

ORIGINAL ARTICLE OPEN ACCESS

Asymmetric Platform Oligopoly

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ABSTRACT

We propose a tractable model of asymmetric platform oligopoly with logit demand in which users from two distinct groups are subject to within-group and cross-group network effects and decide which platform to join. We characterize the equilibrium when platforms manage user access by setting participation fees for each user group. We explore the effects of platform entry, a change of incumbent platforms' quality under free entry, and the degree of compatibility. We show how the analysis can be extended to partial user participation.

JEL Classification: L13, L41, D43

1 | Introduction

Recent decades have seen the emergence of large digital platforms, such as Alphabet, Amazon, Apple, Meta, and Microsoft, that cater to two or more user groups. Some of their activities have been increasingly scrutinized by legislators, competition watchdogs, and regulators. The assessment of competition policy and regulatory interventions requires a framework of oligopolistic platform competition that accommodates platforms of different sizes. What is more, asymmetries are also a common feature in platform markets in which Big Tech is not present. Yet, as Jullien et al. (2021) note, “the literature still lacks a tractable model of platform competition in asymmetric [...] markets.”¹ This article aims to fill this gap by proposing a tractable yet flexible model of asymmetric oligopolistic platform competition.

We model two-sided platforms as firms that bring together users from two groups. Each user cares about the participation of other users in their own group and/or in the other group; for example,

competing software packages are made available to business and private users and each user benefits from improved functionality as the number of other users of the service increases. Every user in the same group obtains an average maximal utility (when network effects play out fully) that is adjusted by the realized network size plus a utility realization of their idiosyncratic taste. Then, each user makes a discrete choice between the different (asymmetric) platforms; in other words, each user single-homes.

Platform competition with single-homing by users of each group is of high theoretical interest because platforms directly compete for users in each group. It formalizes real-world markets when heterogeneous users make a discrete choice between different systems, standards, or applications, and the providers of such offers price discriminate between user groups, as in the example of competing software packages with different offers for business and private users. Another example is competing cloud storage services that are offered to business and private users where network effects arise due to file-sharing possibilities. Yet another

We benefitted from comments by the Editor Gary Biglaiser, two anonymous reviewers, Akifumi Ishihara, Bruno Jullien, Daiki Kishishita, Toshihiro Matsumura, Noriaki Matsushima, Hodaka Morita, Daniel Quint, Nicolas Schutz, Makoto Watanabe, Paul Wegener, Julian Wright, Yusuke Zenny, Junjie Zhou, and several seminar audiences. This article replaces Sato (2021a). Peitz acknowledges support by the German Research Foundation (DFG) through CRC TR 224 (project B05). Sato acknowledges support by the Japan Society for the Promotion of Sciences (JSPS) KAKENHI Grant Number 22K13381.

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is enterprise resource planning softwares (e.g., by Oracle or SAP) that cater to large and small enterprises.

We analyze a multinomial logit demand model augmented by within-group and cross-group network effects. Although, for tractability reasons, most of the theoretical literature assumes linear network effects, we assume that user benefits depend on the logarithm of the sizes of the two user groups; this is a specification widely adopted in the empirical analysis of network effects and platforms (e.g., Ohashi 2003, Rysman 2004, 2007, Zhu and Inasti 2012). In line with our modelling choice, according to practitioners, the incremental benefit of additional users typically declines with the user level; for instance, Chen (2021) writes: "... network effects become less incrementally powerful. In eBay's case, when you search something like 'Rolex vintage daytona,' the product experience (and associated conversion rate) improve dramatically as you add the first few listings. It might even continue with a first few dozen. But you don't need the search to return 1,000 or 5,000 listings ..." (page 256).

Platforms are heterogeneous with respect to their costs and the average value they offer to users (after controlling for network effects). They simultaneously set participation fees for both user groups to maximize own profit. For given fees, users in both groups simultaneously make their participation decision. In the unique interior participation equilibrium, platform market shares have a closed-form solution. A platform's profit function depends on the vector of all the platforms prices for both group; in our setting it can be rewritten as one that depends on two choice variables and their aggregates, which are the sum of the respective choice variables over all platforms. Making use of this structure, we derive three sets of results.

First, we show that there exists an equilibrium in the pricing game and in the case of multiple equilibria, these equilibria are ordered. We also show that in the setting with network goods (i.e., no cross-group network effects) and in the setting with one-sided pricing the equilibrium is always unique.

The second set of results is about equilibrium characterization. In line with earlier work (Armstrong 2006, Tan and Zhou 2021), the fees set by each platform in each group feature a "discount" to attract users in the same or the other group, triggered by within- and cross-group network effects. New to the literature, we establish conditions under which the higher-quality platform sets higher fee for both user groups than a lower-quality platform and conditions under which it does not. We also explore when one subset of platforms subsidizes one user group, whereas another subset subsidizes the other group (and possibly a third subset subsidizes neither).

The third set of results concerns comparative statics in relation to exogenous platform entry, changes in incumbent platforms' quality under endogenous entry, and partial compatibility. Exogenous platform entry necessarily increases user surplus if there are no cross-group network effects. In the presence of cross-group network effects, in our setting, one or both of the user groups benefits from entry; however, it is possible that one of the groups suffers. Platform entry can affect the price structures of incumbent platforms by influencing platform asymmetry and thereby lead to incumbent platforms subsidizing one user group

because of entry. Furthermore, we show by example that platform entry may lead to higher profits of the incumbent platforms. This is because entry may lead incumbent platforms to give up on a large share of users in one group, which then relaxes incumbents' competition for users in the other group and leads to an overall profit increase for the incumbent platforms.

Under endogenous entry, the number of fringe platforms depends on market conditions and the strategic choices of incumbent platforms, such as changes in the quality of their offers for at least one group of users. Under free entry such that some fringe platforms are active, we show that, after a change of quality offered to one or both user groups by one or several incumbent platforms, one of the two user groups is better off, whereas the other group is worse off—his constitutes a strong and novel see-saw property.

With respect to partial compatibility we show that better compatibility in some situations increases and in others decreases user surplus when there are no cross-group network effects. With asymmetric networks, better compatibility is more likely to benefit users by reducing the market power of a larger network. We also address how better compatibility tends to affect the two user groups when they are connected through cross-group network effects.

1.1 | Related Literature

To tackle asymmetric firms, in our analysis we make use of the aggregative game property of our model. Platform competition with two-sided single-homing implies that we cannot resort to a single aggregate in contrast to the oligopoly models analyzed by Anderson et al. (2020) and Nocke and Schutz (2018) as well as the platform models in Anderson and Peitz (2020, 2023). Sato (2021b) uses our framework and shows that market share and profit are not necessarily positively correlated (which is in line with Belleflamme et al. 2022). Anderson and Peitz (2020) consider a competitive bottleneck model with logit demand that can be written as an aggregative game—compared to two-sided single-homing such a model is conceptually simpler because competition plays out on one side only and thus can resort to one aggregate. In our construction, profits can be written as a function of a platform's actions (such that there is a one-to-one relationship between actions and platform fees) and the corresponding aggregates as the sums of the actions over all platforms; thus, we work with a two-dimensional aggregate.

This article contributes to the literature on (two-sided) platform competition. This literature has examined the importance of network effects in platform competition, typically under symmetry—see Jullien et al. (2021) for a review of the literature. Prominent works with two-sided single-homing include Armstrong (2006), Tan and Zhou (2021), and Jullien and Pavan (2019). Armstrong (2006) proposes a model with linear cross-group network effects and two symmetric platforms within a Hotelling setting on each side and examines the pricing implications of cross-group network effects;² Tan and Zhou (2021) examine the welfare property of free entry equilibria in a model with general network effects and symmetric platforms; Jullien and Pavan (2019) examine the pricing implications in duopoly with linear

cross-group network effects when platforms and users face uncertainty about the distribution of users' tastes and derive insights regarding the platforms' information management policies.

Earlier literature focused on platforms catering to a single user group characterized by direct network effects. Contributions within the multinomial logit setting include Anderson et al. (1992) and Starkweather (2003), both of which assumed linear direct network effects. In these settings, there is no explicit solution for the participation game with asymmetric platforms.³ We also contribute to this literature and characterize the unique price equilibrium under asymmetric platform competition in the special case that cross-group network effects are absent.

The article is organized as follows. In Section 2, we present the model. In Section 3, we characterize participation equilibria for any given platform fees and show that there is a unique interior participation equilibrium; we identify this as the unique asymptotically stable participation equilibrium. We write profit functions as functions of two choice variables and their aggregates and express user welfare as a function of the aggregates. The platform pricing game has an equilibrium and all equilibria can be ranked by the surplus of one of the two groups. We establish equilibrium uniqueness in two special cases: in the oligopoly with network goods and under one-sided pricing. In Section 4, we provide characterization results regarding market shares, price-cost margins, and profits. In Section 5, we provide comparative statics results with respect to the set of active platforms (exogenous platform "entry") and incumbent platforms' "quality" under free entry; we also consider partial compatibility (analyzed in more detail in the Online Appendix). In Section 6, we amend our framework in three different ways to allow for partial coverage—that is, some users in each group choose the outside option (details of this analysis are relegated to the Online Appendix). Section 7 concludes. All proofs are relegated to the Online Appendix.

2 | The Platform Oligopoly Model

Consider $M > 1$ platforms competing for users from two groups, A and B . Each platform $i \in \{1, \dots, M\}$ charges a membership or subscription fee $p_i^k \in \mathbb{R}$ to users from group $k \in \{A, B\}$. We consider the game in which, first, platforms simultaneously set fees p_i^A, p_i^B and then a unit mass of users from both groups simultaneously decide which platform to join. We solve for subgame perfect Nash equilibria (applying the selection criterion detailed below). In the following, we describe the platforms' problem and the user demand model.

2.1 | Platforms

Each platform i incurs a constant marginal cost $c_i^k \geq 0$ for serving group- k users. We denote platform i 's number of group- k users by n_i^k and the vector of fees charged to group k by $p^k = (p_1^k, \dots, p_M^k)$. Then, we can write platform i 's profit as $\pi_i(p^A, p^B) = (p_i^A - c_i^A)n_i^A(p^A, p^B) + (p_i^B - c_i^B)n_i^B(p^A, p^B)$, where n_i^A and n_i^B depend on the fees set by all platforms for both groups.

Our main focus is on two-sided pricing—that is, each platform i charges fees p_i^A and p_i^B to each user group. We also consider one-

sided pricing under which each platform i has to set a fee of zero to one group (presuming that the marginal cost is zero for that group) or a fee equal to marginal costs (when allowing for positive marginal costs for that group).

2.2 | Users

A unit mass of users from each group decide which platform to join. Each user's utility from joining a platform consists of a maximal value of the platform, network effects, and an idiosyncratic preference for the platform. Formally, the utility of a group- k consumer from joining platform i is given by

$$u_i^k = a_i^k - p_i^k + \alpha^k \log n_i^k + \beta^k \log n_i^l + \varepsilon_i^k. \quad (1)$$

The first term $a_i^k - p_i^k$ is the expected value of platform i for group- k users in the hypothetical case that all users from both groups were to join this platform, where a_i^k represents the "quality" of platform i for group k . The second and third terms, $\alpha^k \log n_i^k$ and $\beta^k \log n_i^l$, capture within-group and cross-group network effects, where $\alpha^k \in [0, 1)$ and $\beta^k \in [0, 1)$ are the parameters that represent the importance of platform-specific within-group and cross-group network effects, and n_i^k and n_i^l are the number of group- k and group- $l (\neq k)$ users who join platform i . We call n_i^k group k 's network size of platform i . We note that the chosen logarithmic specification of network effects is broadly adopted in the empirical literature (e.g., Ohashi 2003, Rysman 2004, 2007, Zhu and Inasti 2012).⁴

The last term, ε_i^k , is an idiosyncratic taste shock from an i.i.d. type-I extreme value distribution. We assume that network effects are not too strong, that is, $\alpha^k + \beta^l < 1$ hold for any $k, l \in \{A, B\}$. Thus, $\max\{\alpha^A, \alpha^B\} + \max\{\beta^A, \beta^B\} < 1$. Table 1 summarizes the notation.

