Discussion Paper No. 04-53

Semiparametric Estimation of Consumption Based Equivalence Scales – The Case of Germany

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Non-technical Summary

Recent reforms of the social security system in Germany will almost certainly lead to the merger of social benefits (Sozialhilfe) and unemployment assistance (Arbeitslosenhilfe) by the year 2005. When this reform takes effect, up to 1.7 million individuals and their families will obtain new needs-oriented social benefits (Arbeitslosengeld II) in addition to the over 2.3 million employable individuals currently receiving similar social benefits. In contrast to the current benefit system, the system of new social benefits is intended to provide stronger incentives to the unemployed to search for and accept new jobs (Hartz, 2002). However, gross need for social benefits will continue to be calculated on the basis of equivalence scales which determine the equivalent income between different demographic groups of households such that both types of households can achieve the same standard of living. The design of the equivalence scale scheme will therefore essentially drive the incentives for job seekers. This means that, finding appropriate values for the equivalence scales will assume even greater importance in the future.

This paper presents a comprehensive empirical study of the semiparametric estimation of consumption based equivalence scales. Equivalence scales for Germany are estimated by applying Wilke's (2003) estimator for the extended partially linear model suggested by Blundell et al. (1998) to the most recent version of the German income and consumption survey data (EVS 1998). For estimation purposes the data is segmented into homogenous groups of households conditional on employment status of the household head, the west/east issue and on whether the household owns property or not. The estimated consumption based equivalence scales are mostly lower than the equivalence scales of the German social benefits system.

It is difficult to infer policy recommendations from the results because of the large standard errors of the estimates and because of some degree of theoretical arbitrariness involved in the underlying modelling approach. However, the estimations provide some indications that on average the costs for additional persons in a household are *at least* covered by the standard rates of German social benefits. In the light of recent decisions of the Federal Constitutional Court (Bundesverfassungsgericht) concerning the costs of children and growing discussion of demographic transitions in Germany, it is not apparent from the estimation results that equivalence scales need to be increased for households with children.

Semiparametric Estimation of Consumption Based Equivalence Scales - The Case of Germany

Ralf A. Wilke*

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Abstract

Consumption based equivalence scales are estimated by applying the extended partially linear model (EPLM) to the 1998 Income and Consumption Survey (EVS) of Germany. The chosen flexible semiparametric specification is able to capture a large variety of functional forms of household expenditure shares; it yields \sqrt{N} —consistent parameter estimates and is consistent with consumer theory. The model specification seems to be appropriate for many demographic groups of the survey population. The estimated equivalence scales are mostly lower than the expert equivalence scales of the German social benefits system and the OECD scales. The large standard errors of the estimates indicate that there is still unexplained noise in the data even after constructing homogenous data segments for the estimations.

Keywords: semiparametric estimation, wild bootstrapping, equivalence scales, social benefits

JEL: C14, C31, D12, H53

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1 Introduction

Recent reforms of the social security system in Germany will almost certainly lead to the merger of social benefits (Sozialhilfe) and unemployment assistance (Arbeitslosenhilfe) by the year 2005. When this reform takes effect, up to 1.7 million individuals¹ and their families will obtain new needs-oriented social benefits (Arbeitslosengeld II) in addition to the over 2.3 million employable individuals currently receiving similar social benefits. In contrast to the current benefit system, the system of new social benefits is intended to provide stronger incentives to the unemployed to search for and accept new jobs (Hartz, 2002). However, gross need for social benefits will continue to be calculated on the basis of equivalence scales and these will therefore essentially drive the incentive scheme. This means that, finding appropriate values for the equivalence scales will assume even greater importance in the future. This paper provides estimates for this purpose by applying the estimator of Wilke (2003) to the most recent version of the German income and consumption survey.

Equivalence scales are often used in welfare systems to compute households' need for financial support. These scales determine whether and to what extent households are eligible for social benefit transfer payments. To make things more precise, let us state what is usually understood as an equivalence scale:

Equivalence scales deflate household money income [...] according to household type to "calculate the relative amounts of money two different types of households require in order to reach the same standard of living". (Muellbauer, 1977)

The purpose of social benefit transfer payments is to ensure that all households enjoy a minimum standard of living. If equivalence scales are incorrectly codified, the standard rates for social benefits will not coincide with their intended values. If transfer payments are too high, the respective household may receive more money than it needs to reach the minimum standard of living. If some members of the household are unemployed, this might reduce incentives for these members to look for and accept a new job. The high level of social benefits in Germany may therefore be one of the reasons for a the country's large share of long-term unemployment. Indeed, Christensen (2003) observes that low skilled unemployed who have a reservation wage above social benefits often remain unemployed for very long periods, whereas unemployed people with reservation wages below or equal to social benefits tend to find employment more quickly. Since social benefits are financed out of public funds,

¹Note that the set of social benefits recipients and the set of unemployment assistance recipients are not disjoint.

they could result in an unnecessarily high tax burden for the economy as a whole if the rate was too high. On the other hand, if the value of the equivalence scales is too low, the respective household may not be able to achieve the minimum standard of living. The standard rates must therefore be determined with great care.

Many theoretical and empirical contributions have already examined the issue of how to find a *reasonable* equivalence scale for this purpose. In most most cases one of the following three approaches has been adopted.

In the first approach, "expert scales" are devised based on the opinion of social security experts. Table 1 presents scales in this class. In the case of Germany the scales are supported by several example calculations. The main criticism of this approach is its lack of theoretical justification which means that the resulting equivalence scales appear ad hoc to a large extent.

