} - m_s\right)^2}{\frac{1-2\beta}{2d}\left(\overline{\theta} + \underline{\theta} - 2m_s\right)} \\ &= \underline{\theta} - \frac{1-2\beta}{4}\left(\overline{\theta} + \underline{\theta} - 2m_s\right) - \frac{\left(\underline{\theta} - m_s\right)^2}{\overline{\theta} + \underline{\theta} - 2m_s} \\ &= \frac{\underline{\theta}\overline{\theta} + \underline{\theta}^2 - 2\underline{\theta}m_s - \underline{\theta}^2 - m_s^2 + 2\underline{\theta}m_s}{\overline{\theta} + \underline{\theta} - 2m_s} - \frac{1-2\beta}{4}\left(\overline{\theta} + \underline{\theta} - 2m_s\right) \\ &= \frac{\underline{\theta}\overline{\theta} - m_s^2}{(\overline{\theta} - m_s) + (\underline{\theta} - m_s)} - \frac{1-2\beta}{4}[(\overline{\theta} - m_s) + (\underline{\theta} - m_s)]. \end{aligned}$$

Therefore, from Figure 2.2, we conclude that $\theta \in (\underline{\theta}, \overline{\theta})$ pool at $x^* = \frac{1-2\beta}{2d}\left(\overline{\theta} + \underline{\theta} - 2m_s\right)$ with price $p^* = \frac{\underline{\theta}\overline{\theta} - m_s^2}{(\overline{\theta} - m_s) + (\underline{\theta} - m_s)} - \frac{1-2\beta}{4}[(\overline{\theta} - m_s) + (\underline{\theta} - m_s)]$. The other types would stay at the original separating outcome.

□

### 2.6.2 Proof of Lemma 2.3

*Proof.* When the deviating market maker offers the price schedule $(p^*, x^*)$, investors within the interval $(\underline{\theta}, \overline{\theta})$ are attracted to the new contract by Lemma 2.2.

Our first claim is that for such a price schedule to be profitable, the deviating market maker must earn strictly positive profits from the type $\underline{\theta}$ with the lowest cost. In other words, the price $p^*$ he charges must be greater than $\beta\underline{\theta} + (1-\beta)m_s$. Otherwise, if

$$p^* < \beta\underline{\theta} + (1-\beta)m_s,$$

it also holds that

$$p^* < \beta\theta + (1-\beta)m_s$$

for all $\theta \in (\underline{\theta}, \overline{\theta})$ since the cost function of investors $f(\theta) := \beta\theta + (1-\beta)m_s$ is increasing in $\theta$. This implies the market maker would incur losses from all types within $(\underline{\theta}, \overline{\theta})$.

Our second claim is that the deviating market maker cannot make positive profits from all types within $(\underline{\theta}, \overline{\theta})$ that are attracted to the new contract. In other words, the price $p^*$ the deviating market maker charges must be less than the highest cost of those who purchase the assets, so

$$p^* < \beta\overline{\theta} + (1-\beta)m_s$$

holds for $\overline{\theta} > \underline{\theta} > m_s$. To demonstrate this, we start by substituting the expression of $p^*$ from Lemma 2.2 into the equation above, which yields the following equivalent form:

$$\frac{\underline{\theta}\overline{\theta} - m_s^2}{(\overline{\theta} - m_s) + (\underline{\theta} - m_s)} - \frac{1-2\beta}{4}\Big((\overline{\theta} - m_s) - (\underline{\theta} - m_s)\Big) < \beta\bar{\theta} + (1-\beta)m_s$$

$$
\begin{aligned}
&\Leftrightarrow \frac{(\theta - m_s)(\underline{\theta} - m_s)}{(\overline{\theta} - m_s) + (\underline{\theta} - m_s)} < \frac{1+2\beta}{4}(\bar{\theta} - m_s) + \frac{1-2\beta}{4}(\underline{\theta} - m_s)\\
&\Leftrightarrow (\overline{\theta} - m_s)(\underline{\theta} - m_s) < \frac{1+2\beta}{4}(\overline{\theta} - m_s)^2 + \frac{1-2\beta}{4}(\underline{\theta} - m_s)^2 + \frac{1}{2}(\overline{\theta} - m_s)(\underline{\theta} - m_s)\\
&\Leftrightarrow \frac{1}{2}(\overline{\theta} - m_s)(\underline{\theta} - m_s) < \frac{1+2\beta}{4}(\overline{\theta} - m_s)^2 + \frac{1-2\beta}{4}\left(\underline{\theta} - m_s\right)^2\\
&\Leftrightarrow \frac{1}{2}(\overline{\theta}\underline{\theta} - \underline{\theta}m_s - \overline{\theta}m_s + m_s^2) < \frac{1+2\beta}{4}\left(\overline{\theta}^2 + m_s^2 - 2\overline{\theta}m_s\right) + \frac{1-2\beta}{4}\left(\underline{\theta}^2 + m_s^2 - 2\underline{\theta}m_s\right)\\
&\Leftrightarrow 0 < \frac{1}{4}\left(\overline{\theta}^2 + \underline{\theta}^2 - 2\overline{\theta}\underline{\theta}\right) + \frac{\beta}{2}\left(\overline{\theta}^2 - \underline{\theta}^2\right) + \beta m_s\left(\overline{\theta} - \underline{\theta}\right).
\end{aligned}
$$

Note that the above inequality holds because:

$$
\begin{aligned}
RHS &= \frac{1}{4}\left(\overline{\theta} - \underline{\theta}\right)^2 + \frac{\beta}{2}\left(\overline{\theta} - \underline{\theta}\right)\left(\overline{\theta} + \underline{\theta}\right) + \beta m_s\left(\overline{\theta} - \underline{\theta}\right)\\
&= \frac{1}{4}\left(\overline{\theta} - \underline{\theta}\right)^2 + \beta(\overline{\theta} - \underline{\theta})\left(\frac{\overline{\theta} + \underline{\theta}}{2} - m_s\right),
\end{aligned}
$$

since $\overline{\theta} > \underline{\theta} > m_s$, it follows that $RHS > 0$.

Combining the above two claims, for the deviating market maker to make strictly positive profits, it must hold that:

$$
\underbrace{\beta\underline{\theta} + (1-\beta)m_s}_{\text{cost of the lowest type}} < p^* < \underbrace{\beta\overline{\theta} + (1-\beta)m_s}_{\text{cost of the highest type}}.
$$

Given that the cost of investors $f(\theta) := \beta\theta + (1-\beta)m_s$ is continuous and increasing in $\theta$, the intermediate value theorem guarantees the existence of a well-defined cutoff $\theta_{\text{cutoff}} \in (\underline{\theta}, \overline{\theta})$ such that the deviating market maker exactly breaks even at $\theta_{\text{cutoff}}$, i.e., $p^* = \beta\theta_{\text{cutoff}} + (1-\beta)m_s$. □

### 2.6.3 Proof of Lemma 2.4

*Proof.* Since $X$ is standard normally distributed, its density is given by

$$
\phi(x) = \frac{1}{\sqrt{2\pi}}e^{\frac{-1}{2}x^2}, \quad -\infty < x < \infty.
$$

The associated distribution function is

$$\begin{aligned}\Pr[X \le x] &= \int_{-\infty}^{x} \phi(t)dt \\ &= \Phi(x).\end{aligned}$$

Since $Y = \mu X + \sigma$, the distribution of $Y$ is

$$\begin{aligned}\Pr[Y \le y] &= \Pr[\mu + \sigma X \le y] \\ &= \Pr[X \le \frac{y-\mu}{\sigma}] \\ &= \int_{-\infty}^{\frac{y-\mu}{\sigma}} \phi(t)dt \\ &= \Phi(\frac{y-\mu}{\sigma}).\end{aligned}$$

Applying Leibnitz's rule to the second to the integral above, the density of Y is

$$\begin{aligned}f(y) &= \frac{1}{\sigma}\phi(\frac{y-\mu}{\sigma}) \\ &= \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\mu}{2}\left(\frac{y-\mu}{\sigma}\right)^2}, \quad -\infty < y < \infty.\end{aligned}$$

The probability that $Y$ falls within the interval $A = [a_1, a_2]$ is given by $\Phi\left(\frac{a_2-\mu}{\sigma}\right) - \Phi\left(\frac{a_1-\mu}{\sigma}\right)$. Therefore, the conditional density of $Y$ is expressed as:

$$f(y|A) = \frac{\frac{1}{\sigma}\phi(\frac{y-\mu}{\sigma})}{\Phi(\frac{a_2-\mu}{\sigma}) - \Phi(\frac{a_1-\mu}{\sigma})}, \quad a_1 \le y \le a_2.$$

To derive the mean of the doubly truncated normal distribution, consider the moment-generating function (MGF):

$$\begin{aligned}M(t) &= E[e^{tY}|Y \in A] \\ &= \frac{\int_{a_1}^{a_2} e^{ty} f(y)dy}{\Phi(\frac{a_2-\mu}{\sigma}) - \Phi(\frac{a_1-\mu}{\sigma})} \\ &= e^{\mu t + \sigma^2 t^2/2} \frac{\Phi(\frac{a_2-\mu}{\sigma} - \sigma t) - \Phi(\frac{a_1-\mu}{\sigma} - \sigma t)}{\Phi(\frac{a_2-\mu}{\sigma}) - \Phi(\frac{a_1-\mu}{\sigma})}.\end{aligned}$$

The last equality holds because

$$\begin{aligned}
\frac{1}{\sigma\sqrt{2\pi}}\int_{a_1}^{a_2} e^{ty}e^{\frac{-1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}dy &= \frac{1}{\sigma\sqrt{2\pi}}\int_{a_1}^{a_2} e^{\frac{-1}{2\sigma^2}\{|y-(\sigma^2t+\mu)|^2-(\sigma^2t+\mu)^2+\mu^2\}}dy \\
&= e^{\frac{-1}{2\sigma^2}|\mu^2-(\sigma^2t+\mu)^2|}\frac{1}{\sigma\sqrt{2\pi}}\int_{a_1}^{a_2} e^{\frac{-1}{2}(\frac{y-\mu'}{\sigma})^2}dy \\
&= e^{\mu t+\sigma^2t^2/2}\int_{a_1}^{a_2}\frac{1}{\sigma}\phi(\frac{y-\mu'}{\sigma})dy \\
&= e^{\mu t+\sigma^2t^2/2}\left[\Phi(\frac{a_2-\mu'}{\sigma})-\Phi(\frac{a_1-\mu'}{\sigma})\right],
\end{aligned}$$

where $\mu' = \sigma^2 t + \mu$.