Several applications fit as special cases. In e-commerce marketplaces, sellers and buyers constitute the two user groups and parameters β^A and β^B are positive, whereas, in the simplest version, $\alpha^A = \alpha^B = 0$. Here, there are mutual cross-group network effects because buyers are attracted to platforms with many sellers and sellers to platforms with many buyers. Similarly, for two-sided matching platforms such as heterosexual online dating platforms. On ad-funded social networks and media platforms with user-generated content, advertisers and consumers constitute two user groups A and B and network effects are such that in the simplest case, $\beta^A, \beta^B > 0$ and $\alpha^A = \alpha^B = 0$ (advertisers care about consumer participation and consumers care about user-generated content but not advertising). For a discussion, see Belleflamme and Peitz (2021).⁵

For given network sizes $\bar{n} = (\bar{n}_i^A, \bar{n}_i^B)_{i \in \{1, \dots, M\}}$, group- k consumer demand of platform i can be written as

$$n_i^k = \Pr \left(u_i^k \geq u_j^k \text{ for all } j \neq i \right) = \frac{\exp(a_i^k - p_i^k) (\bar{n}_i^k)^{\alpha^k} (\bar{n}_i^l)^{\beta^k}}{\sum_{j=1}^M \exp(a_j^k - p_j^k) (\bar{n}_j^k)^{\alpha^k} (\bar{n}_j^l)^{\beta^k}} =: T_i^k(\bar{n}). \quad (2)$$

TABLE 1 | Notation.

Notation	Meaning
k, l	Indices for the two user groups
a_i^k	Group- k quality of platform i
c_i^k	Marginal cost for group- k participation on platform i
p_i^k	Group- k fee of platform i
n_i^k	Group- k network size of platform i
α^k	Parameter for within-group network effect of group k
β^k	Parameter for cross-group network effect enjoyed by group k

This is the multinomial demand structure with network sizes endogenously determining platform quality.

3 | Equilibrium Analysis

We first characterize the participation equilibrium at stage 2 for given platform fees. We then analyze subgame perfect Nash equilibria of the price-then-participation game.

3.1 | Participation Equilibrium

In a participation equilibrium, network sizes n_i^k on the left-hand side are equal to \bar{n}_i^k on the right-hand side of equation (2) for all $k \in \{A, B\}$ and $i \in \{1, \dots, M\}$.

Due to complementarity in platform choices, there may be multiple participation equilibria, an issue pointed out, for instance, by Tan and Zhou (2021), among others. In the present setting, equation (2) indicates that whenever users expect $\bar{n}_i^k = 0$, such an expectation will be self-fulfilling (for any platform prices). Therefore, one can pick any weak subset of platforms, and there is an equilibrium in which all other platforms have zero participation in equilibrium.⁶

We will first characterize the unique participation equilibrium for a given set, $\mathcal{M} \subseteq \{1, \dots, M\}$, of active platforms (i.e., platforms with strictly positive demand for both groups). We call such an equilibrium an interior participation equilibrium when all platforms are active.

Lemma 1. For any given prices $p = (p_1^A, \dots, p_M^A, p_1^B, \dots, p_M^B)$, there exists a unique participation equilibrium with the set of active platforms $\mathcal{M} \subseteq \{1, \dots, M\}$. Equilibrium participation levels are given by

$$n_i^k(p) = \frac{\exp[\Gamma^{kk}(a_i^k - p_i^k) + \Gamma^{kl}(a_i^l - p_i^l)]}{\sum_{j \in \mathcal{M}} \exp[\Gamma^{kk}(a_j^k - p_j^k) + \Gamma^{kl}(a_j^l - p_j^l)]}, \quad (3)$$

for all $i \in \mathcal{M}$ and $k, l \in \{A, B\}$ with $l \neq k$, where Γ^{kk} and Γ^{kl} are given by

$$\Gamma^{kk} = \frac{1 - \alpha^l}{(1 - \alpha^k)(1 - \alpha^l) - \beta^k \beta^l} \geq 1 \text{ and } \Gamma^{kl} = \frac{\beta^k}{(1 - \alpha^k)(1 - \alpha^l) - \beta^k \beta^l} \geq 0.$$

The demand system given by equation (3) is a logit demand system augmented by within-group and cross-group network effects. To see why the coefficients Γ^{kk} and Γ^{kl} enter into equation (3), first note that under logit demand, the choice probability for one alternative relative to another, $T_i^k(n)/T_j^k(n)$, is log-linear in their values, which is well-known in the empirical IO literature (e.g., Berry 1994). With logarithm network effects, the following log-log linear relationship holds:

$$\begin{pmatrix} \log\left(\frac{T_i^A(n)/T_j^A(n)}{T_i^B(n)/T_j^B(n)}\right) \\ \log\left(\frac{T_i^B(n)/T_j^B(n)}{T_i^A(n)/T_j^A(n)}\right) \end{pmatrix} = \begin{pmatrix} \Delta_{ij}^A \\ \Delta_{ij}^B \end{pmatrix} + \begin{pmatrix} \alpha^A & \beta^A \\ \beta^B & \alpha^B \end{pmatrix} \begin{pmatrix} \log(n_i^A/n_j^A) \\ \log(n_i^B/n_j^B) \end{pmatrix}, \quad (4)$$

where $\Delta_{ij}^k := (a_i^k - p_i^k) - (a_j^k - p_j^k)$ is platform i 's unadjusted advantage or disadvantage in the value offered to group- k users relative to platform j . The log-log linear relationship in equation (4) allows us to exploit the linearity to obtain the closed-form solution with coefficients Γ^{kk} and Γ^{kl} that captures how the relative advantage of a platform ($\Delta_{ij}^A, \Delta_{ij}^B$) is amplified by positive-feedback loops.⁷

Consider two special cases. In the special case of within-group network effects but no cross-group network effects ($\alpha^k > 0, \beta^k = 0$ for $k \in \{A, B\}$), logit choice probabilities are adjusted by within-group network effects:

$$n_i^k = \frac{\exp\left(\frac{a_i^k - p_i^k}{1 - \alpha^k}\right)}{\sum_{j \in \mathcal{M}} \exp\left(\frac{a_j^k - p_j^k}{1 - \alpha^k}\right)}.$$

In the special case of cross-group network effects but no within-group network effects ($\alpha^k = 0, \beta^k > 0$ for $k \in \{A, B\}$), logit choice probabilities are:

$$n_i^k = \frac{\exp\left(\frac{a_i^k - p_i^k + \beta^k(a_i^l - p_i^l)}{1 - \beta^k \beta^l}\right)}{\sum_{j \in \mathcal{M}} \exp\left(\frac{a_j^k - p_j^k + \beta^k(a_j^l - p_j^l)}{1 - \beta^k \beta^l}\right)}.$$

To summarize, we obtain a tractable closed-form expression of user participation when network effects are logarithmic in network size and demand takes the logit form. Because of logarithmic specification of the network effects, any platform can be empty even under logit demand and thus worthless for users.

Hence, there are multiple participation equilibria, one for every non-empty set of active platforms. There are two ways to address this multiplicity. One possibility is to postulate that for reasons outside the model there is a given set of active platforms. Lemma 1 then characterizes equilibrium participation decisions for any set of prices of these platforms.

The other possibility to address the multiplicity of participation equilibria is to propose a particular selection criterion. We do so in the analysis that follows and provide a selection criterion according to which all available platforms are active in equilibrium.

3.2 | Equilibrium Selection

We impose *asymptotic stability* of best-response dynamics as our selection criterion and show that the only equilibrium that meets the selection criterion is the interior participation equilibrium. The notion of best-response dynamics corresponds to that used in the literature of population games (Sandholm 2010), and the notion of asymptotic stability is used to capture the stability of dynamic systems (Luenberger 1979).

Definition 1. Define the best-response dynamics and asymptotic stability of network sizes as follows:

1. A best-response dynamics $\{n_t\}_{t=0}^{\infty}$ from the initial network sizes $n_0 = (n_{i,0}^A, n_{i,0}^B)_{i \in \{1, \dots, M\}}$ is defined by a sequence of network sizes $n_t = (n_{i,t}^A, n_{i,t}^B)_{i \in \{1, \dots, M\}}$ such that $n_{i,t}^k = T_i^k(n_{t-1})$ according to the best-response functions T_i^k defined in equation (2) for all $t \in \{1, 2, \dots\}$, $i \in \{1, \dots, M\}$ and $k \in \{A, B\}$.
2. A network size vector $n = (n_i^A, n_i^B)_{i \in \{1, \dots, M\}}$ is the limit of the best-response dynamics $\{n_t\}_{t=0}^{\infty}$ from the initial network size n_0 if $n = \lim_{t \rightarrow \infty} n_t$.
3. A participation equilibrium with the equilibrium network sizes n is asymptotically stable if for any strictly positive n_0 , n is the limit of the best-response dynamics from the initial network sizes n_0 .

Definition 1 requires that the equilibrium network sizes are the result of best-response dynamics starting from any interior starting point. We call a participation equilibrium with asymptotically stable network sizes an *asymptotically stable participation equilibrium*.

The following remark establishes that the interior participation equilibrium is the only equilibrium that is asymptotically stable.

Remark 1. For any given prices $p = (p_1^A, \dots, p_M^A, p_1^B, \dots, p_M^B)$, the interior participation equilibrium, characterized by equations (3) with $\mathcal{M} = \{1, \dots, M\}$, is the unique asymptotically stable participation equilibrium.

Other selection criteria used in the literature on network effects in industrial organization include: Pareto dominance (Katz and Shapiro 1986, Fudenberg and Tirole 2000), coalitional rationalizability or coalition proofness (Ambrus and Argenziano 2009,

Karle et al. 2020), potential maximization (Chan 2021), and focality advantage or attached consumers (Caillaud and Jullien 2003, Halaburda et al. 2020, Biglaiser and Crémer 2020). This includes dynamic consideration leading to incumbency advantages in the cases of focality and attached consumers. In our model, for any cost-adjusted quality and any prices, a platform facing unfavorable beliefs—in the sense that each user expects the smallest number to join that is compatible with equilibrium—will not become active.⁸

3.3 | Aggregates, Profit Functions, and User Surplus

We will write platform profits as functions of own actions and corresponding aggregates. Furthermore, we will write user surplus of the two groups as functions of these aggregates. To do so, we define a platform's own actions as

$$h_i^A := \exp[\Gamma^{AA}(a_i^A - p_i^A) + \Gamma^{AB}(a_i^B - p_i^B)],$$

$$h_i^B := \exp[\Gamma^{BB}(a_i^B - p_i^B) + \Gamma^{BA}(a_i^A - p_i^A)],$$

and the corresponding aggregates $H^A := \sum_{j=1}^M h_j^A$ and $H^B := \sum_{j=1}^M h_j^B$. Thus, group- k demand on platform i is $n_i^k = h_i^k/H^k$.

There is a one-to-one mapping between (p_i^A, p_i^B) and (h_i^A, h_i^B) . As we show in the following lemma, any (h_i^A, h_i^B) induce prices $(p_i^A(h_i^A, h_i^B), p_i^B(h_i^A, h_i^B))$.

