Stochastic Population Projection for Germany

- based on the Quadratic Spline approach to modelling age specific fertility rates -

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Stochastic Population Projection for Germany
- based on the QS-approach to modelling age specific fertility rates -

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Abstract

This contribution builds upon a former paper by the authors (Lipps and Betz 2004), in which a stochastic population projection for East- and West Germany is performed. Aim was to forecast relevant population parameters and their distribution in a consistent way.
We now present some modifications, which have been modelled since. First, population parameters for the entire German population are modelled. In order to overcome the modelling problem of the structural break in the East during reunification, we show that the adaptation process of the relevant figures by the East can be considered to be completed by now. As a consequence, German parameters can be modelled just by using the West German historic patterns, with the start-off population of entire Germany. Second, a new model to simulate age specific fertility rates is presented, based on a quadratic spline approach. This offers a higher flexibility to model various age specific fertility curves.
The simulation results are compared with the scenario based official forecasts for Germany in 2050. Exemplary for some population parameters (e.g. dependency ratio), it can be shown that the range spanned by the medium and extreme variants correspond to the s-intervals in the stochastic framework. It seems therefore more appropriate to treat this range as a s-interval covering about two thirds of the true distribution.

1. Introduction

Official population projections rest on deterministic models. In these models, today's population and assumptions on the development of demographic rates determine future population. To account for forecast uncertainty, several scenarios are calculated, usually based on "high", "medium" and "low" assumptions of a population rate. This technique - though common practice - suffers mainly from two deficiencies: First, it cannot provide information on the probability of a certain scenario. Second, modelling uncertainty by means of different scenarios is necessarily inconsistent. An extensive methodological discussion is found in Lee (1998).

To overcome these problems, probabilistic approaches to population forecasting have been developed in recent years. The main goal of probabilistic population projections is to obtain prediction intervals of demographic variables and thus to measure projection uncertainty. Probabilistic projections make use of historical forecast errors (Keyfitz 1981, Stoto 1983), rest on expert opinion (Lutz, Sanderson and Scherbov 1996) or rely on time series analysis to project future population parameters (Lee and Carter 1992, Lee 1993, Lee and Tuljapurkar 1994). Lutz and Scherbov (1998) offer a probabilistic population projection for Germany based on expert opinion, while this paper uses time series methods.

In stochastic population projections, forecast errors are propagated over the years. Depending on their correlation structure they may either reinforce each other or cancel out over time. Thus, a correct specification of these correlations is crucial to obtain consistent projection intervals. For stochastic projections on the national level, four types of correlations matter (Keilman et al. 2002): the autocorrelation of demographic rates; the correlation of demographic components, which are fertility, mortality and migration; correlation of adjacent age groups and finally correlation of the sexes.

The higher the autocorrelation of demographic rates, the weaker is the tendency of forecast errors to cancel out over time. The autocorrelation of fertility and mortality is high. Though fertility patterns may change substantially over the course of a decade, there is relatively little change from year to year. Regarding mortality, we have seen a steady decline through the last decades. In contrast, net migration is far less autocorrelated. Migration is highly dependent on the political environment and may respond quickly to policy shocks. Concerning the correlation between components, there is no reason to assume that in developed countries fertility, mortality and migration are correlated (Lee 1998). The correlation of adjacent age groups is usually positive, e.g., if mortality of people aged 69 declines, it declines probably also for people aged 70. Finally, correlation between the sexes is also positive. For instance, if mortality declines, one would expect both sexes to take advantage of the decline.

Lipps and Betz (2004) develop a stochastic population projection framework for Germany based on time series analysis. This paper draws on our previous work and addresses mainly two issues: First, we offer an alternative approach to modelling and forecasting fertility. Second, we propose a different way to deal with the structural break due to German reunification and the resulting lack of suitable time series in East Germany.

Lipps and Betz (2004) parameterise age specific fertility rates by a Gaussian curve defined by three parameters. We use different models to separately project the future course of these parameters. This approach is subject to primarily two problems. First, the Gaussian curve imposes symmetry on future age specific fertility schedules. If the trend towards delayed childbearing continues, then symmetry is unlikely to hold in the future. Second, forecasting the parameters separately simply ignores possible interrelations between them. In this paper, we apply the QS-model developed by Schmertmann (2003). He proposes a system of quadratic splines to parameterise age-specific fertility rates and shows that it fits a variety of international schedules well. Hereby, we avoid imposing symmetry on future fertility schedules. We capture possible interrelations among the parameters defining the splines by projecting them using a Vector Autoregression (VAR).

Lipps and Betz (2004) project the population of West and East Germany separately. Due to unification, however, a time series suitable to project the East German population is lacking. We therefore have either modelled adaptation processes from East rates to West rates or used West rates directly. To obtain the stochastic simulations, independent random numbers were drawn for the East as well as for the West. This, however, contradicts the intention to model an adaptation process, because increasing convergence of demographic rates should also imply increasing correlations between East rates and West rates. By drawing independent random numbers, forecast uncertainty is therefore underestimated. Further, Lipps and Betz (2004) did not account for interregional migration, though many young East Germans still move to the West. Neglecting interregional migration overestimates population in the East while underestimating that of the West. This assumes that migrants exhibit approximately the same fertility and mortality patterns as the population in the target country.