Note that by differentiating the moment generating function (MGF) $i$ times with respect to $t$ and setting $t = 0$, one can obtain the $i$th central moment. Therefore, the expectation of doubly truncated normal is

$$\begin{aligned}
E[Y|Y \in A] &= M'(t)|_{t=0} \\
&= \mu - \sigma\frac{\phi(\alpha_2)-\phi(\alpha_1)}{\Phi(\alpha_2)-\Phi(\alpha_1)}.
\end{aligned}$$

□

### 2.6.4 Proof of Proposition 2.2

*Proof.* The starting point is the profitability condition (2.13), or its equivalent form, equation (2.14). Substituting the expression for $p^*$ as given in Lemma 2.2 into equation (2.14) gives

$$\frac{\underline{\theta}\overline{\theta}-m_s^2}{(\overline{\theta}-m_s)+(\underline{\theta}-m_s)}-\frac{1-2\beta}{4}\Big((\overline{\theta}-m_s)-(\underline{\theta}-m_s)\Big) > (1-\beta)m_s+\beta E\big[\theta|\underline{\theta}\leq\theta\leq\overline{\theta}\big],$$

which is equivalent to

$$\beta\left(\frac{\overline{\theta}+\underline{\theta}}{2}-E\big[\theta|\underline{\theta}\leq\theta\leq\overline{\theta}\big]\right) > \frac{\overline{\theta}+\underline{\theta}}{4}+\frac{m_s}{2}-\frac{\underline{\theta}\overline{\theta}-m_s^2}{\big(\overline{\theta}-m_s\big)+\big(\underline{\theta}-m_s\big)}.$$

Note that

$$
\begin{aligned}
RHS &= \frac{\overline{\theta} - m_s + \underline{\theta} - m_s}{4} + m_s - \frac{\underline{\theta}\overline{\theta} - m_s^2}{\left(\overline{\theta} - m_s\right) + \left(\underline{\theta} - m_s\right)} \\
&= \frac{\overline{\theta} - m_s + \underline{\theta} - m_s}{4} - \frac{(\overline{\theta} - m_s)(\underline{\theta} - m_s)}{(\overline{\theta} - m_s) + (\underline{\theta} - m_s)} \\
&= \frac{1}{2}\left(\frac{\overline{\theta} - m_s + \underline{\theta} - m_s}{2} - \frac{2}{\frac{1}{\overline{\theta} - m_s} + \frac{1}{\underline{\theta} - m_s}}\right).
\end{aligned}
$$

Hence, the profitability condition is transformed to

$$
\beta\left(\frac{\overline{\theta} + \underline{\theta}}{2} - E[\theta|\underline{\theta} \leq \theta \leq \overline{\theta}]\right) > \frac{1}{2}\left(\frac{\overline{\theta} - m_s + \underline{\theta} - m_s}{2} - \frac{2}{\frac{1}{\overline{\theta} - m_s} + \frac{1}{\underline{\theta} - m_s}}\right), \tag{2.16}
$$

where $\overline{\theta} > \underline{\theta} > m_s$. Recall that $\theta$ is defined as the sum of some normally distributed variables, i.e.,

$$
\theta = m - a\sigma^2 W = \frac{1}{\sigma_\varepsilon^2 + \sigma_s^2}(\sigma_\varepsilon^2 m_s + \sigma_s^2 s - a\sigma_s^2\sigma_\varepsilon^2 W).
$$

Therefore, $\theta$ must also follow a normal distribution with mean $m_s$ and variance $\sigma_\theta^2$ defined as

$$
\sigma_\theta^2 = \left(\frac{\sigma_s^2}{\sigma_\varepsilon^2 + \sigma_s^2}\right)^2 (\sigma_\varepsilon^2 + \sigma_s^2 + a^2\sigma_\varepsilon^4\sigma_W^4).
$$

Applying Lemma 2.4 yields

$$
E\left[\theta|\underline{\theta} \leq \theta \leq \overline{\theta}\right] = m_s - \sigma_\theta \frac{\phi\left(\frac{\overline{\theta} - m_s}{\sigma_\theta}\right) - \phi\left(\frac{\underline{\theta} - m_s}{\sigma_\theta}\right)}{\Phi\left(\frac{\overline{\theta} - m_s}{\sigma_\theta}\right) - \Phi\left(\frac{\underline{\theta} - m_s}{\sigma_\theta}\right)}.
$$

The profitability condition can be further expressed as:

$$
\frac{\frac{\bar{\theta} + \underline{\theta}}{2} - m_s + \sigma_\theta \frac{\phi\left(\frac{\bar{\theta} - m_s}{\sigma_\theta}\right) - \phi\left(\frac{\underline{\theta} - m_s}{\sigma_\theta}\right)}{\Phi\left(\frac{\bar{\theta} - m_s}{\sigma_\theta}\right) - \Phi\left(\frac{\underline{\theta} - m_s}{\sigma_\theta}\right)}}{\frac{\bar{\theta} - m_s + \underline{\theta} - m_s}{2} - \frac{2}{\frac{1}{\bar{\theta} - m_s} + \frac{1}{\underline{\theta} - m_s}}} > \frac{1}{2\beta}.
$$

Multiplying the numerator and denominator of $LHS$ simultaneously by $\frac{1}{\sigma_\theta}$ gives

$$LHS = \frac{\frac{\bar{\theta}-m_s}{2\sigma_\theta} + \frac{\underline{\theta}-m_s}{2\sigma_\theta} + \frac{\phi\left(\frac{\bar{\theta}-m_s}{\sigma_\theta}\right)-\phi\left(\frac{\underline{\theta}-m_s}{\sigma_\theta}\right)}{\Phi\left(\frac{\bar{\theta}-m_s}{\sigma_\theta}\right)-\Phi\left(\frac{\underline{\theta}-m_s}{\sigma_\theta}\right)}}{\frac{\bar{\theta}-m_s}{2\sigma_\theta} + \frac{\underline{\theta}-m_s}{2\sigma_\theta} - \frac{2}{\frac{\sigma_\theta}{\bar{\theta}-m_s}+\frac{\sigma_\theta}{\underline{\theta}-m_s}}}.$$

We define $\bar{X} = \frac{\bar{\theta}-m_s}{2\sigma_\theta}$ and $\underline{X} = \frac{\underline{\theta}-m_s}{\sigma_\theta}$ for notation ease. Then, the inequality can be rewritten as:

$$\frac{\frac{\bar{X}}{2} + \frac{\underline{X}}{2} + \frac{\phi(\bar{X})-\phi(\underline{X})}{\Phi(\bar{X})-\Phi(\underline{X})}}{\frac{\bar{X}}{2} + \frac{\underline{X}}{2} - \frac{2}{\frac{1}{\bar{X}}+\frac{1}{\underline{X}}}} > \frac{1}{2\beta}.$$

Note $\phi(\bar{X}) < \phi(\underline{X})$ for $\bar{X} > \underline{X} > 0$, the above inequality can be rearranged to ensure the positivity of each term:

$$\frac{\frac{\bar{X}+\underline{X}}{2} - \frac{\phi(\underline{X})-\phi(\bar{X})}{\Phi(\bar{X})-\Phi(\underline{X})}}{\frac{\bar{X}+\underline{X}}{2} - \frac{2}{\frac{1}{\bar{X}}+\frac{1}{\underline{X}}}} > \frac{1}{2\beta}.$$

□

### 2.6.5 Proof of Proposition 2.3

*Proof.* Below, we demonstrate that the partial-pooling price schedule remains profitable regardless of the smallness of $\beta$. In other words, it's possible to identify suitable values for $\underline{X}$ and $\bar{X}$ that satisfy the profitability condition even for very small $\beta$.

Recall the profitability condition is

$$\frac{\frac{\bar{X}+\underline{X}}{2} - \frac{\phi(\underline{X})-\phi(\bar{X})}{\Phi(\bar{X})-\Phi(\underline{X})}}{\frac{\bar{X}+\underline{X}}{2} - \frac{2}{\frac{1}{\bar{X}}+\frac{1}{\underline{X}}}} > \frac{1}{2\beta}. \tag{2.17}$$

Remark that the denominator of the left-hand side in the above inequality approaches zero as $\underline{X}$ approaches $\bar{X}$, a result evident through the AM-GM-HM inequality. Additionally, the numerator converges to zero through the application of l'Hospital's rule. However, these two distinct functions cannot approach zero at the same rate. We employ Taylor's theorem to explore the rate of convergence.