Lemma 2. Platform fees can be written as functions of (h_i^A, h_i^B) :

$$p_i^A(h_i^A, h_i^B) = a_i^A - (1 - \alpha^A) \log h_i^A + \beta^A \log h_i^B, \quad (5)$$

$$p_i^B(h_i^A, h_i^B) = a_i^B - (1 - \alpha^B) \log h_i^B + \beta^B \log h_i^A. \quad (6)$$

Recall that platform i 's profit as a function of platform fees is $(p_i^A - c_i^A)n_i^A + (p_i^B - c_i^B)n_i^B$. Because $n_i^k = h_i^k/H^k$ and there is a one-to-one mapping between (p_i^A, p_i^B) and (h_i^A, h_i^B) , the profit of platform i can be written as the function of the two action variables h_i^A and h_i^B and their aggregates H^A and H^B :

$$\begin{aligned} \Pi_i(h_i^A, h_i^B, H^A, H^B) &= \Pi_i^A(h_i^A, h_i^B, H^A) + \Pi_i^B(h_i^A, h_i^B, H^B) \\ &= \frac{h_i^A}{H^A} [p_i^A(h_i^A, h_i^B) - c_i^A] + \frac{h_i^B}{H^B} [p_i^B(h_i^A, h_i^B) - c_i^B], \end{aligned} \quad (7)$$

where we define the profit associated with group k as $\Pi_i^k = \frac{h_i^k}{H^k} [p_i^k(h_i^A, h_i^B) - c_i^k]$, $k, l \in \{A, B\}$, $l \neq k$.

Group- k user surplus CS^k is given by the expected indirect utility of users, and the aggregate user surplus CS is given by the sum of the user surplus in both groups:

$$\begin{aligned} CS^k &= \log \left[\sum_{i=1}^M \exp(a_i^k - p_i^k) (n_i^k)^{\alpha^k} (n_i^l)^{\beta^k} \right] \\ &= (1 - \alpha^k) \log H^k - \beta^k \log H^l, \end{aligned}$$

$$CS = (1 - \alpha^A - \beta^B) \log H^A + (1 - \alpha^B - \beta^A) \log H^B.$$

We observe that user surplus of group k , CS^k , is increasing in the aggregate of this group, H^k , and weakly decreasing in the aggregate of the other user group, H^l ; it is strictly decreasing if and only if group l exerts a cross-group network effect. Total user surplus $CS = CS^A + CS^B$ increases in each of the two aggregates H^A and H^B .

3.4 | Price Equilibrium in Asymmetric Platform Oligopoly

Using the demand system obtained from the participation equilibrium, we analyze price competition between platforms using the continuation profits from the participation equilibrium at stage 2.

We establish the following lemma that guarantees that we can restrict attention to the first-order conditions of profit maximization when analyzing platform pricing.

Lemma 3. *For any given $H_{-i}^A = \sum_{j \neq i} h_j^A$ and $H_{-i}^B = \sum_{j \neq i} h_j^B$, there is a unique solution to the first-order conditions of profit maximization of $\Pi_i(h_i^A, h_i^B, h_i^A + H_{-i}^A, h_i^B + H_{-i}^B)$ with respect to h_i^A, h_i^B , and this solution is a global maximizer of platform i 's pricing problem.*

The first-order conditions $\partial \Pi_i / \partial h_i^A = 0$ using the expression for profits given in (7) can be rewritten to have the price-cost margins on the left-hand side:

$$\begin{aligned} p_i^A(h_i^A, h_i^B) - c_i^A &= \frac{1}{1 - \frac{h_i^A}{H^A}} \left(1 - \alpha^A - \beta^B \frac{h_i^B}{H^B} \frac{H^A}{h_i^A} \right) \\ &= \frac{1}{1 - n_i^A} \left(1 - \alpha^A - \beta^B \frac{n_i^B}{n_i^A} \right). \end{aligned}$$

Correspondingly, for group B . In the standard multinomial logit model without network effects ($\alpha^k = \beta^k = 0$, for all $k \in \{A, B\}$), the price-cost margin is equal to $1/(1 - n_i^k)$. In the presence of within-group network effects $\alpha^k > 0$ only, the price-cost margin is reduced by α^k . The lower price-cost margin is due to the larger price elasticity of demand arising from within-group network effects. In the presence of cross-group network effect $\beta^l > 0$, the price-cost margin for group k is reduced by the amount $\beta^l n_i^l / n_i^k$. Here, the lower price-cost margin is due to the cross-subsidization incentive of the platform: it expands participation of group k to attract users in group l ; this is in line with the formulas for price-cost margins in symmetric platform oligopoly reported in Armstrong (2006) and Tan and Zhou (2021).

Substituting for prices using Equations (5) and (6), the system of first-order conditions becomes:

$$\begin{aligned} \alpha_i^A - c_i^A - (1 - \alpha^A) \log h_i^A + \beta^A \log h_i^B &= \frac{1}{1 - \frac{h_i^A}{H^A}} \left(1 - \alpha^A - \beta^B \frac{h_i^B}{H^B} \frac{H^A}{h_i^A} \right) \\ \alpha_i^B - c_i^B - (1 - \alpha^B) \log h_i^B + \beta^B \log h_i^A &= \frac{1}{1 - \frac{h_i^B}{H^B}} \left(1 - \alpha^B - \beta^A \frac{h_i^A}{H^A} \frac{H^B}{h_i^B} \right) \end{aligned}$$

In equilibrium, it must be satisfied for all $i \in \{1, 2, \dots, M\}$. As shown in the following lemma, for each i , this defines implicit best replies $(h_i^A(H^A, H^B), h_i^B(H^A, H^B))$.

Lemma 4. *For any (H^A, H^B) , the system of first-order conditions defines implicit best replies $(h_i^A(H^A, H^B), h_i^B(H^A, H^B))$ for each platform $i \in \{1, \dots, M\}$.*

Summing over all i , an equilibrium satisfies

$$\sum_{i=1}^M h_i^k(H^A, H^B) = H^k, \tag{8}$$

for $k, l \in \{A, B\}, l \neq k$. With the following proposition, we establish that there exists a price equilibrium and that, whenever multiple equilibria exist, these are ordered in terms of surplus of one of the two user groups: if one equilibrium features higher surplus for one group, the other equilibrium features a higher surplus for the other group.

Proposition 1. *There exists a price equilibrium pinned down by aggregates (H^{A*}, H^{B*}) . When there are multiple price equilibria for a given set of active platforms, we obtain the ranking for any pair of equilibrium aggregates given by (H_1^{A*}, H_1^{B*}) and (H_2^{A*}, H_2^{B*}) with associated user surpluses (CS_1^{A*}, CS_1^{B*}) and (CS_2^{A*}, CS_2^{B*}) : $CS_1^{A*} > CS_2^{A*}$ holds if and only if $CS_1^{B*} < CS_2^{B*}$.*

Together with Remark 1, according to Proposition 1 we cannot exclude the possibility that there are multiple pricing equilibria even when selecting the unique asymptotically stable participation equilibrium for any given prices. We note that the equilibrium is always unique if platforms are symmetric.⁹ A price equilibrium is characterized by the pair of aggregates (H^{A*}, H^{B*}) that satisfy the system of equations (8), which implicitly defines functions $\tilde{H}^k(H^l)$. An intersection of these two functions constitutes an equilibrium, as illustrated by the two numerical examples in Figure 1.¹⁰

3.5 | Discussion

Because the surplus of group- k users, $CS^k = (1 - \alpha^k) \log H^k - \beta^k \log H^l$, depends only on aggregates (H^A, H^B) , the characterization of equilibrium aggregates directly characterizes user surplus in equilibrium. We note that in the aggregative-games frameworks of price competition in standard oligopoly (Anderson et al. 2020) and platform competition with competitive bottlenecks (Anderson and Peitz 2020) consumer surplus (i.e., user surplus on the single-homing side) depends on a one-dimensional aggregate. To be able to write user welfare as a function of the aggregates in a differentiated Bertrand oligopoly game, the demand system must satisfy the IIA property (Proposition 1 of Anderson et al. 2020). Logit demand satisfies the IIA property and the aggregate is the denominator of the demand function—as far as we know, this property does not have an economic interpretation (see also Train 2009 on page 56).

As we show, with network size entering with logs in the utility functions, the property that user welfare is a linear function of the logs of the aggregates is inherited by a platform model with two-

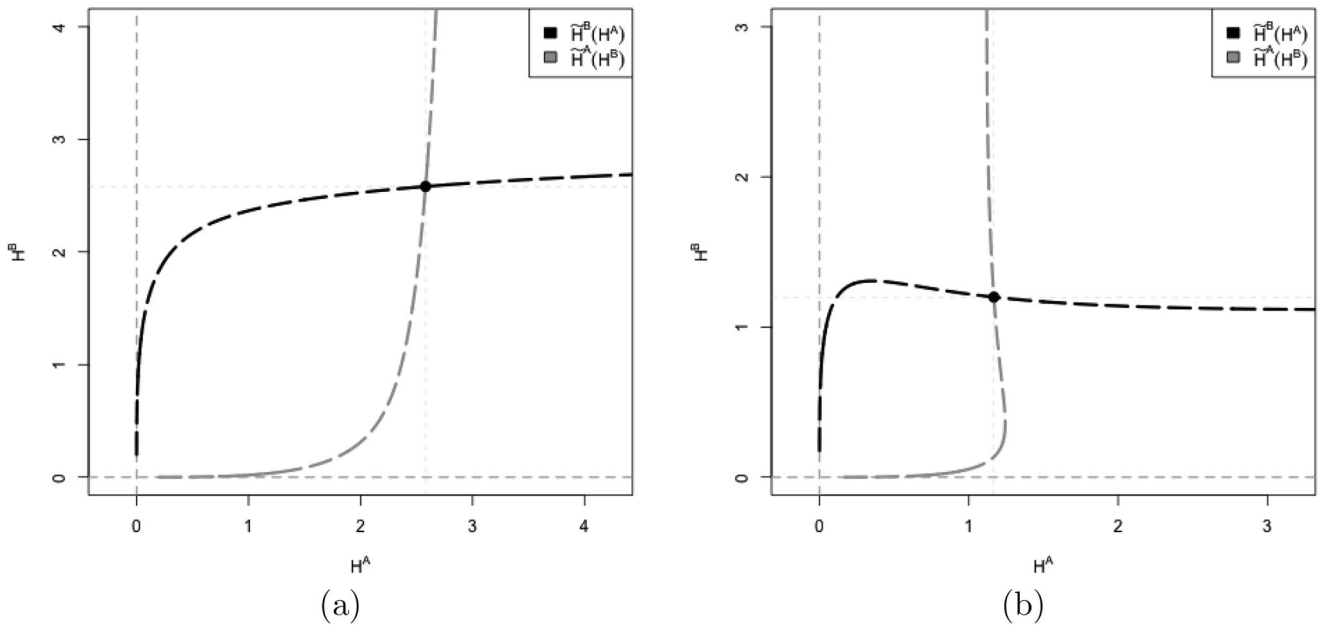


FIGURE 1 | Shapes of $\tilde{H}^A(H^B)$ and $\tilde{H}^B(H^A)$.

sided single-homing: surplus of user group k depends positively on the aggregate of this group and negatively on the aggregate of the other group. To illustrate this property, consider symmetric platforms with symmetric prices across platforms, $p_i^k \equiv p^k$ for all $i \in \{1, \dots, M\}$, $k \in \{A, B\}$ and look at the effect of a decrease of p^A by the same amount for all platforms keeping p^B unchanged. In the case of positive cross-group network effects, this implies that H^A and H^B increase, whereas it continues to hold that $h_i^A/H^A = h_i^B/H^B = 1/M$. The increases of the aggregates are such that CS^A increases and CS^B remains unchanged, which must hold because participation decisions are not affected and thus group- B users' net utilities are not affected by the price reduction experienced by group- A users. Here, the negative dependence of CS^B on H^A must hold for the simultaneous increases in H^A and H^B to keep CS^B unchanged.

A technical issue in the equilibrium existence results with price competition is to show that strategies are chosen from a compact strategy space. In a standard logit model without network effects, Nocke and Schutz (2018, forthcoming) directly show that setting too high prices is always unprofitable, thereby obtaining upper bounds on the strategy space. In the logit demand with within-group network effects (i.e., $\alpha^A, \alpha^B > 0$ but $\beta^A = \beta^B = 0$), we obtain an upper bound on prices in the same way as Nocke and Schutz (2018, forthcoming). With cross-group network effects (i.e., $\beta^A > 0$ or $\beta^B > 0$), we also have to worry about a *lower bound* on prices because, in theory, platforms could choose to turn towards negative infinite fees for one group and positive infinite fees for the other group at the same time. In the proof, we show that this strategy is always dominated as long as $\alpha^k + \beta^l < 1$ for $k, l \in \{A, B\}$ with $l \neq k$.

