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On two different characterizations of bisimulation

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Abstract

[AM89] and [JNW94] give distinct characterizations of bisimulation on labelled transition systems in terms of category theory. This paper discusses the differences between their formalisms and shows how to translate these approaches into one another.

Contents

1 Introduction

Various notions of bisimulation on labelled transition systems have been introduced in order to identify processes that cannot be distinguished by an external agent. These concepts are based on ideas of [Mil80] and [Par81]. Later on they have been carried over to other models of concurrent systems, e. g. on event structures by [GG90].

In [AM89] strong bisimulation on labelled transition systems is characterized in terms of category theory. [AM89] consider a labelled transition system T as a coalgebra (A, α) of a particular endofunctor and a bisimulation on T as a coalgebra (R, γ), $R \subseteq A \times A$, satisfying certain conditions.

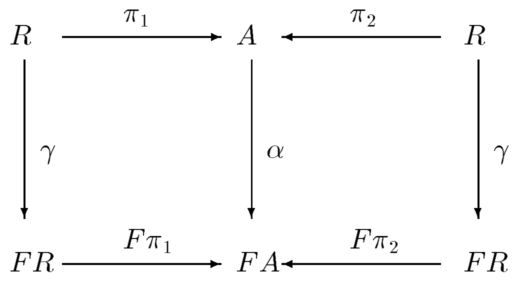

Figure 1: AM-bisimulation

[JNW94] take a more general approach. Their aim is an abstract characterization of strong bisimulation on a arbitrary category **M** in which a full subcategory **P** of so-called path objects is distinguished. Two objects X_1, X_2 in **M** are called bisimular if there is an object X in **M** and so-called **P**-open morphisms $f_i : X \to X_i$, $i = 1, 2$. This approach applies in particular to the category $\mathbf{T}_L$ of L-labelled transition systems and transition preserving mappings.

We show how these two characterizations of strong bisimulation on transition systems relate by translating one into the other.

2 The view of Aczel and Mendler

A *coalgebra* for an endofunctor F on a category **C** is a pair (A, α) where A is an object of **C** and $\alpha : A \to FA$ a morphism. A morphism $\pi : A \to B$ in **C** is called a *homomorphism* between coalgebras (A, α) and (B, β) iff $\beta \circ \pi = (F\pi) \circ \alpha$ holds. The coalgebras and homomorphisms form itself a category denoted by $\mathbf{C}_F$.

In order to provide the existence of final objects in $\mathbf{C}_F$ for a special type of functors F [AM89] use the category **Class**. As we are here interested in transition systems and bisimulation it suffices to deal with the category **Set**.

Let F be an endofunctor on **Set**. We call a coalgebra (R, γ) an *AM-bisimulation* on a coalgebra (A, α) iff $R \subseteq A \times A$ and the projections π_1, π_2 of R on A are homomorphisms $(R, \gamma) \to (A, \alpha)$, i.e. they make the diagram in figure 1 commute.

In the view of [AM89] a labelled transition system over a fixed set of labels L is an object in $\mathbf{Set}_F$, where $F := \mathcal{P}(L \times \_)$. In the rest of this paper $\mathbf{Set}_F$ denotes the category of coalgebras for this special functor.

Each coalgebra (A, α) in $\mathbf{Set}_F$ encodes a labelled transition system $T_{(A,\alpha)} = (A, \Leftrightarrow\!\!\to)$ and vice versa: A is the set of states of $T_{(A,\alpha)}$ and there is a transition $x \overset{a}{\Leftrightarrow\!\!\to} y$ with label

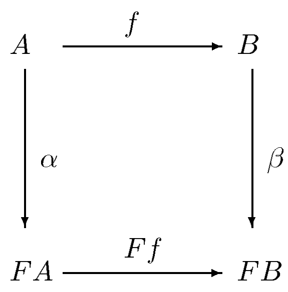

Figure 2: Homomorphism

$a \in L$ between two states $x, y \in A$ in $T_{(A,\alpha)}$ iff $(a, y) \in \alpha(x) \subseteq L \times A$.

