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Preface

Monetary policy plays a central role in stabilizing inflation, mitigating business cycle fluctuations, and responding to financial crises. Its effectiveness derives from the broad influence of nominal interest rates on aggregate economic activity. However, different regions within a country or currency union may react differently to the same monetary policy impulse due to regional heterogeneities. A strong form of regional heterogeneities can be found in housing markets.

Furthermore, it is well known that expectations about asset prices, and house prices in particular, violate the rational expectations hypothesis. Instead, a large empirical literature has documented that asset price expectations. experience extrapolative dynamics.

In this thesis, I make progress in understanding the transmission and conduct of monetary policy in the presence of regional heterogeneities and extrapolative house price expectations.

Chapter (1), which is joint with Ulrich Roschitsch, shows that regional disparities in house price growth are more pronounced during house price busts than during booms. To explain this observation we construct a two-region currency union model incorporating a housing sector and extrapolative belief updating regarding house prices. To solve the model, we propose a new method that efficiently handles extrapolative belief updating in a wide class of structural models. We show that intensified extrapolation in busts and regional housing market heterogeneities jointly explain elevated regional house price growth dispersion in busts and muted dispersion in booms. Consistent with our theory, we provide empirical evidence that house price belief updating is indeed more pronounced in busts and document that regional heterogeneities on the housing supply side affect regional house

prices. Quantitatively, our model can match the empirically observed elevated regional house price growth dispersion in busts. Moreover, we demonstrate that a monetary authority targeting house prices may reduce the volatility of output and house prices, as well as regional house price growth disparities. This policy is welfare-improving relative to an inflation-targeting benchmark.

Chapter (2), which is joint with Lukas Hack, studies monetary policy transmission in the Euro Area. We document that monetary policy has heterogeneous effects on real GDP and inflation across Euro Area member states. To investigate the underlying drivers we construct a two-region currency union model with idiosyncratic risk and cross-region household heterogeneity. The model matches household-level heterogeneity in homeownership rates, mortgage types, and the prevalence of hand-to-mouth households. These features account for 70% of the cross-region differences in GDP responses to monetary policy shocks. This is primarily driven by the interplay of demand amplification through hand-to-mouth households and demand dampening through trade effects.

In Chapter (3), which is joint with Ulrich Roschitsch, we examine the interaction between housing and business cycles in a tractable two-agent New Keynesian model featuring extrapolative house price beliefs. The model includes a saver and a hand-to-mouth borrower who uses housing as collateral. We identify four key transmission channels from housing markets to aggregate output: consumption, residential investment, collateral, and fire sales. Under rational expectations, output volatility is limited and primarily driven by consumption. In contrast, extrapolative beliefs significantly amplify output volatility, mainly through residential investment. Finally, we propose a novel solution approach for two-agent models with asset trade and asset price extrapolation, which is essential for solving models with fire sale motives.

Chapter 1

United in Booms, Divided in Busts: Regional House Price Cycles and Monetary Policy

Joint with Ulrich Roschitsch.

1.1 Introduction

A substantial body of literature connects boom-bust cycles in house prices to over-optimism and over-pessimism regarding the future evolution of house prices.¹ The bust phases of these cycles are often associated with significant economic costs, making them a primary concern for policymakers, particularly central bankers.² However, housing markets are inherently regional, and so too are housing cycles and the economic costs associated with them. While a substantial literature examines housing cycles and their implications for monetary policy at the federal level, there is comparatively little focus on regional heterogeneities in house price dynamics. This paper seeks to address this gap.

¹See e.g., Armona, Fuster, and Zafar (2019), Case, Shiller, and Thompson (2012), Kaplan, Mitman, and Violante (2020), Kuchler and Zafar (2019), and Ma (2020)

²During the Great Financial Crises 20% of household wealth was lost (Detting, Hsu, and Llanes, 2018), and 8 million jobs were lost (Mian and Sufi, 2015). During this time the Fed responded by drastically cutting interest rates.

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This paper contributes to the literature in several ways. Empirically, we document that regional disparities in house price growth are more pronounced during housing busts than during booms. To account for this observation, we develop a two-region currency union model that incorporates regional heterogeneity in housing markets and extrapolative belief updating regarding house prices. To solve this model, we propose a new method that allows us to efficiently solve structural models with extrapolative belief updating. In our model, intensified extrapolation in busts and regional housing market heterogeneities jointly explain cross-regional house price growth divergences in busts.

To validate our theoretical modeling choices we turn to the data. First, we show that households' expectations of future house price growth are formed by extrapolating on realized growth rates, with a stronger extrapolation from past to expected rates during busts than in booms. Second, we provide evidence that cross-regional variation in house prices is associated with structural differences in housing supply. Using these insights, we calibrate our model to the US economy, demonstrating that it quantitatively matches the response of aggregate house prices and its' regional dispersion in growth rates to monetary policy shocks in both booms and busts. Finally, we illustrate that by leaning against aggregate house prices, the monetary authority can mitigate cross-regional disparities in house prices and economic activity, resulting in welfare improvements relative to the baseline case.

We begin by presenting our central empirical finding, namely that cross-regional dispersion in house price growth rates are more pronounced during busts, both unconditionally and conditionally on macroeconomic shocks. First, we document that the cross-regional standard deviation of house price growth among major US cities is 50%-60% larger during bust periods compared to booms.³ We further show that these patterns are also present for the US state levels and the Euro Area at a country level. Second, conditional on monetary policy shocks, we observe a significant and sizable increase in the cross-city standard deviations of house price growth during busts, while the response is muted and far less significant during

³Throughout the paper, a bust period is defined as one in which house prices declined in the previous period.

boom periods. This result is robust regarding other types of shocks.

To rationalize the structural forces giving rise to our empirical findings, we construct a two-region currency union New Keynesian model. We extend the framework of Benigno (2004a) by (i) incorporating a housing sector, and (ii) allowing for subjective expectations about house prices. Housing markets are regional, and regions are symmetric except for the housing supply side. In this model, households form subjective expectations only about house prices. This assumption is motivated by the central role of house prices in our framework. However, households' expected endogenous choices, for instance expected future consumption, depend on future house prices. Therefore, these expected endogenous choice variables are also shaped by subjective expectations. This raises the issue of characterizing subjective beliefs regarding these variables.

To solve this type of model, we propose a novel method, based on first-order approximations, for solving general equilibrium models with time-consistent but non-rational expectations, such as extrapolative asset price beliefs. Our approach offers the advantage of restricting subjective expectations exclusively to house prices while maintaining the rational expectations hypothesis for all other variables exogenous to agents. The ability to confine subjective expectations to specific variables allows us to model expectation formation in a data-consistent way where data is available while adhering to the established benchmark for variables where data is unavailable. This method further enables us to explicitly characterize the expectations of endogenous variables, i.e. household choices. Finally, the first-order approximation renders our model compatible with standard solution methods used in the DSGE literature, making it easy to handle, scale, and serve as a basis for Ramsey-type analyses of optimal policy.⁴

Equipped with our model, we demonstrate analytically that regional differences in housing markets combined with house price belief extrapolation, can account for the asymmetry in house price growth dispersion observed across the house price

⁴Our solution method contrasts with that introduced by Winkler (2020), which, although based on first-order approximation, does not confine subjective expectations to specified asset prices and results in belief spillovers to all other prices. Conversely, Adam, Pfäuti, and Reinelt (2022) solve a similar model using non-linear solution techniques.

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cycle.⁵ First, regional heterogeneity in housing markets creates regional differences in the responsiveness of house prices. Second, subjective expectations about house prices introduce a backward-looking component through extrapolation. This mechanism dynamically amplifies the responsiveness of house prices: when agents observe an increase in house prices, they anticipate further price increases in the future, leading to heightened demand for housing and, subsequently, driving prices even higher. The interaction of these two forces generates the following dynamics: regional heterogeneities in housing markets create differences in regional house price growth rates, which are further amplified by extrapolation. This result allows us to align our theoretical model with the empirically observed patterns. Specifically, our model is capable of generating pronounced regional dispersion in house price growth during busts, provided that extrapolation is stronger in those periods. The intuition is straightforward: extrapolation amplifies regional disparities, so when extrapolation intensifies during busts, it results in greater regional dispersion in house price growth, consistent with the empirical evidence.

The following example illustrates this mechanism. Suppose an expansionary monetary policy shock increases housing demand. House prices rise moderately in a region with an elastic housing supply and more strongly in a region with an inelastic supply. Households extrapolate from these changes, expecting higher future price growth, particularly in the inelastic region where the initial price rise was greater. This optimism increases housing demand and further raises prices, especially in the inelastic region. Thus, extrapolation amplifies the initial regional price differences. This effect is more pronounced during busts, as expectations react more strongly to observed price movements, intensifying cross-regional dispersion in house price growth in these periods.

In a version of the model with fully rational expectations, the ability of cross-regional differences in housing markets to produce differences in house prices in response to a shock is much weaker. Additionally, this version lacks extrapolative belief updating and thus cannot account for amplified cross regional heterogeneity in busts in the absence of any additional frictions. Moreover, this model version is generally less responsive to shocks.

⁵We focus on monetary policy shocks, but the general proposition extends to all shocks that affect housing demand.

To test our theoretical predictions we move to the data. First, we estimate the extent to which US households' house price expectations depend on prior expectations and past house price growth, and find that the pass-through from past house price growth to expectations is stronger during busts than during booms, i.e. households update their beliefs more strongly during busts. Second, we investigate the response of regional house prices to a common monetary policy shock using a panel local projections approach. Furthermore, we differentiate these responses based on regional housing supply conditions by incorporating interaction terms. We find that in more supply-constrained regions house prices exhibit a stronger response to a monetary policy shock. These findings are consistent with our theoretical predictions and provide a basis for calibrating our model with respect to house price belief formation and heterogeneities in housing supply.

We use the calibrated model to analyze the response of house prices relative to an empirical counterpart. Specifically, we target the estimated aggregate house price response to a monetary policy shock, independent of boom-bust asymmetries in belief updating. The model accurately captures the magnitudes of aggregate house price changes in both booms and busts when accounting for asymmetries in the belief updating process, even though these asymmetries were not targeted. It also successfully replicates the regional house price growth dispersion observed during booms and busts in response to a monetary policy shock. In contrast, the rational expectations version of the model fails to match these dynamics, underestimating aggregate house price responses by a factor of three and generating no significant difference in regional house prices.

Finally, we investigate the extent to which leaning against house prices influences economic outcomes. Specifically, we assess the effects of including aggregate house prices in the Taylor rule, compared to a baseline rule focused solely on inflation stabilization in response to a productivity shock. Our findings indicate that adding an additional coefficient targeting house prices to the Taylor rule reduces the volatility of both output and house prices, albeit at the cost of increased inflation volatility. Notably, a stronger response to house prices reduces the regional dispersion for all variables. In our framework, leaning against house prices therefore has the

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additional effect of synchronizing economic activity across regions. The underlying intuition is that regional heterogeneities originate in housing markets and are subsequently propagated throughout the economy via house prices. By targeting house prices, the central bank reduces the extent to which house price beliefs can diverge, thereby mitigating the channel through which regional heterogeneities in housing markets can spill over to the real economy. We show that house price targeting is beneficial under subjective expectations expressing the utilitarian welfare as in Galí (2015).⁶ In booms, welfare is improved by 6% relative to the inflation targeting baseline. In busts, when belief extrapolation is stronger, we observe a welfare gain of 25% relative to the inflation targeting baseline.

Related literature. Our paper relates to a broad empirical literature emphasizing that house price beliefs are not formed according to rational expectations.⁷ This literature highlights momentum and revisions in the belief formation of house prices. On the theoretical side, we connect to the behavioral macro-finance literature, which focuses on deviations from rational expectations in terms of asset price expectations.⁸ More specifically, our paper relates to the literature on capital gains extrapolation.⁹ Regarding house prices Glaeser and Nathanson (2017) and Schmitt and Westerhoff (2019) model house price expectations through certain forms of extrapolation in a partial equilibrium environment. In contrast, we focus on a general equilibrium New Keynesian environment, which is most closely related to Adam, Kuang, and Marcet (2012), Caines and Winkler (2021), and Adam, Pfäuti, and Reinelt (2022). Other studies, such as Burnside, Eichenbaum, and Rebelo (2016), Guren (2018), and Kaplan, Mitman, and Violante (2020), explain house price behavior through optimism and pessimism, concave demand curves faced by sellers in the housing market, or exogenous shifts in house price beliefs. Our contribution

⁶Specifically, the welfare loss is expressed in terms of the equivalent permanent consumption decline, measured as a fraction of steady state consumption.

⁷See, for instance, Armona, Fuster, and Zafar (2019), Case, Shiller, and Thompson (2012), Kuchler and Zafar (2019), and Ma (2020).

⁸Among others, see: Barberis (2018), Bianchi, Ilut, and Saijo (2024), Bordalo, Gennaioli, and Shleifer (2018), Caballero and Simsek (2019, 2020), Krishnamurthy and Li (2020), L’Huillier, Singh, and Yoo (2023), and Maxted (2024).

⁹Adam, Marcet, and Beutel (2017) and Winkler (2020) examine asset price learning in the context of stock markets. Fontanier (2022) and Farhi and Werning (2020) focus on asset price extrapolation more generally.

to this body of literature lies in emphasizing the asymmetric evolution of asset price beliefs during boom and bust episodes. Additionally, we introduce a new solution method for models incorporating capital gains extrapolation.

We also contribute to the literature examining regional housing supply variations in the United States. Studies by Mian, Rao, and Sufi (2013), Mian and Sufi (2014), and Guren, McKay, Nakamura, and Steinsson (2021) leverage housing supply elasticities to explore the housing wealth effect. Our work is closely related to the studies by Aastveit and Anundsen (2022) and Aastveit, Albuquerque, and Anundsen (2023), which demonstrate that house prices in US metropolitan areas with more inelastic housing supply exhibit greater responsiveness to monetary policy shocks. We focus on the state level instead. Furthermore, we show that similar patterns can be observed in the Euro Area. While the aforementioned studies are primarily empirical, our contribution lies in providing theoretical insights to interpret our empirical findings. We also connect to Glaeser, Gyourko, and Saiz (2008), who show both theoretically and empirically that more supply-inelastic regions have a higher probability of experiencing a house price bubble. In contrast to their approach, we highlight asymmetries in house price belief formation, construct a general equilibrium New Keynesian model, and focus on monetary policy.

We focus on a setup involving multiple regions governed by a single monetary authority, thereby connecting to the literature on cross-regional heterogeneities in currency unions. Studies by Benigno (2004a), Galí and Monacelli (2008), and Kekre (2022) examine optimal policy in a currency union setting. Additionally, Calza, Monacelli, and Stracca (2013), Slacalek, Tristani, and Violante (2020b), Bletzinger and Thadden (2021a), Pica (2021), Almgren, Gallegos, Kramer, and Lima (2022b), and Corsetti, Duarte, and Mann (2022a) explore various sources of heterogeneity within currency unions and their effects on economic activity. In this strand of literature, cross-regional heterogeneities include variations in price and wage setting, the share of hand-to-mouth consumers, and mortgage market dynamics. For the US, Beraja, Hurst, and Ospina (2019), Chen (2019), and Gitti (2024) focus on regional differences in wage and price setting, exposure to fiscal policy shocks, housing markets, and migration. To our knowledge, we are the first to study supply-side housing constraints and connect these to asymmetries in the formation of house price beliefs.

1.2. Regional house prices in booms and busts

Finally, we connect to the literature on leaning against asset prices. Stein (2012) argues that leaning against the wind can address externalities arising from over-borrowing. Svensson (2017) argues that the costs of these policies outweigh their benefits, while Weidmann (2018) points out that macroprudential policies are better suited to address these concerns. Gourio, Kashyap, and Sim (2018) quantitatively outlines a trade-off between a lower crisis probability and increased cyclical inflation and output dynamics. However, once rational expectations are abandoned, leaning against the wind generally becomes beneficial (Adam and Woodford, 2021; Caines and Winkler, 2021). This holds true even if macroprudential tools are fully (Fontanier, 2022) or at least partially available (Caballero and Simsek, 2019). Our paper contributes to this literature by emphasizing that targeting asset prices can also mitigate regional dispersion and thus help harmonize an economically integrated area. To our knowledge, we are the first to make this point.

Outline. The rest of this paper is organized as follows. In Section (1.2), we document our main empirical observation: regional house price growth is more dispersed in busts than in booms. Section (1.3) outlines our theoretical model and describes our solution method. In Section (1.4) we analytically show how extrapolation on house price growth and housing market heterogeneity can explain our main empirical observation. Section (1.5) provides evidence of stronger extrapolation in busts, housing markets supply side heterogeneity, and describes the model calibration. Section (1.6) presents our quantitative results. Finally, Section (1.7) contains the policy exercise. Section (1.8) concludes.

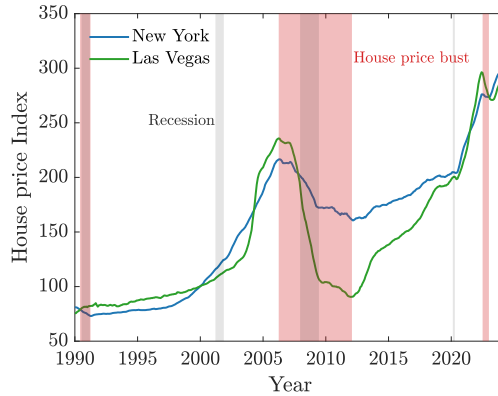
1.2 Regional house prices in booms and busts

House prices experience large regional differences. This is in particular the case in times of house price busts. Figure (1.1) illustrates the house price index for New York and Las Vegas, with the shaded red areas indicating periods of house price busts. For the purposes of analysis, housing booms are defined as periods in which federal house price growth is positive ($\Delta q_t^{agg} > 0$), while busts are characterized

by negative growth ($\Delta q_t^{agg} < 0$).¹⁰

The figure reveals that house prices tend to move in tandem during boom periods; however, during busts, significant divergences emerge. In particular, house prices in Las Vegas experienced a sharp decline during the Great Recession. By contrast, while house prices in New York also declined, the reduction was far less severe. Importantly, the observed divergence in house price growth during busts cannot be attributed solely to economic recessions. For instance, there was minimal divergence during the Dotcom bubble and the COVID-19 crisis, both of which were periods marked by overall positive federal house price growth.

Figure 1.1: Regional house prices in booms and busts



Notes: Case-Shiller house price index for New York and Las Vegas.

Cross-regional house price variation, unconditional. To further investigate and generalize this pattern, we extend the analysis to regional house price growth data on different levels. We utilize regional house price growth data, specifically at the major city or state level for the United States. For each point in time, we compute the cross-regional standard deviation of house price growth rates. We then split the sample into boom and bust episodes, where these episodes are defined based on the aggregate house price growth of the economic entity. Our preferred specification is based on US major cities in the cross-section.¹¹ The data

¹⁰House price growth rates on the aggregate/federal level (Δq_t^{agg}) are based on the Case-Shiller Index.

¹¹We use the Case-Shiller house price index on the city level. The cities include 20 major US cities: Atlanta, Boston, Charlotte, Chicago, Cleveland, Dallas, Denver, Detroit, Las Vegas, Los

1.2. Regional house prices in booms and busts

is given in monthly frequency and therefore allows for sufficient observations in booms and busts. On the US state level, we use quarterly data. Our sample includes all states except Alaska and Hawaii but includes the District of Columbia. Both US data sets span from 1990 until 2024. Table (1.1) reports the mean and median cross-regional standard deviations.

Table 1.1: Cross-regional house price growth standard deviation (σ_c) in booms and busts

	<i>US, cities</i>		<i>US, states</i>	
	<i>mean</i>	<i>median</i>	<i>mean</i>	<i>median</i>
<i>Bust</i>	1.60	1.64	1.23	1.26
<i>Boom</i>				
$\sigma_c : \text{Boom}$	0.47	0.41	0.87	0.80
$\sigma_c : \text{Bust}$	0.74	0.67	1.08	1.01
$p\text{-Val. Bust} > \text{Boom}$	0.000	0.000	0.007	0.007
<i>Number regions</i>		20		51
<i>Obs. boom</i>		331		109
<i>Obs. bust</i>		82		28
<i>Sample</i>	1990M1 – 2024M5		1990Q1 – 2024Q1	

Notes: The Table reports the mean and median, across time, of the estimates of cross-sectional standard deviations of house price growth rates within the given economic entity. The test for $\text{Bust} > \text{Boom}$ is based on a one-sided t-test.

Our findings indicate that the cross-regional standard deviations during busts are larger for both the mean and median estimates than those observed during booms. Specifically, cross-regional standard deviations during busts are 60% larger than booms if one considers the monthly US city data. The ratio decreases to 23% – 30% for the US states. We find that a t-test rejects the H_0 hypothesis of regional standard deviations being larger in booms for both regional specifications. Table (1.C.1) in the Appendix shows that the same holds for the Euro Area on country level.

Cross-regional house price variation, conditional. While the previous analysis focused on unconditional variations in house price growth during booms and

Angeles, Miami, Minneapolis, New York, Phoenix, Portland, San Diego, San Francisco, Seattle, Tampa, Washington DC.

busts, we now move to a conditional approach. Using local projections, we estimate the response of the cross-regional standard deviation of house price growth to a monetary policy shock (MP). We utilize city-level data due to its sufficiently large time series dimension. Equation (1.1) depicts the estimation equation. The left-hand-side variable (y_{t+h}) denotes the three month moving average of the cross-regional standard deviation of house price growth at horizon h after the impact of the shock. α^h denotes constant. The monetary policy shock, ϵ_t^{MP} , is the high frequency identified and orthogonalized shock from Bauer and Swanson (2023). The monetary policy shock further conditions on times of booms ($\Delta q_{t-1}^{agg} > 0$) and busts ($\Delta q_{t-1}^{agg} < 0$). We adopt this approach because monetary policy shocks are relatively small and do not influence the economy's transition between boom and bust phases.¹² We use the absolute values of the shocks, as the cross-regional dispersion should be more responsive in busts independent of the sign of the shock. The controls, x_t , contain 12 lags of the left-hand side variable, the log of the house price index on a federal level, log of industrial production, the log of CPI, the FFR, and the shocks. The sample runs from 1990 to 2019.

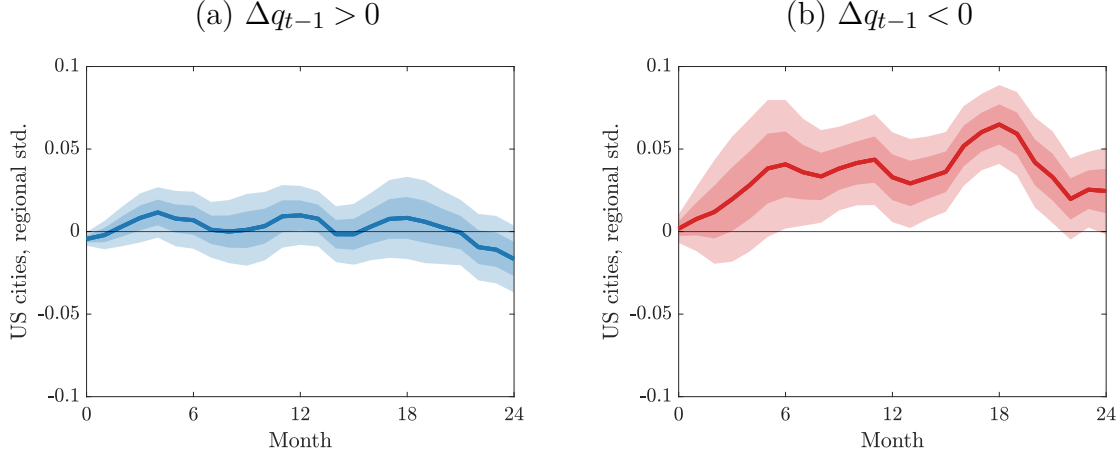
$$y_{t+h} = \alpha^h + \mathbf{1}(\Delta q_{t-1}^{agg} > 0) \times \beta_1 |\epsilon_t^{MP}| + \mathbf{1}(\Delta q_{t-1}^{agg} < 0) \times \beta_2 |\epsilon_t^{MP}| + x_t + u_{t+h} \quad (1.1)$$

Figure (1.2) presents the results. We see no significant change of cross-regional standard deviations in booms, see panel (a). In times of busts, panel (b), we find that a monetary policy shock leads to a significant increase in cross-regional house price growth variation over the first two years. Hence, conditional on monetary policy shocks, regional house price growth is more dispersed in busts relative to booms. This result aligns with our unconditional exercise presented above.

¹²Regressing house price growth on the aggregate level, for the current period and various leads, on the monetary policy shock reveals that the R^2 is never above 0.0007 across various leads. This emphasizes that these shocks do not drive the cycle.

1.2. Regional house prices in booms and busts

Figure 1.2: City-level std. house price growth response to MP shock, boom-bust



Notes: Responses to MP shock (1 std, absolute value) ; Confidence Intervals: 68% and 95% (Newey-West). The lhs variable is the 3 month moving average of the cross-city std. of house price growth.

Robustness. Figure (1.1) raises the question of whether house price booms and busts are simply expansions and recessions if we consider all major US cities in our sample. Figure (1.C.1) shows that this is not the case. It plots the cross-city three-month moving average of house price growth. In particular, it shows that the standard deviations was already elevated before and remained elevated after the Great Recession. We further observe an increase in the standard deviation *after* COVID around 2022-2023. This period is associated with a house price bust, but no Recession. Further, we observe that the dotcom and the COVID recession were not associated with house price busts and did not experience elevated cross-regional heterogeneity in house price growth. Therefore, house price busts and Recessions seem not to be perfectly correlated.

We also study the response of regional house price growth dispersion to a main business cycle shock constructed by Angeletos, Collard, and Dellas (2020). The shock is given in quarterly frequency and we therefore move to state-level data to extend our cross-section. The main idea is that the main business cycle shock should be strong enough to affect the house price cycle.¹³ Consequently, we study

¹³Regressing the house price growth on the shock gives an R^2 of 0.1, meaning that this shock can explain a non-trivial part of the variation in house price growth.

the response of the cross-state standard deviation of house price growth to an expansionary main business cycle shock. We observe that this shock indeed reduces house price growth heterogeneities, see Figure (1.C.2). As a sanity check, we also report the response of house price growth to the shock and observe that it is increasing. Our results are therefore in line with the findings reported above.

1.3 Model

With these empirical impressions in mind, we now describe our two-region currency union model, inspired by Benigno (2004a), which we shall use to connect our empirical evidence and to conduct policy experiments. A notable difference between our model and Benigno's is the incorporation of incomplete bond markets across regions, instead of having perfect consumption insurance. In addition, we incorporate a housing sector into the model, introducing two critical elements.

First, we account for subjective expectations in the formation of house prices. Previous research has demonstrated that house price expectations deviate from rational expectations and play a crucial role in explaining boom-bust dynamics in the housing market.¹⁴ In our model, subjective expectations are applied exclusively to house prices, as the primary focus of this study is on housing market dynamics. Furthermore, the availability of forecast data is limited to a select number of variables, making it feasible to incorporate subjective expectations only for house prices, where empirical data can effectively inform the model.

Second, we allow for regional differences in housing markets. We differentiate between a "home" region and a "foreign" region, with all foreign region variables denoted by an asterisk. The only difference between regions is on the housing supply side. The distinction between regions is the time required to construct houses, referred to as "time-to-build." This parameter enables us to model supply-side heterogeneities in a tractable manner.¹⁵ Below we will also briefly discuss how housing demand side heterogeneities may be modeled.

¹⁴See, for example, Armona, Fuster, and Zafar (2019), Case, Shiller, and Thompson (2012), Kuchler and Zafar (2019), and Ma (2020).

¹⁵In the empirical analysis below we will be more precise on why time-to-build seems a reasonable approximation for supply-side differences from an empirical point of view.

1.3. Model

1.3.1 The Economy

Households. A representative domestic household derives utility from consuming domestic and foreign varieties, leisure, and housing. The preferences are as follows:

$$\begin{aligned} \mathbb{E}_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t & \left(\frac{\xi_{c,t} c_t^{1-\sigma}}{1-\sigma} + \frac{\xi_{h,t} h_t^{1-\nu}}{1-\nu} - \frac{\chi n_t^{1+\varphi}}{1+\varphi} \right) \\ c_t &= \left[\lambda^\varsigma c_{H,t}^{1-\varsigma} + (1-\lambda)^\varsigma c_{F,t}^{1-\varsigma} \right]^{\frac{1}{1-\varsigma}} \\ c_{H,t} &= \gamma \left[\int c_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}, \quad c_{F,t} = (1-\gamma) \left[\int c_{F,t}(j^*)^{\frac{\epsilon-1}{\epsilon}} dj^* \right]^{\frac{\epsilon}{\epsilon-1}} \end{aligned}$$

where $\mathbb{E}_0^{\mathcal{P}}$ denotes the subjective expectations operator discussed below. ξ_j , $j \in \{c, h, a, x, i\}$, denote model-exogenous shock terms, h_t and n_t denote housing and hours worked respectively. $\gamma \in (0, 1)$ denotes the measure of households in the home economy. Following Benigno (2004a), γ is simultaneously the economic size of the home region, i.e. the mass of variety-producing firms. c_t denotes consumption of the domestic basket that is assembled from the home-good and the foreign-good which in turn are CES-aggregates of two groups of varieties. A preference bias for goods produced in the respective region of residence (“home bias”) arises if $\lambda, 1 - \lambda^* \neq \gamma$.¹⁶ The slope of the housing demand curve is governed by the parameter ν . Regional differences on the housing demand side may be modeled by allowing for regional differences in ν .

The household budget constraint is then given by:

$$\begin{aligned} c_t + q_t(h_t - (1-\delta)h_{t-1}) + b_{t+1} + \frac{P_{H,t}}{P_t} x_t = \\ w_t n_t + (1+r_t)b_t + q_t \cdot H(x_{t-\tau}, \xi_{x,t}) - T_t + \Sigma_t + \mathbf{b}_t \end{aligned}$$

The budget constraint is expressed in units of the region- H final consumption basket, c . Σ_t are profits from all domestic firms, which are owned evenly by all domestic households, w_t is the real wage, T_t are government lump-sum taxes, and

¹⁶Throughout the paper we maintain the assumptions that (i) the degree of home bias is symmetric: $\gamma(1-\lambda) = (1-\gamma)(1-\lambda^*)$, and (ii) the bias is such that households favor domestically produced products, $\lambda \geq \gamma$.

b_t is a one-period nominal zero-coupon bond that is traded union-wide. q_t denotes the real house price and x_t is the number of domestic consumption units dedicated to the production of new housing units.

To invest into housing production, households need to purchase domestically produced goods and transform them into housing investment units. Housing production is defined as $H(x_{t-\tau}, \xi_{x,t}) = \xi_{x,t} \frac{x_{t-\tau}^\eta}{\eta}$, $\eta \in (0, 1)$. τ denotes the number of quarters it takes to construct new housing units. Housing investment done in period t will therefore lead to a return in period $t + \tau$. Regional heterogeneities on the housing supply side are modeled by allowing for cross-regional differences in time-to-build (τ).

In the steady state we will calibrate the steady state values for $\xi_h, \xi_h^*, \xi_x, \xi_x^*$ s.t. $b_{ss} = b_{ss}^* = 0$. Hence, in the steady state, there are no net debtor and net creditor regions. This ensures that the only form of regional heterogeneity is situated on the housing side.

It is convenient to express the bond holdings in units of region H 's final basket. The real interest rate r_t is taken as given by households and is determined in equilibrium by the following Fisher-type equation: The value of bond holdings in units of numéraire is $B_t = P_t \cdot b_t$ and the nominal bond pays $i_{t-1} - \psi b_t$ units of currency as interest.¹⁷ The real interest rate is thus given by

$$1 + r_t = \frac{1 + i_{t-1} - \psi b_t}{1 + \pi_t}$$

where $\pi_t := P_t/P_{t-1} - 1$. Finally, $\mathfrak{b}_t := (\beta^{-1} - 1) \left(\gamma + (1 - \gamma) \frac{P_{t-1}^*}{P_{t-1}} \right) (1 + \pi_t)^{-1} \bar{b}$, taken as exogenous by the household, captures payment streams between H and F that guarantee that households are content with holding no bonds in the non-stochastic steady state with zero inflation and real exchange rate parity.¹⁸

¹⁷The nominal interest rate is elastic in the aggregate holdings of bonds by domestic households. We follow Schmitt-Grohe and Uribe (2003) to ensure stationarity of the first-order dynamics. In Appendix 1.A we provide a simple micro-foundation for debt-elastic interest rates.

¹⁸Given that bond holding entails a real cost in equilibrium, see footnote 17, introducing the payments \mathfrak{b}_t is a way to ensure that there are no bond holding costs in the non-stochastic steady state with zero inflation and real exchange rate parity (i.e. $1 + \pi_t = 1 + \pi_t^* = 1 + \pi_{H,t} = 1 + \pi_{F,t} = \frac{P_{H,t}}{P_{F,t}} = 1$). This ensures that this steady state is efficient, given that fiscal policy undoes the monopolistic competition distortion. \mathfrak{b}_t may be interpreted as the real interest rate paid by a non-market nominal consol, that perpetually pays the nominal rate $(\beta^{-1} - 1)(\gamma + (1 -$

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Subjective expectations house price setup. As is standard in the literature on capital gain extrapolation (e.g. Adam and Marcet, 2011; Adam, Marcet, and Beutel, 2017), households are endowed with a set of beliefs in the form of a probability measure over the full sequence of variables that they take as given, henceforth external variables: $(\xi_t, r_t, w_t, \Sigma_t, T_t, \mathbf{b}_t, \pi_t, (P_{H,t}/P_{F,t}), q_t)_{t \geq 0}$. This measure we denote as \mathcal{P} . Rational expectations are a special case of this setup in the form that households' beliefs agree with the objective, or equivalently "true" or "equilibrium-implied", distribution of external variables, $\mathcal{P} = \mathbb{P}$. Although households may hold expectations that are generally inconsistent with the equilibrium-implied (conditional) distribution of external variables, it is worth emphasizing that first they have a time-consistent set of beliefs, and second they behave optimally given their beliefs. That is, households are *internally rational* in the sense of Adam and Marcet (2011). Moreover, the fact that all households are identical in beliefs and preferences is not common knowledge among agents so households cannot discover the misspecification of their beliefs, $\mathcal{P} \neq \mathbb{P}$, by eductively reasoning through the structure of the economy. Given the observed path of external variables up to period t , households then use this information and \mathcal{P} to form a conditional expectation over the continuation sequence of external variables, which we denote as $\mathbb{E}_t^{\mathcal{P}}$. We denote the conditional rational expectations operator as usual by \mathbb{E}_t .

We assume that agents have rational expectations with respect to all external variables, except for house prices, q_{t+s} .¹⁹ Households entertain the idea that house

$\gamma)P_{t-1}^*/P_{t-1})$ and of which the household is endowed with \bar{b} units. The endowments of these consols ensure that nominal payments balance, i.e. $\gamma\bar{b} + (1-\gamma)\bar{b}^* = 0$, see Appendix 1.A. Since we will linearize the model around a steady state with zero bond holding, \mathbf{b}_t will be zero in equilibrium.

¹⁹Formally, $\mathcal{P} := \mathbb{P}_{-q} \otimes \mathcal{P}_q$, where \mathbb{P}_{-q} is the objective measure over sequences of external variables without house prices, \mathcal{P}_q is the measure over sequences of house prices implied by the described perceived model of house prices, and \otimes is the product measure. Since we are interested in a first-order solution to the model, it does not matter what households perceive to be the dependence structure between house prices and the other external variables.

prices follow a simple state-space model:

$$\begin{aligned} \ln \frac{q_{t+1}}{q_t} &= \ln m_{t+1} + \ln e_{t+1} \\ \ln m_{t+1} &= \varrho \ln m_t + \ln v_{t+1}, \quad \varrho \in (0, 1) \\ (\ln e_t \quad \ln v_t)' &\sim \mathcal{N} \left(\begin{pmatrix} -\frac{\sigma_e^2}{2} & -\frac{\sigma_v^2}{2} \end{pmatrix}, \begin{pmatrix} \sigma_e^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix} \right) \end{aligned} \quad (1.2)$$

Hence, agents perceive house price growth rates as the sum of a transitory and a persistent component. Crucially, $\ln e_t$ and $\ln v_t$ are not observable to the agents, rendering $\ln m_t$ unobservable. Agents apply the optimal Bayesian filter, i.e. the Kalman filter, to arrive at the observable system.²⁰

Lemma 1 (House price belief updating). *Applying the Kalman filter to the state-space model and log-linearizing around the non-stochastic steady state gives:*

$$\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s} = \hat{q}_t + \frac{1 - \varrho^s}{1 - \varrho} \varrho \hat{m}_t \quad (1.3)$$

$$\hat{m}_t = (\varrho - g) \hat{m}_{t-1} + g \Delta \hat{q}_{t-1} \quad (1.4)$$

where $\ln \bar{m}_t := \mathbb{E}_t^{\mathcal{P}}(\ln m_t)$ is the posterior mean, $g = \frac{\sigma^2 + \sigma_v^2}{\sigma^2 + \sigma_v^2 + \sigma_e^2}$ is the steady state Kalman filter gain, $\sigma^2 = \frac{1}{2}[-\sigma_v^2 + \sqrt{\sigma_v^4 + 4\sigma_v^2\sigma_e^2}]$ is the steady state Kalman filter uncertainty, and $\ln \hat{e}_t$ is perceived to be a white noise process.

Proof. See Appendix (3.A) for the application of the Kalman filter. Log-linearization around the steady state gives the result. \square

Variables denoted with a " $\hat{\cdot}$ " express the respective variables in percent deviations from its' steady state value. Equation (3.5) shows that future house price beliefs depend on the current house price and today's beliefs. As $\varrho \in (0, 1)$, the weight on the beliefs increases in the forecast horizon. Current house prices translate one-to-one into house price expectations. Hence, today's house price is extrapolated into the future. Turning to the belief updating Equation (3.5), we see that beliefs have an autoregressive component and are updated according to past observed

²⁰We assume agents' prior variance equals the steady state Kalman variance.

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house price changes. House price updating is increasing in the Kalman gain, g , and decreasing in the persistence of the beliefs ϱ .

Asymmetry in house price beliefs. For our analysis, we may also consider time variation in the house price updating process. Specifically, we allow for heterogeneities in the persistence of beliefs (ϱ) and the Kalman gain (g) below and above a certain threshold $\bar{\omega}$. Variations in the persistence of the belief process can be micro-founded by assuming that agents hold different perceptions of belief persistence (ϱ^h, ϱ^l). Differential Kalman gains can be modeled by allowing the relative variances of the transitory components $((\sigma_e^h)^2, (\sigma_e^l)^2)$ and the persistent components $((\sigma_v^h)^2, (\sigma_v^l)^2)$ to differ above and below the threshold. Augmenting the house price belief model (3.4) by incorporating differential belief updating above and below the threshold $\bar{\omega}$, and proceeding analogously to the derivation in Lemma (3), yields a threshold version of the belief updating model:

$$\mathbb{E}_t^P \hat{q}_{t+1} = \mathbb{1}(\Delta \hat{q}_{t-1} > \bar{\omega}) \left[q_t + \varrho^h \widehat{m}_t \right] + \mathbb{1}(\Delta \hat{q}_{t-1} < \bar{\omega}) \left[q_t + \varrho^l \widehat{m}_t \right] \quad (1.5)$$

$$\begin{aligned} \widehat{m}_t = \mathbb{1}(\Delta \hat{q}_{t-1} > \bar{\omega}) & \left[(\varrho^h - g^h) \widehat{m}_{t-1} + g^h \Delta \hat{q}_{t-1} \right] + \\ & \mathbb{1}(\Delta \hat{q}_{t-1} < \bar{\omega}) \left[(\varrho^l - g^l) \widehat{m}_{t-1} + g^l \Delta \hat{q}_{t-1} \right] \end{aligned} \quad (1.6)$$

This threshold framework captures distinct dynamics in belief adjustments, conditional on whether the extrapolation parameter lies above or below the critical value $\bar{\omega}$. Consequently, agents update their beliefs asymmetrically across housing market regimes, reflecting heterogeneity in their responses to house price fluctuations. This extension provides a more nuanced representation of belief formation, particularly under varying market conditions.

Firms and price setting. We assume a continuum of monopolistically competitive firms that produce intermediate good varieties and have the same beliefs as households. Firm beliefs, however, concern only variables over which households have rational expectations. Therefore, firms are rational. Firm j buys labor $n_t(j)$

from the representative labor packer and produces the variety $y_t(j)$ with a linear technology where labor is the only production factor. The variety is bought by households from both regions. The firm sets its retail price $P_{H,t}(j)$ and maximizes the expected discounted stream of profits, subject to Rotemberg-type adjustment costs. Formally the firm solves:

$$\max_{P_{H,t}(j)} \mathbb{E}_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{P_t} \left[P_{H,t}(j) y_{H,t}(j) - (1 - \tau^\ell) W_t n_t(j) - P_{H,t} \frac{\kappa}{2} \left(\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1 \right)^2 y_{H,t} \right]$$

$$\text{with } y_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} y_{H,t}$$

with $y_{H,t}(j) = \xi_{a,t} n_t(j)$. $\Lambda_t = u'_{c,t}/u'_{c,0}$ denotes the stochastic discount factor and τ^ℓ is a wage subsidy paid by the government. It is selected such that the monopolistic competition distortion is offset in the non-stochastic steady state. The subsidy is financed through a lump-sum tax on the firm. In symmetric equilibrium, all firms choose the same price, $P_{H,t}(j) = P_{H,t} \forall j$. The solution of the firm problem results in a standard currency union version of the Phillips-Curve.

Monetary authority. The monetary authority sets the nominal interest rates according to a standard Taylor rule targeting currency union consumer price inflation:

$$i_t = \frac{1}{\beta} (\Pi_t^{cu})^{\phi_\pi} \xi_{i,t} \quad (1.7)$$

Currency union inflation is the average of region-level consumer price inflation, weighted by the country size: $\Pi_t^{cu} = (\Pi_t)^\gamma (\Pi_t^*)^{1-\gamma}$.

Market clearing. To achieve goods market clearing, each goods market for a variety j must clear. Additionally, labor and housing markets must clear within each region. Finally, the balance-of-payments equations must hold. Aggregation and the specific market clearing conditions are relegated to Appendix (1.B.3).

Equilibrium. We adopt the equilibrium concept of Internally Rational Expectations Equilibrium, as defined in Adam and Marcet (2011):

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Definition 1 (Internally Rational Expectations Equilibrium). *An IREE consists of three bounded stochastic processes: shocks $(\xi_t)_{t \geq 0}$, allocations $([c_{H,t}(j)]_{j \in [0,1]}, [c_{F,t}^*(j^*)]_{j^* \in [0,1]}, b_t, b_t^*, h_t, h_t^*, x_t, x_t^*, n_t, n_t^*)_{t \geq 0}$ and prices $(w_t, w_t^*, q_t, q_t^*, i_t, [P_{H,t}(j)]_{j \in [0,1]}, [P_{F,t}(j^*)]_{j^* \in [0,1]})_{t \geq 0}$, such that in all t*

1. *households choose $[c_{H,t}(j)]_{j \in [0,1]}, [c_{F,t}^*(j^*)]_{j^* \in [0,1]}, b_t, b_t^*, h_t, h_t^*, x_t, x_t^*, n_t, n_t^*$ optimally, given their beliefs \mathcal{P} ,*
2. *firms choose $([P_{H,t}(j)]_{j \in [0,1]}, [P_{F,t}(j^*)]_{j^* \in [0,1]})_{t \geq 0}$ optimally, given their beliefs \mathcal{P} ,*
3. *the monetary authority acts according to the Taylor rule (1.7),*
4. *markets for consumption good varieties, hours, and housing clear given the prices, and the balance-of-payments Equation holds.*

Appendix (1.B.4) contains (i) a proof that Walras' law holds for our economy and (ii) the derivation of the Balance-of-Payments condition. Appendix (1.B.5) presents the system of equations that characterizes the IREE.

1.3.2 Solution method

We solve our model to first order around a non-stochastic and efficient steady state. This preserves analytic tractability at many points in the model and allows us to derive results on household behavior under capital gain extrapolation in closed form. The model presented in Section (1.3.1) admits a unique non-stochastic steady state with zero net inflation and parity of the terms of trade:

Lemma 2 (Non-stochastic steady state). *Consider the model economy presented in Section (1.3.1). As the variance of shocks (actual or perceived) fades, $\text{Var}[\xi_t] \rightarrow 0$, $\xi_t = (\xi_{a,t}, \xi_{c,t}, \xi_{h,t}, \xi_{x,t}, \xi_{i,t}, e_t, v_t)^\top$, there exists one and only one steady state in which net inflation is zero, $\pi_{H,ss} = \pi_{F,ss} = \pi_{ss} = \pi_{ss}^* = 0$, and in which the terms of trade are at parity, $s_{ss} = 1$.*

Proof. See Appendix (1.B.6). □

Linearizing models with capital gain extrapolation is not straightforward. In fact, to the best of our knowledge, we are the first to provide a first-order approximation to a model with capital gain extrapolation under the assumption that agents hold rational expectations outside of the asset pricing block.²¹ The entire exposition focuses on the typical linearized household problem in region H with all derivations being analogous for the typical household in region F .²² For expositional clarity, we omit the specification that housing investment occurs in domestically produced goods. This simplification does not affect the fundamental logic of our solution method.

External and internal variables. Standard first-order solution techniques for models with rational expectations rely on a recursive representation of the equilibrium conditions. For instance, the inter-temporal consumption decision is captured by the forward recursion, the Euler equation:

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} \mathbb{E}_t \hat{r}_{t+1}.$$

Under subjective expectations an equivalent formulation exists:

$$\hat{c}_t = \mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+1} - \frac{1}{\sigma} \mathbb{E}_t^{\mathcal{P}} \hat{r}_{t+1}.$$

At this point, it is important to note that first, we can characterize external variables for which households have distorted expectations. In our case this is only the house price, $\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s}$, for which we have derived a subjective expectation model. Second, for all other external variables, for which households have rational ex-

²¹Winkler (2020) proposes the “conditionally model-consistent expectations” (CMCE) concept as a starting point for linearizing models with capital gain extrapolation. Under CMCE, however, beliefs over all external variables are distorted relative to rational expectations. In our approach, linearized decision rules may be obtained under the assumption that belief distortions apply only to asset prices, allowing to confine the deviations from rational expectations to exactly those variables where survey data allows to discipline the expectations-modeling choices.

²²For any variable $\text{var}_t \notin \{b_t, b_t^*, \Sigma_t, \Sigma_t^*\}$ define $\widehat{\text{var}}_t := \frac{\text{var}_t - \text{var}_{ss}}{\text{var}_{ss}} \simeq \ln \text{var}_t - \ln \text{var}_{ss}$ to first order. For b_t, Σ_t (analogously for b_t^*, Σ_t^*) define $\widehat{\text{var}}_t := \frac{\text{var}_t - \text{var}_{ss}}{y_{ss}}$, which allows for the case that $\text{var}_{ss} = 0$. (That is, we scale deviations in bond holdings and profits by GDP.) Note furthermore that $\widehat{1+r_{t+1}} \simeq \ln(1+r_{t+1}) - \ln(1+r_{ss}) \simeq r_{t+1} + \ln \beta$. We abuse notation slightly and write \hat{r} instead of $\widehat{1+r}$.

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expectations, we can formulate all equilibrium conditions recursively in the usual manner. In this case, the equilibrium-implied distribution measure applies and we can drop \mathcal{P} from the expectations operator. The difficulty arises concerning expectations of household choices or internal variables, in our case $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}$. These variables depend on external variables over which households hold subjective and rational expectations. In this case, we need to determine the subjective expectations distribution of \hat{c}_{t+1} .²³ Our solution method allows us to determine these variables in closed form. We provide a detailed derivation in Appendix (1.B.7), and concentrate here on conveying the intuition of our solution approach.²⁴

Characterizing subjective expectations over internal variables. In solving for the subjectively optimal consumption plan we exploit two key insights into how households behave to first-order that are valid irrespective of which set of beliefs they hold.²⁵ First, there is only one inter-temporal trade-off, namely in consumption, c . Given a path for consumption, the first-order conditions for housing, hours worked, and housing investment uniquely pin down a mapping from external variables to decisions for these internal variables. This insight allows us to concentrate on finding the optimal path for consumption.

The second insight is that to first-order, the permanent income hypothesis holds and consumption depends only on the path of real interest rates, an external variable, and the subjectively expected lifetime income. By iterating forward the Euler Equation we receive

$$\hat{c}_t = -\frac{1}{\sigma} \sum_{s \geq 0} \mathbb{E}_t \hat{r}_{t+s+1} + \mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty} \quad (1.8)$$

where we defined $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty} = \lim_{s \rightarrow \infty} \mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}$. This shows that the only subjective

²³The reason is that households have distorted expectations over at least one price sequence and therefore will make distorted choices; in particular they plan to make choices in the future that are inconsistent with what these choices will be in equilibrium. If we ignored this, i.e. exchanged $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}$ for $\mathbb{E}_t \hat{c}_{t+s}$ in the forward iteration above, the computed equilibrium would be different from the IREE in Definition 2.

²⁴For tractability, we set habit formation, \bar{h} , to zero and we assume that housing investment is made out of the aggregate consumption goods bundle c_t instead of only locally produced goods. Relaxing these assumptions does not affect our methodological approach.

²⁵In particular, our solution method can be used to solve RE models.

expectation variable to remain is $\mathbb{E}_t^{\mathcal{P}} \hat{c}_\infty$.²⁶

At this point the question arises whether this term is merely a technical artifact originating from our methodological approach, or if it possesses intrinsic economic meaning. For expositional purposes it is informative to consider a transitory shock to the model. Suppose that the shock raises house prices, which leads households to believe that house prices will rise further. Consequently, they believe that they will be wealthier in the future, leading to an increase in their current and planned future consumption. This expectation is reflected in the limit by an increase in $\mathbb{E}_t^{\mathcal{P}} \hat{c}_\infty$, thereby capturing a subjective expectations wealth effect. In essence, due to extrapolation, households believe they will be richer than they actually will be. Under rational expectations, this term also exists. However, households recognize that the shock is purely transitory and will not influence their long-term wealth. Therefore, the wealth effect under rational expectations is always zero.

Characterizing the subjective expectations wealth effect. We will now focus on deriving an analytical expression for the subjective expectations wealth effect, $\mathbb{E}_t^{\mathcal{P}} \hat{c}_\infty$. Using the insights from above, the household's first-order conditions are given by the equations in (1.9). (For expositional brevity we have dropped the exogenous shocks and set time-to-build to zero.)

Rational Expectations	Subjective Expectations
$\hat{c}_t = -\frac{1}{\sigma} \sum_{s \geq 0} \mathbb{E}_t \hat{r}_{t+s+1} + \mathbb{E}_t \hat{c}_\infty$	$\hat{c}_t = -\frac{1}{\sigma} \sum_{s \geq 0} \mathbb{E}_t \hat{r}_{t+s+1} + \mathbb{E}_t^{\mathcal{P}} \hat{c}_\infty$
$\hat{h}_t = \frac{\sigma}{\nu(1-\beta)} (\hat{c}_t - \bar{\beta} \mathbb{E}_t \hat{c}_{t+1}) - \frac{1}{\nu} \frac{\hat{q}_t - \bar{\beta} \mathbb{E}_t \hat{q}_{t+1}}{1-\beta}$	$\hat{h}_t = \frac{\sigma}{\nu(1-\beta)} (\hat{c}_t - \bar{\beta} \mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+1}) - \frac{1}{\nu} \frac{\hat{q}_t - \bar{\beta} \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+1}}{1-\beta}$
$\varphi \hat{n}_t + \sigma \hat{c}_t = \hat{w}_t$	$\varphi \hat{n}_t + \sigma \hat{c}_t = \hat{w}_t$
$\hat{q}_t = (1-\eta) \hat{x}_t$	$\hat{q}_t = (1-\eta) \hat{x}_t$

(1.9)

where $\bar{\beta} := \beta(1-\delta)$ and $\mathbb{E}_t \hat{c}_\infty = 0$ as explained above. From this representation it becomes clear, that the difference between subjective expectations and rational expectations lies in the measure \mathcal{P} expectations with respect to next periods consumption and house prices, and the existence of the subjective expectations wealth effect, $\mathbb{E}_t^{\mathcal{P}} \hat{c}_\infty$. Subjective expectations with respect to house prices are easy to characterize, as these

²⁶We have directly used the fact that households have rational expectations over external variables other than house prices, $\mathbb{E}_t^{\mathcal{P}} \text{var}_{t+1} = \mathbb{E}_t \text{var}_{t+1}$ for any variable $\text{var}_{t+1} \neq q_{t+1}$.

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are pinned down by the subjective expectations model. Further note, that subjective expectations on next periods consumption can be characterized by leading the Euler Equation (1.8) one period, once we have a representation for the subjective expectations wealth effect. Therefore, in order to solve the subjective expectations model, it suffices to find a representation for the subjective expectations wealth effect. We find this characterization by combining the first-order conditions with the linearized household budget constraint, (BC) in Appendix-equation (1.27). After iterating over this equation, we can find a closed form expression for the subjective expectations wealth effect.