We postulated that within- and cross-group network effects are non-negative. However, in some real-world environments, some network effects are arguably negative. We note that all of our analysis is applicable to the case with negative within-group network effects (i.e., $\alpha^k < 0$).¹¹ However, our analysis fails to

apply with negative cross-group network effects (i.e., $\beta^k < 0$) due to our logarithmic specification. With negative cross-group network effects experienced by one group—for instance, group A —a platform can charge an unboundedly high fee to group- B users to increase $\beta^A \log n_i^B$ without bounds and then enjoy a monopoly profit from group- A users.

3.6 | Network Goods

It is insightful to consider the special case of only within-group network effects (i.e., $\beta^A = \beta^B = 0$). In other words, we analyze the asymmetric logit model with network effects. Users in one group do not care about user participation in the other group and it is sufficient to consider group A . The pricing equation for platform i becomes $p_i^A - c_i^A = \alpha_i^A - c_i^A - (1 - \alpha^A) \log h_i^A$. Thus, the first-order conditions of profit maximization for group A can be written as

$$(1 - \alpha^A) \frac{H^A}{H^A - h_i^A} = (\alpha_i^A - c_i^A) - (1 - \alpha^A) \log h_i^A \quad (9)$$

and the solution to the system of first-order conditions constitutes a price equilibrium.

Uniqueness of the price equilibrium (for any given set of active platforms) can be shown as follows: The right-hand side of equation (9) is decreasing in h_i^A , whereas the left-hand side is increasing in h_i^A . Thus, for any H^A there is a unique $h_i^A(H^A)$. Note also that the right-hand side does not depend on H^A , whereas the left-hand side is shifted downward after an increase in H^A . Hence, $h_i^A(H^A)$ is increasing in H^A and so is $\sum_i h_i^A(H^A)$. Furthermore, because $\sum_i h_i^A(H^A)$ increase with H^A at a slower rate than H^A does, there is at most one equilibrium.

Remark 2. There exists a unique price equilibrium when $\beta^A = \beta^B = 0$.

3.7 | One-Sided Pricing

We also consider one-sided pricing under which participation is free for one user group. For example, shopping malls and flea markets typically charge retailers but often not end users. This may be because platforms would charge negative fees (or fees below costs) and such fees are not feasible. Alternatively, platforms would like to charge end user fees but such positive fees would go hand-in-hand with high transaction costs or are simply not possible (as in traditional free-to-air radio or television broadcasting).

As mentioned in Section 2, we assume that marginal costs are zero for users in the group with a zero fee. If we were to allow positive symmetric marginal costs also for this group, our analysis applies if instead of a zero fee we were to consider a fee equal to marginal costs.

Suppose that group B is the zero-fee group. Using the equations from Lemma 2, we then must have

$$\begin{aligned} p_i^A(h_i^A, h_i^B) &= \alpha_i^A - (1 - \alpha^A) \log h_i^A + \beta^A \log h_i^B, \\ 0 &= \alpha_i^B - (1 - \alpha^B) \log h_i^B + \beta^B \log h_i^A. \end{aligned}$$

We rewrite the second equation as $\log h_i^B = \alpha_i^B / (1 - \alpha^B) + (\beta^B / (1 - \alpha^B)) \log h_i^A$ and substitute into the first equation to obtain (with an abuse of notation, we write p_i^A as a function of h_i^A)

$$p_i^A(h_i^A) = \tilde{\alpha}_i^A - (1 - \tilde{\alpha}^A) \log h_i^A, \quad (10)$$

where $\tilde{\alpha}_i^A := \alpha_i^A + (\beta^A / (1 - \alpha^B)) \alpha_i^B$ and $\tilde{\alpha}^A := \alpha^A + \beta^A \beta^B / (1 - \alpha^B)$. In the special case that users in the group with monetization (group A) do not care about the participation of the other group (i.e., $\beta = 0$), prices are the same as in the model with network goods.

Platform i 's profit as a function of h_i^A and its aggregate is

$$\Pi_i(h_i^A, H^A) = \frac{h_i^A}{H^A} [p_i^A(h_i^A) - c_i^A] = \frac{h_i^A}{H^A} [\tilde{\alpha}_i^A - c_i^A - (1 - \tilde{\alpha}^A) \log h_i^A].$$

Then the analysis for platforms with only direct network effects for group A applies after a change of variables from (α_i^A, α^A) to $(\tilde{\alpha}_i^A, \tilde{\alpha}^A)$, where $\tilde{\alpha}^A < 1$ holds because this is equivalent to $\beta^A \beta^B < (1 - \alpha^A)(1 - \alpha^B)$ and implied by our assumption $\alpha^k + \beta^l < 1$ for $k, l \in \{A, B\}, l \neq k$. Thus, with the change of variables, Remark 2 applies and a unique price equilibrium exists.

Remark 3. There exists a unique price equilibrium under one-sided pricing.

The equivalence between the pricing of network goods and one-sided pricing is reminiscent of and extends the equivalence between direct and “indirect” network effects in the literature on network effects (e.g., Katz and Shapiro 1985 and Church and Gandal 1992 in oligopoly models different from ours), where positive indirect network effects would be the case that $\alpha^A = \alpha^B = 0$ and $\beta^A > 0, \beta^B > 0$.

Note that, in contrast to the setting with two-sided pricing, our analysis under one-sided pricing carries over to the case with negative cross-group network effects (as long as they are not too large), because the model can be translated into the model of network goods. This means that our framework can cover purely ad-funded media platforms under two-sided single-homing.

4 | Equilibrium Characterization Results

In this section, we provide equilibrium characterization results, focusing on how platform asymmetry affects market shares, price-cost margins, and profits.

4.1 | Platform Type and Market Share

The relative position of a platform with respect to the size of its user groups is determined by its “type” (v_i^A, v_i^B) where $v_i^k = \alpha_i^k - c_i^k$ is the cost-adjusted quality that platform i offers to group- k users. Thus, v_i^k stands for the platform’s ability to provide value to group- k users. Proposition 1 allows us to conduct an equilibrium analysis of platform oligopoly with arbitrary heterogeneity of platforms with respect to their cost-adjusted quality on each side. We first take a look at an individual platform (we make use of this lemma in the proofs of several of the following propositions).

Lemma 5.

1. For any given aggregates (H^A, H^B) and network size $(n_i^A, n_i^B) \in (0, 1)^2$, there exists a type (v_i^A, v_i^B) such that $h_i^k(H^A, H^B)/H^k = n_i^k$ for both $k \in \{A, B\}$.
2. For any given type (v_i^A, v_i^B) and network size $(n_i^A, n_i^B) \in (0, 1)^2$ of platform i , there exists a unique pair of aggregates (H^A, H^B) such that $h_i^k(H^A, H^B)/H^k = n_i^k$ for both $k \in \{A, B\}$.

Furthermore, for any market structure, we can find a profile of cost-adjusted qualities that decentralizes any market share allocation as an equilibrium outcome, as we formally establish in the following remark, where we define the aggregate type for group- k users with $\bar{v}^k := \log \sum_{j=1}^M \exp\{v_j^k\}$ for $k \in \{A, B\}$.

Remark 4. Pick any profile of network sizes $(n_i^A, n_i^B)_{i \in \{1, \dots, M\}}$ such that $\sum_{j=1}^M n_j^k = 1$ for $k \in \{A, B\}$.

1. In addition, pick any aggregates $(H^A, H^B) \in \mathbb{R}_{++}^2$. There exists a unique type profile $(v_i^A, v_i^B)_{i \in \{1, \dots, M\}}$ such that the equilibrium network sizes and aggregates in the price equilibrium are $(n_i^A, n_i^B)_{i \in \{1, \dots, M\}}$ and (H^A, H^B) , respectively.
2. In addition, pick any aggregate type $(\bar{v}^A, \bar{v}^B) \in \mathbb{R}^2$. There exists a unique type profile $(v_i^A, v_i^B)_{i \in \{1, \dots, M\}}$ generating aggregate type $(\bar{v}^A, \bar{v}^B) \in \mathbb{R}^2$ such that the equilibrium network sizes are $(n_i^A, n_i^B)_{i \in \{1, \dots, M\}}$.

In the following, we address the question of how market shares, price-cost margins, and profits differ across different platforms when they are asymmetric with respect to what they offer to users in one group. We start with market shares.

4.2 | Comparison of Market Shares

In the following result we establish that the platform with higher cost-adjusted quality for one user group has a strictly larger market share for this user group and a weakly larger market share for the other user group—it is strictly larger if at least one of the cross-group network effects is positive ($\beta^A > 0$ or $\beta^B > 0$).

Proposition 2. *Take any two platforms i and j with $v_i^A > v_j^A$ and $v_i^B = v_j^B$. Then, in equilibrium, $n_i^A > n_j^A$ and $n_i^B \geq n_j^B$. Furthermore, $n_i^B > n_j^B$ if and only if $\beta^A > 0$ or $\beta^B > 0$.*

We also note that if a platform is of higher type for both groups (i.e., $v_i^A > v_j^A$ and $v_i^B > v_j^B$), then $n_i^A > n_j^A$ and $n_i^B > n_j^B$ for any network effects.

Our findings under two-sided single-homing can be contrasted to what happens under competitive bottleneck in Anderson and Peitz (2020). In that setting, platforms are asymmetric regarding the quality offered to single-homing users (say group A). When platforms set the participation level for multi-homing users (group B) and participation fees for single-homing users,¹² the higher-quality platform admits fewer multi-homing users.¹³ In that setting, the higher-quality platform does not admit as many group- B users as its lower-quality competitor and still attracts more single-homing users in equilibrium. Such a reduced number of group- B users is attractive for the higher-quality platform because this raises revenues from the multi-homing group.

4.3 | Comparison of Price-Cost Margins

We next look at the pricing implications for users in one user group (group B in the proposition below) when platforms are asymmetric with respect to the other group. To do so, we consider two polar cases: (i) only the other group benefits from cross-group network effects and (ii) the reverse; that is, the group for which platforms are symmetric with respect to the cost-adjusted quality they offer to that group benefits from cross-group network effects. Price-cost margins of platform i are denoted as $\mu_i^k := p_i^k - c_i^k$, $k \in \{A, B\}$.

Proposition 3. *Take any two platforms i and j with $v_i^A > v_j^A$ and $v_i^B = v_j^B$. (i) Suppose that $\beta^A > 0$ and $\beta^B = 0$. Then, the price-cost margin from group- B users is smaller on the higher-quality platform i than on j ; that is, $\mu_i^B < \mu_j^B$. (ii) Suppose that $\beta^A = 0$ and $\beta^B > 0$. Then, the price-cost margin charged to group- B users is larger on the higher-quality platform i than on j ; that is, $\mu_i^B > \mu_j^B$.*

Thus, it depends on the direction of cross-group network effects whether the user group that considers two platforms to be symmetric in their cost-adjusted quality (say group B) faces a higher or lower price-cost margin on the platform with higher cost-adjusted quality for the other user group (say group A). If only group A benefits from cross-group network effects ($\beta^A > 0$, $\beta^B = 0$), the platform with the higher cost-adjusted quality for group A sets a lower price-cost margin for group B than the competing platform. This lower price-cost margin for group- B users fosters the participation of those users. Because $\beta^A > 0$, this gives an extra push to group- A users to join this platform. The platform with the higher (ex ante) cost-adjusted quality

benefits more from this. This implies that the asymmetry between platforms for group A is amplified.

In the opposite case, in which only group B benefits from cross-group network effects ($\beta^A = 0$, $\beta^B > 0$), the platform with the higher cost-adjusted quality for group A will have more group- A participation, which translates into an endogenous quality advantage for group B , $\beta^B(\log n_i^A - \log n_j^A)$. In equilibrium, this results a higher price-cost margin to group- B users than the one charged by the competing platform.

