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RESEARCH ARTICLE

Biased recommendations and differentially informed consumers

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Abstract
We consider a monopolist selling a product to differentially informed consumers: some consumers are uncertain about their tastes, whereas other consumers are perfectly informed. The monopolist sets a uniform price and can make personalized product recommendations. We characterize conditions under which the monopolist *biases* its recommendations—that is, some consumers with values below the marginal cost follow the recommendation to buy the product or some consumers with values above the marginal cost follow the recommendation not to buy the product.

Keywords Information design · Biased recommendations · Recommender system · Information disclosure

JEL Classification L12 · L15 · D21 · D42 · M37

1 Introduction

When consumers buy certain types of products irregularly, they may lack information that determines how much they value the product. With no or little information available to them, these consumers may base their purchase decision on whether the firm selling the product (or facilitating the sales as an intermediary) recommends it. With advances in data collection and data analytics, firms can often infer values with high precision and make personalized purchase recommendations. For example, e-commerce retailers (and platforms) collect a wealth of information about their customers and frequently make algorithmic purchase recommendations. In this article, we point to the importance that consumers are often heterogeneous with respect to the precision of this ex ante information. For instance, this holds in markets in which

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(i) consumers have heterogeneous skills that help them in assessing the expected value prior to use, (ii) consumers have differential previous exposure to related products, or (iii) consumers arrive exogenously over time at a shop or website and late arrivals obtain information on their value through word-of-mouth. This has important implications for the firms' profit-maximizing recommendation strategies.

We analyze information disclosure and price setting by a monopoly seller under uniform pricing.[1] There are two ex-ante consumer types: some consumers perfectly observe how much they value the product, while the others are uninformed and only know the prior distribution from which the value is drawn. We assume that the monopoly seller can make recommendations based on the true value of each consumer. Because of ex-ante heterogeneity, the monopolist may want to recommend its product to consumers whose valuation is less than the marginal cost. We establish the conditions under which such inflated recommendations are part of the profit-maximizing monopoly strategy. We also provide alternative conditions under which recommendations are socially insufficient—that is, some consumers do not receive the recommendation to buy even though their valuation is strictly larger than the marginal cost.

Related literature Our analysis of ex-ante heterogeneous information complements the analysis of ex-ante heterogeneous tastes in Peitz and Sobolev (2025)—for more details, see the discussion at the end of Sect. 2.[2] In this article, the firm's choice of inflated versus insufficient recommendations depends on the shape of the virtual value function; our Theorem 1 builds on a result by Ivanov (2009), as we explain in the main text. More broadly, this paper belongs to the literature on Bayesian persuasion initiated by Kamenica and Gentzkow (2011)—for recent surveys, see Bagnoli and Morris (2019) and Kamenica (2019). Lewis and Sappington (1994) consider the edge cases in which either all consumers receive a fully informative or a completely uninformative signal. He shows that a monopolist does not have an incentive to bias its recommendations. Rayo and Segal (2010, Section VIII.C) consider a seller who discloses information and sets the product price in a setting where a consumer draws their value. They show that the profit-maximizing disclosure rule is fully revealing. By contrast, we show that a monopoly seller maximizes its profit by partially informing consumers with an uninformative signal and providing biased recommendations.

[1] There are several reasons for which a monopolist may decide not to or not be able to engage in discriminatory pricing. First, discriminatory pricing opens the room for consumer arbitrage. Second, discriminatory pricing may trigger the intervention by a consumer protection agency, a sector regulator, or the legislator. Third, the monopolist may not have information on the consumer's type and value when setting its price. Another reason could be consumer backlash, where consumers stop buying from the firm if they discover discriminatory pricing. It has been reported in the business press that "all the way back in 2000, when Amazon was mostly an online book and media store, it experimented with charging different prices to individual customers for the same DVDs. The customer response was so swift and negative that, nearly 20 years later, the e-tailer still avoids the practice." Ben Unglesbee, 'Why dynamic and personalized pricing strategies haven't taken over retail—yet', Retail Dive, 22 July 2019, available at: https://www.retaildive.com/news/why-dynamic-and-personalized-pricing-havent-taken-over-retail-yet/558975/lastaccessed17April2024.

[2] As mentioned in the conclusion, our results also to speak to the literature on biased intermediaries as we could restrict the firm to set its price and introduce an intermediary making recommendations (following Peitz and Sobolev (2025)). Other work on biased intermediaries includes (Armstrong and Zhou 2011; Hagiu and Jullien 2011), and de Cornière and Taylor (2019).

In our analysis, the monopolist is restricted to set a uniform price. If the monopolist were able to segment consumers and price discriminate, it would not have an incentive to bias its recommendations. Thus, our work complements the work on information design under price discrimination (Bergemann et al. 2015). In a setting with a single buyer and a single seller, Roesler and Szentes (2017) and Kartik and Zhong (2024) characterize the possible allocations under monopoly pricing for any information structure focusing on situations in which trade is always efficient. In their online appendix, Roesler and Szentes (2017) analyze the buyer-optimal selling mechanism when, with positive probability, the production cost is larger than the consumer's value and, thus, trade can be inefficient. They show that trade can occur with positive probability even when there are negative gains from trade, whereas trade always occurs when there are positive gains from trade. However, the seller-optimal selling mechanism always features efficient trade, in contrast to our setting.

Our model is presented in Sect. 2. In Sect. 3, we provide conditions, under which the seller inflates recommendations or provides insufficient recommendations. Section 4 concludes.