As we lack a sufficiently long time series for post “Wende” Germany, a perfect solution to the problems mentioned above does not exist. Yet we do argue that for the time being it is less harmful to project the German population by using West rates and the population vector of entire Germany. As the former GDR adopted largely the institutional setting of Western Germany, we therefore consider the history of the West to be relevant. Since unification, main demographic variables such as the total fertility rate and life expectancy at birth have been converging to West levels. We judge this process to be sufficiently complete by now. It is emphasized that this claim only refers to demographic aggregates relevant for projection purposes. Different patterns in micro-behavior or attitudes may well persist. We will discuss this issue in greater detail when dealing with fertility and mortality. By using this approach, we avoid underestimating the uncertainty of the forecast by drawing independent random numbers. Besides, interregional migration is by definition not an issue in this context.

The paper is organized as follows: Section 2 discusses modelling and forecasting fertility. Section 3 briefly reviews our mortality model. Section 4 deals with migration. Section 5 discusses the projection results in; section 6 concludes.

2. Fertility

East and West

Fertility in post war Germany was subject to structural breaks. In Western Germany, the interval from 1954 to 1966 is characterized by high fertility rates, peaking in 1964 at 2.54 births per woman. The period following the baby boom leads to a pronounced decline in fertility. Since 1973, the Western TFR fluctuates around 1.4 children per woman, as shown in figure 1.

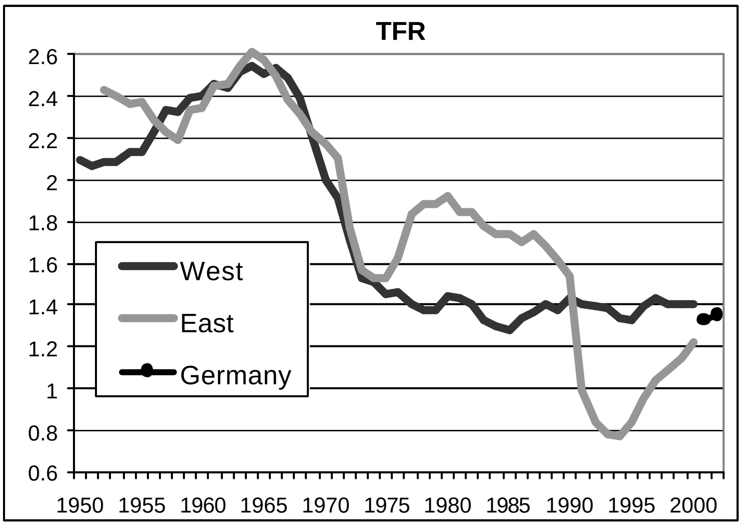

Figure 1: TFR, West and East Germany and aggregate TFR (2001, 2002)

In Eastern Germany, fertility was higher than in Western Germany throughout the 1970s and 1980s, but still below the replacement level of 2.1. This is attributed to a variety of family policies that encouraged early family formation and provided incentives to have large families (Kreyenfeld 2003). After the fall of communism, the TFR dropped sharply from 1.5 in 1990 to 0.8 from 1992 to 1995. In 2000, the latest year for which separate data is available, East TFR increased again to 1.2 while the West TFR remained at 1.4. Since 2001, Statistics Germany only publishes data for unified Germany. For 2001 and 2002, figure 1 therefore depicts the total fertility rate for entire Germany that equals 1.35 in 2002.

Thus, convergence of the period fertility indicators seems almost completed, but Kreyenfeld (2003) points out that this does not imply convergence of actual fertility behavior. She reports that six years after unification East Germans are younger at first birth than their West German counterparts and are less likely to have a second child. She therefore concludes that a rapid convergence of behavior cannot be taken for granted. However, she concedes that the East German TFR will possibly equal the West German TFR by 2005.

There are two implications when assuming that East German fertility can be explained by West German data. As far as the TFR is concerned, it seems justified to assume that convergence is completed. The second implication is that imposing the West German age schedule on the East German population creates a distortion. Since East Germans are generally younger when giving birth, more births occur over the course of time. However, we believe that on the aggregate level, this distortion has no significant impact.

We would like to base our analysis also on data from 2001 and 2002, but unfortunately Statistics Germany no longer publishes data separately for East and West Germany. Hence our time series end at 2000. Decomposing the time series into East and West turned out to be not viable, as start-off population serves the German population on 31.12.2002.

Model

Lipps and Betz (2004) parameterize age-specific fertility rates between 1973 and 2000 by a Gaussian curve. In this context, the mean of the Gaussian curve µ represents the mean age of mothers at childbearing. The standard deviation s refers to the standard deviation of the mean age of mothers while a level parameter is to be interpreted as TFR. To construct future age-specific fertility rate (ASFR) schedules, we project the parameters as follows:

1. μ is fitted by a logistic growth curve, resulting in the three parameters saturation level, expansion parameter (multiplier of $t-t_0$), and inflection point.
2. s is fitted by a vector autoregression model using s, TFR, and μ, with one lag.
3. TFR is modelled as a random walk time series.

One advantage of the Gaussian model is that its defining parameters have a straightforward interpretation and can be easily extrapolated. Also, the goodness of fit increases significantly from 1973 to 2000, because over time age specific fertility rate schedules have become more and more symmetric. However, this approach suffers from two major drawbacks. First, separate projection of the parameters does not take into account possible interrelations. Second, it assumes symmetry of the fertility schedules throughout the forecast horizon. However, we have seen a trend towards delayed childbearing throughout the last decades. If this trend remains effective, it may lead to negatively skewed ASFR schedules, but the more skewed ASFR schedules eventually become, the worse they will be approximated by an inherently symmetric functional form.