Let $\underline{X} = x$ and $\overline{X} = x+\varepsilon$. Applying Taylor's series expansion to the denominator yields:

$$
\begin{aligned}
x + \frac{1}{2}\varepsilon - \frac{2(x+\varepsilon)x}{2x+\epsilon} &= x + \frac{1}{2}\varepsilon - \frac{x+\varepsilon}{1+\frac{\varepsilon}{2x}} \\
&= x + \frac{1}{2}\varepsilon - (x+\varepsilon)\left(1 - \frac{\varepsilon}{2x} + \frac{\varepsilon^2}{4x^2} + o(\varepsilon^2)\right) \\
&= x + \frac{1}{2}\varepsilon - \left(x + \varepsilon - \frac{\varepsilon}{2} - \frac{\varepsilon^2}{2x} + \frac{\varepsilon^2}{4x} + o(\varepsilon^2)\right) \\
&= \frac{\varepsilon^2}{2x} - \frac{\varepsilon^2}{4x} + o(\varepsilon^2) \\
&= \frac{\varepsilon^2}{4x} + o(\varepsilon^2).
\end{aligned}
$$

In the same way, the numerator can also be transformed to

$$
\begin{aligned}
&x + \frac{1}{2}\varepsilon + \frac{\phi' + \frac{1}{2}\phi''\varepsilon + \frac{1}{6}\phi^{(3)}\varepsilon^2 + o(\varepsilon^2)}{\phi + \frac{1}{2}\phi'\varepsilon + \frac{1}{6}\phi''\varepsilon^2 + o(\varepsilon^2)} \\
&= x + \frac{1}{2}\varepsilon + \frac{1}{\phi}\left[\phi' + \frac{1}{2}\phi''\varepsilon + \frac{1}{6}\phi^{(3)}\varepsilon^2 + o(\varepsilon^2)\right]\left[1 - \left(\frac{1}{2}\frac{\phi'}{\phi}\varepsilon + \frac{1}{6}\frac{\phi''}{\phi}\varepsilon^2\right) + \left(\frac{1}{2}\frac{\phi'}{\phi}\varepsilon + \frac{1}{6}\frac{\phi''}{\phi}\varepsilon^2\right)^2 + o(\varepsilon^2)\right] \\
&= x + \frac{1}{2}\varepsilon + \frac{1}{\phi}\left[\phi' + \frac{1}{2}\phi''\varepsilon + \frac{1}{6}\phi^{(3)}\varepsilon^2 + o(\varepsilon^2)\right]\left[1 - \left(\frac{1}{2}\frac{\phi'}{\phi}\varepsilon + \frac{1}{6}\frac{\phi''}{\phi}\varepsilon^2\right) + \frac{1}{4}\left(\frac{\phi'}{\phi}\right)^2\varepsilon^2 + o(\varepsilon^2)\right] \\
&= x + \frac{1}{2}\varepsilon + \frac{1}{\phi}\left[\phi' + \frac{1}{2}\phi''\varepsilon + \frac{1}{6}\phi^{(3)}\varepsilon^2 - \frac{1}{2}\frac{\phi'}{\phi}\varepsilon - \frac{1}{4}\frac{\phi''\phi'}{\phi}\varepsilon^2 - \frac{1}{6}\frac{\phi''\phi'}{\phi}\varepsilon^2 + \frac{1}{4}\left(\frac{\phi'}{\phi}\right)^2\phi'\varepsilon^2 + o\left(\varepsilon^2\right)\right].
\end{aligned}
$$

It can be verified that the density of the standard Normal distribution exhibits the following properties:

$$
\begin{aligned}
\phi'(x) &= \phi(x)(-x) \\
\phi''(x) &= \phi(x)(x^2-1) \\
\phi^{(3)}(x) &= \phi(x)(-x^3+3x).
\end{aligned}
$$

Combining the equations above, the numerator can be further simplified to:

$$
\begin{aligned}
&x + \frac{1}{2}\varepsilon - x + \frac{1}{2}(x^2-1)\varepsilon + \frac{1}{6}(-x^3+3x)\varepsilon^2 - \frac{1}{2}x^2\varepsilon - \frac{5}{12}(x^2-1)(-x)\varepsilon^2 + \frac{1}{4}(-x)^3\varepsilon^2 + o(\varepsilon^2) \\
&= \frac{1}{12}x\varepsilon^2 + o(\varepsilon^2).
\end{aligned}
$$

Therefore, the profitability condition simplifies to:

$$f(x,\varepsilon) = \frac{\frac{1}{12}x\varepsilon^2 + o(\varepsilon^2)}{\frac{\varepsilon^2}{4x} + o(\varepsilon^2)}$$
$$= \frac{\frac{1}{12}x + \frac{o(\varepsilon^2)}{\varepsilon^2}}{\frac{1}{4x} + \frac{o(\varepsilon^2)}{\varepsilon^2}}.$$

As $x$ becomes large and $\varepsilon$ approaches 0, the function $f(x,\varepsilon)$ diverge. This implies that for any arbitrarily small value of $\beta$, it is always possible to identify suitable $\underline{X}$ and $\overline{X}$, or the corresponding deviating price schedule as a function of $\underline{\theta}$ and $\overline{\theta}$ as defined in Lemma 2.2, to satisfy the profitability condition (2.15). □

# Chapter 3

# Limited Attention, Information Choice, and Market Microstructure

## 3.1 Introduction

Classical market microstructure theory typically assumes that investors incorporate all available information into their trading decisions. However, in practice, investors face limited cognitive resources and must exert effort to process information (e.g., Van Nieuwerburgh and Veldkamp (2010); Kacperczyk et al. (2016)). Certain market characteristics, such as price volatility, are empirically associated with investor attention, as shown in Figure 3.1.[1] Additionally, Liu et al. (2023) documents that macro news crowds out retail investors' attention to firm earnings news by as much as 49%. During periods of greater aggregate uncertainty, macro news reduces price responsiveness to firm earnings news. These empirical patterns are difficult to reconcile with traditional models that assume unlimited investor attention. This motivates the need for a framework that explicitly incorporates attention allocation and generates predictions consistent with the observed facts.

This paper addresses the following research question: How does endogenous investor attention allocation influence trading behavior, asset price and market outcomes? We introduce limited attention and information choice using an entropy approach into Kyle (1985). We choose Kyle's continuous auction setting because it

[1] The figure was made by the author. We downloaded the data for VIX from https://www.cboe.com/tradable_products/vix/vix_historical_data/, and the data for Google search trends from https://trends.google.com/trends/explore?date=today%205-y&geo=US&q=S%26P%20500&hl=zh-CN.

generates market characteristics in a tractable way and allows us to explore how investors' information choices affect market characteristics. We relax the assumption that an investor has perfect information about asset value and add an additional stage for information choice before trading under asymmetric information. Our model captures the comovement between investor attention and asset price volatility, provides a microfoundation for the crowding-out effects between macroeconomic news and firm earnings news, and explains varied price volatility responsiveness to different information sources.

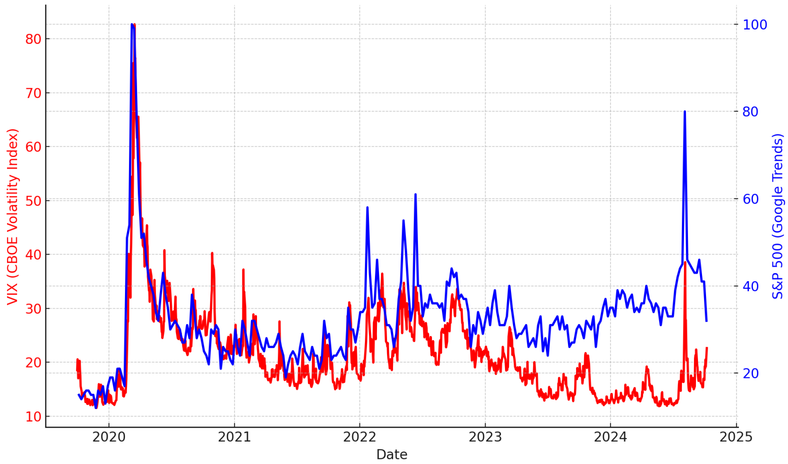


Figure 3.1: The volatility of the U.S. stock market comoves with the investors' attention. The red line shows the CBOE Volatility Index (VIX), which measures the variance of the S&P 500, while the blue line represents normalized Google Search Trends for "S&P 500," a proxy for investor attention (e.g., Da et al. (2011)). Their co-movement suggests that market volatility positively correlates with investor attention.

In our setting, an informed investor allocates her limited attention between macro-level news (e.g., central bank policy, green energy tax changes) and firm-specific news (e.g., earnings announcements, board composition changes). Different information sources exhibit varying degrees of uncertainty. For instance, macro-level uncertainty tends to dominate during recessions, while firm-level information becomes more volatile during earnings seasons (e.g., Kacperczyk et al. (2016); Liu et al. (2023)). Devoting more attention to one source improves its signal precision but leaves less capacity for the other. After selecting her information portfolio, the investor processes signals and submits a market order that reflects her private information. Market makers observe the total order flow, which consists of the informed trades and random orders from liquidity traders. The market makers Bayesian updates beliefs about the asset's value and sets a price to clear the market orders.

To our knowledge, the first model incorporating investor attention into a Kyle-type trading framework is Ruan and Zhang (2016). They assume that an investor observes a noisy signal, $s$, which is a weighted average of the true asset value $v$ and a noise term $\epsilon$, i.e., $s = \rho v + \sqrt{1-\rho^2}\epsilon$, where $\rho$ is exogenously given and interpreted as the correlation between the asset value and the signal. This specification is the static version of time-varying attention in a diffusion process, i.e., $ds_t = \Phi_t dZ_t^f + \sqrt{1-\Phi_t^2} dZ_t^s$ in Andrei and Hasler (2015), who study a consumption-portfolio decision problem with incomplete information.

In contrast, our model adopts an entropy-based approach in which a representative investor with limited capacity $\kappa$ endogenously chooses how to allocate attention across two information sources. This framework allows us to explore how attention decisions shape market outcomes. Our model not only matches stylized empirical facts, such as the co-movement of volatility and attention, but also captures novel features, including crowding-out effects between information sources and asymmetric impacts of different news types on volatility, bid-ask spread, and market depth. These features are not present in previous models.

**Related Literature.** Our paper contributes to the literature on limited attention and its implications for various markets. Several studies incorporate endogenous attention allocation into classical frameworks. For example, Peng and Xiong (2006) explores how category-based attention affects asset prices. Van Nieuwerburgh and Veldkamp (2010) allow investors to learn about asset values before constructing portfolios and show that bounded attention can lead to anomalous investment choices. Kacperczyk et al. (2016) model mutual fund managers choosing which signals to acquire before allocating capital. In a macro-labor context, Gondhi (2023) show that attention misallocation during uncertain times can lead to inefficient resource allocation. Ye (2024) analyze firms competing for investor attention and feedback effects through prices. Our model investigates the trading behavior of rationally inattentive investors in securities markets, analyzing how attention allocation decisions propagate through order-driven mechanisms and ultimately shape market outcomes.

We are also related to a broad literature based on Kyle's trading model, which is widely used in economics and finance due to its tractability. Fishman and Hagerty (1989) add a disclosure stage before the single-period Kyle model and find that managers over-disclose compared to the socially efficient level to compete for investor attention. Edmans et al. (2015) extend the one-round trading model by introducing a post-trading investment stage, where managers observe the price generated during trading and choose an investment level that affects the firm's fundamental value to analyze the feedback effects. A recent work by Cetemen et al. (2022) studies wolf-pack activism, where a leader activist strategically trades to induce a follower trader with correlated information before they jointly exert costly effort to determine the firm's value. Each period's trading is based on the single-period Kyle model. Our extension investigates how the limited-attention investor's endogenous information choice affects their trading behavior and shapes market outcomes based on the one-round trading framework in Kyle.