2 Model and preliminaries

2.1 Model

Consider a monopoly seller offering a single good. There is a unit mass of consumers with heterogeneous values distributed according to the cumulative distribution function $F(v)$ with support $[\underline{v}, \overline{v}]$, where $0 \leq \underline{v} < \overline{v}$ with $\overline{v} = \infty$ or finite. The function $F(v)$ is continuous on $\mathbb{R}_+$, has a continuous strictly positive density $f(v)$ on $(\underline{v}, \overline{v})$, and a finite mean $\mathbb{E}[v] < \infty$. We assume that the constant marginal cost of production satisfies $c \in (\underline{v}, \overline{v})$.[3]

A fraction $\alpha \in [0, 1]$ of consumers are uninformed about how much they value the product and believe that values are distributed according to $F(v)$. As consumers buy only once, it is immaterial whether consumers observe their value after purchase. Thus, we do not need to take a stance on whether the product is an experience good or a credence good for uninformed consumers. All other consumers are informed and know their value before purchase.

The seller sets a uniform price and, in addition, can reveal information to uninformed consumers about their values by providing personalized product recommendations. Uniform pricing may be due to a regulatory requirement or due to other reasons, e.g., free arbitrage.[4]

Let M be a set of messages that contains at least two messages. Then, the product recommendation policy μ consists of sending a random message $m \in M$ that depends on a consumer's value $v \in [\underline{v}, \overline{v}]$.

[3] In this setting, a profit-maximizing price exists and is less than $\overline{v}$ (see van den Berg 2007, Proposition 1).

[4] See also footnote 1. In the case of free arbitrage, we would need to assume that ex-ante uninformed consumers learn their value after consumption.

We start by characterizing the profit-maximizing recommendation policy and price of the seller when all consumers are either fully informed ($\alpha = 0$) or uninformed ($\alpha = 1$).

2.2 Edge cases

First, suppose that $\alpha = 0$ and, thus, all consumers are informed. The profit of the seller setting price p is $\pi^i(p) \equiv (p-c)(1-F(p))$. We assume that $\pi^i(p)$ is strictly quasi-concave. It thus has a unique maximizer denoted by p^i. The first-order condition is given by

$$\frac{d\pi^i}{dp} = \left(\frac{1-F(p)}{f(p)} - (p-c)\right) f(p) = 0,$$

implying that the monopoly price p^i lies within the interval $(c, \overline{v})$ and solves

$$p^i = c + \frac{1-F(p^i)}{f(p^i)}.$$

Second, suppose that $\alpha = 1$ and, thus, all consumers are uninformed about their value. The total surplus is maximal when all consumers with $v \geq c$ buy the good and equals $\int_c^{\overline{v}} (v-c)dF(v)$. Note that the seller can capture the maximal total surplus by revealing to each consumer whether their value v is weakly greater than c and setting price

$$p^u \equiv \mathbb{E}[v|v \geq c],$$

which is finite as follows from $\mathbb{E}[v] < \infty$. As the expected consumer surplus is zero, this strategy maximizes the seller's profit $\pi^u(p)$ (which is defined in Lemma 2 in the Appendix). The following remark shows that in every optimal recommendation policy, the seller recommends buying the product if and only if $v \geq c$ (pools all the types above marginal cost) and sets price p^u.

Remark 1 Suppose that all consumers are uninformed ($\alpha = 1$). The seller sets price $p^u = \mathbb{E}[v|v \geq c]$ and recommends buying the product if and only if $v \geq c$. Consumers follow the seller's recommendations. The equilibrium profit of the seller is given by

$$(1 - F(c))\mathbb{E}[v - c|v \geq c].$$

If all consumers are uninformed about their value, the profit-maximizing pricing and recommendation policy does not feature biased recommendations—this result has already been obtained by Saak (2006). Since only consumers with $v \geq c$ receive the recommendation to buy the product, the recommendation policy of the seller is efficient from the perspective of the social planner maximizing total welfare.

In Fig. 1 we illustrate the shape of the profit functions $\pi^i(\cdot)$ and $\pi^u(\cdot)$ in the case of

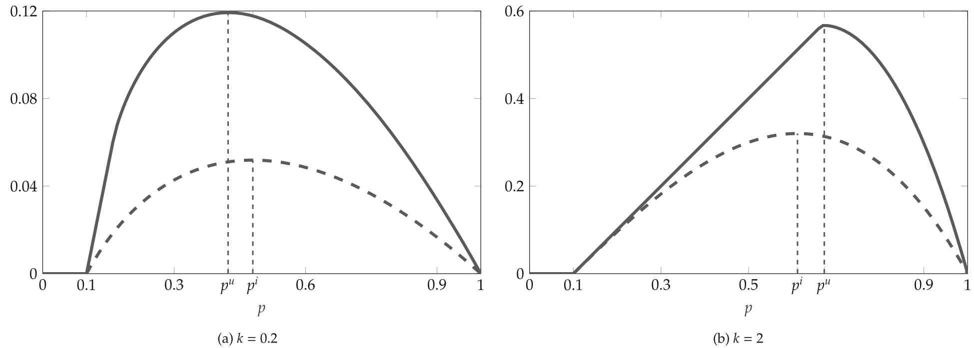

Fig. 1 Profits $\pi^u(p)$ (solid) and $\pi^i(p)$ (dashed) in p for $k \in \{0.2, 2\}$ and $c = 0.1$ if F is a power distribution function

power distributions $F(v) = v^k$, which is further developed in Example 1 below. For a low k, we observe that $p^u < p^i$, whereas for a high k, the ordering is reversed and $p^u > p^i$. The ordering of these two prices will turn out to be crucial for our findings in the following section.