Schmertmann (2003) proposes a system of quadratic splines to parameterize age-specific fertility rate schedules. The shape of the schedule is described in terms of the ages at which fertility reaches some characteristic points and is defined by three parameters. While α refers to the starting age of fertility, P is given by the age at which fertility reaches its peak level. Finally, H marks the youngest age above P at which fertility falls to half of its peak level. Schmertmann (2003) shows that the QS model fits a wide variety of international fertility schedules well and that it outperforms the Coale Trussel model (1974) in the majority of cases. To avoid imposing symmetry on future fertility schedules, we fit the QS-model to West German ASFR schedules since 1950.

The QS-model fits five quadratic polynomials to ASFR schedules. The resulting shape function is continuous with the first derivate also continuous. The shape function requires the specification of five sampling points t_0,.., t_4 and of the age at which fertility ends. By imposing suitable mathematical restrictions on the knot positions, it is uniquely determined by the index ages [α, P, H]. The assumptions used by Schmertmann (2003) are as follows:

1. Knot 0 is given by the starting age of fertility
   $t_0=\alpha$
2. Knot 1 is between the starting age and the peak age of fertility
   $t_1=(1-W)\,\alpha+WP$, where
   $W=\min[0.75,0.25+0.25(P-\alpha)]$
3. Knot 2 is at the peak age of fertility
   $t_2=P$
4. Knot 3 is halfway between P and H
   $t_3=(P+H)/2$
5. Knot 4 is halfway between H and the age at which fertility ends
   $t_4=(H+\beta)/2$
6. In general, childbearing ends at age 50[1]
   $\beta=50$

[1] Schmertmann proposes minor adjustments for very steep or very flat schedules, but in our context they do not apply

To obtain an intuition for the model, the fit to the German 2000 schedule and the five knots of the quadratic spline are depicted in figure 2. Apparently, the five subsections are quadratic. The parameter H is not itself a sampling point. But it serves to derive sampling points t_3 and t_4.

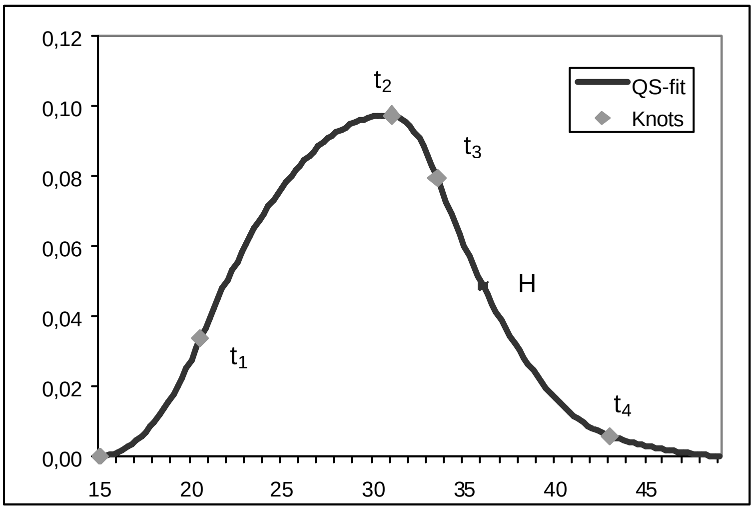

Figure 2: QS approximation to German 2000 ASFR schedule

From 1950 to the late 1980s, the QS-model fits West German fertility schedules remarkably well. Since then, however, we see a relative increase in fertility rates of women aged 18 –22 leading to a small hump in the ASFR schedule. While the QS-model allows for one turning point between α and P, three turning points are necessary to capture this feature of empirical schedules throughout the 1990s. This failure, however, applies also to the Gaussian approach by Lipps and Betz (2004) as well as the Gamma curve proposed by Thompson et al. (1989).

In this environment, the model in its original form overestimates fertility at very young ages and underestimates fertility at ages 18-22. It also leads to implausibly low starting ages of fertility. In 2000, for instance, we end up with fertility starting at age 12 and using the model for projection leads to even smaller values of α at the end of the forecast horizon. It is important to note that this phenomenon does not reflect fertility patterns but regression mechanics. The model is fitted to the empirical fertility schedule in order to minimize the sum of squared residuals. The pattern of over- and underestimation as well as the decrease in α is thus due to the minimization of the loss function.

Different strategies are conceivable to overcome this problem. For instance, one could add two sampling points between α and P. Since we are lacking data on fertility below age 15 anyway, we do not follow this strategy but fix α at 15. Of course, optimization only over the two remaining parameters P and H leads to a worse result. Therefore, we minimize also over W and thus indirectly over t_1. This modification improves the fit below age 25 but also comes at a cost. Fertility around age 30 is slightly underestimated. Since it is important to precisely approximate the empirical schedule at ages where fertility is high, one can additionally modify the loss function. Thus, we no longer minimize the sum of squared residuals, but apply a weighting scheme similar to that of Thompson et al. (1989). This scheme gives more weight to deviations from the empirical schedule at ages where fertility is high.

The West German 2000 schedule and the QS fit are displayed in figure 3. Here, the squared deviations from the empirical schedule between age 25 and 34 enter the loss function multiplied by a factor of four. Still, age-specific fertility rates around age 30 are slightly underestimated, but the model approximates the empirical schedule fairly well.