It is useful to translate the definition of a homomorphism between two coalgebras in terms of their related transition systems:

Lemma 2.1

*A morphism $f : A \to B$ in **Set** is a homomorphism between coalgebras (A, α) and (B, β) iff for the related transition systems $T_{(A,\alpha)}$ and $T_{(B,\beta)}$ holds:*

(i) if $x \stackrel{a}{\Longleftrightarrow} y$ in $T_{(A,\alpha)}$ then $f(x) \stackrel{a}{\Longleftrightarrow} f(y)$ in $T_{(B,\beta)}$ and

(ii) if $r \stackrel{a}{\Longleftrightarrow} s$ in $T_{(B,\beta)}$ and $r = f(x)$ then $s = f(y)$ for some $y \in A$ and $x \stackrel{a}{\Longleftrightarrow} y$ in $T_{(A,\alpha)}$.

Proof: Let f be an homomorphism. Figure 2 shows the commuting diagram. If there is a transition $x \stackrel{a}{\Longleftrightarrow} y$ in $T_{(A,\alpha)}$ then by definition $(a, y) \in \alpha(x)$. As $(Ff) \circ \alpha = \beta \circ f$ we get $(a, f(y)) \in \beta(f(x))$ and therefore $f(x) \stackrel{a}{\Longleftrightarrow} f(y)$ in $T_{(B,\beta)}$. If there is a transition $r \stackrel{a}{\Longleftrightarrow} s$ in $T_{(B,\beta)}$ with $r = f(x)$ for some $x \in A$ then $(a, s) \in (\beta \circ f)(x)$. Thus we get $(a, s) \in ((Ff) \circ \alpha)(x)$. Therefore there exists $y \in A$ with $s = f(y)$ and $(a, y) \in \alpha(x)$. This implies $x \stackrel{a}{\Longleftrightarrow} y$ in $T_{(A,\alpha)}$.

Now let $f : A \to B$ be a morphism in **Set** which fulfills (i) and (ii). If $(a, z) \in ((Ff) \circ \alpha)(x)$ then there exists $y \in A$ with $f(y) = z$ and $(a, y) \in \alpha(x)$. Thus we have $x \stackrel{a}{\Longleftrightarrow} y$ in $T_{(A,\alpha)}$ which implies $f(x) \stackrel{a}{\Longleftrightarrow} f(y)$ in $T_{(B,\beta)}$. This is equivalent with $(a, f(y)) = (a, z) \in \beta(f(x))$. Therefore we know $((Ff) \circ \alpha)(x) \subseteq (\beta \circ f)(x)$. If $(a, s) \in (\beta \circ f)(x)$ then we have $r \stackrel{a}{\Longleftrightarrow} s$ in $T_{(B,\beta)}$ and $r = f(x)$ for some $r \in B$. Thus by (ii) there exists $y \in A$ with $f(y) = s$ and $x \stackrel{a}{\Longleftrightarrow} y$ in $T_{(A,\alpha)}$. This is equivalent with $(a, f(y)) = (a, s) \in (Ff \circ \alpha)(x)$. ∎

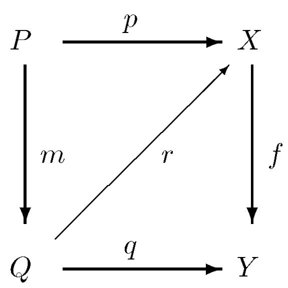

Figure 3: Path lifting condition

3 The view of Joyal, Nielsen and Winskel

To give an abstract characterization of bisimulation [JNW94] choose a category $\mathbf{M}$ of models and a full subcategory $\mathbf{P}$ of $\mathbf{M}$ of "path objects". In $\mathbf{M}$ a morphism $f : X \to Y$ is called ***P**-open*, iff whenever there are objects P, Q and a morphism $m : P \to Q$ in $\mathbf{P}$ and morphisms $p : P \to X$, $q : Q \to Y$, then there exists a morphism $r : Q \to X$ with $r \circ m = p$ and $f \circ r = q$. Figure 3 illustrates this "path lifting condition". $\mathbf{P}$-open morphisms include all the identity morphisms and are closed under composition. Two objects X_1 and X_2 of $\mathbf{M}$ are called ***P**-bisimular*, iff there exists an object X in $\mathbf{M}$ and $\mathbf{P}$-open morphisms $f_1 : X \to X_1$ and $f_2 : X \to X_2$.