2.3 Discussion

We obtain our results under the assumption that $\pi^i(p)$ is strictly quasi-concave. This property is implied by Marshall's second law of demand—that is, the absolute value of the price elasticity of demand, $|\varepsilon(p)| \equiv pf(p)/(1 - F(p))$, is non-decreasing in price for any $p \in [\underline{v}, \overline{v})$.[5]

We also note that for any density function $f(v)$ that is continuously differentiable and log-concave on the open interval $(\underline{v}, \overline{v})$, demand $1 - F(v)$ is also log-concave on $(\underline{v}, \overline{v})$ (Bagnoli and Bergstrom 2005, Theorem 3) and log-concavity of $1 - F(v)$ implies Marshall's second law of demand.[6] This means that demand functions with log-concave densities are a special case of demand functions satisfying Marshall's second law of demand.

In our setting, a seller chooses the profit-maximizing recommendation policy—i.e., we analyze the seller-preferred information design. The personal recommendation is privately observed by each consumer. Since the seller sets a uniform pricing, it is immaterial whether the seller observes whether a consumer receives a recommendation. It is also immaterial whether the seller observes the consumer's value (as long as the consumer value is available to make recommendations). Hence, the seller may set the price before the input to the recommendation policy is obtained.

We note that uninformed consumers benefit if the seller obtains information on the consumers' value that is used to make recommendations. Hence, without any extra benefits, uninformed consumers are willing to grant permission to collect this information. For example, consumers may allow the seller to set cookies or collect relevant user data through other means and use these data to make purchase recommendations.

Peitz and Sobolev (2025) consider the profit-maximizing recommendation policy when consumers have ex ante heterogeneous tastes: Consumers draw their value from a type-dependent distribution, whereas in the present model, all consumers draw their value from the same distribution. In their base model, consumers are either "picky"—i.e., they have either a low or a high value—or are "flexible"—i.e., they have an intermediate value for sure—which is assumed to be higher than the expected value of picky consumers. None of the models covered in Peitz and Sobolev (2025) can be translated into a model of ex ante heterogeneous information. However, we can reinterpret our present setting as one of ex ante heterogeneous consumers: There is a single type of picky consumers who know that their value is drawn from $F(\cdot)$ but

[5] Pigou (1920, chapter 5) is credited with being the first to explicitly write about this property of a demand function.

[6] Log-concavity of $1 - F(v)$ is equivalent to $h(v) \equiv f(v)/(1 - F(v))$ non-decreasing in v; that is $h'(v) \geq 0$. A non-decreasing price elasticity of demand in absolute value, $|\varepsilon(p)| = ph(p)$, is equivalent to $h(p) + ph'(p) \geq$ and is therefore implied by $h'(p) \geq 0$.

do not observe it before purchase and a continuum of flexible consumer types who ex ante know their values $v \sim F(\cdot)$.

3 Biased recommendations

3.1 Main result

We analyze the model in which both consumer groups are present; that is, $\alpha \in (0, 1)$. We explore how the presence of informed consumers changes the seller's profit-maximizing recommendation policy to characterize the conditions under which the seller provides *(i) inflated recommendations*—some consumers with $v < c$ receive purchase recommendations; *(ii) insufficient recommendations*—some consumers with $v > c$ do not receive purchase recommendations; or *(iii) efficient recommendations*—consumers receive purchase recommendations if and only if $v \geq c$.

We define $p^* = p^*(\alpha)$ as a profit-maximizing price of the seller for $\alpha \in (0, 1)$.[7] For any price $p \in (c, \overline{v})$, the profit-maximizing recommendation policy prescribes recommending the good if and only if v is at least as large as the cutoff level $\hat{v}(p)$, which makes the incentive constraint of the uninformed consumers binding—that is, it satisfies $p = \mathbb{E}[v|v \geq \hat{v}(p)]$ (we prove this statement in the Appendix, see Lemma 1). Thus, if $p^* < p^u$, then the corresponding recommendation policy features $\hat{v}(p^*) < c$ and some consumers with $v < c$ are recommended to buy the good—that is, the seller induces *inflated recommendations*. If $p^* > p^u$, the seller provides insufficient recommendations. If instead $p^* = p^u$, the seller's recommendation policy is efficient.

The following theorem establishes that determining whether the seller's profit-maximizing price p^* is higher or lower than p^u, and consequently, whether the seller provides inflated or insufficient recommendations, reduces to comparing p^i with p^u.

Theorem 1 *Suppose that $\alpha \in (0, 1)$. The seller's profit-maximizing strategy entails*

- *inflated recommendations if and only if $p^u > p^i$,*
- *insufficient recommendations if and only if $p^u < p^i$,*
- *efficient recommendations if and only if $p^u = p^i$.*

We prove Theorem 1 by showing that if $p^u > p^i$, then the profit-maximizing price p^* satisfies $p^* < p^u$. If instead $p^u < p^i$, then $p^* > p^u$. For $p^i = p^u$, we have $p^* = p^u$ and the recommendations are efficient. The proof is relegated to the Appendix.