Table 1: Comparison of existing equivalence scales schemes

	German	social ber	nefits	OECD (1982)
	Standard rates*	Average	gross needs**	
		West	East	
Single households				
without children (S0)	1.00	1.00	1.00	1.00
with one child (S1)	1.65	1.64***	1.68***	1.50
Couple households				
without children (C0)	1.80	1.58	1.62	1.70
with 1 child (C1)	2.45	2.04	2.11	2.20
with 2 children (C2)	3.10	2.47	2.58	2.70
with 3 children (C3)	3.75	2.92	3.03	3.20

^{*} Federal Law for Social Benefits (BSGH), children of age 7-13

The second approach uses data about the satisfaction of a household with its income for the determination of subjective equivalence scales. A criticism of this method is that the results depend on subjective valuations. Other more objective criteria would be preferable. However, this method, does allow equivalence scales to be estimated with sophisticated

^{**} Reporting date 1/JAN/2003, source: Federal Ministry of Health and Social Security

^{***} Child aged < 7

econometric methods. See Bellemare, Melenberg and van Soest (2002) for a comparison of different estimators using the German Socio Economic Panel (GSOEP).

While the third approach, -consumption based equivalence scales- is to some extent supported by consumer theory, this approach also involves a degree of arbitrariness (Pollak and Wales, 1992, Kohn and Missong, 2002). These scales are determined on the basis of households' consumption behavior. Empirical consumption based equivalence scales can be estimated using cross section consumption data. The purpose of this paper is to estimate consumption based equivalence scales for Germany using the semiparametric estimator of Wilke (2003) and the 1998 income and consumption survey of Germany. In the past consumption based equivalence scales were mainly estimated using parametric linear demand systems. See for example Blundell and Lewbel (1991) for Britain and Merz and Faik (1995) for Germany. However, empirical evidence has shown that in many cases the demand functions of households are nonlinear, see for example Blundell, Paschardes and Weber (1993). An extension to nonlinear parametric or partially linear expenditure systems is straight and accounts for this misspecification. New developments in consumer theory show that this model choice may also be inappropriate if demographic variation is taken into account (Blundell, Duncan and Penkadur, 1998). In this light the parametric quadratic specification of Kohn and Missong (2003) for Germany appears crucial. Blundell et al. (1998) recently introduced a semiparametric approach, the so called extended partially linear model (EPLM) that is based on the work of Pendakur (1999). Thanks to its nonparametric element it is more flexible than parametric models. Wilke (2003) develops an implementable estimator and derives its theoretical properties. A small application with the British Family expenditure survey indicates that there is empirical evidence. In this paper the EPLM is applied to the 1998 German income and consumption survey (EVS) using Wilke's estimator. The model specification appears to be appropriate for a variety of estimations not only conditional on demographics but also by segmenting the data according to the employment status of the head of the household, depending on whether the household owns property or not and by distinguishing between east and west German households. Most of the estimated average equivalence scales are lower than or equal to the expert scales used for the calculation of the gross needs for social benefits in Germany. However, precise policy recommendations are not derivable because of the philosophical problems involved in the theoretical approach and due to the large standard errors of the estimated parameters. However, the results can be considered to provide the first comprehensive empirical result for this class of models and they clearly suggest that there are economies of scales in consumption for larger households.

The paper is structured as follows: Section 2 presents the theoretical framework of the

underlying consumer theory. Section 3 sketches the system of social benefits in Germany and explains the importance of equivalence scales. Section 4 introduces the econometric model for the estimation of the extended partially linear model. Section 5 describes the data, and the estimation results are presented in section 6. The last section concludes and provides suggestions for further research.

2 Consumer Theory

This section presents the underlying microeconomic framework for the econometric analysis. Since we consider cross section data which should be recorded at a given point of time, we ignore calender time variations. The expenditure shares are given by

$$y = m(x, z, p),$$

where m is the vector of expenditure shares for commodities j = 1, ..., J, x is the log. of total expenditure, z is a household specific finite dimensional vector of observable characteristics and p is the $J \times 1$ vector of log prices. The equivalence scales between two groups z and z_0 is defined as $exp(\alpha(z,p))$. It can be identified from the respective cost functions c(p,u,z) and $c(p,u,z_0)$, which correspond to the minimum expenditures in order to achieve a specific utility level u. More specifically, we have

$$\alpha(z, p, u) = lnc(p, u, z) - lnc(p, u, z_0),$$

where $\alpha(p,z)$ is the log. of the equivalence scale and z_0 denotes the reference group. Then household z requires $exp(\alpha(z,p,u))$ of the reference household's income to reach the same utility level u. Cost functions and expenditure shares are directly related because from Shepard's lemma we have $m(x,z,p) = \partial lnc(p,u,z)/\partial p$. This relationship suggest the identification of equivalence scales from consumption data. However, this approach involves the problem of not observing u but knowledge of the utility level is indispensable for welfare comparisons. This fundamental identification problem is not yet solved (Pollak and Wales, 1992). Stronger assumptions are required in order to induce that the equivalence scale do not depend on the utility level. Unfortunately, stronger assumptions like the independence of the base utility, i.e. $\alpha(z,p,u) = \alpha(z,p)$, are still empirically untestable. Kohn and Missong (2002) therefore conclude that "observed demand quantities do not suffice for a unique identification of equivalence scales – a fact that renders welfare comparisons impossible". Moreover, the utility arising from leisure is ignored by uniquely focusing on utility coming from consumption. Consequently, the leisure related part is not captured by a model that is

solely estimated with consumption data. If we assume that utility can be divided into consumption and leisure elements, we would only model and estimate the consumption-related utility element.