Proposition 1 (subjective expectations wealth effect). *To first-order around the non-stochastic steady state the subjective expectations wealth effect, $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}$, is given by:*

$$\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty} = Q/\sigma \cdot \underbrace{[\hat{q}_t + M\hat{m}_t]}_{=\Lambda_{t,1}} + C \frac{y_{ss}}{c_{ss}} \frac{1-\beta}{\beta} \left[\underbrace{\sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{z_{t+n}^*\}}_{=\Lambda_{t,2}} + \hat{b}_{t+1} \right], \quad (1.10)$$

where $Q, C \in (0,1)$, $M > 0$ are defined below equation (1.33), and z_{t+n}^* is a function of external variables the household has rational expectations about and is specifically stated in Appendix 1.B.7.

Proof. See Appendix (1.B.7). □

The notation “ss” in the subscript of a variable, denotes its’ steady state values. Equation (1.10) shows that the subjective expectations wealth effect consists of three parts. First, variables for which the household has subjective expectations, $\Lambda_{t,1}$. In our case this is only the house price, \hat{q}_t , and posterior beliefs about house prices, \hat{m}_t . Second, it depends on the expectations of all variables the household has rational expectations about, $\Lambda_{t,2}$, which are collected in z_{t+n}^* . Third, it depends on today’s bond choices \hat{b}_{t+1} . Intuitively, households evaluate the wealth effect under subjective expectations by applying the permanent income hypothesis. In doing so they account for their future decisions by using their first-order conditions resulting in a function that depends only on prices and today’s saving choices. This formulation elucidates the previously discussed intuition regarding the role of subjective expectations in wealth effects. Specifically, an increase in house prices (\hat{q}_t) or in beliefs about future house prices (\hat{m}_t) leads to a rise in $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}$, given that $Q, M > 0$. Agents, anticipating higher wealth as a result of these changes, adjust their expectations accordingly and expect an increase in consumption in the long run.

Discussion. Our method has two important advantages over previous approaches to solving asset price learning models. First, we solve the model using a first-order approximation which makes it fast to solve, easily scalable, and amenable to the analysis of Ramsey-optimal policies. The literature has previously relied on non-linear solution techniques (Adam, Marcet, and Beutel, 2017), or hybrid techniques (Adam, Pfäuti, and Reinelt, 2022) to solve these models. Hence, solution procedures are much more involved and limits the complexity of models that numerically can be solved. Second, our solution method confines subjective expectations to house prices. A previously developed method by Winkler (2020) and Caines and Winkler (2021), which also relies on perturbation, assumes household expectations to conform with the concept of conditionally model-consistent expectations. Under this concept, subjective expectations about one variable lead to spillovers to expectations about other variables. Thus, households will form subjective expectations across all model variables. In our approach, households only hold subjective expectations with respect to one variable, while they remain rational with respect to all other variables. This method of explicitly characterizing choices in terms of lifetime income is general in the sense that it allows solving for household decisions under any time-consistent set of beliefs.

1.4 Analytic Insights

We proceed by deriving the primary channel through which house price extrapolation and heterogeneities in the housing market can produce the patterns observed in Section (1.2). Our analysis demonstrates that the results are robust to regional heterogeneities, whether these are located on the supply or demand side of the housing market. Furthermore, we establish that our findings do not hold under a rational expectations framework.

To do so we examine a simplified version of the model introduced above: we consider a one-region, zero liquidity endowment economy with fully sticky prices. We study the path of house prices within an exercise that, while keeping everything tractable, allows us to understand the effects of expansionary monetary policy: the consumption endowment process is selected such that the real rate drops by ε on impact of the shock and returns to 0 afterwards. This is arguably a parsimonious way to model expansionary monetary policy.

We assume that the housing supply is a function of endowment, and moves in response

1.4. Analytic Insights

to the shock by $-\iota\varepsilon$. Hence, in response to an expansionary monetary policy shock housing supply increases. In the following regional differences in housing supply will be simply modelled as regional variations in ι . In Appendix (1.B.9) we also consider the case of demand-side heterogeneities and show that similar results can be attained as under supply-side heterogeneities.

Throughout this section we will consider parameter choices commonly used in the literature. Specifically, we consider parameter choices in line with Section (1.5), which are stated in Table (3.3.1) and (1.C.2). Detailed derivations are relegated to Appendix (1.B.9). Under the given assumptions the house price and housing market clearing can be expressed as:

$$\hat{q}_t = \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+1} - \hat{r}_{t+1} - \nu(1 - \bar{\beta})\hat{h}_t + \nu(1 - \bar{\beta})^2 \sum_{s=0}^{\infty} \bar{\beta}^s \mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+s+1} \quad (1.11)$$

$$\hat{h}_t - (1 - \delta)\hat{h}_{t-1} = -\iota\varepsilon_t \quad (1.12)$$

1.4.1 Supply-side heterogeneity

As we focus on housing supply side heterogeneities we simplify the housing demand side and assume that $\nu = 1$. The house price is pinned down jointly by the equations (1.11) and (1.12). The differences between rational and subjective expectations arise due to the expectation formation on future house prices, $\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+1}$, and future housing choices, $\mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+s+1}$. Under rational expectations, we can simply use the housing market clearing condition (1.12) to pin down house prices. Doing so yields the following proposition.

Proposition 2 (House prices under Rational Expectations). *In the outlined one-region, zero liquidity endowment economy with exogenous housing supply, and considering a shock monetary policy shock such that $\varepsilon_1 < 0$ and $\varepsilon_t = 0$ for $t > 1$. House prices are given by:*

$$\hat{q}_t = \mathbb{E} \hat{q}_{t+1} - \hat{r}_{t+1} - (1 - \bar{\beta})\hat{h}_t + (1 - \bar{\beta})(1 - \delta)^t \left(1 - \frac{1 - \bar{\beta}}{1 - \bar{\beta}\delta}\right)(-\iota\varepsilon_t)$$

And for the first two periods we have:

$$\hat{q}_1 = -\varepsilon_1 + \omega_h \iota \varepsilon_1$$

$$\hat{q}_2 = (1 - \delta)\omega_h \iota \varepsilon_1$$

with $\omega_h \in (0, 1)$.

Proof. See Appendix (1.B.9). □

Proposition (2) makes clear that an expansionary monetary policy shock raises house prices. The shock fades after the first period. If housing supply increases simultaneously ($\iota > 0$), the house price increase is less pronounced in the first period. Due to the increase in housing supply, house prices will be below the steady state level after the first period, converging back to the steady state going forward.

Moving on to the subjective expectation case, house price expectations, $\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+1}$, are pinned down by the subjective expectations model, equations (3.5) and (3.6). Future expected housing choices, $\mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+s+1}$, can be characterized using the solution method described in Section (1.3.2). We arrive at the following proposition:

Proposition 3 (House prices under Subjective Expectations). *In the outlined one-region, zero liquidity endowment economy with exogenous housing supply, and considering a shock monetary policy shock such that $\varepsilon_1 < 0$ and $\varepsilon_t = 0$ for $t > 1$. House prices are given by:*

$$\hat{q}_t = -\hat{h}_t - \omega_r \hat{r}_{t+1} + \omega_m \hat{m}_t$$

And for the first two periods we have:

$$\begin{aligned}\hat{q}_1 &= -\omega_r \varepsilon_1 + \iota \varepsilon_1 \\ \hat{q}_2 &= (1 - \delta) \iota \varepsilon_1 + \omega_m g \hat{q}_1\end{aligned}$$

with $\omega_r > 1$, $\omega_m > 1$, and $\frac{\partial \omega_m}{\partial \rho} < 0$.

Proof. See Appendix (1.B.9). □

Proposition (3) elucidates several key dynamics. First, an expansionary monetary policy shock leads to an increase in house prices, and given that $\omega_r > 1$, this increase is more pronounced than in the rational expectations scenario. The fact that the house price rises by more on impact under subjective expectations is a direct consequence of the fact that house prices not only convey the degree of scarcity of a good but also reveal information about the future path of the price of an asset. Under rational expectations, agents understand that house prices will mean-revert—while under subjective expectations they apply their perceived model to conclude that house prices will stay elevated. Second, a positive response in housing supply mitigates the rise in house prices induced by the monetary policy shock, and this mitigating effect is stronger compared to the

1.4. Analytic Insights

rational expectations scenario, as indicated by $\omega_h < 1$. Third, belief extrapolation introduces a backward-looking component that drives up house prices. An increase in the Kalman gain (g) amplifies the effectiveness of this channel. Similarly, a decrease in the persistence of belief formation (ϱ) also enhances the channel's potency.

Let us now turn towards understanding how these dynamics come together when two regions are involved, A and B . Consider an expansionary monetary policy shock that is symmetric across regions and an increase in housing supply that is more pronounced in region A , $\iota^A > \iota^B$. Using the results from above, we can now conclude:

Proposition 4 (Differential house price growth responses, supply-side). *The differential house price growth response in region A and B under rational expectations for the first two periods are given by:*

$$\begin{aligned}\Delta\hat{q}_1^B - \Delta\hat{q}_1^A &= \omega_h(\iota^B - \iota^A)\varepsilon_1 \\ \Delta\hat{q}_2^B - \Delta\hat{q}_2^A &= -\delta\omega_h(\iota^B - \iota^A)\varepsilon_1\end{aligned}$$

The differential house price growth response in region A and B under subjective expectations for the first two periods are given by:

$$\begin{aligned}\Delta\hat{q}_1^B - \Delta\hat{q}_1^A &= (\iota^B - \iota^A)\varepsilon_1 \\ \Delta\hat{q}_2^B - \Delta\hat{q}_2^A &= (\omega_m g - \delta)(\Delta\hat{q}_1^B - \Delta\hat{q}_1^A)\end{aligned}$$

where $\omega_m g \gg \delta$ generally holds. And house price extrapolation amplifies regional differences:

$$\frac{\partial(\Delta\hat{q}_1^B - \Delta\hat{q}_1^A)}{\partial g} = \omega_m(\Delta\hat{q}_1^B - \Delta\hat{q}_1^A) > 0$$

Proof. Differential house price growth, $\Delta\hat{q}_j^B - \Delta\hat{q}_j^A$ with $j = 1, 2$, are derived by subtracting regional prices from Proposition (2), and Proposition (3) respectively. \square

Proposition (4) summarizes the paper's main argument. In the subjective expectations framework, supply-side differences lead to differential house price growth responses, reflected by $\iota^B - \iota^A$. As we consider an expansionary policy shock, $\varepsilon < 0$, house price growth in period one will be larger in region B . This holds for the rational expectations and the subjective expectations model. However, in the subjective expectations model the regional differences will be more pronounced since $\omega_h \in (0, 1)$. In the subsequent episodes, we observe a qualitative difference between the rational expectations and the

subjective expectations model. Under rational expectations the regional differences flip: $\Delta\hat{q}_2^B - \Delta\hat{q}_2^A < 0$ as $-\delta\omega_h(\iota^B - \iota^A)\varepsilon_1 < 0$. In the subjective expectations model, this is not the case. Extrapolation introduces a backward-looking term, which overturns the dynamic observed under rational expectations: $\Delta\hat{q}_2^B - \Delta\hat{q}_2^A > 0$ since $\omega_m g > \delta$. Extrapolation thereby provides a dynamic amplification mechanism of regional house price growth rates. The higher g , or the lower ϱ , the higher the coefficient $\omega_m g$, and therefore the stronger the amplification channel.²⁷

This mechanism explains why the dispersion in regional house price growth is greater during busts than in booms. As we will demonstrate below, the formation of house price beliefs differs between boom and bust periods. In particular, the Kalman gain (g) is higher during busts than in booms. As a result, the amplification effect of house price extrapolation is stronger in bust periods, leading to increased regional dispersion in house price growth. Importantly, this mechanism is absent in the rational expectations model, rendering it incapable of accounting for the asymmetry in regional dispersion in house price growth rates between booms and busts. Therefore, the rational expectations framework fails to capture the differential effects observed in the dispersion of house price growth across regions during these distinct phases of the housing market cycle observed in the data.

1.5 Model calibration

In this section, we will describe the model calibration. We will start by focusing on the calibration of the subjective expectations belief formation process. To do so, we will turn to the data to directly estimate the belief persistence (ϱ) and the Kalman gain (g): Notably, we find that in busts the Kalman gain is larger than in booms.

We then move to regional heterogeneities and present evidence that housing supply-side differences lead to regional responses of house prices to a monetary policy shock. We use this analysis to calibrate cross-regional supply-side differences in time-to-build.

We calibrate the slope of the Phillips-Curve such that the model matches the peak response of a 25 bp monetary policy shock unconditional on a house price boom or bust. The remaining parameters are chosen in line with the literature.

²⁷Numerical explorations show that $\omega_m g$ can be as large as 73, which constitutes a significant amplification mechanism. Realistic values for the Kalman gain g range between 0.01 – 0.025. In our numerical example we assumed a Kalman gain of 0.02.

1.5. Model calibration

1.5.1 House price beliefs and boom-bust dynamics

In Section (1.3), we construct a model that breaks the rational expectations hypothesis with respect to house prices and introduces a Bayesian belief updating model to describe the house price belief formation. This modeling approach is grounded in a substantial body of empirical literature that examines the dynamics of belief formation in the context of housing markets.²⁸ In appendix (1.C.3), we conduct our own analysis and demonstrate that our modelling approach remains valid when conditioning on monetary policy shocks. Specifically, we examine the response of forecast errors to these shocks. Under the assumption of rational expectations, this response should theoretically be zero; however, our analysis reveals a substantial and statistically significant response. We observe a sluggish adjustment of beliefs following the shock, with evidence of initial over-pessimism in the first month, which subsequently transitions to over-optimism in the longer term. These findings are consistent with existing literature and suggest that a Bayesian belief updating model is well-suited to capturing the observed dynamics in house price expectations.

House price belief formation. Our findings in Section (1.4) suggested that stronger belief updating in busts are essential to explain diverging regional house price growth rates in busts. Therefore, we estimate the belief updating process described in Section (1.3) and allow for potential differences in the belief updating process. Specifically, we test two specifications: a linear one and threshold specification allowing for regime switching. The linear process reads:

$$\mathbb{E}_t^{\mathcal{P}} \Delta q_{t+1} = c + \varrho(\varrho - g) \times \mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t + \varrho g \times \Delta q_{t-1}. \quad (1.13)$$

Equation (1.13) can be derived by combining equations (3.5) and (3.6). $c = \varrho \bar{m}(1 - \varrho)$ is a constant with \bar{m} being long-run house price growth expectations. The belief updating model is in monthly frequency, and the sample is 2007 to 2020. We obtain the month-on-month percentage change in house price expectations, $\mathbb{E}_t^{\mathcal{P}} \Delta q_{t+1}$, by dividing the year-on-year percentage changes from the Michigan Survey by 12. House price growth data is taken from Case-Shiller National House Price Index. To estimate this model, we replicate the belief updating process employed by agents in our model, which follows a recursive updating procedure. Specifically, to estimate this process we feed in realized

²⁸See: Adam, Pfäuti, and Reinelt (2022), Armona, Fuster, and Zafar (2019), Case, Shiller, and Thompson (2012), Kuchler and Zafar (2019), and Ma (2020)

past house price growth data, Δq_{t-1} . For a given tuple (ϱ, g, \bar{m}) and by recursively updating $\mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t$, we obtain a sequence of house price growth expectations. We then use a solver choosing (ϱ, g, \bar{m}) such that the MSE between the data and the fitted values is minimized.²⁹ For the estimation procedure we also impose that $\varrho, g \in (0, 1)$.

Equation (1.14) allows for heterogeneity in ϱ and g below and above a certain threshold. It is based on equations (1.5) and (1.6). The estimation procedure is equivalent to the linear model, only now we choose $(\varrho^h, \varrho^l, g^h, g^l, \bar{m}^h, \bar{m}^l, \omega)$ to minimize the MSE between fitted values and the data. Notice that the threshold is also estimated and that the regime depends on past observed house price growth, Δq_{t-1} .

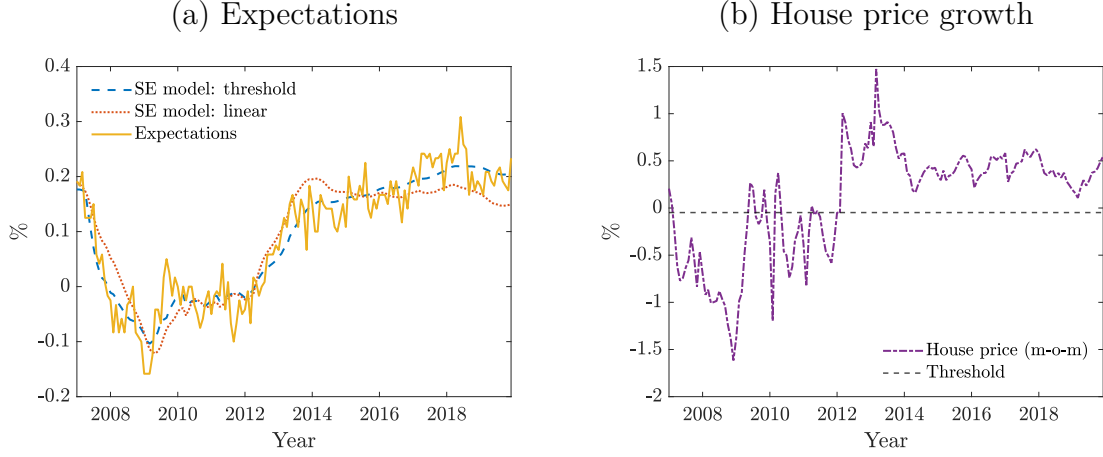
$$\begin{aligned} \mathbb{E}_t^{\mathcal{P}} \Delta q_{t+1} = & \mathbb{1}(\Delta q_{t-1} > \omega) \left(c^h + \varrho^h (\varrho^h - g^h) \times \mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t + \varrho^h g^h \times \Delta q_{t-1} \right) + \\ & \mathbb{1}(\Delta q_{t-1} < \omega) \left(c^l + \varrho^l (\varrho^l - g^l) \times \mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t + \varrho^l g^l \times \Delta q_{t-1} \right) \end{aligned} \quad (1.14)$$

The estimation results are presented in Figure (1.3). The solid yellow line in panel (a) plots the expectations data from the Michigan survey, the blue dashed line represents the threshold model estimates, and the dotted red line shows the linear model. The dashed-dotted purple line in panel (b) depicts the month-on-month house price growth, while the dashed black line indicates the threshold for the threshold model. Turning to the linear model first, we find that it does a reasonable job of explaining the expectation formation process. However, it lags behind during the bust episode and underestimates expectations for house price growth in the recovery following the Recession. In comparison, the threshold model improves on these dimensions. It captures the bust without an obvious lag and performs better in recovery episodes. The estimated threshold for this model is close to zero, suggesting that the updating behavior differs during periods when house prices rise (booms) compared to when they fall (busts).

²⁹This procedure demands a starting value for $\mathbb{E}_{t-1}^{\mathcal{P}} \Delta q_t$ which can affect the outcomes. To minimize the effect of the starting values on the estimated parameters, we start in 1987 to feed in monthly house price growth data, thereby obtaining initial values for 2007M1. Changing the starting value in 1987 has insignificant effects on our results.

1.5. Model calibration

Figure 1.3: House price belief model, US



Notes: SE model, threshold (blue, dashed): fitted values of Equation (1.14); SE model, linear (red, dotted): fitted values of Equation (1.13); Expectations (yellow, solid): mean expectations data from the Michigan Survey; House price (m-o-m) (purple, dashed-dotted): Month on month percentage change in house price. The black dashed line in panel (b) depicts the threshold in the threshold model.

The estimated parameters from the models are shown in Table (1.2). We find a ϱ of 0.97 and a Kalman gain of 0.0175 for the linear model. For the threshold model, we find a ϱ^h of 0.99, and a Kalman gain, g^h , 0.0117 in booms. In busts, ϱ^l decreases to 0.91, while g^l increases to 0.0233. All parameters are roughly in line with the literature. The differences in the parameters across regimes transparently show why the threshold model can fit the data better than the linear model: In busts when house prices and beliefs fall drastically in a short period, a higher Kalman gain, and a lower persistence parameter enable faster pass-through from observed house prices to beliefs. In booms, when house prices and beliefs recover steadily over a longer period, expectations are better matched by a slow-moving process with high persistence. This is achieved by a higher degree of persistence ϱ^h and a lower degree of updating g^h . The inclusion of the threshold component decreases the MSE by roughly 47%.

We also perform a robustness exercise to study whether the parameter differences across regimes affect the model performance. To do so, we impose the estimated parameters from above and below the threshold, subsequently on the linear model. We find a significant increase in the MSE, implying that the differences in the estimated parameters of the threshold model matter for the model performance.

Table 1.2: House price belief model: parameters

<i>Specification</i>	ϱ^{lin}	ϱ^h	ϱ^l	g^{lin}	g^h	g^l	ω	$\frac{MSE^i}{MSE^{lin}}$
<i>Baseline :</i>								
<i>linear</i>	0.97			0.0175				1.00
<i>threshold</i>		0.99	0.91		0.0117	0.0233	-0.048	0.53
<i>Robustness :</i>								
<i>high regime</i>	0.99			0.0117				4.66
<i>low regime</i>	0.91			0.0233				8.57

Notes: linear: estimated parameters from Equation (1.13); threshold: estimated parameters from (1.14); Robustness exercises are explained in the main text.

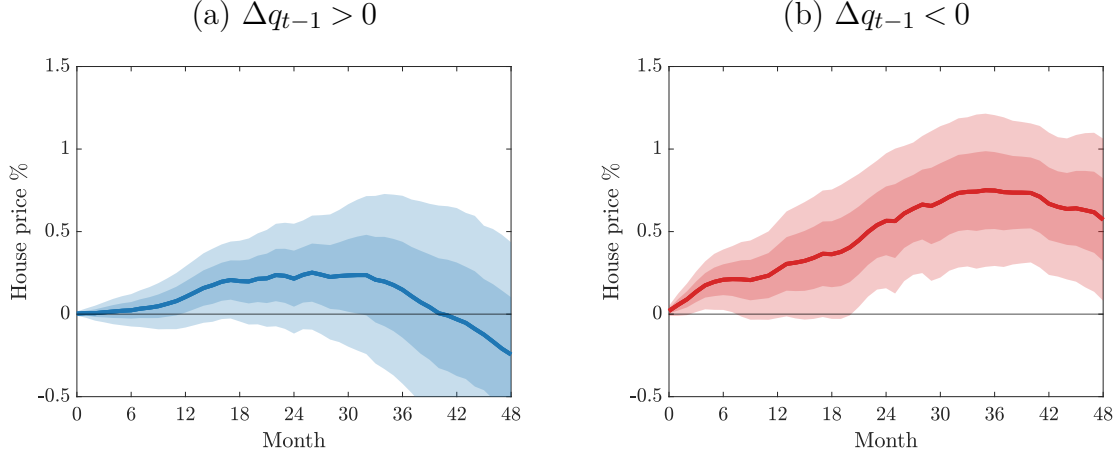
House prices in booms and busts. Our findings indicate that house price expectations updating is more pronounced in busts. This, in turn, can affect house prices themselves: a larger reaction in expectations translates into economic decision-making and will eventually be reflected in prices. To test whether this is the case, we run local projections on house prices responding to a monetary policy shock, conditioning on booms and busts. The estimation equation is given by:

$$q_{t+h} = \alpha^h + \mathbb{1}(\Delta q_{t-1} > 0) \times \beta_1 \epsilon_t^{MP} + \mathbb{1}(\Delta q_{t-1} < 0) \times \beta_2 \epsilon_t^{MP} + x_t + u_{t+h} \quad (1.15)$$

The monetary policy shock, ϵ_t^{MP} , is the high frequency identified and orthogonalized shock from Bauer and Swanson (2023). The left-hand side variable, q_{t+h} , is the log house price. The sample runs from 1990-2019 and is in monthly frequency. The controls, x_t , contain 12 lags of the left-hand side variable, log of industrial production, the log of CPI, the FFR, and the shocks. We will focus on expansionary monetary policy shocks throughout the whole empirical analysis. Figure (1.4) plots the results. The blue line shows the response if house prices were increasing in the past, and the red line if they were decreasing. We find that house prices are notably more responsive to monetary policy shocks in times when they have been decreasing. The peak response of the point estimates almost doubles. Also, for the most part of the dynamic response, the boom and bust confidence intervals measured at one standard deviation, do not overlap, indicating a statistically significant difference at a 32% confidence level. In connection with our previous findings about belief updating in booms and busts, this result indicates that stronger belief updating in busts indeed affects realized prices. In Appendix (1.C.4) we show that this result also extends to forecast errors responding to monetary policy shocks.

1.5. Model calibration

Figure 1.4: House price response to MP shock, boom-bust



Notes: Responses to expansionary MP shock (1 std); Confidence Intervals: 68% and 95% (Newey-West).

1.5.2 Cross-regional heterogeneity and housing supply

Our findings in Section (1.4) indicated that structural regional disparities within the housing market are crucial in explaining why house price growth exhibits greater dispersion during economic downturns than in periods of expansion. In the following, we focus on a specific dimension of regional heterogeneity—variation in the housing supply—which has been shown to influence regional house price dynamics.³⁰ Recent research by Aastveit and Anundsen (2022) has linked regional variations in housing supply elasticities across US metropolitan areas to differential responses in house prices to monetary policy shocks. Building upon their work, we demonstrate that these findings apply at the US state level. Additionally, we provide evidence that analogous patterns are observable in the Euro Area.

Econometric setup. To study the reaction of these regional variables to a common monetary policy shock, we estimate panel local projections to an externally high-frequency identified monetary policy shock. Equation (2.1) represents the empirical specification:

$$q_{n,t+h} = \alpha_n^h + \beta^h \epsilon_t^{MP} + \gamma^h \epsilon_t^{MP} \times z_n + x_{n,t} + u_{n,t+h}, \quad h = 0, 1, \dots, H. \quad (1.16)$$

³⁰See, for example, Glaeser, Gyourko, and Saiz (2008), Guren, McKay, Nakamura, and Steinsson (2021), Mian, Rao, and Sufi (2013), Mian and Sufi (2014), and Saiz (2010).

For the left-hand side variable (q_n) is the log of house prices. ϵ_t^{MP} denotes the monetary policy shock. We further interact this shock with a region-specific variable capturing supply-side heterogeneities, which we denote as z_n . $x_{n,t}$ is a vector of aggregate and regional controls. A time-fixed effect and a region-fixed effect are also included. We use this empirical specification for the US, as well as for the Euro Area. For consistency, we study a one-standard deviation monetary policy shock and standardize the interaction coefficients throughout all exercises.

Results. For the US we focus on state-level data in the cross-section.³¹ The monetary policy shock is the same as before and taken from Bauer and Swanson (2023). The interaction term is the house price sensitivity indicator from Guren, McKay, Nakamura, and Steinsson (2021). It measures the responsiveness of metropolitan area house prices to an increase in house prices at the Census region level controlling for a broad range of local economic conditions. It aims to capture housing supply side heterogeneities.³² Empirically, housing supply elasticities reflect regional geographical or administrative constraints that influence the construction sector (Saiz, 2010). To facilitate the interpretation of this measure and to enable a tractable integration of our empirical estimates into the model, we interpret housing supply elasticities as a broad indicator of the time required to complete housing construction. We show that the housing sensitivity indicator is correlated with time-to-build measured at the Census division level in Appendix (1.C.5). To obtain state-level housing sensitivity measures, we aggregate the metropolitan-level data by weighing them according to population size. The vector of controls includes 8 lags of the left-hand side variable, the log of US GDP, the log of the GDP deflator, the FFR, the shock, and the interacted shock term. The sample runs from 1990 to 2019 and is in quarterly frequency.

Figure (1.5) plots the response of nominal house prices to an expansionary monetary policy shock. We find a sizeable and persistent increase in house prices in the mean coefficient, as seen in panel (a). Further, the interaction coefficient, shown in panel (b), is positive and persistent. Both IRFs are significant at least at a 90% confidence level. Our findings indicate that house prices are increasing in response to an expansionary monetary policy shock and they increase by more in states where supply is more constrained. These findings are in line with Aastveit and Anundsen (2022). Appendix (1.C.6) shows

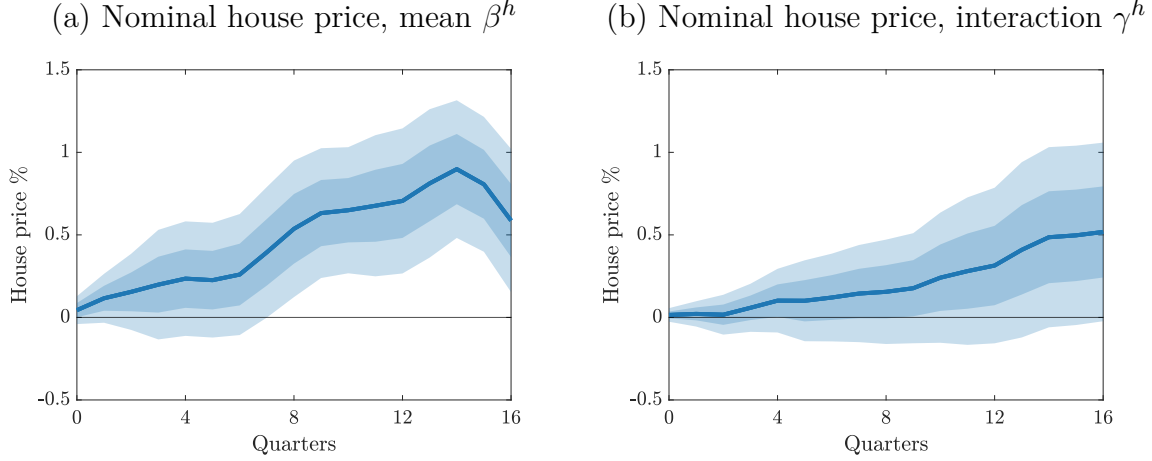
³¹Hawaii and Alaska are not included in the sample due to insufficient data coverage.

³²This indicator can be understood as a proxy for supply-side elasticities in the spirit of Saiz (2010). Contrary to the supply-side elasticities estimated by Saiz (2010), the house price sensitivity indicator is uncorrelated with demand-side characteristics.

1.5. Model calibration

that these results carry over to the Euro Area.

Figure 1.5: House price response to MP shock, US



Notes: House price response to expansionary MP shock (1 std); The interaction term as been standardized; Confidence Intervals: 68% and 95% (Driscoll and Kraay, 1998).

Calibrating supply side heterogeneities. To calibrate the model, we use the time-to-build measure presented in Appendix (1.C.5). A significant proportion of census divisions exhibit an average construction duration of approximately six months, measured from the issuance of building authorization to project completion. Notably, in two densely populated regions on the East Coast, the reported time-to-build extends to nearly ten months. Considering the additional time required for obtaining building permits and completing the sale of the property, we set the time-to-build parameters in the model to $\tau = 2$ and $\tau^* = 4$ quarters.³³

1.5.3 Remaining model parameters

Table (3.3.1) lists the parameter values, all of which are symmetric across regions. First, focusing on the household sector, the labor disutility shifter, the inverse Frisch elasticity, and the intertemporal elasticity of substitution are set to standard values in line with the literature. The discount factor is set to achieve a 1.5% steady state interest rate. We further assume the regions are symmetric in size. We set housing depreciation to

³³The time it takes to obtain a building permit can vary largely across cities. In Dallas, for instance, it may take only a few business days, while in New York it takes between 1-3 months (see: permitflow.com). This difference is reflected in our calibration.

2% per quarter. Moving to the production sector, we set the home bias, the elasticity of substitution across regions and goods in accordance with Bletzinger and Thadden (2021a). We will choose the persistence parameter of house price beliefs and the Kalman gain as presented in Table (1.2). The Taylor rule weight on inflation is 1.5, which is standard. In terms of price adjustment, we choose the price adjustment costs such that the peak response of house prices to a monetary policy shock, unconditional on booms or busts, matches the response in the data. The slope of the Phillips Curve is given by $\frac{\epsilon-1}{\kappa} = 0.018$. This is in line with the recent literature.³⁴

Table 1.3: Model parameters (symmetric parameters)

Parameter		Value	Description	Source/ Target
Households	χ	1.000	labor disutility shifter	standard
	φ	1.250	inverse Frisch elasticity	standard
	σ	2.000	inverse of intertemp. EOS	standard
	ν	1.000	housing utility elasticity	Iacoviello (2005)
	δ	0.020	housing depreciation	2% quart. depreciation
	β	0.995	discount factor	standard for quart. frequency
	γ	0.500	relative region size	symmetric regions
Goods agg. & production	λ	0.800	home bias	
	ς	1.000	EOS across regions	Bletzinger and Thadden (2021a)
	ϵ	6.000	EOS across varieties	
	κ	272.18	price adjustment costs	slope of 0.0125 for the PC
	η	0.800	elasticity of housing prod.	Adam, Pfäuti, and Reinelt (2022)
Policy	ϕ_π	1.500	Taylor coefficient	standard

Notes: All parameters depicted above are equal across countries. One period in the model is one quarter.

Table (1.C.2) in the Appendix shows the allocation and prices in the non-stochastic

³⁴Adam and Billi (2006) set the slope of the Phillips Curve to 0.057. However, it has been argued that the slope of the Phillips Curve has decreased, see i.e Del Negro, Lenza, Primiceri, and Tambalotti (2020) and Hazell, Herreno, Nakamura, and Steinsson (2022). Our parameter choice is in line with their estimates.

1.6. Quantitative Results

steady state. We choose a higher degree of time-to-build in the foreign region. The steady state value for the housing preference shifter is chosen such that we attain a symmetric steady state in the allocation variables. This modeling choice ensures that all cross-regional differences result from the structural heterogeneity on the housing supply side. The steady state value for the house price is the only variable that differs across countries. Symmetric steady state values for bond levels, which are zero for both countries, also imply that there is no net-borrower or net-saver country in the steady state. Changes in monetary policy will therefore not lead to Fisherian debt revaluation effects. We parameterize the model to match the housing sector in the US economy and target a steady state housing investment-to-consumption ratio, $\frac{x_{ss}}{c_{ss}}$, of 6.5% as in Adam, Pfäuti, and Reinelt (2022).

1.6 Quantitative Results

In this section, we will present our findings. First, we show that the model is able to replicate aggregate house price responses to a monetary policy shock, conditional and unconditional on booms and busts. Second, the model captures the documented boom-bust-asymmetry in regional house price growth dispersion.

1.6.1 Aggregate house prices in booms and busts

We will start by studying the model performance on an aggregate level. All aggregate variables are a convex combination of the regional variables weighted by their size. Hence, the aggregate house price is given by $\hat{q}_t^{agg} = \gamma \hat{q}_t + (1 - \gamma) \hat{q}_t^*$. We will study the aggregate house price response in the model and the data, unconditionally and conditionally on being in a boom or a bust. As discussed above, we target the peak response unconditionally of being in a boom or bust. We then study the performance of the rational expectations model and the subjective expectations model conditional on being in a boom or bust. For the unconditional response in the model we simply consider the linear model using the parameterization described above. For the unconditional empirical response we estimate Equation (1.15) and drop the conditionality on the boom-bust regimes with respect to the monetary policy shocks. The conditional empirical response is given by local projections estimated from Equation (1.15). This setup estimates house price responses to a monetary policy shock conditional on being in a boom or a bust. An appropriate model counterpart produces the response to a monetary policy shock

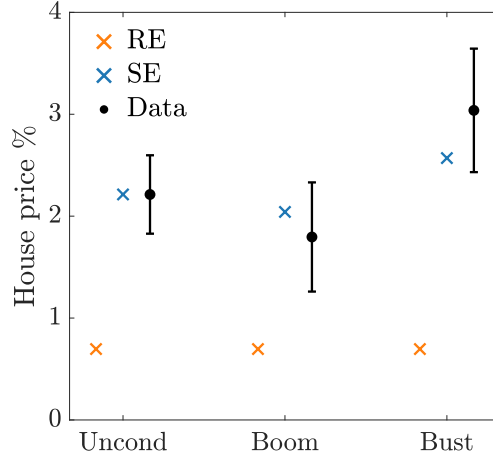
conditional on being in either a boom or bust regime. To construct this conditionality in the model, we use a linear model but adjust the model parameters such that the model captures either boom or bust dynamics in house price updating. We use the parameters estimated in Table (1.2). For the boom we have a persistence parameter of $\varrho^h = 0.99$ and a Kalman gain of $g^h = 0.0117$. For the bust period we have $\varrho^l = 0.91$ and $g^l = 0.0233$. Importantly, the respective parameters have no effect on the steady state. Therefore, the shock will hit both models in the same steady state but the dynamics will be different due to differential updating behavior. We consider a 25 basis points expansionary monetary policy shock across all models and their empirical counterparts.³⁵ As discussed previously, these shocks are empirically too small to affect the transition from one regime to the other, thereby validating our quantitative exercise. We will first focus on the magnitudes of the responses in the model and the data, after which we move on to the dynamics.

Figure (1.6) plots the peak responses in the model and the data, unconditional and conditional on booms and busts. First, the rational expectations model is unable to match the peak response observed in the data. It misses the peak response in the data by roughly a factor of 3. Further, the response is equivalent across boom and bust periods as there is no source of asymmetry in the rational expectations model. In contrast, the subjective expectations models conditioning on booms and busts match their empirical counterparts quite well. For the house price response in a boom, we find a peak response of 2.0 in the model and 1.8 in the data. The model peak response lies within one standard deviation of the empirical estimate. For the bust case, we find a peak response of 2.6 in the model and 3.0 in the data. Accounting for estimation uncertainty, the model lies in the range of the one standard deviation confidence intervals. The results reflect our findings from Proposition (2) and (3). Under subjective expectations, the response to a monetary policy shock is scaled up compared to the rational expectations counterpart. Additionally, extrapolation further dynamically amplifies house price responses. Both of these channels seem to be very important to match the magnitude of the house price response in the data.

³⁵To determine the size of 25 bp monetary policy shock in the data, we estimate a local projection exercise regressing the FFR on the monetary policy shock. We choose the minimum of the IRF from the first 12 months as a reference point to determine the empirical shock size. We use this approach unconditional and conditional on booms/busts.

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Figure 1.6: Peak response of house prices to a MP shock, model vs. data

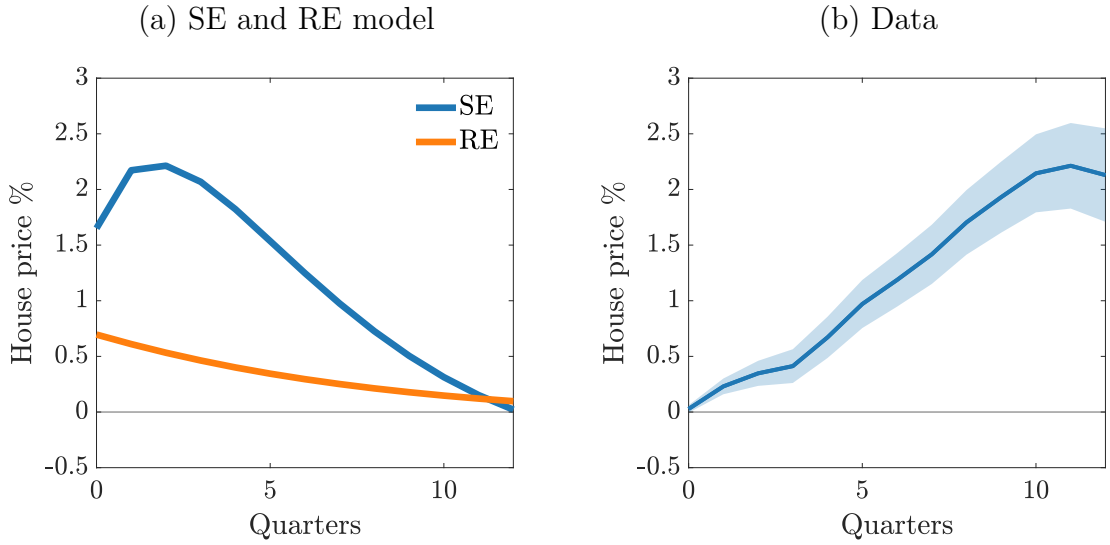


Notes: Response to expansionary MP shock (25 bp); Confidence Intervals: 68% (Newey-West); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

We will now turn to the dynamic responses in the model and the data. Figure (1.7) shows the response of house prices to a 25 basis point expansionary monetary policy shock in the model and in the data unconditional on booms and busts. The empirical response is depicted in panel (b). The model counterparts, in panel (a), show the response of the linear model under subjective expectations and rational expectations. In terms of dynamics we observe a very sluggish response in the data, the peak response is only reached after 10 quarters. In the rational expectations model, we observe no hump shaped pattern at all. House prices peak on impact and return to the steady state thereafter. In the subjective expectations model we do observe some sluggishness in the house price dynamics. House prices respond on impact, increase for the first four quarters, after which the model converges back to its' steady state. The dynamic can be explained through the extrapolative belief structure. After the initial shock, agents believe that house prices will increase further and invest into housing. They will continue to do so until their beliefs are not met, after which they will adjust their behaviour and the model returns to its' steady state. Quantitatively, it turns out that this behaviour is too short-lived to explain the sluggishness observed in the data. The inability of DSGE models to match the persistence in the data is well known. Therefore, medium sized DSGE models such as Smets and Wouters (2007) tend to add backward looking components, for instance habit formation, to match these dynamics. While the extrapolative beliefs do improve the model performance to a certain degree, they cannot

capture the full persistence of the response. This indicates that some sort of sluggish adjustment behaviour in housing demand, for instance habit formation on housing or housing search frictions, are needed to exactly match the data.

Figure 1.7: House price response to MP shock, model vs. data

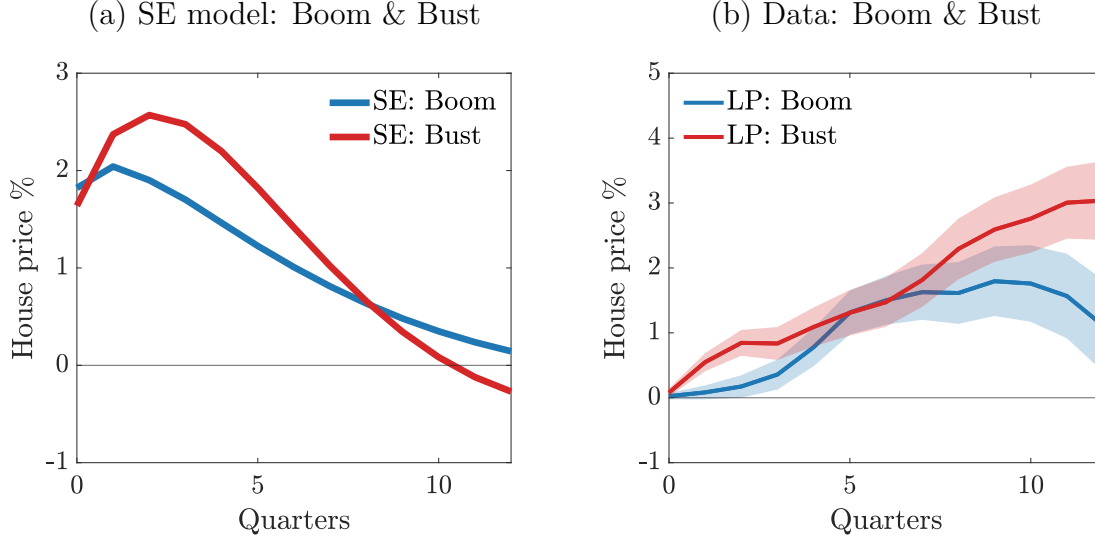


Notes: Responses to expansionary MP shock (25 bp); Confidence Intervals: 68% (Newey-West).

We will now turn to house price responses in booms and busts. Panel (b) in Figure (1.8) plots the empirically estimated impulse responses from the local projections exercise. The dynamics are equivalent to the ones depicted in Figure (1.4), but scaled to match a 25 basis point increase. We only plot 68% confidence bands. Panel (a) shows the subjective expectations model responses for the boom and bust parameterization discussed above. In terms of dynamics, we observe the same short-comings as in the unconditional case. The model is unable to capture the full persistence observed in the data. However, we observe a less persistent response in the boom relative to the bust. The model is able to capture this pattern qualitatively.

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Figure 1.8: House price response to MP shock in booms and busts, model vs. data



Notes: Responses to expansionary MP shock (25 bp); Confidence Intervals: 68% (Newey-West); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

1.6.2 Cross-regional heterogeneity in booms and busts

Having demonstrated that our model can replicate aggregate house price responses during both boom and bust periods, we now turn our attention to analyzing cross-regional heterogeneities. We evaluate the model's performance in response to a monetary policy shock conditional on booms and busts.

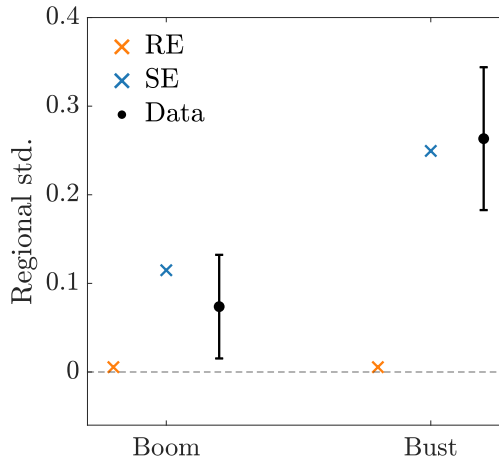
Regional house price growth dispersion. We begin by comparing the cross-regional house price growth standard deviation in the data to those in the model. Empirically, we use the local projections presented in Figure (1.2). As for house prices, we focus on the impulse responses in booms and busts at their respective peaks. We compare the empirically estimated peak responses with their counterparts in a rational expectations and subjective expectations model conditional on being in a booms or busts. Figure (1.9) presents the results.

Focusing on the rational expectations model, we find that the model is unable to generate any sizable regional differences: the peak response is almost zero. As before, the rational expectations model is also unable to generate any differences between booms and busts. Moving to the subjective expectations model, we find that the model slightly overstates the peak response in a boom relative to the data. However, accounting for es-

timination uncertainty, we are well within the one standard deviation confidence intervals. In the bust the model does extremely well, it only marginally understates the empirically estimated peak response.

These results are in line with Proposition (4): Housing supply side heterogeneities create differences in house price growth, which in turn is amplified by extrapolation. Larger extrapolation in busts means stronger amplification and therefore larger cross-regional differences. Under rational expectations the amplification mechanism is absent, therefore the model is only able to create minor cross-regional differences.

Figure 1.9: Peak response of reg. house price growth std. to a MP shock, model vs. data



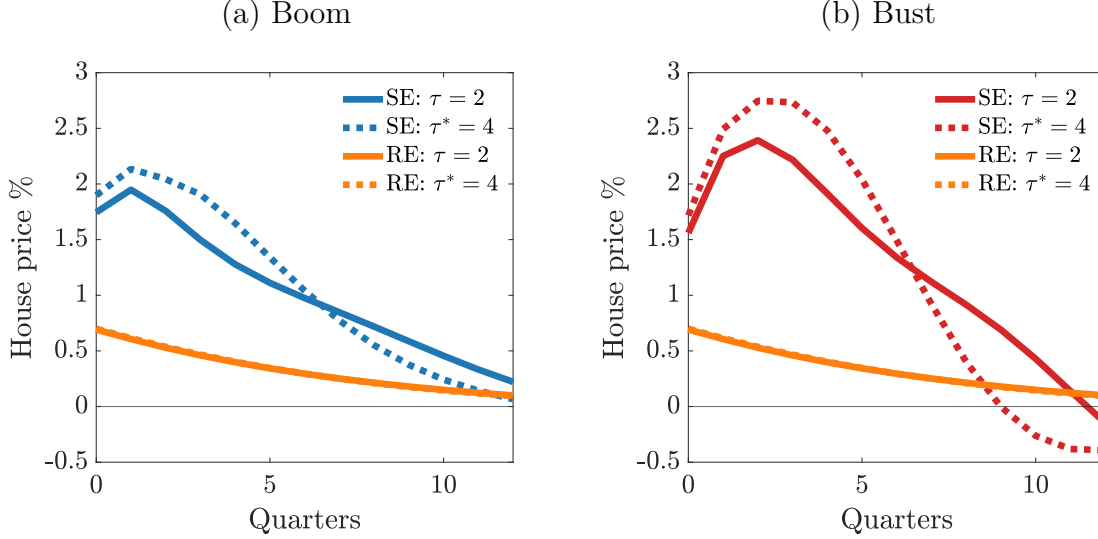
Notes: Response to MP shock (25 bp); Confidence Intervals: 68% (Newey-West); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

House prices and economic activity. After establishing that regional house price growth responses to monetary policy shocks differ between boom and bust periods, we now shift our focus to examining house price responses in levels and, ultimately, their impact on aggregate economic activity. These dynamics are of particular interest due to their implications for welfare and, consequently, their relevance for policy considerations. In the following, we will only focus on the subjective expectations model since our earlier results showed that regional disparities in the rational expectations model are negligible. Figure (1.10) shows the response of house prices to a monetary policy shock conditional on booms and busts. First, we find that generally, the house price is more responsive in regions where time-to-build is larger, hence in the region where $\tau^* = 4$. Second, the

1.6. Quantitative Results

differences between the regions is larger in times of a house price bust. Both of these findings are in line with the results presented in Section (1.4).

Figure 1.10: Regional house price response to MP shock in booms and busts

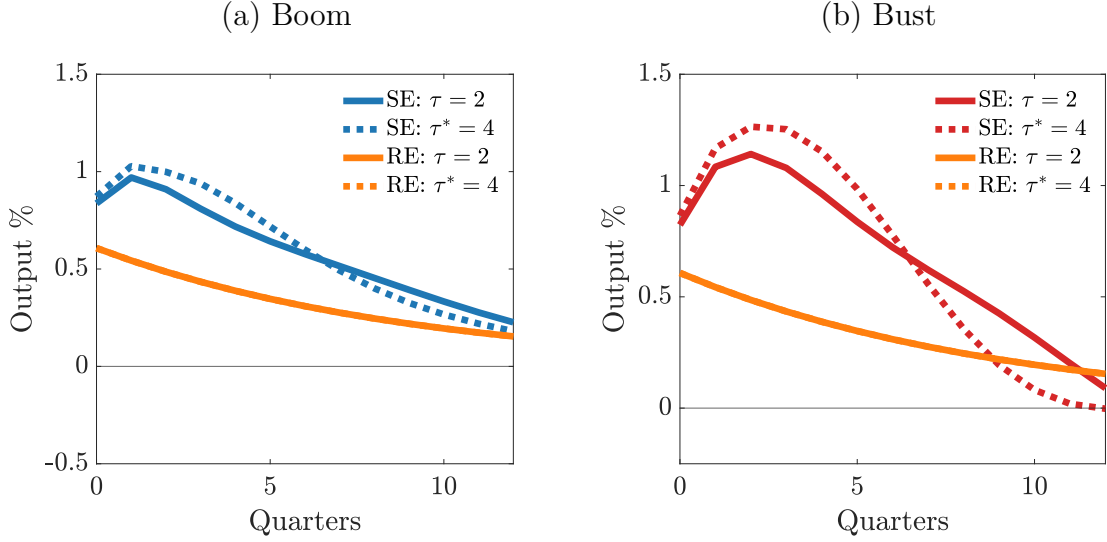


Notes: Responses to expansionary MP shock (25 bp); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

In terms of aggregate activity, we study the response of regional output to a monetary policy shock, again conditioning on booms and busts. Figure (1.11) presents the results. We observe that the region in which house prices are more responsive, the region with a higher time-to-build, also experiences a larger expansion in output. The underlying intuition is as follows: as house prices rise, households expect future price increases and subsequently raise their investment in housing. This surge in housing investment leads to a corresponding increase in output. Due to stronger extrapolation during busts, the output response and regional differences in output are more pronounced in these periods. These findings highlight that regional differences in house price growth translate into house price levels and output variations. As these regional disparities propagate through the model, they eventually affect consumption, labor markets, and inflation. Consequently, these dynamics influence welfare and become of primary importance for monetary policy. In the next section, we will discuss the policy implications of these results. Figure (1.D.1) in the Appendix shows the response of regional house prices and output under rational expectations. We observe that the rational expectations model is unable to create any sizable regional differences. This result is in line with our Propositions

from Section (1.4).

Figure 1.11: Regional output response to MP shock in booms and busts



Notes: Responses to expansionary MP shock (25 bp); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

1.7 Policy analysis

Turning to policy implications, we study how targeting house prices in the Taylor rule changes aggregate variation and cross-regional variation. The linearized Taylor rule we consider is given by:

$$i_t = \phi^\pi \hat{\pi}_t^{agg} + \phi^q \hat{q}_t^{agg} \quad (1.17)$$

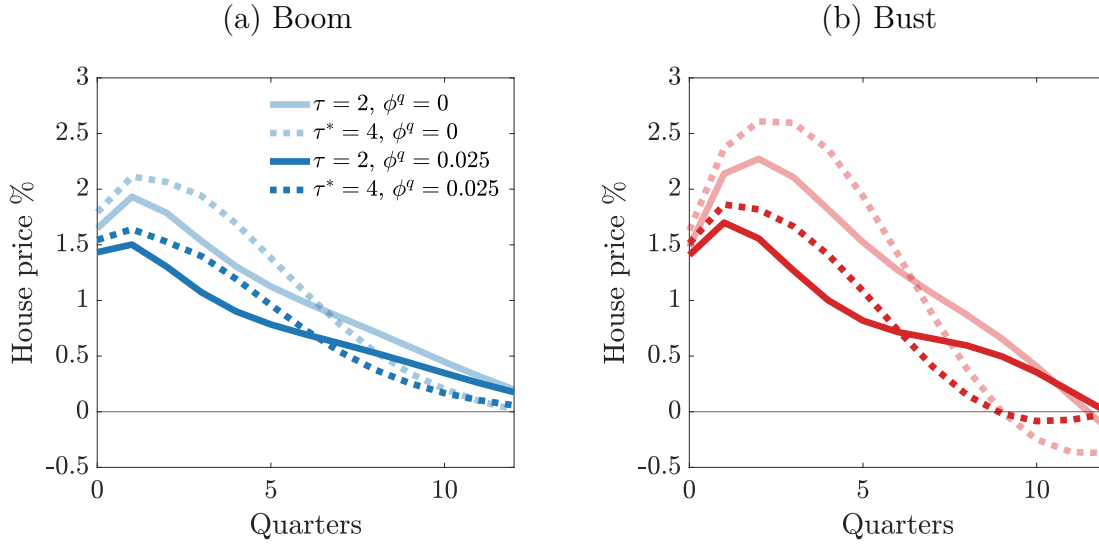
For our baseline case, we consider a rule with $\phi_\pi = 1.5$ and $\phi_q = 0$, as we did for all exercises above. Under house price targeting, we increase the weight on house prices to $\phi_q = 0.025$. We study the response of house prices to an expansionary productivity shock in the baseline case and under house price targeting.