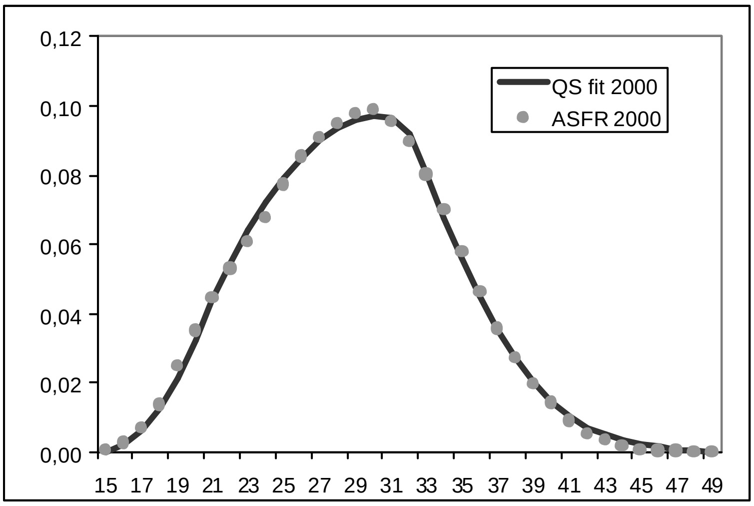

Figure 3: West German 2000 ASFR schedule and weighted QS fit

These modifications do not explicitly model the hump in fertility. Our goal, however, is not to analyze fertility pattern, but to develop a stochastic population projection. Thus, we are primarily interested in a reasonably exact approximation of fertility schedules. This is exactly what the proposed modifications accomplish. Over the course of time, the QS model provides a better approximation of ASFR schedules than the Gamma curve by Thompson et al. (1989) and the Gaussian model by Lipps and Betz (2004).

Forecasting

The above modifications yield a QS shape function that is defined by parameters [W, P, H]. These parameters do not determine the level of fertility which is given by the TFR. Thus, to obtain future age-specific fertility rates one has to estimate W, P, H and the TFR. The projected shape function is then rescaled such that the integral over the curve equals the total fertility rate. We apply a VAR with two lags to the unweighted fit in order to forecast the shape function. In contrast to Lipps and Betz (2004), this model enables us to exploit cross-correlation of the parameters. To project the TFR, the same random walk approach is used as in Lipps and Betz (2004).

According to the VAR forecast, the trend towards delayed childbearing continues, albeit much weaker. The QS-fit in 2000 yields a modal age of fertility P equal to 31.1 years, while the halfway point of descent H is at 36 years. We project P to equal 31.7 years in 2050, and H equal to 37.8 years. This implies an increase in the modal age of fertility of 0.6 years, while the halfway point of descent is projected to increase by 1.8 years. The parameter W lacks a straightforward interpretation, but its development can be expressed in terms of t_1. This sampling point is projected to increase by 1.5 years from 20.2 years in 2000 to 21.7 years in 2050.

For the VAR, the eigenvalue stability criterion is satisfied; all eigenvalues lie strictly inside the unit circle. This implies covariance stationarity[2] of the time series, i.e. the first two moments of the process are independent of t (Hamilton 1994, 46), confirming our intention to model without trend (Lee 2004). Tests on normality of the disturbances are however rejected, and order two lagged disturbances exhibit weak autocorrelations. However, a Wald test of the hypotheses that the variables at lag two are jointly zero is clearly rejected. As a consequence of these analyses, all coefficients of lag one and two are kept in the VAR equations. Due to stationarity of the VAR forecast, the prediction interval does not adequately take into account the increasing uncertainty with increasing forecast time. We modify the simulated trajectories accordingly by letting each trajectory start at the fitted value in 2000.

The VAR, however, is not without problems. Due to permanent change in past fertility behavior, parameter estimates are quite imprecise, that is standard errors of the estimates are fairly large. For instance, the projected standard error of P in 2050 equals 3.5 years and for H the estimated standard error amounts to 2.6 years. Restraining insignificant coefficients to 0 to reduce the sampling variance is not a viable option because it leads the forecast astray. A shorter historical time series containing less variation like the period from 1973 to 2000 does not improve the forecast either.

A second problem emerges from the estimated residual variance that is rather substantial. Hence, when performing stochastic simulations, some iterations may yield a P greater than H. In this case the QS-model is ill-defined, because it requires H to be greater than P. Adding lags of higher order can hardly be justified, since coefficients of the higher order lags are no longer jointly significant. To ensure that the model is well-defined, we therefore put restrictions on the range of simulated Ps and Hs. P is allowed to take on values between 20 and 40 years, whereas H is confined to P+2 and 49 years. If $H_t<P_t+2$, we define $H_t=(H_{t-1}+P_{t-1}+2)/2$, similarly, if $H_t>49$ we define $H_t=(H_{t-1}+49)/2$. Due to the strong serial correlation of P and H, this seems reasonable.

To model the total fertility rate, we do not change the specification used in Lipps and Betz (2004). Thus, the historical times series is considered only from 1973 on. The structural breaks caused by the baby boom and baby bust period are hence ignored. Lipps and Betz (2004) apply a simple random walk to project fertility. This implies perfect autocorrelation of fertility rates. The model can be justified by applying the augmented Dickey-Fuller test. The null hypothesis of a unit root cannot be rejected. Hence, the process may be or may not be stationary.