**Organization of the Paper.** The remainder of the paper is organized as follows. Section 3.2 introduces the model and explains the entropy-based learning technology. Section 3.3 analyzes the game and derives the equilibrium results, with a particular focus on the linear trading equilibrium. We conduct comparative statics to analyze how limited attention allocation affects market outcomes. Section 3.4 concludes and outlines directions for future research.

## 3.2 Model

In this section, we introduce the information structure and the timing of the game, explain the entropy-learning approach, and define the equilibrium.

### 3.2.1 Players and Information Structure

Suppose that there is a single risky asset, such as a stock with value $v$, which is normally distributed with mean $\mu_v$ and variance $\sigma_v^2$, i.e., $v \sim N(\mu_v, \sigma_v^2)$. The asset value $v$ is assumed to be composed of two parts, driven by the macro-level factor and the firm-level factor: $v = v_m + v_f$. There are two types of traders: a representative informed trader with limited attention and uninformed liquidity traders (also known as noise traders), along with market makers.

**Informed Trader.** The limited-attention investor first allocates her attention between macro-level news $s_m$ and firm-level news $s_f$, subject to an information flow constraint (3.3), which is explained in detail in the next section. After that, she observes a vector of two noisy signals: $s = (s_m, s_f)$. Signals are assumed to be linear in values: $s_m = v_m + \epsilon_m$, with $v_m \sim N(\mu_m, \sigma_m^2)$, and $s_f = v_f + \epsilon_f$, with $v_f \sim N(\mu_f, \sigma_f^2)$. All noise terms are independently and normally distributed, i.e., $\epsilon_m \sim N(0, \sigma_{\epsilon_m}^2)$ and $\epsilon_f \sim N(0, \sigma_{\epsilon_f}^2)$. This specification follows Peng and Xiong (2006) and Gondhi (2023).[2]

**Liquidity Trader.** Liquidity traders, who have no information, submit random market orders. The total demand of liquidity traders is represented by an exogenous random order size $u \sim N(0, \sigma_u^2)$. A liquidity trader trades to balance his portfolio or adjust her position, rather than to take advantage of information. The role of liquidity traders is to provide camouflage for the informed trader to earn information rents at their expense.

**Market Maker.** Market makers observe the total order flow $Q$, which consists of both informed trading $q$ and noise trading $u$, i.e., $Q = q + u$. However, they cannot determine the source of the orders. Market makers infer the asset value $v$ from the aggregate order flow $Q$ and intermediate trades by quoting a price $P(Q)$, which is assumed to equal to their best estimate $E[v|Q]$. The pricing rule can be understood as a Bertrand competition among two or more market makers, where they undercut each other until profits are driven down to zero in a competitive setting.

[2]Peng and Xiong (2006) study investor attention allocation and overconfidence problems in a consumption-based equilibrium model. They assume that the dividend $d_{i,j,t}$ is decomposed into an independent market factor $h_t$, a sector factor $f_{i,t}$, and a firm-specific factor $g_{i,j,t}$, i.e., $d_{i,j,t} = h_t + f_{i,t} + g_{i,j,t}$. In Gondhi (2023), an inattentive manager is assumed to receive information about firm-specific shocks $z_i$ and economy-wide shocks $a$. The log output is linear in these factors: $y_i = a + z_i + \alpha n_i$, where $n_i$ is the labor employed by the firm in a standard RBC model. The linear function form is made for traceability, not as the driving force of results. What matters is the substitutability of information flow in the constraint (3.3).

### 3.2.2 Explanation on Entropy-Learning Technology

This section provides a brief introduction to the entropy approach used in this paper. Readers familiar with this topic may skip ahead to the information flow constraint at the end of the section.

Entropy is a measure of the uncertainty associated with a random variable. The entropy of a random variable $x$ with density $p(x)$ is formally defined as

$$H(x) = -E[\ln(p(x))].$$

For example, a constant has entropy 0, a binomial random variable has entropy $\ln(2)$, and a uniform random variable on $[0, a]$ has entropy $\ln(a)$.

Information processing is regarded as a reduction in uncertainty. Given two random variables $x$ and $y$, the reduction in the entropy of $y$ due to knowledge of $x$ is defined as the mutual information between $x$ and $y$:

$$I(x; y) = H(x) - H(x|y),$$

where the second term is the conditional entropy of $x$ on $y$.

One approach to modeling limited attention is to impose a constraint on the mutual information between $x$ and $y$:

$$H(x) - H(x|y) \leq \kappa \tag{3.1}$$

where $\kappa$ represents information processing capacity. The economic interpretation of $\kappa$ is the number of binary signals required to partition the states of the world.[3]

In our linear-Gaussian setting, for a $n$-dimensional normal vector $\mathbf{x} \sim N(\mu, \Sigma)$, its entropy is given by $H(\mathbf{x}) = \frac{1}{2} \ln [(2\pi e)^n |\Sigma|]$, where $|\Sigma|$ denotes the determinant of the covariance matrix $\Sigma$. The conditional entropy has a similar form: $H(\mathbf{x}|\mathbf{y}) = \frac{1}{2} \ln \left[(2\pi e)^n |\hat{\Sigma}|\right]$, where $\hat{\Sigma}$ represents the posterior variance-covariance

[3]See Cover and Thomas (1991) for proof that the entropy of a random variable is the approximation of the number of binary signals needed to convey the same information.

matrix. Substituting this expression into (3.1) yields

$$|\Sigma| \leq \exp(2\kappa)|\hat{\Sigma}|. \tag{3.2}$$

With independent signals, the entropy constraint binds the product of the precision of the signals' precision. In our case, where $n = 2$, the prior variance-covariance matrix $\Sigma$ is given by

$$\Sigma = \begin{pmatrix} \sigma_m^2 & 0 \\ 0 & \sigma_f^2 \end{pmatrix},$$

and the posterior variance-covariance matrix $\hat{\Sigma}$ is

$$\hat{\Sigma} = \begin{pmatrix} \left(\frac{1}{\sigma_m^2} + \frac{1}{\sigma_{\epsilon m}^2}\right)^{-1} & 0 \\ 0 & \left(\frac{1}{\sigma_f^2} + \frac{1}{\sigma_{\epsilon_f}^2}\right)^{-1} \end{pmatrix}.$$

The determinant of the prior variance-covariance matrix is given by $|\Sigma| = \sigma_m^2 \cdot \sigma_f^2$, and the determinant of the posterior variance-covariance matrix is $|\hat{\Sigma}| = \left(\frac{1}{\sigma_m^2} + \frac{1}{\sigma_{\epsilon m}^2}\right)^{-1} \cdot \left(\frac{1}{\sigma_f^2} + \frac{1}{\sigma_{\epsilon_f}^2}\right)^{-1}$. Thus, the information flow constraint (3.2) can be written as

$$\frac{|\Sigma|}{|\hat{\Sigma}|} = \left(1 + \frac{\sigma_m^2}{\sigma_{\epsilon_m}^2}\right)\left(1 + \frac{\sigma_f^2}{\sigma_{\epsilon_f}^2}\right) \leq \exp(2\kappa). \tag{3.3}$$

This is also equivalent to its logarithm form:

$$\frac{1}{2}\ln\left(1 + \frac{\sigma_m^2}{\sigma_{\epsilon_m}^2}\right) + \frac{1}{2}\ln\left(1 + \frac{\sigma_f^2}{\sigma_{\epsilon_f}^2}\right) \leq \kappa.$$

A similar specification is used in Gondhi (2023) to study labor misallocation in macroeconomics.

Below we explain some properties of the information flow constraint. The precision of signals is defined as $\tau_m = \frac{1}{\sigma_{\epsilon m}^2}$ and $\tau_f = \frac{1}{\sigma_{\epsilon_f}^2}$. If an investor does not waste her cognitive resource, the binding constraint is given by

$$\left(1 + \frac{\sigma_m^2}{\tau_m}\right) \cdot \left(1 + \frac{\sigma_f^2}{\tau_f}\right) = \exp(2\kappa),$$

so that the precision of one signal can be expressed in terms of the other as

$$\tau_m = \left( \frac{\exp(2\kappa)}{1 + \frac{\sigma_f^2}{\tau_f}} - 1 \right) \cdot \frac{1}{\sigma_m^2}.$$

The information flow constraint has two properties. First, $\frac{\partial \tau_m}{\partial \tau_f} < 0$, meaning that increasing the precision of one signal comes at the expense of the other. Second, $\frac{\partial \tau_m}{\partial \kappa}$ decreases with $\tau_f$. This implies that the marginal return from exploring one signal, say, $s_m$, diminishes as the precision of the other signal, $s_f$, increases. Since more cognitive resources are already allocated to reducing uncertainty in signal $s_f$, fewer resources remain available for signal $s_m$. Hence, it is as if investors still face higher uncertainty in $s_m$ when allocating one extra resource unit, keeping the marginal return of exploring $s_m$ lower.

### 3.2.3 Structure of the Game and Equilibrium Concept

The game is modeled as an information choice stage by the limited-attention informed trader, followed by a single-round trading as in Kyle. In the trading stage, informed and noise traders act first by submitting market orders to market makers, who then quote a price at which the orders are executed. The sequence of moves unfolds as follows.

1. **Information Choice Stage.** The limited-attention informed trader chooses variance of macro-level news $\sigma^2_{\epsilon_m}$ and firm-level news $\sigma^2_{\epsilon_f}$, and then observes a signal vector $s = (s_m, s_f)$.

2. **Trading Stage.** The informed trader submits a market order $q(s)$. Liquidity traders submit random market orders with a total size $u$. Market makers observe the net total flow $Q = q + u$ and announce a price $P(Q)$ to clear the market.

3. All values and payoffs are realized.

**Definition of Equilibrium.** The equilibrium consists of the information choices ($\sigma^2_{\epsilon_m}$ and $\sigma^2_{\epsilon_f}$), the trading strategy $q(s)$ of the limited-attention trader, and the pricing rule $P(Q)$ set by market makers. The beliefs of market makers and investors are Bayesian updated and consistent with their strategies.