Impulse responses under house price targeting. Figure (1.12) presents the house price responses to a productivity shock in booms and busts comparing house price targeting to our baseline. The response of house prices to a productivity shock under the baseline model closely mirrors the response to a monetary policy shock, as outlined in Section (1.6). Specifically, both house prices and regional disparities in

1.7. Policy analysis

house prices tend to be more pronounced during busts than during booms. When house prices are explicitly targeted, however, the response of house prices is dampened, with a corresponding reduction in regional heterogeneity. Furthermore, a comparison of booms and busts reveals that monetary policy exhibits greater effectiveness during busts, as the relative reduction in both house prices and regional heterogeneity is more substantial in busts than in booms. The increased responsiveness of house prices can be attributed to a larger pass-through effect from house prices to expectations, which arises from a heightened degree of extrapolation. As a result, policy changes are also transmitted to house prices with greater intensity.

Figure 1.12: House price targeting: house price response in booms and busts



Notes: Responses to expansionary productivity shock (100 bp); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

Figure (1.D.2) in the Appendix shows that these results carry over to aggregate activity. House price targeting reduces the response of output and regional heterogeneity in output. As for house prices, house price targeting is more effective in busts. Intuitively, house prices drive housing investment and thereby output. By targeting house prices the central bank implicitly reduces housing investment and consequently output.

Under rational expectations, this result breaks down (see Figure (1.D.3) in the Appendix): the house price and output response are reduced but to a smaller degree than under subjective expectations. Further, regional heterogeneity is unaffected, as the model is unable to create any regional differences in the first place.

To understand how house price targeting affects the economy in a broader sense, we compute standard deviations of producer price inflation, output, and house price over the first 48 quarters after the shock hits the economy. Table (1.4) presents the results for those variables at an aggregate level in booms and busts. It also contrasts the rational expectations model with the subjective expectations model.

Analyzing the subjective expectations model, we find that house price targeting, on the one hand, reduces aggregate volatility in both house prices and output. Moreover, this policy proves to be more effective during busts, as previously discussed. On the other hand, a trade-off emerges between stabilizing house prices and managing inflation. Increasing the emphasis on house price targeting diminishes house price volatility but leads to greater inflation volatility. The underlying intuition is that a greater focus on house prices by the monetary authority reduces the relative weight placed on inflation, thereby increasing inflation.

In the rational expectations model, we observe similar dynamics. In contrast to the subjective expectations model, the changes in volatility between the baseline and house price targeting are smaller, and there is no difference in booms and busts.

Table 1.4: Aggregate standard deviations across policy rules

<i>Model</i>	ϕ^π	ϕ^q	<i>Boom</i>			<i>Bust</i>		
			π^{agg}	y^{agg}	q^{agg}	π^{agg}	y^{agg}	q^{agg}
<i>RE</i>	1.5	0.00	0.03	0.15	0.18	0.03	0.15	0.18
	1.5	0.25	0.04	0.14	0.16	0.04	0.14	0.16
<i>SE</i>	1.5	0.00	0.01	0.30	0.64	0.02	0.36	0.79
	1.5	0.25	0.02	0.22	0.48	0.02	0.24	0.50

Notes: The table shows standard deviations of several variables in response to an expansionary productivity shock (100 bp); The standard deviations are computed over the first 48 quarters; Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

Focusing on regional differences, we compute the cross-regional standard deviations across both policy rules and both models for producer price inflation, output, and house prices. The results are shown in table (1.5). Beginning with the rational expectations model, we observe no cross-regional variation in either inflation or output. However, there are minor regional variations in house prices, which are further diminished by leaning against house price fluctuations. In contrast, the subjective expectations model

1.7. Policy analysis

exhibits cross-regional differences across all variables. Here, targeting house prices significantly reduces these variations, with the policy proving more effective in times of busts. Intuitively, extrapolation amplifies movements in house prices which spill over to the rest of the economy. If house prices are not as volatile, extrapolation is also mitigated, and so is the differential regional response.

Table 1.5: Cross-region standard deviations across policy rules

<i>Model</i>	ϕ^π	ϕ^q	<i>Boom</i>			<i>Bust</i>		
			$\pi^{H,F}$	$y^{H,F}$	$q^{H,F}$	$\pi^{H,F}$	$y^{H,F}$	$q^{H,F}$
<i>RE</i>	1.5	0.00	0.000	0.000	0.0040	0.000	0.000	0.0040
	1.5	0.25	0.000	0.000	0.0037	0.000	0.000	0.0037
<i>SE</i>	1.5	0.00	0.001	0.016	0.055	0.002	0.045	0.135
	1.5	0.25	0.000	0.012	0.040	0.000	0.036	0.110

Notes: The table shows standard deviations of several variables in response to an expansionary productivity shock (100 bp); The standard deviations are computed over the first 48 quarters; Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

Welfare consequences. We have demonstrated that house price targeting leads to a reduction in both the aggregate variance of house prices and output and a universal decrease in regional dispersion across all variables. However, this policy is associated with an increase in inflation volatility. This observation prompts the critical question of whether house price targeting is a desirable policy intervention. To assess this, we derive a welfare-based loss function and perform a comparative analysis across different policy rules. The loss function is based on a second-order approximation of the utility function of each region, with each region being weighted by its size. It is important to note that the steady state is efficient.

Before we turn to the derivation of the loss function, we will briefly review the sources of inefficiency in this model. In the rational expectations version of the model, the only source of inefficiency is inflation which arises due to price adjustment costs. Under subjective expectations, agents' choices are distorted due to their misspecified beliefs. In particular, households may be willing to shift resources from consumption units to housing investment and housing demand to achieve capital gains in the future. Additionally, this will also distort households' labor supply. This provides a motive for the policymaker to lean against house prices. Further, in our setup their welfare losses can arise due to regional heterogeneities. We will discuss this in detail below.

A second-order approximation of the households utility gives the following loss function. We follow Galí (2015) and express the welfare loss in terms of the equivalent permanent consumption decline, measured as a fraction of steady state consumption.

$$\frac{\mathbb{W}_{-1}}{c_{ss}} = -c_{ss}^\sigma \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \cdot \left[\frac{\gamma}{2} \Omega_t + \frac{1-\gamma}{2} \Omega_t^* + \frac{1}{2} \Gamma^s \hat{s}_t^2 + O(3) + \text{t.i.p.} \right] \quad (1.18)$$

Ω_t collects the welfare-relevant choices and is defined as:

$$\Omega_t = \Gamma^\pi \hat{\pi}_{H,t}^2 + \Gamma^c \hat{c}_t^2 + \Gamma^h \hat{h}_t^2 + \Gamma^n \hat{n}_t^2 + \Gamma^x \hat{x}_t^2 + \Gamma^b \hat{b}_t^2 - \Gamma^{\xi_a} \hat{\xi}_{a,t} \hat{n}_t$$

Ω_t^* is defined equivalently. The welfare weights (Γ^j) are explicitly stated in Appendix (1.D.2). From equation (1.18) it becomes apparent that the welfare loss contains three parts: the domestic choices (Ω_t), the foreign choices (Ω_t^*), and relative prices measured by the terms of trades (\hat{s}_t). The formulation further reveals that regional dispersion negatively impacts welfare. This result can be demonstrated in two distinct ways. First, consider a scenario where all variables, except one, are symmetric across regions and the regions have the same size. Suppose the policymaker could implement a policy that alters the heterogeneous variable by shifting variation from one region to another without influencing the other variables. Given the convexity of the loss function, it would be welfare-enhancing to eliminate this regional heterogeneity. Second, the terms of trade reflect relative price differentials between regions. If regions move in tandem, this term becomes zero, improving welfare. Consequently, policymakers are incentivized to promote homogeneous co-movement across regions to optimize welfare.

To illustrate this point, we can further rearrange equation (1.18) in terms of variation at the aggregate level and cross-regional variance.

$$\frac{\mathbb{W}_{-1}}{c_{ss}} = -c_{ss}^\sigma \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \cdot \left[\frac{1}{2} \Omega_t^{agg} + \frac{1}{2} \Omega_t^{var} - 2\Omega_t^{cov} + \frac{1}{2} \Gamma^s \hat{s}_t^2 + O(3) + \text{t.i.p.} \right] \quad (1.19)$$

Ω_t^{agg} collects the aggregate variation on a currency area level, Ω_t^{var} the cross-regional variation, and Ω_t^{cov} covariance terms:

$$\begin{aligned} \Omega_t^{agg} &= \Gamma^\pi (\hat{\pi}_t^{agg})^2 + \Gamma^c (\hat{c}_t^{agg})^2 + \Gamma^h (\hat{h}_t^{agg})^2 + \Gamma^n (\hat{n}_t^{agg})^2 + \Gamma^x (\hat{x}_t^{agg})^2 - \Gamma^{\xi_a} \hat{\xi}_{a,t} \hat{n}_t^{agg} \\ \Omega_t^{var} &= \Gamma^\pi \hat{\sigma}_{\pi,t}^2 + \Gamma^c \hat{\sigma}_{c,t}^2 + \Gamma^h \hat{\sigma}_{h,t}^2 + \Gamma^n \hat{\sigma}_{n,t}^2 + \Gamma^x \hat{\sigma}_{x,t}^2 + \Gamma^b \hat{\sigma}_{b,t}^2 \\ \Omega_t^{cov} &= \Gamma^\pi \lambda (1-\lambda) \hat{\pi}_{H,t}^2 \hat{\pi}_{F,t}^2 \end{aligned}$$

1.7. Policy analysis

The $\sigma_{j,t}^2$ terms denote the cross-regional variance of a variable j . The covariance term (Ω_t^{cov}) appears due to the presence of the home bias in consumption. Equation (1.19) indicates that the policymaker's objective is a function of both the aggregate economic variation and the degree of regional dispersion. By implementing a policy that promotes more synchronized economic dynamics across regions, the policymaker can effectively enhance overall welfare.

We now revisit the productivity shock and calculate the welfare loss under both our baseline model and house price targeting, comparing outcomes for the rational expectations model and the subjective expectations model. Table (1.6) presents the results and a decomposition into welfare loss on the aggregate level (Ω^{agg}) and the cross-regional variation (Ω^{var}). We drop the covariance and the terms of trades parts, as they are quantitatively irrelevant. We find that under rational expectations house price targeting is welfare detrimental, as house price targeting increases the loss by 12% relative to the baseline. However, under subjective expectations, house price targeting proves beneficial, yielding welfare improvements of 7% during booms and 24% during busts. This difference arises because, under subjective expectations, rising house prices lead households to anticipate further price increases. This results in a reallocation of resources from consumption to housing investment, alongside an increase in labor supply. These decisions, driven by overly optimistic projections of future house prices, generate welfare losses. House price targeting helps to stabilize price dynamics, thereby reducing the inefficiencies arising from misguided decision-making. As extrapolation is stronger during busts, the associated welfare losses are more severe, making house price targeting particularly advantageous during these periods. In contrast, under rational expectations, these channels are absent and so are the potential benefits of house price targeting.

Turning to the sources of welfare loss, we find that the welfare loss is mostly accounted for by volatility in the aggregate variables (Ω^{agg}). Under rational expectations, welfare loss through cross-regional variation (Ω^{var}) plays no role. Under subjective expectations, we find small welfare losses in the boom case. During a bust, these losses triple in the baseline case, but are still relatively small compared to the overall welfare loss. The relatively small welfare loss attributed to regional heterogeneities can be explained as follows. The primary driver of cross-regional variation in the welfare loss function arises from differences in housing investment across regions. However, the housing sector represents a relatively small share of the overall economy, resulting in a low welfare weight (Γ^x), which exerts a minimal effect on aggregate welfare. Additionally, the pass-through

from the housing sector to the broader economy is weak, as it operates predominantly through general equilibrium channels. A model that amplifies these linkages, such as one incorporating heterogeneous agents, would likely generate stronger cross-regional variation in the non-housing sector. Consequently, cross-regional disparities would play a more significant role in determining aggregate welfare outcomes.

Table 1.6: Welfare loss across policy rules

<i>Model</i>		<i>RE</i>	<i>SE</i>	
			<i>Boom</i>	<i>Bust</i>
<i>Baseline</i>	Ω^{agg}	−0.172	−0.147	−0.204
	Ω^{var}	0.000	−0.001	−0.003
	<i>total</i>	−0.172	−0.148	−0.207
\hat{q}_t targeting	Ω^{agg}	−0.191	−0.137	−0.154
	Ω^{var}	0.000	−0.001	−0.002
	<i>total</i>	−0.191	−0.138	−0.156
<i>Ratio</i>	Ω^{agg}	1.11	0.94	0.76
	Ω^{var}	0.86	0.57	0.57
	<i>total</i>	1.11	0.94	0.75

Notes: The welfare loss in response to an expansionary productivity shock (100 bp); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust); Ratio denotes the ratio of the welfare loss between house price targeting and the baseline; The welfare loss has been scaled by 100.

1.8 Conclusion

In this paper, we examine regional heterogeneity in housing cycles and their implications for monetary policy. We document that regional dispersion in house price growth is larger in busts compared to booms. We then develop a two-region New Keynesian model and show that including stronger belief updating and regional housing supply side heterogeneities can jointly explain the increase of regional house price growth dispersion in busts. We empirically provide evidence that supports our modeling choices in terms of belief updating and housing supply side heterogeneities. Our findings suggest that placing a greater emphasis on house prices by the monetary authority can reduce volatility in output and house prices, as well as regional dispersion in inflation, output, and house prices. Under subjective beliefs, this policy proves to be welfare-improving.

Appendix

1.A Micro-founding the debt-elastic interest rate

In the model, households in country H receive on their bond holdings the effective nominal interest rate $1 + i_{t-1} - \psi b_t$, with b_t being the real value of the aggregate bond holding in country H ; households in country F receive the effective nominal rate $1 + i_{t-1} - \psi b_t^*$. Moreover, the intermediation of bond positions entails a real cost $\gamma(1 + \pi_t)^{-1} \frac{\psi}{2} (b_t)^2 + (1 - \gamma)(1 + \pi_t)^{-1} \frac{\psi}{2} (b_t^*)^2$ of which $(1 + \pi_t)^{-1} \frac{\psi}{2} (b_t)^2$ is paid by each consumer in H and $(1 + \pi_t)^{-1} \frac{\psi}{2} (b_t^*)^2$ is paid by each consumer in F . In this Appendix we detail how these debt-elastic interest rates and the associated intermediation cost can be parsimoniously micro-founded. We achieve this by introducing two competitive bond clearing houses, one in each country, that represent the only access of households to financial markets and who incur a real cost that is quadratic in the size of their balance sheet. The specific market arrangement is as follows: households hold a consol and may hold liquid bonds.

Consol. Each household in H is endowed with $\bar{b} \in \mathbb{R}$ units of a non-marketable consol³⁶ that pays as a coupon $(\beta^{-1} - 1)(\gamma P_{t-1} + (1 - \gamma)P_{t-1}^*)P_t^{-1}$ units of H 's consumption basket each period, per unit of consol. This implies that the nominal coupon rate, applied to the nominal coupon value $P_{t-1}\bar{b}$, is $(\beta^{-1} - 1)(\gamma P_{t-1} + (1 - \gamma)P_{t-1}^*)/P_{t-1}$; the real coupon rate applied to the real value \bar{b} in turn is $(\beta^{-1} - 1)\left(\gamma + (1 - \gamma)\frac{P_{t-1}^*}{P_{t-1}}\right)(1 + \pi_t)^{-1}$. The situation in country F is symmetric: each household is endowed with \bar{b}^* units of a consol that pays $(\beta^{-1} - 1)(\gamma P_{t-1} + (1 - \gamma)P_{t-1}^*)(P_t^*)^{-1}$ units of F 's consumption basket each period, per unit of consol. \bar{b}, \bar{b}^* are model parameters selected such that (i) $\gamma\bar{b} + (1 - \gamma)\bar{b}^* = 0$ and (ii) all markets clear in the non-stochastic steady state with zero net inflation and terms-of-trade parity *without* households holding any liquid bonds. The latter fact ensures

³⁶A consol is a type of bond that has infinite maturity and just keeps paying a constant or varying coupon perpetually.

that there is no cost of financial intermediation in the steady state, shutting down this particular friction. The specific choice of the coupon payment scheme ensures two facts: (1) condition (i) implies that the nominal payments between H and F associated with the two consols exactly cancel out – whatever H/F receives as coupon payments on its consol endowment is paid for by F/H as a coupon service on its (endowed) short position of consol; and (2) the real coupon rates paid by/ to the consol endowment only depend on the real exchange rate and the inflation rates, not on the price levels. Households cannot trade their consol holdings.

Bonds. Households do have the possibility, though, to vary their position in the liquid bond. This liquid bond is a nominal, one-period, zero-coupon bond and the positions of the representative H -, respectively the representative F -household are denominated b_t, b_t^* . If a household wants to hold a net balance of liquid bonds different from zero, she has to go to one of the clearing houses in her country: In the H -country, there is a continuum of mass γ (respectively mass $1 - \gamma$ in F) of competitive clearing houses buying and selling bonds from and to the government and from and to the respective country's citizens. Households themselves cannot directly buy/sell government bonds without having an account at the clearing house. The clearing house can costlessly buy/sell bonds but incurs an operating cost that is quadratic in the size of its balance sheet, making this a model of costly financial intermediation. Thus, the interest rate that each citizen gets on her bond holdings is determined by the nominal rate paid on government bonds and the aggregate holding of liquid bonds. Each clearing house is owned equally by all citizens of the respective country so that it pays its profits to those citizens.³⁷ Consider an arbitrary clearing house in H (with symmetric arrangements in F). Denoting as $B_{c,t+1}$ the nominal value of the clearing house's net liabilities against H 's citizens and as $B_{g,t+1}$ the nominal value of the clearing house's position in the

³⁷In equilibrium, each clearing house makes a non-negative profit, and along the transition path back to the steady state after some shock, each clearing house makes a strictly positive profit. This fact is in principle incompatible with the notion of competitiveness (there is an incentive to open up more clearing houses or, equivalently, it is strictly profitable to split each clearing house). Therefore, it is better to interpret the program of the clearing house as reflecting capacity constraints: the here-presented program can be thought of as the inner problem of a profit maximization program with an additional factor (say, managerial effort) that which (i) makes the intermediation service production function exhibit constant returns to scale (instead of decreasing RTS), (ii) is provided by households, and (iii) is in perfectly inelastic supply. Under this way of modeling the clearing house, it behaves exactly as modeled here, it always makes zero profits, and households get as remuneration for providing the additional factor the amount that is the profit in the current way of modeling.

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government bond, the profit maximization program is:

$$\max_{B_{c,t+1}, B_{g,t+1} \in \mathbb{R}} -(1 + i_t^b)B_{c,t+1} + (1 + i_t)B_{g,t+1} - \frac{\psi}{2}P_t^{-1}(B_{c,t+1})^2, \text{ s.t. } B_{c,t+1} = B_{g,t+1}$$

where i_t^b is the nominal rate clearing the market for household bond positions and i_t is the nominal rate on government bonds that is set by the monetary authority. $\frac{\psi}{2}P_t^{-1}(B_{c,t+1})^2 = \frac{\psi}{2}P_t(b_{c,t+1})^2$ is the nominal cost of intermediating – crucially, the real cost of intermediation does not directly depend on the price level. The first order conditions for this program are

$$\begin{aligned} 1 + i_t^b + \psi P_t^{-1}B_{c,t+1} &= \mu_t, \\ 1 + i_t &= \mu_t, \\ B_{c,t+1} &= B_{g,t+1}, \end{aligned}$$

where μ_t is the Lagrange multiplier on the balance-sheet constraint $B_{c,t+1} = B_{g,t+1}$. Market clearing in the household bond positions in H requires

$$\gamma B_{c,t+1} = \gamma P_t b_{t+1},$$

and market clearing in the government bond positions requires

$$\gamma B_{g,t+1} + (1 - \gamma)B_{g,t+1}^* = 0,$$

so that by using the balance-sheet constraints $B_{c,t+1} = B_{g,t+1}$, $B_{c,t+1}^* = B_{g,t+1}^*$ and the clearing conditions for household bond positions in H and F we recover the market clearing condition for government bonds in the main model:

$$\gamma P_t b_{t+1} + (1 - \gamma)P_t^* b_{t+1}^* = 0.$$

In sum, the aggregate conditions implied by this market arrangement are:

$$\begin{aligned} 1 + i_t^b + \psi b_{t+1} &= 1 + i_t, \\ 1 + i_t^{b,*} + \psi b_{t+1}^* &= 1 + i_t, \\ \gamma P_t b_{t+1} + (1 - \gamma)P_t^* b_{t+1}^* &= 0. \end{aligned}$$

The nominal profits of the typical clearing house in H in equilibrium are:

$$\begin{aligned} \text{Profit}_{t+1} &= (i_t - i_t^b)B_{c,t+1} - \frac{\psi}{2}P_t^{-1}(B_{c,t+1})^2 \quad \text{with optimal } B_{c,t+1} = \frac{i_t - i_t^b}{\psi}P_t \\ &= (i_t - i_t^b)^2\psi^{-1}P_t - (i_t - i_t^b)^2\psi^{-1}P_t \cdot \frac{1}{2} \\ &= P_t \frac{\psi}{2}(b_{t+1})^2 \quad \text{using market clearing in the household bond positions.} \end{aligned}$$

Of the $1 + i_t\%$ nominal interest collected on (paid for) its position of government bonds, each clearing house withholds $\psi b_{t+1}\%$ of the interest from its customers (respectively, charges $-\psi b_{t+1}\%$ of additional interest if $b_{t+1} < 0$). Half of these $\psi b_{t+1}\%$ are used for covering the operating cost (by buying this amount of H 's final basket and selling it in exchange for numéraire), and the other half is paid as profit to the owners of the clearing house (which, in equilibrium, are its customers).

1.B Proofs and Derivations

1.B.1 Proof of Lemma (3)

Agents apply the optimal Bayesian filter, i.e. the Kalman filter, to arrive at the observable system:³⁸

$$\begin{aligned} \ln \frac{q_{t+1}}{q_t} &= \varrho \ln \bar{m}_t + \ln \hat{e}_{t+1} \\ \ln \bar{m}_t &= \varrho \ln \bar{m}_{t-1} - \frac{\sigma_v^2}{2} + g \cdot \left(\ln \hat{e}_t + \frac{\sigma_e^2 + \sigma_v^2}{2} \right) \end{aligned}$$

where $\ln \bar{m}_t := \mathbb{E}_t^{\mathcal{P}}(\ln m_t)$ is the posterior mean, $g = \frac{\sigma^2 + \sigma_v^2}{\sigma^2 + \sigma_v^2 + \sigma_e^2}$ is the steady state Kalman filter gain, $\sigma^2 = \frac{1}{2}[-\sigma_v^2 + \sqrt{\sigma_v^4 + 4\sigma_v^2\sigma_e^2}]$ is the steady state Kalman filter uncertainty, and $\ln \hat{e}_t$ is perceived to be a white noise process.

To avoid simultaneity in the house price we modify the belief setup following Adam, Marcet, and Beutel (2017).³⁹ We obtain the same observable system but with lagged

³⁸We assume agents' prior variance equals the steady state Kalman variance.

³⁹ q_t appears twice: in the forecast equation, and in the Kalman-updating Equation through $\ln \hat{e}_t$. Since q_t depends on \bar{m}_t , but the latter also depends on the former, it is not assured that at any point an equilibrium asset price exists and whether it is unique. See Adam, Marcet, and Beutel (2017) for the details. The idea of the modification is to alter agents' perceived information setup in that they observe each period one component of the lagged transitory price growth.

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information being used in the posterior mean updating equation:

$$\ln \bar{m}_t = (\varrho - g) \left(\ln \bar{m}_{t-1} - \frac{\sigma_v^2}{2} \right) + g \left(\ln \frac{q_{t-1}}{q_{t-2}} + \frac{\sigma_e^2}{2} \right). \quad (1.20)$$

Under this formulation, the posterior mean is pre-determined. We may now derive the posterior mean on the $s > 0$ periods ahead of price:

$$\mathbb{E}_t^{\mathcal{P}} q_{t+s} = q_t \cdot \exp \left(\ln \bar{m}_t \cdot \varrho \frac{1-\varrho^s}{1-\varrho} + \frac{1}{2} \sigma^2 (\varrho \frac{1-\varrho^s}{1-\varrho})^2 \right) \cdot \exp(V), \quad V \propto \sigma_v^2 \quad (1.21)$$

For the derivation of equation(3.19) see Appendix (1.B.2). This completes the proof.

1.B.2 Derivation of Equation (3.19)

Equation (3.19) is the result of the following calculations:

$$\begin{aligned} \mathbb{E}_t^{\mathcal{P}} q_{t+s} &= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \frac{q_{t+s}}{q_t} = q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp \left(\ln q_{t+s} - \ln q_t \right) = q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp \left(\sum_{n=1}^s \Delta \ln q_{t+n} \right) \\ &= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp \left(\sum_{n=1}^s \ln m_{t+n} \right) \cdot \underbrace{\mathbb{E}_t^{\mathcal{P}} \left[\prod_{n=1}^s e_{t+n} \right]}_{= \prod_n \mathbb{E}_t^{\mathcal{P}} e_{t+n} = 1} \\ &= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp \left(\sum_{n=1}^s \left[\sum_{j=0}^{n-1} \varrho^j \ln v_{t+n-j} + \varrho^n \ln m_t \right] \right) \\ &= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp \left(\ln m_t \cdot \sum_{n=1}^s \varrho^n \right) \cdot \underbrace{\mathbb{E}_t^{\mathcal{P}} \exp \left(\sum_{n=1}^s \sum_{j=0}^{n-1} \varrho^j \ln v_{t+n-j} \right)}_{\sim \mathcal{N}} \\ &= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp \left(\varrho \frac{1-\varrho^s}{1-\varrho} \ln m_t \right) \cdot \exp(V), \quad V \propto \sigma_v^2 \\ \iff \mathbb{E}_t^{\mathcal{P}} q_{t+s} &= q_t \cdot \exp \left(\ln \bar{m}_t \cdot \varrho \frac{1-\varrho^s}{1-\varrho} + \frac{1}{2} \sigma^2 (\varrho \frac{1-\varrho^s}{1-\varrho})^2 \right) \cdot \exp(V), \quad V \propto \sigma_v^2 \end{aligned}$$

1.B.3 Aggregation and market clearing

To achieve goods market clearing, each goods market for a variety j must clear. For notational convenience, we define $y_{H,t}(j) := c_{H,t}(j) + x_{H,t}(j) + \Psi_{t,H}(j)$, as the total demand for good (H, j) coming from one typical H -consumer. $\Psi_t := (1 + \pi_t)^{-1} \frac{\psi}{2} (b_t)^2$ is the real cost of intermediating the position of an H -citizen in the union-wide bond. This cost,

just like consumption and housing investment, gets passed along down to the varieties: $\Psi_{t,H} := (\frac{P_{H,t}}{P_t})^{-\frac{1}{\varsigma}} \lambda \Psi_t$. Goods market clearing across all goods markets requires:

$$y_t := \int y_{H,t}(j) dj = \gamma \int y_{H,t}(j) dj + (1-\gamma) \int y_{H,t}^*(j) dj + \int \Phi_t(j) dj$$

where $\Phi_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} \Phi_t$ and $\Phi_t = \frac{\kappa}{2} (\Pi_{H,t} - 1)^2 y_{H,t}$ account for the price adjustment costs from the firm side. Since housing investment requires domestically produced goods we have that $x_t = x_{H,t}$ and $x_t^* = x_{F,t}^*$. And therefore $x_{H,t}^* = 0$ and $x_{F,t} = 0$. Aggregation and successive substitution eventually yields the domestic and foreign aggregate good market clearing conditions:

$$\begin{aligned} \left(1 - \frac{\kappa}{2} (\Pi_{H,t} - 1)^2\right) y_t \gamma &= \gamma y_{H,t} + (1-\gamma) y_{H,t}^* \\ \left(1 - \frac{\kappa}{2} (\Pi_{F,t} - 1)^2\right) y_t^* (1-\gamma) &= \gamma y_{F,t} + (1-\gamma) y_{F,t}^* \end{aligned}$$

Further, the bond market clearing condition is given by:

$$\gamma P_t b_{t+1} + (1-\gamma) P_t^* b_{t+1}^* = 0.$$

Market clearing in the housing sectors is given by:

$$\begin{aligned} H(x_{t-\tau}, \xi_t) &= (h_t - (1-\delta)h_{t-1}), \\ H(x_{t-\tau}^*, \xi_{x,t}^*) &= (h_t^* - (1-\delta)h_{t-1}^*). \end{aligned}$$

Finally, the balance-of-payments Equation ensures that the household budget constraints hold:

$$\gamma y_{F,t} P_{F,t} - P_{H,t} (1-\gamma) y_{H,t}^* + \gamma (P_t b_{t+1} - (1+i_{t-1}) P_{t-1} b_t - \mathfrak{b}_t) = 0.$$

1.B.4 Walras' law and Balance-of-Payments

Walras' law

To make sure the economics of the model with home bias checks out, we prove that Walras' law holds in our model economy.

We start by providing a list of all market clearing conditions, household budget constraints, and relevant variable definitions (assuming $T_t = T_t^* = 0$), with equations involv-

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ing more than one good being in nominal terms (i.e. units of union-wide currency). In doing so, we make use of the micro-foundation of the debt-elastic interest rate rule that is presented in Appendix (1.A). The set of market clearing-, profit-, and budget-conditions is:

$$\begin{aligned}
(GMC) \quad & \left(1 - \frac{\kappa}{2}\pi_{H,t}^2\right) y_{H,t}\gamma = \gamma y_{H,t} + (1-\gamma)y_{H,t}^*, \\
(GMC^*) \quad & \left(1 - \frac{\kappa}{2}\pi_{F,t}^2\right) y_{F,t}^*(1-\gamma) = \gamma y_{F,t} + (1-\gamma)y_{F,t}^*, \\
(HMC) \quad & H(x_{t-\tau}, \xi_{t-\tau}) = h_t - (1-\delta)h_{t-1}, \\
(HMC^*) \quad & H(x_{t-\tau}^*, \xi_{t-\tau}) = h_t^* - (1-\delta)h_{t-1}^*, \\
(B) \quad & \gamma P_t b_{t+1} + (1-\gamma)P_t^* b_{t+1} = 0, \\
(BC) \quad & (c_{H,t} + x_{H,t})P_{H,t} + (c_{F,t} + x_{F,t})P_{F,t} + P_t q_t (h_t - (1-\delta)h_{t-1}) + P_t b_{t+1} \\
& = W_t n_t + (1 + i_{t-1} - \psi b_t)P_{t-1} b_t + H(x_{t-\tau}, \xi_{t-\tau}) \cdot P_t q_t \\
& + P_t \Sigma_t + (\beta^{-1} - 1) \frac{\gamma P_{t-1} + (1-\gamma)P_{t-1}^*}{P_{t-1}} P_{t-1} \bar{b}, \\
(BC^*) \quad & (c_{H,t}^* + x_{H,t}^*)P_{H,t} + (c_{F,t}^* + x_{F,t}^*)P_{F,t} + P_t^* q_t^* (h_t^* - (1-\delta)h_{t-1}^*) + P_t^* b_{t+1}^* \\
& = W_t^* n_t^* + (1 + i_{t-1} - \psi b_t^*)P_{t-1}^* b_t^* + H(x_{t-\tau}^*, \xi_{t-\tau}) \cdot P_t^* q_t^* \\
& + P_t^* \Sigma_t^* + (\beta^{-1} - 1) \frac{\gamma P_{t-1} + (1-\gamma)P_{t-1}^*}{P_{t-1}^*} P_{t-1}^* \bar{b}^*, \\
(\Sigma) \quad & P_t \Sigma_t = P_{H,t} \left(1 - \frac{\kappa}{2}\pi_{H,t}^2\right) y_t - W_t n_t + P_{t-1} \frac{\psi}{2} [b_t]^2, \\
(\Sigma^*) \quad & P_t^* \Sigma_t^* = P_{F,t} \left(1 - \frac{\kappa}{2}\pi_{F,t}^2\right) y_t^* - W_t^* n_t^* + P_{t-1}^* \frac{\psi}{2} [b_t^*]^2.
\end{aligned} \tag{1.22}$$

These are 9 conditions – Walras’ law now asserts that any one of these nine conditions should be obtainable through the summation of the remaining eight conditions. We show that the collective of all equations, except for (B), implies equation (B). First, plug (HMC) and (Σ) into (BC) to

get

$$\begin{aligned}
 & \stackrel{(HMC\&\Sigma\&BC)}{=} (c_{H,t} + x_{H,t})P_{H,t} + (c_{F,t} + x_{F,t})P_{F,t} + P_t b_{t+1} = P_{H,t} \left(1 - \frac{\kappa}{2} \pi_{H,t}^2\right) y_t \\
 & \quad + (1 + i_{t-1} - \psi b_t) P_{t-1} b_t + P_{t-1} \frac{\psi}{2} [b_t]^2 + (\beta^{-1} - 1)(\gamma P_{t-1} \\
 & \quad + (1 - \gamma) P_{t-1}^*) \bar{b} \\
 & = P_{H,t} \left(1 - \frac{\kappa}{2} \pi_{H,t}^2\right) y_t + (1 + i_{t-1}) P_{t-1} b_t - P_{t-1} \frac{\psi}{2} [b_t]^2 \\
 & \quad + (\beta^{-1} - 1)(\gamma P_{t-1} + (1 - \gamma) P_{t-1}^*) \bar{b} \\
 & \Leftrightarrow \stackrel{(HMC\&\Sigma\&BC)}{=} (c_{H,t} + x_{H,t})P_{H,t} + (c_{F,t} + x_{F,t})P_{F,t} + P_t \underbrace{(1 + \pi_t)^{-1} \frac{\psi}{2} [b_t]^2}_{=\Psi_t} + P_t b_{t+1} = \\
 & \quad (c_{H,t} + x_{H,t} + \Psi_{H,t})P_{H,t} + (c_{F,t} + x_{F,t} + \Psi_{F,t})P_{F,t} + P_t b_{t+1} = \\
 & \quad P_{H,t} \left(1 - \frac{\kappa}{2} \pi_{H,t}^2\right) y_t + (1 + i_{t-1}) P_{t-1} b_t \\
 & \quad + (\beta^{-1} - 1)(\gamma P_{t-1} + (1 - \gamma) P_{t-1}^*) \bar{b} \\
 & \Leftrightarrow \stackrel{(HMC\&\Sigma\&BC)}{=} \gamma(y_{H,t} P_{H,t} + y_{F,t} P_{F,t} + P_t b_{t+1}) = P_{H,t} \gamma \left(1 - \frac{\kappa}{2} \pi_{H,t}^2\right) y_t \\
 & \quad + (1 + i_{t-1}) P_{t-1} \gamma b_t + (\beta^{-1} - 1) \gamma (\gamma P_{t-1} + (1 - \gamma) P_{t-1}^*) \bar{b} \\
 & \Rightarrow \stackrel{(HMC\&\Sigma\&BC\&GMC)}{=} \gamma(y_{H,t} P_{H,t} + y_{F,t} P_{F,t} + P_t b_{t+1}) = P_{H,t} (\gamma y_{H,t} + (1 - \gamma) y_{H,t}^*) \\
 & \quad + (1 + i_{t-1}) P_{t-1} \gamma b_t + (\beta^{-1} - 1) \gamma (\gamma P_{t-1} + (1 - \gamma) P_{t-1}^*) \bar{b},
 \end{aligned}$$

and symmetric derivations deliver

$$\begin{aligned}
 & \stackrel{(HMC^*\&\Sigma^*\&BC^*\&GMC^*)}{=} (1 - \gamma)(y_{H,t}^* P_{H,t} + y_{F,t}^* P_{F,t} + P_t^* b_{t+1}^*) = P_{F,t} (\gamma y_{F,t} + (1 - \gamma) y_{F,t}^*) \\
 & \quad + (1 + i_{t-1}) P_{t-1}^* (1 - \gamma) b_t^* + (\beta^{-1} - 1)(1 - \gamma)(\gamma P_{t-1} + (1 - \gamma) P_{t-1}^*) \bar{b}^*.
 \end{aligned}$$

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Now adding the two equations gives:

$$\begin{aligned}
& (HMC \& \Sigma \& BC \& GMC) + (HMC^* \& \Sigma^* \& BC^* \& GMC^*) \\
& [\gamma P_t b_{t+1} + (1-\gamma)P_t^* b_{t+1}^*] - (1+i_{t-1})[\gamma P_{t-1} b_t + (1-\gamma)P_{t-1}^* b_t^*] \\
& - (\beta^{-1} - 1)(\gamma P_{t-1} + (1-\gamma)P_{t-1}^*) \underbrace{[\gamma \bar{b} + (1-\gamma)\bar{b}^*]}_{=0 \text{ by choice of } \bar{b}, \bar{b}^*} = 0.
\end{aligned}$$

Now if the initial bond levels (which are model parameters) are chosen in agreement with the bond market clearing condition, i.e. $\gamma P_{-1} b_0 + (1-\gamma)P_{-1}^* b_0^* = 0$, a simple induction argument over t establishes $(B), \forall t$.

Balance-of-Payments

In Appendix 1.B.4 we establish that

$$\begin{aligned}
& (HMC \& \Sigma \& BC \& GMC) \quad \gamma \left(y_{H,t} P_{H,t} + y_{F,t} P_{F,t} + P_t b_{t+1} \right. \\
& \quad \left. - (1+i_{t-1})P_{t-1} b_t - (\beta^{-1} - 1) \frac{\gamma P_{t-1} + (1-\gamma)P_{t-1}^*}{P_{t-1}^*} P_{t-1} \bar{b} \right) \\
& = P_{H,t} \left(\gamma y_{H,t} + (1-\gamma)y_{H,t}^* \right) \\
& (HMC^* \& \Sigma^* \& BC^* \& GMC^*) \quad (1-\gamma) \left(y_{H,t}^* P_{H,t} + y_{F,t}^* P_{F,t} + P_t^* b_{t+1}^* - (1+i_{t-1})P_{t-1}^* b_t^* \right. \\
& \quad \left. - (\beta^{-1} - 1) \frac{\gamma P_{t-1} + (1-\gamma)P_{t-1}^*}{P_{t-1}^*} P_{t-1}^* \bar{b}^* \right) \\
& = P_{F,t} \left(\gamma y_{F,t} + (1-\gamma)y_{F,t}^* \right).
\end{aligned}$$

This is equivalent to

$$\begin{aligned}
& (HMC \& \Sigma \& BC \& GMC) \quad \gamma y_{F,t} P_{F,t} - P_{H,t} (1-\gamma) y_{H,t}^* + \gamma \left(P_t b_{t+1} - (1+i_{t-1})P_{t-1} b_t \right. \\
& \quad \left. - (\beta^{-1} - 1) \frac{\gamma P_{t-1} + (1-\gamma)P_{t-1}^*}{P_{t-1}^*} P_{t-1} \bar{b} \right) = 0 \\
& (HMC^* \& \Sigma^* \& BC^* \& GMC^*) \quad (1-\gamma) x_{H,t}^* P_{H,t} - P_{F,t} \gamma y_{F,t} + (1-\gamma) \left(P_t^* b_{t+1}^* \right. \\
& \quad \left. - (1+i_{t-1})P_{t-1}^* b_t^* - (\beta^{-1} - 1) \frac{\gamma P_{t-1} + (1-\gamma)P_{t-1}^*}{P_{t-1}^*} P_{t-1}^* \bar{b}^* \right) = 0.
\end{aligned}$$

Now both equations dictate that the value of net imports in the respective country (imports less of exports) be covered by an equal-sized increase in the level of debt. By Walras' law (see Appendix 1.B.4), the whole list of market clearing conditions, budget constraints and relevant variable definitions, (1.22), is equivalent to (1.22) without $(BC), (BC^*)$ and augmented with the

balance of payments equation

$$(BOP) \quad \gamma(1-\lambda)\left(\frac{P_{F,t}}{P_t}\right)^{1-\frac{1}{\varsigma}}y_t - (1-\gamma)(1-\lambda^*)\left(\frac{P_{H,t}}{P_t^*}\right)^{1-\frac{1}{\varsigma}}\frac{P_t^*}{P_t}y_t^* \\ + \gamma\left(b_{t+1} - (1+i_{t-1})(1+\pi_t)^{-1}b_t - (\beta^{-1}-1)\left(\gamma + (1-\gamma)\frac{P_{t-1}^*}{P_{t-1}}\right)(1+\pi_t)^{-1}\bar{b}\right) = 0.$$

where we have used the demand schedules to substitute out the H and F good variables that do not feature in the MSV-representation of the model, and we have divided by P_t to get the representation in units of country H 's final consumption basket.

1.B.5 Nonlinear Equilibrium Conditions

As a starting point to solving the model, we collect all equilibrium conditions in a parsimonious fashion by performing light substitutions.

Expressing price levels with only inflation rates and terms of trade

Define the terms of trade as

$$s_t := \frac{P_{H,t}}{P_{F,t}}.$$

This entails

$$s_t = \frac{\Pi_{H,t}}{\Pi_{F,t}}s_{t-1},$$

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and allows us to write

$$\begin{aligned}
\Pi_t &= \left[\left((\Pi_{H,t}^{1-1/\varsigma})^{-1} + \frac{1-\lambda}{\lambda} (s_t^{1-1/\varsigma})^{-1} \right)^{-1} + \left(\frac{\lambda}{1-\lambda} s_t^{1-1/\varsigma} + (\Pi_{F,t}^{1-1/\varsigma})^{-1} \right)^{-1} \right]^{\frac{1}{1-1/\varsigma}}, \\
\Pi_t^* &= \left[\left((\Pi_{H,t}^{1-1/\varsigma})^{-1} + \frac{\lambda^*}{1-\lambda^*} (s_t^{1-1/\varsigma})^{-1} \right)^{-1} + \left(\frac{1-\lambda^*}{\lambda^*} s_t^{1-1/\varsigma} + (\Pi_{F,t}^{1-1/\varsigma})^{-1} \right)^{-1} \right]^{\frac{1}{1-1/\varsigma}}, \\
\left(\frac{P_{H,t}}{P_t} \right)^{-\frac{1}{\varsigma}} &= \left[\lambda + (1-\lambda) s_t^{\frac{1}{\varsigma}-1} \right]^{\frac{1}{1/\varsigma-1} \cdot (-\frac{1}{\varsigma})} =: \mathbf{p}_H(s_t)^{-\frac{1}{\varsigma}}, \\
&\quad \mathbf{p}'_H(s) = \mathbf{p}_H(s)^{2-1/\varsigma} (1-\lambda) s^{1/\varsigma-1} > 0, \quad \mathbf{p}'_H(1) = 1-\lambda, \\
\left(\frac{P_{F,t}}{P_t} \right)^{-\frac{1}{\varsigma}} &= \left[\lambda s_t^{1-\frac{1}{\varsigma}} + (1-\lambda) \right]^{\frac{1}{1/\varsigma-1} \cdot (-\frac{1}{\varsigma})} =: \mathbf{p}_F(s_t)^{-\frac{1}{\varsigma}}, \\
&\quad \mathbf{p}'_F(s) = -\mathbf{p}_F(s)^{2-1/\varsigma} \lambda s^{1/\varsigma} < 0, \quad \mathbf{p}'_F(1) = -\lambda, \\
\left(\frac{P_{H,t}}{P_t^*} \right)^{-\frac{1}{\varsigma}} &= \left[(1-\lambda^*) + \lambda^* s_t^{\frac{1}{\varsigma}-1} \right]^{\frac{1}{1/\varsigma-1} \cdot (-\frac{1}{\varsigma})} =: \mathbf{p}_H^*(s_t)^{-\frac{1}{\varsigma}}, \\
&\quad \mathbf{p}_H^{*'}(s) = (\mathbf{p}_H(s)^*)^{2-1/\varsigma} \lambda^* s^{1/\varsigma-1} > 0, \quad \mathbf{p}_H^{*'}(1) = \lambda^*, \\
\left(\frac{P_{F,t}}{P_t^*} \right)^{-\frac{1}{\varsigma}} &= \left[(1-\lambda^*) s_t^{1-\frac{1}{\varsigma}} + \lambda^* \right]^{\frac{1}{1/\varsigma-1} \cdot (-\frac{1}{\varsigma})} =: \mathbf{p}_F^*(s_t)^{-\frac{1}{\varsigma}}, \\
&\quad \mathbf{p}_F^{*'}(s) = -(\mathbf{p}_F^*(s))^{2-1/\varsigma} (1-\lambda^*) s^{1/\varsigma} > 0, \quad \mathbf{p}_F^{*'}(1) = -(1-\lambda^*), \\
\frac{P_t}{P_t^*} &= \left[\frac{\lambda s_t^{1-1/\varsigma} + (1-\lambda)}{(1-\lambda^*) s_t^{1-1/\varsigma} + \lambda^*} \right]^{\frac{1}{1-1/\varsigma}} =: \mathbf{p}(s_t), \\
&\quad \mathbf{p}'(s) = \mathbf{p}(s)^{1/\varsigma} \frac{s^{-1/\varsigma}}{\left((1-\lambda^*) s^{1-1/\varsigma} + \lambda^* \right)^2} \cdot [\lambda \lambda^* - (1-\lambda)(1-\lambda^*)] > 0, \quad \mathbf{p}'(1) = \lambda \lambda^* - (1-\lambda)(1-\lambda^*).
\end{aligned}$$

We have characterized every expression that involves any consumption price level in terms of the inflation rates and the terms of trade. This means that we need not track the price levels which are determined in equilibrium only up to a translation.

Condensing the set of market clearing conditions

Using the expressions above, and the demand schedules for varieties, we can rewrite the goods market clearing condition into

$$\begin{aligned}
\left(1 - \frac{\kappa}{2} (\Pi_{H,t} - 1)^2 \right) \xi_{a,t} n_t \gamma &= \gamma \lambda \mathbf{p}_H(s_t)^{-\frac{1}{\varsigma}} y_t + (1-\gamma) (1-\lambda^*) \mathbf{p}_H^*(s_t)^{-\frac{1}{\varsigma}} y_t^*, \\
\left(1 - \frac{\kappa}{2} (\Pi_{F,t} - 1)^2 \right) \xi_{a,t} n_t^* (1-\gamma) &= \gamma (1-\lambda) \mathbf{p}_F(s_t)^{-\frac{1}{\varsigma}} x_t + (1-\gamma) \lambda^* \mathbf{p}_F^*(s_t)^{-\frac{1}{\varsigma}} x_t^*.
\end{aligned}$$

We can use the expressions above also in the BOP equation (derived in 1.B.4), to receive

$$\begin{aligned} (BOP) \quad & \gamma(1-\lambda)\mathbf{p}_F(s_t)^{1-\frac{1}{\varsigma}}\mathbf{y}_t - (1-\gamma)(1-\lambda^*)\mathbf{p}_H^*(s_t)^{1-\frac{1}{\varsigma}}\mathbf{p}(s_t)^{-1}\mathbf{y}_t^* \\ & + \gamma(b_{t+1} - (1+i_{t-1})(1+\pi_t)^{-1}b_t - (\beta^{-1}-1)(\gamma + (1-\gamma)\mathbf{p}(s_{t-1})^{-1})(1+\pi_t)^{-1}\bar{b}) = 0. \end{aligned}$$

We are now ready to state the set of nonlinear equilibrium conditions.

1.B. Proofs and Derivations

Nonlinear equilibrium conditions

For simplicity we drop time to build here. Using the short-hand notation $y_t := c_t + x_t + (1 + \pi_t)^{-1} \frac{\psi}{2} (b_t)^2$ (analogously for $*$):

Household

$$\begin{aligned}
 (h) \quad q_t &= \frac{\xi_{h,t} h_t^{-\nu}}{\xi_{c,t} c_t^{-\sigma}} + \beta(1-\delta) \mathbb{E}_t^{\mathcal{P}} \left\{ \frac{\xi_{c,t+1}}{\xi_{c,t}} \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} q_{t+1} \right\}, \\
 (n) \quad \frac{\chi n_t^{\varphi}}{\xi_{c,t} c_t^{-\sigma}} &= w_t, \\
 (b) \quad 1 &= \beta \mathbb{E}_t^{\mathcal{P}} \left\{ \frac{\xi_{c,t+1}}{\xi_{c,t}} \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} (1+r_{t+1}) \right\}, \\
 (x) \quad 1 &= q_t \cdot \xi_{x,t} x_t^{\eta-1},
 \end{aligned} \tag{1.23}$$

Household*

$$\begin{aligned}
 (h^*) \quad q_t^* &= \xi_{h,ss}^* \frac{\xi_{h,t}}{\xi_{c,t} (c_t^*)^{-\sigma}} \frac{(h_t^*)^{-\nu}}{\xi_{c,t} (c_t^*)^{-\sigma}} + \beta(1-\delta) \mathbb{E}_t^{\mathcal{P}} \left\{ \frac{\xi_{c,t+1}}{\xi_{c,t}} \left(\frac{c_t^*}{c_{t+1}^*} \right)^{\sigma} q_{t+1}^* \right\}, \\
 (n^*) \quad \frac{\chi (n_t^*)^{\varphi}}{\xi_{c,t} (c_t^*)^{-\sigma}} &= w_t^*, \\
 (b^*) \quad 1 &= \beta \mathbb{E}_t^{\mathcal{P}} \left\{ \frac{\xi_{c,t+1}}{\xi_{c,t}} \left(\frac{c_t^*}{c_{t+1}^*} \right)^{\sigma} (1+r_{t+1}^*) \right\}, \\
 (x^*) \quad 1 &= q_t^* \cdot \xi_{x,ss}^* \frac{\xi_{x,t}}{\xi_{x,ss}^*} (x_t^*)^{\eta-1},
 \end{aligned}$$

Interest rates

$$\begin{aligned}
 (r) \quad 1+r_t &= \frac{1+i_{t-1}-\psi b_t}{1+\pi_t}, \\
 (r^*) \quad 1+r_t^* &= \frac{1+i_{t-1}-\psi b_t^*}{1+\pi_t^*},
 \end{aligned}$$

Firm

$$\begin{aligned}
 (PC) \quad \kappa(\Pi_{H,t}-1)\Pi_{H,t} - (1-\epsilon) - \epsilon(1-\tau^{\ell}) \frac{w_t}{\xi_{a,t}} \mathbf{p}_H(s_t)^{-1} &= \\
 \mathbb{E}_t^{\mathcal{P}} \left\{ \beta \frac{\xi_{c,t+1}}{\xi_{c,t}} \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} \frac{\xi_{a,t+1} n_{t+1}}{\xi_{a,t} n_t \Pi_{t+1}} \kappa(\Pi_{H,t+1}-1)\Pi_{H,t+1}^2 \right\},
 \end{aligned}$$

Firm*

$$\begin{aligned}
 (PC^*) \quad \kappa(\Pi_{F,t}-1)\Pi_{F,t} - (1-\epsilon) - \epsilon(1-\tau^{\ell}) \frac{w_t^*}{\xi_{a,t}} \mathbf{p}_F^*(s_t)^{-1} &= \\
 \mathbb{E}_t^{\mathcal{P}} \left\{ \beta \frac{\xi_{c,t+1}}{\xi_{c,t}} \left(\frac{c_t^*}{c_{t+1}^*} \right)^{\sigma} \frac{\xi_{a,t+1} n_{t+1}^*}{\xi_{a,t} n_t^* \Pi_{t+1}^*} \kappa(\Pi_{F,t+1}-1)\Pi_{F,t+1}^2 \right\},
 \end{aligned}$$

Bond market clearing

$$(B) \quad \gamma b_{t+1} + (1-\gamma) \mathbf{p}(s_t) b_{t+1}^* = 0,$$

Goods market clearing

$$\begin{aligned}
 (GMC) \quad & \left(1 - \frac{\kappa}{2}(\Pi_{H,t} - 1)^2\right) \xi_{a,t} n_t \gamma = \gamma \lambda \mathbf{p}_H(s_t)^{-\frac{1}{\varsigma}} y_t + (1 - \gamma)(1 - \lambda^*) \mathbf{p}_H^*(s_t)^{-\frac{1}{\varsigma}} y_t^*, \\
 (GMC^*) \quad & \left(1 - \frac{\kappa}{2}(\Pi_{F,t} - 1)^2\right) \xi_{a,t} n_t^* (1 - \gamma) = \gamma (1 - \lambda) \mathbf{p}_F(s_t)^{-\frac{1}{\varsigma}} y_t + (1 - \gamma) \lambda^* \mathbf{p}_F^*(s_t)^{-\frac{1}{\varsigma}} y_t^*, \\
 (BOP) \quad & \gamma (1 - \lambda) \mathbf{p}_F(s_t)^{1-\frac{1}{\varsigma}} y_t - (1 - \gamma)(1 - \lambda^*) \mathbf{p}_H^*(s_t)^{1-\frac{1}{\varsigma}} \mathbf{p}(s_t)^{-1} y_t^* \\
 & + \gamma \left(b_{t+1} - (1 + i_{t-1})(1 + \pi_t)^{-1} b_t \right. \\
 & \left. - (\beta^{-1} - 1)(\gamma + (1 - \gamma) \mathbf{p}(s_{t-1})^{-1})(1 + \pi_t)^{-1} \bar{b} \right) = 0,
 \end{aligned}$$

Housing market clearing

$$\begin{aligned}
 (HMC) \quad & \eta^{-1} \xi_{x,t} x_t^\eta = h_t - (1 - \delta) h_{t-1}, \\
 (HMC^*) \quad & (\eta^*)^{-1} \xi_{x,ss}^* \frac{\xi_{x,t}}{\xi_{x,ss}} (x_t^*)^{\eta^*} = h_t^* - (1 - \delta) h_{t-1}^*
 \end{aligned}$$

Price indices

$$\begin{aligned}
 (s) \quad & s_t = \frac{\Pi_{H,t}}{\Pi_{F,t}} s_{t-1}, \\
 (\Pi) \quad & \Pi_t = \left[\left((\Pi_{H,t}^{1-1/\varsigma})^{-1} + \frac{1-\lambda}{\lambda} (s_t^{1-1/\varsigma})^{-1} \right)^{-1} + \left(\frac{\lambda}{1-\lambda} s_t^{1-1/\varsigma} + (\Pi_{F,t}^{1-1/\varsigma})^{-1} \right)^{-1} \right]^{\frac{1}{1-1/\varsigma}}, \\
 (\Pi^*) \quad & \Pi_t^* = \left[\left((\Pi_{H,t}^{1-1/\varsigma})^{-1} + \frac{\lambda^*}{1-\lambda^*} (s_t^{1-1/\varsigma})^{-1} \right)^{-1} + \left(\frac{1-\lambda^*}{\lambda^*} s_t^{1-1/\varsigma} + (\Pi_{F,t}^{1-1/\varsigma})^{-1} \right)^{-1} \right]^{\frac{1}{1-1/\varsigma}}.
 \end{aligned}$$

with the shocks $(\xi_t)_{t \geq 0}$, the allocation variables $(c_t, c_t^*, b_t, b_t^*, h_t, h_t^*, x_t, x_t^*, n_t, n_t^*)_{t \geq 0}$ and the price variables $(w_t, w_t^*, q_t, q_t^*, i_t, \Pi_t, \Pi_t^*, \Pi_{H,t}, \Pi_{F,t}, s_t)_{t \geq 0}$.