Apart from the random walk, different specifications are possible. An AR(1) model fit to the time series from 1973 to 2000 yields comparable results. Taking into account the entire time series from 1950 to 2000, an AR(2) model would be appropriate. This, however, implies a mean TFR equal to 1.78. To be in line with the official forecast of Statistics Germany (Statistisches Bundesamt 2003) that assumes a TFR of 1.4, requires imposing a mean constraint as in Lee (1993).

The random walk yields a point estimate equal to 1.41, which is simply the 2000 West German TFR. The prediction interval increases by the square root of the forecast horizon. Thus, for 2050 we project a σ-interval of [0.995; 1.814].

[2] This holds although tests on stationarity of the univariate time series ln(p), ln(h), and ln(w) are rejected.
[3] The order selected maximises AICC.

3. Mortality

East and West

In recent decades, Germany has seen a constant rise in life expectancy. As displayed in figure 4, life expectancy of West German females increased from 71 years in 1954 to more than 80 years in 2000. Life expectancy of their male counterparts increased from 66 years to slightly less than 75 years. From the late 1970s to the fall of communism, life expectancy in the West grew faster than in the East. By the late 1980s, female life expectancy in the West was around three years higher than in the East.

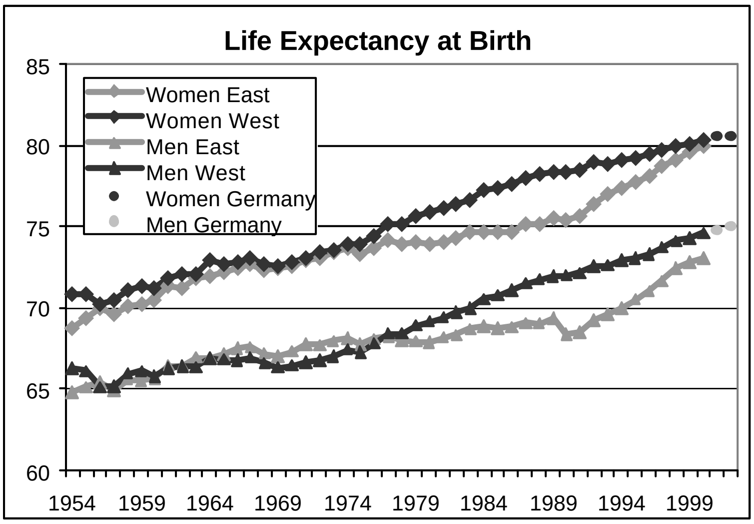

Figure 4: Life expectancy at birth, women and men, East and West Germany, 2001 and 2002 data refer to the entire German population

Since reunification, however, there is a strong tendency of adaptation; East German life expectancy has caught up rapidly. In 2000, the gap in female life expectancy had narrowed to just 0.2 years. East German males did not catch up that quickly to their West German counterparts. However, the figures of 2001 and 2002 seem to continue the trend of former West German life expectancy. The process of adaptation of East life expectancy to West rates appears to be almost complete. Thus, we argue that it is justified to project German life expectancy based on the West German time series alone.

Model

We model mortality based on the well known approach by Lee and Carter (1992). This method allows to describe and to project the development of age-specific mortality rates over time within a parsimonious framework. Basically, the model splits mortality rates into age-specific components that are constant over time and a time varying survival index capturing the development of mortality. Then, one can extrapolate the time series of the mortality index by means of a suitable time series model. Future age-specific mortality rates can be recovered by linking the projected mortality index to the age-specific components. The model reads:

$$\ln(mort_{x,t}) = a_x + b_x k_t + e_{x,t} \quad ,\text{where}$$

$mort_{x,t}$: mortality risk at age x during period [t-1,t].

a_x: age specific mean mortality rate, standardised to $a_x = 1/T * \ln(surv_{x,t})$.

b_x: age specific average change of mortality rate (standardised to $Sb_x = 1$)

k_t: time series factor („time specific mortality rate“)

While a_x maps the age specific “generic” survival rate, b_x measures, to what extent different age groups contribute to changes in the mortality index over time. To avoid a limited range, we take logits of the mortality rates. Essentially, the Lee-Carter model yields a solution by means of the singular value decomposition (SVD), projecting to the first singular value. SVD uses least squares to find the best approximation to the following system of equations (Pedroza 2002):

$$\ln (mort_{x,t}/(1-mort_{x,t})) - a_x = b_x k_t \quad =: f(x,t) \text{ with discrete x ad t.}$$
$$(=A) \qquad\qquad = UDV$$

A decomposition A=UDV into the diagonal matrix D, which contains the ordered[4] singular values, and the orthogonal matrices U and V, is applicable to any rectangular matrix A. The approximation of A by $b_x k_t$ (=D consisting of just the first singular value) is the better, the larger the first singular value is compared to the second. An empirical analysis of the survival rates shows that the first singular value (=18.9) is far greater than the second (=3.1)[5]. Therefore, the approximation, that is the projection into the vector space spanned by the first singular value is appropriate.