A variety of functional forms for expenditure shares are consistent with economic theory. A popular linear specification is the so called Price-Independent Generalized Logarithmic (PIGLOG, see Muellbauer, 1976). This arises from indirect utility functions which are linear in the log. of total expenditure. Complete demand systems such as the AIDS (Deaton and Muellbauer, 1982) and the ELES (Lluch, 1973) are based on linear specifications of the expenditure shares. Nowadays, there is enough empirical evidence that this specification has to be generalized, since for many goods there is a nonlinear relationship. A simple generalization is the partially linear model (PLM) which includes as a special case the quadratic model. This class of models has attractive theoretical properties and there is empirical evidence for the quadratic specification (Blundell et al., 1993). Nevertheless, based on Blundell et al. (1998) -who found that if the expenditure share of one commodity is PIGLOG than consumer theory induces the same property for all the other commodities- demand systems based on expenditure shares belonging to the class of PLMs should not be used to estimate equivalence scales. As a consequence the PLM can drastically restrict the functional forms for all expenditure shares in order to be still consistent with consumer theory. The PLM is therefore a crucial specification, since there is empirical evidence that some expenditure shares do and others do not belong in the PIGLOG class. For example the expenditure share for food is linear in Britain (Blundell et al., 1998). Similarly, we find in this paper that some expenditure shares seem to be linear whereby others are nonlinear in Germany. Inspired by Pendakur (1999) and by the findings for Britain, Blundell et al. (1998) suggest an alternative system of expenditure shares that accounts for demographic decomposition, that is nonlinear in log of total expenditure and consistent with consumer theory. However, it requires the assumption that the equivalence scales are independent of the baseline utility. Given a smooth unknown function g_i , Blundell et al. (1998) state the following lemma for the extended partially linear model (EPLM):

If expenditure shares have the EPLM form:

$$m_j(x, z, p) = \frac{\partial \alpha(z, p)}{\partial p_j} + g_j(x - \alpha(z, p)), \tag{1}$$

then if the reference share equations

$$m_j(x, z_0, p) = g_j(x, p) \tag{2}$$

are consistent with consumer theory and $\exp(\alpha(z,p))$ is weakly concave and homogeneous of degree zero in $\exp(p)$, expenditure shares given by (1) are also consistent with consumer theory.

The derivation of the EPLM and further underlying theory can be found in Pendakur (1999) and in Blundell et al. (1998). It uses the main tools of dual theory and skillfully exploits the definition of baseline utility independent equivalence scales. Interestingly, the class of functionals in equation (1) belongs to the shape invariant models because we have simple vertical and horizontal (due to $\alpha(z,p)$) shifts of an unknown smooth function g_j . Apparently, the shape of the function g_j may differ across the commodities, whereby $\alpha(z,p)$ does not. The horizontal shift $\alpha(z,p)$ is of particular interest because its exponential transformation, i.e. $\exp(\alpha(z,p))$, is the equivalence scale. The EPLM is therefore a general theoretical model for the estimation of equivalence scales that are independent of the base utility. It requires very mild assumptions on the functional form of the reference share equations (2) and it is therefore flexible enough to capture a wide class of functionals in the empirical analysis.

3 Equivalence Scales for Social Benefits

- the Case of Germany

In Germany social benefits for more than 1.23 million households are mainly calculated according to a method based on equivalence scales. See table 2 for a descriptive overview of the year 2001. ² Each household has a defined income requirement in order to achieve a minimum standard of living. The gross needs (Bruttobedarf) for social benefits should meet this amount. The net entitlements in table 2 correspond to the gross needs for social benefits minus the current income of the household. The net entitlements are the amount of money finally paid to the household. The gross needs for social benefits consists of two parts: firstly, the standard rate (Regelsatz) that accounts for the demographic composition, i.e. the number of adults and the number of children living in the respective household, and secondly, payments for housing, heating and other supplementary general costs that are calculated on a case by case basis by a responsible administrator at the social assistance office.

²Table 2 contains information about regular means of subsistence only. Households in specific circumstances, e.g. disabled, who receive social benefits are not included because it is not possible to identify these households in the data.

The demographic composition of a household plays an essential role in determining the standard rates. For the latter the social planner computes the equivalent income between the demographic groups of households on the basis of an equivalence scale that is codified in the Federal Law of Social Benefits (BSHG). Table 1 presents the equivalence scales of the German social security system and the widely accepted "OECD (1982) scales". It also presents the demographic compositions that are supsequently considered for the estimations. It is evident that standard rates in Germany are higher than the OECD rates. If we look at the - empirically relevant - average gross needs, the opposite appears to be the case. The empirical scales computed from the average gross needs are lower than the OECD scales, with the exception of the scale between single person households and single people with a child. This interesting observation has not been noted to date in the related empirical literature about Germany, e.g. Merz and Faik (1995) and Kohn and Missong (2003).

Demographic group	#		average gross needs average net entitlement total gross needs total net entitlements	total gross needs	total net entitlements
All households	1.235.326	098	394	1.062.663 294	486.718.444
Single households					
without children	605.020	581	326	351.705.194	197.312.874
with 1 child $(< 18 \text{ years})$	190.696	919	429	175.219.574	81.790.457
with 2 children $(< 18 \text{ years})$	106.664	1.214	470	129.541.414	50.116.993
Couple households					
without children	120.819	893	407	107.836.320	49.117.174
with 1 child $(< 18 \text{ years})$	67.016	1.101	464	73.778.288	31.103.827
with 2 children $(< 18 \text{ years})$	52 523	1.336	489	70.181.561	25.691.999
more than 2 children (< 18 years)	44.365	1.756	559	77.905.957	24.804.852

Table 2: Some facts about German "Sozialhilfe" in 2001: regular means of subsistence. Gross needs and net entitlements are per month in euros. Source: Federal Statistical Office (2001), own calculations.