1.B.6 Proof of Lemma 2

In this section, we prove that there exists a unique non-stochastic steady state with zero net inflation and parity in the terms of trade $P_{H,t}/P_{F,t}$ (which implies parity in the real exchange rate P_t/P_t^* , see the definition of \mathbf{p}). We prove this first for arbitrary parameters $\eta, \eta^*, \xi_{h,ss}, \xi_{x,ss}, \xi_{h,ss}^*, \xi_{x,ss}^*$, and then show that it is possible to select parameters such that the steady state allocation is symmetric. The non-stochastic steady state with zero net inflation and real exchange rate parity (“SS” for short) obtains by setting $\text{Var}[\xi_t] = 0$, where the shock vector contains both actual shocks and shocks that are only perceived (and never observed) by the household within her perceived house price model: $\xi_t = (\xi_{a,t}, \xi_{c,t}, \xi_{h,t}, \xi_{x,t}, \xi_{i,t}, e_t, v_t)^\top$. Thus, the non-stochastic steady state represents the time-invariant equilibrium that obtains if agents do not expect any shock to ever materialize and indeed no shock ever does materialize, and we have $\xi_t = (1, 1, \xi_{h,ss}, \xi_{x,ss}, 1, 1, 1)^\top$ almost surely where $\xi_{h,ss}, \xi_{x,ss}$ are model parameters. We are interested in, and prove existence and uniqueness of, a non-stochastic steady state in which the net rates of inflation are zero and in which the terms of trade are at parity. The latter assumption

1.B. Proofs and Derivations

implies $\mathbf{p}_H(1) = \mathbf{p}_H^*(1) = \mathbf{p}_F(1) = \mathbf{p}_F^*(1) = \mathbf{p}(1) = 1$.

Beliefs are irrelevant in the non-stochastic steady state. The first relevant insight is that in the non-stochastic steady state, defined as above, the presence and precise parameterization of subjective beliefs over house prices is irrelevant. To see this, recall that the subjective house price expectation dynamics are fully characterized by

$$\begin{aligned} \forall s > 0, \mathbb{E}_t^{\mathcal{P}} q_{t+s} &= q_t \cdot \exp \left(\ln \bar{m}_t \cdot \varrho^{\frac{1-\varrho^s}{1-\varrho}} + \frac{1}{2} \sigma^2 \left(\varrho^{\frac{1-\varrho^s}{1-\varrho}} \right)^2 \right) \cdot \exp(V), \quad V \propto \sigma_v^2 \\ \ln \bar{m}_t &= (1-g) \left(\ln \bar{m}_{t-1} - \frac{\sigma_v^2}{2} \right) + g \left(\ln \frac{q_{t-1}}{q_{t-2}} + \frac{\sigma_e^2}{2} \right), \\ &+ \text{equations determining equilibrium-evolution of price level, } q_t. \end{aligned} \quad (1.24)$$

Now recall that the non-stochastic steady state represents the unique equilibrium of the economy when the variance of actual and perceived external shocks tends to zero and the initial conditions are selected such that they give rise to a constant path of equilibrium variables. Formally, the SS arises by replacing $\text{Var}[\xi_t]$ with $\varphi^2 \cdot \text{Var}[\xi_t]$ in the model, taking the limit $\varphi \rightarrow 0$, and solving for the fixed point of the equilibrium equations. Applying this logic, $\sigma_e, \sigma_v, \sigma \rightarrow 0$, to the equations (1.24) delivers

$$\begin{aligned} q_{ss} &= q_{ss} \cdot \exp \left(\ln \bar{m}_{ss} \cdot \varrho^{\frac{1-\varrho^s}{1-\varrho}} + 0 \right) \cdot \exp(0) \\ \ln \bar{m}_{ss} &= (1-g) \ln \bar{m}_{ss} + g \cdot 0 \end{aligned}$$

from which we see that there exists a unique fixed point which is $\bar{m}_{ss} = 1$, regardless of the level of q_{ss} . This means that the presence of subjective beliefs has no consequences for the non-stochastic steady state: provided that agents have the prior that house prices do not change, $\bar{m}_{ss} = 1$, the absence of perceived and actual shocks means that house prices indeed never change which, in turn, means that the prior of constant house prices is never changed.

In summary, iff a non-stochastic steady state with zero net inflation and terms of trade parity

exists and is unique, then it solves the following system of equations.

Household

$$\begin{aligned}
 (h) \quad q_{ss} &= \xi_{h,ss} h_{ss}^{-\nu} c_{ss}^{\sigma} + \beta(1-\delta)q_{ss}, \\
 (n) \quad \chi n_{ss}^{\varphi} c_{ss}^{\sigma} &= w_{ss}, \\
 (b) \quad 1 &= \beta(1+r_{ss}), \\
 (x) \quad 1 &= q_{ss} \cdot \xi_{x,ss} x_{ss}^{\eta-1},
 \end{aligned} \tag{1.25}$$

Household*

$$\begin{aligned}
 (h^*) \quad q_{ss} &= \xi_{h,ss}^* (h_{ss}^*)^{-\nu} (c_{ss}^*)^{\sigma} + \beta(1-\delta)q_{ss}^*, \\
 (n^*) \quad \chi (n_{ss}^*)^{\varphi} (c_{ss}^*)^{\sigma} &= w_{ss}^*, \\
 (b^*) \quad 1 &= \beta(1+r_{ss}^*), \\
 (x^*) \quad 1 &= q_{ss}^* \cdot \xi_{x,ss}^* (x_{ss}^*)^{\eta-1},
 \end{aligned}$$

Interest rates

$$\begin{aligned}
 (r) \quad 1+r_{ss} &= 1+i_{ss}-\psi b_{ss}, \\
 (r^*) \quad 1+r_{ss}^* &= 1+i_{ss}-\psi b_{ss}^*,
 \end{aligned}$$

Firm

$$(PC) \quad w_{ss} = 1 \quad \text{using that } \tau^{\ell} = 1/\epsilon,$$

Firm*

$$(PC^*) \quad w_{ss}^* = 1 \quad \text{using that } \tau^{\ell} = 1/\epsilon,$$

Bond market clearing

$$(B) \quad \gamma b_{ss} + (1-\gamma)b_{ss}^* = 0,$$

Goods market clearing

$$\begin{aligned}
 (GMC) \quad n_{ss}\gamma &= \gamma \lambda y_{ss} + (1-\gamma)(1-\lambda^*)y_{ss}^*, \\
 (GMC^*) \quad n_{ss}^*(1-\gamma) &= \gamma(1-\lambda)y_{ss} + (1-\gamma)\lambda^*y_{ss}^*, \\
 (BOP) \quad \gamma(1-\lambda)y_{ss} &- (1-\gamma)(1-\lambda^*)y_{ss}^* - \gamma(i_{ss}b_{ss} + (\beta^{-1}-1)\bar{b}) = 0,
 \end{aligned}$$

Housing market clearing

$$\begin{aligned}
 (HMC) \quad \eta^{-1} \xi_{x,ss} (x_{ss})^{\eta} &= \delta h_{ss}, \\
 (HMC^*) \quad (\eta^*)^{-1} \xi_{x,ss}^* (x_{ss}^*)^{\eta^*} &= \delta h_{ss}^*.
 \end{aligned}$$

where \bar{b}, \bar{b}^* are model parameters (see Appendix 1.A for an interpretation) chosen so as to (i) ensure equilibrium existence, and (ii) ensure that (§1) $\gamma\bar{b} + (1-\gamma)\bar{b}^* = 0$. We solve for the SS in 4 steps.

1. First, we solve a number of equations explicitly, thus substituting out a number of variables:

1.B. Proofs and Derivations

- (a) (b) and (b^*) together with (r) and (r^*) and (B) imply $b_{ss} = b_{ss}^* = 0$ and $i_{ss} = \beta^{-1} - 1$ whence it follows $y_{ss} = c_{ss} + x_{ss}$ and analogously for $*$. (§1) then implies $\bar{b} = -\bar{b}^* \cdot (1 - \gamma)/\gamma$ where \bar{b}^* is not pinned down yet. We will solve for it in the very last step.
- (b) (PC) and (PC^*) imply $w_{ss} = w_{ss}^* = 1$;
- (c) together with (n) and (n^*) this implies $n_{ss} = (\chi^{-1})^{1/\varphi} \cdot c_{ss}^{-\sigma/\varphi} =: \phi(c_{ss})$ with $\phi' < 0$ and analogously for n_{ss}^* with the *same* function ϕ ;
- (d) (HMC) and (HMC^*) imply $h_{ss} = (\delta)^{-1} \xi_{x,ss} \eta^{-1} x_{ss}^\eta$ and analogously for h_{ss}^* ;
- (e) (h) and (h^*) imply (with $\bar{\beta} := \beta(1 - \delta)$)

$$q_{ss} = (1 - \bar{\beta})^{-1} \xi_{h,ss} [\delta^{-1} \xi_{x,ss} \eta^{-1} x_{ss}^\eta]^{-\nu} c_{ss}^\sigma$$

$$q_{ss}^* = (1 - \bar{\beta})^{-1} \xi_{h,ss}^* [\delta^{-1} \xi_{x,ss}^* (\eta^*)^{-1} (x_{ss}^*)^{\eta^*}]^{-\nu} (c_{ss}^*)^\sigma$$

- (f) (BOP) now reads $\gamma(1 - \lambda)y_{ss} = (1 - \gamma)(1 - \lambda^*)y_{ss}^* - (1 - \gamma)(\beta^{-1} - 1)\bar{b}^*$, and the symmetric (BOP^*) which is redundant by Walras' law reads $(1 - \gamma)(1 - \lambda^*)x_{ss}^* = \gamma(1 - \lambda)x_{ss} + (1 - \gamma)(\beta^{-1} - 1)\bar{b}^*$; using this in (GMC) , (GMC^*) produces

$$(GMC) \quad \phi(c_{ss}) = y_{ss} + (1 - \gamma)/\gamma \cdot (\beta^{-1} - 1)\bar{b}^*,$$

$$(GMC^*) \quad \phi(c_{ss}^*) = y_{ss}^* - (\beta^{-1} - 1)\bar{b}^*,$$

2. The remaining equations are (x) , (x^*) and (GMC) , (GMC^*) , (BOP) with unknowns x_{ss} , x_{ss}^* , c_{ss} , c_{ss}^* , \bar{b}^* . In this step, we show there are strictly increasing functions that yield x_{ss} , x_{ss}^* given c_{ss} , c_{ss}^* respectively. Start by plugging q_{ss} into (x) :

$$(1 - \bar{\beta})^{-1} \xi_{h,ss} [\delta^{-1} \xi_{x,ss} \eta^{-1} x_{ss}^\eta]^{-\nu} c_{ss}^\sigma \xi_{x,ss} x_{ss}^{\eta-1} = 1$$

(the equation for $*$ is symmetric.) Now since $\eta \in (0, 1)$ and $\nu > 0$, the expression on the left-hand-side is a strictly decreasing function of x_{ss} for any c_{ss} . Moreover, for $x_{ss} \rightarrow 0$, the *LHS* $\rightarrow +\infty$ and for $x_{ss} \rightarrow \infty$, the *LHS* $\rightarrow 0$, whence Bolzano's intermediate value theorem (and continuity) ensures that for each c_{ss} there exists a unique x_{ss} . Call this implicitly defined mapping $x_{ss} = \psi(c_{ss})$. As the implicit function theorem shows, $\eta, \nu, \sigma > 0$ imply $\psi' > 0$. Analogous arguments hold for $*$.

3. We now insert our previous findings into the only remaining equations:

$$(GMC) \quad \phi(c_{ss}) - c_{ss} - \psi(c_{ss}) - (1 - \gamma)/\gamma \cdot (\beta^{-1} - 1)\bar{b}^* =: \zeta(c_{ss}, \bar{b}^*) = 0,$$

$$(GMC^*) \quad \phi(c_{ss}^*) - c_{ss}^* - \psi^*(c_{ss}^*) + (\beta^{-1} - 1)\bar{b}^* =: \zeta^*(c_{ss}^*, \bar{b}^*) = 0,$$

Observe now $c_{ss} \mapsto \zeta$ is continuous and strictly decreasing with $\lim_{c \rightarrow 0} \zeta = +\infty$ (by

$\lim_{c \rightarrow 0} \phi = +\infty$ and $\lim_{c \rightarrow 0} \psi < +\infty$) and $\lim_{c \rightarrow +\infty} \zeta = -\infty$ (by $\lim_{c \rightarrow +\infty} c, \psi = +\infty$ and $\lim_{c \rightarrow \infty} \phi = 0$). Therefore, Bolzano's intermediate value theorem ensures there exists a unique c_{ss} for each \bar{b}^* . The exactly analogous argument ensures existence and uniqueness of c_{ss}^* . Call these mappings $\varpi : \bar{b}^* \mapsto c_{ss}, \varpi^* : \bar{b}^* \mapsto c_{ss}^*$. The implicit function theorem now yields:

$$\partial \varpi / \partial \bar{b}^* < 0 \text{ and } \partial \varpi^* / \partial \bar{b}^* > 0.$$

4. Finally, only one equation remains, (*BOP*), with only one variable, \bar{b}^* :

$$\begin{aligned} & \gamma(1-\lambda)[\varpi(\bar{b}^*) + \psi(\varpi(\bar{b}^*))] - (1-\gamma)(1-\lambda^*)[\varpi^*(\bar{b}^*) + \psi^*(\varpi^*(\bar{b}^*))] \\ & + (1-\gamma)(\beta^{-1} - 1)\bar{b}^* =: \mathcal{H}(\bar{b}^*) = 0 \end{aligned}$$

with $\bar{b}^* \mapsto \mathcal{H}$ continuous. It also holds that $\lim_{\bar{b}^* \rightarrow -\infty} \mathcal{H}(\bar{b}^*) = -\infty$.⁴⁰ On the other hand, as $\bar{b}^* \rightarrow +\infty$, $\mathcal{H} \rightarrow +\infty$.⁴¹ Thus, Bolzano's intermediate value theorem ensures **existence** of a $\bar{b}^* \in \mathbb{R}$ that satisfies the BOP-equation and thus existence of a non-stochastic steady state.

Uniqueness of the steady state can be shown by establishing strict positive monotonicity

⁴⁰Proof: (1) $\gamma(1-\lambda)[\varpi^*(\bar{b}^*) + \psi^*(\varpi^*(\bar{b}^*))] \geq 0$ by non-negativity of consumption & housing investment; (2) $\gamma(1-\lambda)[\varpi + \psi \circ \varpi] + (1-\gamma)(\beta^{-1} - 1)\bar{b}^* = \gamma(1-\lambda)[\phi - \frac{1-\gamma}{\gamma}(\beta^{-1} - 1)\bar{b}^*] + (1-\gamma)(\beta^{-1} - 1)\bar{b}^* = \gamma(1-\lambda)\phi + \lambda(1-\gamma)(\beta^{-1} - 1)\bar{b}^*$, where the second equality is a consequence of (*GMC*); (3) $\lim_{\bar{b}^* \rightarrow -\infty} \varpi = +\infty$ (assuming the contrary will produce a contradiction with (*GMC*)); (4) $\lim_{c \rightarrow 0} \phi = 0$; (5) steps 1–4 now imply $\mathcal{H}(\bar{b}^*) \leq \gamma(1-\lambda)\phi + \lambda(1-\gamma)(\beta^{-1} - 1)\bar{b}^* \rightarrow -\infty$ as $\bar{b}^* \rightarrow -\infty$. \square

⁴¹Proof: (1) $\gamma(1-\lambda)[\varpi(\bar{b}^*) + \psi(\varpi(\bar{b}^*))] \geq 0$ by non-negativity of consumption & housing investment; (2) $-(1-\lambda^*)[\varpi^*(\bar{b}^*) + \psi^*(\varpi^*(\bar{b}^*))] + (\beta^{-1} - 1)\bar{b}^* = -(1-\lambda^*)[\phi(c_{ss}^*) + (\beta^{-1} - 1)\bar{b}^*] + (\beta^{-1} - 1)\bar{b}^* = -(1-\lambda^*)\phi(c_{ss}^*) + \lambda^*(\beta^{-1} - 1)\bar{b}^*$ where the substitution is made using the definition of ϖ^* ; (3) $\lim_{\bar{b}^* \rightarrow +\infty} \varpi^* = +\infty$ (assuming the contrary will produce a contradiction with (*GMC**)); (4) Fact 3 and $\lim_{c \rightarrow +\infty} \phi(c) = 0$ implies $\mathcal{H}(\bar{b}^*) \geq -(1-\lambda^*)\phi(c_{ss}^*) + \lambda^*(\beta^{-1} - 1)\bar{b}^* \rightarrow +\infty$ as $\bar{b}^* \rightarrow +\infty$. \square

1.B. Proofs and Derivations

of \mathcal{H} : (we suppress arguments for brevity)

$$\begin{aligned}
\frac{\partial \mathcal{H}}{\partial \bar{b}^*} &= \gamma(1-\lambda)(1+\psi')\varpi' - (1-\gamma)(1-\lambda^*)(1+\psi^{*'})\varpi^{*'} \\
&\quad + (1-\gamma)(\beta^{-1}-1) \\
\text{step 2, cf. notes below:} \quad &> -(1-\gamma)(1-\lambda)(\beta^{-1}-1) - (1-\gamma)(1-\lambda^*)(\beta^{-1}-1) \\
&\quad + (1-\gamma)(\beta^{-1}-1) \\
&= (1-\gamma)(\beta^{-1}-1)[1-1+\lambda-1+\lambda^*] \\
&\stackrel{\text{sign}}{=} \lambda + \lambda^* - 1 \\
\text{step 3, cf. notes below:} \quad &= \lambda + 1 - \frac{\gamma}{1-\gamma}(1-\lambda) - 1 \\
&\stackrel{\text{sign}}{=} (1-\gamma)\lambda - \gamma(1-\lambda) \\
&= \lambda - \gamma \\
&\geq 0 \text{ by assumption.}
\end{aligned}$$

Step 3 follows by symmetric home bias, i.e. $\gamma(1-\lambda) = (1-\gamma)(1-\lambda^*) \iff \lambda^* = 1 - \frac{\gamma}{1-\gamma}(1-\lambda)$; Step 2 requires slightly more work: First, use the implicit function theorem on (GMC) & $(GMC)^*$, respectively, to obtain

$$\begin{aligned}
\varpi' &= -\frac{\partial \zeta / \partial \bar{b}^*}{\partial \zeta / \partial c_{ss}} = \frac{\frac{1-\gamma}{\gamma}(\beta^{-1}-1)}{\phi' - (1+\psi')} < 0, \\
\varpi^{*'} &= -\frac{\partial \zeta^* / \partial \bar{b}^*}{\partial \zeta^* / \partial c_{ss}^*} = \frac{-(\beta^{-1}-1)}{\phi' - (1+\psi^{*'})} > 0;
\end{aligned}$$

Second, recognize that since $\phi' < 0$ it is

$$\frac{1+\psi'}{1+\psi' - \phi'} < 1 \iff \frac{1+\psi'}{-(1+\psi') + \phi'} > -1$$

and symmetrically for $*$. This shows that $\frac{\partial \mathcal{H}}{\partial \bar{b}^*} > 0$, and the SS is unique.

Finally, notice that since existence and uniqueness follow for arbitrary parameters $\eta \in (0, 1), \eta^* \in (0, 1), \xi_{h,ss}, \xi_{x,ss}, \xi_{h,ss}^*, \xi_{x,ss}^* > 0$, it is possible to set $\eta^* > \eta$ and then choose $\xi_{x,ss}, \xi_{x,ss}^*$ so as to ensure that both (HMC) and $(HMC)^*$ hold if $h_{ss} = h_{ss}^*$ and $x_{ss} = x_{ss}^*$. Given this choice of $\xi_{x,ss}, \xi_{x,ss}^*$, and the symmetry in housing stock and housing investment, it is then possible to select $\xi_{h,ss}, \xi_{h,ss}^*$ such that the equations $(x), (x^*)$ hold. It then follows that $c_{ss} = c_{ss}^*, n_{ss} = n_{ss}^*, \bar{b} = \bar{b}^* = 0$, and the allocation is symmetric. This completes the proof.

1.B.7 Derivation of the household's subjectively optimal plans

In this Appendix, we provide a formal derivation of the linearized subjectively optimal household decision rules presented in equations (1.9) and (1.10). Throughout the derivation, we concentrate on the representative household in H with the understanding that the situation in F is symmetric. For simplicity, we omit domestically produced housing investment goods. This does not affect the layout of the proof below.

Consider the household program presented in Section (1.3.1), which is restated here for convenience.⁴² We first clarify the shape of the underlying probability space to set the appropriate frame for the following derivations: denote $\Omega \ni \omega_t := (\xi_t, r_t, w_t, \Sigma_t, \pi_t, (P_{H,t}/P_{F,t}), q_t)^\top$ the vector of external decision-relevant variables that the household takes as given, denote $\Omega^t \ni \omega^t := (\omega_{t-s})_{s \geq 0}$ the one-sided infinite history of past external variables and denote $\Omega^\infty \ni \omega := (\omega_t)_{t \in \mathbb{Z}}$ the typical element from the set of possible realizations of full sequences of external variables;⁴³ denote \mathcal{B}^∞ the Borel-sigma-algebra over Ω^∞ . Each household is now endowed with a probability measure \mathcal{P} over $(\Omega^\infty, \mathcal{B}^\infty)$ which encodes her subjective beliefs over the realizations of external variables ω . Rational Expectations, denoted $\mathcal{P} = \mathbb{P}$, are a special case of this setup where \mathbb{P} is the (unique) measure generated by the distribution of $\xi := (\xi_t)_{t \in \mathbb{Z}}$ and the equilibrium conditions that allow to compute $\omega \setminus \xi$ as a deterministic function of ξ .⁴⁴ As explained in Section 1.3.1, we assume \mathcal{P} to be of a particular form, that is we assume that $\mathcal{P} = \mathcal{P}_q \otimes \mathbb{P}_{-q}$, where \otimes is the product measure (statistical independence), \mathcal{P}_q is the distribution over $(q_t)_{t \in \mathbb{Z}}$ generated by the unobserved components model (3.4), and \mathbb{P}_{-q} is the rational expectations measure without house prices—i.e. the measure over $\omega \setminus q = (\xi_t, r_t, w_t, \Sigma_t, \pi_t, (P_{H,t}/P_{F,t}))_{t \in \mathbb{Z}}$ that which is consistent with the equilibrium-implied joint probability distribution of $\omega \setminus q$. Consequently, for any two measurable functions $f : \mathbb{R} \rightarrow \mathbb{R}, g : \mathbb{R}^{\dim \Omega - 1} \rightarrow \mathbb{R}$, we have

$$\mathbb{E}^{\mathcal{P}}[f(q_t) \cdot g(\omega_t \setminus q_t)] = \int f \mathcal{P}_q(dq) \cdot \int g \mathbb{P}_{-q}(d\omega \setminus q).$$

Finally, $\mathbb{E}_t^{\mathcal{P}}$ is the expectation implied by \mathcal{P} conditional on the sigma-algebra generated by ω^t .

Now, at each calendar date $t \in \mathbb{Z}$, the household takes as given ω^t and chooses today's consumption, labor, housing, housing investment, and bond levels, $(c_t, n_t, h_t, x_t, b_{t+1})$, as well as contingent plans for the future, $\{(c_{t+s}, n_{t+s}, h_{t+s}, x_{t+s}, b_{t+s+1}) : \Omega^{t+s} \rightarrow \mathbb{R}_+^4 \times \mathbb{R}\}_{s \in \mathbb{N}_+}$, to max-

⁴²We ignore the terms T_{t+s}, b_{t+s} since they will be zero in the equilibrium we analyze. All derivations go through if these terms are non-zero.

⁴³We restrict Ω to be the set of absolutely summable sequences.

⁴⁴For any three vectors x, y, z where $x = (y, z)$, we define $x \setminus y := z$.

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imize

$$\begin{aligned} \mathbb{E}_t^p \sum_{s=0}^{\infty} \beta^s \left(\frac{\xi_{c,t+s} c_{t+s}^{1-\sigma}}{1-\sigma} + \frac{\xi_{h,t+s} h_{t+s}^{1-\nu}}{1-\nu} - \chi \frac{n_{t+s}^{1+\varphi}}{1+\varphi} \right) \text{ subject to} \\ c_{t+s} + q_{t+s} (h_{t+s} - (1-\delta)h_{t+s-1}) + b_{t+s+1} + x_{t+s} = w_{t+s} n_{t+s} \\ + (1+r_{t+s})b_{t+s} + q_{t+s} \cdot \frac{\xi_{x,t+s}}{\eta} x_{t+s-\tau}^\eta + \Sigma_{t+s}, \quad \forall s \geq 0, \mathcal{P}\text{-almost surely,} \end{aligned} \quad (1.26)$$

as well as subject to a standard no-Ponzi-game condition on b . The first-order conditions for program (1.26) (suppressing the transversality conditions) are:

$$\begin{aligned} \forall s \geq 0, \quad (h) \quad & \xi_{c,t+s} c_{t+s}^{-\sigma} q_{t+s} = \xi_{h,t+s} h_{t+s}^{-\nu} + \beta(1-\delta) \mathbb{E}_{t+s}^{\mathcal{P}} \{ \xi_{c,t+s+1} c_{t+s+1}^{-\sigma} q_{t+s+1} \}, \\ (n) \quad & \chi n_{t+s}^\varphi \xi_{c,t+s}^{-1} c_{t+s}^\sigma = w_{t+s}, \\ (b) \quad & \xi_{c,t+s} c_{t+s}^{-\sigma} = \beta \mathbb{E}_{t+s}^{\mathcal{P}} \{ (1+r_{t+s+1}) \xi_{c,t+s+1} c_{t+s+1}^{-\sigma} \}, \\ (x) \quad & \beta^\tau \mathbb{E}_{t+s}^{\mathcal{P}} \left\{ q_{t+s+\tau} \xi_{c,t+s+\tau} c_{t+s+\tau}^{-\sigma} \cdot \xi_{x,t+s+\tau} x_{t+s+\tau}^{\eta-1} \right\} = \xi_{c,t+s} c_{t+s}^{-\sigma}, \\ (BC) \quad & c_{t+s} + q_{t+s} (h_{t+s} - (1-\delta)h_{t+s-1}) + b_{t+s+1} + x_{t+s} = w_{t+s} n_{t+s} \\ & + (1+r_{t+s})b_{t+s} + q_{t+s} \cdot \frac{\xi_{x,t+s}}{\eta} x_{t+s-\tau}^\eta + \Sigma_{t+s}. \end{aligned} \quad (1.27)$$

In close analogy to the standard procedure in a model with fully rational expectations, $\mathcal{P} = \mathbb{P}$, we now derive a linear approximation to (1.27) that – together with the other linearized equilibrium conditions – allows to solve the model to first order in the amplitude of shocks. The specific challenge here, with $\mathcal{P} = \mathcal{P}_q \otimes \mathbb{P}_{-q}$, will be to compute all expectations explicitly that depend on house prices, q . This includes the house prices themselves, $(q_{t+s})_{s>0}$, as well as expectations over future contingent choices, $(c_{t+s}, n_{t+s}, h_{t+s}, x_{t+s}, b_{t+s+1})_{s>0}$.

To first order around the steady state⁴⁵ from Lemma 2, in which we have $1+r_{ss} = \beta^{-1}$, $\chi n_{ss}^\varphi c_{ss}^\sigma = w_{ss}(=1)$, $q_{ss} = \xi_{h,ss} h_{ss}^{-\nu} c_{ss}^\sigma / (1-\bar{\beta})$, $\beta^\tau q_{ss} \xi_{x,ss} x_{ss}^{\eta-1} = 1$, it holds that, $\forall s \geq 0$:

$$\begin{aligned} (h) \quad & \hat{h}_{t+s} = \frac{1}{\nu} \xi_{h,t+s} + \frac{\sigma}{\nu} \frac{\hat{c}_{t+s} - \frac{1}{\sigma} \hat{\xi}_{c,t+s} - \bar{\beta} \mathbb{E}_{t+s}^{\mathcal{P}} \left\{ \hat{c}_{t+s+1} - \frac{1}{\sigma} \hat{\xi}_{c,t+s+1} \right\}}{1-\bar{\beta}} - \frac{1}{\nu} \frac{\hat{q}_{t+s} - \bar{\beta} \mathbb{E}_{t+s}^{\mathcal{P}} \{ \hat{q}_{t+s+1} \}}{1-\bar{\beta}}, \\ (n) \quad & \varphi \hat{n}_{t+s} + \sigma \hat{c}_{t+s} - \hat{\xi}_{c,t+s} = \hat{w}_{t+s}, \\ (b) \quad & \hat{c}_{t+s} - \frac{1}{\sigma} \hat{\xi}_{c,t+s} = -\frac{1}{\sigma} \mathbb{E}_{t+s} \{ \hat{r}_{t+s+1} \} + \mathbb{E}_{t+s}^{\mathcal{P}} \left\{ \hat{c}_{t+s+1} - \frac{1}{\sigma} \hat{\xi}_{c,t+s+1} \right\}, \\ (x) \quad & \mathbb{E}_{t+s}^{\mathcal{P}} \hat{q}_{t+s+\tau} - \mathbb{E}_{t+s}^{\mathcal{P}} \left\{ \sigma \hat{c}_{t+s+\tau} - \hat{\xi}_{c,t+s+\tau} \right\} \\ & + \left(\sigma \hat{c}_{t+s} - \hat{\xi}_{c,t+s} \right) + \mathbb{E}_{t+s} \hat{\xi}_{x,t+s+\tau} = (1-\eta) \hat{x}_{t+s}, \\ (BC) \quad & \hat{b}_{t+s+1} = \frac{1}{\bar{\beta}} \hat{b}_{t+s} + \frac{w_{ss} n_{ss}}{y_{ss}} (\hat{w}_{t+s} + \hat{n}_{t+s}) + \hat{\Sigma}_{t+s} - \frac{c_{ss}}{y_{ss}} \hat{c}_{t+s} \\ & - \frac{q_{ss} h_{ss}}{y_{ss}} [\hat{h}_{t+s} - (1-\delta) \hat{h}_{t+s-1}] + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \left(\hat{x}_{t+s-\tau} - \beta^\tau \hat{x}_{t+s} + \hat{\xi}_{x,t+s} \right). \end{aligned} \quad (1.28)$$

⁴⁵Technically, we are scaling $\text{Var}^{\mathcal{P}}[|\omega|] \rightarrow 0$. This is consistent, however, with the definition of \mathcal{P} and $\text{Var}[|\xi|] \rightarrow 0$.

To characterize the household's expectations over own choice variables, start with the optimality condition for liquid bonds. Iteration over future instances of the condition reveals:

$$(b) \quad \hat{c}_{t+s} - \frac{1}{\sigma} \hat{\xi}_{c,t+s} = -\frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n}\} + \lim_{n \rightarrow \infty} \mathbb{E}_{t+s}^{\mathcal{P}} \hat{c}_{t+s+n}.$$

To keep notation concise, we define for any random process $(k_t)_t$: $\mathbb{E}_{t+s}^{\mathcal{P}} k_{\infty} := \lim_{n \rightarrow \infty} \mathbb{E}_{t+s}^{\mathcal{P}} k_{t+s+n}$. We implicitly assumed here that a law of iterated expectations holds for $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}$ – at the end of the section we verify that the $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}$ we recover does indeed satisfy a law of iterated expectations. Careful inspection of equations (1.28) reveals that $\mathbb{E}_{t+s}^{\mathcal{P}} \hat{c}_{\infty}$ is the only subjective expectation of a choice variable left unknown. This is because the law of iterated expectations for \mathcal{P} allows substituting (b) into

$$\begin{aligned} (h) \quad \hat{h}_{t+s} &= \frac{1}{\nu} \xi_{h,t+s} - \frac{\sigma}{\nu} \frac{\frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n}\} - \bar{\beta} \frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n+1}\}}{1 - \bar{\beta}} - \frac{1}{\nu} \frac{\hat{q}_{t+s} - \bar{\beta} \mathbb{E}_{t+s}^{\mathcal{P}} \{\hat{q}_{t+s+1}\}}{1 - \bar{\beta}} + \frac{\sigma}{\nu} \mathbb{E}_{t+s}^{\mathcal{P}} \hat{c}_{\infty}, \\ (n) \quad \varphi \hat{n}_{t+s} + \sigma \hat{c}_{t+s} - \hat{\xi}_{c,t+s} &= \hat{w}_{t+s}, \\ (b) \quad \hat{c}_{t+s} - \frac{1}{\sigma} \hat{\xi}_{c,t+s} &= -\frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n}\} + \mathbb{E}_{t+s}^{\mathcal{P}} \hat{c}_{\infty}, \\ (x) \quad \mathbb{E}_{t+s}^{\mathcal{P}} \hat{q}_{t+s+\tau} - \sum_{n=1}^{\tau} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n}\} + \mathbb{E}_{t+s} \hat{\xi}_{x,t+s+\tau} &= (1 - \eta) \hat{x}_{t+s}, \\ (BC) \quad \hat{b}_{t+s+1} &= \frac{1}{\beta} \hat{b}_{t+s} + \frac{w_{ss} n_{ss}}{y_{ss}} (\hat{w}_{t+s} + \hat{n}_{t+s}) + \hat{\Sigma}_{t+s} - \frac{c_{ss}}{y_{ss}} \hat{c}_{t+s} \\ &\quad - \frac{q_{ss} h_{ss}}{y_{ss}} [\hat{h}_{t+s} - (1 - \delta) \hat{h}_{t+s-1}] + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \left(\hat{x}_{t+s-\tau} - \beta^{\tau} \hat{x}_{t+s} + \hat{\xi}_{x,t+s} \right). \end{aligned} \tag{1.29}$$

Now to find $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}$ (and characterize the choices at calendar date t), we first use the budget constraint for some $s > \tau$. Start by plugging in the optimality conditions:

$$\begin{aligned} (BC) \quad \hat{b}_{t+s+1} &= \frac{1}{\beta} \hat{b}_{t+s} + \frac{w_{ss} n_{ss}}{y_{ss}} (1 + 1/\varphi) \hat{w}_{t+s} + \hat{\Sigma}_{t+s} - \frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \left(\hat{c}_{t+s}^* - \frac{1}{\sigma} \hat{\xi}_{c,t+s} \right) - \frac{c_{ss}}{y_{ss} \sigma} \hat{\xi}_{c,t+s} \\ &\quad - \frac{q_{ss} h_{ss}}{y_{ss}} [\hat{h}_{t+s}^* - (1 - \delta) \hat{h}_{t+s-1}^*] + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \left(\hat{x}_{t+s-\tau}^* - \beta^{\tau} \hat{x}_{t+s}^* + \hat{\xi}_{x,t+s} \right) \\ &\quad - \frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi + \delta q_{ss} h_{ss} \sigma / \nu}{y_{ss}} \mathbb{E}_{t+s}^{\mathcal{P}} \hat{c}_{\infty} \\ &\quad + \frac{q_{ss} h_{ss}}{y_{ss}} \frac{1}{\nu} \frac{1}{1 - \bar{\beta}} \left(\hat{q}_{t+s} - \bar{\beta} \mathbb{E}_{t+s}^{\mathcal{P}} \{\hat{q}_{t+s+1}\} - (1 - \delta) \left(\hat{q}_{t+s-1} - \bar{\beta} \mathbb{E}_{t+s-1}^{\mathcal{P}} \{\hat{q}_{t+s}\} \right) \right) \\ &\quad + \frac{x_{ss}}{y_{ss}} \frac{1}{\beta^{\tau} (1 - \eta)} \left(\mathbb{E}_{t+s-\tau}^{\mathcal{P}} \{\hat{q}_{t+s}\} - \beta^{\tau} \mathbb{E}_{t+s}^{\mathcal{P}} \{\hat{q}_{t+s+\tau}\} \right) \end{aligned}$$

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where we have defined the auxiliary variables

$$\begin{aligned}\hat{h}_{t+s}^* &:= \frac{1}{\nu} \xi_{h,t+s} - \frac{\sigma}{\nu} \frac{\frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n}\} - \bar{\beta} \frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n+1}\}}{1-\bar{\beta}}, \\ \hat{c}_{t+s}^* - \frac{1}{\sigma} \hat{\xi}_{c,t+s} &:= -\frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n}\}, \\ (1-\eta) \hat{x}_{t+s}^* &:= -\sum_{n=1}^{\tau} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n}\} + \mathbb{E}_{t+s} \hat{\xi}_{x,t+s+\tau}.\end{aligned}$$

Notice that the auxiliary variables are composed only of terms over which the household has rational expectations; thus of terms that fade to zero as $s \rightarrow \infty$.

Next, we apply the operator $\mathbb{E}_t^{\mathcal{P}}$ to both sides of (BC) and consider the limit as $s \rightarrow \infty$. This delivers:

$$\begin{aligned}(BC) \quad \mathbb{E}_t^{\mathcal{P}} \hat{b}_{\infty} &= \frac{1}{\beta} \mathbb{E}_t^{\mathcal{P}} \hat{b}_{\infty} - \frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi + \delta q_{ss} h_{ss} \sigma / \nu}{y_{ss}} \mathbb{E}_{t+s}^{\mathcal{P}} \hat{c}_{\infty} \\ &\quad + \frac{\delta q_{ss} h_{ss} / \nu + x_{ss} (\beta^{-\tau} - 1) / (1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \hat{q}_{\infty} \\ \iff \mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty} &= \frac{y_{ss}}{c_{ss} + n_{ss} w_{ss} \sigma / \varphi + \delta q_{ss} h_{ss} \sigma / \nu} \left[\frac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \hat{b}_{\infty} + \frac{\delta q_{ss} h_{ss} / \nu + x_{ss} (\beta^{-\tau} - 1) / (1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \hat{q}_{\infty} \right].\end{aligned}$$

Now since $\mathbb{E}_t^{\mathcal{P}} \hat{q}_{\infty}$ can be computed from the subjectively perceived house price model, the only unknown left is $\mathbb{E}_t^{\mathcal{P}} \hat{b}_{\infty}$. To find it, we plug $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}$ into (BC). At some $s > 0$ we have

$$\begin{aligned}(BC) \quad \mathbb{E}_t^{\mathcal{P}} \hat{b}_{t+s+1} &= \frac{1}{\beta} \mathbb{E}_t^{\mathcal{P}} \hat{b}_{t+s} + \frac{w_{ss} n_{ss}}{y_{ss}} (1 + 1/\varphi) \mathbb{E}_t \hat{w}_{t+s} + \mathbb{E}_t \hat{\Sigma}_{t+s} - \frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \mathbb{E}_t \left(\hat{c}_{t+s}^* - \frac{1}{\sigma} \hat{\xi}_{c,t+s} \right) \\ &\quad - \frac{c_{ss}}{y_{ss} \sigma} \mathbb{E}_t \hat{\xi}_{c,t+s} - \frac{q_{ss} h_{ss}}{y_{ss}} \mathbb{E}_t [\hat{h}_{t+s}^* - (1-\delta) \hat{h}_{t+s-1}^*] + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \mathbb{E}_t^{\mathcal{P}} \left(\hat{x}_{t+s-\tau} - \beta^{\tau} \hat{x}_{t+s} + \hat{\xi}_{x,t+s} \right) \\ &\quad - \frac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \hat{b}_{\infty} - \frac{\delta q_{ss} h_{ss} / \nu + x_{ss} (\beta^{-\tau} - 1) / (1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \hat{q}_{\infty} \\ &\quad + \frac{q_{ss} h_{ss}}{y_{ss}} \frac{1}{\nu} \frac{1}{1-\beta} \mathbb{E}_t^{\mathcal{P}} \left(\hat{q}_{t+s} - \bar{\beta} \mathbb{E}_{t+s}^{\mathcal{P}} \{\hat{q}_{t+s+1}\} - (1-\delta) \left(\hat{q}_{t+s-1} - \bar{\beta} \mathbb{E}_{t+s-1}^{\mathcal{P}} \{\hat{q}_{t+s}\} \right) \right).\end{aligned}$$

After defining the auxiliary variables

$$\begin{aligned}z_{t+s}^* &:= \frac{w_{ss} n_{ss}}{y_{ss}} (1 + 1/\varphi) \hat{w}_{t+s} + \hat{\Sigma}_{t+s} - \frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \left(\hat{c}_{t+s}^* - \frac{1}{\sigma} \hat{\xi}_{c,t+s} \right) - \frac{c_{ss}}{y_{ss} \sigma} \hat{\xi}_{c,t+s} \\ &\quad - \frac{q_{ss} h_{ss}}{y_{ss}} [\hat{h}_{t+s}^* - (1-\delta) \hat{h}_{t+s-1}^*] + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \hat{\xi}_{x,t+s}, \\ Q_{t+s} &:= \frac{q_{ss} h_{ss}}{y_{ss}} \frac{1}{\nu} \frac{1}{1-\beta} \left(\hat{q}_{t+s} - \bar{\beta} \mathbb{E}_{t+s}^{\mathcal{P}} \{\hat{q}_{t+s+1}\} - (1-\delta) \left(\hat{q}_{t+s-1} - \bar{\beta} \mathbb{E}_{t+s-1}^{\mathcal{P}} \{\hat{q}_{t+s}\} \right) \right),\end{aligned} \tag{1.30}$$

we can rewrite (BC) into

$$\begin{aligned}(BC) \quad \mathbb{E}_t^{\mathcal{P}} \hat{b}_{t+s+1} &= \frac{1}{\beta} \mathbb{E}_t^{\mathcal{P}} \hat{b}_{t+s} + \mathbb{E}_t z_{t+s}^* + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \mathbb{E}_t^{\mathcal{P}} (\hat{x}_{t+s-\tau} - \beta^{\tau} \hat{x}_{t+s}) + \mathbb{E}_t^{\mathcal{P}} Q_{t+s} \\ &\quad - \frac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \hat{b}_{\infty} - \frac{\delta q_{ss} h_{ss} / \nu + x_{ss} (\beta^{-\tau} - 1) / (1-\eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \hat{q}_{\infty}.\end{aligned}$$

Performing backward substitution until $s = 1$ delivers

$$\begin{aligned}
 (BC) \quad \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s+1} &= \beta^{-s} \widehat{b}_{t+1} - \beta^{-s} (1 - \beta^s) \frac{\beta}{1 - \beta} \left[\frac{1 - \beta}{\beta} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{\infty} + \frac{\delta q_{ss} h_{ss} / \nu + x_{ss} (\beta^{-\tau} - 1) / (1 - \eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_{\infty} \right] \\
 &\quad + \beta^{-s} \sum_{n=1}^s \beta^n (\mathbb{E}_t z_{t+n}^* + \mathbb{E}_t^{\mathcal{P}} Q_{t+n}) + \beta^{-s} \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \sum_{n=1}^s \beta^n \mathbb{E}_t^{\mathcal{P}} (\widehat{x}_{t+n-\tau} - \beta^{\tau} \widehat{x}_{t+n}) \\
 \iff \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s+1} - \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{\infty} &= \beta^{-s} \cdot \left[\widehat{b}_{t+1} + \sum_{n=1}^s \beta^n (\mathbb{E}_t z_{t+n}^* + \mathbb{E}_t^{\mathcal{P}} Q_{t+n}) + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \sum_{n=1}^s \beta^n \mathbb{E}_t^{\mathcal{P}} (\widehat{x}_{t+n-\tau} - \beta^{\tau} \widehat{x}_{t+n}) \right. \\
 &\quad \left. - \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{\infty} - (1 - \beta^s) \frac{\beta}{1 - \beta} \frac{\delta q_{ss} h_{ss} / \nu + x_{ss} (\beta^{-\tau} - 1) / (1 - \eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_{\infty} \right].
 \end{aligned}$$

Now since we know that $\lim_{s \rightarrow \infty} \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{t+s}$ exists, it must be that the left side of this equation tends to zero as $s \rightarrow \infty$. But since $\beta^{-s} \rightarrow +\infty$ as $s \rightarrow \infty$, this implies that

$$\begin{aligned}
 \mathbb{E}_t^{\mathcal{P}} \widehat{b}_{\infty} &= \widehat{b}_{t+1} + \sum_{n=1}^{\infty} \beta^n (\mathbb{E}_t z_{t+n}^* + \mathbb{E}_t^{\mathcal{P}} Q_{t+n}) - \frac{\beta}{1 - \beta} \frac{\delta q_{ss} h_{ss} / \nu + x_{ss} (\beta^{-\tau} - 1) / (1 - \eta)}{y_{ss}} \mathbb{E}_t^{\mathcal{P}} \widehat{q}_{\infty} \\
 &\quad + \underbrace{\frac{x_{ss}}{y_{ss}} \beta^{-\tau} \sum_{n=1}^{\infty} \beta^n \mathbb{E}_t^{\mathcal{P}} (\widehat{x}_{t+n-\tau} - \beta^{\tau} \widehat{x}_{t+n})}_{= \beta^{-\tau} \sum_{n=1}^{\tau} \beta^n \widehat{x}_{t+n-\tau}}
 \end{aligned}$$

so that we can characterize $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{\infty}$ entirely in terms of variables that are either pre-determined, current choice variables (i.e. known under $\mathbb{E}_t^{\mathcal{P}}$), or variables of which we can compute expectations in closed form (be they rational or not):

$$\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{\infty} = \frac{y_{ss}}{c_{ss} + n_{ss} w_{ss} \sigma / \varphi + \delta q_{ss} h_{ss} \sigma / \nu} \frac{1 - \beta}{\beta} \left[\widehat{b}_{t+1} + \sum_{n=1}^{\infty} \beta^n (\mathbb{E}_t z_{t+n}^* + \mathbb{E}_t^{\mathcal{P}} Q_{t+n}) + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \sum_{n=1}^{\tau} \beta^n \widehat{x}_{t+n-\tau} \right].$$

In the last step, we explicitly characterize the subjective expectation $\mathbb{E}_t^{\mathcal{P}} \sum_{n=1}^{\infty} \beta^n Q_{t+n}$ in terms of the processes governing subjective house price expectations, namely $\widehat{q}_t, \widehat{m}_t$.

Recall that for any $s > 0$ we have $\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s} = \widehat{q}_t + (1 - \varrho^s) \frac{\varrho}{1 - \varrho} \widehat{m}_t$, so that we have

$$\begin{aligned}
 \mathbb{E}_t^{\mathcal{P}} Q_{t+s} &= \frac{q_{ss} h_{ss}}{y_{ss}} \frac{1}{\nu} \frac{1}{1 - \beta} \left(\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s} - \bar{\beta} \mathbb{E}_t^{\mathcal{P}} \{\widehat{q}_{t+s+1}\} - (1 - \delta) \left(\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s-1} - \bar{\beta} \mathbb{E}_t^{\mathcal{P}} \{\widehat{q}_{t+s}\} \right) \right) \\
 &= \frac{q_{ss} h_{ss}}{y_{ss}} \frac{1}{\nu} \frac{1}{1 - \beta} \left(\delta (1 - \bar{\beta}) \widehat{q}_t + (1 - \varrho^s) \frac{\varrho}{1 - \varrho} \widehat{m}_t - \bar{\beta} (1 - \varrho^{s+1}) \frac{\varrho}{1 - \varrho} \widehat{m}_t \right. \\
 &\quad \left. - (1 - \delta) (1 - \varrho^{s-1}) \frac{\varrho}{1 - \varrho} \widehat{m}_t + (1 - \delta) \bar{\beta} (1 - \varrho^s) \frac{\varrho}{1 - \varrho} \widehat{m}_t \right) \\
 &= \frac{q_{ss} h_{ss}}{y_{ss}} \frac{1}{\nu} \frac{1}{1 - \beta} \left(\delta (1 - \bar{\beta}) \widehat{q}_t + \widehat{m}_t \frac{\varrho}{1 - \varrho} \left[\delta (1 - \bar{\beta}) + (1 - \bar{\beta} \varrho) (1 - \varrho - \delta) \varrho^{s-1} \right] \right)
 \end{aligned}$$

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which in turn implies

$$\sum_{s \geq 1} \beta^s \mathbb{E}_t^{\mathcal{P}} Q_{t+s} = \beta \frac{q_{ss} h_{ss}}{y_{ss}} \frac{1}{\nu} \left(\frac{\delta}{1-\beta} \hat{q}_t + \hat{m}_t \frac{\varrho}{1-\varrho} \left[\frac{\delta}{1-\beta} + \frac{1-\bar{\beta}\varrho}{1-\beta} \frac{1-\varrho-\delta}{1-\beta\varrho} \right] \right). \quad (1.31)$$

Notice that, as claimed in the main text, it is that

$$\frac{\delta}{1-\beta} + \frac{1-\bar{\beta}\varrho}{1-\beta} \frac{1-\varrho-\delta}{1-\beta\varrho} > 0, \quad \forall \beta, \delta, \varrho \in (0, 1),^{46}$$

so that the coefficient of the posterior expected house price growth rate onto the series is positive.