- In trading, the informed trader chooses a market order $q(s)$ to maximize her expected payoff, i.e.,

$$q(s) \in \arg\max E\left[\pi|s\right] := E\left[(v - P(Q))q(s)|s\right].$$

- When acquiring information, the informed investor chooses the shape of signal distribution to maximize her ex-ante profits, subject to the information flow constraint, i.e.,

$$\max_{\sigma^2_{\epsilon_m}, \sigma^2_{\epsilon_f}} \Pi = E_s\left[E\left[\pi|s\right]\right]$$

s.t.

$$\left(1 + \frac{\sigma^2_m}{\sigma^2_{\epsilon_m}}\right) \cdot \left(1 + \frac{\sigma^2_f}{\sigma^2_{\epsilon_f}}\right) \leq \exp(2\kappa).$$

- Market makers quote a price $P(Q)$, which is their best estimate of the asset's value given the total order flow $Q$, i.e.,

$$P(Q) = E[v|Q].$$

## 3.3 Equilibrium Analysis

We analyze the game using backward induction, beginning with the trading stage, followed by the information choice stage. We then summarize the main results and conduct a comparative statics analysis.

### 3.3.1 Trading Stage

Upon observing her signal vector $s = (s_m, s_f)$, the informed investor needs to estimate the asset value $v$ based on these signals. Following Maćkowiak and Wiederholt (2009), we assume that the investor processes different pieces of information separately, and then combines them to learn the asset value. The result is summarized in the following lemma, which is proven using the projection theorem.

**Lemma 3.1.** *Upon observing a signal vector $s = (s_m, s_f)$, the informed trader's best estimate of the asset value $v$ and the variance are given by*

$$E[v|s] = \mu_v + \frac{\sigma_m^2}{\sigma_m^2 + \sigma_{\epsilon_m}^2}(s_m - \mu_m) + \frac{\sigma_f^2}{\sigma_f^2 + \sigma_{\epsilon_f}^2}(s_f - \mu_f)$$
$$Var[v|s] = \sigma_m^2 \cdot \frac{\sigma_{\epsilon_m}^2}{\sigma_m^2 + \sigma_{\epsilon_m}^2} + \sigma_f^2 \cdot \frac{\sigma_{\epsilon_f}^2}{\sigma_f^2 + \sigma_{\epsilon_f}^2}.$$

*Proof.* See the Appendices. □

Following Kyle (1985), we focus on linear equilibrium. We conjecture that the informed investor's order size is linear in signals:

$$q(s) = \beta_m(s_m - \mu_m) + \beta_f(s_f - \mu_f), \tag{3.4}$$

where $\beta_m$ and $\beta_f$ are undetermined coefficients which measure the investor's trading aggression on macro-level news and firm-level news, respectively. The intercept of $q(s)$ is 0, because if the signals $s_m$ and $s_f$ exactly match the respective prior mean values $\mu_m$ and $\mu_f$, there is no informational motive to trade, and thus the order size is zero. Market makers are also assumed to use a linear strategy, meaning that the pricing rule is linear in the total order flow they observe:

$$P = \mu_v + \lambda Q, \tag{3.5}$$

where $\lambda$ is a coefficient determined endogenously in equilibrium. The constant is the prior mean $\mu_v$. This is because if an order size $Q$ is 0, it carries no information about the asset value, and thus the price should match the prior mean value $\mu_v$.

First, market makers need to estimate the asset value $v$ conditional on the order flow $Q$. By the projection theorem, we have that

$$E[v|Q] = E[v] + \frac{Cov(v, Q)}{Var(Q)}(Q - E[Q]).$$

Given the variance and covariance terms:

$$
\begin{aligned}
Var(Q) &= Var(q(s)+u) = \beta_m^2 Var(s_m) + \beta_f^2 Var(s_f) + \sigma_u^2 \\
&= \beta_m^2(\sigma_m^2 + \sigma_{\epsilon_m}^2) + \beta_f^2(\sigma_f^2 + \sigma_{\epsilon_f}^2) + \sigma_u^2, \\
Cov(v, Q) &= Cov(v, q(s)+u) = (\beta_m + \beta_f)\sigma_v^2,
\end{aligned}
$$

the pricing rule is simplified to

$$
P = E[v|Q] = \mu_v + \frac{(\beta_m + \beta_f)\sigma_v^2}{\beta_m^2(\sigma_m^2 + \sigma_{\epsilon_m}^2) + \beta_f^2(\sigma_f^2 + \sigma_{\epsilon_f}^2) + \sigma_u^2} \cdot Q. \tag{3.6}
$$

Comparing the coefficients in the above price schedule (3.6) and the conjectured price schedule (3.5) pins down $\lambda$:

$$
\lambda = \frac{(\beta_m + \beta_f)\sigma_v^2}{\beta_m^2(\sigma_m^2 + \sigma_{\epsilon_m}^2) + \beta_f^2(\sigma_f^2 + \sigma_{\epsilon_f}^2) + \sigma_u^2}, \tag{3.7}
$$

where $\sigma_{\epsilon_m}^2$ and $\sigma_{\epsilon_f}^2$ will later be determined in the information choice stage, and $\beta_m$ and $\beta_i$ will be determined in the investor's payoff maximization problem as follows.

Next, we analyze the informed investor's optimal trading strategy. Since $P = \mu_v + \lambda Q = \mu_v + \lambda(q+u)$, and $E[u] = 0$, the investor's expected payoff if she submits a market order $q(s)$ after observing her signal $s$ is given by

$$
E[\pi|s] = (E[v|s] - \mu_v - \lambda q(s)) \cdot q(s). \tag{3.8}
$$

Taking the derivative with respect to $q(s)$ leads to the first-order condition (FOC):

$$
q(s) = \frac{E[v|s] - \mu_v}{2\lambda}.
$$

The second-order condition requires that $\lambda > 0$.

The investor faces a trade-off between aggressively exploiting private information to increase trade volume and reducing trade size to secure a better price, as a larger order entails a higher per-unit trading cost under the pricing rule set by market makers. Substituting the estimated asset value from Lemma 3.1 into the investor's

FOC gives

$$q(s) = \frac{\frac{\sigma_m^2}{\sigma_m^2+\sigma_{\epsilon_m}^2}(s_m - \mu_m) + \frac{\sigma_f^2}{\sigma_f^2+\sigma_{\epsilon_f}^2}(s_f - \mu_f)}{2\lambda}. \tag{3.9}$$

Comparing (3.9) with the conjectured trading strategy (3.4) pins down trading aggression in terms of $\lambda$:

$$\beta_m = \frac{1}{2\lambda} \cdot \frac{\sigma_m^2}{\sigma_m^2 + \sigma_{\epsilon_m}^2}, \quad \beta_f = \frac{1}{2\lambda} \cdot \frac{\sigma_f^2}{\sigma_f^2 + \sigma_{\epsilon_f}^2}. \tag{3.10}$$

Substituting them back into (3.7) yields an equation in terms of $\lambda$:

$$\lambda = \frac{\frac{1}{2\lambda} \cdot \frac{\sigma_m^2}{\sigma_m^2+\sigma_{\epsilon_m}^2} + \frac{1}{2\lambda} \cdot \frac{\sigma_f^2}{\sigma_f^2+\sigma_{\epsilon_f}^2}}{\frac{1}{4\lambda^2} \cdot \frac{\sigma_m^4}{\sigma_m^2+\sigma_{\epsilon_m}^2} + \frac{1}{4\lambda^2} \cdot \frac{\sigma_f^4}{\sigma_f^2+\sigma_{\epsilon_f}^2} + \sigma_u^2}.$$

Since $\lambda > 0$, the above equation has a unique solution given by

$$\lambda = \frac{\sqrt{\frac{\sigma_m^4}{\sigma_m^2+\sigma_{\epsilon_m}^2} + \frac{\sigma_f^4}{\sigma_f^2+\sigma_{\epsilon_f}^2}}}{2\sigma_u}. \tag{3.11}$$

Given the trading intensity $\beta_m$ and $\beta_f$ in (3.10), as well as the price sensitivity to order size $\lambda$ in (3.11), the conjectured trading strategy (3.4) and the pricing rule (3.5) are determined. The unknowns are the variances ($\sigma_{\epsilon_m}^2$ and $\sigma_{\epsilon_f}^2$), or equivalently, the inverse precision of the signals, which will be determined in the information choice stage.

### 3.3.2 Information Choice Stage

Since the informed investor allocates her attention before observing the signals, we need to express her ex-ante (unconditional) payoff in terms of signal uncertainty. Substituting her trading quantity $q(s)$ in (3.9) back into the investor's interim payoff

(3.8), we obtain

$$E[\pi|s] = \frac{1}{4\lambda} \cdot \left( \frac{\sigma_m^2}{\sigma_m^2 + \sigma_{\epsilon_m}^2}(s_m - \mu_m) + \frac{\sigma_f^2}{\sigma_f^2 + \sigma_{\epsilon_f}^2}(s_f - \mu_f) \right)^2$$
$$= \frac{1}{4\lambda} \cdot (E[v|s] - \mu_v)^2.$$

Hence, her unconditional payoff is given by

$$\Pi = E_s\left[E\left[\pi|s\right]\right] = E_s\left[\frac{1}{4\lambda} \cdot (E[v|s] - \mu_v)^2\right].$$

The unconditional payoff can be further simplified, and one observation is that it is monotone in signal uncertainty. This is summarized in the following lemma.