4 Econometric Model

The foregoing section suggests that the EPLM would be an appropriate framework for the econometric analysis. The advantages of this semiparametric approach are also clear from the viewpoint of an econometrician: the risk of misspecification of the functional form of the expenditure shares is lower than for purely parametric models. At the same time the rate of convergence of the parameter of interest, e.g. of the equivalence scale parameter, is the same as in parametric frameworks $N^{1/2}$, where N is the number of observations. Purely nonparametric estimators are ruled out as possible alternatives as we intend to estimate a parameter of interest. In this paper we use the recently developed estimator of Wilke (2003) which is based on the work of Härdle and Marron (1990), which provides applicable solutions to the identification problems involved in this framework, and which has better finite sample properties.

Assuming the availability of cross section data at a given point of time with given log prices p. Define $m_j^0(x) = m_j(x, z_0, p)$ as the share equation of the reference household type z_0 and $m_j^1(x) = m_j(x, z, p)$ for any $z \neq z_0$. According to the restrictions of the EPLM we may write equation (1) as (Blundell et al., 1998)

$$m_j^1(x) = a_j + m_j^0(x - c),$$
 (3)

where the function m_j^1 is a vertically and horizontally shifted translation of the reference function m_j^0 . Our empirical focus is on the estimation of the parameter c, which corresponds to the log. of the equivalence scale. The parameter a_j reflects the elasticity of the equivalence scale with respect to the commodity price j. For the estimation of equation (3) we always compare two homogeneous subgroups of households. For each subgroup we have a sample of observations with different sample sizes N_0 and N_1 . In order to identify the equivalence scale, we need a consistent estimate of c. Let us therefore introduce the estimation model and the identification conditions as given by Wilke (2003).

Suppose we have samples $(Y_{ji}, X_i)_{i=1,...,N_0}$ and $(S_{ji}, W_i)_{i=1,...,N_1}$ with j = 1,...,J. Let us assume the following functional relationships:

$$Y_{ji} = m_j^0(X_i) + U_{ji}$$

 $S_{ji} = m_j^1(W_i) + V_{ji} \text{ for } j = 1, ..., J$

with $E(U_{ji}|X_l) = E(V_{ji}|W_l) = 0$ for all i, l and j. U_{ji} and V_{ji} have finite fourth moments and the pairs U_{ji}, V_{ji} are mutually independent. $X_i \in \mathcal{X}_1$ and $W_i \in \mathcal{W}$ are i.i.d random variables with realizations on compact sets with twice differentiable densities $f_x(x) > 0$ and $f_w(w) > 0$

for all x and w. Furthermore, let the true parameter values a_{j0} for $j=1,\ldots,J$ and c_0 be in the interior of open subsets in IR. Let us denote the set $\{x-c\} = \mathcal{W}_c$ for all $x \in \mathcal{X}_1$. The following assumptions ensure the identifiability of the parameters: $\mathcal{W} \cap \mathcal{W}_c$ is nonempty for all c. This condition implies that the two nonparametric functions overlap on their support for all c. There exists a j such that the function $m_j^0(x-c)$ is not periodic on $\mathcal{W} \cap \mathcal{W}_c$. This means that for at least one commodity there is no $c \neq c_0$ with $m_j^0(x-c) = m_j^0(x-c_0)$ for all $x-c \in \mathcal{W} \cap \mathcal{W}_c$. This is required for a unique solution in c. Furthermore, there exist a j such that the function $m_j^0(x-c)$ is nonlinear on $\mathcal{W} \cap \mathcal{W}_c$ for all c. This is required for the joint identification of a_j and c. Under some technical assumptions on the nonparametric estimates of m^0 , m^1 and f, the solution to the problem

$$\min_{a,c} L_{N_0,N_1}(a,c) = \sum_{j=1}^{J} \frac{\int_{\mathcal{W} \cap \mathcal{W}_c} [\hat{m}_j^1(x) - a_j - \hat{m}_j^c(x)]^2 dx}{\int_{\mathcal{W} \cap \mathcal{W}_c} \hat{f}_x(x) dx},$$
(4)

yields consistent parameter estimates, where $\hat{m}_{j}^{c}(x)$ denotes the nonparametric estimate of the function m_{j}^{0} after shifting it horizontally by the parameter c.³ Under further technical conditions the parameter estimates converge at rate $N^{1/2}$ and they are normally distributed (Wilke, 2003). We use here the HM4SE⁴ which is an improved version of the Härdle and Marron (1990) estimator.⁵ The estimator is implemented as follows:

- 1. Estimate the nonparametric functions m_j^0 and m_j^1 for j = 1, ..., J. In our applications we use the Nadaraya-Watson estimator and the local linear smoother with constant bandwidths that are chosen according to the plug-in method as given in Fan and Gijbels (1995) and multiplied by a positive constant.
- 2. Estimate the parameters a_i given c by least squares, i.e.

$$\min_{a_j} \int_{\mathcal{W} \cap \mathcal{W}_c} (\hat{m}_j^1(x) - a_j - \hat{m}_j^c(x))^2 dx$$

for any c and all j. Denote the estimate \hat{a}_{j}^{c} .