3.2 Virtual value function

Next, we provide a sufficient condition to determine the ranking of p^i and p^u. We define the virtual value function as

$$\psi(v) \equiv v - \frac{1 - F(v)}{f(v)} = v - \frac{1}{h(v)},$$

[7] We show in the proof of Theorem 1 that a profit-maximizing price p^* exists with $\min\{p^i, p^u\} \leq p^* \leq \max\{p^i, p^u\}$.

where $h(v) = f(v)/(1 - F(v))$ is the hazard rate function. The following proposition establishes that the shape of the virtual value function determines whether the monopolist provides inflated, efficient, or socially insufficient recommendations.

Proposition 1 *Suppose that* $\alpha \in (0, 1)$. *The seller's profit-maximizing strategy entails*

- *inflated recommendations if* $\psi(\cdot)$ *is strictly concave on* $[c, \overline{v}]$,
- *insufficient recommendations if* $\psi(\cdot)$ *is strictly convex on* $[c, \overline{v}]$,
- *efficient recommendations if* $\psi(\cdot)$ *is linear on* $[c, \overline{v}]$.

Proof The derivative of π^i can be rewritten as

$$\frac{d\pi^i}{dp} = \left(\frac{1 - F(p)}{f(p)} - (p - c)\right) f(p) = (c - \psi(p)) f(p).$$

The sign of the derivative of π^i at $p^u = \mathbb{E}[v|v \geq c]$ is determined by the sign of $c - \psi(p^u)$. Note that

$$\begin{aligned} c - \mathbb{E}[\psi(v)|v \geq c] &= \frac{1}{1 - F(c)} \int_c^{\overline{v}} (c - \psi(v)) f(v) dv \\ &= \frac{1}{1 - F(c)} \int_c^{\overline{v}} \frac{d\pi^i}{dv} dv \\ &= \frac{\pi^i(\overline{v}) - \pi^i(c)}{1 - F(c)} = 0. \end{aligned}$$

Therefore, if $\psi(\cdot)$ is strictly concave, by Jensen's inequality, we obtain that $c - \psi(p^u) < c - \mathbb{E}[\psi(v)|v \geq c] = 0$. This implies that the derivative of π^i is negative at $p = p^u$ and hence $p^u > p^i$ (by strict quasi-concavity of $\pi^i(\cdot)$). By Theorem 1, the seller inflates recommendations. Similarly, if $\psi(\cdot)$ is strictly convex, the seller induces insufficient recommendations. Finally, if $\psi(\cdot)$ is a linear function, then the seller provides efficient recommendations. □

Ivanov (2009) compares the monopoly price under full information p^i and the expected value $\mathbb{E}[v]$ for different distributions of v. Theorem 1 of Ivanov (2009) establishes a sufficient condition for $p^i > (<)\mathbb{E}[v]$ that depends on the convexity vs. concavity of the virtual value function and whether or not $\underline{v} > c$. In Proposition 1, we adapt this result to our problem and show that the convexity vs. concavity of the virtual value function (which is equivalent to the concavity vs. convexity of the inverse hazard rate function) is sufficient to determine the ranking of p^i and p^u.

For several families of distribution functions that satisfy Marshall's second law of demand (and thus imply that π^i is strictly quasi-concave in price), the virtual value function $\psi(\cdot)$ is either strictly convex, linear, or strictly concave. These include the power, symmetric Beta, and Weibull families of distribution functions.

Example 1 (Power distribution) Consider the power distribution, $F(v) = v^k$, for $k > 0$ with support [0, 1]. The absolute value of elasticity, $|\varepsilon(p)| = kp^k/(1 - p^k)$, is nondecreasing in p, implying that Marshall's second law of demand is satisfied for any

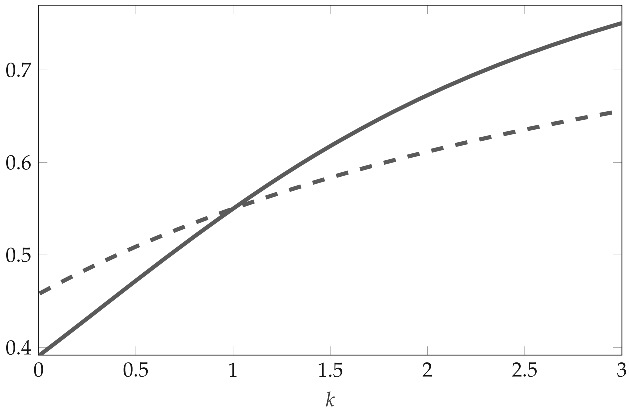

Fig. 2 Prices p^u (solid) and p^i (dashed) in k for $F(v) = v^k$, $k > 0$ and $c = 0.1$

$k > 0$. It follows that $\pi^i(p)$ is strictly quasi-concave. The virtual value function is given by $\psi(v) = v - \frac{1-v^k}{kv^{k-1}}$. The second derivative of $\psi(\cdot)$ is $\psi''(v) = (1-k)/v^{k+1}$. Therefore, if $k > 1$ and the distribution of v is strictly convex, then the virtual value function is strictly concave and the seller inflates recommendations. Otherwise, if $k \in (0, 1)$, then the seller provides insufficient recommendations. Figure 2 shows p^i and p^u as the functions of k. For $k > 1$, we have that $p^u > p^i$, and the seller inflates recommendations. Otherwise, if $k \in (0, 1)$, we have that $p^i > p^u$ and the seller provides insufficient recommendations. In the borderline case $k = 1$, recommendations are efficient.

Example 2 (Beta distribution) As another example, we consider symmetric Beta distributions with the density function given by $f(v) = v^{k-1}(1-v)^{k-1}/B(k, k)$ for $k > 0$ with support $[0, 1]$, where the beta function $B(k, k)$ is a normalization constant.