In the case of transition systems [JNW94] use as category of models the category $\mathbf{T}_L$. Its *objects* are transition systems over a fixed set of labels L. They have the form $(S, s, \Leftrightarrow\!\!\!\rightarrow)$, where S is a set of states, $s \in S$ is the initial state and $\Leftrightarrow\!\!\!\rightarrow \subseteq S \times L \times S$ is the transition relation. The existence of an initial state implies $S \neq \emptyset$. A *morphism* σ between two transition systems $T_1 = (S_1, s_1, \Leftrightarrow\!\!\!\rightarrow_1)$ and $T_2 = (S_2, s_2, \Leftrightarrow\!\!\!\rightarrow_2)$ is a mapping $\sigma : S_1 \to S_2$ which satisfies: $\sigma(s_1) = s_2$ and if $x \overset{a}{\Leftrightarrow\!\!\!\rightarrow}_1 y$ then $\sigma(x) \overset{a}{\Leftrightarrow\!\!\!\rightarrow}_2 \sigma(y)$.

As category of path objects [JNW94] choose $\mathbf{Bran}_L$. This is the full subcategory of $\mathbf{T}_L$, whose objects are those acyclic transition systems which consist only of one finite branch.

There is a characterization of the $\mathbf{Bran}_L$-open morphisms in [JNW94]:

Lemma 3.1
The $\boldsymbol{Bran}_L$-open morphisms in the category $\boldsymbol{T}_L$ are those morphisms $\sigma : T_1 \to T_2$, $T_1 = (S_1, s_1, \Leftrightarrow\!\!\!\rightarrow_1)$, $T_2 = (S_2, s_2, \Leftrightarrow\!\!\!\rightarrow_2)$, with the property that for all reachable states $s \in S_1$ holds: if $\sigma(s) \overset{a}{\Leftrightarrow\!\!\!\rightarrow}_2 u$ then $s \overset{a}{\Leftrightarrow\!\!\!\rightarrow}_1 t$ and $\sigma(t) = u$ for some $t \in S_1$.

4 Comparison of the different views

In both approaches a strong bisimulation is an object of the respective category. In [AM89] bisimulation is defined within one transition system (A, α) and a bisimulation is a transition system (R, γ) where $R \subseteq A \times A$ such that the projections make the diagram in figure 1 commute. In [JNW94] a bisimulation between two transition systems $T_i = (S_i, s_i, \Leftrightarrow\!\!\rightarrow_i)$, $i = 1, 2$, with initial states s_1 resp. s_2 is a transition system T together with two $\mathbf{Bran}_L$-open morphisms $f_i : T \rightarrow T_i$, $i = 1, 2$.

In order to relate the two views we have to be able to switch from the category $\mathbf{Set}_F$ to $\mathbf{T}_L$ and vice versa. This can easily be done in the case of objects introducing respectively omitting initial states.

Looking at the morphisms is slightly more complicated: A morphism in $\mathbf{T}_L$ fulfills only condition (i) of lemma 2.1. A $\mathbf{Bran}_L$-open morphism $\sigma : T_1 \rightarrow T_2$ in $\mathbf{T}_L$ fulfills condition (ii) of lemma 2.1 only for reachable states in T_1 and therefore does not always induce a morphism in $\mathbf{Set}_F$. The situation when we start with a morphism f in $\mathbf{Set}_F$ is described in the next lemma:

Lemma 4.1

Let f be a morphism between two coalgebras (A, α) and (B, β) in $\mathbf{Set}_F$ with $A \neq \emptyset$ and $B \neq \emptyset$. Let $T_{(A,\alpha)} = (A, \Leftrightarrow\!\!\rightarrow_A)$ and $T_{(B,\beta)} = (A, \Leftrightarrow\!\!\rightarrow_B)$ be the related transition systems. Then f is a $\mathbf{Bran}_L$-open morphism in $\mathbf{T}_L$ between transition systems $T'_{(A,\alpha)} := (A, s, \Leftrightarrow\!\!\rightarrow_A)$ and $T'_{(B,\beta)} := (B, f(s), \Leftrightarrow\!\!\rightarrow_B)$ for all $s \in A$.