³In fact the estimation objective function (4) does not involve the shape invariance restriction across all household types z because it is restricted to the comparison of two household types only. The equivalence scales could be estimated for all groups simultaneously by using $\hat{m}_{j}^{0}(x) = a'_{j}z + \hat{m}_{j}(x - c'z)$, where a_{j} and c are column vectors of the length of the total amount of demographic groups and z is a dummy vector of the same length.

⁴We do not consider the PR4SE estimator which is based on the estimator of Pinkse and Robinson (1995) because it seems to have a worse finite sample performance.

⁵Stengos and Wang (2002) and Pendakur (2004) use a penalizing function in order to overcome the finite sample difficulties.

3. Solve problem (4) numerically in c conditional on \hat{a}_{j}^{c} in order to obtain \hat{c} . Denote the function $L_{N_0,N_1}(c|a_c)$ as the loss function in c.

4.
$$\hat{a}_j = \hat{a}_j^{\hat{c}}$$
 for $j = 1, ..., J$.

The least squares estimation in step three is not efficient, since the variance of the non-parametric estimators is a function depending on the location on the support where it is evaluated. This variance function might be estimated by (wild)-bootstrap and used for constructing weights in the least squares estimation.

The standard errors of the parameter estimates are computed from the empirical distribution of the parameter estimates obtained by wild bootstrapping. Wild bootstrapping in the EPLM is described in the appendix A I.

5 Data

The 1998 German Income and Consumption Survey (EVS) is used for the estimations. The survey data is based on 49,720 households from both west and east Germany with more than 900 variables (demographic, consumption and income related). It is a quota sample with voluntary participation and is therefore not representative with regard to the whole population (Kühnen, 1999). Single people and blue-collar workers for example have a lower rate of reply. The same is true for households on either low or high incomes. Projection factors are available to generate representative results. The analysis in this paper does not use these factors as there is no obvious reason for doing so as the analysis is performed for homogenous demographic groups conditional on total household expenditure, employment status and separately for east and west Germany. However, we can only assume that the observed consumption behavior in each of the segments is nevertheless representative for the whole population segment. It is also important to mention that the sample and census design design have significantly changed from previous EVSs used for example by Merz and Faik (1995) and Kohn and Missong (2003). Due to the voluntary participation of the households and the generally long recording period of one year ⁶, attrition was too high in the past (Chlumsky and Ehling, 1997). For this reason the responsible Federal Statistical Office (Statistisches Bundesamt) reduced the recording period from one year to three months. In the author's view, this should also increase the quality of the observed variables. They should become more precise (due to higher motivation of the recording households). Moreover, the probability should be greater that variables such as employment status, demographic decomposition

⁶There are also to some extend records on a monthly basis (Feinaufschriebe).

j	Commodity group		
1	Food	2	Clothing
3	Housing	4	Energy
5	Interior decoration	6	Health care
7	Transport	8	Communication
9	Leisure and travelling	10	Education
11	Board and lodging	12	Other goods

Table 3: The commodity groups used for the estimations. The numbering of the commodity groups coincides with the tables in the appendix.

and prices do not in fact vary, as the former are recorded by interviews at the beginning and at the end of the recording period only. For estimations we only use observations that are recorded in the second or in the third quarter of the year, i.e. during summer time, in order to wipe out calender time effects on the consumption structure of the households. In terms of commodity aggregation we are confronted with the following trade-off: if we use all possible consumption items available in the data (several hundred) there are insufficient observations and in many cases one commodity may substitute a very similar one. There is therefore no alternative but to work with some aggregated commodity groups. However, aggregation must be done carefully if it is not to induce a measurement error which could seriously bias the estimation results. For the estimation we consider 12 aggregated commodity groups which are presented in table 3. These groups are directly taken from the 1998 EVS data. The aggregation of the commodities is performed by the German Federal Statistical Office and the commodity categories are constructed such that each reflects a central need of the households. This categorization is harmonized with international standards, i.e. COICOP 1998.⁷ As a result it seems reasonable to adopt this categorization for our purposes. Kohn and Missong (2003) decrease the number of commodity groups by merging some of them. ⁸ There are many zero entries in the data for some commodity groups. These correspond to either zero expenditures or to missing values. In the following analysis the zero entries are treated as zero expenditures. This assumption can be justified by the fact that participation in the survey is completely voluntary. Some commodity groups contain expenses for durables, e.g. transport expenditures contain expenses for car purchases. The following the

 $^{^{7}}$ There are some minor deviations from the international standard which are mainly for comparative reasons with older issues of the EVS-data.

⁸A sensitivity analysis with respect to the total number of commodity groups might be an interesting extension of this paper.

analysis is restricted to non-durables, since we are interested in transfer payments for regular means of subsistence. Expenditures for durables are therefore subtracted.

The survey population is segmented into demographic groups based on the structure shown in table 1. Furthermore, we distinguish between west and east German households, whether the head of a household is either full time employed or not employed at all, e.g. unemployed, retired and whether the household owns property or not. Again, only observations that are recorded during the summer quarters of the year are used. The construction of the homogenous sub-samples revealed the sample size in some cells decreased such that reliable semiparametric estimations become impossible (see table 7 in the appendix). For this reason the single household with one child (S1) group dropped out completely and it is therefore impossible to make a comparison with other groups.