We show that the absolute value of the price elasticity of demand, which is given by

$$|\varepsilon(p)| = ph(p) = \frac{p^k(1-p)^{k-1}}{\int_p^\infty v^{k-1}(1-v)^{k-1}dv},$$

is non-decreasing in p. The derivative of the absolute value of elasticity is given by

$$|\varepsilon(p)|' = \left(k - (k-1)\frac{p}{1-p} + |\varepsilon(p)|\right)\frac{|\varepsilon(p)|}{p},$$

which is clearly non-negative for $k \in (0, 1]$. Suppose instead that $k > 1$ and note that $(\log f(p))'' = (1-k)\left(\frac{1}{p^2} + \frac{1}{(1-p)^2}\right) < 0$, meaning that $f(\cdot)$ is strictly log-concave. By the strict version of Bagnoli and Bergstrom (2005, Theorem 3) we have that $1 - F(\cdot)$ is also strictly log-concave. It follows that $\pi^i(p)$ is strictly quasi-concave for $k > 0$.

Next, we show that the virtual value function is strictly concave if $k > 1$, linear if $k = 1$, and strictly convex if $k \in (0, 1)$. Note that for $k > 1$, the first derivative of the density function, $f'(v) = (k-1)\frac{1-2v}{v(1-v)}f(k)$, is increasing for values of $v < 1/2$ and decreasing for $v > 1/2$, i.e., single-peaked. To the contrary, for $k \in (0,1)$, the derivative of the density is decreasing for values of $v < 1/2$ and increasing for $v > 1/2$, i.e., u-shaped. By Loertscher and Marx (2022, Proposition 6) a single-peaked density ($k > 1$) implies a strictly concave virtual value function and a u-shaped density implies a convex virtual value function ($k \in (0,1)$). Therefore, by Proposition 1, the seller inflates recommendations for $k > 1$ and provides insufficient recommendations for $k \in (0,1)$. For $k = 1$, the hazard rate function is constant (and, thus, $\psi(\cdot)$ is linear in v), implying that the seller provides efficient recommendations.

Example 3 (Weibull distribution) The Weibull distribution is given by the cumulative distribution function $F(v) = 1 - \exp\{-bv^k\}$ for $b > 0$, $k > 0$ with support $[0, \infty)$. Marshall's second law of demand is satisfied, as the absolute value of elasticity, $|\varepsilon(p)| = bkp^k$, is non-decreasing in p. This implies that $\pi^i(p)$ is strictly quasi-concave. The virtual value function is given by $\psi(v) = v - \frac{1}{bkv^{k-1}}$ and its second derivative is $\psi''(v) = (1-k)/v^{k+1}$. By Proposition 1, we have that the seller inflates recommendations for $k > 1$, provides efficient recommendations for $k = 1$, and provides insufficient recommendations for $k \in (0,1)$.

In our fourth example, we provide a family of cumulative distribution functions that induce the seller to make efficient recommendations.

Example 4 (ρ-linear demand) The class of the so-called ρ-linear demand functions (see Bulow and Pfleiderer, 1983) are derived from distribution functions

$$F(v) = 1 - M\left(1 + \frac{1}{\rho}(a - bv)\right)^{\rho},$$

with support $[\underline{v}, \overline{v}] \subset \mathbb{R}_+$ that depend on the parameters a, b, M and ρ, which all take positive values, such that $F(\underline{v}) = 0$ and $F(\overline{v}) = 1$ hold. Note that the cost pass-through rate is constant (and equals to $\alpha = 1/(1+\rho)$) if and only if the demand function $1 - F(p)$ is ρ-linear. The density function is given by $f(v) = \frac{b(1-F(v))}{1+\frac{1}{\rho}(a-bv)}$. The absolute value of elasticity is given by $|\varepsilon(p)| = \frac{bp}{1+\frac{1}{\rho}(a-bp)}$ and is non-decreasing in p. Moreover, it is straightforward to see that the virtual value function is linear. Proposition 1 then implies that the seller provides efficient recommendations. Hence, for ρ-linear demand functions, the seller's recommendation policy is efficient.

3.3 Cost pass-through rates

Provided that Marshall's second law of demand holds, we establish a connection between the concavity/convexity of the virtual value function and the behavior of the cost pass-through rate, $\frac{dp^i}{dc}$. Differentiating the first-order condition of the seller's profit maximization problem with respect to c when $\alpha = 0$, we obtain $\psi'(p^i)\frac{dp^i}{dc} = 1$. Since $\psi(p) = p(1 - 1/|\varepsilon(p)|)$, we have that, by Marshall's second law of demand,

$\psi'(p) \geq 0$, implying that $\frac{dp^i}{dc} > 0$. Moreover,

$$\frac{d^2 p^i}{dc^2} = -\frac{\psi''(p^i)\frac{dp^i}{dc}}{(\psi'(p^i))^2} = -\psi''(p^i)\left(\frac{dp^i}{dc}\right)^3.$$

We observe that the cost pass-through rate strictly increases (strictly decreases) if and only if the virtual value function is strictly concave (convex). This implies that the sufficient conditions of Proposition 1 can be rewritten in terms of the behavior of the cost pass-through rate.

Remark 2 Suppose that the cost pass-through rate dp^i/dc strictly increases in marginal cost c. Then, the seller induces inflated recommendations. If the cost pass-through rate strictly decreases in c, then the seller provides insufficient recommendations. If the cost pass-through rate is constant in c, then the seller provides efficient recommendations.