Next, we take a look at the user group that experiences different cost-adjusted qualities across platforms. One might expect that the platform that offers the higher cost-adjusted quality always has a higher price-cost margin for the same group (as would happen absent network effects, as shown in Proposition 1 in Anderson and de Palma 2001 and Proposition 4 in Anderson et al. 2020). Although this is correct under a number of conditions (parameter conditions or outcome variables), we show by example that this is not always the case.

Remark 5. Take any two platforms i and j with $v_i^B = v_j^B$. Then, the price-cost margin for group- A users is larger on platform i with the higher cost-adjusted quality $v_i^A > v_j^A$ (i.e., $\mu_i^A > \mu_j^A$) if cross-group network effects are not mutual, that is, (1) $\beta^A = 0$ or (2) $\beta^B = 0$, (3) platforms i and j attract weakly more users from group A than B , or (4) platforms set fees above costs for both user groups. However, there are environments in which the platform with the lower quality has a higher price-cost margin for group- A users (i.e., $\mu_i^A < \mu_j^A$); this can only happen if $\beta^A > 0$, $\beta^B > 0$ and, in equilibrium, $n_i^B > n_j^B$.

We conclude that cost-adjusted quality differences between platforms for one user group (when cost-adjusted quality for the other user group is the same across platforms) give rise to non-trivial differences in user participation across platforms in the presence of cross-group network effects. This hints at platform asymmetry shaping the pricing structures of two-sided platforms. We can write the relation between market shares n_i^A and price-cost margins μ_i^k as

$$\mu_i^A = \frac{1}{1 - n_i^A} \left(1 - \alpha^A - \beta^B \frac{n_i^B}{n_i^A} \right), \quad (11)$$

$$\mu_i^B = \frac{1}{1 - n_i^B} \left(1 - \alpha^B - \beta^A \frac{n_i^A}{n_i^B} \right). \quad (12)$$

Equations (11) and (12) show that it depends on the relative size n_i^A/n_i^B of platform i on the two sides whether the price-cost margin is positive or negative.

The price-cost margins can be related to cost-adjusted qualities, which are the primitives of our model. Suppose that platforms are asymmetric only with respect to the cost-adjusted quality that each platform offers to group- A users. We find that platforms with high group- A net quality regard group A as the money side and group B as the subsidy side because they earn more from the group- A users. By contrast, platforms with low group- A net

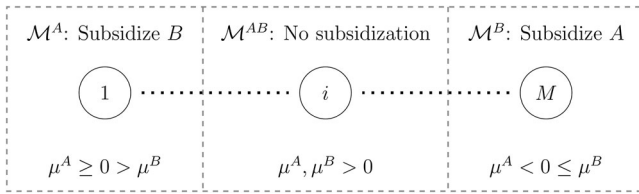


FIGURE 2 | Price structures in asymmetric oligopoly.

quality regard the other group (group B) as the money side and group A as the subsidy side. Those with intermediate group- A net quality charge positive price-cost margins to both groups.

To formally establish this result, we introduce subsets of platforms $\mathcal{M}^A, \mathcal{M}^B, \mathcal{M}^{AB} \subseteq \{1, 2, \dots, M\}$, where the superscript indicates the user group(s) for which the platform charges positive price-cost margins; that is $\mu_i^A \geq 0$ and $\mu_i^B < 0$ for all $i \in \mathcal{M}^A$, $\mu_i^A \geq 0$ and $\mu_i^B \geq 0$ for all $i \in \mathcal{M}^{AB}$, and $\mu_i^A < 0$ and $\mu_i^B \geq 0$ for all $i \in \mathcal{M}^B$.

Proposition 4. *Suppose that $\beta^B \geq \beta^A$, $\beta^B > 0$, $v_i^B = v^B$ for all $i = 1, \dots, M$, and $v_1^A \geq v_2^A \geq \dots \geq v_M^A$. Consider any two platforms i and j belonging to different subsets \mathcal{M}^A , \mathcal{M}^B , and \mathcal{M}^{AB} .*

- i. *If $i \in \mathcal{M}^A$ and $j \in \mathcal{M}^{AB}$, then $i < j$.*
- ii. *If $i \in \mathcal{M}^{AB}$ and $j \in \mathcal{M}^B$, then $i < j$.*
- iii. *If $i \in \mathcal{M}^A$ and $j \in \mathcal{M}^B$, then $i < j$.*

Furthermore, $\mathcal{M}^A \cup \mathcal{M}^B \cup \mathcal{M}^{AB} = \{1, 2, \dots, M\}$ and for any $M \geq 3$ and $\beta^A > 0$, there exist cost-adjusted qualities such that none of the subsets is empty.

Figure 2 graphically illustrates the platforms' price structures given in Proposition 4 for an appropriately chosen i with $1 < i < M$ when all three sets are non-empty.

In monopoly settings, it has been shown that platforms tend to set a high price to the group that is less price sensitive (e.g., Armstrong 2006, Weyl 2010). Our novel result in asymmetric oligopoly is that different price sensitivities endogenously arise due to quality differences, thereby endogenously leading to opposing price structures across competing platforms.

4.4 | Profit Comparison

Our framework also allows for a simple and intuitive characterization of profit rankings: the larger the net quality of a platform on one side, the larger is the equilibrium profit of the platform.

Proposition 5. *Consider two platforms i and j with $v_i^A > v_j^A$ and $v_i^B = v_j^B$. Then, in equilibrium, platform i obtains higher profit than platform j .*

By Proposition 2, the higher-quality platform has larger market shares for both groups and, by Proposition 5, larger profits. The finding that market shares and profits are positively associated has been obtained in oligopoly with price competition and differentiated products absent network effects (Proposition 1 in

Anderson and de Palma 2001 and Proposition 4 in Anderson et al. 2020). It thus extends to platform oligopoly with two-sided single-homing in which user decisions in the two groups are interdependent through cross-group network effects, when platforms are ranked by cost-adjusted quality on one side. More generally, two platforms may be asymmetric with respect to both groups. Then, it might well be the case that a platform with lower market shares in both groups makes a higher profit than a rival (as was mentioned in the introduction and shown in Sato 2021b, confirming Belleflamme et al. 2022, the latter analyzing a linear duopoly setting).

In the proof of Proposition 5, we also establish that platform profit is decreasing in the equilibrium user surpluses (CS^A, CS^B). This fact is used in the proofs of some of the results in Section 5.

4.5 | Network Goods and Two-Sided Platforms with One-Sided Pricing

We ask which of the results in the previous section depend on the platform's ability to charge both user groups. We continue to assume that platforms are asymmetric regarding the primitives of the model with respect to one of the two user groups. We then have to make the case distinction whether or not the asymmetry is on the zero-pricing side.

As a backdrop, let us study asymmetries in the model with a network good.

Remark 6. In the model with network goods, take any two platforms with $v_i > v_j$ for some i, j . Then, in the unique equilibrium, the platform with higher cost-adjusted quality has a larger market share, a higher price-cost margin, and higher profits; that is, $n_i > n_j$, $p_i - c_i > p_j - c_j > 0$, and $\Pi_i > \Pi_j > 0$.

Thus, within-group network effects do not overturn the cross-section result in oligopoly models without any network effects. We now turn to the model with two interdependent user groups and one-sided pricing (i.e., $\max\{\beta^A, \beta^B\} > 0$) under the assumption that marginal costs are zero for the non-paying user group (i.e., the zero pricing side).

First, suppose that the zero pricing side is asymmetric—that is, group A users face platforms that differ in a_i^A —whereas cost-adjusted qualities are the same for all platforms with respect to the paying side—that is $a_i^B - c_i^B = a_j^B - c_j^B \equiv v^B$ for all platforms i, j . We recall that the model is equivalent to the model with network effects in group B after a change of variables to $\tilde{a}_i^B = a_i^B + (\beta^B / (1 - \alpha^A)) a_i^A$ and $\tilde{\alpha}^B = \alpha^B + \beta^A \beta^B / (1 - \alpha^A)$. We write the modified cost-adjusted qualities for group- B users as $\tilde{v}_i^B := v^B + (\beta^B / (1 - \alpha^A)) a_i^A$.

Because marginal costs for group- A users are zero, the platform asymmetry is due to differences in a^A only. Thus, as long as users in group B care about the participation of group- A users ($\beta^B > 0$), any asymmetry of the primitives in group A gives rise to an induced asymmetry in group B —that is, $a_i^A > a_j^A$ implies that $\tilde{v}_i^B > \tilde{v}_j^B$ if and only if $\beta^B > 0$. Then, as follows from Remark 6, the platform with a higher quality on the zero-pricing side has a

larger market share and a higher price-cost margin on the paying side leading to larger profits. Otherwise, if $\beta^B = 0$, the paying side (group B) is isolated from the group- A asymmetry and the outcome for group B will be symmetric: all platforms have the same price-cost margin, the same market share, and the same profit, despite the fact that market shares are asymmetric on the zero pricing side.

Second, suppose that the zero pricing side (group A) is symmetric (i.e., $\alpha_i^A = \alpha_j^A \equiv \alpha^A$), whereas cost-adjusted qualities v_i^B are asymmetric on the paying side. The change of variable in quality then is $\tilde{v}_i^B := v_i^B + (\beta^B / (1 - \alpha^A))\alpha^A$, which leads to a parallel shift of \tilde{v}_i^B compared to the setting with $\beta^B = 0$. As directly follows from Remark 6, the higher-quality platform has a larger market share and a larger price-cost margin on the paying side leading to larger profits. For $\beta^A > 0$, the asymmetry of primitives regarding group B also leads to an asymmetric outcome for group A : The platform that is of higher cost-adjusted quality has a larger market share on the zero pricing side. By contrast, for $\beta^A = 0$, market shares regarding group- A users must be symmetric because the platform does not charge this group of users.

Results on market shares under one-sided pricing are broadly in line with those under two-sided pricing, but there are some differences. As Proposition 2 shows, if cost-adjusted qualities are asymmetric in group B only, then under two-sided pricing, $v_i^B > v_j^B$ implies $n_i^A > n_j^A$ if and only if $\beta^A > 0$ or $\beta^B > 0$. By contrast, under one-sided pricing, we have the same ranking of market shares $n_i^A > n_j^A$ if and only if $\beta^A > 0$; when $\beta^A = 0$, $n_i^A = n_j^A$ holds under one-sided pricing because group- A users enjoy no network benefits and platforms lack a pricing instrument to subsidize group A . Consequently, when $\beta^B = 0$ and $\beta^A > 0$, platform i with higher quality for group B than platform j (i.e., $v_i^B > v_j^B$) obtains a larger group- A market share ($n_i^A > n_j^A$) under two-sided pricing but the same market share ($n_i^A = n_j^A$) under one-sided pricing.

It is obvious that negative price-cost margins for group- B users can not be an equilibrium outcome under one-sided pricing because this necessarily leads to losses of the platform. This implies that the possibility of heterogenous cross-subsidization strategies in equilibrium (as shown in Proposition 4) cannot arise under one-sided pricing.

The restriction to one-sided pricing does not affect the profit ranking with one exception: A platform that provides a higher cost-adjusted quality for one user group (and the same for the other group) makes higher profits under one-sided as well as under two-sided pricing, unless the asymmetry applies to the non-paying group and this group does not exert a positive cross-group effect on the paying group in which case the different platforms make the same profit despite the asymmetry.

5 | Active Platforms, Platform Quality, and Compatibility

In this section, we derive comparative statics results under a given selection rule at the pricing stage (e.g., always selecting the price equilibrium with maximal surplus for group k with $k \in \{A, B\}$). We investigate comparative statics properties of three shocks or

interventions: changes to the set of active platforms, changes to the incumbent platforms' characteristics under free entry, and partial compatibility.

