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When Money Shouldn't Buy

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When Money Shouldn't Buy*

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Abstract

Banning money in markets for goods like education or health is a common policy to prevent unfair access by the wealthy. We investigate whether this policy is well-targeted for its intended goal. For this, we introduce a fairness criterion called *discrimination-freeness* which requires that goods are allocated independently of wealth. Using a model where willingness to pay increases with income, we find the answer depends critically on the level of wealth inequality. When inequality is high, a transfer ban is a well-aligned policy. It is then no more restrictive than requiring discrimination-freeness. The resulting allocations are constrained-efficient, meaning that any Pareto improvement would be discriminatory. When inequality is low, however, a transfer ban can be overly restrictive, as using monetary transfers may improve outcomes without causing discrimination. Our findings suggest that societies with more equitable wealth distribution may have more flexibility to use price mechanisms than those with high inequality.

JEL classification: D47, D63, H42, I00
Keywords: repugnance, inequality, market design, matching markets

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1 Introduction

Why worry that we are moving toward a society in which everything is up for sale? . . . One [reason] is about inequality . . . . Where all good things are bought and sold, having money makes all the difference in the world.

—Michael Sandel, *What Money Can't Buy: The Moral Limits of Markets*

In many markets, monetary transfers are banned or considered undesirable. For example, the sale of human organs is almost universally prohibited, and in many countries, public schooling is free. From a classical utilitarian perspective, the banning of monetary transfers seems counterintuitive, as price mechanisms are known to increase the efficiency of resource allocation. However, many people feel that certain goods should not be for sale. One ethical concern which is brought forward is that monetary transfers should be banned to mitigate the consequences of wealth inequality: "From the egalitarian's angle of vision, what underlies noxious markets. . . is a prior and unjust distribution of resources. . . [The] fairness of the underlying distribution of wealth and income is extremely relevant to our assessment of markets" (Satz, 2010, p. 5).[1] In a society with high wealth inequality, an individual's willingness to pay (WTP) for goods may reflect their wealth rather than the degree to which they would benefit from consuming the goods (however "benefit" may be defined). Consequently, classical market mechanisms may reinforce existing disparities by allocating resources disproportionately to the wealthy.

In this work, we investigate whether the common policy of banning monetary transfers is a well-targeted policy for ensuring wealth-independent access to goods. Our contribution is twofold. First, we formalize the desire for wealth-independent access with a new fairness criterion, discrimination-freeness, which requires that the allocation of goods does not depend on the agents' wealth. We embed this criterion within an assignment model where heterogeneous wealth and positive income effects make wealth-based discrimination a salient concern. Second, we use this framework to analyze when a simple ban on monetary transfers—a widely used tool that guarantees discrimination-freeness—is a well-calibrated instrument, and when it is unnecessarily restrictive. Our analysis reveals that the answer depends critically on the level of wealth inequality and the distribution of utilities for the goods. It suggests that societies may need to reconsider their tools for ensuring wealth-independent access depending on their degree of wealth inequality.

[1]See also, e.g., Kahneman, Knetsch and Thaler (1986), Frey and Pommerehne (1993), Roth (2007), and Ambuehl, Niederle and Roth (2015).

We show that when wealth inequality is high, a ban on monetary transfers serves as a well-aligned proxy for discrimination-freeness. From an implementability perspective, the two constraints are equivalent: any object allocation a market designer can achieve while respecting discrimination-freeness is also achievable without using monetary transfers. While both approaches have the same reach, the resulting allocations are not fully Pareto-efficient, as they leave unrealized gains from trade where a rich agent could compensate a poor agent for a good, making both better off. However, for high wealth inequality these mechanisms are constrained-efficient because realizing any Pareto-improving trade would necessarily violate the discrimination-freeness constraint. A ban on monetary transfers thus achieves the best possible outcome that can be reached without being discriminatory.

When wealth inequality is low, however, the alignment can break down. An optimal transfer-free mechanism (i.e., one with no transfer-free Pareto improvements) now falls into one of two categories. Depending on the span of the distribution of object utilities, it may be fully Pareto-efficient, leaving no gains from trade. Alternatively, it may leave unrealized gains from trade that are independent of the agents' wealth. We show that depending on the convexity of the distribution of object utilities, the set of implementable allocations under the discrimination-freeness constraint can be strictly larger than under a transfer ban. This implies that when wealth inequality is low, a simple ban can be a strictly more restrictive policy than the normative goal it is meant to serve.

By formalizing the concern that wealth should not grant disproportionate access to certain goods, our paper particularly contributes to the emerging literature on inequality-aware market design (e.g., Dworczak, Kominers and Akbarpour (2021); see also Subsection 1.1). While we set aside other arguments for banning monetary transfers—such as concerns about coercion or commodification—our focus is motivated by the particular relevance of wealth-dependent access in markets that shape fundamental life opportunities. For instance, wealth-based access is a defining feature of markets for consumer goods like cars or clothing, but it is widely considered problematic in markets for education and healthcare, where equal access is a cornerstone of the Universal Declaration of Human Rights (UN General Assembly (1948)).[2]

Our formal analysis proceeds as follows. We consider a market designer who

[2]See, e.g., articles 25 and 26, as well as General Comment No. 14, which states, "Health facilities, goods and services have to be accessible to everyone without discrimination" (UN Committee on Economic, Social and Cultural Rights, 2000).

assigns a set of indivisible objects to a group of agents. Each agent's type is defined by two private components: a vector of object utilities and an initial wealth endowment. These components are independently drawn from common object utility and wealth spaces. The market designer chooses a social choice function (SCF)—a mapping from reported types to an allocation—which we require to be implementable in dominant strategies.

We crucially assume that preferences exhibit positive income effects, such that an agent's WTP for an object increases in wealth. This assumption provides the essential channel for studying how wealth can grant disproportionate access. In our model, a high WTP for an object can result from two sources: an agent's object utility and their initial wealth. This has two key implications for trading incentives that depart from the standard quasilinear framework. First, a wealthy agent's WTP can be high enough to compensate a poorer agent for an object, even if the poorer agent's utility for it is higher. Second, income effects create a gap between an agent's willingness to accept (WTA) to part with an object and their WTP to acquire it, meaning agents may not trade even if a richer buyer values an object more.

We define an SCF as *discrimination-free* if its object allocation does not depend on agents' wealth endowments. This criterion builds on the distinction between the two sources of WTP, restricting the influence of wealth on an allocation while allowing it to depend on object utility. Since an implementable transfer-free SCF is inherently discrimination-free, we can analyze the widely used policy of a transfer ban as a tool to achieve this goal. Our analysis then investigates when this policy is a well-calibrated instrument and when it is unnecessarily restrictive. As we show, the answer hinges on the degree of wealth inequality (modeled as the upper bound of the wealth space, holding the lower bound constant) and the structure of the agents' utility space.

Our model setup has direct consequences for the efficiency of a transfer ban as a tool to accommodate discrimination-freeness. To understand these consequences, we analyze an optimal transfer-free SCF—that is, one inducing an allocation where agents would not wish to exchange objects without monetary transfers. Importantly, such a mechanism might not assign an object to the agent with the highest cardinal utility for it. We investigate whether such a mechanism can be fully Pareto-efficient or is at least constrained-efficient (meaning any Pareto improvement would require discrimination).

When wealth inequality is high, we show that any such SCF is inefficient. This is because a wealthy agent's WTP is sufficient to compensate a poorer agent for an object, even if the poorer agent's utility for it is higher. However, this potential trade is fundamentally wealth-driven; if the agents' wealth levels were reversed, the mutually beneficial trade would no longer be possible. Consequently, optimal transfer-free SCFs are constrained-efficient in the high-inequality regime, as no discrimination-free Pareto improvement exists.

When wealth inequality is low, one of the efficiency properties does not continue to hold. An optimal transfer-free SCF can be fully Pareto-efficient, but only if the agents' WTA/WTP gap is robust enough to prevent any potential trading incentives that stem from differences in both wealth and object utility differences. In this case, even the wealthiest agent with the highest object utility cannot compensate the poorest agent with the lowest object utility. We find this requires that objects are sufficiently distinct. Conversely, if objects are not sufficiently distinct, an optimal transfer-free SCF is not only inefficient but also fails to be constrained-efficient. This is because a discrimination-free Pareto improvement exists: an agent who is nearly indifferent between two objects can be compensated by another for whom the utility gain from swapping is much larger, creating a trade that is viable independent of the agents' wealth. In this scenario, a transfer ban precludes desirable trades that are not driven by wealth.

The potential existence of such desirable, non-discriminatory trades leads to our central implementability result. We show that a transfer ban is more restrictive than the discrimination-freeness constraint—in that the set of implementable object allocations is strictly smaller—if and only if wealth inequality is low and the object utility space is non-convex. Otherwise, the two constraints are equivalent regarding implementable object allocations.

To see the intuition, consider a simple price mechanism where receiving an object requires paying a fixed price. For this mechanism to be discrimination-free, an agent's decision to pay must depend only on their object utility, not on their wealth. For high wealth inequality, this is impossible: a low-utility agent will still be willing to pay the price if they are rich enough. For low wealth inequality, however, a price can be set such that high-utility agents are always willing to pay and low-utility agents never are. However, this separation fails to be implementable if the object utility space is convex, as there will always be some intermediate utility level where the decision to pay the price hinges on wealth. Therefore, only a non-convex, or "gappy," utility space allows a designer to use prices to screen for high-utility agents in a discrimination-free way.

Our analysis also considers the case where the designer knows agents' wealth but not the utility of a single object that has to be assigned. The information about wealth allows the designer to use tools like wealth redistribution or wealth-dependent pricing, expanding the set of implementable, discrimination-free allocations. We show, however, that this expanded toolkit has sharp limits if efficiency is desired. If the object utility space is convex, achieving both discrimination-freeness and efficiency is possible only through a mechanism equivalent to first fully equalizing wealth and then using a price system to assign the objects. In other words, as long as any wealth inequality persists, no wealth-dependent pricing scheme can restore full efficiency in a discrimination-free way.

Considering real-world applications, our work suggests that a ban on monetary transfers is a well-calibrated tool for ensuring wealth-independent access to goods when the society's wealth inequality is high. While other rationales for such bans exist, our results show that the goal of preventing wealth-based discrimination is a sufficient justification on its own. Conversely, when inequality is low, a strict ban may be an overly blunt instrument, sacrificing welfare that could be achieved by using monetary transfers without introducing discrimination. This suggests that societies with more equitable wealth distributions may have greater flexibility in designing markets than those with high inequality.

Finally, while our model is stylized, its core logic extends to other settings. In two-sided markets like organ donation, for instance, a potential donor's decision may also depend on their wealth; applying the discrimination-freeness criterion to both sides of the market would yield a similar set of trade-offs. In other settings a formal ban on monetary transfers may be even insufficient to ensure wealth-independent access. If wealth can confer an advantage outside the central mechanism—through bribery, or because private markets co-exist alongside public ones (as in education)—the wealthy may regain their advantage, undermining the goal of a discrimination-free outcome.

1.1 Related work

Our work primarily connects to and contributes to mainly four distinct strands of literature.

Repugnant markets and the role of inequality. Our work is motivated by the literature studying markets where monetary transfers are often considered repugnant (e.g., Kahneman *et al.* (1986), Frey and Pommerehne (1993)). Roth (2007) formalized this concept, analyzing repugnance as a key constraint on market design. Our paper focuses on a central factor driving such repugnance: the view that wealth inequality should not grant disproportionate access to certain goods (Sandel (2012), Satz (2010)). The importance of the financial context is further highlighted by Ambuehl *et al.* (2015), who show that an individual's assessment of a market depends on their financial perspective.

Our work is also connected to the notion of equality of opportunity. Fleurbaey and Maniquet (2012) emphasize that equality of opportunity does not imply equal outcomes. Instead, they distinguish between two sources of outcome differences: those stemming from circumstances beyond one's control, which are deemed objectionable and those resulting from individual preferences, which are considered legitimate. In their view, external resources should be allocated to compensate for inequalities that unfair outcome differences, while they should not respond to differences that reflect fair inequalities. In our model, agents are likewise characterized by two elements—object utilities and endowments. Interpreting endowments as circumstances that can yield unfair differences in outcomes, while viewing heterogeneous object utilities as reflecting fair differences, our requirement of discrimination-freeness goes beyond compensation by demanding that circumstances must not affect outcomes at all.

We acknowledge that monetary transfers may give rise to other concerns that we do not consider. These include "slippery slope" effects causing unintended consequences such as organ commercialism (Bruzzone (2010)). Similarly, Gneezy and Rustichini (2000) argue that the existence of monetary fines can induce unexpected behavior. Monetary transfers may also have unwanted external effects (see, e.g., Jehiel, Moldovanu and Stacchetti (1996) for an example concerning the sale of nuclear weapons, as well as Satz (2008) and Rippon (2014) for discussions of kidney sales). In contrast, Ambuehl (2023) examines the potential harmful effects of undue inducements but finds no support for this concern. In addition, there is a large literature on how incentives affect individuals' moral behavior (Richard (1970); Frey and Oberholzer-Gee (1997); Mellström and Johannesson (2008); Gneezy and Rustichini (2000)). Our work focuses on the orthogonal question of how incentives affect who receives what.