We now have characterized the household's decisions at an arbitrary calendar date t up to first order around the deterministic steady state:

$$\begin{aligned} (h) \quad & \hat{h}_t = \frac{1}{\nu} \xi_{h,t} - \frac{\sigma}{\nu} \frac{\frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_t \{\hat{r}_{t+n}\} - \bar{\beta} \frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_t \{\hat{r}_{t+n+1}\}}{1-\bar{\beta}} - \frac{1}{\nu} \hat{q}_t + \frac{1}{\nu} \frac{\bar{\beta}}{1-\bar{\beta}} \varrho \hat{m}_t + \frac{\sigma}{\nu} \mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}, \\ (n) \quad & \varphi \hat{n}_t + \sigma \hat{c}_t - \hat{\xi}_{c,t} = \hat{w}_t, \\ (b) \quad & \hat{c}_t - \frac{1}{\sigma} \hat{\xi}_{c,t} = -\frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_t \{\hat{r}_{t+n}\} + \mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}, \\ (x) \quad & \hat{q}_t + (1-\varrho^{\tau}) \frac{\varrho}{1-\varrho} \hat{m}_t - \sum_{n=1}^{\tau} \mathbb{E}_t \{\hat{r}_{t+n}\} + \mathbb{E}_t \hat{\xi}_{x,t+\tau} = (1-\eta) \hat{x}_t, \\ (BC) \quad & \hat{b}_{t+1} = \frac{1}{\beta} \hat{b}_t + \frac{w_{ss} n_{ss}}{y_{ss}} (\hat{w}_t + \hat{n}_t) + \hat{\Sigma}_t - \frac{c_{ss}}{y_{ss}} \hat{c}_t \\ & \quad - \frac{q_{ss} h_{ss}}{y_{ss}} [\hat{h}_t - (1-\delta) \hat{h}_{t-1}] + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \left(\hat{x}_{t-\tau} - \beta^{\tau} \hat{x}_t + \hat{\xi}_{x,t} \right), \end{aligned} \quad (1.32)$$

where, after a few rearrangements,

$$\begin{aligned} \mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty} &= \frac{\delta q_{ss} h_{ss} / \nu}{c_{ss} + \sigma / \varphi \cdot n_{ss} w_{ss} + \delta q_{ss} h_{ss} \sigma / \nu} \cdot \left[\hat{q}_t + \hat{m}_t \cdot \frac{\varrho}{1-\varrho} \underbrace{\left(1 + \frac{1-\bar{\beta}\varrho}{1-\bar{\beta}} \frac{1-\varrho-\delta}{1-\beta\varrho} \frac{1-\beta}{\delta} \right)}_{> 0 \quad \forall \beta, \delta, \varrho \in (0, 1)} \right] \\ &+ \frac{y_{ss}}{c_{ss} + \sigma / \varphi \cdot n_{ss} w_{ss} + \delta q_{ss} h_{ss} \sigma / \nu} \frac{1-\beta}{\beta} \cdot \left[\sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{z_{t+n}^*\} + \frac{x_{ss}}{y_{ss}} \sum_{n=0}^{\tau-1} \beta^{-n} \hat{x}_{t-n} + \hat{b}_{t+1} \right]. \end{aligned}$$

Lastly, we verify that the law of iterated expectation holds for the explicit formula given for $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}$ above. Define

$$A_t := \hat{b}_{t+1} + \sum_{n=1}^{\infty} \beta^n (\mathbb{E}_t z_{t+n}^* + \mathbb{E}_t^{\mathcal{P}} Q_{t+n}) + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \sum_{n=1}^{\tau} \beta^n \hat{x}_{t+n-\tau}$$

⁴⁶ *Proof.* This follows from the claim being equivalent to the statement $f(\varrho) := \frac{\delta}{1-\beta} (1-\beta\varrho) + \frac{1-\bar{\beta}\varrho}{1-\beta} (1-\varrho-\delta) > 0, \quad \forall \beta, \delta, \varrho \in (0, 1)$, from f being an upward-open parabola with $f(1) = 0$, and $f'(1) < 0$. \square

and note that the law of iterated expectations holds if and only if $\mathbb{E}_t^{\mathcal{P}} A_{t+1} = A_t$. Using equation (BC) to substitute \hat{b}_{t+2} in $\mathbb{E}_t^{\mathcal{P}} A_{t+1}$ we arrive at:

$$\begin{aligned} \mathbb{E}_t^{\mathcal{P}} A_{t+1} = A_t / \beta - \frac{c_{ss} + \sigma / \varphi \cdot n_{ss} w_{ss}}{c_{ss} + \sigma / \varphi \cdot n_{ss} w_{ss} + \delta q_{ss} h_{ss} \sigma / \nu} \frac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} A_{t+1} - \frac{q_{ss} h_{ss} \sigma / \nu}{c_{ss} + \sigma / \varphi \cdot n_{ss} w_{ss} + \delta q_{ss} h_{ss} \sigma / \nu} \frac{1-\beta}{\beta} \mathbb{E}_t^{\mathcal{P}} A_{t+1} \\ + \frac{(1-\delta) q_{ss} h_{ss} \sigma / \nu}{c_{ss} + \sigma / \varphi \cdot n_{ss} w_{ss} + \delta q_{ss} h_{ss} \sigma / \nu} \frac{1-\beta}{\beta} A_t, \end{aligned}$$

which can be rearranged into $\mathbb{E}_t^{\mathcal{P}} A_{t+1} = A_t$.

1.B.8 Solving the model with Dynare

Although equations (3.20) serve as an explicit solution of the household decision problem at time t ,⁴⁷ they are not in a form that lends itself to easy numerical implementation, i.e. to solving the model. Bringing the equations into a recursive form that – when combined with the other equations describing equilibrium – can be solved by standard methods is the goal of this appendix.

In a first step, we take care of the appearing infinite sums which are not easily recursifiable. By that, we mean the forward summation over expected real interest rates, $\hat{c}_t^* = -1/\sigma \cdot \sum_{n \geq 1} \mathbb{E}_t \hat{r}_{t+n}$. In principle, this variable could be recursified as

$$\hat{c}_t^* = -1/\sigma \cdot \mathbb{E}_t \hat{r}_{t+1} + \mathbb{E}_t \hat{c}_{t+1}^*.$$

This representation would be incomplete, though, without the boundary condition $\lim_{s \rightarrow \infty} \mathbb{E}_t \hat{c}_{t+s}^* = 0$. To the best of our knowledge, imposing such a boundary condition is not possible in Dynare's native `stoch_simul`-command.⁴⁸ Therefore, we employ several rearrangements of the equations (3.20) in order to eliminate \hat{c}_t^* . First, consider the geometric summation over \hat{c}_t^* , which is contained in $\sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{z_{t+n}^*\}$ (z^* is defined in equation (3.C)):

$$\begin{aligned} \sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{\hat{c}_{t+n}^*\} &= -\frac{\beta}{\sigma} \sum_{n \geq 0} \beta^n \sum_{s \geq 0} \mathbb{E}_t \hat{r}_{t+2+n+s} \\ &= -\frac{\beta}{\sigma} \sum_{n \geq 0} \mathbb{E}_t \hat{r}_{t+2+n} \cdot \sum_{s=0}^n \beta^s = \frac{\beta}{1-\beta} \left[\underbrace{-\frac{1}{\sigma} \sum_{s \geq 0} \mathbb{E}_t \hat{r}_{t+2+s} + \frac{1}{\sigma} \sum_{s \geq 1} \beta^s \mathbb{E}_t \hat{r}_{t+1+s}}_{=\mathbb{E}_t \hat{c}_{t+1}^*} \right] \end{aligned}$$

⁴⁷In the sense that they provide an equation system describing the household decisions purely in terms of contemporaneous or pre-determined variables and expectations taken over the objective law.

⁴⁸Although it is possible with the command `perfect_foresight_solver`, a routine we chose not to use due to its inability to automatically check the Blanchard-Kahn condition.

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Next, consider the housing terms included in z^* :

$$\begin{aligned}\mathbb{E}_t[\hat{h}_{t+s}^* - (1-\delta)\hat{h}_{t+s-1}^*] &= \frac{\sigma}{\nu} \mathbb{E}_t \frac{\hat{c}_{t+s}^* - \beta \hat{c}_{t+1+s}^*}{1-\beta} - (1-\delta) \frac{\sigma}{\nu} \mathbb{E}_t \frac{\hat{c}_{t-1+s}^* - \beta \hat{c}_{t+s}^*}{1-\beta} \\ &= -\frac{1}{\nu} \mathbb{E}_t \frac{\hat{r}_{t+s+1} + (1-\beta) \sum_{n \geq 1} \hat{r}_{t+1+s+n}}{1-\beta} + (1-\delta) \frac{1}{\nu} \mathbb{E}_t \frac{\hat{r}_{t+s} + (1-\beta) \sum_{n \geq 1} \hat{r}_{t+s+n}}{1-\beta} \\ &= -\frac{1}{\nu} \frac{\beta(1-\delta)^2 + \delta}{1-\beta} \mathbb{E}_t \hat{r}_{t+s+1} + \frac{1}{\nu} \frac{1-\delta}{1-\beta} \mathbb{E}_t \hat{r}_{t+s} - \frac{\delta}{\nu} \sum_{n \geq 1} \mathbb{E}_t \hat{r}_{t+s+1+n}.\end{aligned}$$

Symmetrically to before, the geometric summation over these terms that is contained in $\sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{z_{t+n}^*\}$ thus reads

$$\begin{aligned}\sum_{s=1}^{\infty} \beta^s \mathbb{E}_t [\hat{h}_{t+s}^* - (1-\delta)\hat{h}_{t+s-1}^*] &= \sum_{s=1}^{\infty} \beta^s \left[-\frac{1}{\nu} \frac{\beta(1-\delta)^2 + \delta}{1-\beta} \mathbb{E}_t \hat{r}_{t+s+1} + \frac{1}{\nu} \frac{1-\delta}{1-\beta} \mathbb{E}_t \hat{r}_{t+s} \right] \\ &\quad + \frac{\delta \sigma}{\nu} \frac{\beta}{1-\beta} \mathbb{E}_t \hat{c}_{t+2}^* + \frac{\delta}{\nu} \frac{\beta}{1-\beta} \sum_{s=1}^{\infty} \beta^s \mathbb{E}_t \hat{r}_{t+s+2}\end{aligned}$$

We can now restate the entire geometric sum:

$$\begin{aligned}\sum_{s=1}^{\infty} \beta^s \mathbb{E}_t z_{t+s}^* &= \sum_{s=1}^{\infty} \beta^s \mathbb{E}_t \left[\frac{w_{ss} n_{ss}}{y_{ss}} (1+1/\varphi) \hat{w}_{t+s} + \hat{\Sigma}_{t+s} - \frac{c_{ss}(1+1/\sigma) + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \hat{\xi}_{c,t+s} + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \hat{\xi}_{x,t+s} \right. \\ &\quad \left. - \frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \hat{c}_{t+s}^* - \frac{q_{ss} h_{ss}}{y_{ss}} [\hat{h}_{t+s}^* - (1-\delta)\hat{h}_{t+s-1}^*] \right] \\ &= \sum_{s=1}^{\infty} \beta^s \mathbb{E}_t \left[\frac{w_{ss} n_{ss}}{y_{ss}} (1+1/\varphi) \hat{w}_{t+s} + \hat{\Sigma}_{t+s} - \frac{c_{ss}(1+1/\sigma) + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \hat{\xi}_{c,t+s} + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \hat{\xi}_{x,t+s} \right. \\ &\quad - \frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \frac{\beta}{1-\beta} \frac{1}{\sigma} \hat{r}_{t+1+s} - \frac{q_{ss} h_{ss}}{y_{ss}} \left[-\frac{1}{\nu} \frac{\beta(1-\delta)^2 + \delta}{1-\beta} \hat{r}_{t+s+1} + \frac{1}{\nu} \frac{1-\delta}{1-\beta} \hat{r}_{t+s} + \frac{\delta}{\nu} \frac{\beta}{1-\beta} \hat{r}_{t+s+2} \right] \\ &\quad \left. - \frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \frac{\beta}{1-\beta} \mathbb{E}_t \hat{c}_{t+1}^* - \frac{\delta q_{ss} h_{ss}}{y_{ss}} \frac{\sigma}{\nu} \frac{\beta}{1-\beta} \underbrace{\mathbb{E}_t \hat{c}_{t+2}^*}_{= \mathbb{E}_t \hat{c}_{t+1}^* + \frac{1}{\sigma} \mathbb{E}_t \hat{r}_{t+2}} \right] \\ &= \mathbb{E}_t \hat{c}_{t+1}^* + \frac{1}{\sigma} \mathbb{E}_t \hat{r}_{t+2}\end{aligned}$$

The remaining geometric sums may easily be recursified:

$$\begin{aligned}\sum_{s=1}^{\infty} \beta^s \mathbb{E}_t z_{t+s}^* &= \beta Z_t^* - \frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi + \delta q_{ss} h_{ss} \sigma / \nu}{y_{ss}} \frac{\beta}{1-\beta} \mathbb{E}_t \hat{c}_{t+1}^* - \frac{\delta q_{ss} h_{ss}}{y_{ss}} \frac{\sigma}{\nu} \frac{\beta}{1-\beta} \frac{1}{\sigma} \mathbb{E}_t \hat{r}_{t+2} \quad \text{where} \\ Z_t^* &= \beta Z_{t+1}^* + \mathbb{E}_t \left[\frac{w_{ss} n_{ss}}{y_{ss}} (1+1/\varphi) \hat{w}_{t+1} + \hat{\Sigma}_{t+1} - \frac{c_{ss}(1+1/\sigma) + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \hat{\xi}_{c,t+1} + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \hat{\xi}_{x,t+1} \right. \\ &\quad \left. - \frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \frac{\beta}{1-\beta} \frac{1}{\sigma} \hat{r}_{t+2} - \frac{q_{ss} h_{ss}}{y_{ss}} \left[-\frac{1}{\nu} \frac{\beta(1-\delta)^2 + \delta}{1-\beta} \hat{r}_{t+2} + \frac{1}{\nu} \frac{1-\delta}{1-\beta} \hat{r}_{t+1} + \frac{\delta}{\nu} \frac{\beta}{1-\beta} \hat{r}_{t+3} \right] \right].\end{aligned}$$

Now, defining $C_t := \mathbb{E}_t^P \hat{c}_{\infty} + \mathbb{E}_t \hat{c}_{t+1}^* + \frac{\delta q_{ss} h_{ss}}{c_{ss} + n_{ss} w_{ss} \sigma / \varphi + \delta q_{ss} h_{ss} \sigma / \nu} \frac{1}{\nu} \mathbb{E}_t \hat{r}_{t+2}$ allows us to arrive at the

final Dynare-ready formulation of the household's decision rules:

$$\begin{aligned}
 (h) \quad & \hat{h}_t = \frac{1}{\nu} \xi_{h,t} - \frac{1}{1-\beta} \frac{1}{\nu} \mathbb{E}_t \hat{r}_{t+1} - Q \frac{1}{\nu} \mathbb{E}_t \hat{r}_{t+2} - \frac{1}{\nu} \hat{q}_t + \frac{1}{\nu} \frac{\bar{\beta}}{1-\beta} \varrho \hat{m}_t + \frac{\sigma}{\nu} \mathcal{C}_t, \\
 (b) \quad & \hat{c}_t - \frac{1}{\sigma} \hat{\xi}_{c,t} = -\frac{1}{\sigma} \mathbb{E}_t \hat{r}_{t+1} - Q \frac{1}{\sigma} \mathbb{E}_t \hat{r}_{t+2} + \mathcal{C}_t, \\
 (C) \quad & \mathcal{C}_t = Q \frac{1}{\sigma} \cdot \left[\hat{q}_t + M \cdot \hat{m}_t \right] + C \frac{y_{ss}}{c_{ss}} \frac{1-\beta}{\beta} \cdot \left[\beta \mathcal{Z}_t^* + \frac{x_{ss}}{y_{ss}} \sum_{n=0}^{\tau-1} \beta^{-n} \hat{x}_{t-n} + \hat{b}_{t+1} \right], \\
 (Z) \quad & \mathcal{Z}_t^* = \beta \mathcal{Z}_{t+1}^* + \mathbb{E}_t \left\{ -\frac{\frac{w_{ss} n_{ss}}{y_{ss}} (1+1/\varphi) \hat{w}_{t+1} + \hat{\Sigma}_{t+1} - \frac{c_{ss} (1+1/\sigma) + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \hat{\xi}_{c,t+1} + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \hat{\xi}_{x,t+1}}{-\frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \frac{\beta}{1-\beta} \frac{1}{\sigma} \hat{r}_{t+2} - \frac{q_{ss} h_{ss}}{y_{ss}} \left[-\frac{1}{\nu} \frac{\beta(1-\delta)^2 + \delta}{1-\beta} \hat{r}_{t+2} + \frac{1}{\nu} \frac{1-\delta}{1-\beta} \hat{r}_{t+1} + \frac{\delta}{\nu} \frac{\beta}{1-\beta} \hat{r}_{t+3} \right]} \right\}, \\
 (n) \quad & \varphi \hat{n}_t + \sigma \hat{c}_t - \hat{\xi}_{c,t} = \hat{w}_t, \\
 (x) \quad & \hat{q}_t + (1 - \varrho^\tau) \frac{\varrho}{1-\varrho} \hat{m}_t - \sum_{n=1}^{\tau} \mathbb{E}_t \{ \hat{r}_{t+n} \} + \mathbb{E}_t \hat{\xi}_{x,t+\tau} = (1-\eta) \hat{x}_t, \\
 (BC) \quad & \hat{b}_{t+1} = \frac{1}{\beta} \hat{b}_t + \frac{w_{ss} n_{ss}}{y_{ss}} (\hat{w}_t + \hat{n}_t) + \hat{\Sigma}_t - \frac{c_{ss}}{y_{ss}} \hat{c}_t \\
 & \quad - \frac{q_{ss} h_{ss}}{y_{ss}} [\hat{h}_t - (1-\delta) \hat{h}_{t-1}] + \frac{x_{ss}}{y_{ss}} \beta^{-\tau} \left(\hat{x}_{t-\tau} - \beta^\tau \hat{x}_t + \hat{\xi}_{x,t} \right),
 \end{aligned} \tag{1.33}$$

where

$$\begin{aligned}
 Q &:= \delta q_{ss} h_{ss} \sigma / \nu (c_{ss} + \sigma / \varphi \cdot n_{ss} w_{ss} + \delta q_{ss} h_{ss} \sigma / \nu)^{-1} \in (0, 1), \\
 M &:= \frac{\varrho}{1-\varrho} \left(1 + \frac{1-\bar{\beta} \varrho}{1-\bar{\beta}} \frac{1-\varrho-\delta}{1-\beta \varrho} \frac{1-\beta}{\delta} \right) > 0, \\
 C &:= (c_{ss} + n_{ss} w_{ss} \sigma / \varphi) (c_{ss} + \sigma / \varphi \cdot n_{ss} w_{ss} + \delta q_{ss} h_{ss} \sigma / \nu)^{-1}.
 \end{aligned}$$

(Notice that $M > 0$ is proven in footnote 46 in Appendix (1.B.7).)

1.B.9 Derivations for Section 1.4

In this Appendix we derive analytical results on the behavior of house prices in a one-region, zero liquidity endowment economy with instantaneous housing production.

The equations describing the economy are:

$$\begin{aligned}
 (h) \quad & \hat{q}_t - \bar{\beta} \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+1} = \sigma (\hat{c}_t - \bar{\beta} \mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+1}) - \nu (1 - \bar{\beta}) \hat{h}_t, \\
 (b) \quad & \hat{c}_t = -\sigma^{-1} \mathbb{E}_t \hat{r}_{t+1} + \mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+1}, \\
 (BC) \quad & \hat{b}_{t+1} = \beta^{-1} \hat{b}_t + \hat{y}_t - \frac{c}{y} \hat{c}_t - \frac{qh}{y} (\hat{h}_t - (1-\delta) \hat{h}_{t-1}) + \frac{qh\delta}{y} \hat{q}_t, \\
 (B) \quad & \hat{b}_{t+1} = 0, \\
 (HMC) \quad & \hat{h}_t - (1-\delta) \hat{h}_{t-1} = -\iota \varepsilon_t
 \end{aligned} \tag{1.34}$$

with the endowment process \hat{y}_t being the only exogenous disturbance to the economy. Iterating

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on (h) and plugging the results into the Euler equation gives our key equation:

$$\hat{q}_t = \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+1} - \hat{r}_{t+1} - \nu(1 - \bar{\beta})\hat{h}_t + \nu(1 - \bar{\beta})^2 \sum_{s=0}^{\infty} \bar{\beta}^s \mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+s+1}$$

Proof of Proposition (2)

Under rational expectations, and using the fact that the shocks only hit in period one, we can use housing market clearing to rewrite the house price equation as:

$$\hat{q}_t = \mathbb{E}\hat{q}_{t+1} - \hat{r}_{t+1} - (1 - \bar{\beta})\hat{h}_t + (1 - \bar{\beta})(1 - \delta)^t \left(1 - \frac{1 - \bar{\beta}}{1 - \bar{\beta}\delta}\right)(-\iota\varepsilon_t)$$

Using the fact that under rational expectations expected house prices have the following form

$$\begin{aligned} \mathbb{E}_1 \hat{q}_2 &= \mathbb{E}_1 \hat{q}_3 - (1 - \bar{\beta})(1 - \delta)\varepsilon_1^h + (1 - \bar{\beta})(1 - \delta)^2 \left(1 - \frac{1 - \bar{\beta}}{1 - \bar{\beta}\delta}\right)(-\iota\varepsilon_t) \\ \mathbb{E}_1 \hat{q}_3 &= \mathbb{E}_1 \hat{q}_4 - (1 - \bar{\beta})(1 - \delta)^2 \varepsilon_1^h + (1 - \bar{\beta})(1 - \delta)^3 \left(1 - \frac{1 - \bar{\beta}}{1 - \bar{\beta}\delta}\right)(-\iota\varepsilon_t) \\ &\dots \end{aligned}$$

Iteration on this equation and using the fact that the economy returns to the steady state in expectations gives:

$$\mathbb{E}_1 \hat{q}_2 = -\frac{1}{\delta}(1 - \bar{\beta})\left((1 - \delta)\frac{1 - \bar{\beta}}{1 - \bar{\beta}\delta} + \delta\right)(-\iota\varepsilon_t)$$

Also note that $\mathbb{E}_1 \hat{q}_3 = (1 - \delta)\mathbb{E}_1 \hat{q}_2$. Finally, we can derive the house price for the first to periods as a function of shocks:

$$\begin{aligned} \hat{q}_1 &= -\varepsilon_1 - \omega_h(-\iota\varepsilon_t) \\ \hat{q}_2 &= -(1 - \delta)\omega_h(-\iota\varepsilon_t) \end{aligned}$$

With $\omega_h = (1 - \bar{\beta})\left[(1 - \delta)\left(\frac{1 - \bar{\beta}}{1 - \bar{\beta}\delta}\right)\left(1 + \frac{1}{\delta}\right) + \delta\right]$. This completes the proof.

Proof of Proposition (3)

Under subjective beliefs we need to specifically characterize $\mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+s+1}$. We can make use of the method described in section (1.3.2). Importantly, we can make use of the fact that the shocks on the endowment, and hence the interest rate only hit in the first period. As households have rational expectations about these processes, they possess knowledge of this fact. Applying our

solution method, and substituting the subjective beliefs housing model, we get:

$$\begin{aligned} \hat{q}_t = \hat{q}_t + \varrho \hat{m}_t - (1 - \bar{\beta}) \hat{h}_t - \hat{r}_{t+1} + (1 - \bar{\beta}) \left[-\hat{q}_t - \frac{y}{y + \delta q h} \frac{1 - \beta}{1 - \bar{\beta}} \hat{r}_{t+1} \right. \\ \left. + \left(\beta \frac{qh}{y} \frac{\varrho}{1 - \varrho} \left(\frac{1 - \bar{\beta} \varrho}{1 - \bar{\beta}} \frac{1 - \varrho - \delta}{1 - \beta \varrho} + \frac{1}{1 - \beta \varrho} \right) + \frac{1 - \varrho}{1 - \varrho \bar{\beta}} - \frac{1}{1 - \varrho} \right) \hat{m}_t \right] \end{aligned}$$

Rearranging yields:

$$\hat{q}_t = -\hat{h}_t - \omega_r \hat{r}_{t+1} + \omega_m \hat{m}_t$$

where $\omega_r = 1 + \frac{y(1-\beta)}{y+hq\delta}$ and $\omega_m = (1 - \bar{\beta})^{-1} \left[\varrho + (1 - \bar{\beta}) \left(\beta \frac{qh}{y} \frac{\varrho}{1 - \varrho} \left(\frac{1 - \bar{\beta} \varrho}{1 - \bar{\beta}} \frac{1 - \varrho - \delta}{1 - \beta \varrho} + \frac{1}{1 - \beta \varrho} \right) + \frac{1 - \varrho}{1 - \varrho \bar{\beta}} - \frac{1}{1 - \varrho} \right) \right]$. Further, we have that

$$\begin{aligned} \frac{\partial \omega_m}{\partial \varrho} = (1 - \bar{\beta})^{-1} \left[1 + (1 - \bar{\beta}) \beta \frac{qh}{y} \left(\frac{\varrho}{1 - \varrho} \left(\frac{(1 - \bar{\beta})(\beta \bar{\beta} \varrho (\varrho + \delta) - 1)}{((1 - \bar{\beta})(1 - \beta \varrho))^2} + \frac{\beta}{(1 - \beta \varrho)^2} \right) \right. \right. \\ \left. \left. \frac{1 - 2\varrho}{(1 - \varrho)^2} \left(\frac{1 - \bar{\beta} \varrho}{1 - \bar{\beta}} \frac{1 - \varrho - \delta}{1 - \beta \varrho} + \frac{1}{1 - \beta \varrho} \right) - \frac{1 - \bar{\beta}}{(1 - \bar{\beta} \varrho)^2} + \frac{1}{(1 - \varrho)^2} \right) \right] \end{aligned}$$

which is smaller than zero for standard parameter choices. The first to periods of the house price are given by:

$$\begin{aligned} \hat{q}_1 &= -\omega_r \varepsilon_1 - (-\iota \varepsilon_t) \\ \hat{q}_2 &= -(1 - \delta)(-\iota \varepsilon_t) + \omega_m g \hat{q}_1 \end{aligned}$$

This completes the proof.

Demand-side heterogeneity

Turning to regional differences on the housing demand side, we focus on differences in the housing demand elasticity (ν). Following the same logic as in Propositions (2) and (3), we can derive closed-form characterizations of the house price with respect to a monetary policy shock.

Under rational expectations, the house price is given by

$$\hat{q}_t = \mathbb{E} \hat{q}_{t+1} - \hat{r}_{t+1} - \nu(1 - \bar{\beta}) \hat{h}_t + \nu(1 - \bar{\beta})(1 - \delta)^t \left(1 - \frac{1 - \bar{\beta}}{1 - \bar{\beta} \delta} \right) (-\iota \varepsilon_t)$$

Using the same steps as described in the Proof of Proposition (1.B.9) we get:

$$\mathbb{E}_1 \hat{q}_2 = -\nu \frac{1}{\delta} (1 - \bar{\beta}) ((1 - \delta) \frac{1 - \bar{\beta}}{1 - \bar{\beta} \delta} + \delta) (-\iota \varepsilon_t)$$

1.B. Proofs and Derivations

The house price response in the first two periods is given by

$$\begin{aligned}\hat{q}_1 &= -\varepsilon_1 - \omega_h \nu(-\iota \varepsilon_t) \\ \hat{q}_2 &= -(1-\delta)\omega_h \nu(-\iota \varepsilon_t)\end{aligned}$$

Given that $\iota > 0$, a higher elasticity of housing demand, hence a lower (ν), leads to a stronger response in house prices to a monetary policy shock.

Moving to the subjective beliefs model, we can use the same approach as in the proof of Proposition (1.B.9) and arrive at:

$$\hat{q}_t = -\nu \hat{h}_t - \tilde{\omega}_r \hat{r}_{t+1} + \tilde{\omega}_m \hat{m}_t$$

where $\tilde{\omega} = 1 + \frac{y(1-\beta)}{y+hq\delta\nu^{-1}}$ and $\tilde{\omega}_m = (1-\bar{\beta})^{-1} \left[\varrho + (1-\bar{\beta})(\beta \frac{qh\nu^{-1}}{y} \frac{\varrho}{1-\varrho} (\frac{1-\bar{\beta}\varrho}{1-\bar{\beta}} \frac{1-\varrho-\delta}{1-\bar{\beta}\varrho} + \frac{1}{1-\bar{\beta}\varrho})) + \frac{1-\varrho}{1-\varrho\bar{\beta}} - \frac{1}{1-\varrho} \right]$. Assuming that $\iota = 0$, the house price response to the shock in the first two periods is given by:

$$\begin{aligned}\hat{q}_1 &= -\tilde{\omega}_r \varepsilon_1 \\ \hat{q}_2 &= \tilde{\omega}_m g \hat{q}_1\end{aligned}$$

Under subjective beliefs, we arrive at the same result without conditioning on a positive supply response ($\iota \geq 0$). As under the supply side differences, the subjective beliefs model is more responsive to a monetary policy shock.

To examine the regional disparities in house price growth driven by variations in housing demand, we make the simplifying assumption that $\iota = 0$. As before, we consider two regions (A, B) facing the same monetary policy shock. The regions differ in their elasticity of housing demand $\nu^A > \nu^B$. The following statement can be made:

Proposition 5 (Differential house price growth responses, demand-side). *The differential house price growth response in region A and B under rational expectations with $\iota = 0$ is zero.*

The differential response of house prices in regions A and B under subjective expectations, with $\iota = 0$ for the first two periods, is given by:

$$\begin{aligned}\Delta \hat{q}_1^B - \Delta \hat{q}_1^A &= (\tilde{\omega}_r^B - \tilde{\omega}_r^A) \varepsilon_1 \\ \Delta \hat{q}_2^B - \Delta \hat{q}_2^A &= g(\tilde{\omega}_m^B - \tilde{\omega}_m^A)(\Delta \hat{q}_1^B - \Delta \hat{q}_1^A)\end{aligned}$$

where $\tilde{\omega}_r^B \geq \tilde{\omega}_r^A > 0$ and $\tilde{\omega}_m^B \geq \tilde{\omega}_m^A > 0$.

Proof. Differential house price growth, $\Delta \hat{q}_j^B - \Delta \hat{q}_j^A$ with $j = 1, 2$, are obtained by differentiating regional house price responses to a monetary policy shock derived above. \square

Proposition (5) demonstrates that under demand-side heterogeneities a similar result can be obtained compared to supply-side heterogeneities. On impact of the shock demand-side heterogeneities creates differential house price growth rates. This difference is dynamically amplified through extrapolation. Additionally, differences in housing demand elasticities affect the subjective beliefs path of housing demand ($\tilde{\omega}_m^B \geq \tilde{\omega}_m^A$). The region with a more elastic housing demand elasticity is more responsive to the shock, as subjective beliefs about future housing demand are more responsive. Under the simplifying assumptions we made a similar channel is absent in the case of supply-side heterogeneities.

1.C Empirical part and model calibration

1.C.1 Data sources

US data. For the monthly house price data on the federal level we use the SP CoreLogic Case-Shiller U.S. National Home Price Index seasonally adjusted from the FRED database (CSUSH-PISA). The FFR (FEDFUNDS), industrial production (INDPRO), and CPI (CPIAUCSL) on a monthly frequency are also taken from the FRED. The house price expectations data are the mean of the expected change in home value during the next year taken from the Survey of Consumers by the Michigan University. For the quarterly state level data, we use all transactions house price indices ("State code"STHPI) from the FRED and seasonally adjust them. The remaining state level data on building permits ("State code"BPPIVSA), employment in construction ("State code"CONS), unemployment ("State code"UR), and employment in retail are all taken from the FRED and already seasonally adjusted. We omit the code for employment in retail, as these vary across states. GDP (GDP) and the GDP deflator (GDPDEF) are also obtained from the FRED. As already mentioned in the main text, the house price sensitivity indicator is taken from Guren, McKay, Nakamura, and Steinsson (2021), and the monetary policy shock from Bauer and Swanson (2023). When aggregating the shocks from a monthly to a quarterly frequency we weigh the shocks according to the time they occurred in the quarter. Giving a higher weight to shocks that occurred at the beginning of the quarter. Hence, the weights are $1, \frac{2}{3}, \frac{1}{3}$. This aggregation method is in line with Gertler and Karadi (2015) and Almgren, Gallegos, Kramer, and Lima (2022b).

Euro Area data. For the Euro Area we obtain the real residential property prices from the BIS. For housing investment we use real fixed capital formation in dwellings (namq_10_an6) from Eurostat. Building permits (sts_cobp_q), real GDP (namq_10_gdp), unemployment (une_rt_q_h), HICP (prc_hicp_midx), and the EONIA (irt_st_m) are also obtained from Eurostat. All series, except the EONIA, are seasonally adjusted. As already mentioned in the main text, the time to obtain a building permit is taken from the World Bank database. And the monetary policy shocks are taken from Altavilla, Brugnolini, Gürkaynak, Motto, and Ragusa

1.C. Empirical part and model calibration

(2019a). For these, we use the same aggregation method as for the US data.

1.C.2 Robustness: Cross-regional heterogeneity in booms and busts

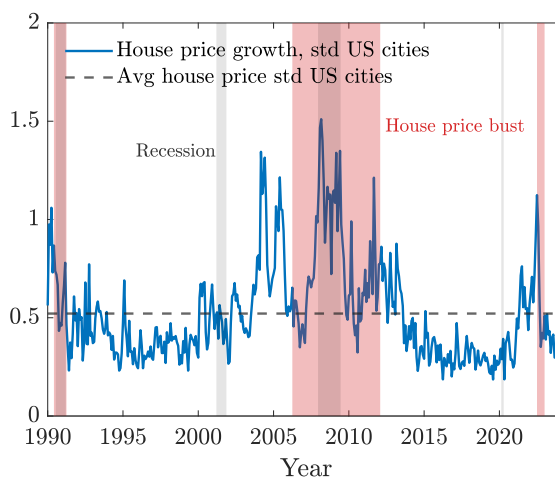
For the Euro Area, the sample covers 2000 to 2019, also on a quarterly basis. The countries included in the Euro Area sample are Austria, Germany, Spain, Finland, France, Ireland, Italy, the Netherlands, and Portugal.

Table 1.C.1: Cross-regional house price growth standard deviation (σ_c) in booms and busts, Euro

	<i>mean</i>	<i>median</i>
<i>Bust</i>	1.30	1.27
<i>Boom</i>		
$\sigma_c : Boom$	1.57	1.59
$\sigma_c : Bust$	2.04	2.02
$p - Val. Bust > Boom$	0.002	0.004
<i>Number regions</i>		8
<i>Obs. boom</i>		55
<i>Obs. bust</i>		24
<i>Sample</i>	2000Q1 – 2019Q3	

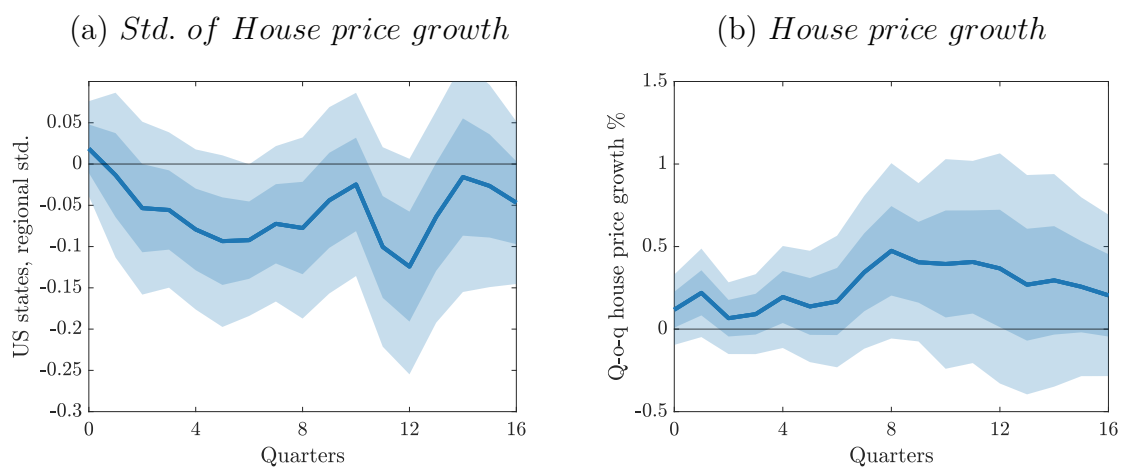
Notes: The Table reports the mean and median, across time, of the estimates of cross-sectional standard deviations of house price growth rates for the Euro Area. The test for $Bust > Boom$ is based on a one-sided t-test.

Figure 1.C.1: Regional house price variations in booms and busts



Notes: 3 Month moving average of cross-city std.

Figure 1.C.2: State-level std. house price growth response to Main business cycle shock



Notes: Responses to a main business cycle shock (1 std) ; Confidence Intervals: 68% and 95% (Newey-West); 3 month moving average of std. across states of quarter-on-quarter house prices growth (left); Q-o-q house price growth at a federal level (right.)

1.C.3 Forecast error response to monetary policy shock

We are first interested in whether house price expectations are formed according to rational expectations. To answer this question, we study the response of forecast errors to a monetary

1.C. Empirical part and model calibration

policy shock. Forecast errors are interesting for two reasons. First, they show whether house price expectations follow rational expectations. Under rational expectations forecast errors should not respond to the shock. This becomes obvious in our local projections setup. We define forecast errors as follows:

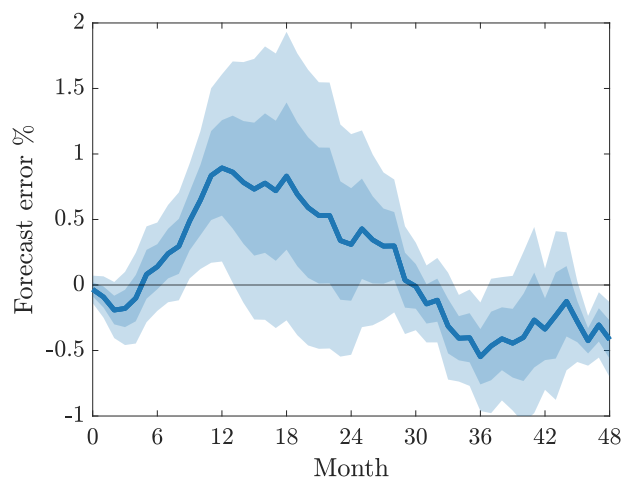
$$fe_t = \Delta q_{t+12}^y - \mathbb{E}_t^{\mathcal{P}} \Delta q_{t+12}^y$$

where Δq_{t+12}^y is the year-on-year percentage change in house prices. $\mathbb{E}_t^{\mathcal{P}} \Delta q_{t+12}^y$ is the expected year-on-year percentage change in 12 month formed today. The local projections we estimate are given by Equation (1.35). At the time the shock hits, in period t , the forecast errors seen on the left-hand side already contain the forecast formed today for 12 month ahead. In other words, the shock is contained in the information set of the agent and should be taken into account. Under rational expectations agents will perfectly use this information and forecast errors should not respond. The second insight we may gain from this exercise is on how house price expectations are formed. Given the response of the forecast errors is non-zero, the dynamics reveal how households' expectations evolve over time. This will enable us to zoom in on a specific belief updating process that explains our observations. Due to data availability we will focus on the US in this exercise. The monetary policy shock is taken from Bauer and Swanson (2023). We use the expectations data from the Michigan Survey on Consumer Sentiment. The frequency is monthly and the sample runs from 2007 to 2019. As controls, we include 6 lags of the forecast error and the monetary policy shock. We estimate the following equation:

$$fe_{t+h} = \alpha^h + \beta^h \epsilon_t^{MP} + x_t + u_{t+h} \quad h = 0, 1, \dots, H \quad (1.35)$$

Figure (1.C.3) shows the results. The impulse response reveals a non-zero and highly significant response in forecast errors. We can therefore reject rational expectations. On the dynamic behaviour we find that for the first 30 months the forecast error is positive, indicating over-pessimism on the side of the agents. After 30 months the forecast error turns negative, indicating over-optimism. For both cases, over pessimism and over optimism, the IRF is significant at a 95% confidence level. Further, the hump-shaped dynamics in the forecast errors indicate sluggish updating in the expectation formation process. These findings are in line with Adam, Pfäuti, and Reinelt (2022). Angeletos, Huo, and Sastry (2021) document similar dynamics in a SVAR setup for unemployment and inflation in response to an unemployment and inflation shock.

Figure 1.C.3: Forecast error response to a monetary policy shock, US

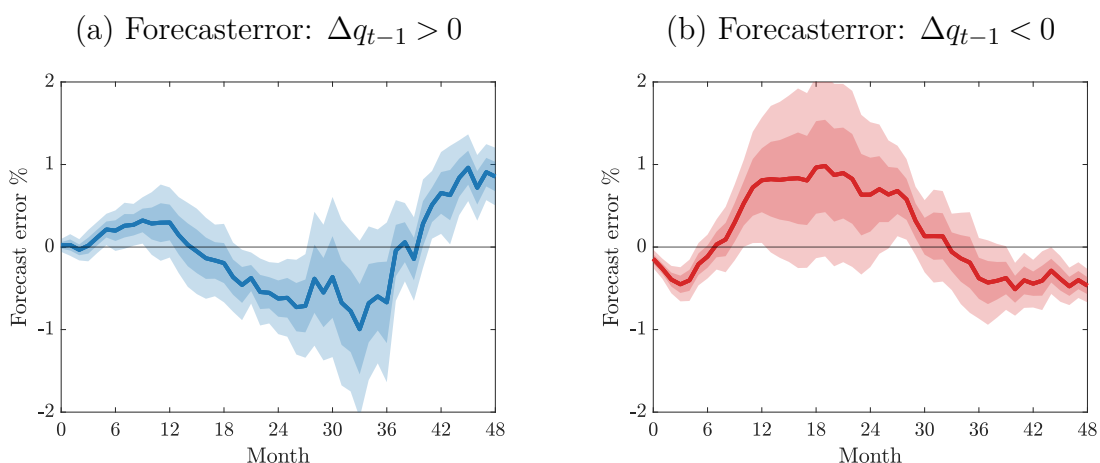


Notes: Response to expansionary MP shock (1 std); Confidence Intervals: 68% and 95% Newey-West).

1.C.4 Forecast error response to a monetary policy shock in booms and busts

In the following we estimate equation (1.15) for forecast errors. Figure (1.C.4) plots the results. We notice, that forecast errors react notably different in booms relative to busts.

Figure 1.C.4: Forecast errors in response to monetary policy shock, boom-bust



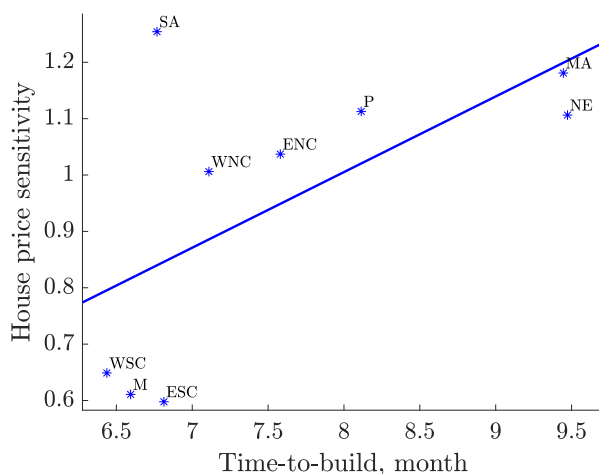
Notes: Responses to expansionary MP shock (1 std); Confidence Intervals: 68% and 95% (Newey-West).

1.C. Empirical part and model calibration

1.C.5 Time-to-build in the US

The time it takes to obtain a building permit, or generally the time it takes to build a house, is not available for the US on a state level. However, the Census Bureau documents construction times for homes on a census division level. Below we aggregated up the housing sensitivity indicator from Guren, McKay, Nakamura, and Steinsson (2021) to a census division level and correlate them with the construction time of housing measured from date of authorization to completion. One can see in figure (1.C.5) that they are clearly positively correlated.

Figure 1.C.5: Correlation of house price sensitivity indicator with time-to-build



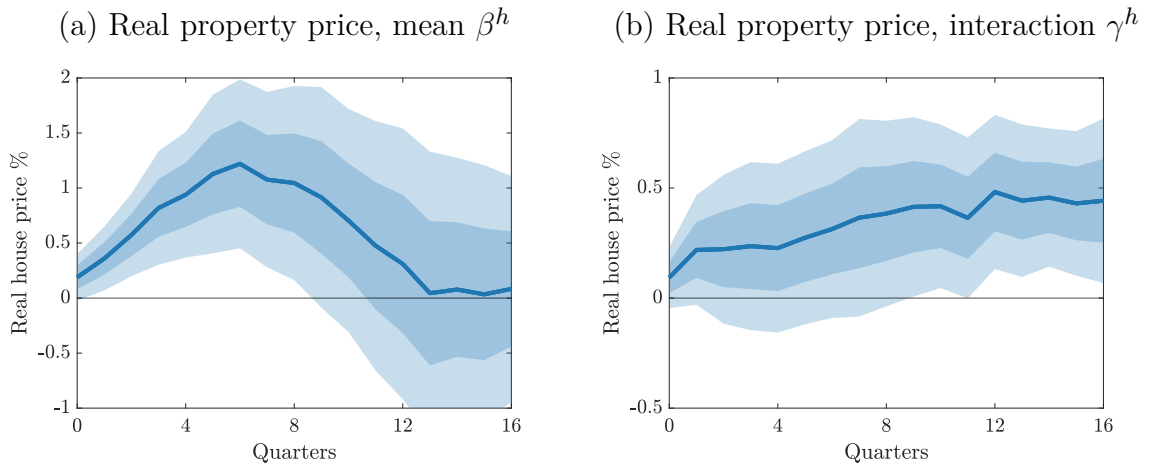
Notes: Correlation of house price sensitivity indicator from Guren, McKay, Nakamura, and Steinsson (2021) with time-to-build a house across census divisions. Time-to-build is measured from the date of the authorization until the completion of the building activities.

1.C.6 House price response and housing supply side heterogeneity in the Euro Area

For the Euro Area we rely on the same setup as for the US: We estimate Equation (2.1) for house prices. The cross-section is taken at a country level. The monetary policy shock is high-frequency identified from overnight interest swaps at a one-year horizon and taken from Altavilla, Brugnolini, Gürkaynak, Motto, and Ragusa (2019a). As an interaction term, we use the days it takes to obtain a building permit in a given country, provided by the World Bank database. The sample runs from 2000 to 2019 and is in quarterly frequency. The countries contained in the sample are Austria, Germany, Spain, Finland, France, Ireland, Italy, the Netherlands, and Portugal. The vector of controls consists of 6 lags of the following variables: The left-hand-side variable, log GDP, log HICP, the EONIA, the shock, and the shock interacted with the interaction term.

Figure (1.C.6) plots the response of real property prices to an expansionary monetary policy shock. We find that house prices increase (panel (a)) and that they increase more in countries where supply is more inelastic (panel (b)). The IRFs are significant at a 95% confidence level. These results mirror the ones we found for the US.

Figure 1.C.6: House price response to monetary policy shock, Euro



Notes: House price response to expansionary MP shock (1 std); The interaction term as been standardized; Confidence Intervals: 68% and 95% (Driscoll and Kraay, 1998).

1.C. Empirical part and model calibration

1.C.7 Model: steady state values

Table 1.C.2: steady state values

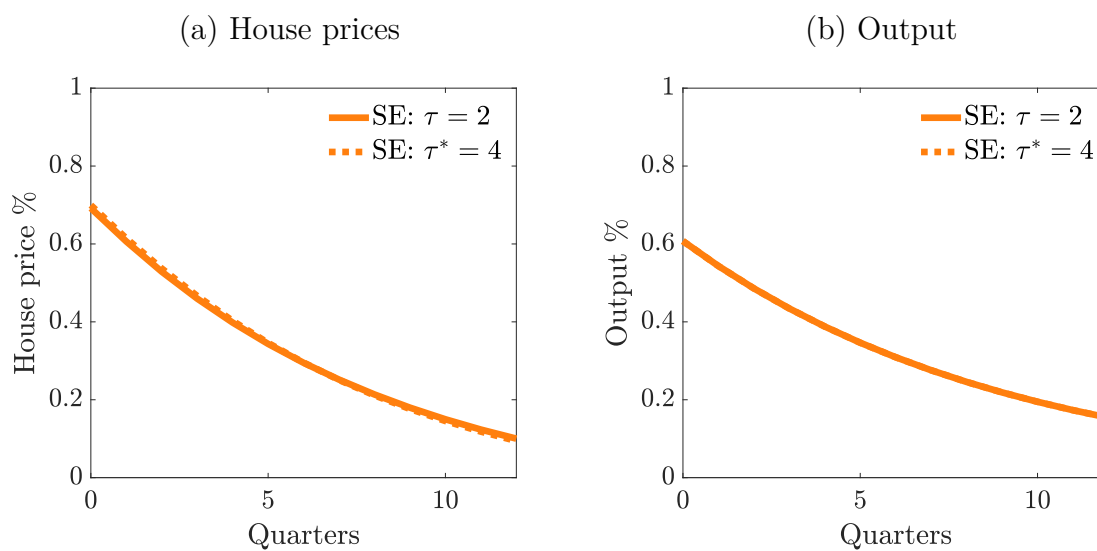
Domestic	Value	Foreign	Value	Description
$\xi_{x,ss}$	0.800	$\xi_{x,ss}^*$	0.800	housing productivity shifter
$\xi_{h,ss}$	0.104	$\xi_{h,ss}^*$	0.105	housing preference shifter
c_{ss}	0.976	c_{ss}^*	0.976	consumption
x_{ss}	0.063	x_{ss}^*	0.063	housing investment
h_{ss}	5.506	h_{ss}^*	5.506	housing
y_{ss}	1.040	y_{ss}^*	1.040	output
b_{ss}	0.000	b_{ss}^*	0.000	bond holdings
q_{ss}	0.735	q_{ss}^*	0.727	house price
w_{ss}	1.000	w_{ss}^*	1.000	wage

Notes: All steady states except the house price are symmetric across countries. Time-to-build and the steady state housing preference shifter are not symmetric across countries. All elements of ξ_{ss} not explicitly mentioned assume the value 1.

1.D Quantitative model

1.D.1 Further results

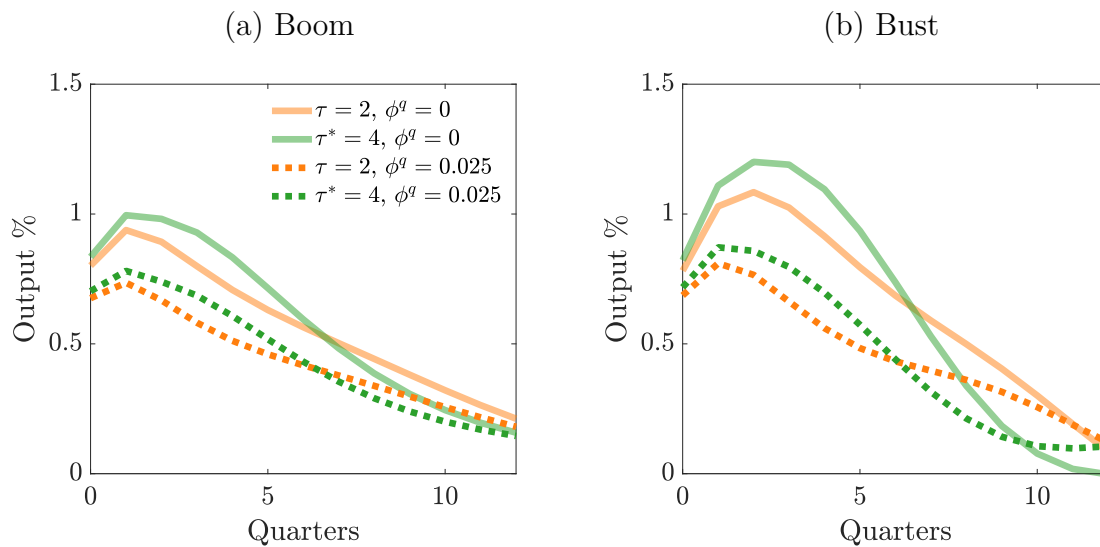
Figure 1.D.1: Regional house price and output response under RE



Notes: Responses to expansionary MP shock (25 bp).

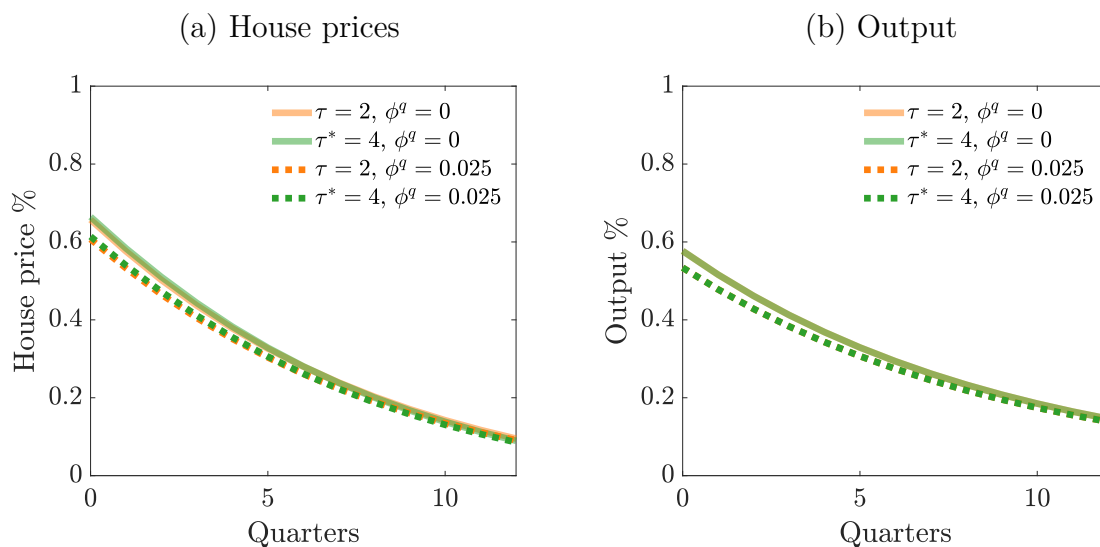
1.D. Quantitative model

Figure 1.D.2: House price targeting: output response in booms and busts



Notes: Responses to expansionary productivity shock (100 bp); Parameterization: $\varrho^h = 0.99$, $g^h = 0.0117$ (boom), $\varrho^l = 0.91$, $g^l = 0.0233$ (bust).

Figure 1.D.3: House price targeting: house price and output response under RE



Notes: Responses to expansionary productivity shock (100 bp).