6 Estimation Results

The equivalence scales are estimated by comparing separately the following demographic groups: (s0,c0), (c0,c1), (c1,c2), $(c2,c3)^9$ for west and for east Germany, conditional on the status of the head of the household (full time employed or non employed) and by distinguishing between households which hold property and which do not. These distinctions are made for the following reasons: the price system and income distribution in west and east Germany differ and we expect that the latter are horizontally shifted. For this reason we should expect different consumption behavior at a given level of total expenditures. A distinction between full time employed and non-employed seems to be appropriate because we should expect that the income of households with a non-employed (retired, unemployed) household head typically depends to a certain extent on social security transfer payments. It is interesting to see whether the consumption behavior of households which rely on a social transfer scheme is different from the consumption behavior of households with working income. Unfortunately, not many households only receive social benefits (all other transfers schemes depend at least to some extent on the prior working income). For this reason the group of non-employed household heads is chosen with the drawback that a large number of these households has income related to prior working income. Nonetheless, this allows us to relate the consumption structure of demographic groups of households that are more homogeneous with respect to leisure time. If leisure and consumption are perfectly separable and if in addition the design of social transfers payment schemes does not affect the behavior

⁹Again, the notation of table 1 is used.

of households, the results should be identical for the two leisure-groups. The analysis in this paper does not consider the age of children as it is the case in the German social security system, where the equivalence scale increases when the children become older. This simplification ensures that there are enough observations in each data segment considered. It is easier to deal with this issue in parametric estimation frameworks because the sample size in each segment can be smaller. See Kohn and Missong (2002) for a possible segmentation which takes account of the age of the children in the households. Other demographic groups are not considered for similar reasons. ¹⁰ The estimations reveal that the model specification is appropriate in most cases. The estimated change in the equivalent income for additional adults or additional children is always in an economically plausible range (0-100%). However, in the case of the first child the estimated equivalence scale is often below this range, i.e. it is negative. There is one plausible explanation for this: parents with a young child reduce total expenditures because they are most likely to stay at home and they therefore consume less by reducing their expenditure on leisure, alcohol etc. A distinction according to the age of children would therefore be interesting. Positive scales should be anticipated if the first child is already a couple of years old. If we turn attention back to all the cases again, the shifted nonparametric functions appear at a glance to fit acceptably in most cases. The loss function possesses in most cases a unique minimum for plausible values of the equivalence scales, i.e. $c \in [0, ln2]^{11}$ Tables 5-6 in the appendix report the detailed estimation results and present an extended coefficient of determination for the parametric part of the EPLM, the \mathbf{eR}^2 which is introduced in appendix A.II. Since the \mathbf{eR}^2 is mostly in the range 0.3-0.6it is evident that the simple transformation with two parameters yields a convincing fit for survey consumer data. This clearly indicates that the large standard errors of the parameter estimates come from the variance of the first step nonparametric estimates. Therefore, it seems that the model (4) is appropriately specified for the EVS 1998. The reported standard errors are computed from the empirical distribution of 500 wild-bootstrap estimates. Note that each 100.000 Nadaraya-Watson and local linear smoothing estimates are performed in the bootstrap estimation of the standard errors. It is observed that the standard errors are often greater when using the local linear smoother as nonparametric estimator. This is in accordance with the fact that the variance of the local linear smoother increases sharply as

¹⁰For example sex of the household head, whereby disability of household members is not observed. A skillful extension of the semiparametric approach that accounts for a variety of regressors remains for future research. Chen, Blundell and Kristensen (2001) move in this direction; however, their identification conditions are subjected to hardly any practical verification.

¹¹Figures which illustrates this are available upon request from the author.

the density of the marginal distribution approaches zero. This efficiency problem is evident at the boundary of the support of the nonparametric function. The choice of the bandwidth and the choice of the support for the nonparametric estimation therefore affect the results. In many cases the estimated parameters possess large standard errors and therefore have to be considered as of limited reliability. However, the sensitivity of the results was checked by weakly varying the boundaries of the support of the nonparametric functions. The chosen bandwidths are obtained by the plug-in method suggested in Fan and Gijbels (1995). The bandwidths used are mostly in the range 0.2 - 0.5.

A summary of the point estimates is given in table 4. This compares them with the expert equivalence scales of the German social benefits system, the OECD equivalence scales, the empirical evident values computed from the gross needs and with the estimation results of Bellemare et al. (2002). The reported ranges of the estimation results are based on the two point estimates obtained and are not therefore based on distributional information. However, the reported intervals can be roughly used as an indication of where the true values may be located. It is apparent that the upper bound of this range is mostly below the value suggested by the expert equivalence scales of the German social security system or by the OECD, whereby the lower bound is slightly below. It is difficult to judge whether there are systematic differences in the results for the homogeneous subgroups. In any case the results suggest that there are economies of scale for larger households, i.e. the percentage increase due to additional households declines with the household size.

Intuitively, the estimated equivalence scales correspond to the average increase in household income (conditional on east/west and on the employment status of the household head) such that the household is able to maintain the same standard of living if an additional member (adult or child) is added. The word average means that it is the empirical mean for all members of the respective data segments. It is therefore an estimate of the mean equivalence scale. According to economic theory, the model of equation (4) assumes that equivalence scales do not depend on the utility level. The estimates cannot provide any information about a reasonable absolute amount of gross needs for social benefits for the single person household (S0).

¹²In a earlier version of this paper the bandwidth was chosen to be three times the optimal bandwidth. This high degree of oversmoothing was conducted in order to reduce the variance of the first step nonparametric estimates which was much greater when including expenditures for durables. As a result the parameter estimates based on the two nonparametric estimators diverged to a greater extent.