5.1 | Active Platforms

We provide comparative statics results about the effects of an additional platform becoming active. In other words, the number of active platforms increases from $M \geq 2$ to $M + 1$ platforms. Under the selection criterion of asymptotic stability at the participation stage, this is equivalent to exogenous entry of a new platform. Taking the set of active platforms as a strict subset of available platforms, adding a platform to the set of active platforms amounts to this additional platform having overcome the curse of unfavorable user beliefs. If an entrant cannot overcome this curse, it cannot successfully use divide-and-conquer strategies according to which it would subsidize one group to make sure that some users from this group join and monetize through the other group (on the use of divide-and-conquer strategies with homogeneous platforms, see, for example, Caillaud and Jullien 2003). Under our logarithmic network effect functions even extreme subsidization does not achieve this (despite the fact that platforms are horizontally differentiated) and active platforms do not adjust their prices in response to entry by an entrant facing unfavorable beliefs. However, such subsidization or other choices outside our model (such as advertising) may shift beliefs.

For ease of exposition, in what follows we speak of platform entry when an additional platform joins the set of active platforms. As a benchmark, consider the case in which all platforms are symmetric.

Remark 7. Suppose platforms are symmetric, that is, $v_i^A = v^A$ and $v_i^B = v^B$ for all $i = 1, \dots, M$. Then, an exogenous platform entry always lowers prices and platform profits, and raises user benefits for each group.

This result is in line with findings in standard oligopoly. By contrast, Tan and Zhou (2021) provide an example in a symmetric setting such that exogenous entry can lead to higher prices, higher platform profits, and lower user benefits.

To understand the difference between our finding and the one in Tan and Zhou (2021) of the effect of entry on prices, consider the special case that $c_i^k = 0$, $k \in \{A, B\}$. However, suppose that the network effect takes the more general form $\gamma^{kl}(n_i^l)$ —in our model, $\gamma^{kk}(n_i^k) = \alpha^k \log n_i^k$ and $\gamma^{kl}(n_i^l) = \beta^k \log n_i^l$ for $l \neq k$. As Tan and Zhou (2021) show, the symmetric equilibrium price p^{k*} can be written as

$$p^{k*} = \frac{M}{M-1} - \frac{1}{M-1} \sum_{k' \in \{A, B\}} \left(\frac{\partial \gamma^{k'k}(n_i^{k*})}{\partial n_i^k} \Big|_{n_i^k = 1/M} \right). \quad (13)$$

The first term is the standard market power term, which is decreasing in M but the second term may be increasing in M depending on the shape of $\gamma^{k'k}(\cdot)$. The second term reflects the fact that network effects drive pricing incentives, which depend on the number of active firms. Entry reduces the relative size

advantage of a platform that attracts an additional unit mass of group- k users because each of the $M - 1$ competitors loses $1/(M - 1)$. Holding marginal network benefits constant, entry lowers the incentives to reduce price. When marginal network benefits are not constant, the extent to which size advantage matters depends on the marginal network benefit functions $(\partial \gamma^{k'l} / \partial n_l^k)_{n_l^k=1/M}$. Tan and Zhou (2021) use an example with linear network benefit functions (i.e., $\gamma^{kk}(x) = \bar{\alpha}^k x$ for $\bar{\alpha}^k \geq 0$ and $\gamma^{kl}(x) = \bar{\beta}^k x$ for $l \neq k$) and show that p^{k*} and Π^* are increasing in M for M sufficiently small and $\bar{\alpha}^k + \bar{\beta}^l$ sufficiently large (see their Example 4). Although the second term continues to be increasing in entry in our setting (as in the linear case), for any parameter values satisfying our assumption $\alpha^k + \beta^l < 1$, it is always dominated by the decrease of the first term, leading to price-decreasing entry.¹⁴ Intuitively, with a strictly concave network benefit function, platforms have a stronger incentive to reduce price after entry than with a linear network benefit function because the marginal network benefit increases with entry.

We turn to the case in which platforms are asymmetric. We establish below that in that case, one user group may be worse off after a new platform enters (whereas the other group is better off).

Proposition 6. *Consider the effect of entry of platform E on user surplus.*

1. *For any entrant platform (v_E^A, v_E^B) , there exists a value $\underline{\beta}$ such that entry increases user surplus for both groups if $\beta^A < \underline{\beta}$ and $\beta^B < \underline{\beta}$.*
2. *For any given $\beta^A > 0$, selecting the CS^k -minimizing equilibrium for group $k \in \{A, B\}$, there always exists a platform type (v_E^A, v_E^B) such that the user surplus of this group k decreases after entry.*
3. *Selecting the CS^k -minimizing equilibrium for group $k \in \{A, B\}$, entry increases the user surplus of at least one user group.*

Proposition 6-1 shows that in the absence of cross-group network effects ($\beta^A = \beta^B = 0$), H^k increases with entry and, thus, user surplus must go up. Although this property is satisfied in standard oligopoly models without network effects, it is a priori not obvious that this result carries over to a model with network effects. The reason is that, under full participation, the entering platforms attract users from the incumbent platforms reducing the network benefits of the users active on incumbent platforms due to reduced participation on those platforms. Nonetheless, in our setting, entry of a new platform always benefits users if cross-group network effects are zero (or sufficiently weak). Proposition 6-1 establishes this result.

In the presence of cross-group network effects, entry of a platform may hurt one of the user groups, as established in Proposition 6-2. The proof of Proposition 6-2 indicates that an instance of entry that lowers the user surplus for one group (group A) is the entry of a platform that primarily caters to the needs of the other user group (v_E^B large and v_E^A small and possibly negative); one may call such a platform “highly focused” on one user group. In such a case, entry will not add surplus to group- A users, but reduces the market shares of the incumbent platforms. This reduces the network benefits that group- A users enjoy from joining

existing platforms or the incumbent platforms’ incentive to attract group- A users. Then, such entry lowers group- A user surplus.

Although entry may harm one user group, Proposition 6-3 establishes that at least one user group benefits from entry. Our results indicate that the welfare effects of entry of a two-sided platform crucially depend on the characteristics of the entrant. Entry of a highly focused platform may hurt the users in the group that the entrant is not focused on.

According to the existing literature, entry may hurt users even under platform symmetry (Tan and Zhou 2021). Other works address welfare effects of platform entry in different market environments. Correia-da-Silva et al. (2019) consider homogeneous-product Cournot platform models and examine the welfare effects of exogenous entry. They find that platform entry may reduce consumer surplus of all groups due to the fragmentation of network benefits; Gama et al. (2020) find such a result when the platform caters to a single user group and this group experiences network effects. Anderson and Peitz (2020) consider an asymmetric platform oligopoly in which one user group multi-homes and the other single-homes (competitive bottleneck) and study the consumer welfare effect of platform entry (see footnote 15).

With mutual cross-group network effects and asymmetric platforms, entry might affect the platforms’ price structure qualitatively differently than for networks (i.e., settings with within-group network effects only), as shown in the following remark for two symmetric incumbent platforms (and proved in the Online Appendix).

Remark 8 (Entry into a previously symmetric market). Suppose that two symmetric platforms with the cost-adjusted quality (v_I^A, v_I^B) were active before entry. Then, in the pre-entry equilibrium, price-cost margins are positive for both groups. If $\beta^A > 0$ and $\beta^B > 0$, there exist entrant types (v_E^A, v_E^B) such that there is a post-entry equilibrium in which $\mu_I^B < 0$ and $\mu_E^A < 0$ and other entrant types such that $\mu_I^A < 0$ and $\mu_E^B < 0$.

Hence, the asymmetry induced by entry or exit qualitatively affects the price structure of competing platforms. Platform entry may lead to negative price-cost margins of incumbent platforms for one user group in situations in which their margins were positive absent entry. Here subsidization of one group is a response to entry of a platform that is more attractive to that group. According to Remark 8, the entry of a platform with (v_E^A, v_E^B) leads to $\mu_I^B < 0$ and $\mu_E^A < 0$. The entrant platform captures a large share of group- B users but a small share of group- A users. This occurs if v_E^A is relatively small and v_E^B is relatively large compared with v_I^A and v_I^B . Conversely, if v_E^A is relatively large and v_E^B is relatively small, then entry leads to $\mu_I^A < 0$ and $\mu_E^B < 0$.

Next, we turn to the effect of platform entry on profit. As we show in the following proposition, entry may increase the profit of incumbent platforms due to the asymmetry it introduces in the market. This also implies that industry profits increase.

Proposition 7. *There exist pre-entry conditions and entrant types such that entry increases the profit of incumbent platforms. In such a case, entry necessarily reduces the user surplus of one group.*

TABLE 2 | A numerical example illustrating Proposition 7 with $\alpha^A = \alpha^B = 0$, $\beta^A = \beta^B = 0.95$, and $(n_I^A, n_I^B) = (0.4, 0.1)$.

	n_I^A	n_I^B	μ_I^A	μ_I^B	Π_I^A	Π_I^B	Π_I
Pre-entry	0.5	0.5	0.10	0.10	0.05	0.05	0.10
Post-entry	0.4	0.1	1.27	-3.11	0.51	-0.31	0.20
Difference	-0.1	-0.4	1.17	-3.01	0.46	-0.36	0.10

Proposition 7 establishes that incumbent platforms may benefit from entry of another platform. For example, when incumbents are symmetric before the entry, they compete rather fiercely with each other for both user groups. Suppose now that a platform focused on group B enters the market. After entry, the incumbent platforms sacrifice a large share of one user group B even though they now subsidize that group because they offer much lower cost-adjusted quality than the entrant to this group. At the same time, because the incumbent platforms have become less inclined to compete for group- B users through an increase of their group- A user base, they increase their margins for group A . In doing so, they lose rather few group- A users to the entrant because the entrant offers a low cost-adjusted quality to that group. This softening of competition for group- A users increases incumbents' profits from that group, which may dominate the profit loss in the market for group- B users. This scenario is likely to arise when there are strong mutual cross-group network effects (i.e., β^A and β^B are large), the entrant is highly focused on one group (i.e., v_E^k is very large and v_E^l is very low), and there is a fierce competition between incumbents (e.g., incumbents being symmetric). The following numerical example illustrates this finding, which is summarized in Table 2.

Example 1. Let $\alpha^A = \alpha^B = 0$, $\beta^A = \beta^B = 0.95$ and suppose that two symmetric incumbents of the same type $(v_I^A, v_I^B) = (0, 0)$ are active before entry. Then, in the pre-entry equilibrium, each incumbent obtains market shares $(n_I^{A*}, n_I^{B*}) = (0.5, 0.5)$, sets price-cost margins $(\mu_I^{A*}, \mu_I^{B*}) = (0.1, 0.1)$, and earns profit $\Pi_I^* = 0.1$. Suppose now that platform E with $(v_E^A, v_E^B) \simeq (-7.43, 9.66)$ enters. In the post-entry equilibrium, the entrant obtains market shares $(n_E^{A**}, n_E^{B**}) = (0.2, 0.8)$ and sets price-cost margins $(\mu_E^{A**}, \mu_E^{B**}) \simeq (-3.5, 3.812)$. In this equilibrium, each incumbent obtains market shares $(n_I^{A**}, n_I^{B**}) = (0.4, 0.1)$, sets price-cost margins $(\mu_I^{A**}, \mu_I^{B**}) \simeq (1.27, -3.11)$, and makes profits $\Pi_I^{**} \simeq 0.2 > \Pi_I^*$. Hence, entry leads to an increase of the incumbents' profits. We also note that entry harms group- A users but benefits group- B users (as the entrant is focused on the latter group): Before entry, user surplus of each of the two groups is $CS^{A*} = CS^{B*} \simeq -0.065$. After entry, we observe that $CS^{A**} \simeq -2.54$ and $CS^{B**} \simeq 4.54$.

This result stands in stark contrast to results in standard oligopoly: Entry increases the competitive pressure and therefore reduces incumbents' price-cost margins and profits. It also stands in contrast to the setting with within-group network effects only, where entry always increases the equilibrium aggregate, as follows from Proposition 6-1. As a result, price-cost margins and platform profits are necessarily lower after entry in the model with network goods. Recall that the setting with cross-group network effects and one-sided pricing is equivalent to a

setting with within-group network effects only. Therefore, price-cost margins and platform profits are also lower after entry under one-sided pricing. Our finding in Proposition 7 relates to the finding by Tan and Zhou (2021): in their more flexible but symmetric setting, platform entry can increase incumbent platforms' profits, whereas in our model, such an increase cannot happen under symmetry but requires asymmetries between platforms.