Proof: First we have to show that f is a morphism in $\mathbf{T}_L$. By construction f maps the initial state of $T'_{(A,\alpha)}$ to the initial state of $T'_{(B,\beta)}$. As f is a homomorphism we know by lemma 2.1 (i) that f fulfills the transition condition of morphisms in $\mathbf{T}_L$. Lemma 2.1 (ii) implies the characterization of $\mathbf{Bran}_L$-morphism from Lemma 3.1. ■

To translate AM-bisimulation into $\mathbf{Bran}_L$-bisimulation we have to introduce suitable initial states:

Theorem 4.2

Let (R, γ) be an AM-bisimulation on a coalgebra (A, α) with $R \neq \emptyset$. Let $T_{(A,\alpha)} = (A, \Leftrightarrow\!\!\rightarrow_A)$ be the related transition system of (A, α). Then for any pair $(s_1, s_2) \in R$ the transition systems $T_1 := (A, s_1, \Leftrightarrow\!\!\rightarrow_A)$ and $T_2 := (A, s_2, \Leftrightarrow\!\!\rightarrow_A)$ are $\mathbf{Bran}_L$-bisimular.

Proof: $R \neq \emptyset$ implies $A \neq \emptyset$. As (R, γ) is an AM-bisimulation on (A, α) the projections π_1 and π_2 are homomorphism. Let $T_R := (R, \Leftrightarrow\!\!\rightarrow_R)$ be the related transition system of (R, γ). With lemma 4.1 we can conclude: for any pair $(s_1, s_2) \in R$ the mappings π_i are $\mathbf{Bran}_L$-open morphisms from $(R, (s_1, s_2), \Leftrightarrow\!\!\rightarrow_R)$ to T_i, $i = 1, 2$. ■

Starting with a $\mathbf{Bran}_L$-bisimulation we have to combine two transition systems $T_1 = (S_1, s_1, \Leftrightarrow\!\!\rightarrow_1)$ and $T_2 = (S_2, s_2, \Leftrightarrow\!\!\rightarrow_2)$ from $\mathbf{T}_L$ to one coalgebra in $\mathbf{Set}_F$. Therefore it is necessary to make the set of states S_1 and S_2 disjoint – otherwise the transition relations $\Leftrightarrow\!\!\rightarrow_1$ and $\Leftrightarrow\!\!\rightarrow_2$ could interfere in the coalgebra.

Theorem 4.3
Let $T_1 = (S_1, s_1, \Leftrightarrow\!\!\rightarrow_1)$ and $T_2 = (S_2, s_2, \Leftrightarrow\!\!\rightarrow_2)$ be $\mathbf{Bran}_L$-bisimular in $\mathbf{T}_L$. Then there exists an AM-bisimulation (R, γ) with $((s_1, 1), (s_2, 2)) \in R$ on the coalgebra (A, α), where

- $A := (S_1 \times \{1\}) \cup (S_2 \times \{2\})$ *and*
- $(a, (y, i)) \in \alpha(x, i) :\Longleftrightarrow\ x \overset{a}{\Leftrightarrow\!\!\rightarrow}_i y,\ i = 1, 2.$

Proof: As T_1 and T_2 are $\mathbf{Bran}_L$-bisimular there exists a transition system $T = (S, s, \Leftrightarrow\!\!\rightarrow)$ and $\mathbf{Bran}_L$-open morphisms $f_i : T \rightarrow T_i$, $i = 1, 2$. We construct the coalgebra (R, γ) as follows:

- $R := \{((f_1(u), 1), (f_2(u), 2)) \mid u \in S \text{ reachable}\}$
- Let $(a, ((f_1(v), 1), (f_2(v), 2))) \in \gamma((f_1(u), 1), (f_2(u), 2)) :\Longleftrightarrow\ u \overset{a}{\Leftrightarrow\!\!\rightarrow} v$ in T.

As $f_1(s) = s_1$ and $f_2(s) = s_2$ we have $((s_1, 1), (s_2, 2)) \in R$. Thus it remains to prove that the projections π_1 and π_2 are homorphism between (R, γ) and (A, α). To do this we use lemma 2.1:

Let $(a, ((f_1(v), 1), (f_2(v), 2))) \in \gamma((f_1(u), 1), (f_2(u), 2))$. Then by definition of γ we have $u \overset{a}{\Leftrightarrow\!\!\rightarrow} v$ in T. As the f_i are morphisms in $\mathbf{T}_L$ this implies $f_i(u) \overset{a}{\Leftrightarrow\!\!\rightarrow}_i f_i(v)$ in T_i, $i = 1, 2$. Therefore by the definition of α we have $(a, (f_i(v), i)) \in \alpha(f_i(u), i)$, $i = 1, 2$, and condition (i) of lemma 2.1 is fulfilled.