		Additional adult	adult	First child	Additional child	child
		C0/S0	C1/S1	C1/C0	C2/C1	C3/C2
German social benefits						
Standard rates [†]		%08	49%	36%	27%	21%
Average gross needs 2003	west	28%	24%	29%	21%	18%
	east	62%	26%	30%	22%	17%
OECD (1982)		%02	47%	29%	23%	19%
Consumption based, EVS 1998	89					
Employed west	owns property	51-56%	*	-1007% 8%	8%	10%
	does not own property	41%	*	%9080 -	13-19%	11-13%
Employed east	owns property	*	*	-34 - $-30%$	12 - 13%	*
	does not own property	50-72%	*	-0704%	3-6%	*
non employed west	owns property	20-25%	*	23-25%	11-12%	*
	does not own property	40 - 47%	*	*	*	*
Subjective, SOEP 1998						
Bellemare et al. (2002)		34-45%**	$5-33\%^{**}$	-20 - 44%**	I	I

Table 4: Increase of equivalent income: comparison of the estimation results to policy rules of table 1. † BSHG, children of age 7-13. * Too few observations. ** This is the range of point estimates spanned by the various methods; children of age 7-12 years.

7 Summary and Outlook

This paper presents a comprehensive empirical study of the semiparametric estimation of consumption based equivalence scales. Equivalence scales for Germany are estimated by applying Wilke's (2003) estimator for the extended partially linear model suggested by Blundell et al. (1998) to cross section EVS survey data for 1998. For estimation purposes the data is segmented into homogenous groups of households conditional on employment status of the household head, the west/east issue and on whether the household owns property or not. The estimated consumption based equivalence scales are mostly lower than the equivalence scales of the German social benefits system.

It is difficult to infer policy recommendations from the results because of the large standard errors and because of the theoretical arbitrariness involved in this modelling approach. However, the estimations provide some indications that on average the costs for additional persons in a household are at least covered by the standard rates of German social benefits. In the light of recent decisions of the Federal Constitutional Court (Bundesverfassungsgericht) concerning the costs of children and growing discussion of demographic transitions in Germany, it is not apparent from the estimation results that equivalence scales need to be increased for households with children. The way net household entitlements change if the standard rates of social benefits are modified needs to be examined before more detailed policy recommendations can be made. Before attempting to infer policy recommendations from this class of models, the assumption that equivalence scales do not depend on household income should be scrutinized. This can be done by segmenting the data according to some quantiles of the income distribution. Moreover, some readjustments in the model specification may help to reduce the noise in the data. For example, it may be worthwhile clustering the data according to the age of the children in a household as is the case in the German social benefits system. This has improved the specification fit of the parametric system of Kohn and Missong (2003) with the EVS 1988 and 1993. In contrast, conditioning on the type of region (urban, rural etc.), where the respective household stays, did not improve the model fit. This has already been checked by the author. While segmenting the data we are directly confronted with the curse of dimensionality, i.e. the problem of running into data cells with low frequency. For this reason the age of the children is not considered in this paper.

It might also be interesting to extend the theoretical framework to a semiparametric quantile regression estimator. An extension to an estimation framework that accounts for endogeneity as in Blundell, Kristensen and Chen (2001) is also desirable.

Appendix:

A I: Wild bootstrapping in the EPLM.

Resample the observations several times and estimate the unknown regression functions with each resample. This yields an empirical distribution for the parameter estimates of interest. However, naive resampling does not work in the EPLM because the conditions E(U|X=x)=E(V|W=w)=0 would not be imposed. Therefore, wild bootstrapping is performed which induces the required conditions.

Let Q be a random variable with a two point probability distribution H:

$$Q=(1-\sqrt{5})U/2$$
 with probability $(1+\sqrt{5})/2\sqrt{5}$ and
$$Q=(1+\sqrt{5})U/2$$
 with probability $(1+\sqrt{5})/2\sqrt{5}$

This implies E(Q|H) = 0, $E(Q^2|H) = U^2$ and $E(Q^3|H) = U^3$.

Compute the residuals of the first step nonparametric estimation, i.e. $\hat{U}_{ji} = Y_{ji} - \hat{m}_{j}^{0}(X_{i})$ and $\hat{V}_{ji} = S_{ji} - \hat{m}_{j}^{1}(W_{i})$. Then Wild bootstrapping is then carried out as follows:

- 1. Compute $U_{ji}^* = Q\hat{U}_{ji}$ and $V_{ji}^* = Q\hat{V}_{ji}$ for all i and j.
- 2. Compute $Y_{ji}^* = \hat{m}_j^0(X_i) + U_{ji}^*$ and $S_{ji}^* = \hat{m}_j^1(W_i) + V_{ji}^*$ for all i and j.
- 3. Estimate m_j^{0*} and m_j^{1*} using the samples (Y_{ij}^*, X_i) and (S_{ij}^*, W_i) for all j.
- 4. Obtain bootstrap parameter estimates \hat{a}^* and \hat{c}^* .
- 5. Repeat steps one to four in order to get finitely many realizations of \hat{a}^* and \hat{c}^* .

The empirical distribution of \hat{a}^* and \hat{c}^* is used to approximate the distribution of \hat{a} and \hat{c} . For further details concerning the wild-bootstrap method see Härdle and Mammen (1993). Härdle and Mammen (1993) suggests choosing a larger bandwidth for the pilot nonparametric estimates and an optimal bandwidth for the bootstrap estimates. In this paper the same bandwidth is used for the estimation of m_j and m_j^* . This is done for the simple reason of computational feasibility.