1.D.2 Loss function

We assume that the policy-maker's objective is the Utilitarian average welfare in the economy, evaluated under rational expectations. This definition leaves open the question of what information the policy-maker possesses. Conditional welfare, \mathbb{W}_{-1} , is the average expected welfare in the union, conditional on (i) the economy being in steady state in the current and all past periods $t \leq 0$, and on (ii) the policy-maker knowing that in $t = 0$ a set of shocks $(\xi_t)_{t \geq 0} \in (\mathbb{R}^n)^\infty$ will realize with a Gaussian probability distribution but not knowing about the exact realization of the shocks. The experiment here is that the policy-maker decides at the end of the period $t = -1$ about her instruments, maximizing the average utility in the union under the expectation that a set of surprise shocks might materialize in the next period according to a known distribution—let \mathbb{E}_{-1} denote the operator drawing the expectation with respect to the rational expectations measure and conditional on (i) and (ii). Specifically,

$$\begin{aligned}\mathbb{W}_{-1} &:= \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t [\gamma u_t + (1-\gamma)u_t^*] \\ \mathbb{W} &:= \mathbb{E}[\gamma u_t + (1-\gamma)u_t^*]\end{aligned}$$

where u_t, u_t^* are the utility functions of the home and foreign household, respectively.⁴⁹ We approximate both criteria to second order around the non-stochastic steady state: Assuming that the allocation in the non-stochastic steady state is symmetric ($c_{ss} = c_{ss}^*, n_{ss} = n_{ss}^*, h_{ss} = h_{ss}^*$ and so on), we have that

$$\begin{aligned}\frac{\mathbb{W}_{-1}}{c_{ss}^{1-\sigma}} &= -\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \cdot \left[\frac{\gamma}{2} \left(\kappa \frac{n_{ss}}{c_{ss}} \pi_{H,t}^2 + \sigma \hat{c}_t^2 + \nu \frac{h_{ss}^{1-\nu}}{c_{ss}^{1-\sigma}} \hat{h}_t^2 + \varphi \chi \frac{n_{ss}^{1+\varphi}}{c_{ss}^{1-\sigma}} \hat{n}_t^2 + \frac{x_{ss}}{c_{ss}} (1-\eta) \hat{x}_t^2 + \frac{n_{ss}}{c_{ss}} \psi n_{ss} \hat{b}_t^2 \right. \right. \\ &\quad \left. \left. - 2 \frac{n_{ss}^{1+\varphi}}{c_{ss}^{1-\sigma}} \hat{\xi}_{a,t} \hat{n}_t \right) \right. \\ &\quad \left. + \frac{1-\gamma}{2} \left(\kappa \frac{n_{ss}}{c_{ss}} \pi_{F,t}^2 + \sigma (\hat{c}_t^*)^2 + \nu \frac{h_{ss}^{1-\nu}}{c_{ss}^{1-\sigma}} (\hat{h}_t^*)^2 + \varphi \chi \frac{n_{ss}^{1+\varphi}}{c_{ss}^{1-\sigma}} (\hat{n}_t^*)^2 + \frac{x_{ss}}{c_{ss}} (1-\eta^*) (\hat{x}_t^*)^2 + \frac{n_{ss}}{c_{ss}} \psi n_{ss} (\hat{b}_t^*)^2 \right) \right. \\ &\quad \left. - 2 \frac{n_{ss}^{1+\varphi}}{c_{ss}^{1-\sigma}} \hat{\xi}_{a,t} \hat{n}_t^* \right. \\ &\quad \left. + \frac{1}{2} \frac{1}{\varsigma^2} v \cdot \hat{s}_t^2 \right] + O(3) + \text{t.i.p.}\end{aligned}$$

where $v = (1-\lambda)(1+\lambda+[2-\lambda]\varsigma) + \lambda(1+[2+\varsigma]\lambda) + \lambda^*(2-\lambda^*+[1+\lambda^*]\varsigma) + (1-\lambda^*)(1+(2+\varsigma)[1-\lambda^*])$, $O(3)$ are terms of third or higher order and “t.i.p.” stands for “terms independent of policy”, i.e. variables that are beyond the policy-maker's control and thus irrelevant for the selection of policy.

⁴⁹Note that it holds that $\mathbb{E}\mathbb{W}_{-1} = \mathbb{W}$.

Chapter 2

Transmission of Monetary Policy in a Currency Area with Heterogenous Households

Joint with Lukas Hack.

2.1 Introduction

The impact of monetary policy on economic outcomes varies significantly across the member states of the Euro Area¹, presenting a challenge to achieving even monetary policy transmission and price stability within the currency area. Therefore, it is crucial to understand the underlying forces driving heterogeneous responses and their relative strength.

Empirical investigations led to suggestive evidence that household heterogeneity in the prevalence of liquidity-constrained, hand-to-mouth households, adjustable-rate mortgages, and homeownership rates, are likely drivers of heterogenous transmission (e.g. Almgren, Gallegos, Kramer, and Lima, 2022a; Corsetti, Duarte, and Mann, 2022b; Pica, 2022). Nevertheless, identification based on reduced-form methods is difficult because cross-sectional variation is limited to Euro Area member states, and the different forms of household heterogeneity tend to be correlated across countries.² Moreover, Euro Area

¹For example, the peak GDP response to monetary policy shocks in Spain is more than two times larger than in Germany, as we show in Section (2.2).

²Studying time variation is not feasible due to lack of microdata at a suitable frequency.

countries cannot be treated as independent regions but rather interact through various economic linkages.

To overcome these challenges, we propose a structural approach. First, we construct a quantitative two-region heterogeneous-agent New Keynesian (HANK) model with a common currency area and cross-region heterogeneity in the number of hand-to-mouth households, housing tenure status as well as in the prevalence of adjustable-rate mortgages. To the best of our knowledge, this is the first two-region HANK model with cross-region household heterogeneity.³ Second, the model rationalizes 70% of the heterogeneity in the GDP responses to monetary policy shocks, primarily through the interplay between hand-to-mouth heterogeneity and trade openness. Finally, we demonstrate that trade is decisive for heterogeneous responses to monetary policy shocks. Policies that increase trade openness can synchronize inflation responses perfectly but at the expense of amplified heterogeneity for GDP, which may even flip signs compared to the baseline.⁴

Empirical facts. We document micro-level cross-country differences in household heterogeneity based on survey data and macro-level cross-country heterogeneity in GDP and inflation responses to high-frequency identified monetary policy shocks.⁵

We focus on the core countries which we group into two regions, France & Germany, the *North*, and Italy & Spain, the *South*. At the household level, we find that a higher fraction of households are hand-to-mouth in the South, even conditional on housing tenure status. Similarly, there are more homeowners in the South and the mortgages held by these owners are more likely to have adjustable rates. These aspects render household demand in the South more responsive to monetary policy because hand-to-mouth households respond stronger to changes in their disposable income and monetary policy may directly affect the disposable income of homeowners through adjustable-rate mortgages. Consistent with this, we find that the GDP response to monetary policy shocks is more pronounced in the South.

³In simultaneous work, Bayer, Seyrich, Müller, and Kriwoluzky (2023) and Chen, Lazarakis, and Varthalitis (2023) study two-region HANK models but focus on cross-country differences in fiscal policies, as we discuss below.

⁴By increased trade openness, we refer to a decrease in the home bias for domestically produced goods and an increase in the elasticity of substitution across goods produced in different regions. Euro Area policy may affect these parameters through, e.g., fostering trade in services across member states.

⁵The micro-level evidence is based on the ECB's Household Finance and Consumption Survey, and the monetary policy shocks are based on the Euro Area Monetary Policy Event-Study Database from Altavilla, Brugnolini, Gürkaynak, Motto, and Ragusa (2019b).

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Model. Motivated by these facts, we construct a two-region HANK model. Households are ex-ante heterogeneous in their housing tenure status which determines whether they are renters, outright owners, or mortgagors, and are subject to idiosyncratic income risk against which they self-insure via a liquid bond. The bond is traded within the currency area. The New Keynesian elements are wage and price stickiness through Rotemberg adjustment costs in conjunction with monopolistic competition in labor and intermediate goods markets. Labor supply and wages are set by unions that sell differentiated labor services to labor packers. Intermediate goods are produced with a linear technology from labor services and transformed to region-specific final goods. These final goods are traded, while households have a home bias for domestically produced goods. To match both, household heterogeneity in hand-to-mouth (and hence, in marginal propensities to consume) and hump-shaped empirical impulse responses, we closely follow Auclert, Rognlie, and Straub (2020): At the micro level, we assume heterogeneity in discount rates across housing tenure status and region to match the differential hand-to-mouth shares. At the macro level, we assume that households infrequently update their private belief about the future aggregate state which allows for generating hump-shaped responses to shocks, as found in the data.

Calibration. We calibrate the model in two steps. First, we match the household-level data moments from the survey data and set certain parameters that are common across households to standard values. Second, we estimate key parameters of the model by impulse response function matching, based on the method of simulated moments. Importantly, we target only the GDP response of the South to a monetary policy shock. The response of the North, and hence, the differential response across regions is not targeted. We do estimate the adjustment costs of wages and prices, interest rate smoothing, and the inflation coefficient in the Taylor rule, as well as the updating frequency of households with regard to the aggregate state.

Results. The model matches the cross-region differences in the GDP responses to monetary policy shocks fairly well. This holds in terms of the level and the shape. At the peak response, the model can account for 70% of the differential effects across both regions. We decompose the differences in impulse responses into the underlying demand components, finding that they are primarily driven by homeowners whereas renters are less important. Conversely, trade provides an important counteracting force that dampens the differential effects substantially. Net exports of the North increase

in response to the shock which puts downward pressure on output in the South but upward pressure on output in the North. Ignoring these trade effects would amplify the differential response by 80% at the peak, relative to the baseline. We further show that realistic heterogeneity in hand-to-mouth is key for the differential demand amplification across regions. When we shut down the discount rate heterogeneity, we obtain hardly any differences across regions. This suggests that the interplay of demand amplification through hand-to-mouth households and the counteraction through trade crucially shape the differential effects across regions. This finding supports our modeling strategy that both, incomplete markets and meaningful trade interaction are crucial ingredients to understanding the heterogeneous responses found in the data.

We further inspect the importance of trade and study a counterfactual with increased trade openness. Specifically, we assume that both regions have no home bias and that the elasticity of substitution across both goods is large. In such a counterfactual, price differences across regions cannot be entertained as an equilibrium outcome. Any price difference is exploited by households substituting domestic and foreign goods until the price difference vanishes. Thus, consumer good inflation rates behave completely symmetric across regions. This qualitatively alters the cross-region differences in output responses because net exports of the North are amplified, relative to the baseline. Therefore, the output response of the North can be substantially larger than the response of the South, despite more demand amplification through household heterogeneity in the South. The underlying mechanism works through labor supply. Southern households increase consumption by more, leading to a higher valuation of leisure, relative to the North. This makes production in the North more competitive. With a high elasticity of substitution, households can exploit the lower cost of production more intensely. In particular, they exploit this relation until consumer prices are completely equalized.

Our results have important implications for policies that aim to foster trade (integration) within the Euro Area. Such policies may help to synchronize inflation rates across regions. This removes an important trade-off currently faced by the European Central Bank. Instead of monitoring various inflation rates, it is sufficient to control Euro Area inflation to achieve price stability not only on average but in each single member country. However such policies come at the cost of potentially larger heterogeneity in production.

Related literature. This paper relates to the literature studying heterogeneity in the transmission of monetary policy in the Euro Area. Early work by Calza, Monacelli, and Stracca (2013) notes that mortgage rate types affect the reaction of aggregate

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consumption to monetary policy shocks in OECD countries. More recent findings by Pica (2022) show how homeownership rates and adjustable rate mortgages affect the transmission of monetary shocks in the Euro Area. Focusing on liquidity constraints, Almgren, Gallegos, Kramer, and Lima (2022a) point out that there is a strong positive correlation between the reaction of GDP to a monetary policy shock and the share of hand-to-mouth households across Euro Area member states. Combining micro and macro level evidence with a tractable HANK model, Slacalek, Tristani, and Violante (2020a) study the different channels through which monetary policy affects household choices in the four largest Euro Area countries. Their channels heavily rely on the presence of hand-to-mouth agents. Finally, Corsetti, Duarte, and Mann (2022b) construct a dynamic factor model consisting of the Euro Area founding members. They find positive correlations between GDP responses to Monetary policy shocks and hand-to-mouth households, adjustable mortgage rates, homeownership rates, and wage stickiness. We add to this literature by integrating various potential driving forces of cross-country heterogeneity in impulse responses into a quantitative currency union HANK model and decomposing the effect of various channels.

This paper is also closely connected to the literature on the transmission of shocks in quantitative open economy HANK models. Auclert, Rognlie, Souchier, and Straub (2021) and Guo, Ottonello, and Perez (2020) study the effect of international shocks in the presence of household heterogeneity in a small open economy setting. Bayer, Seyrich, Müller, and Kriwoluzky (2023) and Chen, Lazarakis, and Varthalitis (2023) propose currency union open economy HANK models to study fiscal policies in a currency union. Our paper in turn focuses on monetary policy, adds important characteristics of household heterogeneity, and points out the importance of cross-country interactions in a currency union.

Further, we relate to a literature that studies structural differences in currency unions in representative agent economies (Benigno, 2004b; Bletzinger and Thadden, 2021b; Kekre, 2022).

Finally, we connect to the fast literature that studies monetary policy and hand-to-mouth agents (Auclert, 2019; Kaplan, Moll, and Violante, 2018; Luetticke, 2021; McKay, Nakamura, and Steinsson, 2016).

Outline. Section (2.2) describes the empirical facts our analysis relies on. In Section (2.3) we describe the model environment. The results are presented in Section (2.4), while Section (2.5) concludes.

2.2 Empirical analysis

In this section, we document important empirical regularities across Euro Area countries. We focus on France & Germany as well as Italy & Spain as the four core countries which account for a large fraction of Euro Area GDP. We refer to these groups as *North* and *South* respectively. We find that the South's GDP responds substantially stronger to monetary policy shocks which is consistent with more hand-to-mouth households, higher homeownership rates, and more adjustable-rate mortgages that we find in the micro data.

2.2.1 Macro-level evidence

Data. We estimate the impulse response of real GDP to high-frequency identified monetary policy shocks via local projections,

$$y_{n,t+h} = \alpha_n^h + t + \beta_n^h \varepsilon_t^{MP} + \zeta_n^h X_{n,t} + v_{t+h}^h, \quad h = 0, 1, \dots, H \quad (2.1)$$

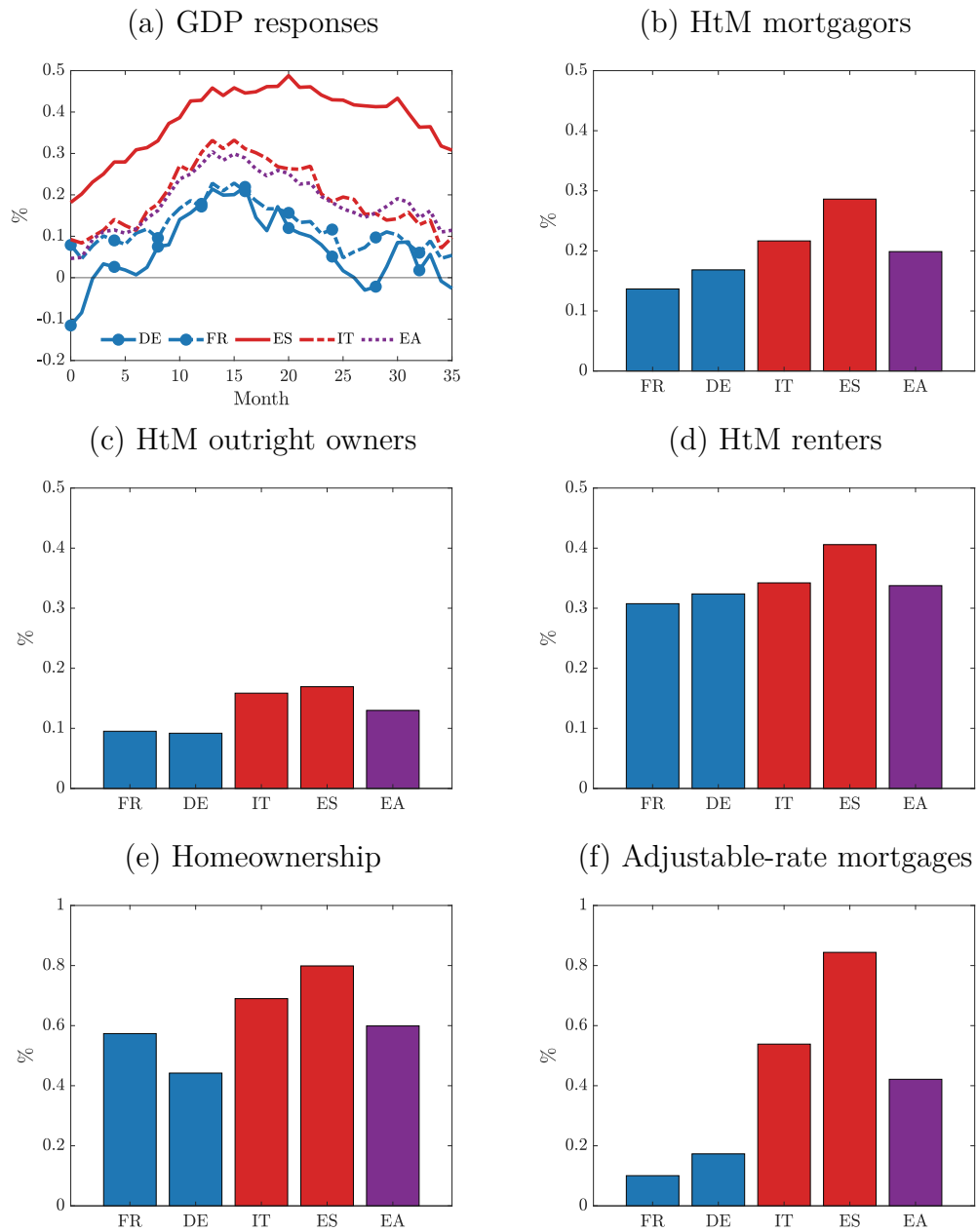
where n indexes the country, $y_{n,t+h}$ refers to the logarithm of real GDP, ε_t^{MP} is a monetary policy shock and $X_{n,t}$ is vector of control variables. We interpolate real GDP to monthly frequency using the Chow and Lin (1971).⁶ The monetary policy shock is a monetary policy surprise from Altavilla, Brugnolini, Gürkaynak, Motto, and Ragusa (2019b). We choose a one-year horizon to incorporate responses in adjustable-rate mortgages which are only updated once or twice per year.⁷ Finally, the control vector includes lags of country-level log GDP, Euro Area log GDP and HICP, the EONIA, and lags of the shock. The sample runs from 2000 to 2019.

⁶Interpolation to monthly frequency is standard (e.g. Almgren, Gallegos, Kramer, and Lima, 2022a; Leeper, Sims, and Zha, 1996). We use the unemployment rate, industrial production, and the turnover and volume of trade index to interpolate real GDP.

⁷We aggregate the surprises from Altavilla, Brugnolini, Gürkaynak, Motto, and Ragusa (2019b) to monthly frequency by using the same weighting scheme as Almgren, Gallegos, Kramer, and Lima (2022a) which assigns more weight to shocks early in the month.

2.2. Empirical analysis

Figure 2.2.1: Empirical evidence for asymmetric responses



Notes: Panel (a) shows estimated impulse responses of GDP to a high-frequency identified monetary policy shock, based on the local projection in (2.1). Panels (b)-(d) show the share of hand-to-mouth (HtM) households, conditional on housing tenure status. Panels (e) and (f) show the number of homeowners, relative to all households and the number of adjustable-rate mortgages, relative to all mortgages.

Results. We show impulse responses $\beta_n^0, \dots, \beta_n^H$ to an expansionary monetary policy shock equal to one percentage point. The results for the four core countries as well as for the average response across all Euro Area members are in Panel (a) of Figure (2.2.1). The Euro Area average displays the point-wise average response across all member states, weighted by the average GDP in our sample. The red lines show that the South tends to respond strongly compared with both, the Euro Area average response (purple dotted line) and the North (lines with markers). For example, at the peak of the German response, we find that the response of Spain is twice as large. This suggests that there is a sizable heterogeneity in the transmission of monetary policy across the core countries.

2.2.2 Micro-level evidence

Data. We compute country-specific statistics based on the Household Finance and Consumption Survey (HFCS), and pool the first three waves. We distinguish households by their housing tenure status, i.e., whether they rent or own their home and whether they still have outstanding mortgages. For each subgroup, we compute the share of hand-to-mouth (HtM) households following the definition of Slacalek, Tristani, and Violante (2020a).⁸ We further measure homeownership rates as the share of homeowners in the total population and the importance of adjustable-rate mortgages as the fraction of mortgages that feature adjustable rates, relative to all mortgages of the respective country.

Results. Panels (b) to (f) in Figure (2.2.1) display average shares for each core country, as well as Euro Area averages which are weighted by average GDP, consistent with the average local projections. Across housing tenure status, we find that there are more hand-to-mouth households in the South. Renters feature most hand-to-mouth households but the cross-country differences are slightly smaller. Among homeowners, we find that mortgage holders are more constrained, perhaps because debt service makes it hard to accumulate liquid savings. Finally, homeowners are counted as wealthy hand-to-mouth because they own a house which is an illiquid asset. Homeownership rates in the South are also higher putting more weight on owners, relative to renters. Finally, there is a stark difference in the prevalence of adjustable-rate mortgages across regions.

⁸They assume that households are liquidity constrained when they hold liquid assets below their biweekly income, or negative liquid assets with less than a biweekly income distance to the credit limit.

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For example, only one out of ten mortgages in France features adjustable rates whereas it is almost nine out of ten in Spain.

Overall, this illustrates that heterogeneous GDP responses tend to be correlated with the prevalence of hand-to-mouth households, homeownership rates, and the importance of adjustable-rate mortgages. This is consistent with Corsetti, Duarte, and Mann (2022b) and suggests that these household-level cross-country heterogeneities are potential channels that drive heterogeneous GDP responses to monetary policy shocks.

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The model features two economic regions, which are populated by a continuum of households of mass α and $1 - \alpha$, respectively. We refer to these regions as North and South, in anticipation of our calibration. We state the model from the perspective of the North because cross-region differences enter only via the model calibration. Both regions are connected through the trade of goods and an integrated asset market. Variables of the South are denoted with an asterisk, and time is discrete.

2.3.1 Households

Households supply labor, save in a liquid bond, and consume region-specific aggregate consumption baskets that follow from a nested constant elasticity of substitution (CES) demand structure, which takes variety goods from North and South producers as inputs.

Dynamic problem. Households are ex-ante heterogeneous with respect to their housing tenure status, which is indexed by $k \in \{o, m, r\}$, referring to outright owners, mortgage holders, and renters, respectively. Each household i solves the following dynamic problem

$$V_t^k(e_{i,t}, b_{i,t-1}, x_{i,t}) = \max_{c_{i,t}, b_{i,t}} \left(\frac{c_{i,t}^{1-\gamma}}{1-\gamma} - \varphi \frac{n_t^{1-\nu}}{1-\nu} \right. \\ \left. + \beta_k \mathbb{E}_t \left[\xi V_{t+1}^k(e_{i,t+1}, b_{i,t}, x_{i,t} + 1) + (1 - \xi) V_{t+1}^k(e_{i,t+1}, b_{i,t}, 0) \right] \right) \quad (2.2)$$

$$\text{s.t. } -\underline{b} \leq b_{i,t}, \quad \text{and} \quad \ln(e_{i,t+1}) = \rho_e \ln(e_{i,t}) + v_{i,t+1}, \quad (2.3)$$

with $v_{i,t} \stackrel{iid}{\sim} N(0, \sigma_e)$, and subject to a budget constraint which we discuss below. Flow utility is additively separable and depends on constant relative risk aversion γ , inverse Frisch elasticity of labor supply ν , and a labor disutility shifter φ . The beginning-of-period bond holdings $b_{i,t-1}$ are given and households allocate disposable income to consumption $c_{i,t}$ and liquid bond holdings $b_{i,t}$. The labor supply decision is delegated to a labor union which we explain in Section (2.3.2). The endowment with labor efficiency units $e_{i,t}$ is subject to idiosyncratic risk. Housing tenure status enters through the budget constraint as we abstract from utility through housing service. This is appropriate because housing tenure status is not a choice variable of the household. Instead, housing tenure status enters through the budget constraints as follows.

Outright homeowner

$$c_{i,t} + b_{i,t} = e_{i,t} w_t n_t + (1 + r_t) b_{i,t-1} + d_{i,t} \quad (2.4)$$

Mortgage holder

$$c_{i,t} + b_{i,t} = e_{i,t} w_t n_t + (1 + r_t) b_{i,t-1} + d_{i,t} - r_{m,t} m_{k,t} \quad (2.5)$$

Renter

$$c_{i,t} + b_{i,t} = e_{i,t} w_t n_t + (1 + r_t) b_{i,t-1} + d_{i,t} - h_{k,t} \quad (2.6)$$

All household types allocate their disposable income to consumption and savings. Variables w_t and r_t denote the aggregate real wage and the real interest rate on bond holdings. All households face disposable income from labor, the previous period's bond holdings, and dividends $d_{i,t}$ which are allocated proportionally to efficiency units $e_{i,t}$. This is a standard choice to avoid meaningful redistribution of resources through firm profits. Outright owners do not need to pay for housing services because they own their homes and hold no mortgage. We abstract from housing depreciation which is likely similar across both regions and therefore of minor importance for cross-region asymmetries. Mortgage holders differ only in the mortgage $m_{k,t}$ for which they have to pay real interest rate $r_{m,t}$. The evolution of real interest rates will depend on the amount of adjustable-rate mortgage that will be governed by an exogenous parameter, see Section (2.3.3). Finally, renters pay real rent $h_{k,t}$. The measure of each household type is denoted by ϕ_k and $\alpha = \sum_k \phi_k$ holds.

We do not model housing tenure status and adjustable- or fixed-rate mortgages as choice variables. Introducing a microfounded theory of these choices is beyond the scope of our paper since the literature is not yet settled on the underlying drivers of the cross-country

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heterogeneity. Instead, our strategy is to carefully account for the empirical heterogeneity when calibrating the model in Section (2.3.4). This may be reasonable because we will take a first-order perturbation around the steady state and hence, consider small shocks that are less likely to affect housing tenure status differentially across regions.

Finally, we follow Auclert, Rognlie, and Straub (2020) in two important dimensions. First, we allow for ex-ante heterogeneity in discount factors β_k which admits matching the hand-to-mouth shares by housing tenure status found in the data. This is important because it implies realistic marginal propensities to consume. Second, we assume that households are inattentive with respect to the aggregate state. With probability ξ , households do not update their belief about the future evolution of the aggregate state.⁹ The idiosyncratic state variable $x_{i,t}$ tracks the period in which the household was updating its beliefs for the last time. Importantly, beliefs refer to the future evolution of the aggregate state and not its contemporaneous values. The latter is always understood by households, ensuring that budget constraints hold. Assuming inattentive households is a convenient modeling device that allows humps in heterogeneous agent economies, as found in the data.¹⁰ This is important in our setting to be able to match the empirically identified impulse response functions.

Consumption bundles. Households spread their aggregate consumption across North and South goods baskets, denoted by $c_{N,t}$ and $c_{S,t}$, respectively. Each goods basket consists of different varieties $c_{N,t}(j)$ and $c_{S,t}(j)$ from the respective region as

$$c_{N,t} = \left(\int_0^\alpha c_{N,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad c_{S,t} = \left(\int_\alpha^1 c_{S,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (2.7)$$

where we assume that the mass of varieties coincides with the number of households that populate the region. The elasticity of substitution across varieties from a given country is ϵ . In the second CES stage, households aggregate the North and South consumption

⁹The beliefs about the aggregate state enter the household problem through the time indexation of the value function. The aggregate state includes the labor compensation per efficiency unit $w_t n_t$, the aggregate dividend payments $d_t = \int_i d_{i,t} di$, and interest rates $r_t, r_{m,t}$.

¹⁰For example, consider the aggregate wage w being above steady state for multiple periods. Households that do not update their beliefs think that the contemporaneous increase in the wage is perfectly transitory. Hence, they save a large fraction of the income gain to spread consumption across multiple periods. This effect decreases as households are accumulating more wealth leading to a sluggish increase in consumption demand which builds up over time.

baskets to their final domestic consumption basket

$$c_t = \left(\lambda^{\frac{1}{\eta}} c_{N,t}^{\frac{\eta-1}{\eta}} + (1-\lambda)^{\frac{1}{\eta}} c_{S,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad (2.8)$$

where the elasticity of substitution across North and South goods baskets is η and households have a bias for domestically produced goods, given by $\lambda \geq 1/2$. The derivation of demand schedules and associated price indices is standard and relegated to Appendix (2.B).

2.3.2 Production

Labor supply is set by labor unions which sell labor services varieties to intermediate goods producers through a representative labor packer. The intermediate good varieties are produced with linear technology and labor is the only production input.

Labor unions. We assume a continuum of monopolistically competitive labor unions, as in Auclert, Rognlie, and Straub (2018). Each household provides $n_{i,t}(u)$ hours of work to labor union u . The union collects labor supply across households $n_t(u) = \int n_{i,t}(u) di$ and differentiates it to a specific labor service variety. A representative labor packer buys all varieties and aggregates them using a CES technology with an elasticity of substitution ϵ_w . The union takes the implied demand schedule as given and sets the nominal wage $W_t(u)$ charged for the labor variety $n_t(u)$. It maximizes the average discounted stream of utility of all union members, subject to Rotemberg-type wage adjustment costs that are denominated in utils and uniform across all members.¹¹ Formally, the unions solves

$$\max_{W_t(u)} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^t \int \frac{c_{i,t}^{1-\gamma}}{1-\gamma} di - \varphi \frac{n_t(u)^{1+\nu}}{1+\nu} - \frac{\kappa_w}{2} \left(\frac{W_t(u)}{W_{t-1}(u)} - 1 \right)^2 \right) \quad (2.9)$$

$$\text{s.t. } n_t(u) = \left(\frac{W_t(u)}{W_t} \right)^{-\epsilon_w} n_t, \quad (2.10)$$

where n_t denotes the aggregate labor demand. The key assumption is that labor supply is equal across all union members. In a symmetric equilibrium, all unions choose the same (aggregate) nominal wage $W_t \equiv (\int W_t(u)^{1-\epsilon_w} du)^{\frac{1}{1-\epsilon_w}} = W_t(u)$ and the first-order condition of the union implies an aggregate wage Phillips curve

¹¹The firm uses the deterministic average discount factor $\beta = \sum_k \phi_k \beta_k$, where ϕ_k denotes the measure of households with housing tenure status k .

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$$(\Pi_{w,t} - 1)\Pi_{w,t} = \beta \mathbb{E}_t((\Pi_{w,t+1} - 1)\Pi_{w,t+1}) + \frac{\epsilon_w}{\kappa_w} \left(\varphi n_t^{1+\nu} - \frac{\epsilon_w - 1}{\epsilon_w} w_t n_t \int c_{i,t}^{-\gamma} e_{i,t} di \right) \quad (2.11)$$

where $\Pi_{w,t} = \frac{W_t}{W_{t-1}}$ denotes wage inflation and $w_t = W_t/P_{c,t}$ is the real wage. In the zero inflation steady state with $\Pi_{w,t} = 1$, the left-hand side is equal to zero and the right-hand side is a standard static labor supply condition, accounting for market power $\epsilon_w < \infty$ and uniform labor supply across households.

Variety producers. We assume a continuum of monopolistically competitive firms that produce intermediate good varieties. Firm j buys labor $n_t(j)$ from the representative labor packer and produces the variety $y_t(j)$ with a linear technology where labor is the only production factor. The variety is bought by households from both regions. The firm sets its retail price $P_{p,t}(j)$ and maximizes the expected discounted stream of profits, subject to Rotemberg-type adjustment costs.¹² Formally, the firm solves

$$\max_{P_{p,t}(j)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{P_{c,t}} \left(P_{p,t}(j) y_t(j) - W_t n_t(j) - \frac{\kappa}{2} \left(\frac{P_{p,t}(j)}{P_{p,t-1}(j)} - 1 \right)^2 P_{p,t} y_t \right) \quad (2.12)$$

$$\text{s.t. } y_t(j) = \left(\frac{P_{p,t}(j)}{P_{p,t}} \right)^{-\epsilon} y_t, \quad \text{and} \quad y_t(j) = a_t n_t(j), \quad (2.13)$$

where y_t denotes aggregate output and a_t is total factor productivity.

The nominal flow profits are converted to real domestic consumption units as we divide with $P_{c,t}$. In a symmetric equilibrium, all firms set the same retail price $P_{p,t} \equiv \left(\int_0^{\alpha} \frac{1}{\alpha} P_{p,t}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} = P_{p,t}(j)$ and the first-order condition of the firm implies an aggregate Phillips curve

$$(\Pi_{p,t} - 1)\Pi_{p,t} = \beta \mathbb{E}_t \left(\frac{y_{t+1}}{y_t \Pi_{c,t+1}} (\Pi_{p,t+1} - 1) \Pi_{p,t+1}^2 \right) + \frac{1}{\kappa} \left((1 - \epsilon) + \epsilon \frac{w_t}{a_t} \frac{P_{c,t}}{P_{p,t}} \right)$$

where $\Pi_{p,t} = P_{p,t}/P_{p,t-1}$ denotes producer price inflation. It is convenient to re-write the

¹²The firm uses the deterministic average discount factor $\beta = \sum_k \phi_k \beta_k$. We abstract from a stochastic discount factor in anticipation of the solution method. Up to first-order, a stochastic discount factor is irrelevant because certainty equivalence holds.

Philips curve as

$$(\Pi_{p,t} - 1)\Pi_{p,t} = \beta \mathbb{E}_t \left(\frac{y_{t+1}}{y_t \Pi_{c,t+1}} (\Pi_{p,t+1} - 1) \Pi_{p,t+1}^2 \right) + \frac{1}{\kappa} \left((1 - \epsilon) + \epsilon \frac{w_t}{a_t} \left[\lambda + (1 - \lambda) T_t^{1-\eta} \right]^{-\frac{1}{1-\eta}} \right) \quad (2.14)$$

where $T_t \equiv \frac{P_{p,t}^*}{P_{p,t}}$ denotes the terms-of-trade.

2.3.3 General equilibrium

We derive the relevant market clearing conditions and discuss the supply of housing services. Finally, we explain how the degree of adjustable-rate mortgages is modeled and close the model with a monetary policy rule.

Goods markets. We have a continuum of α North and $1 - \alpha$ South varieties. All households demand all varieties and all North intermediate good producers pay their respective price adjustment costs in North varieties. This implies the following equation for aggregate demand of variety j

$$y_t(j) = \int_0^\alpha c_{i,t}(j) di + \int_\alpha^1 c_{i,t}^*(j) di + \frac{\kappa}{2} (\Pi_{p,t} - 1)^2 y_t. \quad (2.15)$$

The first term is the demand from domestic households. The second term is the demand from foreign households. The final term is the demand from domestic firms to pay the price adjustment cost, which is proportional to domestic output. Substituting the demand schedules, and relative price expressions, aggregating over all $j \in [0, \alpha]$ and imposing goods market clearing yields

$$a_t n_t = \left(1 - \frac{\kappa}{2} (\Pi_{p,t} - 1)^2 \right)^{-1} \left\{ \lambda \left[\lambda + (1 - \lambda) T_t^{1-\eta} \right]^{\frac{\eta}{1-\eta}} c_t + (1 - \lambda^*) \frac{1 - \alpha}{\alpha} \left[\lambda^* T_t^{1-\eta^*} + (1 - \lambda^*) \right]^{\frac{\eta}{1-\eta}} c_t^* \right\}, \quad (2.16)$$

where the left-hand side is aggregate North production as a function of North labor supply and the right-hand side denotes aggregate demand. The derivation and the symmetric equation for goods produced in the South are given in Appendix (2.B). Finally, we can use the nested CES structure to express consumer price inflation as a function

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of the terms-of-trade and producer price inflation

$$\Pi_{c,t} = \left(\frac{\lambda (\Pi_{p,t})^{1-\eta}}{\lambda + (1-\lambda)(T_{t-1})^{1-\eta}} + \frac{(1-\lambda)(\Pi_{p,t}^*)^{1-\eta}}{\lambda(T_{t-1})^{\eta-1} + (1-\lambda)} \right)^{\frac{1}{1-\eta}}. \quad (2.17)$$

Housing services. The supply of housing services is not modeled explicitly. Instead, we assume that intermediate goods producers own the housing stock. The rent payments from the households thus accrue to these firms. There are no costs to be covered because we abstract from depreciation to be consistent with our assumptions for homeowners. The rent payments are exogenous. Note that a suitably calibrated exogenous housing supply will imply the same steady state rent payments.

Interest rates. Mortgage holders pay interest on their outstanding mortgages. The nominal interest rate payment is given by

$$i_{m,t} = \theta i_{b,t} + (1-\theta) i_{b,ss} + \omega, \quad (2.18)$$

where $i_{b,ss}$ denotes the nominal interest rate on bonds in the steady state. Parameter θ denotes the share of adjustable-rate mortgages and therefore governs the responsiveness of nominal mortgage rates to changes in the nominal interest rate on bonds. The parameter ω is a wedge that allows for a permanent interest rate differential. Finally, the Fisher equation $1 + r_{s,t} = \frac{1+i_{s,t-1}}{\Pi_{c,t}}$ maps nominal rates into (ex-post) real rates for $s = m, b$. For brevity, we define $r_t \equiv r_{b,t}$ and $i_t \equiv i_{b,t}$.

The asset market is integrated across both regions with market clearing condition

$$\int_0^\alpha b_{i,t} di + \chi_t \int_\alpha^1 b_{i,t}^* di = \phi_m m_{m,t} + \chi_t \phi_m^* m_{m,t}^*, \quad (2.19)$$

where $\chi_t = ((\lambda^* T_t^{(1-\eta)} + (1-\lambda^*)) / (\lambda + (1-\lambda) T_t^{1-\eta}))^{\frac{1}{1-\eta}}$ converts South into North consumption baskets. Thus, the left-hand side is the aggregate savings of the region, denominated in consumption baskets of the North and the right-hand side is the total amount of outstanding mortgages held by households. Parameters ϕ_m and ϕ_m^* denote the measure of households that hold mortgages $m_{m,t}$ and $m_{m,t}^*$ respectively.

Dividends. The intermediate good producers accrue profits due to market power and rent payments as described above. There are further profits from financial intermediation

that are given by

$$f_t = \phi_m m_{m,t} r_{m,t} + \chi_t \phi_m^* m_{m,t}^* r_{m,t}^* - \int_0^\alpha b_{i,t} r_t di - \chi_t \int_\alpha^1 b_{i,t}^* r_t^* di, \quad (2.20)$$

where the first two terms capture the debt service paid by mortgage holders whereas the last two terms capture interest rate payments to bondholders. Note that these profits follow mechanically from interest rate differentials as we do not explicitly model a financial sector. We assume that each region receives profits, proportional to the population size. This provides an equalizing force that rules out that cross-region differences are driven by region-specific profits. Each region's profit share is paid in dividends to households, proportional to labor efficiency units. This ensures that within-country redistribution does not drive our results.

Monetary policy. The monetary policy authority responds to union-wide inflation rate $\pi_t = (\Pi_{c,t})^\alpha (\Pi_{c,t}^*)^{1-\alpha} - 1$ based on the following policy reaction function

$$i_t = \rho i_{t-1} + (1 - \rho) \phi \pi_t + \varepsilon_t^{MP}, \quad (2.21)$$

where ρ and ϕ govern the degree of interest rate smoothing and the response to inflation, respectively. Finally, $\varepsilon_t^{MP} \stackrel{iid}{\sim} N(0, \sigma_{\varepsilon^{MP}})$ is a monetary policy shock. We abstract from other policies and focus on monetary policy transmission.

Equilibrium and solution. The decentralized general equilibrium of our model demands that all agents behave optimally, given their information sets, as described above. Both final goods markets, both labor markets, and the common asset market must clear at any date t .

We use standard methods to solve for the steady state numerically. We then compute responses to aggregate shocks based on a first-order perturbation in sequence space, following Auclert, Bardóczy, Rognlie, and Straub (2021).

2.3.4 Calibration

We calibrate the model to the four largest economies of the Euro Area, using household-level survey data to discipline the cross-region household heterogeneity.

2.3. Model

Calibration strategy. We calibrate the model to a quarterly frequency. The North region consists of France & Germany and the South region consists of Italy & Spain. The calibration follows three steps. First, we calibrate the ex-ante household heterogeneity to match household-level data moments from the survey data. Household-level moments are always computed as the average across countries, weighted by average real GDP from 2001 until 2019. In the second step, we take parameter values from the literature which affect the steady state of the economy. Finally, we estimate key parameters that govern the response to aggregate shocks but do not affect the steady state.

Cross-region heterogeneity. The parameters that are heterogeneous across both regions are summarized in Table (2.3.1). The process for the endowment of efficiency units implies a first-order autoregressive process for labor income in the steady state. The GRID database provides the annual standard deviation and first-order autocorrelation of residualized income for many European economies, including the four core countries.¹³ We choose the standard deviation and autocorrelation of the endowment process to match the average moments of each region.

The remaining heterogeneity parameters are based on data moments from the HFCS. We start with the homeownership rate $1 - \phi_r$ which is region specific, as shown in Figure (2.2.1). The amount of mortgage holders out of homeowners $\phi_m/(1 - \phi_r)$ is the average across both regions. Rent payments $h_r = h_{r,t}$ are set to match the region-specific rent-to-income ratio in the steady state. Mortgage holders have a common loan-to-value ratio because there is no cross-region difference in the data. There are two house sizes h_s and h_l that coincide with house values and we choose them to match the average house value-to-income ratios in the data. These variables determine the outstanding mortgages $m_k \equiv m_{k,t}$. The share of adjustable-rate mortgages θ is set as shown in Figure (2.2.1) and governs the response of the mortgage debt service to aggregate shocks.

Hand-to-mouth shares are an endogenous model outcome which makes them harder to match. Indeed, we cannot match the hand-to-mouth shares in levels. Instead, we match the differential hand-to-mouth shares across regions and housing tenure status.¹⁴ We show the results in Table (2.3.2). We understate the cross-country difference by one per-

¹³The GRID database has been compiled by Fatih Guvenen, Luigi Pistaferri, and Gianluca Violante and their collaborators. It provides annual income data moments estimated from high-quality administrative data.

¹⁴We define hand-to-mouth in the model as households with savings below their biweekly income, relative to the borrowing limit. This means that a household is hand-to-mouth if $b_{i,t} \leq -\underline{b} + \frac{1}{6}w_t n_t e_{i,t}$.

Table 2.3.1: Cross-region heterogeneity

South	Value	North	Value	Description	Target
β_o^*	0.978	β_o	0.985	discount factor	differential HtM shares across reg.
β_m^*	0.967	β_m	0.980		
β_r^*	0.957	β_r	0.970		
LTV_m^*	0.45	LTV_m	0.45	loan-to-value	cross-reg. average
h_s^*	9.60	h_s	9.60	small house	cross-reg. avg. of value-to-income
h_l^*	33.28	h_l	33.28	large house	
θ^*	0.66	θ	0.14	mortgage types	reg.-specific share of adj.-rates
h_r^*	0.27	h_r	0.29	rent payments	reg.-specific rent-to-income
ϕ_o^*	0.51	ϕ_o	0.35	mass of households	reg.-specific
ϕ_m^*	0.22	ϕ_m	0.15		
ϕ_r^*	0.27	ϕ_r	0.50		
σ_e^*	0.279	σ_e	0.251	SD of endowment	annual SD of income
ρ_e^*	0.886	ρ_e	0.888	AR of endowment	annual AR of income
\underline{b}^*	3.10	\underline{b}	0.00	borrowing limit	symmetric per capita GDP

Notes: The annual income data moments are taken from the GRID database. The remaining data moments are taken from the pooled first three waves of the HFCS. Region-specific parameters are computed as the GDP-weighted average, using the average GDP from 2001 until 2019. Similarly, common parameters are the GDP weighted average across both regions.

Indices o, m, r refer to outright owners, mortgage holders, and renters respectively.

centage point for each housing tenure status. Thus, our hand-to-mouth calibration can be seen as conservative concerning its explanatory power for the cross-region differences in impulse responses. Finally, we choose the borrowing limits \underline{b} to ensure a steady state that features symmetric per-capita output.¹⁵

Symmetric parameters. The remaining household parameters are given in Table (3.3.1) and set as follows. The Frisch elasticity of $1/\nu = 0.25$ and relative risk aversion equal to $\gamma = 2.00$ are standard choices. Both countries have the same population size

¹⁵A symmetric borrowing limit would imply larger per-capita in the South which is counterfactual. Moreover, our choice of borrowing limits delivers qualitatively consistent net foreign asset positions because the North becomes a net lender in the steady state.

2.3. Model

Table 2.3.2: Calibration of hand-to-mouth

	Owner	Mortgagor	Renter
South			
Model	0.10	0.17	0.18
Data	0.16	0.24	0.37
North			
Model	0.04	0.09	0.14
Data	0.09	0.15	0.32
South-North			
Model	0.06	0.08	0.04
Data	0.07	0.09	0.05

Notes: The table shows hand-to-mouth shares in levels and cross-region differences (South-North), by housing tenure status. The empirical hand-to-mouth shares are computed from the pooled first three waves of the HFCS for each country, following the definition from Slacalek, Tristani, and Violante (2020a). We report region-specific average hand-to-mouth computed as the GDP-weighted average, using the average GDP from 2001 until 2019. Hand-to-mouth in the model are defined as households with savings below their biweekly income, relative to the borrowing limit.

$\alpha = 1 - \alpha = 0.50$. The steady state, which is symmetric in output demands the same equilibrium labor supply in both regions. We set the labor disutility shifter φ to normalize steady state labor supply to unity. The mortgage wedge is set to a preliminary value $\omega = 0.0025$ and is not important for our quantitative findings.

For the production side, we follow Bletzinger and Thadden (2021b), who calibrate their Eura Area economy to a North and South region as well. We depart from their Cobb-Douglas assumption and allow for a slightly larger elasticity of substitution across regions, only for computational purposes. Finally, we assume that the elasticity of substitution across labor service varieties is identical to intermediate good varieties. These parameters determine the steady state of our two-region economy.

Estimation. In the final step, we follow Auclert, Rognlie, and Straub (2020) and estimate key parameters that govern the responses to aggregate shocks. Specifically, we estimate Taylor rule parameters ϕ and ρ , as well as the degree of household inattention ξ and the cost-shifters for price and wage adjustment costs, assuming $\kappa = \kappa_w$ to mirror the

Table 2.3.3: Symmetric parameters

Parameter	Value	Description	Target or source
Households	φ	0.74	labor disutility shifter
	γ	2.00	inverse of intertemp. EOS
	ν	4.00	inverse Frisch elasticity
	ω	0.0025	mortgage wedge
	α	0.50	relative region size
Production	λ	0.80	home bias
	η	1.10	EOS across regions
	ϵ	5.00	EOS across varieties
	ϵ_w	5.00	EOS across labor services
Estimation	ϕ	1.51	Taylor rule inflation coeff.
	ρ	0.93	interest rate smoothing
	ξ	0.90	household inattention
	$\kappa = \kappa_w$	720.66	price/wage adjustment cost

Notes: The first two panels contain calibrated parameters. The last panel contains the parameters that are estimated based on the method of simulated moments to match the empirical output response of the South region to a monetary policy shock.

symmetry in the elasticities. The estimation relies on the method of simulated moments and targets the impulse response function of the South region to a monetary policy shock as displayed in Figure (2.2.1) in Section (2.2).¹⁶ Importantly, the response of the North region is untargeted. The cross-region differences only stem from the discussed cross-region heterogeneities.

The resulting parameter estimates are given in the last panel of Table (3.3.1). An inflation coefficient of $\phi = 1.51$ is standard. The degree of interest rate smoothing and household inattention is sizeable and similar to Auclert, Rognlie, and Straub (2020). Finally, we find $\kappa = \kappa_w = 720.66$ which implies very flat Phillips curves. The reason is simply that the empirical responses are very persistent and the model can replicate them with very slow and small price and wage adjustments. Empirically, inflation responses

¹⁶Naturally, one can match response from the model and the data that correspond to a standard deviation monetary policy shock. The appropriate size of the model shock then simply follows from an appropriate choice for $\sigma_{\epsilon MP}$ which we also estimate, along with the other parameters.

2.4. Results

are quite small which seems to be qualitatively consistent with flat Philips curves.¹⁷

2.4 Results

We study the model responses to a monetary policy shock and provide model-based decompositions. We find that the interplay of demand amplification through hand-to-mouth heterogeneity and demand dampening through trade are the key determinants of cross-region differences in impulse responses. Finally, we study a counterfactual with increased trade openness that suggests that sizing up the trade channel can be particularly powerful.

2.4.1 Baseline

Output responses. We report the responses of output to an expansionary monetary policy shock in Panel (a) of Figure (2.4.1). The solid red line (without markers) displays the response of the South that is targeted in the estimation. The blue solid line (with markers) displays the untargeted response of the North. The corresponding dashed lines contain the empirical impulse responses introduced in Section (2.2). The model matches both, the targeted and untargeted responses quite well. Indeed, at the peak of the empirical impulse responses, the model can account for 70% of the cross-region differences.

We next study the differential output response, defined as the South response minus the North response, $dy_t^* - dy_t$. We provide a demand-based decomposition of this differential response. From the aggregated demand equations of both regions, it follows that

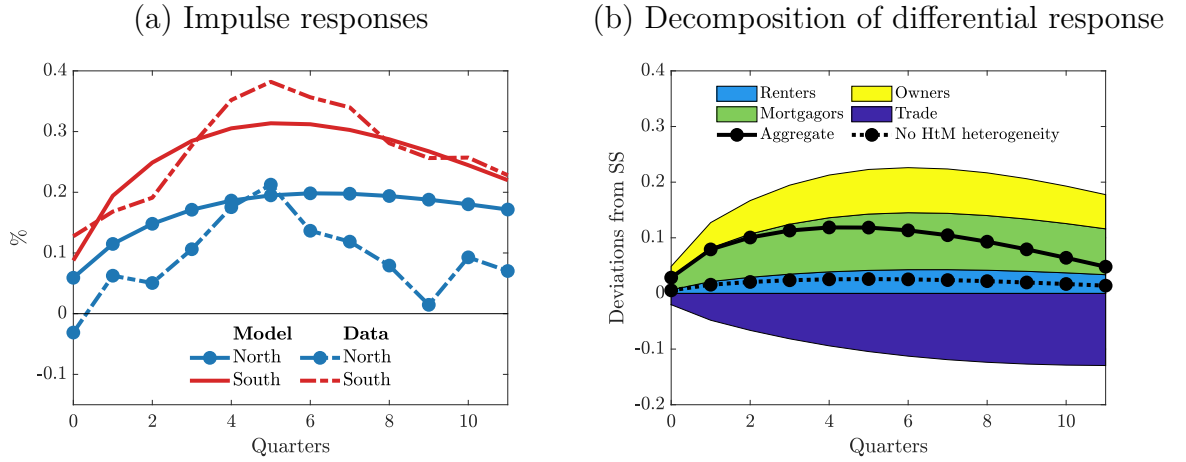
$$dy_t^* - dy_t = \underbrace{dc_{o,t}^* - dc_{o,t}}_{\text{Owners}} + \underbrace{dc_{m,t}^* - dc_{m,t}}_{\text{Mortgagors}} + \underbrace{dc_{r,t}^* - dc_{r,t}}_{\text{Renters}} + \underbrace{dE_t^* - dE_t}_{\text{Trade}}, \quad (2.22)$$

where the first three terms account for differential domestic demand by housing tenure status and the last term denotes differential net exports. Note that all objects on the right-hand side are appropriately defined to account for terms-of-trade. This demand-based composition simply follows from goods market clearing and is an exact decomposition that does not require solving the model with a modified calibration. Panel (b) of Figure (2.4.1) shows the results. The top area in yellow represents the differential

¹⁷Contemporaneous work by Beaudry, Portier, and Preston (2023) shows that the imposition of a Taylor rule may also contribute to estimating flat Phillips curves.

demand contribution coming from outright homeowners. The second green area represents the differential demand contribution from mortgage holders. The third light-blue area provides the same for renters. One can see that homeowners are the crucial group driving the differential response, whereas renters are of less importance. Finally, the dark-blue area at the bottom represents the effect of trade, which delivers a sizeable negative contribution. The four components add up to the differential response that is given as a black solid line.

Figure 2.4.1: Output responses



Notes: Panel (a) shows the impulse responses of GDP to an expansionary monetary policy shock in the data and in the model. The empirical responses are the same as in Section (2.2). Panels (b) provide a decomposition of the differential model responses based on equation (2.22). The solid black line shows the total differential response and the dotted black line shows the total differential response in a counterfactual without discount factor heterogeneity.

The trade channel dampens the differential response across both regions. Prices in the South raise by more because South households prefer domestic goods due to their home bias. Thus, households substitute away from goods produced in the South and consume more North goods instead. This reallocation of demand dampens the differential output effects discussed above.