Inequality-aware Market Design. Our paper contributes most directly to the emerging literature on inequality-aware market design that explores how market distortions like price controls may be justified on welfare grounds. Weitzman (1977) was the first to argue that a price mechanism is not optimal if the WTP does not adequately reflect the agents' needs. Condorelli (2013) shows that non-market mechanisms can be optimal when WTP is negatively correlated with need.

A prominent set of recent papers justifies price-based interventions by studying the trade-off between efficiency and redistribution (see Dworczak *et al.* (2021) and Akbarpour, Dworczak and Kominers (2024b)). This work typically models inequality as heterogeneity in the marginal utility of wealth and shows that redistributive motives can justify price regulations. Following a related approach, Groh and Reuter (2023) demonstrate that it may be optimal not to sell to those willing to pay the most. Akbarpour, Budish, Dworczak and Kominers (2024a) study the problem of vaccine allocation and show that it can be optimal to combine a non-price mechanism (that provides the vaccine based on observable information) with a price mechanism (that allows for the elicitation of unobservable preferences). Other work models inequality through budget constraints, arguing that market-clearing prices may fail to be optimal (Che, Gale and Kim (2013)). Taking a different perspective, Grassi and Ma (2010) compare subsidy policies based explicitly on either wealth or benefit information, establishing conditions under which the two approaches can implement the same allocation.

Our work differs from most works on inequality-aware market design in two fundamental ways. First, our research question is different. Rather than seeking an optimal mechanism, we formalize the goal of preventing wealth-based discrimination with our discrimination-freeness criterion and evaluate how a common real-world policy—a ban on monetary transfers—performs against this goal. Second, our approach of modeling inequality is distinct. We model inequality through heterogeneous wealth endowments and positive income effects. Our framework explicitly separates the two components of the WTP—object utility and wealth. This separation is essential for analyzing our central criterion. While in Che *et al.* (2013), inequality acts as a hard budget constraint on what agents can pay, our model provides an endogenous channel through which inequality affects what agents are willing to pay. This preference-driven approach is crucial, as it leads to a richer set of conclusions across the entire spectrum of inequality. For instance, in the low-inequality regime, our model explains how efficiency can be maintained even when no budget constraints exist or they are not binding, as the WTP/WTA gap persists. In the high-inequality regime, our framework allows for Pareto-improving trades from poorer to wealthier agents, a possibility that is typically ruled out in budget-constraint models with constant marginal utility of money.

Market Design with income effects. Our modeling choice connects our paper to the literature on market design with non-quasilinear preferences. It is well-known that income effects can substantially alter classical results. For instance, they disrupt canonical findings in auction theory (Maskin and Riley (1984)) and can even overturn famous impossibility results, as in the efficient bilateral trade problem of Myerson and Satterthwaite (1983) (see Garratt and Pycia (2023)). More relevant to our setting, several studies show that income effects can make non-market mechanisms, such as random allocation, Pareto-superior to standard price-based mechanisms (e.g., Baisa (2017); Huesmann (2017); see also Che *et al.* (2013) for a related finding under budget constraints). We contribute to this literature by identifying the dual role that positive income effects play in generating the trade-offs discussed above.

On the one hand, they provide the direct channel for the wealth-based discrimination that drives inefficiency when inequality is high. On the other, they create the crucial wedge between an agent's WTP and WTA. As noted previously, this WTP/WTA gap is the key mechanism that can, under certain conditions in the low-inequality regime, prevent inefficient trades and maintain full efficiency.

Fairness constraints in Market Design. Finally, by introducing a new fairness criterion, we contribute to the broad literature on fair allocation. While related to existing fairness constraints, our notion of discrimination-freeness is distinct. For example, it differs from anonymity (Thomson (2011)), which typically requires that the entire outcome be independent of agents' identities. Our criterion applies only to the object allocation and requires this allocation to be independent of wealth endowments. It also offers a distinct perspective on equal treatment of equals (Bogomolnaia and Moulin (2001)). Whereas standard definitions require agents to have identical preferences over entire bundles, discrimination-freeness considers two agents as equals for the purpose of object allocation as long as their object utilities are the same, even if their wealth levels—and thus their full preferences—differ. By focusing on a characteristic, namely wealth, that is deemed morally irrelevant for the distribution of certain goods, our concept provides a new tool for market designers concerned with the source of agents' market power, enriching the existing fairness toolkit.

2 Model

Consider the problem of assigning a set Ω of objects to a set N of n agents. The set Ω contains k distinct objects, plus a null object 0 (which is assigned to an agent by default if he does not receive any other object). Each object $\omega \in \Omega$ has a capacity $\kappa(\omega)$ satisfying $\sum_{\omega \neq \{0\}} \kappa(\omega) < n$ and $\kappa(0) = n$.[3] Each agent receives at most one object, and the assignment of objects must respect their capacities.

Payoff environment. Each agent i has preferences about owning an object ω and wealth e, which are described by an additively separable utility function $u_i : \Omega \times \mathbb{R} \to \mathbb{R}$ of the following form:

$$u_i(\omega, e) = \theta_i(\omega) + h(e). \tag{2.1}$$

We normalize by setting the utility of the null object equal to zero: $\theta_i(0) = 0$ for every i. We assume the marginal utility of wealth is positive (i.e., $h' > 0$) and decreasing (i.e., $h'' < 0$). We do not explicitly assume any budget constraints; wealth may become negative.[4] Furthermore, assume $\lim_{e\to\infty} h(e) = \infty$, $\lim_{e\to\infty} h'(e) = 0$, and $\lim_{e\to-\infty} h'(e) = \infty$.

Each agent i is endowed with some initial wealth $e_i \in \mathbb{R}$, and the utilities he attaches to the k non-null objects in Ω are given by the vector $\theta_i \in \mathbb{R}^k_+$. If not stated otherwise, types are private information to the agents. From the perspective of the market designer and the other agents, each agent's object utilities and wealth endowment are drawn independently from some distributions with supports $\Theta \subset \mathbb{R}^k_+$ and $E \subset \mathbb{R}$. We call $\Theta \times E$ the type space, Θ the object utility space, and E the wealth space. Furthermore, we set

$$\underline{e} = \inf E > -\infty \quad \text{and} \quad \overline{e} = \sup E \leq \infty. \tag{2.2}$$

Agent i's preferences are therefore determined by his $(k + 1)$-dimensional type $t_i = (\theta_i, e_i) \in \Theta \times E$. His utility from being assigned an object ω and a monetary transfer m is $\theta_i(\omega) + h(e_i + m)$.

Given an object utility vector $\theta \in \Theta$, we let $R_\theta : \Omega \to \{1, \dots, k+1\}$ denote the object ranking implied by θ; that is, $R_\theta(\omega) < R_\theta(\omega')$ if and only if $\theta(\omega) > \theta(\omega')$. (Note that since the null object always has utility 0, $R_\theta(0) = k + 1$.) Conversely, given an object ranking $R : \Omega \to \{1, \dots, k + 1\}$, we denote by $\Theta(R)$ the set of all $\theta \in \Theta$ with $R_\theta = R$.

[3]This condition ensures that every object is assigned to some agent and that at least one agent receives the null object. Omitting these assumptions would not alter our results, but it would require us to make additional case distinctions, distracting from the main point of our arguments.

[4]If we required $e > 0$, our qualitative results would not change, but we would need to make some more case distinctions in our arguments. Importantly, our results are driven not by budget constraints but by positive income effects. Incorporating budget constraints but assuming a constant marginal utility of wealth would yield qualitatively different results.

The assumption of additive separability in the agents' preferences enables a clear distinction between phenomena driven by object utilities and those driven by wealth. However, as long as income effects are positive, our core arguments remain valid even for fairly general preference spaces (e.g., with object utilities depending on wealth); for details, see Section 6.2.

Social choice functions. A *social choice function* (SCF) $\varphi = (\sigma, m)$ consists of an *object assignment* σ: $(\Theta \times E)^n \to \Omega^n$ and a *transfer rule* m: $(\Theta \times E)^n \to \mathbb{R}^n$. That is, given a type profile $(\theta_j, e_j)_{j\in N} \in (\Theta \times E)^n$, φ assigns to agent i the object $\sigma_i \in \Omega$ and the monetary transfer $m_i \in \mathbb{R}$.[5] When types are private information, an SCF represents the corresponding direct mechanism that maps reported types to outcomes. We limit our attention to the set Φ of SCFs with $\sum_{i\in N} m_i \leq 0$ (i.e., with no subsidy).[6] By $\Phi_{TF} \subset \Phi$ we denote the set of all SCFs that are *transfer-free*; that is, $\varphi \in \Phi_{TF}$ if and only if φ is of the form $(\sigma, 0)$.

Definitions. An SCF $\varphi' = (\sigma', m') \in \Phi$ *Pareto-dominates* $\varphi = (\sigma, m) \in \Phi$ if for all type realizations $(\theta_i, e_i)_{i\in N} \in (\Theta \times E)^n$ all agents are weakly better off under φ' than under φ, and if for at least one type realization some agent is strictly better off. An SCF $\varphi \in \Phi$ is *(Pareto-)efficient* if there is no $\varphi' \in \Phi$ that Pareto-dominates φ. An SCF $\varphi = (\sigma, m) \in \Phi$ is *ordinally efficient* if there is no object assignment σ' such that (σ', m) Pareto-dominates (σ, m). For any $\Phi' \subset \Phi$, an SCF $\varphi \in \Phi'$ is at the *(Pareto-)efficient frontier* of Φ' if there is no SCF $\varphi' \in \Phi'$ that Pareto-dominates φ.

An SCF $\varphi = (\sigma, m)$ is *implementable* if there exists a mechanism with a dominant strategy equilibrium whose outcome is the outcome of φ for all type profiles.[7] We limit our attention to implementable SCFs for which truth-telling is a dominant strategy. That is, if agent i has type $t_i = (\theta_i, e_i) \in \Theta \times E$ and the other agents' types are given by $t_{-i} = (\theta_j, e_j)_{j\neq i} \in (\Theta \times E)^{n-1}$, then

$$u_i(\sigma_i(t_i, t_{-i}), e_i + m_i(t_i, t_{-i})) \geq u_i(\sigma_i(t_i', t_{-i}), e_i + m_i(t_i', t_{-i})) \quad \forall\, t_i' \in \Theta \times E. \tag{2.3}$$

Tie-breakers. An SCF φ may use tie-breaking rules, such as priorities or lotteries. Such tie-breakers are determined *before* the mechanism is conducted and

[5] For brevity, we let σ, σ_i, m, and m_i denote either maps from type profiles to outcomes, or the outcomes themselves.

[6] Our qualitative results continue to hold if we require $\sum_{i\in I} m_i \leq F$ for some $F \in \mathbb{R}$. A value $F < 0$ corresponds to a fund size that has to be raised, and $F > 0$ corresponds to a budget for subsidies.

[7] The requirement of individual rationality is not relevant for our results.

are fixed for each agent independently of the realization of types. We therefore take the perspective of an interim stage, in which the tie-breakers may introduce a non-anonymous aspect to the SCF even if it is anonymous ex ante. This perspective allows us to focus on deterministic outcomes and is thus more suitable for our analysis, since we are interested in whether monetary transfers can increase efficiency, rather than whether ex-ante efficiency gains can be achieved through probabilistic assignments. Evaluating probabilistic assignments with income effects is not straightforward and requires a separate assessment (see also Section 6.2).

All proofs are provided in the Appendix A.2.

3 Discrimination-free SCFs

We define an SCF as *discrimination-free* (with respect to wealth) if the object assignment does not depend on the agents' wealth endowments.

Definition 1 (discrimination-free)**.** *An SCF $\varphi = (\sigma, m) \in \Phi$ is discrimination-free (with respect to wealth) if and only if*

$$\sigma(\theta, e) = \sigma(\theta, e') \quad \textit{for all} \quad (\theta, e), (\theta, e') \in (\Theta \times E)^n. \tag{3.1}$$

We let $\Phi_{DF} \subset \Phi$ denote the set of all discrimination-free SCFs.

Discrimination-freeness as a constraint is meaningful in our model precisely because positive income effects make wealth a determinant of an agent's WTP, creating a valid concern about wealth-based discrimination. In a discrimination-free SCF, the object allocation may depend on agents' object utilities but not on their wealth endowments. The transfer rule, however, is not restricted in this way. Our definition thus imposes equality only in terms of access to goods, distinguishing it from fairness criteria that typically refer to the entire outcome. Furthermore, it is distinct from classical inequality aversion, as it does not aim to equalize utilities or redistribute wealth, and may even restrict a Pareto-improving trade if that trade's feasibility depends on wealth.

Our analysis aims to study whether a ban on monetary transfers is an appropriate tool to satisfy discrimination-freeness. A crucial link between this policy tool and our normative goal is that any implementable and transfer-free SCF is inherently discrimination-free, as a designer who cannot use monetary transfers has no channel through which to use wealth information for the assignment. We therefore investigate the trade-offs of using this policy. We first analyze the efficiency properties of transfer-free SCFs that arise within our model (Section 4), and then study whether a transfer ban is more restrictive than the discrimination-freeness constraint itself (Section 5).