**Lemma 3.2.** *The limited-attention investor's unconditional payoff is given by*

$$\Pi = \frac{1}{4\lambda} \cdot (Var[v] - Var[v|s])$$
$$= \frac{\sigma_u}{2}\sqrt{\frac{\sigma_m^4}{\sigma_m^2 + \sigma_{\epsilon_m}^2} + \frac{\sigma_f^4}{\sigma_f^2 + \sigma_{\epsilon_f}^2}},$$

*which is decreasing in uncertainty in both macro-level and firm-level signals, i.e.,* $\frac{\partial \Pi}{\partial \sigma_{\epsilon_m}^2} < 0$ *and* $\frac{\partial \Pi}{\partial \sigma_{\epsilon_f}^2} < 0$*, or equivalently increasing in the precision of signals, i.e.,* $\frac{\partial \Pi}{\partial p_1} > 0$ *and* $\frac{\partial \Pi}{\partial p_2} > 0$*, where* $p_1 = \frac{1}{\sigma_{\epsilon_m}^2}$ *and* $p_2 = \frac{1}{\sigma_{\epsilon_f}^2}$*.*

*Proof.* See the Appendices. □

Minimizing uncertainty in signals (i.e., $\sigma_{\epsilon_m}^2$ and $\sigma_{\epsilon_f}^2$) has two opposing effects on the informed investor's payoff. On the one hand, it enables the investor to obtain more precise information about the asset's value, enhancing their informational advantage over noise traders. This effect is reflected in the second term of $\Pi$, which represents the difference between the uninformed trader's uncertainty and the informed trader's uncertainty regarding the asset value, i.e., $Var[v] - Var[v|s]$. On the other hand, anticipating an increase in the informed investor's trading intensity prompts market makers to adjust the pricing rule adversely. The latter effect is second-order, whereas the former is first-order. The magnitude of the latter effect satisfies $\frac{1}{\lambda} \propto \frac{1}{\sqrt{\frac{\sigma_m^4}{\sigma_m^2+\sigma_{\epsilon_m}^2}+\frac{\sigma_f^4}{\sigma_f^2+\sigma_{\epsilon_f}^2}}}$, indicating that it is inversely proportional to the square root of the first effect. Hence, the first effect dominates. Overall, the investor's ex-ante payoff is monotonically decreasing in uncertainty, or equivalently, increasing in the precision of the signals.

In the information choice stage, the investor chooses the precision of signals to maximize her ex-ante payoff, subject to the information flow constraint. The optimization problem is given by

$$\max_{\sigma_{\epsilon_m}^2, \sigma_{\epsilon_f}^2} \Pi = \frac{\sigma_u}{2} \cdot \sqrt{\frac{\sigma_m^4}{\sigma_m^2 + \sigma_{\epsilon_m}^2} + \frac{\sigma_f^4}{\sigma_f^2 + \sigma_{\epsilon_f}^2}}$$

subject to

$$\left(1 + \frac{\sigma_m^2}{\sigma_{\epsilon_m}^2}\right) \cdot \left(1 + \frac{\sigma_f^2}{\sigma_{\epsilon_f}^2}\right) \leq \exp(2\kappa).$$

The solution to this attention allocation problem is summarized in the following lemma.

**Lemma 3.3.** *Assume w.l.o.g. that $\sigma_f > \sigma_m$. The optimal allocation of attention is discussed in the following two cases:*

*Case 1: $\kappa > \ln\left(\frac{\sigma_f}{\sigma_m}\right)$. The interior solution is given by*

$$\sigma_{\epsilon_m}^2 = \frac{\sigma_m^2}{\frac{\exp(\kappa)\cdot\sigma_m}{\sigma_f}}, \quad \sigma_{\epsilon_f}^2 = \frac{\sigma_f^2}{\frac{\exp(\kappa)\cdot\sigma_f}{\sigma_m} - 1}.$$

*Case 2: $\kappa \leq \ln\left(\frac{\sigma_f}{\sigma_m}\right)$. The corner solution is given by*

$$\sigma_{\epsilon_m}^2 = \infty, \quad \sigma_{\epsilon_f}^2 = \frac{\sigma_f^2}{\exp(2\kappa) - 1}.$$

*Proof.* See the Appendices. □

### 3.3.3 Equilibrium Results

In this section, we first state the equilibrium attention allocation, trading strategy, and pricing rule. Based on these, we derive the market characteristics such as price volatility, market depth, and bid-ask spread, and analyse how investor's limited attention and information choice affect these market outcomes.

**Proposition 3.1.** *Assume w.l.o.g. that* $\sigma_f > \sigma_m$. *The equilibrium is discussed in two cases.*

*Case 1.* $\kappa > \ln\left(\frac{\sigma_f}{\sigma_m}\right)$. *The equilibrium asset price is given by* $P(Q) = \mu_v + \lambda \cdot Q$, *where* $\lambda = \frac{\sqrt{\sigma_m^2 + \sigma_f^2 - \frac{2\sigma_m\sigma_f}{\exp(\kappa)}}}{2 \cdot \sigma_u}$. *The attention allocation results are given by* $\sigma_{\epsilon_m}^2 = \frac{\sigma_m^2}{\frac{\exp(\kappa)\cdot\sigma_m}{\sigma_f} - 1}, \sigma_{\epsilon_f}^2 = \frac{\sigma_f^2}{\frac{\exp(\kappa)\cdot\sigma_f}{\sigma_m} - 1}$. *The linear trading strategy is given by* $q(s) = \beta_m (s_m - \mu_m) + \beta_f (s_f - \mu_f)$, *where* $\beta_m = \frac{\sigma_u}{\sqrt{\sigma_m^2 + \sigma_f^2 - \frac{2\sigma_m\sigma_f}{\exp(\kappa)}}} \cdot \left(1 - \frac{\sigma_f}{\exp(\kappa)\sigma_m}\right)$, *and* $\beta_f = \frac{\sigma_u}{\sqrt{\sigma_m^2 + \sigma_f^2 - \frac{2\sigma_m\sigma_f}{\exp(\kappa)}}} \cdot \left(1 - \frac{\sigma_m}{\exp(\kappa)\sigma_f}\right)$.

*Case 2.* $\kappa < \ln\left(\frac{\sigma_f}{\sigma_m}\right)$. *The equilibrium asset price is given by* $P(Q) = \mu_v + \lambda \cdot Q$ *where* $\lambda = \frac{\sigma_f}{2\cdot\sigma_u}\sqrt{\frac{\exp(2\kappa)-1}{\exp(2\kappa)}}$. *The attention allocation results are* $\sigma_{\epsilon_m}^2 = \infty$ *and* $\sigma_{\epsilon_f}^2 = \frac{\sigma_f^2}{\exp(2\kappa)-1}$. *The linear trading strategy is given by* $q(s) = \beta_m (s_m - \mu_m) + \beta_f (s_f - \mu_f)$, *where* $\beta_m = \frac{1}{2\lambda} \cdot \frac{\sigma_m^2}{\sigma_m^2 + \sigma_{\epsilon_m}^2} = 0$ *and* $\beta_f = \frac{1}{2\lambda} \cdot \frac{\sigma_f^2}{\sigma_f^2 + \sigma_{\epsilon_f}^2} = \frac{\sigma_u}{\sigma_f} \cdot \sqrt{\frac{\exp(2\kappa)-1}{\exp(2\kappa)}}$.

*Proof.* Combining the trading strategy and the pricing rule derived in the trading stage with the attention allocation result (See Lemma 3.3) in the information choice stage yields the above proposition. □

This result is symmetric: if $\sigma_m > \sigma_f$, the subscripts $f$ and $m$ should be swapped. The above proposition tells us that when firm-level news is more volatile than macro-level news, a limited-attention investor allocates more attention to firm-level news. In the extreme case of severely limited attention, the investor entirely ignores macro-level news, focusing solely on firm-level news. Conversely, during periods such as recessions, when macro-level news becomes more volatile than firm-specific news (i.e., $\sigma_m > \sigma_f$), the model predicts a shift in investor attention toward the more volatile macro-level news. These findings align with empirical observations and provide a micro-foundation for the crowding-out effect of investor attention between macro- and firm-level information, as noted in Liu et al. (2023).

The key intuition is that the lower an investor's attention level, the more likely she is to experience information overload. As a result, the investor focuses on extracting information from the more valuable signal to save attention. In the case of two signals and scarce attention, the investor concentrates all attention on the one with higher prior uncertainty, ignoring the less important signal.

We first check the limit case in which the investor has unrestricted attention capacity.

**Corollary 3.1.** *As $\kappa \to \infty$, we have that*

$$\sigma_{\epsilon_m}^2 = \frac{\sigma_m^2}{\exp(\kappa) \cdot \left(\frac{\sigma_m}{\sigma_f}\right) - 1} \to 0, \quad \sigma_{\epsilon_f}^2 = \frac{\sigma_f^2}{\exp(\kappa) \cdot \left(\frac{\sigma_f}{\sigma_m}\right) - 1} \to 0,$$

$$\beta_m = \frac{1}{2\lambda} \cdot \frac{\sigma_m^2}{\sigma_m^2 + \sigma_{\epsilon_m}^2} \to \frac{1}{2\lambda}, \quad \beta_f = \frac{1}{2\lambda} \cdot \frac{\sigma_f^2}{\sigma_f^2 + \sigma_{\epsilon_f}^2} \to \frac{1}{2\lambda},$$

$$\lambda = \frac{\sqrt{\sigma_m^2 + \sigma_f^2 - \frac{2\sigma_m \sigma_f}{\exp(\kappa)}}}{2 \cdot \sigma_u} \to \frac{\sigma_v}{2\sigma_u},$$

*and*

$$\Pi = \frac{\sigma_u}{2} \cdot \sqrt{\frac{\sigma_m^4}{\sigma_m^2 + \sigma_{\epsilon_m}^2} + \frac{\sigma_f^4}{\sigma_f^2 + \sigma_{\epsilon_f}^2}} \to \frac{\sigma_u \cdot \sigma_v}{2}.$$

This result says that our model incorporates Kyle's model as a limiting case. As the investor's attention becomes sufficiently large ($\kappa \to \infty$), they can fully process both macro-level and firm-level news, eliminating signal uncertainty ($\sigma_{\epsilon_m}^2 \to 0$, $\sigma_{\epsilon_f}^2 \to 0$). Consequently, trading aggression ($\beta_m$ and $\beta_f$), market depth ($\frac{1}{\lambda}$), and the investor's ex-ante profits ($\Pi$) converge to those in the Kyle model.

We then analyze the effect of limited attention and attention allocation on key market characteristics such as price volatility, market depth, and the bid-ask spread. We assume that $\sigma_f > \sigma_m$ in the following analysis.