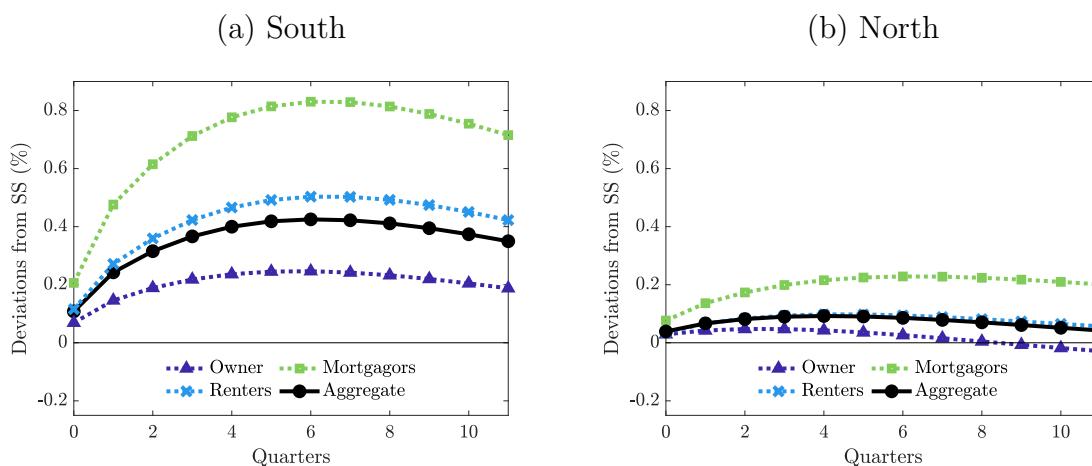
We further run a counterfactual without discount factor heterogeneity that we set to match the hand-to-mouth in our baseline calibration. The counterfactual differential response is given as a dotted black line. The differential response vanishes almost completely. This suggests that the interplay between demand amplification through realistic hand-to-mouth heterogeneity and demand-dampening through trade is key to under-

2.4. Results

standing the heterogeneous responses across Euro Area member states.

Consumption responses. Consumption demand is the key driver behind the cross-region output differences. Thus, we study the consumption responses across space and household types. Figure (2.4.2) shows the consumption response by housing tenure status for the South and North, in Panels (a) and (b) respectively. Mortgage holders respond the strongest, followed by renters and outright owners. This pattern holds in both regions, although the overall amplitudes are substantially larger in the South.

Figure 2.4.2: Consumption responses



Notes: The figure shows the model consumption responses to an expansionary monetary policy shock, in percent deviation of the steady state. We display these responses for each region and for each tenure status, and the region-specific aggregate responses.

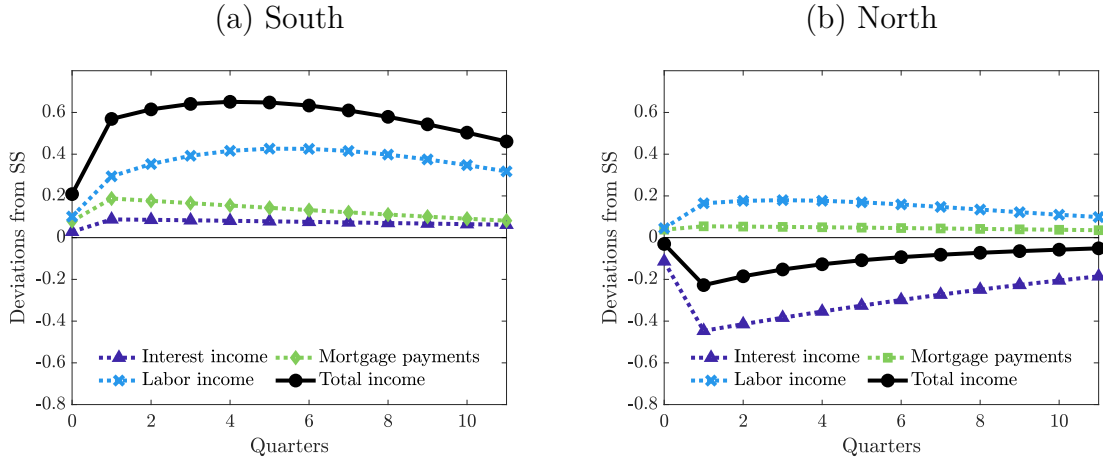
We next study the disposable income response to understand the cross-region consumption differences, averaging across housing tenure status.¹⁸ The results are provided as black solid lines in Figure (2.4.3). While disposable income strongly increases for the South, we find that this actually decreases in the North. This rationalizes why the consumption response is so much stronger in the South. One may wonder why households in the North still consume more, despite the decline in income. The reason is that the interest rate cut induces the household to dis-save, as we discuss below.

We further decompose disposable income into contributions from labor income, mort-

¹⁸We refer to disposable income as the right-hand side of the budget constraint, i.e., the amount of income which the household can allocate to consumption or net savings.

gage payments, and interest income.¹⁹ The key differences is coming from interest income displayed as dotted purple lines (triangular markers). The North is a net lender in the steady state and interest rate income declines in response to the expansionary monetary policy shock, and vice versa for the South. The blue dotted line (cross markers) represents labor income which also responds stronger in the South. This is because the home bias leads to more consumption demand in the South. Firms need to pay higher wages to cater to this demand. Finally, the response of mortgage payments leads to more disposable income response in the South because the pass-through of the rate cuts from the shock to mortgage rates increases in the share of adjustable-rate mortgages.

Figure 2.4.3: Disposable income responses



Notes: The figure shows the model disposable income responses to an expansionary monetary policy shock, in percent deviation of the steady state. We display these responses for each region and for each tenure status, and the region-specific aggregate responses. Disposable income is defined as income including labor income, dividends and interest rate payments.

Finally, we summarize the household responses in Table (2.4.1), distinguishing again by housing tenure status. Specifically, we display the responses of consumption, net savings, and disposable income cumulated over a 12-quarter horizon. The net savings response confirms that North households run down their savings to compensate for income losses. The substitution effect dominates leading North households to increase consumption despite less disposable income. Focusing on housing tenure status, we see that in the North, only mortgage holders see an increase in disposable income because they hold

¹⁹We add dividends and labor income together since dividends are distributed proportionally to labor income.

2.4. Results

little bonds. In contrast, all households in the South do gain disposable income.

Table 2.4.1: Budget constraint decomposition of the household response

	South				North			
	Agg.	Outright	Mortgagor	Renter	Agg.	Outright	Mortgagor	Renter
Consumption	4.22	1.39	1.73	1.10	0.85	0.10	0.35	0.40
Net savings	2.44	0.82	1.11	0.51	-2.10	-1.84	0.05	-0.31
Disp. income	6.66	2.20	2.84	1.62	-1.25	-1.74	0.40	0.08
Labor	4.26	2.18	0.93	1.15	1.68	0.59	0.25	0.84
Savings	0.86	0.02	0.37	0.46	-3.48	-2.32	-0.39	-0.76
Mortgage	1.54	0.00	1.54	0.00	0.54	0.00	0.54	0.00

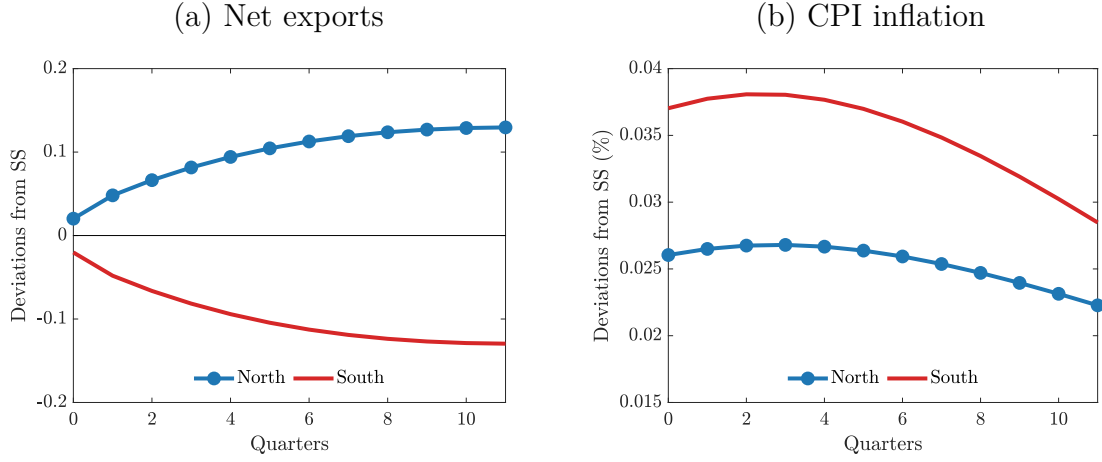
Notes: The table shows the model disposable income responses to an expansionary monetary policy shock, cumulated over a 12-quarter horizon. The top panel shows the household choices, consumption, and net savings, which add up to disposable income. The bottom panel shows the different sources of disposable income. All responses are displayed for each region and for each tenure status, along with the region-specific aggregate responses.

Inflation and trade responses. Next, we study the responses of net exports and inflation to further understand the importance of trade. Panel (a) of Figure (2.4.4) displays net exports for both countries. By construction, this response is symmetric because the net exports of the North (blue solid line with markers) equal the net imports of the South (red solid line). Panel (b) shows the corresponding responses of the consumer price inflation rates $\Pi_{c,t}$, which increase more in the South than in the North. The responses of both outcomes can be explained with a typical expenditure-switching mechanism. Consumption demand increases by more in the South, leading to higher price pressure because households prefer domestic over foreign goods. Consequently, this leads to an improvement in the terms-of-trade from the perspective of the Northern region, resulting in increased exports from the North to the South.

2.4.2 The role of trade openness

Our findings suggest that trade is an important balancing channel to counteract cross-region differential responses to monetary policy shocks. This channel crucially depends on the home bias and elasticity of substitution when aggregating North- and South-specific baskets to final goods. Fundamentally, these parameters reflect, both the taste

Figure 2.4.4: Net exports and inflation responses to a monetary policy shock



Notes: The figure shows the model responses of net exports and CPI inflation to an expansionary monetary policy shock.

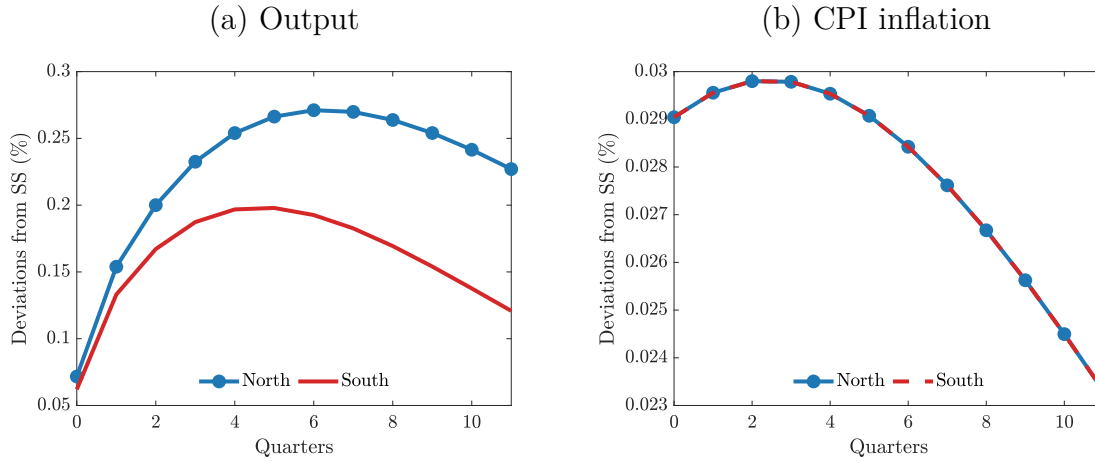
of households but also institutional details that may be influenced by policy choices. For example, policies could foster the trade of services by harmonizing legal requirements. We thus study a counterfactual with increased trade openness to learn how important such policies potentially can be. Specifically, we increase the elasticity of substitution between North and South baskets to $\eta = \eta^* = 10$, and we assume no taste for goods produced in one particular region $\lambda = \lambda^* = 0.5$.

Figure (2.4.5) displays the results for output and inflation in Panels (a) and (b), respectively. In this counterfactual, differential CPI inflation responses completely vanish. In equilibrium, no price differences can be entertained because households would immediately exploit them, as North and South goods are close substitutes. The output response is not equalized, however. In contrast, the differential output response flips its sign. In this counterfactual, the North output responds more strongly, despite having more hand-to-mouth in the South. Since prices are equalized, it follows that output must adjust to clear both markets.

More concretely, net exports increase by more, sizing up the trade channel. Fundamentally, this reflects a labor supply effect. As households in the South increase consumption by more, it follows that the valuation of leisure is larger, compared with the North. Hence, marginal costs are smaller in the North, making this region more competitive. The high elasticity of substitution implies that households exploit such differences in marginal costs by more, which sizes up the trade channel.

2.5. Conclusion

Figure 2.4.5: High degree of trade openness: Output and CPI inflation



Notes: The figure shows the model responses of output and CPI inflation to an expansionary monetary policy shock. We study a counterfactual with increased trade openness defined by no home bias and a large elasticity of substitution, as described in Section (2.4.2).

Overall, our results emphasize the importance of the expenditure-switching mechanism and associated trade channel in currency union HANK models. This has important policy implications. Policies that foster trade openness may reduce or even eliminate differential inflation responses. This removes an important trade-off currently faced by the European Central Bank. Instead of monitoring various inflation rates, it is sufficient to control Euro Area inflation to achieve price stability not only on average but in each single member country. However, increased trade openness can overturn cross-region differences in output responses. Depending on parameters, this effect may even amplify cross-region differences in output response which is highly welfare relevant.

2.5 Conclusion

In this paper, we investigate the sources of heterogeneous monetary policy transmission among Euro Area member states. We constructed a two-region currency Union HANK model that allows for rich cross-region household heterogeneity. The heterogeneous output responses to monetary policy shocks in the Euro Area are driven by the interplay of two forces. On the one hand, regions with more hand-to-mouth households respond stronger because these households lead to demand amplification at the aggregate. On the other hand, within union trade provides a sizeable counteracting force. In the base-

line calibration, we can rationalize 70% of the differential output responses because the former channel dominates. Counterfactual simulations suggest that policies that foster trade openness can size up the trade channel substantially. Quantitatively, this can lead to synchronization of inflation but not of output responses. In fact, the differential output effects may even flip the sign when trade dominates the demand amplification. Overall, the study underscores the importance of cross-region heterogeneity in hand-to-mouth households and trade in a currency union. In future versions of this draft, we will investigate the trade effects empirically and study the policy implications for supply shocks.

Appendix

2.A Data

The macro-level time series have been retrieved from Eurostat. This includes: GDP, HICP, the EONIA, unemployment rates, industrial production, the turnover and volume of trade index. The monetary policy shock has been retrieved from Altavilla, Brugnolini, Gürkaynak, Motto, and Ragusa (2019b).

All micro-level data have been retrieved from the Household Finance and Consumption Survey (HFCFS). We use this database to compute the HtM shares (outright owners and renters), the adjustable-rate mortgage shares, the homeownership rates, and all other household-level data moments we use throughout our analysis (for instance, the ones presented in Table (2.3.1)).

2.B Model derivations

Consumption bundles Households spread their aggregate consumption choices across North and South good baskets, denoted by $c_{N,t}$ and $c_{S,t}$, respectively. Each goods basket consists of different varieties for which the households choose consumption $c_{N,t}(j)$ and $c_{S,t}(j)$. The households then solve the following first-stage consumption decision given a certain maximal expenditure E . $P_{p,t}(j)$ denotes the price of one unit of a domestic variety j (producer price) in units of the domestic currency. α denotes the size of the domestic economy, while ϵ is the elasticity of substitution between goods varieties.

$$\max_{(c_{N,t}(j))_{j \in [0, \alpha]}} \left[\int_0^\alpha c_{N,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \quad s.t. \quad \int_0^\alpha P_{p,t}(j) c_{N,t}(j) dj = E$$

And equivalent for the foreign consumption basket. The optimal variety choice is given

by

$$c_{N,t}(j) = \left[\frac{P_{p,t}(j)}{P_{p,t}} \right]^{-\epsilon} \frac{c_{N,t}}{\alpha}, \quad P_{p,t} = \left[\int_0^\alpha \frac{1}{\alpha} P_{p,t}(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$$

$$c_{S,t}(j) = \left[\frac{P_{p,t}^*(j)}{P_{p,t}^*} \right]^{-\epsilon} \frac{c_{S,t}}{1-\alpha}, \quad P_{p,t}^* = \left[\int_\alpha^1 \frac{1}{1-\alpha} P_{p,t}^*(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$$

In the second stage, the households choose the optimal composition of domestic and foreign consumption baskets within the aggregate consumption expenditure. The aggregate consumption basket weighs the domestic consumption good according to a home bias λ . The same weighting scheme is employed in the consumer price index formulation. η denotes the elasticity of substitution between domestic and foreign good baskets.

$$c_t = \left[\lambda^{\frac{1}{\eta}} c_{N,t}^{\frac{\eta-1}{\eta}} + (1-\lambda)^{\frac{1}{\eta}} c_{S,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad P_{c,t} = \left[\lambda P_{p,t}^{1-\eta} + (1-\lambda) P_{p,t}^{*1-\eta} \right]^{\frac{1}{1-\eta}}$$

A similar maximization problem gives the optimal consumption choice for domestic and foreign consumption goods baskets

$$c_{N,t} = \left(\frac{P_{p,t}}{P_{c,t}} \right)^{-\eta} c_t \lambda, \quad c_{S,t} = \left(\frac{P_{p,t}^*}{P_{c,t}} \right)^{-\eta} c_t (1-\lambda)$$

Goods markets. We have a continuum of α domestic and $1-\alpha$ foreign varieties. All households demand all varieties and all domestic intermediate good producers pay their respective price adjustment costs in domestic varieties. This implies the following equation for aggregate demand of variety j

$$y_t(j) = \int_0^\alpha c_{H,i,t}(j) di + \int_\alpha^1 c_{F,i,t}^*(j) di + \frac{\kappa}{2} (\Pi_{p,t} - 1)^2 y_t. \quad (2.23)$$

The first term is the demand from domestic households. The second term is the demand from foreign households. The final term is the demand from domestic firms to pay the price adjustment cost, which is proportional to domestic output. Substituting the demand schedules of the household side and aggregating over all $j \in [0, \alpha]$ yields

$$y_t = \lambda \left(\frac{P_{p,t}}{P_{c,t}} \right)^{-\eta} c_t + (1-\lambda) \frac{1-\alpha}{\alpha} \left(\frac{P_{p,t}}{P_{c,t}^*} \right)^{-\eta} c_t^* + \frac{\kappa}{2} (\Pi_{p,t} - 1)^2 y_t. \quad (2.24)$$

2.B. Model derivations

Substituting the relative prices with terms-of-trade expressions and imposing goods market clearing yields

$$a_t n_t = \left(1 - \frac{\kappa}{2} (\Pi_{p,t} - 1)^2\right)^{-1} \left\{ \lambda \left[\lambda + (1 - \lambda) T_t^{1-\eta} \right]^{\frac{\eta}{1-\eta}} c_t + (1 - \lambda^*) \frac{1-\alpha}{\alpha} \left[\lambda^* T_t^{1-\eta^*} + (1 - \lambda^*) \right]^{\frac{\eta}{1-\eta}} c_t^* \right\} \quad (2.25)$$

$$a_t^* n_t^* = \left(1 - \frac{\kappa^*}{2} (\Pi_{p,t}^* - 1)^2\right)^{-1}, \quad (2.26)$$

$$\left\{ \lambda^* \left[\lambda^* + (1 - \lambda^*) T_t^{\eta^*-1} \right]^{\frac{\eta^*}{1-\eta^*}} c_t^* + (1 - \lambda) \frac{\alpha}{1-\alpha} \left[\lambda T_t^{\eta-1} + (1 - \lambda) \right]^{\frac{\eta^*}{1-\eta^*}} c_t \right\},$$

where the second equation follows from analogous steps for varieties produced in the South. Note that η and λ are assumed to be symmetric across both regions.

Finally, we can use the nested CES structure to express consumer price inflation as a function of the terms-of-trade and producer price inflation

$$\Pi_{c,t} = \left(\frac{\lambda (\Pi_{p,t})^{1-\eta}}{\lambda + (1 - \lambda) (T_{t-1})^{1-\eta}} + \frac{(1 - \lambda) (\Pi_{p,t}^*)^{1-\eta}}{\lambda (T_{t-1})^{\eta-1} + (1 - \lambda)} \right)^{\frac{1}{1-\eta}}, \quad (2.27)$$

$$\Pi_{c,t}^* = \left(\frac{\lambda^* (\Pi_{p,t}^*)^{1-\eta}}{\lambda^* + (1 - \lambda^*) (T_{t-1})^{\eta-1}} + \frac{(1 - \lambda^*) (\Pi_{p,t})^{1-\eta}}{\lambda^* (T_{t-1})^{1-\eta} + (1 - \lambda^*)} \right)^{\frac{1}{1-\eta}} \quad (2.28)$$

Chapter 3

House Price Extrapolation and Business Cycles: A Two-Agent New Keynesian Approach

Joint with Ulrich Roschitsch.

3.1 Introduction

Housing constitutes arguably the most important asset in household portfolios, functioning both as a durable consumption good and as a key form of collateral in credit markets. At the same time, house prices exhibit pronounced boom-bust cycles, often amplified by extrapolative over-optimism in expectations during expansions and over-pessimism during downturns, which contribute to substantial volatility in housing markets.¹ These characteristics position housing as a central component in the business cycle and underscore its importance for policy.

This paper offers two important contributions to the existing literature. First, we develop a tractable two-agent New Keynesian (TANK) model featuring a housing sector and extrapolative belief formation over house prices. This framework enables a decomposition of the transmission from changes in housing wealth to aggregate output through four

¹See e.g.: Armona, Fuster, and Zafar (2019), Case, Shiller, and Thompson (2012), Kaplan, Mitman, and Violante (2020), Kuchler and Zafar (2019), and Ma (2020).

3.1. Introduction

distinct channels: consumption by savers (Ricardian households), housing investment, collateral constraints, and fire sale dynamics. Second, we introduce a novel solution method for TANK models with active asset trading between heterogeneous agents in the presence of belief-driven house price dynamics.

Our quantitative findings highlight that output responses are generally attenuated under rational expectations (RE) relative to subjective expectations (SE) characterized by extrapolative belief updating. Under RE, output dynamics are primarily driven by the consumption behavior of savers, whereas under SE, fluctuations in housing investment emerge as the dominant force. The collateral and fire sale channels, while theoretically relevant, contribute only marginally to the variation in aggregate output.

We begin our analysis by considering the representative agent New Keynesian (RANK) framework. Throughout the paper, we examine the effects of a contractionary monetary policy shock. In the RANK model, house prices respond to changes in the real rate through multiple channels: intertemporal consumption smoothing, housing supply dynamics, expectations over future house prices, and future expected wealth changes. This decomposition continues to hold under the extended TANK specifications. For the SE case, we assume internal rationality and solve the RANK version of the model as proposed by Roschitsch and Twieling (2024). Under SE, the house price response is substantially amplified relative to the RE benchmark. This amplification is primarily driven by the more pronounced adjustment in house price beliefs. Aggregate output in the RANK model is shaped by two mechanisms: the consumption response of the representative (Ricardian) household and the dynamics of housing investment. We find that output is considerably more responsive under SE than under RE, a result largely attributable to the stronger housing investment response. While consumption is the dominant driver of output fluctuations under RE, housing investment becomes the primary transmission channel under SE.

Extending the analysis to a TANK framework, we incorporate non-Ricardian, or hand-to-mouth (HtM), households. These agents rely on housing as collateral to access credit, subject to an exogenous loan-to-value (LTV) constraint, which limits their borrowing capacity. In the baseline TANK specification, we assume that HtM households hold housing in the steady state but do not participate in housing market transactions outside of it. As a result, they are passive with respect to housing trade in response to shocks. This modeling choice enables us to isolate the collateral channel linking house prices to

HtM consumption.

The output decomposition in this setting builds on the RANK model, now comprising three channels: savers' consumption, housing investment, and the collateral effect on HtM consumption. Consistent with previous findings, the output response to a monetary policy shock is significantly larger under SE than under RE. Under SE, housing investment remains the dominant driver of output fluctuations, whereas under RE, the primary contribution continues to stem from savers' consumption. The collateral channel has only a transitory influence on output, with limited quantitative differences between RE and SE regimes.

Finally, we turn to a version of the TANK model that allows for housing trade on the side of the HtM agents. In this model version, the HtM can sell or buy housing to stabilize consumption variations arising from exogenous shocks. Under SE the actual house price may not be aligned with the fundamental price due to extrapolative belief updating. It is therefore possible that agents sell housing at a price below the fundamental price to increase consumption, which is commonly referred to as a fire sale. A TANK model under SE and housing trade therefore allows for the emergence of fire sale motives.

However, this modeling approach introduces another layer of complexity. By allowing HtM to trade housing, we introduce an intertemporal choice to the HtM problem as housing is durable. As a result, we need to characterize beliefs about future HtM housing and consumption choices under SE. The method developed in Roschitsch and Twieling (2024) is not suitable for this task, as it relies on the consumption Euler equation, which is not binding for the HtM. We therefore propose to solve this problem by relying on a lag polynomial factorization, which allows us to characterize future expected household choices of the HtM as functions of prices.

Under RE, HtM households respond to a contractionary monetary policy shock by selling housing to stabilize consumption. Crucially, these transactions occur at prices consistent with fundamentals, as expectations are model-consistent. In contrast, under SE, HtM households initially increase housing in the quarters immediately following the shock. This behavior is driven by their expectations about future consumption. Given their high marginal propensities to consume (MPCs), anticipated declines in future income lead HtM households to forecast a corresponding drop in future consumption. To insure against this expected decline, they increase their current housing demand to smooth consumption. Due to the high MPCs, this effect is relatively strong in our model. After initially buying housing at overvalued prices, the HtM will sell housing in consecutive

3.1. Introduction

periods at undervalued prices, triggering fire sales.

In terms of aggregate output dynamics, the qualitative patterns are consistent with earlier model variants: output responses are more pronounced under SE relative to RE. Moreover, under SE, housing investment emerges as the primary driver of output fluctuations, whereas under RE, the consumption response of savers dominates. The collateral channel, while present, plays only a limited and transitory role in shaping the overall output response under both RE and SE. Finally, the fire sale channel is fairly transitory, but relative to the impact output response quite sizable.

Literature review. Leamer (2007) famously stated that "housing is the business cycle". Empirically, he documents that residential investment is an important driver of the business cycle. His empirical findings are in line with our findings under extrapolative house price beliefs, but at odds with the RE version of the model.

Our paper contributes to a broad empirical literature that emphasizes the formation of house price beliefs deviating from the rational expectations framework.² This body of work identifies momentum and revisions in belief formation as critical elements in understanding house price dynamics. On the theoretical side, our study is linked to the behavioral macro-finance literature, which explores departures from rational expectations, particularly in the formation of asset price expectations.³ More specifically, we align with the literature on capital gains extrapolation.⁴ In the context of housing markets, Glaeser and Nathanson (2017) and Schmitt and Westerhoff (2019) model house price expectations using forms of extrapolation within partial equilibrium frameworks. In contrast, we adopt a general equilibrium New Keynesian framework, positioning our work closer to studies such as Adam, Kuang, and Marcet (2012), Caines and Winkler (2021), and Adam, Pfäuti, and Reinelt (2022). In addition to that, other relevant contributions, such as Burnside, Eichenbaum, and Rebelo (2016) and Guren (2018) explain house price behavior through mechanisms such as optimism and pessimism, or concave demand curves faced by sellers.

We also connect to the literature studying housing collateral effects. Mian, Rao, and

²See, for example, Armona, Fuster, and Zafar (2019), Case, Shiller, and Thompson (2012), Kuchler and Zafar (2019), and Ma (2020).

³Among others, see: Barberis (2018), Bianchi, Ilut, and Saijo (2024), Bordalo, Gennaioli, and Shleifer (2018), Caballero and Simsek (2019, 2020), Krishnamurthy and Li (2020), L'Huillier, Singh, and Yoo (2023), and Maxted (2024).

⁴Adam, Marcet, and Beutel (2017) and Winkler (2020) investigate asset price learning in stock markets.

Sufi (2013) and Mian and Sufi (2015) document that during the great recession collateral constraints on housing were important drivers in explaining the decline in household consumption. In a theoretical context Guerrieri and Lorenzoni (2017) and Greenwald (2018) study the effects of collateral constraints on demand dynamics. Finally, Kaplan, Mitman, and Violante (2020) develop a large-scale quantitative model that incorporates housing collateral constraints and deviations from rational expectations in house price beliefs to investigate the drivers of the Great Recession. Their analysis highlights the critical role of belief distortions—specifically, deviations from rational expectations—in accounting for the severity of the Great Financial Crisis. While sharing a focus on the macroeconomic implications of non-rational beliefs in housing markets, our approach differs in a key dimension: whereas Kaplan, Mitman, and Violante (2020) impose exogenous belief processes over house prices, our model endogenizes the belief formation.

Outline. The rest of this paper is organized as follows. In Section (3.2) we introduce our general model framework. In Section (3.3), we present our results and the decompositions into the different channels. Finally, Section (3.4) concludes.

3.2 General Model Setup

In this section, we outline the structure of the model. The heterogeneous agent framework builds on the approach developed in Bilbiie (2024). On the household side, the economy comprises two types of agents: borrowers and savers. Borrowers are subject to a collateral constraint that is tied to the value of housing and binds at all times. In contrast, savers correspond to standard Ricardian households who are unconstrained. The production sector consists of monopolistically competitive firms operating under nominal rigidities in the form of price adjustment costs, consistent with the canonical New Keynesian framework

Households. The household block consists of a borrower (h), or HtM, and a saver (s). Borrowers are more impatient than savers, $\beta^s > \beta^h$, and as a result, borrowers will always be on the borrowing constraint, which we define below. Households maximize utility choosing consumption c^i , hours worked n^i , housing h^i , and bonds b^i . The savers can additionally invest into building new housing units, by committing x_t^s consumption units to it, hence $(x_t^h = 0, \forall t)$. Within the group of savers/borrowers, there is perfect insurance, hence, all households within a group make the same decisions. Households

3.2. General Model Setup

stay within their group, and we therefore arrive in a standard TANK setting. $\mathbb{E}_0^{\mathcal{P}}$ denotes the subjective expectations operator, which we will address below in more detail. The utility function is of the usual iso-elastic form:

$$\mathbb{E}_0^{\mathcal{P}} \sum_{t=0}^{\infty} (\beta^i)^t U(c_t^i, n_t^i, h_t^i), \quad U(c_t^i, n_t^i, h_t^i) = \frac{\xi_c^i (c_t^i)^{1-\sigma}}{1-\sigma} + \frac{\xi_h^i (h_t^i)^{1-\nu}}{1-\nu} - \frac{\xi_n^i (n_t^i)^{1+\varphi}}{1+\varphi} \quad (3.1)$$

Households consume, borrow or lend in bonds, buy housing at the price q_t , which is subject to a quadratic adjustment cost, receive/pay interest on bonds, receive income, receive returns on their housing investment if they are savers, and finally receive taxes and transfers. The budget constraint reads:

$$c_t^i + b_{t+1}^i + q_t[h_t^i - (1 - \delta^i)h_{t-1}^i] + \kappa_H^i (h_t^i - h_{t-1}^i)^2 + x_t^i = (1 + r_t)b_t^i + w_t n_t^i + q_t f(x_t^i) + \Sigma_t^i + T_t^i \quad (3.2)$$

$f(x_t^i) = \xi_x \eta^{-1} x_t^\eta$ is the housing production function. We make the same assumption regarding profits and taxes as Bilbiie (2024). Only savers receive profits, which are then taxed and redistributed to borrowers. The taxation schedule is chosen such that counter-cyclical income risk arises, which is the empirically plausible case. Households are subject to a borrowing constraint which is given by:

$$-b_{t+1}^i \leq q_t h_t^i \phi_t \quad (3.3)$$

ϕ_t is exogenous and can be thought of as an exogenous loan to (housing) value constraint.

House price beliefs. Our formulation of house price beliefs follows the setup proposed in Roschitsch and Twieling (2024). As is standard in the literature on capital gains extrapolation (e.g., Adam and Marcet, 2011; Adam, Marcet, and Beutel, 2017), households are endowed with a subjective probability measure over the entire sequence of variables they perceive as exogenous, henceforth referred to as external variables, denoted by $(r_t, w_t, \Sigma_t^i, T_t^i, \pi_t, q_t)_{t \geq 0}$. We denote this belief system by \mathcal{P} . The rational expectations benchmark corresponds to the special case in which this belief measure coincides with the objective (or “true”/equilibrium-implied) distribution over external variables, i.e., $\mathcal{P} = \mathbb{P}$.

While households may hold beliefs that deviate from the equilibrium-implied distribution, two key features are worth emphasizing. First, agents possess a time-consistent

belief system. Second, they make decisions optimally given these beliefs. In this sense, agents are internally rational, as formalized by Adam and Marcet (2011).⁵

Conditional on the observed history of external variables up to period t , households use their belief system \mathcal{P} to form expectations about the future evolution of these variables. We denote this subjective expectations operator by $\mathbb{E}_t^{\mathcal{P}}$, while the standard conditional rational expectations operator is denoted by \mathbb{E}_t . In our framework, agents are assumed to have rational expectations with respect to all external variables except for house prices, q_{t+s} .⁶ Agents are assumed to entertain a simple state-space model for the evolution of house prices:

$$\begin{aligned} \ln \frac{q_{t+1}}{q_t} &= \ln m_{t+1} + \ln e_{t+1} \\ \ln m_{t+1} &= \varrho \ln m_t + \ln v_{t+1}, \quad \varrho \in (0, 1) \\ (\ln e_t \quad \ln v_t)' &\sim \mathcal{N} \left(\begin{pmatrix} -\frac{\sigma_e^2}{2} & -\frac{\sigma_v^2}{2} \end{pmatrix}, \begin{pmatrix} \sigma_e^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix} \right) \end{aligned} \quad (3.4)$$

Equation (3.4) states that agents perceive house price growth rates to be the sum of a transitory and a persistent component. The disturbances $\ln e_t$ and $\ln v_t$ are not observable to the agents, rendering $\ln m_t$ unobservable. Agents apply the optimal Bayesian filter, i.e. the Kalman filter, to arrive at the observable system.⁷

Lemma 3 (House price belief updating). *Applying the Kalman filter to the state-space model and log-linearizing around the non-stochastic steady state gives:*

$$\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s} = \hat{q}_t + \frac{1 - \varrho^s}{1 - \varrho} \varrho \hat{m}_t \quad (3.5)$$

$$\hat{m}_t = (\varrho - g) \hat{m}_{t-1} + g \Delta \hat{q}_{t-1} \quad (3.6)$$

where $\ln \bar{m}_t := \mathbb{E}_t^{\mathcal{P}}(\ln m_t)$ is the posterior mean, $g = \frac{\sigma^2 + \sigma_v^2}{\sigma^2 + \sigma_v^2 + \sigma_e^2}$ is the steady state Kalman

⁵Moreover, it is not common knowledge among agents that all households share identical beliefs and preferences. As a consequence, agents are unable to uncover potential misspecifications in their beliefs—that is, the discrepancy $\mathcal{P} \neq \mathbb{P}$ cannot be resolved through deductive reasoning about the structure of the economy.

⁶Formally, $\mathcal{P} := \mathbb{P}_{-q} \otimes \mathcal{P}_q$, where \mathbb{P}_{-q} is the objective measure over sequences of external variables without house prices, \mathcal{P}_q is the measure over sequences of house prices implied by the described perceived model of house prices, and \otimes is the product measure. Since we are interested in a first-order solution to the model, it does not matter what households perceive to be the dependence structure between house prices and the other external variables.

⁷We assume agents' prior variance equals the steady state Kalman variance.

3.2. General Model Setup

filter gain, $\sigma^2 = \frac{1}{2}[-\sigma_v^2 + \sqrt{\sigma_v^4 + 4\sigma_v^2\sigma_e^2}]$ is the steady state Kalman filter uncertainty, and $\ln \hat{e}_t$ is perceived to be a white noise process.

Proof. See Appendix (3.A) for the application of the Kalman filter. Log-linearization around the steady state gives the result. \square

Variables denoted with " $\hat{\cdot}$ " represent percentage deviations from their steady state values. Equation (3.5) shows that expected future house prices depend on current prices and beliefs. The parameter $\varrho \in (0,1)$ captures belief persistence, implying that the weight on prior beliefs increases with the forecast horizon. Current house prices enter one-to-one into expectations, reflecting extrapolative behavior. The belief-updating equation features an autoregressive component and adjusts based on observed past house price changes. Updating is more responsive when the Kalman gain g is high and belief persistence ϱ is low.

Firms and price setting. We assume a continuum of monopolistically competitive firms that produce intermediate good varieties and have the same beliefs as households. Firm beliefs, however, concern only variables over which households have rational expectations. Therefore, firms are rational. Firm j buys labor $n_t(j)$ from the representative labor packer and produces the variety $y_t(j)$ with a linear technology where labor is the only production factor. The firm sets its retail price $P_t(j)$ and maximizes the expected discounted stream of profits, subject to Rotemberg-type adjustment costs. The log-linearized Phillips-Curve is given by:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\epsilon - 1}{\kappa} \hat{w}_t \quad (3.7)$$

steady state, Market clearing and Equilibrium. To solve the model, we take a first-order approximation around the non-stochastic steady state. This steady state is equivalent under RE and SE. We ensure that the steady state is efficient in terms of the production side by including a firm subsidy, as is standard in the literature.

In equilibrium, labor, goods, and housing markets need to clear. Further, the monetary authority sets the nominal interest rate, i_t , according to a standard Taylor rule targeting only inflation, which includes a monetary policy (MP) shock ϵ_t^{mp} . The log-linearized formulation is given by:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \hat{\epsilon}_t^{MP}$$

Finally, the definition of the equilibrium is as follows:

Definition 2 (Internally Rational Expectations Equilibrium). *An IREE consists of three bounded stochastic processes: shocks $(\phi_t, \epsilon_t^{mp})_{t \geq 0}$, allocations $(c_t^i, b_t^i, h_t^i, n_t^i, x_t^s)_{i=s,h}$ and prices $(w_t, q_t, i_t, [P_t(j)]_{j \in [0,1]})$, such that in all t*

1. *households choose $(c_t^i, b_t^i, h_t^i, n_t^i, x_t^s)_{i=s,h}$ optimally, given their beliefs \mathcal{P} ,*
2. *firms choose $([P_t(j)]_{j \in [0,1]})$ optimally, given their beliefs \mathcal{P} ,*
3. *the monetary authority acts according to a certain rule,*
4. *markets for consumption good varieties, hours, and housing clear given the prices.*

3.3 Results

We report results for three different model cases: first, the RANK case, second, the TANK case without fire sales, and third, the TANK case with fire sales.

3.3.1 Model calibration

Before we turn to the results, we briefly discuss the calibration, which is summarized in table (3.3.1). Our calibration reflects a quarterly frequency, and we set the savers' discount factor accordingly. The HtM discount factor is chosen below the savers' discount factor to ensure that the borrowing constraint binds. The Frisch elasticity, the intertemporal elasticity of substitution, the housing utility elasticity, housing depreciation, the HtM share, and the Taylor rule coefficient are all standard parameter choices in line with the literature. We choose a steady state LTV ratio of 0.4. The LTV ratios on origination are between 0.8 and 1.0 (Greenwald, 2018). Our lower value reflects that not all HtM have mortgages and that some have been partially paid back. The taxation of saver profits is chosen such that the model generates countercyclical inequality as in Bilbiie (2024). Regarding the production side, we chose the price adjustment cost and elasticity of substitution across goods to target the slope of the Phillips Curve in Bilbiie (2024). The housing production elasticity is set as in Adam, Pfäuti, and Reinelt (2022).

3.3. Results

Table 3.3.1: Model parameters

Parameter	Value	Description	Source/ Target	
Households	φ^i	1.500	inverse Frisch elasticity	standard
	σ^i	2.000	inverse of intertemporal EOS	standard
	ν^i	1.000	housing utility elasticity	Iacoviello (2005)
	δ^i	0.010	housing depreciation	1% quarterly depreciation
	β^s	0.995	discount factor	standard for quarterly frequency
	β^h	0.500	discount factor	sufficiently low for LTV to bind
	ϕ	0.400	steady state LTV	calibrated to match average LTV
	λ	0.350	HtM share	standard
	τ^d	0.350	Tax on firm profits	s.t. countercyclical inequality
Production	ϵ	6.000	EOS across varieties	slope of PC as in Bilbiie (2024)
	κ	250.00	price adjustment costs	
	η	0.800	elasticity of housing production	Adam, Pfäuti, and Reinelt (2022)
Policy	ϕ_π	1.500	Taylor coefficient	standard

Notes: One period in the model is one quarter.

Concerning the steady state, we assume that there is no steady state inequality in consumption. This ensures that the aggregate housing stock, output, labor supply, and house prices are equivalent in the RANK and TANK model. Concerning housing, the HtM hold 15% of housing in per capita terms in the steady state. Following Adam, Pfäuti, and Reinelt (2022), the ratio of housing investment to aggregate consumption is 6%. The steady state wage is 1. The aggregate housing stock is roughly one quarter of annual output.

3.3.2 The RANK case

We start with the standard RANK case. To do so, we simply set the share of the HtM to zero ($\lambda = 0$). We also assume that the housing adjustment cost is zero ($\kappa_H^s = 0$). It is important to note that the savers' block, described in this section, remains the same across the different model versions we present, and therefore, the decompositions also remain valid.

Model solution. Solving this type of model under SE is not straightforward. Specifically, we have assumed that households do not have rational expectations about house prices and, in turn, provided them with a law of motion on how to form beliefs. At the same time, future household choices themselves are functions of the house price. Therefore, one needs to characterize these future expected internal variables as functions of future prices.⁸ To solve the model, we employ the method developed in Roschitsch and Twieling (2024). In the following, we provide a brief summary: under SE, the consumption Euler equation takes the following form:

$$\hat{c}_t^s = \sum_{s=0}^{\infty} \hat{r}_{t+s} + \mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty} \quad (3.8)$$

$\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}$ is the subjective expectations wealth effect. It captures the household's expectations of future consumption in the far future as $s \rightarrow \infty$. Intuitively, assume that a transitory shock raises house prices today. Due to extrapolation, households believe that house prices will further rise in the future, which will lead them to believe that they will be able to consume more in the limit, hence $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}$ increases. Under RE this wealth effect is always zero, because agents understand that the economy will transition back to the steady state eventually.

It can be shown that $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}$ has an analytical formulation depending only on external variables. Furthermore, the knowledge of $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}$ is sufficient to characterize all internal variables in expectations. This representation therefore allows us to solve the model under SE. The system of equations representing the households' first-order conditions after applying the solution method is given in Appendix (3.B).

House price dynamics. Next, we illustrate the house price dynamics under RE and SE. Throughout the paper, we consider a 25 bp contractionary monetary policy shock in the SE model. What essentially matters for the model responses is the path of the real rate. To ensure that the RE and SE model are comparable, we always consider a 25 bp shock in the SE model, compute the path of the real rate, and then feed this path into the RE model.⁹ This type of shock is well understood empirically and is well defined across all versions of our model. The house price response to this shock can be

⁸In our framework, $\mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+s}^s$ and $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}^s$ enter directly into the households' first-order conditions. As a result, it is necessary to explicitly characterize these expectations.

⁹This essentially means that in the RE model we are not considering a 25 bp shock but a sequence of shocks that generates the same path of the real rate as in the SE model.

3.3. Results

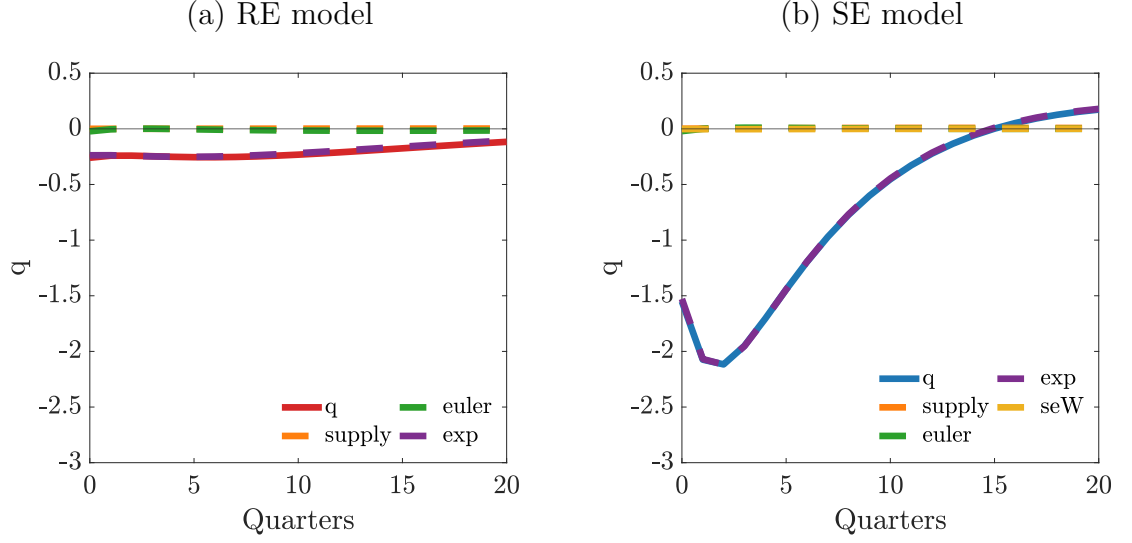
decomposed in the following way:

$$\hat{q}_t = \underbrace{-\mathbb{E}_t \hat{r}_{t+1} - (1 - \bar{\beta}) \sum_{n>0} \mathbb{E}_t \hat{r}_{t+n+1}}_{Euler} - \underbrace{\bar{\nu} \hat{h}_t^s}_{supply} + \underbrace{\bar{\beta} \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+1}}_{expectations} + \underbrace{\sigma(1 - \bar{\beta}) \mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}}_{SE \text{ wealth}} \quad (3.9)$$

This formulation is given by the housing Euler/demand equation from the household problem in which we have defined $\bar{\beta} = \beta^s(1 - \delta)$ and $\bar{\nu} = \nu(1 - \bar{\beta})$. The decomposition consists of four parts: First, an intertemporal or Euler part, which appears because housing is durable and can therefore be used for consumption smoothing. Second, a housing supply part, which states that a more flexible housing supply will lead to less volatility in house prices. Third, if the house prices are expected to increase in the future, that leads to an increase in the price today, reflecting an asset pricing component. And fourth, the subjective expectations wealth effect, which was already discussed above. This part will be zero under RE.

We now turn to the response of house prices to a contractionary monetary policy shock shown in Figure (3.3.1). One can observe that the response in the SE model is roughly four times larger than under RE. This is in line with a large literature documenting that belief extrapolation amplifies the volatility of asset prices. For both models, it is the case that house price expectations drive the majority of the response, although under RE, we observe a small response due to intertemporal substitution once accounting for the small overall responsiveness of the house price.

Figure 3.3.1: House price response to MP shock, decomposition



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

It seems surprising that almost all the variation in house prices is driven by expectations, especially since a large literature empirically documents significant effects from housing supply frictions on house prices.¹⁰ Under RE it is indeed difficult to generate house price volatility through changes in the housing supply side. However, under SE, we can derive an alternative decomposition that does not exist under RE. Specifically, one can solve for the fixed point in the house price by substituting $\mathbb{E}_t^P \hat{q}_{t+1}$ and $\mathbb{E}_t^P \hat{c}_\infty$, which are both functions of the current house price. We arrive at the following decomposition:

$$\hat{q}_t = \frac{1}{1-\bar{Q}} \left[\underbrace{-(1-\bar{\beta})^{-1} \mathbb{E}_t \hat{r}_{t+1} - \sum_{n>0} \mathbb{E}_t \hat{r}_{t+n+1}}_{\text{Euler}} - \underbrace{\nu \hat{h}_t^s}_{\text{supply}} + \underbrace{\frac{\bar{\beta}}{1-\bar{\beta}} \varrho \hat{m}_t}_{\text{posterior mean expectations}} + \underbrace{\sigma \mathbb{E}_t^P \tilde{c}_\infty}_{\text{SE wealth}} \right] \quad (3.10)$$

We observe that instead of house price expectations at $t+1$, we now have the posterior mean expectations. In addition, $\mathbb{E}_t^P \tilde{c}_\infty$ denotes the effect of the SE wealth net of house prices. Importantly, this is still a function of the posterior mean expectations \hat{m}_t . Finally, the whole decomposition is scaled by $(1-\bar{Q})^{-1}$ with $\bar{Q} \in (0,1)$. Just by comparing both decompositions, one can see that in equation (3.10) housing supply is likely to play a

¹⁰See for instance: Aastveit and Anundsen (2022), Guren, McKay, Nakamura, and Steinsson (2021), and Mian, Rao, and Sufi (2013)

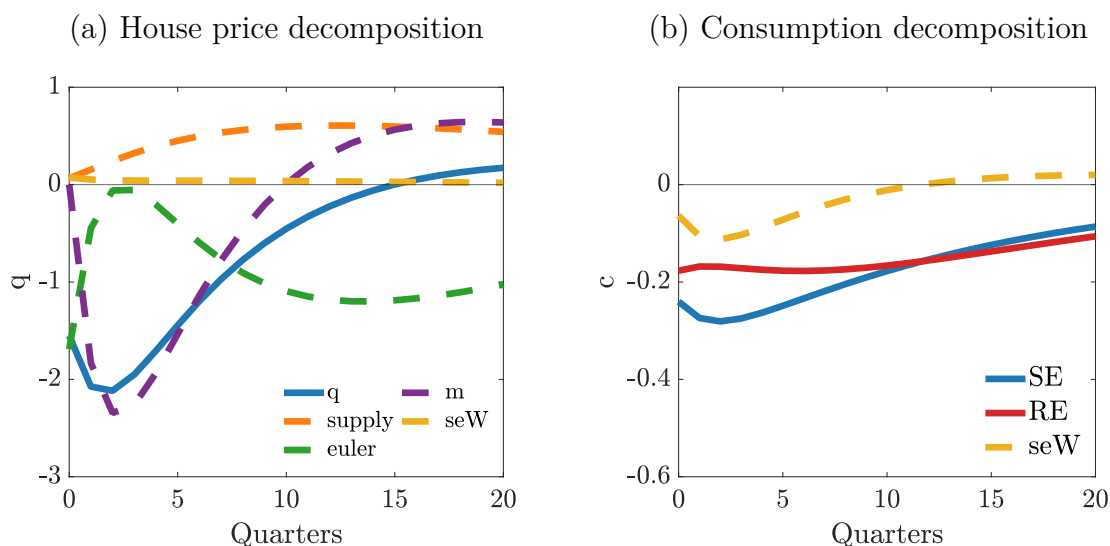
3.3. Results

larger role: in equation (3.9) $\bar{\nu}$ scales the housing supply effect. As β is likely close to one and δ close to zero, $\bar{\nu}$ is fairly small. Under the alternative decomposition in equation (3.10), only ν appears, and therefore the coefficient scaling of housing supply is significantly larger.

This can be seen in panel (a) of Figure (3.3.2). Housing supply plays now a much larger role in balancing the negative response in the other channels, in particular in the long run. This is the case because it takes time for the housing stock to be built up. The posterior expectations respond with a lag to the shock, this is intuitive because they are a backward-looking function of house price growth. Further, the Euler effect is quite important, in particular on impact, which is due to the fact that the coefficient scaling the intertemporal substitution in equation (3.10) is significantly larger. Finally, the SE wealth effect again has only a marginal impact on house prices.

Our findings of both house price decompositions support the conclusion that the SE wealth effect generally plays a minor role. Panel (b) of Figure (3.3.2) shows the consumption decomposition derived in Equation (3.8). It is apparent that the SE wealth effect roughly accounts for one-third of the volatility in consumption for the first few quarters. Therefore, while this effect is negligible for house price responses, it is important for consumption dynamics.

Figure 3.3.2: House price response to MP shock, decomposition



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

Output dynamics. Finally, we can focus on the output response. Given goods market clearing, output is just a function of consumption and investment:

$$\hat{y}_t = \frac{c_{ss}}{y_{ss}} \hat{c}_t^s + \frac{x_{ss}}{y_{ss}} \hat{x}_t^s \quad (3.11)$$

In Figure (3.3.3), we present the decomposition of the output response in both the RE and subjective expectations SE models. The overall output response in the SE framework is nearly four times larger than that observed under RE. This amplification is primarily attributable to the heightened responsiveness of housing investment. From the household optimization problem, we derive the following relationship:

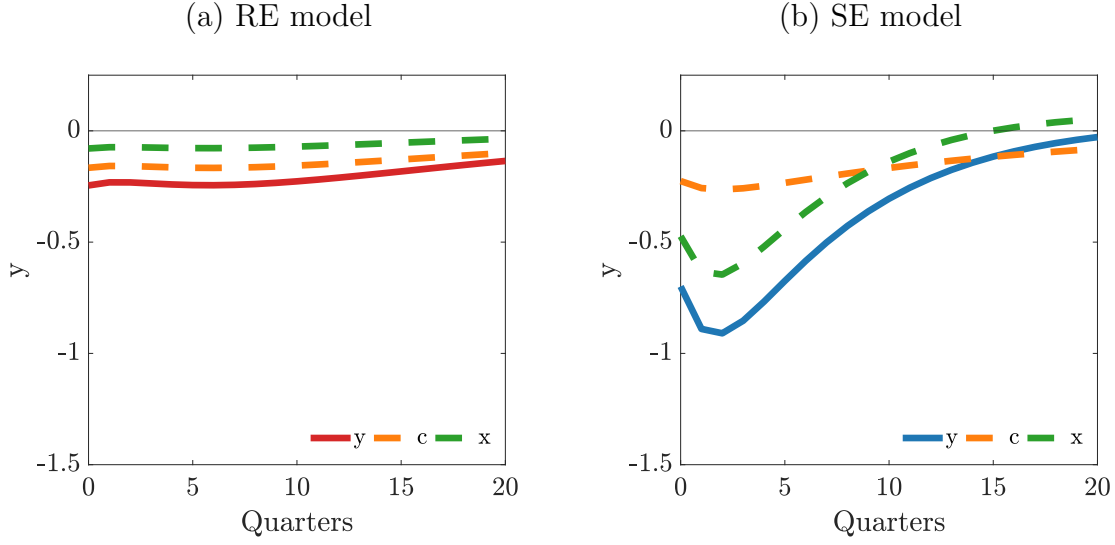
$$\hat{x}_t^s = (1 - \eta)^{-1} \hat{q}_t$$

which indicates that greater volatility in house prices translates directly into amplified fluctuations in housing investment. Consequently, under SE, housing investment emerges as the main driver of output dynamics, whereas under RE, the output response is predominantly driven by consumption. The results under SE are therefore consistent with the empirical evidence documented by Leamer (2007), who emphasizes the central role of housing in the business cycle. In contrast, the RE model fails to replicate this feature, underestimating the cyclical importance of housing. These findings highlight that under subjective expectations housing plays a significantly more prominent role in driving the business cycle.