4 Efficiency of transfer-free SCFs

A central objective for a market designer is to attain Pareto efficiency, where objects are allocated so that no ex-post gains from trade remain. In our model with positive income effects, however, trading incentives can arise from two distinct sources: differences in object utilities and disparities in wealth. Monetary transfers provide a standard way to realize Pareto improvements by trading off these differences, but a policy that bans transfers may preclude such efficiency gains.

This section analyzes the consequences of our modeling choice for the efficiency of transfer-free SCFs. We investigate the conditions under which a ban on monetary transfers can still yield fully efficient outcomes, and when it can at least achieve constrained efficiency by reaching the Pareto frontier of Φ_{DF} (meaning no discrimination-free Pareto improvement exists). As a starting point for our analysis, we first discuss the nature of trading incentives between two agents.

4.1 Trading incentives between two agents

In our model with positive income effects, the conditions for a mutually beneficial trade are fundamentally different from the standard quasilinear benchmark. To see this, we first formally define an agent's willingness to pay (WTP) and willingness to accept (WTA). An agent's WTP for an object that provides utility x, denoted by $k(x, e)$, is defined implicitly by

$$x + h(e - k(x, e)) = h(e), \tag{4.1}$$

while their WTA, denoted by $c(x, e)$, is defined by

$$x + h(e) = h(e + c(x, e)). \tag{4.2}$$

Lemma 2 in the Appendix establishes that both functions are well-defined and outlines their key properties. Crucially, with positive income effects, both $k(x, e)$ and $c(x, e)$ are increasing in utility x and wealth e. By definition it holds that

$k(x, e) = c(x, e - k(x, e))$. It implies that an agent's WTA to sell an object is always strictly greater than their WTP to buy it:

$$k(x, e) < c(x, e). \tag{4.3}$$

In contrast, under quasilinear preferences, WTP and WTA are identical.

This gap between WTP and WTA has two key implications for trading incentives. Suppose a seller S owns object ω and a buyer B owns ω', where both prefer ω to ω' by utility differences x_S and x_B, respectively. A trade is mutually beneficial if the buyer's WTP to swap, $k(x_B, e_B)$, meets the seller's WTA, $c(x_S, e_S)$. In the quasilinear case, trade occurs if and only if $x_B > x_S$. With positive income effects, however, this condition is neither necessary nor sufficient. First, even if the seller's utility loss from the swap exceeds the buyer's utility gain ($x_S > x_B$), a trade can still occur if the buyer is sufficiently wealthy. Second, and conversely, even if the buyer's utility gain is larger than the seller's loss ($x_B > x_S$), a trade may not be mutually beneficial if the buyer's WTP fails to meet the seller's WTA.

School choice example. Suppose a population is served by two schools, A and B. School B has unlimited capacity while school A has limited capacity but is preferred by all students. Consider two students, Ada and Bob. Ada's WTP to attend school A instead of B is \$50,000, while Bob's WTP is \$40,000. Suppose Bob has been assigned to school A and Ada has been assigned to school B.

Now Ada offers Bob \$50,000 to switch places. If positive income effects are present, Bob might refuse this offer. Although his WTP for a place at A is only \$40,000, his WTA to give up his place might exceed the offered \$50,000. This asymmetry arises because, under diminishing marginal utility of wealth, the disutility from paying \$50,000 is greater than the utility gained from receiving the same amount. As a result, Bob may prefer to keep his place at School A, even though Ada's WTP is higher than his own.

4.2 Known types: Maximizing the sum of object utilities

As discussed in Subsection 4.1, trading incentives can stem from both differences in object utilities and wealth disparities. A market designer who knows the agents' types and operates under a transfer ban policy can address the first source by considering an SCF φ^c whose object allocation maximizes the sum of the agents' object utilities:

$$\varphi^c = (\sigma^c, 0) \quad \text{with} \quad \sigma^c(\theta) = \arg\max_{\sigma} \sum_{i \in N} \theta_i(\sigma_i(\theta)). \tag{4.4}$$

By construction, φ^c is discrimination-free and eliminates potential trading incentives arising purely from differences in object utilities. However, wealth disparities might still induce mutually beneficial trades, meaning φ^c is not necessarily efficient. This contrasts with the quasilinear setting, where maximizing the sum of utilities is necessary and sufficient for efficiency. As a benchmark, we first discuss how our modeling choices affect the efficiency of φ^c, before turning to implementable SCFs in Subsection 4.3.

School choice example. For illustration, consider again the school choice example from Subsection 4.1. School B is the null object, and school A is the only non-null object ($\Omega = \{A, B\}$); let $\Theta \subset \mathbb{R}_+$ denote the set of possible utilities from school A. The utility-maximizing SCF φ^c assigns the limited places at school A to students with the highest utility for A. Such an SCF is efficient if and only if

$$k(\theta, e) - c(\theta', e') \leq 0 \quad \text{for all} \quad \theta \leq \theta' \in \Theta \quad \text{and} \quad e, e' \in E. \tag{4.5}$$

Since both $k(\cdot,\cdot)$ and $c(\cdot,\cdot)$ are increasing in their arguments, condition (4.5) is equivalent to requiring that the WTP of the wealthiest agent does not exceed the WTA of the poorest agent, even when they have the same object utility:

$$k(\theta, \overline{e}) - c(\theta, \underline{e}) \leq 0 \quad \text{for all} \quad \theta \in \Theta, \tag{4.6}$$

where $\overline{e} = \sup E$ and $\underline{e} = \inf E$. If $\underline{e} = \overline{e}$, the inequality holds strictly, while it fails if $\overline{e} = \infty$. Thus, φ^c is efficient if and only if wealth inequality is sufficiently small (measured by $\overline{e}$ while keeping $\underline{e}$ fixed). Furthermore, Lemma 3 in the Appendix shows that for $\overline{e} < \infty$, the function $k(\theta, \overline{e}) - c(\theta, \underline{e})$ is concave in θ and negative if and only if θ exceeds some threshold $\theta^* > 0$. This implies that efficiency requires, ceteris paribus, that either wealth inequality is small enough or that the utility provided by school A is high enough for all students.

As noted earlier, under φ^c, any remaining trading incentives are driven solely by wealth inequality. This means that any potential Pareto improvement would require reassigning the object based on wealth, thereby violating discrimination-freeness. In other words, φ^c is always at the efficient frontier of the set of discrimination-free SCFs, Φ_{DF}.

General case. The proposition below generalizes the intuition from this example to a setting with an arbitrary number of objects. When there are multiple objects, the degree of similarity possible between objects becomes relevant. We measure this using the minimum utility difference between any two distinct objects across all possible utility profiles:

$$\inf \Theta := \inf_{\theta \in \Theta,\ \omega, \omega' \in \Omega} |\theta(\omega) - \theta(\omega')|. \tag{4.7}$$

Proposition 1. *There exists* $e^c \geq \underline{e}$ *such that* $\varphi^c = (\sigma^c, 0) \in \Theta_{TF}$ *is efficient if and only if* $\overline{e} \leq e^c$*. As a function of* $\inf \Theta$, $e^c := e^c(\inf \Theta, \underline{e})$ *is strictly increasing, with* $e^c(0, \underline{e}) = \underline{e}$*, and* $\lim_{\inf \Theta \to \infty} e^c(\inf \Theta, \underline{e}) = \infty$*. Furthermore,* φ^c *is at the efficient frontier of* Φ_{DF}*.*

For all proofs see the Appendix. Proposition 1 implies that whether maximizing the sum of object utilities suffices for efficiency depends on both wealth inequality (measured by $\overline{e}$ for fixed $\underline{e}$) and the potential degree of similarity between objects ($\inf \Theta$). If two objects can be arbitrarily similar ($\inf \Theta = 0$), then φ^c is efficient only if there is no inequality ($\overline{e} = \underline{e}$). However, if objects are sufficiently distinct ($\inf \Theta > 0$), φ^c can be efficient despite some wealth inequality, because the WTP/WTA gap persists when inequality is small (see Eq. (4.3)). Conversely, for any given level of wealth inequality ($\underline{e} < \overline{e} < \infty$), φ^c is efficient if $\inf \Theta$ is large enough. Importantly, the proposition also confirms that φ^c always lies at the constrained-efficient frontier, regardless of the parameters.

4.3 Unknown types: Ordinal efficient SCFs

If agents' types are private information, the market designer cannot implement the utility-maximizing SCF φ^c. Dominant-strategy implementability without transfers requires that an agent's assignment depend only on their ordinal preferences over objects (their object ranking), not on the intensity of these preferences (the cardinal values of θ_i) or on their wealth endowment.[8] This straightforward implication is captured by the following lemma.

Lemma 1. *Let* $\varphi = (\sigma, 0) \in \Phi_{TF}$ *be an implementable SCF. For a given agent* i*, fix the types* $(\theta_j, e_j) \in \Theta \times E$ *of all agents* $j \neq i$*. Then,* $\sigma_i(\theta_i, e_i) = \sigma_i(\theta'_i, e'_i)$ *for all* $e_i, e'_i \in E$ *and* $\theta_i, \theta'_i \in \Theta(R)$ *for some ranking* R*.*

[8] In contrast, if monetary transfers can be used, preference intensities can be incorporated, for instance by assigning objects via an auction.

Lemma 1 implies that any implementable transfer-free SCF is discrimination-free, as the object allocation cannot depend on wealth. The efficient frontier within this class consists of the ordinally efficient SCFs (e.g., serial dictatorship). However, unlike the benchmark φ^c, an ordinally efficient SCF may now generate ex-post trading incentives due to both wealth inequality and differences in preference intensities. Therefore, besides $\underline{e}$, $\overline{e}$, and $\inf \Theta$, the potential variation in object utilities also becomes relevant for efficiency. We measure this variation using

$$\sup \Theta = \sup_{\theta \in \Theta,\ \omega, \omega' \in \Omega} |\theta(\omega) - \theta(\omega')|. \tag{4.8}$$

School choice example. As before, we explain the intuition using the example of school choice. Recall that school A (which has limited capacity) is preferred by all students over school B (the null object), and $\Theta \subset \mathbb{R}_+$ represents the space of possible utilities from school A. Suppose students are randomly assigned via a lottery (this mechanism is implementable and ordinally efficient). The lottery is efficient if and only if the WTP of the richest student with the highest valuation for A is not enough to compensate the WTA of the poorest student with the lowest valuation for A. Clearly, the lottery is inefficient if wealth can be arbitrarily high ($\overline{e} = \infty$), A can be arbitrarily similar to B ($\inf \Theta = 0$), or the utility gain from A can be arbitrarily large ($\sup \Theta = \infty$).

Beyond these extreme cases, the formal condition for efficiency is

$$k(V \cdot \inf \Theta, \overline{e}) - c(\inf \Theta, \underline{e}) \leq 0, \tag{4.9}$$

where $V = \sup \Theta / \inf \Theta$ is the maximal relative variation of Θ.[9] Thus, efficiency depends on (i) wealth inequality (measured by $\overline{e}$ for fixed $\underline{e}$), (ii) the minimum possible utility difference between A and B ($\inf \Theta$), and (iii) the maximum possible variation in the utility of A across students (V). For fixed $\underline{e}$, V, and $\inf \Theta$, let e^* be the unique wealth level satisfying $k(V \cdot \inf \Theta, e^*) - c(\inf \Theta, \underline{e}) = 0$. The lottery is then efficient if and only if $\overline{e} \leq e^*$. Note that if $\inf \Theta$ is sufficiently low, e^* can be less than $\underline{e}$, meaning the lottery is inefficient for any $\overline{e} \geq \underline{e}$. Conversely, if $\inf \Theta$ is sufficiently high, it implies $e^* > \underline{e}$. Then, the lottery can be efficient despite some wealth inequality.

If the lottery is inefficient, we ask if it is at least constrained-efficient (i.e., on the Pareto frontier of Φ_{DF}). This depends on whether potential trades are driven by

[9] Keeping V fixed instead of $\sup \Theta$ is more convenient for comparative statics, since if V is fixed, $\inf \Theta$ can attain any value, while if $\sup \Theta$ is fixed, $\inf \Theta$ is bounded above by $\sup \Theta$.

wealth inequality or by differences in θ. A discrimination-free Pareto improvement exists if and only if trading incentives remain even when the potential buyer is poor and the potential seller is rich. Formally, the lottery is constrained-efficient if and only if

$$k(V \cdot \inf\Theta, \underline{e}) - c(\inf\Theta, \overline{e}) < 0. \tag{4.10}$$

This condition differs from (4.9) primarily in the reversal of $\underline{e}$ and $\overline{e}$. It essentially checks if trades are only possible when the buyer is sufficiently rich and the seller sufficiently poor. Let $\hat{e}$ be the unique wealth level satisfying $k(V \cdot \inf\Theta, \underline{e}) - c(\inf\Theta, \hat{e}) = 0$. The lottery is then constrained-efficient if and only if $\overline{e} > \hat{e}$.