**Price Volatility.** Price volatility, defined as the variance of price, is related to attention allocation. Asset price volatility is given by

$$Var(P) = \begin{cases} \frac{1}{2}\left(\sigma_m^2 + \sigma_f^2 - \frac{2\sigma_m\sigma_f}{\exp(\kappa)}\right), & \text{if } \kappa > \ln\left(\frac{\sigma_f}{\sigma_m}\right), \\ \frac{1}{2}\sigma_f^2\left(1 - \frac{1}{\exp(2\kappa)}\right), & \text{if } \kappa < \ln\left(\frac{\sigma_f}{\sigma_m}\right). \end{cases}$$

It holds that

$$\frac{\partial Var(P)}{\partial \kappa} > 0, \quad \frac{\partial Var(P)}{\partial \sigma_f} > \frac{\partial Var(P)}{\partial \sigma_m}.$$

This result shows that price volatility comoves with investor attention level, consistent with the findings in Aouadi et al. (2013) and Andrei and Hasler (2015). Moreover, during earnings announcements period when firm-level news is more volatile (i.e., $\sigma_f > \sigma_m$), price volatility responds more strongly to firm-level news. During periods when macro-level uncertainty is higher than firm-level uncertainty (i.e., $\sigma_m > \sigma_f$), price fluctuations respond more to the macro news, i.e., $\frac{\partial \mathrm{Var}(P)}{\partial \sigma_m} > \frac{\partial \mathrm{Var}(P)}{\partial \sigma_f}$. This is consistent with the empirical findings in Liu et al. (2023).

**Market Depth.** The sensitivity of the price $P$ to the order size $Q$ is represented by $\lambda$, and its inverse, $\frac{1}{\lambda}$, is called market depth which measures market liquidity. It is given by

$$\frac{1}{\lambda} = \begin{cases} \frac{2\sigma_u}{\sqrt{\sigma_m^2 + \sigma_f^2 - \frac{2\sigma_m\sigma_f}{\exp(\kappa)}}}, & \text{if } \kappa > \ln\left(\frac{\sigma_f}{\sigma_m}\right), \\ \frac{2\cdot\sigma_u}{\sigma_f}\sqrt{\frac{\exp(2\kappa)}{\exp(2\kappa)-1}}, & \text{if } \kappa < \ln\left(\frac{\sigma_f}{\sigma_m}\right). \end{cases}$$

It holds that

$$\frac{\partial(1/\lambda)}{\partial \kappa} < 0, \quad \frac{\partial(1/\lambda)}{\partial \sigma_f} > \frac{\partial(1/\lambda)}{\partial \sigma_m}.$$

Market depth represents the order flow required to move the price by one unit, as $\frac{1}{\lambda} = \frac{\partial Q}{\partial P}$. The above result first predicts that market depth decreases with the informed trader's increasing attention. A higher level of attention ($\kappa$) gives the informed trader a greater informational advantage over liquidity traders. As a result, informed trading constitutes a larger share of total trading volume. Anticipating this, market makers adopt a pricing rule that is more sensitive to trading volume in order to offset the increased adverse selection cost. Consequently, a smaller quantity of trade causes a one-unit change in price, implying a reduction in market depth. A second intuitive prediction is that when firm-level news exhibits greater volatility than macro-level news, market depth responds more strongly to firm-level information, and vice versa.

**Bid-Ask Spread.** The bid price is given by $B(Q) = P(Q) - \mu_v = \lambda \cdot Q$ for $Q > 0$, and the ask price is given by $A(Q) = \mu_v - P(Q) = \lambda \cdot |Q|$ for $Q < 0$. The bid-ask spread $S(Q)$ is thus given by

$$S(Q) = \frac{1}{2}\left(B(Q) - A(Q)\right) = \lambda \cdot Q = \begin{cases} \frac{\sqrt{\sigma_m^2 + \sigma_f^2 - \frac{2\sigma_m\sigma_f}{\exp(\kappa)}}}{2\sigma_u} \cdot Q, & \text{if } \kappa > \ln\left(\frac{\sigma_f}{\sigma_m}\right), \\ \frac{\sigma_f}{2\cdot\sigma_u}\sqrt{\frac{\exp(2\kappa)-1}{\exp(2\kappa)}} \cdot Q, & \text{if } \kappa < \ln\left(\frac{\sigma_f}{\sigma_m}\right). \end{cases}$$

It holds that

$$\frac{\partial S(Q)}{\partial \kappa} > 0, \qquad \frac{\partial S(Q)}{\partial \sigma_f} > \frac{\partial S(Q)}{\partial \sigma_m}.$$

The above result implies that the bid-ask spread comoves with the informed trader's level of attention. A higher attention capacity leads to a larger share of informed trading, holding the total order flow $Q$ constant. As a result, market makers charge a higher price to those willing to buy, or a lower price to those willing to sell, in order to recoup expected losses from trading against better-informed investors. This leads to a wider spread. Moreover, the spread responds more strongly to news with higher volatility.

## 3.4 Concluding Remarks

We study the impact of investors' endogenous attention allocation on their trading behavior and market characteristics. Our model shows that investor attention does influence trading and market characteristics. Furthermore, during the earnings announcement period, when firm-level news is more volatile, investors allocate more attention to it, while less volatile macro news may be ignored due to limited attention. Conversely, when aggregate uncertainty is higher than that of firm-level news, attention shifts accordingly from firm-level news to macro-level news. These shifts cause price fluctuations, market depth, and the bid-ask spread to be more responsive to information sources with higher prior uncertainty. Several model predictions align with recent empirical findings, and our framework offers a microfoundation to interpret such patterns. In addition, some predictions lend themselves to empirical verification using microstructure data.

One interesting extension is the characterization of equilibrium in a multi-period trading model. In the standard $N$-period model without limited attention, the asset's value is fixed and known to the informed investor across all trading periods. The linear recursive equilibrium features a trading strategy in each period that depends on the history of pricing rules. However, in our setting with limited attention, the informed investor receives private signals prior to each round of trading. As a result, her trading strategy may depend not only on the price history but also on anticipated future signals. This dynamic information structure poses challenges for equilibrium characterization and is left for future research.

## 3.5 Appendices to Chapter 3

### 3.5.1 Proof of Lemma 3.1

*Proof.* By the projection theorem, the conditional value on the macro-level signal takes the form

$$E[v_m|s_m] = E[v_m] + \rho_{v_m,s_m}\frac{\sigma_m}{\sigma_{s_m}}(s_m - E[s_m]),$$

where $\rho_{v_m,s_m}$ is the correlation between $v_m$ and $s_m$, and the conditional variance is given by

$$Var[v_m|s_m] = (1 - \rho^2_{v_m,s_m}) \cdot Var[v_m].$$

Given $Var[s_m] = \sigma^2_m + \sigma^2_{\epsilon_m}$ and $\rho_{v_m,s_m} = \frac{Cov(v_m,s_m)}{\sqrt{Var(v_m)Var(s_m)}} = \frac{\sigma^2_m}{\sigma_m(\sigma^2_m+\sigma^2_{\epsilon_m})}$, the conditional value is reduced to

$$E[v_m|s_m] = \mu_m + \frac{\sigma^2_m}{\sigma^2_m + \sigma^2_{\epsilon_m}}(s_m - \mu_m),$$

and the conditional variance is reduced to

$$Var[v_m|s_m] = \sigma^2_m \cdot \frac{\sigma^2_{\epsilon_m}}{\sigma^2_m + \sigma^2_{\epsilon_m}}.$$

Similarly, the conditional value on firm-specific news is given by

$$E[v_f|s_f] = \mu_f + \frac{\sigma^2_f}{\sigma^2_f + \sigma^2_{\epsilon_f}}(s_f - \mu_f),$$

and the conditional variance is

$$Var[v_f|s_f] = \sigma^2_f \cdot \frac{\sigma^2_{\epsilon_f}}{\sigma^2_f + \sigma^2_{\epsilon_f}}.$$

Thus, the corresponding conditional asset value on $s$ is given by

$$\begin{aligned} E[v|s] &= E[v_m|s_m] + E[v_f|s_f] \\ &= \mu_m + \frac{\sigma^2_m}{\sigma^2_m + \sigma^2_{\epsilon_m}}(s_m - \mu_m) + \mu_f + \frac{\sigma^2_f}{\sigma^2_f + \sigma^2_{\epsilon_f}}(s_f - \mu_f) \\ &= \mu_v + \frac{\sigma^2_m}{\sigma^2_m + \sigma^2_{\epsilon_m}}(s_m - \mu_m) + \frac{\sigma^2_f}{\sigma^2_f + \sigma^2_{\epsilon_f}}(s_f - \mu_f). \end{aligned}$$

The first equality holds due to the independence assumption, and the last equality holds as $\mu_v = \mu_m + \mu_f$ holds by definition.

The conditional variance of the asset value given the signals is given by

$$\begin{aligned} Var[v|s] &= Var[v_m|s_m] + Var[v_f|s_f] \\ &= \sigma_m^2 \cdot \frac{\sigma_{\epsilon_m}^2}{\sigma_m^2 + \sigma_{\epsilon_m}^2} + \sigma_f^2 \cdot \frac{\sigma_{\epsilon_f}^2}{\sigma_f^2 + \sigma_{\epsilon_f}^2}. \end{aligned}$$

The first equality holds due to the independence of the signals. This completes the proof. □

### 3.5.2 Proof of Lemma 3.2

*Proof.* The ex-ante payoff of the informed investor is given by

$$\Pi = E_s \left[ \frac{1}{4\lambda} \cdot (E[v|s] - \mu_v)^2 \right],$$

where the coefficient is

$$\lambda = \frac{\sqrt{\frac{\sigma_m^4}{\sigma_m^2 + \sigma_{\epsilon_m}^2} + \frac{\sigma_f^4}{\sigma_f^2 + \sigma_{\epsilon_f}^2}}}{2\sigma_u}.$$

For notational ease, we denote the normalized de-meaned value as $\bar{v} = v - \mu_v$. The unconditional payoff reads

$$\begin{aligned} E_s \left[ \frac{1}{4\lambda} \cdot (E[v|s] - \mu_v)^2 \right] &= E_s \left[ \frac{\sigma_u}{2\sqrt{\frac{\sigma_m^4}{\sigma_m^2 + \sigma_{\epsilon_m}^2} + \frac{\sigma_f^4}{\sigma_f^2 + \sigma_{\epsilon_f}^2}}} \cdot (E[\bar{v}|s])^2 \right] \\ &= \frac{\sigma_u}{2\sqrt{\frac{\sigma_m^4}{\sigma_m^2 + \sigma_{\epsilon_m}^2} + \frac{\sigma_f^4}{\sigma_f^2 + \sigma_{\epsilon_f}^2}}} E_s \left[ (E[\bar{v}|s])^2 \right]. \end{aligned}$$