We prove condition (ii) only for π_1. Let $(a, (y, 1)) \in \alpha(x, 1)$ and $((x, 1), (z, 2)) \in R$. Then by the definition of R exists $u \in S$ with $f_1(u) = x$ and $f_2(u) = z$. By the definition of α we know that $x \overset{a}{\Leftrightarrow\!\!\rightarrow}_1 y$. As u is reachable in T and f_1 is $\mathbf{Bran}_L$-open there exists by lemma 3.1 $v \in S$ with $u \overset{a}{\Leftrightarrow\!\!\rightarrow} v$ in T and $f_1(v) = y$. Thus $(a, ((f_1(v), 1), (f_2(v), 2))) = (a, ((y, 1), (f_2(v), 2))) \in \gamma((f_1(u), 1), (f_2(u), 2)) = \gamma((x, 1), (z, 2))$. ■

Theorem 4.2 and 4.3 translate an AM-bisimulation into a $\mathbf{Bran}_L$-bisimulation and vice versa but this relates not the “information content” of the original bisimimulation with the one of the translated bisimulation. To do this we apply successively both theorems on a bisimulation. This leads to a new bisimulation of the same type which we can compare with the original one.

First we deal with AM-bisimulation. As theorem 4.3 throws away all states of the $\mathbf{Bran}_L$-bisimulation which are not reachable we study just those coalgebras (R, γ) as AM-bisimulations where all states are "reachable":

Theorem 4.4

Let (R, γ) be an AM-bisimulation on a coalgebra (A, α). Let all elements of R be reachable from an element $(s_1, s_2) \in R$. If we

- *first apply theorem 4.2 and translate the AM-bisimulation (R, γ) into a $\boldsymbol{Bran}_L$-bisimulation consisting of a transition system T with initial state (s_1, s_2) and $\boldsymbol{Bran}_L$-open morphisms π_1 and π_2 and*
- *second apply theorem 4.3 and translate this $\boldsymbol{Bran}_L$-bisimulation back into an AM-bisimulation (R', γ')*

then (R, γ) and (R', γ') are isomorphic.

Proof: Let $T_{(R,\gamma)} = (R, \Leftrightarrow\!\!\!\rightarrow_R)$ be the related transition system of (R, γ). As $(s_1, s_2) \in R$ we may apply theorem 4.2. This leads to a transition system $T := (R, (s_1, s_2), \Leftrightarrow\!\!\!\rightarrow_R)$. Applying theorem 4.3 results in a coalgebra (R', γ') with

- $R' := \{((r, 1), (s, 2)) \mid (r, s) \text{ reachable in } T\}$ and
- $(a, (u, 1), (v, 2))) \in \gamma'((r, 1), (s, 2)) :\Longleftrightarrow \ (r, s) \overset{a}{\Leftrightarrow\!\!\!\rightarrow} (u, v) \text{ in } T \Leftrightarrow (a, u, v) \in \gamma(r, s)$.

By the definition of γ' the mappings

$$f : \begin{cases} R & \rightarrow & R' \\ (x, y) & \mapsto & ((x, 1), (y, 2)) \end{cases} \quad \text{and} \quad g : \begin{cases} R' & \rightarrow & R \\ ((x, 1), (y, 2)) & \mapsto & (x, y) \end{cases}$$

are homomorphism in $\mathbf{Set}_F$ and it holds obviously: $f \circ g = id_{R'}$ and $g \circ f = id_R$. ■

Theorem 4.5

Let $T = (S, s, \Leftrightarrow\!\!\!\rightarrow)$, T_1 and T_2 be transition systems and $f_i : T \rightarrow T_i$, $i = 1, 2$, be $\boldsymbol{Bran}_L$-open morphisms, i.e. T_1 and T_2 are $\boldsymbol{Bran}_L$-bisimular. If we

- *first apply theorem 4.3 and translate this $\boldsymbol{Bran}_L$-bisimulation into an AM-bisimulation (R, γ) and*
- *second apply theorem 4.2 and translate this AM-bisimulation (R, γ) back into a $\boldsymbol{Bran}_L$-bisimulation consisting of transition systems $T' = (R, ((f_1(s), 1), (f_2(s), 2)), \Leftrightarrow\!\!\!\rightarrow_R)$, T_1', T_2' and $\boldsymbol{Bran}_L$-open morphisms π_1 and π_2*

then T' is $\textbf{Bran}_L$-bisimular to T. If we walk around once more, i. e. first apply theorem 4.3 on T' to get a coalgebra (R', γ') and second apply theorem 4.2 to get a transition system T'', then T' and T'' are isomorphic.