5.2 | Shocks to Incumbent Platforms Under Free Entry

To study long-run competition, we consider platform competition under free entry of "fringe" platforms. To this end, we extend the baseline framework by incorporating symmetric entrants as in Anderson et al. (2013).

Suppose that, along with $M_I \geq 1$ incumbents $\{1, \dots, M_I\}$, $\bar{M}_E \geq 1$ (potential) entrants $\mathcal{E} := \{M_I + 1, \dots, M_I + \bar{M}_E\}$ choose whether to enter. Entrants $e \in \mathcal{E}$ all have the same characteristics $(a_e^A, a_e^B, c_e^A, c_e^B)$ and incur entry cost $K > 0$ to become active. Incumbent platform $i \in \{1, \dots, M_I\}$ has characteristics $(a_i^A, a_i^B, c_i^A, c_i^B)$ that may differ from those of other platforms. We assume that entry costs are such that some of the potential entrants become active and the number of potential entrants \bar{M}_E is sufficiently large to ensure that the number of active entrants M_E is less than \bar{M}_E . In our analysis we ignore integer constraints.

Let $\pi_E(H^A, H^B)$ be the post-entry profit of an entrant when it optimally chooses the action variables (h_E^A, h_E^B) and the values of the aggregates are given by (H^A, H^B) . Specifically, the post-entry profit with aggregates (H^A, H^B) , $\pi_E(H^A, H^B)$, is given by

$$\pi_E(H^A, H^B) := \Pi_E(h_E^A(H^A, H^B), h_E^B(H^A, H^B), H^A, H^B).$$

Using this notation, we define the free-entry equilibrium as follows.

Definition 2. The number of active entrants M_E constitutes a free-entry equilibrium if the triple (H^A, H^B, M_E) satisfies the following conditions:

$$\pi_E(H^A, H^B) - K = 0, \tag{14}$$

$$\sum_{i=1}^{M_I} h_i^A(H^A, H^B) + M_E h_E^A(H^A, H^B) = H^A,$$

$$\sum_{i=1}^{M_I} h_i^B(H^B, H^A) + M_E h_E^B(H^B, H^A) = H^B.$$

The definition of free-entry equilibrium endogenizes the number of active entrants M_E through the zero-profit condition (14). Entrants sequentially enter as long as the post-entry profit exceeds the entry cost, and the entry stops once additional entry becomes unprofitable. Using Definition 2, we examine the welfare effects of a shock to the incumbent platforms' characteristics, which is captured by a change in $(a_i^A, a_i^B, c_i^A, c_i^B)$ for $i \in \{1, \dots, M_I\}$.

In the aggregative game analysis of standard oligopoly, the zero-profit condition of entrants uniquely pins down the value of single aggregate (e.g., Davidson and Mukherjee 2007, Ino and Matsumura 2012, Anderson et al. 2013, 2020). Because consumer surplus is determined solely by the value of the aggregate, any change in the competitive environment, such as incumbents' investment and platform mergers does not affect consumer surplus, as long as there is at least some entry. By contrast, with two-sided platforms, the zero profit condition (14) only pins down the relation between the two aggregates (H^A, H^B). Therefore, the competitive environments are no longer necessarily neutral to the user surplus in each group and the total user surplus. In a particular setting, we establish a *strong see-saw property*: any change in the competitive environment that increases user surplus of one group reduces user surplus of the other group.

For instance, suppose that an incumbent invests in group- A benefit a_i^A so that entrants' network size on group A decreases. In a standard oligopoly, competition for group- A users becomes more intense due to the incumbent's investment. As an equilibrium response, fewer entrants will join, so the competition for group- A users becomes weaker. In two-sided markets, a more subtle strategic interaction may exist due to network effects and implied changes in the two-sided pricing structure.

Proposition 8. *Consider a free-entry equilibrium with a non-empty set of entrants. Then, any change in competitive environments that increases the surplus of one user group decreases the surplus of the other user group. Formally, holding the parameters ($\alpha^A, \alpha^B, \beta^A, \beta^B, a_E^A, a_E^B, c_E^A, c_E^B, K$) fixed, compare two free-entry equilibria that differ in other parameters. Denoting the equilibrium surplus of the two user groups under the two settings by (CS^{A*}, CS^{B*}) and (CS^{A**}, CS^{B**}) , we have that*

$$(CS^{A*} - CS^{A**})(CS^{B*} - CS^{B**}) < 0.$$

Because the post-entry profit of Π_E is decreasing in user surpluses (CS^A, CS^B) , to keep Π_E constant, any increase in CS^A must be compensated by a corresponding decrease in CS^B . Hence, Proposition 5 establishes a strong see-saw property in user surplus.

The strong see-saw property poses a challenge to competition authorities evaluating business practices of large incumbent platforms in an environment with fringe platforms. Because an incumbent platform's practice generically benefits users in one group at the expense of those in the other group, the competition authority must decide which group to protect (or which weights to give them in an overall consumer welfare ranking). In the context of e-commerce, some authorities focus on private consumers, which is in line with a narrow interpretation of the consumer welfare standard. For instance, Khan (2017) argues that such an approach fails to recognize other harms of incumbent platforms' practices, including the harm to third-party sellers, which can be included under a broader interpretation of the consumer welfare standard. Proposition 8 establishes that there is a conflict between what benefits users of one group and what benefits the other. This conflict is inevitable in two-sided platform competition with free entry of the type studied in this article.¹⁵

Regarding the welfare property of free entry, note that Tan and Zhou (2021) show the following: When taste shocks follow the type-I extreme value distribution, platform entry is socially excessive (see their Lemma 2 and the following paragraph). Thus, platform entry is socially excessive in our model when platforms are symmetric.¹⁶

5.3 | Partial Compatibility

We address how an increase of the degree of compatibility affects market shares, prices, and user surplus. Suppose that there are only within-group network effects and, thus, each user group can be analyzed in isolation. Partial compatibility implies that a fraction of network effects are industry-wide. It is gained if some of the functionalities are available to all users, not only those on the same platform, but also those on competing platforms. The fraction of functionalities available to all users is denoted by λ , the degree of compatibility. An example of a regulatory intervention with the goal to increase compatibility is Article 7 in the Digital Markets Act (DMA) in the European Union. According to this regulation, a gatekeeper of a number-independent interpersonal communications service must "make the basic functionalities of its number-independent interpersonal communications services interoperable with the number-independent interpersonal communications services of another provider." The provision applies only to gatekeeper platforms and interoperability has to be offered upon the request of another provider. As a caveat, our model does not accommodate the situation that some but not all of the competing providers ask for interoperability.

Partial compatibility allows users to benefit from the presence of users on different platforms.

$$\begin{aligned} u_i^k &= a_i^k - p_i^k + \lambda \alpha^k \log \sum_{j=1}^M n_j^k + (1 - \lambda) \alpha^k \log n_i^k + \varepsilon_i^k \\ &= a_i^k - p_i^k + (1 - \lambda) \alpha^k \log n_i^k + \varepsilon_i^k. \end{aligned}$$

By Remark 2 there exists a unique price equilibrium for any value of $\lambda \in [0, 1]$.

How does the equilibrium depend on the degree of compatibility λ ? The general answer is the following and has been formalized by Crémer et al. (2000) in the Katz-Shapiro model: A decrease in compatibility increases the quality differentiation between two asymmetric platforms. The larger platform, which relies relatively less on access to the other platform's users, gains a competitive advantage, and competition between the two platforms is softened.¹⁷

Our framework provides related insights (for more details and additional insights see Online Appendix A.2).¹⁸ When the degree of compatibility is increased, lower-quality platforms gain market share whereas higher-quality platforms lose and, thus, industry concentration (e.g., measured by the HHI) goes down (Proposition A.1 in Online Appendix A.2). If the asymmetry between platforms is sufficiently small, increased compatibility reduces the intensity of price competition and platforms set higher prices. Nevertheless, users benefit from increased compatibility because the direct effect dominates the effect on prices. As

we establish in the duopoly case, if the asymmetry is sufficiently large, the platform with the higher cost-adjusted quality sets a lower price after an increase in compatibility (Proposition A.2 in Online Appendix A.2).

Restricting attention to the duopoly case, we also address the effect of compatibility on industry concentration under cross-group network effects, where we consider the case that partial compatibility applies to both user groups. Confirming the result derived under within-group network effects only, we show that compatibility mitigates industry concentration—that is, for each user group $k \in \{A, B\}$, an increase of the degree of compatibility decreases the group- k market share of the platform that is of higher cost-adjusted quality for each user group (see Proposition A.3 in Online Appendix A.2).

Multi-homing is an alternative way for each user to “better” interact with other users. The more users multi-home, the larger is the number of users any single-homing user has access to (for given relative market shares of platforms among single-homing users). However, our model of partial compatibility does not translate into a model in which a fraction λ of users multi-home.¹⁹ A single-homing user has then access to all single-homing users on the same platform and all multi-homing users and the network benefit function becomes $\alpha^k \log(\lambda + (1 - \lambda)n_i) = \alpha^k \log(\lambda \sum_{j=1}^M n_j^k + (1 - \lambda)n_i)$, which is different from the function under partial compatibility, $\lambda \alpha^k \log \sum_{j=1}^M n_j^k + (1 - \lambda) \alpha^k \log n_i = (1 - \lambda) \alpha^k \log n_i$. Furthermore, we can not use aggregative games tools in such a model. We note that under linear network effects, the two functions would be the same.²⁰

6 | Partially Covered Markets

In our analysis, we postulated that the market is fully covered for both user groups. However, if some users choose not to participate, the market is only partially covered. Below, we present three versions with partial coverage in decreasing order of tractability.

In our first version of partial coverage, the outside option is subject to the same network effects and idiosyncratic taste shocks as the for-profit platforms (for details, see Online Appendix A.3.1). This applies if choosing the outside option consists in choosing a non-commercial offer that is free of charge. For example, this could be an open-source software platform that is provided free of charge to two different user groups or a programming language that is provided free of charge and brings together users and developers. It could also be a content platform that brings together content providers and consumers at no charge.

Our model in Section 2 can easily be generalized and accommodate such a free platform by adding platform 0 that offers quality a_0^k to side $k \in \{A, B\}$ at zero price, $p_0^k = 0$. Following our change of variables, platform 0 then offers (h_0^A, h_0^B) , which is independent of the choices offered by the for-profit platforms, and we write $H^k = \sum_{i=0}^M h_i^k$. The equilibrium characterization of the participation game (Remark 1) and the existence of a non-empty ordered set of price equilibria (Proposition 1) generalize to the introduction of such an outside option. Also, the characterization results of a price equilibrium in Section 4 continue to hold. In the presence of outside options for each user group, it is of interest to consider

comparative statics in the attractiveness of the outside options: As the outside option becomes more attractive for group- k users, user surplus of this group will increase, whereas user surplus of the other group will (weakly) decrease.

In our second version of partial coverage, users make an opt-in decision upfront—that is, they decide whether to participate before observing their taste realization and before observing the prices set by the platforms. After learning their taste realization and prices charged by the platform, users decide which platform to join. Thus, users do not observe prices at the opt-in stage, but correctly predict equilibrium prices, given the parameters of the model (for details, see Online Appendix A.3.2). The idiosyncratic outside option is given by $a_0^k + \theta \varepsilon_0^k$, where, for each group- k user, ε_0^k is an i.i.d. draw from some distribution function.