Note that

$$\begin{aligned} E_s \left[ (E[\bar{v}|s])^2 \right] &= E_s \left[ E[\bar{v}^2|s] - Var[\bar{v}|s] \right] \\ &= Var[\bar{v}] - Var[\bar{v}|s] \\ &= Var[v] - Var[v|s]. \end{aligned}$$

The first equality follows from the law of total variance. The second holds because the conditional variance $Var[\bar{v}|s]$ (or $Var[v|s]$) is a constant and irrelevant for observation $s$ (See Lemma 3.1). The third equality holds because the prior mean $\mu_v$ is a constant. Hence, the unconditional payoff is reduced to

$$\Pi = \frac{1}{4\lambda} \cdot (Var[v] - Var[v|s]).$$

Substituting $Var[v] = \sigma_m^2 + \sigma_f^2$, the conditional variance $Var[v|s] = \sigma_m^2 \cdot \frac{\sigma_{\epsilon_m}^2}{\sigma_m^2 + \sigma_{\epsilon_m}^2} + \sigma_f^2 \cdot \frac{\sigma_{\epsilon_f}^2}{\sigma_f^2 + \sigma_{\epsilon_f}^2}$ from Lemma 3.1, and the expression (2.12) for $\lambda$ into the above expression for $\Pi$ and simplifying terms gives

$$\Pi = \frac{\sigma_u}{2} \sqrt{\frac{\sigma_m^4}{\sigma_m^2 + \sigma_{\epsilon_m}^2} + \frac{\sigma_f^4}{\sigma_f^2 + \sigma_{\epsilon_f}^2}},$$

which is clearly decreasing in $\sigma_{\epsilon_m}^2$ and $\sigma_{\epsilon_f}^2$. This completes the proof.

□

### 3.5.3 Proof of Lemma 3.3

*Proof.* Since the objective payoff function is monotone decreasing in $\sigma_{\epsilon_m}^2$ and $\sigma_{\epsilon_f}^2$, the information flow constraint must be binding. Otherwise, one could reduce $\sigma_{\epsilon_m}^2$ or $\sigma_{\epsilon_f}^2$ by a small amount to improve the payoff while keeping the constraint satisfied. The binding constraint reads

$$\left(1 + \frac{\sigma_m^2}{\sigma_{\epsilon_m}^2}\right) \cdot \left(1 + \frac{\sigma_f^2}{\sigma_{\epsilon_f}^2}\right) = \exp(2\kappa).$$

To simplify notation, denote $M = 1 + \frac{\sigma_m^2}{\sigma_{\epsilon_m}^2} \geq 1$ and $I = 1 + \frac{\sigma_f^2}{\sigma_{\epsilon_f}^2} \geq 1$. Then, the information flow constraint reads $M \cdot I = \exp(2\kappa)$ and the objective reads

$$\begin{aligned} \frac{\sigma_u}{2} \sqrt{\frac{\sigma_m^4}{\sigma_m^2 + \sigma_{\epsilon_m}^2} + \frac{\sigma_f^4}{\sigma_f^2 + \sigma_{\epsilon_f}^2}} &= \frac{\sigma_u}{2} \sqrt{\frac{M-1}{M}\sigma_m^2 + \frac{I-1}{I}\sigma_f^2} \\ &= \frac{\sigma_u}{2} \left(\sigma_m^2 + \sigma_f^2 - \frac{1}{M}\sigma_m^2 - \frac{1}{I}\sigma_f^2\right). \end{aligned}$$

Therefore, the optimization problem transforms to choosing $M$ and $I$ to minimize $\frac{1}{M}\sigma_m^2 + \frac{1}{I}\sigma_f^2$ subject to $M \cdot I = \exp(2\kappa)$, $M \geq 1$ and $I \geq 1$. We substitute $I = \frac{\exp(2\kappa)}{M}$ into the objective $\frac{1}{M}\sigma_m^2 + \frac{M}{\exp(2\kappa)}\sigma_f^2$ to reduce dimensionality, which yields

$$\frac{1}{M}\sigma_m^2 + \frac{M}{\exp(2K)}\sigma_f^2 := g(M).$$

Note that $g(M)$ is a convex function (with a U shape) for $M > 0$ and it is minimized at $M_{\text{cutoff}} = \exp(\kappa) \cdot \frac{\sigma_m}{\sigma_f}$. Since $\sigma_f > \sigma_m$ and $\kappa > 0$, $M_{\text{cutoff}}$ can be greater than or less than 1. We need to discuss two cases.

Case 1: If $\kappa > \ln\left(\frac{\sigma_f}{\sigma_m}\right)$, the solution is interior:

$$M^* = \exp(\kappa) \cdot \frac{\sigma_m}{\sigma_f}, \quad I^* = \exp(\kappa) \cdot \frac{\sigma_f}{\sigma_m},$$

or equivalently,

$$\sigma_{\epsilon_m}^2 = \sigma_m^2 \left(\exp(\kappa) \cdot \frac{\sigma_m}{\sigma_f} - 1\right), \quad \sigma_{\epsilon_f}^2 = \sigma_f^2 \left(\exp(\kappa) \cdot \frac{\sigma_f}{\sigma_m} - 1\right).$$

Case 2: If $\kappa \leq \ln\left(\frac{\sigma_f}{\sigma_m}\right)$, the solution is at the corner:

$$M^* = 1, \quad I^* = \exp(2\kappa),$$

or equivalently,

$$\sigma_{\epsilon_m}^2 = \infty, \quad \sigma_{\epsilon_f}^2 = \frac{\sigma_f^2}{\exp(2\kappa) - 1}.$$

This completes the proof.

$\square$

### 3.5.4 Derivation of Price Volatility

*Proof.* We first examine the case where $\kappa > \ln\left(\frac{\sigma_f}{\sigma_m}\right)$. The price volatility is given by

$$\begin{aligned}
\mathrm{Var}(P) &= \lambda^2 \mathrm{Var}(Q) \\
&= \lambda^2 \left(\mathrm{Var}(q_s) + \sigma_u^2\right) \\
&= \lambda^2 \left(\beta_m^2 \mathrm{Var}(s_m) + \beta_f^2 \mathrm{Var}(s_f) + \sigma_u^2\right) \\
&= \lambda^2 \left(\beta_m^2(\sigma_m^2 + \sigma_{\epsilon_m}^2) + \beta_f^2(\sigma_f^2 + \sigma_{\epsilon_f}^2) + \sigma_u^2\right).
\end{aligned}$$

To analyze the monotonicity of $\mathrm{Var}(P)$, we substitute the expressions for $\lambda$, $\beta_m$, $\beta_f$, $\sigma_{\epsilon_m}$, and $\sigma_{\epsilon_f}$ into $\mathrm{Var}(P)$ and simplify terms to get

$$\mathrm{Var}(P) = \frac{1}{2}\left(\sigma_m^2 + \sigma_f^2 - \frac{2\sigma_m\sigma_f}{\exp(\kappa)}\right).$$

The derivative with respect to $\kappa$ is given by

$$\frac{\partial \mathrm{Var}(P)}{\partial \kappa} = \frac{\sigma_m\sigma_f}{\exp(\kappa)} > 0,$$

and the sensitivity of price volatility to firm-level and macro-level news is given by

$$\frac{\partial \mathrm{Var}(P)}{\partial \sigma_f} = \sigma_f - \frac{\sigma_m}{\exp(\kappa)}, \quad \frac{\partial \mathrm{Var}(P)}{\partial \sigma_m} = \sigma_m - \frac{\sigma_f}{\exp(\kappa)}.$$

To show that

$$\frac{\partial \mathrm{Var}(P)}{\partial \sigma_f} > \frac{\partial \mathrm{Var}(P)}{\partial \sigma_m},$$

note that this is equivalent to

$$\sigma_f - \frac{\sigma_m}{\exp(\kappa)} > \sigma_m - \frac{\sigma_f}{\exp(\kappa)}.$$

This inequality holds under the assumption that $\sigma_f > \sigma_m$. For the case where

$\kappa < \ln\left(\frac{\sigma_f}{\sigma_m}\right)$, the derivative with respect to $\kappa$ is

$$\frac{\partial \text{Var}(P)}{\partial \kappa} = \frac{\sigma_f^2}{\exp(2\kappa)} > 0.$$

This completes the proof.

$\square$

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# Author’s Declaration

**Eidesstattliche Versicherung gemäß §8 Absatz 2 Buchstabe a) der Promotionsordnung der Universität Mannheim zur Erlangung des Doktorgrades der Wirtschaftswissenschaften (Dr. rer. pol.)**

1. Bei der eingereichten Dissertation mit dem Titel „Essays in Financial Markets and Market Design“ handelt es sich um mein eigenständig erstelltes Werk, das den Regeln guter wissenschaftlicher Praxis entspricht.

2. Ich habe nur die angegebenen Quellen und Hilfsmittel benutzt und mich keiner unzulässigen Hilfe Dritter bedient. Insbesondere habe ich wörtliche und nicht wörtliche Zitate aus anderen Werken als solche kenntlich gemacht.

3. Die Arbeit oder Teile davon habe ich bislang nicht an einer Hochschule des In- oder Auslandes als Bestandteil einer Prüfungs- oder Qualifikationsleistung vorgelegt.

4. Die Richtigkeit der vorstehenden Erklärung bestätige ich.

5. Die Bedeutung der eidesstattlichen Versicherung und die strafrechtlichen Folgen einer unrichtigen oder unvollständigen eidesstattlichen Versicherung sind mir bekannt. Ich versichere an Eides statt, dass ich nach bestem Wissen die reine Wahrheit erklärt und nichts verschwiegen habe.

Die eingereichten Dissertationsexemplare sowie der Datenträger gehen in das Eigentum der Universität über.

Mannheim, 2025

Chang Liu

# Curriculum Vitae

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| 2019–2025 | University of Mannheim (Germany)<br>*Ph.D. in Economics* |
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