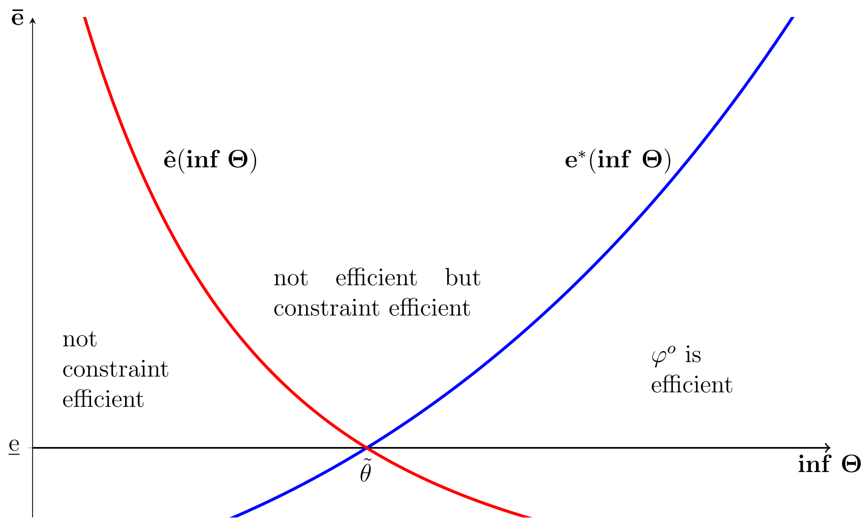

Figure 1: e^* and $\hat{e}$ for fixed $V < \infty$

Figure 1 illustrates the thresholds e^* and $\hat{e}$ as functions of $\inf\Theta$ (keeping V fixed). There exists a critical value $\tilde{\theta}$ such that $\hat{e}(\tilde{\theta}) = e^*(\tilde{\theta}) = \underline{e}$. For $\inf\Theta < \tilde{\theta}$ (school A can be very similar to B for some students), we have $e^* < \underline{e}$ and $\hat{e} > \underline{e}$. In this case, the lottery is inefficient for any $\overline{e} > \underline{e}$. It is constrained-efficient if wealth inequality is large ($\overline{e} > \hat{e}$) but not if it is small ($\overline{e} \leq \hat{e}$). For $\inf\Theta > \tilde{\theta}$ (school A is sufficiently distinct from B for all students), we have $e^* > \underline{e}$ and $\hat{e} < \underline{e}$. Here, the lottery is constrained-efficient for any $\overline{e} > \underline{e}$. It remains inefficient if inequality is large ($\overline{e} > e^*$) but becomes fully efficient if inequality is small ($\overline{e} \leq e^*$).

These relationships imply that, for a fixed level of inequality $\overline{e} > \underline{e}$, the lottery transitions through different efficiency categories as $\inf\Theta$ changes: it is efficient for large $\inf\Theta$, inefficient but constrained-efficient for intermediate $\inf\Theta$, and neither efficient nor constrained-efficient for low $\inf \Theta$.

Finally, the higher the utility variation V, the larger $\tilde{\theta}$ becomes. This expands the parameter region where the lottery is not constrained-efficient and shrinks the region where it is fully efficient. When $V = 1$ (all students value A identically), the conditions simplify to those of Proposition 1, with $e^* = e^c$ and $\tilde{\theta} = 0$.

General case. We now generalize the insights from the school choice example. First, we treat the extreme cases where wealth inequality may be arbitrarily high ($\overline{e} = \infty$), objects may be arbitrarily similar ($\inf \Theta = 0$), or utility differences between objects may be arbitrarily large ($\sup \Theta = \infty$). While optimal transfer-free SCFs are inefficient in all three cases, they are constrained-efficient in one of them.

Proposition 2. *If $\overline{e} = \infty$, any implementable SCF at the efficient frontier of Φ_{TF} is not efficient but at the efficient frontier of Φ_{DF}. If $\overline{e} < \infty$, and if $\inf \Theta = 0$ or $\sup \Theta = \infty$, then an implementable SCF at the efficient frontier of Φ_{TF} is neither efficient nor at the efficient frontier of Φ_{DF}.*

For the non-extreme cases, we establish how wealth inequality affects both efficiency and constrained efficiency for any implementable SCF on the efficient frontier of transfer-free SCFs.

Proposition 3. *Let $\overline{e} < \infty$, $\inf \Theta > 0$, and $\sup \Theta < \infty$. Consider an implementable SCF $\varphi^o = (\sigma^o, 0)$ that is at the efficient frontier of Φ_{TF}. There exist thresholds e^* and $\hat{e}$ such that*

- *φ^o is efficient if and only if $\overline{e} \leq e^*$*
- *φ^o is at the efficient frontier of Φ_{DF} if and only if $\overline{e} > \hat{e}$.*

Furthermore, there exists a strictly increasing function $f : \mathbb{R} \to \mathbb{R}$, which is independent of the wealth space E, such that $e^* = f(\underline{e})$ and $\hat{e} = f^{-1}(\underline{e})$.

Proposition 3 implies that when wealth inequality is high ($\overline{e} > \max\{e^*, \hat{e}\}$), any SCF on the efficient frontier of Φ_{TF} is inefficient but constrained-efficient. When wealth inequality is low, however, at least one of these properties fails. Specifically, if $e^* < \underline{e}$, the SCF is inefficient for any $\overline{e} \geq \underline{e}$. Furthermore, if wealth inequality is sufficiently low ($\underline{e} \leq \overline{e} < \hat{e}$), the SCF is not constrained-efficient either. Conversely, if $e^* \geq \underline{e}$, then for sufficiently low wealth inequality ($\overline{e} \leq e^*$), the SCF is fully efficient. In this case, the SCF also remains constrained-efficient whenever it is inefficient ($\overline{e} > e^*$).

Corollary 1. *For any implementable SCF $\varphi^o = (\sigma^o, 0)$ at the efficient frontier of Φ_{TF}, it holds that $e^* < \underline{e}$ if and only if $\hat{e} > \underline{e}$.*

The thresholds e^* and $\hat{e}$ determined in Proposition 3 depend on $\underline{e}$ and the function f, which itself depends on Θ and the specific SCF φ^o. To gain sharper insights, we now develop bounds for e^* and $\hat{e}$ based only on key characteristics of the utility space Θ, building on the intuition from the school choice example.

For this, we use two measures of utility variation. The first is the *maximal relative variation* V_Θ, defined as

$$V_\Theta = \frac{\sup \Theta}{\inf \Theta}. \tag{4.11}$$

The second is the *minimal relative variation* v_Θ, defined as

$$v_\Theta = \inf_R \inf_{\omega,\omega' \in \Omega} \sup_{\theta,\theta' \in \Theta(R)} \frac{|\theta(\omega) - \theta(\omega')|}{|\theta'(\omega) - \theta'(\omega')|}, \tag{4.12}$$

where the outermost infimum is taken over all object rankings R. In words, v_Θ measures the minimum possible ratio of utility differences between two objects across agents who share the same ranking. Note that $\sup \Theta < \infty$ and $\inf \Theta > 0$ imply $V_\Theta < \infty$ and $v_\Theta < \infty$.

Proposition 4. *Assume $\overline{e} < \infty$, $\inf \Theta > 0$, and $\sup \Theta < \infty$. There exist functions $\delta(\inf \Theta, V_\Theta, \underline{e})$ and $\rho(\inf \Theta, v_\Theta, \underline{e})$ such that, for all implementable SCFs φ^o at the efficient frontier of Φ_{TF},*

$$\delta(\inf \Theta, V_\Theta, \underline{e}) \leq e^* \leq \rho(\inf \Theta, v_\Theta, \underline{e}) \leq e^c(\inf \Theta, \underline{e}). \tag{4.13}$$

$\delta(\cdot)$ and $\rho(\cdot)$ are strictly increasing in $\inf \Theta$, with

$$\lim_{\inf \Theta \to \infty} \delta(\inf \Theta, V_\Theta, \underline{e}) = \infty \quad \text{and} \quad \lim_{\inf \Theta \to 0} \rho(\inf \Theta, v_\Theta, \underline{e}) \leq \underline{e}. \tag{4.14}$$

They are strictly decreasing in V_Θ and v_Θ, respectively, with

$$\lim_{v_\Theta \to \infty} \rho(\inf \Theta, v_\Theta, \underline{e}) < \underline{e}. \tag{4.15}$$

Furthermore, defining $\delta_{\inf \Theta, V_\Theta}(\underline{e}) := \delta(\inf \Theta, V_\Theta, \underline{e})$ and $\rho_{\inf \Theta, v_\Theta}(\underline{e}) := \rho(\inf \Theta, v_\Theta, \underline{e})$, it holds that

$$\rho^{-1}_{\inf \Theta, v_\Theta}(\underline{e}) \leq \hat{e} \leq \delta^{-1}_{\inf \Theta, V_\Theta}(\underline{e}). \tag{4.16}$$

If $\delta(\inf \Theta, V_\Theta, \underline{e}) > \underline{e}$, then any implementable SCF at the efficient frontier of

Φ_{TF} is efficient, provided $\overline{e}$ is small enough. Equation (4.14) further implies that for fixed $V_\Theta < \infty$ and any given level of wealth inequality ($\underline{e} < \overline{e} < \infty$), efficiency is guaranteed whenever $\inf \Theta$ is sufficiently large. Conversely, if $\overline{e} \geq \rho(\inf \Theta, v_\Theta, \underline{e})$, then any implementable transfer-free SCF is inefficient. In particular, by (4.14) and (4.15), inefficiency is guaranteed if, ceteris paribus, v_Θ is large enough or $\inf \Theta$ is small enough.

Estimates for $\hat{e}$ follow from those for e^* using (4.16). These bounds imply that the greater v_Θ or the smaller $\inf \Theta$, the higher the lower bound for $\hat{e}$. This aligns with the intuition that the more agents' object utilities can differ, the less their potential trading incentives depend on wealth, and thus the more opportunities exist for discrimination-free Pareto improvements.

This section's results show that when wealth inequality is high, implementable transfer-free SCFs on the efficient frontier of Φ_{TF} are inefficient but constrained-efficient (i.e., they lie on the efficient frontier of Φ_{DF}). In this regime, monetary transfers are necessary to achieve full efficiency, but not to reach the constrained-efficient frontier. However, when wealth inequality is low, an implementable transfer-free SCF that is inefficient may also fail to be constrained-efficient. In this case, a ban on transfers is more restrictive than discrimination-freeness from an efficiency perspective, as transfers could potentially allow for discrimination-free Pareto improvements. We note, however, that these potential improvements may not themselves be implementable. Section 5 examines the implementability question directly.

Remark. The measures of relative variation v_Θ and V_Θ are equal if there is only one non-null object, but $V_\Theta > v_\Theta$ generally holds. V_Θ is useful for establishing a sufficient condition for efficiency (the lower bound δ on e^*) because it captures the most extreme potential utility gain from trade. If no trade occurs even in this case, efficiency is guaranteed. However, V_Θ is not suitable for establishing a necessary condition for efficiency (the upper bound ρ) because a large V_Θ does not guarantee inefficiency (e.g., if all agents have identical preferences). In contrast, v_Θ captures the minimum relative utility difference between objects across agents with the same ranking. Therefore, if v_Θ is sufficiently large, we can guarantee that some pair of agents will have large enough utility differences to induce trading incentives, making v_Θ suitable for the upper bound ρ which provides a sufficient condition for inefficiency.

5 Implementability of discrimination-free SCFs with monetary transfers

In the previous section, we took the perspective of a market designer who cannot use monetary transfers and discussed how the type space determines whether she can achieve an efficient or at least constrained-efficient object allocation - with discrimination-freeness as a constraint.

Now consider a market designer who *can* use monetary transfers but faces the constraint of discrimination-freeness. If the agents' types are private information, then using transfers can help account for differences in the intensities of their preferences. However, linking the assignment of an object to the payment of a price may lead to discrimination, as each agent's WTP for an object depends on their wealth. Therefore, the central question in this setting is whether transfers can be employed *without discrimination* to implement object allocations that are not implementable without transfers. In other words, do discrimination-free SCFs provide a broader toolkit for the market designer to allocate the objects than transfer-free SCFs?

School choice example. Consider again the example of school choice discussed in Section 4.3, where school A has limited capacity while school B (the null object) does not. Suppose an SCF $\varphi = (\sigma, m)$ uses a fixed fee p to assign places at school A, and assume this allocation σ is not implementable without the fee (implying some students get A only if their utility for A is high enough). For this mechanism to be discrimination-free, the decision to pay p must depend only on a student's utility $\theta \in \Theta$, not on their wealth e. This requirement is difficult to satisfy when wealth inequality is high: even a student with low utility for A might be willing to pay p if they are sufficiently wealthy. However, if wealth inequality is low enough, a price p might successfully separate students: those with high utility pay, and those with low utility do not, regardless of their wealth. Crucially, this separation becomes impossible if the object utility space Θ is convex (an interval). If Θ is convex, there will always exist some intermediate utility θ^{ID} such that a student with this utility is indifferent at some wealth level e^{ID}. For this utility, richer students ($e > e^{ID}$) would pay p, while poorer students ($e < e^{ID}$) would not, violating discrimination-freeness. Thus, using transfers (like a price) in a discrimination-free way is possible only if wealth inequality is low and the utility space Θ is non-convex, or "gappy," allowing the price to cleanly separate types without interference from wealth effects. In the context of the school choice example, such non-convexity means that each student either values school A high or low, but not at an intermediate level.