It is also striking that the output response is quite persistent under RE. The reason is that the real rate response generated by the monetary policy shock under SE and which we feed into the RE model, is itself very persistent. Under RE, the consumption response is simply the sum of the future real interest rate path. If the real rate is persistent, so is the consumption response, and this spills over to the rest of the model.

3.3. Results

Figure 3.3.3: Output response to MP shock, decomposition



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

3.3.3 The TANK case: collateral channel

We now turn to the TANK case of the model with only collateral effects being present. In this model formulation, the share of HtM is $\lambda = 0.35$. Further, we set the housing adjustment costs κ_H^i for savers to zero, and for the HtM we set $\kappa_H^h \rightarrow \infty$. This ensures that only the savers buy or sell housing units in response to shocks hitting the economy, while HtM agents stick with their steady state level of housing. Thereby, we can isolate the collateral effect on the HtM households.

The HtM consumption response is given by their budget constraint, in which we have substituted the labor supply, the borrowing constraint, and market clearing conditions to arrive at the following equation:

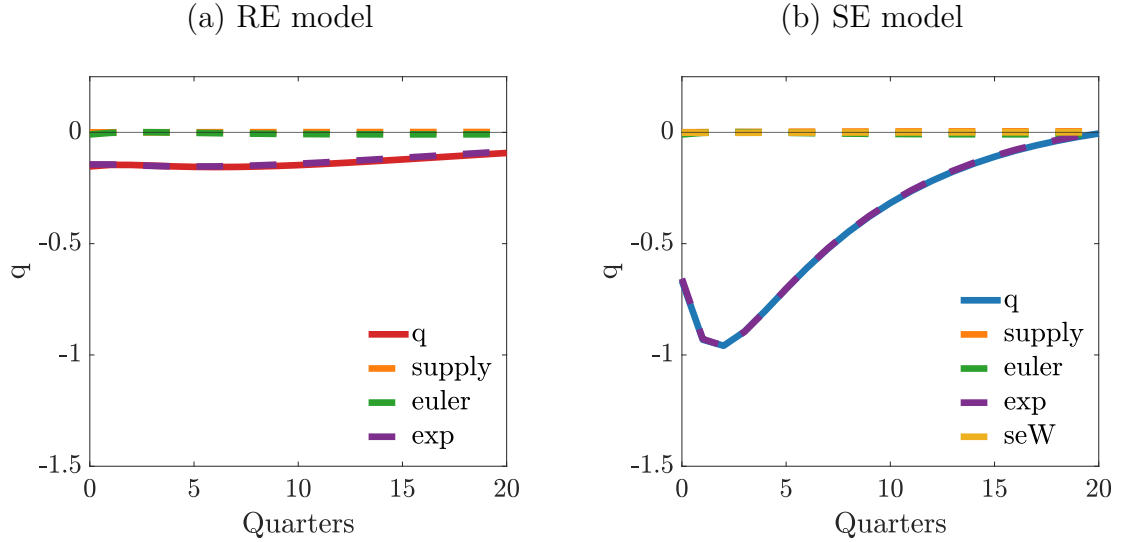
$$\hat{c}_t^h = \underbrace{\chi_1^c \hat{y}_t + \chi_2^c \hat{x}_t}_{GE} + \underbrace{\chi_3^c \hat{q}_t - \chi_4^c (\hat{r}_t + \hat{q}_{t-1})}_{Collateral(\phi)} - \underbrace{\chi_5^c \delta \hat{q}_t}_{depreciation(h)} \quad (3.12)$$

The consumption behavior of HtM households can be decomposed into three components. First, a general equilibrium (GE) component operates through conventional channels: increasing aggregate output and housing investment raise labor demand and hence wages, thereby stimulating consumption. Since $\chi_1^c > 1$ inequality is countercyclical in

the sense of Bilbiie (2024).¹¹ Second, a collateral channel captures the impact of fluctuations in house prices on borrowing constraints. When house prices rise, the relaxation of these constraints enables higher borrowing and thus higher consumption. Importantly, the level of debt incurred also influences future repayment obligations, and higher real interest rates similarly alter the repayment schedule, thus shaping HtM consumption. Finally, there exists a housing depreciation effect, which arises purely for accounting reasons. To maintain their steady state level of housing, HtM households must repurchase a fraction δ of housing each period to offset depreciation. Given the typically small magnitude of δ , this effect is quantitatively negligible.

House price dynamics. As before, we start by focusing on the house price dynamics, which we plot in Figure (3.3.4). The picture that emerges is similar to the RANK case: house prices are much more volatile under SE compared to RE. Again, the decomposition shows that most of the response is driven by house price expectations. It is important to note that the path of the real rate in the TANK model is not the same as in the RANK model. Therefore, one should be cautious with a quantitative comparison between the two models.

Figure 3.3.4: House price response to MP shock, decomposition



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

¹¹For the remaining coefficients we have $\chi_j^c > 0, j \in \{2, 3, 4, 5\}$. For the definitions see Appendix (3.C).

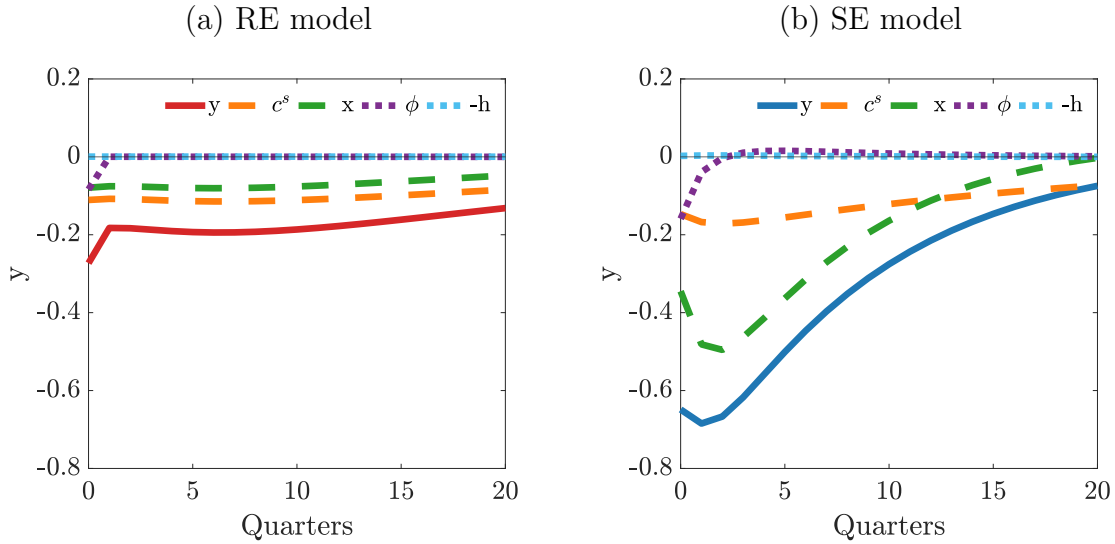
3.3. Results

Output dynamics. In terms of output dynamics, we can derive a similar decomposition as in the RANK case. Equation (3.13) shows that it consists of saver consumption and housing investment, as in RANK. Additionally, the collateral channel and the depreciation parts now also show up.¹²

$$\hat{y}_t = \chi_1^y \hat{c}_t^s + \chi_2^y \hat{x}_t + \underbrace{\chi_3^y \hat{q}_t - \chi_4^y (\hat{r}_t + \hat{q}_{t-1})}_{\text{Collateral}(\phi)} - \underbrace{\chi_5^y \delta \hat{q}_t}_{\text{depreciation}(h)} \quad (3.13)$$

Turning to the responses to the monetary policy shock, we show in Figure (3.3.5) that output is much more responsive under SE. As before, this is mainly driven by the stronger response in housing investment. Under RE, as before, saver consumption dominates the remaining channels. The collateral channel has only a marginal effect and is quite short-lived; this holds for both models. Housing depreciation has no impact, as discussed before.

Figure 3.3.5: Output response to MP shock, decomposition



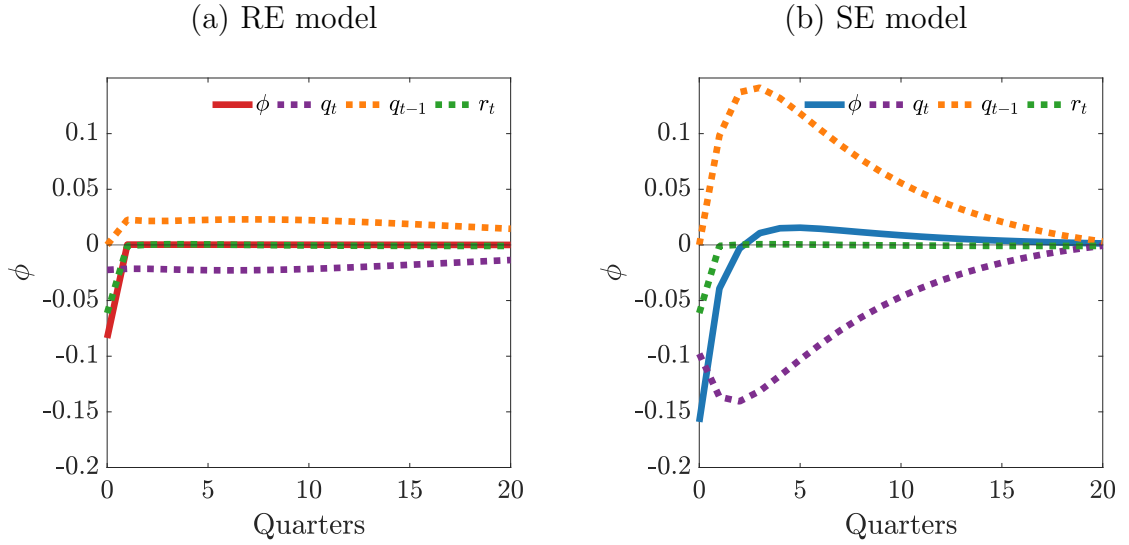
Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

Focusing on the collateral response, we decompose it in Figure (3.3.6) into its sub-components $\chi_3^y \hat{q}_t$, $\chi_4^y \hat{r}_t$, and $\chi_4^y \hat{q}_{t-1}$. One can observe that the initial drop is driven by the increase in the real rate, \hat{r}_t is predetermined, but $\hat{\pi}_t$ will respond, and the decline in the house prices. Following the initial drop, the collateral effect will quickly converge

¹²The coefficients are defined in Appendix (3.C). All coefficients are positive.

towards zero. That is because, as the HtM were able to borrow less initially, they will also have to repay less in the consecutive periods. In other words, as the real rate effect returns towards zero relatively quickly: $\chi_3^y \hat{q}_t$, and $\chi_4^y \hat{q}_{t-1}$ cancel each other out.¹³ Under SE, the collateral response is roughly twice as strong on impact and is more persistent. As the real rate effect is the same in both models, the differential response needs to be driven by the stronger and more hump-shaped response of the house prices under SE.

Figure 3.3.6: Collateral decomposition in response to MP shock



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

3.3.4 The TANK case: fire sale channel

We now turn to our final channel, the fire sale channel. This channel occurs if HtM agents actively participate in the housing market. In our model, this means that $\kappa_H^h = 0$. We call this channel the fire sale channel because, under SE, house prices may fall below their fundamental value. If in this period HtM agents sell housing to stabilize the goods consumption, the sale takes place in an environment where housing is undervalued, and hence a fire sale occurs. Importantly, these sales may lead to a further decline in house prices, which could lead to further downward pressure on output through a decline in housing investment and a tightening of borrowing constraints.

¹³Algebraically this is also since $\chi_4^y = \beta^s \chi_3^y$, and therefore the coefficients are almost the same as β^s is close to one.

3.3. Results

The budget constraint of the HtM, after several substitutions, is given by:

$$\hat{c}_t^h = \underbrace{\chi_1^c \hat{y}_t + \chi_2^c \hat{x}_t}_{GE} + \underbrace{\chi_3^c (\hat{q}_t + \hat{h}_t) - \chi_4^c (\hat{r}_t + \hat{q}_{t-1} + \hat{h}_{t-1})}_{Collateral(\phi)} - \underbrace{\chi_5^c (\delta \hat{q}_t + \hat{h}_t^h - (1 - \delta) \hat{h}_{t-1}^h)}_{fire\ sales(h)} \quad (3.14)$$

As in the TANK model without fire sales, the transmission mechanism operates through both a GE channel and a collateral channel. In the collateral channel, housing purchases now play an active role: increases in housing purchases expand the stock of collateral, thereby enhancing households' borrowing capacity. The final component is the fire sale channel, which enters with a negative sign. Specifically, when HtM households sell housing—i.e., when $\hat{h}_t^h - (1 - \delta) \hat{h}_{t-1}^h$ declines—they are able to increase consumption by reallocating resources from housing to consumption goods.

Model solution. In a model in which the HtM are allowed to trade housing, we need to pin down the housing demand of these agents. As for the savers, this is done by the HtM housing Euler/demand equation (3.15). This equation is similar to the one from the savers, only that it now accounts for the binding LTV constraint. Here we have defined $\beta^\Delta = (\beta^s - \beta^h)/\beta^s$. Housing demand depends on consumption today and tomorrow, house prices today and tomorrow, as well as tomorrow's real rate.

$$\bar{\nu} h_t^h = \sigma(1 - \phi) \hat{c}_t^h - \sigma(\bar{\beta}^h - \phi(1 - \beta^\Delta)) \mathbb{E}_t^P \hat{c}_{t+1}^h - (1 - \phi \beta^\Delta) \hat{q}_t + \bar{\beta}^h \mathbb{E}_t^P \hat{q}_{t+1} - \phi(1 - \beta^\Delta) \mathbb{E}_t \hat{r}_{t+1} \quad (3.15)$$

Crucially, the HtM housing Euler/demand equation (3.15) depends on future expected HtM consumption $\mathbb{E}_t^P \hat{c}_{t+1}^h$. Under RE, this is not an issue, and we can continue in the standard way. But, as already discussed in the saver problem, under SE, we need to characterize future expected internal variables as a function of prices. For the saver, we heavily relied on the consumption Euler equation, which allowed us to characterize the SE wealth effect and thereby solve the model. In contrast to the saver, this is not possible for the HtM agent as the consumption Euler equation is not binding.

It is possible to characterize the future expected HtM consumption using the HtM budget constraint at $t + 1$ in which we did not substitute for the market clearing conditions:

$$\begin{aligned} (c_{ss}^h + n_{ss}^h w_{ss} \sigma / \varphi) \mathbb{E}_t^P \hat{c}_{t+1}^h &= \left(n_{ss}^h w_{ss} * \left(\frac{1}{\varphi} + 1 \right) - \frac{\tau^d}{\lambda} \right) \mathbb{E}_t \hat{w}_{t+1} + q_{ss} h_{ss}^h \phi (\mathbb{E}_t^P \hat{q}_{t+1} + \mathbb{E}_t^P \hat{h}_{t+1}) \\ &\quad - \beta^{-1} q_{ss} h_{ss}^h \phi (\mathbb{E}_t \hat{r}_{t+1} + \hat{q}_t + \hat{h}_t) - q_{ss} h_{ss}^h \left(\delta \mathbb{E}_t^P \hat{q}_{t+1} + \mathbb{E}_t^P \hat{h}_{t+1}^h - (1 - \delta) \hat{h}_t^h \right) \end{aligned} \quad (3.16)$$

House Price Extrapolation and Business Cycles: A Two-Agent New Keynesian Approach

However, this equation in turn depends on future expected housing choices of the HtM $\mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+1}^h$, which again depends on $\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+2}^h$. To solve this model, we therefore propose a method relying on lag polynomial factorization as outlined in Proposition (1).

Proposition 1 (Expected HtM consumption under Subjective Expectations). *For any $s > 1$, the HtM budget constraint and the HtM housing Euler/demand equation can be presented in the following lag polynomial formulation:*

$$\wp(\mathbb{L})\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}^h = \mathbb{E}_t Z_{t+s} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s}$$

Where $\wp(\mathbb{L})$ and $Q(\mathbb{L})$ are lag polynomials of order two. $\mathbb{E}_t Z_{t+s}$ is a function of the prices over which the agent holds rational expectations. The solution of the lag polynomials yields the roots:

$$\gamma_1 := \frac{b - \sqrt{b^2 - 4ac}}{2a} \text{ and } \gamma_2 := c/(a\gamma_1).$$

Where a, b, c are the coefficients in $\wp(\mathbb{L})$. Further, the roots are real valued and $0 < \gamma_1 < 1$ and $\gamma_2 > 1$. Expected future consumption is therefore given by:

$$\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}^h = \left(\frac{a\gamma_1}{c}\right)^s \hat{c}_t^h + \sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{s+n-1} [(a\gamma_1)/c]^k (\mathbb{E}_t Z_{t+s-k+n} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s-k+n})$$

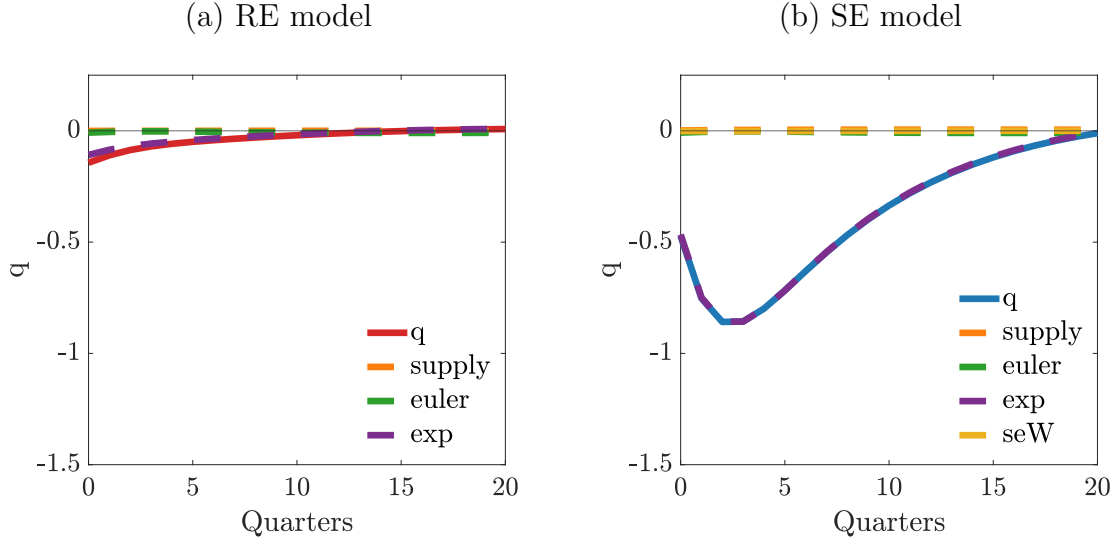
Proof. See Appendix (3.D). □

Proposition (1) illustrates that for any period $t + s$ and $s > 1$ we can derive a representation of future expected HtM consumption that only depends on current HtM consumption \hat{c}_t^h , future expected prices the households have rational expectations about, $\mathbb{E}_t Z_{t+s-k+n}$, and expectations about house prices $\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s-k+n}$. Hence, we can derive a formulation of expected HtM consumption that only depends on current internal and expected external variables. Finally, the HtM choices are characterized by the HtM budget constraint at t and $t + 1$, the HtM housing Euler/demand equation at t and $t + 1$, as well as the HtM consumption expectations formulation derived in Proposition (1) at $t + 2$ ($s = 2$).

House price dynamics. We begin once again by examining the dynamics of house prices as depicted in Figure (3.3.7). Under SE, house price fluctuations are substantially amplified relative to RE. This amplification is primarily driven by the dynamics of house price expectations. These findings closely resemble the results obtained under the RANK and TANK formulation with only the collateral channel.

3.3. Results

Figure 3.3.7: House price response to MP shock, decomposition



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

Output dynamics. Turning to the dynamics of output, the decomposition is presented in equation (3.17). In contrast to previous specifications, this decomposition now includes a fire-sale channel. Analogous to the decomposition of HtM consumption, housing sales by HtM households can serve as a stabilizing force for aggregate output. The intuition is that the HtM will use the proceeds of the sale to consume, which eventually stabilizes output.

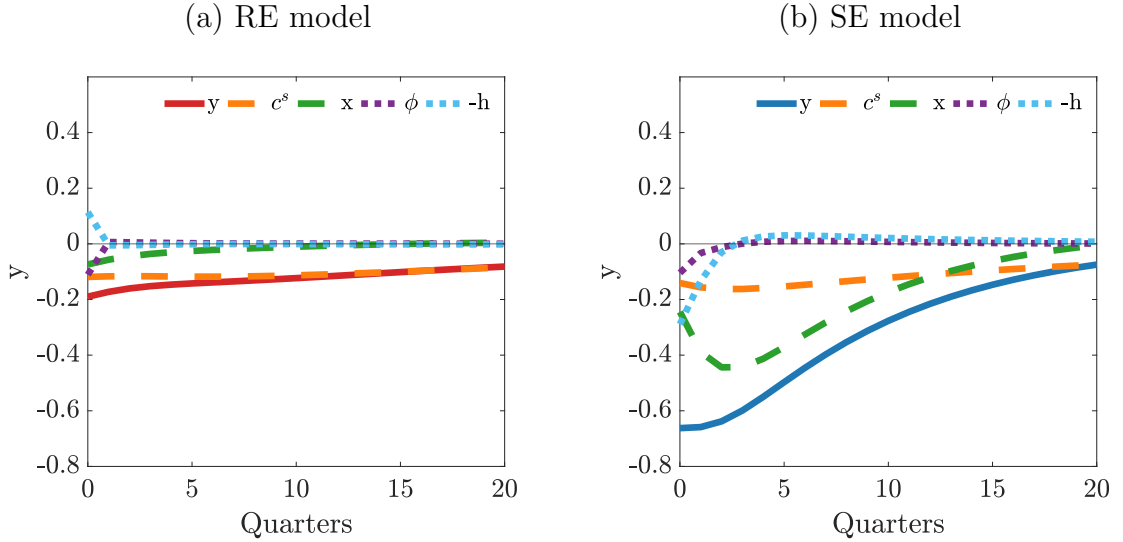
$$\hat{y}_t = \chi_1^y \hat{c}_t^s + \chi_2^y \hat{x}_t + \underbrace{\chi_3^y (\hat{q}_t + \hat{h}_t^h) - \chi_4^y (\hat{r}_t + \hat{q}_{t-1} + \hat{h}_{t-1}^h)}_{\text{Collateral}(\phi)} - \underbrace{\chi_5^y (\delta \hat{q}_t + \hat{h}_t^h - (1 - \delta) \hat{h}_{t-1}^h)}_{\text{fire sales}(h)} \quad (3.17)$$

The output decomposition is shown in Figure (3.3.8). As before in the SE model, the output response is largely amplified. Similarly, in the SE model, this amplification is driven by housing investment while saver consumption is relatively muted. Further, under RE saver consumption dominates housing investment as already discussed above. The collateral channel, as before, operates only in the short term. Relative to the output response, it only plays a minor role under SE but a larger role under RE.

Turning to the fire sale channel, under RE, HtM households sell housing in order to stabilize consumption, thereby contributing to the stabilization of aggregate output. Moreover, the stronger impact of the collateral channel can also be attributed, at least

in part, to these housing sales, which tighten borrowing constraints. Under SE the opposite is the case, the HtM buy housing, exerting downward pressure on output, but stabilizing house prices. In both cases, under RE and SE, the effect is relatively short-lived but relative to the output response sizable.

Figure 3.3.8: Output response to MP shock, decomposition



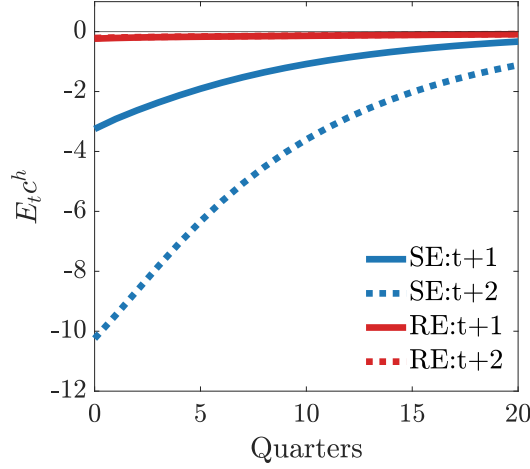
Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

The dynamics of the fire sale channel in the SE model seem surprising. One would expect the HtM, who have a high marginal propensity to consume (MPC), to sell housing in order to stabilize consumption and not the opposite. Based on equation (3.15), this type of behavior could be explained through one of the following dynamics: First, real rates strongly decrease, which we can rule out. Second, accounting for their relative coefficients, house price expectations react by less than the current house price. Third, again accounting for the relative coefficients, HtM consumption expectations drop by more than the current HtM consumption. The latter turns out to be the main driving force.

Figure (3.3.9) plots the consumption expectations one and two periods ahead. While under RE, the drop in both expectations is relatively small, under SE the drop is large for the $t+1$ expectations, but almost increases by a factor of four for expectations about $t+2$. This sharp decline in future expected consumption leads HtM households to buy housing to mitigate future consumption losses.

3.3. Results

Figure 3.3.9: HtM consumption expectations in response to MP shock



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

This raises the question of why HtM households' consumption expectations exhibit such pronounced sensitivity. Recall that Proposition (1) is used to derive $\mathbb{E}_t^P \hat{c}_{t+2}^h$, where the representation explicitly incorporates rational expectations about future wages, as captured by the term $\mathbb{E}_t Z_{t+s-k+n}$. Given the MPCs characteristic of HtM households, their consumption is highly responsive to fluctuations in income, both in realized and expected terms. As demonstrated in Appendix (3.D), the expression for $\mathbb{E}_t^P \hat{c}_{t+2}^h$ includes an infinite sum of rational expectations over future wages embedded in the $\mathbb{E}_t Z_{t+s-k+n}$ component. Due to the structure of the HtM environment, future wages receive substantial weight in this summation, thereby amplifying the sensitivity of consumption expectations.

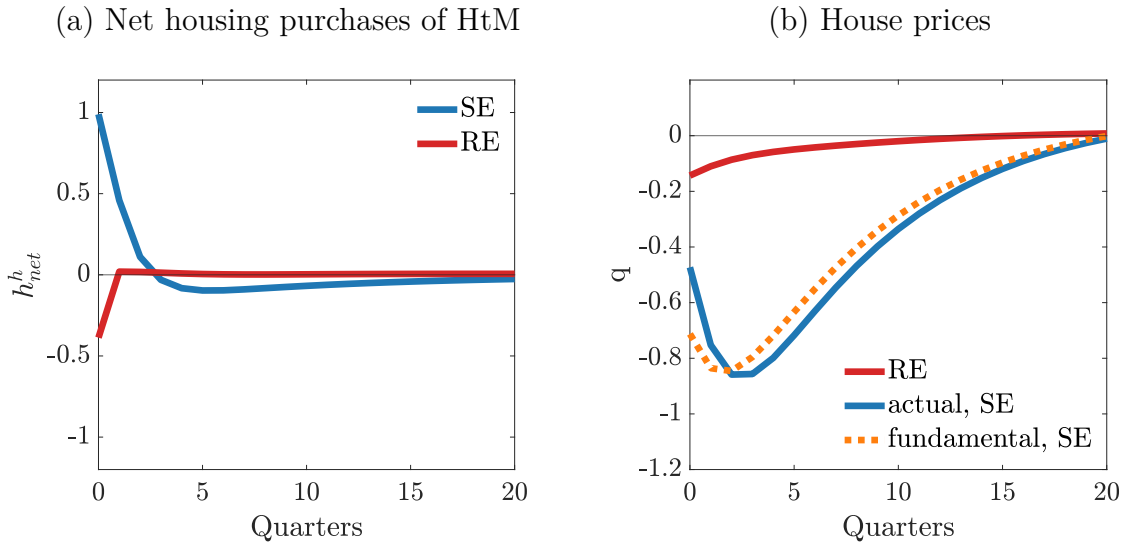
To mitigate this excessive sensitivity, one could consider model extensions in which wage dynamics are less volatile, agents hold SE over future income, or additional constraints, such as payment-to-income limits.

Finally, we want to return to the notion of fire sales. In the SE model, it is theoretically possible to generate this phenomenon. To understand whether they occur in our example, we plot the HtM net purchases of housing in panel (a) of Figure (3.3.10). It shows the housing stock net of depreciation of the HtM, i.e., if it is positive, the HtM purchase housing relative to the steady state. Panel (b) plots the house prices, and for the SE model, the fundamental house prices. This is the house price that would arise if the

value of housing is evaluated using the actual laws of motion, contrary to the perceived laws of motion.

Within this framework, a fire sale arises when HtM households engage in housing sales while the actual house price lies below its fundamental value. Turning to the figure, we observe that in the initial period under SE, HtM households purchase housing, despite the fundamental price being lower than the prevailing market price. This implies that HtM households are acquiring housing at overvalued prices. After approximately three periods, the behavior reverses: HtM households begin to sell housing while the fundamental price exceeds the actual price. It is during these periods that fire sales materialize, as housing is sold at a discount relative to its fundamental valuation.

Figure 3.3.10: Fire sales in response to MP shock



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

3.4 Conclusion

In this paper, we investigate the interaction between housing and business cycles within a tractable heterogeneous agent New Keynesian model with extrapolative house price beliefs. In this model, the interaction between housing and business cycles can be broken down into consumption, housing investment, collateral effects, and fire sales. We show that in the SE model, output is much more volatile, which is largely driven by residential investment. Under RE, consumption dominates housing investment in every model type

3.4. *Conclusion*

we investigate. Finally, we propose a new method on how to solve TANK models with housing trade under extrapolative house price beliefs.

Appendix

3.A Proof of Lemma (3)

Agents apply the optimal Bayesian filter, i.e. the Kalman filter, to arrive at the observable system:¹⁴

$$\begin{aligned}\ln \frac{q_{t+1}}{q_t} &= \varrho \ln \bar{m}_t + \ln \hat{e}_{t+1} \\ \ln \bar{m}_t &= \varrho \ln \bar{m}_{t-1} - \frac{\sigma_v^2}{2} + g \cdot \left(\ln \hat{e}_t + \frac{\sigma_e^2 + \sigma_v^2}{2} \right)\end{aligned}$$

where $\ln \bar{m}_t := \mathbb{E}_t^{\mathcal{P}}(\ln m_t)$ is the posterior mean, $g = \frac{\sigma^2 + \sigma_v^2}{\sigma^2 + \sigma_v^2 + \sigma_e^2}$ is the steady state Kalman filter gain, $\sigma^2 = \frac{1}{2}[-\sigma_v^2 + \sqrt{\sigma_v^4 + 4\sigma_v^2\sigma_e^2}]$ is the steady state Kalman filter uncertainty, and $\ln \hat{e}_t$ is perceived to be a white noise process.

To avoid simultaneity in the house price we modify the belief setup following Adam, Marcet, and Beutel (2017).¹⁵ We obtain the same observable system but with lagged information being used in the posterior mean updating equation:

$$\ln \bar{m}_t = (\varrho - g) \left(\ln \bar{m}_{t-1} - \frac{\sigma_v^2}{2} \right) + g \left(\ln \frac{q_{t-1}}{q_{t-2}} + \frac{\sigma_e^2}{2} \right). \quad (3.18)$$

¹⁴We assume agents' prior variance equals the steady state Kalman variance.

¹⁵ q_t appears twice: in the forecast equation, and in the Kalman-updating Equation through $\ln \hat{e}_t$. Since q_t depends on \bar{m}_t , but the latter also depends on the former, it is not assured that at any point an equilibrium asset price exists and whether it is unique. See Adam, Marcet, and Beutel (2017) for the details. The idea of the modification is to alter agents' perceived information setup in that they observe each period one component of the lagged transitory price growth.

3.B. Equilibrium equations saver block under SE

Under this formulation, the posterior mean is pre-determined. We may now derive the posterior mean on the $s > 0$ periods ahead of price:

$$\mathbb{E}_t^{\mathcal{P}} q_{t+s} = q_t \cdot \exp \left(\ln \bar{m}_t \cdot \varrho \frac{1-\varrho^s}{1-\varrho} + \frac{1}{2} \sigma^2 \left(\varrho \frac{1-\varrho^s}{1-\varrho} \right)^2 \right) \cdot \exp(V), \quad V \propto \sigma_v^2 \quad (3.19)$$

This completes the proof.

3.B Equilibrium equations saver block under SE

The saver's decisions at an arbitrary calendar date t up to first order around the deterministic steady state is:

$$\begin{aligned} (h) \quad \hat{h}_t &= -\frac{\sigma}{\nu} \frac{\frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_t \{\hat{r}_{t+n}\} - \bar{\beta} \frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_t \{\hat{r}_{t+n+1}\}}{1-\bar{\beta}} - \frac{1}{\nu} \hat{q}_t + \frac{1}{\nu} \frac{\bar{\beta}}{1-\bar{\beta}} \varrho \hat{m}_t + \frac{\sigma}{\nu} \mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}, \\ (n) \quad \varphi \hat{n}_t + \sigma \hat{c}_t &= \hat{w}_t, \\ (b) \quad \hat{c}_t &= -\frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_t \{\hat{r}_{t+n}\} + \mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty}, \\ (x) \quad \hat{q}_t &= (1-\eta) \hat{x}_t, \end{aligned} \quad (3.20)$$

For notational ease, we have dropped the superscript s in the savers' choices. The SE wealth effect is given by,

$$\begin{aligned} \mathbb{E}_t^{\mathcal{P}} \hat{c}_{\infty} &= \frac{\delta q_{ss} h_{ss} / \nu}{c_{ss} + \sigma / \varphi \cdot n_{ss} w_{ss} + \delta q_{ss} h_{ss} \sigma / \nu} \cdot \left[\hat{q}_t + \hat{m}_t \cdot \frac{\varrho}{1-\varrho} \underbrace{\left(1 + \frac{1-\bar{\beta}\varrho}{1-\bar{\beta}} \frac{1-\varrho-\delta}{1-\beta\varrho} \frac{1-\beta}{\delta} \right)}_{> 0 \forall \beta, \delta, \varrho \in (0,1)} \right] \\ &+ \frac{y_{ss}}{c_{ss} + \sigma / \varphi \cdot n_{ss} w_{ss} + \delta q_{ss} h_{ss} \sigma / \nu} \frac{1-\beta}{\beta} \cdot \left[\sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \{z_{t+n}^*\} + \hat{b}_{t+1} \right]. \end{aligned}$$

And we have defined the following auxiliary variables and assumed that the savers' transfers are zero outside of the steady state:

$$\begin{aligned} z_{t+s}^* &:= \frac{w_{ss} n_{ss}}{y_{ss}} (1 + 1/\varphi) \hat{w}_{t+s} + \hat{\Sigma}_{t+s} - \frac{c_{ss} + n_{ss} w_{ss} \sigma / \varphi}{y_{ss}} \hat{c}_{t+s}^* - \frac{q_{ss} h_{ss}}{y_{ss}} [\hat{h}_{t+s}^* - (1-\delta) \hat{h}_{t+s-1}^*], \\ \hat{h}_{t+s}^* &:= -\frac{\sigma}{\nu} \frac{\frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n}\} - \bar{\beta} \frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n+1}\}}{1-\bar{\beta}}, \\ \hat{c}_{t+s}^* &:= -\frac{1}{\sigma} \sum_{n \geq 1} \mathbb{E}_{t+s} \{\hat{r}_{t+s+n}\} \end{aligned}$$

3.C Definition of coefficients for decompositions

In this section, we define the coefficients in the consumption and output decompositions. The decompositions are derived by substituting the labor supply equation, goods-, and labor market clearing into the budget constraint of the HtM and solving for consumption. All coefficients, for consumption and output, are positive under our calibration. The definitions for coefficients in the consumption decompositions are given by:

$$\begin{aligned}\chi_1^c &= (1 + \varphi(n_{ss}^h w_{ss} - \frac{\tau^d}{\lambda})(\varphi + y_{ss}\sigma)/(\varphi + \sigma)) \\ \chi_2^c &= (1 - \varphi(n_{ss}^h w_{ss} - \frac{\tau^d}{\lambda})(1 - \lambda)\sigma x_{ss}^s) \\ \chi_3^c &= \frac{\varphi}{\varphi + \sigma} q_{ss} h_{ss}^h \phi \\ \chi_4^c &= \frac{\varphi}{\varphi + \sigma} q_{ss} h_{ss}^h \phi / \beta^s \\ \chi_5^c &= \frac{\varphi}{\varphi + \sigma} q_{ss} h_{ss}^h\end{aligned}$$

The definitions for coefficients in the output decompositions are given by:

$$\begin{aligned}\chi_1^y &= \frac{1 - \lambda}{\lambda} / (y_{ss}\lambda^{-1} - \chi_1^c) \\ \chi_2^y &= (\lambda^{-1} - (n_{ss}^h w_{ss} - \frac{\tau^d}{\lambda})\varphi\sigma + \sigma)(1 - \lambda)x_{ss}^s / (y_{ss}\lambda^{-1} - \chi_1^c) \\ \chi_3^y &= \chi_3^c / (y_{ss}\lambda^{-1} - \chi_1^c) \\ \chi_4^y &= \chi_4^c / (y_{ss}\lambda^{-1} - \chi_1^c) \\ \chi_5^y &= \chi_5^c / (y_{ss}\lambda^{-1} - \chi_1^c)\end{aligned}$$

3.D Proof of Proposition (1)

We have that for $s > 1$:

$$\begin{aligned}\bar{\nu}\mathbb{E}_t^{\mathcal{P}} h_{t+s}^h &= \sigma(1 - \phi)\hat{c}_{t+s}^h - \sigma(\bar{\beta} - \phi(1 - \beta^\Delta))\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s+1}^h \\ &\quad - (1 - \phi\beta^\Delta)\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s} + \bar{\beta}\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s+1} - \phi(1 - \beta^\Delta)\mathbb{E}_t \hat{r}_{t+s+1}\end{aligned}$$

3.D. Proof of Proposition (1)

$$\begin{aligned} (c_{ss}^h + n_{ss}^h w_{ss} \sigma / \varphi) \mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}^h &= \left(n_{ss}^h w_{ss} * \left(\frac{1}{\varphi} + 1 \right) - \frac{\tau^d}{\lambda} \right) \mathbb{E}_t \hat{w}_{t+s} + q_{ss} h_{ss}^h \phi (\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s} + \mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+s}) \\ - \beta^{-1} q_{ss} h_{ss}^h \phi (\mathbb{E}_t \hat{r}_{t+s} + \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s-1} + \mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+s-1}) &- q_{ss} h_{ss}^h \left(\delta \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s} + \mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+s}^h - (1 - \delta) \mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+s-1}^h \right) \end{aligned}$$

Plugging the housing demand equation into the HTM budget, rearranging, and expressing in terms of the lag operator yields:

$$\begin{aligned} &\left[c_{ss}^h + n_{ss}^h w_{ss} \sigma / \varphi + q_{ss} h_{ss}^h \bar{\nu}^{-1} \sigma ((1 - \phi)^2 + (\bar{\beta} - \phi)(1 - \beta^\Delta)(1 - \delta - \phi/\beta)) \right. \\ &- q_{ss} h_{ss}^h \bar{\nu}^{-1} \sigma (1 - \phi)(\bar{\beta} - \phi)(1 - \beta^\Delta) \mathbb{L}^{-1} - q_{ss} h_{ss}^h \bar{\nu}^{-1} \sigma (1 - \delta - \phi/\beta)(1 - \phi) \mathbb{L} \left. \right] \mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}^h = \\ &\quad \left(n_{ss}^h w_{ss} * \left(\frac{1}{\varphi} + 1 \right) - \frac{\tau^d}{\lambda} \right) \mathbb{E}_t \hat{w}_{t+s} \\ &- q_{ss} h_{ss}^h \phi \left(\beta^{-1} + \bar{\nu}^{-1} (1 - \beta^\Delta)(1 - \delta - \phi/\beta) \right) \mathbb{E}_t \hat{r}_{t+s} + q_{ss} h_{ss}^h \phi \bar{\nu}^{-1} (1 - \phi)(1 - \beta^\Delta) \mathbb{E}_t \hat{r}_{t+s+1} \\ &\quad + q_{ss} h_{ss}^h \left[\phi - \delta + \bar{\nu}^{-1} \{ (1 - \phi)(1 - \phi\beta^\Delta) + \bar{\beta}(1 - \delta - \phi/\beta) \} \right. \\ &\quad \left. + \{ \phi/\beta + \bar{\nu}^{-1} (1 - \beta^\Delta \phi)(1 - \delta - \phi/\beta) \} \mathbb{L} - \bar{\nu}^{-1} \bar{\beta}(1 - \phi) \mathbb{L}^{-1} \right] \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s} \end{aligned}$$

Hence, the equation has the following form:

$$\wp(\mathbb{L}) \mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}^h = \mathbb{E}_t Z_{t+s} + Q(\mathbb{L}) \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s}$$

We can now define:

$$\begin{aligned} \wp : \mathbb{C} &\rightarrow \mathbb{C} : z \mapsto -az + b - cz^{-1} \\ a &:= q_{ss} h_{ss}^h \bar{\nu}^{-1} \sigma (1 - \delta - \phi/\beta)(1 - \phi) \\ b &:= c_{ss}^h + n_{ss}^h w_{ss} \sigma / \varphi + q_{ss} h_{ss}^h \bar{\nu}^{-1} \sigma ((1 - \phi)^2 + (\bar{\beta} - \phi)(1 - \beta^\Delta)(1 - \delta - \phi/\beta)) \\ c &:= q_{ss} h_{ss}^h \bar{\nu}^{-1} \sigma (1 - \phi)(\bar{\beta} - \phi)(1 - \beta^\Delta) \\ \mathbb{E}_t Z_{t+s} &:= \chi_1^z \mathbb{E}_t \hat{w}_{t+s} - \chi_2^z \mathbb{E}_t \hat{r}_{t+s} + \chi_3^z \mathbb{E}_t \hat{r}_{t+s+1} \\ Q : \mathbb{C} &\rightarrow \mathbb{C} : z \mapsto q_{ss} h_{ss}^h \left[\chi_1^m + \chi_2^m z - \chi_3^m z^{-1} \right] \end{aligned}$$

The polynomial $z \mapsto -\wp(z)z$ has the roots

$$\gamma_1 := \frac{b - \sqrt{b^2 - 4ac}}{2a} \text{ and } \gamma_2 := c/(a\gamma_1).$$

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One can show that $b^2 - 4ac > 0$ and therefore the roots are real valued. Also, given realistic calibrations of the model, $0 < \gamma_1 < 1$ and $\gamma_2 > 1$. Now we can factorize:

$$-\wp(z)z = c(1 - z/\gamma_1)(1 - z \cdot (a\gamma_1)/c)$$

so that

$$\wp(z) = c/\gamma_1(1 - z^{-1}\gamma_1)(1 - z \cdot (a\gamma_1)/c)$$

And we finally get:

$$\wp^{-1}(z) = \gamma_1/c \left(\sum_{s \geq 0} \gamma_1^s z^{-s} \right) \left(\sum_{s \geq 0} ((a\gamma_1)/c)^s z^s \right)$$

Now we can apply this to our expected consumption formulation

$$\begin{aligned} \wp(\mathbb{L})\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}^h &= \mathbb{E}_t Z_{t+s} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s} \\ \iff \frac{c}{\gamma_1}(1 - \gamma_1 \mathbb{L}^{-1})\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}^h &= \frac{a\gamma_1}{c} \frac{c}{\gamma_1}(1 - \gamma_1 \mathbb{L}^{-1})\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s-1}^h + \mathbb{E}_t Z_{t+s} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s} \\ \iff \mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+s}^h &= \left(\frac{a\gamma_1}{c}\right)^s \hat{c}_t^h + \sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{s+n-1} [(a\gamma_1)/c]^k (\mathbb{E}_t Z_{t+s-k+n} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s-k+n}) \end{aligned}$$

Finally, the following system of equations determines the choices of the HTM agent:

$$\bar{\nu} h_t^h = \sigma(1 - \phi)\hat{c}_t^h - (\sigma\bar{\beta} - \phi(1 - \beta^\Delta))\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+1}^h - (1 - \phi\beta^\Delta)\hat{q}_t + \bar{\beta}\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+1} - \phi(1 - \beta^\Delta)\mathbb{E}_t \hat{r}_{t+1}$$

$$\begin{aligned} \bar{\nu}\mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+1}^h &= \sigma(1 - \phi)\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+1}^h - (\sigma\bar{\beta} - \phi(1 - \beta^\Delta))\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+2}^h \\ &\quad - (1 - \phi\beta^\Delta)\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+1} + \bar{\beta}\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+2} - \phi(1 - \beta^\Delta)\mathbb{E}_t \hat{r}_{t+2} \end{aligned}$$

$$\hat{c}_t^h = \chi_1^c \hat{y}_t + \chi_2^c \hat{x}_t + \chi_3^c (\hat{q}_t + \hat{h}_t) - \chi_3^c (\hat{r}_t + \hat{q}_{t-1} + \hat{h}_{t-1}) - \chi_5^c (\delta \hat{q}_t + \hat{h}_t^h - (1 - \delta)\hat{h}_{t-1}^h)$$

$$\begin{aligned} (c_{ss}^h + n_{ss}^h w_{ss} \sigma / \varphi) \mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+1}^h &= \left(n_{ss}^h w_{ss} * \left(\frac{1}{\varphi} + 1 \right) - \frac{\tau^d}{\lambda} \right) \mathbb{E}_t \hat{w}_{t+1} + q_{ss} h_{ss}^h \phi (\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+1} + \mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+1}) \\ &\quad - \beta^{-1} q_{ss} h_{ss}^h \phi (\mathbb{E}_t \hat{r}_{t+1} + \hat{q}_t + \hat{h}_t) - q_{ss} h_{ss}^h \left(\delta \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+1} + \mathbb{E}_t^{\mathcal{P}} \hat{h}_{t+1}^h - (1 - \delta)\hat{h}_t^h \right) \end{aligned}$$

$$\mathbb{E}_t^{\mathcal{P}} \hat{c}_{t+2}^h = \left(\frac{a\gamma_1}{c}\right)^2 \hat{c}_t^h + \sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{n+1} [(a\gamma_1)/c]^k (\mathbb{E}_t Z_{t+2-k+n} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+2-k+n})$$

3.D. Proof of Proposition (1)

Now first derive $\sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{n+1} [(a\gamma_1)/c]^k \mathbb{E}_t Z_{t+2-k+n}$

For $n = 0$: $\mathbb{E}_t Z_{t+2} + \frac{a\gamma_1}{c} \mathbb{E}_t Z_{t+1}$

For $n = 1$: $\gamma_1 \left(\mathbb{E}_t Z_{t+3} + \frac{a\gamma_1}{c} \mathbb{E}_t Z_{t+2} + \left(\frac{a\gamma_1}{c} \right)^2 \mathbb{E}_t Z_{t+1} \right)$

For $n = 2$: $\gamma_1^2 \left(\mathbb{E}_t Z_{t+4} + \frac{a\gamma_1}{c} \mathbb{E}_t Z_{t+3} + \left(\frac{a\gamma_1}{c} \right)^2 \mathbb{E}_t Z_{t+2} + \left(\frac{a\gamma_1}{c} \right)^3 \mathbb{E}_t Z_{t+1} \right)$

...

Hence, we get

$$\frac{a\gamma_1}{c} \times \sum_{n \geq 0} \gamma_1^n \left(\frac{a\gamma_1}{c} \right)^n \times \mathbb{E}_t Z_{t+1} + \sum_{n \geq 0} \gamma_1^n \left(\frac{a\gamma_1}{c} \right)^n \times \mathbb{E}_t Z_{t+2} + \gamma_1 \sum_{n \geq 0} \gamma_1^n \left(\frac{a\gamma_1}{c} \right)^n \times \mathbb{E}_t Z_{t+3} + \dots$$

And finally, we get:

$$\begin{aligned} \sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{n+1} [(a\gamma_1)/c]^k \mathbb{E}_t Z_{t+2-k+n} &= \mathbb{E}_t \tilde{Z}_t + \frac{a\gamma_1}{c} \left(1 - \frac{a\gamma_1^2}{c} \right)^{-1} \mathbb{E}_t Z_{t+1} \\ \mathbb{E}_t \tilde{Z}_t &= \gamma_1 \mathbb{E}_t \tilde{Z}_{t+1} + \left(1 - \frac{a\gamma_1^2}{c} \right)^{-1} \mathbb{E}_t Z_{t+2} \end{aligned}$$

Now consider $\sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{n+1} [(a\gamma_1)/c]^k Q(\mathbb{L}) \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+2-k+n}$

Note that $\mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+s} = \hat{q}_t + \frac{1-\varrho^s}{1-\varrho} \hat{m}_t$. So we have:

$$\begin{aligned} \sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{n+1} [(a\gamma_1)/c]^k q_{ss} h_{ss}^h \left[(\bar{\phi} - \delta) + \bar{\nu}^{-1} (1 + (1 - \delta) \bar{\beta}) \right. \\ \left. - (\beta^{-1} \bar{\phi} + \bar{\nu}^{-1} (1 - \delta)) \mathbb{L} - \bar{\nu}^{-1} \bar{\beta} \mathbb{L}^{-1} \right] \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+2-k+n} \end{aligned}$$

We therefore get:

$$\begin{aligned} &\sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{n+1} [(a\gamma_1)/c]^k q_{ss} h_{ss}^h \left[\chi_1^m - \chi_2^m - \chi_3^m \right] \hat{q}_t \\ &+ \sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{n+1} [(a\gamma_1)/c]^k q_{ss} h_{ss}^h \frac{\varrho}{1-\varrho} \left[\chi_1^m - \chi_2^m - \chi_3^m \right] \hat{m}_t \\ &- \sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{n+1} [(a\gamma_1)/c]^k q_{ss} h_{ss}^h \frac{\varrho}{1-\varrho} \left[\chi_1^m \varrho^{2-k+n} - \chi_2^m \varrho^{1-k+n} - \chi_3^m \varrho^{3-k+n} \right] \hat{m}_t \end{aligned}$$

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First, one can show that

$$\sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{n+1} [(a\gamma_1)/c]^k = \sum_{n \geq 0} \gamma_1^n \times \sum_{n \geq 0} [(a\gamma_1)/c]^n = \frac{1}{1-\gamma_1} \frac{1}{1-a\gamma_1/c}$$

Further, we have that

$$\begin{aligned} \sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{n+1} [(a\gamma_1)/c]^k q_{ss} h_{ss}^h \frac{\varrho}{1-\varrho} & \left[\chi_1^m \varrho^{2-k+n} - \chi_2^m \varrho^{1-k+n} - \chi_3^m \varrho^{3-k+n} \right] = \\ q_{ss} h_{ss}^h \frac{\varrho}{1-\varrho} & \left\{ [(a\gamma_1)/c] \sum_{n \geq 0} \gamma_1^n [(a\gamma_1)/c]^n \left[\chi_1^m \varrho^1 - \chi_2^m \varrho^0 - \chi_3^m \varrho^2 \right] \right. \\ & + \sum_{n \geq 0} \gamma_1^n [(a\gamma_1)/c]^n \times \sum_{j \geq 0} \gamma_1^j \varrho^j \times \left[\chi_1^m \varrho^2 - \chi_2^m \varrho^1 - \chi_3^m \varrho^3 \right] \Big\} = \\ q_{ss} h_{ss}^h \frac{\varrho}{1-\varrho} & \frac{1}{1-a\gamma_1^2/c} \left(\frac{1}{1-\varrho\gamma_1} + \frac{a\gamma_1}{c\varrho} \right) \left[\chi_1^m \varrho^2 - \chi_2^m \varrho^1 - \chi_3^m \varrho^3 \right] \end{aligned}$$

Therefore, we finally have that:

$$\begin{aligned} \sum_{n \geq 0} \gamma_1^n \sum_{k=0}^{n+1} [(a\gamma_1)/c]^k Q(\mathbb{L}) \mathbb{E}_t^{\mathcal{P}} \hat{q}_{t+2-k+n} = & \\ \frac{1}{1-\gamma_1} \frac{1}{1-a\gamma_1/c} q_{ss} h_{ss}^h & \left[\chi_1^m - \chi_2^m - \chi_3^m \right] \hat{q}_t \\ + \frac{1}{1-\gamma_1} \frac{1}{1-a\gamma_1/c} q_{ss} h_{ss}^h \frac{\varrho}{1-\varrho} & \left[\chi_1^m - \chi_2^m - \chi_3^m \right] \hat{m}_t \\ - \frac{1}{1-a\gamma_1^2/c} \left(\frac{1}{1-\varrho\gamma_1} + \frac{a\gamma_1}{c\varrho} \right) q_{ss} h_{ss}^h \frac{\varrho}{1-\varrho} & \left[\chi_1^m \varrho^2 - \chi_2^m \varrho^1 - \chi_3^m \varrho^3 \right] \hat{m}_t \end{aligned}$$

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