Proof: Applying theorem 4.3 yields a coalgebra (R, γ) with

- $R := \{((f_1(u), 1), (f_2(u), 2)) \mid u \in S \text{ reachable}\}$ and
- $(a, ((f_1(v), 1), (f_2(v), 2))) \in \gamma((f_1(u), 1), (f_2(u), 2)) :\iff u \overset{a}{\Leftrightarrow\!\!\rightarrow} v$ in T.

Let $T_{(R,\gamma)} = (R, \Leftrightarrow\!\!\rightarrow_R)$ be the related transition system. Theorem 4.2 transforms (R, γ) into the transition system $T' = (R, ((f_1(s), 1), (f_2(s), 2)), \Leftrightarrow\!\!\rightarrow_R)$. Denote the set of reachable states in transition system T by U. To establish the $\textbf{Bran}_L$-bisimulation we consider the transition system $T_{reach} := (U, s, \Leftrightarrow\!\!\rightarrow \cap (U \times L \times U))$ of all reachable states of T. The mapping $g_1 : U \to S$, $u \mapsto u$, is obviously $\textbf{Bran}_L$-open. Let $g_2 : U \to R$, $u \mapsto ((f_1(u), 1), (f_2(u), 2))$. If $u \overset{a}{\Leftrightarrow\!\!\rightarrow} v$ in T_{reach} then by definition of γ we have $(f_1(u), f_2(u)) \overset{a}{\Leftrightarrow\!\!\rightarrow}_R ((f_1(v), f_2(v))$. As the set R is just the image of U under g_2 the converse is also true. Thus T and T' are $\textbf{Bran}_L$-bisimular.

As we know from theorem 4.4 the coalgebras (R, γ) and (R', γ') are isomorphic, i. e. in $\textbf{Set}_F$ exist morphisms $f : R \to R'$ and $g : R' \to R$ with $f \circ g = id_{R'}$ and $g \circ f = id_R$. Lemma 4.1 translates f and g into $\textbf{Bran}_L$-open morphims between T' and T''. ■

It is not possible to establish an isomorphism between T and T' in theorem 4.5: Let $T_1 = T_2$ be the transition systems with just one state x and one transition $x \overset{a}{\Leftrightarrow\!\!\rightarrow} x$. Then the transition system T with $S := \mathbb{N}$, initial state $s = 0$ and transitions $i \overset{a}{\Leftrightarrow\!\!\rightarrow} i + 1$ for all $i \in \mathbb{N}$ together with $f_1 = f_2 : \mathbb{N} \to \{x\}$, $i \mapsto x$, is a $\textbf{Bran}_{\{a\}}$-bisimulation between T_1 and T_2. Applying first theorem 4.3 and then theorem 4.2 leads to a transition system T' which consists of the state $((x, 1), (x, 2))$ and the transition $((x, 1), (x, 2)) \overset{a}{\Leftrightarrow\!\!\rightarrow} ((x, 1), (x, 2))$. As we proved in theorem 4.5 T' is $\textbf{Bran}_{\{a\}}$-bisimular to T but these transistion systems are obviously *not* isomorphic.

This shows a difference between $\textbf{Bran}_L$-bisimulation and AM-bisimulation: using *functions* f_i instead of *projections* π_i gives more freedom in the choice of the transition system representing the bisimulation. In the above example we may choose both T and T' as $\textbf{Bran}_L$-bisimulation between T_1 and T_2 but only an equivalent to T' as AM-bisimulation.

Another difference is that AM-bisimulation may relate more states than a $\textbf{Bran}_L$-bisimulation: we could establish theorem 4.4 only under the condition that all states of (R, γ) are reachable from one state $(s_1, s_2) \in R$. In AM-bisimulation the statement that two states cannot be distinguished by an external agent is possible for any states x and y. In $\textbf{Bran}_L$-bisimulation this can be done just for reachable states.

As we showed with theorems 4.2 and 4.3 the capability to distinguish transition systems is the same for $\mathbf{Bran}_L$-bisimulation and AM-bisimulation: If there is a bisimulation of one type then there is also one of the other type. Theorems 4.4 and 4.5 provide that the translation processes lead not to trivial bisimulations.

5 Conclusion

We presented how to translate AM-bisimulation into **P**-bisimulation and vice versa in the case of transition systems. It is an open problem if similar results can be obtained for other models of concurrency.

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