Forecast

Having estimated the only time dependent variable k_t its time series is analysed by a univariate ARIMA model. As the k_t increase almost linearly, we use a simple “random walk with drift” model to forecast future values. Then age-specific mortality rates are recovered. The stochastic simulation yields trajectories for male life expectancy at birth as displayed in figure 5. The mean life expectancy at birth for males amounts to 80.3 years in 2050, with a standard deviation of 1.2 years. In contrast to Lipps and Betz (2004), we take into account[6] the high correlation of the error terms in the forecast of k_t for men and women and assume equal forecast errors. This is reasonable since the mechanisms responsible for mortality decline, especially medical progress, apply in general equally to males and females.
For women, we simulate a mean of 86.7 with a standard deviation of 1.5 years. These figures, together with the official forecast (Statistisches Bundesamt 2003) are depicted in table 1.

Source	Males	females
Simulation	80.3 (std=1.2)	86.7 (std=1.5)
Official forecast: low life expectancy	78.9	85.7
Official forecast: median life expectancy	81.1	86.6
Official forecast: high life expectancy	82.6	88.1

Table 1: Life expectancy at birth in 2050, mean and standard deviation of simulated and officially forecasted figures, Germany

Our simulations are close to the median official forecasts, but only women’s high and low variants are enclosed by the simulated mean +/- one s-interval. For males, the extreme variants are within the simulated mean +/- two s-intervals.

[4] I.e. $D_{ii} > D_{kk}$ for i<k.
[5] This holds for West German males, with the time period from 1954-2000 used.
[6] The correlation coefficient of the residual time series of k_t for men and women is .91.

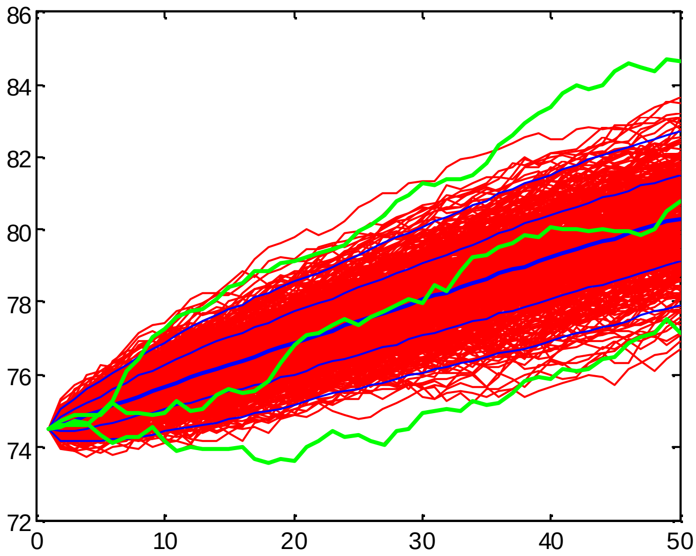

Figure 5: Simulated trajectories of male life expectancy at birth: all (red), maximum, minimum and median added up trajectories (green), and cross sectional mean, +/- one, and +/- two s-intervals (blue). Forecast horizon 2002-2050. West Germany

4. Migration

The treatment of migration is essentially the same as in Lipps and Betz (2004). The only difference is that we no longer assign migrants to either West or East Germany. We model net migration assuming an AR(1) process. The mean forecast amounts to 244,000 net immigrants per year. This is close to the medium variant of the official population forecast (Statistisches Bundesamt 2003) that assumes a net migration of 200,000 people per year. We assume the actual age distribution of the yearly migration vector to be constant throughout the forecast horizon. Hence, the only stochastic element is net migration.

We project that from the forecast horizon for 2002 to 2050, around 15 million inhabitants have a migration history. The standard error of the projection equals 4.77 million. Table 2 compares these figures to the official forecasts (Statistisches Bundesamt 2003), which are based on the assumption of 100,000, 200,000 and 300,000 people per year, respectively.

Source	Immigrants (million)
Simulation	14.96 (std=4.77)
Official forecast: low migration	5.66 (cumulated)
Official forecast: median migration	10.46 (cumulated)
Official forecast: high migration	14.46 (cumulated)

Table 2: Net Migration until 2050, mean and standard deviation of simulated and officially forecasted figures, Germany

When interpreting the figures, it is important to keep in mind that the official numbers are cumulated net migrations, i.e. disregarding mortality and fertility.

5. Results

Total Population

For 2050, we project a mean population of 75.8 million people. The standard deviation of the forecast equals 7.5 million. As the distribution of the total population can be treated as normal, this implies a s –interval of [68.3; 83.3] million inhabitants. The 2002 start-off population is 82.5 million people. Comparing this figure to our mean forecast implies that the German population can be expected to decrease by 6.7 million people until 2050. Figure 6 shows the simulated trajectories of the total population. They illustrate how forecast uncertainty increases over the forecast horizon. Also, the tendency of the total population to decline becomes apparent, though an increase in population cannot be ruled out.

We first compare our simulated results to the official forecast. Statistics Germany forecasts a population of 67.0 million in the variant „minimum population“ in 2050, 75,1 million in the variant „medium population“, and 81.3 million in the variant „maximum population“ (Statistisches Bundesamt 2003). Thus, our mean forecast is a little higher that the medium variant of the official forecast. This might be due to our slightly higher TFR and that we project higher net migration. The difference between the official maximum and minimum forecast amounts to 14.3 million people, while the difference between the upper and lower bounds of our s–interval equals 15 million people. This suggests that the extreme variants of the official forecast can be treated roughly as a s–interval containing 68 % of the probability mass. However, this in turn implies that a substantial probability mass is outside the official prediction interval and thus that forecast uncertainty is higher than suggested by the official forecast.