General case. The following propositions generalize the intuition derived from the school choice example to settings with multiple objects. Proposition 5 establishes conditions under which discrimination-freeness and transfer-freeness are equivalent constraints from an implementability perspective.

Proposition 5. *Let $\underline{e} < \overline{e}$. If $\overline{e} = \infty$ or if the closure $\overline{\Theta}$ of Θ is convex, then for any implementable SCF $\varphi = (\sigma, m) \in \Phi_{DF}$, the transfer-free version $\varphi_0 = (\sigma, 0) \in \Phi_{TF}$ is implementable as well.*

Thus, when wealth inequality is high ($\overline{e} = \infty$) or the utility space $\overline{\Theta}$ is convex, the discrimination-freeness constraint does not expand the set of implementable object allocations beyond what is achievable with a ban on monetary transfers. Any object allocation that can be implemented without discrimination can also be implemented without monetary transfers. Note that this does not preclude the use of monetary transfers entirely; for instance, a designer could still charge fees that are independent of the object allocation (e.g., to cover costs).

Proposition 5 shows that non-convexity of $\overline{\Theta}$ is a necessary condition for a discrimination-free SCF with transfers to implement an allocation that is not implementable without transfers (assuming $\overline{e} < \infty$). Proposition 6 provides a partial converse, showing that non-convexity is sufficient if $\overline{\Theta}$ is a Cartesian product.

Proposition 6. *Let $\overline{\Theta}$ be non-convex and satisfy $\overline{\Theta} = \prod_{\omega \in \Omega} \overline{\Theta}(\omega)$ (i.e., it is a Cartesian product). Then for any $\underline{e} > -\infty$, there exists $\overline{e} \in (\underline{e}, \infty)$ such that if the wealth space satisfies $E \subset [\underline{e}, \overline{e}]$, there is some implementable SCF $\varphi = (\sigma, m) \in \Phi_{DF}$ for which the transfer-free version $\varphi_0 = (\sigma, 0) \in \Phi_{TF}$ is not implementable.*

Proposition 6 establishes that when wealth inequality is sufficiently low and the utility space is non-convex (and has a product structure), monetary transfers strictly expand the set of implementable discrimination-free allocations. The Cartesian product assumption reflects environments where utilities for different objects are drawn independently; relaxing it would restrict the generality of this possibility result.

5.1 Observable wealth

So far, we have assumed that types—including both object utilities and wealth—are private information. However, the market designer may have access to information about agents' wealth. This knowledge can potentially expand the space of implementable discrimination-free assignments in two ways. First, the designer could redistribute wealth, thus reducing inequality. Second, prices could be designed to depend on wealth in such a way that the decision to acquire an object does not depend on wealth.

In the following, we concentrate on the case where Ω contains only one non-null object and assume the object utility space Θ is convex (an interval). According to Proposition 5, this implies that if wealth inequality exists ($\underline{e} < \overline{e}$) and types are unknown, the object assignment of any implementable, discrimination-free SCF is also implementable without transfers. Now, suppose wealth is observable. The designer could first redistribute wealth and then assign the object using a price. As an extreme case, consider fully equalizing wealth ($e_i = e$ for all i). The designer could then assign the object via a second-price auction. The resulting SCF is discrimination-free since the assignment depends only on object utilities. We next consider whether it is efficient. Note that the agent with the highest object valuation θ receives the object, and efficiency requires that their WTA after paying the price exceeds any other agent's WTP. Since $k(\theta, e) = c(\theta, e - k(\theta, e))$, and the winner pays a price less than or equal to their WTP $k(\theta, e)$, while all other agents have WTP less than $k(\theta, e)$, the allocation is indeed efficient.

However, full redistribution might not be feasible or desired. If agents still differ in wealth after some partial redistribution, the designer might try wealth-dependent pricing to ensure implementability and discrimination-freeness. For instance, consider a mechanism where agent i with the highest object utility θ_i receives the object and pays a price $p_i = k(\theta_j, e_i)$, where θ_j is the second-highest object utility and e_i is agent i's endowment after any potential partial redistribution). This SCF is implementable and discrimination-free, but it is not efficient. Trading incentives can exist, for example, if $\theta_i = \theta_j$. If agent i receives the object but agent j is richer ($e_j > e_i$), then

$$k(\theta_j, e_j) > c(\theta_j, e_j - k(\theta_j, e_j)) > c(\theta_i, e_i - k(\theta_j, e_i)), \tag{5.1}$$

implying an inefficiency.

The following proposition shows more generally that full redistribution is necessary for efficiency when using a price mechanism, even if prices are wealth-dependent.

Definition 2. *Let $\Omega = \{\omega, 0\}$. An SCF φ represents a price mechanism if for any type profile $(\theta_i, e_i)_{i \in I}$, there is at most one agent i with $\varphi_i = (\omega, e_i - p_i)$ and*

$p_i \geq 0$, and $\varphi_j = (0, e_j)$ for all agents $j \neq i$.

Proposition 7. *Let $\Omega = \{\omega, 0\}$. Assume $\Theta \times E$ is such that $\overline{\Theta}$ is convex and a pure lottery is inefficient. Assume the market designer can observe agents' wealth. There exists an implementable, discrimination-free, and efficient SCF φ that represents a price mechanism if and only if $\underline{e} = \overline{e}$.*

Therefore, while a market designer can use wealth information to adjust prices and ensure discrimination-freeness, efficiency cannot be achieved via a price mechanism unless wealth is fully equalized. As long as wealth disparities persist, trading incentives driven by these disparities create potential inefficiencies.

Note that Proposition 7 considers settings where the pure lottery is inefficient. If the pure lottery is efficient (which can occur even with $\underline{e} < \overline{e}$, as discussed in Section 4), then the lottery itself is an implementable, discrimination-free, and efficient SCF. In such cases, price mechanisms with sufficiently small prices might also achieve all three properties.

6 Discussion

In the first part of this section, we discuss how banning monetary transfers may still be insufficient to ensure discrimination-freeness if money can be used outside the centralized procedure. In the second part, we discuss some of the assumptions of our model.

6.1 Using money outside the mechanism

Our analysis focuses on the direct use of monetary transfers within a mechanism. However, even if transfers are formally banned, wealth disparities may still affect outcomes if agents can use money outside the mechanism to influence the allocation, undermining the goal of discrimination-freeness.

One way to formalize this is through the concept of bribery (Schummer, 2000b). A bribe occurs when one agent pays another to misreport their preferences to achieve a mutually beneficial outcome different from truthful reporting. Even if an SCF is discrimination-free, wealthier agents might have greater capacity or incentive to engage in bribery, potentially reintroducing wealth-based discrimination. For example, under serial dictatorship (which is implementable and discrimination-free), a wealthier agent might bribe someone with higher priority to misreport their choices. A real-world instance occurred at Emory University, where students with course enrollment priority reportedly misreported their preferences by registering for popular courses, not to attend them, but specifically to sell their secured spots to other students (Koenig, 2019).

Preventing such external uses of money might require stronger conditions than just banning transfers within the mechanism. One such condition is bribe-proofness, which requires that no incentives for bribery exist (Schummer, 2000b). However, bribe-proofness is highly restrictive, often requiring an agent's allocation to be independent of others' types, which severely limits efficiency (Schummer, 2000a). For instance, if there are as many objects as agents, bribe-proofness essentially requires a constant allocation, rendering preference information irrelevant.

Similar concerns arise in other real-world scenarios where wealth can influence outcomes indirectly through mechanisms akin to bribery. *Investing in priority* allows wealthier agents to gain advantages in systems based on criteria correlated with costly actions. For instance, in many school choice systems, priority is given based on proximity. Wealthier families can afford housing in neighborhoods with better schools, effectively using money to gain priority (see Black (1999) on the correlation between house prices and school quality). Similarly, in organ allocation systems based on waiting lists, wealthier individuals may gain an advantage by registering on multiple lists in different locations, requiring the resources to travel on short notice, as reportedly occurred in the case of Steve Jobs' liver transplant.[10] *Coexisting private markets* offer another channel for wealth to affect access. When private options (e.g., private schools charging tuition) exist alongside a transfer-free public system, wealth differences can undermine the intended allocation of the primary mechanism, creating outcomes functionally similar to bribery.

Fully addressing wealth-based discrimination may therefore require considering not just the mechanism itself, but also the broader environment in which it operates.

6.2 Model assumptions

In the following, we discuss some key assumptions of our model.

Preference space. Our model incorporates several assumptions regarding agents' utility functions, the most restrictive being additive separability between object utility and wealth utility ($u_i(\omega, e) = \theta_i(\omega) + h(e)$). Additive separability allows

[10] See, e.g., Ray (2009).

us to clearly isolate the impact of wealth on the marginal utility of money, which simplifies the identification of the drivers of our results. However, in some applications, an agent's wealth might also influence the utility derived directly from an object. For instance, a poor student might benefit more from attending a good school than a rich student, as the latter could more easily compensate for a less desirable school's shortcomings.

The core assumption driving our results is that an agent's WTP increases with their wealth (positive income effects). Modifying the specific utility representation does not change the qualitative nature of our findings, as long as WTP continues to rise with wealth. In particular, it remains true that high wealth inequality will imply strong potential trading incentives under a transfer-free SCF (even if the poorer agent has higher object utility), and these trades cannot be realized without violating discrimination-freeness. Conversely, when wealth inequality is low, SCFs on the efficient frontier of Φ_{TF} can still be efficient, because an agent's WTA exceeds their WTP due to positive income effects.

Assigning probability shares. Our analysis adopts an ex-interim perspective, focusing on deterministic outcomes after any tie-breakers (like lotteries) are resolved. However, considering a probabilistic model where the designer assigns probability shares π of objects might allow for ex-ante efficiency improvements. In such a context, an SCF could be defined as discrimination-free if each agent's vector of probability shares is independent of their wealth.

While the basic insight that higher wealth inequality leads to stronger trading incentives likely persists in a probabilistic model, analyzing efficiency becomes more complex. An agent's WTP for receiving an object with probability π is typically concave in π. This implies that smoothing access by assigning shares to multiple agents might increase efficiency compared to assigning the object deterministically to one agent (see Huesmann (2017)). Since our goal is not to identify the optimal SCF but to evaluate the policy of a transfer ban against the discrimination-freeness criterion, we maintain the simplification of the ex-interim perspective.

Two-sided markets. Our model considers a one-sided assignment problem where only recipients are strategic. However, our notion of discrimination-freeness could be straightforwardly extended to two-sided markets (e.g., organ donation), where the providers of the objects (donors) might also be strategic players whose decisions are influenced by wealth. In such settings, discrimination-freeness might be relevant for both sides. For organ donation, one might require that richer patients do not have disproportionate access to organs, and that poorer individuals are not disproportionately likely to become donors.

7 Conclusion

In many markets where concerns arise that money shouldn't buy access, monetary transfers are banned. We investigate this common policy by formalizing one underlying goal—preventing wealth from determining access—using a new criterion, *discrimination-freeness*. Analyzing this in an assignment model with heterogeneous wealth and positive income effects, we find that the effectiveness of a transfer ban depends crucially on the level of wealth inequality. When wealth inequality is high, a transfer ban aligns well with discrimination-freeness. When wealth inequality is low, however, this alignment can break down, and a transfer ban can be unnecessarily restrictive.

Following Li (2017), our aim is not to ultimately say whether money should be banned or not but to clarify the consequences of different design choices. Specifically, we research the link between the policy of banning transfers and a desire to avoid richer individuals having better access to goods.

Our results suggest a nuanced perspective on using a transfer ban to avoid wealth-based discrimination. In societies with high wealth inequality, such bans appear well-calibrated, even if efficiency is sacrificed. Conversely, in societies with low inequality or strong redistribution mechanisms, transfer bans may be overly blunt, and allowing transfers within a discrimination-free framework could improve welfare.

In applying our results, it is important to note that designers and participants might use tools beyond monetary transfers. For instance, a designer could use tokens as "play money" to elicit preferences, or rely on agents signaling preferences through waiting times (e.g., assuming those queuing earlier for congressional hearings desire attendance more, Kliff (2019)).

However, even if transfers are formally banned, wealth can influence outcomes externally, meaning a ban may not fully eliminate discrimination concerns. The congressional hearings example again illustrates this point: lobbyists reportedly pay professional "line-standers"—and sometimes homeless people—to queue for them (Kliff, 2019), undermining the waiting time mechanism. Similarly, in education markets, wealthier families often gain better access to desirable schools by affording housing in more expensive neighborhoods (Black, 1999). Coexisting private markets also provide avenues for wealth to affect access. Fully addressing discrimination thus requires considering the broader environment, not just the formal mechanism.

Our results suggest avenues for future research. Examining the specific markets where discrimination-freeness is most normatively compelling and quantifying the efficiency trade-offs involved are important directions. Further work could also analyze other ethical concerns, such as coercion, commodification, or potential participant regret, as illustrated by the high rate of regret Zargooshi (2001) found among people in Iran who sold a kidney.

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A Appendix: Proofs

A.1 Preliminaries

We start with some useful characteristics of the WTP and the WTA.