3.4 The special cases of high and low marginal costs

We provide approximation results for high $c \leq \overline{v} < \infty$ and low $c \geq \underline{v}$. This allows us to obtain conditions that do not rely on the virtual value function, under which the seller gives inflated or insufficient recommendations for high and low marginal cost, assuming that $\overline{v} < \infty$. To determine whether or not the seller inflates recommendations, we derive Taylor approximation results for $p^u - p^i$.

The case of high c. We explore the sign of $p^u - p^i$ in the neighborhood of $c = \overline{v}$, assuming that $f'(\overline{v})$ exists.

Remark 3 In the neighborhood of $c = \overline{v}$, the seller inflates recommendations if $f'(\overline{v}) > 0$ and provides insufficient recommendations if $f'(\overline{v}) < 0$.

The proof is relegated to the Appendix. For illustration, consider again the power distribution $F(v) = v^k$ (see Example 1). We have that $f'(\overline{v}) = k(k-1)\overline{v}^{k-2}$ is positive for $k > 1$, and the seller inflates recommendations. If instead $k \in (0, 1)$, $f'(\overline{v})$ is negative, and the seller provides insufficient recommendations.

The case of low c. We explore the sign of $p^u - p^i$ in the neighborhood of $c = \underline{v}$. The profit-maximizing price that the seller would set if all consumers were uninformed about their value tends to $\tilde{p}^u = \mathbb{E}[v]$ as $c \to \underline{v}$. The profit-maximizing price when all consumers are informed tends to $\tilde{p}^i$ as $c \to \underline{v}$, where $\tilde{p}^i$ solves $\tilde{p}^i - \underline{v} = \frac{1-F(\tilde{p}^i)}{f(\tilde{p}^i)}$. For symmetric $f(\cdot)$, the next remark provides a necessary and sufficient condition on $f(\cdot)$ under which $\tilde{p}^u - \tilde{p}^i > (<)0$. By Theorem 1, this condition guarantees that the seller provides inflated recommendations (insufficient recommendations) in the neighborhood of $c = \underline{v}$.

Remark 4 Suppose that f is symmetric, i.e., $f\left(\frac{\overline{v}+\underline{v}}{2} - x\right) = f\left(\frac{\overline{v}+\underline{v}}{2} + x\right)$, for all $x \in \left[0, \frac{\overline{v}-\underline{v}}{2}\right]$. Then, in the neighborhood of $c = \underline{v}$, the seller provides inflated recommendations (insufficient recommendations) if and only if $f\left(\frac{\overline{v}+\underline{v}}{2}\right) < (>)\frac{1}{\overline{v}-\underline{v}}$.

The proof is relegated to the Appendix.

4 Conclusion

In this article, we analyze a monopoly seller's price and recommendation policy where the monopolist cannot price discriminate. Consumers draw their values from the same distribution, but some consumers are perfectly informed about the realization, while others are uninformed. We provide conditions such that the monopolist maximizes its profits by inflating recommendations—that is, some consumers receive a purchase recommendation and buy even though the marginal cost is larger than how much they value the product—or making insufficient recommendations—that is, some consumers do not receive a purchase recommendation and do not buy even though the marginal cost is less than how much they value the product.

Our finding appears to be robust to a more general setting in which consumers receive informative but noisy signals about how much they value the product, as long as the level of noise is heterogeneous across consumers. Our finding also extends to a setting in which the firm setting the retail price is different from the firm making the purchase recommendation. For example, digital platforms may provide purchase recommendations and charge sellers for their intermediation service (that includes the recommendation service). In such a setting the intermediary biases its recommendations if it receives a fraction of the seller's profits (and, thus, the seller's and the intermediary's incentives are aligned).

Appendix

Lemma 1 *For any price $p \in (c, \overline{v})$, the firm maximizes its profit by recommending to buy the good if and only if v is greater than some cutoff level $\hat{v} = \hat{v}(p)$. Moreover,*

- *if $\mathbb{E}[v] > c$ and $p \in (c, \mathbb{E}[v]]$, then $\hat{v}(p) = \underline{v}$;*
- *if $p \in (\max\{c, \mathbb{E}[v]\}, \overline{v})$, then $\hat{v}(p)$ belongs to $(\underline{v}, \overline{v})$ and is chosen such that consumers are indifferent between buying and taking the outside option; it solves*

$$p = \mathbb{E}[v | v \geq \hat{v}(p)]. \tag{1}$$

Proof First, suppose that $\mathbb{E}[v] > c$ and $p \in (c, \mathbb{E}[v]]$. As $p < \mathbb{E}[v]$, the uninformed consumers buy even if the seller always recommends to buy and thus $\hat{v}(p) = \underline{v}$.