We adopt the concept of fulfilled expectation equilibrium in the spirit of Katz and Shapiro (1985), where users make opt-in decisions by forming an expectation over preferences and prices, and platforms take the aggregate user base (N^A, N^B) as given when they set prices. The opt-in decisions determine the total demand for each of the two user groups. Given any platform prices, we characterize the unique interior participation equilibrium according to which each platform and the outside option attract some users from each group (Proposition A.4) and illustrate for the case that outside options are exponentially distributed. We also show the existence of a non-empty ordered set of price equilibria and that (under some condition) price equilibria are ordered by group- k user surplus such that the CS^k -maximal equilibrium is the CS^l -minimal equilibrium for $k, l \in \{A, B\}$, $l \neq k$ (Proposition A.5). The characterization results of a price equilibrium in Section 4 continue to hold, but comparative statics analysis in this setting is generally complicated. Nevertheless, if users enjoy within-group network effects alone, we provide a comparative statics result with respect to the base attractiveness of the outside option a_0^k for group- k users. A higher a_0^k reduces user participation and thereby the network benefits that can be obtained. As we show for the case in which the idiosyncratic component of the outside option is exponentially distributed, for an increase of a_0^k sufficiently large, this may end up hurting users overall (Remark A.3) because users who decide not to opt in exert a negative externality on users who opt in. We note that this result is not driven by platform asymmetry and can also be obtained under symmetry.

In our third version of partial coverage, users simultaneously decide whether and which platform to join, after observing platform prices (for details, see Online Appendix A.3.3). We characterize the interior participation equilibrium in this setting, in which the aggregates from the setting with full coverage are replaced by augmented aggregates that account for the fact that some users abstain from joining a platform (Proposition A.6). The existence of a price equilibrium holds for sufficiently unattractive outside options. Despite being a natural way to introduce outside options, this version turns out to be rather intractable: Although we can show the existence of a unique interior participation equilibrium, we do not obtain closed-form demand functions. Turning to the profit-maximization problem of the platforms, it becomes harder to establish the existence of implicit best-response functions. One could proceed with computational methods to study the properties of solutions of

the system of first-order conditions (and also numerically check the validity of the identified solution). We leave such numerical explorations for future work.

7 | Discussion and Conclusion

We propose a two-sided single-homing model of platform competition that features differences between platforms with respect to (i) marginal costs incurred for users of the two groups and (ii) the utility that platforms offer to their users (for given participation rates by both groups). Incorporating platform asymmetries provides a rich setting that allows us to explore the relative outcomes of platforms in equilibrium and the impact of exogenous shocks on the performance of different platforms. After establishing the existence and uniqueness of the participation equilibrium for a given set of active platforms, we characterize the equilibrium outcome under price competition and obtain insights with respect to exogenous platform entry, incumbent platform investments under free entry, and mandated partial compatibility. Our analysis makes use of the IIA structure of the demand systems of both groups. Platform profits can be written as functions of two action variables and their aggregates (as the sum of action variables across platforms).

We follow the seminal work on platform competition and focus on the platform's pricing decisions. Our analysis can be extended to cover other design decisions if these decisions are taken concurrently with the pricing decision.²¹ It is also interesting to extend the analysis to environments in which platforms do not charge any fees to one user group, but can use non-price strategies that directly affect the attractiveness of the platform for that group. For example, social media platforms typically charge advertisers but do not charge end users and devise non-price strategies to attract end users. We leave extensions in this direction for future work, as they are outside the canonical platform competition model.

We make the functional form assumption that network effects enter as logarithmic functions of participation numbers of each group into user utility and that users experience taste shocks that lead to a logit structure. This specification can be seen as a special case of the model of Tan and Zhou (2021). Although such a logarithmic specification of network effects is popular in empirical work, most previous theoretical work assumed linear network effects and few theoretical studies allow for more general forms of network effects (Hagiu 2009, Weyl 2010, Belleflamme and Peitz 2019, Tan and Zhou 2021). Within the logit demand setting, any generalization beyond logarithmic network effects would make it impossible to obtain closed-form solutions for the participation equilibrium and to subsequently write the platforms' profit functions as a function of their action variables and the aggregates thereof.

In our framework, users draw idiosyncratic taste shocks that enter their utility function as a stand-alone value. Users may also be heterogeneous regarding their sensitivity to network size (i.e., group- k users may differ in their network effect parameters α^k and β^k). Unfortunately, the aggregative game framework is not sufficiently malleable to accommodate such a heterogeneity. We can think of our analysis as analyzing the model in which all users

are of the "average" type ($\mathbb{E}[\alpha^k], \mathbb{E}[\beta^k]$). Presuming that there is an equilibrium also with heterogeneous network effects, we conjecture that our characterization results hold by continuity in a setting close to the limit when the heterogeneity disappears. We also conjecture that introducing heterogeneous network effects would lead to composition effects: In the case of heterogeneous cross-group network effects, a platform that has a higher share of group- A users than another platform will attract relatively more group- B users that are particularly sensitive to network effects. Such a setting is related to Ambrus and Argenziano (2009) who show the emergence of asymmetric equilibria in a symmetric duopoly model in which users are heterogeneous with respect to their sensitivity to network size.²²

Arguably, the canonical model of platform competition features two-sided single-homing. This specification is widely adopted by the literature, including by Armstrong (2006), Jullien and Pavan (2019), and Tan and Zhou (2021). In various real-world environments, however, some users in one or both groups can multi-home (see, e.g., Armstrong 2006 and Anderson and Peitz 2020, for the former and Bakos and Halaburda 2020, Adachi et al. 2023, and Teh et al. 2023, for the latter).²³ As pointed out in Section 5, when a fraction of users multi-homes, our model loses the aggregative game property and our analysis does not extend to such more-complex homing decisions.

Acknowledgments

Open access funding enabled and organized by Projekt DEAL.

Endnotes

- ¹We removed the words "and/or partially covered" from this quote, which is taken from page 522. In Section 6, we address partial coverage.
- ²For an empirical application to the German magazine market, see Kaiser and Wright (2006). The model with asymmetric platforms is used to analyze platform taxation (Belleflamme and Toulemonde 2018) and the relationship between profits and market shares (Belleflamme et al. 2022).
- ³The operations research literature has looked at monopoly pricing and assortment problems in the presence of direct network effects and multinomial logit demand; see, for example, Du et al. (2016) and Wang and Wang (2017). Wang and Wang (2017) include an explicit solution of the participation game when network effects are logarithmic.
- ⁴Most of the existing theoretical literature postulates linear network effects (e.g., Armstrong 2006). However, in many real-world applications, a strictly concave function looks more plausible.
- ⁵Our model also nests the standard logit oligopoly model without an outside option (see, e.g., Anderson et al. 2020)—in this case, $\alpha^A = \alpha^B = \beta^A = \beta^B = 0$.
- ⁶We note that under logit demand, linear network effects would always lead to participation equilibria being interior. However, this model does not allow for closed-form demand functions.
- ⁷This amplification can be formally expressed as follows:

$$\left[I + \begin{pmatrix} \alpha^A & \beta^A \\ \beta^B & \alpha^B \end{pmatrix} + \begin{pmatrix} \alpha^A & \beta^A \\ \beta^B & \alpha^B \end{pmatrix}^2 + \dots \right] \times \begin{pmatrix} \Delta_{ij}^A \\ \Delta_{ij}^B \end{pmatrix} = \left[I - \begin{pmatrix} \alpha^A & \beta^A \\ \beta^B & \alpha^B \end{pmatrix} \right]^{-1} \begin{pmatrix} \Delta_{ij}^A \\ \Delta_{ij}^B \end{pmatrix} \\ = \begin{pmatrix} \Gamma^{AA} & \Gamma^{AB} \\ \Gamma^{BA} & \Gamma^{BB} \end{pmatrix} \begin{pmatrix} \Delta_{ij}^A \\ \Delta_{ij}^B \end{pmatrix}.$$

- ⁸If such unfavorable beliefs are associated with the status of being an entrant, entry will not be possible unless the entrant platform is able to shift beliefs by taking further actions (see our analysis of entry in Section 5). For further work on incumbency advantage as a result of dynamic user choice in the presence of network effects, see Biglaiser et al. (2022).
- ⁹Although we could not rule out multiple price equilibria with asymmetric platforms, all the numerical examples that we looked at have a unique equilibrium.
- ¹⁰The figures illustrate the shape of $(\tilde{H}^A, \tilde{H}^B)$ for parameter values $\alpha^A = \alpha^B = 0.1$ and $\beta^A = \beta^B = 0.3$. Panel (a) does so with $M = 2$, $(v_1^A, v_1^B) = (3, 0)$, and $(v_2^A, v_2^B) = (0, 3)$, where $v_i^k = a_i^k - c_i^k$, panel (b) with $M = 3$, $(v_1^A, v_1^B) = (0, 1)$ and $(v_2^A, v_2^B) = (v_3^A, v_3^B) = (0, 0.5)$.
- ¹¹In this case, we strengthen our assumption that $\alpha^k + \beta^l < 1$ to $|\alpha^k| + \beta^l < 1$ for all $k, l \in \{A, B\}$, which implies that $|\alpha^k| < 1$, $k \in \{A, B\}$, and ensures the asymptotic stability of the interior participation equilibria. See also footnote 1 of Online Appendix A.1.
- ¹²We take note that the models differ not only with respect to the homing assumption. Most importantly, Anderson and Peitz (2020) do not allow for setting the participation fee on the multi-homing side, which would complicate their analysis because of feedback loops, but instead assume that platforms set participation levels.
- ¹³See Proposition 11 in the online appendix of Anderson and Peitz (2020). The relevant case for comparison is the one with positive cross-group network effects, which means that $\gamma < 0$ according to their notation.
- ¹⁴With a logarithmic network benefit function, our assumption $\alpha^k + \beta^l < 1$ is needed to obtain compact strategy sets and prevent a platform from setting infinitely low price on one side to enjoy monopoly power on the other side.
- ¹⁵Anderson and Peitz (2020) establish a strong see-saw property of exogenous entry for purely ad-funded media platforms in competitive bottleneck with advertisers exerting a negative effect on consumers. As they show, their competitive bottleneck model fails to give rise to the see-saw property if advertisers exert a positive effect on consumers or if media platforms set a fee on the consumer side.
- ¹⁶For conditions on the distribution function of the taste shocks that lead to excessive entry, see also Tan and Zhou (2024).
- ¹⁷Crémer et al. (2000) study connectivity between asymmetric internet backbone providers in the Katz-Shapiro model (Katz and Shapiro 1985). Our statement is a minor rephrasing of their explanation that “connectivity creates a quality differentiation between the two networks. The larger backbone, which relies relatively less on access to the other backbone’s customers, gains a competitive advantage, and competition between the two backbones is softened.” (Crémer et al. 2000 at page 435)
- ¹⁸There is an important difference in the nature of the asymmetry in our framework compared to the one considered by Crémer et al. (2000). In the latter, full compatibility makes platforms symmetric, as the asymmetry between platforms is due to size differences in the installed base. By contrast, in our framework, even under full compatibility, platforms are asymmetric.
- ¹⁹For example, such partial multi-homing may be the result of users having installed a multi-homing device such a meta search engine that allows them to access all platforms or may reflect an environment in which a fraction of users has chosen the option to be visible on different messaging services, whereas others declined the offer.
- ²⁰Even with linear network effects, there may be an interesting interplay between multi-homing and compatibility; for an analysis in symmetric Hotelling duopoly with linear network effects, see Doganoglu and Wright (2006).
- ²¹In this case, platforms compete in utilities $\bar{u}_i^k = a_i^k - p_i^k$ for users and platforms may increase value a_i^k . In particular, suppose that there is a one-to-one relationship between value a_i^k and per-user cost c_i^k that depends on the user group and the identity of the platform. Thus, we can write $c_i^k(a_i^k)$, and platforms set a_i^k such that $c_i^k(a_i^k)' = 1$.
- ²²They find that platform 1 sets a lower price for group- k users than platform 2 and a higher price for uses of the other group l . Less-sensitive group- k users then buy from platform 1, whereas more-sensitive group- k users from platform 2. Because of the heterogeneity, there is endogenous differentiation between platforms, which allows them to make positive profits in equilibrium.
- ²³Work on ad-funded media platforms has also looked at the effects of viewer multi-homing; see, e.g. (2016) and Anderson et al. (2019).

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Supporting Information

Additional supporting information can be found online in the Supporting Information section.

Data S1