Lemma 2. *For any $x > 0$ and $e \in \mathbb{R}$, there exist unique $k(x,e)$ and $c(x,e)$ with*

$$x + h(e - k(x,e)) = h(e) \quad \text{and} \quad x + h(e) = h(e + c(x,e)). \tag{A.1}$$

Furthermore, $k(x,e)$ is strictly concave in x, $c(x,e)$ is strictly convex in x, $\lim_{e\to\infty} k(x,e) = c(x,e) = \infty$, and $\lim_{e\to-\infty} k(x,e) = c(x,e) = 0$.

Proof of Lemma 2. By assumption $h' > 0$ with $\lim_{e\to\infty} h(e) = \infty$. $h'' < 0$ implies that for any e^*, $h(e)$ is below the tangent at e^*. Therefore, $\lim_{e\to-\infty} h(e) = -\infty$. This implies that $k(x,e)$ and $c(x,e)$ are well defined.

By definition, $k(x,e) = e - h^{-1}(h(e) - x)$ and $c(x,e) = h^{-1}(h(e) + x) - e$. Since $h'' < 0$, $k(x,e)$ is strictly concave in x and $c(x,e)$ is strictly convex in x.

$h'' < 0$ implies that for all $K > 0$

$$\frac{h(e) - h(e-K)}{K} < h'(e-K). \tag{A.2}$$

Since $\lim_{e\to\infty} h'(e-K) = 0$, the left hand side converges to zero as well for $e \to \infty$. Therefore, for any $x > 0$ there exists some $e^* \in \mathbb{R}$ such that $h(e) - h(e-K) < x$ for all $e > e^*$. This is equivalent to

$$x + h(e-K) > h(e) \quad \text{for all} \quad e > e^*. \tag{A.3}$$

By definition of $k(x,e)$, it implies that $k(x,e) > K$ for all $e > e^*$. Since $K > 0$ was arbitrary, $\lim_{e\to\infty} k(x,e) = \infty$ holds. $k(x,e) = c(x, e - k(x,e))$ implies that $\lim_{e\to\infty} c(x,e) \to \infty$ holds as well.

$h'' < 0$ implies $(h^{-1})'' > 0$. For any $x > 0$ and $y = h(e)$ it holds that

$$\frac{h^{-1}(y) - h^{-1}(y-x)}{x} < (h^{-1})'(y). \tag{A.4}$$

This is equivalent to

$$\frac{e - h^{-1}(h(e) - x)}{x} < \frac{1}{h'(e)}. \tag{A.5}$$

By definition, $k(x,e) = e - h^{-1}(h(e) - x)$. $\lim_{e\to-\infty} h'(e) = \infty$ implies that $k(x,e) \to 0$. $k(x,e) = c(x, e - k(x,e))$ implies that $\lim_{e\to-\infty} c(x,e) = 0$ as well.

Lemma 3. *For any $n \geq 1$, $V \geq 1$ and $e_1 > e_2$ consider the function $F : \mathbb{R}_+ \to \mathbb{R}_+$ defined as*

$$F(x) = nk(xV, e_1) - c(x, e_2). \tag{A.6}$$

It holds that $F'(0) > 0$, $F''(x) < 0$ for all $x \geq 0$. Furthermore, there exists a unique $x^* > 0$ such that $F(x) < 0$ if and only if $x > x^*$.

Proof of Lemma 3. By definition of k and c,

$$\frac{\partial c(x,e)}{\partial x} = \frac{1}{h'(e+c)} \quad \text{and} \quad \frac{\partial c(x,e)}{\partial e} = \frac{h'(e) - h'(e+c)}{h'(e+c)} \tag{A.7}$$

$$\frac{\partial k(x,e)}{\partial x} = \frac{1}{h'(e-k)} \quad \text{and} \quad \frac{\partial k(x,e)}{\partial e} = \frac{h'(e-k) - h'(e)}{h'(e-k)} \tag{A.8}$$

This implies that

$$F'(x) = \frac{Vn}{h'(e_1 - k(Vx, e_1))} - \frac{1}{h'(e_2 + c(x, e_2))} \tag{A.9}$$

Since $h'' < 0$ and $e_1 > e_2$ it implies $F'(0) > 0$. Furthermore, $\frac{Vn}{h'(e_1 - k(Vx, e_1))}$ decreases in x and $\frac{1}{h'(e_2 + c(x, e_2))}$ increases in x. This implies that $F'(x)$ decreases in x.

For the last part first note that $F'(x) < 0$ for x large enough. Convexity of $F(x)$ implies that $F(x)$ is smaller than the tangent at x with $F'(x) < 0$. Therefore, $\lim_{x\to\infty} F(x) = -\infty$. Together with $F'(0) > 0$ and $F''(x) < 0$ it implies that there is a unique $x^* > 0$ with $F(x^*) = 0$ and that $F(x) < 0$ if and only if $x > x^*$.

A.2 Proofs of the main document

Proof of Proposition 1

First we show that for any $x > 0$ and $\overline{e} < \infty$ it holds that

$$x < h(\overline{e}) - h(\underline{e}) \Leftrightarrow k(x, \overline{e}) > c(x, \underline{e}). \tag{A.10}$$

To show (A.10), use that $k(x,e) = e - h^{-1}(h(e) - x)$ and

$$x < h(\overline{e}) - h(\underline{e}) \Leftrightarrow \underline{e} < h^{-1}(h(\overline{e}) - x) \tag{A.11}$$

It implies that $x < h(\overline{e}) - h(\underline{e})$ is equivalent to

$$\begin{aligned} c(x, \underline{e}) &= c(x, \overline{e} - (\overline{e} - \underline{e})) && (A.12)\\ &< c(x, \overline{e} - (\overline{e} - h^{-1}(h(\overline{e}) - x))) && (A.13)\\ &= c(x, \overline{e} - k(x, \overline{e})) && (A.14)\\ &= k(x, \overline{e}) && (A.15)\end{aligned}$$

Now define $e^c = h^{-1}(h(\underline{e}) + \inf \Theta)$. By definition,

$$\overline{e} > e^c \Leftrightarrow \inf \Theta < h(\overline{e}) - h(\underline{e}). \tag{A.16}$$

For $\overline{e} > e^c$ the SCF φ^c is inefficient: By using (A.16), for $\overline{e} > e^c$ there exists $\theta \in \Theta$, $\omega', \omega \in \Omega$ with $\theta(\omega') - \theta(\omega) < h(\overline{e}) - h(\underline{e})$. Assume that $\theta_i = \theta$ for all agents i and let i be the agent receiving ω' and j be the agent receiving ω. By (A.10),

$$k(\theta(\omega') - \theta(\omega), \overline{e}) > c(\theta(\omega') - \theta(\omega), \underline{e}). \tag{A.17}$$

By (A.17), there exist $e_j, e_i \in E$ such that agents i and j have an incentive to trade. This implies that φ^c is inefficient. For $\overline{e} = \infty$, by the very same arguments trading incentive occur once agent j is rich enough.

For $\overline{e} \le e^c$ the SCF φ^c is efficient: $\overline{e} \le e^c$ is equivalent to $\inf \Theta \ge h(\overline{e}) - h(\underline{e})$. $\varphi^c = (\sigma^c, 0)$ is efficient if and only if for all object utility profiles $\theta_N = (\theta_i)_{i \in N}$ and object allocations σ

$$\sum_{i \in N^+} k(\theta_i(\omega_i) - \theta_i(\omega_i^c), \overline{e}) \le \sum_{i \in N^-} c(\theta_i(\omega_i^c) - \theta_i(\omega_i), \underline{e}). \tag{A.18}$$

Here, $\omega_i^c = \sigma_i^c(\theta_N)$ and $\omega_i = \sigma_i(\theta_N)$. N^+ and N^- are the sets of agents for whom the object assignment under σ (compared to σ^c) improves and worsens, respectively.

Define $X_i = |\theta_i(\omega_i) - \theta_i(\omega_i^c)|$. Since σ^c maximizes the sum of object utilities we have $\sum_{i \in N^+} X_i \le \sum_{i \in N^-} X_i$. Furthermore, for $\delta = h(\overline{e}) - h(\underline{e})$ it holds that $k(\delta, \overline{e}) = c(\delta, \underline{e})$. It implies

$$\sum_{i \in N^+} X_i k(\delta, \overline{e}) \le \sum_{i \in N^-} X_i c(\delta, \underline{e}). \tag{A.19}$$

Since $\inf \Theta \ge \delta$ we have $X_i \ge \delta$ for all i. By Lemma 2, $k(x, e)$ is strictly concave in x and $c(x,e)$ is strictly convex in x. By Jensen's inequality, $k(X_i, \overline{e}) \leq \frac{X_i}{\delta} k(\delta, \overline{e})$ as well as $c(X_i, \underline{e}) \geq \frac{X_i}{\delta} c(X_i, \underline{e})$. Combining it with A.19 yields

$$\sum_{i \in N^+} k(X_i, \overline{e}) \leq \sum_{i \in N^+} \frac{X_i}{\delta} k(\delta, \overline{e}) \leq \sum_{i \in N^-} \frac{X_i}{\delta} c(\delta, \underline{e}) \leq \sum_{i \in N-} c(X_i, \underline{e}). \tag{A.20}$$

This corresponds to (A.18) which implies that $\varphi^c = (\sigma^c, 0)$ is efficient.

$e^c(\inf \Theta, \underline{e}) = h^{-1}(h(\underline{e}) + \inf \Theta)$ strictly increases in $\inf \Theta$ since $h' > 0$. Furthermore, $\lim_{\inf \Theta \to \infty} e^c(\inf \Theta, \underline{e}) = \lim_{\inf \Theta \to \infty} h^{-1}(h(\underline{e}) + \inf \Theta) = \infty$ and $e^c(0, \underline{e}) = h^{-1}(h(\underline{e}) + 0) = \underline{e}$.

Finally, we show that φ^c is at the efficient frontier of Φ_{DF}. Assume the contrary, i.e., for any type profile a set of agents has an incentive to trade. Since φ^c maximizes the sum of object utilities, the trading incentive disappears once the buyers are poor and the sellers are rich. Therefore, any Pareto improvement of φ^c discriminates.

Proof of Lemma 1

First, fix any $e_i \in E$. Assume that $\varphi_i(\theta_i) \neq \varphi(\theta_i')$ for some $\theta_i, \theta_i' \in \Theta$ with $R(\theta_i) = R(\theta_i')$. Since $\varphi \in \Theta_{TF}$ it implies that $\sigma_i(\theta_i) \neq \sigma_i(\theta_i')$. Since $R(\theta_i) = R(\theta_i')$ agent i then has either an incentive to misreport for θ_i or θ_i'. Therefore, φ is not implementable.

Now fix any $\theta_i \in \Theta$ and consider $e_i, e_i' \in E$ with $e_i \neq e_i'$. Assume that Assume that $\varphi_i(e_i) \neq \varphi(e_i')$ which implies that $\sigma_i(e_i) \neq \sigma_i(e_i')$. Since the rank order of objects does not depend on the endowment, agent i either has an incentive to misreport for e_i or e_i'.

Proof of Proposition 2

Let $\varphi^o = (\sigma^o, 0)$ be an implementable SCF at the efficient frontier of Φ_{TF}. By Lemma 1,φ^o is independent of wealth endowments and an agent's assignment depends only on their rank order of objects.

Case 1: $\overline{e} = \infty$. Assume that all agents have the same type $(\theta, e) \in \Theta \times E$ and φ^o assigns object ω to agent i and object ω' to agent j with $\theta(\omega) - \theta(\omega') > 0$. Since $\overline{e} = \infty$ and $k(\theta(\omega) - \theta(\omega'), e) \to \infty$ for $e \to \infty$ (see Lemma 2), there exists some $e' \in E$ such that

$$k(\theta(\omega) - \theta(\omega'), e') > c(\theta(\omega) - \theta(\omega'), e) > 0. \tag{A.21}$$

Now assume all agents except agent j have the type (θ, e) while agent j has the type (θ, e'). Since φ^o does not depend on wealth endowments, the object allocation does not alter. Then, agent i and agent j have an incentive to trade which implies that φ^o is inefficient.

To proof that φ^o is at the efficient frontier of Φ_{DF} assume that for the type profile $t_N = (\theta_i, e_i)_{i \in N}$ a Pareto-improvement $\varphi = (\sigma, m)$ of $\varphi^o = (\sigma^o, 0)$ exists. We show that it implies that φ is not discrimination-free. Since φ is a Pareto-improvement,

$$\sum_{i \in N^+} k(\theta_i(\omega_i) - \theta_i(\omega_i^o), e_i) > \sum_{i \in N^-} c(\theta_i(\omega_i^o) - \theta_i(\omega_i), e_i). \tag{A.22}$$

Here, $\omega_i^o = \sigma_i^o(t_N)$ and $\omega_i = \sigma_i(t_N)$. N^+ and N^- are the sets of agents for whom the object assignment under σ (compared to σ^c) improves and worsens, respectively.

Now consider a type profile t'_N that differs from t_N only with respect to the wealth endowments of agents that are part of N^-. Since both φ^o and φ are discrimination-free, their object allocations are the same for t_N and t'_N. $c(x, e) \to \infty$ for $e \to \infty$ (see Lemma 2) and $x > 0$ implies that there exists some $e' \in E$ such that if t'_N is such that all agents $i \in N^-$ have wealth e', keeping all other type parameters fixed, inequality (A.22) does not hold any more which implies that φ cannot be a discrimination-free Pareto improvement of φ^o. Therefore, φ^o is at the efficient frontier of Φ_{DF}.