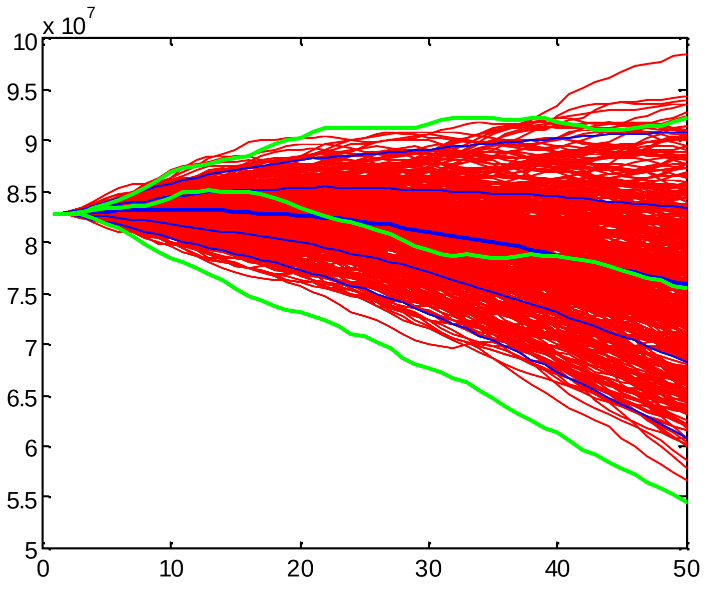

Figure 6: Simulated trajectories of the population number: all (red), maximum, minimum and median added up trajectories (green), and cross sectional mean, +/- one, and +/- two s-intervals (blue). Forecast horizon 2002-2050, Germany

Lipps and Betz (2004) project a mean population of 75.2 million for 2050. The projected standard deviation is equal to 6.4 million people. The difference in the mean forecast amounts to 0.6 million people and is surprisingly large, given that the model projecting the TFR is the same. Part of the difference can be explained by an increase in the start-off population. In

Lipps and Betz (2004), the start-off population is given by the population of 31.12.2001, while in our new model we use the population of 31.12.2002. From 2001 to 2002, however, the German population has increased by almost 100,000 people. Further, East German fertility and survival rates tend to be lower in Lipps and Betz (2004) in the beginning of the forecast horizon. This is due to the explicit modelling of adjustment paths from East levels to West levels. Explaining the 1.1 million increase in the standard deviation is relatively straightforward. First, the QS-parameterisation leads to a higher standard deviation than the Gaussian model in Lipps and Betz (2004). Second, we no longer simulate the East and West German population independently. Assuming perfect correlation between the regions increases forecast uncertainty.

Simulating the population by sex and age yields the stochastic population pyramid for 2050 as depicted in figure 7. The green, yellow and blue trajectories refer to the minimum, median and maximum population in 2050. They need not be identical with the green trajectories in figure 6, which represent the minimum, median and maximum population summed up over the entire forecast horizon.

The population pyramid shows how forecast uncertainty is related to the age structure. Uncertainty above age 80 can be attributed almost only to mortality, while uncertainty between age 50 and age 80 largely stems from migration. Uncertainty below age 50 is due to fertility as well as migration, though fertility is the dominant source. Apparently, uncertainty is the greatest at the bottom of the pyramid. It then systematically decreases as the age of the people increases. This is intuitively appealing, since in 2050 not only the number of potential children is uncertain, but also the number of potential mothers.

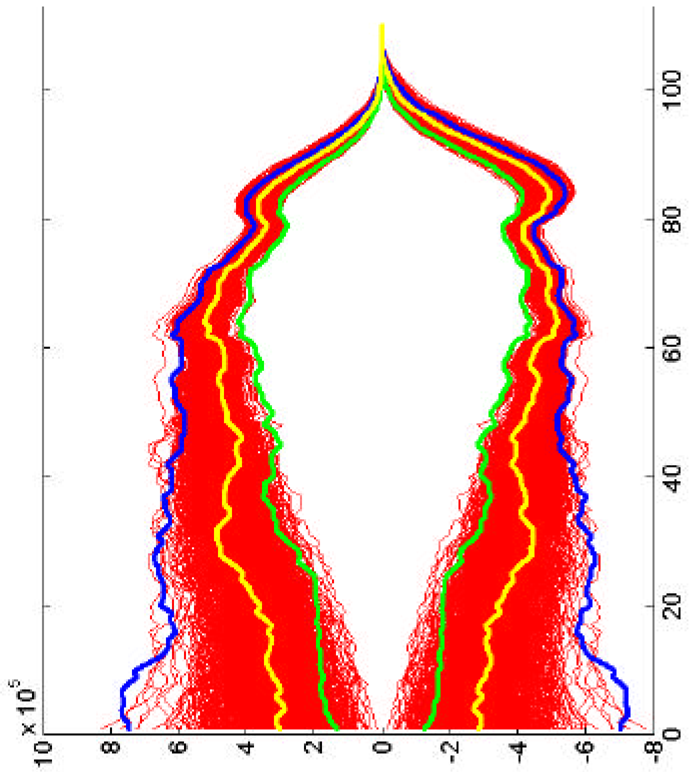

Figure 7: Stochastic Population pyramid in 2050 (red). Trajectory with the maximum population is printed blue, with minimum population (green), and median population (yellow). Germany

Total Dependency Ratio

A prominent application of population projections are future dependency ratios. They serve to assess the future financial burden put on the potentially employed population by the people too young or too old to work. As an example, we analyse the development of the „Total dependency ratio“ (TDR), which we define as the ratio of the potentially employed (approximated by the 20-59 year old population) to the „dependent“ population (under the age of 20 or over 59). The course of the TDR is depicted in figure 8.