A II: Second stage R² in the EPLM

This appendix introduces the extended coefficient of determination for the parametric transformation in the EPLM, the \mathbf{eR}^2 . It determines how well the differences between the two sets of nonparametric functions \hat{m}_0^j and \hat{m}_1^j are explained by the parametric model. However, it only incorporates the point estimates and ignores information about higher moments of the distribution of \hat{m}_0^j and \hat{m}_0^j . Since a large part of the variance of the estimated coefficients is due to the variance of the first stage nonparametric estimates, the suggested \mathbf{eR}^2 cannot be seen as a general goodness of fit measure for the EPLM.

Let us denote $\bar{m}_0^j = \sum_i \hat{m}_0^j(x_i)$ and $\bar{m}_1^j = \sum_i \hat{m}_1^j(x_i)$ as the mean expenditure shares for commodity j. Then the coefficient of determination for commodity j is given by

$$\mathbf{R}_{j}^{2} = \frac{\left[\sum_{i} \left(\hat{m}_{1}^{j}(x_{i}) - \bar{m}_{1}^{j}\right) \left(\hat{m}_{c}^{j}(x_{i}) - \bar{m}_{c}^{j}\right)\right]^{2}}{\left[\sum_{i} \left(\hat{m}_{1}^{j}(x_{i}) - \bar{m}_{1}^{j}\right)^{2}\right] \left[\sum_{i} \left(\hat{m}_{c}^{j}(x_{i}) - \bar{m}_{c}^{j}\right)^{2}\right]}$$

which has the standard properties of the \mathbf{R}^2 , i.e. it is the squared correlation between the nonparametric function \hat{m}_1^j and its predicted value \hat{m}_c^j both evaluated at the observations. Note that the constant \hat{a}_j cancels out. The $\mathbf{e}\mathbf{R}^2$ is simply an average over the \mathbf{R}_j^2 , i.e.

$$\mathbf{e}\mathbf{R}^2 = \frac{1}{J}\sum_{i}\mathbf{R}_{j}^2.$$

A III: Tables and Figures

	Additional adult		First child	
	C0/S0		C1/C0	
	NW	LLS	NW	TLS
West, empl=1, prop=1				
\hat{c} , s.e.	$0.4150 \ (0.0678)$	$0.4150 \ (0.0678) 0.4461 \ (0.0899)$	$-0.0774 \; (0.0687) 0.1032 \; (0.0741)$	$0.1032\ (0.0741)$
$ m eR^2$	0.31	0.31	0.57	0.57
West, empl=1, prop=0				
\hat{c} , s.e.	$0.3439 \; (0.0584)$	$0.3439 \ (0.0584) 0.3439 \ (0.0797)$	$-0.0882 \ (0.0706) -0.0661 \ (0.0822)$	-0.0661 (0.0822)
$ m eR^2$	0.38	0.34	0.37	0.35
West, empl=0, prop=1				
\hat{c} , s.e.	$0.2202 \ (0.2803)$	$0.2202\ (0.2803) 0.1813\ (0.1835) 0.2205\ (0.0781)$	0.2205 (0.0781)	0.2082 (0.0905)
$ m eR^2$	0.27	0.26	0.30	0.33
West, empl=0, prop=0				
\hat{c} , s.e.	$0.3358\; (0.3740) 0.3852\; (0.3197)$	0.3852 (0.3197)	I	I
$ m eR^2$	0.32	0.28		
East, $empl=1$, $prop=0$				
\hat{c} , s.e.	$0.4027 \ (0.2919)$	$0.4027 \ (0.2919) 0.5411 \ (0.1983)$	$-0.0686\; (0.2800) -0.0381\; (0.2122)$	-0.0381 (0.2122)
${ m eR}^2$	0.13	0.12	0.32	0.38

Table 5: Semiparametric estimation results, consumption based, EVS 1998, part I

	Additional child			
	C2/C1		C3/C2	
	NW	LLS	NW	LLS
West, empl=1, prop=1				
\hat{c} , s.e.	0.0812 (0.0466)	0.0812 (0.0602)	$0.0812\; (0.0466) 0.0812\; (0.0602) 0.0954\; (0.0694) 0.0954\; (0.0691)$	$0.0954 \ (0.0691)$
$ m eR^2$	0.80	0.78	0.60	0.59
West, empl=1, prop=0				
\hat{c} , s.e.	0.1241 (0.1091)	0.1758 (0.1231)	$0.1241 \; (0.1091) 0.1758 \; (0.1231) 0.1238 \; (0.2193) 0.1019 \; (0.1609)$	0.1019 (0.1609)
$ m eR^2$	0.57	0.53	0.39	0.34
West, empl=0, prop=1				
<i>ĉ</i> , s.e.	$0.1209\ (0.1966) 0.1093\ (0.1472)$	0.1093 (0.1472)	ı	I
$ m eR^2$	0.40	0.39		
East, empl=1, prop=1				
\hat{c} , s.e.	$0.1096\; (0.1282) 0.1196\; (0.1556)$	0.1196 (0.1556)	I	I
$ m eR^2$	0.55	0.51		
East, $empl=1$, $prop=0$				
<i>ĉ</i> , s.e.	$0.0444 \ (0.0833)$	$0.0444 \ (0.0833) \ \ 0.0710 \ (0.0951)$	I	I
${ m eR}^2$	0.46	0.38		

Table 6: Semiparametric estimation results, consumption based, EVS 1998, part II

Table 7: Sample sizes

		W	est		East	
	Full-time e	employed	Nonemp	oloyed	Full-time e	employed
	owns property	no property	owns property	no property	owns property	no property
S0	652	1.452	864	1.702	61	295
C0	1.577	1.128	2.228	1.084	282	362
C1	1.424	788	330	189	343	331
C2	2.322	805	390	122	489	352
C3	778	207	154	49	62	44

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