Case 2: $\inf \Theta = 0$ **(and** $\overline{e} < \infty$**).** For $\inf \Theta = 0$ there exists a sequence $(\theta_n)_{n \in \mathbb{N}}$ with $\theta_n \in \Theta$ and $R(\theta_n) = R(\theta_m)$ for all $n, m \in \mathbb{N}$ such that

$$|\theta_n(\omega') - \theta_n(\omega)| \to 0 \quad \text{for some} \quad \omega', \omega \in \Omega \tag{A.23}$$

Such a sequence exists because Ω is finite and therefore the number of rank orders that can exist is finite as well. Without loss of generality we assume that $\theta_n(\omega') > \theta_n(\omega)$. (A.23) implies that there exist $\theta, \theta' \in \Omega$ with $R(\theta) = R(\theta')$ such that

$$k(\theta(\omega') - \theta(\omega), \underline{e}) - c(\theta'(\omega') - \theta'(\omega), \overline{e}) > 0 \tag{A.24}$$

Let $\varphi^o = (\sigma^o, 0)$ be an implementable SCF at the efficient frontier of Φ_{TF}. Let the type profile $t_N = (theta_i, e_i)_{i \in N}$ be such that $\theta_i = \theta$ for all agents i. Assume that $\sigma_i^o(t_N) = \omega$ and $\sigma_j^o(t_N) = \omega'$. Note that implementability of φ^o implies that φ^o does not depend on wealth. Now consider t'_N that differs from t_N only by the object utilities of agent j, which is now θ'. $R(\theta) = R(\theta')$ implies that $\sigma^o(t_N) = \sigma^o(t'_N)$. By (A.24), agents i and agent j have an incentive to trade independent of their wealth realizations. Therefore, φ^o is not efficient. Furthermore, providing i with ω' and providing j with ω plus a transfer for compensation from i to j is a Pareto improvement. While the transfer may depend on wealth realizations, the new object allocation does not. This implies that the Pareto improvement is discrimination-free and φ^o is therefore not at the efficient frontier of Φ_{TF}.

Case 3: $\sup \Theta = \infty$ (and $\overline{e} < \infty$). For $\sup \Theta = \infty$ analogous arguments as for $\inf \Theta = 0$ hold. More specifically, there exists a sequence $(\theta_n)_{n \in \mathbb{N}}$ with $\theta_n \in \Theta$ and $R(\theta_n) = R(\theta_m)$ for all $n, m \in \mathbb{N}$ such that

$$|\theta_n(\omega') - \theta_n(\omega)| \to \infty \quad \text{for some} \quad \omega', \omega \in \Omega \tag{A.25}$$

Again, without loss of generality, $\theta_n(\omega') > \theta_n(\omega)$. It implies that there some $\theta, \theta' \in \Omega$ with $R(\theta) = R(\theta')$ such that (A.24) holds. The remainder of the proof is then the same as for $\inf \Theta = 0$.

Proof of Proposition 3

Proof of the proposition. Let $\varphi^o = (\sigma^o, 0)$ be an implementable SCF at the efficient frontier of Φ_{TF}. By Lemma 1, φ^o then only depends on the rank order of objects. φ^o is efficient if and only if for all object utility profiles $\theta_N = (\theta_i)_{i \in I}$ and object allocations σ it holds that

$$\sum_{i \in N^+} k(\theta_i(\omega_i) - \theta_i(\omega_i^o), \overline{e}) \leq \sum_{i \in N^-} c(\theta_i(\omega_i^o) - \theta_i(\omega_i), \underline{e}). \tag{A.26}$$

Here, $\omega_i^o = \sigma_i^o(t_N)$ and $\omega_i = \sigma_i(t_N)$. N^+ and N^- are the sets of agents for whom the object assignment under σ (compared to σ^c) improves and worsens, respectively.

Since σ^o is ordinally efficient, $N^- \neq \emptyset$. For every object allocation σ with $N^+ \neq \emptyset$ define $\tilde{e}(\theta_N, \sigma, \underline{e})$ such that for $\overline{e} = \tilde{e}(\theta_N, \sigma, \underline{e})$, (A.26) holds with equality. $\tilde{e}(\theta_N, \sigma, \underline{e})$ exists and is well defined since $k(x, e)$ strictly increases in e with

$k(x, e) \to \infty$ for $e \to \infty$ and $k(x, e) \to 0$ for $e \to -\infty$. Then define

$$f(\underline{e}) = \inf_{\sigma} \inf_{\theta_N} \tilde{e}(\theta_N, \sigma, \underline{e}) \tag{A.27}$$

Then, by (A.26), φ^o is efficient if and only if $\overline{e} \leq e^*$ with $e* = f(\underline{e})$. Note that f is strictly increasing since $\inf \Theta \neq 0$ and $c(x, e)$ is strictly increasing in e for $x > 0$. By definition, f depends on φ^o and Θ but does not depend on the wealth space E.

Define $\hat{e} = f^{-1}(\underline{e})$. Then, $\overline{e} > \hat{e}$ if and only if for all object utility profiles $\theta_N = (\theta_i)_{i \in I}$ and object allocations σ

$$\sum_{i \in N^+} k(\theta_i(\omega_i) - \theta_i(\omega_i^o), \underline{e}) < \sum_{i \in N^-} c(\theta_i(\omega_i^o) - \theta_i(\omega_i), \overline{e}). \tag{A.28}$$

If (A.28) holds, φ^o is at the efficient frontier Θ_{DF} because any potential trading incentives disappear once the buyers are poor enough and the seller are rich enough. Conversely, if (A.28) does not hold, there exists some type profile θ_N and object allocation σ such that allocating the object according to σ and compensating those receiving worse objects on the cost of those receiving better objects is a discrimination-free Pareto improvement.

Proof of Proposition 4

Definition and characteristics of ρ. Define $\rho(\inf \Theta, v_\Theta, \underline{e})$ implicitly by

$$k(v_\Theta \inf \Theta, \rho) - c(\inf \Theta, \underline{e}) = 0 \tag{A.29}$$

ρ is well defined because $k(x, e)$ strictly increases in e and x with $\lim\limits_{e \to \infty} k(x, e) = \infty$ and $\lim\limits_{e \to -\infty} k(x, e) = 0$ (see Lemma 2). It furthermore implies that $\rho(\inf \Theta, v_\Theta, \underline{e})$ strictly decreases in v_Θ and strictly increases in $\underline{e}$. Also, $k(v_\Theta \inf \Theta, \underline{e}) > c(\inf \Theta, \underline{e})$ holds for for v_Θ large enough such that $\lim\limits_{v_\Theta \to \infty} \rho(\inf \Theta, v_\Theta, \underline{e}) < \underline{e}$. For $v_\Theta = 1$ the definition of ρ equals the definition of e^c such that $\rho(\inf \Theta, 1, \underline{e}) = e^c(\inf \Theta, \underline{e})$. This implies $\rho(\inf \Theta, v_\Theta, \underline{e}) \leq e^c(\inf \Theta, \underline{e})$.

For how ρ depends on $\inf \Theta$ note that Lemma 3 implies $\frac{\delta \rho(v_\Theta \inf \Theta, \underline{e}}{\delta \inf \Theta} > 0$. Furthermore, $\rho(\inf \Theta, v_\Theta, \underline{e}) \leq e^c(\inf \Theta, \underline{e})$ implies that $\lim_{\inf \Theta \to 0} \rho(\inf \Theta, v_\Theta, \underline{e}) \leq \lim_{\inf \Theta \to 0} e^c(\inf \Theta, \underline{e}) = \underline{e}$.

We proof that $e^* \leq \rho(\inf \Theta, v_\Theta, \underline{e})$ by showing that $\overline{e} > \rho(\inf \Theta, v_\Theta, \underline{e})$ implies inefficiency of any implementable SCF at the efficient frontier of Φ_{TF}. By definition of ρ, $\overline{e} > \rho$ implies

$$k(v_\Theta \inf \Theta, \overline{e}) - c(\inf \Theta, \underline{e}) > 0 \tag{A.30}$$

By definition of $\inf \Theta$, there exist two objects $\omega, \omega' \in \Omega$ and $\theta \in \Theta$ such that

$$k(v_\Theta \inf \Theta, \overline{e}) - c(\theta(\omega') - \theta(\omega), \underline{e}) > 0 \tag{A.31}$$

By definition of v_Θ, there exists some θ' with the same rank order as θ such that $\theta'(\omega') - \theta'(\omega) > v_\Theta \inf \Theta$ holds. It implies that

$$k(\theta'(\omega') - \theta'(\omega), \overline{e}) - c(\theta(\omega') - \theta(\omega), \underline{e}) > 0 \tag{A.32}$$

Consider any implementable SCF $\varphi^o = (\sigma^o, 0)$ at the efficient frontier of Φ_{TF}. Assume that type realizations are such that $\theta_i = \theta$ for all agents. Let agent i be the agent receiving object ω and let agent j be the agent receiving ω'. Now change the type profile such that all agents keep their type except for agent j who now has object utilities θ'. Since θ' implies the same rank order of objects as θ does, implementability of φ^o implies that the object allocation is the same as for σ^o. By (A.32), there exist wealth realizations such that agent j and agent i have an incentive to trade and φ^o is not efficient.

Definition and characteristics of δ. Define $\delta(\inf \Theta, V_\Theta, \underline{e})$ implicitly by

$$(n-1)k(V_\Theta \inf \Theta, \delta) - c(\inf \Theta, \underline{e}) = 0 \tag{A.33}$$

The monotonicity properties of δ follow from the same arguments as those used for ρ.

To show that $e^* \leq \delta$ we show that if $\overline{e} \leq \delta$, any implementable SCF at the efficient frontier of Φ_{TF} is efficient. Consider an implementable SCF $\varphi^o = (\sigma^o, 0)$ at the efficient frontier of Φ_{TF}. φ^o then only depends on the rank order of objects. φ^o is efficient if for all object utility profiles $\theta_N = (\theta_i)_{i \in N}$ and object allocations σ the following is satisfied

$$\sum_{i \in N^+} k(\theta_i(\omega_i) - \theta_i(\omega_i^o), \overline{e}) \leq \sum_{i \in N^-} c(\theta_i(\omega_i^o) - \theta_i(\omega_i), \underline{e}). \tag{A.34}$$

Here, $\omega_i^o = \sigma_i^o(t_N)$ and $\omega_i = \sigma_i(t_N)$. N^+ and N^- are the sets of agents for whom the object assignment under σ (compared to σ^c) improves and worsens, respectively.

Since σ^o is ordinal efficient, $N^- \neq \emptyset$.

Now fix any σ and θ_N. It holds that

$$\sum_{i\in N^-} c(\theta_i(\omega_i^o) - \theta_i(\omega_i), \underline{e}) \geq c(\inf\Theta, \underline{e}). \tag{A.35}$$

Since $\theta_i(\omega) - \theta_i(\omega') \leq V_\Theta \inf\Theta$ for any two objects ω, ω' and N^+ contains at most $n-1$ agents we have

$$\sum_{i\in N^+} k(\theta_j(\omega_i) - \theta_i(\omega_i^o), \overline{e}) \leq (n-1)k(V_\Theta \inf\Theta, \overline{e}) \tag{A.36}$$

Now consider $\overline{e} \geq \delta(\inf\Theta, V_\Theta, \underline{e})$. It then implies

$$\sum_{i\in N^+} k(\theta_j(\omega_i){-}\theta_i(\omega_i^o), \overline{e}) \leq (n{-}1)k(V_\Theta \inf\Theta, \overline{e}) \leq c(\inf\Theta, \underline{e}) \leq \sum_{i\in N^-} c(\theta_i(\omega_i^o){-}\theta_i(\omega_i), \underline{e}). \tag{A.37}$$

Estimates for $\hat{e}$. By Proposition 3, it holds that $\hat{e} = f^{-1}(\underline{e})$. The same argumentation can be used to show that the estimates for $\hat{e}$ are the inverse functions of the estimates of e^*.

Proof of Proposition 5

Assume $\varphi = (\sigma, m) \in \Phi_{DF}$ is implementable but $\varphi_0 = (\sigma, 0)$ is not. We show that this leads to a contradiction if E is unbounded or if $\overline{\Theta}$ is convex.

Discrimination-freeness of φ implies that σ does not depend on wealth realizations. Consider any $e \in E$. If $\varphi_0 = (\sigma, 0)$ is not implementable there is an agent i and $\theta^1, \theta^2 \in \Theta$ and $\omega^1, \omega^2 \in \Omega$ such that for a fixed announcement of the other agents (omitted in the following)

$$\varphi_i(\theta^1, e) = (\omega^1, m^1) \quad \text{and} \quad \varphi_i(\theta^2, e) = (\omega^2, m^2)$$

while $\theta^1(\omega^2) > \theta^1(\omega^1)$. Furthermore, we have $m^1 > m^2$ because φ is implementable. Differently said, if announcing θ^2 agent i pays a price for receiving a more preferred object.