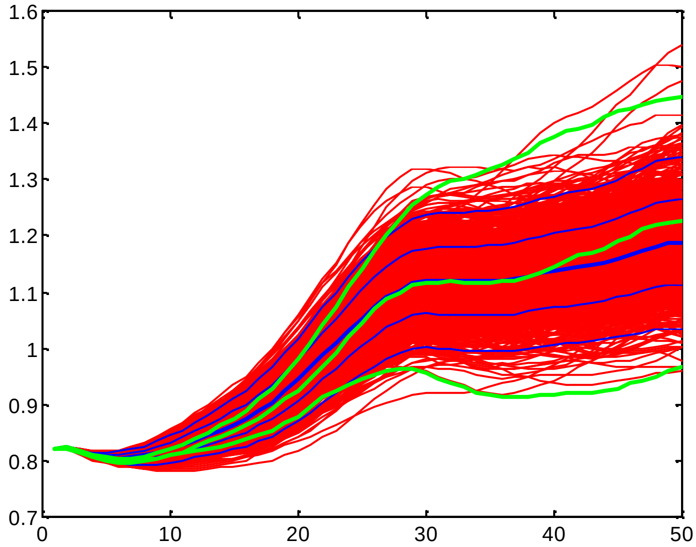

Figure 8: Simulated trajectories of the TDR: all (red), maximum, minimum and median added up trajectories (green), and cross sectional mean, +/- one, and +/- two s-intervals (blue). Forecast horizon 2002-2050. Germany

The mean TDR in Germany is simulated to be around 1.19 in 2050, with a standard deviation of 0.07. This implies a s-prediction interval of [1.12; 1.26]. Statistics Germany projects the following (table 3) range of TDRs, under the assumptions of different life expectancy and migration scenarios (Statistisches Bundesamt 2003). As noted earlier, the TFR is assumed to be 1.4 throughout the forecast period.

Variant	Assumption life expectancy	Assumption net migration	Total dependency ratio (0-19 plus 60+ / 20-59 year old pop), in 2050
2	low	high (> 300 000)	1.05
5	medium	medium (~ 200 000)	1,12
8	high	low (< 100 000)	1.22

Table 3: Scenarios and forecasts for TDR in 2050 by Statistics Germany (Statistisches Bundesamt 2003)

Our simulated dependency ratio is one standard deviation higher than the official “medium” variant. The range enclosed by the minimum TDR scenario and the maximum TDR scenario equals 0.17. It is thus only slightly greater than the range of our s -prediction interval equal to 0.14. However, all official variants are inside our 2s -prediction interval.

The simulated total dependency ratios differ only marginally from those in Lipps and Betz (2004), where the projected mean TDR equals 1.18 for 2050. The standard deviation of 0.07 is not affected by the differences in model specification. Though the new forecast leads to an increase in both, mean population and its standard deviation, it does not seem to affect the age-structure as measured by the total dependency ratio.

6. Conclusion

In this paper, we have developed a stochastic population projection for Germany based on time series analysis. To fit age-specific fertility rates, we replace the Gaussian model used in Lipps and Betz (2004) by a system of quadratic splines as proposed by Schmertmann (2003). The QS-model fits West German ASFR schedules remarkably well. During the 1990s, a small increase in fertility rates of women aged 18 to 22 cannot be captured by the model. However, this failure applies also to its rivals.

We then project the parameters defining the QS-model by a vector autoregression. Unfortunately, standard errors of the estimated parameters are relatively large. This applies also to the residual variance. Consequently, the simulation of the VAR forecast requires restrictions on the course of parameters for the QS-model to be defined in all cases. The remaining question is whether this problem can be alleviated by applying more advanced time series techniques or whether it simply reflects the limits to time series forecasting.

Second, we no longer project the population of West and East Germany separately. Instead, our projection of demographic rates is based solely on West German time series. The future population is then obtained by multiplying the entire German population by these rates. As we have shown in the sections on fertility and mortality, the convergence of East rates to West rates is sufficiently advanced to justify this strategy.

These modifications increase the 2050 mean population forecast by 0.6 million people from 75,2 million in Lipps and Betz (2004) to 75.8 million. The standard deviation increases by 1.1 million people to 7.5 million. The total dependency ratio, however, remains largely unaffected. The mean forecast increases only marginally by 0.01 to 1.19 in 2050, while the standard deviation is still at 0.07. Hence, the above modifications influence population size but not its age structure as measured by the total dependency ratio.

Comparing these results to those of the official forecast (Statistisches Bundesamt 2003) shows that the range enclosed by the official maximum and minimum variants does not cover the entire distribution of a population parameter derived from our model. It seems more appropriate to treat the range spanned by the official variants as a s -interval covering roughly two thirds of the distribution.

Further research should focus on applications of stochastic population projections. Their main advantage is to yield probability distributions of population parameters and thus a measure of projection uncertainty. Promising applications include all areas that are strongly related to demography and that require a precise assessment of uncertainty. An overview of potential applications is given in Lee (2004). For instance, one could model the impact of a pension reform on the social security system. Uncertainty of future costs is given by the uncertainty in the number of the elderly. Here, a stochastic model can provide a meaningful estimate of the financial uncertainty.

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