Second, suppose that $p \in (\max\{c, \mathbb{E}[v]\}, \overline{v})$. In this case, recommendations must be informative; otherwise, the uninformed consumers will not purchase at price p. Note that two messages are sufficient for the optimal recommendation strategy. Let m_1 and m_0 be the recommendation to buy and not to buy, respectively. Define $\mu(v)$ as the probability of sending message m_1 to a consumer with the value v. Then, after receiving m_1, the consumer's incentive compatibility constraint to buy at price p is given by

$$\frac{\int_{\underline{v}}^{\overline{v}} \mu(v) v dF(v)}{\int_{\underline{v}}^{\overline{v}} \mu(v) dF(v)} - p \geq 0 \iff \int_{\underline{v}}^{\overline{v}} \mu(v)(v - p) dF(v) \geq 0. \tag{2}$$

Then, the problem of the monopolist is to maximize $(p-c)\mathbb{E}[\mu(v)]$ subject to (2). The Lagrangian of that problem is:

$$\max_{\mu} \int_{\underline{v}}^{\overline{v}} \mu(v)(p-c+\lambda(v-p))dF(v),$$

where $\lambda > 0$ is the Lagrange multiplier. Therefore, for any $p \in (\max\{c, \mathbb{E}[v]\}, \overline{v})$, there is a cutoff level $\hat{v}(p) \in (\underline{v}, \overline{v})$ such that $\mu(v) = 1$ for $v \geq \hat{v}(p)$ and $\mu(v) = 0$ for $v < \hat{v}(p)$. □

Lemma 2 *For any price $p \in [c, \overline{v}]$, the seller's expected profit per uninformed consumer under the profit-maximizing recommendation policy is given by:*

$$\pi^u(p) \equiv \begin{cases} \int_{\hat{v}(p)}^{\overline{v}} (v-c)dF(v), & \text{if } p \in (\max\{c, \mathbb{E}[v]\}, \overline{v}] \\ p-c, & p \in [c, \max\{c, \mathbb{E}[v]\}], \end{cases} \tag{3}$$

where $\hat{v}(p)$ solves equation (1). The function $\pi^u(p)$ is continuous and strictly quasi-concave on $[c, \overline{v}]$ and maximized at $p^u = \mathbb{E}[v|v \geq c]$.

Proof The seller setting price $p \in (\max\{c, \mathbb{E}[v]\}, \overline{v})$ and adopting the profit-maximizing recommendation policy given in Lemma 1 earns a profit per uninformed consumer equal to

$$\pi^u(p) \equiv (p-c)(1-F(\hat{v}(p))),$$

where $\hat{v}(p)$ solves $p = \mathbb{E}[v|v \geq \hat{v}(p)]$. Substituting for $p = \mathbb{E}[v|v \geq \hat{v}(p)]$, the seller's profit is written as

$$\pi^u(p) = (1-F(\hat{v}(p)))\mathbb{E}[v-c|v \geq \hat{v}(p)] = \int_{\hat{v}}^{\overline{v}} (v-c)dF(v).$$

If $p \in [c, \max\{c, \mathbb{E}[v]\}]$, then the seller always recommends to buy, and the uninformed consumers follow the recommendations. The resulting profit is $p-c$.

Continuity of $\pi^u(p)$ follows immediately. It remains to show that $\pi^u(p)$ is strictly quasi-concave on $[c, \overline{v}]$. Note that if $\mathbb{E}[v] > c$, then $\pi^u(\cdot)$ strictly increases on $[c, \mathbb{E}[v])$. Suppose that $p \in (\max\{c, \mathbb{E}[v]\}, \overline{v})$. Then,

$$\frac{d\pi^u}{dp} = -(\hat{v}(p)-c)f(\hat{v}(p))\hat{v}'(p).$$

By the implicit functional theorem, we have

$$\hat{v}'(p) = \frac{(1-F(\hat{v}(p)))^2}{f(\hat{v}(p))\int_{\hat{v}(p)}^{\overline{v}}(v-\hat{v}(p))dF(v)} > 0.$$

This shows that $\pi^u(\cdot)$ strictly increases in p for $p \in (\max\{c, \mathbb{E}[v]\}, \mathbb{E}[v|v \geq c])$ and strictly decreases in p for $p \in (\mathbb{E}[v|v \geq c], \overline{v})$. It follows that $\pi^u(\cdot)$ is quasi-concave on $[c, \overline{v}]$ and maximized at $p = \mathbb{E}[v|v \geq c]$. □

Proof of Remark 1 Note that if all consumers are uninformed, then the seller's profit function is given by $\pi^u(p)$ (defined in equation (3)). The proof follows directly from Lemma 2. □

Proof of Theorem 1 Suppose that $p^u > p^i$. The seller's profit at price $p \in [c, \overline{v}]$ is given by

$$\pi(p) = \alpha\pi^i(p) + (1-\alpha)\pi^u(p).$$

Consider the price p^* that maximizes the seller's profit $\pi(p)$. By strict quasi-concavity of $\pi^i(\cdot)$ and, as shown in Lemma 2, $\pi^u(\cdot)$, it follows that for $p \geq p^u$, we have that

$$\pi'(p) = \alpha\frac{d\pi^i}{dp} + (1-\alpha)\frac{d\pi^u}{dp} \leq \alpha\frac{d\pi^i}{dp} < 0.$$

Therefore, p^* cannot be weakly higher than p^u, as the seller could then raise his profits from both consumer groups by slightly lowering the price and adjusting the recommendation strategy accordingly. Similarly, p^* cannot be strictly lower than p^i. This implies that $p^* \in (p^i, p^u)$. Correspondingly, for $p^u < p^i$. □

Proof of Remark 3 We seek a second-order approximation of $\overline{v} - c$ with respect to $\overline{v} - p^i$ in the neighborhood of $c = \overline{v}$. The first-order condition determining price p^i can be rewritten as

$$\overline{v} - c = \overline{v} - p^i + \frac{1 - F(p^i)}{f(p^i)}.$$

The first and the second derivatives of the inverse hazard rate are respectively given by

$$\begin{aligned}
\left(\frac{1-F}{f}\right)' &= -1 - \frac{f'(1-F)}{f^2}, \\
\left(\frac{1-F}{f}\right)'' &= -\frac{f'}{f}\left(\frac{1-F}{f}\right)' - \left(\frac{f'}{f}\right)'\frac{1-F}{f} \\
&= \frac{f'}{f} + \frac{1-F}{f}\left(\left(\frac{f'}{f}\right)^2 - \left(\frac{f'}{f}\right)'\right).
\end{aligned}$$