Case I: $\overline{e} = \infty$. Implementability and discrimination-freeness of φ implies that $\theta^1(\omega^1) + h(e + m^1) \geq \theta^1(\omega^2) + h(e + m^2)$ for all $e \in E$. However, if $\overline{e} = \infty$, $m^1 - m^2$ is not enough to compensate the agent once he is rich enough. This is a contradiction.

Case II: Θ is convex. We call a bundle $(\omega, m) \in \Omega \times \mathbb{R}$ *reachable* for agent i if there exists $\theta \in \Theta$ with $\varphi_i(\theta, e) = (\omega, m)$. Let $\mathcal{S}$ be the set of all reachable bundles (still keeping the other agents' types fixed). Implementability implies that $|\mathcal{S}| \leq k+1$. By announcing type (θ, e), agent i is assigned to a bundle that type (θ, e) (weakly) prefers most among all reachable bundles in $\mathcal{S}$.

We now seek to show that φ is not discrimination-free by finding $\theta \in \overline{\Theta}$ with $\varphi(\theta, e_L) \neq \varphi(\theta, e_H)$ for some $e_L < e_H \in E$. The main step is to construct some $\theta^* \in \overline{\Theta}$ such that agent i with type (θ^*, e) with $e = \frac{e_H - e_L}{2}$ and $e_L, e_H \in E$ is indifferent between two distinct bundles in $\mathcal{S}$ while preferring both over all other reachable bundles in $\mathcal{S}$. Positive income effects and implementability then imply that $\varphi(\theta^*, e_L) \neq \varphi(\theta^*, e_H)$. If $\theta^* \in \Theta$ take $\theta = \theta^*$. If $\theta^* \notin \Theta$ there is some θ close enough to θ^* such that $\varphi(\theta, e_L) \neq \varphi(\theta, e_H)$ holds as well.[11]

We construct θ^* by using the convexity of $\overline{\Theta}$. Convexity of $\overline{\Theta}$ implies that there exists some $\alpha \in [0, 1]$ such that for $\theta^3 = \alpha\theta^2 + (1-\alpha)\theta^1 \in \overline{\Theta}$ we have

$$\theta^3(\omega^2) + h(e + m^2) = \theta^3(\omega^1) + h(e + m^1). \tag{A.38}$$

If all other reachable bundles $(\omega, m) \in \mathcal{S}$ are (weakly) less preferred by type (θ^3, e), take $\theta^* = \theta^3$. Otherwise, let (ω^3, m^3) be such that

$$\theta^3(\omega^3) + h(e + m^3) > \theta^3(\omega^2) + h(e + m^2) = \theta^3(\omega^1) + h(e + m^1) \tag{A.39}$$

Using the same arguments we then can find θ^4 as a convex combination of θ^3 and θ^2 such that

$$\theta^4(\omega^3) + h(e + m^3) = \theta^4(\omega^2) + h(e + m^2) \geq \theta^4(\omega^1) + h(e + m^1). \tag{A.40}$$

Again, if all other reachable bundles $(\omega, m) \in \mathcal{S}$ are (weakly) less preferred by type (θ^4, e), take $\theta^* = \theta^4$. Otherwise, let (ω^4, m^4) be such that

$$\theta^4(\omega^4) + h(e + m^4) > \theta^4(\omega^3) + h(e + m^3) = \theta^4(\omega^2) + h(e + m^2) \geq \theta^4(\omega^1) + h(em^1) \tag{A.41}$$

By repeating this procedure, we ultimately find some θ^a with $a \leq |\mathcal{S}| \leq k+1$ such that

$$\theta^a(\omega^a) + h(e + m^a) = \theta^a(\omega^{a-1}) + h(e + m^{a-1}) \geq \theta^a(\omega) + h(e + m) \tag{A.42}$$

[11] Note that it is not necessary that $e \in E$ since e is only needed to construct indifference for e while wealth endowments $e_L, e_H \in E$ imply the contradiction to discrimination-freeness.

for all reachable bundles $(\omega, m) \in \mathcal{S}$. $\theta^* = \theta^a$ then satisfies the desired criteria which shows that convexity of $\overline{\Theta}$ implies that φ cannot be discrimination-free and implementable if φ_0 is not.

Proof of Proposition 6

If $\overline{\Theta}$ is not convex, there exist $\theta, \theta' \in \Omega$ and $\alpha \in (0,1)$ such that $\theta^* = \alpha\theta + (1-\alpha)\theta' \notin \overline{\Theta}$. It implies that for some $\omega \in \Omega$ we have $\theta^*(\omega) = \alpha\theta(\omega) + (1-\alpha)\theta'(\omega) \notin \overline{\Theta}(\omega)$. Since $\overline{\Theta}^C$ is an open set, and $k(\theta^*(\omega), e)$ is continuous in e and in $\theta^*(\omega)$, there exist $\epsilon > 0$ and $\delta > 0$ such that for all $\theta \in \Theta$, $e \in B_\delta(e^*)$ it either holds that

$$k(\theta(\omega), e) < k(\theta^*(\omega), e^*) - \epsilon \quad \text{or} \quad k(\theta(\omega), e) > k(\theta^*(\omega), e^*) + \epsilon.$$

Therefore, if $E \subset B_\delta(e^*)$, the WTP is always either below $k(\theta^*(\omega), e^*) - \epsilon$ or above $k(\theta^*(\omega), e^*) + \epsilon$ but never inbetween. Furthermore, by construction of θ^*, both cases occur for some object utilities.

Now consider serial dictatorship mechanism: one after another, each agent picks an object. Transfers are zero for all objects except for ω, for which the price $k(\theta^*(\omega), e^*)$ must be paid. This mechanism is implementable. It is also discrimination-free, because whether or not an agent selects ω depends only on their object utility θ, not on their wealth. However, this mechanism is not implementable without transfers. The reason is that all agents prefer ω over the null object 0 (that is assigned to some agent). An agent who does not select ω because their valuation is too low to justify paying the price would nonetheless choose ω if no payment were required. Thus, this mechanism is both implementable and discrimination-free, while the same allocation is not implementable without transfers. This proves the proposition.

Proof of Proposition 7

Let $\varphi = (\sigma, m)$ be an implementable, discrimination-free and efficient SCFs that represents a price mechanism. Since there is only one good, we can, without loss of generality, focus on only two agents, agent 1 and agent 2.

Note that agents are not necessarily anonymous because we take an ex-interim perspective where potential priorities or lotteries may exist that do not depend on types and are determined ex-ante. This allows us to focus on deterministic outcomes (see also Section 2). Thereby we implicitly assume that the outcomes of a potential ex-ante lottery or priorities are known to the agents. However, the result and the general approach do not change if the lottery outcome might be unknown to the agents.

Since φ represents a price mechanism, for any two types $t_1 = (\theta_1, e_1)$ and $t_2 = (\theta_2, e_2)$, $\varphi_i(t_1, t_2) = (\pi_i, e_i - p_i)$ with $\pi_i \in \{0, 1\}$. $\pi_i = 1$ if any only if agent i receives the object. Furthermore, $p_i \geq 0$ and $p_i > 0$ implies that $\pi_i = 1$.

Monotonicity properties: Consider $t_i = (\theta_L, e_i)$, $t'_i = (\theta_H, e_i)$ with $\theta_L \leq \theta_H$ and $t_j = (\theta_j, e_j)$ with $i \neq j$ and $i, j \in \{1, 2\}$. Denote $\pi_L = \pi_i(t_i, t_j)$, $\pi_H = \pi_i(t'_i, t_j)$, $p_L = p_i(t_i, t_j)$ and $p_H = p_i(t'_i, t_j)$. We show that

$$\pi_L \leq \pi_H \quad \text{and} \quad p_L \leq p_H. \tag{A.43}$$

By implementability of φ,

$$\pi_L\theta_L + h(e_i - p_L) \geq \pi_H\theta_L + h(e_i - p_H) \tag{A.44}$$

$$\Leftrightarrow \quad (\pi_L - \pi_H)\theta_L + h(e_i - p_L) - h(e_i - p_H) \geq 0. \tag{A.45}$$

Now assume that that A.43 does not hold such that $\pi_L > \pi_H$. Since $\theta_L < \theta_H$ it implies

$$(\pi_L - \pi_H)\theta_H + h(e_i - p_L) - h(e_i - p_H) > 0. \tag{A.46}$$

This contradicts implementability since agent i with type $t'_i = (\theta_H, e_i)$ has an incentive to report $t_i = (\theta_L, e_i)$ instead. Therefore, $\pi_L \leq \pi_H$ has to hold. By (A.44), $\pi_L \leq \pi_H$ implies that $p_L \leq p_H$ holds as well, implying A.43.

Efficient SCFs. To prove the proposition we show that $\underline{e} > \overline{e}$ either implies that φ is inefficient or that a pure lottery is efficient.

First note that discrimination-freeness of φ implies that π_i does not depend on wealth endowments but only on the object valuations. For any object utility θ^*_j of agent j define

$$\hat{\theta}_i(\theta^*_j) = \inf_{\theta_i \in \Theta} (\theta_i | \pi(\theta_i, \theta^*_j) = 1). \tag{A.47}$$

as the lower bound of object valuations for which agent $i \neq j$ receives the object. By the monotonicity properties derived above agent i receives the object if $\theta_i > \hat{\theta}_i(\theta^*_j)$ for some given θ^*_j and does not receive the object if $\theta_i < \hat{\theta}_i(\theta^*_j)$.

Case 1: Assume that there exists some θ^*_j with $\inf \Theta < \hat{\theta}_i(\theta^*_j) < \sup \Theta$.

Let $p_i(\theta_i, \theta^*_j, e_i, e_j)$ be the price agent i has to pay. It holds that

$$p_i(\theta_i, \theta^*_j, e_i, e_j) = 0 \quad \text{for all} \quad \theta_i < \hat{\theta}_i(\theta^*_j) \tag{A.48}$$

Implementability requires

$$p_i(\theta_i, \theta_j^*, e_i, e_j) = p_i(\theta_j^*, e_i, e_j) \quad \text{for} \quad \theta_i > \hat{\theta}_i(\theta_j^*). \tag{A.49}$$

Furthermore, by implementability,

$$p_i(\theta_j^*, e_i, e_j) \leq k(\hat{\theta}_i(\theta_j^*), e_i). \tag{A.50}$$

This is because otherwise, agent i with type $t_i = (\theta_i^*, e_i)$ and $\theta_i^* > \hat{\theta}_i(\theta_j^*)$ but close enough to $\hat{\theta}_i(\theta_j^*)$ has an incentive to misreport. Also, by implementability,

$$p_i(\theta_j^*, e_i, e_j) \geq k(\hat{\theta}_i(\theta_j^*), e_i). \tag{A.51}$$

Otherwise, agent i with type $t_i = (\theta_i^*, e_i)$ and $\theta_i^* < \hat{\theta}_i(\theta_j^*)$ but close enough to $\hat{\theta}_i(\theta_j^*)$ has an incentive to misreport. Therefore,

$$p_i(\theta_j^*, e_i, e_j) = k(\hat{\theta}_i(\theta_j^*), e_i). \tag{A.52}$$

We now show that efficiency of φ implies that $\underline{e} = \overline{e}$. If φ is efficient, for all $\theta_i > \hat{\theta}_i(\theta_j^*)$

$$k(\theta_j^*, \overline{e}) \leq c(\theta_i, \underline{e} - k(\hat{\theta}_i(\theta_j^*), \underline{e})). \tag{A.53}$$

If $\hat{\theta}_i(\theta_j^*) = \theta_j^*$, by definition of the WTP k and the WTA c, this inequality is satisfied if and only if $\underline{e} = \overline{e}$. If $\hat{\theta}_i(\theta_j^*) < \theta_j^*$, even for $\underline{e} = \overline{e}$, the left hand side is strictly larger than the right hand side. This contradicts efficiency of φ. If $\hat{\theta}_i(\theta_j^*) > \theta_j^*$, it implies that $\hat{\theta}_j(\hat{\theta}_i(\theta_j^*)) < \hat{\theta}_i(\theta_j^*)$. Then, φ is inefficient for the same reason as discussed for the case $\hat{\theta}_i(\theta_j^*) < \theta_j^*$.

Case 2: Let $\hat{\theta}_i(\theta_j^*) \in \{\inf \Theta, \sup \Theta\}$ for all $\theta_j^* \in \Theta$ and all $i \neq j \in \{1, 2\}$. Assume $\hat{\theta}_i(\theta_j^*) = \inf \Theta$. Then, if agent j's object utility is θ_j^*, agent i receives the good independent of θ_i. Since $\hat{\theta}_j(\sup \Theta) < \theta_j^*$ and $\hat{\theta}_j(\sup \Theta) \in \{\inf \Theta, \sup \Theta\}$ it has to holds that $\hat{\theta}_j(\sup \Theta) = \inf \Theta$ (if $\sup \Theta \notin \Theta$ consider the limit). This implies that agent i receives the good independent of θ_i and θ_j. If φ is efficient, it implies that the pure lottery is efficient as well (if agent i does not want to sell the good after a price is paid he won't sell the good without having paid the price).

If, for some θ_j^*, $\hat{\theta}_i(\theta_j^*) = \sup \Theta$, the same argumentation can be used to see that then agent i does not receive the object independent of the object utilities.

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