Therefore, the second-order Taylor approximation of the inverse hazard rate function at $p = \overline{v}$ is given by

$$\frac{1 - F(p^i)}{f(p^i)} = -(p^i - \overline{v}) + \frac{1}{2}\frac{f'(\overline{v})}{f(\overline{v})}(\overline{v} - p^i)^2 + o((\overline{v} - p^i)^2),$$

where $o(\cdot)$ is Landau's little-o: $f(x) = o(g(x))$ in the neighborhood of $x = x_0$ if $f(x)/g(x) \underset{x \to x_0}{\longrightarrow} 0$. Plugging this into the first-order condition, which determines price p^i, we obtain

$$\overline{v} - c = 2(\overline{v} - p^i) + \frac{1}{2}\frac{f'(\overline{v})}{f(\overline{v})}(\overline{v} - p^i)^2 + o((\overline{v} - p^i)^2).$$

Next, we derive a second-order approximation of $\overline{v} - p^u$ with respect to $\overline{v} - c$ in the neighborhood of $c = \overline{v}$. The equation determining p^u can be rewritten as

$$\overline{v} - p^u = \overline{v} - c - \frac{\int_c^{\overline{v}}(1 - F(v))dv}{1 - F(c)}.$$

We derive the second-order approximation of the right-hand side with respect to $\overline{v} - c$. Note that

$$\int_c^{\overline{v}}(1 - F(v))dv = \frac{1}{2}f(\overline{v})(\overline{v} - c)^2 - \frac{1}{6}f'(\overline{v})(\overline{v} - c)^3 + o((\overline{v} - c)^3).$$

Moreover, we have that

$$\frac{\overline{v} - c}{1 - F(c)} = \frac{1}{f(\overline{v})} + \frac{1}{2}\frac{f'(\overline{v})}{f^2(\overline{v})}(\overline{v} - c) + o(\overline{v} - c).$$

Thus, we obtain that

$$\begin{aligned}\frac{\int_c^{\overline{v}}(1 - F(v))dv}{1 - F(c)} &= \frac{\int_c^{\overline{v}}(1 - F(v))dv}{\overline{v} - c}\frac{\overline{v} - c}{1 - F(c)} \\ &= \left(\frac{1}{2}f(\overline{v})(\overline{v} - c) - \frac{1}{6}f'(\overline{v})(\overline{v} - c)^2\right)\left(\frac{1}{f(\overline{v})} + \frac{1}{2}\frac{f'(\overline{v})}{f^2(\overline{v})}(\overline{v} - c)\right) \\ &\quad + o((\overline{v} - c)^2) \\ &= \frac{1}{2}(\overline{v} - c) + \frac{1}{12}\frac{f'(\overline{v})}{f(\overline{v})}(\overline{v} - c)^2 + o((\overline{v} - c)^2).\end{aligned}$$

Plugging this back into the equation determining p^u and using the approximation of $v - c$ with respect to $v - p^i$, we obtain:

$$\begin{aligned}\overline{v} - p^u &= \frac{1}{2}(\overline{v} - c) - \frac{1}{12}\frac{f'(\overline{v})}{f(\overline{v})}(\overline{v} - c)^2 + o((\overline{v} - c)^2) \\ &= \overline{v} - p^i - \frac{1}{12}\frac{f'(\overline{v})}{f(\overline{v})}(\overline{v} - p^i)^2 + o((\overline{v} - p^i)^2).\end{aligned}$$

Thus, we obtain

$$p^u - p^i = \frac{1}{12}\frac{f'(\overline{v})}{f(\overline{v})}(\overline{v} - p^i)^2 + o((\overline{v} - p^i)^2).$$

We conclude that in the neighborhood of $c = \overline{v}$, $p^u > p^i$ ($p^u < p^i$) if and only if $f'(\overline{v}) > 0$ ($f'(\overline{v}) < 0$). Therefore, by Theorem 1, the seller provides inflated recommendations (insufficient recommendations) if $f'(\overline{v}) > 0$ ($f'(\overline{v}) < 0$). □

Proof of Remark 4 The expected value for the symmetric distribution is given by $\tilde{p}^u = \mathbb{E}[v] = \frac{\overline{v}+\underline{v}}{2}$. The derivative of π^i at $p = \tilde{p}^u$ when $c = \underline{v}$ is given by

$$\begin{aligned}\left.\frac{d\pi^i}{dp}\right|_{p=\tilde{p}^u} &= -(\tilde{p}^u - \underline{v})f(\tilde{p}^u) + (1 - F(\tilde{p}^u)) \\ &= -\frac{\overline{v}-\underline{v}}{2}f\left(\frac{\overline{v}+\underline{v}}{2}\right) + \left(1 - F\left(\frac{\overline{v}+\underline{v}}{2}\right)\right).\end{aligned}$$

Since the distribution is symmetric, we have that $F\left(\frac{\overline{v}+\underline{v}}{2}\right) = \frac{1}{2}$, implying that $\tilde{p}^u - \tilde{p}^i > (<)\ 0$ if and only if

$$f\left(\frac{\overline{v}+\underline{v}}{2}\right) > (<) \frac{1}{\overline{v}-\underline{v}}.$$

□

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